ACTIVE CONTROL OF PIPE-BORNE PUMP NOISE

by

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Abstract

A water loop that simulates part of the cooling system of a submarine exists in an MIT Laboratory. The loop consists of a water holding tank, rubber hose, two sections of steel pipes, a throttle, and a centrifugal pump. The hose simulates the ocean acoustically. This project explores a method for minimizing water-borne noise in the loop, using sensors and actuators with feedforward and feedback control.

A mathematical model of the broadband response of the loop was developed. It uses the plane wave approximation, and neglects the reflection from the centrifugal pump and elbows of the piping system. The reflection coefficient from the steel rubber interface was estimated to be -0.6. The rubber hose was modeled to be infinite in length and the coupling between the walls of the pipe and the flowing fluid was approximated as quasi-static. Predictions correlated well with the broadband response of the experiment, suggesting that the simplifying assumptions are valid, and that active control of the acoustic mode is a sensible technique to implement.

To absorb the acoustic output of the pump operating at a static pressure of 150 psi, the actuator must vary 0.148 cm$^3$ at 30 Hz and exert a pressure perturbation of 0.75 psi. Using this specification, possible actuators were considered. The actuator of choice is a flex-tension transducer, using a piezoceramic stack.

The control algorithm of choice is adaptive feedforward to eliminate the narrowband disturbances and feedback or adaptive feedforward to eliminate the broadband noise. Very preliminary design of such a control system is considered.

Thesis Supervisor: Dr. Andreas von Flotow
Title: Associate Professor of Aeronautics and Astronautics
Dedication

To MOM and DAD
with Love
Acknowledgment

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List of Variables

a = radius of the steel pipe in the experiment = 2 inches
A = cross sectional area of the pipe = \( \pi a^2 \)
\( A_{act} = 2\pi aL \)
A_1 = actuator design cross sectional area
c = speed of sound in water in a pipe
c_0 = speed of sound in water = 1500 m/s.
c_1 = speed of compressional waves in steel pipe
c_r = speed of sound in water in the rubber hose
C = capacitor
C_1 = actuator bulk compliance
C(s) = compensator
d = diameter of the steel pipe in the experiment = 4 inches
d_{max} = maximum displacement
d_{31} = piezoelectric constant in the 31 direction
d_{33} = piezoelectric constant in the 33 direction
d(s) = input from the centrifugal fan pump
E = Young's modulus of the pipe's material
E_p = Young's modulus of the piezoelectric material
E_r = Young's modulus rubber
E_s = Young's modulus steel
f_0 = resonance or ring frequency of the pipe
F = force
F_b = force exerted by the constrained piezoelectric stack.
G(s) = transfer function of the actuator
H(s) = plant transfer function from disturbance to output
I = current
k_p = stiffness of the piezoelectric spring
K = wave number
= \( \omega/c \)
$K_0$ = wave number with the speed of sound in water as reference
  = $\omega/c_0$

$K_1$ = ratio of change in volume per unit change in voltage of an
  actuator

$l$ = length of piezoelectric stack = $Nt$

$\Delta l$ = elongation of the piezoelectric stack

$m$ = mass

$N$ = number of piezoelectric disks

$p$ = pressure inside a ring

$p_{\text{max}}$ = maximum excess pressure

$P$ = pressure

$P_0$ = reference pressure = $1\times10^{-6}$ Pa

$P_b$ = pressure actuator feels if it is constrained

$P_r$ = pressure in the rubber hose

$r_1$ = outer radius of spherical actuator

$R$ = average radius of spherical actuator

$t$ = thickness

$T$ = kinetic energy

$u$ = total flow velocity in the pipe

$u'$ = acoustic velocity perturbation in the pipe

$u_0$ = the mean flow velocity in the pipe.

$U$ = strain energy

$u(s)$ = input from actuator

$v$ = velocity

$v_r$ = velocity in the rubber hose

$V$ = volume

$V_{\text{act}}$ = volume change caused by the actuator

$\Delta V$ = change in volume

$\Delta V_f$ = change of the piezoelectric with no external force

$w$ = thickness of steel pipe wall = .25 inch

$Y$ = admittance

$y(s)$ = transmitted pressure

$\varepsilon$ = strain

$\varepsilon_r$ = permittivity of the material

$\kappa$ = compressibility of the material.

$\rho$ = mass density of water
\( \rho' \) = perturbed mass density of water
\( \rho_0 \) = unperturbed mass density of water = 1000 kg/m\(^3\)
\( \rho_1 \) = density of pipe's material
\( \rho_p \) = density of piezoelectric material
\( \sigma \) = stress
\( \nu \) = Poisson's ratio for the steel pipe
\( \omega \) = frequency
1. Introduction

Noise reduction techniques for pipe structures can be beneficial to both commercial and military sectors. Imagine a Porche 914 racing down the freeway with a "noiseless" exhaust system and a factory operating with "silent" machineries. When implemented correctly, noise reduction devices result in a quieter atmosphere and a more efficient system.

For military applications, both efficiency and silence are at a premium to avoid the possibility of detection by the adversary. As the technology of sensors progresses, the demand for quieter transport vehicles increases. This demand is especially apparent in submarines.

A prominent source of detection for a submarine that is attempting to conceal itself originates from the cooling system. This system regulates temperature and therefore, unlike the engine of the vehicle, can never be turned off. The pipes that carry cooling water to and from the machinery have obvious prominence as potential 'noise shorts' simply because they are so large relative to the other connectors. The proposed problem is to find a method to minimize the noise that is radiated underwater.

A cooling system in a typical submarine is driven by a centrifugal pump. This pumps sea water through the necessary machinery for the purpose of temperature regulation before discharging the water back out to the sea. The pipes in the cooling system are open to the sea at both ends. Consequently, they must be isolated mechanically to insure that vibration of the pipes do not result in acoustic noise. The pipes also provide a direct path through which water-borne noise from the propagating fluid can be transmitted. Another potential problem is that when excitation frequency coincides with a natural frequency of the fluid contained in the pipes, the fields both
within and radiated from the pipe are amplified. The most prominent water bourne sound that must be minimized is the shaft and blade passage frequency of the fan and their harmonics.

A large literature exists on the mitigation of noise propagating in air ducts [1,2,3,4,5], and fluid filled ducts [7,8,9,10,11]. The studies conducted by Burgess [10], and Snyder and Hansen [11] were theoretical. Successful demonstrations [6,21,22,23,24,25,26,27] and several commercial products were also demonstrated. Reducing the shaft and blade passage frequencies in an air duct has been explored [1,5]. Sound propagation in water-filled pipes, with or without mean flow, shares many characteristics with this air duct situation. However, there are sufficient different issues to warrant a new study. One difference is that in water-filled pipes the elastic wall motion generally participates significantly in the acoustic response. Another difference is that the wavelength-to-diameter ratio is generally much greater in water-filled pipes than in air ducts. Culbreth, Hendricks, and Hansen, [8], demonstrated that periodic sound in a fluid filled plastic pipe may be reduced significantly by active means.

The two common methods of changing the acoustic output of a system are passive and active approaches; each have its own advantages and disadvantages. Both techniques are described in the following sections.

1.1 Passive Techniques for minimizing noise propagation

Passive techniques are typically achieved with an intrinsic geometric reconfiguration with no addition of energy. Some examples are the modification of the walls of the pipe and the insertion of a different material into the piping system. Both of these approaches can change the acoustic response of the system significantly. The impedance mismatch of the system attenuates the sound. Neise, and Koopmann explored the effect of the addition of resonators [5] and
Purshouse added baffles to a system [7]. Passive approaches are widespread, and introduce no risk of instability. However, tuned passive techniques are generally only applicable to some predetermined acoustic output, and would not be effective if operating conditions were changed. Another limitation is that for a variety of flow conditions, cutoff geometries, and inlet configurations often encountered, passive techniques are difficult and sometimes impossible to implement effectively, particularly for low frequencies. These limitations make active control of noise reduction an attractive alternative.

1.2 Active Techniques for minimizing noise propagation

Unlike passive techniques, active noise cancellation requires additional energy. One advantage of active control is that the existing piping system may be used. This could preserve the invested capitals and minimize the changes in future designs. The only necessary modifications are to mount actuators and a control system in the pipe. More specifically, the active control of noise suppression may be robust to changes in performance due to natural system deterioration, unanticipated environmental changes, and varying operating conditions. The controller senses the acoustic output far downstream of the system, then reacts by transmitting a signal, equal in magnitude but 180 degrees out of phase, into the water and cancels the incoming noise. Active techniques promise better results, but introduce the risk of unstable interaction with system dynamics.

Because the level of sound is a mission critical specification for military application, it was decided that active control, as opposed to passive control, of noise suppression was the more advantageous technique to use.

In chapter 2, the experimental setup for simulating the cooling system of the submarine is described. The piping system is shown in
Fig. (1.1). Chapter 3 discusses the mathematical models of the experimental setups and shows some simulations of the models. The predictions from the models were compared to the results obtained on the experimental setups. Good correlations between the two were observed. Consequently, it was decided that active control of noise cancellation is a practical application for the experimental setups. Actuator choices and configurations were explored in detail in chapter 4. A design and mounting scheme were chosen. Some simulations of the actuator with the experimental setup are presented in the chapter 5. Chapter 6 focuses on control algorithms. It was decided to use an adaptive feedforward notch filter to cancel the narrow band noise and a feedback or an adaptive feedforward loop to cancel the broadband noise. This thesis will then offer some possible areas for future research in chapter 7.
Fig. (1.1) Closed water loop setup at MIT Gas Turbine Laboratory.
2. Experimental Setup

A closed water loop, shown in Fig. (1.1), was designed to simulate the cooling system in a submarine. The water flows from a 600 gallon pressurized tank into two consecutive rubber hoses that are each 100 feet long. The 4.5 inch diameter hose is connected to a steel pipe. The steel pipe's inner diameter is 4 inches and the wall is 0.25 inches thick. The pipe is connected to the seven blade centrifugal fan. At the outlet of the centrifugal fan, the flow continues through a two feet steel pipe and then propagates to two more consecutive rubber hoses that are also each 100 feet long. The water is then constricted to a throttle line of 1.5 inch diameter steel pipe before it is discharged into the tank.

The hose is significantly more elastic than the pipe. The increase in elasticity causes a near pressure release boundary condition. This is a partial simulation of the water flowing out of the actual cooling system and into the sea. The throttle was placed in the loop to adjust the pressure rise across the pump.

Currently, the water loop is set up at the MIT Gas Turbine Laboratory. A concurrent project [15] on the experimental setup involves the investigation of the influence of inlet distortion on the unsteady pressure field produced by the water pump. Hydrophones mounted flush with the interior surface of the steel pipe are used for data acquisition. Experimental data presented here were measured by Barton in the course of his SM work [15].
Fig. (2.1) Path of water from the rubber hose through the steel pipe to the pump. This is configuration one. Ma, Mb, and Md indicate the locations of the hydrophones.
Fig. (2.2) Path of water from the rubber hose through the steel pipe to the pump. This is configuration two. $Ma$, $Mb$, and $Md$ indicate the locations of the hydrophones.
Fig. (2.3) Energy spectral density of the pressure at the inlet of the pump, Ma, for configuration one. The reference pressure is 10⁻⁶ Pa.
Fig. (2.4) Energy spectral density of the pressure further upstream at the inlet of the pump, Mb, for configuration one. The reference pressure is $10^{-6}$ Pa.
Fig. (2.5) Energy spectral density of the pressure at the outlet of the pump, Md, for configuration one. The reference pressure is 10⁻⁶ Pa.
Fig. (2.6) Energy spectral density of the pressure at the inlet of the pump, $M_a$, for configuration two. The reference pressure is $10^{-6}$ Pa.
Fig. (2.7) Energy spectral density of the pressure further upstream at the inlet of the pump, Mb, for configuration two. The reference pressure is $10^{-6}$ Pa.
Fig. (2.8) Energy spectral density of the pressure at the outlet of the pump, Md, for configuration two. The reference pressure is $10^{-6}$ Pa.
2.1 Energy Spectral Density

In order to determine the energy spectral density of the pipe at the inlet and outlet of the centrifugal fan, three hydrophones were mounted on different configurations. Fig. (2.1) shows the first configuration and Fig. (2.2) shows the second configuration. For the first configuration, a eight foot long steel pipe is connected to the rubber hose through a 90 degree elbow on the inlet side. For the second configuration, a 34 inch long steel pipe is connected to the centrifugal fan with a 90 degree elbow. On the inlet side of both configurations, one sensor was placed one foot from the centrifugal pump, Ma, and the another sensor was placed one foot from the rubber hose, Mb. A third hydrophone, Md, located one foot from the pump, was placed on the outlet side of the fan. The output spectrum is broadband with "spikes" at some characteristic frequency, usually that of the shaft and blade passage frequency and harmonics. From the measurements collected by the three hydrophones, the Fourier transforms of the pressures were computed. The energy spectral density vs. frequency was calculated using eq. (2.1). The energy spectral density level (ESD), in units of decibels, is defined as:

\[ E_{dB} = 10 \log_{10} \frac{P^2}{P_0^2} \]

(2.1)

The reference pressure, \( P_0 \), is 10\(^{-6} \) Pa. With the centrifugal fan operating at 1780 rpm, typical values of the energy spectral density vs. the frequency for the first configuration are shown in Fig. (2.3), (2.4), and (2.5). For the second configuration, typical values of the energy spectral density vs. the frequency are shown in Fig. (2.6), (2.7), and (2.8). The pump generates periodic noise with dominant harmonics at shaft and blade rate. The pump shaft passage frequency occurs at 30 Hz. For the seven blade centrifugal fan, the blade passage frequency occurs at 210 Hz. As evident from the figures, the greatest amount of "noise" generated by centrifugal pumps occur at the blade harmonics. As also evident from Fig. (2.3)
to (2.8), the broadband response of the systems at the two configurations are very different. Each system has a clear pole, zero configuration. However, judging from the difference in placements of the poles and zeros for each system, a resonant system was suspected.

2.1.1 Turbulent Pressures

It is unclear the amount of pressure spectra in Fig. (2.3) to (2.8) that are attributable to turbulent eddies convecting past the hydrophones. A very useful test would be to measure the coherence between the pressures at two widely spaced hydrophones, since this would suggest the fraction of the measured pressure due to an acoustic field. For such plots of coherence, refer to reference 15.

2.2 Reflection Coefficient

Because of the impedance mismatch at the steel rubber interface, there is a reflection at the intersection. Barton simultaneously measured the pressure fields at \( x = 0 \) and \( x = -s \). Assuming the presences of a single pair of plane waves, the pressures may be approximated with eq. (2.2).

\[
p(x,\omega) = A(e^{ikx} + Re^{-ikx})
\]

(2.2)

where \( e^{ikx} \) is the transmitted wave and \( Re^{-ikx} \) is the reflected wave with the reflection coefficient of \( R \). Substituting the locations where the pressures were measured yields eq. (2.3).

\[
P(-s,\omega) = A(e^{-iks} + Re^{iks})
\]

\[
P(0,\omega) = A(1 + R)
\]

(2.3)

Let \( H_{12} \) be equal to the ratio of the measured value of \( P(0,\omega)/P(-s,\omega) \). The reflection coefficient, \( R \), may be calculated with eq. (2.4).
\[ R = \frac{H_{12} e^{-iks} - 1}{1 - H_{12} e^{iks}} \]  

(2.4)

Measuring pressures and taking the fourier transform of the signals yields the cross spectrum \( H_{12} \), which is a function of frequency. Using eq. (2.4), the reflection coefficient as a function of frequency was determined. The reflection coefficient for the first and second configuration are shown in Fig. (2.9) and (2.10), respectively. Ignoring the high values of \( R(\omega) \), the reflection coefficients for the system was assumed to be a constant for frequencies between 20 and 1000 Hz, and approximated to be equal to -0.6 for both configurations.

2.2.1 Reflection Coefficient at the Pump

The acoustic impedance of the pump is unknown. Consequently, the reflection and transmission coefficients are also unknown. The same method that was used to measure the reflection coefficient at the steel rubber interface may also be used to measure the reflection coefficient at the pump. From the reflection coefficient, the percentage of transmitted wave may be calculated. Knowing the reflection coefficient of the pump would improve the model.
Fig. (2.9) Reflection coefficient at steel-rubber interface for configuration one.
Fig. (2.10) Reflection coefficient at steel-rubber interface for configuration two.
3. Simple Model of Experimental Setup

A simple mathematical model of the experimental setup was formulated to determine which effects are dominant, and which effects may be neglected. The plan was to begin modelling with the simplest assumptions that could possibility capture all the important aspects of the piping system. More complicated effects would then be considered if necessary. The experimental setup can be mainly disturbed by three different means. The excitation of the pipe structure may be due to (1) acoustic waves generated by the pump and traveling through the internal medium; (2) turbulent internal flow; and (3) mechanical forces exerted by the pump onto the pipe. The simplest model neglects turbulent internal flow, and external mechanical forces.

3.1 Plane Wave Approximation

Irrespective of how the waves are generated by the pump, it was assumed that the waves are essentially planar after only a few diameters of propagation along the pipe. This is a valid assumption, because in a circular pipe, higher order modes can only propagate if the acoustic wavelength, \( \lambda \), is smaller than 1.7 times the diameter of the pipe [12]. For the pipe in the experiment, the wavelength must be less than 0.173 meter. The corresponding frequency when higher order modes propagate is calculated with eq. (3.1).

\[
f = \frac{c_0}{\lambda}
\]  

(3.1)
The frequency must be greater than 8,337 Hz before higher order modes propagate down the pipe. Since the frequencies of interest are below 5 kHz, plane wave approximation is valid.

3.2 Resonance Frequency of the Pipe

Another critical parameter to determine is the resonance or the ring frequency, \( f_0 \), of the pipe. This frequency may be calculated with eq. (3.2)

\[
    f_0 = \frac{\sqrt{\frac{E}{\rho_1}}}{2\pi a}
\]  

(3.2)

The quantity \( \sqrt{\frac{E}{\rho_1}} \) is also known as the longitudinal wave speed in the wall material. For steel it is approximately 5,130 m/s. For a four inch diameter pipe, the ring resonance frequency is 16 kHz. Therefore, for frequencies below 5 kHz, the hoop behavior of the pipe was modeled with quasi-static stiffness.

3.3 Coupling Between the Pipe Wall and the Fluid

It also had to be determined whether the pipe was "rigid" or if it expands radially (wall is locally reacting) with each pressure pulse. This assumption is valid if the operating frequencies of interest are well below the ring frequency of the pipe. The elastic coupling of the pipe wall with the fluid flow may slow down the speed of sound in water significantly. Ingard [12] and Junger and Feit [18] determined the effect of wall compliance on sound propagation in the water filled pipe. The mass conservation equation is given by eq. (3.3).

\[
\frac{\partial \rho}{\partial t} + \text{div} (\rho u) = 0
\]  

(3.3)
Using the linearized version of eq. (3.3) and substituting \( \rho = \rho_0 + \rho' \) and \( u = u_0 + u' \) results in eq. (3.4).

\[
\frac{\partial \rho'}{\partial t} + \rho_0 \text{div} \; u' + u_0 \text{div} \rho' = 0
\]  

(3.4)

For Mach number, \( U/c, << 1 \), the third term of eq. (3.4) is insignificant and therefore may be neglected. The equation of state is as follows:

\[
\frac{1}{\rho_0} \frac{\partial \rho'}{\partial t} = \kappa \frac{\partial \rho'}{\partial t}
\]  

(3.5)

Combining eq. (3.4) and (3.5) yield eq. (3.6).

\[
\kappa \frac{\partial \rho'}{\partial t} + \text{div} \; u' = 0
\]  

(3.6)

---

![Diagram](image)

**Fig. (3.1)** Control volume of water in pipe.

Assuming a harmonic time dependence, eq. (3.6) may be rewritten as eq. (3.7).

\[
-i\omega \kappa \rho' + \text{div} \; u' = 0
\]  

(3.7)
The admittance is defined as the ratio of velocity over the pressure. Therefore the velocity is equal to the pressure times the admittance. The mass into the control volume is equal to \( \rho'u(x)A \) and the mass out is equal to \( \rho'u(x + \Delta x)A + \rho'u_12\pi a\Delta x \). The rate of change of mass inside the control volume is equal to \( i\omega \rho' \rho' \kappa A \). Therefore the mass flux is equal to eq. (3.8).

\[-i\omega \kappa Ap' + 2\pi \rho' Y + A \frac{\partial u'}{\partial x} = 0 \quad (3.8)\]

Rearranging the variables yield eq. (3.9).

\[-i\omega K_e p' = - \frac{\partial u'}{\partial x} \quad (3.9)\]

where \( K_e = \kappa(1 + i2Y/(a\omega\kappa)) \)

Combining eq. (3.9) with the linearized momentum equation given in eq. (3.10) yields eq. (3.11).

\[-i\omega \rho_0 u' = - \frac{\partial p'}{\partial x} \quad (3.10)\]

\[\frac{\partial^2 p'}{\partial x^2} + K^2 p' = 0 \quad (3.11)\]

Eq. (3.11) is also known as the wave equation. \( K \) is the wave number and is also known as the propagation constant. Eq. (3.12) shows the relationship between \( K \) and \( K_0 \).

\[\frac{K}{K_0} = \sqrt{1 + \frac{\frac{d\rho_0 c^2}{\partial p_1 c^2(1 - \frac{\omega^2}{\omega_0^2})}}}{(3.12)}\]
where $c_1 = \sqrt{\frac{E}{(1 - v^2) \rho_1}}$ and $\omega_0^2 = c_1^2/a^2$. $\omega_0^2$ is equal to $1.13E10$ (1/sec$^2$) for the experimental setup. For steel $c_1 = 5400$ m/s. Substituting $K = \omega/c$, $K_0 = \omega/c_0$, and $\omega << \omega_0$, eq. (3.12) may be rearranged to show

$$\frac{c_0}{c} = \sqrt{1 + \frac{d \rho_0 c_0^2}{\mu_1 c_1^2}}$$  \hspace{1cm} (3.13)

The ratio $c_0/c$ shows the speed of sound in water to the speed of sound in water in a pipe. For a steel pipe with diameter 4 inches, and wall thickness .25 inch, the ratio $c/c_0$ is equal to 93%. Therefore the velocity of sound in the fluid filled pipe is 1395 m/s. The calculation shows that the coupling between the walls of the steel pipe and the water is not very strong.

The hoop stiffness of the rubber hose is unknown. However, it may be calculated from the reflection coefficient obtained from the experiment. Reflection coefficient and the wave velocity of the water-borne noise in the rubber hose is related by eq. (3.14).

$$R = \left| \frac{c_s - c_t}{c_s + c_t} \right|$$  \hspace{1cm} (3.14)

From the experimentally determined reflection coefficient of -0.6, the velocity of the water-borne noise in the rubber hose was calculated to be 349 m/s. Therefore, the hoop stiffness of the rubber was approximated to be $1.5E9$ N/m$^2$.

3.4 Model of the System

Figure (2.3) to (2.8) show that the broadband response have a clear pole, zero configuration. The poles are also more highly damped at higher frequencies. The spacing between the poles and the zeros differs significantly for the two configurations. A resonant system
was suspected; this model reveals the resonance to be in the pipe axial water acoustics.

3.4.1 Mathematical Model of the Piping System

The experimental apparatus was divided into two portions, the inlet and the outlet to the centrifugal fan, for modeling purposes. Fig. (3.2) shows the model. The bends in the actual piping system were neglected. As a result, the same model was used for the two configurations. However, different lengths of the inlet steel pipes were accounted for. The steel pipe on the inlet side of the fan have coordinate y, which varies from 0 to L₁, and the steel pipe on the outlet side of the fan has coordinate x. x varies from 0 to L₂. Assuming plane waves, the general solutions for the pressures are given by eq. (3.15).

\[
\begin{align*}
P_1 &= [A_1 \ e^{i ky} + B_1 \ e^{-i ky}] \ e^{i \omega t} \\
P_2 &= [A_2 \ e^{i kx} + B_2 \ e^{-i kx}] \ e^{i \omega t}
\end{align*}
\]  

3.4.2 Boundary Conditions

The general equations have four unknowns, therefore four boundary conditions are needed. The steel rubber interface at the inlet portion of the fan occurs at y = 0 and Z = 0. The rubber hose was modeled to be semi-infinite. Therefore, the pressure in the rubber is given by eq. (3.16). The coordinates are shown in Fig. (3.3).

\[
P_r (Z,t) = P_r (Z=0,t) \ e^{i(kZ-\omega t)}
\]  

33
Fig. 3.2 Simplified model of piping system
Fig. (3.3) Close up at the steel-rubber boundary.

At the interface, the linearized momentum equation yields eq. (3.17).

\[
\rho \frac{\partial v_1}{\partial t} = -\frac{\partial P_1}{\partial y} \\
\rho \frac{\partial v_r}{\partial t} = -\frac{\partial P_r}{\partial Z}
\]

(3.17)

However, from Fig. (3.3), at the boundary, \( P_r(Z=0,t) = P_0^- \) and \( \frac{\partial v_r}{\partial t} = \frac{\partial v_0^+}{\partial t} \). Take the derivative of eq. (3.16) and combine it with eq. (3.17) to yield eq. (3.18).

\[
\rho \frac{\partial v_0^+}{\partial t} = -ikP_0^- e^{-i\omega t}
\]

(3.18)

Assuming a time harmonic dependence, eq. (3.18) may be reduced to eq. (3.19).

\[
v_0^- = \frac{1}{\rho c_r} P_0^-
\]

(3.19)

Fig. (3.3) shows that \( v_0^- = v_0^+ \) and \( P_0^- = P_0^+ \). Consequently,
eq. (3.19) may be rewritten as eq. (3.20).

\[ v_{0+} = \frac{1}{\rho c_r} p_{0+} \]  \hspace{1cm} (3.20)

Taking the time derivative of eq. (3.20) yields eq. (3.21).

\[ \frac{\partial v_{0+}}{\partial t} = \frac{1}{\rho c_r} \frac{\partial p_{0+}}{\partial t} \]  \hspace{1cm} (3.21)

Combining eq. (3.17) and eq. (3.21) yields eq. (3.22).

\[ \frac{1}{c_r} \frac{\partial p_1}{\partial t} = \frac{\partial p_1}{\partial y} \]  \hspace{1cm} (3.22)

At \( x = 0 \), which is at the rubber and steel interface at the outlet of the centrifugal fan, the rubber was again modeled as semi-infinite. Using the same reasoning as above the equation, at \( x = 0 \), is as follows:

\[ \frac{1}{c_r} \frac{\partial p_2}{\partial t} = \frac{\partial p_2}{\partial x} \]  \hspace{1cm} (3.23)

The boundary conditions at the centrifugal fan, which corresponds to \( x = L_2 \) and \( y = L_1 \) are given in eq. (3.24) and (3.25).

\[ p_1 = p_2 \]  \hspace{1cm} (3.24)

\[ v_1 = -v_2 - \frac{\dot{V}_{\text{pump}}}{A} \]  \hspace{1cm} (3.25)

The volume velocity of the pump causes motion in the water. The pressures were assumed to be equal at the interface. For a plane wave, the volume velocity is related to the pressure as given by eq. (3.26).
\[ \rho \frac{\partial v_1}{\partial t} = -\frac{\partial P_1}{\partial y} \]
\[ \rho \frac{\partial v_2}{\partial t} = -\frac{\partial P_2}{\partial x} \]

(3.26)

Multiply by the cross sectional area and take the derivative of eq. (3.25). Then combine eq. (3.25) and eq. (3.26) to obtain eq. (3.27).

\[ \frac{\partial P_1}{\partial y} + \frac{\partial P_2}{\partial x} = \rho \frac{\dot{V}_{\text{pump}}}{A} \]

(3.27)

As evident from the formula given above, the centrifugal fan was modeled as a volume acceleration source (as opposed to a pressure source). Eq. (3.22) to (3.24) and eq. (3.27) were solved simultaneously for frequencies from 0 to 5000 Hz. The ratio squared of the pressures at the locations of the hydrophones over the pump volume acceleration were determined. Comparing the calculated results with the experimental data shows good correlation of pole and zero frequencies. This is described in Fig. (3.4) to (3.9). The magnitudes of the two curves are not equal because the experimental data was referenced to 10\(^{-6}\) Pa while the calculated results were referenced to the unknown volume acceleration of the fan. For purposes of generating these curves, the volume acceleration of the pump was assumed to be white noise. The values obtained in the model predicted the broadband response of the experimental results well. The model did not attempt to account for the shaft and blade passage frequencies. Consequently, it was concluded that the simple model assumptions are valid and active control of the acoustic pipe mode is a sensible technique to consider.

Some unmodeled dynamics that might possibly have a large effect on the piping system are the pump impedance and reflection coefficient at the steel rubber interface. The pump was modeled as a velocity source with no reflection and one hundred percent transmission. This is the simplest possible model; a better model would be to
Fig. (3.4) For configuration one, at the inlet of the pump, $Ma$.
(a) energy spectral density obtained by experiment. The reference is $10^{-6}$ Pa. (b) square of the calculated transfer function. The transfer function is the ratio of the output pressure and the acceleration of the pump.
Fig. (3.5) For configuration one, close to the rubber, at the inlet of the pump, Mb. (a) Energy spectral density obtained by experiment. The reference is $10^{-6}$ Pa. (b) Square of the calculated transfer function. The transfer function is the ratio of the output pressure and the acceleration of the pump.
Fig. (3.6) For configuration one, at the outlet of the pump, Md, (a) energy spectral density obtained by experiment. The reference is $10^{-6}$ Pa. (b) square of the calculated transfer function. The transfer function is the ratio of the output pressure and the acceleration of the pump.
Fig. (3.7) For configuration two, at the inlet of the pump, Ma.
(a) energy spectral density obtained by experiment. The
reference is $10^{-6}$ Pa. (b) square of the calculated
transfer function. The transfer function is the ratio of the
output pressure and the acceleration of the pump.
Fig. (3.8) For configuration two, close to the rubber, at the inlet of the pump, Mb. (a) energy spectral density obtained by experiment. The reference is $10^{-6}$ Pa. (b) square of the calculated transfer function. The transfer function is the ratio of the output pressure and the acceleration of the pump.
Fig. (3.9) For configuration two, at the outlet of the pump, Md.
(a) energy spectral density obtained by experiment. The reference is $10^{-6}$ Pa. (b) square of the calculated transfer function. The transfer function is the ratio of the output pressure and the acceleration of the pump.
include the reflection and transmission at the pump location. Other important effects might be the curvatures, the bends of the pipe, the very slow flow velocity, or higher speed pipe modes (bending, ovaling, etc).

The measured reflection coefficient at the steel rubber interface shows that it is a function of frequency. This implies that either higher order modes play a significant role or the hydrophones are not sensing a signal representative of the acoustic wave propagating along the pipe. To determine what the hydrophones are sensing, simple calculations were carried out. An axisymmetric radial acceleration due to 0.4 psi sinusoidal pressure at 210 Hz is $10^{-6}$ g. Therefore, the acceleration sensitivity of the pressure sensor is not an issue. Lateral translation of the pipe due to bending is related to wall pressure by eq. (3.28).

$$P = \rho d \frac{\partial v}{\partial t}$$

(3.28)

For a lateral acceleration of $10^{-4}$ g, the corresponding wall pressure would be $10^{-3}$ psi. Wall pressure of significance is of the order $10^{-3}$ psi. To measure wall accelerations that might be responsible for significant wall pressures, the accelerometers must be sensitive to the order $10^{-4}$ g. The experiment that should be carried out is to mount accelerometers to determined the magnitude of wall accelerations.
4. Design of an Actuator for Active Control

4.1 Actuator Requirements

From the experimental data, the greatest pressure peaks occur at shaft and blade rate. A successful active control will send a signal equal in magnitude but 180 degrees out of phase to the incoming wave to cancel the noise. The method of generating additional signals is to physically inject a volume perturbation. As a result, the size of the actuator needed to generate the necessary pressure wave must be determined.

The maximum pressure perturbation occurs at a blade rate harmonic and has an amplitude of approximately half a percent of the pump static operating pressure[19]. This agrees reasonably well with experimental data; it was found that when the water loop was subjected to a static pressure of 60 psi, a 0.3 psi pressure fluctuation existed. The experimental setup at MIT is currently limited to a maximum static loop pressure of 150 psi. Therefore, the maximum pressure perturbation, $p_{\text{max}}$, can at most be 0.75 psi. To determine the corresponding volume perturbation that the actuator must implement, the characteristic impedance of a plane wave in water, eq. (4.1) was used.

$$d_{\text{max}} = \frac{p_{\text{max}}}{\rho_0 c_0 \omega}$$

(4.1)

The equation shows that the pressure amplitude, $p_{\text{max}}$, will result in a peak particle displacement of $d_{\text{max}}$. The volume displacement is equal to $d_{\text{max}}$ times the cross sectional area. As evident from eq. (4.1), the displacement is a function of frequency. The volume displacement at the shaft passage, blade passage, and maximum
frequency of interest, for a pressure perturbation of 0.75 psi, are given in Table (4.1).

Table (4.1) Values of volume displacement, for a pressure perturbation of 0.75 psi, at different frequencies.

<table>
<thead>
<tr>
<th>Static Loop Pressure (psi)</th>
<th>Frequency (Hz)</th>
<th>Maximum Volume (cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>30</td>
<td>0.148</td>
</tr>
<tr>
<td>150</td>
<td>210</td>
<td>0.021</td>
</tr>
<tr>
<td>150</td>
<td>1000</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

The offending noise is broadband, therefore it was decided to use electro-mechanical sources (instead of resonators) so that amplitudes and phases of the secondary signals may be controlled. Other requirements dictate that the actuators are compact, symmetrical, affordable, and do not require unreasonable drive currents or voltages. They must also be able to withstand a static pressure up to 150 psi, and have a wide frequency range. Two types of electro-mechanical sources were considered; they are the electrodynamic devices and the piezo-actuators.

4.2 Quasi-Static Model of Actuators and Fluid Loading

4.2.1 A Quasi-Static Actuator Model

For all actuators, the maximum volume perturbation, at a given applied voltage, occurs for the unloaded case. At the other extreme, a sufficient surface pressure can constrain the actuator from moving. Below the resonance of the actuator, the relationship between the surface pressure and the volume perturbation is illustrated with Fig. (4.1).
Fig. (4.1) The surface pressure vs. volume perturbation for different applied voltages. Volts1, Volts2, and Volts3 are the applied voltage.

The governing equation is given as follows:

$$
\Delta V = \Delta V_f \left(1 - \frac{P}{P_b}\right)
$$

(4.2)

$\Delta V_f$ corresponds to the unloaded case and $P_b$ is the pressure the actuator feels if it is constrained from movement. $P$ is the pressure load the actuator is driving. Since $\Delta V_f$ is a linear function of the input voltage, eq. (4.2) may be rearranged to yield eq. (4.3).

$$
\Delta V = K_1 \Delta \text{volts} - C_1 \Delta P
$$

(4.3)

$K_1$ is equal to $\Delta V/\Delta \text{volts}$ and $C_1$ is equal to $\Delta V/\Delta P$. $\Delta V_f$ is equal to $K_1 \Delta \text{volts}$. $K_1$ and $C_1$ differ for different geometries and they will be derived in subsequent sections. In the above development, the mass of the actuator is neglected. This is a reasonable approximation if the actuator is operating below resonance.
As the applied voltage increases, the surface pressure and volume perturbation also increases. However, for a given material, the amount of voltage that may be applied is limited.

4.2.2 Acoustic Loading of a Quasi-Static Actuator

The actuator will be sending a wave in two directions along the pipe. Therefore, the volume velocity of the actuator is related to the velocity of the plane waves by eq. (4.4).

\[ 2v = \frac{\dot{V}_{act}}{A} \]  

(4.4)

However, the change in volume of the actuator is given by eq. (4.3). Combining the two equations yield eq. (4.5).

\[ 2vA = K_1v_{volts} - C_1 \dot{P} \]  

(4.5)

If \( C_1 \), the actuator bulk compliance, is much less than the inverse of the pipe impedance divided by the area, the second term on the right hand side of eq. (4.5) will be much less than the first term and may therefore be neglected. This signifies that the actuator can be approximated to be a stroke actuator and the back loading of the water may be neglected. The pipe impedance is given by eq. (4.6).

\[ \frac{\dot{P}}{v} = \rho_0c_0 \]  

(4.6)

Substituting for pressure in eq. (4.5) yields

\[ C_1\rho_0c_0v + 2Av = K_1v_{volts} \]  

(4.7)

This equation is a model of how a quasi-static actuator described by eq. (4.3) creates velocity perturbations, \( v \), when driving a pipe that is infinite in both directions. Since the actuator is coupled to the pipe
model in terms of volume perturbations, eq. (4.7) may be rewritten as eq. (4.8).

\[
C_1 \frac{\rho_0 c_0}{2A} \psi + v = K_1 \text{volts}
\]  

(4.8)

Taking a Laplace transform of eq. (4.8) yields the transfer function between actuator voltage and injected volume, which is given by the following equation.

\[
\frac{V}{\text{volts}} = \frac{K_1}{1 + s \frac{C_1 \rho_0 c_0}{2A}}
\]  

(4.9)

Therefore the critical frequency, \(\omega_{\text{crit}}\), is equal to \(\frac{2A}{C_1 \rho_0 c_0}\). For

![Graph showing the response of a quasi-static actuator model.](image)

**Fig. 4.2** The response of a quasi-static actuator model.

Frequencies greater than or equal to \(\omega_{\text{crit}}\), the water back loading the actuator must be taken into account. This is illustrated in Fig. (4.2). The figure ignores the resonance of the actuator. The ratio of a
volume change per volt is constant for a given pressure until the
cutoff frequency is reached. It is desired for this cutoff frequency to
be as large as possible so that the back loading of the water may be
neglected for a greater frequency range. The cutoff frequencies for
all actuators are derived in subsequent sections.

4.2.3 Actuator Resonance

A more realistic model than the one illustrated in Fig. (4.2) would
include the resonant frequency of the actuator. A more complete
transfer function of the ratio of the volume and the input voltage of
an actuator is given by eq. (4.10).

\[
\frac{\text{volume}}{\text{volts}} = \frac{K_1 \omega_{\text{crit}} \omega_n^2}{(s + \omega_{\text{crit}})(s^2 + 2\zeta \omega_n s + \omega_n^2)}
\]  

(4.10)

\(\omega_n\) is the natural frequency and \(\zeta\) is the damping ratio of the actuator.

4.3 Custom Actuator Designs

In order to determine the size of the actuators needed, the amount of
volume change a piezoelectric element can exert must be found. Two
geometric shapes were originally under consideration. They are a
fully wetted block, and an air-backed piston.

4.3.1. Fully Wetted Block of Piezoelectric

To obtain a volume change, a block of piezoelectric material may be
immersed in an enclosed medium. It is necessary to determine the
amount of piezoelectric material needed to cause a volume change of
0.148 cm\(^3\). The volume of a rectangle with sides X, Y, and Z are
shown in Fig. (4.3). The volume is equal to XYZ. For the unloaded
Fig. (4.3) The deformation that occurs when voltage is applied to piezoelectric block.

In this case, the derivative of the volume is given by eq. (4.11).

\[ \Delta V_f = \frac{\Delta V_f}{\Delta x} \Delta x + \frac{\Delta V_f}{\Delta y} \Delta y + \frac{\Delta V_f}{\Delta z} \Delta z \]  

\( (4.11) \)

By definition, \( \Delta x = \varepsilon_x x \), \( \Delta y = \varepsilon_y y \), and \( \Delta z = \varepsilon_z z \). Therefore eq. (4.11) may be rewritten as eq. (4.12).

\[ \Delta V_f = V (\varepsilon_x + \varepsilon_y + \varepsilon_z) \]  

\( (4.12) \)

However, it is known that for piezoelectric material polarized in the \( x \) direction, \( \varepsilon_y \) and \( \varepsilon_z \) are equal and are negative quantities. Consequently, eq. (4.12) is given by eq. (4.12a).

\[ \Delta V_f = V (\varepsilon_x - 2\varepsilon_y) \]  

\( (4.12a) \)

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Combining eq. (4.2), (4.3) and (4.12a) \( K_1 \) was determined. \( C_1 \) is the volume divided by the bulk modulus of a block of material. The equations governing \( K_1 \) and \( C_1 \) for the block are given as follows:

\[
K_1 = V \left( \frac{d_{33}}{t} - 2 \frac{d_{31}}{t} \right) \\
C_1 = \frac{3XYZ}{E} (1-2\nu)
\]  

(4.13)

Values for eq. (4.12a) must be obtained in order to determine the size of material required. The commercially available material that is able to withstand the most voltage per unit thickness is EC-98 as given by the EDO catalog. It is 900 volts/mm. The piezoelectric constant in the 33 direction, \( d_{33} \), is equal to 730 E-12 m/volt. In the 31 direction, \( d_{31} \) is equal to -312 E-12 m/volt. Therefore the maximum volumetric strain, \( \Delta V/V \), as calculated with eq. (4.12a) is equal to 9.54 E-5. For \( \Delta V = 0.148 \text{ cm}^3 \), \( V \) must be 1552 cm\(^3\). If the sides of the rectangle are equal, each side must be 4.6 inches! The corresponding \( C_1 \) is 1.74E-14 m\(^3\)/N and the cutoff frequency is 78 KHz. Piezoelectric material is very expensive and this may not be a practical approach to implement. To maximize the change in volume, it is best not to immerse the entire block but to utilize the strain in the x direction without the minimizing effects in the other two directions. It was proposed to use the piezoelectric material as an air-backed plunger.

4.3.2 Air-Backed Plunger

To modify the response of a single element, the piezoelectric elements may be manufactured as disks. The disks may then be stacked mechanically in series and electrically in parallel so as to provide maximum strain for a given applied field. Fig. (4.4) is a graphical representation of this. An air-backed plunger modifies the volume change for a given change in length of the piezoelectric elements, as illustrated in Fig. (4.5).
Using the relationships $\sigma = E\varepsilon$, $F = \sigma A$, and $\varepsilon = \Delta l/l$ yields $F = EA\Delta l/l$. Therefore the piezoelectric stack may be seen as a spring with the stiffness given in eq. (4.14).

$$k_p = \frac{EA}{l} \quad (4.14)$$

The equilibrium length is dependent upon the applied voltage. The actuator displacement for the free or maximum deflection for the unloaded case at an applied voltage is given by eq. (4.15).

$$\Delta V_f = NA_1d_{33}\text{volts} \quad (4.15)$$

Where $N$ is the number of disks used. The length of the piezo-stack is equal to $Nt$. $K_1$ and $C_1$ for the plunger are given as follows:

$$K_1 = NA_1d_{33}$$
$$C_1 = \frac{NA_1^2t}{EA} \quad (4.16)$$

Fig. (4.4) A stack of piezoelectric disks mechanically connected in series and electrically in parallel.
Fig. (4.5) Air backed plungers. (a) Plunger before a voltage was applied. (b) Plunger after an applied voltage.

Using piezoelectric disks that are commercially available from EDO Corp., the thickness $t$ is equal to 0.02 inch and $A$ is equal to $4.9E-2$ in$^2$. Using $A_1$ equal to $6.36E-3$ m$^2$, $N$ must be 70 in order to obtain a volume change of $0.148$ cm$^3$ at 30 Hz. $C_1$ for the air-backed plunger is equal to $8.3E-13$ m$^5$/N and the cutoff frequency is 13 KHz. A major drawback of the air-backed plunger is the difficulty in constructing linear side walls that are able to withstand up to 150 psi of static pressure. The walls are symbolically drawn in Fig. (4.5).
They may assume any geometric shape. Consequently, commercial actuators were considered.

4.4. Commercial Actuators

Several companies such as INTERNATIONAL TRANSUDER CORP. (ITC), EDO CORP., PIEZO-SYSTEMS, INC., KYNAR PIEZO FILM, ARGOTEC, INC., and PCB PIEZOTRONICS have piezoelectric actuators available commercially.

4.4.1 Interpretation of Commercial Specifications

The performance of commercially available actuators are given in terms of pressure into an infinite fluid medium at one meter from the surface of the transducer. For purpose of evaluation of suitability, we must convert this specification into a volume perturbation achieved. For example, some of the most promising actuators are manufactured by ITC. The transducers under consideration, 1042 and 4001, are spherical and therefore omnidirectional. It was assumed that the velocity have a harmonic time dependence, and the radius was equal to a. Assuming compactness, \((\omega a/c \ll 1)\), the magnitude of the pressure perturbation is related to the surface velocity, \(u\), by eq. (4.17).

\[
p(r) = \frac{\mu \rho \omega a^2}{r}
\]  

(4.17)

The pulsating sphere undergoes a volume perturbation equal to eq. (4.18).

\[
V = \frac{\mu 4\pi a^2}{\omega}
\]  

(4.18)

Therefore, the magnitude of the pressure is related to the volume perturbation through eq. (4.19).

\[
|p| = \frac{\rho_0 \omega^2}{4\pi r} |V|
\]  

(4.19)
The magnitude of the pressure perturbation varies proportionally as the magnitude of the volume and as the square of the frequency. It varies inversely as the distance from the pulsating sphere. Extrapolating the curves from the ITC catalog, the greatest volume perturbation a single actuator is able to exert, assuming the maximum amount of voltage that can be applied is 1500 volts is 1.9E-3 cm³ for the 1.4 inch diameter sphere and 7.4E-2 cm³ for the 6 inch diameter sphere. Dividing the values obtained into 0.15 cm³ shows that it is necessary to use seventy-eight 1.4 inch diameter or two 6 inch diameter spherical transducers. The 1.4 inch diameter sphere was ruled out immediately because of the projected cost. The 6 inch diameter sphere offers a possibility but is not ideal because of its large volume. The flex-tensional actuator describe in appendix A has a volume perturbation of 5.7E-11 m³/volt. With a maximum driving voltage of 2000 volts, the maximum volume perturbation of 1.1 E-7 m³. This actuator may just meet our volume specification.

4.4.2 Commercial Electrodynamic Actuators

Some commercial electro-mechanical acoustic transducers that meet our specification are manufactured by Argotec, Inc. The actuators are standard underwater sound projectors and are known as the J series. The units have a drum-like diaphragm which are supported by a rubber suspension system that permits large linear movement of the diaphragm, but offers a high acoustic impedance to the region around it. The transducers' driving force for the diaphragm is a moving coil positioned in the field of a permanent magnet. At 30 m below the surface of water, the J-9 transducer is able to exert a pressure of 0.708 Pa at 30 Hz. One volt of electricity into a J-9 would cause a volume perturbation of 0.35 cm³. Therefore one J-9 might do the job. However, it weighs 91 kg; it has a diameter of 11.4 cm. and a length of 28 cm. Other transducers, manufactured by the same company, that are able to exert the required pressure and volume displacement are the 214, 216, and 219 models. These projectors
use the same rubber suspension as the J series but have internal and external mechanical stops. They use a linear motion ball bushing to align the diaphragm coil within a permanent magnet. However, the 214, 216, and 219 models weigh 18, 320, and 62 kg. respectively. The diameters of the diaphragms are 14.1 cm, 68.6 cm, and 21.9 cm respectively. Even though commercial electrodynamic devices may be able to do the job, the large size and heavy weight are a deterrent. Therefore, the electro-mechanical source of choice will be made of piezoceramic.

4.4.3 Analysis of Commercial Piezoelectric Actuators

4.4.3.1 Hollow Spherical Piezoelectric

![Diagram](image)

Fig. (4.6) The coordinates of the thin wall sphere.

A thin walled sphere is made of piezoelectric material and filled with air. As given by Fig. (4.6), at the neutral axis of the sphere, the radius is \( R = r - \frac{t}{2} \). The equation may be rewritten as eq. (4.20)

\[
\Delta R = \Delta r - \frac{\Delta t}{2}
\]  

(4.20)

By definition \( \varepsilon_{11} = \frac{\Delta R}{R} \) and \( \varepsilon_{33} = \frac{\Delta t}{t} \). Combining the above three equations yield eq. (4.21).
\[ \Delta r = R \varepsilon_{11} + \frac{1}{2} \varepsilon_{33} \]  \hspace{1cm} (4.21)

The volume of a sphere is equal to \(4\pi r^3/3\). Therefore the volume perturbation per unit change of radius is equal to eq. (4.22).

\[ \frac{\Delta V}{\Delta r} = 4\pi r^2 \]  \hspace{1cm} (4.22)

For the unloaded piezoceramic sphere, combining eq. (4.21) and (4.22) yields eq. (4.23).

\[ \Delta V_f = V \left( 3 \varepsilon_{11} - \frac{3t}{2r} \varepsilon_{11} + \frac{3t}{2r} \varepsilon_{33} \right) \]  \hspace{1cm} (4.23)

Because \(t \ll r\) for a sphere with thin walls, eq. (4.23) may just be approximated with eq. (4.24).

\[ \Delta V_f = 3V \varepsilon_{11} \]  \hspace{1cm} (4.24)

\(K_1\) and \(C_1\) for the sphere are given as follows:

\[ K_1 = V \left( 3 \frac{d_{31}}{t} \frac{3}{2r} d_{31} + \frac{3}{2r} d_{33} \right) \]
\[ C_1 = \frac{2\pi r^3}{Et} (1-v^2) \]  \hspace{1cm} (4.25)

Eq. (4.25) is used to predict the values of \(K_1\) and \(C_1\). For the 1.4 inch diameter sphere, the \(K_1\) obtained from the experimental data from the catalog is equal to \(1\text{E}-12\) m\(^3\)/volts. This agrees reasonably well with the predicted value of \(7.23\text{E}-12\) m\(^3\)/volts. \(C_1\) and the cutoff frequencies for spheres of different diameters are tabulated in Table (4.2).
Table (4.2) Values of $C_1$ and the cutoff frequencies of spheres.

<table>
<thead>
<tr>
<th>Diameter of Spheres inches</th>
<th>$C_1$ m$^5$/N</th>
<th>Cutoff Frequency KHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>3.78 E-15</td>
<td>456</td>
</tr>
<tr>
<td>4.25</td>
<td>1.69 E-13</td>
<td>10.18</td>
</tr>
<tr>
<td>6</td>
<td>4.24E-13</td>
<td>4.05</td>
</tr>
</tbody>
</table>

At the resonant frequency of the sphere, the volume change per unit volt of input is much greater than off resonance. In order to model the actuator, it is necessary to determine the resonant frequency of the sphere so as to limit the operating frequency to remain below resonance. The value may be found with eq. (4.26).

$$\omega_{res} = \sqrt{\frac{\text{strain energy}}{\text{kinetic energy}}}$$

(4.26)

The strain energy is equal to eq. (4.27).

$$U = \frac{V}{2} (\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3)$$

(4.27)

where $V = 4\pi R^2 t$. For a sphere $\varepsilon_1 = \varepsilon_2$. Using the identity $\sigma = E \varepsilon$, eq. (4.21), and eq. (4.27) may be reduced to eq. (4.28).

$$U = 4\pi t R^2 E \frac{(\Delta R)^2}{R}$$

(4.28)

The kinetic energy is equal to eq. (4.29).

$$T = \frac{1}{2} m v^2$$

(4.29)

The mass is equal to $\rho_p 4\pi R^2 t$ and the velocity is equal to $dr$. Therefore, eq. (4.29) may be written as eq. (4.30).

$$T = 2\rho_p t R^2 (dr)^2$$

(4.30)
Combining eq. (4.26), (4.28) and (4.30) results in eq. (4.31).

$$\omega_{res} = \sqrt{\frac{2E}{\rho_p R^2}}$$  \hspace{1cm} (4.31)

The resonant frequency varies inversely as the outer radius of the sphere. Note that it does not depend on the thickness of the sphere. Some calculated values as well as some values from the ITC catalog are shown in Table (4.3). The values are shown graphically in Fig. (4.7).

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Model</th>
<th>Resonant Frequency as given in catalog</th>
<th>Resonant Frequency as calculated from eq. (4.15)</th>
</tr>
</thead>
<tbody>
<tr>
<td>inches</td>
<td>number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>N/A</td>
<td>N/A</td>
<td>56.8 KHz</td>
</tr>
<tr>
<td>1.4</td>
<td>ITC-1042</td>
<td>80 KHz</td>
<td>40 KHz</td>
</tr>
<tr>
<td>2</td>
<td>N/A</td>
<td>N/A</td>
<td>28.4 KHz</td>
</tr>
<tr>
<td>4.25</td>
<td>ITC-1001</td>
<td>16.5 KHz</td>
<td>13.4 KHz</td>
</tr>
</tbody>
</table>

The equation predicts the resonant frequencies for the larger spheres more accurately than those for spheres with smaller diameters. Discrepancies between results obtained from these equations and experiments may be due to simplifying assumptions, particularly unmodeled mass or stiffness. As given previously, seventy-eight 1.4 inch diameter spheres and two 6 inch diameter spheres are required to inject the necessary volume perturbation. Since the material used to construct the spheres remain constant, a number between two and seventy-eight spheres with diameters between 1.4 inches and 6 inches are needed to meet the specifications.
Fig. (4.7) Resonant frequencies for spheres with different diameters.

4.4.3.2 Flex-Tension Actuator

The flex-tension actuator is manufactured by EDO Corp. This is shown in Fig. (4.8). As the stack of piezoelectric material elongates, the sides bulge outwards, maximizing the volume displacement. Using the variables given in Fig. (4.8), the volume change for the unloaded case at an applied voltage is given by eq. (4.32)

$$\Delta V_f = (A_1 L_1 + \pi \left( \frac{RL_1^2}{2\theta} \cdot \frac{L_1^3}{6} \right)) \frac{d_{33} \text{ volts}}{t}$$  \hspace{1cm} (4.32)

The term containing $L_1^3$ may be neglected due to its small contribution. $K_1$ and $C_1$ are given by eq. (4.33).

$$K_1 = (A_1 L_1 + \pi \left( \frac{RL_1^2}{2\theta} \cdot \frac{L_1^3}{6} \right)) \frac{d_{33}}{t}$$

$$C_1 = \frac{2\theta}{EA} (A_1 + \pi RL_1) L_1$$  \hspace{1cm} (4.33)
Fig. (4.8) Flex tension actuators. (a) before voltage was applied. (b) after voltage was applied.

The term containing $L_1^3$ may be neglected due to its small contribution. $K_1$ and $C_1$ are given by eq. (4.33).

$$K_1 = (A_1 L_1 + \pi \left( \frac{RL_1^2}{2\theta} - \frac{L_1^3}{6} \right) \frac{d_{33}}{t} \left( A_1 + \frac{\pi RL_1}{2\theta} \right)^2 L_1$$

$$C_1 = \frac{2\theta L_1}{E A}$$  (4.33)

The lever effect will be limited by the amount of pressure that needs to be exerted. Using the dimensions as given in Appendix A, $K_1$ was calculated to be 6.5E-11 m$^3$/volt. From the experimental data given in the catalog, $K_1$ is equal to 5.7E-11 m$^3$/volt. $C_1$ was calculated to be
and thickness of the material is defined, the driving voltage is also defined. To determine the driving current, \( I \), eq. (4.34) may be used.

\[
|I| = C|\text{volts}|
\]  
(4.34)

Assuming a time harmonic dependence, eq. (4.34) may be rewritten as eq. (4.35).

\[
|I| = C\omega|\text{volts}|
\]  
(4.35)

\( C \) is the capacitance of the actuator. Approximating the geometry of the actuator to be parallel plates, the capacitance of the transducer may be calculated with eq. (4.36).

\[
C = \frac{\varepsilon A}{t}
\]  
(4.36)

\( A \) is the area of the plate and \( t \) is the distance separating the two plates. For example, the voltage requirement for the flex tension at 30 Hz, is 2000 volts and 0.0045 amps. These are not very big current and voltage requirements. Therefore the utilization of the flex tension is feasible. The voltage and current requirements are given in Table (4.4).

### 4.6 Actuator Selection

Table (4.4) summarizes the actuators considered.

Too many 1.4 inch diameter spherical transducers are needed to meet the specifications. The most promising design is the lever configurations and the 6 inch diameter sphere. The flex tension actuator is the best choice because it requires the least amount of power to operate. The commercially available actuator is able to withstand a maximum static pressure of 200 psi. The operating frequency range is from 0 Hz to the resonant frequency of the
<table>
<thead>
<tr>
<th>Design</th>
<th>Cutoff Frequency KHz</th>
<th>Weight of one Actuator kg</th>
<th>Volume of one Actuator m³</th>
<th># of Actuators needed</th>
<th>Driving Voltage volts</th>
<th>Driving Current at 30 Hz amps</th>
<th>Capacitance farads</th>
<th>Driving Field volts/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flex</td>
<td>2.46</td>
<td>4.1</td>
<td>1.145E-3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.49E-9</td>
<td>160</td>
</tr>
<tr>
<td>Block</td>
<td>78</td>
<td>12.5</td>
<td>1.55E-3</td>
<td>1</td>
<td>not calculated</td>
<td>not calculated</td>
<td>not calculated</td>
<td>not calculated</td>
</tr>
<tr>
<td>Plunger</td>
<td>13</td>
<td>1.63</td>
<td>6.46E-3</td>
<td>1</td>
<td>457</td>
<td>900</td>
<td>3.03E-9</td>
<td>600</td>
</tr>
<tr>
<td>Air Backed Plunger</td>
<td>456</td>
<td>.55</td>
<td>2.35E-5</td>
<td>1</td>
<td>78</td>
<td>1500</td>
<td>7.60E-8</td>
<td>196</td>
</tr>
<tr>
<td>Sphere Diameter = 1.4 inch</td>
<td>1.34</td>
<td>1.85E-3</td>
<td>1.34</td>
<td>2</td>
<td>1.45</td>
<td>1500</td>
<td>4.66E-7</td>
<td>0.132</td>
</tr>
<tr>
<td>Sphere Diameter = 6 inch</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table (4.4) The critical values of actuators under consideration.
actuator. For frequencies above the resonant frequency, the system may no longer be modeled as quasi-static.

4.7 Mechanical Design of Pipe Modification

![Diagram of a mechanical design of a pipe modification with labels for Actuator, Sensor, Rubber, Flange, and z]

Fig. (4.9) Insert to add to the existing piping system.

The walls of the existing pipe must be modified to accommodate the actuator. It is desired that the actuator displaces the water axisymmetrically. Therefore, two actuators should be used and they
will be placed at opposing sides of the walls of the pipe. The actuator should be immersed in a rubber filled section that has the same inner diameter as the steel pipe. To ensure that the rubber does not undergo large deformations under static pressures, it is necessary to find rubber with the same or greater bulk modulus as water. Bulk modulus of a liquid is the reciprocal of the compressibility. The modulus is defined as the change in pressure times volume divided by the change in volume. It is a function of temperature. At 20 C, the bulk modulus of water is equal to 220 E7 N/m². There are several rubber within the range of this bulk modulus. The range of vulcanized rubber is from 9.8E8 Pa to 1.3E9 Pa [17].

It is desired for a volume change in the actuator to cause a corresponding volume change in the fluid. However, the actuator should not deform the outer walls of the enclosed structure. In order for this assumption to be a valid, it is necessary to ensure that the outer walls of the enclosing pipe be much stiffer than that of the inner walls. Then the outer walls may be modeled as infinitely stiff. It is envisioned that the outer walls are made of steel and the pocket filled with rubber. The structure that holds the piezoelectric material is designed to have the shape of a thick washer. Taking an element Δx, assuming thin walled cylinder, the stress exerted on the θ direction is equal to eq. (4.37).

$$\sigma = \frac{pR}{t} \quad (4.37)$$

Combining eq. (4.34), σ=Eε, and ε = Δθ/2π yields eq. (4.38).

$$\frac{EΔθ}{2π} = \frac{pR}{t} \quad (4.38)$$

Assume unit axial length, the force is equal to eq. (4.39).

$$F = EτΔθ \quad (4.39)$$
Therefore the hoop stiffness may be approximated to be $E_t$. The stress in the axial direction is given by eq. (4.40).

$$\sigma = \frac{pR}{2t}$$  \hspace{1cm} (4.40)

Combining eq. (4.40), $\sigma = E \varepsilon$, and $\varepsilon = \Delta z / z$ yield eq. (4.41)

$$E \frac{\Delta z}{z} = \frac{pR}{2t}$$  \hspace{1cm} (4.41)

The force is given by eq. (4.42).

$$F = \frac{2E \pi rR}{z}$$  \hspace{1cm} (4.42)

The stiffness of the end caps is equal to $2\pi r t E / z$. It is desirable for the stiffness of the steel structure to be much greater than the stiffness of the rubber. Therefore the dimensions of the structure must be given by eq. (4.43).

$$E_s > \frac{2\pi r E_r}{z}$$  \hspace{1cm} (4.43)

Rearranging eq. (4.43) yields eq. (4.44).

$$z > \frac{2\pi r E_r}{E_s}$$  \hspace{1cm} (4.44)

Since the flex tension actuator has a length of 0.18 m, and it is placed vertically in the rubber, $r$ must be at least 0.09 m. Setting $r = 0.12$ m, from eq. (4.44), $z$ must be at least 7.5E-3 m. However, the diameter of the flex tension is 0.09 m, therefore the dimensions of $z$ is not dictated by the stress on the walls of the mounting scheme but by the physical geometry of the actuator. The bulge in the wall of the original pipe will be small. This is a practical and elegant design with minimum modification to the existing structure.
An alternative method of mounting the flex-tensinal actuator is sketched in Fig. (4.10).
Fig. (4.10) Alternative mounting scheme.
5. Simulation of System Driven by Actuators

5.1 Model of the System with Actuators

It was proposed to place the actuators as shown in Fig. (5.1). A simplified model of the centrifugal pump and steel pipes with actuators is shown in Fig. (5.2). With an additional actuator, the model derived in chapter 3 must be modified. Assuming the pressure generated by the actuator is also planar, it will have the same form as those given in eq. (3.1c). The general solution for the pressure is given by eq. (5.1).

\[ P_3 = [A_3 e^{ikz} + B_3 e^{-ikz}] e^{i\omega t} \]  \hspace{1cm} (5.1)

Using the same boundary conditions as given by eq. (3.22) to (3.24) and eq. (3.27), two additional boundary conditions must be found. The pressures \( P_1 \) and \( P_3 \) at \( y = L_1 \) and \( z = 0 \), are equal. Consequently, one boundary condition is given by eq. (5.2).

\[ P_3 = P_1 \]  \hspace{1cm} (5.2)

At the location of the actuator, eq. (5.3) must hold.

\[ v_1 = v_3 - \frac{\Delta v}{A} \]  \hspace{1cm} (5.3)

For a plane wave the volume velocity is related to the pressure as given by eq. (5.4).
Fig. (5.7) Simplified diagram to indicate a possible location for the actuators.
Fig. (5.2) Simplified model of the piping system.
\[ \rho \dot{v}_1 = -\frac{\partial P_1}{\partial y} \]
\[ \rho \dot{v}_3 = -\frac{\partial P_3}{\partial z} \]  
(5.4)

Take the derivative of eq. (5.3) and combine the equation with eq. (5.4) to obtain eq. (5.5).

\[ \frac{\partial P_1}{\partial y} \cdot \frac{\partial P_3}{\partial z} = \frac{\rho \dot{V}_{\text{act}}}{A} \]  
(5.5)

This equation considers the actuator to be a volume source, with volume acceleration, \( \dot{V}_{\text{act}} \). Eq. (4.10) gives a relationship between actuator volume and driving voltage, assuming an acoustic loading of an infinite pipe. It was assumed that this relationship also governs the actuator driving the finite pipe, leading to the concatenation of eq. (5.5) and (4.10) to obtain the following equation in the frequency domain.

\[ \frac{\partial P_1}{\partial y} \cdot \frac{\partial P_3}{\partial z} = \frac{\rho}{A} \frac{K_1 \omega_{\text{crii}} \omega_n^2 \text{s}^2 \text{volts}}{(s + \omega_{\text{crii}})(s + 2\zeta \omega_n s + \omega_n^2)} \]  
(5.6)

This equation holds for frequencies below the critical frequency for a particular actuator.

Eq. (3.22) to (3.24), eq. (3.27), eq. (5.2), and eq. (5.6) were solved simultaneously for frequencies between 0 and 1000 Hz. Setting the velocity of the pump equal to zero, the transfer functions from the actuator to the sensors were found. The transfer function is the ratio of the output pressure and the input voltage. Some simulations for configuration one are shown in Fig. (5.3) to Fig. (5.4). As evident from the graphs, the pipe has an alternating pole zero configuration with collocated pressure sensor and actuator.
Fig. (5.3) For configuration one, at the inlet of the pump, Ma, (a) the ratio of the output pressure and the input voltage vs. frequency. (b) The corresponding phase in degrees vs. frequency.
Fig. (5.4) For configuration one, close to the rubber, at the inlet of the pump, Mb, (a) the ratio of the output pressure and the input voltage vs. frequency. (b) The corresponding phase in degrees vs. frequency.
6. Control Synthesis

6.1 Architecture of the Proposed Control System

6.1.1 Position of Actuators

It was proposed to modify the water loop so that the control may be implemented. One design is shown in Fig. (6.1) as evident from the figure, it is proposed to use two actuators. If only one actuator is used, it can at most reflect an oncoming wave so that a standing wave between the noise source and the actuator is produced. The actuator does not exchange energy with the noise field in the duct, but is analogous to an impedance change, and instead of being absorbed, the noise is effectively reflected back up the duct. For example, for our experiment, if only one actuator is used, the pressure on one side of the transducer will be zero but on the other side of the transducers the pressure will be twice the original amplitude. Therefore, as proved by Swinbanks [16], a pair of secondary sources are necessary to cancel a primary wave in one direction while not affecting the wave in the other direction. A pair of secondary source will be unidirectional and will not reflect the unwanted sound back to the source rather than absorbing it. If the source strength of the upstream actuator at time t is \( m_1(t) \) and the downstream actuator is \( m_2(t) \), then Swinbanks showed that there will be zero output from the combination of the two transducers in the upstream direction if eq. (6.1) is satisfied.

\[
m_1(t) = -m_2( t - \frac{b}{c_0 (1-M)})
\]  

(6.1)
Fig. (6.1) Modified experimental setup.
where $b$ is the axial distance between the two transducers and $M$ is the Mach number. For our experiment, the Mach number is much less than one, therefore eq. (6.1) may be reduced to eq. (6.2).

$$m_1(t) = -m_2(t - \frac{b}{c_0})$$ \hspace{1cm} (6.2)

The equation shows that the amplitude of $m_1$ must be the inverse of $m_2$, but delayed by the amount $b/c_0$. Therefore Fig. (6.1) shows that the actuators differ by a lag.

### 6.1.2 Position of Sensors for Narrowband Control

For the narrowband signals, it was proposed to use an adaptive feedforward scheme to cancel the noise. Adaptive feedforward refers to controlling the plant in an open loop configuration after the adaptation has reached steady state. This method is applicable when an auxiliary reference input is correlated to the disturbance. In the experimental setup, the reference is the shaft signal in the centrifugal fan pump. Therefore, one sensor is needed at the shaft of the centrifugal pump. An additional sensor is needed far upstream for feedforward adaptation. A good sensor to use would be the one located at $M_b$.

### 6.2 Adaptive Feedforward for Narrowband Control

From the experimental data, the signal of the water-borne noise is primarily periodic, with the dominant harmonics being at shaft and at blade rate; 30 Hz and 210 Hz. These harmonic disturbances challenge existing sensors and actuators because they are at the threshold of the actuator's dynamic range. Therefore, it was decided to remove the harmonic disturbances before attempting to control the remaining broadband signal. There are many options for control algorithm design and implementation. It was proposed to use adaptive feed forward to eliminate the interference.
The system was approximated to be a multivariate linear time invariant system. Detailed analysis by Elliott et al [13] and others proved that when the reference signal is a synchronously sampled sinusoid, the control system dynamics between the error sensor and the actuator signal are linear time invariant. The sensor output, $y(s)$, may be described by eq. (6.3) and illustrated in Fig. (6.2).

$$y(s) = G(s)u(s) + d(s)$$  \hspace{1cm} (6.3)

where $y(s)$ is the output pressure the hydrophones record, $d(s)$ is the periodic narrowband disturbance, $u(s)$ is the input actuator voltage, and $G(s)$ converts voltage applied to the piezo-actuator into a pressure at the sensor. The pressure is added to the pressure from the centrifugal fan pump to produce a transmitted pressure. The desired transmitted pressure is equal to zero. The spectrum of the narrowband disturbance, $d(s)$, is assumed to be made of a discrete number of harmonics at shaft and blade passage frequencies. The shaft passage frequency is 30 Hz. The compensator for a sum of discrete harmonics is given by eq. (6.4).
\[ C(s) = \sum_{i=1}^{p} \epsilon \frac{s + a_i}{s^2 + \omega_i^2} \]  

(6.4)

where \( \omega_i \) is given by eq. (6.5)

\[ \omega_i = n2\pi(30) \quad i=1,2,3,\ldots,p \]  

(6.5)

The frequencies of interest are between 30 and 1000 Hz, therefore \( p \) is at most 34. However, inspection of Fig. (2.3) to (2.8) suggests that only 5 or 6 harmonics are important.

Fig. (6.3) Magnitude vs. frequency of one of the terms in the series of the compensator.

Adaptive feedforward may be analyzed as a feedback loop. The open loop poles of \( C(s) = +/- j\omega_n, n = 1,2,\ldots,p \). The compensator poles provide disturbance attenuation since they manifest themselves as zeros in the closed loop configuration. The bode gain characteristics of the compensator is shown in Fig. (6.3). \( \omega_b \) is the width of the passband with amplitudes above 0 dB and is also known as the bandwidth of the filter. The compensator is fed the output of the
transmitted pressure and provides the input to the piezo-actuator. The diagram is shown in Fig. (6.4).

\[ u(s) = \text{applied voltage} \]

\[ G(s) = \text{transfer function} \]

\[ -C(s) = \text{narrowband compensator} \]

\[ d(s) = \text{narrowband disturbance} \]

\[ y(s) = \text{output pressure} \]

**Fig. (6.4)** Block diagram of closed loop system.

A method of implementation is to use the filtered-x least mean square (LMS) algorithm. LMS is an iterative gradient descent algorithm that uses an estimate of the gradient on the mean-square error surface to approximate the optimum weight vector at the minimum mean-square error point. The adaptation constant determines the step size taken at each iteration along the estimated negative-gradient direction. If the adaptation constant is small enough, so that the step size is not too large, adaptation noise due to error in the gradient estimate is averaged out.
6.2.1 Closed Loop Stability for Narrowband Control

The zero in the compensator transfer function is chosen such that the open loop transfer function has the appropriate stability margins. \( a_n \) controls the location of the compensator zero. The \( \varepsilon \) and \( a_n \) in the compensator must be chosen such that the closed loop system is stable. The magnitude of \( \varepsilon \) also controls the bandwidth of the compensation. The closed loop output is given by eq. (6.6).

\[
y(s) = \frac{d(s)}{1 + C(s)G(s)}
\]  

(6.6)

In the closed loop form, it is a notch filter as given by the above equation. From the equation, the sensitivity is zero at \( s = \omega_n \), which implies a complete rejection of a sinusoidal disturbance at the particular frequencies. To find the closed loop poles, set the denominator equal to zero. As analyzed by Mercadal [14], assume \( \mu_n \) to be the closed loop poles. Then \( \mu_n \) can be written as follows.

\[
\mu_n = j\omega_n + \alpha_n
\]  

(6.7)

where \( j\omega_n \) is the open loop poles, and \( \alpha_n \) is the of the order \( \varepsilon \). Rewriting \( C(s)G(s) \) as fractions yield eq. (6.8).

\[
G(s) = \frac{N_G(s)}{d_G(s)}; \quad C(s) = \frac{N_d(s)}{d_d(s)}
\]  

(6.8)

\( N(s) \) and \( d(s) \) are polynomials. The closed loop characteristic equation may be rewritten as eq. (6.9).

\[
d_G(s)d_d(s) + \varepsilon N_G(s)N_G(s) = 0
\]  

(6.9)

Substituting eq. (6.7) into (6.9) and expanding in a Taylor series to first order in \( \varepsilon \) yields eq. (6.10).

\[
\alpha_n = \left\{\varepsilon \text{Res}(C(j\omega_n)G(j\omega_n)) \right\}_{mn}^{-1}
\]  

(6.10)
Therefore, \( \alpha_n \) is of the order of the gain, \( \varepsilon^{1/m} \), where \( m \) is the number of multiple roots. \( \alpha_n \) is negative if the phase of \( C(j\omega_n) \) is chosen appropriately. This is done by appropriate selection of \( a_n \). The phase margin is +/- 90 degrees for each compensator pole (each harmonic). For our experiment \( m \) is equal to 1. For small enough values of \( \varepsilon \), the closed loop pole is assumed to remain in the left half plane of the complex plane for appropriate choices of \( C(j\omega_n) \). Therefore, the plant poles will not become unstable. Small \( \varepsilon \) would imply a narrow bandwidth for the filter and also cause the system to adapt very slowly. This is desirable so that the nearby harmonics will not be disturbed.

6.3 Broadband Control

6.3.1 Feedforward for Broadband Control

A large literature exists on the topic of control of sound propagation in ducts via adaptive feedforward of the signal of an "acoustically upstream" sensor to an actuator. This approach has used sensor and actuator arrays with selective sensitivity to sensing (or actuating) waves propagating in only one direction [16,21,22,23,24,25,26,27] but has also used omni-directional point actuators and sensors. In all cases, there will be some "acoustic feedback", some of the actuator signal will be sensed by the sensor which provides the reference signal for feedforward. To the degree this acoustic feedback is important, stability will be an issue, and electronic compensation for this feedback must be used. Fig. (6.5) illustrates the sensor configuration under consideration.

Impressive results have been attained with this approach; e.g. to 20 dB reduction in the performance sensor signal over two or three octaves of frequency. It may be possible to adapt one of these published approaches, or even to use a commercial unit for the problem being considered here. However, due to the potentially
Fig. (6.3) Feedback dynamics in a pipe.
very strong, time-varying acoustic feedback dynamics anticipated in the submarine application, conventional approaches may not be effective. Fig. (6.5) suggests that the feedback is due to the direct field from the actuators to the sensors. In reality, much stronger feedback is due to the reverberant field; Fig. (5.3) suggests that at resonant frequencies of the pipe system, the reverberant contribution is perhaps 10 times as great as the direct contribution. (The damping ratio is about 0.1). This reverberant acoustic feedback is very dependent upon details of the dynamics of the system, and can be expected to change significantly with operating conditions. A very real risk of instability therefore exists with the implementation of Fig. (6.5). Further work needs to be done in this area.

6.3.2 Feedback

In this discussion of feedback, the simplest actuation possibility, a single point actuator, is considered. Therefore the closed loop behavior is the reflection of incoming sound back toward the pump. The focus in this section is on the design of the sensors for use in feedback. A primary criterion is that the dynamics from actuator to sensor be such that robustly stable feedback control is possible.

6.3.2.1 Collocated Point Sensor and Point Actuator for Feedback

A point sensor will be placed as close to the actuator as possible. This is presented in Fig. (4.9). The output of the sensor will be fed into the feedback control loop. Diagrams of the transfer functions from actuator to collocated sensor are given in Fig. (6.5) and (6.7). Fig. (6.6) shows the ratio of the pressure sensed by the hydrophone at the location of the actuator over the volume perturbation of the actuator vs. frequency. This does not include the dynamics of the actuator as given in chapter 4. The transfer function plotted in Fig. (6.7) is the ratio of pressure at the actuator per unit volt. This figure considers the dynamics of the actuator.
Fig. (6.6) For configuration one the ratio of the output pressures and the volume perturbation of the actuator vs. frequency. (b) The corresponding phase in degrees vs. frequency.
Fig. (6.7) For configuration one the ratio of the output pressures and the input voltage vs. frequency. (b) The corresponding phase in degrees vs. frequency.
One method of cancelling the broadband noise is to invert the plant transfer function in feedback. The block diagram of the narrowband and broadband control scheme is shown in Fig. (6.8). Feedback compensation of the plant transfer function of Fig. (6.7) is not impossible. The compensator should add some lag at low frequency (below about 500 Hz) and some lead at high frequency (above about 1000 Hz);

\[
C(s) = g \frac{(s+1000\pi)^2}{s} \tag{6.11}
\]

Such compensation is infinite bandwidth and undesirable. A practical design will rolloff the compensation. It is desirable for the rolloff to be at approximately 2 kHz. This will require plant inversion in the cross-over region. Inverting the plant of Fig. (6.7) is very dangerous, since both the compensator and the plant will have lightly damped (\(\zeta = 0.1\)) poles and zeroes. If the plant dynamics are mis-modeled near cross-over, the inversion will be unstable in closed loop.

6.3.2.2 Closed Loop Stability for Broadband Control

The stability of the closed loop can be determined by looking at magnitude vs. frequency and phase vs. frequency plots of the open loop transfer function. It is common practice to represent the closeness to the threshold of instability in terms of phase margin and gain margin. For a stable minimum phase system, the gain margin indicates how much the gain can be increased before the system becomes unstable. For an unstable system, the gain margin in indicative of the amount of gain that must be decreased to make the system stable. A zero gain margin indicates that the system is at the verge of instability. The system will exhibit substantial oscillations. The gain margin is the reciprocal of the magnitude of the open loop transfer function at the frequency where the phase angle is -180 degrees. The gain crossover frequency is at unity loop gain. This helps define the phase margin, which is the amount of additional
Actuator input

Plant

Pressure

$\xi \frac{s + a}{s^2 + w_i^2}$

Shaft harmonic

Blade rate

Broadband compensator

Fig. (6.8) Feedback control loop
phase lag required to bring the system to the verge of instability. Phase margin is equal to 180 degrees plus the phase angle of the open loop transfer function at the gain crossover frequency. Gain and phase margins are illustrated in Fig. (6.9). Superimposed on this generic figure are gain and phase excursions near cross-over due to inexact inversion of a resonant plant. These excursions limit the practicality of plant inversion.

It is not only necessary to have a stable system, safety margins must also be included. Designing a proper phase and gain margin ensures us against variations in the system components and are specified for definite values of frequency. Unmodeled dynamics will cause the system to become unstable if there is no safety gain margin. Some unmodeled dynamics are higher order modes, the reflection coefficient at the pump, and nonlinear effects. Furthermore, it was proposed to also add a controller at the outlet of the pipe. The interaction of the two active systems is yet another unmodeled effect.

Another major problem with a point sensor and point actuator pair is that the sensor may be sensing turbulent pressure. Consequently, it will feedback the sensed turbulence and thereby create unwanted noise. One possible method of obtaining a better performance is to use a distributed sensor.

6.3.2.3 Feedback with Distributed Sensor and Point Actuator

The advantages of using distributed sensors for feedback stability in a resonant waveguide was analyzed by Collins [20]. A point sensor can only sense past and present information; it cannot anticipate future information. Consequently, for a point sensor, there is always a phase lag associated with any gain rolloff. The phase lag will reduce the feedback control system's phase margin or may cause the system to become unstable. The proposed distributed sensor would allow the gain to rolled off without introducing additional phase lag,
Fig. (6.9)  Gain and phase margin.
thereby avoiding the addition of new instabilities to the closed-loop system.

Another advantage of a distributed sensor is that it will reduce the amount of turbulence sensed. Consequently, there is less probability of creating unwanted noise.

The proposed distributed sensor will be constructed of discrete point sensors that combine spatially distributed measurements with a weighting function. A weighting function under consideration is the sinc function. The distributed sensor will consist of twelve discrete point sensors which are placed 0.25 m apart. The total length of the pipe is 3.048 m and an actuator is placed at the circumference of the axial center of the pipe. Two disadvantages associated with using a truncated sinc function is that it does not roll off very quickly and a sign change occurs at each node [20]. The change of sign can destroy any phase margin that may have been preserved. Therefore, it was proposed to modify the sinc function by taking the convolution of it with a Bartlett Window. Since it is desired for the functions to relate space and frequency as opposed to space and time, the Fourier transform of both functions must be computed. The Fourier transform of the sinc function and the Bartlett Window yields a rectangular window and a sinc² respectively. The cutoff frequency for the rectangular window is shown by eq. (6.11).

$$f_{cut} = \frac{cL}{2L} \quad (6.11)$$

where $L$ is the distance between all nodes of the sinc function except the center lobe which has length $2L$. It is desired for $f_{cut}$ to be 1395 Hz, therefore $L$ must be 0.5 m. A sinc function is infinite in domain. However, for experimental studies, it must be truncated. Truncation introduces additional errors. The sinc function considered was truncated after a total width of 3 m. The transfer function, plotted in Fig. (6.10), is the ratio of the sum of the weighted sensors and the volume perturbation of the pump drive. The spike at 2790 Hz is due
Fig. (6.10) For configuration one, at the actuator, the ratio of the sum of 12 output pressures weighted with the Bartlett Window and the sinc function and the volume perturbation of the pump vs. frequency. (b) The corresponding phase in degrees vs. frequency.
to sampling at discrete points; the response above 2790 Hz is due to spatial aliasing. If it is desired for the spike to occur at a higher frequency, the sensors must be placed closer together and the distance between nodes must also increase.

If the actuator, which is located at the center of the sensors, is operating, the sensors will be measuring the direct field response of the pressure wave as well as the reverberant field. The result is shown in Fig. (6.11) and (6.12). Fig. (6.11) shows the transfer function to be the ratio of the sum of the weighted sensors and the volume perturbation of the actuator. This transfer function does not account for the dynamics of the actuator. As evident from the figure, the spatial nyquist frequency is 2790 Hz. Any signal above the nyquist frequency must be removed by pre-filtering to prevent aliasing. Fig. (6.12) accounts for the actuator dynamics. The transfer function is the ratio of the sum of the weighted sensors and the input voltage.

Feedback compensation of the plant transfer function plotted in Fig. (6.12) is possible without inverting difficult to predict resonant dynamics near cross-over. The spatially distributed sensor has made these dynamics unobservable. The dynamics above 3500 Hz are due to spatial aliasing, and can be suppressed by using more point sensors, placed closer together. If this is not done, and if the dynamics above 3500 Hz are real and not just a modeling artifact, feedback compensation of Fig. (6.12) will lead to disappointing performance; the loop gain must be kept low to gain stabilize the high frequency dynamics.

These dynamics above 3500 Hz are highly suspicious. At that frequency, the wavelength in the pipe is 40 cm, comparable to the pipe diameter and actuator length. Fig. (6.13) was generated by suppressing these dynamics through the use of 30 point pressure sensors with a spacing of 10 cm and a weighting of the sinc function and a Bartlett Window. Fig. (6.13) shows the loop transfer function
Fig. (6.11) For configuration one, at the actuator, the ratio of the sum of 12 output pressures weighted with the Bartlett Window and the sinc function and the volume perturbation of the actuator vs. frequency. (b) The corresponding phase in degrees vs. frequency.
Fig. (6.12) For configuration one, at the actuator, the ratio of the sum of 12 output pressures weighted with the Bartlett Window and the sinc function and the input voltage vs. frequency. (b) The corresponding phase in degrees vs. frequency.
Fig. (6.13) For configuration one, the ratio of the sum of 30 output pressures weighted with the Bartlett Window and the sinc function and the input voltage times the compensator vs. frequency. The compensator is \((s+2000\pi)^2/s\). (b) The corresponding phase in degrees vs. frequency.
(the product of the plant and the compensation) for a compensation as given by eq. (6.12).

\[ C(s) = g \frac{(s+2000\pi)^2}{s} \]  

(6.12)

Inspection of Fig. (6.13) shows the entire frequency range to be phase stabilized; there is infinite upward gain margin. Loop gain and disturbance rejection is not limited by feedback stability.

In reality, it is highly unlikely that the dynamic model used in generating Fig. (6.13) is correct to the level that was predicted. Little progress can be made without some idea of model fidelity.

6.3.3 Summary of Broadband Control

This thesis has only begun to address the issues of broadband control. Even with the analytical model being used, much remains to be done. A more careful design of distributed sensors for feedback, potentially with selected sensitivity to waves propagating in only one direction, should be carried out. Distributed actuators need to be considered. The first consideration is a pair of actuators as sketched in Fig. (6.1), with no wave output towards the pump. Other actuator arrays can be imagined, particularly for robust feedback control. A detailed analysis of the implementation and feedback issues in adaptive feedforward as discussed in section 6.3.1 should be carried out.

Ultimately, the broadband control design and analysis is only as good as the dynamics upon which it is based. Little knowledge of the fidelity of our dynamic models are known. Further work in this area will become practically senseless without a strong parallel experimental program quantifying plant dynamics from actuator to sensor. The current pump loop will not suffice for this; no actuators are in place. A minimum of one actuator must be added for progress in this area.
7. Conclusions/Recommendations

The model of the experimental setup neglected the bends in the pipe, the reflection coefficient from the centrifugal fan pump, higher order modes, turbulence in the fluid and external mechanical forces. It also approximated the reflection coefficient from the steel rubber interface to be equal to -0.6. Although the model was simple, it predicted the broadband response of the experimental setup well, as illustrated in Fig. (3.4) to (3.9). If more accurate predictions are desired, future models should include the effect of higher order modes, vibrations from external forces on the structure, and a reflection coefficient for the pump. Such a modeling effort should not be undertaken without a strong laboratory testing program. An actuator must be added to make such testing valuable. The tentative conclusion was that active control is a sensible technique to consider.

To change the acoustic output of the system, it was determined that an actuator must displace 0.148 cm³ at 30 Hz and exert a pressure perturbation of 0.75 psi, while operating at a static pressure of 150 psi. A sensible actuator choice is to purchase the flex tension transducer from EDO Corp. The actuator would then be mounted in the scheme given in Fig. (4.9) or Fig. (4.10).

It was proposed to use adaptive feedforward of a shaft signal to eliminate the periodic disturbances, and feedback or adaptive feedforward of pressure transducers to suppress broadband response. Adaptive feedforward of pressure signals was not investigated in any detail; the point was made that this system is dynamically quite different from those upon which this approach has been successfully demonstrated. This piping system exhibits strong and complex (probably highly variable) resonant response, providing complex acoustic feedback from actuation to pressure sensing.

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This thesis has focused somewhat upon feedback of sensed pressure to actuated volume perturbation. It was demonstrated that use of a single point sensor would lead to a complex and highly sensitive feedback compensation, and have sketched out an array of pressure sensors that permit robust feedback. Little more can be done in this area without a more thorough knowledge of plant dynamics, which can only be obtained through laboratory measurement.
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Appendix A

ABSTRACT

Barrel stave flextensional transducers are potentially useful as compact, low-frequency, high-power projectors. An equivalent circuit model that includes a higher mode, extensional compliance is used to estimate the maximum radiated power. Because the mechanical quality factor $Q$ is low ($\approx 3$ to $4$), the source level of such a projector is limited by the maximum electric field that the piezoceramic ring stack driver can safely handle without depolarization or significant dielectric losses ($\approx 400$ kV/m for Navy Type III lead zirconate titanate). A barrel stave flextensional projector 18 cm long and 9 cm in diameter with a mass of 4.1 kg in air was tested to 200 psig (1.4 MPa) in the pressure vessel at NUSC's Dodge Pond Field Station. A source level of 194.7 dB/1$\mu$Pa-m was obtained at 1.56 kHz for an applied rms voltage of 5 kV. The projector figure-of-merit was about 14 W/kg-kHz-$Q$, and this number would be expected to apply to a larger, lower frequency projector of commensurate dimensions.

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GLOSSARY

A Cross-sectional area of piezoceramic rings (m²)
b Mean width of stave (m)
C_b Blocked capacitance of ring stack (F), Eq. (4)
C_m^E Short-circuit compliance of ring stack (m/N), Eq. (7)
C_0 Combined compliance (m/N), Eq. (9)
C_1 Fundamental modal compliance of staves, Eq. (3)
C_2 Extensional-mode compliance of staves, Eq. (2)
d_{33} Piezoelectric constant (m/V)
E_{max} Maximum allowable rms electric field intensity (V/m)

f_0 Fundamental resonance frequency (Hz)
h Mean thickness of stave (m)
k_{33} Piezoelectric coupling factor
l Length (m) of ring stack (= unsupported length of stave)

M Fundamental, water-loaded, modal mass (includes radiation mass) (kg)
N Electromechanical turns ratio (N/V), Eq. (6)
n Number of piezoceramic rings in stack
P_f Maximum field-limited radiated power (W), Eq. (12)
P_s Maximum stress-limited radiated power (W), Eq. (13)
Q Mechanical quality factor, Eq. (11)
Q_{opt} Optimum Q for maximum radiated power, Eq. (14)
R_m Mechanical loss resistance (N·s/m)
R_r Radiation resistance (N·s/m)

s_{33}^E Short-circuit piezoceramic compliance coefficient (Pa⁻¹)
T_{max} Maximum allowable rms stress in ring stack (Pa)
Y Young's modulus of stave material (Pa)
\alpha Mechanical transformer turns ratio, Eq. (1)

\varepsilon_{33}^T Free dielectric permittivity (F/m)
\eta Electromechanical efficiency, Eq. (10)
\omega_0 2\pi f_0 (rad/s)
INTRODUCTION

The barrel stave flexextensional transducer has been developed at the Defence Research Establishment Atlantic (DREA), Canada, by Jones and McMahon,\(^1,2\) although its origins can be traced\(^3\) to transducers patented by Hayes\(^4\) in 1936 and by Merchant\(^5\) in 1966. The barrel stave flexextensional transducer concept is illustrated in figure 1. A piezoceramic ring stack drives two end pieces in opposite directions along the axis of the device. Attached to the end pieces are a number of curved staves that are driven at their ends into flexure because of their curvature. Cavanagh\(^6\) calls his version of this device an “end-driven bar” projector. Except for the fact that the staves in present day configurations are concave (as in figure 1) rather than convex, the barrel stave is essentially a Class I flexextensional transducer in the nomenclature of Royster.\(^7\) With concave staves, the radial motion of the staves is in phase with the axial motions of the ends, whereas in the original Class I flexextensional,\(^7,8\) the axial and radial displacements were 180 degrees out of phase. Furthermore, whereas external pressure on convex staves will tend to unload the compressive prestress on the ring stack, pressure on concave staves will enhance the prestress.

In a flexextensional transducer, the placement of the flexing staves (or shell, as in the case of the more common Class IV flexextensional transducer) between the ring stack drive and the water radiation load introduces a lever that transforms the relatively small radiation impedance of the water to a larger impedance more compatible with that of the stiff ring stack. The transformed radiation mass of the water is often large enough to be the dominant mass loading the ceramic stack spring—that large enough to result in a low frequency resonance from a relatively small projector.

A high-power test of a barrel stave flexextensional transducer constructed in a joint Navy Industrial Cooperative Research and Development (NICRAD) venture between NUSC Code 8223\(^9\) and Edo Corporation’s Western Division\(^10\) is reported herein. The device, depicted in figure 1, is nearly identical to the DREA transducer described by Jones and McMahon.\(^1\) It is now commercially available as Edo Model 6993. The overall length is about 18 cm and the outer diameter is 9 cm. There are 8 aluminum staves with a radius of curvature of about 19 cm that span the ring stack height of 10 cm. The stave thickness varies from a minimum value of 3 mm to a maximum of 6 mm; the staves are flat on the inside (octagonal) diameter of the transducer and rounded on the outside (circular) diameter. The ring stack consists of eight Edo EC-69 lead zirconate titanate rings, each 51 mm OD, 38 mm ID, by 12.7 mm thick. The transducer has a total mass of 4.1 kg.

![Figure 1. Cross-Section of Edo Barrel Stave Flexextensional Transducer\(^10\)](image-url)
THEORY

To estimate the maximum obtainable source level, the approach taken by Woollett\textsuperscript{11} is used. He used an equivalent circuit model (that of Van Dyke\textsuperscript{12}) and assumed that the response was linear up to failure. The Van Dyke circuit is suitable for tonpilz, bender bar, and many other projector types. Although it can be made to fit flextensional transducer behavior in the vicinity of resonance,\textsuperscript{13} we prefer to use an equivalent circuit that more closely fits the flextensional response over a broader frequency range. Such an equivalent circuit is shown in figure 2. This is a simplified version of the circuit model that Brigham\textsuperscript{14} used for Class IV flextensional transducers.

![Figure 2. Simplified Equivalent Circuit for Flextensional Transducers](image)

In figure 2, $C_b$, $N$, and $C_m^E$ are the blocked capacitance, the electromechanical turns ratio, and the short-circuit compliance, respectively, of the piezoceramic ring stack. The transformer of turns ratio $\alpha$ represents the lever action that converts the extensional ring stack motion into radial motion of the staves. If we use the fundamental modal velocity as the velocity of the staves represented to the right of the mechanical transformer in figure 2 (so that $M$ is the modal mass, i.e., the actual mass of the staves plus a radiation mass), then

$$\alpha = 0.9 \frac{l}{r},$$  \hspace{1cm} (1)

where $l$ is the stave length and $r$ the stave radius of curvature. The series resonator, consisting of compliance $C_1$, mass $M$, mechanical loss $R_m$, and radiation resistance $R_r$, represents the staves as resonant flexural bars loaded by the radiation impedance of the cylindrical radiating surface. In parallel with this fundamental mode flexural resonator is a higher mode resonator\textsuperscript{14} but, if attention is confined to frequencies below the higher mode resonance, the higher mode resonator can be represented simply by compliance $C_2$. It is the presence of this compliance that makes the equivalent circuit of figure 2 qualitatively different from the Van Dyke\textsuperscript{12} circuit commonly used for nonflextensional transducers. For Class IV flextensional devices, $C_2$ is a shell breathing-mode compliance.\textsuperscript{14,15} For the barrel stave transducer, $C_2$ is the extensional compliance of the staves; i.e.,

$$C_2 = \nu Y b h,$$ \hspace{1cm} (2)

where $\nu$ is the number of staves, $Y$ their Young's modulus, $b$ their width, and $h$ their thickness. The barrel stave higher mode involves a stretching of the staves. The staves become longer and are displaced radially inward as the ring stack expands; whereas, in the fundamental mode, the staves bend radially outward as the stack expands. The modal compliance\textsuperscript{16} is

$$C_1 = (0.024/\nu Y b) (l/h)^3,$$ \hspace{1cm} (3)

if it is assumed that the staves behave like flat bars with clamped ends (i.e., the ends are not permitted to rotate).
The ring stack blocked capacitance is\(^{17}\)

\[ C_b = n^2 (1-k^{33})^2 \varepsilon^{33T} A/l , \]  

where

\[ k^{33} = d^{33} / \varepsilon^{33} l s^{33E} , \]  

is the square of the piezoceramic material coupling factor, \( d^{33} \) is the piezoelectric constant, \( \varepsilon^{33T} \) the free dielectric permittivity, \( s^{33E} \) the short-circuit compliance coefficient (reciprocal of Young's modulus), \( A \) the ring stack cross-sectional area, \( l \) the stack length (assumed equal to the stave length, and \( n \) the number of rings in the stack. The electromechanical turns ratio is

\[ N = \frac{n d^{33} A}{s^{33E} l} . \]  

The ring stack short-circuit compliance is

\[ C_mE = \frac{s^{33E} l}{A} . \]  

The resonance frequency is \( f_0 = \omega_0 / 2\pi \), where

\[ \omega_0 = \frac{1}{\sqrt{MC_0}} \]  

and

\[ C_0 = \frac{C_1 (C_2 + C_mE)}{a^2 C_1 + C_2 + C_mE} . \]  

The electromechanical efficiency is

\[ \eta = \frac{R_r}{R_r + R_m} , \]  

and the mechanical quality factor is

\[ Q = \frac{1}{\omega_0 C_0 (R_r + R_m)} . \]  

Following Woollett,\(^{11}\) we consider two kinds of limitation of the radiative power. The field limit is the largest power that can be radiated without depolarizing the piezoceramic or producing significant dielectric losses, and is expressed in terms of a maximum allowed electric field rms amplitude, \( E_{\text{max}} \). The stress-limited power is that produced when the piezoceramic rms stress amplitude is the maximum allowable value, \( T_{\text{max}} \). The corresponding peak value \( \sqrt{2} T_{\text{max}} \) is taken to be equal to the compressive prestress, i.e., the ring stack should not be allowed to undergo tension during any part of its motion.

The field-limited power at resonance is

\[ P_f = \frac{\eta \omega_0 Q k^{33} (\varepsilon^{33T} E_{\text{max}}^2) A l}{\left( 1 + \frac{C_2}{C_mE} \right) \left( 1 + \frac{C_2 + C_mE}{a^2 C_1} \right)} . \]  

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where the quantity \( \varepsilon_{33} \sigma E_{\text{max}}^2 \) will be recognized as an electrical energy density. Multiplied by \( k_{33}^2 \), this quantity becomes the maximum electromechanical stored energy density,\(^{11}\) a quantity dependent entirely on the properties of the piezoceramic material. The numerator of equation (12) is identical to Woollert's result.\(^{11}\) The additional factors in the denominator are due to the extra compliances that exist in the flexextensional equivalent circuit of figure 2. The stress-limited power at resonance is

\[
P_s = \frac{\eta_0 Q \left( \varepsilon_{33} \sigma E_{\text{max}}^2 \right)}{\left[ 1 + \left( \frac{Q}{1 + \frac{C_2 + C_m E}{\alpha^2 C_1}} \right)^2 \right] \left[ 1 + \frac{C_2 + C_m E}{\alpha^2 C_1} \right]},
\]

where the quantity \( \varepsilon_{33} \sigma E_{\text{max}}^2 \) is the maximum mechanical energy density of the piezoceramic ring stack.

If the mechanical quality factor \( Q \) is small enough, the field-limited power \( P_f \) is less than the stress-limited power \( P_s \), and the transducer is field-limited. On the other hand, for large \( Q \) the transducer is stress-limited. Woollert\(^{11}\) defined an optimum quality factor, \( Q_{\text{opt}} \), for maximum power such that \( P_f = P_s \). From equations (12) and (13),

\[
Q_{\text{opt}} = \left( 1 + \frac{C_2 + C_m E}{\alpha^2 C_1} \right) \sqrt{\left( 1 + \frac{C_2 + C_m E}{C_m E} \right)^2 - 1}.
\]

(14)

Inspection of equation (14) reveals that the optimum quality factor for flexextensional transducers will always be larger than the corresponding value for nonflexextensional transducers—obtained by setting \( C_2 = 0 \) and \( \alpha^2 C_1 \gg C_m E \). As an example, it is worth noting that the latter value for Navy Type III ceramic is \( Q_{\text{opt}} = 4.5 \) (assuming \( E_{\text{max}} = 10 \text{ Vrms} \text{ per mil} = 400 \text{ kV/m} \), and \( T_{\text{max}} = 30 \text{ MPa} \approx 4.4 \text{ ksi} \)), and so a flexextensional transducer having a Type III ring stack driver is expected to be field limited if the quality factor \( Q \) is less than this value.

**EXPERIMENT**

A simplified block diagram of the experimental test arrangement is shown in figure 3. Because the quality factor \( Q \) for the Edo barrel stave flexextensional is approximately 3, the test transducer was assumed to be field-limited as explained in the previous paragraph. A commonly accepted\(^{11,18}\) value for \( E_{\text{max}} \) is 10 Vrms/mil = 4 kV/cm. Because the EC-69 rings are 0.5 inch in thickness, a 5000 Vrms drive was required to produce the maximum allowable source level. A 5-kVA CML amplifier was used to drive a 4.3:1 step-up transformer which, in turn, drove the untuned transducer with 75 ft of DSS-3 cable attached. The transformer was actually an output transformer (from another CML amplifier) driven backward. (The primary winding was the conventional 256 \( \Omega \) output winding, and the secondary was the normally balanced input winding.) A Tektronix 1000:1 voltage probe was used in conjunction with a Tektronix 454 oscilloscope to measure the transducer voltage.
Figure 3. Experimental Test Arrangement

Because the transducer had previously been shown to be omnidirectional\textsuperscript{19} at resonance, beam patterns were not obtained at Dodge Pond. The USRD H-52 hydrophone\textsuperscript{20} (Serial No. 017) was placed at a 2-meter horizontal range from the projector. Both free-field and pressure-vessel source level measurements were made at Dodge Pond. The free-field measurements were made at a depth of 30 ft (9.1 m). The pressure vessel was an epoxy/glass fiber cylindrical tank that was pressurized to 200 psig. Measurement of the source level with the tank pressure at zero psig provided a comparison with free-field source levels at the lower drive levels. The rms drive voltage was varied from 100 volts to 5000 volts. Oscilloscope traces of the hydrophone output waveforms were photographed with a Tektronix C-30A camera.

RESULTS AND DISCUSSION

A typical low-level, free-field transmitting voltage response (TVR) is illustrated by the square symbols plotted in figure 4. The transducer resonance occurred at 1560 Hz. The result of driving the transducer to successively higher voltages at the 30 ft operating depth is shown by the squares in figure 5, which is a plot of source level versus applied voltage. At rms voltages above 2300 V, the source level does not increase beyond 188 dB/μPa-m. Such a ceiling on source level can be an indication of cavitation\textsuperscript{21,22} (as the cavitation bubbles act as a screen between the projector and its far field). However, examination of the radiating surface while the transducer was brought to within a few inches of the surface and driven at high levels revealed no visible bubbles, but rather indicated that the boat was flapping, i.e., not conforming to the convex surface of the staves. The boat had not been bonded to the staves during construction of the transducer.\textsuperscript{10}
**Figure 4. Transmitting Voltage Responses**

**Figure 5. Source Level as a Function of rms Drive Voltage**
Further evidence that boot flapping, rather than cavitation, was occurring is indicated by the waveforms of figure 6 for operation at the 30 ft depth. Figure 6A shows the received signal for a drive of 1400 Vrms (below the threshold), while figure 6B is for a 2800 Vrms drive (above the threshold). If cavitation were occurring, one would expect to see significant nonlinear distortion in figure 6B that was not present in figure 6A. It should be noted that the waveforms of figure 6 have been lowpass-filtered by the 50-kHz rolloff of the Scientific-Atlanta 1116 preamplifier. However, this filter would have allowed many harmonics of the fundamental transducer resonance frequency to pass had they been present. The difference in the shapes of the pulse envelopes of figures 6A and 6B is attributed to the boot-flapping phenomenon which, apparently, takes several cycles to reach a steady state.

![Waveform Images](Image)

**Figure 6. Free-Field Waveforms at 2 Meters, 1.6 kHs**

To inhibit boot flapping and to attain source levels greater than 188 dB/1μPa-m, the projector was mounted inside the epoxy/glass fiber pressure vessel. The solid TVR in figure 4 was obtained at low level (100 Vrms) with no pressurization of the water in the tank. Comparison of the solid TVR with that indicated by the squares (free-field case, 75 Vrms drive) gives an indication of the effect of the tank wall reflections. At frequencies up to 2 kHz, the differences are 1 dB or less. Above 2 kHz, they become greater, amounting to about 3 dB at 3 kHz. In the vicinity of the transducer resonance, the tank wall effect on the TVR is less important than the effect of drive level (indicated by the circles in figure 4) and still less important than the effect of hydrostatic pressure (dashed curve in figure 4). Because of the mechanical amplification provided by the transducer assembly, a 200 psi increase in hydrostatic pressure results in a 4 to 5 ksi increase in compressive prestress on the piezoceramic ring stack. Such a change in prestress is not sufficient, however, to account for the 3.5 dB reduction in TVR seen at resonance—the difference between the solid and dashed curves in figure 4. McMahon has suggested that under pressure, increased mechanical losses result from boot intrusion into the spaces between the strakes. He has further suggested that the boot effect is nonlinear, becoming less important at higher drive levels. This phenomenon is illustrated by the 200 psig data, indicated by the circles in figure 5. At 100 Vrms drive, the result of 200 psig pressurization is 3.5 dB less source level than with no pressurization. At higher levels, but still below the 2300-V threshold, the source levels converge. (A “play” mechanism may be involved—at low amplitude, when displacements are on the order of the play, significant losses occur; at high
amplitudes, displacements are much larger than the play, and it is no longer important.) The nonlinearity is also indicated by the steeper slope (versus 20 dB/decade) of the 200 psig data in figure 5.

The minimum source level, field-limited at 10 Vrms/mil, i.e., for 5000 V drive, was 194.7 dB//1µPa-m. To compare this with the theoretical value from equation (12), estimates are required for a number of parameters. The efficiency η was measured to be 50 percent at the resonance frequency f₀ of 1560 Hz, and the mechanical quality factor Q was approximately 3. The EC-69 lead zirconate titanate nominal material properties are

\[ k_{33} = 0.62, \quad e_{33}^E = 13.5 \times 10^{-12} m^2/N, \quad e_{33}^T = 1050 \varepsilon_0 \]

(where \( \varepsilon_0 \) is the free space permittivity, \( 8.85 \times 10^{-12} F/m \), and \( E_{\text{max}} = 394 \text{ kV/m} \) (10 Vrms/mil)). The ring stack had a cross-section area \( A \) of 8.9 cm² and a length \( l \) of 10 cm. From equation (1), with a stave length \( l \) of 10 cm and radius of curvature \( r \) of 19 cm, the mechanical transformer turns ratio \( \alpha \) was approximately 0.5. The eight aluminum staves had a mean width \( b \) of 2.8 cm and a mean thickness \( h \) of 4.5 mm. With a Young's modulus for aluminum of \( 7.0 \times 10^{10} \text{ Pa} \), equation (2) yields \( C_1 = 1.4 \text{ nm/N} \) for the extensional compliance of the staves, and equation (3) yields \( C_1 = 18 \text{ nm/N} \) for their flexural compliance. From equation (7), the ring stack compliance \( C_m^E \) is 1.6 nm/N. The field-limited power is then calculated from equation (12) to be \( P_f = 230 \text{ W} \). The corresponding maximum source level is approximately 194 dB//1µPa-m, which is in good agreement with the measured value. On the other hand, Woollett's result for a nonflexextensional transducer containing the same amount of piezoceramic material (based on the Van Dyke equivalent circuit) would predict a maximum field-limited source level 5 dB higher, i.e., 199 dB//1µPa-m.

Using the equivalent circuit of figure 2, scaling laws for flexextensional transducers can be established. For example, if all dimensions of the test barrel stave flexextensional were doubled, masses would increase by a factor of 8 while all compliances would be reduced by a factor of 2. Thus the resonance frequency, inversely proportional to the square root of a mass and compliance product, would decrease by a factor of 2. The radiation resistance would quadruple, and the mechanical loss resistance would also quadruple if it could be perceived as a constant loss factor associated with the stave compliance \( C_1 \). The mechanical quality factor \( Q \), from equation (11), would remain constant. The field- and stress-limited powers would each quadruple, while the optimum quality factor would not change. Thus the maximum source level would increase by 6 dB if all dimensions were simply doubled. A projector figure-of-merit (FOM) in common use is the maximum radiated power per unit (mass × frequency × quality factor), a quantity which remains invariant with scale changes. For the tested barrel stave flexextensional transducer, the FOM was approximately 14 W/kg-kHz-Q.

**CONCLUSION**

Maximum radiated power calculations for flexextensional projectors require that Woollett's results be modified to account for a nonzero, higher mode compliance \( C_2 \), and a finite flexural-mode compliance \( C_1 \). The maximum (10 Vrms/mil) field-limited source level obtained from the tested barrel stave flexextensional projector was 194.7 dB//1µPa-m at its resonance frequency of 1560 Hz. This is in good agreement with the predicted value of 194 dB//1µPa-m. Because the mechanical quality factor was about 3, the projector figure-of-merit was approximately 14 W/kg-kHz-Q. Scaling laws suggest that the latter number would apply to larger, lower frequency barrel stave flexextensional transducers that are scaled by increasing all dimensions by a common factor.
REFERENCES


REFERENCES (Cont'd)


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