NUMERICAL SIMULATION OF THE CRITICAL
IONIZATION VELOCITY MECHANISM

by

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The 'critical ionization velocity' (CIV) of a neutral gas is related to a form of anomalous ionization first proposed by Alfvén. Although the CIV phenomenon has been verified in laboratory experiments, space-based experiments have provided only inconclusive or negative results as to the existence of CIV in the space environment. If the existence of CIV can be confirmed in space plasmas, there may be wide applications of the theory to astrophysical models such as cometary coma formation and Io's plasma torus, as well as engineering implications regarding space shuttle glow, MPD thruster operation, and thruster firings from spacecraft. The purpose of this thesis is to investigate the mechanism of CIV, develop an estimate for the rate of CIV, and apply these rate estimates to explain the discrepancy between laboratory and space experiments. Many of the results are achieved through particle-in-cell (PIC) simulations. Unlike previous PIC simulations of CIV, the current work employs an implicit PIC code to allow the use of realistic mass ratios and collisional cross sections in the simulation. The inclusion of realistic mass ratios and collisional cross sections results in realistic estimates of the characteristic times required for CIV to develop.

The results of the simulations and analyses indicate that CIV operates through the initiation of an ion beam through some form of seed ionization. This ion beam is unstable to the modified two-stream instability (M2SI). The M2SI efficiently transfers energy from the beam ions to electrons. The electrons heat to energies above the ionization energy of the neutral gas. Electron impact ionization of the neutral gas then reinforces the ion beam and leads to a positive feedback loop resulting in an anomalous form of ionization occurring on a time scale much faster than classical ionization processes.

For the laboratory experiments, the seed ionization is provided through hot electrons that are present in the initial electron distribution function. In the space experiments, the seed ionization must be provided through charge exchange reactions with the ambient oxygen plasma. The results indicate that in the laboratory experiments, the seed ionization rate is much faster than the transit time of the neutral cloud. Hence, CIV may proceed. In the space experiments, however, the charge exchange rate is on the same order as the transit time of the neutral cloud. In this case, CIV may not develop. The discrepancy between the laboratory and space experiments, therefore, is related to relative efficiencies of the seed ionization rate.
The results are used to recommend an improved experimental design for space experiments, as well as to investigate the possibility of CIV during an arcjet thruster firing. If CIV occurs during an arcjet thruster firing, the plasma density near the vehicle may be greatly enhanced. This enhancement of the plasma cloud density will adversely affect spacecraft charging, sensor operation, and other aspects of the interaction between the spacecraft and the ionosphere.

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Chapter 1

Introduction

The goal of this thesis is to develop a comprehensive theory of the ‘critical ionization velocity’ (CIV)\(^1\) of a neutral gas. This will be achieved through numerical and analytical results.

The concept of CIV has its origins in the work of Alfvén[4]. In conjunction with his theory on the formation of solar systems, Alfvén hypothesized that a neutral gas propagating across a plasma background in a magnetic field will undergo a rapid, anomalous ionization when the relative speed between the plasma and neutrals exceeds a critical value given by

\[
v_c = \sqrt{\frac{2e\phi_{ion}}{m_n}}
\]

(1.1)

where \(\phi_{ion}\) and \(m_n\) are the ionization potential and mass of the neutral atom, respectively, and ‘rapid’ and ‘anomalous’ refer to ionization that occurs on a time scale much faster than can be accounted for by classical collision theory. The result of the ionization will be to reduce the relative plasma-neutral velocity.

Although first introduced as a somewhat ‘ad hoc’ artifice for the theory of the formation of the solar system, the existence of the ‘critical velocity’ was later verified in a series of laboratory experiments. As predicted by Alfvén’s hypothesis, neutral gases

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\(^1\)The acronym CIV is defined as ‘Critical Ionization Velocity’. However, throughout this thesis, as a matter of convenience, the term ‘CIV’ will be used with some license. In general, ‘CIV’ will be used when ‘the critical velocity ionization’, ‘the critical ionization velocity phenomenon’ or ‘the critical ionization velocity mechanism’ would perhaps be better syntactically. The precise meaning should be clear from the context.
propagating across a magnetic field were indeed observed to undergo a rapid ionization once the relative plasma-neutral velocity exceeded the critical velocity for the neutral gas. Since the ionization occurred at a rate much faster than could be accounted for through classical impact ionization, it was clear collective plasma effects must play an important role in the CIV mechanism. At the time, the precise physical mechanism of the collective effects was not well understood.

Since the experimental confirmation of its existence, CIV has continued to receive much attention. Experimental, theoretical and computational results have all played an important part in increasing the understanding of the mechanism behind CIV, yet none of these techniques have succeeded in providing a comprehensive understanding of CIV:

- Experimental results have confirmed CIV’s existence in the laboratory, but ionospheric releases have failed to prove conclusively CIV’s existence in the space plasma. The discrepancies between laboratory and space-based experimental results have not been adequately explained, and the cost and difficulties of performing space-based CIV experiments make it difficult to develop a large data base of experimental results for the ionosphere. In addition, an adequate understanding of the mechanisms operating during an ionospheric gas release requires a high degree of spatial and temporal resolution in the diagnostics, something difficult to obtain in space based experiments.

- The theoretical understanding of CIV is in general based on linear or quasilinear theories. Although these theories do well in explaining the initial stages of CIV, the saturation mechanisms of CIV usually involve nonlinear plasma processes such as ion trapping. In addition, realistic cases of CIV involve complex sets of collisional interactions between different chemical species. Source/sink terms associated with the collisional effects substantially complicate the analysis of CIV. Together, the nonlinear effects associated with CIV and the complex collisional interactions make a complete theory of CIV difficult to develop.

- Realistic simulations for CIV are complicated by the wide range of time and length scales important in correctly modeling CIV. In addition, three-dimensional plasma
simulations are still possible only with a large investment of CPU time on the fastest supercomputers available. The results of these complications have been simulations using a reduced number of dimensions (one or two) and reduced mass ratios (to reduce the disparity in time and length scales in the problem). Furthermore only simple geometric situations have so far been amenable to the simulations.

Although experimental, analytic, and computational methods all have their drawbacks, the recent growth of computational power and maturation of simulation techniques make computational methods the most likely to provide insight into the full nonlinear mechanism of CIV at a reasonable cost. It is for this reason that the focus of this thesis is to investigate the CIV mechanism through numerical simulation.

1.1 The Basics of CIV

Before continuing further, it is important to provide a definition of the CIV process and to at least outline the basic hypothesis of the CIV mechanism.

In line with the original CIV hypothesis of Alfvén, CIV is defined as an anomalous ionization of a neutral gas propagating across an ambient plasma embedded in a magnetic field. The anomalous ionization occurs when the kinetic energy of the neutral gas (which forms a beam) exceeds its ionization energy (Cf. Equation 1.1), and "anomalous" means at a rate much faster than be accounted for through classical processes. The free energy source for the ionization comes from the kinetic energy of the propagating neutral beam, implying that as energy is spent in ionization, the relative plasma-neutral velocity must decrease.

Although the theory of CIV will be discussed in depth in the following chapters, the basics of the CIV mechanism as proposed by several researchers [2, 26] are stated here to help clarify the discussion in the following sections. CIV is hypothesized to occur through collective plasma processes. More specifically, a six step process is believed to be involved: 1) A neutral beam is propagating perpendicular to a magnetic field, 2) charge exchange, photo-ionization, or ionization by pre-existing hot electrons provides a source of seed ionization which creates an ion beam propagating across the magnetic

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field, 3) the ion beam is unstable to the modified two-stream instability (M2SI), 4) the instability effectively heats electrons parallel to the magnetic field, 5) the heated electrons gain enough energy to impact ionize the neutral beam, 6) the electron impact ionization of the neutral beam reinforces the ion beam, creating a positive feedback loop leading to rapid ionization of the neutral beam.

This six step process will be analyzed extensively in the following chapters. Before proceeding to this, the significance of CIV to various astrophysical processes and a review of the literature is presented.

1.2 Significance of CIV

The existence of CIV may have implications in a variety of astrophysical situations. In addition, plasma density enhancements that result from CIV may have important implications for the design and development of plasma devices and space vehicles. In this section, a brief discussion is provided of possible applications of the CIV theory as suggested by various authors. The details of each application may be found in the given references. A comprehensive review of many of these possible applications is also provided by Newell [57].

Formation of the Solar System

The roots of CIV lie in Alfvén’s theory of the formation of the solar system. As early as 1942[3], Alfvén pointed out that when the logarithm of a satellite’s gravitational potential energy is plotted versus the logarithm of the mass of the central body, the satellites fall into bands. This is true for both the sun-planets system as well as for the satellite systems of the outer planets. To explain this, Alfvén proposed that neutral gas, drawn by the gravitational attraction of the central mass, would begin to fall inward until its kinetic energy equaled its ionization energy. At this point, when the gas reached its ‘critical velocity’, the neutrals would quickly ionize, and the magnetic field would stop further inward collapse. Initially, Alfvén did not suggest a mechanism of the ionization.

Several objections were quickly raised to Alfvén’s theory. The first objection was
that thermal ionization, the only ionization mechanism considered at first, would neither occur fast enough nor would it be related to the 'critical velocity' [73]. This criticism was answered by Alfvén in 1954 [4] when he suggested a collective plasma mechanism could effectively transfer energy to electrons. Electrons could then impact ionize neutrals, reinforcing the collective mechanism and leading to a positive feedback loop. Later work by others suggested the modified two-stream instability as a possible source of the collective mechanism.

A second, and more serious, objection was that the planets were not composed of the elements whose critical velocity would fall in the band in which they are located (e.g. the critical velocity of O, C, N and Ne are near 13 km/s, which would suggest they would form the band in which Jupiter, Saturn, Uranus and Neptune lie). Such criticisms have been discussed by Alfvén as late as 1983[5], although most researchers remain skeptical.

Overall, the role of CIV in the formation of the solar system remains controversial at best. However, as one final point, it is interesting to note that De [20] predicted rings at Uranus in 1972 based on CIV. At the time, his manuscript was rejected for publication. The reviewers cited insufficient evidence of CIV for such a hypothesis of ring formation. After the discovery of the the rings at Uranus, the manuscript was published in a different journal.

Space Shuttle Glow

Observations of the space shuttle during flight have shown the existence of a significant illumination of surfaces facing in the ram direction. This illumination has a major practical significance in that it may interfere with optical and IR observations performed from the shuttle. The source of the illumination is usually associated with the production and excitation of NO ions on the space shuttle surfaces. Papadopoulos [58] has proposed that plasma effects related to CIV phenomenon may also be important. Papadopoulos points out that a small fraction of incoming ambient ions will be reflected by the space shuttle surfaces. Relative to the incoming ions, the reflected ions are counterstreaming at twice the orbital velocity (≈ 16 km/s). The critical velocity of the ambient oxygen neutrals is 12.7 km/s. The argument of Papadopoulos is that the counterstreaming ion
populations will excite lower hybrid waves. The waves will in turn heat electrons causing a rapid ionization in a manner similar to CIV.

The theoretical analysis of Papadopoulos leads to several predictions which agree with observations obtained from the plasma diagnostic package flown on STS-3. Namely, measurements of plasma densities in the vicinity of the shuttle an order of magnitude larger than the ambient density, energetic electrons (20-100 eV), ion fluxes with energies up to 30 eV, and intense, broadband electrostatic noise near the lower hybrid frequency all agree with the hypothesis that CIV processes are operating in the space shuttle environment.

Whether the counterstreaming ion population and resulting instabilities plays an important part in shuttle glow is yet unresolved. While Papadopoulos predicts many of the measurements actually obtained from the shuttle, the correlation between these observations and the existence of the space shuttle glow is still unclear. Most researchers still neglect collective plasma events in favor of direct excitation of NO by the ram flux of ambient plasma.

**Cometary Coma Ionization**

Observations of comets suggest that the formation of cometary ions occurs in a relatively small region near the comet nucleus. Considering the typical speed of gas expansion from the comet, the conclusion is reached that the ionization rate of the cometary gas must be substantially higher than can be accounted for through classical processes such as photoionization. Formisano et al. [23] have suggested that the interaction between the solar wind and cometary atmosphere present the possibility for CIV. Using quasilinear theory to estimate the efficiency of energy transfer from neutrals to electrons, they were able to derive ionization rates. The derived rates agreed well with observations of Comet Kohoutek. Galeev et al. [28] have also compared a theory incorporating CIV with measurements taken at Comet Halley. They conclude that two density enhancements measured in the inner coma of Comet Halley are manifestations of CIV.

To further investigate the role of CIV in coma formation, Chang et al. [12] have performed a laboratory simulation of a comet. In this laboratory experiment, a water
ice ball was placed in a vacuum chamber. A coaxial plasma gun was then used to create and accelerate a plasma to simulate the solar wind. The results indicated a velocity dependent secondary ionization of the water ablated from the ice ball. Chang et al. assign this velocity dependent anomalous ionization to CIV processes occurring in the experiment. Further measurements concerning the variation of the magnetic field strength also agree with the CIV hypothesis.

Although measurements of actual cometary atmospheres remain sketchy, both the theory of Formisano et al. and the laboratory simulations of Chang et al. provide evidence that CIV may play an important role in the formation of cometary coma formation. Further in situ measurements will be required to confirm the actual occurrence of CIV in comets.

**Io Plasma Torus**

Jupiter is known to have a plasma torus surrounding it at the orbit of its moon Io. It is believed that the plasma torus is produced from neutrals expelled from Io by volcanic activity. Galeev and Chabibrachmanov[27] have suggested that CIV may play an important part in the formation of the plasma torus. The difference between the orbital velocity of Io and the corotational magnetospheric plasma of Jupiter at the orbit of Io is 57 km/s. Neutral particles of the main constituents of the plasma torus (oxygen and sulfur) ejected from Io are therefore well above their critical velocities. Using the quasi-linear theory of Formisano et al.[23], Galeev and Chabibrachmanov assumed 2.5% of the neutrals’ energy was transferred to heating electrons, which is still sufficient for CIV to proceed. Furthermore, in addition to the analytic calculations, some direct observations of suprathermal electrons (a characteristic of CIV) have been made.

The importance of CIV processes in the formation of the Io plasma torus again seems plausible. Conclusive evidence however would require substantially more detailed data than is currently available.
CIV near a Space Vehicle

Thrusters fired from on-board a space vehicle provide a neutral cloud moving with a velocity of several kilometers per second with respect to the vehicle. Assuming the thruster is fired from a vehicle in... low earth orbit in the ram direction, the thrusters create a neutral beam traveling at greater than 10 km/s with respect to the ambient plasma. The critical velocity of many molecules typical of thruster firings is near this velocity (as one example, N₂, a major product of hydrazine thruster firings, has a critical velocity of 10.3 km/s) leading to the possibility of CIV during thruster firings.

Recently, Mogstad and Hastings[53] have investigated the possible effects of CIV during a thruster firing. More specifically, they consider the possibility of a thruster fired in the vicinity of a solar array. Using preliminary results of this thesis, Mogstad and Hastings have determined that a thruster fired over a solar array can lead to increases of several orders of magnitude in the plasma density if CIV occurs. Such high plasma densities near the solar array lead to increased arcing rates and loss of efficiency and lifetime of the array.

In addition to degradation of high voltage surfaces due to increased arcing resulting from increased plasma densities, the possibility of CIV during thruster firings has important implications in terms of the visible and IR spectrum of the vehicle. This is important both for observations performed from the vehicle and observations of the vehicle. CIV leads to the production of hot electrons which both excite and ionize ambient neutrals. The excitation and de-excitation of the neutrals will lead to increased visible and IR radiation from the vicinity of the space vehicle during a thruster firing. Similar to space shuttle glow, this increased radiation may affect astrophysical or other optical observations being performed. In fact, CIV during a thruster firing leads to the possibility of the excitation of a wide variety of neutral molecules. These excitations will produce a broad spectrum of optical emissions, making optical or IR observations difficult to correct for CIV by filtering. The increased radiation from the vicinity of the vehicle will also significantly change its optical and IR signature. The change in the radiation signature may have important implications for tracking and military applications.
MPD Thrusters

Magneto Plasma Dynamic (MPD) thrusters operate by accelerating a plasma by a Lorentz force. In a typical self field MPD device (i.e. an MPD device with no externally applied B-field), an electric field is applied transverse to the gas flow. The electric field causes transverse and parallel currents that generate a self-consistent magnetic field. The overall effect of this configuration is a net accelerating force on the plasma. Choueiri et al.[13] have suggested that the currents in an MPD thruster operating at less than full ionization represent a situation in which CIV may be possible. Choueiri et al.'s work focuses on proving that the modified two-stream instability may operate in the MPD thruster. Their results indicate that, under certain situations, CIV may indeed be possible in MPD thrusters and may place a limitation on the exhaust velocity.

Hastings and Niewood [32] have also studied the operation of the M2SI in MPD thrusters. This work did not focus on CIV per se, but only on the possibility of sustaining the M2SI. Using linear and non-linear theory, they conclude that the M2SI may indeed exist in MPD thrusters. The results of the operation of the M2SI will be seen as an increase in the plasma resistivity and a growth of voltage fluctuations at the lower hybrid frequency.

Although no experiments on MPD thrusters have focused on experimentally investigating CIV, Choueiri et al. do cite some indirect experimental evidence for CIV in MPD thrusters. The focus of their remarks is on the limitation of exhaust velocity if CIV occurs. According to Choueiri et al., anomalous ionization due to CIV in an MPD thruster will decrease the the electric fields as the newly formed ion-electron pairs separate in the magnetic field. (Although the ion gyroradius is on the order of the length of the thruster, under the assumptions used by Choueiri et al., enough separation of the ion-electron pairs will occur to significantly affect the electric field.) The decrease in the electric field will also decrease the velocity of primary (as opposed to 'newly born') ions. For operation of an MPD thruster at less than full ionization of the neutral gas, CIV should therefore limit the exhaust velocity to the critical velocity. Once full ionization is achieved, the limitation is removed. Evidence of this velocity limitation is cited by Choueiri et al. in operation of both the half-scale benchmark thruster and the half-scale
flared anode thruster at Princeton. The results of these experiments indicate a velocity limiting mechanism for operating currents which provide less than full ionization of the neutrals. Singular behavior and the onset of voltage fluctuations occur as the current is increased to near the point at which full ionization is achieved. Unfortunately, the indirect evidence provided by Choueiri et al. is not sufficient to positively identify the singular behavior near the point of full ionization with CIV mechanisms.

Finally, Choueiri et al. point out that the onset of voltage fluctuations in early thrusters led to limitations on the current corresponding to exhaust velocities of approximately the critical velocity. However, improved geometries have been developed which allow for operation at current levels giving exhaust velocity substantially in excess of the critical velocity. The conclusion of Choueiri et al. is, therefore, that CIV does not constitute a limitation to MPD operation. The precise effect of energy losses to anomalous ionization during low current operation are not clear, however.

Further experimental work is clearly needed to determine the precise role of CIV in MPD operation. Several points which need investigation are the relative drifts of different species inside the thruster, the frequency spectrum of voltage fluctuations near onset, and the correlation between ionization level and exhaust velocity. Together, further investigation of these points should help determine if CIV is indeed being observed in MPD thrusters.

1.3 CIV Experiments

Having discussed the relevance of CIV, attention now returns to the CIV process itself. In the next several sections, a review is provided of experimental, theoretical and numerical work performed on CIV. In this section, the review begins by considering the experiments that have been performed in both the laboratory and space environment.

1.3.1 Laboratory Experiments

Laboratory experimental investigations of CIV began as early as 1960. The real “core” of the experimental basis of the CIV mechanism involves the experiments of Danielsson [18],
Danielsson and Brenning [19], and Brenning [10]. In the following, these experiments will be referred to as I, II, and III, respectively.

An identical experimental set-up was used in I and II, a schematic of which is shown in Figure 1-1. The only difference between I and II was the use of some additional diagnostics in II. In these experiments a hydrogen plasma was created and accelerated down a drift tube along a magnetic field. As the plasma continued down the drift tube, the orientation of the magnetic field was slowly rotated from axial to transverse. The rotation of the magnetic field caused ions and electrons to move in opposite directions a distance on the order of their gyroradius. As the ions and electrons separated spatially in the rotating magnetic field, a self-consistent $E$-field was established perpendicular to the magnetic field. Although much of the plasma was lost, the remainder was therefore polarized and continued to propagate across the magnetic field. As the plasma propagated across the magnetic field, it impinged upon a cloud of neutral gas (usually helium). The situation in the test section is therefore similar to the hypothesized scenario for CIV: a neutral cloud drifting relative to a plasma in a magnetic field (note that in the hypothesis of Alfvén the neutrals are drifting with respect to a plasma which is stationary in the frame of the magnetic field, which is different from these experiments).

The parameters of the experiment were chosen so that the plasma-neutral gas interaction would be collisionless (Table 1.1). Hence, classically, no interaction should be expected between the drifting plasma and the neutral cloud. The experimental results, however, show something quite different.

Figure 1-2 shows a plot of the initial velocity of the impinging plasma versus the 'final' velocity of the plasma (as measured 1 cm behind the center of the neutral cloud). As shown in Figure 1-2, if the initial plasma drift velocity was below the critical velocity ($v_{\text{init}} < v_c$), no interaction was observed between the plasma and neutral gas, and the plasma simply drifted through the neutral cloud. However, for initial plasma drift velocities in excess of the critical velocity ($v_{\text{init}} > v_c$), the plasma was observed to brake to near the critical velocity. In these cases, the final velocity was almost independent of the initial plasma drift velocity.

A more complete picture of the velocity of the plasma is given by Figure 1-3. Figure 1-
3a shows an actual recording of the plasma velocity as a function of time obtained in the experiments at a position behind the center of the neutral cloud. The recording shows the initial arrival of the plasma as a quick increase (note the recording is inverted; ‘increasing velocity’ is downward) in the plasma velocity to a sharp peak. After the peak is reached, the velocity quickly drops to near the critical velocity where it is maintained until the last plasma traverses the probe. Figure 1-3b uses the velocity recordings from different positions in the cloud to develop a profile of the velocity through the neutral cloud. (The center of the helium cloud is denoted as $z = 0$, and the helium cloud has an axial extent of about 5 cm.) This figure from Danielsson and Brenning [19] shows the peak velocity as obtained from the recordings for the first three points and the plateau velocity for the other points. Clearly, a strong plasma-neutral interaction occurs within the first several centimeters of the front of the neutral cloud, braking the plasma velocity to near the critical velocity within about 5 cm.

Several other diagnostics provide useful insights into the plasma-neutral interaction observed in these experiments:

- **Change in the Polarization.** Electrostatic probes were placed in the test section to investigate the charge distribution. As the plasma first reached the test section, the expected distribution of charge, consistent with the polarization of the drifting plasma, was measured by the probes. As the plasma-neutral interaction begins, the signal from the positively charged portion of the test section decreases while the negatively charged portion even changes polarity. Danielsson [18] has interpreted this decrease in polarity as an effect of ionization of the neutral helium. Assuming the ion gyroradius is small compared to the radius of the experimental setup, as the helium ionizes, newly formed ions will be displaced one gyroradius toward the electron side, while the electrons remain in essentially the same position. This effectively "shorts out" the polarization and leads to a decrease in both the polarization of the cloud and the drift velocity of the plasma.

- **Radiation Emission.** Four emission lines of helium were observed during the plasma-neutral interaction in order to gain some understanding of the characteristic energy of the electrons. The results of the emission study indicate that during
the interaction, a population of hot electrons develops. The characteristic energy of these electrons is approximately 100 eV. The polarization of the helium lines suggests that the electron distribution function is highly anisotropic with most of the heating occurring parallel to the magnetic field.

Since the initial energy of the electrons was 5-10 eV, it is clear from these results that a strong heating mechanism must be operating during the interaction. Since the free energy source to heat the electrons must be the relative plasma-neutral drift energy, it is also clear that some form of collective mechanism must operate which effectively transfers energy from the drifting ions to the electrons. This mechanism heats electrons mainly parallel to the magnetic field.

- Plasma density increase: The results for the radiation emission also led Danielsson and Brenning[19] to an estimate of the increase in the plasma density during the interaction. At the peak of the interaction, they estimate that the density of hot electrons is seven times the original density of the plasma. Clearly, a strong anomalous ionization occurred which led to increases in the plasma density.

- Effect of the magnetic field: In order to judge the effect of the magnetic field, several test runs were made without an applied magnetic field. In these runs, none of the diagnostics recorded evidence of an anomalous interaction between the plasma and neutrals: the plasma simply collisionlessly drifted through the neutrals.

This last observation established that a magnetic field is necessary for CIV to operate, however the lower limit on the strength of the magnetic field is not clear. The purpose of experiment III was to further investigate the effect of magnetic field strength on the CIV process. In III, a device similar to I and II was again employed. Experiments similar to I and II were performed but with lower magnetic field strength and higher plasma density. This effectively changed the ratio of the electron cyclotron to electron plasma frequency ($\Omega_c/\omega_p$) from about 1.0 in I and II to about .05 in III.

For the lower magnetic field case, the results were substantially different from I and II. In fact, neither the plasma velocity nor the electron energies indicate that CIV occurred. The lower magnetic field strength in these experiments increases the beam ion gyroradius.
substantially. In fact, the magnetic field strength was low enough in these experiments that the ion gyroradius was of the same order as the diameter of the experimental test section (the ions were effectively unmagnetized). Because of this, the production of beam ions would not be expected to change the polarization and reduce the plasma-neutral velocity as in the high magnetic field case. Effects such as an increase in ionization would still be expected, however, if CIV was operating. The lack of a measurable increase in ionization or radiation emission indicates that the M2SI was not operating in these experiments. The results of the experiments were discussed in terms of the stabilization of the modified two-stream instability (M2SI) by electromagnetic effects by Brenning [11]. Under the hypothesis that the M2SI is the basic mechanism operating during CIV, CIV will not be observed if the M2SI is stabilized by electromagnetic effects. (The effect of electromagnetic waves and a finite \( \beta \) are discussed more fully in Section 2.1.2.) The stabilization of the M2SI by electromagnetic effects leads to a criterion for CIV of

\[
\frac{\omega_{pe}}{\Omega_e} < \frac{c}{U} \left( \frac{m_e(1 + \beta_e)}{m_i} \right)^{1/2}
\]

(1.2)

where \( U \) is the beam ion velocity, \( \omega_{pe} \) and \( \Omega_e \) are the electron plasma and cyclotron frequencies, respectively, \( c \) is the speed of light, \( m_e \) and \( m_i \) are the mass of the electron and ion respectively, and \( \beta_e = n_e T_e / (B_0^2 / 2 \mu_0) \) is the electron beta. This criterion may also be written as \( U < v_a \sqrt{1 + \beta_e} \) where \( v_a = B_0 / \sqrt{\mu_0 \rho_m} \) is the Alfvén velocity ( \( \rho_m \) is the mass density). This expression indicates that the M2SI will be stabilized if the phase speed of the dominant wave of the M2SI is greater than the Alfvén speed (the wave phase speed \( v_p \sim U \)). Physically, when the wave phase speed exceeds the Alfvén speed, the electrons can no longer coherently interact with the waves in the M2SI. Since electrons can no longer coherently interact, the M2SI is stabilized.

Figure 1-4 shows a comparison of this criterion with results from a variety of CIV experiments. Reasonable agreement with observations of CIV is obtained, although a wide variety of experimental and diagnostics techniques were employed in the experiments compared. In general, these results of Brenning indicate that the magnetic field must be strong enough to inhibit the electromagnetic stabilization of the M2SI. This is
also consistent with the theoretical understanding of CIV as well as the numerical results of Machida and Goertz that will be discussed below.

1.3.2 Space Experiments

As discussed in the previous section, experiments have confirmed the existence of the CIV mechanism in the laboratory. Because of the possible importance of CIV in astrophysics, a natural extension of the experimental process is to consider experiments in the ionosphere. This section describes a series of space-based experiments that have been performed.

Space-based experiments for CIV have consisted mainly of either radial or conical shaped charge releases of barium or strontium performed from sounding rockets. Barium and strontium are good choices for these releases because of their low critical velocities ($v_c = 2.7 \text{ km/s for Ba, and 3.5 km/s for Sr}$). Ground based optical diagnostics (filtered TV) is the most prevalent method of observing the experiments, although some experiments have also included in situ measurements of densities and electric fields. The general outline of the experiments is similar (e.g. see Figure 1-5). A sounding rocket is launched carrying a payload of a shaped charge release. Since photoionization will contaminate the results of the experiment, but sunlight is desirable for optical diagnostics, the charge release explodes just below the solar terminator with the explosion directed at an angle to the magnetic field. By having the release occur just below the solar terminator, ions formed through CIV should quickly travel up the magnetic field line along which they were created and emerge into sunlight where they can be detected through filtered TV diagnostics. Ions formed through slower processes such as charge exchange should appear as a more diffuse ion population rising into sunlight. Finally, as the remaining neutrals reach sunlight, they should undergo photoionization. Overall, then, the expected results for the optical diagnostics are an intense streak near the magnetic field line connected to the release point (the ‘CIV’ portion), a less intense, more diffuse signature representing ions created through charge exchange, and finally, a large population photoionizing as they rise into sunlight well away from the release point magnetic field line.

The first experiment specifically designed to test the CIV hypothesis in space was
"Porcupine" [31]. This experiment was a barium shaped charge release carried out on March 19, 1979 from ESRANGE/Kiruna. The geometry of the release is shown in Figure 1-5. The barium release took place approximately 100 km below the solar terminator, and the angle between the axis of the release and the magnetic field direction was 28°. Although the velocity distribution of the released neutrals was not known precisely, Haerendel[31] estimated that a "large fraction" of the ions had transverse velocities in excess of the critical velocity of 2.3 km/s.

Figure 1-6 presents the main results of the Porcupine experiment. The relative intensity of two densitometer tracings of the ionized barium cloud are shown. The first tracing was taken 9 s after the release, the second at 30 s after the release. The tracings show a large initial ionization occurring within 15 km (horizontal distance) of the release point. Beyond 15 km, a substantial component of the ionization occurs through photoionization. An integration of the data led Haerendel to an initial estimate that 30% of the ions with transverse velocities in excess of the critical velocities ionized within the initial 15 km[31]. Subsequent analysis led to an improved estimate of 20% ± 10%.

In addition to the densitometer tracings, two in situ packages provided diagnostics for the Porcupine experiment. The purpose of these packages was to record the $E$ and $B$ fields, as well as to measure the hot electron population that is known from laboratory experiments to be present during CIV. The configuration of the in situ packages was such that they did not interact directly with the expanding neutral cloud. Instead, the in situ packages were only able to measure fringe effects from the experiment that were several orders of magnitude reduced from the estimated peak effects. The most revealing measurements from the in situ packages concern the $E$ and $B$ field components. Significant perturbations of all six components of $E$ and $B$ were observed. The magnitude of the perturbations were in the range 6 – 20 mV/m and 10 – 30 nT, respectively. The perturbations are assigned by Haerendel to restraining of hot electrons along the magnetic field ($E_{\parallel}$), diamagnetic effects ($B_{\parallel}$), and the excitation of shear Alfvén waves ($E_{\perp}$ and $B_{\perp}$). Since only fringe effects were measured it is hard to conclude too much from these field measurements. However, in consideration of the electric field measurements, Haerendel does conclude that the ions produced during Porcupine could not be produced near the
release point and then redistributed over the 15 km. Instead, the drift velocities were small enough to conclude that the ions were produced along the field line at which they appeared above the terminator.

Overall, Haerendel concluded that CIV provided the only possible mechanism capable of producing the ions within 15 km of the release. Neither hot electrons produced during the explosion nor pre-existing hot electrons in the ambient plasma could account for the ionization rate observed. In reaching this conclusion, however, Haerendel did not include the possibility of contamination of the results by charge exchange between the fast barium neutrals and ambient oxygen ions. In these experiments, ions produced through this charge exchange reaction would be indistinguishable from ions formed through CIV. Many researchers have assumed the charge exchange cross section to be too small ($\mathcal{O}(10^{-17})$ cm$^2$) to account for any substantial ionization during the experiments. Recently, however, this assumption has been questioned by Swenson et al[72]. Although the barium-oxygen charge exchange cross section has not been directly measured in the laboratory, based on arguments using the calcium-oxygen charge exchange cross section, Swenson et al. suggest that the barium-oxygen charge exchange cross section may be as large as $10^{-14}$ cm$^2$. Such a large cross section would lead to severe contamination of the Porcupine results. More will be discussed about this possible contamination in Chapter 5 when simulations are compared to experimental results. For the moment, it is simply stated that the results obtained by Haerendel may not be as clear an example of CIV as first determined.

Although not designed specifically to test the CIV hypothesis, an experiment subsequent to Porcupine was also expected to show evidence of CIV. "Bubble Machine" was a radial Ba charge doped with approximately 1% strontium. It was launched on March 25, 1981 from Poker Flats. The release was performed in full sunlight so that photoionization of the barium could be expected. Strontium does not photoionize. Since the explosion of the charge should lead to Sr velocities in excess of the Sr critical velocity, this experiment was expected to lead to CIV of the Sr. Due to malfunction of the nose cone, most instrumentation did not acquire the release until about 10 s after burst. Photometer passes across the strontium/barium cloud 13 s after the release showed a definite peak
at the 4077 Å line of SrII. The intensity of the measurement led to an estimate that as much as 50% of the strontium may have ionized. Due to uncertainties in the viewing geometry and the low absolute yield of Sr ions, however, the error in the estimate may be as large as a factor of three. Hence, although the observed ionization of the strontium was consistent with CIV, the large error range in the measurements and lack of in situ field or particle measurements make it hard to accept this experiment as definitive proof of CIV.

The next experiment designed specifically to test for CIV was “Star of Lima” [74]. Based on the belief of positive results from Porcupine, Star of Lima was a conical Ba release launched March 21, 1983 to an altitude of 428 km from Punta Lobos, Peru. Diagnostics included TV observation as well as an in situ daughter package located approximately 2 km from the release point. The barium release was directed downward along the magnetic field lines toward the instrumented payload. Roughly 50% of the $10^{24}$ Sr atoms released were estimated to have transverse velocities in excess of the critical velocity.

The sounding rocket overperformed and exposed the release to higher levels of sunlight than planned. An integration of the brightness from the TV diagnostics indicated that about 1/40th of the Ba was ionized, easily accounted for by photoionization from the unexpected level of sunlight. Although the ionization rate could be accounted for through mechanisms other than CIV, the in situ electric field measurements did indicate the generation of waves near the lower hybrid frequency of Ba. Such waves would be expected to be a characteristic sign of CIV. In addition, electrons with energies up to 50 eV were detected, indicating that some form of collective plasma process must have been present since photoionization could not account for these energetic electrons.

The conclusions reached for Star of Lima are that although conditions were designed to be ideal for CIV, none of the optical or in situ measurements provided evidence of anomalous ionization. Electric field and energetic electron measurements, however, suggest that some elements of the CIV mechanism were operating.

The “Star of Condor” release was performed in conjunction with Star of Lima[77]. Star of Condor was a radial strontium release designed to be similar to Bubble Machine,
producing Sr moving at all angles to the magnetic field. Unlike Bubble Machine, this experiment did not use any barium. The release occurred at an altitude of 571 km above Punta Lobos, Peru on March 23, 1983.

Since Sr does not photoionize on the time scales of concern for these experiments and roughly 50% of the released Sr was estimated to have velocities in excess of the critical velocity, this release was expected to provide an excellent example of the operation of CIV in the ionosphere. The results, however, indicate virtually no ionization occurred. As with Star of Lima, this release provided no evidence of anomalous ionization. Since no in situ package was present in this experiment, no data is available for electric field or energetic electron measurements.

Further attempts to repeat the results of Porcupine were carried out during the "CRIT-I" releases [71]. CRIT-I consisted of two conical barium releases performed at an altitude of about 380 km off of Wallops Island on May 13, 1986. The releases were performed approximately 45 km below the solar terminator directed upward toward sunlight. The angle between the axis of the jet and the magnetic field was 45°. Diagnostics consisted of a variety of ground based optical observations as well as two in situ instrumented payloads. The main payload was located along the axis of the neutral release several kilometers from the release point. A subpayload was located above and outside the jet along a magnetic field line approximately 1 km from the release point. Hence, ions formed 1 km from the release would travel along the magnetic field and reach the subpayload.

Instead of a peak in ion production observed near the release point as in 'Porcupine', this experiment observed an ion cloud of roughly uniform brightness between the release point and the intersection with the solar terminator. The number of ions produced were estimated to follow

\[ N_i = N_n(1 - \exp^{-t/\tau}) \]  

(1.3)

where \( N_n \) is the number of neutral particles. The time constant \( \tau \) was estimated at 1800 s.

Although there is evidence that ions were formed below the solar terminator, it is not clear that CIV was operating in this experiment. The diffuse nature of the ion production
seems to indicate charge exchange with background oxygen ions may be important in this experiment. In the analyses of the experiment, Stenbaek-Nielsen et al. [71] assumed a barium-oxygen charge exchange cross section of $2 \times 10^{-18}$ cm$^2$ to obtain a time constant for charge exchange of 70000 s. Their conclusion therefore was that charge exchange could not account for the observed ionization rate. As mentioned in conjunction with Porcupine, however, this charge exchange cross section has recently been suggested to be as large as $10^{-14}$ cm$^2$. Using this cross section leads to a time constant for charge exchange of 1400 s. Hence, if the supposition of the large charge exchange cross section is correct, the observed diffuse ionization can be accounted for without invoking the CIV mechanisms.

In conclusion, CRIT-I can not be considered as proof of CIV. The diffuse nature of the ionization seems to indicate processes other than CIV were operating. As will be discussed in Chapter 5, the observed non-solar produced ions could likely be a results of Ba-O$^+$ charge exchange reactions.

The "Sr90" release was carried out at the same time as CRIT-I[76]. Sr90 was launched from Wallop’s Island on May 13, 1986. The conical Sr release occurred at an altitude of 539.6 km, at an angle of 45° to the magnetic field. Diagnostics for the experiment were provided through ground based optical observations. A faint field aligned streak was detected, but it could be accounted for through photoionization. If ions produced by CIV were present, the maximum yield was estimated to be 0.18% of the Sr. Like other Sr releases, Sr90 failed to provide evidence of the existence of CIV.

“CRIT-II” was launched May 4, 1989 from Wallop’s Island [70]. Like CRIT-I, it also consisted of two barium shaped charges. Similar results were obtained in each release. The releases were performed 100 km below the terminator at an angle of 58° to the magnetic field. Roughly $10^{23}$ atoms were estimated to have velocities in excess of the critical velocity. Diagnostics included ground based optical monitoring.

In these experiments, the diagnostics indicated some enhanced ionization near the point of release. Ionization rates of about 0.8%/s were observed. Approximately 0.6%/s of this was attributed to ionization processes other than classical ionization. This provides a time constant of roughly 170 s. As mentioned above in relation to other barium
releases, however, the charge exchange cross section for barium and oxygen is highly uncertain. With the new proposed cross sections or $10^{-14} \text{ cm}^2$, Swenson et al. [72] have shown that the observed ionization could be accounted for by charge exchange.

As with the other barium releases, the diffuse nature of the ionization observed in CRIT-II and the possibility of contamination of the results by charge exchange leave the interpretation of the mechanism responsible for the production of the observed ions in some doubt. It is not at all clear that CIV was the mechanism through which "enhanced" ionization occurred.

Finally, some of the most recent ionospheric releases have been performed as part of the CRRES program. CRRES is a satellite designed to perform magnetospheric studies as well as to test the CIV hypothesis. Over a several month period, barium releases were performed at different altitudes and different ambient conditions. On board diagnostics supplemented by ground based observations were used to analyze the results of the releases. The final results of these experiments are not yet available. Some preliminary data does however indicate that CIV was not observed.

Up to this point, all of the experiments discussed have been performed through shaped charge releases from sounding rockets. A very different methodology was used in the recent IBSS-CIV experiments. The IBSS releases used the space shuttle as an experimental platform. The concept was to release four different neutral gases from canisters in the shuttle bay. Since the shuttle travels at approximately 8 km/s with respect to the ambient plasma, and a gas release would give neutral gas velocities of about 2 km/s with respect to the shuttle, the neutral velocity should be about 10 km/s with respect to the ambient plasma. Such a high velocity would be in excess of the critical velocity of several simple molecules. The gases chosen for these experiments were NO, Xe, Ne and CO$_2$. CO$_2$ and NO were chosen because of their potential for polymerization during expansion through the release nozzle. Murad et al. [55] have suggested that the polymers make good candidates for CIV since the increased effective molecular mass decreases the critical velocity. Xe has a low critical velocity and is a good candidate for a gas likely to undergo CIV during space releases. Ne was a control gas that was not expected to ionize.
IBSS-CIV flew on board STS-39 in April, 1991. Gas releases were carried out both parallel and perpendicular to the shuttle velocity vector in both darkness and full sunlight. The releases continued for a duration of approximately 10 s with typical release rates of about 1/8 to 1/2 mole/s. Diagnostics consisted of optical observation from the shuttle as well as optical observations and field measurements from an instrumentation platform positioned out board of the shuttle bay.

Unfortunately, due to hardware malfunctions during the shuttle flight, the data obtained from IBSS is limited. Currently, there is some indication of negative results, although a more conclusive result is not yet available.

Although the concept of using the shuttle as a platform for performing CIV experiments seems a valuable approach, the hardware problems of IBSS-CIV limit the use of this experiment in drawing conclusions about the existence of CIV in the ionosphere.

1.3.3 Summary of CIV Experiments

A variety of both laboratory and space based experiments have given insight into the CIV process. The results of these experiments are now briefly summarized.

Laboratory Experiments:

The laboratory experiments have indicated that a neutral gas (usually hydrogen, although some heavier gases have been tested [6]) propagating across a magnetized plasma background will undergo an anomalous ionization in agreement with the hypothesis of Alfvén. The main characteristics indicative of this anomalous interaction are

- braking of the relative plasma-neutral velocity to the critical velocity of the neutral gas,
- heating of the electrons parallel to the magnetic field, and
- rapid increases in the plasma density.

In addition to the characteristics of CIV, laboratory experiments have indicated that a limit exists on the strength of the magnetic field: the strength of the magnetic field must be such that it allows the M2SI to operate in the experiment.
Space Experiments:

Space based experiments have not been as successful in providing evidence of CIV in the ionospheric plasma. None of the experiments performed provides conclusive evidence of CIV. Tables 1.2 and 1.3 summarize the parameters and results of the different experiments.

The results of space based experiments indicate

- barium releases are more effective in producing "enhanced" ionization. Other than perhaps Bubble Machine, strontium releases have not produced any signs of CIV.

- many of the experiments do produce characteristic signs of CIV such as lower hybrid waves and hot electrons even when the experiment does not lead to explosive ionization.

- enhanced ion production rates seem to be empirically related to higher ambient plasma densities. The releases observing the highest degree of ionization (Porcupine and CRIT-II) were performed at the highest background densities.

Finally, as mentioned in conjunction with several of the releases, much of the diffuse ion production observed in the barium releases may be attributable to charge exchange reactions. Unfortunately, the barium-oxygen charge exchange cross section has not been measured.

1.4 Comparison of Laboratory and Space Experiments

Laboratory experiments provide examples of the existence of CIV while space experiments have in general shown inconclusive evidence of CIV. In considering the reasons for this inconsistency, it is important to realize that substantial differences exist in the laboratory and space experiments.

The most obvious difference is the reference frame change between the laboratory and space experiments. In laboratory experiments, the neutral gas is stationary in the reference frame of the magnetic field. The plasma is polarized and drifts across the magnetic field as it impinges on the neutral gas. For the space experiments, the ambient
plasma is stationary in the frame of the magnetic field. The shaped charge explosions create a neutral gas propagating across the ambient plasma.

Although there is clearly a difference in the reference frame for the two processes, the effect of this difference on at least the initial CIV process is not obvious. As mentioned in the overview of CIV, CIV should initiate through a seed ionization process which creates a weak ion beam driving a M2SI. At least at the initial stages of CIV, before sufficient ionization to "short out" the polarization occurs in the laboratory experiments, the process is equivalent to the space experiment reference frame, but with a steady $E$ field applied in the direction perpendicular to the velocity vector. The application of this steady $E$ field should not affect the operation of the M2SI or the response of the electrons. Initially, then, the electrons should heat equivalently in either reference frame.

The main effect of the different reference frames will probably concern the development of the plasma-neutral relative velocity after ionization begins. The braking of the relative velocity in the laboratory experiments involves the "shorting" of the charge distribution which creates the polarization of the plasma. As long as ionization is proceeding, the polarization of the plasma and the drift velocity across the magnetic field continue to decrease: the plasma will brake towards the "critical velocity" where ionization is no longer possible. An analogous situation in the space experiments involves the generation of a "pickup" current across the field lines. The pickup current results from the separation of newly formed ion-electron pairs. When the ion-electron pairs are formed, they separate in the plane perpendicular to the magnetic field at a distance on the order of the ion gyroradius. As long as ionization is occurring, the offset of the ions with respect to the electrons effectively creates a current flowing perpendicular to the magnetic field. This pickup current creates a $j \times B$ force which tends to accelerate the bulk of the plasma and reduces the relative plasma-neutral velocity. Like the "shorting" of the polarization in the laboratory experiments, this accelerating force continues until the ionization stops, i.e. until the relative plasma-neutral velocity reaches the critical velocity.

Because of the complex coupling between the newly created and ambient plasmas in the space experiments, it may be much easier to short out the polarization of the
laboratory experiments than to create a polarized cloud in the space experiments. If polarization of the plasma cannot be achieved, one would expect heating of the electrons and an initial anomalous ionization of the neutrals, but unlike the laboratory experiments, the relative neutral-plasma velocity would not be observed to asymptote to the critical velocity. However, since measurements taken during the space experiments are designed to look for hot electrons and anomalous ionization and not the critical velocity per se, the results of the space experiments should not be seriously affected by the different reference frame.

Overall, the precise role of the reference frame change in explaining the discrepancy between laboratory and space experiments remains somewhat sketchy. However, considering the large amount of analytic and simulation results which predict CIV in the space environment, it does not seem that the reference frame change is the main inhibition against CIV occurring in the ionosphere.

Another major difference between the experiments involves the neutral density. In the laboratory experiments, the neutral gas cloud fills a region of the experimental test section with a uniform density. Hence, as the plasma traverses the test section, it experiences a volume of constant neutral density. In the space experiments, the situation is substantially different. When released from a point source, the neutral gas of the space experiments will expand outward in a conic section. During this expansion the neutral density drops as $1/r^2$ where $r$ is the radial distance from the release point. As will be discussed in Chapter 5, this drop in neutral density in the space experiments will limit the distance from the release point at which CIV can be initiated. When coupled with other limitations on CIV, there may not be a region in which CIV can reach and sustain a stage of explosive growth.

A third major difference between the two types of experiments involves the seed ionization process necessary to initiate the M2SI. In the space experiments, seed ionization of the neutrals must be provided through charge exchange reactions with the ambient plasma (assuming photoionization and pre-existing hot electrons do not contribute significantly to the initial ionization). As again will be discussed more fully in Chapters 4 and 5, the time for sufficient charge exchange reactions to take place to heat electrons to
the ionization energy of the neutrals may be near the transit time of the neutral cloud. If the time to establish electron impact ionization does exceed the transit time, CIV will not occur.

Seed ionization provided through charge exchange reactions is substantially different than the process in the laboratory experiments. In the laboratory experiments, the seed ionization is provided by direct electron impact ionization of the neutrals by pre-existing hot electrons. This process is in general much faster than the charge exchange processes required for the space experiments. In addition, the electrons in the laboratory experiments begin at an energy much nearer the ionization energy of the neutrals \( T_{e0} \approx 10 \text{ eV} \) than in the space experiments \( T_{e0} = 0.1 \text{ eV} \). As a consequence, the electrons in the laboratory experiments require a much shorter heating time in order to establish a substantial anomalous ionization. This point will also be expanded on in later chapters.

Finally, Table 1.4 shows a comparison of other experimental parameters. Several important differences are highlighted by the table values. First, the values given for the neutral/plasma density ratio \( n_n/n_e \), the ratio of electron-neutral elastic collision time to neutral transit time \( \tau_{en}/\tau_{tr} \), and the ratio of the ion-neutral charge exchange time to the neutral transit time \( \tau_{pn}/\tau_{tr} \) indicate the effect of the density gradient in the space experiments. The neutral density is shown for three different values: \( n_n/n_e = 10^7, 200 \) and 1. These values represent a typical value near the release point, a value comparable to the laboratory experiment, and equal neutral-plasma densities. The effect of the different neutral densities is seen in the range of \( \tau_{en}/\tau_{tr} \) and \( \tau_{pn}/\tau_{tr} \). Near the release point, for the highest neutral densities, the ratio of collision times to transit time is below the equivalent values for the laboratory experiments. As the neutral densities fall, the collision times quickly exceed equivalent laboratory experiment values.

Overall, because of the different scenarios and parameters involved, it is difficult to directly compare laboratory and space experiments. However, it is not immediately obvious that any of the differences in the space experiments should stop CIV from developing. As will be investigated in future chapters, however, the drop in neutral density and different seed ionization mechanisms encountered in the space experiments may explain much of the discrepancy between the laboratory and space experiments.
1.5 Theory of CIV

After Alfvén first proposed a critical velocity, the question was quickly raised as to the mechanism of CIV. der Taar raised two objections to Alfvén’s hypothesis of the formation of the solar system shortly after the hypothesis was proposed[73]. The first objection was that the ionization rate invoked by Alfvén could not be accounted for through classical collisions. (The second objection, that the composition of the planets do not correspond that the composition suggested by Alfvén's theory, does not concern us here.) To answer this criticism, Alfvén first suggested that a collective plasma process such as an instability might lead to a positive feedback loop producing an avalanche discharge. This concept of an instability driving a positive feedback loop remains the underlying theory of CIV. Much of the work done since the pioneering work of Alfvén has been a matter of filling in the details of the theory. In the next paragraphs, the most important of the many papers regarding the theory of CIV are briefly reviewed.

After Alfvén suggested a collective plasma process could lead to an anomalous ionization of a neutral beam, Sherman [68] investigated the M2SI as a possible mechanism for the collective plasma process. The M2SI has the properties of effectively transferring energy from the ion beam to the electrons and anisotropically heating the electrons parallel to the magnetic field [51]. Both of these features are consistent with the laboratory experiments.

Using the framework of the M2SI, Sherman was able to develop the scenario outlined near the beginning of this Introduction: seed ionization of the neutral beam initiates the unstable ion beam. The propagating ion beam is unstable to the M2SI. As the M2SI operates, it effectively transfers energy to the electrons. As electrons are heated to above the ionization energy of the neutrals, they begin to electron impact ionize further neutrals. Finally, as further neutrals are ionized, the ion beam is reinforced, leading to a positive feedback loop. The details of this model will be discussed more extensively in Chapter 2.

Piel et al. [63] have objected to the use of homogeneous models in explaining the CIV process in laboratory experiments. They have argued that laboratory experiments show evidence of the ionization of CIV occurring within a small sheath-like area. Because of
this, Piel et al. have argued that a more appropriate model of the laboratory experiments is an "ionizing front". In the ionizing front model, a strong inhomogeneity is assumed to exist in the plasma. Figure 1-7 shows a schematic of the ionizing front in the frame of the front which is moving with velocity $U$. In this frame, the neutrals enter the interaction region with a velocity $-U$. A neutral ionized in the front will create a new ion-electron pair. The new electron will be restrained by the magnetic field, while the newly born ion will penetrate into the plasma. The result of the ionization, then, is to set up a strong electrostatic sheath near the plasma-neutral interface. The potential drop through this sheath is estimated by Piel et al. to be

$$
eq \Delta \phi = \frac{1}{2} m_i U^2$$

Also assuming the sheath thickness, $D$, is much less than the new ion gyroradius, i.e.

$$D \ll \frac{U}{\Omega_i}$$

where $\Omega_i$ is the cyclotron frequency of the ion, an estimate of the electric field in the sheath can be obtained:

$$E \gg \frac{m_i U^2}{2eD}$$

If the sheath thickness is large compared to the electron gyroradius, the electrons will behave differently than the ions in the sheath. The ions will penetrate into the plasma, but the strong electric field in the sheath region will impose a secondary drift velocity on the electrons parallel to the sheath. The magnitude of this drift velocity can be expressed as

$$v_d = \frac{E}{B} \gg \frac{m_i U^2}{2eDB}$$

The result of this secondary drift is to set up a relative velocity between the electrons and ions in the sheath region. This secondary drift will be unstable to the M2SI. Piel et al. argue that, in fact, it is this secondary drift, not the primary drift of $U$, which is the most important drift in exciting the M2SI in the laboratory experiments.

Piel et al. point out one of the advantages of this model is that in the sheath region,
the entire electron population is drifting with respect to the entire ion population. This is somewhat different from the theory of Sherman in which the electron population is drifting only with respect to the newly born ions. The drift of the entire electron population leads to a more unstable distribution and larger growth rates of the instability. Faster operation of the instability is advantageous in describing the results of laboratory experiments.

As will be discussed below, computer simulations performed by Machida et al. [49] have confirmed the existence of an ionizing front in the idealized model of a sharp edged neutral cloud. However, the precise roles of the two drifts invoked by Sherman and Piel, respectively, are not clear from the experimental evidence. In fact, it is difficult from the experimental evidence to judge which drift is dominant, or even whether one drift is dominant throughout the entire interaction. Overall, the drift invoked by Piel et al. leads to larger growth rates of the instability, but also requires a more idealized situation in which to operate. In a more realistic situation, the plasma-neutral interface will not be an idealized sharp edge, but may be a gradual transition over several ion gyroradii. In this more realistic case, it is more difficult to estimate the strength of the electric field and secondary drift. It is likely however that in this case the electric field and secondary drift will be much lower than in the idealized case. This lower drift velocity will lead to slower growth rates and perhaps even the stabilization of the instability in the sheath region.

Throughout this thesis, the primary drift as invoked by Sherman will be considered to be the dominant mode of sustaining CIV. It is important to keep in mind however that the possibility of a secondary drift capable of increasing the rate of CIV exists in the laboratory experiments.

The analysis of Sherman [68] and Piel et al. [63] invoked the M2SI as the main mechanism for transferring energy from ions to electrons. Both of these works relied on the computer simulations of McBride et al.[51] for estimates of the efficiency of this energy transfer. In the computer simulations of McBride et al., it was found that 50% of the ion drift energy could ultimately be changed into electron kinetic energy. In addition, much of this energy went into the formation of a suprathermal electron tail. (In a CIV
scenario with ionization processes included, the suprathermal electrons would be quite effective in ionizing neutrals. As suprathermal electrons above the ionization energy ionize neutrals, the suprathermal tail is "truncated". In order to further investigate the efficiency of energy transfer between ions and electrons, Formisano et al. [23] have considered the quasilinear theory of the CIV process. Although linear theory indicates when the instability operates, it provides no information as to the fraction of energy transferred from ions to electrons, i.e. the saturation state is not treated by linear theory. By treating the instability in the framework of quasilinear theory, Formisano et al. determined the fractional energy transferred.

By considering the relaxed distribution function, Formisano et al. have determined that for unmagnetized ions (i.e. when the ionization rate is faster than the ion cyclotron frequency), 2/3 of the kinetic energy of the new ions is transferred to the electrons. The situation is dramatically changed for highly magnetized ions (i.e. when the ionization rate is much slower than the ion cyclotron period). In this case only 2.5% of the energy is transferred to the electrons.

Using these estimates of the efficiency of the energy transfer process, Formisano et al. determine the ionization rate in the coma of Halley's comet. Although the details of the cometary model are not of interest here, the quasilinear results are important since they have been used by several subsequent authors.

The details of the estimates derived by Formisano et al. will be discussed in detail in the next chapter. The point here is simply to mention several short-comings of the theory. The major inaccuracy in the theory comes from the quasilinear formulation itself. The M2SI instability, in most cases, saturates by trapping of the beam ions. This non-linear effect is not modeled in the quasilinear theory. The result of the strong non-linear interaction between the waves and particles is to substantially change the saturation level of the wave spectrum. As a result, the energy transfer between beam ions and background electrons is substantially different than proposed. As will be shown later, simulations indicate that an efficiency of 2/3 is optimistic for unmagnetized ions. A more likely figure seems to be roughly 30-40%. The estimates for highly magnetized ions seem to be considerably better since ion trapping is not as important in this case (i.e. the
beam ions never have a resonant interaction with the waves). Also for the CIV process, the assumption of unmagnetized ions may be difficult to achieve. Two conditions are required for unmagnetized ions to be used effectively. The first is that the ionization rate is higher than the ion cyclotron frequency (i.e. $\nu_{\text{ion}}/\Omega_i > 1$). The second condition is that the quasilinear diffusion time from the beam in velocity space is much shorter than the characteristic time for ion creation (i.e. $\nu_{\text{ion}} \tau_{\text{diff}} \ll 1$). This condition assures that the ionization process may be ignored on the plasma time scale. Together, these two assumptions imply that a newly born ion dumps all of its available energy into the waves before the next beam ion is born. As will be discussed later, it may be difficult to achieve a result in which this is an appropriate assumption.

Despite its shortcomings, the theory of Formisano et al. probably provides the best estimates of the efficiency yet developed. The fact that the theory does not always provide highly accurate results testifies to the difficulties associated with analyzing the highly non-linear mechanisms at work in CIV.

So far, the theories discussed have considered the development of relative ion-electron drifts and the excitation of the M2S1. Attention now turns to some of the macroscopic models used to investigate the CIV process.

Abe [1] developed a macroscopic rate model of CIV that reproduces the critical velocity condition. By assuming the ion population consists of newly born ions and thermalized ions (i.e. $n_i = n^* + n_t$, where $n_i$ is the total ion density, $n^*$ is the density of the newly born ions, and $n_t$ is the density of the thermalized ions), and that the electron population is made up of a hot electron tail and thermal electrons, Abe was able to write down the macroscopic rate equations for the development of the densities:

$$\frac{dn^*}{dt} = \nu_{\text{ion}} n_H - (dn^*/dt)_D - (dn^*/dt)_e$$  \hfill (1.8)

$$\frac{dE_H}{dt} = (dn^*/dt)_D(\eta m_e U^2/2) - \nu_{\text{ion}} n_H e \phi_{\text{ion}}$$ \hfill (1.9)

where $n^*$ and $n_H$ are the densities of the newly born ions and hot electrons, respectively, $\nu_{\text{ion}} = \sigma_{\text{ion}} v_e n_n$ is the rate of anomalous ionization ($\sigma_{\text{ion}}$ is the collisional cross section for ionization, $v_e$ is the relative neutral-electron velocity and $n_n$ is the neutral density), the
subscript ‘D’ and ‘c’ refer to velocity space diffusion and collisions, respectively, \( E_H \) is the energy density of the hot electron population, \( \eta \) is the efficiency of the energy transfer between newly born ions and electrons, \( m_u U^2/2 \) is the energy of a newly born ion, and \( e\phi_{ion} \) is the ionization potential of the neutrals. Equation (1.8) relates the change in the newly born ion density to increases from ionization and losses due to velocity space diffusion and collisions. The energy equation, Equation (1.9), considers the energy of the hot electron population to increase due to energy transfer from ions via the M2SI and decrease due to ionization. As will be discussed in more detail in Chapter 2, the solution of Equations (1.8) and (1.9) produces exponential growth only if the condition

\[
e\phi_{ion} < \eta m_u U^2/2
\]

(1.10)

is satisfied. Hence, this simple macroscopic model reproduces a condition similar to the critical velocity condition. In this model, however, the definition of critical velocity is modified by the addition of an efficiency factor related to the fraction of energy transferred from beam ions to electrons.

Several other authors have employed macroscopic models similar to Abe’s. As shown by this work of Abe, the macroscopic models do reproduce some of the characteristics of CIV. The main short-comings of these models is that of incorporation of microscopic information. For example, in the work of Abe, the diffusion coefficient \( \nu_D \), the efficiency factor \( \eta \), and anomalous ionization rate \( \nu_{ion} \) are not constants, but must develop self consistently with the problem. Although formulations for the microscopic parameters (such as Formisano’s estimates for \( \eta \) discussed above) do provide some guidelines for use in macroscopic models, the estimates usually are not very robust. Considering the wide variations of parameters that are likely to occur in space and time during CIV, it is difficult to determine the applicability of these macroscopic methods.

Another aspect of the macroscopic problem associate with CIV has been considered by Goertz et al [30]. The main emphasis of this work is on the coupling of the newly created plasma with the background plasma. Much of the work mentioned in the previous paragraphs has concerned the energy balance between newly created beam ions, electrostatic waves, and electrons. Not considered, however, has been the momentum.
In CIV, the relative velocity of the neutrals and plasma is decreased. The energy is absorbed into ionization and heating of the ions and electrons. It is not clear from this picture what happens to the momentum. Haerendel has suggested that the magnetic field and surrounding plasma absorbs the momentum. The work of Goertz et al. treats this question of the momentum balance in more detail. More specifically, Goertz et al. assume the momentum of the plasma cloud is increased by the addition of new ions at a given rate and decreased by a volume $j \times B$ force. The resulting momentum balance equation is closed by phenomenological assumptions about the microscopic processes underlying CIV.

Again as will be discussed further in Chapter 2, the results of Goertz et al.'s analysis show that for a constant neutral gas density, CIV reaches an asymptotic state in which the ionization rate depends only on the neutral cloud velocity and a single coupling constant with the surrounding plasma. The coupling constant is proportional to the square root of the background plasma density multiplied by the magnetic field strength.

The theories put forth by the authors mentioned above have clarified many of the details of the CIV process. Most of them suggest how CIV occurs. Unfortunately, none of them suggest why CIV has not been observed in space based experiments. As negative or inconclusive results have been compiled in relation to space based experiments, mechanisms that might damp the CIV process have been considered. One of the mechanisms most extensively studied is the effect of collisional processes other than impact ionization. In general, line excitation, metastable excitation, dissociation and other reactions act as an energy sink from the hot electron population. These subsidiary reactions drain energy from the electrons, making it unavailable for ionization reactions. The general tendency of these subsidiary reactions is to slow or stop the anomalous ionization of the neutrals.

In the work of Newell and Torbert [56], the effect of reactions other than ionization were used to determine an efficiency factor including collisional effects. The equation expressing the critical velocity condition was modified to include the effects of collisional processes, i.e.

$$\eta_c \eta \frac{1}{2} m_n U^2 = e \Phi_{ion}$$  \hspace{1cm} (1.11)
where $\eta$ is the efficiency as derived by Formisano et al. and $\eta_{c,i}$ is now an efficiency to account for energy draining collisional processes. For example, Newell and Torbert considered the line excitation of Ba. The collisional efficiency is defined as the ratio of the energy spent by the electrons in ionizing neutrals to the total energy spent by the electrons in exciting and ionizing neutrals. This ratio may be found by noting that the number of excitation and ionization collisions performed by an electron is related to the cross section of the process. Taking the energy level of the excitation as 2.24 eV and the ionization energy as 5.2 eV, the efficiency of the process may be expressed as

$$
\eta_{c,i} = \frac{(5.2eV)\sigma_{ion}}{(2.24eV)\sigma_{exc} + (5.2eV)\sigma_{ion}}
$$

(1.12)

where $\sigma_{exc}$ and $\sigma_{ion}$ are the cross sections for line excitation and ionization, respectively. Using typical cross sections, Newell and Torbert find that $\eta_{c,i} \approx .4$ over most of the range of electron energies of interest. Hence, nearly 60% of the electron's energy is lost to processes other than ionization.

Lai et al. [40] have also considered the effects of collisional processes other than ionization. The main focus of this work is the effect of metastable states on the rate of anomalous ionization. Lai et al. developed a rate model assuming an electron energy distribution similar to the results of simulations performed by Machida and Goertz. Using this assumed energy distribution, the rates of metastable and line excitation collisions and ground and metastable impact ionization could be determined. Models with and without metastable states were calculated and the resulting rates of anomalous ionization determined. The results showed that the metastable states acted as an important energy pooling mechanism. Electrons de-energized through metastable collisions, but unlike line excitation where the energy was subsequently lost to radiation, further collisions could drive the metastably excited neutral to ionization. Lai et al. conclude that for low electron energies metastable ionization may provide the dominant mechanism for ionization. This is important in considering the onset of the CIV mechanism.

In recent work which is still unpublished, Goertz has also suggested why CIV might not be observed in the ionosphere. Goertz argues that the phase velocities of the waves associated with the M2SI are such that no electrons are initially resonant with the waves.
The result is that electrons must first substantially bulk heat to become resonant with the waves. Once the electrons become resonant with the waves, a suprathermal tail evolves on the electron distribution function. It is this suprathermal tail which leads to CIV. A comparison of the time scales, however, shows that the time to bulk heat electrons and form the suprathermal tail may be on the same order as the transit time of the neutral cloud in the ionospheric experiments. If the time to initiate CIV is longer than the transit time of the neutral cloud, CIV will not be observed. In many respects, the hypothesis of Goertz agrees well with the conclusions of this thesis.

Finally, Papadopoulos[59] has investigated some aspects of CIV using linear and nonlinear analysis. Papadopoulos analyzes the dispersion relation of the lower hybrid beam instability included a term to represent beaming $O^+$. The beaming $O^+$ is included to represent ambient ionospheric ions which have been snowplowed or reflected by the expanding Ba in the space experiments. Papadopoulos' analysis leads to an expression for the anomalous ionization rate of CIV. By further assuming that the ion loss rate is known, Papadopoulos can predict when CIV will lead to rapid ionization, i.e. when ionization rate exceeds ion loss rate. Since the ion loss rate is chosen in a somewhat 'ad hoc' fashion in this work, the precise relation between the analytic and experimental results is not yet clear. Some agreement is shown however between the analysis and experiments.

Summary of Theoretical Work on CIV

The linear and quasilinear analyses of CIV have provided a framework for the basic mechanism of CIV. The linear analysis of the M2SI provides information as to the regime in which CIV may be expected to operate. Quasilinear analysis provides further information as to the efficiency of energy transfer from ions to electrons. The estimation of the efficiency of this energy transfer has been important in macroscopic models that have been useful in demonstrating the exponential growth of the CIV process. In addition, considerations of electromagnetic effects and subsidiary collision paths have provided insight into limits on the CIV mechanism. Overall, however, a complete, self-consistent theory of CIV has not yet been formulated. In addition, present theories do not ade-
quately explain the lack of CIV observed in space based experiments.

1.6 Simulation of CIV

After experimental investigations supported the CIV hypothesis of Alfvén and the basic theoretical underpinnings of CIV had been proposed, a series of numerical simulations began to study in more detail the non-linear behavior of the CIV process. Almost all of these numerical simulations have employed explicit particle-in-cell (PIC) codes. To date, the emphasis of simulating CIV has not been on understanding the CIV experiments in an experimental sense. Instead, most simulations have adopted as their goal developing an understanding of the non-linear regimes of CIV on long time scales, where theory is lacking. Because of this emphasis, explicit PIC simulations of CIV have adopted the use of unphysical mass ratios. The use of unphysical mass ratios (e.g. $m_i/m_e = 100$) has the advantage of reducing the computational time necessary to perform the simulation. The technique of using artificial mass ratios is quite common in PIC simulations. As long as the emphasis is on simply producing an understanding of non-linear physics and not on producing 'real' engineering numbers, an artificial mass ratio sufficiently retains the proper physical disparity in the mass ratio (i.e. $m_i/m_e \gg 1$) to produce useful results.

The series of simulations discussed here begins with a simple one-dimensional simulation of an ion beam streaming across a plasma background in a magnetic field. Later simulations added collision models to simulate the formation of a positive feedback loop and initiate CIV. These collision models have increased in their complexity, as have the simulations in general. The latest simulations of CIV involve two-dimensional bounded systems incorporating several collisional processes.

The earliest numerical modeling of CIV is probably the work of Abe and Machida [2]. The goal of this earliest work was not to directly model the CIV process, per se, but to confirm numerically the ability of instabilities to heat electrons. In their work, Abe and Machida performed an explicit PIC code simulation of an initial background plasma embedded in an externally applied magnetic field. A streaming ion beam was propagating across the plasma. Since the simulations were one dimensional, only the direction perpendicular to the beam was considered. As predicted by theoretical consid-
erations, the situation was observed to be unstable. A strong electrostatic lower-hybrid wave developed in the system leading to strong heating of the electrons parallel to the magnetic field.

A slightly later paper of Machida, Abe, and Terasawa [46] provided an improvement over their earlier work by incorporating a simple collision operator to model electron impact ionization of a neutral beam. The results indicated that for neutral velocities below the critical velocity, a very low rate of ionization of the neutral beam was observed. As the neutral velocity exceeded the critical velocity, however, the rate of ionization quickly increased, in accordance with Alfvén’s hypothesis.

These results were again limited to one-dimension, and they only incorporated a single, simple collision path. They did, however, provide the first numerical evidence that a positive feedback loop is formed when the neutral velocity exceeded the critical velocity.

The work of Machida and Goertz [47] continued to increase the complexity and realism of one-dimensional simulations. This study included several collision paths in addition to electron impact ionization. Photoionization, charge exchange, electron and ion elastic, and electron impact excitation collisions were all included in addition to electron impact ionization.

The results of this work identified two regimes for CIV. In the first regime (denoted as ‘resistive CIV’), resistive heating of the electrons raises their temperature to energy levels capable of initiating the positive feedback loop of CIV. This regime of CIV can only occur at high neutral density where resistive heating occurs on a fast time scale. In the second regime (‘collective CIV’), beaming instabilities lead to the formation of a suprathermal electron tail (“truncated” at energies above the ionization energy) similar to theoretical suggestions.

The work of Machida and Goertz [47] was further improved in 1988 by performing two-dimensional simulations including electromagnetic effects [48]. Linear theory suggests that electromagnetic effects should stabilize the modified two-stream instability (Cf. Section 2.1.2). Since the M2SI is believed to be the most important mechanism of energy transfer from beam ions to electrons, electromagnetic effects should also act to
quench CIV.

The results of this study indeed show the quenching of CIV by electromagnetic stabilization of the M2SI. It was found that the relationship

$$U < (1 + \beta_e)^{1/2} v_a$$

must be satisfied in order for the instability to exist. In these relations, $U$ is the neutral velocity, $v_a$ is the Alfvén velocity ($v_a = B_0/\sqrt{\mu_0 \rho_m}$, $\rho_m$ is the mass density) and $\beta_e = n_e T_e/(B_0^2/2\mu_0)$ is the electron plasma beta.

This work is useful in identifying when electromagnetic effects are important in the CIV process. The results indicate that for most space release experiments (where $\beta_e \ll 1$ and $\omega_{pe}/kc \ll 1$), electromagnetic effects are not an important consideration. Since the work was doubly periodic, no striking differences were seen due to the multiple dimensions used in the simulation.

The latest study in the series of simulations performed by Machida and his co-workers was a two-dimensional, bounded, electrostatic model used to investigate the formation of an ‘ionizing front’ [49]. In this simulation, a magnetized plasma was initially loaded into the right hand side of the simulation region. On the left hand side, only neutrals were present. The neutrals were assumed to be propagating to the right, into the plasma, with a constant velocity. As the neutrals impacted the plasma filled region, ionization began along the neutral-plasma interface. The momentum of the newly born ions caused them to ‘overshoot’ with respect to newly born electrons, setting up a strong region of polarization near the plasma-neutral interface. This situation is the ‘ionizing front’ described by the theory of Piel et al [63]. The strong electric field in the polarized region caused an $E \times B$ drift of the newly formed plasma. This $E \times B$ drift was shown in the simulation to be great enough to stimulate the M2SI in the polarized region. The instability then led to energy transfer to electrons and further electron impact ionization of the neutrals. Unfortunately, because the simulation used unphysical mass ratios, it could not be determined whether the primary or secondary (i.e. $E \times B$) drift would dominate in a realistic situation.

Although a somewhat idealized simulation, with a plasma in one portion of the
simulation region, and a neutral region in the other portion, this simulation did confirm
the possibility of the existence of an ionizing front and the importance it may have in
CIV. Whether this ionizing front is the most important mechanism for initializing the
M2SI could not be determined from these simulations.

One suggestion for the absence of observed CIV in space experiments involves sub-
sidiary reactions that may quench the CIV mechanism. The test of this hypothesis
requires a rather detailed collision model compared to the previously mentioned studies.
The work of Person et al. [61] considered the effect of charge exchange and electron-
neutral elastic collisions by including a detailed collision model in a one-dimensional,
electrostatic PIC code. Unlike the previous studies that assumed isotropic scattering
angles and simple energy dependencies of the cross sections, Person et al. included the
differential cross sections as a function of energy.

The two main results of this work indicated the importance of charge exchange, elas-
tic and excitation collisions. First, the charge exchange reactions were found to lower the
threshold for CIV by providing an additional means of forming beam ions. Second, es-
pecially at high neutral pressures, elastic collisions and excitation collisions provide both
a means of isotropizing the electron distribution function, destroying the suprathermal
tail of the electron distribution function, and an energy loss mechanism. Both of these
effects tend to slow the rate of CIV. Above neutral densities of roughly $10^{12}$ cm$^{-3}$, the
lower energy excitation collisions are favored over ionization events, tending to quench
CIV.

Further collision processes were considered in the work of McNeil et al [52]. In this
work, line excitation, excitation of metastable states, and ionization from metastable
states were all included.

The results showed that line excitation processes act as an energy sink for the elec-
trons. The loss of energy to these excitation states removes energy from being available
for ionization. Metastable states however act as a 'pooling' mechanism. Energy is stored
in the metastable states, but is not lost to future ionization as in line excitation. Future
collisions may act to ionize the neutral in the metastable state.
Summary of Simulations of CIV

Simulation work to date has focused mainly on the underlying physics of the CIV mechanism. Simulations have been performed in one- and two-dimensional, bounded and unbounded systems and have succeeded in showing the importance of the M2SI in CIV and demonstrating the basic non-linear mechanisms involved in sustaining a CIV discharge. Because these simulations have all used explicit PIC codes, unphysical mass ratios have universally been used. Unfortunately, simplifications necessary to minimize the required computational power have only allowed results to be produced which cannot be directly compared to experimental results.

Simulations of the collisional processes involved in CIV have also been used to identify the possibility of quenching of CIV through energy loss mechanisms associated with excitational processes. Here again however, the use of unphysical mass ratios and other simplifications makes it difficult to judge the precise effect these other collisional processes may have in an actual CIV experiment.

1.7 Outline of Current Work

The previous sections have discussed the analytical, numerical and experimental work previously done on CIV. As described in these sections, several features of CIV are still not well understood. It is the goal of this thesis to attempt to clarify these inconsistencies. Specifically, three main questions may be posed that a full theory of CIV needs to answer:

- Is the basic mechanism of CIV as outlined by Sherman and developed more fully by others correct and consistent with the experimental evidence? This question has actually been addressed through previous simulation work with explicit PIC codes. The current work is to extend these simulations to physical mass ratios, verify the operation of the basic mechanism with physical mass ratios, and discuss the differences between simulation results using physical and unphysical mass ratios.

- Assuming the hypothesis of the mechanism is correct, do simulation results lead to ionization rates with length and time scales consistent with experimental evidence?
Since past simulations of CIV have employed unphysical mass ratios, this question has not been previously addressed. The current work will provide a means of determining actual reaction rates for the CIV process. These reaction rates can then be compared to experimental evidence to determine if the model is reasonable.

- Why do the majority of space experiments provide inconclusive or negative results? The space experiments performed have been designed to provide conditions thought to be conducive to CIV, yet most results have not shown conclusive evidence of CIV. This discrepancy between the expected and actual results for CIV experiments is the biggest question still remaining in CIV work.

To attempt to answer these questions, the following chapters will present analytic and numerical results. When possible, the results will be compared to experimental data. To begin the analysis of CIV, Chapter 2 provides a more detailed discussion of the theory of the CIV mechanism. An integral part of this thesis is the development of numerical methods capable of simulating the CIV process with realistic mass ratios and collision cross sections. This has been accomplished through the inclusion of a probabilistic collision model in an implicit particle in cell code. The development of the code is discussed in detail in Chapter 3. Chapter 4 presents simulation results aimed at verifying the theoretical discussion of Chapter 2 and developing an understanding of the effect of non-linear and collisional processes on CIV. Chapter 5 applies the analytic and simulation results to the experimental evidence available to determine why CIV is observed in the laboratory but not in space. Chapter 6 draws conclusions from the previous chapters and provides suggestions for future work. Finally, a series of test cases to verify the correct operation of the code is included in Appendix A.
Figure 1.1: Experimental setup of the CIV laboratory experiments (reproduced from Danielsson, 1970 [18]). A hydrogen plasma is created and accelerated along a drift tube. The magnetic field is slowly rotated to polarize the plasma. The polarized plasma continues to drift across the magnetic field as it impacts a neutral helium cloud near z=0.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plasma Velocity</td>
<td>$2 \times 10^4$ to $4 \times 10^6$ m/s</td>
</tr>
<tr>
<td>Plasma Density</td>
<td>$10^{17}$ to $10^{18}$ m$^{-3}$</td>
</tr>
<tr>
<td>Initial electron temperature</td>
<td>5 eV</td>
</tr>
<tr>
<td>Axial spread of neutral gas</td>
<td>5 cm</td>
</tr>
<tr>
<td>Average neutral density</td>
<td>$10^{20}$ m$^{-3}$</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>0.2 Tesla</td>
</tr>
<tr>
<td>e-n collision time (5 ev)</td>
<td>0.1 μsec</td>
</tr>
<tr>
<td>e-n collision time (85 ev)</td>
<td>0.3 μsec</td>
</tr>
<tr>
<td>e-e coulomb collision time (5 ev)</td>
<td>1.0 μsec</td>
</tr>
<tr>
<td>p-n charge transfer time</td>
<td>2.0 μsec</td>
</tr>
<tr>
<td>transit time</td>
<td>0.15 μsec</td>
</tr>
</tbody>
</table>

Table 1.1: Typical parameters for the laboratory CIV experiments. The plasma-neutral interaction is designed to be collisionless.
Figure 1-2: Plasma velocity 1 cm behind the center of the gas cloud as a function of the original velocity (reproduced from Danielsson, 1970 [18]). \( v_c \) is the critical velocity (3.5 \( \times 10^4 \) m/sec). \( B = 0.44V \) sec/m².

<table>
<thead>
<tr>
<th>Release</th>
<th>Shape/Neutrons</th>
<th>Alt(km)</th>
<th>Ang(°)</th>
<th>( n_e ) (cm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porcupine (3/19/79)</td>
<td>Conical Ba</td>
<td>470</td>
<td>28</td>
<td>( 2 \times 10^5 )</td>
</tr>
<tr>
<td>Bubble Machine (3/25/81)</td>
<td>Radial Sr</td>
<td>500</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>Star of Lima (3/21/83)</td>
<td>Conical Ba</td>
<td>429</td>
<td>0</td>
<td>( 2 \times 10^4 )</td>
</tr>
<tr>
<td>Star of Condor (3/23/83)</td>
<td>Radial Sr</td>
<td>571</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>CRIT-I (5/13/86)</td>
<td>Conical Ba</td>
<td>380</td>
<td>45</td>
<td>( 6 \times 10^4 )</td>
</tr>
<tr>
<td>Sr90 (5/13/86)</td>
<td>Conical Sr</td>
<td>540</td>
<td>45</td>
<td>( 1.5 \times 10^4 )</td>
</tr>
<tr>
<td>CRIT-II (5/4/89)</td>
<td>Conical Ba</td>
<td>422</td>
<td>58</td>
<td>( 6 \times 10^5 )</td>
</tr>
</tbody>
</table>

Table 1.2: Summary of CIV experimental space releases.

<table>
<thead>
<tr>
<th>Release</th>
<th>Evidence of CIV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porcupine (3/19/79)</td>
<td>20%±10% ionization</td>
</tr>
<tr>
<td>Bubble Machine (3/25/81)</td>
<td>Maybe 50% Ionization, but large uncertainty</td>
</tr>
<tr>
<td>Star of Lima (3/21/83)</td>
<td>Lower hybrid waves; hot electrons</td>
</tr>
<tr>
<td>Star of Condor (3/23/83)</td>
<td>None</td>
</tr>
<tr>
<td>CRIT-I (5/13/86)</td>
<td>Diffuse ion production</td>
</tr>
<tr>
<td>Sr90 (5/13/86)</td>
<td>None</td>
</tr>
<tr>
<td>CRIT-II (5/4/89)</td>
<td>Some enhanced ionization</td>
</tr>
</tbody>
</table>

Table 1.3: Summary of CIV experimental space release results
Figure 1-3: (a) Recording of the plasma velocity obtained during the CIV experiments (reproduced from Figure 3 of Danielsson and Brenning, 1975 [19]). The velocity shows a quick rise to a peak as the plasma initially arrives at the probe. This is followed by a sharp decrease to a plateau velocity. The plateau velocity corresponds to the critical velocity of the neutral gas. (b) Plasma velocity profile in the helium cloud (z=0 is the center of the cloud) showing the relation between the spread of the measurements in different positions. $v_0$ is the average undisturbed plasma velocity. For the first three points $v$ is the peak velocity; for the remaining points it is the plateau velocity.
Figure 1-4: Limits on the magnetic field strength (reproduced from Brenning, 1985 [11]). The figure shows a comparison of the criterion based on stabilization of the M2SI with results from a variety of experiments on CIV. (×=strong interaction, -=weak interaction, o=no interaction) Reasonable agreement is achieved between the theory and experiments.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Laboratory</th>
<th>Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_n/v_c$</td>
<td>.6 to 11.</td>
<td>1.6</td>
</tr>
<tr>
<td>$T_{e0}/e\phi$</td>
<td>.2</td>
<td>.04</td>
</tr>
<tr>
<td>$n_m/n_e$</td>
<td>200</td>
<td>$10^7 - 200 - 1$</td>
</tr>
<tr>
<td>$\omega_{pe}/\Omega_{ce}$</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$\tau_{en}/\tau_{tr}$</td>
<td>.7</td>
<td>.02 - 900 - 170000</td>
</tr>
<tr>
<td>$\tau_{pm}/\tau_{tr}$</td>
<td>13</td>
<td>.35 - 18000 - $3\times10^6$</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>0° to 58°</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.4: Comparison of Laboratory and Space Experiment Non-dimensional Parameters. For the space experiments, the neutral density dependent parameters are shown for three different values of neutral density.
Figure 1.5: The geometry of the Porcupine release (reproduced from Haerendel, 1982 [31]). The release occurs just below the solar terminator. Ions produced through CIV should appear above the terminator as a field aligned streak near the magnetic field line through the release point.
Figure 1-6: Densitometer tracings of the Porcupine release (reproduced from Haerendel, 1982 [31]). Ions representing approximately 20% of the ions capable of undergoing CIV are observed within 15km of the release point. Haerendel has attributed these ions to CIV processes.
Figure 1-7: The ionizing front model (see Piel et al, 1980 [63]). A strong inhomogeneity exists in the plasma. Newly created electrons are restrained by the magnetic field, while newly created ions penetrate deep into the plasma. The results is a strong $E \times B$ drift which leads to the M2SI operating in the region of strong inhomogeneity.
Chapter 2

The Critical Ionization Velocity Mechanism: Theory

Before discussing the simulation results, an analytic model of CIV is developed. Several researchers [2, 26] have proposed that CIV occurs along the general lines described in the Introduction and restated here, namely: 1) A neutral beam is propagating perpendicular to a magnetic field, 2) charge exchange, photo-ionization, or ionization by pre-existing hot electrons provides a source of seed ionization which creates an ion beam propagating across the magnetic field, 3) the ion beam is unstable to the modified two-stream instability (M2SI), 4) the instability effectively heats electrons parallel to the magnetic field, 5) the heated electrons gain enough energy to impact ionize the neutral beam, 6) the electron impact ionization of the neutral beam reinforces the ion beam, creating a positive feedback loop leading to rapid ionization of the neutral beam. This general scenario for CIV is accepted in the following theoretical development. With the hypothesis that this general mechanism for CIV is correct, the analysis aims at examining the following questions:

- What region of parameter space leads to operation of the M2SI? Is this region of parameter space consistent with the experimental data?

- Given that the M2SI is operational, does it effectively heat electrons to energies above the ionization energy of the neutral gas?
• Do the hot electrons ionize neutrals on a time scale consistent with the anomalous ionization observed in laboratory experiments?

• Does the ionization of neutrals establish a feedback mechanism leading to an explosive growth of the plasma density?

• Does a rapid ionization of the neutrals account for the drop in the relative plasma-neutral velocity?

The following sections discuss these questions.

2.1 Modified Two-stream Instability

The first two questions mentioned in the previous paragraph require an understanding of the M2SI. The first question involves the region of parameter space in which the M2SI operates. This question can be addressed through linear theory. The second question concerns the energy transfer between ions and electrons. Since an answer to this question requires knowing the saturation state of the instability, linear theory is insufficient. Instead, quasilinear and nonlinear theory are used to investigate this second question.

2.1.1 Linear Dispersion Relation

The first concern is to develop the linear dispersion relation for an ion beam propagating across an ion/electron background plasma. To be more specific, a geometrical situation is considered in which a uniform magnetic field is applied in the $z$-direction, $\mathbf{B} = B_0 \hat{z}$. An ion beam of mass $m_i$, density $n_i$ and velocity $U$ propagates perpendicular to the magnetic field, $U = U \hat{x}$. The background ions which do not drift with respect to the electrons have mass $m_i$ and density $n_i$. Electron-neutral elastic collisions are included in the analysis through a number conserving Krook-type operator, which is a good model for these collisions [39].

The waves of interest propagate at an angle to the magnetic field, hence $k = k_x \hat{x} + k_z \hat{z}$. Furthermore, the waves are assumed to have wavelengths much longer than the electron
gyroradius but much shorter than the ion or beam gyroradius, and frequencies small compared to electron gyrofrequencies but large compared to ion or beam gyrofrequencies, specifically

\[ k \rho_i \gg 1 \]
\[ k \rho_b \gg 1 \]
\[ k_s \rho_e \ll 1 \]
\[ \Omega_e \gg |\omega| \gg \Omega_i \]
\[ \Omega_e \gg |\omega| \gg \Omega_b \] (2.1)

where \( \Omega_j = q_j B_0 / m_j \) is the gyrofrequency of species \( j \), \( \rho_e = v_e / \Omega_e \) is the electron gyroradius, \( \rho_i = v_i / \Omega_i \) is the ion gyroradius, \( \rho_b = U / \Omega_b \) is the gyroradius of the beam particles, and \( v_j^2 = T_j / m_j \) is the thermal velocity of species \( j \).

The equations to be solved are the electrostatic, linearized Vlasov equation and Poisson's equation. The initial conditions are assumed to be a uniform distribution of all species leading to an unperturbed electric field of \( E_0 = 0 \). The equations linearized about \( E_0 = 0 \) are \(^1\)

\[
\left( \frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial x} - \frac{e}{m_e c} v \times B_0 \cdot \frac{\partial}{\partial v} \right) \delta f_e = \frac{e}{m_e} E \cdot \frac{\partial f_{0e}}{\partial v} - \nu (\delta f_e - \frac{N_i}{N_0} f_{0e})
\] (2.2)

for the electrons, and

\[
\left( \frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial x} \right) \delta f_j = -\frac{e_j}{m_j} E \cdot \frac{\partial f_{0j}}{\partial v}
\] (2.3)

for the ions. Note that the ions have been treated as unmagnetized and collisionless since the growth rate is assumed to be much greater than the ion cyclotron and collision.

---

\(^1\)As mentioned, electron-neutral collisions are accounted for through a number conserving 'Krook' operator. The 'Krook' operator assumes that collisions will cause the perturbed distribution function to relax to a Maxwellian distribution function on a time scale \( \nu^{-1} \), i.e. \( (\partial f / \partial t)_e = -\nu (f - f_{MAX}) \) where \( f_{MAX} \) is the Maxwellian distribution. Defining number density in the usual way, \( N(x, t) = \int f(x, v, t) dv \), with similar definitions for \( N_0 \) and \( N_1 \) in terms of the unperturbed distribution, \( f_0 \), and perturbation, \( \delta f \), the Maxwellian distribution as a function of position and time may be expressed as \( f_{MAX}(x, v, t) = \frac{N(x, v, t)}{N_0} f_0(v) = \left[ 1 + \frac{N(x, v, t)}{N_0} \right] f_0(v) \). With this, the collision term then becomes \( (\partial f / \partial t)_e = -\nu \left[ \delta f(x, v, t) - \frac{N(x, v, t)}{N_0} f_0(v) \right] \), which is the term used in Equation 2.2.
frequencies. The fields are found from Poisson’s equation,

\[ \nabla^2 \delta \phi = - \sum_j 4\pi e_j \int dv \delta f_j = - \sum_j 4\pi e_j N_{1,j}. \quad (2.4) \]

In the above, \( f_{0,j} \) is the unperturbed distribution function of species \( j \), \( \delta f_j \) is the linear perturbation of the distribution function, \( \nu \) is the electron-neutral elastic collision frequency, \( \delta \phi \) is the perturbed potential, \( N_1 = \int dv \delta f_e \), \( N_{1,j} = \int dv \delta f_j \), and \( N_0 = \int dv f_{0,e} \).

The solution of these equations is carried out in the well known fashion [39]. Equations (2.2) and (2.3) are solved through the method of characteristics by integrating over the unperturbed orbits of the particles. The solution for \( \delta f_j \) is then substituted into Equation (2.4). The result to be obtained is a dispersion relation of the form

\[ 1 + \chi^e + \chi^i + \chi^b = 0 \quad (2.5) \]

where \( \chi^e \), \( \chi^i \), and \( \chi^b \) are the electron, ambient ion, and beam ion susceptibilities, respectively. The next sections detail the derivation of these susceptibilities.

Much of the analysis will rely of Laplace-Fourier transforms. The Laplace-Fourier transform of a quantity \( \psi \) is defined as

\[ \psi(x, t) = \int_{-\infty}^{\infty} d^3 k \exp(ik \cdot x) \int_C \frac{d\omega}{2\pi} \exp(-i\omega t) \hat{\psi}(k, \omega) \quad (2.6) \]

where \( \omega \) is the frequency and \( k \) is the wavenumber. \( \text{Im}(\omega) \) is large enough that the transform integral

\[ \hat{\psi}(k, \omega) = \int_0^\infty dt \exp(i\omega t) \hat{\psi}(k, t) \quad (2.7) \]

converges. Finally, the contour of integration, \( C \), represents

\[ \int_C d\omega = \int_{-\infty+i\sigma}^{\infty+i\sigma} d\omega \quad (2.8) \]

where \( \sigma \) is chosen such that the contour is above any singularities in \( \hat{\psi}(k, \omega) \).
Electron Susceptibility: The solution of Equation (2.2) begins by Laplace-Fourier transforming to get

\[
\left[ i(-\omega - i\nu + k \cdot v) - \frac{e}{m} v \times B \frac{\partial}{\partial v} \right] \delta f_e = \left[ \frac{e}{m} \hat{E}_k \cdot \frac{\partial}{\partial v} + \nu \frac{\hat{N}_1}{\hat{N}_0} \right] f_{0,e}. \tag{2.9}
\]

where the carets denote transformed quantities.

The left hand side of Equation (2.9) is a general differential operator which can be denoted by \( D \delta f_e \). Letting \( D^{-1} \) be the formal inverse of this operator,

\[
\delta f_e = D^{-1} \left[ \frac{e}{m} \hat{E}_k \cdot \frac{\partial}{\partial v} + \nu \frac{\hat{N}_1}{\hat{N}_0} \right] f_{0,e}. \tag{2.10}
\]

Equation (2.10) is not yet the solution of Equation (2.2) since \( \hat{N}_1 \) is a function of the perturbation \( \delta f_e \). By introducing the definition of \( \hat{N}_1 \), namely \( \hat{N}_1 \equiv \int dv \delta f_e \), \( \hat{N}_1 \) may be expressed as

\[
\hat{N}_1 = \int dv D^{-1} \left[ \frac{e}{m} \hat{E}_k \cdot \frac{\partial}{\partial v} + \nu \frac{\hat{N}_1}{\hat{N}_0} \right] f_{0,e}
= \frac{e}{m} \hat{E}_k \cdot \int dv D^{-1} \frac{\partial f_{0,e}}{\partial v} + \nu \frac{\hat{N}_1}{\hat{N}_0} \int dv D^{-1} f_{0,e}. \tag{2.11}
\]

Solving for \( \hat{N}_1 \),

\[
\hat{N}_1 = \frac{\frac{e}{m} \hat{E}_k \cdot \int dv D^{-1} \frac{\partial f_{0,e}}{\partial v}}{1 - \frac{\nu \hat{N}_1}{\hat{N}_0} \int dv D^{-1} f_{0,e}} \tag{2.12}
\]

which is what will be needed later for Poisson’s equation.

The integrals needed in Equation (2.12), namely \( \int D^{-1} f_{0,e} dv \) and \( \int D^{-1} \frac{\partial f_{0,e}}{\partial v} dv \), will now be calculated. The inversion of an operator of the form \( D f_1 = g(v) \) is discussed by Dougherty [22]. Dougherty notes that the solution of \( D f_1 = g(v) \) may be found by integrating over the unperturbed orbits, similar to the well known solution of the Vlasov equation using the method of characteristics [39]. Dougherty gives

\[
D^{-1} g(v) = \int_{-\infty}^{t} g(v(t')) e^{i[k(x'(t') - x) + (\omega + i\nu)(t' - t)]} dt'
\]

where \( x'(t') \) and \( v'(t') \) are the position and velocity along the unperturbed orbits at time

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subject to the boundary conditions that \( x'(t' = t) = x \) and \( v'(t' = t) = v \). The exponential factor may be regarded as a Green's function and the unperturbed orbits are the characteristic curves of the differential equation. Using this inversion, the first integral may be written as

\[
\int D^{-1} f_{0,e} d\nu = \int d\nu \int_{-\infty}^{t'} dt' e^{-i[k(x'(t') - x)(\omega + i\nu)(t' - t)]} f_{0,e}
\]

(2.14)

To perform the integration in Equation (2.14), \( x'(t') \) and \( v'(t') \) are replaced by the particle orbits in the unperturbed \((E = 0, B = B\hat{z}, \nu = 0)\) fields. These orbits are easily found to be

\[
\begin{align*}
v'_x &= v_\perp \cos(\phi - \Omega_e \tau) \\
v'_y &= v_\perp \sin(\phi - \Omega_e \tau) \\
v'_z &= v_\parallel \\
x' &= z - \frac{v_\perp}{\Omega_e} \sin(\phi - \Omega_e \tau) + \frac{v_\perp}{\Omega_e} \sin(\phi) \\
y' &= y + \frac{v_\perp}{\Omega_e} \cos(\phi - \Omega_e \tau) - \frac{v_\perp}{\Omega_e} \cos(\phi)
\end{align*}
\]

(2.15)

where \( \tau = t' - t \) and \( \phi = \tan^{-1}(v_y/v_z) \) is the phase angle of the velocity in the plane perpendicular to the magnetic field. Also, the unperturbed electron distribution is chosen to be an isotropic Maxwellian:

\[
f_{0,e} = \frac{N_0}{(2\pi v_e)^{3/2}} e^{-v^2/(2v_e^2)}. \]

(2.16)

Using Equation (2.15) to replace the term \( k \cdot (x' - x) \) along with the identity

\[
e^{i\lambda \sin \alpha} = \sum_m J_m(\lambda) e^{im\alpha}
\]

(2.17)

where \( J_m \) is the Bessel function of order \( m \), Equation (2.14) may be written

\[
\int d\nu D^{-1} f_{0,e} = \int d\nu f_{0,e} \int_{-\infty}^{t'} d\tau e^{-i(\omega + i\nu)\tau} e^{-ik_\perp v_\perp/\Omega_e[\sin(\phi - \Omega_e \tau) - \sin(\phi)]} e^{ik_\parallel v_\parallel \tau}
\]

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\[ \int dv f_{0,e} \int_{-\infty}^{0} d\tau \sum_{m} \sum_{l} J_m(\lambda)J_l(\lambda)e^{i(m-l)\phi}e^{i(k_x v_x - \omega - i\nu - m\Omega_e)\tau} \]  

(2.18)

where \( \lambda = \left( \frac{k_x v_x}{\Omega_e} \right) \). Since Equation (2.1) gives

\[ \lambda \ll 1, \]  

(2.19)

only the \( m = l = 0 \) term of Equation (2.1.1) needs to be retained:

\[ \int dv D^{-1} f_{0,e} = \int dv f_{0,e} \int_{-\infty}^{0} d\tau J_0^2(\lambda)e^{i(k_x v_x - \omega - i\nu)\tau} \]

\[ = \int dv \frac{if_{0,e}J_0^2(\lambda)}{-k_x v_x + \omega + i\nu}. \]  

(2.20)

Letting \( dv = \int_0^{2\pi} d\phi f_{0,e} \int_{-\infty}^{\infty} dv_\perp v_\perp \int_{-\infty}^{\infty} dv_z \) and performing the integration over \( \phi \),

\[ \int dv D^{-1} f_{0,e} = 2\pi \int_0^{\infty} dv_\perp v_\perp \int_{-\infty}^{\infty} dv_z \frac{if_{0,e}J_0^2(\lambda)}{-k_x v_x + \omega + i\nu}. \]  

(2.21)

Using the identity [39],

\[ \frac{1}{v_e^2} \int_0^{\infty} dv_\perp v_\perp J_l^2(\lambda)e^{-v_x^2/2v_e^2} = I_l(b) e^{-b} \]  

(2.22)

where \( I_l \) is the modified Bessel function of order \( l \), \( Z(\zeta) \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dt \exp(-t^2) \) is the definition of the tabulated plasma dispersion function[24], and

\[ b \equiv \left( \frac{k_x v_x}{\Omega_e} \right)^2, \]  

(2.23)

Equation (2.21) can be simplified to \(^2\)

\[ \int dv D^{-1} f_{0,e} = iN_0 \int_{-\infty}^{\infty} \frac{dv_z}{\sqrt{2\pi(-k_x v_x + \omega + i\nu)}} e^{-v_x^2/2v_e^2} I_0(b) e^{-b} \]

\[ = \frac{-iN_0 I_0(b) e^{-b}}{\sqrt{2k_x v_x}} Z(\zeta_e) \]  

(2.24)

\(^2\)Actually, at this point a good approximation may be \( I_0(b) e^{-b} \approx 1 \), since \( b \ll 1 \). However, in order to be consistent with previous derivations [48, 32, 51], the \( I_0(b) e^{-b} \) term is retained for the time being.
where $\zeta_e = \frac{\omega + iv}{\sqrt{2n_e k_e}}$.

The second integral, namely $\int dv D^{-1} \frac{\partial f_{0,e}}{\partial v}$ is now calculated. Premultiplying by $ik$ and replacing $f_0$ by its assumed Maxwellian gives

$$
\int dv D^{-1} ik \cdot \frac{\partial f_{0,e}}{\partial v} = - \int dv D^{-1} \frac{ik \cdot v}{v_e} f_{0,e}
$$

$$
= \frac{-1}{v_e^2} \int dv f_{0,e} \int_{-\infty}^{t} dt'C e^{-i(\omega + iv)(t-t')} + ik \cdot (x-x(t'))ik \cdot v'(t').
$$

(2.25)

Replacing $v' = \frac{dx'}{dt'}$, and integrating by parts results in

$$
\int dv D^{-1} ik \cdot \frac{\partial f_{0,e}}{\partial v} = \frac{-1}{v_e^2} \int dv f_{0,e} \left[ 1 + i(\omega + iv) \int_{-\infty}^{t} dt' e^{-i(\omega + iv)(t-t')} + ik \cdot (x-x(t')) \right].
$$

(2.26)

The last integral in Equation (2.26) is precisely the same as in Equation (2.14). Using $\int dv f_{0,e} = N_0$, Equation (2.26) becomes immediately,

$$
\int dv D^{-1} ik \cdot \frac{\partial f_{0,e}}{\partial v} = \frac{-N_0}{v_e^2} \left[ 1 + I_0(b) e^{-b} \zeta_e Z(\zeta_e) \right].
$$

(2.27)

Equations (2.24) and (2.27) are now substituted into Equation (2.11) to obtain $N_1$. After replacing $\mathbb{E}_k = -ik \delta \phi_k$, using $N_1$ in Poisson’s equation (Equation 2.4) and simplifying in a straightforward manner, the electron susceptibility is found to be

$$
\chi_e = \frac{\omega_p^2}{v_e^2} \frac{1 + I_0(b) e^{-b} \zeta_e Z(\zeta_e)}{1 + iv \zeta_e \frac{I_0(b) e^{-b} Z(\zeta_e)}{\sqrt{2n_e v_e}}}.
$$

(2.28)

**Ambient Ion Susceptibility:** Since the ion gyroradius and gyroperiod are larger than the electron gyroradius and gyroperiod by the mass ratio, the ambient and beam ions may both be treated as un magnetized. Hence, from Equation (2.3),

$$
\frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial x} \delta f_i = \frac{n_e}{m_i} \nabla \delta \phi \cdot \frac{\partial f_{0,i}}{\partial v}.
$$

(2.29)

Laplace-Fourier transforming, Equation (2.29) can be immediately solved for $\delta f_i$:

$$
\delta \dot{f}_i = - \frac{e}{m} \frac{k \cdot \delta f_{0,i}}{\delta v} \delta \phi_k.
$$

(2.30)

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Substituting into Equation (2.4) gives

$$\chi^i = -\omega_{pi}^2 \int d\nu^3 \frac{k \cdot \frac{\partial f_{0,i}}{\partial \nu}}{\omega - k \cdot v}. \quad (2.31)$$

Taking the unperturbed ion distribution to be Maxwellian,

$$f_{0,i} = \frac{1}{(2\pi v_i)^{3/2}} e^{-\nu^2/(2v_i^2)}, \quad (2.32)$$

Equation (2.31) may be solved to obtain

$$\chi^i = \frac{-\omega_{pe}^2}{2v_i^2} Z'(\zeta_i) \quad (2.33)$$

where $\zeta_i = \frac{\omega}{\sqrt{2kv_i}}$ and $Z'(\zeta) = dZ(\zeta)/d\zeta$.

**Beam ion susceptibility:** The beam ions are also treated as unmagnetized. The result is the same as Equation (2.31) with the $i$ subscript changed to $b$. However, the unperturbed state $f_{0,b}$ must now be treated as a drifting Maxwellian:

$$f_{0,b} = \frac{1}{(2\pi v_b)^{3/2}} e^{-(\nu^2 - U^2 + v_b^2)/(2v_b^2)}. \quad (2.34)$$

After substituting into Poisson’s equation and performing the velocity integration, the result is

$$\chi^b = \frac{-\omega_{pe}^2}{2v_b^2} Z'(\zeta_b) \quad (2.35)$$

where now $\zeta_b = \frac{\omega - kU}{\sqrt{2kv_b}}$.

**Dispersion Relation:** Substituting Equations (2.28), (2.33) and (2.35) into (2.5) gives the final dispersion relation for the modified two-stream instability:

$$1 + \frac{\omega_{pe}^2}{k^2 v_e^2} \frac{1 + I_0(b)e^{-\nu \zeta_e}Z(\zeta_e)}{1 + i\nu I_0(b)e^{-\nu \zeta_e}Z(\zeta_e)} - \frac{\omega_{pe}^2}{2k^2 v_i^2} Z'(\zeta_i) - \frac{\omega_{pe}^2}{2k^2 v_b^2} Z'(\zeta_b) = 0 \quad (2.36)$$

The above result should be compared to that in McBride et al. [51]. For the collisionless case ($\nu = 0$), it is identical except for the background ion term, which is similar.
in form to the beam ions but was not included by McBride et al. The next sections discuss the solutions of the dispersion relation and the nature of the modified two-stream instability.

2.1.2 Results of Linear Dispersion Relation

Equation (2.36) can be rewritten in terms of nondimensional groupings:

\[
1 + \frac{1}{\kappa^2} \frac{\omega_{pe}^2}{\Omega_e^2} \left[ 1 + \frac{i \nu}{\omega_{pi} \kappa \sin(\theta)} \sqrt{\frac{m_i n_i}{m_e n_e}} I_0(b) e^{-b \zeta_e Z(\zeta_e)} \right] - \frac{1}{2 \kappa^2} \frac{\omega_{pe}^2}{\Omega_e^2} \frac{T_e n_i}{T_i n_e} Z'(\zeta_i) - \frac{1}{2 \kappa^2} \frac{\omega_{pe}^2}{\Omega_e^2} \frac{T_e n_b}{T_b n_e} Z'(\zeta_b) = 0
\]

with the parameters expressed as

\[
\sin \theta = \frac{k_z}{k}
\]

\[
\kappa = \frac{k v_e}{\Omega_e}
\]

\[
b = \kappa^2 \cos^2(\theta)
\]

\[
\zeta_e = \sqrt{\frac{1}{2} \frac{\omega + i \nu \omega_{pe}}{\omega_{pi} \kappa \sin(\theta)}} \sqrt{\frac{m_e n_i}{m_i n_e}}
\]

\[
\zeta_i = \sqrt{\frac{1}{2} \frac{\omega \omega_{pe}}{\omega_{pi} \kappa \sqrt{T_e n_i}}}
\]

\[
\zeta_b = \sqrt{\frac{1}{2} \frac{\omega \omega_{pe}}{\omega_{pi} \kappa \sqrt{T_b n_e}}}
\]

The solution of the dispersion relation \(\omega(\kappa)\) is seen to depend on the 10 nondimensional groupings \(\theta, \nu/\omega_{pi}, \omega_{pe}/\Omega_e, m_e/m_i, n_i/n_e, T_e/T_i, m_b/m_i, U/v_b, T_b/T_e,\) and \(n_b/n_e\). This section provides an analysis of the dispersion relation in terms of these parameters.

To begin, the dispersion relation may be compared to the work of McBride et al. [51] which did not include the effects of background ions or collisions. The dispersion relation of McBride et al. may be obtained from Equation (2.36) by letting \(\nu \rightarrow 0\) and \(\frac{n_b}{n_e} \rightarrow 0\). Because of the similarity in the form of the dispersion relations, several of the results of McBride et al. are applicable to the current dispersion relation. First, the important scaling of the angle still holds: the variable \(\theta\) appears to lowest order only in
the grouping \( \theta \sqrt{m_i/m_e} \) (note that here the scaling involves the mass of the ambient ions, not the beam ions). Hence, for small values of \( \theta \) (the parameter range of interest since waves at large values of \( \theta \) are heavily damped by electron Landau damping) the results of the dispersion relation are the same for constant \( \theta \sqrt{m_i/m_e} \). A second important scaling pointed out by McBride et al. also holds for the current dispersion relation in the collisionless limit. This scaling applies for \( \kappa \ll 1 \) and \( \theta \ll 1 \), i.e. for wavelengths long compared to the electron Larmor radius, which is the regime of interest. In this case the electron susceptibility may be simplified:

\[
\frac{1}{\kappa^2 \Omega_e^2} \left[ 1 + I_0(b)e^{-b} \zeta_e Z'(\zeta_e) \right] = \frac{1}{\kappa^2 \Omega_e^2} \left[ 1 - I_0(b)e^{-b} \left( 1 + \frac{1}{2} Z' Z'' \right) \right] \tag{2.39}
\]

Expanding \( I_0(b)e^{-b} \) for \( b \ll 1 \),

\[
I_0(b)e^{-b} = (1 - b - \cdots)(1 + \frac{b^2}{4} + \cdots) \approx 1 - b \tag{2.40}
\]

Substituting \( b = \kappa^2 \) gives

\[
\frac{1}{\kappa^2 \Omega_e^2} \left[ 1 - I_0(b)e^{-b} \left( 1 + \frac{1}{2} Z' Z'' \right) \right] \approx \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{1}{2\kappa^2} Z''(\zeta_e) \tag{2.41}
\]

Defining new variables

\[
\bar{\theta} = \theta \sqrt{m_i/m_e} \tag{2.42}
\]

\[
\bar{\kappa} = \frac{\kappa \Omega_e}{\omega_{pe}} \sqrt{1 + (\omega_{pe}/\Omega_e)^2} \]

\[
\bar{\omega} = \frac{\omega}{\omega_{pi}} \sqrt{1 + (\omega_{pe}/\Omega_e)^2}
\]

Equation (2.36) may be rewritten as

\[
1 - \frac{1}{2\bar{\kappa}^2} Z''(\zeta_e) - \frac{1}{2\bar{\kappa}^2} \frac{T_e}{T_i} n_i Z'(\zeta_i) - \frac{1}{2\bar{\kappa}^2} \frac{T_e}{T_b} n_e Z'(\zeta_b) = 0 \tag{2.43}
\]
with the parameters expressed as

\[ \zeta_e = \frac{1}{\sqrt{2}} \frac{\omega}{R} \sqrt{\frac{n_i}{n_e}} \]

\[ \zeta_i = \frac{1}{\sqrt{2}} \frac{\omega}{R} \sqrt{\frac{T_i n_i}{T_e n_e}} \]

\[ \zeta_b = \frac{1}{\sqrt{2}} \frac{\omega}{R} \sqrt{\frac{T_b n_i m_i}{T_e n_e m_i}} - \frac{U}{\sqrt{2} v_b} \]  (2.44)

By use of the scalings, the space of nine parameters for the collisionless dispersion relation has been reduced to seven. This may be further reduced by assuming overall neutrality of the system, i.e. \((n_i + n_b)/n_e = 1\), which may be used to remove \(n_b\) from Equation (2.43) and reduce the number of parameters to six.

Before proceeding to analyze the effect of the remaining six parameters on the dispersion relation, the validity of the scalings is checked in Figure 2-1. In subfigure (a), the growth rate maximized over \(\kappa\), \(\gamma^* = \frac{\gamma_{\text{max}}}{\omega_{pi}} \sqrt{1 + (\omega_{pe}/\Omega_e)^2}\), is shown as a function of the angle \(\delta\). The parameters are set to the 'nominal' values given below in Equation (2.65).

The plot contains the solution for two mass ratios, \(m_i/m_e = 7344\) and 29736 (helium and oxygen, respectively). As predicted by the scaling, the results appear identical. Subfigure (b) presents the results of the \(\kappa\) scaling. In this case, the growth rate is shown as a function of \(\kappa\) for a mass ratio of \(m_i/m_e = 29376\), \(i\thetaeta = 1\) and two values of \(\omega_{pe}/\Omega_e\) (.5 and 2). Again, as predicted from the scaling arguments, no perceptible difference is noticeable in the two cases. Physically, this result corresponds to the fact that the plasma response is insensitive to electron gyromotion since \(k \rho_e \ll 1\).

**Dispersion Relation for a Cold Plasma**

One more limit will be useful in the discussion of the behavior of the dispersion limit. In this limit, all the arguments of the \(Z\) functions are assumed \(\zeta \gg 1\). This limit will apply when (assuming \(T_e \approx T_i \approx T_b\)):

\[ \frac{1}{\theta v_e} \gg 1 \]

\[ \frac{v_p}{v_i} \gg 1 \]  (2.45)
\[
\frac{v_p - U}{v_b} \gg 1
\]

where \(v_p = \omega / k\) is the phase velocity. As seen in these equations, this limit applies when the wave phase speed is much greater than the thermal speeds and beam velocities of all the species, i.e. this is the cold plasma approximation. It is also seen that the restrictions are most easily satisfied when \(\theta \ll 1\). Because of this restriction, the following analysis will continue to consider the case of \(\theta \ll 1\). Physically, the restrictions in Equation (2.45) correspond to requiring that Landau damping be unimportant. The ambient or beam ions will become resonant with the waves and heavily damp them if either of the last two restrictions of Equation (2.45) is violated, respectively. Similarly, if the first restriction is violated, the electrons, which are restricted to moving along the B-field (hence the introduction of the angle \(\theta\) in the restriction) will become resonant with the waves and damp them. Thermal effects are therefore most important (1) at large \(\theta\) and (2) for phase velocities near the thermal velocity of the ambient ions or the beam velocity of the beam ions.

The cold plasma limit must be used with caution in the current situation since it is not applicable in many cases of interest in CIV. However, because the simplifications the approximation affords still provide useful insight, the analysis is included here. In the cold plasma approximation, the \(Z\) function may be expanded

\[
Z'(\zeta) \approx \zeta^{-2} \quad \zeta \gg 1
\]

Equation (2.43) then becomes (after removing the normalizations which are substantially simplified since all the thermal velocities go to zero):

\[
1 + \frac{\omega_p^2}{\Omega_e^2} = \theta^2 \frac{\omega_{pe}^2}{\omega^2} + \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pb}^2}{(\omega - kU)^2}
\]

(2.47)

Equation (2.47) is seen to be of the same form as the dispersion relation for the well known two-stream instability in a cold, unmagnetized plasma [39], hence the name 'modified two stream instability'.

As a first point in analyzing Equation (2.47), an exact solution can be found in the
case $\theta^2 \omega_{pe}^2 + \omega_{pi}^2 = \omega_{pb}^2$, i.e. when the ambient plasma response is similar to the beam ion response. In this case, Equation (2.47) has solution

$$\left( \frac{\omega}{\omega_{LH,b}} \right)^2 = \frac{1}{4} \left( \frac{kU}{\omega_{LH,b}} \right)^2 + 1 \pm \sqrt{\left( \frac{kU}{\omega_{LH,b}} \right)^2 + 1}$$

(2.48)

where $\omega_{LH,b} = \omega_{pb} / \sqrt{1 + \omega_{pe}^2 / \Omega_e^2}$ is the lower hybrid frequency based on the beam. Differentiation of Equation (2.48) leads to the values of the maximum growth rate, $\gamma^*$, and the real frequency, $\omega^*$ and wavenumber, $k^*$, corresponding to maximum growth. The result is

$$\gamma^* = \frac{1}{2} \omega_{LH,b}$$

$$\omega^* = \frac{1}{2} \sqrt{3} \omega_{LH,b}$$

$$k^* = \sqrt{3} \omega_{LH,b} / U$$

(2.49)

Equations (2.49) indicate that the real and imaginary frequencies at maximum growth rate are expected to be near the lower hybrid frequency of the beam ions. In addition, the phase velocity of the fastest growing wave is seen to be $\omega^* / k^* = U / 2$. Since the phase velocity is approximately half the beam velocity, $U$, Equation (2.45) is now seen to place a limit on the beam velocity for the cold plasma approximation to be correct: $U \gg v_b$ is required for thermal effects to be unimportant. If this criterion is violated, ion Landau damping must be included.

The previous paragraphs have discussed the exact solution of Equation (2.47) which can be found when $\theta^2 \omega_{pe}^2 + \omega_{pi}^2 = \omega_{pb}^2$. The solution of the dispersion relation when this condition is not satisfied is now considered. Since Equation (2.47) is of the same form as the two-stream instability, a similar analysis may be applied. Figure 2-2 shows a plot of the left and right hand sides of Equation (2.47). The figure shows a stable situation, i.e. four real roots exist. The situation will be unstable when the local minimum is no longer less than $1 + \omega_{pe}^2 / \Omega_e^2$. The frequency corresponding to the local minimum (labeled
\[ \omega_A \text{ in the figure) is given by } \]
\[ \omega_A = kU \left[ \frac{W_1^{2/3}}{W_1^{2/3} + 1} \right] \tag{2.50} \]
where
\[ W_1 = \frac{\theta^2 \omega_{pe}^2 + \omega_{pi}^2}{\omega_{pb}^2} \tag{2.51} \]

For instability to occur, the right hand side of Equation (2.47) evaluated at \( \omega_A \) must be greater than \( 1 + \frac{\omega_{pe}^2}{\Omega_e^2} \). The condition for instability to occur is therefore
\[ \frac{|kU|}{\omega_{LH,b}} < \left[ W_1^{2/3} + 1 \right]^{3/2} \tag{2.52} \]

Equation (2.52) is most easily satisfied for long wavelength waves or for small ion velocity, \( U \). For small \( U \), however, thermal effects become important and the current analysis breaks down.

In order to determine the values at the maximum growth rate, Equation (2.47) is differentiated with respect to \( \kappa \). After some manipulation,
\[ \left[ \left( \frac{\omega}{k} - U \right)^3 \left( \theta^2 \omega_{pe}^2 + \omega_{pi}^2 \right) + \frac{\omega_{pb}^2 \omega^3}{k^3} \right] \frac{d\omega}{dk} = \omega_{pb}^2 U \frac{\omega^3}{k^3} \tag{2.53} \]

By letting \( \omega = \omega_r + i\gamma \), the maximum growth rate is found by letting \( d\gamma/dk = 0 \). A solution for \( d\gamma/dk = 0 \) is obtainable when
\[
\text{Im} \left\{ \frac{\omega^3}{k^3} \left[ \left( \frac{\omega}{k} - U \right)^3 \left( \theta^2 \omega_{pe}^2 + \omega_{pi}^2 \right) + \frac{\omega_{pb}^2 \omega^3}{k^3} \right] \ast \right\} = 0, \tag{2.54} \]

where, in this expression, the asterisk denotes the complex conjugate. More manipulation shows that Equation (2.54) is satisfied if the frequency satisfies
\[ \text{Im} \left\{ \left( 1 - \frac{kU}{\omega} \right)^3 \right\} = 0 \tag{2.55} \]
indicating that the frequency at the maximum growth rate (\( \omega^* \), maximized over \( \frac{1}{k} \)) must be of the form
\[ \frac{\omega^*}{\omega_{LH,b}} = \frac{k^* U}{\omega_{LH,b}} \left( \frac{1}{1 - S_e \pm i \pi / 3} \right) \tag{2.56} \]
where \( S = S(W_1) \) is a real number and \( k^* \) is the wavenumber corresponding to maximum growth. Substitution of Equation (2.56) into Equation (2.47) also reveals

\[
\left( \frac{k^* U}{\omega_{LH,b}} \right)^2 = (1 - S e^{i\pi/3}) \left[ W_1 + (S e^{i\pi/3})^{-2} \right]
\]

(2.57)

As seen in Equations (2.56) and (2.57), the solution of the dispersion relation \( (\omega^*/\omega_{LH,b} \text{ and } k^* U/\omega_{LH,b}) \) relies only on \( W_1 \) and \( S(W_1) \). Unfortunately, unlike the usual two-stream instability, a suitable simplification cannot be found to provide a simple approximation for \( S \). The dispersion relation is, however, easily solved numerically to obtain \( S(W_1) \), as well as \( \omega(W_1) \) and \( k(W_1) \). The results are shown in Figure 2-3. At values of \( W_1 = 1 \), the solution to the dispersion relation are seen to satisfy the exact solution which was found in Equation (2.49). For the entire range of \( W_1 \) shown, \( \omega^* \) and \( \gamma^* \) are both seen to be on the order of the beam's lower hybrid frequency. The real frequency \( \omega_r \) continues to increase with increasing \( W_1 \), but if increasing \( W_1 \) is due to increasing \( \theta \), the cold plasma approximation must begin to break down for large \( W_1 \). The thermal effects which cause the current approximation to break down will be discussed more fully below.

Before proceeding to consider more complex forms of the dispersion relation, it is useful to summary the results for the cold plasma approximation:

- In order for the cold plasma approximation to be valid, Equations (2.45) must be satisfied. Thermal effects are most important at large \( \theta \) and for phase velocities near the thermal velocity of the ambient ions or the beam velocity of the beam ions.

- For \( W_1 = 1 \), an exact solution of the dispersion relation may be found. The result for \( \omega(k) \) is given by Equation (2.48).

- For the exact solution, the maximum frequencies and corresponding wavenumber are given by Equation (2.49).

- A condition on \( kU \) for instability can be developed. This condition is given by Equation (2.52). The instability condition is most easily satisfied for long wavelength waves.
• For \( W_1 \neq 1 \), the solution of the dispersion relation is given by Equations (2.56) and (2.57). The function \( S(W_1) \) is shown in Figure (2-3) along with the solution of the dispersion relation.

• For the values of \( W_1 \) investigated, the real and imaginary parts of the frequency of the fastest growing wave are near the lower hybrid frequency of the beam.

**Electromagnetic and Finite \( \beta \) Effects:**

Before returning to a discussion of the electrostatic dispersion relation with thermal effects included, it is useful to investigate electromagnetic and finite \( \beta \) effects. For simplicity, the cold plasma limit is again considered. In the cold plasma limit, the inclusion of electromagnetic and finite \( \beta \) effects leads to a modification of Equation (2.47). The result is [48]

\[
1 + \frac{\omega_p^2}{\omega_e^2} \left( 1 + \frac{\omega_p^2}{k^2c^2(1 + \beta_e)} \right) - \frac{\omega_p^2}{\omega_e^2} - \frac{\omega_p^2}{(\omega - k \cdot U)^2} - \frac{\omega_p^2}{\omega_e^2} \frac{\theta^2}{1 + \omega_p^2/k^2c^2} = 0 \tag{2.58}
\]

where \( c \) is the speed of light and \( \beta_e = n_e T_e/(B_0^2/2\mu_0) \). Equation (2.58) reduces to Equation (2.47) in the limit \( \omega_p/kc \to 0 \) and \( \beta_e \to 0 \). Equation (2.58) shows that the inclusion of electromagnetic waves and a finite \( \beta \) results in additional wavenumber dependencies in the electron and beam ion responses. The effect of these additional wavenumber dependencies is to stabilize the M2SI [51]. In fact, since Equation (2.58) is again similar in form to the well known Bunemann two-stream instability, a similar stability analysis may be performed. The condition for instability in this case is

\[
\left| \frac{kU}{\omega_B} \right| < \left[ W_2^{2/3} + 1 \right]^{3/2} \tag{2.59}
\]

where

\[
W_2 = \frac{\omega_p^2}{\omega_e^2} \frac{1 + (\omega_p/kc)^2}{1 + \omega_p^2/(k^2c^2(1 + \beta_e))} \tag{2.60}
\]

\[
\omega_B^2 = \frac{\omega_p^2}{\omega_e^2} \left[ 1 + \frac{(\omega_p/kc)^2}{1 + \beta_e} \right] \tag{2.61}
\]
In the limit \( \omega_{pe} / kc \ll 1 \), Equation (2.59) reduces to the stability condition for the electrostatic limit, Equation (2.52). However, for \( \omega_{pe} / kc \gg 1 \) (i.e. when electromagnetic effects are most important) and assuming \( \theta \ll 1 \), the requirement for instability becomes

\[
\frac{kU}{\omega_B} < \left[ \left( \frac{\omega_{pe}}{\omega_{ph}} \right)^{4/3} + 1 \right]^{3/2}
\]

In the initial stages of CIV, \( n_b \ll n_i \), i.e. the case of interest is a very weak beam. In this case the stability condition can be further simplified to

\[
U < v_a \sqrt{1 + \beta_e}
\]

where \( v_a = B_0 / \sqrt{\mu_0 \rho_m} \) (\( \rho_m \) is the mass density). This is the stability requirement used by several previous authors\[1, 48\]. Physically, the limitation imposed by Equation (2.63) states that when the wave phase speed exceeds the Alfvén speed, the electrons can no longer coherently interact with the waves in the M2SI. Since electrons can no longer coherently interact, the M2SI is stabilized.

For the laboratory and space experiments of CIV, \( \beta_e \ll 1 \). In these experiments, then, for the instability to occur the Alfvén speed places an upper limit on the beam velocity. Equation (2.63) can also be written

\[
\frac{\omega_{pe}}{\Omega_e} < \frac{c}{U} \left( \frac{m_e(1 + \beta_e)}{m_i} \right)^{1/2}
\]

showing the dependency of the instability on the strength of the magnetic field.

In both laboratory and ionospheric experiments involving CIV, \( \beta_e \ll 1 \) is a good approximation. In addition, the ionospheric experiments satisfy \( \omega_{pe} / kc \ll 1 \), indicating electromagnetic effects are not important. Most laboratory experiments also satisfy \( \omega_{pe} / kc \ll 1 \); the exceptions are the experiments of Brenning [11] using a small magnetic field strength (Cf. the Introduction). Since the cases of interest satisfy both \( \beta_e \ll 1 \) and \( \omega_{pe} / kc \ll 1 \), the electrostatic approximation is applicable.

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2.1.3 Numerical Solution of the Dispersion Relations:

The analysis now returns to the full electrostatic dispersion relation, Equation (2.37). Equation (2.37) will be analyzed numerically by performing a parametric study of six parameters: the angle, mass, density, velocity and temperature of the ion beam and the temperature of the ambient ions. Each of the parameters $\bar{\theta}$, $n_b/n_e$, $U/v_b$, $T_b/T_e$, and $T_i/T_e$ will be independently varied from a nominal value and the results of the dispersion relation discussed. The effect of the parameter $m_b/m_i$ will be analyzed by performing each of the parametric searches for three ambient/beam ion mass ratios. The nominal values chosen for the parameters are

\[
\begin{align*}
\bar{\theta} & = 1 \\
n_b/n_e & = 0.5 \\
U/v_b & = 15 \quad \text{(2.65)} \\
T_b/T_e & = 1.0 \\
T_i/T_e & = 1.0
\end{align*}
\]

As stated, the effect of each of these parameters will be studied for three values of $m_b/m_i$. The chosen values are

\[
\frac{m_b}{m_i} = \begin{cases} 
0.25 & \text{Helium/Oxygen} \\
1.00 & \text{Oxygen/Oxygen} \\
5.24 & \text{Kryton/Oxygen}
\end{cases} \quad \text{(2.66)}
\]

Figures (2-4) through (2-8) present the results of the parametric search for each parameter. In each figure, the results are shown as four subfigures. Subfigures (a) and (b) present the real and imaginary parts of the frequency of the most unstable wave (maximized over the wavenumber $\bar{k}$) as a function of the parameter being studied. The frequencies are normalized to the lower hybrid frequency based on the background ions. Hence $\bar{\omega}^* = 1$ implies a frequency equal to the lower hybrid frequency, $\omega_{LH} = \omega_{pi}/\sqrt{1 + (\omega_{pe}/\Omega_e)^2}$. The wavenumber ($\bar{k}^*$) of the most unstable wave is shown in Subfigure (c). The wavenumber
is normalized such that \( \bar{\kappa}^* = k \rho_e \frac{\Omega_e}{\omega_{pe}} \sqrt{1 + \left( \frac{\omega_{pe}}{\Omega_e} \right)^2} \). Since the cases of interest here have \( \omega_{pe}/\Omega_e \approx 1 \), \( \bar{\kappa}^* \approx 1 \) implies a wavenumber \( k \rho_e \approx 1/\sqrt{2} \). For wavelengths large compared to electron Larmor radius, \( \bar{\kappa}^* \ll 1 \) should be expected. Finally, Subfigure (d) shows the phase velocity, \( \bar{\omega}_r/\bar{\kappa}^* \), of the most unstable wave. The normalization on the phase velocity gives

\[
\frac{\bar{\omega}_r}{\bar{\kappa}^*} = \frac{\omega_{pe}}{\bar{\omega}_{pe}} \frac{\omega}{\omega_{pe}} \frac{1}{\omega_{pe}} \frac{\Omega_e}{\omega_{pe}} \sqrt{\frac{n_e m_i}{n_i m_e}} \frac{1}{\omega_{pe}} \frac{1}{k}.
\]

For weak beams \( (n_i \approx n_e) \) with \( \Omega_e/\omega_{pe} = 1 \), the normalization reduces to the ion acoustic speed, \( C_s = \sqrt{T_e/m_i} \). In each of the subfigures, the red lines are included to represent the three ambient/beam ion mass ratios. The solid lines on the figures represent the numerical solution of the dispersion relation. The solid dots are values obtained from simulations of the M2SI. These simulation results will be discussed more fully in Chapter 4.

**Angle:** The first parameter to be considered is the angle. Figure 2.4 shows the results for the most unstable waves as a function of the angle. The growth rate of the instability reaches a maximum at an angle slightly greater than \( \bar{\theta} = 1 \) for all three beam gases. This is qualitatively the same as found in McBride et al.[51] for a similar, but simplified, situation. The maximum occurs at greater values of \( \bar{\theta} \) for the lighter gases. In fact, the maximum in the growth rate occurs at roughly constant \( \bar{\theta} \sqrt{m_b/m_i} \) for the different mass ratios. The ‘effective electron mass’ as described by McBride is related to the beam ion mass, not the ambient ion mass. The growth rate decreases quickly above the maximum. This is a result of electron Landau damping of the waves. Waves with large \( \bar{\theta} \) are resonant with electrons moving along the magnetic field line. These resonant electrons heavily damp waves at large \( \bar{\theta} \). At small angles, the growth rate saturates as \( \bar{\theta} \to 0 \). Maximum values are also obtained for the real frequency and wavenumber, although the maximums do not correspond to the maximum in growth rate. For both real frequency and wavenumber, the lighter beam gases reach the maximum at larger values of \( \bar{\theta} \). The phase velocity of the wave increases slightly with increasing \( \bar{\theta} \). The waves in the lighter gases have higher phase velocities. In summary, low beam/ambient mass ratios provide a larger region of unstable modes in \( \bar{\theta} \) space and also provide waves.
with higher phase velocities. The system is stable at large \( \theta \) due to electron Landau damping.

**Beam velocity:** Figure 2-5 presents the solution of the dispersion relation as a function of the beam velocity. For about \( U/v_b > 15, \omega/k \gg 1 \), i.e. this is the cold plasma approximation. In this case, the scalings indicated by the approximate solution of the dispersion are evident. The wavenumber falls off as \( v_b/U \), while the real and imaginary frequencies become independent of \( U/v_b \). The limiting values of \( U/v_b \gg 1 \) are seen to agree well with the results of the cold dispersion relation \( W_1 = 1.5 \), cf. Figure 2-3). The linear relation between \( \omega^* \) and \( \kappa^* \) \( (\omega^*/\kappa^* \sim U, \text{see Equation 2.56}) \) is also shown in Part (d) of the figure.

**Beam Temperature:** The effects of both the beam and ion temperatures are shown in Figure 2-6. First, both the real and imaginary parts of the frequency are roughly independent of the temperature ratios. The results for \( T_i/T_e \) and \( T_b/T_i \) are also seen to be nearly identical; an indication that the dispersion relation is most sensitive to the ratio \( T_b/T_e = \frac{T_b}{T_i} \frac{T_i}{T_e} \).

**Beam Density:** The results of parametrically varying the beam density are shown in Figure 2-7. In order to remove the density dependence from the normalization, it is noted the the frequencies in Parts (a) and (b) of the figure have been renormalized by multiplying by \( \sqrt{n_b/n_e} \).

Several features are notable in the figure. First, as \( n_b/n_e \to 0, \gamma^* \to 0 \). This should be obvious, as no instability would be expected without a beam as a source of free energy. Also, in the limit \( n_b/n_e \to 1 \), the results become the same as those considered by McBride, since in this case, no background plasma is present. Considering the other results, the real frequency and wavenumber are only weakly dependent on the beam density. The real frequency decreases with increasing beam density while wavenumber increases. The phase velocity is seen to slowly decrease with increasing beam density.

**Collisional Effects:** In this section, the effect of collisions on the dispersion relation solution is examined. The parameter \( \nu \) is independently varied while the rest of the parameters retain their nominal values. The scaling on \( \kappa \) no longer holds for this section,
so care must be taken in interpreting the results. Although the approximations made in the discussions above no longer apply, certain statements can be made about the behavior of the dispersion relation on a physical basis. First, for low collisionality, i.e. \( \nu / \omega_{pi} \ll 1 \), the solution must revert to the collisionless case. Second, an important physical point in the operation of the M2SI is that the electrons accelerate along the magnetic field lines as they stay in phase with the lower hybrid wave. For \( \nu / \omega_{pi} \approx 1 \), the particles collide before they can stay in phase with the wave for an extended period. Since the collision will knock the electrons out of phase with the wave, the growth rate is expected to decrease quickly. The actual solution of the dispersion relation is shown in Figure 2-8. As expected, for small \( \nu / \omega_{pi} \), the results do indeed approach the collisionless case. As the collision rate increases, the growth rate decreases quickly. The real component of the frequency and the wavenumber exhibit complicated behavior as \( \nu / \omega_{pi} \) nears 1.0. This behavior is not at all obvious from an examination of the dispersion relation. A strong dependence on the mass ratio may also be noted in the figure.

Summary: In the past paragraphs, the dispersion relation has been solved parametrically for six different parameters. At this point, the salient features are summarized:

- a review of Figures 2-4 to 2-8 indicates that for the parameter range of interest, the real frequency and growth rate of the most unstable wave are both \( \mathcal{O}(\omega_{LH}) \). Also, \( k \rho_e \ll 1 \), where \( \rho_e \) is the electron Larmor radius.

- a maximum in the growth rate occurs at an angle roughly in the range \( 1 < \theta < 10 \),

- a cutoff angle, \( \theta_c \), exists above which the M2SI is stabilized due to electron Landau damping,

- for large \( U / v_b \) (say roughly \( U / v_b > 10 \)), the real and imaginary parts of the frequency are independent of \( U / v_b \) (physically, this is analogous to the "Mach number independence principle" of hypersonic flows). Physically, this regime also corresponds to the cold plasma approximation,

- the M2SI does not operate for \( U / v_b \) below a cutoff value due to stabilization by ion Landau damping,
• the real and imaginary parts of the frequency are roughly independent of the temperature ratios $T_b/T_i$ and $T_i/T_e$,

• the growth rate of the instability increases with increasing density ratio, $n_b/n_e$, and

• the growth rate decreases rapidly near $\nu/\omega_{pi} \approx 1$, since at this value the electrons can no longer stay in phase with the wave.

Implications for CIV: If the M2SI is accepted as the basic mechanism operating in CIV, the above results have several implications for the CIV process. First, two conditions must be met in the experiments for CIV to occur: (1) the velocity must exceed the cutoff value, and (2) the electron-neutral collision frequency must be below the ion plasma frequency. If either of these conditions is violated, the M2SI and CIV will not operate. The first condition is easily satisfied in most experiments since the more stringent restriction $U > v_c$ is applied, where $v_c$ is the critical velocity. The second condition may have consequences for CIV in regions at high neutral densities. In most experiments, the neutral density is low enough that $\nu/\omega_{pe} \ll 1$. The exception is in space experiments near the point of release. In this region, the densities may be high enough to violate the $\nu/\omega_{pi} < 1$ restriction. This implies that CIV may be slowed in high density regions. This concept will be explored more fully in Chapter 4 in relation to the simulation results.

The results also indicate that the wave spectrum observed during CIV should be peaked near the lower hybrid frequency ($\omega \sim \omega_{LH}$), and that the most unstable waves should propagate almost perpendicular to the magnetic field (unstable waves only exist for $\theta \ll 1$). Indeed, both of these observations are consistent with the measurements made in laboratory experiments.

2.1.4 Quasilinear and Nonlinear Theory

Before considering more CIV-like situations, it is useful to look briefly at the nonlinear and quasilinear development of the M2SI in order to investigate two points. The first point deals with the mass ratio scaling observed in the linear stability analysis. This is
an important point in a numerical sense, since if it can be shown that the scaling holds for the nonlinear system, simulations performed with a reduced mass ratio can be used to determine the solution at realistic mass ratios. The second point is to provide physical insight into the development of the M2SI.

The analysis begins by considering the nonlinear system. The full nonlinear system is described by the Vlasov-Poisson system

\[
\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \frac{\partial f_i}{\partial \mathbf{x}} + \frac{q_i}{m_i} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_i}{\partial \mathbf{v}} = 0 \tag{2.67}
\]

\[
\nabla^2 \phi = -\frac{1}{\varepsilon_0} \sum_j q_j \int f_j d\nu \tag{2.68}
\]

Consistent with the linear analysis, the beam and ambient ions are treated as unmagnetized. Using this assumption,

\[
\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i - \frac{q_i}{m_i} \nabla \phi \cdot \frac{\partial f_i}{\partial \mathbf{v}} = 0 \tag{2.69}
\]

and

\[
\frac{\partial f_b}{\partial t} + \mathbf{v} \cdot \nabla f_b - \frac{q_b}{m_b} \nabla \phi \cdot \frac{\partial f_b}{\partial \mathbf{v}} = 0 \tag{2.70}
\]

The electrons are treated in the drift approximation giving

\[
\frac{\partial f_e}{\partial t} + v_z \frac{\partial f_e}{\partial z} - \frac{q_e}{m_e} \frac{\partial \phi}{\partial z} \frac{\partial f_e}{\partial v_z} + \nabla \cdot (\mathbf{v}_d f_e) = 0 \tag{2.71}
\]

where \( \mathbf{v}_d \) is the electron drift velocity given by

\[
\mathbf{v}_d = \frac{\mathbf{E} \times \mathbf{B}}{B_0^2} - \frac{m_e}{eB_0^2} \left\{ \frac{\partial \mathbf{E}_\perp}{\partial t} + \mathbf{v}_d \cdot \nabla \mathbf{E}_\perp + v_z \frac{\partial \mathbf{E}_\perp}{\partial z} \right\} \tag{2.72}
\]

Under the further assumption that variations only exist in the U-B plane and that the variations in the electric field are much greater across the magnetic field (the direction denoted by \( z \)) than along the magnetic field (the direction denoted by \( x \)), the equations may be written as

\[
\frac{\partial f_i^z}{\partial t^z} + v_z^z \frac{\partial f_i^z}{\partial z^z} - \frac{q_i^z}{m_i^z} \frac{\partial \phi^z}{\partial z^z} \frac{\partial f_i^z}{\partial v_z^z} = 0 \tag{2.73}
\]
\[ \frac{\partial f^*_b}{\partial t^*} + v^*_z \frac{\partial f^*_b}{\partial x^*} - \frac{q^*_e}{m^*_b} \frac{\partial \phi^*}{\partial x^*} \frac{\partial f^*_b}{\partial v^*_z} = 0 \] (2.74)

and

\[ \frac{\partial f^*_e}{\partial t^*} + v^*_z \frac{\partial f^*_e}{\partial x^*} - \frac{q^*_e}{m^*_e} \frac{\partial \phi^*}{\partial x^*} \frac{\partial f^*_e}{\partial v^*_z} + \frac{\partial (v^*_p f^*_e)}{\partial x^*} = 0 \] (2.75)

where \( t^* = \omega_{pi} t, \ x^* = x \omega_{pi} / U, \ m^* = m / m_b, \ \phi^* = e \phi / m_b U^2, \ v^* = v / U \) and \( U \) is the initial velocity of the beam ions. Only the polarization drift, \( v_p \), is retained in the electron drift velocity, \( v_d \) (i.e. the \( E \times B \) drift drops out of the equation under the current assumptions\(^3\)):

\[ v^*_p = \frac{v_p}{U} = \frac{m_b \omega_{pe}^2}{m_e \Omega_e^2} \left\{ \frac{\partial}{\partial t^*} + v^*_p \frac{\partial}{\partial x^*} + v^*_z \frac{\partial}{\partial v^*_z} \right\} \frac{\partial \phi^*}{\partial x^*} \] (2.76)

In non-dimensionalized form, the Poisson Equation is also written as

\[ \frac{\partial^2 \phi^*}{\partial z^*} = - \sum q^*_j n^*_j \int f^*_j dv \] (2.77)

The point regarding the mass ratio scaling that was seen in the linear analysis is again highlighted by Equations (2.73) to (2.77). The mass ratio \( m^*_e / m_b \) may be removed from the problem by rescaling in the \( z \)-direction.

\[ \bar{z} = z^* \sqrt{\frac{m_e}{m_b}} \]

\[ \bar{v}_z = v^*_z \sqrt{\frac{m_e}{m_b}} \] (2.78)

Hence, the mass ratio scaling pointed out in the linear analysis is also true for the full non-linear problem (within the assumption of no \( E \times B \) drift). This will allow one- and two- dimensional simulations (in the U-B plane) to be performed with unrealistic mass ratios. This will be shown numerically in Chapter 4.

The second point of this section is to provide physical insight into the M2SI by considering the quasilinear diffusion equations. The application of the standard quasilinear

---

\(^3\)It should be emphasized that the purpose here is to investigate the M2SI physically. The assumptions leading to the neglect of the \( E \times B \) drift may not be applicable to CIV situations involving ionization mechanisms. Certainly, these assumptions eliminate Piel's mechanism from the current analysis.
procedure to Equations (2.73) to (2.77) is given by McBride et al., and the details are omitted here. The resulting quasilinear equations for the beam and ambient ions are

$$\frac{\partial f_{0,i}}{\partial t} = \frac{\partial}{\partial v_i} \left[ D_j \frac{\partial f_{0,i}}{\partial v_a} \right]$$

(2.79)

where

$$D_j = \frac{q_i^2}{m_i^2} \sum_k \frac{k_z^2 |\phi_k|^2 \gamma_k}{(\omega_k - k_z v_z)^2 + \gamma_k^2}$$

(2.80)

whereas for the electrons, the equation is

$$\frac{\partial f_{0,e}}{\partial t} = \frac{\partial}{\partial v_z} \left[ D_e \frac{\partial f_{0,e}}{\partial v_z} \right]$$

(2.81)

where

$$D_e = \frac{q_e^2}{m_e^2} \sum_k \frac{k_z^2 |\phi_k|^2 \gamma_k}{(\omega_k - k_z v_z)^2 + \gamma_k^2}$$

(2.82)

An interesting physical interpretation can be deduced from Equations (2.79) through (2.82). As indicated by the results of the linear dispersion relation, the most unstable waves of the M2SI have $k_z/k \sim \sqrt{m_e/m_b}$. Taking also $v_z/v_x \sim \sqrt{m_b/m_e}$, the electron response may be rewritten as

$$\frac{\partial f_{0,e}}{\partial t} \sim \frac{\partial}{\partial v_z} \left[ D'_e \frac{\partial f_{0,e}}{\partial v_z} \right]$$

(2.83)

where

$$D'_e \sim \frac{q_e^2}{m_e^2} \sum_k \frac{k_z^2 |\phi_k|^2 \gamma_k}{(\omega_k - k_z v_z)^2 + \gamma_k^2}$$

(2.84)

Equations (2.83) and (2.84) indicate that the electrons respond to the waves not as a low mass species that is highly magnetized, but as an unmagnetized species with an effective mass near $m_b$. This occurs since the electrons are closely tied to the magnetic field lines which are nearly perpendicular to the wave vector. Hence, the electrons only experience a small component of the wave electric field.

With the viewpoint that the electrons act as if they were unmagnetized and have a high effective mass ratio, it becomes substantially easier to explain the quasilinear development of the M2SI. Figure 2-9 shows a series a sketches to explain this development.
The situation considered here is that of an ion beam propagating across a magnetized ambient plasma. The first panel of the figure indicates the initial distribution functions of the ambient and beam ions and the ambient electrons assuming an effective mass of $m_e = m_i = m_b$ and equal ambient and beam ion densities (i.e. $n_b = n_i = 0.5$). Also indicated on the figure is the range of phase velocities of unstable wave as obtained from linear theory. The region of unstable waves coincides with the positive gradient of the ion distribution function in agreement with the results of the well known Penrose stability criterion [39]. As sketched in the figure, only the beam ions are initially resonant with the unstable waves. This initial situation is quite similar to bump-on-tail instabilities which have been studied extensively in plasma physics. These bump-on-tail instabilities saturate through a process of plateau formation, which is thus the expected saturation mode for the current case. The second panel of Figure 2-9 shows the initial stages of this plateau formation. The ion beam distribution begins to fill in the unstable region. The ambient ions and electrons simultaneously begin to heat in bulk as they interact in a nonresonant manner with the waves. As the plateau begins to form, the unstable region of the wave spectrum also begins to broaden. Eventually, the growing plateau region begins to interact with the ambient ions and electrons, i.e. ambient ions and electrons begin to become resonant with the unstable waves. This point is portrayed in the third panel of the figure. As the ambient ions and electrons become resonant with the unstable waves, they also begin to build plateaus. The final state, shown in the last panel of the figure, is one in which the entire unstable region has been filled in by the ion distribution function.

In addition to providing insight into the development of the M2SI as discussed in the previous paragraphs, the quasilinear equations developed here can be used to derive estimates of the energy transfer between ions and electrons in the M2SI. In the following section, the method of Formisano et al. [23] will be used to develop estimates of the energy transferred from beam ions to electrons. This is the "constant ionization rate model".
2.2 Constant Ionization Rate Model

Further understanding into the CIV process may be made by introducing a "constant ionization rate model" into the M2SI model described in the previous sections. The previous examples assumed a 'pre-formed' ion beam. The collisional processes to form the beam are not included at all. This is, of course, something of an unrealistic simulation. CIV proceeds by initializing a beam through some type of seed process. In addition, through this formation process, many of the ions have time to complete one or more gyroperiods, hence the unmagnetized ions assumed in the previous section may no longer by valid.

In fact, Formisano et al. [23] have shown that the amount of energy transferred from the ions to the electrons is a strong function of the gyroperiod of the ion. Their work considers an ion beam propagating across a background plasma and magnetic field in a manner similar to that described in the previous section. Now, however, the beam ions are created at a rate \( \nu_{\text{ion}} \) such that

\[
\frac{dn_e}{dt} = \nu_{\text{ion}} n_e. \tag{2.85}
\]

By defining the fraction of energy transferred from the ion beam to the electrons as \( \eta \), the energy equation for the electrons may be written in a simple form as

\[
n_e \frac{dT_e}{dt} = \nu_{\text{ion}} n_e \left( \eta \frac{1}{2} m_a U^2 - T_e \right) \tag{2.86}
\]

where the first term on the right represents energy added to the electrons via waves from the ion beam. The temperature is seen to saturate \( (dT_e/dt = 0) \) when

\[
T_{e,\text{sat}} = \eta \frac{1}{2} m_a U^2 \tag{2.87}
\]

Although in this section the parameter \( \nu_{\text{ion}} \) will be treated as a fixed input parameter, this anomalous ionization rate is in actuality determined self-consistently by the balance of energy added to the electrons via waves and lost by collisions. In these more realistic situations, the anomalous ionization rate is a function of the saturation temperature.
The parameter \( \eta \), then, plays an important role in the overall development of CIV.

Analytic estimates of the fraction of energy transferred (\( \eta \)) may be obtained in the case of unmagnetized ions (where now unmagnetized means \( \nu_{\text{ion}} > \Omega_i \)) and highly magnetized ions (\( \nu_{\text{ion}} \ll \Omega_i \)) by considering the quasilinear relaxation of the distribution function. This has been done by Formisano et al. [23] and Galeev [26]. The development is briefly restated below in non-dimensional form in order to produce estimates of \( \eta \) as well as to provide insight into the mass ratio scaling of the CIV problem. Further details on the analysis may be found in the references.

For unmagnetized ions, the non-dimensionalization of the quasilinear equations is carried out by defining \( t^* = \omega_LHt \), \( k^* = kV/\omega_L \), \( \phi^* = e\phi/m_iU^2 \), \( n^* = n/n_{e,0} \), \( v^* = v/U \), \( \omega^* = \omega/\omega_L \), and \( \nu_{\text{ion}}^* = \nu_{\text{ion}}/\omega_L \), where \( U \) is the initial velocity of the newly born ions. Dropping the asterisks for notational convenience, the quasilinear equation for the development of the beam ion distribution function, \( f_+(\vec{v}, t) \), is similar to that found in the previous section:

\[
\frac{\partial f_+}{\partial \tau} = \sum k \cdot \frac{\partial}{\partial \vec{v}} \left[ k^2 |\phi_k|^2 \right] (\omega_k - k \cdot \vec{v})^2 + \gamma_k^2 \right] + \nu_{\text{ion}} n_e F_0(\vec{v}) \tag{2.88}
\]

where \( k^2 |\phi_k|^2/8\pi \) is the electric field energy of mode \( \vec{k} \), and \( F_0(\vec{v}) \) is the distribution function of the undisturbed beam ions. \( \omega_k \) and \( \gamma_k \) are the real and imaginary parts of the solution to the dispersion relation. Here, a simplified form of Equation (2.43) is used for the dispersion relation:

\[
1 - \frac{1}{\omega^2} - \frac{m_i}{m_e n_i} \sin^2(\theta) \frac{1}{\omega^2} + \frac{m_i}{m_e n_i} \frac{1}{k^2} \int_{-\infty}^{\infty} dv \frac{\vec{k} \cdot \partial f_+/\partial \vec{v}}{\omega - k \cdot \vec{v}} = 0 \tag{2.89}
\]

For an ionization rate low compared to the quasilinear relaxation time, the shape of the distribution function may be considered to be independent of time. Also, since the magnitude will be proportional to \( n_+(t) \), the distribution function may be expressed as

\[
f_+(\vec{v}, t) = n_+(t) F_+(\vec{v}) \tag{2.90}
\]

By further assuming the newly formed ions have a delta function distribution, and the
relaxed distribution function is one-dimensional, i.e.

\[ F_0 = \delta(v_z - 1)\delta(v_x)\delta(v_y) \]  \hspace{1cm} (2.91)

and

\[ F_+(\vec{v}) = F_+(v_x)\delta(v_x)\delta(v_y) \]  \hspace{1cm} (2.92)

the distribution function may be seen to satisfy the diffusion equation

\[ \frac{\partial}{\partial \vec{v}_z} D \frac{\partial}{\partial v_x} F_+(v_x) = \nu_{\text{ion}} [F_+(v_x) - \delta(v_x - 1)] \]  \hspace{1cm} (2.93)

where

\[ D = \pi \sum_k k^2 |\phi_k|^2 \delta(\omega_k - \vec{k} \cdot \vec{v}_z) \]  \hspace{1cm} (2.94)

In the limit of low collisionality, when the right hand side of Equation (2.93) can be ignored, the solution to Equation (2.93) for the relaxed distribution function is a one-dimensional plateau as shown in Figure 2-10. By integrating over the relaxed distribution function, the energy transferred from the ions to the electrons may be obtained,

\[ \eta = 1 - \int_{-\infty}^{\infty} v_x^2 F_+(v_x) dv_x = \frac{2}{3} \]  \hspace{1cm} (2.95)

For magnetized ions, the value of \( \eta \) may change dramatically. Again, Formisano et al. [23] may be referred to for the quasilinear equation for the development of the beam distribution function. After non-dimensionalizing as for unmagnetized ions, the result is

\[ \frac{\partial f_+}{\partial t} = \sum_{k \perp v_\perp > \omega_k} \frac{\partial}{\partial \vec{v}_\perp} |\phi_k|^2 \frac{\omega_k^2}{\sqrt{(k_\perp \vec{v}_\perp - \omega_k)^2 \vec{v}_\perp \cdot \vec{v}_\perp}} \frac{\partial f_+}{\partial \vec{v}_\perp} + 2\pi \nu_{\text{ion}} n_e F_0(\vec{v}) \]  \hspace{1cm} (2.96)

where

\[ \gamma_k = \frac{2\pi \omega_k^2}{k^2 n_e I} \int_{-\infty}^{\infty} dv_\parallel \int_{-\infty}^{\infty} dv_\perp \frac{\partial f_+}{\partial v_\perp} \left[ v_\perp^2 - \frac{\omega_k^2}{k_\perp^2} \right]^{-1/2} \]  \hspace{1cm} (2.97)

\[ \omega_k^2 = \frac{1}{2} \left( 1 + \frac{m_i}{m_e} \sin^2 \theta \right) \]  \hspace{1cm} (2.98)
and
\[
\mathcal{I} = \left[ 1 + \frac{2\pi}{k^2 n_i} \int_{-\infty}^{\infty} dv_\parallel f_+(v_\parallel^2 = 0, v_\parallel) \right]
\]  
(2.99)

Assuming now that the ions are highly magnetized \((\nu_{\text{ion}}/\Omega_i \ll 1)\), the newly ionized atoms form a ring type distribution function,
\[
F_0(\vec{v}) = \frac{1}{\pi} \delta(v_\perp^2 - 1) \delta(v_\parallel)
\]  
(2.100)
The relaxed distribution function may also be considered isotropic in the plane perpendicular to the magnetic field leading to
\[
F_+(\vec{v}) = \frac{1}{\pi} \psi(v_\perp^2) \delta(v_\parallel)
\]  
(2.101)
The distribution function is found to relax to
\[
\psi(v_\perp) = (5a^2/4) v_\perp^{3/4} \left\{ I_{3/5}(av_\perp^{5/4}) K_{3/5}(a) \Theta(1 - v_\perp) + K_{3/5}(av_\perp^{5/4}) I_{3/5}(a) \Theta(v_\perp - 1) + 2K_{3/5}(av_\perp^{5/4}) K_{3/5}(a)/\Gamma(2/5)\Gamma(3/5) \right\}
\]  
(2.102)
where \(\Theta(v_\perp)\) is the step function, \(I\) and \(K\) are Bessel functions and \(\Gamma\) is the gamma function. The parameter \(a\) is the ratio of quasilinear diffusion time to ionization time
\[
\frac{25}{16} a^2 = \nu_{\text{ion}} / \sum_k \frac{4\omega_k^2}{c} |\phi_k|^2
\]  
(2.103)
Marginal stability is found to exist at a value \(a = 1.7\). The relaxed distribution function is shown in Figure 2-11. Integration over the relaxed distribution function at this value of \(a\) then leads to
\[
\eta = 1 - \int_0^\infty v_\perp \psi(v_\perp) dv_\perp = .025
\]  
(2.104)
Physically, the much lower efficiency is related to the fact the ion distribution function now spreads towards both larger velocities \((v_\perp < U)\) and smaller velocities \((v_\perp > U)\).
Reviewing the quasilinear equations presented above, the scaling with the mass ratio is again notable. For unmagnetized ions, scaling time by $\omega_{LH}$ again removes the mass ratio from the problem if $\bar{\theta}$ is chosen correctly. Significantly, even in the case of highly magnetized ions, the quasilinear equations are independent of the mass ratio if time and $\nu_{ion}$ are again scaled by $\omega_{LH}$ and $\bar{\theta}$ is kept constant. In these two limits, the ion cyclotron period is not an important time scale in the problem: the beam ion distribution function develops either much faster (highly magnetized) or much slower (unmagnetized) than the waves. This leaves only the wave time scale ($\omega_{LH}$) in the problem. With only $\omega_{LH}$ as a time scale, the problem may be adjusted by correcting $\bar{\theta}$ to remove the mass ratio dependence.

Although the mass ratio dependence may be eliminated in the unmagnetized and highly magnetized regimes, it is not obvious that the same can be said for the intermediate regime. In the region in which the ions are neither unmagnetized nor highly magnetized, the anisotropic form of the distribution function makes analysis difficult. An analytic approximation for $\eta$ over the whole range of $\nu_{ion}/\Omega_i$ is not currently known. In fact, since the intermediate regime must account for the anisotropy in the distribution function of the beam ions in the plane perpendicular to the magnetic field (and the amount of anisotropy must be a strong function of the ion gyroperiod), it is reasonable to expect that both $\omega_{LH}$ and $\Omega_i$ are important in the problem. In this case, the scaling on $\bar{\theta}$ is not sufficient to eliminate the effects of the mass ratio. Hence, it is not clear how to remove the effects of the mass ratio in the intermediate regime: physical mass ratios may be necessary to produce accurate results.

Implications for CIV:

Like the linear analysis of the M2SI, the quasilinear theory of Formisano et al. has several implications for the CIV process. The most important implication is simply that the energy transfer mechanism between beam ions and electrons is inefficient if the newly formed ions produce a ring-type distribution. The low efficiency of the energy transfer mechanism (resulting from the spread of the distribution function to velocities both higher and lower than the injection velocity $U$) will lead to low electron saturation
temperatures and will not be able to sustain CIV. On the other hand, if the ionization rate is sufficient to prevent the formation of a ring type distribution function, the efficiency of the transfer remains high ($\eta \approx 2/3$). Overall, CIV is only likely to be observed if an ionization rate can be obtained which prevents the formation of a ring distribution.

2.3 A Simple Rate Model

The previous sections have discussed the linear and nonlinear development of the M2SI. These sections showed the M2SI is efficient in heating electrons parallel to the magnetic field. The efficiency of the heating was determined through the quasilinear analysis of the constant ionization rate model. In order to continue the development of a theory of CIV, it remains to be shown that the heating of the electrons can lead to an explosive growth in the electron number density and a decrease in the relative plasma-neutral velocity. This section will describe a simple rate model from Abe[1] which illustrates an explosive growth regime for CIV. A discussion of the plasma-neutral relative velocity decrease is reserved for the global models of the next section.

For the rate model considered, the only components of the plasma considered are the ions and hot electrons. In addition, the ion component is divided into two portions: thermal background ions and newly born, beam ions. Simple, global conservation of particles may be used to arrive at the equation governing the particle densities:

$$\frac{dn_b}{dt} = \nu_{ion} \alpha_{ion} n_e - \nu_D n_b - (dn_b/dt)_c \tag{2.105}$$

$$\frac{dE_H}{dt} = \nu_D n_b (\eta m_e V^2 / 2) - \nu_{ion} \alpha n_e e \phi_{ion} \tag{2.106}$$

where $n_b$ and $n_e$ represent the density of the beam ions and electrons, respectively. The factor $\alpha$ is the fraction of electrons above the ionization energy (the fraction of "hot" electrons). The first equation, then, expresses the rate of change of beam ions as increasing from ionization, decreasing to velocity space diffusion out of the beam with some diffusion coefficient $\nu_D$, and further decreasing to other classical collisional processes. The second equation is an expression for the energy density of the hot electrons,
$E_H$. The energy density increases due to energy transfer from the beam through wave heating and decreases due to ionization.

To simplify the analysis as much as possible, it is further assumed that the production of new ions is proportional to their energy density of the electron tail, i.e.

$$\nu_{\text{ion}} an_e \sim E_H,$$  \hspace{1cm} (2.107)

and that the diffusion coefficient $\nu_D$ and the classical collision term $(dn_b/dt)$ are independent of $n_b$ and $E_H$.

With these assumption, Equations (2.105) and (2.106) become

$$\frac{dn_b}{dt} = AE_H - \nu_D n_b - C$$  \hspace{1cm} (2.108)

$$\frac{dE_H}{dt} = \nu_D n_b (\eta m_\alpha V^2/2) - A e\phi_{\text{ion}} E_H$$  \hspace{1cm} (2.109)

where $A$ and $C$ are taken as constants.

The solution to these equations is given by Abe as

$$n_b(t) = \left[ k_2 \left\{ (n_b(0) + k_1 E_H(0) - C/S_1) e^{S_1 t} + C/S_1 \right\} - k_1 \left\{ (n_b(0) + k_2 E_H(0) - C/S_2) e^{S_2 t} + C/S_2 \right\} \right] / (k_2 - k_1)$$  \hspace{1cm} (2.110)

$$E_H(t) = \left[ \left\{ (n_b(0) + k_1 E_H(0) - C/S_1) e^{S_1 t} + C/S_1 \right\} - \left\{ (n_b(0) + k_2 E_H(0) - C/S_2) e^{S_2 t} + C/S_2 \right\} \right] / (k_2 - k_1)$$  \hspace{1cm} (2.111)

The parameters $k_1$ and $k_2$ are the solutions of

$$k^2 \nu_D (\eta m_\alpha V^2/2) + (A e\phi_{\text{ion}} - \nu_D) k - A = 0$$  \hspace{1cm} (2.112)

with $k_1 > 0 > k_2$. The other parameters $S_1$ and $S_2$ satisfy

$$S_{1,2} = \left[ k_{1,2} (\eta m_\alpha V^2/2) - 1 \right] \nu_D$$  \hspace{1cm} (2.113)
with

\[ S_1S_2 = \nu_D A (\epsilon \phi_{ion} - \eta m_e V^2 / 2) \]

\[ S_1 + S_2 = -(2 \nu_D + A \epsilon \phi_{ion}) < 0 \]  
(2.114)

The growth or decay of \( n_b \) and \( E_H \) is governed by the signs of \( S_1 \) and \( S_2 \). Examination of Equation (2.114) leads to two classifications of the solutions:

Case I: \( \epsilon \phi_{ion} > \eta m_e V^2 / 2 \Rightarrow S_1, S_2 < 0 \)

Case II: \( \epsilon \phi_{ion} < \eta m_e V^2 / 2 \Rightarrow S_1 > 0 > S_2 \)

These relations show that only Case II can lead to explosive growth of \( n_b \) and \( E_H \), leading to the conclusion that CIV requires

\[ \eta m_e V^2 / 2 > \epsilon \phi_{ion} \]  
(2.115)

Equation (2.115) represents a modified form of Alfvén’s original hypothesis, Equation (1.1). The current equation has been used by several authors as a more realistic estimate of the critical velocity. It accounts for incomplete energy transfer from the ions to electrons in the positive feedback loop. This equation again highlights the importance of understanding the energy transfer efficiency as derived in the constant ionization rate model.

Although the model shown here is quite simple, it does indicate that assumptions for CIV can lead to explosive growth in the plasma density.

### 2.4 Global Models

The final point to be discussed in the theoretical development of CIV is the decrease observed in relative plasma-neutral velocity. This is perhaps the most difficult aspect of CIV to treat analytically. The problem requires a macroscopic solution of the forces acting on the plasma cloud drifting with respect to neutrals consistent with the microscopic physics detailed in previous sections. To date, such a complete model has not been formulated. Because of this, the purpose of this section is simply to outline the mechanisms affecting the relative plasma-neutral velocity and to discuss their relations to the critical
velocity. The laboratory and space experiments present two different scenarios, and each will be discussed in turn.

In the laboratory model, the initial state resembles that shown in Figure 2-12. The polarized plasma has positive and negative charge layers leading to a polarization electric field, $E_y$ and a drift across the magnetic field with velocity

$$ V = E_y / B_z. \quad (2.116) $$

As neutrals are ionized at a rate $\nu_{ion}$, the newly born ions are displaced in the direction of the electric field a distance of one ion gyroradius (assuming the ion gyroradius, $\rho_i < D$, where $D$ is the size of the experimental cloud), while the electron remains essentially stationary. This results in a current [29]

$$ j_y = q_i \nu_{ion} R_g = q_i \nu_{ion} V / \Omega_i \quad (2.117) $$

The direction of the current is such that it reduces the charge contained in charge layer, thus reducing the polarization electric field and drift velocity, $V$. A qualitative estimate of this may be stated by estimating the electric field in the plasma from Gauss' law as

$$ E_y A_y = Q_{tot} \quad (2.118) $$

where $A_y$ is a unit area and $Q_{tot}$ is the total charge contained in the charge layer. The change in $Q_{tot}$ as a function of time due to the current can be found from the continuity equation,

$$ \frac{\partial Q_{tot}}{\partial t} + j_y A_y = 0 \quad (2.119) $$

Combining Equations (2.116) through (2.119) yields

$$ \frac{\partial V}{\partial t} + \frac{\omega_{pi0}^2 \nu_{ion}}{\Omega_i^2 n_{i0}} V = 0 \quad (2.120) $$

Although the derivation of this equation relied on arguments too simple to provide quantitative estimates of $V(t)$, the physical content is clear. As long as ionization proceeds,
the charge layer is diminished, reducing the drift velocity of the plasma. Since the basic hypothesis of CIV is that ionization only occurs for drift velocities which exceed the critical velocity, Equation (2.120) implies that the drift velocity must be reduced towards the critical velocity.

The model of space experiments is taken from Goertz et al. [30] and considers a neutral cloud moving with velocity \( U \) in the positive \( z \)-direction. The ‘microphysics’ of the processes described in the previous sections are treated in this model in a heuristic way by assuming that the neutral cloud undergoes ionization at a rate \( \dot{Q}_b = dn_e/dt \). The momentum of the created plasma cloud is given by \( Nmv \), where \( v \) is the mean velocity and \( m \) is the mean mass of the plasma cloud, and is increased at a rate \( \dot{Q}_b mU \) and decreased by a \( j \times B \) force acting on the cloud. In order to calculate the \( j \times B \) force, the currents are needed. A model of the currents is shown in Figure 2-13, taken from Goertz et al. A pickup current flows in the \( y \)-direction and connects to field line current sheets. The current loop is closed through perpendicular flowing polarization currents. The total polarization current may be found from

\[
I_{pv} = \int (nm/B^2)(dE_y/dt)dz
\]  

(2.121)

or integrating over \( z = v_A t \) where \( v_A \) is the Alfvén speed,

\[
I_{pv} = (2/\mu_0 v_A) [E(t) - E(0)]
\]  

(2.122)

Since \( I_{pv} = j_{PU,y} h \), and letting \( E_y = -UB \), the \( j \times B \) force is then given by

\[
j \times B_z = -\frac{2v_A n_0 N_0}{h} m_0 v
\]  

(2.123)

The equation for the momentum balance of the cloud is then

\[
\frac{d}{dt}(n_e v) = \dot{Q}_b U - \frac{2V_{a0} n_0 m_0 m_0 v}{h} = Q_b U - Kv
\]  

(2.124)

where \( K = 2v_{a0} n_0 m_0^2/(hm) \). The factor \( K \) represents a mass loading factor, relating
the amount of surrounding plasma to which the newly created plasma couples. An asymptotic state for the CIV process is given by Equation (2.124) as

$$v_\infty = \frac{U}{1 + K/Q_b}$$  \hspace{1cm} (2.125)

In the case of the space experiments, then, the asymptotic velocity is related to both the coupling with the surrounding medium and the ionization rate. Haerendel has used similar arguments and the results of the Porcupine release to conclude that in the space experiments, the asymptotic drift velocity remains small. Hence, as in Porcupine, the drift of the plasma is not an important point in space releases.

2.5 Summary

This chapter has attempted to show that the M2SI is a good candidate for the basic mechanism operating during CIV. In doing this, linear theory has been used to determine the region of parameter space in which CIV can develop, while quasilinear theory has been used in conjunction with the constant ionization rate model to determine the saturated state of the M2SI instability and the efficiency of energy transfer between beam ions and electrons. In addition, models have been presented to explain the explosive growth of CIV and to investigate the drop in relative plasma-neutral velocity which occurs during CIV. The results of this chapter indicate that

- operation of the M2SI requires that both the velocity of the ion beam and the electron-neutral collision frequency satisfy criteria given by linear theory. The criterion should be satisfied in both the laboratory and space experiments.

- the wave spectrum of the M2SI should be peaked near the lower hybrid frequency of the ion beam and the waves should propagate nearly perpendicular to the magnetic field.

- if the ionization rate is sufficient to maintain an unmagnetized beam ion distribution, the efficiency of energy transfer from ions to electrons can be high ($\eta \approx 2/3$).
If, on the other hand, the newly born ions form a ring type distribution, the efficiency of energy transfer will drop to $\eta \approx .025$.

- the assumption of energy transfer to electrons through the M2SI lead to explosive growth of the plasma density if Equation (2.115) is satisfied.

- consideration of the momentum coupling of the newly born ions and ambient plasma lead to expressions showing that the relative plasma-neutral velocity will decrease toward the critical velocity as ionization proceeds.
Figure 2-1: Scaling effects on the dispersion relation. Part (a) compares the maximum growth rate (maximized over $\kappa$) as a function of $\bar{\theta}$ for mass ratios of $m_i/m_e = 7344$ and $m_i/m_e = 29736$. The two solutions are indistinguishable. Part (b) shows $\gamma$ as a function of $\kappa$ for a mass ratio of $m_i/m_e = 29376$, $\bar{\theta} = 1$ and two values of $\omega_{pe}/\Omega_e$. Again, the solutions are indistinguishable.

Figure 2-2: Plot of the left and right hand sides of Equation (2.47). For the case shown, four real roots exist, indicating a stable situation.
Figure 2-3: Cold Plasma dispersion relation results for the frequency and wavenumber of the fastest growing wave. The top panel shows $S(W_1)$, the bottom panel shows the solution of the dispersion relation as a function of the parameter $W_1$. 

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Figure 2-4: Dispersion relation results as a function of the angle, $\tilde{\theta}$. For large angles, the M2SI is stabilized by electron Landau damping. (a) $\omega^{*}$, (b) $\gamma^{*}$, (c) $\kappa^{*}$, (d) $\omega^{*}/\kappa^{*}$
Figure 2-5: Dispersion relation results as a function of $U/v_b$. For $U/v_b \gg 1$, the dispersion relation reduces to the cold plasma approximation ($W_1 = 1.5$). (a) $\omega^*$, (b) $\gamma^*$, (c) $\kappa^*$, (d) $\omega^*/\kappa^*$
Figure 2-6: Dispersion relation results as a function of the temperature ratios $T_b/T_e$ and $T_i/T_e$. The real and imaginary frequencies are insensitive to the temperature ratios. (a) $\omega^*$, (b) $\gamma^*$, (c) $\kappa^*$, (d) $\omega^*/\kappa^*$
Figure 2-7: Dispersion relation results as a function of the beam density $n_b/n_e$. As $n_b/n_e \to 0$, the M2SI is stabilized since no beam exists. For $n_b/n_e = 1$, the results are the same as McBride, et al. (a) $\omega^*$, (b) $\gamma^*$, (c) $\kappa^*$, (d) $\omega^*/\kappa^*$.
Figure 2-8: Dispersion relation results as a function of the collision frequency $\nu/\omega_{pi}$. When the collision frequency becomes near the lower hybrid frequency, the growth rate of the instability decreases substantially since electrons can no longer stay in resonance with the waves. (a) $\omega_\gamma^*$, (b) $\gamma^*$, (c) $\kappa^*$, (d) $\omega_\gamma^*/\kappa^*$
Figure 2-9: Quasilinear development of the M2SI. The beam ions form a plateau in velocity space to saturate the instability. As the electrons become resonant with the unstable waves, they form a suprathermal tail.
Figure 2-10: One-dimensional relaxed distribution function predicted from quasilinear theory. The relaxed distribution function is valid for unmagnetized ions ($v_{ion}/\Omega_i > 1$).

Figure 2-11: Two-dimensional relaxed distribution function predicted from quasilinear theory. The distribution function is valid for highly magnetized ions ($v_{ion}/\Omega_i \ll 1$).
Figure 2-12: Schematic of the laboratory experiments. As ionization proceeds, ions and electrons are offset by a gyroradius. The result is a 'shorting' of the charge layers that cause the $E \times B$ drift of the plasma.

Figure 2-13: The system of currents assumed in the work of Goertz et al. [30] A pickup current due to ionization processes causes a $j \times B$ force to act on the newly formed plasma.
Chapter 3

Particle in Cell Codes

The previous chapter discussed the theory of CIV. Most of the chapter relied on linear and quasilinear theory to develop ionization rates and efficiency estimates for use in rate and global models. Numerical simulations provide a means of obtaining the full nonlinear behavior of the plasma. The results of numerical simulations will be used in the following two chapters to verify the accuracy and applicability of the theoretical findings, as well as to develop some empirical rules for CIV where the theory is not applicable. The purpose of this chapter is to develop the numerical theory of both explicit and implicit PIC codes, discuss their properties, and describe the implementation of the algorithms used in the following chapters.

3.1 Hierarchy of Plasma Simulation Codes

Simulation of plasma systems grew out of fluid simulations in the early 1960s when the addition of electromagnetic body forces into fluid codes led to the development of MHD simulations. As computational power grew, a desire also developed to study in more detail the actual kinetic processes occurring in a plasma. This desire led to the development of the earliest particle methods which were not PIC codes, but instead involved a direct calculation of the Columbic forces acting on a particle. These N-body simulations assume N point particles interacting with each other through a Columbic
force law. The force on a particle, $F_i$, is calculated from

$$F_i = \sum_j F_{ij}$$  \hspace{1cm} (3.1)

where $F_{ij}$ is determined from Coulomb's law

$$F_{ij} = \frac{q_i q_j}{4\pi\epsilon_0} \frac{x_i - x_j}{|x_i - x_j|^3}$$  \hspace{1cm} (3.2)

and $x_{i,j}$ are the positions of two interacting particles, $q_{i,j}$ are the electric charges of the particles and $\epsilon_0$ is the permittivity of free space. The force is used to update the particles' positions and velocities through use of the equations of motion

$$\dot{x}_j = v_j$$  \hspace{1cm} (3.3)

$$\dot{v}_j = (A_j + v_j \times \Omega_j)$$  \hspace{1cm} (3.4)

where $A_j = \frac{F_j}{m_j}$ is the acceleration and $\Omega_j$ is the cyclotron vector of the particle. Early simulations of this type met with limited success due to the lack of sufficient computational power and the $O(N^2)$ nature of calculating all the interacting forces which should be evident from Equation (3.1).

Particle in cell codes replaced the direct N-body simulations in the mid-1960s. The concept of PIC codes is to treat electrons and ions not as point particles, but as finite sized particles moving under the influence of electromagnetic forces. The term 'finite sized' refers to weighting the particles via some interpolation function onto a grid. The particle is therefore 'smeared' over a finite area, instead of being represented by a point. Instead of calculating the force by a direct summation of Columbic forces, a great increase in efficiency is achieved by interpolating the finite sized particles to determine a continuous charge density. By superimposing a grid on the simulation region and letting the charge density and currents exist at discrete nodes, Maxwell's equations can be efficiently solved to determine the electric and magnetic fields. An interpolation of the fields from the grids to the particles then determines the force acting on a particle.

Although the introduction of PIC methods provides a great increase in efficiency over
$N$-body codes, several severe restrictions still exist which limit their universal application. Most of these restrictions are related to the wide disparity of length and time scales that exist in a typical plasma problem. In fact, the greatest challenge to plasma simulation comes from the wide variety of time and length scales of interest (e.g. see Table 3.1). To date, no single computational method is capable of completely resolving all the length and time scales of a plasma. Instead, a hierarchy of computational methods has been developed to efficiently simulate the desired physics in the different regimes of time and length scales. At the most basic physical level are explicit, electromagnetic PIC codes (even possibly including relativistic effects, although these will not be considered here). These codes carefully resolve the electron plasma and gyro frequency as well as solve the complete set of Maxwell's equations to follow electromagnetic plane waves. Although quite complete in the physics included, the use of explicit, electromagnetic PIC codes is limited to short length and time scales. The speed of light sets a Courant condition limitation on the allowable time step:

$$c\Delta t/\Delta x < 1.$$  \hspace{1cm} (3.5)

Furthermore, the typical implementation of explicit PIC codes requires grid cell sizes on the order of the Debye length, $\lambda_{De}$, to prevent unacceptable levels of numerical heating and places a further restriction on the time step regarding plasma waves, namely

$$\omega_{pe} \Delta t < 2 \hspace{1cm} (3.6)$$

The restriction placed by either Equation (3.5) or (3.6) limits these codes to length scales on the order of Debye lengths and time scales on the order of the electron plasma period.

In many cases, the propagation of electromagnetic plane waves is not important to the physics of the problem. In this case, a commonly adopted assumption is the 'Darwin' approximation. In the Darwin approximation, the displacement current is dropped from Amperes Law. The physical result is the removal of electromagnetic, plane waves from the simulation while retaining other electromagnetic components. The restriction of Equation (3.5) is then removed, although a similar Courant condition based on other
fast electromagnetic waves (e.g. the Alfvén speed) may still be present.

Courant conditions based on electromagnetic waves can be removed altogether by accepting an electrostatic approximation. In the electrostatic approximation, the B-field is assumed to remain unperturbed. This approximation also removes electromagnetic plane waves from the problem, hence Equation (3.5) is no longer applicable. For the electrostatic case, the remaining stability restriction (Equation 3.6) relates to the electron plasma frequency.

Although electrostatic approximations remove the Courant conditions based on the velocity of electromagnetic waves, explicit, electrostatic codes still remain quite restrictive in the range of physics that can be simulated in a reasonable amount of CPU time. For example, a typical two-dimensional explicit PIC code run on a supercomputer (circa 1990) will require approximately 10 μs/(particle-time step) to update a particle (including field solve and interpolation). Furthermore, even a “small” simulation will contain 10^4 particles. Hence, following the particles for a single cyclotron period for a hydrogen ion would require approximately 15 minutes of CPU time (taking ω_pe Δt = 0.2). Larger or longer simulations can quickly become prohibitively costly.

In many interesting cases, the physical processes develop on ion plasma period or ion cyclotron period time scales. Often, in these cases, the solution is insensitive to the precise electron trajectories. Long time scale methods are appropriate for such cases. The goal of these codes is to separate the electron and ion time scales in the simulations. The analytic equivalent of this is a multi-time scale solution to the governing equations. By averaging over the fast (electron) time scales, the electron motion can be replaced by drift equations. The drift motion of the electrons is the desired response on the ion time scales.

The direct simulation of the drift equations while using the full Lorentz force law for the ions is possible with gyrokinetic codes. In gyrokinetic codes, the multiple time-scale analysis is performed analytically to develop a governing set of drift equations for the electrons. Unfortunately, the resulting set of drift equations is complex and difficult to implement numerically. This complexity has prevented gyrokinetic codes from becoming widely used. Along the same line but substantially simpler to implement are guiding
center methods. These codes are similar to the gyrokinetic codes but only retain the lowest order drifts. The simplest case is when only the $E \times B$ drift is retained. Although simpler to implement, these codes have also substantially reduced the physical content of the simulation.

Gyrokinetic and guiding center codes both average over the gyromotion of the electrons, retaining only the low frequency drifts. However, in the parallel direction, plasma waves are retained. When gyrokinetic or guiding center codes are used in conjunction with the standard leapfrog integrator, the restriction on the plasma frequency still exists. The final removal of this restriction requires implicit methods which are the next step up the hierarchy of time scales.

In implicit codes, forward time information is introduced into the particle equations of motion to obtain a (linearly) absolutely stable algorithm for all time steps. Proper choice of the differencing also gives a second order accurate algorithm. The forward information adds complexity to the solution however. In most implicit methods, the desired forward information is the electric field at time level $n + 1$, i.e. $E^{n+1}$. However, this forward electric field is a function of the particle positions at the forward time step, $x^{n+1}$, which are themselves a function of the forward electric field. The overall result is a large set of coupled nonlinear equations. The efficient solution of this set of nonlinear equations is the central issue in implicit PIC codes.

Implicit methods can in general be divided into two classes based on the solution method of the nonlinear equations: moment methods and direct methods. Both the moment and direct methods start by predicting the particle positions at time level $n + 1$ in order to obtain $E^{n+1}$. The forward electric field at $n + 1$ based on the predicted particle positions is then used to increase the accuracy of the prediction of the particle positions at time $n + 1$. The iteration of (predicted particle positions) $\rightarrow E^{n+1} \rightarrow$ (better predicted particle positions) continues until convergence. The difference in the moment and direct methods involves the approximation to the particle positions at time $n + 1$. In moment methods, a prediction of the forward particle positions is obtained from velocity moments of the equations of motion. The moment method however requires some type of closure assumption which may severely affect the efficiency and accuracy of the method. Direct
methods solve the nonlinear set of equations by a direct linearization of the particle-field equations. Since no velocity moments are introduced, the direct method also removes the difficulty of obtaining appropriate closure assumptions.

Although the typical implicit PIC algorithms are theoretically stable for all time steps, this may be difficult to achieve in practice. In many cases, the relevant electron physics are suitably modeled by fluid approximations. Such an approximation is the next step up the hierarchy of simulation techniques and is implemented in hybrid codes. In hybrid codes, ions are treated as particles, similar to explicit PIC codes. The electron response however is replaced by the fluid equations. These codes usually require an assumption of adiabatic or isothermal electrons.

The final step up the hierarchy is a return to fluid codes. MHD codes are widely used in large scale simulations where individual particle motion is unimportant. MHD codes are well developed and widely applied. However, since the individual particle motion is important to the current work, MHD and other fluid code variants will not be further discussed.

3.2 Choice of Algorithms

The previous section briefly described the hierarchy of plasma simulation techniques which is available. This section states the choice of algorithm for the current work.

A review of Chapter 2 leads to three broad conclusions concerning the appropriate simulation method to be used for CIV simulations:

- The process can be considered electrostatic, i.e. \( B \) is unperturbed.

- The electron plasma frequency and electron gyrofrequency are not important time scales in the problem. Instead, the relevant physical processes occur at the lower hybrid frequency.

- Detailed information must be retained about the electron distribution function. The development of a high energy electron tail is an important mechanism in CIV.

These conclusions lead to a choice of the appropriate simulation method. Since
the detailed electron distribution must be retained, MHD and hybrid codes are not appropriate; also, explicit PIC codes cannot be applied efficiently on the lower hybrid period time scale. The choices remaining are gyrokinetic, guiding center, and implicit methods. Since some parallel component of the magnetic field is probably desired in the simulation plane, gyrokinetic and guiding center methods still place limitations on the step size related to the electron plasma period. This severely limits their application to the CIV simulation. The correct choice, then, is the implicit method which retains the physical content of the guiding center methods, but provides an absolutely stable algorithm for all time steps.

Finally, the choice of moment or direct methods must be considered. Ultimately, moment and direct methods are closely related. Recently, direct methods have been obtaining greater acceptance. This is mainly because direct methods do not require the introduction of auxiliary fluid equations and ad hoc closure assumptions. For this study, direct methods are chosen, although it is acknowledged that moment methods could also likely be applied successfully. The next section reviews previous work on direct implicit codes, followed by a detailed discussion of the algorithm and its implementation.

3.3 Review of Direct Implicit PIC Codes

Implicit PIC codes have only been developed in the last decade. When compared to the number of simulations employing explicit PIC codes, the number of implicit simulations is quite small. In the next few paragraphs, the main papers of the still small literature regarding implicit PIC codes are reviewed.

The concept of an implicit PIC code to obtain long time scale algorithms was first proposed by Langdon [43]. In this work, he developed the set of nonlinear equations to be solved, but concluded that the solution of the coupled equations was not feasible. At the time, Langdon only considered a direct inversion of the nonlinear operator and did not look for approximation techniques which might allow a solution in a reasonable amount of CPU time.

The first major advance in implicit codes occurred in the early 1980s when Mason [50] showed that the moment method could be used to achieve a stable iteration to solve
the coupled, nonlinear equations. The basic idea of this moment method was, in the first approximation, to replace the differenced equations of motion by their fluid analog. By updating the Maxwell’s equations using the fluid source terms (i.e. the current and charge density as predicted from the fluid equations), an approximation of $E^{n+1}$ can be found. The prediction of $E^{n+1}$ leads to an iteration which can be used to solve the full set of coupled, nonlinear equations.

At roughly the same time the moment method was being developed, work was also starting on the direct linearization of the set of equations. This direct method was outlined in two articles [14, 44] in the early 1980s. These two papers introduced the implicit algorithm, analyzed its convergence, accuracy and stability properties, and provided several simple examples of its application.

One of the undesirable properties of implicit codes is unphysical electron cooling which results from a secular acceleration term of order $(\Delta t)^2$. This secular acceleration arises from the damping of high frequency modes, i.e. a plasma particle does not interact with the full wave spectrum. The result is an acceleration of the particle toward the nearest value of $k v_0 \Delta t = 2\pi N$ where $v_0$ is the particle velocity, $k$ is the wavenumber and $N = 0, 1, 2, \ldots$. For the usual case of $k v_0 \Delta t \ll 1$, the result is a deceleration of the particles which appears as electron cooling. Further information on this point may be found in Cohen et al. [14]. Much of the later work on implicit codes has focused on reducing the effect of this secular acceleration. Spatial smoothing is one way to reduce the unphysical electron cooling. The consistent inclusion of spatial smoothing into implicit codes was the subject of several papers [15, 7] as was the optimization of the numerical parameters to achieve other properties [16, 25].

Recently, several variations of implicit codes have been developed which hold promise for future development. These include integral centering of $z$ and $v$ in the algorithm [25] which will allow the time step to be changed as the simulation develops, and multi-scale particle simulations [60] in which different groups of particles move with different $\Delta t$'s.

As this short review should indicate, implicit PIC codes are not yet well developed. Most applications of implicit PIC codes have been restricted to one or two dimensional simulations and have been restricted to periodic boundary conditions. As
will be discussed below, boundary conditions remain a problem in implicit codes once spatial smoothing is introduced. Additional physical processes such as discrete collisions and ionization have also not previously been implemented in implicit PIC codes, and much still needs to be achieved in analyzing the numerical implications of integrating particle motion on time scales large compared to the plasma frequency. The numerical damping inherent in implicit PIC codes can introduce undesirable effects into the simulation. Proper control of these errors remains a challenging task. The next sections will discuss the implicit PIC algorithm in detail and extend the algorithm to include discrete collisional effects.

3.4 Governing Equations and Normalization

The motion of charged particles in an electrostatic field is governed by the Lorentz force law and Gauss' law:

\[
\dot{x}_j = v_j
\]

\[
\dot{v}_j = (A_j + v_j \times \Omega_j) + A_{\text{ext}}
\]

\[
\vec{E} = -\nabla \phi
\]

\[
\nabla^2 \phi = -\rho/\varepsilon
\]

\[
\rho = \sum_j q_j \delta(x - x_j)
\]

where the dot superscript represents the time derivative, the \( j \) subscript refers to the particle, \( A_j \) is the acceleration term due to the electric field \( ((\frac{e}{m})_j \vec{E}) \), \( A_{\text{ext}} \) is the (given) acceleration due to an external force such as gravity, \( \Omega_j \) is the cyclotron vector \( ((\frac{e}{m})_j \vec{B}) \), and the rest of the symbols have their usual meaning. With appropriate initial and boundary conditions, the solution of these equations constitutes the desired solution of the problem at hand.

The following normalizations are introduced to non-dimensionalize the governing equations:
\[
\begin{align*}
    m^* &= \frac{n}{m_e} \\
    v^* &= \frac{v}{v_{Te,0}} \\
    x^* &= \frac{x}{\lambda_{De}} \\
    t^* &= \omega_{pe} t \\
    B^* &= \frac{B}{B_0} \\
    q^* &= \frac{q}{q_e} \\
    \phi^* &= \frac{\phi}{kT_{e,0}} \\
    E^* &= \frac{\lambda_{De} E}{E} \\
    n^* &= n \lambda_{De}^3
\end{align*}
\]

where \( \omega_{pe}, v_{Te}, \) and \( \lambda_{De} \) are the initial plasma frequency, electron thermal velocity, and Debye length, respectively. \( B_0 \) is a reference magnetic field, and \( T_{e0} \) is the initial electron temperature. The acceleration on a particle due to the electric field is now
\[
\mathbf{A}^* = \frac{\mathbf{A}_j}{\omega_{pe} v_{Te,0}} = \frac{q^*}{m^*} \mathbf{E}^*.
\]

For the rest of the work, the asterisks are dropped for notational convenience. With these normalizations and introducing \( R = \Omega_e / \omega_{pe0} \), Equations (3.7) through (3.11) may be written

\[
\begin{align*}
    \dot{x}_j &= v_j \quad (3.12) \\
    \dot{v}_j &= (A_j + R v_j \times \Omega_j) \quad (3.13) \\
    E &= -\nabla \phi \quad (3.14) \\
    \nabla^2 \phi &= -\rho \quad (3.15) \\
    \rho &= \sum_j q_j \delta(x - x_j) \quad (3.16)
\end{align*}
\]

The PIC method corresponds to solving Equations (3.12) through (3.16) by the method of characteristics; the characteristics of the equations being simply the trajectory of particles moving in the self-consistent electric and magnetic fields.

The numerical solution of the governing equations by the method of characteristics can be thought of as comprised of two parts: time integration of the equations of motion (Equations 3.12 and 3.13) and determination of the forces acting on a particle (Equations 3.14 through 3.16). The following two sections describe these parts in turn. In these sections, a typical explicit algorithm is developed and analyzed in parallel with the implicit method. This should provide a way to compare and contrast the implicit method with the better understood explicit methods.
3.4.1 Differencing of the Equations of Motion

The equations of motion are differenced using the standard second-order accurate leapfrog integrator. The particle position, \( x \), and electric field, \( E \), are maintained at integral levels of the time step \((n, n+1, n+2, ...)\), while the velocity is known at half-increments of the time step \((n-1/2, n+1/2, n+3/2, ...)\). With superscripts referring to the time step and the subscript \( j \) referring to the particle, the discretized equations of motion are

\[
x_{j}^{n+1} = x_{j}^{n} + v_{j}^{n+1/2} \Delta t
\]  \hspace{1cm} (3.17)

\[
v_{j}^{n+1/2} = v_{j}^{n-1/2} + (A_{j}^{n} + A_{ex t}^{n} + \mathcal{R} v_{j}^{n} \times \Omega_{j}) \Delta t
\]  \hspace{1cm} (3.18)

where \( v^{n} \) is an approximation to the velocity at time level \( n \). This approximation is taken as a linear interpolation of the known quantities at time levels \( n-1/2 \) and \( n+1/2 \), which is expressed as

\[
v^{n} = \gamma_{+} v_{n+1/2}^{n} + \gamma_{-} v_{n-1/2}^{n}
\]  \hspace{1cm} (3.19)

where \( \gamma_{+} + \gamma_{-} = 1 \). Specific values of \( \gamma_{+} \) and \( \gamma_{-} \) will be discussed below.

For an explicit code, the quantity \( \bar{A}_{j}^{n} \) is simply the acceleration at time level \( n \), hence

\[
\bar{A}_{j}^{n} = A_{j}^{n}
\]  \hspace{1cm} (3.20)

In the implicit code, this quantity is a time filtered acceleration comprising some combination of past and future, filtered and unfiltered, time levels, hence

\[
\bar{A}_{j}^{n} + \alpha_{1} \bar{A}_{j}^{n-1} + \alpha_{2} \bar{A}_{j}^{n-2} + \ldots = cA_{j}^{n+1} + \beta_{0} A_{j}^{n} + \beta_{1} A_{j}^{n-1} + \beta_{2} A_{j}^{n-2} + \ldots
\]  \hspace{1cm} (3.21)

The determination of the coefficients (\( \alpha \)'s, \( \beta \)'s and \( c \)) to produce a stable algorithm for all time steps will be discussed below.

Finally, the formulation becomes somewhat clearer if Equation (3.18) is rewritten in the equivalent form

\[
v_{j}^{n+1/2} = \bar{R} \cdot v_{j}^{n-1/2} + \bar{S} \cdot (\bar{A}^{n} + A_{ex t}^{n})
\]  \hspace{1cm} (3.22)
where $\tilde{\mathbf{K}}$ and $\mathbf{S}$ are matrices due to the magnetic field. Letting

$$\zeta^2 = \mathcal{R}^2(\Delta t)^2 \quad (3.23)$$

and defining $d_j = (1 + \gamma_+^2 \Omega_j^2 \zeta^2)^{-1}$, the matrix $\tilde{\mathbf{K}}$ is

$$\begin{vmatrix}
R_{x,x} & R_{x,y} & R_{x,z} \\
R_{y,x} & R_{y,y} & R_{y,z} \\
R_{z,x} & R_{z,y} & R_{z,z}
\end{vmatrix}

= d_j \quad (3.24)$$

with elements given by

$$R_{x,x} = 1 + \zeta^2(\gamma_+^2 \Omega_{j,x}^2 - \gamma_- \gamma_+ (\Omega_{j,y}^2 - \Omega_{j,z}^2)) \quad (3.25)$$

$$R_{x,y} = \gamma_+ \zeta^2 \Omega_{j,y} \Omega_{j,x} + \zeta \Omega_{j,z} \quad (3.26)$$

$$R_{x,z} = \gamma_+ \zeta^2 \Omega_{j,z} \Omega_{j,x} - \zeta \Omega_{j,y} \quad (3.27)$$

$$R_{y,x} = \gamma_+ \zeta^2 \Omega_{j,x} \Omega_{j,y} - \zeta \Omega_{j,z} \quad (3.28)$$

$$R_{y,y} = 1 + \zeta^2(\gamma_+^2 \Omega_{j,y}^2 - \gamma_- \gamma_+ (\Omega_{j,x}^2 + \Omega_{j,z}^2)) \quad (3.29)$$

$$R_{y,z} = \gamma_+ \zeta^2 \Omega_{j,z} \Omega_{j,y} + \zeta \Omega_{j,x} \quad (3.30)$$

$$R_{z,x} = \gamma_+ \zeta^2 \Omega_{j,x} \Omega_{j,z} + \zeta \Omega_{j,y} \quad (3.31)$$

$$R_{z,y} = \gamma_+ \zeta^2 \Omega_{j,y} \Omega_{j,z} - \zeta \Omega_{j,x} \quad (3.32)$$

$$R_{z,z} = 1 + \zeta^2(\gamma_+^2 \Omega_{j,z}^2 - \gamma_- \gamma_+ (\Omega_{j,x}^2 + \Omega_{j,y}^2)) \quad (3.33)$$

The matrix $\mathbf{S} = (\gamma_- I + \gamma_+ \tilde{\mathbf{K}})$ where $I$ is the unit tensor.

In order to solve Equations (3.17) and (3.22), the acceleration due to the electric field must be found from the field equation. The next sections discuss the process by which this is done.

3.4.2 The Field Equations

With the use of a grid, the determination of the force on a particle may be thought of as a multistep process. The stages of this process are
1. From the known particle positions, determine the charge density at discrete points, \( \rho \).

2. Determine the potential from \( \rho \) by solving Poisson's equation on the grid,

3. Difference the potential to determine the electric field at the grid nodes,

4. Interpolate to determine the electric field at the particle position.

Each of these steps is discussed in the following paragraphs.

1. Determining the Charge Density

The first challenge is to determine a charge density from the particle point positions. To accomplish this a grid is introduced as is usually done in PIC codes [9, 34]. The field quantities \((E, \phi)\) are stored on the grid. Particle quantities \((m, v)\) are interpolated to the grid, and field quantities are interpolated from the grid to the particles using some interpolation function \(i(x_g - x_j)\), where \(x_g\) is the grid point. The interpolation function corresponds to the ‘particle shape’ as described by Birdsall and Langdon [9]. The ‘raw’ charge density of species \(\alpha\) at grid point \(g\) is then given by

\[
    n_\alpha = \sum_j q_j i(x_g - x_j) \tag{3.34}
\]

At this point, a smoothing function \(h(x_g - x)\) may also be introduced. The smoothing function is discussed below and will be used to minimize the effect of the grid on the numerical results. The smoothing operator is applied as a convolution over the grid points to obtain the charge density from the ‘raw’ densities of each species. Hence,

\[
    \rho_\alpha = \sum_{g'} h(x_g - x_{g'}) n_\alpha(x_{g'}) \equiv H * n_\alpha \tag{3.35}
\]

2. Solving Poisson's Equation

Having determined the source term for Poisson's equation, the next task is to actually invert the \(\nabla^2\) operator to determine \(\phi\). This is straightforward for the explicit case, but the need to determine \(\phi^{n+1}\) in the implicit case leads to complications.
Explicit Case: In the explicit case, the field solution must only be obtained at time level \( n \), for which the source term is known. The field equation is simply

\[
\nabla^2 \phi^n = -\sum_\alpha \rho_\alpha^n
\]

(3.36)

Methods for solving Poisson’s equation are well developed; the most efficient algorithms being conjugate gradient methods or transform techniques if appropriate boundary conditions are applicable.

Implicit Case: As will become clear when the implicit algorithm is outlined below, the field equation in the implicit scheme must be solved at time level \( n + 1 \). Unfortunately, the source term, \( \rho_\alpha \), is itself dependent on the future field. Because of this, the field equation must be simultaneously solved along with the particle equations of motion through a predictor-corrector procedure. In order for the predictor-corrector method to converge, Poisson’s equation must first be linearized about a predicted value of the potential.

In order to perform the linearization, the particles are first pushed to a predicted position and velocity, denoted by \( \bar{x}^{l,n+1} \) and \( \bar{v}^{l,n+1/2} \), by letting \( \bar{\mathbf{A}} = \frac{1}{2}(\bar{\mathbf{A}}^{n-1} + \mathbf{A}^{l,n+1}) \) in the equations of motion. (\( \mathbf{A}^{l,n+1} \) is an initial guess to the forward acceleration. The \( l \) superscript refers to an iteration number, while \( n \) and \( n + 1 \) refer to time levels.) The field equation, Equation (3.10), is expanded to get

\[
\nabla^2 (\bar{\phi} + \delta \phi) = -H \sum_\alpha (\bar{n}_\alpha^{l,n+1} + \delta n_\alpha)
\]

(3.37)

where \( \bar{n}_\alpha^{l,n+1} \) is the density calculated from the predicted positions \( \bar{x}_\alpha^{l,n+1} \), \( \bar{\phi}^{l,n+1} \) is the potential calculated from the predicted density distribution, and \( \delta n_\alpha \) and \( \delta \phi \) are the correction to the densities and potentials resulting from the acceleration term \( \delta \mathbf{A} = \mathbf{A}^{l+1,n+1} - \mathbf{A}^{l,n+1} \). The goal is to solve Equation (3.37) for \( \delta \phi \) in order to determine a better approximation \( \bar{\phi}^{l+1,n+1} = \bar{\phi}^{l,n+1} + \delta \phi \). This leads to an improved guess on the acceleration, \( \bar{\mathbf{A}} = \frac{1}{2}(\bar{\mathbf{A}}^{n-1} + \mathbf{A}^{l+1,n+1}) \), where \( \mathbf{A}^{l+1,n+1} \) refers to the acceleration determined from the potential, \( \phi^{l+1,n+1} \). The improved acceleration can be used in the
equations of motion to improve the estimates of particle positions $x^{l+1,n+1}$ and velocities $v^{l+1,n+1/2}$, and the iteration can proceed with a new solution of the Equation (3.37). For a convergent predictor-corrector iteration, $\delta \phi \to 0$ giving $\nabla^2 \phi^{n+1} = -\sum_\alpha \rho^{n+1}_\alpha$ at the end of the iteration.

The solution of Equation (3.37) requires expressing $\delta n_\alpha$ in terms of $\delta \phi$. To begin, $\delta n_\alpha$ may be expressed in terms of the corrections to the particle positions, i.e.

$$
\delta n_\alpha = \sum_j q_j \nabla i(x^{l,n+1}_j - x_j) \cdot \delta x_j
$$

(3.38)

where $\delta x_j \equiv x^{l+1,n+1}_j - x^{l,n+1}_j$. From the equation of motion,

$$
\delta x_j = L \cdot \left( E^{l+1,n+1}(x^{l+1,n+1}_j) - E^{l,n+1}(x^{l,n+1}_j) \right)
$$

(3.39)

where L is a general tensor operator relating changes in the electric field to changes in the particles position, and the spatial difference in E between iteration $l+1$ and $l$ has been retained. Expanding $E^{l+1,n+1}(x^{l+1,n+1}_j)$ in a Taylor series leads to

$$
E^{l+1,n+1}(x^{l+1,n+1}_j) = E^{l+1,n+1}(x^{l,n+1}_j) + \nabla E^{l+1,n+1}(x^{l,n+1}_j) \cdot \delta x_j
$$

(3.40)

Equation (3.39) may then be rewritten as

$$
\delta x_j - \left[ L \cdot \nabla E^{l,n+1}(x^{l,n+1}_j) \right] \cdot \delta x_j = L \cdot \delta E(x^{l,n+1}_j)
$$

(3.41)

where $\delta E(x^{l,n+1}_j) \equiv E^{l+1,n+1}(x^{l,n+1}_j) - E^{l,n+1}(x^{l,n+1}_j)$.

Defining $\varepsilon^{-1} = \left( I - [L \cdot \nabla E^{l,n+1}(x^{l,n+1}_j)] \right)^{-1}$,

$$
\delta x = \varepsilon^{-1} \cdot \left( L \cdot \delta E(x^{l}_j) \right)
$$

(3.42)

Combining Equations (3.41), (3.42), and (3.38), using $E = -\nabla \phi$, and noting (see the
section on interpolating the E-field below) that
\[
\delta \mathbf{E}(\mathbf{x}_j^{l,n+1}) = H \sum_{g'} i(\mathbf{x}_{g'}^{l,n+1} - \mathbf{x}_j^{l,n+1}) \delta \mathbf{E}(\mathbf{x}_{g'}^{l,n+1}),
\] (3.43)

\(\delta n_\alpha\) can be expressed in terms of \(\delta \phi\),
\[
\delta n_\alpha = \sum_j q_j \nabla i(\mathbf{x}_g - \mathbf{x}_j) \cdot \varepsilon^{-1} \cdot \left[ \mathbf{L} \cdot H \sum_{g'} i(\mathbf{x}_{g'}^{l} - \mathbf{x}_j^{l}) \nabla \delta \phi(\mathbf{x}_{g'}^{l}) \right] \] (3.44)

Substitution of this expression into Equation (3.37) leads to
\[
\nabla^2 \delta \phi - H \sum_j q_j \nabla i(\mathbf{x}_g - \mathbf{x}_j) \cdot \varepsilon^{-1} \cdot \left[ \mathbf{L} \cdot H \sum_{g'} i(\mathbf{x}_{g'}^{l} - \mathbf{x}_j^{l}) \nabla \delta \phi(\mathbf{x}_{g'}^{l}) \right] = -\bar{\rho} - \nabla^2 \tilde{\phi} \] (3.45)

Finally, for a magnetized plasma, the equations of motion give
\[
\mathbf{L} = c \frac{g(\Delta t)^2}{m} [\gamma_- \mathbf{I} + \gamma_+ \mathbf{R}] = c \frac{g(\Delta t)^2}{m} \mathbf{S} \] (3.46)

where \(c\) is the same parameter used in Equation (3.21). Combining Equations (3.45) and (3.46) gives
\[
\nabla^2 \delta \phi - H \sum_j q_j \nabla i(\mathbf{x}_g - \mathbf{x}_j) \cdot \varepsilon^{-1} \cdot \left[ \left( c \frac{g(\Delta t)^2}{m} \mathbf{S} \right) \cdot H \sum_{g'} i(\mathbf{x}_{g'}^{l} - \mathbf{x}_j^{l}) \nabla \delta \phi(\mathbf{x}_{g'}^{l}) \right] = -\bar{\rho} - \nabla^2 \tilde{\phi} \] (3.47)

Equation (3.47) completes the formal development of the field equation. Several simplifications can now be made. First, under the assumption of slowly varying fields, the spatial correction term should be small, i.e. \(\mathbf{L} \cdot \nabla \mathbf{E}^{l,n+1}(\mathbf{x}^{l,n+1}) \ll \mathbf{I}\), hence
\[
\varepsilon^{-1} \approx \mathbf{I} \] (3.48)

Secondly, assuming that the interpolation is a "local" operator, only the nearest grid
point contributes significantly in the summation over $g'$. Therefore,

$$
\sum_{g'} i(x_{g'}^j - x_j^i) \nabla \delta \phi(x_{g'}^j) \approx i(x_g^i - x_j^i) \nabla \delta \phi
$$

(3.49)

Likewise, the operator involving the gradient of the interpolation function acts as a finite difference operator on the rest of the function. Hence

$$
\sum_{g'} \nabla i(x_{g'}^j - x_j^i) \rightarrow -\nabla
$$

(3.50)

Combining these simplifications into Equation (3.47) leads to

$$
\nabla^2 \delta \phi + H \ast \nabla \cdot \left[ \sum_j \left( \frac{c q_j (\Delta t)^2 S_j}{m_j} \right) \cdot H \ast i(x_g^i - x_j^i) \nabla \delta \phi \right] = -\bar{\rho} - \nabla^2 \bar{\phi}
$$

(3.51)

The summation over particles may now be simplified to a summation over species,

$$
\nabla^2 \delta \phi + H \ast \nabla \cdot \left[ \sum_{\alpha} \left( \frac{c q_\alpha (\Delta t)^2 S_\alpha}{m_\alpha} \right) \cdot H \ast \nabla \delta \phi \right] = -\bar{\rho} - \nabla^2 \bar{\phi}
$$

(3.52)

Finally, defining

$$
\bar{\chi} = \sum_{\alpha} \left( \frac{c q_\alpha (\Delta t)^2 S_\alpha}{m_\alpha} \right)
$$

(3.53)

noting that the convolution and the gradient operator commute, and renormalizing $\phi' = H \ast \phi$, the field equation can be put in its final form:

$$
\nabla^2 \delta \phi' + \nabla \cdot H^2 \ast [\bar{\chi} \cdot \nabla \delta \phi'] = -H \ast \bar{\rho} - \nabla^2 \bar{\phi}'
$$

(3.54)

An interesting point to note here is that the unsmoothed potential, $\phi$, is never used in the implicit method. Only the smoothed potential, $\phi'$, appears. Because of this, the unsmoothed potential is dispensed with for the rest of this work and $\phi'$ is simply replaced in Equation (3.54) with the notation $\phi$.

**Solution of the Implicit Field Equation:** The field equation for use in the implicit method was developed in the previous section and led to Equation (3.54). The actual
solution of Equation (3.54) still remains to be discussed.

Equation (3.54) represents a large sparse matrix, the efficient inversion of which requires an inner iterative calculation using a conjugate gradient or other method suitable to large, sparse matrices. As a practical point, however, the smoothing operator, $H^2\ast$, greatly increases the band width of the matrix. Even for a convolution over a few of the nearest points, the band width becomes prohibitively large. To get around this problem, the solution here is restricted to simple boundary conditions, i.e. periodic, $\phi = 0$, or $E_{\text{normal}} = 0$. In these cases, transform methods are applicable, and the efficiency of the solution is greatly increased. Note, however, that the second term of Equation (3.54) does not lend itself directly to a transform method, hence an iterative method related to the 'global' iterative methods of Concus and Golub[17] is introduced.

To begin, Equation (3.54) may be written

$$\nabla^2 \delta \phi + \nabla \cdot H^2 \ast [\vec{\kappa} \cdot \nabla \delta \phi] = -\epsilon$$

(3.55)

where $\epsilon \equiv \bar{\rho} + \nabla^2 \bar{\phi}$. The Fourier transform of Equation (3.55) results in

$$k^2 \hat{\delta \phi}_k + |h_k| \sum_{k'} k \cdot \vec{\kappa}_{k-k'} \cdot k' \hat{\phi}_{k'} = \hat{\epsilon}_k$$

(3.56)

or, separating the $k = k'$ mode,

$$(k^2 + |h_k| k \cdot \vec{\kappa}_0 \cdot k) \delta \phi_k = \hat{\epsilon}_k - \sum_{k' \neq k} k \cdot \vec{\kappa}_{k-k'} \cdot k' \delta \phi_{k'}.$$

(3.57)

The portion of the left hand side of Equation (3.57) in parenthesis represents an approximate inverse of Equation (3.56). An iterative method can then be written as (using the superscript $m$ for this iteration)

$$\delta \phi_k^{m+1} = \frac{\hat{\epsilon}_k - \sum_{k' \neq k} k \cdot \vec{\kappa}_{k-k'} \cdot k' \delta \phi_k^m}{(k^2 + |h_k| k \cdot \vec{\kappa}_0 \cdot k)}$$

(3.58)

The direct computation of the summation in the numerator of Equation (3.57) requires summing each $k$ with every other $k$ mode, i.e. the summation is an $\mathcal{O}(N^2)$ process, where
$N$ is the number of $k$-modes. Equation (3.58) can be more efficiently solved through a mix of physical and transform space calculations. For example, begin by calculating $\delta \hat{\phi}^n$. Multiply by $ik$ and transform back to physical space. Multiply by $\tilde{\chi}$, transform back to $k$ space, and multiply by $ik$. Subtract from $\hat{\delta}_k$ and divide by $(k^2 + |h_k| k \cdot \chi_0 \cdot k)$ to obtain $\delta \hat{\phi}^{m+1}$. Continue the iteration until $\delta \hat{\phi}^{m+1} - \delta \hat{\phi}^m$ is reduced to the desired level. A final inversion of $\delta \hat{\phi}_k$ gives $\delta \phi$ for use in the outer iteration on the $l$ variable as described in the previous section.

3. Differencing of the Potential

At several points in the previous discussion, the relation $E = -\nabla \phi$ has been introduced without mentioning the actual numerical implementation. In the current code, the differencing of the potential is simply accomplished through the use of a centered difference operator, i.e.

$$E_x = (\phi_{i+1,j} - \phi_{i-1,j})/(2\Delta x)$$

(3.59)

and similar expressions for $E_y$ and $E_z$ in multiple dimensions.

4. Interpolation of Field Quantities to the Particles

Similar to the interpolation of the particle shape to determine the charge density, the acceleration of a particle is found by interpolating the electric field from the grid back to the particle position,

$$A_j = \left(\frac{q}{m}\right)_j \sum_g i(x_j - x_g)H \ast E$$

(3.60)

where

$$H \ast E \equiv \sum_{g'} h(x_g - x_{g'})E$$

(3.61)

Note that it has been assumed that the smoothing function has also been applied to the electric field and that the same interpolation and smoothing functions have been used in both determining the charge density and determining the acceleration at the particle. Although the use of the same interpolation and smoothing functions is not
required in both of these steps, algorithms using different interpolation and smoothing functions will not result in strict momentum conservation. In the work done here, the same interpolation and smoothing functions were used both to and from the grid.

3.5 Algorithms

The previous sections have discussed the differencing of the equations of motion and the solution of the field equation. This section describes the overall algorithm for updating particles positions and fields in both the explicit and implicit codes.

3.5.1 Explicit method

The explicit leap-frog scheme proceeds in the following manner:

- Given $x^n, v^{n-1/2}$.

- Interpolate the particle quantities onto the grid to determine the 'raw' densities (Equation 3.34).

- Solve Poisson's equations, with the smoothing operator applied to the density (Equations 3.35 and 3.36).

- Calculate the electric field (Equation 3.59).

- Interpolate the electric field to find the acceleration of each particle (Equations 3.60 and 3.61).

- Advance the velocity to get $v^{n+1/2}$ (Equation 3.18).

- Step forward the position to find the positions $x^{n+1}$ (Equation 3.17).

3.5.2 Implicit method

Since the implicit method requires a predictor-corrector method to establish the forward acceleration, the algorithm is substantially more complicated than the explicit method. It proceeds as follows (see Figure 3-1):
- Given $z^n, v^{n-1/2}, A^{n-1}$.

- Approximate $A^n = \frac{1}{2} A^{n-1}$ (Other choices are possible for this initial guess such as $A^n = A^n$).

- Advance Equation (3.18) to get the approximation $\psi^{l,n+1/2}$

- Step forward Equation (3.17) to find an approximation to the positions $x^{l,n+1}$.

- Interpolate the particle quantities onto the grid to determine the 'raw' predicted densities, $n^{l,n+1}_d$ (Equation 3.34).

- Solve the linearized form of Poisson's equations (Equation 3.54) via the global iterative method outlined to determine $\delta \phi$

- Update the predicted potential $\phi^{l+1,n+1} = \phi^{l,n+1} + \delta \phi$

- Calculate the electric field $E^{l+1,n+1}$ (Equation 3.59)

- Interpolate the electric field to find the acceleration of each particle, $A^{l+1,n+1}$ (Equation 3.60)

- Using the better approximation $A^n = \frac{1}{2}(A^{n-1} + A^{l+1,n+1})$, iterate through the loop beginning at re-pushing the particles with Equation (3.18) until $\delta \phi < \epsilon$, where $\epsilon$ is some small number representing convergence.

3.6 $\alpha, \beta, c, \gamma_+, i, H, k$ and Other Practical Concerns

The previous sections have outlined the numerical algorithms used in both explicit and implicit codes. In this discussion, however, several functions and parameters were introduced but not specifically defined. These include the weighting of the time levels in determining the implicit acceleration ($\alpha$'s, $\beta$'s and $c$ in Equation 3.21), the decentering parameter, $\gamma_+$ used in Equation (3.19), the interpolation function, $i$, the smoothing function, $H$, and the $k$-vector. The purpose of this section is to discuss the choices of these parameters and functions and to describe the effect of the choices made.
3.6.1 $\alpha$'s, $\beta$'s, $c$, and the Stability of Implicit Methods

The proper selection of the free parameters in Equation (3.21) will lead to an absolutely stable numerical algorithm in the implicit code. To determine these parameters, then, the stability of both explicit and implicit codes is discussed.

Explicit Case: To determine the stability properties of the explicit method, the equations of motion can be replaced by the model equations for a simple harmonic oscillator. This is equivalent to checking the stability of the natural modes of the plasma. The differenced leap-frog scheme for the model equation is:

$$z^{n+1} = z^n + v^{n+1/2} \Delta t \tag{3.62}$$

$$v^{n+1/2} = v^{n-1/2} - \omega_m^2 z^n \Delta t \tag{3.63}$$

where $\omega_m$ is the mode frequency of the oscillation. For a plasma, $\omega_m$ may be identified with the plasma frequency $\omega_{pe}$ or other natural modes.

Putting Equations (3.63) and (3.62) in terms of positions only, the leap frog scheme becomes

$$z^{n+1} - 2z^n + z^{n-1} = -\omega_m^2 z^n (\Delta t)^2 \tag{3.64}$$

Substituting in the eigenmode $\lambda = \exp(i\omega n \Delta t)$ into Equation (3.64) leads to the characteristic equation

$$\lambda^2 - 2\lambda + 1 = -\omega_m^2 \lambda \tag{3.65}$$

To remain stable requires $|\lambda| \leq 1$. This is satisfied for $\omega_m \Delta t \leq 2$ which is the well known stability limit of explicit PIC codes.

Implicit Case: The stability limit of the implicit method is determined in a manner similar to the explicit algorithm. The forcing term is replaced by the expression for a simple harmonic oscillator.

$$z^{n+1} = z^n + v^{n+1/2} \Delta t \tag{3.66}$$

$$v^{n+1/2} = v^{n-1/2} - \omega_m^2 z^n \Delta t \tag{3.67}$$

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where now $\bar{x}$ is a combination of past and future $z$ values expressed as
\[ x_j^n + \alpha_1 x_j^{n-1} + \alpha_2 x_j^{n-2} + \ldots = c x_j^{n+1} + \beta_0 x_j^n + \beta_1 x_j^{n-1} + \beta_2 x_j^{n-2} + \ldots \]  \hspace{1cm} (3.68)

Again, substituting the eigenmode $\lambda = \exp(i \omega n \Delta t)$ into Equations (3.66), (3.67) and (3.68) leads to a characteristic equation for $\lambda$. Keeping $I$ time levels leads to
\[ (\lambda^{I-1} + \alpha_1 \lambda^{I-2} + \ldots + \alpha_{I-1})(\lambda - 1)^2 + \omega_m^2 \lambda (c \lambda_I + \beta_0 \lambda^{I-1} + \ldots + \beta_{I-1}) = 0 \]  \hspace{1cm} (3.69)

The roots of this equation determine the stability of the algorithm. An analyses of the roots can be found in the paper by Barnes et al. [7]. The conclusions reached in this analyses are

- $c \neq 0$ is required for absolute stability.

- For the simplest choice of filter ($I=2$), second order accuracy may be obtained for
\[ \bar{z}^n = \frac{1}{2} (z^{n+1} + z^{n-1}) \]  \hspace{1cm} (3.70)

i.e. $c = .5$, $\alpha_1 = -.5$, and all other $\alpha$'s and $\beta$'s are zero.

- For $I=3$, second order accuracy leads to the choice
\[ \bar{z}^n = \frac{2}{5} (z^{n+1} + 2z^{n-1} - \frac{1}{2} \bar{z}^{n-2}) \]  \hspace{1cm} (3.71)

i.e. $c = .4$, $\alpha_1 = -.8$, $\alpha_2 = .2$ and all other $\alpha$'s and $\beta$'s are zero.

The choice of either Equation (3.70) or (3.71) represents the choice of a recursive filter on the electric field. The purpose of this filter is to eliminate high frequency, unstable wave modes from the simulation. If the filter were to work ideally, the response would be $1 - |\lambda| = 1$ for $\omega_m \Delta t > 1$ and $1 - |\lambda| = 0$ for $\omega_m \Delta t < 1$, where again $\lambda = \exp(i \omega n \Delta t)$. This ideal filter removes all underresolved modes. The actual response of the filters represented by Equations (3.70) and (3.71) is shown in Figure 3-2. As seen in the figure, both filters heavily damp modes with $\omega_m \Delta t < 1$. The more accurate filter of
Equation (3.71) is slightly more "ideal", although as a practical point, experience using both of these filters has indicated that the choice of the \( I = 2 \) or \( I = 3 \) filter does not greatly affect the results. In general, the \( I = 2 \) filter has been used for most of the results in this work.

### 3.6.2 The Decentering Parameter, \( \gamma_+ \)

The previous section has shown that the implicit algorithm can be made stable for time steps \( \omega_{pe} \Delta t \gg 1 \). Another concern is the electron cyclotron period. The electron cyclotron period does not in general impose stability restrictions on the algorithm. However, for time steps \( \Omega_e \Delta t \gg 1 \), the electron cyclotron period is not resolved. The effect of underresolving the electron cyclotron period is aliasing into longer periods. In fact, Parker [60] derives an expression for the numerical "oscillation frequency". In the \( v \times B \) mover used here with \( \gamma_+ = \gamma_- = .5 \) (which is correct for explicit codes), the angle of rotation in one time step is given by

\[
\omega_{os} \Delta t = \theta = 2 \arctan \left( \frac{1}{2} \Omega_e \Delta t \right)
\]  

(3.72)

For \( \Omega_e \Delta t \gg 1 \),

\[
\theta \approx \pi - \frac{4}{\Omega_e \Delta t}
\]  

(3.73)

indicating that as \( \Omega_e \Delta t \rightarrow \infty \), \( \omega_{os} \Delta t \rightarrow \pi \). Hence, at large time steps, the electron orbit is simply a rapid odd-even bounce with a slow precession about the guiding center. Also, for large time steps, the numerical oscillation frequency is much lower than the correct electron gyrofrequency.

One result of the incorrect oscillation period is a numerical increase in the electron gyroradius. As given by Birdsall and Langdon, the numerical gyroradius is given by

\[
\rho_{os} = \frac{v_\perp}{2 \sin(\frac{1}{2} \theta)} = \rho_e \left[ 1 + \left( \frac{\Omega_e \Delta t}{2} \right)^2 \right]^{1/2}
\]  

(3.74)

where \( \rho_{os} \) is the numerical gyroradius and \( \rho_e \) is the physical electron gyroradius. For
\( \Omega_e \Delta t \gg 1 \), Equation (3.74) becomes

\[
\rho_{os} = \rho_e \frac{\Omega_e \Delta t}{2} \tag{3.75}
\]

Hence, the numerical gyroradius may become much greater than the physical gyroradius. If the numerical gyroradius is on the order of the scale length of the waves in a problem, a resultant loss of accuracy is observed in the simulations as the electrons incorrectly sample the wave spectrum.

A simple way to combat the problem of large numerical gyroradii is through the decentering of the equations of motion. The effect of decentering the equations of motion is to eliminate all gyrations due to the magnetic field while retaining the correct guiding center drifts. In fact, taking \( \gamma_+ = 0.5 + \epsilon \), Barnes et al.\[7\] have shown that the leapfrog differenced equations, expanded in a Taylor series for \( \Omega_e \Delta t \gg 1 \), are consistent with the differential equation

\[
v_\perp = \frac{E \times B}{B^2} + \frac{\rho_e \dot{E}}{qB^2} - \epsilon \Delta t \frac{d}{dt} \left[ \frac{E \times B}{B^2} + 2 \frac{\rho_e \dot{E}}{qB^2} \right] + O(\Delta t^2) \tag{3.76}
\]

\[
\dot{v}_\parallel = \frac{qE_\parallel}{m} + O(\Delta t^2) \tag{3.77}
\]

These differential equations indicate that for \( \Omega_e \Delta t \gg 1 \), the guiding center motion of the electrons retains the zero gyroradii drift motions. In addition, for modes such that

\[
\gamma \leq |\omega| \Delta t < 1 \tag{3.78}
\]

second order accuracy is maintained.

The arguments laid out in this section are illustrated in Figure 3-3. The figure shows the orbit of a single particle moving in crossed \( E \) and \( B \) fields. The particle trajectory is calculated in the plane perpendicular to a magnetic field. The electric field is applied so that the particle should drift across the grid at a 45° angle. Part (a) of the figure shows the orbit calculated using an explicit time step of \( \Omega_e \Delta t = 0.2 \) and \( \gamma_+ = \gamma_- = 0.5 \). The explicit code carefully calculates the electron orbit, hence the
result shown corresponds closely to the analytic solution. In Part (b), the time step is increased to $\Omega_x \Delta t = 5$ while maintaining $\gamma_+ = \gamma_- = .5$. As predicted by the remarks above, the numerical gyroradius has grown substantially. A measurement will show that the numerical gyroradius is roughly the size indicated by Equation (3.75). It should also be noted that the correct drift motion has still been retained. Finally, Part (c) of the figure shows the results at the same time step of $\Omega_x \Delta t = 5$, but now $\epsilon = .1$ in order to heavily damp the gyration effects. As seen in the figure, after a short transient, the particle does not exhibit gyrations, but simply follows the guiding center trajectory.

For the current work, the electron cyclotron radius is small compared to the characteristic wavelengths of the electrostatic waves. An increased numerical gyroradius tends to adversely effect the simulation. Because of this, most of the results presented in this work have adopted $\epsilon = .1$, heavily damping the electron gyromotion to the guiding center drift.

3.6.3 The Interpolation Function

The interpolation function $i(x_j - x_p)$ is used to both interpolate the particles to determine the charge density on the grid and to interpolate the electric field to determine the force on a particle. In the manner used in this work, the interpolation function corresponds to the well known 'particle shape' as used by Birdsall and Langdon [9]. A thorough discussion of the use of nearest grid point (NGP) and first and second order weighting (referred to in [9] as "cloud-in-cell", CIC, and "quadratic spline", QS, weighting, respectively) is included in the reference. Since the use of these interpolation functions is widely understood, the details will not be presented here.

NGP, CIC and QS weighting have all been implemented in the implicit code. Experience with the code has shown that little difference exists between results with CIC and QS weighting. Because of this, the simpler of these two, namely first order or CIC weighting, has been used for most of the results presented.
3.6.4 The Smoothing Function

The algorithms as developed above allow for the introduction of a smoothing function $h(x_g - x_g')$ in order to reduce numerical noise at short wavelengths and increase the convergent rate of the implicit field solver.

The purpose of this section is to describe the selection of the spatial smoothing function used in the simulations. Details of the implications of spatial smoothing for PIC codes can be found in Birdsall and Langdon.

In general, implicit PIC codes require substantially more spatial smoothing than explicit PIC codes. This does not present a problem if implicit PIC codes are accepted as being appropriate for cases where the physics is dominated by low frequency and long wavelength processes.

A typical explicit PIC code will add no additional spatial smoothing beyond the interpolation function, which is usually CIC. If spatial smoothing is applied, a common spatial filter is a Gaussian, i.e.

$$h(x_g - x_g') = \frac{\Delta x}{a\sqrt{2\pi}} \exp \left( - \frac{((x_g - x_g')^2)}{2a^2\Delta x^2} \right)$$  \hspace{1cm} (3.79)

where $a$ is the width of the Gaussian. For an explicit code, $a = 1$ is a common value. As pointed out by Barnes et al., however, tolerable numerical cooling in implicit codes requires $a > 2$ with a value of $a = 3$ recommended. This is a large particle size. The important point, of course, is that the particle size with $a = 3$ be smaller than the wavelength of the dominant modes in the problem. If this is the case, the particle accurately samples the wave field, and the correct physics are retained in the simulation. The result is that a restriction is placed on the grid size, $\Delta z$; namely $a\Delta z \ll \lambda$, where $\lambda$ is the important wavelength in the problem.

Throughout most of this work, the recommendation of Barnes et al. is accepted. Most of the simulation results presented were produced using a spatial smoothing function of a Gaussian with width $a = 3$.

The adoption of a Gaussian smoothing function leads to a question of the efficient application of the smoothing function. Clearly, performing the convolution on the grid
in physical space will be a costly operation. In fact, experience with the current code indicates it is prohibitively costly. The alternative is to apply the smoothing function in $k$-space, in which case the convolution becomes a simple multiplication. Although this increases the efficiency and allows spatial smoothing to be adopted, it also restricts the implicit method to cases in which transform methods can be applied: i.e. it restricts the grid to constant and rectangular and the boundary conditions to periodic, $\phi = 0$ or $E_{\text{normal}} = 0$. These are severe restrictions when compared to the versatility of explicit codes.

In fact, experience gained through the current work has tended to impress the idea that the amount of spatial smoothing required to achieve adequate results with implicit codes is their most severe shortcoming. Convergence of the field equation and the desired level of accuracy in the simulations could not be achieved without using Gaussian smoothing with at least $a = 2$. The application of this necessary smoothing restricts the code to simple boundary conditions (usually periodic). In many cases of interest, periodic boundary conditions are simply not sufficient. Much work needs to be done in this area before implicit codes become more widely applicable to a variety of plasma problems.

3.6.5 Operations in $k$-space

As should be obvious from both the discussion of the solution of the field equation and the previous section relating to spatial smoothing, implicit codes rely heavily on transform space operations. In this section, several points are made about the implementation of $k$-space portions of the implicit code.

The first point is simply to define $k$. When Fast Fourier Transform (FFT) methods are employed on discrete meshes, the $k$ spectrum is also discrete. A ‘raw’ $k$-value can be defined as simply

$$k_{z,g} = \frac{2\pi (g - 1)}{N_z - 1}$$

(3.80)

where $g$ refers to the grid point and $N_z$ is the number of grid points in the $z$-direction. Similar relations exist for $k_{\nu,g}$ and $k_{\tau,g}$.

Using the ‘raw’ $k$ values, expressions can be determined for any desired derivative
operator. For example, take the simple centered differencing of the potential in order to determine \( E_z \). In physical space, the differencing to be performed is

\[
E = -\frac{\partial \phi}{\partial z} = -\frac{\phi_{i+1} - \phi_{i-1}}{2\Delta z}
\]  

(3.81)

(only one dimension is of concern here). The appropriate \( k \)-space operator for this equation can be derived by Fourier transforming Equation (3.81). The result is

\[
\hat{\mathcal{E}}_k = -\frac{e^{ik\Delta s} - e^{-ik\Delta s}}{2\Delta s} \hat{\phi}_k
\]  

(3.82)

which may be rewritten in the revealing form

\[
\hat{\mathcal{E}}_k = -\frac{i}{\Delta x} \sin(k\Delta x) \hat{\phi}_k
\]  

(3.83)

from which the following relation may be deduced

\[
\frac{\partial}{\partial z} \rightarrow \frac{i}{\Delta x} \sin(k\Delta x)
\]  

(3.84)

The \( ik \) operator implied from continuous Fourier analysis is replaced by the operator shown by Equation (3.84) in discrete space.

Another useful operator comes from

\[
\frac{\partial^2 u}{\partial z^2} = \frac{1}{\Delta x^2} (u_{i+1} - 2u_i + u_{i-1})
\]  

(3.85)

which leads to

\[
\frac{\partial^2}{\partial z^2} \rightarrow -\frac{4}{\Delta x^2} \sin^2\left(\frac{k\Delta x}{2}\right)
\]  

(3.86)

Other operators may be developed for higher order accuracy or other derivatives. It is the trigonometric forms of the \( k \)-space operators which are used in the solution of the field equation and elsewhere when \( k \)-space differencing is needed.
3.7 Initial and Boundary Conditions

The implementation of a particle code requires initial and boundary conditions. This section discusses these conditions.

3.7.1 Initial Conditions

The initial conditions used in both implicit and explicit PIC codes are similar. For the current work, the appropriate initial conditions are almost always a stationary Maxwellian plasma in a magnetic field. This is a commonly used initial condition. To reduce the numerical noise in the simulations, quiet starts are used. The concept and implementation of quiet starts are well covered in the text by Birdsall and Langdon [9], and the interested reader is referred there for details.

The only complication to the initial conditions introduced by implicit codes involves the implementation of the first time step. The implicit code uses the value of acceleration at time level $n - 1$ in the particle pusher. At the first step, however, the $n - 1$ information is not known. This problem is overcome by simply using a fully forward implicit first step (i.e. $\bar{A}^n = A^{n+1}$).

3.7.2 Boundary conditions

As mentioned above, the proper implementation of boundary conditions remains an unresolved problem in multi-dimensional implicit PIC codes. The main difficulty arises from the necessity of adding substantial spatial smoothing in order to control unphysical numerical cooling. The large particle sizes needed to adequately control the numerical errors imply a coupling between many grid points in the problem. Once this coupling is introduced into the field equation, the inversion of Equation (3.54) becomes prohibitively expensive (the bandwidth of the matrix has increased unacceptably). The way to avoid inverting the wide bandwidth matrix of Equation (3.54) is through the introduction of transform methods as described in a previous section. Although transform methods lead to an efficient solution method of Equation (3.54), they also restrict the solution to simple boundary conditions. Three possible boundary conditions are implemented for the fields
in the current code. The three boundary conditions (periodic, \( \phi = 0 \), and \( E_{\text{normal}} = 0 \)) represent solving Equation (3.54) using Fourier, sine and cosine transforms, respectively. Each of these methods offers the advantage of transforming the convolution over grid points into a multiplication and allows the efficient solution of the field equation.

### 3.8 Collision Model

The modeling of CIV requires collisional processes be included in the simulation. No previous implicit PIC codes have included discrete collisional effects. The purpose of this section is to describe the implementation of the collision model in the current work.

To begin, since neutral particles are not modeled in the current work, the assumption used in previous CIV PIC simulations is adopted. This assumption is that the neutral density is much greater than the plasma density, i.e.

\[
n_n \gg n_e
\]

This assumption is valid for the laboratory experiments (where \( n_n/n_e \approx 100 \)) and in the space experiments near the neutral release point. In the space experiment, the neutral density drops as \( 1/r^2 \), where \( r \) is the distance from the release point, but for typical release parameters, the assumption is good for at least several kilometers. The assumption of high neutral densities allows two further key assumptions. These assumptions are

- the neutral density is uniform and constant. Neutral density variations will not be considered on the order of the wavelengths of the lower hybrid waves, the dominant wave mode in CIV simulations. Also, no time variation of the neutral density will be considered in the simulations.

- electron-neutral and ion-neutral collisions are the dominant collisional modes. Discrete charged particle-charged particle collisions are not considered. This is valid if the neutral density is high enough that electron-neutral collisions dominate over electron-electron Coulomb collisions. This is true in both the laboratory and space experiments. (For example, in the laboratory experiments, the electron-electron
Coulomb collisions time, \( \tau_{ee} = 1 \mu\text{sec} \), while the electron neutral collision time, \( \tau_{en} = 0.1 \mu\text{sec} \).

The following paragraphs describe the implementation of the collision model under the previously stated assumptions.

For collisional process \( j \), the collision frequency of a charged particle with a neutral is given by

\[ \nu_j = \langle \sigma_j v \rangle N_n \]  

(3.88)

where \( \sigma_j \) is the cross section of the \( j^{th} \) reaction as a function of relative energy, \( v \) is the relative velocity of the two particles and \( N_n \) is the density of the neutral species. In the case of neutral densities low enough so that the criterion

\[ \sum_j \nu_j / \omega_{pe} \ll 1 \]  

(3.89)

is satisfied, the collisions may be treated in an explicit manner in the overall implicit scheme. This is done by using a method commonly used in explicit codes. For a particle with a collision frequency \( \nu_j \), the probability of having a collision of type \( j \) in time \( \Delta t \) is

\[ P_j = 1 - \exp^{-\nu_j \Delta t} \]  

(3.90)

Non-dimensionalizing the frequency and time with the plasma frequency,

\[ P_j = 1 - \exp^{-\nu_j \omega_{pe} \Delta t} \]  

(3.91)

At each time step, for each particle, the collision algorithm is implemented in the following manner:

1. For a known relative velocity and energy, look up the cross section of each type of reaction and calculate the collision frequencies \( \nu_j \),

2. Calculate the total collision frequency, \( \sum_j \nu_j \),

3. Test that \( \nu_j \Delta t \ll 1 \). This must be true for the current algorithm to be accurate.

   If this condition is violated, the time step, \( \Delta t \), must be reduced.
4. From the collision frequencies, calculate the probability of a collision of any type in the time step,

$$P_{\text{total}} = 1 - \exp\left(-\sum_{j} \frac{\omega_{j}}{\omega_{p}} \cdot \omega_{p} \Delta t\right)$$  
(3.92)

5. Chose a random number, $p_{\text{ran}}$, evenly distributed between 0 and 1. If $p_{\text{ran}} < P_{\text{total}}$, the particle collided during the time step,

6. Chose a second random number evenly distributed between 0 and 1, $p_{\text{ran2}}$. If $0 < p_{\text{ran2}} < \sigma_{1}/\sum_{j} \sigma_{j}$, the particle underwent a collision of type 1; if $\sigma_{1}/\sum_{j} \sigma_{j} < p_{\text{ran2}} < (\sigma_{1} + \sigma_{2})/\sum_{j} \sigma_{j}$, the particle underwent a collision of type 2; etc.

7. If a collision occurred, determine the new velocity of the particle in the manner discussed below, otherwise continue to the next particle.

Having determined a collision occurs, the equations for conservation of mass, momentum and energy must be solved in order to determine the particle’s velocity after the collision. This process is discussed for each of the typical reaction types included in the current code.

**Electron Ionising collisions**

The initial velocity of the colliding electron, $V_{1e}$, is known. A velocity is chosen for a characteristic neutral,

$$V_{n} = v_{\text{beam},n} + v_{t,n}$$  
(3.93)

where $v_{\text{beam},n}$ is the beam velocity of the neutrals and $v_{t,n}$ is a random vector chosen to fulfill the distribution function of the neutral thermal velocity. A two dimensional collision plane is defined by the two vectors $V_{1e}$ and $V_{n}$. The collisions will be assumed to be head on, so the particle velocities after the collision will also lie in this plane. Noting that the normal unit vector to the plane is given by

$$n = \frac{V_{1e} \times V_{n}}{|V_{1e} \times V_{n}|},$$  
(3.94)
the angle $\theta$ may be defined as the angle between the $n$ and the original $z$-axis, and $\phi$ as the angle between the projection of $n$ onto the $x$-$y$ plane and the $x$-axis. The velocities are first transformed into this collision plane (denote the new coordinate system by carets):

$$\hat{V}_{1e} = \hat{U} \cdot V_{1e}$$  \hspace{1cm} (3.95)
$$\hat{V}_n = \hat{U} \cdot V_n$$  \hspace{1cm} (3.96)

where $\hat{U}$ is a transformation matrix

$$\hat{U} = \begin{bmatrix}
\cos \phi \cos \theta & -\sin \theta & \sin \theta \cos \phi \\
\sin \phi \sin \theta & \cos \theta & \sin \theta \sin \phi \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}$$  \hspace{1cm} (3.97)

Conservation of momentum in the center of mass frame requires

$$m_e \hat{V}_{1e} + m_e \hat{V}_{2e} + m_i \hat{V}_i = 0$$  \hspace{1cm} (3.98)

In addition, conservation of energy may be written as

$$\frac{1}{2} m_n \hat{V}_n^2 + \frac{1}{2} m_e \hat{V}_{1e}^2 - \frac{1}{2} (m_n + m_e) V_C^2 = \frac{1}{2} m_i \hat{V}_i^2 + \frac{1}{2} m_e \hat{V}_{1e}^2 + \frac{1}{2} m_e \hat{V}_{2e}^2 + e\phi_{ion}$$  \hspace{1cm} (3.99)

where $m_n = m_i + m_e \approx m_i$, the primes represent the velocities after the collision in the center of mass frame, and

$$V_C = \frac{m_n \hat{V}_n + m_e \hat{V}_{1e}}{m_n + m_e}$$  \hspace{1cm} (3.100)

is the velocity of the center of mass.

In order to solve Equations (3.98) and (3.99), three random angles ($\alpha$, $\beta$ and $\gamma$) are chosen to represent the scattering angle of each of the particles in the collision plane. The scattering angles are chosen uniformly between 0 and $2\pi$. Equations (3.98) and (3.99) then lead to expressions for the magnitude of the velocities after the collision:

$$\hat{V}_{1e}' = \frac{m_i \sin(\beta - \gamma)}{m_e \sin(\alpha - \beta)} \hat{V}_i' = A \hat{V}_i'$$  \hspace{1cm} (3.101)
\[
\dot{V}'_{2e} = -\frac{m_i \sin(\alpha - \gamma)}{m_e \sin(\alpha - \beta)} \dot{V}'_i = B \dot{V}'_i \\
\dot{V}'_i = \left\{ \frac{\dot{V}'_{1e}^2 + \frac{m_i}{m_e} \dot{V}'_{1n}^2 - (1 + \frac{m_i}{m_e})V_C^2 - \frac{2e\phi_i}{m_e}}{A^2 + B^2 + \frac{m_i}{m_e}} \right\}^{1/2}
\]
(3.102)
(3.103)

The velocity vectors in the collision plane, center of mass frame are

\[
\dot{V}'_j = (\dot{V}'_j \cos \theta_j, \dot{V}'_j \sin \theta_j)
\]
(3.104)

where \( j = 1e, 2e, i \) and \( \theta = \alpha, \beta, \gamma \). Then, moving back to the laboratory frame from the center of mass frame:

\[
\dot{V}'_{j,L} = \dot{V}'_j + V_C
\]
(3.105)

where \( V'_L \) is the velocity in the laboratory frame. Finally,

\[
\begin{align*}
\dot{V}'_{1e,L} &= \dot{U}^T \cdot \dot{V}'_{1e,L} \\
\dot{V}'_{2e,L} &= \dot{U}^T \cdot \dot{V}'_{2e,L} \\
\dot{V}'_{n,L} &= \dot{U}^T \cdot \dot{V}'_{n,L}
\end{align*}
\]
(3.106)
(3.107)
(3.108)

where \( \dot{U}^T \) is the transpose of the matrix given in Equation (3.97).

**Electron Elastic and Excitational Collisions:**

In the case no new particles are produced, conservation of momentum and energy in the collision plane give

\[
m_e \dot{V}'_{1e} + m_i \dot{V}'_i = 0
\]
(3.109)

\[
\frac{1}{2}m_n \dot{V}'_{1n}^2 + \frac{1}{2}m_e \dot{V}'_{1e}^2 - \frac{1}{2}(m_n + m_e)V_C^2 = \frac{1}{2}m_i \dot{V}'_i^2 + \frac{1}{2}m_e \dot{V}'_{1e}^2 + e\phi_j
\]
(3.110)

where \( \phi_j \) is the excitational energy of the \( j^{th} \) collisional process and \( \phi_j = 0 \) for elastic collisions. The speed of the electron after the collision is found from

\[
\dot{V}'_e = \left\{ \frac{\dot{V}'_{1e}^2 + \frac{m_a}{m_e} \dot{V}'_{1n}^2 - (1 + \frac{m_a}{m_e})V_C^2 - \frac{2e\phi_j}{m_e}}{1 + \frac{m_a}{m_e}} \right\}^{1/2}
\]
(3.111)
Choosing a random scatter angle $\alpha$, the velocity is

$$\mathbf{V}_{1e} = \hat{\mathbf{v}}_e \begin{pmatrix} \cos \alpha \\ \sin \alpha \\ 0 \end{pmatrix}$$

(3.112)

The velocity is then transformed back to the laboratory frame similar to the ionization case.

**Charge Exchange Collisions:**

For charge exchange reactions, the velocity of the new ion is simply set equal to the velocity of a particle chosen randomly from the neutral distribution.

### 3.9 Effects of the Implicit Code on the Collision Model

The only effect of the implicit code on the collision model is that the implicit scheme requires time centered accelerations, $\ddot{A}^n = \frac{1}{2}(\ddot{A}^{n-1} + \ddot{A}^{n+1})$, for the particles at each step. If a particle experiences a collision, the past information ($\ddot{A}^{n-1}$) is not available. This is dealt with by advancing a particle on the time step following a collision with a fully forward acceleration ($\ddot{A}^n = A^{n+1}$).
<table>
<thead>
<tr>
<th>Frequency</th>
<th>$\Omega_e$</th>
<th>$\omega_{pe}$</th>
<th>$\omega_{LH}$</th>
<th>$\omega_{pi}$</th>
<th>$\Omega_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric Value (s$^{-1}$)</td>
<td>$10^7$</td>
<td>$10^7$</td>
<td>$10^5$</td>
<td>$10^5$</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Numerics</td>
<td>Explicit PIC</td>
<td>Implicit PIC</td>
<td>Gyrokinetic</td>
<td>Guiding Center</td>
<td>Fluid</td>
</tr>
</tbody>
</table>

Table 3.1: Representative time scales encountered in plasma simulations. Typical values are listed for the ionosphere. The type of numerical code commonly used for the different regimes is also indicated. Note that the time scales vary over four orders of magnitude.
Figure 3-1: Flowchart for the main program and the field solver.
Figure 3-2: Amplitude response of D1 and D2 schemes. The D1 scheme refers to the I=2 filter; the D2 scheme is the I=3 filter. Both the D1 and D2 schemes heavily damp modes with $\omega_n \Delta t < 1$. The D2 scheme is slightly more "ideal", although the practical difference is minimal.
Figure 3-3: Gyroradii effects. Part (a) shows an integration of an electron orbit in crossed E and B fields using an explicit code. The results agrees well with the analytic result. In Part (b), a large time step is used without decentering the equations of motion. The gyroradius has increased substantially. Part (c) shows the same integration performed with a large $\Delta t$, but with decentered equations of motion. The electron motion quickly damps to the drift motion.
Chapter 4

The Existence and Rate of CIV: Simulation Results

Chapter 2 discussed in detail the theory of the CIV mechanism. Much of this analysis relied on linear or quasilinear analysis and simple rate models. The full nonlinear problem including detailed collision physics is not amenable to analysis. In light of this, this chapter has two goals. The first of these goals is to present a verification of the previously developed theory through a series of particle in cell simulations. When linear and quasilinear theory are valid, the simulation results should be expected to agree closely with theory. The second of the goals is to develop some empirical relations involving the time scales for initiation and growth of CIV in realistic gases. This is achieved through a series of one-dimensional PIC simulations using physical mass ratios and collision cross sections for a variety of gases. These results provide an extension of the theory of CIV to regimes not treatable by analytic methods.

The chapter begins with simulations of the M2SI. These results are compared to the analytic estimates developed in Chapter 2. Attention then turns to the constant ionization rate model. This model is used to show the efficiency of energy transfer from the ions to electrons. Here again, the results can be compared to the analytic solutions of Chapter 2. Finally, simulations of CIV using physical mass ratios and collision cross sections are presented.
4.1 Modified Two Stream Instability Simulations

Much of the analysis of Chapter 2 focused on the linear and non-linear development of the M2SI, the instability hypothesized to be the basic mechanism of CIV. Accordingly, the initial focus of the simulation results presented here will be a simple case of the M2SI. For these simulations, an ion beam, assumed to be formed by some seed ionization such as charge exchange or photoionization, propagates in the z direction across a stationary plasma background. A magnetic field is applied in the \( z - z \) plane. The small angle between the magnetic field and the \( z \)-direction is denoted by \( \theta \). The magnetic field is assumed nearly perpendicular to the direction of propagation of the ion beam. The beam ions have temperature \( T_b \), density \( n_b \), mass \( m_b \), and velocity \( U \). The background ions and electrons are denoted by subscripts \( i \) and \( e \), respectively. The simulations are one-dimensional and periodic. The conditions in the simulations describe a situation as close as possible to the situation described analytically in Chapter 2.

The methodology of the next sections will be to first present the results of a typical M2SI simulation. The point here will be to show the phase space development of the instability in order to orient the reader. The linear regime will then be analyzed and the results quantitatively compared to the results of the linear dispersion relation found in Chapter 2. After discussing the linear regime, the discussion will turn to the nonlinear regime. Here, a more qualitative comparison will be made with the results of Chapter 2. Finally, a comparison of runs with different mass ratios is made in order to investigate the mass ratio scaling discussed in Chapter 2.

Typical M2SI Simulation

To begin, the results of a typical M2SI simulation are presented. Figures 4-1 and 4-2 show a typical run of the M2SI simulation. The simulation is that described above: an ion beam propagating across a background plasma. The simulation run here takes \( \bar{\theta} = 1, \omega_{pe}/\Omega_e = 1.0, n_b/n_e = n_i/n_e = 0.5, U/v_b = 15.0, T_b/T_e = T_i/T_e = 1.0, m_b/m_i = 1.0 \) and \( m_i/m_e = 29736 \) (Oxygen). Numerical parameters used are \( \Delta x/\lambda_{De} = 1.0 \) and \( \omega_{pe}\Delta t = 20.0 \). The D-2 method is used with the decentered equation of motion for the electrons \( (\varepsilon = 0.1) \), and a Gaussian smoothing operator of width \( \alpha = 1.0 \) is applied.
Both the ambient and beam ions are treated as unmagnetized species. There are 243 grid cells, 6562 electrons, 3821 ambient ions and 3821 beam ions.

The left hand column of Figure 4-1 shows the $z - v_z$ phase space of the electrons, while the right hand column shows $z - v_z$ phase space of the ions. Each of the subfigures shows the phase space at a particular point in time. The topmost panels represent phase space at the initial conditions. The electrons and ions are loaded with Maxwellian distributions. The beam ions also have a positive mean velocity. The next figures depict the phase space position of the particles during the linear phase of wave growth. The nearly sinusoidal nature of the perturbation in the particle velocities is indicative of the wavelength of the most unstable wave mode. The next panel is after the end of the linear growth phase. In this figure, the beam ions are beginning to trap in the potential wells. This trapping of the beam ions is the most prevalent form of saturation for the M2SI. The electrons are also seen to heat substantially. The most highly heated electrons are those with perpendicular velocity components near the wave phase speed, i.e. resonant electrons. Although not shown here, the $z - v_z$ phase space of the electrons shows little heating of the electrons perpendicular to the magnetic field. The final panel represents a time substantially after saturation of the instability. The final state indicates that the mean motion of the beam ions has decreased substantially, both beam ions and ambient ions have heated in the perpendicular direction, and electrons have heated in the parallel direction.

The heating of the different species is perhaps shown more clearly in Figure 4-2. This figure displays the energy of the mean motion and thermal energy of the different species and the electrostatic field energy as a function of time. Initially, nearly all the system's energy is represented as drift energy of the beam ions. As the instability begins to operate, the drift energy is transferred to thermal energy of the species. In this case, the energy is roughly equally partitioned among the species.

**Linear Regime Comparison**

An analysis of the modal field energy in the system can lead to a determination of the growth rate and spatial wavenumber of the fastest growing wave. For example, Figure 4-
3 depicts the field energy in modes \( m = 3, 4 \) and 5 \( (k = \frac{2\pi m}{L}, \text{ where } L \text{ is the system length}) \) and the total field energy for the simulation described in the previous paragraph. Most of the energy is seen to reside in Mode 4, i.e. Mode 4 is the dominant wavenumber. Mode 4 corresponds to \( k \lambda_{De} = .051 \). Also, by plotting the mode 4 field energy on a semi-log plot against time, the growth rate, \( \gamma \), can easily be measured. For Figure 4-3, \( \gamma = .82 \). The wavenumber and growth rate of the most unstable wave in the linear region can similarly be found for other simulation runs. The results of such an analysis of the linear regime were included in Figures 2-4 through 2-7. As shown in those figures, the growth rates and wavenumbers agree quite well with linear theory, although the growth rates are somewhat lower than predicted from theory. Most of the discrepancy is due to the finite \( k \)-spectrum allowed in gridded PIC methods. Referring back to Figure 2-1(b), the growth rate is seen to have a strong peak as a function of the wavenumber. However, in a gridded PIC simulation, the wavenumber can only attain certain values depending on the resolution of the grid, namely \( k = \frac{2\pi m}{L}, m = 0, 1, 2, \ldots \). Hence, the fastest growing wave predicted by theory may not be resolved on the grid. If the allowed values of \( k \) lie on either side of the peak shown in Figure 2-1(b), the observed growth rate of the strongest wave will be less than predicted for the peak growth rate. This is the discrepancy that leads to slightly lower values of growth rate in the simulations.

**Nonlinear Regime Comparison**

The nonlinear development of the M2SI was also described in Chapter 2. The simulations provide a means of confirming the predicted nonlinear behavior of the M2SI. Figure 4-4 shows the development of the electron, ambient ion, and beam ion distribution functions as determined from the simulation. This figure is similar to Figure 2-9 discussed in relation to the nonlinear theory of Chapter 2. The first panel of the figure indicates the initial conditions. The ions and electrons all have Maxwellian distributions, with the beam propagating across the background plasma. As in Figure 2-9 the figure here shows the \( z \)-direction projection of the electron velocities. Not shown on this figure is the unstable wave spectrum which is predicted to be resonant initially only with the beam ions. The second panel of the figure shows the distribution functions in the linear growth
phase of the instability. At this time, the beam ions have begun to form a plateau in the resonant region of velocity space. This is seen as the growth of the plateau on the left hand side of the beam ion distribution. The nonresonant ambient ions and electrons show only minor bulk heating at this time. The beam ion plateau and unstable wave spectrum have just begun to reach the ambient ion and electron distribution functions in the third panel. The beam ions show a substantial plateau at this time, while the ambient ions and electrons have still only undergone some bulk heating. The situation changes rapidly as the ambient ions and electrons become resonant with the growing waves. This is shown in the fourth panel of the figure. In this panel, the beam ions now fill in most of phase space between zero and the initial mean velocity. In addition, the ambient ions and electrons have also grown significant tails at large velocities. The final panel in the figure shows the final saturated condition of the instability. The beam ions now have a nearly uniform density in velocity space between 0 and 1, while the ambient ions and electrons show both bulk heating and high energy tails.

As a comparison of Figures 2-9 and 4-4 will show, the results of the simulation are in good agreement with the predictions of nonlinear theory for the development of the M2SI.

Mass Ratio Scaling

As a final point in regards to the M2SI, this section considers the mass ratio scaling of the linear and nonlinear dispersion relations. In addition to providing insight into the nature of the M2SI, this comparison also provides a way of verify the proper operation of the implicit PIC code.

In the theory of the M2SI, it was suggested that for $\theta \ll 1$, the angle only appeared in the combination $\tilde{\theta} = \theta \sqrt{m_i/m_e}$. Because of this scaling, simulations performed using different mass ratios but equivalent $\tilde{\theta}$ should lead to nearly identical results when time is scaled by the lower hybrid frequency.

Figures 4-5 and 4-6 show simulation results for different mass ratios with the time scaled to the lower hybrid frequency. The simulations were performed by varying the mass ratio and angle in order to maintain the same $\tilde{\theta}$ in all the runs. The other ratios
appearing in Equation (2.43) were also held constant. Values chosen in these simulations were \( \omega_{pe}/\Omega_e = 1.0 \), \( T_e/T_i = T_e/T_b = 1.0 \), \( n_i/n_e = 0 \), \( n_b/n_e = 1.0 \), \( \tilde{\theta} = 1.0 \), and \( U/v_b = 15.0 \). The solid line in the figure is the solution for the maximum growth rate obtained from the linear dispersion relation. The different symbols represent the results from the different simulation runs. In order to present a verification of the current code the results shown include explicit runs of the current code at the lower mass ratios as well as one case using the well known ES1 code.

The agreement between the various simulation results and the analytic result for the growth rate is quite good. The results between the different simulation runs is also excellent, especially considering that in addition to different mass ratios, the results essentially represent three different codes. The amount of energy transferred from the ion beam to the electrons is seen to be approximately 0.5, in agreement with the argument given by McBride.

**Summary of M2SI Simulations**

In this section, the results of simulations of the M2SI have been presented and compared to analytic predictions. Good agreement has been achieved between the simulation and analytic results in both the linear and nonlinear regime of the instability. More specifically, the following points, determined from theory, have been confirmed by the simulations:

- dispersion relation Equation (2.43) correctly predicts the frequency and wavelengths of the most unstable wave,

- the basic nonlinear mechanism is given correctly by nonlinear theory, and

- the mass ratio scaling \( \tilde{\theta} = \sqrt{n_i/n_e} \theta \) removes the mass ratio from the problem allowing a single M2SI simulation to predict results accurately for different mass ratios.
4.2 Constant Ionization Rate Simulations

The next series of simulations will be used to confirm the points discussed in relation to the 'constant ionization model' described in Chapter 2. In these simulations, instead of having a 'pre-formed' ion beam as in the M2SI simulations, new ion-electron pairs are injected evenly in space at each time step in order to satisfy Equation (2.85). In order to most closely match the theory presented in Chapter 2, in these simulations, the injected ions and electrons are cold, as are the original background particles. Other parameters are $m_i/m_e = 6400$, $m_e/m_i = 1.0$, and $\bar{s} = 1.0$. The two main objectives of these simulations are to investigate the efficiency of energy transfer from ions to electrons and to explore the effect of using unphysical mass ratios. The last objective, exploring the effect of using unphysical mass ratios, is clearly related to the numerics, as opposed to the physics, of studying CIV. It is an important point, however, considering that all previous simulations of CIV have been performed with unphysical mass ratios.

Energy Transfer Efficiency

The first point to be discussed involves the efficiency of energy transfer between the beam ions and electrons. This is done by examining plots of electron temperature vs. time from the simulations. The energy transfer efficiency, $\eta$, may be determined for each simulation run from the saturation electron temperature and Equation (2.87). As discussed in Chapter 2, the value of $\eta$ should depend strongly on the rate of ion creation. For the case of unmagnetized ions ($\nu_{\text{ion}}/\Omega_i > 1$), quasilinear theory predicts $\eta = 2/3$, while for highly magnetized ions ($\nu_{\text{ion}}/\Omega_i \ll 1$), the theory predicts $\eta = .025$. The simulations will also provide the advantage over theory of estimating the energy transfer efficiency in the intermediate regime ($\nu_{\text{ion}}/\Omega_i \approx 1$).

Figure 4-7 shows the value of $\eta$ as a function of the plasma creation rate, $\nu_{\text{ion}}/\Omega_i$, as determined by a series of simulations. In the highly magnetized region ($\nu_{\text{ion}}/\Omega_i \ll 1$), $\eta$ approaches .025 as suggested by theory. It should be noted, however, that this figure may only be accurate to approximately 50% due to the limitations of numerical accuracy. For cases where only .025% of the energy is transferred from ions to electrons, the transferred energy represents only 1-2% of the total energy of the system. It is unlikely that either
implicit or explicit codes can predict the saturation energies to within 1-2% of the total system energy. Hence, errors in $T_{e,sat}$ as large as 50% should be kept in mind when considering the values of $\eta$ given for highly magnetized ions.

In the unmagnetized limit, the energy transfer is substantially lower than given by the analysis. Even using unmagnetized ions, $\eta$ only approaches between .3 and .4, depending on the value of $\nu_{ion}/\omega_{LH}$. To explain the discrepancy, Figure 4-8 compares the distribution function at the end of a simulation using unmagnetized ions to the distribution predicted by considering quasilinear relaxation. Two major discrepancies between the simulation and analytic results are noticeable. First, the simulation results contain a large peak at the ion beam velocity near $v/U = 1$ where new ions are being created. The analytic results where derived only in the limit of low collisionality, i.e. in the limit where the diffusion time from the peak is small compared to the ion creation time. This assumption effectively means ignoring the peak. The results shown in Figure 4-8, however, were obtained with a creation rate $\nu_{ion}/\omega_{LH} = .004$. This creation rate seems reasonably low if the other constraint in the analytic theory, i.e. $\nu_{ion}/\Omega_i > 1$ is considered. Although the results shown were produced using numerically unmagnetized ions, they would correspond to $\nu_{ion}/\Omega_i \approx 25$. Creation rates much lower than $\nu_{ion}/\omega_{LH} = .004$ would violate the $\nu_{ion}/\Omega_i > 1$ restriction. Hence, the present results indicate it may be difficult to achieve a regime where both unmagnetized ions and low collisionality are both appropriate assumptions.

The second discrepancy between analytical and numerical results is seen near $v/U = 0$ and $v/U = 1$. Near these velocities, the simulated distribution shows a somewhat broader spread in velocity space than the analytic result. This spread seems to arise from the non-linear terms not present in the quasilinear theory. Together, the two discrepancies lead to lower values of $\eta$ than the 2/3 suggested by the theory.

**Physical Mass Ratio Effects in the Constant Ionisation Rate Model**

As shown previously, the M2SI could be simulated accurately while using reduced mass ratios if the ions were assumed unmagnetized. As mentioned in Chapter 2, however, when ionization is introduced, the choice of unmagnetized ions may no longer be appropriate.
In certain regimes, this may reduce the effectiveness of using artificial mass ratios in obtaining accurate simulation results. There are three time scales of importance in this model of CIIV. The first time scale, the lower hybrid frequency, $\omega_{ LH}$, is the fundamental time scale of the M2SI. The second time scale is the ion cyclotron frequency, $\Omega_i$. $\Omega_i$ is important in determining the efficiency of energy transfer from the beam ions to the electrons. The final time scale is the collision rates for collision type $j$, $\nu_j$. In general, in order to correctly model the CIIV process, the ratios $\nu_j/\omega_{ LH}$ and $\nu_j/\Omega_i$ must be correct in the simulation. Unfortunately, both these ratios cannot be correctly incorporated using unphysical mass ratios since $\omega_{ LH} \sim \sqrt{m_i}$ but $\Omega_i \sim m_i$. This inapplicability of unphysical mass ratios holds when both $\omega_{ LH}$ and $\Omega_i$ are important in the problem. In two limits, however, $\Omega_i$ is not important. The first limit is when the ionization rate is higher than the ion cyclotron frequency ($\nu_{ion}/\Omega_i > 1$). In this limit, the ions may be assumed to be unmagnetized, and $\Omega_i$ is not relevant to the problem. The second limit is when the ionization rate is much lower than the ion cyclotron frequency ($\nu_{ion}/\Omega_i \ll 1$). In this limit, the ions form a ring distribution in velocity space. Although the formation of the ring distribution is important to the problem, the precise value of $\Omega_i$ is unimportant. Again in this limit, then, $\Omega_i$ drops out of the problem. The result of these arguments is that unphysical mass ratios are applicable when $\nu_{ion}/\Omega_i > 1$ and when $\nu_{ion}/\Omega_i \ll 1$. In the intermediate regime, however, unphysical mass ratios will not provide correct results.

In order to reinforce the remarks made in the previous paragraph, results produced with $m_i/m_e = 400$ and $m_i/m_e = 6400$ are compared in Figures 4-9 to 4-12. Figure 4-9 compares the runs at different mass ratio for unmagnetized ions. The parameters $\bar{\theta}$ and $\nu_{ion}/\omega_{ LH}$ are the same (as are the other ratios in the problem) for the two runs. As suggested by the analytic remarks presented above, in this example, only the $\omega_{ LH}$ time scale should be present, and the results should be nearly identical when $\bar{\theta}$ is chosen correctly. The top panel of the Figure shows the field energy as a function of time, while the bottom panel shows the time history of the electron kinetic energy (both the field energy and electron kinetic energy are normalized by the beam energy, $\frac{1}{2} m_i U^2$). With the time scaled to $\omega_{ LH}$, the results of the runs agree quite well.

Figure 4-10 shows data for ions in the ‘intermediate’ regime ($\left( \frac{\nu_{ion}}{\Omega_i} \right)_{6400} = 1/2$).
As in the unmagnetized example, \( \bar{\theta} \) and \( \nu_{\text{ion}}/\omega_{\text{LH}} \) are chosen to be the same for the two mass ratios (which implies \( \nu_{\text{w,1}}/\Omega_i \approx 400 = (\nu_{\text{ion}}/\Omega_i)_{400}/64 = 1/128 \)). Now, although the time history of the field energy is seen to compare well for the two mass ratios, a substantial difference appears in the electron saturation temperature. The discrepancy in \( T_{e,\text{sat}} \) is a result of the importance of the \( \Omega_i \) time scale to the problem. In the high mass ratio case, the ions only complete a portion of an ion cyclotron period on the \( \nu_{\text{ion}} \) time scale, while the low mass ratio ions have already formed a ring distribution. The ions in the low mass ratio case may be thought of as 'more magnetized' than the heavier ions. 'More magnetized' ions leads to a lower value of \( \eta \) and a lower electron saturation temperature. This difference is clearly seen in Figure 4-10, where the electrons in the low mass ratio case saturate at a temperature substantially below the heavy ion case.

Since scaling \( \nu_{\text{ion}} \) by \( \omega_{\text{LH}} \) does not eliminate the mass ratio in the 'intermediate' regime, another possibility might be to keep \( \bar{\theta} \) and \( \nu_{\text{ion}}/\Omega_i \) constant. For these simulations, \( (\nu_{\text{ion}}/\Omega_i)_{400} = (\nu_{\text{ion}}/\Omega_i)_{400} = 1/2 \), but now \( (\nu_{\text{ion}}/\omega_{\text{LH}})_{400} = 1/160 \), while \( (\nu_{\text{ion}}/\omega_{\text{LH}})_{400} = 1/20 \). The results of this are shown in Figure 4-11. Here, although the ionization is correct on the ion gyroperiod time scale, on the wave (i.e. \( \omega_{\text{LH}} \)) time scale, the ion production rate is incorrect. The evidence is seen in the plot of field energy vs. time. Ultimately, the distribution function, and hence the solution to \( \omega \) in the dispersion relation, has been changed between the two cases. The field energy in the low mass ratio case grows on a different time scale than the higher mass ratio case. This is also seen in the growth of the electron kinetic energy. Although the two cases saturate near the same values (which might be expected since \( \nu_{\text{ion}}/\Omega_i \) is the same for the cases), the temporal development is different indicating the wave spectrum has changed in the two cases.

Finally, in Figure 4-12, a comparison is shown in a case with highly magnetized ions \( ((\nu_{\text{ion}}/\Omega_i)_{400} = 0.1) \). Here again, the analysis of the quasilinear equations suggests that the results should be independent of the mass ratio if \( \bar{\theta} \) is selected carefully and \( \nu_{\text{ion}}/\omega_{\text{LH}} \) is kept constant. As seen in the figure, with time and \( \nu_{\text{ion}} \) scaled by \( \omega_{\text{LH}} \), the field energy saturates at the same value for the different mass ratios. The electron saturation temperature also agrees well for the different mass ratios.

As the above examples should show, the use of unphysical mass ratios in CIV sim-
ulations do not always lead to accurate results. For the simple constant ionization rate model, unphysical mass ratios can be scaled appropriately in the unmagnetized and highly magnetized regimes. In more realistic simulations, however, two problems arise in regards to scaling the unphysical mass ratio results. First, in more realistic simulations, the value of $\nu_{\text{ion}}/\Omega_i$ is not known a priori but develops self consistently through the interplay of wave heating and collisional events. An incorrect assumption as to whether the ions are unmagnetized or highly magnetized may lead to misleading results. Secondly, the mass ratio scaling has only been shown to be correct in the unmagnetized and highly magnetized regimes for a single reaction path. In CIV simulations involving multiple reactions and complex wave interactions, it is not clear that the same scaling laws hold. Overall, it is concluded here that, although unphysical mass ratios may be a useful artifice for the purpose of studying the basic physical properties of CIV, more realistic estimates of the anomalous ionization rates to be expected from CIV require simulations incorporating realistic mass ratios and cross sections. The next section turns to investigating some of these simulations.

Summary of Constant Ionisation Rate Model

In this section, the results of simulations have been compared with the theoretical predictions involving the ‘constant ionization rate’ model of CIV. The salient points are as follows:

- the energy transfer efficiency, $\eta$, agrees well with theoretical predictions for highly magnetized ions,

- for unmagnetized ions, simulations suggest the value of $\eta$ may be substantially less than the value of $2/3$ predicts by theory. The main discrepancies come from the assumption of a small quasilinear diffusion rate which may be difficult to achieve, and from nonlinear effects not considered by the quasilinear theory,

- artificial mass ratios may not result in accurate simulations for plasma creation rates near the ion cyclotron frequency.
4.3 CIV Simulations of the IBSS Gases

Up to this point, the simulations have been used to confirm the theory as discussed in Chapter 2 and to point out deficiencies in the use of unphysical mass ratios in CIV simulations. The final sections of this chapter turn to performing full simulations of the CIV process. In this section, situations similar to the IBSS CIV experiments are considered while the next section discusses a simulation more applicable to the laboratory experiments.

In this section, the gases simulated are neon, xenon, carbon dioxide, and nitric oxide (with an ambient oxygen plasma). The gases Ne, Xe, NO and CO$_2$ represent the gases released during the recent IBSS CIV experiments on board the space shuttle (STS-39, April 1991). For this experiment, CO$_2$ and NO were chosen because of their potential for polymerization during expansion through the release nozzle. Murad et al. [55] have suggested that the polymers make good candidates for CIV since the increased effective molecular mass decreases the critical velocity. Xe has a low critical velocity and is a good candidate for a gas likely to undergo CIV during space releases. Ne was a control gas that was not expected to ionize. The simulations include electron elastic collisions, non-resonant and resonant charge exchange and electron impact ionization.

The methodology adopted here is to first present a typical simulation of CIV in these gases and to discuss some definitions of characteristic time scales that will be useful in describing the CIV process. After discussing a typical run, each portion of the CIV process will be described in turn. The emphasis here will be in developing some semi-empirical estimates of the time scales involved in CIV. These estimates will be used in the subsequent chapter to compare the simulation results with experimental measurements.

4.3.1 Typical Simulation of CIV

Figure 4-13 shows typical results of the simulation. The top panel presents the time history of the electron number and the bottom panel shows the electron temperature as a function of time. As the simulation begins, an ion beam begins to form through charge exchange reactions between the beam and ambient ions. This beam quickly transfers energy to the electrons through the M2SI. The energy transferred heats the
electrons. The process continues with the beam being formed and re-energized through charge exchange. The electrons continue to heat until they reach energies sufficient for ionization. At this point, the beam is quickly reinforced through ground ionization while electrons are heated by the M2SI but lose energy to ionization. The electron energy saturates at a point where the energy transferred from the waves is equal to the energy lost to ionization collisions.

Several definitions will be useful in describing the simulations in the following section. The first important time scale is the time to first ionization $\tau_1$. $\tau_1$ is simply the point at which the first ionization event takes place in the simulation. Ultimately, it represents an electron heating time since at least some electrons must heat to the ionization energy of the neutrals before ionization begins. Another useful time scale is the "ignition time", $\tau_{ign}$. The ignition time is defined as the time to reach the "break" point of the exponential growth phase of the electron density. This time represents a measure of the time necessary for the CIV process to reach an exponential growth state. In addition to the two time scales, the 'anomalous ionization rate' is also an important parameter. After the temperature saturates, the growth in the electron number becomes exponential and leads to a definition of the 'anomalous ionization rate', $\nu_{ion}$:

$$\frac{N_e}{N_{e,0}} = \exp(\nu_{ion}t)$$  \hspace{1cm} (4.1)

The following sections will be concerned with describing the simulations during these processes and developing estimates and scaling laws for the different time scales and anomalous ionization rate. For purposes of clarity, the NO simulations will be discussed most thoroughly, although the remarks apply equally well to simulations of the other gases. The CIV process is divided into four regimes: (1) an initial stage involving the development of the lower hybrid waves, (2) a stage comprising the time up to $\tau_1$, (3) a transient regime between $\tau_1$ and $\tau_{ign}$, and (4) an asymptotic state regime in which Equation (4.1) is satisfied. Each of these regimes is discussed in turn in the following sections.
Initial Stage of CIV

The initial conditions for the current simulation are a quiet, Maxwellian oxygen plasma in a magnetic field. The temperature of the plasma is .2eV. A neutral beam with a density $n_n/n_e = 10^6$ is propagating across the simulation region.

As the simulation starts, charge exchange reactions between the NO neutrals and $O^+$ ions initiate an ion beam. At this stage, the beam is being created at a linear rate of

$$\frac{dn_b}{dt} = \nu_{nrces} n_{O^+} \quad (4.2)$$

where $\nu_{nrces}$ is the non-resonant charge exchange rate given by

$$\nu_{ces} = <\sigma_{nrces} v_r > n_{neut} \quad (4.3)$$

and $\sigma_{nrces}$ is the cross section, $v_r$ is the neutral-$O^+$ relative velocity and $n_{neut}$ is the neutral density.

As the beam begins to build, it also begins to transfer energy to the electrons. A conceptual picture of the initial stages can be constructed by considering the remarks made in the previous sections. As the first few ions enter the beam, some amount of time will be required to establish the lower hybrid wave spectrum that eventually acts to heat electrons. During this time, the energy transfer rate from ions to electrons should be quite low since there is no coherent wave structure to act as the energy transfer mechanism. The linear theory of the M2SI has shown that the characteristic time scale for the waves in the M2SI is the lower hybrid period. Hence, the time required to establish the wave spectrum can be expected to be on the order of the lower hybrid period.

After the initial establishment of the wave spectrum, the ions begin to transfer energy to the electrons. At least during the very initial stages, this energy transfer process can also be expected to be linear in nature. The total electron kinetic energy can be found by assuming that the fraction of energy transferred from the newly created ions to the electrons is a constant, and that the time to transfer energy from the beam to electrons is small compared to the time scale of interest (e.g. $\omega_{LH} \ll \Omega_i$). Since the number of newly born beam ions is simply given by $n_i \nu_{nrcest} t$, the electron kinetic energy is given
\[ n_e(T_e - T_{e0}) = \nu_{\text{vcess}} n_i(t - t_0) \eta \frac{1}{2} m_n V_n^2 \]  

where \( \eta \) is the fraction of energy transferred from ions to electrons, and \( t_0 = O(\omega_{\text{LH}}) \). \( \eta \) can be predicted by again considering the constant ionization rate model discussed previously. If the time of interest is restricted to times at which the beam is still weak (i.e. only a small fraction of the background ions have undergone a charge exchange collision), the beam ion distribution function must still be in an ‘unmagnetized’ regime. In this case, the efficiency should be expected to be quite high. In fact, a review of the constant ionization model simulation results would lead to expectations that more than 40% of the energy may be transferred from ions to electrons.

To verify these predictions, Figure 4-14 shows the results for the initial stages of the NO simulation. The top panel of the figure shows the beam ion density, while the bottom panel shows the kinetic energy as a function of time. As seen in the top panel, the ion beam density increases linearly in agreement with the remarks made above. The solid line on the figure is the value predicted by the charge exchange rate using \( v_r = V_n \). The actual number of beam ions is somewhat above the value predicted by the charge exchange rate. The discrepancy is most likely due to statistical deviations since the number of particles created is still low (the absolute number of particle created in the simulations for this region is below 100).

The electron kinetic energy starts with a region of no heating. This constant electron temperature lasts on the order of 5000\( \omega_{\text{pe}}^{-1} \), which is consistent with the local lower hybrid frequency of the beam (\( \omega_{\text{LH}} \approx 2000 \omega_{\text{pe}} \) assuming \( n_b/n_e = .01 \)). After about 5000\( \omega_{\text{pe}}^{-1} \), the electron temperature begins to increase roughly linearly (there is substantial noise in the system since the number of particles in the beam is still low). The rate of increase of the electron temperature is consistent with an efficiency of \( \eta = .54 \), in agreement with the argument that on the current time scale the ion beam remains unmagnetized. Further evidence of the beam ion distribution function is shown in Figure 4-15. This figure shows the ion velocities perpendicular to the magnetic field at a time \( \Omega_i t = 1 \). The large central concentration represents the Maxwellian background oxygen ions. The more diffuse population of ions with positive velocities is the ion beam. At \( \Omega_i t = 1 \),
the ions have only completed about 1/6 of a cyclotron orbit. Note that although the ions are introduced in velocity space at \( v_b/v_e = 0.0613 \) with a thermal spread of only \( v_{thi}/v_e = 0.004 \), they have filled in a substantial portion of velocity space between 0.0613 and 0. This is a result of interaction with the waves and is equivalent to the plateau formation as described by quasilinear theory. The energy lost by the ions has been transferred mainly to the electrons.

The velocity dependence of the initial rise in the electron temperature is shown in Figure 4-16. This figure was produced by measuring the slope of the temperature in the linear regime found in the simulations and dividing by the non-resonant charge exchange rate, \( \nu_{ncee} \). In agreement with Equation (4.4), this should give \( \eta \frac{3}{2} m_e V_n^2 \). The points in the figure represent measurements from the simulations while the solid line is \( \eta \frac{3}{2} m_e V_n^2 \) with \( \eta = 0.52 \). The good agreement indicates that \( \eta \) is nearly a constant during this initial phase of CIV.

Ignition of CIV

As the previous section discussed, charge exchange reactions initiate an ion beam which begins to transfer energy to the electrons. Once electrons are heated to above the ionization energy of the neutrals, ionization by electron impact will begin. The purpose of this section is to develop an estimate of the time needed to begin the initial ionization.

In the previous section, the rate of increase of the electron kinetic energy was determined. However, the determination of the start of ionization requires information about the heating of the tail electrons, not just the bulk. Figure 4-17 shows the electron energy distribution function near the time of first ionization. The interest here is in the “tip” of the distribution function since these will be the first electrons to ionize. The needed parameter is the time required to heat the electrons in the tip of the distribution function to the ionization energy of the neutrals.

The time to heat the electrons to the ionization energy may be estimated by assuming that the “tip” of the distribution function will reach the ionization energy when the bulk
electron temperature is equal to some fraction of the ionization energy,

\[ T_{e\phi} = f(e\phi_{ion}) \]  

(4.5)

independent of the beam velocity. Along with Equation (4.4), this observation gives the time required for the tip to reach \( e\phi_{ion} \). After the tip of the distribution function reaches the ionization energy of the neutrals, ionization will not be observed for some period after this related to the ionization frequency, i.e. \( \tau_{ion} = \nu_{ion}^{-1} \). Since the electrons will continue to heat during this short time of \( \tau_{ion} \), at the start of ionization, the electron bulk temperature will be given by

\[ T_{e,ion} = T_{e\phi} + \tau_{ion} \nu_{ces} \eta \frac{1}{2} m_e \nu_e^2. \]  

(4.6)

This result can be compared directly with the simulation results, with one caution. The “start” of ionization is a statistical process in the PIC code. Therefore, the first ionization has some statistical fluctuation in its measured value. In order to account for this, a range of temperature will be given for the temperature of initial ionization as determined from the simulations. The range is taken to be the temperature range covered between the first and third ionization events occurring in the simulation run. These results from the NO simulation are shown in Figure 4-18. The vertical bars represent the times determined from the simulation corresponding to the first and third ionization events. The points are the results of Equation (4.6) assuming \( T_{e\phi}/(e\phi_{ion}) = .09 \) and \( \tau_{ion} = 9000\omega_{pe}^{-1} \). The value of \( \tau_{ion} \) is consistent with the ionization time \( \tau_1 \) of an electron with energy near the ionization energy. There is fairly good agreement between the simulation results and Equation (4.6) indicating that the assumption made in Equation (4.5) is reasonable.

A more useful quantity than the temperature is the time it takes to reach initial ionization, \( \tau_1 \). This can be found by again assuming the efficiency remains constant and the temperature is increasing linearly (combining Equation 4.4 and 4.6 and taking
\[ n_e \approx n_i \]

\[ \tau_1 = \tau_s + \frac{T_{e\phi} - T_0}{\nu_{n(e\phi)} \eta \frac{3}{2} m_n V_n^2} \]

(4.7)

where \( \tau_s \) is a constant term on the order of the lower hybrid period.

The efficacy of Equation (4.7) is indicated in Figure 4-19 which compares the results from the equation with the results obtained from the simulations for all four gases as a function of the neutral velocity. The parameters \( \tau_s \) and \( T_{e\phi} \) are seen to be fairly constant for the different gases indicating the usefulness of the approximation for different situations. The parameters chosen for each of the gases are included on the figures. In each case, the efficiency is about 50%, showing that a beam distribution is maintained in each case. The constant \( \Omega_i \tau_s \) can also be seen to vary between approximately .06 and .2. An interesting parameter is \( T^* = \frac{T_{e\phi}}{e\phi_{ion}} \). This parameter represents the electron kinetic energy as a percentage of the ionization energy of the neutrals. As can be seen in the figures, the bulk electron kinetic energy of the electrons needs to reach about 10\% of the neutral ionization energy in order to start ionization. For Xe, NO, and Ne, \( \tau_1 \) is on the order of the \( \Omega_i^{-1} \). The situation is somewhat different for CO\(_2\) where \( \tau_1 \approx .1\Omega_i^{-1} \). The smaller value of \( \tau_1 \) for CO\(_2\) is a result of the large CO\(_2\)-O\(^+\) charge exchange cross section. This cross section is roughly an order of magnitude larger than the corresponding neutral-oxygen charge exchange cross sections for the other neutrals.

Figure 4-20 shows similar results as a function of the neutral density for NO and CO\(_2\). The parameters are the same as used in Figure 4-19. For the case of NO, good agreement is seen over four orders of magnitude. The situation is somewhat different for CO\(_2\). The agreement is reasonable for \( n_n/n_e < 10^7 \). Above this value however, the measured values of the ignition time exceed the predicted values. This is the first evidence of the ‘windowing effect’, which will be more in evidence when the anomalous ionization rates are considered below. The ‘windowing effect’ is a result of energy losses to modes other than ionization. This presents a modification of Equations (4.6) and (4.7) and gives a weak density dependence above \( n_n/n_e < 10^7 \). More will be said about this when the anomalous ionization rate is considered below.
Transition Phase

After the first ionization events, the electron kinetic energy will continue to increase towards a saturation value. An estimate of the time needed to enter the exponential phase can be obtained by defining the 'ignition time' as the time needed to reach the break point as illustrated in Figure 4-13. The ignition time is a function of the detailed collisional processes and energy gain/loss mechanisms occurring after ionization begins. In this regime, the linear temperature increase assumption used is the previous section is no longer valid. A specific expression for the ignition time is difficult to obtain, however an estimate can be obtained by considering the ratio of the ignition time to the time for first ionization, \( \tau_{\text{ign}} / \tau_1 \). As shown in Figure 4-21, for most cases the ignition time is roughly 3 to 5 times the time for first ionization. This estimate is roughly true as a function of both velocity and density. The CO\(_2\) values as a function of density do show a strong dependence on the density. This is also related to the windowing effect.

Exponential Growth State

The electrons continue to heat until energy losses to ionization and excitation are equal to energy added from wave transfer. Once the exponential growth state is reached the plasma density increases as

\[
\frac{dn_e}{dt} = \nu_{\text{ion}} n_e
\]  \hspace{1cm} (4.8)

The precise magnitude of the anomalous ionization rate, \( \nu_{\text{ion}} \), is difficult to estimate a priori because it is sensitive to the details of the electron distribution function. However, the scaling of \( \nu_{\text{ion}} \) with the neutral velocity and density can be suggested through simple arguments. First, Equation (4.8) may be rewritten as

\[
\frac{dn_e}{dt} = \alpha_{\text{ion}} \nu'_{\text{ion}} n_e
\]  \hspace{1cm} (4.9)

where \( \alpha_{\text{ion}} \) is the fraction of the electron population with energies above the ionization energy, and \( \nu'_{\text{ion}} \) is just a redefinition of the anomalous ionization rate, \( \nu'_{\text{ion}} = \nu_{\text{ion}} / \alpha_{\text{ion}} \). The rearranging is a simple acknowledgment of the fact that only electrons with energies above the ionization energy of the neutrals are capable of ionizing. The redefined
anomalous ionization rate can be written as

$$\nu'_{\text{ion}} = <\sigma_{\text{ion}} v > n_n$$ (4.10')

where the angle brackets represent an average taken over the portion of the distribution function with $E > e\phi_{\text{ion}}$. Over the energy ranges of interest here, the quantity $<\sigma_{\text{ion}} v >$ is assumed to be nearly constant.

The remaining portion of the anomalous ionization rate, $\alpha_{\text{ion}}$, can be found by considering the idealized saturated electron distribution function as given by Formisano et al. [23] and used subsequently by Lai et al. [42] and others. In the idealized case, the hot tail of the electron distribution function simply has a constant density from below the ionization energy to the beam energy (see Figure 4-22). For this distribution function, the fraction of electrons with energies above the ionization energy is simply proportional to the beam energy. Combining this with Equation (4.10) gives the scaling law for the anomalous ionization rate,

$$\nu_{\text{ion}} \sim n_n (V_n - v_c)^2$$ (4.11)

where $v_c$ is the critical velocity. Although derived through very simple arguments, the simulations as described below confirm the efficacy of this result.

The anomalous ionization of the gases is first considered as a function of the neutral beam velocity. For these runs the neutral density is chosen such that $N_n/N_{e,0} = 10^6$. Figure 4-23 shows $\nu_{\text{ion}}/\Omega_i$ after electron temperature saturation for the gases as a function of the neutral velocity $(V_n/V_e)$. The markers shown in the figure represent data points from the simulation results. The solid lines are quadratic fits to the data points. As seen in the figure, for $V_n/V_e < 1$, $\nu_{\text{ion}}/\Omega_i \approx 0$. This is expected since below the critical velocity no anomalous ionization should be observed. Above the critical velocity, however, the anomalous ionization rate quickly increases. The excellent agreement between the quadratic fits and the data indicates the ionization rate's dependence on the energy of the neutral beam.

Also of interest is the effect of the neutral density on the ionization rate. Figure 4-24 shows $\nu_{\text{ion}}/\Omega_i$ as a function of the neutral density, $N_n/N_e$, for CO$_2$ and NO. In this
figure, the neutral beam velocity was held constant at $V_n/V_e = 1.5$ while the neutral beam density was varied.

For NO, the ionization rate is seen to scale linearly with density up to a density of $N_{neut}/N_{e0} \approx 10^7$. That is, for $N_{neut}/N_{e0} < 10^7$

$$\frac{\nu_{ion}}{\Omega_i} \sim \left( \frac{N_n}{N_{e,0}} \right)$$

(4.12)

Above $N_{neut}/N_{e0} \approx 10^7$, the ionization rate begins to level off. At these high densities, the growth in the ionization rate decreases because elastic collisions destroy the hot tail of the electron distribution function. This corresponds to a transition between the 'collective' and 'resistive' forms of CIV discussed by Machida and Goertz [47].

A more dramatic example of the effect of the neutral density is shown in the CO$_2$ runs. In the panel showing CO$_2$, two sets of data are presented corresponding to runs with and without the vibrational modes included (no vibrational modes were included for the NO runs). For low neutral densities, the results with and without the vibrational modes are nearly identical. As in the NO runs, at the low neutral densities, the ionization rate scales linearly with the neutral density. However, as the collision time becomes near the lower hybrid period ($\tau_h \sim \omega_{lh}^{-1}$), the ionization rate begins to decrease. In the simulations without the vibrational modes, the growth in the ionization rate decreases because elastic collisions destroy the hot tail of the electron distribution function. For the cases including the vibrational modes, the ionization rate quickly decreases as the electron energy lost to the vibrational modes becomes significant. Near $N_n/N_{e,0} = 10^8$, the ionization rate goes to zero, as CIV is quenched by the vibrational modes. This loss of energy to the vibrational modes is the 'windowing' effect seen previously in the simulations of McNeil et al. [52]. In CIV, an electron has some characteristic heating time due to energy transfer from the waves, $\tau_h$. It also has some mean collision time $\tau_c$. As the electron heats, it must pass through a region where it is energetically possible to excite neutrals but not ionize them. For low neutral densities (Figure 4-25a), $\tau_h \ll \tau_c$. In this case, the electron is likely to heat to ionization energies without losing energy to the neutral's excitation modes. For high neutral densities (Figure 4-25b), $\tau_h \gg \tau_c$. In
this case, the particle is likely to collide and de-energize before it is heated sufficiently to ionize. This can slow or even quench CIV. For carbon dioxide, the vibrational modes act to de-energize the electrons. In this case, only small amounts of energy are lost in a collision. However, the electron is capable of vibrationally exciting a molecule over most of its energy range. A similar windowing effect can be found in other gases. The windowing effect operates when either vibrational modes or electronic states can be excited by the energetic electrons.

It is interesting to note that Person et al. [61] using an explicit simulation with unphysical mass ratios arrived at the conclusion that the windowing effect becomes significant at a neutral density of approximately $10^{12} \text{ cm}^{-3}$. Since the current simulations assumed $N_{e0} = 10^6 \text{ cm}^{-3}$, and the windowing effect becomes significant at $N_n/N_{e0} > 10^7$, the criterion arrived at here is the same as used by Person et al.

The 'windowing' effect has received some attention lately as a possible explanation for the lack of CIV observed in space experiments. For the case shown here, CIV is not seriously inhibited by the 'windowing' effect until $N_n/N_{e0} > 10^7$. Such high densities only occur in space experiments quite near the release nozzle. The implications of this for CIV in the space experiments will be described in the next chapter.

**Summary of IBSS Simulations**

This section has discussed in detail the simulations performed using Ne, Xe, NO, and CO$_2$. The results have indicated several important scaling laws involving the time scales of CIV:

- A weak ion beam is formed through charge exchange collisions. This leads to the growth of lower hybrid waves. During this time, little heating of the electrons occurs. This regime lasts on the order of the lower hybrid frequency ($t_0 \sim \omega_{LH}^{-1}$)

- After the lower hybrid wave spectrum has been established, the electrons begin to heat as energy is transferred from the beam ions to the electrons via the waves. During this time, the growth in the electron temperature is seen to be roughly linear. The regime lasts until the beginning of ionization of the neutral beam. This time is denoted by $\tau_1$ and can be estimated from Equation (4.7).
• After the beginning of ionization, the electron temperature continues to increase until the energy lost to ionization becomes on the same order as the energy transferred to the electrons from the waves. The time to reach the “break point” (roughly the point at which the electron temperature saturates) is denoted by $\tau_{\text{ion}}$ and is found to be between approximately $3\tau_l$ and $5\tau_l$.

• After the electron temperature saturates, the plasma density increases exponentially. The scaling of the “anomalous ionization rate” with neutral velocity and density is found to agree with Equation (4.11).

• At high densities, the “windowing effect” can seriously slow the anomalous ionization.

4.4 CIV Simulations of the H⁺-He System

The previous section dealt with the gases released during the IBSS experiments. The scaling results are also applicable to other ionospheric gas releases. An underlying assumption of the previous results was that the ion beam is initiated through charge exchange reactions with the background plasma. In this section, a somewhat different case is considered. This is the case similar to the laboratory experiments of Danielsson. In the experiments of Danielsson, a hydrogen plasma is created and accelerated down a drift tube where it impacts a neutral helium cloud. As it impacts the helium cloud, seed ionization begins not by charge exchange but more directly by electron impact ionization by electrons in the tail of the initial distribution function. The time scales in this situation will be found to be substantially shorter than found in the previous section.

The simulations in the H⁺-He system are similar to those performed for the IBSS releases. The choices made to perform these simulations need to be carefully stated since they do not correspond precisely to the experimental conditions. The simulations begin with a H⁺ plasma embedded in a magnetic field with a He neutral cloud moving in the positive $z$-direction with a velocity $V_n$. The magnitude of the neutral velocity is chosen so that the neutral energy is 1 keV, a typical value used in the experiments. Resonant and non-resonant charge exchange, electron elastic, and electron impact ionization collisions
are included in the system. Other parameters chosen to be consistent with the laboratory values are \( n_m/n_0 = 200 \), \( \Omega_e/\omega_{pe} = 0.333 \), \( T_e/T_i = T_e/T_b = 1.0 \), and \( \bar{\theta} = 1.0 \). Since in the laboratory experiments the electron temperature was only measured at the plasma gun and not at the start of the interaction region, some uncertainty exists in the initial electron temperature. Because of this, the simulations will be performed for several different initial electron temperatures.

The most important point in this choice of simulation parameters is that the reference frame of the laboratory experiments has not been incorporated into the simulations. In the laboratory experiments, the neutrals were stationary in a magnetic field, with the plasma drifting as it impacted the neutrals. In the simulations, the plasma is stationary with the neutrals drifting. This is adopted since in a one-dimensional simulation it is not possible to achieve the polarization of a plasma to create a drift across the magnetic field. In addition, the application of an external E-field to artificially represent the polarization led to unphysical results as the newly created plasma picked up too much energy in the applied E-field. Finally, two-dimensional simulations required the resolution of a sharp charge layer to correctly model the polarization. The implicit methods used in order to implement realistic mass ratios make it difficult to model this sharp charge layer. The implicit code requires long wavelength processes, an assumption violated in the polarization process. Because of these restrictions, stationary plasma and drifting neutrals are adopted. At least for the initial stages of CIV, when the polarization is not seriously affected, the model should provide reasonable results.

The top panel of Figure 4-26 shows the plasma density increase as a function of time for the hydrogen-helium system. As mentioned above, the results are shown for several different values of initial electron temperature. The time history of the plasma density is seen to be substantially different from the case of the IBSS releases. In the IBSS releases, CIV started through a slow build up of an ion beam through non-resonant charge exchange reactions. During this period of charge exchange reactions, ionization did not occur, and hence the plasma density did not increase. For the current case, some tail electrons are initially hot enough to immediately begin ionization of the neutrals. The ratio of the saturation temperature to the electron temperature is also smaller leading
to more rapid saturation in the current case. The current case begins immediately in the transient stage as described in the previous section. Because of this, an estimate of the time to enter the saturated exponential growth phase is difficult to determine. As seen in the figure, however, for the laboratory experiment parameters, the exponential growth phase is reached within 1 μs for an initial electron temperature of 20 eV and at about 2 μs for $T_{e0} = 7.5$ eV. In addition, substantial ionization of the neutrals occurs before the exponential phase is reached. For example, in the 20 eV case, the plasma density nearly doubles before the saturated exponential growth phase is reached.

The electron temperature as a function of time is shown in the bottom panel of Figure 4-26. As seen in the figure, the electron temperature initially stays nearly constant. In this region, the initial hot electrons are losing energy to ionization while only a small amount of energy is being transferred to the electrons via the waves. In fact, although it may not be clear from the figure, the electron temperature actually initially drops slightly as the electrons lose energy to ionization at a faster rate than is replenished through waves. As the beam and wave structure develops and the initial hot electron population is depleted, the energy addition from waves exceeds energy losses to ionization, and the electrons heat slightly. Only a small amount of heating is necessary to raise tail electrons above the ionization energy of the neutrals. Hence, further electron impact ionization quickly occurs, reinforcing the beam and increasing the energy transfer from the beam. The electrons saturation temperature is about 40% of the beam energy for each of the cases. (The saturation level would ideally be somewhat clearer from the figure, however the particle number was increasing so quickly that the simulations ended before the electron saturation temperature was very well defined). The same saturation temperature is expected in each case since neutral density and velocity are constant between the runs. The high efficiency is a result of the ionization rate ($\nu_{ion}/\Omega_i \approx .5$) meaning the ions are not highly magnetized.

The most important point of these simulations is that the initial electron temperatures used in the laboratory simulations will quickly lead to electron impact ionization of the neutrals. The time to first ionization discussed in the previously section is not relevant to the laboratory experiments. The implication of these statements will be
discussed more fully in the next chapter.

4.5 Summary

This chapter has investigated the CIV mechanism through a series of simulations. The simulations have focused on the M2SI mechanism, the constant ionization rate model, and simulations of CIV in several different gases using realistic mass ratios and cross sections. Simulation results applicable to both space experiments and laboratory experiments were presented. The major results indicate

- the results of linear theory and simulations agree well. The predictions of linear theory presented in Chapter 2 provide an accurate representation of when the M2SI is unstable, and the real and imaginary frequencies that occur when the M2SI operates.

- the nonlinear mechanism proposed in Chapter 2 qualitatively predicts the behavior of the M2SI in the nonlinear regime.

- the mass ratio scaling, $\tilde{\theta} = \theta \sqrt{m_i/m_e}$ is a useful way of simulating the M2SI using artificial mass ratios, but more realistic simulations of CIV using artificial mass ratios may not produce accurate results.

- the constant ionization model of Formisano provides reasonable estimates of the energy transfer efficiency of the M2SI. The simulations indicate however that the highest efficiency to be expected is probably $\approx 50\%$, not the $2/3$ predicts by Formisano.

- for models involving ionization processes, artificial mass ratios provide accurate results in the case of $\nu_{\text{ion}}/\Omega_i > 1$ or $\nu_{\text{ion}}/\Omega_i < 1$. In the intermediate case, $\nu_{\text{ion}}/\Omega_i \approx 1$, realistic mass ratios may be necessary in order to correctly model the process.

- for space experiments, CIV is initiated through charge exchange reactions. Initially, the electron temperature increases nearly linearly as the energy from the beam (which is being created through charge exchange) is transferred to the electrons at a nearly constant efficiency.
• the initially linear increase in electron temperature can be used to estimate the time for first electron impact ionization of neutrals (Equation 4.4).

• after reaching first ionization, CIV goes through a transition phase to exponential growth. A rule of thumb is shown indicating that the transition time lasts until 3 to 5 times the time to first ionization.

• after the transition phase, the electron temperature reaches saturation and the plasma density increases exponentially,

• the anomalous ionization rate scales linearly with density and as a quadratic with velocity

• for the laboratory experiments, hot electrons initiate CIV by immediately electron impact ionizing neutrals. The process immediately enters the transition phase. Exponential growth of the plasma density is quickly reached.
Figure 4-1: Phase space evolution of the M2Sl. Electron phase space is shown on the left, ion phase space on the right. (a) $\omega_{pe}t = 0$ (b) $\omega_{pe}t = 1600$ (c) $\omega_{pe}t = 3500$ (d) $\omega_{pe}t = 10000$
Figure 4-2: Energy history for M2SI. (a) Drift Energy. The beam ion drift energy acts as the free energy source driving the instability. (b) Thermal Energy. All of the species are seen to heat substantially. (c) Field Energy. The linear growth rate is seen to apply to $\approx \omega_{pe}t = 4000$. After this point, the field energy slowly decreases.
Figure 4-3: Field energy for various wave modes. The total field energy for the simulation is compared with the modal energy for $m=3, 4$ and 5. Clearly, $m=4$ is the dominant wave mode in this simulation.
Figure 4-4: Nonlinear distribution function development during the M2SI, (a) \( \omega_{pc}t=0 \) (b) \( \omega_{pc}t=2700 \) (c) \( \omega_{pc}t=3000 \) (d) \( \omega_{pc}t=3500 \) (e) \( \omega_{pc}t=5000 \)
Figure 4-5: Time history of the field energy for the M2SI from simulations using different mass ratios. The results for the different mass ratios are nearly identical when scaled by the lower hybrid frequency. The solid line is the solution for the growth rate obtained from the dispersion relation.

Figure 4-6: Time history of the kinetic energy of the ions and electrons for the M2SI from simulations using different mass ratios. Symbols are the same as in Figure 4-5.
Figure 4-7: The parameter $\eta$ as calculated from the simulation results as a function of $\nu_{ion}/\Omega_i$. The lower limit is near that predicted by quasilinear theory. The upper asymptote is substantially below the prediction of quasilinear theory.

Figure 4-8: Comparison of saturated distribution functions for unmagnetized ions. The numerical solution is substantially broader than the predicted distribution function, leading to a lower value of $\eta$. 

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Figure 4-9: Time history of the field energy and electron kinetic energy. Unmagnetized Ions. (○)\(m_i/m_e = 6400\), (+)\(m_i/m_e = 400\). Good agreement is seen in this example since only the \(\omega_{LH}\) time scale is present.
Figure 4-10: Time history of the field energy and electron kinetic energy. Magnetized Ions. Scaled by lower hybrid frequency. \((\odot)m_i/m_e = 6400\), \((+ \odot)m_i/m_e = 400\). The unphysical mass ratio leads to the incorrect saturation of the electron kinetic energy.
Figure 4-11: Time history of the field energy and electron kinetic energy. Magnetized Ions. Scaled by the cyclotron frequency. (o)\(m_i/m_e = 6400\), (+)\(m_i/m_e = 400\). The unphysical mass ratio leads to the incorrect saturation of both the field energy and electron kinetic energy.
Figure 4-12: Time history of the field energy and kinetic energy. Highly Magnetized Ions. Scaled by lower hybrid frequency. (o)\(m_i/m_e = 6400\), (+)\(m_i/m_e = 400\). After an initial transient, the two results agree well indicating relaxation to an isotropic distribution function.
Figure 4-13: Typical simulation results. (a) Electron number vs time shows an exponential growth. (b) Electron thermal energy vs time displays saturation.
Table 4.1: Parameters used for the different gases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CO₂</th>
<th>Ne</th>
<th>NO</th>
<th>Xe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_n/m_e$</td>
<td>80800</td>
<td>37049</td>
<td>55098</td>
<td>241067</td>
</tr>
<tr>
<td>$e \phi_{ion}$ (eV)</td>
<td>12.1</td>
<td>21.6</td>
<td>9.2</td>
<td>12.13</td>
</tr>
<tr>
<td>$v_c$ (km/s)</td>
<td>7.7</td>
<td>14.3</td>
<td>7.7</td>
<td>4.2</td>
</tr>
</tbody>
</table>

Figure 4-14: Initial stages of CIV. (a) Beam density. During the initial stage of CIV, charge exchange reactions increase the beam density linearly with time. (b) Electron kinetic energy. Since the efficiency of energy transfer to the beam is constant, the electron kinetic energy also increases linearly.
Figure 4-15: Velocity space near beginning of run. The cluster around $v = 0$ represents the ambient ions. The diffuse population on the right are the ions that have been created through charge exchange, but have diffused in velocity space as they interact with the waves.
Figure 4-16: $\eta$ as a function of the neutral velocity, $V_n/V_c$, where $V_c$ is the critical velocity. The results agree well with Equation (4.4), indicating that the efficiency of energy transfer is constant during the initial stages of electron heating.

Figure 4-17: The electron distribution function near the time of first ionization. The tip of the distribution function has just reached the ionization energy.
Figure 4-18: A comparison of the simulation results for the temperature at first ionization with the prediction of the analytic model.
Figure 4-19: Time to first ionization. The vertical bars represent the results of the simulations, the markers are the prediction of the equation in the text. For the simulation results, the values plotted represent the range between the time between the first electron impact ionization event and the third electron impact ionization event.
Figure 4-20: Dependence of the time for first ionization on the neutral density. (a) NO, (b) CO₂. The NO results agree well over four orders of magnitude. For CO₂, the windowing effect affects the results for $n_n/n_e > 10^7$
Figure 4-21: Ratio of time to reach the break point and the time for first ionization. 
○ = Ne, + = Xe, ◊ = NO, × = CO2 (a) Velocity Dependence. (b) Neutral density dependence.
Figure 4-22: An idealized model of the electron distribution function, $F_e(E)$, after electron temperature saturation in CIV as a function of the energy. The idealized distribution has a constant density hot electron tail. (After [42])
Figure 4-23: Anomalous ionization rate vs neutral beam velocity for the four different gases. Below $V_n/V_c = 1$, $\nu_{ion}/\Omega_i \approx 0$. The anomalous ionization rate increases quickly as the neutral velocity exceeds the critical velocity.
Figure 4-24: Effect of Density on the anomalous ionization. The top panel shows NO, the bottom panel CO$_2$. For the lower values of density, $\nu_{\text{ion}}/\Omega_i \sim N_n/N_e$. For higher densities, the 'windowing effect' is evident.
Figure 4-25: Electron energy vs time. (a) For high densities, an electron will collide and de-energize before it reaches ionization energies. (b) For low densities, the electron will not de-energize.
Figure 4-26: Results for helium-hydrogen system for different $T_e$. The electrons are initially hot enough to begin ionization through electron impact. The system quickly transitions to self-sustaining CIV.
Chapter 5

Simulation and Experimental Results: A Comparison

In laboratory experiments, the interaction region of the neutral cloud with the plasma extended over a region of thousands of Debye lengths. The interaction lasted on the order of hundreds of thousands of plasma periods. Likewise, the diagnostics of space experiments show evidence of CIV on length scales of millions of Debye lengths and time scales on the order of hundreds of millions of plasma periods. Although the implicit numerical methods used to generate the results of the last chapter have provided a gain over traditional explicit codes, an upper practical limit in 1-D is a simulation region of thousands of Debye lengths with time scales on the order of hundreds of thousands of plasma periods. In 2-D, the practical limit is length scales of hundreds of Debye lengths in each coordinate direction and time scales of tens of thousands of plasma periods. Because of this, the full, realistic simulation of the entire CIV interaction region in at least 2-D is not yet practical. As the results of the previous chapter have indicated, however, small scale simulations of the microphysics can lead to estimates of the ignition times needed to start CIV and the anomalous ionization rates once CIV has started.

The purpose of this chapter is to use the results of the numerical simulations in simple rate models to suggest time and length scales necessary for CIV to occur. Three situations will be discussed. The first rate model assumes a point release of neutral gas
in the ionosphere and is meant to represent the sounding rocket experiments. A second model is introduced involving a neutral gas source moving with respect to the ambient plasma. This model corresponds to the IBSS experiments performed from the shuttle bay. The final model considers the neutrals to be at constant density and stationary in the frame of the magnetic field. The plasma is polarized and drifting across the magnetic field. This model, of course, is designed to represent the laboratory experiments. These models for the neutral cloud will be used in conjunction with the results of the previous chapters to suggest where and why CIV might be expected to occur in both the laboratory and space experiments.

The simple 1-D calculations presented in the previous chapter are a great simplification of the actual experimental scenarios. Because of the limitations of the simulations, the focus of attention here is on upper bound estimates of CIV: the assumptions introduced through the following sections should lead to an estimated increase in plasma density larger than should be experimentally observed. As the following discussion will show, even the upper bound predictions for the occurrence of CIV will lead to severe restrictions on where CIV should be observed.

## 5.1 A Simple Rate Model

CIV is modeled as the simple convection of a neutral gas across a magnetic field. Once a sufficient time has passed to heat electrons to the ionization energy, the neutral gas will begin to undergo ionization at the anomalous ionization rate, \( \nu_{\text{ion}} \). Based on the results of the previous section, the anomalous ionization rate will be assumed to scale linearly with the neutral density. With this assumption, the anomalous ionization rate may be expressed as the product of a reaction rate, \( p_0 \), and the neutral density, i.e.

\[
\nu_{\text{ion}} = p_0 n_n
\]

where \( p_0 \) is independent of the neutral density. The undetermined factor in the equations is the reaction rate, \( p_0 \). This reaction rate is taken from the simulation results of the previous chapter.
The macroscopic properties of the neutral gas may be found from taking proper moments of the distribution function, $f_n$. The governing equation for $f_n$ is the Vlasov equation, hence the macroscopic properties of the neutrals may be found from a solution of

$$\frac{\partial f_n}{\partial t} + \mathbf{v}_n \cdot \nabla f_n = \left( \frac{\partial f_n}{\partial t} \right)_c$$  \hspace{1cm} (5.2)

where $\mathbf{v}_n$ is the neutral velocity and $(\partial f_n/\partial t)_c$ represents a source/sink term due to ionization of the neutrals.

The fastest production of electrons that can reasonably be expected for the CIV process occurs in the case when electron losses along the magnetic field are ignored. For this case, it can be assumed that electron-ion pairs will be created at a point in space at a rate $p_0 n_n n_e$. The newly created charged particles do not propagate with velocity $\mathbf{v}_n$ but instead are tied to the magnetic field line on which they are created. The plasma density at a point, then, may be assumed to increase as a result of the ionization process, i.e.

$$\frac{\partial n_e}{\partial t} = p_0 n_n n_e + q_0$$  \hspace{1cm} (5.3)

where $q_0$ is the rate of seed ionization such as photoionization or ionization by initial hot electrons. The assumptions leading to Equation (5.3) are consistent with the approach of producing upper bound estimates for the occurrence of CIV.

Solving the above equations requires three steps: (1) determining the neutral density as a function of time for given initial and boundary conditions, (2) finding when the ignition time criterion is satisfied, and (3) solving for the electron density as a function of $r$ and $t$ using the anomalous ionization rates determined in the previous chapter. Each of these steps will be considered in turn for the Porcupine release, IBSS releases, and laboratory experiments.

### 5.2 Neutral Models

At least for the initial stages of CIV, a reasonable assumption is that only a small fraction of the neutrals will be ionized. With this assumption, the right hand side of
Equation (5.2) can be ignored giving simply

\[
\frac{\partial f_n}{\partial t} + \mathbf{v}_n \cdot \nabla f_n = 0
\]  \hspace{1cm} (5.4)

5.2.1 "Porcupine" Release

A good approximation to the shaped charge velocity distribution [31] is

\[
f(v) = \begin{cases} 
1/v_h & 0 < v < v_h \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (5.5)

where \( v_h \) is some maximum velocity. An initial condition is also needed. The initial condition chosen here is that the neutrals are initially located between \( r_0 \) and \( r_1 \) and exhibit a \( 1/r^2 \) dependence on the radial distance, i.e.

\[
n_n(r, t = 0) = \begin{cases} 
C_0/r^2 & r_0 < r < r_1 \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (5.6)

In order to match the experiments, \( C_0 \) is chosen as \( 4 \times 10^{22} \text{ m}^{-1} \) to achieve \( n_n = 10^{18} \text{ m}^{-3} \) at \( r_1 = 200 \text{ m} \). The choice of \( 10^{18} \text{ m}^{-3} \) is related to the cutoff due to the windowing effect as discussed above, while the \( r_1 = 200 \text{ m} \) length is consistent with the start of ionization in the Porcupine data. The lower length of \( r_0 = 56 \text{ m} \) is chosen to provide \( 10^{24} \) neutral particles within the initial volume assuming a conic half-angle of \( \Theta = 13.5^\circ \). The velocity \( v_h \) is taken as \( 13 \text{ km/s} \). With these initial conditions, the neutral density as a function of position and time can be found by solving Equation (5.4) for \( f(r, v, t) \) and integrating over velocity space. The result is

\[
n_n(r > r_1, t) = \begin{cases} 
\frac{C_0 r_1^2 - r}{r_1 v_h t} & \max(r_1, r'_0) < r < r'_1 \\
\frac{C_0 r_1 v_h t}{r_1^2} & r_1 < r < \max(r_1, r'_0) \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (5.7)

where \( r'_1 = r_1 + v_h t \) and \( r'_0 = r_0 + v_h t \). This expression is simply a geometric expansion into the spherical solid angle \( 1/r^2 \), with a correction term \( (r_1 - r_0)/(v_h t) \) to account for
the velocity distribution. A pictorial version of the neutral density at several different times is shown in Figure 5.1.

### 5.2.2 IBSS Releases

For the space shuttle release, the gas is assumed to be cold and have a single velocity relative to the shuttle, $v_i$. The IBSS releases are assumed to be fired in the ram direction for a duration of 10 sec. The situation is shown pictorially in Figure 5.2. The expression for the neutral velocity in the plasma frame can easily be determined to be

$$
\begin{align*}
\text{if } t < t_f : \quad n_n &= \begin{cases} 
\frac{C_0}{(r-V_n t)^2} & V_n t < r - r_0 < V_n t \\
0 & \text{otherwise}
\end{cases} \\
\text{if } t \geq t_f : \quad n_n &= \begin{cases} 
\frac{C_0}{(r-V_n t_f)^2} & V_n t_f + V_n (t - t_f) < r - r_0 < V_n t \\
0 & \text{otherwise}
\end{cases}
\end{align*}
$$

(5.8)

where $V_s$ is the shuttle velocity and $V_n = V_s + V_i$ is the neutral velocity in the plasma frame. This expression represents the expansion of a gas from a source moving with velocity $V_s$. The parameters are chosen as $C_0 = 4 \times 10^{22} \text{ m}^{-1}$, $r_0 = 200 \text{ m}$, $V_s = 8 \text{ km/s}$ and $V_n = 10 \text{ km/s}$.

### 5.2.3 Laboratory Experiments

In the laboratory experiments, the neutral gas is simply stationary, i.e.

$$
n_n = n_0 \quad 0 < r < L
$$

(5.9)

where $L$ is the spatial dimension of the neutral cloud. In this model, the plasma is initially streaming in the positive $z$-direction with a single velocity $V_n$. 

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5.3 Ignition Time Analysis

As shown in the previous chapter, for the space based experiments, the time to start CIV (i.e. the ignition time) is about $3\tau_1$ to $5\tau_1$, where, for the parameters of Figure 4-19, $\tau_1$ is given by Equation (4.7). In the laboratory experiments, the initial electron population is already hot enough to immediately begin ionizing electrons. In this section, the time to first ionization, $\tau_1$, is determined for the Porcupine and IBSS releases. This time is compared to the transit time, $T_{tr}$, of the neutral gas past a given element of plasma. For CIV to ignite, the transit time of the neutral gas must exceed the time to first ionization.

The simulations in the previous chapter were all performed at a given neutral velocity and density. However, as the results for the neutral model indicate, the density and velocity at a point in space will vary in time. In order to expand the results of the previous chapter to the current situation, a simple integral representation is adopted: the electron energy at a point in space is assumed to follow

$$T_e = T_{e0} + \int \eta \frac{1}{2} m_n V_n^2 < \sigma_{nrcez} v > n_n dt$$

(5.10)

over the linear regime. The time to first ionization is given by $\tau_1$ where $\tau_1$ satisfies

$$T_{e\phi} - T_{e0} = \int_{t_0}^{t_0 + \tau_1} \eta \frac{1}{2} m_n V_n^2 < \sigma_{nrcez} v > n_n dt$$

(5.11)

and $t_0$ is the time at which neutrals first arrive at the point (the small constant $\tau_s \sim O(\omega_{LB})$ has been ignored here). Equation (5.11) is somewhat more clearly written as

$$T_{e\phi} - T_{e0} = \eta \left( \frac{V_n}{V_c} \right)^2_{max} < \sigma_{nrcez} v >_{max} \int_{t_0}^{t_0 + \tau_1} n_n dt$$

(5.12)

where $T^* = T/(m_n V_c^2/2) = T/(e\phi_{ion})$, and $(V_n/V_c)$ and $< \sigma_{nrcez} v >$ are evaluated at their maximum values (again consistent with looking for the upper bound for the occurrence of CIV).

The solution to this equation for the different neutrals models is now discussed.
5.3.1 “Porcupine” Release

For the Porcupine case, the integral of Equation (5.12) is simplified by taking \((V_n/V_c)_{\text{max}} = (v_h \sin \theta/V_c)\), where \(\theta\) is the angle of the neutral release with respect to the magnetic field. The inclusion of \(\theta\) in the definition of \((V_n/V_c)_{\text{max}}\) states that the component of the neutral velocity perpendicular to the magnetic field is important for CIV, not the full velocity, \(v_h\). Using the model of the neutrals from the previous section and performing the integration gives

\[
E = \frac{r_0v_h}{C_0} \int_{t_0}^{t^* = t_0 + \tau_1} n_n \, dt = \begin{cases} 
\frac{r_0^2}{r_1} \left[ \frac{r_1 - r}{r_0} \log \left( \frac{r_0 t^*}{r_1 - r_1} \right) + \frac{r_0 t^*}{r_0} - \frac{r - r_1}{r_0} \right] & t^* < \frac{r - r_0}{v_h} \\
\frac{r_0^2}{r_1} \frac{r_1 - r_0}{r_0} \left[ \frac{r_1 - r}{r_1 - r_0} \log \left( \frac{r - r_0}{r_1 - r_1} \right) + \log \left( \frac{r_0 t^*}{r_1 - r_0} \right) + 1 \right] & t^* \geq \frac{r - r_0}{v_h} 
\end{cases}
\]

where

\[
E \equiv \frac{r_0v_h}{C_0} \frac{T_{c\phi}^* - T_{c0}}{\eta \left( \frac{V_c}{\text{max}} \right)^2 < \sigma_{\text{nrcev}} v >_{\text{max}}} \quad (5.14)
\]

and \(t_0 = (r - r_1)/v_h\). Physically, the parameter \(E\) represents the (nondimensionalized) characteristic time necessary to heat electrons to energies sufficient to begin ionization.

The solution to Equation (5.13) can be found through a simple iterative procedure: calculate \(t^*\) using the second condition of Equation (5.13); if the criterion \(t^* \geq \frac{r - r_0}{v_h}\) is violated, calculate \(t^*\) from the first condition of Equation (5.13) (which requires a Newton-Raphson iteration or other root finder since this equation is transcendental).

Once the time to first ionization is known, the criterion for CIV to start is that the transit time, \(T_{tr}\), of the neutrals through a plasma element be longer than the time to first ionization.

\[
\tau_1 < T_{tr} \quad (5.15)
\]

The transit time can be considered as the time between the arrival at a point in space of the first neutrals and the arrival of the neutrals moving at the critical velocity, i.e.

\[
T_{tr} = \frac{r - r_0}{v_c/\sin \theta} - \frac{r - r_1}{v_h} \quad (5.16)
\]
The desire is to compare the results of Equation (5.13) with the results of Equation (5.16) for parameters similar to those of the Porcupine release. The previous section did not simulate the anomalous ionization of barium directly. The results do however suggest reasonable number which may be used to estimate the time for CIV to start in barium. The values used here are $T_{e\phi} = .1e\phi = .5$ eV, $T_{eO} = .2$ eV, and $\eta = .5$. In addition, for Porcupine, $v_h = 13$ km/s, and $\theta=28^\circ$.

Figure 5-3 compares the time to first ionization, $\tau_1$, and the transit time, $T_{tr}$. Since the value of the barium-oxygen charge exchange cross section is controversial, the solution is shown for a series of values of $<\sigma_{nrxz}v>$. The cross section used varies from $10^{-17}$ cm$^2$ to $10^{-14}$ cm$^2$ in order to cover the range of cross sections assumed by various authors. The corresponding values of $E$ ($\approx 3 \times 10^{-2}$ to $3 \times 10^{-5}$, respectively) are also included on the figure. The exponentially increasing curves shown on the figure are the time to reach first ionization for the stated value of $E$, as given by the solution of Equation (5.13). The times are shown as a function of the downrange distance from the release point ($s_\perp = r \sin \theta$). The linearly increasing line is the transit time as a function of $s_\perp$ (Equation 5.16). As shown in the figure, the criterion for CIV to ignite ($T_{tr} > \tau_1$) is only satisfied within several hundred meters of the release point for $\sigma_{nrxz} = 10^{-17}$ cm$^2$, while CIV ignites out to a horizontal range of over 8 km for $\sigma_{nrxz} = 10^{-14}$ cm$^2$. This is an important result in relation to the Porcupine experiment. The results of Porcupine indicate enhanced ionization out to 15 km from the release point. Even with the largest charge exchange cross section assumed here ($\sigma_{nrxz} \approx 10^{-14}$ cm$^2$), CIV cannot be achieved beyond 9 km. The results here indicate that CIV should be much more localized than the results of Porcupine indicate. As mentioned in the Introduction, one factor not considered in the original analysis of the Porcupine data was the charge exchange contamination of the results. For the $\sigma_{nrxz} = 10^{-14}$ cm$^2$ results shown here, charge exchange will produce more ions than CIV over most of the 15 km range of the Porcupine data. As the following sections will make clear, consideration of the charge exchange products can make the results of the Porcupine experiment questionable.
5.3.2 IBSS Releases

An analysis similar to above can be performed using the neutral density model for the IBSS releases, Equation (5.8). The needed integration is

$$E' = \int_{t_0}^{t_0 + \tau_1} \frac{C_0}{(r - V_s t)^2}$$

(5.17)

where

$$E' \equiv \frac{r_0 V_s}{C_0} \frac{T_{ee}' - T_0'}{\eta \sigma_{neceless} v > (V_n/V_c)^2}$$

(5.18)

(Note that $E'$ is the same as $E$ in the previous section, but defined using $V_s$ for the normalizing velocity). For this case, $t_0 = (r - r_0)/V_n$.

After performing the integration, the result for the time to first ionization may be expressed as

$$\frac{V_s \tau_1}{r_0} = \frac{E' \xi^2}{1 + E' \xi}$$

(5.19)

where

$$\xi = \frac{r}{r_0} - \frac{V_s}{V_n} \left( \frac{r}{r_0} - 1 \right)$$

(5.20)

A non-dimensionalized transit time can also be expressed in terms of $\xi$. The transit time for this case is the difference between the time for the arrival of the first neutrals and the arrival of the shuttle. The result for the transit time is

$$\frac{V_s T_{tr}}{r_0} = \xi - 1.$$  

(5.21)

As for the Porcupine release, CIV can begin if the time to first ionization is less than the transit time of the neutrals, i.e. $\tau_1 < T_{tr}$.

As Equation (5.19) indicates, the characteristics of whether the system reaches first ionization are governed by the parameter $E'$, a non-dimensional heating time. The results of the previous chapter can be used to suggest representative numbers for this heating time. For all the gases studied in the previous chapter, the values of $E'$ turn out to be in the range $E' \approx 10^{-2}$ to $10^{-1}$.

Figure 5-4 compares the results of Equations (5.19) and (5.21) as a function of the
position for a range of $E' = 10^{-3}$ to 1.0. The spatial distance considered is the region covered by the shuttle while the release is occurring. This region is the region most likely to experience CIV. Beyond 80 km, after the firing, the neutral density drops quickly, stopping CIV.

As shown in Figure 5-4, first ionization is reached during nearly the entire firing for the values considered here. This is a result of the high densities encountered near the source, where first ionization is reached quickly. (For very small values of $s_\perp$, ignition isn't achieved since $T_{tr}$ goes to zero. This region is too small to be visible on the figure, and is not of any real importance.) As in the Porcupine releases, the highest values of $\sigma_{nrccez}$ (lowest $E'$) satisfy the ignition criterion most easily. For $E' = 10^{-3}$, the time to reach first ionization is much lower the gas transit time at $s_\perp = 80$ km, while for $E' = 1.0$, $\tau_1$ and $T_{tr}$ are almost identical. Clearly, the higher values of $\sigma_{nrccez}$ mean the process ignites sooner, leaving additional time for ionization in the exponential growth phase. This production of ions during the exponential growth phase is the subject of the next section.

5.4 Exponential Growth Phase

The previous section determined when first ionization would occur for the different neutral models. In this section the exponential growth phase of CIV is examined, and the fractional increase in the plasma density is determined.

Ionization begins at the time $t_0 + \tau_1$. Consistent with the strategy of producing an upper bound, the gas is assumed to enter its exponential phase immediately upon reaching $\tau_1$, i.e. the transient phase described in the previous chapter is ignored. Equation (5.3) gives the fractional increase in the plasma density as

$$\frac{n_e}{n_{e0}} = \exp \left[ \int_{t_0 + \tau_1}^{t_0 + T_{tr}} p_0 n_m dt \right]$$  \hspace{1cm} (5.22)

The results for $n_m$, $\tau_1$ and $T_{tr}$ from the previous sections can be used to solve this equation for the fractional increase in plasma density.
5.4.1 “Porcupine” Release

For the Porcupine release, the density model is given by Equation (5.7). $\tau_1$ and $T_{tr}$ are found from Equations (5.13) and (5.16), respectively. Performing the integration gives

$$
\frac{n_e}{n_{e0}} = \begin{cases} 
\left( \frac{T_{tr}}{\tilde{\tau}_1} \right)^b & 1 < \tilde{\tau}_1 < \tilde{T}_{tr} \\
\tilde{\tau}_1 \tilde{T}_{tr}^b e^{c(1-\tilde{\tau}_1)} & \tilde{\tau}_1 < 1 < \tilde{T}_{tr} \\
\left( \frac{T_{tr}}{\tilde{\tau}_1} \right)^c e^{c(\tilde{T}_{tr}-\tilde{\tau}_1)} & \tilde{\tau}_1 < \tilde{T}_{tr} < 1
\end{cases}
$$

(5.23)

where

$$
b \equiv \frac{p_0 C_0}{r_0 v_h} \left( \frac{\tau_1}{r_0} - 1 \right) \frac{r_0^2}{r^2}; \quad c \equiv b \frac{r_1 - r}{r_1 - r_0}
$$

$$
\tilde{\tau}_1 \equiv \frac{v_h \tau_1}{r - r_0}; \quad \tilde{T}_{tr} \equiv \frac{v_h T_{tr}}{r - r_0}
$$

and $\tau_1$ and $T_{tr}$ are given by the results of the previous section.

Although Equation (5.23) is too complex to be very revealing, the plot of $n_e/n_{e0}$ as a function of $s_\perp$ shown in Figure 5-5 is informative. The figure shows the results for various values of the electron heating rate, $E$, and the anomalous ionization reaction rate, $p_0$, in order to account for uncertainties in the analysis. The figure shows two values of $E$, $3 \times 10^{-3}$ and $3 \times 10^{-5}$, corresponding to charge exchange cross sections of $10^{-16} \text{ cm}^2$ and $10^{-14} \text{ cm}^2$, respectively. Some uncertainty also exists in the anomalous reaction rate, $p_0$, since barium has not been simulated directly and some controversy exists as to the cross sections. The results of the previous chapter do suggest however that the anomalous ionization rate should be in the range of $p_0 = 10^{-15}$ to $10^{-17} \text{ m}^3/\text{s}$.

The results shown range over these values of $p_0$.

In agreement with the ignition time analysis, the results here show that CIV does not exist except within several kilometers of the release point. In fact, although first ionization will be achieved within 9 km for the $E = 3 \times 10^{-5}$ case, substantial increases in the plasma density due to CIV are only observed within about 2 km. Similar results are obtained for other values of $E$: increases in plasma density attributable to CIV are
confined to within at most several kilometers of the release point even for the largest anomalous reaction rates.

Considering the results of the previous chapter and the "upper bound" assumptions that have been built into the current model, the most likely anomalous reaction rate to be achieved in an actual experimental release will probably be less than $10^{-16} \text{ m}^3/\text{s}$. As seen in Figure 5-5, for an anomalous reaction rate $p_0 \approx 10^{-16}$, a peak increase of about 30% in the plasma density is achieved near the release point. The enhanced plasma density also decreases to negligible values within several hundred meters of the release point. For even lower anomalous reaction rates, only a small increase in plasma density is seen even near the release point.

These results again have important implications for sounding rocket releases. Because of the decreasing neutral density as the neutral cloud expands from the release point, the time necessary to heat electrons to begin ionization, and the magnitude of the reaction rates likely to be achieved, CIV will not be an efficient ion production mechanism in point releases. Instead, fractional plasma density increases of $\approx 30\%$ may occur within the first several hundred meters of the release point. Beyond several hundred meters, the ionization rate due to CIV is negligible.

The relation of these results to the actual experimental evidence obtained from Porcupine is discussed below. First, however, the plasma density increases are discussed for the IBSS releases and laboratory experiments.

### 5.4.2 IBSS Releases

For the IBSS releases, the fractional ionization can be found from the integration of Equation (5.8):

$$\frac{n_e}{n_{e0}} = \exp \left[ \frac{p_0 C_0}{V_s r_0} \left( 1 - E' - \frac{1}{\xi} \right) \right]$$

(5.24)

Using the values for the release and the results of the previous chapter gives $p_0 \approx 1 \times 10^{-15}, 4 \times 10^{-16}, 3 \times 10^{-15}$ and 0.0 for the IBSS release of CO$_2$, NO, Xe, and Ne, respectively.

Figure 5-6 shows the results of Equation (5.24) for a range of $p_0 = 10^{-17}$ to $8 \times 10^{-17} \text{ m}^3/\text{s}$ and $E' = .01$. As seen in the figure, for these values of $p_0$, which are smaller
than the $p_0$'s predicted by the results of the previous chapter, significant increases in ionization should be observed during the release. The values shown in Figure 5-6 are a result of the high neutral density encountered near the nozzle as the shuttle passes through a given plasma element. In fact, the fractional increase in the plasma density obtained for $p_0 > 10^{-16} \text{ m}^3/\text{s}$ seem unreasonably high considering the simplicity of the model used here. In regions of greatly increased plasma density, neither the assumption of no electron losses nor the assumption of no appreciable density depletion due to collisions is valid. Clearly, though, significant increases in plasma density are likely to be observed in the IBSS experiments.

5.4.3 Laboratory Experiments

The results of the previous chapter indicate that in the laboratory experiments ionization will begin almost immediately because of pre-existing hot electrons. This form of seed ionization leads to an exponential growth in the plasma density within 1-2 $\mu$s, depending on the initial electron temperature. As shown in the results of the experimental work discussed in Chapter 1, a time scale of 1-2 $\mu$s is consistent with the experimental measurements.

5.5 A Comparison of "Porcupine" Results

Up to this point, the electron density as a function of the radial distance from the release point has been determined. In this section, these results are related to the experimental measurements obtained during the Porcupine release.

In Porcupine, two main results were obtained: (1) An estimated 10% of the total neutral population was ionized within 9 sec of the release, and (2) a densitometer tracing taken above the terminator gave the relative intensities shown in Figure 1-6.

The densitometer measurements are an indication of the total number of ions appearing above the terminator. The results in the previous section focused on the electron density as a function of radial distance. For purposes here, the number of beam ions appearing above the terminator is assumed to be related to the number of ions pro-
duced at a radial position, \( r \), integrated over the spherical volume and projected to a perpendicular distance:

\[
I(s_\perp / \sin \theta) = I(r) = 2\pi (1 - \cos \Theta) \int_{r - \Delta r/2}^{r + \Delta r/2} r^2 n_b dr
\]  
\[\text{(5.25)}\]

The ion beam density at a point is related to the electron density and ambient ion densities through

\[
n_b + n_{O^+} = n_e
\]  
\[\text{(5.26)}\]

For the determination of the time to first ionization, the variation in the ambient ion density was ignored. On the time scale of seconds of interest here, this is no longer a good assumption. Instead, the ambient ion density is calculated from

\[
\frac{\partial n_{O^+}}{\partial t} = -\nu_{nrce} n_{O^+}
\]  
\[\text{(5.27)}\]

giving

\[
\frac{n_{O^+}}{n_{O^+0}} = \exp \left[ -\int_{t_0}^{t} p_{nrce} n_e dt \right]
\]  
\[\text{(5.28)}\]

This equation is similar to Equation (5.22), but with different integration limits and \( p_0 \) replaced with \( -p_{nrce} \). The result is

\[
\frac{n_{O^+}}{n_{O^+0}} = \begin{cases} 
\frac{\hat{r}_0 \hat{b} e^{(1-\hat{\tau}_0)}}{\hat{r}_0} & 1 < \hat{t} \\
\left(\frac{\hat{t}}{\hat{\tau}_0}\right)^c e^{(\hat{t}-\hat{\tau}_0)} & \hat{t} < 1
\end{cases}
\]  
\[\text{(5.29)}\]

where

\[
b \equiv \frac{-p_{nrce} C_0}{r_0 \nu_h} \left(\frac{r_1}{r_0} - 1\right) \frac{r_0^2}{r^2}; \quad c \equiv \delta \frac{r_1 - r}{r_1 - r_0}
\]
\[
\hat{\tau}_0 \equiv \frac{r - r_1}{r - r_0}; \quad \hat{t} \equiv \frac{\nu_h t}{r - r_0}
\]

The beam density can be calculated from Equation (5.26) using Equation (5.29) and (5.23), and the intensity as a function of \( s_\perp \) can be calculated from Equation (5.25). These last steps are carried out numerically, and the results of \( I \) vs. \( s_\perp \) are presented in Figure 5-7 for several different assumptions on the cross sections.
The top panel of Figure 5-7 shows the results assuming a non-resonant charge exchange cross section of $10^{-14}$ cm$^2$ for Ba-O$^+$ ($E \approx 3 \times 10^{-5}$). Results are included for three different values of the anomalous ionization reaction rate: $p_0 = 0$ (no CIV), $10^{-16}$ cm$^2$ and $10^{-15}$ cm$^2$. The figure clearly shows that the effect of CIV at the assumed values of anomalous ionization reaction rates is negligible. Little difference is seen between the results with and without CIV. In these results, nearly all newly formed ions occur through charge exchange; the CIV portion is not significant.

Also included in the figure are markers representing the results obtained by Haerendel during Porcupine. The shape of the relative intensity curve is seen to agree well with the Porcupine results except near the release point. In the current model, the decrease in ion production near the release point is an artifact of the model. Near the release point, the current model results in depletion of the ambient ions. The entire background ion population has been converted into beam ions in this region. In a more realistic model, some portion of these ions should be replaced by ions diffusing into the region from the surrounding plasma.

The given value of total ionization for Porcupine was 20% of the ions with velocity in excess of the critical velocity. Equivalently, this represents roughly 10% of the total ion population. The current results assuming $E = 3 \times 10^{-5}$ and $p_0 = 0$ gives 4% ionization within 15 km of the release. Considering the simplicity of the model, this is good agreement.

The lower panel of Figure 5-7 shows similar results, but assuming $E = 3 \times 10^{-3}$. Here, a difference does appear between the results with and without CIV, but this difference is only significant within approximately 1 km of the release point. According to this result, which corresponds more closely to the charge exchange cross section assumed by Haerendel, the results of Porcupine should have produced a more intense streak within 1 km of the release point than was actually observed. In addition, this model produces only .04% ionization.

The conclusion drawn from these results is that CIV did not occur in Porcupine, or, if it did, its effect was much less than calculated by Haerendel. The ionization that was observed was most likely produced by charge exchange reactions between the
neutral beam and ambient oxygen plasma. The cross section for this reaction may be $\mathcal{O}(10^{-14})$ cm$^{-2}$, several orders of magnitude larger than assumed by Haerendel.

One final point of clarification should be added here. Charge exchange was assumed to be the only source of seed ionization in the analysis. However, since the Ba-O$^+$ charge exchange cross section in not known, it cannot be stated with certainty that charge exchange is the dominant source of seed ionization in the space experiments. For example, Lai and Murad[41] have suggested the $Ba + O \rightarrow -Ba^+ + O + e$ stripping reaction may also be important. This reaction requires that the energy of the Ba in the center of mass frame be above its ionization energy. Hence, only a fraction of the Ba neutrals released in a space experiment undergo this reaction. In addition, the stripping reaction cross section is believed to be $\approx 10^{-16}$ cm$^2$. If the charge exchange cross section is small, i.e. $\sigma_{n\text{ceex}} \approx 10^{-17}$ cm$^2$, the stripping reaction would dominate over the charge exchange reaction. The stripping reaction could then increase the seed ionization rate over that assumed in the analysis. Very roughly this additional reaction path can be thought of as increasing the "effective" seed ionization cross section: $\sigma_{\text{eff}} \approx \sigma_{n\text{ceex}} + \sigma_{\text{str}}$ where $\sigma_{\text{str}}$ is the stripping cross section. As seen however, even an "effective" cross section of $10^{-16}$ cm$^2$ does not increase the seed ionization rate to sufficient levels to produce significant ionization by CIV. For the proposed charge exchange cross section of $10^{-14}$ cm$^2$, the stripping reaction is a negligible source of seed ionization. Since the results here indicate that the space experiment results are consistent with the large charge exchange cross section, stripping reaction are considered negligible and will not be considered further in the analysis.

5.6 Comparison with other releases

The results discussed so far have been applied to Porcupine. In this section, the implications of these results for the other barium and strontium releases are discussed. More specifically, this section discusses the hypothesis that Porcupine did not observe CIV, but instead that the observed ionization was due to charge exchange. The effect on charge exchange is consistent with many of the other CIV results observed in space based experiments.
**Bubble Machine:**

Bubble Machine was a charge release performed in full sunlight consisting of both Ba and Sr. The Ba was expected to photoionize, while the Sr was observed for evidence of CIV. Bubble Machine observed up to 50% ionization of the Sr. As mentioned in the Introduction, the error in this figure may be up to a factor of three. Unfortunately, the large uncertainty in the experimental results makes it difficult to judge this release. The observed ionization of the Sr, however, is consistent with the conclusions drawn for Porcupine. In Bubble Machine, the photoionizing Ba would act as a rapid seed ionization mechanism. This seed ionization through photoionization of Ba would likely lead to a more rapid electron heating than observed in Porcupine. The increase in electron heating would decrease the time to first ionization of the Sr, and ultimately lead to enhanced values of Sr ionization. Hence, "pumping" of the CIV process through photoionization of Ba likely led to the enhanced ionization of Sr observed in this experiment.

**Star of Condor, Sr90:**

These two strontium releases should not observe charge exchange contamination of the results. Indeed, virtually no ionization was observed in either release.

**Star of Lima:**

Star of Lima was a barium release carried out from a sounding rocket that overperformed. The overperforming rocket exposed the released barium to more sunlight than expected. The results of the experiment indicate that approximately $5 \times 10^{20}$ of the $10^{24}$ released ions were ionized. Torbert and Newell [74] point out that this level of ionization can be accounted for through photoionization in this experiment. However, using a charge exchange cross section $\approx 10^{-14}$ cm$^2$ and the measured ambient plasma density of $2 \times 10^4$ cm$^{-3}$, about .1% of the ions would be expected to ionize through charge exchange. The number of ions produced should be expected to be $\approx .001 \times 10^{24} = 10^{21}$. Hence, charge exchange reactions can also account for the number of ions produced in Star of Lima. In fact, the problem is not in explaining how the ions were produced, but in explaining why so few ions were produced. The answer is most likely that the charge
exchange cross section is somewhat lower than the assumed value of $10^{-14}$ cm$^2$

The relative roles of the photoionization and charge exchange mechanisms in producing the ions is not clear from the experimental data available in Star of Lima. It is clear, however, that CIV does not need to be invoked in order to account for the ionization observed.

**CRIT-I:**

In CRIT-I, diffuse ionization was observed several 10’s of kms away from the release point. The analysis of the data led Stenbaek-Nielsen et al. to determine a time constant of ionization of 1800 sec. In their work, Stenbaek-Nielsen et al. also assumed a barium-oxygen charge cross section of $10^{-16}$ cm$^2$, giving a time constant for charge exchange of 70000 sec. However, using the assumed value of $10^{-14}$ cm$^2$, the time constant for charge exchange is 1400 sec. Clearly, if the large Ba-O$^+$ charge exchange cross section is correct, charge exchange can account for all the ionization observed in CRIT-I.

**CRIT-II:**

Charge exchange contamination of CRIT-II has been examined by Swenson et al. CRIT-II observed diffuse ion production at a calculated rate of .8%/sec. Here, the charge exchange cross section was assumed to be $10^{-17}$ cm$^2$. Using this value, charge exchange is not a significant ionization source. However, using a value of $10^{-14}$ cm$^2$, charge exchange is shown by Swenson et al. to account for the entire .8%/sec ionization.

**5.6.1 Other Observations:**

The predictions here that space releases of Ba will lead to substantial charge exchange reactions, but not CIV are also consistent with the in situ measurements of hot electrons and lower hybrid waves made during Porcupine and Star of Lima. According to the theories developed here, the charge exchange reactions should form an unstable beam which will act to heat electrons through the M2SI. In the case of sounding rocket releases, the time scales are such that explosive growth in the plasma density is not achieved in CIV, yet the lower hybrid waves of the M2SI and the hot electrons should still be
observed. This is indeed the case in Porcupine and Star of Lima.

The review of the space releases given in Chapter 1 indicates some empirical correlation between increased ambient plasma density and CIV-like processes (enhanced ionization, hot electrons and lower hybrid waves). The releases observing the highest levels of enhanced ionization (Porcupine and CRIT-II) were performed at the highest ambient densities. The hypothesis that charge exchange reactions are the source of the observed enhanced ionization is also consistent with this empirical correlation. Higher ambient plasma densities will lead to an increased rate of charge exchange reactions.

Summary

Considering the number of releases considered here, the conclusion that CIV is inhibited by a combination of the time necessary to heat electrons and rapidly falling neutral density agrees well with experimental results. Instead of invoking a CIV mechanism, experimental observations of “enhanced” ionization of Ba can be accounted for by assuming a Ba-O⁺ charge exchange cross section 10⁻¹⁴ cm².

5.7 Remarks on IBSS

The IBSS releases recently performed from the space shuttle present a slightly different situation than the sounding rocket releases discussed above. In the case of CIV, a given plasma element will experience an increasing neutral density as the shuttle approaches. As seen in results above, this allows for ignition of CIV and significantly enhanced plasma densities in the vicinity of the space shuttle. The largest increases should occur during the Xe release. This experiment seems a much more promising means of observing CIV in the space environment than sounding rocket releases. Unfortunately, because of hardware problems, data is not yet available to conclude whether CIV actually was observed during this experiment.
5.8 Remarks on the Laboratory Experiment

For the laboratory experiments, CIV is inhibited neither by the time necessary to heat electrons nor by falling neutral densities. CIV in the laboratory experiments is initiated through seed ionization by pre-existing hot electrons.

As the results of the previous chapter showed, the predicted time scales for CIV to occur in the laboratory are consistent with the experimental observations.

5.9 Arcjet Firings: An Engineering Application

The previous sections have analyzed laboratory and space experiments based on results achieved from simulations of CIV. The goal of this section is to evaluate the possibility of CIV during a thruster firing from an orbiting vehicle. During a thruster firing in the ram direction, the velocity of the thruster effluents, relative to the ambient plasma, may exceed the critical velocity. If CIV does ignite, the plasma cloud that results may adversely affect spacecraft charging, sensor operation, and other aspects of the interaction between the spacecraft and the ionosphere. Hence, it is important to recognize that the possibility of CIV exists and to analyze its possible effect on the space vehicle's environment.

In the present case, a vehicle is considered to be traveling in LEO through an ambient plasma at 8 km/s perpendicular to the Earth's magnetic field. The ambient plasma is taken to be $O^+$ at $.2$ eV. The concern here is with a hydrazine arcjet firing in the ram direction. The arcjet is analyzed since it is a good candidate to experience CIV. Since an arcjet will produce a partially ionized effluent, the M2SI will immediately heat electrons. The requirement for charge exchange to heat electrons to reach first ionization is therefore removed. This will be idealized in the current section by letting the time to first ionization go to zero, i.e. $\tau_1 \rightarrow 0$. As an example, consider a hydrazine arcjet producing effluents of $N_2$, $H_2$, and $NH_3$. Taking an $I_{sp} \approx 500$ sec, the gas exit velocity is $u_e \approx 5$ km/s relative to the vehicle, or when fired in the ram direction, $u'_e \approx 13.$ km/s relative to the ambient plasma. Such a velocity would be above the critical velocity of the nitrogen ($v_e = 10.3$ km/s) and ammonia ($v_e = 10.7$ km/s).
The arcjet firing can be analyzed in a manner similar to the IBSS releases. Like IBSS, the neutral gas is assumed to obey

\[
\begin{align*}
\text{if } t < t_f : \quad n_n &= \begin{cases} \frac{C_0}{(r-V_n t)^2} & V_n t < r - r_0 < V_n t \\ 0 & \text{otherwise} \end{cases} \\
\text{if } t \geq t_f : \quad n_n &= \begin{cases} \frac{C_0}{(r-V_n t)^2} & V_n t_f + V_n (t - t_f) < r - r_0 < V_n t \\ 0 & \text{otherwise} \end{cases}
\end{align*}
\]

(5.30)

where $V_s$ is the spacecraft velocity and $V_n = V_s + V_i$ is the neutral velocity in the plasma frame. Here, $C_0$ is chosen as $4 \times 10^{22} \text{m}^{-1}$, $r_0 = 200 \text{m}$, $V_s = 8 \text{km/s}$ and $V_i = 5 \text{km/s}$.

As mentioned above, since the arcjet effluents are partially ionized, the time to first ionization will be ignored. The electron density as a function of position is then given by

\[
\frac{n_e}{n_{e0}} = \exp \left[ \int_{t_0}^{t_0 + T_{ir}} p_0 n_n dt \right]
\]

(5.31)

where $t_0 = (r - r_0)/V_n$ and $T_{ir} = (r - r_0)/V_s - t_0$. The result of the integration is

\[
\frac{n_e}{n_{e0}} = \exp \left[ p_0 C_0 \frac{1}{V_s r_0} \left( 1 - \frac{1}{\xi} \right) \right]
\]

(5.32)

which is the same as Equation (5.24), but with $E' = 0$.

Figure 5-8 shows the density after the spacecraft has traversed 80 km. The results are almost identical to the IBSS release. The sensitivity of the results to the value of the reaction rate is evident from the figure. Even for small values of the anomalous ionization rate, significant enhancements are seen in the plasma density. The plasma enhancements will continue as long as the thruster fires. Such density increases could significantly affect operation of on-board systems such as solar arrays and optical detectors. Hence, if CIV does ignite, the impact on the spacecraft may be large.
Figure 5-1: Neutral Density in Porcupine. The neutral cloud expands outward from the release point. The neutral density quickly drops during the expansion.

Figure 5-2: Neutral Density during IBSS. The neutral cloud expansion for the IBSS release is treated as a moving point source.
Figure 5-3: Ignition time and transit time for "Porcupine" release. The exponential-like curves are the time to reach first ionization. The linear curve is the transit time. For CIV to ignite, the ignition time must be less than the transit time.

Figure 5-4: Ignition time and transit time for IBSS release. Because of the high neutral density near the release nozzle, ignition is reached in IBSS during almost the entire release.
Figure 5-5: Fractional Ionization for Porcupine. The plasma density is seen to be enhanced by CIV only within 1-2km of the release point. For $p_0 = 10^{-16} \text{ m}^3/\text{s}$, the plasma density is increased by a maximum of only 30%.
Figure 5-6: Fractional Ionization for IBSS. IBSS should have observed highly enhanced plasma densities near the releases point. ($E' = 0.01$)
Figure 5-7: Comparison of Charge Exchange and Anomalous Ionization in Porcupine. The curves represent the relative intensity calculated from the current results. The markers are the data of Porcupine. (a) $E = 3 \times 10^{-5}$. In this case, the contribution to ionization from CIV is almost insignificant. Most of the beam ions created are produced by charge exchange. (b) $E = 3 \times 10^{-3}$. Here, CIV can significantly increase the number of ions produced, but only within 1 km of the release point. Case (a) results in 4% ionization of the neutrals, Case (b) gives .04%
Figure 5-8: Fractional Ionization for an arcjet. An arcjet may observed highly enhanced plasma densities near the nozzle.
Chapter 6

Conclusions and Recommendations

In this chapter, the results of the previous chapters are summarized. The main results of the previous chapters involve the existence of a CIV mechanism, the anomalous ionization rate to be expected from CIV, the implications of these anomalous ionization rates for observations of CIV in space based experiments, and the lack of CIV observed in space based experiments. After discussing and summarizing these results, recommendations are made for further laboratory and space based experiments to verify the stated conclusions.

6.1 Result: Mechanism of CIV

Chapter 2 discussed in depth the theory of the CIV mechanism. The situation covered in the chapter is a neutral beam propagating across a stationary background plasma. The plasma is assumed to be magnetized, and the neutrals have a velocity component perpendicular to the magnetic field in excess of the critical velocity as defined in Equation (1.1).

Chapter 2 considered CIV as a multi-step process. These steps are

- seed ionization of the neutral beam to establish an ion beam across a plasma background,
• operation of the M2SI in the parameter space of interest,

• heating of the electrons to energies above the neutral ionization energy,

• ionization of the neutral beam to form a positive feedback loop,

• polarization of the plasma to explain the braking of the relative neutral-plasma velocity to the critical velocity.

The first step is the seed ionization of the beam. This process is accomplished through charge exchange between fast neutrals and oxygen ions in space experiments and through ionization of neutrals by the hot electron tail in the laboratory experiments. This difference in seed ionization mechanisms plays a large role in the discrepancy between the observed CIV in laboratory experiments and the apparent lack of CIV in space experiments. The seed ionization in laboratory experiments is fast enough to establish the positive feedback loop. In space experiments, the charge exchange rates may be too slow to allow enough energy to be transferred to the electrons to start the CIV process. These concepts were analyzed in the simulation results of Chapters 4 and 5, where it was shown that the transit time of the neutral cloud in the space experiments may be shorter than the ignition time required for CIV to start.

The second step of the CIV process is the operation of the M2SI. The theory of this step was also considered in Chapter 2. Linear and nonlinear theory show that the M2SI is consistent with the experimental evidence of laboratory experiments of CIV. The hypothesis taken for this thesis therefore is that the M2SI forms the basis of CIV by acting as the energy transfer mechanism between beam ions and electrons. Since the M2SI is the main transfer mechanism for the beam energy, its operation is essential for CIV to occur. The parameter space in which the M2SI operates was determined through linear theory. The parametric search is summarized at the end of Section 2.1.2. The results indicate that the velocity and electron-neutral elastic collision frequency must both satisfy certain criterion for the M2SI to operate and that the space and laboratory experiments do satisfy these criterion. It was also shown that the growth rate of the M2SI is on the order of the lower hybrid frequency, fast enough to account for the laboratory data.
Although linear theory is capable of producing the conditions which must be met for the M2SI to occur, it does not provide information as to the fraction of energy that can be transferred to the electrons from the ion beam. In order to analysis this point, the third step in the CIV process, quasilinear and nonlinear theory were used to derive estimates for the efficiency of the energy transfer between the ion beam and electrons. The results indicate that up to half the beam energy can be transferred to electrons, with the electrons heating parallel to the magnetic field. This efficiency of energy transfer is sufficient to heat electrons to energies above the neutral ionization energy.

The next step in the CIV process is the completion of the positive feedback loop to establish a discharge-like process. In Chapter 2, a simple rate model was used to suggest that this was possible. This simple model was later verified through simulation. The simulations clearly show the establishment of an exponential plasma growth through a positive feedback loop.

The final step was the one with received the least attention in this thesis. This step is the braking of the relative plasma-neutral velocity to the critical velocity. In Chapter 2, simple theories were presented to suggest how the braking motion may occur. In the space experiments, it was suggested that the establishment of a polarized cloud establishes a pickup of the plasma. A force balance on the plasma cloud shows that an asymptotic value near the critical velocity can be obtained. In the laboratory experiments, the plasma is initially polarized and drifts across the magnetic field. For this model, it was suggested that the ionization process 'shorts out' the polarization and drops the relative velocity to the critical velocity where ionization no longer occurs.

In summary, theory and simulations all show that the five steps necessary for CIV can occur. These results however do not guarantee that all the necessary conditions can be met simultaneously. To establish that CIV can occur through the outlined mechanism a series of simulations were run to determine the anomalous ionization rates and ignition times of various gases.
6.2 Result: Verification of Theory by Simulation

Chapter 2 relied on a series of partial theories of the CIV mechanism to develop a coherent whole. Several of the parts of this coherent whole however relied on parameters such as efficiency factors which could not be determined, or could be determined only very approximately, from theory. The purpose of the simulations was to provide more realistic estimates of the possibility of CIV occurring. In order to make the simulations as realistic as possible, the current work used an implicit PIC code to allow the use of realistic mass ratios and collision cross sections. This improvement over previous studies allowed the first realistic estimates of the ignition time and anomalous ionization rates of CIV in real gas systems.

In Chapter 2, the results of simulations were compared against linear theory to provide mutual confirmation as to the correctness of both the assumptions of the theory and the proper operation of the simulation code. In general, the linear theory and simulation results agreed well. The simulations provided linear growth rates slightly below the predictions of linear theory. This is an artifact of the discreteness of $k$-space in the simulations. The smaller growth rates are not so inaccurate as to seriously affect the simulation results however.

The simulations were also shown to confirm the nonlinear saturation mechanism of the M2SI as outlined in Chapter 2. The M2SI saturates through plateau formation of the beam ions. As this plateau grows, the unstable region of velocity space becomes resonant with the electrons, causing electron tail formation. This tail formation is important in CIV, as it leads to suprathermal electrons which can effectively ionize the neutral beam.

Finally, many previous studies of CIV have used artificial mass ratios. The rationale for the use of artificial mass ratios is the mass ratio scaling of the M2SI instability as discussed in Chapter 2. The simulations were used in Chapter 4 to examine the efficacy of the artificial mass ratio assumption. The results of this section showed that in more realistic CIV simulations, artificial mass ratios may not produce realistic results.
6.3 Result: Ignition Times

Results of a typical CIV simulation were used to define a characteristic time and rate for CIV. The characteristic time was defined as the ‘ignition time’ and is related to the time it takes for a CIV process to achieve an exponential growth. Ultimately, the ignition time is a function of the seed ionization and wave processes operating in the plasma. For the space experiments, simulation results indicate that the ignition time is closely related to the charge exchange time. An estimate is made that the ignition time is roughly 3 to 5 times the time to first ionization, and that $\gamma_1$ may be estimated through Equation (4.7).

In the laboratory experiment, the ignition time is related to the electron impact ionization time at the initial electron temperature. Ionization of the neutrals is begun immediately by pre-existing hot electrons. This leads to a rapid achievement of an explosive growth of the neutral beam.

6.4 Result: Anomalous Ionization Rates

If the ignition time is shorter than the transit time of the neutral cloud, a self-sustaining discharge-like exponential increase in plasma density can be achieved in CIV. The rate of plasma increase has been defined in the current work as the ‘anomalous ionization rate’, $\nu_{\text{ion}}$. This anomalous ionization rate can be found from the simulations by examining the rate of increase of the number of particles in the simulation once the electron temperature has saturated. $\nu_{\text{ion}}$ has been calculated for a number of gases as a function of velocity and density, and the results are presented in Chapter 4. The important points of these simulation are

- Near $V_n/V_e = 1$, the anomalous ionization rate goes to zero. This is consistent with the basic concept of the critical velocity. Near or below the critical velocity, a self-sustaining process is not energetically possible.

- For a neutral density of $n_n/n_e = 10^6$, the anomalous ionization rates are greater than the ion cyclotron frequency for $V_n/V_e \approx 2$. 

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• The anomalous ionisation rate is shown to be a function of the neutral beam energy, i.e. \( \nu_{\text{ion}} \sim (V_n/V_e)^2 \).

• Below \( n_n/n_e \approx 10^7 \), the anomalous ionisation rate shows a linear dependence on the neutral density.

• For \( n_n/n_e > 10^7 \), the 'windowing effect' is noticeable. The windowing effect occurs when an excitational level of the neutral exists such that the electrons can energetically excite a neutral can ionize the neutral. For high neutral densities, electrons are more probable to collide and excite a neutral without ionizing. The loss of energy to the excitational modes of the electrons can slow or even quench CIV. As mentioned, the density at which the windowing effect becomes important seems to be \( \approx 10^7 \). Such high densities will only occur in space experiments quite near the release point.

6.5 Result: Explanation Laboratory/Space Experiment Discrepancy

The theory and simulations both show that CIV should be an achievable result. Yet space experiments have failed to provide conclusive proof that CIV can exist in the ionosphere. Chapter 5 addresses this point.

The results of a simple rate model indicate that beyond several kilometers from the release point the transit time of the neutral cloud generated in the chemical releases may be shorter than the ignition time. Hence, CIV cannot become a self-sustaining, exponential process. Such a result is consistent with the in situ data collected during CIV experiments that lower hybrid waves are present and that some some hot electrons are produced. The initial seed ionization process is beginning to work, but by the time the electrons are sufficiently hot to complete the feedback loop, the neutral density and velocity have dropped to where CIV is no longer sustainable.

Within several kilometers of the release point, the results indicate that ignition of CIV will be achieved. However, CIV may still not produce a significant number of ions.
when compared to charge exchange reactions. Quantitative estimates indicate that the plasma density near the release point may be increased by 30% or less by the CIV process.

The results of the simulations are consistent with the empirical correlation of the space experiments showing higher levels of CIV-like processes (hot electrons, some enhanced ionization, lower hybrid waves) occurring when the ambient plasma density is highest. Higher ambient plasma densities will increase the rate of charge exchange reactions. This in turn will bring the experiment closer to achieving ignition.

For the laboratory experiments, the seed ionization process is quite different from the space experiments. The laboratory experiments begin with a hot electron tail capable of initiating a neutral beam. In addition, the electron temperature is initially near the ionization potential. Hence, only a small amount of additional heating of the electrons is required to begin the CIV process. In the space experiments, the cold ambient plasma must be heating by almost two orders of magnitude (taking ionization potentials on the order of 10eV, and initial electron temperatures of .1eV). Also, in the laboratory experiments the neutral density is spatially uniform, as opposed to the $1/r^2$ expansion of the neutrals in the space experiments. Hence, in the laboratory experiments, newly created plasma does not drift into regions of lower neutral densities where it is more difficult to sustain CIV.

6.6 Result: No CIV was Observed in “Porcupine”

If the above arguments concerning the lack of CIV in space experiments are correct, an alternative mechanism must be provided to account for the results of Porcupine and other barium releases in which enhanced ionization was observed. This alternative mechanism was discussed in Chapter 5. The discussion focused on the possibility of contamination of the results of the barium releases by charge exchange reactions with the background plasma. It has recently been suggested that the Ba-O+ charge exchange cross section may be much larger than previously assumed. In fact, Swenson et al. [72] have argued that the barium-oxygen charge exchange cross section may be as large as $10^{-14}$ cm$^2$. Chapter 5 showed that this assumption of a large charge exchange cross section can be used to account for the ion production rate observed in Porcupine and other barium
shaped charge releases.

6.7 Result: Usefulness of Implicit PIC Methods

The summary of the results given above has concentrated on the achievements made in understanding CIV. Yet, simultaneously, this research has shown the usefulness of employing implicit PIC methods in simulating CIV and other low frequency processes. The simulation results given here represented the first simulations of CIV performed with realistic mass ratios. Because of this, these results also represent some of the first realistic estimates of the ignition times and anomalous ionization rates. These estimates have led to the suggestions above explaining the lack of CIV seen in space experiments. Such conclusions would not be possible without using realistic mass ratios.

6.8 Recommendation: Laboratory Experiments

The experiments usually referred to in providing proof for CIV are those performed by Danielsson and Danielsson and Brenning. These experiments were performed in the early 1970s. Since space experiments have failed to prove the existence of CIV in the ionosphere, it would be worthwhile to repeat experiments similar to those of Danielsson with improved, modern plasma diagnostics.

One specific point regarding the plasma diagnostics is highly recommended. The simulation results indicate that in order to achieve CIV in the laboratory experiments, the initial electron temperature must be somewhat higher than the 5-10eV suggested by previous experimental work. However, experiments done by Danielsson and Danielsson and Brenning did not measure the electron temperature near the neutral interaction region, but only near the plasma generator. After the electrons are generated in the plasma gun, they travel along a magnetic field which is slowly rotated. The rotation of the magnetic field, in addition to polarizing the plasma, can easily lead to some types of streaming instabilities which would act to heat the electrons. It does not seem unlikely therefore that the electron temperature entering the interaction region may be about 15eV or higher. Improved diagnostics should allow the measurement of the
electron temperature near the interaction region. Since this is a key parameter in the interpretation of the results, the proper measurement of this quantity seems vital.

As a second point, a more detailed description of the plasma density as a function of position and time would be useful. The diagnostics of Danielsson focus on measuring the plasma-neutral relative velocity. This velocity should be related to the state of polarization of the gas, which in turn should be related to the degree of ionization of the neutral cloud (shorting out the polarization). In order to properly understand this process, more information is needed regarding the plasma density.

6.9 Recommendation: Determination of the Ba-O\(^+\) Charge Exchange Cross Section

As described in Chapters 4 and 5, an important value needed to understand the results obtained from space experiments is the Ba-O\(^+\) charge exchange cross section. Values used for this cross section in the literature vary from 10\(^{-17}\) to 10\(^{-14}\) cm\(^2\). For proper interpretation of the experimental results, it is imperative that this cross section be measured.

6.10 Recommendation: Space Experiments

On the basis of the results of Chapter 4 and 5, an improved design of CIV space-based experiments can be suggested. A schematic of an idealized experiment is shown in Figure 6-1. The experiment consists of a Xe release performed from a source moving at orbital velocity (similar to IBSS). The release should be performed at a high altitude (\(\approx 800\) km), with Xe a good choice of the release gas. The choice of a high altitude and an inert gas are made to reduce the possibility of contamination of the experiment by interaction with the ambient plasma. In addition to reducing contamination through collisions with the background plasma particles, using Xe at a high altitude will not provide enough seed ionization to ignite CIV. To overcome this, it is proposed that an ionization source be included in the experiment to control the seed ionization process. Instead of releasing a neutral beam, the beam should be partially ionized as it is released.
The ions produced at the source will initiate the M2SI and heat ambient electrons which can then ignite CIV. Ideally, the fraction of the beam ionized should be variable in order to study the properties of CIV as a function of the seed ionization rate. An important point is, as much as possible, to keep the beam from diverging. The ideal situation would be to produce a cylindrical neutral beam which propagates across the magnetic field with constant density. The density of the beam should be near \(10^{12} \text{ cm}^{-3}\). Suggested diagnostics for the experiment include measurement of wave and particle properties on the axis of the beam several hundred meters in front of the source and along the field line several hundred meters above the source.

The results of the previous chapters indicate that the experiment outlined here should lead to CIV. The results of the experiment should indicate generation of lower hybrid waves, production of hot electrons, and a sharp increase in the plasma density as CIV ignites.

### 6.11 Recommendation: Simulations

Although the implicit PIC method allowed simulations to be performed with realistic mass ratios, it also imposed several severe restrictions on the simulation, the greatest of which concerned the boundary conditions. Several recommendations are made for future development of implicit PIC methods:

- Further work must be done on understanding the numerical cooling of electrons in the simulation. Presently, the numerical cooling is combatted by increasing the numerical smoothing. As seen in Chapter 3, however, increased numerical smoothing unacceptably increases the bandwidth of the field equation matrix. This in turn restricts the boundary conditions to those which are amenable to transform solutions.

- The stability properties of implicit methods should be further investigated. There has been some suggestion in the literature that the implicit methods can lead to nonlinear numerical instabilities. The present code has shown evidence of instability in some situations which should be linearly stable. The cause of these
instabilities should be better understood.

- currently, very large spatial regions cannot be adequately modeled using implicit PIC methods. For $\lambda_{D_e} \Delta z \gg 1$, the particles heat numerically due to the lack of quasineutrality. Ways of controlling this numerical heating need to be devised.
Figure 6-1: A suggested experimental set-up. The release takes place at an altitude of 800km to remove the possibility of charge exchange contamination. The release gas is partially ionized to ignite CIV.
Appendix A

Verification Tests

This appendix includes a series of test runs performed to verify the proper operation of the computer simulation software. Each of the test cases is taken from the literature. Test cases are

- ion-acoustic fluctuations of a uniform, thermal, unmagnetized, two-temperature, one-dimensional plasma,

- modified two-stream instabilities,

- periodic, one-dimensional reduced-mass simulations of CIV, and

- periodic, two-dimensional reduced-mass simulations of CIV.

Each of these test cases will be discussed below. Differences in results between the current simulation and the test cases will be discussed. It is important to remember throughout that particle-in-cell codes are statistical in nature and will experience substantial “shot noise” in each run. The results are not expected to match the published results exactly.

Ion-acoustic Fluctuations

The first test case employed is the measurement of the thermal spectrum in a two-temperature, one-dimensional plasma. This case also illustrates the energy conservation properties of the code.
A Maxwellian plasma is loaded uniformly at $t = 0$ with $N_0 = 9216$ ions and electrons, $\Delta z/\lambda_{De} = 20$, $N_e = 128$, $T_e/T_i = 20$, and $m_i/m_e = 100$. The plasma is unmagnetized. Gaussian smoothing with $a = 3$ is used. The time step is fixed at $\omega_{pe}\Delta t = 10$, and the total time of the simulation is $\omega_{pe}\Delta T = 10^5$.

Figure A.1 compares the energy spectrum of the simulated plasma with analytic results for the Maxwellian plasma. As given by Krall and Trivelpiece [39], the fluctuation spectrum of the two-temperature Maxwellian plasma is

$$\frac{<E^2_k/8\pi>}{T_e/2} = \frac{T_i}{T_e} \frac{k^2 \lambda_{De}^2 |h(k)|^2}{1 + k^2 \lambda_{De}^2 |h(k)|^2} \cdot \left\{ \frac{\Theta + \sqrt{(m_e/m_i)(T_i/T_e)}}{\Theta + \sqrt{(m_e/m_i)(T_i/T_e)^{3/2}}} \right\} \quad (A.1)$$

where

$$\Theta = \exp \left\{ -\frac{T_e/2T_i}{1 + k^2 \lambda_{De}^2 |h(k)|^2} \right\} \quad (A.2)$$

In this expression, the term

$$\frac{T_i}{T_e} \frac{k^2 \lambda_{De}^2 |h(k)|^2}{1 + k^2 \lambda_{De}^2 |h(k)|^2} \quad (A.3)$$

represents the fluctuation spectrum of the plasma if the electrons are treated adiabatically. The correction term,

$$\left\{ \frac{\Theta + \sqrt{(m_e/m_i)(T_i/T_e)}}{\Theta + \sqrt{(m_e/m_i)(T_i/T_e)^{3/2}}} \right\} \quad (A.4)$$

represents the contribution of Landau damping to the fluctuation energy from resonant electrons and ions. Hence, if Landau damping is correctly modeled in the simulations, the result should agree with Equation (A.1). Incorrect modeling of Landau damping would result in a fluctuation spectrum given by the expression in (A.3). The figure presents the results from Equation (A.1) (the upper curve) and the results neglecting the Landau damping term (lower analytic curve). The simulation results match the analytic results over most of the spectrum, including the correct calculation of the Landau damping of the plasma. At the right hand side of the figure, the theory used for the analytic curve breaks down as $k\lambda_{De} \sim 1$. This explains the difference in the analytic and numerical solutions for the larger values of $k\lambda_{De}$.

Figure A.2 shows the total energy conservation of the code. Over approximately $10^4$
time steps, the simulation looses approximately 6% of the total energy. This is somewhat better than the results achieved by Barnes, where approximately 10% of the energy was lost in the same number of time steps.

Modified Two-Stream Instability:

The behavior of the M2SI was discussed extensively in Chapter 4. The results of one-dimensional M2SI were shown to agree well with results from the linear dispersion relation developed in Chapter 2. In addition, the mass ratio scaling discussed in Chapter 2 was exploited to compare implicit simulations using realistic mass ratios with reduced mass ratio results. A run of the ES1 code was included for comparative purposes. The results agreed well.

One-dimensional CIV: Machida and Goertz, 1986

One of the most complete set of one-dimensional, periodic simulations of CIV has been performed by Machida and Goertz [47]. In these section, the results of one of their simulations is compared to an implicit PCIVIC run.

The one-dimensional simulations begin with a Maxwellian, magnetized (both electrons and ions) plasma \((m_i/m_e = 100)\) uniformly distributed along the \(z\)-axis. A magnetic field is applied at an angle \(\theta_0 = \sqrt{m_e/m_i}\) to the \(z\)-axis. The strength of the magnetic field is such that \(\omega_{pe}/\Omega_e = 0.5\). A constant, uniform neutral background propagates in the \(z\)-direction. The current comparison will consider ‘Case C’ of the reference. For this case, the neutral velocity \(v_n/v_e = 2.0\). Electron elastic, ion elastic, electron impact excitation, electron impact ionization, charge exchange and photoionization collisions are all included in the model. Other parameters are summarized in Table A.1.

The results from PCIVIC run with \(\omega_{pe0}\Delta t = 2.0\) are compared with the results of Machida and Goertz in Figure A-3. The subfigures show the total number of particles in the simulation, the electron temperature, and the field energy as a function of time.

The agreement between the current results and Machida and Goertz for the plasma density as a function of time is quite good over most of the run. Near the end of the run, the implicit results do begin to fall somewhat below the Machida and Goertz results.
This is most likely due to the stringent test of using an implicit code with the artificial mass ratio. The artificial mass ratio of $m_i/m_e = 100$ leads to an initial lower hybrid frequency of $\omega_{LH} \approx 0.08$. It is desirable to maintain $\omega_{LH} \Delta t \ll 1$ but also $\Omega_e \Delta t \gg 1$. The chosen value of $\omega_{pe0} \Delta t = \Omega_e \Delta t = 4$ (giving $\omega_{LH0} \Delta t \approx 0.16$) is a reasonable compromise between these criteria. Near the end of the simulation, however, with $N_e/N_{e0} = 8$, the lower hybrid frequency has shifted until $\omega_{LH} \Delta t \approx 0.45$, which is somewhat higher than desirable. At this point, several strong wave modes are not being resolved. This is noticeable in the density plot as the divergence between the explicit and implicit results.

By not resolving the strong wave modes, the energy transfer mechanism between the ions and electrons is being damped, leading to a drop in the rate of plasma production in the implicit simulation.

The electron temperature also shows a slightly lower saturation level than the results of Machida and Goertz. This is again probably related to the shift in the lower hybrid frequency. One point should also be made about the electron temperature near the beginning of the run. The plot of electron temperature versus time in the paper clearly shows an initial electron temperature near $T_e = 0.01$. Unfortunately, this value is inconsistent with the stated initial conditions. The conditions shown in Table A.1 lead to an initial electron temperature of $T_e = 0.046$. Since the rest of the run seems consistent with both the calculations performed here and the theoretical discussion given in the reference, it is unclear why this discrepancy exists near the beginning of the simulation.

The field energy initially rises at roughly the same rate in the implicit and explicit simulations. Saturation in the field energy is reached at $\Omega_e T \approx 2000$. At this point, the explicit simulations have reached a somewhat higher value than the implicit simulations. As mentioned before, this is quite common for implicit codes since some high frequency modes have been damped out. The divergence in the results for $\Omega_e T > 4000$ is most likely again the result of the shift in the lower hybrid frequency. For later times, some of the stronger wave modes are not being resolved. Overall, field energy is difficult to compare directly since it is highly sensitive to spatial and temporal smoothing applied in the code. A better quantitative measure is probably the power spectrum. The spectrum for the PCIVIC run is shown in Figure A-4. This figure may be compared to Figure 8 of
the reference. Upon comparing the figure, good agreement is seen in the lower frequency range. The strongest modes are those associated with the lower hybrid frequency at about $\omega/\Omega_c = .1$. The results of Machida and Goertz also contain some modes near the upper hybrid frequency. These modes are of course not seen in the implicit results since these high frequencies are not resolved.

Two-dimensional CIV: Machida and Goertz, 1988

In their 1988 paper, Machida and Goertz presented results for a two-dimensional, doubly-periodic, electromagnetic simulation of CIV. Although the electromagnetic simulations are not comparable to the electrostatic code used throughout this thesis, one case included in Machida and Goertz's work was electrostatic (Case E of the reference). This electrostatic result is now compared with the results from the similar case run using PCIVIC.

This simulation begins with a Maxwellian plasma ($m_i/m_e = 100$) in the $x - y$ simulation plane. A uniform, constant neutral background propagates across the simulation region with $V_n/V_e = 2.5$. The magnetic field lies in the positive $y$ direction and has a strength such that $\omega_{pe}/\Omega_c = 1$. Initially, there are 4096 each of electrons and ions. The ions are treated as unmagnetized particles. Photoionization and electron impact ionization are the only collision processes included. Other parameters are summarized in Table A.2.

A comparison of the current code with the results of Machida and Goertz are presented in Figure A-5. The three subfigures display the total number of particles in the simulation, the mean electron kinetic energy, and the electrostatic field energy as a function of time. The current code was run in both and explicit ($\omega_{pe0}\Delta t = 0.2$) and implicit ($\omega_{pe0}\Delta t = 2.0$) mode. Both of these results are included in the figure with the results of Machida and Goertz.

The results of the current code in explicit and implicit mode and the results of Machida and Goertz agree well for the plasma production rate. Some divergence in the plasma production is noticeable near the end of the simulation. This is similar to the divergence seen in the one-dimensional simulations. $\omega_{LH}T$ is somewhat larger than
desirable near the end of the run.

The agreement in the mean electron kinetic energy as a function of time is also good. The results of Machida and Goertz show a slightly higher electron temperature before $\omega_{pe}T \approx 200$, but this is probably not significant. (It should perhaps be noted that the values given for Machida and Goertz’s data were simply read off of the rather small figure given in their paper. Hence, the accuracy of the results of Machida and Goertz given here is simply the resolution of a pencil, straightedge and eyeball).

The final panel of the figure shows the field energy as a function of time for the explicit and implicit code. (The results of Machida and Goertz are not included on this figure since the results in their paper were presented on too small of scale to reproduce accurately here). The field energy in the implicit code is smaller than in the explicit code. This is common for implicit codes and is a result of the damping of the high frequency waves. Qualitatively, the results also agree well with Machida and Goertz, with a gradual rise until $\omega_{pe0}\Delta t \approx 200$ and saturation beyond that point.
Figure A-1: Ion acoustic fluctuations: Calculated energy spectrum

Figure A-2: Ion acoustic fluctuations: Energy Conservation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_i/m_e$</td>
<td>100</td>
</tr>
<tr>
<td>$V_{ce0}/V_{i0}$</td>
<td>10</td>
</tr>
<tr>
<td>$V_n/V_{i0}$</td>
<td>8</td>
</tr>
<tr>
<td>$\omega_{pe0}/\omega_{ce}$</td>
<td>.5</td>
</tr>
<tr>
<td>$q_{ext}/(n_0\omega_{pe0})$</td>
<td>.000434</td>
</tr>
<tr>
<td>$\nu_{ion}\omega_{pe0}$</td>
<td>.002</td>
</tr>
<tr>
<td>$\nu_{ce}/\nu_{ion}$</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table A.1: Parameters used in the 1-d simulation of Machida and Goertz [47]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_i/m_e = 100 )</td>
<td></td>
</tr>
<tr>
<td>( V_{e0}/V_{i0} = 10 )</td>
<td></td>
</tr>
<tr>
<td>( V_n/V_{i0} = 13.3 )</td>
<td></td>
</tr>
<tr>
<td>( \omega_{pe0}/\omega_{ce} = 1)</td>
<td></td>
</tr>
<tr>
<td>( q_{ext}/(n_0\omega_{pe0}) = .0049 )</td>
<td>( L_x/\Delta x = 32 )</td>
</tr>
<tr>
<td>( \nu_{ion}\omega_{pe0} = .04 )</td>
<td>( L_y/\Delta y = 32 )</td>
</tr>
<tr>
<td>( \Delta x/\lambda_{De} = 4.7 )</td>
<td></td>
</tr>
<tr>
<td>( \Delta y/\lambda_{De} = 6.6 )</td>
<td></td>
</tr>
</tbody>
</table>

Table A.2: Parameters used in the 2-d simulation of Machida and Goertz, 1988 [48]
Figure A.3: One-dimensional CIV test against Machida and Goertz, 1986 [47]: Density, Temperature, Field Energy. The solid lines are the results of the current code. The points represent the results of Machida and Goertz.
Figure A-4: One-dimensional CIV test against Machida and Goertz, 1986: Power Spectrum. The power spectrum is peaked near the lower hybrid frequency. This agrees with the results of the reference.
Figure A-5: Two-dimensional CIV test against Machida and Goertz, 1988: [48] Density, Temperature, Field Energy. The solid lines represent the current code run in explicit and implicit mode. The points represent the results of the reference.
Appendix B

Collision Cross Section

References

The probabilistic collision model included in the particle in cell code discussed in the previous two appendices requires the collisional cross section as a function of particle energy. This section discusses the values chosen for the cross sections. A search of the literature has shown that several of the cross sections desired for the present study have not been measured or have only been measured at energy ranges outside that needed for this work. In these cases, estimates have been made for the cross sections.

In general, the assumption is made that the neutral density of the beam species is much greater than any ambient neutral density. Under this assumption, collisions with ambient oxygen neutrals are not included in the model. Collisional processes included consist of (1) electron elastic collisions with the neutral beam species, (2) electron impact excitation of the neutral beam species (line excitation for monatomic species, vibrational modes for diatomic species), (3) electron impact ionization, (4) non-resonant charge exchange between between ambient ions and beam neutrals, and (5) resonant charge exchange between beam ions and beam neutrals.
B.1 Carbon Dioxide-Oxygen

The three dominant vibrational modes with resonances at 0.08, 0.3, and 0.6 eV are included for the CO₂ runs. The ionization energy of CO₂ is 13.8eV. The dissociative reaction \( O^+ + CO_2 \rightarrow O_2^+ + CO \) is not included here since its cross section drops quickly above 0.2eV. The resonant charge exchange cross section is an estimate.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>e + CO₂ elastic</td>
<td>[75]</td>
</tr>
<tr>
<td>e + CO₂ vibrational excitation</td>
<td>[62]</td>
</tr>
<tr>
<td>e + CO₂ \rightarrow 2e + CO₂⁺</td>
<td>[64]</td>
</tr>
<tr>
<td>CO₂ + O⁺ \rightarrow CO₂⁺ + O</td>
<td>[67]</td>
</tr>
<tr>
<td>CO₂ + CO₂⁺ \rightarrow CO₂⁺ + CO₂</td>
<td>[67]*</td>
</tr>
</tbody>
</table>

*=estimated cross section

B.2 Neon-Oxygen

A single line excitation reaction is assumed at \( \Delta E = 16.9 \text{ eV} \); the ionization energy is 21.56eV. The non-resonant charge exchange reaction is estimated from the data for argon and krypton given in the reference.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Reference</th>
</tr>
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<tbody>
<tr>
<td>e + Ne elastic</td>
<td>[21]</td>
</tr>
<tr>
<td>e + Ne line excitation</td>
<td>[21]</td>
</tr>
<tr>
<td>e + Ne \rightarrow 2e + Ne⁺</td>
<td>[37]</td>
</tr>
<tr>
<td>Ne + O⁺ \rightarrow Ne⁺ + O</td>
<td>[35]*</td>
</tr>
<tr>
<td>Ne + Ne⁺ \rightarrow Ne⁺ + Ne</td>
<td>[65]</td>
</tr>
</tbody>
</table>

*=estimated cross section
B.3 Nitric Oxide-Oxygen

No excitational modes are included for NO. Although the data in the literature is somewhat sketchy, all vibrational mode cross sections are small ($\sigma < 10^{-17}$ cm$^{-2}$). The elastic cross section and nonresonant charge exchange cross sections are extrapolated above 10eV. The ionization energy of NO is taken as 9.2eV.

<table>
<thead>
<tr>
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<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e +$ NO elastic</td>
<td>[78]$^\dagger$</td>
</tr>
<tr>
<td>$e +$ NO $\rightarrow 2e + NO^+$</td>
<td>[38]</td>
</tr>
<tr>
<td>NO + O$^+$ $\rightarrow NO^+ + O$</td>
<td>[8]$^\dagger$</td>
</tr>
<tr>
<td>NO + NO$^+$ $\rightarrow NO^+ + NO$</td>
<td>[69]</td>
</tr>
</tbody>
</table>

$^\dagger$=extrapolated above 10eV

B.4 Nitrogen-Oxygen

The ionization energy of nitrogen is taken to be 15.58eV. The $\nu = 0 - 1$ vibrational excitation transition is included with $\Delta E = .289eV$.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Reference</th>
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<tbody>
<tr>
<td>$e + N_2$ elastic</td>
<td>[36]</td>
</tr>
<tr>
<td>$e + N_2$ line excitation</td>
<td>[36]</td>
</tr>
<tr>
<td>$e + N_2$ $\rightarrow 2e + N_2^+$</td>
<td>[36]</td>
</tr>
<tr>
<td>$N_2 + O^+ \rightarrow N_2^+ + O$</td>
<td>[54]$^\dagger$</td>
</tr>
<tr>
<td>$N_2 + N_2^+ \rightarrow N_2^+ + N_2$</td>
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</tr>
</tbody>
</table>

$^\dagger$=extrapolated cross section
B.5 Xenon-Oxygen

The ionization energy of xenon is taken as 12.13eV; a single line excitation reaction is included with $\Delta E=8.32eV$. The non-resonant charge exchange reaction is estimated from data for Argon.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Reference</th>
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<tbody>
<tr>
<td>$e + Xe$ elastic</td>
<td>[66, 33]</td>
</tr>
<tr>
<td>$e + Xe$ line excitation</td>
<td>[33]</td>
</tr>
<tr>
<td>$e + Xe \rightarrow 2e + Xe$</td>
<td>[33]</td>
</tr>
<tr>
<td>$Xe + O^+ \rightarrow Xe^+ + O$</td>
<td>[35]*</td>
</tr>
<tr>
<td>$Xe + Xe^+ \rightarrow Xe^+ + Xe$</td>
<td>[65]</td>
</tr>
</tbody>
</table>

* = estimated cross section
Bibliography


