MODELLING OF SHEAR-OFF FAILURE OF CONCRETE:
Fracture Mechanics Approach

by

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ABSTRACT

The objective of this research work is to develop the fracture mechanics based modelling strategy of shear failure of concrete through analyzing the shear-off failure of shear key joints. A very simple mechanical model for the analysis and design of plain concrete and fiber reinforced concrete shear key joints is developed. The method makes use of well-known results of fracture mechanics and truss model theory, combined in a simple model.

The analysis employs a single discrete crack model under wedging force and a smeared crack model under remote shear force. The proposed formulation identifies two main fracture mechanisms for shear-off failure of shear key joints: single curvilinear cracking and development of multiple diagonal cracks. The mechanical model is in good agreement with push-off shear test results reported in the literatures. Furthermore, a nonlinear finite element analysis is carried out for the push-off shear test to supplement the assessment of the proposed model, indicating that the mechanical model developed in this study is quite reliable.

The elegance of the proposed mechanical model lies not only in the simplicity and accuracy of the model but also in the fact that the necessary parameters (e.g., constitutive laws) are kept in the model so that design sensitivities can be generated, parametric studies can be carried out, and reasonable first estimates for the member sizes can be made. Thus, the mechanical model is viewed as an efficient design tool, and therefore it gives a rational basis for developing a suitable design methodology for shear key joints of plain and fiber reinforced concrete or similar materials.

The procedures developed in this study would be useful in designing shear key joints according to the precast concrete segmental method of construction, which is becoming increasingly popular. In addition, the present modelling strategy has been applied to the preliminary modelling of shear failure in membrane elements and deep beams. There is a reasonable agreement between the model prediction and experimental results showing that the present methodology is potentially a powerful tool for the analysis of general shear failure of concrete.
DEDICATION

to my daughter Mai
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MAJOR NOTATION

\( a_0 \) = crack size;
\( C \) = compliance;
\( D \) = depth of shear key;
\( E \) = Young's modulus;
\( E_c \) = Young's modulus of concrete;
\( E_s \) = secant modulus of concrete;
\( E_{st} \) = Young's modulus of steel;
\( e \) = element size;
\( F \) = vertical wedging force;
\( F' \) = lateral wedging force;
\( f_t \) = tensile strength of concrete;
\( f'_c \) = compressive strength of concrete;
\( f_{tu} \) = residual tensile strength of fiber reinforced concrete;
\( G_I \) = mode I energy release rate;
\( G_f \) = fracture energy;
\( H_{max} \) = maximum tensile stress factor;
\( h \) = band width;
\( K \) = stress intensity factor;
\( K_C \) = fracture toughness;
\( K_I \) = mode I stress intensity factor;
\( K_{II} \) = mode II stress intensity factor;
\( K_{Ia} \) = mode I stress intensity factor for wedging force;
\( K_{Ib} \) = mode I stress intensity factor for fiber bridging force;
\( K_{IC} \) = mode I fracture toughness;
\( k \) = function of crack length;
\( k' \) = function of fiber characteristics;
\( L \) = length of shear key;
\( \ell \) = crack length;
\( \ell_{ch} \) = characteristic length;
\( \ell_c \) = critical fiber length;
\( \ell_f \) = fiber length;
\( Q \) = load;
\( q \) = vertical load or vertical displacement;
\( t \) = thickness of composite;
\( V_f \) = fiber volume fraction;
\( W \) = strain energy;
\( Z \) = material parameter;
\( \alpha \) = threshold angle;
\( \beta \) = shear retention factor;
\( \gamma_{xy} \) = shear strain in x-y system;
\( \Delta \) = vertical shear slip displacement;
δ = crack opening displacement;
δ_u = ultimate opening displacement;
ε_c = compressive strain in c-direction;
ε_c0 = compressive strain at maximum compressive stress;
ε_c1 = compressive strain at 80% of maximum compressive stress;
ε_c2 = compressive strain at maximum compressive stress;
ε_cr = cracking strain;
ε_cu1,ε_cu2 = post-crushing compressive strain;
ε_t = tensile strain in t-direction;
ε_u1,ε_u2 = post-cracking tensile strain;
ε_x = normal strain in x-direction;
ε_y = normal strain in y-direction;
η0 = fiber orientation factor;
η_f = fiber length efficiency factor;
θ,Φ = angles;
θ_0 = angle corresponding to maximum tensile stress factor;
ν = Poisson's ratio;
ν_s = apparent Poisson's ratio;
ρ = angle;
λ = material parameter;
Π = potential energy;
σ_c = compressive stress in c-direction;
σ_fu = ultimate fiber stress;
σ_p = lateral prestress;
σ_t = tensile stress in t-direction;
σ_x = normal stress in x-direction;
σ_y = normal stress in y-direction;
τ_max-cal = calculated maximum shear stress;
τ_max-lo = maximum shear stress corresponding to lower limit of test data;
τ_max-up = maximum shear stress corresponding to upper limit of test data;
τ_ct = shear stress in c-t system;
τ_xy = shear stress in x-y system;
τ UF = ultimate bond strength of fiber;
ϕ_f = equivalent fiber diameter; and
Ω = complementary energy.
CHAPTER 1

INTRODUCTION

1-1. BACKGROUND AND IMPORTANCE OF THE PROBLEM

Precast segmental construction
The construction industry is comprised of many small firms which specialize in local markets, lack vertical integration, are not heavily capitalized, have low overhead and profit margins, rely on a floating labor force, and do little mass production. Current increase of labor cost and shortage of skilled workers spur these problems in the construction industry [Kaneko and Li, 1990]. To overcome these problems, reduction of required labor force in construction site by an efficient systematic construction method is desired. One of those methodologies is a precast segmental construction method.

The typical application of this methodology is precast concrete segmental bridges. A nice review on the precast concrete segmental bridges was provided by Bakhoun et al. [1989] and Bakhoun [1991], and is summarized briefly as follows. In recent decades, the precast concrete segmental method of construction has become increasingly popular. Increased speed of erection, improved aesthetics, and mitigation of environmental disturbances are among the factors which have made precast segmental construction the choice in many recent bridge projects. In addition, the various techniques within segmental construction are adaptable to a variety of span lengths, and quality control is improved. To ensure proper fit of the precast segments in the completed bridges, casting of the segments is carried out according to the 'match-casting method' whereby the faces of adjoining segments are cast against one another at the factory, so a nice fit is obtained when the segments are later re-assembled at the bridge site [Hurd, 1986].

The advantages of this method are efficiency, standardized mass production, speed of erection, the elimination of expensive formwork, and particularly to afford solutions for restricted construction access in congested urban or environmentally sensitive areas [Moreton, 1989]. Precast segments are fabricated away from the construction site in a casting yard or factory. Rapid production of high quality segments is achieved under the controlled factory manufacturing environment [Muller, 1975]. Completed segments are transported to the construction site via truck or barge, and they are assembled to the complete structures with cranes or gantries.

In the precast segmental construction method, the segments are precast, assembled in the appropriate position, and tied together by post-tensioning to form the completed
structures. This construction process creates contact joints between segments. Contact joints can be epoxied (having a thin layer of epoxy between the segments) or dry (without any epoxy). However, the major concern of using epoxy is that the transfer of stresses in epoxied sections may not be uniform because field conditions do not allow for precise application of the epoxied layer [e.g., Richard, 1988]. In addition, improper mixing in the application of epoxy has actually caused some unexpected failure. In contrast, the use of dry joints is easier in construction, and relieves the epoxy from any structural role thus avoiding problems associated with epoxied joints. However, it may be less durable in general.

The segments are usually cast with one or more integral shear keys at the segment ends. The shear keys are used for alignment during construction. With dry joints, the shear keys ensure mechanical interlock and transmit the shear force between the segments during construction and in the completed structures. On the other hand, with epoxied joints, the function of shear keys is more pronounced during construction because epoxy cannot resist any loads before its polymerization. In early bridges, a single large key was used in each web. The current trend, however, is to use joints in which several small keys, 5 to 20 keys, are distributed over the height of the web. The dimensions of these keys are much smaller than those used in single key joints, and they are usually not reinforced. Although they are more prone to damage during construction, the use of multiple keys is expected to ensure a more uniform transfer of stresses between the segments, thus providing a better mechanical interlock [FIP 1978, Podolny and Muller 1982, Mathivat 1983].

New structural systems

Similar to the adoption of the precast segmental construction method, the reduction of required labor force in construction site mentioned above may also be resolved by light-weight and small size of structural members [Kaneko and Li, 1990].

Present wide spread infrastructural decay could make the construction industry reconsider their usage of conventional construction materials. In the short term, urgency in rehabilitation creates a market for materials suitable for patching, resurfacing and other restoration work. In the long term, replacement of aged structures creates a market for materials with much longer life time which require minimal maintenance.

In unresolved engineering challenges such as containment of hazardous waste, structures used in tackling such problems will demand high performance materials with better controlled microstructures.
Existing construction materials such as steel and concrete are inadequate for applications in severe environments which involve a combination of high temperature, high corrosion potential, and highly localized loading. Underground, marine, and space construction are especially difficult to engineer and construct, and present unique opportunities for advanced structural concepts based on new more durable and lighter weight materials. Especially, the development of space structure could be one of the hottest topic which has been carried out in the civil engineering research [ASCE, 1988].

We presently have severe urban problems such as a big city concentration of every function in economy or culture and the related overpopulation. Saturated city functions are threatening our living space and environment. There might also exist in the future the demand to construct huge transportation stations including airports in cities. Thus, the size of cities will expand not only vertically, but also horizontally. To construct these huge city space, huge structures with ultra-long span and ultra-multi stories will be required to reorganize the city space. These engineering challenges may place additional demands on material performance evaluated on a unit weight basis.

The need for infrastructure renewal, the demand of specialized structures subject to severe environments, and the expansion of urban centers all suggest significant future contributions of advanced materials to the construction industry. Kaneko [1990] and Kaneko and Li [1990 and 1991a] carried out a qualitative analysis on applicability of advanced ceramics to construction based on a set of criteria, and investigated the feasible structural systems for ceramics. Most of structural systems introduced by them consist of shear keys, and some of the systems are precast segmental structures. Based on the fact that the fracture behavior of the shear key could control the behavior of the whole structure, they studied analytically the fracture behavior of the shear key structure by means of nonlinear fracture mechanics approach [Kaneko and Li, 1991b]. Based on their initial efforts, new structural systems for advanced materials may need further extensive works on the fracture behavior of shear keys.

1-2. OBJECTIVES

Based on the background discussed above, it could be expected that the precast segmental structures integrated through shear key joints will be used on a larger scale in the future. However, despite the importance very little information is at present available on the shear behavior and strength of shear key joints in precast segmental structures, and essentially no established methodology exists for the design. Thus, the research in this area is in general limited [Bakhoum et al. 1989, Bakhoum 1991]. Therefore, the prediction
of the shear failure behavior of shear key joints is studied as the present research target. In particular, shear key joints without steel reinforcement is considered. This is because such joints are used in both the multiple keys and the new structural systems for advanced materials mentioned above.

The addition of fiber reinforcement is also considered in this thesis based on the fact that fibers can contribute to the improvement of both shear strength and ductility, especially significant structural integrity of whole structures after the failure. This feature is also very significant if the structures are subjected to an earthquake. In addition, fiber reinforcement might ease the construction process in shear keys compared with other techniques such as the use of longitudinal steel rebars. This is because it requires less labor during casting and at the construction site. Specifically, steel fibers are mixed with the concrete, and used for casting the ends of the precast segments and shear keys.

The transfer of shear stresses in precast segmental structures is one of the most critical issues since the shear forces have to be transmitted through a weak plane with properties different from those of the rest of the structures, and the resistance of the section might lead to failures by shearing off the shear keys. Thus, the joints between the precast segments require special attention in design and construction. The deformations and load transfer mechanism of the joints will affect the structural integrity of the whole structures, its deflections and its ultimate load resistance capacity.

Bakhoum et al. [1989] and Bakhoum [1991] indicated that there was currently no specification for the design of joints in precast concrete segmental structures in the USA or elsewhere except the Post-Tensioning Institute [PTI, 1988] specification which recommended the use of different strength reduction factors for bridges with different joint types based on relatively limited test results. The current design for shear resistance of concrete is thus still based on empirical results despite the decades of research which have been carried out on the evaluation of the shear performance of concrete members.

Materials such as cement paste, mortars, concrete and all cementitious composites are heterogeneous and multi-phase systems. Their fracture process is extremely complex, and the relationship between fracture theory and engineering design is not clearly defined. This argument is especially true for shear failure. Agreement exists among researchers and practising engineers over the behavior of concrete under bending moments. Design formulae have been developed for the analysis of the bending capacity of reinforced concrete members. However, as soon as shear enters the scene the scientific world is divided and many different formulae were developed to predict shear capacity. The reason for this situation is that the formulae are mostly empirical and not based on clear physical understanding, which is opposite to the case of bending [Reinhardt, 1989].
Thus, the theoretical formulation for shear failure is often desired in most of concrete structures. Regarding the diagonal shear failure strength of reinforced concrete beams, nonlinear fracture mechanics has been applied, and the mostly theoretical formula was proposed [e.g., Bazant and Kim 1984, Bazant and Sun 1987]. However, no theoretical proposal for shear load deformation characteristics was provided so far. This argument is also applicable to the shear-off failure of shear key joints. As mentioned, the deformation and load transfer mechanisms of the shear key joints will affect the structural integrity of the whole structure, its deflections and its ultimate load resistance capacity. Thus, a theoretical approach which gives such information is necessary in the design of shear key joints.

On the other hand, there exist many numerical models by means of Finite Element Method or Boundary Element Method to simulate the fracture behavior of concrete. The currently available fracture models as the basis for the numerical approaches are not developed well for the mixed mode problem due to the lack of a sound theoretical model as will be discussed in Chapter 3. Therefore, one needs simplified or easily-handled approaches which can provide a good approximation.

Based on the above-mentioned discussion, the objective of the research work summarized in this thesis is to develop the fracture mechanics based modelling strategy of shear failure of concrete through the shear-off failure of shear key joints. Specifically:

1. To provide the interpretation of complex cracking phenomena in the shear-off failure of plain concrete or fiber reinforced concrete shear key joints;

2. To develop a simple mechanical model to predict the load deformation characteristics in the shear-off failure of plain concrete or fiber reinforced concrete shear key joints;

Here, the terminology 'Mechanical Model' mentioned above is only used for the proposed model in this research work. The mechanical model is defined in this thesis such that the model consists of both the discrete crack model (Wedge Crack Model) formulated by linear elastic fracture mechanics (LEFM) results and the smeared crack model (Rotating Smeared Crack Band Model) formulated by the truss model theory, based on the physical description of the shear failure of key joints. Thus, this mechanical model is developed by means of theoretical approaches which are different from numerical approaches such as Finite Element Method (FEM). To confirm, although many analytical models are reviewed
in this thesis, only the model developed in Chapter 4 is defined as the mechanical model in this thesis. This mechanical model is thus the main proposal in this thesis.

The verification of the proposed mechanical model is achieved by the comparison with both test results reported in the literatures [e.g., Bakhoum et al., 1989; Beattie et al., 1989] and the nonlinear Finite Element Method (FEM) analysis results. From the interpretation of the present analytical studies, a fundamental understanding of the physical behavior of the shear key joints under shear loading can be achieved. As a first step in developing design aids for the shear key joints, a simple design formula in a closed form is proposed based on the proposed mechanical model for evaluating the shear-off strength of the plain concrete or fiber reinforced concrete shear key joints under given normal stress.

1-3. RESEARCH SIGNIFICANCE

Design of shear key joints
The main achievement in this research work is the contribution to the design of shear key joints. As mentioned, the current available specification for the design of shear key joints [e.g., PTI, 1988] is still based on the relatively limited empirical results. Furthermore, such design code considers only the shear strength, but not the load-displacement characteristics. The reason for the use of empirical formula for the shear strength and for the lack of a theoretical proposal on the load-displacement characteristics at the present stage is that a clear physical understanding of the shear failure in the shear key joints is not available. Actually, the precise prediction of the deformation behavior is necessary in the design since the deflections in the shear key joints will affect the structural integrity of the whole structure. The influence of the load-displacement characteristics of shear key joints on the behavior of the whole structure will be quantitatively demonstrated in this thesis.

To resolve these problems, the mechanical model is formulated and verified to predict the load-displacement characteristics of shear key joints in this thesis. In addition, as a first step in developing design aids for the shear strength in shear key joints, a very simple design formula is also formulated and verified based on the analytical study by this mechanical model. Thus, the author believes that these achievements in this thesis can provide an impact on the design of shear key joints such that one can design the load-displacement behavior of shear key joints based on the clear physical understanding, and also employ easily the parametric study for different material properties or geometries. Furthermore, economical design can be achieved since one does not have to resort to abstract safety factor in the empirical formula.
Fracture modelling

The contrast between the previous fracture models and the proposed mechanical model in this thesis is described as follows:

1. Previously, one employs a discrete crack model or a smeared crack model, separately. In the present work, the discrete crack model and the smeared crack model are combined into the simple mechanical model proposed in this thesis. This combination is especially important in the case of direct shear failure of concrete structures in which we observe both a single discrete crack and multiple smeared cracks. Previously, one does not pay attention to the discrete crack since such cracking would not contribute to the shear strength of the structure. However, this discrete crack becomes important if one needs to predict the load-displacement characteristics in the structures. The detail will be discussed in this thesis.

2. Previously, a representative tension crack model for a wing crack propagation from a preexisting flaw under biaxial remote compression is proposed [e.g., Horii & Nemat-Nasser, 1986], which can predict such wing crack propagation with considerable accuracy. In addition, a rotating smeared crack model is implemented in the numerical FEM analysis [e.g., Rots and Blaauwendraad, 1989], which can predict the softening behavior of structures without having to resort to abstract theories on the shear stress strain relation along the smeared crack. These two models are combined into the simple mechanical model proposed in this thesis as a representative rotating smeared crack model. This modelling strategy makes the present complex problem of shear failure of shear key joints easier and make the proposed mechanical model a very practical engineering model.

3. The fracture process of the shear failure of concrete is extremely complex resulting in a nonlinear problem. Therefore, in general one needs to employ a numerical iteration to obtain the analytical solution. To eliminate this complexity, an apparent Poisson's ratio is proposed in this thesis. This quantity is implemented in the proposed mechanical model to express the relationship between the tensile principal strain and compressive principal strain in the cracked element. Therefore, no numerical iteration is necessary in the calculation by the proposed mechanical model, and thus the model increases the practical significance without reducing the accuracy.
1-4. THESIS ORGANIZATION

The organization of the present research consists of four parts (see Fig.1.1): (1) to review literatures on the physical basis of shear fracture; (2) to propose a mechanical model; (3) to verify the proposed model; (4) to assess the proposed mechanical model by the examination of the applicability to other geometries and of the transition phenomenon of two distinctive types of cracking; (5) to show the implications of the proposed mechanical model in design of segmental structures.

The first part provides the interpretation of experimentally observed complex cracking phenomena in shear failure of concrete key joints and currently available fracture modelling strategy, which are used as the basis of the proposed mechanical model.

The second part provides a very simple mechanical model of the shear-off failure of shear key joints. The proposed mechanical model is the main finding in this research and consists of two distinctive crack models: Wedge Crack Model and Rotating Smear Crack Band Model. In addition, as a first step in developing design aids for the shear strength in shear key joints, a very simple design formula is also formulated based on the analytical study by this mechanical model.

The third part provides the verification study on the proposed mechanical model by means of comparison with both experimental results in push-off shear key tests and a nonlinear FEM analysis. In the nonlinear FEM analysis, both smeared crack approach and discrete crack approach are employed with the simple constitutive models used in the proposed mechanical model. The intent to employ the FEM analysis is to supplement the verification study of the proposed mechanical model by the comparison with experimental results in push-off shear key tests which number is relatively limited.

The fourth part provides an additional discussion on both the applicability of the proposed mechanical model to other geometries such as reinforced concrete membrane elements and reinforced concrete deep beams, and the transition phenomenon of two distinctive types of cracking observed in shear failure of key joints.

The fifth part provides the implications of the proposed mechanical model in the design of segmental structures through the quantitative analysis of the behavior of a segmental concrete beam with shear key joints.

Specifically, the following contents are discussed in this thesis:

In Chapter 1, a general background to the study is provided. The objectives of the present research work, the research significance and the thesis organization are also presented.
In Chapter 2, the macroscopic shear failure phenomena of plain concrete or fiber reinforced concrete shear key-type structures are first reviewed. Two distinctive fracture mechanism in shear-off failure, a single curvilinear crack (hereafter called $S$ crack) and multiple diagonal cracks (hereafter called $M$ cracks), are then identified. To investigate detailed mechanism of the cracking phenomena on shear-off failure in key joints, the similarity of cracking is looked for between key joints and other geometry through the review of shear failures of: a) plain concrete and rock under direct shear tests; b) rock under compressive stress field; c) reinforced concrete element in the push-off shear test. Based on the reviewed phenomena, a physical description of the shear-off failure of key joints is provided.

In Chapter 3, the currently available fracture modelling strategy which can be applicable to the shear-off failure of key joints is reviewed based on the fracture mechanics approach. Basic crack models and fracture parameters are first reviewed. Next, main numerical approaches to implement the basic crack models, and the application of those approaches to mixed mode problem like shear-ff failure are briefly reviewed. In addition, theoretical approaches to simulate the similar fractures to both $S$ crack and $M$ cracks in shear key joints are also reviewed. These approaches will become the basis of the mechanical model proposed in this thesis for shear-off failure of shear key joints.

In Chapter 4, a very simple mechanical model for shear-off failure in plane and fiber reinforced concrete shear key joints using a fracture mechanics approach is developed. The analysis is based on a single discrete crack model under wedging force, and a smeared crack model under remote shear force. The proposed formulation identifies two main fracture mechanisms for shear-off failure of key joints: a single curvilinear crack ($S$ crack) and multiple diagonal cracks ($M$ cracks). Then, the two models are combined to predict the entire load-displacement behavior. The method is an approximate one, using well-known results of fracture mechanics and truss model theory, combined in a simple mechanical model.

In Chapter 5, the verification of mechanical model is examined through comparison of the prediction by the proposed mechanical model with both experimental results and FEM analysis results. In the comparison with experiments, two kinds of comparisons are achieved here: entire load-displacement relations and the shear strength. These comparison can show the accuracy of the proposed mechanical model in the prediction of macroscopic behavior. The second comparison with FEM analysis results is also employed to obtain load-displacement relations. The reliability of the FEM analysis is examined by comparison with the experimental observation in terms of the cracking status.
In Chapter 6, the applicability of the proposed mechanical model to the macroscopic shear failure of two types of shear-dominated structures: reinforced concrete membrane element subjected to in-plane shear; and reinforced concrete deep beams is examined. Since these structures do not show a distinctive S crack formation like shear key joints, only the smeared crack model developed for M cracks in the mechanical model is applied to these geometric/loading configurations. Based on this analysis, the adequacy of the proposed mechanical model to other but similar structures can be examined.

In Chapter 7, the transition between S crack and M cracks in shear key joints is investigated. Well-known elastic solutions are introduced here to analyze the principal strain distribution along the key base based on the fact that this transition is caused by the changed boundary condition induced due to the distinctive S crack formation. Then, these elastic solutions are compared with the principal strain distribution along the key base obtained by the nonlinear FEM analysis. Based on this comparison, the cause of the transition between S crack and M cracks is examined.

In Chapter 8, the current design approaches for shear capacity of segmental structures with joints are discussed, and the possible contribution of the proposed mechanical model to the design is qualitatively specified. To demonstrate the specified contribution of the mechanical model, a nonlinear FEM analysis is carried out, emphasizing the effects of the load-displacement characteristics of joints on the fracture behavior of segmental structures. Based on this investigation, the implications of the mechanical model in the current design of segmental structures is discussed.

In Chapter 9, the important results are summarized, and conclusions and suggestions for future work are presented.
(1) REVIEW OF LITERATURES
   a. Observation:
      - Shear-off Failure of Shear Key Joints
      - Similar Shear Failure
   b. Physical Description of Two Distinctive Cracking
      - S crack
      - M cracks
   c. Fracture Modelling Strategy
      - Basic Crack Model
      - Numerical Approach
      - Theoretical Approach

(2) DEVELOPMENT OF MECHANICAL MODEL
   a. Wedge Crack Model (WCM)
      - LEFM Formulation
   b. Rotating Smeared Crack Band Model (RSCBM)
      - Rotating Smeared Crack
      - Truss Model Formulation

(3) VERIFICATION STUDY
   a. Comparison with Push-off Shear Key Test Results
   b. Comparison with Nonlinear FEM Analysis

(4) ADDITIONAL DISCUSSION
   a. Applicability to Other Geometry
   b. Transition Phenomenon between S crack and M cracks

APPLICATION TO DESIGN

(5) INVESTIGATION OF IMPLICATIONS IN DESIGN

Fig. 1.1: Thesis Organization
CHAPTER 2

OBSERVATION OF SHEAR FAILURE

2-1. INTRODUCTION

In this chapter, the macroscopic shear failure phenomena of plain concrete or fiber reinforced concrete shear key-type structures are first reviewed, especially focussing on the experiments carried out at MIT [Bakhoum et al. 1989, Bakhoum 1991, Beattie 1989, Beattie et al. 1989] which results will be simulated in this thesis. Two distinctive fracture mechanism in shear-off failure, a single curvilinear crack and multiple diagonal cracks, are then identified.

To investigate detailed mechanism of the cracking phenomena on shear-off failure in key joints, the similarity of cracking is looked for between key joints and other geometry through the review of shear failures of: a) plain concrete and rock under direct shear tests; b) rock under compressive stress field; c) reinforced concrete element in the push-off shear test.

Based on the reviewed phenomena, a physical description on the shear-off failure of key joints is identified.

2-2. FAILURE OF CONCRETE SHEAR KEY

Three kinds of keyed joints are reviewed here, especially focussing on the macroscopic cracking behavior of key joints: (1) in precast segmental bridges; (2) in large panel concrete building and in corbel structures.

2-2-1. Dry Keyed Joints in Precast Segmental Bridges

As mentioned in Chapter 1, segments of the bridge are precast, assembled in the appropriate position, and tied together by post-tensioning to form the bridge superstructure. This construction process creates joints between the segments in which both cast-in-place and contact joints between the segments are used. Contact joints can be epoxied (having a thin layer of epoxy between the segments) or dry (without any epoxy). In most bridges with contact joints, shear keys are used in the webs, and sometimes the slabs, at both ends of precast segments [Bakhoum et al. 1989, Bakhoum 1991]. The functions of the keys are
to ensure proper alignment during construction and to transmit shear forces between the segments.

In contrast to the tests of dry and epoxied flat joints, very few tests have been performed on keyed joints. In this section, three such investigations are reviewed.

**Koseki and Breen [1983]**

Koseki and Breen [1983] studied the shear strength of segmental concrete bridge by testing specimens having a cross-section of 3 in. (76.2 mm) by 20 in. (508 mm) which was approximately one forth the size of a web in a typical segmental bridge. As a model concrete, microconcrete was used. The basic design mix proportion used was: 2/8 in. aggregate, #1 blast sand, Ottawa silica sand, Type III cement, water, admixtures (ASTM C494 Type B) and the water/cement ratio of 0.7. The average of the concrete strength was 6550 psi (45.2 MPa). The shear test arrangement is shown in Fig.2.1. The applied average prestress was 424 psi (2.92 MPa). The types of joints considered included no key, single large key (reinforced with 10 gage wire) and multiple keys (no reinforcement) shown in Fig.2.2. Both dry and epoxy joints were tested. Here, the epoxied joints are outside the topic area in this thesis, and we will concentrate on the dry key joints.

Results of these limited exploratory tests indicated that designing keys as corbels using either ACI or PCI specifications was conservative, and the predicted values were only 60% of the actual test results for both the single key and multiple key dry joints.

The following cracking pattern at failure of both single and multiple keys without epoxy was reported. In the specimen with single key joint shown in Fig.2.3, three types of major cracking characteristics were observed: (1) major flexural cracks starting at the junction of the top face of the male key and the end face of the segment; (2) shear cracks in the shear plane of the male key; (3) the splitting cracks in the shear key reinforcement planes of both adjacent segments which were accompanied by spalling-off in the male key region. The flexural cracks extended into the specimen with a downward angle of about 45°. The shear cracks were observed just after the flexural cracks.

In the specimen with multiple key joint shown in Fig.2.4, short diagonal cracks were observed in each key just before the major direct shear failure (keys on one side of the segments were completely sheared off). Each key showed complete shear-off plane, and the failure pattern was therefore defined as direct shear failure. Due to the characteristic of direct shear failures, the damage was well-confined within the key portion and it did not extend to other regions (in contrast with the case of a single key joint). In other word, the cracks were localized in the neighborhood of the keys.
Fig. 2.1: Shear Test Arrangement [after Koseki and Breen, 1983]

Fig. 2.2: Configuration of Joints [after Koseki and Breen, 1983]
Fig. 2.3: Crack Pattern of Single Key at failure [after Koseki and Breen, 1983]

Fig. 2.4: Crack Pattern of Multiple Key at Failure [after Koseki and Breen, 1983]
Fig.2.5: Comparison of Behavior of Joints [after Koseki and Breen, 1983]

Fig.2.6: Test Set-up for Flat Specimen [after Bakhoum et al., 1989]
The relationship between the applied load and the joint slip observed in the test was explained as shown in Fig.2.5. The specimen without key joint slipped at the first stage. However it continued to carry the load up to the failure load with increasing slip at the joint. The load vs slip relationship of single key specimen was essentially tri-linear. First, considerable slip took place in order for a single large key to act and develop high shearing stresses. After the frictional slip, the stiffness decreased. At a higher load, flexural cracking occurs at the key base. After the flexural failure of the key, the joint continued to carry the shearing load, but the slope of the load vs slip curve was much flatter than in the case of the dry no-key joint. It was supposed that the reinforcement in the vicinity of the key helped to maintain the load-carrying capacity. After the shear failure of the male key, which might have been accompanied by the splitting cracks in the key, the load dropped rapidly. The slip at the joint of the specimen with multiple key joint occurred gradually as the load increased, showing substantially larger joint stiffness at higher load levels than the other joints, and the curve was multi-linear. The slope of the load vs slip curve decreased continuously until the failure load was reached, when direct shear failure finally took place in the multiple keys.

From the experiment of Koseki and Breen [1983], it is obvious that the main cracking mechanism of the single shear key is the sequence of a flexural crack from the corner of the key, shear cracks along the key base and the splitting cracks in the reinforcement plane. Here, it should be noted that the splitting cracks could be caused by the reinforcement near the concrete face. Thus, if there is no reinforcement, such phenomenon may not be critical. On the other hand, the detailed cracking observation in the multiple keys is not provided although we see the overall shear-off mechanism along the key base. The following review will provide such observation.

Bakhoun et al. [1989] and Bakhoun [1991]
Bakhoun et al. [1989] and Bakhoun [1991] carried out push-off tests on both plain concrete flat joints and plain concrete keyed joints with different parameters. The test setup for flat specimen and key specimen are shown in Fig.2.6 and 2.7, respectively. The parameters considered in the tests were the joint type (flat or keyed joints), confining pressure (equivalent to prestressing in segmental bridges), surface preparation (dry or epoxied joints) and epoxy thickness. Here, the epoxied joints are outside the topic area in this thesis, and therefore we will concentrate on non-epoxied ones. The basic water, cement, fine aggregate, and coarse aggregate ratio, by weight, was reported as 0.40:1.00:1.25:1.60. The fine aggregate was mortar sand, and the coarse aggregate

-35-
consisted of 1/4 in., 3/8 in. and 1/2 in. The average concrete compressive strength from all the mixes was 7100 psi (49 MPa).

The monotonic loading tests were carried out for 3 dry flat specimens and 14 dry key ones using four levels of normal prestressing stresses: \( \sigma_v = 100 \) psi (0.69 MPa), 200 psi (1.38 MPa), 300 psi (2.07 MPa) and 500 psi (3.45 MPa). The vertical loading was applied at the upper portion of the specimen while the lateral prestressing loading was kept constant. The specimen of key joints had dimensions of 10 in. along the base and 15 1/2 in. along the outside edge and was 3 in. deep. The total height of the specimen was 21 in. The keyed specimens had one shear key which was a 1:1 model of a shear key, representing one of a series of multiple keys, used in actual bridges. The bases of the trapezoidal shaped key were 3 7/8 in. and 2 5/8 in., and the depth was 1 1/4 in. The centerline-to-centerline dimension of the keys was 6 in. The projected contact area on the shear plain was 6 in. by 3 in. or 18 square inches. It was emphasized that this dimensions were determined to guard against a bearing failure near the loading plates, crushing of concrete at the sharp corners, and flexural failure at the end of the concrete block. Mild reinforcement was also placed in the specimen to eliminate the flexural, bearing, or splitting failure. The specimen of the flat joints tests was slightly smaller. The outside dimensions for the flat joints were 7 in. on the base and 9 1/2 in. along the outer edge, and the specimen was 3 in. deep. The total height of the specimen was 15 in. The contact area on the shear plane was 9 square inches; this area was 3 in. high by 3 in. deep.

The introduced shear stress-slip curves for the dry flat joints (epoxy thickness of 0 mm) under different degrees of confining pressure are shown in Fig.2.8. Data were presented by plots in which the abscissa gave the values of the vertical slip and ordinate represented the shear stress of the joint. Shear stress was defined here as the load divided by a projected area of 9 sq. in. It was indicated that the relation was nearly linear up to the load level at which the joint slipped. Here, it seems to the author that the initial slippage occurred at the vertical displacement of about 0.02 mm to 0.03 mm, and this value could be identical to the linear limit of the shear stress-slip curves for dry keyed joints shown in Fig.2.9. After slippage, the slope of the curve dramatically decreased and the displacement increased gradually and infinitely under the constant ultimate load. They reported that no crack was observed on the specimen and the joint surfaces were not damaged.

Typical curves of introduced shear stress-slip for dry keyed joints with different degrees of confining pressure are shown in Fig.2.9. Data were presented by plots in which the abscissa gave the values of the vertical slip and ordinate represented the shear stress of the joint normalized with \( \sqrt{f'_c} \), where \( f'_c \) is the compressive strength of concrete.
Fig. 2.7: Test Set-up for Key Specimen [after Bakhoum et al., 1989]

Fig. 2.8: Shear Stress-Slip Curve for Dry Flat Joints at Different Confinement Levels [after Bakhoum et al., 1989]
Fig. 2.9: Shear Stress-Slip Curve for Dry Key Joints at Different Confinement Levels [after Bakhoum et al., 1989]

Fig. 2.10: Cracking Sequence in Shear keys [after Bakhoum et al., 1989]
Shear stress was defined here as the load divided by a projected area of 18 sq. in. The ultimate shear strength was defined as the maximum shear stress that the specimen carried. The introduced cracking sequence for dry key joints is shown in Fig.2.10. It was observed that the load increased continuously up to almost 70% of the maximum load, then a drop occurred in the curve. It was explained that this drop was accompanied by the formation of a curvilinear crack at the bottom corner of the key which propagated away from the shear plane at approximately a $45^\circ$-angle as shown in Fig.2.10. As the load increased, short diagonal cracks started to form and join along the root of the key. At the maximum load, the cracks at the root of the key joined together (shearing-off fracture), and the confining pressure kept the two parts of the specimen from separating. It was thus suggested that the failure plane had formed in the concrete leaving only aggregate interlock and friction to resist the load. At this stage, the initial curvilinear crack closed, and a sudden slip between the two parts of the specimen was observed. Then, the shear stress-slip curve became horizontal at a slowly decreasing rate as the aggregates on the shear plane were smoothed out.

At higher confining stresses, the drop in the load due to cracking at the bottom corner of the key was partially overcome. Specifically, the initial curvilinear crack at the root of the key was not as large, and smaller load occurs drop until the maximum load was observed. This may be interpreted as that this initial curvilinear crack would be arrested by the compressive stress perpendicular to the crack orientation.

The test results by Bakhoum et al. will be used to verify the mechanical model developed in this thesis.

Beattie et al. [1989] and Beattie [1989]
Beattie et al. [1989] and Beattie [1989] conducted an exploratory experimental program which utilized steel fibers in the concrete mix in an effort to improve the strength and safety of joint regions in precast concrete segmental bridges. In that program, push-off tests on steel fiber reinforced concrete (SFRC) keyed joints with different parameters were carried out. The geometric-loading configuration of the experiment and the parameters considered in the tests were basically the same as those of plain concrete [Bakhoum et al. 1989, Bakhoum 1991]. The basic water, cement, fine aggregate, and coarse aggregate ratio, by weight, was 0.38:1.00:1.14:1.46. The fine aggregate was standard mortar sand, and the coarse aggregate consisted of 1/4 in., 3/8 in. and 1/2 in. The average concrete compressive strength from all the mixes was about 7500 psi (52 MPa).

The monotonic tests were carried out for 24 dry key specimens using three levels of normal prestressing stresses: $\sigma_x=100$ psi (0.69 MPa), 300 psi (2.07 MPa) and 500 psi
(3.45 MPa). The vertical loading was applied at the upper portion of the specimen while the lateral prestressing loading was kept constant. Two types of steel fibers were used in the experiments: straight (Flexten) and deformed (Dramix ZL 30/0.50 hooked-end fiber). The Flexten fibers were 0.28x0.56x25.4 mm carbon steel rectangular fibers (ε_t=25.4 mm, φ_t=0.447 mm) made from steel sheet with a tensile strength of approximately 345 MPa. The Dramix ZL 30/0.50 hooked-end fibers were 30 mm long with a 0.5 mm diameter (ε_t=30.0 mm, φ_t=0.5 mm) and were manufactured from a low carbon, cold drawn steel wire, with a minimum tensile yield strength of 1034 MPa.

The introduced shear stress-slip curves for different volume fraction of straight fiber reinforced dry keyed joint under 500 psi confinement are shown in Fig.2.11. Data were presented by plots in which the abscissa gave the values of the vertical slip and ordinate represented the shear stress of the joint normalized with \( \sqrt{f_c} \). Shear stress was defined here as the load divided by a projected area of 18 sq. in. The ultimate shear strength was defined as the maximum shear stress that the specimen carried. The toughness parameter which qualified the energy absorption of the materials was defined as the area under the shear stress-slip curve up to a 2.5 mm vertical slip. It was indicated that the improved post-peak behavior was evident through a 47.9 % toughness increase with a 11.5 % strength increase for the 1 % V_f (fiber volume fraction) specimen over corresponding values for the plain concrete. Similarly, the 2 % V_f specimen showed a 19.7 % increase in strength and a 64.3 % increase in toughness. Specifically, the SFRC keys were able to maintain significantly higher post-peak load levels.

The obtained cracking sequence for the 1 % V_f specimen is shown in Fig.2.12. From this figure, it was indicated that the cracking behavior of SFRC specimens was more extensive in the pre-peak loading curve than was seen in the plain concrete specimens. For the 1 % V_f specimen, the first crack occurred at 89 % of the ultimate load, and this crack formed at the root of the key and propagated at 30° to the direction of loading (see Fig.2.12a). A nonlinear trend in the curves was also presented at an earlier stage as a result of extended cracking behavior. Additional inclined cracks formed in the key body at an angle of 20° to 40° to the direction of loading (see Fig.2.12b). Final failure of the section occurred when these cracks coalesced as in Fig.1.12c. For 2 % V_f specimen, the first crack load was 83 % of the ultimate load while the plain concrete specimen did not develop the root crack until it had reached 96 % of the ultimate load [Bakhoum et al. 1989, Bakhoum 1991]. It was also reported that at lower confinement the benefits from SFRC become even more apparent in terms of strength and toughness. Here, it should be noted that for the lower confinement SFRC specimens showed more significant curvilinear crack
Fig. 2.11: Shear Stress-Slip Curve for Dry Key Joints with at 500 psi Confinement, Straight Fibers [after Beattie et al., 1989]

Fig. 2.12: Cracking Sequence for Dry Key Joint at 500 psi Confinement, 1% $V_f$ Straight Fibers [after Beattie et al., 1989]
Fig. 2.13: Shear Displacement for Dry Key Joints at 100 psi Confinement, Crimped-end Fibers [after Beattie et al., 1989]

Fig. 2.14: Cracking Pattern of a Key Joint [after Cholewcki, 1971]

Fig. 2.15: Modes of Failure for Keys [after Lacombe and Pommeret, 1974]
propagating from the lower corner of the key with a slight stress drop which was also observed in the plain concrete [Bakhoum et al., 1989, Bakhoum 1991].

The introduced shear stress-slip curves for different volume fraction of crimped-end fiber reinforced dry keyed joint under 100 psi confinement are shown in Fig.2.13. It was indicated that the improved post-peak behavior was evident through a 71.9 % toughness increase with a 23.5 % strength increase for the 1 % $V_f$ specimen over corresponding values for the plain concrete. It was then described that the 2 % $V_f$ specimen showed the highest strength and ductility gains such as a 67.5 % increase in strength and a 200 % increase in toughness. For the 1 % $V_f$ specimen, the first crack occurred at 77 % of the ultimate load, but no stress drop was observed. Thus, the crimped-end fibers displayed the ability to control crack growth where the straight fibers were unable to do so. For 2 % $V_f$ key, the first crack load was 50 % of ultimate.

It was concluded that in general cracking patterns for SFRC keys were similar to those for plain concrete; the pattern in Fig.2.12a would be observed at 80-90 % of the ultimate load, and Fig.2.12b was typical of cracking at roughly 95 % of the ultimate load. Then, the primary difference between the plain and fiber reinforced concrete key joints was suggested to be the typically smaller crack widths for SFRC specimens.

The test results by Beattie et al. will be also used to verify the mechanical model developed in this thesis.

2-2-2. Joint in Large Precast Concrete Panel and Corbel Structure

Large panel concrete buildings are constructed of large precast concrete panels, which may be used either vertically in wall members, or horizontally in slabs and floors. The panels are connected by joints or connections. Loads on joints in large panel structures and their cracking behavior are similar, in some respects, to those of joints in precast concrete segmental bridge.

Cholewicki [1971]
Cholewicki [1971] studied the behavior of vertical joints (without reinforcement) in large panel buildings. He concentrated on the multiple-key joints with in-situ concrete between the panels (wet-connection). Cholewicki indicated that under the action of shear forces, the joints could behave in two characteristic ways: (1) when the bond assures the structural homogeneity of the wall in place of joints up to failure of the joints, and (2) when the bond is damaged due to very small forces or it does not exist at all. Then, the joint behavior was classified into two phases: phase-I- up to the moment when splitting (destruction of bond
along joints) appeared along at least one edge of the prefabricate; phase IIa- after splitting but before diagonal cracking; phase IIb- after diagonal cracking and up to the failure of the joints. In these phases, the following patterns of cracking were observed (see Fig.2.14): diagonal cracks (skew trajectoryal crack) corresponding to direction of principal stresses or longitudinal cracks caused by shearing in plane of action of longitudinal shear stresses. These diagonal cracks appeared after the slipping along the contact surfaces just at the moment bond was destroyed (observed as a dislocation in the joint), and marked the beginning of a progressive failure of the whole joint.

Lacombe and Pommeret [1974]
Lacombe and Pommeret [1974] identified three failure modes for the shear keys (without reinforcement) in large panel buildings (see Fig.2.15). Failure mode (a), which was observed in most of the tests, occurs by shearing-off of the shear keys. The displacement, parallel to the joint, causing this shear-off of the keys is in the order of 0.3 mm. Failure mode (b) occurs when d/h is large. In such cases, compression failure (crushing) occurs at the edge of the concrete shear key. This happens because high shearing forces are required to shear off the key at its base. The compressive stresses resulting from this force at the edge of the key is higher than the compressive strength of the concrete, resulting in crushing. Failure mode (c) occurs by dislocation of the panel accompanied by slipping. This phenomena happens when the angle of inclination is about 30° to 45°.

It was concluded that the second and third modes mentioned above were not desired and dimensioning of the shear keys should be made to prevent these modes.

As indicated by Koseki and Breen [1983], the shear strength of shear key joints are much higher than that of corbel structures (usually reinforced by steel bars) through the examination of design code of corbel structures. Specifically, the fracture mechanism of the latter could be different from that of the former. Thus, the study of this difference through the stress distribution and cracking patterns could be instructive for the interpretation of the fracture mechanism of shear key joints.

Niedenhoff [1963]
Niedenhoff [1963] carried out the experiments on full-scale "one-sided" corbels to obtain some idea of the stress-distribution in this type of structures. The dimensions of the specimen and the details of the reinforcement used in the tests are given in Fig.2.16. The used test set-up and the obtained crack pattern at failure are shown in Fig.2.17. In the tests, the following cracking behavior was reported.
Fig. 2.16: Detail of Corbel Specimen [after Niedenhoff, 1963]

Dimensions of the corbel.

Reinforcement of the corbel.

Fig. 2.17: Test Set-up & Crack Pattern at Failure [after Niedenhoff, 1963]

Test set-up.

Crack pattern at failure.
Fig.2.18: Photo-elastic results [after Niedenhoff, 1963]
During the experiment, load-steps of 50 kN were applied. The first crack (No.1 in Fig.2.17) was reported in the second load-step (Pt=100kN). This crack developed into a main vertical crack at a load level of 150 kN. At Pt=300kN, this crack was fully developed, and the horizontal crack (No.2 in Fig.2.17) was developed (may be due to the horizontal reinforcement). Cracks now occur in the column, but more cracks also develop in the corbel. Finally, failure occurred at a load level of 585 kN when the main reinforcement started yielding. At this stage, crack No.1 widened and the compressive zone in the corner failed (No.3 in Fig.2.17). Thus, the compressive failure could be accompanied by large opening displacement of the No.1 crack.

The stress distribution of the corbel at failure were also studied by means of a photoelastic investigation shown in Fig.2.18. From this picture, it was indicated that a concentration of tensile stresses appeared in the upper corner between the column and the corbel, a concentration of compressive stresses developed simultaneously in the lower corner, and the stress concentrations were visible under the loading platen. In addition, it was found that the resultant force of the principal compressive stresses followed the diagonal from the loading point to the lower inner corner of the corbel. Since tensile stresses developed perpendicular to this diagonal, the diagonal cracks following from the loading point to the lower inner corner of the corbel were observed in Fig.2.17.

From the cracking phenomena and the stress distributions of the corbel structure, it is obvious that the bending effect on the cracking of the corbel is quite large, and results in the difference of stress field at the corner between the corbel and shear key. This is because the corbel has a larger key length than the usual shear key joints, and also it has no normal stress or prestress.

2-2-3. Summary of Problem Issues

In the several cracking behaviors of the shear-key-type structures reviewed in the previous section, the shear-off failure was observed in most of the failures. The basic shear-off failure of key joints may be then to some extent classified into two distinctive fractures (see Fig.2.19a): a preceding single curvilinear/linear crack propagating diagonally (about 45°) from the corner of the key (S crack) and the subsequent multiple diagonal cracks along the key base (M cracks). After small slippage along the contact surface, S crack can occur at the load level of 70-90% of the shear strength [Bakhoum et al. 1989, Bakhoum 1991, Beattie et al. 1989 and Beattie 1989] or just after the slippage [Cholewicki 1971]. This crack would not contribute to the maximum shear strength as shown so far,
Fig. 2.19: Cracking Behavior of (a) Typical Shear Key; (b) Different Key Length; (c) Different Prestress
and the final failure can be constructed by the coalescence of M cracks. The crack opening of these cracks are effectively constrained by fiber reinforcement. This basic cracking behavior can be changed into different cracking behavior if the key length or the confinement are changed as shown in Fig.2.19b,c.

Regarding the key length (see Fig.2.19b), the long key may show somehow corbel-type cracking consisting of both a single crack propagating vertically from the upper corner of the key and compression strut following the diagonal from the loading point to the lower inner corner of the key. In this case, S crack may dominate the final failure mode and M cracks may not appear. In addition, to allow formation of the compression strut which could increase the shear strength, we need certain reinforcement at the upper corner to arrest S crack propagation. This is outside the topic area in this thesis. On the other hand, the short key may show crushing failure on the loading point or dislocation failure mentioned previously. The crushing failure can happen because high shearing forces are required to shear off the key at the base, and this behavior could be very brittle since the cracking is very localized, and therefore is not desirable. The dislocation mode is also not desirable and would be easily eliminated by arranging the angle of the key face [Lacombe and Pommeret, 1974].

Regarding the confinement (see Fig.2.19c), highly confined key joints may show evenly distributed M cracks without a distinctive S crack. Specifically, the higher confining stress could suppress the unstable growth of the multiple cracks, and induce the quasi-plastic macroscopic shear deformation uniformly distributed throughout the failure plane (shear ductile flow). This behavior would be supported by the increased ductility under higher normal stresses introduced previously, and be also explained in the review of the research by Petit [1988] discussed later. On the other hand, poorly confined key joints somehow show distinctive S crack propagation and M cracks which number would decrease and which length would increase (like splitting fracture). Specifically, poor confinement can not arrest S crack propagation or M cracks propagation. These phenomena will be examined by reviewing the researches of Ingraffea et al. [1985a] and Petit [1988] discussed later.

As mentioned, the physical interpretation of the cracking behavior of S crack and M cracks could thus lead to the basis of shear failure of plain/fiber reinforced concrete key joints. So far, we have discussed these two cracking phenomenally, but not the detailed physical mechanism. Under what stress or strain filed are S crack and M cracks nucleated and how do they propagate? How do M cracks coalesce leading to the final failure? Since few study on such investigation is currently available, in the next section we will look for the similar cracking patterns to S crack and M cracks of other type of geometries in which
detailed physical mechanism of cracking behavior based on a fracture mechanics concept is currently available.

2-3. SIMILAR TYPE OF SHEAR FAILURE

In this section, the similarity of the shear failure between key joints and other type of geometries is looked for through the currently available literatures on shear failure of: (a) plain concrete and rock in direct shear tests; (b) rock under compressive stress field; (c) reinforced concrete element in push-off shear tests. Here, we especially focus on a fracture mechanics consideration to find the physical cracking mechanism.

It seems to the author that the cracking behaviors of (a) and (b) are analogous to S crack and M cracks. Fig.2.20 shows such analogy between the direct shear beam test, compression test and the shear key configuration. Here, the shear key base could be represented by the region between the two notch-tips in the direct shear beam, and could also be represented by the prolongation region of the pre-existed crack. Since many researchers have studied the detailed cracking behavior of (a) and (b) based on the fracture mechanics concept, these review can be very instructive for the study of the physical mechanism of S crack and M cracks. In addition, the review of (c) will also be instructive for the study of macroscopic shear failure of concrete where the effect of reinforcement should be considered.

2-3-1. Direct Shear Tests of Plain Concrete and Rock

Cracks in concrete or mortar have been generally assumed to propagate in the direction normal to the maximum principal stress, which represents the tensile, opening fracture mode, designated as mode I. This type of cracking has been observed even for the failure of many structures loaded in shear, e.g., the diagonal shear failure of beams, the punching shear failure of slabs, the torsional failure of beams, the shear failure of panels, etc. Arrea and Ingraffea [1981] recently showed that in a shear-loaded beam with a starter notch in the mode II fracture direction the crack did not propagate in this direction but run to the side in the direction normal to the maximum principal stress. On the other hand, Bazant et al.[1985a,b, 1986] insisted that the possibility of shear-produced fractures was of considerable concern for all problems where the shear resistance of cracks due to surface roughness (aggregate interlock) was an issue. In these problems, Bazant et al. assumed that the shear-loaded cracks had previously somehow formed due to tensile loading (mode I
Fig. 2.20: Analogy between Direct Shear Test, Compression Test and Shear Key Configuration
loading). More likely, however, the preformed tensile cracks would be only partial, discontinuous, and the final continuous cracks would be produced by the shear loading itself. The investigation of these two arguments could identify an analogous physical mechanism for S crack and M cracks in shear key joints. Therefore, in this section, this type of shear fracture is mainly investigated.

**Bazant et al. [1985a,b, 1986]**

Bazant et al. [1985a,b, 1986] tested symmetrically notched beam specimens of concrete and mortar, loaded near the notches by concentrated forces as shown in Fig.2.21. A pair of symmetric notches, of depth d/6 and thickness 2.5 mm was cut with a diamond saw into the hardened specimens using a concrete mix with water-cement ratio of 0.6 and cement-sand-gravel ratio of 0:2:2 (all by weight). The maximum gravel size was 12.7 mm, and the maximum sand grain size was 4.83 mm. Mineralogically, the aggregate consisted of crushed limestone and siliceous river sand. Portland cement C150, ASTM Type I, with no admixtures, was used. The mean compressive strength was 37.9 MPa for concrete specimens, and 49.0 MPa for the mortar specimens. They reported that a shear fracture occurred at the plane connected the two notch tips as shown in Fig.2.22. To explain the fractures observed in their tests, they used a shear fracture model shown in Fig.2.23. It was assumed that a shear fracture connecting the notch tips was likely to form as a zone of tensile microcracks with a predominantly 45°-inclination which was only later connected by shearing. They called the connecting link between the microcracks an inclined strut. The final damage mechanism in their model was a shear-compression failure of these struts.

This macroscopic shear fracture contrasted with that observed by Arrea and Ingraffea [1981] sketched in Fig.2.24a. Bazant et al. explained that their test differed by its wider separation of the loading points from that of Arrea and Ingraffea. For comparison, the symmetrically notched specimen with a wider separation of the loading points was also repeated. In that case, it was indicated that the cracks propagated from the notch tip basically in the direction normal to the maximum principal stress, same as the observation by Arrea and Ingraffea. This phenomena was hypothesized as: (a) In both the test shown in Fig.2.21 and tests with the wider shear zone shown in Fig.2.24b, the stress fields near the fracture front are similar; (b) So the crack somehow senses the stress field remote from the cracks, and responds to it; (c) Consequently, the stress field near the fracture front, as well as the strain and strain energy density fields near the fracture front, does not govern the direction of fracture propagation; (d) A mode I crack propagating sideways from the notch tip would quickly run into a low stress zone of the material, and would, therefore, release little energy. On the other hand, a vertically running crack (mode II) continued to
Fig. 2.21: Shear Test Specimen [after Bazant et al., 1986]

Fig. 2.22: Photograph of a Specimen of 6 in. Depth after Failure [after Bazant et al., 1986]
Fig. 2.23: Band Model [after Bazant et al., 1986]

Fig. 2.24: Crack Path Observed in Notched Beams with a Wide Zone of Shear Force [after Bazant et al., 1986]
remain in the highly stressed zone of the material (along the shear failure plane), and could, therefore, cause a large release of strain energy; (c) Thus, the crack propagation direction could be governed by the criterion of the 'maximum energy release rate'.

Based on the hypothesis mentioned above, Bazant et al. simulated the crack propagations shown in both Fig.2.22 and 2.24 by means of finite element calculation, and obtained the agreement with the observed crack propagation. They then insisted that the fracture energy due to the compressive stress parallel to the inclined multiple cracks should be considered and the energy of this compression failure, which is absent in the mechanism of mode I fracture, was far larger than the energy dissipated by tensile cracking.

Because of both the similar geometry and phenomenally similar cracking pattern, we may see some analogy between S crack propagation in key joints and the mode I crack propagating sideway from the notch tip and running into a low stress zone of the material discussed here. The analogy between M cracks in key joints and a zone of tensile microcracks which is later connected by shearing could be also obvious. In addition, the concept of maximum energy release rate may lead to physical interpretation on the transition between S crack and M cracks. Regarding the argument on the mode II for the shear fracture with a zone of tensile microcracks, it seems to the author that observed shear fracture consisting of both a zone of tensile microcrack nucleation and crushing failure of the struts are in one sense macroscopically mode II fracture. However, the clear definition of mode II fracture in concrete is not currently available. Thus, this argument will be placed after such study. In addition, regarding their shear fracture model, it could be doubtful to assume the predominantly 45°-inclination in a zone of tensile microcracks. The exact principal stress distribution or tensile cracking will be discussed in the next literature.

Ingraffea et al. [1985a]

Ingraffea et al. [1985a] provided the alternative explanation for the cracking behavior observed and modeled by Bazant et al. [1985a,b, 1986] mentioned above. In their linear finite element analysis, the principal stresses were plotted as vectors at the element Gauss points shown in Fig.2.25, and they indicated that the major principal stress in this region was tensile and the direction of this stress was nearly perpendicular to a line between the notch tips. They then specified that the stress normal to Line BAC (see Fig.2.26a) was very nearly the principal tensile stress, and that this stress was fairly uniform over this line. They also indicated that the shear stress along Line BAC (see Fig.2.26b) did not show the parabolic distribution and showed high shear stress region near the notch tips, which was supported by the classical problem of the theory of elasticity shown in Fig.2.27 [Timoshenko and Goodier, 1970]: The elementary beam theory was inadequate to describe
the stress field in this configuration. It was then claimed that the notch tips were located in a region of decreasing shear stress. Based on the above mentioned analysis, Ingraffea et al. predicted well the maximum loads for the Iosipescu specimen of Bazant et al., by proposing a hypothetical circular specimen inscribed in the beam shown in Fig.2.28, which could comprise the Brazilian test assuming that both small tractions distributed along its circumference and the two notches cut into it have little effect.

Finally, they hypothesized the following tensile fracture mechanism in the Iosipescu specimen tested by Bazant et al.: (1) Quite load-stable tensile cracks first propagate from the notch tips, and a substantial increase in load keeps them propagating in the direction which minimize the mode II component; (2) At a load level higher than that which causes cracking from the notch tips, a new tensile crack is nucleated near the center of the beam, just as it does in the Brazilian test. This crack would be nucleated at an angle of only about 10° from the vertical; (3) The tips of this central crack propagate upwards and downwards, toward the notch tips; (4) There will remain a short ligament between the tip of the central crack and notch tips which will be in high, nearly vertical compression, and crushes (through intensive microcracking) to complete the fracture process.

This hypothesis was assessed by nonlinear discrete crack analysis with a linear tensile stress-crack opening displacement model. The highly magnified displaced shape both at 90% of the maximum load and at the maximum load are shown in Fig.2.29 and 2.30, respectively. The darkened area is a damaged zone or macroscopically discrete crack. The crack at the notch tips had progressed and had begun to turn as indicated in Fig.2.29. This cracking phenomenon seems to be forced under 'Wedge Force' which can directly open the crack mouth. Near the maximum load, the notch-tip cracks had not lengthened. After this stage, a new crack in the central region of the beam was nucleated and propagated in the direction of colinear line connecting the load support point.

The hypothesis discussed here is different from that by Bazant et al. [1985a,b, 1986]. However, one can see the following similarity between them: (a) The mode I tensile crack is nucleated at the both notches, propagates in the mixed mode direction, and stops propagating after some load stage; (b) The central region of the beam is damaged as a tensile crack or cracks and this damaged process zone develops as the load increases. Ingraffea et al. modelled this region by a single discrete crack like a Brazilian splitting crack, while Bazant et al. modelled it by a zone of multiple diagonal cracks. On the other hand, one can see the following difference on the damage process in the central region between them: Ingraffea et al. insisted that such damage nucleated near the central region and propagated toward the notch tips in the direction normal to the principal tensile stress dissipating a mode I fracture energy, while Bazant et al. insisted that such damage
Fig. 2.25: (a) Tensile Principal Stress; (b) Compressive Principal Stress  
[after Ingraffea et al., 1985a]

Fig. 2.26: (a) Stress Normal to BAC; (b) Shear Stress along BAC  
[after Ingraffea et al., 1985a]
Fig. 2.27: Shear Stress Distribution [after Timoshenko and Goodier, 1970]

Fig. 2.28: Diametrically Loaded Disk [after Ingraffea et al., 1985a]
Fig.2.29: Highly Amplified Displaced Shape around Upper Notch after Two Increments of Cracking [after Ingraffea et al., 1985a]

Fig.2.30: Deflected Shape at Maximum Load [after Ingraffea et al., 1985a]
nucleated at the notch tips and propagated toward the central region in the direction maximizing the released strain energy at both tensile microcracks and the inclined struts of the material between the cracks. These argument will be further discussed in the following review.

**Barr and Derradj [1990]**

A numerical investigation was carried out on a compact shear test specimen geometry (cube specimen and prismatic specimen shown in Fig.2.31) based on the original Iosipescu geometry [Barr and Derradj, 1990] to determine the most suitable test specimen geometries. They reported that tensile stresses were created at the roots of the two notches, these tensile zone might result in the mode I type of failure and probably accounted for the mode I or mixed mode fracture observed in tests using the Iosipescu geometry. It was also shown that shallow notches gave a stress distribution between the roots of the two notches similar to that developed in the indirect tensile test (Brazilian test) and hence tensile failure was likely to precede shear failure in such cases. On the other hand, it was shown that deep notches gave a greater shear stress than the corresponding tensile stress between the roots of the two notches, and increased the likelihood of shear fracture prior to tensile failure.

The numerical study of Barr and Derradj may show that we can probably observe the phenomena of both Bazant et al. [1985a,b, 1986] and Ingraffea et al. [1985a] in the different geometries which have a different depth of shear plane.

**Ballatore et al. [1990]**

Ballatore et al. [1990] interpreted the failure in four point shear specimens (see Fig.2.32) of concrete as comprising of more than one failure mechanism contributing to the final collapse of such a specimen: (1) mixed mode crack propagation which is favoured for small distances c and/or for large sizes b shown in Fig.2.33; (2) flexural failure on the supports which is favoured for large distances c and/or for small sizes b; (3) tensile splitting failure in the middle of the specimen which is favoured for smaller distance c [Ingraffea et al.,1985a]; (4) pure shear failure between the central supports. It was indicated that all three failure mechanisms (1), (2) and (3) could be imputed to tensile stress, and not to local shear deformations.

They carried out the shear test of two different concretes with maximum aggregate size \( D_{\text{max}} \) of 10 and 20 mm respectively to study the size-scale transition between mechanism (1) and (2) mentioned above. It was demonstrated that the mechanism (1) represented the Linear Elastic Fracture Mechanics (LEFM) instability, whereas the mechanism (2) indicated the attaining of ultimate bending strength at the supports. Regarding LEFM phenomenon,
it was explained as follows. LEFM was a valid crack branching criterion for large concrete structures. For mode I crack propagation, this has already been demonstrated. For mixed mode crack propagation, this is a logical consequence since the mode II stress-singularity power was still 1/2 as for mode I. The experimental results were reported on this basis, where the maximum load divided by the beam area was plotted against the inverse root b^{1/2}, as shown in Fig.2.34. The experimental points describe the transition between an inclined straight line (LEFM instability) and a horizontal asymptote (ultimate bending strength). It was also indicated that concrete 2 appeared brittler than concrete 1 since the horizontal asymptote was absent in Fig.2.34(b).

In addition, they calculated the mixed mode fracture energy, which was given by the total dissipated energy, divided by the total fracture area. Specifically, the total energy was calculated by the integration of 'the loading force F' times 'the loading points deflection δ'. In this calculation, they deliberately neglected the remarkable amount of energy dissipated in the volume by both punching at the supports and compression inside the specimen, and obtained the fact that such mixed mode fracture energy was very close to the value of mode I fracture energy G_I. Finally, they concluded that each crack propagation step was always produced by mode I (opening) mechanism, and the fracture toughness of concrete was defined by the unique parameter G_I, even for mixed mode problems. Based on this conclusion, Ballatore et al. indicated that the mechanism (4) did not activate locally, but only globally and was always produced by local tensile stresses.

Ballatore et al. [1990] seem to support the hypothesis of Ingraffea et al. [1985a] on the tensile splitting failure of direct shear tests through the calculation of the mixed mode fracture energy which was comparable with the mode I fracture energy. This may be explained by the fact that the principal stress directions do not change significantly as the crack propagates and hence the crack remains quite straight. If there is a change in the principal stress direction, such as when the loading points are closer to each other, the crack will kink a lot. Then, when the process zone has been completely developed, there should be significant dissipation of energy by shear displacement along the crack face to lead to a much higher mode II fracture energy. Indeed, they did not study quantitatively the case of a narrow shear force zone like the geometry of Bazant et al. [1985a,b, 1986]. Moreover, Bazant et al. reported that the fracture energy (they called mode II fracture energy) in their shear fracture was about 30 times larger than mode I fracture energy (G_II = 1049 N/m for concrete, 578 N/m for mortar). This point will be discussed later.

However, it is very interesting that the mixed mode crack propagation like the crack trajectories shown in Fig.2.33 can be expressed by LEFM for large structures, and also the
Fig. 2.31: Test Specimen [after Barr and Derradj, 1990]

Fig. 2.32: Loading Configuration [after Ballatore et al., 1990]
Fig. 2.33: Fracture Trajectories [after Ballatore et al., 1990]

Fig. 2.34: Size-scale Transition from LEFM instability to Tensile Strength Collapse of (a) Concrete 1; (b) Concrete 2 [after Ballatore et al., 1990]
mode I fracture energy can be used for such crack propagation as the mixed mode fracture energy. This could be instructive for the modelling of S crack.

Melin [1989]
In order to investigate the influence of material on the shear fracture experiments by Bazant et al. [1985a,b, 1986], interesting experiments were carried out in PMMA (poly methyl methacrylate), which could be considered to be isotropic and linearly elastic, with similar geometry to that used by Bazant et al., and a numerical simulation was made assuming mode I crack growth [Melin, 1989]. The crack pattern paths obtained from the experiments on PMMA showed excellent agreement with finite-element calculations based on the assumption that crack growth occurred in the direction that maximized mode I stress intensity factor for the incipient crack formation and nullified mode II stress intensity factor for the other (see Fig.2.35). This crack propagation criterion was adopted based on the fact that the conditions $K_I = K_{I_{\text{max}}}$ and $K_{II} = 0$ were equivalent after, not at, incipient crack formation [Melin, 1985], and the former condition appeared to be the most logical one.

On the other hand, the crack paths found in concrete and mortar by Bazant et al. [1985a,b, 1986] differed significantly from those in PMMA shown in Fig.2.36. Melin suggested that these differences lay in the fact that small-scale yielding condition was not realized well enough in concrete and mortar, due to their comparatively large process zone, which consisted of a fairly narrow band of small cracks, most probably opened by tensile stresses. He concluded that the direction of this narrow band, at least to some extent, was governed by the location of large tensile stresses in the vicinity of the crack tip shown in Fig.2.37, assuming a crack path similar to what was obtained in tests on the concrete specimen. From the two types of crack shown in Fig.2.36, it is obvious that precise propagation of S crack cannot be expressed by the criterion of $K_I = K_{I_{\text{max}}}$ and $K_{II} = 0$. Besides, maximum tensile stress criterion may be applicable for S crack propagation.

The difference in behavior between PMMA and concrete may be explained as follows. In concrete, the shear resistance along the crack due to aggregate interlocking will tend to promote kinking of the crack towards a vertical direction and once that direction is approached, there is little energy release associated with crack growth. The tensile stresses caused by combined shearing/compression in the zone between the notches may then lead to multiple diagonal cracks. In PMMA, the crack do not kink as much and also the material is not as weak in tension as in concrete and so the tension in the region between the notches may not be high enough to introduce failure in the form of multiple diagonal cracks.
Fig. 2.35: Crack Path of PMMA [after Melin, 1989]

Fig. 2.36: Crack Path of PMMA and Concrete [after Melin, 1989]

Fig. 2.37: Largest Principal Stress [after Melin, 1989]
Fig. 2.38: Short Beam Shear Specimen [after Watkins and Liu, 1985]

Fig. 2.39: Failure Mechanism [after Watkins and Liu, 1985]

Fig. 2.40: Loading Arrangement and the Specimen [after Davies et al., 1987]
Watkins and Liu [1985]
The in-plane shear strength tests were carried out using modified standard concrete with the configuration of a specimen (100 x 100 x 200mm) shown in Fig.2.38 [Watkins and Liu, 1985]. The specimen size was kept constant throughout the tests, and the slot separation between the opposite double notches of the specimen ranged from 20 mm to 100mm. The notches were produced by means of a clipper masonry saw which was fitted with a 350 mm diameter carborundum blade.

It was reported that the specimens with a/w between 0.1 and 0.25 failed between the two slots as shown in Fig.2.39. It was observed that very narrow crush zones, particularly at the crack tips, developed and material crumbled away, and when this was cleaned off it was noted that the two parts did not fit together. This phenomenon was different from the split cube tests [Liu, 1982] producing a mode I fracture and the split-cylinder test in which the fracture planes were clean and both parts fitted together very well. They then concluded that two different modes of failure had taken place; one associated with tensile mode I and the other with shear mode II, from the distinct difference between the appearance of fracture planes in these two types of tests. It was also explained based on FEM analysis that the stress field at the crack tip vicinity was compressive, tensile stress was remote from this region and shear stress along the line of loading was predominant.

Regarding the applicability of LEFM concept (K, G) to concrete, Watkins and Liu [1985] indicated that altogether 46 experimental results had been analyzed by means of LEFM and the coefficient of variation for $K_{IIc}$ was found to be 11.7% whereas for the average shear stress over the failure plane the variation was found to be 12.4%. Then, they concluded that the LEFM approach gave slightly more consistent results. Toughness was calculated by means of “Compliance Method”, and the fracture toughness for concrete specimens in shear edge sliding mode, mode II was found to be $K_{IIc} = 0.53 \text{ MNm}^{1.5}$ compared to $K_{Ic} = 0.71 \text{ MNm}^{1.5}$ obtained from the split-cube test [Liu, 1982] for the same matrix.

Mode II fracture in shear test by Watkins and Liu seems to be the same mechanism observed by Bazant et al. [1985a,b, 1986]. Specifically, the shear zone may consist of tensile cracks which will finally be crushed as shown in Fig.2.39. The results that the mode II fracture toughness was comparable with mode I fracture toughness in concrete is similar to results by Ballatore et al. [1990] if LEFM condition is valid. In addition, the applicability of LEFM to mode II crack propagation in concrete could be also similar to that in mixed mode propagation by Ballatore et al. [1990].
Davies et al. [1987]

A shear fracture mechanism in mortar with the average compressive strength and modulus of elasticity of 41 MPa and 33 GPa respectively was studied [Davies et al., 1987]. The used specimen geometry and the fracture test arrangement are shown in Fig.2.40. The reported typical load-displacement (vertical displacement of the compressive platen) graph is shown in Fig.2.41. For all the samples tested, a hairline inclined crack (see Fig.2.40) first developed at one of the bottom notches corresponding to the 'pop-in' load P (see Fig.2.41) and subsequently the shear (shear-compressive) cracks, parallel with the line of loading developed rapidly before sudden catastrophic failure. It was indicated that the brittle fracture in compression is due to tensile microstresses, however the tensile mechanism is not in itself sufficient to cause the failure of material, and another failure mechanism, most probably controlled by shear stresses, becomes active at some stage of the fracture process: Shearing mechanism could be initiated in zones of high compressive or tensile stress concentration. Specifically, with decreasing notch spacing H and increasing a/W ratio, the shear failure mechanism assumed greater significance. This phenomenon is the same as the previous discussion by Barr and Derradj [1990].

Davies et al. determined the fracture toughness value in this geometry based on the 'pop-in' load. It was argued that extensive micro-cracking occurred beyond the 'pop-in' load P and the corresponding energy is not associated with the material fracture toughness, but to the latent strength in the materials. From the fact that the obtained fracture toughness value seemed to be independent of specimen geometry, $K_{IC} = 1.90 \text{ MNm}^{-1.5}$ based on the stress intensity factor approach and $K_{IC} = 1.78$ to 2.09 MNm$^{-1.5}$ based on the strain energy release rate approach, they concluded that LEFM approach may be applied to this material and that both the stress intensity factor and the energy approaches yield almost identical results.

Davies et al. indicated the applicability of LEFM to mortar in mode II as did Watkins and Liu [1985] for concrete. The calculation of the work done during fracture was based on the first crack load corresponding to the observed hairline inclined crack. This crack seems to be similar to S crack. Therefore, it may be also concluded that S crack propagation can be described by LEFM concept.

Davies [1987]

To assess the previous experimental studies [Davies et al., 1987], Davies [1987] carried out the plain strain finite element analysis of the specimen shown in Fig.2.42. It was indicated that the high shear stress concentration was seen in the region of both notches. The plots of principal stress vectors $\sigma_1$ and $\sigma_2$ are shown in Fig.2.43. This figure shows
Fig. 2.41: Load Deflection Graph
[after Davies et al., 1987]

Fig. 2.42: Geometric Configuration
[after Davies, 1987]

Fig. 2.43: Principal Stress Vectors [after Davies, 1987]
Fig. 2.44: Load-Displacement Curve [after Davies, 1987]

Fig. 2.45: Test Arrangement [after Petit, 1988]
that some degree of bending takes place therein, with the tensile stress field generated at the bottom notch and the compressive stress field generated at the top notch. Davies indicated that the pure shear state of stress was nearly achieved in the crack tip region, and that the principal tensile stresses $\sigma_1$ generated by a shear force may cause the formation of tensile microcracking along the planes about $52^\circ$ inclined from the horizontal axis, which are planes perpendicular to the direction of the maximum principal tensile stress. Thus, it was concluded that the shear fracture would be associated with presence of tensile microcracks developing along the planes approximately $52^\circ$ inclined from the horizontal axis.

Based on the numerical study mentioned above, it was explained for the obtained load-displacement curve shown in Fig.2.44 as follows. The load $P_1$ was the first crack load based on the visually and acoustically observed cracks at the top or bottom notches. With further increase in loading cracks propagated along several planes from the areas of high stress concentration at the top and bottom notches, and finally inclined and crushing zones fully developed between the top and bottom notches, which corresponded to the load $P_{\text{max}}$.

Two kinds of mode II fracture toughness were calculated: fracture toughness $K_{\text{IC}}$ corresponding to the load $P_1$ or first crack load and the latent toughness $K_{\text{IL}}$ corresponding to the maximum load $P_{\text{max}}$. The range of the former value was 1.8 to 2.0 MNm$^{-1.5}$ and that of the latter was 4.8 to 5.8 MNm$^{-1.5}$. Thus, the latter was about three times higher than the former. These values were compared with the fracture toughness in mode I, $K_{\text{IC}} = 0.27 - 1.3$ MNm$^{-1.5}$, reported in the literature [Swamy, 1979], and significant difference was indicated. The difference was explained as follows. The fracture process in tension involved only the energy to create tensile cracks and microcracks. The fracture process in shear compression, however, involved not only the energy to create the tensile microcracks usually inclined, but also the energy required to break the shear resistance due to interlocking of matrix.

Davies showed the same physical mechanism for the macroscopic shear failure as that by Bazant et al. [1985a,b, 1986]. It is also noted that the latent fracture toughness $K_{\text{IL}} (= 4.8$ to $5.8$ MNm$^{-1.5}$) is identical to $G_{\text{IL}} (= 700$ to $1000$ N/m in plane stress with the modulus of elasticity of 33 GPa introduced in the previous review) if LEFM is valid. This value is the same order of that by Bazant et al. [1985a,b, 1986]. As discussed previously, Ballatore et al. [1990] and Watkins and Liu [1985] indicated that mode II fracture toughness or energy was comparable with that of mode I. On the other hand, Bazant et al. and Davies showed the large difference between them. Thus, the clear definition of mode II fracture parameters is not available at the present stage, and further investigation will be required.
Petit [1988]

A set of shear box tests on intact sandstone specimens were carried out as shown in Fig.2.45 [Petit, 1988]. The specimens were dry low porosity (less than 2%) fine grained red sandstone with the dimensions of 85 mm thick, 130 mm long and 55 mm wide. A 5 mm wide gap was left around the specimen between the two parts of the shear box so that fracturing could be partly observed during loading and rupture in the enforced shear zone. For each experiment, normal stress was first imposed and the shear stress was then applied. As the dilatation of the specimen tended to increase the normal stress, the normal stress was maintained constant by slight normal pressure released with the hand-pump.

The failure envelope (see Fig.2.46) was established with a schematic cracking behavior. Here, it should be noted that this loading/geometric configuration and cracking phenomena are quite similar to those of key joints. This schematic cracking behavior shows that the lower the normal stress, the lower the number of cracks observed. This point was discussed previously. The main stress dependent morphological features were shown in Fig.2.47 where the numbers on left indicate the value of normal stress at rupture in MPa, the thick lines indicated striated/frictional surface (P, R), the broken lines indicated partially striated surface (PO) and the thin lines indicated non-striated surface (T). The main results were reported as follows: An increasing normal stress led to (i) an increasing striated (P, R or PO) / non-striated (T) surface ratio; (ii) a tendency for rare P-type striated surface to be replaced by tight R-type striated shear surfaces; (iii) the tendency of the front half of both frictional surfaces to be more striated than the rear. Fig.2.48 shows the typical sequential development of rupture as deduced from direct (during loading) and post-rupture observations. Fig.2.48a,b shows respectively the cases for low and high applied normal stress. In the figure, number '1' on left indicates the first formed fractures, '2' does the intermediate stage before failure and '3' does the failure stage. In addition, the terms I and II indicate the fracture mode, and the small arrows indicate the tendency of specimens to rotate during loading 1, 2 and collapse at rupture.

Based on the above-mentioned observation, the following sequence was explained. For any normal stress, the first formed fractures consisted of one or two tensile (mode I) fractures dipping in the direction of movement at angles ranging from 10° to 45° with the enforced shear direction, with a smaller angle for a lower normal stress (see Fig.2.48). These fractures were initiated right across the width of the specimen from the upper rear edge of the sample at the contact with the blocking wedge, and appeared during the last 20% shear before peak value. With increasing stress, the first formed fractures progressively opened, while a shear component in the same sense as the enforced shear developed (mode I + II or mixed mode). For low normal stress, only one low angle fracture tended to
develop, but for higher normal stress (> 50 MPa), similar fractures, first tensile and then shear, generally developed towards the front edge, making an en echelon-like structure whose elements tended to be at a higher angles than those for low normal stress. In some cases, very low angle minor en echelon arrays were observed, connecting the first formed open fracture with a later one. After offset it was seen that this connecting structure corresponded either to a fully striated surface inside the block, or to a succession of tight minor shear fracture. All these features appeared on both sides. Final rupture at low normal stress was catastrophic, whereas with higher normal stress, rupture was generally less brutal and preceded by a certain amount of accelerating creep (horizontal displacement at constant load).

It was indicated that the tensile fractures were likely to have formed from one stressed edge to another, more or less comparable to Brazilian test rupture (provided low normal stress conditions prevail which will not inhibit the formation of long tensile fractures). The primary fractures were mainly localized in the rear in this experiments. This was explained by the tendency of the upper part of the specimen to rotate (see Fig.2.49), in spite of the precautions taken to block it. Indeed, mode I crack formation was promoted, and as soon as the first fractures were initiated, this tendency was encouraged. Thus, it was concluded that the initiation clearly seemed to be a mode I (plain strain) phenomena.

As the shear strain increased, it led to a mixed mode (opening + shear) relative movement of the first formed fracture (Fig.2.48, part 1 and 2). After the specimen had been released from the blocking system, it could be seen that such fractures tended to follow either a curvilinear trajectory or to turn into a branch kinked crack, both tending towards a vertical position.

With increasing normal stress (see Fig.2.48), the formation of further fractures frontwards could be induced by the progressing mode I component linked to rupture, but it was necessarily limited as the already fractured dilatant rear part only supported a small amount of normal and shear forces. Thus, normal and shear stresses were concentrated in the front part (see Fig.2.49). This explained the more marked striation here. Moreover, because of the local highly confined conditions, the rupture more or less followed the enforced direction (mode II macroscopic propagation). Petit insisted the analogy of this macroscopic shear fracture with the mode II macroscopic propagation of a large defect having mode II initiation geometry [Petit et al, 1988]. Specifically, it was explained that the already splitted part of the direct shear specimen could roughly represent the pre-existing defect; by analogy the repeated shear surfaces formed at the front of the specimen could be initially tensile (mode I) en echelon fractures submitted to shear (mode II
Fig.2.46: Failure Envelope [after Petit, 1988]

Fig.2.47: Profiles of the Rupture Surfaces [after Petit, 1988]
Fig. 2.48: Sequential Development of Rupture [after Petit, 1988]

Fig. 2.49: Inhomogeneous Stress Distribution [after Petit, 1988]
movement) because of the front-wards progressing shear strain. This will be discussed further in the next section.

The study of Petit can give us a physical concept of direct shear failure under normal stress. It seems to support the hypothesis of Ingraffea et al.[1985a] under lower normal stress, and that of Bazant et al. [1985a,b, 1986] under higher normal stress. Specifically, there may exist the transition between both phenomena in terms of the normal stress and also depth of shear plane discussed previously. However, it is obvious that the macroscopic shear fracture propagation of Petit is different from that of Bazant et al. The former one is aligned more uniformly due to normal stress than the latter. The former is then more similar to that of shear key joints. His study may also suggest that S crack is initiated in mode I condition, and propagates in the mixed mode condition. Moreover, M cracks propagation may be explained by mode I en echelon type fracture which will be later coalesced by mode II movement.

2-3-2. Shear Failure of Rock under Compressive Stress Field

In this section, the fracture phenomenon in the pre-existing crack under compressive stress field is studied. The fracture sequence of single pre-existing crack under compressive stress field may show the similar cracking phenomenon to S crack and M cracks in key joints if we imagine that the crack tip of such pre-existing crack could be analogous to the corner of the shear key under direct shear loading (see Fig.2.20b). In addition, macroscopic shear fractures produced by coalescence of many tensile cracks under compressive stress field may also show the similar cracking phenomenon to the propagation of M cracks as discussed previously. The detailed study of such failure modes could be instructive to the physical mechanism of shear-off failure of key joints.

Petit et al. [1988]
In the context of mode II initiation geometry, the experiments for PMMA and sandstone were carried out [Petit et al., 1988]. The dimensions of the specimens were 5 mm thick, 50 mm long and 32 mm wide for PMMA, and 50 mm square and 5 mm thick for sandstones with the case of 4 % (low porosity) and 18 % (high porosity) porosity. The dimensions of the slot oriented at 30° to the applied load $\sigma_1$ were 8 mm length and 0.3 mm width. For PMMA, ethyl alcohol was added as a slight solvent, and rocks were tested in wet conditions. Uniaxial loading tests ($\sigma_3 = 0$) were done on all samples and biaxial loading ones ($\sigma_3 > 0$, compressive) only on PMMA. The major principal stress $\sigma_1$ was applied at a constant strain rate of 0.2 mm/min. In biaxial tests, the lateral pressure $\sigma_3$ was
applied by an independent hydraulic system so that the $\sigma_3/\sigma_1$ ratio could be kept more or less constant during loading and unloading. The experimental results are schematically shown in Fig.2.50.

In the case of PMMA, when the principal stress ratio $\sigma_3/\sigma_1 < 1/10$, the jumplike (localized more or less rapid propagation) formation of an axisymmetric branch (curvilinear trajectory approaching asymptotically the central loading axis) occurred for high stress values. After unloading, the branch fracture remained partly open. During most of the loading, the progressive development of a shear zone marked by en echelon mode I microcracks initiated from the preexisting slot tip could be observed. These en echelon mode I microcracks formed a rather wide swarm which progressed slowly in the prolongation of the slot. They were preceded and then accompanied by an elongated zone of high stress differences marked by the concentration of isochromes (lines of equal value of maximum shearing stress) (see Fig.2.51a). This may suggest that tensile fractures were induced for a certain shear stress value. These microcracks were initiated from the surface. After initiation, each deepened and lengthened but remained superficial. It is obvious from the figure that all the fractures follow $\sigma_1$ isostatics, and the branch crack and en echelon microcracks develop along the local principal stress trajectories due to local $\sigma_3 < 0$ (see Fig.2.51b). In biaxial tests with an increasing $\sigma_3/\sigma_1$ ratio (see Fig.2.50), the development of the branch fracture was progressively inhibited and disappeared completely when $\sigma_3/\sigma_1$ was about 1/10; the en echelon shear zone persisted but was distributed in a shear zone narrower than that formed under a low $\sigma_3/\sigma_1$ ratio.

In the case of sandstones, it was reported that a branch fracture (roughly curvilinear trajectory but with oscillating path) with similar bulk geometry to that of PMMA was always observed as rather slow jumplike propagation at the end of the test and it was initiated before any other structures, but less developed in the high porosity sandstone.

During the slow propagation of the branch fracture in low-porosity sandstone, the microcrack formation at the slot tip was marked by the whitening of quartz grains, and this fracture did not always propagate along exactly the same path, but more often it propagated as a fracture with shearing component within the angle between the defect axis and the branch crack. It generally tended to rejoin the loading direction. When the load was increased to maximum stress, the previous structures could still appear, but at maximum stress, the formation of a shear zone roughly aligned in the slot direction occurred in a catastrophic way. In the high-porosity sandstone, such propagation was slow and so could be seen to initiate from the slot tip.
Fig. 2.50: Stress/Strain Curves and aspect of Plaquettes (a) for PMMA; (b) for Sandstone [after Petit et al., 1988]

Fig. 2.51: Stress Distribution linked to Pre-existing Flaw in Uniaxial Compression in PMMA, (a) Distribution of Isochromes (lines of equal value of maximum shearing stress); (b) Major Principal Stress Isostatics [after Petit et al., 1988]
Regarding the mode II macroscopic propagation in the shear zone, it is obvious that this shearing structure is not the mere prolongation of the defect by a single coplanar fracture. Specifically, mode I fracturing is possibly a mechanism of the loss of cohesion in the shear zone as well as shearing movement. Petit et al. explained the physical meaning of this phenomenon as follows. Fracture initiation mainly results from ionic and covalent bond rupture initiated from mainly immobile flaw populations. In glass which is too homogeneous and highly brittle, cohesion prevents any flaws from being mobilized even at high shearing stress levels. However, both PMMA and sandstone are semibrittle, presenting linear defects which can be more easily mobilized. In PMMA, these defects behave like dislocations, explaining the plastic deformation, while in sandstones they are mainly grain boundaries or pores. These flaws are easily mobilized by shear stress concentrations at slot tips inducing intergranular and intragranular ruptures. Thus, the shear propagation from a major defect needs a high density of minor flaws which can be mobilized to give macroscopic ductility.

From the study of Petit et al., it is obvious that in spite of numerous differences between PMMA and sandstones, the branch fracture as well as the shearing structures are always present in both materials. Similar phenomena are found in shear key structures. The roughly curvilinear branch crack found in PMMA and that with oscillating path found in the sandstone seems to be equivalent to S crack in shear key joints. Since these branch cracks are initiated and propagate due to local principal tensile stress, S crack can be explained in the same manner. In addition, as shown in PMMA, it was indicated that biaxial loading at a suitable $\sigma_2 / \sigma_1$ ratio would inhibit branch fracture formation in rocks, thus leading to the occurrence of shear structures alone. This phenomenon is also identical to that in shear key joints under higher normal stress.

The shear zone of M cracks in key joints can be inferred by analogy with the shearing structure discussed by Petit et al. (see Fig.2. 20b). The upper edge of the shear key could be analogous with the major defect, and M cracks be analogous with a high density of minor flaws which are initiated by the loss of cohesion or mode I initiation. This description can be equivalent to that discussed in direct shear fracture of concrete.

Einstein et al. [1990]
The fracture genesis of the tensile and shear fractures created in predominantly compressive stress fields (two dimensional stress states shown in Fig.2.52 which represent predominantly or purely compressive stress states) was proposed [Einstein et al., 1990].

It was reviewed that the tensile crack propagation took place around an elliptical opening under the stress states between circles 3 to 5. Specifically, crack propagation
Fig. 2.52: Mohr Failure Envelope [after Einstein et al., 1990]

Fig. 2.53: Genesis of Tensile and Shear Fracture [after Einstein et al., 1990]
occurs if the tangential tensile stresses at some locations of the circumference exceed the local tensile strength, and these propagating cracks align themselves with the major principal stress. These propagating cracks were called 'wing cracks', while the combination of original and wing cracks 'winged crack'. It was suggested that this crack propagation was influenced by the material properties, by any material in the crack and by the stress state around the crack, and the latter in turn was affected by the crack shape. For elliptically shaped cracks, the particular combination of a 'far field' stress state and ellipse aspect ratio will provide a rough idea under which circumstances tensile tangential stresses exist and crack propagation continues even if compressive stresses in the minor direction exist. In addition, it was mentioned that the propagating wing cracks with a sharp leading edge will stop to propagate once they reach the major principal direction and if compressive stresses exist in the minor direction.

Based on the physical concept mentioned above and several experimental reviews, Einstein et al. conceptually classified the following three types of fractures shown in Fig.2.53: (A) For stress states corresponding to circle 2 to 5 shown in Fig.2.52, individual tensile fractures propagate as wing cracks from existing cracks, and the tensile fracture can extend through the entire block depending on the magnitude of $\sigma_t$; (B) For stress states like those of circle 6, shear fracture is produced by coalescence of many winged cracks, the combination of original and wing cracks, and there can be other winged cracks in addition to the winged cracks making up the shear fracture (note that the shear zone may initially consist of a lot of tension fractures individually parallel to the direction of the major principal stress); (C) For stress states like those of circle 6 and under higher stresses, shear fracture is produced directly, and individual winged cracks also exist. The difference between (B) and (C) depends on the relationship between shearing resistance and tensile strength of a material.

The macroscopic shear fracture (zone of winged cracks) introduced by Einstein et al. can represent the shear zone of M cracks in the key joints. To study the final shear-off failure of key joints, the coalescence of such winged cracks then becomes the important issue. This point will be studied in the next literature.

Reyes et al. [1991]
The coalescence of preexisting non-persistent fractures in uniaxial compression experiments with a rock-like model material was carefully studied [Reyes et al., 1991]. The uniaxial compression tests were run on prismatic specimens as shown in Fig.2.54. The material used was a hydrated gypsum made from a mixture of gypsum (Hydrocal B-11), celite (diatomaceous earth) and water at ratios water : gypsum = 0.4 and water : celite
Fig. 2.54: Geometry of Specimen with Pre-existing Fractures [after Reyes et al., 1991]

Fig. 2.55: Non-overlapping (left) and Overlapping (right) Fractures [after Reyes et al., 1991]

Fig. 2.56: Maximum Tensile Principal Stress Contours, Dotted Areas Indicate Regions of High Tensile Stress [after Reyes et al., 1991]

Fig. 2.57: Maximum Tensile Principal Strain Contours, Dotted Areas Indicate Regions of High Tensile Strain [after Reyes et al., 1991]
35. The pre-existing fractures inclined at 30°, 45° and 60° to the horizontal were obtained by inserting steel shims in the mold. Two 12.7 mm long pre-existing fractures separated by a rock bridge of 12.7 mm length were used, and the fractures had a finite open width of about 0.25 mm which stayed open during compression testing. The fracture geometries were characterized as either overlapping or non-overlapping, depending on the inclination of the rock bridge with respect to the direction of the applied uniaxial compressive load as shown in Fig.2.55. It was observed that if the pre-existing fractures overlapped they coalesced through interconnection of the developing wing cracks; if the pre-existing fractures did not overlap coalescence occurred through secondary cracks which occurred in addition to and after the wing cracks. It was also reported that the wing crack surface mainly showed the hackle marks which characterized tensile fractures, and the coalescence crack surface showed the pulverized materials which characterized shearing.

Regarding the mechanisms underlying the fracture coalescence observed in the experiments, a linear elastic finite element calculation was performed. It was mentioned that there were regions of high tensile stresses in areas outside of the rock bridge where no cracking was observed (see Fig.2.56), and on the other hand a contour plot of major principal strains showed regions of high tensile strains only in areas where cracking occurred, such as close to the fracture tips and within the rock bridge (see Fig.2.57). It was then explained that principal strains were higher for regions where there was a combination of large tensile and compressive principal stresses. Then, Reyes et al. developed a fracture coalescence model based on the combination of a smeared crack/damage mechanics approach and a strain-based failure criterion, and obtained an agreement between the experimental and computed coalescence loads. This result is in contrast to the prediction from some other crack propagation criteria in fracture mechanics such as the maximum tensile stress criterion [Erdogan and Sih, 1963] or the minimum strain energy density criterion [Sih, 1974].

The coalescence phenomena through secondary cracks between the pre-existing cracks discussed here could be analogous to the coalescence between M cracks. Specifically, the crack sequence in the shear key introduced previously did not show the interconnection of the developing wing cracks from each M crack, but did show the similar secondary cracks discussed here. So far, most of reviewed observation indicated such phenomenon as a crushing. The crushing is physically caused by intense microcracking, and can be defined as highly strain localization. Thus, a careful observation of principal strain distribution in the shear zone of M cracks could be also instructive to the modelling of the coalescence of M cracks.
2-3-3. Push-off Shear Test of Reinforced Concrete

The push-off shear test of reinforced concrete element shows similar cracking phenomenon to M cracks in shear key joints. The use of reinforcement in such test can align multiple diagonal cracks more uniformly. Although there is no reinforcement in the shear key joints we would like to study, the steel reinforcement could be in some extent represented by the prestressing in the key joints, and therefore the study of such behavior will give us macroscopic comprehension on the mechanism of M cracks. In this section, the push-off shear test of reinforced concrete will be reviewed briefly based on this point.

Hsu et al. [1987]

Hsu et al. [1987] summarized the cracking behavior of shear transfer test specimens of reinforced concrete based on the extensive researches on this problem in the past 15 years. It was described that the problem of shear transfer across a plane in concrete could be basically divided into two kinds of distinctively different behavior in shear transfer problems: shear transfer across an initially cracked plane, and shear transfer across an initially uncracked plane. The behavior in the former case was governed largely by the shear-slip characteristics of the cracked plane. Aggregate interlock, dowel action, and constraints in a direction normal to the shear plane affected the resistance to shear. The final failure occurred along the existing crack (see Fig.2.58a) with little or no additional cracks formed across the existing crack, except in cases with a high percentage of steel crossing the initial crack.

It was also introduced that, in contrast, shear failure across an initially uncracked plane occurred after numerous cracks formed in a direction inclined to the shear plane (see Fig.2.58b). The final failure was usually due to the crushing of concrete in the compression struts formed approximately parallel to the direction of the cracks. The compression in the struts and the tension provided by the reinforcing bars across and parallel to the shear plane constitute a truss-like action. The region of numerous cracks mentioned above was explained based on the examination of the typical test specimen shown in Fig.2.59. Before cracking, the shear stress along the shear plane should be considerably larger near the two ends of the shear plane where the open slot disrupted the smooth geometry and introduced local stress concentration. After diagonal cracking, a cracked region was observed in the vicinity of the shear plane and eventually leads to failure (this cracked region was called a 'critical zone', see the shaded area in Fig.2.59). A typical width of this zone was observed to be about 2 to 3 in. for a 10 in. wide specimen [Mattock 1974]. Within this zone, the extensive cracking of the concrete had an effect of
Fig. 2.58: Shear Transfer Specimens [after Hsu et al., 1987]

Fig. 2.59: Push-off Test Specimen and Critical Zone [after Hsu et al., 1987]
redistributing the shear stress and the transverse normal stress more evenly along the shear plane. The cracking also reduced stiffness in this zone as compared to that outside of the zone. This would cause a redistribution of the compression stress in the transverse direction to become more evenly distributed across sections perpendicular to the shear plane. Thus, within this critical zone, the stresses could be assumed uniform.

Based on the observation mentioned above, Hsu et al. [1987] developed the analytical model of shear transfer strength in initially uncracked concrete. This will be discussed in Chapter .... The most important finding here is that the critical zone mentioned above may be a simple representation of the shear zone of M cracks in shear key joints although the width is much larger than that of M cracks. The horizontal steel reinforcement across the shear plane in the former case could be replaced by the prestress in the latter case. Then, it may be macroscopically explained that M cracks develop evenly along the shear failure plane and propagate with oscillating path near shear failure plane, and the crushing of the struts between M cracks would lead to the final shear-off failure.

2-4. PHYSICAL DESCRIPTION OF SHEAR-OFF FAILURE

As discussed in the previous section, the shear-off failure is typical in the shear failure of plain concrete or fiber reinforced shear key joints. The basic shear-off failure of key joints may be to some extent classified into two distinctive fracture mechanisms: a preceding single curvilinear/linear crack propagating diagonally (about 45°) from the corner of the key (S crack) after small slippage along the contact surface, and the subsequent multiple diagonal cracks along the key base (M cracks). In this section, the physical mechanisms of such fractures in the shear key joints are identified based on the previous reviews. Then, the basis of mechanical model developed in this thesis is identified.

2-4-1. Fracture Process of S & M Cracks

The fracture process of S crack and M cracks are identified as follows and schematically described in Fig.2.60. S crack is nucleated in the slow jumplike manner [e.g., Petit et al. 1988] by tensile stresses induced due to high shear stress concentration (pure shear state of stress) [e.g., Davies 1987] or wedge-like force [e.g., Ingraffea et al. 1985a] at the corner of the key joints, and this crack is thus a mode I crack [e.g., Bazant et al. 1986, Ingraffea et al. 1985a and Petit 1988]. The load level of this initiation has a large variation from 50% to 90% of the peak load [e.g., Bakhour et al. 1989, Beattie et al. 1989 and Petit 1988]. With further shear loading, S crack is loaded in the state of mixed
mode [e.g., Ingraffea et al. 1985a and Petit 1988] and propagates in two ways: a) roughly curvilinear crack path tending to rejoin the shear loading direction [e.g., Petit et al. 1988]; b) about 45°-inclined linear path [e.g., Koseki and Breen 1983, Bakhoun et al. 1989, Beattie et al. 1989, Cholewicki 1971 and Lacombe & Pommeret 1974]. Both crack trajectories are oscillated since S crack microscopically consists of a fairly narrow band of small tensile cracks in the vicinity of the crack tip: process zone [e.g., Petit et al. 1988]. Under high normal stress, the propagation of S crack tends to be arrested [e.g., Bakhoun et al. 1989 and Beattie et al. 1989]. Most of the reviewed literatures indicated that this mixed mode propagation was governed by the local maximum principal tensile stresses [e.g., Ingraffea et al. 1985a and Melin, 1989]. Another explanation was that the crack propagation was governed by a maximum energy release rate direction [e.g., Bazant et al. 1986]. Both are typical for concrete-like materials. This phenomenon is certainly different from that of PMMA which shows pure mode I crack path with zero mode II component [e.g., Melin, 1989].

At a certain load level, S crack tends to stop propagating [e.g., Bakhoun et al. 1989, Beattie et al. 1989 and Ingraffea et al. 1985a]. This can be explained by the fact that this crack runs into a low stress zone of the material [e.g., Bazant et al. 1986] or is arrested by a local principal compressive stresses. The reduced propagation of S crack under high normal stresses is also explained by this reason. This phenomenon induces another cracking mechanism which is exposed under higher stress zone, and causes a larger release of strain energy. Such fracture occurs in the shear zone of M cracks.

The fracture process of M cracks are more complex than that of S crack. At the present stage where the understanding of the actual propagation process is limited, the following process could be identified. M cracks are initiated on the local principal stress trajectories by tensile stresses under the changed stress field due to the distinctive S crack formation, and form a shear zone roughly aligned along the key base. Such initiation of M cracks can be under mode I condition [e.g., Bazant et al. 1986, Ingraffea et al. 1985a, Petit 1988 and Petit et al. 1988].

Specifically, the distinctive formation of S crack induces the rotation of the key specimens and also changes the boundary condition for the stress field along the key base. These phenomena can produce high stress field along the key base which allows M cracks to be initiated, while the S crack is running into low stress zone and the propagation of S crack is constrained. Then, the propagation of M cracks would cause a large release of strain energy while the propagation of S crack would release little strain energy [e.g., Bazant et al. 1986]. Thus, the main fracture process is moved to the zone of M cracks, and S crack becomes inactive.
Microscopically, the following mechanism could also be described [e.g., Petit et al. 1988]. Fracture initiation mainly results from ionic and covalent bond rupture initiated from many immobile flaw populations. Since concrete is in one sense semibrittle, it has linear defects like grain boundaries or pores which can be easily mobilized. At the first stage of loading, the flaws near the corner of the key are easily mobilized by shear stress concentration, and S crack is thus initiated. With further loading, S crack propagates and the flaws at the S crack tip front are not mobilized since the shear stress concentration is then moved to the key base especially near the corner (see Fig.2.51a) but is approximately distributed uniformly along the base [e.g., Hsu et al. 1987] due to the normal stress. At that stage, the flaws along the key base are easily mobilized by such shear stress concentration [e.g., Petit et al. 1988]. Then, intensive multiple cracking or M cracks formation occurs.

Under high normal stresses or in the case of shallow depth (see Fig.2.60a), M cracks tend to be initiated as multiple tensile cracks [e.g., Barr and Derradj 1990 and Davies et al. 1987] with a certain predominant inclination or have en echelon-like structures [e.g., Petit 1988], and approximately distribute evenly along the base due to the normal stresses. Specifically, normal and shear stresses are concentrated on the relatively lower part of the key due to the bending effect [e.g., Petit 1988]. With further shear loading, tensile cracks propagate as branch cracks or wing cracks from each M crack tip [e.g., Einstein et al. 1990] resulting in the formation of winged cracks (propagated M cracks). These winged cracks (M cracks) are finally coalesced due to the local highly confined condition [e.g., Petit 1988 and Reyes et al. 1991]. This highly localized strain distribution would lead to the crushing or intense microcracking between each winged crack due to the enforced shear movement [e.g., Bazant et al. 1986, Davies et al. 1987, Reyes et al. 1991 and Hsu et al. 1987].

On the other hand, under low normal stresses or in the case of deep depth (see Fig.2.60b), M cracks tend to reduce in number [e.g., Petit 1938] and there may only be a few large discrete cracks like splitting cracks [e.g., Ingraffea et al. 1985a]. With further shear loading, few splitting cracks propagate under mixed mode condition [e.g., Ingraffea et al. 1985a and Petit 1988]. The final shear-off failure in that case is completed in the catastrophic manner by crushing the remained short ligament between the tip of the splitting cracks and the corner of the key [e.g., Ingraffea et al. 1985a].

The fracture process of S crack and M cracks mentioned above will be examined analytically later in this thesis, focussing on both the cracking behavior and the transition process between S crack and M cracks.
Fig. 2.60: Sequential of S & M cracks in the case of:
(a) High Normal Stress or Shallow Depth; (b) Low Normal Stress or Deep Depth
2-4-2. Basis of Modelling for S & M cracks

Since the nature and orientation of the process zone for a pure mode II or mixed mode case are at present not well understood, LEFM description in terms of $K_I$ and $K_II$ can be a powerful approach if the process zone is small compared to the fracture width and other characteristic dimensions. In several literatures reviewed previously, LEFM concept was applied to shear fractures of concrete structures under the mixed mode or mode II condition [e.g., Ballatore et al. 1990, Davies 1987 and Watkins and Liu 1985], and like under mode I condition satisfactory results are obtained if the small scale yielding (SSY) condition was satisfied. In that case, the material parameter of fracture energy $G_f$ was used. Based on this approach, the propagation of S crack can be governed only by the linear elastic stress-singularity in the crack tip region under SSY condition and the toughness of concrete here is defined by the unique parameter $G_f$. In this case, 'maximum tensile stress criterion' [e.g., Melin, 1989] or 'maximum energy release rate criterion' [e.g., Bazant et al. 1986] could be applicable to the propagation of S crack.

As well as S crack, macroscopic shear fracture like the propagation of M cracks was also described based on LEFM concept as macroscopic mode II fractures in several literatures reviewed in the previous section [e.g., Watkins and Liu 1985]. However, the clearly defined mode II fracture toughness or mode II fracture energy are not currently available [e.g., Bazant et al. 1986, Ballatore et al. 1990, Watkins and Liu 1985 and Davies 1987] and further experimental and analytical works are necessary. Moreover, the shear zone of M cracks dissipates large energy by crushing the compression strut between M cracks in addition to tensile cracking [e.g., Bazant et al. 1986 and Davies et al. 1987]. Therefore, a rough macroscopic simplification based on LEFM (stress-based criterion) may not be suitable at present, and careful treatment of the energy dissipating process is necessary based on the nonlinear strain localization concept [e.g., Reyes et al. 1991]. Specifically, we should consider both the fracture represented by the tensile stress-strain diagram of material with M cracks, and that represented by the compressive stress-strain diagram for compression strut (including the strain-softening portion of the diagram) between M cracks.

2-5. SUMMARY AND CONCLUSION

In this chapter, the macroscopic shear failure phenomena of plain concrete or fiber reinforced concrete shear key-type structures are first reviewed, and two distinctive fracture mechanisms are identified: a preceding single curvilinear/linear crack propagating
diagonally (about 45°) from the corner of the key (S crack) after small slippage along the contact surface, and the subsequent multiple diagonal cracks along the key base (M cracks).

The physical mechanism of these two distinctive cracking behaviors was investigated based on the fracture mechanics concept through the review of shear failures of: a) plain concrete and rock under direct shear tests; b) rock under compressive stress field; c) reinforced concrete element in the push-off shear test.

Based on the investigated phenomena, a physical description of the shear-off failure of key joints is identified. This description will be utilized as the basis for the subsequent mechanical model developed in this thesis.
CHAPTER 3

REVIEW OF MODELLING STRATEGY

3-1. INTRODUCTION AND BACKGROUND

In this chapter, the modelling strategy based on the fracture mechanics approach which will be applicable to the shear-off failure of key joints is reviewed. Basic crack models and fracture parameters are first reviewed. Next, main numerical approaches to implement the crack models, and the application of those approaches to mixed mode problem like shear-off failure are briefly reviewed. In addition, theoretical approaches to simulate features such as S crack and M cracks in the shear key joints are also reviewed. These approaches will be the basis of the mechanical model developed in this thesis for shear-off failure of shear key joints.

Before review of modelling, it is necessary to make the definition of crack in concrete clear. Most work in fracture mechanics so far deals with metals. Some of the analytical methods used for metals are not suitable for concrete, as the fracture behavior of the two materials is very different [Hillerborg, 1983]. In metals, fracture is mostly preceded by yielding, giving a typical stress-elongation relation in tension. This has the following two consequences: (1) The stresses in the fracture zone in front of a crack tip remain constant or show a slight increase with an increasing deformation until local fracture occurs and the crack advances; (2) The yielding is accompanied by a tendency to contraction perpendicular to the stress direction. If this contraction is not prevented (plane stress conditions), significant yield deformations can take place with a corresponding high energy absorption. If the contraction is prevented (plane strain conditions), the amount of yield deformation and thus also the energy absorption are substantially smaller. Near the surface of a specimen, there is a zone where plane stress conditions prevail, while the interior of a thick specimen is mainly under plane strain conditions. Therefore, the width of the specimen (the length of the crack front) is of importance for the average energy absorption per unit increase in crack area, and thus the difference between plane stress and plane strain has to be taken into account in design and tests of metals.

In contrast, the fracture behavior of concrete is very different from that of metals. This is because the fracture of concrete is preceded by microcracking instead of yielding: The stresses decrease with increasing microcracking and thus with increasing deformation. Due to the microcracking, the stresses in front of a crack tip may have a typical distribution. Here, we have no well defined crack tip for concrete (like metals), but rather a fracture
zone, within which the cracking increases and the stress decreases as the deformation increases. The microcracking of concrete is not accompanied by any substantial contraction corresponding to that which occurs when metals yield. For this reason, no essential difference exists between plane stress and plane strain conditions for concrete. Specifically, the width of the specimen (the length of the crack front) is not expected to be of significance for concrete, like it is for metals which has also been confirmed by tests [Mindess and Nadeau, 1976].

3-2. BASIS OF FRACTURE MECHANICS MODELLING

In this section, basic crack models are first reviewed. Then, fracture parameters in the crack models of normal, high strength and fiber reinforced concretes are also reviewed. Both crack models and fracture parameters reviewed here will be the basis of the mechanical model developed in this thesis. Based on the reviews on modelling, the limitation of the present fracture mechanics modelling will then be identified.

3-2-1. Crack Models

The main problem of Fracture Mechanics, in the classical approaches, was to obtain conditions for the propagation of a single crack, modelized as a single surface discontinuity. Such a problem was essentially handled by means of a fracture criterion added to the classical field equations of Continuum Mechanics. This treatment led to a conceptual and computational distinction between fracture methods and methods derived from strength concept, which apply, respectively, when the dominant collapse mode is crack propagation or when the collapse is dominated by yielding. However, for most practical situations, the actual structural behavior of concrete was not dominated by these extreme modes, and crack fracture models were developed to provide a better description of the actual behavior.

Some of them did not include a global fracture criterion and were able to describe a progressive fracturing of the material without stress singularities. Such models permit a description of a smooth transition from the continuous medium to the discontinuous (fully fractured) one. They rely on a basic concept: strain localization. It seems plausible that strains are localized in a gradual way, as shown in Fig.3.1a. Simpler criteria on implementing this general hypothesis lead to simpler criteria (Fig.3.1b and 3.1c), which describe localization along a line [Hillerborg et al., 1976] or within a band [Bazant and Oh, 1983]. In this section, two basic crack models are reviewed: Cohesive Crack Model where
the former simplification (strain localization along the line) is used, and Band Model where the latter simplification (strain localization within the band) is used. These models are developed for cracking of concrete at macro-level, where concrete is treated as a continuum and homogeneous medium.

Cohesive crack model
The theory of cohesive cracks, arising from different physical motivations, has been used for a long time [Barenblatt 1962, Dugdale 1960]. A new interpretation of this concepts was made by Rice [1968] who assumed for elastic materials the restraining stress as a function of the separation distance, relating both values to 'J'. It is worth noting, however, that despite the efforts made in the theoretical and experimental fields, a fully general theory of cohesive crack development is still lacking. The theory is well established for isotropic solids, for pure opening (mode I), and for monotonic loading (continuously increasing crack opening), but the cohesive crack behavior for anisotropic media, mixed modes, or non-monotonic opening cases, is still subject to conjecture.

The crack initiation and crack evolution of a cohesive crack model are described as follows [Manuel Elices and Jaime Planas, 1989]. For isotropic materials, the behavior at a point is usually assumed to be elastic until the moment when the maximum principal stress reaches the tensile strength $f_t$. At this moment, fracture starts as a cohesive (stress transferring) crack normal to the maximum tensile stress direction, hence the condition for crack initiation is taken to be independent of triaxiality. This supposition could, obviously, be modified to take into account the influence of the transverse stresses on the strength, but such a refinement has not been introduced in practice. A generalization to anisotropic materials is still lacking. Once the cohesive crack has formed, the stress transferred through the crack faces is assumed to depend on the relative displacement of the crack faces. For pure opening, the transferred stress 'σ' is normal to the crack faces and is usually assumed to depend only on the evolution of normal crack opening 'w'. Hence, triaxiality is again ignored. For monotonic crack opening, the transferred stress is often assumed to be uniquely defined by the crack opening:

$$\sigma = f(w)$$  \hspace{1cm} (3.1)

where function $f(w)$ describes the softening behavior of the material, and is a material function.

The application of the underlying concepts to concrete fracture was pioneered by Hillerborg and co-workers [Fictitious Crack Model: Hillerborg et al., 1976]. Since then, a
good deal of theoretical as well as applied work has been performed in which cohesive crack models play a paramount role. The basis of the fictitious crack model can be demonstrated by means of a tension test as shown in Fig.3.2 [Hillerborg, 1983]. The test was assumed to be deformation-controlled and stable to follow the descending branch of the stress-deformation curve. If the specimen is homogeneous and has a constant area, the curves A and B coincide until the maximum load is reached. On further deformation, a fracture zone forms somewhere in the specimen, e.g., the whole fracture zone falls within gauge length A. As the fracture zone develops, the force will decrease due to the formation of microcracks and the corresponding weakening of the material. The deformation within gauge length B can then be described by means of a stress-strain curve including an unloading branch. The deformation within gauge length A includes also the deformation of the fracture zone. The additional deformation 'w' due to the fracture zone is the difference between the descending branches of curves A and B. Regarding the width of the fracture zone, it was assumed that the original width of the fracture zone was zero. The total width of the fracture zone then equals 'w'. Thus, the fracture zone can be described as a tied crack with width 'w', i.e. a crack which can transfer a stress $\sigma$ according to the $\sigma$-w curve when its width is w. As the fracture zone in reality has a certain width, the tied crack is not a real crack, and has been therefore called a 'fictitious crack'. Here, the area below the $\sigma$-w curve was defined as the fracture energy 'Gf', which thus was the absorbed energy per unit crack area for the complete separation of the crack surfaces, and this crack area in question is the projected area, not the total area of the irregular crack surface. The energy absorption outside the fictitious crack is determined as the volume of the specimen times the area below the $\sigma$-e curve. From the elastic and fracture parameters, two magnitudes having dimensions of length were defined. They are called the characteristic crack opening '$w_{ch}$' and the characteristic length '$l_{ch}$' [Hillerborg et al., 1976], and are defined as

$$w_{ch} = \frac{G_f}{f_t}$$  \hspace{1cm} (3.2a)

$$l_{ch} = \frac{EG_f}{f_t^2}$$  \hspace{1cm} (3.2b)

These two parameters are advantageously used to reduce structural lengths to dimensionless forms.

Regarding the link between Linear Elastic Fracture Mechanics (LEFM) and the fictitious crack model, LEFM may be considered a valid approximation in two extreme cases: (a) as a limit of cohesive crack behavior for large sizes; (b) as a limit for softening.
Fig. 3.1: Modes of strain localization. a) Smooth, arbitrary shape strain localization. b) Localization within a band. c) Localization into a crack-line.

Fig. 3.2: The principles for division of the deformation properties into $\sigma$-$e$ diagram and $\sigma$-$w$ diagram [after Hillerborg, 1983].
curves approaching a delta function. In Case (a), for infinite bodies it may be shown that
the solution does indeed converge to LEFM, so that the instability (critical) condition is
given by either of the following equations:

\[ K_{IC} = \sqrt{E'G_f} \]  
(3.3a)
\[ G_C = J_C = G_f \]  
(3.3b)
\[ CTOD_{C} = w_C \]  
(3.3c)

where, \( E' = E \) for plane stress, \( E' = E/(1-v^2) \) for plane strain, and the subscript 'c' refers to a
critical or instability condition. Then, it can be concluded that for large enough bodies the
only relevant fracture parameter is the fracture energy \( G_f \) and the maximum load is
independent of the shape of the softening curve. In Case (b), LEFM can be approached for
a given size when the softening curve is steep enough. Specifically, the condition of
applicability of LEFM is that the size of the specimen should be large in relation to \( \ell_{ch} \).
This condition is satisfied in Case (a), and also satisfied when the softening curve is such
that the characteristic length tends to zero or the softening curve approaches a delta function
so that the tensile strength becomes infinite while maintaining a finite area under the
softening curve. LEFM approach can be extended a little further with the concept of
effective, or equivalent elastic crack [Hillerborg, 1983]. In this approximation, it is
assumed that the stress field in the elastic zone surrounding the fracture process zone is still
dominated by a single parameter singularity, but the intensity of the singularity is given by
an effective stress intensity factor which is the stress intensity factor computed for the
actual load and an equivalent crack length equal to the initial crack length plus an effective
crack extension.

While the fictitious crack model mentioned above does not allow a stress singularity to
happen at the cohesive crack tip, Jenq and Shah [1985] considered such stress singularity
and proposed a 'Two Parameter Fracture Model' where the singularity at the cohesive
 crack tip was permitted while retaining the remaining features of the cohesive crack model;
in particular, the stress-transferring capacity of the newly formed crack. Attempts to apply
linear elastic fracture mechanics (LEFM) to concrete have been made for several years.
Several investigators have reported that when fracture toughness, \( K \), is evaluated from
notched specimens using conventional LEFM (measured peak load and initial notch
length), a significant size effect is observed. This size effect has been attributed to
nonlinear slow crack growth occurring prior to the peak load. Jenq and Shah included this
nonlinear slow crack growth. Critical stress intensity factor, \( K_{IC} \), is calculated at the tip of
the effective crack (see Fig.3.3). The critical effective crack extension is dictated by the elastic critical crack tip opening displacement, CTOD_0. The proposed model can be used to calculate the maximum load (for mode I fracture) of a structure of an arbitrary geometry. The concept behind this model can be explained from load P, versus crack mouth opening displacement, CMOD, relationships shown in Fig.3.3. Initially, the load-CMOD plot is linear up to about the load corresponding to approximately half the maximum load. At this stage, crack tip opening displacement is negligible, and K_I (mode I stress intensity factor) is less than 0.5 K_{IC}^s (see Fig.3.3a). Significant inelastic displacement and slow crack growth occur during the nonlinear range (see Fig.3.3b). At the critical point (see Fig.3.3c), the crack tip opening displacement reaches a critical value and K_I=K_{IC}^s. The only essential difference between the fictitious crack model and the two parameter fracture model is that the former involves the crack formation condition. In the two parameter fracture model, new cohesive crack surfaces can be created only from a preexisting crack tip and, consequently this model are suited for crack growth analysis rather than for a description of crack nucleation processes.

Cohesive crack models mentioned above are subject to the following limitations [Manuel Elices and Jaime Planas, 1989]: (a) It is very difficult to make a direct determination of the softening curve; (b) Experimental difficulties in mixed mode loading are well known and the limited results are, moreover, very difficult to analyze due to the lack of a sound theoretical model; (c) There are intrinsic limitations arising from the general hypothesis such as time independence and non dissipative bulk behavior; (d) Cohesive crack models are well suited to handle a single crack, or even a discrete crack system formed by cracks neatly separated, but not suited to multiple cracking or diffuse cracking; (e) A cohesive crack cannot develop in pure sliding mode because of the requirement of a pure opening mode for the initial stages of crack development. This could be relieved by postulating a different condition for crack formation involving both normal and tangential stresses on the potential fracture plane.

**Band model**

At first glance, band models could be considered as a formulation of the smeared crack approach. But the smeared crack concept in itself was initially introduced as an approximate expedient to handle cracking in the framework of numerical methods rather than as a consistent approach to model fracture behavior. On the other hand, the band model described here takes shape as a theory advocated as independent of the numerical method used to solve a particular problem. Band theory models the strain localization as a softening band of constant thickness. The crack band formation and band evolution are
Fig. 3.3: Fracture Resistance Stages of Plain Concrete: a) $K_t < 0.5K_{IC}$ b) Nonlinear Range; c) Critical Point, $K_t = 0.5K_{IC}$ [after Jenq and Shah, 1985]

Fig. 3.4: Stress-deformation Relation and Fracture Mechanics Parameters
described as follows [Manuel Elices and Jaime Planas 1989]. For isotropic materials, the behavior at a point is usually assumed to be elastic until the moment where the maximum principal stress reaches the tensile strength \( f_t \). At this moment, the fracture starts as an array of densely and uniformly distributed cracks, normal to the maximum tensile stress direction, and distributed over the band thickness 'h'. Hence, in this model, as in the cohesive crack model, the condition for band initiation is independent of triaxiality. Once the crack band has formed, the crack orientation remains fixed and it is assumed that the stress tensor and strain tensor remain uniform through the band thickness. Stress and strain are related through a constitutive equation, or stress-strain relationship, displaying softening. As in a cohesive crack model, it is usually assumed that the traction vector acting on crack planes depends on the average crack opening per unit band thickness.

In this model, it is assumed that the softening zone can be described as a band in which multiple parallel cracking has occurred. For pure opening mode, the traction vector is normal to the crack faces. The softening curve is then expressed as a stress-strain curve where the area under the curve is the volumic fracture energy '\( g_f \)' which is related to the fracture energy \( G_f \) and the band width 'h' by

\[
G_f = h g_f
\]  

(3.4)

From this equation, characteristic crack opening and characteristic length may be defined as for a cohesive crack.

A band model strictly parallels a cohesive crack model. A band model will then display all the limitations of cohesive crack models discussed previously, and some more as follows: (a) There is no direct experimental evidence of a softening band appearing to have constant thickness and uniform strain throughout its thickness. The band thickness is not a magnitude of deep physical significance. Namely, the band model is no more than a numerical expedient to handle a discrete crack problem by smearing the actual displacement over a fixed length; (b) Essential hypotheses of the band model do not fit into the framework of classical continuum mechanics.

Both cohesive crack and band models studied previously display the linear elastic bulk behavior. For concrete, the bulk behavior is generally assumed to be isotropic linear elastic, hence defined by an elastic modulus \( E \) and a Poisson' ratio \( \nu \). However, in general bulk energy dissipation and strain irreversibility are presented as shown in Fig.3.2. Since unloading always occurs in some region of the bulk after strain localization, statement of the unloading behavior of the bulk is essential. The cohesive crack and band models displaying bulk dissipation could be formulated by retaining the hypothesis regarding the
formulation and evolution, while relaxing the hypothesis of linear elastic behavior of the bulk. However, in doing so, one of the most appealing features of the previous models, the coincidence of $G_f$ with the mean energy dissipation per unit area, is lost, and computational difficulties are heavily increased. Moreover, in the formulation of the previous reviewed models, it is clear that a hypothesis of non-dissipative bulk behavior is a useful and probably reasonable simplification. Therefore, the models with bulk dissipation are not discussed in this thesis. However, it is worth trying to set up a general framework for cohesive crack and band models with bulk dissipation since they could explain, at least partially, the apparent size dependence of $G_f$ as measured by the RILEM methodology for concrete.

3-2-2. Fracture parameters

In this section, main characteristics of normal, high strength and fiber reinforced concrete are discussed with emphasis on the fracture parameters used in the crack models discussed in the previous section.

Normal concrete
In the stress-deformation relation under tensile loading as shown in Fig.3.4, the following fracture mechanics parameters are included [Hordijk, Van Mier and Reinhardt, 1989]: the tensile strength $f_t$, the Young's modulus $E$, the fracture energy $G_f$, the shape of the descending branch and the maximum crack opening $w_o$ at which stress can no longer be transferred (stress-crack opening can be derived from the stress-deformation relation). The Young's modulus for tension is mostly taken to be equal to that for compression. The Poisson's ratio generally varies between 0.15 and 0.25. The tensile strength $f_t$ was already considered to be a material parameter on the classical strength theory. Standard test methods are available to determine the tensile strength: the uniaxial tensile test (direct tension test), the splitting test (Brazilian test) and the bending test. The fracture energy $G_f$ is considered to be a material parameter only since the introduction of the fictitious crack model. It is defined as the amount of energy necessary to create one unit of crack area and equals the area under the $\sigma$-$w$ relation. For a ordinary concrete and maximum aggregate size varying between 8 and 32 mm, $G_f$ values between 65 and 200 N/m were found. Specifically, in uniaxial tensile tests, $G_f$ values for ordinary concrete in the order of 100 N/m have been found [Cornelissen et al. 1986].

The importance of the shape of the descending branch in the stress-crack opening curve has been demonstrated by many researchers. The simple approximations of the
Fig. 3.5: (a) Approximation of the $\sigma$-w Curve by means of a Straight Line; (b) Bi-linear Representation of $\sigma$-w Curve [after Hillerborg, 1985a]
stress-crack opening curve are a linear model and a bi-linear model (Fig.3.5) which can achieve a good approximation [e.g. Hillerborg, 1985a]. More complex expressions are derived as follows.

[ Cornelissen et al. 1986]:

\[
\frac{\sigma}{f_t} = \left(1 + \left(c_1 \frac{w}{w_o}\right)^3\right) \exp\left(-c_2 \frac{w}{w_o}\right) - \frac{w}{w_o} \left(1 + c_1^3\right) \exp(-c_2)
\]

\[c_1 = 3, \ c_2 = 6.93, \ w_o = 160 \ \text{for a certain normal concrete,}\]

\[w \ \text{and} \ w_o \in 10^{-6} \ \text{m}\] (3.5)

[ Gopalaratnam and Shah, 1985]:

\[
\frac{\sigma}{f_t} = \exp(-kw^\lambda) \quad (\lambda = 1.01 \ \text{and} \ k = 0.063, \ w \in 10^{-6} \ \text{m})
\] (3.6)

The tensile strength $f_t$ and the fracture energy $G_f$ are based partially on different material characteristics. The tensile strength depends on the undamaged material, while the fracture energy depends mainly on interlocking, which in turn depends on the strength and stiffness of the crack surfaces.

In band models, an additional parameter 'h', the width of the softening zone is required. By fitting test results from different experiments, Bazant and Oh [1983] found the optimum value for this parameter to be about three times the maximum aggregate size. However, since it is still difficult to demonstrate the existence of a process zone it may be obvious that there is not much information about experimentally determined values for the parameter 'h'.

Using the equipment of Fig.3.6, a series of tests under mixed mode loading were performed on connection with a cube compression strength of 48.4 N/mm², a cube splitting strength of 3.12 N/mm² and a maximum grain size of 8 mm [Reinhardt et al. 1987]. First, uniaxial tensile tests were carried out in which a softening zone was created by uniaxial tension first up to a deformation of about 15 to 20 mm. Then, the uniaxial deformation was fixed and a shear force was applied gradually up to a certain value. Stress-deformation diagram of two mixed mode tests are shown in Fig.3.7. In these tests, a softening zone was created first which had a normal deformation of about 15 mm in test number 02-2 and about 20 mm in test number 02-4. Then a shear force was applied gradually which was 0.4 N/mm² in the test 02-2 and 0.8 N/mm² in the test 02-4 respectively, which caused a small dip in the normal stress versus normal deformation
Fig. 3.6: Mixed Mode Testing Equipment: a) Specimen, b) View of Testing Equipment [after Reinhardt et al., 1987]

Fig. 3.7: Normal Stress versus Normal Deformation of Two Tests with Additional Shear Stress [after Reinhardt et al., 1987]
Fig. 3.8: a) Normal Stress versus Normal Deformation, b) Shear Stress, and c) Normal Stress versus Shear Deformation during Cycling at Constant Normal Deformation of 58 mm. [after Reinhardt et al., 1987]

Fig. 3.9: a) Complete Plot of Four Interrelated Quantities: Shear Stress, Normal Stress, Shear Deformation and Normal Deformation [after Reinhardt et al., 1987]
curve. Then, loading in the normal direction was resumed. It appeared that the shape of the σ-δ line was almost the same with or without simultaneous shear stress.

Subsequently, two tests were performed in which a crack opening was created by uniaxial tension. Then, cyclic shear was applied with an upper shear stress level of 2 N/mm² and a lower one of zero. After three manually controlled cycles, the shear stress was increased till 8 N/mm² and decreased to zero again. Then the specimen was unloaded completely. Fig.3.8a shows the normal stress versus normal deformation relation during the complete test. Uniaxial loading is applied first until the normal deformation reaches the value of 58 mm. Then a shear stress is applied at the normal deformation which was intended to be constant. When shear is applied, the normal stress had to be changed from tensile to compressive in order to keep the normal displacement constant which was the desired boundary condition of the test. Fig.3.8b shows the shear stress versus shear displacement relation for the same test. We observe that the shear stiffness varies from the first to the second cycle, but that the second and the third cycle are almost the same. Obviously, there is energy dissipation which seems to be due to partly friction and partly damage. The following cycle with higher stress shows an almost linear relation between shear stress and deformation. Fig.3.8c gives the normal stress versus shear deformation relation for the four cycles. It can be seen that a small normal stress develops during the first small shear cycles with negligible energy dissipation. The last cycle however shows a large degradation of the material. Fig.3.9 shows the complete plot of the quantities involved, i.e., normal stress, normal displacement, shear stress, and shear displacement. It can be seen that the shear stiffness is reduced by the normal deformation and that normal compression forces have to be applied in order to keep the normal displacement at constant values.

Regarding the mixed mode behavior mentioned above, the following modeling is available. The relation between stress increment and crack displacements can be given in general terms [Hordijk, Van Mier and Reinhardt, 1989]:

\[
\begin{bmatrix}
\Delta \sigma_{nn} \\
\Delta \sigma_{nt}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta \delta_n \\
\Delta \delta_t
\end{bmatrix}
\]  

(3.7)

The index n denotes normal and t denotes tangential to the crack. Eq.(3.7) is connected to strain if the crack displacement is smeared out over a certain width which is mostly the size of an element mesh in a FE-model. Eq.(3.7) then becomes
\[
\begin{bmatrix}
\Delta \sigma_{mn} \\
\Delta \sigma_{nt}
\end{bmatrix} = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}\begin{bmatrix}
\Delta \epsilon^c_{mn} \\
\Delta \gamma^c_{nt}
\end{bmatrix}
\]

(3.8)

The subscript \(cr\) denotes cracking. Experiments [Millard and Johnson 1984 & 1985, Walraven 1980, Mattock et al. 1975, Laible et al. 1977] have shown that the stiffness matrix is not symmetric which causes some numerical inconvenience. Walraven and Keuser [1987] have shown that \(B_{12}\) can be put to zero if \(\delta_y/\delta_n < 2/3\). \(B_{21}\) takes account of a decreasing shear stiffness with increasing crack width. If this phenomenon is formulated by making \(B_{22}\) crack width dependent, \(B_{21}\) can also be put to zero. Then, the stiffness matrix \(B_{ij}\) becomes symmetric and numerically simpler.

The total shear strain increment which is composed of the contribution of the uncracked concrete and the crack displacement, reads

\[
\Delta \gamma_{nt} = \Delta \gamma^c_{nt} + \Delta \gamma^c_{nt} = \left( \frac{2(1 + \nu)}{E} + \frac{1}{B_{22}} \right) \Delta \sigma_{nt}
\]

(3.9)

Multiplied by the shear modulus, it accounts for the continuously decreasing shear stiffness due to cracking:

\[
\Delta \gamma_{nt} = \frac{\Delta \sigma_{nt}}{\beta G}
\]

(3.10)

Combining Eq.(3.9) and Eq.(3.10) yields the shear retention factor:

\[
\beta^{-1} = 1 + \frac{G}{B_{22}}
\]

(3.11)

There are several expressions found in the literature which give shear retention factor \(\beta\) as a function of normal strain or crack width. Rots et al. [1984] have suggested

\[
\beta^{-1} = 1 + 4447 \epsilon_{nn}
\]

(3.12)

Bazant and Gambarova [1980] derived the following relation
\[
\beta = \frac{1}{4762\varepsilon_{nn}} - \frac{1}{1346\sqrt{\varepsilon_{nn}}}
\]  \hspace{1cm} (3.13)

Pruijssers [1988] proposed a rather complex expression

\[
\beta^{-1} = 1 + p\varepsilon_{nn}
\]

where

\[
p = \frac{2500}{\max^{0.14} \left[ 0.76 - 0.16 \varepsilon_{nn} \gamma_{nt} \left( 1 - \exp \left( -6 \frac{\gamma_{nt}}{\varepsilon_{nn}} \right) \right) \right]}
\]  \hspace{1cm} (3.14)

where \( \max \) is the maximum aggregate grain size.

It should be kept in mind that crack propagation under mixed mode loading is due to tensile stresses and perpendicular to the direction of the principal tensile stress but that, after a crack has developed, both shear and normal stresses can act on the crack. At the moment, there are theoretical and experimental studies under way which will improve mixed mode modelling and extend the models to cyclic loading and cases under other mechanical and physical influences.

**High strength concrete**

The following two categories of high strength concrete could be distinguished [Hordijk, Van Mier and Reinhardt 1989]: (1) High Strength Concrete (HSC) with a compressive strength between 40 and 80 N/mm\(^2\). These concretes are generally obtained by the simple addition of a super-plasticizer and by the use of very good cement and suitable aggregates; (2) Very High Strength Concrete (VHSC) with a compressive strength higher than 80 N/mm\(^2\). To obtain such concretes, it is necessary to add ultrafines with a particle size smaller than that of the cement. Among the ultrafines most commonly used, silica fume may be mentioned.

In the uniaxial compressive and tensile tests, the post-peak behavior of HSC is less stable (steeper downward slope) than in normal concrete. There is still very little theoretical research done on the mechanical behavior of HSC. Most models for normal concrete can in principle be applied to HSC. At the present time, two theoretical approaches have dealt with the study of crack propagation in high strength concrete: one concerning the application of linear elastic fracture mechanics [De Larrard et al. 1987] and the other that of nonlinear fracture mechanics. The main results obtained by such researches can be
Table 3.1: Values of $K_{IC}$ as a Function of Compressive Strength [after De Larrard et al., 1987]

<table>
<thead>
<tr>
<th></th>
<th>$f'_C$ N/mm$^2$</th>
<th>$K_{IC}$ N/mm$^{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Concrete</td>
<td>49</td>
<td>68.3</td>
</tr>
<tr>
<td>HSC</td>
<td>75</td>
<td>80.3</td>
</tr>
<tr>
<td>VHSC</td>
<td>105</td>
<td>90.1</td>
</tr>
</tbody>
</table>

Fig.3.10: Notch Sensitivity vs. Relative Notch Depth of 100 X 100 X 400 mm Beams [after Tognon and Cangiano, 1989]
Fig. 3.11: Fracture Toughness of Beams with Different Geometries in Relation to Their Compressive Strength [after Tognon and Cangiano, 1989]
Fig. 3.12: Fracture Energy $G_f$ as Influenced by Mean Slenderness [after Tognon and Cangiano, 1989]
summarized as: The toughness value $K_{IC}$ increases with the compressive strength of the concrete. De Larrard et al. [1987] obtained the quantitative results shown in Table 3.1. This fracture parameters of LEFM was derived based on the fact that the microcracked zone which precedes macrocracking is smaller and has a smaller microcrack density in the case of HSC than in the case of normal concrete [Carrasquillo et al. 1981]. The same approach was done for HSC and VHS as follows.

Tognon and Cangiano [1989] carried out the 'three-point bend beam test' to obtain typical fracture mechanics parameters of high strength and very high strength concrete. Different types of concrete have been tested, namely normal-strength (NS / gel-like structure; compressive strength of 50-60 N/mm$^2$), high-strength (HS / gel-like structure; compressive strength of 50-120 N/mm$^2$) and very high strength (VHS / quasi-crystalline structure; compressive strength of 170 N/mm$^2$) concretes. The applicability of LEFM to beams with span of 400 mm and made up of three classes of concrete was investigated by measuring the notch sensitivity. This notch sensitivity was defined by the ratio of the flexural tensile strength of notched beams, referred to the net resisting section, to the flexural tensile strength of unnotched beams. The results indicated that VHS concretes prove to be noticeably notch sensitive at all a/d ratios considered (Fig.3.10). Here, 'a' indicates the notch size, and 'd' the depth of beams. It was indicated that the considerable notch sensitivity of the autoclaved VHS concretes makes it possible to apply LEFM to them with sufficient accuracy. In this assumption, fracture toughness expressed by critical factor $K_{IC}$ was evaluated by the method of ASTM [1974]. By expanding this assumption to NS and HS concrete, the corresponding values of $K_{IC}$ were also obtained and shown in Fig.3.11. In these figures, one can see some size effect on the fracture toughness in all three materials. Fig.3.12 shows the obtained mean trends of the fracture energy $G_f$ based on the method of RILEM [1985] referred to strength classes as a function of the three slenderness values ($=1/d$) of the beams, having a relative notch depth a/d=0.45.

**Fiber reinforced concrete**

Three stages are generally distinguished in the crack process of normal concrete: (1) The first stage concerns the creation of microcracks, distributed randomly within the concrete volume; (2) The second stage is translated by the location of these microcracks at one or more sites in the concrete volume. These locations lead to the creation of one or more macrocracks; (3) Finally, the last stage corresponds to the propagation of macrocracks.

Contrast to these stages, the fibers intervene in different ways [Rossi 1988]: (1) During the first stage corresponding to uniformly distributed microcracking, the fibers have a stitching action on the microcracks, preventing them from propagation. This intervention
of the fibers retards the microcracking location phase and hence the creation of macrocracks; (2) When macrocracking occurs, fibers will play the same role as reinforcement in reinforced concrete with respect to these macrocracks.

Thus, the fibers will intervene at two different levels: at the structural level (location phase) and at the material (or macrocrack) level. This will require the optimization of fiber dimensions and also of the incorporated percentage according to the mechanical properties to be improved. For workability, two possibilities can be considered for the mix design of a fiber reinforced concrete (with the exception of continuous fibers): (1) It is possible either to incorporate a high percentage of short fibers or; (2) a low percentage of long fibers. In the first case, the strength of the material will increase (if the dimensional scale of the fibers is the same as that of the microcracks), and in the second case, the ductility of the structure will be improved (to be effective with regard to macrocracking, the fiber must necessarily have a sufficiently long anchoring length). Abundant literature exists on the mechanical characterization of fiber reinforced concrete, for almost all types of fibers that are available at the present. In general, the values of strength for fiber reinforced concretes (compressive, tensile, bending, etc.) are higher than those for normal concrete. Figures 3.13 and 3.14 show some experimental data introduced by IJordijk, Van Mier and Reinhardt [1989].

As a nonlinear fracture mechanics approach for fiber reinforced concrete, two models could be presented: the fictitious crack model and two-parameter model. As these two models were dealt with previously, only the introduction of fiber action to these models is discussed here. The fictitious crack model is based on knowledge of the stress-deformation relationship obtained during a direct tensile test (determination of tensile strength \( f_t \), energy dissipated within the nonlinear domain \( G_f \), and the shape of the \( \sigma-w \) curve after the stress peak). A simple theoretical model is proposed on the behavior of a fiber going through a crack [Hillerborg 1980]. This model makes it possible to obtain different \( \sigma-w \) curves according to the type of fiber used as shown in figures 3.15 and 3.16. After a stepwise linear approximation as shown in Fig.3.17, these curves can be used in numerical studies, for example by means of the finite element method [e.g. Hillerborg et al. 1976]. Fig.3.18 provides an example of an analysis carried out on the basis of the curves in Fig.3.17. It considers a classical notched beam loaded in three point bending.

In the two-parameter model [Jeng and Shah 1985, Jeng and Shah 1986], two intrinsic properties govern the propagation of a crack in concrete: \( K_{IC} \) (toughness of concrete in mode I) and CTODc (critical value of crack tip opening displacement). The purpose of using these two parameters is to obtain a model independent of the scale effect and also of
Fig. 3.13: Stress-strain Curves of FRC in Uniaxial Compression [after Hordijk, Van Mier and Reinhardt 1989]

Fig. 3.14: Stress-strain Curves of FRC in Uniaxial Tension [after Hordijk, Van Mier and Reinhardt 1989]
Fig. 3.15: Examples of Possible $\sigma$-w-curves for Steel-fiber Reinforced Concrete [after Hillerborg, 1980]

Fig. 3.16: Examples of Possible $\sigma$-w-curves for Plastic-fiber Reinforced Concrete [after Hillerborg, 1980]
Fig. 3.17: Stepwise Linear Approximations of Some $\sigma$-$w$ curves [after Hillerborg, 1980]

Fig. 3.18: Theoretical Load-deflection Curves for Concrete Beams with $\sigma$-$w$-curves [after Hillerborg, 1980]
Fig. 3.19: Composition of External Load Applied on a Fiber-reinforced Structure [after Jenq and Shah, 1986]

Fig. 3.20: Comparison of theoretical prediction and Experimental Results of Load-deflection Curves [after Jenq and Shah, 1986]
the boundary conditions on the structure under investigation. In the case of fiber reinforced concrete, crack propagation in the matrix is governed by the same criterion as for normal concrete, namely: $K_{Ic} = K_c$. The CTOD parameter allows the calculation of effective crack length (real crack + microcracked zone) during propagation. For the calculation of $K_{Ic}$, this crack length must be known. The total stress acting on fiber reinforced concrete is broken down as follows:

$$P = P^m + P^K + P_S$$

(3.15)

where $P^m$ is the contribution of the matrix and is related to $K_I$, $P^K$ is related to $K_I$ and intervenes on the singularity at the crack tip (stitching of microcracks), and $P_S$ is the stress produced by the fibers which have the stitching action on the macrocrack. Fig.3.19 shows schematically this breakdown of the stress $P$. An example of results obtained with the two-parameter model is presented in Fig.3.20. In this figure, experimental results obtained in three-point bending are compared with predictions of the two-parameter model showing good agreement between them.

3-2-3. Summary

So far, basic crack models and fracture parameters were reviewed, which are currently available for the modelling of the shear-off failure of key joints. Then, the following limitations and features are identified which should be considered in the fracture modelling.

<table>
<thead>
<tr>
<th>Crack models</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. There is no well defined crack tip for concrete as crack growth is preceded by microcracking in front of a crack tip.</td>
</tr>
<tr>
<td>2. Cohesive and band crack models were well established for isotropic solids, for pure opening (mode I), and for monotonic loading (continuously increasing crack opening), but not for anisotropic media, mixed modes, or non-monotonic opening cases which are still subject to conjecture. In addition, there are intrinsic limitations arising from the general hypothesis such as time independence and non-dissipative bulk behavior.</td>
</tr>
<tr>
<td>3. Cohesive crack models are well suited to handle a single crack, or even a discrete crack system formed by cracks neatly separated, but not suited to multiple cracking or diffuse...</td>
</tr>
</tbody>
</table>
cracking. On the other hand, band models are well suited to handle such multiple cracking, but lack direct experimental evidence of a softening band appearing to have constant thickness and uniform strain throughout its thickness. The band thickness is thus not a magnitude of deep physical significance.

4. Cohesive cracks cannot develop in pure sliding mode because of the requirement of a pure opening mode for the initial stages of crack development.

5. For large enough bodies, the only relevant fracture parameter is the fracture energy $G_f$ and the maximum load is independent of the shape of the softening curve.

Fracture parameters
1. $G_f$ values for ordinary concrete is in the order of 100 N/m varying between 65 and 200 N/m.

2. Although several shapes of the descending branch in the stress-crack opening curve have been presented, a bi-linear curve could be suitable to achieve a good approximation.

3. The crack propagation under mixed mode loading is due to tensile stresses and perpendicular to the direction of the principal tensile stress but that, after a crack has developed, shear and normal stresses may act on the crack. Under such condition, the shape of the $\sigma$-$\delta$ line is almost the same with or without simultaneous shear stress if such quantity is relatively small. Meanwhile, the shear stiffness of the crack is reduced by the normal deformation, and an almost linear relation between shear stress and deformation is observed.

4. The microcracked zone which precedes macrocracking is smaller and has a smaller microcrack density in the case of very high or high strength concrete than in the case of normal concrete. The use of LEFM is then a reasonable approximation for very high strength concrete or high strength concrete.

5. Fibers in concrete play two roles as: (1) providing a stitching action on the microcracks, preventing them from propagation which leads to the increase of the strength of materials; (2) providing a stitching action on the macrocracks as reinforcement in reinforced concrete which leads to improvement of the ductility of the structure. Specifically, the former case can be achieved effectively by a high percentage of short
fibers (if the dimensional scale of the fibers is the same as that of the microcracks) and the latter by a low percentage of long fibers (necessarily have a sufficiently long anchoring length).

3-3. NUMERICAL APPROACHES

In this section, two main numerical approaches to implement the crack models discussed previously are briefly reviewed: discrete crack approach and smeared crack approach. Then, the application of those two approaches to mixed mode problem is investigated. Based on these reviews, the features and limitations of the current available numerical approaches are identified.

3-3-1. Discrete Crack Approach

The discrete crack approach is a direct application of the crack models. Under certain conditions, a band model can be also looked upon as a discrete crack approach [Hillerborg and Rots 1989]. When crack growth is analyzed on the assumption that cohesive forces are acting in the process zone, this is normally done by means of some type of numerical technique, typically by means of finite element method, which is hitherto by far the most common.

The complexity of the finite element analysis depends very much on whether the crack path is assumed in advance or not. If the crack path is assumed in advance, the finite element mesh is arranged in such a way that the crack either follows element boundaries or is described by 'cracked' element along its path. In the former case, the fracture zone is modelled as a separation of the elements along the crack path. This is a pure crack model, and it is often referred to as the fictitious crack model [Hillerborg et al. 1976]. In the latter case, the fracture zone is modeled as a change in stiffness of a row of elements along the crack path. This is what is referred to as the crack band model [Bazant and Oh 1983]. These models are illustrated by means of the beam shown in Fig.3.21, with a fracture zone in the tensile side of the beam. Fig.3.21a illustrates the separation of elements with the introduction of closing stresses, which depend on the fracture zone deformation w, which is equal to the node separation distance. Fig.3.21b illustrates the change of stiffness of a row of elements. Thus, Fig.3.21a corresponds to the fictitious crack model and Fig.3.21b to the crack band model. These two models mainly differ in the finite element formulation, but the numerical results are practically identical. The crack band model can be looked
upon as a type of discrete crack approach only in the case where the crack follows along a row of elements, where element boundaries are parallel to the crack. If not, the crack band model is a smeared approach.

The analysis of the formation and propagation of a fracture zone by means of the fictitious crack model is illustrated in Fig.3.22, which shows the development of the fracture zone through the finite element mesh of the beam in Fig.3.21. The fracture zone in this case can be expected to follow a vertical line at the center of the beam. The element mesh is arranged in such a way that element boundaries are situated along this line. Each node point along this line is separated into two, belonging to the elements on each side. The internal forces between the two node points in each pair are calculated during the analysis. At the beginning of the analysis, the two node points in each pair coincide. The first step in the analysis is to calculate the load which makes the internal force between a pair of node points equal to the tensile strength. Then, a process zone starts forming and a closing force $F_1$ is acting between two node points, corresponding to the cohesive stress. Next step in the analysis is to find the condition for the propagation of the process zone, i.e. the load that makes the force $F_2$ (Fig.3.22) reach a value corresponding to the tensile strength. By means of this step by step analysis, the complete behavior of the beam can be followed as the fracture zone propagates.

To install the fictitious crack into the element boundaries, interface elements are used which are incorporated within the original mesh and of which the initial stiffness is assigned a large dummy value in order to simulate the uncracked state with rigid connection between overlapping nodes [Rots and Blaauwendraad 1989]. Such a model links the traction $t^{cr}$ across the crack to the relative displacement $u^{cr}$ across the crack via $C^{cr}$ which represents phenomena like tension-softening and aggregate interlock:

$$\Delta t^{cr} = C^{cr} \Delta u^{cr}$$  \hspace{1cm} (3.16)

A distinction can be made between lumped interface elements [Ngo and Scordelis 1967] which evaluate the tractions and displacements at isolated node-sets, and continuous interface elements [Goodman et al. 1968] which smooth the behavior along an interpolated field. It has been suggested that the latter class of elements is superior. However, Rots [1988] insisted that this was not generally true. This relates to the question of how large the dummy stiffness can and should be made. Ideally, it should be made extremely large to keep the initial dummy separation negligible. It was indicated that with the continuous interface elements, such high stiffness values turned out to produce significant flutter in the
Fig. 3.21: Application of the Discrete Crack Approach by means of Finite Element Analysis and a) the Fictitious Crack Model; b) The Crack Band Model
Fig. 3.22: Development of a Fracture Zone according to the Fictitious Crack Model. Only the node points along the predicted crack path are shown, together with the closing forces. [after Hillerborg and Rots, 1989]

Fig. 3.23: Example of Automatic Remeshing. The upper figure shows a part of the original mesh with the present and assumed next crack tip position. In a)-d) the steps in the automatic remeshing are shown. [after Ingraffea and Saouma, 1985b]
traction profiles, whereas with the lumped interface elements, the results for increasing dummy stiffness correctly converged towards the rigid-connection solution.

If the crack path is not known in advance, the problem is much more complicated. When the discrete crack approach is applied in this case, two different strategies can be followed. One strategy is to assume different fixed crack paths, and make one analysis for each path, each time with the finite element mesh arranged with respect to the chosen path. It is then possible to check if a path is close to the correct one by comparing the crack propagation direction with the principal stress directions. The other strategy is to study a crack which does not follow the original finite element mesh, but finds its own way through the structure, by a successive regeneration of the mesh in the vicinity of the crack path. The successive regeneration has some numerical problems regarding the band width of the global stiffness matrix, however this difficulty was overcome by means of special programs, which automatically perform the necessary changes. An illustration of this technique is given in Fig.3.23, and the detail can be referred to the work of Ingraffea et al. [1985b].

3-3-2. Smeared Crack Approach

The counter part of the discrete crack approach is the smeared crack approach, in which a cracked solid is imagined to be a continuum with the notion of stress and strain [Rots 1988, Rots and Blaauwendraad 1989]. The behavior of cracked concrete can then be described in terms of stress-strain relations and it is sufficient to switch from the initial isotropic stress-strain relation to an orthotropic stress-strain relation upon cracking. As a consequence, the topology of the original finite element mesh remains preserved which is computationally convenient. It is for this reason that the method came into widespread use and quickly replaced the early discrete crack models.

**Standard fixed smeared crack concept**

Once the principal tensile stress exceeds the tensile strength, a fixed crack can be initiated perpendicular to the direction of principal stress. Traditionally, the isotropic stress-strain law has then been changed into an orthotropic law with respect to the fixed n, t axes of orthotropy, where n refers to the direction normal to the crack (mode I) and t refers to the direction tangential to the crack (mode II). Considering the mode I tension softening and mode II shear softening, one may have the following constitutive relation:
\[
\begin{bmatrix}
\Delta \sigma_{nn} \\
\Delta \sigma_{tt} \\
\Delta \sigma_{nt}
\end{bmatrix} =
\begin{bmatrix}
\frac{\mu E}{1 - \nu^2 \mu} & \frac{\nu \mu E}{1 - \nu^2 \mu} & 0 \\
\frac{\nu \mu E}{1 - \nu^2 \mu} & \frac{\nu \mu E}{1 - \nu^2 \mu} & 0 \\
0 & 0 & \beta G
\end{bmatrix}
\begin{bmatrix}
\Delta \varepsilon_{nn} \\
\Delta \varepsilon_{tt} \\
\Delta \varepsilon_{nt}
\end{bmatrix} 
\] (3.17)

where \( \mu \) is a negative normal retention factor and \( \beta \) is a shear retention factor.

It has been shown that fixed smeared crack results still tend to remain too stiff because of excessive shear transfer [Rots and de Borst 1987, Rots 1988]. The problems relate to the fact that the shear retention function controls the amount of stress rotation and principal stress rebuild after cracking. Further improvements can be achieved by using shear softening for the fixed plane [Rots and de Borst 1987] which will be discussed later.

**Fixed smeared crack concept with strain-decomposition**

The strain-vector in Eq.(3.17) represents an overall strain of the cracked solid. A more transparent model is obtained by decomposing the total strain increment \( \Delta \varepsilon \) into a part \( \Delta \varepsilon^{cr} \) of the crack and a part \( \Delta \varepsilon^{co} \) of the concrete between the cracks [e.g. de Borst and Nauta 1985, Rots et al. 1985]:

\[
\Delta \varepsilon = \Delta \varepsilon^{co} + \Delta \varepsilon^{cr} 
\] (3.18)

The local crack strain vector is

\[
\Delta \varepsilon^{cr} = \begin{bmatrix} \Delta \varepsilon_{nn}^{cr} \\ \Delta \gamma_{nt}^{cr} \end{bmatrix}^T 
\] (3.19)

where \( \Delta \varepsilon_{nn}^{cr} \) is the mode I crack normal strain, \( \Delta \gamma_{nt}^{cr} \) is the mode II crack shear strain and T denotes a transpose. The relation between local and global crack strain is

\[
\Delta \varepsilon^{cr} = N \Delta \varepsilon^{cr} 
\] (3.20)

with \( N \) being a 3 x 2 transformation matrix that reflects the orientation of the crack.

The local incremental tractions are

\[
\Delta t^{cr} = \begin{bmatrix} \Delta t_n^{cr} \\ \Delta t_t^{cr} \end{bmatrix}^T 
\] (3.21)
in which $\Delta t^\sigma$ is the mode I normal traction and $\Delta t^\sigma$ is the mode II shear traction increment. The relation between the global stress increment $\Delta \sigma$ and the local traction increment $\Delta t^\sigma$ can be derived to be

$$\Delta t^\sigma = N^T \Delta \sigma$$

(3.22)

The constitutive relation of concrete is

$$\Delta \sigma = D^\omega \Delta e^\omega$$

(3.23)

with $D^\omega$ containing the instantaneous moduli of the concrete. For the crack, a local stress-strain relation can be employed

$$\Delta t^\sigma = D^\sigma \Delta e^\sigma$$

(3.24)

with $D^\sigma$ a 2 x 2 matrix incorporating the mode I, mode II and eventually, mixed-mode properties of the crack. Then, the overall stress-strain law for the cracked concrete was developed:

$$\Delta \sigma = \left[ D^\omega - D^\omega N [D^\sigma + N^T D^\omega N]^{-1} N^T D^\omega \right] \Delta e$$

(3.25)

Criteria of closing and re-opening are generally defined in terms of total local crack stress or total local crack strain. As the fixed crack concept assumes the local crack axes to remain unaltered, these quantities are readily available in the form of an accumulation of previous increments. This permanent memory of damage orientation is the prominent feature of fixed smeared crack concepts.

There are three main advantages of this decomposition model. A first advantage is that the behavior of the crack and the behavior of the intact concrete between the cracks can be treated separately. A second advantage is that the mixed mode crack matrix (off-diagonal terms are zero) can be inserted as follows.

$$D^\sigma = \begin{bmatrix} D^I & 0 \\ 0 & D^\Pi \end{bmatrix}$$

(3.26)
where $D^t$ is the mode I tensile softening modulus and $D^u$ is the mode II shear modulus. The relation between the $\mu$ factor and the $\beta$ factor of the orthotropic model of Eq.(3.17) and parameters $D^t$ and $D^u$ of the present model are

\[
\frac{1}{E} + \frac{1}{D^t} = \frac{1}{\mu E}
\]

(3.27)

\[
\frac{1}{G} + \frac{1}{D^u} = \frac{1}{\beta G}
\]

(3.28)

**Multi-directional fixed smeared crack concept**

A third advantage of strain-decomposition is that it allows for a sub-decomposition of the crack strain into the separate contribution of a number of multi-directional cracks that simultaneously occur at a sampling point [Litton 1974, de Borst and Nauta 1985, Riggs and Powell 1986], i.e.

\[
\Delta \varepsilon^{cr} = \Delta \varepsilon_1^{cr} + \Delta \varepsilon_2^{cr} + \cdots \cdots
\]

(3.29)

where $\Delta \varepsilon_1^{cr}$ is the global crack strain increment owing to a primary crack, $\Delta \varepsilon_2^{cr}$ is the global crack strain increment owing to a secondary crack and so on. Thus, by assembling the individual vectors and matrices, the overall stress-strain relation for the multiply cracked solid can be set up analogously to the relation for a single cracked solid.

The significance of the multi-directional crack concept is obvious in conditions of biaxial and triaxial tension. Here, one can expect two or three orthogonal cracks and the behavior of each of which can be monitored separately keeping record of memory. This option is particularly relevant to axi-symmetric and plane-strain analysis, where numerous points may be cracked longitudinally as well as transversely. The second field of application is given by conditions where the fracture starts in tension and subsequently proceeds in tension-shear. This behavior generally implies that the axes of principal stress rotate after crack formation. For such cases, the use of fixed single crack leads to an increasing discrepancy between the axes of principal stress and the fixed crack axes. The fixed multi-directional crack concept provides an alternative. Whenever the angle of inclination between the existing crack(s) and the current direction of principal stress exceeds the value of a certain threshold angle $\alpha'$, a new crack may be initiated.
Rotating smeared crack concept

Consistent use of rotating principal stress-strain relations requires principal stress and strain to be coaxial. It has been shown that this coaxiality can be achieved only via an implicit shear term in the rotating principal 1, 2 reference frame [Bazant 1983, William et al. 1987]:

\[
G_{12} = \frac{(\sigma_{11} - \sigma_{22})}{2(e_{11} - e_{22})}
\]  

(3.30)

The tangential stiffness matrix should include this term and then the implementation of rotating principal stress-strain relations is consistent. It is intriguing to examine the parallels between the fixed multi-directional crack concept and the rotating crack concept. While the fixed multi-directional concept controls the formation of subsequent cracks via the threshold angle, the rotating concept assumes the crack orientation to change continuously. Assuming the threshold angle for multi-directional cracks to vanish, a new fixed crack arises at the beginning of each stage of the incremental process. In doing so, one can observe that the fixed multi-directional concept reduces to the rotating concept, provided that: (1) The condition of a vanishing threshold angle is the only condition that controls the orientation of subsequent cracks; (2) Previous cracks are rigorously made inactive and erased from memory on activation of the new crack; (3) The local traction-strain law Eq.(3.24) of the active crack is filled in such a way that (a) the memory of previous defects is accounted for, (b) the overall shear modulus ensures coaxiality according to Eq.(3.30). Then, the rotating crack approach could be considered as the limiting case of the fixed multi-directional crack approach. In this approach, a collection of fixed tiny defects of different orientation are considered, each of them having its own local traction-strain law, rather than a single rotating crack with a rotating principal stress-strain relation.

Bazant [1983] raised the objection against rotating crack concepts that rotating defects against the material was unacceptable from a physical point of view. However, with distributed fracture, the notion of fixed defects of gradually rotating orientation is supported by experimental evidence [Vecchio and Collins 1986, Bhide and Collins 1987, Kollegger and Mehlhorn 1987]. Rots and Blaauwendraad [1989] indicated that with localized fracture, such experimental justification did not seem to exist, but there was little argument that mixed-mode crack tip processes involved fixed defects of gradually rotating orientation.
3-3-3. Discrete Versus Smeared Approaches

Ever since cracking has been modelled, the applicability of discrete versus smeared approaches has been the subject of much controversy. Initially, it was thought that the smeared approach was better suited for engineering analyses of distributed fracture, while the discrete approach had its strength in detailed analyses of localized fracture. In the early 1980’s, it became clear that the smeared approaches were also capable of predicting localization, at least in a qualitative sense. To date, there is not yet consensus on the question of which type of approach should be preferred for a given problem. At the present stage, the following comments on this problem can be identified [Rots 1988, Hillerborg and Rots 1989, Rots and Blaauwendraad 1989].

Even the best possible smeared crack results are not free from stress-locking which is due to the fundamental disagreement between the assumption of displacement continuity and the realism of a geometrical discontinuity. In addition, the smeared crack approach may suffer from the problem of spurious mechanism. These difficulties may hamper convergence and even blow up the entire iterative procedure. However, the discrete approaches do not suffer from this phenomena. Here, the problem lies in the required change in the topology of the mesh. To sum up, both approaches presently seem to have their own merits and demerits. These arguments change when we consider distributed fracture. Examples are the diffuse crack patterns in large-scale shear walls or panels due to the presence of densely distributed reinforcement. Such cases provided a true physical basis for smeared concepts, at least if the scale of the representative continuum is large compared to the crack spacing. Thus, the smeared concept seems to be the only rational approach towards distributed fracture since the use of a discrete concept, which considers each individual crack then becomes clearly unwieldy.

In conclusion, it can be seen that the discrete crack approach provides a physical basis for single crack in the numerically stable manner, but not rational approach for distributed fracture. Therefore, this approach is suitable for a single large crack such as S crack in the key joints. On the other hand, the smeared crack approach suffers from numerical complexity, but provides a physical basis for distributed fracture, and therefore is suitable for M cracks in key joints.

3-3-4. Numerical Simulation for Mixed Mode Problem

In this section, several numerical simulations for mixed mode problems by means of both discrete crack and smeared crack approaches are reviewed, mainly tension and
tension-shear dominated problems. These numerical simulations will provide both positive points and negative points in modelling of the fracture of concrete, and therefore will be instructive to the modelling of the shear-off failure of key joints.

**Elastic-softening material description**

For tension and tension-shear dominated problems, an adequate crack model was constructed by assuming elasticity for the concrete and softening for the crack [Rots 1988, Rots and Blaauwendraad 1989]. Prior to cracking, concrete was represented sufficiently accurately as an isotropic, linear-elastic material. Then, the stiffness matrices in the constitutive relations were assumed to be of the form:

\[
C^{cr} = \begin{bmatrix}
C^I & 0 & 0 \\
0 & C^{II} & 0 \\
0 & 0 & C^{III}
\end{bmatrix}
\]  
(3.31)

\[
D^{cr} = \begin{bmatrix}
D^I & 0 & 0 \\
0 & D^{II} & 0 \\
0 & 0 & D^{III}
\end{bmatrix}
\]  
(3.32)

where \(C^I, C^{II}, C^{III}\) and \(D^I, D^{II}, D^{III}\) are the mode I, mode II and mode III stiffness moduli for a discrete single crack and a smeared single crack respectively.

The assumption of zero off-diagonal terms in Eq.(3.31) and Eq.(3.32) implies that direct shear-normal coupling has been ignored. A salient feature of the model components presented here is that they assume fracture to be initiated in mode I, whereas mode II or mode III shear effects only enter upon subsequent rotation of the principal stresses. This hypothesis is supported by experimental evidence with respect to static loading conditions of tension and tension-shear [e.g. Kobayashi et al. 1985, Jenq and Shah 1988]. The used mode I parameters are described as follows. The crack stiffness moduli must be expressed in terms of the strength parameter, the energy parameter and the shape of the softening diagram. For a fixed single crack, the definition of \(G_f\) gives

\[G_f = \int t_n^{cr} du_n^{cr}\]  
(3.33)
which corresponds to the area under the softening curve for a discrete crack. Here, \( t_n^{cr} \) is the crack normal traction and \( u_n^{cr} \) is the crack normal displacement. Evaluation of Eq.(3.33) simply results in a tangential softening modulus for a single discrete crack

\[
C^I = -\frac{1}{k} \frac{f_{eq}^2}{G_f}
\]

(3.34)

with \( k \) reflecting the shape of the softening diagram.

For smeared cracks, the fracture is distributed over a crack band width 'h', which is related to the particular finite element configuration. Consequently, the energy should be related over this width in order to obtain results that are objective with regard to mesh refinement [Bazant and Oh, 1983]. Assuming a constant strain distribution over the crack band gives \( u_n^{cr} = h e_{mn}^{cr} \). This leads to

\[
D^I = -\frac{1}{k} \frac{f_{eq}^2 h}{G_f}
\]

(3.35)

Here, it should be noted that the elastic-softening approach introduced here is adequate for the mixed mode problem of tension-shear dominated configurations. Therefore, this approach is applicable to the nucleation and propagation of S crack and M cracks in shear key joints. However, regarding the final failure of shear key joints which is achieved by the crushing of the compression strut between each M crack, it is obvious that the tension-softening concept should be modified by the additional constitutive model such as plasticity in compression. This requirement would give us the numerical complication such that a number of integration points show tension with significant lateral compression, so that these points show cracking as well as plasticity [Reinhart, 1989]. Strictly speaking, the tensile strain-softening formulation is valid only for pure tensile fracture modes of failure but does not apply to situations with significant lateral compression. On the other hand, the plasticity formulation applies primary to the compression-compression region within the principal stress space. For the compression-tension region within this space, a smooth transition between these extreme modes is still to be developed. Therefore, in this section, only the elastic-softening approach is discussed. However, this cracking-plasticity approach will be briefly examined as well as the elastic-softening approach in Chapter 5.
CLWL-DCB specimen

A Crack-Line-Wedge-Loaded Double-Cantilever-Beam, which had been tested by Kobayashi et al. [1985] (see Fig.3.24b) was analyzed by means of both smeared crack approach and discrete crack approach [Rots 1988, Rots and Blaauwendaal 1989]. The specimen of 50.8 mm thickness was assumed to be in a state of plane stress, and subjected to a wedge load \( F_1 \) as well as a diagonal compression load \( F_2 \). The ratio of the diagonal force to the wedge force was kept approximately constant at 0.6 until a predetermined diagonal force of 3.78 kN was reached, whereafter the diagonal force was kept constant and only the wedge force was altered. The finite element mesh consists of four-point integrated four-node quadrilaterals, but it was reported that the use of different element types did not affect the conclusions [Rots 1988].

In this analysis, the objective match between the smeared crack results and the discrete crack results was examined. The elastic-softening parameters have been taken as: Young's modulus \( E=30000 \) N/mm², Poisson's ratio \( v=0.2 \), tensile strength \( f_t=3.0 \) N/mm², fracture energy \( G_f=100 \) N/m and the concave softening diagram of Fig.3.25. The crack band width 'h' was estimated to be \( \sqrt{2} \) times the element size [Rots 1988], i.e. 14 mm. Five computations were compared: (1) fixed smeared cracks \( \alpha=60^\circ, \beta=0 \); (2) fixed smeared cracks \( \alpha=60^\circ, \beta=0.05 \); (3) fixed smeared cracks \( \alpha=60^\circ \), variable \( \beta \) according to Eq.(3.36) below; (4) coaxial rotating smeared cracks \( \alpha=0^\circ, \beta \) enforcing coaxiality; (5) predefined discrete crack in which both shear traction and shear stiffness after cracking were set equal to zero.

\[
\beta = \left[ 1 - \frac{\varepsilon_{\text{mm}}}{\varepsilon_{\text{u}}} \right]^2
\]  

(3.36)

Fig.3.26 presents solutions in terms of the wedge load versus Crack Mouth Opening Displacement. It is apparent that all smeared crack results are too stiff in the post-peak regime. Only the rotating smeared cracks and the fixed smeared cracks with \( \beta=0 \) appear capable of producing a limit point with subsequent softening. Figures 3.27 and 3.28 illustrate the fracture localization for these two cases. One can observe that with fixed cracks of almost zero shear retention, the experimental direction of fracture propagation (see Fig.3.24) is captured surprisingly well, whereas with the coaxial rotating crack concept, the localization prefers to follow the lines of the mesh. This suggests that the fixed crack concept suffers less from directional bias than the coaxial rotating crack concept. Fig.3.29 shows the genuine separation obtained for the predefined discrete crack
Fig. 3.24: a) Finite Element Idealization of Notched CLWL-DCB Specimen, Dimensions mm [after Rots and Blaauwendraad, 1989]; b) Experimental Results [after Kobayashi et al., 1985]

Fig. 3.25: Nonlinear Mode I Tensile Softening Relation between Crack Normal Traction and Crack Normal Strain [after Rots and Blaauwendraad, 1989]
Fig.3.26: Load-CMOD Response of CLWL-DCB Specimen for Different Crack Models [after Rots and Blaauwendraad, 1989]
Fig. 3.27: Fracture Localization for Fixed Smeared Cracks with $\beta=0$ [after Rots and Blaauwendraad, 1989]

Fig. 3.28: Fracture Localization for Rotating Smeared Cracks (coaxial) [after Rots and Blaauwendraad, 1989]

-135-
Fig. 3.29: Genuine Separation for Predefined Discrete Crack [after Rots and Blaauwendraad, 1989]
Fig. 3.30: Principal Tensile Stress Trajectories for CLWL-DCB; a) severe stress locking for fixed smeared cracks with $\beta = 0.05$, b) stress locking for fixed smeared cracks with $\beta = 0$, c) stress locking for rotating smeared cracks (coaxial), d) correct stress relief at either side of discrete crack [after Rots and Blaauwendraad, 1989]
Table 3.2: Recommendations for Estimating the Crack Band Width [after Rots, 1988]

<table>
<thead>
<tr>
<th>element type and integration</th>
<th>effective band width $h$ for zig-zag fracture propagation through regular mesh (found by trial-and-error for present problem)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cdot$</td>
<td>$e$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$e\sqrt{2}$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$\frac{1}{2}e\sqrt{2}$</td>
</tr>
<tr>
<td>$\cdot$</td>
<td>$e$</td>
</tr>
</tbody>
</table>

Fig. 3.31: Mesh-(in)sensitivity of Fracture Localization for Three-node Triangles; a) Non-preferential Mesh, b) Cross-diagonal Mesh [after Rots, 1988]
analysis. This analysis produces the most flexible load-CMOD response and it is surprising that even the two best possible smeared crack results are significantly stiffer (see Fig. 3.26). To investigate the underlying causes, the principal stresses have been plotted in Fig. 3.30 for the various computations. For fixed cracks, one can observe that the tensile stresses in the vicinity of the localization entirely refuse to decrease. Rather, the plot exhibits locked-in stresses at locations where the stress should actually drop to zero. This provides an explanation for the overstiff response. For higher values of $\beta$ or the variable $\beta$, the plots revealed severe stress-locking. This stress-locking is a fundamental consequence of finite element displacement continuity in smeared softening approaches mentioned previously.

Rots [1988] also studied the fracture behavior of CLWL-DCB specimen with several meshes, composed of four-node quadrilaterals with two-by-two Gaussian integration, three-node constant-strain triangles with single-point integration, six-node linear-strain triangles with seven-point Gaussian integration and eight-node quadrilateral serendipity elements with three-by-three Gaussian integration. He concluded that all element types were capable of simulating the fracture localization, whereby for similar computational costs, the lower order elements produced the most narrow failure band, and the constant-strain triangles were adequate in this respect, because they could be integrated using a single-point scheme and because they permitted the fracture to jump to adjacent rows of elements without spreading the corresponding deformation to a substantial neighboring area. Four-point quadrilaterals are slightly less attractive, owing to coupling between integration points and interlocking between the squares. The same holds for quadratic elements. These differences relate to the fact that the smeared concept is an artifice that aims at imitating the limit of the discrete concept. Since a discrete crack involves 'infinite strain over zero length', it is approximated more closely by constant strain over narrow bands (fine mesh with linear interpolations) than by linear strain over wider bands (coarse meshes with quadratic interpolations). A crack band width 'h' depends on the finite element configuration. Rots [1988] recommended the rough estimation of the crack band width 'h' as a function of element size and type shown in Table 3.2. Rots [1988] also indicated that the fracture preferentially follows the lines of the mesh. Directional bias is due to the fact that strain discontinuities are represented at the element boundaries, i.e., at the lines of the mesh. A mesh composed of quadrilaterals provides only two lines, while a mesh composed of triangles provides three lines, the skew one of them being highly arbitrary however. Improvements are achieved by placing the triangles in a cross-diagonal pattern instead of a bisectonal pattern, as shown in Fig.3.31. In doing so, symmetry is
preserved and the number of lines increases up to four, which minimizes bias in locations of strain discontinuities.

**Single-notched shear beam**

A single-notched shear beam which failed in curved mode I fracture [Arrea and Ingraffea 1981] was analyzed by means of discrete crack approach [Ingraffea 1984, Ingraffea et al. 1985b]. Since this specimen involves both crack opening and crack sliding displacement, COD and CSD, respectively, there exist not only normal stress transfer across the process zone but shear stress as well. Consequently, the so-called 'aggregate interlock' model of Fenwick and Paulay [1968] in which shear transfer across a crack was related to the COD was employed. In that relationship, the transferred shear stress varies non-linearly with COD and linearly with CSD as shown qualitatively in Fig.3.32. Based on the parametric study in which the crack trajectory observed in testing was modelled in the mesh, the σ-COD constitutive model A in Fig.3.33 was chosen for the cast-in starter crack modeled in the initial mesh in their simulation. A discrete propagation analysis was performed with automatic remeshing occurring at each crack increment. The final mesh configuration and displaced shape are shown in Fig.3.34. The predicted trajectory of the crack was very close to the observed one, and the computed load versus CMSD response as shown in Fig.3.35, was very similar to the experimental data. Finally, Ingraffea [1984] suggested that the response was sensitive to the shear transfer model across the process zone and across the true crack itself, although the shear and normal stress transfer models used in his simulation were uncoupled.

The same single-notched shear beam was also studied by using shear softening for the fixed smeared cracks [Rots and de Borst, 1987]. The shear softening modulus DII might also be related to three parameters: an ultimate shear stress τu transferred across the crack, a fracture energy GIIC (defined as the amount of energy required to create one unit of area of a pure mode II crack) and a specific shape of the shear softening diagram. Here, applications are restricted to a bi-linear shape of the shear softening diagram, in particular to a diagram linearly ascending to the ultimate stress τu, followed by a linear descending branch, as shown in Fig.3.36. This formulation assumes fracture to be initiated normal to the axis of the principal tensile stress. A direct consequence of this crack initiation criterion is that the shear stress across the crack is zero at the onset of cracking. Only on subsequent rotation of the principal stress axes, a shear stress may develop across the crack faces until its maximum level τu whereafter the shear softening branch is entered. Assuming the fracture energy released in mode II fracture to be a material property, a stress-free crack shear strain γu was defined:
\[ \gamma_u = \frac{2G_I^u}{\tau_u h} \]  

The concrete was modeled as linearly elastic in compression with a Young's modulus \( E = 24800 \text{N/mm}^2 \) and Poisson's ratio \( \nu = 0.18 \). The mode I crack parameters were assigned the following values: tensile strength \( f_t = 2.8 \text{N/mm}^2 \), mode I fracture energy \( G_I^t = 75 \text{J/m}^2 \) and intercrack threshold-angle \( \alpha = 60^\circ \) [de Borst and Nauta 1985]. The crack band width was estimated as \( h = 20.3 \text{mm} \). Three analyses with different shear softening feature were performed: (1) a constant shear retention factor of \( \beta = 0.2 \); (2) a shear softening model with the initial shear retention factor of \( \beta_0 = 0.2 \), \( G_I^{II} = 75 \text{J/m}^2 \), \( \tau_u = 0.5 \text{N/mm}^2 \); (3) a shear softening model with the initial shear retention factor of \( \beta_0 = 0.2 \), \( G_I^{II} = 10 \text{J/m}^2 \), \( \tau_u = 0.5 \text{N/mm}^2 \). Fig.3.37 shows the finite element idealization. The beam was analyzed using eight-noded plane stress elements, which were integrated using nine-point Gaussian quadrature. In the transition region between the coarse part and the fine part of the mesh, three-point integrated six-noded triangles were used. The beam had a thickness of 156 mm. Fig.3.38 shows that the computational result for the load-CMSD response nicely fails within the experimental scatter when shear-softening is added. The low value for the mode II fracture energy \( G_I^{II} = 10 \text{J/mm}^2 \) yields an even more pronounced structural softening than \( G_I^{II} = 75 \text{J/mm}^2 \). However, the peak load does not seem to be markedly affected, which indicates that mode I effects prevail before peak. The same trend was observed upon an increase of \( G_I^{II} \) (e.g., \( G_I^{II} = 500 \text{J/mm}^2 \)) and upon an increase of the ultimate crack shear stress \( \tau_u \). The crack pattern of \( G_I^{II} = 75 \text{J/mm}^2 \) is shown in Fig.3.39.

The rotational smeared crack concept was added to the previous analysis for the same single-notched shear beam [Rots 1988, Rots and Blauwendraad 1989]. The mesh is shown in Fig.3.40 and consists of three-node triangles in cross diagonal pattern. The steel beam ACB has not been included in the mesh. Instead, the loading has been applied at the points A and B. The parameters were taken as: \( E = 24800 \text{N/mm}^2 \), \( \nu = 0.18 \), \( f_t = 2.8 \text{N/mm}^2 \), \( G_I = 100 \text{J/m}^2 \), concave softening (see Fig.3.25) and \( h = 12 \text{mm} \). Five solutions are presented, corresponding to those of the CLWL-DCB specimen mentioned previously [Rots 1988, Rots and Blauwendraad 1989]. The solutions are summarized in Fig.3.41, giving the load F versus the Crack Mouth Sliding Displacement. It was indicated that all smeared crack results were too stiff in the post-peak regime. From Fig.3.41, one can observe that the experimental scatter also shows a somewhat stiff behavior at the end of the softening regime.
\[ \tau = E_s \times CSD \text{ where:} \]
\[ E_s = 4.7 \times 10^5 \text{ psi/in} \quad \text{for COD} < 0.001 \text{ in} \]
\[ E_s = \left[ \frac{470}{\text{COD}} - 8400 \right] \text{psi/in} \quad \text{for COD} > 0.001 \text{ in} \]

Fig.3.32: Typical Shear Stress versus COD and CSD Relationship for Process Zone in Concrete [after Ingraffea and Saouma, 1985b]

\[ \sigma = \beta \left[ \frac{0.0542}{(\text{COD} + 0.0001)} + 58.2 - 3760 \times \text{COD} \right] \times 10^{-3} \]

Fig.3.33: Various Constitutive Models for a Discrete Representation of the Process Zone in Concrete [after Ingraffea, 1984]
Fig. 3.34: Amplified Displace Shape after 5 Crack Increments through the Mesh, Amplification Factor = 438 [after Ingraffea, 1984]

Fig. 3.35: Comparison of Crack Propagation Model Prediction with Experimental Results [after Ingraffea, 1984]
Fig. 3.36: Mode II Shear Softening Relation between Shear Stress and Crack Shear Strain [after Rots and Borst, 1987]

Fig. 3.37: Finite Element Idealization for Single-Notched Specimen [after Rots and Borst, 1987]
Fig. 3.38: Load versus CMSD Computed for Three Different Crack Shear Representations (Shaded Area Denote Range of Experimental Results) [after Rots and Borst, 1987]

Fig. 3.39: Crack Pattern at Ultimate Residual Load [after Rots and Borst, 1987]
Fig. 3.40: Element Mesh for Smeread Crack Analysis of Single-Notched Shear Beam [after Rots and Blaauwendraad, 1989]

Fig. 3.41: Load F versus CMSD of Single-Notched Shear Beam [after Rots and Blaauwendraad, 1989]
Double-notched shear beam

The double-notched shear beam, tested by Bazant et al. [1985a,b, 1986] was analyzed by using a shear-softening for the fixed smeared cracks [Rots and de Borst 1987, Rots, Kusters and Blaauwendraad 1987]. It should be noted that the cracking behavior in this test was extensively discussed in Chapter 2, and this cracking status could be similar to S crack and M cracks in shear key joints. The finite element mesh is shown in Fig.3.42. The concrete was modeled as linearly elastic in compression with a Young's modulus E=25000N/mm² and Poisson's ratio ν=0.2. The mode I crack parameters were assigned the following values: tensile strength $f_t=3.0$ N/mm², mode I fracture energy $G_f=75$ J/m² and intercrack threshold-angle $\alpha=60^\circ$ [de Borst and Nauta 1985]. The crack band width was estimated as $h=12.7$ mm. Three analyses with different shear softening feature were performed: (1) a constant shear retention factor of $\beta=0.2$; (2) a shear softening mode with the initial shear retention factor of $\beta_0=0.2$, $G_f=75$ J/m², $\tau_u=0.5$ N/mm²; (3) a shear softening model with the initial shear retention factor of $\beta_0=0.2$, $G_f=10$ J/m², $\tau_u=0.5$ N/mm².

Fig.3.43 shows the computational results in terms of the load-deflection curves for the loading point C of steel beam AB. It was indicated that the numerical prediction of the peak load appeared to correspond well with the experimental value ($F=44.8$ kN) and did not seem to be sensitive to the adopted representation for the shear transfer across the crack. The crack pattern at ultimate residual load is shown in Fig.3.44, which reveals that the fracture discontinuities first propagate slightly sideways from the notches and then continue to propagate in the vertical direction towards the opposite notch. The crack pattern exhibits quite a number of multiply cracked sampling points, which indicates that stress rotations after primary cracking have been significant, even though shear softening has been added. It was finally concluded that mode I effects prevailed up to peak, while mode II effects became gradually more important during the post-peak regime.

Here, it seems to the author that the cracking status shown in Fig.3.44 does not express the macroscopic shear fracture at the plane connected the two notch tips indicated by Bazant et al. [1985a,b, 1986] and discussed in Chapter 2. Specifically, most of the crack status seem to show the mode I crack propagating sideways from the notch tip which can be differentiated by the vertically-running shear fracture (see Chapter 2). This discrepancy may arise from the fact that no plasticity is considered in this simulation. As mentioned above, the modelling of a smooth transition between cracking and plasticity modes is yet to be done.
Fig. 3.42: Finite Element Discretization for Double-Notched Specimen [after Rots and Borst, 1987]

Fig. 3.43: Load-Deflection Curves of Point C for Three Different Crack Shear Representations [after Rots and Borst, 1987]

Fig. 3.44: Crack Pattern at Ultimate Residual Load [after Rots and Borst, 1987]
3-3-5. Summary

In this section, two main numerical approaches to implement the crack models discussed in the previous section are briefly reviewed: discrete crack approach and smeared crack approach. Then, the application of those two approaches to mixed mode problem is investigated. Based on these reviews, the following features and limitations of the current available models can be identified.

**Numerical approaches**

1. The discrete crack approach provides a physical basis for single crack in the numerically stable manner, but not rational approach for distributed fracture. Therefore, this approach is suitable for a single large crack such as S crack in the key joints. On the other hand, the smeared crack approach suffers from numerical complexity, but provides a physical basis for distributed fracture, and therefore is suitable for M cracks in key joints.

**Numerical simulations for mixed mode problem**

1. The predictions by both discrete and smeared crack approaches for the load-displacement curves before the peak load agree well with the experimental data. However, in the post-peak regime most of smeared crack results are stiff while discrete crack results can agree well with the test data. Only the rotating smeared cracks and the fixed smeared cracks with $\beta=0$ appear capable of producing a limit point with subsequent softening. In addition, the experimentally observed direction of fracture propagation is captured well by fixed smeared cracks of almost zero shear retention.

2. In most of the simulations for mixed mode problems reviewed here, mode I effects prevailed up to peak, while mode II effects became gradually more important during the post-peak regime.

3. Shear-compression fracture such as coalescence of M cracks along the key base requires the numerical complication such that a number of integration points show tension with significant lateral compression, so that these points show cracking as well as plasticity. However, for the compression-tension region within the principal stress space a smooth transition between the cracking and plasticity modes is still to be developed.
From the literature review on numerical approaches discussed in this section, it is obvious that the current available numerical approaches are under way which will improve mixed mode modelling. This is because the currently available crack models as the basis for the numerical approaches are not developed well for the mixed mode problem resulting due to the lack of a sound theoretical model. Therefore, the computational difficulties in the numerical approaches are heavily increased. Therefore, one needs simplified or easily-handled approaches which can achieve a good approximation. In the next section, currently available theoretical approaches will be reviewed which may be instructive for the modelling of shear-off failure of key joints.

3-4. THEORETICAL APPROACHES

Two theoretical approaches, LEFM and a truss model, are reviewed. These approaches were used to simulate the similar fractures to S crack and M cracks in the shear key joints. Then, the features and limitations of such reviewed approaches are identified.

3-4-1. LEFM Approach

Applicability of LEFM to concrete
The linear elastic fracture mechanics (LEFM) techniques cover the great majority of all applications of fracture mechanics at present. While nonlinear techniques beyond those of LEFM are increasingly being developed and used, it is unlikely that this trend will obviate LEFM. If nothing else, one will often find that LEFM techniques are useful for a first cut at a given problem before resorting to the generally more complex procedures inherent in a nonlinear fracture mechanics treatment. LEFM is based on the assumption that the material is fully elastic everywhere and that no fracture zone exists. LEFM can therefore never describe the behavior of a real material correctly. It is always an idealization, and thus an approximation, as it assumes infinite stresses (and thus infinite strength) and no fracture zone (or fracture zone of zero length). This is because in real material there is always a finite stress, and a fracture zone of finite length in front of a crack with tensile stresses perpendicular to the crack plane. Compared to a real material, LEFM can be looked upon as the limiting case when the length of the fracture zone approaches zero for a linear elastic material. In the practical application of LEFM, a calculated value of the stress intensity factor 'K' or the energy release rate 'G' is compared to the material property 'K_c' or 'G_c'. K_c and G_c are the values of K and G when the crack starts growing and are correctly determined by means of tests.
However, it is not possible to determine the load for which a crack starts in a test of concrete since there does not exist any well-defined crack tip for concrete [Hillerborg, 1983]. Normally, it is therefore assumed that this load coincides with the maximum load, which should be the case according to LEFM. As there does not seem to be any better alternative definition, $K_C$ and $G_C$ are usually taken to be the values of $K$ and $G$ corresponding to the maximum loads in the tests.

The degree of approximation involved in the application of LEFM was examined through the three-point bend test of a notched beam with the depth of 'd' [Hillerborg, 1983]. It was indicated that the value of $K_C$ or $G_C$ from a test depends very much on the ratio $d/l_{ch}$ if $d/l_{ch}$ is small. Thus, $K_C$ and $G_C$ show large 'size effect' and are not material properties in this sense. With increasing $d/l_{ch}$, $K_C$ and $G_C$ approach constant values which may be regarded as material properties. In other words, if the length of the fracture zone approaches zero, the energy absorption per unit growth in crack area ($G_C$) then approaches the fracture energy $G_f$, as that unit crack area will absorb all the energy associated with the deformation of the fracture zone until complete separation. Then, one can write

$$G_C \to G_f \quad \Rightarrow \quad \left\{ \begin{array}{c} K_C \to \sqrt{EG_f} \\ \left( \frac{K_C}{f_t} \right)^2 \to l_{ch} \end{array} \right.$$

(3.38)

Although the applicability of LEFM to concrete is largely limited mentioned above, the use of LEFM is quite attractive to us. This is because with LEFM it is often possible to find general analytical solutions, which are much easier to handle than numerical solutions such as FEM. Therefore, to find the valid condition of LEFM is quite significant, especially from the engineering point of view. As mentioned, the existence of the process zone affects the distribution of stresses and may invalidate the linear elastic analysis in LEFM. However, if the size of the process zone is sufficiently small relative to the crack length and to the other dimensions of the cracked body being analyzed, then its effects on the stress distribution can be neglected and the linear elastic analysis of the crack tip zone is valid. Thus, small scale yielding (SSY) at the crack tip zone is the basic premise of LEFM. Before the LEFM approach can be used, the process zone size must be therefore estimated and compared with expected crack lengths and structural dimensions. If small scale yielding conditions are satisfied within the structure, then the analyst can assume that LEFM analysis is valid. Otherwise, the effect of the process zone on the crack tip must be considered and a model which takes into account the process zone must be incorporated.
Fig. 3.45: Effective Crack Length, Fracture Zone Length, and Corresponding Stress Distribution [after Hillerborg, 1983]
into the stress analysis. Furthermore, an appropriate failure criterion must also be developed to replace the $K_{IC}$ concept.

Regarding this validity of LEFM, Hillerborg [1983] suggested the alternative idea. It was indicated that the accuracy of LEFM can be increased by using an 'effective crack length'. As mentioned, the assumption of LEFM that the stresses may approach infinitely is unrealistic, as a plastic or fracture zone will always develop where the strains get too high. The influence of the limited stresses in the fracture zone may be approximately taken into account by 'cutting off' the peak in the stress curve. In order to retain the equilibrium, it is then necessary to make corrections to the stress curve. Fig. 3.45 shows a LEFM stress curve for a 'effective crack length' $a_i=a+\Delta a$, greater than the real crack length 'a'. The curve has been cut off at $\sigma=f_i$ and the stress has been assumed to equal $f_i$ between the crack tip and this point. This stress distribution may be reasonable for a yielding material with the yield stress equal to $f_i$ [Dugdale, 1960]. According to LEFM, the following relations were derived:

$$\Delta a = x_1 = \frac{1}{2\pi} \left( \frac{K_{IC}}{f_i} \right)^2 = \ell_{ch} \frac{\ell_{ch}}{2\pi}$$

(3.39)

The corresponding length $\ell_F$ of the fracture zone is therefore

$$\ell_F = a + x_1 = \ell_{ch}$$

(3.40)

These values are only valid as long as the size of the specimen is large compared to $\ell_F$. Hillerborg [1983] indicated that even if this assumption was fulfilled, however, the given values of $\Delta a$ and $\ell_F$ must be regarded as lower limits, at least for the fundamental case of a crack in an infinite plate. Then, it was recommended to use an effective crack length 'a_e' instead of 'a' in analysis of test results and in design based on LEFM, and to choose the value of $\Delta a$ for plain concrete as about 0.2 $\ell_{ch}$ at crack advance if no more correct value was known. It was emphasized that the correction would be presumably always have a positive effect on the accuracy.

**Brittle failure under complex stress state**

In Chapter 2, the fracture phenomena under compressive stress field were studied which could be similar to S crack and M cracks in the shear key fracture. This cracking behavior for concrete has never been successfully modelled by LEFM approach because of the
limitation of LEFM for concrete mentioned so far. On the other hand, such modelling for brittle materials like PMMA (poly methyl methacrylate) has been successively studied in several literatures. If we can construct the validity of LEFM for concrete mentioned above, such theoretical approach for brittle failure under complex stress states could be instructive to S crack and M cracks propagation.

Cracks, holes and inclusions in an elastic solid can interact with a compressive stress field in a way which causes new cracks to grow from them. If these cracks extend to the sample surface, or if they interact with each other so that they grow unstably, then a macroscopic failure may follow. Crystalline ceramics, rocks and minerals often contain a distribution of fine cracks with a size about equal to the grain size: They are caused by thermal or elastic stress during earlier thermal or mechanical loading. Acoustic and dilatometric studies show that they start to propagate when the axial compressive stress reaches about one half of the ultimate failure stress, and microscopic observations show that they extend parallel to the compression-axis. Their subsequent behavior depends on the confining pressures, as shown in Fig.3.46 [Ashby and Hallam 1986]. Simple axial or radial compression, shown at (a) and (d), causes a few cracks to propagate and combine to give failure on plates parallel to the maximum compressive stress. A modest confining pressure prevents this unlimited crack growth; failure then occurs by the interaction of cracks to give the macroscopic shear failure shown at (b). Larger confining pressures limit the growth of individual cracks even further, and the sample deforms in a quasi-ductile way with large-scale deformation taken up by many short, homogeneously distributed microcracks, shown at (c).

Based on microscopic observations, various mathematical models have been introduced in an effort to analyze the failure process of brittle materials in compression. Most of these models are grounded on the idea that frictional sliding of a preexisting crack produces, at the crack tips, tension cracks that grow in the direction of maximum compression [e.g., McClintock & Walsh 1963, Kachanov 1982, Steif 1984, Ashby & Hallam 1986 and Horii & Nemat-Nasser 1986]. These authors showed analytically and verified experimentally that, under axial compression, tension cracks nucleated from the tips of the preexisting flaw at an angle close to 70° with respect to the flaw orientation, and grew with increasing axial load, curving toward the direction of the maximum axial compression. If the axial compression is accompanied by any amount of lateral tension, the crack growth becomes unstable after a certain crack length is attained, resulting in axial splitting. If, on the other hand, some lateral compression accompanies the axial load, the tension cracks grow to a certain length and then stop. To model faulting observed in the presence of a confining pressure, these authors considered a row of flaws and calculated
the length of the tension cracks emanating from the tips of these preexisting flaws and growing under an increasing axial load. Depending on the overall orientation of the row of flaws, it turns out that such a crack growth process can also become unstable, leading to the formation of a fault. Here, the two theoretical approaches according to the literatures [Ashby and Hallam 1986, Horii and Nemat-Nesser 1986] are reviewed as follows.

**Ashby and Hallam [1986]**

Ashby and Hallam [1986] analyzed the growth of wing cracks from an initial inclined flaw (sharp starter cracks) in PMMA plates and examined the conditions under which an array of such cracks interact. The method was an approximate one, using well-known results of fracture mechanics and beam theory, combined in simple models. They considered an infinite elastic plate containing an initial crack of length '2a' and subjected to principal stress \( \sigma_1 \) and \( \sigma_3 \) as shown in Fig.3.47. The stresses are treated as positive when tensile, negative when compressive; then \( \sigma_1 \) is the most negative (most compressive) and \( \sigma_3 \) the most positive (least compressive) principal stress. The crack lies at an angle \( \psi \) to the \( x_1 \) axis, and contains the \( x_3 \) direction. The remote stress field generates shear (\( \sigma_{xy} \)) and normal (\( \sigma_{xx} \)) stresses on the crack plane, where

\[
\sigma_{xy} = \frac{\sigma_3 - \sigma_1}{2} \sin 2\psi = \tau \sin 2\psi \tag{3.41}
\]

and

\[
\sigma_{xx} = \frac{\sigma_3 + \sigma_1}{2} + \frac{\sigma_3 - \sigma_1}{2} \cos 2\psi \\
= \frac{\sigma_3 + \sigma_1}{2} + \tau \cos 2\psi \tag{3.42}
\]

The shear stress tends to make the cracks surfaces slide; but because the cracks are closed, a frictional stress \( \mu \sigma_{xx} \) opposes the sliding (\( \mu \) is the coefficient of friction). Then, the effective sliding stress is

\[
\sigma_{xy}^e = \sigma_{xy} + \mu \sigma_{xx} \tag{3.43}
\]

This stress is intensified by the crack, so that a singular stress field, characterized by the quantity \( \sigma_{xy} \sqrt{\pi a} \) appears at its tips. In the plane of the initial crack, this field is
Fig. 3.46: Failure Modes in Compression [after Ashby & Hallam 1986]

Fig. 3.47: Schematic of Wing Crack Growth [after Ashby & Hallam 1986]

Fig. 3.48: The Co-ordinates, Stresses and Angles Involved in Analyzing Wing-Crack Initiation [after Ashby & Hallam 1986]
Fig. 3.49: Wing Crack Growth in an Infinite Plate with $\mu = 0.6$ [after Ashby & Hallam 1986]

Fig. 3.50: Deformation Configuration when the sample is narrow [after Ashby & Hallam 1986]

Fig. 3.51: Comparison of theory with experiments for PMMA [after Ashby & Hallam 1986]
predominantly shear in character; but on planes which lies at an angle $\theta$ of the crack tip (see Fig.3.48), a normal stress appears, tending to cause a mode I wing crack to grow from the tips of the initial angled flaw. Using the results of Williams [1957] and Cotterell and Rice [1980], they obtained the mode I stress intensity $K_I$ on a very small wing of length $\ell$ and at an angle $\theta$ to the main crack as:

$$K_I = \frac{3}{2} \frac{\sigma_{xy} \sqrt{\pi a}}{\sin \theta \cos \frac{\theta}{2}}$$

(3.44)

The orientation, $\theta = \theta_c$, of the wing crack is obtained by maximizing $K_I$ with respect to $\theta$ and results in $\theta_c = 0.392\pi = 70.5^\circ$. The angle $\psi$ for the most dangerous crack is obtained by maximizes $K_I$ and results in $\psi = 0.5 \tan^{-1}(1/\mu)$. The condition for crack initiation is given by setting $K_I$ equal to $K_{IC}$.

Once wing cracks have initiated, further sliding of the main crack wedges them open, causing them to grow further. When the sample is loaded, the crack slides, allowing the remote stresses $\sigma_1, \sigma_3$ to do work. The sliding generates an internal stress field in which elastic energy is stored. Ashby and Hallam [1986] calculated the equilibrium sliding displacement ($\delta$) by minimizing the potential energy of the body (the work done minus the energy stored) with respect to $\delta$. They indicated that the expression for the stress intensity factor driving wing crack growth in an infinite plate has two contributions: one from the stress field of the initial, angled crack, dominant when $\ell$ is small; the other from the wedging action caused by the sliding displacement $\delta$, dominant where $\ell$ is large. Then, the stress intensity factor driving wing crack growth in an infinite plate is

$$K_I = \frac{-\sigma_1 \sqrt{\pi a}}{3^{1/4}} \left\{ 1 - \lambda - \mu(1 + \lambda) - 4.3\lambda L \right\}$$

$$\times \left\{ 0.23L + \frac{1}{\sqrt{3}\sqrt{(1 + L)}} \right\}$$

(3.45)

where $L = \ell/a$. The wing cracks grow until $K_I$ falls to $K_{IC}$.

Their calculation on the wing crack growth is plotted as full lines for $\mu = 0.6$ in Fig.3.49. Here $\lambda$ is the ratio of the principal stresses: $\lambda = \sigma_3/\sigma_1$. In the figure, they compared the results with those of Nemat-Nasser and Horii [1982] who used a Green's-function method (broken line; discussed later). It is obvious that the approximate calculation by Ashby and Hallam is in good agreement with the exact formulation by
Nemat-Nasser and Horii throughout the entire range of interest. Ashby and Hallam then concluded that crack growth is stable if \( \lambda \) is zero or positive, but even a very small lateral tension (\( \lambda \) is negative) leads to a peak beyond which crack growth becomes unstable.

Ashby and Hallam indicated that crack growth accelerated when additional bending displacements appear in the plate. These bending displacements are shown schematically in Fig.3.50. They studied the crack surface interaction and showed how it explained the experimental observation. First, consider a plate containing a central angled crack with principal stress \( \sigma_1, \sigma_3 \), as shown in Fig.3.50. On loading the plate, sliding at the initial crack wedges open the wing cracks as before. But, the beams, being narrow, bend outwards—the narrower they are, the more easily they bend—allowing a larger sliding displacement than was possible in the infinite plate. This extra displacement increases the stress intensity at the crack tip and makes it grow further (at the same load) than before. If the sample is first loaded with the crack faces pinned together, and then the crack faces are released so that they slide and separate, the potential energy of the body and its loads change. If this potential energy is minimized with respect to sliding displacement, \( \delta \), the stable value of \( \delta \) can be found. The stable value of \( \delta \) is then used to evaluate the elastic energy in one beam, \( U \), and the crack driving force \( G \) is calculated from

\[
G = -\frac{1}{b} \frac{dU}{d\ell} \tag{3.46}
\]

where \( b \) is the sample thickness. Finally, the contribution to \( K_1 \) is calculated from \( K_1 = \sqrt{EG} \), and added to Eq.(3.45).

Ashby and Hallam also considered the fact that PMMA can become plastic if the compressive stress is large enough. When a beam of the thickness \( b \) and depth \( t \) is subjected to an axial stress and a bending moment, it starts to yield when the maximum surface stress reaches the yield strength. Then, the crack is wedged open further, and \( K_1 \) increases greatly.

Fig.3.51 shows the test data for PMMA compared with their theory (full lines) mentioned above. The linear elastic theory explains the stable crack growth up to a length of about \( L = 2 \). In addition, their combined elastic-plastic analysis accounts fairly well for the magnitude and shape of the crack-growth data, and explains the dependence of the plateau level on plate width, \( t/a \).
Horii and Nemat-Nasser [1986]

Horii and Nemat-Nasser [1986] studied the inelastic deformations associated with low temperature, essentially rate-independent processes that lead to the following failure modes, depending on the magnitude of the confining pressure: (1) axial splitting of the sample by macroscopic cracks extending in the direction of axial compression, in the absence of any lateral confining pressure; (2) faulting or macroscopic shear failure, when axial compression is accompanied by moderate confining pressure and finally; (3) ductile flow in the presence of a suitably large confining pressure. They proposed the model for splitting and faulting consisting of an isolated thin straight flaw with cohesive and frictional resistance, which under far-field compression, may nucleate tension cracks at its tips, as well as produce plastic deformation there.

Their model consists of a straight 'flaw' endowed with frictional resistance and cohesion, embedded in a linearly elastic homogeneous solid. In Fig.3.52, the preexisting flaw PP' is sketched together with the associated curved tension cracks PQ and P'Q', under axial and lateral compressions $\sigma_1$ and $\sigma_2$, $|\sigma_1| > |\sigma_2|$; compression is viewed negative. With a coordinate system x, y, as shown, the conditions on the preexisting 'frictional and cohesive' flaw PP', are

$$u^+_y = u^-_y, \quad \tau_{xy} = -\tau_e + \mu \sigma_y$$

(3.47)

and on the curved tension cracks PQ and P'Q', one must have

$$\sigma_\theta = \tau_\theta = 0$$

(3.48)

where $\tau_e$ is the cohesive stress, $\mu$ is the coefficient of friction, $\sigma_y$ is the normal and $\tau_{xy}$ is the shear stress on PP', $u_y$ is the displacement in the y-direction, and $\sigma_\theta$ and $\tau_\theta$ are the polar components of the hoop and shear stresses on PQ. In Eq.(3.47), superscripts '+' and '-' refer to the value of the corresponding quantity calculated immediately above and below the x-axis, along the y-direction.

Nemat-Nasser and Horii [1982] and Horii and Nemat-Nasser [1985] presented the exact formulation of this boundary-value problem in terms of singular integral equations. In addition, they proposed a closed-form, but approximate, and estimated the stress intensity factors $K_1$ and $K_\Pi$ at the tips of the extended tension cracks PQ and P'Q', when these cracks are regarded as straight lines (see Fig.3.53) as follows. In the figure, they considered a representative single crack QQ' of length '2l', subjected at its center to a pair

-160-
Fig. 3.52: Preexisting Flaw PP' and Curved Cracks PQ and P'Q' [after Horii & Nemat-Nasser 1986]

Fig. 3.53: (a) Preexisting Flaw PP' and Straight Cracks PQ and P'Q'; (b) a Representative Tension Crack QQ' with Splitting Forces F [after Horii & Nemat-Nasser 1986]
Fig. 3.54: (a) Normalized Maximum Opening-mode Stress Intensity Factor and (b) Optimal Crack Orientation angle as functions of Initial Flaw Orientation; (c) Normalized Maximum Opening-mode Stress Intensity Factor as a Function of Crack Length [after Horii & Nemat-Nasser 1986]
of collinear splitting forces of common magnitude ‘F’, which make an angle γ with the σ₁-direction. These forces represent the effect on the representative crack QQ’ of the sliding of the preexisting flaw PP’, under the action of the resolved shear stress. ‘F’ was calculated by estimating the driving shear stress τ* on the preexisting flaw, as follows.

\[
F = 2c\tau^*
\]

\[
\tau^* = -\frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\gamma - \tau_c + \mu \frac{1}{2}[(\sigma_1 + \sigma_2 - (\sigma_1 - \sigma_2)\cos 2\gamma]
\]

(3.49a)

(3.49b)

The representative crack QQ’ is also subjected to the far-field stresses σ₁ and σ₂. The stress intensity factors at Q and Q’, produced by the splitting forces ‘F’, acting alone, are

\[
K_I = \frac{F\sin \theta}{\sqrt{\pi \ell}}
\]

(3.50a)

\[
K_{II} = \frac{F\cos \theta}{\sqrt{\pi \ell}}
\]

(3.50b)

Although these estimates are good when ℓ is large, they break down, becoming unbounded at crack initiation, when ℓ is vanishingly small. The stress intensity factor KІ at crack initiation was estimated by

\[
K_I = \frac{3}{4}\sqrt{\pi c}\tau^*\left(\sin \frac{1}{2}\theta + \sin \frac{3}{2}\theta\right)
\]

(3.51)

where tension is taken to be positive. The maximum value of KІ occurs at θ=θc=0.392π.

To match KІ given by Eq.(3.50a) with that given by Eq.(3.51), they introduced an effective crack length 2(ℓ+ℓ*') in place of 2ℓ and set Eq.(3.50a) equal to Eq.(3.51) at ℓ=0 and θ=0.392π. Then, including the effect of the far-field stresses σ₁ and σ₂, the stress intensity factors at Q and Q’ were estimated by

\[
K_I = \frac{2c\tau^*\sin \theta}{\sqrt{\pi (\ell + \ell^*)}} + \sqrt{\pi \ell} \frac{1}{2}[(\sigma_1 + \sigma_2 - (\sigma_1 - \sigma_2)\cos 2(\theta - \gamma)]
\]

(3.52a)

\[
K_{II} = -\frac{2c\tau^*\cos \theta}{\sqrt{\pi (\ell + \ell^*)}} - \sqrt{\pi \ell} \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2(\theta - \gamma)
\]

(3.52b)

\[
\ell^*/c \approx 0.27
\]

(3.52c)
The orientation, $\theta=\theta_0$, of these tension cracks is obtained by maximizing $K_I$ in Eqs. (3.52) with respect to $\theta$. The common length, $\ell$, of PQ and P'Q' is obtained by equating the maximum value of $K_I$ with the opening-mode fracture toughness, $K_C$.

Horii and Nemat-Nasser verified the accuracy of these estimation with the exact (numerical) solution of the elasticity problem associated with Fig. 3.53 [Nemat-Nasser and Horii 1982, Horii and Nemat-Nasser 1985]. The results are shown in Fig. 3.54 to check the accuracy of the analytic estimation: Eqs. (3.52). The estimation given by Eqs. (3.52) are shown by broken lines, and the exact numerical solutions are presented by solid lines. Considering the simplicity of Eqs. (3.52), they concluded that these analytic estimation were remarkably accurate. Here, it should be mentioned that the accuracy of this exact solution was also verified by the comparison with a series of tests on plates 6 mm thick of Columbia resin CR39 (which is rather brittle at room temperature).

3-4.2. Truss Model

A model of shear transfer in initially uncracked concrete (push-off shear test) based on the truss model theory is reviewed here. The model incorporated a softened compression stress-strain relation along the concrete struts. Shear failure across an initially uncracked plane occurs after numerous cracks formed in a direction inclined to the shear plane (see Fig. 2.58 in Chapter 2). After these cracks formed, there was a relative longitudinal movement of the two halves of the initially uncracked specimens. This was due to rotation of the short concrete struts formed by the diagonal tension cracks, when the shear transfer reinforcement stretched as shown in Fig. 3.55 [Mattock and Hawkins, 1972]. The final failure is usually due to the crushing of concrete in the compression struts formed approximately parallel to the direction of the cracks. The compression in the struts and the tension provided by the reinforcing bars across and parallel to the shear plane constitute a truss-like action.

The truss mechanism is based on the analogy between the shear resistance of a parallel chord truss and a web-reinforced concrete beam such that the web of the equivalent truss consists of stirrups acting as tension members and concrete struts running parallel to diagonal cracks. The flexural concrete compression zone and the flexural reinforcement form the top and bottom chords of this analogous pin-jointed truss. The forces in the truss can be only determined from consideration of equilibrium. It should be then noted that the truss mechanism in beams can function only after the formation of diagonal cracks.

The truss-like action of shear transfer in initially uncracked concrete was well investigated by Mattock and Hawkins [1972] through the following experimental
observation. When the shear load is increased after the formation of diagonal cracks, a truss action develops as shown in Fig.3.55a. Diagonal struts of concrete are formed by the short, parallel diagonal tension cracks. When the shear acts on the truss, the struts tend to rotate and so stress the transverse reinforcement as mentioned above. Because the diagonal struts are continuous with the concrete on both sides of the shear plane, there will be both compression and transverse shear in the strut. The applied shear is therefore resisted by the components of the strut compression and shear forces acting parallel to the shear plane as shown in Fig.3.55b. The reinforcement crossing the shear plane will eventually develop its yield strength, provided a failure of the concrete does not occur first. Failure will finally occur when the concrete struts fail under the combined action of compression and shear in the struts, while the reinforcement continues to develop its yield strength.

Hsu et al. [1987] indicated that the direct application of the truss model mentioned above would result in a much higher prediction of the shear strength, and the shear stress in the compression strut considerably would complicate the truss model theory. It was then emphasized that the fundamental difficulty in predicting the shear transfer strength of initially uncracked concrete was in the uncertainty of the compressive strength of the strut. On the other hand, it has been reported that compressive strength of the diagonal struts formed after the cracking of the reinforced concrete was much lower than the standard cylinder strength and this phenomenon was called the 'softening of concrete strut' which was related to the tensile strain in a direction perpendicular to the struts [e.g., Vecchio and Collins, 1981]. Here, it should be mentioned that the 'softening of concrete strut' does indicate the overall reduction of the compressive stress associated with the constitutive compressive stress-strain curve including the reduction of the compressive strength. Thus, the 'softening of concrete strut' does not indicate only the softening branch after the peak load in the constitutive compressive stress-strain curve which can be usually observed in the standard cylinder uniaxial compression test. Based on this phenomenon, Hsu et al. predicted the shear strength of various reinforced concrete member with good accuracy as follows.

**Hsu et al. [1987]**

A concrete element is reinforced with longitudinal bars in the \( \ell \)-direction and with transverse bars in the \( t \)-direction as shown in Fig.3.56. It is subjected at its edges to the in-plane normal stresses \( \sigma_\ell \) and \( \sigma_t \), as well as the shear stresses \( \tau_\ell \). After diagonal cracking, a series of diagonal compression struts is formed in the diagonal or \( d \)-direction, resulting in a truss-like action. It was assumed that the element took only compressive stress \( \sigma_d \) in the direction of the compressive struts, and tensile stress \( \sigma_t \) in the \( r \)-direction transverse to the
Fig. 3.55: Shear Transfer in Initially Uncracked Concrete [after Mattock and Hawkins, 1972]

Fig. 3.56: Truss Model for Reinforced Concrete Element [after Hsu et al., 1987]
(a) Compression Stress–Strain Relationship

\[ \varepsilon_p = \frac{\varepsilon_0}{\lambda} \]

(b) Tension Stress–Strain Relationship

Fig. 3.57: Softened stress-strain relationships for concrete [after Hsu et al., 1987]
Fig. 3.58: Shear stress-shear strain curves for Specimen M2 [after Hsu et al., 1987]

Fig. 3.59: Shear stress-shear strain curves for Specimen M6 [after Hsu et al., 1987]
compression struts. The shear stress $\tau_a$ was then assumed zero. The angle between the $\ell$-t and d-r coordinate systems is designated as $\alpha$. This angle is also that of inclination of the compressive struts with respect to the longitudinal axis. It was formulated that the stresses $\sigma_t$, $\sigma_n$, and $\tau_a$ in the reinforced concrete element were resisted jointly by the concrete and the steel reinforcement. They constructed the equilibrium and compatibility for the reinforced concrete element by means of both 'Mohr’s stress circle' and 'Mohr’s strain circle'.

The used constitutive laws in their model were originally suggested by Vecchio and Collins [1981]. The stress-strain relation in the direction of the compression strut in their model was represented by the following two equations and shown graphically in Fig.3.57a.

**Ascending branch:**

$$\sigma_d = -f_0' \left[ 2 \left( \frac{\epsilon_d}{\epsilon_o} \right) - \lambda \left( \frac{\epsilon_d}{\epsilon_o} \right)^2 \right] \quad \text{if} \quad |\epsilon_d| \leq |\epsilon_p| \quad (3.53a)$$

**Descending branch:**

$$\sigma_d = -f_0' \left[ 1 - \left( \frac{\epsilon_d}{\epsilon_o} - \frac{1}{\lambda} \right)^2 \right] \quad \text{if} \quad |\epsilon_d| > |\epsilon_p| \quad (3.53b)$$

where $\epsilon_d$ is the normal strain in the d-r coordinate system, $\epsilon_p = \epsilon_o / \lambda$ is defined as the peak strain, with $\epsilon_o$ taken as 0.002, and $\lambda$ was defined as a coefficient to take care of the softening phenomena and was expressed by

$$\lambda = \sqrt{\frac{\epsilon_t + \epsilon_i - 2\epsilon_d}{\epsilon_d} - 0.3} \quad (3.53c)$$

$$= \sqrt{0.7 - \frac{\epsilon_t}{\epsilon_d}}$$

where $\epsilon_i$ and $\epsilon_t$ are normal strain in the vertical and horizontal direction of specimens, respectively, and $\epsilon_t$ is the normal strain in the d-r coordinate system.
The stress-strain relation in a direction perpendicular to the compression strut is represented by the following equations and shown in Fig.3.57b.

Ascending branch:

\[ \sigma_r = E_c \varepsilon_r \quad \text{if} \quad \varepsilon_r < \varepsilon_{cr} \]  \hspace{1cm} (3.54a)

Descending branch:

\[ \sigma_r = \frac{f_{cr}}{1 + \sqrt{\frac{\varepsilon_r - \varepsilon_{cr}}{0.005}}} \quad \text{if} \quad \varepsilon_r > \varepsilon_{cr} \]  \hspace{1cm} (3.54b)

where \( E_c \) is initial modulus of elasticity of concrete, \( f_r \) is concrete cracking stress and \( \varepsilon_{cr} \) is concrete cracking strain.

To apply their model to a shear transfer test as shown in Fig.2.58, Hsu et al. considered the following cracking phenomenon as discussed in Chapter 2 in their model. After diagonal cracking, a cracked region is observed in the vicinity of the shear plane and eventually leads to failure. This cracked region is called the 'critical zone' and is the shaded area in Fig.2.59. A typical width of this zone was observed to be about 2 to 3 in. for a 10 in. wide specimen [Mattock 1974]. Within this zone, the extensive cracking of the concrete had an effect of redistributing the shear stress and the transverse normal stress more evenly along the shear plane. The cracking also reduced stiffness in this zone as compared to that outside of the zone. This would cause a redistribution of the compression stress in the transverse direction to become more evenly distributed across sections perpendicular to the shear plane. Thus, within this critical zone, the stresses might be assumed to be uniform. Specifically, they estimated the shear stress \( \tau_{\ell t} \) (see Fig.2.59) to be the average stress over the entire shear plane, and the compressive stress in the transverse direction \( \sigma_t \) to be the average stress over a cross-sectional plane perpendicular to the shear plane.

In their model, Hsu et al. introduced the ratio of maximum transverse stress to maximum shear stress 'K'. It was indicated that under the condition that the stresses are distributed uniformly over the whole specimen, 'K' was equated to \( \ell/h \) where \( \ell \) is the length of shear plane in the transverse direction, and \( h \) is the width of test specimen in the longitudinal direction. The following iterative procedures were then presented: (1) Select a value of the strain of compression struts \( \varepsilon_d \); (2) Assume a value of \( \sigma_t \); (3) Solve for the strain of diagonal cracks \( \varepsilon_r \); (4) Find \( \lambda \); (5) Find \( \sigma_d \); (6) Solve for \( \alpha \); (7) Solve for \( \sigma_t \); (7)
Compare the assumed value and calculated value of $\sigma_r$; (7-1) If both values are close enough, a set of solutions are obtained; (7-2) If not, a new $\sigma_r$ is calculated by a bisection method and steps 2-7 are repeated.

The comparison of their prediction and experimental data for push-off test are shown in Fig.3.58 and Fig.3.59 which show the shear stress versus shear strain curves obtained from the literatures [Mattock 1974, Mattock et al. 1976]. It was described that for convenience, the starting point was taken at the zero stress state and successive tracing was done from uncracked state to cracked state of the concrete. They concluded that the predicted curves using the softened compression stress-strain curve agreed well with the experimental curve regarding the maximum stress and the strain at maximum stress. In contrast, the predicted curves based on the non-softened compression stress strain curve overestimate considerably the maximum stress as well as the strain at maximum stress. It was also indicated that the truss model theory was not intended for the prediction of behavior before cracking. Specifically, tests in Fig.3.58 and Fig.3.59 show that the specimens before cracking are considerably stiffer than those predicted, and only when the ultimate strength is approached can the predicted shear stresses and shear strains become valid. This is a limitation of their truss model. Specifically, their model is valid for the prediction of the shear strength, but not for the entire shear failure behavior. They also compared their model with 32 test results reported in the literatures in terms of the shear strength and obtained the good agreement. It should be mentioned that their softened truss model was applied successfully to the structures where shear behavior predominated such as low-rise shear walls, framed wall panels, deep beams, as well as push-off shear test, and also to the structures subjected to torsion [e.g. Mau and Hsu 1987, Hsu 1988, Mau and Hsu 1989].

3.4.3. Summary

In this section, two theoretical approaches, LEFM and Truss model, are reviewed. These approaches were used to simulate fracture due to S crack and M cracks in the shear key joints. Based on these reviews, the following features and limitations of the current available models can be identified.

<table>
<thead>
<tr>
<th>LEFM model</th>
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<tr>
<td>1. Generally, the fracture parameters of LEFM of concrete such as $K_C$ and $G_C$ show large 'size effect' in the case of small specimen compared with the process zone size and are therefore not material properties in this sense.</td>
</tr>
</tbody>
</table>
2. Small scale yielding (SSY) at the crack tip zone is the basic premise of LEFM.

3. The accuracy of LEFM for concrete structure can be increased by using an 'effective crack length'. Possible additional crack length may be about 0.2 \( \ell_m \) for concrete.

4. LEFM approach has been successively applied to brittle materials like PMMA (not concrete) in compression. Most of these models are grounded on the idea that frictional sliding of a preexisting crack produces, at the crack tips, tension cracks that grow in the direction of maximum compression. At that time, the fracture instability was estimated by equating the maximum value of \( K_i \) with the opening-mode fracture toughness \( K_{ic} \) and the wing crack orientation was thus determined by the maximum \( K_i \) direction.

5. Simply approximated closed-forms of stress intensity factor at the tips of wing cracks were proposed for brittle materials like PMMA in compression. It was shown that these solutions agreed quite well with the exact (numerical) solutions in terms of singular integral equations.

**Truss model**

1. A truss model was successively applied to model analytically shear transfer in initially uncracked concrete (push-off shear test). This truss model can predict the shear strength of various reinforced concrete member with good accuracy.

2. The present truss model is valid for the prediction of the shear strength in initially uncracked concrete (push-off shear test), but not for the entire shear failure behavior, which arises from the fact that the truss model theory is not intended for the prediction of behavior before cracking.

3-5. SUMMARY AND CONCLUSION

In this chapter, basic crack models such as cohesive crack models and band models, and the fracture parameters were first reviewed. Then, the limitations and features of such models which should be considered in modelling of shear-off failure of key joints are identified. Next, two main numerical approaches to implement the crack models are briefly reviewed: discrete crack approach and smeared crack approach. Then, the application of those two approaches to mixed mode problem is investigated. In addition, two theoretical
approaches, LEFM and Truss model, are reviewed. These approaches were used to simulate fracture due to S crack and M cracks in the shear key joints mentioned previously.

Based on these reviews, the features and limitations of the current available modelling were identified. This description will form the basis for the mechanical model for the shear-off failure of key joints developed in Chapter 4.
CHAPTER 4

PROPOSED MECHANICAL MODEL

4-1. INTRODUCTION

In the previous chapters, several literature reviews were carried out on both shear failure observations and the analytical models to develop the mechanical model for shear-off failure of key joints. These reviews are first summarized briefly as follows.

The problem of shear-off failure of plain or fiber reinforced concrete shear key joints has been studied in many experimental studies [e.g., Cholewicki 1971, Lacombe and Pommeret 1974, Koseki and Breen 1983, Bakhoum et al., 1989 and Beattie et al. 1989]. These studies have identified two kinds of distinctively different cracking mechanisms for shear key joints, involving: (a) a large single curvilinear crack (S crack); and (b) short or long diagonal multiple cracks (M cracks). A typical cracking sequence of S crack and M cracks reported in the literatures [Bakhoum et al. 1989, Bakhoum 1991] is shown in Fig.4.1. The reported cracking sequence is: (1) The single curvilinear crack is nucleated at the corner of the key and propagates away from the shear-off plane at an approximately 45° angle; (2) In contrast, the multiple cracks are nucleated along the root of the key after the formation of the single crack, join together, and finally shear off the root of the key. Despite the experimental studies, very limited analytical work of the shear key problem is currently available.

The problem of direct shear failure of plain concrete and rock, which is similar to the shear failure of key joints, has been studied both experimentally and analytically [e.g., Bazant et al. 1985a,b and 1986, Ingraffea et al. 1985a, Watkins and Liu 1985, Davies et al., 1987, Davies 1988, Petit 1988, Solveig 1989, Ballatore et al., 1990, Barr and Derradj 1990]. The macroscopic shear failure of rock and PMMA under a compressive stress field has been studied as well [e.g., Petit et al. 1988, Einstein et al. 1990]. In most of these studies, fracture mechanics concepts have been used extensively to develop a deep insight of the shear failure of brittle materials like plain concrete and rock. The mode of fracture, whether it is mode I, II or mixed mode, has always been an argument in these studies.

Numerical formulations of the cracking behavior of concrete structures in finite element analysis (FEM) based on nonlinear fracture mechanics have been developed by means of primarily two models: the discrete crack model and the smeared crack model [e.g., Hillerborg 1980 and 1985, Bazant and Oh 1983, Ingraffea 1984, Jenq and Shah 1985, de Borst and Nauta 1985, Ingraffea and Saouma 1985, Rots and de Borst 1987,
Fig. 4.1: Cracking Sequence [after Bakhoun et al., 1989]
Fig. 4.2: Shear-off Fracture Sequence of (a) Observed Crack; (b) Idealized Crack and (c) Analogy between Winged Crack and Representative Rotated Crack
Rots and Blaauwendraad 1989]. Some of the formulations described in these references have given satisfactory solutions to the shear failure phenomena of plain concrete structures under direct shear loading.

Based on microscopic observations, various mathematical models have been introduced in an effort to analyze the failure process of brittle solids in compression. Most of these models are grounded on the idea that frictional sliding of a preexisting crack produces tension cracks at the preexisting crack tips that grow in the direction of maximum compression [e.g., Ashby and Hallam 1986, Horii and Nemat-Nasser 1986].

Analytical models for shear transfer in initially uncracked concrete have been presented based on a truss model [e.g., Hsu et al., 1987, Mau and Hsu 1987, Mansur and Ong 1991]. These models incorporate a softening stress-strain relation along the concrete compression struts.

Based on the ground of the above studies, the present work aims at developing a very simple mechanical model for the analysis and design of plain concrete and fiber reinforced concrete shear key joints. The present mechanical model describes the macroscopic cracking behavior for the shear-off failure mechanism of shear keys based on fracture mechanics concepts. The method is an approximate one, using well-known results of fracture mechanics and truss model theory, combined in a simple model.

4-2. FRACTURE SEQUENCE OF SHEAR-OFF FAILURE

Based on the physical description of shear-off failure in shear key joints established from experimental observations (see Chapter 2), the following simple fracture sequence of S crack and M cracks is macroscopically constructed in this study and is schematically shown in Fig.4.2. In the figure, the number of sequence is conjugated with that of description below. Here, it should be kept in mind that some of fracture sequences mentioned below are modified simply to make the mechanical model and others keep its physical concept described in Chapter 2. Therefore, both observed crack sequence and idealized one are shown in Fig.4.2.

<table>
<thead>
<tr>
<th>Phase 1 (dominated by S crack):</th>
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<td>(1-1) At the first stage of loading, the flaws near the corner of the key are easily mobilized by tensile stresses induced due to high shear stress concentration, and S crack is thus nucleated in the slow jumplike manner at the corner of the key joints. Such nucleation of S</td>
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crack is thus macroscopically a mode I crack. The load level of this initiation has a large variation such as 50% to 90% of the peak load.

(1-2) With further shear loading, S crack is loaded in the state of mixed mode or alternatively mode I, and propagates along the inclined linear path. This mixed mode propagation of S crack is governed by the local maximum principal tensile stresses. Here, it should be noted that the linear path is assumed in this study for simplicity of modelling. This simplification is not therefore in accord with the fact that the propagation of S crack was in reality observed as roughly curvilinear crack path tending to rejoin the shear loading direction, and the crack trajectory was oscillated since S crack microscopically consisted of a fairly narrow band of small tensile cracks in the vicinity of the crack tip: process zone. However, this simplification could be supported by the fact that approximately straight S crack was observed in some experimental works [e.g., Cholewicki 1971, Lacombe and Pommeret 1974, Koseki and Breen 1983]. In addition, mode I propagation is also alternatively considered in this study for simplicity of modelling. This simplification could be supported by the fact that S crack propagation is similar to that of a wing crack which usually propagates in the manner of mode I mentioned previously.

(1-3) At the certain load level, S crack tends to stop propagating or is constrained. This is because this crack runs into a low stress zone of the material or is arrested by a local principal compressive stresses. Then, S crack would release little strain energy.

Phase 2 (transition between S crack and M cracks):

(2-1) The distinctive formation of S crack induces the rotation of the key specimens and also changes the boundary condition for the stress field along the key base. These phenomena can rotate significantly the principal axis and produce high stress field along the key base which allows M cracks to be mobilized. At that stage, the flaws along the key base are easily mobilized by such tensile stresses.

(2-2) Then, intensive multiple cracking or M cracks occur on the local principal stress trajectories, and form a shear zone roughly aligned. Such initiation of M cracks thus occur under mode I condition.

(2-3) With further shear loading, M cracks approximately distribute evenly along the key base with a predominantly certain degree-inclination. Here, it should be noted that the distribution of M cracks are assumed uniformly for simplicity of modelling. This
simplification is not therefore in accord with the fact that in reality such distribution is not evenly and specifically normal and shear stresses are concentrated on the relatively lower part of the key base due to the bending effect. However, this simplification could be supported by the 'critical zone' concept along the direct shear plane [Hsu et al., 1987] as follows. After diagonal cracks initiate, a cracked region is formed in the vicinity of the shear plane. Within this region, the extensive cracking of the concrete has the effect of redistributing the shear stress and the transverse normal stress more evenly along the shear plane. The cracking also results in a stiffness reduction in this zone which causes a redistribution of the compression stress in the transverse direction, so that it becomes more evenly distributed across sections perpendicular to the shear plane. Thus, the stresses in the cracked zone can be assumed to be uniform.

**Phase 3 (dominated by M cracks):**

(3-1) With further shear loading, M cracks rotate following the principal stress axis, and hence are always under mode I condition. The shear stress along them is thus assumed zero. Here, it should be noted that representative rotating tension cracks are assumed here for simplicity of modelling. This concept is attractive from an engineering point of view since the analysis can suffice for specifying nonlinear stress-strain curves for the principal directions without having to resort to abstract theories. This assumption is not therefore in accord with the fact that multiple tensile cracks propagate in reality as winged cracks with wing cracks from each multiple crack tip mentioned previously. However, this assumption could show the analogy between the winged crack and the representative rotated crack along the principal axis as shown in Fig.4.2c. In addition, it was reported by Rots [1989] that the rotating crack concept introduced here for M cracks could describe the distributed defects of gradually rotating orientation, supported by considerable experimental evidence [Vecchio and Collins 1986, Bhide and Collins 1987, Kollegger and Mehlhorn 1987, Mattock and Hawkins [1972].

(3-2) With further shear loading, both the M crack-opening displacement and the compressive strain of the struts between each M crack increase continuously.

**Phase 4 (failure):**

(4-1) Finally, crushing failure of the compression struts occurs as a result of further increase of the shear load, leading to final failure. Specifically, M cracks are coalesced
caused by the highly localized strain distribution or intense microcracking (crushing) between each M crack due to the enforced shear movement.

In the present approach, the fracture process of S crack is modelled by means of linear elastic fracture mechanics (LEFM) formulation. The intent of this approach is based on the fact that it is often possible to find general analytical solutions, which are much easier to handle than numerical solutions such as FEM as mentioned in Chapter 3. In addition, many analytical works based on LEFM formulation were done for S crack-like phenomena mentioned previously. The fracture process of M cracks is modelled by means of a combination of the smeared crack concept [e.g., Bazant et al. 1985b 1986, Rots 1988, Rots and Blaauwendraad 1989] and a truss model [e.g., Hsu et al. 1987, Mau and Hsu 1987, Mansur and Ong 1991]. This is because these approaches can be easily handled by theoretical formulation and can show an acceptable accuracy in the M crack-like fracture.

It should be also kept in mind that we make the assumption in this study that the fracture process of shear key joints under shear loading is dominated by macroscopic cracking (visible by naked eye), and thus we neglect microcracking (not visible by naked eye). In addition, the term crack is not used here in its classical sense, that is as a complete discontinuity in both traction and displacement fields. Instead, it is used to describe an effective crack which consists of a true crack (in the classical sense) preceded by its process zone. The process zone is that area associated with crack propagation in which inelastic material behavior takes place.

4-3. MODELLING STRATEGY FOR SINGLE CRACK

In this section, the modelling strategy for S crack formation is developed by means of LEFM formulation. First, the valid condition of LEFM for S crack propagation is discussed. Based on this condition, LEFM formulation for strength and displacement is introduced.

4-3-1. Condition Of Validity For LEFM Approach

The fracture behavior of tension-softening materials like concrete, rock and fiber reinforced composite is dominated by the process zone: highly localized zone of straining which eventually forms a through crack. Li & Liang [1986] analyzed the behavior of fiber reinforced cementitious composites in terms of the transition of a strength-based failure
criterion to a linear elastic brittle crack failure criterion (LEFM). The former assumes no crack in the structure and the latter assumes that the crack tip is dominated by a K-field (i.e., a stress field whose intensity is characterized by a stress intensity factor $K_I$). They showed that between these two limiting conditions, the process zone is of comparable size with the traction free crack length and its presence must be explicitly accounted for. They also concluded that process zone length depends on the stress-separation constitutive behavior, the loading configuration and the structural geometry.

On the other hand, by increasing the size-scale of a concrete or composite element, the influence of heterogeneity disappears and the body may be considered as macroscopically homogeneous. Moreover, by increasing the size-scale of a cracked element, the influence of the non-linear softening material behavior vanishes, the cohesive crack tip forces disappear and the crack propagation is governed only by the linear elastic stress-singularity in the crack tip region.

Based on the phenomena mentioned above, Kaneko [1990], Kaneko and Li [1991b] employed a hybrid boundary element formulation to derive numerically the condition of validity for LEFM approach of a simple shear key model made of fiber reinforced ceramics for a fixed loading configuration, but for different stress-separation constitutive law and different structural geometry (structural size). The geometric and loading configuration is shown in Fig.4.3. The external load tends to open the corner which has a surface flaw $a_{0}$. For this configuration, only mode I crack may result and the direction of crack propagation is always 45°-inclination from the vertical line. As shown, the behavior of crack propagation in the analysis of Kaneko et al. could be thus similar to that of S crack in this study. In their analysis, three linear stress-deformation constitutive curves with different pre-peak stress-strain curves in tension were used as shown in Fig.4.4. Here, it should be noted that the softening model 2 in Fig.4.4b is identical to the linear model shown in Fig.3.5a introduced in Chapter 3.

Kaneko et al. obtained numerically the residual strength diagram for the case of $L/\ell_{ch}$ = 0.5, 1.0 and 2.0 ($\ell_{ch}$ is a characteristic length) as shown in Fig.4.5. LEFM and nonlinear elastic fracture mechanics results in the figures were calculated with the same fracture energy $G_f$, and the toughness in LEFM was obtained by the following formulae.

$$K_{IC} = \frac{EG_f}{\sqrt{(1-\nu^2)}}$$ for plane strain 

$$K_{IC} = \sqrt{EG_f}$$ for plane stress

(4.1a) (4.1b)
where $K_{IC}$ and $G_I$ are a fracture toughness and a fracture energy, respectively. $E$ and $\nu$ are Young’s modulus and Poisson’s ratio, respectively. Although this figure shows the structural strength for cracked structures corresponding to the crack extension, one can also know the condition of validity for LEFM approach. Rough estimation from these figures may indicate that LEFM approach for S crack propagation in the shear key joints could be valid under the following condition.

$$\frac{a_0}{\ell_{eh}} \geq 0.01$$  \hspace{1cm} (4.2)

If the above condition is not satisfied, the predicted value will be overestimated and some error will be introduced in the prediction. Strictly speaking, the above condition of validity for LEFM may not be always correct. It must be emphasized that the limiting normalized crack length depends on the structural configuration, the size of the structure, and the stress-separation curve; the value of 0.01 is just an average one. Moreover, the analysis of Kaneko et al. is based on a traction-free crack tip, and their results may not be directly translatable for uncracked structures. However, this condition can approximately show the accuracy of the present modelling, and this formula will be therefore used to check the validity of the present modelling based on LEFM formulation.

Regarding the fiber reinforced concrete structures, LEFM approach is largely limited. This is because fiber reinforced concrete structures have generally a greater non-elastic zone than plain concrete structures. In addition, the fracture energy of fiber reinforced concrete is not well defined although some literatures introduced such values, while the toughness $K_C$ in plain concrete structures can be directly derived from the well-defined fracture energy $G_f$ of concrete matrix as mentioned. Therefore, a special treatment is required for the use of LEFM approach in this case. As mentioned in Chapter 3, randomly oriented short fibers intervene at the following two stages of the crack formation process in concrete: (a) Fibers have a stitching action on the microcracks distributed randomly within the concrete volume, and the crack nucleation is thus retarded; (b) When macrocracking occurs, fibers play the same role as conventional steel reinforcement does in reinforced concrete. To use LEFM approach in a simple manner for fiber reinforced concrete key joints, the first intervention mechanism of fibers is ignored, and the second intervention mechanism is considered as an external load. It is then assumed that: (1) Fibers act only a closing spring force along the path of a matrix crack like steel reinforcement in conventional reinforced concrete structures; (2) The stress ahead of the crack tip can be described by the linear elastic crack tip field with intensity given by $K_I$ corresponding to the toughness of the
Fig. 4.3: Geometric and Loading Configuration of Analytical Shear Key Model [after Kaneko and Li, 1991b]
Fig.4.4: Tension Softening Model: (a) model 1 ($f_t^*/f_t=0.5$); (b) model 2 ($f_t^*/f_t=1.0$); (c) model 3 ($f_t^*/f_t=2.0$) [after Kaneko and Li, 1991b]
Fig. 4.5: Residual Strength Diagrams of: (a) $L/L_{ch}=5.0$; (b) $L/L_{ch}=1.0$; (c) $L/L_{ch}=0.5$ [after Kaneko and Li, 1991b]
matrix or plain concrete; (3) Fiber bridging stress distributes uniformly along the path of a matrix crack with a value equal to the residual tensile strength of the fiber $f_u$ (this quantity will be explained later); and (4) A traction free crack does not exist along the considered crack path. Thus, the contribution of the fiber reinforcement to the fracture process is to produce a negative stress intensity at the crack tip and to reduce the crack opening displacement. Since the fibers act only as an external loading and the toughness of the composite can be assumed as that of the matrix, the LEFM validity assumptions are the same as those for the case of plain concrete. The main frame of the assumption mentioned above is derived (see Chapter 3) from the fact that (with the exception of continuous fibers): (1) A high percentage of short fibers can increase the strength of the material (the dimensional scale of the fibers is the same as that of the microcracks) and; (2) A low percentage of long fibers can improve the ductility of the structure (to be effective with regard to macrocracking, the fiber must necessarily have a sufficiently long anchoring length). In general, short fibers especially short steel fibers used in concrete structures have relatively larger dimension rather than that of microcracks in concrete, and therefore the latter contribution of fibers usually prevail. Thus, the present assumption could be applicable in this sense.

4-3-2. Stress Intensity Factors Of Discrete Crack

As discussed, S crack initiation and propagation could be analogous to wing cracks from an initial inclined flaw under compression stress field. Ashby and Hallam [1986] and Horii and Nemat-Nasser [1982, 1985, 1986] developed the theoretical formulation for the propagation of wing cracks, and indicated that the stress intensity at the wing crack tip had two contributions: one from the stress field of the initial crack, dominant when the wing crack length $\ell$ is small; the other from the wedging action caused by the sliding displacement, dominant when $\ell$ is large. On the other hand, we do not have any preexisting macroscopic flaw or crack in the shear key configuration before the shear loading. In addition, for the geometric with relatively small S crack size LEFM is not applicable in the crack tip region. Therefore, the argument on this region when the S crack length is small is lack of significance in this study. Thus, only the situation after S crack propagation with certain length is considered here, and the latter contribution (wedging action) mentioned above is treated. Therefore, such calculation of the stress intensity factor will be always be an underestimation, but closer to the actual value for larger crack length. Here, the present model will be hereafter called 'Wedge Crack Mode (WCM)' in this thesis.
Wedge Crack Model describes the formation of S crack in analogy to a short crack propagation emanating from the edge of a semi-infinite body under a wedging force as shown in Fig. 4.6a. In this figure, the distributed load \( Q_t \) indicates the internal pressure which will be considered as the fiber bridging in the case of fiber reinforced concrete structures. Now, we apply this representative crack model to the actual wedging action on the shear key shown in Fig.4.6b. Here, it is assumed that both the vertical and the horizontal stresses acting on the shear key can be lumped to the edge of the crack (loads \( F \) and \( F' \), respectively), thus neglecting a moment which tends to increase the stress intensity factors. This increase is assumed small in the present modelling.

In the case of plain concrete key joints, the stress intensity factors at S crack tip, produced by the wedging forces \( F \) and \( F' \), are expressed by the following equations.

\[
K_I = 2(F\sin\theta - F'\cos\theta)\sqrt{\frac{\pi}{(\pi^2 - 4)\ell}} \tag{4.3a}
\]

\[
K_{II} = 2(F\cos\theta + F'\sin\theta)\sqrt{\frac{\pi}{(\pi^2 - 4)\ell}} \tag{4.3b}
\]

where \( F' = \sigma_p \ell \cos\theta \tag{4.3c} \)

and the subscripts I and II imply mode I and II, respectively. The equations \( 4.3a \) and \( 4.3b \) are identical to the formula of the stress intensity factor for a short crack propagation emanating from the edge of a semi-infinite body under a wedging force \( Q_t \) as shown in Fig.4.6a, which was derived theoretically by Freund [1978] by means of the so-called 'M-integral conservation law'. The similar expressions (numerical results) are available in the literatures [e.g., Bueckner 1971, Hartranft and Sih 1973].

As mentioned, it should be kept in mind that these estimation break down, becoming unbounded at wedge crack initiation, when \( \ell \) is vanishingly small.

Similarly, Wedge Crack Model is applied to fiber reinforced key joints here considering the additional force due to a fiber bridging \( f_{iu} \) which is distributed uniformly along the crack path as shown in Fig.4.6c. The stress intensity factor produced by the wedging forces \( F \) and \( F' \) (\( K_{Iu}, K_{IIu} \)), and the stress intensity factor produced by the fiber bridging stress or residual tensile strength of fibers \( f_{iu} (K_{Ib}) \) at the S-crack tip are given by the following equations:
\[ K_{1a} = 2(F \sin \theta - F' \cos \theta) \frac{\pi}{\sqrt{(\pi^2 - 4)\ell}} \] (4.4a)

\[ K_{1b} = -1.1215 f_m \sqrt{\pi \ell} \] (4.4b)

\[ K_{\Pi a} = 2(F \cos \theta + F' \sin \theta) \frac{\pi}{\sqrt{(\pi^2 - 4)\ell}} \] (4.4c)

\[ K_{\Pi b} = 0 \] (4.4d)

while

\[ K_I = K_{1a} + K_{1b} \] (4.4e)

\[ K_{\Pi} = K_{\Pi a} + K_{\Pi b} \] (4.4f)

The equation (4.4b) is identical to the formula of the stress intensity factor for a short crack propagation emanating from the edge of a semi-infinite body under uniform internal pressure \(Q_2\) as shown in Fig.4.6a, which was derived numerically in the literatures [e.g., Hartranft and Sih 1973]. Here, it should be noted that the contribution of fibers to mode II stress intensity factor is ignored in this formula for simplicity of the modelling. This is because the consideration of such contribution will considerably complicate the present modelling. In addition, it should be considered that S crack propagation may be largely dominated by mode I condition since such propagation is quite similar to the mode I crack like wing cracks mentioned previously.

4-3-3. Crack Propagation Criteria

The fracture instability of wing cracks in the case of PMMA was estimated by equating the maximum value of \(K_I\) with the opening-mode fracture toughness \(K_{IC}\), and the wing crack orientation was thus determined by the maximum \(K_I\) direction [Ashby and Hallam 1986, Horii and Nemat-Nasser 1982 1985 1986]. This approach for mixed mode problem is however not appropriate for concrete, and the Maximum Tensile Stress criterion could be suitable as discussed in Chapter 2. This criterion has been considered to be appropriate strength criterion for brittle materials whose main mode of failure is tensile separation. In the case of LEFM, the stresses at the fracture tip are infinite and the strength criterion cannot be applied. However, Erdogan and Sih [1963] proposed a propagation criterion expressed in terms of stress intensity factors which is a modified form of the Maximum Tensile Stress criterion. In the present model, such approach is adopted (Approach SA). In addition, to make a simple model, mode I crack propagation with assumed crack orientation is also treated here (Approach SB). Approach SB is adopted based on the fact
Fig. 4.6: (a) Short Crack Emanating from the Edge of Semi-infinite Body; and Schematic of S Crack Growth by Wedging Action of (b) Plain Concrete; (c) Fiber Reinforced Concrete
Fig. 4.7: Asymptotic Stress Field near Fracture Tip
that mode I approach is easily handled, and S crack propagation may be largely dominated
by mode I condition since such formation is quite similar to the mode I crack like wing
cracks. In addition, the mode II fracture toughness of concrete was not well defined.

In Approach SA, the condition for S crack growth is given by setting maximum tensile
stress factor 'H_{max}' to K_{IC} (the fracture toughness of the material). The formulation is
described based on the work of Chan [1986] as follows. The asymptotic stress fields near
the fracture tip (see Fig.4.7) are given by

\[ \sigma_{rr} = A \left[ K_I \left( 1 + \sin^2 \frac{\theta}{2} \right) + \frac{3}{2} K_{II} \sin \theta - 2K_{II} \tan \frac{\theta}{2} \right] \] (4.5a)

\[ \sigma_{\theta\theta} = A \left[ K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] \] (4.5b)

\[ \sigma_{r\theta} = A \left[ K_I \sin \theta + K_{II} (3\cos \theta - 1) \right] \] (4.5c)

where

\[ A = -\frac{1}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \text{ (r is small)} \] (4.5d)

It should be noted that these equations assume that the tip is opened (i.e., K_I > 0) and there
is no stress acting on the fracture surfaces near tip. Constants in this formula should be
added to \( \sigma_{\theta\theta} \) and \( \sigma_{r\theta} \) if there are stresses on the fracture surfaces near tip. The former
assumption could be true for S crack since the crack is always initiated under mode I
condition (open crack), but the latter assumption is not true for fiber reinforced concrete
key joints in which fibers may induce stresses acting on the fracture surfaces near tip.
Thus, the constants in Eqs. (4.5) are necessary for fiber reinforced shear key joints.
However, these constants do not affect the following derivations for fracture initiation and
direction.

Let the Tensile Stress Factor 'H' be defined by

\[ H(\theta) = -\sqrt{2\pi r} \sigma_{\theta\theta} \text{ (for Small r)} \]

\[ = \cos \frac{\theta}{2} \left[ K_I \cos^2 \frac{\theta}{2} - \frac{3}{2} K_{II} \sin \theta \right] \] (4.6)

Maximum value of 'H' occurs when [Erdogan et al. 1963]

\[ K_I \sin \theta + 3K_{II} \cos \theta = K_{II} \] (4.7)

Let \( \theta_0 \) be the angle at which H is at its maximum, namely

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\[ H_{\text{max}} = H(\theta_0) \] (4.8)

Then, maximum tensile stress factor states that the fracture tip will propagate if \( H_{\text{max}} = K_{IC} \). To find \( \theta_0 \), let \( B = K_{II}/K_1 \) and substitute this into Eq.(4.7). Then, \( \theta_0 \) is expressed by

\[ \sin \theta_0 + 3B \cos \theta_0 = B \] (4.9)

Using the variable 'a' described by the following relation,

\[ \cos a = \frac{1}{\sqrt{1 + 9B^2}} \] (4.10a)

\[ \sin a = \frac{3B}{\sqrt{1 + 9B^2}} \] (4.10b)

we obtain the following relation.

\[ \sin(\theta_0 + a) = \frac{B}{\sqrt{1 + 9B^2}} \] (4.11)

Finally, we obtain the expression for \( \theta_0 \) as follows.

\[ \theta_0 = \sin^{-1} \left[ \frac{B}{\sqrt{1 + 9B^2}} \right] - a \]

\[ = \sin^{-1} \left[ \frac{B}{\sqrt{1 + 9B^2}} \right] - \sin^{-1} \left[ \frac{3B}{\sqrt{1 + 9B^2}} \right] \] (4.12)

Substituting \( B = K_{II}/K_1 \) into Eq.(4.12) and using the fact of \( H_{\text{max}} = K_{IC} \) at \( \theta = \theta_0 \), we finally obtain the following relation.
\[ \theta_0 = \sin^{-1} \left[ \frac{\frac{K_{II}}{K_I}}{\sqrt{1 + 9 \left( \frac{K_{II}}{K_I} \right)^2}} \right] - \sin^{-1} \left[ \frac{3 \frac{K_{II}}{K_I}}{\sqrt{1 + 9 \left( \frac{K_{II}}{K_I} \right)^2}} \right] \]

(4.13a)

\[ K_{IC} = \cos \frac{\theta_0}{2} \left[ K_I \cos^2 \frac{\theta_0}{2} - \frac{3}{2} K_{II} \sin \theta_0 \right] \]

(4.13b)

To calculate the shear strength of the key joint corresponding to S crack propagation or the crack length \( \ell \) under fixed normal loading \( F' \), the following solution procedure should be followed.

1. Select a value of \( \ell \).
2. Assume the wedging force \( F \).
3. Assume the value of \( \theta \).
4. Calculate \( K_I \) and \( K_{II} \) from Eq.(4.3) or (4.4).
5. Calculate \( \theta_0 \) from Eq.(4.13a).
6. If the calculated value of \( \theta_0 \) is close enough to the assumed value of \( \theta \), go to the next step; otherwise, go back to the third step. (caution: the sign of \( \theta_0 \) and \( \theta \) is opposite).
7. Calculate \( K_{IC} \) from Eq.(4.13b).
8. If the calculated value of \( K_{IC} \) is close enough to the material toughness \( K_{IC} \), go to the next step; otherwise, go back to the second step.
9. Calculate the shear strength corresponding to a S crack (length \( \ell \)) propagation by dividing the force \( F \) by the depth \( D \) of the shear key.

In Approach SB, the condition for S crack growth is given by setting \( K_I \) to \( K_{IC} \) (the fracture toughness of the material). In the case of plain concrete key joints, we have

\[ F = \frac{K_{IC} \sqrt{\left( \frac{\pi^2}{2} - 4 \right) \ell}}{2 \sqrt{\pi} \sin \theta} + F' \cos \theta \]

(4.14)

Substituting Eq.(4.3c) into Eq.(4.14),

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\[
F = \frac{\frac{K_{IC}\sqrt{(\pi^2-4)}\ell}{2\sqrt{\pi}} + \sigma_p \ell \cos^2 \theta}{\sin \theta}
\]

(4.15)

Similarly, in the case of fiber reinforcement, we have;

\[
F = \frac{\left(\frac{K_{IC} + 1.1215 f_u \sqrt{\pi} \ell}{2\sqrt{\pi}}\right)\sqrt{(\pi^2-4)}\ell}{\sin \theta} + \sigma_p \ell \cos^2 \theta
\]

(4.16)

Based on substantial experimental evidence, \( \theta \) is assumed to be 45° [e.g., Cholewicki 1971, Lacombe and Pommeret 1974, Koseki and Breen 1983, Bakhoun et al., 1989] and we obtain the final expression.

For plain concrete:

\[
F = \frac{\frac{K_{IC}\sqrt{(\pi^2-4)}\ell}{2\sqrt{\pi}} + 0.5 \sigma_p \ell}{0.707}
\]

(4.17a)

For fiber reinforced concrete:

\[
F = \frac{\left(\frac{K_{IC} + 1.1215 f_u \sqrt{\pi} \ell}{2\sqrt{\pi}}\right)\sqrt{(\pi^2-4)}\ell}{0.707} + 0.5 \sigma_p \ell
\]

(4.17b)

The shear strength of shear key model corresponding to the S crack length \( \ell \), can be calculated by dividing '\( F \)' by the depth 'D' of the shear key. Here, it should be noted that Approach SB can give a simple closed solution without any numerical iteration.

4-3-4. Calculation Of Displacement

The vertical displacement of the key joint corresponding to S crack propagation could be calculated from the S crack opening/sliding displacement (COD/CSD). This calculation can be carried out by means of 'Compliance Method'. In the present modelling, two load
sets are considered for both plain concrete and fiber reinforced concrete key joints as follows.

Consider a planar body of unit thickness containing a crack of length $\ell$, subjected to two load sets '1' and '2', each of which produces the same 'pure crack tip mode'. Let the two generalized loads be $Q_1$ and $Q_2$ with work conjugate displacements $q_1$ and $q_2$, respectively (see Fig.4.6a). There will generally be a 'coupling' of the two load sets with a corresponding linear compliances $C(\ell)$ so that

$$q_i = C_{ij}(\ell)Q_j$$  \hspace{1cm} (4.18)

where $i=1, 2$ and a sum on $j$ from 1 to 2 is implied. From the Betti-Maxwell reciprocity theorem, the compliance terms are symmetric, so that $C_{ij}=C_{ji}$. Here, the compliances depend on the crack length $\ell$. The displacements $q_i$ can be calculated as

$$q_1 = C_{11}Q_1 + C_{12}Q_2$$  \hspace{1cm} (4.19a)

$$q_2 = C_{21}Q_1 + C_{22}Q_2$$  \hspace{1cm} (4.19b)

If the load $Q$ is specified, the 'dead load' potential energy of the load point must be reckoned in calculation of the potential energy of the system:

$$\Pi = W - Qq$$  \hspace{1cm} (4.20)

where $W$ is the strain energy and can be calculated by

$$W = \frac{1}{2}Qq$$  \hspace{1cm} (4.21)

Then, Substituting Eq.(4.18) and Eq.(4.21) into Eq.(4.20),

$$\Pi = -\frac{1}{2}Q^2C(\ell)$$  \hspace{1cm} (4.22)

The complementary energy of the body is thus calculated as
\[ \Omega = \frac{1}{2} Q_i q_i = \frac{1}{2} Q_i C_{ij} Q_j \]  

(4.23) 

while the energy release rate \( G_I \) and the stress intensity factor \( K_I \) are related by the following in the case of plane stress:

\[ G_I = \frac{K_I^2}{E} = \frac{\partial \Omega}{\partial \ell} \bigg|_Q = \frac{1}{2} Q_i \frac{dC_{ij}}{d\ell} Q_j \]  

(4.24) 

The \( K_I \) calibration must be linear in \( Q \) and dependent on the crack length. To this end, we write by superposition:

\[ K_I = Q_i k_i(\ell) = Q_i k_1(\ell) + Q_2 k_2(\ell) \]  

(4.25) 

Substituting Eq.(4.25) into Eq.(4.24),

\[ \frac{dC_{ij}}{d\ell} = \frac{2}{E} k_i(\ell) k_j(\ell) \]  

(4.26) 

Integrating Eq.(4.26), we obtain the compliance of the body:

\[ C_{ij}(\ell) = C_{ij}(0) + \frac{2}{E} \int_0^\ell k_i(\ell) k_j(\ell) d\ell \]  

(4.27) 

where \( C_{ij}(0) \) is the compliance of the uncracked body.

In the case of plain concrete key joints, \( k_i(\ell) \) can be obtained from Eq.(4.3) as

\[ k_1(\ell) = 2 \sqrt{\frac{\pi}{(\pi^2 - 4)\ell}} \]  

(4.28a) 

\[ k_2(\ell) = 0 \]  

(4.28b) 

Substituting Eq.(4.28) into Eq.(4.27), we obtain

\[ C_{11}(\ell) = C_{11}(0) + \frac{8\pi}{E(\pi^2 - 4)} \ln \ell \]  

(4.29)
Finally, substituting Eq.(4.29) into Eq.(4.18) and ignoring \( C_\text{0}(0) \) with the known values of \( Q \) from Eq.(4.3), we calculate COD and CSD as follows:

\[
\text{COD: } q_{\text{COD}}(\ell) = \left(F\sin \theta - \sigma_p \ell \cos^2 \theta \right) \frac{8 \pi}{E(\pi^2 - 4)} \ln \ell
\]
\[
\text{CSD: } q_{\text{CSD}}(\ell) = \left(F\cos \theta + \sigma_p \ell \cos \theta \sin \theta \right) \frac{8 \pi}{E(\pi^2 - 4)} \ln \ell
\]

Then, we finally obtain the vertical shear slip displacement due to \( S \) crack propagation in the plain concrete shear key joints as follows.

For Approach SA:

\[
\Delta_s(\ell) = \frac{8 \pi}{E(\pi^2 - 4)} \ln(\ell) \left( F\sin \theta - \sigma_p \ell \cos^2 \theta \right) \sin \theta
\]
\[
+ \left( F\cos \theta + \sigma_p \ell \cos \theta \sin \theta \right) \cos \theta
\]

For Approach SB:

\[
\Delta_s(\ell) = \sin 45^\circ \frac{8 \pi}{E(\pi^2 - 4)} \left( F\sin 45^\circ - \sigma_p \ell \cos^2 45^\circ \right) \ln(\ell)
\]

Similarly, in the case of fiber reinforcement, \( k_\ell(\ell) \) can be obtained from Eq.(4.4) as

\[
k_1(\ell) = 2 \sqrt{\frac{\pi}{(\pi^2 - 4)\ell}}
\]
\[
k_2(\ell) = 1.1215 \sqrt{\pi \ell}
\]

Substituting (4.32) into (4.27), we obtain

\[
C_{11}(\ell) = C_{11}(0) + \frac{8 \pi}{E(\pi^2 - 4)} \ln \ell
\]
\[
C_{12}(\ell) = C_{12}(0) + \left( \frac{4.486 \pi}{E\sqrt{\pi^2 - 4}} \right) \ell
\]
Finally, substituting Eq.(4.33) into Eq.(4.18) and ignoring C_y(0) with the known values of Q from Eq.(4.4), we calculate COD and CSD as follows:

\[
\text{COD: } q_{\text{COD}}(\ell) = \left( F \sin \theta - \sigma_p \cos^2 \theta \right) \frac{8\pi}{E(\pi^2 - 4)} \ln \ell - f_{\text{uw}} \left( \frac{4.486\pi}{E\sqrt{\pi^2 - 4}} \right) \ell \quad (4.34a)
\]

\[
\text{CSD: } q_{\text{CSD}}(\ell) = \left( F \cos \theta + \sigma_p \ell \cos \theta \sin \theta \right) \frac{8\pi}{E(\pi^2 - 4)} \ln \ell \quad (4.34b)
\]

Then, we finally obtain the vertical shear slip displacement due to S crack propagation in the fiber reinforced concrete shear key joints as follows.

For Approach SA:

\[
\Delta_s(\ell) = \frac{8\pi}{E(\pi^2 - 4)} \ln(\ell) \left( F \sin \theta - \sigma_p \ell \cos^2 \theta \right) \sin \theta
\]

\[
+ \left( F \cos \theta + \sigma_p \ell \cos \theta \sin \theta \right) \cos \theta - f_{\text{uw}} \left( \frac{4.486\pi}{E\sqrt{\pi^2 - 4}} \right) \ell \sin \theta \quad (4.35a)
\]

For Approach SB:

\[
\Delta_s(\ell) = \sin 45^\circ \left[ \frac{8\pi}{E(\pi^2 - 4)} \ln \ell \left( F \sin 45^\circ - \sigma_p \ell \cos^2 45^\circ \right) - f_{\text{uw}} \left( \frac{4.486\pi}{E\sqrt{\pi^2 - 4}} \right) \right] \quad (4.35b)
\]

4-4. MODELLING STRATEGY FOR MULTIPLE CRACKS

In this section, the modelling strategy for M cracks formation in shear key joints is developed by means of both a rotating smeared crack approach and a truss model. The model is then developed to satisfy three basic requirements: equilibrium, compatibility and material constitutive laws. Here, the present model will be hereafter called 'Rotating Smeared Crack Band Model (RSCBM)' in this thesis.
4-4-1. Equilibrium And Compatibility Conditions

Stress transformation conditions (equilibrium) in a cracked element are shown in Fig.4.8. In this modelling, the shear-off stress along the key root is assumed to be the average stress over the entire shear-off plane, based on the diagonal cracking phenomenon within the critical zone of push-off direct shear tests [Hsu et al., 1987] as mentioned previously. The stresses and strains are thus assumed uniformly distributed as averaged ones. After diagonal cracking occurs, a series of diagonal compression struts is formed in the diagonal or c-direction. As mentioned, the element takes only compressive stresses $\sigma_c$ in the direction of the compression struts, and only tensile stresses $\sigma_t$ in the t-direction transverse to the compression struts. The shear stress $\tau_{ct}$ along the cracked element is assumed zero. Thus, $\sigma_c$ and $\sigma_t$ are always the principal stresses of this system. The angle between the x-y and c-t coordinate systems is designated as $\theta$. This angle is also the angle of inclination of the compression struts with respect to the x-axis.

The average stresses in the two coordinate systems, x-y and c-t, are transformed according to the following equations:

$$\sigma_x = \sigma_c \cos^2 \theta + \sigma_t \sin^2 \theta$$  \hspace{1cm} (4.36a)
$$\sigma_y = \sigma_c \sin^2 \theta + \sigma_t \cos^2 \theta$$  \hspace{1cm} (4.36b)
$$\tau_{xy} = (\sigma_c - \sigma_t) \sin \theta \cos \theta$$  \hspace{1cm} (4.36c)

where $\sigma_x$ and $\sigma_y$ are the normal stresses in the x-y coordinate system, $\tau_{xy}$ is the shear stress in the x-y coordinate system, and $\sigma_c$ and $\sigma_t$ are the normal stresses in the c-t coordinate system (principal stresses). Similar expressions can be obtained in terms of average strains as follows.

$$\varepsilon_x = \varepsilon_c \cos^2 \theta + \varepsilon_t \sin^2 \theta$$  \hspace{1cm} (4.37a)
$$\varepsilon_y = \varepsilon_c \sin^2 \theta + \varepsilon_t \cos^2 \theta$$  \hspace{1cm} (4.37b)
$$\gamma_{xy} = 2(\varepsilon_c - \varepsilon_t) \sin \theta \cos \theta$$  \hspace{1cm} (4.37c)

where $\varepsilon_x$, $\varepsilon_y$ are normal strains in the x-y coordinate system, $\gamma_{xy}$ is shear strain in x-y coordinate system and $\varepsilon_c$ and $\varepsilon_t$ are normal strains in the c-t coordinate system (principal strains).
Fig. 4.8: (a) Crack Formation and (b) Stress Transformation Systems in Rotating Smeared Crack Band Model
4-4-2. Constitutive Models

Plain concrete
The assumed tensile stress-strain relation in the direction perpendicular to the compression struts is shown schematically in Fig. 4.9a.

Ascending branch:

\[ \sigma_t = E_c \varepsilon_t \quad \text{if} \quad \varepsilon_t \leq \varepsilon_{cr} \tag{4.38a} \]

where \( E_c \) is the initial Young's modulus of concrete. The bi-linear tensile stress-deformation relation proposed by Hillerborg [1985a] is adopted for the descending branch (see Fig. 4.9b).

Descending branch:

\[ \sigma_t = \frac{f_t}{3} \left( \frac{\varepsilon_{u1} - \frac{1}{3} \varepsilon_{cr} - \frac{2}{3} \varepsilon_t}{\varepsilon_{u1} - \varepsilon_{cr}} \right) \quad \text{if} \quad \varepsilon_{cr} < \varepsilon_t \leq \varepsilon_{u1} \tag{4.38b} \]

\[ \sigma_t = \frac{f_t}{3} \left( \frac{\varepsilon_{u2} - \varepsilon_t}{\varepsilon_{u2} - \varepsilon_{u1}} \right) \quad \text{if} \quad \varepsilon_{u1} < \varepsilon_t \leq \varepsilon_{u2} \tag{4.38c} \]

where

\[ \varepsilon_{cr} = \frac{f_t}{E_c} \tag{4.38d} \]

\[ \varepsilon_{u1} = \varepsilon_{cr} + \frac{4 G_f}{5 f_t h} \tag{4.38e} \]

\[ \varepsilon_{u2} = \varepsilon_{cr} + \frac{18 G_f}{5 f_t h} \tag{4.38f} \]

where \( G_f \) is the fracture energy, \( f_t \) is the tensile strength, \( \varepsilon_{cr} \) is the cracking strain, \( \varepsilon_{u1} \) and \( \varepsilon_{u2} \) are the post-cracking characteristic strains and \( h \) is the band width. Here, the band width 'h' is approximated as the width of the area dominated by M cracks (see Fig.4.8a). Strictly speaking, \( h \) should be estimated as the width of each crack normal to the crack orientation. However, it is quite difficult to find such value from the practical point of
Fig. 4.9: (a) Uniaxial Tension Constitutive Model; (b) Tensile Stress-Deformation Curve [after Hillerborg 1985a]
Fig. 4.10: Uniaxial Compression Constitutive Models:
(a) Model C1 [after Soroushian et al., 1986];
(b) Model C2 [after Mansur and Ong, 1991]
view. This is because the width of each crack has a large variation as shown in Fig.4.1. In addition, the width of each crack decreases as shear loading increases. On the other hand, the width of the area dominated by M cracks could be approximately constant, and is also the only length that can be practically determined for multiple cracking. In addition, rough estimation could show that the width of the area dominated by M cracks was close to the width of each M crack (see Fig.4.1). It will be also shown later that the influence of h on the shear stress-slip relation of key joints is relatively small for the entire load-displacement history.

The assumed compressive stress-strain relation in the direction of the compression struts is based on two compressive stress-strain curves reported in the literatures [Soroushian et al. 1986, Mansur and Ong 1991]. The former (hereafter called model C1) was derived from the study of confined and unconfined plain concrete subjected to dynamic compression, and the latter (hereafter called model C2) was derived by modifying a softening truss model (softening of concrete strut) describing the behavior of steel fiber reinforced concrete (with steel reinforcement) deep beams in shear. As mentioned, the softening of concrete strut does indicate the overall reduction of the compressive stress associated with the constitutive compressive stress-strain curve including the reduction of the compressive strength. Strictly speaking, the softened truss model in the latter model was developed for conventional steel reinforced concrete element [Hsu et al., 1987] based on the study of the behavior of reinforced concrete panels under predominantly shear stress. Thus, this softening of concrete was observed in reinforced concrete element not for plain concrete elements. Therefore, addition of the softening coefficient may be conservative for plain concrete or fiber reinforced concrete shear key joints, and this point will be assessed in comparison with the former (non-softened) model C1. Both models introduced in the literatures are slightly modified here to fit the present modelling and also incorporated by the often quoted idealized stress-strain curve for concrete in uniaxial compression [Hognestad 1951]. Thus, the assumed stress-strain relations shown in Fig.4.10 are described by the following equations:

**Model C1**

Ascending branch in compressive constitutive model C1:

\[
\sigma_c = f_c \left[ \frac{2\varepsilon_c}{\varepsilon_{c0}} - \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right] \quad \text{if } \varepsilon_c \leq \varepsilon_{c0} \quad (4.39a)
\]
where \( f'_c \) is the compressive strength and \( \varepsilon_{c0} \) is the associated strain.

**Descending branch in compressive constitutive model C1:**

\[
\sigma_c = f'_c \left[ 1 - Z(\varepsilon_c - \varepsilon_{c0}) \right] \quad \text{if } \varepsilon_{c0} < \varepsilon_c \leq \varepsilon_{cul}
\]

\[
Z = \frac{0.5}{\frac{3 + 145 \varepsilon_{c0} f'_c}{145 f'_c - 1000} - \varepsilon_{c0}}
\]

**Horizontal branch in compressive constitutive model C1:**

\[
\sigma_c = 0.2 f'_c \quad \text{if } \varepsilon_{cul} < \varepsilon_c
\]

\[
\varepsilon_{c0} = \frac{2 f'_c}{E_c}
\]

\[
\varepsilon_{cul} = \frac{0.8}{Z} + \varepsilon_{c0}
\]

**Model C2**

**Ascending branch in compressive constitutive model C2:**

\[
\sigma_c = f'_c \left[ \frac{2 \varepsilon_c}{\varepsilon_{c0}} - \lambda \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right] \quad \text{if } \varepsilon_c \leq \frac{\varepsilon_{c0}}{\lambda}
\]

\[
\lambda = \sqrt{0.7 - \frac{\varepsilon_{c0}}{\varepsilon_c}}
\]

**Descending branch in compressive constitutive model C2:**

\[
\sigma_c = f'_c \left[ 1 - 0.8 \left( \frac{\varepsilon_c - \varepsilon_{c0}}{\lambda} \right) \right] \quad \text{if } \frac{\varepsilon_{c0}}{\lambda} < \varepsilon_c \leq \varepsilon_{cul}
\]

**Horizontal branch in compressive constitutive model C2:**
\[ \sigma_c = \frac{0.2f_c'}{\lambda} \quad \text{if } \varepsilon_{cu2} < \varepsilon_c \] (4.40d)

\[ \varepsilon_{cu2} = \frac{0.041 - 2\varepsilon_c f_c'}{f_c - 6.896} + \varepsilon_{c0} \] (4.40e)

**Fiber reinforced concrete**

The tensile stress-strain relation adopted here for the direction perpendicular to the compression struts in the fiber reinforced concrete shear key joints is shown schematically in Fig. 4.11a.

Ascending branch:

\[ \sigma_t = E_c \varepsilon_t \quad \text{if } \varepsilon_t \leq \varepsilon_{cr} \] (4.41a)

where \( E_c \) is the initial Young's modulus of concrete and \( \varepsilon_{cr} \) is the cracking strain. The post-peak regime is described by both the fracture energy of the matrix \( G_f \) [Hillerborg 1985b] and the residual tensile strength of fiber reinforced concrete \( f_{tu} \) reported in the literature (e.g., Lim et al. 1987a,b, Mansur and Ong 1991]. Specifically, the descending branch A-B in Fig. 4.11a corresponds to fracture of the matrix (plain concrete) and the area of the triangle A-D-E is defined by the fracture energy of plain concrete \( G_f \):

Descending branch:

\[ \sigma_t = \frac{(\varepsilon_{wu2} - \varepsilon_t)f_t}{\varepsilon_{wu2} - \varepsilon_{cr}} \quad \text{if } \varepsilon_{cr} < \varepsilon_t \leq \varepsilon_{wu1} \] (4.41b)

\[ \sigma_t = f_{tu} \quad \text{if } \varepsilon_{wu1} < \varepsilon_t \] (4.41c)

\[ f_{tu} = 2\eta_t \eta_0 \tau_{uf} V_f \frac{\ell_f}{\phi_f} \] (4.41d)

\[ \varepsilon_{cr} = \frac{f_t}{E_c} \] (4.41e)

\[ \varepsilon_{wu1} = \varepsilon_{wu2} - \frac{f_{tu}}{f_t}(\varepsilon_{wu2} - \varepsilon_{cr}) \] (4.41f)

\[ \varepsilon_{wu2} = \varepsilon_{cr} + \frac{2G_f}{h f_t} \] (4.41g)
where $f_1$ is the tensile strength, $e_{u1}$ and $e_{u2}$ are the post-cracking characteristic strains and $h$ is the bond width as introduced previously. $\eta_1$ is the length efficiency factor for the fibers, $\eta_0$ is the fiber orientation factor, $\tau_{uf}$ is ultimate bond strength of fibers, $V_f$ is the fiber volume fraction, $l_f$ is the fiber length and $\phi_f$ is the equivalent fiber diameter. Regarding the fiber bond strength $\tau_{uf}$, several researchers have performed pullout tests in which an individual fiber was embedded in a mortar matrix. Typical bond versus slip curves for both straight and deformed fibers with different embedded lengths are for example given by Lim et al. [1987a] and shown in Fig.4.12. The length efficiency factor $\eta_1$ accounts for the varying fiber stress at the end portions of the fibers and is given as [Lim et al. 1987a,b]

$$\eta_1 = \begin{cases} 0.5 & \text{for } l_f \leq l_c \\ 1 - \frac{l_c}{2l_f} & \text{for } l_f > l_c \end{cases}$$ (4.42)

where $l_c$ is the critical fiber length, denoting a length twice that required to develop the ultimate fiber stress $\sigma_{fu}$ in the fiber when a uniform ultimate bond stress $\tau_{uf}$ is assumed at the fiber matrix interface. $l_c$ is given as follows [Lim et al. 1987a,b]:

$$l_c = \frac{0.5\sigma_{fu}\phi_f}{\tau_{uf}}$$ (4.43)

Using the thickness $t$ and the width $b$ of the composite and the fiber length $l_f$, the following simple formulae for the fiber orientation factor $\eta_0$ are given by Lim et al. [1987a,b] and Soroushian et al. [1990].

$$\eta_0 = \frac{\int \int \cos \theta \cos \rho \, d \theta d \rho}{\int \int d \theta d \rho} = \frac{\sin \bar{\theta} \sin \bar{\rho}}{\bar{\theta} \bar{\rho}}$$ (4.44a)

$$\bar{\theta} = \sin^{-1} \left( \frac{t}{l_f} \right) \leq \frac{\pi}{2}$$ (4.44b)

$$\bar{\rho} = \sin^{-1} \left( \frac{b}{l_f} \right) \leq \frac{\pi}{2}$$ (4.44c)
Fig. 4.11: (a) Uniaxial Tension Constitutive Model; (b) Uniaxial Compression Constitutive Model
Lim et al. [1987a,b] also indicated that if the fibers can be considered as randomly oriented in a three-dimensional manner, the two dimensions t and b should be greater than or equal to $\ell_f$. Hence, $\overline{\theta}$ and $\overline{p}$ are both equal to $\pi/2$, while $\eta_0$ is calculated simply as

$$\eta_0 = \frac{\int_{0}^{\pi/2} \int_{0}^{\pi/2} \cos \theta \cos \rho \, d\theta d\rho}{\pi/2 \pi/2} = \frac{\sin(\pi/2)\sin(\pi/2)}{(\pi/2)^2} = 0.405$$  \hspace{1cm} (4.45)

Here, it should be mentioned that Eqs. (4.44) are strictly speaking rough estimates [e.g., see comment by Stroeven, 1989]. Moreover, the effect of fiber inclination. The effect of fiber inclination cannot be accounted for by just considering the geometrical effect. Indeed, the bridging action of an inclined fiber is quite different from that of a fiber perpendicular to the crack. Effects such as fiber bending, fiber snubbing (i.e., fiber passing through a frictional pulley at the exit point) and matrix spalling all affect the crack bridging stress and these effects are still under investigation. However, this thesis deals more on the structural behavior of shear key joints rather than the deviation of precise stress-strain behavior. Therefore, the approximate expressions for the orientation factor by Eqs. (4.44) are adopted to study the structural behavior. However, the same overall analytical framework is still applicable when a more accurate constitutive model is incorporated.

The compressive stress-strain relation in the direction of the compression strut in the present model is constructed by slightly modifying the compressive stress-strain curves reported in the literature by Mansur and Ong [1991], to obtain a better representation of the fiber reinforced shear key behavior. This is done by neglecting the softening coefficient. Thus, the assumed stress-strain relation (Fig. 4.11b) is described by the following equations.

**Ascending branch:**

$$\sigma_c = f'_c \left[ \frac{2\varepsilon_c}{\varepsilon_{c0}} - \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right] \quad \text{if } \varepsilon_c < \varepsilon_{c0} \quad (4.46a)$$

**Descending branch:**

\[ \sigma_c = k'f'_{c} + \frac{(1-k')f'_{c}(\varepsilon_{cul} - \varepsilon_{c})}{(\varepsilon_{cul} - \varepsilon_{c0})} \quad \text{if } \varepsilon_{c0} \leq \varepsilon_{c} < \varepsilon_{cul} \]  

(4.46b)

Horizontal branch:

\[ \sigma_c = k'f'_{c} \quad \text{if } \varepsilon_{cul} \leq \varepsilon_{c} \]  

(4.46c)

where

\[ k' = 0.38V_f \left( \frac{\ell_f}{\phi_f} \right) \]  

(4.46d)

\[ \varepsilon_{cul} = \frac{0.041 - 2\varepsilon_{c0}f'_{c}}{f'_{c} - 6.896} + \varepsilon_{c0} \]  

(4.46e)

where \( f'_{c} \) is the compressive strength, \( \varepsilon_{c0} \) is the associated strain and \( \varepsilon_{cul} \) is the strain corresponding to the end of the descending branch.

4-4-3. Model Of Apparent Poisson’s Ratio

In this modelling, the preceding equations contain 11 variables (except material properties): \( \sigma_x, \sigma_y, \tau_{xy}, \varepsilon_x, \varepsilon_y, \gamma_{xy}, \sigma_t, \varepsilon_t, \varepsilon_c \) and \( \theta \). Among these unknown variables, \( \sigma_x \) is specified as the prestress of the shear key. Furthermore, \( \varepsilon_t \) and \( \varepsilon_c \) may be related to each other. Here, It is assumed that the relation between the tensile and compressive strain of a compression strut can be constructed using the apparent Poisson's ratio \( \nu_a \). This is a salient feature of RSCBM. The quantitative value of the apparent ratio between \( \varepsilon_t \) and \( \varepsilon_c \) is established here by the following formulation and shown schematically in Fig.4.13.

\[ \nu_a = 0.2 \text{(elastic behavior; volume change)} \quad \text{if } \varepsilon_c \leq \varepsilon_{c1} \]  

(4.47a)

\[ \nu_a = 0.2 - 0.5 \text{(linear transition)} \quad \text{if } \varepsilon_{c1} < \varepsilon_c \leq \varepsilon_{c2} \]  

(4.47b)

\[ \nu_a = 0.5 \text{(plastic behavior; no volume change)} \quad \text{if } \varepsilon_{c2} < \varepsilon_c \]  

(4.47c)

where \( \varepsilon_{c1} \) and \( \varepsilon_{c2} \) are the strains at \( \sigma_c = 0.8f'_{c} \) and \( \sigma_c = f'_{c} \), respectively. In the constitutive models used in this thesis, such quantities are expressed from Eqs. (4.39a) and (4.46a) as:
\[ \varepsilon_{c1} = 0.553 \varepsilon_{c0} \]  \hspace{1cm} (4.47d)\\
\[ \varepsilon_{c2} = \varepsilon_{c0} \]  \hspace{1cm} (4.47e)

Eqs. (4.47d) and (4.47e) are also applied to the softened compressive constitutive model C2 by Eqs.(4.40).

The above formulation is first established based on the experimental observation given in the literature by Chen [1982] for the relation between the stress-strength ratio and the apparent Poisson's ratio as shown in Fig.4.14. This figure shows the apparent Poisson's ratio between \( \varepsilon_t \) and \( \varepsilon_c \) in a uniaxial compression test. Chen described this quantity such that: Under uniaxial loading, the apparent Poisson's ratio remained constant until approximately 80 percent of \( f_c \), at which stress the apparent Poisson's ratio began to increase; In the unstable crushing phase the apparent Poisson's ratio even becomes larger than 0.5.

Here, it should be noted that this apparent Poisson's ratio was derived from the uniaxial compression test in which the compressive strain controls the deformation of the structure and this apparent Poisson's ratio is thus identical to the value of \( \varepsilon_c/\varepsilon_c \). However, since the coalescence of M cracks may be achieved by highly strain localization between each M crack as mentioned previously, the apparent Poisson's ratio controlled by the tensile strain should be also considered in the modelling of M cracks. To quantify such apparent Poisson's ratio in the tensile strain-dominated field, the following comparison is employed.

Fig.4.15 shows the relation between the apparent Poisson's Ratio \( \nu_s = \varepsilon_s/\varepsilon_c \) and the normalized compressive strain \( \varepsilon_c/\varepsilon_{c0} \) where the strain \( \varepsilon_{c0} \) is associated with the compressive strength. Here, it should be noted that in this case the tensile strain controls the deformation of the structures. In the figure, the broken line stands for the proposed model which is identical to Fig.4.13. The scattered data stand for both experimental data and theoretical predictions introduced in literatures. The black circular points are the results from tests of reinforced concrete elements subjected to in-plane shear and normal stresses [Vecchio and Collins, 1986] and the normalized compressive strain was associated with the measured strain in concrete cylinder at peak stress \( f_c \). The white circular points are the result of BEAM B3 test (shear span ratio = 0.865) which was reinforced fiber concrete deep beam [Mansur and Ong, 1991], and the normalized compressive strain was obtained by assuming \( \varepsilon_{c0} \) is -0.002. The black rectangular points are obtained from the parameters calculated by Hsu et al. [1987] for the comparison of their theoretical prediction with the test data of reinforced concrete elements under push-off shear loading by Mattock [1974] and Mattock et al. [1976]. The normalized compressive strain was obtained by assuming
Fig. 4.12: Experimental Bond-slip Relationships of Fibers and Idealization [after Lim et al., 1987a]

Fig. 4.13: Idealized Relationship between Stress-strength Ratio and Apparent Poisson's Ratio
Fig. 4.14: Relation between Stress-strength Ratio and Poisson's Ratio [after Chen, 1982]

Fig. 4.15: Apparent Poisson's ratio Measured in Experiments
\( \varepsilon_{co} \) is -0.002. The white rectangular points are obtained from the parameters calculated by Vecchio and Collins [1986] for the comparison of their theoretical prediction with their test data of PV 20 specimen which was reinforced concrete element subjected to in-plane shear stress. From the figure, it is obvious that the apparent Poisson’s ratio controlled by the tensile strain could be applicable for tensile strain dominated field.

The above two different apparent Poisson’s ratios \( (\nu_s = \varepsilon_t / \varepsilon_c \) and \( \nu_s = \varepsilon_c / \varepsilon_t \) \) will be applied to the modelling of M cracks propagation as follows. Here, it should be also mentioned that the employment of the apparent Poisson’s ratio would eliminate numerical iteration in the solution procedures in the present modelling.

4-4-4. Solution Procedures

Using the apparent Poisson’s ratio and a specified \( \sigma_x \), the preceding 11 unknowns \( (\sigma_x, \sigma_y, \tau_{xy}, \varepsilon_x, \varepsilon_y, \gamma_{xy}, \sigma_t, \sigma_c, \varepsilon_t, \varepsilon_c \) and \( \theta \) \) are reduced to 9. By selecting one of them as a known value, the remaining 8 unknowns can be obtained from the set of the 8 equations consisting in equilibrium equations, compatibility ones and constitutive ones. Based on the failure sequence described previously, solution procedures in this modelling are established by two approaches such that: (1) The fracture behavior of the key base in shear key joints is controlled by the tensile crack strain transverse to the compression struts (hereafter called Approach MA). In this case, \( \varepsilon_t \) and \( \varepsilon_c \) are related to each other by Eq.(4.38a). As mentioned, this approach is presumably appropriate since the coalescence of M cracks may be achieved by highly strain localization between each M crack. In addition, the alternative approach is also employed such that: (2) The fracture behavior of the key base in shear key joints is controlled by the compressive strain in the compression struts (hereafter called Approach MB). In this case, \( \varepsilon_t \) and \( \varepsilon_c \) are related to each other by Eq.(4.38b). The verification of these two approaches will be examined through the comparison with experimental results in Chapter 5.

Approach MA:
\[
\varepsilon_c = \nu_s \varepsilon_t \text{(controlled by tensile strain in the crack)} \tag{4.48a}
\]

Approach MB:
\[
\varepsilon_t = \nu_s \varepsilon_c \text{(controlled by compressive strain in the strut)} \tag{4.48b}
\]

Hence, one can develop the relation between the average shear stress \( \tau_{xy} \) and the average shear slip displacement given next.
**Approach MA:**

1. Select a value of $\varepsilon_c$.
2. Calculate $\nu$ from Eq.(4.47) using the previously calculated value of $\varepsilon_c$, or assume $\nu = 0.2$ in the first calculation.
3. Calculate $\varepsilon_c$ from (4.48a).
3'. Calculate $\lambda$ from (4.40b) in the case of constitutive model C2.
4. Calculate $\sigma_t$ and $\sigma_c$ from Eqs.(4.38)-(4.41) and Eq.(4.46).
5. Solve for $\theta$ from Eq.(4.36a) with $\sigma_x$ known.
6. Calculate $\tau_{xy}$, $\varepsilon_y$ and $\gamma_{xy}$ from Eqs.(4.36c), (4.37b) and (4.37c).

**Approach MB:**

1. Select a value of $\varepsilon_c$.
2. Calculate $\nu$ from Eq.(4.47).
3. Calculate $\varepsilon_t$ from Eq.(4.48b).
4. Follow steps (3')-(6) described above.

The vertical displacement $\Delta$ at the edge of the key joint (shear slip displacement) is calculated by assuming that it is the result of both compressive strains in the cracked zone and shear strains along the shear key length:

$$\Delta = \varepsilon_y D + \gamma_{xy} L \tag{4.49}$$

where $\varepsilon_y$ and $\gamma_{xy}$ are the normal and shear strains in the $x$-$y$ coordinate system shown in Fig. 4.8b, and $D$ and $L$ are the depth and length of the key joint, respectively. It should be noted that the second term of the right hand side of Eq.(4.49) is added to the slip displacement for vertical displacements measured at the edge of shear keys. Note that in Eq.(4.49), the shear strain should actually be multiplied by the length of the localized shear zone rather than the length of the key. However, since the shear cracks extend into both sides of the shear plane, the size of the shear zone is assumed to be comparable to that of the key length.
4-4-5. Formula For Shear Strength Of Shear Key Joints

As mentioned, the peak load or shear strength of shear key joints is governed by M cracks formulation. Therefore, the shear strength of key joints is determined by only RSCBM. In this case, WCM is not considered since both models are numerically independent of each other. Thus, if one would like to know the shear strength, one needs to carry out the numerical process of only RSCBM which is basically developed for the calculation of load-displacement behavior for the entire load range. Although this process is quite simple without any numerical iteration, it does not provide a closed form formula which is much simpler from the engineering point of view. In this section, a simple design formula for shear strength of key joints as a closed form is developed. This formula will be verified by the comparison with experimental data.

First, the following condition is considered in this formulation, and this condition was obtained from the calculation by RSCBM, which will be shown in the model verification study in the next chapter: The compressive stress $\sigma_c$ of compression strut becomes the compressive strength $f'_c$ when the shear stress reaches the maximum value. Then, the compressive stress of compression strut is equal to the compressive strength as

$$\sigma_c = f'_c \quad (4.50)$$

The associated compressive strain of compression strut is calculated by Eq.(4.39e) as

$$\varepsilon_{c0} = \frac{2f'_c}{E_c} \quad (4.51)$$

Then, the tensile crack strain $\varepsilon_t$ is calculated by Eq.(4.47c) and Eq.(4.48a) as

$$\varepsilon_t = 2.0 \times \varepsilon_{c0} = \frac{4f'_c}{E_c} \quad (4.52)$$

Here, it should be noted that Approach MA (tensile strain control) is adopted. This adoption will be verified in Chapter 5. Substituting Eq.(4.52) into Eqs.(4.38) and (4.41), then one can obtain the tensile stress as follows.
For plain concrete:

\[ \sigma_t = 4f'_c \]
\[ \sigma_t = \frac{5f_t^2h}{6E_cG_f} (f_t - 4f'_c) + f_t \quad \text{if} \quad f'_c \leq \frac{f_t}{4} \]  \hspace{1cm} (4.53a)
\[ \sigma_t = \frac{5}{66} \frac{f_t^2h}{E_cG_f} (f_t - 4f'_c) + \frac{3}{11} f_t \quad \text{if} \quad \frac{f_t}{4} < f'_c \leq \frac{f_t}{4} + \frac{1}{5} \frac{E_cG_f}{f_t h} \]  \hspace{1cm} (4.53b)
\[ \sigma_t = \frac{5}{66} \frac{f_t^2h}{E_cG_f} (f_t - 4f'_c) + \frac{3}{11} f_t \quad \text{if} \quad \frac{f_t}{4} + \frac{1}{5} \frac{E_cG_f}{f_t h} < f'_c \leq \frac{f_t}{4} + \frac{9}{10} \frac{E_cG_f}{f_t h} \]  \hspace{1cm} (4.53c)
\[ \sigma_t = 0 \quad \text{if} \quad \frac{f_t}{4} + \frac{9}{10} \frac{E_cG_f}{f_t h} < f'_c \]  \hspace{1cm} (4.53d)

For fiber reinforced concrete:

\[ \sigma_t = 4f'_c \quad \text{if} \quad f'_c \leq \frac{f_t}{4} \]  \hspace{1cm} (4.54a)
\[ \sigma_t = \frac{hf_t^2}{2E_cG_f} (f_t - 4f'_c) + f_t \quad \text{if} \quad \frac{f_t}{4} < f'_c \leq \frac{f_t}{4} + \frac{E_cC_t}{2 f_t h} \left( 1 - \frac{f_m}{f_t} \right) \]  \hspace{1cm} (4.54b)
\[ \sigma_t = f_m \quad \text{if} \quad \frac{f_t}{4} + \frac{E_cG_f}{2 f_t h} \left( 1 - \frac{f_m}{f_t} \right) < f'_c \]  \hspace{1cm} (4.54c)

From the equations above, it is obvious that the situations by both Eqs.(4.53a) and (4.54a) have never occur in the concrete elements. \( E_c \) is here estimated based on the following ACI 318-89 [1989] equation:

\[ E_c = 4733 \sqrt{f'_c} \quad \text{(MPa)} \]  \hspace{1cm} (4.55)

Since the direct tensile strength of concrete is difficult to measure, \( f_t \) is here estimated based on the following approximate equation [Chen 1982]:

\[ f_t = 0.332 \sqrt{f'_c} \quad \text{(MPa)} \]  \hspace{1cm} (4.56)

Substituting Eqs.(4.55) and (4.56) into Eqs. (4.53b), (4.53c), (4.54b) and (4.54c), and using Eqs.(4.36a) and (4.36c), one can obtain the closed form of shear strength as follows.
\[ \tau_{\text{max}} = \frac{f'_e - \sigma_t}{2} \sin 2 \left\{ \cos^{-1} \sqrt{\frac{\sigma_x - \sigma_t}{f'_e - \sigma_t}} \right\} \]  

(4.57)

where, for plain concrete:

\[
\begin{align*}
\sigma_t &= \frac{f'_e h}{117700G_f} \left( 0.332 - 4\sqrt{f'_e} \right) + 0.332\sqrt{f'_e} \\
&\text{if } \frac{\sqrt{f'_e}}{12} < f'_e \leq \frac{\sqrt{f'_e}}{12} + 2850 \frac{G_f}{h} \\
\sigma_t &= \frac{f'_e h}{568000G_f} \left( 0.332 - 4f'_e \right) + \frac{\sqrt{f'_e}}{11} \\
&\text{if } \frac{\sqrt{f'_e}}{12} + 2850 \frac{G_f}{h} < f'_e \leq \frac{\sqrt{f'_e}}{12} + 12830 \frac{G_f}{h} \\
\sigma_t &= 0 \\
&\text{if } \frac{\sqrt{f'_e}}{12} + 12830 \frac{G_f}{h} < f'_e \\
\end{align*}
\]  

(4.58a)

(4.58b)

(4.58c)

For fiber reinforced concrete:

\[
\begin{align*}
\sigma_t &= \frac{hf'_e}{224850 \sqrt{f'_e G_f}} \left( 0.332 \sqrt{f'_e} - 4f'_e \right) + 0.332\sqrt{f'_e} \\
&\text{if } \frac{\sqrt{f'_e}}{12} < f'_e \leq \frac{\sqrt{f'_e}}{12} + 7130 \frac{G_f}{h} \left( 1 - \frac{f_u}{0.332\sqrt{f'_e}} \right) \\
\sigma_t &= f_u \\
&\text{if } \frac{\sqrt{f'_e}}{12} + 7130 \frac{G_f}{h} \left( 1 - \frac{f_u}{0.332\sqrt{f'_e}} \right) < f'_e \\
\end{align*}
\]  

(4.59a)

(4.59b)

where

\[ f_u = 2n_f n_0 T_{uf} V_f \frac{f_t}{\phi_f} \]  

(4.59c)

where \( f'_e \) is the compressive strength of concrete, \( \sigma_t \) is the tensile stress, \( \sigma_x \) is the normal or prestressing stress, \( h \) is the band width, \( G_f \) is the fracture energy and \( f_u \) is the residual tensile strength of fiber reinforced concrete.
The equations for shear strength mentioned above are still complex, and can be simpler as follows. The situations by Eqs.(4.58a), (4.58c), and (4.59a) usually do not occur in concrete elements. This is easily obtained by substituting typical values of $f'_c$, $G_f$, and $h$. Then, the formula can be described as follows.

$$\tau_{\text{max}} = \frac{f'_c - C}{2} \sin 2 \left\{ \cos^{-1} \sqrt{\frac{\sigma_s - C}{f'_c - C}} \right\} \quad \text{(MPa)} \quad (4.60a)$$

where for plain concrete:

$$C = \frac{f'_c h}{568000 G_f} \left( \frac{1}{3} - 4 \sqrt{f'_c} \right) + \frac{\sqrt{f'_c}}{11} \quad \text{(MPa)} \quad (4.60b)$$

For fiber reinforced concrete:

$$C = 2\eta_c \eta_0 \tau_u f \frac{f'_f}{\phi_f} \quad \text{(MPa)} \quad (4.60c)$$

Eqs. (4.60) will be assessed in Chapter 5.

4-5. COMBINATION OF PROPOSED MODELLING

So far, two models have been developed: Wedge Crack Model (WCM) and Rotating Smeared Crack Band Model (RSCBM). The former can predict S crack formation, and the latter M cracks formation. Both models can be treated, separately. As mentioned, WCM could predict the early stage of shear-off failure in shear key joints, and RSCBM could predict the late stage including the peak load (shear strength). This feature is in accordance with the assumed fracture sequence described in the previous section. To develop the complete mechanical model which can predict the entire load-displacement characteristics in the shear key joints, we need to combine these two models. This is the main objective in this thesis. However, such conjunction and the transition between S crack and M cracks are very complex, and it is not therefore easy to estimate such mechanism quantitatively and objectively at the present stage. Therefore, the following simple combination sequence (see Fig.4.16) is proposed here, and this will be verified by comparison with both experimental data and FEM analysis in Chapter 5.
Fig. 4.16: Schematic Transition Between Wedge Crack Model and Rotating Smeared Crack Band Model
1. S crack propagation (associated with Curve 1 in Fig.4.16) according to the Wedge Crack Model dominates the overall fracture mechanism of key joints until the shear slip displacement according to M cracks propagation becomes apparent. The size of S crack becomes certain critical value $\ell_{sc}$ at this stage. Up to this stage, M cracks propagation is ignored.

2. After this stage, M cracks propagation (associated with Curve 2 in Fig.4.16) according to the Rotating Sineared Crack Band Model dominates the overall fracture mechanism of key joints, and S crack propagation is then ignored. However, the previous shear slip displacement corresponding to S crack propagation is added to that corresponding to M cracks.

In the simple sequence mentioned above, it is necessary to determine the critical size of S crack $\ell_{sc}$ to complete the entire shear stress-slip curve. Such quantity may be specified by the two conditions: (1) $\ell_{sc}$ is identical to the maximum S crack size observed in the experiment; and (2) $\ell_{sc}$ is associated with the point in which both Curve 1 and Curve 2 can be conjugated with each other in the relatively smooth manner or the tangents of both curves are relatively close to each other. These conditions will be examined as the model verification study in Chapter 5.

Here, the first condition is an objective approach based on the actual observation, and therefore should be preferred. However, if one has no such observation which is usual in design, one can refer to the second condition which is however not the objective approach, but subjective one. To develop the objective approach in the second condition, further analytical and experimental works on the interaction between S crack and M cracks are necessary, and such research will be placed as the future work.

4-6. SUMMARY AND CONCLUSION

In this chapter, the mechanical model for shear-off failure in plane and fiber reinforced concrete shear key joints using a fracture mechanics approach was developed based on the physical description of the shear failure of key joints. The proposed formulation identified two main fracture mechanisms for shear-off failure of key joints: single curvilinear crack and multiple diagonal cracks.

The mechanical model developed here consists of both a single discrete crack model under wedging force (Wedge Crack Model) formulated by LEFM results and the rotating
smeared crack band model under remote shear force (Rotating Smeared Crack band Model) formulated by the truss model theory. Then, the two models were combined to predict the entire shear stress-slip displacement behavior of shear-off failure.

The verification of the proposed model will be carried out by the comparison with both the test results reported in the literature, and also with nonlinear FEM analysis results in the next chapter.
CHAPTER 5

MODEL VERIFICATION STUDY

5-1. INTRODUCTION

In this chapter, the verification of proposed mechanical model is carried out through the comparison of the prediction with both experimental measurements and Finite Element Method (FEM) analysis results. In the comparison with experiments, two kinds of comparison are achieved here: entire load-displacement relations and the maximum shear stress or the shear strength. These comparison can show the accuracy of the proposed mechanical model in the prediction of macroscopic behavior. The second comparison with nonlinear FEM analysis results is also carried out in terms of the load-displacement relations to supplement the verification study of the proposed mechanical model by the experimental results in push-off sheer key tests which number is relatively limited. The reliability of the FEM analysis is examined by the comparison with the experimental observation of the cracking status.

5-2. COMPARISON WITH EXPERIMENTS

The proposed mechanical model consisting of both Wedge Crack Model (WCM) and Rotating Smeared Crack Band Model (RSCBM) is verified in terms of both the load-displacement curves in the entire load range and the shear strength. To achieve this comparison, the experimental works of both plain concrete shear keys [Bakhoum et al. 1989 and Bakhoum 1991] and steel fiber reinforced concrete shear keys [Beattie 1989 and Beattie et al. 1989] are adopted. This is because these test data could be typical of concrete shear keys. In addition, these tests were carried out by the same group of researchers. Therefore, the experimental program is basically identical to each other except the material properties, and the typical difference between plain and fiber reinforced concrete keys can be therefore examined.

5-2-1. Interpretation Of Shear Key Test Results

Before the comparison between the prediction by the proposed mechanical model and test data of Bakhoum et al. and Beattie et al., the detailed interpretation of the obtained data in their experiments is examined here as follows.
As introduced in Chapter 2, the geometric and loading configurations for dry keyed joints in their experiment are shown in Fig.2.7, and these configurations are identical in the case of both plain concrete specimens and steel fiber reinforced concrete specimens. Thus, the following description is always applied to both cases. The vertical loading was applied at the upper portion of the specimen while the lateral prestressing loading was kept constant. Here, the specimen can be assumed to be in a state of plane stress. This is justified because the thickness of the specimen (76.2 mm) is relatively small with respect to its height (533.4 mm) and its width (254 mm). The length L and depth D of the shear key are 31.75 mm and 98.4 mm, respectively.

To compare the proposed mechanical model with the test results, the shear stress-slip relations measured in the test are appropriately modified because the shear stress reported in the experimental study was defined as the average shear stress along the entire projected area (depth of 6 inches (152.4 mm) shown in Fig.2.7), while of interest to us in the present study is the shear stress along the root of the key joints (depth of 3.875 inches (98.4 mm) shown in Fig. 2.7). The shear stress-slip curves reported in their tests are shown in Figures 2.9, 2.11 and 2.13. These shear stress-slip curves consist of mainly two fracture mechanisms: frictional fracture along the straight surfaces of the joints and shear-off fracture at the roots of keys. Here, the shear-off fracture mechanism can be simulated by proposed mechanical model, and is therefore interesting point. Then, if we ignore the frictional fracture in their data and make the shear-off stress be the total load divided by the area of key root (between A point and B point in Fig.2.7), we can obtain the upper limit of shear stress-slip curve. On the other hand, if we consider the frictional fracture on the whole area contacted by two specimens except the key root in their data (between C point and A point, and also between B point and D point along the shear failure line in Fig.2.7) and make the shear-off stress be the total load minus the frictional load, all divided by the area of key root (between A point and B point in Fig.2.7), we can obtain the lower limit of shear stress-slip curve.

The frictional fracture mentioned above can be quantified by the test of flat dry joints which geometric and loading configuration are shown in Fig.2.6. The obtained shear stress-slip curves are also shown in Fig.2.8. Here, it should be noted that the initial slippage occurred at the vertical displacement of about 0.02 mm to 0.03 mm shown in Fig.2.8 could be identical to the linear limit of the shear stress-slip curves for dry keyed joints shown in Fig.2.9. From these curves, it is obvious that the quasi-plastic behavior shown in Fig.2.8 indicates that the frictional resistance can exist in shear stress-slip curves for dry keyed joints during the almost whole fracture procedures. From these two
experiments of dry keyed joints and flat joints, both upper limit and lower limit of shear-off stress-slip relations can be constructed as follows.

The upper limit shear-off stress \( \tau_{\text{upp}} \) is calculated as

\[
\tau_{\text{upp}} = \frac{6''}{3\frac{7}{8}} \tau_{\text{orig}} = 1.548 \tau_{\text{ori}}
\]  

(5.1)

where \( \tau_{\text{orig}} \) is the original shear stress obtained by translating the value of normalized shear stress to the shear stress in terms of MPa, and 6" is the length of the projected area.

The frictional shear stress \( \tau_{\text{fric}} \) in the flat joints was approximately estimated from Fig.2.8 as

\[
\begin{align*}
\tau_{\text{fric}} &= 0.57 \text{ MPa} & \text{for } \sigma_p = 0.69 \text{ MPa} \\
\tau_{\text{fric}} &= 1.47 \text{ MPa} & \text{for } \sigma_p = 2.07 \text{ MPa} \\
\tau_{\text{fric}} &= 1.72 \text{ MPa} & \text{for } \sigma_p = 3.45 \text{ MPa}
\end{align*}
\]

(5.2a)  (5.2b)  (5.2c)

Then, the total frictional load \( P_{\text{fric}} \) along the flat surfaces of the key joints except for the key root is

\[
P_{\text{fric}} = \tau_{\text{fric}} \left(10\text{''} - 3\text{''} \frac{7}{8}\right) \times 3\text{''} = \tau_{\text{fric}} (6\text{''} \times 3\text{''})
\]

\[
= \tau_{\text{fric}} (152.4 \text{ mm} \times 76.2 \text{ mm})
\]

\[
= 6.65 \text{ KN} \quad \text{for } \sigma_p = 0.69 \text{ MPa}
\]

\[
= 17.05 \text{ KN} \quad \text{for } \sigma_p = 2.07 \text{ MPa}
\]

\[
= 19.94 \text{ KN} \quad \text{for } \sigma_p = 3.45 \text{ MPa}
\]

(5.3)

where 10" (254 mm) is the length of the contact path between the two specimens or the distance between C point and D point in Fig.2.7.

Finally, the lower limit shear-off stress \( \tau_{\text{low}} \) is calculated as

\[
\tau_{\text{low}} = \frac{\tau_{\text{orig}} (6\text{''} \times 3\text{''} = 152.4 \text{ mm} \times 76.2 \text{ mm}) - P_{\text{fric}}}{\frac{3\text{''} \times 3\text{''} = 98.425 \text{ mm} \times 76.2 \text{ mm}}{3\frac{7}{8}}} - P_{\text{fric}}
\]

(5.4)
Thus, Eqs.(5.1) and (5.4) for both the upper limit and lower limit of shear-off stress in whole test data respectively will be used in this model verification study. Here, the quantity of the friction stress $P_{\text{frie}}$ is also used for the test data of steel fiber reinforced concrete key joints by Beattie [1989] and Beattie et al. [1989], and considered in the same way.

5-2-2. Prediction Of Entire Load-Displacement Curves

Plain concrete specimens
The mechanical model proposed in Chapter 4 is applied to the case of push-off shear key tests on plain concrete dry key joints reported by Bakhoun et al. [1989] and Bakhoun [1991]. As introduced previously, the average concrete compressive strength in plain concrete specimens was 49 MPa. The tests were carried out for 12 dry key specimens using 3 normal prestressing stresses: $\sigma_x=0.69$ MPa, 2.07 MPa and 3.45 MPa.

In the proposed modelling, the uniaxial tension stress-strain relationship parameters are taken as follows: Young's modulus $E_c=33$ GPa; tensile strength $f_t=2.3$ MPa; fracture energy $G_f=0.1$ N/mm; and tension softening constitutive model as shown in Fig.4.9a. The tensile Young's modulus is assumed to be the initial tangent stiffness $E_c$ in compression. $E_c$ was estimated based on the ACI 318-89 [1989] equation (see Eq.(4.55)). The tensile strength $f_t$ was estimated based on Eq. (4.56) [Chen 1982]. The fracture energy $G_f$ was estimated as 0.1 N/mm, a value often used by many researchers [e.g., Rots and Blaauwendaal 1989]. In addition, Balakrishnan and Murray [1988] suggested values between 0.05 N/mm and 0.25 N/mm, with 0.1 N/mm being the most effective. The crack band width $h$ was estimated from the cracking sequence shown in Fig.2.10 as 10 mm. Using these values, one can calculate the fracture toughness needed in the Wedge Crack Model as follows:

$$K_{IC} = \sqrt{E_c G_f} = \sqrt{33000 \times 0.1} = 57 \text{ N/mm}^2$$

(5.5)

Here, it should be mentioned that Eq.(5.5) is derived for the plane stress condition. As mentioned in Chapter 3, no essential difference exists between plane stress and plane strain conditions for concrete. This is because we have no well defined crack tip for concrete (like metals), but rather a fracture zone, within which the cracking increases and the stresses decreases as the deformation increases. The microcracking of concrete is not
accompanied by any substantial contraction corresponding to that which occurs when metals yield. In addition, the geometry of test specimen which is used in this simulation can be assumed in the plane stress condition. Thus, Eq.(5.5) is adopted in this simulation.

The uniaxial compression stress-strain relationship parameters are taken as follows: Young's modulus $E_c=33$ GPa; compressive strength $f_c=49$ MPa; and constitutive models as shown in Fig.4.10a and 4.10b.

First, the validity of LEFM is estimated here. From the previous discussion in Chapter 4, it was concluded that the LEFM approach could be an appropriate approximation for nonlinear fracture mechanics problems when the normalized crack size $a_0/E_c$ is larger than approximately 0.01 (see Eq.(4.2)). Here, $a_0$ is the physical crack size and $E_c$ is the material characteristic length, defined by $E_c = E_c f_t / f_t^2$, where $E_c$ is the Young's modulus and $f_t$ is the tensile strength. Using Eqs.(4.55)-(4.56) for $E_c$ and $f_t$, the valid condition for the minimum crack size could be expressed as follows ('N' and 'mm' units are used):

$$a_0 \geq 430 \frac{G_f}{\sqrt{f_c}} \left( = 0.01 E_c = 0.01 \frac{4733\sqrt{f_c G_f}}{0.332 \sqrt{f_c}} \right)$$

Substituting the values of 0.1 N/mm and 49 MPa for $G_f$ and $f_c$, respectively, one can obtain the minimum crack size of 6 mm for LEFM to be valid. If the crack size is more than roughly 6 mm, LEFM in the Wedge Crack Model is acceptable. However, if the crack size is less than 6 mm, some error will be introduced in the prediction. As mentioned, the above condition of validity for LEFM may not be always correct. It must be emphasized that the limiting normalized crack length depends on the structural configuration, the size of the structure, and the stress-separation curve; the value of 0.01 is just an average one.

Next, we compare the shear stress-slip displacement curves predicted by the Wedge Crack Model (Approaches SA and SB) and the Rotating Smear Crack Band Model (Approaches MA and MB) with the experimental data for a prestress of 2.07 MPa (K300-DR-3), to get a feeling of the overall behavior. Here, Approach SA is based on the mixed mode S crack propagation by means of 'maximum tensile stress factor', and SB on the mode I propagation with assumed crack path, while MA is based on the tensile strain control approach and MB on the compressive strain control one (see Chapter 4). The comparison is shown in Fig.5.1. The calculation by WCM was achieved up to the S crack size of 40 mm which was estimated as an observed maximum crack length (see Fig.2.10). In the figure, the upper and lower limits of the experimental results are obtained from the
Fig. 5.1: Preliminary Study on Shear Stress versus Shear Slip Displacement Curves for K300-DR3 Test

Fig. 5.2: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for K100-DR2 Test
original test curves. In addition, for example, SA and MA-C1 in Fig.5.1 stand for the predictions by Approach SA in WCM and Approach MA with the compressive constitutive model C1 in RSCBM, respectively. Predictions of the other approaches are obtained in the same manner.

It can be observed that the predictions by WCM agree well with the early stage of the behavior of the experimental results. This is because the WCM simulates $S$ crack propagation. Specifically, Approach SB predicts the upper limit of the experiments, while Approach SA predicts the lower limit. It is thus seen that the mixed mode result underestimates the load for a given displacement. A plausible explanation is that the crack follows a curved path and the propagation is accompanied by both shear and opening displacement. With shear displacement, the energy dissipation and hence the value of $K_C$ can be significantly increased. Thus, assuming a constant $K$ equal to the value at pure mode I may be the reason for the underestimation. At the present, this discrepancy can not be completely resolved since there are not enough data on material behavior (such as the effect of mode mixity on toughness and softening behavior) to provide an accurate quantitative solution to the problem. Before a better method for the prediction of mixed mode crack propagation can be developed, the present formulation is quite useful as an engineering approximation.

On the other hand, the RSCBM gives a reasonable prediction of the entire behavior except for the early stage. In particular, Approach MA-C1 predicts most accurately the maximum stress and the overall slope of the curve. Here, it should be noted that the addition of the softened compressive strut concept (MA-C2) underestimates the load for a given displacement. This discrepancy may indicate that the softened compressive strut concept is not applicable to the plain or fiber reinforced concrete shear keys. In addition, it was observed that the maximum shear load point was attained when the compressive stress of compressive strut was close to the compressive strength in this calculation. It can be hence concluded from this preliminarily analysis such that:

1. Approach MA-C1 is the most accurate approach in RSCBM. Specifically, the shear-off failure of shear key joints is controlled by the tensile strain in M cracks.

2. Approach SB in WCM can be conjugated with Approach MA-C1 in terms of both stresses and displacements.
3. Approach SB in WCM can satisfy the condition of the critical size \( \ell_{sc} \) of S crack as:

(1) \( \ell_{sc} \) is identical to the maximum S crack size observed in the experiment mentioned above and

(2) \( \ell_{c} \) is associated with the point in which both curves by WCM and RSCBM can be conjugated with each other in the relatively smooth manner (see Chapter 4).

4. The maximum load point was attained when the compressive stress of compressive strut was close to the compressive strength.

Therefore, these two approaches 'Approach SB in WCM' and 'Approach MA-C1 in RSCBM' are adopted here and the overall load-displacement predictions according to the combination sequence developed in Chapter 4 (see Fig. 4.15) are constructed as follows.

### Table 5.1: Summary of Test Results and Comparison of Experiments With Analysis

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>( \sigma_p ) (MPa)</th>
<th>( f_c' ) (MPa)</th>
<th>( \tau_{\text{max-up}} ) (MPa)</th>
<th>( \tau_{\text{max-lo}} ) (MPa)</th>
<th>( \tau_{\text{max-cal}} ) (MPa)</th>
<th>( (5)/(7) )</th>
<th>( (6)/(7) )</th>
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<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
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<tr>
<td>K100DR</td>
<td>1</td>
<td>0.69</td>
<td>47.2</td>
<td>8.27</td>
<td>7.38</td>
<td>7.23</td>
<td>1.144</td>
<td>1.021</td>
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<td>K100DR</td>
<td>2</td>
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<td>7.29</td>
<td>1.198</td>
<td>1.077</td>
</tr>
<tr>
<td>K100DR</td>
<td>3</td>
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<td>46.9</td>
<td>8.14</td>
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<td>7.25</td>
<td>1.123</td>
<td>1.001</td>
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<td>0.69</td>
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<td>K300DR</td>
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<td>13.60</td>
<td>1.114</td>
<td>0.919</td>
</tr>
</tbody>
</table>

\( \tau_{\text{max-up}} \): maximum shear stress corresponding to upper limit of test data

\( \tau_{\text{max-lo}} \): maximum shear stress corresponding to lower limit of test data

\( \tau_{\text{max-cal}} \): maximum shear stress corresponding to proposed model

A summary of the testing program considered here is provided in Table 5.1. As evaluation of test results for epoxied joints and cyclic loading is outside the scope of this
Fig. 5.3: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for K300-DR3 Test

Fig. 5.4: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for K500-DR3 Test
study and the associated values are omitted from the table. Note that the table shows the modified maximum shear stress of the upper and lower limits obtained in the experiments.

Figures 5.2, 5.3 and 5.4 show comparisons between experimental shear stress-slip displacement relations and the prediction by the proposed mechanical model for different values of prestressing (the test series of K100-DR2, K300-DR3 and K500-DR3). The predictions by the proposed mechanical model were obtained according to the procedures described above. It is realized that from the engineering point of view the predictions by the proposed mechanical model are in good agreement with the experimental results for the entire range of load. Specifically, we may conclude the following:

1. The proposed mechanical model gives a good prediction of the stress-slip curve in the pre-peak load region, especially for the upper limit values.

2. The mechanical model captures the unstable stress drop after the maximum stress is reached (negative slope) and predicts the post-peak stable deformation behavior in a reasonable manner.

3. The mechanical model predicts the maximum shear-off stress with considerable accuracy. The quantitative comparison will be introduced in the next section.

4. According to the proposed modelling, the compressive constitutive law plays an important role in determining the shear strength, and so do both the tensile and the compressive constitutive laws in determining the entire shear stress-slip displacement curve.

5. The predictions by the mechanical model show a post-peak plateau while the experimental results show a continued drop. This is due to the fact that a plateau is assumed in the compressive constitutive model.

**Fiber reinforced concrete specimens**

The proposed mechanical model is also applied next to the case of push-off shear key tests on steel fiber reinforced concrete dry key joints reported by Beattie [1989] and Beattie et al. [1989]. As introduced previously, the average concrete compressive strength was 52 MPa. The tests were carried out for 24 dry key specimens and 9 epoxied ones using three levels of normal prestressing stresses: $\sigma_x=0.69$ MPa, 2.07 MPa and 3.45 MPa.
Two types of steel fibers were used in the experiments: straight (Flexten) and deformed (Dramix ZL 30/0.50 hooked-end fiber). The Flexten fibers were 0.28x0.56x25.4 mm carbon steel rectangular fibers ($\ell_f$=25.4 mm, $\phi_f$=0.447 mm) made from steel sheet with a tensile strength of approximately 345 MPa. The Dramix ZL 30/0.50 hooked-end fibers were 30 mm long with a 0.5 mm diameter ($\ell_f$=30.0 mm, $\phi_f$=0.5 mm) and were manufactured from a low carbon, cold drawn steel wire, with a minimum tensile yield strength of 1034 MPa.

In the prediction by the mechanical model, the uniaxial tension stress-strain relationship parameters are taken as follows: fracture energy of matrix $G_c$=0.1 N/mm; and tension softening relation as shown in Fig. 4.11a. The tensile Young's modulus is assumed to be the initial tangent stiffness $E_c$ in compression. $E_c$ is estimated by Eq.(5.5). The tensile strength $f_t$ is estimated by Eq.(5.6). The crack band width $h$ is approximately estimated from the cracking pattern shown in Fig.2.12 as 10 mm. Using these values, one can calculate the fracture toughness used in the Wedge Crack Model similarly to the case of plain concrete by Eq. (5.5).

Regarding the fiber bond strength $\tau_{uf}$, no information is available in the test of Beattie [1989] and Beattie et al. [1989]. Therefore, in this study, the bond versus slip curves for both straight and deformed fibers with different embedded lengths given by Lim et al. [1987a] are adopted (ass Fig. 4.12). From these curves, it is assumed that $\tau_{uf}$=3.0 MPa for straight fibers and $\tau_{uf}$=6.5 MPa for deformed fibers. The length efficiency factor is calculated from Eq.(4.42) as $\eta_\ell$=0.5 for both types of fibers. The fiber orientation is here simply assumed as 0.405 which is described by Eq. (4.45). Strictly speaking, this quantity can be obtained in the case of randomly oriented fibers in a three-dimensional manner as mentioned. On the other hand, the fiber distribution in the test of Beattie [1989] and Beattie et al. [1989] may be somewhere between a two-dimensional and three-dimensional distribution due to the way of specimen preparation. Thus, a more accurate derivation will give us a higher value for the orientation factor. In addition, the effect of fiber inclination is not considered in this modelling and that is still under present investigation as mentioned previously. Furthermore, the fiber interfacial strengths are not measured by Beattie [1989] and Beattie et al. [1989] and therefore they are floating parameters. Because of the undeveloped fiber behavior and the floating parameters in their test, the quantity of the fiber orientation factor is simply determined as 0.405 to study the structural behavior.

As mentioned, the validity of LEFM for fiber reinforced key joints is in this modelling considered only by the matrix parameters. Specifically, the contribution of the fiber reinforcement to the fracture process is assumed to produce a negative stress intensity at the
crack tip and to reduce the crack opening displacement. Since the fibers act only as an external loading, the toughness of the composite can be assumed to be that of the matrix, and the LEFM validity assumptions are the same as those for the case of plain concrete. Then, the valid condition for LEFM in this case can be described by Eq. (5.6) and is almost identical to that of plain concrete shear keys since the quantities of $G_f$ and $f'_c$ are almost identical.

**Table 5.2: Summary of Test Results and Comparison of Experiments With Analysis**

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\sigma_p$ (MPa)</th>
<th>Fiber Type</th>
<th>$V_f$ (%)</th>
<th>$f'_c$ (MPa)</th>
<th>$\tau_{\text{max-up}}$ (MPa)</th>
<th>$\tau_{\text{max-lo}}$ (MPa)</th>
<th>$\tau_{\text{max-cal}}$ (MPa)</th>
<th>(7)/(9)</th>
<th>(7)/(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>B</td>
<td>3.45</td>
<td>FT</td>
<td>1</td>
<td>45.2</td>
<td>14.55</td>
<td>11.90</td>
<td>13.12</td>
<td>1.109</td>
<td>0.907</td>
</tr>
<tr>
<td>KF1</td>
<td>C</td>
<td>3.45</td>
<td>FT</td>
<td>2</td>
<td>45.2</td>
<td>15.59</td>
<td>12.93</td>
<td>14.18</td>
<td>1.099</td>
<td>0.912</td>
</tr>
<tr>
<td>KF1</td>
<td>B</td>
<td>3.45</td>
<td>FT</td>
<td>1</td>
<td>49.4</td>
<td>15.71</td>
<td>13.06</td>
<td>13.47</td>
<td>1.166</td>
<td>0.969</td>
</tr>
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<td>C</td>
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<td>FT</td>
<td>2</td>
<td>49.4</td>
<td>16.76</td>
<td>14.11</td>
<td>14.55</td>
<td>1.152</td>
<td>0.969</td>
</tr>
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<td>KF3</td>
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<td>0.69</td>
<td>FT</td>
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<td>46.9</td>
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<td>8.48</td>
<td>7.90</td>
<td>1.186</td>
<td>1.073</td>
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<td>FT</td>
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<td>9.89</td>
<td>9.68</td>
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<td>3.45</td>
<td>FT</td>
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<td>49.6</td>
<td>16.38</td>
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<td>13.49</td>
<td>1.214</td>
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<tr>
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<td>A</td>
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<td>FT</td>
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<td>52.7</td>
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<td>1.161</td>
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<tr>
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<td>FT</td>
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<td>13.32</td>
<td>13.70</td>
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</tr>
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<td>KF8</td>
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<td>3.45</td>
<td>FT</td>
<td>2</td>
<td>58.7</td>
<td>19.27</td>
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<td>1.195</td>
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</tr>
<tr>
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<td>DM</td>
<td>1</td>
<td>56.3</td>
<td>11.10</td>
<td>10.22</td>
<td>10.22</td>
<td>0.989</td>
<td>0.911</td>
</tr>
<tr>
<td>KF10</td>
<td>C</td>
<td>0.69</td>
<td>DM</td>
<td>2</td>
<td>56.3</td>
<td>15.06</td>
<td>14.18</td>
<td>14.61</td>
<td>1.031</td>
<td>0.970</td>
</tr>
<tr>
<td>KF11</td>
<td>B</td>
<td>3.45</td>
<td>DM</td>
<td>1</td>
<td>53.6</td>
<td>19.85</td>
<td>17.19</td>
<td>15.15</td>
<td>1.310</td>
<td>1.135</td>
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<tr>
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<td>C</td>
<td>3.45</td>
<td>DM</td>
<td>2</td>
<td>53.6</td>
<td>18.95</td>
<td>16.29</td>
<td>17.37</td>
<td>1.091</td>
<td>0.937</td>
</tr>
</tbody>
</table>

- $\tau_{\text{max-up}}$: maximum shear stress corresponding to upper limit of test data
- $\tau_{\text{max-lo}}$: maximum shear stress corresponding to lower limit of test data
- $\tau_{\text{max-cal}}$: maximum corresponding to proposed model
- FT: Flexten fibers
- DM: Dramix fibers

Next, we construct the overall load-displacement prediction for the behavior of fiber reinforced shear keys by Approach SB in WCM and MA-C1 in RSCBM according to the procedure described in the case of plain concrete shear key joints. A summary of the testing program considered here is provided in Table 5.2. As mentioned, evaluation of test results for epoxied joints and cyclic loading is outside the scope of this study and the
associated values are omitted from the table. The Flexten fibers are denoted by FT, and the Dramix fibers are denoted by DM. Note that the table shows the modified maximum shear stress of the upper and lower limits obtained in the experiments.

Figures 5.5-5.9 show comparisons between experimental shear stress-slip displacement relations and predictions by the mechanical model for the test series of KF7A, KF7B, KF7C, KF4C and KF10B, respectively. It is seen that the predictions by the proposed mechanical model are in relatively good agreement with the experimental results for different prestressing levels and fiber reinforcement in the entire range of load. Here, it should be noted that the test result for specimen KF4C (Vf=2 %) shows a brittle behavior after the peak load, and deviates from the prediction. In the literature [Beattie 1989, Beattie et al. 1989], it was reported that KF4C showed a lower post-peak strength (stress plateau in the curve) than KF4B (Vf=1 %). It was explained that this brittle behavior might have been the result of both deficient compaction of the concrete and many minute stress concentrations due to the higher volume fraction of fibers. This condition cannot be considered in the present mechanical model, and hence the rather large discrepancy between experiment and prediction for this particular specimen. The following are then concluded:

1. The proposed mechanical model gives a good prediction of the stress-slip curve in the pre-peak load region, especially for the upper limit values.

2. The mechanical model captures the unstable stress drop after the maximum stress is reached and predicts the post-peak stable deformation behavior in a reasonable manner.

3. The mechanical model predicts the maximum shear-off stress with considerable accuracy. The quantitative comparison will be introduced in the next section.

4. According to the proposed modelling, the compressive constitutive law plays an important role in determining the shear strength, and so do both the tensile and the compressive constitutive laws in determining the entire shear stress-slip displacement curve.

5. The predictions by the mechanical model show a post-peak plateau while the experimental results show a continued drop. This is due to the fact that a plateau is assumed in the tensile constitutive model and so the bridging stress does not drop with crack opening. This argument also applies to the compressive constitutive model.
Fig. 5.5: Predicted and Experimental Shear Stress Versus Shear Slip Displacement Curves for KF7A Test

Fig. 5.6: Predicted and Experimental Shear Stress Versus Shear Slip Displacement Curves for KF7B Test
Fig. 5.7: Predicted and Experimental Shear Stress Versus Shear Slip Displacement Curves for KF7C Test

Fig. 5.8: Predicted and Experimental Shear Stress Versus Shear Slip Displacement Curves for KF4C Test
Fig. 5.9: Predicted and Experimental Shear Stress Versus Shear Slip Displacement Curves for KF10B Test

Fig. 5.10: Comparison of Maximum Shear Stress Between Test Data and Prediction in the Case of Plain Concrete Specimen
Fig. 5.11a: Comparison of Maximum Shear Stress Between Test Data and Calculation by Proposed Formula in the Case of Plain Concrete Specimen

Fig. 5.11b: Comparison of Maximum Shear Stress Between Test Data and Calculation by Proposed Formula by Bakhoun et al. [1989]
Fig. 5.12: Comparison of Maximum Shear Stress Between Test Data and Prediction in the Case of Fiber Reinforced Concrete Specimen

Fig. 5.13: Comparison of Maximum Shear Stress Between Test Data and Calculation by Proposed Formula in the Case of Fiber Reinforced Concrete Specimen
5-2-3. Prediction Of Shear Strength

One of the most important information from fracture tests is the maximum load measured. Therefore, the comparison of the maximum shear stress or shear strength between test data and the predictions by the proposed mechanical model is discussed here.

Plain concrete specimens
A comparison of the maximum shear stress of test data and the prediction for 12 test series is schematically shown in Fig.5.10. The agreement between the measured and calculated shear strength is indeed good. The mean value of the ratio of the upper limit shear strength of test data to calculated shear strength is 1.1039 and the standard deviation is 0.082485. The mean value of the ratio of the lower limit shear strength of test data to calculated shear strength is 0.94281 and the standard deviation is 0.096564. In the figure, ±10 percent error range is also introduced. Since the actual test data may exist between the upper and lower limit, the actual test data may be inside of the ±10 percent error range. Thus, the present mechanical model predicts the maximum shear-off stress with considerable accuracy.

In Chapter 4, a simple design formula for shear strength of key joints is developed such as Eqs.(4.60). The same comparison above is carried out here for this simple closed solution, and the results are shown in Fig.5.11a. The agreement between the measured and calculated shear strength is good as well as RCSBM. The mean value of the ratio of the upper limit shear strength of test data to calculated shear strength is 1.1315 and the standard deviation is 0.075375. The mean value of the ratio of the lower limit shear strength of test data to calculated shear strength is 0.95171 and the standard deviation is 0.10295. In the figure, ±10 percent error range is also introduced. Since the actual test data may exist between the upper and lower limit, the actual test data may be inside of the ±10 percent error range. Thus, the proposed simple design formula of Eqs (4.60) can predict the maximum shear-off stress in plain concrete shear key joints with considerable accuracy.

On the other hand, Bakhoun et al. [1989] and Bakhoun [1991] proposed the following shear strength formula in dry shear key joints obtained from regression analysis of the test data (in units of psi).

\[
\tau = 7.80\sqrt{f_c} + 1.36\sigma_c
\] (5.7)
where \( \tau \) is the shear strength in the projected area consisting of both keyed portion and flat portion (see Fig.2.7), and \( \sigma_e \) is the prestress. The comparison between the calculated shear strength by Eq.(5.7) and the test data is shown in Fig.5.11b. Here, it should be noted that the test data in the figure is not modified as was previously. This is because Eq.(5.7) is applied to the projected area in the key joints, which is different from the prediction by the proposed mechanical model. The mean value of the ratio of the test data to calculated shear strength is 1.0142 and the standard deviation is 0.055681. It is seen that most of data are inside of the \( \pm 10 \) percent error range. This results is reasonable since the formula of Eq.(5.7) is obtained from the regression of test data which are used in this comparison. The importance of this comparison is that the accuracy of the simple proposed design formula of Eq.(4.60) seems to be comparable with that of the regression analysis despite the simple formulation of Eq.(4.60).

**Fiber reinforced concrete specimens**

A comparison between the maximum shear stress of the test data and the prediction for 15 test series of fiber reinforced concrete shear key is given in Fig.5.12. The agreement between the measured and calculated shear strengths is good. The mean value of the ratio of the upper limit shear strength of the test data to the calculated shear strength is 1.15 and the standard deviation is 0.079. The mean value of the ratio of the lower limit shear strength of the test data to the calculated shear strength is 0.99 and the standard deviation is 0.065. In the figure, \( \pm 10 \) percent error range is also introduced. Since the actual test data may exist between the upper and lower limit, the actual test data may be inside of the \( \pm 10 \) percent error range. Thus, the present mechanical model predicts the maximum shear-off stress with considerable accuracy. In fact, good agreement can be found for the lower limit of the test results, while slight underestimation of the upper limit data is obtained.

Similar to the plain concrete key joints, a simple design formula for shear strength of key joints expressed by Eqs.(4.60) is examined here. The results are shown in Fig.5.13. The agreement between the measured and calculated shear strength is good as well as RCSBM. The mean value of the ratio of the upper limit shear strength of test data to calculated shear strength is 1.1524 and the standard deviation is 0.061487. The mean value of the ratio of the lower limit shear strength of test data to calculated shear strength is 0.99844 and the standard deviation is 0.057297. In the figure, \( \pm 10 \) percent error range is also introduced. Since the actual test data may exist between the upper and lower limit, the actual test data may be inside of the \( \pm 10 \) percent error range. As well as RCSBM, good agreement was found for the lower limit of the test results, while slight underestimation of the upper limit data was obtained. Thus, the simple formulae of Eqs (4.60) can predict the
maximum shear-off stress in fiber reinforced concrete shear key joints with considerable accuracy.

5-2-4. Sensitivity Of RSCBM

In this section, the sensitivity of several parameters which could be the necessary in analysis and design by Rotating Smeared Crack Band Model (RSCBM) is examined with the same geometric and loading configuration used above under constant normal stress of 2.07 MPa: band width \( h \); tensile strength \( f_t \); fracture energy \( G_f \); Young's modulus \( E_c \); Compressive strength \( f_c \). This is because many parameters are used in RSCBM, and it is important to find which parameter is critical for the fracture behavior.

Sensitivity for band width \( h \)
Band width is one of the most difficult parameter to estimate in RSCBM. As mentioned, this parameter has no physical meaning except the case which the width of fracture zone is close to the width between each M crack. Therefore, it is important to know the sensitivity for this parameter.

In the previous comparison between the present mechanical model and test data, quantity of 10 mm was used as 'h'. Here, the band width of 5.0, 10.0, 20.0 mm are used in the calculation of the shear stress-slip relation, with tensile strength \( f_t \) of 2.3 MPa, fracture energy \( G_f \) of 0.1 N/mm, Young’s modulus \( E_c \) of 33 GPa, compressive strength \( f_c \) of 49 MPa.

The results are shown in Fig.5.14. Here, it can be observed that the overall behavior or shape of shear stress-slip curves including the shear slip displacement at the maximum shear stress is almost identical. However, it is also observed that the larger band width shows the smaller the shear stress. This could be explained such that smaller band width indicated larger tensile strain capacity, and resulted in large shear resistance.

The difference of maximum shear-off stress for 5 mm difference of band width is estimated to be about 5 %.

Sensitivity for tensile strength \( f_t \)
Tensile strength is also difficult parameter to estimate in the experiment, especially direct tensile strength. Therefore, it is important to know the sensitivity of RSCBM for the tensile strength.

In the previous comparison, quantity of 2.0-3.0 MPa was used as the tensile strength. The shear stress-slip relations for tensile strength \( f_t = 2.0, 3.0, 4.0 \) MPa are calculated,
with band width h of 10.0 mm, fracture energy $G_f$ of 0.1 N/mm, Young’s modulus $E_c$ of 33 GPa, compressive strength $f'_c$ of 49 MPa.

The results are shown in Fig.5.15. Here, it can be observed that the overall behavior or shape of shear stress-slip curves including the shear slip displacement at the maximum shear stress is almost identical. However, it is also observed that the larger tensile strength indicates the larger shear stress at the early loading stage, but the smaller tensile strength then indicates the larger shear stress at the late loading stage.

The difference of maximum shear stress for 1.0 MPa difference of tensile strength is estimated to be about 6 %.

**Sensitivity for fracture energy $G_f$**

In the previous comparison, quantity of 0.1 N/mm was used as the fracture energy. The shear stress-slip relations for fracture energy $G_f$ of 0.05, 0.1, 0.15 N/mm are calculated here, with tensile strength $f_t$ of 2.3 MPa, band width h of 10.0 mm, Young’s modulus $E_c$ of 33 GPa, compressive strength $f'_c$ of 49 MPa. The results are shown in Fig.5.16. Here, it can be observed that the overall behavior or shape of shear stress-slip curves including the shear slip displacement at the maximum shear stress is almost identical. However, it can be observed that the larger fracture energy shows the larger shear stress.

The difference of maximum shear stress for 0.05 N/mm difference of fracture energy is estimated to be about 10 %.

**Sensitivity for Young’s modulus $E_c$**

In the previous comparison, quantity of 30-35 GPa was used as the Young’s modulus. The shear stress-slip relations for Young’s modulus $E_c$ of 30, 40, 50 GPa are calculated, with tensile strength $f_t$ of 2.3 MPa, band width h of 10.0 mm, fracture energy $G_f$ of 0.1 N/mm, compressive strength $f'_c$ of 49 MPa. The results are shown in Fig.5.17.

Here, it can be observed that the overall behavior or shape of shear stress-slip curves including the shear slip displacement at the maximum shear stress deviates from each other. Specifically, the larger Young’s modulus indicates the larger shear stress at the early loading stage, but the smaller Young’s modulus then indicates the larger shear stress at the late loading stage. Here, it should be also noted that the slope of ascending curves is different, but that of descending curves is relatively identical.

The difference of maximum shear stress for 10 GPa difference of Young’s modulus is estimated to be about 3 %.
Sensitivity for compressive strength $f'_c$

In the previous comparison, quantity of 45-55 MPa was used as the compressive strength. The shear stress-slip relations for compressive strength $f'_c$ of 40, 50, 60 MPa, are calculated, with tensile strength $f_t$ of 2.3 MPa, band width $h$ of 10.0 mm, fracture energy $G_f$ of 0.1 N/mm, Young’s modulus $E_x$ of 33 GPa.

The results are shown in Fig.5.18. Here, it can be observed that the overall behavior or shape of shear stress-slip curves including the shear slip displacement at the maximum shear stress deviates from each other except the early loading stage. Specifically, the larger compressive strength indicates the larger shear stress especially after the peak load.

The difference of maximum shear stress for 10 MPa difference of compressive strength is estimated to be about 10%.

Based on the observation above, the following conclusions are drawn:

1. The band width 'h' is relatively insensitive to the load-displacement relation at entire load range, and therefore, the rough estimation of this quantity in the proposed mechanical model could be acceptable.

2. The tensile strength is relatively insensitive to the load-displacement relation at entire load range, and therefore, the rough estimation of this quantity in the proposed mechanical model could be acceptable.

3. The fracture energy is sensitive to the load-displacement relation at entire load range especially near the peak load, and therefore, the estimation of this quantity should be considered carefully in the proposed mechanical model.

4. The Young’s modulus is sensitive to the load-displacement relation at early load range, especially the deformation characteristics. Therefore, the estimation of this quantity should be considered carefully in the proposed mechanical model if the displacement is an important issue.

5. The compressive strength is largely sensitive to the load-displacement relation at late load range, and therefore, the estimation of this quantity should be considered carefully in the proposed mechanical model.
Fig. 5.14: Shear Stress versus Shear Slip Displacement Curves in the Variation of Band Width

Fig. 5.15: Shear Stress versus Shear Slip Displacement Curves in the Variation of Tensile Strength
Fig. 5.16: Shear Stress versus Shear Slip Displacement Curves in the Variation of Fracture Energy

Fig. 5.17: Shear Stress versus Shear Slip Displacement Curves in the Variation of Young's Modulus
Fig. 5.18: Shear Stress versus Shear Slip Displacement Curves in the Variation of Compressive Strength
5-3. COMPARISON WITH FEM ANALYSIS

In this section, the proposed mechanical model is also compared with nonlinear FEM analysis results in addition to the comparison with experimental results of both plain concrete shear keys [Bakhoum et al. 1989 and Bakhoum 1991] and steel fiber reinforced concrete shear keys [Beattie 1989 and Beattie et al. 1989]. The objective of this comparison is to supplement the previous verification study of the proposed mechanical model by the comparison with experimental results in push-off shear key tests which number is relatively limited. The reliability of the FEM analysis is examined by the comparison with the experimental observation in terms of the cracking status.

The FEM analysis is carried out by using the 'DIANA Finite Element Program' developed at the TNO Institute for Building Materials and Structures. The program uses an approach based on the multi-directional fixed smeared crack concept [e.g., Rots 1988, Rots and Blaauwendaal 1989]. This is because as mentioned in Chapter 3 the smeared crack approach provides a physical basis for distributed fracture like M cracks in key joints while the discrete crack approach provides a physical basis for single crack in the numerically stable manner, but is not a rational approach for distributed fracture. In this FEM analysis, the simple constitutive models used in the proposed mechanical model are employed.

5-3-1. Smeared Crack Analysis

Elastic softening FEM model
The FEM model employed here was determined to describe the test configuration shown in Fig.5.19a reported in literatures [Bakhoum et al. 1989, Bakhoum 1991, Beattie 1989 and Beattie et al. 1989]. After some trials in preliminarily numerical studies, the geometric/loading configuration shown in Fig.5.19b is adopted as the FEM model. This FEM model represents the vertical cantilever portion of the upper specimen including the elastic 'region A' as shown in Fig.5.19a. Here the specimen is assumed to be in a state of plane stress as was in the previous comparison between test data and predictions by the mechanical model. The vertical displacement of the upper portion of the key is restrained. The vertical prescribed displacement is applied at the bottom portion of the shear key, while a constant lateral prestressing load is applied from the right side. It should be noted that the lateral displacement of the left side in the FEM model is restrained at two points, the central and the bottom, because the cantilever portion of the upper specimen in the test should be allowed to rotate. This FEM model can be applied for both plain concrete and fiber reinforced concrete shear key tests.
Fig. 5.19: (a) Considered Modeling Area in Test Specimen; (b) Geometry of Numerical Model; (c) Finite Element Discretization
Plain concrete

The simulation by the FEM model of the push-off shear test of plain concrete specimen is carried out here for the test of K300-DR3 by Bakhoun et al. The material is idealized as elastic with a softening behavior in tension [e.g., Rots and Blaauwendraad 1989]. This assumption indicates a linear elastic behavior under compressive stresses and a nonlinear tension softening behavior under tensile stresses. The elastic softening parameters are taken as described in the previous comparison (see Fig. 4.9a) between the predictions by the mechanical model and experimental data, except for the Young's modulus. Here, the secant modulus of elasticity described by the following equation is adopted in the FEM analysis (see Figures 4.10 and 4.11).

\[ E_s = \frac{E_c}{2} \]  
(5.8)

This reduction is justified because while the FEM model is based on linear elastic behavior under compressive loading, the initial tangent stiffness is quite high to describe the entire behavior to failure. The tensile Young's modulus is also assumed to be \( E_s \). In the calculation, a smeared cracking formulation with fixed multiple smeared cracks is employed. A shear retention factor \( \beta = 0 \) and a threshold angle \( \alpha = 60^\circ \) are assumed. This is because the experimental load-displacement and the direction of fracture propagation were captured well by fixed smeared cracks of almost zero shear retention as mentioned in Chapter 3.

The finite plane stress elements consist of three-node linear triangles (3-point integration scheme) in cross diagonal pattern and four-node linear quadrilaterals (2x2 integration scheme). The mesh profile is shown in Fig. 5.19c. The crack band width \( h \) in the FEM model is assumed to be 5.66 mm for the triangles and 11.31 mm for the quadrilaterals. These values are determined by the following equations [Rots 1988]:

\[ h = \frac{1}{2} e \sqrt{2} \text{ for triangles} \]  
(5.9a)

\[ h = e \sqrt{2} \text{ for quadrilaterals} \]  
(5.9b)

where 'e' is the element size.

First, the cracking status predicted by the FEM analysis is examined to assess the reliability. The cracking patterns for vertical displacements of 0.3 mm, 0.4 mm and 0.58 mm are shown in Fig. 5.20. In this figure, only cracks associated with tensile strains of
$e_{\mu 2}/2$ (see Fig.4.9a) or higher are shown. Here, it should be noted that the size of lines for cracks is relative to the element size (not relative to the crack strain). In addition, the highly amplified deformed configuration for the vertical displacement of 0.58 mm is shown in Fig.5.20d. The crack pattern reveals that the fracture discontinuities first propagate sideways from the shear key corner and then continue to propagate along the key base in the vertical direction. Thus, it appears that the cracking pattern is representative of both $S$ crack and $M$ cracks nucleation and propagation. Specifically, Fig.5.20a shows that the $S$ crack has propagated, and figures 5.20b,c show the propagation of the $M$ cracks while the $S$ crack has stopped. Figures 5.20d also shows the regions where both $S$ crack and $M$ cracks localize. It is then concluded that the present FEM model can predict the cracking behavior of push-off shear tests reliably, and therefore be acceptable.

Next, the calculation of the load-displacement characteristics by the FEM model is examined. Figure 5.21 shows a comparison of the shear stress-slip displacement relationships for the test result, the prediction by the proposed mechanical model and the prediction by the FEM model, at the normal stress level of 2.07 MPa (K300-DR3 of Bakhoum et al.). Here, the prediction by the mechanical model is identical to that calculated in the previous section. The FEM model agrees well with both the mechanical model and the test results in the pre-peak region, especially those corresponding to an upper limit. In addition, the FEM model predicts a stress drop before the maximum stress is reached, which is also typical in the test results. Thus, the present FEM model can predict satisfactorily the fracture behavior of test results before the peak load.

On the other hand, it is also observed that the FEM model does not trace the post-peak behavior of the test. This may indicate that the stress drop after the peak stress is the result of compressive crushing, which is properly accounted for in the mechanical model. However, the present elastic softening FEM model can not show the compressive crushing between cracks. This is because the compression field in the FEM model is assumed as elastic. To eliminate this discrepancy, the plasticity concept should be incorporated into the present FEM model. However, since the numerical modelling of a smooth transition between cracking and plasticity modes is still to be developed as mentioned in Chapter 3, we need further analytical works on the successful modelling of this problem. This is outside the topic area in this thesis. However, the incorporation of the plasticity into the present elastic-softening FEM model will be examined later.

As reference, the CPU time by FEM model was recorded as 31500 seconds up to the vertical displacement of 0.6775 mm. This is quite time-consuming compared with the proposed mechanical model which could need a few minutes.
Fig. 5.20: Cracking Pattern of K300-DR3 Test for Vertical Displacement of (a) 0.3 mm; (b) 0.4 mm; (c) 0.58 mm; and (d) Deformed Configuration for Vertical Displacement of 0.58 mm
Fig. 5.21: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for K300-DR3 Test.
Fiber reinforced concrete

Next, the simulation by the FEM model of the push-off shear test of steel fiber reinforced concrete specimen is carried out here for the test series of KF7B and KF10B by Beattie et al. The material idealization is the same as those in the case of plain concrete. The elastic softening parameters are taken as described in the previous comparison (see Fig. 4.11a) between the predictions by the mechanical model and experimental data, except for the Young's modulus. As well as the plain concrete, the secant modulus of elasticity is adopted. In addition, the finite plane stress elements and the crack band width are also identical.

First, the cracking status predicted by the FEM model is examined to assess the reliability. The cracking patterns in the KF7B test for vertical displacements of 0.3 mm, 0.4 mm and 0.6 mm are shown in Figures 5.22a-c. In these figures, only cracks associated with tensile strains of $\varepsilon_{tu1}$ (see Fig. 4.11a) or higher are shown. Here, it should be noted that the size of lines for cracks is relative to the element size (not relative to the crack strain). In addition, the highly magnified deformed configuration for a vertical displacement of 0.6 mm in the KF7B test is shown in Fig. 5.22d. The crack pattern reveals that the fracture discontinuities first propagate sideways from the shear key corner and then continue to propagate along the key base in the vertical direction. The crack pattern exhibits quite a number of multiply cracked sampling points along the S crack path, which indicates that stress rotations after primary cracking have been significant. Thus, the cracking pattern is representative of both S crack and M cracks nucleation and propagation. Specifically, Fig. 5.22a shows that the S crack has propagated, and figures 5.22b,c show propagation of the M cracks. Fig. 5.22d indicates the regions where both S crack and M cracks localize. It is then concluded that the FEM model can predict the cracking behavior of push-off shear tests of fiber reinforced concrete shear key reliably as well as the plain concrete one.

Next, the calculation of the load-displacement characteristics by the FEM model is examined. Figures 5.23 and 5.24 show a comparison of the shear stress-slip displacement relationships for the experimental result, the predictions by the proposed mechanical model and the prediction by the FEM model for test series KF7B and KF10B. While the FEM model agrees reasonably well with both the proposed mechanical model and the test results in the pre-peak regime of the behavior, it fails to trace the post-peak response indicating that the stress drop after the peak stress is the result of compressive crushing, which is properly accounted for in the mechanical model. This results are also identical to those of plain concrete tests.
Fig. 5.22: Cracking Pattern of KF7B Test for Vertical Displacement of (a) 0.3 mm; (b) 0.4 mm; (c) 0.6 mm; and (d) Deformed Configuration for Vertical Displacement of 0.6 mm
Fig. 5.23: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for KF7B Test

Fig. 5.24: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for KF10B Test
Fig. 5.25: Comparison of Predicted Average Principal Strain at Shear Key Base between RSCBM and FEM
Finally, we investigate the principal strain distribution along the key base predicted by both smeared crack FEM results and Rotating Smeared Crack Band Model in the proposed mechanical model. Fig.5.25 shows the comparison of the averaged principal strain and averaged shear stress $\tau_a$ between the RSCBM and the FEM model. The averaged principal strain predicted by the FEM model is calculated from 12 point along the key base (the key base is divided by 12 element in the FEM model; see Fig.5.19c). In the figures, EP1, EP2 stand for the tensile principal strain and compressive principal strain, respectively. From this figure, it can be concluded that RSCBM result can be in good agreement with the FEM result in the compression regime, but somehow deviates in the tension regime. However, this deviation in tension is finally overcome by the rapid increase of tensile principal strain predicted by the FEM results.

Based on the above comparison, it can be concluded that:

1. The present elastic-softening FEM model can predict both the cracking behavior and the stress-slip relations (in the pre-peak load region) of push-off shear tests successfully. Thus, this model can be acceptable.

2. The proposed mechanical model is also in reasonably good agreement with the FEM analysis results in terms of the stress-slip curve in the pre-peak region, especially for the upper limit values.

3. The elastic-softening FEM model can not trace the post-peak behavior of the test. This may indicate that the stress drop after the peak stress is the result of compressive crushing, which is properly accounted for in the proposed mechanical model.

4. The present comparison shows that the FEM approach is time-consuming to obtain results identical to the proposed mechanical model in the prediction of shear-off failure of shear key joints. Thus, it could be indicated that the proposed mechanical model is an efficient design tool which gives a reasonable first estimation.

5. The averaged principal strain along the key base predicted by the proposed mechanical model is good agreement with that by the FEM analysis in the compression region, but somehow deviates from each other in the tension region.
Plastic FEM model
As shown, the elastic-softening FEM model does not trace the post-peak behavior of the test since the compression field in this FEM model is assumed as elastic. To eliminate this discrepancy, the plasticity concept is incorporated into the previous FEM model. Regarding the plasticity modelling, Mohr-Coulomb criterion is considered in this section. Here, it should be noted that in soil mechanics Coulomb criterion is widely used, in applied mechanics Mohr’s criterion has been widely used and for concrete Mohr-Coulomb criterion appears to be most popular [Chen, 1982]. The failure surface in the Mohr-Coulomb criterion is not a smooth surface, and the corners or singularities are then known to be difficult to handle in numerical analysis. To obtain a smooth approximation to the Mohr-Coulomb surface, Drucker-Prager model [Drucker and Prager, 1952] is used in this calculation.

The Mohr-Coulomb strength criterion postulates, in analogy with the law of dry friction between two sliding surfaces for any particular surface element (see Fig.5.26a),

$$|\tau| = c - \sigma \tan \phi$$  \hspace{1cm} (5.10a)

where $\tau$ is the limiting shearing stress in a plain, $c$ is the cohesion and $\phi$ is the internal-friction angle of the material. This criterion simply means that all possible stress circles are bounded by the envelope. This linear envelope could provide a good approximation for the failure of brittle-ductile materials like concrete in the intermediate stress level [Chen, 1982].

From Fig.5.26b, it can be seen that Eq.(5.10a) is identical with the following criterion [Vermeer P.A. and de Borst R. 1984, de Bosrt R. 1986]:

$$\tau^* - \sigma^* \sin \phi - c \cos \phi = 0$$  \hspace{1cm} (5.10b)

where

$$\sigma^* = -\frac{1}{2}(\sigma_{xx} + \sigma_{yy})$$  \hspace{1cm} (5.10c)

$$\tau^* = \sqrt{\frac{1}{4}(\sigma_{xx} - \sigma_{yy})^2 + \sigma_{xy}^2}$$  \hspace{1cm} (5.10d)

Then, the yield function for the Mohr-Coulomb criterion can be defined as:

$$f = \tau^* - \sigma^* \sin \phi - c \cos \phi$$  \hspace{1cm} (5.11)

Concrete elements in reality show nonlinear constitutive law including the softening branch [e.g., Fig.4.10]. Regarding this point, Vermeer P.A. and de Borst R. [1984] and
de Bosrt R. [1986] proposed the hardening-softening model in compression. The hardening-softening model in compression consists of two strain-dependent quantities: a mobilized internal-friction angle $\phi^*$ and a mobilized cohesion $c^*$ instead of constant internal-friction angle $\phi$ and cohesion $c$. The mobilized internal-friction angle increases as a function of the effective strain increases, and the mobilized cohesion decreases as a function of the effective strain increases. The proposed yield function is as follows;

$$ f = \tau^* - \sigma^* \sin \phi^* - c^* $$

(5.12)

The mobilized internal-friction angle and mobilized cohesion are defined by the following equations and shown schematically in Fig.5.26b.

$$ \sin \phi^* = 2 \sqrt{\bar{e}^p e^f} \sin \phi \quad \text{for} \quad \bar{e}^p < e^f $$

(5.13a)

$$ \sin \phi^* = \sin \phi \quad \text{for} \quad \bar{e}^p \geq e^f $$

(5.13b)

$$ c^* = c \exp \left[ - \left( \frac{\bar{e}^p}{\bar{e}^c} \right) \right] $$

(5.13c)

where $\bar{e}^p$ is a plastic strain, and $e^f$ and $e^c$ are constants.

It was suggested that the hardening in the internal-friction angle was obtained based on the experimental curves from sand testing, and the softening on the cohesion can be made plausible by considering that when a specimen of intact rock or concrete is sheared, micro-cracks first develop, and that at failure the specimen is heavily cracked. Hence, the cementation of the material gradually decreases so that also the cohesive strength of the material vanishes. Regarding the verification on this hardening-softening model in compression, there is no attempt to fit the hardening-softening model on any existing data accurately. Therefore, simple elastic perfectly plastic model is used in this thesis.

Since tensile stress also occurs in the shear key joints, the Mohr-Coulomb criterion is combined with a maximum-tensile-strength cut-off [e.g., Cowan 1953, van Mier 1987]. Here, the uniaxial tensile strength is used as the tensile strength for cut-off. This criterion can be schematically shown in Fig.5.27. The constant cut-off stress (tension cut-off no.1) in the figure is used for the tension-softening law. Here, it should be mentioned that this approach is not well developed and the numerical modelling of a smooth transition between cracking and plasticity modes is still to be developed as mentioned in Chapter 3.
Fig. 5.26: (a) Coordinate system and stress circle for a material element in plane state of strain; and (b) Quasi-empirical relations in the hardening-softening model [after Vermeer and Borst 1984]
Elastic perfectly plastic FEM model is applied here to the same geometry and finite plane stress elements shown in Fig.5.19 based on Drucker-Prager criterion (modified Mohr-Coulomb criterion) with tension cut-off criterion mentioned above. Test of K300-DR3 by Bakhoum et al. is simulated here.

Regarding the dilatancy problem, the associated plasticity is adopted: The internal-friction angle $\phi$ is equal to the dilatancy angle $\psi$ or no plastic volume change. This is because there is currently very little experimental evidence to decide in favor of either associated or non-associated plasticity [Chen, 1982]. The friction angle $\phi$ and dilatancy angle $\psi$ are assumed to be 30°, and the cohesion $c$ is calculated by the following equation.

$$ c = f' = \frac{1 - \sin(\phi)}{2 \cos(\phi)} $$

(5.14)

First, the cracking status and plasticity status predicted by the plastic FEM model are examined to assess the reliability. First, Figures 5.28a,b show only cracks associated with tensile strains of $\epsilon_{m2}/2$ (see Fig.4.9a) or higher for the vertical displacement of 0.3 mm and 0.6 mm, respectively. The size of lines for cracks is relative to the element size (not relative to the crack strain). The crack pattern reveals that the fracture discontinuities propagate along the key base in the vertical direction. Thus, it is found that there is no obvious S crack formation, and only M cracks can exist.

Figures 5.29c,d show the plasticity status for the vertical displacement of 0.2 mm and 0.6 mm, respectively. Here, it should be noted that the size of the triangle is relative to the magnitude of the plastic strain. From these figures, it is found that the compressive crushing could precede the S crack formation at the upper corner of the key, and constrain the crack formation and propagation. In addition, it can be observed that the concentration of plastic strain moved from the upper corner of the key to the center of the key base with further shear loading. This represents the crushing behavior between M cracks.

Next, the calculation of the load-displacement characteristics by the plastic FEM model is examined. The obtained shear stress-slip curve by the plastic FEM model is shown in Fig.5.29 with the test data and the prediction by elastic-softening FEM model used previously. Here, it should be mentioned that the curve of elastic perfectly plastic FEM model was obtained up to the shear-slip of around 0.7 mm, but not after the value due to the numerical divergence problem. From this figure, it is obvious that the prediction by elastic perfectly plastic FEM model deviates from that by the elastic-softening FEM model after the vertical displacement of around 0.3 mm is reached, and the former FEM model predicts the maximum shear stress of lower limit of experiment.
Fig. 5.27: Mohr-Coulomb yield criterion with "tension cut-off" in the biaxial principal stress plane [after Van Mier, 1987]
Fig.5.28: Cracking Pattern of K300-DR3 Test for Vertical Displacement of (a) 0.3 mm; (b) 0.6 mm; and Plasticity Status for Vertical Displacement of (c) 0.3 mm; (d) 0.6 mm
Fig. 5.29: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for K300-DR3 Test
Finally, it can be concluded that:

1. The elastic perfectly plastic FEM model underestimates the shear stress at the late loading stage compared with the elastic-softening model.

2. The plastic FEM model does not fit the experimental curves as closely as the elastic-softening model.

3. The plastic FEM model can not predict the formation of S crack.

4. The plastic FEM model predicts the crushing sequence along the key base.

The plastic FEM model consists of both plasticity in compression and softening in tension, while the elastic-softening FEM model consists of only softening in tension. Since in reality there should always be plastic behavior in a local element, the plastic FEM model should be more accurate than the elastic-softening FEM model. However, the first two conclusions above indicate that the elastic-softening model is better. This may be caused by the poor treatment of both plasticity and cracking in the present FEM formulation as mentioned previously.

The more important conclusion is the third one. The formation of both S crack and M cracks is one of the most important issue in this thesis. Therefore, the elastic perfectly plastic model may not be suitable for this point in the present FEM formulation. In addition, the CPU time for both the elastic-softening FEM model and elastic perfectly plastic FEM model are: 31500 seconds up to the vertical displacement of 0.6775 mm by the former model; and 57400 seconds up to the vertical displacement of 0.74 mm by the latter model. The plastic FEM model needs roughly two times as much CPU as the elastic-softening FEM model.

Based on the above discussion on the plastic FEM model, the plasticity concept will not be considered in the subsequent sections in this thesis.

5-3-2. Discrete Crack Analysis

In the previous section, smeared crack analysis was employed to assess the proposed mechanical model. This is because overall fracture sequence in shear key joints consists of both S crack (single crack) and M cracks (multiple cracking), and therefore the smeared
crack approach is appropriate in that case. On the other hand, the discrete crack approach provides a physical basis for single crack in the numerically stable manner, and is therefore applicable to a single crack like S crack, as mentioned previously. Thus, the fracture sequence of the formation of S crack and also of transition between S crack and M cracks could be predicted by the discrete crack approach. Therefore, in this section the fracture process in terms of strain distribution before the mature formation of M cracks is examined by the discrete crack model, emphasizing on the transition between S crack and M cracks.

Two discrete crack FEM models are considered here to both assess the assumption of Approach SB in the Wedge Crack Model such as 45°-inclined path of S crack, and know the stress or strain distribution along the key base or the zone of M cracks during the formation of S crack. Two kinds of discrete crack models are: 1) discrete crack FEM model D1 with the prescribed crack path assumed from the smeared crack analysis (see Fig.5.20); 2) discrete crack FEM model D2 with the prescribed crack path of 45°-inclination from the key base.

Test of K300-DR3 by Bakhoum et al. is simulated here. The material parameters and geometric/loading configurations are the same as that of the smeared crack FEM model. The mesh profile is the same as that of smeared crack FEM model except the discrete crack element. The tension-softening constitutive law in the discrete crack is assumed to be a bilinear model (see Fig.4.9b). The other part is assumed elastic. Similarly to the approach reviewed in Chapter 3 [e.g., Rots and Blaauwendraad], the zero shear traction and zero shear stiffness along the discreet crack are adopted, and thus only mode I crack results. This is because in most of the simulations reviewed in Chapter 3, mode I effects prevailed up to peak, while mode II effects became gradually more important during the post-peak regime. Since the transition between S crack and M cracks is before the peak load, this modelling could be appropriate in this simulation.

The discrete crack path is modelled by linear line interface element in which linear interpolation is carried out in the interface between two lines in 2D configuration. The integration is carried out by nodal-lumping scheme with two Gauss point (lumped interface element: see Chapter 3). Using these discrete models, test of K300-DR3 by Bakhoum et al. is simulated here as follows.

First, the principal stresses/strains predicted by the discrete crack FEM model D1 are examined to see the stress or strain distribution along the key base or the zone of M cracks during the formation of S crack. Figures 5.30a-c and 5.31a-c show the enlarged principal stress and strain vector sequence by discrete model D1 for the vertical displacement of 0.03 mm, 0.1 mm and 0.3 mm, respectively. The size of the lines is relative to the magnitude of the stress and strain, and the orientation is the actual direction to the stress and strain,
respectively. Positive values are plotted with a solid line, negative values with a dashed line. From these figures, it is obvious that the principal stress vectors cannot show very well the stress concentration along the key base, while the strain vectors show well the strain concentration resulting in the strain localization along the key base. Specifically, in the stress vectors, we cannot observe the clear change of stress concentration between the vertical displacement of 0.03 mm and 0.1 mm. In addition, the stress concentration along the key base at the vertical displacement of 0.3 mm is not clear.

On the other hand, the strain vectors show the clear change of strain concentration between the vertical displacement of 0.03 mm and 0.1 mm, and also show the larger strain localization along the key base than near S crack path at the vertical displacement of 0.3 mm. Figures 5.32a-c also show the transition for S crack and M cracks between the vertical displacement of 0.1 mm and 0.3 mm. Specifically, at the vertical displacement of 0.1 mm, the strain concentration near S crack path is higher than that along the key base. However, at the vertical displacement of 0.3 mm, the strain concentration along the key base is higher than that near S crack path. Thus, the principal strain distribution could be more instructive to the fracture of the shear key joints than the principal stress distribution. This observation is also identical to the physical description developed in Chapter 2. In addition, this observation could be analogous to the hypothesis in the proposed mechanical model. Specifically, Approach MA in the Rotating Smeared Crack Band Model is based on the principal strain control, and the prediction is always influenced by the principal strain distribution along the key base.

Figures 5.32a-c show the principal strain vectors for the vertical displacement of 0.03 mm, 0.1 mm and 0.2 mm by the discrete crack FEM model D1, respectively. Figures 5.33 show the principal strain vectors for the vertical displacement of 0.03 mm, 0.1 mm and 0.2 mm by the discrete crack FEM model D2, respectively. The size of the lines is relative to the magnitude of strain, and the orientation is the actual direction to the strain. Positive values are plotted with a solid line, negative values with a dashed line. From these figures, it is obvious that the principal strain vectors along the key base up to the vertical displacement of 0.1 mm predicted by both discrete crack FEM models could be approximately identical. However, at the vertical displacement of 0.2 mm, the discrete crack FEM model D1 shows the strain localization at the upper region of the key base, while the discrete crack FEM model D2 shows the strain localization at the center of the key base. This observation could indicate that the assumption of $45^\circ$-inclination from the key base in Wedge Crack Model (Approach SB) could be acceptable in this configuration if we only consider mode I condition, and the simpler discrete crack FEM model D2 is thus
Fig. 5.30: Enlarged Principal Stress Sequence for the Vertical Displacement of (a) 0.03 mm; (b) 0.1 mm; (c) 0.3 mm
Fig. 5.31: Enlarged Principal Strain Sequence for the Vertical Displacement of (a) 0.03 mm, (b) 0.1 mm, (c) 0.3 mm.
Fig 5.32: Principal Strain Vectors Predicted by Model D1 for the Vertical Displacement of
(a) 0.03 mm, (b) 0.1 mm, (c) 0.2 mm
Fig. 5.3: Principal Strain Vectors Predicted by Model D2 for the Vertical Displacement of
(a) 0.03 mm, (b) 0.1 mm, (c) 0.2 mm
Fig. 5.35: Predicted and Experimental Shear Stress versus Shear Slip Displacement Curves for K300-DR3 Test
acceptable to predict the fracture of shear key joints up to the transition between S crack and M cracks.

Figures 5.34a-b show the highly magnified deformed configurations for the vertical displacement of 0.3 mm by both discrete crack FEM models, respectively. One can observe the discrete cracking along the S crack path. Here, it should be noted that the two deformed configurations show the almost identical shape in the key portion, but the different shape above the key portion.

Next, the calculation of the load-displacement characteristics by both discrete crack FEM models is examined. Fig.5.35 shows the shear stress-slip relation predicted by the two discrete crack FEM models, Wedge Crack Model (identical to SB curve in Fig.5.1) and the experimental results. The curves by the two discrete crack FEM models start to deviate around the shear stress of 5.0 MPa. Up to this loading point, a good agreement can be observed between the Wedge Crack Model and two discrete crack FEM models. It is then concluded that the assumption of 45°-inclination from the key base in Wedge Crack Model (Approach SB) could be acceptable in this configuration if we only consider mode I condition. In addition, the shear stress of 5.0 MPa is associated with the vertical displacement of around 0.1 mm which is identical to the point in which transition between S crack and M cracks is observed in terms of the principal strain concentration (see Fig. 5.31).

Finally, it can be concluded that:

1. Wedge Crack Model (Approach SB: mode I fracture with 45°-inclined path of S crack) in the mechanical model developed in Chapter 4 could be appropriate for the fracture of shear key joints up to the transition between S crack and M cracks

2. The principal strain distribution is more instructive to the fracture of the shear key joints than the principal stress distribution. Therefore, Rotating Smeared Crack Band Model (Approach MA: tensile strain control approach)) in the proposed mechanical model could be appropriate for the strain localization of M cracks.

5-3-3. Sensitivity For Key Length

Since the proposed mechanical model in this thesis is based on both a single discrete crack model under wedging loading and a smeared crack model under remote shear loading, the exact boundary conditions are not considered in the formulation. Therefore, to
investigate the sensitivity of shear-off failure of shear key joints to geometric dimension is instructive to see the limitation of the proposed mechanical model.

As discussed in Chapter 2, the shear-off failure mechanism in the shear key joints is influenced by both geometric and loading configurations (see Fig.2.60): (a) high normal stress or shallow depth; (b) low normal stress or deep depth. In these two extreme case, the geometric parameter is examined by both the long key which is representative of long length or shallow depth, and the short key which is representative of short length or deep depth. The analysis is performed by the smeared crack FEM model based on elastic-softening concept used previously to see the entire load-displacement characteristics.

Key length considered here is the half of and two times as much as that of the previous FEM model (see Fig.5.19c). The former is called as a short key and the latter as a long key, and the previously used model is called here as an intermediate key. The other parameters are the same as the previous ones used in the simulation for Test of K300-DR3 by Bakhroum et al. The mesh profiles of the short and long key are shown in Fig.5.36. Here, it should be mentioned that the element size in each key portion is different from each other. However, from the previous studies, it was observed that the fractured zone was concentrated on the root of the key inside the main body (e.g., see Figures 5.20 and 5.22). Therefore, finer or coarser element size may not differentiate the fracture behavior of three keys.

First, the cracking status for these different key configurations is examined. The cracking patterns of the short key for the vertical displacement of 0.2 mm, 0.29 mm and 0.34 mm are shown in figures 5.37a-c. In this figure, only cracks associated with tensile strains of $\varepsilon_{tu}/2$ (see Fig.4.9a) or higher are shown. The size of lines for cracks is relative to the element size (not relative to the crack strain). The calculation stopped here because of the numerical divergence problem. This may be caused by the limited boundary condition as shown in Fig.5.37c. It is interesting that S crack propagates almost straight with the inclination of about 45° up to the vertical displacement of 0.29 mm without the formation of M cracks. The deviation from 45°-inclination may be caused by the element shape: quadrature element may restrict the direction of crack propagation compared with the triangle element. However, at the point of 0.34 mm, the formation of M cracks is observed. This fracture mechanism seems to be identical to the observation of S crack in large precast concrete panel as discussed in Chapter 2 [e.g., Cholewicki 1971, Lacombe and Pommeret 1974].

The cracking patterns of the long key for the vertical displacement of 0.2 mm, 0.3 mm, 0.4 mm and 0.5 mm are shown in figures 5.38a-d. In these figures, only cracks associated with tensile strains of $\varepsilon_{tu}/2$ (see Fig.4.9a) or higher are shown. The size of
Fig. 5.36: Geometry and Finite Element Discretization of (a) Short Key Model; (b) Long Key Model
Fig. 5.37: Cracking Pattern Predicted by Short Key Model for Vertical Displacement of (a) 0.2 mm; (b) 0.29 mm; (c) 0.34 mm
Fig. 5.38: Cracking Pattern Predicted by Long Key Model for Vertical Displacement of (a) 0.2 mm; (b) 0.3 mm; (c) 0.4 mm; (d) 0.5 mm
Fig. 5.39: Predicted Shear Stress versus Shear Slip Displacement Curves for Different Key Length
lines for cracks is relative to the element size (not relative to the crack strain). It is also obvious that S crack propagates curvilinearly up to the vertical displacement of 0.4 mm without the formation of M cracks. However, at the point of 0.5 mm, the formation of M cracks is observed. Compared with the short key configuration, the fracture mechanism of the long key configuration seems to be largely dominated by the bending effect. This fracture mechanism could be also identical to the observation of S crack in corbel structures as discussed in Chapter 2 [e.g., Niedenhoff, 1963].

In the calculation, it was observed that the extremely large strain in the short key configuration existed at the upper corner of the key after the vertical displacement of 0.2 mm. This large strain concentration at the corner of the short key may eliminate the formation of M cracks. On the other hand, such large strain concentration at the key corner was not observed in both the long key and the intermediate key.

Finally, the shear stress-slip curves predicted by three models are shown in Fig.5.39. In these three curves, only the curve of the short key configuration is stopped due to the numerical divergence after certain loading stage as mentioned. From this figure, it may be concluded that the stress-slip curves of three different geometries are essentially identical up to a certain loading stage. Therefore, it could be concluded that the mechanical model proposed in Chapter 4 is applicable to different shear key length with relatively small error, if the cracking behavior relatively similar.

5-4. SUMMARY AND CONCLUSION

The proposed mechanical model for shear key joints is shown to be sound. Good agreement between predicted and measured shear strength as well as entire load-displacement history was obtained. However, there is still some discrepancy, which should be eliminated in future improvements of the mechanical model.

From the verification study of the proposed mechanical model by means of the comparison with both experimental results and the nonlinear FEM analysis results reported in this chapter, the following conclusions could be drawn:

| 1. The proposed mechanical model identifies two main fracture mechanisms for shear-off failure of key joints: single curvilinear cracking and multiple diagonal cracks. The model predicts reasonably well not only the ultimate shear strength but also the complete load-displacement response for the entire range of loads reported in the literature. Another simulation with a modified constitutive law could have resulted in an even better agreement between test data and the predictions. However, the current results already show that the |
fracture behavior of plain concrete and fiber reinforced concrete shear key joints was reliably captured by the mechanical model proposed in Chapter 4, and any additional parameter refinement would be just an exercise in curve fitting.

2. According to the proposed modelling, the compressive constitutive law plays an important role in determining the shear strength, and so do both the tensile and the compressive constitutive laws in determining the entire shear stress-slip displacement.

3. The proposed simple design formula for the shear-off strength of shear key joints predicts the experimental results well.

4. The load-displacement relation through the entire load range is relatively insensitive to the band width 'h' used in the proposed mechanical model, and therefore, the rough estimation of this quantity in the proposed mechanical model could be acceptable.

5. The nonlinear FEM analysis employed in this chapter can predict the cracking behavior observed in the experiment, and agrees closely with results from the proposed mechanical model up to the point where the peak shear stress is reached.

6. The nonlinear FEM model cannot trace the post-peak behavior of the test. This may indicate that the stress drop after the peak stress is the result of compressive crushing, which is properly accounted for in the proposed mechanical model.

7. Wedge Crack Model (Approach SB: mode I fracture with 45°-inclined path of S crack) in the mechanical model developed in Chapter 4 could be appropriate for the fracture of shear key joints up to the transition between S crack and M cracks.

8. The principal strain distribution is more important to the fracture of the shear key joints than the principal stress distribution. Therefore, Rotating Smeared Crack Band Model (Approach MA: tensile strain control approach) in the proposed mechanical model is appropriate for the strain localization of M cracks.
CHAPTER 6

APPLICATION OF PROPOSED MECHANICAL MODEL

6-1. INTRODUCTION

In this chapter, the applicability of the mechanical model proposed in Chapter 4 to macroscopic shear failure of both reinforced concrete membrane elements subjected to in-plane shear and reinforced concrete deep beams is examined in terms of load-displacement relations.

The analogy of cracking behavior between membrane element, deep beam and shear key configuration is shown in Fig.6.1. This figure shows diagonal multiple cracks along the shear failure plane of both membrane elements and deep beams which are similar to M cracks in shear key joints. In addition, membrane elements and deep beams do not show a distinctive S crack formation. Therefore, only Rotating Smeared Crack Band Model (RSCBM) in the proposed mechanical model can be applied to the shear fractures in membrane elements and deep beams.

6-2. MODIFIED RSCBM

First, RSCBM developed in Chapter 4 is modified based on the truss model of Hsu et al. [1987] here to apply to these reinforced concrete structures. Specifically, the contribution of steel bars should be incorporated into RSCBM as follows.

6-2-1. Basic Equations

Stress transformation conditions (equilibrium) in a cracked element are shown in Fig.6.2. As shown in Fig.6.2a, a concrete element is reinforced with longitudinal bars in x-direction and with transverse bars in the y-direction. In this modelling, the shear stress along the shear failure plane is assumed to be the average stress over the entire shear plane as employed in the modelling for shear key joints. The stresses and strains are thus assumed uniformly distributed as averaged ones. After diagonal cracking occurs, a series of diagonal compression struts is formed in the diagonal (or c-direction in Fig.6.2b). Then, the element takes only compressive stresses $\sigma_c$ in the direction of the compression struts, and only tensile stresses $\sigma_t$ in the direction transverse to the compression struts (or t-direction in Fig.6.2b). This assumption is identical to that in RSCBM which describes the
Fig. 6.1: Analogy between Membrane Element, Deep Beam and Shear Key Configuration
Fig. 6.2: (a) Crack Formation; (b) Stress Transformation Systems in Concrete and (c) Equilibrium of Reinforced Concrete in x-y Coordinate
representative rotating tension cracks. The shear stress $\tau_{ct}$ along the cracked element is assumed zero. Thus, $\sigma_c$ and $\sigma_t$ are always the principal stresses of this system. The angle between the x-y and c-t coordinate systems is designated as $\theta$ as shown in Fig.6.2b. This angle is also the angle of inclination of the compression struts with respect to the x-axis.

As shown in Fig.6.2c, the average stresses $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ in the reinforced concrete element are resisted jointly by the concrete element and the steel reinforcement. The stresses contributed by concrete are $\sigma_{xc}$, $\sigma_{yc}$ and $\tau_{xy}$. The average stresses of concrete in the two coordinate systems, x-y and c-t, are transformed according to the following equations (see Fig.6.2b):

\[
\sigma_{xc} = \sigma_c \cos^2 \theta + \sigma_t \sin^2 \theta \tag{6.1a}
\]
\[
\sigma_{yc} = \sigma_c \sin^2 \theta + \sigma_t \cos^2 \theta \tag{6.1b}
\]
\[
\tau_{xy} = (\sigma_c - \sigma_t) \sin \theta \cos \theta \tag{6.1c}
\]

The steel reinforcement is assumed here to contribute to only normal stresses as shown in Fig.6.2c. Here, $\sigma_{sx}$ and $\sigma_{sy}$ are the steel stresses in x and y-direction, respectively. $\rho_x$ and $\rho_y$ are the reinforcement ratios in x and y-direction, respectively. Then, the total averaged stresses in a reinforced concrete element in the two coordinate systems, x-y and c-t, are transformed by the following superposition.

\[
\sigma_x = \sigma_c \cos^2 \theta + \sigma_t \sin^2 \theta + \rho_x \sigma_{sx} \tag{6.2a}
\]
\[
\sigma_y = \sigma_c \sin^2 \theta + \sigma_t \cos^2 \theta + \rho_y \sigma_{sy} \tag{6.2b}
\]
\[
\tau_{xy} = (\sigma_c - \sigma_t) \sin \theta \cos \theta \tag{6.2c}
\]

Since the averaged stress $\sigma_x$ in x-direction in the reinforced concrete element can be specified as zero in the membrane or deep beam structures (no prestressing), Eq.(6.2a) is considered here. Then, only the stress of the longitudinal reinforcement of the web reinforcement $\rho_x \sigma_{sx}$ is necessary to calculate the averaged shear stress and the associated quantities in this system. In this case, the contribution of transverse reinforcement of the web reinforcement $\rho_y \sigma_{sy}$ is not considered and the averaged stress $\sigma_y$ in Eq.(6.2b) is remained unknown. This treatment is supported by the fact that the longitudinal reinforcement had a more important role than that of the transverse reinforcement in shaping the shear resistance of the shear plane under the evenly distributed longitudinal and transverse reinforcement [Hsu et al. 1987]. Thus, the present calculation is in one sense only valid for the case of evenly distributed longitudinal and transverse reinforcement.
The average strains are described similarly by:

\[ \varepsilon_x = \varepsilon_c \cos^2 \theta + \varepsilon_t \sin^2 \theta \]  \hspace{1cm} (6.3a)
\[ \varepsilon_y = \varepsilon_c \sin^2 \theta + \varepsilon_t \cos^2 \theta \]  \hspace{1cm} (6.3b)
\[ \gamma_{xy} = 2(\varepsilon_c - \varepsilon_t) \sin \theta \cos \theta \]  \hspace{1cm} (6.3c)

where \( \varepsilon_x \) and \( \varepsilon_y \) are normal strains in the \( x-y \) coordinate system, \( \gamma_{xy} \) is the shear strain in \( x-y \) coordinate system and \( \varepsilon_c \) and \( \varepsilon_t \) are normal strains in the \( c-t \) coordinate system (principal strains).

6-2-2. Constitutive Relations

The constitutive law for the concrete element introduced here can be basically referred to the constitutive models introduced in Chapter 4 except the softening of the cracked concrete in compression (softened compression strut; see equations (4.38)-(4.46)). As mentioned in Chapter 3, the 'softening of concrete strut' does indicate the overall reduction of the compressive stress associated with the constitutive compressive stress-strain curve including the reduction of the compressive strength. It has been reported that the softening of concrete strut should be considered in the reinforced concrete structures [e.g., Vecchio et al., 1986]. Since this softening concept of the cracked concrete was developed from the intensive experiment of reinforced concrete elements, this softening of concrete strut was not applied well to shear key joints because of no steel reinforcement as discussed in Chapter 5. However, in the case of reinforced concrete structures discussed in this chapter, this concept may be applicable and therefore such concept is incorporated into the previous mechanical model as follows.

Regarding the tensile stress-strain relation in the direction perpendicular to the compression struts, Eq.(4.38) is used for steel reinforced plain concrete and Eq.(4.41) for steel reinforced fiber concrete.

Regarding the compressive stress-strain relation in the direction of the compression strut, Eq. (4.40) is used for steel reinforced plain concrete. For steel reinforced fiber concrete, Eq.(4.46) is used with slight modification based on the softened compressive stress-strain curves reported in the literature [Mansur & Ong 1991]. Then, the assumed stress-strain relations are represented by the following equations.

Ascending branch for steel reinforced plain concrete:
\[
\sigma_c = f'_c \left[ 2 \varepsilon_c - \lambda \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right] \quad \text{if } \varepsilon_c \leq \frac{\varepsilon_{c0}}{\lambda} \tag{6.4a}
\]

\[
\lambda = \sqrt{0.7 - \frac{\varepsilon_1}{\varepsilon_c}} \tag{6.4b}
\]

where \(\varepsilon_{c0}\) is the concrete yield strain and \(f'_c\) is the compressive strength.

Descending branch for steel reinforced plain concrete:

\[
\sigma_c = \frac{f'_c}{\lambda} \left[ 1 - 0.8 \left( \frac{\varepsilon_c - \frac{\varepsilon_{c0}}{\lambda}}{\varepsilon_{c2} - \frac{\varepsilon_{c0}}{\lambda}} \right) \right] \quad \text{if } \frac{\varepsilon_{c0}}{\lambda} < \varepsilon_c \leq \varepsilon_{c2} \tag{6.4c}
\]

Horizontal branch for steel reinforced plain concrete:

\[
\sigma_c = \frac{0.2f'_c}{\lambda} \quad \text{if } \varepsilon_{c2} < \varepsilon_c \tag{6.4d}
\]

\[
\varepsilon_{c2} = \frac{0.041 - 2\varepsilon_{c0}f'_c}{f'_c - 6.896} + \varepsilon_{c0} \tag{6.4e}
\]

Ascending branch for steel reinforced fiber concrete:

\[
\sigma_c = f'_c \left[ 2 \varepsilon_c - \lambda \left( \frac{\varepsilon_c}{\varepsilon_{c0}} \right)^2 \right] \quad \text{if } \varepsilon_c < \frac{\varepsilon_{c0}}{\lambda} \tag{6.5a}
\]

\[
\lambda = \sqrt{0.7 - \frac{\varepsilon_1}{\varepsilon_c}} \tag{6.5b}
\]

Descending branch for steel reinforced fiber concrete:

\[
\sigma_c = \frac{f'_c}{\lambda} \left( 1 - (1 - k') \left( \frac{\varepsilon_c - \frac{\varepsilon_{c0}}{\lambda}}{\varepsilon_{c2} - \frac{\varepsilon_{c0}}{\lambda}} \right) \right) \quad \text{if } \frac{\varepsilon_{c0}}{\lambda} \leq \varepsilon_c < \varepsilon_{c2} \tag{6.5c}
\]
\[
\begin{aligned}
&k' = 0.38V_f \frac{f_f}{\phi_f} \\
&\varepsilon_{cu1} = \frac{0.041 - 2\varepsilon_{c0}f_c'}{f_c' - 6.896} + \varepsilon_{c0}
\end{aligned}
\] (6.5d)

Horizontal branch for steel reinforced fiber concrete:

\[
\sigma_c = k' \frac{f_c'}{\lambda} \quad \text{if} \quad \varepsilon_{cu1} < \varepsilon_c
\] (6.5e)

The stress-strain relation for the longitudinal reinforcement is assumed to be elastic-perfectly plastic. Thus,

\[
\begin{aligned}
&\sigma_{xx} = E_{st}\varepsilon_x \quad \text{if} \quad \varepsilon_x < \varepsilon_{ty} \\
&\sigma_{xx} = f_y \quad \text{if} \quad \varepsilon_x \geq \varepsilon_{ty} \\
&\varepsilon_{ty} = \frac{f_y}{E_{st}}
\end{aligned}
\] (6.6a, 6.6b, 6.6c)

where \( E_{st} \) is the Young's modulus, \( \varepsilon_{ty} \) is the yield strain and \( f_y \) is the yield strength of the steel reinforcement.

6-2-3. Solution Procedure

The solution procedures are based on the tensile strain control approach (Approach MA in RSCBM) described by Eq.(4.48a). The model of apparent Poisson's ratio in Eq.(4.47) is used in the same manner.

Then, one can develop the relation between the average shear stress \( \tau_{xy} \) and the associated quantities:

1. Select a value of \( \varepsilon_c \).
2. Calculate \( \nu \) from Eq.(4.47) using the previously calculated value of \( \varepsilon_c \), or assume \( \nu = 0.2 \) in the first calculation.
3. Calculate \( \varepsilon_c \) from (4.48a).
4. Calculate \( \lambda \) from (6.4b) or (6.5b).
5. Calculate \( \sigma_t \) from Eq.(4.38) or (4.41).
6. Calculate $\sigma_c$ from Eq.(6.4) or (6.5).
7. Assume the value of $\theta$.
8. Calculate $\epsilon_x$ from Eq.(6.3a).
9. Calculate $\sigma_{ax}$ from Eq.(6.6).
10. Calculate $\sigma_x$ from Eq.(6.2a).
11. If the calculated value of $\sigma_x$ is close enough to the specified value (zero prestress in this geometries), go to the next step. Otherwise, go back to the seventh step.
12. Calculate $\tau_{xy}$, $\gamma_{xy}$ and $\epsilon_y$ from Eqs.(6.2c), (6.3b) and (6.3c).

The vertical displacement $\Delta$ (shear slip displacement) is calculated by assuming that it is the result of both compressive strains in the cracked zone and shear strains along the shear key length:

$$\Delta = \epsilon_y D + \gamma_{xy} L$$  \hspace{1cm} (6.7)

where $\epsilon_y$ and $\gamma_{xy}$ are the normal and shear strains in the x-y coordinate system shown in Fig. 6.2b, and D and L are the depth and length of the fractured region of the specimens, respectively.

6-3. VERIFICATION OF MODIFIED MODEL

6-3-1. Comparison With Concrete Membrane Element

The first simulation is carried out for the reinforced concrete membrane element subjected to in-plane shear which was reported by Vecchio and Collins [1986]. The geometric and loading configuration of the test by Vecchio and Collins are shown in Fig.6.3. The specimen is 890 mm square x 70 mm thick. The cracking pattern after failure in the specimen PV 20 is also shown in Fig.6.4.

The concrete compressive strength was reported as 19.6 MPa. The yield strength of the steel reinforcement was reported as 460 MPa and the longitudinal reinforcement ratio $\rho_x$ is calculated as 1.79%. The tensile strength and Young's modulus are determined from Eqs.(4.55)-(4.56) as 1.5 MPa, 21 GPa, respectively. The fracture energy is assumed as 0.1 N/mm. The band width 'h' is assumed to be 160 mm from the cracking pattern shown in Fig.6.4 which could be identical to the width between diagonal cracks. The shear stress
Fig. 6.3: Geometric and Loading Configurations of Test [after Vecchio and Collins, 1986]

Fig. 6.4: Appearance of the Specimen PV 20 after Failure [after Vecchio and Collins, 1986]
Fig. 6.5: Comparison of Calculated and Observed Response of Specimen PV20 [after Vecchio and Collins, 1986]
versus shear strain relation of specimen PV 20 reported in the literature is simulated by the modified RSCBM with these parameters.

The obtained shear stress-strain curves including the test data are shown in Fig.6.5. From this figure, it can be concluded that the modified RSCBM gives a good prediction of the stress-strain curve at entire loading stage. In addition, the modified model predicts the maximum shear stress with considerable accuracy. The ratio of the test data to the predicted one is 0.92.

6-3-2. Comparison With Deep Beam

The next simulation is carried out for the reinforced concrete deep beam or the reinforced fiber concrete deep beam. The test data of reinforced concrete deep beam is referred to the work by Leonhardt and Walther [1966], and the test data of reinforced fiber concrete deep beam is referred to the work by Mansur & Ong [1991].

Reinforced concrete deep beam
The geometric and loading configuration of the deep beam test on three supports by Leonhardt and Walther [1966] are shown in Fig.6.6. The specimen DWT2 is a beam on three supports with a 360 mm thickness supporting member in the middle. The thickness of the panel is 100 mm, with overall dimensions of 3040 mm X 1600 mm. Four 8 mm diameter reinforcing bars were installed in the lower part of the beam, and a double square grid of 5 mm bars was placed over the entire panel. Furthermore, six reinforcing bars of 6 mm diameter were installed over the middle support. The cracking patterns recorded during the test are shown in Fig.6.7.

The concrete compressive strength was reported as 30.2 MPa. The yield strength of the steel reinforcement was reported as 430 MPa and the longitudinal reinforcement ratio $\rho_s$ is calculated as 0.259%. Here, this quantity includes the six reinforcing bars of 6 mm diameter mentioned above as averaged value. The tensile strength and Young's modulus are determined from Eqs.(4.55)-(4.56) as 1.8 MPa, 26 GPa, respectively. The fracture energy is assumed as 0.1 N/mm. The band width 'h' is assumed to be the quarter of the shear span (=a/4=120 mm). This quantity could be also identical to the width between diagonal cracks.

The shear stress versus deflection in the middle of each of the two spans relation of specimen DWT2 (shear span ratio = 0.3) reported in the literature is simulated by the modified RSCBM with these parameters. Since no quantity in terms of shear stress was introduced in the literature, the shear stress of the test data is here calculated by dividing the
Fig. 6.6: Geometric and Loading Configurations of Test [after Leonhardt and Walther, 1966]
Fig. 6.7: Observed Cracking Patterns [after Leonhardt and Walther, 1966]
Fig. 6.8: Comparison of Calculated and Observed Response of Specimen DWT2
measured load acting on the shear plane by the total shear plane area, 160000 mm$^2$ (=100 mm x 1600 mm). The quantities of 'L' and 'D' for Eq.(6.7) are determined as the half of the shear span (a/2=240 mm) and 1600 mm, respectively.

The obtained shear stress-deflection curves including the test data are shown in Fig.6.8. In the figure, the broken line indicates the maximum load observed in the experiment, but the associated deflection was not measured in the test. From this figure, it can be concluded that the modified RSCBM gives a good prediction of the stress-deflection curve at entire loading stage although the post-peak load-deflection relation was not measured in the experiment. In addition, the modified model predicts the maximum shear stress with considerable accuracy. The ratio of the test data to the predicted one is 1.06.

**Reinforced fiber concrete deep beam**

The geometric and loading configuration of the reinforced fiber concrete deep beam test by Mansur & Ong [1991] are shown in Fig.6.9. The cracking pattern for Beam B1 (shear span ratio = 0.285) is shown in Fig.6.10. In the test, Mansur & Ong used the straight but slightly twisted steel fibers with the length of $l_f=30$ mm and the cross section of 0.5 mm square ($\phi_f=0.56$ mm).

First, the shear stress-shear strain relation of Beam B3 (shear span ratio = 0.865) is simulated by the modified RSCBM with the following parameters. The concrete compressive strength was reported as 35.5 MPa. The yield strength of the steel reinforcement was reported as 440 MPa and the longitudinal reinforcement ratio $\rho_x$ is calculated as 0.5%. Here, this quantity includes only the web reinforcement in the test. The tensile strength and Young's modulus are determined from Eqs.(4.55)-(4.56) as 2.0 MPa, 28.2 GPa, respectively. The fracture energy is assumed as 0.1 N/mm. The band width 'h' is assumed to be the quarter of the shear span (a/4=108 mm). This quantity could be also identical to the width between diagonal cracks. The fiber volume fraction in Beam B3 was 1%, and the associated parameters were reported as the strength efficiency factor for the fiber $\mu_x$ of 0.5, the fiber orientation factor $\mu_0$ of 0.33, the ultimate bond strength of fibers $\tau_{cf}$ of 4.12 MPa.

The shear stress in the test is calculated by dividing the load by the total shear plane area of 45000 mm$^2$ (=500 mm x 90 mm) similarly to the case of Leonhardt and Walther. The quantities of 'L' and 'D' for Eq.(6.7) are determined as the half of the shear span (a/2=216.3 mm) and 500 mm, respectively.

The obtained shear stress-strain curves including the test data are shown in Fig.6.11. From this figure, it can be concluded that the modified RSCBM gives a good prediction of the stress-deflection curve at entire loading stage although the posk-peak load-deflection
Fig. 6.9: Geometric and Loading Configuration of Test [after Mansur and Ong, 1991]

Fig. 6.10: Observed Cracking Patterns [after Mansur and Ong, 1991]
Fig. 6.11: Comparison of Calculated and Observed Response of Specimen B3

Fig. 6.12: Comparison of Calculated and Observed Response of Specimen B1
relation was not measured in the experiment. In addition, the modified model predicts the maximum shear stress with considerable accuracy. The ratio of the test data to the predicted one is 1.1.

Next, the shear stress-shear strain relation of Beam B1 (shear span ratio = 0.285) is simulated by the modified RSCBM with the following parameters. The concrete compressive strength was reported as 35.7 MPa. The yield strength of the steel reinforcement was reported as 440 MPa and the longitudinal reinforcement ratio $\rho_\ell$ is calculated as 0.5%. Here, this quantity includes only the web reinforcement in the test. The tensile strength and Young's modulus are determined from Eqs.(4.55)-(4.56) as 2.0 MPa, 28.3 GPa, respectively. The fracture energy is assumed as 0.1 N/mm. The band width 'h' is assumed to be the quarter of the shear span (=a/4=35.6 mm). This quantity could be also identical to the width between diagonal cracks. The fiber volume fraction in Beam B1 was 1%, and the associated parameters are reported as the strength efficiency factor for the fiber $\mu_\ell$ of 0.5, the fiber orientation factor $\mu_\phi$ of 0.33, the ultimate bond strength of fibers $\tau_uf$ of 4.12 MPa.

The shear stress in the test is calculated by dividing the load by the total shear plane area of 45000 mm$^2$ (=500 mm x 90 mm) similarly to the case of Leonhardt and Walther. The quantities of 'L' and 'D' for Eq.(6.7) are determined as the half of the shear span (a/2=71.25 mm) and 500 mm, respectively.

The obtained shear stress-strain curves including the test data are shown in Fig.6.12. In the figure, two predictions by the modified mechanical model are introduced as: (1) softening of concrete strut with $\lambda$ estimated by Eq.(6.4b); (2) non-softening of concrete strut with $\lambda$ of 1.0 which is identical to the compressive constitutive model shown in Fig.4.11b. The latter model is identical to the RSCBM for shear key joints except the addition of steel reinforcement. As shown in the figure, the prediction by the softening compression strut model largely deviate from the test results. On the other hand, non-softening compression strut model gives a relatively good prediction of the stress-deflection curve at entire loading stage although the posk-peak load-deflection relation was not measured in the experiment. In addition, the latter model predicts the maximum shear stress with considerable accuracy. The ratio of the test data to the predicted one is 0.88.

From the above observation, it can be concluded that the softening compression strut concept may not be applicable to the deep beams with very short shear span ratio (e.g., less than 0.2-0.3). In the test, Mansur & Ong reported that Beam B1 had no flexural cracking while Beam B3 had flexural cracking after the formation of the diagonal cracks. In addition, at a load ranging from 80 to 90% of the ultimate load for Beam B3, one of the diagonal cracks began to grow excessively wide, and finally leading to failure. In the case
of Beam B1, final failure occurred by crushing of the concrete between the diagonal cracks. Thus, the cracking behavior of Beam B1 is not influenced largely by the steel reinforcement because of the relatively small tensile crack opening. In result, the cracking phenomenon is quite similar to the M cracks in shear key joints, and both are predicted by the non-softening compression strut concept. This discussion needs further both analytical and experimental works and therefore it is recommended as the future research.

6-4. SUMMARY AND CONCLUSION

In this chapter, the applicability of the mechanical model proposed in Chapter 4 to macroscopic shear failure of both reinforced concrete membrane elements subjected to in-plane shear and reinforced concrete deep beams is examined in terms of the load-displacement relations. The mechanical model is then modified by incorporating the softening compression strut concept into RSCBM.

Based on this analysis, the following conclusions may be drawn:

1. The modified RSCBM predicts reasonably well not only the ultimate shear strength but also the complete load-displacement response for the entire range of loads in the reinforced concrete membrane element and reinforced concrete deep beam.

2. The proposed mechanical model could be applicable to other but similar geometries. However, further analytical and experimental works are necessary to improve the present formulation. Especially, better formulation for the softening in the compression strut should be developed for different shear span ratios of deep beams.
CHAPTER 7

ADDITIONAL DISCUSSION ON CRACK TRANSITION

7-1. INTRODUCTION

The objective of this additional discussion in this chapter is to study analytically the transition phenomenon between S crack and M cracks in shear key joints. Well-known elastic solutions are introduced here to analyze the principal strain distribution along the key base after S crack propagation. Then, these elastic solutions are compared with the principal strain distribution along the key base obtained by both the discrete crack FEM analysis and the smeared crack FEM analysis.

7-2. ANALYSIS OF TRANSITION PROBLEM

In Chapter 5, the fracture behavior of transition between S crack and M cracks was examined in terms of the principal strain distribution by the discrete crack FEM analysis. Specifically, during S crack formation the strain concentration moves from the shear key corner to the key base. Then, one can ask under which condition this movement of strain concentration is observed. In this chapter, further discussion on this point will be addressed for the plain concrete shear key of K300 DR3 Test often used in this thesis [Bakhoum et al. 1989 and Bakhoum 1991].

As mentioned in Chapter 2, the transition between S crack and M cracks is caused by the changed boundary condition. Specifically, S crack propagation could change the boundary condition for the stress field along the key base, and the strain localization along the key base or M cracks strain localization is thus influenced by the S crack path. To study this condition, elastic solutions for a two-dimensional elasticity problem are introduced here.

7-2-1. Concentrated Load on Vertex of Wedge

The stress fields in the geometric configurations of shear key joints before and after S crack propagation are schematically shown in Fig. 7.1. In the figure, the S crack propagation is assumed 45°-inclination from the key base. This assumption was verified in Chapter 5 such that the 45°-inclination of S crack could be applicable up to the transition between S crack and M cracks. This figure shows that the distinctive S crack formation
Fig. 7.1: Change of Boundary Condition by S Crack Propagation
Fig. 7.2: Concentrated Loading on the Vertex of a Wedge
changes the boundary condition for the stress field along the key base. In result, the stress concentration at the corner of the key moved to the area along the key base. Then, one can see the analogy between this geometric configuration of shear key joints after S crack formation and a two-dimensional elasticity problem of 'Concentrated Load and Moment acting on the Vertex of a Wedge' [Volterra and Gaines, 1971] shown in Fig.7.2. Here, It should be also mentioned that the predicted strain vectors by discrete crack FEM results overall distribute in the tangential direction θ in the Polar Coordinates of the wedge or the corner of the key as shown in figures.5.31-5.33. This phenomenon is identical to that shown in Fig.7.2, and could also support the present study based on the two-dimensional elasticity problem. The elastic solutions for this hypothesized geometry shown in Fig.7.2 are therefore applied to the calculation of the principal strain distribution along the key base after S crack propagation shown in Fig.7.1 as follows [Volterra and Gaines, 1971].

Consider first the concentrated load of intensity P1 acting on the vertex of the wedge of unit thickness (Michell's Problem). For this case, the stress function is assumed to be

$$\Phi(r, \theta) = C \cdot P1 \cdot r \cdot \theta \cdot \sin \theta \quad (7.1)$$

where C is a constant to be determined from the boundary conditions. Eq.(7.1) satisfies the following compatibility equation.

$$\left[ \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \left[ \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} \right] = 0 \quad (7.2)$$

The components of stress in this case are given as follows.

$$\sigma_r = \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} = 2C \cdot P1 \frac{\cos \theta}{r} \quad (7.3a)$$

$$\sigma_\theta = \frac{\partial^2 \Phi}{\partial r^2} = 0 \quad (7.3b)$$

$$\tau_{r\theta} = \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial r \partial \theta} = 0 \quad (7.3c)$$

The boundary conditions which must be satisfied are as follows.

$$\sigma_\theta = \tau_{r\theta} = 0 \quad \text{for } \theta = \pm \alpha \quad (7.4a)$$

$$2 \int_0^\alpha \sigma_r r \cos \theta d\theta = -P1 \quad (7.4b)$$
The first two boundary conditions are identically satisfied by Eqs.(7.3b)-(7.3c). Eq.(7.4b) determines the constant C. By substituting the expression for $\sigma_r$ into Eq.(7.4b), one obtains

$$C = -\frac{1}{2\alpha + \sin 2\alpha} \tag{7.5}$$

By substituting this value of $C$ into Eq.(7.3a), one gets for the components of stress:

$$\sigma_r = -\frac{2P1}{2\alpha + \sin 2\alpha} \frac{\cos \theta}{r} \tag{7.6a}$$
$$\sigma_\theta = 0 \tag{7.6b}$$
$$\tau_{r\theta} = 0 \tag{7.6c}$$

Similarly, the stress components for the case of a horizontal force $P2$ are obtained as follows.

$$\sigma_r = -\frac{2P2}{2\alpha - \sin 2\alpha} \frac{\sin \theta}{r} \tag{7.7a}$$
$$\sigma_\theta = 0 \tag{7.7b}$$
$$\tau_{r\theta} = 0 \tag{7.7c}$$

Consider next the moment $M$ acting on the vertex of the wedge (Inglis' Problem). For this case, the stress function is assumed to be

$$\Phi(r, \theta) = -\frac{M(\sin 2\theta - 2\theta \cos 2\alpha)}{2(\sin 2\alpha - 2\alpha \cos 2\alpha)} \tag{7.8}$$

Eq.(7.8) satisfies Eq.(7.2), and the components of stress are obtained as follows.

$$\sigma_r = \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} = \frac{-M}{2(\sin 2\alpha - 2\alpha \cos 2\alpha)} \frac{4}{r^2} \sin 2\theta \tag{7.9a}$$
$$\sigma_\theta = \frac{\partial^2 \Phi}{\partial r^2} = 0 \tag{7.9b}$$
\[ \tau_{r\theta} = \frac{1}{r^2} \frac{\partial \Phi}{\partial \theta} - \frac{1}{r} \frac{\partial^2 \Phi}{\partial \theta \partial r} = \frac{M}{2(\sin 2\alpha - 2\alpha \cos 2\alpha)} \frac{2}{r^2} \cos 2\theta - \cos 2\alpha \]  

(7.9c)

Eq.(7.9) satisfies the following boundary conditions.

\[ \sigma_\theta = \tau_{r\theta} = 0 \quad \text{for} \ \theta = \pm \alpha \]  

(7.10)

Now, we apply the elastic solutions mentioned above to the shear key configuration shown in Fig.7.3. Fig.7.3a shows the relation between the shear key loading configuration and the representative loads acting on the vertex of the wedge or the corner of the shear key. The relations are expressed as follows.

\[ V = F_1 = \tau_s D \]  

(7.11a)

\[ H = F_2 = \sigma_p D \]  

(7.11b)

\[ M_0 = F_1 \frac{L}{2} + F_2 \frac{D}{2} \]  

(7.11c)

where \( \tau_s \) is the averaged shear stress along the key base, and \( \sigma_p \) is the prestressing and assumed to be 2.07 MPa in this calculation. In the figure, three point of A, B and C are introduced. These points stand for the location in the calculation of principal strain, and are identical to the location of the triangular elements in FEM analysis(see Fig.5.19c). From the previous discrete crack FEM analysis, it was found that the significant strain localization was observed around these three points as shown in figures 5.32-5.33. Fig.7.3b shows the wedge forces acting on cracked configuration of the shear key, and these forces are related to the shear key loading shown in Fig.7.3a by the following equations.

\[ P_1 = V \cos \theta_2 - H \sin \theta_2 \]  

(7.12a)

\[ P_2 = -V \sin \theta_2 - H \cos \theta_2 \]  

(7.12b)

\[ M = M_0 \]  

(7.12c)

As concluded in Chapter 5, the principal strain vectors could show well the transition between S crack and M cracks rather than the principal stress vectors. Therefore, we calculate the principal strain by the following translation between principal stress and strain.
Fig. 7.3: Wedge Forces Acting on Cracked Shear Key
\[ \varepsilon_r = \frac{1}{E} (\sigma_r - \nu \sigma_\theta) \]  
(7.13a)

\[ \varepsilon_\theta = \frac{1}{E} (\sigma_\theta - \nu \sigma_r) \]  
(7.13b)

\[ \gamma_{r\theta} = \frac{2(1+\nu)}{E} \tau_{r\theta} \]  
(7.13c)

where \( \nu \) is assumed as 0.2, and

\[ \varepsilon_1 = \frac{\varepsilon_r + \varepsilon_\theta}{2} + \frac{1}{2} \sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + \gamma_{r\theta}^2} \]  
(7.14a)

\[ \varepsilon_2 = \frac{\varepsilon_r + \varepsilon_\theta}{2} - \frac{1}{2} \sqrt{(\varepsilon_r - \varepsilon_\theta)^2 + \gamma_{r\theta}^2} \]  
(7.14b)

7-2-2. Comparison With FEM Results

Now, we calculate the principal strain along the key base using the above-mentioned formulation, and compare it with the FEM results by the discrete crack FEM model D2. Here, it should be noted that the area along the key base in the discrete crack FEM model is assumed as a linear elastic material as mentioned in Chapter 5. Here, the discrete crack path in the FEM analysis is not completely separated because of the tension-softening constitutive element. However, this effect is assumed relatively small in this calculation. Therefore, the present elastic solutions are directly applied to this comparison. Figures 7.4-7.6 show the comparison of the principal strain versus averaged shear stress relations between the present elastic solutions and the discrete crack FEM results (see Fig.5.34) for the point A, B and C, respectively. In the figures, EP1, EP2 stand for the tensile principal strain and compressive principal strain, respectively. From these figures, we may conclude that the present formulation could predict the principal strain distribution of the cracked shear key joint up to the averaged shear stress of 5.0 MPa while there is relatively large deviation after that point. This point of 5.0 MPa was the transition between S crack and M cracks as discussed in Chapter 5. Thus, the present elastic solution based on the changed boundary condition concept could express the transition in the shear key configuration. It is also mentioned that the small deviation at the lower shear stress may indicate the incomplete formation of the discrete crack or the boundary condition of FEM analysis.

In reality, we may observe a stress singularity near the discrete crack tip. Such phenomenon is expressed by the following equations, and schematically in Fig.7.7.
Fig. 7.4: Comparison of Predicted Principal Strain at Point A

Fig. 7.5: Comparison of Predicted Principal Strain at Point B
Fig. 7.6: Comparison of Predicted Principal Strain at Point C
Fig. 7.7: Stresses near Crack Tip
Fig. 7.8: Calculation of Tensile Principal Strain
\[ \sigma_x = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\psi}{2} \left(1 - \sin \frac{\psi}{2} \sin \frac{3\psi}{2} \right) \]  
(7.15a)

\[ \sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\psi}{2} \left(1 + \sin \frac{\psi}{2} \sin \frac{3\psi}{2} \right) \]  
(7.15b)

\[ \tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos \frac{\psi}{2} \sin \frac{\psi}{2} \cos \frac{3\psi}{2} \]  
(7.15c)

However, the key base is located far from the crack tip field, and therefore these stress acting on the key base is small compared with the stresses induced by the concentrated load on the vertex of the wedge. Specifically, the value of \( r \) in Eq.(7.15) is large, and therefore the stresses are small. Thus, we ignore the influence on the stress filed along the key base by the crack tip stress singularity in the present analysis.

Since the above comparison is applicable to only the case that the stress field along the key base is overall linear elastic, it can not identify the transition between S crack and M cracks if the tensile stresses in some elements along the key base exceed the tensile strength at the transition. Therefore, next we consider the principal strain distribution obtained by the smeared crack FEM analysis in Chapter 5. Here, the area along the key base in the smeared crack FEM model is assumed as an elastic-softening material as mentioned in Chapter 5. Specifically, the tensile stress in this region can not exceed the tensile strength in this FEM calculation. Therefore, the present elastic solutions can not be directly applied to this comparison. To apply the present elastic solutions to such non-linear problem, the following calculation procedures are carried out: (1) The tensile and compressive principal stresses, \( \sigma_1 \) and \( \sigma_2 \), are first calculated from the present elastic solutions; (2) The tensile principal strain \( \varepsilon_p \) is calculated from the tensile principal stress \( \sigma_1 \) based on the 'principle of the conservation of strain energy' as shown schematically in Fig.7.8; This figure shows that the area under linear stress-strain curve obtained by the elastic solutions is equated to the area under the bi-linear tension softening curve (see Fig.4.9a) up to the assumed principal tensile strain; (3) The compressive constitutive law is linear up to the compressive strength (calculation will be stopped at the compressive strength). Here, it should be noted that the second calculation can not satisfy the equilibrium condition. However, the total energy is equilibrated. Since the elastic solutions are used here, this approach is adopted to calculate the principal strain as an elasticity-based solutions. Using the principal stress calculated from Eqs.(7.6), (7.7) and (7.9), we calculate the principal strain as follows.
Fig. 7.9: Comparison of Predicted Principal Strain at Point A

Fig. 7.10: Comparison of Predicted Principal Strain at Point B
Fig. 7.11: Comparison of Predicted Principal Strain at Point C
\[ \varepsilon_1 = \varepsilon_p - \frac{v}{E} \sigma_2 \]  
(tension) \hfill (7.16a)

\[ \varepsilon_2 = \frac{1}{E} \sigma_2 \]  
(compression) \hfill (7.16a)

where \( \varepsilon_p \) is the calculated tensile strain as shown in Fig.7.8. Here, it should be mentioned that Poisson’s effect by the tensile stress on the compressive strain is ignored. This is because the tensile stress \( \sigma_1 \) calculated from the elastic solutions is not a correct tensile stress which should be actually less than the tensile strength, and therefore the tensile stress \( \sigma_1 \) loses the physical meaning in this calculation.

Then, we calculate the principal strain along the key base using the above-mentioned formulation, and compare it with the prediction by the smeared crack FEM model. Figs.7.9-7.11 show the comparison of the principal strain versus averaged shear stress relations between the present elasticity-based solutions and the smeared crack FEM results; for the point A, B and C (see Fig.7.3), respectively. In the figures, EP1, EP2 stand for the tensile principal strain and compressive principal strain, respectively. From these figures, it can be concluded that the present elasticity-based solutions could predict the principal strain distribution of the cracked shear key joint up to the averaged shear stress of 8.0 MPa while there is relatively large deviation after that point. This point of 8.0 MPa is larger than the transition point between S crack and M cracks. Thus, present elasticity-based solutions based on the changed boundary condition concept could express the transition between S crack and M cracks in which the tensile softening phenomenon is allowed. It is also mentioned that the small deviation at the lower shear stress may indicate the incomplete formation of the discrete crack or the boundary condition of FEM analysis.

From the above analyses by both the elastic solutions and the elasticity-based solutions, the most important finding here can be identified such that the transition between S crack and M cracks is caused by the changed boundary condition. Specifically, S crack propagation changes the boundary condition for the stress field along the key base, and the strain localization along the key base or M cracks strain localization is thus influenced by the S crack path.

7.3. SUMMARY AND CONCLUSION

In this chapter, the transition phenomenon between S crack and M cracks in shear key joints is studied analytically. Well-known elastic solutions are introduced here to analyze the principal strain distribution along the key base after S crack propagation. Then, these
elastic solutions are compared with both the discrete crack FEM analysis and the smeared crack FEM analysis in terms of the principal strain distribution along the key base.

Based on this investigation, the following conclusions may be drawn:

1. The transition between S crack and M cracks is induced by the change of the boundary condition.

2. Specifically, S crack propagation changes the geometry of the shear key, and induces the intensive strain localization along the key base resulting in the formation of M cracks.
CHAPTER 8

IMPLICATIONS IN DESIGN OF SEGMENTAL STRUCTURES

8-1. INTRODUCTION

In this chapter, the implications of the mechanical model proposed in Chapter 4 for the current design of segmental structures is investigated.

First, the current design approaches for shear capacity of segmental structures with shear key joints are discussed, and the possible contribution of the mechanical model to the design is qualitatively specified.

To demonstrate the specified contribution of the mechanical model, a nonlinear FEM analysis is carried out, emphasizing the effects of the load-displacement characteristics of joints on the fracture behavior of segmental structures.

8-2. STATUS QUO OF DESIGN FOR JOINTS

As mentioned in Chapter 1, there is currently no specific provision for the design of joints in precast concrete segmental structures in the USA or elsewhere [Bakhoum et al. 1989]. This implies that the joints are treated similarly to the adjacent concrete segments. In other words, the current design for segmental structures is employed based on the assumption of a structural monolith rather than the consideration of structural integrity of segmental structures.

The effects of joints on the structural behavior are thus not considered. For instance, the dimension of multiple shear keys in the concrete highway bridges is determined as a structural detail (see Fig.8.1) without quantitative calculation [JRA, 1990]. This is because designers strongly rely on the effect of prestressing which may possibly eliminate the shear failure of key joints. Therefore, the design approach for shear failure of segmental structures is almost identical to that of non-segmental structures, especially in the case of prestressed members. The current design for shear failure of segmental structures is then aimed at the prediction of diagonal cracking load like the design provisions for shear strength of prestressed members. The diagonal cracks can form in the web near the centroid of the member in concrete that has not been previously cracked (web shear cracks) or develop as extension of previously existing flexural cracks (flexural-shear cracks) [Collins and Mitchell, 1987].
$5 \text{ cm} \leq H \leq 10\text{ cm}$

$3H \leq V \leq 5H$

Fig. 8.1: Structural Detail of Multiple Shear Key [JRA, 1990]
To account for the possible effect of the joints on the overall strength, the Post-Tensioning Institute [PTI, 1988] recommended the use of different strength reduction factors as follows: For flexure, \( \phi_f = 0.95 \) for type A joints (cast in-situ, wet concrete or epoxy joints), \( \phi_f = 0.90 \) for type B joints (dry joints); For shear, \( \phi_v = 0.85 \) for type A joints, \( \phi_v = 0.70 \) for type B joints. Lower values are used for type B joints to account for the larger slip that might occur in these joints. It was indicated [PTI, 1988] that values of \( \phi_f \) and \( \phi_v \) were based on consideration of relatively limited test results and were considered interim provisions until further comprehensive tests, analyses and experience with completed structures were obtained.

Another example is the designing of prestressed precast concrete structures of buildings [AIJ, 1987]. It is recommended to design the joints by evaluating the ultimate shear strength of joints as follows;

\[
\tau = \mu \sigma_c \quad \text{with} \quad \mu = 0.5
\]  

(8.1)

where \( \tau \) is the average shear strength of joints (kg/cm²), \( \mu \) is the coefficient of friction and \( \sigma_c \) is the confining stress perpendicular to the joints. Here, it is recommended that the effect of the concrete/steel shear key on the shear strength of joints is ignored in the design except for a case with the exact shear strength of joints experimentally certified.

The strength reduction factors recommended by PTI seem to be able to account for the effects of the load-displacement characteristics of joints on the overall load-displacement characteristics of segmental structures. However, these factors only provide rough estimates. This is so because the fracture behavior of joints depends also on the material properties or the type of joints (flat joints or key joints) and on prestressing as has been shown in this thesis. Therefore, the prediction may or may not give conservative results.

The evaluation of shear strength of joints recommended by AIJ also seems to be rough estimates for the same reason mentioned above. In addition, only the strength of joints is considered in the design of segmental structures, but not the effects of the load-displacement characteristics of joints on the overall load-displacement characteristics of segmental structures. Therefore, the predicted overall load-bearing capacity of the whole structures may or may not be different from the actual one.

The lack of a comprehensive design approach indicated above comes from the assumption of a structural monolith in the current design. Therefore, the consideration of structural integrity by accounting for the load-displacement characteristics of joints is highly desired.
8-3. CONTRIBUTION OF MECHANICAL MODEL TO DESIGN

As shown in this thesis, the mechanical model for shearing-off failure of key joints is developed based on the observed fracture behavior. The mechanical model gives good predictions of the shear-off strength as well as the entire shear stress-slip displacement behavior in the shear key joints. From these points of view, the possible contribution of the proposed mechanical model to the design of precast concrete segmental structures mentioned in the previous section could be specified as follows.

The mechanical model can be used to identify the actual safety factor of segmental structures designed by means of the current design approach. As mentioned, the current design is based on the assumption of a structural monolith rather than the consideration of structural integrity of segmental structures. However, in reality, there exists an effect of joints on the behavior of segmental structures. By considering the load-displacement characteristics of joints and the effects on structures integrated by segments, one can see the actual safety factor of segmental structures. Specifically, the mechanical model can be incorporated in the analytical model of segmental structures for the response under the design loading, and the actual shear capacity can be obtained. This analysis will be demonstrated in the next section.

Thus, the mechanical model potentially suggests a new design approach by considering the structural integrity of segmental structures.

8-4. ANALYSIS OF SEGMENTAL BEAM

In this section, the fracture behavior of a prestressed concrete segmental beam is analyzed to examine the effect of shear key joints on the segmental structures. As mentioned, the design of segmental structures is carried out by assuming a structural monolith rather than considering the structural integrity. Therefore, two analyses are employed here: (1) segmental beam without joints; (2) segmental beam with joints. The former is identical to the situation predicted by the current design, and the latter actual situation of the segmental structure. From these analyses, one can see how the structural behavior predicted by the current design approach is different from the actual one.

8-4-1. Beam Model

The FEM analysis is carried out by using the 'DIANA Finite Element Program' used previously (see Chapter 5). The geometric and loading configuration of the beam are
shown in Fig.8.2a. The segmental beam is integrated by the prestressing tendon and loaded on two points at the center of the beam. Two kinds of beam span are considered here as shown in Fig.8.2b,c. The former is a long span beam (Beam Model 1) which failure mode could be dominated by flexure, and the latter is a short span beam (Beam Model 2) which failure mode could be dominated by shear.

Fig.8.3 shows the details of a segment of the beam. These details are also typical of precast segmental beams in general. It can be seen that in addition to the prestressing, there is transverse and longitudinal mild reinforcement which stop before the joints. There is thus no mild reinforcement crossing the joint between the segments. Each joint has two shear keys which dimension is identical to that in the shear key test by Bakhoum et al. [1989] except the thickness of 1.74 in. The thickness of the flange and web is 6 in. and 1.745 in., respectively. The material properties of concrete are identical to those used in the FEM analysis for the test of K300-DR3 by Bakhoum et al. (see Chapter 5).

The deformed prestressing bar of Grade 157 (the ultimate strength of 157 ksi or 1083 MPa and the area of 0.28 square inches) and the deformed mild reinforcing bar of Grade 60 (the yield strength of 60 ksi or 414 MPa) are used for prestressing tendon and transverse/longitudinal reinforcement, respectively. The reinforcement ratios of both transverse and longitudinal reinforcement are assumed to be 1.0 %. In the FEM analysis, both the prestressing tendon and the mild steel reinforcement are embedded in concrete with perfect bond. The prestressing tendon is initially stressed. The stress-strain relation of both prestressing tendon and mild reinforcement is assumed to be elastic-perfectly plastic.

In the case of the beam with joints, only one joint near the support shown in Fig.8.2b,c is considered, in which bending stress is relatively small. Therefore, the prestressing at the joint can be assumed to be constant at 500 psi (3.45 MPa) during the entire loading. This prestress is identical to the test of K300-DR3 by Bakhoum et al. At the joint, both the shear stress-slip displacement relation for shear key joints obtained by the proposed mechanical model and that for flat joints obtained by push-off test by Bakhoum et al. are adopted, and simplified (solid line) as shown Figures 8.4-8.5 to be implemented along the line between two adjacent elements (see Fig.8.2b,c). The curve with broken line in Fig.8.4 is identical to that in Fig.5.4, and that in Fig.8.5 is identical to that in Fig.2.8.

The beam is assumed to be in a state of plane stress. In the calculation, a smeared cracking formulation with fixed multiple smeared cracks is employed. A shear retention factor \( \beta=0 \) and a threshold angle \( \alpha=60^\circ \) are assumed (see Chapter 5). The mesh profile is shown in Fig.8.2b,c. The finite plane stress elements consist of eight-node quadratic quadrilaterals (3x3 integration scheme). The joints are modelled using quadratic line
Fig. 8.2: (a) Elevation of Segmental Beam and Mesh Profile of (b) Beam Model 1; (c) Beam Model 2
Fig. 8.3: Detail of Elevation and Cross Section of Segmental Beam
Fig. 8.4: Assumed Shear Stress-Slip Curves for Key Joints

Fig. 8.5: Assumed Shear Stress-Slip Curves for Flat Joints
interface elements (nodal-lumping scheme), where normal traction-normal relative displacement relation is assumed linear elastic (monolithic) and the tangential traction-tangential relative displacement relation is modelled by the curves (solid line) in figures 8.4-8.5.

8-4-2. Results Of Analysis

Based on the above-mentioned condition, a nonlinear FEM analysis is carried out, and the results are shown in figures 8.6-8.11. Figures 8.6, 8.7 and 8.10 show the results of Beam Model 1 (long beam), and figures 8.8, 8.9 and 8.11 show those of Beam Model 2 (short beam).

Figures 8.6a and 8.7a show the highly amplified deformed configuration for the vertical displacement of 10.0 mm in the Beam Model 1. Here, the small vertical slip at the joint is observed in Fig.8.7a.

Figures 8.6b and 8.7b show the principal strain vectors for the vertical displacement of 10.0 mm. The size of the lines is relative to the magnitude of the strain, and the orientation is the actual direction to the strain. Positive values are plotted with a solid line, negative values with a dashed line. From these figures, it can be seen that the tensile strain of the Beam Model 1 without joints (predictions by the current design approach) is concentrated both at the centroid of the web in the middle span and at the bottom of web and flange in the loading point, while that of Beam Model 1 with joints only at the bottom of web and flange in the loading point. This can be explained such that the vertical slip of the joint reduces the stiffness of the beam, and induces the flexural strain concentration due to stress redistribution.

This phenomenon is associated with the obtained cracking patterns for vertical displacement of 10.0 mm shown in figures 8.6c and 8.7c. In these figures, only cracks associated with tensile strains of $\varepsilon_{tu1}$ (see Fig.4.9a) or higher are shown. Here, it should be noted that the size of lines for cracks is relative to the element size (not relative to the crack strain). It can be seen that the diagonal shear crack at the centroid of the web in Beam Model 1 without joints is much larger than that in Beam Model 1 with joints. Thus, the failure mode of the Beam Model 1 without joints is close to the flexural-shear crack mode, while that with joints is flexural crack mode.

Figures 8.6d and 8.7d show the plasticity status of reinforcement for the vertical displacement of 10.0 mm. Here, it should be noted that the size of the triangle is relative to the magnitude of the plastic strain of reinforcement. It can be seen that plastic strain of
Fig. 8.6: (a) Deformed Configuration; (b) Principal Strain Vectors; (c) Cracking Pattern and (d) Plasticity Status of Reinforcement for Vertical Displacement of 10 mm in the Beam Model 1 without Joints
Fig. 8.7: (a) Deformed Configuration; (b) Principal Strain Vectors; (c) Cracking Pattern and (d) Plasticity Status of Reinforcement for Vertical Displacement of 10 mm in the Beam Model 1 with Joints
Fig. 8.8: (a) Deformed Configuration; (b) Principal Strain Vectors; (c) Cracking Pattern and (d) Plasticity Status of Reinforcement for Vertical Displacement of 3 mm in the Beam Model 2 without Joints
Fig. 8.9: (a) Deformed Configuration; (b) Principal Strain Vectors; (c) Cracking Pattern and (d) Plasticity Status of Reinforcement for Vertical Displacement of 3 mm in the Beam Model 2 with Joints
reinforcement is more concentrated at the bottom of web and flange in the loading point in the Beam Model 1 with joints.

From the above-mentioned observation, it can be concluded that even though the structure is dominated by flexural failure, the load-displacement characteristics of the joints can influence the fracture behavior of segmental structures or may possibly change the failure mechanism.

Figures 8.8a and 8.9a show the highly amplified deformed configuration for the vertical displacement of 3.0 mm in the Beam Model 2. Here, the large vertical slip at the joint is observed in Fig.8.9a.

Figures 8.8b and 8.9b show the principal strain vectors for the vertical displacement of 3.0 mm. From these figures, it can be seen that the tensile strain of the Beam Model 2 without joints (prediction by the current design approach) is concentrated at the centroid of the web in the middle span, while that of Beam Model 2 with joints is concentrated near the joint. This can be explained such that the large vertical slip of joint induces the shear stress concentration near the joint like direct shear test, and reduces the tensile strain at the centroid of the web in the middle span.

This phenomenon is associated with the obtained cracking patterns for vertical displacement of 3.0 mm shown in figures 8.8c and 8.9c. In these figures, only cracks associated with tensile strains of $e_{u1}$ (see Fig.4.9a) or higher are shown. It can be seen that the fracture of the Beam Model 2 without joints is dominated by the diagonal shear crack at the centroid of the web, while that of the Beam Model 2 with joints by the diagonal shear crack along the joint. Thus, the failure mode of the Beam Model 2 without joints is a web shear crack mode, while that of the Beam Model 2 with joints is a shear-off failure mode.

Figures 8.8d and 8.9d show the plasticity status of reinforcement for the vertical displacement of 3.0 mm. It can be seen that plastic strain of reinforcement in the Beam Model 2 without joints is concentrated at the bottom in the loading point, while that in the Beam Model 2 with joints is concentrated at the upper portion in the joint.

From the above-mentioned observation, it can be concluded that as in the structure dominated by flexural failure, the joints can influence largely the fracture behavior of segmental structures dominated by shear failure and can change the failure mechanism.

Fig. 8.10 shows the load-displacement curves of the Beam Model 1 both without and with joints. In addition, the reduced shear strength recommended by PTI is introduced in the figure. This quantity is calculated by multiplying the maximum shear force of the Beam Model 1 without joints by the reduction factor of $\phi_f = 0.90$ (dry joints for flexure). From the figure, one can see that the flexural strength of the Beam Model 1 with joints is smaller.
Fig. 8.10: Load-Displacement Curves for Beam Model 1

Fig. 8.11: Load-Displacement Curves for Beam Model 2
than that of the Beam Model 1 without joints, and also smaller than the PTI recommendation in this case.

Regarding the slippage of the joint, at the vertical displacement of 4.5 mm (see point A in Fig.8.10), the shear stress of the shear key is observed 6.87 MPa which is on the first branch of the shear stress-slip curve shown in Fig.8.4. At the vertical displacement of 10.0 mm (see point B in Fig.8.10), the shear stress of the shear key is observed 7.58 MPa which is still on the first branch of the shear stress-slip curve shown in Fig.8.4. Thus, the shear stress of shear key does not reach the maximum value during the entire loading.

Fig. 8.11 shows the load-displacement curves of the Beam Model 2 both without and with joints. In addition, the reduced shear strength recommended by PTI is introduced in the figure. This quantity is calculated by multiplying the maximum shear force of the Beam Model 2 without joints by the reduction factor of $\phi_f = 0.70$ (dry joints for shear). From the figure, one can see the shear strength of the Beam Model 2 with joints is much smaller than that of the Beam Model 2 without joints. It should also be noted that the maximum shear force of the Beam Model 1 with joints is almost identical to the PTI recommendation. However, after the maximum shear force is reached, the Beam Model 2 with joints shows a very brittle behavior. Thus, PTI recommendation may not be appropriate in this case.

Regarding the slippage of the joint, at the vertical displacement of 1.0 mm (see point A in Fig.8.11), the shear stress of the shear key is observed 9.6 MPa which is on the first branch of the shear stress-slip curve shown in Fig.8.4. At the vertical displacement of 2.775 mm (see point B in Fig.8.11), the shear stress of the shear key is observed 13.2 MPa which is just on the maximum stress point of the shear stress-slip curve shown in Fig.8.4. At the vertical displacement of 4.0 mm (see point C in Fig.8.11), the shear stress of the shear key is observed 5.0 MPa which is on the horizontal branch, and the shear slip displacement is larger than the value of 2.0 mm at the point C. Thus, the shear key is fully fractured during the loading. After the point C in the figure, the brittle failure of shear key could be observed.

Finally, the following conclusions are drawn.

1. For flexural failure-dominated geometries, the load-displacement characteristics of joints reduces the flexural strength of segmental structure which is smaller than the PTI recommendation for the case considered in this thesis. In addition, a flexural stress concentration is induced. In this case, the shear stress of the shear key does not reach the maximum value, and the nonlinear shear stress-slip displacement relation of the key before the shear-off strength becomes important for the behavior of segmental structures.
2. For shear failure-dominated geometries, the load-displacement characteristics of joints reduces largely the shear strength of segmental structure which is close to the PTI recommendation for the case considered in this thesis. In addition, the failure mode changes from a diagonal shear crack mode to a shear-off failure mode of joints. Here, although the PTI recommendation is close to the analytical prediction, the segmental structures with shear failure-dominated geometries show a very brittle behavior, and in this sense the PTI recommendation might not be appropriate. In this case, the shear stress of the shear key does reach the maximum value or the post-peak horizontal branch, and the nonlinear shear stress-slip displacement relation of the key after the shear-off strength becomes important for the behavior of segmental structures.

3. The above-mentioned analytical results demonstrate the potential contribution of the mechanical model to the current design approach mentioned in the previous section.

8-5. SUMMARY AND CONCLUSION

In this chapter, the involvement of the proposed mechanical model in the current design of segmental structures is investigated.

The current design approaches for shear capacity of segmental structures with shear key joints are discussed, and the possible contribution of the mechanical model to the design is qualitatively specified.

Then, a nonlinear FEM analysis is carried out, emphasizing the effects of the load-displacement characteristics of joints on the behavior of segmental structures to demonstrate the specified contribution of the mechanical model.

Based on this investigation, the following implications of the mechanical model in the current design of segmental structures may be drawn:

1. The mechanical model can be used to identify the actual safety factor segmental structures designed by means of the current design approach.

2. The mechanical model potentially suggests a new design approach by considering the structural integrity of segmental structures.
CHAPTER 9

CONCLUSION AND RECOMMENDATION

9.1. CONCLUSION

This thesis focuses on the development of the fracture mechanics based modelling strategy of shear failure of concrete through the shear-off failure of shear key joints. Specifically, based on intensive studies on the shear failure of concrete-like materials in many investigations reported in the literature, the present work aims at developing a very simple mechanical model for the analysis and design of plain concrete and fiber reinforced concrete shear key joints. Here, the terminology 'Mechanical Model' mentioned above is only used for the proposed model in this research work. The present mechanical model describes the macroscopic cracking behavior for the shear-off failure mechanism of shear keys based on fracture mechanics concepts. The method makes use of well-known results of fracture mechanics and truss model theory, combined in a simple model. Unlike numerical methods based on finite element computations, the mechanical model does not consider the exact boundary condition, and is therefore approximate. However, it gives good predictions of the shear-off strength as well as the entire shear stress-slip displacement behavior observed in push-off shear tests reported in the literatures [Bakhoum et al. 1989, Bakhoum 1991, Beattie 1989, Beattie et al. 1989] and in similar geometries such as membrane elements and deep beams [Vecchio and Collins 1986, Leonhardt and Walther 1966, Mansur & Ong 1991]. The proposed approach is thus a powerful tool for the analytical prediction of the behavior of shear keys and associated structural systems (potentially a powerful tool for the analysis of general shear failure of concrete), despite the relatively simple assumptions it is based on.

The elegance of the proposed mechanical model lies not only in the simplicity and accuracy of the model but also in the fact that the necessary parameters (e.g., constitutive laws) are kept in the model so that design sensitivities can be generated, parametric studies can be carried out, and reasonable first estimates of the member sizes can be made. Thus, the mechanical model is viewed as an efficient design tool, and therefore it gives a rational basis for developing a suitable design methodology for shear key joints of plain and fiber reinforced concrete or other similar materials.

In summary, the following results and conclusions are obtained in this thesis.
Proposal of mechanical model
Based on both the physical description of the shear-off failure of key joints and the fracture mechanics based modelling strategy, which are identified from the literature review, a very simple mechanical model for shear-off failure in plane and fiber reinforced concrete shear key joints using a fracture mechanics approach was developed.

The formulation in the proposed mechanical model identifies two main fracture mechanisms for shear-off failure of key joints: single curvilinear crack and multiple diagonal cracks. The analysis employs both the discrete crack model (Wedge Crack Model) formulated by linear elastic fracture mechanics (LEFM) results and the smeared crack model (Rotating Smeared Crack band Model) formulated by the truss model theory, based on the physical description of the shear failure of key joints. Then, the two models are combined to predict the entire shear stress-slip displacement behavior of shear-off failure. Thus, this mechanical model is developed by means of theoretical approaches which are different from numerical approaches such as Finite Element Method (FEM).

As a first step in developing design aids for the shear key joints, a simple design formula for the shear-off strength of shear key joints is also proposed.

Verification of proposed mechanical model
The verification of proposed mechanical model is carried out by means of the comparison of the prediction by the mechanical model with both experimental data and nonlinear FEM analysis results.

In the comparison with experiments, two kinds of comparison are achieved here: entire load-displacement relations and the shear-off strength. The mechanical model predicts reasonably well not only the shear-off strength but also the complete load-displacement response for the entire range of loads reported in the literature. Another simulation with a modified constitutive law could have resulted in even better agreement between test data and the predictions. However, the current results already show that the fracture behavior of plain and fiber reinforced concrete shear key joints was reliably captured by the mechanical model proposed in this thesis, and any additional parameter refinement would be just an exercise in curve fitting.

The proposed simple design formula for the shear-off strength of shear key joints predicts well the experimental results.

In the comparison with nonlinear Finite Element Method (FEM) analysis results, the nonlinear FEM model employed in this thesis can predict the cracking behavior observed in the experiment, and agrees closely with the load-displacement results from the proposed mechanical model up to the point where the peak shear stress is reached. However, the
nonlinear FEM model can not trace the post-peak behavior of the test. This may indicate that the stress drop after the peak stress is the result of compressive crushing, which is properly accounted for in the proposed mechanical model.

**Applicability to other geometries**

The applicability of the mechanical model proposed in Chapter 4 to macroscopic shear failure of both reinforced concrete membrane elements subjected to in-plane shear and reinforced concrete deep beams is examined in terms of the load-displacement relations. The mechanical model is then modified by incorporating the softening compression strut concept into Rotating Smeared Crack Band Model.

It is then concluded that the modified mechanical model predicts reasonably well not only the ultimate shear strength but also the complete load-displacement response for the entire range of loads in the reinforced concrete membrane elements and reinforced concrete deep beams.

**Transition between S crack and M cracks**

The transition phenomenon between S crack and M cracks in shear key joints is studied analytically. Well-known elastic solutions are introduced to analyze the principal strain distribution along the key base after S crack propagation. Then, these elastic solutions are compared with both the discrete crack FEM analysis and the smeared crack FEM analysis in terms of the principal strain distribution along the key base.

It is then concluded that the transition between S crack and M cracks is induced by the change of the boundary condition. Specifically, S crack propagation changes the geometry of the shear key, and induces the intensive strain localization along the key base resulting in the formation of M cracks.

**Implications in the design of segmental structures**

The implications of the proposed mechanical model in the current design of segmental structures is investigated. The current design approaches for shear capacity of segmental structures with shear key joints are discussed, and the possible contribution of the mechanical model to the design is qualitatively specified. Then, a nonlinear FEM analysis is carried out, emphasizing the effects of the load-displacement characteristics of joints on the behavior of segmental structures to demonstrate the specified contribution of the mechanical model.

It is then concluded that the mechanical model can be used to identify the actual safety factor of designed segmental structures by means of the current design approach, and can...
suggest a new design approach by considering the structural integrity of segmental structures.

9-2. RECOMMENDATION

The analytical investigation described in this thesis present an initial effort to develop the fracture mechanics based modelling strategy of shear failure of concrete. Possible areas for future research work are described as follows.

One can find in the literature reports of a certain number of test results to measure the shear stress and the slip across shear-off planes in shear key joints. Additional experimental work for the push-off shear test would provide more information with regard to the deviation between the test results and the prediction by the proposed mechanical model. Specifically, the different dimensions in terms of length or depth of the shear key specimens and the different material parameters such as the compressive strength should be included in these additional experimental works. In addition, the exact measurement of shear stress along the key base should be carried out to compare the proposed mechanical model with the test results. In this case, the friction in the contact face between two specimens except the trapezoidal shaped key surface should be removed.

The shear-off failure of shear key joints is a very complicated process and the present mechanical model leaves many aspects of the problem unexplained, but the approach appears to have much promise and to be capable of further development. To improve the model further, the interaction between S crack M cracks should be examined both analytically and experimentally to give an objective criterion to determine the transition between Wedge Crack Model and Rotating Smeared Crack Band Model.

In this thesis, the proposed mechanical model was applied to other but similar geometries. There is generally good agreement between prediction and experimental data but some aspects such as the ambiguous softening compression strut concept is not fully developed and understood. For further improvement, additional simulation based on the physical basis for such cracking behavior should be carried out.

As shown in this thesis, the proposed mechanical model can suggest a new design approach for segmental structures by considering the structural integrity. To develop such new design methodology, further comparison with the current code is desired.

Finally, the fracture mechanics based modelling strategy of shear failure of concrete introduced in this thesis may hopefully simulate studies of new structural systems made of advanced materials. As an initial effort to develop such structural systems, the present mechanical model could be used to assess the fracture behavior of structural members.
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