CHARACTERIZATION AND APPLICATION OF A RESPONSYN STEPPING MOTOR

by

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ABSTRACT

Tests were made on the Responsyn actuator stepping motor to determine its characteristics and limitations. An analysis was attempted to try to explain the observed behavior.

The results of the testing can be briefly summarized as follows. The motor will drive an inertia load of ten ounce-inch² at eight hundred pulses per second (each pulse corresponds to a step of 0.45 degrees). Unloaded at this speed the motor has a useful torque of one hundred ounce-inches. The motor will accelerate and decelerate to this speed immediately from standstill without losing steps. The motor will run on nonuniformly spaced pulses such as those from a binary rate multiplier but operation is noisier and rough. The motor is at best about 15% efficient. Its life expectancy looks good.

The driving circuits developed for the motor proved adequate and did not limit performance. Logic was developed which allowed the motor to run continuously, in bursts, or reversing.

At low speeds the response to a single step is second order with parameters varying with loading. At higher speed the individual step performance disappears and the total response is like a ramp. The motor is running slightly behind the pulses but will stop correctly after a slight delay. A delay between input pulse and the start of response of one millisecond was observed.

Thesis Supervisor: George C. Newton, Jr.
Title: Professor of Electrical Engineering
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Definition of Symbols and Abbreviations

\( f \) - force on rotor, or frequency where applicable

\( F \) - magnetomotive force

\( W_e \) - electrical energy

\( W_m \) - mechanical energy

\( W_{sr} \) - energy stored in rotor

\( W_s \) - energy stored in stator

\( W_{sa} \) - energy stored in air gap

\( N \) - number of turns

\( I \) - winding current

\( \phi \) - magnetic flux

\( x \) - air gap

\( V \) - volume

\( H \) - magnetic field

\( B \) - magnetic flux density

\( \mu_0 \) - permeability of free space

\( l \) - length, magnetic path

\( \Theta \) - angle

\( T \) - torque

\( R \) - rotor radius

\( J \) - inertia

\( \theta \) - friction factor

\( s \) - LaPlace Operator

\( \omega_n \) - natural frequency
Definition of Symbols and Abbreviations (Continued)

\(v_p\) - peak velocity

\(K.E\) - kinetic energy

\(M_f\) - mass of flex spline

\(D_6\) - inner diameter of stator

\(r\) - binary fraction

\(pp/s\) - pulses per second

\(cps\) - cycles per second

\(oz-in^2\) - inertia in ounce-(inch)^2

\(oz-in\) - torque in ounce-inches

\(mm^f\) - magnetomotive force
CHAPTER I - INTRODUCTION AND GENERAL SUMMARY

Since the advent of the digital computer in control systems, one of the most difficult problems has been mating the computer to the analogue devices which are the conventional outputs of the system. To do this, elaborate digital to analogue converters have been developed. A device which could take a digital input and produce an analogue output directly is attractive. The stepping motor or step servo does exactly this and does it well. Its output is an angular rotation for a pulse input. Since a precise increment of rotation occurs for each input pulse, feedback is not necessary to determine the shaft position.

At present there are many types of stepping motors commercially available. These range from solenoid-ratchet mechanical types to electromagnetic ones. At the present, the electric ones are much faster than the mechanical and are approaching them in power. Essentially, the operation of all of these types consists of pulsing consecutively the phases of a multiphase winding. The rotor follows the stator either to minimize reluctance or to align a permanent magnet. Many schemes of split or offset poles have been developed in order to increased the resolution of the motor without external gearing. However, the current motors are far from optimum. Faster, more precise, more powerful motors are needed.
The electromagnetic action of the Responsyn* actuator is basically that of a variable reluctance motor. The operation of a variable reluctance motor is explained with reference to figures 1 and 2. Start with winding A excited and pole EG aligned. When winding B is excited the pole FD will move to align with it. Similarly, the motor will rotate in steps as the windings are excited in sequence. The process is illustrated for a sequence ABCA in figure 2. In order to obtain this action it is necessary that all the rotor poles do not align with the stator poles at one time.

The Responsyn actuator differs from a simple variable reluctance motor by its application of the Harmonic Drive principle developed by the United Shoe Machinery Corporation. Therefore, before discussing the operation of the Responsyn actuator it is necessary to describe the Harmonic Drive. Figure 3 illustrates a simple harmonic actuator. Essentially it consists of an inner flexible cup attached to an output shaft. Bonded to this cup around the circumference is an external gear ring. An internally geared ring of the same diametral pitch but slightly larger radius is cut into the stator. The input shaft is connected to a mechanical actuator in the form of an elliptical cam. The major axis of the ellipse is such that at the tips the cup is stretched so that the teeth engage. As the input shaft turns, the contact

*TM
SCHEMATIC REPRESENTATION
OF A TYPICAL STEPPING MOTOR

Figure 1 Winding Configuration

<table>
<thead>
<tr>
<th>Excited Winding</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Position</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>E</td>
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<td></td>
<td>D</td>
<td>D</td>
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<tr>
<td></td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Figure 2 Rotor positions
point is moved around the stator. Because of the difference in diameter between the stator and the rotor (flexspline), the stator has 202 teeth to the flexspline's 200. Thus, a complete revolution of the point of contact relative to the rotor means 200/202 revolutions relative to the stator. Thus the rotor has fallen back 1/100 of a revolution relative to the stator. The net result is that the output velocity is 1/100 of the input velocity and in the opposite direction.

To utilize the Harmonic Drive for a stepping motor the flexspline was made of magnetic steel and placed in a multipole stator. A cutaway view of this is shown in figure 4. A schematic of the stator winding is shown in figure 5. In place of the elliptical cam the magnetic action of the stator is used. When a winding is excited the flexspline stretches toward it to minimize reluctance. Thus the gear teeth are brought into mesh. As the current is switched from winding to winding the point of contact moves around the stator as in the mechanical actuator. The output shaft turns 1/100 of the speed of the magnetic field, in the opposite direction. One revolution of the shaft requires that one of the windings be kept excited. During stepping it is necessary that the current to one winding not be turned off before the other is turned on.
Figure 3 Flex spline Drive

Figure 4 Responsyn Cutaway

Figure 5 Winding Configuration

These pictures taken from Reference No. 6
This thesis is an attempt to examine the properties of the Responsyn actuator and test and evaluate its performance. The factors which need to be considered are those that limit its torque and speed - the two most important properties. These factors are: friction, damping, inertia, gear lash, and torsional rigidity. Also electrical factors such as inductance and resistance must be considered. The effect of these properties has been evaluated in the body of this report.

In order to obtain information about these parameters, tests are needed to gather the information. The information thus gained is also useful for defining motor specifications and limitations. The tests decided on were maximum pulsing rate versus inertia loading, torque versus pulsing rate, efficiency, and step response. To examine the motor's behavior under possible working conditions the motor was driven by non-uniform pulsing from a binary rate multiplier. Feedback is also a consideration for these devices. Although ideally they do not require it, in practice it can sometimes be beneficial.

Work that was done concurrently with my thesis by the United Shoe Machinery Corporation showed that the characteristics of the motor could be improved by modifying the magnetic properties of the rotor. Inside the cup, originally
there had been placed a roll of forty feet of magnetic shim stock to increase the rotor permeability. The modification consisted of reducing this to sixteen feet. Tests were performed on the motor which I used with both amounts of shim stock. In order to differentiate between the results, I refer to them as the modified motor and original motor. If no designation is made, I refer to the original motor.

**General Summary**

(a) **Circuitry**

The logic and driving circuits for the Responsyn actuator were originally developed by Professor G. C. Newton, Jr. Those basic circuits with a few modifications I introduced during the course of the testing were used throughout. A general block diagram of the circuit is shown in figure 6. The logic circuit consists basically of three flip flops - the first driving the other two. The outputs of these two are the inputs to the driver stage. All the flip flops are buffered by emitter followers at each output. For reliable triggering, the input is fed into a Schmitt trigger. Because both of the flip flops will turn on one driver - therefore one winding - the motor will operate with two windings excited instead of one. The diametrically opposite windings in the motor are connected in parallel so there are effectively only four windings. One flip flop drives windings 1 and 3.
Figure 6 Block Diagram of Logic and Driving Circuit for the Responsyn
The other drives 2 and 4. The resulting switching sequence with two windings excited is

\[ \begin{align*}
1 & 3 & 3 & 1 & 1 & 3 \\
2 & 2 & 4 & 4 & 2 & 2
\end{align*} \]

and so on. One pair of windings changes state per pulse. The resultant magnetic field occurs between poles and rotates 45° per step. The field rotation is through 180° only as the variable reluctance action is insensitive to field polarity.

The reversing circuit acts by shorting the inputs of the two flip flops to ground. This creates a signal at the input of the flip flop which was previously above ground. This flip flop will complement without affecting the other. The sequence is thus reversed and the motor reversed. However, the reversing pulse causes the motor to move one step in the direction of original motion before reversing and this must be taken account in the logic. The monostable at the reversing circuit input was necessary to prevent coincidence of input and reversing pulses under certain experimental inputs.

Chapter II of this thesis deals in detail with the circuitry and discusses each component in detail. For further information see this section.

(b) **Experiments**

The basic experimental set up is shown in the block diagram of figure 7 and illustrated in the picture figure 8.
Figure 7  Block Diagram of Experimental Setup

Figure 8  Inertia Load Setup
The diagram illustrates the set up as used for inertia loading
tests. The modifications necessary for other tests will be
described with the test description. Essentially there is
the motor with the inertia load attached to one end of the
shaft and a potentiometer connected to the other end. A
second potentiometer is used to form a bridge and the shaft
position can be determined relative to a reference set by
this potentiometer. A counter driven by the driving circuit
is used to count the pulses into the motor.

Using the set up as in figure 7 the motor was tested for
its ability to step inertia loads of various sizes. The
procedure was to feed the input with pulses of a given
repetition rate gated into groups of twenty pulses. Usually
forty groups of this type were applied to give a total of
800 pulses. The motor was considered to have stepped success-
fully if one complete revolution of the shaft, as indicated
by the potentiometer.bridge, occurred. The results of this
testing are summarized in the graph (figure 9). Results are
shown for both the original and the modified motor.

The graph shows that for the original motor there is a
fairly flat plateau from 0-25 oz.in.\(^2\) of inertia where the
motor will maintain from 750 to 800 pulses per second. In
the same region the modified motor is capable of higher
Figure 3
PLOT OF MAXIMUM PULSING
RATE VERSUS INERTIA LOAD
FOR RESPONSYN NO. HD2141
speeds - up to 1300 pps - but drops off very rapidly with load. Above 25 oz.in.\(^2\) both variations of the motor drop off quickly with load to 50 oz.in.\(^2\). The maximum rate of the modified motor drops below that of the original and stays there. From 50 oz.in.\(^2\) the two variations both have a slow decrease in rate with load.

From the graph it would seem apparent that the modification is desirable for inertia loads under 25 oz.in.\(^2\) since it offers higher speed. However, if the load was not purely inertial and were subject to any time variation the maximum rate might have to be set low enough that very little advantage would be gained. The flat plateau of the original design might be more desirable in this case. Further rotor modification might produce a motor with a flat plateau at a higher average pulsing rate.

An important parameter to measure in any motor is the torque it develops. In a stepping motor which is constantly stopping and starting it is not sufficient to measure the static torque or the torque at one speed. Therefore the torque must be determined throughout the speed range to determine the effective torque at that speed. The set up was basically that of figure 7 but a torsion bar was attached in place of the inertia load. The other end of the torsion bar was clamped. This is illustrated in the picture (figure 10.)
Figure 10  Torque Measurement Setup
The motor was then run forward and back from equilibrium a variable number of steps at a given rate. As the shaft angle increased the counter torque of the torsion bar increased until finally the motor would miss. The running torque could then be calculated from the shaft angle and the torque constant of the torsion bar.

The results of these tests on both the original motor and the modification are shown in the graph (figure 11). The graph shows both the torque when running as described above and the torque when running into a dead stall. The stall torque can be seen to be generally greater than the running torque. The situation is somewhat confused by the presence of resonance effects in the rotor from 300 - 400 pps. However, the trends are obvious. The original design produces about 10% more torque throughout most of the speed range. Also the available torque drops about 30% from low speed to 800 pps on both versions. In the case of the modified motor the situation was complicated because the current through the windings began dropping off with frequency above about 600 pulses per second. However, the torque drops off mainly due to inertia effects, that is the torque required to accelerate the rotor at higher speeds.

The holding torque of the motor was hard to measure because the rotor was sensitive to a slight disturbance.
Figure 11
PLOT OF TORQUE VERSUS PULSING RATE FOR THE REACTORSYN

W. B. R.

WESTERN D.F.

TORQUE (oz-in)

0 200 400 600 800 1000 1200 1400

Pulse Frequency (cps)

Original Running torque
Responsyn Stall torque
Modifed Running torque
Responsyn Stall torque
However, the holding torque with the windings energized, is at least three times the stall torque at low speed. This amounts to 700 oz.in. for the original motor and 600 oz.in. for the modified motor. The holding torque with the windings unexcited is low and variable. It is not of much importance since disengaging the flex spline creates an uncertainty in the position. More details of the torque testing are given in Chapter III of this report.

In order to evaluate the motor under less ideal input conditions the motor was tested using an input from a binary rate multiplier. The binary rate multiplier is described in appendix A. Essentially, it provides non-uniformly space pulses based on a given clock frequency. This is done by gating which passes some of the clock pulses and excludes others. The gating is variable and any number from all to none of the gate pulses are retrievable. The average frequency is a binary fraction of the clock frequency. For the experiments the procedure was to set the clock frequency and then make runs with various fractions.

It was found that the motor would run effectively on pulses whose average frequency did not exceed 800 pulses per second. This is a limit. The higher the clock frequency went, the lower the average frequency. The behavior is summarized in the graphs (figures 12 & 13). The shape of the
BINARY RATE MULTIPLIER DRIVE

MAXIMUM AVERAGE PULSING RATE
VERSUS PULSE CLOCK FREQUENCY
FOR VARIOUS INERTIA LOADS

Figure 12

MAXIMUM AVERAGE PULSING RATE
VERSUS INERTIA LOAD FOR
VARIOUS CLOCK FREQUENCIES

Figure 13
curve of maximum average pulsing rate versus inertia load for one clock frequency is similar to the uniform pulsing curve, up to 25 oz.in.\(^2\) of inertia load. However, above 25 oz.in.\(^2\) the motor behavior became erratic. At any given clock frequency there was some fraction that excited a rotor resonance and caused loss of steps.

In general the operation on non-uniform pulses was noisy but precise below 25 oz.in.\(^2\). This rough running might have an adverse effect on the lifetime of the motor. This was not evaluated. More information about this test is given in Chapter III of this report.

Using the potentiometer on the Responsyn actuator output shaft the response of the motor to steps was displayed. These responses for two inertia loads are displayed in the figures 14 a, b, c, d, e, f. As is shown here, at low speeds the response is roughly second order to any one pulse with parameters depending on the inertia load. With no load the frequency of oscillation is approximately 200 cps and with 11.5 oz.in.\(^2\) of load it drops to about 125 cps. The overshoot increases with load and the damping appears to vary with rotor position. However the operation is definitely underdamped. As the speed is increased the response loses its step character and by high speeds it is essentially a straight line to a group of pulses.
STEP RESPONSE OF THE RESPSYN

(a) Position 10 mV/div 44 mV/div

2.75 pps inertia = 0

(b) Position 10 mV/div 44 mV/div

2.75 pps inertia = 11.5 oz-in²

(c) Position 20 mV/div 44 mV/div

133 pps inertia = 0

(d) Position 20 mV/div 44 mV/div

133 pps inertia = 11.5 oz-in²

(e) Position 20 mV/div 44 mV/div

500 pps inertia = 0

(f) Position 20 mV/div 44 mV/div

500 pps inertia = 11.5 oz-in²

Figure 14
Analysis of the motor predicts a second order response with higher order effects. Disregarding the obvious nonlinearities the qualitative result of the analysis is a transfer function of the form:

\[
\Theta_{\text{output}} = \frac{C \Theta_0}{s^2 J + s B + K} \sum_{j=0}^{\infty} e^{-T_j s}
\]

- \( C \) = constant
- \( \Theta_0 \) = angle for 1 step
- \( J \) = inertia load
- \( B \) = damping coeff.
- \( K \) = torque constant
- \( T_j \) = times of pulse occurrence

This corresponds to an infinite sum of delayed step responses. Details of this analysis and more experimental results are given in Chapter IV of this report.

The Responsyn actuator is capable of 800 pps and over 100 oz.in. of torque when modified. As inertia load is added the maximum speed of the motor is reduced, but in both forms it can maintain at least 750 pps at inertias up to 25 oz.in.².

The behavior of the motor at low speeds is that of a series of second order responses. As the speed is increased individual step behavior disappears. The rotor follows the stator as well as possible. There is some lag apparent in the high-speed pulsing but this is caught up when the motor stops. Because at high speeds the motor is not responding to the steps individually, non-uniformly spaced pulses do not present too great a handicap. Essentially the close spaced
pulses are compensated by subsequent larger spaced pulses. Under uniform pulsing there is an initial time lag and then a lag which is a function of the pulsing rate. The total delay must not be allowed to exceed 2 steps or else the rotor can align itself by falling back 2 steps. This is the ultimate limiting factor on the speed.

Over 300 hours of use have been put on the motor during the experiments. The motor performance has not deteriorated appreciably. The driving and logic circuits have been equally durable.
CHAPTER II - CIRCUITRY FOR THE RESPONSEN ACTUATOR

The block diagram of the complete circuit was shown earlier in figure 6 and discussed in Chapter I. In this section I will discuss each component in detail. Also the circuits necessary for performing the experiments will be described.

(a) DRIVER STAGE

Most important of the circuits in the general system is the driver stage which is shown in figure 15. The drivers are transistor switches operating in pairs with the diode interconnection as shown. This circuit was developed by Professor G. C. Newton, Jr. In order to utilize this system it was necessary to make both ends of the windings available and to eliminate the common lead.

The action of the circuit can be described by considering one pair of drivers. Assume \( Q_5 \) and \( Q_6 \) are on. Thus current is flowing through the diode \( D_3 \) into winding #3. At time \( t=0 \) the transistor \( Q_5 \) is turned off, turning off \( Q_6 \). Simultaneously \( Q_1 \) is turned on, turning on \( Q_2 \). Current will start to flow through winding #1 but will be inhibited by the winding inductance. However, due to the sudden turn off of \( Q_6 \) there will be a large induced voltage on the winding #3. This drives current through the diode \( D_7 \) into winding #1. The switching time is thus considerably decreased and the circuit response sped up considerably. The 20Ω
resistors were included in the diode interconnection to reduce the time constant of the inductive delay. This value of resistance was chosen as a good compromise between a fast time constant and excessive power loss in the resistors. When winding #3 is turned on again the same action occurs in the other direction. One additional advantage of this circuit is that it provides the necessary overlap of the current in the windings to prevent rotor disengagement.

Because of the inductive voltage the power transistors need to have a high voltage rating. The collector voltage waveform (figure 16) shows a peak voltage of about 250V. So far 2N3585 transistors and DTS-423 transistors have met the requirements. In order to supply sufficient base current to these transistors the 2N1323-1 transistors drive the power transistors.

This circuit provides the necessary two amperes to the windings, which have a d.c. impedance of 5 ohms, from about fifteen supply volts. This circuit can drive the motor quite successfully up to 1300 pulses per second. However, in this range the average winding current has dropped by about 30% to 1390 pps. Because the motor is operated at twice rated power (two windings at rated current), it was not possible to compensate by increasing the supply voltage since this would endanger the motor when the pulsing stopped. Two possibilities
VOLTAGE WAVEFORM AT THE COLLECTOR
OF A DRIVING STAGE POWER TRANSISTOR
AT VARIOUS PULSING RATES

Figure 16
could be considered to improve this. The first of these would be to speed up the circuit. The other would be to create a circuit to increase the voltage if the average current drops.

(b) **LOGIC CIRCUIT**

The logic necessary for the driving circuit is obtained basically from a single flip flop driving a pair of flip flops. The requirements on the flip flops were not stringent with a maximum rate of less than 2000 cycles per second. The circuit used for the flip flops is shown in figure 17. 2N3415 Silicon NPN transistors were used throughout.

Basically the operation of the flip flop can be described as follows. Assume Q₂ is on and Q₁ is off. The circuit is designed such that this situation or the inverse must be true in the steady state. A negative pulse at the input turns off Q₂ and raises its collector voltage. This increase is coupled to the base of Q₁ and it turns on. Regeneration in the circuit completes the process and a new steady state is created with Q₁ on and Q₂ off. The driver circuit is operated by the voltage at the collector of the off transistor.

In order to buffer the flip flop stage outputs emitter followers were coupled to each flip flop output. The circuit for these is shown in figure 18. 2N3415 transistors are also used here. Originally no emitter follower was required on the output of flip flop number one. It became necessary, however,
Figure 17 Flip Flop

Figure 18 Emitter Follower
for proper isolation of that stage when the reversing circuit was added. Essentially the emitter followers provide high input impedance and low output impedance with greater drive capabilities.

The input to the logic was fed through a Schmitt trigger shown in figure 19. This circuit helps eliminate some of the noise problems at the input. Also it produces a sharp uniform pulse regardless of the form of the input.

In order to allow reversing of the motor for logic operations a second input was required. Basically the operation necessary to produce a reversal is to change the state of one of the secondary flip flops with the other unchanged. Also the lead flip flop must not be changed. This was accomplished by connecting a transistor switch at point A of the input of flip flops 2 and 3. Point A is shown on figure 17 between the 2.2K resistor and the 150 pf capacitor at the input. Since this point will be at zero volts on one flip flop and above ground on the other the simultaneous closing of the switches would produce a negative going step on the above ground one. This step would be differentiated through the capacitor and thus provide the negative pulse to complement the flip flop. The other flip flop would be unaffected.

The circuit used to implement this is shown in figure 20. The input to the switch is a positive pulse. As mentioned earlier it was necessary to use emitter followers in the outputs
Figure 19 Schmitt Trigger

Figure 20 Reversing Circuit
of the first or lead flip flop to prevent undesired state changes induced by the reversing pulse.

In order to allow the use of a mechanical switch as the input to the reversing circuit, a monostable multivibrator was placed between the switch and the reversing circuit input. This circuit (figure 21) is a simple collector coupled one shot. The circuit is designed such that the transistor Q2 is normally on. A negative pulse through the 1300pf capacitor turns Q2 off. The collector rise is coupled through the network and turn on Q1 at its base. Feedback of the current through the capacitor C causes Q2 to be turned on after a delay given by RC in 2. The circuit as constructed gave a delay time of about 5 milliseconds. This was sufficient to eliminate the switching noise due to contact bounce from a microswitch. However, this produced a limitation on how soon after applying a reversing pulse that stepping could be started.

This did not present a problem in the tests as performed for this thesis. For a normal programmed input with clean pulses the monostable would be unnecessary.

(c) TEST CIRCUITS

In order to suitably test the motor it was necessary to count the input pulses to the motor. It was decided that the truest reading could be obtained from the driving circuit where it would be relatively insensitive to stray pulses.
Figure 21 Monostable Multivibrator

Figure 22 Pulse Pickoff Circuit
The collector voltage spike was ideal for this purpose being an extremely large amplitude pulse. The circuit chosen to do this is shown in figure 22. Essentially the voltage spike produces a pulse through the capacitor and triggers the switch. The switch turns on and the negative going level change is differentiated by the output capacitor. This negative pulse operates the counter.

One of the most important of the inputs necessary for the experiments is groups of pulses of a given frequency with a given number in the group. This is used to test stepping reliability. It was decided to do this by using a General Radio Unit Pulser as a gated supply voltage for a unijunction transistor sawtooth oscillator. This pulser produces only positive going pulses which go to ground from a negative level. In order that the system could be grounded it was necessary to use a battery in series with the pulser output. A block diagram of this is shown in figure 23. Effectively the pulser voltage is set to the negative of the battery voltage. When a pulse occurs the voltage goes to ground and the battery can drive the sawtooth oscillator for the duration of the pulse.

For the sawtooth oscillator the simple unijunction transistor circuit of figure 24 was used. The emitter follower across the output buffered the sawtooth oscillator from the
Figure 23 Circuit for Pulse Packet Input

Figure 24 Sawtooth Oscillator
logic circuit input. The circuit operation can be described as follows. A unijunction transistor turns on when the base voltage exceeds about 60% of the voltage applied across the other two lead. The capacitor charges up through the resistor R until the turn on voltage is reached. At this point the capacitor discharges quickly through the transistor until the transistor turns off. The process then repeats to produce a sawtooth wave. The frequency is variable with either R or C. For the experiments a range from 30-1500 cps was achieved by using a variable 125K potentiometer in conjunction with either a 0.1uf. or a 0.6uf. capacitor.

(d) **CONCLUSIONS**

All of the circuitry produced for driving and testing of the Responsyn actuator worked well. There was no problem with failures with the exception of the failure of one of the 2N3585 transistors after about 300 hours. At this point all of the 2N3585's were replaced by Delco DTS 423 transistors. The logic circuits were relatively insensitive to supply voltage and although twelve volts were specified it would operate satisfactorily from ten to fifteen volts. Speed was certainly no problem with the logic circuit and so the design of additional similar units should cause little problem. Standard logic elements could be used in place of this logic circuit but it would not be very difficult to produce a logic card specifically for the Responsyn actuator.
In the prototype which was a breadboard layout noise was somewhat of a problem. Stray pulses could cause the motor to step. Since this problem was not serious in the experiments the circuit was not redesigned to minimize this. However, to get good reliability from the motor-circuit combination, care should be taken with circuit layout to eliminate this problem.

Generally, the circuit as described for the motor is well suited to the task. With this circuit two amperes can be switched through the windings at a high rate with only about 15V supply voltage. However, with two amperes into two windings at once the power input into the motor is twice the rated value. Thus a fan is needed to keep the motor cool. This extra power increases the torque but reduces efficiency and increases losses. If a good circuit could be developed that would operate a single winding with the necessary speed it might be superior.
CHAPTER III - EXPERIMENTS AND TESTS ON THE RESPONSYN

Several experiments and tests were developed for the Responsyn to give the most worthwhile information about this machine. Among these were the measurement of torque and maximum speed versus inertia load. In addition the circuit was tested with the non-uniform pulsing of the binary rate multiplier. Initially the tests were performed on the Responsyn as delivered. However, some concurrent tests by the United Shoe Machinery Corporation had shown that by reducing the size of shimstock roll in the flex spline the speed could be increased. They suggested reducing the length from 40 feet to 16 feet as a near optimum figure. This modification was made and some of the tests re-performed. The motor with the shimstock reduced will be designated as the modified motor in the test procedures and results.

The general experimental set up has been illustrated previously in figure 17 and discussed briefly at that time. Some of the details and difficulties were omitted there for brevity. These will be discussed in more detail here and in each section as they become pertinent.

The logic and driving circuits are discussed in detail in Chapter II. It was necessary to have both the gated input pulses and the reversing circuits for the testing. These were provided by the sawtooth oscillator gated by the pulse generator and by the monostable switch reversing circuit. The counter was also very valuable as it gave an exact count of the pulses.
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fed to the motor for comparison. Also in steady running it could be used to determine the frequency of the pulsing.

The motor was mounted rigidly on a servo mounting plate attached securely to a solid base (see figure 8). The main end of the shaft was used for attaching inertia loads and was suspended beyond the base to allow clearance for the load. The standard inertia loads were a brass disc of 11.5 oz.in.\(^2\) inertia and a weight arm with aluminum and brass weights. The weight arm allowed the inertia to be varied from 25 oz.in.\(^2\) to 500 oz.in.\(^2\) in steps.

The potentiometer was mounted on a second upright and joined to the other end of the motor shaft through couplings and constant velocity joints. A second potentiometer was required to allow the zero position to be set. This potentiometer was mounted back to back with the first to shorten lead length. The bridge connection of the two potentiometers as shown in figure 7 allowed the detector (oscilloscope) to be zeroed at any shaft position to use as reference. It was necessary to use deposited film potentiometers instead of wirewound. This was because the resolution of the stepper was greater than that of the wirewound potentiometer. Twenty-two and a half volts was chosen as the maximum voltage the potentiometers could safely handle. This gave good resolution and no heating problems. The output of the potentiometer was fed directly into the oscilloscope.
(a) PULSING RATE VERSUS INERTIA LOAD TESTS

Prior to performing the actual experiments to determine the maximum pulsing rate versus inertia load the motor was run in the steady state with various inertia loads to test performance. It was found that the motor would run up to almost 800 pulses per second regardless of the inertia load. Above this speed the motor would suddenly run roughly and then stop. However, with large inertia loads a slight disturbance would cause the motor to run badly or stop. However, this mode of operation is not a good test of the motor performance. To get a good evaluation of the motor it is necessary to have the motor stop and start. A test that would cause the motor to run in bursts would determine the ability of the motor to accelerate and decelerate.

In order to do this it was decided that the motor should run on bursts or groups of twenty equally spaced pulses. The frequency of the pulsing would be given by the spacing of the pulses. In order to do this the circuit of Chapter II shown in figures 23 and 24 was used. The pulser was triggered manually and the pulse duration set to give exactly twenty pulses of a given frequency. Usually forty groups of pulses were fed into the input giving a total of 800 pulses. Reliable stepping at the given speed and load was determined if the potentiometer bridge indicated that one revolution had been completed. If this did not happen the frequency or the load would be changed to test this different set of parameters. The procedure was
repeated until the threshold between good stepping performance and inaccurate stepping was found for each load. Also for some loads the motor performance was tested at lower speeds to find if any poor speed ranges could be found.

The results of the tests are reproduced in the graph figure 9 shown earlier. This shows the results for the motor in both the original and modified forms. As can be seen from the graph with no load the modified version is considerably faster at 1300 pps compared to 800 pps for the original version with no load. However, as inertia load is added the speed drops off extremely rapidly for the modified version and is down to 950 pps by an inertia of 11.5 oz.in.\(^2\). The original version has dropped only to 790 pps. At 25 oz. in.\(^2\) they are both down to 750 pps. As the load increases beyond this amount, the modified version drops below the original by about 50-100 pps and remains there as the load increases. At 50 oz.in.\(^2\) the speed of the original version has halved. The speed decreases slowly as the inertia load is increased up to 500 oz.in.\(^2\).

The behavior of the motor above its maximum level, for some load, is of interest. When the speed is slightly exceeded the motor will require just a few extra pulses to complete a revolution. As the pulsing rate is increased further it falls back very rapidly. For example at about 10% over the maximum speed it may require 50% more pulses per revolution. Eventually it will quit entirely. When it is running above the speed
the motor is not only falling back but also running badly. This would probably damage the machine if continued for any period of time.

When the motor is running at speeds below the maximum, it is subject to resonances which cause vibration of the whole Responsyn unit. The amplitude and frequency of these oscillations vary with the inertia load. There are a large number of harmonics and in the range 100-500 pps almost any speed is at or near resonance for some inertia load. However, as far as I could ascertain the presence of the vibration did not affect the stepping performance of the machine as long as the frequency was below the maximum for that load. Operation for prolonged periods of time in this region is not recommended as the vibrations might affect the life of the Responsyn. There are probably some speeds in the range which would do the job well in a particular case.

Assuming that the most important uses for the Responsyn call for its operation at as high a speed as possible, this is the best range of operation for the device. The modified version of the device certainly produces a much higher stepping rate potential but does so at the expenses of high load sensitivity. For any inertia load over 15 oz.in.\(^2\) it offers little advantage over the original. On the other hand for lighter loads it is superior. Which version of the motor would be most suitable would depend mainly on the load in the particular case. If the
load could be considered purely inertial and the value known then the choice would be clear. However, if the load had some friction or torque character to it the picture would change. A motor with an amount of shimstock between 40 and 16 feet might provide a higher stepping rate than the original with better load sensitivity.

(b) TORQUE MEASUREMENTS

For a stepping motor such as the Resopsonyn it is important to know the torque output of the motor. At first one might consider that the torque could be measured on some form of static or steady state basis and thus the torque determined. However, this is not the case. A stepping motor accelerates and deaccelerates every step. Thus some torque is required to accelerate the inertia that will not be available for the load. Therefore, the conventional measures of motor torque are not of much use. To try to stall the motor against a weight on an arm would give only the standstill torque. To drive a generator and measure the torque from the power output would not give the useful torque due to the generator's inertia. Therefore a low inertia method of measurement was needed.

It was decided that a torsion bar could be used to do the job. The torsion bar would provide an opposing torque that increases with shaft angle. Thus by running the motor with the torsion bar as a load a torque could be obtained which would stop the motor from stepping. This torque is independent of
speed and acceleration. Therefore it satisfies the criteria for the test.

The set up for this test was shown in figure 10 in the first chapter. The torsion bar is clamped at one end and attached to the motor shaft at the other end. In order to get an estimate of the running torque it was decided to operate the motor in a reversing mode. The motor was run forward n steps and then brought back to equilibrium at a given frequency. The number n was variable and the running torque was estimated by the maximum number of steps the motor would make in this mode without slipping. The stall torque was estimated from the number of steps the motor would make in one direction without being stopped by the torsion bar.

The torsion bar was calibrated using weights on a lever arm. This calibration curve is shown in figure 25. With the exception of the area in the immediate vicinity of the origin (which is of little interest) the curve is linear with slope:

\[ m = \frac{T}{\theta} = \frac{230 \text{ oz-in}}{14^\circ} = 16.4 \text{ in-oz/deg} \]

Since one step = 0.45° therefore:

\[ m = 16.4 \text{ in oz/deg} \times 0.45^\circ/\text{step} \]

\[ m = 7.38 \text{ in oz/step} \]
Figure 25
CALIBRATION CURVE FOR THE TORSION BAR

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Torque (oz-in)

Angle of twist (degrees)

-46-
Most of the readings were in the region for $\Theta > 6^\circ$ so that the approximation of linearity is valid.

The torque measurements are limited in precision to the nearest step. This gives about 10% accuracy. However, this is more than adequate to estimate the torque and will not mask the trends. Measurements were taken at three points on the rotor spaced about 60 degrees apart. This was done to determine if torque varied with rotor position. No variation was found so the three readings were averaged to minimize errors. The three readings and the average are presented in table 1. The graph (figure 11) contains the average values. The tests were performed on the original motor and on the modified version.

The results as shown in the table and graph are a bit confused by effects, which I ascribe to resonance, in the 300-400 pps range. In this range low erratic readings were obtained. However, ignoring this region the trends are clear. At slow speeds up to about 150 pps the torque was 190 oz.in. for the original motor and 170 oz.in. for the modified motor. Above this speed the torque of the motor drops off. In the case of the original motor as the speed approached 800 pps the torque levels off at about 140 oz.in. For the modified motor the torque continues to drop to about 90 oz.in. by 1200 pps. The torque required to stall the motor is about 10 to 20% higher than the running torque. The running torque is a measure of the maximum torque the motor could drive without losing a step.
Table 1
Torque vs. Speed
(a) Original Responsyn

<table>
<thead>
<tr>
<th>Pulsing Rate</th>
<th>Running Torque</th>
<th>Stall Torque</th>
<th>Stall Torque</th>
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<tr>
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<td>-</td>
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<tr>
<td>30</td>
<td>185</td>
<td>-</td>
<td>192</td>
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<tr>
<td>60</td>
<td>192</td>
<td>192</td>
<td>192</td>
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<tr>
<td>100</td>
<td>177</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>200</td>
<td>141</td>
<td>155</td>
<td>162</td>
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<tr>
<td>300</td>
<td>124</td>
<td>140</td>
<td>148</td>
</tr>
<tr>
<td>400</td>
<td>124</td>
<td>140</td>
<td>148</td>
</tr>
<tr>
<td>500</td>
<td>148</td>
<td>140</td>
<td>162</td>
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<tr>
<td>600</td>
<td>148</td>
<td>140</td>
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<tr>
<td>700</td>
<td>133</td>
<td>156</td>
<td>133</td>
</tr>
<tr>
<td>750</td>
<td>141</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(b) Modified Responsyn

<table>
<thead>
<tr>
<th>Pulsing Rate</th>
<th>Running Torque</th>
<th>Stall Torque</th>
<th>Stall Torque</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pos 1</td>
<td>Pos 2</td>
<td>Pos 3</td>
</tr>
<tr>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>30</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>170</td>
<td>185</td>
<td>163</td>
</tr>
<tr>
<td>100</td>
<td>170</td>
<td>170</td>
<td>163</td>
</tr>
<tr>
<td>200</td>
<td>165</td>
<td>163</td>
<td>148</td>
</tr>
<tr>
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<td>125</td>
<td>141</td>
<td>133</td>
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<tr>
<td>400</td>
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<td>125</td>
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<td>111</td>
<td>126</td>
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<td>111</td>
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<tr>
<td>900</td>
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<td>111</td>
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<td>96</td>
<td>96</td>
<td>96</td>
</tr>
<tr>
<td>1100</td>
<td>96</td>
<td>96</td>
<td>92</td>
</tr>
<tr>
<td>1200</td>
<td>89</td>
<td>92</td>
<td>89</td>
</tr>
</tbody>
</table>

Note: All torques in oz-in
The torque measurements show some agreement with the
textory of operation of the Responsyn which will be discussed in
the next chapter. Up to about 150 pps the motor response is in
the form of distinct steps of similar nature. Especially in the
lower vicinity of this speed range the motor has plenty of time
to recover from one step before the next one is begun. Thus the
torque is inherent to a step and is unaffected by speed. As the
speed increases beyond 150 pps the steps begin to overlap. Thus
the maximum acceleration is increased and the torque available
is reduced. Thus there will be less useful torque from the
motor.

Certainly as the motor stands it can produce plenty of
torque for most stepping motor applications at high speeds.
Part of the torque fall off at high speeds is attributable to
current fall off portion of it. If greater torque were needed
at high speeds a circuit redesign could provide some of it.
(c) **EFFICIENCY**

Although when thinking about stepping motor design one
rarely considers efficiency, I felt that this might be of
concern to someone in a device which draws an average current
of four amperes.

The efficiency was calculated as the ratio of the maximum
average power output to the average power input to the motor.
The maximum torque was used in this figure since the load does
not appreciably affect the input power. Thus the efficiency is
given by: \[ \eta = \frac{T \times \omega}{2I^2R_w} \]

\(T\) = torque
\(\omega\) = shaft speed
\(I\) = winding current
\(R_w\) = winding resistance

The factor of 2 is necessary because two windings are excited at once. The resulting efficiencies are shown in figure 26.

As can be seen the original motor is more efficient than the modified version throughout. However, since neither efficiency exceeds 15\%, the device can be considered inefficient and the argument need go no further. In actual fact the efficiency of the whole system is lower due to losses in the driving circuit.

(d) **OPERATION WITH THE BINARY RATE MULTIPLIER**

For machine tool control and other forms of control the binary rate multiplier provides a useful output device for the electronics. It is programmable to produce a variable average output frequency based on a constant clock frequency pulse. Because of the possible mating of the two and because as a non-uniform pulse train it offered a different type of input to the Responsyn, this input was used for another test.

The details of binary rate multipliers and the circuit of the actual device employed are shown in Appendix A. However, in order to understand the input it gives, I will give some discussion of its behavior at this time. Consider the pulses of clock frequency as shown in figure 27(a). Suppose we divide
Figure 26: PLOT OF EFFICIENCY VERSUS PULSING RATE FOR THE RESPONSYN

- Efficiency %
- Pulse Frequency (cps)

ORIGINAL
MODIFIED

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Figure 27 Operation of the Binary Rate Multiplier
these pulse into four categories as shown in figure 27 b, c, d, e. Label these groups 1, 2, 3, 4. Note that pulse number 16 is not included in any of the groups. The pulses in each group retain their time relationship as in the clock pulses. Suppose we can add the pulses of the groups together we will get combinations such as figure 27(f) which is 1, 3, 4; figure 27(g) which is 1&2; and figure 27(h) which is 2 and 3. For the combination 1, 3, 4, we get eleven of the original sixteen pulses. Thus the average frequency is 11/16 of the clock frequency. Similarly for the second and third groups the average frequencies are 12/16 and 6/16, respectively, of the clock frequency. There are fifteen such combinations of the pulses giving average frequencies from 1/16 to 15/16 of the clock frequency. The spacing between pulses can be from $\frac{1}{T_c}$ to $2n \frac{1}{T_c}$ where $n_{max}$ is 4. Thus we have a variable average frequency with non-uniformly spaced pulses. The implementation of this is discussed in Appendix A.

It was decided that the most meaningful test to perform with the binary rate multiplier was the maximum pulsing rate versus inertia load. The set up used was essentially that of the uniform pulsing test with the addition of the binary rate multiplier. The pulses could not be grouped by gating the clock pulses. The binary rate multiplier was gated by shutting off the output, thus it was impossible to insure getting exactly 20 pulses in a group due to the non-uniformity of the pulses.
The result was 20±1 pulses which meant that after 40 groups the result might be more or less than 800 pulses. However, by knowing the scale of a step on the oscilloscope it was possible to overcome this difficulty.

The combinations giving fractions from 8/16 to 15/16 of the clock frequency were chosen. Below 8/16 the results are similar to those given by a higher fraction on half the clock frequency and as such were considered of little value. The testing was carried out by setting the clock frequency and the load and measuring the stepping performance as the fraction was varied.

The interpretation of the data for binary rate pulsing for comparison to the uniform pulsing presented a problem. I finally decided that the maximum average pulsing rate was a function of both inertia load and clock frequency. In order to present the data effectively the data is presented in two forms. In graph figure 12 the maximum average pulsing rate versus clock frequency with the load as a parameter. In figure 13 the maximum average frequency versus load was presented with clock frequency as parameter.

The limits of the two graphs are $f_c=1200$ pps and inertia load of 25 oz.in.$^2$. The maximum clock frequency represents the closest spaced pulses the Responsyn can react to even irregularly. As far the inertia limit, it was the point at which the
motor behavior became erratic. Even below this limit occasional erratic behavior was apparent. However, above 25 oz.in.$^2$ of inertia it was difficult to find a clock frequency for which more than a few fractions gave good stepping behavior. Table 2 lists some frequencies for which the motor will operate correctly for all fractions with inertia loads up to 40 oz.in.$^2$. The only reasonable explanation for this behavior is that a certain sequence of pulses at a given frequency excite a resonant mode of the motor. As the inertia gets larger these resonances get more violent and affect the performance of the motor.

The question of comparison to the motor under uniform pulsing is a difficult one. From figures 12 and 13 if the inertia load is under 15 oz.in.$^2$ and the clock frequency was under 1000 pps the performance is comparable. However, in order to be able to utilize all fractions the maximum clock frequency would have to be limited to 800 pps. Thus on the average the motor would be running below this frequency. The other consideration is the durability of the motor. The motor was considerably noisier and ran rougher under the binary rate pulsing. I, therefore, assume that this would cause more wear than uniform pulsing. Also, there is this resonance problem that occurs at some frequencies even with small inertia loads. It would be difficult to specify a good operating frequency of the motor for any particular applications without extensive testing. Because of these considerations I would suggest that although operation
Table 2

Results of Binary Rate Pulsing –
Maximum Pulsing Rate vs Inertia Load

(a) Zero inertia load

<table>
<thead>
<tr>
<th>Clock freq.</th>
<th>Fraction</th>
<th>Avg. Pulse Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>15/16</td>
<td>800</td>
</tr>
<tr>
<td>860</td>
<td>14/16</td>
<td>800</td>
</tr>
<tr>
<td>900</td>
<td>12/16</td>
<td>788</td>
</tr>
<tr>
<td>1000</td>
<td>9/16</td>
<td>750</td>
</tr>
<tr>
<td>1200</td>
<td></td>
<td>675</td>
</tr>
</tbody>
</table>

(b) Inertia load = 11.5 oz-in²

<table>
<thead>
<tr>
<th>Clock freq.</th>
<th>Fraction</th>
<th>Avg. Pulse Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>15/16</td>
<td>796</td>
</tr>
<tr>
<td>900</td>
<td>14/16</td>
<td>788</td>
</tr>
<tr>
<td>1000</td>
<td>12/16</td>
<td>750</td>
</tr>
<tr>
<td>1200</td>
<td>9/16</td>
<td>675</td>
</tr>
</tbody>
</table>

(c) Inertia load = 25 oz-in²

<table>
<thead>
<tr>
<th>Clock freq.</th>
<th>Fraction</th>
<th>Avg. Pulse Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>15/16</td>
<td>750</td>
</tr>
<tr>
<td>850</td>
<td>14/16</td>
<td>743</td>
</tr>
<tr>
<td>900</td>
<td>12/16</td>
<td>675</td>
</tr>
<tr>
<td>1050</td>
<td>9/16</td>
<td>590</td>
</tr>
</tbody>
</table>

(d) Inertia load = 28 oz-in²
Acceptable performance at all fractions for clock frequency 300 & 600 p.p.s.

(e) Inertia load = 30 oz-in²
Acceptable performance at all fractions for clock frequency 200 p.p.s

(f) Inertia load = 40 oz-in²
Acceptable performance at all fractions for clock frequency 100 or 200 p.p.s.
with binary rate multiplier pulsing is possible it is less desirable than that from uniform pulsing.

(e) CONCLUSIONS

The experiments performed and written up in this section give us a good idea of what the motor can do and its limitations. At this point a short summary is worthwhile. To be conservative I will say the motor can drive an inertia load of 10 oz.in.\(^2\) at 800 pps. At this speed unloaded it has a useful torque of 100 oz.in. The motor will accelerate and decelerate to this speed immediately from standstill without losing any steps. The motor will run on non-uniformly spaced pulses although it prefers the uniform variety. The motor is at best about 15% efficient and the complete system as tested is about 10% efficient. From all evidence presented so far it is a durable machine. It has survived 300 hours of strenuous testing without serious damage.
CHAPTER IV - THEORY OF OPERATION OF THE RESPONSYN

The physical operation of the Responsyn is very complex and as such analysis is difficult. However, in order to try to understand its behavior it is necessary to make an attempt. Therefore, in this chapter, I intend to develop a simple transfer function which is applicable to the motor. Numbers for the parameters will be evaluated from Responsyn tests and specifications. The results of evaluating the transfer function in terms of these parameter values will be compared to experimental results. Qualitatively, the discrepancies between the results of the theory and experiments will be discussed.

The rotor and stator configuration for the Responsyn are shown in figure 28 with the rotor engaged. The actual winding configuration is shown in figure 29. Not indicated on the diagram is the fact that the windings 1 and 2, and so on, are connected in parallel but produce flux in the opposite sense. The complete flux path is through one pair of poles, into the rotor, and out the diametrically opposite pair of poles. Each tooth has 165 turns around it and the current in all the windings is one ampere under normal operating conditions.

(a) MATHEMATICAL ANALYSIS

The torque action of the Responsyn can be considered in terms of a force, located at the next rotor equilibrium, acting on the rotor. The gear teeth act as a pivot which progresses
Figure 28 Position of flex spline and tooth engagement with windings excited.

Figure 29 Winding Configuration for the Responsyn

Figure 30a-b Progression of Flex spline Pivot Point Along the Stator During Stepping
towards the equilibrium point. This idea is illustrated in figures 30 (a and b). In figure 30a the motor is shown at the instant the current was switched. The force tending to close the gap to minimize the reluctance is now acting from position 2. Eventually the contact point of the rotor will reach position 2 and the torque will go to zero. Beyond this point the torque will be negative and will tend to return the rotor to equilibrium.

The force $f$ tending to close the rotor gap can be computed by considering the simple magnetic circuit of figure 31. Assume that the flux is essentially determined by the iron not the air gap. A simple relationship can be developed for the force. The stator mmf is given by:

$$ F = NI $$

The energy differentials are:

$$ dW_e = NI d\phi $$

(electrical energy)

$$ dW_m = f dx $$

(mechanical energy)

$$ dW_{sR} = V_R H_R dB_R $$

(energy stored in rotor)

$$ dW_{sS} = V_S H_S dB_S $$

(energy stored in stator)

$$ dW_{sa} = \frac{B_e A_e dx}{2 \mu_0} + \frac{V_e B_o dB_o}{\mu_0} $$

(energy stored in air gap)

Any change in energy input must be reflected in the stored energy. Therefore:

$$ NI d\phi + f dx = V_R H_R dB_R + V_S H_S dB_S + \frac{B_e A_e dx}{2 \mu_0} + \frac{V_e B_o dB_o}{\mu_0} $$

$$ NI = H_R l_R + H_S l_S + H_o x $$
Figure 31 Magnetic Circuit for developing the Force Equation
The last equation expresses the fact that the ampere turns must equal the line integral around the closed loop. The flux is given by:

$$d\phi = A_R dB_R + A_s dB_s + A_a dB_a$$  \(\text{\textcopyright}\)  

Substituting from (8) and (9) for NIdφ in equation (7) and noting \(V_R = A_R \frac{dL}{dR}\) etc. therefore:

$$V_R H_R dB_R + V_s H_s dB_s + V_a H_a dB_a + f dx$$

$$= V_R H_R dB_R + V_s H_s dB_s + B_a^2 A_a \frac{dx}{2\mu_o} + V_a B_a dB_a$$  \(\text{\textcopyright}\)

Cancelling terms and remembering that \(B_a = \mu_o H_a\) equation (10) reduces to

$$f dx = \frac{B_a^2 A_a}{2\mu_o}$$  \(\text{\textcopyright}\)

or

$$f = \frac{B_a^2 A_a}{2\mu_o}$$  \(\text{\textcopyright}\)

The use of this equation for the Responsyn will be shown later in the chapter.

Now, assuming that the force developed can be calculated, how can the torque be determined. In order to do this I will make two simplifying assumptions. First I will assume that the force, which is actually distributed acts at one point on the stator. Second, it will be assumed that the rotor is sufficiently flexible that motion on one side of the flexspline does not affect the other side. This is reasonable because both sides are moving in the direction they are pulled anyway. The
initial position of the rotor will be the reference point for measuring the angle \( \Theta \). The force will be applied at the position \( \Theta_0 \) (see figure 32). \( \Theta \) is measured at the rotor contact position. The torque is given by:

\[
T = fR \sin (\Theta_0 - \Theta) \quad (3)
\]

This equation is based on the force turning the rotor around the contact point. The torque developed by the motor is actually twice this amount since both sides experience this torque.

\[
T = 2fR \sin (\Theta_0 - \Theta) \quad (4)
\]

However, for most of this range the error will not be too large (less than 10\%) if \( \sin \Theta \) is replaced by \( \Theta \). Therefore, for the calculations I will assume:

\[
T = 2fR(\Theta_0 - \Theta) \quad (5)
\]

This is illustrated in figure 33. This torque equation is valid only in the region of an equilibrium.

Now consider the response of the rotor to a single step i.e. the magnetic field moved through one increment. At slow speeds the electrical system is much faster than the mechanical system. Therefore, it is safe to assume that the position of force application moves instantaneously. In order to determine the response, inertia and frictional effects must be included. Therefore the differential equation is

\[
T = J \frac{d^2\Theta}{dt^2} + B \frac{d\Theta}{dt} \quad (6)
\]
Figure 32. Definition of terms for torque calculation

Figure 33. Torque vs. Position around equilibrium at $\frac{\pi}{4}$

Figure 34. Flex spline considered as a lever for determining the effective force at the teeth.
Substituting for the torque
\[ 2I_0 R(\theta_0 - \theta) = J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} \]
(7)
Rearranging and letting \(2I_0 R = K\) then
\[ J \frac{d^2 \theta}{dt^2} + B \frac{d\theta}{dt} + K\theta = K\theta_0 \]
(8)
Taking transforms on both sides we get
\[ \Theta(s) = \frac{K\theta_0/s}{s^2 J + sB + K} \]
(9)
This will give us a second order response with frequency
\[ \omega_n = \sqrt{\frac{K}{J}} \]
(10)
and a damping ratio
\[ \zeta = \frac{B}{2J} \]
(11)
Because of the gearing action of the Responsyn there is no torque multiplication at the output but the output angular increment will be \(\frac{1}{100}\) of the rotor movement. Therefore the transfer function at the output shaft is
\[ \Theta_s(s) = \frac{K}{100} \frac{\theta_0/s}{s^2 J + sB + K} \]
(22)
For a series of steps, this response is repeated for every input pulse. The result is a series of second order transfer functions occurring repetitively. Thus for a large number of pulses we have an infinite sum of delayed second order responses. Thus an overall transfer function for many pulses is:
\[ \Theta_s(s) = \frac{K}{100} \frac{\theta_0/s}{s^2 J + sB + K} \sum_{j=0}^{\infty} e^{-T_j s} \]
(23)
Here the \(T_j\)'s are the time of the occurrence of the input pulse.
In order to evaluate the performance and to compare the model to the experimental results it is necessary to obtain values for the parameters. Some of these will be estimates and others numbers based on test information supplied by Professor G. C. Newton, Jr.

From the equation (12) the force can be calculated if the flux density can be determined. To do this I will rely on some flux measurements made by Professor G. C. Newton, Jr. The mmf per pole for the Responsyn is given by:

\[ F = NI \]

\[ = 165 \text{ amp turns/pole} \]

At this level of mmf the measurements showed that the average flux per tooth was \( \approx 6 \times 10^4 \text{ webers} \). The area of the air gap per tooth is the same as the area of the tooth face. This is 1.38 in. x 0.25 in. or \( A_a = 0.345 \text{ in}^2 = 2.22 \times 10^{-4} \text{ m}^2 \). The measurements also showed that the flux was about 30% lower in the front than in the back. The average flux density is then

\[ B_a = \frac{\phi}{0.85 A_a} \]

\[ \therefore \ B_a = \frac{1.6}{0.85 \times 2.22} \approx 0.85 \text{ webers/m}^2 \]

To calculate the effective force at the gears the rotor is treated as a lever as in figure 34. The magnetic force is divided into two forces corresponding to the two different flux levels acting at the centre of their own region. Since four teeth are active on each side of the rotor at one time then a
factor of 4 appears in the forces. The front force \( f_f \) is given by:

\[
f_f = \frac{4 \beta_e^2 \times A_e}{2\mu_e} = \frac{4 \times 0.85^2 \times 2.22 \times 10^{-4}}{4 \times 4 \pi \times 10^{-7}} = 127.5 \text{ newtons or 454 oz.}
\]

Similarly

\[
f_b = \frac{4 \times (0.7 \times 0.85)^2 \times 2.22 \times 10^{-4}}{4 \times 4 \pi \times 10^{-7}} = 62.3 \text{ newtons or 223 oz.}
\]

Using the lever idea and resolving the forces to the position of the gear ring.

\[
f_t = \frac{454 \times 2.35 + 223 \times 1.65}{1.90} = 755 \text{ oz.}
\]

The torque constant \( K \) is given by the equation \( K = 2f_tR \)

\[
\therefore K = 2 \times 755 \text{ oz} \times 0.82 \text{ in} = 1240 \text{ oz-in/rad}
\]

In order to calculate the rotor inertia it is necessary to refer to motor action. The inertia \( J \) can be considered to be made up of two portions a portion due to the flexing \( J_f \) and the non-flexing or normal inertia of the unit \( J_{nf} \). Therefore \( J = J_f + J_{nf} \). Consider the ellipsoidal shape of the rotor in its engaged position as shown in figure 28. There is a space \( \frac{d}{2} \) shown between the stator circumference and the minor axis of the ellipse. The distance \( d \) can be shown to be the stator diameter \( D_s \) divided by the gear ratio \( R_g \). (\( d = D_s/R_g \)). Now at
45° from the initial position the gap is approximately \( \frac{R_3}{2} \).

Part of the rotor has to close this gap in a time given by the shaft velocity.

\[
t = \frac{2\pi}{8 \omega_3 R_3}
\]

The velocity at peak of the rotor is given by

\[
v_p = \frac{\pi d}{4} \cdot \frac{2\pi}{\omega_3 R_3} = \frac{\sqrt{3}}{2} \omega_3 R_3 d
\]

The mean square velocity is given by \( v_p^2 / 2 \) and thus the Kinetic energy is

\[
K.E. = 2 \omega_3^2 R_3^3 d \cdot \frac{\pi^2}{2} M_f
\]

Recognizing that the \( K.E. = J_f \omega_3^2 \) and that \( R_3 \cdot d = D_3 \), then \( J_f \cdot \frac{R_3^3 M_f}{\pi^2} \)

However, this is only half the inertia so the actual flexing portion of the inertia, therefore is

\[
J_f = \frac{2 D_3^2 M_f}{\pi^2}
\]

The non-flexing portion of the inertia is given by \( m r_g^2 \), where \( r_g \) is the radius of gyration of the flexspline.

\[
J = \frac{2 \times 1.64^2 \times 4.75}{\pi^2} + 4.75 \times 0.75^2
\]

\[
= 5.25 \text{ oz.-in}^2 = 1.36 \times 10^{-2} \text{ oz.-in.-sec}^2
\]

We are now in a position to calculate the oscillation frequency of the step response. This frequency is given by

\[
\omega_n = \sqrt{K/J}
\]

Values of the frequency are calculated for no inertia load, 11.5 oz. in.\(^2\) inertia load, and 25 oz. in.\(^2\).
inertia load. These values and the values obtained from the experiments are shown in table 3.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>oz. in.²</th>
<th>oz. in.²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 inertia load</td>
<td>11.5</td>
<td>25</td>
</tr>
<tr>
<td>Calculated frequency</td>
<td>48 c.p.s.</td>
<td>27 c.p.s.</td>
</tr>
<tr>
<td>Experimental frequency</td>
<td>200 c.p.s.</td>
<td>125 c.p.s.</td>
</tr>
</tbody>
</table>

As for the damping ratio, to ascertain a single value for this is difficult at best. The operation of the flexspline is such that any frictional forces measured without the motor running would be invalid. The best that can be done is to make an estimate from the experimental results. With no inertia load the damping ratio is roughly 0.5 which is a fair estimate. Thus \( \omega_n = 2\pi \times 48 \times 0.5 = 161 \). The friction constant \( B \) can be determined by using the relation \( 2\omega_n = B/f \)

\[
B = 302 \times 1.86 \times 10^{-2} \text{ oz.-in.-sec}
\]

\[
B = 4.1 \text{ oz.-in.-sec.}
\]

A complete second order response with these parameters is shown in figure 35. This can be compared to the experimental results shown later in this chapter.

Looking at figure 35 and table 3 it becomes apparent that this simple theory is only qualitatively correct. The general shape of the curve and the order of magnitude of the result agree all right but the numbers for frequency are out by a
factor of 4 to 5. I do not believe this invalidates the general idea of the theory or makes it valueless. However, it does show that it is not as simple as this. In this thesis it is not of sufficient value to go into the complex non-linear mathematics to see whether a better result could be achieved. For one thing the values of the parameters could not be evaluated that correctly. However, if we examine the assumptions that can be made in deriving the simple theory, certain suggestions can be made towards accounting for the discrepancies. First of all since the ratio of the calculated frequency to the actual frequency stays constant as the inertia is varied, this does not seem to be the problem. Thus, we must look at the torque constant K. There are two possibilities to consider. First of all the force is definitely not located at a point but is distributed over 90° of the stator. The effect of this could tend to make the torque curve rise steeply from the origin and then level off. In particular in the vicinity of equilibrium the torque for a small displacement might be much larger than the simple linear theory would predict. This is likely to cause the frequency of the ringing to increase. Secondly, the idea of the two sides of the rotor not interacting is limited also. Obviously if the rotor were midway between the two active poles there would be equal forces to both directions. Thus there is an unstable equilibrium for a rotor angle relative to the stator of 2θ. This will cause
some change in the form of the force versus position curve that could tend to increase the calculated frequency. Some of the higher order effects can be accounted for by the non-linearities of the system.

(b) **Experimental Response Measurements**

The actual Resonsyn responses are shown in figures 36-42. These were photographed from the oscilloscope trace from the potentiometer bridge as the motor was stepped. Responses were taken with three inertia loads 0, 11.5 oz.in.², and 25 oz.in.² and with various speeds were taken for both the original and the modified motor.

The low speed response of the motor is essentially similar to that predicted in the theory. However, the curves were not as smooth as the theory predicted and there was considerable disagreement from step to step. The frequencies were determined as shown in table 3 and range from 100-200 cycles/second. At this speed the rise time of the response is approximately 2 milliseconds. For the modified motor the risetime is essentially the same but the frequencies were somewhat higher. This is attributable to the lighter rotor. Not too much other difference was noticed.

As the speed is increased the second order response gets lost and only part of it shows. In the region from 80-200 pps the result is a rise, an overshoot, a slight fall, and then
STEP RESPONSE OF THE RESPONSEYN

Shaft Position
5 mV/div
44 mV = 1°

\[ t \rightarrow (2 \text{ ms/div}) \]
\[ 27.5 \text{ pps} \quad \text{inertia} = 0 \]

Shaft Position
5 mV/div
44 mV = 1°

\[ t \rightarrow (4 \text{ ms/div}) \]
\[ 27.5 \text{ pps} \quad \text{inertia} = 0 \]

Shaft Position
5 mV/div
44 mV = 1°

\[ t \rightarrow (4 \text{ ms/div}) \]
\[ 27.5 \text{ pps} \quad \text{inertia} = 11.5 \text{ oz-in}^2 \]

Shaft Position
20 mV/div
44 mV = 1°

\[ t \rightarrow (20 \text{ ms/cm}) \]
\[ 27.5 \text{ pps} \quad \text{inertia} = 25 \text{ oz-in}^2 \]

Shaft Position
5 mV/div
44 mV = 1°

\[ t \rightarrow (10 \text{ ms/cm}) \]
\[ 27.5 \text{ pps} \quad \text{inertia} = 25 \text{ oz-in}^2 \]

Figure 36
STEP RESPONSE OF THE RESPONSYN
(MODIFIED ROTOR)

Shaft Position
20mv/div
62mv = 1°

\[ t \rightarrow (20\text{ms/div}) \]
27.5 pps  inertia = 0

Shaft Position
5mv/div
62mv = 1°

\[ t \rightarrow (2\text{ms/div}) \]
27.5 pps  inertia = 0

Shaft Position
20mv/div
62mv = 1°

\[ t \rightarrow (20\text{ms/div}) \]
27.5 pps  inertia = 11.5 oz-in²

Shaft Position
5mv/div
62mv = 1°

\[ t \rightarrow (4\text{ms/div}) \]
27.5 pps  inertia = 11.5 oz-in²

Shaft Position
20mv/div
62mv = 1°

\[ t \rightarrow (20\text{ms/div}) \]
27.5 pps  inertia = 25 oz-in²

Shaft Position
10mv/div
62mv = 1°

\[ t \rightarrow (10\text{ms/div}) \]
27.5 pps  inertia = 25 oz-in²

Figure 37
STEP RESPONSE OF THE RESPONSYN

Shaft Position
10 mV/div
44 mV = 1°

\( t \rightarrow (5 \text{ms/div}) \)
80 pps inertia = 0

Shaft Position
10 mV/div
44 mV = 1°

\( t \rightarrow (5 \text{ms/div}) \)
80 pps inertia = 11.5 oz-in²

Shaft Position
10 mV/div
44 mV = 1°

\( t \rightarrow (1 \text{ms/div}) \)
80 pps inertia = 0

Shaft Position
5 mV/div
44 mV = 1°

\( t \rightarrow (2 \text{ms/div}) \)
80 pps inertia = 0

Shaft Position
5 mV/div
44 mV = 1°

\( t \rightarrow (2 \text{ms/div}) \)
80 pps inertia = 11.5 oz-in²

Shaft Position
10 mV/div
44 mV = 1°

\( t \rightarrow (5 \text{ms/div}) \)
80 pps inertia = 25 oz-in²

Shaft Position
10 mV/div
44 mV = 1°

\( t \rightarrow (5 \text{ms/div}) \)
80 pps inertia = 25 oz-in²

Figure 38
STEP RESPONSE OF THE RESPONSYN

Shaft Position
10 mv/div
44 mv = 1°

$t \rightarrow (1 \text{ ms/div})$
133 pps  \hspace{1em} \text{inertia} = 0$
end of the pulse train

Shaft Position
20 mv/div
44 mv = 1°

$t \rightarrow (2.5 \text{ ms/div})$
133 pps  \hspace{1em} \text{inertia} = 0$

Shaft Position
10 mv/div
44 mv = 1°

$t \rightarrow (1 \text{ ms/div})$
133 pps  \hspace{1em} \text{inertia} = 11.5 \text{ oz-in}^2$

Shaft Position
20 mv/div
44 mv = 1°

$t \rightarrow (5 \text{ ms/div})$
133 pps  \hspace{1em} \text{inertia} = 25 \text{ oz-in}^2$

Shaft Position
10 mv/div
44 mv = 1°

$t \rightarrow (1 \text{ ms/div})$
133 pps  \hspace{1em} \text{inertia} = 25 \text{ oz-in}^2$

Initial Delay Phenomenon

Shaft Position
10 mv/div
44 mv = 1°

$t \rightarrow (1 \text{ ms/div})$
100 pps  \hspace{1em} \text{inertia} = 0$

Shaft Position
10 mv/div
44 mv = 1°

$t \rightarrow (1 \text{ ms/div})$
800 pps  \hspace{1em} \text{inertia} = 0$

Figure 39
STEP RESPONSE OF THE RESPONSYN

Shaft Position
10 mV/div
44 mV = 1°

Figure 40
STEP RESPONSE OF THE RESPONSE SYN (MODIFIED ROTOR)

**Shaft Position**
- 20mv/div
- 62mv=1°
- t \rightarrow \frac{10ms}{div}
- 100pps
- inertia = 0

**Shaft Position**
- 10mv/div
- 62mv=1°
- t \rightarrow \frac{2ms}{div}
- 100pps
- inertia = 0

**Shaft Position**
- 20mv/div
- 62mv=1°
- t \rightarrow \frac{10ms}{div}
- 100pps
- inertia = 11.5 oz-in²

**Shaft Position**
- 10mv/div
- 62mv=1°
- t \rightarrow \frac{2ms}{div}
- 100pps
- inertia = 11.5 oz-in²

**Shaft Position**
- 20mv/div
- 62mv=1°
- t \rightarrow \frac{2ms}{div}
- 500pps
- inertia = 0

end of pulse train

**Shaft Position**
- 20mv/div
- 62mv=1°
- t \rightarrow \frac{5ms}{div}
- 500pps
- inertia = 11.5 oz-in²

end of pulse train

Figure 11
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STEP RESPONSE OF THE RESPONSEYN (MODIFIED ROTOR)

Shaft Position
50mv/div
62mv=1°

\[ t \rightarrow (5 \text{ ms/div}) \]
900 pps inertia = 0

Shaft Position
20 mv/div
62 mv=1°

\[ t \rightarrow (2.5 \text{ ms/div}) \]
900 pps inertia = 0
end of pulse train

Shaft Position
50mv/div
62mv=1°

\[ t \rightarrow (5 \text{ ms/div}) \]
900 pps inertia = 11.5 oz-in²

Shaft Position
20mv/div
62mv=1°

\[ t \rightarrow (2.5 \text{ ms/div}) \]
900 pps inertia = 11.5 oz-in²
end of pulse train

Shaft Position
20mv/div
62mv=1°

\[ t \rightarrow (1 \text{ ms/div}) \]
900 pps inertia = 0
initial delay phenomenon

Shaft Position
5mv/div
62mv=1°

\[ t \rightarrow (1 \text{ ms/div}) \]
100 pps inertia = 0
initial delay phenomenon

Figure 42
the occurrence of the next step. This shows up more precisely in the smaller scale pictures than at larger scale where the detail obscures the general result. Usually in these cases the last step is considerably different than the others since the motor must be stopped here. This speed range is also prone to resonance since the natural frequencies of the system fall into this range.

In the high speed range above 200 pulses per second the motor response no longer shows any distinct step response but rather is a somewhat irregular ramp with a ringing after the last pulse. It should be noted that the ringing frequency at the end of a pulse train is the same frequency as that for low speed pulsing. However, the amplitude is greater. The same thing is true of the modified motor as is true for the original version. In general in this region the average rise time of the ramp is that necessary to maintain stepping at that rate.

I was surprised to find in these pictures that there was a delay of about 1 millisecond between the firing of the first pulse and the start of the motor response. These pictures are shown in figure 39 for the original motor and figure 42 for the modified version. No good explanation for this phenomenon is readily available. The best guess is that some slack in the flexspline has to be taken up before the shaft will move. It was impossible to tell at high speed whether this delay occurred
on every step, but at low speed there was definitely a delay between the input pulse to the motor and the first shaft movement. This delay was definitely in the motor not in the driving circuit.

The high frequency region is of most practical importance and as such should be studied more thoroughly. It would be difficult to ascertain exactly what limits the motor. However, a few suggestions and a brief description of the probable action will be given. Essentially, the motor tries the best it can to follow the pulses that are fed into it. Once it is underway at the proper speed it is not too difficult for it to keep up because inertia is helping it. The difficult thing here is the acceleration to this speed. Here the limit is a total delay equivalent to the time for two pulses. If the motor falls back further than this it will be easier to fall back two more steps than to go forward the two steps. Since the initial delay discussed previously uses one millisecond of this time. This leaves just the rest of the time available for acceleration. In actual fact this limitation is more of a position limit than a time limitation as the criterion is to not let the rotor fall back far enough that it will be pulled backward. It would appear that this initial delay is the most severe problem to increasing the speed. If this could be shortened the maximum pulsing rate could be increased proportionally.
The outstanding feature of the Responsyn is its ability to stop on the pulse even at higher speeds. It is safe to assume that the motor is essentially free wheeling at this speed, running as fast as it can trying to catch the pulsing. It will always be behind the pulses but never by a large distance. When the pulsing stops the rotor searches out the desired final position and stops. The secret of the success is three elements. First, the stopping force of the motor on the rotor is large. Second, there is a certain amount of slowing friction in the gears. Third, the flexspline must take up the proper final position. It could do this with \( \pm 4n \) steps from the desired position. However, since the motor is running towards the desired position and is only a couple of steps behind, the easiest position for it to take up is the correct one.

In general the behaviour of the Responsyn is extremely complicated but can be approximated at low speeds by a second order response. In the higher speed regions it loses its step character but still is able to stop and start correctly with slight delays. The motor will not likely be in the exact position dictated by the pulses but the delay will be most a few milliseconds. There is definitely some room for further study of the mechanical properties of this device in the high speed range. This would be done best if a special mock-up could be made so that rotor action could be observed. This testing
might allow improvements to be made that would considerably speed up the motor.
CHAPTER V - GENERAL CONCLUSIONS

From the tests and analysis it can be seen that as a stepping motor the Responsyn actuator has many desirable features. It will accelerate easily to speeds beyond 800 pps and handle inertia loads of about 15 oz.in.\(^2\) at that speed. Also its running torque at 800 pps is 100 oz.in. Therefore, it is capable of relatively high speed operation with appreciable load. The motor will run on non-uniform pulses but prefers the uniform variety.

Because of its properties and low angular output velocity the Responsyn actuator is best suited to precise positioning at high speed. Perhaps for this purpose a gear ratio of about 125 to 1 which would give 1000 pulses per revolution and allow a better match to the decimal system. However, this is not a serious problem. Examples of the use of the machine would be for machine tool positioning, mechanical plotter drives, and memory drum drives. Each of these requires that the motor move precise increments as quickly as possible. Thus they would use the high speed capabilities of the motor.

The other use of the Responsyn actuator would be for open loop positional control of many types. At present the industry is skeptical about the idea of open loop control and reluctant to attempt it. However, the Responsyn actuator shows excellent ability to stop on command and may eventually
be considered sufficiently reliable to use in an open loop system. This may be its greatest advantage over the competition. The use of feedback at intervals may be deemed necessary to prevent major errors due to a system disturbance or failure.

Originally it had been planned to use feedback on the Responsyn actuator to try to improve its performance. This was abandoned on three grounds. First there was little to be gained at low loads because the motor would step nearly as quickly as it would run without meeting mechanical difficulties. Second, at higher loads the motor would run faster once going but would run very roughly on start up and most likely would cause severe motor wear. Third, because of the extreme precision of the rotation - one part in 800 - a sufficiently accurate feedback device would be expensive and difficult to obtain. It is definite that the cost of the feedback package would be many times that of the motor. For these three reasons feedback was deemed impracticable and no attempts in this direction were made.

Over three hundred hours of operation of the motor and driving circuits occurred during the tests. This operation was generally under severe conditions. However, no appreciable deterioration of flex spline was noticeable. Thus the life expectancy of the motor under normal operating conditions looks good. The driving circuit worked well throught except
for the failure of one power transistor. Since no subsequent failures occurred it is safe to assume that this failure was not due to circuit design.

The Responsyn actuator is already a good stepping motor but I feel that a better one is possible. Already the effect of the amount of shimstock in the rotor has been tested and improvements made to advantage. This high-speed region of the operation is the area of interest. More examination of the rotor action in this range might allow a valuable design change. In particular, this anomalous initial delay phenomenon which I observed should be tracked down and its effect on performance evaluated. Also in the high-speed region, some performance increase could be gained from better circuit performance.
APPENDIX A

BINARY RATE MULTIPLIERS

A binary rate multiplier is a digital device which produces a pulse output rate which is a variable fraction of the input pulse rate. A simple block diagram for a binary rate multiplier is shown in figure 43. It consists of a data register and a counter register plus "and" gating between the corresponding bits in the two registers.

In binary counting there can be many 1 to 0 transitions for one count but only one 0 to 1 transition can occur. This will occur in the most significant bit changed. A counter of m bits will count to $2^m$ before recycling, and during that time there will be $2^m$ 0-1 transitions. Now the least significant digit will make $2^{m-1}$ 0-1 transitions at the $f_0/2$ where $f_0$ is the input clock frequency. The second least significant digit will make $2^{m-2}$ evenly spaced 0-1 transitions at frequency $f_0/4$ and so on. The most significant digit will make 1 transition.

Now if we insert 1's in certain bits of the data register and apply appropriate "and" gating then we will get an output pulse if, and only if, the digit making the 0-1 transition has as its counterpart in the data register are treated as an m bit binary fraction r then the output rate is precisely $rf_0$. Thus, the name binary rate multiplier. This fraction can be changed by changing the contents of the data register.
However, one limitation is apparent. The effect of the binary rate multiplier is to give a non-uniform pulse train whose average frequency is \( rf_0 \) but instantaneously may be quite different. This brings about the phenomenon of round off error. If the binary rate multiplier runs for a complete \( 2^m \) count before changing, the average is exactly \( rf_0 \). However, for smaller periods, and longer periods not a multiple of \( 2^m \), it is in error. This is especially significant when rapid changes are desired.

For the purpose of the experiments on the Responsyn actuator it was decided that a four bit binary rate multiplier would be sufficient. The circuit was built up from Digital Equipment Corporation logic cards and is shown in figure 44. The data register was synthesized from toggle switches and a three-volt battery. The slower output pulse from the pulse amplifier was necessary to activate the driving circuit. Also, in order to reliably trigger the motor logic circuit, it was necessary to drive a transistor inverter from the output of the binary. The inverter drove the logic circuit input. In order to gate the binary rate multiplier to get 20 pulse groups, the three-volt battery was replaced by a pulse of suitable duration from the General Radio pulse generator.
Figure 43 Generalized Block Diagram of a Binary Rate Multiplier

Figure 44 Binary Rate Multiplier used in the test made from D.E.C. logic modules
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