STOCHASTIC AND TOPOLOGICAL
FRACTURE GEOMETRY MODEL

by

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Jun-Suk Lee

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Abstract

A hierarchical model is developed to represent the geometry of fracture sets. The most important characteristics of this model are the possibility to model sequential fracture genesis as it usually occurs in nature and to model clustering. This model uses the inhomogeneous or the doubly stochastic (Cox) point process model to represent the midpoints of the primary set. In a second step, the fracture trace lengths of the primary set are modelled. This is done using Maximum Likelihood Estimates (MLE) of the observed data. In a third step, the secondary set is considered; in particular the independence/dependence of the two sets. The correlation of location with trace length is considered with the line-kernel function method and the nearest neighbor fiber distance method.

The hierarchical model has been applied to two cases in which detailed fracture patterns have been observed, one with a single set and one with two sets. The validity of the model was checked by visual comparison between model prediction and mapped patterns and most importantly, by statistical tests such as the second-moment analysis and the Monte Carlo test. A satisfactory fit of the predicted pattern was obtained in both cases.

Finally, a topological application of the stochastic geometry model is developed. Using a fully persistent block model, the kinematic as well as the kinetic stability of a rock slope is investigated. Through sensitivity studies, the most significant parameters affecting slope stability are found.

It is important to note that the combined hierarchical and topological model can be applied to any problem involving stability or failure of fractured bodies or masses, thus not only slope stability but also tunnel stability can be considered. Although limited at present to fully persistent fractures, the model concept is also applicable to non-persistently fractured masses/bodies and can thus include concrete structures.

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Table of Contents

Abstract 2
Acknowledgements 3
Table of Contents 4
List of Figures 7
List of Tables 11

1. INTRODUCTION 12

2. BASIC CHARACTERISTICS OF POINT PROCESSES 15
   2.1 Test of Complete Spatial Randomness (CSR) 17
       2.1.1 Inter-event Distances 18
       2.1.2 Nearest Neighbor Distances 21
   2.2 Second Moment Properties of Point Processes 22
   2.3 Point Process Models 27
       2.3.1 Homogeneous Poisson Point Process 27
       2.3.2 Inhomogeneous Poisson Point Process 28
       2.3.3 Poisson Cluster Process 28
       2.3.4 Cox Process (Doubly Stochastic Process) 30
       2.3.5 Simple Inhibition Process [Hard-core Process] 32
       2.3.6 Thinned Process 33
       2.3.7 Soft-core Process 35
       2.3.8 Discussion 37

3. APPLICATION OF POINT PROCESSES TO ROCK FRACTURE MODELING 39
   3.1 Introduction 39
   3.2 Inhomogeneous Poisson Point Model 39
   3.3 Marked (or Weighted) Point Process 44
   3.4 Doubly Stochastic Point Model 50
       3.4.1 Doubly Stochastic Point Process 50
       3.4.2 Spectral Density Function 53
   3.5 Conclusion 62

4. FIBER (or LINE - SEGMENT) PROCESSES 66
   4.1 Introduction 66
   4.2 Recent Trend in Fiber Modeling 66
       4.2.1 Homogeneous Poisson Fiber Process (Baecher) Model 67
       4.2.2 Parent - Daughter Model (Witherspoon and Long, 1987) 71
       4.2.3 Fractal Model 71
       4.2.4 Branching Model (Watanabe, 1986) 73
       4.2.5 Hierarchical Model (Conrad & Jacquin, 1975) 74
       4.2.6 Percolation Model 76
       4.2.7 Crack Tessellation Model (Gray, et al., 1976) 76
   4.3 Proposed Hierarchical Model 77
       4.3.1 Background 77
       4.3.2 Independence Test 77
Table of Contents

Abstract 2
Acknowledgements 3
Table of Contents 4
List of Figures 7
List of Tables 11

1. INTRODUCTION 12

2. BASIC CHARACTERISTICS OF POINT PROCESSES 15
   2.1 Test of Complete Spatial Randomness (CSR) 17
   2.1.1 Inter-event Distances 18
   2.1.2 Nearest Neighbor Distances 21
   2.2 Second Moment Properties of Point Processes 22
   2.3 Point Process Models
      2.3.1 Homogeneous Poisson Point Process 27
      2.3.2 Inhomogeneous Poisson Point Process 28
      2.3.3 Poisson Cluster Process 28
      2.3.4 Cox Process (Doubly Stochastic Process) 30
      2.3.5 Simple Inhibition Process [Hard-core Process] 32
      2.3.6 Thinned Process 33
      2.3.7 Soft-core Process 35
      2.3.8 Discussion 37

3. APPLICATION OF POINT PROCESSES TO ROCK FRACTURE MODELING 39
   3.1 Introduction 39
   3.2 Inhomogeneous Poisson Point Model 39
   3.3 Marked (or Weighted) Point Process 44
   3.4 Doubly Stochastic Point Model
      3.4.1 Doubly Stochastic Point Process 50
      3.4.2 Spectral Density Function 53
   3.5 Conclusion 62

4. FIBER (or LINE - SEGMENT) PROCESSES 66
   4.1 Introduction 66
   4.2 Recent Trend in Fiber Modeling
      4.2.1 Homogeneous Poisson Fiber Process (Baecher) Model 67
      4.2.2 Parent - Daughter Model (Witherspoon and Long, 1987) 71
      4.2.3 Fractal Model 71
      4.2.4 Branching Model (Watanabe, 1986) 73
      4.2.5 Hierarchical Model (Conrad & Jacquin, 1975) 74
      4.2.6 Percolation Model 76
      4.2.7 Crack Tessellation Model (Gray, et al., 1976) 76
   4.3 Proposed Hierarchical Model
      4.3.1 Background 77
      4.3.2 Independence Test 77
4.3.3 Correlation Measure 79
4.4 Trace Length Distribution Function 81
4.5 Discussion 83

5. APPLICATION OF THE HIERARCHICAL MODEL 84
5.1 Introduction 84
5.2 Step 1 - Midpoint Generation of Set 1 92
5.3 Step 2 - Traces of Set 1 94
5.4 Step 3 - Independence Test 99
5.5 Step 4 - Generation of Set 2 Midpoints 102
5.6 Step 5 - Generation of Set 2 Traces 110
5.7 Step 6 - Termination 117
5.8 Concluding Comments 121

6. TOPOLOGICAL APPLICATION OF FRACTURE GEOMETRY MODEL: SLOPE STABILITY ANALYSIS 122
6.1 Introduction 122
6.1.1 Previous Research on Slope Stability 122
6.1.2 Proposed Work on Slope Stability Analysis 123
6.2 Generation of Fracture Pattern 124
6.3 Kinematic Analysis 126
6.3.1 Construction of Connectivity Matrices 126
6.3.2 Kinematic Consideration 129
6.4 Kinetic Analysis 130
6.4.1 Assumptions used in Stability Analysis 131
6.4.2 Simplified Mechanical Model for Sliding 132
6.4.3 Program TRACESIM (see Appendix D for details) 134
6.5 An Example of Slope Stability Analysis 134
6.6 Parametric Studies of a Slope with Fully Persistent Fractures 135
6.6.1 Case 1: Variation of Cohesion 138
6.6.2 Case 2: Variation of Friction Angle 147
6.6.3 Case 3: Variation of Tensile Cut-off Stress 149
6.6.4 Case 4: Variation of Concentration Factor of Orientation 152
6.6.5 Case 5: Variation of Mean Orientation 155
6.6.6 Case 6: Variation of Mean Trace Length 155
6.6.7 Case 7: Variation of Number of Fracture Sets 158
6.6.8 Case 8: Variation of Midpoint Model of Fractures 159
6.7 Discussion 164

7. SUMMARY, CONCLUSIONS AND OUTLOOK 166

Appendix A. TESTS FOR RANDOMNESS 169
A.1 Distance-based [ Plotless ] Tests 169
A.2 Quadrat Count Tests 175
A.3 Discussion 176

Appendix B. METHODS FOR EDGE-CORRECTION 177
B.1 Edge-correction with the Graphical Method 177
B.2 Edge-correction by Toroidal Shift Method 178

Appendix C. USER MANUAL AND LIST FOR "POINT" AND "FIBER" 179
C.1 Introduction 179
C.2 Input Manual for Program POINT 179
C.3 Input Manual for Program FIBER 182
C.4 Input Manual for Program TDIST 184
C.5 Input Manual for Program TERMN 184
C.6 Program Listings : POINT, FIBER, TDIST, TERMN 185

Appendix D. USER MANUAL AND LIST FOR "TRACESIM" 260

D.1 Introduction 260
D.2 Input Manual for Program TRACESIM 260
D.3 Program Listing : TRACESIM 263
List of Figures

Figure 2-1:  Map of Florence Lake Outcrop (After Segall & Pollard, 1983; Only Traces within Rectangular Region are Considered)  
Figure 2-2:  CSR test with Inter-event Distance  
Figure 2-3:  CSR Test with Nearest Neighbor Distance  
Figure 2-4:  Second-order Moment Analysis of Mapped Data with Homogeneous Poisson Point Process Assumption  
Figure 2-5:  An Example of the Homogeneous Poisson Point Pattern  
Figure 2-6:  An Example of the Inhomogeneous Poisson Point Pattern  
Figure 2-7:  An Example of a Poisson Cluster Process  
Figure 2-8:  An Example of the Cox Process  
Figure 2-9:  Covariance Density of the Process Used in Fig. 2-8  
Figure 2-10:  An Example of the Simple Inhibition Process  
Figure 2-11:  An Example of the Thinned Process  
Figure 2-12:  An Example of the Soft-core Process, After Upton & Fingleton (1985)  
Figure 3-1:  Mid Point Representation of Traces of Florence Lake Outcrop  
Figure 3-2:  An Example of Schematic Kernel Estimate with Window Width 0.4 and Standard Deviation = 1.0  
Figure 3-3:  Kernel Estimator using Normal Distribution Function with Inhomogeneous Poisson Point Pattern  
Figure 3-4:  Inhomogeneous Poisson Point Pattern  
Figure 3-5:  Inter-event Analysis of Inhomogeneous Poisson Point Process  
Figure 3-6:  Nearest Neighbor Distance Analysis of Inhomogeneous Poisson Point Process  
Figure 3-7:  Second-moment Analysis of Inhomogeneous Poisson Point Process  
Figure 3-8:  An Example of the Marked (Weighted) Point Process  
Figure 3-9:  Correlations of Trace Length  
Figure 3-10:  Directional Second Moment of Mapped Data  
Figure 3-11:  Covariance Densities for Each Case  
Figure 3-12:  Correlation Function for X and Y Direction  
Figure 3-13:  X-dir. Spectral Density Function  
Figure 3-14:  Y-dir. Spectral Density Function  
Figure 3-15:  Calculation of Spectral Density Function  
Figure 3-16:  Mid Point Pattern Using Cox Process  
Figure 3-17:  Second Moment Analysis of Cox Process  
Figure 4-1:  Schematic Procedures for Constructing the Homogeneous Poisson Fiber Process Model (After Long, et al., 1982)  
Figure 4-2:  Enhanced Baecher Model  
Figure 4-3:  Parent-Daughter Model in Space, after Chiles (1988)  
Figure 4-4:  Branching Model of Watanabe (1986)  
Figure 4-5:  Hierarchical model of Conrad & Jacquin (1975)  
Figure 4-6:  Simulated Rock Fractures Network, After Conrad Jacquin (1975)  
Figure 4-7:  Schematic Estimation of Trace length  
Figure 5-1:  Trace Map of PA100 (From Barton & Larson, 1985)  
Figure 5-2:  Digitized Trace Map of PA100  
Figure 5-3:  Digitized Trace Map of Set 1 (PA100)  
Figure 5-4:  Digitized Trace Map of Set 2 (PA100)  
Figure 5-5:  Mid Point Map of PA100: Mid Points of All Traces
Figure 5-6: Mid Point Map of PA100 : Mid Points of Set 1 Traces Only 90
Figure 5-7: Mid Point Map of PA100 : Mid Points of Set 2 Traces Only 91
Figure 5-8: Directional Second Moment of Set 1. Characteristic directions of 93
30-40° and 120-130° are used to calculate the directional second moment. Isotropic case is also shown for comparison.
Figure 5-9: Simulation of Set 1 Mid Points with Cox Process 95
Figure 5-10: Second Moment Analysis of Set 1. Acceptance band (MIN to 96
MAX) and average value (AVRG) obtained with model simulations are shown. The second moment of the mapped data (MAPPED) is also given
for comparison.
Figure 5-11: Log-Likelihood of Mean Trace Length for Set 1, assuming 97
Exponential Distribution
Figure 5-12: Simulation of Trace Lengths for Set 1 98
Figure 5-13: Independence Test with Simulated Homogeneous Poisson Point 100
Process for Sets 1 and 2. Acceptance band (MIN to MAX) and average value (AVRG) are constructed through perturbations. When the data is
from an isotropic Poisson point process (ISOTROPIC), it will be inside the
band.
Figure 5-14: Independence Test of Mapped Pattern. Symbols are the same as
those in Fig. 5-13. The independence of the actual mapped pattern (MAPPED) is tested. It shows the dependence of set 2 on set 1 traces.
Figure 5-15: One of the Simulated Mid Point Patterns of Set 2 Using the Line-
Kernel Function Method to Express Correlation.
Figure 5-16: Bivariate Second Moment Analysis for Sets 1 and 2 - Mid Point 105
Correlation Based on the Line-Kernel Function Method. Acceptance band
(MIN to MAX) and average value (AVRG) are obtained with model simulations, while the mapped pattern (MAPPED) is from actual data.
Figure 5-17: (Univariate) Second Moment Analysis for Set 2 Mid Points Using
the Line-Kernel Function Method. Symbols are the same as those in Fig.
5-16.
Figure 5-18: One of the Simulated Mid Point Patterns of Set 2 Using the 107
Nearest Neighbor Function Method to Express Correlation
Figure 5-19: Bivariate Second Moment Analysis for Sets 1 and 2 - Mid Point
Correlation Based on the Nearest Neighbor Function Method. Acceptance
band (MIN to MAX) and average value (AVRG) are obtained with model simulations, while the mapped pattern (MAPPED) is from actual data.
Figure 5-20: (Univariate) Second Moment Analysis for Set 2 Mid Points Using
the Nearest Neighbor Function Method. Symbols are the same as those in
Fig. 5-19.
Figure 5-21: Orientation Distribution of Set 2 on a Circle 112
Figure 5-22: Orientation Distribution of Set 2 Based on a Nearest Fiber
Distance of Set 1. Orientations are measured with increasing distances (1m, 2m, 3m), also Fig. 5-21 (GLOBAL = actual orientation data) is shown
for comparison.
Figure 5-23: Wrapped Normal and Von Mises Orientation Distribution Fitted to
Orientation Data of Set 2
Figure 5-24: Trace Pattern of Set 2 with Von Mises Orientation Distribution 115
Figure 5-25: Trace Pattern of Set 2 with Wrapped Normal Distribution 116
Figure 5-26: Calculation of Termination Probability 118
Figure 5-27: One of the Simulated Patterns of PA100 with the Von Mises
Orientation Distribution Function
Figure 5-28: One of the Simulated Patterns of PA100 with the Wrapped Normal
Orientation Distribution function 120
Figure 6-1: Iterative Searching Scheme for Effective Fractures (a) Initial Fracture Network (b) Iterative Searching Scheme for Effective Fractures

Figure 6-2: Typical Rock Slope Configuration

Figure 6-3: Example of a Connectivity Matrix \([C_x]\) (a) A Simulated Effective Fracture Pattern (b) Construction of a Connectivity Matrix (x-coordinate) (c) Searching Scheme for a Fracture Path

Figure 6-4: Kinematically Inadmissible Fracture Paths

Figure 6-5: Rigid Body Motion of a Typical Rock Block Along a Face of Minimum Dip angle \(\alpha\)

Figure 6-6: Configuration of a Slope Example

Figure 6-7: Initial Fracture Pattern in a Slope

Figure 6-8: Effective Fracture pattern in a Slope

Figure 6-9: One of the Two Unstable Fracture Paths

Figure 6-10: One of the Two Unstable Fracture Paths

Figure 6-11: Critical Fracture Path

Figure 6-12: A Plot of Sliding Angle v.s. Volume of Rock Block with All Data Set

Figure 6-13: Simplification of Fig. 6-12 with Maximum Volume

Figure 6-14: Randomness Test of Occurrences of Kinematically Admissible Rock blocks

Figure 6-15: Calculation of Mean Sliding Angle Given Volume of Rock Block

Figure 6-16: Calculation of No. of Kinematic Mechanisms

Figure 6-17: Distribution of Contact Length Given Volume of Rock Block

Figure 6-18: Fitting Contact Length to the Beta Distribution

Figure 6-19: \(P_f\) Given Volume of Rock Block: Variation of Cohesion

Figure 6-20: Rate of Failure Occurrences in a Volume Interval: Variation of Cohesion

Figure 6-21: \(P_f\) in a Volume Interval: Variation of Cohesion

Figure 6-22: \(P_f\) Given Volume of Rock Block: Variation of Friction Angle

Figure 6-23: Rate of Failure Occurrences in a Volume Interval: Variation of Friction Angle

Figure 6-24: \(P_f\) in a Volume Interval: Variation of Friction Angle

Figure 6-25: \(P_f\) Given Volume of Rock Block: Variation of Tensile Cut-off Stress

Figure 6-26: Typical Fracture Pattern with Increasing \(\kappa\)

Figure 6-27: \(P_f\) Given Volume of Rock Block: Variation of Concentration Factor with Cohesion = 1000

Figure 6-28: \(P_f\) Given Volume of Rock Block: Variation of Concentration Factor with Cohesion = 100

Figure 6-29: \(P_f\) Given Volume of Rock Block: Variation of Mean Orientation

Figure 6-30: Rate of Failure Occurrences in a Volume Interval: Variation of Mean Orientation

Figure 6-31: \(P_f\) in a Volume Interval: Variation of Mean Orientation

Figure 6-32: \(P_f\) in a Volume Interval: Variation of Mean Trace Length

Figure 6-33: Changes of Mean Sliding Angle: Variation of Mean Trace Length

Figure 6-34: Typical Fracture Pattern with Two Sets of Fractures (a) \(\alpha = 20^\circ\), \(40^\circ\) (b) \(\alpha = 40^\circ\), \(40^\circ\) (c) \(\alpha = 130^\circ\), \(40^\circ\)

Figure 6-35: \(P_f\) Given Volume of Rock Block: Variation of Number of Fracture Sets

Figure 6-36: Rate of Failure Occurrences in a Volume Interval: Variation of Number of Fracture Sets
Figure 6-37: \( P \) in a Volume Interval: Variation of Number of Fracture Sets 162
Figure 6-38: \( P_f \) Given Volume of Rock Block: Variation of Midpoint Model of Fractures 163
Figure 6-39: Rate of Failure Occurrences in a Volume Interval: Variation of Midpoint Model of Fractures 163
Figure 6-40: \( P \) in a Volume Interval: Variation of Midpoint Model of Fractures 164
Figure A-1: Various Nearest-neig\( \bar{b} \)or Measurements 170
Figure A-2: Semisystematic Sampling Scheme of Byth & Ripley (1980) 174
Figure B-1: Graphical Method for Edge-correction 178
List of Tables

Table 6-1: Initial and Changing Values of Parameters 139
Table 6-2: Investigated Cases 140
Chapter 1

INTRODUCTION

Since rock masses are usually discontinua, one needs to consider fracture geometry when considering rock mass behavior. The MIT rock mechanics group has developed idealized stochastic models: Baecher, et al. (1977) assumed a disk shaped fracture in a space. The Baecher model can be constructed by generating a homogeneous Poisson point pattern for fracture centers and by adopting suitable fracture size distribution functions as well as fracture orientation distribution functions. Veneziano (1979), in his model, used a Poisson plane process in space followed by a Poisson line process in each plane. Using a persistence parameter, polygonal fractures can be constructed in space. Dershowitz (1985) modified the Veneziano model by considering fracture intersection behavior. With a Poisson plane process, he generates intersections of various fracture planes in space which in turn bound polygonal fractures. These simplified stochastic fracture models have been applied in a number of cases. They are limited, however, since they cannot represent many fracture patterns encountered in the field. Specifically:

- They do not account for spatial nonhomogeneities such as fracture clustering.
- The models are only loosely tied to the geologic genesis of the fractures. In particular, most models assume independence among fracture sets. From a geologic viewpoint, this assumption is often incorrect since the fracture sets are, in almost all cases, sequentially generated with inter-relations among themselves.
- Only in a few cases have the models been validated using actual fracture data.

In this research project, advanced stochastic geometry models are created which better represent reality.

Stochastic geometry can handle both the geometric and the stochastic nature of a structure (i.e., rock mass). In general, stochastic geometry can be considered in different ways:
1. Based on a geometry (shape), which can be a point or line process in a plane, or a three-dimensional Random Closed Sets (RACS) process.

2. Based on data sets for the different characteristics; for each characteristic, the data set can be univariate or multivariate processes.

Application of point processes can be found in the field of forestry. Trees in a forest can be represented as points in a region. From this simplified process, we can find the inter-relations among trees. Examples of line processes come from material science. A defect in the metal surface can be regarded as a line (or a fiber with finite length). In the field of microscopy, one often uses a RACS model where the closed sets consist of spheres or disks in a space. If the characteristics of the model are not uniform, one employs bivariate or multivariate point, fiber or RACS processes. For example, if two kinds of trees exist in a region, the bivariate point process becomes a plausible model.

Within the framework of rock fracture modeling, a point process can be used if the distribution of fracture traces on a rock surface is simple. Each trace is actually a segment (fiber) of finite length, but if the trace length, trace orientation and trace position are independently distributed, we can describe the segment as a point, usually the mid point. If the above mentioned trace characteristics are correlated with each other, we use the fiber process for modeling fractures in a plane. A RACS process seems to be best suited for the three dimensional situations we are actually interested in. However, these models rely on information which is not readily available such as size and shape of fractures. Eventually, RACS process models will have to be developed.

We, at this stage, want to concentrate on the point and fiber processes. Specifically, we will develop alternative models to the conventional homogeneous Poisson point and Poisson fiber processes. The main features of these models are the hierarchical description of fracture sets which allows one to reproduce the sequential genesis of fracture sets, and the consideration of dependencies among fractures of the same set or of different sets. The
sequential generation and correlation of fracture sets correspond to what happens in nature. Equally important as the modelling principle is the availability of statistical procedures to estimate parameters and validate the model. These will provide the basis for future work on the RACS processes.

To apply the hierarchical fracture geometry model, a two dimensional rock slope will be considered. The fracture patterns simulated with the hierarchical model can be used in any rock mass problem such as flow through fractured rock masses, deformability of fractured rock masses and stability of fractured rock masses. The last of these problems is chosen here because it has great practical significance both regarding rock slope stability and stability of any fractured mass (rock mass around tunnel, concrete structures with cracks, etc). For such applications, existing models are not suitable. Consequently, new models need to be developed which, in our case, will consist of a combination of the hierarchical fracture model and a topological model.

In Chapter 2, we will discuss the basic characteristics of the point process and of some alternative models which may be used in rock fracture modeling. In particular, we will consider the Complete Spatial Randomness (CSR) as an indicator of the homogeneous, isotropic point process. In Chapter 3, the appropriate point process models which can be best fit to our mapped data are specified and developed. Additional mathematical expressions are introduced which make the unknown functions (in our case, the trace intensity function) easy to formulate. In Chapter 4, we will introduce the basic characteristics of the fiber process. Also traditional fiber process models will be reviewed and criticized. In Chapter 5, we will expand our point process model to a fiber process model. Topological application of the stochastic fracture geometry model is considered in Chapter 6. Finally, in Chapter 7, conclusions will be drawn as to advantages and limitations of the fracture geometry models.
Chapter 2

BASIC CHARACTERISTICS OF POINT PROCESSES

Point processes can be regarded as statistical models for point-like objects in a plane. Traditionally trees in a forestry or earthquake occurrences in the space-time domain have been represented by point patterns. The reason for using a point processes is to analyze patterns of points and to suggest a plausible mechanism by which they might have been generated. In our case, the traces on a rock outcrop are considered as segments, and can be represented as points, usually mid points, in a plane. (This is a first approximation; more complex processes will be introduced later.) Fig. 2-1 is an example of a simple trace pattern and we will consider such trace data with point processes. Usually, one will assume a Poisson point process as a null hypothesis when generating center points of rock fractures in a plane or space.

In this Chapter, we, first of all, check the above mentioned null hypothesis using mapped data such as the fracture traces of Fig. 2-1. There are many tests which can be used to check randomness assumptions. Two relatively powerful tests are described here, and a brief history of various test methods is given in Appendix A. Next, the second moment properties of the point processes are considered since, in most cases, point patterns can be characterized by second moments. Finally, we will discuss various point process models, including the homogeneous Poisson point process model, which may be applied to rock fracture modelling.
Figure 2-1: Map of Florence Lake Outcrop (After Segall & Pollard, 1983; Only Traces within Rectangular Region are Considered)
2.1 Test of Complete Spatial Randomness (CSR)

The simplest assumption of point pattern characteristics is no interaction between the points. This is called Complete Spatial Randomness (CSR) (Diggle, 1983). If the intensity of points changes from place to place, the pattern is not homogeneous and we call it inhomogeneous or heterogeneous. Directional patterns are called anisotropic.

The hypothesis of CSR for a point pattern asserts that,
1. The number of events in any planar region A with area $|A|$ follows a Poisson distribution with mean $\lambda|A|$, where $\lambda$ is the intensity.
2. Given $n$ events $x_i$ in a region A, the $x_i$ are an independent random sample from the uniform distribution on A.

There are two reasons for considering CSR.
1. CSR is a minimal prerequisite for randomness.
2. CSR operates as a dividing hypothesis between regular and aggregate patterns.

Therefore, if the given set of points does not follow CSR (because there is an attractive or inhibitive correlation among points), our aim will be to find non-Poisson models and to establish a method for testing the fit of any proposed model.

To test CSR of a given data set, we use two different methods. One is based on distance measures, the other uses a quadrat count. Only the distance measure method will be discussed here, specifically, inter-event distances and nearest neighbor distances. This will be done because, in some cases, the quadrat count method cannot provide much information, for example, on clustering or inhibitory behavior, and it is not easy to visualize. Furthermore, if the number of data points is small, the quadrat count method may not be useful. The specific distance measure methods described here are the inter-event distances and nearest neighbor distances.
2.1.1 Inter-event Distances

The theoretical distribution of the distance \( t, H(t) \), between two events independently and uniformly distributed in a region \( A \) depends on the size and shape of \( A \), but can be expressed in closed form for the most common cases of a square or circular area \( A \) (Bartlett, 1964). Assuming that for the particular region \( A \) in question, \( H(t) \) is known, we can calculate the empirical distribution function (EDF) of inter-event distances. This function, \( \hat{H}_1(t) \) say, represents the observed proportion of inter-event distances \( t_{ij} \) which are at most \( t \). Thus,

\[
\hat{H}_1(t) = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} \mathbb{I}(t_{ij} \leq t)
\]

(2.1)

where \( \mathbb{I} \) means "the number of" and \( \hat{H}_1(t) \) is the empirical distribution function. A plot of \( \hat{H}_1(t) \) on the ordinate against \( H(t) \) on the abscissa should be roughly linear if the data are compatible with CSR. To assess the significance and the deviations from linearity, the conventional approach would be to find the sampling distribution of \( \hat{H}_1(t) \) under CSR. Since this is complicated by the dependence between inter-event distances with a common endpoint, one, therefore, proceeds as follows:

1. Calculate EDF \( \hat{H}_i(t) \), \( i = 2, 3, \ldots, s \), from each of \( s-1 \) independent simulations of \( n \) events independently and uniformly distributed on \( A \).

2. Define upper and lower simulation envelopes,

\[
U(t) = \max \left( \hat{H}_i(t) \right), \quad L(t) = \min \left( \hat{H}_i(t) \right), \quad i = 2, 3, \ldots, s.
\]

(2.2)

these simulation envelopes can also be plotted against \( H(t) \), and have the property that under CSR, and for each \( t \),

\[
P \left( \hat{H}_1(t) > U(t) \right) = P \left( \hat{H}_1(t) < L(t) \right) = s^{-1}
\]

(2.3)

since we assume that each simulation is distributed independently and uniformly on \( A \). The simulation envelopes are intended to help in the interpretation of the plot of \( \hat{H}_1(t) \) against \( H(t) \). That is, if the theoretical pattern lies between \( U(t) \) and \( L(t) \) throughout its range, it means acceptance of CSR.

3. If the region \( A \) is one for which the theoretical distribution function \( H(t) \) is unknown as in our case (rectangular region), a test can still be carried out if \( H(t) \) is replaced by
\[
\bar{H}_i(t) = (s-1)^{-1} \sum_{j \neq i} \hat{H}_j(t)
\]  
(2.4)

Similarly, the graphical procedure then consists of plotting \(\hat{H}_i(t), U(t)\) and \(L(t)\) against \(\bar{H}_i(t)\) or, to make it easy to visualize, plotting \(\hat{H}_i(t), U(t), L(t)\) and \(\bar{H}_i(t)\) against distances. Note that because \(\bar{H}_i(t)\) is a simulated value under the assumption of CSR and does not include the original data, it provides an unbiased estimate of \(H(t)\). Checking CSR with simulation envelopes can also be applied here, i.e., if \(\bar{H}_i(t)\) lies between maximum (U(t)) and minimum (L(t)) envelopes throughout its range, it means acceptance of CSR.

Fig. 2-2 shows an inter-event distance test of our mapped pattern from Fig. 2-1, where the averaged value is given in Eqn (2.4), and for clarity, U(t) and L(t) are replaced by maximum and minimum value, respectively. From this figure, we do not accept the CSR assumption for the data of Fig. 2-1. This means, our mapped pattern is not random and there seems to be a certain interaction among mid points. However, with this test, it is not easy to see the degree of interaction. In other words, we see the clustered behavior of the traces from Fig. 2-1, but we cannot use this test to quantitatively describe the clustering behavior.

Also from a simple Monte Carlo test (Barnard, 1963), we can decide on the acceptance of CSR for given data. When we use the Monte-Carlo test, it provides a useful check on the applicability of the asymptotic theory. Let \(u_1\) be the observed value of a statistic U and let \(u_i, i=2, \cdots, s\) be corresponding \(s\) values generated by independent random sampling from the distribution of U under a simple hypothesis H. Let \(u_{(j)}\) denote the \(j\)-th largest amongst \(u_i, i=1, \cdots, s\). Then, under H,

\[
P \{ u_j = u_{(j)} \} = s^{-1}, \quad j=1, \cdots, s
\]  
(2.5)

and rejection of H on the basis that \(u_1\) ranks \(k\)-th largest or higher gives an exact, one-sided test of size \(\frac{k}{s}\). For example, if \(u_1\) is the highest among 20 sampling values, one rejects the hypothesis of the 5% significance level. This assumes that the values of the \(u_i\) are all
Figure 2-2: CSR test with Inter-event Distance
different, so that the ranking of \( u_i \) is unambiguous. In our case, define \( u_i \) to be a measure of
the discrepancy between \( \hat{H}_i(t) \) and \( H(t) \) over the whole range of \( t \),
\[
\text{\( u_i = \int (\hat{H}_i(t) - H(t))^2 \, dt \) (2.6)}
\]
and proceed with the above mentioned Monte Carlo test based on the rank of \( u_i \).

In our case, we tried 49 simulations (i.e., \( s = 50 \)) and, from the one-sided test at the
2% significance level, we reject CSR for our mapped pattern.

2.1.2 Nearest Neighbor Distances

For \( n \) events in a region \( A \), let \( y_i \) denote the distance from the \( i \)-th event to the nearest
other event in \( A \). The \( y_i \) are called nearest neighbor distances. We can calculate the EDF,
\( \hat{G}_1(y) \) say, of the nearest neighbor distances, i.e.,
\[
\text{\( \hat{G}_1 = n^{-1} \) \( \text{no.} (y_i \leq y) \) (2.7)}
\]
In many practical situations, interactions between events exist, if at all, only on a small
physical scale: for example, trees would be expected to compete for sunlight or nutrient
within an area roughly confined to their crowns or root systems, respectively. In such a
case, nearest neighbor distances provide an objective means of concentrating on small inter-
event distances when a precise threshold distance cannot be specified in advance. The
theoretical distribution of nearest neighbor distance \( Y \) under CSR depends on the number of
events \( n \) and on \( A \), and is not expressible in closed form because of complicated edge
effects. An approximation which ignores these edge effects is obtained by noting that if \( |A| \)
denotes the area of \( A \), then \( \pi y^2 |A|^{-1} \) is the probability under CSR that an arbitrary event is
within distance \( y \) of a specified event. Since the events are located independently, the
approximate distribution function of \( Y \) is
\[
G(y) = 1 - (1 - \pi y^2 |A|^{-1})^{n^{-1}} \quad (2.8)
\]
The EDF of nearest neighbor distances, \( \hat{G}_i(y) \), can be compared with upper and lower
simulation envelopes from simulated EDFs \( \hat{G}_i(y) : i=2,3, \cdots, s \) in a similar way to the
inter-event method. The approximate result, Eqn (2.8), can be used to suggest a suitable range of tabulation but, because it is approximate, it is generally preferable to use the sample mean \( \overline{G}_i(y) \) of simulated EDFs.

\[
\overline{G}_i(y) = (s-1)^{-1} \sum_{j \neq i} \hat{G}_j(y)
\]  

(2.9)

Here again, \( \overline{G}_i(y) \) involves only the simulation of CSR and it provides an unbiased estimate of \( G(t) \).

Now we can construct extreme envelopes like those in Fig. 2-3 in a similar way as we did in Fig. 2-2. We can plot \( \hat{G}_i(t) \), \( U(t) \) (max.), \( L(t) \) (min.) and \( \overline{G}_i(t) \) against distance t where \( U(t) \) and \( L(t) \) can be derived as follows,

\[
U(t) = \max \left( \hat{G}_i(t) \right),
\]

\[
L(t) = \min \left( \hat{G}_i(t) \right), \quad i = 2, 3, \ldots, s.
\]  

(2.10)

Fig. 2-3 shows rejection of CSR. In addition, we clearly see the interaction of data points. Near the distance 1.0 and 3.0, we see the clustering phenomenon among traces, whereas near 2.5, there seems to be inhibitory interactions.

Again a Monte-Carlo test, which is analogous to Eqn (2.6), shows rejection of CSR at the 2% significance level. In this case, we replace the H function of Section 2.1.1. by G function shown above.

2.2 Second Moment Properties of Point Processes

To compare any candidate model with a set of data, we often use first and second moment properties of a point process. Here, \( \mathbb{E}(x) \) is the expectation of a random variable \( x \); \( N(A) \) represents the number of events in a region \( A \); \( |A| \) is the area of \( A \); \( dx \) is an infinitesimal region which contains the point \( x \); \( b_x(t) \) represents the disc with center \( x \) and radius \( t \); \( \mathbb{R}^2 \) is the entire plane where \( \mathbb{R}^2 = b(R) \times b(R) = \{ (x,y) : x, y \in \mathbb{R} \} \) which is the co-ordinatization of the plane and generally \( \mathbb{R}^d \) is referred to as the \( d \)-dimensional Euclidean space.
Figure 2-3: CSR Test with Nearest Neighbor Distance
First-order properties are described by an intensity function,

\[
\lambda(x) = \lim_{|dx| \to 0} \left( \frac{E[N(dx)]}{|dx|} \right) \tag{2.11}
\]

For a stationary process (i.e., the process in any region A of the plane is invariant under arbitrary translation of A), \( \lambda(x) \) assumes a constant value \( \lambda \), the mean number of events per unit area. The second-order intensity function is similarly defined as

\[
\lambda_2(x,y) = \lim_{|dx||dy| \to 0} \left( \frac{E[N(dx)N(dy)]}{|dx||dy|} \right) \text{ for } x,y \text{ in } \mathbb{R}^2 \tag{2.12}
\]

For a stationary process, \( \lambda_2(x,y) = \lambda_2(x-y) \); for a stationary, isotropic (i.e., invariant under rotation) process, \( \lambda_2(x-y) \) reduces further to \( \lambda_2(t) \), where \( t \) is the distance between \( x \) and \( y \).

In statistical mechanics, the scaled quantity \( \lambda_2(t) / \lambda^2 \) is referred to as the radial distribution function, although it is not a distribution function in the accepted statistical sense.

An alternative characterization of the second-order properties of a stationary, isotropic process is provided by the function \( K(t) \) (Ripley, 1977), which can be defined as

\[
K(t) = \lambda^{-1} E \left[ \text{number of further events within distance } t \text{ of an arbitrary event} \right] \tag{2.13}
\]

Thus, \( \lambda^2 K(t) \) can be interpreted as the expected number of pairs of points less than distance \( t \) apart with the first point in a given set of unit area. In order to establish a link between \( K(t) \) and \( \lambda_2(t) \), we shall assume that our process is orderly, by which we mean that multiple coincident events cannot occur. The expected number of further events within distance \( t \) of an arbitrary event is,

\[
\lambda K(t) = \int_0^{2\pi} \int_0^t \left( \frac{\lambda_2(x)}{\lambda} \right) x \, dx \, d\theta \\
= 2\pi \lambda^{-1} \int_0^t \lambda_2(x) \, dx \tag{2.14}
\]

or

\[
\lambda_2(t) = \lambda^2 \left( 2\pi t \right)^{-1} K'(t) \tag{2.15}
\]

where \( \theta \) is the angle of the arc in a circle with center \( x \) and radius \( t \). The covariance density \( \gamma \) is then
\gamma(t) = \lambda_2(t) - \lambda^2 \tag{2.16}

The above mentioned second moment characteristics play an important role when we consider the relative positions of two points of the process and quantify the apparent deviation from Poisson randomness to clustering or inhibition between points. For this, we, again, test CSR of our pattern with the second moment properties. If the mapped pattern follows the homogeneous Poisson point process, and if we use Monte-Carlo simulations, edge-corrected second moments of given data sets would exist between extreme envelopes. The extreme envelopes can be plotted by adopting a maximum and minimum \( \hat{K}(t) \) value of the null hypothesis, and edge-corrections can be performed either by graphical concepts (Ripley, 1977 or Diggle, 1983) or by toroidal shifts (Lotwick & Silverman, 1982). The idea of the edge-correction can be understood, if we think of small areas near the boundaries, i.e., there may exist some additional points which do not lie inside the region, but the influence that these points have needs to be considered. We will discuss edge-correction methods in Appendix B.

Fig. 2-4 shows a second moment analysis of data given in Fig. 2-1. It is constructed by assuming that the data follow a homogeneous Poisson point process; the simulation envelopes are the realizations of the homogeneous Poisson point pattern. From the figure, we see the discrepancies of the second moments between the simulated and the mapped values. As a result, we conclude that the mapped pattern does not follow the homogeneous Poisson point pattern, i.e. CSR. This was confirmed previously when we tested CSR with the inter-event and the nearest neighbor distance method. Also the Monte-Carlo test shows rejection of the homogeneous Poisson point process.

In some cases, we need to include the relative positions of three and more points; Hanisch (1983) considers this case in detail. He shows that the third moment measure is determined by a function \( K(t_1, t_2, \theta) \) of three variables, where \( t_1, t_2 \) are distances of two points from the origin and \( \theta \) is the angle between these two points. Meanwhile, Baddeley &
Figure 2-4: Second-order Moment Analysis of Mapped Data with Homogeneous Poisson Point Process Assumption
Silverman (1984) showed that investigation of the second moment properties alone cannot, sometimes, detect differences between point patterns with identical K functions.

2.3 Point Process Models

In this section, we discuss various models which can be applied to rock fracture modeling. We already mentioned the application of the homogeneous Poisson point process and saw that it is often inadequate to model rock fracture patterns. In order to provide a systematic treatment, the discussion below will nevertheless start with the definition of the homogeneous Poisson point process.

2.3.1 Homogeneous Poisson Point Process

This process represents the simplest possible stochastic mechanism for the generation of a spatial point process. It corresponds exactly to the definition of CSR given in Section 2.1.

1. For a given intensity \( \lambda > 0 \) and any finite planar region \( A \), \( N(A) \) has a Poisson distribution with mean \( \lambda |A| \).

2. Given \( N(A) = n \), the \( n \) events in \( A \) form an independent random sample from the uniform distribution on \( A \).

3. For any two different regions \( A \) and \( B \), \( N(A) \) and \( N(B) \) are independent.

so from Eqn (2.12),
\[
\lambda_2(t) = \lambda^2, \quad t > 0
\]  
(2.17)
and from Eqn (2.14) and Eqn (2.17),
\[
K(t) = \pi t^2, \quad t > 0
\]  
(2.18)
This result is quite clear when we refer to the definition of the second moment function (see, Eqn (2.13)) since, in the homogeneous case, the expected number of further events within distance \( t \) become \( \lambda \pi t^2 \), thus leading to Eqn (2.18). Fig. 2-5 shows an example of the homogeneous Poisson point pattern.
2.3.2 Inhomogeneous Poisson Point Process

A class of non-stationary point processes is obtained if the constant intensity $\lambda$ of the Poisson point process is replaced by a variable intensity function $\lambda(x)$, where,

1. $N(A)$ has a Poisson distribution with mean $\lambda \int_A \lambda(x) dx$.

2. Given $N(A) = n$, the $n$ events in $A$ form an independent random sample from the distribution on $A$ with probability density function proportional to $\lambda(x)$.

However, difficulties remain when the intensity function, $\lambda(x)$, is hard to obtain. Fig 2-6 shows a realization of the inhomogeneous Poisson point pattern where $\lambda(x_1, x_2) = \exp(-2x_1 - x_2)$.

2.3.3 Poisson Cluster Process

The Poisson cluster process (Neyman & Scott, 1958) is often called parent-daughter model. This kind of process provides a satisfactory basis for the modeling of aggregated planar point patterns.
Figure 2-6: An Example of the Inhomogeneous Poisson Point Pattern

1. Parent events form a Poisson process with intensity $\rho$.

2. Each parent produces a random number $s$ of offspring, realized independently and identically for each parent according to a probability distribution $\{ P_s, s = 0, 1, \ldots \}$.

3. The positions of the offspring relative to their parents are independently and identically distributed according to a probability density function (PDF) $h(\cdot)$.

Poisson cluster processes are stationary, with intensity $\lambda = \rho \mathbb{E}[s]$. They are isotropic if $h(\cdot)$ is a radially symmetric function such as a bivariate normal distribution function. To express the second-order properties, it is convenient to define

\[
h_2(x) = \int h(x) h(x-z) \, dx
\]

(2.19)

the PDF of the vector difference between two offsprings from the same parent. The expected number of ordered pairs of such offspring is $\mathbb{E}[S(S-1)]$, so that

\[
\lambda_2(x-y) = \lambda^2 + \rho \mathbb{E}[S(S-1)] h_2(x-y)
\]

(2.20)

in which the first term arises from a consideration of two offsprings at $x$ and $y$ from
different parents, and the second from two offsprings of the same parent. Fig. 2-7 illustrates an example of a Poisson cluster process with 25 parents and four offsprings per parent.

![Figure 2-7: An Example of a Poisson Cluster Process](image)

**Figure 2-7: An Example of a Poisson Cluster Process**

2.3.4 Cox Process (Doubly Stochastic Process)

The Cox process may be appropriate if the observed pattern reflects underlying environmental variation. The source of the environmental heterogeneity might itself be stochastic in nature. This suggests investigation of a class of doubly stochastic processes formed as inhomogeneous Poisson processes with stochastic intensity function. Thus,

1. \( \{ \Lambda(x); x \in R^2 \} \) is a non-negative-valued stochastic process.

2. Conditional on \( \{ \Lambda(x) = \lambda(x); x \in R^2 \} \), the events form an inhomogeneous Poisson process with intensity function \( \lambda(x) \).

The difference between the Cox process and the inhomogeneous Poisson point process is that the intensity function of the Cox process is a stochastically varying function, whereas the intensity function of the inhomogeneous Poisson point process is deterministic.

First- and second-order properties are obtained from those of the inhomogeneous Poisson process by taking expectations with respect to \( \{ \Lambda(x) \} \). Thus, in the stationary case, the intensity is,
\[ \lambda = E[\Lambda(x)] \]  
\text{(2.21)}

Also, the conditional intensity of a pair of events at \( x \) and \( y \) given \( \{ \Lambda(X) \} \) is \( \Lambda(x) \Lambda(y) \), so that,

\[ \lambda_{2}(x,y) = E[\Lambda(x) \Lambda(y)] \]  
\text{(2.22)}

In the stationary, isotropic case, this can be written as

\[ \lambda_{2}(t) = \lambda^2 + \gamma(t) \]  
\text{(2.23)}

where,

\[ \gamma(t) = Cov \{ \Lambda(x), \Lambda(y) \} \]  
\text{(2.24)}

and \( t \) is the distance between \( x \) and \( y \). Note that the covariance function \( \gamma(t) \) of the intensity process is also the covariance density of the point process. From a statistical point of view, the distinction between clustering and heterogeneity can only be made if additional information is available and Diggle (1977) suggested a two-phase test for distinction. Fig. 2-8 and Fig. 2-9 show a realization of the Cox process and corresponding covariance function, respectively (see Matern, 1971).

\[ \text{Figure 2-8: An Example of the Cox Process} \]
2.3.5 Simple Inhibition Process [Hard-core Process]

The alternatives to the Poisson processes described in Section 2.3.3. and 2.3.4. share a tendency to produce aggregated patterns. Regular patterns arise most naturally by the imposition of a minimum permissible distance, δ, between any two events. For example, as Matern (1986) described, assuming a random point representing a cell, specifically the core of the cell, then each point must be surrounded by a certain region where no other point can exist. Processes of this sort, which incorporate no further departure from complete spatial randomness, will be called simple inhibition (or repulsion) processes (or hard-core processes). The packing intensity of a simple inhibition process is defined as:

\[ \tau = \lambda \pi \delta^2 / 4 \]  

(2.25)

where \( \lambda \) is the intensity. Thus, \( \tau \) is the proportion of the plane covered by non-overlapping discs of diameter \( \delta \), or the expected proportion of coverage for a finite region \( A \).

Matern (1986) described two types of simple inhibition processes,

1. A Poisson process \( \rho \) is thinned by the deletion of all pairs of events a distance less than \( \delta \) apart. That is, if two events have a mutual distance less than \( \delta \), both of them are deleted. The probability that an arbitrary event survives is therefore \( \exp(-\pi \rho \delta^2) \), and the intensity of the simple inhibition process is,
\[ \lambda = \rho \exp(-\pi \rho \delta^2) \]  \hspace{1cm} (2.26)

2. In case of a dynamic scheme, the events of a Poisson process are marked with times of birth and an event is removed if it lies within a distance \( \delta \) of an older event. An expression analogous to Eqn (2.26) can be obtained, but only by ignoring any consideration of whether or not the older event in question has itself previously been removed.

Recognition of this last aspect leads to a simple sequential inhibition process, defined on any finite region \( A \) as follows. Consider a sequence of \( n \) events \( x_i \) in \( A \) and let \( d(x,y) \) denote the distance between two points \( x \) and \( y \). Then,

1. \( x_i \) is uniformly distributed in \( A \).
2. Given \( \{ x_i = x_i, j = 1, \ldots, i-1 \} \), \( x_i \) is uniformly distributed on the intersection of \( A \) with \( \{ y ; d(y,x_j) \geq \delta, j = 1, \ldots, i-1 \} \).

Simple sequential inhibition is parameterized most naturally by its packing intensity, \( \tau = n \pi \delta^2 / 4A \), where \( n \) is the number of events. Note that if too high a value of \( \tau \) is prescribed, the sequential procedure may terminate prematurely. The maximum attainable packing intensity is a random variable whose distributional properties appear to be largely intractable. Stoyan & Stoyan (1985) modified Matern’s two models by considering random hard-core distances. Fig 2-10 shows an example of the simple inhibition process.

2.3.6 Thinned Process

A thinned point process is defined by a primary point process \( \{ N_\circ(dx) \} \) and a thinning field \( \{ Z(x) \} \), which is a stochastic process with realized values \( 0 \leq Z(x) \leq 1 \) for all \( x \). Given realization of \( \{ N_\circ(dx) \} \) and \( \{ Z(x) \} \), the events \( x_i \) of \( \{ N_\circ(dx) \} \) are retained, independently, with probabilities \( z(x_i) \), and the corresponding realization of the thinned point process \( \{ N(dx) \} \) consists of the retained events of \( \{ N_\circ(dx) \} \).

The second-order properties of \( \{ N(dx) \} \) are obtainable from those of \( \{ N_\circ(dx) \} \) and \( \{ Z(x) \} \). In particular, in the stationary, isotropic case, let \( p \) and \( \gamma(t) \) denote the mean and covariance function of \( \{ Z(x) \} \). Then the second-order intensity function of \( \{ N(dx) \} \) is

\[ \lambda_\circ(t) = \lambda_\circ(t) \{ \gamma(t) + \rho^2 \} \]  \hspace{1cm} (2.27)
Figure 2-10: An Example of the Simple Inhibition Process

where $\lambda_{02}(t)$ is the corresponding second-order intensity function of $\{ N_0(dx) \}$. The $K$-function of $\{ N(dx) \}$ and $\{ N_0(dx) \}$ are therefore related by

$$K(t) = K_0(t) + \int_0^t \gamma(u)K'_0(u)du$$  

(2.28)

a pattern of the thinned process is shown in Fig. 2-11.

Stoyan (1979) showed the relationship between the thinned process and the Cox process and derived radial distribution functions for various cases.
2.3.7 Soft-core Process

If there is an interaction between the points, then it may not be "hard" or abruptly ending, but "soft" or continuously decreasing. In contrast to the hard-core process (i.e., inhibition process), the interaction of points is such that the inhibitory forces increase continuously with decreasing inter-point distance (Stoyan, 1987). Let a stationary Poisson process of intensity \( \lambda_n \) be given. Its points get independent marks which are distributed
according to the continuous distribution function $F$ on $[0, \infty)$. Then the initial point process is thinned as follows. A point $x$ with mark $r(x)$ survives if and only if there is no other point $y$ with mark $r(y)$ such that:

1. $x$ is in the circle centered at $y$ with radius $r(y)$
2. $r(y) > r(x)$.

If $F$ is discontinuous, then the points get an additional independent mark $u$ uniformly distributed on $[0, 1]$. If $r(x) = r(y)$, then condition 2. alone is replaced by $u(x) \geq u(y)$. Of course, the thinned point process is stationary and isotropic. Its intensity $\lambda$ is given by

$$\lambda = \lambda_b p_r$$

(2.29)

where $p_r$ is the probability that a point of the basic Poisson process is retained. It satisfies

$$p_r = \int_0^1 \exp(-\lambda_b V(s)) ds$$

(2.30)

Here $V(s)$ is the volume of the three-dimensional set $M(0,s)$, where for $x = (x_1, x_2)$

$$M(x,s) = \left\{ (\xi, \eta, \zeta) ; \sqrt{(\xi-x_1)^2 + (\eta-x_2)^2} \leq F^{-1}(\zeta), \right\}$$

(2.31)

$F^{-1}$ denotes the inverse of $F$. The product-density $\rho(r) = \lambda^2 g(r)$ is given by

$$\rho(r) = \lambda_b^2 k(r)$$

(2.32)

where $k(r)$ is the probability that both members of a point-pair of distance $r$ of the basic Poisson Process are retained. It satisfies

$$k(r) = 2 \int_0^1 \int_{T(s)} \exp(-\lambda_b V(r,s,t)) dt ds$$

(2.33)

Here $V(r,s,t)$ is the volume of $M(0,s) \cup M(r,t)$, where $r = (r_1, r_2)$ is a point of $\mathbb{R}^2$ with distance $r$ to origin 0. $T(s)$ is the set of all real numbers $t$ in $[s,1]$ with $(r_1, r_2, t) \notin M(0,s)$.

Fig. 2-12 is a pattern of beadlet anemones on a rock in which the positions and sizes of the anemones are indicated. Stoyan (1987) used a soft-core process when he analyzed this pattern.
Figure 2-12: An Example of the Soft-core Process, After Upton & Fingleton (1985)

2.3.8 Discussion

We showed that our mapped pattern (Fig. 2-1) does not follow the CSR assumption using both the inter-event distance test and the nearest neighbor distance test. We checked it with the second moment measure and it also shows rejection of a CSR. To find a proper model, we discussed various possible point process models, including a homogeneous Poisson point process model, which can be applied to rock fracture modelling.

If we think of the large scale, the rock fracture model based on point processes can be perceived as stationary (Ripley, 1988). In such a case, CSR would probably satisfied. However, if we confine our interests to a rather small portion of the rock surface, the
pattern is probably nonstationary. It is in situations like this where the one of the nonhomogeneous models, which were discussed above, will have to be used. We will now examine if some of these models can represent patterns such as traces in Fig. 2-1.
Chapter 3
APPLICATION OF POINT PROCESSES TO ROCK FRACTURE MODELING

3.1 Introduction

Many fracture modeling studies choose a homogeneous Poisson point process as a basic assumption when generating mid-point patterns. In Chapter 2, we tested the randomness of the mapped pattern (Fig. 2-1) with three methods and concluded that it did not follow the homogeneous Poisson point process. Hence, we need to find an alternative model which has non-random properties. In our case, the inhomogeneous Poisson point processes and doubly stochastic point (Cox) processes seem to be suitable alternatives even though no clear rule exists as to which of these alternatives should be chosen.

In the following, we will consider the above mentioned two alternative models and discuss fitting the proposed model to the mapped data.

3.2 Inhomogeneous Poisson Point Model

As discussed in detail in Section 2.3.2, when using the inhomogeneous Poisson process, we should know an intensity function \( \lambda(x,y) \) in advance. If we have an intensity function \( \lambda(x,y) \), we can generate a point pattern with an Acceptance / Rejection (A/R) scheme as proposed by Lewis & Schedler (1979). This A/R scheme consists of simulating a Poisson process on a region \( A \) with intensity \( \lambda_o \), equal to the maximum value of \( \lambda(x) \) within \( A \) and retaining an event at \( x \) with probability \( \lambda(x)/\lambda_o \). The A/R scheme plays an important role when we generate a point pattern, especially a non-homogeneous pattern.

Fig. 3-1 is a midpoint representation of our trace map of Fig. 2-1. Considering such a
pattern, it is hard to find the intensity function \( \lambda(x,y) \). With regression, we can only find the y-directional intensity function like \( \lambda(y) = \exp(0.36y - 0.028y^2 - 3.25) \) since significant clustering as well as inhibitory behavior exists simultaneously in the x-direction. To derive an x-directional intensity function, we use a kernel function \( f \). Usually \( f \) is a symmetric probability density function, for example the normal density function, with the conditions,

\[
\begin{align*}
    f(x) &= f(-x) \\
    \int_{-\infty}^{\infty} f(x) \, dx &= 1 \\
    \int_{-\infty}^{\infty} f(x) x^2 \, dx &= 1
\end{align*}
\]

(3.1)

The kernel estimator with kernel function \( f \) is defined by

\[
\hat{\lambda}(x) = \frac{1}{nh} \sum_{i=1}^{n} f\left(\frac{x-X_i}{h}\right)
\]

(3.2)

where \( h \) is the window width and often called the smoothing parameter or bandwidth.

The intuitive properties are as follows. Just as the moving average (see Silverman (1986) for details) can be considered as the height of the histogram (i.e., frequency) centered at the observation, the kernel estimator is a sum of bumps, where the window width \( h \) determines their width. This property of the kernel function, therefore, makes an intensity variation easy to derive. Fig. 3-2 shows a schematic representation of the kernel estimates. Provided the kernel \( f \) is everywhere non-negative and satisfies the condition, Eqn (3.1), it will follow at once from the definition that \( \hat{\lambda} \) will itself be a probability density. Furthermore, \( \hat{\lambda} \) will inherit all the continuity and differentiability properties of the kernel \( f \), so that if \( f \) is the normal density function, then \( \hat{\lambda} \) will be a smooth curve having derivatives of all orders. Particularly, one of the edge-corrected kernel estimators of \( \lambda(r) \) with efficient kernel function \( f_\delta \) is (see Fiksel, 1988)

\[
\hat{\lambda}_\delta(r) = \sum_{x \neq y} \frac{f_\delta(|x-y|-r)}{2\pi|x-y|a(W_x \cap W_y)}
\]

(3.3)
Figure 3-1: Mid Point Representation of Traces of Florence Lake Outcrop
Figure 3-2: An Example of Schematic Kernel Estimate with Window Width 0.4

where, the summation goes over all points $x$ and $y$ of the pattern in the window of observation $W$. $||$ is the Euclidean distance; $f_\delta$ is a kernel function; $W_x$ is a set of all points $z$ of the plane having the form $z = w + x$, $w \in W$; $a$ = area; $\delta$ is a band-width parameter; $r$ is a distance between two points and

$$f_\delta(r) = \frac{3}{4\sqrt{5} \delta} \left(1 - \frac{r^2}{5\delta^2}\right), \quad -\delta\sqrt{5} \leq r \leq \delta\sqrt{5}$$

$$0 \quad \text{otherwise} \quad (3.4)$$

The above kernel function is a general form of the Epanechnikov kernel (Epanechnikov, 1969). Though efficient, it is not easy to implement. However, a kernel estimator using normal distribution function provides a good estimate of the $x$-directional intensity function $\lambda(x)$.

We now have the intensity function both in the $x$- and $y$- direction. Hence, we can
Figure 3-3: Kernel Estimator using Normal Distribution Function with Standard Deviation = 1.0
simulate the inhomogeneous Poisson point pattern with the aforementioned A/R scheme. Fig. 3-4 is one of the simulations of the inhomogeneous Poisson point process assuming a normal distribution as a kernel function. Comparing this simulated inhomogeneous Poisson point pattern with our mapped pattern (Fig. 3-1), we see some similarity. To check the inhomogeneous Poisson point process with our mapped data, we use the inter-event distance, nearest neighbor distance and second moment procedure (see Fig. 3-5 to Fig. 3-7). From these figures, we see good correspondence of our mapped pattern with the inhomogeneous Poisson point process. Again Monte-Carlo tests for each the above three procedures show satisfactory results, also.

3.3 Marked (or Weighted) Point Process

Before proceeding to the doubly stochastic point process, we introduce the marked (or weighted) point process. Fig. 3-8 illustrates an example of a marked point process where a point can be further characterized either by an $\times$-mark or by an o-mark. Since a point process itself does not give us any information on trace lengths or orientations, a marked point process can be used when considering additional characteristics of fractured rock if these are not too complicated. These characteristics can be included if we give marks or weights to each point assuming that the mark represents a trace length or orientation.

Considering mark-correlation functions and the cross-mark-correlation functions, which are similar to the second-moment properties, we can obtain information on marks such as trace length correlations and orientation correlations. Assume that there are two marks for each point such as the $l$-mark for a trace length and the $m$-mark for an orientation. Now calculate the mean of the product of the $l$-mark of the point in one of the infinitesimal areas and of the $m$-mark of that in the other one. This mean, denoted by $E_{lm}(r)$, is written as,
Figure 3-4: Inhomogeneous Poisson Point Pattern
Figure 3-5: Inter-event Analysis of Inhomogeneous Poisson Point Process
Figure 3-6: Nearest Neighbor Distance Analysis of Inhomogeneous Poisson Point Process
Figure 3-7: Second-moment Analysis of Inhomogeneous Poisson Point Process
Figure 3-8: An Example of the Marked (Weighted) Point Process

\[ E_{lm}(r) = k_{lm}(r) \lambda^2 g(r) dF_1 dF_2 \]  

(3.5)

where, \( k_{lm} \), cross-mark-correlation function, is a conditional mean of the product of the \( l \)-mark and \( m \)-mark of the members of a point-pair of distance \( r \); \( \lambda \) is intensity; \( dF_1 \) and \( dF_2 \) are infinitesimal areas; the radial distribution function, \( g(r) \), is related to the second-order property \( K \) by

\[ g(r) = \frac{d}{dr} K(r) / 2\pi r \]  

(3.6)

The definition of mark-correlation functions \( k_{ll} \) and \( k_{mm} \) is analogous. The statistical determination of \( k_{lm}(r) \) is as follows.

1. Estimate the edge-corrected quantity \( \rho_{lm}(r) \) given by
\[
\rho_{lm}(r) = k_{lm}(r) \rho(r)
\]
\[
\hat{\rho}_{lm}(r) = \sum_{[x_j \in W_x]} \sum_{[y_j \in W_y]} \frac{f_\delta(|x-y|-r) l(x)m(y)}{2\pi|x-y|a(W_x \cap W_y)}
\] (3.7)

here, \(l(x)\) is the \(l\)-mark of \(x\), and \(m(y)\) is the \(m\)-mark of \(y\).

2. Compute \(\hat{k}_{lm}(r)\) by

\[
\hat{k}_{lm}(r) = \frac{\hat{\rho}_{lm}(r)}{\hat{\rho}_s(r)}
\] (3.8)

here, \(\hat{\rho}_s(r)\) is the same as the edge-corrected second-moment and can be calculated using kernel estimator.

3. Transformed correlation functions are

\[
g_{il}(r) = k_{il}(r)/\bar{\rho},
\]
\[
g_{mm}(r) = k_{mm}(r)/\bar{m}^2,
\]
\[
g_{lm}(r) = k_{lm}(r)/\bar{m}
\] (3.9)

where, \(\bar{\cdot}\) denotes a mean \(l\)-mark and so on. These functions tend towards 1 if \(r\) goes to infinity.

Fig. 3-9 shows trace length correlations of our mapped pattern. As can be seen, this function approaches 1 as distance increases and has an approximate value 1.8 near the distance 0.2. From this result, we conclude that traces are heavily correlated with each other when they get closer, and at greater distance, they become independent.

3.4 Doubly Stochastic Point Model

3.4.1 Doubly Stochastic Point Process

In this section, we use the doubly stochastic (Cox) point process for our rock fracture model. The original idea of the Cox process (see Section 2.3.4) is as follows,

1. \(\{\Lambda(x) : x \in \mathbb{R}^2\}\) is a non-negative-valued stochastic process.

2. Conditional on \(\{\Lambda(x) : x \in \mathbb{R}^2\}\), the events form an inhomogeneous Poisson process with intensity function \(\lambda(x)\).

The point process is stationary if and only if the intensity process \(\{\Lambda(x)\}\) is stationary, and it is isotropic if and only if the intensity process is isotropic. A convenient
Figure 3-9: Correlations of Trace Length
and expressive terminology is to refer to the Cox process driven by \{ \Lambda(x) \}. First- and second-order properties are obtained from those of the inhomogeneous Poisson process by taking expectations with respect to \{ \Lambda(x) \}. Thus, in the stationary case, the intensity is,
\[ \lambda = E[\Lambda(x)] \]  
(3.10)
Also the conditional intensity of a pair of events at \( x \) and \( y \), given \{ \Lambda(x) \}, is \( \Lambda(x)\Lambda(y) \), so that
\[ \lambda_2 = E[\Lambda(x)\Lambda(y)] \]  
(3.11)
In the stationary, isotropic case this can be written as
\[ \lambda_2(x,y) = \lambda^2 + \gamma(t) \]  
(3.12)
where
\[ \gamma(t) = \text{Cov} \{ \Lambda(x), \Lambda(y) \} \]  
(3.13)
and \( t \) is the distance between \( x \) and \( y \), \( \gamma(t) \) is the covariance density of the point process.

A simple example of the Cox process is to use the bivariate kernel function as an expansion form of the univariate kernel function which was used in Section 3.2. in the inhomogeneous point process. Assume \( h(.) \) is a bivariate probability density function and define an intensity process \{ \Lambda(x) \} such that
\[ \Lambda(x) = \mu \sum_{i=1}^{\infty} h(x - X_i) \]  
(3.14)
for a non-negative value \( \mu \), where the \( X_i \) are the points of a Poisson process. One of the possible ways to construct the bivariate kernel function is to use the bivariate normal distribution function. Thus the Cox process driven by Eqn (3.14) is also a kind of Poisson cluster process. The above mentioned kernel function method can replicate the local pattern of the fracture sets, but if we need to extrapolate a regional pattern from the local mapped data, this method becomes impractical since the kernel function needs actual data sets and, in addition, simulation of the pattern is limited to a region where the data sets are available.

One of the promising ways to construct a regional pattern from the mapped data is to
use the spectral density function which is frequently used in simulating the random function (see Shinozuka & Jan, 1972). We will discuss this function in the following subsection.

3.4.2 Spectral Density Function

As mentioned before, the main efforts in the doubly stochastic Poisson point process should be directed to finding the planar intensity function $\lambda(x)$. In Section 3.2, we adopted the inhomogeneous point process with one-dimensional (or univariate) kernel functions and showed the corresponding modeling process. The other way to estimate the intensity function is to use the spectral density function; this is useful if the explicit form for the intensity measures is hard to obtain.

Briefly saying, what we want to find with spectral density functions is the intensity function, $\lambda(x,y)$. We assume that the intensity function can be derived from the $x$- and $y$-directional intensity measure since the mid point pattern in Fig. 3-1 shows clustering behavior in the $x$- and $y$-direction, and we also assume that each directional intensity measure is independent of the other. Then we have,

$$\lambda(x,y)=\hat{\lambda}+\sqrt{Var}\lambda_x(x)\lambda_y(y)$$

(3.15)

where, $\hat{\lambda}$ is a mean value of the intensity measure and, in our case, the sampling average can be used. In other words, the intensity of a region is assumed to follow a homogeneous Gaussian distribution. Since the sampling average can be obtained from data, we need to enumerate the functions $\lambda_x(x)$ and $\lambda_y(y)$.

To find $\lambda_x(x)$ and $\lambda_y(y)$, we adopt the spectral density functions. Before employing spectral density functions for each direction, we calculate some additional properties.

1. Modify the second moment in order to include the directional characteristics.

The definition of the second moment measure $K(t)$ is

$$K(t)=\lambda^{-1}E[\text{no. of further events within distance } t \text{ of an arbitrary event}]$$

(3.16)

or equivalently,
\[ \lambda K(t) = \int_0^{2\pi} \int_0^t \left\{ \frac{\lambda_2(x)}{\lambda_1} \right\} dx d\theta \]
\[ = 2\pi \lambda^{-1} \int_0^t \lambda_2(x) x \, dx \]  
\hfill (3.17)

Conversely,
\[ \lambda_2(t) = \lambda^2 (2\pi t)^{-1} K'(t) \]  
\hfill (3.18)

Similarly, we can define a directional second moment measure as
\[ K(t, \theta) = \lambda^{-1} E[ \text{no. of further events within distance } t \text{ and within angle } \theta \text{ of an arbitrary event} ] \]  
\hfill (3.19)

or
\[ \lambda_2(t, \theta) = \lambda^2 (2\theta t)^{-1} K'(t, \theta) \]  
\hfill (3.20)

In our case, \( \theta \) is measured between the line connecting two selected points and the x-axis, and is assumed to vary within 0 to 10 deg. for the x-direction and 90 to 100 deg. for the y-direction. Fig. 3-10 shows the directional second-moment measure for the x- and y-directions, as well as the \( K(t) \) values of the mapped pattern and the homogeneous, isotropic case. From Fig. 3-10, we see the clustering behavior in the y-direction and short-range clustering behavior in the x-direction since the second moments of these two cases are greater than that of the homogeneous, isotropic case. However, as relative distance between points increases, we can see the inhibitory behavior along the x-direction. This behavior can be seen in Fig. 3-10, also.

2. Find directional covariance density for each direction. A directional covariance density function can be defined as
\[ \gamma(t, \theta) = \lambda_2(t, \theta) - \lambda^2 \]  
\hfill (3.21)

Fig. 3-11 shows the calculated covariance densities for each direction. For comparison, a covariance density for the non-directional mapped case is also plotted.

3. Calculate correlation function \( \rho(t, \Theta) \) from
\[ \gamma(t, \theta) = \rho(t, \theta) \gamma(0) \]  
\hfill (3.22)

where \( \gamma(0) \) is a variance of the intensity measure. If the pattern is homogeneous and isotropic, this value becomes \( \gamma(0) = \lambda(1 - \lambda) \).

4. For later use, derive an exponential form for the correlation function. From curve fitting, we estimate the x-directional correlation function as \( \exp(-0.33x) \) and that for the y-direction as \( \exp(-0.1y) \). Fig. 3-12 shows the exponential curve fitting form of the correlation function. Since \( t \) is a distance between two points (in our case, a point represents the mid point of a trace) and the correlation function \( \rho \) is an even function with respect to \( t \),
\[ \rho(t, \theta) = \rho(-t, \theta) \]  
\hfill (3.23)

In the general case, we assume that the \( n \)-fold Fourier transform of \( \rho(t) \) exists
Figure 3-10: Directional Second Moment of Mapped Data
Figure 3-11: Covariance Densities for Each Case
Figure 3-12: Correlation Function for X and Y Direction
(Shinozuka & Jan, 1972). The spectral density function, $S(\omega)$, of the intensity measure is then defined as

$$S(\omega) = \frac{1}{(2\pi)^n} \int_{-\infty}^{\infty} \rho(t) e^{-i\omega t} \, dt \tag{3.24}$$

where, $\omega$ is the frequency (wave number) vector and $\omega \cdot t$ is the inner product of $\omega$ and $t$. Then from the properties of Eqn (3.23),

$$\int_{-\infty}^{\infty} \rho(t) \sin(\omega \cdot t) \, dt = 0 \tag{3.25}$$

and from Eqn (3.24),

$$S(\omega) = S(-\omega) \tag{3.26}$$

Now, if we confine our interests to a mapped pattern, $n$ becomes 1, $S(\omega)$ becomes $S_x(\omega)$ in the $x$-direction and $S_y(\omega)$ in the $y$-direction. Then $x$-directional spectral density function, $S_x(\omega)$, becomes

$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_x(t) \cos(\omega \cdot t) \, dt \tag{3.27}$$

and $y$-directional spectral density function, $S_y(\omega)$, is

$$S_y(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \rho_y(t) \cos(\omega \cdot t) \, dt \tag{3.28}$$

and is real.

Since we have an exponential form of the correlation function, we can obtain a simplified form of Eqn (3.27) and Eqn (3.28). That is, if $\rho_x(t) = e^{-at}$, then $S_x(\omega) = a/\pi(1+\omega^2a^2)$. Our spectral density functions according to this form are plotted in Fig. 3-13 and Fig. 3-14.

As a result, we can estimate the $x$- and $y$-directional intensities as follows.

1. Consider the $x$-directional intensity function, $\lambda_x(t)$, and assume that the spectral density function becomes negligible outside the range $(-\tilde{\omega}, \tilde{\omega})$ ($\tilde{\omega}$ is a cut-off frequency)
Figure 3-13: X-dir. Spectral Density Function
Figure 3-14: Y-dir. Spectral Density Function
2. Divide the area under the spectral density function into small areas \( S_x(\omega) \Delta \omega \) at equal frequency intervals \( \frac{2\bar{\omega}}{N} \).

3. Now define the x-directional intensity function, \( \lambda_x(t) \), as (Augusti, et al., 1984)

\[
\lambda_x(t) = \sum_{i=0}^{N} \left[ 2S_x(-\bar{\omega} + i\Delta\omega) \Delta\omega \right]^{1/2} \cos \left[ (-\bar{\omega} + i\Delta\omega)t + \phi_i \right]
\] (3.29)

Here, if angle \( \phi_i \) is a realization of a random phase angle \( \bar{\phi}_i \), uniformly distributed over \((0,2\pi)\), Eqn (3.29) becomes an ergodic periodic random function \( \hat{\lambda}_x(t) \), with period \( T = \pi / \bar{\omega} \).

Fig. 3-15 shows the schematic procedure to calculate the spectral density function.

In our case, we assume that the cut-off frequency is \( \pi/2 \) and the number of subdivision \( N \) is 20. The same procedure can be applied to the y-directional intensity function \( \lambda_y(t) \). With the aforementioned procedure, we can find the intensity function, Eqn (3.15), in a region which is a realization of the Cox process.

Simulation of the Cox process driven by the spectral density functions can again be done with the Acceptance / Rejection scheme which was discussed in Section 3.2. But in this case, the A / R process should be used twice for the x- and y-direction (two-phase A/R scheme) because the directional intensity function derived from the spectral density function may have negative values. One of the simulated patterns is shown in Fig. 3-16 where we can see the clustering behavior of the simulated point pattern which is similar to that in the original pattern (see Fig. 3-1). Again, the second-moment analysis as well as the Monte-Carlo test show that the Cox model fits our mapped pattern (Fig. 3-17).
3.5 Conclusion

We conclude from the above results that:

1. Our mapped pattern (Fig. 2-1) does not follow a homogeneous Poisson point pattern even though our case has a rather simple fracture pattern.

2. An inhomogeneous Poisson point process driven by a kernel function method is proposed as a model for our mapped pattern. However, the kernel functions summarize all the variations on the x-axis and, therefore, intensity variations in the y-direction cannot be expressed.
Figure 3-16: Mid Point Pattern Using Cox Process
Figure 3-17: Second Moment Analysis of Cox Process
3. Nevertheless, using a kernel function with normal distributions, shows the regional inhibitory behavior in the x-direction. In the general case, one could further expand the kernel function to two-dimensional bivariate kernel functions; this is not done because the Cox process (see 5 below) provides a better approach.

4. Trace lengths are correlated as they get closer. That is to say, a clustering mechanism among traces can be predicted.

5. With the doubly stochastic (Cox) process, we are using a directional second-moment measure and find the directional correlation function. Also, we adopt a spectral density function with which we are able to transform the random function into the frequency domain. From the assumption of the stationary Gaussian random distribution, we derive the intensity distribution and perform simulations using a two-phase A/R process. The resulting doubly stochastic point pattern also shows a directional distribution. The doubly stochastic point process model fits the mapped pattern with satisfactory goodness of fit both using the second-moment analysis and using the Monte-Carlo test.

6. As just stated, using a doubly stochastic process, we transform the intensity measure into frequency domain and derive a spectral density function. This means that, once we have a spectral density function for a particular area, we can directly reproduce a similar pattern in a larger area. This is an important advantage of the Cox process compared to the inhomogeneous Poisson point process.

7. From the point of view of rock fracture modelling, we prefer the doubly stochastic point process model to the inhomogeneous Poisson point process model. This is so because the doubly stochastic point process model is based on the correlation measures of the mapped data while the inhomogeneous Poisson point process model is not.
Chapter 4

FIBER (or LINE-SEGMENT) PROCESSES

4.1 Introduction

As described in Chapter 1, the fiber process is required for fracture geometry modelling if the traces have complex inter-actions such as intersections. In this chapter, we only consider, for the time being, two dimensional cases. Applications of the fiber processes to two-dimensional planar rock surfaces can be deduced from the three-dimensional Baecher model (Baecher, et al., 1977) or can be found in Long, et al. (1982). However, these models assume that the mid point of each trace follows a homogeneous Poisson point process with a lognormal or an exponential trace length distribution and with a uniform (or uni-directional) orientation distribution. In some cases, these assumptions are not appropriate and some modified models have been developed. In the following, we briefly introduce these recent developments in modeling of rock fractures in two dimensions and discuss their feasibility. Also, a new rock fracture modeling scheme which accounts for sequential generation of fractures is proposed and a new method to evaluate the trace length distribution function is considered.

4.2 Recent Trend in Fiber Modeling

The following developments of two dimensional fracture configuration modeling have taken place in the recent past:
4.2.1 Homogeneous Poisson Fiber Process (Baecher) Model

This model is identical to that of the homogeneous Poisson point process model except that it includes fiber length distribution and orientation distribution functions. Assuming more than one fracture set, each set of fractures is generated independently using a homogeneous, isotropic Poisson point process. Then the individual sets are superimposed. The location of each fracture in a set is designated by assuming that the centers of the fractures are randomly distributed (Poisson type) within the generation region. For each set, a density is supplied to determine the total number of fracture centers to be generated. The orientation of each fracture in a set is given next. The length of each fracture is chosen independently. Fracture lengths within a set are assumed to be distributed lognormally or exponentially. As a mark or weight, apertures can be assigned to each fracture with assumed distributions (e.g., lognormal distribution). Fig. 4-1 shows schematic procedures for constructing the homogeneous Poisson fiber process.

Some mathematical characteristics for the homogeneous Poisson fiber process can be found for each fracture set (see, Parker & Cowan, 1976, Cowan, 1979 and Stoyan, et al., 1987);

1. Intensity of fibers \( L_A \): The total length \( \Phi(w) \) of all fiber pieces in a plane W can be measured easily if an image-analyzer is employed, or if the fibers are all segments or straight lines. An unbiased estimator for \( L_A \) is given by

\[
\hat{L}_A = \frac{\Phi(w)}{\nu_2(w)} \tag{4.1}
\]

where, \( \nu_2 \) is a Lebesgue measure on \( \mathbb{R}^2 \). Otherwise, intersections with sampling lines or circles can be used.

2. Second-order Characteristics \( K(t) \): Define \( L_A K(t, \alpha) \) analogous to that of a point process. \( L_A K(t, \alpha) \) is the mean total fiber length in a sector with radius \( t \) and angle \( \alpha \)
centered at a typical fiber point. If the fiber process is motion-invariant, the typical fiber point can be explained by Palm distribution (Stoyan, et al., 1987). The second-order characteristics of the motion-invariant Poisson fiber processes are defined as

\[ K(t) = L_A E[\text{total lengths of further fibers within distance } t \text{ of a typical fiber point}] \]  

and, especially,

- If we consider the Poisson infinite line process with assumed fiber intensity, \( L_A \),

\[ K(r) = \pi r^2 + 2r/L_A, \quad r \geq 0 \]  

This sum has two main components; \( 2r/L_A \) is the contribution from the infinite line containing the typical point and \( \pi r^2 \) is formed from the remainder of the process.
If we consider the Poisson constant unit segment process,

\[ K(r) = \pi r^2 + 2r(1-\frac{r}{2})/L_A \quad \text{if} \quad r \leq 1 \]
\[ = \pi r^2 + 1/L_A \quad \text{if} \quad r > 1 \]  \hfill (4.4)

If we consider the Poisson segment process where segments have random lengths,

\[ L_A K(r) = L_A \pi r^2 + \int_0^r x dF_0(x) + \int_r^\infty (2r - \frac{r^2}{x^2}) dF_0(x), \quad r \geq 0 \]  \hfill (4.5)

where

\[ F_0(x) = \int_0^x z dF(z)/m, \quad x \geq 0 \]

\[ F(z); \text{ length distribution function} \]
\[ m; \text{ mean length} \]  \hfill (4.6)

Even though the above cases have explicit forms, applications of these characteristics to mapped data are not easy since, from a practical point of view, the typical fiber point lies at random on a fiber. Hence to test the homogeneity of given mapped data is almost impossible. Here, the meaning of the typical fiber point is different from that of the mid point in that the typical point is an arbitrary point in a fiber and is only used when calculating the second-order characteristics, whereas the mid point is a fixed point which becomes a center of a fracture. Stoyan, et al. (1987) suggested approximate test schemes by introducing a test system of parallel lines or of a Poisson line process. However, this scan line method is also impractical because the spacing of the test system or the fiber intensity of the Poisson line process influence the second-order characteristics of mapped data. This particular property of the fiber processes makes a homogeneous Poisson fiber process model hard to check against mapped data.

In rock fracture modeling, Some modifications appeared after the original homogeneous Poisson fiber (two-dimensional Baecher) model. Dershowitz (1988) showed an Enhanced Baecher Model by using a termination probability. If the traces intersect the pre-existing traces, these latter traces are terminated at the intersection point
with given probability. The termination probability is a ratio of the termination points and the total number of traces in a map. Dershowitz also suggested a non-circular fracture shape such as polygons and ellipses. Fig. 4-2 shows an Enhanced Baecher model with non-circular fracture shape.

Figure 4-2: Enhanced Baecher Model
4.2.2 Parent - Daughter Model (Witherspoon and Long, 1987)

Witherspoon & Long (1987) chose a Poisson cluster process (see Section 2.3.3.) when they modeled fractures in swarms or zones. They worked in three dimensional space assuming disk-shaped fractures, but a two dimensional parent-daughter model can be deduced from their work. The location of the parents may be purely random, i.e., a fixed rate Poisson process, or it may be a regional variation in the density of the parents. Once the parents have been determined, the daughters are found at some distances from the parents with certain distribution (e.g., Gaussian). Fig. 4-3 is a three dimensional parent-daughter model. The implementation of the model requires knowledge of the distribution of the daughters per parent and the daughters around the parents as well as the density of the parents.

There are, however, certain problems when using this model. First, several swarms may overlap. Thus, it will be uncertain as to which parent a trace daughter belongs. Hence, dividing the overall density into that of parents and daughters becomes ambiguous. Second, the parent - daughter model, in most cases, shows clustering behavior near the parents and inhibitory behavior at greater distances from the parents according to a specified distribution form of daughters. Thus a fitting procedure for a model is not easy to apply.

4.2.3 Fractal Model

The usual method when considering fractal geometry is to use the fractal dimension as a main tool. If the given data follow linear relations between the number of data and the sampling size in log-log plot, the fracture network is a self-similar fractal and the fractal dimension is the value of slope in log-log plot whether the data are the trace lengths (see Barton & Larson, 1985) or the polygons made by trace intersections (La Pointe, 1988). But, the fractal dimension itself does not give us any information on the model and there is little progress in modeling rock fractures using fractal geometry.
Figure 4-3: Parent-Daughter Model in Space, after Chiles (1988)

Chiles (1988) suggested one- and two-dimensional simulation methods with fractal geometry. But, as he mentioned, it is not clear whether the simulation procedure is related to the fractal dimension or not.
4.2.4 Branching Model (Watanabe, 1986)

This model is probabilistic rather than stochastic as far as generating the branches is concerned. The main idea is to calculate the branching probabilities at each branching point and check the results against the total existing length. As seen in Fig. 4-4, each fracture is divided into many branches of constant length $L_0$. Then, step numbers are assigned at each branch. Using branching probabilities $P_n$, clustered fractures are generated.

Figure 4-4: Branching Model of Watanabe (1986)
4.2.5 Hierarchical Model (Conrad & Jacquin, 1975)

Conrad & Jacquin (1975) used the terminology 'hierarchical' when they simulate fractures by using Poisson-type polygons and Boolean-type fissures (see also Serra, 1980). Their hierarchical model which was applied in rock fracture modeling is used to explain the sequential generation of a Boolean model in a domain. The concept is as follows.

1. The major network, which we name the first set, is a network of Poisson straight lines (or the realization of the Poisson line process) of density $\lambda_1(d\alpha)$ determining convex polygons: the matrix blocks. The infinite lines in Fig. 4-5 represent the matrix blocks.

2. Intersect the interior of each polygon with a Boolean diagram of segments with density $\lambda_2$ and distribution function $F(dl, d\alpha)$, i.e., in each polygon we have a realization of a single Boolean diagram of segments in which their lengths are extended to the perimeter of the polygon. In two distinct polygons the Boolean diagrams are independent and have the same characteristics. The finite segments in Fig. 4-5 are the realization of the Boolean diagram of segments.

3. The matrix block generated by the above two steps is defined by the first set, but the second set is important when describing the minor network.

4. The parameters introduced in this model are
   - density $\lambda_1(d\alpha)$ of the Poisson line process
   - density of the Poisson segment process $\lambda_2$
   - distribution function of the primary grain $F(dl, d\alpha)$ which represent the length and orientation of segments.

Fig 4-6 shows a simulated rock fractures network using the hierarchical model of Conrad & Jacquin (1975). This model differs from the Veneziano model (1979) in that the former considers polygons made by Poisson line processes as rock blocks and remaining fibers made by a Boolean diagram of segments as rock fractures, whereas the latter one, according to the persistence parameters, chooses some of the polygons made by Poisson plane processes and Poisson line processes as rock blocks.
Figure 4-5: Hierarchical model of Conrad & Jacquin (1975)

Figure 4-6: Simulated Rock Fractures Network, After Conrad Jacquin (1975)
4.2.6 Percolation Model

In percolation theory, a medium is considered as an infinite set of sites and a bond is assumed to be a path which connect certain pairs of sites. Therefore, two different approaches are possible when modeling fracture systems with percolation theory. The one is site percolation theory by Robinson (1983). In this model, segments of specified length and orientation distributions are uniformly generated in a square. By way of the trial and error method, one can find the critical value of segment intensity and length scale with which percolation is to occur. Thus we can idealize fracture systems in a plane.

The other is bond percolation by Watanabe (1986). Assume an original network composed of many sites linked together by bonds. The network is essentially defined by the number of bonds $z$ initiating from each site. Next, by taking out some of the bonds from the original network, the modified networks can be constructed. In this course of modification, the decision as to whether each bond may remain or be removed is independently established using a fixed existence probability $P_b$, where $P_b$ can be calculated from a map or can be assumed. If $P_b$ is large, many bonds remain and some large clusters linked with each other will remain.

4.2.7 Crack Tessellation Model (Gray, et al., 1976)

One of the interesting models of random tessellation is to adopt the crack model. With some modification, a non-homogeneous Poisson model can be developed. A marked point process $\Phi$ is constructed on the plane, each point of $\Phi$ being marked with the orientation of a line. The edges are produced by a growth process: each edge starts at one of the points and grows in the two opposing directions specified by the marks at a constant rate. Growth in a particular direction continues until further edges are hit. This process is reminiscent of crack growth.
4.3 Proposed Hierarchical Model

4.3.1 Background

The original hierarchical model which was applied in rock fracture modeling (Conrad & Jacquin, 1975) was used to explain the sequential generation of a Boolean model in a domain. We are now trying to modify the idea of the original hierarchical model and apply it to the rock fractures, where the rock fracture is assumed to be generated by geological sequences. When we deal with the modified model, we assume that the fracture generation sequence is known a priori. We can, therefore, model the hierarchical fracture sequence as follows.

1. We divide the fractures into several groups according to the geological history. For simplicity, we assume that there are two sets such as set 1 and set 2.

2. From the assumed or calculated intensity, orientation and trace length distribution, generate set 1 of fractures on the basis of the point process.

3. Do an independence test between set 1 and the set 2. For this, we replace a trace by a point (i.e., mid point) and introduce the bivariate point processes. If we find that these two sets have no dependent relations, ignore step 4.

4. Before generating set 2, measure the interrelations (i.e., correlations) between two sets, where each set of traces is equivalent to a set of points.

5. If there are no such correlations, set 2 is obviously independent of set 1, and the superimposition of set 2 is possible. If we find the correlations between two sets, the A/R procedure can again be used to simulate set 2. If we want to impose a termination of set 2 when it crosses set 1, we can define a termination condition at this stage.

6. Iterate step 2 to step 5 according to the number of fracture sets.

In the following, we will discuss our proposed hierarchical model in detail.

4.3.2 Independence Test

Assume that the process consists of two sets of traces classified as type 1 events and type 2 events by their geological history. If the mid points of each set are distributed independently of each other, we can superimpose traces set by set. But if there are
correlations between the two sets, we need to include these correlations into our hierarchical model. For this, we use the bivariate point process when performing the independence test.

Extension of the univariate point process (Chapter 2) to the bivariate case is relatively straightforward and will now be discussed. Define a bivariate point process which generates events classified as type \( j \) for \( j = 1, 2 \). First-order properties are determined by each intensity.

\[
\lambda_j = \lim_{|dx| \to 0} \left( \frac{E[N_j(dx)]}{|dx|} \right)
\]

and second-order properties are

\[
\lambda_{ij}(u) = \lim_{|dx||dy| \to 0} \left( \frac{E[N_j(dx)] E[N_i(dy)]}{|dx||dy|} \right)
\]

where the infinitesimal regions \( dx \) and \( dy \) are centered on points \( x \) and \( y \) a distance \( u \) apart. Stationarity and isotropy imply that \( \lambda_{ij}(u) = \lambda_{ji}(u) \).

The bivariate extension of second moment \( K(t) \) can be defined as

\[
K_{12}(t) = \lambda_2^{-1} E \left[ \text{number of type 2 events within distance } t \right. \nonumber \left. \text{of an arbitrary type 1 event} \right]
\]

(4.9)

To relate the \( K_{12}(t) \) to the corresponding \( \lambda_{12}(u) \), note that the conditional intensity of a type 2 event at a specified location a distance \( u \) from the origin, given a type 1 event at the origin, is \( \lambda_{12}(u)/\lambda_1 \). Like in the univariate case,

\[
K_{12}(t) = 2\pi (\lambda_1 \lambda_2)^{-1} \int_0^t \lambda_{12}(u) u \, du
\]

(4.10)

It follows that \( K_{12}(t) = K_{21}(t) \).

Cross-covariance densities are defined as

\[
\gamma_{12}(u) = \lambda_{12}(u) - \lambda_1 \lambda_2
\]

(4.11)

In practice, if we assume two independent components, type 1 and type 2, the dual
intensity of a type 1 event and a type 2 event at any two specified locations is \( \lambda_1 \lambda_2 \). In other words, the second-order intensity function \( \lambda_{12}(u) \) assumes a constant value \( \lambda_1 \lambda_2 \) for all distance measure \( u \), then Eqn (4.10) gives

\[
K_{12}(t) = \pi t^2
\]

and Eqn (4.11) becomes zero.

Here, independence is analogous to complete spatial randomness for a univariate pattern in that it provides a convenient reference point for the characterization of the more interesting bivariate structure. Generally speaking, we say that components 1 and 2 are positively (negatively) correlated at range \( t \), if the derivative of \( K_{12} - \pi t^2 \) with respect to \( t \) is positive (negative) (Diggle, 1983).

4.3.3 Correlation Measure

As mentioned before, if two sets of traces are not independent, correlation measures between sequential steps become important since two sets of traces are inter-correlated and, thus, there exist correlations which need to be included when using the hierarchical model. Here, we will discuss two basic ideas which can be used when calculating the correlations.

We can generate the hierarchical sequences of the two fracture sets by conditioning the mid points pattern of set 2 on the traces of set 1. This is done by using the line-kernel function method for representing traces of set 1 and by using mid points for representing set 2. For use of a line-kernel function, we divide each trace into a set of segments in which all segments have the same length. For each segment, the center point is obtained. Then we can include the length effect of a fiber process by evaluating the line-kernel function over all center points of set 1. The resulting line-kernel function can be visualized as a contour plot around the fiber which has a peak value at the mid point of the fiber. This method requires complicated numerical work when calculating the distances between the mid points of set 2 and the evaluated center points of set 1. From the measurement of the distances, we have
\[ \lambda(t|T_1) = \sum_{T_1} \int_{T_1i} h(t,l) dL(l) = C_1(\sigma)[F(t|\sigma) + C_2] \]  

(4.13)

where, \( h(t,l) \) is a line-kernel function with segment length \( l \) and parameter \( \sigma \); \( C_1(\sigma) \) and \( C_2 \) are the coefficients of the line-kernel function method. Once we have the conditional distributions for the midpoints of set 2 with respect to the fiber lengths of set 1, we can also include the orientation parameter in the line-kernel function.

The other way to measure the correlations is to modify the nearest neighbor distance method of Chapter 2. Here one uses the nearest fiber distance from the midpoints of set 2. Any position of a fiber can be chosen when calculating the nearest neighbor distance, and this flexible position of the nearest fiber method makes it possible to include the fiber length effect in a correlation measure. Measuring the nearest neighbor distance, we have

\[ \lambda(t|T_1) = C_3(\sigma)[G(t|\sigma) + C_4] \]  

(4.14)

where, \( G(t|\sigma) \) is an assumed nearest neighbor distance function with parameter \( \sigma \); \( C_3(\sigma) \) and \( C_4 \) are the coefficients of the nearest neighbor method. The parameters and coefficients of these two methods can be found if we use MLE for a given data set.

Stoyan & Ohser (1984) proposed another method to calculate the correlations between two sets of fibers and developed a mathematical correlation formula for the fiber processes which are based on the second-order characteristics. But their method uses a scan line method when choosing a typical point and also requires measurement of the intersection angle between a scan line and a fiber. Thus, estimation of the correlations is further complicated.
4.4 Trace Length Distribution Function

Before we discuss the application of the hierarchical model, we need to develop a trace length distribution. In general one knows that three sampling biases for trace lengths exist when we adopt a scan line method (Einstein and Baecher, 1983).

- **Proportional Length Bias**: Longer trace lengths have a proportionally larger probability of intersecting the line and therefore of being sampled.

- **Censoring Bias**: Many of the traces observed in the excavation run off into the rock walls or the overburden, and cannot be observed in their entirety. Since this censoring occurs with proportionally higher probability to longer traces, the sample is biased toward shorter lengths and the extreme upper tail disappears completely. If we assume exponential distributions, we have a closed form for the mean trace length using maximum likelihood.

\[
\hat{\lambda}_{ML} = \frac{\sum l_{x_i} + \sum l_{z_j}}{r}
\]

(4.15)

where, \( l_{x_i} \) is a set of completely observable traces (\( i = 1, \ldots, r \)) and \( l_{z_j} \) is a set of traces for which only one or neither end is observable (\( j = 1, \ldots, t \)).

- **Truncation Bias**: Traces shorter than some cut-off length are not recorded.

As a modification of this existing bias correction, we developed a new approach to correct the first two biases:

The following Maximum Likelihood Estimators (MLE) can be used where a trace is partitioned into three groups.

- \( u = \{ u_1, \ldots, u_a \} \): Trace lengths with both ends observable
- \( v = \{ v_1, \ldots, v_b \} \): Trace lengths with one end observable
- \( w = \{ w_1, \ldots, w_c \} \): Trace lengths with no ends observable

Now our MLE for the density function of trace length distribution \( f_x(x) \) becomes

\[
I(\{ u_i \}, \{ v_j \}, \{ w_k \} | f_x(x)) = \prod_{i=1}^{a} P[u_i|L_i]f_{u_i|x_i}(u_i) \prod_{j=1}^{b} P[v_j|L_j]f_{v_j|x_j}(v_j) \prod_{k=1}^{c} P[w_k|L_k]f_{w_k|x_k}(w_k)
\]

(4.16)

Here,
\[ P[u|L] \propto \int_0^L f_x(x)(L-x)dx \]
\[ P[v|L] \propto 2\int_0^L f_x(x)x \, dx + 2\int_L^\infty f_x(x)L \, dx \]
\[ = 2\int_0^\infty \min(x,L)f_x(x) \, dx \]
\[ P[w|L] \propto \int_L^\infty f_x(x)(x-L) \, dx \tag{4.17} \]

and
\[ f_w(u_i) \propto (L-u_i)f_x(u_i) \]
\[ f_w(v_j) \propto 1 - F_x(v_j) \]
\[ f_w(w_k) \propto 1 \tag{4.18} \]

Here, \( L \) is a window length of the map. Fig 4-7 shows schematic estimation of our MLE.

\[ \lambda(x, \xi) = \lambda_x f_x(x) \]

Figure 4-7: Schematic Estimation of Trace length
4.5 Discussion

In this Chapter, we have shown various stochastic fracture models including a homogeneous Poisson fiber process and proposed our hierarchical model in which geological history of the fracture genesis can be represented. As was predicted, the hierarchical model needs the fiber density function as well as length and orientation distribution functions for the sequential sets. We, therefore, have discussed the possible measurement statistics of the independence test and correlation measurements in order to find the fiber density functions. The main difference between this modified hierarchical model and the existing models is the realization of the correlated behavior of the sequential sets which is not fully implemented in the existing models. In addition, we developed a new technique to measure the mean trace length by utilizing MLE.

In the next Chapter, we will apply the above mentioned model and related statistics to real mapped data and discuss the feasibility of our model.
Chapter 5

APPLICATION OF THE HIERARCHICAL MODEL

5.1 Introduction

In applying the point process for modeling of fracture patterns, we have chosen a simple mapped pattern (Fig. 2-1) and developed appropriate fracture models based on the doubly stochastic point process and based on the inhomogeneous Poisson point process (see Chapter 3 for details). On the other hand, if the mapped pattern involves complex interactions among traces or fracture termination points, the point process is not an appropriate method for modeling fracture patterns. A further step is therefore to introduce a fiber process model, and to incorporate it in a hierarchical model. Fig. 5-1 is chosen as the mapped pattern (see Barton & Larson, 1985) to be modeled. It shows intensely fractured zones and complex interrelations among traces. Fig. 5-2 is a digitized trace map of Fig. 5-1 without the intensely fractured zones (shaded in Fig. 5-1). For the use of the hierarchical model, we divide the pattern in Figs. 5-1 and 5-2 into two characteristic sets. Set 1 is the group of fractures in which all traces have uni-directional orientations and have no common intersection points with each other (see, Fig. 5-3), while set 2 represents the remaining traces (see, Fig. 5-4) excluding, however, the intensely fractured zones (shaded in Fig. 5-1). Incidentally, our assumption for the two sub-sets is identical to the classification used by Barton and Larson (1985) who grouped the traces according to their apparent aperture width. The mid point patterns of the traces shown in Fig. 5-2 to Fig. 5-4 are shown in Fig. 5-5 to Fig. 5-7, respectively, assuming that the half length point is identical to the mid point.
Figure 5-1: Trace Map of PA100 (From Barton & Larson, 1985)
Figure 5-2: Digitized Trace Map of PA100
Figure 5-3: Digitized Trace Map of Set 1 (PA100)
Figure 5-4: Digitized Trace Map of Set 2 (PA100)
Figure 5-5: Mid Point Map of PA100: Mid Points of All Traces
Figure 5-6: Mid Point Map of PA100: Mid Points of Set 1 Traces Only
Figure 5-7: Mid Point Map of PA100: Mid Points of Set 2 Traces Only
5.2 Step 1 - Midpoint Generation of Set 1

As can be seen in mid point map of set 1 (Fig. 5-6), both clustering and inhibitory behavior exist. Therefore, a possible alternative to a homogeneous, isotropic Poisson point pattern for set 1 might be a parent-daughter model. However, if we use the parent-daughter model, we might not be able to simultaneously represent the inhibitory and the clustered behavior, unless we use one more parameter such as inhibitory distance or we use a regionalized intensity function \( \lambda(x) \) instead of a constant value \( \lambda \) as explained before in Chapter 4. We, therefore, use the doubly stochastic (Cox) point process to model fracture set 1 with the hierarchical model.

Fig. 5-8 shows a directional second moment of set 1 in which characteristic directions of 30-40\(^\circ\) and 120-130\(^\circ\) (counter-clockwise from the East) are used. In this particular case, if we confine our interests to small distances (e.g., up to 5m in Fig. 5-8), we cannot see any significant deviations from the isotropic case which is also shown in Fig. 5-8. The isotropic case is plotted for comparison and is identical to a homogeneous Poisson point process where the second moment, \( K(t) \), is identical to \( \pi t^2 \). This special situation of non-deviation from the isotropic case is quite probable since our mapped pattern has only 39 points, and we, therefore, cannot see any interactive behavior using the second moment. It also means that it is not possible to derive a correlation function directly because the correlation function becomes zero in an isotropic case. For this particular case, we modify the intensity function as follows.

\[
\ln \lambda(x, y) = \ln \lambda_0(y) + \varepsilon(x, y) \\
- \sigma_0^2 \rho_0(\Delta y) + \sigma_\varepsilon^2 \rho_\varepsilon(r)
\]  

(5.1)

and

\[
\ln \lambda_0(y) \sim C_0(\Delta x, \Delta y) = \sigma_0^2 \rho_0(\Delta y) \\
\varepsilon(x, y) \sim C_\varepsilon(\Delta x, \Delta y) = \sigma_\varepsilon^2 \rho_\varepsilon(r) \\
r = \sqrt{\Delta x^2 + \Delta y^2}
\]  

(5.2)
Figure 5-8: Directional Second Moment of Set 1.
Characteristic directions of 30-40° and 120-130°
are used to calculate the directional second moment.
Isotropic case is also shown for comparison.
We assume that, due to the properties of a spectral density function, the mean intensity is governed only by the y direction and the clustering behavior is controlled by distance r (see Eqn (5.2)), where x and y are the transformed coordinates with the axes rotated by 40° (counter-clockwise from the East) and C(.) is an assumed covariance function.

With the Acceptance / Rejection scheme (Lewis and Shedler, 1979), we performed 50 simulations; one of the simulations is shown in Fig. 5-9 where we assume that $\sigma_0^2 = \sigma_c^2 = 0.5$, $\rho_e(t) = 0.33t$ and $\rho_0(t) = 0.5t$. The second moment analysis of set 1 shows that a good similarity between the simulated and actual (mapped) point pattern exists (Fig. 5-10). The Monte-Carlo test also shows a satisfactory result. The Cox process can thus be used in a first step of the hierarchical model, particularly where both clustering and inhibitory behavior exist at the same time.

5.3 Step 2 - Traces of Set 1

The next step is to generate the traces for the simulated mid points of set 1. This simulation uses trace length statistics corrected for biases using the maximum likelihood formulations as derived in Chapter 4. In this case, set 1 has 15 u type traces (both ends observable), 22 v type traces (one end observable) and 2 w type traces (no ends observable). For simplicity, we neglect the w type traces.

The log likelihood of each trace type and the log likelihood of all traces are shown in Fig. 5-11, while a realization of the simulation is shown in Fig. 5-12.
Figure 5-9: Simulation of Set 1 Mid Points with Cox Process
Figure 5-10: Second Moment Analysis of Set 1. Acceptance band (MIN to MAX) and average value (AVRG) obtained with model simulations are shown. The second moment of the mapped data (MAPPED) is also given for comparison.
Figure 5-11: Log-Likelihood of Mean Trace Length for Set 1, assuming Exponential Distribution
Figure 5-12: Simulation of Trace Lengths for Set 1
5.4 Step 3 - Independence Test

In step 3, dependence or independence between sets 1 and 2 has to be established. If there is no dependence between set 1 and set 2, we can generate each set by simply superimposing the traces set by set. One of two independence tests either the perturbation method or toroidal shift method can be used.

In the perturbation method, we calculate the (bivariate) second moment of the mapped pattern. We, next, perturb the mid points of set 2 by a small amount. This perturbation distance can be obtained using a random number generator. Then the second moment for the perturbed data is calculated. After a sufficient number of iterations, the significance (i.e., rank) of the second moment using the mapped pattern is estimated among those from perturbed data. Instead of perturbation, toroidal shifting of set 2 can be used as suggested by Lotwick and Silverman (1982). That is, instead of a small distance on a plane, a random distance on a toroid is used when perturbing the second set. Otherwise, The remaining procedure is the same as the perturbation method.

If the mapped pattern is composed of two independent Poisson point processes, where a Poisson process implies the stationary, isotropic behavior, the second moment measure will be as shown Fig. 5-13. For calculations in Fig. 5-13, we assume that the number of traces in set 1 is 39 and 138 in set 2. These numbers are the same as those in the mapped pattern of Fig. 5-5. As can be seen, the shifted bivariate second moment, $K(t) - \pi t^2$, is almost zero (i.e., independence) and satisfies the Monte-Carlo test since we assumed two independent Poisson point processes. However, with real mapped data, the shifted second moment is not zero and set 2 is positively correlated with set 1 (see Fig. 5-14). The Monte-Carlo test also rejects the independence assumption. From this result, we conclude that our mapped data set is neither homogeneous nor isotropic, and that it shows clustering. That is, set 2 has a tendency to get closer to set 1, or, in a rock mechanical sense, if there is a pre-existing fracture, the next one tends to be near the pre-existing fracture.
Figure 5-13: Independence Test with Simulated Homogeneous Poisson Point Process for Sets 1 and 2.

Acceptance band (MIN to MAX) and average value (AVRG) are constructed through perturbations. When the data is from an isotropic Poisson point process (ISOTROPIC), it will be inside the band.
Figure 5-14: Independence Test of Mapped Pattern. Symbols are the same as those in Fig. 5-13. The independence of the actual mapped pattern (MAPIED) is tested. It shows the dependence of set 2 on set 1 traces.
5.5 Step 4 - Generation of Set 2 Midpoints

In the preceding steps, we have found that set 2 is not independent of set 1 and that there are positive correlations between the two sets. Thus, we use correlation measures when generating the mid point pattern of set 2. For this, we propose two possible correlation measure methods namely the line-kernel function method and the nearest neighbor fiber method.

When using the line-kernel function method, we divide each trace of set 1 into a number of segments of identical length, and we measure the center point of each segment. Next, we calculate the distance between a center point of a set 1 segment and a mid point of set 2. Using a bivariate normal distribution function, we derive a two-dimensional kernel estimate which has a circular distribution. This is done for all center points; it produces the line-kernel estimates for our mapped data. To describe the relative uniformity of the pattern, we use a mixed distribution when calculating the MLE for conditional intensity:

\[ f_k(x,y|\sigma,C) = \frac{C}{A} + \frac{1-C}{n_2} \sum_{i=1}^{n_1} \int K(x,y|\sigma) dL \] (5.3)

where, \( f_k(x,y|\sigma,C) \) is a conditional density function mixing the uniform distribution and the line-kernel function, \( C \) and \( \sigma \) are parameters which can be estimated using MLE, \( A \) is an area, \( n_1 \) is the number of traces of set 1 and \( n_2 \) is the number of the mid points of set 2. The first part of the right-hand side of Eqn (5.3) comes from the uniform distribution of the pattern, and the second part is from the line-kernel function with which local clustering can be described.

Using MLE, we find \( C \) as 0.65 and \( \sigma \) as 0.75, and perform 50 simulations. One of the simulated mid point patterns is shown in Fig. 5-15. The goodness of fit is checked with two different methods. One is to use the bivariate second moment (\( K_{12} \)) with which the mid point patterns of set 1 and set 2 are compared. The bivariate second moment is shown
In Fig. 5-16; good agreement between the mapped data and the simulated pattern is obtained. The other check is based on the (univariate) second moment \( K_{22} \) in which only the mapped data and simulated patterns of set 2 are compared (Fig. 5-17). This also shows acceptance of the simulated pattern. From these results, we conclude that the conditional intensity measures driven by the line-kernel function method are satisfactory. The mixture model using the line-kernel function is thus a suitable procedure to generate the mid point pattern of set 2 correlated with set 1.

Instead of the line-kernel function method, the nearest neighbor distance method can be used as a correlation measure. The main idea is to find the nearest fiber from a mid point of set 2. The nearest fiber distance can be measured between a mid point of set 2 and an arbitrary point in a fiber of set 1. Since the nearest distance can involve any point on a fiber of set 1, this scheme makes it possible to include the fiber length effect. To find the conditional intensity function, we again use a mixed distribution, i.e.,

\[
f_n(x,y|C,\sigma) = \frac{C}{A} + \frac{1-C}{n_2} G(x,y|\sigma)
\]  

(5.4)

where \( f_n(x,y|C,\sigma) \) is a conditional intensity function with parameters \( C \) and \( \sigma \). \( G(x,y|\sigma) \) is a nearest neighbor distance distribution function which is assumed to be normally distributed. Again, we assume that the conditional intensity function is mixed with the uniform distribution. From MLE, we found \( C = 0.65 \) and \( \sigma = 0.2 \). One of the simulated mid point patterns of set 2 is shown in Fig. 5-18. As we did with the line-kernel function method, we compared the simulated patterns with mapped data. Fig. 5-19 is a bivariate second moment analysis of sets 1 and 2, and Fig. 5-20 is a (univariate) second moment analysis of set 2. These results also show an acceptable goodness of fit of the mixture model driven by the nearest neighbor fiber method.

The advantage of the nearest neighbor function method is that it only considers a point within a fiber which becomes a nearest neighbor, while the line-kernel function
Figure 5-15: One of the Simulated Mid Point Patterns of Set 2 Using the Line-Kernel Function Method to Express Correlation.
Figure 5-16: Bivariate Second Moment Analysis for Sets 1 and 2 - Mid Point Correlation Based on the Line-Kernel Function Method. Acceptance band (MIN to MAX) and average value (AVRG) are obtained with model simulations, while the mapped pattern (MAPPED) is from actual data.
Figure 5-17: (Univariate) Second Moment Analysis for Set 2 Mid Points Using the Line-Kernel Function Method. Symbols are the same as those in Fig. 5-16.
Figure 5-18: One of the Simulated Mid Point Patterns of Set 2 Using the Nearest Neighbor Function Method to Express Correlation
Figure 5-19: Bivariate Second Moment Analysis for Sets 1 and 2 - Mid Point Correlation Based on the Nearest Neighbor Function Method. Acceptance band (MIN to MAX) and average value (AVRG) are obtained with model simulations, while the mapped pattern (MAFPED) is from actual data.
Figure 5-20: (Univariate) Second Moment Analysis for Set 2 Mid Points Using the Nearest Neighbor Function Method. Symbols are the same as those in Fig. 5-19.
method requires an additional step when calculating the line-kernel function. That is, we need to calculate the kernel estimates of all the segments in a fiber.

5.6 Step 5 - Generation of Set 2 Traces

Once we have a mid point pattern of set 2, we determine the trace length and orientation for each point of set 2:

First, the mean trace length can be calculated using MLE as described before for set 1. With MLE, we estimate the mean trace length to be 2.5m assuming that there are no trace length correlations between sets 1 and 2.

To find the orientation correlations, we again consider the nearest fiber concept. Since we have assumed that the orientation of set 1 is constant (fixed at 40°, see Fig. 5-3), we calculate orientation distributions of set 2 conditional on set 1. Fig. 5-21 shows the orientation distribution of set 2 without considering the correlation measures. When we calculate the orientation distributions of set 2 as a function of the nearest neighbor distance to set 1, we find that there are no correlations between the sets. This is evident from Fig. 5-22 where the orientation distributions of set 2 obtained by the nearest neighbor distance to set 1 (1m, 2m and 3m) are plotted. Also, the distribution of Fig. 5-21 is plotted for comparison. Thus, we only need an orientation distribution function for set 2 without any conditions. For this, we applied two distribution functions which have distribution forms on a circle. One is the Von Mises distribution (Eqn (5.5)) and the other is the wrapped normal distribution (Eqn (5.6)) (see Mardia, 1972 or Fisher, et al, 1987):

\[ f(\theta) = \frac{1}{2\pi I_o(\kappa)} \exp(\kappa \cos(\theta - \alpha)), \quad 0 \leq \theta < 2\pi \]  

(5.5)

With

\[ I_o(\kappa) = 1 + \frac{(0.5 \kappa)^2}{1!1!} + \frac{(0.5 \kappa)^4}{2!2!} + \frac{(0.5 \kappa)^6}{3!3!} + \cdots \]

where, \( \kappa \) is a shape parameter and \( \alpha \) is a location parameter.
\[ f(\theta) = \left( \frac{\sigma \sqrt{2\pi}}{\sqrt{\pi}} \right)^{-1} \sum_{r=0}^{\infty} \exp\left( -\frac{(\theta - \alpha + 2r\pi)^2}{2\sigma^2} \right), \quad 0 \leq \theta < 2\pi \] (5.6)

where, \( \alpha \) is a location parameter and \( \sigma^2 \) is a shape parameter.

To fit the data, we use the bi-modal form of the orientation distribution function, i.e.,

for the Von Mises distribution,

\[
f(\theta) = \frac{C}{2\pi I_0(\kappa)} \exp(\kappa \cos(\theta - \alpha)) \\
+ \frac{1-C}{2\pi I_0(\kappa)} \exp(\kappa \cos(\theta - \alpha - \pi))
\] (5.7)

and for the wrapped normal distribution,

\[
f(\theta) = C \left( \frac{\sigma \sqrt{2\pi}}{\sqrt{\pi}} \right)^{-1} \sum_{r=0}^{\infty} \exp\left( -\frac{(\theta - \alpha + 2r\pi)^2}{2\sigma^2} \right) \\
+ (1-C) \left( \frac{\sigma \sqrt{2\pi}}{\sqrt{\pi}} \right)^{-1} \sum_{r=0}^{\infty} \exp\left( -\frac{(\theta - \alpha - \pi + 2r\pi)^2}{2\sigma^2} \right)
\] (5.8)

In our case, \( C = 0.5 \). Using MLE, we find the parameters for the Von Mises distribution to be \( \alpha = 105^\circ \) and \( \kappa = 1.9 \), and for the wrapped normal distribution \( \alpha = 105^\circ, \sigma = 0.8 \). Fig. 5-23 shows the orientation distribution forms of these two functions as well as the orientation distribution of set 2. To evaluate the appropriateness of these models relative to the mapped data, we performed a Kolmogorov-Smirnov test for each case. Both distributions fit the data at the 5% significance level. Both models can thus be used to simulate the pattern of set 2. Fig. 5-24 is a simulated pattern of set 2 with the mid point pattern obtained by conditional intensity functions (see, Section 5.4) and with the Von Mises orientation distribution. Fig. 5-25 is the same except that the orientation distribution is the wrapped normal distribution.
Figure 5-21: Orientation Distribution of Set 2 on a Circle
Figure 5-22: Orientation Distribution of Set 2 Based on a Nearest Fiber Distance of Set 1. Orientations are measured with increasing distances (1m, 2m, 3m), also Fig. 5-21 (GLOBAL = actual orientation data) is shown for comparison.
Figure 5-23: Wrapped Normal and Von Mises Orientation Distribution Fitted to Orientation Data of Set 2
Figure 5-24: Trace Pattern of Set 2 with Von Mises Orientation Distribution
Figure 5-25: Trace Pattern of Set 2 with Wrapped Normal Distribution
5.7 Step 6 - Termination

We now have the mid point pattern, mean trace length and orientation distribution of sets 1 and 2. The remaining issue is to define a termination probability among traces. Dershowitz and Einstein (1988) defined the termination probability by counting both the intersection points and the termination points among traces. An intersection point can be a crossing point or a termination point as illustrated in Fig. 5-26 (a) and (b). The Dershowitz/Einstein termination probability is simply the ratio of termination points / intersection points. However, in our mapped pattern, this method is not appropriate because traces of set 2 cross traces of set 1 at many intersection points, i.e., an intersection of a set 2 trace with a set 1 trace is not necessarily a termination. To express the termination probability more appropriately, a new definition is introduced here. All termination points and crossing points are counted except the points between the center of a set 2 trace and the farthest intersection point. Thus, in Fig. 5-26 (d), each trace of set 2 \( t_{[2]} \) has a maximum of two intersection (crossing or termination) points (A and B); it can also have only one or none. Using only these corrected intersection points, we derive termination probability as the ratio of termination points / intersection points. This procedure introduces some distortion in the distribution of set 2 trace lengths; however, the resulting bias does not appear to be serious. Fig. 5-26 illustrates the determination of this termination probability. If the pattern is not complicated, the termination probability with our new termination scheme is the same as that of Dershowitz and Einstein (1988) as in Fig. 5-26 (c), but if the pattern is complex, the new termination probability will be different (Fig. 5-26 (d)). This termination probability is used when simulating the pattern.

We have now reached the point where the modeled fracture pattern can be completed. The simulated patterns of PA100 with the Von Mises orientation distribution function is shown in Fig. 5-27 and with the wrapped normal distribution function in Fig. 5-28.
Figure 5-26: Calculation of Termination Probability
Figure 5-27: One of the Simulated Patterns of PA100 with the Von Mises Orientation Distribution Function
Figure 5-28: One of the Simulated Patterns of PA100 with the Wrapped Normal Orientation Distribution function
5.8 Concluding Comments

In applying the fiber process to model fracture trace distributions, we did the following:

1. We divide all the traces of a map into two sets according to orientation and we define sets 1 and 2.

2. A doubly stochastic point (Cox) process is used as a model for the mid point pattern of set 1.

3. With a new MLE scheme, we calculate the mean trace length for each set assuming an exponential trace length distribution.

4. Using the bivariate point process, we find that mid points of set 1 and set 2 are dependent on each other and that they are positively correlated.

5. To calculate the conditional intensity of set 2, two correlation measure methods are used. One is the line-kernel function method and the other is the nearest neighbor fiber method. Using these two schemes, we simulate the mid point pattern of set 2 (the nearest neighbor fiber method is easier to use than the line-kernel function method).

6. The Von Mises and the wrapped normal orientation distributions on a circle are proposed as models for set 2, whereas a fixed orientation of 40° is used in set 1.

7. To consider the terminations among traces a new termination probability is proposed such that a trace is terminated at the farthest end with a prescribed probability.

In Chapter 4 and 5, we proposed our hierarchical fiber model and studied its feasibility. It showed that fracture traces of several dependent or independent sets can be modelled with the hierarchical procedure. The models can represent clusters as well as all the other geometrical characteristics. The application discussed in this chapter related to only two sets but the hierarchical model can be just as easily applied to a larger number of sets.
Chapter 6

TOPOLOGICAL APPLICATION OF FRACTURE GEOMETRY
MODEL : SLOPE STABILITY ANALYSIS

6.1 Introduction

When modelling the performance of fractured rock, be that slope stability, deformation under foundations and around tunnels or flow through the rock mass, both the geometry and the mechanics need to be represented. As was shown in preceding chapters, it is now possible to represent the two dimensional stochastic geometry by the hierarchical fracture model. Also, developments toward a three dimensional hierarchical fracture modelling are under way.

What will be discussed in this Chapter is the combination of the fracture geometry model with suitable mechanical models to represent slope stability. As will be discussed in more detail below, the mechanical models for slope stability involve models representing sliding along discontinuities (fractures) and models representing the creation of new fractures in intact rock which interconnect existing fractures.

6.1.1 Previous Research on Slope Stability

Several studies on rock slope stability have been performed using stochastically generated fracture patterns and applying simplified or idealized mechanical models for fracturing of intact rock.

Glynn (1979) generated a parallel fracture pattern with the two dimensional Veneziano model (Veneziano, 1979) and used an idealized mechanical model. Specifically, stress increments both in the horizontal and vertical direction were created so that an en echelon fracturing could be reproduced. O'Reilly (1980) modified Glynn's idealized
mechanical model by introducing both tensile and shear failure mechanisms. A further step was accomplished by Shair (1981) who expanded O’Reilly’s geometric model by considering two fracture sets which are not necessarily parallel. Einstein, et al. (1983) summarized the above mentioned models and performed parametric studies on geometric and on mechanical variables.

All these models are restricted to two dimensions by assuming an infinite extension in the third direction. A solution to the three dimensional stability problems using stochastic fracture geometry was described by O’Reilly (1980); however, his application was restricted to a special case. Numerous applications dealing with three dimensional sliding failure and deterministic geometry exist (e.g., Einstein, et al., 1979; Hoek and Bray, 1981), but they are not directly related to the problem at hand.

6.1.2 Proposed Work on Slope Stability Analysis

In this preliminary study, we will first consider the fully persistent model of a slope. The fully persistent model is useful when rock block behavior dominates slope stability. Later, fracturing through intact rock and between existing fractures will be included. For simplicity, our work will be confined to two dimensional case only. Slope stability modelling will be approached as follows:

1. Geometric construction of fracture paths: Depending on the actual fracture pattern, either the homogeneous Poisson point (or fiber) process model or the hierarchical fiber process model is used to generate the network of fractures in a given two dimensional slope.

2. Kinematic updating: All the possible rock blocks (i.e., fully persistent blocks) that can be produced with the fracture pattern are simulated and those which are kinematically admissible are identified.

3. Kinetic (or mechanical) testing: Using an appropriate mechanical model, the kinetic stability of the kinematically movable blocks is determined.

4. Parametric Studies: By changing values of both the mechanical and the geometric parameters, the significance and the influence of these variables on overall behavior of the slope stability is studied.
6.2 Generation of Fracture Pattern

Generation of planar fracture patterns in a given rock slope is basically the same as that in a two dimensional planar surface. Therefore, various fiber process models can be used (see Chapter 4). Conventional models (e.g., the two dimensional Baecher model and the Dershowitz model, see Dershowitz and Einstein, 1988) used the homogeneous Poisson process model for fracture locations and an exponential or lognormal distribution of fracture lengths. Very often, in slope stability problems, fracture traces are assumed to be parallel to each other. However, in this study, a modified orientation distribution is also used; either the von Mises or the wrapped normal orientation distribution is adopted, in addition to the parallel orientation distribution (As we will discuss later, the non-parallel orientation will cause a more complicated rock block behavior. That is, the generated rock blocks consisting of one or two sets of parallel fractures have a well defined prescribed face on which sliding occurs, whereas the blocks constructed from the non-parallel orientations do not have unique sliding faces. The sliding faces can only be determined after the simulation of the particular fracture pattern is performed. Also, the kinematic admissibility becomes much more complicated to check). Finally, one can include the fracture termination probability as in Dershowitz and Einstein (1988) or by using our new concept discussed in Chapter 5. At present, this is not done here.

Once the slope boundaries and fracture network are generated, one needs to find the effective fractures. An effective fracture is defined as a fracture which can be a part (i.e., a face) of a rock block: It must, therefore, have at least two intersection points with other fractures or with the slope boundaries. In essence, effective fractures are interconnected fractures. Effective fractures are thus relevant both in slope stability problems and in flow through rock masses. One can imagine a few algorithms to find the effective fractures; we will use an iterative searching scheme with which the non-effective fractures are sequentially eliminated during iterations. As shown in Fig. 6-1, in each iteration, the non-
effective fractures are eliminated, and this in turn induces further eliminations of the fractures which are originally connected to them. Our results show that, within at most 5

Figure 6-1: Iterative Searching Scheme for Effective Fractures
(a) Initial Fracture Network
(b) Iterative Searching Scheme for Effective Fractures
iterations, all the non-effective fractures are eliminated. The elimination of the non-effective fractures is also an important step in relation to the program storage design since the effective fractures rather than total number of fractures or intersection points are used to define kinematically admissible fracture paths. As a consequence, one can improve the efficiency of the path-searching algorithm.

6.3 Kinematic Analysis

After removing the non-effective fractures and constructing all possible rock blocks, one needs to check whether a constructed fracture path is kinematically admissible or not. Since the fracture orientations (i.e., dip angles) are not necessarily parallel, one cannot predict sliding faces or the block shapes beforehand. Therefore, a systematic kinematic analysis is necessary.

6.3.1 Construction of Connectivity Matrices

A connectivity matrix is used to find all the appropriate fracture paths (or fully persistent rock blocks) among effective fractures.

Assume that the intersection points between effective fractures and slope boundaries are known. One can, then, construct a connectivity matrix by employing effective fractures and slope boundaries (i.e., slope face and free surface, see Fig. 6-2). Since all the fracture paths are composed of either face-to-face paths (paths starting and ending at the slope face) or face-to-surface paths (paths starting in slope face and ending in surface), one can consider the slope boundaries as effective fractures. For this reason, the corresponding dimension of the connectivity matrix, \( N \), is

\[
N = \text{Number of Effective fractures} + 2
\]  \hspace{1cm} (6.1)

To construct a connectivity matrix, we assume that each element of the matrix
represents either the x- or the y-coordinate of the intersection point and each row or column of the matrix represents an effective fracture or a slope boundary. Therefore, the connectivity matrix becomes a square matrix with dimension \( N \). To simplify the searching algorithm, one can assume that the first row/column is the slope face and the last row/column is the surface of the slope. One can then store the intersection points in the appropriate places of the connectivity matrix according to their relationship to the effective fractures or slope boundaries. Since an effective fracture and a slope boundary cannot intersect itself, the connectivity matrix has a zero diagonal and is symmetric. An example connectivity matrix is shown in Fig. 6-3. In Fig. 6-3 (a), effective fractures including slope boundaries are numbered such that the slope face is effective fracture 1; the surface is effective fracture 5. The remaining fractures are numbered arbitrarily. Fig. 6-3 (b) is the complete matrix for this case. For example, effective fracture 1 intersects effective fracture 2 at x-coordinate 1, and effective fracture 3 intersects effective fractures 2 as well as 4 at x-coordinates 2 and 3, respectively.

![Figure 6-2: Typical Rock Slope Configuration](image-url)
Figure 6-3: Example of a Connectivity Matrix \([C_x]\)
(a) A Simulated Effective Fracture Pattern
(b) Construction of a Connectivity Matrix (x-coordinate)
(c) Searching Scheme for a Fracture Path
6.3.2 Kinematic Consideration

Once the connectivity matrix constructed, the next step is to find the kinematically possible fracture paths using the connectivity matrix. The information on the fracture paths are the coordinates of intersection points which become vertices of the blocks. For this, either the x- or y-coordinate system can be employed in the connectivity matrix and in the searching scheme. The other x- or y-coordinates of the intersection points which are not used in the connectivity matrix can be found easily if the intersection points are numbered sequentially, and the coordinates of the intersection points are stored correspondingly.

The searching scheme for the kinematically admissible rock blocks is similar to the dynamic programming scheme used by Glynn (1979). It searches for a fracture path and kinematic admissibility of the path simultaneously. In the following, we will explain these separately for reasons of clarity:

The search for a fracture path which defines a fully persistent rock block follows the sequence of steps cited below (see also Fig. 6-3 (c)):

1. Start always at row 1 of the connectivity matrix, where row 1 represents the slope face.

2. Search for the non-zero element in row 1. If the non-zero element is encountered in the \( j \)-th column, store the information (i.e., the corresponding effective fracture and intersection coordinate) and go to row \( j \). It means that an effective fracture, designated as the \( j \)-th fracture, is intersecting the slope face at the stored value of the connectivity matrix.

3. Starting from column 1 of fracture \( j \), find the non-zero elements of the \( j \)-th row of the matrix. If the non-zero element is the same as the one stored before, neglect it since this is an intersection point which was previously found. If the column 1 has a non-zero element, store the information. It represents a face-to-face path. If the last column has a non-zero element, store it also. In this case, the corresponding fracture path will be a face-to-surface path (Recall that row/column 1 represents a slope face and row/column \( N \) represents a free surface, where \( N \) is the total number of the effective fractures including slope boundaries, see Eq. (6.1)).

4. In step 3, if the \( k \)-th column of row \( j \) has a non-zero element, go to row \( k \), i.e., fracture \( k \).

5. Iterate steps 3 and 4 for row \( k \) to find a complete fracture path. Whenever a
fracture path is detected in row $l$ and non-zero elements still exist in row $l$, go back to the previous step (i.e., row $l - 1$) and search for another non-zero element (i.e., another fracture path) until no more non-zero elements are detected (see steps 5 of Fig. 6-3(c)).

To eliminate the kinematically inadmissible fracture paths, the following assumptions are made during the searching algorithm:

1. If the relative location of the next intersection point compared to the position of the current intersection point lies in region IV (see, Fig. 6-4a), neglect the corresponding fracture path since it is kinematically inadmissible. Unless this assumption is made, a sliding rock block will penetrate into the rock mass.

2. If row $j$ of the connectivity matrix is encountered again during a path searching process, ignore the corresponding path. It means that a concave or convex sub-block within a rock block is generated and, which is kinematically unacceptable unless a breakdown of concave/convex sub-block occurs (Fig. 6-4b).

3. Concerning the face-to-face path, even if kinematically admissible, whenever the location of the starting intersection point of a path is higher (i.e., nearer to the slope surface) than that of the ending point, neglect that path (Fig. 6-4c). In this manner, the face-to-face path will be only counted once, namely going upward.

4. To eliminate a stair-shaped path (Fig. 6-4d), do not consider paths when the advancing fracture path has negative orientation (see Fig. 6-4a for symbols) and the absolute value of it, $|\beta|$, is greater than the minimum dip angle, $|\alpha|$, on which a rock block is sliding down.

6.4 Kinetic Analysis

In the preceding steps, we have found the kinematically admissible fracture paths. The remaining task is to analyze whether the kinematically admissible fracture paths are kinetically or mechanically stable or not. For this, we will use Coulomb’s shear failure criterion as a mechanical model (other models can be easily implemented).

As mentioned in the introduction, we assume that the fracture path making up a rock block is fully persistent and that the dip angles of the fractures are not necessarily the same (i.e., parallel). One cannot, therefore, predict the minimum dip angle on which the rock block is to slide until the kinematic analysis is performed. The minimum dip angle plays an
Figure 6-4: Kinematically Inadmissible Fracture Paths

important role when one predicts the block behavior. As shown in Fig. 6-5, if the rock
block is assumed to behave as a rigid body, i.e., if an intact rock failure is not allowed, there
exists only one contact face on which the block can slide. This is a special case since we
have assumed that the fracture orientations are not parallel such that a fracture which has
the minimum dip angle becomes the contact (i.e., sliding) face. In the following, we will
discuss this particular block behavior in detail.

6.4.1 Assumptions used in Stability Analysis

To solve the slope stability problem, the following assumptions are made in the
kinetic analysis:

- The problem is two dimensional, i.e., slope boundaries and fractures are
  assumed to extend infinitely in the third dimension.
Figure 6-5: Rigid Body Motion of a Typical Rock Block
Along a Face of Minimum Dip angle $\alpha$

- Failure of a rock block is assumed to occur along the fracture with the minimum dip angle. Two failure mechanisms are possible; sliding and separation along the fracture faces.
- Failure occurs when the driving force exceeds the resisting force and we will apply the Coulomb criterion in this case.
- A tensile cut-off stress is used when considering the separation mechanism.
- Water pressure, rock bolt force and seismic effects are neglected.
- Toppling is assumed to be impossible, and, therefore, rotational behavior is not considered.
- A rock block is assumed to be fully persistent and rigid such that an intact rock failure or an en echelon failure mode (i.e., failure of intact rock which connects the adjacent existing fractures) is not possible.

These assumptions are idealized representation of field conditions and future work will eliminate some of these assumptions.

6.4.2 Simplified Mechanical Model for Sliding

As explained above, sliding will only be considered on the fracture plane with the minimum dip angle (one may include the tensile cut-off stress, $\sigma_{\text{ct}}$, when considering separation between blocks). As is well known, the shearing resistance on a sliding fracture face is
\[ \tau_j = C_j + \sigma_n \tan \phi_j \]  

(6.2)

and correspondingly the resisting force, \( R_j \), is

\[ R_j = \tau_j A \]  

(6.3)

where, \( \sigma_n \) is the average normal stress which can be derived from the total block weight \( W \); \( C_j \) and \( \phi_j \) are the cohesion and the friction angle of a fractured face, respectively; \( A \) is the initial contact area of the fracture with the minimum dip angle. Since a two dimensional plane failure case is considered, the rock block is assumed to have a unit length in the third dimension.

The resisting force due to separation can also be included such that the total resisting forces on a rock block can consist of the shearing force on a sliding face and the tensile cut-off force on separating faces. The tensile cut-off force, \( R_{oj} \), which is conservatively a scalar sum of all the components along the separating faces can be written as

\[ R_{oj} = \sum_{i=1}^{N} \sigma_{oj} A_i \]  

(6.4)

where, \( N \) and \( A_i \) are the total number of separation faces and the area of the separating face of a rock block, respectively. The total resisting force is therefore,

\[ R = R_j + R_{oj} \]  

(6.5)

The resulting safety margin, SM (Einstein, et al., 1983), and the safety factor, SF, are calculated as

\[ SM = D - R \]

\[ SF = \frac{R}{D} \]  

(6.6)

where, \( D \) is the total driving force and can be calculated from Fig. 6-5 as \( D = W \sin \alpha \) along with the minimum dip angle \( \alpha \).
6.4.3 Program TRACESIM (see Appendix D for details)

To generate a fracture pattern in a slope, which does not necessarily follow a homogeneous Poisson fracture process, and to perform the kinematic as well as the kinetic analysis of the fully persistent rock block model, a new computer code, TRACESIM, was developed. The main features of the program TRACESIM are:

1. Up to 10 effective fractures can be included in one rock block; at present, a rock block which has more than 10 faces is not considered. However, modification for more complex block patterns which can include more than 10 effective fractures is possible.

2. Both the homogeneous and the non-homogeneous Poisson fracture pattern can be simulated. However, among non-homogeneous fracture patterns, only the Poisson cluster model (Diggle, 1983) can be considered. For more accurate modelling of the fracture pattern, the existing program POINT (see, Chapter 4 and Appendix D for details) which was developed to simulate the non-homogeneous (hierarchical) fracture process model should be employed.

3. An exponential or a lognormal fracture length distribution can be simulated.

4. The von Mises, wrapped normal, uniform and unidirectional orientation distributions can be simulated.

5. Analysis of two or three fracture sets which have different mechanical (i.e., $C_j$, $\phi_j$ and $\sigma_{ij}$) or geometric (i.e., trace length distribution and orientation distribution) properties is possible.

6. An auxiliary plotting program TRACESIMP is available for post-processing and plotting of the results.

7. It can consider not only the conventional face-to-surface rock block which may be convex or concave, but also face-to-face blocks.

6.5 An Example of Slope Stability Analysis

To test the applicability of the concept developed in Sections 6.3 and 6.4, and of the program TRACESIM, we will analyze the example shown in Fig. 6-6. As illustrated in Figs. 6-7 and 6-8, there are initially 40 fractures in a slope, and subsequently 6 effective fractures with 10 intersection points. A von Mises orientation distribution with a high concentration factor and an exponential trace length distribution are used in Fig. 6-7. As can be detected visually in Fig. 6-8, there exist 3 kinematically possible fracture paths. The
kinetic analysis shows that two of the paths are unstable (see, Figs. 6-9 and 6-10). Finally, the safety factors, SF, are calculated for these two unstable paths and the failure path for the case with the minimum SF is plotted in Fig. 6-11.

Through Monte-Carlo simulations, in which one parameter is varied at one time while the others are held constant, one can determine the influence of the geometric and mechanical parameters.

6.6 Parametric Studies of a Slope with Fully Persistent Fractures

So far an example for a fully persistent set of fractures with fixed geometric and mechanical parameters has been given. In this section, parametric studies will be performed to establish relationships between various parameters and the safety factor (i.e., the probability of failure), and to find the most significant parameter. Initially, the mechanical parameters, such as cohesion \( C_j \), friction angle \( \phi_j \) and tensile cut-off stress \( \sigma_{oj} \) of the fractures, are changed one by one. Later, the geometric configurations such as mean orientations, mean trace length and the midpoint model of the fractures are also varied and their influence on the safety factor is investigated.
Figure 6-7: Initial Fracture Pattern in a Slope

Figure 6-8: Effective Fracture pattern in a Slope
Figure 6-9: One of the Two Unstable Fracture Paths

Figure 6-10: One of the Two Unstable Fracture Paths
Figure 6-11: Critical Fracture Path

Table 6-1 shows initially fixed and subsequently varied values of all the parameters. As was the case in the previous example, we do not specify the units of the mechanical or geometric parameters. Any consistent system of units can be used. Also, slope configuration is the same as shown in Fig. 6-6 and the initial midpoint model is the homogeneous Poisson process with 50 fractures. Table 6-2 lists the cases which are considered in the present study.

6.6.1 Case 1: Variation of Cohesion

Before proceeding to detailed analyses of the parametric studies, we calculate the probability of failure, $P_f$. Calculation of $P_f$ is composed of several steps;

1. For a given case, we perform 50 simulations per parameter state of the varied parameter (the parameter states are listed in Tab. 6-1). The resulting data set has hundreds of possible rock blocks, most of which come from the small variations of fracture paths. To simplify the data set, we choose the maximum
### Table 6-1 Initial and Changing Values of Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fixed Value</th>
<th>Varied Range</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td>Homogeneous or Non-homogeneous Poisson Model</td>
</tr>
<tr>
<td>No. of Fracture Sets, $N_j$</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Cohesion, $C_j$</td>
<td>1000</td>
<td>0, 500, 1000, 1500</td>
<td></td>
</tr>
<tr>
<td>Friction Angle, $\phi_j$</td>
<td>30°</td>
<td>10°, 20°, 30°, 40°</td>
<td></td>
</tr>
<tr>
<td>Tensile Cut-off Stress, $\sigma_{oj}$</td>
<td>0</td>
<td>0, 10, 50, 100</td>
<td></td>
</tr>
<tr>
<td>Orientation Concentration Factor, $\kappa$</td>
<td>20</td>
<td>10, 20, 30, 40</td>
<td></td>
</tr>
<tr>
<td>Mean Fracture Orientation (Dip Angle), $\alpha$</td>
<td>40°</td>
<td>20°, 30°, 40°, 50°</td>
<td></td>
</tr>
<tr>
<td>Mean Trace Length, $m_l$</td>
<td>6</td>
<td>4, 6, 8, 10</td>
<td></td>
</tr>
<tr>
<td>Unit Weight of Rock Mass, $\gamma$</td>
<td>2200</td>
<td></td>
<td>Fixed</td>
</tr>
</tbody>
</table>

or the minimum volume among rock blocks which have the same sliding angle. We already know that the sliding angle is the minimum dip angle which becomes one of the faces of the fully persistent rock block, and on which the rock block is sliding down. We expect that choosing either the maximum or the minimum volume does not affect the overall properties of the data (e.g., randomness of the number of occurrences of kinematically admissible rock blocks and mean sliding angle, etc) since, even though different in shapes due to path variations, the fracture paths which retain the same sliding angle have essentially the same properties. Fig. 6-12 is a plot of sliding angle v.s. volume of rock blocks for case 1 with parameter state, $C_j = 1000$ (recall that 50 simulations are run per parameter state). Fig. 6-13 is a simplification of the data in Fig. 6-12, in that only the maximum rock block volume is considered.
Table 6-2 Investigated Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Model</th>
<th>$N_j$</th>
<th>$C_j$</th>
<th>$\phi_j$</th>
<th>$\sigma_{oj}$</th>
<th>$\kappa$</th>
<th>$\alpha$</th>
<th>$m_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>H. P.</td>
<td>1</td>
<td>Varied</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 2</td>
<td>H. P.</td>
<td>1</td>
<td>Fixed</td>
<td>Varied</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 3</td>
<td>H. P.</td>
<td>1</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Varied</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 4</td>
<td>H. P.</td>
<td>1</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Varied</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 5</td>
<td>H. P.</td>
<td>1</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Varied</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 6</td>
<td>H. P.</td>
<td>1</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Varied</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 7</td>
<td>H. P.</td>
<td>2</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case 8</td>
<td>N. P.</td>
<td>1</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
</tbody>
</table>

2. Check the randomness of the number of occurrences of kinematically admissible rock blocks, or kinematic mechanisms. If the kinematically admissible rock blocks occur randomly, i.e., if the number of kinematic mechanisms follow the Poisson distribution during simulations, the corresponding probability of failure, due to the Poisson process properties, can be calculated easily. In the following, we will discuss the procedure to derive the probability of failure in detail. To check the randomness, a $\chi^2$-test is employed. The resulting goodness-of-fit is satisfied at the 10% significance level, see Fig. 6-14. One can, therefore, assume that, during simulations, the kinematically admissible rock blocks occur randomly.

3. Calculate mean ($m_{\alpha|p}$) and standard deviation ($\sigma_{\alpha|p}$) of the sliding angle for a given volume of rock block. The mean sliding angle can be estimated by regression, whereas the standard deviation is calculated from the moving average of the data with respect to the estimated mean sliding angle. We do this because the standard deviation derived from the moving average gives a relatively constant value and, therefore, only one variable, i.e., ($m_{\alpha|p}$), is necessary to calculate the probability of failure for a given volume of rock block. A window width of 10 volume units is used in our case. Fig. 6-15 shows the curve obtained from nonlinear regression which is used to find the mean sliding angle for a given volume of rock block.

4. Calculate the number (or the frequency) of kinematically admissible rock blocks, $\lambda(V)$, as a function of the volume of rock blocks. In our case, either an Exponential or a Gamma distribution can be fitted to the data. However, to prevent the fitted frequencies from approaching zero as the volume of rock block increases, and thus to make it possible to calculate the probability of failure of a very large volume, a Gamma distribution is used. Fig. 6-16 is an
example of the fitting the data with the Gamma distribution to find the frequency of kinematic mechanisms.

5. To include the cohesion effects on a sliding face of the rock block, one needs to calculate the contact lengths for given volumes of rock blocks. In our case, since the contact lengths vary widely and fluctuate inconsistently for a given volume of rock block, they cannot be estimated directly, nor precisely (see, Fig. 6-17). However, one can derive a distribution form for the contact length indirectly as follows:

- Assume a reference point (see, Fig. 6-17). This reference point can be connected with any of the data points in Fig. 6-17. The reference point, however, does not connect with every data point, e.g., if the volume is too small, one can ignore the corresponding data points since they do not affect the overall relationship between the contact length and the volume of rock block. This is evident when we consider the following steps where the relationship between length and volume is represented by a distribution form.

- Calculate slopes of lines which connect the reference point and the data points. Also, calculate the maximum and the minimum slope (Fig. 6-17).

- A Beta distribution is, then, fitted to the slope data to estimate the distribution of the slopes for given volume of rock blocks, Fig. 6-18. In Fig. 6-18, ε is calculated as:

\[ \varepsilon = \alpha_{\text{max}} - \alpha \]  \hspace{1cm} (6.7)

where, \( \alpha_{\text{max}} \) is the maximum slope found in Fig. 6-17, and \( \alpha \) is a currently considered slope. Fitting the data to a Beta distribution is performed with the method of moments (the first and the second moment are used to find the parameters of a Beta distribution).

- Finally, relate the contact length and the volume of rock block since one of the points (i.e., the reference point) and the distribution of the slopes are now known.

Assuming the slope angle is distributed normally for a given volume of rock block, the probability of failure for a given volume of rock block, \( P_{f|v} \), is calculated as:

\[ P_{f|v} = P(\{ \alpha > \alpha_{\text{crit}} \} | V) = 1 - P(\{ \alpha < \alpha_{\text{crit}} \} | V) = 1 - \int_{-\infty}^{\alpha_{\text{crit}}} \frac{1}{\sqrt{2\pi}\sigma_{\alpha|v}} \exp \left[-\frac{1}{2\sigma_{\alpha|v}^2} (\alpha - m_{\alpha|v})^2\right] d\alpha \]  \hspace{1cm} (6.8)
Figure 6-12: A Plot of Sliding Angle v.s. Volume of Rock Block with All Data Set

Figure 6-13: Simplification of Fig. 6-12 with Maximum Volume
Figure 6-14: Randomness Test of Occurrences of Kinematically Admissible Rock blocks

Figure 6-15: Calculation of Mean Sliding Angle Given Volume of Rock Block
where, $\alpha_{\text{crit}}$ is a critical angle for which the safety factor, S. F., becomes 1. $m_{\alpha_{lv}}$ and $\sigma_{\alpha_{lv}}$ are the mean and the standard deviation of the sliding angle for a given volume of rock block, respectively.

From Step 4 above and from Eq. (6.8), one can estimate the rate of failure occurrences (i.e., the expected number of kinetically unstable rock blocks) within any volume interval; specifically, the rate of failure occurrences within a volume interval $[V_1, V_2]$ can be calculated as:

$$E \left[ \text{No. Failures in } (V_1, V_2) \right] = \int_{V_1}^{V_2} \lambda(V) P_{f_{lv}} dV$$  \hspace{1cm} (6.9)
Figure 6-17: Distribution of Contact Length Given Volume of Rock Block

Figure 6-18: Fitting Contact Length to the Beta Distribution
where, $\lambda(V)$ is the frequency of kinematically admissible rock blocks derived in Step 4 in the above. Finally, since the occurrences of kinematically admissible rock blocks are random (see, Step 2 in the above), the probability of failure within a volume interval $[V_1, V_2]$ is estimated from the following equation:

$$P_f(V_1 - V_2) = 1 - e^{-E[\text{No. Failures in } (V_1, V_2)]} \tag{6.10}$$

Up to now, we derived the equation for the probability of failure for a given volume of rock block and, from this, obtained the probability of failure as well as the rate of failure occurrences within a volume interval. Using these relations, one can now investigate the influence of various parameters. We begin with the cohesion effect. To investigate the cohesion effect, the other mechanical and geometric parameters are set constant. As can be imagined, if the cohesion of a fracture is increased from 0 to 1500, the probability of failure for a given volume of rock block significantly decreases, see Fig. 6-19. Note that, when the cohesion has no influence on overall behavior (i.e., $C_j = 0$), the probability of failure will be a monotonically decreasing function, whereas when the cohesion is included in estimating probabilities, one can define a critical or maximum probability of failure which is a function of the volume of rock block. This is an expected result since the slope has a fixed height. It can also explained when one considers the limit state equilibrium. The safety factor for cases affected by friction only will be a function of the sliding angle which is generally inversely proportional to the volume of rock block (see, Fig. 6-15). However, if the cohesion is included in the analysis, the cohesion and the contact length along which cohesion acts will influence the stability. In other words, when the volume is small, the cohesiveal resistance is more significant than the frictional resistance, while the frictional resistance is dominating the overall behavior of the rock block as the volume of rock block increases. This is because the unit weight of rock mass, $\gamma$, is a relatively large value (2200 units) such that, as the volume is increased, its effect on overall behavior (i.e., frictional
resistance) greatly increases. One can, therefore, define the critical volume or the maximum probability of failure at the junction of these two effects. Another fact to notice is that, as cohesion increases, the critical volume at which the maximum probability of failure occurs also increases.

The rate of failure occurrences in a volume interval, Eq. (6.9), is plotted in Fig. 6-20, where the interval is set to 1 unit. In Fig. 6-20, one cannot define a maximum rate of failure occurrences with cohesion value 100 (i.e., the maximum rate occurs at 0 volume). This is because the rate of change of the number of kinematic mechanisms in Fig. 6-16 is more rapid than the rate of change of the probability of failure for given small volumes in Fig. 6-19. The probability of failure in a volume interval (1 unit) is depicted in Fig. 6-21.

6.6.2 Case 2: Variation of Friction Angle

When the friction angle increases from 10° to 40°, dramatic changes in overall behavior of the probabilities are induced; specifically the probability of failure for a given volume of rock block is diminished as the friction angle increases, Fig. 6-22. However, the critical volume at which the maximum probability of failure occurs decreases as the friction angle is increased. This is different from the effect of varying cohesion and it can be explained by the fact that, when the friction angle is small, the resisting force due to the friction angle together with the volume (actually, weight) is relatively less significant than the resisting force due to cohesion; whereas, if the friction angle increases, the frictional resistance increases rapidly (recall that the unit weight of rock mass, γ, is a relatively large value, 2200) and it will dominate the overall probabilities. The critical volume is, therefore, decreased.

The rate of failure occurrences and the probability of failure in a volume interval (1 unit) are illustrated in Figs. 6-23 and 6-24, respectively. These two figures clearly show the critical volume interval at which the maximum rate and the maximum probability of failure
Figure 6-19: $P_f$ Given Volume of Rock Block: Variation of Cohesion

Figure 6-20: Rate of Failure Occurrences in a Volume Interval: Variation of Cohesion
Figure 6-21: $P_f$ in a Volume Interval: Variation of Cohesion

occur. In our slope geometry case, the critical volume interval is roughly in 4 to 5 volume units.

6.6.3 Case 3: Variation of Tensile Cut-off Stress

The tensile cut-off stress is varied between 0 to 100. The probability of failure for a given volume of rock block decreases slightly, Fig. 6-25, as one goes from 0 to 100. One cannot, however, observe any significant changes in overall behavior of the probabilities. This means that, when cohesion is relatively large (i.e., $C_j = 1000$) and the maximum tensile cut-off stress is less than one tenth of the cohesion value, the resisting force due to the tensile cut-off stress does not dominate the resisting forces and, therefore, the probability of failure.
Figure 6-22: $P_f$ Given Volume of Rock Block: Variation of Friction Angle

Figure 6-23: Rate of Failure Occurrences in a Volume Interval: Variation of Friction Angle
Figure 6-24: $P_f$ in a Volume Interval: Variation of Friction Angle

Figure 6-25: $P_f$ Given Volume of Rock Block: Variation of Tensile Cut-off Stress
6.6.4 Case 4: Variation of Concentration Factor of Orientation

Various orientation (dip angle in two dimensional slope stability analysis) models are used when generating a fracture pattern. In our case, we have used the Von Mises distribution with two parameters; one is the mean orientation which is set to $40^\circ$, the other is the concentration factor $\kappa$ which determines the concentration around the mean orientation. If $\kappa$ is relatively high, the fracture pattern is concentrated (peaked) around the mean orientation; if not, the pattern is dispersed about the mean orientation and many of kinematically admissible rock blocks can be generated. In the parametric study, concentration factors ranging from 10 up to 40 have been used. Also, to include the influence of cohesion, two different cohesion values are compared.

First, when the cohesion value is relatively large ($C_j = 1000$), the probability of failure for a given volume of rock block increases with $\kappa$ up to $\kappa = 20$ because the mean sliding angle becomes steep (recall the mean sliding angle is different from the mean orientation in that the mean sliding angle is the minimum dip angle which constitutes the sliding face of a rock block), see Fig. 6-26. However, as the concentration factor is further increased, the mean sliding angle as well as the contact length of the sliding face increase. (Fig. 6-26). At this point, the resisting force due to cohesion effect dominates the overall behavior and, correspondingly, the probability of failure for a given volume of rock block decreases. Eventually, when the fractures becomes parallel (very high $\kappa$), the probability of failure for a given volume of rock block will be zero, Fig. 6-27. Second, if the cohesion value is relatively small ($C_j = 100$), the probability of failure for a given volume of rock block gradually increases and finally becomes 1 as the pattern becomes parallel, Fig. 6-28. This is because the cohesive resistance is not so significant compared with the frictional resistance of the rock block. From the comparison of these two cases, one can conclude that, when the cohesion is large, the probability of failure for a given volume of rock block increases up to a favorable concentration factor (in our case, $\kappa = 20$) where the contact
length (i.e., the cohesion effect) does not contribute to the overall behavior. However, as
the fractures approach a parallel pattern (in our case, when $\kappa$ is 30 or more), the contact
lengths are getting longer and finally all the blocks have triangular shapes where the faces
of a triangle are composed of two slope boundaries (slope face and surface) and a contact
face, Fig. 6-26. It means that the resisting force due to cohesion becomes significant and
the probability of failure for a given volume of rock block rapidly decreases and finally
becomes zero. When the cohesion effect is negligible, the probability of failure for a given
volume of rock block generally increases and finally becomes 1 as the pattern becomes
parallel. One more fact to notice is that, when the pattern is parallel, a maximum limit
volume exists since our model of a slope has a toe; in the present simulations, the maximum
is 11 volume units.

Figure 6-26: Typical Fracture Pattern with Increasing $\kappa$

The rate of failure occurrences as well as the probability of failure in a volume
interval have the same tendencies as the probability of failure for a given volume of rock
block.
Figure 6-27: $P_f$ Given Volume of Rock Block: Variation of Concentration Factor with Cohesion = 1000

Figure 6-28: $P_f$ Given Volume of Rock Block: Variation of Concentration Factor with Cohesion = 100
6.6.5 Case 5: Variation of Mean Orientation

When the mean orientation (dip angle) of the fractures is changed from 20° to 50°, the overall probability of failure increases. However, since the slope angle, θ, is set to 50° (see Fig. 6-6), the probability of failure for a given volume of rock block sharply decreases if the mean orientation (dip angle) is 50° and if the volume is increased, see Fig. 6-29. This can be explained by the fact that, for a mean orientation 50°, a chance to construct a fully persistent rock block is rarely possible except for small volumes. Figs. 6-30 and 6-31 also show the rate of failure occurrences and the probability of failure in a volume interval.

6.6.6 Case 6: Variation of Mean Trace Length

Using the exponential trace length distribution model and varying the mean trace length from 4 to 10 units, one can find the critical volume and the maximum probability of failure, Fig. 6-32. However, when the mean trace length is less than 4 units, one cannot find any trend since there is little chance to construct a fully persistent rock block. As can be foreseen, when the mean trace length is increased, the probability of failure for a given volume of rock block increases since the mean sliding angle is getting steeper as the mean trace length is increased, see Fig. 6-33. The main reason for the increase in the fitted mean sliding angle is that, since the fracture patterns with different trace length cases are generated with the same random number sequences, the blocks made with large mean trace length include the blocks which were previously made with small mean trace length, see Fig. 6-33. As a result, the fitted mean sliding angle has a possibility to be steeper as the mean trace length is increased. Also, the same behavior occurs both in the rate of failure occurrences and in the probability of failure in a volume interval.
Figure 6-29: \( P_f \) Given Volume of Rock Block: Variation of Mean Orientation

Figure 6-30: Rate of Failure Occurrences in a Volume Interval: Variation of Mean Orientation
Figure 6-31: $P_f$ in a Volume Interval: Variation of Mean Orientation

Figure 6-32: $P_f$ in a Volume Interval: Variation of Mean Trace Length
6.6.7 Case 7: Variation of Number of Fracture Sets

To study the fracture set effect, we divide the fractures into two sets. Set 1 is a set of 25 fractures with mean orientation of 40°; set 2 is a set of 25 fractures with mean orientations ranging from 20° to 130°. Considering the absolutely very low probabilities of failure in Fig. 6-35, one can state that the probability of failure for a given volume of rock block is not greatly influenced by the second set when $C_f = 1000$. This is quite probable since, in the case of 20°, the dip angle on which the rock block is sliding down is dominated by set 2 orientation (i.e., the mean sliding angle decreases, see Fig. 6-34(a)), and therefore, the overall probability of failure is diminished. Also, in the case of set 2 of 40°, combination of two sets determines the sliding dip angle (i.e., the fitted mean sliding angle becomes the steepest if the mean orientations of two sets are equal to 40°, Fig. 6-34(b)) and the probability of failure increases (recall that the sliding dip angle is determined as the
minimum dip angle among fractures which constitute the faces of a rock block). However, in the remaining cases, the dominating dip angle on which the block is sliding down is governed by set 1 (i.e., mean sliding angle = 40°) and no significant changes in probabilities are expected, Fig. 6-34(c).

When it comes to the rate of failure occurrences and the probability of failure in a volume interval (Figs. 6-36 and 6-37, respectively), the above mentioned two fracture set cases, i.e., 20° and 40°, as well as the case with 130° have a significant influence on the overall behavior. We may deduce the reason for these results as follows; First, when the mean orientations of two sets are orthogonal (i.e., 40° and 130°), more chances to make fully persistent blocks are expected in small volumes where the mean trace length was fixed at 6 units; however, as the volume is increased, these chances to construct the fully persistent blocks are rarely possible. Second, when the mean orientations of the two sets are 20° and 40°, one has the greatest possibility to construct a fully persistent block except in the small volume region where the orthogonal sets (see above) have greatest probabilities of failure. One can explain this by considering the connectivity matrix concept. That is, if fractures intersect the slope face at many points, more chances to construct the fully persistent rock blocks are possible, and if the mean orientation of fracture set is relatively flat, it will intersect the slope face at many points. Finally, for the same reason as above, when the mean orientations of two sets are equal and are set to 40°, one also has much more chances to make blocks except the second case.

6.6.8 Case 8: Variation of Midpoint Model of Fractures

Up to now, we have utilized the homogeneous Poisson process model when generating the midpoints of fractures. For modelling non-homogeneous Poisson processes, we will employ the Poisson clustering model (rather than the Cox process model) in which the number of seeds (i.e., parents) and the number of daughters are predetermined. In our
Figure 6-34: Typical Fracture Pattern with Two Sets of Fractures
(a) $\alpha = 20^\circ, 40^\circ$
(b) $\alpha = 40^\circ, 40^\circ$
(c) $\alpha = 130^\circ, 40^\circ$
Figure 6-35: $P_f$ Given Volume of Rock Block: Variation of Number of Fracture Sets

Figure 6-36: Rate of Failure Occurrences in a Volume Interval: Variation of Number of Fracture Sets
case, we will use 3 midpoints as seeds, and the remaining 47 midpoints as daughter fractures. To compare the result, we also use the homogeneous Poisson process model in which 50 fractures are generated randomly over the slope.

The probability of failure for a given volume of rock block increases in the case of non-homogeneous Poisson process model compared to the case of homogeneous Poisson process model since the fractures are clustered around the parent, and therefore, make it easier to construct a fully persistent block. Fig. 6-38 shows the differences in the probability of failure between the non-homogeneous and the homogeneous Poisson process model. Also shown in Figs. 6-39 and 6-40 are the rate of failure occurrences and the probability of failure in a volume interval, respectively.
Figure 6-38: $P$, Given Volume of Rock Block: Variation of Midpoint Model of Fractures

Figure 6-39: Rate of Failure Occurrences in a Volume Interval: Variation of Midpoint Model of Fractures
6.7 Discussion

In this chapter, we investigated blocky rock mass behavior by using stochastically generated fracture patterns and by employing a topological concept. At present, the model is limited to fully persistent fractures, but one can vary mechanical properties of fractures such as cohesion, friction angle and tensile cut-off stress. Also, geometric properties such as mean orientation, concentration factor, mean trace length, number of fracture sets and the midpoint model of fractures can be varied. The effect of varying these parameters, which amounts to a sensitivity study, showed that the number of kinematically admissible rock blocks was largely controlled by the number of effective fractures which intersect the slope face, and that the most important parameter in slope stability analysis was the cohesion as
well as the friction angle of a contact face. These two parameters influenced the critical volume (or the maximum probability of failure). Less importantly, the orientation distribution (i.e., mean orientation and concentration factor) affected the overall behavior of the slope stability. However, in our cases, the tensile cut-off stress did not dominate the overall probabilities.

The present limitation of the model in addition to the assumption of persistent fractures is that Program TRACESIM can detect only up to 10 different fractures which become the faces of a rock block. This needs to be modified if the pattern becomes complicated (i.e., more than 10 fracture paths in a rock block). The limitation can be overcome by expanding the connectivity matrix and by adding iteration loops into program TRACESIM. Future research will also make it possible to consider different slope shapes such as slopes with benches and with tension crack developed at the slope surface. Meanwhile, modelling of non-persistent failure requires representation of fracture coalescence.
Chapter 7

SUMMARY, CONCLUSIONS AND OUTLOOK

The main objective of this research was to develop a model and simulation procedure for rock fractures that can reflect the sequential mechanism of fracture formation and, thus, includes dependencies and spatial nonhomogeneities that are often seen in rock outcrops.

Modelling of sequential fracture sets has been accomplished using the hierarchical fracture geometry model. Specifically, the main features of the model are:

1. The spatial variation of trace density is represented by an inhomogeneous Poisson point process or by a doubly stochastic (Cox) point process.
2. A new method, based on maximum likelihood, has been developed for the unbiased estimation of trace length distribution.
3. When several trace sets are present, these sets are analyzed sequentially, according to a hierarchical order. Each set is represented by a conditional stochastic process, conditioning being with regard to the lower-order sets.
4. Two correlation measure methods, the line-kernel function method and the nearest neighbor distance method, have been used to account for the dependencies among fracture sets.
5. Methods from multivariate point processes (Diggle, 1983) have been adapted to the estimation and validation of multivariate fiber processes.
6. A modified termination probability was developed to describe the complex pattern of fracture intersections.

The hierarchical model has been applied to two cases in which detailed fracture patterns have been obtained, one with a single set and one with two sets. The validity of the model was checked by visual comparison between model prediction and mapped pattern and most importantly, by statistical checks such as the second-moment analyses and the Monte Carlo test. A satisfactory fit of the predicted pattern was obtained in both cases.

The hierarchical fracture model has been combined with a topological model to investigate the behavior of a blocky rock mass. The application of the combined hierarchical and topological model proceeds as follows;
1. A stochastic fracture pattern is generated with the hierarchical fracture geometry model.

2. Kinematic as well as kinetic analyses on fully persistent rock blocks are conducted.

3. Finally, using sensitivity analyses, the mechanical and geometric fracture parameters which have the greatest influence are determined.

Slope stability analysis using a fully persistent block model shows that the number of effective fractures which intersect the slope face are the key element for constructing a fully persistent blocky rock mass. Also, in most cases, we could define the critical volume which gives the maximum probability of failure for a given rock block volume.

So far the hierarchical model has been applied to two dimensions only, i.e., to fracture traces in a plane. Further work on the stochastic fracture geometry model will have to be focused on the three dimensional expansion of the existing model. One of the possible three dimensional models can be based on the homogeneous, isotropic Poisson RACS (RAndom Closed Set) process. This idealized model can be expanded to a three dimensional hierarchical model. Clearly, this requires establishing relationships between two dimensional trace length distributions and three dimensional fracture size distributions as well as the consideration of three dimensional orientation distributions.

Concerning topological modelling of a rock mass, two issues will have to be considered in the future work; one is the implementation of a more realistic mechanical model for rock fracturing, notably the mechanism of coalescence of non persistent fractures; the other is an expansion of the two dimensional slope stability problem into a three dimensional one.

It should be noted that, while this research has concentrated on an application to rock slope stability, the basis has been laid for a much broader application. The hierarchical fracture model can be used in any problem involving fractured rock masses. The topological model is similarly useful in other rock mass problem such as flow through
fractured rock masses. Most importantly, by expanding the mechanical aspects to include coalescence of existing fractures, the combined hierarchical and topological model can be used in any problem involving solid bodies with discontinuities. Notably, it will be possible to model fractured (cracked) concrete structures. On the other hand, a better understanding of basic small and large scale geologic processes will also be possible.
Appendix A

TESTS FOR RANDOMNESS

Various test methods exist for checking Complete Spatial Randomness. The approaches can be divided into two groups; one is based on the quadrat count method, while the other uses a distance measure schemes. We will mainly discuss the latter since the quadrat count method, in some cases, cannot satisfactorily describe the point pattern.

A.1 Distance-based [ Plotless ] Tests

We will discuss the distance-based test assuming the Poisson point process as a null hypothesis $H_0$. For later use, we define various distances which have their own characteristics. Define an event as a real point pattern such as a tree or mid point of a trace and a point as an artificial point selected randomly. We denote a event-event distance by $W$, a point-nearest event distance by $X$, a point-second nearest event by $X_2$, a distance from an event selected using a random point to the nearest event on a same half plane by $Y$ and, lastly, a distance from an event selected using a random point to the nearest event on the other half plane by $Z$ (see Fig A-1 for details). Here we assume $m$ as the total number of events and $\rho$ as the population intensity. The summation ($\sum$) in the following equations is for $i = 1,...,m$.

Clark and Evans (1954) developed a test based on the inter-event distances. Assuming a point population intensity $\rho$ in a region, the mean observed distance can be represented as

$$\overline{W}_A = \frac{\sum W}{m} \quad (A.1)$$

The mean distance which would be expected if the pattern is following CSR,
Figure A-1: Various Nearest-neighbor Measurements

\[ W_E = \frac{1}{2 \sqrt{\rho}} \]  \hspace{1cm} (A.2)

The ratio

\[ h_{CE} = \frac{\overline{W_A}}{W_E} \]  \hspace{1cm} (A.3)

can be used as a measure of the degree of departure from CSR. In CSR, \( h_{CE} = 1 \), under conditions of maximum aggregation, \( h_{CE} \) becomes 0.

Hopkins (1954) used X and W distances. His test requires complete enumeration of events in the study region. Under CSR,

\[ h_H = \frac{\sum X^2}{\sum W^2} \sim F_{2m,2m} \]  \hspace{1cm} (A.4)

Here, relatively small or large values of \( h_H \) indicate a regular or aggregated pattern, respectively.
Pielou (1959) used an index of non-randomness by calculating

\[ h_p = \pi \rho \frac{\sum X^2}{m} \sim N\left(1, \frac{1}{m}\right) \]  \hspace{1cm} (A.5)

In an aggregated population, one would expect a large value of \( X^2 \), giving a higher value of \( h_p \). Mountford (1961) considered the sampling errors and corrected Pielou’s index.

Holgate (1965) adopted nearest and second nearest distance \( X, X_2 \) and suggested a ratio test, based on the sample mean of the ratio

\[ h_{HO} = \left\{ \frac{\sum X^2}{X_2^2} \right\} / m \sim N\left(\frac{1}{2}, \frac{1}{12m}\right) \]  \hspace{1cm} (A.6)

We reject the \( H_0 \) if the observed value of \( h_{HO} \) differs from its expectation by more than \( k \) times its sampling standard deviation, where \( k \) is the appropriate percentage point of the normal distribution. If, instead of the mean of ratios, the ratio of means is considered

\[ h'_{HO} = \left( \frac{\sum X^2}{X_2^2} \right) \sim F_{2m,2m} \]  \hspace{1cm} (A.7)

These test methods are efficient whether a pattern is distributed regularly or not. Here, a regularity means that the points are evenly distributed over a region.

Eberhardt (1967) considered the moment ratio which is easy to calculate

\[ h_E = \frac{E(X^2)}{[E(X)]^2} = (\text{coefficient of variation})^2 + 1 = \frac{n \sum X^2}{(\sum X)^2} \]  \hspace{1cm} (A.8)

where, \( h_E \) increases with increasing tendency for aggregation. Since the sampling distributions of the ratio have not been worked out, an exact test of deviation from CSR is not available. But Eqn (A.8) doesn’t require a knowledge of density.

Besag & Gleaves (1973) introduced a T-square test. This test aims to preserve the intuitive appeal of Hopkins’ (1954) test, and it proved more successful than Holgate’s (1965) test. It consists of the following statistic
\[ h_{BG1} = \frac{1}{m} \sum \frac{2X^2}{2X^2 + Z^2} \sim N\left(\frac{1}{2}, \frac{1}{12m}\right) \]
\[ h_{BG2} = \frac{2 \sum X^2}{\sum Z^2} \sim F_{2m, 2m} \]  
(A.9)

Both tests are comparatively powerful against regular or aggregated alternatives since relatively small or large values of \( h_{BG1} \) or \( h_{BG2} \) indicate a regular or aggregated pattern, respectively. Thus these tests can be used as three-way characterization of spatial point pattern as regular, random or aggregated corresponding to a small, non-significant or large value for \( h_{BG1} \) and \( h_{BG2} \).

Diggle, et al. (1976) suggested CSR test with \( X \) and \( Z \) distance. Their statistic is as follows,
\[ h_{DB} = \sum 2 \left[ \min(2X^2, Z^2) \right] / (2X^2 + Z^2) / m \sim N\left(\frac{1}{2}, \frac{1}{12m}\right) \]  
(A.10)

It only detects nonrandomness and cannot distinguish between aggregation and regularity.

Cox & Lewis (1976) used the conditioned distance ratio method. They considered \( X \) and \( W \) distance and collected pairs \( m_0 \leq m \) for which \( W_i \leq X_i \). Transforming these two distances into \( r_i \), they showed for regularity,
\[ M = \min(r_1, \ldots, r_m) \sim \text{Beta}(1, m_0) \]  
(A.11)

and for aggregation,
\[ R = \frac{\sum r}{m_0} \sim N\left(\frac{1}{2}, \frac{1}{12m_0}\right) \]  
(A.12)

Diggle (1977) suggested a generalized likelihood ratio test. This test is based on the method of Besag & Gleaves (1973) when he considered the difference between clustering and random heterogeneity in the environment.
\[ h_D = 48m \left( m \log \left( \sum 2X^2 + Z^2 / m \right) - \sum \log (2X^2 + Z^2) \right) / (13m + 1) \]
\[ \sim \chi^2_{m-1} \]  
(A.13)
It can be used as four-way characterization of spatial point patterns as regular, random-homogeneous, random-heterogeneous, or aggregated. This test can be used to test heterogeneity if the test of Besag & Gleaves (1973) doesn't have a satisfactory result.

Ripley (1977) considered an edge-corrected second-moment as a test based on the idea of Besag (1977). The second moment is

$$\hat{K}(t) = m^{-2} \sum k(x, y)$$

(A.14)

where \(k(x, y)^{-1}\) is the proportion of the circumference of the circle centered \(x\) passing through \(y\). Besag (1977) suggested the plot of the stabilizing function \(L(t) = \sqrt{k(t)/\pi}\) vs \(t\) and \(L_m = \sup |L(t) - t|\) as a test statistic for CSR.

Brown & Rothery (1978) considered three statistics,

\[
D = \frac{\sum X^2}{m} \\
S = \frac{1}{m-1} \sum (X^2 - D)^2/D^2 \\
G = (\prod_{i=1}^{m} X^2)^{1/m}/D
\]

(A.15)

here, \(D\) is the mean squared distance, \(S\) is the square of the coefficient of variation of the squared nearest neighbor distances. Small values indicate local regularity, for complete regularity \(S = 0\). \(S\) is an equivalent form of the Eberhardt (1967) test except for the squared distances. \(G\) is the ratio of the geometric mean to the arithmetic mean of the squared distances. It lies in the interval \((0, 1)\) and large values of \(G\) indicate local regularity. When points are chosen at random, the distribution of \(S\) and \(G\) depend on the number of points and on the shape, but not the size, of the area.

Hines & Hines (1979) modified the Eberhardt (1967) test and adapted it to T-square sampling,

$$h_{HH} = \frac{2m \sum (2X^2 + Z^2)}{(\sum [X \sqrt{2} + Z])^2}$$

(A.16)

They proved \(h_H\) is powerful when detecting aggregation.
Byth & Ripley (1980) used a semi-systematic sampling scheme as a modification of the Hopkins' (1954) test. Fig A-2 shows the schematic sampling of their procedure. Set up a grid of $2m$ points. From $m$ sampling points, measure the squared distance to nearest event, $X_i^2, i=1,...,m$ with remaining points. Layout a small plot of a size which would contain about five events on average and count the events in each plot. Select $m$ events at random from those enumerated and measure the squared distance $W_i^2$. Thus,
\[ h_{BR1} = \frac{\sum X^2}{\sum W^2} \sim F_{2m, 2m} \]
\[ h_{BR2} = \frac{1}{m} \sum \left( \frac{X^2}{X^2 + W^2} \right) \sim N \left( \frac{1}{2} \cdot \frac{1}{12m} \right) \] (A.17)

and use these statistic as tests of CSR. They suggested \( h_{BR1} \) for suspecting clustered pattern and \( h_{BR2} \) for regular alternatives.

Heinrich (1984, 1986) modified the Ripley’s (1977) second moment test and included the variance, assuming the normally distributed random variable of \( \hat{K}(t) \). For testing the null hypothesis with given significance level \( \alpha \), he suggested
\[ h_{HE}(t, \rho, a) = \frac{\rho a [\hat{K}(t) - \pi t^2]}{t \sqrt{2\pi(1 + 2\pi t^2 \rho)}} \] (A.18)

where \( a \) is a length of a squared area.

A.2 Quadrat Count Tests

It is well known that the quadrat count test is a weak test compared with the distance-based test since (1) the size of the quadrat has effect on the results, (2) several different patterns may have same test results and (3) the counts lack spatial autocorrelations.

To test CSR with quadrat count method, we assume that the data are made up of independent counts \( n_1, \ldots, n_m \) in \( m \) quadrats, each with area \( B \), and with the mean of counts is \( \bar{n} \).

Fisher, et al. (1922) used the sample variance to mean ratio or index of dispersion as a test
\[ I = \frac{\sum (n_i - \bar{n})^2}{(m - 1) \bar{n}} \] (A.19)

Under CSR,
\[(m-1)I \sim \chi^2_{m-1}\]  
\hspace{1cm} \text{(A.20)}

where, usually \(m > 6\) and \(\rho B > 1\). Significantly large or small values indicate aggregate or regular pattern, respectively.

Mead (19974) showed various alternative tests based on the hierarchical classification of quadrats. Upton & Fingleton (1985) described the use of quadrat count test in detail.

A.3 Discussion

We have discussed many tests which can be used to test CSR. To find the most efficient test set is not simple because each test has its own merit. In general, one needs to include edge effects. For this reason, we advocate the use of Ripley's (1977) test. In this type of test, any shape of region can be used and no correction for edge effects is required if we use the Monte-Carlo simulation suggested by Barnard (1963).

Since the CSR test can only be used to check the null hypothesis, we need an alternative model (i.e., non-Poisson point model) if the null hypothesis is not satisfied.
Appendix B

METHODS FOR EDGE-CORRECTION

When we calculate the second moment of given data points, we need to consider edge effects. Here, we discuss two popular methods which can be used to correct the edge effects.

B.1 Edge-correction with the Graphical Method

The original idea for this correction is Ripley's (1977); it was simplified by Ohser & Stoyan (1981) and by Diggle (1983). One definition of the second moment, $K(t)$, is

$$\hat{K}(t) = n^{-2} A \sum k(x_1, x_2)$$  \hspace{1cm} (B.1)

where, $n$ is the number of events in a region $A$ and the sum is over all pairs $(x_1, x_2)$ of events which are apart up to distance $t$. $k(x_1, x_2)$ is a correction factor for edge effects. Consider the circle centered on $x_1$ passing through $x_2$, see Fig. B-1. If this circle is completely within region $A$ then $k = 1$. Otherwise, $1/k(x_1, x_2)$ is the proportion of the circumference of the circle within region.

Particularly, when a region $A$ is rectangle with side $a$ and $b$ ($a < b$), Ohser & Stoyan (1981), considering the whole region, classified the distance measure as belonging to one of four cases, whereas, Diggle (1983), considering practical uses, grouped the distance measure in two cases. Ripley (1985) expanded these cases to irregular shaped regions and suggested a generalized graphical method.
B.2 Edge-correction by Toroidal Shift Method

Original toroidal shifting (Lotwick & Silverman, 1982) was used when evaluating an independence test of bivariate point processes. We can use this idea when we calculate the second moment. Assume that the region of interest is rectangle with sides $a$ and $b$ $(a < b)$. Wrap the region onto a torus both in the $x$- and $y$-direction. Thus there are no edges. Calculate distance between two events as follows where we assume $x(x_1, x_2), y(y_1, y_2)$

$$t(x, y) = \sqrt{[f_a(x_1 - y_1)]^2 + [f_b(x_2 - y_2)]^2}$$  \hspace{1cm}  (B.2)

where,

$$f_h(s) = \min(|s|, h - |s|)$$  \hspace{1cm}  (B.3)

and calculate the second moment, $K(t)$, with derived distance $t$. Here, $K(t)$ is unbiased for all $t < t_0 = \min(a, b)/2$. 

Figure B-1: Graphical Method for Edge-correction
Appendix C

USER MANUAL AND LIST FOR "POINT" AND "FIBER"

C.1 Introduction

For the analysis of mapped fracture patterns, two different programs are needed:

- **POINT**: a program with which mid point behavior of mapped patterns and various point processes are analyzed.
- **FIBER**: a program which analyzes trace length behavior using a hierarchical concept.

In addition, a mathematical subroutine package called IMSL is used when generating random numbers and simulating basic distribution functions. Calculation of trace length distributions is done with program TDIST. Lastly, simulations of trace patterns using a termination probability are performed with program TERMIN. These programs are installed both on the JVN super computer CYBER 205 and on the Micro-VAX II at MIT. All programs are based on FORTRAN 77.

C.2 Input Manual for Program POINT

Program POINT generates pseudo-random numbers, and from these, calculates inter-event and nearest-neighbor distances. It also computes second-moments of mapped data and simulation patterns. Free-format is used for all input data except for the title. Except for a default input data file (unit 5), an additional input data file is required when dealing with an inhomogeneous Poisson point process modelling and a doubly stochastic process modelling. In this case, the input file unit is 8.

1. **TITLE** (20A4)
2. **IOPTN**: analysis options
   1: Inter-event distance analysis
   2: Nearest-neighbor distance analysis
3: Both 1 & 2
4: Second-moment analysis
5: Inhomogeneous Poisson point process model
6: Marked point process model
7: Doubly stochastic (Cox) point process model
3. If IOPTN = 7, then read ANGLE (in degrees). If not, skip, where ANGLE is a direction with which the angular second-moment is calculated.
4. NOPNT, NOSIM, NSTEP, IMAPP
   • NOPNT: No. of points generated
   • NOSIM: No. of simulations including analysis of mapped pattern
   • NSTEP: No. of calculation steps when evaluating the distance-based measurement
   • IMAPP = 0: consider simulations only when checking validity of a model
   • IMAPP = 1: consider simulations and mapped pattern when checking validity of a model
5. DSTEP: distance step length
6. KXCLS, KYCLS: No. of divisions in X- and Y-direction, respectively
7. XBOT, XTOP, YBOT, YTOP: boundary coordinates for X- and Y-direction
8. If IOPTN = 5, read values of curve-fitting parameter and options
   XVAL1, XVAL2, YVAL1, YVAL2, CONST: parameters
JOPTN, IKERN
   • JOPTN: analysis options
     1: inter-event distance analysis
     2: nearest-neighbor distance analysis
     3: second-moment analysis
   • IKERN
     1: fixed kernel function measure
     2: linear kernel function measure
SIGMA: standard deviation of a fixed kernel function (input unit = 8)
COORD(IOPNT,1) : X-coord. ; COORD(IOPNT,2) : Y-coord. (input unit = 8)

where, IOPNT = 1 through NOPNT

9. If IOPTN = 6, read IDUMM, COORD(IOPNT,1), COORD(IOPNT,2), TRACE(IOPNT)
   • IDUMM : dummy value
   • COORD(IOPNT,1) : X-coord.
   • COORD(IOPNT,2) : Y-coord.
   • TRACE(IOPNT) : trace length

10. If IOPTN = 7, read option NOJOB and coefficients
    • NOJOB
      NOJOB = 1 : Cox process with bivariate normal distribution function
      NOJOB = 2 : simulation of Cox process using angular second-moment analysis
      NOJOB = 3 : calculation of angular second-moment
    • XCOF2, YCOF2, FMULT, XVARI, YVARI, IPRIN (input unit = 8)
      XCOF2 : X-dir. spectral density coefficient
      YCOF2 : Y-dir. spectral density coefficient
      XVARI, YVARI : standard deviation for X- and Y-direction
      IPRIN = 0 : no printout ; = 1 print the results
    • DUMMY, DUMM1, IDUMM : dummy values
    • COORD(IOPNT,1) : X-coord.
      COORD(IOPNT,2) : Y-coord.

11. If IMAPP ≠ 0 and IOPTN ≤ 3, read coordinates of mapped pattern ; If not skip

    COORD(IOPNT,1) : X-coord. ; COORD(IOPNT,2) : Y-coord.

    IOPNT = 1 through NOPNT
C.3 Input Manual for Program FIBER

Program FIBER evaluates the hierarchical fiber (Line-segment) process which is a realization of the given mapped pattern.

1. TTITLE (20A4)
2. NOPNT, NOFPT, NOLPT, NOSIM, NSTEP, DSTEP, SIGMA, TSTEP
   - NOPNT : No. of total fibers (or mid points)
   - NOFPT : No. of fibers in the 1st set
   - NOLPT : No. of fibers in the 2nd set
   - NOSIM : No. of simulations iterated
   - NSTEP : No. of steps when calculating the statistic
   - DSTEP : distance step size
   - SIGMA : standard deviation of the line-kernel function
   - TSTEP : unit of segment in a fiber
3. XBOT, XRANG, YBOT, YRANG
   - XBOT : X-coord. of starting boundary
   - XRANG : range of X-coord.
   - YBOT : Y-coord. of starting boundary
   - YRANG : range of Y-coord.
4. IOPTN, JOPTN, LOPTN

   IOPTN = 0 : do not enter the point process
   IOPTN = 1 : bivariate kernel function method
   IOPTN = 2 : independence test
   IOPTN = 3 : orientation correlation option
   IOPTN = 4 : MLE for orientation data
   JOPTN = 0 : do not enter the fiber process
   JOPTN = 1 : MLE for line-kernel function method
   JOPTN = 2 : MLE for nearest-neighbor fibers
   JOPTN = 3 : simulation of set 2 with line-kernel function method
   JOPTN = 4 : simulation of set 2 with nearest-neighbor distance method
LOPTN = 1 : calculate the bivariate second-moment

LOPTN = 2 : calculate the univariate second-moment

5. If IOPTN = 2 ; read coordinates of mid points of all fibers
   - COORD(IOPTN,1), COORD(IOPTN,2) : X- and Y-coord. of the mid point
   - ISHFT ; analysis option
     ISHFT = 1 : small perturbation test
     ISHFT = 2 : toroidal shift test

6. If IOPTN = 3 ; read ANGLE and coordinates of fibers
   - ANGLE : step angle magnitude in degree
   - COORD(IOPTN,1), COORD(IOPTN,2) : X- and Y-coord. at starting point of fiber
     COORD(IOPTN,3), COORD(IOPTN,4) : X- and Y-coord. at ending point of fiber

7. If IOPTN = 4 ; read coordinates of fibers of 2nd set and option MOPTN
   - COORD(IOPTN,1), COORD(IOPTN,2) : X- and Y-coord. at starting point of fiber
     COORD(IOPTN,3), COORD(IOPTN,4) : X- and Y-coord. at ending point of fiber
   - MOPTN : orientation distribution option
     MOPTN = 1 : Von Mises distribution on a circle
     MOPTN = 2 : wrapped normal distribution on a circle

8. If IOPTN ≤ 2, read XLAMO : distance measure at starting point

9. If IOPTN ≠ 0, read coordinate of 1st set of fibers
   - COORD(IOPTN,1), COORD(IOPTN,2) : X- and Y-coord. at starting point of fiber
     COORD(IOPTN,3), COORD(IOPTN,4) : X- and Y-coord. at ending point of fiber

10. If IOPTN = 1, = 2 or = 3 : read coord. of 2nd set of fibers
    - COORD(IOPTN,1), COORD(IOPTN,2) : X- and Y-coord. at starting point of fiber
      COORD(IOPTN,3), COORD(IOPTN,4) : X- and Y-coord. at ending point of fiber
C.4 Input Manual for Program TDIST

Program TDIST estimates the trace length distribution using MLE. Currently, only the exponential form of the distribution is considered.

1. TRACE(IOPNT,1), TRACE(IOPNT,2), NCONF(IOPNT)
   - TRACE(IOPNT,1) : trace length for IOPNT-th trace
   - TRACE(IOPNT,2) : boundary length for IOPNT-th trace
   - NCONF(IOPNT) = 1 : trace length with both ends observable
     NCONF(IOPNT) = 2 : trace length with one end observable
     NCONF(IOPNT) = 3 : trace length with no end observable

2. NITER : No. of iterations (currently input as 1)

3. NOPTN : analysis option (currently input as 1)

C.5 Input Manual for Program TERMN

Program TERMN generates the trace pattern according to the simulated typical points. Also, it evaluates the termination points from calculated termination probabilities.

1. NOPNT, NOFPT, NOSIM, VMEAN, AMEAN, SIGMA, TRUNL
   - NOPNT : No. of mid points generated
   - NOFPT : No. of mid points of 1st set
   - NOSIM : No. of simulation performed
   - VMEAN : mean trace length of set 2
   - AMEAN : mean (strike) angle of set 2
   - SIGMA : standard deviation of angle of set 2
   - TRUNL : truncation length of set 2

2. XBOT, XTOP, YBOT, YTOP : coordinate of simulation boundary

3. MOPTN : orientation distribution option
   - MOPTN = 1 : Von Mises distribution
   - MOPTN = 2 : wrapped normal distribution

4. read coordinate of 1st set CORFP
CORFP(IOPNT,1), CORFP(IOPNT,2) : coordinate at starting point of fiber
CORFP(IOPNT,3), CORFP(IOPNT,4) : coordinate at ending point of fiber

5. IDUMM, COORD(IOPNT,1), COORD(IOPNT,2)
   • IDUMM : dummy value
   • COORD(IOPNT,1) : X-coord. of mid point of 2nd set
   • COORD(IOPNT,2) : Y-coord. of mid point of 2nd set

C.6 Program Listings : POINT, FIBER, TDIST, TERMN

PROGRAM POINT
CCC
C PROGRAM POINT GENERATES PSEUDO-RANDOM NUMBERS &
C CALCULATE INTER-EVENT, NEAREST EVENT DISTANCES
C AND SECOND-MOMENT OF THE
C SIMULATION DATA AS WELL AS MAPPED DATA
C
CCC
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C COMMON/CONTR/NDOFN, DSTEP, NTOIL, NOPNT, NOSIM, NSTEP,
*       NTOL, IMAPP, IOPTN, XBOT, XRANG, YBOT,
*       YRANG, XKCLS, YKCLS, KTCLS, ANGLE, NOANG
C DIMENSION KK(20000), AA(30000), TITLE(20)

C
C COORD : COORDINATE OF MID-POINT (NOPNT,2)
C RSCLE : NO. OF GENERATED RANDOM NUMBER (2*NOPNT)
C NOFRE : NO. OF DISTANCE FREQUENCIES IN A SIMULATION (NOSIM,NSTEP)
C DISTS : DISTANCE MEASURE ( 0.5*NOPNT*(NOPNT-1) )
C
C NRVAR = 30000
C NIVAR = 20000
C
C NDOFN = 2
C READ (5,800) TITLE
C 800 FORMAT (20A4)
C WRITE(6,800) TITLE

C 1 ANALYSIS OPTION : IOPTN
C 1 : INTER-EVENT DISTANCES
C 2 : NEAREST NEIGHBOR DISTANCES
C 3 : BOTH 1 & 2
C 4 : SECOND MOMENT MEASURE DISTANCE
C 5 : INHOMOGENEOUS POISSON P.P.
C 6 : MARKED POINT PROCESS
C 7 : DOUBLY STOCHASTIC P.P.
1.1 ANGLE OF MEASUREMENT IN DEGREES : ANGLE

( OPTIONAL WHEN IOPTN = 7 )

2 NO. OF MID-POINTS : NOPNT

3 NO. OF SIMULATIONS INCLUDING MAPPED DATA : NOSIM

4 NO. OF DISTANCE STEPS : NSTEP

5 FLAG INDEX IMAPP
   0 : SIMULATIONS ONLY
   1 : INCLUDE A MAPPED PATTERN

6 DISTANCE STEP USED IN STATISTIC : DSTEP

NOANG = 1
READ (5,*) IOPTN
WRITE(6,900) IOPTN
IF ( IOPTN.EQ.7 ) THEN
   READ (5,*) ANGLE
   NOANG = 360 / ANGLE
   WRITE(6,980) ANGLE
ENDIF
READ (5,*) NOPNT, NOSIM, NSTEP, IMAPP
WRITE(6,910) NOPNT, NOSIM, NSTEP, IMAPP
READ (5,*) DSTEP
WRITE(6,920) DSTEP

KXCLS = NO. DIVISIONS IN X-DIR.
KYCLS = NO. DIVISIONS IN Y-DIR.
USED IN INHOMOGENEOUS POISSON POINT PROCESS
IF IOPTN.NE.5, SET KXCLS = KYCLS = 1

READ (5,*) KXCLS,KYCLS
WRITE(6,970) KXCLS,KYCLS
NTOTL = NDOFN * NOPNT
NTOVL = NOPNT * (NOPNT-1) / 2
NTOST = NOSIM * NSTEP
NTOSI = NOSIM * NOPNT
KTCLS = KXCLS * KYCLS
NSEC = NSTEP * NOANG

DYNAMIC DIMENSIONING

NR1 = 1
NR2 = NR1 + NTOTL
NR3 = NR2 + NTOTL
NR4 = NR3 + NTOVL

DFREQ = ( NOSIM,NOPNT )
SIMUL = ( KTCLS,2 )

NR5 = NR4 + NTOSI
NR6 = NR5 + KTCLS*2

SVALU = ( NOSIM,NSTEP ) : USED IN SECOND MOMENT MEASURE

NR7 = NR6 + NTOST
NR8 = NR7 + NOPNT
NR9 = NR8 + NSECT

NI1 = 1
NI2 = NI1 + NTOST

NRTOT = NR9 - 1
NITOT = NI2 - 1
IERRO = 0

WRITE(6,930) NRTOT, NRVAR
IF ( NRVAR.GT.NRTOT ) GO TO 20
WRITE(6,940)
IERRO = IERRO + 1

20 CONTINUE
WRITE(6,950) NITOT, NIVAR
IF ( NIVAR.GT.NITOT ) GO TO 30
WRITE(6,960)
IERRO = IERRO + 1

30 CONTINUE
IF ( IERRO.GT.0 ) STOP

DO 40 IVARI = 1,NRVAR

40 AA(IVARI) = 0.
DO 50 IVARI = 1,NIVAR

50 KK(IVARI) = 0

IF BOTH INTER-EVENT AND NEAREST NEIGHBOR DISTANCE
METHOD ARE USED, OPEN TAPE10

IF ( IOPTN.GE.3 ) THEN
OPEN (10,FILE='TAPE10',FORM='UNFORMATTED',STATUS='NEW')
ENDIF

CALL MAIN OPTION

CALL MAINS (AA(NR1), AA(NR2), AA(NR3), AA(NR4), AA(NR5),
* AA(NR6), AA(NR7), AA(NR8), KK(NI1))

900 FORMAT (//,5X,'ANALYSIS OPTION' = ',I3,/,7X,'(1:INTER-EVENT DISTANCES)' = ',I3,/,7X,'(2:NEAREST EVENT DISTANCE)' = ',I3,/,7X,'(3:BOOTH 1 & 2 OPTIONS)' = ',I3,/,7X,'(4:2nd MOMENT MEASURE)' = ',I3,/,7X,'(5:INHOMOGENEOUS POISSON P)' = ',I3,/,7X,'(6:MARKED POINT PROCESS)' = ',I3,/,7X,'(7:DUBLY STOCHASTIC P.P.)' = ',I3,/
910 FORMAT (//,5X,'NO. OF MID-POINT GENERATED' = ',I3,/,5X,'NO. OF SIMULATIONS' = ',I3,/,5X,'NO. OF DISTANCE MEASURES' = ',I3,/,5X,'MAPPED PATTERN INDICATOR' = ',I3,/,7X,'(0:S IMPLE SIMULATIONS)' = ',I3,/,7X,'(1: MAPPED DATA INCLUDED)' = ',I3,/
920 FORMAT (//,5X,'DISTANCE STEP SIZE' = ',F7.3,/
930 FORMAT (//,5X,'REAL STORAGE REQUIRED' = ',I7,'
* 5X,'REAL STORAGE SPECIFIED' = ',I7
940 FORMAT (5X,'** INCREASE STORAGE FOR REAL ARRAYS')
950 FORMAT (5X,'INTEGER STORAGE REQUIRED' = ',I7,
* 5X,'INTEGER STORAGE SPECIFIED' = ',I7
960 FORMAT (5X,'** INCREASE STORAGE FOR INTEGER ARRAYS')
970 FORMAT (5X,'NO. OF SUBSET IN X-DIRECTION' = ',I3,
* 5X,'NO. OF SUBSET IN Y-DIRECTION' = ',I3
980 FORMAT (5X,'ANGLE OF MEASUREMENT' = ',F5.1,' DEG.'
C
STOP
C
END

SUBROUTINE MAINS  (COORD, RSCALE, DIS'SS, DFREQ, SIMUL, SVALU,
* TRACE, ANOMON, NOFRE)
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTR/NOFNT, DSTEP, NTOIL, NOPNT, NOSIM, NSTEP,
* NTOVL, IMAPP, IOPTN, XBOT, XRANG, YBOT,
* YRANG, KXCLS, KYCLS, KTCLS, ANGLE, NOANG
DLEMION COORD(NOPNT,2), RSCALE(NTOIL), DISTS(NTOVL),
* NOFRE(NOSIM,NSTEP), DFREQ(NOSIM,NOPNT),
* SIMUL(KTCLS,2), SVALU(NOSIM,NSTEP), TRACE(NOPNT),
* ANOMON(NOANG,NSTEP)
C
READ SIMULATION PATTERN SIZE
C
READ (5,*) XBOT,XTOP,YBOT,YTOP
XRANG = XTOP - XBOT
YRANG = YTOP - YBOT
C
ANALYSIS OPTION
C
GO TO (100,200,300,400,500,600,700) IOPTN
C
INTER-EVENT DISTANCE ANALYSIS
C
100 CONTINUE
CALL INTER (COORD, RSCALE, DISTS, NOFRE )
GO TO 800
C
NEAREST NEIGHBOR DISTANCE ANALYSIS
C
200 CONTINUE
CALL NEARS (COORD, RSCALE, DISTS, NOFRE, DFREQ )
GO TO 800
C
BOTH INTER-EVENT & NEAREST NEIGHBOR DISTANCES
C
300 CONTINUE
CALL INTER (COORD, RSCALE, DISTS, NOFRE )
CALL NEARS (COORD, RSCALE, DISTS, NOFRE, DFREQ )
GO TO 800
SECOND-MOMENT MEASURE METHOD

400 CONTINUE
CALL SMSTA ( COORD, RSCLE, SVALU )
GO TO 800

INHOMOGENEOUS POISSON POINT PROCESS

500 CONTINUE
CALL IHPPS ( COORD, RSCLE, DISTS, SIMUL, NOFRE, DFREQ, SVALU )
GO TO 800

MARKED POINT PROCESS
MARK : TRACE LENGTH OF EACH JOINT

600 CONTINUE
CALL MPSS ( COORD, TRACE, SVALU )
GO TO 800

DOUBLY STOCHASTIC POINT PROCESS

700 CONTINUE
CALL DOUBL ( COORD, RSCLE, DISTS, SIMUL, NOFRE, DFREQ, SVALU, ANMOM )

800 CONTINUE
RETURN
END

SUBROUTINE INTER ( COORD, RSCLE, DISTS, NOFRE )

SUBROUTINE INTER CALCULATES THE INTER-EVENT DISTANCE ANALYSIS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTR/NDOFN, DSTEP, NTOTL, NOPNT, NOSIM, NSTEP,
* NYOVL, IMAPP, IOPTN, XBOT, XBOT, XRANG, YRANG,
* YRANG, KYCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NOPNT,2), RSCLE(NTOTL), DISTS(NYOVL),
* NOFRE(NOSIM,NSTEP)

JOSIM = 1

NPSIM = NOSIM
IF ( IMAPP.GT.0 ) NPSIM = NOSIM - 1
IOSIM = 1
DSEED = SECNDS(0.0) * 100.

GENERATE MID-POINT COORDINATE USING RANDOM NUMBER GENERATOR OF IMSL (GGUBS)

10 CONTINUE

CALL GGUBS ( DSEED, NTOTL, RSCLE )
DO 20 IDOFN = 1, NOPNT
JDOFN = IDOFN * 2 - 1
KDOFN = IDOFN * 2
COORD(IDOFN,1) = XBOTM + RSCLE(JDOFN) * XRANG
COORD(IDOFN,2) = YBOTM + RSCLE(KDOFN) * YRANG

C IF ( IOSIM.EQ.1 ) THEN
    WRITE(6,930) (COORD(IDOFN, IODOF), IODOF=1,2)
ENDIF

C 20 CONTINUE
C
C CALCULATE INTER-EVENT DISTANCES
C
CALL INDIS ( JOSIM,COORD, NOPNT, DSTEP, NSTEP, NOFRE, DISTS,
             * NOSIM, NTOVL )
IF ( IOSIM.LT.NPSIM ) THEN
    IOSIM = IOSIM + 1
    JOSIM = JOSIM + 1
    GO TO 10
ENDIF

C MAPPED PATTERN ANALYSIS
C
IF ( IMAPP.GT.0 ) THEN
    DO 50 IOPHT = 1, NOPNT
    DO 50 IDOFN = 1, NDOFN
    COORD(IOPHT, IDOFN) = 0.
50 CONTINUE
    DO 60 IOPHT = 1, NOPNT
    READ (5,*) (COORD(IOPHT, IDOFN), IDOFN=1,NDOFN )
60 CONTINUE

C IF IOPHT = 3, SAVE COORD. DATA FOR LATER USE
C
C IF ( IOPHT.EQ.3 ) WRITE(10) COORD(IOPHT,1),COORD(IOPHT,2)
C
C 60 CONTINUE
C
IOSIM = NOSIM
CALL INDIS ( IOSIM,COORD, NOPNT, DSTEP, NSTEP, NOFRE,
             * DISTS,NOSIM,NTOVL )
ENDIF
C
C CALCULATE MIN. & MAX NO. OF DISTANCE MEASURE
C
IF ( NOSIM.EQ.1 ) STOP

mtotl = 0
WRITE(6,940)
DO 40 ISTEP = 1, NSTEP
    minno = nofre(1,ISTEP)
    maxno = nofre(1,ISTEP)
    mttotl = nofre(1,ISTEP)
    DO 30 IOSIM = 2, NPSIM
        IF ( NOFRE(IOSIM,ISTEP).LT.MINNO )
        * MINNO = NOFRE(IOSIM,ISTEP)
IF ( NOFRE(IOSIM,ISTEP).GT.MAXNO )
*    MAXNO = NOFRE(IOSIM,ISTEP)
MTOTL = MTOTL + NOFRE(IOSIM,ISTEP)
30 CONTINUE
C
HMINV = MINNO
HMAXV = MAXNO
HAVRG = MTOTL
HMINV = HMINV / NTOL
HMAXV = HMAXV / NTOL
HAVRG = HAVRG / ( NTOL * NPSIM )
C
DSTNS = DSTEP * ISTEP
C
EMPirical SOLUTION
C
HEXAT = 0.
IF ( XRANG.EQ.YRANG ) THEN
   HEXAT = 3.1415 * DSTNS**2 - 8*DS**3 /3 + DSTNS**4/2
ELSE
   HEXAT = HAVRG
ENDIF
C
MAPPED PATTERN CASE
C
IF ( IMAPP.GT.0 ) THEN
  PAMAP = NOFRE(NOSIM,ISTEP)
  PAMAP = PAMAP / NTOL
  WRITE(6,950) DSTNS, HMINV, HMAXV, HEXAT, PAMAP
  GO TO 40
ENDIF
C
WRITE(6,950) DSTNS, HMINV, HMAXV, HEXAT
40 CONTINUE
C
930 FORMAT ( 5X,2(D15.7,2X))
940 FORMAT (/, 10X, 'INTER-EVENT DISTANCE METHOD', /,
*        5X, 'DISTANCE MIN. VALUE MAX. VALUE EXACT',
*        ' MAPPED', /
950 FORMAT ( 5X,F7.3,5X,4(F7.5,3X) )
RETURN
END
C
SUBROUTINE INDIS ( IOSIM,COORD,NOPNT,DSTEP, NSTEP,NOFRE,
*                  DISTS,NOSIM,NTOVL )
C
SUBROUTINE INDIS calculates inter-event distances of
mid-points
C
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION COORD(NOPNT,2), NOFRE(NOSIM,NSTEP), DISTS(NTOVL)
C
NPOIN = NOPNT - 1
ICOUN = 0
MCOUN = NOPHT * (NOPHT-1) / 2
FSTEP = DSTEP

DO 10 IPOIN = 1,NPOIN
   XCOORD = COORD(IPOIN,1)
   YCOORD = COORD(IPOIN,2)
   KPOIN = IPOIN + 1
   DO 10 JPOIN = KPOIN,NOPHT
      ICOUN = ICOUN + 1
      XCOORD1 = COORD(JPOIN,1)
      YCOORD1 = COORD(JPOIN,2)
      DISTS(ICOUN) = SQRT((XCOORD-XCOORD1)**2+(YCOORD-YCOORD1)**2)
   10 CONTINUE

CALCULATES THE FREQUENCIES OF DISTANCE CATEGORIES

20 CONTINUE
   JCOUN = 0
   KCOUN = 0
   KCOUN = KCOUN + 1
   FSTEP = DSTEP * KCOUN
   DO 30 ICOUN = 1,NCOU
      IF ( DISTS(ICOUN),LE,FSTEP ) THEN
         JCOUN = JCOUN + 1
      ENDFI
30 CONTINUE
   NOFRE(IOSTM,KCOUN) = JCOUN
   IF ( HCOUN.LT.NSTEP ) GO TO 20
   RETUR
   END

SUBROUTINE NERES ( COORD, RSCLE, DISTS, NOFRE, DFREQ )

SUBROUTINE NERES COMPUTE NEAREST NEIGHBOR DISTANCE
OF GIVEN EVENT

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTR/NDOPH, DSTEP, NT社L, NOPHT, NOSIM, NSTEP,
*  NTOVL, IMAPF, IOPTN, XBOT, XRANG, YBOT,
*  YRANG, KXCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NOPHT,2), RSCLE(NSTOL), DISTS(NTOVL),
*  NOFRE(NOSIM,NSTEP), DFREQ(NOSIM,NOPHT)

INITIALIZE

DO 110 ICOOR = 1,NOPHT
DO 110 IDOFN = 1,NDOFN
   COORD(ICOOR,IDOFN) = 0.
110 CONTINUE
DO 120 ITOTL = 1,NTOTL
   RSCLE(ITOTL) = 0.
120 CONTINUE
DO 130 ITOVVL = 1,NTOVL
   DISTS(ITOVVL) = 0.
130 CONTINUE
   DO 140 LOSIM = 1, NOSIM
   DO 140 LSTEP = 1, NSTEP
   NOFRE(LOSIM, LSTEP) = 0
140   CONTINUE
C
   REWIND 10
C
   JOSIM = 1
C
   IF ( IMAPP.GT.0 ) NPSIM = NOSIM - 1
   IOSIM = 1
   DSEED = SECONDS(0.0) * 100.0
C
   GENERATE MID-POINT COORDINATE USING RANDOM NUMBER
   GENERATOR OF IMSL (GGUBS)
C
10  CONTINUE
   CALL GGUBS( DSEED, NTOTL, RSCLE )
   DO 20 IDOFN = 1, NOPFT
     JDOFN = IDOFN * 2 - 1
     KDOFN = IDOFN * 2
     COORD(IDOFN,1) = XBOT + RSCLE(JDOFN) * XRANG
     COORD(IDOFN,2) = YBOT + RSCLE(KDOFN) * YRANG
20  CONTINUE
C
   CALCULATE NEAREST NEIGHBOR DISTANCE
C
   CALL NEDIS( IOSIM, COORD, NOPFT, DSTEP, NSTEP, NOFRE, DISTS,
                   NOSIM, NTOVL, XRANG, YRANG, DFREQ, AVRGD )
   IF ( IOSIM.LE.NPSIM ) THEN
     IOSIM = IOSIM + 1
   GO TO 10
   ENDIF
C
   MAPPED PATTERN ANALYSIS
C
   IF ( IMAPP.GT.0 ) THEN
     DO 30 IOPNT = 1, NOPFT
     DO 30 IDOFN = 1, NDOFN
     COORD(IOPNT,IDOFN) = 0.
30   CONTINUE
     IF ( IOPNT.EQ.2 ) THEN
       DO 40 IOPNT = 1, NOPFT
       READ (5,*)(COORD(IOPNT,IDOFN), IDOFN=1,NDOFN)
40   CONTINUE
     ELSE
       WRITE(6,992)
       DO 45 IOPNT = 1, NOPFT
       READ (10) (COORD(IOPNT,IDOFN), IDOFN=1,NDOFN)
45   CONTINUE
   ENDIF
C
   IOSIM = NOSIM
   CALL NEDIS( IOSIM, COORD, NOPFT, DSTEP, NSTEP, NOFRE,
* DISTS, HOSIM, NTOL, XRANG, YRANG, DFREQ,
*      AVRGD 
AVRGD = AVRGD / ( NOPT * ( NOPT - 1 ) )
ENDIF

CALCULATE MIN. & MAX NO. OF DISTANCE MEASURE

IF ( HOSIM.EQ.1 ) STOP

WRITE(6,940)
DO 60 IOSM = 1,NPSM
MINNO = NOFRE(1,ISTEP)
MAXNO = NOFRE(1,ISTEP)
DO 50 IOSM = 2,NPSM
IF ( NOFRE(IOSM,ISTEP).LT.MINNO )
* MINNO = NOFRE(IOSM,ISTEP)
   IF ( NOFRE(IOSM,ISTEP).GT.MAXNO )
* MAXNO = NOFRE(IOSM,ISTEP)

50 CONTINUE

HMINV = MINNO
HMAXV = MAXNO
HMINV = HMINV / NOPT
HMAXV = HMAXV / NOPT

DSTNS = DSTEP * ISTEP

AVERAGE VALUE OF SIMULATED DATA

PAVRG = 0.
DO 70 IOSM = 1,NPSM
PAVRG = PAVRG + NOFRE(IOSM,ISTEP)
70 CONTINUE
PAVRG = PAVRG / ( NPSM * NOPT )

MAPPED PATTERN

IF ( IMAPP.GT.0 ) THEN
PAMAP = NOFRE(HOSIM,ISTEP)
PAMAP = PAMAP / NOPT
WRITE(6,950) DSTNS, HMINV, HMAXV, PAVRG, PAMAP
GO TO 60
ENDIF

WRITE(6,950) DSTNS, HMINV, HMAXV, PAVRG
60 CONTINUE

CALCULATE MONTE-CARLO STATISTIC

IF ( IMAPP.GT.0 ) THEN
RMINV = AMAX1(XRANG,YRANG)
RMAXV = 0.
WRITE(6,960)
WRITE(6,965) AVG, D
DO 100 IOSM = 1,NOSIM
CSTAT = 0.
RSTAT = 0.
DO 90 ISTEP = 1,NSTEP
JSTAT = 0
ISTAT = NOFRE(IOSIM,ISTEP)
DO 80 JOSIM = 1,NOSIM
IF ( JOSIM.EQ.IOSIM ) GO TO 80
JSTAT = JSTAT + NOFRE(JOSIM,ISTEP)

80 CONTINUE
XSTAT = JSTAT
XSTAT = XSTAT / ( NOPNT - 1 )
CSTAT = ( ( ISTAT - XSTAT ) / NOPNT )*2
RSTAT = RSTAT + CSTAT

90 CONTINUE
IF ( IOSIM.NE.NOSIM.AND.RSTAT.LE.RMINV ) RMINV = RSTAT
IF ( IOSIM.NE.NOSIM.AND.RSTAT.GE.RMAXV ) RMAXV = RSTAT
WRITE(6,970) IOSIM,RSTAT

100 CONTINUE
WRITE(6,980) RMINV,RMAXV,RSTAT
ENDIF

C

940 FORMAT (//, 10X,'NEAREST NEIGHBOR DISTANCE METHOD',//,
* 5X,'DISTANCE MIN. VALUE MAX. VALUE AVERAGE',
* ' MAPPED',
950 FORMAT ( 5X,F7.3,5X,4(F7.5,5X) )
960 FORMAT (//, 3X,'*** MONTE-CARLO STATISTIC',//)
965 FORMAT ( 5X,'SAMPLE MEAN',6X,F15.7,//)
970 FORMAT ( 5X,'STATISTIC .',15,('=','F15.7)' )
980 FORMAT (//, 3X,'EXTREME VALUES OF SIMULATION',//,
* 5X,'MIN. VALUE OF STATISTIC = ','F15.7',/,
* 5X,'MAX. VALUE OF STATISTIC = ','F15.7',/,
* 5X,'MAPPED VALUK = ','F15.7 ')
992 FORMAT (//,' MAPPED COORD IN NEARS ',//)
RETURN
END

C

SUBROUTINE NEDIS ( IOSIM,COORD,NOPNT,DSTEP,NSTEP,NOFRE,
* DISTS,NOSIM,NTOVL,XRANG,YRANG,DFREQ,
* AVRGD )

C

SUBROUTINE NEDIS MEASURES THE NEAREST OTHER EVENT

WITHIN A DISTANCE OF GIVEN EVENT

IMPICLIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION COORD(NOPNT,2), NOFRE(NOSIM,NSTEP), DISTS(NTOVL),
* DFREQ(NOSIM,NOPNT)

C

NPOIN = NOPNT - 1
NCOUN = NOPNT - 1
FSTEP = DSTEP

C

DISMN = AMAX1(XRANG,YRANG)
DISMN = 0.
AVRGD = 0.
DISTANCE CALCULATION

DO 10 IPOIN = 1,NOPNT
   ICOUN = 1
   XCOOR = COORD(IPOIN,1)
   YCOOR = COORD(IPOIN,2)
   DO 20 JPOIN = 1,NOPNT
      IF ( JPOIN.EQ.IPOIN ) GO TO 20
      XCOR1 = COORD(JPOIN,1)
      YCOR1 = COORD(JPOIN,2)
      DISTS(ICOUN) = SQRT((XCOOR-XCOR1)**2+(YCOOR-YCOR1)**2)
10 CONTINUE

CALCULATE AVERAGE NEAREST NEIGHBOR DISTANCE

IF ( IOSIM.EQ.NOSIM ) THEN
   AVERG = AVERG / DISTS(ICOUN)
ENDIF

IF ( DISTS(ICOUN).LE.DISMN ) DISMN = DISTS(ICOUN)
IF ( DISTS(ICOUN).GE.DISMX ) DISMX = DISTS(ICOUN)
   ICOUN = ICOUN + 1
20 CONTINUE
   DDFREQ(IOSIM,IPOIN) = DISMN
   DISMN = AMAX1(XRANG,YRANG)
10 CONTINUE

CALCULATE THE FREQUENCIES OF DISTANCE CATEGORIES

JCOUNT = 0
30 CONTINUE
   JCOUNT = JCOUNT + 1
   KCOUNT = 0
   FSTEP = DSTEP * JCOUNT
   DO 40 IPOIN = 1,NOPNT
      IF ( DDFREQ(IOSIM,IPOIN).LE.FSTEP ) THEN
         KCOUNT = KCOUNT + 1
      ENDIF
40 CONTINUE
   NDFRE(IOSIM,JCOUNT) = KCOUNT
   IF ( JCOUNT.LT.NSTEP ) GO TO 30
RETURN
END

SUBROUTINE IHPPS (COORD,RSCLE,DISTS,SIMUL,NDFRE,DFREQ,SVALU )

SUBROUTINE IHPPS SIMULATES NON-HOMOGENEOUS POISSON POINT PROCESS
WHEN MAPPED POINT PATTERN DO NOT FOLLOW HOMOGENEOUS POISSON
POINT PROCESS

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTR/HDOF, DSTEP, NTOL, NOPNT, NOSIM, NSTEP,
   NTOL, IMAPP, IOPTH, XBOT, XRANG, YBOT,
   YRANG, KXCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD (NOPNT, 2), RSCLE (NTOTL), SIMUL (NTCLS, 2),
*    DISTS (NTOTL), NOFRE (NOSIM, NSTEP), DFREQ (NOSIM, NOPNT),
*    SVALU (NOSIM, NSTEP)

C
REWIND 10

C
XCRIT = 0.5

C
READ CURVE-FITTING PARAMETERS

C
READ (5, *) XVAL1, XVAL2, YVAL1, YVAL2, CONST

C
JOPTN = 1 : INTER-EVENT DISTANCE METHOD
  2 : NEAREST NEIGHBOR DISTANCE METHOD
  3 : SECOND-MOMENT MEASURE METHOD

C
IKERL = 1 : FIXED KERNEL FUNCTION MEASURE
  2 : LINEAR KERNEL FUNCTION

C
READ (5, *) JOPTN, IKERL
WRITE (6, 990) JOPTN, IKERL

C
JITRT IS USED IN FIXED KERNEL FUNCTION OPTION
JITRT = 1 : READ COORDINATES OF GIVEN MAP
  > 1 : SKIP

C
JITRT = 1

C
IXREG = ( XRANG ) / KKCLS
  IYREG = ( YRANG ) / KYCLS

C
NPSIM = NOSIM - 1
IOSIM = 0

C
ICOUN = 0
JCOUN = 0
DO 35 IYCLS = 1, KYCLS
    JCOUN = JCOUN + 1
    YRSUS = IYREG * IYCLS
    YPSUS = IYREG * ( IYCLS - 1 )
    VALUE = ( XVAL1 + XVAL2*IYCLS)*IYCLS + (YVAL1+YVAL2*IYCLS)*IYCLS
    RAMBD = EXP ( VALUE + CONST ) * NOPNT
    ILAMB = RAMBD
    XLAMB = RAMBD - ILAMB
    IF ( XLAMB.GE.XCRIT) ILAMB = ILAMB + 1
    ICOUN = ICOUN + ILAMB
    KCOUN = ICOUN - ILAMB + 1
  35 CONTINUE

C
SIMUL (JCOUN, 1) = JCOUN
SIMUL (JCOUN, 2) = XLAMB

C
NOSUS = ILAMB * NDOFN

C
CALL RANDP (NOSUS, RSCLE, IXCLS, IYCLS, IOSIM)
IITER = 1
DO 20 JLAMB = KCOUN, ICOUN
IDOFN = IITER * 2 - 1
JDOFN = IDOFN + 1
COORD(JLAMB,1) = XBOT + RSCLE(IDOFN) * XRANG
COORD(JLAMB,2) = YPSUS + RSCLE(JDOFN) * IYREG
IITER = IITER + 1
20 CONTINUE
35 CONTINUE
C
IF ( ICOUN.EQ.NOPNT ) GO TO 120
IREMG = NOPNT - ICOUN
NTOJM = KYCLS - 1
DO 50 K = 1, NTOJM
SUMMA = SIMUL(K, 2)
IRESI = K
KP = K + 1
DO 60 L = KP, KTCLS
IF ( SUMMA.GT.SIMUL(L, 2) ) GO TO 60
SUMMA = SIMUL(L, 2)
IRESI = L
60 CONTINUE
DIST1 = SIMUL(IRESI, 1)
DIST2 = SIMUL(IRESI, 2)
SIMUL(IRESI, 1) = SIMUL(K, 1)
SIMUL(IRESI, 2) = SIMUL(K, 2)
SIMUL(K, 1) = DIST1
SIMUL(K, 2) = DIST2
50 CONTINUE
DO 70 I = 1, IREMG
SIMUL(I, 2) = SIMUL(I, 2) + 100.
70 CONTINUE
C
10 CONTINUE
C
120 CONTINUE
IOSIM = IOSIM + 1
ICOUN = 0
MCOUN = 0
DO 90 IYCLS = 1, KYCLS
JCOUN = 0
MCOUN = MCOUN + 1
YPUS = IYREG * ( IYCLS - 1 )
DO 85 LCOUN = 1, KYCLS
XCOUN = MCOUN
IF ( SIMUL(LCOUN, 1).EQ.XCOUN ) GO TO 86
85 CONTINUE
86 CONTINUE
VALUE = (XVAL1+XVAL2*IXCLS)*IXCLS+(YVAL1+YVAL2*IXCLS)*IYCLS
RAMBD = EXP ( VALUE + CONST ) * NOPNT
ILAMB = RAMBD
XLAMB = RAMBD - ILAMB
IF ( XLAMB.LE.XCRIT ) ILAMB = ILAMB + 1
IF ( SIMUL(LCOUN, 2).GE.100. ) ILAMB = ILAMB + 1
C
MODIFY DENSITY ACCORDING TO KERNEL FUNCTION

IF ( IKERL.EQ.1 ) THEN

    DENSE = FLOAT(ILAMB) / ( XRANG*ITYREG )
ELSE
    DENSE = FLOAT(ILAMB) / FLOAT(NOPNT)
ENDIF

ICOUN = ICOUN + ILAMB
KCOUN = ICOUN - ILAMB + 1

NOSUS = ILAMB * NDOFN

NOSUS = 2
88 CONTINUE
CALL RANDP ( NOSUS, RSCLE, IXCLS, IYCLS, IOSIM )

CALCULATE X-DIR. INTENSITY FUNCTION

XVALU = RSCLE(1) * XRANG

IF ( IKERL.EQ.1 ) THEN
    CALL KERNEL ( NOPNT, XVALU, VALUE, JTRT )
    JTRT = JTRT + 1
ELSE
    CALL CURVE ( XVALU, VALUE )
ENDIF

PINTE = VALUE / DENSE

IF ( RSCLE(1).LE.PINTE ) THEN
    COORD(KCOUN,1) = XBOT + RSCLE(1) * XRANG
    COORD(KCOUN,2) = YPSUS + RSCLE(2) * IYREG
    KCOUN = KCOUN + 1
    JCOUN = JCOUN + 1
ENDIF

IF ( JCOUN.LT.ILAMB ) GO TO 88

90 CONTINUE

IF ( IOSIM.EQ.1 ) THEN
    KCONT = KCOUN - 1
    WRITE(6,910) KCONT
    DO 40 IOPNT = 1, NOPNT
    WRITE(6,920) ( COORD(IOPNT, IDOFN), IDOFN=1, NDOFN)
40       CONTINUE

ENDIF

GO TO ( 210, 220, 230 ) JOPTN
210    CALL INDIS ( IOSIM, COORD, NOPNT, DSTEP, NSTEP, NOFRE,
                  *       DISTS, NOSIM, NTOVL )
      GO TO 240
220    CALL NEDIS ( IOSIM, COORD, NOPNT, DSTEP, NSTEP, NOFRE,
                  *       DISTS, NOSIM, NTOVL, XRANG, Xrang, DFRE(?),
* AVRGD 
GO TO 240
230 CALL SMSTT ( IOSIM,COORD,SVALU )
C
240 CONTINUE
IF ( IOSIM.LT.NPSIM ) GO TO 10
C
C MAPPED PATTERN ANALYSIS
C
REWIN 8
READ (8,*) SIGMA
DO 110 IOPNT = 1,NOPNT
READ (8,*) ( COORD(IOPNT,IDOFN),IDOFN=1,NDOFN)
110 CONTINUE
IOSIM = NOSIM
GO TO ( 250, 260, 270 ) JOPTN
250 CALL INDIS ( IOSIM,COORD,NOPNT,DSSTEP,NSTEP,NOFRE,
* WRITE(6,950)
GO TO 280
260 CALL NEDIS ( IOSIM,COORD,NOPNT,DSSTEP,NSTEP,NOFRE,
* AVRGD = AVRGD / ( NOPNT * ( NOPNT - 1 ) )
WRITE(6,930)
GO TO 280
270 CALL SMSTT ( IOSIM,COORD,SVALU )
280 CONTINUE
C
GO TO SUB. KSTAT IN SECOND-MOMENT MEASURE CASE
C
IF ( JOPTN.EQ.3 ) THEN
CALL KSTAT ( SVALU,NOPNT,NOSIM,NSTEP,DSSTEP )
GO TO 300
ENDIF
C
DO 150 ISTEP = 1,NSTEP
MTOTL = 0
MINNO = NOFRE(1,ISTEP)
MAXNO = NOFRE(1,ISTEP)
MTOTL = NOFRE(1,ISTEP)
DO 130 IOSIM = 2,NPSIM
IF ( NOFRE(IOSIM,ISTEP).LT.MINNO ) MINNO = NOFRE(IOSIM,ISTEP)
IF ( NOFRE(IOSIM,ISTEP).GT.MAXNO ) MAXNO = NOFRE(IOSIM,ISTEP)
MTOTL = MTOTL + NOFRE(IOSIM,ISTEP)
130 CONTINUE
C
HMINV = MINNO
HMAXV = MAXNO
IF ( JOPTN.EQ.1 ) THEN
NSTST = NTOVL
ELSE
NSTST = NOPNT
ENDIF
HEXAT = 0.
HAVRG = MTOVL
HMINV = HMINV / NSTST
HMAXV = HMAXV / NSTST
HAVRG = HAVRG / ( NTOVL * NPSIM )
HEXAT = HAVRG

DSTNS = DSTZP * ISTEP

AVERAGE VALUE OF SIMULATED DATA

PAVRG = 0.
DO 140 IOSIM = 1,NPSIM
   PAVRG = PAVRG + NOFRE(IOSIM,ISTEP)
140  CONTINUE

PAVRG = PAVRG / ( NPSIM * NOPNT )

MAPEP PATTERN

PAMAP = NOFRE(NOSIM,ISTEP)
PAMAP = PAMAP / NSTST
IF ( JOPTN.EQ.1 ) THEN
   WRITE(6,940) DSTNS,HMINV,HMAXV,HEXAT,PAMAP
ELSE
   WRITE(6,940) DSTNS,HMINV,HMAXV,PAVRG,PAMAP
ENDIF
150  CONTINUE
300  CONTINUE

MONTE-CARLO STATISTICS

WRITE(6,960)
DO 180 IOSIM = 1,NOSIM
   CSTAT = 0.
   RSTAT = 0.
DO 170 ISTEP = 1,NSTEP
   JSTAT = 0
   IF ( JOPTN.EQ.3 ) THEN
      ZSTAT = SVALU(IOSIM,ISTEP)
   ELSE
      ZSTAT = NOFRE(IOSIM,ISTEP)
   ENDIF

DO 160 JOSIM = 1,NOSIM
   IF ( JOSIM.EQ.IOSIM ) GO TO 160
   IF ( JOPTN.EQ.3 ) THEN
      XSTAT = XSTAT + SVALU(JOSIM,ISTEP)
   ELSE
      JSTAT = JSTAT + NOFRE(JOSIM,ISTEP)
   ENDIF
160  CONTINUE

IF ( JOPTN.NE.3 ) XSTAT = JSTAT
XSTAT = XSTAT / ( NOPNT - 1 )
CSTAT = ((( ZSTAT - XSTAT ) / NOPNT )**2
RSTAT = RSTAT + CSTAT
170  CONTINUE
IF ( IOMIG.EQ.1 ) THEN
    RMINV = RSTAT
    RMAXV = RSTAT
ENDIF

IF ( IOMIG.NE.NOSIM.AND.RSTAT.LE.RMINV ) RMINV = RSTAT
IF ( IOMIG.NE.NOSIM.AND.RSTAT.GE.RMAXV ) RMAXV = RSTAT
WRITE(6,970) IOMIG, RSTAT
180 CONTINUE
WRITE(6,980) RMINV, RMAXV, RSTAT
RETURN

910 FORMAT (/**,5X,'INHOMOGENEOUS POISSON POINT PATTERN',/**,
             *      5X,'TOTAL NO. OF MID-POINT GENERATED =',I5,/**)
920 FORMAT (7X,2(F15.7,3X))
930 FORMAT (/**,7X,'INHOMOGENEOUS POISSON POINT PROCESS',/**,
             *      9X,'NEAREST NEIGHBOR DISTANCE STATISTIC',/**,
             *      5X,'DISTANCE MIN. VALUE MAX. VALUE AVERAGE',
             *      ' MAPPED',/**)
940 FORMAT (5X,F7.3,5X,4(F7.5,5X))
950 FORMAT (/**,7X,'INHOMOGENEOUS POISSON POINT PROCESS',/**,
             *      9X,'INTER-EVENT DISTANCE STATISTIC',/**,
             *      5X,'DISTANCE MIN. VALUE MAX. VALUE AVERAGE',
             *      ' MAPPED',/**)
960 FORMAT (/**,7X,'*** MONTE-CARLO STATISTICS ***',/**)
970 FORMAT (5X,'STATISTIC',I5,' = ',F15.3)
980 FORMAT (/**,3X,'EXTREME VALUES OF SIMULATION',/**,
             *      5X,'MIN. VALUE OF STATISTIC = ',F15.3,/**,
             *      5X,'MAX. VALUE OF STATISTIC = ',F15.3,/**,
             *      5X,'MAPPED VALUE = ',F15.3)
990 FORMAT (/**,5X,'ANALYSIS OPTION = ',I3,
             *      7X,'( 1 : INTER-EVENT DISTANCE METHOD )',
             *      7X,'( 2 : NEAREST NEIGHBOR DISTANCE METHOD )',
             *      7X,'( 3 : SECOND MOMENT MEASURE METHOD )',
             *      5X,'KERNEL FUNCTION OPTION = ',I3,
             *      7X,'( 1 : FIXED KERNEL FUNCTION )',
             *      7X,'( 2 : LINEAR KERNEL FUNCTION )')
END

SUBROUTINE RANDP (NOSUS, RSCLE, IXCLS, IYCLS, IOSIM)
SUBROUTIN RANDP GENERATE PSUDO-RANDOM NUMBER
USING IMSL LIBRARY
DIMENSION RSCLE (NOSUS)

IF ( IYCLS.EQ.1.AND.IOSIM.EQ.0 )
    DSEED = SECONDS(0.0) * 100.0
CALL GGUBS (DSEED, NOSUS, RSCLE)
RETURN
END
SUBROUTINE CURVE ( XVALU, VALUE )

SUBROUTINE CURVE CALCULATE CURVE FITTING COEFFICIENTS

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

FUNC1 = EXP ( -0.5* ( XVALU - 3 )/1.2 )**2
FUNC1 = 0.368*FUNC1/( 1.2*2.5 )
FUNC2 = EXP ( -0.5* ( XVALU - 9.2 )/2.03 )**2
FUNC2 = 0.86*FUNC2/( 2.03*2.5 )
FUNC3 = EXP ( -0.5* ( XVALU - 13.4 )/0.86 )**2
FUNC3 = 0.14*FUNC3/( 0.86 * 2.5 )
FUNC4 = EXP ( -0.5* ( XVALU - 22.65 )/0.93 )**2
FUNC4 = 0.42*FUNC4/( 0.93 * 2.5 )
FUNC5 = EXP ( -0.5* ( XVALU - 34.35 )/4.3 )**2
FUNC5 = 1.42*FUNC5/( 4.3*2.5 )
FUNC6 = EXP ( -0.5* ( XVALU - 44.7 )/4.4 )**2
FUNC6 = 0.41*FUNC6/( 4.4*2.5 )
VALUE = FUNC1 + FUNC2 + FUNC3 + FUNC4 + FUNC5 + FUNC6
RETURN
END

SUBROUTINE KERNEL ( NOPNT, XVALU, VALUE, JITRT )

SUBROUTINE KERNEL EVALUATE FIXED KERNEL FUNCTION OF
ANY GIVEN DATA POINT IN X-DIRECTION

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
DIMENSION COORD(106,2)

IF ( JITRT.EQ.1 ) THEN

READ STANDARD DEVIATION OF THE KERNEL FUNCTION

READ (8,*) SIGMA

READ COORDINATES OF GIVEN MAP

DO 10 IOPNT = 1, NOPNT
   READ (8,*) (COORD(IOPNT, IDOFN), IDOFN=1,2)
10   CONTINUE

WRITE (6,920) SIGMA
ENDIF

PITWO = 6.283185
DO 20 IOPNT = 1, NOPNT
   XMEAN = COORD(IOPNT, 1)
   FUNCT = EXP ( -0.5* ( XVALU - XMEAN )/SIGMA )**2
   VALUE = VALUE + FUNCT / ( SQRT(PITWO)*SIGMA )
20     CONTINUE
CONSIDER EDGE EFFECTS

IF ( XMEAN.LE.20..OR.XMEAN.GE.30. ) THEN
  FUNCT = EXP ( -0.5*( (XVALU + XMEAN)/SIGMA)**2 )
  VALUE = VALUE + FUNCT / ( SQRT(PITWO)*SIGMA )
ENDIF
20 CONTINUE
VALUE = VALUE / NOPNT

920 FORMAT(/,'XSTD. DEV. IN KERNEL FUNCTION = ',F5.3,/) RETURN
END

SUBROUTINE SMSTT ( IOSIM, COORD, SVALU, ANMOM, NOJOB )

SUBROUTINE SMSTT EVALUATE SECOND-MOMENT STATISTIC OF MAPPED DATA

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

COMMON/CONTR/MDOFN, DSTEP, NTOL, NOPNT, NOSIM, NSTEP,
  * NTOL, IMAPP, IOPTN, XBOT, XRANG, YBOT,
  * XRANG, XKCLS, YKCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NOPNT,2), SVALU(NOSIM,NSTEP),
  * ANMOM(NOANG,NSTEP)

NPSIM = NOSIM - 1
NPNT = NOPNT - 1
CNSTV = 1.

DO 30 ISTEP = 1,NSTEP
SVALU(IOSIM,ISTEP) = 0.
DO 30 IOANG = 1,NOANG
ANMOM(IOANG,ISTEP) = 0.
30 CONTINUE

CALCULATE WEIGHTING COEFFICIENT w(x,u)

DO 20 IPOIN = 1,NOPNT
XCOORD = COORD(IPOIN,1)
YCOORD = COORD(IPOIN,2)
DIST1 = AMIN1 ( XCOORD, (XRANG - XCOORD) )
DIST2 = AMIN1 ( YCOORD, (YRANG - YCOORD) )
SQDIS = DIST1**2 + DIST2**2
DO 10 JPOIN = 1,NOPNT
IF ( JPOIN.EQ.IPOIN ) GO TO 10
XCOORD1 = COORD(JPOIN,1)
YCOORD1 = COORD(JPOIN,2)
DIST0 = SQRT((XCOORD-XCOORD1)**2+(YCOORD-YCOORD1)**2)
WEIGHT = 0.
ISTEP = DIST0 / DSTEP + 1
IF ( ISTEP.GT.NSTEP ) GO TO 10
IF ( DIST0**2.LE.SQDIS ) THEN
  DIST3 = ACOS ( CNSTV * AMIN1(DIST1,DIST0) / DIST0 )
  DO 20
  DO 20
  DO 20
DIST4 = ACOS( CNSTV * AMIN(DIST2, DIST0) / DIST0 )
WEIGT = 1.0 - ( DIST3 + DIST4 ) / 3.14159
ELSE
DIST3 = ACOS( CNSTV * DIST1 / DIST0 )
DIST4 = ACOS( CNSTV * DIST2 / DIST0 )
WEIGT = 0.75 - ( DIST3 + DIST4 ) / 6.28318
ENDIF
WEIGT = XRANG * YRANG / ( WEIGT * (NOPNT)**2 )

CALCULATE K FUNCTION

SVALU(IOSIM, ISTEP) = SVALU(IOSIM, ISTEP) + WEIGT

CALCULATE ANGULAR K-FUNCTION

IF ( NOJOB.NE.3 ) GO TO 10
IF ( IOSIM.EQ.NOSIM ) THEN
ANGL2 = ATAN( (YCOR1 - YCOOR) / (XCOR1 - XCOOR) )
IF ( ANGL2.LT.0.0 ) THEN
ANGL1 = 360.0 + ANGL2
ELSE
ANGL1 = ANGL2
ENDIF
IOANG = ANGL1 / ANGLE + 1
WEIGT = XRANG * YRANG / ( NOPNT**2 )
ANIANM(IOANG, ISTEP) = ANIANM(IOANG, ISTEP) + WEIGT
ENDIF

10 CONTINUE
20 CONTINUE

CUMULATIVE K FUNCTION

90 CONTINUE
DO 40 ISTEP = 1, NSTEP
ADDTV = 0.
IF ( ISTEP.EQ.1 ) GO TO 40
KSTEP = ISTEP - 1
C
ADDTV = SVALU(IOSIM, KSTEP)
SVALU(IOSIM, ISTEP) = SVALU(IOSIM, ISTEP) + ADDTV
40 CONTINUE

IF ( IOSIM.NE.NOSIM ) RETURN

CUMULATIVE ANGULAR K-FUNCTION

IF ( NOJOB.NE.3 ) RETURN
DO 70 IOANG = 1, NOANG
DO 60 ISTEP = 1, NSTEP
ADDTV = 0.
IF ( ISTEP.EQ.1 ) GO TO 60
KSTEP = ISTEP - 1

ADDTV = ANMOM(JOANG, KSTEP)
ANMOM(JOANG, ISTEP) = ANMOM(JOANG, ISTEP) + ADDTV

60 CONTINUE
70 CONTINUE
RETURN
END

SUBROUTINE KSTAT(SVALU, NOPNT, NOSIM, NSTEP, DSTEP, XRANG, YRANG, ANMOM, *
          ANGLES, NOANG, NOJOB)

SUBROUTINE KSTAT INTERPRET THE K FUNCTION

IMPLICIT DOUBLE PRECISION (A-H, O-Z)

DIMENSION SVALU(NOSIM, NSTEP), ANMOM(NOANG, NSTEP), ANCOV(100)

FIND MIN. & MAX. VALUE OF SIMULATED DATA

AREAT = XRANG * YRANG
XLAMB = NOPNT / AREAT
XLAM2 = XLAMB**2

WRITE(6, 910)
NPSIM = NOSIM - 1
PHIVU = 3.141592

DO 20 ISTEP = 1, NSTEP
STOTL = 0.
IF (NOJOB.EQ.3) GO TO 15
SMINV = SVALU(1, ISTEP)
SMAVX = SVALU(1, ISTEP)
STOTL = SVALU(1, ISTEP)
DO 10 IOSIM = 2, NPSIM
IF (SVALU(IOSIM, ISTEP).LT.SMINV) SMINV = SVALU(IOSIM, ISTEP)
IF (SVALU(IOSIM, ISTEP).GT.SMAVX) SMAVX = SVALU(IOSIM, ISTEP)
STOTL = STOTL + SVALU(IOSIM, ISTEP)
10 CONTINUE
SAVRG = STOTL / NPSIM

MAPIED PATTERN

15 CONTINUE

SMAFP = SVALU(NOSIM, ISTEP)

STABILIZE THE K VALUE

DSTNS = DSTEP * ISTEP
DSTS2 = DSTNS **2
SMINV = SMINV - PHIVU*DSTS2
SMAVX = SMAVX - PHIVU*DSTS2
SAVRG = SAVRG - PHIVU*DSTS2
SMAFP = SMAFP - PHIVU*DSTS2
IF (NOJOB.EQ.3) THEN
  WRITE (6,920) DSTNS, SVALU(NOSIM, ISTEP)
ELSE
  WRITE (6,920) DSTNS, SMINV, SMAXV, SAVRG, SMAPP
ENDIF
20 CONTINUE
IF ( NOJOB.NE.3 ) RETURN
C
C CALCULATE ANISOTROPIC COVARIANCE FUNCTION
C
WRITE(6,930)
NOAN1 = NOANG / 2
NSTP1 = NSTEP - 1
C
DO 25 ISTEP = 1,NSTEP
DO 26 IOANG = 1, NOAN1
JOANG = IOANG + NOAN1
ANMOM(IOANG,ISTEP) = ANMOM(IOANG,ISTEP) + ANMOM(JOANG,ISTEP)
25 CONTINUE
DO 27 KANGL = 1,18
ANMOM(KANGL,ISTEP) = ANMOM(KANGL,ISTEP) * 18.
27 CONTINUE
DSTS1 = DSTEP * ISTEP
SVUTN = SVALU(NOSIM,ISTEP)
THERY = PHIVU * DSTS1**2
25 CONTINUE
C
WRITE(6,950)
WRITE(6,970)
XLAMB = NOPNT / AREAT
ANGCT = 1.
DO 70 ISTEP = 2, NSTEP
JSTEP = JSTEP - 1
DSTNS = JSTEP * DSTEP
C
ANCO1 = ANMOM(4,ISTEP) - ANMOM(4,JSTEP)
ANCO2 = ANMOM(13,ISTEP) - ANMOM(13,JSTEP)
DKFUN = SVALU(NOSIM,ISTEP) - SVALU(NOSIM,JSTEP)
COEF2 = PHIVU * DSTNS * 2. * DSTEP
C
ANCO1 = ANCO1 / COEF2
ANCO2 = ANCO2 / COEF2
DKFUN = DKFUN / COEF2
C
COVA1 = ANCO1 - 1.
COVA2 = ANCO2 - 1.
COVA3 = DKFUN - 1.
WRITE (6,980) DSTNS, ANCO1, ANCO2, DKFUN, COVA1, COVA2, COVA3
70 CONTINUE
C
910 FORMAT(’/’,7X, ’INHOMOGENEOUS POISSON POINT PROCESS’, ’/’,
* ’/’,7X, ’SECOND-MOMENT MEASUREMENT STATISTIC’,’/’,
* ’3X,’ ’DISTANCE MIN. VALUE MAX. VALUE’,
* ’2X,’ ’AVERAGE MAPPED K-ft COVARIANCE’,
920 FORMAT( 1X,F9.3,1X,5(F9.3,1X),F15.6 )
930 FORMAT(//,7X,'ANGULAR K - FUNCTION',/,'5X,'DIST.',*
  2X,'K(40) K(130) K(MAPPED) K(ISO.)')/
950 FORMAT(//,7X,'ANGULAR K-MOMENT MEASURE',/)
970 FORMAT(//,'5X,'COVARIANCE FUNCTION',/,
  * 7X,' DIST. COV(x,40.) COV(x,130) COV(x,360.)')/
980 FORMAT(2X,F6.3,6(F9.5,1X))
C
RETURN
END

SUBROUTINE SMSTA ( COORD,RSCALE,SVALU )

SUBROUTINE SMSTA CALCULATE SECOND MOMENT MEASURE DISTANCE
OF MID-POINT MAP

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTR/NDOFN, DSTEP, NTOVL, NOPNT, NOSIM, NSTEP,
* NTOVL, IMAPP, IOPTN, XBOT, XRANG, YBOT,
* YRANG, KKCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NOPNT,2), RSCALE(NTOVL), SVALU(NOSIM,NSTEP)

NPSIM = NOSIM
IF ( IMAPP.GT.0 ) NPSIM = NOSIM - 1
IOSIM = 1
DSEED = SECNDS(0.0) * 100.

GENERATE MID-POINT COORDINATE USING RANDOM NUMBER
GENERATOR OF IMSL ( GGUBS )
WRITE(6,900)
10 CONTINUE
CALL GGUBS ( DSEED, NTOVL,RSCALE )
DO 20 IDOFN = 1,NOPNT
JDOFN = IDOFN * 2 - 1
KDOFN = IDOFN * 2
COORD(IDOFN,1) = XBOT + RSCALE(JDOFN) * XRANG
COORD(IDOFN,2) = YBOT + RSCALE(KDOFN) * YRANG
C
IF ( IOSIM.EQ.1 ) THEN
  WRITE(6,910) (COORD(IDOFN,IODOF),IODOF=1,2)
ENDIF
20 CONTINUE

CALCULATE SECOND-MOMENT MEASURE

CALL SMSTT ( IOSIM, COORD, SVALU )
C
IF ( IOSIM.LT.NPSIM ) THEN
  IOSIM = IOSIM + 1
  GO TO 10
ENDIF
C
MAPPED PATTERN ANALYSIS
IF ( IMAPP.GT.0 ) THEN
DO 50 IOPNT = 1,NOPNT
DO 50 IDOFN = 1,NDOFN
COORD(IOPNT,IDOFN) = 0.
50 CONTINUE
DO 60 IOPNT = 1,NOPNT
READ (5,*) ( COORD(IOPNT,IDOFN),IDOFN=1,NDOFN )
60 CONTINUE

IOSIM = NOSIM
CALL SMSTT ( IOSIM, COORD, SVALU )
ENDIF

CALCULATE MIN. & MAX. NO. OF DISTANCE MEASURE

IF ( NOSIM.EQ.1 ) RETURN
CALL KSTAT ( SVALU,NOPNT,NOSIM,NSTEP,DSTEP,XRANG,XRANG)

900 FORMAT (1/5X, 'SECOND MOMENT MEASURE DISTANCE METHOD',1/)
910 FORMAT (12F15.7,5X )

RETURN
END

SUBROUTINE MPHSS ( COORD,TRACE,SVALU )

SUBROUTINE MPHSS EVALUATE MARKED POINT PROCESS AND
CALCULATE K-FUNCTION OF MARKS

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTR/NDOFN, DSTEP, NTOTL, NOPNT, NOSIM, NSTEP,
* NTOTL, IMAPP, IOPNT, XBOT, XRANG, YBOT,
* XRANG, KKCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NOPNT,2),TRACE(NOPNT),
* SVALU(NOSIM,NSTEP)

READ MID-POINT COORDINATE & TRACE LENGTH

TMEAN = 0.
DO 5 IOPNT = 1,NOPNT
READ (7,*) IDUMM, (COORD(IOPNT,IDOFN),IDOFN=1,NDOFN),TRACE(IOPNT)
TMEAN = TMEAN + TRACE(IOPNT)
5 CONTINUE
TMEAN = TMEAN / NOPNT
WRITE(6,999) TMEAN
TMEAN2 = TMEAN**2

INITIALIZE K-FUNCTION & M.P.P. K-FUNCTION

DO 30 ISTEP = 1,NSTEP
SVALU(1,ISTEP) = 0.
SVALU(2,ISTEP) = 0.
30 CONTINUE
CONSIDER EDGE EFFECT

DO 20 IPOINT = 1,NOPNT
XCOOR = COORD(IPOINT,1)
YCOOR = COORD(IPOINT,2)
DIST1 = AMIN1 ( XCOOR, (XRANG - XCOOR) )
DIST2 = AMIN1 ( YCOOR, (YRANG - YCOOR) )
SQDIS = DIST1**2 + DIST2**2
DO 10 JPOINT = 1,NOPNT
IF ( JPOINT .EQ. IPOINT ) GO TO 10
XCOR1 = COORD(JPOINT,1)
YCOR1 = COORD(JPOINT,2)
DIST0 = SQRT((XCOOR-XCOR1)**2+(YCOOR-YCOR1)**2)
WEIGT = 0.
ISTEP = DIST0 / DSTEP + 1
IF ( ISTEP .GT. NSTEP ) GO TO 10
IF ( DIST0**2 .LE. SQDIS ) THEN
   DIST3 = ACOS( AMIN1(DIST1,DIST0)/DIST0 )
   DIST4 = ACOS( AMIN1(DIST2,DIST0)/DIST0 )
   WEIGT = 1. - ( DIST3 + DIST4 ) / 3.14159
ELSE
   DIST3 = ACOS( DIST1 / DIST0 )
   DIST4 = ACOS( DIST2 / DIST0 )
   WEIGT = 0.75 - ( DIST3 + DIST4 ) / 6.28318
ENDIF

SVALU(1,ISTEP) = SVALU(1,ISTEP) + 1. / WEIGT
SVALU(2,ISTEP) = SVALU(2,ISTEP) + 1. / WEIGT *
* TRACE(IPOINT) * TRACE(JPOINT)
10 CONTINUE
20 CONTINUE

CUMULATIVE R & MARKED R FUNCTION

WRITE(6,900)
DO 40 ISTEP = 1,NSTEP
SVALU(2,ISTEP) = SVALU(2,ISTEP) / SVALU(1,ISTEP)
DSTNS = DSTEP * ISTEP
PAIRF = SVALU(2,ISTEP) / TMEN2
PAIRC = SVALU(2,ISTEP) / SVALU(1,ISTEP)
WRITE(6,910) DSTNS, PAIRC, PAIRF
40 CONTINUE

900 FORMAT(/,7X,'MARKED POINT PROCESS',/,
* /,5X,' DISTANCE CORRELATION F STABILIZED F ')
910 FORMAT(5X,3(F10.3,3X) )
999 FORMAT(/,5X,'TMEAN = ',F15.7)
RETURN
END

SUBROUTINE DOUBL ( COORD, RSCALE, DISTS, SIMUL, NOFRE, DFRQ, SVALU, *
* ANMOM )
SUBROUTINE DOUBL ESTIMATES THE DOUBLY STOCHASTIC POINT
PROCESS.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTR/NDOFN, NSTEP, NTTL, NNPNT, NOSIM, NSTEP,
  NTOVL, IMP, IOPTN, XBOT, XRANG, YBOT,
  YRANG, KXCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NNPNT,2), RSCALE(NTTL), SIMUL(KTCLS,2),
  DIST(NTOVL), NOFRE(NOSIM,NSTEP),
  DFREQ(NOSIM,NNPNT), SVALU(NOSIM,NSTEP),
  ANMOA(NOANG,NSTEP)
DIMENSION RSCOL1(5000), HCOOR(5000,2)

REWRIND 10
IFLAG = 0
JITRT = 1
NPSIM = NOSIM - 1

ANALYSIS OPTION
  NOJOB = 1 : DO COX PROCESS WITH BIVARIATE NORMAL KERNEL FUNCTION
  NOJOB = 2 : USING ANGULAR K-FUNCTION SIMULATE THE COX PROCESS
  NOJOB = 3 : CALCULATE THE ANGULAR K-FUNCTION

READ(5,*) NOJOB
GO TO ( 5, 100, 35, 5 ), NOJOB

ITERATE FOR EACH SIMULATIONS

5 CONTINUE
  DO 30 IOSIM = 1,NPSIM
  IF (NOJOB.EQ.4) GO TO 6
  IF ( IOSIM.EQ.1 ) THEN

  CALCULATE THE MAX. DENSITY COEFFICIENT IN AN AXI-SYMMETRIC
  NORMAL DENSITY FUNCTION.

  CALL MAXCF ( COEMX, NNPNT, XRANG, YRANG, XBOT, YBOT )

  GENERATE THE HOMOGENEOUS POISSON POINT PATTERN ACCORDING TO
  THE MAX. DENSITY FUNCTION.
  NOGPT : NO. OF POINTS GENERATED

    AREAT = XRANG * YRANG
    NOGPT = COEMX * AREAT
    NOGPT2 = 2 * NOGPT
    DSEE = 123456.0D0
  ENDIF

10 CONTINUE
  CALL GGUBS ( DSEE, NOGPT2, RSCOL1 )
  DO 20 IDOFN = 1, NOGPT
    JDOFN = IDOFN + NOGPT
    HCOOR(IDOFN,1) = XBOT + RSCOL1(IDOFN) * XRANG
    HCOOR(IDOFN,2) = YBOT + RSCOL1(JDOFN) * YRANG
  END
DO THINNING PROCESS AND GET A COX PATTERN

CALL THINP ( NOGPT, HCOORD, COORD, NOPNT, COEMY, IOSIM,
* XRANG, YRANG, XBOT, YBOT )

2nd MOMENT MEASURE

6 CALL PARENT ( COORD, IOSIM )
CALL SMSST ( IOSIM, COORD, SVALU, ANMOM, NOJOB )
30 CONTINUE
GO TO 85

MAPPED PATTERN ANALYSIS

35 CONTINUE
REWIND 8
READ (8,*) DUMMY, DUMMI, IDUMM
DO 40 NOPNT = 1, NOPNT
READ (8,*) ( COORD(IOPNT,IDOYN), IDOFN=1,NDOFN )
40 CONTINUE
IOSIM = NOSIM
CALL SMSST ( IOSIM, COORD, SVALU, ANMOM, NOJOB )

CALL KSTAT ( SVALU, NOPNT, NOSIM, NSTEP, DSTEP, XRANG, YRANG, ANMOM,
* ANGLE, NOANG, NOJOB )

IF (NOJOB.EQ.3) RETURN
100 CONTINUE
DO 80 IOSIM = 1, NPSIM
CALL SPECT ( NSTEP, NOPNT, XRANG, YRANG, COORD, IOSIM, DSTEP )
CALL SMSST ( IOSIM, COORD, SVALU, ANMOM, NOJOB )
80 CONTINUE

MAPPED PATTERN ANALYSIS

85 CONTINUE
DO 90 IOPNT = 1, NOPNT
READ (8,*) ( COORD(IOPNT,IDOYN), IDOFN=1,NDOFN )
90 CONTINUE
IOSIM = NOSIM
CALL SMSST ( IOSIM, COORD, SVALU, ANMOM, NOJOB )

CALL KSTAT ( SVALU, NOPNT, NOSIM, NSTEP, DSTEP, XRANG, YRANG, ANMOM,
* ANGLE, NOANG, NOJOB )
WRITE(6,910)

MONTE-CARLO STATISTIC

WRITE(6,920)
DO 70 IOSIM = 1, NOSIM
CSTAT = 0.
RSTAT = 0.
DO 60 ISTEP = 1, NSTEP
JSTAT = 0
XSTAT = 0.
ZSTAT = SVALU(IOSIM,ISTEP)
DO 50 JOSIM = 1,NOSIM
   IF ( JOSIM.EQ.IOSIM ) GO TO 50
   XSTAT = XSTAT + SVALU(JOSIM,ISTEP)
50 CONTINUE
   XSTAT = XSTAT / ( NOSIM - 1 )
   CSTAT = ( ZSTAT - XSTAT )**2
   RSTAT = RSTAT + CSTAT
60 CONTINUE
   IF ( IOSIM.EQ.1 ) THEN
      RMINV = RSTAT
      RMAXV = RSTAT
   ENDIF
   IF ( IOSIM.NE.NOSIM.AND.RSTAT.LE.RMINV ) RMINV = RSTAT
   IF ( IOSIM.NE.NOSIM.AND.RSTAT.GE.RMAXV ) RMAXV = RSTAT
   WRITE(6,930) IOSIM, RSTAT
70 CONTINUE
   WRITE(6,940) RMINV, RMAXV, RSTAT
C
910 FORMAT(//,'DOUBLY STOCHASTIC POINT PROCESS',// )
920 FORMAT(//,'*** MONTE - CARLO STATISTIC ***',// )
930 FORMAT( 5X,'STATISTIC',I5, ' = ',F15.3 )
940 FORMAT(//,'EXTREME VALUES OF SIMULATION',//,
* 5X,'MIN. VALUE OF STATISTIC = ',F15.3,/',
* 5X,'MAX. VALUE OF STATISTIC = ',F15.3,/',
* 5X,'MAPPED VALUE = ',F15.3 )
C
RETURN
END

SUBROUTINE COXPP ( NOPNT,XVALU,YVALU,VALUE,JITRT,XRANG,YRANG,
* XBOT, YBOT )
SUBROUTINE COXPP EVALUATE THE DOUBLY STOCHASTIC PROCESS
(COX PROCESS ) IN A PLANE
INTENSITY FUNCTION IS ASSUMED TO SYMMETRIC NORMAL
DENSITY FUNCTION.
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION COORD(39,2), RSCALE(1000)
READ COORDINATES OF THE DATA AT THE FIRST ITERATION
REPEAT 8
NOPN2 = NOPNT * 2
PITWO = 6.283185
IF ( JITRT.EQ.1 ) THEN
READ MEAN VALUE AND STANDARD DEVIATION OF THE NORMAL FUNCTION
LPOTP : 1 : INHOMOGENEOUS INTENSITY FUNCTION
        2 : HOMOGENEOUS INTENSITY FUNCTION
C
READ (8,*) VMEAN, SIGMA, LOPTN

C

IF ( LOPTN.EQ.1 ) THEN
  DO 10 IOPNT = 1, NOPNT
  READ (8,*) ( COORD(IOPNT,IDOFN), IDOFN=1, 2 )
  CONTINUE
10 ELSE
  DSEED = SECONDS(0.0) * 100.
  CALL GGUBS ( DSEED, NOPN2, RSCLE )
  DO 15 IDOFN = 1, NOPNT
    JDOFN = IDOFN + NOPNT
    COORD(IDOFN,1) = XBOT + RSCLE(IDOFN) * XRANG
    COORD(IDOFN,2) = YBOT + RSCLE(JDOFN) * YRANG
  CONTINUE
15 ENDIF
C

WRITE(6,910) VMEAN, SIGMA
ENDDIF
C
C AXI-SYMMETRIC BIVARIATE NORMAL DISTRIBUTION FUNCTION
C
VALUE = 0.
DO 20 IOPNT = 1, NOPNT
  XMEAN = COORD(IOPNT,1)
  YMEAN = COORD(IOPNT,2)
  XVAL1 = ( XVALU - XMEAN )**2
  YVAL1 = ( YVALU - YMEAN )**2
  FUNCT = EXP ( -0.5*( XVAL1 + YVAL1 ) / SIGMA**2 )
  VALUE = VALUE + FUNCT / ( PIMO*SIGMA**2 )
20 CONTINUE
C
VALUE = VALUE * VMEAN
C
910 FORMAT(/,,5X,'MEAN VALUE IN NORMAL FUNCTION = ',F5.3,,/
      *      5X,'STD. DEV. IN NORMAL FUNCTION = ',F5.3,/)
RETURN
END
C
C SUBROUTINE MAXCF ( COEMX, NOPNT, XRANG, YRANG, XBOT, YBOT )
C
C CALCULATES THE MAX. DENSITY COEFFICIENT OF GIVEN DENSITY
C FUNCTION
C
C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C
DIMENSION COEF(5000)
C
LXDIR = XRANG + 1
LYDIR = YRANG + 1
JITRT = 1
C
READ (9,*) IXSTP,IYSTP
C
COEMX = 0.
DO 10 IXCON = 1, LXDIR, IXSTP
C
DO 20 IXCON = 1, LYDIR, IYSTP
XVALU = IXCON
YVALU = IYCON
CALL COXPP ( NOPNT, XVALU, YVALU, VALUE, JITRT, XRANG, YRANG,
* XBOT, YBOT )
COEFT(JITRT) = VALUE

C C C

FIND THE MAX. COEFFICIENT

IF ( COEFT(JITRT) .GT. COEMX ) THEN
COEMX = COEFT(JITRT)
IXREG = IXCON
IYREG = IYCON
ENDIF
JITRT = JITRT + 1
20 CONTINUE
10 CONTINUE
C
WRITE(6,910) COEMX
WRITE(66,920) IYSTP, IYSTP, IXREG, IYREG, COEMX
910 FORMAT(//,5X,'MAX. DENSITY COEFFICIENT = ',F10.4,/) 
920 FORMAT(//,5X,'X-DIR. STEP SIZE = ',I3,/,
* 5X,'Y-DIR. STEP SIZE = ',I3,/,
* 5X,'APPROX. X-COORD = ',I3,/,
* 5X,'APPROX. Y-COORD = ',I3,/,
* 5X,'MAX. DEN. COEFF. = ',F10.4,/) 
C
RETURN
END
C C
SUBROUTINE THINP ( NOGPT, HCOORD, COORD, NOPNT, COEMX, IOSIM,
* XRANG, YRANG, XBOT, YBOT )
C C
SUBROUTINE THINP EVALUATES THE THINNING PROCESS
C C IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION HCOORD(5000,2), RSCE(1), COORD(NOPNT,2)
C C CALL COXPP TO CALCULATE THE DENSITY COEFFICIENT
C
KCOUN = 0
JITRT = 1
40 CONTINUE
JITRT = JITRT + 1
DO 10 IOGPT = 1, NOGPT
XVALU = HCOORD(IOGPT,1)
YVALU = HCOORD(IOGPT,2)
IF ( XVALU.LT.0.) GO TO 10
CALL COXPP ( NOPNT, XVALU, YVALU, VALUE, JITRT, XRANG, YRANG, XBOT, YBOT )
C C
CARE FOR A DSEED VALUE IN RADOP ROUTINE
C
IF ( JITRT.EQ.2 ) THEN
   CALL RANDP ( 1, RSCE, 0, IOGPT, 0 )
ELSE
    CALL RANDP ( 1, RSCLE, 0, 0, 0 )
ENDIF

C

DENSE = VALUE / COEMK
IF ( RSCLE(1) .GT. DENSE ) GO TO 10
KOUN = KOUN + 1
DO 20 IDOFN = 1,2
    COORD(KOUN, IDOFN) = HCOORD(IOGPT, IDOFN)
20 CONTINUE
    HCOORD(IOGPT, 1) = -HCOORD(IOGPT, 1)
IF ( KOUN .EQ. NOPNT ) GO TO 30
10 CONTINUE
30 CONTINUE

C

CALCULATE THE EXACT NO. OF POINTS THAT SHOULD BE GENERATED

WRITE(66,910) JITRT-1, KOUN
IF ( JITRT.GE.5 ) THEN
    WRITE(66,920) JITRT-1
    RETURN
ENDIF
IF ( KOUN .NE. NOPNT ) GO TO 40
IF ( IOSIM .EQ. 1 ) THEN
    WRITE(6,930) KOUN
    WRITE(6,940)
    DO 60 JOPNT = 1, KOUN
    WRITE(6,950) JOPNT, (COORD(JOPNT, IDOFN), IDOFN=1,2)
60 CONTINUE
ENDIF

C

910 FORMAT(//,5X,'NO. OF ITERATIONS TO GENERATE THE POINTS = ',I3,/, * 5X,'NO. POINTS GENERATED = ',I3,/) 920 FORMAT(//,5X,'*** TOO MANY ITERATIONS (',I2,' )',//) 930 FORMAT(//,5X,'NO. OF POINTS GENERATED = ',I5,///) 940 FORMAT(//,5X,'DOUBLY STOCHASTIC POINT PATTERNS',//) 950 FORMAT(3X,I3,5X,2(F15.7,3X))
RETURN
END

CC

SUBROUTINE NORM ( NSTEP, NOPNT, XRNG, XRNG, COORD, IOSIM )

THE CORRELATION COEFFICIENTS WITH GGNML ( IMSL ).

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION COORD(106,2), RSCLE(1000)

REWIND 10
ICOUN = 1
NOTPT = 0
NOGOR = 1
ROGOR = 1.
SIGMA = 0.5
SIGM1 = 1.
NXSTP = XRANG / 2
NYSTP = YRANG / 4
60 CONTINUE
   NOTPT = 0
   DO 20 IYSTP = 1, NYSTP
   DO 10 IXSTP = 1, NXSTP
      IF ( IOSIM.EQ.1.AND.ICOUN.EQ.1 ) DSEED = 123457.D0
      ICOUN = 2
   CALL NORMAL RANDOM DEVIATE GENERATOR OF IMSL
   CALL GGNML ( DSEED, NOGOR, ROGOR )
   COEF1 = ROGOR*SQR(0.5)
   CALL GGNML ( DSEED, NOGOR, ROGOR )
   COEF2 = ROGOR*SQR(0.5)
   CALL XPLASM ( DSEED, NOGOR, ROGOR )
   DSEED = DSEED * DSEED
   XPLAM1 = XPLAM1 + SIGMA**2 * COEF1 * COEF2
   XLAM1 = N0PT1 / ( XRANG * YRANG )
   XLAM2 = XLAM1 + SIGMA**2 * COEF1 * COEF2
   IF ( XLAM1.LT.0 ) XLAM1 = 0.
   XNOPT = XLAM1 * 8.
   NOPT1 = XNOPT
   IF ( NOPT1.GT.0 ) THEN
      NOSUS = NOPT1 * 2
      CALL GGUBS ( DSEED, NOSUS, R3CLE )
      XBCOR = ( IXSTP - 1 ) * 2.
      YBCOR = ( IYSTP - 1 ) * 4.
      DO 40 ILOOP = 1, NOPT1
         IDOFN = ILOOP * 2 - 1
         JDOFN = ILOOP * 2
         IPOIN = NOTPT + ILOOP
         IF ( IPOIN.GT.NOPT1 ) THEN
            NOTPT = NOPNT
            GO TO 30
         ENDIF
         COORD(IPOIN,1) = XBCOR + RSCLE(IDOFN) * 2.
         COORD(IPOIN,2) = YBCOR + RSCLE(JDOFN) * 4.
   40 CONTINUE
   NOTPT = NOTPT + NOPT1
   ENDIF
   ICOUN = ICOUN + 1
10 CONTINUE
20 CONTINUE
30 CONTINUE
   IF ( NOTPT.LT.NOPT1.OR.IYSTP.LT.(NYSTP-1) ) GO TO 60
   IF ( IOSIM.EQ.1 ) THEN
      WRITE(6,920)
      DO 50 IPOIN = 1, NOTPT
         WRITE(6,930) IPOIN, ( COORD(IPOIN,IDOFN), IDOFN=1,2 )
      50 CONTINUE
   ENDIF
SUBROUTINE PAREN (COORD, IOSIM )

SUBROUTINE PAREN EVALUATE THE INTENSITY FUNCTION
OF THE COX PROCESS USING SYMMETRIC NORMAL
DISTRIBUTION FUNCTION FOR PARENT - DAUGHTER MODEL.
( CURRENTLY, NOT AVAILABLE )

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONTN/NDOFN, DSTEP, NTOTL, NCPNT, NOSIM, NSTEP,
*     NTOLV, IMAPP, IOPNT, XBOT, XRANG, YBOT,
*     YRANG, KXCLS, KYCLS, KTCLS, ANGLE, NOANG
DIMENSION COORD(NOPNT,2), RSCALE(80)
DIMENSION HCOORD(10,2), VALUE(50,40)

HCOORD(i,j) : i = total no. of parent points
           : j = ndofn

VALUE(i,j) : i = no. of simulations
             : j = no. of steps

TWOPI = 6.28318
XRAN1 = 14.
YRAN1 = 14.

IF ( IOSIM.EQ.1 ) READ (5,*) NOPAR, NOSON, STDEV, DISTL, PADLL

CALCULATE THE INTENSITIES WITH POISSON DISTRIBUTION
OF THE DATA

IF ( IOSIM.EQ.1 ) READ (5,*) DSEEP, MAITH
1 CONTINUE
   JPOIN = 0
   MODPT = 0
   NODAU = 2
   NOITN = 0
   GO TO 10

10 CALL GGUBS ( DSEEP, NODAU, RSCALE )
   NOITN = NOITN + 1
   IF ( NOITN.GT.MAITH ) GO TO 1

GENERATE THE PARENT POINTS

HCORX = RSCALE(1) * XRAN1
HCORY = RSCALE(2) * YRAN1
HCOXX = HCORX * 1.5 + 5.5
HCOYY = HCORY - 5.5
HCOR1 = 0.76 * HCOXX - 0.64 * HCOYY
HCOR2 = 0.64 * HCOXX + 0.76 * HCOYY
IF ( HCOR1.LE.XBOT.OR.HCOR1.GE.XRANG ) GO TO 10
IF ( HCOR2.LE.YBOT.OR.HCOR2.GE.YRANG ) GO TO 10

C

IF ( MODPT.GT.0 ) THEN
  DO 100 I = 1, MODPT
    PADIS = SQRT( (HCOX-HCOR(I,1))**2+(HCORY-HCOR(I,2))**2 )
    IF ( PADIS.LE.PADLL ) GO TO 10
  CONTINUE
ENDIF
C

MODPT = MODPT + 1
HCOOR(MODPT,1) = HCORX
HCOOR(MODPT,2) = HCORY
IF ( MODPT.LT.NOPAR ) GO TO 10

C

GENERATE THE DAUGHTER POINTS

20 CONTINUE
CALL GGUBS ( DSEEP, 2, RSCLE )
COORX = RSCLE(1) * XRAN1
COORY = RSCLE(2) * YRAN1
CORXX = COORX * 1.5 + 5.5
CORYY = COORY - 5.5
CORK2 = 0.76 * CORXX - 0.64 * CORYY
CORY2 = 0.64 * CORXX + 0.76 * CORYY
IF ( CORK2.LE.XBOT.OR.CORK2.GE.XRANG ) GO TO 20
IF ( CORY2.LE.YBOT.OR.CORY2.GE.YRANG ) GO TO 20
DO 30 IPOIN = 1, NOPAR
  DSTRI = SQRT( (COORX-HCOOR(IPOIN,1))**2 +
                (COORY-HCOOR(IPOIN,2))**2 )
  IF ( IPOIN.EQ.1 ) DSMIN = DSTRI
  IF ( DSTRI.LT.DSMIN ) DSMIN = DSTRI
30 CONTINUE
IF ( DSMIN.GT.DISTL ) GO TO 20
C

JPOIN = JPOIN + 1
COORD(JPOIN,1) = COORX
COORD(JPOIN,2) = COORY
IF ( JPOIN.LT.NOSON ) GO TO 20
WRITE(6,920)
C

CONSTRUCT THE BIVARIATE NORMAL DISTRIBUTION FUNCTION

DO 40 ISTEP = 1, NSTEP
VALUE(IGSIM,ISTEP) = 0.
40 CONTINUE
DO 60 IPONT = 1, NOSON
  DO 50 JPONT = 1, NOSON
    DIST2 = SQRT( (COORD(IPONT,1)-HCORJ(JPONT,1))**2 +
                   (COORD(IPONT,2)-HCORJ(JPONT,2))**2 )
  C

ISTEP = DIST2 / DSTEP
ISTEP = ISTEP + 1
IF ( ISTEP.GT.NSTEP ) ISTEP = NSTEP
XMEAN = HCOOR(JPONT,1)
YMEAN = HCOOR(JPONT,2)
XVAL1 = (COORD(IPONT,1) - XMEAN)**2
YVAL1 = (COORD(IPONT,2) - YMEAN)**2
FUNCT = EXP(-0.5*(XVAL1 + YVAL1)/STDEV**2)
VALFT = FUNCT/(TWOPI*STDEV**2)
VALUE(IOSIM,ISTEP) = VALUE(IOSIM,ISTEP) + VALFT
50 CONTINUE
60 CONTINUE

C
IF (IOSIM.EQ.1) THEN
DO 70 ISTEP = 1,NSTEP
DSTAS = ISTEP*DSTEP
WRITE(6,910) IOSIM,DSTAS,VALUE(IOSIM,ISTEP)
70 CONTINUE
ENDIF
DO 80 I = 1,NOPAR
J = I + NOSON
COORD(J,1) = HCOOR(I,1)
COORD(J,2) = HCOOR(I,2)
80 CONTINUE
WRITE(6,920)
DO 90 I=1,NOPNT
C0R3 = COORD(I,1) * 1.5 + 5.5
C0R3 = COORD(I,2) - 5.5
C0R4 = C0R3*0.76 - C0R3*0.64
C0R4 = C0R3*0.64 + C0R3*0.76
COORD(I,1) = C0R4
COORD(I,2) = C0R4
IF (IOSIM.LE.5) THEN
WRITE(6,960) I, (COORD(I,J),J=1,2)
ENDIF
90 CONTINUE
C
WRITE(6,920)

910 FORMAT(5X,'SIM. = ',I5,5X,2(F15.7,5X))
920 FORMAT(/)
960 FORMAT(5X,I5,5X,2(F15.7,3X))
C
RETURN
END

CC
SUBROUTINE SPECT ( NSTEP,NOPNT,XRANG,YRANG,COORD,IOSIM,DSTEP )

SUBROUTINE SPECT CALCULATES THE SPECTRAL DENSITY FUNCTION
AND DIRECTIONAL INTENSITY FUNCTION.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION COORD(39,2),RSCLE(60000),COOR(30000,2),
* XINTE(200), YINTE(400)

IF (IOSIM.EQ.1) THEN
REVERSE 8
READ(8,*) XCOF2, YCOF2, FMULT, XVARI, YVARI, IPRIN
ENDIF
COEFF = 3.14159
FREQ = COEFF / 2.

C
LXLEN = 50

C
SUMX = 0.
SUMY = 0.
MCOUN = 0
MMCNT = 0
NOTPT = 0

C
X & Y-DIRECTIONAL RANGES OF THE MAP
SHOULD BE CHANGED ACCORDING TO A SPECIFIC MAP CONCERNED

C
X = XRAN1 = 22.
Y = YRAN1 = 14.
IF (IOSIM.EQ.1) JCUNT = 0
XLAM1 = NOPNT / (XRA1 * YRAN1)
IF (IOSIM.GT.1) GO TO 100

C
X- & Y-COORDINATE

C
DELOM = COEFF / LXLEN

C
DO 20 IXSTP = 1,10
XCOORD = FLOAT(IXSTP-1) * 3. + 1.5

C
MMCNT = MMCNT + 1

C
SUMX = 0.

C
DO 10 KSTEP = 0,LXLEN
MCOUN = MCOUN + 1
OMEGA = -FREQ + KSTEP*DELOM
XSPEC = XCOF2 / (COEFF * (OMEGA**2 + XCOF2**2))
INUMB = 1
IF (IOSIM.EQ.1.AND.MCOUN.EQ.1) READ (8,*) DSEEK
CALL GGUBS (DSEEK, INUMB, RSCLE)
THET1 = RSCLE(1) * 2.*COEFF
XOEF5 = SQRT(2.*XSPEC*DELOM) * COS(OMEGA*XCOORD + THET1)
SUMX = SUMX + XOEF5
10 CONTINUE

C
XDEN = SUMX * XVARI
XINTI(IXSTP) = XDEN
IF (IXSTP.EQ.1) XDMAX = XDEN
IF (IXSTP.EQ.1) MAXXD = 1
IF (XDEN.GT.XDMAX) THEN
XDMAX = XDEN
MAXXD = IXSTP
ENDIF
DO 40 IYSTP = 1,7  
YCOORD = FLOAT(IYSTP-1)*3. + 1.5  
SUMMY = 0.

DO 30 KSTEP = 0,LYLEN  
OMEGA = -FREQL + KSTEP*DELM  
YSPEC = YCOORD / ( COEFF * ( OMEGA**2 + YCOORD**2 ) )  
INUMB = 1  
CALL GGUBS ( DSEE, INUMB, RSCLE )  
THET2 = RSCLE(1) * 2. * COEFF  
YOEF5 = SQRT(2.*YSPEC*DELM) * COS(OMEGA*YCOORD + THET2)  
SUMMY = SUMMY + YOEF5

30 CONTINUE

YDENS = SUMMY * YVARI  
YINTE(IYSTP) = YDENS
IF ( IYSTP.EQ.1 ) YDMAX = YDENS  
IF ( IYSTP.EQ.1 ) MAXYD = 1  
IF ( YDENS.GT.YDMAX ) THEN  
  YDMAX = YDENS  
  MAXYD = IYSTP
ENDIF

40 CONTINUE

DO 700 IISTP = 1,15  
CORNK = (IISTP-1.) * 3. + 1.5  
CORNY = (IISTP-1.) * 3. + 1.5  
XINT1 = XINTE(IISTP) / XDMAX  
YINT1 = YINTE(IISTP) / YDMAX  
WRITE(6,710) IISTP, CORNK,CORNY,  
* XINT1, YINT1

700 CONTINUE

710 FORMAT(5X,’STEP Coord X- Y- ’,I5,4F10.4)

DENMAX = ( XDMAX + YDMAX ) * FMULT  
DENMX1 = XDMAX + YDMAX

READ(9,*) DENCR  
WRITE(6,940) XDMAX,MAXXD,YDMAX,MAXYD

100 CONTINUE

NOGPT = XRAN1 * XRAN1 * DENMX  
NOGPT2 = NOGPT * 2  
IF ( IOSIM.EQ.1 ) READ (8,*) DSEE1  
CALL GGUBS ( DSEE1,NOGPT2,RSCLE )  
JOGPT = 1  
DO 110 JOGPT = 1, NOGPT  
IDOFN = JOGPT * 2 - 1  
JDOFN = JOGPT * 2

CORX1 = RSCLE(IDOFN) * XRAN1 + 5.5  
CORY1 = RSCLE(JDOFN) * YRAN1 - 5.5  
CORX2 = 0.76 * CORX1 - 0.64 * CORY1
CORY2 = 0.64 * CORX1 + 0.76 * CORY1
IF ( CORX2.LE.0. OR.CORX2.GE.XRANG ) GO TO 110
IF ( CORY2.LE.0. OR.CORX2.GE.YRANG ) GO TO 110
COOR1(JOGPT,1) = RSCLE(IDOFN) * XRN1
COOR1(JOGPT,2) = RSCLE(JDOFN) * YRN1
JOGPT = JOGPT + 1
110 CONTINUE
WRITE(6,950) JOGPT - 1

C
C THINNING PROCESS
C
DENST = 39. / ( 25. * 17. )
JNUMB = 2
DO 120 JOGPT = 1,JOAPT
XTRI1 = COOR1(JOAPT,1)
YTRI1 = COOR1(JOAPT,2)
XADD1 = SQRT( XTRI1**2 + YTRI1**2 )
C
IXADD = XADD1 * 0.33
IXADD = IXADD + 1
IYADD = YTRI1 * 0.333
IYADD = IYADD + 1
XXDEN = XINTE(IXADD)
YDEN = YINTE(IYADD)
IF ( YDEN.LE.0. OR.XXDEN.LE.0. ) GO TO 120
XXDEN = XXDEN / XDMIN
YDEN = YDEN / YDMIN
XXDEN = XXDEN + YDEN
C
CALL GGUBS ( DSEE1, JNUMB, RSCLE )
C
IF ( XXDEN.GE.DENCR ) THEN
   NOTPT = NOTPT + 1
   IF ( NOTPT.GT.NCPT ) GO TO 150
   CX1 = COOR1(JOAPT,1) + 5.5
   CY1 = COOR1(JOAPT,2) - 5.5
   COORD(NOTPT,1) = CX1*0.76 - CY1*0.64
   COORD(NOTPT,2) = CX1*0.64 + CY1*0.76
ENDIF
120 CONTINUE
C
150 CONTINUE
WRITE(6,910) JOGPT
IF ( IOSIM.LE.IPRIN ) THEN
   WRITE(6,920)
   DO 160 IOPNT = 1,ICPT
   WRITE(6,930) IOPNT, ( COORD(IOPNT,IDOFN),IDOFN = 1,2 )
160 CONTINUE
ENDIF
C
910 FORMAT(5X,' NO. OF ITERATIONS = ',/I5)
920 FORMAT(//,5X,'COORD. OF POINTS GENERATED BY COX PROCESS',/,
   * X-COORD. Y-COORD.',/)
930 FORMAT(5X,I5,2(F10.3,3X) )
940 FORMAT(//,5X,'MAX. X-DIR. INTENSITY = ',F12.5,' AT ',/I5,
* /, 5X, 'MAX. Y-DIR. INTENSITY = ', F12.5, ' AT ', I5, //)
950 FORMAT (/, 5X, 'TOTAL NO. OF POINTS GENERATED IN SPECT. = ', I5)
RETURN
END

PROGRAM HIERA

CCCCCCC

PROGRAM HIERA EVALUATES THE HIERARCHICAL FIBER
CC ( LINE - SEGMENT ) PROCESS WHICH IS A REALIZATION
CC OF THE GIVEN MAP.

CC

CC

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMM/NCONFN/NOPNT, NOPT, NOLPT, XRANG, YRANG, XBOT, YBOT, NOSIM,
* NTOTAL, NDOP, NSTEP, DSTEP, NDOPN, SIGMA, TSTEP, MAXNW,
* MAXW, IOPT, JOPT,LOPTN, DFACT
DIMENSION KK(20000), AA(30000), TITLE(20)

CC

CC INITIAL SETS OF THE DIMENSION

CC

NRVAR = 30000
NIVAR = 20000
NDOPN = 2
NDOP = NDOPN * 2
DFACT = 0.

CC

READ (5, 800) TITLE
WRITE (6, 800) TITLE

CC 10 CONTINUE

CC READ VARIABLES

CC NOPNT : NO. OF TOTAL MID POINTS IN A MAP
CC NOPT : NO. OF TOTAL MID POINTS IN A FORMER SET
CC NOLPT : NO. OF TOTAL MID POINTS IN A LATTER SET
CC NOSIM : NO. OF SIMULATIONS
CC NSTEP : NO. OF STEPS IN CALCULATING THE STATISTIC
CC DSTEP : DISTANCE STEP SIZE
CC SIGMA : STD. DEV. OF THE LINE - KERNEL FUNCTION
CC TSTEP : UNIT OF SEGMENT IN LINE - KERNEL FUNCTION
CC OR THE PERTURBATION DISTANCE IN INDEPENDENCE TEST

CC READ (5, *) NOPNT, NOPT, NOLPT, NOSIM, NSTEP, DSTEP, SIGMA, TSTEP
WRITE (6, 910) NOPNT, NOPT, NOLPT, NOSIM, NSTEP, DSTEP, SIGMA, TSTEP

CC READ BOUNDARY OF THE SIMULATION REGION

CC READ (5, *) XBOT, XRANG, YBOT, YRANG
XTOP = XBOT + XRANG
YTOP = YBOT + YRANG
WRITE (6, 920) XBOT, XTOP, YBOT, YTOP

CC MAXNW = 2
MAXXW = 2
READ (5,*) IOPTN, JOPTN, LOPTN
IF ( JOPTN GE 3 ) THEN
   READ (5,*) XWIND, YWIND, DFACT
   MAXXW = XRANG / XWIND + 1
   MAXYW = YRANG / YWIND + 1
ENDIF

DYNAMIC ALLOCATION

NR1 : COORD : COORD. OF THE TOTAL TRACES ( 4 FOR EACH TRACE )
NR2 = CORFB : COORD. OF THE FORMER SET ( )
NR3 = SECON : SLOPE OF ALL FIBERS ( NO. OF FIBERS )
NR4 = TRACE : TRACCE LENGTH OF THE FORMER SET
NR5 = CORLF : COORD. OF THE LATTER SET ( )
NR6 = COORL : WORKING ARRAY FOR THE LATTER SET ( 2 FOR TRACE )
NR7 = COORF : WORKING ARRAY FOR THE LATTER SET ( )
NR8 = SVALU : VALUES OF THE STATISTIC ( NOSIM, NSTEP )
NR9 = CPFST : CENTER POINTS OF THE WHOLE SET ( NOPNT, 2 )
NR10 = CPSND : CENTER POINTS OF LATTER SET ( NOLPT, 2 )
NR11 = DENTY : INTENSITY VARIATION OF LATTER SET ( JOPTN=2 )

NRTOTL = NDOFN * NOPNT
NRTOT2 = NDOF2 * NOPNT
NRTFT = NDOF2 * NOFFT
NRTLFT = NDOF2 * NOLFT
NOSAS = NOSIM * NSTEP
NOWIN = MAXXW * MAXYW

NR1 = 1
NR2 = NR1 + NRTOT2
NR3 = NR2 + NRTFT
NR4 = NR3 + NOPNT
NR5 = NR4 + NOFFT
NR6 = NR5 + NTLFT
NR7 = NR6 + NOLFT * 2
NR8 = NR7 + NOFFT * 2
NR9 = NR8 + NOSAS
NR10 = NR9 + NOPNT * 2
NR11 = NR10 + NOLPT * 2
NR12 = NR11 + NOWIN

N11 = 1

NRTOT = NR12 - 1
NITOT = N11 - 1
IERRO = 0

WRITE(6,930) NRTOT, NRTOT
IF ( NRTOT.GT.NRTOT ) GO TO 20
WRITE(6,940)
IERRO = IERRO + 1
20 CONTINUE
WRITE(6,950) NITOT, NITOT
IF ( NITOT.GT.NITOT ) GO TO 30
WRITE (6, 960)
IERRO = IERRO + 1
30 CONTINUE
IF ( IERRO.GT.0 ) STOP

C

C INITIALIZE

DO 40 IVARI = 1, NRVAR
40 AA(IVARI) = 0.
DO 50 IVARI = 1, NIVAR
50 KK(IVARI) = 0

C

CALL MAIN

CALL MAINS ( AA(NR1), AA(NR2), AA(NR3), AA(NR4), AA(NR5),
* AA(NR6), AA(NR7), AA(NR8), AA(NR9), AA(NR10),
* AA(NR11) )

C

800 FORMAT (20A4)
910 FORMAT (/5X, 'NO. OF TOTAL MID POINTS IN A MAP' = ',T60,I5,
* /5X, 'NO. OF TOTAL MID POINTS IN A FORMER SET' = ',T60,I5,
* /5X, 'NO. OF TOTAL MID POINTS IN A LATTER SET' = ',T60,I5,
* /5X, 'NO. OF SIMULATIONS' = ',T60,I5,
* /5X, 'NO. OF INTERPRETATION STEPS' = ',T60,I5,
* /5X, 'DISTANCE STEP SIZE' = ',T60,F6.3,
* /5X, 'STD. DEV. OF LINE-KERNEL FUNCTION' = ',T60,F6.3,
* /5X, 'UNIT OF SEGMENT IN LINE-KERNEL FUNCTION' = ',T60,F6.3)
920 FORMAT (/5X, 'BOUNDARY OF THE SIMULATION',
* /7X, 'X-DIR. BOUNDARY ( FROM : TO )' = ',T60,2(F10.3,2X),
* /7X, 'Y-DIR. BOUNDARY ( FROM : TO )' = ',T60,2(F10.3,2X) )
930 FORMAT (/5X, 'REAL STORAGE REQUIRED' = ',I5,
* /5X, 'REAL STORAGE SPECIFIED' = ',I5,/)
940 FORMAT (/5X, '*** INCREASE STORAGE FOR REAL ARRAYS ***')
950 FORMAT (/5X, 'INTEGER STORAGE REQUIRED' = ',I5,
* /5X, 'INTEGER STORAGE SPECIFIED' = ',I5,/)
960 FORMAT (/5X, '*** INCREASE STORAGE FOR INTEGER ARRAYS ***')

C

STOP
END

SUBROUTINE MAINS ( COORD, CORFB, SECON, TRACE, CORLF, COORL,
* COORF, SVALU, CPFST, CPSND, DENTY )

SUBROUTINE MAINS CONTROLS THE MAIN OPTION

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONFL/NOPNT, NOPPT, NOLPT, XANG, YANG, XBOT, YBOT, NOSIM,
* NTOTL, NDOS2, NSTEP, DSTEP, NDOMN, SIGMA, TSTEP, MAXXX,
* MAXXW, IOPTM, JOPTN, LOPTM, LOPTN, DFACT
DIMENSION CORDF (NOPNT, 4), CORFB (NOPPT, 4), SECON (NOPNT),
* CORL (NOLPT, 4), COORL (NOLPT, 2), TRACE (NOPPT),
* COORF (NOPPT, 2), CPFST (NOPNT, 2), CPSND (NOLPT, 2)
DIMENSION SVALU (NOSIM, NSTEP), DENTY (MAXXX, MAXXW)
READ OPTION VARIABLES

IOPTN : MAIN OPTION FOR THE POINT PROCESS
( 0 : DO NOT ENTER THE POINT PROCESS )
( 1 : BIVARIATE KERNEL FUNCTION METHOD )
( 2 : INDEPENDENCE TEST FOR SETS )
( 3 : ORIENTATION CORRELATION OPTION )
( 4 : MLE FOR ORIENTATION DATA )

JOPTH : MAIN OPTION FOR THE FIBER PROCESS
( 0 : DO NOT ENTER THE FIBER PROCESS )
( 1 : MLE FOR LINE - KERNEL FUNCTION )
( 2 : MLE FOR NEAREST NEIGHBOR FIBERS )
( 3 : SIMULATION OF SET 2 WITH L - K )
( 4 : SIMULATION OF SET 2 WITH NN F )

LOPTN : ANALYSIS OPTION
( 1 : CALCULATE BIVARIATE SECOND MOMENT )
( 2 : CALCULATE UNIVARIATE SECOND MOMENT )

WRITE(6,910) IOPTN, JOPTH, LOPTN

FIRST, SIMULATE THE FORMER SET USING POINT PROCESS

IF ( IOPTN.EQ.0 ) GO TO 99
GO TO ( 10, 20, 30, 30 ), IOPTN

IOPTN = 1 : BIVARIATE KERNEL FUNCTION METHOD

10 CONTINUE
GO TO 90

IOPTN = 2 : BIVARIATE INDEPENDENCE TEST

20 CONTINUE
CALL INDEP ( COORD, CORFB, CORLF, SVALU, CPFST, CPSND )
GO TO 90

IOPTN = 3 : ORIENTATION CORRELATION OPTION ( BIVARIATE )
IOPTN = 4 : MLE FOR ORIENTATION DATA

30 CONTINUE
CALL ORINT ( COORD, TRACE, SECON )

90 CONTINUE

( 1 : LOG DISTRIBUTION )
( 2 : EXPONENTIAL DISTRIBUTION )

CALL LENGT ( KOPTN, COORF )

99 CONTINUE

IF ( JOPTH.EQ.0 ) RETURN

SECOND, SIMULATE THE LATTER SET USING FIBER PROCESS
IF ( JOPTN.LE.2 ) READ (5,*) XLAMO
DO 60 IOPNT = 1, NOFFT
   READ (5,*) ( COORD(IOPNT,IDOFN), IDOFN = 1, NDOP2 )
60 CONTINUE
C
GO TO ( 100, 200, 300, 300 ), JOPTN
C
JOPTN = 1 : MLE OF LINE - KERNEL FUNCTION
C
100 CONTINUE
   CALL KMLE ( COORD, CORFB, SECON, TRACE, CORLF, COORL, SVALU,
               * XLAMO );
RETURN
C
JOPTN = 2 : MLE OF NEAREST NEIGHBOR FIBERS
C
200 CONTINUE
   CALL NNMLE ( COORD, TRACE, SECON, XLAMO )
RETURN
C
JOPTN = 3 : SIMULATION OF SET 2 WITH L - K FUNCTION
C
JOPTN = 4 : SIMULATION OF SET 2 WITH N.N. FUNCTION
C
300 CONTINUE
   CALL FIBRE ( COORD, CPFST, SECON, TRACE, CPSND, COORL, SVALU,
               * DENTY );
RETURN
C
910 FORMAT(/,,5X,'MAIN OPTIONS :',
* /,,7X,'MAIN OPTION FOR THE POINT PROCESS' && ',',T60,15,
* /,,9X,'( 0 : DO NOT ENTER THE POINT PROCESS )',
* /,,9X,'( 1 : BIVARIATE KERNEL FUNCTION METHOD )',
* /,,9X,'( 2 : INDEPENDENT TEST FOR SETS )',
* /,,9X,'( 3 : CORRELATION MEASURE OF ANGLE )',
* /,,9X,'( 4 : MLE OF ORIENTATION DATA )',
* /,,7X,'MAIN OPTION FOR THE FIBER PROCESS' && ',',T60,15,
* /,,9X,'( 0 : NOT CONSIDER THE FIBER PROCESS )',
* /,,9X,'( 1 : MLE OF LINE - KERNEL FUNCTION )',
* /,,9X,'( 2 : MLE OF NEAREST NEIGHBOR FIBERS )',
* /,,9X,'( 3 : SIMULATION WITH L - K FUNCTION )',
* /,,9X,'( 4 : SIMULATION WITH N. N. FUNCTION )',
* /,,7X,'ANALYSIS OPTION FOR POINT PATTERN' && ',',T60,15,
* /,,9X,'( 1 : BIVARIATE SECOND MOMENT OPTION )',
* /,,9X,'( 2 : UNIVARIATE SECOND MOMENT OPTION )')
C
END
C
SUBROUTINE FIBRE ( COORD, CPFST, SECON, TRACE, CPSND, COORL, SVALU,
               * DENTY )
C
SUBROUTINE FIBRE EVALUATES THE HIERARCHICAL FIBER MODEL
WITH LINE - KERNEL FUNCTION.
SIMULATE THE SECOND SET USING THE DATA FROM MLE

JITRT = 1
TWOPI = 6.28318
NPLU1 = NOFPT + 1
NPLU2 = NOFPT + NOLPT
Xlam2 = NOLPT / (XRANG * YRANG)

STORE THE MID POINTS DATA FOR SECOND MOMENT ANALYSIS

DO 5 IOPNT = 1, NOFPT
    XCOORD = (COORD(IOPNT,1) + COORD(IOPNT,3)) / 2.
    YCOORD = (COORD(IOPNT,2) + COORD(IOPNT,4)) / 2.
    CPFST(IOPNT,1) = XCOORD
    CPFST(IOPNT,2) = YCOORD
5 CONTINUE

READ MID POINT OF THE LATTER SET (MAPPED DATA)

DO 10 IOPNT = NPLU1, NPLU2
    READ (5,*) (COORD(IOPNT,IDOFN), IDOFN=1,NDOF2)
    XCOORD = (COORD(IOPNT,1) + COORD(IOPNT,3)) / 2.
    YCOORD = (COORD(IOPNT,2) + COORD(IOPNT,4)) / 2.
    JOPT = IOPNT - NOFPT
    COORN(JOPT,1) = XCOORD
    COORN(JOPT,2) = YCOORD
10 CONTINUE

FIND INTENSITY VARIATION WITH BIVARIATE NORMAL DENSITY FUNCTION

MAXXW = XRANG / XWIND + 1
MAXYW = YRANG / YWIND + 1

XWIND = XRANG / (MAXXW - 1.)
YWIND = YRANG / (MAXYW - 1.)
WRITE(6,950) XWIND, YWIND, DFACT

DO 20 IXWIN = 1, MAXXW
    DO 20 IYWIN = 1, MAXYW
        DENTY(IXWIN, IYWIN) = 0.
20 CONTINUE

JITRT = 1
DO 30 IOPNT = 1, NOLPT
    XCOORD = COORN(IOPNT,1)
    YCOORD = COORN(IOPNT,2)
IXMIN = XCOORD / XWIND + 1
IXWIN = YCOORD / YWIND + 1

JOPTN = 3 : SIMULATION WITH L - K FUNCTION

IF ( JOPTN.EQ.3 ) THEN
   CALL LKERN (JITRT, XCOORD, YCOORD, TRACE, VALUE, COORD, SECON,
               *       NTOFB)
   JITRT = 2
   DENTY(IXWIN,IXWIN) = DENTY(IXWIN,IXWIN) +
   *          VALUE * 0.35 / FLOAT(HOLPT) + 0.65 / (22.*14.)
ELSE

JOPTN = 4 : SIMULATION WITH N. N. FUNCTION

CALL NEARF (JITRT, XCOORD, YCOORD, FDIST, COORD, TRACE, SECON)
JITRT = 2
DENTY(IXWIN,IXWIN) = DENTY(IXWIN,IXWIN) +
*      EXP ( -0.5 * (FDIST/SIGMA)**2 ) / ( SIGMA*SQRT(TWOPI)
*      * FLOAT(HOLPT) ) * 0.35 + 0.65 / (22.*14.)
ENDIF

IF ( JOPTN.EQ.1 ) DENMX = DENTY(IXWIN,IXWIN)
IF ( DENTY(IXWIN,IXWIN).GT.DENMX ) DENMX = DENTY(IXWIN,IXWIN)
30 CONTINUE

DENMX = DENMX * DFACT
KOPTN = 1

SIMULATION STEP

NPSIM = NOSIM - 1
IF ( NPSIM.EQ.0 ) RETURN

DO 50 IOSIM = 1, NPSIM
   IF ( IOSIM.LE.5 ) WRITE(6,910) IOSIM
50

SIMULATION BY NORMAL DISTRIBUTION FUNCTION

ICONT = 0
KCONT = 0

220 CONTINUE
   KCONT = KCONT + 1

GENERATE RANDOM POINT WHICH REPRESENTS THE MID POINT OF THE
   LATTER SET.

NOSUS = 2
CALL RANDF ( NOSUS, RSCALE, KOPTN )
KOPTN = 2

XCOORD = RSCALE(1) * XRANG
YCOORD = RSCALE(2) * YRANG
IXWIN = XCOOR / XWIN
IYWIN = YCOOR / YWIN
IXWIN = IXWIN + 1
IYWIN = IYWIN + 1
VALUE = DENTY(IXWIN, IYWIN)

CALL RANDF ( 1, RSCLE, KOPTN )
IF ( KCONT.GT.10000 ) RETURN
IF ( RSCLE(1).GT. (VALUE/DENMK) ) GO TO 220
ICONT = ICONT + 1
COORL(ICONT,1) = XCOOR
COORL(ICONT,2) = YCOOR

IF (IOSIM.LE.5) WRITE(6,920) ICONT, COORL(ICONT,1), COORL(ICONT,2)
IF ( ICONT.LT.KOLPT ) GO TO 220
WRITE(6,930) KCONT

CALCULATE APPROPRIATE SECOND MOMENT

LOPTN = 1 : BIVARIATE SECOND MOMENT OPTION
2 : UNIVARIATE SECOND MOMENT OPTION

IF ( LOPTN.EQ.2 ) THEN

CALCULATE THE SECOND MOMENT

CALL SMSTT ( IOSIM, COORL, SVALU )
ELSE

CALCULATE THE BIVARIATE SECOND MOMENT

DO 230 KOPNT = 1, NOFPT
   XCOOR = CPFST(KCPNT,1)
   YCOOR = CPFST(KOPNT,2)
   CALL BVSSMS ( XCOOR, YCOOR, COORL, SVALU, IOSIM )
230  CONTINUE
ENDIF

50 CONTINUE

CALCULATE THE SECOND MOMENT FOR REAL DATA SET 2

IOSIM = NOSIM

IF ( LOPTN.EQ.2 ) THEN
   CALL SMSTT ( IOSIM, COORN, SVALU )
ELSE

CALCULATE THE BIVARIATE SECOND MOMENT FOR REAL DATA SET 2

DO 240 KOPNT = 1, NOFPT
   XCOOR = CPFST(KCPNT,1)
   YCOOR = CPFST(KOPNT,2)
   CALL BVSSMS ( XCOOR, YCOOR, COORN, SVALU, IOSIM )
240  CONTINUE
ENDIF
CALCULATE THE CUMULATIVE $K$ FUNCTION

DO 260 IOSIM = 1, NOSIM
DO 250 ISTEP = 1, NSTEP
ADTV = 0.
IF ( ISTEP.EQ.1 ) GO TO 250
KSTEP = ISTEP - 1
ADTV = SVALU(IOSIM,KSTEP)
SVALU(IOSIM,ISTEP) = SVALU(IOSIM,ISTEP) + ADDTV
250 CONTINUE
260 CONTINUE

CALL KSTAT ( SVALU )

MONTE - CARLO STATISTICS

WRITE(6,890)
DO 90 IOSIM = 1, NOSIM
CSTAT = 0.
RSTAT = 0.
DO 80 ISTEP = 1, NSTEP
JSTAT = 0
XSTAT = 0.
ZSTAT = SVALU(IOSIM,ISTEP)
DO 70 JOSIM = 1, NOSIM
IF ( JOSIM.EQ.IOSIM ) GO TO 70
XSTAT = XSTAT + SVALU(JOSIM,ISTEP)
70 CONTINUE
XSTAT = XSTAT / ( NOSIM - 1 )
CSTAT = ( ZSTAT - XSTAT )**2
RSTAT = RSTAT + CSTAT
80 CONTINUE
IF ( IOSIM.EQ.1 ) THEN
RMINV = RSTAT
RMAXV = RSTAT
ENDIF
IF ( IOSIM.NE.NOSIM.AND.RSTAT.LE.RMINV ) RMINV = RSTAT
IF ( IOSIM.NE.NOSIM.AND.RSTAT.GE.RMAXV ) RMAXV = RSTAT
WRITE(6,990) IOSIM, RSTAT
90 CONTINUE
WRITE(6,990) RMINV, RMAXV, RSTAT

890 FORMAT(//,5X,'MONTE - CARLO TEST',//)
910 FORMAT(//,5X,'SIMULATIONS OF THE SET 2 WITH LINE - KERNEL FT.

* /,5X,'ITERATION STEP = ',I5,
* /,9X,'X - COORD'//)
920 FORMAT( 5X,I5,3X,2(F15.7,3X) )
930 FORMAT(//,5X,'NO. OF ITERATIONS = ',I5,//)
950 FORMAT(//,5X,'X-DIR. WINDOW WIDTH = ',F10.5,

* /,5X,'Y-DIR. WINDOW WIDTH = ',F10.5,
* /,5X,'DENSITY FACTOR = ',F10.5,
* //)
980 FORMAT( 5X,'STATISTIC',I5,' = ',F15.3)
990 FORMAT(//,5X,'MIN. VALUE OF STATISTIC = ',F15.3,
* /,5X,'MAX. VALUE OF STATISTIC = ','F15.3,
* /,5X,'MAPPED VALUE = ','F15.3 )
C
RETURN
END

SUBROUTINE SMSTT ( IOSIM, COORL, SVALU )
SUBROUTINE SMSTT EVALUATE SECOND-MOMENT STATISTIC OF
MAPPED DATA
IMPLICIT DOUBLE PRECISION (A-H,O-Z)

COMMON/CONFL/NOPNT, NOLPT, XBOT, XBOT, NOSIM,
* NTOTL, NDOF2, NSTEP, DSTEP, NDOFN, SIGMA, TSTEP, MAXXW,
* MAXXW, ILOPTN, JOPTN, LOPTN, DFACT
DIMENSION COORL(NOLPT, 2), SVALU(NOSIM, NSTEP)

NPSIM = NOSIM - 1
CNSTV = 1.

DO 30 ISTEP = 1,NSTEP
SVALU(IOSIM,ISTEP) = 0.
30 CONTINUE

CALCULATE WEIGHTING COEFFICIENT w(x, u)

DO 20 IPOIN = 1, NOLPT
XCOORD = COORL(IPOIN,1)
YCOORD = COORL(IPOIN,2)
DIST1 = AMIN1( XCOORD, (XRANG - XCOORD) )
DIST2 = AMIN1( YCOORD, (YRANG - YCOORD) )
SQDIS = DIST1**2 + DIST2**2
DO 10 JPOIN = 1,NOLPT
IF ( JPOIN.EQ.IPOIN ) GO TO 10
XCOORD1 = COORL(JPOIN,1)
YCOORD1 = COORL(JPOIN,2)
DIST0 = SQRT((XCOORD-XCOORD1)**2+(YCOORD-YCOORD1)**2)
10 CONTINUE

WEIGT = 0.
ISTEP = DIST0 / DSTEP + 1
IF ( ISTEP.GT.NSTEP ) GO TO 10
IF ( DIST0**2.LE.SQDIS ) THEN
   WEIGT = 1.
ELSE
   DIST3 = ACOS( CNSTV * DIST1 / DIST0 )
   DIST4 = ACOS( CNSTV * DIST2 / DIST0 )
   WEIGT = 0.75 - ( DIST3 + DIST4 ) / 6.28318
ENDIF

WEIGT = XRANG * YRANG / ( WEIGT * (NOLPT)**2 )

CALCULATE K FUNCTION

SVALU(IOSIM,ISTEP) = SVALU(IOSIM,ISTEP) + WEIGT
10 CONTINUE
20 CONTINUE

RETURN
END

SUBROUTINE KSTAT ( SVALU )

SUBROUTINE KSTST INTERPRET THE K FUNCTION

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

COMMON/CONFL/NOPNT, NOPPT, NOLPT, XRANG, YRANG, XBOT, YBOT, NOSIM,
* NTOFL, NDOF2, NSTEP, DSTEP, NDOWN, SIGMA, TSTEP, MAXW,
* MAXXW, IOPNT, JOPTN, LOPTN, DFAC

DIMENSION SVALU(NOSIM, NSTEP)

FIND MIN. & MAX. VALUE OF SIMULATED DATA

AREAT = XRANG * YRANG
XLAMB = NOLPT / AREAT
XLAM2 = XLAMB**2

WRITE(6,910)

NPSIM = NOSIM - 1
PHIVU = 3.141592

DO 20 ISTEP = 1, NSTEP
STOTL = 0.
SMINV = SVALU(1, ISTEP)
SMAKV = SVALU(1, ISTEP)
STOTL = SVALU(1, ISTEP)
DO 10 IOSIM = 2, NPSIM
IF ( SVALU(IOSIM, ISTEP).LT.SMINV ) SMINV = SVALU(IOSIM, ISTEP)
IF ( SVALU(IOSIM, ISTEP).GT.SMAKV ) SMAKV = SVALU(IOSIM, ISTEP)
STOTL = STOTL + SVALU(IOSIM, ISTEP)
10 CONTINUE
SAVRG = STOTL / NPSIM

MAPPED PATTERN

15 CONTINUE

SMAPP = SVALU(NOSIM, ISTEP)

STABILIZE THE K VALUE

DSTNS = DSTEP * ISTEP
DSTS2 = DSTNS **2
SMINV = SMINV - PHIVU*DSTS2
SMAKV = SMAKV - PHIVU*DSTS2
SAVRG = SAVRG - PHIVU*DSTS2
SMAPP = SMAPP - PHIVU*DSTS2
C
WRITE (6, 920) DSTNS, SMINV, SMAV, SAVRG, SMAPP
20 CONTINUE
C
910 FORMAT (' ', X, ' INHOMOGEONEOUS POISSON POINT PROCESS ', '
  
  ', X, ' SECOND-MOMENT MEASUREMENT STATISTIC ', '
  
  ', X, ' DISTANCE MIN. VALUE MAX. VALUE ', 
  
  ', X, ' AVERAGE MAPPED ' )
920 FORMAT ( 4X, F9.3, 3X, 4(F12.4, 2X) )
C
RETURN
END
C
SUBROUTINE LKERN ( JITRT, XCOOR, YCOOR, TRACE, VALUE, COORD,
  
  SECON, NTOFB)
C
SUBROUTINE LKERN CALCULATE THE LINE KERNEL FUNCTION
OF THE FIBER PROCESS.
AS AN ASSUMPTION, WE ONLY CONSIDER THE LINE LENGTH.

IMPLICIT DOUBLE PRECISION ( A-H, O-Z )
C
COMMON/COFL/NOPNT, NOFPT, NOLPT, XRANG, YRANG, XBOT, YBOT, NOSIM,

*   NTOFB, NDF2, NSTEP, DSTEP, NDFN, SIGMA, TSTEP, MAXRN,

*   MAXRN, IOFTR, IOFB, LOTTI, LOFR, NFACT
DIMENSION COORD (NDF2), TRACE (NOFPT), SECON (NOPNT)
C
PITRNO = 6.283185
VALUE = 0.

IF ( JITRT.EQ.1 ) THEN
C
READ END COORDINATES OF THE FORMER SET.
COORD (IOPNT,1) & COORD (IOPNT,2) : STARTING POINT
COORD (IOPNT,3) & COORD (IOPNT,4) : END POINT
C
DO 10 IOPNT = 1, NOFPT
IF ( COORD (IOPNT,1) .GT. COORD (IOPNT,3) ) THEN
    CORX = COORD (IOPNT,3)
    CORY = COORD (IOPNT,4)
    COORD (IOPNT,3) = COORD (IOPNT,1)
    COORD (IOPNT,4) = COORD (IOPNT,2)
    COORD (IOPNT,1) = CORX
    COORD (IOPNT,2) = CORY
ENDIF
C
CALCULATE THE LINE EQUATION FOR THE GIVEN FIBER
y = secod * ( x - Xi ) + Yi
trace = trace length
C
SECOD = ( COORD (IOPNT,4) - COORD (IOPNT,2) ) / 
      ( COORD (IOPNT,3) - COORD (IOPNT,1) )
SECON (IOPNT) = SECOD
```
TRACE(IOPNT) = SQRT((COORD(IOPNT, 3) - COORD(IOPNT, 1))**2 + 
                   (COORD(IOPNT, 4) - COORD(IOPNT, 2))**2 )

10 CONTINUE
ENDIF

LOOKING FOR A DISTANCE BETWEEN FIBER AND DATA POINT

NTOFB = 0
DO 20 IOPNT = 1, NOFPT
FTLEN = TRACE(IOPNT)
NITER = FTLEN / TSTEP
IF ( NITER.EQ.0 ) NITER = 1
NTOFB = NTOFB + NITER

20 CONTINUE

VALU1 = 0.
DO 30 IITER = 1, NITER
TLITN = TSTEP * ( IITER - 1 ) + TSTEP / 2.
XTRPT = COORD(IOPNT, 1) + TLITN * ( COORD(IOPNT, 3) - 
               COORD(IOPNT, 1) ) / TRACE(IOPNT)
YTRPT = COORD(IOPNT, 2) + TLITN * ( COORD(IOPNT, 4) - 
               COORD(IOPNT, 2) ) / TRACE(IOPNT)
IF ( NITER.EQ.1 ) THEN
   XTRPT = ( COORD(IOPNT, 3) + COORD(IOPNT, 1) ) / 2.
   YTRPT = ( COORD(IOPNT, 4) + COORD(IOPNT, 2) ) / 2.
ENDIF

DTRPT = SQRT((XCOORD-XTRPT)**2 + (YCOORD-YTRPT)**2 )

VALU1 = VALU1 + EXP(-0.5 * (DTRPT/SIGMA)**2 ) / 
         ( SIGMA **2 * PITWO )
30 CONTINUE

CALCULATE THE TOTAL INFLUENCE OF THE FIBER PROCESS

VALUE = VALUE + VALU1

20 CONTINUE

RETURN
END

SUBROUTINE RANDF ( NOSUS, RSCLE, KOPTN )

SUBROUTINE RANDF GENERATE PSEUDO-RANDOM NUMBER
USING IMSL LIBRARY

DIMENSION RSCLE(NOSUS)

IF ( KOPTN.EQ.1 ) DSEED = SECNDS(0.0) * 100.0
CALL GGUBS ( DSEED, NOSUS, RSCLE )

RETURN
END
```
SUBROUTINE LENGT ( KOPTN, COORF )

RETURN
END

SUBROUTINE INDEP (COORD, CORFB, CORLF, SVALU, CPFST, CPSND)

SUBROUTINE INDEP EVALUATE THE INDEPENDENCE TEST
FOR BIVARIATE POINT PROCESS
USING EITHER SMALL PERTURBAION METHOD OR TOROIDAL SHIFT SCHEME

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONF1/NOPNT, NOFFT, NOLPT, XRANG, XRANG, XBOT, YBOT, Nosim,
*  NTOXT, NDOP2, NSTEP, DSTEP, NDOPN, SIGMA, TSTEP, MAXNN,
*  MAXXN, IOPTN, IOPTN, IOPTN, DFACT
DIMENSION COORD(NOPNT, 4), CORFB(NOFFT, 4), CORLF(NOLPT, 4),
*  SVALU(NOSIM, NSTEP)
DIMENSION RSCLE(2)
DIMENSION CPFST(177, 2), CPSND(138, 2)

INITIATE WITH THE ORIGINAL MID POINT DATA OF FORMER &
LATTER SETS

NPLU1 = NOFFT + 1
NPLU2 = NOFFT + NOLPT

DO 10 IOPNT = 1, NOPNT
READ (5,*) (COORD(IOPNT, IDOFN), IDOFN=1, NDOPF2)
XCOOR = (COORD(IOPNT, 1) + COORD(IOPNT, 3)) / 2.
YCOOR = (COORD(IOPNT, 2) + COORD(IOPNT, 4)) / 2.
CPFST(IOPNT, 1) = XCOOR
CPFST(IOPNT, 2) = YCOOR
10 CONTINUE

CALCULATE 2nd MOMENT OF BIVARIATE POINT PROCESS

SELECT ANALYSIS OPTION
   ISHFT = 1 : SMALL PERTURBATION TEST
   ISHFT = 2 : TOROIDAL SHIFT TEST

READ (5,*) ISHFT

NFSIM = NOSIM - 1
DO 20 IOSIM = 1, NFSIM

DO 25 LSTEP = 1, NSTEP
SVALU(IOSIM, LSTEP) = 0.
25 CONTINUE

IF ( ISHFT.EQ.1 ) THEN

DEFINE PERTURBATION DISTANCE AND USE RANDOM DISTANCE

   IF ( IOSIM.EQ.1 ) DSEED = 123457.0
CALL GGUBS ( DSEED, 2, RSCALE )
XCOTR = RSCALE(1) * TSTEP * 2. - TSTEP / 2.
YCOTR = RSCALE(2) * TSTEP * 2. - TSTEP / 2.
DO 30 IPLUS = NPLU1, NPLU2
JPLUS = IPLUS - NOFPF
CPSND(JPLUS,1) = CPFST(IPLUS,1) + XCOTR
CPSND(JPLUS,2) = CPFST(IPLUS,2) + YCOTR
30 CONTINUE
ELSE
IF (IOSIM.EQ.1) DSEED = 123457.0
CALL GGUBS( DSEED, 2, RSCALE )
XCOTR = RSCALE(1) * XRANG
YCOTR = RSCALE(2) * XRANG
DO 40 IPLUS = NPLU1, NPLU2
JPLUS = IPLUS - NOFPF
CPSND(JPLUS,1) = CPFST(IPLUS,1) + XCOTR
CPSND(JPLUS,2) = CPFST(IPLUS,2) + YCOTR
IF (CPSND(JPLUS,1).LT.XBOT ) CPSND(JPLUS,1) = CPSND(JPLUS,1) + XRANG
IF (CPSND(JPLUS,1).GT.XRANG) CPSND(JPLUS,1) = CPSND(JPLUS,1) - XRANG
IF (CPSND(JPLUS,2).LT.YBOT ) CPSND(JPLUS,2) = CPSND(JPLUS,2) + YRANG
IF (CPSND(JPLUS,2).GT.YRANG) CPSND(JPLUS,2) = CPSND(JPLUS,2) - YRANG
40 CONTINUE
ENDIF
ADJUST THE TRANSFORMED COORDINATE
DO 50 IOPNT = 1, NOFPF
IF (CPSND(IOPNT,1).LT.XBOT ) CPSND(IOPNT,1) = XBOT
IF (CPSND(IOPNT,1).GT.XRANG) CPSND(IOPNT,1) = XRANG
IF (CPSND(IOPNT,2).LT.YBOT ) CPSND(IOPNT,2) = YBOT
IF (CPSND(IOPNT,2).GT.YRANG) CPSND(IOPNT,2) = YRANG
50 CONTINUE
CALCULATE THE 2nd MOMENT
DO 60 IOPNT = 1, NOFPF
XCOORD = CPFST(IOPNT,1)
YCOORD = CPFST(IOPNT,2)
CALL BVSUMM ( XCOORD, YCOORD, CPSND, SVALU, IOSIM )
60 CONTINUE
CONTINUE
MAPPED PATTERN CASE
IOSIM = NOSIM
DO 80 JOPNT = NPLU1,NPLU2
KOPNT = JOPNT - NOFPF
CPSND(KOPNT,1) = CPFST(JOPNT,1)
CPSND(KOPNT,2) = CPFST(JOPNT,2)
80 CONTINUE
DO 75 LSTEP = 1, NSTEP
SVALU(NOSIM,LSTEP) = 0.
75 CONTINUE
    DO 70 IOPTN = 1, NOOPT
    XCOOR = CPFST(IOPTN, 1)
    YCOOR = CPFST(IOPTN, 2)
    CALL BVSSIM ( XCOOR, YCOOR, CPSND, SVALU, IOSIM )

70 CONTINUE

C
C CUMULATIVE K FUNCTION
C
DO 300 IOSIM = 1, NOSIM
DO 200 ISTEP = 1, NSTEP
ADDTV = 0.
IF ( ISTEP.EQ.1 ) GO TO 200
KSTEP = ISTEP - 1
ADDTV = SVALU(IOSIM, KSTEP)
SVALU(IOSIM, ISTEP) = SVALU(IOSIM, ISTEP) + ADDTV

200 CONTINUE
300 CONTINUE

C
CALL KSTAT ( SVALU )

C
MONTE - CARLO STATISTICS

WRITE(6,900)
DO 110 IOSIM = 1, NOSIM
CSTAT = 0.
RSTAT = 0.
DO 100 ISTEP = 1, NSTEP
JSTAT = 0
XSTAT = 0
ZSTAT = SVALU(IOSIM, ISTEP)
DO 90 JOSIM = 1, NOSIM
IF ( JOSIM.EQ.IOSIM ) GO TO 90
XSTAT = XSTAT + SVALU(JOSIM, ISTEP)

90 CONTINUE
XSTAT = XSTAT / ( NOSIM - 1 )
CSTAT = ( ZSTAT - XSTAT )**2
RSTAT = RSTAT + CSTAT

100 CONTINUE
IF ( IOSIM.EQ.1 ) THEN
    RMINV = RSTAT
    RMAXV = RSTAT
ENDIF
IF ( IOSIM.NE.NOSIM.AND.RSTAT.LE.RMINV ) RMINV = RSTAT
IF ( IOSIM.NE.NOSIM.AND.RSTAT.GE.RMAXV ) RMAXV = RSTAT
WRITE(6,910) IOSIM, RSTAT

110 CONTINUE
WRITE(6,920) RMINV, RMAXV, RSTAT

C
900 FORMAT(/,5X,'MONTE - CARLO TEST',/)
910 FORMAT( 5X,'STATISTIC',I5,,' = ',F15.3 )
920 FORMAT(/,5X,'MIN. VALUE OF STATISTIC = ',F15.3,
          *, /,5X,'MAX. VALUE OF STATISTIC = ',F15.3,
          *, /,5X,'MAPPED VALUE = ',F15.3 )

C
RETURN
END

SUBROUTINE BVSM ( XCOORD, YCOORD, CPSND, SVALU, IOSIM )

SUBROUTINE BVSM EVALUATE THE SECOND MOMENT MEASURE FOR
BIVARIATE POINT PROCESS

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONT/NOPT,NOPT,NOPT,NLOPT,XRANG,YRANG, XBOT, YBOT,NOSIM,
*       NTO1,NDOF2,NSTEP,DSTEP,NDOFN,SIGMA,TSTEP,MAXIN,
*       MAXYN,LOPTN,LOPTN,LOPTN,DFACT
DIMENSION CPSND(138,2), SVALU(NOSIM,NSTEP)

CNSTV = 1.

CALCULATE THE WEIGHTING COEFFICIENT W(X,U)

DIST1 = AMIN1 ( XCOORD, (XRANG - XCOORD) )
DIST2 = AMIN1 ( YCOORD, (YRANG - YCOORD) )
SQRDIS = DIST1**2 + DIST2**2
DO 20 IPOIN = 1, NLOPT
XCOR1 = CPSND(IPOIN,1)
YCOR1 = CPSND(IPOIN,2)
DIST0 = SQRT( (XCOORD-XCOR1)**2+(YCOORD-YCOR1)**2 )

NUMERICAL ERROR TERM

WEIGT = 0.
IF ( DIST0.LE.0.1 ) THEN
  WEIGT = 1.
  GO TO 15
ENDIF

ISTEP = DIST0 / DSTEP + 1
IF ( ISTEP.GT.NSTEP ) GO TO 20
IF ( DIST0**2.LE.SQRDIS ) THEN
  DIST3 = ACOS( CNSTV * AMIN1(DIST1,DIST0) / DIST0 )
  DIST4 = ACOS( CNSTV * AMIN1(DIST2,DIST0) / DIST0 )
  WEIGT = 1. - ( DIST3 + DIST4 ) / 3.14159
ELSE
  DIST3 = ACOS( CNSTV * DIST1 / DIST0 )
  DIST4 = ACOS( CNSTV * DIST2 / DIST0 )
  WEIGT = 0.75 - ( DIST3 + DIST4 ) / 6.28318
ENDIF

15 CONTINUE
WEIGT = XRANG * YRANG / ( WEIGT * (NLOPT*NLOPT) )

CALCULATE K FUNCTION

SVALU(IOSIM,ISTEP) = SVALU(IOSIM,ISTEP) + WEIGT

20 CONTINUE

RETURN
END

SUBROUTINE NNMLE ( COORD, TRACE, SECON, XLAMO )

SUBROUTINE NNMLE EVALUATES THE HIERARCHICAL FIBER MODEL
WITH NEAREST NEIGHBOR FIBERS.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)

COMMON/CONFL/NOPNT, NOFPT, NOLPT, XRANG, YRANG, XBOT, YBOT, NOSIM,
*   NTOPT, NDOF2, NSTEP, DSTEP, NDOPN, SIGMA, TSTEP, MaxxW,
*   MaxxW, JOPTN, JOPTH, LOPHT, DFACT
DIMENSION COORD(NOPNT,4), TRACE(NOFPT), SECON(NOFPT)
DIMENSION COORN(138,2)

TWOPI = 6.28318
JITRT = 1
VALTC = 0.
NPLU1 = NOFPT + 1
NPLU2 = NOFPT + NOLPT
WRITE(6,910)

CALCULATE THE MLE OF THE NEAREST NEIGHBOR FIBER FUNCTION

DO 10 IOPNT = NPLU1,NPLU2

READ MID POINT OF THE LATTER SET ( MAPPED DATA )

READ (5,*) ( COORD(IOPNT,IDOFN), IDOFN=1,NDOF2)
XCOR = ( COORD(IOPNT,1) + COORD(IOPNT,3) ) / 2.
YCOR = ( COORD(IOPNT,2) + COORD(IOPNT,4) ) / 2.
JOPNT = IOPNT - NOFPT
COORN(JOPNT,1) = XCOR
COORN(JOPNT,2) = YCOR

10 CONTINUE

JITRT = 1

EVALUATE LOG SUM OF MLE

SIGMA = SIGMA
DO 40 JSTEP = 1, NSTEP
XLAMO = XLAMO + DSTEP
DO 30 ISTEP = 1, NSTEP
VALUE = 0.
IF ( ISTEP.EQ.1 ) SIGMA = SIGMA
SIGMA = SIGMA + DSTEP

DO 20 IOPNT = 1, NOLPT

FIND THE SHORTEST DISTANCE BETWEEN A POINT AND A FIBER

XCOR = COORN(IOPNT,1)
YCOR = COORN(IOPNT,2)
CALL NEARF ( JITRT, XCOOR, YCOOR, FDIST, COORD, TRACE, SECON )
JITRT = 2

EVALUATE MLE FOR NEAREST NEIGHBOR DISTANCE USING MIXED DISTRIBUTION

VALUE = VALUE + ALOG(XLAMO / (22.*14.) + *(1.-XLAMO) * EXP( -0.5 * (FDIST/SIGMA)**2 ) / *(SIGMA * SQRT(TWOPI) * FLOAT(NOLPT) ) )

20 CONTINUE
WRITE(6,920) SIGMA, XLAMO, VALUE
IF ( ISTEP.EQ.1.AND.JSTEP.EQ.1 ) THEN
  VALMX = VALUE
  SIGMX = SIGMA
  XLAMX = XLAMO
ENDIF
IF ( VALUE.GT.VALMX ) THEN
  VALMX = VALUE
  SIGMX = SIGMA
  XLAMX = XLAMO
ENDIF
30 CONTINUE
40 CONTINUE
WRITE(6,930)
WRITE(6,920) SIGMX, XLAMX, VALMX

910 FORMAT(//,5X,'MLE OF NEAREST NEIGHBOR FIBER FUNCTION',//)
920 FORMAT(//,2X,'SIGMA , XLAMO AND LOG SUM OF ML VALUE = ',
        *2F10.5,5X,F15.7)
930 FORMAT(//,5X,'MAX. VALUES',//)

RETURN
END

SUBROUTINE KFMLE ( COORD, CORFB, SECON, TRACE, CORLF, COORL, SVALU, *

SUBROUTINE KFMLE EVALUATES THE HIERARCHICAL FIBER MODEL WITH LINE - KERNAL FUNCTION.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONFL/NOPNT, NOFPT, NOLPT, XANG, YRANG, XBOT, YBOT, NOSIM, *
        NTOPL, NDOF2, NSTEP, DSTEP, NDOFN, SIGMA, TSTEP, MAXXW,
        MAXXY, IOPTN, JOPTN,LOPTN, DFACT
DIMENSION COORD(NOPNT,4), CORFB(NOFPT,4), SECON(NOPNT), *
        CORLF(NOLPT,4), COORL(NOLPT,2), TRACE(NOFPT)
DIMENSION SVALU(NOSIM,NSTEP), RSCALE(2)
DIMENSION COORW(138,2)

CALCULATE THE SHAPE OF THE LINE - KERNAL FUNCTION WITH MAPPED DATA.

TWOPI = 6.28318
JITRT = 1
VALTC = 0.
NPLU1 = NOFP + 1
NPLU2 = NOFP + NOLPT
WRITE(6,910)

CALCULATE THE MLE OF THE LINE - KERNEL FUNCTION

DO 10 IOPNT = NPLU1,NPLU2

READ MID POINT OF THE LATTER SET ( MAPPED DATA )

READ (5,*) ( COORD(IOPNT,IDOFN), IDOFN=1,NDOF2)
XCOOR = ( COORD(IOPNT,1) + COORD(IOPNT,3) ) / 2.
YCOOR = ( COORD(IOPNT,2) + COORD(IOPNT,4) ) / 2.
JOPNT = IOPNT - NOFP
COORN(IOPNT,1) = XCOOR
COORN(IOPNT,2) = YCOOR
10 CONTINUE

CALCULATE THE SUM OF ML

SIGMA = SIGMA
DO 40 JSTEP = 1, NSTEP
XLAMO = XLAMO + DSTEP
DO 20 ISTEP = 1, NSTEP
VALTC = 0.
VALUE = 0.
IF ( ISTEP.EQ.1 ) SIGMA = SIGMA
SIGMA = SIGMA + DSTEP
DO 30 IOPNT = 1, NOLPT
XCOOR = COORN(IOPNT,1)
YCOOR = COORN(IOPNT,2)
CALL LKERN ( JITRT,XCOOR,YCOOR,TRACE,VALU1,COORD,SECON, *
* NTOFB )
JITRT = 2
VALUE = VALUE + ALOG( XLAMO / ( 22. * 14. ) + *
* (1.-XLAMO) * VALU1 / FLOAT(NOLPT) )
30 CONTINUE
WRITE(6,920) SIGMA, XLAMO, VALUE
IF ( ISTEP.EQ.1.AND.JSTEP.EQ.1 ) THEN
VALMX = VALUE
SIGMX = SIGMA
XLAMX = XLAMO
ENDIF
IF ( VALUE.GT.VALMX ) THEN
VALMX = VALUE
SIGMX = SIGMA
XLAMX = XLAMO
ENDIF
20 CONTINUE
40 CONTINUE

WRITE(6,930)
WRITE(6,920) SIGMX, XLAMX, VALMX
CC

SUBROUTINE KERNL ( NOLPT, XCOR2, YCOR2, VLUKL, COORN, SIGMA )

SUBROUTINE KERNL EVALUATE THE BIVARIATE KERNEL FUNCTION WHEN
GENERATING THE 2nd SET.

DIMENSION COORN(138,2)

TWOPI = 6.28318
VLUKL = 0.

DO 10 IOPNT = 1, NOLPT
XCORD = COORN(IOPNT,1)
YCORD = COORN(IOPNT,2)
DIST = ( XCOR2-XCORD )**2 + ( YCOR2-YCORD )**2
VLUKL = VLUKL + EXP( -0.5 * DIST / SIGMA**2 )
10 CONTINUE
VLUKL = VLUKL / FLOAT(NOLPT)

RETURN
END

CC

SUBROUTINE NEARF ( JITRT, XCOORD, YCOORD, FDIST, COORD, TRACE, *
SECON )

SUBROUTINE NEARF CALCULATES THE NEAREST NEIGHBOR FIBER
DISTANCE FROM A GIVEN POINT IN A MAP

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONFIL/XOPNT, NOFFT, NOLPT, XRANG, YRANG, XBOT, YBOT, NOSIM,
*   NTOVL, NDOP2, NSTEP, DSTEP, NOFNS, SIGMA, TSTEP, MAXK,
*   MAXY, IOPTH, JOPTH, IOPTN, DTACT
DIMENSION COORD(NOPNT,4), TRACE(NOFFT), SECON(NOPNT)

FDIST = 0.

IF ( JITRT.EQ.1 ) THEN
   DO 10 IOPNT = 1, NOFPT
      IF ( COORD(IOPNT,1).GT.COORD(IOPNT,3) ) THEN
         CORXP = COORD(IOPNT,3)
         CORYP = COORD(IOPNT,4)
         COORD(IOPNT,3) = COORD(IOPNT,1)
         COORD(IOPNT,4) = COORD(IOPNT,2)
         COORD(IOPNT,1) = CORXP
      END IF
   10 CONTINUE
   FDIST = FDIST + DSTEP
ENDIF
COORD(IOPNT, 2) = CORYP

ENDIF

CALCULATE THE LINE EQUATION FOR THE GIVEN FIBER

\[ y = \text{secod} \times (x - Xi) + Yi \]
\[ \text{trace} = \text{trace length} \]

\[ \text{SECOD} = (\text{COORD(IOPNT, 4)} - \text{COORD(IOPNT, 2)}) / \]
\[ (\text{COORD(IOPNT, 3)} - \text{COORD(IOPNT, 1)}) \]

\[ \text{SECOD(IOPNT)} = \text{SECOD} \]

\[ \text{TRACE(IOPNT)} = \text{SQRT}((\text{COORD(IOPNT, 3)} - \text{COORD(IOPNT, 1)})^2 + \]
\[ (\text{COORD(IOPNT, 4)} - \text{COORD(IOPNT, 2)})^2 \]

10 CONTINUE

ENDIF

LOOKING FOR A NEAREST NEIGHBOR FIBER FROM A GIVEN POINT

DO 20 IOPNT = 1, NOFPPT
IF ( SECOD(IOPNT) .LT. 0.001 ) SECOD(IOPNT) = 0.001
SECOD = SECOD(IOPNT)

\[ \text{XCROS} = (\text{SECOD} \times \text{COORD(IOPNT, 1)} + \text{XCOORD/SECOD} + \text{YCOORD} \]
\[ - \text{COORD(IOPNT, 2)}) / (\text{SECOD} + 1./\text{SECOD}) \]

IF ( XCROS.LE.COORD(IOPNT, 1) ) XCROS = COORD(IOPNT, 1)
IF ( XCROS.GE.COORD(IOPNT, 3) ) XCROS = COORD(IOPNT, 3)

\[ \text{YCRS} = (\text{XCROS} - \text{COORD(IOPNT, 1)}) \times \text{SECOD} + \text{COORD(IOPNT, 2)} \]

\[ \text{FDISI} = \text{SQRT}((\text{XCOORD} - \text{XCROS})^2 + (\text{YCOORD} - \text{YCRS})^2) \]

FIND THE SHORTEST DISTANCE

IF ( IOPNT.EQ.1 ) FDIST = FDISI
IF ( FDISI.LT.FDIST ) FDIST = FDISI
20 CONTINUE

RETURN
END

SUBROUTINE ORINT ( COORD, TRACE, SECOD )

SUBROUTINE ORINT MEASURE THE ORIENTATION
CORRELATIONS BETWEEN SET 1 AND 2.

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON/CONFL/HOPNT, NOFPPT, NLOPT, XRANG, YRANG, XBOT, YBOT, WOSIM,
* NTOLT, NDF02, NSTEP, DSTEP, NDOPM, SIGMA, TSTEP, MAXXN,
* MAXYN, IOPTN, IOPTH, LOPTN, DFACT
DIMENSION COORD(NOPNT, 4), TRACE(NOFPT), SECOD(NOPNT)
DIMENSION CROMN(40, 18), INTN (20)

TWOPI = 6.28318
NPLU1 = NOFPPT + 1

IF ( IOPTN.EQ.4 ) GO TO 110
READ (5,*) ANGLE  
NOANG = 180. / ANGLE

C READ COORDINATES OF FIBERS  
C
DO 10 IOPNT = 1, NOPNT  
READ (5,*) ( COORD(IOPNT, JOPNT), JOPNT=1, NDOF2 )
10 CONTINUE

C LOOKING FOR A NEAREST NEIGHBOR FIBER

C JITRT = 1
DO 20 IOPNT = NPLU1, NOPNT  
XCOOR = ( COORD(IOPNT,1) + COORD(IOPNT,3) ) / 2.
YCOOR = ( COORD(IOPNT,2) + COORD(IOPNT,4) ) / 2.
CALL NEARFF ( JITRT, XCOOR, YCOOR, FDIST, COORD, TRACE, SECON )
JITRT = 2

C IDIST = FDIST / DSTEP  
IDIST = IDIST + 1
CORDD = COORD(IOPNT,3) - COORD(IOPNT,1)
IF ( CORDD.LE. 0.01.AND.CORDD.GE.0. ) CORDD = 0.01
IF ( CORDD.GE.-0.01.AND.CORDD.LE.0. ) CORDD =-0.01
STRA = ( COORD(IOPNT,4) - COORD(IOPNT,2) ) / CORDD
STRA = ATAN(STRA) * 180. / 3.14159
IF ( STRA.LT.0. ) STRA = STRA + 180.
IORTN = STRA / ANGLE
IORTN = IORTN + 1
CORN(IDIST, IORTN) = CORN(IDIST, IORTN) + 1
20 CONTINUE

C WRITE(6,910)  
DO 30 ISTEP = 1, NSTEP  
DIST = ISTEP * DSTEP
DO 40 JSTEP = 1, NOANG  
STRA = JSTEP * ANGLE
IFREQ = CORN(ISTEP, JSTEP)
FREQY = FLOAT(IFREQ) / FLOAT(NOLPT)
WRITE(6,920) DIST, STRA, IFREQ, FREQY
30 CONTINUE

C WRITE(6,930)
DO 50 JSTEP = 1, NOANG
STRA = JSTEP * ANGLE
DO 60 ISTEP = 1, NSTEP
IRINT(ISTEP) = CORN(ISTEP, JSTEP)
60 CONTINUE
WRITE(6,940) STRA, (IRINT(ISTEP), ISTEP=1, NSTEP)
50 CONTINUE
RETURN

C IOPTN = 4 : MLE OF ORIENTATION DISTRIBUTION

C WRAPPED NORMAL DISTRIBUTION IS ADOPTED IN CURRENT VERSION
C 110 CONTINUE
DO 115 IOFPNT = 1, NOLP2
READ (5,*) DUMM1, DUMM2, DUMM3, DUMM4
115 CONTINUE
DO 120 IOFPNT = 1, NOLP2
READ (5,*) (COORD(IOFPNT, JOPNT), JOPNT=1, NDOF2)
DIFFS = COORD(IOFPNT, 3) - COORD(IOFPNT, 1)
IF (DIFFS.LE.0.01.AND.DIFFS.GE.0.0.) DIFFS = 0.01
IF (DIFFS.GE.-0.01.AND.DIFFS.LE.0.) DIFFS = -0.01
SECOD = (COORD(IOFPNT, 4) - COORD(IOFPNT, 2)) / DIFFS
SECON(IOFPNT) = ATAN(SECOD)*57.2958
IF (SECON(IOFPNT).LT.0.) SECON(IOFPNT) = SECON(IOFPNT) + 180.
120 CONTINUE
C
C READ OPTION ( MOPTN )
C 1 : Von Mises DISTRIBUTION ON A CIRCLE
C 2 : WRAPPED NORMAL DISTRIBUTION
C
NOLP2 = NOLP2 * 2
READ (5,*) MOPTN
WRITE (6,990) MOPTN
IF (MOPTN.EQ.2) GO TO 200
C
MOPTN = 1
C CALCULATE THE MLE OF GIVEN DATA USING Von Mises DISTRIBUTION
C
XXVAL = TSTEP
XANGL = SIGMA
DO 150 JSTEP = 1, NSTEP
XANG1 = XANGL * JSTEP
XANG2 = XANG1 + 180.
DO 140 ISTEP = 1, NSTEP
XXVAL = TSTEP + (ISTEP - 1) * DSTEP
C
BEssl = 1. + (0.5*XXVAL)**2 + (0.5*XXVAL)**4 / 4. +
* (0.5*XXVAL)**6 / 36. + (0.5*XXVAL)**8 / (24.)**2 +
* (0.5*XXVAL)**10 / (120.)**2
BESSL = 1. / ( TWOPI * BESSL )
C
VALUE = 0.
DO 130 IOFPNT = 1, NOLP2
IF (IOFPNT.LE.NOLP2) THEN
  ANGLE = SECON(IOFPNT)
ELSE
  JOPNT = IOFPNT - NOLP2
  ANGLE = SECON(JOPNT) + 180.
ENDIF
C
VALUE = VALUE + ALOG(0.5 * BESSL * (EXP(XXVAL *
* COS((ANGLE-XANG1)*0.01745)) + 0.5 * BESSL
* (EXP(XXVAL * COS((ANGLE-XANG2)*0.01745))))
130 CONTINUE
C
WRITE (6,950) XXVAL, XANG1, VALUE
IF ( ISTEP.EQ.1.AND.JSTEP.EQ.1 ) THEN
  XM2MAX = XKVAL
  XM2MAX = XANG1
  VALM2X = VALUE
ENDIF
IF ( VALUE.GT.VALM2X ) THEN
  XM2MAX = XKVAL
  XM2MAX = XANG1
  VALM2X = VALUE
ENDIF
140 CONTINUE
150 CONTINUE
GO TO 300

C
C      MOPTN = 2
C      WRAPPED NORMAL DISTRIBUTION
C
200 CONTINUE
XANG2 = SIGMA
KSTEP = -NSTEP / 2
DO 230 ISTEP = 1, NSTEP
  XANG1 = XANG2 + ( ISTEP-1) * 5.
  XANG2 = XANG1 + 180.

C
DO 220 JSTEP = 1, NSTEP
  VALUE = 0.
  XKVAL = TSTEP + ( JSTEP - 1 ) * DSTEP
  DO 210 IOPNT = 1, NOLP2
    IF ( IOPNT.LE.NOLPT ) THEN
      ANGLE = SECON(IOPNT)
    ELSE
      JOPNT = IOPNT - NOLPT
      ANGLE = SECON(JOPNT) + 180.
    ENDIF
    VALU1 = 0.
    VALU2 = 0.

C
DO 240 LSTEP = 1, NSTEP
  RSTEP = KSTEP + LSTEP
  VALU1 = VALU1 + EXP( -0.5 * ( (ANGLE-XANG1-360.*RSTEP) *
                      0.01745 ) **2 / XKVAL**2 )
  VALU2 = VALU2 + EXP( -0.5 * ( (ANGLE-XANG2-360.*RSTEP) *
                      0.01745 ) **2 / XKVAL**2 )
240 CONTINUE
  VALUE = VALUE +
  * ALOG( 0.5 / (XKVAL*SQR(TWPI)) ) * (VALU1+VALU2)

C
210 CONTINUE
WRITE(6,950) XKVAL, XANG1, VALUE
IF ( ISTEP.EQ.1.AND.JSTEP.EQ.1 ) THEN
  XM2MAX = XKVAL
  XM2MAX = XANG1
  VALM2X = VALUE
ENDIF
IF ( VALUE.GT.VALM2X ) THEN
\begin{verbatim}
XKMAX = XKVAL
XAMAX = XANG1
VALMX = VALUE
ENDIF
220 CONTINUE
230 CONTINUE
C
300 CONTINUE
C
WRITE(6, 960)
WRITE(6, 950) XKMAX, XAMAX, VALMX
C
CALCULATE FREQUENCIES OF THE WRAPPED NORMAL DISTRIBUTION
WITH CALCULATED MAX. VALUE OF SIGMA
C
WRITE(6, 970)
VALUE = 0.
VALMX = 0.
SIGMA = XKMAX
XKVAL = XKMAX
XANG1 = XAMAX
XANG2 = XANG1 + 180.
DO 330 IANGL = 1, 36
    ANGLE = IANGL * 10.
    VALUE = 0.
    VALU1 = 0.
    VALU2 = 0.
    IF ( MOPTN.EQ.2 ) THEN
        DO 320 JSTEP = 1, NSTEP
            RSTEP = KSTEP + JSTEP
            VALU1 = VALU1 + EXP( -0.5 * ( (ANGLE-XANG1-360.*RSTEP)
                             * 0.01745 )**2 / SIGMA**2 )
            VALU2 = VALU2 + EXP( -0.5 * ( (ANGLE-XANG2-360.*RSTEP)
                             * 0.01745 )**2 / SIGMA**2 )
        320 CONTINUE
        VALUE = 0.5 / (SIGMA*SQRT(TWOPI)) * (VALU1+VALU2)
        VALMX = VALMX + VALUE
    ELSE
        BESSL = 1. + (0.5*XKVAL)**2 + (0.5*XKVAL)**4 / 4. +
            (0.5*XKVAL)**6 / 36. + (0.5*XKVAL)**8 / (24.)**2 +
            (0.5*XKVAL)**10 / (120.)**2
        BESSL = 1. / (TWOPI * BESSL)
        C
        VALUE = BESSL * ( 0.5 * EXP( XKVAL *
                          COS( (ANGLE-XANG1)*0.01745 ) ) + 0.5 *
                          EXP( XKVAL * COS( (ANGLE-XANG2)*0.01745 ) )
                          )
        VALMX = VALMX + VALUE
    ENDIF
    WRITE(6, 980) ANGLE, VALUE
330 CONTINUE
    VALMX = VALMX * 0.17453
    WRITE(6, 890) MOPTN, VALMX
C
890 FORMAT(//, 5X, 'OPTION = ', I5,
           '*', //, 5X, 'INTEGRATION OF FREQUENCY = ', F15.7)
\end{verbatim}
910 FORMAT( '///,5X,'Orientation correlation measure',
         '///,5X,'Distance angle frequency',
         '///,5X,'Angle distance : (dstep * integer) m',
         '///,3X,F7.3,3X,10I5)
920 FORMAT( 5X,2(F7.3,3X),I3,3X,F9.6)
930 FORMAT( '///,5X,'Mle option : Moptn ',10X,15,
         '///,8X,'1 : Von Mises distribution on a circle',
         '///,8X,'2 : Wrapped normal distribution',
         '///)
RETURN
END

PROGRAM TDIST

PROGRAM TDIST estimate the trace length distribution
using maximum likelihood estimate

Three possible distributions are examined
1. Exponential distribution
2. Truncated normal distribution
3. Lognormal distribution

TRACE(i,1) : Trace length data
TRACE(i,2) : Boundary of the mapped area
NCONF(i) : Characteristics of the trace
   1 for the trace with both ends observable
   2 for the trace with one end observable
   3 for the trace with no end observable

DIMENSION TRACE(39,2), NCONF(39)

OPEN (5, FILE='tdist.dat5', STATUS='UNKNOWN')
OPEN (6, FILE='tdist.out')

NOPY = 39

DO 10 IOFPT = 1,NOPY
READ (5,*) TRACE(IOFPT,1), TRACE(IOFPT,2), NCONF(IOFPT)
TRACE(IOFPT,1) = 0.833 * TRACE(IOFPT,1)
TRACE(IOFPT,2) = 0.833 * TRACE(IOFPT,2)
10 CONTINUE

Iterate for each case

SUMAX = 0.
SUMMS = 0.
READ (5,*) NITER
DO 20 ITER = 1,NITER

READ THE OPTION

NOPY = 1 : Exponential distribution with mean value
2 : TRUNCATED NORMAL DISTRIBUTION WITH TWO VARIABLES
3 : LOGNORMAL DISTRIBUTION WITH TWO VARIABLES

READ (5,*) NOPTN
GO TO ( 100, 200, 300 ), NOPTN

EXponential DISTRIBUTION WITH ASSUMED MEAN VALUE

100 CONTINUE
WRITE (6,900)
READ (5,*) RMEAN, STEPS, NSTEP
DO 40 ISTEP = 1,NSTEP
PROB3 = 0.
PROB6 = 0.
PROB9 = 0.
XMEAN = RMEAN + (ISTEP - 1) * STEPS
DO 30 IOFPT = 1, NOPTN
JOPTN = NCONF(IOFPT)
RTRCE = TRACE(IOFPT,1)
BTRCE = TRACE(IOFPT,2)
CONST = 1. / ( 3.*XMEAN + BTRCE - 3.*XMEAN*EXP(-BTRCE/XMEAN) )
CONST = 1. / ( XMEAN + BTRCE - XMEAN * EXP(-BTRCE/XMEAN) )
IF ( JOPTN.EQ.1 ) THEN
PROB1 = BTRCE - XMEAN + XMEAN * EXP(-BTRCE/XMEAN)
PROB1 = PROB1 * CONST
PROB2 = (BTRCE - RTRCE) * EXP(-RTRCE/XMEAN) / XMEAN
PROB3 = PROB3 + ALOG(PROB1*PROB2)
ENDIF
IF ( JOPTN.EQ.2 ) THEN
PROB4 = 2.*XMEAN*(1. - EXP(-BTRCE/XMEAN))
PROB4 = PROB4 * CONST
PROB5 = EXP(-RTRCE/XMEAN) / (XMEAN*(1. -
* EXP(-BTRCE/XMEAN)))
PROB6 = PROB6 + ALOG(PROB4*PROB5)
ENDIF
IF ( JOPTN.EQ.3 ) THEN
PROB7 = XMEAN*EXP(-BTRCE/XMEAN)
PROB7 = PROB7 * CONST
PROB8 = 1.
PROB9 = PROB9 + ALOG(PROB7*PROB8)
ENDIF
PROBQ = PROB3 + PROB6
30 CONTINUE

FIND maximum LIKELIHOOD ESTIMATES

WRITE(6,910) XMEAN, PROB3, PROB6, PROB9
IF ( ISTEP.EQ.1 ) THEN
SUMM4 = PROBQ
SUMM3 = PROB3
SUMM6 = PROB6
SUMM9 = PROB9
VMEA3 = XMEAN
VMEA6 = XMEAN
VMEA9 = XMEAN
ENDIF
IF ( PROB3.GT.SUMM3 ) THEN
  SUMM3 = PROB3
  VMEEA3 = XMEAN
ENDIF
IF ( PROB6.GT.SUMM6 ) THEN
  SUMM6 = PROB6
  VMEEA6 = XMEAN
ENDIF
IF ( PROB9.GT.SUMM9 ) THEN
  SUMM9 = PROB9
  VMEEA9 = XMEAN
ENDIF
IF ( PROBQ.GT.SUMMA ) THEN
  SUMMA = PROBQ
  VMEEAQ = XMEAN
ENDIF
40 CONTINUE
WRITE(6,920) VMEEA3, SUMM3, VMEEA6, SUMM6, VMEEA9, SUMM9, VMEEAQ, SUMMA

C
C
COMPARE THE RESULT WITH EPSTEIN'S FORMULA FOR SAMPLING LINE
C
ISUMM = 0
TLENG = 0.
DO 50 IOFPT = 1,NOFPT
KOPTN = NCONF(IOFPT)
TLENG = TLENG + TRACE(IOFPT,1)
IF ( KOPTN.EQ.1 ) ISUMM = ISUMM + 1
50 CONTINUE
VMEEAN = TLENG / FLOAT(ISUMM)
WRITE(6,930) VMEEAN

C
GO TO 400

C
TRUNCATED NORMAL DISTRIBUTION WITH ASSUMED MEAN AND STD. DEVIATION
C
200 CONTINUE
READ (5,*) XMEAN, DEVIA, STEP1, STEP2
GO TO 400

C
LOGNORMAL DISTRIBUTION WITH ASSUMED MEAN STD. DEVIATION
C
300 CONTINUE
READ (5,*) XMEAN, DEVIA, STEP1, STEP2

C
400 CONTINUE
20 CONTINUE

C
900 FORMAT(\ 5X,'EXPONENTIAL DISTRIBUTIONS OF TRACES'
  * 8X,'MEAN VALUE',10X,'LOG SUM FOR TYPE 1 2 & 3',/)
910 FORMAT(7X, F12.5,5X,3(F10.5,3X) )
920 FORMAT(\ 5X,'MAXIMUM LIKELIHOOD ESTIMATE FOR EXPONENTIAL CASE'
  * 5X,'FOR TYPE 1 ( BOTH ENDS VISIBL E)'',
  * 7X,'MEAN VALUE = ',T20,F10.4,
  * 7X,'MAX. VALUE = ',T20,F10.4,
* //,5X,'FOR TYPE 2 ( ONE END VISIBLE )',
  * //,7X,'MEAN VALUE =',T20,F10.4,
  * //,7X,'MAX. VALUE =',T20,F10.4,
  * //,5X,'FOR TYPE 3 ( NO END VISIBLE )',
  * //,7X,'MEAN VALUE =',T20,F10.4,
  * //,7X,'MAX. VALUE =',T20,F10.4,
  * //,5X,'FOR SET 1 ( ALL TRACES )',
  * //,7X,'MEAN VALUE =',T20,F10.4,
  * //,7X,'MAX. VALUE =',T20,F10.4,//)
930 FORMAT(//,5X,'MLE FOR MEAN VALUE BY EPSSTEIN EQUATION =',F15.7)
C
STOP
END

PROGRAM TERM

PROGRAM TERMN EVALUATE THE TRACE LENGTH DISTRIBUTIONS
ACCORDING TO THE GIVEN SIMULATED TYPICAL POINTS.

DIMENSION COORD(138,2), TDIST(138), RSCLE(1), TSEND(138),
  TRCPT(138,4), TFREQ(30), RSCL1(1)
DIMENSION CORRFF(39,4), TRACE(39), SECON(39)
C
NL = 30

FIRST, READ THE BEST FIT OF THE TYPICAL POINTS

READ (5,*) NOLPT, NOFFT, NOSIM, VMEAN, AMEAN, SIGMA, TRUKN
NOLPT = NOLPT - NOPT
READ (5,*) XBOT, XTOP, YBOT, YTOP
WRITE (6,280) NOLPT, NOFFT, NOLPT, VMEAN, AMEAN, SIGMA
820 FORMAT(//,5X,'NO. OF TOTAL POINTS = ',I5,
  //,5X,'NO. OF POINTS IN SET 1 = ',I5,
  //,5X,'NO. OF POINTS IN SET 2 = ',I5,
  //,5X,'MEAN TRACE LENGTH OF SET 2 = ',F15.5,
  //,5X,'MEAN ANGLE OF SET 2 = ',F15.5,
  //,5X,'STANDARD DEVIATION OF ANGLE = ',F15.5,//)
XRANG = XTOP - XBOT
YRANG = YTOP - YBOT
C
READ (5,*) MOPFTN
WRITE (6,990) MOPFTN
990 FORMAT(//,5X,'ORIENTATION DISTRIBUTION OPTION = ',I10,
  //,8X,'( 1 : Von Mises DISTRIBUTION )',
  //,8X,'( 2 : WRAPPED NORMAL DISTRIBUTION )',//)
DO 10 IOPTN = 1, NOPTN
READ (5,*) ( CORPFF(IOPTN, JOPTN), JOPTN=1,4)
CORRFF = CORPFF(IOPTN,3) - CORPFF(IOPTN,1)
IF ( CORRFF.LT.0 ) THEN
  CORF = CORPFF(IOPTN,1)
  CORY = CORPFF(IOPTN,2)
  CORPFF(IOPTN,1) = CORPFF(IOPTN,3)
  CORPFF(IOPTN,2) = CORPFF(IOPTN,4)
  CORPFF(IOPTN,3) = CORF
  CORPFF(IOPTN,4) = CORY
ENDIF
TRACE(IOPNT) = SQRT( (CORFP(IOPNT,3)-CORFP(IOPNT,1))**2 +
*                  (CORFP(IOPNT,4)-CORFP(IOPNT,2))**2 )
IF ( CORRS.LE.0.01.AND.CORRS.GE.0. ) CORRS = 0.01
IF ( CORRS.GE.0.01.AND.CORRS.LT.0. ) CORRS = -0.01
SECON(IOPNT) = ( CORFP(IOPNT,4)-CORFP(IOPNT,2) ) / CORRS
10 CONTINUE
C
DO 20 IOPNT = 1, NOLPT
READ (5,*) IDUMM, COORD(IOPNT,1), COORD(IOPNT,2)
20 CONTINUE
C
TWOP1 = 6.28318
XANGL = 105.
IF ( MOPTN.EQ.1 ) THEN
   XKVAL = 1.9
   BESSL = 1. + (0.5*XKVAL)**2 + (0.5*XKVAL)**4 / 4. +
*          (0.5*XKVAL)**6 / 36. + (0.5*XKVAL)**8 / (24.)**2
*          + (0.5*XKVAL)**10 / (120.)**2
   VALMX = BESSL1 * ( 0.5 * EXP(XKVAL) + 0.5 * EXP(-XKVAL) )
ELSE
   XKVAL = 0.8
   VALU1 = 0.
   VALU2 = 0.
   NSTEP = 10
   KSTEP = -NSTEP / 2
   DO 500 JSTEP = 1, NSTEP
      RSTEP = KSTEP + JSTEP
      VALU1 = VALU1 + EXP(-0.5 * (((-360.*RSTEP)*0.01745)**2
          / XKVAL**2 ) )
      VALU2 = VALU2 + EXP(-0.5 * (((-180.-360.*RSTEP)*0.01745)**2
          / XKVAL**2 ) )
500 CONTINUE
   VALMX = 0.5 * ( VALU1 + VALU2 ) / ( XKVAL * SQRT(TWOP1) )
ENDIF
C
JOPNT = 0
DSEED = 12345.D0
30 CONTINUE
CALL GGENN ( DSEED, VMEAN, 1, RSCLE )
IF ( RSCLE(1).LT.TRUNL ) GO TO 30
C
35 CONTINUE
CALL GGUBS ( DSEED, 1, RSCLE )
ORINT = RSCLE(1) * 180.
IF ( MOPTN.EQ.1 ) THEN
   VALUE = BESSL1 * ( 0.5 *EXP( XKVAL *
*                     COS( (ORINT-XANGL)*0.01745 ) ) +
*                     0.5*EXP(XKVAL*COS((ORINT-XANGL-180.)*0.01745)))
ELSE
   VALU1 = 0.
   VALU2 = 0.
   DO 510 ISTEP = 1, NSTEP
      RSTEP = KSTEP + ISTEP
      VALU1 = VALU1 + EXP( -0.5 * ((( ORINT-XANGL-360.*
* RSTEP) * 0.01745 )**2 / XXVAL**2 )
VALU2 = VALU2 + EXP( -0.5 * ((( ORINT-XANGL-180.-360.*
* RSTEP) * 0.01745 )**2 / XXVAL**2) )

510 CONTINUE
VALUE = 0.5*(VALU1+VALU2) / ( XXVAL * SQRT(TWOP1) )
ENDIF
CALL GGUBS( DSEED, 1, RSCL1 )
IF ( RSCL1(1).GT.(VALUE/VMAX) ) GO TO 35

C

JOPNT = JOPNT + 1
TDIST(JOPNT) = RSCL1(1)
TDSTH = TDIST(JOPNT) / 2.
TDST1 = SQRT( 1. + (TAN(ORINT))**2 )
TRCPT(JOPNT,1) = COORD(JOPNT,1) + TDSTH / TDST1
TRCPT(JOPNT,2) = COORD(JOPNT,2) + TDSTH * TAN(ORINT) / TDST1
IF ( TRCPT(JOPNT,1).GT.XRANG ) THEN
TRCPT(JOPNT,1) = XTOP
TRCPT(JOPNT,2) = COORD(JOPNT,2) +
* ( TRCPT(JOPNT,1) - COORD(JOPNT,1) ) * TAN(ORINT)
ENDIF
IF ( TRCPT(JOPNT,2).GT.XRANG ) THEN
TRCPT(JOPNT,2) = YTOP
TRCPT(JOPNT,1) = COORD(JOPNT,1) +
* ( TRCPT(JOPNT,2) - COORD(JOPNT,2) ) / TAN(ORINT)
ENDIF

C

IF ( TRCPT(JOPNT,1).LT.XBOT ) THEN
TRCPT(JOPNT,1) = XBOT
TRCPT(JOPNT,2) = COORD(JOPNT,2) -
* ( COORD(JOPNT,1) - XBOT ) * TAN(ORINT)
ENDIF
YCOOR = -1.19 * TRCPT(JOPNT,1) + 7.78
IF ( TRCPT(JOPNT,2).LT.YCOOR ) THEN
TRCPT(JOPNT,1) = ( 7.78 + TAN(ORINT) ) * COORD(JOPNT,1)
* - COORD(JOPNT,2) ) / ( 1.19 + TAN(ORINT) )
TRCPT(JOPNT,2) = -1.19 * TRCPT(JOPNT,1) + 7.78
ENDIF
IF ( TRCPT(JOPNT,2).LT.YBOT ) THEN
TRCPT(JOPNT,2) = YBOT
TRCPT(JOPNT,1) = COORD(JOPNT,1) -
* ( COORD(JOPNT,2) - YBOT ) / TAN(ORINT)
ENDIF

C

TRCPT(JOPNT,3) = COORD(JOPNT,1) - TDSTH / TDST1
TRCPT(JOPNT,4) = COORD(JOPNT,2) - TDSTH * TAN(ORINT) / TDST1
IF ( TRCPT(JOPNT,3).LT.XBOT ) THEN
TRCPT(JOPNT,3) = XBOT
TRCPT(JOPNT,4) = COORD(JOPNT,2) -
* ( COORD(JOPNT,1) - TRCPT(JOPNT,3) ) * TAN(ORINT)
ENDIF
IF ( TRCPT(JOPNT,3).GT.XRANG ) THEN
TRCPT(JOPNT,3) = XTOP
TRCPT(JOPNT,4) = COORD(JOPNT,2) +
* ( TRCPT(JOPNT,3) - COORD(JOPNT,1) ) * TAN(ORINT)
ENDIF
YCOOR = -1.19 * TRCPT(JOPNT, 3) + 7.78
IF ( TRCPT(JOPNT, 4) .LT. YCOOR ) THEN
  TRCPT(JOPNT, 3) = ( 7.78 + TAN(ORINT) * COORD(JOPNT, 1)
  - COORD(JOPNT, 2) ) / ( 1.19 + TAN(ORINT) )
  TRCPT(JOPNT, 4) = -1.19 * TRCPT(JOPNT, 3) + 7.78
ENDIF

IF ( TRCPT(JOPNT, 4) .LT. YBOT ) THEN
  TRCPT(JOPNT, 4) = YBOT
  TRCPT(JOPNT, 3) = COORD(JOPNT, 1) -
  ( COORD(JOPNT, 2) - TRCPT(JOPNT, 4) ) / TAN(ORINT)
ENDIF

IF ( TRCPT(JOPNT, 4) .GT. YTOP ) THEN
  TRCPT(JOPNT, 4) = YTOP
  TRCPT(JOPNT, 3) = COORD(JOPNT, 1) +
  ( TRCPT(JOPNT, 4) - COORD(JOPNT, 2) ) / TAN(ORINT)
ENDIF

YCOOR = 0.698 * TRCPT(JOPNT, 1) - 4.56
IF ( TRCPT(JOPNT, 2) .LT. YCOOR ) THEN
  TRCPT(JOPNT, 1) = ( -4.56 + TAN(ORINT) * COORD(JOPNT, 1)
  - COORD(JOPNT, 2) ) / ( -0.698 + TAN(ORINT) )
  TRCPT(JOPNT, 2) = 0.698 * TRCPT(JOPNT, 1) - 4.56
ENDIF

YCOOR = 0.698 * TRCPT(JOPNT, 3) - 4.56
IF ( TRCPT(JOPNT, 4) .LT. YCOOR ) THEN
  TRCPT(JOPNT, 3) = ( -4.56 + TAN(ORINT) * COORD(JOPNT, 1)
  - COORD(JOPNT, 2) ) / ( -0.698 + TAN(ORINT) )
  TRCPT(JOPNT, 4) = 0.698 * TRCPT(JOPNT, 3) - 4.56
ENDIF

YCOOR = 0.698 * TRCPT(JOPNT, 1) + 11.
IF ( TRCPT(JOPNT, 2) .GT. YCOOR ) THEN
  TRCPT(JOPNT, 1) = ( 11. + TAN(ORINT) * COORD(JOPNT, 1)
  - COORD(JOPNT, 2) ) / ( -0.698 + TAN(ORINT) )
  TRCPT(JOPNT, 2) = 0.698 * TRCPT(JOPNT, 1) + 11.
ENDIF

YCOOR = 0.698 * TRCPT(JOPNT, 3) + 11.
IF ( TRCPT(JOPNT, 4) .GT. YCOOR ) THEN
  TRCPT(JOPNT, 3) = ( 11. + TAN(ORINT) * COORD(JOPNT, 1)
  - COORD(JOPNT, 2) ) / ( -0.698 + TAN(ORINT) )
  TRCPT(JOPNT, 4) = 0.698 * TRCPT(JOPNT, 3) + 11.
ENDIF

IF ( TRCPT(JOPNT, 3) .LT. TRCPT(JOPNT, 1) ) THEN
  TRCTX = TRCPT(JOPNT, 1)
  TRCTY = TRCPT(JOPNT, 2)
  TRCPT(JOPNT, 1) = TRCPT(JOPNT, 3)
  TRCPT(JOPNT, 2) = TRCPT(JOPNT, 4)
  TRCPT(JOPNT, 3) = TRCTX
  TRCPT(JOPNT, 4) = TRCTY
ENDIF

TDIST(JOPNT) = SQRT( ( TRCPT(JOPNT, 3) - TRCPT(JOPNT, 1) )**2 +
  ( TRCPT(JOPNT, 4) - TRCPT(JOPNT, 2) )**2 )
IF ( TDIST(JOPNT) .LT. TRUNKL ) THEN
  JOPNT = JOPNT - 1
GO TO 30

ENDIF

CORRS = TRCPT(JOPNT,3) - TRCPT(JOPNT,1)
IF ( CORRS.LE. 0.01.AND.CORRS.GE.0. ) CORRS = 0.01
IF ( CORRS.GE.-0.01.AND.CORRS.LT.0. ) CORRS =-0.01
TSEND (JOPNT) = ( TRCPT(JOPNT,4)-TRCPT(JOPNT,2) ) / CORRS

CALCULATE TERMINATION POINTS

IF TRACLEN LENGTH IS SMALLER THAN MEAN LENGTH, TERMINATE AT
THE NEAREST NEIGHBOR FIBER, IF IT IS GREATER THAN THAT OF MEAN
TERMINATE AT THE SECOND NEAREST NEIGHBOR FIBER.

XMEAN = ( TRCPT(JOPNT,3) + TRCPT(JOPNT,1) ) / 2.
YMEAN = ( TRCPT(JOPNT,4) + TRCPT(JOPNT,2) ) / 2.

IFLAG = 0
JFLAG = 0
XMAXL = ( TRCPT(JOPNT,3) - TRCPT(JOPNT,1) ) / 2.
XMINL = ( TRCPT(JOPNT,1) - TRCPT(JOPNT,3) ) / 2.
XMAXO = XMAXL
XMINO = XMINL

DO 100 IOPNT = 1, NOFPT
XCROS = (SECON(IOPNT)*CORFP(IOPNT,1)-TSEND(IOPNT)*TRCPT(IOPNT,1)
* + TRCPT(IOPNT,2) - CORFP(IOPNT,2) ) /
* ( SECON(IOPNT) - TSEND(IOPNT) )
IF ( XCROS.GT.TRCP(T(JOPNT,3) OR XCROS.LT.TRCP(T(JOPNT,1) )
* GO TO 100
YCROS = SECON(IOPNT) * ( XCROS - CORFP(IOPNT,1) ) +
* CORFP(IOPNT,2)
IF ( TRCP(T(JOPNT,4).GE.TRCP(T(JOPNT,2) ) THEN
IF ( XCROS.GT.TRCP(T(JOPNT,4) OR XCROS.LT.TRCP(T(JOPNT,4) )
* GO TO 100
ELSE
IF ( XCROS.GT.TRCP(T(JOPNT,2) OR XCROS.LT.TRCP(T(JOPNT,4) )
* GO TO 100
ENDIF

TRILH = SQRT((XMEAN-XCROS)**2 + (YMEAN-YCROS)**2 )
IF ( TRILH.TGT.TDSTH ) GO TO 100

IF ( XCROS.GE.XMEAN ) IFLAG = IFLAG + 1
IF ( XCROS.LT.XMEAN ) JFLAG = JFLAG + 1
XRESI = XCROS - XMEAN
IF ( IFLAG.EQ.1 ) XMAXL = XRESI
IF ( JFLAG.EQ.1 ) XMINL = XRESI
IF ( IFLAG.GT.1.AND.XRESI.GT.XMAXL ) XMAXL = XRESI
IF ( JFLAG.GT.1.AND.XRESI.LT.XMINL ) XMINL = XRESI
100 CONTINUE

CONSIDER SET 2 EFFECT

IF ( JOPNT.GT.1 ) THEN
KOPNT = JOPNT - 1
DO 110 KPOINT = 1, KOPNT
XCROS = (TSEND (KPONT) * TRCPT (KPONT, 1) - TSEND (JOPNT) * TRCPT (JOPNT, 1)
         + TRCPT (JOPNT, 2) - TRCPT (KPONT, 2) ) /
         ( TSEND (KPONT) - TSEND (JOPNT) )
IF ( XCROS.GT.TRCP (JOPNT, 3).OR.XCROS.LT.TRCP (JOPNT, 1) )
  GO TO 110
YCROS = TSEND (KPONT) * ( XCROS - TRCP (KPONT, 1) ) +
         TRCP (KPONT, 2)
IF ( TRCP (JOPNT, 4).GE.TRCP (JOPNT, 2) ) THEN
  IF ( YCROS.GT.TRCP (JOPNT, 4).OR.YCROS.LT.TRCP (JOPNT, 2) )
    GO TO 110
ELSE
  IF ( YCROS.GT.TRCP (JOPNT, 2).OR.YCROS.LT.TRCP (JOPNT, 4) )
    GO TO 110
ENDIF

C
TRUTH = SQRT((XMEAN-XCROS)**2 + (YMEAN-YCROS)**2 )
IF ( TRUTH.GT.TDSTH ) GO TO 110
XRESI = XCROS - XMEAN
IF ( XRESI.GT.0..AND.XRESI.GT.XMAXL ) XMAXL = XRESI
IF ( XRESI.LT.0..AND.XRESI.LT.XMINL ) XMINL = XRESI
110 CONTINUE
ENDIF

C
CALL GGUBS( DSEED, 1, RSCLE )
IF ( RSCLE(1).LE.0.232 ) THEN
  TRCP (JOPNT, 1) = XMEAN + XMNO
  TRCP (JOPNT, 3) = XMEAN + XMNO
ELSE
  TRCP (JOPNT, 1) = XMEAN + XMINL
  TRCP (JOPNT, 3) = XMEAN + XMAXL
ENDIF
TRCP (JOPNT, 2) = TSEND (JOPNT) * ( TRCP (JOPNT, 1) - XMEAN )
         + YMEAN
TRCP (JOPNT, 4) = TSEND (JOPNT) * ( TRCP (JOPNT, 3) - XMEAN )
         + YMEAN
TDIST (JOPNT) = SQRT(( TRCP (JOPNT, 3)-TRCP (JOPNT, 1))**2 +
         (TRCP (JOPNT, 4)-TRCP (JOPNT, 2))**2 )
IF ( TDIST (JOPNT) .LT.TRUBL ) THEN
  JOPNT = JOPNT - 1
  GO TO 30
ENDIF
C
IF ( JOPNT.LT.NOLPT ) GO TO 30
C
WRITE(6,900)
DO 45 KOPNT = 1, NOFPT
WRITE(6,910) (CORFP (KOPNT, I), I = 1, 4)
45 CONTINUE
WRITE(6,900)
DO 40 KOPNT = 1, NOLPT
WRITE(6,910) (TRCP (KOPNT, I), I = 1, 4)
40 CONTINUE
C
900 FORMAT(///,5X,'SIMULATIONS OF THE TRACE LENGTHES',//)
910 FORMAT( 5X,2(8F15.7,5X),//,5X )
STOP
END
Appendix D
USER MANUAL AND LIST FOR "TRACESIM"

D.1 Introduction

Program TRACESIM is used to analyze a blocky rock mass behavior. For this, the fracture pattern is first generated in a rock slope, followed by a topological analysis of fully persistent rock blocks. Finally, the stability analysis for a given rock block is performed using Coulomb's failure criterion. Output data can be stored on a tape so that plotting of the results is possible.

Program TRACESIM can be run both on VMS and on Unix operating systems. In the case of VMS system, since one can attach a random number generator such as IMSL mathematical package, minor changes in the program TRACESIM are needed (in subroutines HPOIS, NHPOI and PATEN).

D.2 Input Manual for Program TRACESIM

Free format is used for all input data except for the title. According to the input option (MOPTN), maximum three plotting data files are generated to plot the fracture pattern, effective fractures and kinetically unstable rock blocks.

1. TITLE (20A4)
2. Analysis option data : NPOIN, NOSET, NOSIM, MOPTN, NSTAT, NCASE, NSYST

- NPOIN : No. of fractures generated in a slope
- NOSET : No. of fracture sets in a slope
- NOSIM : No. of simulations for a given data
- MOPTN : Printout option
a. MOPTN = 1: Standard output of the result
b. MOPTN = 2: Store simulated fracture pattern in file 7
c. MOPTN = 3: Store effective fractures and intersection points in file 8
d. MOPTN = 4: Store cohesive and frictional forces of rock blocks in file 10
e. MOPTN = 5: Store simulated fracture pattern, effective fractures and kinetically unstable rock blocks in files 7, 8 and 9, respectively.

- NSTAT: Option for statistical analysis
  a. NSTAT = 0: Do not consider
  b. NSTAT = 1: Store the number of kinematically admissible rock blocks according to volume.
  c. NSTAT = 2: Store the number of kinematically admissible rock blocks according to dip angle.

- NCASE: No. of analysis cases

- NSYST: Operating system
  a. NSYST = 1: VMS system
  b. NSYST = 2: Unix system

3. Read statistical analysis data (skip if NSTAT = 0): VARMN, DSTEP, NSTEP, NREG1, NREG2

- VARMN: Minimum volume or dip angle according to NSTAT
- DSTEP: Size of increment
- NSTEP: Total no. of increments
- NREG1: Maximum number of simulations used to store the data in the plotting file (NREG1 \leq NOSIM)
- NREG2: Maximum number of data case used to store the data in the plotting file (NREG2 \leq NCASE)

4. Data for slope geometry: ANGLE, HEIGHT, RANGE, WEIGT, SFMAX (use any consistent units)

- ANGLE: Slope angle in degrees
- HEIGHT: Slope height
- RANGE: Distance between slope toe and maximum slope face (see Fig. 6-2 in the text)
- WEIGT: Unit weight of rock material
5. Read geometric option and mechanical properties of rock material (Total no. of data sets = NOSET) : IOSET, NPROP(IOSET,4), PROPT(IOSET,7), DSEED, STAND (use any consistent units)

- **IOSET** : No. of fracture sets
- **NPROP(IOSET,1)** : No. of fractures generated
- **NPROP(IOSET,2)** : Options for midpoint models of fractures
  - a. NPROP(IOSET,2) = 1 : Homogeneous Poisson point process model
  - b. NPROP(IOSET,2) = 2 : Non-homogeneous Poisson point process model
- **NPROP(IOSET,3)** : Options for trace length distribution models
  - a. NPROP(IOSET,3) = 1 : Exponential distribution model
  - b. NPROP(IOSET,3) = 2 : Lognormal distribution model
- **NPROP(IOSET,4)** : Options for fracture orientation models
  - a. NPROP(IOSET,4) = 1 : Von Mises distribution model
  - b. NPROP(IOSET,4) = 2 : Wrapped normal distribution model (currently ignored)
  - c. NPROP(IOSET,4) = 3 : Uniform distribution model
  - d. NPROP(IOSET,4) = 4 : Fixed (i.e., parallel) orientation model
- **PROPT(IOSET,1)** : Mean trace length
- **PROPT(IOSET,2)** : Standard deviation of trace length (input 0 when NPROP(IOSET,3) = 1)
- **PROPT(IOSET,3)** : Mean fracture orientation in degrees
- **PROPT(IOSET,4)** : Concentration factor of the Von Mises orientation distribution. For other distribution, i.e., NPROP(IOSET,4) = 2, 3 or 4, set PROPT(IOSET,4) = 0.
- **PROPT(IOSET,5)** : Cohesion of fracture
- **PROPT(IOSET,6)** : Friction angle of fracture
- **PROPT(IOSET,7)** : Tensile cut-off stress of fracture
- Read random number seed : DSEED or ISEED (input integer value when NSYST = 2)
- Input max. radius of influence when NPROP(IOSET,2) = 2 : STAND
D.3 Program Listing : TRACESIM

PROGRAM TRACESIM

PROGRAM TRACESIM IS INITIALLY INTENDED TO SIMULATE THE
MECHANICAL FRACTURING BEHAVIOR OF JOINTS ON A SLOPE
WHICH INCLUDES WING CRACK, COALESCENT CRACK AND SECOND
CRACK USING STOCHASTIC JOINT GEOMETRY MODEL.

VERSION 1 IS DEVELOPED IN THE ROCK MECHANICS GROUP AT
MIT, FEB. 1990, USING JVNC SUPER COMPUTER.
THIS VERSION IS USED FOR THE SLOPE STABILITY ANALYSIS
CONSIDERING FULLY PERSISTENT ROCK BLOCK MODEL

VERSION 1.1 IS UPDATED AND ADAPTED TO UNIX SYSTEM

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/NPOIN, NTRCE, HOSIM, NDOFN, MOPTN, IOSTIM, NSYST,
*     NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEIGT, NCASE
COMMON/CONTROL/NMODE, SFMAX, HOSIM
COMMON/STATIC/NSTAT, VARMN, DSTEP, NSTEP, ICASE, NREG1, NREG2
DIMENSION RR(10000), AA(40000), TITLE(20)

NRVAR = 40000
NIVAR = 10000
NDOFN = 2

C*** OPEN STATEMENT
C
OPEN (5, FILE='tracesim.dat5', STATUS='UNKNOWN')
OPEN (6, FILE='slope.out6', STATUS='UNKNOWN')
C
READ (5,900) TITLE
WRITE(6,900) TITLE

1. NO. OF SIMULATED JOINTS IN A SLOPE : NPOIN
2. NO. OF JOINT SETS : NOSET
3. NO. OF SIMULATIONS : NOSIM

4. PRINTOUT OPTION : MOPTN
   1 = PRINT A SIMULATED TRACE PATTERN IN STANDARD OUTPUT FILE 6
   2 = PRINT A SIMULATED TRACE PATTERN BOTH
       IN STANDARD OUTPUT FILE 6 AND IN PLOTTING FILE 7
   3 = PRINT A DETECTED TRACE PATH IN PLOTTING FILE 8
   4 = STORE SAFETY FACTOR IN FILE 10
   5 = STORE ALL DATA IN PLOTTING FILES 7, 8 AND 9

5. OPTION FOR STATISTICAL ANALYSIS : NSTAT
   0 = DO NOT CONSIDER
   1 = STATISTICAL ANALYSIS OF S.F. ACCORDING TO VOLUME
2 = STATISTICAL ANALYSIS OF S.F. ACCORDING TO DIP

6. OPTION FOR NO. OF CASES : NCASE
7. OPTION FOR OPERATING SYSTEM : NSYST
   1 = RUNNING ON VMS SYSTEM
   2 = RUNNING ON Unix SYSTEM

8. ANALYSIS OPTION : NOPTN
   1 = HOMOGENEOUS POISSON MID-POINT PATTERN
   2 = NON-HOMOGENEOUS POISSON MID-POINT PATTERN

9. TRACE LENGTH DISTRIBUTION OPTION : NOPLT
   1 = EXPONENTIAL DISTRIBUTION WITH MEAN TRACE LENGTH
   2 = LOGNORMAL DISTRIBUTION WITH MEAN AND STD. DEVIATION

10. ORIENTATION DISTRIBUTION OPTION : NOPDP
    1 = Von Mises Distribution Function With Mean Orientation
        And Concentration Factor
    2 = WRAPPED NORMAL DISTRIBUTION FUNCTION WITH MEAN
        ORIENTATION AND CONCENTRATION FACTOR
    3 = UNIFORM DISTRIBUTION ON [0, 180 deg]
    4 = FIXED ORIENTATION ON THETA

11. NO. OF EFFECTIVE JOINTS IN A SLOPE : NTRACE

12. PRESCRIBED SAFETY FACTOR : SFMAX

READ (5,*) NPOIN, NOSET, NOSIM, MOPTN, NSTAT, NCASE, NSYST
WRITE(6,910) NPOIN, NOSET, NOSIM, MOPTN, NSTAT, NCASE, NSYST
MOSIM = NOSIM + 1

** OPEN STATEMENTS **

IF ( MOPTN.GE.2 ) OPEN (7, FILE='slope.out7', STATUS='UNKNOWN' )
IF ( MOPTN.GE.3 ) OPEN (8, FILE='slope.out8', STATUS='UNKNOWN' )
IF ( NSTAT.GT.0 ) OPEN (9, FILE='slope.out9', STATUS='UNKNOWN' )
IF ( NSTAT.GT.0 ) OPEN (10, FILE='slope.out0', STATUS='UNKNOWN' )

READ PARAMETERS FOR STATISTICAL ANALYSIS
VARMN : MINIMUM VALUE OF VOLUME OR DIP ANGLE
DSTEP : SIZE OF INCREMENT
NSTEP : NO. OF INCREMENTS

IF ( NSTAT.NE.0 ) THEN
  READ (5,*) VARMN, DSTEP, NSTEP, NREG1, NREG2
  WRITE(6,920) VARMN, DSTEP, NSTEP
ENDIF

READ (5,*) ANGLE, HEIGT, RANGE, WEGIT, SFMAX
WRITE(6,930) ANGLE, HEIGT, RANGE, WEGIT, SFMAX

MOPNR = 4
MPARA = 7
NOPNR = NOSET * MOPNR
NTOTL = NDOFN * NPOIN
NTOVL = NTOTL * 2
NPARAM = NOSET * MPARAM
NLIST = MOSIM * NSTEP
MSTOR = NCASE * NSTEP
IF ( NSTAT.EQ.0 ) THEN
   NLIST = 1
   MSTOR = 1
ENDIF

DYNAMIC DIMENSIONING

NR1 : OUTPUT OF STABILITY PROBABILITY 1 = CASE0(NSTEP,NCASE)
NR2 : OUTPUT OF STABILITY PROBABILITY 2 = CASE1(NSTEP,NCASE)

NR1 = 1
NR2 = NR1 + MSTOR
NR3 = NR2 + MSTOR
JVAR1 = NR3 - 1
DO 1 IVARI = 1, JVAR1
AA(IVARI) = 0.
1 CONTINUE

DO LOOP FOR EACH PARAMETER CASE

DO 500 ICASE = 1, NCASE

DYNAMIC DIMENSIONING

NR3 : MATERIAL PARAMETERS = PROPT(NOSET,MPARAM)
NR4 : KINEMATIC UNSTABLE JOINT PATHS = SVALU(MOSIM,NSTEP)
      ( OPTIONAL )
NR5 : MECHANICAL UNSTABLE JOINT PATHS = TVLAU(MOSIM,NSTEP)
      ( OPTIONAL )
NR6 : MPO POINTS DIMENSION = COORM(NPOIN,2)
NR7 : END POINTS DIMENSION = COORE(NTOTL,2)

NR4 = NR3 + NPARAM
NR5 = NR4 + NLIST
NR6 = NR5 + NLIST
NR7 = NR6 + NTOTL
NR8 = NR7 + NTOVL

NII1 : NO. OPTION PARAMETERS IN TOTAL JOINT SETS = NPROP(NOSET,4)
NII2 : NO. INTERSECTION POINT AT EACH TRACE = NINPT(NPOIN)

NII1 = 1
NII2 = NII1 + NOPNR
NII3 = NII2 + NPOIN

LVAR1 = NR6 - 1
LVAR2 = NII2 - 1
DO 5 IVARI = NR3, LVAR1
AA(IVARI) = 0.
DO 6 IVARI = 1, IVAR2
KK(IVARI) = 0

MAIN LOOP

DO 100 IOSIM = 1, NOSIM

INITIALIZE ARRAYS

DO 10 IVARI = NR6, NRVAR
AA(IVARI) = 0.
DO 20 IVARI = NI2, NIVAR
KK(IVARI) = 0

CALL MAIN OPTION

CALL MAINS (AA(NR3), AA(NR6), AA(NR7), KK(NI1), KK(NI2),
* MAXPT, NOEFJ, MAXBT)

IF (NOEFJ.EQ.0.OR.MAXBT.LT.2) THEN
  WRITE(6,980) NOEFJ, MAXBT
  GO TO 100
ENDIF

CALL GEOMETRIC SCREENING OPTION

NTRCE = NOEFJ + 2
NTOCY = NTRCE * NTRCE
NTOCO = NOEFJ * 4

NR8 : TRACE CONNECTIVITY MATRIX = XCONN(NTRCE,NTRCE)
NR9 : TRACE CONNECTIVITY MATRIX = YCONN(NTRCE,NTRCE)
NR10 : COORDINATE MATRIX OF EFFECTIVE TRACES = COORF(NOEFJ,4)
NI3 : NO. INTERSECTION POINTS AT EFFECTIVE JOINT = NINEP(NOEFJ)
NI4 : MATERIAL PROPERTY AT EACH EFFECTIVE JOINT = NMTRL(NOEFJ)

NR9 = NR8 + NTOCY
NR10 = NR9 + NTOCY
NR11 = NR10 + NTOCO

NI4 = NI3 + NOEFJ
NI5 = NI4 + NOEFJ

NRTOT = NR11 - 1
NITOT = NI5 - 1
IERRO = 0

IF (NIVAR.GT.NRTOT) GO TO 30
WRITE(6,950)
IERRO = IERRO + 1

CONTINUE
IF (NIVAR.GT.NITOT) GO TO 40
WRITE (6, 970)
IERSO = IERSO + 1
40 CONTINUE
IF ( IERSO.GT.0 ) STOP

C
CALL KINEM ( AA(NR1), AA(NR2), AA(NR3), AA(NR4), AA(NR5),
/ * AA(NR6), AA(NR7), AA(NR8), AA(NR9), AA(NR10),
/ * KK(NI1), KK(NI2), KK(NI3), KK(NI4),
/ * MAXPT, NOEFJ )

C
100 CONTINUE
500 CONTINUE
C
900 FORMAT (20A4)
910 FORMAT (/, 5X, 'NO. JOINTS IN A SLOPE = ', 3X, I5,
/ * /, 5X, 'NO. JOINT SETS IN A SLOPE = ', 3X, I5,
/ * /, 5X, 'NO. SIMULATIONS = ', 3X, I5,
/ * /, 5X, 'PRINTOUT OPTION = ', 3X, I5,
/ * /, 7X, '( 0 : NO PRINTOUT )',
/ * /, 7X, '( 1 : STANDARD PRINTOUT )',
/ * /, 7X, '( 2 : CREATE PLOTTING FILE 7 )',
/ * /, 7X, '( 3 : CREATE FILES 7 AND 8 )',
/ * /, 7X, '( 4 : INCLUDE PERTURBED DATA )',
/ * /, 5X, 'STATISTICAL ANALYSIS OPTION = ', 3X, I5,
/ * /, 7X, '( 0 : DO NOT CONSIDER )',
/ * /, 7X, '( 1 : VOLUME CRITERION )',
/ * /, 7X, '( 2 : MIN. DIP CRITERION )',
/ * /, 5X, 'NO. OF SIMULATION CASES = ', 3X, I5,
/ * /, 5X, 'OPERATING SYSTEM = ', 3X, I5,
/ * /, 7X, '( 1 : RUNNING ON VMS SYSTEM )',
/ * /, 7X, '( 2 : RUNNING ON Unix SYSTEM )'
)
920 FORMAT (/, 5X, 'STATISTICAL ANALYSIS',
/ * /, 7X, 'STARTING VOLUME OR DIP = ', F10.3,
/ * /, 7X, 'SIZE OF INCREMENT = ', F10.3,
/ * /, 7X, 'NO. OF INCREMENTS = ', I10 )
930 FORMAT (/, 5X, 'SLOPE SHAPE CONFIGURATION',
/ * /, 7X, 'SLOPE ANGLE ( Degrees ) = ', F10.3,
/ * /, 7X, 'SLOPE HEIGHTS = ', F10.3,
/ * /, 7X, 'FREE SURFACE RANGES = ', F10.3,
/ * /, 5X, 'UNIT WEIGHT OF ROCK MASS = ', F10.3,
/ * /, 5X, 'PRESCRIBED SAFETY FACTOR = ', F10.3 )
950 FORMAT (/, 5X, '*** INCREASE STORAGE FOR REAL ARRAY' )
970 FORMAT (/, 5X, '*** INCREASE STORAGE FOR INTEGER ARRAY' )
980 FORMAT (/, 5X, '*** NOT APPROPRIATE JOINT PATTERN ***',
/ * /, 7X, 'TOTAL INTERSECTION POINTS = ', I5,
/ * /, 7X, 'TOTAL INTERSECTING BOUNDARY = ', I5)
C
STOP
END
CCC
C
SUBROUTINE MAINS ( PROPT, COORM, COORE, NPROP, NINPT,
/ * MAXPT, NOEFJ, MAXBT )
C
C
SUBROUTINE MAINS CONTROLS THE OPTIONS
IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOFN, MOPTN, IOSIM, NSYST,
    *   NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEIGT, NCASE
COMMON/CONTRO/NMODE, SFMIN, NOSIM
COMMON/STATIC/NSTAT, VARMIN, DSTEP, NSTEP, ICASE, NREG1, NREG2
DIMENSION COORM(NPOIN,2), COORE(NPOIN,4), PROPT(NOSET,7)
DIMENSION NPROP(NOSET,4), NIMPT(NPOIN)

FIRST, PROGRAM TRACESIM SIMULATES THE APPROPRIATE JOINT
PATTERN IN A SLOPE ACCORDING TO THE OPTIONS OF
NPOIN, NDOFL AND NDOFP.

CALL GENES ( COORM, COORE, PROPT, NPROP )

SECOND, CONSIDER THE GEOMETRICALLY ADMISSIBLE JOINT PATHS

CALL GEOMETRY ( COORM, COORE, NIMPT, MAXPT, NOEPJ, MAXPT )

RETURN
END

SUBROUTINE GENES ( COORM, COORE, PROPT, NPROP )

SUBROUTINE GENES GENERATES THE JOINT PATTERN IN A SLOPE
ACCORDING TO THE OPTIONS OF NPOIN, NDOFL AND NDOFP.

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOFN, MOPTN, IOSIM, NSYST,
    *   NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEIGT, NCASE
DIMENSION COORM(NPOIN,2), COORE(NPOIN,4)
DIMENSION NPROP(NOSET,4), PROPT(NOSET,7)

PIRAD = 3.1415926 / 180.

READ GEOMETRY OPTIONS

IF ( IOSIM.EQ.1 ) THEN
    NTRPT = 0
    DO 10 NOSET = 1, NOSET
        READ (5,*) NOSET
        READ (5,*) ( NPROP(NOSET,JOSET), JOSET= 1, 4 )
        READ (5,*) ( PROPT(NOSET,JOSET), JOSET= 1, 7 )
        WRITE(6,910) NOSET, ( NPROP(NOSET,JOSET), JOSET= 1, 4 )
        WRITE(6,915) ( PROPT(NOSET,JOSET), JOSET= 1, 7 )
        PROPT(NOSET,3) = PROPT(NOSET,3) * PIRAD
        PROPT(NOSET,6) = PROPT(NOSET,6) * PIRAD
        NTRPT = NTRPT + NPROP(NOSET,1)
    10   CONTINUE
    IF ( NTRPT.NE.NPOIN ) THEN
        WRITE(6,920)
        RETURN
    ENDIF
ENDIF
GENERATE THE MID-POINT PATTERN OF JOINTS IN A SLOPE : NOPTN

IPOIN = 0
DO 400 IOSET = 1, NOSET
NOPTN = NPROP(IOSET, 2)
GO TO (100, 200), NOPTN

A HOMOGENEOUS POISSON POINT PATTERN

100 CONTINUE
CALL NPOIS ( COORM, ISEED, DSEED, NPROP, IOSET, IPOIN )
GO TO 300

A NON-HOMOGENEOUS POISSON POINT PATTERN

200 CONTINUE
CALL NHPOI ( COORM, ISEED, DSEED, NPROP, IOSET, IPOIN )
300 CONTINUE

GENERATE A TRACE LENGTH AND AN ORIENTATION OF A JOINT
AT EACH MID-POINT ACCORDING TO NOPLT AND NOPDP

CALL PATEN ( COORM, COORE, ISEED, DSEED, NPROP, IOSET,
* PROPT, IPOIN )

400 CONTINUE

910 FORMAT(//, 5X, '*** JOINT SET NO. = ', I5,
* /, 7X, 'NO. OF JOINTS = ', I5,
* /, 7X, 'OPTION FOR JOINT PATTERN DISTRIBUTION = ', I5,
* /, 9X, '(' 1 : HOMO. POISSON POINT PATTERN )',
* /, 9X, '(' 2 : NON-HOMO. POISSON POINT PATTERN )',
* /, 7X, 'OPTION FOR JOINT LENGTH DISTRIBUTION = ', I5,
* /, 9X, '(' 1 : EXPONENTIAL LENGTH DISTRIBUTION )',
* /, 9X, '(' 2 : LOGNORMAL LENGTH DISTRIBUTION )',
* /, 7X, 'OPTION FOR ORIENTATION DISTRIBUTION = ', I5,
* /, 9X, '(' 1 : VON MISES ORIENTATION DISTRIBUTION )',
* /, 9X, '(' 2 : WRAPPED NORMAL DISTRIBUTION )',
* /, 9X, '(' 3 : UNIFORM ORIENTATION DISTRIBUTION )',
* /, 9X, '(' 4 : FIXED ORIENTATION )')

915 FORMAT(//, 7X, 'GEOMETRIC PARAMETERS OF JOINTS',
* /, 9X, 'MEAN JOINT LENGTH = ', F10.3,
* /, 9X, 'STD. DEVIATION (LOGNORMAL) = ', F10.3,
* /, 9X, 'MEAN ORIENTATION (Degree) = ', F10.3,
* /, 9X, 'CONCENTRATION FACTOR = ', F10.3,
* /, 7X, 'MECHANICAL PROPERTIES OF JOINTS',
* /, 9X, 'COHESION OF JOINT FACE = ', F10.3,
* /, 9X, 'FRICTION ANGLE OF JOINT FACE (Deg.) = ', F10.3,
* /, 9X, 'TENSILE CUT-OFF STRESS OF JOINT = ', F10.3 )

920 FORMAT(//, 5X, '*** WARNING *** : NO. POINT NOT MATCHED')

RETURN
END
SUBROUTINE HPOIS ( COORM, ISEED, DSEED, NPROP, IOSET, IPOIN )

SUBROUTINE HPOIS GENERATES THE HOMOGENEOUS POISSON POINT
PATTERN IN A SLOPE

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/HPOIN, NTRCE, NOSIM, NDOPH, NOPTH, IOSIM, NSYST,
* NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEIGT, NCASE
DIMENSION COORM(NPOIN,2), TMPRY(2)
DIMENSION NPROP(IOSET,4)

CALL RANDOM NUMBER GENERATOR
ACCORDING TO UNIX OR VMS MODE

IF ( IOSIM.EQ.1.AND.IOSET.EQ.1 ) THEN
    IF ( NSYST.EQ.1 ) THEN

*** REAL SEED FOR VMS SYSTEM ( USE IMSL ROUTINE )

READ (5,*) DSEED
ELSE

*** INTEGER SEED FOR UNIX SYSTEM FUNCTION RANDOM

READ (5,*) ISEED
ENDIF
ENDIF

NOPNT = NPROP(IOSET,1)
IOFNT = 0
PIVAL = 3.1415926
ANRAD = ANGLE * PIVAL / 180.

CONTINUE
IF ( NSYST.EQ.2 ) THEN

*** CALL INTRINSIC FUNCTION IN UNIX SYSTEM

TMPRY(1) = RANDOM (ISEED)
ISKED = IRANDM (ISEED)
TMPRY(2) = RANDOM (ISEED)
ISKED = IRANDM (ISEED)
ELSE

*** CALL IMSL IN VMS SYSTEM

CALL GGUBS (DSEED, 2, TMPRY)
ENDIF

TMPCX = TMPRY(1) * RANGE
TMPSF = TMPCX * TAN(ANRAD)
TMPCY = TMPRY(2) * HEIGT

CHECK THE SLOPE FACE BOUNDARY
IF ( TMPCY.GT.TMPSR ) GO TO 10
IPOIN = IPOIN + 1
IOPNT = IOPNT + 1
COORM(IPOIN,1) = TMPCX
COORM(IPOIN,2) = TMPCY
IF ( IOPNT.LT.NOFNT ) GO TO 10
RETURN
END

SUBROUTINE NHPOI ( COORM, ISEE, DSEE,
* NPROF, ISET, IPOIN, ICASE )

SUBROUTINE NHPOI Generates the Non Homogeneous
Hierarchical Point Pattern in a Slope.

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOFN, MOPTN, IOSIM, NSYST,
* NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEGT, NCASE
DIMENSION COORM(NPOIN,2), TMRPY(2)
DIMENSION NPROF(NOSET,4)

CURRENTLY, CLUSTERING PATTERN OF THE SECOND SET AROUND
THE FIRST SET CAN BE REALIZED.

NOPTN = NPROF(ISET,1)
IOPNT = 0
PIVAL = 3.1415926
ANRAD = ANGLE * PIVAL / 180.
NOPRS = NPOIN - NOPTN

IF ( IOSIM.EQ.1.AND.ISET.NE.1 ) THEN
  READ (5,*) STAND
  WRITE(6,100) STAND
ENDIF

DO 50 IOPNT = 1, NOPTN
  CONTINUE
50 IF ( NSYST.EQ.2 ) THEN

*** CALL RANDOM IN UNIX SYSTEM

  TMRPY(1) = RANDOM (ISEED)
  ISEE = IRANDM (ISEED)
  TMRPY(2) = RANDOM (ISEED)
  ISEE = IRANDM (ISEED)
ELSE

*** VMS SYSTEM

  CALL GGUBS (DSEE, 2, TMRPY)
C KPOIN = IPOIN - NOPNT
JPOIN = KPOIN + 1
10 CONTINUE
KPOIN = KPOIN + 1
C
IF ( NSYST.EQ.1 ) THEN
  IF ( NOPLT.EQ.1 ) THEN
    CALL GGEXN ( DSEE, TRCEM, 1, TMPA1 )
  ELSE
    CALL GGNLG ( DSEE, 1, TRCEM, STDTN, TMPA1 )
  ENDIF
ELSE
ENDIF
C
*** EXponential Trace LENGTH GENERATOR WITH UNIX SYSTEM
C
TMPA1(1) = RANDOM (ISEED)
ISEED = IRANDM (ISEED)
TMPA1(1) = -ALOG ( 1. - TMPA1(1) ) * TRCEM
ENDIF
C
TCLEN = TMPA1(1) / 2.
C
VON MISES ORIENTATION DISTRIBUTION
C
IF ( NOPDP.EQ.1 ) THEN
  xindx = 1. + ( 0.5*conft )**2 + ( 0.5*conft )**4 / 4.
  * + ( 0.5*conft )**6 / 36.
  xindx = 1. / ( xindx * 2. * pirad )
C
*** UNIX SYSTEM FUNCTION
C
IF ( NSYST.EQ.2 ) THEN
  FREQX = EXP ( CONFT )
  CONTINUE
  TMPA2(1) = RANDOM (ISEED)
  ISEED = IRANDM (ISEED)
  YINDEX = TMPA2(1) * PIRAD * 2.
  FREQX = EXP ( CONFT * COS( YINDEX - THETA ) )
  FREQQ = FREQX / FREQX
  TMPA2(1) = RANDOM (ISEED)
  ISEED = IRANDM (ISEED)
  IF ( TMPA2(1).GT.FREQQ ) GO TO 99
  ORINT = YINDEX
ELSE
C
*** VMS MODE
C
  CALL GGVMS ( DSEE, CONFT, 1, TMPA2 )
  ORINT = TMPA2(1) + THETA
ENDIF
ENDIF
C
C CURRENTLY (NOPDP.EQ.2) OPTION (WRAPPED NORMAL DISTRIBUTION)
C IS IGNORED
ENDIF

CPP

KRNDM = TMPRY(1) * NOPRS + 1
COORX = COORM(KRNDM, 1)
COORY = COORM(KRNDM, 2)
TMPCX = COORX + STAND * TMPRY(1) - STAND / 2.
TMPCY = COORX + STAND * TMPRY(2) - STAND / 2.
TMPSP = TMPCX * TAN(ANRAD)

CPP

IF ( TMPCX.LE.0. OR. TMPCX.GE.RANGE ) GO TO 10
IF ( TMPCY.LE.0. OR. TMPCY.GE.HEIGT ) GO TO 10
IF ( TMPCY.GT.TMPSP ) GO TO 10

CPP

IPOIN = IPOIN + 1
IOPNT = IOPNT + 1
COORM(IPOIN, 1) = TMPCX
COORM(IPOIN, 2) = TMPCY
IF ( IOPNT.LE.NOPNT ) GO TO 10

CPP

100 FORMAT(5X, 'INFLUENCE ZONE OF CLUSTERING PATTERN = ', 3X, F12.2)
CPP

RETURN

END

CPP

SUBROUTINE PATEN ( COORM, COORE, ISEED, DSEED, NPROP, IOSET, *

CPP

* FROPT, IPOIN )

CPP

SUBROUTINE PATEN EVALUATES THE TRACE LENGTH AND ORIENTATION

CPP

OF A TRACE ACCORDING TO NOPLT AND NOPDP

CPP

IMPLICIT DOUBLE PRECISION (A-Z, 0-Z)
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOPN, MOPTN, IOSIM, NSYST,

CPP

* NOSET, ANGLE, HEIGT, RANGE, SYMAX, WEIGT, HCASE
COMMON/CONTOL/NMODE, SFMIN, MOSIM
DIMENSION COORM(NPOIN, 2), COORE(NPOIN, 4), TMPA1(1), TMPA2(1)
DIMENSION NPROP(NOSET, 4), FROPT(NOSET, 7)

CPP

PIRAD = 3.1415926
RADIN = PIRAD / 180.
ANRAD = ANGLE * RADIN

CPP

NOPNT = NPROP(IOSET, 1)
NOPLT = NPROP(IOSET, 3)
NOPDP = NPROP(IOSET, 4)

CPP

TRCEM = PROPT(IOSET, 1)
STDTN = PROPT(IOSET, 2)
THETA = PROPT(IOSET, 3)
CONF = PROPT(IOSET, 4)

CPP

USE MID-POINT COORDINATES EITHER FROM SUBROUTINE HPOIS OR

CPP

FROM HPOI.
FROM IMSL, CHOOSE SUBROUTINES GGEKN OR GGMLG FOR EXPONENTIAL

CPP

AND LOGNORMAL TRACE LENGTH DISTRIBUTION, RESPECTIVELY.
IF ( NODPF.EQ.2 ) THEN

UNIFORM ORIENTATION DISTRIBUTION

IF ( NODPF.EQ.3 ) THEN
  IF ( NSYST.EQ.1 ) THEN

*** VMS

CALL GGUBS ( DSEED, 1, TMFA2 )
ELSE

*** UNIX SYSTEM FUNCTION

TMFA2(1) = RANDOM (ISEED)
ISEED = IRANDM (ISEED)
ENDIF
ORINT = TMFA2(1) * PIRAD
ENDIF

FIXED ORIENTATION

IF ( NODPF.EQ.4 ) ORINT = THETA

CALCULATE END POINTS OF EACH TRACE

XCOORD = COORM(KPOIN,1)
YCOORD = COORM(KPOIN,2)
XCOORD1 = XCOORD + TCLEN * COS(ORINT)
YCOORD1 = YCOORD + TCLEN * SIN(ORINT)
XCOORD2 = XCOORD - TCLEN * COS(ORINT)
YCOORD2 = YCOORD - TCLEN * SIN(ORINT)

ANDSET THE BOUNDARY CONDITIONS

IF ( YCOORD1.GE.HEIGHT ) THEN
  YCOORD1 = HEIGHT
  IF ( ORINT.EQ.PIRAD/2. ) THEN
    XCOORD1 = XCOORD
  ELSE
    XCOORD1 = XCOORD + ( YCOORD1-YCOORD ) / TAN(ORINT)
  ENDIF
ENDIF
IF ( YCOORD2.GE.HEIGHT ) THEN
  YCOORD2 = HEIGHT
  IF ( ORINT.EQ.PIRAD/2. ) THEN
    XCOORD2 = XCOORD
  ELSE
    XCOORD2 = XCOORD + ( YCOORD2-YCOORD ) / TAN(ORINT)
  ENDIF
ENDIF
IF ( YCOORD1.LE.0. ) THEN
  YCOORD1 = 0.
  IF ( ORINT.EQ.PIRAD/2. ) THEN
    XCOORD1 = XCOORD
  ENDIF

ELSE
  XCOR1 = XCOOR + ( YCOR1 - YCOOR ) / TAN(ORINT)
ENDIF
ENDIF
IF ( YCOR2 .LE. 0 ) THEN
  YCOR2 = 0.
  IF ( ORINT .EQ. PIRAD/2. ) THEN
    XCOR2 = XCOOR
  ELSE
    XCOR2 = XCOOR + ( YCOR2 - YCOOR ) / TAN(ORINT)
  ENDIF
ENDIF
ENDIF
IF ( XCOR1 .GE. RANGE ) THEN
  XCOR1 = RANGE
  YCOR1 = TAN(ORINT) * ( XCOR1 - XCOOR ) + YCOOR
ENDIF
IF ( XCOR1 .LE. 0. ) THEN
  XCOR1 = 0.
  YCOR1 = TAN(ORINT) * ( XCOR1 - XCOOR ) + YCOOR
ENDIF
IF ( XCOR2 .GE. RANGE ) THEN
  XCOR2 = RANGE
  YCOR2 = TAN(ORINT) * ( XCOR2 - XCOOR ) + YCOOR
ENDIF
IF ( XCOR2 .LE. 0. ) THEN
  XCOR2 = 0.
  YCOR2 = TAN(ORINT) * ( XCOR2 - XCOOR ) + YCOOR
ENDIF
C
SLOP1 = XCOR1 * TAN(ANRAD)
SLOP2 = XCOR2 * TAN(ANRAD)
IF ( YCOR1 .GT. SLOP1 ) THEN
  YCOR3 = YCOOR - YCOR1
  XCOR3 = XCOOR - XCOR1
  IF ( ABS(XCOR3) .LT. 0.0001 ) THEN
    YCOR1 = TAN(ANRAD) * XCOOR
  ELSE
    TANJTC = YCOR3 / XCOR3
    XCOR1 = ( TANJTC * XCOR1 - YCOR1 ) / ( TANJTC - TAN(ANRAD) )
    YCOR1 = TAN(ANRAD) * XCOR1
  ENDIF
ENDIF
ENDIF
IF ( YCOR2 .GT. SLOP2 ) THEN
  YCOR4 = YCOOR - YCOR2
  XCOR4 = XCOOR - XCOR2
  IF ( ABS(XCOR4) .LT. 0.0001 ) THEN
    YCOR2 = TAN(ANRAD) * XCOOR
  ELSE
    TANJTC = YCOR4 / XCOR4
    XCOR2 = ( TANJTC * XCOR2 - YCOR2 ) / ( TANJTC - TAN(ANRAD) )
    YCOR2 = TAN(ANRAD) * XCOR2
  ENDIF
ENDIF
ENDIF
STORE THE END POINTS OF EACH TRACE INTO COORE

COORE(KPOIN,1) = XCOR1
COORE(KPOIN,2) = YCOR1
COORE(KPOIN,3) = XCOR2
COORE(KPOIN,4) = YCOR2
IF ( XCOR1.GT.XCOR2 ) THEN
    COORE(KPOIN,1) = XCOR2
    COORE(KPOIN,2) = YCOR2
    COORE(KPOIN,3) = XCOR1
    COORE(KPOIN,4) = YCOR1
ENDIF

IF ( KPOIN.LT.IPOIN ) GO TO 10

IF ( MOPTN.GE.1 ) , WRITE THE END POINT COORDINATES

IF ( MOPTN.EQ.1 ) THEN
    WRITE(6,900) IOSET
    DO 20 KPOIN = JPOIN, NPOIN
        WRITE(6,910) ( COORE(KPOIN,JDOFN), JDOFN = 1,4 )
    20 CONTINUE
ENDIF

IF ( MOPTN.GE.5 ) THEN
    IF ( IOSTM.EQ.1.AND.IOSET.EQ.1 )
        WRITE(7,920) NPOIN, ANGLE, HEIGT, RANGE, NOSIM, MOPTN
    DO 30 KPOIN = JPOIN, IPOIN
        WRITE(7,910) ( COORE(KPOIN,JDOFN), JDOFN = 1,4 )
    30 CONTINUE
ENDIF

900 FORMAT('///,5X, 'SIMULATED TRACE PATTERN OF JOINT SET = ', I5, //)
910 FORMAT(2(5X,F10.3, 5X,F10.3, //))
920 FORMAT(I5, 3F10.3, I5)

RETURN
END

SUBROUTINE GEOTY ( COORM, COORE, NINPT, MAXPT, NREFJ, MAXBT )

SUBROUTINE GEOTY EVALUATES THE GEOMETRICALLY CONTINUOUS JOINT PATHS

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOFN, MOPTN, IOSIM, NSYST,
    * HOSET, ANGLE, HEIGT, RANGE, SMAX, WEIGT, NCASE
DIMENSION COORM(NPOIN,2), COORE(NPOIN,4), NINPT(NPOIN)

TO FIND THE EFFECTIVE JOINT PATHS, CALCULATE THE EFFECTIVE INTERSECTION POINTS AMONG JOINTS AND SLOPE BOUNDARIES

PIRAD = 3.1415926
TANAN = TAN(ANGLE*PIRAD / 180.)
XAPEX = HEIGT / TANAN
MAXFP = 0
IIITER = 0

C
5 CONTINUE
MAXPT = 0
MAXBT = 0
NOEFJ = 0
IIITER = IIITER + 1

C
DO 30 IPOIN = 1, NPOIN
   IF ( IIITER.GT.1 .AND. NINPT(IPOIN).LT.2 ) GO TO 30
   NINPT(IPOIN) = 0
   XCOR1 = COORE(IPOIN,1)
   YCOR1 = COORE(IPOIN,2)
   XCOR2 = COORE(IPOIN,3)
   YCOR2 = COORE(IPOIN,4)
   IF ( XCOR2.EQ.XCOR1 ) THEN
      SLOP1 = 9.E+10
   ELSE
      SLOP1 = ( YCOR2-YCOR1 ) / ( XCOR2-XCOR1 )
   ENDIF

C
DO 10 JPOIN = 1, NPOIN
   IF ( JPOIN.EQ.IPOIN ) GO TO 10
   IF ( IIITER.GT.1 .AND. NINPT(JPOIN).LT.2 ) GO TO 10
   XCOR3 = COORE(JPOIN,1)
   YCOR3 = COORE(JPOIN,2)
   XCOR4 = COORE(JPOIN,3)
   YCOR4 = COORE(JPOIN,4)

C
C
   CALCULATE THE INTERSECTION POINTS AND CHECK THEIR ADMISSIBILITY

C
COEF1 = XCOR4 - XCOR3
COEF2 = YCOR4 - YCOR3
   IF ( COEF1.EQ.0. ) THEN
      SLOP2 = 9.E+10
   ELSE
      SLOP2 = COEF2 / COEF1
   ENDIF
   IF ( SLOP1.EQ.SLOP2 ) GO TO 10
IF ( SLOP1.EQ.9.E+10 ) THEN
   XINPT = XCOR1
   IF ( (XINPT.GE.XCOR3.AND.XINPT.LE.XCOR4) THEN
      YINPT = SLOP2 * ( XINPT-XCOR3 ) + YCOR3
      IF ( ( YINPT.GE.MIN1(YCOR1,YCOR2) ).AND.
*          YINPT.LE.MAX1(YCOR1,YCOR2) ) THEN
         NINPT(IPOIN) = NINPT(IPOIN) + 1
         MAXPT = MAXPT + 1
      GO TO 10
   ENDIF
   ENDIF
   ENDIF
   IF ( SLOP2.EQ.9.E+10 ) THEN
      XINPT = XCOR3

IF (XINPT.GE.XCOR1.AND.XINPT.LE.XCOR2) THEN
   YINPT = SLOP1 * (XINPT-XCOR1) + YCOR1
   IF (YINPT.GE.AMIN1(YCOR3,YCOR4).AND.
       YINPT.LE.AMAX1(YCOR3,YCOR4)) THEN
      NINPT(IPOIN) = NINPT(IPOIN) + 1
      MAXPT = MAXPT + 1
   GO TO 10
ENDIF
ENDIF
C
COEF3 = SLOP1 * XCOR1 - SLOP2 * XCOR3
COEF4 = YCOR3 - YCOR1
XINPT = (COEF3 + COEF4) / (SLOP1 - SLOP2)
YINPT = SLOP1 * (XINPT-XCOR1) + YCOR1
C
IF (XINPT.LT.XCOR1.OR.XINPT.GT.XCOR2) GO TO 10
IF (XINPT.LT.XCOR3.OR.XINPT.GT.XCOR4) GO TO 10
IF (YINPT.LT.AMIN1(YCOR1,YCOR2).OR.
    YINPT.GT.AMAX1(YCOR1,YCOR2)) GO TO 10
IF (YINPT.LT.AMIN1(YCOR3,YCOR4).OR.
    YINPT.GT.AMAX1(YCOR3,YCOR4)) GO TO 10
IF (YINPT.GT.HEIGT) GO TO 10
C
NINPT(IPOIN) = NINPT(IPOIN) + 1
MAXPT = MAXPT + 1
CONTINUE
C
SEARCH FOR INTERSECTION POINTS WITH SLOPE BOUNDARIES
C
IF (AMAX1(YCOR1,YCOR2).EQ.HEIGT) THEN
   NINPT(IPOIN) = NINPT(IPOIN) + 1
   MAXBT = MAXBT + 1
ENDIF
IF (SLOP1.EQ.TANAN) GO TO 20
IF (SLOP1.EQ.9.E+10) THEN
   XINPT = XCOR1
   YINPT = TANAN * XINPT
ELSE
   XINPT = (SLOP1*XCOR1 - YCOR1) / (SLOP1 - TANAN)
   YINPT = TANAN * XINPT
ENDIF
IF (XINPT.EQ.0..AND.YINPT.EQ.0.) THEN
   XINPT = 1.E-10
   YINPT = 1.E-10
ENDIF
IF (ABS(XINPT-XCOR1).GT.0.01.AND.
    ABS(XINPT-XCOR2).GT.0.01) GO TO 20
IF (ABS(YINPT-AMIN1(YCOR1,YCOR2)).GT.0.01.AND.
    ABS(YINPT-AMAX1(YCOR1,YCOR2)).GT.0.01) GO TO 20
IF (YINPT.GT.HEIGT) GO TO 20
NINPT(IPOIN) = NINPT(IPOIN) + 1
MAXBT = MAXBT + 1
C
COUNT THE EFFECTIVE TRACES
CONTINUE
IF ( NINPT(IPOIN).GE.2 ) NOEFJ = NOEFJ + 1
CONTINUE

IF ( IITER.EQ.1 ) WRITE(6,900) IOSIM

MAXPT = MAXPT / 2 + MAXBT

FIND FURTHER ELIMINATIONS

IF ( MAXPT.EQ.MAXPP ) GO TO 40
MAXPP = MAXPT
IF ( IITER.GT.10 ) THEN
  WRITE(6,930)
ENDIF
GO TO 5

CONTINUE

DATA FORTHOLE

IF ( MOPTN.GE.5 ) THEN
  WRITE(7,950) NOEFJ
  DO 50 IPOIN = 1, NPOIN
    IF ( NINPT(IPOIN).LT.2 ) GO TO 50
    WRITE(7,940) ( COORE(IPOIN,JDOWN), JDOWN = 1,4 )
  CONTINUE
ENDIF

FORMAT(//, 3X, '*** SIMULATION NUMBER ***', I5)
FORMAT(//, 5X, '*** TOO MANY ITERATIONS ***')
FORMAT(2(5X,F10.3,5X,F10.3,/) )
FORMAT(I5)

RETURN
END

SUBROUTINE KINEM ( CASEQ, CASET, PROPT, SVALU, TVALU, COORM,
*                    COORE, XCONN, YCONN, COORE, NPROP,
*                    NINPT, NINEP, NMTRL, MAXPT, NOEFJ )

SUBROUTINE KINEM EVALUATES THE TOPOLOGICALLY AND KINEMATICALLY
ADMISSIBLE JOINT PATHS USING MATRIX CONNECTIVITY METHOD

IMPLICIT DOUBLE PRECISION (A-H, O-Z)
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOWN, MOPTN, IOSIM, NSYST,
*          NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEGIT, NCASE
COMMON/CONTRO/NMODE, SMIN, MOSIM
COMMON/CTRL/CRITX(10), CRITY(10), NCRIT, YCRIT
COMMON/CTRL/NCTMX(10,2)
COMMON/STATIC/NSTAT, VARMN, DSTEP, NSTEP, ICASE, NREG1, NREG2
DIMENSION COORM(NPOIN,2), COORE(NPOIN,4), PROPT(NOSET,4)
DIMENSION XCONN(NTRCE,NTRCE), YCONN(NTRCE,NTRCE)
DIMENSION CCTMX(10), CCTMY(10)
DIMENSION MMP1(10), MMP2(10)
DIMENSION COORF(NOEFJ,4), NMTRL(NOEFJ)
DIMENSION NPROP(NOSET,4), HINPT(NPOIN), HINEP(NOEFJ)
DIMENSION SVALU(MOSIM,NSTEP), TVALU(MOSIM,NSTEP)
DIMENSION CASEO(NCASE,NSTEP), CASET(NCASE,NSTEP)
DIMENSION PROBY(30), TPRBY(100)

C
C FIRST, CALCULATES THE INTERSECTION POINTS AMONG JOINTS AND
SLOPE BOUNDARY USING EFFECTIVE TRACES

PIRAD = 3.1415926
TANAN = TAN(ANGLE*PIRAD/180.)
NOMTX = 0
KPOIN = 0

C
DO 5 ITRCE = 1, NTRCE
DO 5 JTRCE = 1, NTRCE
XCONN(ITRCE,JTRCE) = 0.
YCONN(ITRCE,JTRCE) = 0.
5 CONTINUE

C
JCONT = 0
DO 6 IOSET = 1, NOSET
NOPNT = NPROP(IOSET,1)
JCONT = JCONT + NOPNT
MMP1(IOSET) = JCONT
MMP2(IOSET) = JCONT - NOPNT + 1
6 CONTINUE

C
DO 420 IPOIN = 1, NPOIN
IF ( HINPT(IPOIN).GE.2 ) THEN
KPOIN = KPOIN + 1
HINEP(KPOIN) = HINPT(IPOIN)
C
DO 400 IOSET = 1, NOSET
   IF ( IPOIN.GE.MMTP2(IOSET).AND.IPOIN.LE.MMTP1(IOSET) )
      NMTRL(KPOIN) = IOSET
400 CONTINUE
C
DO 410 IDOFN = 1, 4
   COORF(KPOIN,IDOFN) = COORE(IPOIN,IDOFN)
410 CONTINUE

ENDIF
420 CONTINUE

C
KPOIN = 1
DO 50 IPOIN = 1, NOEFJ
ICOUN = 0
KPOIN = KPOIN + 1
XCSR1 = COORF(IPOIN,1)
YCSR1 = COORF(IPOIN,2)
XCSR2 = COORF(IPOIN,3)
YCSR2 = COORF(IPOIN,4)
IF ( XCSR1.EQ.XCSR2 ) THEN
SLOP1 = 9.0E10
ELSE
  SLOP1 = ( YCOR2 - YCOR1 ) / ( XCOR2 - XCOR1 )
ENDIF

C
FIRST, CALCULATE INTERSECTION POINT COORD. (XINPT, YINPT)
WITH SLOPE FACE
C
IF ( SLOP1.NE.TANAN ) THEN
  IF ( SLOP1.EQ.9.0E+10 ) THEN
    XINPT = XCOR1
    YINPT = TANAN * XINPT
  ELSE
    XINPT = ( SLOP1*XCOR1 - YCOR1 ) / ( SLOP1-TANAN )
    YINPT = TANAN * XINPT
  ENDIF
  IF ( XINPT.EQ.0..AND.YINPT.EQ.0. ) THEN
    XINPT = 1.0E-10
    YINPT = 1.0E-10
  ENDIF
  IF ( ABS(XINPT-XCOR1).GT.0.01.AND.
  *        ABS(XINPT-XCOR2).GT.0.01 ) GO TO 10
  IF ( ABS(YINPT-AMIN1(XCOR1,YCOR2)).GT.0.01.AND.
  *        ABS(YINPT-AMAX1(XCOR1,YCOR2)).GT.0.01 ) GO TO 10
  IF ( YINPT.GT.HEIGT.OR.YINPT.LT.0. ) GO TO 10
C
  XCONN(1,KPOIN) = XINPT
  XCONN(KPOIN,1) = XINPT
  YCONN(i,KPOIN) = YINPT
  YCONN(KPOIN,1) = YINPT
  NOMTX = NOMTX + 2
  ICOU1 = ICOU1 + 1
ENDIF

10 CONTINUE
LPOIN = 1
C
CALCULATE THE INTERSECTION POINTS AMONG JOINTS
C
DO 30 JPOIN = 1, NOEFJ
LPOIN = LPOIN + 1
IF ( JPOIN.EQ.IPOIN ) GO TO 30
XCOR3 = COORF(JPOIN,1)
YCOR3 = COORF(JPOIN,2)
XCOR4 = COORF(JPOIN,3)
YCOR4 = COORF(JPOIN,4)
C
CALCULATE THE INTERSECTION COORDINATES AND CHECK THEIR
ADMISSIBILITY
C
COEF1 = XCOR4 - XCOR3
COEF2 = YCOR4 - YCOR3
IF ( COEF1.EQ.0. ) THEN
  SLOP2 = 9.0E10
ELSE
  SLOP2 = COEF2 / COEF1
ENDIF
IF ( SLOP1.EQ.SLOP2 ) GO TO 30
IF ( SLOP1.EQ.9.E+10 ) THEN
  XINPT = XCOR1
  IF ( XINPT.GE.XCOR3.AND.XINPT.LE.XCOR4 ) THEN
    YINPT = SLOP2 * ( XINPT-XCOR3 ) + YCOR3
    IF ( YINPT.GE.AMIN1(YCOR1,YCOR2).AND.
        * YINPT.LE.AMIN1(YCOR1,YCOR2).AND.YINPT.LE.HEIGT ) GO TO 20
  ENDIF
GO TO 30
ENDIF
IF ( SLOP2.EQ.9.E+10 ) THEN
  XINPT = XCOR3
  IF ( XINPT.GE.XCOR1.AND.XINPT.LE.XCOR2 ) THEN
    YINPT = SLOP1 * ( XINPT-XCOR1 ) + YCOR1
    IF ( YINPT.GE.AMIN1(YCOR3,YCOR4).AND.
        * YINPT.LE.AMAX1(YCOR3,YCOR4).AND.YINPT.LE.HEIGT ) GO TO 20
  ENDIF
GO TO 30
ENDIF
C
COEF3 = SLOP1 * XCOR1 - SLOP2 * XCOR3
COEF4 = YCOR3 - YCOR1
XINPT = ( COEF3 + COEF4 ) / ( SLOP1 - SLOP2 )
YINPT = SLOP1 * ( XINPT-XCOR1 ) + YCOR1
C
IF ( XINPT.LE.XCOR1.OR.XINPT.GT.XCOR2 ) GO TO 30
IF ( XINPT.LE.XCOR3.OR.XINPT.GT.XCOR4 ) GO TO 30
IF ( YINPT.LE.AMIN1(YCOR1,YCOR2).AND.
    * YINPT.GT.AMAX1(YCOR1,YCOR2) ) GO TO 30
IF ( YINPT.LE.AMIN1(YCOR3,YCOR4).OR.
    * YINPT.GT.AMAX1(YCOR3,YCOR4) ) GO TO 30
IF ( YINPT.GT.HEIGT ) GO TO 30
C
20 CONTINUE
NOMTX = NOMTX + 1
ICOUN = ICOUN + 1
XCONN(KPOIN,LPOIN) = XINPT
YCONN(KPOIN,LPOIN) = YINPT
30 CONTINUE
C
CALCULATE THE INTERSECTION COORDINATE WITH FREE SURFACE
C
IF ( AMAX1(YCOR1,YCOR2).NE.HEIGT ) GO TO 40
IF ( SLOP1.EQ.9.E+10 ) THEN
  XINPT = XCOR1
ELSE
  XINPT = XCOR1 + ( HEIGT-XCOR1 ) / SLOP1
ENDIF
C
IF ( ABS(XINPT-XCOR1).GT.0.01.AND.
    * ABS(XINPT-XCOR2).GT.0.01 ) GO TO 40
YINPT = HEIGT
XCONN(KPOIN,NTRCE) = XINPT
XCONN(NTRCE,KPOIN) = XINPT
YCONN(KPOIN,NTRCE) = YINPT
YCONN(NTRCE,KPOIN) = YINPT
NOMTX = NOMTX + 2
ICOUN = ICOUN + 1

40 CONTINUE
IF ( ICOUN.NE.NINEP(IPOIN) ) WRITE(6,970) ICOUN, NINEP(IPOIN)

C
50 CONTINUE
C
STORE THE INTERSECTION POINTS FOR PLOTTING
C
IF ( MOPTN.GE.5 ) THEN
MJOIN = 0
DO 80 ITRCE = 1, NTRCE
KTRCE = ITRCE
DO 70 JTRCE = KTRCE, NTRCE
IF ( XCONN(ITRCE,JTRCE).LE.0. ) GO TO 70
WRITE(8,960) XCONN(ITRCE,JTRCE), YCONN(ITRCE,JTRCE)
MJOIN = MJOIN + 1
70 CONTINUE
80 CONTINUE
C
WRITE(7,940) MJOIN, MAXPT
ENDIF
C
DO A MATRIX CONNECTIVITY SEARCHING
C CURRENTLY, UP TO 10 (TEN) INTERSECTION POINTS CAN BE CONSIDERED
C AS A JOINT PATH
C
NPATR = 10
NPATH = 0
NMODE = 0
LCOUN = 0
TPATH = 0.
SFMIN = SFMAX
C
DO 300 ICOL1 = 1, NTRCE
C
DO 120 IPATR = 1, NPATR
DO 110 JDOFN = 1, NDOFN
NCTMX(IPATR,JDOFN) = 0
110 CONTINUE
CCTMX(IPATR) = 0.
CCTMY(IPATR) = 0.
120 CONTINUE
C
START SEARCHING : 1ST
C
IF ( XCONN(1,ICOL1).EQ.0. ) GO TO 300
NCTMX(1,1) = 1
NCTMX(1,2) = ICOL1
CCTMX(1) = XCONN(1,ICOL1)
CCTMY(1) = YCONN(1,ICOL1)
C
2ND
DO 290 ICOL2 = 1, NTRCE
IF ( XCONN(ICOL1, ICOL2) .EQ. 0. ) GO TO 290
IF ( XCONN(ICOL1, ICOL2) .EQ. CCTMX(1) ) GO TO 290
IF ( ICOL2 .EQ. NTRCE .AND. XCONN(ICOL1, NTRCE) .NE. 0. ) THEN
   NCTMX(2,1) = ICOL1
   NCTMX(2,2) = NTRCE
   NPATH = NPATH + 1
   CCTMX(2) = XCONN(ICOL1, NTRCE)
   CCTMX(2) = YCONN(ICOL1, NTRCE)
   NITER = 2
   CALL KINET( 2, CCTMX, CCTMX, KOPTN, YPERT )
   CALL FMODE( NITER, CCTMX, CCTMX, JFLAGS, NMTRL, NOEXF, PROPT,
               * SVALU, TVALU )
   IF ( JFLAGS.EQ.1 )
   * CALL PORTH( NITER, CCTMX, CCTMX, NCTMX, YPERT )
GO TO 290
ENDIF
C
C *** MODIFY THE KINEMATIC JOINT PATH BY NOT ALLOWING THE DECLINING
2ND INTERSECTION POINT

C
IF ( CCTMX(2) .LE. CCTMX(1) ) GO TO 290
C
C
3RD

DO 280 ICOL3 = 1, NTRCE
IF ( XCONN(ICOL2, ICOL3) .EQ. 0. ) GO TO 280
IF ( XCONN(ICOL2, ICOL3) .EQ. CCTMX(1) ) GO TO 280
IF ( ICOL3 .EQ. ICOL1 ) GO TO 280
NCTMX(3,1) = ICOL2
NCTMX(3,2) = ICOL3
CCTMX(3) = XCONN(ICOL2, ICOL3)
CCTMX(3) = YCONN(ICOL2, ICOL3)
IF ( CCTMX(3) .GE. CCTMX(2) .AND. CCTMX(3) .LE. CCTMX(2) )
* GO TO 280
C
IF ( ICOL3 .EQ. 1 .AND. CCTMX(3) .NE. 0. ) THEN
   IF ( XCONN(ICOL2,1) .GT. CCTMX(1) ) THEN
      KOPTN = 0
      CALL KINET( 3, CCTMX, CCTMX, KOPTN, YPERT )
      IF ( KOPTN.EQ.1 ) GO TO 280
      NPATH = NPATH + 1
      TPATH = TPATH + 1.
      NITER = -3
      CALL FMODE( NITER, CCTMX, CCTMX, JFLAGS, NMTRL, NOEXF,
                  * PROPT, SVALU, TVALU )
      IF ( JFLAGS.EQ.1 )
      * CALL PORTH( NITER, CCTMX, CCTMX, NCTMX, YPERT )
   ENDIF
GO TO 280
ENDIF

C

NFLAG = 0
IF ( CCTMY(3).LT.CCTMY(2) ) CALL KINES ( 3, CCTMX, CCTMY, NFLAG )
IF ( NFLAG.EQ.1 ) GO TO 280
C

IF ( ICOL3.EQ.NTRCE.AND.XCONN(ICOL2,NTRCE).NE.0. ) THEN
  KOPTN = 0
  CALL KINET ( 3, CCTMX, CCTMY, KOPTN, YPERT )
  IF ( KOPTN.EQ.1 ) GO TO 280
  NPATH = NPATH + 1
  NITER = 3
  CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ, PROPT, 
  *           SVALU, TVALU )
  IF ( JFLAG.EQ.1 )
  *     CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
  GO TO 280
ENDIF
C

4TH

DO 270 ICOL4 = 1, NTRCE
IF ( XCONN(ICOL3,ICOL4).EQ.0. ) GO TO 270
IF ( XCONN(ICOL3,ICOL4).EQ.CCTMX(1) ) GO TO 270
IF ( XCONN(ICOL3,ICOL4).EQ.CCTMX(2) ) GO TO 270
IF ( ICOL4.EQ.ICOL1.OR.ICOL4.EQ.ICOL2 ) GO TO 270
NCTMX(4,1) = ICOL3
NCTMX(4,2) = ICOL4
CCTMX(4) = XCONN(ICOL3,ICOL4)
CCTMY(4) = YCONN(ICOL3,ICOL4)
IF ( CCTMX(4).GE.CCTMX(3).AND.CCTMY(4).LE.CCTMY(3) )
  *     GO TO 270
C

IF ( ICOL4.EQ.1.AND.CCTMX(4).NE.0. ) THEN
  IF ( XCONN(ICOL3,1).GT.CCTMX(1) ) THEN
    KOPTN = 0
    CALL KINET ( 4, CCTMX, CCTMY, KOPTN, YPERT )
    IF ( KOPTN.EQ.1 ) GO TO 270
    NPATH = NPATH + 1
    TPATH = TPATH + 1.
    NITER = 4
    CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ, 
    *           PROPT, SVALU, TVALU )
    IF ( JFLAG.EQ.1 )
    *     CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
  ENDIF
  GO TO 270
ENDIF

C

NFLAG = 0
IF ( CCTMY(4).LT.CCTMY(3) ) CALL KINES ( 4, CCTMX, CCTMY, NFLAG )
IF ( NFLAG.EQ.1 ) GO TO 270
C

IF ( ICOL4.EQ.NTRCE.AND.XCONN(ICOL3,NTRCE).NE.0. ) THEN
  KOPTN = 0
CALL KINET ( 4, CCTMX, CCTMY, KOPTN, YPERT )
IF ( KOPTN.EQ.1 ) GO TO 270
NPATH = NPATH + 1
NITER = 4
CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ, PROPT, 
* SVALU, TVALU )
IF ( JFLAG.EQ.1 )
* CALL FORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
GO TO 270
ENDIF

C C
5TH

C DO 260 ICOL5 = 1, NTRCE
IF ( XCONN(ICOL4,ICOL5).EQ.0. ) GO TO 260
IF ( XCONN(ICOL4,ICOL5).EQ.CCTMX(1) ) GO TO 260
IF ( XCONN(ICOL4,ICOL5).EQ.CCTMX(2) ) GO TO 260
IF ( XCONN(ICOL4,ICOL5).EQ.CCTMX(3) ) GO TO 260
IF ( ICOL5.EQ.ICOL1.OR. 
* ICOL5.EQ.ICOL2.OR.ICOL5.EQ.ICOL3 ) GO TO 260
NCTMX(5,1) = ICOL4
NCTMX(5,2) = ICOL5
CCTMX(5) = XCONN(ICOL4,ICOL5)
CCTMY(5) = XCONN(ICOL4,ICOL5)
IF ( CCTMX(5).GE.CCTMX(4).AND.CCTMY(5).LE.CCTMY(4) )
* GO TO 260

C IF ( ICOL5.EQ.1.AND.CCTMX(5).NE.0. ) THEN
IF ( XCONN(ICOL4,1).GT.CCTMX(1) ) THEN
KOPTN = 0
CALL KINET ( 5, CCTMX, CCTMY, KOPTN, YPERT )
IF ( KOPTN.EQ.1 ) GO TO 260
NPATH = NPATH + 1
TPATH = TPATH + 1.
NITER = -5
CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ, 
* PROPT, SVALU, TVALU )
* CALL FORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
ENDIF
GO TO 260
ENDIF

C NFLAG = 0
IF ( CCTMY(5).LT.CCTMY(4) ) CALL KINES ( 5, CCTMX, CCTMY, NFLAG )
IF ( NFLAG.EQ.1 ) GO TO 260

C IF ( ICOL5.EQ.NTRCE.AND.XCONN(ICOL4,NTRCE).NE.0. ) THEN
KOPIN = 0
CALL KINET ( 5, CCTMX, CCTMY, KOPIN, YPERT )
IF ( KOPIN.EQ.1 ) GO TO 260
NPATH = NPATH + 1
NITER = 5
CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ, PROPT, 
* SVALU, TVALU )
IF ( JFLAG.EQ.1 )
  CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
  GO TO 260
ENDIF

C
C
C
6TH
C

DO 250 ICOL6 = 1, NTRCE
  IF ( XCONN(ICOL5, ICOL6).EQ.0. ) GO TO 250
  IF ( XCONN(ICOL5, ICOL6).EQ.CCTMX(1) ) GO TO 250
  IF ( XCONN(ICOL5, ICOL6).EQ.CCTMX(2) ) GO TO 250
  IF ( XCONN(ICOL5, ICOL6).EQ.CCTMX(3) ) GO TO 250
  IF ( XCONN(ICOL5, ICOL6).EQ.CCTMX(4) ) GO TO 250
  IF ( ICOL6.EQ.ICOL1.OR.ICOL6.EQ.ICOL2.OR.
      ICOL6.EQ.ICOL3.OR.ICOL6.EQ.ICOL4 ) GO TO 250
  NCTMX(6,1) = ICOL5
  NCTMX(6,2) = ICOL6
  CCTMX(6) = XCONN(ICOL5, ICOL6)
  CCTMY(6) = YCONN(ICOL5, ICOL6)
  IF ( CCTMX(6).GE.CCTMX(5).AND.CCTMY(6).LE.CCTMY(5) )
      GO TO 250
ENDIF
C

IF ( ICOL6.EQ.1.AND.CCTMX(6).NE.0. ) THEN
  IF ( XCONN(ICOL5,1).GT.CCTMX(1) ) THEN
    KOPTH = 0
    CALL KINET ( 6, CCTMX, CCTMY, KOPTH, YPERT )
    IF ( KOPTH.EQ.1 ) GO TO 250
    NPATH = NPATH + 1
    TPATH = TPATH + 1.
    NITER = -6
    CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ,
                      PROPT, SVALU, TValu )
    IF ( JFLAG.EQ.1 )
        CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
    ENDIF
    GO TO 250
ENDIF
C

NFLAG = 0
IF ( CCTMY(6).LT.CCTMY(5) ) CALL KINES ( 6, CCTMX, CCTMY, NFLAG )
IF ( NFLAG.EQ.1 ) GO TO 250

C

IF ( ICOL6.EQ.NTRCE.AND.XCONN(ICOL5,NTRCE).NE.0. ) THEN
  KOPTH = 0
  CALL KINET ( 6, CCTMX, CCTMY, KOPTH, YPERT )
  IF ( KOPTH.EQ.1 ) GO TO 250
  NPATH = NPATH + 1
  NITER = 6
  CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ, PROPT,
                  SVALU, TValu )
  IF ( JFLAG.EQ.1 )
      CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
  GO TO 250
ENDIF
C
DO 240 ICOL7 = 1, NTRCE
    IF ( XCONN(ICOL6, ICOL7) .EQ. 0. ) GO TO 240
    IF ( XCONN(ICOL6, ICOL7) .EQ. CCTXM(1) ) GO TO 240
    IF ( XCONN(ICOL6, ICOL7) .EQ. CCTXM(2) ) GO TO 240
    IF ( XCONN(ICOL6, ICOL7) .EQ. CCTXM(3) ) GO TO 240
    IF ( XCONN(ICOL6, ICOL7) .EQ. CCTXM(4) ) GO TO 240
    IF ( XCONN(ICOL6, ICOL7) .EQ. CCTXM(5) ) GO TO 240
    IF ( ICOL7 .EQ. ICOL1.OR.ICOL7 .EQ. ICOL2.OR. 
         * ICOL7 .EQ. ICOL3.OR.ICOL7 .EQ. ICOL4.OR. 
         * ICOL7 .EQ. ICOL5 ) GO TO 240
    NCTMX(7,1) = ICOL6
    NCTMX(7,2) = ICOL7
    CCTXM(7) = XCONN(ICOL6, ICOL7)
    CCTXM(7) = XCONN(ICOL6, ICOL7)
    IF ( CCTXM(7) .GE. CCTXM(6) AND. CCTXM(7) .LE. CCTXM(6) )
         * GO TO 240
    IF ( ICOL7 .EQ. 1 .AND. CCTXM(7) .NE. 0. ) THEN
        IF ( XCONN(ICOL6,1) .GT. CCTXM(1) ) THEN
            KOPTH = 0
            CALL KINET ( 7, CCTXM, CCTXM, KOPTH, YPERT )
            IF ( KOPTH.EQ.1 ) GO TO 240
            NPATH = NPATH + 1
            TPATH = TPATH + 1.
            NITER = -7
            CALL FMODE ( NITER, CCTXM, CCTXM, JFLAG, NMTRL, NOEFJ, 
                         * PROPT, SVALU, TVALU )
            IF ( JFLAG.EQ.1 )
                * CALL PORTH ( NITER, CCTXM, CCTXM, NCTMX, YPERT )
        ENDIF
        GO TO 240
    ENDIF
    NFLAG = 0
    IF ( CCTXM(7) .LT. CCTXM(6) ) CALL KINES ( 7, CCTXM, CCTXM, NFLAG )
    IF ( NFLAG.EQ.1 ) GO TO 240
    IF ( ICOL7 .EQ. NTRCE .AND. XCONN(ICOL6,NTRCE) .NE. 0. ) THEN
        KOPTH = 0
        CALL KINET ( 7, CCTXM, CCTXM, KOPTH, YPERT )
        IF ( KOPTH.EQ.1 ) GO TO 240
        NPATH = NPATH + 1
        NITER = 7
        CALL FMODE ( NITER, CCTXM, CCTXM, JFLAG, NMTRL, NOEFJ, PROPT, 
                     * SVALU, TVALU )
        IF ( JFLAG.EQ.1 )
            * CALL PORTH ( IITER, CCTXM, CCTXM, NCTMX, YPERT )
            GO TO 240
        ENDIF
    8TH
    DO 230 ICOL8 = 1, NTRCE
IF ( XCONN(ICOL7, ICOL8) .EQ. 0. ) GO TO 230
IF ( XCONN(ICOL7, ICOL8) .EQ. CCTMX(1) ) GO TO 230
IF ( XCONN(ICOL7, ICOL8) .EQ. CCTMX(2) ) GO TO 230
IF ( XCONN(ICOL7, ICOL8) .EQ. CCTMX(3) ) GO TO 230
IF ( XCONN(ICOL7, ICOL8) .EQ. CCTMX(4) ) GO TO 230
IF ( XCONN(ICOL7, ICOL8) .EQ. CCTMX(5) ) GO TO 230
IF ( XCONN(ICOL7, ICOL8) .EQ. CCTMX(6) ) GO TO 230
IF ( ICOL8.EQ.ICOL1.OR.ICOL8.EQ.ICOL2.OR.
*     ICOL8.EQ.ICOL3.OR.ICOL8.EQ.ICOL4.OR.
*     ICOL8.EQ.ICOL5.OR.ICOL8.EQ.ICOL6   ) GO TO 230
NCTMX(8, 1) = ICOL7
NCTMX(8, 2) = ICOL8
CCTMX(8) = XCONN(ICOL7, ICOL8)
CCTMX(8) = YCONN(ICOL7, ICOL8)
IF ( (CCTMX(8).GE.CCTMX(7).AND.CCTMX(8).LE.CCTMX(7) )
*     GO TO 230
C IF ( ICOL8.EQ.1.AND.CCTMX(8).NE.0. ) THEN
IF ( XCONN(ICOL7, 1).GT.CCTMX(1) ) THEN
   KOPTN = 0
   CALL KINET ( 8, CCTMX, CCTMX, KOPTN, YPERT )
   IF ( KOPTN.EQ.1 ) GO TO 230
   NPATH = NPATH + 1
   TPATH = TPATH + 1.
   NITER = -8
   CALL FMODE ( NITER, CCTMX, CCTMX, JFLAG, NMTRL, NOEFJ,
*               PROPT, SVALU, TVALU )
*     IF ( JFLAG.EQ.1 )
*     CALL PORTH ( NITER, CCTMX, CCTMX, NCTMX, YPERT )
ENDIF
GO TO 230
ENDIF
C NFLAG = 0
IF ( CCTMX(8).LT.CCTMX(7) ) CALL KINES ( 8, CCTMX, CCTMX, NFLAG )
IF ( NFLAG.EQ.1 ) GO TO 230
C IF ( ICOL8.EQ.NTRCE.AND.XCONN(ICOL7,NTRCE).NE.0. ) THEN
   KOPTN = 0
   CALL KINET ( 8, CCTMX, CCTMX, KOPTN, YPERT )
   IF ( KOPTN.EQ.1 ) GO TO 230
   NPATH = NPATH + 1
   NITER = 8
   CALL FMODE ( NITER, CCTMX, CCTMX, JFLAG, NMTRL, NOEFJ, PROPT,
*               SVALU, TVALU )
*     IF ( JFLAG.EQ.1 )
*     CALL PORTH ( NITER, CCTMX, CCTMX, NCTMX, YPERT )
GO TO 230
ENDIF
C C 9TH
C DO 220 ICOL9 = 1, NTRCE
IF ( XCONN(ICOL8, ICOL9).EQ.0. ) GO TO 220
DO 180 JITER = 1, 7
IF ( XCONN(ICOL8, ICOL9).EQ.CCTMX(JITER) ) GO TO 220

180 CONTINUE
  *  ICOL9.EQ.ICOL7 ) GO TO 220
   NCTMX(9,1) = ICOL8
   NCTMX(9,2) = ICOL9
   CCTMX(9) = XCONN(ICOL8, ICOL9)
   CCTMY(9) = YCONN(ICOL8, ICOL9)
 IF ( CCTMX(9).GE.CCTMX(8).AND.CCTMY(9).LE.CCTMY(8) )
  *  GO TO 220

C

IF ( ICOL9.EQ.1.AND.CCTMX(9).NE.0. ) THEN
   IF ( XCONN(ICOL8,1).GT.CCTMX(1) ) THEN
     KOPTN = 0
     CALL KINET ( 9, CCTMX, CCTMY, KOPTN, YPERT )
     IF ( KOPTN.EQ.1 ) GO TO 220
     NPATH = NPATH + 1
     TPATH = TPATH + 1.
     NITER = -9
     CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, HOEFJ,
  *                     PROPT, SVALU, TVALU )
  *  IF ( JFLAG.EQ.1 )
  *  CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
 ENDIF
 GO TO 220
ENDIF

C

NFLAG = 0
 IF ( CCTMY(9).LT.CCTMY(8) ) CALL KINES ( 9, CCTMX, CCTMY, NFLAG )
 IF ( NFLAG.EQ.1 ) GO TO 220

C

IF ( ICOL9.EQ.NTRCE.AND.XCONN(ICOL8,NTRCE).NE.0. ) THEN
   KOPTN = 0
   CALL KINET ( 9, CCTMX, CCTMY, KOPTN, YPERT )
   IF ( KOPTN.EQ.1 ) GO TO 220
   NPATH = NPATH + 1
   NITER = 9
   CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTRL, HOEFJ, PROPT,
  *                     SVALU, TVALU )
  *  IF ( JFLAG.EQ.1 )
  *  CALL PORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
 GO TO 220
ENDIF

C

10TH

C

DO 210 ICOL0 = 1, NTRCE
 IF ( XCONN(ICOL9, ICOL0).EQ.0. ) GO TO 210
 DO 190 JITER = 1, 8
  IF ( XCONN(ICOL9, ICOL0).EQ.CCTMX(JITER) ) GO TO 210
190 CONTINUE
 IF ( ICOL0.EQ.ICOL1.OR.ICOL0.EQ.ICOL2.OR.ICOL0.EQ.ICOL3.OR.
  *  ICOL0.EQ.ICOL4.OR.ICOL0.EQ.ICOL5.OR.ICOL0.EQ.ICOL6.OR.
  *  ICOL0.EQ.ICOL7.OR.ICOL0.EQ.ICOL8 ) GO TO 210

DO 210 ICOL0 = 1, NTRCE
 IF ( XCONN(ICOL9, ICOL0).EQ.0. ) GO TO 210
 DO 190 JITER = 1, 8
  IF ( XCONN(ICOL9, ICOL0).EQ.CCTMX(JITER) ) GO TO 210
190 CONTINUE
 IF ( ICOL0.EQ.ICOL1.OR.ICOL0.EQ.ICOL2.OR.ICOL0.EQ.ICOL3.OR.
  *  ICOL0.EQ.ICOL4.OR.ICOL0.EQ.ICOL5.OR.ICOL0.EQ.ICOL6.OR.
  *  ICOL0.EQ.ICOL7.OR.ICOL0.EQ.ICOL8 ) GO TO 210
NCTMX(10,1) = ICOL9
NCTMX(10,2) = ICOL0
CCTMX(10) = XCONN(ICOL9,ICOL0)
CCTMY(10) = YCONN(ICOL9,ICOL0)

IF ( CCTMX(10).GE.CCTMX(9).AND.CCTMY(10).LE.CCTMY(9) )
   * GO TO 210
C

IF ( ICOL0.EQ.1.AND.CCTMX(10).NE.0.) THEN
   IF ( XCONN(ICOL9,1).GT.CCTMX(1) ) THEN
      KOPTH = 0
      CALL KINET ( 10, CCTMX, CCTMY, KOPTH, YPERT )
      IF ( KOPTH.EQ.1 ) GO TO 210
      NPATH = NPATH + 1
      TPATH = TPATH + 1.
      NITER = -10
      CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTLR, NOEFJ,
                   * PROPT, SVALU, TVALU )
      IF ( JFLAG.EQ.1 )
      * CALL FORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
   ENDFI
   GO TO 210
ENDIF

C

IF ( ICOL0.EQ.NTRCE.AND.XCONN(ICOL9,NTRCE).NE.0.) THEN
   KOPTH = 0
   CALL KINET ( 10, CCTMX, CCTMY, KOPTH, YPERT )
   IF ( KOPTH.EQ.1 ) GO TO 210
   NPATH = NPATH + 1
   NITER = 10
   CALL FMODE ( NITER, CCTMX, CCTMY, JFLAG, NMTLR, NOEFJ, PROPT,
                * SVALU, TVALU )
   IF ( JFLAG.EQ.1 )
   * CALL FORTH ( NITER, CCTMX, CCTMY, NCTMX, YPERT )
   GO TO 210
ENDIF
LCOUN = LCOUN + 1
C

210 CONTINUE
220 CONTINUE
230 CONTINUE
240 CONTINUE
250 CONTINUE
260 CONTINUE
270 CONTINUE
280 CONTINUE
290 CONTINUE
300 CONTINUE

IF ( LCOUN.GT.0 ) WRITE(6,950) LCOUN
C

C STORE THE TOTAL NUMBER OF EFFECTIVE JOINT PATHS
C

MPATH = TPATH
WRITE(6,920) NPATH, MPATH
IF ( MOPTH.GE.5 ) WRITE(7,940) NPATH, MPATH
TOTAL JOINT PATHES WHICH ARE MECHANICALLY UNSTABLE : NMODE

WRITE (6,930) NMODE
WRITE (9,925) ICASE, IOSIM, NPATH, NMODE
WRITE (7,945) NMODE
IF ( NMODE.GE.1 ) THEN
  WRITE (6,990) SFMIN
  CALL PORMT ( NCRT, CRITX, CRITY, NCTMX, YCRIT )
  WRITE (8,965) GMIN
ENDIF

STORE THE STATISTICAL INFORMATION INTO TAPE 9

IF ( NSTAT.GT.0.AND.MOPTN.GE.5 ) THEN
  DO 500 ISTEP = 1, NSTEP
      VALUE = VARMN + (ISTEP-1) * DSTEP
  WRITE (9,955) VALUE, SVALU(IOSIM,ISTEP), TVALU(IOSIM,ISTEP)
  CONTINUE
ENDIF

IF ( IOSIM.EQ.NOSIM.AND.NSTAT.GT.0 ) THEN
  IF ( ICASE.EQ.NCASE ) WRITE (6,975)
  DO 600 ISTEP = 1, NSTEP
      SVALE = 0.
      TVALE = 0.
      DO 550 JOSIM = 1, NOSIM
          SVALE = SVALE + SVALU(JOSIM,ISTEP)
          TVALE = TVALE + TVALU(JOSIM,ISTEP)
      550 CONTINUE
      SVALU(MOSIM,ISTEP) = SVALE
      TVALU(MOSIM,ISTEP) = TVALE
      VALUE = VARMN + (ISTEP - 1) * DSTEP
  CASEO(ICASE,ISTEP) = SVALE
  CASET(ICASE,ISTEP) = TVALE
  IF ( ICASE.EQ.NCASE )
      * WRITE(6,956) VALUE, (CASEO(JCASE,ISTEP), CASET(JCASE,ISTEP),
      * JCASE=1,NCASE)
  CONTINUE
ENDIF

STORE THE FINAL STABILITY ANALYSIS INTO FILE 9

IF ( ICASE.EQ.NCASE.AND.IOSIM.EQ.NOSIM.AND. NSTAT.NE.0 ) THEN
  WRITE (9,940) NCASE, NSTEP, NOSIM, NSTAT
  WRITE (9,935)
  WRITE (6,980)
  DO 710 ISTEP = 1, NSTEP
      VALUE = VARMN + (ISTEP-1) * DSTEP
  DO 715 JCASE = 1, NCASE
      IF ( CASEO(JCASE,ISTEP).EQ.0. ) THEN
          PROSB(JCASE) = 0.
      ELSE
          PROSB(JCASE) = CASET(JCASE,ISTEP) / CASEO(JCASE,ISTEP)
      ENDIF
  ENDIF
715 CONTINUE
WRITE (6, 955) VALUE, (PROBY (JCASE), JCASE=1, NCASE)
WRITE (9, 955) VALUE, (PROBY (JCASE), JCASE=1, NCASE)
710 CONTINUE

CALCULATE THE PROBABILITY OF FAILURE OF EACH VOLUME OR DIP

WRITE (9, 935)
WRITE (6, 985)
DO 730 JSTEP = 1, NSTEP
TPRBY (JSTEP) = 0.
DO 720 JCASE = 1, NCASE
TPRBY (JSTEP) = TPRBY (JSTEP) + CASET (JCASE, JSTEP)
730 CONTINUE
720 CONTINUE
DO 750 JSTEP = 1, NSTEP
VALUE = VARMN + (JSTEP-1) * DSTEP
DO 740 JCASE = 1, NCASE
IF ( TPRBY (JSTEP) .EQ. 0. ) THEN
    PROBY (JCASE) = 0.
ELSE
    PROBY (JCASE) = CASET (JCASE, JSTEP) / TPRBY (JSTEP)
ENDIF
740 CONTINUE
WRITE (6, 960) VALUE, (PROBY (KCASE), KCASE=1, NCASE)
WRITE (9, 960) VALUE, (PROBY (KCASE), KCASE=1, NCASE)
750 CONTINUE

STORE THE INFORMATION ONTO TAPE 9

WRITE (9, 935)
DO 760 KSTEP = 1, NSTEP
VALUE = VARMN + (KSTEP-1) * DSTEP
WRITE (9, 956) VALUE, (CASEO (JCASE, KSTEP), CASET (JCASE, KSTEP),
JCASE=1, NCASE)
760 CONTINUE
ENDIF

920 FORMAT (/, 5X, 'TOTAL NO. OF REAL EFFECTIVE PATHS = ', I5,
* /, 5X, 'TOTAL NO. OF FACE TO FACE PATHS = ', I5 )
925 FORMAT (4(5X))
930 FORMAT (/, 5X, 'TOTAL NO. OF UNSTABLE JOINT PATHS = ', I5 )
935 FORMAT (/
940 FORMAT (4(5X))
945 FORMAT (I5)
950 FORMAT (/, 5X, '*** 10 CONSECUTIVE SEARCHING ORDER IS NOT ',
* 'SUFFICIENT ***',
* /*, 7X, 'TOTAL PATHS OVER 10 JOINTS ARE = ', I5 )
955 FORMAT (6X, F7.2, 5(F7.2,3X))
956 FORMAT (6X, F7.2, 5(F7.2,3X,F7.2,3X))
960 FORMAT (6F10.3)
965 FORMAT (F10.3)
970 FORMAT (/, 5X, '*** WARNING ***',
* /*, 7X, 'CALCULATED INTERSECTION POINTS = ', I5,
* /*, 7X, 'STORED INTERSECTION POINTS = ', I5 )
975 FORMAT(//, 5X, 'TOTAL NUMBER OF JOINT PATHS', 
         * /, 4X, 'VALUE TOTAL PATHS UNSTABLE PATHS', 
         * /)
980 FORMAT(//, 5X, 'PF AT EACH VOLUME CATEGORY', 
         * /, 3X, 'INCREMENT', 3X, 'PF FOR EACH CASE', /)
985 FORMAT(//, 5X, 'PF OF EACH VOLUME AMONG UNSTABLE PATHS', 
         * /, 4X, 'INCREMENT', 6X, 'PF FOR EACH CASE', /)
990 FORMAT(//, 5X, 'MIN. FACTOR OF SAFETY = ', F10.3, 
         * /, 7X, '( RESISTING FORCE / DRIVING FORCE )' )
C
RETURN
END
C
SUBROUTINE PORTH ( LITER, CCTMX, CCTMY, NCTMX, YPERT )
C
SUBROUTINE PORTH STORES THE JOINT PATH COORDINATES
IN FILE 8
C
COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDOFN, MOPTN, IOSIM, NSYST,
* NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEGIT, HCASE
DIMENSION CCTMX(10), CCTMY(10), NCTMX(10,2)
C
ANRAD = 3.1415926 / 180.
XPERT = -0.5
XRANG = HEIGT / TAN(ANGLE*ANRAD)
NITER = ABS(LITER)
C
IF ( MOPTN.GE.5 ) THEN
   WRITE(8,910) NITER
   DO 10 IITER = 1,NITER
   WRITE(8,920) CCTMX(IITER), CCTMY(IITER)
     CONTINUE
 10 ENDIF
IF ( MOPTN.GE.5 ) THEN
  IF ( LITER.LT.0 ) THEN
    KITER = NITER + 1
  WRITE(8,910) KITER
  DO 20 IITER = 1, NITER
    CCTMX1 = CCTMX(IITER) + XPERT
    CCTMY1 = CCTMY(IITER) + XPERT
    WRITE(8,920) CCTMX1, CCTMY1
  20 CONTINUE
    CCTMX1 = CCTMX(1) + XPERT
    CCTMY1 = CCTMY(1) + XPERT
    WRITE(8,920) CCTMX1, CCTMY1
 ELSE
    KITER = NITER + 2
  WRITE(8,910) KITER
  DO 30 IITER = 1, NITER
    CCTMX1 = CCTMX(IITER) + XPERT
    CCTMY1 = CCTMY(IITER) + XPERT
    WRITE(8,920) CCTMX1, CCTMY1
  30 CONTINUE
    CCTMX1 = XRANG + XPERT
CCTMX1 = HEIGT + XPERT
WRITE(8,920) CCTMX1, CCTMY1
CCTMX1 = CCTMX(1) + XPERT
CCTMY1 = CCTMY(1) + YPERT
WRITE(8,920) CCTMX1, CCTMY1
ENDIF
ENDIF
C
910 FORMAT(I5)
920 FORMAT(2F10.3)
C
RETURN
END

CCC
C
SUBROUTINE KINES ( NITER, CCTMX, CCTMY, NFLAG )
C
SUBROUTINE LINES TESTS WHETHER THE ASSUMED JOINT PATH IS
KINEMATICALLY ADMISSIBLE OR NOT.
C
DIMENSION CCTMX(10), CCTMY(10)

NFLAG = 0
C
ITER1 = NITER - 1
ITER2 = NITER - 2
XCOR1 = CCTMX(ITER1)
YCOR1 = CCTMY(ITER1)
XCOR2 = CCTMX(ITER2)
YCOR2 = CCTMY(ITER2)
XCOOR = CCTMX(NITER)
YCOOR = CCTMY(NITER)
C
IF ( YCOR1.EQ.YCOR2 ) THEN
  IF ( YCOOR.LT.YCOR1 ) THEN
    NFLAG = 1
    GO TO 50
  ENDIF
ELSE
  IF ( XCOR1.EQ.XCOR2 ) THEN
    IF ( XCOOR.GT.XCOR1 ) THEN
      NFLAG = 1
      GO TO 50
    ENDIF
  ENDIF
ENDIF
SLOPE = ( YCOR2 - YCOR1 ) / ( XCOR2 - XCOR1 )
YCOR3 = ( YCOR2-YCOR1 ) * ( XCOOR-XCOR1 ) / ( XCOR2-XCOR1 )
+ YCOR1
IF ( SLOPE.GT.0..AND.YCOR3.GT.YCOOR ) NFLAG = 1
IF ( SLOPE.LT.0..AND.YCOR3.LT.YCOOR ) NFLAG = 1
ENDIF
C
50 CONTINUE
C
RETURN
END

SUBROUTINE KINET ( MITER, CCTMX, CCTMY, KOPTN, YPERT )

SUBROUTINE KINET CHECKS THE ADMISSIBILITY OF
AN ASSUMED JOINT PATH USING MINIMUM DIP AND
PERTURBATION BEHAVIOR

DIMENSION CCTMX(10), CCTMY(10)
DIMENSION SLOPE(10)

PIRAD = 3.1415926
XPERT = -0.5
KOPTN = 0

FIND THE MINIMUM DIP

MIPTR = MITER - 1
IF ( MITER.EQ.2 ) GO TO 30
DO 10 IITER = 1, MIPTR
   JITER = IITER + 1
   XCOR1 = CCTMX(JITER)
   YCOR1 = CCTMY(JITER)
   XCOR2 = CCTMX(JITER)
   YCOR2 = CCTMY(JITER)
   IF ( XCOR1.EQ.XCOR2 ) XCOR2 = XCOR2 + 1.E-10
   DIPAN = ( YCOR2 - YCOR1 ) / ( XCOR2 - XCOR1 )
   SLOPE(IITER) = ATAN(DIPAN)
   IF ( SLOPE(IITER).LT.0.) SLOPE(IITER) = SLOPE(IITER) + PIRAD
   IF ( XCOR2.LT.XCOR1.AND.YCOR2.LT.YCOR1 )
      * SLOPE(IITER) = SLOPE(IITER) + PIRAD

DETERMINE MIN. DIP ANGLE ( RAD. )

IF ( IITER.EQ.1 ) THEN
   SLMIN = SLOPE(1)
   INDSP = 1
ELSE
   IF ( SLOPE(IITER).GT.0..AND.SLOPE(IITER).LT.SLMIN ) THEN
      SLMIN = SLOPE(IITER)
      INDSP = IITER
   ENDIF
ENDIF
10 CONTINUE

CHECK THE MOBILITY OF A JOINT PATH

DO 20 IITER = 1, MIPTR
   IF ( SLOPE(IITER).LT.PIRAD ) GO TO 20
   SLTRY = SLOPE(IITER) - PIRAD
   IF ( SLTRY.GT.SLMIN ) THEN
      KOPTN = 1
   RETURN
ENDIF
CONTINUE
C
FIND PURRUBED Y-COORDINATE
C
CONTINUE
IF ( MITER.EQ.2 ) THEN
    YPERT = ( CCTMX(2) - CCTMX(1) ) / ( CCTMX(2) - CCTMX(1) )
    ELSE
    YPERT = SLMIN * ( XPERT )
ENDIF
C
RETURN
END

SUBROUTINE FMODE ( LITER, CCTMX, CCTMY, JFLAG, NMTRL, NOEFJ,
* PROPT, SVALU, TVALU )

SUBROUTINE FMODE EVALUATES THE MECHANICAL FAILURE MODE
OF AN ASSUMED JOINT PATH USING COULOMB’S SHEAR FAILURE
CRITERION

COMMON/CONTRO/NPOIN, NTRCE, NOSIM, NDQFN, MOPTN, IOSIM, NSYST,
* NOSET, ANGLE, HEIGT, RANGE, SFMAX, WEIGT, NCAPE
COMMON/CONTROL/WMODE, SFINH, MOSIM
COMMON/CNTROL/CRITX(10), CRITY(10), NCRIT, YCRIT
COMMON/CONTROL/NCTMX(10, 2)
COMMON/STATIC/NSTAT, VARMN, DSTEP, NSTEP, ICASE, NREG1, NREG2
DIMENSION NMTRL(NOEFJ), PROPT(NOSET, 7)
DIMENSION CCTMX(10), CCTMY(10), TLENG(10), DIPTH(10)
DIMENSION SVALU(MOSIM, NSTEP), TVALU(MOSIM, NSTEP)
C
PIRAD = 3.1415926
ANRAD = PIRAD / 180.
XRANG = HEIGT / TAN(ANGLE*ANRAD)
SLOPF = TAN(ANGLE*ANRAD)
JFLAG = 0

CALCULATES THE WHOLE AREA

KITER = 0
AREAT = 0.
NITER = ABS(LITER) - 1
DO 40 IITER = 1, NITER
  DIPTH(IITER) = 0.
  JITER = IITER + 1
  XCOR1 = CCTMX(IITER)
  XCOR2 = CCTMX(JITER)
  YCOR1 = CCTMY(IITER)
  YCOR2 = CCTMY(JITER)
  XDIFF = XCOR2 - XCOR1
  YDIFF = YCOR2 - YCOR1
  TLENG(IITER) = SQRT( XDIFF**2 + YDIFF**2 )

  40 CONTINUE
IF ( XDIFF.EQ.0. ) GO TO 40
SLOPE = YDIFF / XDIFF
IF ( XCOR2.LT.XCOR1.AND.YCOR2.LE.YCOR1 ) SLOPE = -SLOPE
IF ( XCOR2.GT.XCOR1.AND.YCOR2.LT.YCOR1 ) THEN
  WRITE(6,910)
  RETURN
ENDIF

C

IF ( SLOPE.LT.0. ) GO TO 20
KITER = KITER + 1
IF ( KITER.EQ.1 ) THEN
  SLMIN = SLOPE
  MINJT = 1
ELSE
  IF ( SLOPE.LT.SLMIN ) THEN
    SLMIN = SLOPE
    MINJT = IITER
  ENDIF
ENDIF

DIPTH(IITER) = SLOPE
JOINT = NCTMX(KMINJT,2) - 1

20 CONTINUE

C

IF ( XCOR1.GE.XRANG.AND.XCOR2.GE.XRANG ) GO TO 30
IF ( XCOR1.LT.XRANG.AND.XCOR2.LT.XRANG ) THEN
  AREA1 = 0.5 * ( 2.* HEIGT - YCOR1 - YCOR2 ) * ABS(XDIFF)
  YCOR1 = SLOFF * XCOR1
  YCOR2 = SLOFF * XCOR2
  AREA2 = 0.5 * ( 2.* HEIGT - YCOR1 - YCOR2 ) * ABS(XDIFF)
ELSE
  AREA1 = 0.5 * ( 2.* HEIGT - YCOR1 - YCOR2 ) * ABS(XDIFF)
  IF ( SLOPE.GE.0. ) THEN
    XCOR = XCOR1
  ELSE
    XCOR = XCOR2
  ENDIF
  YCOR = SLOFF * XCOR
  AREA2 = 0.5 * ( XRANG - AMIN1(XCOR1,XCOR2) ) * ( HEIGT - YCOR )
ENDIF

AREA3 = AREA1 - AREA2
IF ( AREA3.LE.0. ) WRITE(6,900)
IF ( SLOPE.LT.0. ) AREA3 = -AREA3
AREAT = AREAT + AREA3
GO TO 40

C

30 CONTINUE

C

AREA1 = 0.5 * ( 2.* HEIGT - YCOR1 - YCOR2 ) * ABS(XDIFF)
IF ( SLOPE.LT.0. ) AREA1 = -AREA1
AREAT = AREAT + AREA1

40 CONTINUE

IF ( AREAT.LE.0. ) WRITE(6,900)
USING COULOMB'S LAW ON A MIN. DIP JOINT FACE, CHECK THE
POSSIBILITY OF SLIDING OF ROCK BLOCK

TOWGT = WEIGHT * AREAT
DIPMN = ATAN(SLMIN)
DRIVE = TOWGT * SIN(DIPMN)

TOTCH = 0.
TENCU = 0.
DO 50 IITER = 1, NITER

INCLUDE THE POSSIBILITY OF THE PARALLEL DIP ANGLE

JTSPT = NCTMX(IITER,2) - 1
NOMAT = NMTRL(JTSPT)
IF ( DIPTH(IITER).EQ.SLMIN ) THEN
   COHTJ = PROPT(NOMAT,5) * TLENG(IITER)
   TOTCH = TOTCH + COHTJ
ELSE
   TENJT = PROPT(NOMAT,7) * TLENG(IITER)
   TENCU = TENCU + TENJT
ENDIF

50 CONTINUE
NOMAT = NMTRL(JOINT)
FRIJT = PROPT(NOMAT,6)
RESIT = TOTCH + TOWGT * COS(DIPMN) * TAN(FRIJT) + TENCU

SAFEF = RESIT / DRIVE

STATISTICAL ANALYSIS FOR VOLUME ( NSTAT = 1 )

IF ( NSTAT.EQ.2 ) GO TO 200

ICTGY = ( AREAT - VARMN ) / DSTEP + 2
IF ( AREAT.LT.VARMN ) ICTGY = 1
IF ( ICTGY.GT.NSTEP ) ICTGY = NSTEP
SVALU(IOSIM,ICTGY) = SVALU(IOSIM,ICTGY) + 1.

IF ( NSTAT.NE.1 AND ICASE.LE.NREG2.AND.IOSIM.LE.NREG1 ) THEN
   WRITE(10,940) ICASE, IOSIM, AREAT, DIPMN, TOTCH, TENCU,
*       RESIT, DRIVE
ENDIF

200 CONTINUE

STATISTICAL ANALYSIS FOR MIN. DIP ANGLE ( NSTAT = 2 )

IF ( NSTAT.NE.2 ) GO TO 300
DIPDG = DIPMN / ANRAD
ICTGY = ( DIPDG - VARMN ) / DSTEP + 2
IF ( DIPDG.LT.VARMN ) ICTGY = 1
IF ( ICTGY.GT.NSTEP ) ICTGY = NSTEP
SVALU(IOSIM,ICTGY) = SVALU(IOSIM,ICTGY) + 1.
300 CONTINUE
   IF ( SAFEF.GT.SFMAX ) RETURN
C
C STORE THE UNSTABLE JOINT PATH INTO TVALU
C
   IF ( NSTAT.EQ.1.AND.AREAT.LT.VARMN ) GO TO 400
   IF ( NSTAT.EQ.2.AND.DIPDG.LT.VARMN ) GO TO 400
   IF ( NSTAT.GT.0.AND.ICTGY.LE.NSTEP ) THEN
     TVALU(IOSIM,ICTGY) = TVALU(IOSIM,ICTGY) + 1.
   ENDIF
C
400 CONTINUE
   JFLAG = 1
   NMODE = NMODE + 1
   IF ( NMODE.EQ.1 ) THEN
     SFMIN = SAFEF
   ELSE
     IF ( SAFEF.LT.SFMIN ) SFMIN = SAFEF
   ENDIF
C
C STORE THE COORDINATES OF THE CRITICAL JOINT PATH FOR PLOTTING
C
   IF ( SFMIN.EQ.SAFEF ) THEN
     NITER = ABS(LITER)
     DO 60 IITER = 1, NITER
       CRITX(IITER) = CXTMX(IITER)
       CRITY(IITER) = CTTY(IITER)
     CONTINUE
     NCRIT = LITER
     XPERT = -0.5
     YCRIT = SLMIN * XPERT
   ENDIF
C
900 FORMAT(/,5X,'*** NEGATIVE AREA ***' )
910 FORMAT(/,5X,'*** ENCOUNTERED A KINEMATICALLY INADMISSIBLE PATH'
940 FORMAT(2X,2I3,2X,6(F10.3,1X))
C
RETURN
END
References

Probabilistic Methods in Structural Engineering.

Statistical Description of Rock Properties and Sampling.

contribution to the Discussion of Prof. Bartlett’s Paper.

Spectral Analysis of Two-dimensional Point Processes.

Fractal Geometry of Two-dimensional Fracture Networks at Yucca Mountain,
Southwest Nevada.
Fundamentals of Rock Joints : Proc. of Int. Symp. on Fundamentals of Rock Joints,

Contribution to the Discussion of Ripley’s Paper.

On the Detection of Spatial Pattern in Plant Communities.

Simple Monte Carlo Tests for Spatial Pattern.

Randomness and Local Regularity of Points in a Plane.

On Sampling Spatial Patterns by Distance Methods.

Fractal and Geostatistical Methods for Modeling of a Fractured Network.

Distance to Nearest Neighbor as a Measure of Spatial Relationships in Populations.

Homogeneous Line-Segment Processes.  

A Conditioned Distance Ratio Method for Analyzing Spatial Patterns.  

[16] W.S. Dershowitz.  
*Rock Joint Systems.*  

Personal Communication.  

Characterizing Rock Joint Geometry with Joint System Models.  

The Detection of Random Heterogeneity in Plant Populations.  

Some Graphical Methods in the Analysis of Spatial Point Patterns.  

*Statistical Analysis of Spatial Point Patterns.*  

Statistical Analysis of Spatial Point Patterns by Means of Distance Methods.  

Some Developments in Distance Sampling.  

*Risk Analysis for Rock Slopes in Open Pit Mines.*  

The Effect of Discontinuity Persistence on Rock Slope Stability.

[27] V.A. Epanechnikov.
Non-parametric Estimation of a Multivariate Probability Density.

Edge-corrected Density Estimators for Point Processes.

The Accuracy of the Plating Method of Estimating the Density of Bacterial Populations, with Particular Reference to the Use of Thornton’s Agar Medium with Soil Samples.

*Statistical Analysis of Spherical Data.*

[31] Glynn, E. F.

Topological Properties of Random Crack Networks.

Reduction of n-th Moment Measures and the Special Case of the Third Moment Measure of Stationary and Isotropic Planar Point Processes.

[34] L. Heinrich.
On a Test of Randomness of Spatial Point Patterns.

Asymptotic Normality of a Random Point Field Characteristics.

The Eberhardt Statistic and The Detection of Nonrandomness of Spatial Point Distribution.

*Rock Slope Engineering.*

[38] P. Holgate.
Some New Tests of Randomness.

A New Method for Determining the Type of Distribution of Plant Individuals.

[40] P.H.S.W. Kulatilake.

[41] P.R. La Pointe.
A Method to Characterize Fracture Density and Connectivity Through Fractal Geometry.

Simulation of Non-homogeneous Poisson Processes by Thinning.

Porous Media Equivalents for Networks of Discontinuous Fractures.

Methods for Analysing Spatial Processes of Several Types of Points.

*Statistics of Directional Data.*

*Spatial Variation.*
Springer-Verlag., 1986, 2nd Ed.

A test for Spatial Pattern at Several Scales using Data from a Grid of Contiguous Quadrats.

On E. C. Pielou’s Index of Non-Randomness.

Statistical Approach to Problems of Cosmology.

On the Second-order and Orientation Analysis of Planar Stationary Point Processes.
[51] O'Reilly, K. J.
The Effect of Joint Plane Persistence on Slope Reliability.

[52] P. Parker and R. Cowan.
Some Properties of Line Segment Processes.

[53] L. Paterson.
Serrated Fracture Growth with Branching.

The Use of Point-to-Plant Distances in the Study of the Pattern of Plant Populations.

Modelling Spatial Patterns.

[56] B.D. Ripley.
Analyses of Nest Spacings.

[57] B.D. Ripley.
Point Processes for the Earth Sciences.

Connectivity of Fracture Systems - A Percolation Theory Approach.

Joint Formation in Granitic Rock of the Sierra Nevada.

The Boolean Model and Random Sets.

[61] Shair, A. K.
The Effect of Two Sets of Joints on Rock Slope Reliability.

Digital Simulation of Random Processes and Its Applications.

*Density Estimation for Statistics and Data Analysis.*
[64] D. Stoyan.
Interrupted Point Processes.  

[65] D. Stoyan.

Cross-correlation Measures of Weighted Random Measures and Their Estimation.  

On one of Matern's Hard-Core Point Process Models.  

Stochastic Geometry and Its Application.

Spatial Data Analysis by Example.  

[70] D. Veneziano.
Probabilistic Model of Joints in Rock.

[71] K. Watanabe.
Stochastic Evaluation of the Two Dimensional Continuity of Fractures in a Rock Mass.  

Some Recent Developments in Understanding the Hydrology of Fractured Rocks.  