DYNAMICS OF A REED TYPE VALVE

by

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Signature of Author

Signature of Professor
in charge of research

Signature of Chairman of
Department Committee on
Graduate Students
May 9, 1949.

Professor Joseph S. Newell  
Secretary of the Faculty  
Massachusetts Institute of Technology  

Dear Sir:

In partial fulfillment of the requirements for the degree of Doctor of Science from the Massachusetts Institute of Technology, I hereby submit my thesis entitled: DYNAMICS OF A REED TYPE VALVE.

Respectfully yours,

Michael Costagliola
ACKNOWLEDGEMENTS

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Note: The original experimental data for this thesis has been filed in the Sloan Laboratory Library.
ABSTRACT

"Reed" or "feather" valves have their most important applications in reciprocating compressors. The object of this thesis is to study the dynamics of this type of valve and consequent effects on compressor performance. After the theory had been developed to a certain point it became apparent that all spring loaded compressor valves of the automatic type operate in essentially the same manner. The analysis presented will be applicable to any type of automatic valve which consists of an elastically restrained mass, regardless of the details of mechanical design.

The theory involves simultaneous solution for the valve motion and cylinder pressure during inlet or discharge processes. Thereby a theoretical "indicator card" is obtained from which the valve losses and thermal and volumetric efficiencies are calculated. These quantities are found as functions of four basic dimensionless parameters. This required the solution of a great many cases in order to obtain a sufficient spread of values of the basic parameters.

It was found that with the present design of reed valves the dynamics of the valve itself have little effect on valve losses. For a given compressor, the most important factor is not the weight or spring stiffness of the valve, but rather the area for flow that is provided.

The main effects of valve design on thermal and volumetric efficiencies are summarized as follows:
1) The area for flow and discharge coefficients of the valves should be as large as possible for low losses.

2) Valve dynamics are not too important (as regards losses) when the area for flow is very small compared to the area of piston.

3) When the flow area is large, the dynamics of the valve are important. With a very stiff spring and large flow area the valve may make several cycles of opening and closing in one discharge period, thereby increasing losses.

4) For dynamic considerations, the optimum valve would have no weight and a very small spring constant.

5) Generally speaking, the reed valve as used at present is dynamically close to the optimum or "ideal" valve.

6) For low losses, the volumes of inlet and discharge receivers and piping should be as generous as possible.

The calculated valve losses agree very closely with the experimental values. However, the actual volumetric efficiency is consistently lower than the theoretical value. Therefore, other factors omitted from the theory, such as leakage, heat transfer, pressure waves in the discharge and inlet piping, etc., have important effects on the volumetric efficiency.

The various limitations on improvement of valve performance include impact stresses as an important consideration. A formula for the maximum impact stress is derived and
is suggested as a criterion for design. It is found that increasing valve lift and decreasing valve weight, both of which tend to increase efficiency, also result in higher impact stresses. For the reed valve, a reduction in thickness increases the impact stress.

For any given design conditions and for a fixed reed length, the theoretical optimum reed thickness can be found, but the variation in efficiency with small changes of thickness is slight.

It is apparent that the performance of reciprocating compressors at high speeds is bound to be poor, due to the high valve losses and low volumetric efficiency. Future improvements in valve performance can be accomplished by developing a design having the favorable dynamic characteristics of the reed valve combined with a large flow area and a high discharge coefficient.
## Definitions and Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Units</th>
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<tr>
<td>( A_0 ) = Maximum area for flow (either inlet or discharge valves), with valves fully open.</td>
<td>ft²</td>
</tr>
<tr>
<td>( A ) = Area for flow through valves, at any time (with valves partially open).</td>
<td>ft²</td>
</tr>
<tr>
<td>( x ) = valve lift at any time</td>
<td>ft</td>
</tr>
<tr>
<td>( x_0 ) = Maximum possible opening of valves</td>
<td>ft</td>
</tr>
<tr>
<td>( W_v ) = weight of one valve</td>
<td>lbs</td>
</tr>
<tr>
<td>( A_v ) = face area of one valve</td>
<td>ft²</td>
</tr>
<tr>
<td>( \omega_n ) = natural frequency of valve and spring</td>
<td>( \frac{1}{\text{sec}} )</td>
</tr>
<tr>
<td>( K_c ) = discharge coefficient associated with &quot;A&quot;</td>
<td></td>
</tr>
<tr>
<td>( C_D ) = drag &quot; &quot; &quot; &quot;</td>
<td></td>
</tr>
<tr>
<td>( p ) = absolute pressure in cylinder at any time</td>
<td>( \frac{\text{lb}}{\text{ft}^2} )</td>
</tr>
<tr>
<td>( p_d ) = discharge receiver pressure (constant)</td>
<td>( \frac{\text{lb}}{\text{ft}^2} )</td>
</tr>
<tr>
<td>( p_i ) = inlet &quot; &quot; ( &quot; )</td>
<td>( \frac{\text{lb}}{\text{ft}^2} )</td>
</tr>
<tr>
<td>( T ) = Absolute temperature in cylinder at any time</td>
<td>°F (abs.)</td>
</tr>
<tr>
<td>( T_i ) = Absolute temperature in inlet receiver</td>
<td>°F (abs.)</td>
</tr>
<tr>
<td>( m ) = weight of air in cylinder at any time</td>
<td>lbs</td>
</tr>
<tr>
<td>( w ) = mass flow</td>
<td>lbs./sec.</td>
</tr>
<tr>
<td>( v ) = specific volume</td>
<td>ft³/lb</td>
</tr>
<tr>
<td>( C ) = velocity of sound in air</td>
<td>ft/sec.</td>
</tr>
<tr>
<td>( k ) = ratio of specific heats</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------</td>
</tr>
<tr>
<td>s</td>
<td>piston position from end of cylinder</td>
</tr>
<tr>
<td>s</td>
<td>&quot; stroke</td>
</tr>
<tr>
<td>d</td>
<td>clearance distance plus half stroke</td>
</tr>
<tr>
<td>$A_p$</td>
<td>cross sectional area of piston</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>angular speed of crank</td>
</tr>
<tr>
<td>$\theta$</td>
<td>crank angle measured from top dead center</td>
</tr>
<tr>
<td>t</td>
<td>time, measured from top dead center</td>
</tr>
<tr>
<td>$E$</td>
<td>modulus of elasticity</td>
</tr>
<tr>
<td>$I$</td>
<td>moment of inertia of cross section</td>
</tr>
<tr>
<td>$l$</td>
<td>length of reed valve</td>
</tr>
<tr>
<td>$b$</td>
<td>width &quot; &quot; &quot;</td>
</tr>
<tr>
<td>$t_i$</td>
<td>thickness &quot; &quot; &quot;</td>
</tr>
<tr>
<td>$\sigma_0$</td>
<td>initial impact stress</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant</td>
</tr>
<tr>
<td>$Q$</td>
<td>quantity of heat</td>
</tr>
<tr>
<td>$W$</td>
<td>&quot; &quot; work</td>
</tr>
<tr>
<td>$E', E''$</td>
<td>internal energy of system, initial and final</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$c_v$</td>
<td>&quot; &quot; &quot; &quot; volume</td>
</tr>
<tr>
<td>$\eta_v$</td>
<td>volumetric efficiency (actual)</td>
</tr>
<tr>
<td>$\eta_{i}$</td>
<td>&quot; &quot; &quot; (ideal)</td>
</tr>
</tbody>
</table>
Subscripts:
1 refers to beginning of compression
2 " " of discharge \{See Figure 2\}
3 " " end of discharge
4 " " beginning of intake
d " " discharge process
1 " " inlet process

Dimensionless Ratios

\[ \frac{x}{x_0} = \frac{A}{A_0} = \alpha \]

\[ 1 - \alpha = \sigma \]

\[ \frac{d}{s} = \delta \]

\[ \frac{p}{p_2} = \frac{p}{p_d} = \Phi \]

\[ \frac{p}{p_1} = \frac{p}{p_4} = \frac{p}{p_1} = \Psi \]

\[ \left( \frac{8}{k-1} \right) \frac{A_0 K_c C_2}{sA_p \omega_n} = B_d \]

for discharge valves

\[ \frac{C_D A_v P_2}{\frac{WV \omega_n^2 x_0}{g}} = J_d \]

\[ \left( \frac{8}{k-1} \right) \frac{A_0 K_c C_1}{sA_p \omega_n} = B_1 \]

for inlet valves

\[ \frac{C_p A_v P_1}{\frac{WV \omega_n^2 x_0}{g}} = J_1 \]

\[ \frac{\Omega}{\omega_n} = q \]
\[ M_d = \frac{k^{\frac{1}{k}} \sqrt{\phi \left( \frac{k^{\frac{1}{k}} - 1}{2 \delta + 0.056 - \cos \theta - 0.056 \cos 2 \theta} \right)}}{\text{For discharge process}} \]

\[ N_d = \frac{-k(\sin \theta + 0.112 \sin 2 \theta) \phi}{2 \delta + 0.056 - \cos \theta - 0.056 \cos 2 \theta} \]

\[ M_1 = \frac{1}{k^{\frac{1}{k}} \sqrt{1 - \frac{k-1}{1 - \psi k}}} \frac{1}{2 \delta + 0.056 - \cos \theta - 0.056 \cos 2 \theta} \text{ For inlet process} \]

\[ N_1 = \frac{k(\sin \theta + 0.112 \sin 2 \theta \psi)}{2 \delta + 0.056 - \cos \theta - 0.056 \cos 2 \theta} \]

\[ H = \int_0^\gamma \left(1 - \cos \gamma \right) d \gamma = \gamma - \sin \gamma \]

where \( \gamma \) = any angle, in radians
CHAPTER I

Introduction

In the past, most applications of reciprocating compressors seem to have been for purposes where efficiency was secondary to other considerations. With the advent of the Free Piston Gas Generator or Compressor, however, the problem of efficiency of the device has necessitated a thorough study of losses in reciprocating compressors. The valve losses are particularly worthy of study since they so seriously lower the thermal and volumetric efficiencies. Since the Free Piston Gas Generator is intended for use as part of a gas turbine plant, its efficiency must be the very best that can be obtained.

The work undertaken in this thesis had its inception in the problem of reed-type valves developed for the Free Piston Gas Generator being designed and built by the Lima-Hamilton Company. However, the theory has been developed to cover the whole question of reciprocating compressor performance as affected by valve dynamics. The main discussion will be confined to valves of the automatic type; that is, spring loaded valves which open automatically when subjected to a pressure difference.

Figure (1) shows the type of valve, variously known as the "reed", "feather" or "strip" valve, which is used in the Free Piston Gas Generator as well as some commercial compressors. It consists of a flat strip of steel, covering a slot in the seat plate. The reed is held in position over the slot by a retainer plate which is notched for the ends of the reed. When the valve is opened by an excess of pressure on the lower side, it
FIGURE 1
TYPICAL REED VALVE ASSEMBLY

CURVED STOPS
NOTCHES FOR REEDS
REEDS
INLET SLOTS
SEAT PLATE

STOP PLATE
RETAILER
AIR FLOW
SEAT PLATE
TRANSVERSE SECTION
(VALVES CLOSED)

STOP PLATE
RETAILER
SEAT PLATE
REED (PARTLY OPEN)

FIGURE 2
TYPICAL P-V DIAGRAM FOR COMPRESSOR
flexes against a curved stop plate. This stop plate may be cut
to a uniform arc profile, or to a shape made up of straight
lines and arcs. The retainer plate and stop plate are usually
designed so as to give a small amount of uniform lift, before
the reed begins to bend.

There are numerous other types of valves which have
been developed, (see reference 1), but they all consist essen-
tially of two parts; a valve body and a spring. It will be
noticed that the reed valve body forms its own spring and there-
fore is about the lightest form of valve that can be devised.
In the past, mechanically operated valves were sometimes used,
but have been almost entirely superseded by the automatic type.
The significance of the valve losses can be seen from the P-V
diagram (Figure 2). The numbered points correspond to the sub-
scripts defined in the "Table of Symbols". For comparison, the
"ideal" diagram, with no losses is included. The shaded areas
constitute the added work required to force the gas through the
inlet and discharge valves, and are therefore termed inlet and
discharge losses since they reduce the thermal efficiency. The
"ideal" P-V diagram would be obtained in a compressor traveling
at infinitely low speed if it had valves which could be opened
by an infinitesimal pressure difference. Thus, the gas would
flow in and out of the cylinder at constant pressure along the
lines 1-4 and 2-3 producing no valve losses.

The main purpose of this thesis is to investigate the
effects on the valve losses of the most significant variables
concerned.
CHAPTER II
Theory of the Discharge and Inlet Processes

Part I  Purposes of the Analysis and Assumptions made.

One cannot expect such a complicated process as actually occurs during flow through the valves of Figure 1 to yield to a rigorous mathematical analysis. Therefore, an approximate solution is all that can be hoped for.

The purpose of the analysis is twofold:
1) To determine the relative importance of the various variables concerned and the general trends of their effects on performance.
2) To present a basis for estimating in advance, the performance of a projected design of compressor.

Both 1 and 2 could be accomplished experimentally if a sufficiently wide range of very accurate tests were made. However the difficulties of such an experimental program are very great. In the first place, exact duplication of test conditions must be possible in successive tests since the variations in performance for some of the variables is very slight. Secondly, the measurements made must be extremely precise to detect the small improvements in efficiency made by some of the variables. Thirdly, experimental results will apply only to other machines of a design similar to that being tested, whereas the theory does not depend on the details of the design and can be applied so long as certain essential factors are known for the design in question.
With all the difficulties presented by the experimental approach, it seemed that the logical way to attack the problem was to work out a theory which, though simplified, would take all the significant variables into account. The work of Tsu (reference 11) showed that the inlet and exhaust processes of an internal combustion engine could be successfully calculated and were in very good agreement with the experimental results. Our problem is in some respects more complicated, since we are dealing with valves whose motion depends on the applied forces, which are different for every operating condition, while the internal combustion engine has mechanically operated valves, the motions of which are known for every part of the process.

The basic problem consists in determining the valve motion and cylinder pressure simultaneously, during the inlet process or the discharge process. Neither valve motion nor cylinder pressure can be found independently as each depends on the other.

As the problem is extremely complex, one must make a few simplifying assumptions to obtain any solution at all. It was felt that the first attempt at a solution should consider a simple system in which a compressor takes air from one very large reservoir, compresses it, and delivers it to another large reservoir. The volumes of these reservoirs are so large that the pressures in them may be considered constant during the cycle. The valve system assumed is that of a single degree of freedom system consisting of an elastically restrained mass. This is represented in the drawing (Figure 3), by a poppet type
FIGURE 3
SIMPLIFIED SYSTEM

INLET RECEIVER
PRESS. = p_i
TEMP. = T_i

INLET VALVES
(ONLY ONE SHOWN)

DISCHARGE RECIIVER
PRESS. = p_d

DISCHARGE VALVES
(ONLY ONE SHOWN)

CLEARANCE SPACE

S = STROKE

CYLINDER PRESS. = p
TEMP. = T

AP = AREA OF PISTON

ANGULAR VELOCITY \( \Omega \)

\[ \Theta = \Omega \cdot t \]
valve held down by a helical spring, but the results are appli-
cable to any valve which approximates this system regardless
of the details of mechanical design.

The list of assumptions made is given herewith:

1) No heat transfer throughout the cycle.
2) Processes within the cylinder are considered
isentropic except during the inlet period.
3) Piston speed is low (compared to the velocity
of sound at discharge conditions) so that the
air in the cylinder is essentially at stagnation
conditions.
4) Valve construction.
   a) All inlet valves are identical and all
discharge valves are identical.
   b) A valve consists of an elastically re-
strained mass. With the valve closed
there is no compression in the spring.
The axis of the valve is taken to be in
a horizontal plane so that gravity has
no effect.
   c) Valve spring constant is linear, so that
the spring force is directly proportional
to the lift.
   d) Valve area for flow is a linear function
of the lift of the valve.
   e) Flow through the valves is computed by
standard orifice formula for one dimen-
sional flow.
f) The force on a valve due to fluid flow past it is proportional to the face area of the valve times the pressure difference across it. (The application of this assumption to reed valve is explained later.)
g) No damping is assumed in valve motion.
h) The valve has stops to limit the travel to a definite amount.

5) There is no leakage past the piston or past the valves when closed.

6) The discharge coefficient for the valves is constant and the same as for steady flow conditions.

7) Reynold's number effects are ignored.

8) Inlet and discharge receivers are very large so that the pressures in those spaces are constant.

9) The valves do not rebound from either the seat or the stops when they strike.

10) Perfect gas laws are used with $k = 1.4$ for simplicity.

The validity of these assumptions will be discussed later, when the experimental results are presented.

Part 2 Analysis of the Discharge Process

Figure 3 shows the arrangement of the system analyzed, with the notation used. The piston is on the up stroke and compresses the air in the cylinder adiabatically and isentropically. Meanwhile some air is flowing out so that the weight of air in the cylinder is constantly changing. With "$m" = weight of air
in cylinder at any time during discharge we write the equation of state for the air in the cylinder as follows:

\[ pA_p z = mRT \]  \hspace{1cm} (1)

For point 2 at the beginning of discharge (see Figure 2), we may write:

\[ p_2A_p z_2 = m_2RT_2 \]  \hspace{1cm} (2)

Dividing 1 by 2 we get:

\[ \frac{m}{m_2} = \frac{z}{z_2} \frac{p}{p_2} \frac{T}{T_2} \]  \hspace{1cm} (3)

From assumptions 2 and 11 we have

\[ pv^k = \text{constant, during discharge} \]  \hspace{1cm} (4)

so

\[ pv^k = p_2v_2^k \]  \hspace{1cm} (5)

Where \( v \) = specific volume at any time

From 1, 2, and 4 we get

\[ \frac{T}{T_2} = \left( \frac{p}{p_2} \right)^{\frac{k-1}{k}} = \phi^{\frac{k-1}{k}} \]  \hspace{1cm} (6)

Where \( \phi = \frac{p}{p_2} \)

We use the pressure ratio defined as \( \phi \), rather than the cylinder pressure in order to make the final equations dimensionless. Replacing the value of \( T/T_2 \) in (3) by that given in (6) we arrive at

\[ m = m_2 \frac{z}{z_2} \frac{\phi^{\frac{k-1}{k}}}{\phi^{\frac{k-1}{k}}} = \frac{z}{z_2} m_2 \phi^{\frac{1}{k}} \]  \hspace{1cm} (7)
By definition \( w = -\frac{dm}{dt} \) (8)

where "w" is the mass flow in lbs/sec out of the cylinder. The negative sign is necessary as "m" is decreasing, while \( w \) is considered positive for outward flow. Differentiating (7) and replacing it in (8) we find:

\[
w = -\frac{m_2}{z_2} \left[ \frac{1}{k} \frac{dz}{dt} + \frac{z}{k\phi} \frac{k-1}{k} \frac{d\phi}{dt} \right]
\]

(9)

Now we compute the value of "w", using the well-known formula for one dimensional sub-critical flow. (For the usual cases the values of \( \phi \) are far below the critical pressure ratio, but in some of the extreme cases considered, the flow was critical and the calculation procedure changed accordingly).

With our notation the formula is

\[
w = K_c A \sqrt{\frac{2gk}{R(k-1)}} \frac{p_z}{T} \left( \frac{p_z}{p} \right)^{\frac{1}{k}} \sqrt{1 - \left( \frac{p_z}{p} \right)^{\frac{k-1}{k}}} \]

(10)

where \( K_c \) is the discharge coefficient for the valves as found from steady flow tests.

Putting \( \frac{p}{p_z} = \phi \), \( T/T_z = \phi \frac{k-1}{k} \) we obtain:

\[
w = K_c A \sqrt{\frac{2gk}{R(k-1)T_z}} \ p_z \sqrt{\phi \frac{k-1}{k} - 1}
\]

(11)

But, \( \sqrt{kgRT_z} = C_z(12) \), where \( C_z \) is the velocity of sound in air, in the state existing at point 2. Also, put \( A = A_0 x_0 \) (13), in accordance with assumption (4d).

Here \( x \) = lift of valve at any time and \( x_0 \) = maximum lift of valve when fully open.
We define the second dimensionless ratio \( \alpha \) as

\[
\alpha = \frac{x}{x_0}
\]

(13b)

so that (13) becomes:

\[
A = A_0 \alpha
\]

(13c)

Placing this value back in (11) we get:

\[
w = K_c A_0^{\alpha} \sqrt{\frac{2}{k-1}} \frac{\varepsilon^k p^2}{C_z} \sqrt{\phi \frac{k-1}{k}} - 1
\]

(14)

In order to get completely dimensionless equations we will take the crank angle as the independent variable rather than the time "t". Assuming uniform rotational speed of the crank "\( \Omega \)" radians per second we write:

\[
\theta = \Omega t.
\]

(15)

Where \( \theta \) = crank angle measured from top dead center on \( t \) = time in seconds after top dead center.

Then

\[
\frac{d}{d\theta} [\cdots] = \frac{1}{\Omega} \frac{d}{dt} [\cdots]
\]

(16)

Changing to the new independent variable in equation (9) we arrive at:

\[
w = -\frac{m_a}{z_2 \Omega} \left[ \phi \frac{1}{k} \frac{dz}{d\theta} + \frac{z}{k} \frac{d\phi}{d\theta} \left( \frac{k-1}{k} \right) \frac{d\phi}{d\theta} \right]
\]

(17)

Now we equate (14) and (17):

\[
K_c A_0^{\alpha} \sqrt{\frac{2}{k-1}} \frac{\varepsilon^k p^2}{C_z} \sqrt{\phi \frac{k-1}{k}} - 1 = \frac{m_a}{z_2 \Omega} \left[ \phi \frac{1}{k} \frac{dz}{d\theta} + \frac{z}{k} \frac{d\phi}{d\theta} \left( \frac{k-1}{k} \right) \frac{d\phi}{d\theta} \right]
\]

(18)
Rearranging and using the relations:

\[ p_2 A_p z = m_2 R T_2 \quad (2) \]
\[ C_2 = k g R T_2 \quad (12) \]

We obtain:

\[ \frac{1}{k} \frac{d \varphi}{d \theta} + \frac{z}{k \varphi^{k-1}} \frac{d \varphi}{d \theta} = -\sqrt{\frac{2}{k-1}} \frac{C_2 K C_o}{A_p} \sqrt{\frac{k-1}{k}} \varphi - 1 \quad (19) \]

Rearranging we get:

\[ \frac{d \varphi}{d \theta} = -\frac{k}{z/s^2} \left\{ \sqrt{\frac{8}{k-1}} \frac{C_2 K C_o}{A_p s \sqrt{\frac{k-1}{k}}} \varphi - \frac{k-1}{k} \sqrt{\frac{k-1}{k}} \varphi - 1 + \frac{2 \varphi}{s} \frac{dz}{d \theta} \right\} \quad (20) \]

Where "s" is stroke of the piston.

The dimensionless discharge parameter "B_d" is defined as:

\[ B_d = \frac{8}{\sqrt{k-1}} \left( \frac{C_2 K C_o}{A_p s \sqrt{\frac{k-1}{k}}} \right) \quad (21) \]

(20) now becomes:

\[ \frac{d \varphi}{d \theta} = -\frac{k}{z/s^2} \left\{ B_d \varphi - \frac{k-1}{k} \sqrt{\frac{k-1}{k}} \varphi - 1 + \frac{2 \varphi}{s} \frac{dz}{d \theta} \right\} \quad (22) \]

Thus we arrive at a single dimensionless equation for the pressure ratio in terms of the stroke, crank angle on lift of the valve. If the valves are mechanically operated so that the value of \( \varphi \) is known for every value of \( \theta \), the equation is easily solved for any particular case by a point to point integration. However, we do not know how the valve will lift and must calculate the motion by considering the applied forces on the valve itself. Figure 4 shows the forces on the valve (according to the simplifying assumptions made previously).
The equation of motion is therefore:

\[ \frac{m}{g} \frac{d^2x}{dt^2} = F - Kx \quad (23) \]

Note that this equation holds only during opening and closing of the valve. While the valve is fully opened or closed it is of course invalid (and is not needed).

By assumptions (4-f) and (4-g) the form of "F" is given by:

\[ F = C_D A_v (p - p_a) \quad (24) \]

Here "A_v" is the "face" area of the valve exposed to flow. "C_D" is an experimentally obtained drag coefficient which is associated with the area "A_v". For geometrically similar valves any representative area on the valve could be used so long as the drag coefficient is calculated on the basis of that area. The above equation would be an exact statement of pressure drag on the valve if the flow were incompressible, (although C_D would be a function of the Reynolds's number).
which we are neglecting). For any case of practical interest the flow is close to incompressible during opening and closing of the valve.

The equation of motion now becomes:

\[
\frac{W_v}{g} \frac{d^2 x}{dt^2} + K_1 x - C_D A_v (p - p_2) = 0
\]  
(25)

As before we put: \( \frac{x}{x_0} = \alpha \)

\[
\frac{p}{p_2} = \phi
\]  
(26)

Also we change the independent variable from "t" to "\( \theta \)",

getting:

\[
\frac{d^2 x}{dt^2} = x_0 \frac{d^2 \phi}{dt^2} = \omega^2 a x_0 \frac{d^2 \alpha}{d\theta^2}
\]  
(26a)

(25) now becomes:

\[
\frac{W_v \omega^2 a x_0}{g} \frac{d^2 \phi}{d\theta^2} + K_1 \alpha x_0 - C_D A_v p_2 (\phi - 1) = 0
\]  
(27)

Dividing through by \( \frac{gK_1}{W_v} x_0 \), and noticing that:

\[
\sqrt{\frac{gK_1}{W_v}} = \omega_n
\]  
(28)

Where \( \omega_n \) = natural frequency of valve system.

The equation of motion then reduces to:

\[
\frac{-\omega_n^2}{\omega_n^2} \frac{d^2 \phi}{d\theta^2} + \alpha - C_D A_v p_2 (\phi - 1) = 0
\]  
(29)
The dimensionless ratio \( J_d \) is defined as:

\[
J_d = \frac{C_D A_v P_2}{\frac{W_v}{\omega_n^2 x_o} \frac{K_l x_o}{g}}
\]

Also put \( q = \frac{\Omega}{\omega_n} \) \( \tag{31} \)

The final expression for the equation of motion is:

\[
q^2 \frac{d^2 \alpha}{d\theta^2} + \alpha - J_d (\rho - 1) = 0 \quad \tag{32}
\]

It will be noticed that \( J_d \) is the ratio of two forces.

The numerator is the pressure force which must be overcome in opening the valve, while the denominator is the spring force resisting opening of the valve.

Now we have two equations, (22) and (32) for our two unknowns \( \rho \) and \( \alpha \), which are to be determined as functions of the crank angle \( \theta \). The values of \( z \) and \( dz/d\theta \) can be computed for every value of \( \theta \) from the geometry of the compressor studied, while the values of \( B_d \) and \( J_d \) must be calculated from the known valve properties and empirical values of \( K_c \) and \( C_D \).

**Solution of the Equations**

We have the following system of differential equations to solve:
\[ q^2 \frac{d^2 \alpha}{d\theta^2} + \alpha - \int_d (\phi - l) = 0 \tag{32} \]

\[ \frac{d\phi}{d\theta} = -\frac{k}{z^2} \left\{ -\frac{1}{k} \frac{d^2 \phi}{d\theta^2} - \frac{1}{k^2} \frac{d\phi}{d\theta} \right\} \tag{22} \]

The unknowns are \( \alpha \) and \( \phi \), which must be found as functions of the independent variable \( \theta \), (it being assumed that \( z \) and \( \frac{dz}{d\theta} \) are known functions of \( \theta \)).

The initial conditions (at point 2, Figure 2) are:

\[ \theta = \theta_2 \]
\[ \alpha = 0 \]
\[ \phi = 1 \]
\[ \frac{d\alpha}{d\theta} = 0 \]

In the solution we will first consider (22). This is the equation of motion of a single degree of freedom system when acted on by a force which is a function of the time. The general solution of such a system has been worked out by the methods of Operational Calculus, (See Reference 3, pp. 397-404).

The elements of the solution are as follows:

A single degree of freedom system consisting of a weight \( W \), attached to a spring of constant \( K_1 \), is acted upon by a force "F which varies with the time "t". (See Figure 5)

First one calculates the response of the system to a "unit step function", \( F(t) = 1 \), which is applied at \( t = 0 \) and
a) SIMPLE SYSTEM

\[ F = F(t) \]

b) UNIT STEP FUNCTION

\[ x = \frac{1}{k} - \frac{1}{k} \cos \sqrt{\frac{gk}{W}} t = A(t) \]

c) DETERMINING RESULTING MOTION, USING STEP FUNCTION

\[ x = F(0) A(t) + \int_{\gamma=0}^{\gamma=t} \frac{dF}{d\gamma} [A(t-\gamma)] d\gamma \]

d) TYPICAL CURVE OF VALVE LIFT DURING DISCHARGE
continues indefinitely.

The general equation of motion is:

\[
\frac{W}{g} \frac{d^2x}{dt^2} + K_1x = F(t)
\]  \hspace{1cm} (33)

During application of the unit step function, \( F = 1 \). This gives:

\[
\frac{W}{g} \frac{d^2x}{dt^2} + K_1x = 1
\]  \hspace{1cm} (34)

The solution of this equation is given by:

\[
x = \frac{1}{K} + c_1 \sin \sqrt{\frac{gK}{W}} t + c_2 \cos \sqrt{\frac{gK}{W}} t
\]

\[= \frac{1}{K} + c_1 \sin \omega_n t + c_2 \cos \omega_n t
\]  \hspace{1cm} (35)

where \( c_1 \) and \( c_2 \) are arbitrary constants. Using the initial conditions that the mass is at rest and that its displacement is zero we get:

\[c_1 = 0, \ c_2 = -\frac{1}{K_1}
\]  \hspace{1cm} (36)

Thus the response to the unit step function, (defined as the "indicial admittance", \( A(t) \)), is given by:

\[A(t) = \frac{1}{K_1}(1 - \cos \omega_n t)
\]  \hspace{1cm} (37)

The response of the system to the force \( F \), which varies with time, may be found by considering "\( F \)" as applied in incremental step functions, and then calculating the resulting motion as an integral of the increments. Referring to Figure 5, for a mass originally at rest, the response of
the system is given by what is known as Duhamel's Integral:

\[
x = F(0)A(t) + \int_{\gamma = 0}^{\gamma = t} \frac{dF}{d\gamma} A(t-\gamma) \, d\gamma
\]  

(38)

Here \( F(0) \) is the value of \( F \) when \( t = 0 \). The displacement at the time "\( t \)" is calculated as a function of the forces acting at all previous times "\( \gamma \)" from the time when \( \gamma = 0 \) until \( \gamma = t \). Thus "\( \gamma \)" is only a variable of integration and does not appear in the solution when the limits of the integral are applied.

Comparing our equation with the general one:

\[
q^2 \frac{d^2x}{dt^2} + \alpha = J_d(\phi - 1)
\]

\[
\frac{W}{g} \frac{d^2x}{dt^2} + K_1x = F(t)
\]

We see that the function \( J_d(\phi - 1) \) corresponds to the force "\( F \)" in the general equation.

Further:

\[
q^2 \text{ corresponds to } \frac{W}{g}
\]

\[
l \quad \text{"} \quad K_1
\]

or

\[
\frac{1}{q^2} \text{ "} \quad \frac{gK_1}{W} = \omega_n
\]

\[
\phi \quad \text{"} \quad t
\]

In writing the integral we consider the motion at time "\( t \)" as a function of the forces that were acting at all previous times \( \phi = \phi' \). Also our initial condition at point (2) is that
\( \alpha = 0 \), and \( \phi = 1 \). Thus the initial value of the forcing function \( J_d(\phi - 1) \) is zero. We now simply write the solution as given by Duhamel's Integral Formula:

\[
\alpha = \int_{\theta' = \theta_2}^{\theta'} J_d \left( \frac{d\phi}{d\theta} \right) \left[ 1 - \cos \left( \frac{\theta - \theta'}{q} \right) \right] d\theta' \tag{39}
\]

We can get a single equation for the solution by substituting (38) and (22):

\[
\frac{d\phi}{d\theta} = -\frac{k}{2s} \left\{ J_d B \phi^{k-1} \sqrt{\left( \frac{k-1}{k} \right)^{k-\frac{k-1}{k}} - 1} \right\} \int_{\theta = \theta_2}^{\theta'} \left[ -\cos \left( \frac{\theta - \theta'}{q} \right) \right] d\theta' + \frac{\phi}{s} \frac{dz}{d\theta} \tag{40}
\]

This equation holds true only during opening of the valve.

The solution of this equation can be achieved by a point to point integration process, but in general it is best to keep (39) and (22) separate rather than to use equation (40). The numerical methods of solution will be discussed later.

The solution for the opening process is carried out until \( \alpha = 1 \). From this point on, while the valve is fully open, equation (39) is no longer valid, and \( \alpha = 1 \). While the valve is fully open the solution is much simpler as we just put \( \alpha = 1 \) in (22) and calculate the curve of \( \phi \) versus \( \theta \). Referring to Figure 5-D, "a" is the point where the valve starts to open, corresponding to point (2) on the P-V diagram, "b" is where the valve strikes the stops and \( \alpha = 1 \). From "b" to "c", the valve is fully open. At "C" the valve starts to close and at "d" it is fully closed. This is a typical curve which illustrates what happens in the majority of cases, but there
are other cases which are much different. These aberrations will be discussed later.

As described, the curve of \( \phi \) in the interval "b-c" is calculated from (22) with \( \alpha = 1 \). To find point "c" the calculation is carried out until the value of \( \phi \) falls to such a value that the pressure difference will no longer hold the valve open. At point "c" the pressure force on the valve just equals the spring force. The value of \( \phi \) for the point "c" is determined from (32).

\[
q^2 \frac{d^2 \phi}{de^2} + \alpha - J_d(\phi - 1) = 0
\]  

(32)

At point "c" the acceleration and velocity of the valve are zero and \( \alpha = 1 \). Since the acceleration is zero \( \frac{d^2 \alpha}{de^2} = 0 \). Putting these values in (32) gives:

\[
1 = J_d(\phi_c - 1) \quad \text{or} \quad \phi_c = 1 + \frac{1}{J_d}
\]  

(41)

The closing of the valve is calculated in an analogous way to the opening of the valve. However we cannot use (39) as it stands because it was derived with the initial condition that \( \alpha = 0 \), while for the closing of the valve \( \alpha = 1 \), initially. By a simple transformation the equation can be put in a form so that the same formula is applicable.

Put \( 1 - \alpha = \sigma \)  

(42)

Then \( \frac{d^2 \sigma}{de^2} = - \frac{d^2 \alpha}{de^2} \)  

(43)
(32) now becomes:

\[-q^2 \frac{d^2 \sigma}{d \phi^2} + 1 - \sigma - J_d (\phi - 1) = 0\]

or

\[q^2 \frac{d^2 \sigma}{d \phi^2} + \sigma - (1 - J_d (\phi - 1)) = 0 \quad (44)\]

This is the same as (32) except that we have the term

\[1 - J_d (\phi - 1) \quad \text{instead of} \quad J_d (\phi - 1)\]

The solution is:

\[\theta' = \theta\]

\[\sigma = 1 - \alpha = -J_d \int \left\{ \left( \frac{d \phi}{d \theta'} \right) \left[ 1 - \cos \left( \theta - \frac{\theta'}{q} \right) \right] d \theta' \right\} \quad (45)\]

\[
\theta' = \theta_c
\]

Solving for \(\alpha\)

\[\alpha = 1 + J_d \int_{\theta' = \theta_c}^{\theta' = \theta} \left( \frac{d \phi}{d \theta'} \right) \left[ 1 - \cos \left( \theta - \frac{\theta'}{q} \right) \right] d \theta' \quad (46)\]

Since \(\frac{d \phi}{d \theta'}\) is predominantly negative during this process, \(\alpha\) is always less than unity. Using (46) and (32), the solution is completed from "c" to "d".

**The Special Case of the Reed Valve**

The following discussion will show that, with certain simplifying assumptions, the dynamics of the reed type valve are the same as for the single degree of freedom valve. In this analysis the reed will be assumed held at the ends, with no "end lift". Since the end lift is usually only a small part of the total lift, this assumption should not lead to any serious errors. Also, it will be assumed that the valve while opening has the shape of its fundamental mode
of vibration as a simply supported beam. This assumption seems fairly reasonable since the frequency of the exciting force is ordinarily low compared with the first natural frequency of the reed. This means that higher modes should not be excited.

Using these assumptions, the area for flow is directly proportional to the lift \( \alpha \). Therefore, the flow equation (22) is valid for the reed valve. Now the dynamics of the reed itself will be described.

Taking the coordinate system shown in Figure (6), with the following notation:

\[ x \] is the lift of any point along the reed.
\[ x_m \] is the lift of the center of the reed.
\[ y \] is the distance of any point from the center of the reed.

"I" is the moment of inertia of the cross section of the reed about its neutral axis.

\[ b = \text{breadth of the reed} \]
\[ l = \text{length of the reed} \]
\[ x_0 = \text{maximum lift at the center of the reed} \]
\[ P = \text{Fluid drag on reed per unit length along the reed} \]
(P is in general a function of "y" and "t")

\[ W_v = \text{total weight of reed} \]
We take \( P \) in the form

\[
P = b (p - p_a) f(y)
\]  \quad (47)

Where \( f(y) \) is an unknown function of \( y \). This is equivalent to assumption (4-f). The differential equation of motion of the reed for this value of the loading is:

\[
EI \frac{\partial^4 x}{\partial y^4} + \frac{w_v}{gl} \frac{\partial^2 x}{\partial t^2} - b(p - p_a)f(y) = 0
\]  \quad (48)

If only the first mode is present, then:

\[
x = \begin{bmatrix} \cos \frac{ny}{l} \end{bmatrix} x_m(t)
\]  \quad (49)

Where \( x_m \) is a function of "t" only,

\[
\frac{\partial^4 x}{\partial y^4} = \frac{w_v}{l^4} \begin{bmatrix} \cos \frac{ny}{l} \end{bmatrix} x_m(t)
\]  \quad (50)

\[
\frac{\partial^2 x}{\partial t^2} = \begin{bmatrix} \cos \frac{ny}{l} \end{bmatrix} \frac{d^2 x_m}{dt^2}
\]  \quad (51)

Now (48) becomes:

\[
\frac{w_v}{l^4} \begin{bmatrix} \cos \frac{ny}{l} \end{bmatrix} x_m + \frac{w_v}{gl} \begin{bmatrix} \cos \frac{ny}{l} \end{bmatrix} \frac{d^2 x_m}{dt^2} - b(p - p_a)f(y) = 0
\]  \quad (52)

We notice that if \( f(y) \) is taken in the form:

\[
f(y) = C_D \cos \frac{ny}{l}
\]  \quad (53)
Then "y" will disappear from the equation.

With this assumption (52) reduces to:

\[
\frac{n^4EI}{l_4} x_m + \frac{Wv}{gl} \frac{d^2x_m}{dt^2} - C_D(p-p_2) = 0
\]  

(54)

Put \( x_m = x_o \alpha \) and divide through by \( \frac{Wv}{gl} x_o \)

\[
\frac{n^4gEI}{Wv^3} \alpha + \frac{d^2\alpha}{dt^2} - C_D \frac{blp_2(\theta-1)}{Wv \omega n^2 x_o} = 0
\]  

(55)

But \( \frac{n^4gEI}{Wv^3} = \omega_n^2 \)  

(56) (Fundamental frequency)

\[
\frac{d^2\alpha}{dt^2} = -n^2 \frac{d^2\alpha}{de^2}
\]  

(57)

Using these facts and \( q = \frac{n}{\omega_n} \), we get:

\[
q^2 \frac{d^2\alpha}{de^2} + \alpha - C_D \frac{blp_2(\theta-1)}{Wv \omega_n^2 x_o} = 0
\]  

(58)

Noticing that \( bl = A_v \)  

(59) (Face area of valve)

becomes:

\[
q^2 \frac{d^2\alpha}{de^2} + \alpha - C_D A_v \frac{p_2(\theta-1)}{Wv \omega_n^2 x_o} = 0
\]  

(60)
or \[ q^2 \frac{d^2 \alpha}{d \theta^2} + \alpha - J_d(\theta - 1) = 0 \] (61)

Where \( J_d = \frac{C_d A_v \rho}{W_v \omega_n^2} \) as before

Thus the equation of motion for the reed (assuming that only the first mode is present) is the same as for the single degree of freedom system.

The assumption that the reed moves only in the first mode seems borne out by the experimental results, where records of valve lift versus time were obtained. There was nothing in the curves which would indicate that the reeds were bending in any other than the normal manner of the first mode. There is a possibility of "flutter" at some operating condition, but none was observed. Reference (8) gives the analysis for a strip placed edgewise to the flow but no analysis was found in the literature of the case where the strip is normal to the flow. If flutter did occur it would probably be only in the case of a reed too thin and flexible to be of practical use. For the reeds normally used, the natural frequency is fairly high making \( q << l \). Under these circumstances, it seems unlikely that flutter could exist.

**Part 3 Inlet Process Theory**

The analysis of the inlet process is somewhat similar to that of the discharge process. However, while the discharge process can be considered as very close to isentropic (as regards changes inside the cylinder), the inlet process is not, since
irreversible throttling occurs when air flows from the inlet receiver into the cylinder. To get a relation equivalent to equation (22), we must consider the energy changes involved, by using the First Law of Thermodynamics.

Figure (7) shows the thermodynamic system considered. At "a" is shown the boundary of the system at point (4) in the cycle of Figure (2), just as the cylinder pressure becomes equal to the inlet pressure. At (b) a certain amount of gas, "m", has flowed into the cylinder, while the piston has moved to a new position, and the state of the gas in the cylinder has changed.

Using the notation of Reference (2)

\[ \Delta Q = \text{heat added to system in process between "a" and "b".} \]

\[ \Delta W = \text{work done by "" between ""a" and "b".} \]

\[ E' = \text{initial energy of system initially (at "a")} \]

\[ E'' = \text{finally (at "b")} \]

\[ \Delta E = E'' - E' = \text{Increase in internal energy of system during process between "a" and "b".} \]

Since the final mass of the system is the same as the initial:

\[ m = m_4 + \Delta m \quad (62) \]

The statement of the First Law (Ref. 2) is

\[ \Delta E = \Delta Q - \Delta W \quad (63) \]
FIGURE 7 INLET PROCESS

(a) CONDITION AT START OF INLET

SYSTEM CONSIDERED IS THAT BOUNDED BY DOTTED LINE

(b) CONDITION AFTER MASS OF AIR $\Delta m$ HAS PASSED INTO CYLINDER

$m = m_4 + \Delta m$
Since we have assumed no heat transfer, then $\Delta Q = 0$ and (63) becomes:

$$\Delta E = -\Delta W \quad (64)$$

Now we compute the values of $\Delta E$ and $\Delta W$.

With: $c_v = \text{specific heat at constant volume}$

$T = \text{absolute temperature, } ^\circ\text{F abs.}$

$$E' = m_4 c_v T_4 + \Delta mc_v T_1 \quad (65)$$

$$E'' = mc_v T \quad (66)$$

$W = \text{work done by system} = \int_z^{z_4} pA_p dz - p_1 v_1 \Delta m \quad (67)$

(The work done in lifting the valve is very slight and is ignored.)

Then (64) becomes

$$\Delta E = -\Delta W$$

or,

$$c_v [mT - m_4 T_4 - (m - m_4)T_1] = - \int_{z_4}^{z} pA_p dz + p_1 v_1 (m - m_4) \quad (68)$$

We also use the equation of state:

$$pA_p z = mRT \quad 69$$

$$p_1 v_1 = RT_1$$

or:

$$pA_p = \frac{mRT}{z} \quad (70)$$
Using 70 and dividing through by $RT_4$ we get:

$$
\frac{c_v}{R} \left[ \frac{mT}{T_4} - m_4 - (m-m_4) \frac{T_4}{T_4} \right] = - \int_{z_4}^{z_1} \frac{mT}{T_4 z} \, dz + \frac{T}{T_4} (m - m_4) \quad (71)
$$

but $R = \frac{c_p}{c_v} - c_v$, $k = \frac{c_p}{c_v}$

$$
\frac{c_v}{R} \frac{c_p}{c_v} = \frac{1}{c_p/c_v - 1} = \frac{1}{k - 1} 
$$

Also from the equation of state:

$$
\frac{m}{m_4} = \frac{p_4}{p_z} \frac{z_4}{z_4} \quad (74)
$$

Using these relations and the identity:

$$
dz = \frac{dz}{dt} \, dt
$$

As well as the definition of the inlet pressure ratio:

$$
\psi = \frac{p}{p_4} \quad (75) \quad 0 < \psi < 1
$$

Then equation 71 reduces to:

$$
\frac{1}{k-1} \left[ \frac{z_4}{z_4} m_4 \psi - m_4 - (m-m_4) \frac{T_4}{T_4} \right] = - \int_{t_4}^{t} \psi \frac{m_4}{z_4} \left( \frac{dz}{dt} \right) \, dt
$$

$$
+ \frac{T_4}{T_4} (m - m_4) \quad (76)
$$
Now we differentiate with respect to "t" and solve for 
\[ \frac{dm}{dt} \]
\[ \frac{dm}{dt} = \frac{m_4}{z_4} \frac{T_4}{T_1} \left( \psi \frac{dz}{dt} + \frac{z}{k} \frac{d\psi}{dt} \right) \]  
(77)

As before put \( \theta = \Omega t \)
\[ \frac{dz}{dt} = \Omega \frac{dz}{d\theta} \]  
(78)
\[ \frac{d\psi}{dt} = \Omega \frac{d\psi}{d\theta} \]

77 now becomes:
\[ \frac{dm}{dt} = \frac{m_4}{z_4} \frac{T_4-\Omega}{T_1} \left( \psi \frac{dz}{d\theta} + \frac{z}{k} \frac{d\psi}{d\theta} \right) \]  
(79)

As before we use the equation for one dimensional flow through an orifice:
\[ w = K_c A \sqrt{\frac{2gk}{R(k-1)}} \frac{p_1}{\sqrt{T_1}} \left( \frac{p}{p_1} \right)^\frac{1}{k} \sqrt{1 - \left( \frac{p}{p_1} \right)^\frac{k-1}{k}} \]  
(80)

In this analysis \( p_1 = p_1 = p_4 \)

Using (75) \( \psi = \frac{p}{p_4} = \frac{p}{p_1} \)

and \( \sqrt{kgRT_1} = C_1 \)  
(81)
This is the velocity of sound at inlet conditions. Also \( A = A_0 \) as assumed before.

Then (80) becomes:
\[ w = K_c A_0 \alpha \sqrt{\frac{2}{k-1}} \frac{gk p_1}{C_1} \psi \frac{1}{k} \sqrt{1 - \psi \frac{k-1}{k}} \]  
(82)
But \( w = \frac{dm}{dt} \) for the inlet process. \( \tag{83} \)

Here \( \frac{dm}{dt} \) has a plus sign as we are taking the flow inward as positive.

Equating (79) to (82) we get
\[
\frac{z}{k} \frac{d\psi}{d\theta} + \psi \frac{dz}{d\theta} = k_c A_o \sqrt{\frac{2}{k-1}} \frac{p_1 z T_1}{C_1 m_4 T_4} \psi \sqrt{1 - \psi^k} \tag{84}
\]

But, since \( p_1 = p_4 \), and by the equation of state:
\[
\frac{p_4 z_4}{m_4 T_4} = \frac{R}{A_p} \tag{86}
\]

We now get:
\[
\frac{z}{k} \frac{d\psi}{d\theta} + \psi \frac{dz}{d\theta} = k_c A_o \sqrt{\frac{2}{k-1}} \frac{kgRT_1}{A_p s \Omega C_1} \psi \sqrt{1 - \psi^k} \tag{87}
\]

Also from (81) \( C_1 = \sqrt{kgRT_1} \)

Transposing and rearranging
\[
\frac{d\psi}{d\theta} = \frac{k}{z/2} \left( \frac{8}{k-1} \frac{k_c A_o C_1 \psi}{A_p s \Omega} \sqrt{1 - \psi^{k-1}} - \frac{2\psi}{s} \frac{dz}{d\theta} \right) \tag{88}
\]

Now defining the dimensionless ratio:
\[
B_i = \frac{k_c A_o C_1}{A_p s \Omega} \tag{89} \quad \text{Note that here } K_c \text{ and } A_o \text{ are the values for the inlet valves.}
(98) Becomes
\[
\frac{d\psi}{d\theta} = \frac{k}{z^{3/2}} \left\{ B_1 \alpha \psi \frac{1}{k} \sqrt{1 - \psi^{k-1}} - \frac{2}{s} \psi \frac{dz}{d\theta} \right\} \tag{90}
\]

The valve dynamics for the inlet valves are similar to those of the discharge valves. The only difference is in the expression for the fluid drag term.

The equation of motion is:
\[
\frac{W \cdot d^2 \chi}{g} = C_D A_v (p - p) - K_1 x = C_D A_v p_1 (1 - \psi) - K_1 x \tag{91}
\]

Using the same procedure as before we arrive at the following equation:
\[
\frac{q^2 d^2 \chi}{d\theta^2} + \alpha - J_1 (1 - \psi) = 0 \tag{92}
\]

Where \( q \) and \( J_1 \) refer to the inlet valves:
\[
J_1 = \frac{C_D A_v p_1}{\omega^2 n x_o} = \frac{C_D A_v p_1}{K_1 x_o} \tag{93}
\]

The solution of the equation proceeds the same as for the discharge valves.

The result is:
\[
\alpha = - J_1 \int_{\theta_4}^{\theta} \left[ \frac{d\psi}{d\theta} \right] \left[ 1 - \cos \left( \theta - \frac{\theta_4}{q} \right) \right] d\theta \tag{94}
\]

Equation (94) holds true during the opening of the valve (points "a" to "b" in Figure (5d)). In this region (94) and (90) give the complete solution.
From "b" to "c", the solution is obtained from (90) with \( \alpha = 1 \).

At point "c", \( \frac{d\alpha}{d\psi} = 0 \), \( \alpha = 1 \) and \( \psi = 1 - \frac{1}{J_{1}} \).

From "c" to "d" we have:

\[
\alpha = 1 - J_{1} \int_{\theta' = \theta_{c}}^{\theta' = \theta} \frac{d\psi}{d\theta'} \left[ 1 - \cos \left( \frac{\theta - \theta'}{q} \right) \right] d\theta'.
\] (95)

In this period the solution is given by (95) and (90).

**Part 4. Numerical Solution of the Equations**

The first step is to find the values of \( z \) and \( \frac{dz}{d\theta} \) as functions of \( \theta \) for the machine under consideration. Since the experiments were carried out on a standard CFR engine, the calculations are based on the geometry of this engine.

Plate I shows the dimensions of this design. The exact values of \( z \) and \( \frac{dz}{d\theta} \) are given as infinite series, but it will be sufficient to include only terms up to the second order.

With the notation of plate I, the values are:

\[
z = \frac{s}{2} \left( 2\delta + \frac{\beta}{4} - \cos \theta - \frac{\beta}{4} \cos 2\theta \cdots \right)
\] (96)

\[
\frac{dz}{d\theta} = \frac{s}{2} \left( \sin \theta + \frac{\beta}{2} \sin 2\theta \cdots \right)
\]

Where \( \beta = \) ratio of crank/connecting rod.
For our compressor $\beta = .225$, giving

$$z = \frac{3}{2} (2\delta + .056 - \cos \theta - .056 \cos 2\theta)$$

(97)

$$\frac{dz}{d\theta} = \frac{3}{2} (\sin \theta + .112 \sin 2\theta)$$

Inserting these values in (22) we obtain an equation of the form:

$$\frac{d\phi}{d\theta} = -M_D B_D \alpha + N_D$$

(98)

where

$$M_D = \frac{k-1}{k\delta^k \sqrt{\delta}} \frac{k-1}{k^k} \frac{1}{\delta + .056 - \cos \theta - .056 \cos 2\theta}$$

(99)

$$N_D = -k(\sin \theta + .112 \sin 2\theta) \frac{\phi}{2\delta + .056 - \cos \theta - .056 \cos 2\theta}$$

(Discharge process)

Equation (90) becomes:

$$\frac{d\psi}{d\theta} = +M_1 B_1 \alpha - N_1$$

(100)

where

$$M_1 = \frac{1}{k\psi^k} \sqrt{1 - \psi^k} \frac{k-1}{k^k} \frac{1}{2\delta + .056 - \cos \theta - .056 \cos 2\theta}$$

(101)

$$N_1 = k(\sin \theta + .112 \sin 2\theta) \frac{\psi}{2\delta + .056 - \cos \theta - .056 \cos 2\theta}$$

(Inlet process)

Curves of $M$ and $N$ are given on plates 2 and 3, calculated for $\delta = .605$, corresponding to a clearance volume of 10 1/2% of piston displacement.

The numerical solution will be described for the discharge process only, but the inlet process is carried out in an exactly similar manner.
The problem is essentially to solve the following
two equations simultaneously:

\[ \frac{d\phi}{d\phi} = -mB \alpha + N_d \tag{98} \]

\[ \alpha = J_d \int_{\theta_1}^{\theta_2} \left( \frac{d\phi}{d\phi} \right) \left[ 1 - \cos \left( \frac{\theta - \theta'}{q} \right) \right] d\theta' \tag{39} \]

The initial conditions are \( \alpha = 0, \phi = 1, \theta = \theta_2 \). We could
take the value of \( \alpha \) from (39) and put it in (98), thus giving
a single equation for \( \phi \), but it is more convenient to keep the
equations separate.

The usual method of solving a differential equation
which has no explicit algebraic solution is to use a point to
point integration by finite differences. In our case we must
calculate \( \phi \) and \( \alpha \) during the lift of the valve. The initial
conditions are: \( \phi_0 = 1, \alpha_0 = 0 \) and \( \left( \frac{d\phi}{d\phi} \right)_0 = N_0 \), where the
subscript "o" refers to conditions at the start of discharge
(Point 2 on the P-V diagram). Subscripts for subsequent points
in the calculation are 1, 2, 3, etc.

For a first approximation we will consider

\[ \frac{d\phi}{d\phi} = \left( \frac{d\phi}{d\phi} \right)_0 \]

over the whole interval \( a - b \).

When \( \theta = \theta_b, \alpha = 1 \). Putting these values in (39) we get:

\[ 1 = J_d q \left( \frac{d\phi}{d\phi} \right)_0 \int_{\theta_1}^{\theta_2} \left[ 1 - \cos \left( \frac{\theta - \theta'}{q} \right) \right] \frac{d\theta'}{\phi} \tag{102} \]
The quantity $\left( \frac{d\phi}{d\theta} \right)$ is taken outside the integral as it is considered constant over the interval.

Evaluating the integral gives:

$$
\left[ 1 - \cos(\theta_b - \theta') \right] \frac{d\theta'}{q} = \frac{\theta_b - \theta_2}{q} - \sin \left( \frac{\theta_b - \theta_2}{q} \right)
$$

(103)

We define the function "H" such that:

$$
H(\gamma) = \gamma - \sin \gamma \quad (103A)
$$

where "\gamma" is any angle, in radians. Values of the function H are plotted on Plate (4). From (102) and (103) we solve for the value of "H" for the angular interval a-b:

$$
H_{a-b} = \frac{1}{\frac{d}{d\theta} \left( \frac{d\phi}{d\theta} \right)_{0}}
$$

(104)

Knowing H, we get $(\theta_b - \theta_2)$ from plate (4). This gives only a first approximation for the angle during which the valves lift. For a closer approximation we can make the calculation in several steps, considering $\frac{d\phi}{d\theta'}$ constant over each interval.

Figure (8) shows the setup for a four step calculation.

Let $\Delta \theta = \frac{\theta_b - \theta_2}{4}$, with the value of $\theta_b$ from the first approximation. From Plate (4) get values $H_1, H_2, H_3, H_4$ where,

$$
\begin{align*}
H_1 &= \frac{\Delta \theta}{q} - \sin \frac{\Delta \theta}{q} \\
H_2 &= \frac{2\Delta \theta}{q} - \sin \frac{2\Delta \theta}{q} \\
&\vdots \\
&\text{ETC}
\end{align*}
$$

(105)

Also calculate

$$
\begin{align*}
\Delta H_1 &= H_2 - H_1 \\
\Delta H_2 &= H_3 - H_1 \\
\Delta H_3 &= H_4 - H_1
\end{align*}
$$

(106)
Figure 8
Arrangement for Step by Step Integration
The step calculation proceeds as follows:

**Step 1** Calculate $\alpha = \int d\theta \frac{d\theta}{d\theta_1} H_1$ \hspace{1cm} (107)

With this value of $\alpha$, get $\frac{d\theta}{d\theta_1}$ from (98) and calculate the mean value of the slope over the first interval:

$$\left( \frac{d\theta}{d\theta} \right)_m = \frac{1}{2} \left[ \left( \frac{d\theta}{d\theta} \right)_o + \left( \frac{d\theta}{d\theta} \right)_1 \right]$$ \hspace{1cm} (108)

The change in $\theta$, $\Delta \theta$, is given by:

$$\Delta \theta = \Delta \theta_1 = 1 + \Delta \theta_1 = 1 + \left( \frac{d\theta}{d\theta} \right)_m \Delta \theta$$ \hspace{1cm} (109)

**Step 2** Calculate $\alpha_2$:

$$\alpha_2 = J_0 \left\{ \left( \frac{d\theta}{d\theta} \right)_1 H_1 + \left( \frac{d\theta}{d\theta} \right)_o \Delta H_2 \right\}$$ \hspace{1cm} (110)

Get $\frac{d\theta}{d\theta}_o$ as before using from (110), and calculate $\frac{d\theta}{d\theta}_m$, $\theta_2$ and $\theta_2$.

For step 2:

$$\frac{d\theta}{d\theta}_m = \frac{1}{2} \left[ \left( \frac{d\theta}{d\theta}_1 \right)_1 + \left( \frac{d\theta}{d\theta}_2 \right)_2 \right]$$ \hspace{1cm} (111)

$$\theta_2 = \theta_1 + \left( \frac{d\theta}{d\theta}_m \right) \Delta \theta$$ \hspace{1cm} (112)

**Step 3.** Calculate $\alpha_3$

$$\alpha_3 = J_0 \left\{ \left( \frac{d\theta}{d\theta}_2 \right)_1 H_1 + \left( \frac{d\theta}{d\theta}_4 \right) \Delta H_1 + \left( \frac{d\theta}{d\theta}_o \right) \Delta H_2 \right\}$$ \hspace{1cm} (113)

Get $\frac{d\theta}{d\theta}_o$, $\frac{d\theta}{d\theta}_m$ and $\theta_3$ as before.
Step 4  Calculate \( \alpha' \)

\[
\alpha' = \frac{q}{4}\left\{ \frac{d\phi}{d\theta} H_1 \right\} + \left( \frac{d\phi}{d\theta} H_2 \right) + \left( \frac{d\phi}{d\theta} H_3 \right) \tag{114}
\]

Get \( \left( \frac{d\phi}{d\theta} \right)_4 \) and \( \phi_4 \) as before.

The value of \( \alpha' \) will be close to unity. Plot \( \alpha' \) versus \( \theta \) and interpolate or extrapolate for the value of \( \theta \) when \( \alpha = 1 \). Then get \( \left( \frac{d\phi}{d\theta} \right)_4 \) from (98) with \( \alpha = 1 \), and recalculate the value of \( \phi_4 \) as in step (4) with the new value of \( \left( \frac{d\phi}{d\theta} \right)_4 \).

This is a long process and tedious work. However, the form of the function "H" is such that a simpler method can be used. A range of 27 cases was calculated by the four step method and it was found that the values of \( \theta_b \) obtained were within a fraction of a degree of that obtained by the first approximation.

This is due to the fact that in all cases studied, \( \frac{d\phi}{d\theta} \) remains fairly constant and close to the value of \( \frac{d\phi}{d\theta} \) for the major part of the valve opening. Referring to Figure (8), we see that the function \( H \) is very large at the start of lift near point (a), and counts less and less as we approach \( \theta_b \). This means that \( \frac{d\phi}{d\theta} \) may change appreciably near \( \theta_b \) without affecting valve motion to any extent. Therefore, for the whole range calculated subsequently, the first approximation was used to get \( \theta_b \). Then the curve of \( \alpha \) is given immediately by

\[
\alpha = q \int_{\theta}^{\theta_2} \left( \frac{d\phi}{d\theta} \right)_0 H \quad \tag{115}
\]

Where \( H \) is a function of \( \theta_2 - \theta \), and is found from Plate (4). The curve of \( \phi \) must be calculated by the step process described.
It will be noticed that the lift curve approximates a cubic parabola since:

\[ H(\lambda) = \lambda - \sin \lambda = \lambda - (\lambda - \frac{\lambda^3}{3!} + \frac{\lambda^5}{5!} + \ldots) \]

or \[ H(\lambda) \approx \frac{\lambda^3}{6} \quad (116) \]

This is a good approximation up to \( \lambda = 60^\circ \) or \( 70^\circ \)

The discharge process with the valve fully open is comparatively easy to solve as we can use a convenient graphical method. With \( \alpha = 1 \), (98) becomes

\[ \frac{d\phi}{d\theta} = -B_d M_d + N_d \quad (117) \]

This is a first order differential equation of the form:

\[ \frac{dy}{dx} = f(y,x) \quad (118) \]

Since a large number of cases were to be solved, it was considered desirable to get a general solution of (117) for various values of \( B \). This can be done by what is called the "isoclinic" method (see Reference 3, Chapter 1). Taking a set of values of \( \phi \) and \( \theta \) we calculate \( \frac{d\phi}{d\theta} \) from (117). Then we plot curves of \( \frac{d\phi}{d\theta} \) for constant values of \( \phi \), versus \( \theta \). From these intermediate curves we pick off points for constant values of \( \frac{d\phi}{d\theta} = \) constant in the \( \phi - \theta \) plane. These are called "isoclinics" as they are lines along which all integral curves have the same slope.

Along each curve of \( \frac{d\phi}{d\theta} = \) constant we draw short lines having the slope of \( \frac{d\phi}{d\theta} \) for that curve. For example, along the curve \( \frac{d\phi}{d\theta} = 0 \), we draw short lines having a slope of zero (horizontal lines). Figure (9a) illustrates the procedure. When sufficient
FIGURE 9

(a) ILLUSTRATION OF ISOCLINIC METHOD

(b) TYPICAL CURVES OF "Φ"

POINT WHERE VALVE BECOMES FULLY OPEN
lines are drawn they form a network of integral curves giving the complete solution of (117). These are the correct integral curves since their slope at any point equals $\frac{d\varphi}{d\theta}$ as given by equation (117). Since (117) is of the first order there will be one arbitrary constant associated with each integral curve. Thus we have to know only one initial condition which will put us on one of the integral curves. This initial condition is the final condition for the opening of the valve or the value calculated for the point "b" as previously described. We merely plot $\varphi_b$ at $\theta = \theta_b$ and this puts us on one of the integral curves. We follow this curve until we reach a point where the valve will start to close. This is the point where $\varphi = 1 + \frac{1}{d}$, as previously shown. The solution for the closing of the valve cannot be gotten as easily as the opening, in most cases. The actual method used was an extension of formulas, (108), (110), (113) and (114) in tabular form. Table 3 shows a typical calculation with an explanation of the entries. It was found that the angular interval for $\theta$ had to be $2^\circ$ or less to get sufficient accuracy for the calculations to converge. The calculation is carried out until $\varphi = 1$, which is at $\theta = 360^\circ$ or later.

For many cases, the closing of the valve is not as shown in Figure (5d). Instead, the valve makes several opening and closing cycles during the discharge period. To handle these cases we must modify the equations (39) and (46). The first case to be considered is when the valve is found to close before
top dead center and where the value of $\phi$ at closing is greater than unity. Then the valve will reopen but we cannot use (39) as the initial condition for that equation was $\phi = 1$. Thus we modify (39) in accordance with (38), giving

$$\alpha = J_d (\phi_o - 1) \left[ 1 - \cos \left( \frac{\phi - \phi'}{q} \right) \right] + J_d \int_{\phi'}^{\phi} \left[ 1 - \cos \left( \frac{\phi - \phi'}{q} \right) \right] d\phi' \quad (39a)$$

Here $\phi_o$ is the initial value of $\phi$ for the second opening of the valve and $\phi_o$ is the initial value of the crank angle. The integration is carried out by the usual step process.

The equation (39A) is also useful for investigating cases where the valve spring is initially in compression before the valve starts to lift. In these cases, the valve does not start to lift until $\phi$ is somewhat greater than unity. The second case to be discussed is where the valve strikes the stops and starts to close when $\phi < 1 + \frac{1}{J_d}$. Equation (46) is modified to:

$$\alpha = 1 + \left[ J_d (\phi_o - 1) - 1 \right] \left[ 1 - \cos \left( \frac{\phi - \phi'}{q} \right) \right] + J_d \int_{\phi'}^{\phi} \left[ 1 - \cos \left( \frac{\phi - \phi'}{q} \right) \right] d\phi' \quad (46a)$$

Where $\phi_o$ and $\phi_o$ are the initial values at the start of closing. This equation is integrated by the usual step processes.

In rare instances, the valve never becomes fully open. For these cases (39) can be used throughout, with a step by step integration.

In order to cover the wide range of conditions that may occur in practice, a great many separate cases had to be investigated. There are four parameters in the equations: $B, J, q$ and $\delta$. There is an additional variable, the pressure ratio, or
what amounts to the same thing, the initial angle for discharge, $\theta_2$. It seemed that the clearance ratio was the least important variable so it was held fixed and $\delta$ taken as .605. This leaves 4 parameters, $B$, $J$, $q$ and $\theta_2$. A compressor operating at a given speed and pressure has definite values for these parameters. At another operating condition some or all of the parameters will be different.

The values of $\theta_2$ selected were 270°, 295° and 320°, corresponding to compression ratios from about 2 to 8. The other values of parameters selected were estimated from present practice in valve design. They are as follows:

$$B_D = 1, 3, 6, 9$$
$$J_d = 10, 40, 70$$
$$q_d = .20, .0833, .050$$

The combinations of these 4 parameters resulted in 108 separate cases. (See Table 1). These were calculated and are plotted in Plates 12 and 13.

In order to get the valve losses for these cases the $\phi$ curves were replotted versus $Z/s$ and the areas under them obtained by a planimeter.

There is one additional note on these discharge calculations. From the curves of $\phi$ versus $\theta$ for the valves fully opened, at each value of $\theta_2$, the curve of $\phi$ which starts at $\theta = \theta_2$ and $\phi = 1$ corresponds to the case of a valve which opens instantaneously and remains fully open for the whole discharge period. We define
a valve which does this as the "ideal" valve since its dynamics are the most favorable possible for a given flow area. Figure (9b) illustrates the curve of \( \phi \) for a typical case of an ideal valve. It will be noticed that for a given \( \theta_2 \) and \( B \), the ideal valve always has the minimum loss possible.

The inlet process was calculated in a similar manner to the discharge process. The values of the parameters used were:

\[
\begin{align*}
\theta_2 &= 30^\circ, 50^\circ, 70^\circ \\
B_1 &= 1, 3, 6 \\
J_1 &= 10, 20, 30 \\
(q)_1 &= .05, .0633, .20
\end{align*}
\]

The curves of \( \psi \) are plotted on Plate (14).

Calculation of Cycle Efficiency and Volumetric Efficiency

With the calculated curves of \( \phi \) and \( \psi \), the thermal and volumetric efficiencies of the cycle can be calculated. Referring to Figure (10a), the calculated cycle efficiency will be the ratio of:

\[
\text{Total area of P-V diagram — loss areas} \\
\text{Total area of P-V diagram.}
\]

The volumetric efficiency is defined as the ratio:

Actual volume flow of air per unit time (at inlet conditions) through the compressor, divided by the piston displacement per unit time. The "ideal" volumetric efficiency with no valve loss\( \eta_v \) is given by the ratio: \( \frac{z_1-z_4}{z_{1-4}} \), as given on Figure (10-b). The value of \( \eta_v \) with valve losses will always be lower than the ideal.
CALCULATED P-V DIAGRAM

FOR NO HEAT TRANSFER OR LEAKAGE, THERMAL EFF =

\[
\text{AREA INSIDE DOTTED LINE} \quad \text{AREA INSIDE SOLID LINE}
\]

IDEAL P-V DIAGRAM

\[
\eta_{vi} = \frac{z_i - z_4}{s}
\]
The ideal volumetric efficiency in terms of the clearance and compression ratios is calculated as follows:

\[ \eta_{v1} = \frac{z_1 - z_4}{S} \quad (119) \]

where \( z_1 = (\delta + .50) S \)

By the adiabatic relation

\[ \frac{z_4}{z_3} = \left( \frac{p_3}{p_4} \right)^{\frac{1}{k}} = r^{\frac{1}{k}} \]

where "r" is the pressure ratio: \( p_d/p_i \)

And: \( z_3 = (\delta - .50) S \)

\[ \therefore \eta_{v1} = (\delta + .5) - (\delta - .5)r^{\frac{1}{k}} \quad (120) \]

To get the loss in efficiency due to discharge we want the ratio of:

\[ \frac{\text{Discharge loss area (on P-V diagram)}}{\text{Area of ideal P-V diagram}} \]

This will be the same as the ratio of "mean effective pressures" or:

\[ \text{Discharge loss ratio} = \frac{\text{Discharge loss m.e.p.}}{\text{Ideal m.e.p.}} \]

The ideal m.e.p. is given by:

\[ \text{Ideal m.e.p.} = \frac{k}{k-1} \eta_{v1} \left[ r^{\frac{k-1}{k}} - 1 \right] p_i \quad (121) \]

The loss m.e.p. for the discharge process is:

\[ p_d \int (\phi - 1) \frac{dz}{S} \text{ or, } r p_i \int (\phi - 1) \frac{dz}{S} \]
Therefore, the discharge loss ratio becomes:

\[
\text{Discharge loss ratio} = \frac{r \int (\phi-1) \, dz}{S} \frac{k}{k-1} \eta v \left[ \frac{k-1}{k} - 1 \right]
\]  
(122)

For any value of \( \Theta_e \), there is a certain pressure ratio, assuming compression starts at bottom dead center. With this value of "r", the above ratio is calculated for each case.

Note that for any pressure ratio, the area of the diagram (without losses) is proportional to the volumetric efficiency. As will be shown later, the actual volumetric efficiency is a consequence of the inlet process and little affected by the discharge. Thus for the discharge process, we compare the discharge loss to the ideal cycle area since we do not yet know how much the ideal area is reduced due to loss in volumetric efficiency which is a function of the inlet valves.

As the inlet valves are directly responsible for the change in volumetric efficiency we get the desired ratio directly as

\[
\text{Inlet loss ratio} = \frac{\int (1-\psi) \, dz}{S} \frac{k}{k-1} \eta v \left[ \frac{k-1}{k} - 1 \right]
\]  
(123) where \( \eta_v \) = actual volumetric efficiency
The calculated volumetric efficiency is found by going back to equation (76).

From the definition of volumetric efficiency, we need the quantities:

\[
\text{Volume flow per unit time} = (m_1-m_4)v_1 \frac{\Omega}{2\pi} \tag{124}
\]

\[
\text{Piston displacement per unit time} = A_p S \frac{\Omega}{2\pi} \tag{125}
\]

Thus the calculated volumetric efficiency is given by:

\[
\eta_v = \frac{(m_1-m_4)v_1}{A_p S} \tag{126}
\]

Putting \( m = m_1 \) and \( \psi = \psi_1 = 1 \) in (76), and solving for \( m_1-m_4 \)
we get:

\[
m_1-m_4 = T_4 \left( \frac{m_4}{z_4} \right) \left[ \frac{z_1-z_4}{k} + \frac{k-1}{k} \int_{z_4}^{z_1} \psi \frac{dz}{s} \right] \tag{127}
\]

But \( (m_1-m_4)v_1 = (m_1-m_4) \frac{RT_4}{p_1} = (m_1-m_4) \frac{RT_4}{p_4} \tag{128} \)

(since \( p_1 = p_4 \))

This gives:

\[
\eta_v = \frac{m_1-m_4}{A_p S} \frac{RT_4}{p_4} = \frac{RT_4 T_4 m_4}{T_1 A_p S z_4 p_4} \left[ \frac{z_1-z_4}{k} + \frac{k-1}{k} \int_{z_4}^{z_1} \psi \frac{dz}{s} \right] \tag{129}
\]

Also from the equation of state:

\[
p_4 A_p z_4 = m_4 R T_4 \tag{130}
\]

The calculated volumetric efficiency reduces to:

\[
\eta_v = \left[ \frac{z_1-z_4}{Sk} + \frac{k-1}{k} \int_{z_4}^{z_1} \frac{\psi}{S} \frac{d(z)}{s} \right] \tag{131}
\]
Adding and subtracting \( \left( \frac{z_1-z_4}{S} \right) \left( \frac{k-1}{k} \right) \) to the term in brackets, we arrive at

\[
\eta_v = \left[ \frac{z_1-z_4}{Sk} + \frac{k-1}{k} \left( \frac{z_1-z_4}{S} \right) - \frac{k-1}{k} \int_{z_4}^{z_1} (1-\psi) \frac{dz}{S} \right] \]

\[
\eta_v = \frac{z_1-z_4}{S} - \frac{k-1}{k} \int_{z_4}^{z_1} (1-\psi) \frac{dz}{S} \tag{132}
\]

Comparing the formulas for ideal and calculated volumetric efficiencies, we see that the volumetric efficiency with inlet valve losses will be less than ideal due to two factors:

1) The value of \( \frac{z_1-z_4}{S} \) is less than that for the ideal because in general the cylinder pressure does not rise to the inlet pressure until after bottom dead center. Thus \( z_1 \) is always less than for the ideal case.

2) Due to the inlet valve loss there is an additional term

\[
= \frac{k-1}{k} \int_{z_1}^{Z_4} (1-\psi) \frac{dz}{S}.
\]

The physical explanation for this term is that the throttling associated with the intake is an irreversible process, resulting in an increase of entropy and specific volume of the air over what it is at the inlet condition. This term ordinarily accounts for only a small part of the loss in volumetric efficiency.

Plate 23 gives the results of the calculations for volumetric efficiency.
The volumetric efficiency has been analyzed only as regards the inlet process. However, the discharge process may also affect the volumetric efficiency for very low values of $B_d$. From the $\phi$ curves for $B_d = 1$ we can see that $\phi$ became unity somewhat after top dead center. This shifts the starting point for the expansion line slightly, increasing the value of $z_4$ and reducing the volumetric efficiency.

In practice, a compressor should never be operated in this range as the losses are extremely high for $B_d = 1$. If desired, however, the effects of discharge valves on the volumetric efficiency for very low values of $B_d$ can be calculated from the curves for $\phi$, but they have not been considered here.
CHAPTER III

Results of Theoretical Investigation

Part 1 Discharge Process

The curves of $\phi$ versus $\theta$ for the 108 cases analyzed are given on Plates 12 and 13. The most obvious feature of these curves is that the variation of "B" has far greater effects than either J or q. Also, for small values of B, variation of J and q has practically no effect. For $B = 1$, all the cases investigated lie very close to the curve of the "ideal" valve. However, for large values of B, variations in J and q have important effects. We see that a small J combined with a large B, (corresponding to a very stiff spring combined with large flow area), the valve goes through several cycles of opening and closing during discharge. This greatly increases valve losses over what they would be for the ideal valve.

It is also evident that the discharge valves, in general, have little or no effect on volumetric efficiency, since $\phi$ drops to unity either at, or very soon after, top dead center. Thus, the beginning of the expansion line (point (3), Figure (2)) is at top dead center for all cases except when $B_d$ is very small. In the region of practical interest (where $B_d > 3$), there is practically no effect on volumetric efficiency. For $B_d = 1$, the expansion line starts slightly after top dead center, thus shifting point (4) in Figure (2) to the right, reducing the volumetric efficiency.
When the valve losses are calculated as described in the preceding chapter, some noteworthy features become evident. The first to be discussed is the question of how valve losses vary with compressor speed, everything else being constant. Using elementary reasoning, one would analyze the problem as follows:

Doubling the speed means that the average velocity of flow through the valves would be double the initial value. (This assumes the same amount of air pumped per stroke in both cases, and neglects the change in volumetric efficiency with speed). Since the flow velocity is approximately proportional to the pressure difference causing flow, the average pressure difference (the difference between cylinder pressure and discharge receiver pressure) will be four times the initial value. Consequently, the power loss per stroke, which is the excess area on the P-V diagram, will be four times the original value. This crude line of reasoning leads one to believe that valve losses are proportional to the square of the speed.

However, the above line of reasoning is not strictly correct. First of all the volumetric efficiency decreases slightly with speed, reducing the actual loss area compared to what it would be for constant volumetric efficiency, but this effect also reduces the total diagram area in approximately the same proportion. Thus the ratio of loss area to that of the total area minus losses should be almost constant when varying the volumetric efficiency alone. We have taken care of this by plotting the ratio
of the loss to the area of the calculated P-V diagram minus losses. The most important error of the elementary reasoning is in assuming that each ordinate of the pressure difference curve for the doubled speed is four times that for the original speed. Actually, the shapes of the two curves are quite different. Referring to Figure (11), the peak of the pressure curve moves closer to top dead center as the speed is increased until at infinite speed it would be at top dead center. When these curves are plotted versus crank position, in a P-V diagram (see Figure 11), the valve losses are seen in their true proportions. Since the peak values of $\Phi$ for the high speeds are closer to top dead center, their contribution to the loss area is thereby diminished. The net result is that calculated valve losses are in general proportional to a power of the speed which is less than two.\(^{(1)}\) The Log-Log plots, Plates 40 and 41, show the value of this power for various values of "B", which is inversely proportional to speed. For very large B, or very low speed the curves for the ideal valves show a slope of two, giving losses proportional to the square of the speed. For values of B closer to the usual range, the slope goes down until at very low values it is unity or less. These results can be interpreted in the same way for variations in flow area. From the results, it seems that the most important consideration is to provide a large effective area.

\(^{(1)}\) This discussion is confined to the theoretical losses only. In Chapter V it will be shown that there are additional reasons why the actual valve losses depart from the trend suggested by the elementary theory.
FIGURE 11
TYPICAL $\phi$ CURVES WITH VARYING "$B$", AND $\theta_2=$ CONSTANT

$\phi-\theta$ DIAGRAM

$B=0$

VERY SMALL "$B$"

SMALL "$B$"

VERY LARGE "$B$"

$\phi=1.0$

$B=\infty$

$\theta_2$

$\theta=360^\circ$

$\phi-Z$ DIAGRAM

$B=0$

VERY SMALL "$B$"

SMALL "$B$"

LARGE "$B$"

VERY LARGE "$B$"

$\phi$

$B=\infty$

$Z_2$

$Z$

1.0

0
for flow. This makes "B" large and the valve losses small. In order to use a large "B" effectively, however, the valves must be very light and have very flexible springs. The optimum design would be the "ideal" valve which opens and closes instantaneously. To fulfill these requirements, one would have to have:

$$ J = \infty \quad , \quad q = 0 $$

To make $$q = 0$$, $$\omega_n = \infty$$ (for $$n$$ finite)

$$ J = \infty \quad , \quad K_1 = 0 \quad ("p_2 \text{ and } x_0 \text{ finite}) $$

Since $$\omega_n = \sqrt{\frac{K_1}{W_v}}$$, we cannot make $$K_1 = 0$$ and still get $$\omega_n = \infty$$. What we can do is make $$W_v = 0$$ and make $$K_1$$ very small, but not zero. Thus $$J \to \infty$$ and $$q \to 0$$ simultaneously. Therefore, the ideal valve would have no weight and a very small spring constant. The problem of designing efficient valves reduces to the attaining of the following objectives:

a) A very large area for flow, combined with a good flow coefficient.

b) Very little weight of valve combined with:

c) A small spring constant.

The net result of (b) and (c) should be such as to give a high natural frequency, compared with engine speed. The limitations on design preventing realization of the above objectives will be discussed in Chapter VI.

It is believed that the theory gives a fairly accurate picture of valve losses themselves, as will be shown in
Chapter V. However, during the closing of the valves, there is another phenomenon which may be of comparable importance to the valve losses in the performance of the compressor. Since, in general, the valve does not become closed until after the cylinder pressure has fallen to the discharge pressure, the valve will be open for a small period during which the cylinder pressure is dropping below the discharge pressure. Thus, there is a possibility of more or less "back flow" of discharged air into the cylinder. This will tend to reduce both thermal and volumetric efficiencies. Since this phenomena occurs shortly after top dead center, while the piston is moving very slowly on the first part of the expansion stroke, the cylinder pressure is not falling very rapidly. Therefore, if the valve closes within about 2° or 3° after top dead center there will be a negligible pressure difference and consequently little back flow.

This effect cannot be satisfactorily investigated by the theory presented, for practical reasons. Since the closing process was calculated by a point to point integration, the accuracy of the process is poorer the longer the calculation becomes, as errors are cumulative. Therefore, the point where the valve closes cannot be calculated with any degree of accuracy. To calculate the back flow, one would have to know the valve travel quite exactly in the vicinity of top dead center and after. As this was not the case, the effects of back flow could not be analyzed. It will be noticed that the inaccuracy of the point to point integration for the closing of the valve
does not lead to any significant errors in the valve losses, for the following reasons. The calculations for the early part of the closing process will naturally be quite accurate as the cumulative error will be small. This part of the process is somewhat removed from top dead center and the area contributed by it will be of some importance in the P-V diagram. On the other hand, the end of the calculation will be in the vicinity of top dead center, where its effect on the area of the P-V diagram is infinitesimal. Thus one concludes that the calculations for the closing process are fairly accurate as regards losses, but poor as regards valve travel.

Part 2

Inlet Process

The 81 cases analyzed are presented on Plate (14). As in the discharge process, the parameter "B" is much more significant than J or q. Here too, the values of J and q have more effect for high "B_1" than for low "B_1". It will be noticed that in general the curves for ψ do not reach unity until after bottom dead center. Thus, the start of compression is delayed considerably after bottom dead center. This amount increases radically with decreasing "B_1". Therefore, decreasing "B_1" reduces the volumetric efficiency. Within the range investigated, however, the variation of J and q have no appreciable effect on volumetric efficiency.

The variation of valve loss with speed and flow area is similar to that for the discharge process and for
the same reasons. Also, the requirements for the "ideal" valves are the same as before. It appears that inlet and discharge losses are of about the same order of magnitude. However, the inlet losses are more important, as they affect both the thermal and volumetric efficiencies, while the discharge valves influence only the thermal efficiency.

As in the discharge valves, there is a possibility of "back flow" with the inlet valves. In the latter case, some of the air which has already been brought into the cylinder, is forced out through the inlet valves on the compression stroke. This, of course, lowers the volumetric efficiency, and to some extent, the thermal efficiency. Here again, it was not considered feasible to investigate this effect theoretically, as the point to point integration becomes more and more inaccurate as it proceeds. Therefore the valve motion is not known with sufficient precision to make any calculation of back flow. It will be noticed that the equations would have to be modified if the back flow process were to be investigated. It is obvious that back flow for the discharge process is analogous to the intake process and back flow for the inlet valves is analogous to the discharge process. Thus, the general flow equation for the intake process (Equation 90) can be used for back flow in the discharge process, while (22) can be used for back flow in the inlet process. In these equations, of course, the appropriate values of the constants must be used; $B_d$ for discharge and $B_i$ for inlet.
The theoretical inlet losses are given on Plate (20-22) and the volumetric efficiency on Plate (23). The volumetric efficiency varies very little with the values of J and "q" assumed, and is therefore plotted only as a function of $B_1$. All cases calculated will lie within 1% of these average curves.
CHAPTER IV

The Experimental Program

Part I. Requirements for Dynamic Similarity Between Model and Prototype.

In order to make model tests which will give results that can be applied to the full size prototype, one must ensure that the two cases are dynamically similar. The requirements for similarity are found immediately from the differential equations (22), (32), (30), and (92). From these equations we see that if the following dimensionless parameters are the same for model and prototype, the valve dynamics and P-V diagrams will be the same for both, on a dimensionless basis:

1) B, J, and q for inlet valves.
2) B, J, and q for discharge valves.
3) Clearance ratio.
4) Ratio of crank length to connecting rod length.
5) Compression ratio.

Requirements (3) and (4) are necessary to get the same curves of z/s for both model and prototype. Requirement (5) actually comes from the boundary conditions of the problem rather than the differential equations, but it is an obvious requirement which should need no proof. It is, of course, understood that the valves and valve passages are geometrically similar for model and prototype. Also, the inlet and discharge systems (receivers, pipes, tanks, etc.) should be similar, but this is not always easy or convenient to do. From these considerations, both model and prototype should have the same thermal and volumetric efficiencies (disregarding heat transfer and leakage).
For convenience, it is usually more convenient to test full size valves in the model compressor rather than "scaled-down" valves. We can do this and still achieve dynamic similarity as follows. Since the value of "q", or \( \omega_n/N \) is to be made the same, and we have fixed \( \omega_n \) by using the full size valves, then the model must be tested at the same R. P. M. as the full size prototype. As the values of \( J \) must correspond, we make them the same by choosing the same values of \( p_1 \) and \( p_d \) for model and prototype. This will also give the same compression ratio for both. Naturally, the clearance ratio and crank-connecting rod ratio should be the same for both also. Finally, to make the values of "E" correspond, we have essentially to satisfy the relation:

\[
\left( \frac{A_o k_c C}{A_p S N} \right)_{\text{model}} = \left( \frac{A_o k_c C}{A_p S N} \right)_{\text{prototype}}
\]

This must be true for both inlet and discharge processes. We can make the velocity of "C" the same in both cases by using air at the same inlet temperature for both. Also, we have fixed \( N \) as shown above. Thus, we are left with the relation

\[
\left( \frac{A_o}{A_p S} \right)_{\text{model}} = \left( \frac{A_o}{A_p S} \right)_{\text{prototype}}
\]

Therefore, the ratio of valve flow area to cylinder volume (for both inlet and discharge) must be the same for model and prototype. This is done by choosing the proper number of inlet and discharge valves to satisfy the above relation.
Part 2. Test Apparatus

In order to investigate the problem experimentally, a model valve assembly was designed to fit the head of a standard C. F. R. single cylinder engine. The particulars of the design are given on Plate 1 and the test set-up on Plate 24. Due to difficulties of mechanical design, there was room for only two inlet valves and two discharge valves in the head. The valve design selected was that actually used in the Free Piston Gas Generator designed by the Lima-Hamilton Company. Since only an integral number of valves can be installed, it is ordinarily impossible to satisfy the requirements of Part (1) exactly. However, the intention of the tests was to get fundamental information on valve performance, rather than data applicable to one particular design. For this purpose, any reasonable ratio of valve area to piston area is suitable. The results are plotted on the basis of dimensionless parameters so that they can be used for other designs.

Since the actual machine has a very large number of valves (about 800), there is one valve slot for each reed. In our case there are two reeds, but three outlet slots (see Plate 1). It would be more correct to make the outer slots half the width of the center one, but this is of infinitesimal importance. The maximum constriction in flow is through the valve itself, so making the slot area larger should not affect the flow.

Plate (24) shows the layout of the test equipment. A standard C.F.R. engine with its dynamometer and other equipment was furnished through the courtesy of the Sloan Automotive Laboratory. The cylinder used was one of the old style, open head,
cast iron models.

The essentials of the arrangement are as follows. Air from the atmosphere is taken through the inlet pipe and passed through a metering orifice into the inlet quiething tank, which evens out suction pressure fluctuations. From here it goes to the inlet receiver, is compressed and passes out the discharge line into another tank. From the tank it is throttled down to atmospheric pressure by a hand operated valve. The compressor is driven by a variable speed D.C. motor which serves as a dynamometer. Torque is measured by the usual M. I. T. hydraulic scale. Steam and cooling water are supplied for controlling the temperatures of the jacket and lubricating oil. It was found impractical to design the special head to carry cooling water, so only the cylinder is jacketed. The orifice and inlet pressure differentials are measured by water manometers while the discharge pressure is determined by a Bourdon type gauge. Other measurements are described in Part 3.

The study of inlet and discharge phenomena is made more complicated by the presence of pressure waves in the inlet and discharge systems. The inlet pulsations were found to be rather small, but the discharge waves were especially troublesome. These waves originate when a pressure impulse is sent down the discharge pipe at the beginning of discharge. This wave is re- flected back and passes into the cylinder since the valve is still open. The wave may be reflected back and forth several times during the discharge process, thereby superimposing a more or less sinusoidal pressure variation on the cylinder pres-
sure curve. Since these pressure variations obscure the study of the discharge process, an attempt was made to eliminate them. The frequency of the pulsations corresponded approximately to the fundamental frequency of an open ended pipe of the length between the cylinder and the discharge tank. Varying the length of pipe gave frequencies corresponding to the length used, but did little to change the amplitude. It was decided that the pulsations might be partially damped by the arrangement shown in Plate (24). By adjusting the damping valve while watching the pressure gauge, one can reduce the pressure oscillations considerably. However, they cannot be completely eliminated. Plates (25), (26) and (27) shows indicator cards with the inlet and discharge receiver pressures superimposed.

Tests were run at various discharge pressures and speeds, with different clearance volumes. The C. F. R. engine is conveniently provided with a hand crank which moves the cylinder up and down to give a wide range of clearance volumes. A few tests were run with only one discharge reed, giving half the usual discharge area. One of the discharge slots was blocked off by driving an aluminum block into the slot. The results of the tests with this blocking strip are probably not very reliable, as there may have been some leakage by the strip. Since the effect of clearance volume was not investigated in the theoretical work, three different clearance volumes were used in the tests to see if there would be any discernible trends in valve losses due to this factor.
The limitations on the range of the experiments were set by the mechanical construction of the compressor. This engine was provided with a cast iron flywheel which was unsafe above 2000 R. P. M. The field rheostat for the motor gave a minimum speed of 800 R. P. M. The pressure ratio was limited due to structural weaknesses of the cast iron cylinder, restricting the allowable tension in the head bolts. This caused leakage and gasket failures at high pressures, so a maximum discharge pressure of 80 pounds per square inch, gage, was decided on.

The arrangement described has the disadvantage that the inlet pressure will be slightly different for every test, depending on the pressure drop through the orifice and inlet system. This is not a serious drawback compared with the alternative of measuring the discharged air after it has been compressed. The latter method requires an absolutely tight discharge system, so that leaks would not lead to errors. It is much easier to make the inlet line air tight since the pressure within it is only slightly below atmospheric, so that leakage into it is negligible. It would have been possible to use the supercharging air line to bring the inlet pressure up to atmospheric, but this would require manual adjustment of inlet pressure, discharge pressure, and speed, simultaneously. The slight advantages to be gained by such an arrangement are too small to justify the additional work.
The important results of the tests are listed in Tables (4-11). Two of the tests showed very poor volumetric efficiencies, evidently due to gasket leaks. These tests were omitted from the tabulations.

Part 3. Instrumentation

The various measuring instruments are shown on the arrangement of plate (33). All the essential data for finding air flow and power were taken. Indicator diagrams were taken with the M. I. T. Balanced Diaphragm Indicator, Reference (12), using a "free diaphragm" pickup. This type of unit is susceptible to errors when oil is present, so considerable care must be taken in its use. There is always some oil in the cylinders of reciprocating compressors, and this oil finds its way into the unit. If the unit is frequently removed for cleaning, the oil troubles will be minimized. Typical indicator cards are shown on Plates 25-31.

Brake readings were taken, but not used. Theoretically, the indicated horsepower should be the difference between the brake horsepower and the friction horsepower. The difficulty arises in obtaining a true representation of the friction horsepower. A number of tests were made with the head off and with a false head having no valves, in order to arrive at the friction torque at various speeds. It was concluded that the use of these friction torques combined with brake readings during the tests would be of insufficient accuracy to give more than a rough check on the indicated horsepower. Thus, the whole
analysis of valve losses herein is obtained from indicator cards alone. This is the most satisfactory method, providing considerable care is taken in obtaining the indicator cards.

To study the valve dynamics experimentally, an apparatus was devised to record the valve motion while in operation. This apparatus is shown in Figure 12, and operates as follows. A very light stylus is attached to the center of the reed by a pivoted joint. The hole in the reed (about .025" diameter) was cut by grinding in a drill press using a small brass rod and grinding compounds. (The stainless steel of which the reed is made was too hard to drill in the usual way.) The stylus scribes a line on a revolving drum in the receiver, giving a curve of valve motion versus crank angle. There are two drums, one in the inlet receiver and the other in the discharge receiver. They rotate at 1/5 crankshaft speed and are directly coupled to the crankshaft by gearing. The stylus is held away from the drum until the record is taken, by means of a holder at the end of a push rod. The push rods are operated by a specially cut helical cam in the form of a worm. When a release pin is withdrawn, a follower is pulled toward the helical cam and enters the groove. When the follower reaches the part of the cam where the cam diameter is enlarged, it is pushed back, thereby operating the pushrods through a lever. Each stylus is pressed lightly against its drum by means of a fine hair spring which is adjusted to give the lightest possible pressure that will make a scratch. The
ARRANGEMENT OF VALVE LIFT RECORDING DEVICE

WORM & WHEEL
(RATIO OF 5 TO 1)

ENLARGED SECTION
OF WORM

BEVEL GEARS

RECEIVE PIN

STUFFING BOX

PIVOT

STYLiUS

SPRING

REED

STUFFING BOX

PUSH ROD

CRANK SHAFT

WORM

FOllOWER

DETALLE
OF CAM

DRUM SHAFT
(\frac{1}{5} CRANKSHAFT SPEED)

RECORDING DRUM (ONLY ONE SHOWN)
IN INLET OR DISCHARGE RECEIIVER

DETALLE OF STYLiUS

STYLiUS

HAIR SPRING

PUSH ROD

HOLDEr

PIVOT

SCEPTER

(REVIETED THROUGH
HOLE IN REED)
stylus is held against the drum for about 3/5 of one revolution of the drum thus getting records of about three cycles of valve operation. After this point the follower leaves the cam and a spring returns the push rod to its original position.

The record is actually scribed on a strip of shim brass wrapped around the drum. The cam is so positioned that the stylus does not begin to scribe until the joint in the record strip has moved past the stylus. The surface of the brass strip is first lead plated and then oxidized black with a solution of "Liver of Sulphur." The stylus thus scratches through the oxide into the lead, leaving a shiny groove. After the tests, the record strips are photographed and greatly enlarged, since the actual records are too small to measure except by a microscope.

In order to correlate the motion of the valve lift with the indicator cards, a reference mark must be made on each record strip at some definite crank angle. This is accomplished as follows. Before making a run, the engine is rotated slowly by hand in the direction it turns when running. (This takes up the backlash in the gears.) The cam is engaged and the engine rotated until the push rods move in. Then the shaft is stopped at top dead center, and air blown through the compressor from the supercharging line. This opens the valves making small vertical lines on the record strips. The engine is then rotated to bottom dead center and the process repeated.
It is believed that the dynamics of the valve itself are not materially altered by the presence of the pickup. It is true that a certain amount of force is required to move the stylus and make a scratch, but the forces involved are so slight that there should not be much error. As the stylus is very light, it adds only a small percentage to the weight of the valve.

It turned out that the pickups were so delicate that their endurance limit was short, especially at high speeds. The usual failure was by the sockets pulling out of the reeds but sometimes the stylus would break at the pivot. There were no cases of reed breakage, either in the plain reeds or those drilled for the pickups. Since the pickup life was so short, the tests for valve lift diagrams were made in as short a time as possible, consistent with steady conditions of flow. These tests were separate from those for getting thermodynamic data. The latter tests were run with the plain reeds.

Typical curves of valve lift are superimposed on the indicator diagrams, Plates (25) to (31), while Figure (13) gives photographs of some actual record strips.

Part 4. Static Tests

To get experimental values of the discharge coefficient \( K_c \) and drag coefficient \( C_d \), the valve assembly was tested statically. Since the inlet and discharge valves were identical, only the inlet valves were tested. This was accomplished by removing the head bolts and blocking the head up above the cylinder so that the inlet valves would discharge air from the
FIGURE 13
SAMPLE VALVE LIFT RECORD STRIPS
(ENLARGED ABOUT 4 DIAMETERS)

(A) DISCH. PRESS. 20 LBS/SQ.IN.
RPM 900 RPM

(B) DISCH. PRESS. 60 LBS/SQ.IN.
900 RPM

(C) DISCH. PRESS. 80 LBS/SQ.IN.
900 RPM
inlet receiver into the atmosphere. Then air was admitted to the inlet system from the supercharging line. The pressure in the inlet receiver was varied by means of manually operated reducing valves in the supercharging line. Readings were taken of the inlet receiver pressure and temperature as well as the usual temperature and pressures for the measuring orifice. These quantities and the barometric pressure are all that is required for finding the coefficients for the valves, if the characteristics of the metering orifice are known. From the orifice data the actual air flow can be calculated. With the inlet receiver pressure and temperature, and the atmospheric pressure, the theoretical flow through the valves is computed based on the nominal area for flow with valves fully open. The ratio of these quantities gives a discharge coefficient for each pressure difference across the valves. This value increases as pressure difference increases since the valve lift is proportional to the pressure difference.

Plate (42) shows the results of the test. Point "A" represents the condition of "free lift" without bending. At point "B" the valve is fully open, after which the increase of discharge coefficient is slight. The line "A-B" is approximately straight, substantiating the assumptions (4-d) and (4-f).

It will be noticed that the discharge coefficient continues to increase after the valves are fully open. Thus assumption (6) of Chapter II is not strictly correct. Apparently, the valve passages correspond roughly to a converging-diverging
nozzle. As such, their discharge coefficient would increase with increasing Reynold's Number, and consequently, with the pressure difference.

The value of the "drag coefficient" $C_D$, is also found from the discharge coefficient curve. Equation (32) gives the starting point. As the test is static, the acceleration term drops out and we have

$$J_T (\varphi_T - 1) = \alpha_T = 1$$

or

$$J_T = \frac{1}{\varphi_T - 1}$$

where the subscript "T" refers to the values for the static test. This is for the condition where the valves first become fully open. The value of $\varphi_T$ is simply:

$$\varphi_T = \frac{P_o + \Delta p}{P_o}$$

where $P_o$ = Atmospheric pressure

$\Delta p$ = Pressure difference to open valves

(Found from Plate 42.).

The drag coefficient can then be found by simple substitution in the formula for $J$, which in our case is:

$$J_T = C_D A_v (P_o + \Delta p)$$

$$\frac{W v a h \pi x_o}{G}$$

since the actual valve construction permits some free lift, the above analysis actually assumes an approximate curve of lift versus $\Delta p$ as shown in Figure (14).

(1) In cases where a static test is not obtainable and a preliminary estimate of valve performance is desired, one can take a value of $C_D$ in the neighborhood of 1.3.
This assumption should make little difference where the free lift is only a small part of the total lift, but when the free lift is large, the approximation would not be very good. The results of the static tests for the reed valves give the following formulas for B and J.

\[ J = 1.16 \, p \]
\[ B = 1.69 \, \frac{T}{N} \]

where \( p = \) absolute pressure (inlet or discharge) lbs/in.\(^2\)

\( T = \) absolute temperature, °F, (inlet or discharge conditions)

\( N = \) R. P. M.

Part 5. **Results of Tests**

The results will be discussed more fully in the next chapter, in conjunction with the theoretical results. The main difficulty in the experimental method seems to be the attainment of exactly similar conditions among various tests. Even with carefully controlled conditions, volumetric and thermal
efficiencies may vary several percent among supposedly identical tests. Some of the discrepancies are probably due to leakage since the variations are usually larger at the higher pressures. Some of the other sources of error may be: poor performance of the indicator, (due to oil on the diaphragm, etc.), leaks in manometer lines, accumulations of carbon on the discharge valves and errors in measurements. The actual performance of the test compressor was materially affected by heat transfer from the hot discharge receiver to the cold inlet receiver. Since the temperature difference between the two may be almost 400°F, there is bound to be a lot of heat transfer. The inlet temperature was measured about 2" above the reeds, and was found to be from 30° to 60° higher than room temperature. In considering the overall performance of the machine, one would base the efficiencies on the room temperature, but since we are mainly interested in valve losses, the effect of other losses such as heat transfer should be eliminated as far as possible. Thus, the actual measured inlet temperature was used for calculating thermal and volumetric efficiencies. The calculations were done according to the A.S.M.E. Standards (Reference (13), with the exception of the choice of inlet temperature just mentioned. It is probable that considerably more heat transfer takes place after the inlet air passes the thermometer. The air velocity is quite high through the valves and valve passages, so convective heat transfer at these points would probably be large.
It was thought desirable to get a check on the dynamic flow through the valves, by analyzing the indicator cards in conjunction with the valve lift curves. Plates 25, 26, and 27 give the necessary information for calculating the flow, in conjunction with the test measurements of temperatures, air flow, etc. The flow is computed by equation (10) for discharge and equation 80 for the inlet process. Plotting the curve of mass flow versus time and getting the area under it gives us the theoretical flow in or out per stroke. Comparing this with the actual air flow from the test measurements gives a "dynamic" discharge coefficient for the valves. The average value calculated for the three tests (giving six values), is 1.575 which is fairly close to the static values of Plate (42). This is in agreement with the findings of Reference (9), where it is shown that the dynamic flow coefficient for poppet valves is practically the same as the static value. These calculations are none too accurate, considering the small pressure differences that must be measured on the indicator cards. However, the average of the calculations should give a fairly good representation of the dynamic flow coefficient.
CHAPTER V

CORRELATION BETWEEN THEORY AND EXPERIMENTS

Part 1. Valve Losses

The losses are compared on the basis of "loss ratios". The inlet or discharge loss ratio is the loss area on the p-v diagram divided by the area of the total diagram minus losses. For the theoretical curves this would be given by:

\[
\frac{\text{Loss area}}{\text{Ideal Cycle Area}} \times \frac{\text{Ideal Volumetric Efficiency}}{\text{Calculated "}}
\]

Since the discharge process has little effect on the volumetric efficiency, the discharge ratio is plotted as the loss area divided by ideal cycle area. These are shown on Plates 16-19. In computing the valve losses one must find the calculated volumetric efficiency (depending on the inlet valve performance) and multiply the plotted values by the ratio of ideal \(\frac{\text{calculated volumetric efficiencies}}{\text{as shown above}}\). As the inlet valves are actually responsible for the lowering of the volumetric efficiency, we can compute the volumetric efficiency for each case of the inlet valves directly. The loss ratio for the inlet valves has already been multiplied by the ratio of volumetric efficiencies in the plotted curves of Plates (20)-(22).

The loss ratios are compared on Plates (32)-(39). Since the inlet conditions varied between tests, the pressure ratios were not the same for corresponding discharge pressures. Therefore, the theoretical curves are drawn for the average pressure ratio for each of the four discharge pressures. As the variation in loss ratio with "q" are small, only the extreme values for the
range of the tests are plotted for the discharge losses. In the inlet valves the variation due to \( q \) was negligible and only the average curve was drawn. Each of the theoretical curves is therefore the predicted loss ratio based on the value of \( J \) for the average pressure ratio of the tests and is plotted against the parameter \( B \). The correlation is quite good considering the number of simplifying assumptions made in the theory. The points for the tests with half the valve area are somewhat low compared with the others, but these tests are not very trustworthy as the blocking strip in the slot may have leaked.

The theoretical calculations were all done for a clearance volume of 10 1/2% of the piston displacement. The test points for three different clearances shows that the clearance ratio is not a very significant parameter for valve losses. Within the accuracy of the experiments, there is no observable variation of valve loss with clearance ratio. Therefore the theory should give a fairly close representation of valve losses for any clearance ratio of from 5% to 10%, which is in the region of present design practice.

It was not considered worthwhile to attempt to correlate the curves of \( \phi \) and \( \psi \) with the actual indicator cards. The theoretical curves were made for integral values of \( B \), \( J \), \( q \) and \( \sigma \), while these quantities in the tests were not integral values. The only way to make a comparison is through a complicated interpolation process among four variables. Besides, the presence of waves in the receivers superimposes pressure variations on the indicator cards, having nothing to do with
the valve operation. However, it is interesting to compare the general shapes of the indicator cards with the theoretical curves on Plates 12 to 14. The experimental curves have the same characteristics as the theoretical, particularly in the inlet process, where pressure waves do not obscure the results.

It is difficult to compare the theoretical valve motion with the actual, except in a general way. The only part of the process amenable to checking is the crank angle during which the valves lift. This is a fairly small quantity and difficult to measure with precision on the valve lift diagrams. For example at 900 R.P.M., the crank angle during which the valves are opening is calculated at 2.5° for 80 pounds per square inch discharge pressure and 4.2° for 20 pounds discharge pressure. The actual valve lift records when enlarged show a width of line of about 2° of crank angle and cannot be measured accurately. However, the lift diagrams show that the actual time of lift is very short and of about the value calculated by theory.

A more significant type of comparison is given on Plate 43, where predicted cycle efficiencies calculated by theory are plotted together with the ratio of: actual p-v diagram area minus losses divided by the actual p-v diagram area. The latter ratio should be the actual cycle efficiency if the valve losses were the only ones. This plot shows that the valve losses themselves can be predicted with very good accuracy (within 1 or 2% of cycle efficiency). This theoretical calculation is based only on the results of static tests of the valve and on the average measured inlet temperature for each
operating condition. From these quantities and the known
valve and engine dimensions, the values of B, J, and q
for every condition are calculated and the valve losses
found from Plates (16-22).

However, when the theoretical cycle efficiency is
compared to the actual cycle efficiency as in Plate (44),
the agreement is not as good, especially at the higher speeds.
Apparently the other losses associated with compressor opera-
tion are greater at high speeds and negligible at the lower
speeds.

In Chapter III there was a discussion of the varia-
tion of theoretical valve losses with flow area and speed.
It showed that theoretical valve losses were not proportional
to the square of speed, except at infinitely slow speeds. A
similar result holds true for flow area. The losses are not
proportional to the inverse square of the flow area except
when the area becomes very large. There are practical rea-
sons why the actual losses cannot be indefinitely reduced
in line with this "inverse square" theory.

The indicator cards with superimposed receiver pres-
sures (Plates 25, 26, 27) give a graphical illustration of this
fact. During discharge, the pressure builds up in the discharge
receiver, so that this space has a higher average pressure dur-
ing discharge than the mean discharge pressure. Similarly,
the inlet receiver pressure drops below its mean value during
inlet. Thus the pressure drop causing flow is actually greater
than it would be if the receiver pressures were constant.
Therefore, the actual valve losses should be greater than the
theoretical values based on constant receiver pressures. This effect is naturally greater at higher speeds (as the pressure has less time to become equalized by flow in or out of the quieting tanks), leading to greater discrepancies between theoretical and actual losses at the higher speed. This is well shown by the comparison of actual and predicted curves of Plate (44).

Increasing the flow area will not reduce this effect and would in fact increase it. Therefore, the pressure drop causing valve losses will always have to be greater than the pressure difference caused by this temporary pressure variation in the receiver during discharge. As valve area is continually increased, the losses would probably approach a constant value, beyond which they could no longer decrease. This amount depends on the volume of receiver space and size of the discharge and inlet pipes.

Therefore, to reduce valve losses, one must not only provide large flow area but also large receiver capacity. 

Part 2. Volumetric Efficiency.

The theoretical volumetric efficiency is compared to the actual in Plates (36-39). Since the tests were run at various clearance ratios, the volumetric efficiency would be different for each clearance. Therefore the plotted quantity is the ratio of actual to ideal volumetric efficiency.

The experimental values are generally lower than the theoretical. Evidently, the effects of leakage and heat transfer amount to from 1% to 5% of the loss. The scatter in the experimental points shows that there is considerable
variation among similar tests. This may be due to inaccuracies in measurement or to variable amounts of leakage. Since the effects seem to be worse at high pressure ratios, leakage is probably the main factor. This includes not only leakage back through the valves when closed, but also that past the sealing gaskets in the cylinder head and valve assemblies. Possibly a better way of testing the theory would be to build a head completely closed except for valve ports. This could not be done with reed valves but might be accomplished by using disc or plate type valves. Another source of variation in volumetric efficiency is the presence of wave phenomena in the discharge and inlet lines. From the plotted indicator cards with superimposed receiver pressures (Plates 25-26-27), we see that the pressure at the end of inlet or discharge may be higher or lower than the mean inlet or discharge pressure, depending on the pressure waves in the system. In Plate (26), for example, the cylinder pressure is about 3 pounds per square inch higher than the mean discharge pressure at top dead center. Thus the expansion line for this test is shifted slightly to the right compared with what it would be if the pressure at top dead center were the mean discharge pressure. Therefore, the volumetric efficiency is slightly lower than it should be. Similar conditions prevail in the inlet system. The result of this effect is that curves of volumetric efficiency versus speed will be "bumpy" due to wave phenomena occurring in inlet and discharge systems.
Plate (45) shows predicted curves of volumetric efficiency compared with the experimental values. The curves have the same trends and shapes, but the test points are everywhere lower than the theoretical due to the factors discussed previously.

Therefore, the theory will give somewhat optimistic values of volumetric efficiency. Then the prediction of actual performance can be based on the theory with an additional deduction estimated from empirical values. This would require statistical analyses of a wide range of tests on various compressors with different types of valves, intake systems, etc., to give reliable empirical factors.
CHAPTER VI
GENERAL DISCUSSION OF VALVE PERFORMANCE

Part I. Possible Improvements in Valve Design.

It was shown in Chapters II and III that the most important factor in valve design is to provide a maximum of effective area for flow. This can be done in various ways.

1) Installing as many valves as possible
2) Increasing the valve lift.
3) Improving the flow coefficient by variation in design of flow passages.
4) Design of radically different types of valves which will give larger flow areas.

To utilize a large flow area, however, one must approach the characteristics of the ideal valve, which has no weight and a very small spring force. Obviously, if we provide a large flow area, but use very stiff springs, the valves may never open all the way, so that the valve area is not completely utilized. Similarly, if the valve is very heavy, it will open and close very sluggishly, causing high losses.

Reference (1) discusses merits of many commercial valves, including various types of plate, channel and disc valves. It would appear that the "reed" valve is closer to the "ideal" valve than any other type in use, since it is so light. Other types are bound to be heavier and require stiff springs to get fast response in closing. The possibilities of improvement in the reed valve itself can be
studied by consideration of the ideal valve. By varying
the proportions of the reed we can change its natural fre-
quency and weight. For the reed valve:

\[ J = \frac{C_D A_y P_2}{W_V} = C_D \frac{b l p_2}{g m^2 x_0} \frac{1}{n^2 x_0} \tag{133} \]

For a given \( C_D \), \( P_2 \) and material ("E" fixed), \( J \) varies as
follows:

\[ J \propto \frac{b l^{1/4}}{I x_0} \tag{34} \]

Substituting for \( I \), \( I = \frac{t_i^3}{l^2} \),

\[ J \propto \frac{l^{1/4}}{t_i^3 x_0} \tag{135} \]

where \( b = \) width of reed
\( t_i = \) thickness of reed
\( l = \) length of reed

In the same way, \( q \propto \frac{1/4}{t_i^2} \), for a fixed "\( \Omega \)". By
reducing the thickness "\( t_i \)" of the reed, as \( t_i \rightarrow 0 \)

\[ J \rightarrow \infty \]

\[ q \rightarrow \infty \]

However, for the ideal valve \( J = \infty \), \( q = 0 \).

Therefore, by reducing the valve thickness indefinitely,
one would not approach the ideal valve. However, there must be
an optimum thickness, since making the valve thicker and thicker
would also not approach the conditions of the ideal valve. The
optimum thickness for any operating condition can be found
from the curves of theoretical valve losses. Some typical calculations of this type were done and the results shown in Plate 46. The calculations are for a reed 2.98" x .212", operating at 1200 r.p.m. The curves are drawn for pressure ratios of 3.85, 5.2, and 6.6, and for three different values of \( B_d \), corresponding to various flow areas. For each curve, the loss ratio for the ideal valve is given for comparison.

Plate (46) shows that for a given reed length and fixed operating conditions, there is an optimum reed thickness. However, the variation in total cycle efficiency are slight over the range studied. It is interesting to note that the optimum thickness is about .025" for all the cases, although the optimum thickness increases slightly with pressure ratio. We conclude from this study that the present design of reed is dynamically close to the ideal valve and gives cycle efficiencies only one or two percent lower than the ideal.

**Part 2. Stresses**

As a limitation on design, the stresses should receive due consideration. Regardless of the details of the design, the stresses are of two types. The first is the static stress in the spring, which is a maximum with the valve fully opened, and can be calculated from the standard formulas. The maximum stress in the spring when operating would probably not exceed this value as long as the compressor frequency is much lower than the natural frequency of the engine \( \omega_1 \). The second type of stress is due to impact of the valve against the seat, or against the stops. The importance of the impact
stresses was well illustrated by tests with various thicknesses of reeds performed by the Lima Hamilton Company. It was found that reducing the valve thickness resulted in more frequent reed failures. Since reducing the valve thickness (for the same lift and stop plate curvature), results in a lower static bending stress, (because the outer fibers are stretched less), one would expect fewer failures with thinner reeds. The fact that the opposite is true leads one to the conclusion that impact stresses play an important part in valve endurance. The effects of impact stresses can be analyzed as follows:

The classic theory of impact, (Reference (5), page 389) tells us that the initial impact stress in a body striking an infinite mass is given by:

\[ \sigma_o = V_o \sqrt{\frac{E \rho}{g}} \]  \hspace{1cm} (137)

Where \( V_o \) = velocity of impact ft/sec.

\( E \) = Young's Modulus, lb/ft\(^2\)

\( \rho \) = weight density, lb/ft\(^3\)

\( g \) = 32.2 ft/sec\(^2\)

\( \sigma_o \) = impact stress, lb/ft\(^2\)

This approximates the conditions of our problem. However, since the valve does not ordinarily strike the stops uniformly over its whole area (as assumed in the theory), the exact stress is not given by the formula. But, for similar valves we may
write:

\[ \sigma_o \propto V_o \sqrt{EP \over g} \]  \hspace{1cm} (138)

Thus, \( \sigma_o \) as calculated by (138) above will be a nominal stress giving an index of the severity of impact for similar valve systems. The only unknown in the formula is the striking velocity \( V_o \). It appears from the theoretical investigation that the most severe impact is when the valve strikes the stops on opening. The closing of the valve is much slower. This will be the only case described, and will be worked out for the discharge valves. The inlet valve analysis would be practically identical.

It was shown in Chapter II, that the curve of valve lift during opening is approximately a cubic parabola. With the notation of Chapter II, let \( \theta \) = crank angle during which valve opens. To get the value of \( \theta \) we put \( \alpha = 1 \) in equation 115.

\[ H(\Delta \theta) = \frac{1}{qJ(\theta)} \]  \hspace{1cm} (139)

The function \( "H(\Delta \theta)" \) is found to be approximately given by:

\[ H(\Delta \theta) \approx \frac{1}{6} (\Delta \theta)^3 \]  \hspace{1cm} (140)

Putting this in (139), we have, solving for \( \Delta \theta \),

\[ \Delta \theta = 3^{1/2} \frac{6q^2}{J(\theta)} \]  \hspace{1cm} (141)

Now to find the striking velocity refer to Figure (15).
The equation for \( \alpha \), assuming a cubic parabola is given by:

\[
\alpha = \left(\frac{\theta - \theta_a}{\Delta \theta}\right)^3 \quad (142)
\]

The slope at point \( b \), \( \left(\frac{d\alpha}{d\theta}\right)_b \) is:

\[
\left(\frac{d\alpha}{d\theta}\right)_b = 3 \left(\frac{\theta_b - \theta_a}{\Delta \theta}\right)^2 \left(\frac{\Delta \theta}{\Delta \theta}\right)^2 = \frac{3 (\Delta \theta)^2}{\Delta \theta} \quad (143)
\]

The striking velocity is \( V_o = \left(\frac{dx}{dt}\right)_b = \theta_b \quad (144) \)

But:

\[
\frac{dx}{dt} = x_o \cdot \frac{d\alpha}{d\theta} \quad (144)
\]

\[
\therefore V_o = x_o \cdot \frac{d\alpha}{d\theta}_b = 3x_o \cdot \frac{\Delta \theta}{\Delta \theta} = \frac{3x_o \cdot \Delta \theta}{3} = \frac{3x_o \cdot \Omega}{\sqrt[3]{6q^2 \cdot \int_0^a \left(\frac{d\phi}{d\theta}\right)_a}} \quad (145)
\]

Or:

\[
V_o = 3x_o \cdot \Omega \sqrt[3]{\frac{\int_0^a \left(\frac{d\phi}{d\theta}\right)_a}{6q^2}} \quad (146)
\]
From this equation we see at once the impossibility of attaining the characteristics of the ideal valve. Since for the ideal valve we have $J \to \infty$, $q \to 0$, both of which effects tend to make the impact velocity infinite, the impact stress will be infinite. We can make the results more useful by substituting for $J$ and $q$ in terms of the design conditions. The result is:

$$
\sigma_0 = \frac{3}{\sqrt{6}} \sqrt{\frac{E}{g}} \sqrt{\frac{\sqrt{D} \nu \rho x_0^2 \frac{d\phi}{d\theta}}{\frac{W_V}{g}}}
$$

(147)

Thus we see that the stress is increased by the following

1) Increasing the face area.
2) " " discharge pressure.
3) " " valve lift.
4) " " compressor R.P.M.
5) Reducing the valve weight.

Note: The value of $\frac{d\phi}{d\theta}$ increases with pressure ratio, increasing the stress. It therefore appears that increasing the valve lift and reducing the weight, both of which tend to improve the efficiency, increase the impact stress. Making the lift larger increases "B" and decreases "J". The former improves efficiency while the latter reduces it, but the effect of "B" is more pronounced. Therefore, increasing the lift will always increase the efficiency until a point is reached where the effective flow area equals the port area. For a poppet
type valve, this amount of lift is about 25% of the valve diameter. To give good efficiencies with this type one would require a very light valve body and a large lift. This would result in enormous impact stresses, as has been shown recently by the disc valve failures in the German Free Piston Compressors developed in the late war. A broken valve is a very serious matter and reduces the efficiency of the compressor to a great extent.

Part 3 Suggestions for Future Investigations

From the results of the preceding discussion, it appears that the reed valve is practically ideal with respect to its dynamics. It is poor from the standpoint of discharge coefficient, however. Even with this fault, this type can be arranged to give considerable flow area, since the slots can be placed very close together. With circular valves one would have to make the diameter very small to get a comparable area and satisfactory dynamic qualities. There are various valve arrangements in use which permit large flow areas, but they all have one or more of the following defects.

a) The design is complicated and expensive.

b) The impact stresses are large, due to the large lift used, causing frequent breakage.

c) If the valve is made heavy enough for satisfactory stresses, the response of the valve is too slow.

The theory shows that valve losses are bound to be high for large piston speeds. There does not seem to be any hope of designing satisfactory spring loaded valves for extremely high speeds. For high speeds one would have to decrease
the lift or increase the valve weight to keep stresses down, both of which effects cause greater valve losses.

One possible expedient which might be investigated is to use mechanically operated poppet valves with a large lift. The problem here would be mainly one of machine design. It might be worthwhile to mention here that the "isoclinic" method of Chapter II is ideally suited to the study of processes of flow through the mechanically operated type valves. Since the valve motion is known in advance, one knows the curve of $\alpha$ versus crank angle and the flow equation is of the form $\frac{d\phi}{d\theta} = f(\phi, \theta)$, which is easily solved by the isoclinic method(1). Of course, mechanically operated valves can be designed for only one pressure ratio, unless one provides interchangeable cams for varying the points of valve opening and closing. The discharge valve gear would be particularly difficult to design because of the very small crank angle for discharge at high pressure ratios. This would require very high acceleration and deceleration of the valve.

Another aspect of the problem worth investigating is the possibility of reducing impact stresses by providing air cushions or other damping behind the valve. Such devices are employed now (see Reference (1)) but little is known of their effects, except for manufacturers claims of "noiseless operation".

(1) The isoclinic method can also be used to study exhaust and suction processes in the internal combustion engine.
The fact that the actual volumetric efficiency is a few percent lower than the theoretical seems to be due to effects that are independent of valve dynamics. A thorough study of these effects, including heat transfer during the inlet process, and leakage, would be a worthwhile project. Elimination of these troubles is almost as important as improving the valve performance.
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### Table 3: Typical Calculation for Closing of Valve Discharge Process

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**Explanation of Columns**

1. Value of H from Plate (4) for angle of column (2)
2. (ΔH) = Difference of successive values of H in column (3)
3. Initial value of Δq/Δθ for each step times .00088
4. To (6) column 4 times value of Δq/Δθ for each step
5. Algebraic summation of values in each "box" denoted by Σ₁, Σ₂, etc.
6. 8 = Σ₁ + Σ₂ = 8 x (7)
7. Φ = 1 + (8)
8. G₁ = η(Δθ) = Final value of θ for each step
9. Assumed final value of Δq/Δθ
10. G₂ = Final value from Plate (2)

11. (dθ)/dφ = dθ/2π = dθ/2π (Mean value of Δθ/Δφ over interval)
12. G₃ = 25 x .035
13. Final value of G for step G₄ = G₁ + G₃

Note: G₄ for step 1 is G₁, G₂, G₃, G₄. G₄, G₅, G₆, G₇.
### Table: Sing of Valve

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<th>$x_n$</th>
<th>$\theta_n$</th>
<th>$N_n$</th>
<th>$N_n'$</th>
<th>$d\theta / d\theta_m$</th>
<th>$\Delta \phi$</th>
<th>$\phi_n$</th>
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### Diagram

- $\theta = \theta_n$
- $\phi = \phi_n$
- $\Delta \theta = 2^\circ$
- $\Delta \phi$ for step 1 = $\phi_0$
- $\phi_1$ = $\phi_0$
- $\phi_2$ = $\phi_1$ + $\phi_1$
- $\phi_3$ = $\phi_2$ + $\phi_2$
- $\phi_4$ = $\phi_3$ + $\phi_3$
- $d\theta / d\theta_m$ over interval

---

**Note:** $\phi$ for step 1 = $\phi_0$, $\phi_1 = \phi_0$, $\phi_2 = \phi_0 + \phi_0$, $\phi_3 = \phi_0 + \phi_0 + \phi_0$, $\phi_4 = \phi_0 + \phi_0 + \phi_0 + \phi_0$. $\Delta \phi$ over interval.
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**TABLE 7**

**TEST 9 (FULL VALVE AREA)**

CLEARANCE VOL. 2.80 Cu. In.

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<td>900</td>
<td>1200</td>
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<td>abs.</td>
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<td>34.78</td>
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<td>583</td>
<td>591</td>
<td>591</td>
<td>596</td>
<td>596</td>
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<td>.0144</td>
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<td>.930</td>
<td>.879</td>
<td>.875</td>
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<td>.826</td>
<td>.785</td>
<td>.781</td>
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<td>.929</td>
<td>.920</td>
<td>.912</td>
<td>.904</td>
<td>.904</td>
<td>.880</td>
<td>.880</td>
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<td>3.320</td>
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<td>1.602</td>
<td>2.100</td>
<td>2.560</td>
<td>2.560</td>
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<td>.809</td>
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<td>.106</td>
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### TABLE 9

**TEST 11 (ONE DISCHARGE REED)**

CLEARANCE VOLUME 2.80
" RATIO .075

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<tr>
<td>RPM</td>
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<td>1800</td>
<td>1500</td>
<td>1800</td>
<td>1500</td>
<td>1800</td>
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<td>34.70</td>
<td>34.70</td>
<td>54.70</td>
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<td>74.70</td>
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<td>Inlet Temp, °F Abs.</td>
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<td>--</td>
<td>592</td>
<td>597</td>
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<td>.934</td>
<td>.881</td>
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<td>--</td>
<td>.834</td>
<td>.789</td>
<td>.788</td>
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**NOTE:** Runs 11-4 and 11-5 omitted.
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<td>595</td>
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<td>Inlet Loss Ratio</td>
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<td>.128</td>
<td>.074</td>
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PLATE 1 GENERAL DATA FOR TESTS

VALVE DIMENSIONS (INLET AND DISCHARGE IDENTICAL)

1" FLAT

16" R .050" CENTER LIFT

0.3" END LIFT

3/6" 0.22" 2.25"

FULL SIZE

REED

2.98" x .216" x .025"

VALVE AREA (FULLY OPEN) .413 SQ. IN. .413 SQ. IN.

NATURAL FREQUENCY (AS SIMPLY SUPPORTED BEAM) 256 VIBRATIONS/SEC.

COMPRRESSOR DATA

STANDARD C.F.R. ENGINE (SINGLE CYLINDER)

BORE--------------------------------- 3 7/8"

STROKE : "S"---------------------- 4 5/8"

RATIO OF CRANK : CONNECTING ROD "B"--- .225

PISTON DISPLACEMENT "A_p X S"--------- 37.33 CU. IN.

PISTON AREA "A_p"------------------ 8.29 SQ. IN.

CLEARANCE VOLUME---------------- VARIABLE

SPEEDS------------------------------ 800-2000 RPM.

INLET PIPE SIZE------------------- 1 1/2"

DISCHARGE PIPE SIZE--------------- 1 1/2"
DISCHARGE PROCESS

\[ M_d = \frac{k \phi^{n} \sqrt{\phi^{m} - 1}}{26 + 0.56 - \cos \theta - 0.56 \cos 2\theta} \]

\[ N_d = \frac{k (\sin \theta + 1.1(\sin \theta \cos \theta) \phi)}{26 + 0.56 - \cos \theta - 0.56 \cos 2\theta} \]
PLATE 4

VALUE OF $H(y) = y - \sin y$

WHERE $y$ = ANY ANGLE, IN RADIANS
PLATE 12
DISCHARGE PROCESS

FOR CASE NUMBERS
SEE TABLE 1

φ CURVES FOR $F_d = 1$
(ONLY EXTREME CASES DRAWN)

- VALVE BECOMES FULLY OPEN
- " STARTS TO CLOSE

φ CURVES $B_d = 3$
PLATE 13 DISCHARGE PROCESS

(SEE TABLE I)

- VALVE BECOMES FULLY OPEN
- " STARTS TO CLOSE
- x " CLOSED

B = 6, θ₂ = 270°

B = 6, θ₂ = 295°

B = 6, θ₂ = 320°

B = 9, θ₂ = 270°

B = 9, θ₂ = 295°

B = 9, θ₂ = 320°
PLATE 14  INLET PROCESS
SEE TABLE 2

* VALVE BECOMES FULLY OPEN
STARTS TO CLOSE

\( B_1 = 1 \)

ONLY EXTREME CASES SHOWN

\( B_1 = 3 \)

\( B_1 = 6 \)
\( \theta_a = 30^\circ \)

\( B_1 = 6 \)
\( \theta_a = 50^\circ \)

\( B_1 = 6 \)
\( \theta_a = 70^\circ \)
PLATE 15

PISTON POSITION & VELOCITY VS. CRANK ANGLE

\[ \frac{dz}{\theta} \]

\[ \frac{1}{\theta} \cdot \frac{dz}{\theta} \]

POSITIVE FROM 0° TO 180°
NEGATIVE FROM 180° TO 360°

\[ R = \frac{z}{2} \]

\[ \frac{R}{L} = 0.225 \]

\[ \frac{d}{L} = 0.605 \]

(For theoretical calculations)
THEORETICAL DISCHARGE LOSS RATIO

\[ B_d = 3 \]

(SEE TABLE 1 FOR CASE NUMBERS)
THEORETICAL INLET LOSSES $B_i = 6$

(SEE TABLE 2 FOR CASE NUMBERS)
DIAGRAMMATIC LAYOUT OF TEST APPARATUS
PRESS.
LBS./SQ. IN
ABS
(HEAVY SPRING)

INDICATOR CARD AND VALVE LIFT CURVES
20 LBS./IN² DISCH. PRESS., 1200 RPM.
5.65% CLEARANCE

INLET RECEIVER PRESS.
(LIGHT SPRING)

CYL. INLET PRESS. (LIGHT SPRING)
5#./IN

CYL. PRESS. DISCH. RECEIVER PRESS.
HEAVY SPRING

INLET VALVE LIFT CURVE

DISCH. VALVE LIFT CURVE

CRANK ANGLE Θ

PRESS.
LBS./IN²
LIGHT SPRING

PLATE 25
Press.
LB/IN² ABS.
(Heavy Spring)

Cylinder Press.

Disch. Receiver Press.

Heavy Spring

Indicator Card and Valve Lift Curves

Disch. Press 40 LB/IN² Gage
900 RPM
8.65 % Clearance

Inlet Receiver Press.
(Light Spring)

Cylinder Press.
(Light Spring)

Press.
LB/IN² ABS.
(Light Spring)

Plat 26
INDICATOR CARD AND VALVE LIFT CURVES

TEST 6

20 LBS./IN² DISCHARGE PRESS.
900 RPM.
5.65 % CLEARANCE

PRESS., LB./IN² ABS.
(HEAVY SPRING)

PRESS., LB./IN² ABS.
(LIGHT SPRING)

CYLINDER PRESS.
(HEAVY SPRING)

MEAN DISCH. PRESS.

MEAN INLET PRESS

CYLINDER PRESS.
LIGHT SPRING

DISCH. VALVE LIFT CURVE

INLET VALVE LIFT CURVE

CRANK ANGLE

0 200° 240° 280° 320° 360° 40° 80° 120° 160° 200°
INDICATOR CARD AND VALVE LIFT CURVES

TEST 7-1
20 LBS/IN² DISCH. PRESS.
1500 R.P.M.
5.65% CLEARANCE

PRESS., LB/IN² ABS
(HEAVY SPRING)

CYL. PRESS.
(HEAVY SPRING)

MEAN INLET PRESS

MEAN DISCH. PRESS

CYL. PRESS.
(LIGHT SPRING)

15°

10°

5°

200° 240° 280° 320° 360°

CRANK ANGLE

INLET VALVE LIFT

DISCH. VALVE LIFT

PLATE 29
INDICATOR CARD AND VALVE LIFT CURVES

PRESS: LB/IN² ABS.
(HEAVY SPRING)

TEST 6-6
60 LB/IN² DISCH. PRESS.
1200 RPM.
.00657% CLEARANCE

PRESS: LB/IN² ABS.
(LIGHT SPRING) 20

MEAN DISCH.
(HEAVY SPRING)
PRESS.

MEAN INLET PRESS.

CYLINDER PRESS.
(LIGHT SPRING)

DISCH. VALVE LIFT CURVE

INLET VALVE LIFT CURVE

PLATE 30-B
COMPARISON OF THEORY WITH EXPERIMENTS

AVERAGE PRESS. RATIO 2.45

DISCH. LOSS RATIO
(CALCULATED)

q = 0.117 (1600 RPM)
q = 0.0585 (900 RPM)

PLATE 32
COMPARISON OF THEORY WITH EXPERIMENTS

AVERAGE PRESS. RATIO .385

CALCULATED DISCH. LOSS RATIO

TESTS
6 & 7
8 & 9
10 & 11
12 & 13

q = .117 (1800 RPM)
q = .0585 (900 RPM)
Comparison of theory with experiments

Average press. ratio 5-20

Calculated
Disch. loss ratio

Tests
- 6 & 7
- 8 & 9
- 10 & 11
- 12 & 13

q = 0.17 (1800 RPM)
q = 0.0585 (900 RPM)
COMPARISON OF THEORY WITH EXPERIMENTS

DISCH. LOSS RATIO

AVERAGE PRESS. RATIO 6.60

TESTS

+ 6 & 7
0 8 & 9
* 10 & 11
A 12 & 13

DISCH. LOSS RATIO (CALCULATED)

$q = 0.117$ (1800 RPM)
$q = 0.0585$ (900 RPM)

$B_d$
COMPARISON OF THEORY WITH EXPERIMENTS

AVERAGE PRESS. RATIO 2.45

TESTS
+ 6 & 7
o 8 & 9
• 10 & 11
△ 12 & 13

CALCULATED INLET LOSS RATIO
PLATE 37

Comparison of Theory with Experiments

Average Press Ratio 3.85

Calculated Inlet Loss Ratio

CALC VOL EFF
IDEAL VOL EFF.

COMPARISON OF THEORY WITH EXPERIMENTS

AVERAGE PRESS. RATIO 3.2

TESTS
6 & 7
8 & 9
10 & 11
12 & 13

CALCULATED INLET LOSS RATIO
Comparison of Theory with Experiments

Average Press Ratio 6.60


Inlet Loss Ratio (Calculated)

Tests:
- 6 & 7
- 8 & 9
- 10 & 11
- 12 & 13
PLATE 40
LOG-LOG PLOT OF DISCHARGE LOSS RATIO (THEORETICAL)

$B_4$

$B_2 = 270^\circ$

$J = 10$
$q = 2.0$

$B_2 = 295^\circ$

$J = 10$
$q = 20$

$B_2 = 320^\circ$

$J = 10$
$q = 20$

IDEAL
PLATE 41
LOG-LOG PLOT OF INLET LOSS RATIO
(THEORETICAL)

\[ B_i \]

\[ \theta_4 = 30^\circ \]

\[ J = 10, \theta = 20 \]

\[ \theta_4 = 50^\circ \]

\[ J = 10, \theta = 20 \]

\[ \theta_4 = 70^\circ \]

\[ J = 10, \theta = 20 \]
RESULTS OF STATIC TESTS ON REED VALVES

DISCHARGE COEFFICIENT BASED ON AREA WHEN FULLY OPEN (1.413 SQ. IN)

PRESSURE DIFFERENCE ACROSS VALVES, INCHES OF MERCURY
Pressure Ratio

Comparision of Ratio

P-V Diagram Area - Loss Areas

P-V Diagram Area

Theoretical

Experimental

(Tests 12 & 13)

900 RPM

1000 RPM

1500 RPM

1800 RPM
Comparison between theoretical and experimental cycle efficiencies
COMPARISON OF THEORETICAL AND EXPERIMENTAL VOLUMETRIC EFFICIENCIES

THEORETICAL CURVES

TEST POINTS (TESTS 12313)

+ 900 RPM
○ 1200 RPM
△ 1500 RPM
● 1800 RPM

PRESSURE RATIO

VOLUMETRIC EFFICIENCY
VARIATION OF DISCHARGE LOSS WITH REED THICKNESS

REED 2.38" x .212"
1200 RPM
Biographical Note

The author was born in Providence, Rhode Island on March 17, 1919, and educated in the public schools of Providence. He attended M. I. T. for one year in 1937 but was forced to leave because of lack of funds. In the summer of 1938 he took a competitive examination for entrance to the Webb Institute of Naval Architecture, a philanthropic institution in New York City. He was admitted and spent the next four years there, graduating in 1942. In 1942-43 he was a Naval Architect for the Mechanical Division of the Panama Canal at Balboa, Canal Zone, engaged in engineering work on the repair and alteration of ships and dredges. From 1944 to 1946 he was employed by the Cramp Shipbuilding Company which was building Naval vessels. Part of the time he was an engineer in the Technical Division doing calculations of strength, stability, launching, etc. The remainder he spent as assistant to the Welding Engineer who also was head of the Test Laboratory. Much of the work here involved experimental stress analysis. After Cramp's contracts were cancelled he resigned and joined the firm of Kindlund and Drake, Naval Architects and Engineers, New York. In September, 1946, he entered M. I. T. as a graduate student receiving the degree of Master of Science in Marine Engineering in June 1947. After a summer spent as a Research Engineer for the DeLaval Company, he was accepted as a candidate for the degree of Doctor of Science in Mechanical Engineering.