THE MECHANICAL PROPERTIES OF Nb₃Sn
MULTIFILAMENTARY COMPOSITES

by

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ABSTRACT

THE MECHANICAL PROPERTIES OF Nb$_3$Sn MULTIFILAMENTARY COMPOSITES

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STUART FORSTER COGAN

Submitted to the Department of Materials Science and Engineering on August 10, 1979 in partial fulfillment of the requirements for the degree of Doctor of Science.

The mechanical and superconducting properties of Nb$_3$Sn multifilamentary composites fabricated by the external diffusion technique have been investigated. Initial problems with Kirkendall porosity were successfully overcome in small diameter wires (~ 0.025 cm), by high temperature recrystallization prior to the tin plating process.

Mechanical properties of bronze at 4.2° K were determined as a function of tin content. The 0.02% offset yield strength varied from 76 MPa for pure copper to 241 MPa for an 8.2 w/o Sn bronze. The work hardening modulus varied correspondingly from 1.5 GPa to 9.6 GPa. Electrical resistivity at 4.2° K was also measured as a function of tin content and found to vary from $1.27 \times 10^{-8}$ ohm-cm to $1.3 \times 10^{-5}$ ohm-cm. The extent of anisotropy from texturing effects was evaluated by determining the preferred orientation in a variety of composites. In the external diffusion processed composite in the as-reacted condition, texturing does not seem to be a significant factor.

Exceptionally high critical currents of up to $2 \times 10^4$ amps/cm$^2$ at 15.4 T, were attained in several 25 v/o Nb composites. The strain tolerance of critical current was high in some low volume fraction composites, only 4% degradation in the zero strain critical current after 2% strain. The mechanical properties of ultrafine filament (~ 0.3 μ) composites were not as good as expected. This is attributed to collective behavior of the filaments in the primary colonies, giving an effective filament size of 1-2 μ.

A fatigue model has been developed which makes qualitative predictions of low cycle fatigue behavior and identifies the matrix work hardening modulus and yield strength as the most important controlling parameters.

Thesis Supervisor: Robert M. Rose
Title: Professor of Metallurgy
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1.1. **INTRODUCTION**

The full potential of Nb$_3$Sn as a superconductor was first realized in 1961 by J. E. Kunzler et al. (1) who found that sustained current densities in excess of 100,000 amp/cm$^2$ in magnetic fields of 8.8 T were possible. This discovery generated considerable interest in Nb$_3$Sn as a superconductor for high field magnet applications, and as such it has become the candidate conductor for use in fusion reactors, transmission lines, magnets for high energy physics, and a variety of other applications (2, 3, 4).

Not until the late sixties, however, did Nb$_3$Sn become practical for magnet applications, because of magnetic instabilities or flux jumps which severely limited its performance. This problem was avoided by fabrication of fine filamentary composites of the superconducting compound in a matrix of a high conductivity normal metal. This reduced the magnetic energy available for flux jumping and greatly simplified the construction of magnet devices. A great deal of research is now underway on the development of practical multifilamentary superconductors of the Nb$_3$Sn type.

One of the major problems in the interpretation of mechanical properties data has been the lack of attention paid to the metallurgy of the composites. Of particular concern in this thesis are the effects of plastic deformation in the bronze matrix, preferred orientation effects, fatigue behavior, and Kirkendall porosity. For large device applications, of course, the most important properties of the composite
will be the critical current, $J_c$ at the operating field, and as substantial a strain tolerance as possible. The U.S. Department of Energy has recently set a goal of 2% strain tolerance with no degradation in $J_c$, and a $J_c$ of $10^5$ amps/cm$^2$ at 15T.

In order to achieve these properties it was decided that filament sizes much below 1μ might be desirable. It would be expected from elementary brittle fracture theory (5) and the Hall-Petch relationship (6, 7), which applies to a variety of microstructures, that increased fineness of the filaments should lead to increased strength. This may also be true for the matrix between the filaments, since the closeness of the filament spacing will tend to support the matrix in the radial direction. During loading along the fiber axis this causes a hydrostatic tensile stress in the matrix which retards the onset of plastic deformation, so increasing the local matrix yield strength. By proper control of the filament size, and consideration of the metallurgical factors previously mentioned, the fabrication of composites with well understood mechanical behavior and excellent strain tolerance may be feasible.

In order to accomplish this objective a large number of composites were fabricated by the external diffusion technique with a variety of niobium contents and filament sizes (see Section 2.1). The external fabrication process was chosen in this laboratory for two reasons in particular: first the ease with which pure niobium and copper can be drawn; and secondly the fact that the continuous filament processes,
i.e. the bronze and external diffusion processes, are the most amenable to large scale fabrication.

After problems encountered with Kirkendall porosity in the tin diffusion stage of fabrication were successfully overcome the composites were heat treated to optimize the $J_c$ versus magnetic field behavior. Attention was also paid to the properties of the bronze matrix, since this is probably the most important factor in controlling the overall mechanical properties of the composite. In order to properly model the mechanical behavior it was also necessary to know the matrix yield strength and work hardening modulus at 4.2°K. The importance of accounting for texturing in the matrix is best evidenced by the dramatic variation in the elastic modulus of copper with crystallographic direction, ranging from 200 GPa in the <111> direction to 69 GPa in the <100> direction. The preferred orientation in all the components of a number of composites was measured using a pole figure goniometer, and conclusions regarding the "as-reacted" matrix modulus and filament shape were made.

An extensive series of tensile tests at 4.2°K on reacted composites in zero magnetic field were made, and effects of the residual tin content in the bronze matrix, filament size, and initial niobium content observed. Measurements of $J_c$ versus stress-strain were made at the FrancisBitter National Magnet Laboratory at fields up to 9 T, and again correlations were made between strain sensitivity, initial niobium content, and residual tin content of the matrix. Finally a model was
devised that describes the fatigue behavior of a composite, in connection
with the influence of plastic deformation on the residual stress state.
The model has had limited success in predicting the composite behavior
under cyclic loading.

Various composites fabricated in this laboratory have produced some
of the best $J_c$ versus field behavior yet measured. Some have sustained
2% strain with minimal ($\leq 4\%$) degradation in $J_c$. Unfortunately those
composites with the best strain tolerance tend to have inferior $J_c$
behavior, and the ultrafine filament composites, although they generally
have equivalent $J_c$ versus field properties to the coarser filament
composites, have not achieved the desired strain tolerances. These
problems may be attributed to difficulties encountered in the fabrication
process. Regardless, the outlook for composites fabricated by the
external diffusion technique with ultrafine filaments is extremely good,
and the likelihood of meeting or exceeding the Department of Energy
specifications is high.
1.2. PREVIOUS WORK

A variety of competing fabrication technologies for multifilamentary composites have emerged over the last decade, the most important of which to date, has been the bronze process, which was first utilized by Kaufman and Picket in 1970 (8). Many fine filaments of niobium are prepared in copper-tin bronze by an extrusion and drawing process. The $\text{Nb}_3\text{Sn}$ is then formed by reaction between the niobium and tin at temperatures above 600°C. In 1972, Suenaga and Sampson (9) proposed a somewhat different process in which niobium rods in a pure copper matrix were extruded and drawn to make multifilamentary wire. The wires were tin plated by a dipping process, and tin diffused into the copper to react with the niobium filaments. This became known as the "external diffusion process." The external diffusion process has some advantages over the bronze process because of the prohibitively high work hardening rate of the bronze matrix, which necessitates frequent annealing. Pure niobium and pure copper can be drawn to very fine sizes without any intermediate annealing steps. Other more recent technologies being investigated for $\text{Nb}_3\text{Sn}$ composite fabrication include the in-situ method pioneered by Tsuei (10) and Roberge (11) and the cold powder process (12).

The in-situ method involves the rapid solidification of Cu-Nb or Cu-Nb-Sn alloys. For alloys containing more than 10 w/o Nb the resulting microstructure comprises a fine dispersion of niobium rosettes in a copper or bronze matrix. A significant increase in the
critical current of these alloys occurs, both in as-cast ingots and
drawn wire, for niobium contents greater than 10-12 w/o. This has
been attributed to the connectivity of the superconducting phase in the
higher niobium content alloys, and has been explained in terms of site
percolation theory (13). In the powder process a mixture of elemental
niobium and copper powders (∼40μ diameter) is extruded and subsequently
drawn to wire. Both hot and cold extrusion have been employed (14).
For either the in-situ or powder process numerous discontinuous
filaments of niobium are formed in the copper matrix, and there is no
essential difference in the micro-geometry between these and the
continuous filament processes.

Although Nb₃Sn has a good high field critical current, Jc, and a
high critical temperature, Tc, it is a brittle intermetallic compound
and has inherently poor mechanical properties. Since Nb₃Sn is formed
at temperatures in excess of 900°K, and is then cooled to 4.2°K, the
high thermal expansion coefficient of the bronze matrix causes large
compressive stresses in the reaction layer; this has been an important
factor in understanding the effects of stress on multifilamentary
composites (15, 16). The most important superconducting property to be
considered, in terms of strain sensitivity, is the critical current.
It is well established that the application of stress to Nb₃Sn, and
other A-15 compounds, will decrease both Jc and Tc (17, 18, 19). In
multifilamentary composites it has been consistently observed that an
initial increase in Jc with strain occurs, because of the relief of
the residual compressive stresses caused by differential thermal
contraction. When the stress becomes tensile $J_c$ decreases rapidly. The extent to which $J_c$ is recoverable varies considerably with particular composites, and permanent degradation has often been observed to occur well below 1% strain. Irreversible degradation has been attributed primarily to filament fracture (16). Bulk Nb$_3$Sn, with no supporting matrix, appears to have a fracture strain of about 0.2%, but experimentally in multifilamentary composites the fracture strain has been found to be quite in excess of this (20, 21).

Reasonable data on the tensile behavior of multifilamentary superconducting composites has become available only over the last few years. Old and Charlesworth (20) performed room temperature tensile tests on a range of bronze processed composites, having between 11 v/o and 25 v/o initial niobium contents. They observed two types of stress-strain behavior, that were markedly different, depending on the initial volume fraction of niobium. Above 15 v/o niobium the composites failed in tension at strains usually below 1%, whilst those composites with less than 15 v/o niobium behaved in a ductile manner sustaining strains of approximately 10% before failure, see Figure 1.1. Acoustic emission studies accompanying the stress-strain testing indicated that, for type B composites, figure 1.1, significant filament fracture did not occur at strains below 0.7%. The same type of behavior has also been observed by Larbalestier et al. (21) for an 11 v/o niobium commercial composite at 4.2°K, where fracture strains ranged from 4.6 to 13.8%. In any case, from metallurgical examination and $J_c$ versus strain measurements in multifilamentary composites with filament sizes of 2µ
Figure 1.1 - Variation in the stress-strain behavior of bronze processed composites with differing volume fractions of niobium (A-25 v/o Nb, B-12 v/o Nb). After Old and Charlesworth (20).
or above, it appears that fracture of the Nb$_3$Sn compound occurs at strains between 0.2-0.7% (16).

An extensive volume of data on $J_c$ versus strain behavior has been reported, and considerable differences in the strain sensitivity have been observed. Ekin (22) tested a number of commercial composites of both the Nb$_3$Sn and NbTi type and found that the $J_c$ decreased immediately upon the application of strain. In the Nb$_3$Sn conductor, 10-60% degradation in initial $J_c$ might occur for strains up to 0.4%. He also observed that the first 20-30% degradation is reversible in both types of conductor. Very much the opposite behavior has been observed by Rupp (23, 24) who found significant increases in $J_c$, and that the peak in the $J_c$ versus strain behavior might occur anywhere between 0.5 and 0.9% strain. The variation in $J_{cm}/J_{co}$ (maximum $J_c$/zero strain $J_c$) was found to be solely dependent on the strain to maximum $J_c$ and that the largest peak strains occur for composites with the smallest fraction of Nb$_3$Sn, in keeping with the qualitative behavior expected from differential thermal contraction effects. Most $J_c$ versus strain data has typically fallen between these extremes; Figure 1.2, after D.W. Deis et al. (25), shows some of the typical behavior for a variety of composites.

There are two basic mechanisms in operation which cause a degradation in the critical current. First, after sufficient plastic deformation has occurred in the matrix there will be a residual tensile stress in the filaments, which should be limited by the local matrix yield strength (26). Secondly, beyond approximately 0.6% real strain in the filaments, major cracks occur which traverse completely the
Figure 1.2 - Comparison of $J_c$ versus strain behavior obtained by various investigators. Deis (25), Rupp (23), Easton (46) and Ekin (22). After Deis et al. (25).
reaction layer thickness. Below 0.6% real strain many minor cracks occur, and it is interesting to note that in low volume fraction composites, <20% Nb, preexisting cracks occur even in unstrained specimens (27). Luhman (27) et al. has observed in monofilament composites that the rate of degradation of $J_c$ under tensile strain is strongly dependent on the bronze to niobium ratio. He found that the strain to onset of permanent degradation varies from 0.8% for a 50 v/o Nb composite to 1.3% for a 12 v/o Nb composite. The strain at which the maximum $J_c$ occurs also increases with decreasing niobium content, scaling as would be expected assuming that it is entirely a function of the prestress from differential thermal contraction.

Depending upon whether the composite conductor is intended for low field, <10 T, or high field applications, the heat treatments must be different. For high low-field $J_c$ the most important factor is flux pinning. This is generally enhanced by reducing the grain-size in the superconducting compound by reaction heat treatments at temperatures that are as low as practically possible. For $\text{Nb}_3\text{Sn}$, 650° C has been a standard reaction heat treatment temperature that has produced good results. Small grain sizes on the order of 0.1μ or less are produced in the solid state diffusion reaction, and appreciable grain growth does not occur if the heat treatment is continued for an extended length of time. Initially doping the niobium with additions of $\text{ZrO}_2$ or Zr has also been employed in an attempt to improve flux pinning. The results obtained are not promising in that the measure $J_c$'s appear low. These additions, particularly the $\text{ZrO}_2$ do, however, retard the grain
growth (28). The high field critical current is controlled primarily by the upper critical field, $H_{c2}$. Enhancement of the $H_{c2}$ is usually accomplished by improving the long range order and stoichiometry of the superconductor through reaction heat treatments at higher temperatures, 700°C (2).
2. EXPERIMENTAL PROCEDURES

2.1. Composite Fabrication

The first stage in the fabrication process begins with the swaging of an OFHC copper tube, cleaned and etched with nitric acid, onto a niobium rod, acid cleaned and degreased with trichloroethylene. The necessary volume fraction of niobium is achieved by etching excess copper from the surface of the rod. This monofilament composite is then drawn through hexagonal dies until it is 0.26 cm (0.102 inch) flat-to-flat, reduction in cross-sectional area of 20-30% occurring between successive dies. The hexagonal wire is then cut into 15.2 cm (6 inch) sections, and bundled into a 2.54 cm (1 inch) o.d. OFHC copper can, which contains 61 such pieces and a smaller number of OFHC copper hexagonal pieces. This results in a 1 inch diameter composite with 61 filaments, the overall volume fraction of niobium being reduced by approximately 30% each time the composite is bundled. The can is then electron beam welded shut with copper end caps in a vacuum of $10^{-5}$ torr, and annealed at 300°C for 1/2-1 hour. Further reduction of the composite continues with a single cold extrusion giving a 75% reduction in area, after which the composite is given the same pre-extrusion anneal. After drawing round to 0.25 inches the composite is divided into two sections; one designated for drawing to 0.102 inches hexagonal, and the other for drawing to 0.010 inches round. The wire drawn to 0.102 inches hexagonal is cut and bundled into an OFHC copper can, producing a composite with 61² (3721) filaments. The composite is then extruded and drawn, part being drawn to 0.010 inch wire. In this way a variety of composites
can be fabricated with a substantial range of niobium contents, with either 61, 61^2, or 61^3 filaments; a complete list is presented in Table 2.1.

The nominal final filament size in each composite was expected to be: 0.1 μ in the 61^3; 1 μ in the 61^2; and 10 μ in the 61 filament. Due to the necessity of annealing at intermediate stages in the drawing process the final filament sizes were actually: 0.3 μ in the 61^3; 2-3 μ in the 61^2; and 10-15 μ in the 61. This is attributed to end effects that occur during the extrusion and early drawing stages when the composite is short in length. The softer copper tends to be pushed to the end of the billet, and the niobium filaments are deformed less than the overall composite reduction would indicate. This effect will be more exacerbated as the niobium becomes successively work hardened.

2.2. Tin Plating

Wires were tin plated using a commercial stannous fluoborate plating solution. Prior to plating they were boiled in trichloroethylene, recrystallized at 650°C for 16 hours and vapor degreased in trichloroethylene. The necessity of the recrystallization anneal will be discussed fully in a later section. Tin plating was carried out at current levels of 1/2-1 ampere, a cell voltage of 300 mV and a temperature of 45°C. This procedure normally produced uniform and adherent coatings of the required thickness, usually 2 μ to 16 μ, in times ranging from 5 minutes to 25 minutes. Failure to produce good coatings could usually be remedied by replacing the plating solution. The quantity of tin plated was determined by measuring thickness with a
Table 2.1

A compilation of the composites fabricated in this laboratory, with their niobium contents, in volume percent, and number of filaments

<table>
<thead>
<tr>
<th>Volume percent niobium</th>
<th>Number of filaments</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>61²</td>
</tr>
<tr>
<td>40</td>
<td>61</td>
</tr>
<tr>
<td>28</td>
<td>61</td>
</tr>
<tr>
<td>25</td>
<td>61³</td>
</tr>
<tr>
<td>25</td>
<td>61²</td>
</tr>
<tr>
<td>19.6</td>
<td>61³</td>
</tr>
<tr>
<td>17.5</td>
<td>61</td>
</tr>
<tr>
<td>13</td>
<td>61³</td>
</tr>
<tr>
<td>12</td>
<td>61²</td>
</tr>
<tr>
<td>10.5</td>
<td>61</td>
</tr>
<tr>
<td>7.35</td>
<td>61²</td>
</tr>
<tr>
<td>5.15</td>
<td>61³</td>
</tr>
</tbody>
</table>
micrometer, accurate to 0.0001 inches (2.5 μ), or by measuring the increase in weight with a sensitive electronic balance.

2.3. Homogenization and Reaction Heat Treatment

Once electroplated the tin must be diffused into the bulk of the wire and reacted with the niobium to form \( \text{Nb}_3\text{Sn} \). It is necessary to begin the homogenization at low temperatures, about 200°C, in 1 atm. of argon, and increase the temperature slowly, usually in steps of 100°C evacuating the furnace to \( 10^{-5} \) torr when the temperature exceeds 300°C. An example of a complete homogenization schedule is shown in Table 2.2. The times and temperatures may be varied considerably with success, the essential features being a slow initial increase in temperature, and the maintenance of 1 atm of argon below 300°C.

Reaction heat treatments were carried out in a titanium gettered vacuum of better than \( 10^{-5} \) torr. The wires were placed in a vycor tube sealed at one end, with the open end connected to the vacuum system. A 24-inch tube furnace was used. A chromel-alumel thermocouple was attached to the outside of the vycor tube and located at the middle of the wire sections, which were then placed at the center of the furnace.

2.4. Measurement of Critical Current versus Critical Field

Critical current, \( J_c \), versus critical field, \( H_c \), was measured using the standard four point probe technique, in a transverse magnetic field. Samples were mounted in a special "header" constructed for a 2 inch bore magnet capable of fields to 15.5 T. Current was supplied from a 12 V truck battery regulated through a reostat. Voltage drop across a
### Table 2.2
Homogenization schedule for tin plating

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Temperature (°C)</th>
<th>Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>1 atm argon</td>
</tr>
<tr>
<td>8</td>
<td>232</td>
<td>1 atm argon</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>1 atm argon</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>2 x 10^{-6} torr</td>
</tr>
<tr>
<td>25</td>
<td>500</td>
<td>2 x 10^{-6} torr</td>
</tr>
</tbody>
</table>
nominal 2 cm of the sample, was measured using a Keithly nanovoltmeter. Both current and voltage were displayed simultaneously on a x-y recorder. The standard procedure for such measurements was to vary the current at constant field, beginning with the highest field first to minimize the possibility of damage during a quench.

2.5. Mechanical Testing

Performing mechanical tests on superconductors in high magnetic fields introduces technical problems due to the small working area available. Tensile testing and simultaneous measurement of $J_c$ were performed at fields up to 9 T in a 6 inch bore magnet. The dewar and requirement of transverse magnetic field limits the working distance for sample and grips to slightly less than 4 inches. A special tensile testing apparatus was designed and constructed, allowing wires of an initial 2 inch gage section to be tested to slightly beyond 2% strain. A schematic of the apparatus is shown in Figure 2.1. In order to avoid problems with friction, that occur frequently during mechanical testing at cryogenic temperatures, the sample is loaded against a fixed pin, the cruciform section, (Figure 2.1) which undergoes pure torsion. Wires are loaded by turning the micrometer which pushes a titanium tube against the brass loading arm. The load is measured as the output from a linear voltage differential transformer (LVDT) fixed inside a steel proof ring. Deflection is measured by another LVDT located below the load cell. By careful calibration of the apparatus; measuring load versus deflection of the cruciform section at 4.2°K, the compression of the titanium shaft during loading, and outputs of the LVDT's as a func-
Figure 2.1 - Schematic of the tensile test apparatus. For clarity the current and voltage leads are not shown.
tion of load and deflection, accurate and reproducible data can be obtained. The information necessary to determine the stress-strain curve can be determined solely from the outputs of the LVDT's; this information is normally plotted directly on an x-y recorder.

Copper, silver and nickel wires were used to check the calibration; very repeatable values of the initial yield strength of copper were obtained, and the elastic modulii of silver and nickel determined to be 69GPa and 172GPa psi respectively at 4.2°K. Of course, uncertainties in the modulii of the wires due to texturing effects must be considered. Later tensile data on bronze wires gave initial elastic modulii in the range of 69-103 GPa at 4.2°K, in reasonable agreement with what would be expected from room temperature data. Figures 2.2 and 2.3 show photographs of the testing apparatus. In these figures the current and voltage connections for measuring \( J_c \) can also be seen. The \( J_c \)'s were measured using the standard four point probe technique as described in the previous section.

2.6. Preferred Orientation Measurement

The preferred orientation in a number of external diffusion and bronze processed composites were measured using a Siemens pole figure goniometer in the reflection configuration (29). Wire samples were normally prepared by tightly packing parallel lengths of wire on a stiff plastic sheet and cementing them in place. The sample array was then ground and polished to approximately one half the wire thickness, and etched in nitric acid. The longitudinal axis of the wire was always
Figure 2.2 - Load and displacement measuring module on the tensile test apparatus.

Figure 2.3 - Loading arm on the tensile test apparatus, showing current and voltage leads.
aligned in the goniometer such that it coincided with the prime meridian of the projection used to display the pole density. Nickel filtered Cu Kα radiation was employed throughout.
3. RESULTS

3.1. Recrystallization and Kirkendall Porosity

It has been recognized in the last few years that Kirkendall porosity can be a serious detriment to the superconducting and mechanical properties of multifilamentary wires produced both by the bronze process and external diffusion process. Void formation occurs in both types of composites because of a net vacancy flux in the opposite direction to the tin diffusion. Such a flux causes a supersaturation of vacancies in some parts of the composite. The vacancies then nucleate heterogeneously to form voids at some preferred site.

In bronze processed composites void formation occurs primarily in the region between the filaments where they are most densely packed. Figure 3.1.1 shows the Kirkendall porosity in an Airco #148 composite; it is typical of all bronze processed composites. Void formation adjacent to reacted filaments has been cited as a major cause of crack initiation in the filaments, and has been correlated with conductor instabilities, because of poor thermal and electrical transfer properties (30). A somewhat different situation occurs in external diffusion processed composites since the greatest vacancy flux will occur near the outside of the wire during the initial stages of the homogenization heat treatment. Void nucleation of almost disastrous proportions occurs in the exterior portions of the wire, rendering it useless for practical applications. An example is shown in Figure 3.1.2 for a 25 v/o Nb 612 filament composite fabricated in this laboratory. It should be
Figure 3.1.1 - Kirkendall porosity in a commercial bronze processed AIRCO # 148 composite.
Figure 3.1.2 - Kirkendall porosity in a 25 v/o Nb 61^2 composite fabricated by the external diffusion process. Not recrystallized, 800 X.
noticed from Figure 3.1.2 that the majority of the void formation occurs at the cold welded interface that separates the bulk of the composite from the outer can of the final extrusion. This is dramatically demonstrated in Figure 3.1.3. This is in keeping with what has been previously observed for nucleation of Kirkendall voids in experiments involving diffusion couples (31). Voids will nucleate heterogeneously at cold welded interfaces, probably at impurities trapped at the interface (32). Baluffi (31) observes that a vacancy supersaturation of 1% or less is adequate for void nucleation, and that dislocations and subgrain boundaries are not likely to be effective as nucleation sites.

Given these circumstances, there are two avenues to suppression of Kirkendall voids: management of the diffusion profile (and, therefore, the vacancy-supersaturated zone), or suppression of the heterogeneous nucleation sites. In practice the former approach would consist of altering the composition of the tin coating. The latter approach, which in terms of classical nucleation theory should be more effective, depends on the nature of the second phase particles which nucleate the voids. Although these have not been identified, it is expected that at least some of the particles can be dissolved partially or completely at elevated temperatures. This approach has now been confirmed to be highly effective in the suppression of Kirkendall void formation during the external diffusion process. Niobium-copper composites were annealed for 16 hours at 450°, 550°, 650°, and 750°C before the tin plating. The porosity is reduced in the final composite as the annealing temperature is increased. As figures 3.1.4 and 3.1.5 show, the voids are
Figure 3.1.3 - Void nucleation at a cold welded interface in an external diffusion processed composite. Note recrystallized, 2480 X.
Figure 3.1.4 - A 25 v/o Nb 61\(^2\) composite fabricated by the external diffusion technique. Recrystallized at 550\(^\circ\)C prior to tin plating. As-reacted.
Figure 3.1.5a - An as-reacted 25 v/o Nb 61 composite recrystallized at 650°C prior to tin plating, 300 X.
Figure 3.1.5b - An as-reacted 25 v/o Nb 61\(^2\) composite recrystallized at 750°C prior to tin plating, 300 X.
almost completely eliminated by using the higher annealing temperatures.

Recrystallization and tin diffusion experiments were also conducted on large diameter wires, where large net vacancy fluxes at the surface will be maintained for greater periods of time. A 25 v/o Nb $61^3$ composite was drawn to wire diameters of .050, 0.040, and 0.030 inches, recrystallized for 16 hours at 650°C and tin plated with a sufficient amount of tin for complete reaction of the filaments. After a homogenization and reaction heat treatment for 140 hours at 650°C metallography revealed the presence of significant porosity at the wire surfaces, as shown in Figures 3.1.6 - 3.1.8. Further work with large diameter wires will be necessary to overcome these problems.

It is not possible to use this method in the internal diffusion technique, where voids nucleate in the depletion regions between the Nb-Nb$_3$Sn filaments, because diffusion of the tin will immediately occur in the matrix. In this respect at least, the external diffusion method is the more flexible of the two.

3.2. Preferred Orientation Effects

Texturing in the matrix of a bronze processed Airco #148 composite is shown in Figures 3.2.1 and 3.2.2, before and after reaction heat treatment, respectively. The as-drawn texture of the bronze matrix (approximately 13 w/o Sn) shows a somewhat weak texture that is approximately <211> or <221>, having a maximum (200) pole density about 60° from the wire axis. The same composite reacted (Figure 3.2.2), exhibits a generally broad intensity band along the equator, indicating
Figure 3.1.6 - Void nucleation in an as-reacted .030 inch (.076 cm) wire recrystallized at 650°C for 16 hrs. prior to tin plating. 242 X.

Figure 3.1.7 - Void nucleation in an as-reacted .050 inch (0.127 cm) wire recrystallized at 650°C for 16 hrs. prior to tin plating. 64 X.
Figure 3.1.8 - Severe porosity in an as-reacted .050 inch. (0.127 cm) wire recrystallized at 650°C for 16 hrs. prior to tin plating. 63 X.
Figure 3.2.1 - Bronze matrix (200) pole figure of an as-drawn bronze processed composite, Airco # 148.
Figure 3.2.2 - Bronze matrix (200) pole figure of an as-reacted bronze processed composite, Airco # 148. Approximately a <100> fiber texture.
a possible <100> texture. The absence of intensity contours at the poles is due to excessive geometric defocusing, which cannot be corrected for at angles below 10° from the wire axis. Some additional difficulties occur in the interpretation of the pole figures from the commercial composite, since these wires have been twisted in order to improve their electrical stability. In any case both these textures are fairly weak, in keeping with the observation that small quantities of tin considerably reduce texturing in copper (33).

The as-drawn fiber texture of a 19.6% Nb 612 composite, fabricated by the external diffusion process is shown in Figure 3.2.3. It displays a strong <111> component and a weak <100> component, as expected for pure copper. After tin plating, homogenization, and a reaction heat treatment for 60 hours at 700°C, the same composite develops a distinct <100> + <111> duplex structure, Figure 3.2.4, <100> component predominating. The development of the duplex texture makes interpretation of the elastic properties, particularly in terms of the variation of elastic modulus with crystallographic direction, more difficult. The <100> texture is more heavily developed, however, and it would be expected that the matrix should contribute less to the composite stiffness than the isotropic elastic modulus would indicate. The Nb3Sn reaction layer exhibited no measurable preferred orientation in this composite; the initial filament size was approximately 2-3 μ.

A series of texture measurements were performed on a 13 v/o 613 composite to assess the effects of recrystallization prior to tin plating, and to determine if the finer filament size, in this case
Figure 3.2.3 - Copper-matrix (200) pole figure of an as-drawn 19.6 v/o Nb 61\textsuperscript{2} composite. It is a duplex \textless 111\textgreater  + \textless 100\textgreater  fiber texture.
Figure 3.2.4 - Bronze matrix (200) pole figure of an as-reacted 19.6 v/o Nb 61\textsuperscript{2} external diffusion processed composite. It is a duplex $<$111$>$ + $<$100$>$ fiber texture.
0.2-0.3 μ, would induce preferred orientation in the Nb₃Sn. Since it is certain that no texturing effects occur in the plane perpendicular to the wire axis, proper fiber textures are obtained. This allows the complete extent of preferred orientation to be determined by scanning along the prime meridian of the stereographic projection. The application of quantitative x-ray diffraction techniques can then be used to measure the relative fractions of each orientation present. Figure 3.2.5 shows results obtained for the copper matrix of an as-drawn 13 v/o 61 ³ composite, which has undergone a 99% reduction in area since it was last annealed at 300°C. It shows the expected <111> + <100> duplex structure; the background intensity has been removed, and the intensities corrected for geometrical defocusing. The expression for the intensity diffracted by a single phase powder (or polycrystalline) specimen in a diffractometer (34), is

\[
I = \frac{I_o e^4}{m^2 c^4} \frac{\lambda^3 A}{32\pi r} \frac{1}{V^2} \left| F \right|^2 p \frac{1+\cos^2 2\theta}{\sin^2 \theta \cos \theta} e^{-2m \mu} \tag{3.2.1}
\]

where \(I = \) integrated intensity, \(I_o = \) intensity of the incident beam, \(e, m = \) charge and mass of the electron, \(c = \) velocity of light, \(\lambda = \) wavelength of radiation, \(r = \) radius of the diffractometer, \(A = \) area of the incident beam, \(V = \) volume of the unit cell, \(F = \) structure factor, \(p = \) multiplicity, \(\theta = \) Bragg angle, \(e^{-2m} = \) a temperature factor, and \(\mu = \) linear absorption coefficient. This expression can be used to determine the relative volume fractions of each orientation present. By measuring the same (200) reflection for either orientation, \(\theta, \mu\), and all the other parameters are kept constant. This leads to the
Figure 3.2.5 - Copper matrix (200) pole density of an as-drawn 13 v/o Nb 61³ composite. It is 55% <111> and 45% <100>.

Figure 3.2.6 - Copper matrix (200) pole density of a 13 v/o Nb 61³ composite, recrystallized for 16 hrs. at 650°C. It is 70% <111> and 30% <100>.
simple relationship for the volume fraction of each component present,

\[ \frac{I_{<100>}}{I_{<111>}} = \frac{C_{<100>}}{C_{<111>}} \]  

(3.2.2)

where \( I_{<100>} \) and \( I_{<111>} \) are the integrated intensities of the \(<100>\) and \(<111>\) reflections respectively, and \( C_{<100>} \) and \( C_{<111>} \) are their respective volume fractions. In the as-drawn composite this indicates 55% \(<111>\) and 45% \(<100>\) are present. After recrystallization for 16 hours at 650°C, prior to tin plating, the basic duplex structure remains the same; the volume fractions changing to 70% \(<111>\) and 30% \(<100>\), Figure 3.2.6. After tin plating, a reaction heat treatment for 40 hours at 650°C eliminated most of the preferred orientation leaving vestiges of the original \(<111>\) and \(<100>\), and traces of other orientations, which were not readily identifiable. Again no preferred orientation was observed in texture measurements of the \((321)\) and \((200)\) poles of the \(\text{Nb}_3\text{Sn}\) reaction layer.

The essential conclusion from these measurements is, that, for the external diffusion process as performed on the 13 v/o Nb 613 composite, matrix anisotropy is unlikely to be a significant factor affecting the mechanical properties. However other fabrication processes, even if only slightly different, may be expected to develop different textures, because of the sensitivity of texture development to prior history, and processing variables.

Niobium has a simple \(<110>\) fiber texture on cold-drawing, as do all b.c.c. metals. Figure 3.2.7 shows the as-drawn texture for the \((200)\) pole of niobium in a 19.6 v/o Nb 612 filament composite; it is strongly,
Figure 3.2.7 – Niobium filament (200) pole figure. It shows a distinct <110> fiber texture.
Figure 3.2.8 - Asymmetric deformation morphology of a niobium filament. The filament width in the shorter direction in \( \sim 1 \mu \).
and exclusively, <110>. Recrystallization of the niobium does not occur at the reaction heat treatment temperature, and in any case <110> deformation textures are retained on primary recrystallization. An interesting consequence of the <110> deformation texture results from the peculiar orientation of the slip systems, which tends to produce plain strain rather than axially symmetric flow of the individual grains during drawing (35). The result of the plain strain restriction is to cause considerable non-uniform plastic flow, and the originally cylindrical niobium filaments become ribbon-like with a highly irregular surface, Figure 3.2.8. The same effects are also observed in in-situ composites (36).

3.3. Mechanical and Electrical Properties of Bronze at 4.2°K

The mechanical and electrical properties of bronze are important in determining the behavior of Nb₃Sn multifilamentary composites. The yield strength and work hardening modulus are the most significant factors influencing the low cycle fatigue behavior (see Section 3.6), and the resistivity of the bronze is important in determining the electrical stability of the composite.

Utilizing the low temperature tensile testing apparatus described in Section 2.5 the yield strength and work hardening modulus of a variety of bronzes were measured at 4.2°K. The tin contents varied between 1.4 and 10.8 w/o. Bronze wires, .010 inches in diameter, were fabricated by tin plating copper wires (see Section 2.2) and homogenizing at 800°C for 140 hours. Since prior mechanical and thermal history may be important influencing factors, the "as received" copper
55.

wire was recrystallized at 700°C for one hour, cold drawn 98% and annealed for 16 hours at 650°C before tin plating. This should ensure a uniform and reproducible recrystallization texture.

The yield strength, defined at the 0.02% offset, is shown in Figure 3.3.1 as a function of tin content. It increases from 76 MPa for pure copper to 241 MPa for an 8.2 w/o bronze, and appears to vary linearly with tin content up to approximately 3 w/o Sn. The work hardening modulus showed rather unusual behavior increasing linearly from 1.5 GPa for a 1.4 w/o alloy to 2.9 GPa for a 5.15 w/o alloy, and then rising sharply to 9.6 GPa for a 6.32 w/o alloy. If this effect is real, significant differences in the mechanical behavior of composites with more or less than 6 w/o residual tin in the bronze would be expected. Reliable values for any mechanically determined modulus are difficult to obtain, but careful calibration of the inherent "softness" of the tensile test apparatus was made (Section 2.5), and close agreement was obtained between experimental and published data in tests with standard materials. This phenomenon therefore bears further investigation.

Representative stress-strain curves for two bronze alloys containing 2.08 w/o Sn and 8.2 w/o Sn are shown in Figures 3.3.2 and 3.3.3 respectively.

The modulus of elasticity, as measured from the stress-strain curves, tended to vary considerably, the 2.75 w/o, 5.15 w/o, and 8.2 w/o alloys had values all of approximately 103 GPa, close to the expected value of 110 GPa (37). Whereas the 1.4 w/o, 2.08 w/o, and 6.32 w/o bronzes had values ranging from 55 GPa to 76 GPa.
Figure 3.3.1 - The 0.02% offset yield strength of bronze as a function of tin content at 4.2°K.

Bronze 2.08 wt% Sn
T = 4.2 K

Figure 3.3.2 - Stress-strain data for a 2.08 w/o Sn bronze at 4.2°K. Points indicated were used to convert instrument output to values of stress and strain.
Figure 3.3.3 - Stress-strain data for an 8.2 w/o Sn bronze at 4.2°K. Points indicated were used to convert instrument output to values of stress and strain.

Figure 3.3.4 - The resistivity of bronze as a function of tin content at 4.2°K.
This variation is quite likely due to the experimental difficulties associated with slight bowing in the samples prior to testing. The measured yield strength of copper, 76 MPa (0.02% offset), was close to the value of 90 MPa (0.2% offset) which was measured at approximately 20°K with a clamp-on strain gauge extensometer (38). De Haas and Hadfield (39) obtained a value of 310 MPa for 10 w/o Sn bronze at 20°K, and, although their yield criterion was not reported, this appears to agree reasonably well with the extrapolated value of 276 MPa from Figure 3.3.1.

Resistivity measurements at 4.2°K were made, using the standard four point probe technique, on .010 inch bronze wires, fabricated in the previously outlined manner. Three orders of magnitude of variation in resistivity was observed between pure copper, $1.27 \times 10^{-8}$ ohm-cm, and a 10.7 w/o Sn bronze, $1.2 \times 10^{-5}$ ohm-cm, Figure 3.3.4. This compares with a resistivity of $3 \times 10^{-8}$ ohm-cm for pure copper at 20°K (40). The large variation in resistivity is convincing evidence for the necessity of regions of high purity copper, normally incorporated into commercial multifilamentary composites, to provide a high conduction path in the normal matrix.

3.4. Optimization of Critical Current

Measurements of the critical current, $J_c$, in fields up to 15.4 T were performed in a 2 inch bore magnet at the Francis Bitter National Magnet Laboratory. A selection of the best results obtained is shown in Figures 3.4.1 - 3.4.4 for a variety of composites. The highest $J_c$
Figure 3.4.1 - $J_c$ versus applied field for a variety of 25 v/o Nb 612 composites.

Figure 3.4.2 - $J_c$ versus applied field for the 12 v/o Nb 612 composite that gave the best results.
Figure 3.4.3 - $J_c$ versus applied field for a variety of 25 v/o Nb 61\(^3\) composites.
at 15.4 T, \(2 \times 10^4\) \text{amps/cm}^2 was obtained in a 25 v/o Nb composite with 6\(^1\)\(^2\) filaments, reaction heat treated 100 hours at 650°C, Figure 3.4.1. A surprisingly high \(J_c\) of \(1.2 \times 10^4\) \text{amps/cm}^2 was also obtained in a 12 v/o Nb 6\(^1\)\(^2\) composite, Figure 3.4.2. The critical currents are all based on the overall composite cross-section, including conductor and normal bronze matrix. These values compare favorably with those obtained for commercial bronze processes and \textit{in-situ} processed composites. At 12 T a \(J_c\) of \(1 \times 10^5\) \text{amps/cm}^2 in a 25 v/o Nb 6\(^1\)\(^2\) composite, Figure 3.4.1, is slightly better than that obtained in commercial multifilamentary composites (3), and at 14 T a \(J_c\) of \(3-4 \times 10^4\) \text{amps/cm}^2 is substantially better than the \(1 \times 10^4\) \text{amps/cm}^2 achieved by a 36 v/o Nb \textit{in-situ} composite (4). Of the 25 v/o Nb 6\(^1\)\(^2\) and 6\(^3\) composites that were recrystallized for 16 hrs at 650°C prior to tin plating, 50% in each case, had a \(J_c\) of better than \(10^4\) \text{amps/cm}^2 at 15.4 T. This includes all samples, no distinction being made between tin contents and reaction heat treatments.

In order to optimize the critical current versus magnetic field behavior a number of different approaches were taken, involving variation of the individual filament size, reaction heat treatment times and temperatures, including multiple heat treatments, and variations in the tin content of the matrix. Nb\(_3\)Sn layer thickness was measured as a function of time at 600°C to determine the feasibility of performing heat treatments at that temperature, in order to reduce the grain size and so increase the low field \(J_c\). Layer thickness was determined by scanning electron microscopy (SEM), and is shown in Figure 3.4.5. The
Figure 3.4.4 - $J_c$ versus applied field for a variety of 12 v/o Nb 61\(^2\) composites. Note the effect of initial tin content.

Figure 3.4.5 - Nb\(_3\)Sn reaction layer thickness versus time at 600°C in a bronze processed composite. The 700°C and 800°C data is after Farrell et al. (47).
$J_c$ of a 25 v/o Nb composite with 61$^3$ filaments, approximately 0.3 μ in diameter, reaction heat treated at 600°C is shown in Figure 3.4.6. The ultrafine filament composite was chosen in order to reduce the time necessary to react the niobium filaments, and because the 0.3 μ filaments may physically limit grain size. The results obtained were disappointing the best $J_c$ obtained at 15.4 T was $2 \times 10^3$ amps/cm², and the low field $J_c$'s were correspondingly poor.

High field critical currents are controlled more by the upper critical field, $H_{c2}$, than flux pinning. So, that for high field applications, it is necessary to improve the long range order and stoichiometry of the Nb$_3$Sn. This is done by performing the reaction heat treatment at higher temperatures, 700°C or above. Short times, 10 hours, at 700°C proved particularly effective in increasing the $J_c$ of composites with prior heat treatments at 600°C, Figure 3.4.7, but did not produce high field properties to match those obtained by heat treating at 650°C for 100 hours. A standard commercial heat treatment of 30 hours at 700°C produced tolerably good $J_c$ behavior, Figure 3.4.8. Heat treatments at 750°C produced generally very poor results, particularly composites with low niobium contents and ultrafine filaments. Somewhat better results at 750°C were obtained in 25 v/o Nb 61$^3$ and 19.6 v/o Nb 61$^2$ composites.

It is possible that the initial tin content of the matrix might influence the $J_c$ behavior. During the tin homogenization process and reaction heat treatment a higher tin gradient should enhance diffusion
Figure 3.4.6 - $J_c$ versus applied field for a variety of 25 v/o Nb 613 composites.

Figure 3.4.7 - $J_c$ versus applied field for a variety of 25 v/o Nb 613 composites, including those with double heat treatments.
Figure 3.4.8 - $J_c$ versus applied field for a variety of 19.6 v/o Nb 61 2 composites.

Figure 3.4.9 - $J_c$ versus applied field for composites given cyclic heat treatments.
and aid in reacting the filaments as completely as possible, in a given
time and temperature. Higher tin concentrations may also promote more
rapid growth of the Nb₃Sn, and improve stoichiometry and ordering for
high field properties. It was found that varying the tin content of the
composite did not appreciably influence the Jₐ behavior, one way or other,
provided the initial tin concentration in the matrix ranged above,
approximately, 13 v/o Sn. If the tin content was too low for reasonably
complete reaction of the niobium, then the Jₐ behavior declined. This
effect was observed by Foner et al. (41) for in-situ composites were
the maximum Jₐ values (degradation by prestress compensated by an applied
stress) at 14T and 15T varied from 1.5 x 10⁴ amps/cm² for an 12 v/o Sn
matrix to 2 x 10³ amps/cm² for a 6 v/o Sn matrix.

At a given reaction heat treatment temperature a maximum in the Jₐ
will occur with time, with the maxima at higher temperatures occurring
in shorter times. No significant peak effect appears to occur for 650°C
reaction temperatures, presumably due to lack of grain growth. The
differences between 100 hr and 200 hr heat treatments are shown in
Figure 3.4.1 for a 25 v/o Nb 61² composite. The 200 hr heat treatment
composite is only very marginally lower. The composites had the same
initial tin concentrations in the matrix.

Since the ultrafine filament composites have initial 61-filament
bundles with very high niobium contents, often 73 v/o, or higher, there
was a possibility that diffusion of tin to the center at these bundles
would be retarded by local depletion around the outer niobium filaments
as the reaction heat treatment proceeded. In order to test this
hypothesis cyclic heat treatments were performed on a number of ultrafine filament composites, utilizing two temperatures, one at which Nb$_3$Sn should form, either 600 or 650°C, and the other, 550°C or less, at which tin diffusion should predominate. Normally, four 25 hr heat treatments at 600°C, interspersed with 20 hr diffusion anneals at 500°C were used. The results, Figure 3.4.9 show that no advantage was gained over a single 25 hr heat treatment at 650°C, Figure 3.4.3.

The overall critical current versus applied magnetic field behavior of these composites, fabricated by the external diffusion process, is extremely promising. Some of the composites reported are superior to both commercial multifilamentary composites and in-situ composites.

3.5. Mechanical Properties and Their Influence on Superconducting Properties

3.5.1. Stress-Strain Relations at 4.2°K.

A series of tensile tests was performed at 4.2°K on composite wires using the mechanical testing apparatus described in Section 2.5. Fracture strains, yield strengths, elastic modulii and work hardening modulii were obtained from the stress-strain data for composites with volume fractions between 12 and 40%, and with varying initial tin contents.

Typically fracture occurs between 1-2% strain for almost all the composites tested, including those with volume fractions below 16%, see Figure 3.5.1a. This was not the result expected since previous studies (20, 21) have indicated that, for composites with initial niobium volume fractions below 16%, ductile behavior should occur, with
Figure 3.5.1a - Stress-strain data for an as-reacted 12 v/o Nb 612 composite at 4.2°K. Recrystallized for 16 hrs. at 650°C, 9μ tin plate, and 100 hrs. at 650°C reaction heat treatment. Yield strength (0.02% offset) 255 MPa.

Figure 3.5.1b - Stress-strain data for an as-reacted 25 v/o Nb 612 composite at 4.2°K. Recrystallized for 16 hrs. at 650°C, 23 w/o initial tin content, and 100 hrs. at 650°C reaction heat treatment. Yield strength (0.02% offset) 214 MPa.
failure strains on the order of 10%. However, a number of 13 v/o and 7 v/o Nb composites that were tested at high fields did sustain strains in excess of 2% without failure, reaching the testing capability of the tensile test apparatus.

The yield strengths of a series of 25 v/o Nb 61\(^2\) composites, reacted at 650°C for 100 hours, ranged from 207 MPa to 276 MPa, with initial tin contents in the bronze varying from 18 w/o to 24 w/o. The yield strength was defined by an 0.02% offset. No regular relationship between tin content and yield strength existed in this range, probably for several reasons: the uncertainty as to the true residual tin content of the matrix after reaction; heterogeneities in tin distribution, and difficulties associated with making accurate tensile measurements on 0.025 cm wires. A 25 v/o Nb 61\(^2\) composite with an initial tin content of only 9 v/o had a yield strength of 120 MPa, Figure 3.5.2. The elastic modulii, measured from the initial slopes of the stress-strain curves, were around 69 GPa for all the composites tested. The work hardening modulii varied slightly with the initial niobium content. The highest value obtained was 48 GPa for a 40 v/o Nb 61\(^2\) composite (Figure 3.5.3). The modulus dropped to 34 GPa for the 25 v/o 61\(^2\) and 61\(^3\) composites, and for composites with less than 25 v/o Nb, was generally around 24 GPa. A 17 v/o Nb 61 filament composite (Figure 3.5.4) showed serrated stress-strain behavior beyond 1.4% strain. Such behavior is indicative of filament fracture.
Figure 3.5.2 - Stress-strain data for an as-reacted 25 v/o Nb 612 composite at 4.2°K. Recrystallized for 16 hrs. at 650°C, 9 w/o initial tin content, and 100 hrs. at 650°C reaction heat treatment. Yield strength (0.02% offset) 121 MPa.

Figure 3.5.3 - Stress-strain data for an as-reacted 40 v/o Nb 612 composite at 4.2°K. Recrystallized for 16 hrs. at 650°C, 15μ tin plate, and 100 hrs. at 650°C reaction heat treatment. Yield strength (0.02% offset) 276 MPa.
Figure 3.5.4 - Stress-strain data for an as-reacted 17 v/o Nb 61 composite at 4.2°K. Recrystallized for 16 hrs. at 650°C, 9 μ tin plate, and 50 hrs. at 750°C reaction heat treatment. Yield strength (0.02% offset) 276 MPa.
3.5.2. Degradation of $J_c$ by Mechanical Loading

Those composites with the best $J_c$ versus strain behavior, that is, those composites which suffer the least degradation in $J_c$ under load, appear to be the 13 v/o Nb 613 composites. Figure 3.5.5 shows the $J_c$ behavior concurrently with the stress-strain data for the composite that gave the best results; it was reaction heat treated for 40 hours at 750°C and had a zero strain critical current, $J_o$, of $1.85 \times 10^4$ Amp/cm$^2$ at 8 T. The composite sustained a strain of 3.2\% at 689 MPa, without failure, and 96\% of $J_o$ was recoverable on unloading from 2.25\% strain. Prestress effects were significant in the composite: the maximum $J_c$ was 1.4 times $J_o$ at 0.78\% strain. The effect of plastic deformation on the residual stress state can be clearly seen in the $J_c$ behavior. On unloading from 0.4\% strain, the unloaded $J_c$ was 1.1 times $J_o$, and on unloading from 0.83\% strain a somewhat higher $J_c$ of 1.15 $J_o$ results, implying further relief of internal residual stress. Beyond 0.83\% strain $J_c$ begins to decrease, dropping to 0.95 $J_o$ at 1.75\% strain and 613 MPa stress. At 2.25\% strain the $J_c$ fell to 0.36 $J_o$, however, on unloading the $J_c$ continuously increased to 0.96 $J_o$ ($1.78 \times 10^4$ Amp/cm$^2$), at 1.45\% residual strain. Other 13 v/o Nb 613 composites behaved in a similar manner, although they appear considerably less strain tolerant, Figures 3.5.6 - 3.5.7.

The same basic features are clearly evident in all the $J_c$ versus strain behavior. A prestress effect is present that causes $J_o$ to increase by a factor of 1.1 and 1.2 at 0.5\% strain and 0.35\% strain.
Figure 3.5.5 - Stress-strain data shown concurrently with $J_c$ for a 13 v/o Nb 613 composite. $J_c$ data was taken at the indicated points and is in units of $10^4$ amps/cm$^2$. 
Figure 3.5.6a - $J_c$ versus strain for a 13 v/o Nb 613 composite. Points indicate strain at which $J_c$ was measured and dashed lines indicate unleading.

Figure 3.5.6b - Stress-strain data for the composite of figure 3.5.6a, points indicate critical current measurements.
Figure 3.5.7 - Stress-strain data shown concurrently with $J_c$ for a 13 v/o Nb 613 composite. The $J_c$ at point (A) is uncertain due to possible premature quench. Points indicate strain at which critical currents were measured.
respectively (Figures 3.5.6 and 3.5.7), and 65%-72% of \( J_0 \) is recoverable on unloading from 1.8% strain. Some inconsistencies occur in the \( J_0 \)'s of these composites, which vary from 9.9 \( \times 10^2 \) amps/cm\(^2\) to 8.5 \( \times 10^4 \) amps/cm\(^2\) at 8 T. As evidenced by the generally poor \( J_c \) versus field behavior obtained in low volume fraction composites with 61\(^3\) filaments (compare with the 12 v/o Nb 61\(^2\) composites), it appears likely that some filament fracture has occurred in these composites during the fabrication process. It is also likely that the higher volume fraction, 25 v/o Nb 61\(^3\), composites with ultrafine filaments have also been damaged during fabrication. Their \( J_c \) versus field behavior is not severely degraded, however, possibly due to percolation effects on which in-situ composites rely (13).

In many of the 25 v/o Nb composites tested, with both 61\(^2\) and 61\(^3\) filaments, very little effect due to an initial prestress was observed. The 25 v/o Nb 61\(^2\) composite, Figure 3.5.8, displays typical behavior observed in those composites. The \( J_c \) decreased to 50% of \( J_0 \) at 0.9% strain, and 93% of \( J_0 \) was recoverable on unloading from 0.68% strain. A 25 v/o Nb 61\(^3\) composite, Figure 3.5.9, also showed no prestress effect, and only moderate degradation in \( J_c \), suffering a 14% decrease over the zero strain value at 1.13% strain, at which point the composite fractured. The zero strain critical current, \( J_0 \), in that composite was poor, only 7 \( \times 10^3 \) amp/cm\(^2\) at 8T. Those 25 v/o Nb composites which did show a prestress effect generally failed at low strains.

Since by the nature of fabrication process, in the multifilamentary composites the distribution of filaments is not uniform. At the center
Figure 3.5.8a – $J_c$ versus strain for a 25 v/o $61^2$ composite. Points indicate strain at which $J_c$ was measured. Dashed line indicates unloading.

Figure 3.5.8b – Stress-strain data that was measured concurrently with $J_c$ for the composite of figure 3.5.8a.
Figure 3.5.9 - Stress-strain data shown concurrently with $J_c$ for a 25 v/o Nb 613 composite. Points indicate strain at which critical currents were measured.
of the initial bundle of 61 filaments in the ultrafine 25 v/o Nb 61\(^3\) composite the volume percent niobium it is close to 75% and in the 25 v/o Nb 61\(^2\) composite it is 51%. The 13 v/o Nb 61\(^3\) composite, Figure 3.5.6, has 40 v/o Nb in the original 61 filament bundles giving a bronze to niobium ratio of 3:2. The 0.35% strain to maximum \(J_c\) in that composite agrees very closely with the measurements of Luhman \textit{et al.} (27) for a 3:2 ratio monofilament composite. The 12 v/o Nb 61\(^2\) composite, Figure I.1, in Appendix I, which has 25 v/o Nb in the initial 61 filament colonies, also has similar \(J_c\) behavior to a 3:1 bronze to niobium monofilament composite.

It is clear that the heterogeneous nature of the distribution of filaments in multifilamentary composites may have important consequences concerning their mechanical and superconducting properties. By asserting that the stress in the bronze matrix cannot be uniform across the entire section of the composite, it would be expected that the regions of higher filament density would behave as if they were in a higher volume fraction niobium composite. This effect can be used to explain the absence of prestress effects in the 25 v/o Nb composites of Figures 3.5.8 and 3.5.9. A common characteristic of all stress-strain data for all composites is a hysteresis effect that occurs on cyclic loading, see for instance Figure 3.5.6. The width of the hysteresis loop becomes progressively larger at higher strain levels. This phenomenon is due to the fact that, during unloading, as the filaments contract elastically, the matrix undergoes plastic deformation before the load is completely relaxed. In an ideal composite this would manifest itself as an abrupt
change in slope during the relaxation half of the stress-strain curve. When the composite is reloaded a hysteresis loop has opened up, and becomes wider at higher strain levels because of the increased amount of plastic deformation that occurs on unloading. In a real composite the ideal behavior is smoothed out by the heterogeneities in the bronze yield strength, filament density and so on.

The ultrafine filament composites with higher niobium contents had disappointingly poor strain tolerance. This may be due, in part, to the absence of prestress, discussed previously. It is likely, however, that the lack of adequate mechanical strength is due more to the collective behavior that appears to occur in the primary 61 filament colonies. Essentially, since there is so little bronze between the filaments they tend to support an applied stress as a group, and any advantage in strength due to fineness of the filament size is lost.

The fracture surface of a 25 v/o Nb 61³ composite, Figure 3.5.10, shows this effect clearly. The primary colonies are flat areas which appear to have sheared during fracture as a single unit. Since those colonies are about 2 μ in diameter they cannot be expected to improve J_c versus strain behavior over 25 v/o Nb 61² composites, which have 2-3 μ diameter filaments.

3.6. The Fatigue Model

Since the initial prestress is considered particularly important in achieving high strain tolerances, a model has been developed which determines changes in the residual stress state due to plastic deformation during cyclic loading. It is able to predict, qualitatively, the fatigue
Figure 3.5.10a - SEM micrograph of the fracture surface of a 25 v/o Nb 61 composite, 2700 X.

Figure 3.5.10b - A primary 61 filament colony in the composite of Figure 3.5.10a showing indications of collective behavior, 27000 X.
behavior. It considers the influence of yield stress and work hardening modulus of the bronze matrix, and may give reasonable quantitative information. The results of a number of investigations (42, 43) which indicated that there was little effect from cyclic stress on $J_c$ behavior can be explained in terms of a large work hardening modulus for the bronze matrix. Some cyclic stress measurements on an in-situ composite (41) have been found to fit the model within the limited range of the experimental data. A detailed description of the model is given in Appendix II.

The essential results are that there exist three regions of cyclic behavior. At high stress or strain levels the matrix work hardens substantially and reaches a predetermined failure stress, possibly the ultimate tensile strength. At intermediate levels the matrix undergoes cyclic workhardening and the yield strength eventually increases to the point at which no more plastic deformation occurs. This effect is known as shakedown. At low stress or strain levels plastic deformation occurs only on the first cycle, and matrix shakedown occurs immediately. The expected fatigue behavior for a composite with material parameters appropriate to the in-situ composite of Foner (41) is shown in Figure 3.6.1. It can be seen that for applied stresses in the range of 330 MPa (48 ksi) to 410 MPa (60 ksi), cyclic effects are observed in the shakedown behavior. The residual stress state in the filaments changed from the initial value of -124 MPa to 610 MPa at the 345 MPa stress level, before shakedown occurred after 192 cycles. This indicates that the initial $J_c$ for the composite should increase with cycling until
the residual stress in the filaments becomes zero and then decrease
as it becomes positive. The experimental observations of Foner (41) on
this composite at the 345 MPa stress level indicated that $J_c$ increased
some 25% over 20 cycles. They noticed the largest increase in the first
5 cycles, 17%, in keeping with the fact that the amount of plastic
deformation on each cycle and, accordingly, change in residual stress
state will tend to decrease quickly as the number of cycles increases.

The work hardening modulus of the bronze matrix (Figure 3.6.1) was
chosen as $K = 7,000$ MPa in accordance with results obtained at 4.2° K
for the mechanical properties of low tin bronzes. As Figure 3.6.2 shows,
the value of the work hardening modulus is critical to the shakedown
behavior; large values of $K$ can be seen to substantially reduce the
number of cycles necessary to achieve shakedown. This is in keeping
with absence of cyclic stress effects reported for a number of composites
(43). For the $K = 7,000$ MPa curve in Figure 3.6.2, only a very narrow
stress range gives rise to cyclic behavior. The failure stress is
reached for the composites with the lower $K$ values at lower stress
levels, because of the increased plastic deformation in the matrix
associated with the cyclic behavior. The initial yield strength of
the matrix has no particular influence on the number of cycles or width
of the stress range in which shakedown occurs, but higher yield stresses
will move the range of cyclic behavior to higher stress levels.

A series of simulations were also made for a 25% Nb composite with
$K$ values of 7,000 MPa, 14,000 MPa, and 21,000 MPa. At the $K = 21,000$
MPa level, it was found that, no region of cyclic shakedown behavior
Figure 3.6.1 - The shakedown behavior expected for an in-situ composite with the same material parameters as that tested by Foner et al. (41).

Figure 3.6.2 - The effect of varying the work hardening modulus $K$ on the shakedown behavior of a 25 v/o $\text{Nb}_3\text{Sn}$ composite.
occurred for applied stresses and strains up to 700 MPa and 2.6% respectively; the composite had an initial yield strength of 210 MPa and a matrix failure stress of 560 MPa. A lower work hardening modulus, K = 14,000 MPa, showed no cyclic behavior for the applied strain analysis, but exhibited a very narrow region of shakedown behavior, between 486 MPa and 532 MPa for the applied stress analysis. The number of cycles to shakedown was essentially constant over this narrow region, varying between 104 and 106 cycles, Figure 3.6.3. For the K = 7,000 MPa simulation a broad band of cyclic shakedown behavior was observed in the applied strain analysis, from 1.2 - 2.1% strain. No cyclic behavior was observed in the applied stress analysis although it is possible that a narrow region could be overlooked because of the 35 MPa step size used to traverse the stress range of interest.

Further analysis of the in-situ composite, with K = 14,000 MPa, demonstrated the dramatic effects of K on the shakedown behavior. The cyclic shakedown region, Figure 3.6.4, was narrowed to 406-427 MPa, compared with 330-410 MPa for the composite of Figure 3.6.1. The number of cycles to shakedown was reduced approximately by a factor of two, and the onset stress for cyclic behavior increased from 330 MPa to 406 MPa. No shakedown behavior was observed at the K = 21,000 MPa level.

The fatigue model, although simple in nature, has been successful in demonstrating the importance of the matrix yield strength and work hardening modulus on fatigue behavior. By suitable choice of yield strength and work hardening modulus it has agreed with the behavior of an in-situ composite within a limited range of experimental data, and has
Figure 3.6.3 - The shakedown behavior of a 25 v/o Nb₃Sn with a work hardening modulus $K = 14$ GPa ($2 \times 10^6$ psi).

Figure 3.6.4 - The effect of an increased work hardening modulus on the shakedown behavior of the in-situ composite of Foner et al. (41).
shown that large $K$ values are probably responsible for the absence of observable fatigue effects in many composites.
4. **CONCLUSION**

A wide variety of properties mechanical and electrical, pertinent to a successful understanding of the behavior of $\text{Nb}_3 \text{Sn}$ multifilamentary composites have been measured. The important properties of the bronze matrix, the variation in yield strength and work-hardening modulus as a function of tin content were determined and used in the fatigue model.

Due to the successful elimination of Kirkendall porosity, the external diffusion process has been found to be a valid fabrication method for multifilamentary wires, at least for small diameter wires (0.010 inches). The extent to which preferred orientation in the bronze matrix might affect the mechanical properties has been considered. Pole figure goniometry was used to measure preferred orientation at various points during the fabrication process and, it was determined, in the as-reacted condition that the bronze matrix exhibited negligible texturing.

The $J_c$ versus magnetic field properties of the composites displayed considerable variation. Some 12 v/o Nb 61$^2$ and 25 v/o Nb 61$^2$ composites had excellent critical currents at 15.4 T, exceeding those of the better commercial and *in-situ* composites. The 25 v/o 61$^3$ composites with the ultrafine filaments were unpredictable in their $J_c$ behavior, the better samples had $J_c$'s similar to the 25 v/o Nb 61$^2$ composites, others were often an order of magnitude lower at a given field. The low volume fraction niobium ultrafine filament composites had uniformly poor $J_c$ behavior. We attribute this to filament breakage during the
fabrication process. Although the ultrafine filament low volume fraction composites, particularly the 13 v/o Nb 61\(^3\), had poor \(J_c\) versus magnetic field behavior, they did exhibit the best strain tolerance. As has been discussed in Section 3.5.2, it appears likely that the high volume fractions of niobium in the initial 61 filament colonies adversely affect the strain behavior of the composites, particularly those with the ultrafine filaments. This is exemplified by the lack of any prestress effect in the 25 v/o Nb composites, and by the collective behavior observed in 25 v/o Nb 61\(^3\) composites.

A model for low cycle fatigue behavior has been developed which predicts qualitatively the effects of cyclic loading, and identifies the yield strength and work hardening modulus of the matrix as the most important controlling parameters. The model works equally well for both continuous filament and in-situ composites, and is in agreement with the limited experimental data available.
5. SUGGESTIONS FOR FURTHER WORK

There are a variety of topics stemming from this work which merit further study. One of the principal goals of developing composites with ultrafine filaments (≈1 μ), and characterizing their mechanical and superconducting properties has not yet been met. It has become obvious that the ultrafine composites manufactured in this laboratory have too high a niobium content in the areas local to the filaments. In order to avoid this problem it will be necessary to fabricate composites from large diameter billets, and extrude with high reduction ratios. Such a composite must be designed so that the final spacing between the filaments is large enough that no collective behavior occurs. It is still an open question as to what final niobium content would be most suitable. Other less immediate avenues of investigation include the following:

1. For proper characterization of the 4.2°K mechanical properties of the composite wires uniacial testing in equipment capable of large elongations is necessary. This is essential for low volume fraction composites which may sustain strains in excess of 10%.

2. The problem of persistent Kirkendall porosity in larger diameter wires should be tackled by recrystallizing prior to tin placing at higher temperatures and longer times, and developing reliable multiple plating techniques.

3. As an alternative to tin plating with an aqueous stannous fluoroborate solution at 45°C, it is suggested that a fused salt
(probably SnCl₂, KOH, ZnCl₂) tin plating solution be used at temperatures in excess of 500°C. In this way it may be possible to control the tin deposition to the point at which diffusion into the bulk of the copper wire maintains the tin surface concentration at less than 14 w/o.

4. Scanning transmission electron microscopy (STEM) can be used to identify the impurities which cause void nucleation at the cold welded interfaces. Knowledge of what these impurities are may facilitate the choice of a more appropriate grade of copper, and may suggest changes in the fabrication procedure.

5. The fatigue model may be augmented by modelling the matrix behavior as continuously work hardening in the plastic region, instead of perfectly plastic. Further experimental verification of the model would be beneficial.
6. REFERENCES


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APPENDIX I

Additional $J_c$ Versus Strain Behavior

In this section additional data on the strain tolerance of the critical current for several composites is presented. The 12% Nb 61\% composite, Figure I.1, had a maximum $J_c$ of 1.26 $J_o$ at 0.49\% strain, although the strain value should probably be corrected to 0.36\% to account for some initial straightening of the wire during the test. Unloading this composite from 0.78\% strain resulted in a zero load $J_c$ that was 1.14 $J_o$. Higher strains, 0.9\% and above, caused severe degradation in $J_c$, and at 0.97\% strain the $J_c$ had fallen to 0.65 $J_o$. Quite remarkable behavior was observed in a 7 v/o Nb 61\% composite, Figure I.2, in which the zero strain critical current was less than 40 amp/cm$^2$ at 8T, stretching the lower limits of the current measuring equipment. Immediately on loading the $J_c$ jumped to $7.4 \times 10^3$ amp/cm$^2$, a remarkable effect, difficult to attribute entirely to the relief of prestress. Beyond 0.2\% strain the $J_c$ began to decrease rapidly, and the conductor was resistive beyond 1.6\% strain. Again by unloading the composite periodically some fraction of $J_o$ could be obtained. In this case since $J_o$ was so small, comparisons are better made with the $J_c$ at 0.2\% strain. On unloading the composite from 0.87\% strain 40\% of $J_o$ (0.02\%) was recoverable, and on unloading from 1.33\% strain 15\% of $J_o$ (0.2\%) was recoverable. The sample sustained 2.26\% strain without failure, as would be expected for a composite with a low initial niobium content.
Figure I.1 - Stress-strain data taken concurrently with $J_0$ for a 12 v/o Nb 612 composite. Dashed line indicates unloading.

Figure I.2 - Stress-strain data taken concurrently with $J_c$ for a 7 v/o Nb 612 composite. Critical current indicate at the points.
Those composites with only 61 filaments, each filament approximately 10-15 μ in diameter, had very poor $J_c$ versus strain behavior. Figure I.3 shows the results obtained from a 17 v/o Nb 61 composite. It was reaction heat treated 50 hours at 750°C in order to promote the growth of a reasonably thick Nb$_3$Sn reaction layer. A small increase in $J_0$, by a factor of 1.02, with a maximum at 0.2% strain, is the only evidence of prestress effects, and beyond 0.4% strain the $J_c$ drops rapidly to 15% of $J_0$ at 0.6% strain, with only 16% of $J_0$ recoverable on loading.
Figure I.3a - $J_c$ versus strain for a 17 v/o Nb 61 filament composite. Dashed line indicates unloading.

Figure I.3b - Stress-strain data taken concurrently with $J_c$ for the composite of Figure I.3a.
APPENDIX II

Section 1 - Introduction to the Fatigue Model

A two-component composite with perfectly elastic fibers and a work-hardening matrix can be expected to undergo significant changes during cyclic loading, due to the plastic deformation and increased yield strength of the matrix. The most important consequences are:

1. removal of the initial residual stress state,
2. superposition of a new stress state dependent on the loading conditions,
3. the occurrence of entirely elastic strains in the matrix (shakedown), and
4. the possible failure of the matrix if the yield stress exceeds a predetermined failure limit.

The model described below determines the occurrence of shakedown or failure, the final stress states of the matrix and fiber, and the number of cycles to reach that stress state. This is accomplished through the repetitive analysis of the stress conditions of each cycle, and the effect this has on the mechanical properties of the matrix for the following cycle. The model can be applied strictly to two-component, unidirectional composites with a work-hardening matrix and perfectly elastic fibers. It is based on the Voigt assumption of equal strains in the fiber and matrix, and although this assumption leads to a discontinuous stress distribution at the fiber-matrix interface, it has been used successfully in many applications (44, 45). Only those
stresses along the fiber axis are considered since the radial stresses are generally negligible for axial displacements that are not too large (45). The work hardening of the matrix has been accounted for by assuming the matrix to be elastic-perfectly plastic on each half-cycle and modifying the yield stress as necessary based on some functional relationship between plastic deformation and yield strength.

1.1. Some Preliminaries

Before a detailed description of the model is given some basic assumptions and equations are outlined. The principal assumption is that the composite follows the Voigt approximation:

\[ \varepsilon_c = \varepsilon_f = \varepsilon_m \]  \hspace{1cm} (II.1)

where \( \varepsilon_c, \varepsilon_f, \) and \( \varepsilon_m \) are the strains in the axial direction in the composite, fiber and matrix respectively. This leads to a volume averaged equation for the composite modulus,

\[ E_c = A_m E_m + A_f E_f \]  \hspace{1cm} (II.2)

\( E_c, E_f, \) and \( E_m \) are the elastic moduli of the composite, fiber and matrix respectively, and \( A_m \) and \( A_f \) are the area fractions of the matrix and fiber respectively (in this case identical with the volume fractions). The stress in the composite is given in a similar manner,

\[ \sigma_c = A_m \sigma_m + A_f \sigma_f \]  \hspace{1cm} (II.3)

where \( \sigma_c, \sigma_m \) and \( \sigma_f \) are the composite, matrix and fiber stress respectively, provided that no plastic deformation has occurred in the matrix. The work hardening of the matrix is described by the equation,
where $\sigma_{ym}$ is the matrix yield stress, $K$ and $n$ are coefficients determined experimentally from the stress-strain behavior of the matrix in the plastic region, and $\varepsilon_p$ the plastic deformation. An initial residual stress state is assumed to exist in the composite, obeying the equilibrium condition,

$$\sigma_{mr}A_m + \sigma_{fr}A_f = 0$$  \hspace{1cm} (II.5)

$\sigma_{mr}$ and $\sigma_{fr}$ are the residual stresses in the matrix and fiber respectively.

1.2. Constant Strain Amplitude Analysis

A detailed analysis for the case of constant applied strain fatigue is now presented. The initial yield strength of the composite is given by $(\sigma_{ym1} - \sigma_{mr})$ where $\sigma_{ym1}$ is the initial yield strength of matrix and $\sigma_{mr}$ the residual stress in the matrix. The initial elastic limit of the composite, $\varepsilon_{LMT}$, is then given by,

$$\varepsilon_{LMT} = (\sigma_{ym1} - \sigma_{mr})/E_m$$  \hspace{1cm} (II.6)

since the composite will yield at the same strain level as the matrix. If the applied strain has a value $\varepsilon_r$, the plastic strain on the first half-cycle, $\varepsilon_{pl1}$, will be,

$$\varepsilon_{pl1} = \varepsilon_r - (\sigma_{ym1} - \sigma_{mr})/E_m$$  \hspace{1cm} (II.7)

and the new yield strength of the matrix, $\sigma_{ym2}$, is obtained from Equation (II.4) by,
If $\varepsilon_T \leq \varepsilon_{LMT}$ no plastic deformation occurs and shakedown is guaranteed. The stress limits of the matrix and fiber are given in Section 2, under Termination Type 3.

On the load removal, or relaxation, half-cycle the composite will return to some equilibrium state, the problem then being to determine the extent of plastic deformation and the equilibrium stresses in the matrix and fibers. To determine the amount of plastic deformation that occurs, it is assumed that once the yield strength of the matrix in compression is reached, the matrix offers no increased resistance to further contraction of the fibers. Therefore, the equilibrium state will be reached when the loads on the fiber and matrix are balance, i.e.,

$$\sigma_{ym2} A_m + \sigma_{fr} A_f = 0$$

Hence, the residual strain both fiber and matrix, $\varepsilon_r$, based on the original length, $\ell_o$, is

$$\varepsilon_r = \frac{\sigma_{ym2} A_m}{\varepsilon_f A_f} + \frac{\sigma_{fr}}{E_f}$$

The plastic deformation on the relaxation half-cycle, is now given by

$$\varepsilon_{p2} = \varepsilon_T - \frac{(\sigma_{ym1} + \sigma_{ym2})}{E_m} - \varepsilon_r$$

and since $\varepsilon_T = \varepsilon_{pl} + (\sigma_{ym1} - \sigma_{mr})/E_m$, then

$$\varepsilon_{p2} = \varepsilon_{pl} - \frac{(\sigma_{mr} + \sigma_{ym2})}{E_m} - \varepsilon_r$$
The new yield strength on the second tensile half-cycle, will now be,

\[ \sigma_{ym3} = \sigma_{ym2} + K (\varepsilon_{p2})^n \]  \hspace{1cm} (II.13)

In order to always know the length of the matrix, it is necessary to keep track of the total amount of plastic deformation, being careful to distinguish between that causing an elongation, and that causing a contraction of the matrix. Hence, after the first relaxation half-cycle, the matrix will have a length,

\[ l_m = l_o \left(1 - \frac{\varepsilon_{pl}}{E_m} + \varepsilon_{p2} - \varepsilon_{p2} \right) \]  \hspace{1cm} (II.14)

At this point, a check is made to see if the new yield strength of the matrix, \( \sigma_{ym2} \), exceeds the failure stress, \( \sigma_f \). If the failure stress has been exceeded, or if the amount of plastic deformation, \( \varepsilon_{p2} \), is negligibly small, i.e. \( \varepsilon_{p2} \leq 0.0001\% \), then it becomes necessary to determine the final stress states of both fiber and matrix. In order to accomplish this the matrix and fibers are considered separately, and their lengths if debonded determined by,

\[ l_m = l_o \left(1 - \frac{\varepsilon_{pl}}{E_m} + \varepsilon_{p1} \right) \]  \hspace{1cm} (II.15)

\[ l_f = l_o \left(1 - \frac{\varepsilon_{fr}}{E_f} \right) \]  \hspace{1cm} (II.16)

where \( l_m \) and \( l_f \) are the lengths of the matrix and fiber respectively, and \( l_o \) is the initial length of the composite, considered large compared to the matrix deformation.

It may be pointed out that since the fiber is always elastic, \( l_f \)
given by Equation II.16 will always be the debonded fiber length. It is a straightforward problem to determine the stresses in the fiber and matrix necessary to maintain equilibrium,

\[ \Delta = \delta_f - \delta_m \]  

(II.17)

\[ \Delta = \delta_f \left( \frac{\sigma_{mr}}{E_m} - \frac{\sigma_{fr}}{E_f} - \varepsilon_{pl} \right) \]

Now, \( \Delta \) constitutes the total deformation in both fiber, \( \delta_f \), and matrix, \( \delta_m \), i.e.,

\[ \Delta = \delta_m + \delta_f \]  

(II.18)

where \( \delta_m \) and \( \delta_f \) are also given by,

\[ \delta_m = \frac{P_m \delta_o}{A_m E_m} \quad \text{and} \quad \delta_f = \frac{P_f \delta_o}{A_f E_f} \]  

(II.19)

and \( P_m \) and \( P_f \) are the loads on the matrix and fiber respectively, equal and oppositely directed. Using Equations (II.17, II.18, and II.19), we obtain,

\[ \left( \frac{\sigma_{mr}}{E_m} - \frac{\sigma_{fr}}{E_f} - \varepsilon_{pl} \right) = \frac{A_f E_f + A_m E_m}{A_m A_f E_f E_m} \]  

(II.20)

and, since, \( E_c = A_f E_f + A_m E_m \)

\[ P = \left( \frac{\sigma_{mr}}{E_m} - \frac{\sigma_{fr}}{E_f} - \varepsilon_{pl} \right) \left( \frac{A_m A_f E_f}{E_c} \right) \]  

(II.21)

So the corresponding stresses in the matrix and fiber become,

\[ \sigma_m = \frac{|P|}{A_m}, \]

104.
\[ \sigma_f = \frac{|P|}{A_f} \]

and which of these stresses are tensile or compressive can be decided in the following manner. If, after the plastic deformation on the first half-cycle, the length of the matrix debonded is less than or equal to the length of the fiber,

\[ \lambda_0 \left(1 - \frac{\sigma_{mr}}{E_m} + \varepsilon_{pl} \right) \leq \lambda_0 \left(1 - \frac{\sigma_{fr}}{E_f} \right) \]  \hspace{1cm} (II.22)

then the matrix remains in tension throughout the first relaxation half-cycle, and since no further plastic deformation occurs at the given load level, shakedown has been achieved. If \( \gamma_{ym2} \) exceeds the failure stress, the stresses in the fibers and matrix are calculated in the same way. The equations derived for the final stress state for all the different criteria are listed in Section 2.

Now, provided the plastic deformation, \( \varepsilon_{p2} \), is not negligible, and the matrix failure stress has not been reached, we can continue with the analysis. The plastic strain on the next tensile half-cycle, \( \varepsilon_{p3} \), is given by,

\[ \varepsilon_{p3} = \varepsilon_r - \frac{(\sigma_{ym3} + \sigma_{ym2})}{E_m} - \varepsilon_r \]  \hspace{1cm} (II.23)

The yield strength of the matrix for the following relaxation half-cycle is now,

\[ \sigma_{ym4} = \sigma_{ym3} + K(\varepsilon_{p3}) \]  \hspace{1cm} (II.24)

and the length of the matrix,

\[ \lambda_m = \lambda_0 \left(1 - \frac{\sigma_{mr}}{E_m} + \varepsilon_{pl} - \varepsilon_{p2} + \varepsilon_{p3} \right) \]  \hspace{1cm} (II.25)
Again, it is necessary to calculate the load in the matrix and fiber during the ensuing relaxation half-cycle by assuming that both are perfectly elastic. We have

\[ \Delta = \ell_f - \ell_m \]

and,

\[ \Delta = \frac{1}{2} \left( \frac{\sigma_{mr}}{E_m} - \frac{\sigma_{fr}}{E_f} - X_m \right) \]

The total plastic deformation, \( X_m \), is given by,

\[ X_m = \varepsilon_p 1 - \varepsilon_p 2 + \varepsilon_p 3 \]

and the load on the second relaxation half-cycle is given by,

\[ P = \frac{\sigma_{mr}}{E_m} - \frac{\sigma_{fr}}{E_f} - X_m \left( \frac{A_m A_f E_f}{E_c} \right) \]

The plastic strain on this half-cycle, \( \varepsilon_p 4 \), can be determined. The total strain on the previous tensile half-cycle was,

\[ \varepsilon_r = \frac{1}{2} \left( \frac{\sigma_{y m 2} + \sigma_{y m 3}}{E_m} + \varepsilon_p 3 \right) \]

therefore, \( \varepsilon_p 4 \) is given by:

\[ \varepsilon_p 4 = \varepsilon_r + \frac{(\sigma_{y m 2} + \sigma_{y m 3})}{E_m} + \varepsilon_p 3 - (\sigma_{y m 3} + \sigma_{y m 4})/E_m - \varepsilon_r \]

\[ \varepsilon_p 4 = \varepsilon_p 3 \left( \frac{(\sigma_{y m 2} - \sigma_{y m 4})}{E_m} + \varepsilon_r \right) - \varepsilon_r \]

\[ \varepsilon_p 4 = \varepsilon_p 3 + (\sigma_{y m 2} - \sigma_{y m 4})\left( \frac{E_c A_f E_f}{A_m} \right) \]
and the new yield stress is,

$$\sigma_{ym5} = \sigma_{ym4} + K(\varepsilon_p^n)$$

From a programming point of view, for a multicycle analysis, it is convenient to repeat variable names, so that $\sigma_{ym5}$ acquires the name $\sigma_{ym3}$ and $\sigma_{ym4}$ becomes $\sigma_{ym2}$. The cyclic analysis of this nature is continued until some termination criteria is reached, either

1. the yield stress of the matrix exceeds the failure stress, or
2. no further plastic deformation occurs and shakedown has been achieved.

1.3. Constant Stress Amplitude Analysis

The essential features of the constant stress analysis are the same as those of the constant strain analysis although the forms of a number of equations are altered. The initial yield stress of the composite is the same, $\sigma_{yml} - \sigma_{mr}$, and the elastic stress limit is given by,

$$\sigma_{LMT} = (\sigma_{yml} - \sigma_{mr})E_c/E_m$$

(II.30)

If the applied stress, $\sigma_a$, is less than $\sigma_{LMT}$ no plastic deformation occurs. Now, to begin the calculation of the stress state on the first tensile half-cycle it is necessary to calculate the amount of plastic deformation, $\varepsilon_{p1}$, that occurs. From geometrical considerations, it can be determined that the plastic deformation on the first half-cycle will be,

$$\varepsilon_{p1} = (\sigma_a - \sigma_{ycl})/A_fE_f$$

(II.31)
where \( \sigma_{ycl} \) is the initial yield strength of the composite given by,

\[
\sigma_{ycl} = (\sigma_{ym1} - \sigma_{mr})E_c/E_m \quad (II.32)
\]

and \( E_f A_f \) is an approximation to the composite modulus after the yield stress of the matrix has been exceeded. This approximation being based on the assumption that \( A_mK \ll A_f E_f \). The new matrix yield strength is,

\[
\sigma_{ym2} = \sigma_{ym1} + nK(\varepsilon_{pl}) \quad (II.33)
\]

The residual load in the matrix and fibers, after the relaxation half-cycle is determined in precisely the same manner as the constant strain amplitude analysis. Hence,

\[
P = \left( \frac{\sigma_{mr}}{E_m} - \frac{\sigma_{fr}}{E_f} - \varepsilon_{pl} \right) \left( \frac{A_m A_f E_f}{E_c} \right) \quad (II.34)
\]

The plastic deformation, \( \varepsilon_{p2} \), is given by,

\[
\varepsilon_{p2} = \varepsilon_{p1} - (\sigma_{ym2} + \sigma_{mr})/E_m - \varepsilon_r
\]

and the new yield stress is,

\[
\sigma_{ym3} = \sigma_{ym2} + nK(\varepsilon_{p2}) \quad (II.35)
\]

On reapplication of the load the plastic deformation on the tensile half-cycle, \( \varepsilon_{p3} \), is given by,

\[
\varepsilon_{p3} = \frac{\sigma_a - \sigma_{ym3}E_c/E_m}{E_f A_f} \quad (II.36)
\]

where \( \sigma_{ym3}E_c/E_m \) is the new yield strength of the composite.
The total plastic deformation, \( X_{\text{ml}} \), is given by,

\[
X_{\text{ml}} = \varepsilon_{p1} - \varepsilon_{p2} + \varepsilon_{p3}
\]  \hspace{1cm} (II.37)

and the load on the second relaxation half-cycle is,

\[
P = \left( \frac{\sigma_{\text{mr}}}{E_m} - \frac{\sigma_{fr}}{E_c} - X_{\text{ml}} \right) \left( \frac{A_m A_f E_m}{E_c} \right)
\]  \hspace{1cm} (II.38)

The plastic strain on this half-cycle, \( \varepsilon_{p4} \), can then be determined. The total strain on the previous tensile half-cycle was,

\[
\varepsilon_r = \frac{1}{E_m} + \frac{(\sigma_{ym2} + \sigma_{ym3})}{E_m} + \varepsilon_{p3}
\]

therefore \( \varepsilon_{p4} \) is given by,

\[
\varepsilon_{p4} = \varepsilon_{p3} + \frac{(\sigma_{ym2} - \sigma_{ym4})}{E_c} \left( \frac{A_m A_f E_m}{E_c} \right)
\]  \hspace{1cm} (II.39)

and the new yield stress is,

\[
\sigma_{ym5} = \sigma_{ym4} + K(\varepsilon_{p3})^n
\]

In an entirely analogous manner to the constant strain analysis, this cyclic process is continued until either the matrix reaches the failure stress or shakedown occurs.
Section 2 - Termination Criteria

A list of the different criteria for completing an analysis at a given stress or strain level is presented. The following nomenclature is used:

- $\sigma_{f_{\text{max}}}$ = maximum stress in the fiber;
- $\sigma_{f_{\text{min}}}$ = minimum stress in the fiber;
- $\sigma_{m_{\text{max}}}$ = maximum stress in the matrix;
- $\sigma_{m_{\text{min}}}$ = minimum stress in the matrix.

The equations are those used to determine the extreme points of the stress ranges for the fiber and matrix.

Termination Type 1 - After the first tensile half-cycle, the matrix remains in tension and no plastic deformation occurs:

**Constant Strain**

\[
\begin{align*}
\sigma_{f_{\text{max}}} &= (\varepsilon_{pl} + (\sigma_{yml} - \sigma_{mr})/E_m)E_f + \sigma_f \\
\sigma_{f_{\text{min}}} &= -|P|/A_f \\
\sigma_{m_{\text{max}}} &= \sigma_{yml} \\
\sigma_{m_{\text{min}}} &= |P|/A_m
\end{align*}
\]
Constant Stress

\[ \sigma_{f_{\text{max}}} = (\varepsilon_{\text{p1}} + (\sigma_{\text{yml}} - \sigma_{\text{mr}})/E_m)E_f + \sigma_{\text{fr}} \]

\[ \sigma_{f_{\text{min}}} = -|P|/A_f \]

\[ \sigma_{\text{mmax}} = \sigma_{\text{yml}} \]

\[ \sigma_{\text{mmmin}} = |P|/A_m \]

Termination Type 2 - After the first tensile half-cycle the matrix is in compression, but the stress does not exceed the yield stress in compression nor the failure stress.

Constant Strain

\[ \sigma_{f_{\text{max}}} = (\varepsilon_{\text{p1}} + (\sigma_{\text{yml}} - \sigma_{\text{mr}})/E_m)E_f + \sigma_{\text{fr}} \]

\[ \sigma_{f_{\text{min}}} = |P|/A_f \]

\[ \sigma_{\text{mmax}} = \sigma_{\text{yml}} \]

\[ \sigma_{\text{mmmin}} = -|P|/A_m \]

Constant Stress

\[ \sigma_{f_{\text{max}}} = (\varepsilon_{\text{p1}} + (\sigma_{\text{yml}} - \sigma_{\text{mr}})/E_m)E_f + \sigma_{\text{fr}} \]

\[ \sigma_{f_{\text{min}}} = |P|/A_f \]

\[ \sigma_{\text{mmax}} = \sigma_{\text{yml}} \]

\[ \sigma_{\text{mmmin}} = |P|/A_m \]
Termination Type 3 - The elastic limit of the composite is not reached:

**Constant Strain**

\[ \sigma_{f_{\text{max}}} = \varepsilon T_f + \sigma_{fr} \]

\[ \sigma_{f_{\text{min}}} = \varepsilon T_m + \sigma_{mr} \]

\[ \sigma_{m_{\text{max}}} = \sigma_{fr} \]

\[ \sigma_{m_{\text{min}}} = \sigma_{mr} \]

**Constant Stress**

\[ \sigma_{f_{\text{max}}} = \sigma E_f/E_c + \sigma_{fr} \]

\[ \sigma_{f_{\text{min}}} = \sigma E_m/E_c + \sigma_{mr} \]

\[ \sigma_{m_{\text{max}}} = \sigma_{fr} \]

\[ \sigma_{m_{\text{min}}} = \sigma_{mr} \]

Termination Type 4 - The plastic deformation on the tensile half-cycle is negligible, and the stress in the matrix may or may not have exceeded the failure stress:

**Constant Strain**

\[ \sigma_{f_{\text{max}}} = \varepsilon p_{f} E_f + \sigma_{fr} \]

\[ \sigma_{f_{\text{min}}} = (\varepsilon p_m - (\sigma m y^2 A_m/A_f - \sigma_{fr})/E_f) E_m - \sigma_{my^2} \]

\[ \sigma_{m_{\text{max}}} = \sigma m y^2 A_m/A_f \]

\[ \sigma_{m_{\text{min}}} = -\sigma m y^2 \]
Constant Stress

\[ \sigma_{f_{\max}} = \left( \left( \frac{\sigma_{ym} A_m}{A_f} - \sigma_{fr} \right) + \frac{\sigma_a}{E_c} \right) E_f \]
\[ \sigma_{f_{\min}} = \frac{\sigma_{ym} A_m}{A_f} \]
\[ \sigma_{m_{\max}} = \left( \sigma_a - \frac{\sigma_{f_{\max}} A_f}{A_m} \right) / A_m \]
\[ \sigma_{m_{\min}} = -\sigma_{ym2} \]

Termination Type 5 - On the first relaxation half-cycle plastic deformation occurs but the failure stress in the matrix is reached:

Constant Strain and Stress

\[ \sigma_{f_{\max}} = \left( \left( \frac{\sigma_{ym1} - \sigma_{mr}}{E_m} + \epsilon_{p2} \right) E_f + \sigma_{fr} \right) \]
\[ \sigma_{f_{\min}} = \sigma_{ym1} \]
\[ \sigma_{m_{\max}} = \frac{\sigma_{ym2} A_m}{A_f} \]
\[ \sigma_{m_{\min}} = -\sigma_{ym2} \]

Termination Type 6 - On the tensile half-cycle, of the multicycle analysis, exceeds the failure stress:

Constant Strain

\[ \sigma_{f_{\max}} = \epsilon_{pT} E_f + \sigma_{fr} \]
\[ \sigma_{f_{\min}} = \frac{\sigma_{ym2} A_m}{A_f} \]
\[ \sigma_{m_{\max}} = \sigma_{ym4} \]
\[ \sigma_{m_{\min}} = -\sigma_{ym2} \]
Termination Type 7 - On the relaxation half-cycle during the multicycle analysis the plastic deformation, $\varepsilon_{p4}$, becomes negligible. The matrix failure stress may or may not have been exceeded.

**Constant Strain**

\[
\sigma_{f_{\text{max}}} = \frac{E_f}{P/A_f} + \sigma_{fr} \\
\sigma_{f_{\text{min}}} = \frac{P}{A_f} \\
\sigma_{m_{\text{max}}} = \sigma_{ym3} \\
\sigma_{m_{\text{min}}} = \frac{-P}{A_m}
\]

**Constant Stress**

\[
\sigma_{f_{\text{max}}} = \left(\frac{E_f}{P/A_f} + \frac{\sigma_{ym3} + \sigma_{ym2}}{E_m} + \frac{\sigma_{ym2}A_m/A_f - \sigma_{fr}}{E}\right) \\
\sigma_{f_{\text{min}}} = \frac{P}{A_f} \\
\sigma_{m_{\text{max}}} = \sigma_{ym3} \\
\sigma_{m_{\text{min}}} = \frac{-P}{A_m}
\]

Termination Type 8 - The plastic deformation on the multicycle relaxation half-cycle is not negligible, but the matrix failure stress has been exceeded.

**Constant Strain**

\[
\sigma_{f_{\text{max}}} = \frac{E_f}{P/A_f} + \sigma_{fr} \\
\sigma_{f_{\text{min}}} = \frac{\sigma_{ym2}A_m/A_f}{P} \\
\sigma_{m_{\text{max}}} = \sigma_{ym3} \\
\sigma_{m_{\text{min}}} = -\sigma_{ym4}
\]
Constant Stress

\[ \sigma_{f\text{max}} = \left( \varepsilon_{p3} + (\sigma_{ym3} + \sigma_{ym2})/E_m + (\sigma_{ym2}A_m/A_f - \sigma_{fr})/E_f \right)E_f \]

\[ \sigma_{f\text{min}} = \sigma_{ym2}A_m/A_f \]

\[ \sigma_{m\text{max}} = \sigma_{ym2} \]

\[ \sigma_{m\text{min}} = -\sigma_{ym2} \]

Termination Type 9 - The maximum cycle cutoff has been reached.

Constant Strain

\[ \sigma_{f\text{max}} = \varepsilon_{pT}E_f + \sigma_{fr} \]

\[ \sigma_{f\text{min}} = \sigma_{ym2}A_m/A_f \]

\[ \sigma_{m\text{max}} = \sigma_{ym3} \]

\[ \sigma_{m\text{min}} = -\sigma_{ym2} \]

Constant Stress

\[ \sigma_{f\text{max}} = \left( (\sigma_{ym2} + \sigma_{ym3})/E_m + (\sigma_{ym2}A_m/A_f - \sigma_{fr}/E_f) + \varepsilon_{p3} \right) \]

\[ \sigma_{f\text{min}} = \sigma_{ym2}A_m/A_f \]

\[ \sigma_{m\text{max}} = \sigma_{ym3} \]

\[ \sigma_{m\text{min}} = -\sigma_{ym2} \]
A computer model to predict the fatigue behaviour of a unidirectional composite with a work-hardening matrix and perfectly elastic fibers.

100 FORMAT(2F12.1,2F12.3)
101 FORMAT(4F12.2)
200 FORMAT(1X,'MATRX MODULUS',13X,F12.3,1X,'FIBR AREA',19X,F12.3,5X,'FIBR FRACTION',13X,F12.3)
210 FORMAT(1X,'INITIAL YIELD STRESS',7X,F12.1,1X,'MATRX FAILURE STRESS',6X,F12.1,1X,'WORK HARDENING MODULUS',5X,F12.1,1X,'WORK HARDENING COEF',8F7.3)
300 FORMAT(1X,'MATERIAL CONSTANTS',//)
400 FORMAT(1X,'LOADING VARIABLES',//)
500 FORMAT(1X,'CONSTANT STRAIN AMPLITUDE ANALYSIS',//)
600 FORMAT(1X,'CONSTANT STRESS AMPLITUDE ANALYSIS',//)
700 FORMAT(1X,'CONSTANT STRESS AND STRAIN AMPLITUDE ANALYSIS',//)
800 FORMAT(1X,'GUARANTEED LOWER SHAKEDOWN LIMIT',F10.6)
900 FORMAT(1X,'RESIDUAL STRESS STATE',1X,'FIBER STRESS LIMITS',F12.2,2X,'MATRIX STRESS LIMITS',F12.2,2X)
1000 FORMAT(1X,'MAXIMUM 400000 CYCLE CUTOFF REACHED')

READ INITIAL MATERIAL CONSTANTS
WRITE(6,300)
READ(5,100) EM,EF,AM,AF
READ(5,101) SYM,SYM,AK,AN
WRITE(6,200) EM,EF,AM,AF,SYM,SYM,AK,AN

CALCULATE THE COMPOSITE MODULUS IN THE VOIGHT APPROXIMATION.
EC = AM*EM*AF*EF
WRITE(6,230) EC

THE RESIDUAL STRESSES IN THE MATRIX AND FIBER MUST SATISFY
THE EQUILIBRIUM CONDITION SMR*AM + SFR*AF = 0.

READ(5,330) SMR, SFR
WRITE(6,340) SMR, SFR

ITER IS AN INTEGER INDICATING THE NUMBER OF DIFFERENT LOADING
CONDITIONS TO BE EVALUATED. A VALUE OF 'SELECT' AND THE
APPROPRIATE STRESS AND STRAIN AMPLITUDES MUST BE SUPPLIED IN
THE DATA FOR EACH ITERATION.

READ(5,350) ITER
ITEST = 1
IF (ITEST > ITER) GO TO 1001
WRITE(6,170)

SELECT IS A FLOATING POINT NUMBER; NEGATIVE FOR CONSTANT STRAIN
AMPLITUDE; POSITIVE FOR CONSTANT STRESS AMPLITUDE; ZERO FOR BOTH

WRITE(6,400)
READ(5,180) SELECT
IF (SELECT) 1, 2, 3

READ IN LOADING VARIABLES BASED ON VALUE OF SELECT
1 WRITE(6,190)
   READ(5,500) EPT
   WRITE(6,600) EPT
   GO TO 4
2 WRITE(6,220)
   READ(5,240) EPT, SA
   WRITE(6,250) EPT, SA
   GO TO 4
3 WRITE(6,210)
   READ(5,260) SA
   WRITE(6,270) SA
   GO TO 5

BEGIN CONSTANT STRAIN AMPLITUDE ANALYSIS
4 WRITE(6,280)
   ELMT = (SYM - SMR) / EM
   WRITE(6,160) ELMT
   IF (EPT .LE. ELMT) GO TO 18

BEGIN CONSTANT STRESS AMPLITUDE ANALYSIS
C

ELMT = (SYM - SMR) / EM
WRITE(6,160) ELMT
IF (EPT .LE. ELMT) GO TO 18

CALCULATE STRESS STATE ON FIRST CYCLE
C

IF (EP2 .LE. 0.0001) GO TO 80
IF (SYM2 .GT. SFM) GO TO 30
BEGIN CALCULATION FOR MULTI-CYCLE STRESS STATES

16 EP3=EP1-(SYM2*AM/AF-SFR)/EF-(SYM3+SYM4)/EM
    IF(EP3.LE.0.000001) GO TO 15
    IF(SYM3.GT.SFM) GO TO 11
    TN=N
    IF(TN.GT.800000.0) GO TO 57
    XML=XML+EP3
    N=N+1
TTDFM=TTDF4+EP3
SYM4=SYM3+AK*EP3**AN
SMLST=SYM3

PF=(SMR/EM-SF/EF-XMLi*(AM*EM*AF*EF)/EC
EP4=EPT-(SYM3+SYM4)/EM-(SYM4*AM/AF-SFR)/EF
    IF(EP4.LE.0.000001) GO TO 82
    IF(SYM4.GT.SFM) GO TO 53
XML=XML-EP4
N=N+1
TTDFM=TTDF4+EP4
SYM3=SYM4+AK*EP4**AN
SMLST=SYM4
SYM2=SYM4
GO TO 16

BEGIN CONSTANT STRESS AMPLITUDE ANALYSIS

5 WRITE(6,290)
    SELECT=SELECT+1
    SLMT=(SYM-SMR)*EC/EM
    WRITE(6,320) SLMT
    IF(SA.LE.SLMT) GO TO 91

BEGIN CALCULATE STRESS STATE ON FIRST CYCLE

SYC1=(SYM-SMR)*EC/EM
EP1=(SA-SYC1)/(EF*AF)
XML=EP1
N=1
TTDFM=EP1
SYM2=SYM+AK*EP1**AN
SMLST=SYM
PF=(SMR/EM-EP1-SFR)/EF-(AM*EM*AF*EF)/(AM*EM*AF*EF)
EP2=EP1-(SYM2+SYM)/EM-(SYM2*AM/AF-SFR)/EF
    IF(EP2.LE.0.000001) GO TO 80
    IF(SYM2.GT.SFM) GO TO 30
XML=XML-EP2
N=N+1
TTDFM=TTDF4+EP2
SYM3=SYM2+AK*EP2**AN
SMLST=SYM2

BEGIN CALCULATION FOR MULTI-CYCLE STRESS STATE

19 EP3=(SA-SYM3*EC/EM)/EF*AF:
IF (EP3.LE.0.000001) GO TO 25
IF (SYM3.GT.SFM) GO TO 12
TN=N
IF (TN.GT.800000.O) GO TO 55
XML=XML+EP3
N=N+1
TTDFM=TTDFM+EP3
SYM4=SYM3+AK*EP3**AN
SMLST=SYM3
PF=(SMR/EM-SFR/EF-XML) *(AM*EM*AF*EF)/EC
EP4=EP3*(SYM2-SYM4)*(FC/(AF*EF*EM))
IF (EP4.LE.0.000001) GO TO 85
IF (SYM4.GT.SFM) GO TO 31
XML=XML-EP4
N=N+1
TTDFM=TTDFM*EP4
SYM3=SYM4+AK*EP4**AN
SMLST=SYM4
SYM2=SYM4
GO TO 19

C OUTPUT SECTION OF PROGRAM

C TERMINATION TYPES 1 AND 2.
80 SFMAX=((SYM-SMR)/EM+EP1)*EF+SFR
SMMAX=SMLST
IF ((EP1-SMR/EM).LE.-(SFR/EF)) GO TO 71
SMMIN=ABS(PF)/AM
SFMIN=ABS(PF)/AF
IT=1
IF ((ABS(PF)/AM).GT.SFM) 30 TO 81
GO TO 20
71 SMMIN=ABS(PF)/AM
SFMIN=ABS(PF)/AF
IT=1
IF ((ABS(PF)/AM).GT.SFM) 30 TO 81
GO TO 20
C TERMINATION TYPE 7, CONSTANT STRESS.
85 SFMAX=(EP3+(SYM3+SYM2)/EM+(SYM3*AM/AF-SFR)/EF)*EF
SMMIN=ABS(PF)/AM
SMMAX=SMLST
IT=7
GO TO 86
C TERMINATION TYPE 7, CONSTANT STRAIN.
82 SFMAX=SFR+EF*EP
SFMIN=ABS(PF)/AF
SMMAX=SMLST
SMMIN=ABS(PF)/AM
IT=7
86 IF ((ABS(PF)/AM).GT.SFM) GO TO A1
GO TO 29
20 NCYCLE=0
WRITE(6,390) IT
WRITE(6,140) NCYCLE,SYM2,TTDFM
WRITE(6,360) SFMAX,SFMIN,SMMAX,SMMIN
IF (SELECT.NE.0.0) GO TO 999
GO TO 5
C TERMINATION TYPE 3, CONSTANT STRAIN.
18 \text{SFMAX} = \text{EPT} \times \text{EF} + \text{SFR}
\text{SMAX} = \text{EP} \times \text{EM} + \text{SMR}
\text{GO TO 26}

C TERMINATION TYPE 3, CONSTANT STRESS.
21 \text{SFMAX} = \text{SFR} + \text{SA} \times \text{EF}/\text{EC}
\text{SMAX} = \text{SMR} + \text{SA} \times \text{EM}/\text{EC}
26 \text{SFMIN} = \text{SFR}
\text{SMIN} = \text{SMR}
\text{IT} = 3
\text{WRITE(6,390) IT}
\text{WRITE(6,370) SFMAX, SFMIN, SMAX, SMIN}
\text{IF(SELECT.NE.0.0) GO TO 999}
\text{GO TO 5}

C TERMINATION TYPE 8, CONSTANT STRAIN.
53 \text{SFMAX} = \text{EPT} \times \text{EF} + \text{SFR}
\text{IT} = 8
50 \text{SMIN} = \text{SYM4}
51 \text{SFMIN} = \text{SYM2} \times \text{AM}/\text{AF}
\text{SMAX} = \text{SMLST}
81 \text{NCYCLE} = \text{N}/2
\text{WRITE(6,390) IT}
\text{WRITE(6,130) NCYCLE, SMLST, TTDIFM}
\text{WRITE(6,380) SFMAX, SFMIN, SMAX, SMIN}
\text{IF(SELECT.NE.0.0) GO TO 999}
\text{GO TO 5}

C TERMINATION TYPE 8, CONSTANT STRESS.
31 \text{SFMAX} = \text{EP3} + (\text{SYM3} + \text{SYM2})/\text{EM} + (\text{SYM2} \times \text{AM}/\text{AF} - \text{SFR})/\text{EF} \times \text{EF}
\text{IT} = 8
\text{GO TO 50}

C TERMINATION TYPE 5, CONSTANT STRAIN AND STRESS.
30 \text{SFMAX} = ((\text{SYM} - \text{SMR})/\text{EM} \times \text{EP1}) \times \text{EF} + \text{SFR}
\text{IT} = 5
52 \text{SMIN} = -\text{SYM2}
\text{GO TO 51}

C TERMINATION TYPE 6, CONSTANT STRAIN.
11 \text{SFMAX} = \text{EPT} \times \text{EF} + \text{SFR}
\text{IT} = 6
\text{GO TO 52}

C TERMINATION TYPE 6, CONSTANT STRESS.
12 \text{SFMAX} = (\text{EP3} + (\text{SYM3} + \text{SYM2})/\text{EM} \times (\text{SYM2} \times \text{AM}/\text{AF} - \text{SFR})/\text{EF} \times \text{EF}
\text{IT} = 6
\text{GO TO 52}

C TERMINATION TYPE 4, CONSTANT STRAIN.
15 \text{SFMAX} = \text{EPT} \times \text{EF} + \text{SFR}
\text{SMAX} = (\text{EP} - (\text{SYM2} \times \text{AM}/\text{AF} - \text{SFR})/\text{EF}) \times \text{EM} - \text{SYM2}
\text{IT} = 4
27 \text{SFMIN} = \text{SYM2} \times \text{AM}/\text{AF}
\text{SMIN} = -\text{SYM2}
\text{IF(SMAX.GT.SFM) GO TO 81}
29 \text{NCYCLE} = \text{N}/2
\text{WRITE(6,150)}
56 \text{WRITE}(6,390) IT
\text{WRITE}(6,140) NCYCLE, SMLST, TTDIFM
\text{WRITE}(6,380) SFMAX, SFMIN, SMAX, SMIN
\text{IF(SELECT.NE.0.0) GO TO 999}
\text{GO TO 5}

C TERMINATION TYPE 4, CONSTANT STRESS.
25 \text{SFMAX} = (\text{SYM2} \times \text{AM}/\text{AF} - \text{SFR})/\text{EF}
\text{SMAX} = (\text{SA} - \text{SFMAX}/\text{AM})/\text{AM}
\text{IT} = 4
GO TO 27
C TERMINATION TYPE 9, CONSTANT STRESS.
   55 WRITE (6,410)
       SFMAX = ((SYM3*SYM2)/EM + (SYM2*AM/AF)/EF - SFR/EF + EP3)*EF
       SMMAX = SYM3
       SFMIN = SYM2*AM/AF
       SMMIN = -SYM2
       NCYCLE = N/2
       IT = 9
   GO TO 56
C TERMINATION TYPE 9, CONSTANT STRAIN.
   57 WRITE (6,410)
       SFMAX = EPT*EF + SFR
       SMMAX = SYM3
       SFMIN = SYM2*AM/AF
       SMMIN = -SYM2
       NCYCLE = N/2
       IT = 9
   GO TO 56
999 ITEST = ITEST + 1
   GO TO 6
1001 WRITE (6,170)
STOP
END
//GO,FT06FOO1 DD SYSOUT=(A,UNL,PROFILE=PRINT),COPIES=3
//GO,SYGIN DD *
1700000. 2000000. .75 .25
30000. 80000. 2000000. 1.0
30000. -90000. 11
1.0
69200.
1.0
69400.
1.0
69600.
1.0
69800.
1.0
69900.
1.0
70100.
1.0
70200.
1.0
70400.
1.0
70600.
1.0
70800.
1.0
70000.
/
Section 4 - Data Input Format

The first block of input data is comprised of the material constants. The first card containing the matrix modulus (EM), fiber modulus (EF), matrix area fraction (AM), and fiber area fraction (AF), in that order. The corresponding format statement is,

100 FORMAT (2F12.1, 2F12.3)

The second card inputs the initial matrix yield strength (SYM), failure stress (SFM), work hardening modulus (AK) and coefficient (AN) respectively, in the format

101 FORMAT (4F12.2)

The initial residual stress state is supplied on the third card. The matrix residual stress (SMR) and fiber residual stress (SFR) respectively given by the format

330 FORMAT (2F10.2)

The number of loading conditions (ITER) to be evaluated is specified on the fourth card. It is an integer and must be right justified. The format is

350 FORMAT (I15)

The fifth card determines which type of analysis will be performed. A negative number for constant strain, positive for constant stress, and zero for both. It may vary for different loading conditions, and must be specified for each set of loading variables. The format is

180 FORMAT (1F10.2)
DISCLAIMER NOTICE

MISSING PAGE(S)

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BIOGRAPHICAL NOTE

The author was born in Bangor, North Wales in the United Kingdom on September 6, 1953. He is a citizen of Great Britain, and presently a permanent resident of the United States.

After receiving most of his early education in the British school system he came to the United States, and completed one year of high school in Durham, N.C. He then entered Duke University in 1971, and graduated with an M.S. degree in 1977, receiving his B.S. in 1975. Since June 1977 he has been a graduate student at M.I.T. in the Department of Materials Science and Engineering.