TECHNIQUES OF VISUAL OBSERVATION OF PARTICLES
IN A SOUND FIELD

by

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Abstract

An apparatus has been designed by the author to measure directly the displacement amplitude of a plane progressive sound wave and to observe visually a sound field in and around an orifice. The explicit relations between displacement amplitude and pressure amplitude, velocity amplitude, sound intensity, and energy in a plane progressive sound wave will be developed and presented in a form most useful for this investigation. Inasmuch as this thesis is a part of a larger over-all program, it will include recommendations for the development and the improvement of the apparatus.
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Table of Contents

Part I. Introduction..............................................5
Part II. Previous Experimental Work.........................7
Part III. Description of the Apparatus......................10
Part IV. Theoretical Results.................................22
Part V. Preliminary Trials..................................28
Part VI. Development and Improvement of the Apparatus...........29
Part VII. Conclusion........................................31

Illustrations

Fig. 1. The Precision Impedance Tube.......................11
Fig. 2. Top View of the Extension..........................12
Fig. 3. The Extension and the End Section...............14
Fig. 4. Rear View of the Microscope Sleeve.................16
Fig. 5. Front View of the Microscope Sleeve.............17
Fig. 6. The Microscope Tube and Bracket...............18
Fig. 7. The Particle Observing Unit......................20

Working drawings appear between pp. 11 and 12.
Part I. Introduction

A direct method for determining the properties of a sound field is to observe the displacement of the sound field particles. To investigate the vibrational motion of air in a tube it is necessary to use some visible particle as trace points which will match exactly the motion of the air. Particles like smoke or dust meet this requirement within certain limits. Under the condition that the particles take up the motion of the oscillating medium, a suitable means of illumination, and a suitable means of observation will enable measurements of the displacement amplitude. These measurements will, in turn, serve as a check to the measurements of pressure level now possible on the Precision Impedance Tube at the M. I. T. Acoustics Laboratory.

Recently some introductory investigations at the Acoustics Laboratory have shown that vortex motions emanating from an orifice attached to the end of the Impedance Tube are present provided that the vibration exceeds a certain limiting intensity. These vortices may also be observed when suitable particles are introduced as trace points.

The purpose of this investigation, then, is to design and develop an apparatus which will allow visual
observation of sound field phenomena, and in particular, provide for the measurement of the displacement amplitude.

In Part II, previous experiments reported in the literature relating to the measurement of displacement amplitude in sound waves are briefly reviewed.

In Part III, the apparatus is described with the aid of photographs and working drawings, and the observing and measuring methods are outlined.

Part IV contains the development and the summary of theoretical results which are applicable to this investigation.

The results of preliminary trials with the apparatus are stated in Part V and the recommendations for the improvement and development of the apparatus are contained in Part VI.
Part II. Previous Experimental Work

In 1931, E.N. da C. Andrade\textsuperscript{1} used tobacco smoke particles to measure the amplitude of the particle displacement in a sound field in order to determine the sound energy in an experiment on circulation in a tube. A water-jacketed glass tube, 3.7 cm square in cross-section contained the smoke under observation. The axis of the beam used for illumination was at right angles to the reading microscope and the source of light was a 12-volt lamp, taking 6-7 amperes, placed about 1 meter from the glass tube. By means of a photographic lens of working aperture 2.9 and focal length 4 inches, the lamp was imaged at the spot where the amplitude was to be measured. The illuminating beam passed through a water cell containing ammoniated copper sulfate in order to decrease the infra-red radiation. The microscope used a 9.6 x power eyepiece and a Beck 40 mm apochromatic objective with a numerical aperture (N.A.) of 0.16. Andrade's attempts to photograph the displacement amplitudes were not successful because of drift caused by convection and circulation. In a frequency range of 89 to 1200 cps he observed displacement amplitude which, in turn, were used for calculating sound energy. In order to determine the degree to which the smoke particles
matched the motion of the sound wave, he estimated their size by taking the rate of settling under gravity. With a value for the radius of $5 \times 10^{-5}$ cm and the density assumed to be 1 gm/cc, he found that the ratio of smoke velocity to air velocity at a frequency of 2,000 cps was 0.9988, while at 1,200 cps, the ratio was 0.99958.

In 1936, E. N. da C. Andrade and R. C. Parker$^2$ used magnesium oxide smoke as tracer points to determine particle displacement amplitude in order to calibrate a standard source for tests on minimum audibility. A tube, of inner radius 1.75 cm, was enclosed in a water bath. Light from a 30 ampere arc was focused on a point close to the inner surface of the tube by means of a parabolic mirror, two condensing lenses, and two microscope objectives, one 23 mm and the other 16 mm. The viewing objective was placed directly above the light focus and was perpendicular to the illuminating beam. Fresh tobacco smoke was found to consist of particles of average radius $0.8 \times 10^{-5}$ cm and after one-half hour of fanning the radius increased to $1.3 \times 10^{-5}$ cm. The fanned smoke was very homogeneous and the coagulation was slow, but tobacco smoke was discarded in favor of magnesium oxide smoke because of the greater luminosity of the latter. The average radius of magnesium oxide smoke that had been fanned for 20 minutes and then left at rest for 10 minutes was given as $3 \times 10^{-5}$ cm.
In order to measure the oscillatory velocity of the field in which a Rayleigh disk was situated, R. A. Scott\(^3\), in 1944, used the smoke particle method for displacement amplitude measurements. The particles were illuminated with an arc lamp focused by a large aperture projection lens. A water cell was placed in the path of the beam in order to reduce convection currents of the air in the tube. Two microscope objectives in series, one a 1 inch Davon of N.A. 0.26 and the other a 16 mm Beck of N.A. 0.28, formed a composite objective of focal length 2.5 cm and N.A. 0.11 which allowed measurements at the longitudinal axis of the tube. He used magnesium oxide smoke and under the assumption that its radius is \(3 \times 10^{-5}\) cm, he calculated that the particles follow the motion of the air to within 1% up to a frequency of about 4,000 cps.
Part III. Description of the Apparatus

The tube which contains the particles under observation is an extension of the Precision Impedance Tube. Figure 1 is a photograph of the entire impedance tube, with the particle observing unit in the foreground. The impedance tube is a 3-inch re-inforced, water-jacketed steel tube with a speaker attached at one end and a crystal microphone fitted internally which travels the length of the tube in order to measure sound field pressure at any point.

The particle observing unit is completely detachable from the Precision Impedance Tube. A universal collar, which was constructed to adapt to other auxiliary units, joins the particle observing unit to the Impedance Tube.

The particle observing unit is made up of three main sections; the extension, the microscope sleeve, and the end section. Working drawings of these sections appear between pages 11 and 12, and their descriptions follow in parts A, B, and C. The source of illumination is described in Part D and the method of observation and measurement is described in Part E.
Figure 1. The Precision Impedance Tube with the Particle Observing Unit in the Foreground
Figure 2. Top View of the Extension and its Split Collar
A. The Extension

The extension fits up against the impedance tube at one end and is fastened by a split collar as shown in Figure 2. A plate containing an orifice may be clamped between the faces of the impedance tube and the extension. A particle inlet is provided near one end of the extension. At the other end, two ports are cut into the tube 90° apart, one at the top to give access to a microscope, and one at the rear to give access to a light source. The ports are windowed by glass plates 1 mm in thickness. A rectangular slot cut in the front of the extension forms a key-way for the microscope sleeve.

B. The End Section

This section rests on two supporting brackets attached to the main impedance tube support. One end is drawn flush to the extension by the two smaller thumb screws on lugs attached to the universal collar. When they are joined, the two sections form one continuous tube through which the sound field passes and over which the microscope sleeve rides. Two ports and a key-way are arranged to match the ports and the key-way in the extension when the two sections are drawn together. (See Figure 3). A circular plate containing an orifice may be
Figure 3. The Extension and the End Section Drawn Apart
placed between the extension and the end section as they are drawn together to enable observation of the field on both sides of the orifice. At the far end of the end section, another particle inlet is provided, and a brass piston is available to adjust the total length of the impedance tube.

C. The Microscope Sleeve

The microscope sleeve, shown in Figure 4, travels over the joint between the extension and the end section. It is fitted with two round ports, 90° apart, which coincide with the ports in the extension and the end section. A bracket, pivoted longitudinally at the top of the sleeve, holds the microscope tube. The microscope achieves longitudinal motion by means of the two larger thumb screws attached to the lugs of the universal collar which fit into slots on the microscope sleeve. (See Figure 5). Vertical focusing motion is achieved by the rack and pinion attached to the microscope tube, and angular motion about the longitudinal axis is achieved by a screw set in the microscope bracket as shown in Figure 6. The angular motion about the longitudinal pivot set in the top of the microscope sleeve provides for observation of the sound field phenomena off the center of the axis of the impedance tube.
Figure 4. Rear View of the Microscope Sleeve
Figure 5. Front View of Microscope Sleeve in Operating Position
Figure 6. The Microscope Tube and Bracket
A Bausch and Lomb 40 mm objective is used in a Spencer microscope tube. The objective has a magnification of 2.6 x, a N.A. of 0.08, and a working distance of 43.5 mm. A 10 x eyepiece, which has a scale of 100 divisions etched in the plane of vision, is used in conjunction with the objective.

D. Illumination

The illuminant is a Burton Projection Lamp which consumes 75 watts at 115 volts. A condensing lens brings the light to a focus at a point in the tube at right angles to the reading microscope. The beam passes through 1 cm of water contained in a glass cell, in order to reduce the effects of heating.

E. Method of Observation and Measurement

In order to measure the particle displacement amplitude in a standing wave, the brass piston in the end section is fixed to give any desired tube length. For the displacement amplitude of a wave made up of incident and partially reflected waves, the piston is replaced by an absorptive impedance. Particles are introduced into the observing unit through the rubber inlet tubes visible in Figure 7. The particles are
Figure 7. The Particle Observing Unit
allowed to stand until the initial influx agitation has died out. The sound field is then applied and the particles are allowed to come into steady state oscillation. When the light source is turned on, the particles appear as short faint lines against a dark field with brighter disks at the ends. These lines represent double the displacement amplitude of the particles in sinusoidal vibration, and are measured by fitting them against the scale in the microscope eyepiece.

There are two alternative positions in which to observe the field around an orifice placed in the impedance tube. With the orifice clamped between the interfaces of the impedance tube proper and the extension of the particle observing unit, the field from 5 3/4 to 6 1/2 inches from the back of the orifice falls under the range of microscopic examination. With the orifice clamped between the interfaces of the extension and the end section, the field directly in front of and directly behind the orifice is observable microscopically.
Part IV. Theoretical Results

A. Particles in an Oscillating Medium

The theory of the motion of a sphere in an oscillating sound field has been worked out by Stokes and others, and the following result is presented by Andrade.\textsuperscript{4}

\[
\frac{X}{X_0} = \left[ \frac{1 + 3b + \frac{9}{2}b^2 + \frac{9}{2}b^3 + \frac{9}{4}b^4}{a^2 + 3ab + \frac{9}{2}b^2 + \frac{9}{2}b^3 + \frac{9}{4}b^4} \right]^{1/2}
\]

(1)

where \( \frac{X}{X_0} \) is the ratio of velocity or displacement amplitudes of the particle to the surrounding medium.

\( a \) is equal to \( \frac{1}{3} + \frac{2}{3} \frac{\rho_o}{\rho} \) where \( \rho_o \) and \( \rho \) are the densities of the particle and the medium respectively.

\( b \) is equal to \( \frac{1}{R} \sqrt{\frac{n}{\nu}} \) where \( R \) is the radius of the particle, \( n \) is the viscosity of the medium, and \( \nu \) is the frequency of the field.

Another formula, derived in the following way, has been presented by Bergmann.\textsuperscript{5} A resistive force given by Stokes' Law is assumed for the particle being driven by the sound field. The equation of motion is
\[ m \frac{d^2 x}{dt^2} + 6\pi \eta R \frac{dx}{dt} = F_0 \epsilon \mathbf{e}^{j\omega t} \]

with a solution

\[ x = \frac{F_0 \epsilon \mathbf{e}^{j\omega t}}{j\omega (6\pi \eta R + j\omega \eta) } \]

As \(\omega\) approaches zero the particle approaches the motion of the oscillating medium. Hence \(x\) is given by

\[ x_0 = \frac{F_0 \epsilon \mathbf{e}^{j\omega t}}{j\omega (6\pi \eta R) } \]

and

\[ \frac{x}{x_0} = \frac{6\pi \eta R}{6\pi \eta R + j\omega \eta} = \frac{1}{1 + j \frac{\omega \eta}{6\pi \eta R}} \]

The ratio of the amplitude is then

\[ \left[ \frac{x}{x_0} \right] = \left[ \frac{x}{x_0} \right] = \left[ \frac{1}{1 + \left( \frac{\omega \eta}{6\pi \eta R} \right)^2} \right]^{1/2} = \left[ \frac{1}{1 + \left( \frac{h\pi \rho R^2 \nu}{9\eta} \right)^2} \right]^{1/2} \] (2)

In order for the particles to serve as good trace points in a sound field, they must have according to equations 1 and 2, a small radius and a low density. However, the other criteria for good trace points are that they be large enough to reflect sufficient light, that they have a large coefficient of reflectivity, that they be reproducible, and that they do not coagulate too rapidly.
B. The Displacement Amplitude

The equations relating the displacement amplitude and pressure amplitude, velocity amplitude, sound intensity, and energy in a plane wave will now be developed. The wave equation in one dimension is

\[ \frac{\partial^2 \xi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \xi}{\partial t^2} \]

where \( \xi \) is the average displacement
\( c \) is the velocity of sound.

For a long cylindrical tube of length \( L \) containing plane waves driven by a harmonic force of frequency \( \nu \), a solution of the wave equation is the real part of

\[ \xi = (A_1 e^{ikx} + A_2 e^{-ikx}) e^{-i\omega t} \]

where \( A_1 \) is a constant of the plus traveling incident wave
\( A_2 \) is a constant of the minus traveling reflected wave

\( k \) is the propagation constant equal to \( \frac{2\pi}{\lambda} = \frac{\omega}{c} \).

According to a method by P. M. Morse\(^6\), \( A_2 \) may be written as

\[ A_2 = A_1 e^{-2\Psi} \]

where \( \Psi \) is in general a complex number equal to

\[ \pi (\alpha - i\beta) \].
Provided that dissipation along the tube is neglected, it can be shown that

\[ \xi = 2A_1 e^{-\pi \alpha_0} \cosh \pi \left[ \alpha_0 - i\beta_x \right] e^{i(\pi \beta_0 - \omega t)} \]

where \( \beta_x = \beta_0 - \frac{kx}{l} = \beta_x + \frac{2}{\lambda} (l - x) \)

The absolute value of \( \xi \) is then

\[ |\xi| = 2A_1 e^{-\pi \alpha_0} \left[ \cosh^2 \pi \alpha_0 - \sinh^2 \pi \beta_x \right] \]

Since \( p = -\rho c^2 \frac{d}{dx} \frac{d}{dx} \)

where \( p \) is the differential pressure,

\[ p = ik\left[ \rho c^2 \right] \left[ -A_1 e^{ikx} + A_2 e^{-ikx} \right] e^{-i\omega t} \]

which develops into

\[ p = -i\rho c \omega A_1 e^{-\pi \alpha_0} \sinh \pi \left[ \alpha_0 - i\beta_x \right] e^{i(\pi \beta_0 - \omega t)} \]

whose absolute value is

\[ |p| = 2\rho c \omega A_1 e^{-\pi \alpha_0} \left[ \cosh^2 \pi \alpha_0 - \cos^2 \pi \beta_x \right] \]

then

\[ |\xi| = \frac{|p|}{\rho c \omega} \left[ \frac{\cosh^2 \pi \alpha_0 - \sin^2 \pi \beta_x}{\cosh^2 \pi \alpha_0 - \cos^2 \pi \beta_x} \right]^{1/2} \]  \hspace{1cm} (3)

The velocity \( \mathbf{u} = \frac{\partial \xi}{\partial t} = -i\omega \xi \). Hence

\[ |\xi| = \frac{1}{\omega} |\mathbf{u}| \]  \hspace{1cm} (4)
From Morse\textsuperscript{7}, the intensity $I'$ of the sound wave, and the average energy density $W$ is given by

$$ I' = 2\pi^2 \nu^2 \rho c \left[ A_1^2 - A_2^2 \right] = 2\pi^2 \nu^2 \rho c A_1^2 \left[ 1 - e^{-4\pi\alpha_o} \right] $$

$$ W = 2\pi^2 \nu^2 \rho \left[ A_1^2 + A_2^2 \right] = 2\pi^2 \nu^2 \rho c A_1^2 \left[ 1 + e^{-4\pi\alpha_o} \right] $$

from which it is easy to obtain

$$ |\xi| = \frac{2}{\alpha} \left[ \frac{I'}{\rho c} \right]^{1/2} \left[ \frac{\cosh^2 \pi\alpha_o - \sin^2 \pi\beta x}{\sinh 2\pi\alpha_o} \right]^{1/2} \quad (5) $$

$$ |\xi| = \frac{2}{\alpha} \left[ \frac{W}{\rho} \right]^{1/2} \left[ \frac{\cosh^2 \pi\alpha_o - \sin^2 \pi\beta x}{\cosh 2\pi\alpha_o} \right]^{1/2} \quad (6) $$

The ratio of minimum to maximum displacement amplitude determines the quantity $\alpha_o$ through the relation

$$ \frac{|\xi|_{\text{min}}}{|\xi|_{\text{max}}} = \tanh \pi\alpha_o $$

while the distance of a maximum displacement amplitude from the termination determines the quantity $\beta_\xi$ through the relation

$$ \beta_\xi = n - \frac{2}{\lambda} d_{\text{max}} $$

where $n$ is an integer taken to make $\beta_\xi$ less than unity, and which corresponds to the number of maxima from the
termination.

For completeness, the absolute value of the specific acoustic impedance is given by

$$|Z| = \frac{|p|}{u} = \rho c \left[ \frac{\cosh^2 \pi \alpha_0 - \cos^2 \pi \beta_x}{\cosh^2 \pi \alpha_0 - \sin^2 \pi \beta_x} \right]^{1/2}$$

Equations 3, 4, 5, and 6 are useful in this investigation for they explicitly relate other sound field quantities to the measured quantity of amplitude displacement. The determination of any two sound field quantities uniquely determines the properties of the field.
Part V. Preliminary Trials

In a first attempt to test the particle observing unit, a piece of cotton thread was fastened in the tube in the field of the microscope. One of the short fibers was clearly in focus and with the application of a suitable field it was observed to vibrate with a double amplitude of about 5 divisions on the eyepiece scale.

Next, cigarette smoke was introduced into the unit. Short light lines against a dark field were visible when the light source was snapped on, but almost immediately thereafter, the particles under observation left the microscope field and no meaningful patterns were observed. Repeated tests of this nature led to the conclusion that the light entering the particle observing unit was of sufficient intensity to cause disturbing convection currents.
Part VI. Development and Improvement of the Apparatus

The first problem to solve before the particle observing unit is put into useful operation is the problem of convection currents caused by the light source. The light beam must be focused in a very narrow pencil, and experiments should be conducted to determine the minimum light intensity required at the observing plane of the microscope. A slit-lamp, of the type used for eye examination,\(^8\) may prove to be a satisfactory source. Alternative methods for focusing and restricting the beam include the use of microscope objectives, condensing lenses, and slits. In addition, the beam should pass through a path of water considerably greater than lcm, for the absorption, which is strongest for long wave lengths, varies exponentially with the distance traversed.

The resolving power of a microscope, which is defined as the minimum separation of two points in the observing plane which can just be resolved, is inversely proportional to the numerical aperture (N.A.) of the microscope objective. Unfortunately, standard objectives which embody the long working distance required for this apparatus, have small numerical apertures. A scheme whereby a long working distance would be combined with a
large numerical aperture, would prove useful. To accomplish this, R. A. Scott devised a method which is described in Part II.

The eyepiece scale should be calibrated by taking measurements of pressure level and relating them to the displacement amplitude through equation 3. The calibration may be checked against an absolute standard by placing a ruled grating, of a known number of lines per centimeter, in the observing plane of the microscope.

Finally photomicrographic equipment should be adapted to the apparatus to secure photgraphic records of the particle displacement amplitudes and sound field phenomena around orifices.
Part VII. Conclusion

The preliminary tests of the particle observing unit indicate that it will measure the displacement amplitude of particles in a plane progressive sound wave provided that convection currents can be minimized or eliminated. Once the eyepiece is calibrated, the measurements of particle displacement amplitude will serve as a measure of the pressure amplitude, velocity amplitude, sound intensity, and energy density, according to the relations in Part IV. The measurements will be subject, however, to the condition expressed in equations 1 and 2, which determine the degree to which the particles match the motion of the oscillating medium.

The observation of the sound field around an orifice will be restricted to a qualitative investigation. The vortex motions around an orifice are of a rotational nature (as opposed to the irrotational nature of a plane progressive sound wave) and the particles will trace out flow lines in the plane of observation. The observation of this will not be possible, however, until convection currents are reduced or eliminated.
Footnotes


6. Morse, *Vibration and Sound*, p. 239.

7. Ibid. p. 224 and p. 228.

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