ANALYSIS, ESTIMATION AND CONTROL FOR
PERTURBED AND SINGULAR SYSTEMS AND FOR
SYSTEMS SUBJECT TO DISCRETE EVENTS

for the period
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Submitted to: Dr. Marc Q. Jacobs
Program Advisor
Directorate of Mathematical and Information Sciences
Air Force Office of Scientific Research
Building 410
Bolling Air Force Base
Washington, D.C. 20332

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I. Summary

In this report we summarize our accomplishments in the research program presently supported by Grant AFOSR–88–0032 over the period from October 1, 1989 to September 30, 1990. The basic scope of this program is the analysis, estimation, and control of complex systems with particular emphasis on (a) multiresolution modeling and signal processing; (b) the investigation of theoretical questions related to singular systems; and (c) the analysis of complex systems subject to or characterized by sequences of discrete events. These three topics are described in the next three sections of this report. A full list of publications supported by Grant AFOSR–88–0032 is also included.

The principal investigator for this effort is Professor Alan S. Willsky, and Professor George C. Verghese is co-principal investigator. Professors Willsky and Verghese were assisted by several graduate research assistants as well as additional thesis students not requiring stipend or tuition support from this grant.
II. Multiresolution Modeling and Signal Processing

One of the major directions of our research in the past year has been motivated by recent results on multiresolution representations of signals and the related notion of wavelet transforms. Our objective has been to develop probabilistic counterparts to this deterministic theory that can then be used as the basis for signal modeling, optimal estimation, and related algorithms. During the past year we have built on our earlier work to establish the foundation of a theory for multiscale stochastic processes and their estimation [32,33,41,42,49,53-56,58-60].

The basic form of a multi-scale representation in 1-D is

$$x_m(t) = \sum_{n=-\infty}^{+\infty} x(m,n)\phi(2^nt - n)$$

where $\phi(t)$ is the basic scaling function defining the representation and $m$ denotes the resolution of the representation. Our work has focused on developing and exploiting stochastic models for the coefficients $x(m,n)$ as the scale, $m$, evolves. As we show in [32,33,41,42], there is a natural structure that arises in examining wavelet transforms from this dynamic synthesis perspective. In particular the $(m, n)$ index set should be thought of as a weighted lattice, where each level of the lattice corresponds to a particular scale and the lattice structure and weights from scale to scale are determined by the wavelet transforms structure. For example, the most basic such structure is the dyadic tree, with a point $(m,n)$ having two descendents, $(m + 1, 2n)$ and $(m + 1, 2n + 1)$, at the next scale.

Starting from this perspective we have developed several lines of investigation aimed at developing a theory and statistical analysis framework for stochastic processes defined on such structures. In one part of our work [33, 41,54,55] we have developed a theory of auto-regressive modeling for processes on dyadic trees. The results here are far more complex than their time series counterparts because of the tree structure, but we have been able to develop Levinson-like efficient algorithms for constructing such models. In another part of our work [32, 42,44,56,59] we have studied a class of “state” processes evolving from coarse-to-fine scales. Much of our work in this area has focused on the analysis of processes on trees, but we have also obtained some results on the more general class of weighted lattices. In particular, we have investigated the estimation of such processes based on multiresolution, noisy measurements and have developed several different approaches to such problems. The first of these involves the use of fast wavelet transforms to map the multiresolution measurements into a set of coordinates in which optimal multiscale fusion can be accomplished by independent processing of the information at each scale. A second algorithm structure takes advantage of the fact that our
models have a Markovian structure on the weighted lattice. This directly leads to an iterative relaxation algorithm, taking advantage of the local structure of the lattice, which resembles the pyramidal iterative structures common in multigrid methods for solving partial differential equations.

The third algorithmic structure, developed for processes on dyadic trees, consists of a fine-to-coarse processing sweep followed by a coarse-to-fine processing step. The statistical operations being performed in this algorithm are reminiscent of the operations performed in the Rauch-Tung-Striebel algorithm for the optimal smoothing of data from temporal state space models. In particular the fine-to-coarse sweep can be viewed as a generalization of the Kalman filter to the problem of multiscale fusion. This algorithm gives rise to a new class of Riccati equations involving three steps—a measurement update step, a “predict” step for moving up the tree, and a “fusion” step for combining information from different subtrees. The analysis of this Riccati equation requires the introduction and development of system-theoretic properties for processes on trees. In [49,59] we present several basic elements of such a system theory and use them to develop asymptotic results for our multiscale Riccati equations and estimation algorithms. In [56] we present the foundation of a multiscale system and realization theory, which should provide the basis for a theory of multiscale statistical modeling and “spectral factorization.”

The Markovian interpretation of our multiscale models has also provided motivation for another element of our work [58,60] which provides a different piece of the statistical modeling methodology we are developing. Specifically, the basic property of statistical models, such as ours, based on wavelet transforms is that the wavelet transform performs a scale-to-scale decorrelation. One such formulation is developed in [58,60] and, more importantly, this and our other work have given us not only results indicating the promise and potential broad applicability of multiscale statistical techniques but also an analytical framework in which we can develop such techniques and assess their value.

Some of our study of multiresolution questions is being carried out in the context of estimation problems for systems of the form
\[ Lx = Bu, \quad y = Cx + r \]
with boundary conditions \( Vx_b = v \) and \( y_b = C_b x_b + r_b \), where \( L, B, \) and \( C \) are general linear operators (possibly 2D or higher), \( x \) denotes a “partial state”, \( y \) represents observations, and \( r \) denotes measurement noise.
The Ph.D. thesis of M. Adams (Aero-Astro Department, MIT, 1983) treated such systems in the case where $B$ and $C$ were constant matrices. The (apparently) more general case where $B$ and $C$ are operators is of interest when, for instance, the observations, involve blurring or averaging, corresponding to using a sensor of low resolution. The natural way to handle extension is by some sort of augmentation that captures the dynamics associated with $B$ and $C$. A simple augmentation (but one that eluded us until recently) is the following:

\[
\begin{pmatrix}
  I & 0 & 0 \\
  -B & L & 0 \\
  0 & -C & I
\end{pmatrix}
\begin{pmatrix}
  \eta \\
  x \\
  \xi
\end{pmatrix}
= 
\begin{pmatrix}
  I \\
  0 \\
  0
\end{pmatrix}
\begin{pmatrix}
  u
\end{pmatrix}
\]

with associated augmented boundary conditions. We are using this augmented model to study the effect on estimation of different levels of blurring in the observations.

We have also made considerable progress in our study of multiresolution modeling for control in an interesting class of discrete-event, continuous-time systems, namely power electronic circuits. These circuits are usually modeled as interconnections of linear, time invariant circuit components and ideal switches. The switching events are determined by the relationship between time varying control inputs and periodically varying clocking waveforms. In steady state, the behavior of a power convertor is periodic. To design controllers that regulate departures from steady state, we need appropriate dynamic models.

The main focus of our research so far has been on averaged models, [34-37], [47], [62]. This focus reflects the fact that in many power circuits - high frequency PWM converters in particular- we are interested in controlling the local average of circuit waveforms, not the instantaneous values. If the clocking waveform in the circuit has period $T$, the local average of interest for a variable $x(t)$ is defined by

\[
\bar{x}(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau) d\tau
\]

This average is \textit{not} the one produced by the classical averaging methods developed in nonlinear mechanics.
Using the fact that the derivative of the local average equals the local average of the derivative, and making reasonable approximations, we can obtain continuous-time dynamic models for the averaged variables by taking the local average of dynamic models for the instantaneous variables. It is also possible to obtain these averaged dynamic models directly in circuit form by direct averaging of the instantaneous circuit, [36]. The continuous-time averaged models are usually far simpler than the instantaneous switched models to analyze and simulate, and are more fruitful in developing controller designs, [62].

The converter considered in [62] is driven by a periodically varying voltage source, and thus has two natural averaging periods associated with it: the long period $T_L$ of the voltage source, and the short period $T_S$ of the clocking or switching waveform. We demonstrate in [62] the advantages of a multiresolution approach that uses averaged models at both these time-scales. The $T_S$-averaged model is periodic with period $T_L$, because it is driven by the periodic voltage source. The $T_L$-averaged model is time invariant, and is obtained by averaging the $T_S$-averaged model. We are not aware of a similar two-stage procedure being treated in the classical averaging literature, and intend to pursue this possibility.

The time invariant controllers designed in [62] are derived using the $T_L$-averaged model. By using the $T_S$-averaged model instead, we obtain periodically varying controllers that enable much faster recovery from transients, [63]. We are now studying the possibility of tuning the parameters in the $T_S$-controller on the basis of computations that involve the $T_L$-model. Such hierarchical control based on aggregation at successively larger time scales is an important theme in both the multiresolution and discrete-event aspects of our research.

In many other situations in power electronics, it is not the local average but the component at the switching frequency (or some other frequency) that is of primary interest. This is the case with so-called resonant converters, for instance. Also, even with PWM converters, we often wish to refine the predictions of an averaged model by computing the "ripple", which is the switching-frequency component. This motivates the definition of the local $\omega$-component as the local average of $x(t)e^{-j\omega t}$. If $x(t)$ is periodic, this is just the Fourier series coefficient at the frequency $\omega$. With this definition, we can again obtain dynamic models for the local $\omega$-component from the instantaneous dynamic models. The value of this approach for resonant converters is demonstrated in [28].
III. Singular Systems

During the past year we have continued our work on the analysis of two point boundary-value descriptor systems (TPBVDS's):

\[ Ex(k + 1) = Ax(k) + Bu(k) \] (3.1)

with boundary conditions

\[ v = Vix(0) + Vfx(N) \] (3.2)

and output

\[ y(k) = Cx(k) \] (3.3)

We have also extended our work into the examination of two-dimensional (2-D) extensions of these models, e.g. to systems of the form

\[ E_{11}x(k+1,j+1) + E_{10}x(k+1,j) + E_{01}x(k,j+1) + E_{00}x(k,j) = Bu(k) \] (3.4)

Such models are natural choices for the description of non-causal (e.g. spatial) phenomena and signal processing tasks.

Our early work in this area [50,61] has focused on estimation problems for models such as these. In particular we have completed several studies of the structure of estimation algorithms for the model (3.1)-(3.4). In one [50] we introduce and develop a complete theory for a new class of generalized Riccati equations. Several additional papers on this topic are in preparation, including one describing in detail a two-stage forward/backward-sweep solution to the optimal smoothing problem. In addition, we have recently [61] extended these results to a broader class of singular estimation problems in which neither the error covariance nor its inverse are well-defined. This work also includes an “inward-outward” implementation of the optimal estimator for TPBVDS’s, which we are also in the process of extending to 2-D systems such as (3.4).

Finally, an important issue in such noncausal estimation problems, particularly in 2-D, is computational complexity. In 2-D the complexity of optimal smoothing algorithms depends explicitly on the size of the data array being processed (since the size of the 2-D boundary depends on the array size, in contrast to what occurs in the 1-D case). This leads directly to the consideration of parallel processing algorithms, in which large data sets are partitioned, with parallel processors assigned to each set in the partition. Parallel algorithms in such a situation would involve the separate processing of these data subsets by individual processors, followed by some type of inter-processor information exchange, and a final parallel processing step in which each processor uses the information from other processors to update its estimates. The algorithms we have developed, both for 1-D TPBVDS’s and their 2-D extensions suggest several alternate structures for such a parallel
processing structure. We are developing these at present and expect not only to be able to provide a clear picture of parallelizability for such estimation problems but also to uncover precise methods, not only for determining the optimal partitioning of data sets (and thus the optimal number processors) for optimal estimation but also for computing a tradeoff curve for suboptimal estimation vs. computational complexity.

Also, the development of Selective Modal Analysis (SMA) techniques for singular systems continues to be of interest. Though we have not progressed far in this direction since our last report, there has been some consolidation of understanding, and further experimental verification on very large power system models. These results are to be presented in an invited talk in the next few weeks, [64].
IV. Systems Subject to Discrete Events

During the past few years there has been considerable interest in the development of control concepts and algorithms for complex processes that are characterized more by the occurrence of discrete events than by differential equations representing the laws of physics. Such processes are typically man-made—flexible manufacturing systems, computer networks, etc.—and are often best described in symbolic, rather than numeric form. Our work in this area is aimed at combining concepts from computer science and from control in order to develop a meaningful theory of control for such systems. In particular, the models and formalisms used in such a study come from the field of computer science (automata, synchronous processes, etc.) while the problems and design paradigms come from control (stability, regulation, robustness...).

Our research [16,17,29–31,43–46,48,51-52,57] has dealt with systems modeled as finite-state automata in which control is effected by enabling or disabling particular events and in which observations consist either of state measurements or the intermittent information provided when any of a particular set of observable events occurs. Our work has had as its objective the development of stability, regulation, and servo theories for such discrete event dynamic systems (DEDS), described by automata and for other more powerful models.

To this point essentially all of our work has focused on the finite-state automaton framework. In this context we have developed a stability theory for DEDS, characterizing the system's ability to return to normal status after an excursion. We have used this theory to develop methods for stabilization of DEDS by state feedback. Also we have developed a theory of observability for DEDS. Given the fact that in many complex systems our sensed information concerns events rather than states, we have considered an intermittent measurement model—i.e. we sense when one of a subset of events has occurred but do not receive direct information about other events. This leads to what we think is a useful notion of observability, in which there are points in time at which we know the state of the system, but they are intermittent.

Another important component of our work is the synthesis of our stabilization and observability constructs in order to develop a methodology for dynamic output compensation. An important element of our theory is that it highlights an important component of real complex systems, namely the criticality of the timing of information and control. In particular, the notions of state feedback stabilization and observability that we use do not together imply stabilizability by output feedback. Specifically since the level of our state knowledge fluctuates
thanks to our event-driven observation model, the critical issue becomes one of ensuring that the requisite level of state information is available at a point at which we have control options that can use it.

With this stabilization theory in hand, we have also been able to study and solve problems of tracking and restrictability for DEDS, i.e. the control of DEDS to follow specified strings of particular marked events or to produce strings with specified properties. This is a critical component of the servo problem. In this context we have also been able to study the question of reliable control, i.e. the ability to recover from one or more failures or errors. In addition, these results have allowed us to develop a framework for task following, aggregation, and higher-level control. Specifically, we have developed a theory for controlling DEDS so that one of a specified set of tasks is performed, where a task is specified as the completion of one of a set of strings. This leads directly to the task-level modeling of a DEDS, which greatly reduces model complexity and thus our ability to consider higher-level control questions. In particular at a higher level we replace an entire string of lower-level events corresponding to a task by a single macro-event. This leads to a comparatively simple and very regular model. Since many real DEDS consist of interconnections of systems, that interact only at the task level, our aggregation results should allow us to reduce considerably the complexity of analysis problems for large systems.

This last point leads directly into our most recent research area, namely the investigation of more structured classes of DEDS models, in order, in part, to deal with the apparent computational explosion arising in DEDS analysis. In particular, many algorithms in [16,17,29–31,43–46]) have computational complexity that is polynomial in the cardinality of the automaton's state space. On the other hand, in many cases a DEDS consists of an interconnection of many smaller DEDS so that the Cartesian product state space can be enormous. However in many cases (e.g. in flexible manufacturing systems), although these subsystems interact strongly, they do so relatively infrequently — i.e. the systems evolve independently except for intermittent coordination times. Taking advantage of structure should lead to considerable reduction in complexity. In addition, we have also initiated investigations of other modeling frameworks for DEDS in order to develop a more natural and convenient setting in which to formulate control problems for complex systems involving the concurrent operation of many interacting subprocesses. This endeavor involves a blending of the modeling paradigms of computer science with the problem formulations and perspectives of control theory.
Publications

The publications listed below represent papers, reports, and theses supported in whole or in part by the Air Force Office of Scientific Research under Grant AFOSR–88–0032:


