Time Dependent Magnetization and Magnetic Loss Tangents

by

Gary Roy Olhoeft

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Abstract

The existence of magnetic loss mechanisms has been known for some time. Those losses involved in time dependent relaxation with time constants greater than 1 microsecond are reviewed, and an extensive bibliography is provided from Ewing in 1885 to date. Complex permeabilities using Debye, Cole-Cole, and Cole-Davidson type of relaxation mechanisms are inserted into the induced dipole moment function (M-jN) for a sphere. Experimental results and airborne electromagnetic applications are discussed.

THESIS SUPERVISOR: George W. Pratt, Jr.
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1.0 Introduction

It is the purpose of this thesis to attempt a joining of magnetic loss mechanisms (in addition to eddy-current loss) to the electromagnetic prospecting problem. Four bibliographies have been included to enable readers of various backgrounds to find literature most relevant to their requirements. As it has been over a decade since the last comprehensive review of the literature on magnetic relaxation, that reference section is the most extensive. Usage of this thesis with the reviews of time dependent magnetic effects, airborne electromagnetics, and dielectric effects (referred to below) is the intent of the author.

After explaining the basic electromagnetic exploration problem (though from an airborne perspective, it is equally well applicable to ground surveys), general relaxation phenomena will be discussed to lead in the topic of magnetic loss phenomena. Having thus displayed evidence for complex magnetic permeabilities, the induced response of a sphere in an uniform alternating magnetic field will be discussed with the addition of the complex permeability. To date, most geophysical interpretation of electromagnetic surveys has neglected the magnetic component of the response
of an ore body, and the interpretation has been based solely on eddy-current conductivity response mechanisms. A case will be presented where such eddy-current interpretation does not fit the data, and the possible magnetic interpretation will be examined. The response function in a frequency-spectrum format contains all of the electromagnetic information necessary to derive the electric and magnetic properties of the sphere material.

The discussion of magnetic loss phenomena will touch upon all of the basic relaxation types while ignoring those due to resonance. A brief history of the phenomena will be presented from eddy-current phenomena studied before 1830 through "magnetic viscosity" with Ewing in 1885 to the most recent results obtained with lunar samples. The primary concentration will be towards applicability in geological contexts.

Unless otherwise specified, all units of measurement are in the MKS system.
2.0 EM Exploration Problem

Electricity and magnetism appear in geophysical prospecting as several basic tools. Perturbations of the earth's magnetic field due to the proximity of ore deposits are usually measured with passive magnetometer systems in total field, component field, or gradient field configurations. Redford and Sumner (1964) have capably reviewed aeromagnetic systems; Royer (1967) presents specific examples of the need for high sensitivity (reaching down into the noise) to obtain sufficient resolution for the definition of certain types of anomaly; Otala (1969) and Pomerantsev et alia (1969) present gradient aeromagnetic techniques using quantum-level superconducting magnetometers.

Electrical measurements of the dielectric constant and loss in the laboratory (see Saint-Amant and Strangway, 1970; Alvarez, 1972; Collett, 1972; and Zonge, 1972) are being applied to the interpretation of data taken with the IP (Induced Polarization) method as described by Madden and Cantwell (1967) and Van Voorhis et alia (1972).

Ground and airborne electromagnetic systems, however are the primary application of the magnetic loss. The
electromagnetic systems appear in many forms — as single frequency, multiple frequency, and in the time domain. Figure 2.02 depicts the typical airborne configuration of wingtip-tail-wingtip dipole coil source, a towed magnetometer as a receiver, and a conducting body. An older review by Pemberton (1962) is still applicable today, though more recent reviews by Wagg (1970) and Smith and Fountain (1970) and Ward (1967b) are of more immediate interest. The single frequency continuous wave electromagnetic system is in little use today, the multiple frequency systems having proven to obtain more significant information. Paterson (1961) describes a two-frequency phase-shift system which is the minimum equipment and number of parameters capable of defining the complex response function of a body. Ideally however, many frequencies would be used, with amplitude and phase of response being measured at each frequency. The result would then plot as a response spectrum (to be discussed below). Such systems using multiple frequencies may be of the artificial source or natural source variety. The natural source system uses the earth currents and sferics (see Figure 2.01) (sometimes called magnetotellurics) as the signal (see Strangway, Swift, and Holmer, 1971). Magnetotellurics using artificial sources have been described by Goldstein and Strangway (1971).
The general technique of artificially transmitting a signal (usually a current loop to produce a magnetic dipole) at several frequencies and then measuring the responses is known as electromagnetic sounding. This has been exploited of late by Dey and Ward (1970), Vanyan (1967), Duckworth (1970), and Ryu et alia (1970, 1971). Also, Simmons et alia (1972) report upon an Apollo 17 experiment which will electromagnetically sound the moon at megahertz frequencies to attempt to find layering.

In typical terrestrial environments, the surface and subsurface presence of water causes conductivities which limit depth penetration in the above methods to approximately 100 meters at up to 3000 Hertz. The problem when prospecting over the ocean (see Coggon and Morrison, 1970) is thus extremely difficult (much less than 100 meter depth penetration). Kraichman (1970) is an excellent guide in estimating depth penetration in conducting media.

Measurement in the time-domain using pulse techniques also may produce the information necessary for a response function spectrum plot. The basic system is that as developed by Barringer in 1959 (see Barringer, 1970 advertising brochure). In brief, the system produces a
magnetic dipole pulse (see Figure 2.02 for a typical airborne configuration) followed by a period of no signal, then a second pulse of opposite polarity, a second no-signal period, and repeats. During the pulse, conductive bodies under the influence of the field generate eddy currents by Lenz's Law (discussed below) which in turn create a secondary magnetic field. When the pulse is turned off, these eddy-currents decay in a quasi-exponential manner. This decay is measured in the quiet period between pulses. To obtain the frequency response spectrum a Fourier transform is then performed. The advantages of this system over the multiple frequency CW systems are the complete spectrum (within limits of the pulse) in one measurement, measurement of the body's response while the primary field is off, and deeper depth penetration (penetrations in excess of 200 meters have been reported).

Consequently, there has been considerable research of late into time-domain airborne electromagnetic systems (Becker, 1969; Morrison et alia, 1969; Nabighian, 1970; Nelson and Morris, 1969; and Verma and Singh, 1970). Palacky and West (1971) also have reported upon a digital technique for routine analysis of Barringer INPUT (Induced Pulse Transient) data. Hohmann et alia (1970) have reported upon
an attempt at airborne IP (see above) using an inductive system; they found no conclusive evidence of any inductively originating IP.

As reported by Nelson (1971) and in communication with Bonniwell and McDougal (1970), the standard pulsed, time domain, airborne electromagnetic system is usually flown uncalibrated and without reference (the operator optimizes the signal recording amplitude for each channel by "feel"). This is extremely unfortunate. Two otherwise identical systems will not produce identical responses to known deposits, the transformed decay information will be meaningless when compared to standard model studies, and theoretical studies will be difficult to verify.

The paper by Colani and Aitken (1966) on the uses of magnetic viscosity in archaeological prospection for ancient hearth sites (by their magnetite) originally initiated the possibility of a magnetic response in electromagnetic systems. Conversations with Bonniwell and McDougal (1970) indicated that there were certain Barringer INPUT surveys whose data could not be interpreted using eddy-current response theory (these were calibrated surveys). Thus, the present thesis which will show that magnetic losses will
alter the eddy-current response spectrum. However, in the application of the results below, it would be wise to have a calibrated system. Fortunately, most of the magnetotelluric and electromagnetic sounding systems are calibrated.

In general, an electromagnetic system of one of the above types will see an anomaly of the form: (Ward, 1967a)

\[ A = (IF)(SF)(PPF)(GF) \]

where

- \( A \) = anomaly condition
- \( IF \) = inducing field
- \( SF \) = size factor
- \( PPF \) = physical property factor
- \( GF \) = geometrical factor

For a sphere this anomaly becomes:

\[ \Delta H = (H_0)(R^3)(M-jN) \left( \frac{(2x^2-y^2-z^2)\hat{i} + 3xy\hat{j} + 3xz\hat{k}}{r^5} \right) \]

The \((M-jN)\) function contains all eddy-current and magnetic response data for the material. Further, the magnetic and induced polarization (as well as gravity and resistivity) survey anomalies are volume dependent, whereas the electromagnetic method is dependent upon the surface area (skin depth limited volume) normal to the incident inducing field.
FIGURE 2.01

Mean geoelectromagnetic spectrum.
(after Bleil, 1964)
FIGURE 2.02

Typical airborne EM system.
(after Ward, 1967b)
3.0 Relaxation Distributions

In this section, relaxation will be discussed in general terms (with specific examples cited as necessary) using the parameter $X$ as a function of frequency and time. Thus, $X$ may represent dielectric constant, magnetic permeability, or some other parameter (elastic modulus, etc.). In several of the early papers on the magnetic time dependence, this type of analysis was exploited to exhibit the parallels between various processes (e.g., between the magnetic decay with time and the diffusion of atoms, or the elastic relaxation); for one example see Snoek, 1938.

Many physical processes may account for these relaxation distributions and will not be detailed here; however, in general, the relaxation is the result of excessive damping in the classical physics equation of motion, and the Debye relaxation of a polar molecule in a viscus medium is an excellent example.
3.1 Debye Single Relaxation

The Debye single relaxation is the classic case of dispersion in an excessively damped medium. Usually, polar molecules are considered rotating to align with an applied electric field in a medium of excessive friction causing neglect of the acceleration term in the classical dispersion formula for the equation of motion due to over-damping. However, diffusion processes of certain types, some magnetic decay phenomena, and some elastic phenomena may also exhibit the single exponential decay characteristic of the Debye relaxation. Hence, we will discuss all of these in general under the Debye type of relaxation. Similarly for distributions of relaxations, we will neglect the particular process involved, for the general mathematical description.

The Debye type of single exponential relaxation is of interest as it occurs frequently in homogeneous, solid dielectric materials, and it is the limiting case for several of the distributed types of decay. The Debye relaxation is well known and has been discussed in numerous publications (see Poole and Farach, 1971; Van Vleck, 1932). In general, geological samples are not homogeneous and exhibit smeared distributions of Debye type relaxations.
The Debye relaxation is described by the following formula:

$$X = X_r - jX_i = X_\infty + \frac{X_0 - X_\infty}{1 + j\omega \tau}$$

where

$$X = X_\infty, \quad \omega = \infty$$

$$X = X_0, \quad \omega = 0$$

$$X_r = X_\infty + \frac{X_0 - X_\infty}{1 + \omega^2 \tau^2}$$

$$X_i = \frac{(X_0 - X_\infty) \omega \tau}{1 + \omega^2 \tau^2}$$

and the loss tangent (which is a measure of the phase lag $\delta$ between the induced signal and the applied) is:

$$\tan \delta = \frac{X_i}{X_r} = \frac{(X_0 - X_\infty) \omega \tau}{X_0 + X_\infty \omega^2 \tau^2}$$

Transforming to the time domain, we see the characteristic single relaxation:

$$X(t) = X_0 - X_\infty u(t) - (X_0 - X_\infty) (1 - e^{-t/\tau})$$

where the actual decay is in the last term. These functions are shown in Figure 3.11.
$X(\omega) = X' - jX''$

FIGURE 3.11
If we adopt the notation of Chikazumi (1967; from Tomono, 1952) we may rewrite the loss tangent as:

\[
\tan \delta = \frac{\xi \omega \tau}{1 + \xi + \omega^2 \tau^2}
\]

where we see

\[
\xi = \frac{X_\infty - X_0}{X_\infty}
\]

Thus we see that the magnetic after-effect as measured by Tomono in cold-rolled iron is a Debye relaxation (this will be further discussed below). We may thus solve for the frequency of maximum loss tangent to find:

\[
\omega_m = \frac{\sqrt{1 + \xi}}{\tau}
\]
3.2 Cole-Cole Distribution

As mentioned above, geological materials do not usually exhibit simple Debye relaxations. Rather distributions of Debye relaxations have been found to fit the data. Excellent examples of this have been documented in the field of dielectrics (see Alvarez, 1972; Poole and Farach, 1971; Garton, 1946). The most useful in this regard has been the Cole-Cole distribution. In particular, when water has been introduced into geological specimens, the Cole-Cole type of arc (to be described below) has produced excellent data fits (see Saint-Amant and Strangway, 1971; Alvarez, 1972; Chung et alia, 1972; and Strangway et alia, 1972). These distributions have been attributed to statistical distributions of grain size (Neel, 1949), activation energies (Whitehead, 1946), and other factors.

The function as documented (Cole and Cole, 1941) is:

\[ X = X_r - jX_i = X_\infty + \frac{X_0 - X_\infty}{1 + (j\omega \tau)^{1-\alpha}} \]

where the limiting cases as alpha approaches zero and one are the Debye single relaxation and a continuous distribution of Debye relaxations, respectively.
The real and imaginary parts of this function are:

\[ X_r = X_\infty + \frac{(X_0 - X_\infty)[1 + (\omega \tau)^{1-\alpha} \cos(1-\alpha)\pi/2]}{1 + 2(\omega \tau)^{1-\alpha} \cos(1-\alpha)\pi/2 + (\omega \tau)^2(1-\alpha)} \]

\[ X_i = \frac{(X_0 - X_\infty)(\omega \tau)^{1-\alpha} \sin(1-\alpha)\pi/2}{1 + 2(\omega \tau)^{1-\alpha} \cos(1-\alpha)\pi/2 + (\omega \tau)^2(1-\alpha)} \]

These are displayed on the real and imaginary axes in the characteristic semicircular arc with a depressed center. As the Figure 3.21 shows, alpha approaching zero produces the Debye limiting case.

The loss tangent is thus:

\[ \tan \delta = \frac{(X_0 - X_\infty)(\omega \tau)^{1-\alpha} \sin(1-\alpha)\pi/2}{X_0 + (X_0 + X_\infty)(\omega \tau)^{1-\alpha} \cos(1-\alpha)\pi/2 + X_\infty(\omega \tau)^2(1-\alpha)} \]

and we see that as alpha approaches one and the continuous distribution limit, the loss tangent goes to zero. Gevers and du Pre (Gevers, 1945; Gevers and du Pre, 1946) discuss some interesting measurements in this limit. Also, in this limit are found some of the interesting logarithmic types of response found in the IP (Induced Polarization; see Madden and Cantwell, 1967) as discussed by Van Voorhis et alia (1972). This, however is more evident in the Cole-Davidson distribution.
\[ \frac{X_i}{X_0 - X_\infty} \]

\[ \frac{X_r - X_\infty}{X_0 - X_\infty} \]

**FIGURE 3.31**
(after Hill et al., 1969)

\[ X_i \]

\[ X_r \]

**FIGURE 3.21**
(after Poole and Farach, 1971)
3.3 Cole-Davidson Distribution

In the Cole-Davidson distribution (Davidson and Cole, 1951; Alvarez, 1972) we see the following:

\[ X = X_r - jX_i = X_\infty + \frac{X_o - X_\infty}{(1 + j\omega \tau)\alpha} \]

where we may use the identity:

\[ \frac{1}{(1 + j\omega \tau)\alpha} = (\cos \phi)^\alpha (\cos \alpha \phi - j \sin \alpha \phi) \]

in which \[ \phi = \tan^{-1} \omega \tau \]

to see the real and imaginary parts:

\[ X_r = X_\infty + (X_o - X_\infty)(\cos \phi)^\alpha \cos \alpha \phi \]

\[ X_i = (X_o - X_\infty)(\cos \phi)^\alpha \sin \alpha \phi \]
Thus, the loss tangent is:

\[ \tan \delta = \frac{(X_\infty - X_0)(\cos \phi)^\alpha \sin \alpha \phi}{X_\infty + (X_\infty - X_0)(\cos \phi)^\alpha \cos \alpha \phi} \]

As before, the limiting cases of this function are the Debye single relaxation and the continuous distribution. However, \( \alpha \) (by convention) is the reverse of the Cole-Cole; hence \( \alpha \) approaching one becomes the Debye relaxation, and \( \alpha \) approaching zero becomes the continuous distribution. Figure 3.31 depicts on the real and imaginary axes the Cole-Davidson distribution as a function of \( \alpha \). The major difference between this and the Cole-Cole is the predominant heavy weighting of the low frequencies.

Transforming to the time domain using standard tables (Erdelyi, 1954; Gradshteyn and Ryzhik, 1965) we see:

\[ X(t) = X_\infty - X_0 \ u_0(t) - (X_\infty - X_0) \ f(t) \]

where

\[ f(t) = 1 + \frac{1}{\tau^\alpha \Gamma(\alpha)} \int t^{\alpha - 1} e^{-t/\tau} dt \]
Which we may rewrite (from the tables above) as:

\[
f(t) = 1 - (\frac{t}{\tau})^{\alpha-1} \frac{e^{-t/\tau}}{\Gamma(\alpha)} \left[ 1 + \sum_{k=1}^{\infty} \left(\frac{t}{\tau}\right)^k (\alpha-1) \cdots (\alpha-k) \right]
\]

As alpha approaches one, this reduces properly to the Debye relaxation, and as alpha approaches zero we see:

\[
f(t) = 1 + \log t + \sum_{k=1}^{\infty} (-1)^k \left(\frac{t}{\tau}\right)^k \frac{1}{k \cdot k!}
\]

where we recall for comparison:

\[
e^{-t/\tau} = 1 + \sum_{k=1}^{\infty} (-1)^k \left(\frac{t}{\tau}\right)^k \frac{1}{k!}
\]

Thus the latter limit has a term decaying faster than an exponential, and it has a logarithmic term as described by Neel (1949) for the magnetic viscosity type of decay (this will be discussed below).
4.0 Magnetic Loss Phenomena

4.1 Literature Review and Theory

The reader is first referred to the several general references available with excellent reviews of magnetic loss phenomena (Chikazumi, 1967; Ratheneau, 1959; Bozorth, 1951; and Becker and Doring, 1939). Early investigations of magnetic hysteresis and eddy current loss may be found in Ewing (1890), Russel (1914), and Faraday (1831).

If we ignore resonance loss (see Janak, 1964) as of no interest geologically (due to inadequate penetration as discussed above), there remain five major types of magnetic loss. They are eddy-current, hysteresis, the fluctuation after-effect, the diffusion after-effect, and the Jordan lag. Eddy current loss is a strong function of frequency and material conductivity and permeability (see Russel, 1914 and Bozorth, 1951); it is adequately modeled after a brief period by a single exponential; and it is the principle loss described by the (M-jN) function (above) as it has been normally used in geophysics (we will see the effect of complex permeability on this function below). Hysteresis loss was first discovered by Warburg in 1881 and independently by Ewing in 1882 (see discussion in Ewing, 1890); Williams, Schockley, and Kittel (1950), Pry and Bean
(1958), and Becker (1959) have shown that hysteresis and eddy-current loss are inter-related as they both involve the motion of domain-walls, are influenced strongly by the impurity and imperfection pinning of domain-walls, and involve the high permeability contrasts in the material due to the existence of the domain-walls.

The fluctuation after-effect (Neel, 1949, 1950, 1951; Barbier, 1954; Street and Woolley, 1949; and Brissonneau, 1958) is due to statistical thermal agitation of domain-walls over potential barriers; it is a weak function of temperature and frequency, and a strong function of impurity content. These factors and the characteristic log t (see Cole-Davidson above) dependence of the decay indicate a broad spectra of relaxation phenomena (usually interpreted as a broad series of potential barriers). The phenomenon is sometimes called the irreversible thermal after-effect.

The diffusion after-effect is subdivided into two basic types, that of magnetic induction lag (Ewing, 1885, 1889; Lord Rayleigh, 1887; Neel, 1949; Barbier, 1954; and le Borgne, 1960) and the time decrease of initial permeability after demagnetization (disaccommodation)(Snoek, 1938, 1939, 1941; Polder, 1945). Ratheneau (1959) and Brissonneau (1958)
give reviews of these effects. Tomono (1952) investigated the magnetic after-effect of the induction type in cold-rolled iron and found single relaxations of the Debye type. Richter (1937, 1938) found for similar investigations in iron and steel, a distribution of the form:

$$X(t) = (X_0 - X_\infty) \left[ 1 - \int_0^\infty \phi(\tau) e^{-t/\tau} d\tau \right]$$

where

$$\phi(\tau) = \begin{cases} 0 & \tau < \tau_1 \\ (\tau \log \tau_2/\tau_1)^{-1} & \tau_1 < \tau < \tau_2 \\ 0 & \tau > \tau_2 \end{cases}$$

The Richter decay is shown in Figure 4.01. This type of effect has been found to be strongly dependent upon temperature, frequency, stress, and impurity content. Neel attributes as the cause of this effect in rocks the superparamagnetism of small grain interactions; whereas, Snoek and Polder attribute the effect in iron and steel to the diffusion of carbon and nitrogen in interstitial spaces.

Jordan (1924) and others (Preisach, 1935; Kindler, 1936; Becker and Doring, 1939) have found a magnetic loss which is very weakly temperature dependent, independent of frequency, and strongly dependent upon stress. Due to temperature and frequency response, it is thought to have a very broad distribution of time constants, but the loss is still very
little understood (see Bozorth, 1951). This loss is termed the Jordan lag.

The studies of interest geologically began with Thellier in 1938 (see review in Nagata, 1961) through Neel (1949), Barbier (1954), le Borgne (1960), and Shimizu (1960). These studies primarily investigated the fluctuation after-effect and the diffusion after-effect (magnetic induction lag) which were all lumped under the general term "magnetic viscosity" (first coined by Ewing in 1885). These studies dealt with correlations between grain size and the magnetic viscosity coefficient (see below) for log t types of decay and with the viscous magnetization as part of the remnant magnetization studied in paleomagnetism.

More recently, Zhilyaeva and Minibaev (1965) have studied the soft paleomagnetic component as due to magnetic viscosity effects from sample handling conditions, but have drawn no clear conclusions. Sholpo and Pechnikov (1966), Tropin and Vlasov (1966), Trukhin (1966), Zhilyaeva and Kolesnikov (1966), Pechnikov (1967), Aver'yanov (1967), and Sholpo (1967) have made similar magnetic viscosity studies with geological interpretations.
Colani and Aitken (1966) reported upon the use of magnetic viscosity in uncovering ancient hearthsites (due to their magnetic content) as a tool for archaeological prospection using electromagnetic means. Tropin (1967) studied the effective magnetic viscosity field and the effects of exchange anisotropy. Tropin (1969) also studied the magnetic viscosity effect in the spin wave approximation and (Tropin, 1970) the use of distribution functions to fit theoretical curves to data. Belokon, Kochegura, and Sholpo (1969) have further developed the Preisach-Neel model of magnetic viscosity in small fields. Shashkanov and Metallova (1970) have studied the temperature dependence and Brodskaya (1970) has studied the effect of gamma radiation on the magnetic viscosity effect.

Nagata and Carleton (1970), Runkorn et alia (1970), Larochelle and Schwarz (1970), Nagata et alia (1971), Gose et alia (1972), Nagata et alia (1972), Pearce et alia (1972), and Strangway et alia (1970, 1972) have published magnetic viscosity measurements of lunar samples. The point of interest in these samples is the determination of any paleomagnetic remanence from an ancient lunar magnetic field, hence it is necessary to be able to estimate the effect of the returning spacecraft fields and the earth's magnetic
field on the acquired remnance of the lunar samples. Magnetic viscosity effects under the influence of small fields over periods of time may cause the acquisition of soft paleomagnetic components which would need to be subtracted.

4.2 Experimental
Proceeding with examples of magnetic viscosity in rocks, we will look at some decays from natural and artificial samples of magnetite, from some lunar samples (data from Gose et alia, 1972), and (for comparison) from some samples of steel. (The measurement techniques are outlined in the appendix.)

First, for the short time constant, single relaxation of the Debye type as measured by Tomono (1952), we see the parameter (see also Chikazumi, 1967):

\[ \xi = \frac{X_\infty - X_\infty}{X_\infty} \]

We have plotted this as a function of magnetite content by volume in Figure 4.02. The data is as measured without corrections for eddy-current loss or other effects. At 100% we have a natural massive magnetite from Ishpeming, Michigan with a high conductivity; the eddy-current contribution for this sample was calculated and found to be of the order of
size of the deviation from the straight line depicted. The remaining points are artificial samples of 200 mesh ground magnetite (also from Ishpeming) (with the exception of the sample at 50% which was of 15 mesh) contained in a matrix of very high resistivity Secar cement (see appendix). Thus, the eddy-current contribution to the loss was considered negligible. The deviation of the 50% sample from the straight line behavior is thought to be due to interactions between grains (see Strangway, 1967). For comparison, Figure 4.03 from Strangway (1967) has been reproduced.

To within 5% the magnetite decay curves could be fitted to a single exponential of time constant near 300 microseconds producing (using the analysis above) a maximum loss tangent near 500 Hertz. We will use the data from Figure 4.03 to estimate the effect of this loss on the (M-jN) function in the discussion (below).

For descriptions of multiple exponential relaxations, we turn to the Richter description of the diffusion after-effect. Such broad distributions are described by Richter (see Figure 4.01):

\[
\phi(\tau) = \begin{cases} 
0 & \tau < \tau_1 \\
(\tau \log \frac{\tau_2}{\tau_1})^{-1} & \tau_1 < \tau < \tau_2 \\
0 & \tau > \tau_2
\end{cases}
\]
Nagata et alia (1972) have modified this description to be more general as:

\[ \phi(\tau) = C \frac{e^{-t_o/\tau}}{\tau^{1+\nu}} \]

where

\[ X(t) = (X_o-X_\infty) \left[ 1 + \int_0^\infty \phi(\tau) e^{-t/\tau} \, d\tau \right] \]

This then becomes upon integrating (using standard tables, the integral is a standard Laplace transform):

\[ X(t) = (X_o-X_\infty) + \frac{C}{(t+t_o)^\nu} \]

which is shown normalized in Figure 4.05.

This is similar to the Cole-Davidson distribution. Figures 4.04 to 4.08 depict curves fitted to data points using the Nagata et alia description above (the recursive computer program used for this is described in the appendix).

The lunar sample data was provided by Gose et alia (1972), and the steel data was measured on the same equipment in a similar manner by the author. This procedure was to demagnetize the sample in an AC demagnetization unit, apply a field of 1.5 gauss for 1 minute (2.5 to 5.0 gauss for 8 minutes in the lunar samples), and measure the magnetic
decay from 8 seconds out using the NASA/MSC Geophysics Branch Princeton Applied Research SM-2D spinner magnetometer (the steel results were also duplicated on a Foner magnetometer, see Foner, 1959).

It may be readily seen that the steel sample in Figure 4.04 has a distribution peaking well within the range of recorded decay. This is also true of lunar sample 14321 in Figure 4.07. However, as is also shown in the data analysis by Nagata et alia (1972), lunar sample 14313 in Figure 4.06 has a peak distribution at the short time limit of measurement. One data point shift would push the distribution off range, and the resultant analysis would not then be unique. Using Neel's formula (1949) to compute a grain size from the time constant of peak distribution as Nagata et alia have done could then be in error; the best that could be placed would be a limiting value of the peak time constant, and a limiting value of the calculated grain size (this is discussed further in Gose et alia, 1972).

Figure 4.08 shows the typical logarithmic decay of the form:

\[ M(t) = M_0 + bS \log t \]

where \( S \) is the magnetic viscosity coefficient and the other terms are associated constants (see Neel, 1949 and Nagata,
1961). This decay is characteristic of the fluctuation after-effect, and it may be remembered from above that the logarithmic dependence is also the limiting case of the Cole-Davidson distribution. This indicates that over the measurement range shown, the sample exhibits a continuous spectrum of relaxation mechanisms.

Further significance of these phenomena will be discussed after we inspect the behavior of the (M-jN) response function with the addition of complex permeabilities.
Richter after-effect (after Becker and Doring, 1939)
MAGNETITE CONTENT

FIGURE 4.02
FIGURE 4.03

Susceptibility ($K$) of samples from Persberg, Sweden against volume ($V$) fraction of magnetic mineral content. (after Strangway, 1967)

\[ K = \frac{K_M V}{1 + K_M (1-V)} \]

$K_M = 1.5$
Magnetic viscosity decay and distribution function for steel with 0.90% C. $v = 1.2$; $t_0 = 297$ sec.
Modified Richter after-effect normalized as shown.
Decay of lunar sample $^{14}N$ after 2.5 and 5.0 oee applied field with modified Richter curves fitted to data. (Data from Gose et al., 1972)
Decay of lunar sample 14321,78-2 and distribution function from modified Richter curve fitted to data. ν = 0.38; τo = 57.5 sec.
(Data from Gose et al., 1972)
FIGURE 4.08
(after Gose et al., 1972)
5.0 Complex Permeability in \((M-jN)\)

The \((M-jN)\) induced response of a sphere in a uniform
alternating field as derived by Ward (1967, p. 74) and others
is found to be (discussed above):

\[
(M-jN) = \frac{2\mu_2(\tan \alpha - \alpha) - \mu_1(\alpha - \tan \alpha + \alpha^2 \tan \alpha)}{2\mu_2(\tan \alpha - \alpha) + 2\mu_1(\alpha - \tan \alpha + \alpha^2 \tan \alpha)}
\]

where the subscripts 0, 1, 2 refer respectively to free
space, the imbedding medium, and the sphere. The plane wave
propagation constants are:

\[
k_i = \mu_i \varepsilon_i \omega^2 + j \mu_i \sigma_i \omega \quad i = 0, 1, 2
\]

We will follow the assumptions of Ward as they do not
destroy the generality of the solution in any airborne
electromagnetic situation:

1) that the imbedding medium has the permeability of
free space

2) that displacement currents are negligible in the
sphere:

\[
\alpha = k_2 R = R \sqrt{\mu_2 \sigma_2 \omega} = (j) \theta
\]

3) that the applied field is purely magnetic and
uniform in the vicinity of the sphere:

\[
|k_2 R| \ll 1
\]
Figure 5.01 displays the \((M-jN)\) function against:

\[
\theta = R \sqrt{\mu \sigma \omega}
\]

We now introduce the complex permeability:

\[
\mu_2 = \mu = (\mu_r - j\mu_i) \mu_0
\]

in which both real and imaginary terms may be frequency dependent. Thus, we see:

\[
\alpha = A + jB = R \sqrt{j \mu_0 (\mu_r - j\mu_i) \sigma_2 \omega}
\]

and we may solve for \(A\) and \(B\) to see:

\[
A = R \sqrt{\sigma_2 \mu_0 \omega / 2} \sqrt{\mu_i + \sqrt{\mu_i^2 + \mu_r^2}}
\]

\[
B = R \sqrt{\sigma_2 \mu_0 \omega / 2} \sqrt{-\mu_i + \sqrt{\mu_i^2 + \mu_r^2}}
\]

Taking standard trigonometric identities:

\[
\tan(A + jB) = \frac{\tan A + \tan jB}{1 - \tan A \tan jB}
\]

and

\[
\tan jB = j \tanh B
\]

we find:

\[
\tan \alpha = \frac{\tan A (1 - \tanh^2 B) + j \tanh B (1 + \tan^2 A)}{1 + \tan^2 A \tanh^2 B}
\]
We may thus set:

\[ X + jY = \tan \alpha - \alpha \]

and find:

\[ X = \frac{\tan A (1 - \tanh^2 B)}{1 + \tan^2 A \tanh^2 B} - A \]
\[ Y = \frac{\tanh B (1 + \tan^2 A)}{1 + \tan^2 A \tanh^2 B} - B \]

Similarly,

\[ E + jG = \alpha - \tan \alpha + \alpha^2 \tan \alpha \]
\[ = (A + jB)^2 \left[ (X + jY) + (A + j) \right] - (X + jY) \]

to thus see

\[ E = (A^2 - B^2)(X + A) - 2AB(Y + B) - X \]

and

\[ G = 2AB(X + A) + (A^2 - B^2)(Y + B) - Y \]
We then have the function \((M-jN)\) becoming:

\[
(M-jN) = \frac{2(\mu_r-j\mu_i)(X+jY)-(E+jG)}{2(\mu_r-j\mu_i)(X+jY)+2(E+jG)}
\]

If we now set

\[
D+jF = 2(\mu_r-\mu_i)(X+jY)
\]

to see

\[
(M-jN) = \frac{(D+jF)-(E+jG)}{(D+jF)+2(E+jG)}
\]

we thus resolve:

\[
M = \frac{(D+2E)(F-G)-(D-E)(F+2G)}{(D+2E)^2+(F+2G)^2}
\]

and

\[
N = \frac{3(DG-EF)}{(D+2E)^2+(F+2G)^2}
\]

To these functions we apply the complex, frequency dependent permeability using the Debye and Cole-Davidson distributions described above.

Figure 5.01 displays the \((M-jN)\) versus a parameter \(\theta\).
with only eddy current loss and with permeability differing from free-space. As the parameter theta contains the permeability which we wish to be frequency dependent, we will replot (M-jN) as in Figure 5.02. This figure also shows the frequency dependence of the function as the parameter of the sphere-radius-square-root-conductivity is changed. For fixed sphere radius, a change in conductivity from low to high will raise the eddy-current decay time constant (the inverse of the frequency at which the imaginary term reaches a maximum) as we would expect.

Figure 5.03 depicts the (M-jN) function with a complex permeability as it appears with Debye relaxation at several values of Debye time constant. As would be expected, for magnetic time constants smaller than the eddy-current decay time constant, the eddy currents dominate (the skin-effect does not allow sufficient penetration to excite the magnetic component). The high frequency permeability value of 1.5 is an extreme case used to exhibit the effect; tabulated below are more realistic situations.

Figure 5.04 displays the variation of the function as a Cole-Davidson distribution broadens and eventually disappears.
\[ \theta = R \sqrt{\sigma \mu \omega} \]

FIGURE 5.01

In-phase (M) and quadrature (N) components of the induced dipole moment of a sphere in an uniform alternating magnetic field. Signs of M and N changed to follow convention (as in all following figures). Sphere permeabilities of 1, 3, and 300 are shown (relative to the imbedding medium). (after Ward, 1967a)
(M-JN) with change in sphere radius or conductivity.
Debye magnetic loss for several time constants in (M-jN).
Varying distributions of Cole-Davidson magnetic loss in (M-JN).
6.0 Discussion

We have seen the configuration of the typical airborne electromagnetic system and the functional form of the anomaly measured in a typical survey. We have investigated the performance of the typical relaxation system as described by the Debye, Cole-Cole, and Cole-Davidson distributions. Magnetic loss phenomena have been presented, and the magnetic viscosity effects observed in geologically interesting samples discussed. We have mentioned the paper of Colani and Aitken (1956) and the conversation with Bonniwell and McDougal (1970) dealing with magnetic loss in electromagnetic systems. The functional dependence of the complex permeability in the (M-jN) response of a sphere has been presented. This presentation in the figures of Chapter 5 has been exaggerated to enhance the effect. We wish to now observe the relative magnitude of the magnetic loss as represented in a complex permeability for a sample of magnetite as compared to the eddy-current response.

Table 6.1 represents a typical magnetite (massive) complex permeability inserted into the function (M-jN). The radius-square-root-conductivity parameter has remained 50.0 to allow comparison to Figures 5.02 and 5.03. The curves
could be shifted for other values as indicated above in Figure 5.02. The time constant chosen is the only unrealistic term (100 seconds as opposed to 300 microseconds mentioned above), but as Figure 5.03 shows, the only effect of this is to move the region of magnetic perturbation (otherwise the eddy-current effect would obscure the magnetic completely for radius-square-root-conductivity chosen).

Thus we see that magnetite has an effect of about 5% on the real component of the \((M-jN)\) response, and a similar effect on the imaginary component (except where the sign reversal occurs). We also observe that (as mentioned above with respect to Figure 5.03) as the time constant (inverse radial frequency of the peak imaginary loss) of the magnetic after-effect loss becomes comparable to or smaller than the time constant of the eddy-current loss, the magnetic effect is completely obscured and unobservable.

With the above in mind, we would thus expect (as Strangway has predicted, private communication, 1970) that any observation of decays that are unexplainable on the basis of conductivity, eddy-current calculations for magnetite deposits (or other minerals with magnetic loss) due to very
low conductivity, could be possibly due to magnetic after-effect decays. Indeed, as communicated to us by Bonniwell and MacDougal (1970), such surveys have been observed and from data from sample surveys given us by Barringer (1970), we have calculated decay times of measurements over two deposits:

Leveaniemi, Sweden
High conductivity deposit of a 50:50 combination of 65% Fe and 35% Fe magnetite having an observed decay time of 389 microseconds

Mertainen, Sweden
Low conductivity deposit of 35% Fe average (all textural types) magnetite in a syenite porphyry with an observed decay time of 285 microseconds.

The natural magnetite sample from Ishpeming, Michigan that was mentioned in Chapter 4 had a decay time of 362 microseconds (including the eddy-current contribution) and the artificial magnetite samples with 1% to 50% magnetite had decay times of 292 to 311 microseconds. This information is suggestive of an eddy-current response in the Leveaniemi deposit (obscuring any magnetic response) and a magnetic after-effect response in the Mertainen deposit. Both of these deposits were 1200 meters long by 100 meters wide by 550 meters deep (drilled). Therefore if the factor of 1.36 between the two observed time constants is caused by eddy-current effects, then the two deposits must differ in
their conductivities by the square of that amount (1.86); however, communication with Barringer Research Ltd. indicates that the two deposits differ in conductivity by much more than 1.86 (magnetite deposits may vary over several orders of magnitude in their conductivity due to water content and other factors, see Parkhomenko, 1967). Similarly, aeromagnetic survey results for these two deposits indicate a maximum factor of 1.02 between the permeabilities. Unfortunately, calibration was not available for the two recordings processed so that comparison between relative signal strengths could be obtained (the eddy-current signal is surface dependent while the magnetic after-effect is volume dependent).

As observed in Figure 5.04, if the loss is not of the Debye form but rather a distribution of the Cole-Davidson type, the magnetic contribution may be smoothed, and the magnetic relaxation will be correspondingly difficult to observe in an airborne electromagnetic survey. This would also be the case in attempting to observe the wide distribution of relaxations characteristic of the IP response at low frequencies, and that may be an additional reason for the difficulties encountered by Hohmann et alia (1970) in the investigation of the inductive IP system.
Table 6.1

(M-jN) Function for $\xi = 0.0525$

$X_0 = \mu_0 = 3.0$

$R\sqrt{\sigma} = 50.$

$\tau = 100.$ (exaggeration, see text)

<table>
<thead>
<tr>
<th>radian frequency</th>
<th>magnetic loss no loss</th>
<th>Debye loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>N</td>
</tr>
<tr>
<td>0.63</td>
<td>0.40</td>
<td>-0.00028</td>
</tr>
<tr>
<td>6.3</td>
<td>0.40</td>
<td>-0.0021</td>
</tr>
<tr>
<td>63.0</td>
<td>0.40</td>
<td>-0.021</td>
</tr>
<tr>
<td>630.</td>
<td>0.33</td>
<td>-0.18</td>
</tr>
<tr>
<td>6300.</td>
<td>-0.12</td>
<td>-0.23</td>
</tr>
</tbody>
</table>

and remaining terms identical between the two cases of only eddy-current loss and both eddy current and magnetic loss
7.0 Summary

We have investigated the electromagnetic prospecting system and calculated the effect of magnetic as well as strictly electrical (eddy-current) loss on the anomaly response function for a sphere. We have seen the various types of magnetic loss phenomena which could contribute to such a perturbation of the (M-jN) response function. From the above discussion, it is apparent that only losses closely approximating Debye relaxation phenomena could cause any discernable change of the (M-jN) response. We have also seen that the eddy-current effect is dominant when the radius-square-root-conductivity is sufficiently high to make the eddy-current time constant greater than the magnetic decay time constant. The effect of magnetite on the (M-jN) function has been shown to be of the order of 5%. A situation in which the eddy-current loss would allow observation of the magnetic decay has been presented (magnetite disseminated in a very low conductivity matrix like the typical volcanic), and a possible Barringer recording with such a situation has been remarked upon.

In conclusion, magnetic loss may effect the interpretation of airborne and ground electromagnetic systems, and further
work is clearly indicated, particularly in terms of airborne survey interpretation and in mineral characterization in terms of magnetic loss phenomena.
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9.0 Appendix

Artificial Magnetite Sample Preparation

These samples were constructed by Mr. D. Kennedy of Ledgemont Laboratory for the author using the following procedure:

Synthetic samples were prepared with the objective of simulating a natural material consisting of an homogeneous, highly resistive, low porosity, ground mass with uniformly disseminated particles of various magnetic and conductive properties. Preliminary experimentation was carried out on matrix materials using quartz and muscovite with sodium silicate as a binder (to simulate Bingham porphyry ground mass). Different combinations of silica sand and a number of high alumina cements were investigated. The matrix material finally used was a refractory cement manufactured by Ciment Fondu LaFarge Corporation and designated Secar 250. It consists of approximately 74% Al2O3, 25% CaO, and small amounts of other material (including less than 0.1% iron compounds of negligible magnetic contribution). Initial samples were made from a 50/50 mixture of Secar and silica sand. It was found that the magnetic and electrical properties of this mixture did not differ appreciably from
the pure Secar. Consequently there seemed little need to model the matrix by using the mixtures first investigated whose mechanical properties were not as satisfactory and which took considerably longer to prepare than a pure Secar matrix.

The cement was combined with the desired materials and sufficient distilled water to make a thick paste. This was placed on a one-inch diameter mold and subjected to a pressure of 10,000 pounds for 15-minutes. The sample was then removed and allowed to cure at room temperature for one week. When the curing process was completed, the samples were cut to the desired length, and the magnetic measurements were made.

Samples were produced from magnetite crushed to 100-200 mesh and of 95% or better purity. Mesh sizes from 20-200 were constructed at 10% magnetite to investigate the effect on the magnetic response, but no difference was observed due to mesh size. The sample at 50% magnetite required mesh 15-20 to hold together and also a Secar/silica-sand mixture for the matrix.
9.0 Appendix

Laboratory Measurement of Magnetic Loss

Magnetic loss having long time constants in the region 10 to 10,000 seconds was measured by two methods. The first involved the spinner magnetometer as described in the text, and the second used the vibrating sample magnetometer described by Foner (1959). Both techniques are standard in the measurement of magnetization and will not be elaborated upon here.

Measurement of loss in the region from 1 microsecond to 10 seconds involved the standard technique of measuring the output voltage of a coil when subjected to a time changing magnetic field. This is as used by Tomono (1952) in his studies of the magnetic after-effect in iron. Also, some measurements were performed in the frequency domain using the standard inductance measuring apparatus.

The time domain measurement of pulse decays using coils was found to be satisfactory over the region 50 microseconds to 3 milliseconds; shorter times than 50 microseconds developed
problems with the coil relaxation itself, and longer times than 3-5 milliseconds developed sensitivity problems. Two coil sets were used for pickup, each in a balanced pair arrangement to cancel signals common to both but produce a voltage proportionate to the dB/dt seen by only one. The first set of 250 turns each was used with the one-inch mineral samples, and the second set of 4500 turns was used with the 0.080 by 0.160 inch steel samples. The procedure was as follows: the sample was inserted into the top coil of the balanced pair; the pair were inserted into the center of a solenoid cylindrical coil; the cylindrical coil was pulsed in a manner like the Barringer INPUT system of two 1.6 millisecond pulses of opposite polarity, separated by 2.0 milliseconds to record the decay. The pulse coil was driven by a Chronetics PG-31 pulse generator through a Krohn-Hite DC-50(R) amplifier; a Tektronix 545A scope with 1A7A plug-in set at DC to 30 kHz filter -3dB points was used to receive the output of the balanced coil pair; the resultant trace was photographed. The trace was then converted into digital form and the decay data was computed on the basis of the Tomono analysis assuming a Debye relaxation.
Computer Algorithm for Modified Richter Decay

In the following algorithm, these formulae are necessary:
(a and b are data point pairs Xa and Xb at time Ta and Tb)

\[ t_o = \frac{t_a (X_a - X_o)^{1/v} - t_b (X_b - X_o)^{1/v}}{(X_b - X_o)^{1/v} - (X_a - X_o)^{1/v}} \]

\[ C = \frac{1}{2} \left[ (X_a - X_o)(t_a + t_o)^v + (X_b - X_o)(t_b + t_o)^v \right] \]

\[ 2v = \frac{\ln(C/(X_a-X_o))}{\ln(t_a+t_o)} + \frac{\ln(C/(X_b-X_o))}{\ln(t_b+t_o)} \]

\[ X_o = \frac{X_l (t_l + t_o)^v - X_m (t_m + t_o)^v}{(t_l + t_o)^v - (t_m + t_o)^v} \]
Algorithm, continued

The formula to be solved is:

\[ X(t) = X_0 + \frac{C}{(t + t_0)^\nu} \]

The following algorithm requires approximately 6 seconds of CPU time on an IBM 360/67 virtual 65 to produce a fit as is shown in Figure 4.07.

1.0 enter data, select a tolerance level (desired curve fitting tolerance in percent divided by 200)

2.0 set initial conditions: \( \nu = 1.0; X_0 \) equals value of data point measured at greatest time

3.0 scan data to find data point nearest maximum slope \( (X_m) \).

4.0 initially set \( t = to' = \) average of times associated with 3 data points nearest \( X_m \)

5.0 start with the third data point (in time) below \( X_m \) and create all possible permutations of data point pairs \( X_a \) and \( X_b \) at times \( t_a \) and \( t_b \) through the third data point above \( X_m \) and:

5.1 calculate a new \( t_o \) with the formula given above, if the new \( t_o \) is less than zero or greater than 1.5*\( t_o' \) return to 5.0 and select another data point pair

5.2 calculate \( C \) from formula above

5.3 calculate new \( \nu \) from formula above, if the difference between the new \( \nu \) and the old \( \nu \) is greater than the tolerance chosen, select a new data point pair and return to 5.1

5.4 add all values of \( t_o, \nu, \) and \( C \) that reach this step into averages to obtain average \( t_o, \nu, \) and \( C \); average through all pair combinations to reach this point

6.0 with average values of \( t_o, \nu, \) and \( C \) and values of data points \( X_1 \) and \( X_m \), calculate a new \( X_0 \) from formula above, if the new \( X_0 \) differs from the old \( X_0 \) by more than the tolerance return to step 5.0 with the present values of \( t_o, \nu, C, \) and \( X_0 \), which are otherwise the solution.
9.0 Appendix

Computer Program Listing

The following program is the result of the preceding algorithm. It is written in Fortran IV and was run on an IBM 360/67.

```
DIMENSION TM(20), DEL(20), DT(20)
READ(2,10) N, (TM(I), DT(I), I=1, 20), TOL
10 FORMAT(1X, 12/20(1X, 2E11.4), 1X, E11.4)
WRITE(1, 10) N, (TM(I), DT(I), I=1, 20), TOL
TOLA=TOL*100.
XIO=DT(N)
NS=N-1
DEL(1)=0.
DELMAX=0.
DEL(2)=0.
DO 210 NI=2, NS
  DELDM=(DT(NI-1)-DT(NI+1))
  DEL(NI)=DELM/(ALOG(TM(NI-1))-ALOG(TM(NI+1)))
  DELH=ABS(1.1*DEL(NI-1))
  DELL=ABS(0.9*DEL(NI-1))
  IF(DELH.EQ.DELL) GO TO 210
  IF(ABS(DEL(NI)).LT.DELMAX) GO TO 210
  DELMAX=ABS(DEL(NI))
  NIS=NI
210 CONTINUE
TOT=1.5*((TM(NIS-1)*TM(NIS)*TM(NIS+1))**(0.33))
NST=NIS-3
NEN=NIS+3
IF(NST.LE.1) NST=1
IF(NEN.GE.N) NEN=N-1
NEU=NEN-1
K=1
DNX=1.0
500 ATO=0.
  DN=DNX
  ADN=0.
  MADN=0
  AC=0.
  M=0
```
DO 100 I=NST,NEU
   J=I+1
DO 100 J=I,J,NEN
NDN=0

200 TAA=(DT(I)-XIO)**(1./DN)
    TBA=(DT(J)-XIO)**(1./DN)
    TO=(TM(I)*TAA-TM(J)*TBA)/(TBA-TAA)
    IF(TO.GT.TOT.OR.TO.LE.0.) GO TO 100
    CA=(DT(I)-XIO)*((TM(I)+TO)**DN)
    CB=(DT(J)-XIO)*((TM(J)+TO)**DN)
    C=(CA+CB)/2.
    IF(C.LE.0.) GO TO 100
    DNA=(ALOG(C/(DT(I)-XIO)))/ALOG(TM(I)+TO)
    DNB=(ALOG(C/(DT(J)-XIO)))/ALOG(TM(J)+TO)
    DNS=DN
    DN=(DNA+DNB)/2.
    DND=(DN-DNS)/DN
    IF(DN.GT.DNS.AND.NDN.GT.5) GO TO 100
    NDN=NDN+1
    DND=ABS(DND)
    IF(DND,GT,TOL) GO TO 200
    ATO=ATO+TO
    AC=AC+C
    ADN=ADN+DN
    MADN=MADN+1
    M=M+1

100 CONTINUE
    TO=ATO/FLOAT(M)
    DN=ADN/FLOAT(MADN)
    C=AC/FLOAT(M)
    XIOS=XIO
    XIOA=(TM(K)+TO)**DN
    XIOB=(TM(N)+TO)**DN
    XIO=(DT(K)*XIOA-DT(N)*XIOB)/(XIOA-XIOB)
    XIOD=(XIO-XIOS)/XIO
    DNS=DN
    DNX=(ALOG(C/(DT(1)-XIO)))/ALOG(TM(1)+TO)
    DND=ABS(DNX-DNS)/DNX
    WRITE(1,22) XIO,TO,DN,C,XIOD
    WRITE(6,55) XIOD
    IF(ABS(XIOD).GT.TOL) GO TO 500

55 FORMAT(8X,F10.8)
22 FORMAT(' XIO=' ,E11.4,' TO=' ,E11.4,' DN=' ,F8.6,
1' C=' ,E11.4,' TOL=' ,F8.6)
    DSCRMX=0.
    ADSCR=0.
DO 600 IT=1,N
XI=XI0+(C/((TM(IT)+TO)**DN))
DSCR=(DT(IT)-XI)/DT(IT)
IF(ABS(DSCR),GT,ABS(DSCRMX)) DSCRMX=DSCR
ADSCR=ADSCR+DSCR
DSCRO=100.*DSCR
PHI=C*EXP(-1.*TO/TM(IT))/((TM(IT))**(1.+DN))
WRITE(1,33) TM(IT),DT(IT),PHI,XI,DSCRO
WRITE(6,44) DSCRO

 FORMAT(F10.4)
 600 CONTINUE
 ADSCR=ADSCR/FLOAT(N)
 IF(ADSCR.GT.0.) TOT=0.9*TOT
 IF(ADSCR.LT.0.) TOT=1.1*TOT
 IF(ABS(ADSCR).GT.TOLA) GO TO 500

 FORMAT(6X,'T=",F10.2," SEC, DATA=",F10.6/
 END
9.0 Appendix

Undergraduate Research Summary

The following is a summary of some of the mineral magnetic decay measurements that were made using the procedure described above during the author's undergraduate studies. These measurements were follow up on the work of Tomono (1952) and Cclani and Aitken (1966) to determine the extent of magnetic viscosity in minerals at short time constants.

Mineral specimens were obtained from Ward's Catalog through Kennecott Copper Corporation; characterization was performed by Dr. A. N. Mariano of Ledgemont Laboratory. All measurements were performed under the supervision of Dr. S. Foner at the Francis Bitter National Magnet Laboratory, and the results have been previously reported in several informal reports to Ledgemont Laboratory and in undergraduate research term-papers.

No magnetic response was observed in the following samples:

- bornite
- cassiterite
- chalcocite
- chalcopyrite
- chromite
- cuprite
- malachite
- pyrite
- quartzite
- smaltite
- sphalerite

The following minerals exhibited single relaxation decays of the form shown:

<table>
<thead>
<tr>
<th>sample</th>
<th>origin</th>
<th>time constant</th>
<th>( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>hematite</td>
<td>Brazil</td>
<td>375 microseconds</td>
<td>0.0013</td>
</tr>
<tr>
<td>magnetite</td>
<td>Michigan</td>
<td>362</td>
<td>0.052</td>
</tr>
<tr>
<td>(without eddy currents)</td>
<td>300</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>niccolite</td>
<td>Ontario</td>
<td>300</td>
<td>0.00043</td>
</tr>
<tr>
<td>pyrrhotite</td>
<td>Ontario</td>
<td>320</td>
<td>0.015</td>
</tr>
</tbody>
</table>