PROPAGATION OVER KNIFE EDGE OBSTACLES
ON A
SPHERICAL EARTH

by

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May 1990

Submitted in Partial Fulfillment
of the Requirements for the Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 1992

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ABSTRACT

Radio wave propagation over the earth's surface is a classical field in electromagnetics with applications that include communications and air defense. In order to evaluate radar performance, for instance, the signal strength is predicted as a function of receiver position using a computer model. At low altitudes and VHF frequencies, the dominant propagation effects are diffraction over a conducting sphere, knife edge diffraction from discrete terrain obstacles such as hills, and specular reflections.

In this thesis, the effect of the earth's surface with knife edge obstacles is investigated by solving the problem of propagation over a knife edge on a sphere for all configurations of source and receiver. The results obtained in this thesis are used to improve the diffraction algorithm in the SEKE (Spherical Earth with Knife Edges) computer model developed at MIT Lincoln Laboratory, in which the spherical earth and knife edge diffraction mechanisms are treated separately and then weighted using an empirical scheme.

A comprehensive review of the available solutions to smooth earth propagation is given, and then related work in the area of knife edge diffraction over ground is presented to sharpen the problem statement. Accurate solutions are available in limiting cases, when the source and receiver are both close to the knife edge (four-ray approximation) and both far from the knife edge (Wait's solution). In between, and for asymmetric configurations, modified approaches are sought.

The existing four-ray method can be extended to include a first-order approximation to the curvature of the earth. However, the general approach for calculating propagation over a knife edge on a sphere is based on Huygens' principle applied to the knife edge aperture. Assuming an equivalent current distribution on the
aperture, the field at the receiver is expressed as an integral of smooth earth fields over the aperture. The integral converges numerically for all antenna configurations of interest. The formulation is extended to take into account two knife edges on a sphere in terms of a double integral.

The double knife edge solution is implemented in a diffraction algorithm for actual terrain, and results are compared with SEKE predictions. SEKE performs well because its weighting scheme is based on propagation statistics, whereas the new algorithm captures the physics of the problem more rigorously. It is found that the new algorithm tends to improve on the SEKE results for very low receiver heights, particularly when there is knife edge masking. For this reason, use of the new model for ground clutter prediction at VHF is investigated and is found to give consistent (though modest) improvements over the use of SEKE. Because it is computationally slow, it is recommended that the new algorithm be selectively used as an alternative branch in the SEKE decision tree.

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ACKNOWLEDGEMENTS

There are many people who have helped me out in my two years at MIT, but I want to keep this section shorter than Chapter 5. Prof. Jin Au Kong, my graduate advisor, thesis supervisor, and teacher, has given me the opportunity to explore the world of EM from within his distinguished research group and has always been there for friendly support, sage advice, and comic relief. Dr. Robert T. Shin, my thesis supervisor and mentor, has given me countless ideas, sparked my interest when motivation was low, led me by the hand when my understanding was weak, opened doors to projects at Lincoln Laboratory, and generally kept me going for the last couple of years. Dr. Y. Eric Yang got me started on propagation studies for the ILS project and has always been there to wake me from computer-related nightmares.

At Lincoln Lab, I thank Jen Jao for his support, ideas, and objective advice. In Group 106, I appreciate David (Tweety) for his caring, concern, and guidance, and for his thoughts on life, work, and that series that doesn’t converge because of those #%@%%&* Airy functions. I thank John E. for his substantial help in the clutter work and for leading measurements that unfortunately did not make it into this thesis. Thanks to Elayne and Ollie for their help in getting my code to run on the Suns, making polar plots, and taking time to answer 5 million questions every day. Much gratitude to Serpil for allowing me to mess around with SEKE, and for her constant encouragement. Also, thanks to David Briggs, Al Bernard, and Serpil for allowing me to give the workshop talk. I appreciate Joe F. down at Pubs for his helpfulness, and John A. at the Flight Facility for his straight talk. And thanks to the shuttle drivers who were not obsessed with easy-listening tunes or running over the car in front of them (or both).

Back on campus, I acknowledge Murat, whose hairdo alone introduced such key concepts as spherical earth diffraction and isotropic antenna pattern. Murat has been my friend, my teacher, and a contributor to this thesis. Ali has given me support and commentary on everything from dipoles to the U. of Mich. (Go Blow).
Thanks to Bob and Kevin for the nontechnical discussions and for introducing me to volleyball. John, who can derive every single physical principle known to man, did so for me on many occasions, and I greatly appreciate that. Thanks to David A. for his work on knife edge stuff and for his plotting routine. Thanks also to Chih, Han, Jake, Kit, Lam, Lawrence, Mr. Gu, and William for their friendly help around the office. (And Li-Fang for her unfriendly help around the office.) I should also mention Joel on trombone, and Pierre's Man-with-the-Moustache play. Lastly, sad farewells to Barb and her attitude, Tony and his old hair which I never knew, and Dr. Ali and his multilayer Green's functions.

Outside the group and Lincoln, thanks go to Prof. Lee for his advice and counseling, and Prof. Weiss for making the OQE as enjoyable as possible. Mark, my roommate on guitar at Tang (we ∜ you Trudy), has helped keep me sane for over a year. Get well soon, Mark. I also appreciate my friends far away who have kept in touch, particularly Chris (no relation), Darshan (Señor Frog), Niels (skål), and Özlem (n’aber). Lots of love and appreciation for Brooke (smartie), who has put up with a moody telephone for the last 4 months. Oh, I mustn't forget Baby, Lucky, Charlie, Robby, LTB, Malawi, Zimbabwe, and Baloo.

Finally, my deepest respect and love to my parents, and Caroline, Margie, and Tommy. Some home cooking would sure taste good right around now.
To Robby
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Chapter 1

Introduction

Radio wave propagation over the earth’s surface is a classical and well established area in electromagnetics. The problem is to calculate the vector electromagnetic fields excited by a source radiating above the ground. Solutions to this problem are applied in areas such as communications and radar systems analysis. To evaluate radar performance or predict clutter returns, for instance, the signal’s propagation loss along a given path must be predicted according to a mathematical model. In a site-specific computer propagation model, the strength of the signal is predicted as a deterministic quantity (statistical bounds may be calculated as well), based on loss mechanisms for the specific propagation path. Propagation cannot be completely modeled deterministically given a limited number of input parameters (e.g., terrain elevation profile and ground parameters), but such a model may be used to quantitatively demonstrate the effects of varying terrain features in the propagation path. In this thesis, the improvement of an existing propagation model developed at MIT Lincoln Laboratory serves as the motivation for investigation of the theoretical problem of propagation over knife edge obstacles on a spherical earth.
1.1 Background

A site-specific propagation model may be analyzed in two parts: modeling of the terrain, and solution of the resulting theoretical problem. That is, first of all, the terrain geometry is simplified using approximations, and then the simplified configuration is solved.

The first step is the modeling of the terrain geometry. In this thesis, only the terrain along the direct path is considered (a two-dimensional approximation), and significant terrain obstacles are modeled as knife edges, perfect black masks, which is an optics approximation [7] that is still reasonable at higher radio frequencies [38]. A knife edge acts to zero the fields along its body without giving rise to reflections and without affecting the incident wavefront above it. Other possible models for terrain obstacles include wedges [27] and rounded humps [15]. Clearly, tradeoffs must exist between modeling the terrain as knife edges on a smooth sphere (with radius of the earth) versus other approximations. The choice of the model depends on the types of terrain effects that one wishes to capture. (For wedge comparisons, see, for example, [9].)

In this research, the interest is in the accurate combination of (and distinction between) signal losses due to (1) discrete obstacles in the terrain profile and (2) the bulk curvature of the earth's surface. The assertion is that the factors limiting the accuracy of the overall propagation model are of a more fundamental nature than is the specific two-dimensional representation of a terrain obstacle. One fundamental limiting factor is the two-dimensional approximation itself; over actual terrain, the
waves propagate in three dimensions and hence are affected by terrain on both sides of the direct path. The advantage of using the knife edge approximation is that it is conceptually simple, computationally fast to solve, and yields fairly accurate results at the frequencies of interest (VHF and higher). The use of wedge diffraction, for example, would not necessarily improve the overall accuracy of results in this study, although it can be a better model for a given two-dimensional profile [26].

Given that the earth's surface is modeled as knife edge obstacles on a sphere, the next step is the solution of the theoretical problem of one-way propagation, for given source and receiver positions over the simplified profile. This is the focus of the thesis. In fact, a complete solution to the problem of a single knife edge on a conducting sphere has been given by Wait [46] in the form of a double summation of infinite extent. As will be discussed in detail in Chapter 2, Wait's solution has numerical convergence problems when either the source or receiver approaches the knife edge, because of the smooth earth field expansion on which it is based. Nevertheless, Wait's solution has been extended to treat two knife edges [3], with the same limitations. The fundamental contribution of this thesis is a complete solution for a knife edge on a conducting sphere (illustrated in Figure 1.1) in the form of a radiation integral, with no limitations on the source or receiver position. The formulation is then extended to solve the problem of two knife edges on a sphere in the form of a double integration. Along the way, basic solutions to smooth earth propagation and knife edge diffraction problems are reviewed, and related work is discussed for perspective.
Figure 1.1: Problem configuration.
1.2 Motivation: The SEKE Model

Much of the motivation for this thesis comes from the study of a computer propagation model named SEKE (Spherical Earth with Knife Edges) developed at MIT Lincoln Laboratory [1,2]. The SEKE model is used to predict low-altitude radar wave propagation loss over general terrain profiles, and it has been extensively validated for frequencies ranging from VHF to X-Band.

The propagation loss is computed in SEKE as a weighted sum of three separate and independently treated types of losses: (1) multipath, which takes into account reflections from the ground, (2) spherical earth diffraction, which treats propagation over the bulk curvature of the earth, and (3) multiple knife edge diffraction, which incorporates the effect of discrete terrain obstacles such as hills. These loss mechanisms are modeled as illustrated in Figure 1.2, where S represents the source and R the receiver. This notation will be freely used throughout the thesis for one-way propagation. Multipath is dominant when the terrain is reflective (e.g., water or grassland) and there is clear line-of-sight between the source and receiver. Spherical earth diffraction is dominant when the terrain is smooth and the receiver is beyond the geometrical horizon. Knife edge diffraction is dominant when the terrain is mountainous and non-reflective. It is important to note that the knife edge contribution is calculated without a ground plane, meaning the solution is to the problem of diffraction over semi-infinite screens. Again, it is emphasized that the three loss mechanisms are treated independently.

Figure 1.3 gives an impressionistic view of the SEKE propagation loss weight-
ing algorithm. The inputs to the program include the terrain elevation profile (above mean sea level) between the source and receiver, the heights of the source and receiver above the ground, the frequency, and ground parameters. Based on the height of the obstacle (above a curve fit to the terrain) resulting in minimum clearance of the direct ray, the clearance itself, and the extent of the first Fresnel zone at that location (a spatial measure of the main lobe of energy flow), SEKE moves through a decision tree and empirically weights the three separately computed types of losses. The weighting scheme is based on a large amount of propagation data, so it generally yields good results.

A sample run of the SEKE program is illustrated in Figure 1.4 and will be used to introduce relevant issues of propagation loss computation. The site is Beiseker in Alberta, Canada, and the terrain is classified as rolling farmland [1,5]. Beiseker is one of many sites for which data has been collected using Lincoln Laboratory’s Phase-One radar [6]. The upper plot shows the terrain elevation profile with earth curvature included (with respect to the source at range zero). The source is fixed at range zero and height 17 meters above the ground, and the receiver range is varied as indicated by the dotted path at height 10 meters above the ground. The frequency is 168 MHz. The lower plot shows the magnitude of the one-way propagation loss factor squared computed by SEKE, versus receiver range. The propagation loss factor is defined as

$$F = \frac{E}{E_0}$$  \hspace{1cm} (1.1)

where $E$ is the received electric field and $E_0$ is the free space electric field. $|F|$ is
1.2. **MOTIVATION: THE SEKE MODEL**

a measure of field strength loss with respect to free space loss, and hence $|F|^2$ is a measure of power loss. The receiver is stepped along the profile, and SEKE calculates $|F|^2$ for each point independent of the values for the other points. Within 20 km, there is clear line-of-sight to the receiver, and the corresponding loss values are calculated by weighting contributions from multipath and spherical earth diffraction routines. Over most of the range beyond 20 km, the receiver is masked due to both discrete terrain features and earth curvature; the loss values are calculated by weighting the knife edge diffraction and spherical earth diffraction results.

Typically, the spherical earth routine yields larger loss values (i.e., weaker signal) than does the knife edge routine. In Figure 1.4, the extreme dips in signal strength at ranges of 35 km and 59 km are due to the model's decision to calculate the loss as purely spherical earth diffraction at those points. Although propagation measurements along the profile are not available for comparison, it is argued that the sudden fluctuations in calculated propagation loss values are due to switching in the SEKE code between dominant diffraction routines and hence are not necessarily physical. The trend of high signal strength for ranges greater than 60 km is due to the knife edge factor and is not physically explainable either. This is one example in which the knife edge factor is weighted too strongly. SEKE usually gives reasonable results, but when it does not, the diffraction algorithm tends to be the problem. One general class of terrain profiles for which the diffraction weighting scheme has trouble is profiles in which a significant hill lies close to the source, and the receiver is beyond the smooth earth geometrical horizon. In such a case, both knife edge diffraction and spherical earth diffraction need to be taken into account, but interpolation
between the two separate solutions is a poor approximation to the true physics of the problem. The end result is that SEKE tends to overpredict the signal strength, because the knife edge factor is too strong.

Improvement of the existing diffraction algorithm in SEKE serves as the real-world motivation for the formulation of a rigorous solution to the knife edge over a sphere problem. Instead of weighting separate knife edge and spherical earth losses empirically, the two mechanisms are to be combined rigorously into a single problem. At the expense of computational time, results should be more physically correct compared to the existing SEKE diffraction algorithm, particularly for cases in which both the terrain obstacles and the curvature of the earth are significant.
1.2. MOTIVATION: THE SEKE MODEL

Figure 1.2: Propagation loss mechanisms in SEKE.
Figure 1.3: The SEKE decision algorithm [1].
1.2. MOTIVATION: THE SEKE MODEL

Latitude: 51 23 24
Longitude: -113 16 18
Azimuth: 180.0 deg
Model: seke1

Antenna Height: 17.0 meters
Target Height: 10.0 meters
Radar Frequency: 1.680x10^8 Hz
Range Increment: 500.00 meters
Initial Range: 0.10 km
Final Range: 70.00 km
Range Resolution: 100.00 meters
K-factor: 1.33330
Reflection Co-ef: 0.70000

Figure 1.4: Sample SEKE run for Beiseker site.
1.3 Description of the Thesis

In this introductory chapter, background is given for site-specific radar propagation modeling, and motivation is presented for the research to be presented in this thesis. The earth's surface is modeled as a smooth sphere with knife edge obstacles, and the propagation loss factor is computed for given source and receiver positions. In Lincoln Laboratory's SEKE computer model, the propagation loss is calculated as an empirically weighted sum of multipath, spherical earth diffraction, and knife edge diffraction losses. The weighting of diffraction losses leads to incorrect results for some cases, and this motivates the development of a more rigorous diffraction routine. Hence, the problem of propagation over knife edges on a spherical earth is to be solved rigorously using a radiation integral approach.

Chapter 2 introduces fundamental solutions for smooth earth propagation and knife edge diffraction over ground. The smooth earth solutions are the building blocks of the whole project. These solutions consist of the Fock series, the geometrical optics approximation, and the Uniform Theory of Diffraction. For given computational time, each smooth earth solution has its own region (i.e., range of antenna positions) of validity. Taken together, the solutions accurately span the entire region of interest. Then, related work in the area of knife edge diffraction over ground is presented to clarify the overall problem and provide perspective. The flat plane four-ray solution and Wait's double summation solution are discussed in detail, the former as a limiting case in which both antennas are very close to the knife edge, and the latter as a complete solution that is limited by convergence problems when either antenna is close to the knife edge. Results show that the flat plane
1.3. DESCRIPTION OF THE THESIS

four-ray solution and Wait's solution do not overlap in a smooth transition region as the antennas are moved in range symmetrically away from the knife edge. Also, neither of these available solutions works well for the case of a highly asymmetrically located knife edge. These gaps must be filled using a different approach.

In Chapter 3, the problem of propagation over a knife edge on a sphere is solved completely. First, the flat plane four-ray solution is extended using a first-order correction to the curvature of the earth. This technique enables the four-ray approach to be accurate enough for larger ranges that it overlaps with Wait's double summation in a transition region, for symmetric cases. Then, a general radiation integral formulation for the problem is given. The approach is based on Huygens' principle, making use of an equivalent electric current distribution on the knife edge aperture proportional to the incident smooth earth fields. This current distribution in turn radiates smooth earth fields to the receiver. The smooth earth solutions of Chapter Two are the key to the accuracy of this approach. The formulation is then extended to treat two knife edges on a sphere, with the solution containing a double integration. Results show that the integral solution agrees with the four-ray approximation and Wait's solution in the appropriate limits. It also converges accurately for asymmetric cases, because the smooth earth field calculations are flexible. Cases shown demonstrate variation with antenna position, knife edge size, and frequency.

In Chapter 4, the implementation of the integral solution for two knife edges on a sphere for real terrain is discussed and compared with SEKE for one-way propagation and ground clutter predictions. First, results are shown for an ideal-
ized terrain profile, i.e., knife edges on a sphere, to isolate the diffraction algorithms for comparison. Then, cases are shown for actual terrain profiles at VHF, with propagation measurements from the Phase-One radar experiments as the reference. Lastly, since the new model is shown to accurately predict the propagation null for very low elevations more consistently than does SEKE, prediction of VHF ground clutter returns is explored. Limited testing indicates that using the new model for the propagation portion of VHF clutter modeling results in a modest but uniform improvement over using SEKE, with Phase-One clutter measurements as the reference.

Finally, in Chapter 5, the main ideas and results of the thesis are discussed and summarized. New questions are raised, and further work is proposed that will hopefully be of interest to the radar propagation community.
Chapter 2

Fundamental Smooth Earth and Knife Edge Solutions

2.1 Introduction

The foundation of this research is smooth earth propagation and knife edge diffraction. In this chapter, the basic propagation solutions to be used in later formulations are introduced, and the problem statement is sharpened through the presentation of related work. A review of the available solutions to the smooth earth problem will be given, and then some relevant knife edge solutions will be discussed.

The theoretical basis for modern propagation models is the set of classical equations derived for radiation in the presence of smooth boundaries. Early in this century, Sommerfeld (summarized later in [41]) derived an analytical solution for the fields of electric and magnetic dipoles on a boundary between free space and a conducting half space; the integral form of the solution was evaluated using asymptotic methods. At around the same time, Watson [54] worked on the problem of a radially oriented dipole above a homogeneous sphere. Building on these
ideas, Bremmer (summarized in [8]), Fock [12], and later Wait [47] worked with field solutions to this problem in the form of a canonical contour integral of Legendre polynomials weighted by spherical Hankel functions. The canonical integral was expressed as a residue series in terms of Hankel functions of fractional order. Fock expressed this series in terms of the more familiar Airy functions. This last version will be referred to as the Fock series, but it is understood that there were many contributors to this fundamental solution. Usage of the Fock series and approximations to it will be discussed with regard to the source and receiver geometry.

The theory developed to treat propagation over discrete obstacles is somewhat more recent. Furutsu [14,15] derived a series solution to the problem of diffraction by smooth rounded obstacles. Letting the radius of such an obstacle tend to zero results in the knife edge approximation for the obstacle. The Fresnel-Kirchhoff approximation is typically made, meaning that the knife edge acts as a perfectly absorbing black mask but does not affect the portion of the incident wavefront propagating above it. This is a fairly good model for prominent terrain obstacles such as hills and ridges, especially at higher (e.g., microwave) frequencies. Qualitatively speaking, as the frequency is increased, masking becomes the dominant loss mechanism. Deygout [11], Vogler [45], Giovaneli [17], and Whitteker [55] are among those who have presented solutions for multiple semi-infinite knife edge diffraction with no ground plane. These solutions give fairly accurate predictions when applied to mountainous terrain for which reflections from the ground can be neglected. Known solutions to knife edge problems with a surface boundary include the flat plane four-ray solution (see [30]) based on the Fresnel integral and image theory, and Wait's
double summation solution for a knife edge on a curved boundary [46]. Results generated using these approaches will show that there exists a need for a modified solution to the knife edge on a sphere problem for intermediate source and receiver geometries; this observation will motivate the formulation in Chapter 3.
2.2 Smooth Earth Solutions

In this section, the smooth earth propagation formulas are presented, and their usage is demonstrated. Three separate solutions for the smooth earth problem will be discussed. The first is the Fock series mentioned above; the other two, the geometrical optics solution and the Uniform Theory of Diffraction solution, are in fact approximations to the Fock series.

2.2.1 Propagation Formulas

The Fock series, the geometrical optics solution, and the Uniform Theory of Diffraction solution are the available smooth earth formulas. For given computational time, each solution has a region of validity, i.e., range of configurations of source and receiver for which the solution works well. Taken together, the different solutions cover all relevant positions of source and receiver, and there is a fair amount of overlap between the regions of validity. Figure 2.1 shows, in an impressionistic sense, the regions of propagation in which the solutions are employed. The heavily shaded area is known as the diffraction (or shadow, or masked) region, the unshaded area is known as the interference (or visible, or line-of-sight) region, and the lightly shaded area is the transition region between the two. Note that the effects of diffraction are significant even above the geometrical horizon. The effects of atmospheric refraction are somewhat crudely taken into account using an effective earth radius that is 4/3 times the actual mean radius [22,51,32].

The Fock series [13] is an expression for the smooth earth propagation loss
factor $F$, the received field normalized with respect to the free space field. (Note: The SEKE model uses the Fock series for the spherical earth diffraction calculation.) It represents a rigorous solution to Maxwell’s equations for an oscillating electric dipole in the presence of a conducting sphere of permittivity $\epsilon$, permeability $\mu_0$, conductivity $\sigma$, and effective radius $a_e$. The Fock series has the form

$$ F = \sqrt{x} e^{ik_0(r-d)} \sum_{n=1}^{\infty} F_n e^{is_n} w_1(t_n - y_1)w_1(t_n - y_2) $$  \hfill (2.1)

Here, $k_0 = \omega \sqrt{\mu_0 \varepsilon_0} = 2\pi/\lambda_0$ is the free space wave number ($\omega$ is the radian oscillation frequency and $\lambda_0$ is the free space wavelength), $d$ is the absolute distance between the antennas, $r$ is the range along the earth’s surface between them, $x$ is the scaled range, and $y_1$ and $y_2$ are the scaled heights of the source and receiver. The scaled parameters are given by

$$ x = \left(k_0/2a_e^2\right)^{1/3} r \hfill (2.2) $$

$$ y_1 = \left(2/k_0 a_e\right)^{1/3} k_0 z_1 \hfill (2.3) $$

$$ y_2 = \left(2/k_0 a_e\right)^{1/3} k_0 z_2 \hfill (2.4) $$

where $z_1$ and $z_2$ are the actual source and receiver heights above the surface. In Eqn. (2.1), $w_1$ is an Airy function satisfying

$$ w_1''(z) = zw_1(z) $$  \hfill (2.5)
and is given in terms of the more familiar function \( Ai(z) \) (reviewed in [4]) by

\[
    w_1(z) = 2\sqrt{\pi} e^{i\pi/6} Ai(z e^{i2\pi/3})
\]  

(2.6)

The parameters \( \{t_n(q)\}_{n=1}^{\infty} \) satisfy

\[
    w_1'(t_n) - qw_1(t_n) = 0
\]  

(2.7)

where \( q \) is the scaled surface impedance given by

\[
    q = -(k_0^a a_z/2)^{1/3} \frac{\sqrt{(k_s^2 - k_0^2)}}{k_s^2}
\]  

(2.8)

with \( k_s = \omega \sqrt{\mu_0(\epsilon + i\sigma/\omega)} \), the complex wave number in the sphere. A good treatment of the impedance boundary condition used in formulating the Fock series is found in [20], and related arguments are reviewed in [37]. Back to Eqn. (2.1), \( F_n \) is a coefficient that depends on \( t_n \) and \( q \) explicitly and is given by

\[
    F_n = \frac{-2\sqrt{\pi} e^{i\pi/4}}{(1 - t_n/q^4) [w_1'(t_n)]^2}
\]  

(2.9)

In this research, \( q \) is allowed to approach \( \infty \), which is a particularly good approximation for horizontal polarization and fairly high conductivity [13,32]. In that case, the \( t_n \) parameters are the zeroes of the function \( w_1 \), as seen from Eqn. (2.7), and their values are tabulated in [33]. The zeroes of \( w_1 \) lie along the ray \( e^{i\pi/3} \) in the complex plane. Hence, the exponential factor in each term of Eqn. (2.1) damps the magnitude of the term. The method discussed in [39] is used to evaluate
2.2. *SMOOTH EARTH SOLUTIONS*

the Airy functions.

The Fock series converges theoretically for all source-receiver positions, and in fact in the shadow region only a few terms need to be computed for good results. However, convergence becomes slow and the computation involves term-by-term cancellation as the receiver is moved well into line-of-sight. This is because, in that region, the Airy functions become highly oscillatory. (Additional background information on Airy functions can be found in [37,10]. Therefore, the series solution is employed only in the diffraction region and part of the transition region.

The geometrical optics (G.O.) solution (reviewed in [22,43]) is used as an approximation to the Fock series in the interference region, although it is not usually introduced as such. More generally, G.O. is a high-frequency approximate solution to Maxwell's equations, with the basic premise that energy propagates along rays. As applied to interference, G.O. takes the theory of flat plane reflections of plane waves and generalizes it to curved surface reflections of arbitrary wavefronts. A local planar approximation is made at the specular point, which is the point on the surface from which the main contribution to the reflected signal comes. In general, the specular point is found using the stationary phase method. The G.O. solution is written as a sum of the direct and the reflected ray contributions,

$$F = 1 + R_{h,v} e^{ik_0(\Delta r)} \sqrt{s/d}$$  \hspace{1cm} (2.10)

where $d$ is again the absolute distance between source and receiver, $s$ is the total reflected path length, $\Delta r$ is the path difference $(s - d)$, and $R_{h,v}$ is the Fresnel reflection coefficient at the specular point. The subscripts $h$ and $v$ correspond to
horizontal and vertical polarization. The reflection coefficients are given by

$$R_h = \frac{\cos(\theta) - \sqrt{[(\varepsilon + i\sigma/\omega)/\varepsilon_0] - \sin^2(\theta)}}{\cos(\theta) + \sqrt{[(\varepsilon + i\sigma/\omega)/\varepsilon_0] - \sin^2(\theta)}}$$  \hspace{1cm} (2.11)$$

and

$$R_v = \frac{[(\varepsilon + i\sigma/\omega)/\varepsilon_0] \cos(\theta) - \sqrt{[(\varepsilon + i\sigma/\omega)/\varepsilon_0] - \sin^2(\theta)}}{[(\varepsilon + i\sigma/\omega)/\varepsilon_0] \cos(\theta) + \sqrt{[(\varepsilon + i\sigma/\omega)/\varepsilon_0] - \sin^2(\theta)}}$$  \hspace{1cm} (2.12)$$

where \(\theta\) is the incident angle at the specular point with respect to the normal, and the ground parameters are as before. In Appendix A, the calculation of the specular point location and path difference is discussed. Since the G.O. solution is very fast to compute, it is used whenever possible. G.O. works better as the receiver is moved higher for a fixed range or closer for a fixed height. It agrees closely with the Fock series down to around the transition region. Then it becomes inaccurate because of diffraction effects.

The Uniform Theory of Diffraction (UTD) solution applied to a sphere is also an approximation to the Fock series. The use of UTD in the literature [28] is much broader than the purpose it serves in this work, which is to approximate the smooth earth series solution in the transition region. The UTD solution in fact is designed to be valid in all scattering regions. In particular, it is valid in the regions where the Geometrical Theory of Diffraction (GTD) (reviewed in [21]) fails, while reducing to the GTD solution in the regions where the latter has been shown to perform well. For this application, a UTD formula is employed that is valid when the receiver is close to the surface and the surface below it is masked [34,35,36]. In that case, the
2.2. SMOOTH EARTH SOLUTIONS

UTD solution takes the form

\[ F = \frac{e^{i\kappa(r-d)}}{\sqrt{r_{h_1}/d}} \Omega_{h,e}(r_{h_2}) \tag{2.13} \]

where \( r_{h_1} \) is the range along the earth's surface from the source to its horizon, \( r_{h_2} \) is the range along the earth's surface from the horizon to the receiver, and \( \Omega \) is a rapidly convergent series expanded in terms of a Fock function and its derivatives. To within an error term of order \( y_2^5 \), where \( y_2 \) is from Eqn. (2.4),

\[ \Omega_{h}(r_{h_2}) = y_2 \tilde{g}_F(r_{h_2}) - \frac{i y_2^2}{3!} \tilde{g}_F'(r_{h_2}) - \frac{2y_2^4}{4!} \tilde{g}_F''(r_{h_2}) + O(y_2^5) \tag{2.14} \]

and

\[ \Omega_{e}(r_{h_2}) = g_F(r_{h_2}) - \frac{i y_2^2}{2!} g_F'(r_{h_2}) - \frac{y_2^3}{3!} g_F(r_{h_2}) - \frac{y_2^4}{4!} g_F''(r_{h_2}) + O(y_2^5) \tag{2.15} \]

where

\[ \tilde{g}_F(\alpha) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dz}{w_1(z)} e^{i\alpha z} \tag{2.16} \]

\[ g_F(\alpha) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dz}{w_1(z)} e^{i\alpha z} \tag{2.17} \]

are the Fock radiation functions [28]. This UTD solution is particularly useful for configurations in which the receiver is visible but very low to the ground, in which case it tends to converge more quickly than the Fock series. This case corresponds to a portion of the transition region in which G.O. fails because diffraction effects are significant, and the Fock series may exhibit slow convergence.
Figure 2.1: Smooth earth propagation solutions.
2.2. SMOOTH EARTH SOLUTIONS

2.2.2 Results

Some results are now presented to illustrate the relevant properties of the smooth earth solutions. Figures 2.2–2.5 show the magnitude of the propagation loss factor squared in decibels plotted versus receiver height and range, for a sphere of $\varepsilon=6\varepsilon_0$, $\sigma=0.006 \ \mu$m, and $a_e=8470$ km. It is noted that these ground parameters and effective earth radius are used for all results shown in this thesis, unless otherwise indicated.

Figure 2.2 shows the behavior of the Fock series versus receiver range for three different frequencies. The source and receiver heights are both fixed at 10 m. Within the visible region (up to the geometrical horizon of 26 km), the higher frequency signals are stronger, whereas in the masked region (beyond the horizon), the lower frequency signals become stronger. In general, at high frequencies, diffraction is less significant, and line-of-sight is the dominant propagation mechanism. In this particular case, it is seen that line-of-sight dominates for the 3000 MHz wave, whereas diffraction over the curvature of the earth is significant for the lower frequency waves, especially the 100 MHz wave.

Figure 2.3 illustrates the error involved in truncating the Fock series. At 100 MHz, the source is fixed at height 20 m, the range is 15 km, and the receiver is moved up in height. Note the null for horizontal polarization near the ground, a consequence of the boundary condition. It turns out that the Fock series with 35 terms gives accurate results up to 200 m. The series with 20 terms is accurate up to 140 m, and the series with 10 terms is accurate only up to 60 m. Above 200 m,
many more terms need to be kept, as the Airy functions become intractable. The choice of the number of terms to retain is always a problem. In this case, if the source height were greater or the range smaller, more terms would be needed in order to obtain the respective degrees of accuracy shown. For the remainder of this thesis, a maximum of 35 terms are kept in the Fock series, and the other smooth earth solutions are used to support it.

Figure 2.4 shows the smooth transition between the geometrical optics solution and the Fock series, for varying range. The frequency is 100 MHz, and the source and receiver heights are both 50 m. Within 10 km, the Fock series with 35 terms does not converge well, but the G.O. solution captures the nulling pattern and the final crest before the monotonic decrease in signal strength. Between 10 km and 20 km, both solutions are accurate, and this makes up the transition region. Beyond 20 km, diffraction effects become significant, and the G.O. approximation is no longer good, even though the horizon is not until almost 60 km. The Fock series accurately captures the smooth earth diffraction mechanism in this region.

Figure 2.5 demonstrates the use of all three smooth earth propagation solutions discussed in this section. The frequency is 100 MHz, and the source height is fixed at 20 m, the range is 25 km, and the receiver height is varied. The Fock series does not converge accurately above 250 m, but there the G.O. solution is used instead. The UTD solution is accurate for heights below 15 m and is faster to compute than the Fock series, although the Fock series converges quickly in that region as well. Hence, smooth earth propagation can be efficiently calculated for all antenna configurations of interest.
2.2. SMOOTH EARTH SOLUTIONS

Figure 2.2: Fock series for 100 MHz, 500 MHz, and 3000 MHz.
Figure 2.3: Convergence of Fock series for 10, 20, and 35 terms.
Figure 2.4: Transition from geometrical optics to Fock series.
Figure 2.5: Use of Fock series, geometrical optics, and UTD.
2.3 Knife Edge Solutions

In this section, related work in the area of single knife edge diffraction over smooth ground is presented for perspective. Practical solutions are available in a couple of limiting cases. These cases are when the source and receiver are (1) both very close to the knife edge (well into line-of-sight) and (2) both very far from the knife edge (in the diffraction region).

2.3.1 Propagation Formulas

Case (1) (close range) is solved approximately by what is called the four-ray method. Case (2) (far range) is solved using Wait's double summation form, which is in fact analytically exact for all source-receiver positions. However, it will be shown that for configurations other than case (2), Wait's solution is not practical in the computational sense. Hence, these two solutions do not overlap in a smooth transition region, i.e., a gap exists between them for which there is no accurate solution as the source and receiver are moved in a symmetric manner with respect to the knife edge. Furthermore, neither solution handles the case of an asymmetrically located knife edge in a satisfactory manner.

If the source and receiver are both close enough to the knife edge, i.e., both enough into line-of-sight of the obstacle, the earth's surface may be approximated by a flat plane, giving rise to the flat plane four-ray solution [30,23]. Although the ground plane is not assumed to be perfectly conducting, image theory is applied to express the field at the receiver as a sum of four separate field contributions,
corresponding to the various rays reflected on either side of the knife edge and diffracted over its tip, as shown in Figure 2.6. The received field is calculated as

\[ E_{\text{rec}} = E_{11} + E_{21} + E_{12} + E_{22} \]  

(2.18)

where each contribution \( E_{ij} \) is calculated by solving the semi-infinite edge diffraction problem in the form of a Fresnel integral (see Appendix B), with the ground plane removed and the source and receiver in positions \( i \) and \( j \), respectively. The three contributions involving images are the result of specular reflections and hence are scaled by the appropriate Fresnel reflection coefficients. In the limit as the knife edge height approaches zero, the four-ray result approaches the solution for a source over a conducting planar halfspace, as it should. The four-ray approach becomes a poor approximation to the curved ground problem as the source or receiver moves away from the knife edge, because the curvature of the earth affects reflections, and diffraction effects become significant.

For any configuration of source and receiver with a knife edge on a spherical earth, Wait has presented an analytical solution in the form of a double summation [46,48,52]. The solution is based on the Fock series for smooth earth propagation. The field at the receiver is expanded in a series of Airy function modes of the same form as the smooth earth series, but with different (unknown) coefficients. The field along the aperture above the knife edge is assumed to be the incident field (Fresnel-Kirchhoff approximation), and hence it has the form of the smooth earth series with known coefficients. Zero fields are assumed on the knife edge itself. Then the received field is matched to the aperture field in the limit as the receiver
approaches the knife edge. From there, the orthogonality properties of the Airy function are used along the aperture to solve for the coefficients of the received field. Hence, this is sometimes referred to as Wait's mode-matching solution. The range and height parameters involved are shown in Figure 2.7 and are all scaled dimensionless quantities. The propagation loss factor is written

$$F = \sqrt{2}e^{ik_0(r-d)} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn}e^{iz_1t_m}w_1(t_m - y_1)e^{i(z-z_1)t_n}w_1(t_n - y_2)$$

(2.19)

where the propagation from the source to the aperture and from the aperture to the receiver is explicit. The ranges \( r \) and \( d \) are defined as in Eqn. (2.1), and the \( t_n \) parameters are the roots of Eqn. (2.7), as before. \( A_{mn} \) converts an incident mode at the aperture to a sum of transmitted modes to the receiver and has dependence on \( t_m, t_n, q, \) and \( y_0 \) (scaled knife edge height) given by

$$A_{mn} = 2\sqrt{\pi}e^{iz/4} \frac{w_1(t_m - y_0)w_1'(t_n - y_0) - w_1(t_n - y_0)w_1'(t_m - y_0)}{(t_n - t_m)((1 - t_m/q^2)[w_1(t_m)]^2)((1 - t_n/q^2)[w_1'(t_n)]^2)}$$

(2.20)

for \( n \neq m \), and

$$A_{mn} = 2\sqrt{\pi}e^{iz/4} \frac{(t_m - y_0)[w_1(t_m - y_0)]^2 - [w_1'(t_m - y_0)]^2}{(1 - t_m/q^2)^2[w_1'(t_m)]^4}$$

(2.21)

for \( n = m \). The series is rapidly convergent when the antennas are in the diffraction region with respect to the knife edge. However, since this double summation is based on the Fock series, it has the same computational limitations. That is, as either the source or receiver moves closer and higher into line-of-sight of the knife edge obstacle, the solution requires more and more terms to converge. Equivalently,
for a fixed number of terms, the result is accurate only for antennas outside a certain minimum range from the knife edge.

It will be shown that, for cases with the knife edge centrally located, Wait's solution with a fixed number of terms does not converge well enough to overlap with the four-ray solution. That is, as the source and receiver are moved closer to the knife edge symmetrically, there is no smooth transition from one solution to the other. Hence, there is a gap that needs to be filled in the symmetric case. In addition, a reliable solution must be found for the asymmetric case, where the source is in the diffraction region but the receiver is well in line-of-sight of the knife edge (or vice versa).
2.3. KNIFE EDGE SOLUTIONS

Figure 2.6: Flat plane four-ray solution.

Figure 2.7: Parameters for Wait's knife edge solution.
### 2.3.2 Results

Results are now presented to demonstrate the behavior of the basic knife edge solutions. Figures 2.8–2.10 show the magnitude of the propagation loss factor squared versus receiver height and total range between the antennas, for 100 MHz.

Figure 2.8 shows the flat plane four-ray solution and its direct term contribution for source height 10 m, knife edge height 50 m, total range between the source and receiver 20 km, and the knife edge centrally located between the antennas. For the range of receiver heights shown, there is never line-of-sight between the antennas, so the direct contribution (no ground plane) monotonically increases with the height. The total solution, however, is a sum of the direct term and three other terms involving reflections. As such, it appears as a complex interference pattern. Note the difference between the diffraction (shadow) null of the direct term and the multipath (reflection after diffraction) null of the total solution.

Figure 2.9 shows the error in truncating Wait's double summation for antenna heights 20 m and a centrally located knife edge of height 10 m. As the antennas approach the knife edge on both sides, it turns out the solution with 400 terms converges accurately up to about 6 km total range. The solution with 225 terms works well until 10 km, and the solution with 100 terms works well until 14 km. Because Wait's solution effectively requires the Fock series to converge on both sides of the knife edge, its region of validity for a fixed maximum number of terms lies outside some fixed range from the knife edge.

Lastly, Figure 2.10 shows the gap between the four-ray solution and Wait's
solution for a symmetric case. The source and receiver are at height 50 m, the knife edge height is 80 m, and the antennas are moved symmetrically away from the knife edge. Within 15 km, the curvature of the earth is not significant (for these antenna heights), and the flat plane four-ray approximation is quite good. Beyond that range, though, the approximation breaks down, and Wait’s solution should be used. The problem is that Wait’s solution (using 225 terms) converges well only beyond 30 km. Hence, there is no transition region between the two solutions; a gap exists between 15 km and 30 km, where a new approach must be taken.
Figure 2.8: Four-ray solution and direct term contribution.
Figure 2.9: Convergence of Wait's solution for 100, 225, and 400 terms.
Figure 2.10: Four-ray solution and Wait's solution for symmetric case.
2.4 Discussion

In this chapter, the foundation has been laid for the research in the thesis. The smooth earth propagation solutions that will be used in the next chapter have been presented, and available knife edge solutions have been reviewed to provide background and to clarify the problem.

In the knife edge over a sphere problem, there are two specific configurations for which the available solutions perform poorly: (1) the symmetric intermediate case, with the antennas equidistant from the knife edge such that the earth curvature is significant but a truncated Wait's series does not converge, and (2) the asymmetric case, with one antenna very close to the knife edge and the other very far from the knife edge. Wait's solution has trouble with case (2), due to the imbalance in the number of terms needed for Fock's series to converge on both sides. Results for case (2) are deferred to Chapter 3, as it represents a limiting case without a "truth" reference as of yet. Although case (1) has no "truth" reference yet either, a result was shown because it represents a visibly intermediate stage between two established references.

Cases (1) and (2) will be treated in Chapter 3. The approach to solving case (2) is general and valid for all configurations. The solution hinges on the calculation of smooth earth fields, namely, the use of the Fock series, geometrical optics, and UTD. It was shown in Section 2.2 that the solutions taken together accurately cover the entire range of interest. The stage is now set for the development of the complete solution for a knife edge on a spherical earth.
Chapter 3

Propagation over a Knife Edge on a Sphere

3.1 Introduction

In Chapter 2, it was established that solutions to the problem of propagation over a knife edge on a sphere are still sought for intermediate cases. In this chapter, the symmetric intermediate case as well as the asymmetric case (both described earlier) are solved, the former by extending the existing four-ray solution, and the latter by using a general approach that is in fact accurate for all antenna geometries. The solution for the problem will then be complete, and an extension will be given to treat multiple knife edges.

The first topic to be discussed is the bridging of the gap between Wait’s double summation results and the flat plane four-ray solution, for symmetric cases. The new method will be referred to as the modified four-ray solution, because it is a correction to the original flat plane approach. As will be shown, the modified four-ray solution serves to extend the region of validity of the ray-tracing Fresnel integral
idea such that it overlaps smoothly with results from Wait's solution while maintaining good computational speed. The method still breaks down when diffraction by the earth's curvature becomes significant, i.e., when the antennas are far enough apart.

The main focus of the chapter is the general approach for solving the proposed problem of a knife edge on a sphere. The starting point is that the fields above the knife edge are approximated as just the incident smooth earth fields. Since these fields are known over the entire knife edge aperture (and assumed to be zero on the knife edge itself), the field at the receiver can be expressed as a radiation integral over the aperture according to Huygens' principle. The simplest version of Huygens' principle states that each infinitesimal element of a wavefront acts as a source of new wavefronts, and summing up contributions from all elements results in the propagation of the wave. Huygens' principle can be used in conjunction with the equivalence principle, which states that a field problem can be transformed into an equivalent problem in which the sources are replaced by a current or field distribution over a surface that encloses the region of interest [18,25]. Of course, the solution is then equivalent only in the region of interest.

For the knife edge problem, the region of interest is bounded by the knife edge aperture, so an equivalent problem is to calculate the field at the receiver due to an equivalent aperture current distribution (which is actually not unique). The current, itself based on smooth earth factors, radiates in the presence of the spherical surface, so the propagation to the receiver is also based on smooth earth factors. The overall approach, then, is to calculate smooth earth propagation on
either side of the knife edge, and to take into account the presence of the knife edge by setting the equivalent currents along its body to zero. This approach is valid for all antenna geometries, as long as the proper smooth earth factors are employed. The advantage of this solution over Wait's double summation is that its convergence is not sensitive to the location of the antennas.
3.2 Modified Four-Ray Solution

In this section, the modified four-ray approximate solution is presented as an extension of the known flat plane method. This somewhat heuristic approach nevertheless gives good results and is computationally far superior to Wait's summation for antennas within close range of the knife edge. The extension is that its results overlap with Wait's summation in a smooth transition region.

3.2.1 Approach

It was demonstrated that, for symmetric cases, Wait's double summation formula (with a fixed number of terms) does not overlap with the flat plane four-ray formula. The four-ray approach is much simpler than Wait's solution, both conceptually and computationally, and is a natural point of extension. Hence, a correction to account for the curvature of the earth is sought for the four-ray solution. Clearly, for very short antenna ranges from the knife edge, the modified results should agree with the flat plane results.

The extended solution still breaks down into four terms, and each term is calculated by solving a Fresnel integral (semi-infinite edge diffraction with no ground plane). The direct ray term is identical to the corresponding flat plane solution term, but the method for locating the source and receiver images changes from the flat plane case. In the flat plane case, the image antenna locations are simply the reflections of the actual antennas through the planar boundary, because in the case of the perfectly conducting ground plane, those images define an equivalent
problem in which the plane is removed. (The boundary condition is satisfied.) However, when the surface is curved, an infinite number of images is needed to satisfy the boundary condition even if the surface is perfectly conducting.

Instead of trying to satisfy the boundary condition, a stationary phase argument is used to select the reference point for reflections, and the geometrical optics approximation is made for the curved surface. The configuration is illustrated in Figure 3.1. The image for an antenna is located by first finding its specular point on the surface with respect to the knife edge tip or the other antenna. Either the knife edge tip or the other antenna is chosen as the reference point, depending on which corresponding reflected ray gives the greater contribution to the field at the receiver. If the reflected ray to the other antenna is masked by the obstacle, then the tip of the knife edge becomes the reference. (This is the case shown in Figure 3.1.) For the double-bounce contribution, the knife edge tip is the reference for both antennas, by definition. In any case, a local planar approximation is made at the specular point, and the image location is found by reflecting the real antenna through that tangent plane. As in the flat plane case, the field at the receiver is a sum of four terms:

$$E_{rec} = \tilde{E}_{11} + \tilde{E}_{21} + \tilde{E}_{12} + \tilde{E}_{22}$$  \hspace{1cm} (3.1)

$\tilde{E}_{ij}$ is computed as an edge diffraction Fresnel integral with source and receiver positions $i$ and $j$, respectively. The three contributions that involve images are scaled by the proper Fresnel reflection coefficients at the specular points.

The modified four-ray solution is a first-order correction to account for the
curvature of the earth. It is effectively just a pair of flat plane approximations to the curvature. However, it gives good results for what it is designed to do. As will be shown next, the modified four-ray solution extends the range of validity of the original flat plane four-ray method so as to provide a smooth transition region between the four-ray solution and Wait’s solution.

Figure 3.1: Modified four-ray solution.
3.2. MODIFIED FOUR-RAY SOLUTION

3.2.2 Results

One representative result is shown to illustrate the extension of the four-ray approach. It is the same case as that plotted in Figure 2.10 from Section 2.3.2. In that plot, the point was made that the flat plane four-ray solution fails to make a smooth transition to Wait's solution in the symmetric case.

To review, the frequency is 100 MHz, the antennas are both at 50 m above the ground, and the knife edge height is 80 m, for this symmetric case. In Figure 3.2, it is seen that the modified four-ray results agree with the flat plane four-ray solution for very close ranges (within roughly 15 km), then deviate as the source and receiver spread out from the knife edge and the slope of the curvature becomes significant. At that point, the results bridge the gap between the flat plane four-ray results and Wait's solution, which does not converge properly at that point for a fixed number of terms. As the antennas spread out further, the modified four-ray results overlap with Wait's for a range of values (from 30 km to 50 km), then deviate as diffraction effects become dominant.

For lower antenna heights and a smaller knife edge, it is expected that Wait's solution (with the same number of terms) would converge accurately for a region beginning closer to the knife edge. Correspondingly, the four-ray approximation would be valid for a smaller region near the knife edge, and the transition region to Wait's solution would be shorter in duration. The four-ray region of validity is limited by increased diffraction effects due to lower antennas. However, the structure of the results would be identical to the case shown.
Figure 3.2: Modified four-ray improvement for symmetric case.
3.3. **GENERAL INTEGRAL SOLUTION**

### 3.3 General Integral Solution

The need to solve the asymmetric case motivates the formulation of a general expression for the received field. The integral expression is an alternative to Wait's double summation, the latter requiring a variable number of terms to be kept, depending on the positions of the antennas with respect to the obstacle. The integral solution is accurate for all configurations, and the dependence on antenna positions is through the choice of the appropriate smooth earth propagation formulas described in Section 2.2. The solution is formulated such that the value of the integral (over a region corresponding to the body of the knife edge) gives the effect of the knife edge on the otherwise smooth earth field. This general solution, together with the discussed approximations in limiting cases, completes the treatment of propagation over a knife edge on a sphere. The extension to multiple knife edges on a sphere is straightforward, and the solution for two knife edges will be presented.

#### 3.3.1 Formulation

The approach ultimately reduces to solving the smooth earth propagation problem. The main conceptual difference between the following formulation and Wait's solution is that what follows assumes free use of different smooth earth formulas, whereas Wait's solution is based fundamentally on the Fock series representation. Instead of a double summation, the resulting expression will contain an integration.

The general problem is illustrated in Figure 3.3, where the origin of the coordinate system is at the base of the knife edge. The source is a radiator of transverse
electric fields. The knife edge of height $H$ is defined to be along the $y$-axis between $y = 0$ and $y = H$. The obstacle is assumed wide enough to be considered effectively infinite in the transverse direction. Hence, this is a two-dimensional treatment. The idea is to calculate the propagation from the source to the knife edge aperture and then from the aperture to the receiver, integrating over the aperture according to Huygens' principle and the equivalence principle. The critical assumption in this formulation is that the only effect of the knife edge is to zero the fields along the portion of the wavefront that is masked by the obstacle. The aperture can thus be treated by assuming an equivalent electric current sheet (and zero magnetic current)

$$\bar{J}_s(y') = 2\hat{n} \times \bar{H}(y') $$

(3.2)

on the aperture, where $\hat{n} = \hat{z}$ is the unit normal, and $\bar{H}$ is the incident smooth earth magnetic field and is forced to zero for $y < H$ (mask condition). Primed coordinates correspond to the position of the equivalent sources, in this case the aperture. Using the far field (plane wave) approximation $\bar{H} = \bar{k}_0 \times \bar{E}/\omega \mu_0$, the current distribution on the aperture can be written as

$$\bar{J}_s(y') = -\frac{2}{\eta_0} \bar{E}(y') = -\frac{2}{\eta_0} E(y')$$

(3.3)

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free space wave impedance, and $\bar{E}$ is the incident smooth earth electric field and is zero for $y < H$. $\bar{J}_s$ is written in terms of $\bar{E}$ for later use in Green's function expressions. The electric and magnetic currents are impressed (not induced) such that in the absence of any masking, they radiate the incident propagating fields. The choice of zero magnetic current is made only for simplicity.
3.3. \textit{GENERAL INTEGRAL SOLUTION}

In fact, there are infinitely many ways to specify electric and magnetic currents over the aperture.

With the position vector in \(x\)-\(y\) space denoted \(\bar{\rho}\), the field at the receiver (for \(\bar{\rho}\) outside the sphere) for one knife edge on a sphere takes the form

\[
\overline{E}_1(\bar{\rho}) = i \omega \mu_0 \int_H dy' \overline{G}(\bar{\rho}, \bar{\rho}') \cdot \overline{J}(y')
\]  

(3.4)

where

\[
\overline{G}(\bar{\rho}, \bar{\rho}') = (\bar{I} + \frac{1}{k_0^2} \nabla \nabla)g_{\text{eff}}(\bar{\rho}, \bar{\rho}')
\]  

(3.5)

is the two-dimensional dyadic Green's function taking into account the earth's surface through dependence on

\[
g_{\text{eff}}(\bar{\rho}, \bar{\rho}') = F(\bar{\rho}, \bar{\rho}') g(|\bar{\rho} - \bar{\rho}'|)
\]  

(3.6)

which is the two-dimensional effective scalar Green's function for radiation in the presence of the sphere. In Eqn. (3.6), \(F\) is the smooth earth propagation loss factor, and \(g\) is the two-dimensional free space scalar Green's function given by

\[
g(|\bar{\rho} - \bar{\rho}'|) = \frac{i}{4} H_0^{(1)}(k_0|\bar{\rho} - \bar{\rho}'|) \sim \frac{i}{\sqrt{8\pi k_0 |\bar{\rho} - \bar{\rho}'|}} e^{i k_0 |\bar{\rho} - \bar{\rho}'|}
\]  

(3.7)

where the far field asymptotic form for the Hankel function is used.

Substituting Eqn. (3.3) into Eqn. (3.4) and using \(\frac{\partial}{\partial z} = 0\) yields the received
where \( A = -2ik_0 \). To review, \( E \) is the incident smooth earth electric field on the aperture, \( F \) is the smooth earth propagation loss factor from the aperture to the receiver, and \( g \) is the free space Green’s function. The bracketed part of the integrand represents the Green’s function for smooth earth to the right side of the knife edge. The field from Eqn. (3.8) can be rewritten in terms of a definite integral as

\[
\overline{E}_1(\rho) = \hat{z}A \int_0^{\infty} dy' E(y')[F(\rho, \rho')g(|\rho - \rho'|)]
\]

(3.9)

\[
- \hat{z}A \int_0^H dy' E(y')[F(\rho, \rho')g(|\rho - \rho'|)]
\]

\[= [\overline{E}_1(\rho)]_{H=0} - \hat{z}A \int_0^H dy' E(y')[F(\rho, \rho')g(|\rho - \rho'|)]
\]

(3.10)

where the first term is the smooth earth field at the receiver (i.e., in the absence of the knife edge) and the second term is the subtracted field contribution from the portion of the wavefront that is masked by the knife edge.

The effect of adding a knife edge obstacle to the smooth earth propagation problem is explicit from Eqn. (3.10), and the physics are clear as well. The second term is the perturbation of the smooth earth field by the knife edge. In the limit as the obstacle height \( H \) approaches zero, the received field approaches the smooth earth field. The presence of the knife edge can either increase or decrease the signal
strength compared to that of smooth earth (e.g., [49,50]). Knife edge gain results when the obstacle's reaching up into a region of stronger field and diffracting it toward the receiver more than compensates for its masking of part of the incident wavefront. A typical example is when the receiver is beyond the smooth earth horizon, and the tip of the knife edge serves to diffract the wave into the shadow region more effectively. In many other cases, though, the masking effect is dominant, and the knife edge results in a loss compared to the smooth earth field strength.

Eqn. (3.10) is written in terms of smooth earth fields and smooth earth propagation loss factors. To calculate the solution, the Fock series, the geometrical optics solution, or the UTD solution is used for each integration point, and the integration is done numerically. The criteria for selecting the proper solution is as follows. Given two points above the sphere, separated by range \( r \) along the surface and at heights \( z_1 \) and \( z_2 \), between which the propagation is to be calculated, the geometrical horizon range \( r_H \) is

\[
r_H = \sqrt{2a_z z_1} + \sqrt{2a_z z_2}
\]

\( r_H \) represents the minimum separation range along the surface such that the line-of-sight between the two points is masked by the curvature. The Fock series is used if

\[
r > \alpha r_H
\]

where \( \alpha \) is an empirical parameter that varies inversely with the free space wave-
length $\lambda_0$. At VHF, good results are generally obtained using $\alpha$ in the range 0.2 to 0.4.

If the Fock series is not used, a choice is made between the geometrical optics solution and the UTD solution according to the smaller of the heights $z_1$ and $z_2$. Basically, the geometrical optics solution has trouble when one of the points is very near the surface and the specular point approaches it. The geometrical optics solution is used if

$$\min\{z_1, z_2\} > \beta \lambda_0 \quad (3.13)$$

where $\beta$ is an empirical parameter ranging from 0.5 to 1.2 at VHF. If the inequality (3.13) is not satisfied, the UTD solution is employed.
Figure 5.3: General solution for a knife edge on a sphere.
3.3.2 Extension to Two Knife Edges

The formulation in Section 3.3.1 may be extended in a straightforward manner to handle multiple knife edges on a sphere. Although the extension is simple conceptually, the notation and computation for the extended solution become quite messy and involved. The solution for two knife edges on a sphere is presented here.

The problem configuration is shown in Figure 3.4. Now there are two apertures of interest, corresponding to knife edge 1 (of height $H_1$) and knife edge 2 (of height $H_2$). Note that there are two coordinate systems. The $x$-$y$ coordinate system with position vector $\rho$ has its origin at the base of knife edge 2, while the $\bar{x}$-$\bar{y}$ coordinate system with position vector $\bar{\rho}$ has its origin at the base of knife edge 1. Following the derivation of Eqn. (3.8), the field at the receiver for the double knife edge problem is written

$$\bar{E}_2(\bar{\rho}) = \hat{z} A \int_{H_2}^{\infty} dy' E_1(y') [F(\bar{\rho}, \bar{\rho}') g(|\bar{\rho} - \bar{\rho}'|)]$$  \hspace{1cm} (3.14)

where $A = -2ik_0$, $F$ is a smooth earth propagation loss factor, and $g$ is a free space Green's function, as before. Now, however, in Eqn. (3.11), $E_1(y')$ is the field along aperture 2 taking into account the presence of knife edge 1. Hence, $E_1(y')$ is found by solving a single knife edge (knife edge 1) on a sphere, and it contains an integration over knife edge 1 according to Eqn. (3.8). Thus the solution for two knife edges on a sphere includes a double integration.

Eqn. (3.14) will now be written out explicitly in terms of definite integrals along the knife edges, using the same simple trick used to obtain Eqn. (3.10). Eqn.
(3.14) is rewritten as

\[
\bar{E}_2(\bar{\rho}) = \hat{z}A \int_0^\infty dy' E_1(y')[F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]

\[-\hat{z}A \int_0^{H_2} dy' E_1(y')[F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]  

\[= [\bar{E}_2(\bar{\rho})]_{H_1=H_2=0} - \hat{z}A \int_0^{H_1} dy' E(\bar{y}')[F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]

\[-\hat{z}A \int_0^{H_2} dy' \{[E_1(y')]_{H_1=0} - A \int_0^{H_1} dy' E(\bar{y}')
\]  

\[\cdot [F(\bar{\rho'}, \rho')g(|\bar{\rho'} - \rho'|)][F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]  

where Eqn. (3.16) comes from noticing that the first term of Eqn. (3.15) is the field with knife edge 1 alone, and from writing \(E_1(y')\) in the second term of Eqn. (3.15) as a smooth earth field minus an integral along knife edge 1. Finally, then, the double knife edge over a sphere solution is written

\[
\bar{E}_2(\bar{\rho}) = [\bar{E}_2(\bar{\rho})]_{H_1=H_2=0} - \hat{z}A \int_0^{H_1} dy' E(\bar{y}')[F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]

\[-\hat{z}A \int_0^{H_2} dy' [E_1(y')]_{H_1=0} [F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]  

\[+ \hat{z}A^2 \int_0^{H_2} dy' \int_0^{H_1} dy' E(\bar{y}')[F(\bar{\rho'}, \rho')g(|\bar{\rho'} - \rho'|)]
\]  

\[\cdot [F(\bar{\rho}, \rho')g(|\bar{\rho} - \rho'|)]
\]  

in terms of only smooth earth fields and smooth earth propagation loss factors.

To evaluate Eqn. (3.17) numerically, then, the same criteria described in Section (3.3.1) are used to select the appropriate smooth earth propagation solutions.
Figure 3.4: Extended solution for two knife edges on a sphere.
3.3.3 Results

The propagation loss factor $F$ magnitude squared is plotted in Figures 3.5–3.8, to illustrate the behavior of the integral solution for a knife edge on a sphere with respect to Wait's solution and the four-ray approximation. The solution reduces to these correct limiting cases for both symmetric and asymmetric configurations and breaks new ground for geometries not handled well by the limiting solutions.

Figure 3.5 shows a symmetric case that is familiar by now, a continuation of Figure 3.2. Again, the antenna heights are 50 m, the knife edge height is 100 m, and the frequency is 100 MHz. The antennas move symmetrically apart from the knife edge. The aperture integral solution curve is plotted only between ranges 5 km and 70 km, because it exactly overlays the limiting solutions at both ends. The integration routine correctly makes the transition between the near-near solution (agreeing with the modified four-ray results) and the far-far solution (agreeing with Wait's results), by choosing the proper smooth earth solutions on either side of the knife edge.

Figure 3.6 shows an asymmetric case at 100 MHz for which the source height is 10 m, the receiver height is 30 m, the knife edge height is 50 m, the source is 30 km from the knife edge, and the receiver is moved away in range from the knife edge. When the receiver is close to the knife edge, the integration picks up the multipath nulling (reflection after diffraction), whereas Wait's solution (225 terms) does not converge. As the receiver moves farther away from the knife edge, the integration result agrees with Wait's solution, as it should. The corresponding loss
for smooth earth is plotted for comparison. The knife edge masking reduces the
signal strength with respect to smooth earth when the receiver is right behind the
obstacle, but when the receiver moves farther out, the knife edge tip diffracts the
signal into the shadow region, resulting in a small gain over smooth earth.

Figure 3.7 shows a complementary asymmetric case at 100 MHz for which the
source and receiver heights are again 10 m and 30 m, respectively, and the knife edge
is again 50 m, but the source is 5 km from the knife edge. As above, the receiver is
moved away in range from the knife edge. When the receiver is close to the knife
dge, the integration picks up the multipath nulling and agrees with the modified
four-ray result, as it should. As the receiver moves away, the four-ray approximation
ceases to be valid, but the rigorous integration routine stays accurate. Again, it
is seen from the comparison with smooth earth propagation that the knife edge
reduces the signal strength when the receiver is directly behind it and enhances the
signal strength when the receiver moves out of the nulling region.

Lastly, Figure 3.8 shows results for the extended double knife edge solution,
for frequencies 100 MHz and 500 MHz. The source is fixed at 10 m height at range
5 km from the first knife edge. The first knife edge is 20 m high, the second knife
dge is 10 m high, and the separation between the two is 10 km. The receiver
is fixed at height 10 m and is moved away from the second knife edge. Over the
ranges shown, the solution behaves quite smoothly as spherical earth loss perturbed
by a fairly uniform double knife edge factor. The higher frequency signal strength
drops off more rapidly, which is reasonable since propagation via diffraction becomes
dominant at greater ranges and lower frequencies.
3.3. GENERAL INTEGRAL SOLUTION

Figure 3.5: General solution symmetric transition between limiting cases.
Figure 3.6: General solution asymmetric transition to Wait solution limit.
Figure 3.7: General solution asymmetric transition to four-ray limit.
Figure 3.8: Double knife edge solution for 100 MHz and 500 MHz.
3.4 Discussion

In this chapter, the solution to the problem of propagation over a knife edge obstacle on a spherical earth has been completed. Motivated by the asymmetric configuration of antennas about the knife edge, a general solution has been formulated in terms of a radiation integral along the knife edge aperture, according to Huygens' principle and the equivalence principle.

The symmetric configuration of antennas about the knife edge is in fact solved without the need for the integral expression. The modified four-ray solution has been developed to extend the region of validity of the original flat plane four-ray solution such that it exhibits a smooth transition to Wait's double summation solution. The search for a suitable solution for the asymmetric case led to the radiation integral formulation, in which the smooth earth propagation between two given points is calculated using one of the three formulas given in Section 2.2. This general solution agrees with the modified four-ray approach and Wait's solution in the appropriate limiting cases. The extension to two knife edges on a sphere has been presented as well.

The knife edge serves only to block a portion of the incident wavefront. It has been shown that the presence of such an obstacle can either increase or decrease the strength of the received signal with respect to the smooth earth signal, depending on the geometry. Physically, when the receiver is in the shadow of the knife edge (i.e., the knife edge masks the line-of-sight), the signal strength tends to be reduced, whereas when the receiver is past the smooth earth horizon, the knife edge tip acts
to effectively reradiate the field into the shadow region, and the signal tends to be enhanced.

Recall that the overall motivation for this research is the need to improve an existing propagation model for real terrain. It remains to be seen if this rigorous diffraction routine will yield improved results over the original algorithm in the Lincoln Laboratory SEKE model, for actual surface profiles. This will be the topic of Chapter 4.
Chapter 4

Propagation over Actual Terrain

4.1 Introduction

In the preceding chapters, a solution was developed for the theoretical problem of propagation over knife edges on a spherical earth. Recall that the need to improve the available computer model SEKE (for actual terrain) serves as much of the motivation for this research. In this chapter, the implementation of the solution in a computer propagation model for real terrain is discussed, and comparisons with the SEKE model are made with respect to one-way propagation and ground clutter prediction.

As discussed in Chapter 1, the SEKE model computes diffraction loss by empirically weighting spherical earth loss and knife edge loss. The goal, then, is to investigate the use of the double integral solution as a possible replacement for the ad hoc weighting scheme. The integral solution is to be implemented in propagation code that models the terrain profile in the same manner as SEKE. That is, the only difference is that the new model computes the loss rigorously due to knife edges on a
spherical earth for every point. It is emphasized that only the diffraction algorithm in SEKE is in question here. Hence, cases are to be run for which the dominant propagation mechanism is diffraction. It will be shown that the new model compares quite well with SEKE for cases dominated by diffraction and tends to pick up the null at the surface more consistently.

Currently, one of the main applications of SEKE is the prediction of ground clutter. First of all, clutter modeling involves two-way propagation, and it is argued from reciprocity that the propagation loss from the radar to the target (clutter) is the same as the propagation loss from the target back to the radar. Thus, the factor $|F|^4$ represents the two-way power loss. Clutter modeling also involves the intrinsic clutter reflectivity or cross-sectional density $\sigma^0$ which is dependent on terrain class, angle of incidence, frequency, etc. Because of the success of the new propagation model for very low elevations, its use in VHF ground clutter prediction is investigated in the last part of the chapter.
4.2 Theoretical Comparison of Algorithms

In this section, the rigorous double integral solution is compared with the SEKE diffraction algorithm for a profile consisting of two knife edges on a spherical earth. Results will show that the weighting scheme employed in SEKE can be a poor approximation to the actual physics of the problem configuration.

The comparison to be made isolates the difference between the diffraction loss calculation in SEKE and the new integral solution, for the case of two knife edges on a sphere. The significance of this configuration is that SEKE effectively reduces any given terrain profile to such a representation. Hence, this comparison gives an example of the differences found between the weighting approximation and the rigorous solution, for modeled terrain.

The specific case to be presented is shown in Figure 4.1. The source is fixed at height 10 m and range 10 km from the first knife edge. The second knife edge is 20 km from the first knife edge, and both obstacles are 40 m in height. The receiver height is fixed at 10 m, and the receiver range from the second knife edge is varied. For this case, the receiver is always masked, meaning that the loss is purely diffraction loss, and both the knife edges and the earth curvature are important. Intuitively, one would expect the knife edge effect to be most significant when the receiver is in the shadow of the second knife edge. Similarly, one would expect the spherical earth effect to be most significant when the receiver is well beyond the smooth earth horizon.

The plotted curves corresponding to the knife edge contribution and the spher-
ical earth contribution are the results given by the individual diffraction routines in SEKE. That is, the knife edge curve is computed without a ground plane (semi-infinite edge diffraction), and the spherical earth curve is computed using the Fock series. When the receiver is very close to the second knife edge, SEKE predicts the loss as purely knife edge loss. When the receiver is very far from the second knife edge, SEKE predicts the loss as purely spherical earth loss. For middle ranges, SEKE interpolates between the two solutions according to an empirical scheme based on the clearance of the first Fresnel zone and the height of the knife edges. The reason for the jump in the SEKE curve at range 10 km is that at that point the first knife edge becomes the more significant one according to the SEKE criteria. The “truth” reference for this case is the rigorous integral solution. The rigorous solution indicates that the propagation is spherical earth dominated.

For ranges less than 10 km from the second knife edge, the weighting scheme used in SEKE is a poor approximation to the physics of this configuration. The knife edge contribution is weighted too strongly. For larger ranges, the approximation is better. This case is representative of the results obtained from SEKE when knife edges exist in fairly close proximity to either the source or receiver. If the source were located closer to the first knife edge, for example, the weighting algorithm would give worse results.
4.2. THEORETICAL COMPARISON OF ALGORITHMS

Figure 4.1: Comparison of integral solution and SEKE.
4.3 New Model with Rigorous Diffraction

In this section, the implementation of the integral solution in a propagation model is discussed and compared with SEKE for actual terrain profiles. The new test model has no decision tree and always computes the propagation loss as that due to two knife edges on a spherical earth. Results show that the new model agrees with SEKE most of the time while performing better for very low receiver heights.

4.3.1 Design

The test model is a straightforward implementation of the rigorous double integral solution, for use in calculating propagation loss over real terrain. The immediate goal is to see, for real terrain, whether the rigorous diffraction algorithm gives more accurate results than the existing SEKE diffraction algorithm. Hence, no decision tree is followed, no multipath routine is included, and no weighting is performed.

In order to make a fair comparison with SEKE, the same knife edge searching algorithm is employed. That is, the same knife edges are chosen as SEKE for any given terrain profile and antenna geometry. As discussed in [1], the clearance $\Delta$ of the direct ray above the terrain is calculated at every point along the profile, as is the extent of the first Fresnel zone $\Delta_0$.

$$\Delta_0 = \sqrt{\frac{\lambda_0 d_1 d_2}{d_1 + d_2}} \quad (4.1)$$

where $d_1$ is the distance from the source to the point of interest, and $d_2$ is the distance from the point of interest to the receiver. Keeping in mind that $\Delta$ is negative for
a given terrain point if that point masks the direct ray, the two most significant obstacles correspond to the two points for which the ratio $\Delta / \Delta_0$ is smallest. The knife edges are placed at those locations, their heights given by the terrain heights measured above a best-fit curve to the terrain profile. This best-fit curve is used in SEKE and the new model to compute effective antenna heights and radius of curvature, as well.

The curve fitting and knife edge placement for the terrain is illustrated in Figure 4.2 for a fictitious terrain profile. The profile represents ground elevations above mean sea level, i.e., earth curvature is not taken into account. The line fit to the terrain serves as a reference above which to measure effective heights. The radius of curvature for this profile would simply be the effective earth radius. The new model then puts knife edges of the measured effective heights at the appropriate ranges on a sphere of effective earth radius and computes the propagation loss using the double integration.
Figure 4.2: Modeling of the terrain profile.
4.3. NEW MODEL WITH RIGOROUS DIFFRACTION

4.3.2 Results

Results are shown in Figures 4.3–4.7 for VHF one-way propagation over actual terrain profiles. The site location for these cases is Beiseker in Alberta, Canada, one of the sites visited by the Phase-One radar of Lincoln Laboratory. The terrain is classified as rolling farmland.

Figure 4.3 shows propagation for the case introduced in Chapter 1 (Figure 1.4). The source height is 17 m, and the receiver is moved away at a fixed height of 10 m above the terrain. There is general agreement between the new model and SEKE. The important features of the new model predictions include (1) more nulling (because it takes into account reflections after diffraction), (2) greater sensitivity to effective receiver height, and (3) reduced signal strength beyond 60 km. The curve generated by the new model is less smooth, because it attempts to take into account the effects of the ground after diffraction, so it is highly sensitive to fluctuations in the effective receiver height. Although measurements are not available for comparison, the point is that the new model gives results that make sense physically.

Figures 4.4 and 4.5 show a case for which horizontal polarization propagation measurements at 168 MHz are available for comparison. The path profile is labeled W35 because it extends 35 km to the west of the radar location. The terrain elevation profile (measured above mean sea level) is plotted in Figure 4.4. Note that there is less than 100 m of elevation variation over a range of 35 km — the terrain is quite flat. The source (radar) is fixed at height 17 m and range zero,
and the receiver height is varied (lowered by helicopter for measurements) at fixed range 35 km. In Figure 4.5 the propagation loss predicted by the new model and SEKE is compared with the measurements. Both models do a good job for heights above 20 m, with SEKE performing a bit better, due to its excellent multipath routine. Below 20 m, however, SEKE shows an unrealistic gain in signal strength due to the knife edge factor. The new model captures the null near the surface more accurately.

Figures 4.6 and 4.7 show another case at 168 MHz. This path profile extends 55 km to the north of the radar. The terrain elevation profile is plotted in Figure 4.6. Here, the scale is very deceptive. Although there is greater dynamic range in elevations compared to the previous profile, it is only 150 m over a range of 55 km. The source (radar) is again at height 17 m and range zero, and the receiver height is varied at range 55 km. Figure 4.7 shows the predictions of the new model and SEKE compared with the measurements. Over the range of receiver heights shown, the receiver is always masked. Hence, the propagation is computed in SEKE as purely diffraction. Above 40 m, both the new model and SEKE agree well with the data. Below 40 m, SEKE again raises the signal strength (knife edge) and misses the null at the surface. On the other hand, the new model captures the trend of the null quite nicely, although the absolute numbers do not quite agree.

The fact that the new model captures the null near the ground consistently brings up the possibility of using it to predict ground clutter returns, where the clutter height would be fixed at some small value such as 1 m for predictions. This is the topic of the next section.
Figure 4.3: New model and SEKE along Beiseker profile.
Figure 4.4: Terrain profile for Beiseker W35.
Figure 4.5: New model, SEKE, and measurements for Beiseker W35.
Figure 4.6: Terrain profile for Beiseker N55.
Figure 4.7: New model, SEKE, and measurements for Beiscker N55.
4.4 VHF Ground Clutter Prediction

In this section, the use of the new propagation model for VHF ground clutter prediction is investigated and compared with results obtained from the use of SEKE. Limited testing has been performed for 5 sites, and representative results from the Beiseker site are presented.

4.4.1 Usage of Clutter Model

Clutter strength is modeled as $\sigma^o|F|^4$, where $\sigma^o$ is the clutter reflectivity and $F$ is the familiar propagation loss factor [40]. The modeling of $\sigma^o$ is quite involved, as it depends on many parameters, including the type of terrain [44]. In this work, the focus is on the modeling of the propagation factor $|F|^4$.

Figure 4.8 shows clutter strength measurements obtained from the Phase-One radar at the Beiseker site, at X-Band and VHF. All polar plots are mapped out to a maximum range (radius) of 24 km, with the top corresponding to due north. The terrain masking plot shows in white the portions of the ground that are visible from the radar fixed in the center at height 17 m. Note that there is very strong correlation between the X-Band regions of high clutter return and the visible regions. At X-Band, the reflectivity $\sigma^o$ is the dominant factor in modeling clutter, and the propagation factor is not that important; simply put, either the clutter is masked, or it is not. At VHF, there is still some correlation between high clutter and visibility, but propagation effects (multipath and diffraction) become significant as well. Hence, SEKE is especially useful in predicting ground clutter at
VHF.

The model for $\sigma^o$ is developed using terrain class information and the X-Band clutter measurements, since the propagation can essentially be factored out of the clutter measurements. The currently best available model for $\sigma^o$ is used, Lincoln Laboratory's CMT-85 random draw model [5] employing X-Band statistics. Then, assuming a fixed clutter (receiver) height of 1 m above the terrain, the propagation models (SEKE and the new model) are run along radials, sweeping in azimuth, for the whole site. It is emphasized that the same $\sigma^o$ values are used on a spatial point-by-point basis to generate the two sets of clutter predictions. Therefore, any differences between the clutter predictions are due solely to the differences in propagation loss calculations.
Terrain masking

Figure 4.8: Clutter measurements ($\sigma^o|F|^4$) at Beiseker, max. range 24 km.
4.4.2 Results

Figures 4.9–4.11 show full-site results for Beiseker: propagation loss plots, clutter strength plots, and clutter strength cumulative distribution plots. Again, the radar (source) is located at the center of the polar plots at a height of 17 m, and the clutter (receiver) is varied in range and azimuth at a height of 1 m. The radar frequency is 168 MHz.

Figure 4.9 shows the terrain elevation of the site (above mean sea level) and the magnitude squared of the propagation loss factor calculated by SEKE and the new model. The terrain is rolling, with the highest elevations in the southern region, and clear visibility for a distinct patch up north as well. The high elevation visible regions have the highest signal strength. SEKE and the new model predict similar trends. Overall, the new model predicts lower signal strength, especially in the western regions, and more nulling after diffraction. That is, the new model takes into account the close proximity of the ground surface more consistently than does SEKE.

Figure 4.10 shows a comparison between the clutter strength predicted by the CMT-85 (X-Band statistics) model together with (1) SEKE propagation and (2) new model propagation, with Phase-One clutter data as the reference. Azimuthal smearing is performed by the clutter model to match the smearing of the data due to the radar beamwidth. Both overall clutter models capture the major regions of high returns fairly well, i.e., to the north and south. Neither one predicts the discreties to the northeast and east; it is possible that these returns are from man-
made structures (e.g., a building or telephone pole) rather than terrain features. The model with SEKE overpredicts the clutter to the west and also the clutter very close to the radar. The model with the rigorous diffraction algorithm correctly reduces the predictions in those regions. The model with SEKE does a better job for the patch of clutter directly east of the radar, probably due to the superior SEKE multipath routine. Overall, the predicted clutter strength with the new propagation model is slightly lower than that with SEKE, which is a good result.

Figure 4.11 quantifies the improvement made by using the new propagation model instead of SEKE, in the clutter model. The cumulative probability for clutter strength is plotted, i.e., the probability that the clutter strength lies below the values on the horizontal axis. Note that the probability is plotted on a Rayleigh scale. The forking of the Phase-One clutter data curve indicates the uncertainty in the measurements due to noise. Within the region containing the upper 50 percent of the data (but not counting the upper 1 percent), the results using the new propagation model show a fairly uniform improvement over the results using SEKE. Also, in the lower region of mostly noise, the results using the new model are between the bounds on the data, whereas the results using SEKE lie outside. This again shows the tendency of SEKE to overpredict the signal strength and shows that the new model does a bit better in that respect.
Figure 4.9: VHF one-way propagation ($|F|^2$) for Beiseker, max. range 24 km.
Figure 4.10: VHF clutter predictions ($\sigma^o|F|^4$) for Beiseker, max. range 24 km.
Figure 4.11: VHF clutter cumulative distributions for Beiseker.
4.5 Discussion

In this chapter, a propagation model based on the rigorous double knife edge on a sphere solution has been compared with SEKE for real terrain. The site Beiseker was selected to show representative results for both one-way propagation and ground clutter prediction. The new model generally compares quite well with SEKE.

A limited amount of testing indicates that the new model captures the null near the surface more consistently than SEKE, because the surface is taken into account rigorously using the smooth earth solutions in Chapter 2. The new model also takes into account reflections after diffraction, resulting in more nulling when the receiver is masked. When applied to VHF ground clutter prediction, the new model gives a modest but consistent improvement over SEKE. In comparing the model predictions to the measurements, trends are more important than absolute numbers. The fundamental modeling limitations such as the two-dimensional terrain profile and the neglecting of smaller local terrain features (most significantly, those near the low receiver) result in unavoidable errors.

All in all, the new diffraction algorithm gives more physical results, but the cost of computation is high. Because of the double numerical integration of complicated smooth earth factors, the new model runs on the order of 10 times slower than SEKE. Thus, by no means is the new model intended to replace SEKE. Rather, this "brute force" comparison has been performed in order to get an idea of the possible improvements that come out of making the diffraction algorithm more rigorous. The rigorous routine gives better results than SEKE for certain cases,
4.5. DISCUSSION

e.g., when the receiver is low to the ground and behind a hill. Ideally, the SEKE decision tree should be modified to allow the rigorous calculation for these certain cases only.

Moreover, the rigorous diffraction calculation can be sped up by using faster solutions such as Wait's series and the four-ray approximation, when such solutions are applicable. Also, the calculation and storage of smooth earth factors can be made more efficient, and the integration itself can be made faster by better coding.
Chapter 5

Conclusions

The need to improve the Lincoln Laboratory propagation model SEKE has motivated the development of a new diffraction algorithm to rigorously take into account the presence of both the smooth curvature of the earth and significant knife edge obstacles, at radio frequencies. In this thesis, the theory of smooth earth propagation and knife edge diffraction has been reviewed, and a complete solution to the problem of propagation over knife edges on a sphere has been formulated. A new rigorous diffraction algorithm has been applied to real terrain profiles and the results compared with SEKE and measurements.

The solution for propagation over a knife edge on a sphere is based on a radiation integral along the knife edge aperture. An equivalent problem is defined in terms of an equivalent electric current distribution on the aperture, and then Huygens' principle is used to express the propagated field as an integration over the aperture. The integrand includes smooth earth propagation on either side of the knife edge, which is calculated using either the Fock series, geometrical optics approximation, or UTD solution. The primary conceptual difference between
this approach and Wait's mode-matching approach is that here the smooth earth approximate formulas are substituted for the Fock series when applicable. The tradeoff is, surefire convergence (insensitive to antenna geometry) for speed and elegance (numerical integration instead of double summation). The solution for two knife edges on a sphere contains a double numerical integration.

A new diffraction algorithm based on the double knife edge over a sphere solution has been compared with the SEKE diffraction algorithm with respect to one-way propagation and ground clutter prediction for real terrain. Because the solution method employed in the new model is more rigorous than the weighting scheme used in SEKE, the predictions of the new model tend to be more physically correct. That is, the effects of knife edge obstacles and the effects of the earth curvature are combined in the physics rather than empirically. For example, the new model incorporates reflections after diffraction and tends to be more accurate for low receiver heights.

The limitation of the new algorithm is computational speed. For the better physics, the price paid is on the order of 10 times the computer time required by SEKE. However, the new algorithm is not to be considered as a replacement for SEKE. Rather, it is considered as an alternative branch in the diffraction portion of the SEKE decision tree. The cases in which the new algorithm has been observed to perform better are when (1) either the source or receiver is closely shadowed by an obstacle, (2) the receiver is masked primarily by earth curvature but small obstacles exist, and (3) the receiver is masked by a combination of obstacles and earth curvature and is low to the ground. It is for these cases that the new algorithm
may be considered practical.

In the propagation model, an issue that has not been fully addressed is that of the effective antenna heights above the curve fit to the profile. For some cases, the curve fit may fail to capture the local trend of the terrain near the receiver, resulting in an effective receiver height that is quite different from the height above the actual ground level. With the new diffraction algorithm, this can be detrimental, because the ground surface is rigorously taken into account. However, it is not correct to always force the effective height to be the height above the terrain, either, particularly if the local terrain shows considerable fluctuation. Because of the sensitivity of the new algorithm to effective antenna heights, a new scheme should be developed to determine these heights as a function of local terrain variation as well as the overall curve fit.

On the theoretical side, there are many topics for further study. The most obvious problem is to find a more efficient method of computing the knife edge integrals. Wait's series and the four-ray approximation can be used when applicable, but there may be a more general approximate method based on the ability to readily predict smooth earth fields as a function of height. For certain height regions, it may be possible to use polynomial or rational function approximations. A more fundamental issue is that of the convergence of the Fock series. The currently used criterion for convergence is ad hoc and needs to be formalized. Once that is done, the convergence of Wait's knife edge solution will be more tractable as well.

To give some perspective, consider that the work in this thesis concerns only direct-path terrain effects on propagation. For propagation over real terrain, more
fundamental modeling issues also need to be addressed. For example, the effects of
scatterers from outside the direct path should be incorporated. Also, the refractive
index of the air is currently neglected, even though it may have significant effects
at large ranges. Finally, the antenna modeling could be made more sophisticated.
Bibliography


References


Appendix A

Specular Point and Path Difference Calculations

In this appendix, methods for calculating (1) the location of the specular point and (2) the path difference for a smooth sphere are discussed. These calculations apply to the geometrical optics solution of Section 2.2.

For the determination of the specular point, consider Figure A.1, where the scale is distorted such that all lengths and angles can be labeled. Given are the antenna heights $z_1$ and $z_2$, the range $r$, and the radius $a_e$. The problem is to find $r_1$ (and hence $r_2$) such that $\theta_1$ is equal to $\theta_2$. Two approaches will be discussed.

The first approach is described in [22]. A cubic equation is derived for $r_1$,

$$2r_1^3 - 3rr_1^2 + [r^2 - 2a_e(z_1 + z_2)]r_1 + 2a_ex_1r = 0$$  \hspace{1cm} (A.1)

which has the solution

$$r_1 = (r/2) - p\cos[(\Phi + \pi)/3]$$  \hspace{1cm} (A.2)
where

\[ p = \frac{2}{\sqrt{3}} \sqrt{a_e (z_1 + z_2) + (r/2)^2} \]  \hspace{1cm} (A.3)

and

\[ \Phi = \cos^{-1} [2a_e (z_2 - z_1) r / p^2] \]  \hspace{1cm} (A.4)

The second approach is purely geometrical. Using the law of cosines,

\[ R_1 \cos(\alpha) = a_e + s_1 \cos(\theta_1) \]  \hspace{1cm} (A.5)

\[ R_2 \cos(\phi - \alpha) = a_e + s_2 \cos(\theta_2) \]  \hspace{1cm} (A.6)

From the law of sines,

\[ \sin(\theta_1) = \frac{R_1}{s_1} \sin(\alpha) \]  \hspace{1cm} (A.7)

\[ \sin(\theta_2) = \frac{R_2}{s_2} \sin(\phi - \alpha) \]  \hspace{1cm} (A.8)

Then, setting \( \theta_1 = \theta_2 \) gives

\[ \frac{R_1 \cos(\alpha) - a_e}{s_1} = \frac{R_2 \cos(\phi - \alpha) - a_e}{s_2} \]  \hspace{1cm} (A.9)
and

\[ \left( \frac{R_1}{s_1} \right) \sin(\alpha) = \left( \frac{R_2}{s_2} \right) \sin(\phi - \alpha) \]  \hspace{1cm} (A.10)

Dividing Eqn. (A.10) by Eqn. (A.9), clearing fractions, and rearranging terms gives

\[ R_1 R_2 [\sin(\alpha) \cos(\phi - \alpha) - \cos(\alpha) \sin(\phi - \alpha)] = a_e [R_1 \sin(\alpha) - R_2 \sin(\phi - \alpha)] \]  \hspace{1cm} (A.11)

which may be written as

\[ \frac{\sin(\alpha)}{R_2} = \frac{\sin(\phi - \alpha)}{R_1} - \frac{\sin(\phi - 2\alpha)}{a_e} \]  \hspace{1cm} (A.12)

Eqn. (A.12) is then solved iteratively for \( \alpha \) (equivalently, \( r_1 \)). Yet another approach is to minimize the total reflected path length (\( s_1 + s_2 \)) as a function of \( \alpha \), according to Fermat’s principle. This approach, however, is not discussed here.

Once the specular point is located, the path difference is calculated exactly using effective distances referenced to a plane tangent to the specular point. The geometry is shown in Figure A.2. The effective heights and range are given by

\[ z_{1\text{eff}} = (a_e + z_1) \cos(r_1/a_e) - a_e \]  \hspace{1cm} (A.13)

\[ z_{2\text{eff}} = (a_e + z_2) \cos(r_2/a_e) - a_e \]  \hspace{1cm} (A.14)

\[ r_{\text{eff}} = (a_e + z_1) \sin(r_1/a_e) + (a_e + z_2) \sin(r_2/a_e) \]  \hspace{1cm} (A.15)
APPENDIX A. SPECULAR POINT AND PATH DIFFERENCE

The path difference $\Delta r$ is defined to be the difference between the direct path length and the reflected path length, i.e.,

$$\Delta r = \sqrt{(z_{1\text{eff}} + z_{2\text{eff}})^2 + r_{\text{eff}}^2} - \sqrt{(z_{1\text{eff}} - z_{2\text{eff}})^2 + r_{\text{eff}}^2} \quad (A.16)$$

Typically, for low-altitude antennas, this expression is a difference of two very large numbers of similar magnitudes, and so it is susceptible to catastrophic numerical cancellation. In the geometrical optics solution (Eqn. (2.10)), the reflection term is highly sensitive to errors in $\Delta r$, and thus it needs to be calculated in a more numerically stable manner.

The trick is to write Eqn. (A.16) as a quotient,

$$\Delta r = \frac{(z_{1\text{eff}} + z_{2\text{eff}})^2 + r_{\text{eff}}^2 - [(z_{1\text{eff}} - z_{2\text{eff}})^2 + r_{\text{eff}}^2]}{\sqrt{(z_{1\text{eff}} + z_{2\text{eff}})^2 + r_{\text{eff}}^2} + \sqrt{(z_{1\text{eff}} - z_{2\text{eff}})^2 + r_{\text{eff}}^2}} \quad (A.17)$$

$$\Delta r = \frac{4z_{1\text{eff}}z_{2\text{eff}}}{\sqrt{(z_{1\text{eff}} + z_{2\text{eff}})^2 + r_{\text{eff}}^2} + \sqrt{(z_{1\text{eff}} - z_{2\text{eff}})^2 + r_{\text{eff}}^2}} \quad (A.18)$$

where the denominator consists of a sum of the total reflected path length and the direct path length. This quotient form is numerically stable.
Figure A.1: Determination of specular point location.

Figure A.2: Calculation of path difference.
Appendix B

Half-Plane Knife Edge Diffraction

In this appendix, the semi-infinite half-plane knife edge diffraction solution is reviewed. It is the most fundamental knife edge solution and is used in the evaluation of the four-ray solutions in Sections 2.3 and 3.2, as well as in the knife edge routine in SEKE.

The problem configuration is shown in Figure B.1. It is assumed that the distances $d_1$ and $d_2$ are much greater than the wavelength $\lambda_0$ and the effective masking height $M$. In the diagram, $M$ is shown negative, as the direct ray clears the knife edge. With the $y$-axis defined along the knife edge, and the origin on the line-of-sight, the field at the receiver takes the form

$$E = A \int_M^\infty dy \frac{e^{ik_0d_1(y)}}{\sqrt{d_1(y)}} \frac{e^{ik_0d_2(y)}}{\sqrt{d_2(y)}}$$

(B.1)

according to Huygens' principle. The stationary point of the integrand corresponds to the direct ray, i.e., $y = 0$. Since the main contribution to the integral comes from
about the stationary point, the field is approximated as

\[ E = \frac{Ae^{ik_0(d_1 + d_2)}}{\sqrt{d_1 d_2}} \int_{-\epsilon}^{\epsilon} dy \, e^{ik_0 \delta_d(y)} \]  \hspace{1cm} (B.2)

where \( \epsilon \) is a small parameter, and \( \delta_d(y) \) is proportional to \( y^2 \) and is a quadratic approximation to the path length deviation. Note that the second order term is kept in the phase factor but not in the amplitude factor.

After scaling the integration variable, extending the limits of integration, normalizing to free space (where \( M \to -\infty \)), and writing \( k_0 = 2\pi/\lambda_0 \), the propagation loss factor takes the form

\[ F = \frac{(1 - i)}{2} \int_{y_0}^{\infty} dy \, e^{i(\frac{\pi}{2})y^2} \]  \hspace{1cm} (B.3)

where

\[ y_0 = M \sqrt{\frac{2(d_1 + d_2)}{\lambda_0 d_1 d_2}} \]  \hspace{1cm} (B.4)

Eqn. (B.3) is evaluated using the extensively tabulated Fresnel integral. If the direct ray is masked, then \( M \) is a positive quantity, the main contribution comes from about the tip of the knife edge, and the expression for the propagation loss factor is unchanged.
Figure B.1: Half-plane knife edge problem.