A FINITE DIFFERENCE
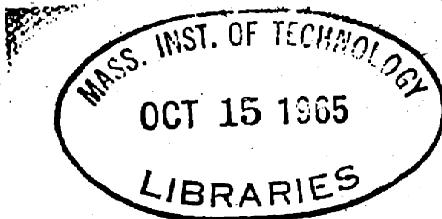# TECHNIQUE FOR THE ANALYSIS
OF THIN, ELASTIC PLATES

by

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(1961)

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(1962)

Submitted in partial fulfillment

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Civil Engineer

at the

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(1965)

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ABSTRACT

A FINITE DIFFERENCE TECHNIQUE
FOR THE ANALYSIS OF THIN, ELASTIC PLATES

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Submitted to the Department of Civil Engineering on August 22, 1965 in partial fulfillment of the requirements for the degree of Civil Engineer.

In the past, the analysis of plates using finite differences has utilized artificial points lying off the surface of the plate when assembling molecules for gridpoints in the vicinity of the boundary. This method of accounting for the boundary conditions introduces serious errors in the resulting molecules.

The primary purpose of the finite difference plate analysis presented herein is to provide an accurate, systematic method of accounting for boundary conditions when assembling molecules for gridpoints in the vicinity of the boundary. This is done by expressing $\nabla^4 w$ at the gridpoint as a linear combination of the boundary conditions for the side, the displacements at neighboring gridpoints, and, if necessary, the values of $\nabla^4 w$ at neighboring gridpoints. Each of these quantities is expanded in a Taylor series about the central gridpoint, and the coefficients of like derivatives on each side of the above expression for $\nabla^4 w$ at the central gridpoint are equated. This will result in a system of equations which can be solved for the molecule at the central gridpoint.

In addition to the above method of accounting for boundary conditions, this analysis also presents a method for computing the bending and twisting moments at the interior gridpoints, the principal moments at the interior gridpoints, and the bending moments, transverse shear force, and/or vertical displacement, (depending on the particular boundary condition), at up to 1000 points along the boundary.

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ACKNOWLEDGMENT

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A. INTRODUCTION

One of the primary problems faced when analyzing thin elastic plates using standard finite differences methods is that of properly accounting for the boundary conditions in assembling the molecule for gridpoints near the boundary. In the standard method, artificial gridpoints lying outside of the plate are created. As will be shown, this could conceivably result in an inaccurate molecule for the interior gridpoints in the vicinity of the boundary.

To illustrate this, let us consider a gridpoint such as point 0 in figure (1) which is located near the boundary of a simply supported plate. The numbers in parenthesis in figure (1) are the molecule for point 0 obtained using the standard finite difference approach.

In order to evaluate the accuracy of the standard finite difference molecule $\nabla^4 w$ at point 0 can be expressed as a linear combination of the surrounding points lying within the plate. (The deflection at point 11 is dependent on that of the interior gridpoints and therefore does not have to be considered). This is expressed mathematically as follows:

$$(1) \qquad A_0 \omega_0 + \sum_{\substack{i=1 \\ i \neq 11}}^{12} A_i \omega_i = \nabla^4 \omega_0$$

where the A's are arbitrary constants. Each of the above
displacements can now be expanded in a Taylor Series about
point. In this example, these expansions will be up to the
sixth derivative of the displacement at 0, and are as follows:

$$W_1 = W_0 + hW_{0,x} + \frac{h^2}{2}W_{0,xx} + \frac{h^3}{6}W_{0,xxx} + \frac{h^4}{24}W_{0,xxxx} + \frac{h^5}{120}W_{0,xxxxx}$$

$$W_2 = W_0 + hW_{0,y} + \frac{h^2}{2}W_{0,yy} + \frac{h^3}{6}W_{0,yyy} + \frac{h^4}{24}W_{0,yyyy} + \frac{h^5}{120}W_{0,yyyyy}$$

$$W_5 = W_0 + hW_{0,x} + hW_{0,y} + \frac{h^2}{2}W_{0,xx} + h^2W_{0,xy} + \frac{h^2}{2}W_{0,yy} + \frac{h^3}{6}W_{0,xxx}$$
$$+ \frac{h^3}{2}W_{0,xxy} + \frac{h^3}{2}W_{0,xyy} + \frac{h^3}{6}W_{0,yyy} + \cdots\cdots + \frac{h^5}{120}W_{0,yyyyy}$$

(2)

$$W_{12} = W_0 - 2hW_{0,y} + 2h^2W_{0,yy} - \frac{4}{3}h^3W_{0,yyy} + \frac{2h^4}{3}W_{0,yyyy} - \frac{32\,h^5}{120}W_{0,yyyyy}$$

After substitution of (2), (1) is written in the
following form:

$$W_0\left[A_1 + A_2 + \cdots\cdots + A_5 + \cdots\cdots + A_{12}\right] + hW_{0,x}\left[A_1 + \cdots\cdots + A_5 + \cdots\cdots\right]$$

$$+ hW_{0,y}\left[A_2 + \cdots\cdots + A_5 + \cdots\cdots - 2A_{12}\right] + \cdots\cdots$$

$$+ \frac{h^5}{120}W_{0,yyyyy}\left[A_2 + \cdots\cdots + A_5 + \cdots\cdots - 32A_{12}\right] = \nabla^4 W_0$$

or, in matrix form:

		w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{12}	$\nabla^4 w$
w_0	1	1	1	1	1	1	1	1	1	1	1	1	1	0
$w_{0,x}$	h	0	1	0	-1	0	1	-1	-1	1	2	0	0	0
$w_{0,y}$	h	0	0	1	0	-1	1	1	-1	-1	0	2	-2	0
$w_{0,xx}$	$\dfrac{h^2}{2!}$	0	1	0	1	0	1	1	1	1	4	0	0	0
$w_{0,xy}$		0	0	0	0	0	2	-2	2	-2	0	0	0	0
$w_{0,yy}$		0	0	1	0	1	1	1	1	1	0	4	4	0
$w_{0,xxx}$	$\dfrac{h^3}{3!}$	0	1	0	-1	0	1	-1	-1	1	8	0	0	0
$w_{0,xxy}$		0	0	0	0	0	3	3	-3	-3	0	0	0	0
$w_{0,xyy}$		0	0	0	0	0	3	-3	-3	3	0	0	0	0
$w_{0,yyy}$		0	0	1	0	-1	1	1	-1	-1	0	8	-8	0
$w_{0,xxxx}$	$\dfrac{h^4}{4!}$	0	1	0	1	0	1	1	1	1	16	0	0	1
$w_{0,xxxy}$		0	0	0	0	0	4	-4	4	-4	0	0	0	0
$w_{0,xxyy}$		0	0	0	0	0	6	6	6	6	0	0	0	2
$w_{0,xyyy}$		0	0	0	0	0	4	-4	4	-4	0	0	0	0
$w_{0,yyyy}$		0	0	1	0	1	1	1	1	1	0	16	16	1
$w_{0,xxxxx}$	$\dfrac{h^5}{5!}$	0	1	0	-1	0	1	-1	-1	1	32	0	0	0
$w_{0,xxxxy}$		0	0	0	0	0	5	5	-5	-5	0	0	0	0
$w_{0,xxxyy}$		0	0	0	0	0	10	-10	-10	10	0	0	0	0
$w_{0,xxyyy}$		0	0	0	0	0	10	10	-10	-10	0	0	0	0
$w_{0,xyyyy}$		0	0	0	0	0	5	-5	-5	5	0	0	0	0
$w_{0,yyyyy}$		0	0	1	0	-1	1	1	-1	-1	0	32	-32	0

Now, if the standard finite difference molecule were accurate up to the sixth derivative of the series expansions, the solution of equation (3) would yield:

$$A_0 = 19 \qquad A_6 = 2$$
$$A_1 = -8 \qquad A_7 = 2$$
$$A_2 = -8 \qquad A_8 = 2$$
(4)
$$A_3 = -6 \qquad A_9 = 1$$
$$A_4 = -8 \qquad A_{10} = 1$$
$$A_5 = 2 \qquad A_{12} = 1$$

which corresponds to the molecule given in Fig. (1).

The constants given in (4) can be shown to identically satisfy the first 10 equations in (3). However, upon substitution into the equation for $\omega_{0,xxxx}$; we have:

(5)
$$1 \times (A_1 = -8) + 1 \times (A_3 = -6) + 1 \times (A_5 = 2) + 1 \times (A_6 = 2) + 1 \times (A_7 = 2)$$
$$+ 1 \times (A_8 = 2) + 16 \times (A_9 = 1) = 10 \neq 1 \times \frac{4!}{h^4}$$

Therefore, the standard finite difference molecule leads to a contradiction in the fourth derivative of the series expansion for the displacement at point 0. Since the quantity being solved for is also a fourth derivative term, ($\nabla^4 \omega_0$), it is seen that the standard finite difference method gives molecules

for gridpoints in the vicinity of the boundary that are inherently inaccurate.

In view of the above falacies, the need for a modification of the standard finites difference method is readily apparent. Therefore, in what follows, a new type of finite difference technique for the analysis of plates is proposed. This new technique considers only points within the plate domain and, as will be shown, is well within the accuracy required for a satisfactory solution of the plate problem.

B. ASSUMPTIONS AND LIMITATIONS

1. Shape of Plate

This analysis will be valid for plates with up to 100 straight sides, but is not valid for curved plates.

2. Interior Conditions

Any holes that exist in the interior of the plate are made up of straight sides. Also, the theory is valid for any transverse loading distribution.

3. Boundary Conditions

The following boundary conditions can be handled by this analysis:

a) Fixed edges

b) Simply supported edges

c) Free edges

d) Edges elastically restrained against rotation

e) Edges elastically restrained against vertical displacement

f) Edges elastically restrained against both rotation and displacement

g) Edge of plate connected to free edgebeam

h) Edge of plate connected to simply supported edgebeam

In boundary conditions (g) and (h), the centroid of the edge beams is assumed to coincide with the center of flexure

4. <u>Assumptions in Small Deflection Plate Theory</u>

    a)   Effect of deflections on bending of plate is negligible

    b) σ_z , (normal stress in transverse direction), is negligible.

    c)   The plate thickness is negligible compared to any representative length or width of the plate.

    d)   The thickness of the plate is constant.

    e)   An infinite transverse shear rigidity is assumed.

    f)   St. Vennant beam theory is assumed valid.

5. <u>Orientation of Plate</u> (see figure (2))

    a)   The plate will be in the x-y plane and all displacements, w, will be in the z direction.

    b)   The positive direction along the tangent to each edge of the plate will be such that the interior of the plate will be on the left (+s direction)

    c)   The positive direction of the normal to each edge of the plate will be pointing away from the interior of the plate. (+n direction)

6. <u>Direction of Stress Resultants and Bending Moments</u>

    The positive directions of all stress resultants and bending moments is given by figure (3).

CHAPTER I

INPUT

An example problem will be used to explain the input.
It is seen that, although the exterior boundary and interior
contours can each have an unlimited number of sides, each side
must be straight, and not curved. This particular example has
one exterior, and two interiorboundaries, and is shown in
figure (4).

1. Title Cards

The first card to be read in as input will contain the
name of the deck, "Plate Analysis", and will be in the form
shown in figure (5a).

The next card will be a title card and will contain the
name of the particular problem being considered. This name is
completely arbitrary; for example, the name of this example
problem was chosen to be "Example Thesis Problem, Superplate".
The form by which the problem name is to be punched on the
data card is shown in figure (5b).

2. **Plate Data And Connectivity Data**

The user will next read in the following constants for the plate:

a) DP =   flexural rigidity of plate (lb. in^2/in)

b) POIS = poissons ratio

c) NS =   total number of sides (= cornerpoints) (up to 100 sides)

d) NC =   total number of contours (no limit to this number)

This will all be on one card, as shown in figure (6).

The user then numbers all cornerpoints and sides arbitrarily and, for each side, reads in the side number, initial and terminal points, and boundary conditions of the side. There will be one such card for each side in the plate, and the form of these cards is shown in figure (7).

Internally, this set of side data cards will be stored as a connectivity matrix MCONS (I,J) , I = 1, -- 4, J = 1 --- NS, in which the first row contains the side numbers, the second row contains the initial cornerpoints for each side, the third row contains the terminal cornerpoints, and the fourth row contains the boundary conditions for each side. Note that only the first three rows are actually concerned with the connectivity of the various contours.

The method of boundary condition input and internal storage are indicated in detail in Table (1), (2), and (3).

Table 1

Boundary Condition Input

BOUNDARY CONDITION	READ IN
1) Fixed ended	FIXED
2) Simply supported	SIMPLE SUPPORT
3) Free edges	FREE
4) Elastically restrained against displacement and rotation	ELASTIC
5) Edge of plate connected to free edgebeam	FREE EDGEBEAM
6) Edge of plate connected to simply supported edgebeam	SIMPLY SUPPORTED EDGEBEAM

Table 2

Internal Storage of Boundary Conditions

BOUNDARY CONDITION INPUT	STORED IN COMPUTER AS:
FIXED	1
SIMPLE SUPPORT	2
FREE	3
ELASTIC	4
FREE EDGEBEAM	5
SIMPLY SUPPORTED EDGEBEAM	6

Table 3

Formation of Connectivity Matrix for Superplate

READER INPUT:

Side 10	From 4 to 10	FREE
Side 3	From 8 to 4	SIMPLE SUPPORT
Side 8	From 10 to 9	FIXED
Side 6	From 5 to 11	ELASTIC
Side 4	From 1 to 3	FREE
Side 1	From 2 to 1	FIXED
Side 2	From 2 to 6	ELASTIC
Side 5	From 13 to 11	SIMPLE SUPPORT
Side 9	From 9 to 8	FREE EDGEBEAM
Side 14	From 3 to 6	FIXED
Side 7	From 5 to 13	SIMPLY SUPPORTED EDGEBEAM

INTERNAL STORAGE IN COMPUTER: CONNECTIVITY MATRIX

Side No.	10	3	8	6	4	1	2	5	9	14	7
FROM CORNER PT	4	8	10	5	1	2	2	13	9	3	5
TO CORNER PT	10	4	9	11	3	1	6	11	8	6	13
B. C.		3	2	1	4	3	1	4	2	5	1	6

3. <u>Initial Rearrangement of Connectivity Matrix</u>

The computer will now determine successive cornerpoints
and sides using MCONS(4,NS) for each contour, check the
numbers of contours with NC, and will check that all sides
and cornerpoint numbers have been used.

In the rearranged connectivity matrix, neighboring columns
(of the same contour) correspond to consecutive sides. The
second and third rows are rearranged so that the direction of
travel about each contour is consistent; the second row contains
all negative adjoint corners, and the third row contains all
positive adjoint corners. The actual direction of travel for
an interior or exterior contour (ie, whether clockwise or counter-
clockwise) is arbitrary. This initial rearrangement of the
connectivity matrix for this example problem is shown in
Table(8a).

After rearranging MCONS(4,NS), the computer will form the
vector NSC(NC) (= number of sides per contour). For this
example:

$$NSC(1) = 4$$
$$NSC(2) = 3$$
$$NSC(3) = 4$$

4. <u>Corner Point Input and Final Rearrangement of Connectivity
Matrix</u>

The user will now input the cornerpoint numbers MC(I),
I =1,----NS, along with their absolute coordinates in inches,
CPC(K,I), K=1,2, I=1,---NS. The order in which these corner-
points and their coordinates is read in is completely arbitrary.

A typical cornerpoint data card is shown in figure (8).

The manner in which this matrix of cornerpoints and their coordinates is stored internally is as indicated in Table (4).

The connectivity matrix will now be rearranged once again, in order to coincide with a consistent direction of travel along exterior contours and interior contours. These assumed directions of travel and the corresponding rearrangement of the connectivity matrix will now be examined in detail.

Table 4

Internal Storage of Corner Point Data for Superplate

MC(I)	1	2	6	3	4	8	9	10	5	11	13
CPC(1,I)	4.8	18.	8.7	59.	46.8	31.2	30.	48.	60.	60.5	47.
CPC(2,I)	24.	4.	18.	74.5	34.8	39.	24.	24.2	55.7	44.4	51.6

a) The positive direction of a boundary is such that the interior lies on the left as one traverses the boundary. This corresponds to the counterclockwise direction for an exterior contour and clockwise for an interior boundary. Using this as a guide, the computer will rearrange the columns of the connectivity matrix such that:

(1) The exterior boundary occupies the first few columns of MCONS(4,NS).

(2) The second and third rows will be rearranged, if necessary, so that the direction of travel about each contour will be positive. This will correspond to a counterclockwise arrangement of cornerpoints for the exterior boundary, and a clockwise arrangement of corner-points for the interior boundaries.

The criteria used in examining the direction consists of determining the sign of the area inside a contour, as described below.

b) Positive Direction of Travel about Contours.

(1) Determine area under each contour (An)

$$A_n = \frac{YP+YN}{2} (XP-XN)$$

where

P = cornerpoint positive adjoint (row 3 in MCONS)

N = cornerpoint negative adjoint (row 2 in MCONS)

(2) Determine total area of contour: $A_r(I)$, $I = 1,--NC$

$$AC = \sum_n A_n$$

(3) We now examine the sign of the area:

If AC(-), direction of travel is counterclockwise

If AC(+), direction of travel is clockwise

(4) We now examine the relative magnitudes of the areas
of the contours. The contour with the largest
absolute magnitude will correspond to the external
boundary. This contour will be moved to the left
so that it occupies the first few columns of MCONS(4,NS).

Example:

AC(1) = +20

AC(2) = +15

AC(3) = +100

Since   AC(3)   is largest, it is the exterior boundary. The
direction of travel about this external boundary is clockwise
since AC(3) is positive. However, since the positive direction
of travel about an exterior contour is counterclockwise, rows
2 and 3 of contour 3 will have to be interchanged.

AC(2) and AC(1) are also clockwise since the sign of their
area is positive. However, since they are interior points,
the clockwise sence is positive. Therefore, rows 2 and 3 of
contours 1 and 2 do not have to be interchanged.

See table (8b) for the rearranged matrix MCONS(4,NS) for
the plate of figure (1).

Table 5a

Initial Rearrangement of Connectivity Matrix

Side No.	10	8	9	3	6	5	7	4	14	2	1
From Corner Pt.	4	10	9	8	5	11	13	1	3	6	2
To Corner Pt.	10	9	8	4	11	13	5	3	6	2	1
B.C.	3	1	5	2	4	2	6	3	1	4	1

	First contour			Second contour			Third contour				

Table 5b

Final Rearrangement of Connectivity Matrix

Side No.	4	1	2	14	10	8	9	3	6	5	7
From Corner Pt.	3	1	2	6	4	10	9	8	5	11	13
To Corner Pt.	1	2	6	3	10	9	8	4	11	13	5
B.C.	3	1	4	1	3	1	5	2	4	2	6

	external contour			internal contour				internal contour			

(c) After rearranging the rows and columns as shown
in Table (8b), the computer will now compute the
direction cosine and sine for each side in each
contour.

For Side I:

$$\cos \alpha_i = COSS(I) = \frac{XP-XN}{[(XP-XN)^2 + (YP-YN)^2]^{1/2}}$$

$$\sin \alpha_i = SINS(I) = \frac{YP-YN}{[(XP-XN)^2 + (YP-YN)^2]^{1/2}}$$

where XN, YN are the second and third rows in CPC(2,NS)
for the cornerpoint corresponding to the second row
in the rearranged matrix, MCONS (4,NS), of Table (8b).
XP, YP, are the second and third rows in CPC(2,NS)

for the cornerpoint corresponding to the third
row of the rearranged connectivity matrix.

5. <u>Additional Information Data for Boundary Conditions</u>

If an edge of the plate is either elastically restrained,
connected to a free edgebeam, or connected to a simply supported
edgebeam, additional structural constants must be read in. This
is done as follows:

    a) On the first card in this sequence, the total number
of sides that are either elastically restrained, connected
to a free edgebeam, or connected to a simply supported
edgebeam are read in. This total number of <u>s</u>ides for
<u>a</u>dditional <u>i</u>nformation is needed, (NSAI), is read in as
shown in figure (10a).

    b) Two cards must now be read in for each side. The
first card will have the side number on it, and the
second card will have whatever additional information
is needed for that side.

    c) The additional information for each type boundary
condition is read into the computer in the following
manner:

        1) If the sides is fixed, simply supported, or free,
no additional information is needed.

        2) For an elastically restrained edge, the quantities
SCR and SCD must be read in on the second card, where:

SCR = rotational spring constant - $\frac{(lb-in)}{in}$

SCD = translational spring constant - (lb/in^2)

3) If the side is connected to a free edgebeam,

EI, GJ, and HOR must be read in on the second card,

where:

EI = flexural rigidity of edgebeam about n axis $(lb-in^2)$

GJ = torsional rigidity of edgebeam $(lb-in^2)$

HOR = horizontal distance from shear center of edgebeam

to junction of plate and beam (inches).

4) For a side connected to a simply supported edgebeam,

the quantities GJ and HOR, defined as for a free

edgebeam, are read in.

See figures (10b)-(10d) for the format of these data

cards.

6. Interior Grid Information

The interior grid information will now be read in. This

input will be generated in the following manner:

a) On the first card, the grid size in inches, H, the

total number of internal points, NP, and the total number

of loading conditions, NL, are punched. This program has

a capacity of 1000 gridpoints and 3 loading conditions.

b) There will be one subsequent card for each internal

gridpoint. The first item on a given card will be the

absolute number of the given gridpoint, where the absolute

number is a four digit co-ordinate expressed as a multiple

of the grid size, H. The first two digits in the absolute

number correspond to the y coordinate, and the
second two digits correspond to the x coordinate of
the gridpoint. (Example: An absolute number of
0203 means that the gridpoint has a y coordinate of
2xH and and x coordinate of 3xH.)

c) The next 3 items on the gridpoint card will be
vertical load at the point per unit area (lb/in^2)
corresponding to each loading condition.

See figure (11) for typical interior grid information
cards.

7. Rearrangement of Gridpoint List

The computer now rearranges the gridpoints so that they
are now listed parallel to the smallest side of the rectangle
enclosing the plate. This will find to minimize the width of
the band matrix in the final system of equations.

Now, let us define MDIM as the maximum number of points
spaced a distance H apart that can be placed along the shortest
side of the enclosing rectangle ab. Since this shortest side
corresponds to the maximum possible width of the actual plate,
it follows that MDIM is the maximum number of gridpoints that
can be spaced along the width of the actual plate. Since MDIM
will be important when solving the final system of equations,
it is now computed and stored.

8. Computation of Boundary Points

The computer now determines the points along the boundary

(their coordinates) at which the reactions will later be calculated.

    a) Determination of number of cornerpoints for a given side.

        1) The computer first determines the distance between the 2 cornerpoints of the side, by using their coordinates which have been previously read in.

        2) This distance is then divided by the gridpoint spacing H, and the result is truncated.

        3) The number of equally spaced boundary points = truncated result - 1.

        4) This process is repeated for each side.

        Example: length of side = 9.0 ft; grid spacing = 7.0 in.

        a) Divide by H: $\dfrac{9.0 \times 12}{7.0}$ = 15.4

        b) Truncated result = 15

        c) Number of equally spaced boundary points on the side = 15-1 = 14

    b) Each interior boundary point (except cornerpoints) is assigned the number zero. As an example, we consider the plate of figure (12).

c) The computer first lists the boundary points as follows:

LBPE = (3,0,0,0,5,5,0,0,0,0,0,0,14,14,0,0,----etc.)

where

LBPE = list of boundary points expanded

Note that each corner point is listed twice- once for each side that frames into it (see Fig. 13)

d) The computer then lists the side number corresponding to each successive point number

LBPSE = (11,11,11,11,11,12,12,12,12,12,12,12,12,19,19,19,etc.)

where

LBPSE = list of boundary points with respect to a side expanded

e) The computer then lists the x and y coordinates of each boundary point.

XBPE = (--,--,--,--,--, etc.) = x coordinates

YBPE = (--,--,--,--,--, etc.) = y coordinates

The successive numbers in XBPE and YBPE are the x,y coordinates respectively of each successive boundary point given in LBPE

9. Problem Specified Card

The final input card in the data reads "PROBLEM SPECIFIED", as shown in figure (13). This tells the computer that the complete set of data has been read in and that the computations can now begin.

CHAPTER II

BOUNDARY CONDITIONS

The various boundary conditions discussed in Chapter I
will now be formulated.

1) Fixed ended case

The boundary conditions for a fixed edge are:

$$(2\text{-}1) \qquad \omega = \frac{d\omega}{dn} = 0$$

where, from Fig. (14a), we obtain:

$$\frac{d\omega}{dn} = \frac{d\omega}{dx}\frac{dx}{dn} + \frac{d\omega}{dy}\frac{dy}{dn} = \omega_{,x}\sin\alpha - \omega_{,y}\cos\alpha$$

In Cartesian coordinates, the boundary conditions corresponding
to a fixed edge are:

$$(2\text{-}2) \qquad \omega = 0$$

$$(2\text{-}3) \qquad \omega_{,x}\sin\alpha_i - \omega_{,y}\cos\alpha_i = 0$$

2) Simply supported case

The boundary conditions corresponding to a simply
supported edge are:

$$(2\text{-}4) \qquad \omega = 0$$

$$(2\text{-}5) \qquad M_n = \frac{d^2\omega}{dn^2} + \nu\frac{d^2\omega}{ds^2} = 0$$

where

$$\frac{\partial^2 w}{\partial s^2} \text{ is identically zero at the boundary.}$$

Now

$$\frac{\partial^2 w}{\partial n^2} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial n}\right)\frac{\partial x}{\partial n} + \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial n}\right)\frac{\partial y}{\partial n} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial n}\right)\sin\alpha_i - \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial n}\right)\cos\alpha_i$$

Performing differentiation

$$\frac{\partial^2 w}{\partial n^2} = w_{,xx}\sin^2\alpha_i - 2w_{,xy}\sin\alpha_i\cos\alpha_i + w_{,yy}\cos^2\alpha_i$$

Utilizing figure (14b) to evaluate $\frac{\partial^2 w}{\partial s^2}$,

we get:

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} = w_{,x}\cos\alpha_i + w_{,y}\sin\alpha_i$$

$$\frac{\partial^2 w}{\partial s^2} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial s}\right)\frac{\partial x}{\partial s} + \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial s}\right)\frac{\partial y}{\partial s}$$

or, upon substituting for $\frac{\partial w}{\partial s}$:

$$\frac{\partial^2 w}{\partial s^2} = w_{,xx}\cos^2\alpha_i + 2w_{,xy}\sin\alpha_i\cos\alpha_i + w_{,yy}\sin^2\alpha_i$$

Now, since $\frac{\partial^2 w}{\partial s^2} = 0$:

$$-2w_{,xy}\sin\alpha_i\cos\alpha_i = w_{,xx}\cos^2\alpha_i + w_{,yy}\sin^2\alpha_i$$

Substituting into the expression for $\frac{\partial^2 w}{\partial n^2}$:

$$\frac{\partial^2 w}{\partial n^2} = w_{,xx} + w_{,yy}$$

Therefore, the boundary conditions in Cartesian coordinates are:

(2-6) $\qquad \omega = 0$

(2-7) $\qquad \omega_{,xx} + \omega_{,yy} = 0$

3) Free Edges

The boundary conditions for a free edge are:

(2-8) $\qquad M_n = DP\left[\dfrac{\partial^2 \omega}{\partial n^2} + \nu \dfrac{\partial^2 \omega}{\partial s^2}\right] = 0$

(2-9) $\qquad Q_{ne} = DP\left[\dfrac{\partial^3 \omega}{\partial n^3} + (2-\nu)\dfrac{\partial^3 \omega}{\partial n \partial s^2}\right] = 0$

It has been shown that:

$$\frac{\partial^2 \omega}{\partial n^2} = \omega_{,xx} \sin^2 \alpha_i - 2\omega_{,xy} \sin \alpha_i \cos \alpha_i + \omega_{,yy} \cos^2 \alpha_i$$

$$\frac{\partial^2 \omega}{\partial s^2} = \omega_{,xx} \cos^2 \alpha_i + 2\omega_{,xy} \sin \alpha_i \cos \alpha_i + \omega_{,yy} \sin^2 \alpha_i$$

Therefore, the boundary condition in Cartesian coordinates corresponding to Mn=0 takes the form:

(2-10) $\qquad \omega_{,xx}\left[\sin^2 \alpha_i + \nu \cos^2 \alpha_i\right] + \omega_{,yy}\left[\cos^2 \alpha_i + \nu \sin^2 \alpha_i\right]$

$\qquad\qquad - 2\omega_{,xy} \sin \alpha_i \cos \alpha_i (1-\nu) = 0$

In the boundary condition for Q_{ne} , we evaluate $\dfrac{\partial^3 \omega}{\partial n^3}$

as:

$$\frac{\partial^3 w}{\partial n^3} = \frac{d}{dx}\left(\frac{\partial^2 w}{\partial n^2}\right)\frac{dx}{dn} + \frac{d}{dy}\left(\frac{\partial^2 w}{\partial n^2}\right)\frac{dy}{dn}$$

where

$$\frac{\partial x}{\partial n} = \sin\alpha_i \; ; \qquad \frac{dy}{dn} = -\cos\alpha_i$$

Substituting for $\frac{\partial^2 w}{\partial n^2}$:

$$\frac{\partial^3 w}{\partial n^3} = w_{,xxx}\sin^3\alpha_i - 3w_{,xxy}\sin^2\alpha_i\cos\alpha_i + 3w_{,xyy}\sin\alpha_i\cos^2\alpha_i - w_{,yyy}\cos^3\alpha_i$$

Now, evaluating $\frac{\partial^3 w}{\partial n \partial s^2}$:

$$\frac{d}{dn}\left(\frac{\partial^2 w}{\partial s^2}\right) = \frac{d}{dx}\left(\frac{\partial^2 w}{\partial s^2}\right)\frac{\partial x}{\partial n} + \frac{d}{dy}\left(\frac{\partial^2 w}{\partial s^2}\right)\frac{dy}{\partial n}$$

Substituting for $\frac{\partial^2 w}{\partial s^2}$, $\frac{\partial x}{\partial n}$, and $\frac{dy}{\partial n}$:

$$\frac{\partial^3 w}{\partial n \partial s^2} = w_{,xxx}\sin\alpha_i\cos^2\alpha_i + w_{,xxy}\left[2\sin^2\alpha_i\cos\alpha_i - \cos^3\alpha_i\right]$$
$$+ w_{,xyy}\left[\sin^3\alpha_i - 2\sin\alpha_i\cos^2\alpha_i\right] - w_{,yyy}\sin^2\alpha_i\cos\alpha_i$$

The effective shear now becomes:

$$Q_{ne} = DP\Big[w_{,xxx}\sin\alpha_i\left(\sin^2\alpha_i + (2-\nu)\cos^2\alpha_i\right)$$
$$+ w_{,xxy}\cos\alpha_i\left(-3\sin^2\alpha_i + (2-\nu)(2\sin^2\alpha_i - \cos^2\alpha_i)\right)$$
$$+ w_{,xyy}\sin\alpha_i\left(3\cos^2\alpha_i + (2-\nu)(\sin^2\alpha_i - 2\cos^2\alpha_i)\right)$$
$$+ w_{,yyy}\cos\alpha_i\left(-\cos^2\alpha_i - (2-\nu)\sin^2\alpha_i\right)\Big]$$

where:

$$\sin^2\alpha_i + 2\cos^2\alpha_i = 1 + \cos^2\alpha_i$$

$$-3\sin^2\alpha_i + 4\sin^4\alpha_i - 2\cos^2\alpha_i = 1 - 3\cos^2\alpha_i$$

$$3\cos^2\alpha_i + 2\sin^2\alpha_i - 4\cos^2\alpha_i = 3\sin^2\alpha_i - 1$$

$$-\cos^2\alpha_i - 2\sin^2\alpha_i = -1 - \sin^2\alpha_i$$

Therefore, the boundary condition in Cartesian Coordinates corresponding to $Q_{ne} = 0$ becomes:

(2-11)
$$W_{,xxx}\sin\alpha_i(1+\cos^2\alpha_i(1-\nu)) + W_{,xxy}\cos\alpha_i(1-3\cos^2\alpha_i-\nu(3\sin^2\alpha_i-1))$$
$$+ W_{,xyy}\sin\alpha_i(3\sin^2\alpha_i-1-\nu(1-3\cos^2\alpha_i)) + W_{,yyy}\cos\alpha_i(-1-\sin^2\alpha_i(1-\nu))$$
$$= 0$$

4) **Elastic Support**

The boundary conditions corresponding to an elastically restrained edge are:

(2-12) $Mn = (SCR)\dfrac{\partial w}{\partial n}$

(2-13) $Q_{ne} = -(SCD)w$

Substituting for Mn & $\dfrac{\partial w}{\partial n}$ and dividing through by DP, the

first boundary condition becomes:

$$\omega_{,xx}[\sin^2\alpha_i + \upsilon\cos^2\alpha_i] + \omega_{,yy}[\cos^2\alpha_i + \upsilon\sin^2\alpha_i]$$

(2-14)
$$-2\omega_{,xy}\sin\alpha_i\cos\alpha_i(1-\upsilon) + \frac{SCR}{DP}(\omega_{,x}\sin\alpha_i - \omega_{,y}\cos\alpha_i)$$

$$= 0$$

Substituting for Q_{ne} and dividing through by DP ; the second boundary condition becomes:

(2-15)
$$\omega_{,xxx}\sin\alpha_i(1+\cos^2\alpha_i(1-\upsilon)) + \omega_{,xxy}\cos\alpha_i(1-3\cos^2\alpha_i-\upsilon(3\sin^2\alpha_i-1))$$

$$+\omega_{,xyy}\sin\alpha_i(3\sin^2\alpha_i-1-\upsilon(1-3\cos^2\alpha_i)) + \omega_{,yyy}\cos\alpha_i(-1-\sin^2\alpha_i(1-\upsilon))$$

$$+\left(\frac{SCD}{DP}\right)\omega = 0$$

5) <u>Edge of Plate Connected to Free Edgebeam</u>

The boundary conditions corresponding to this case are:

1) Twist of Beam = $M_n + Q_{ne}$ (HOR)

2) Distributed Load on Beam = Q_{ne}

We will work with the positive face of the beam when formulating these B.C's.   Now, it was previously stated that S is positive when the interior of the plate is on the left, and n is positive when pointing outward from the plate interior. Therefore, the positive face of the edge beam is as shown in Figure (16a).

a) **Equilibrium Equations** (See figure(16b))

$\sum F_z = 0$:

$$\frac{\partial F_z}{\partial S} + b_z = 0$$

or

(2-16)

$$\frac{\partial F_z}{\partial S} - Q_{ne} = 0$$

$\sum M_A = 0$:

(2-17)

$$\frac{\partial M_{nb}}{\partial S} - F_z = 0$$

$\sum T = 0$:

$$\frac{\partial M_{sb}}{\partial S} + t = 0$$

where $t = M_{np} + Q_{ne}(HOR)$

(2-18)

$$\frac{\partial M_{sb}}{\partial S} + M_{np} + Q_{ne}(HOR) = 0$$

Differentiate (2-17) and substitute (2-16):

(2-19)

$$\frac{\partial^2 M_{nb}}{\partial S^2} - Q_{ne} = 0$$

b) **Force displacement Relations:**

where, for compatibility to be
satisfied, ω(beam) = ω(plate) at
their line of intersection

(2-20) $M_{nb} = - EI_n \dfrac{\partial^2 \omega}{\partial S^2}$

From eq'ns (2) and (5):

$$(2\text{-}21) \qquad F_z = -EI_n \frac{\partial^3 w}{\partial s^3}$$

Torsional moment:

$$(2\text{-}22) \qquad M_{sb} = GJ \frac{\partial \eta}{\partial s} \qquad\qquad \text{where } \eta = \text{angle of twist of beam}$$

and

for compatibility to be satisfied

$$\eta(\text{beam}) = \frac{\partial w}{\partial n} (\text{plate}) \text{ at boundary}$$

$$(2\text{-}23) \qquad M_{sb} = GJ \frac{\partial^2 w}{\partial n \partial s}$$

where

  GJ = torsional rigidity of beam

c) <u>Formulation of Boundary Conditions</u>

  Differentiate (2-20) twice and substitute (2-19):

$$(2\text{-}24) \qquad EI_n \frac{\partial^4 w}{\partial s^4} + Q_{ne} = 0$$

where

$$\frac{\partial}{\partial s}\left(\frac{\partial^2 w}{\partial s^2}\right) = \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial s^2}\right)\frac{\partial x}{\partial s} + \frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial s^2}\right)\frac{\partial y}{\partial s}$$

$$= \cos \alpha_j \left[w_{,xxx}\cos^2\alpha_j + 2 w_{,xxy}\sin\alpha_j\cos\alpha_j + w_{,xyy}\sin^2\alpha_j\right]$$

$$+ \sin \alpha_j \left[w_{,xxy}\cos^2\alpha_j + 2 w_{,xyy}\sin\alpha_j\cos\alpha_j + w_{,yyy}\sin^2\alpha_j\right]$$

or
$$\frac{\partial^3 w}{\partial s^3} = w_{,xxx} \cos^3\alpha_j + 3w_{,xxy}\cos^2\alpha_j \sin\alpha_j + 3w_{,xyy}\sin^2\alpha_j \cos\alpha_j$$
$$+ w_{,yyy}\sin^3\alpha_j .$$

$$\frac{\partial}{\partial s}\left(\frac{\partial^3 w}{\partial s^3}\right) = \frac{\partial}{\partial x}\left(\frac{\partial^3 w}{\partial s^3}\right)\frac{\partial x}{\partial s} + \frac{\partial}{\partial y}\left(\frac{\partial^3 w}{\partial s^3}\right)\frac{\partial y}{\partial s}$$

Substituting for $\frac{\partial x}{\partial s}$, $\frac{\partial y}{\partial s}$, and $\frac{\partial^3 w}{\partial s^3}$:

$$\frac{\partial^4 w}{\partial s^4} = \cos\alpha_j \left[w_{,xxxx}\cos^3\alpha_j + 3w_{,xxxy}\cos^2\alpha_j \sin\alpha_j + 3w_{,xxyy}\sin^2\alpha_j\cos\alpha_j \right.$$
$$\left. + w_{,xyyy}\sin^3\alpha_j \right]$$

$$+ \sin\alpha_j \left[w_{,xxxy}\cos^3\alpha_j + 3w_{,xxyy}\cos^2\alpha_j \sin\alpha_j + 3w_{,xyyy}\sin^2\alpha_j \cos\alpha_j \right.$$
$$\left. + w_{,yyyy}\sin^3\alpha_j \right]$$

or
$$\frac{\partial^4 w}{\partial s^4} = w_{,xxxx}\cos^4\alpha_j + 4w_{,xxxy}\cos^3\alpha_j \sin\alpha_j + 6w_{,xxyy}\cos^2\alpha_j \sin^2\alpha_j$$
$$+ 4w_{,xyyy}\sin^3\alpha_j \cos\alpha_j + w_{,yyyy}\sin^4\alpha_j$$

Substituting for Q_{ne} and dividing through by DP, the boundary condition corresponding to (2-24) becomes:

$$w_{,xxx}\sin\alpha_j\left[1 + \cos^2\alpha_j(1-v)\right] + w_{,xxy}\cos\alpha_j\left[1 - 3\cos^2\alpha_j - v(3\sin^2\alpha_j - 1)\right]$$

(2-25)
$$+ w_{,xyy}\sin\alpha_j\left[3\sin^2\alpha_j - 1 - v(1 - 3\cos^2\alpha_j)\right] + w_{,yyy}\cos\alpha_j\left(-1 - \sin^2\alpha_j(1-v)\right)$$

$$+ EI/DP\left[w_{,xxxx}\cos^4\alpha_j + w_{,xxxy}(4\cos^3\alpha_j \sin\alpha_j) + w_{,xxyy}(6\sin^2\alpha_j \cos^2\alpha_j)\right.$$

$$\left. + w_{,xyyy}(4\cos\alpha_j \sin^3\alpha_j) + w_{,yyyy}\sin^4\alpha_j\right] = 0$$

Differentiate (2-23) and substitute (2-18):

$$(2\text{-}26) \qquad GJ \frac{\partial^3 \omega}{\partial n \partial s^2} + M_{np} + Q_{nc}(HOR) = 0$$

where

$$\frac{\partial^3 \omega}{\partial n \partial s^2} = \omega_{,xxx} \sin\alpha_j \cos^2\alpha_j + \omega_{,xxy} \cos\alpha_j (2 - 3\cos^2\alpha_j)$$
$$+ \omega_{,xyy} \sin\alpha_j (3\sin^2\alpha_j - 2) - \omega_{,yyy} \sin^2\alpha_j \cos\alpha_j$$

Substitute for $\frac{\partial^3 \omega}{\partial n \partial s^2}$ and divide through by DP to obtain boundary condition in Cartesian Coordinates corresponding to (2-26):

$$(2\text{-}27)$$
$$\omega_{,xx}[\sin^2\alpha_j + \upsilon\cos^2\alpha_j] + \omega_{,yy}[\cos^2\alpha_j + \upsilon\sin^2\alpha_j] - 2\omega_{,xy}\sin\alpha_j \cos\alpha_j (1-\upsilon)$$
$$+ \omega_{,xxx}\left[GJ/_{DP} \sin\alpha_j \cos^2\alpha_j + HOR \sin\alpha_j (1 + \cos^2\alpha_j (1-\upsilon)) \right]$$
$$+ \omega_{,xxy}\left[GJ/_{DP} \cos\alpha_j (2 - 3\cos^2\alpha_j) + (HOR)\cos\alpha_j (1 - 3\cos^2\alpha_j - \upsilon(3\sin^2\alpha_j - 1)) \right]$$
$$+ \omega_{,xyy}\left[GJ/_{DP} \sin\alpha_j (3\sin^2\alpha_j - 2) + (HOR)\sin\alpha_j (3\sin^2\alpha_j - 1 - \upsilon(1 - 3\cos^2\alpha_j)) \right]$$
$$+ \omega_{,yyy}\left[-GJ/_{DP} \sin^2\alpha_j \cos\alpha_j + (HOR)\cos\alpha_j (-1 - \sin^2\alpha_j (1-\upsilon)) \right]$$
$$= 0$$

6) Edge of Plate Connected to Simply Supported Edgebeam.

The boundary conditions for this case are:

(2-28) $\omega = 0$

(2-29) $M_{n \, (plate)}$ = Twist (beam)

where, since $\dfrac{d^2 \omega}{dS^2} = 0$ along boundary of plate :

$$M_{np} = -DP \left[\omega_{,xx} + \omega_{,yy} \right]$$ (see B.C's for simply supported edge conditions)

From free edgebeam formulation:

$$GJ \frac{\partial^3 \omega}{\partial n \partial S^2} + M_{np} + Q_{ne}(HOR) = 0$$

Second B.C. becomes (after dividing through by DP)

(2-30)

$$\omega_{,xxx} \sin \alpha_j \left[GJ/DP \cos^2 \alpha_j + (HOR)(1 + \cos^2 \alpha_j (1 - \upsilon)) \right]$$

$$+ \omega_{,xxy} \cos \alpha_j \left[GJ/DP (2 - 3\cos^2 \alpha_j) + (HOR)(1 - 3\cos^2 \alpha_j - \upsilon(3\sin^2 \alpha_j - 1)) \right]$$

$$+ \omega_{,xyy} \sin \alpha_j \left[GJ/DP (3\sin^2 \alpha_j - 2) + (HOR)(3\sin^2 \alpha_j - 1 - \upsilon(1 - 3\cos^2 \alpha_j)) \right]$$

$$+ \omega_{,yyy} \cos \alpha_j \left[-GJ/DP \sin^2 \alpha_j + (HOR)(-1 - \sin^2 \alpha_j (1 - \upsilon)) \right]$$

$$- \left[\omega_{,xx} + \omega_{,yy} \right] = 0$$

CHAPTER III

AUTOMATIC ASSEMBLAGE OF

DIFFERENCE EQUATIONS AT THE GRIDPOINTS

The assemblage of the molecules for the gridpoints on the plate can be thought of as being generated by:

a) The boundary conditions (for those points in the vicinity of the boundary.)

b) The surrounding points in the interior of the plate.

The manner in which the boundary conditions and internal gridpoints combine to form the molecule will now be described in detail.

1. Internal Points and Boundary Points

Each gridpoint in the plate is assumed to be affected by 36 other gridpoints with relative numbers shown in figure (17). The absolute number of each of these surrounding gridpoints relative to that of the central gridpoint is given in Table 6.

The computer will first classify each gridpoint in the plate as to whether it is an interior point or a boundary point. A boundary point is defined as a central gridpoint with one or more of its relative points shown in figure (17) lying off the surface of the plate. Analogously, an interior point is a central gridpoint that has all 36 of its relative points lying on the surface of the plate.

If a point is classified as interior, the standard operators shown in figure (18) will be used. The remainder of this chapter will be devoted to obtaining the finite difference molecule for a boundary point.

TABLE 6

ABSOLUTE COORDINATES OF POINTS RELATIVE TO CENTRAL GRIDPOINT

Relative Point	Absolute Coordinate	Relative Point	Absolute Coordinate
1	LP(I)	19	LP(I)-201
2	LP(1)+1	20	LP(I)-98
3	LP(I)-1	21	LP(I)+98
4	LP(I)+100	22	LP(I)+202
5	LP(I)-100	23	LP(I)-202
6	LP(I)+101	24	LP(I)+198
7	LP(I)-101	25	LP(I)-198
8	LP(I)+99	26	LP(I)+3
9	LP(I)-99	27	LP(I)-3
10	LP(I)+2	28	LP(I)+300
11	LP(I)-2	29	LP(I)-300
12	LP(I)+200	30	LP(I)+103
13	LP(I)-200	31	LP(I)-103
14	LP(I)+102	32	LP(I)+299
15	LP(I)-98	33	LP(I)-299
16	LP(I)+199	34	LP(I)+301
17	LP(I)-199	35	LP(I)-301
18	LP(I)+201	36	LP(I)+97
		37	LP(I)-97

2. <u>Boundary Conditions</u>

The method in which the boundary conditions are taken into account will now be explained.

Since the basic plate equation($\nabla^4 w = \frac{q}{DP}$) is a 4th order differencial equation, 2 boundary conditions are needed for each side. These boundary conditions (for a given side), are classified as follows:

1) <u>Simple Boundary Condition</u> - Boundary Condition with lower derivative

2) <u>Complicated Boundary Condition</u> - Boundary Condition with higher derivative

It is seen that each side will have one simple boundary condition and one complicated boundary condition.

The governing criteria as to how many boundary conditions will used in formulating the operator for a given central point, is the perpendicular distance from that point to the side of any contour. (see figure (19a)).

Since the coordinates of the corner points of each side, and the coordinates of the central point are given, d_i can be easily computed.

The magnitude of   governs just how much influence the boundary conditions will have on the operator for the central gridpoint. Four cases are considered:

(1) $d_i > 2H$:

No boundary condition is formulated here.

(2) $\sqrt{2}H < d_i < 2H$:

The simple boundary condition is formulated at the closest boundary point only. There is a maximum of 1 boundary condition for this case (figure (19b)).

(3). $H < d_i < \sqrt{2}H$:

The simple boundary condition is formulated at the closest point and at points on either side of the closest point, a distance H away. There is a maximum of 3 boundary conditions for this case. (see figure (19c)).

(4) $d_i < H$:

The simple and complicated boundary conditions are formulated at the closest point, and the simple boundary condition is formulated at points on either side of the closest point, a distance H away. There is a maximum of 4 boundary conditions for this case. (see figure (19d)).

Note that in cases (2), (3), and (4), the boundary conditions are formulated at points a, b, and c only if these points lie on the side. To illustrate, consider the central point shown in figure (20).

Now, point c_i does not fall on side i. Therefore, in assembling the operator for central point x, we would consider the simple B.C of side i at points a_i and b_i. Also, for side j, we would consider the simple B.C.'s at points a_j, b_j, and c_j, and the complicated B.C. at b_j. This results in a total of 6 B.C.'s to be considered in the formulation of the operator at point x.

c. <u>Method of Formulation of Molecules for Boundary Points.</u>

The method of determining the molecule for a boundary point can best be illustrated by the example shown in figure (21). In this the boundary conditions and distances to the sides are given below:

a) <u>BOUNDARY CONDITIONS:</u>

Side 1:   Fixed

Side 2:   Simply supported

b) <u>DISTANCES TO SIDES</u>

$d_1 = 0.62 H$

$d_2 = 0.40 H$

From the criteria of the previous section, the boundary conditions to be considered in the formulation of the $\nabla^4 \omega$ operator at point 1 are:

<u>Side 1:</u>

b_1 : $\omega = 0$

$\omega_{,x} \sin f_1 - \omega_{,y} \cos f_1 = 0$

c_1 : $\omega = 0$

<u>Side 2:</u>

b_2 : $\omega = 0$

$\omega_{,xx} + \omega_{,yy} = 0$

c_2 : $\omega = 0$

Since points Q_1 and Q_2 do not fall on sides 1 and 2 respectively, no boundary conditions are considered at these points. Therefore, 6 boundary conditions to be incorporated

into the operator for point 1.

$\nabla^4 \omega_1$ will be expressed as a linear combination of the boundary conditions, the displacements, w, at adjacent points, and if necessary, the values of $\nabla^4 \omega$ at adjacent points (called multi-point operators). Each of these quantities will be expanded in a Taylor Series about central point 1. The number of these quantities that are to be considered depends on the desired accuracy, that is, on the order of the .... non-vanishing derivative contained in the .... ion.

For the plate proble.... ...acy up to the sixth derivative is desired. Therefore, the first 21 terms in the Taylor Series expansion for ω_i about the central gridpoint 1 must be identically satisfied. In order to satisfy these 21 conditions, $\nabla^4 \omega_1$ must be expressed as a linear combination of a total of 21 boundary conditions, displacements, and multi-point operators.

In general, there will be N boundary conditions in the formulation of the operator for point 1. Therefore, we need 21-N interior conditions, in general. However, the case may arise in which there are less than 21-N points lying in the interior of the plate. For this case, the computer will procede in the following manner:

    1) Determine the expansion for the displacement at all points adjacent to central point 1 lying in the interior of the plate.

    2) <u>Multi-Point Operators</u>: Determine the expansion for $\nabla^4 \omega$ at as many points adjacent to point 1 as are needed

to bring the total number of interior conditions to 21-N. The computer first expands $\nabla^4 w$ at the adjacent point closest to 1, then at successively further points until enough interior points are considered. (We know from the plate equation that at any point m in the interior of the plate, $\nabla^4 w_m = \frac{q_m}{DP}$ where q_m is the load per unit at point m).

Now this example has a total of 6 boundary conditions. Therefore, an additional 15 interior conditions are needed in order that the error be in the sixth derivative of the expansion. However, from Fig. 21, it is seen that there are only 9 points adjacent to point 1 lying on the plate surface. Therefore, expansions for $\nabla^4 w$ at the 6 adjacent points nearest to point 1 will also have to be made. In equation (1) below, the coefficients of $A_{16} - A_{21}$ are the multi-point operators.

The equation for $\nabla^4 w_1$ can be written as:

(3-1)

$$A_1 (\text{SIMPLE B.C. AT B1}) + A_2 (\text{SIMPLE B.C. AT C1}) + A_3 (\text{COMPL. B.C. AT B1})$$

$$+ A_4 (\text{SIMPLE B.C. AT B2}) + A_5 (\text{SIMPLE B.C. AT C2}) + A_6 (\text{COMPL. B.C. AT B2})$$

$$+ A_7 w_1 + A_8 w_5 + A_9 w_7 + A_{10} w_{13} + A_{11} w_{17} + A_{12} w_{19} + A_{13} w_{29}$$

$$A_{14} w_{33} + A_{15} w_{35} + A_{16}(q_5/DP) + A_{17}(q_7/DP) + A_{18}(q_{13}/DP)$$

$$+ A_{19}(q_{17}/DP) + A_{20}(q_{19}/DP) + A_{21}(q_{29}/DP)$$

$$= \frac{\partial^4 w_1}{\partial x^4} + 2\frac{\partial^4 w_1}{\partial x^2 \partial y^2} + \frac{\partial^4 w_1}{\partial y^4} \left(= \frac{q_1}{DP}\right)$$

where A_1 ----------A_{21} are constants. Note that since the boundary conditions are homogeneous, the coefficients of $A_1, A_{2}, \cdots A_6$ in equation (3-1) above, are zero.

The left side of equation (1) is now expanded in a series about point 1 and the coefficients of like derivatives of ω_1 on each side of the equation are equated. This results in the required 21 equations necessary for the operator to be accurate up to the sixth derivative of the series expansion.

Expanding the left side, we get

(3-2)

$$A_1[\cdots\cdots] + A_2[\cdots\cdots] + \cdots\cdots$$

$$+A_{21}[\cdots\cdots] = \frac{\partial^4\omega_1}{\partial x^4} + 2\frac{\partial^4\omega_1}{\partial x^2\partial y^2} + \frac{\partial^4\omega_1}{\partial y^4}$$

which can be written in matrix form as

(3-3)

$$\begin{bmatrix} S \\ \text{(CO-EFFICIENT MATRIX)} \end{bmatrix} \begin{Bmatrix} A_1 \\ \vdots \\ \vdots \\ A_{21} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 2 \\ 0 \\ \vdots \end{Bmatrix}$$

Note that the first 6 columns of the coefficient matrix

are the Taylor Series expansions for the boundary conditions,
the next 9 columns are the expansions for the displacements
at the interior points, and the final 6 columns are the expansions
for $\nabla^4 w$ at the 6 adjacent points nearest to w.

In addition to deflections, it is also desirable to
determine the bending moments, M_{xx} and M_{yy}, and the
twisting moment, M_{xy}, at each gridpoint. This is done in the
same way as previously described for deflections. We determine
M_{xx}, M_{yy}, and M_{xy} at point 1 as a linear combination of
the boundary conditions, the displacements at adjacent grid-
points, and, if necessary, the values of $\nabla^4 w$ at adjacent grid-
points. From this it is seen that the equations for the A'S
for these 3 cases will be of the same form as for the determination
of deflections. In fact the coefficient matrix for the A'S
will be exactly the same as in equation (3-3); only the right
hand sides of the equations will be different.

In summary, we see that for each interior gridpoint, we
have 4 different sets of eq'ns for the constants A_i , 1
each for $\nabla^4 w$, $\dfrac{M_{xx}}{DP}$, $\dfrac{M_{yy}}{DP}$, and $\dfrac{M_{xy}}{DP}$. However the left
side of all 4 of these sets of equations is the same, only the
right side is changed.

In matrix form we have:

$$(3\text{-}4) \qquad [C]\{A\} = \left\{ \begin{array}{cccc} \text{(a)} & \text{(b)} & \text{(c)} & \text{(d)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \vdots & 1 & 0 & 0 \\ 0 & 0 & 0 & -(1-\nu) \\ 0 & 0 & 1 & 0 \\ \vdots & \nu & \vdots & \vdots \\ 0 & 0 & 0 & \vdots \\ 1 & 1 & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots \\ 2 & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots \\ 1 & \vdots & \vdots & \vdots \\ 0 & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{array} \right\}$$

where

(a) is the right hand side corresponding to $\nabla^4 w$.

(b) is the right hand side corresponding to M_{xx}/DP.

(c) is the right hand side corresponding to M_{yy}/DP.

(d) is the right hand side corresponding to M_{xy}/DP.

D. **Method of Obtaining Displacements and Bending Moments at Gridpoints.**

1) For boundary points, solve equation (4) for the constants A. This corresponds to the molecule for these boundary points. As previously discussed, the standard 13 point operator of figure (18a) will correspond to the molecule for the interior gridpoints.

2) Since the molecules are now known for all gridpoints, a difference equation for $\nabla^4 w_i = \frac{q_i}{DP}$ can be written at each point. This will result in a system of NP equations in NP unknown displacements, which can be solved using the methods of Chapter IV.

3) Now, for boundary points, equation (3-4) must be solved for a set of A'S corresponding to each of M_{xx}, M_{yy}, and M_{xy}. The standard operators for these moments at interior points are given in figures (18b)-(18d).

4) One of the molecules for the moments are known. The actual values of M_{xx}, M_{yy}, and M_{xy} at each gridpoint can be obtained by back substituting the displacements.

Error Analysis

It has been stated previously that the plate analysis described herein will be accurate up to the sixth derivative. This accuracy will be satisfied except for the following limitations of the method.

a) It is possible that, for a central point located very close to a corner of the plate, the 37 points given in figure (17) will not give a sufficient number of interior conditions. For example, consider central point 1 as pictures in figure (22).

Assuming that the perpendicular distance from point 1 to sides 1 and 2 is less than H in each case, we have 6 boundary conditions. Then, 15 additional interior conditions are needed. However, we see that only 6 points adjacent to point 1 lie in the interior of the plate. Thus even if we consider both the displacement expansion and many point operator at each of these interior points, we get a total of 17 conditions - while 21

are actually needed. Therefore, for the case of figure
(22), the error in the analysis will occur in the fifth
derivative.

NOTE:

The case in figure (22), is quite extreme and probably will
not occur often in practice.

b) It is possible that one or more of the columns of the
coefficient matrix $\mathcal{C}$ (in the determination of the $A'S$)
is linearly dependent on the other columns. If this occurs,
it is possible that a solution for the $A'S$ will not be
able to be found. To safeguard against this, the computer
will determine 30 columns in $\mathcal{C}$, and will consider the
first 21 linearly independent columns when solving for .
Since each column in $\mathcal{C}$ corresponds to a Taylor Series
expansion for either the boundary conditions, displacements
at interior points, or multi-point operators at the interior
points, this means that the computer is making 30-N expansions
at interior points, instead of 21-N (Where N = number of
boundary conditions).

CHAPTER IV

DETERMINATION OF FINITE DIFFERENCE EQUATIONS FOR REACTIONS AND/OR DISPLACEMENTS AT POINTS ALONG THE BOUNDARY OF THE PLATE

A. **Quantities to be Computed at Boundary**

The reactions and/or displacements at the boundary will be computed at points listed in LBPE, whose coordinates are given in XBPE and YBPE. The side on which each point is located is given by LBPSE.

The actual quantities to be computed at each boundary point depends on the boundary condition of the side on which the boundary point is located. The boundary condition for each side of the plate is given as input in MCONS(4,NS). For each boundary condition, the quantities to be computed at each point along the boundary are as follows:

1) **Fixed Edge**

   a) **Bending Moment, M_n :**

   $$M_n = -DP\left[\frac{\partial^2 w}{\partial n^2} + \nu\frac{\partial^2 w}{\partial s^2}\right]$$

   where, along the fixed edge, $\frac{\partial^2 w}{\partial s^2} = 0$.

   Final expression for M_n becomes:

   (4-1) $M_n = -DP\left[w_{,xx} + w_{,yy}\right]$

b) <u>Transverse shear,</u>

$$Q_{ns} = DP\frac{\partial}{\partial n}\left[\frac{\partial^2 \omega}{\partial n^2} + (2-\nu)\frac{\partial^2 \omega}{\partial s^2}\right]$$

Again setting $\frac{\partial^2 \omega}{\partial s^2} = 0$ along the boundary and substituting

for $\frac{\partial^3 \omega}{\partial n^3}$, the expression for transverse shear

becomes:

$$(4-2) \quad Q_{ns} = DP\left[\omega_{,xxx}\sin^3\alpha_i - 3\omega_{,xxy}\sin^2\alpha_i\cos\alpha_i + 3\omega_{,xyy}\sin\alpha_i\cos^2\alpha_i\right.$$
$$\left. - \omega_{,yyy}\cos^3\alpha_i\right]$$

2) <u>Simply Supported Edge</u>

The transverse shear, Q_{ns} , will be the only quantity computed

for this boundary condition. The expression is exactly the same

as that in the fixed edge.

3) <u>Free Edge</u>

The vertical displacement, ω , will be the only quantity

computed for this boundary condition.

4) <u>Elastically Restrained Edge</u>

(free edge with rotational spring and translational spring)

a) <u>Restrained Against Rotation</u>

The vertical displacement ω , and bending moment M_n

will be computed, where:

$$(4-3) \quad M_n = SCR\left[\omega_{,x}\sin\alpha_i - \omega_{,y}\cos\alpha_i\right]$$

b) Restrained Against Translation

The transverse shear, Q_{n_e} is to be computed and is given by:

(4-4) $Q_{n_e} = -(SCD) \omega$

5) Edge of Plate Attached to Free Edgebeam

a) Torsional Moment of Beam

$$M_{sb} = GJ \frac{\partial^2 \omega}{\partial n \partial s} = GJ \left[\frac{\partial}{\partial x} \left(\frac{\partial \omega}{\partial n} \right) \frac{\partial x}{\partial s} + \frac{\partial}{\partial y} \left(\frac{\partial \omega}{\partial n} \right) \frac{\partial y}{\partial s} \right]$$

or, upon substituting for $\frac{\partial \omega}{\partial n}$, $\frac{\partial x}{\partial s}$, and $\frac{\partial y}{\partial s}$:

(4-5) $M_{sb} = GJ \left[\omega_{,xx} \sin \alpha_i \cos \alpha_i + \omega_{,xy} (2 \sin^2 \alpha_i - 1) - \omega_{,yy} \sin \alpha_i \cos \alpha_i \right]$

b) Bending Moment of Beam:

$$M_{nb} = -EI_n \frac{\partial^2 \omega}{\partial s^2}$$

or upon substitution:

(4-6) $M_{nb} = -EI_n \left[\omega_{,xx} \cos^2 \alpha_i + 2 \omega_{,xy} \sin \alpha_i \cos \alpha_i + \omega_{,yy} \sin^2 \alpha_i \right]$

c) Shear of Beam:

$$F_z = -EI_n \frac{\partial^3 \omega}{\partial s^3}$$

(4-7) $\quad F_z = -EI_\eta \left[\omega,_{xxx} \cos^3\alpha_i + 3\omega,_{xxy} \cos^2\alpha_i \sin\alpha_i + 3\omega,_{xyy} \sin^2\alpha_i \cos\alpha_i \right.$
$$\left. + \omega,_{yyy} \sin^3\alpha_i \right]$$

    d)   compute <u>transverse deflection</u>, ω.

6) <u>Edge of Plate Connected to Simply-Supported Edgebeam</u>

    a) <u>Torsional moment of beam</u>:

(4-8) $\quad M_{sb} = GJ \left[\omega,_{xx} \sin\alpha_i \cos\alpha_i + \omega,_{xy}(2\sin^2\alpha_i - 1) - \omega,_{yy} \sin\alpha_i \cos\alpha_i \right]$

    b) Shear/unit width of plate = Q_{np}.

(Distributed load transmitted from plate to edge beam)

(4-9) $\quad Q_{np} = DP \left[\omega,_{xxx} \sin^3\alpha_i - 3\omega,_{xxy} \sin^2\alpha_i \cos\alpha_i + 3\omega,_{xyy} \sin\alpha_i \cos^2\alpha_i \right.$
$$\left. - \omega,_{yyy} \cos^3\alpha_i \right]$$

B. <u>Method of Computation of Boundary Quantities</u>

This can best be illustrated by an example. Consider point I in Figure 23, which lies along a fixed side J.

Since side J is fixed, the bending moment, M_n, and the transverse shear, Q_{ne}, will be computed at point I. This is done in a manner similar that employed in analyzing the interior gridpoints in Chapter III.

First, the force displacement relations are utilized to express the moment and shear at point I as a function of derivatives of the displacement at I.

a) $M_n = -DP[\omega_{,xx} + \omega_{,yy}]$

b) $Q_n = DP[\omega_{,xxx} \sin^3\alpha_i - 3\omega_{,xxy} \sin^2\alpha_i \cos\alpha_i + 3\omega_{,xyy} \sin\alpha_i \cos^2\alpha_i$
$- \omega_{,yyy} \cos^3\alpha_i]$

Q_n and M_n at point I will be determined as a linear combination of the boundary conditions, the displacements, ω, at the neighboring interior gridpoints, and, if necessary, the multi-point operators, ($\nabla^4\omega$), at as many interior gridpoints as are needed to give the desired accuracy. The boundary conditions, displacements, and multi-point operators are then expanded in a Taylor Series <u>about point</u> <u>I</u>, and coefficients of like derivatives of ω_I on each side of the equations for Q_n & M_n are equated.

To be consistent with the formulation of Chapter III, final accuracy is required up to the sixth derivative of the Taylor Series expansions.

a) Boundary Conditions:

For a fixed edge, the boundary conditions are:

Simple boundary condition: $\omega = 0$

Complicated boundary condition: $\omega_{,x} \sin \alpha_i - \omega_{,y} \cos \alpha_i = 0$

Now, for all points on all sides, the following
B.C.'s will be considered:

1) Simple boundary condition at point I.

2) Simple boundary condition at point A, a distance
   above point I on side J

3) Simple boundary condition at point C, a distance
   below point I on side J.

4) Complicated boundary condition at point I.

However, inasmuch as point A lies off of side J, as shown
in figure (23), we cannot use the boundary condition at
this point. Therefore, we have 3 boundary conditions, and
need 18 additional interior conditions to result in the
total of 21 required for the desired accuracy.

b) Interior Conditions

The numbering of the points shown in figure (23)
is determined by choosing the central gridpoint (relative
number = 1) as that interior gridpoint closest to the
boundary point I. The other gridpoints to be considered
are those with the relative numbering given in figure (17)
which lie in the interior of the plate.

From figure (23), it is seen that a total of 9 of
the gridpoints of figure (17) lie in the plate. Therefore
expansions are needed for both ω and $\nabla^4 \omega$ at all 9
of these points in order to result in the required 18

interior conditions. These expansions will be in terms
of the displacement at point I and all derivatives of the
displacement at I up to the sixth derivative. It can be
seen that, in general, the difference in X and y
coordinates between point I and the various internal
gridpoints (ΔX and Δy) will not be integral multiples
of the grid size H , as was the case in Chapter III.

The expansion for the moment at point I can be written
as follows:

$$A_1[\omega \text{ AT } I] + A_2[\omega \text{ AT } C] + A_3[(\omega_{,x} \sin\alpha_L - \omega_{,y} \cos\alpha_L) \text{ AT } I]$$

$$+ A_4 \omega_1 + A_5 \omega_5 + A_6 \omega_7 + A_7 \omega_{13} + A_8 \omega_{17} + A_9 \omega_{19} + A_{10} \omega_{29}$$

(4-10) $$+ A_{11} \omega_{33} + A_{12} \omega_{35} + A_{13}(q_1/DP) + A_{14}(q_5/DP) + A_{15}(q_7/DP)$$

$$+ A_{18}(q_{19}/DP) + A_{19}(q_{29}/DP) + A_{20}(q_{33}/DP) + A_{21}(q_{35}/DP)$$

$$= -DP[\omega_{I,xx} + \omega_{I,yy}]$$

where, since the boundary conditions are homogeneous, the
coefficients of A_1, A_2, and A_3 are zero. Also note
that the coefficients of $A_{13} - A_{21}$ are multi-point operators.
A similar expression can be written for the transverse shear
force.

As in Chapter III, the left side of equation (4-10)
is expanded in a Taylor Series and coefficients of like

derivatives of the displacement at I on the left and right

sides are equated. From this it is seen that the left

side of the matrix equation will have exactly the same

coefficient matrix for both the moment and the shear - and

only the right side of the equation will be different.

In matrix form, the series of equations correspond-

ing to equating coefficients of like displacement derivatives

can be written in the following form:

$$
(4\text{-}11) \qquad [\underset{\sim}{C}]\{\underset{\sim}{A}\} =
\begin{Bmatrix}
(a) & (b) \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 0 \\
1 & 0 \\
0 & \sin^3 \alpha_j \\
0 & -3\sin^2 \alpha_j \cos \alpha_j \\
0 & 3\sin \alpha_j \cos^2 \alpha_j \\
0 & -\cos^3 \alpha_j \\
0 & 0 \\
\vdots & \vdots \\
0 & 0
\end{Bmatrix}
$$

in which column (a) is the right hand side corresponding

to M_n , and column (b) is the right hand side

corresponding to Q_{ne} .

The coefficient matrix $\underset{\sim}{C}$ in equation (4-11) can

now be inverted in order to determine the A's for each

right hand side. Once the A's are known, the bending

moment and effective shear at point I can be written

as:

$$M_n \text{ AT } I = \overset{\text{displ.}}{\underset{i}{\sum}} A_i \omega_i + C_1$$

(4-12)
$$Q_{ne} \text{ AT } I = \overset{\text{displ.}}{\underset{j}{\sum}} A_j \omega_j + C_2$$

where C_1 and C_2 are the effects of the multi-point operators, and, once the A's are determined, are known constants.

(4-13)
$$A_{13}\left[\frac{q_1}{DP}\right] + A_{14}\left[\frac{q_5}{DP}\right] + A_{15}\left[\frac{q_7}{DP}\right] + A_{16}\left[\frac{q_{13}}{DP}\right]$$
$$+ A_{17}\left[\frac{q_{17}}{DP}\right] + A_{18}\left[\frac{q_{18}}{DP}\right] + A_{19}\left[\frac{q_{20}}{DP}\right] + A_{20}\left[\frac{q_{33}}{DP}\right]$$
$$+ A_{21}\left[\frac{q_{35}}{DP}\right]$$

The A's in equation (4-13) correspond to the first hand side in equation (4-11). The expression for C_2 will be of the same form as (4-13), except that the A's will correspond to the second right hand side in (4-11).

It is obvious that as soon as the deflections at the interior gridpoints are determined, the moment and shear at point I can be computed directly. This will be demonstrated in Chapter IV.

As in Chapter III, it is desirable to have more than 21 columns in C in case any of the columns are linearly dependent. In fact, the computer will calculate a maximum of 30 columns for $\underset{\sim}{C}$, and will consider the first 21 linearly independent

columns in computing the A's (as was done in Chapter II).

For this particular example, there is a maximum of 21 columns

in $\underset{\sim}{C}$. Therefore, if any of these columns are linearly

dependent, the error will result in the fifth derivative, rather

than the sixth.

Finally, it is interesting to note that as the boundary

point approaches a corner, this plate analysis will give

results that are somewhat inconsistent with experimental

results. This is because of the fact that the standard plate

theory predicts a concentrated corner force, while a more

gradual increase in reaction intensity with decreasing

distance to the corner actually occurs.

CHAPTER V

SOLUTION, BACKSUBSTITUTION AND OUTPUT FOR PLATE ANALYSIS

A. Solution of System of Finite Difference Equations

The large system of equations involving the displacements at each gridpoint as the unknowns will be solved using a Gauss elimination technique on the partitioned band-matrix. This large system of equations is given by RDE(I) in the flowchart for Link II. A detailed description of this method of solution now follows:

1) Band Matrix Character of System of Equations.

In part I, it was stated that, in the solution of the large system of equations, the gridpoints would be rearranged parallel to the smallest side of the rectangle enclosing the plate (upward & to the left). The reason for this will now be explained.

Consider the plate shown in figure (24), and assume we are writing the operator for point I in the interior of the plate.

It is seen that the shaded portion of figure (24) corresponds to the maximum zone of influence of point I; that is, the gridpoints contained in this shaded region could be included in the difference operator for point I, but all gridpoints outside of the shaded region will definitely not be

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included in the operator for point I.

Now, let us consider the form of the difference operator for point I. It was shown in Part I, that a maximum of MDIM gridpoints can be placed along the plate parallel to the smaller sides of the enclosing rectangle. Therefore, in figure (24), a maximum of MDIM gridpoints can be placed along the distances IA+BC, DE+FG, and HJ+KL, and 2 additional gridpoints can be placed along LN. Since the gridpoints are to be listed in the final system of equations from IA-BC-DE--FG-HJ-KL-LN, we see that there will be a maximum number of 3MDIM+2 gridpoints to the right of point I in the final difference eq'ns. Due to the symmetry of the shaded area of figure (13), there will also be a maximum total of 3MDIM+2 gridpoints to the left of point I, resulting in a total of 6MDIM+4 terms in the difference equation for point I. Therefore, we see that the final set for equations will be in the form of a band matrix, with the maximum width of the band = 6MDIM+4. See figure (25) for an illustration of the band matrix character of the difference equations.

Now, it is seen that by listing the gridpoints parallel to the shortest side of the enclosing rectangle, we have made the final system of equations into a band matrix, and have minimized the width of the band. This will greatly diminish the complexity of solving this large system of simultaneous equations.

Observe that within the above band there will originally
be 3 corridors of zeros (corresponding to regions AA'+BB',
DE'=FG', and H'J+KL in figure (24)). However, since these
corridors fill up once the Gauss Elimination procedure begins,
they cannot be utilized to shorten the amount of computation
required in solving the equations.

2) Gauss Elimination Procedure in Solving a Partitioned

Band Matrix

Due to the extremely large size of the system, it is
necessary to partition the resulting band matrix. A detailed
discussion of the mathematics involved is given in reference
(4).

B. Output for Interior Gridpoints (for each loading condition)

As previously described, the computer will first solve
the large system of equations for the displacements at the
interior points.

After determining the displacements, the moments M_{xx},
M_{yy} and M_{xy} at each gridpoint can be computed by back sub-
stitution into the appropriate difference equations. These
moments will enable the engineer to determine the state of
stress at any point in the plate.

Finally, the principal moments M_1 and M_2 and the angle
the maximum principal moment makes with the positive X axis,

ϕ, can be determined at any point on the surface of the
plate. At any point on the plate, the stresses are given by:

$$\sigma_x = \pm \frac{6M_{xx}}{t^2} \ ; \quad \sigma_y = \pm \frac{6M_{yy}}{t^2} \ ; \quad \tau_{xy} = \pm \frac{6M_{xy}}{t^2}$$

Since t (plate thickness) is constant, the bending and
twisting moments at any point on the plate are constant multiples
of the normal and shear stresses respectively at that point on
the plate surface.

From this, the Mohr Circle Theory for combined stresses
at a point on an elastic body can be readily extended to a
consider a point of "combined moment" at a point on a plate,
and can be utilized to determine the principal moments at
that point on the plate.

. It is now desirable to determine a sign convention for the
Mohr Circle for moments. The positive direction for moments
that has been used in the plate analysis is shown in figure
(3). These moments will be assumed to be represented by the
Mohr Circle of figure (26a), and by the moment vectors of
figure (26b).

The positive moment vector sign convention of Fig. (26b)
corresponds to the directions of the stresses on the bottom
fiber of the plate, caused by the moments shown in Fig. (3).

From Fig. (26b), the following equations for principal moments are obtained:

$$M_1 = \frac{M_{xx} + M_{yy}}{2} + \left[\left(\frac{M_{xx} - M_{yy}}{2}\right)^2 + M_{xy}^2\right]^{1/2}$$

$$M_2 = \frac{M_{xx} + M_{yy}}{2} - \left[\left(\frac{M_{xx} - M_{yy}}{2}\right)^2 + M_{xy}^2\right]^{1/2}$$

Now, from Fig. (26c), we define the angle ϕ as the angle that M_1 makes with the positive x axis. ϕ is computed from:

$$M_{xx} > M_{yy}: \qquad \phi = \frac{1}{2} TAN^{-1}\left[\frac{2M_{xy}}{M_{xx} - M_{yy}}\right]$$

$$M_{yy} > M_{xx}: \qquad \phi = \frac{1}{2} TAN^{-1}\left[\frac{2M_{xy}}{M_{yy} - M_{xx}}\right]$$

where ϕ is positive if clockwise.

For interior gridpoints the output will have the following

form:

LOADING CONDITION 1

COORDINATE	DISPL. (In)	MXX (lb-$\frac{in}{in}$) M	MYY ($\frac{lb-in}{in}$)	MXY ($\frac{lb-in}{in}$)	M1 ($\frac{lb-in}{in}$)	M2 ($\frac{lb-in}{in}$)	PHI (RAD)
0103	xxx	xxx	xxx	xxx	xxx	xxx	xxx
xxxx	xxx	xxx	xxx	xxx	xxx	xxx	xxx

LOADING CONDITION 2

COORDINATE	DISP. (IN)	MXX (lb-in/in)	MYY ($\frac{lb-in}{rn}$)	MXY ($\frac{lb-in}{in}$)	M1 ($\frac{lb-in}{in}$)	M2 ($\frac{lb-in}{in}$)	PHI (RAD)
0103	xxx	xxx	xxx	xxx	xxx	xxx	xxx
xxxx	xxx	xxx	xxx	xxx	xxx	xxx	xxx

b) Boundary Points (for each loading condition)

Having determined the displacements at each interior

gridpoint, the required reactions and/or displacements at

each boundary point can be determined by back substitution

into RBP1(I), RBP2(I), RBP3(I), RBP4(I). Since the output

for each point depends on the boundary condition of the side

where the point is located, the output will be of the following

form:

SIDE 15   FIXED EDGE

COORDINATE	(lb/in)	MN (1n-1n/in)
XXX	XXX	XXX
XXX	XXX	XXX

SIDE 16   FREE EDGE BEAM

COORDINATE	MSB (lb-in)	MNB (lb-in)	FZ (lb)	DISPL (in)
XXX	XXX	XXX	XXX	XXX
XXX	XXX	XXX	XXX	XXX

D. Computer Programming of the Plate Analysis

In addition to the formulation of the plate analysis,
it was also attempted to program the Input Phase and
Assemblage of Molecules Phase of the problem on an I.B.M.
7040 computer. The results of this programming effort will
now be described in detail.

PART I - INPUT, SORTING, AND PRELIMINARY CALCULATIONS

This phase of the programming effort is as described in
Chapter I. The flowchart for this link is given in figure (27-1)
and the source program for this link is given in Section IX-A.

It is important to note that this link, termed LINK1, makes
use of subroutine MATCH, a previously developed program, in
converting the boundary condition input (in MCONS) to numerical
data. Therefore, it is important to include a binary deck of
MATCH when loading LINK1 on the computer.

A feature of LINK1 is that it prints out all data
previously fed in as input. This enables the user to check
his data for numerical mistakes.

PART 2 - AUTOMATIC ASSEMBLAGE OF DIFFERENCE EQUATIONS

Originally, it was intended to have this portion of the
program assembled as one link, the flowchart for which is
given in figure (27-2). However, early programming attempts
revealed that this original link far exceeded the storage
capacity of the computer. Therefore, it was decided to break
the link given in figure (27-2) into two separate links.

The first of these new links, termed LINK2,, will perform
the following calculations:

a) The computer first classifies each gridpoint on the
plate as to whether it is a boundary point or an interior
point. As described in Chapter II, the standard operators
can be used for interior points, while new molecules will
have to be formulated for the boundary points.

b) The perpendicular distance from the gridpoint to each
side in the plate is then computed.

c) The computer then assembles that portion of the
matrix corresponding to the series expansions for the
boundary conditions of sides less than 2H away from the
gridpoint. It is assumed that each gridpoint has up to
8 boundary condition expansions, The total number of
these boundary condition expansions, NBC, is also
computed.

d) Finally, that portion of the AW matrix corresponding
to the boundary conditions and the number of boundary
conditions, NBC, is stored on tape.

e) In this link, if sense switch 5 is down, the computer
will print out the distance from the gridpoint to each
side, the number of boundary conditions, NBC, and the
first 8 columns of the   matrix (containing the
boundary condition expansions.) If sense switch 5 is
left up, this printout will be deleted.

The other portion of Part 2, termed LINK3 in the computer,
performs the remaining calculations involved in assembling
the gridpoint molecules as follows:

a) The computer assembles the standard molecules for all
interior gridpoints.

b) For boundary points, the portion of the AW matrix
corresponding to the boundary conditions is read in from
the tape on which it was stored. The remaining displace-
ment and multi-point operator expansions necessary to fill
30 columns in the AW matrix are then carried out.

c) The right hand sides corresponding to $\nabla^4 w$, M_{xx}, M_{yy},
and M_{xy} are then filled in.

8   d) The AW matrix is inverted to obtain the molecules for
$\nabla^4 w$, M_{xx}, M_{yy}, and M_{xy} . These molecules are then
stored on tape.

e) Steps (b) - (d) are repeated for each boundary point.

f) Finally, the band matrix corresponding to the calculation
of the gridpoint deflections is assembled and stored. Due
to the large size of this matrix, it is necessary to store
it on 2 separate tapes.

g) In LINK3, if sense switch 6 is down, the computer
will print the complete AW matrix prior to its inversion.
If sense switch 6 is left up, this printout will not
occur.

The output for Part 2 consists of a listing of the molecules
for $\nabla^4 w$, M_{xx}, M_{yy}, and M_{xy} for each gridpoint in the plate.
This program also computes and prints out the effective load for
each loading condition to which the plate is subjected. The
effective load at point I is computed as:

$$q_{eff}(I) = q(I) - \sum_{\substack{multi-\\point\\operators}} A_j (\nabla^4 w)_j$$

The source programs for LINK2 and LINK3 are given in Section
IX-D.

It remains to program the portion of the plate analysis
corresponding to the computation of reactions and displacements
along the boundary, and that portion of the analysis that
inverts the band matrix and back substitutes to obtain the
moments at each gridpoint. However, the flowcharts corresponding
to these phases of the analysis have been worked out and are
given in figures (27-3) and (27-4).

E. Conclusions

The following conclusions and suggestions for future work can now be stated as follows:

1) The finite difference plate analysis presents a far more accurate method of obtaining molecules for gridpoints in the vicinity of the boundary than does the standard method. The actual criteria for distance from the gridpoint to each side used in this particular analysis is somewhat arbitrary; however the method itself is systematic and theoretically sound.

2) The generality of plate shapes, loadings, and boundary conditions that can be handled by this method of analysis should render it invaluable as a research and analysis tool. Therefore, it is strongly recommended that the programming of this plate analysis, begun as a part of this thesis, be continued to completion.

VI

FIGURES

SIMPLY SUPPORTED EDGE

AT POINT 3:

$$w_{,xx} = \frac{w_{11} - 2w_3 + w_0}{H^2} = 0$$

$$\therefore w_{11} = 2w_3 - w_0$$
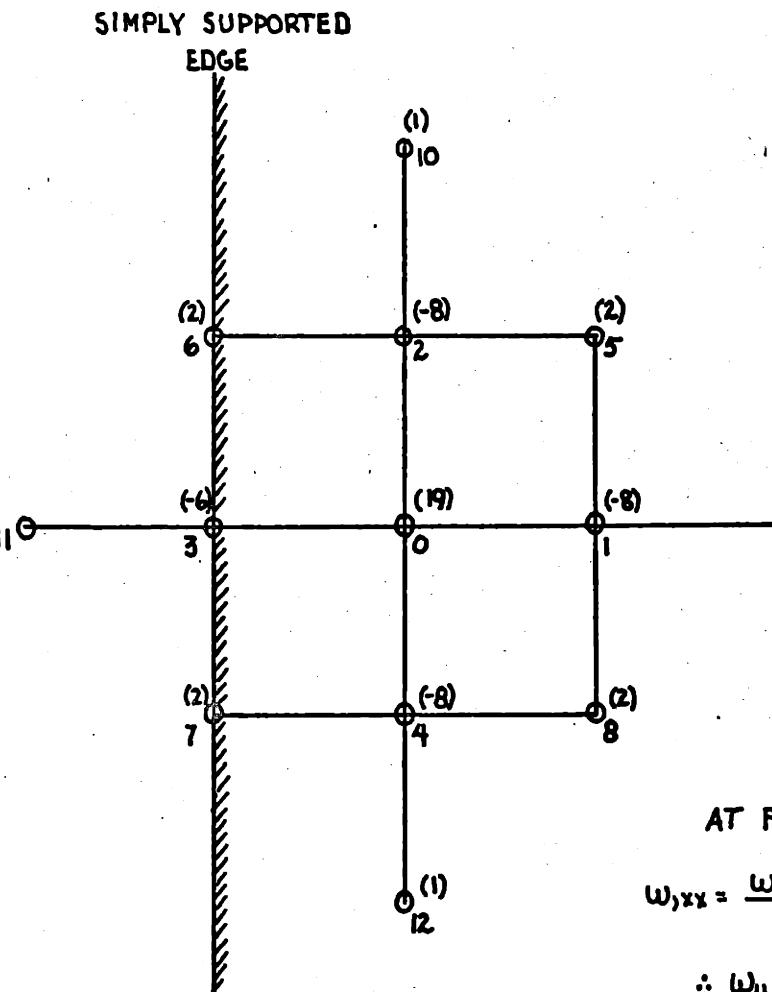
FIGURE (1)
EXAMPLE PROBLEM
ILLUSTRATING INACCURACIES OF
STANDARD DIFFERENCE METHOD
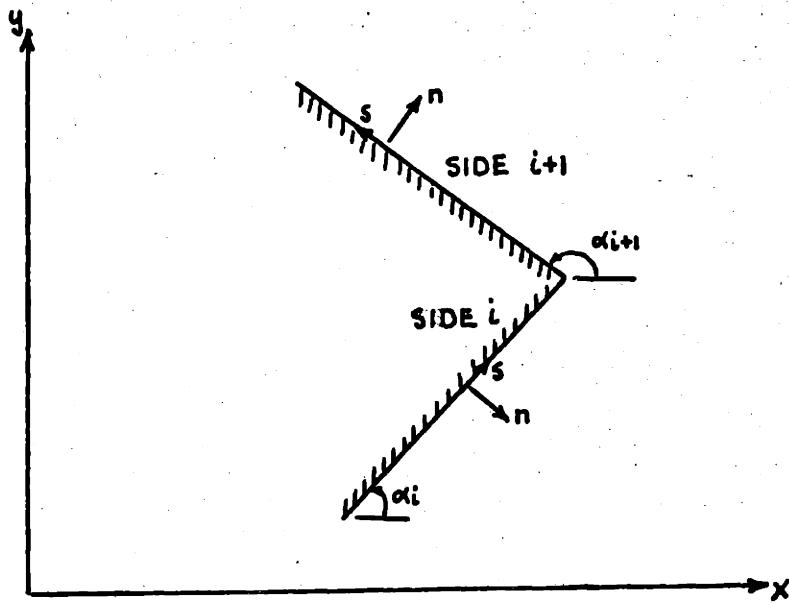
a) POSITIVE DIRECTION
   FOR EXTERIOR CONTOUR
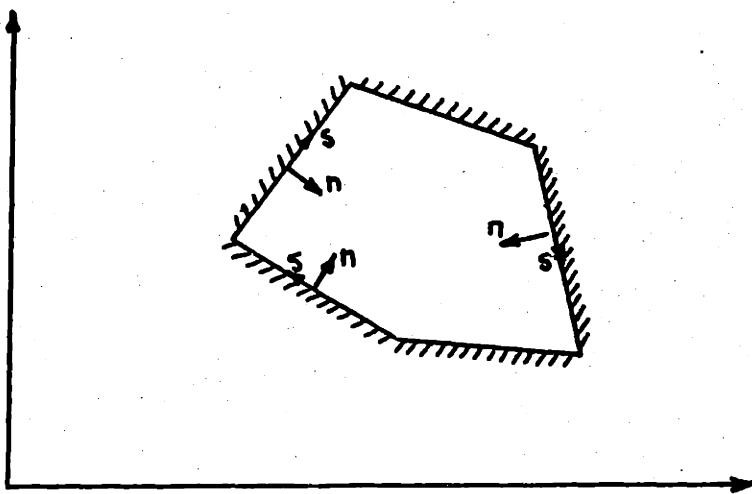
b) POSITIVE DIRECTION
   FOR INTERIOR CONTOUR

FIGURE 2

ASSUMED POSITIVE DIRECTIONS
FOR PLATE CONTOURS
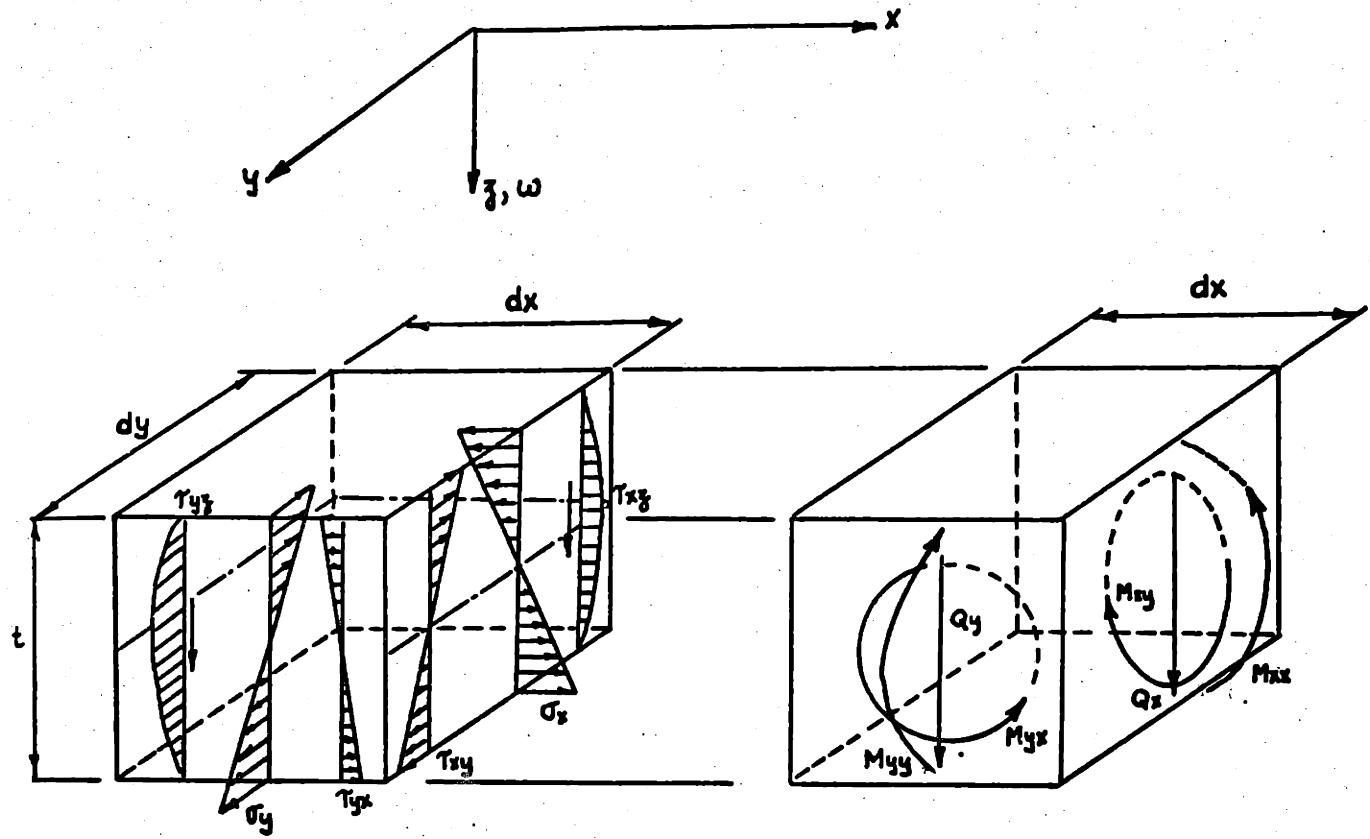
FIGURE 3

POSITIVE DIRECTION FOR
MOMENTS, SHEARS, & DISPLACEMENTS
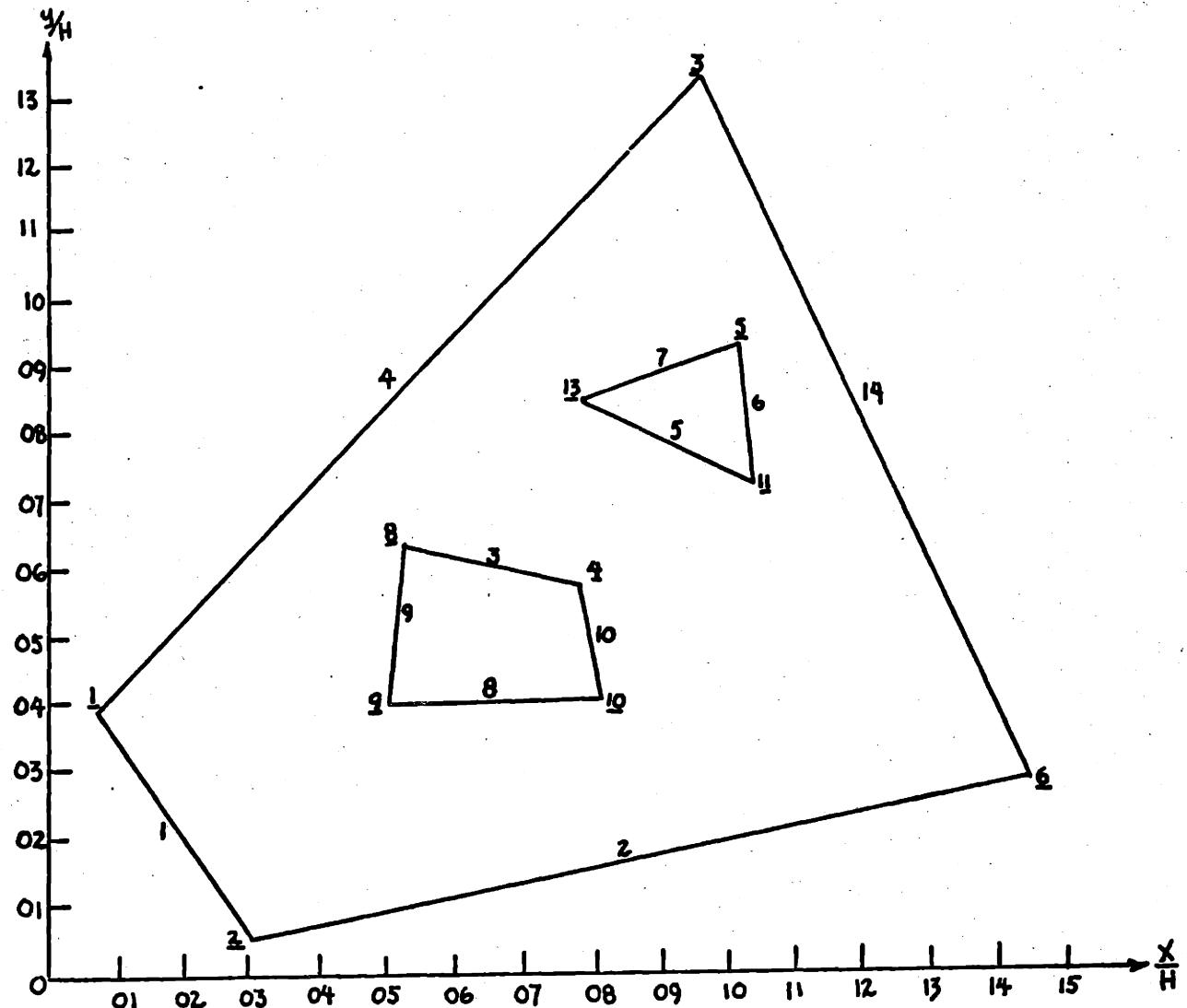
FIGURE 4

EXAMPLE PLATE TO

EXPLAIN INPUT —

SUPERPLATE

col 2

PLATE ANALYSIS
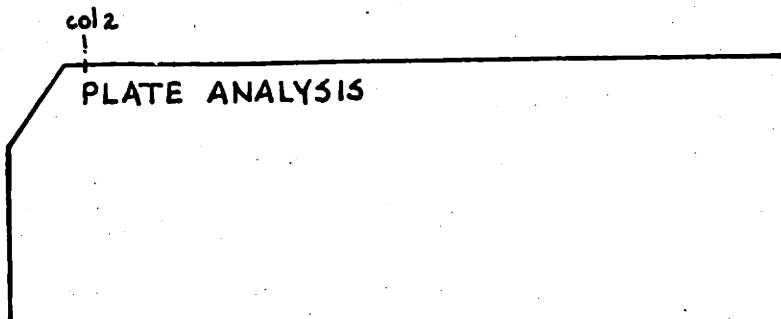
FIGURE (5a)

NAME OF DECK CARD

col 2

EXAMPLE THESIS PROBLEM, SUPERPLATE

FIGURE (5b)

TITLE CARD

col 2 5   19   25   30 33   40 43 48   52 55 60

DP = 594000000!   POIS = 0.3   NS = 8   NC = 2
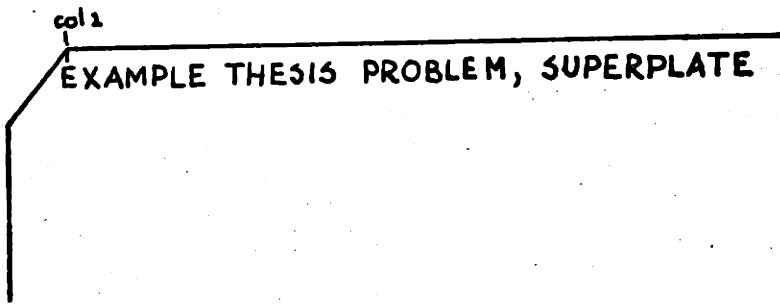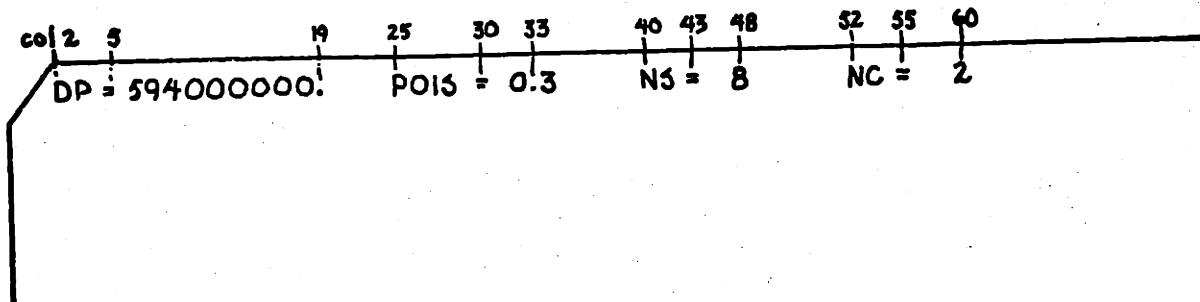
FIGURE (6)

PLATE CONSTANTS

col 1   6   10   15 17 20   25

SIDE 1   FROM 4 TO 2   FIXED
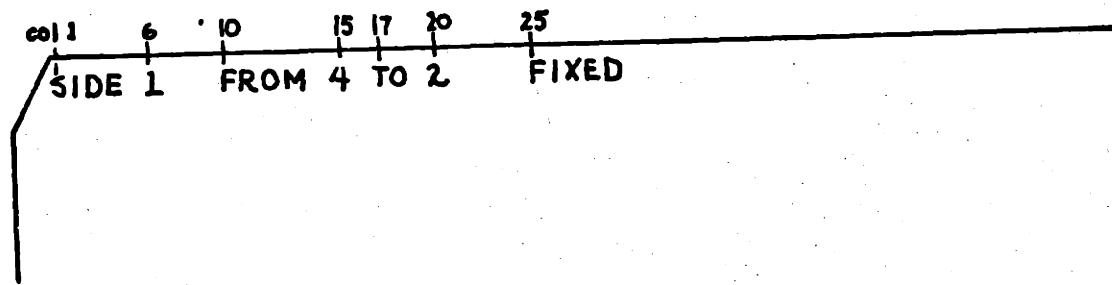
FIGURE (7)

DATA CARD FOR

CONNECTIVITY MATRIX

col 1   11   15 16   23   32 33   41

POINT   9   X =   30.0000   Y=   24.0000
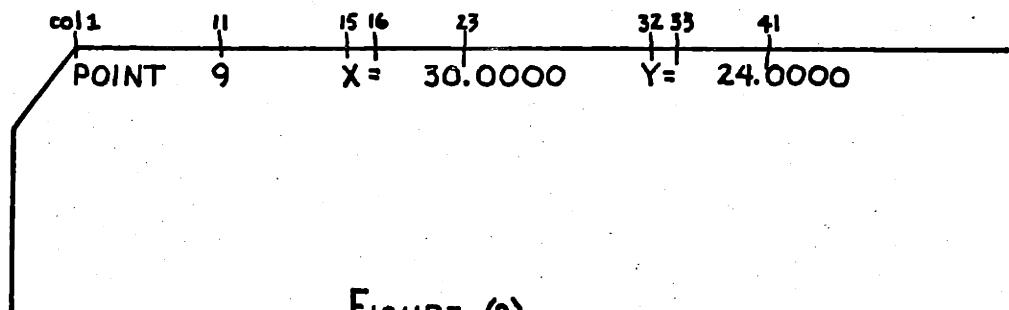
FIGURE (8)

CORNER POINT DATA CARD
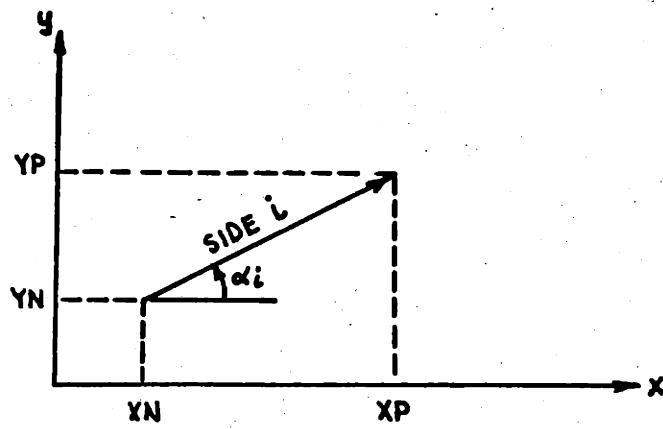
FIGURE (9)

SKETCH DEFINING α_i, XN, YN, XP, & YP

col 2 7 11

NSAI = 3
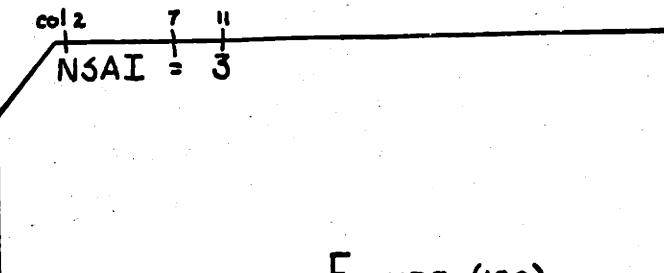
FIGURE (10a)

INITIAL CARD FOR BOUNDARY CONDITION INPUT

col 2 6 14 23 27 35

SCR = 5000.000 SCD = 2500.000

col 1 9

SIDE 4
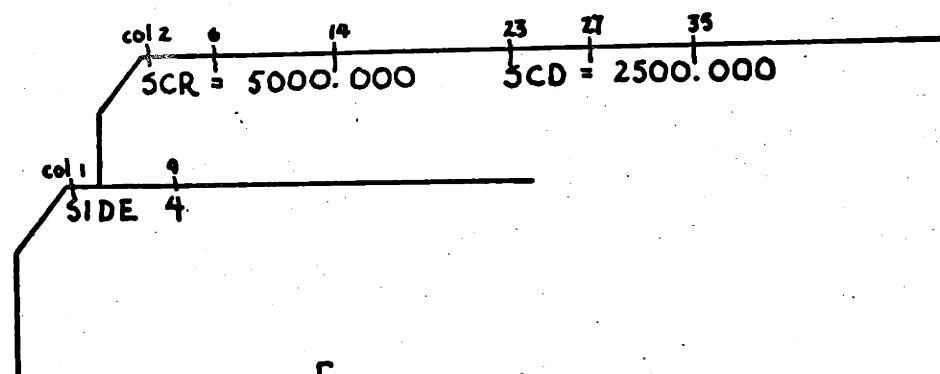
FIGURE (10b)

SEQUENCE OF CARDS FOR ELASTICALLY RESTRAINED EDGE
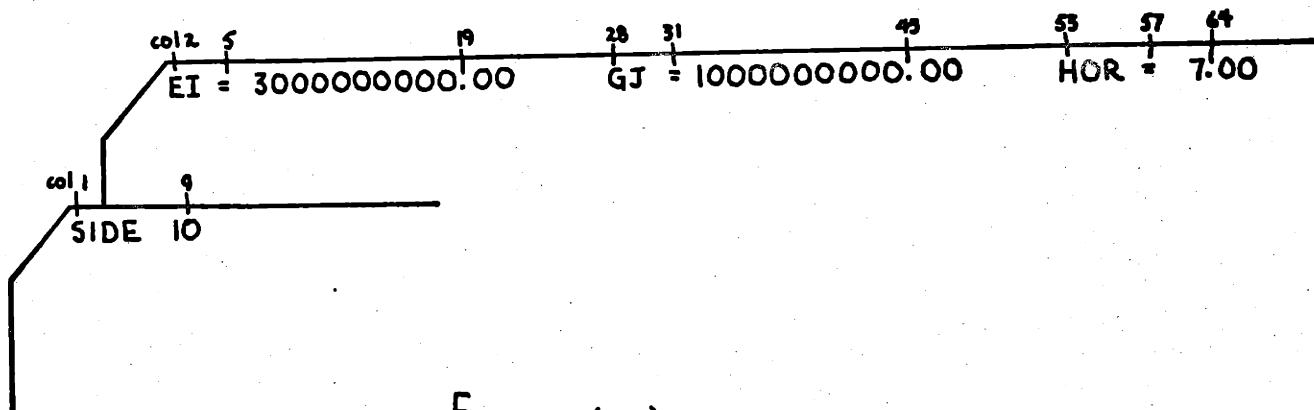
col 2 5 19 28 31 45 53 57 64

EI = 3000000000.00 GJ = 1000000000.00 HOR = 7.00

col 1 9

SIDE 10

FIGURE (10c)

SEQUENCE OF CARDS FOR EDGE OF PLATE

CONNECTED TO FREE EDGEBEAM

col 2 5   19   27   31 38
GJ = 1000000000.00   HOR = 7.00

col 1   9
SIDE 12

FIGURE (10c)
SEQUENCE OF CARDS FOR EDGE OF PLATE
CONNECTED TO SIMPLY SUPPORTED EDGEBEAM

GJ = 15000000000.   HOR = 10.25

SIDE 7

SCR = 10000.   SCD = 5550.28

SIDE 6

EI = 3000000000.   GJ = 1000000000.   HOR = 7.0

SIDE 9

SCR = 5000.   SCD = 2500.

SIDE 2

NSAI = 4
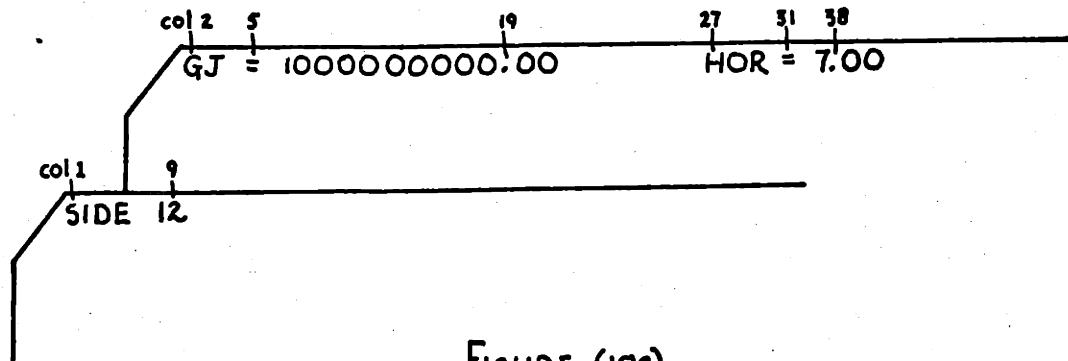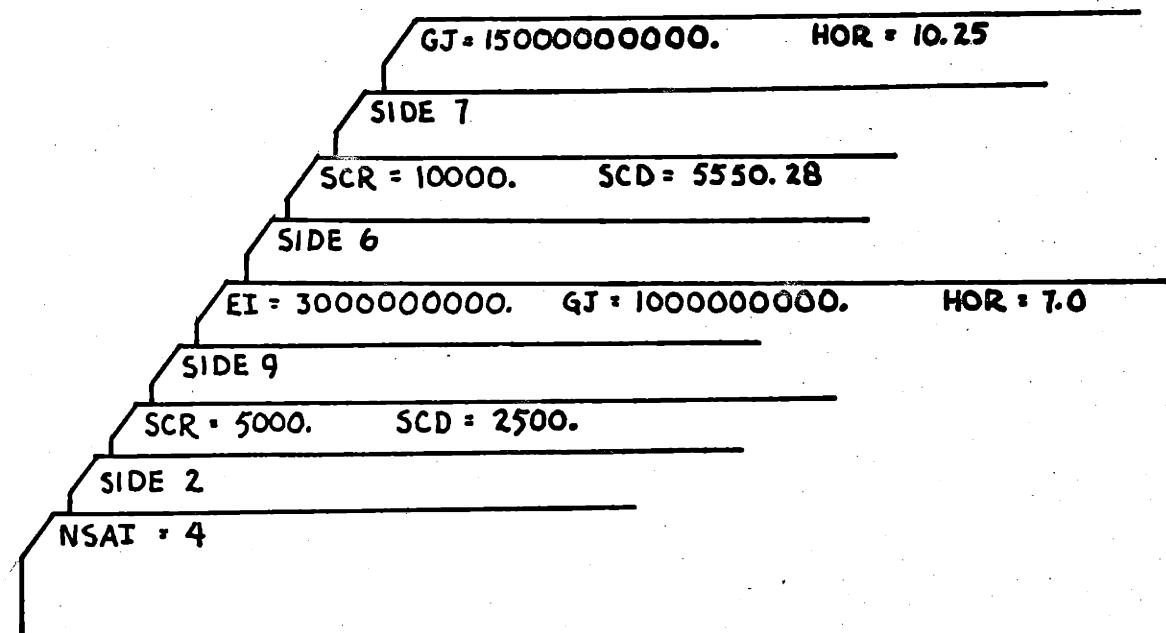
FIGURE (10d)
SEQUENCE OF ADDITIONAL INFORMATION
CARDS FOR SUPERPLATE

col 1   3   7   11 13 15   19 20 24

NP = 86   NL = 3   H = 6.2750

FIGURE (11a)
FIRST INTERNAL GRIDPOINT CARD

col 6   14   26   38

0203   50.3752 25.4800   72.0000

FIGURE (11b)
TYPICAL GRIDPOINT CARD
(ONE FOR EACH GRIDPOINT)
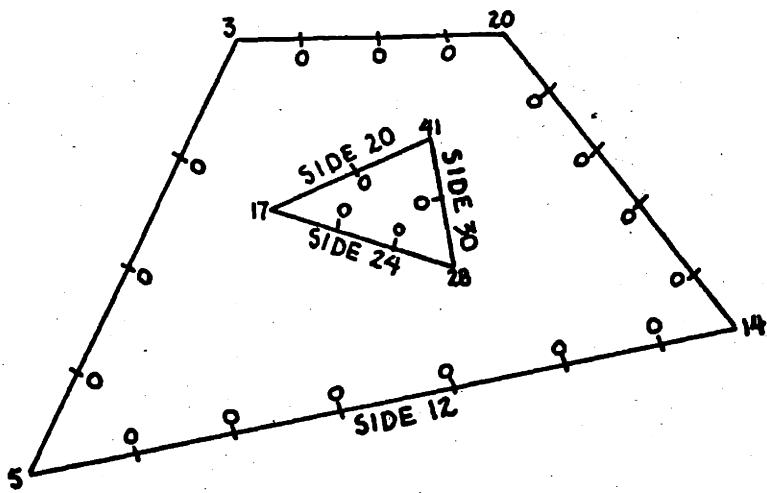
FIGURE (12)
NUMBERING OF BOUNDARY
POINTS AS GENERATED BY COMPUTER

PROBLEM SPECIFIED

FIGURE (13)
PROBLEM SPECIFIED CARD

$$\text{SIN } \alpha_i = \frac{dx}{d\eta}$$

$$\cos \alpha_i = -\frac{dy}{d\eta}$$
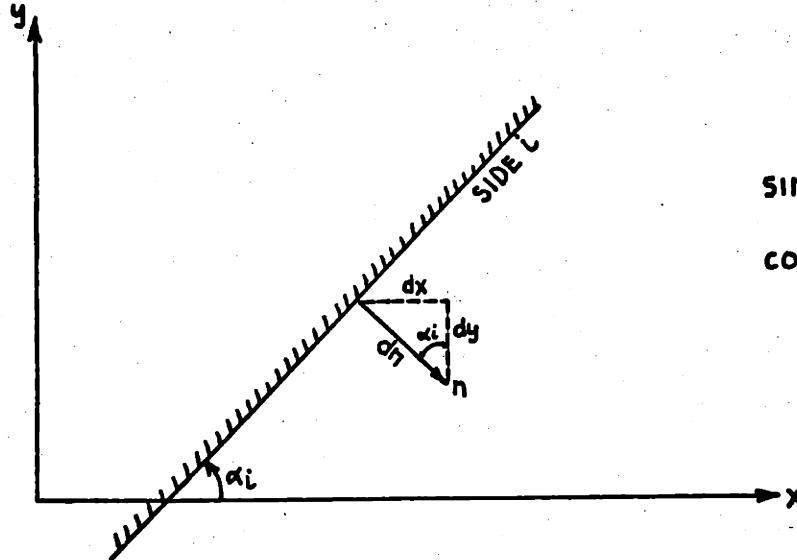
FIGURE (14a)
ORIENTATION OF SIDE i AS
DEFINED BY NORMAL DERIVATIVE

$$\sin \alpha_i = \frac{dy}{ds}$$

$$\cos \alpha_i = \frac{dx}{ds}$$
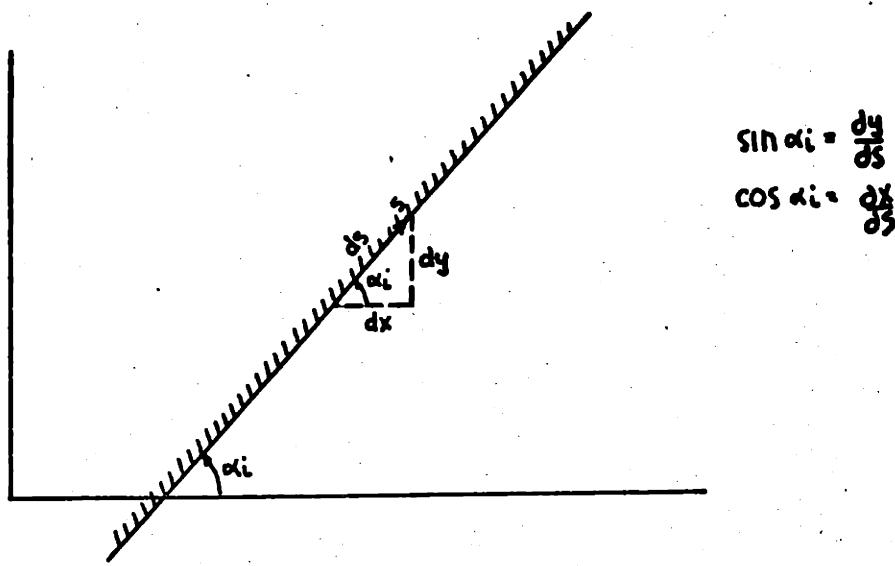
FIGURE (14b)
ORIENTATION OF SIDE i AS
DEFINED BY TANGENT DERIVATIVE

SCD	SCR	
∞	∞	
∞	0	
0	∞	
0	0	
finite	0	
finite	∞	
0	finite	
∞	finite	
finite	finite	
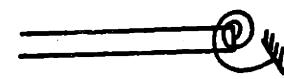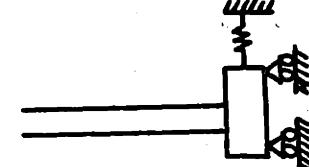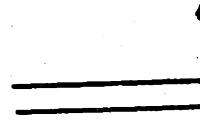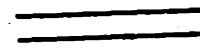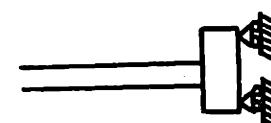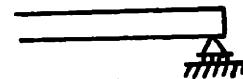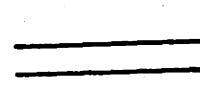
FIGURE (15)
LIMITING VALUES OF SPRING CONSTANTS
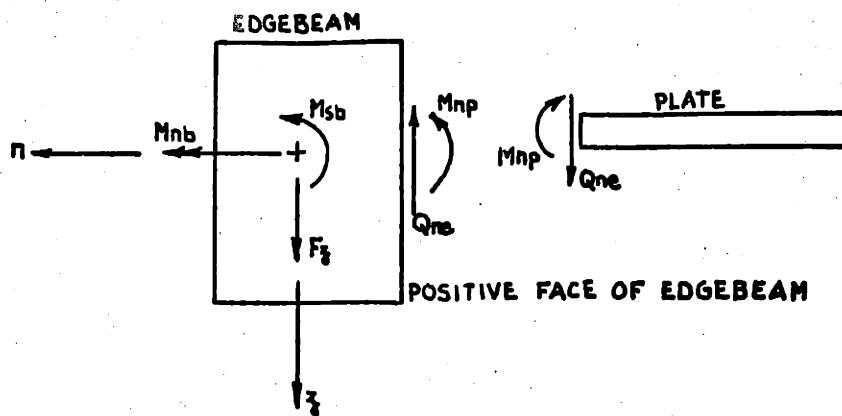
FIGURE (16a)

ASSUMED DIRECTIONS OF STRESS RESULTANTS
AND COUPLES ACTING ON POSITIVE FACE OF
FREE EDGEBEAM
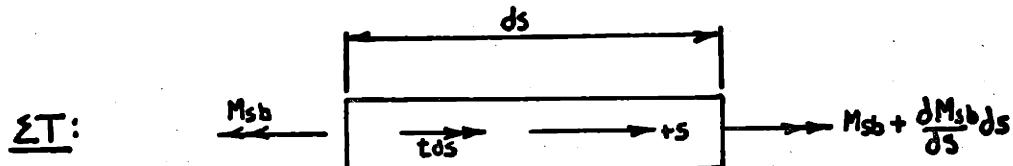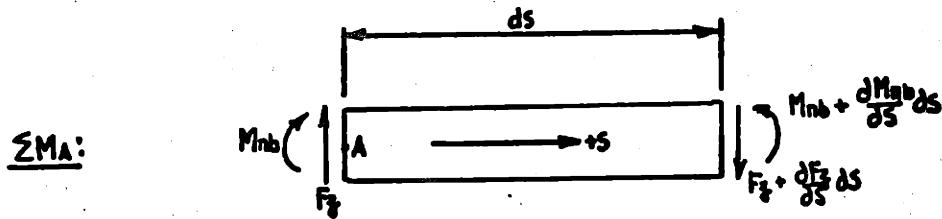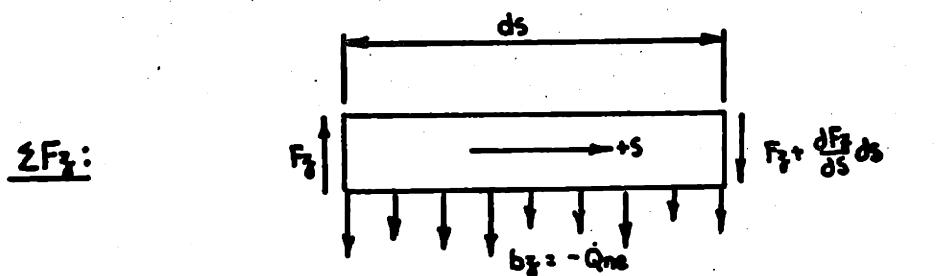
t = EXTERNALLY APPLIED TWIST = $Mnp + Qne$ (HOR)

FIGURE (16b)

ORIENTATION OF FORCES AND COUPLES USED
IN FREE EDGEBEAM EQUILIBRIUM EQUATIONS
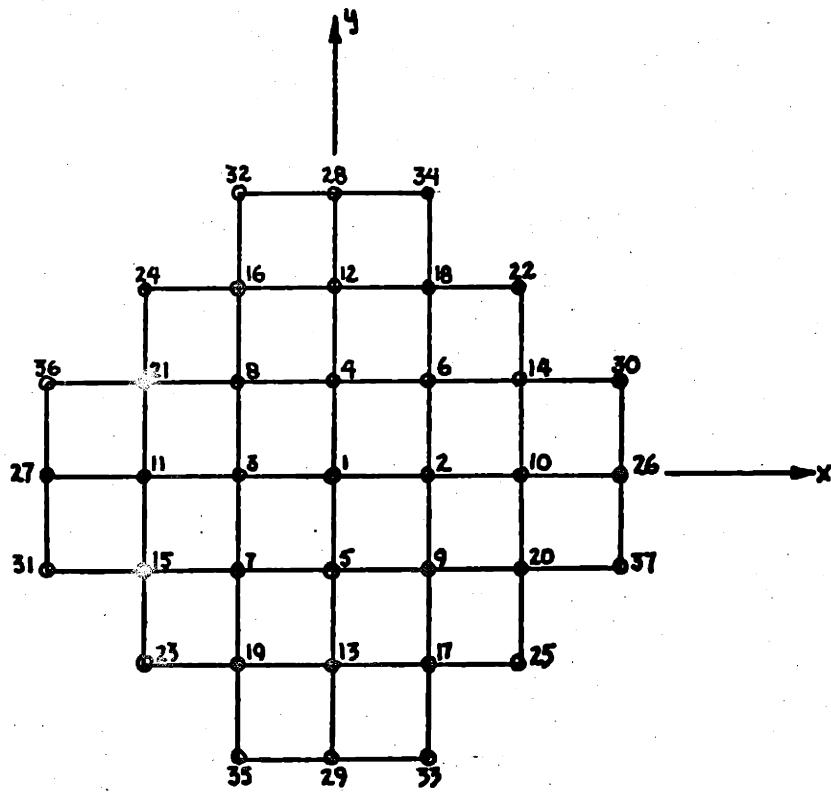
FIGURE (17)
RELATIVE NUMBERING OF GRIDPOINTS
(CENTRAL POINT = 1)
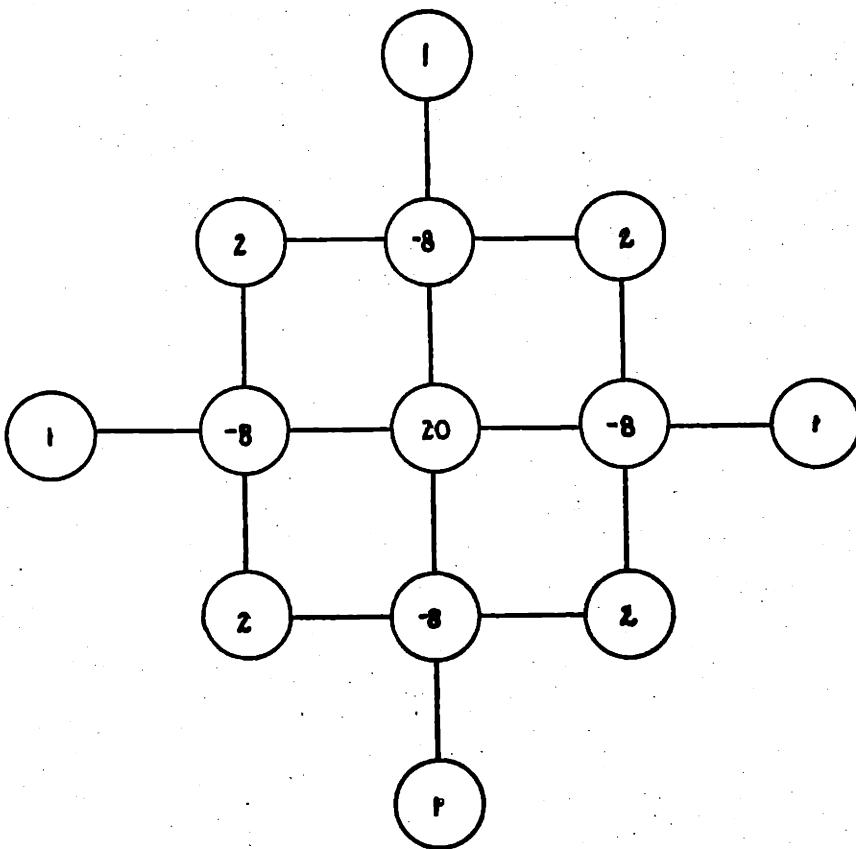
FIGURE (18a)
STANDARD MOLECULE FOR $\nabla^4 \omega$
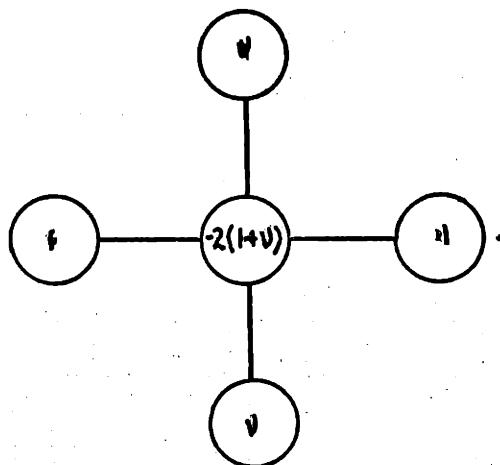
FIGURE (18b)
STANDARD MOLECULE FOR M_{xx}/D_P
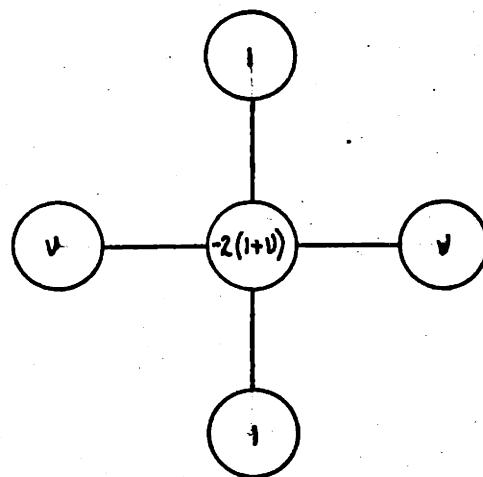
FIGURE (18c)
STANDARD MOLECULE FOR M_{yy}/DP
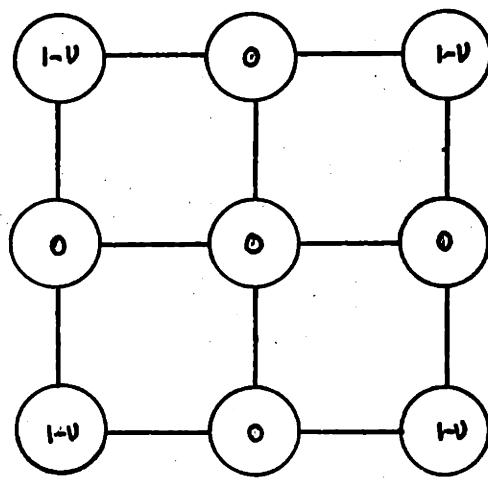
FIGURE (18d)
STANDARD MOLECULE FOR M_{xy}/DP
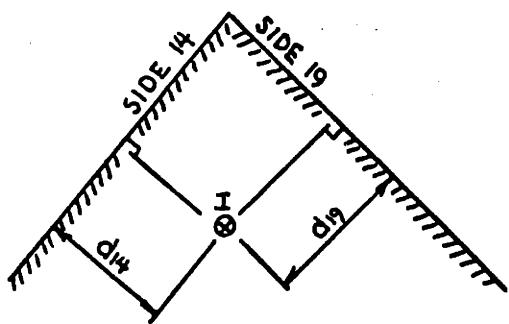
FIGURE (19a)
DISTANCE FROM CENTRAL GRIDPOINT
TO EACH SIDE - d_j
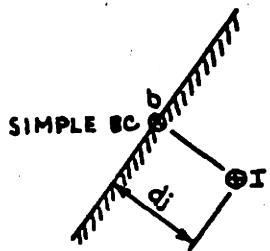
FIGURE (19b)
BOUNDARY CONDITIONS - $\sqrt{2}H < d_j < 2H$
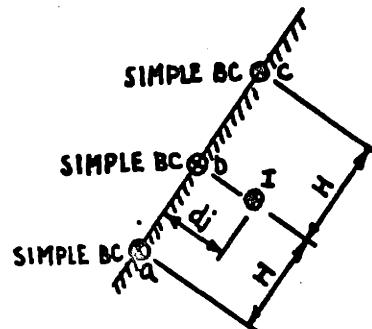
FIGURE (19c)
BOUNDARY CONDITIONS - $H < d_j < \sqrt{2}H$
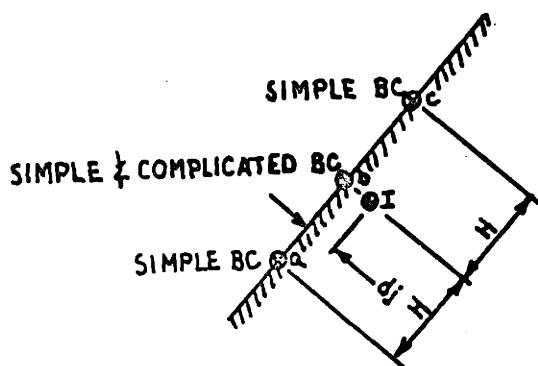
FIGURE (19d)
BOUNDARY CONDITIONS - $d_j < H$

$$H < d_i < \sqrt{2}H$$

$$d_j < H$$
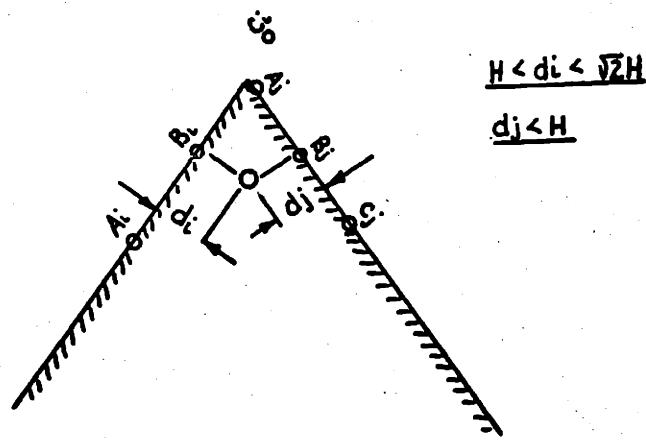
FIGURE (20)

EXAMPLE OF CASE WHERE SOME
BOUNDARY POINTS DO NOT FALL ON
THE SIDE OF THE PLATE
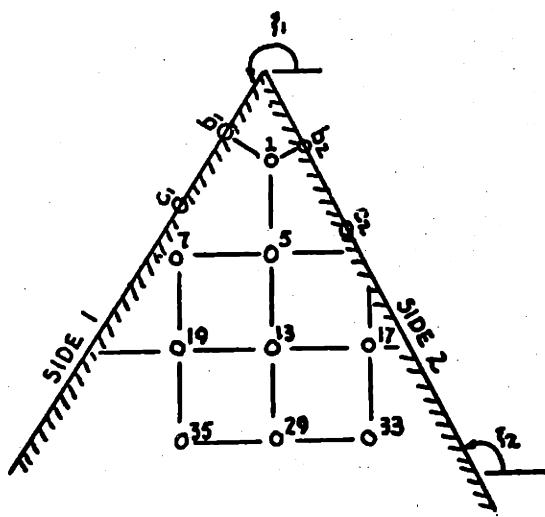
FIGURE (21)
EXAMPLE PROBLEM USED IN ILLUSTRATING
METHOD OF FORMULATING MOLECULE
FOR GRIDPOINT NEAR BOUNDARY
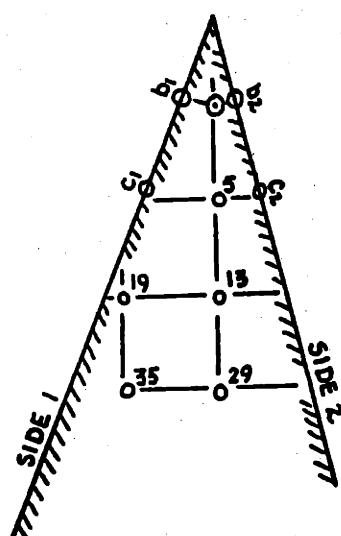
FIGURE (22)

EXAMPLE OF MOLECULE IN WHICH
ACCURACY WILL NOT BE UP TO SIXTH
DERIVATIVE OF SERIES EXPANSION
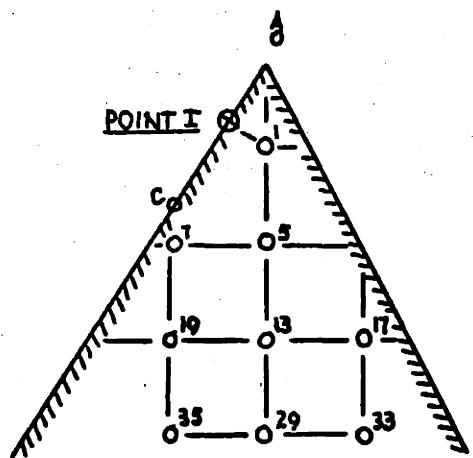
FIGURE (23)
EXAMPLE PROBLEM TO ILLUSTRATE
COMPUTATION OF BOUNDARY QUANTITIES
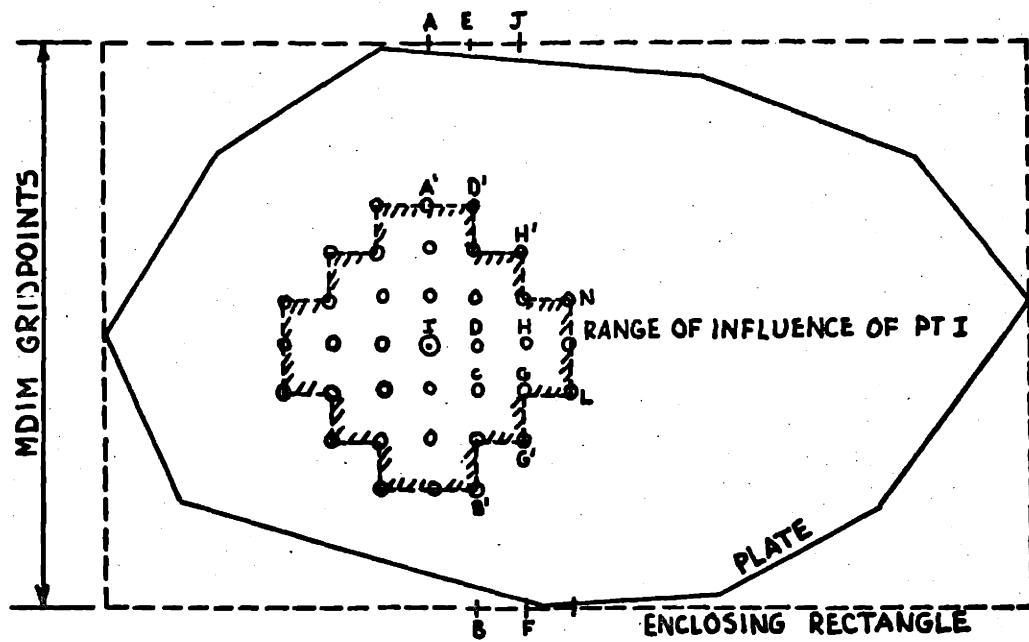
FIGURE (24)
PLATE ILLUSTRATING BAND
MATRIX CHARACTERISTICS
OF FINITE DIFFERENCE EQUATIONS
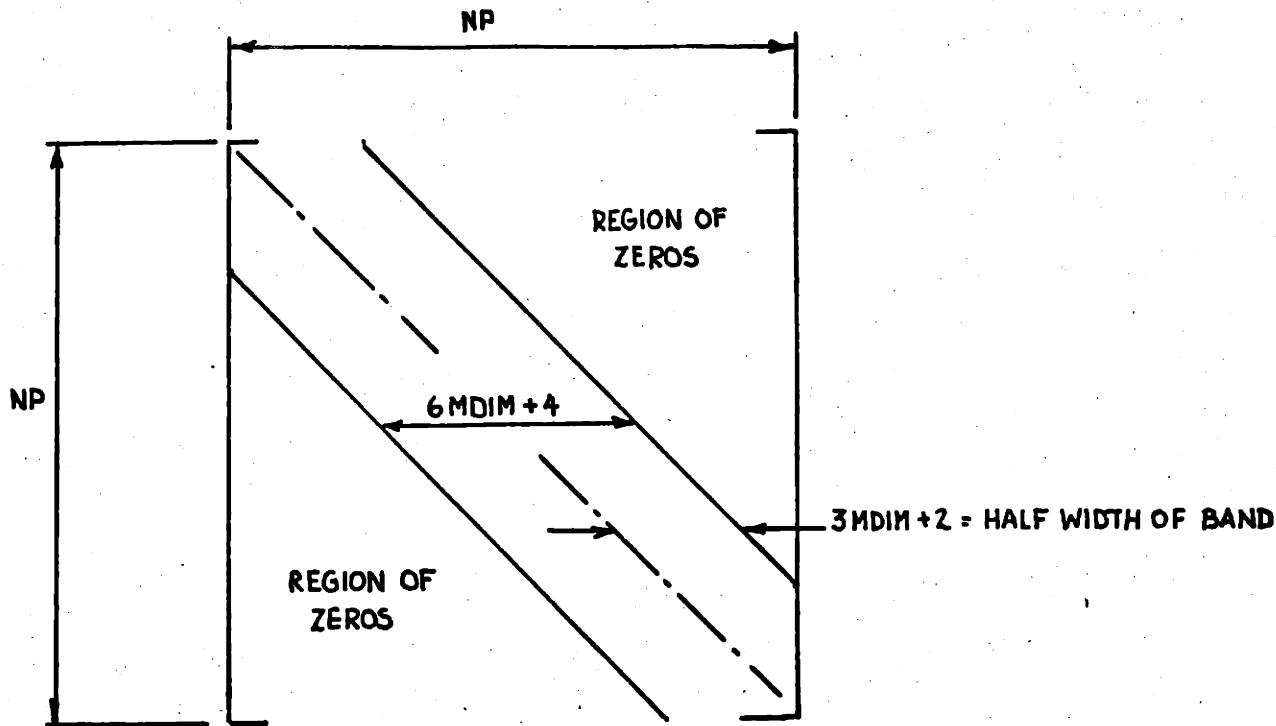
FIGURE (25)
COEFFICIENT MATRIX, $\underset{\sim}{C}$, IN
FINAL SET OF SIMULTANEOUS
DIFFERENCE EQUATIONS:
$$[\underset{\sim}{C}]\{w\} = \{q\}$$
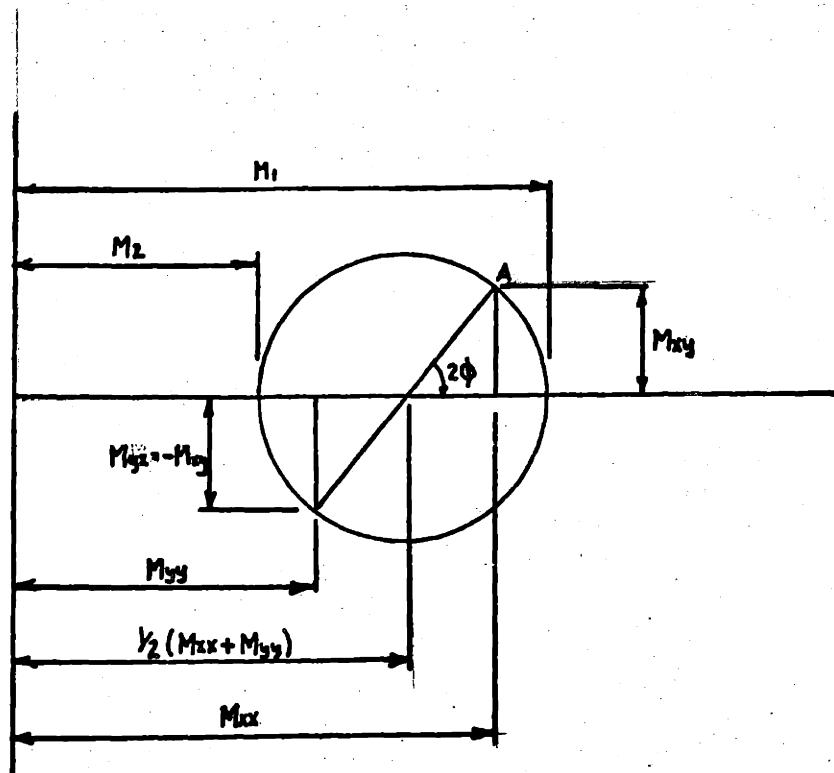
FIGURE (26a)
POSITIVE MOMENTS-
MOHR CIRCLE REPRESENTATION
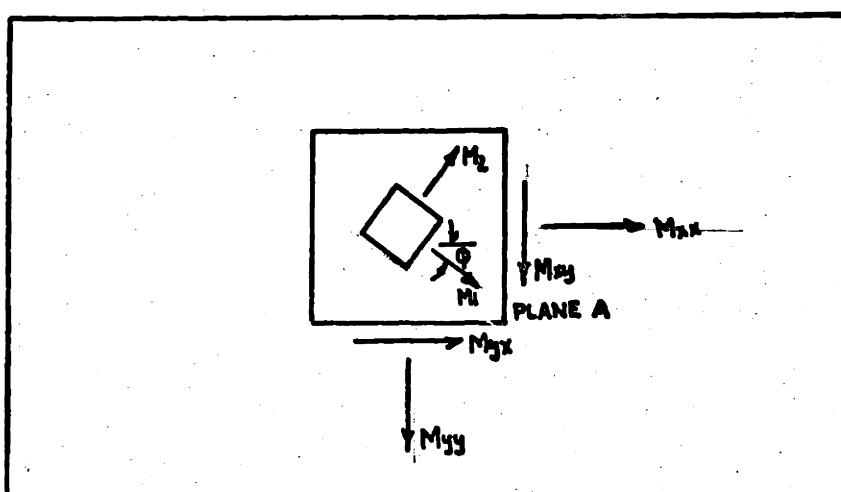
FIGURE (26b)
POSITIVE MOMENTS
VECTOR REPRESENTATION

```
┌─────────────────────────────────┐
│ INPUT: PROGRAM NAME, TITLE,      │
│ DP, POIS, NS, NC, MCONS(4, NS)   │
└─────────────────────────────────┘
                 │
┌─────────────────────────────────┐
│ CHECK AND REARRANGEMENT OF       │
│ CONNECTIVITY MATRIX. CHECK NC.   │
│ FORM NSC(NC).                    │
└─────────────────────────────────┘
                 │
┌─────────────────────────────────┐
│ INPUT: CORNER POINT NUMBERS &    │
│ COORDINATES. CHECK CORNERPOINT   │
│ NUMBERS WITH CONNECTIVITY MATRIX.│
└─────────────────────────────────┘
                 │
┌─────────────────────────────────────┐
│ SECOND REARRANGEMENT OF CONNECTIVITY │
│ MATRIX.   CALCULATE COSS(I), SINS(I) │
└─────────────────────────────────────┘
                 │
┌─────────────────────────────────┐
│ INPUT: ADDITIONAL INFORMATION.   │
│ DEPENDS ON BOUNDARY CONDITIONS.  │
└─────────────────────────────────┘
                 │
┌──────────────────────────────────────┐
│ INPUT: GRID SIZE, H; NUMBER OF POINTS, NP; │
│ INPUT: NL, LP(NP), Q(NP, NL)         │
└──────────────────────────────────────┘
                 │
┌─────────────────────────────────┐
│ REARRANGE LIST OF POINTS VECTOR. │
│ COMPUTE MDIM.                    │
└─────────────────────────────────┘
                 │
┌────────────────────────────────────────┐
│ COMPUTE NUMBER OF BOUNDARY POINTS- MFIN │
│ FORM LBPE(MFIN), LBPSE(MFIN), XBPE(MFIN).│
│ YBPE(MFIN).                             │
└────────────────────────────────────────┘
                 │
┌─────────────────────────────────┐
│ INPUT: PROBLEM SPECIFIED CARD    │
└─────────────────────────────────┘
```

LINK 1

INPUT, SORTING, PRELIMINARY CALCULATIONS

FIGURE 27-1

Link 3 (left diagram)

I=1

DETERMINE RELATIVE GRIDPOINT CLOSEST TO POINT I. USE RELATIVE NUMBERING SCHEME WITH THIS CLOSEST GRIDPOINT AS POINTS.

K=1
KO=1

KO-S =0

<0

KO =3

=2,4

XD=XD+HCOSα_j
YD=YD+HSINα_j

=1

XD,YD

XD=XD-2HCOSα_j
YD=YD-2HSINα_j

XD,YD ON SIDE?

ALFA=0.
BETA=0.

XD,YD ON SIDE?

ALFA,BETA

ALFA,BETA

KD =4

=1,2,3

TAYLOR SERIES FOR COMPLICATED BC

TAYLOR SERIES FOR SIMPLE BC

KO=KO+1

K=K+1

KSTAR(K)=0

ASSEMBLE COLUMN K OF AW(21,K)

MI=0
NBC=K-1

M=1

CONVERT RELATIVE GRIDPOINT NO. TO ABSOLUTE NO. (LPRA)

M=M+1

No

IS LPRA ON PLATE?

Yes

ALFA,BETA

MI =0 / =1

TAYLOR SERIES, TSW

TAYLOR SERIES, $\nabla^4\omega$

KSTAR(K)=ABSOLUTE NO. OF GRIDPOINT

KSTAR(K)=-(ABSOLUTE NO. OF GRIDPOINT)

K=K+1

M-3? <0 / =0   K-3? <0 / =0

MI=1 =0

MI =1

MOVE RIGHTHAND SIDE INTO AW(21,31...34), (DEPENDING ON BOUNDARY CONDITION OF SIDE)

CALL JUDY ON AW(21,34). CALCULATE A's FOR EXTERNAL REACTIONS AND/OR FOR DISPLACEMENT AT POINT I.

LD=1

$TOT(LD)=\sum_j A_j \nabla^4\omega_j$
$TOT1(LD)=\sum_K A_K \nabla^4\omega_K$
$TOT2(LD)=\sum_m A_m \nabla^4\omega_m$
$TOT3(LD)=\sum_n A_n \nabla^4\omega_n$

WRITE FINITE DIFFERENCE EQUATIONS FOR ALL BC'S (UP TO 4):
$RBP1(I)=\sum_j A_j\omega_j + TOT(LD)$
$RBP2(I)=\sum_K A_K\omega_K + TOT1(LD)$
$RBP3(I)=\sum_m A_m\omega_m + TOT2(LD)$
$RBP4(I)=\sum_n A_n\omega_n + TOT3(LD)$

I=I+1   <0 I-MFIN =0 LD-NL <0 LD=LD+1

=0

TO LINK 4

LINK 3
DIFFERENCE EQUATIONS FOR

Link 2 (right diagram)

I=1   DO I=1,NP

CLASSIFICATION OF GRIDPOINTS
LPC(I)=1   INTERIOR POINT
LPC(I)=2   BOUNDARY POINT

LPC(I)-I

J=1   DO J=1,NS

K=1

J-(NS+1)

DISTANCE d_j TO SIDE J

J=J+1

D-2H >0

=0

KO=1

KD =3

=2,4

XD=XD+HCOSα_j
YD=YD+HSINα_j

=1

XD,YD

XD=XD-2HCOSα_j
YD=YD-2HSINα_j

XD,YD ON SIDE J?

NO

YES

ALFA,BETA

D-H <0

KO-4 <0

D-S2H >0

KD =4

=1,2,3

COMPL BC FOR ALFA,BETA

SIMPLE BC FOR ALFA,BETA

TAYLOR SERIES FOR ABOVE BC

ASSEMBLE COLUMN K OF AW$\binom{1}{2}$,K)

KSTAR(K)=0   K=K+1   KO=KO+1

MI=0
NBC=K-1
M=1

CONVERT RELATIVE NO. TO ABSOLUTE NUMBER-LPRA

M=M+1

No

LPRA ON PLATE

Yes

ALFA,BETA

MI =0 / =1

TAYLOR SERIES, TSW

TAYLOR SERIES, $\nabla^4\omega$

KSTAR(K)=ABSOLUTE NO. OF GRIDPOINT

KSTAR(K)=-(ABSOLUTE NO. OF GRIDPOINT)

K=K+1

M=2
MI=1

M-3? <0 / =0   K-3? <0 / =0

MI =1

MOVE RIGHT HAND SIDES FOR $\nabla^4\omega$, M_{xx}, M_{yy}, M_{xy} INTO AW(21,31...34)

USE STANDARD OPERATORS FOR $\nabla^4\omega$, M_{xx}, M_{yy}, M_{xy} AT POINT I. A's ARE KNOWN.

CALL JUDY ON AW(21,34) CALCULATE A's FOR $\nabla^4\omega$, M_{xx}, M_{yy}, M_{xy}

LD=1

SUM(LD)=0.
SUM1(LD)=0.
SUM2(LD)=0.
SUM3(LD)=0.

$\sum_j A_j \nabla^4\omega_j = SUM(LD) - \nabla^4\omega$ CASE
$\sum_K A_K \nabla^4\omega_m = SUM1(LD) - M_{xx}$ CASE
$\sum_m A_m \nabla^4\omega_m = SUM2(LD) - M_{yy}$ CASE
$\sum_n A_n \nabla^4\omega_n = SUM3(LD) - M_{xy}$ CASE

MODIFY LOAD: $Q(I) = Q(I) - SUM(LD)$

DIFFERENCE EQN AT POINT I:
$\sum_j A_j\omega_j = Q(I)$
MOMENT EQNS:
$RMXX(I)=\sum_m A_m\omega_m + SUM1(LD)$
$RMYY(I)=\sum_n A_n\omega_n + SUM2(LD)$
$RMXY(I)=\sum_p A_p\omega_p + SUM3(LD)$

LD=LD+1

LD-NL <0

<0 I-NP =0 LD-NL <0 LD=LD+1

TO LINK 3

LINK 2
AUTOMATIC ASSEMBLAGE OF DIFFERENCE EQUATIONS

```
┌─────────────────────────────────┐
│ SOLVE LARGE SYSTEM OF EQUATIONS │
│ BY GAUSS-ELIMINATION ON PARTITIONED │
│ BAND MATRIX. (AUGMENTED MATRIX IS │
│ OF ORDER  NP×(NP+NL).           │
│                                 │
│ SOLVE FOR DISPLACEMENTS AT      │
│ INTERIOR GRID POINTS .- W(NP,NL)│
└─────────────────────────────────┘
               │
           ┌───────┐
           │ I = 1 │
           └───────┘
               │
     ┌─────────────────────────────┐
     │ OUTPUT:  DISPLACEMENT, W     │
     └─────────────────────────────┘
               │
   ┌───────────────────────────────────┐
   │ MAKING USE OF KSTAR(K), BACK-      │
   │ SUBSTITUTE DISPLACEMENTS INTO:     │
   │                                    │
   │   RMXX(I) TO GET Mxx AT I          │
   │   RMYY(I) TO GET Myy AT I          │
   │   RMXY(I) TO GET Mxy AT I          │
   └───────────────────────────────────┘
               │
   ┌───────────────────────────────────┐
   │ OUTPUT: BMxx(I), BMyy(I), BMxy(I)  │
   └───────────────────────────────────┘
               │
   ┌───────────────────────────────────┐
   │ OUTPUT: PRINCIPAL MOMENTS:         │
   │    BM1(I), BM2(I)                  │
   │         ANGLE OF ORIENTATION:      │
   │         PHI(I)                     │
   └───────────────────────────────────┘
               │
   ┌─────────┐        ◇
   │ I = I+1 │◄─────── I-NP
   └─────────┘        ◇
               │
           ┌───────┐
           │ I = 1 │
           └───────┘
               │
   ┌───────────────────────────────────┐
   │ BACK SUBSTITUTION:                 │
   │ DETERMINE RBP1(I), RBP2(I),        │
   │ RBP3(I), RBP4(I)                   │
   └───────────────────────────────────┘
               │
   ┌───────────────────────────────────┐
   │ OUTPUT:   UP TO 4 REACTIONS,       │
   │ MOMENTS, DISPLACEMENTS             │
   │ (DEPENDS ON BOUNDARY CONDI-        │
   │   TIONS FOR EACH SIDE)             │
   └───────────────────────────────────┘
               │
   ┌─────────┐        ◇
   │ I = I+1 │◄─────── I-MFIN
   └─────────┘        ◇
               │
         ┌────────────┐
         │ END OF JOB │
         └────────────┘
```

LINK 4
SOLUTION, BACK-SUBSTITUTION
AND OUTPUT

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VIII

APPENDICES

A. Definitions of Symbols

AC(I) = Total area enclosed by contour I.

A_1 = Value of molecule at point 1 for a gridpoint.

AN = Area enclosed by one side of a contour and the x axis.

A_w = Augmented matrix of order 21x34 used when solving for gridpoint molecules.

C = Coefficient matrix to be inverted when obtaining gridpoint molecules.

COSS(I) = Cosine of angle between positive direction of side I and the positive x axis.

CPC = Corner point coordinates (in.).

d_1 = Perpendicular distance from the gridpoint to side 1.

DP = Flexural rigidity of plate $(lb-in^2/in)$.

EI_n = Flexural rigidity of edgebeam about n axis $(lb-in^2)$.

F_z = Transverse shear force on positive face of edgebeam (lb).

GJ = Torsional rigidity of edgebeam $(lb-in^2)$.

H = Gridpoint spacing (in).

HOR = Horizontal distance from shear center of edgebeam to function of plate and beam (inches).

LBPE = List of boundary points (at which reaction, moment, and/or displacement will be computed.)

LBPSE = List of sides on which each boundary point in LBPE is located.

LP = List of gridpoints.

LPRA = Absolute number of gridpoint as obtained from relative number of central gridpoint.

MC = List of cornerpoint numbers

MCONS = Connectivity matrix of plate; contains each side, its initial and terminal cornerpoint, and its boundary condition.

MDIM = Maximum number of points spaced a distance H apart that can be placed along the shortest side of the rectangle enclosing the plate.

M_{nb} = Bending moment acting on positive face of edgebeam.

M_{np} = Bending moment of plate at junction of plate and edgebeam.

M_{sb} = Twisting moment acting on positive face of edgebeam.

M_{xx}, M_{yy} = Bending moment acting on the side of a plate element that is perpendicular to the x, y axis respectively.

M_{xy} = Twisting moment acting on above plate element.

M_1, M_2 = Principal moments acting on plate element.

NC = Total number of contours in plate.

NL = Total number of loading conditions to be considered by the analysis (up to 3)

NP = Total number of gridpoints on the plate (up to 1000).

NS = Total number of sides of the plate (up to 100)

NSAI	=	Number of sides for which additional information about boundary conditions is to be read in.
NSC(1)	=	Number of sides in contour 1.
POIS, U	=	Poissons ratio of plate.
Q_n, Q_{ne}	=	Transverse shear stress resultant of plate.
q_r	=	Distributed force per unit area acting on gridpoint r. (lb/in^2).
SCD	=	Translational spring constant ($lb/in/in$).
SCR	=	Rotational spring constant ($lb-in/in$).
SINS(1)	=	Sine of angle between positive direction of side 1 and positive x axis.
t	=	Thickness of plate (inches).
w	=	Vertical displacement of plate.
XBPE, YBPE	=	Cartesian Coordinates of points along boundary at which reactions,, moments, and/or displacements will be computed.
XN, YN	=	Cartesian coordinates of negative cornerpoint of a side of the plate.
XP, YP	=	Cartesian coordinates of positive cornerpoint of a side of the plate.
α_i	=	Angle of inclination at negative end of side 1 between side 1 and positive x axis.
ϕ	=	Angle that principal moment M_1 makes with positive x axis.
σ_i	=	Stress at point in plate in the 1 direction.
η	=	Angle of twist of positive face of edgebeam

B. List of Figures

C. List of Tables

IX

SOURCE PROGRAMS

FOR

PLATE ANALYSIS

LINK1.........INPUT, SORTING, AND PRELIMINARY CALCULATIONS.

```
        DIMENSION TITLE(12)
        DIMENSION MCONS(4,100)
        DIMENSION MC(100)
        DIMENSION CPC(100,2)
        DIMENSION LBPE(1000)
        DIMENSION LBPSE(1000)
        DIMENSION XBPE(1000)
        DIMENSION YBPE(1000)
        DIMENSION COSS(100)
        DIMENSION SINS(100)
        DIMENSION LP(1000)
        DIMENSION LPC(1000)
        DIMENSION Q(1000,3)
        COMMON TITLE,MCONS,MC,CPC,LBPE,LBPSE,XBPE,YBPE,COSS,SINS,LP,LPC,
       1Q,INTSW,IEXQ,INEXT,NS,NC,NP,NL,MDIM,H,MFIN
        DIMENSION AREA(33)
        DIMENSION NSC(33)
        DIMENSION SCR(100)
        DIMENSION SCD(100)
        DIMENSION EI(100)
        DIMENSION GJ(100)
        DIMENSION HOR(100)
        DIMENSION JSIDE(100)
        EXTERNAL LIST,LIST1
        INEXT=1
    800 GO TO (901,1001),INEXT
C       INPUT OF PROGRAM NAME AND TITLE OF PROBLEM.
    901 I=MATCH(LIST,K,1)
        IF(I-5) 901,902,901
    902 IF(K-3) 1001,1001,901
   1001 IEXQ=0
        INEXT=1
        IERR=0
        READ 35,(TITLE(I),I=1,12)
     35 FORMAT (12A6)
        PRINT 36
     36 FORMAT(1H1,15X,12HPROBLEM NAME//)
        PRINT 37,(TITLE(I),I=1,12)
     37 FORMAT (20X,12A6//)
C       INPUT OF PLATE DATA.
   1002 READ 1, DP,POIS,NS,NC
      1 FORMAT (6H DP = F15.2,3X,7HPOIS = F5.3,3X,5HNS = I4,3X,5HNC = I4)
   1003 PRINT 1, DP,POIS,NS,NC
C       USE SUBROUTINE MATCH TO READ IN MCONS(I,J), AND TO CHANGE
C       MCONS(4,J) FROM ALPHABETIC TO NUMERICAL DATA
        DO 600 J=1,NS
        I=MATCH(LIST1,K,1)
        IF(I-5) 401,601,401
    601 IF(K-7) 402,602,402
    602 DO 603 L=1,3
        I=MATCH(LIST1,K,2)
    603 MCONS(L,J)=K
        I=MATCH(LIST1,K,0)
```

```
      IF(K-3) 606,605,606
  605 I=MATCH(LIST1,K,0)
      GO TO (607,401,401,401,608,401),I
  607 K=3
      GO TO 606
  608 IF(K-5) 401,606,401
  606 MCONS(4,J)=K
      GO TO 600
  401 PRINT 63, J
   63 FORMAT(25H SIDE DATA ON CARD NUMBER, I5 , 19HIS IN WRONG FORMAT.)
      STOP
  402 PRINT 64
   64 FORMAT(62H CARDS FOR SIDE DATA ARRANGED IMPROPERLY.   EXECUTION DE
     1LETED.)
      STOP
  600 CONTINUE
      PRINT 56
   56 FORMAT(/////15X,19HCONNECTIVITY MATRIX//)
      IF(NS-15) 1005,1005,1006
 1005 PRINT 1009, (MCONS(1,I), I=1,NS)
 1009 FORMAT(/11X,11HSIDE NUMBER,8X,15I6)
      PRINT 1007, (MCONS(2,I), I=1,NS)
 1007 FORMAT(/5X,25HFROM CORNER POINT NUMBER 15I6)
      PRINT 1008, (MCONS(3,I), I=1,NS)
 1008 FORMAT(/6X,22HTO CORNER POINT NUMBER,2X,15I6)
      PRINT 2001, (MCONS(4,I), I=1,NS)
 2001 FORMAT(/7X,18HBOUNDARY CONDITION,5X,15I6)
      PRINT 21
   21 FORMAT (1X,/////)
      GO TO 1011
 1006 PRINT 1012
 1012 FORMAT(10X,11HSIDE NUMBER,15X,19HCORNER POINT NUMBER,15X,
     18HBOUNDARY)
      PRINT 2002
 2002 FORMAT(36X,4HFROM,13X,2HTO,15X,9HCONDITION//)
      DO 1013 I=1,NS
 1013 PRINT 1014, (MCONS(J,I), J=1,4)
 1014 FORMAT(16X,I3,13X,I3,18X,I3//)
C     FIRST REARRANGEMENT OF CONNECTIVITY MATRIX
 1011 NCCHEC=0
      NSIDEC=1
      DO 1010 JSORT=1,NS
      IF(JSORT-NS) 1020,1031,1020
 1020 ISERCH=JSORT+1
      DO 1030 ISORT=ISERCH,NS
      IF(MCONS(3,JSORT)-MCONS(2,ISORT)) 1035,1040,1035
 1035 IF(MCONS(3,JSORT)-MCONS(3,ISORT)) 1030,1036,1030
 1030 CONTINUE
 1031 NCCHEC=NCCHEC+1
      NSC(NCCHEC)=NSIDEC
      IF(NSIDEC-2) 1900,1900,1050
 1050 NSIDEC=1
      GO TO 1010
 1036 ITEMP=MCONS(2,ISORT)
      MCONS(2,ISORT)=MCONS(3,ISORT)
      MCONS(3,ISORT)=ITEMP
 1040 DO 1060 ISWICH=1,4
```

```
      ITEMP=MCONS(ISWICH,ISERCH)
      MCONS(ISWICH,ISERCH)=MCONS(ISWICH,ISORT)
 1060 MCONS(ISWICH,ISORT)=ITEMP
      NSIDEC=NSIDEC+1
 1010 CONTINUE
      DO 1802 I=1,NC
 1802 PRINT 31, I,NSC(I)
   31 FORMAT(/9H CONTOUR I4,3X,17H NUMBER OF SIDES=I4)
      IF(NCCHEC-NC) 1910,1070,1910
 1070 DO 1100 ITWICE=1,3
      DO 1101 JTWICE=1,NS
      DO 1102 KTWICE=1,NS
      IF(KTWICE-JTWICE) 1110,1102,1110
 1110 IF(MCONS(ITWICE,JTWICE)-MCONS(ITWICE,KTWICE)) 1102,1115,1102
 1115 IF(ITWICE-1) 1940,1930,1940
 1102 CONTINUE
 1101 CONTINUE
 1100 CONTINUE
      LCLOSE=0
      KCLOSE=1
      DO 1120 JCLOSE=1,NC
      LCLOSE=NSC(JCLOSE)+LCLOSE
      IF(MCONS(3,LCLOSE)-MCONS(2,KCLOSE))1950,1120,1950
 1120 KCLOSE=LCLOSE+1
      DO 1130 ICPRED=1,NS
C     INPUT OF CORNER POINT NUMBERS AND COORDINATES
 1140 READ 4,MC(ICPRED),(CPC(ICPRED,JCPRED),JCPRED=1,2)
    4 FORMAT(6HPOINT I6,3X,2HX=F12.4,3X,2HY=F12.4)
      JMCHEK=0
      DO 1150 IMCHEK=1,NS
      IF(MC(ICPRED)-MCONS(3,IMCHEK)) 1150,1160,1150
 1160 JMCHEC=1
 1150 CONTINUE
      IF(JMCHEC)1960,1960,1130
 1130 CONTINUE
      PRINT 57,NS
   57 FORMAT(////15X,21HLIST OF CORNER POINTS,//5X,23HNUMBER OF CORNER P
     10INTS,I4/)
      PRINT 59
   59 FORMAT(10X,12HCORNER POINT,13X,1HX,18X,1HY,18X,/13X,6HNUMBER,11X,
     111HCOORDINATES,8X,11HCOORDINATES///)
      PRINT 58,(MC(I),(CPC(I,J),J=1,2),I=1,NS)
   58 FORMAT(14X,I3,10X,F12.4,7X,F12.4//)
C     FINAL REARRANGEMENT OF CONNECTIVITY MATRIX
      AMAX=0.0
      JAREA2=0
      DO 1170 IAREA=1,NC
      AREA(IAREA)=0.0
      JAREA=NSC(IAREA)
      DO 1185 KAREA=1,JAREA
      JAREA2=JAREA2+1
      DO 1181 IFIND=1,NS
      IF(MC(IFIND)-MCONS(3,JAREA2))1182,1183,1182
 1183 MAREA=IFIND
 1182 IF(MC(IFIND)-MCONS(2,JAREA2))1181,1184,1181
 1184 NAREA=IFIND
 1181 CONTINUE
```

```
1185 AREA(IAREA)=AREA(IAREA)+(CPC(MAREA,1)-CPC(NAREA,1))*(CPC(MAREA,2)+
    1CPC(NAREA,2))/2.0
     IF(ABS (AREA(IAREA))-AMAX) 1170,1170,1190
1190 IMAX=IAREA
     AMAX=ABS (AREA(IAREA))
1170 CONTINUE
     IF(IMAX-1) 1180,1180,1195
1195 ISUM=0
     JSUM=NSC(IMAX)
     DO 1200 I=1,IMAX
1200 ISUM=ISUM+NSC(I)
     LSUM=ISUM-1
     DO 1210 I=1,4
     DO 1220 J=1,JSUM
     ITEMP=MCONS(I,ISUM)
     DO 1230 K=1,LSUM
     KSUM=ISUM-K
1230 MCONS(I,KSUM+1)=MCONS(I,KSUM)
1220 MCONS(I,1)=ITEMP
1210 CONTINUE
     ISUM=IMAX-1
     DO 1250 I=1,ISUM
     LSUM=IMAX-I
1250 NSC(LSUM+1)=NSC(LSUM)
     NSC(1)=JSUM
1180 DO 1260 IFINAL=1,NC
     IF(IFINAL-1) 1270,1280,1270
1280 IF(AREA(1)) 1260,1260,1290
1270 IF(AREA(IFINAL)) 1290,1290,1260
1290 JFINAL=NSC(IFINAL)
     IFTOT=0
     DO 1300 KFINAL=1,IFINAL
1300 IFTOT=IFTOT+NSC(KFINAL)
     MFINAL=IFTOT-NSC(IFINAL)+1
     JBEEP=MFINAL
     KBEEP=IFTOT
1310 ITEMP=MCONS(1,JBEEP)
     JTEMP=MCONS(4,JBEEP)
     MCONS(1,JBEEP)=MCONS(1,KBEEP)
     MCONS(4,JBEEP)=MCONS(4,KBEEP)
     MCONS(1,KBEEP)=ITEMP
     MCONS(4,KBEEP)=JTEMP
     JBEEP=JBEEP+1
     KBEEP=KBEEP-1
     IF(KBEEP-JBEEP) 1320,1320,1310
1320 KBEEP=IFTOT
     DO 1330 I=MFINAL,IFTOT
     ITEMP=MCONS(3,KBEEP)
     MCONS(3,KBEEP)=MCONS(2,I)
     MCONS(2,I)=ITEMP
1330 KBEEP=KBEEP-1
1260 CONTINUE
     DO 1350 IANGLE=1,NS
     DO 1351 I=1,NS
     IF(MC(I)-MCONS(3,IANGLE)) 1352,1353,1352
1352 IF(MC(I)-MCONS(2,IANGLE)) 1351,1354,1351
1353 JANGLE=I
```

```
      GO TO 1351
 1354 KANGLE=I
 1351 CONTINUE
      COSS(IANGLE)=(CPC(JANGLE,1)-CPC(KANGLE,1))/(SQRT ((CPC(JANGLE,1)-C
     1PC(KANGLE,1))**2+(CPC(JANGLE,2)-CPC(KANGLE,2))**2))
 1350 SINS(IANGLE)=(CPC(JANGLE,2)-CPC(KANGLE,2))/(SQRT ((CPC(JANGLE,1)-
     1CPC(KANGLE,1))**2+(CPC(JANGLE,2)-CPC(KANGLE,2))**2))
C     ADDITIONAL INFORMATION
      READ 2020, NSAI
 2020 FORMAT(8H NSAI = I3)
      PRINT 2020, NSAI
      DO 2050 I=1,NSAI
      READ 2021, JSIDE(I)
 2021 FORMAT(6HSIDE ,I3)
      PRINT 2021, JSIDE(I)
      K=1
 2022 IF(MCONS(1,K)-JSIDE(I)) 2023,2024,2023
 2024 IF(MCONS(4,K)-4) 2028,2026,2025
 2026 READ 2027, SCR(K), SCD(K)
 2027 FORMAT(7H SCR = ,F10.3,5X,6HSCD = ,F10.3)
      PRINT 2027, SCR(K), SCD(K)
      GO TO 2050
 2028 PRINT 2029, I
 2029 FORMAT(37H ADDITIONAL INFORMATION DATA IN CARD ,I3,13HIS INCORRECT
     1.,3X,18HEXECUTION DELETED.)
      CALL CHNXIT
 2025 IF(MCONS(4,K)-5) 2031,2031,2033
 2031 READ 2032, EI(K),GJ(K),HOR(K)
 2032 FORMAT(6H EI = F15.2,5X,6H GJ = F15.2,5X,6HHOR = F8.2)
      PRINT 2032, EI(K),GJ(K),HOR(K)
      GO TO 2050
 2033 READ 2034, GJ(K), HOR(K)
 2034 FORMAT(6H GJ = F15.2,5X,6HHOR = F8.2)
      PRINT 2034, GJ(K), HOR(K)
      GO TO 2050
 2023 IF(K-NS) 2040,2028,2028
 2040 K=K+1
      GO TO 2022
 2050 CONTINUE
C     INPUT OF GRIDPOINT DATA
      READ 5, NP,NL,H
    5 FORMAT(3HNP=I4,3X,3HNL=I2,3X,2HH=F8.4)
      PRINT 60
   60 FORMAT (1H1,15X,23HLIST OF INTERNAL POINTS,//)
      PRINT 41,NP,NL
   41 FORMAT (5X,25HNUMBER OF INTERNAL POINTS,I4,10X,28HNUMBER OF LOADIN
     1G CONDITIONS,I3//)
      PRINT 42,H
   42 FORMAT (5X,13HGRID SPACING,F8.3//)
      DO 1360 LSTRED=1,NP
 1360 READ 6, LP(LSTRED),(Q(LSTRED,MSTRED),MSTRED=1,NL)
    6 FORMAT(I6,3F12.4)
C     REARRANGEMENT OF GRIDPOINT LISTING TO MINIMIZE WIDTH OF BAND
C     IN FINAL BAND MATRIX
 1729 MINLP1=9999
      MINLP2=9999
      MAXLP1=0000
```

```fortran
      MAXLP2=0000
      IDIR3=1
      DO 1730 I=1,NP
      IF(MAXLP1-LP(I)) 1731,1732,1732
 1731 MAXLP1=LP(I)
 1732 IF(MINLP1-LP(I)) 1733,1733,1734
 1734 MINLP1=LP(I)
 1733 KTEMP=LP(I)/100
      JTEMP=LP(I)-KTEMP*100
      LTEMP=JTEMP*100+KTEMP
      IF(MAXLP2-LTEMP) 1735,1736,1736
 1735 MAXLP2=LTEMP
 1736 IF(MINLP2-LTEMP)1730,1730,1737
 1737 MINLP2=LTEMP
 1730 CONTINUE
      MIN3=MAXLP1-MINLP1
      MIN2=MAXLP2-MINLP2
      IF(MIN3-MIN2) 1738,1739,1739
 1739 IDIR3=2
      MDIM=((MAXLP1/100-MINLP1/100)+1)*3+2
      GO TO 1362
 1738 DO 1740 I=1,NP
      MDIM=((MAXLP2/100-MINLP2/100)+1)*3+2
      KTEMP=LP(I)/100
 1740 LP(I)=(LP(I)-KTEMP*100)*100+KTEMP
 1362 DO 1720 JSORT=1,NP
      DO 1720 I=1,ITEMP
      IF(LP(I)-LP(I+1)) 1720,1720,1701
 1701 KTEMP=LP(I)
      DO 1703 IFT=1,NL
 1703 QTEMP(IFT)=Q(I,IFT)
      LP(I)=LP(I+1)
      DO 1704 IFT=1,NL
 1704 Q(I,IFT)=Q(I+1,IFT)
      LP(I+1)=KTEMP
      DO 1706 IFT=1,NL
 1706 Q(I+1,IFT)= QTEMP(IFT)
 1720 CONTINUE
      GO TO (1749,1751),IDIR3
 1749 DO 1750 I=1,NP
      KTEMP=LP(I)/100
 1750 LP(I)=(LP(I)-KTEMP*100)*100+KTEMP
 1751 DO 1702 J=1,NP
 1702 PRINT 6, LP(J),(Q(J,MSTRD),MSTRD=1,NL)
C     DETERMINATION OF NUMBER OF EQUALLY SPACED POINTS ALONG THE
C     BOUNDARY, THEIR SIDENUMBERS, AND THEIR COORDINATES
      M=1
      DO 2500 J=1,NS
      DO 2499 IJ=1,NS
      IF(MCONS(2,J)-MC(IJ)) 2499,2498,2499
 2499 CONTINUE
 2498 X1=CPC(IJ,1)
      Y1=CPC(IJ,2)
      DO 2497 IK=1,NS
      IF(MCONS(3,J)-MC(IK)) 2497,2496,2497
 2497 CONTINUE
 2496 X2=CPC(IK,1)
```

```
      Y2=CPC(IK,2)
C     COMPUTATION OF LENGTH OF SIDE J IN FEET
      DISTJ=SQRT((X1-X2)**2 + (Y1-Y2)**2)
C     DETERMINATION OF NUMBER OF EQUALLY SPACED POINTS ALONG SIDE J
      XNUM=DISTJ*12.0/H
      NUM=XNUM
      NMPTJ=NUM-1
      IF(NMPTJ) 2407,2407,2408
 2407 NMPTJ=0
 2408 KTERM=NMPTJ+2
      BNM=NMPTJ
      DPTJ=DISTJ/(BNM+1.0)
C     DETERMINATION OF LBPE, LBPSE, XBPE, YBPE
      DO 2400 K=1,KTERM
      IF(K-1) 2398,2398,2397
 2398 LBPE(M)=MCONS(2,J)
      LBPSE(M)=MCONS(1,J)
      XBPE(M)=X1
      YBPE(M)=Y1
      GO TO 2390
 2397 IF(K-KTERM) 2396,2395,2395
 2396 LBPE(M)=0
      LBPSE(M)=MCONS(1,J)
      XBPE(M)=XBPE(M-1)+DPTJ*COSS(J)
      YBPE(M)=YBPE(M-1)+DPTJ*SINS(J)
      GO TO 2390
 2395 LBPE(M)=MCONS(3,J)
      LBPSE(M)=MCONS(1,J)
      XBPE(M)=X2
      YBPE(M)=Y2
 2390 M=M+1
 2400 CONTINUE
 2500 CONTINUE
      MFIN=M-1
 1699 I=MATCH(LIST,K,1)
      IF(I-5) 1651,1652,1651
 1652 IF(K-3) 1654,1653,1653
 1654 PRINT 1656
 1656 FORMAT (/13X,88HTHE FINAL CARD,WHICH SHOULD BE A PROBLEM SPECIFIED
     1 CARD,IS MISSING OR WRONGLY FORMULATED//15X,27HHOWEVER EXECUTION C
     2ONTINUES//)
      INEXT=2
      GO TO 1653
 1651 IEXQ=1
      PRINT 1657
 1657 FORMAT (/10X,36HMORE DATA IS SUPPLIED THAN REQUIRED.//)
      PRINT 12
   12 FORMAT(10X,54HERRORS,AS LISTED,HAVE BEEN FOUND IN THE SUPPLIED DAT
     1A.//34H PLEASE CORRECT THESE AND RESUBMIT//57H EXECUTION PHASE DEL
     2ETED IN ANTICIPATION OF YOUR NEXT RUN//11H THANK YOU.////////1H1)
 1658 I=MATCH(LIST,K,1)
      IF(I-5) 1658,1659,1658
 1659 IF(K-3) 1661,1658,1658
 1661 GO TO 1001
 1653 IF(IEXQ) 1700,1700,1710
 1700 PRINT 21
      PRINT 11
```

```
   11 FORMAT(1H1,10X,50HNO CONTRADICTIONS ENCOUNTERED IN THE SUPPLIED DA
      1TA//5X,31HEXECUTION PHASE WILL NOW BEGIN.////////10X,8HRESULTS./)
      CALL CHNXIT
 1710 PRINT 12
      GO TO 800
 1900 IEXQ=1
      PRINT 13
   13 FORMAT(5X,59HTHE FOLLOWING BOUNDARY HAS AN INSUFFICIENT NUMBER OF
      1SIDES.//)
      DO 1901 I=1,NSIDEC
      ICR=JSORT-I+1
 1901 PRINT 18,MCONS(1,ICR)
      GO TO 1050
 1910 IEXQ=1
      PRINT 14
   14 FORMAT(5X,67HTHE NC GIVEN IS INCONSISTENT WITH THE SUPPLIED CONNEC
      1TIVITY MATRIX.//////)
      GO TO 1070
 1930 IEXQ=1
      PRINT 15,MCONS(ITWICE,JTWICE)
   15 FORMAT(5X,21HTHE SAME SIDE NUMBER,I2,24H,IS USED MORE THAN ONCE.//
      1///)
      GO TO 1100
 1940 IEXQ=1
      PRINT 16,MCONS(ITWICE,JTWICE)
   16 FORMAT(5X,29HTHE SAME CORNER POINT NUMBER,I2,24H,IS USED MORE THAN
      1 ONCE.//////)
      GO TO 1100
 1950 IEXQ=1
      PRINT 17
   17 FORMAT(5X,39HTHE FOLLOWING BOUNDARY IS DISCONTINUOUS//5X,43HTHIS B
      1OUNDARY IS COMPRISED OF SIDE NUMBERS-//)
      IE=NSC(JCLOSE)
      IBCE=0
      DO 1951 I=1,IE
 1951 IBCE=IBCE+NSC(I)
      IBCS=IBCE-NSC(IE)+1
      DO 1952 J=IBCS,IBCF
 1952 PRINT 18,MCONS(I,J)
   18 FORMAT(30X,I6/)
      GO TO 1120
 1960 IEXQ=1
      PRINT 19,MC(ICPRED)
   19 FORMAT(5X,17HTHE SIDE NUMBERED,I4,84H MENTIONED IN THE LIST OF BOU
      1NDARY POINTS,DOES NOT APPEAR IN THE CONNECTIVITY TABLE.//)
      GO TO 1130
      END

           LISTS FOR SUBROUTINE MATCH.....LINK1

$IBMAP LISTS    REF,LIST
       ENTRY    LIST
       ENTRY    LIST1
 LIST1 DEC      7
       BCI      1,0FIXED
       BCI      1,SIMPLE
       BCI      1,00FREE
```

LINK2.........EXPANSIONS OF BOUNDARY CONDITIONS.

```
      DIMENSION MCONS(4,100)
      DIMENSION COSS(100)
      DIMENSION SINS(100)
      DIMENSION AW(21,12)
      DIMENSION GJ(100)
      DIMENSION SCR(100)
      DIMENSION SCD(100)
      DIMENSION EI(100)
      DIMENSION HOR(100)
      DIMENSION TITLE(12)
      DIMENSION MC(100)
      DIMENSION CPC(100,2)
      DIMENSION LBPE(1000)
      DIMENSION LBPSE(1000)
      DIMENSION XBPE(1000)
      DIMENSION YBPE(1000)
      DIMENSION LP(1000)
      DIMENSION LPC(1000)
      DIMENSION Q(1000,3)
      DIMENSION KSTAR(30)
      DIMENSION MJ(100)
      COMMON TITLE,MCONS,MC,CPC,LBPE,LBPSE,XBPE,YBPE,COSS,SINS,LP,LPC,Q,
     1INTSW,IEXQ,INEXT,NS,NC,NP,NL,MDIM,H,MFIN,SCR,SCD,DP,POIS,EI,GJ,HOR
      REWIND 1
      CALL CLGRD(LP,NP,LPC)
      DO 3089 I=1,NP
      NOBC=0
      DO 3044 IT=1,21
      DO 3044 JT=1,8
 3044 AW(IT,JT)=0.
      IF(LPC(I)-2) 4445,4501,4445
 4501 K=1
      DO 3090 JSIDE=1,NS
      CALL DIST(CPC,H,LP,I,MCONS,JSIDE,D,XD,YD,X1,X2,Y1,Y2,X3,Y3,NS,MC)
      CALL SSWTCH(5,KK)
      IF(KK-2) 550,553,553
  550 PRINT 551, LP(I),MCONS(1,JSIDE),D
  551 FORMAT(//20H DISTANCE FROM POINT,I5,8H TO SIDE,I3,2H =,F12.4//)
      PRINT 552, XD,YD,X1,X2,Y1,Y2
  552 FORMAT(19H XD,YD,X1,X2,Y1,Y2=6F12.4//)
  553 IF(D-2.0*H) 4508,3091,3091
C     SINCE D LESS THAN 2H,BOUNDARY CONDITIONS FOR SIDE JSIDE ARE NEEDED
 4508 KD=1
 3110 IF(KD-2) 3120,4509,4510
 4509 XD=XD+H*COSS(JSIDE)
      YD=YD+H*SINS(JSIDE)
      GO TO 3120
 4510 IF(KD-4) 4511,4509,4509
 4511 XD=XD-2.0*H*COSS(JSIDE)
      YD=YD-2.0*H*SINS(JSIDE)
C     CHECK TO SEE IF XD,YD ARE ON SIDE
 3120 IF(ABS(SINS(JSIDE))-0.707) 3180,3180,3190
 3180 IF(XD-X1) 3200,3210,3210
```

```
 3200  IF(XD-X2) 3220,3230,3230
 3210  IF(XD-X2) 3230,3220,3220
 3190  IF(YD-Y1) 3240,3250,3250
 3240  IF(YD-Y2) 3220,3230,3230
 3250  IF(YD-Y2) 3230,3220,3220
C      SINCE XD,YD ARE ON SIDE, EXPAND B.C. AT THIS POINT
 3230  ALFA=(XD-X3)/H
       BETA=(YD-Y3)/H
C      EXPAND BOUNDARY CONDITIONS FOR SIDE AND INSERT INTO AW MATRIX
       DO 4800 LM=1,21
 4800  AW(LM,K)=0.
       C=COSS(JSIDE)
       S=SINS(JSIDE)
       LYNN=MCONS(4,JSIDE)
       IF(KD-4) 4600,4700,4700
 4600  GO TO (4601,4601,4603,4604,4605,4601), LYNN
C      SIMPLE BC FOR FIXED EDGES, SIMPLY SUPPORTED EDGES, AND
C      FOR SIMPLY SUPPORTED EDGEBEAM
 4601  AW(1,K)=1.
       AW(2,K)=ALFA
       AW(3,K)=BETA
       AW(4,K)=ALFA*ALFA/2.
       AW(5,K)=ALFA*BETA
       AW(6,K)=BETA*BETA/2.
       AW(7,K)=(ALFA**3)/6.
       AW(8,K)=ALFA*ALFA*BETA/2.
       AW(9,K)=ALFA*BETA*BETA/2.
       AW(10,K)=(BETA**3)/6.
       AW(11,K)=(ALFA**4)/24.
       AW(12,K)=(ALFA**3)*BETA/6.
       AW(13,K)=(ALFA**2)*(BETA**2)/4.
       AW(14,K)=ALFA*(BETA**3)/6.
       AW(15,K)=(BETA**4)/24.
       AW(16,K)=(ALFA**5)/120.
       AW(17,K)=(ALFA**4)*BETA/24.
       AW(18,K)=(ALFA**3)*(BETA**2)/12.
       AW(19,K)=(ALFA**2)*(BETA**3)/12.
       AW(20,K)=ALFA*(BETA**4)/24.
       AW(21,K)=(BETA**5)/120.
       GO TO 4710
C      SIMPLE BC FOR FREE EDGE
 4603  AW(4,K)=(S**2)+POIS*(C**2)
       AW(5,K)=-2.0*S*C*(1.0-POIS)
       AW(6,K)=(C**2)+POIS*(S**2)
       AW(7,K)=ALFA*AW(4,K)
       AW(8,K)=BETA*AW(4,K)+ALFA*AW(5,K)
       AW(9,K)=BETA*AW(5,K)+ALFA*AW(6,K)
       AW(10,K)=BETA*AW(6,K)
       AW(11,K)=(ALFA**2)*AW(4,K)/2.
       AW(12,K)=ALFA*BETA*AW(4,K) + (ALFA**2)*AW(5,K)/2.
       AW(13,K)=(BETA**2)*AW(4,K)/2. + (ALFA**2)*AW(6,K)/2.
      1+ALFA*BETA*AW(5,K)
       AW(14,K)=(BETA**2)*AW(5,K)/2. + ALFA*BETA*AW(6,K)
       AW(15,K)=(BETA**2)*AW(6,K)/2.
       AW(16,K)=(ALFA**3)*AW(4,K)/6.
       AW(17,K)=(ALFA**2)*BETA*AW(4,K)/2. + (ALFA**3)*AW(5,K)/6.
       AW(18,K)=ALFA*(BETA**2)*AW(4,K)/2. + (ALFA**3)*AW(6,K)/6.
```

```
      1+(ALFA**2)*BETA*AW(5,K)/2.
      AW(19,K)=(BETA**3)*AW(4,K)/6. + (ALFA**2)*BETA*AW(6,K)
      1+ALFA*(BETA**2)*AW(5,K)/2.
      AW(20,K)=(BETA**3)*AW(5,K)/6. + ALFA*(BETA**2)*AW(6,K)/2.
      AW(21,K)=(BETA**3)*AW(6,K)/6.
      GO TO 4710
C     SIMPLE BC FOR EDGE WITH ELASTIC RESTRAINT
 4604 SPR=SCR(JSIDE)/DP
      TM1=S*S+POIS*C*C
      TM2=-2.0*S*C*(1.-POIS)
      TM3=C*C+POIS*S*S
      AW(2,K)=SPR*S
      AW(3,K)=-SPR*C
      AW(4,K)=TM1+ALFA*SPR*S
      AW(5,K)=TM2+BETA*SPR*S-ALFA*SPR*C
      AW(6,K)=TM3-BETA*SPR*C
      AW(7,K)=ALFA*TM1+(ALFA**2)*SPR*S/2.
      AW(8,K)=BETA*TM1+ALFA*TM2+ALFA*BETA*SPR*S-(ALFA**2)*SPR*C/2.
      AW(9,K)=ALFA*TM3+BETA*TM2+(BETA**2)*SPR*S/2.-ALFA*BETA*SPR*C
      AW(10,K)=BETA*TM3-(BETA**2)*SPR*C/2.
      AW(11,K)=(ALFA**2)*TM1/2.+(ALFA**3)*SPR*S/6.
      AW(12,K)=ALFA*BETA*TM1+(ALFA**2)*TM2/2.+(ALFA**2)*BETA*SPR*S/2.
      1-(ALFA**3)*SPR*C/6.
      AW(13,K)=(BETA**2)*TM1/2.+(ALFA**2)*TM3/2.+ALFA*BETA*TM2
      1+ALFA*(BETA**2)*SPR*S/2.-(ALFA**2)*BETA*SPR*C/2.
      AW(14,K)=ALFA*BETA*TM3+(BETA**2)*TM2/2.+(BETA**3)*SPR*S/6.
      1-ALFA*BETA*BETA*SPR*C/2.
      AW(15,K)=(BETA**2)*TM3/2.- (BETA**3)*SPR*C/6.
      AW(16,K)=(ALFA**3)*TM1/6.+(ALFA**4)*SPR*S/24.
      AW(17,K)=(ALFA**2)*BETA*TM1/2.+(ALFA**3)*TM2/6.
      1+(ALFA**3)*BETA*SPR*S/6.-(ALFA**4)*C*SPR/24.
      AW(18,K)=ALFA*(BETA**2)*TM1/2.+(ALFA**3)*TM3/6.+(ALFA**2)*BETA*TM2
      1/2. + ((ALFA*BETA)**2)*SPR*S/4.-(ALFA**3)*BETA*SPR*C/6.
      AW(19,K)=(BETA**3)*TM1/6.+(ALFA**2)*BETA*TM3/2.+ALFA*(BETA**2)*TM2
      1/2.+ALFA*(BETA**3)*SPR*S/6.-((ALFA*BETA)**2)*SPR*C/4.
      AW(20,K)=ALFA*(BETA**2)*TM3/2.+(BETA**3)*TM2/6.+(BETA**4)*SPR*S/24
      1.-ALFA*(BETA**3)*SPR*C/6.
      AW(21,K)=(BETA**3)*TM3/6.-(BETA**4)*SPR*C/24.
      GO TO 4710
C     SIMPLE BC FOR FREE EDGEBEAM AND COMPLICATED BC FOR SIMPLY
C     SUPPORTED EDGEBEAM
 4605 T1=-(S*S+POIS*C*C)
      T2=2.0*S*C*(1.-POIS)
      T3=-(C*C+POIS*S*S)
      T4=GJ(JSIDE)*S*C*C/DP +HOR(JSIDE)*S*(1.+C*C*(1.-POIS))
      T5=GJ(JSIDE)*C*(2.-3.0*C*C)/DP +HOR(JSIDE)*C*(1.-3.0*C*C
      1-POIS*(3.0*S*S-1.0))
      T6=GJ(JSIDE)*S*(3.0*S*S-2.)/DP +HOR(JSIDE)*S*(3.0*S*S-1.0
      1-POIS*(1.-3.0*C*C))
      T7=-GJ(JSIDE)*S*S*C/DP +HOR(JSIDE)*C*(-1.-S*S*(1.-POIS))
      AW(4,K)=T1
      AW(5,K)=T2
      AW(6,K)=T3
      AW(7,K)=ALFA*T1+T4
      AW(8,K)=BETA*T1+ALFA*T2+T5
      AW(9,K)=BETA*T2+ALFA*T3+T6
      AW(10,K)=BETA*T3+T7
```

```fortran
      AW(11,K)=ALFA*ALFA*T1/2.+ALFA*T4
      AW(12,K)=ALFA*BETA*T1+(ALFA**2)*T2/2.+BETA*T4+ALFA*T5
      AW(13,K)=(BETA**2)*T1/2.+ALFA*BETA*T2+(ALFA**2)*T3/2.+BETA*T5
     1+ALFA*T6
      AW(14,K)=(BETA**2)*T2/2.+ALFA*BETA*T3+BETA*T6+ALFA*T7
      AW(15,K)=(BETA**2)*T3/2.+BETA*T7
      AW(16,K)=(ALFA**3)*T1/6.+(ALFA**2)*T4/2.
      AW(17,K)=ALFA*ALFA*BETA*T1/2.+(ALFA**3)*T2/6.+ALFA*BETA*T4
     1+ALFA*ALFA*T5/2.
      AW(18,K)=ALFA*BETA*BETA*T1/2.+ALFA*ALFA*BETA*T2/2.+(ALFA**3)*T3/6.
     1+BETA*BETA*T4/2.+ALFA*BETA*T5+ALFA*ALFA*T6/2.
      AW(19,K)=(BETA**3)*T1/6.+ALFA*BETA*BETA*T2/2.+ALFA*ALFA*BETA*T3/2.
     1+BETA*BETA*T5/2.+ALFA*BETA*T6+ALFA*ALFA*T7/2.
      AW(20,K)=(BETA**3)*T2/6.+ALFA*BETA*BETA*T3/2.+BETA*BETA*T6/2.
     1+ALFA*BETA*T7
      AW(21,K)=(BETA**3)*T3/6.+BETA*BETA*T7/2.
      GO TO 4710
 4700 GO TO (4701,4702,4704,4705,4707,4605), LYNN
C     COMPLICATED BC FOR FIXED EDGE
 4701 AW(2,K)=S
      AW(3,K)=-C
      AW(4,K)=ALFA*S
      AW(5,K)=BETA*S-ALFA*C
      AW(6,K)=-BETA*C
      AW(7,K)=(ALFA**2)*S/2.
      AW(8,K)=ALFA*BETA*S-(ALFA**2)*C/2.
      AW(9,K)=(BETA**2)*S/2. -ALFA*BETA*C
      AW(10,K)= -(BETA**2)*C/2.
      AW(11,K)= (ALFA**3)*S/6.
      AW(12,K)=(ALFA**2)*BETA*S/2.-(ALFA**3)*C/6.
      AW(13,K)=(BETA**2)*ALFA*S/2.-(ALFA**2)*BETA*C/2.
      AW(14,K)=(BETA**3)*S/6.-(BETA**2)*ALFA*C/2.
      AW(15,K)=-(BETA**3)*C/6.
      AW(16,K)=(ALFA**4)*S/24.
      AW(17,K)=(ALFA**3)*BETA*S/6.-(ALFA**4)*C/24.
      AW(18,K)=((ALFA*BETA)**2)*S/4.-(ALFA**3)*BETA*C/6.
      AW(19,K)=ALFA*(BETA**3)*S/6.-((ALFA*BETA)**2)*C/4.
      AW(20,K)=(BETA**4)*S/24.-ALFA*(BETA**3)*C/6.
      AW(21,K)=-(BETA**4)*C/24.
      GO TO 4710
C     COMPLICATED BC FOR SIMPLY SUPPORTED EDGE
 4702 AW(4,K)=1.
      AW(6,K)=1.
      AW(7,K)=ALFA
      AW(8,K)=BETA
      AW(9,K)=ALFA
      AW(10,K)=BETA
      AW(11,K)=(ALFA**2)/2.
      AW(12,K)=ALFA*BETA
      AW(13,K)=(ALFA**2)/2.+(BETA**2)/2.
      AW(14,K)=ALFA*BETA
      AW(15,K)=(BETA**2)/2.
      AW(16,K)=(ALFA**3)/6.
      AW(17,K)=(ALFA**2)*BETA/2.
      AW(18,K)=0.5*ALFA*(BETA**2)+(ALFA**3)/6.
      AW(19,K)=(BETA**3)/6.+(ALFA**2)*BETA/2.
      AW(20,K)=ALFA*(BETA**2)/2.
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```
          AW(21,K)=(BETA**3)/6.
          GO TO 4710
C         COMPLICATED BC FOR FREE EDGE
   4704   AW(7,K)=S*(1.0+(C**2)*(1.-POIS))
          AW(8,K)=C*(1.0-3.0*(C**2)-POIS*(3.0*(S**2)-1.))
          AW(9,K)=S*(3.0*(S**2)-1.0-POIS*(1.0-3.0*(C**2)))
          AW(10,K)=C*(-1.-(S**2)*(1.-POIS))
          AW(11,K)=ALFA*AW(7,K)
          AW(12,K)=BETA*AW(7,K)+ALFA*AW(8,K)
          AW(13,K)=BETA*AW(8,K)+ALFA*AW(9,K)
          AW(14,K)=BETA*AW(9,K)+ALFA*AW(10,K)
          AW(15,K)=BETA*AW(10,K)
          AW(16,K)=(ALFA**2)*AW(7,K)/2.
          AW(17,K)=ALFA*BETA*AW(7,K)+(ALFA**2)*AW(8,K)/2.
          AW(18,K)=(BETA**2)*AW(7,K)/2.+ALFA*BETA*AW(8,K)+(ALFA**2)*AW(9,K)/
         12.
          AW(19,K)=(BETA**2)*AW(8,K)/2.+ALFA*BETA*AW(9,K)+(ALFA**2)*AW(10,K)
         1/2.
          AW(20,K)=(BETA**2)*AW(9,K)/2.+ALFA*BETA*AW(10,K)
          AW(21,K)=(BETA**2)*AW(10,K)/2.
          GO TO 4710
C         COMPLICATED BC FOR ELASTICALLY RESTRAINED EDGE
   4705   SPD=SCD(JSIDE)/DP
          TM1=S*(1.+C*C*(1.-POIS))
          TM2=C*(1.-3.0*C*C-POIS*(3.0*S*S-1.))
          TM3=S*(3.0*S*S-1.-POIS*(1.-3.0*C*C))
          TM4=C*(-1.-S*S*(1.-POIS))
          AW(1,K)=SPD
          AW(2,K)=ALFA*SPD
          AW(3,K)=BETA*SPD
          AW(4,K)=(ALFA**2)*SPD/2.
          AW(5,K)=ALFA*BETA*SPD
          AW(6,K)=(BETA**2)*SPD/2.
          AW(7,K)=(ALFA**3)*SPD/6.+TM1
          AW(8,K)=(ALFA**2)*BETA*SPD/2.+TM2
          AW(9,K)=ALFA*(BETA**2)*SPD/2.+TM3
          AW(10,K)=(BETA**3)*SPD/6. +TM4
          AW(11,K)=(ALFA**4)*SPD/24.+ALFA*TM1
          AW(12,K)=(ALFA**3)*BETA*SPD/6. +BETA*TM1+ALFA*TM2
          AW(13,K)=((ALFA*BETA)**2)*SPD/4. +BETA*TM2+ALFA*TM3
          AW(14,K)=ALFA*(BETA**3)*SPD/6. +BETA*TM3 +ALFA*TM4
          AW(15,K)=(BETA**4)*SPD/24. +BETA*TM4
          AW(16,K)=(ALFA**5)*SPD/120. +(ALFA**2)*TM1/2.
          AW(17,K)=(ALFA**4)*BETA*SPD/24.+ALFA*BETA*TM1+(ALFA**2)*TM2/2.
          AW(18,K)=(ALFA**3)*(BETA**2)*SPD/12.+(BETA**2)*TM1/2.
         1+ALFA*BETA*TM2+(ALFA**2)*TM3/2.
          AW(19,K)=(ALFA**2)*(BETA**3)*SPD/12.+(BETA**2)*TM2/2.
         1+ALFA*BETA*TM3+(ALFA**2)*TM4/2.
          AW(20,K)=ALFA*(BETA**4)*SPD/24.+(BETA**2)*TM3+ALFA*BETA*TM4
          AW(21,K)=(BETA**5)*SPD/120. +(BETA**2)*TM4/2.
          GO TO 4710
C         COMPLICATED BC FOR FREE EDGEBEAM
   4707   T1=S*(1.+C*C*(1.-POIS))
          T2=C*(1.-3.0*C*C-POIS*(3.0*S*S-1.))
          T3=S*(3.0*S*S-1.-POIS*(1.-3.0*C*C))
          T4=C*(-1.-S*S*(1.-POIS))
          T5=EI(JSIDE)*(C**4)/DP
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      T6=4.0*EI(JSIDE)*(C**3)*S/DP
      T7=6.0*EI(JSIDE)*S*S*C*C/DP
      T8=4.0*EI(JSIDE)*C*(S**3)/DP
      T9=EI(JSIDE)*(S**4)/DP
      AW(7,K)=T1
      AW(8,K)=T2
      AW(9,K)=T3
      AW(10,K)=T4
      AW(11,K)=ALFA*T1+T5
      AW(12,K)=BETA*T1+ALFA*T2+T6
      AW(13,K)=BETA*T2+ALFA*T3+T7
      AW(14,K)=BETA*T3+ALFA*T4+T8
      AW(15,K)=BETA*T4+T9
      AW(16,K)=(ALFA**2)*T1/2. +ALFA*T5
      AW(17,K)=ALFA*BETA*T1+(ALFA**2)*T2/2.+BETA*T5+ALFA*T6
      AW(18,K)=(BETA**2)*T1/2.+ALFA*BETA*T2+(ALFA**2)*T3/2.+BETA*T6
     1+ALFA*T7
      AW(19,K)=(BETA**2)*T2/2.+ALFA*BETA*T3+(ALFA**2)*T4/2.+BETA*T7
     1+ALFA*T8
      AW(20,K)=(BETA**2)*T3/2.+ALFA*BETA*T4+BETA*T8+ALFA*T9
      AW(21,K)=(BETA**2)*T4/2.+BETA*T9
 4710 KSTAR(K)=0.
      K=K+1
 3220 KD=KD+1
      IF(D-1.414*H)  4550,4550,3090
 4550 IF(KD-4)  3110,4551,3090
 4551 IF(D-H)  3110,3110,3090
 3091 NOBC=NOBC+1
 3090 CONTINUE
      IF(NOBC-NS)  4444,4445,4445
 4445 PRINT 17, LP(I)
   17 FORMAT(/28H NO BCS FORMULATED FOR POINT, I6//)
      GO TO 3089
 4444 NBC=K-1
      CALL SSWTCH(5,LL)
      IF(LL-2)  4450,4460,4460
 4450 PRINT 100, LP(I),NBC
  100 FORMAT(////29H EXPANSIONS FOR BCS FOR POINT,I7,3X,4HNBC=,I5//)
      PRINT 101, ((AW(IS,JS),JS=1,8),IS=1,21)
  101 FORMAT(8F10.3)
 4460 WRITE (1) ((AW(IP,JP),JP=1,8),IP=1,21),NBC
 3089 CONTINUE
      CALL CHNXIT
      END
$IBSYS
$IBFTC DIST
      SUBROUTINE DIST (CPC,H,LP,I,MCONS,IDIST,D,XD,YD,X1,X2,Y1,Y2,X3,Y3,
     1NS,MC)
      DIMENSION CPC(100,2)
      DIMENSION LP(1000)
      DIMENSION MCONS(4,100)
      DIMENSION MC(100)
      ISIDE=MCONS(1,IDIST)
      JSID2=MCONS(2,IDIST)
      KSID2=MCONS(3,IDIST)
      DO 2310 IS=1,NS
      IF(MC(IS)-JSID2)2320,2330,2320
```

```
2320  IF(MC(IS)-KSID2)2310,2340,2310
2330  JSIDE=IS
      GO TO 2310
2340  KSIDF=IS
2310  CONTINUE
      X1=CPC(JSIDF,1)
      Y1=CPC(JSIDE,2)
      X2=CPC(KSIDF,1)
      Y2=CPC(KSIDE,2)
      CONX=LP(I)
      ICONX=LP(I)/100
      CON2=ICONX
      CONX=(CONX-CON2*100.)
      CONY=CON2
      X3=CONX*H
      Y3=CONY*H
      XD=(X3*((X1-X2)**2)+X1*((Y1-Y2)**2)-(X1-X2)*(Y1-Y2)*(Y1-Y3))/(((X1
     1-X2)**2)+((Y1-Y2)**2))
      YD=((Y3*(Y2-Y1)**2)+(Y1*(X2-X1)**2)+(X3*(Y2-Y1)*(X2-X1))-(X1*(X2-X
     11 )*(Y2-Y1)))/(((Y2-Y1)**2)+((X2-X1)**2))
      D=SQRT((X3-XD)**2+(Y3-YD)**2)
      RETURN
      END
$IRFTC CLGRD
      SUBROUTINE CLGRD(LP,NP,LPC)
      DIMENSION LP(1000)
      DIMENSION Q(1000)
      DIMENSION LPC(1000)
      DO 4000 I=1,NP
      DO 4001 M=1,37
      CALL RELABS(M,I,LP,LPRA)
      DO 4003 J=1,NP
      IF(LP(J)-LPRA) 4003,4001,4003
4003  CONTINUE
      LPC(I)=2
      PRINT 4008, LP(I),LPC(I)
      GO TO 4000
4001  CONTINUE
      LPC(I)=1
      PRINT 4008, LP(I),LPC(I)
4008  FORMAT(7H POINT=,I7,3X,4HLPC=,I4)
4000  CONTINUE
      RETURN
      END
$IRFTC RELABS
      SUBROUTINE RELABS(M,I,LP,LPRA)
      DIMENSION LP(1000)
      GO TO (80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99
     1,100,101,102,103,104,105,106,107,108,109,110,111,112,113,114,115,1
     216),M
 80   LPRA=LP(I)
      GO TO 120
 81   LPRA=LP(I)+1
      GO TO 120
 82   LPRA=LP(I)-1
      GO TO 120
 83   LPRA=LP(I)+100
```

```
        GO TO 120
   84 LPRA=LP(I)-100
        GO TO 120
   85 LPRA=LP(I)+101
        GO TO 120
   86 LPRA=LP(I)-101
        GO TO 120
   87 LPRA=LP(I)+99
        GO TO 120
   88 LPRA=LP(I)-99
        GO TO 120
   89 LPRA=LP(I)+2
        GO TO 120
   90 LPRA=LP(I)-2
        GO TO 120
   91 LPRA=LP(I)+200
        GO TO 120
   92 LPRA=LP(I)-200
        GO TO 120
   93 LPRA=LP(I)+102
        GO TO 120
   94 LPRA=LP(I)-102
        GO TO 120
   95 LPRA=LP(I)+199
        GO TO 120
   96 LPRA=LP(I)-199
        GO TO 120
   97 LPRA=LP(I)+201
        GO TO 120
   98 LPRA=LP(I)-201
        GO TO 120
   99 LPRA=LP(I)-98
        GO TO 120
  100 LPRA=LP(I)+98
        GO TO 120
  101 LPRA=LP(I)+202
        GO TO 120
  102 LPRA=LP(I)-202
        GO TO 120
  103 LPRA=LP(I)+198
        GO TO 120
  104 LPRA=LP(I)-198
        GO TO 120
  105 LPRA=LP(I)+3
        GO TO 120
  106 LPRA=LP(I)-3
        GO TO 120
  107 LPRA=LP(I)+300
        GO TO 120
  108 LPRA=LP(I)-300
        GO TO 120
  109 LPRA=LP(I)+103
        GO TO 120
  110 LPRA=LP(I)-103
        GO TO 120
  111 LPRA=LP(I)+299
        GO TO 120
```

```
112 LPRA=LP(I)-299
    GO TO 120
113 LPRA=LP(I)+301
    GO TO 120
114 LPRA=LP(I)-301
    GO TO 120
115 LPRA=LP(I)+97
    GO TO 120
116 LPRA=LP(I)-97
120 RETURN
    END
```

```
                    LINK3.........ASSEMBLAGE OF DIFFERENCE MOLECULES.


      $IBFTC LINK3
            DIMENSION MCONS(4,100)
            DIMENSION COSS(100)
            DIMENSION SINS(100)
            DIMENSION AW(21,34)
            DIMENSION GJ(100)
            DIMENSION SCR(100)
            DIMENSION SCD(100)
            DIMENSION EI(100)
            DIMENSION HOR(100)
            DIMENSION SUM(3)
            DIMENSION SUM1(3)
            DIMENSION SUM2(3)
            DIMENSION SUM3(3)
            DIMENSION TITLE(12)
            DIMENSION MC(100)
            DIMENSION CPC(100,2)
            DIMENSION LBPE(1000)
            DIMENSION LBPSE(1000)
            DIMENSION XBPE(1000)
            DIMENSION YBPE(1000)
            DIMENSION LP(1000)
            DIMENSION LPC(1000)
            DIMENSION Q(1000,3)
            DIMENSION KSTAR(30)
            DIMENSION RDFL(21)
            DIMENSION RMXX(21)
            DIMENSION RMYY(100)
            DIMENSION RMXY(100)
            DIMENSION A1(63,63),A2(63,60)
            COMMON TITLE,MCONS,MC,CPC,LBPE,LBPSE,XBPE,YBPE,COSS,SINS,LP,LPC,Q,
           1INTSW,IEXQ,INEXT,NS,NC,NP,NL,MDIM,H,MFIN,JSIDE,KD,DP,POIS,EI,GJ,HO
           2R,SCR,SCD
            REWIND 1
            REWIND 2
            REWIND 3
            REWIND 4
      C     CALCULATE NUMBERS OF ROWS OF SUBMATRICES
            NOM=NP/MDIM+1
      C     CLEAR BAND MATRIX
            DO 8200 MAT=1,NOM
            DO 8000 I=1,MDIM
            DO 8000 J=1,MDIM
            A1(I,J)=0.
      8000 A2(I,J)=0.
            DO 8170 MROW=1,MDIM
      C     CLEAR RIGHT HAND SIDE OF JUDY
            DO 3010 J=1,21
            RDFL(J)=0.
            RMXX(J)=0.
            RMYY(J)=0.
      3010 RMXY(J)=0.
      C     DETERMINE ROW IN LARGE BAND MATRIX AT WHICH WE ARE NOW WORKING
```

```
        ITWO=MDIM*(MAT-1)+MROW
        IF(ITWO-NP) 8005,8005,8210
C       CLEAR LEFT HAND SIDE OF JUDY
 8005 DO 3040 IS=1,30
 3040 KSTAR(IS)=0
        DO 3031 IR=1,21
        DO 3031 JR=1,34
 3031 AW(IR,JR)=0.
C       IS GRIDPOINT AN INTERIOR POINT OR A BOUNDARY POINT
        IF(LPC(ITWO)-2) 4800,4799,4799
C       DETERMINATION OF STANDARD OPERATORS
C       STANDARD OPERATOR FOR RDEL(J)
 4800 I=ITWO
        DO 4850 M=1,13
        CALL RELABS(M,I,LP,LPRA)
        DO 4851 J=1,NP
        IF(LPRA-LP(J)) 4851,4872,4851
 4851 CONTINUE
 4872 IF(M-2) 4852,4853,4854
 4852 AL=20.
        GO TO 4860
 4853 AL=-8.
        GO TO 4860
 4854 IF(M-6) 4853,4855,4856
 4855 AL=2.
        GO TO 4860
 4856 IF(M-10) 4855,4857,4857
 4857 AL=1.
C       DETERMINE WHETHER TO READ OPERATOR INTO A1,A2, OR A3 SUBMATRICES
 4860 IF(J-MDIM*(MAT-1)) 4861,4861,4862
 4861 K=J-MDIM*(MAT-2)
        A1(MROW,K)=AL
        GO TO 4850
 4862 IF(J-MDIM*MAT) 4863,4863,4864
 4863 K=J-MDIM*(MAT-1)
        A2(MROW,K)=AL
        GO TO 4850
 4864 K=J-MDIM*MAT
        A1(MROW,K)=AL
 4850 CONTINUE
        DO 4869 LYNN=1,NL
        SUM1(LYNN)=0.
        SUM2(LYNN)=0.
        SUM3(LYNN)=0.
 4869 SUM(LYNN)=0.
        GO TO 9000
C       DETERMINATION OF OPERATORS FOR BOUNDARY POINTS
 4799 READ (1) ((AW(I,J),J=1,8),I=1,21),NBC
C       DETERMINE COORDINATES OF RELATIVE POINT M
        I=ITWO
        MI=0
        M=1
        K=NBC+1
 4900 CALL RELABS(M,I,LP,LPRA)
C       DOES RELATIVE POINT M LIE ON PLATE SURFACE
        DO 4901 J=1,NP
        IF(LPRA-LP(J)) 4903,4902,4903
```

```
      4903 IF(J-NP) 4901,4908,4908
      4901 CONTINUE
C          POINT LIES ON PLATE.    GET SERIES EXPANSION FOR POINT
      4902 CALL ALBETA(M,ALFA,BETA)
           IF(MI) 4906,4905,4906
      4905 CALL TSW(ALFA,BETA,K,AW)
           KSTAR(K)=LPRA
           GO TO 4907
      4906 CALL TSMPO(ALFA,BETA,AW,K)
           KSTAR(K)=-LPRA
      4907 K=K+1
           IF(K-31) 4908,4910,4908
      4908 IF(M-37) 4904,4909,4909
      4904 M=M+1
           GO TO 4900
      4909 IF(MI) 4911,4911,4910
      4911 MI=1
           M=2
           GO TO 4900
C          MOVE RIGHT HAND SIDES INTO AW MATRIX
      4910 IF(K-32) 4914,4915,4916
      4914 AW(11,K)=1.
           AW(13,K)=2.
           AW(15,K)=1.
           GO TO 4920
      4915 AW(4,K)=-1.
           AW(6,K)=-POIS
           GO TO 4920
      4916 IF(K-34) 4918,4917,4917
      4918 AW(4,K)=-POIS
           AW(6,K)=-1.
      4920 K=K+1
           GO TO 4910
      4917 AW(5,K)=-(1.0-POIS)
           CALL SSWTCH(6,MM)
           IF(MM-2) 4777,4778,4778
      4777 PRINT 553, LP(ITWO)
       553 FORMAT(//9H AT POINT,I5,2X,11HBEFORE JUDY//)
           DO 554 JT=1,30
       554 PRINT 555, (AW(IT,JT),IT=1,21)
       555 FORMAT(1X,21F5.1)
      4778 CALL JUDY (AW,21,30,KSTAR,NBC,LP,ITWO,IEXQ,4)
           PRINT 556, LP(ITWO)
       556 FORMAT(//20H AFTER JUDY AT POINT,I5///)
           PRINT 599
       599 FORMAT(2X,6HDISPL.,6X,3HMXX,7X,3HMYY,7X,3HMXY,12X,5HPOINT///)
           DO 557 J=1,21
       557 PRINT 558, (AW(J,I),I=31,34),KSTAR(J)
       558 FORMAT(1X,4F10.2,8X,I6//)
           DO 4919 I=1,21
           RDFL(I)=AW(I,31)
           RMXX(I)=AW(I,32)
           RMYY(I)=AW(I,33)
           RMXY(I)=AW(I,34)
      4919 CONTINUE
           DO 3711 LYNN=1,NL
           SUM1(LYNN)=0.
```

```
        SUM2(LYNN)=0.
        SUM3(LYNN)=0.
  3711  SUM(LYNN)=0.
C       READ RDEL(I) INTO BAND MATRIX
        DO 5000 I=1,21
        IF(KSTAR(I)) 5008,5000,5011
  5008  LYNN=-KSTAR(I)
        GO TO 5009
  5011  LYNN=KSTAR(I)
  5009  DO 5001 K=1,NP
        IF(LYNN-LP(K)) 5001,5012,5001
  5001  CONTINUE
C       ELEMENT GOES INTO COLUMN K OF BAND MATRIX
C       DETERMINE WHETHER TO READ ELEMENT INTO A1,A2, OR A3 SUBMATRICES
  5012  IF(K-MDIM*(MAT-1)) 5002,5002,5003
  5002  L=K-MDIM*(MAT-2)
        A1(MROW,L)=RDEL(I)
        GO TO 5010
  5003  IF(K-MAT*MDIM) 5004,5004,5005
  5004  L=K-MDIM*(MAT-1)
        A2(MROW,L)=RDEL(I)
        GO TO 5010
  5005  L=K-MAT*MDIM
        A1(MROW,L)=RDEL(I)
C       DETERMINE EFFECTIVE LOAD AT GRIDPOINT(RHS OF BAND MATRIX)
  5010  IF(KSTAR(I)) 5006,5000,5000
C       ONLY MULTI-POINT OPERATOR AFFECTS LOAD AT GRIDPOINT
  5006  DO 5007 LYNN=1,NL
        SUM1(LYNN)=SUM1(LYNN)+RMXX(I)*Q(K,LYNN)
        SUM2(LYNN)=SUM2(LYNN)+RMYY(I)*Q(K,LYNN)
        SUM3(LYNN)=SUM3(LYNN)+RMXY(I)*Q(K,LYNN)
  5007  SUM(LYNN)=SUM(LYNN)+RDEL(I)*Q(K,LYNN)
  5000  CONTINUE
  9000  DO 3712 KM=1,NL
        Q(ITWO,KM)=Q(ITWO,KM)-SUM(KM)
        PRINT 560, KM,LP(ITWO)
   560  FORMAT(18H LOADING CONDITION,I3,5X,5HPOINT,I5)
  3712  PRINT 561, Q(ITWO,KM)
   561  FORMAT(15X,15HEFFECTIVE LOAD=F7.2//)
        WRITE (3) (KSTAR(I),RMXX(I),RMYY(I),RMXY(I),I=1,21),(SUM1(L),SUM2(
       1L),SUM3(L),L=1,NL)
  8170  CONTINUE
        GO TO 8175
C       IF LAST SUBMATRIX NOT OF ORDER MDIM, ADJUST BAND MATRIX
  8210  MBIG=NOM*MDIM
        NP1=NP+1
        DO 8211 I=NP1,MBIG
        K=I-MDIM*(NOM-1)
  8211  A2(K,K)=1.0
        DO 8071 I=IROW,MBIG
        DO 8071 K=1,NL
  8071  Q(I,K)=0.0
  8175  WRITE (4) ((A2(I,J),J=1,MDIM),I=1,MDIM)
        DO 8176 I=1,MDIM
        DO 8176 J=1,MDIM
  8176  A2(I,J)=0.0
        DO 8177 I=1,MDIM
```

```
        J=1
        DO 8177 K=1,J
        A2(I,K)=A1(I,K)
 8177   A1(I,K)=0.0
        WRITE (2) ((A1(I,J),J=1,MDIM),I=1,MDIM)
 8200   WRITE (2) ((A2(I,J),J=1,MDIM),I=1,MDIM)
        REWIND 2
        REWIND 4
        CALL CHNXIT
        END
$IBFTC JUDY      LIST
        SUBROUTINE JUDY(A,JIM1,JIM4,KVECT,NBC,LP,ITWO,IEXQ,NRHS)
        DIMENSION A(21,42),KVECT(37),LP(1000)
        IF(NRHS-3) 2560,2570,2570
 2570   NCHEC=3
        GO TO 2580
 2560   NCHEC=NRHS
 2580   JIM2=JIM4+NRHS
        DO 2529 I=1,JIM1
        DO 2529 K=1,NRHS
        JIM3=JIM4+K
 2529   A(I,JIM3)=-A(I,JIM3)
        IF(NBC) 2524,2524,2525
 2525   NPS=NBC
        GO TO 2527
 2524   NPS=JIM4
 2527   J=1
 2515   PIVOT=0.0
        DO 2500 IP=J,NPS
        DO 2500 JP=J,JIM1
        IF(ABS(PIVOT)-ABS(A(JP,IP))) 2501,2500,2500
 2501   PIVOT=A(JP,IP)
        IPIV=JP
        JPIV=IP
 2500   CONTINUE
        IF(ABS(PIVOT)-.00100)2541,2541,2502
 2502   DO 2504 IEXC=1,JIM1
        ATEMP=A(IEXC,JPIV)
        A(IEXC,JPIV)=A(IEXC,J)
 2504   A(IEXC,J)=ATEMP
        DO 2505 JEXC=1,JIM2
        ATEMP=A(IPIV,JEXC)
        A(IPIV,JEXC)=A(J,JEXC)
 2505   A(J,JEXC)=ATEMP
        ITEMP=KVECT(J)
        KVECT(J)=KVECT(JPIV)
        KVECT(JPIV)=ITEMP
        DO 2506 IKELR=1,JIM2
 2506   A(J,IKELR)=A(J,IKELR)/(-1.*PIVOT)
        K=J+1
        DO 2507 IELSE=1,JIM1
        IF(IELSE-J)2508,2507,2508
 2508   DO 2509 JELSE=K,JIM2
 2509   A(IELSE,JELSE)=A(IELSE,JELSE)+(A(IELSE,J)*A(J,JELSE))
 2507   CONTINUE
        DO 2510 ICOL=1,JIM1
 2510   A(ICOL,J)=A(ICOL,J)/PIVOT
```

```
      A(J,J)=0.0
      A(J,J)=+1.0/PIVOT
      J=J+1
      IF(J-JIM1-1)2511,2512,2512
2511  IF(J-NPS-1)2513,2514,2514
2514  NPS=NPS+1
2513  IF(NPS-1-JIM4) 2515,2503,2503
2503  DO 2516 I=J,JIM1
      DO 2516 K=1,NCHEC
      JIM3=JIM4+K
      IF(ABS(A(I,JIM3))-.000100) 2516,2516,2521
2516  CONTINUE
2520  DO 2522 I=J,JIM4
2522  KVECT(I)=0
      DO 2517 I=J,JIM1
      DO 2517 K=1,NRHS
      JIM3=JIM4+K
2517  A(I,JIM3)=0.0
2512  RETURN
2541  DO 2543 I=1,JIM1
      DO 2542 K=1,NCHEC
      JIM3=JIM4+K
      IF(ABS(A(I,JIM3))-.0001) 2542,2542,2514
2542  CONTINUE
2543  CONTINUE
      GO TO 2520
2521  PRINT 2550,LP(ITWO)
      PRINT 389,J,IPIV,JPIV
      DO 289 I=1,JIM1
289   PRINT 290,(A(I,M),M=1,JIM2)
      PRINT 400,(KVECT(I),I=1,37)
      RETURN
290   FORMAT(/20F6.2)
389   FORMAT (1H1,2HJ=,I4,//6H IPIV=,I4,5HJPIV=I4//)
400   FORMAT (//20I6)
2550  FORMAT (5X,59HTHERE ARE INSUFFICIENT POINTS IN THE NEIGHBOURHOOD O
     1F POINT,I5,46H TO ASSEMBLE THE REQUIRED DIFFERENCE EQUATION ,/50X1
     23HAT THAT POINT//9X,31HHOWEVER EXECUTION WILL CONTINUE/////)
      END
$IBFTC ALBETA   LIST
      SUBROUTINE ALBETA(M,ALFA,BETA)
      GO TO (2601,2602,2603,2604,2605,2606,2607,2608,2609,2610,2611,2612
     1,2613,2614,2615,2616,2617,2618,2619,2620,2621,2622,2623,2624,2625,
     22626,2627,2628,2629,2630,2631,2632,2633,2634,2635,2636,2637,2638,2
     3639,2640,2641,2642,2643,2644,2645,2646,2647,2648,2649),M
2601  ALFA=0.0
      BETA= 0.0
      RETURN
2602  ALFA=1.0
      BETA= 0.0
      RETURN
2603  ALFA=-1.
      BETA=0.0
      RETURN
2604  ALFA=0.0
      BETA= 1.0
      RETURN
```

```
2605 ALFA=0.0
     BETA=-1.0
     RETURN
2606 ALFA=1.0
     BETA= 1.0
     RETURN
2607 ALFA=-1.0
     BETA=-1.0
     RETURN
2608 ALFA=-1.0
     BETA= 1.0
     RETURN
2609 ALFA=1.0
     BETA=-1.0
     RETURN
2610 ALFA=2.0
     BETA= 0.0
     RETURN
2611 ALFA=-2.0
     BETA= 0.0
     RETURN
2612 ALFA=0.0
     BETA= 2.0
     RETURN
2613 ALFA=0.0
     BETA=-2.0
     RETURN
2614 ALFA=2.0
     BETA= 1.0
     RETURN
2615 ALFA=-2.0
     BETA=-1.0
     RETURN
2616 ALFA=-1.0
     BETA= 2.0
     RETURN
2617 ALFA=1.0
     BETA= -2.0
     RETURN
2618 ALFA=1.0
     BETA= 2.0
     RETURN
2619 ALFA=-1.0
     BETA=-2.0
     RETURN
2620 ALFA=2.0
     BETA=-1.0
     RETURN
2621 ALFA=-2.0
     BETA= 1.0
     RETURN
2622 ALFA= 2.0
     BETA= 2.0
     RETURN
2623 ALFA=-2.0
     BETA=-2.0
     RETURN
```

```
2624  ALFA=-2.0
      BETA= 2.0
      RETURN
2625  ALFA=+2.0
      BETA=-2.0
      RETURN
2626  ALFA=+3.0
      BETA=+0.0
      RETURN
2627  ALFA=-3.0
      BETA=+0.0
      RETURN
2628  ALFA=+0.0
      BETA=+3.0
      RETURN
2629  ALFA=-0.0
      BETA=-3.0
      RETURN
2630  ALFA=+3.0
      BETA=+1.0
      RETURN
2631  ALFA=-3.0
      BETA=-1.0
      RETURN
2632  ALFA=-1.0
      BETA=+3.0
      RETURN
2633  ALFA=+1.0
      BETA=-3.0
      RETURN
2634  ALFA=+1.0
      BETA=+3.0
      RETURN
2635  ALFA=-1.0
      BETA=-3.0
      RETURN
2636  ALFA=-3.0
      BETA=+1.0
      RETURN
2637  ALFA=+3.0
      BETA=-1.0
      RETURN
2638  ALFA=+3.0
      BETA=+2.0
      RETURN
2639  ALFA=-3.0
      BETA=-2.0
      RETURN
2640  ALFA=-2.0
      BETA=+3.0
      RETURN
2641  ALFA=+2.0
      BETA=-3.0
      RETURN
2642  ALFA=+2.0
      BETA=+3.0
      RETURN
```

```
2643 ALFA=-2.0
     BETA=-3.0
     RETURN
2644 ALFA=-3.0
     BETA=+2.0
     RETURN
2645 ALFA=+3.0
     BETA=-2.0
     RETURN
2646 ALFA=+4.0
     BETA=+0.0
     RETURN
2647 ALFA=-4.0
     BETA=+0.0
     RETURN
2648 ALFA=-0.0
     BETA=+4.0
     RETURN
2649 ALFA=+0.0
     BETA=-4.0
     RETURN
     END
$IBFTC TSMPO   LIST
     SUBROUTINE TSMPO(ALFA,BETA,AW,K)
     DIMENSION AW(21,34)
     DO 3269 I=1,21
3269 AW(I,K)=0.
     AW(11,K)=1.
     AW(13,K)=2.
     AW(15,K)=1.
     AW(16,K)=ALFA
     AW(17,K)=BETA
     AW(18,K)=2.0*ALFA
     AW(19,K)=2.0*BETA
     AW(20,K)=ALFA
     AW(21,K)=BETA
     RETURN
     END
$IBFTC TSW     LIST
     SUBROUTINE TSW (ALFA,BETA,K,AW)
     DIMENSION AW(21,34)
     AW(1,K)=1.0
     AW(2,K)=ALFA
     AW(3,K)=BETA
     AW(4,K)=ALFA*ALFA/2.0
     AW(5,K)=ALFA*BETA
     AW(6,K)=BETA*BETA/2.0
     AW(7,K)=(ALFA**3)/6.0
     AW(8,K)=ALFA*ALFA*BETA/2.0
     AW(9,K)=ALFA*BETA*BETA/2.0
     AW(10,K)=(BETA**3)/6.0
     AW(11,K)=(ALFA**4)/24.0
     AW(12,K)=(ALFA**3)*BETA/6.0
     AW(13,K)=(ALFA**2)*(BETA**2)/4.0
     AW(14,K)=ALFA*(BETA**3)/6.0
     AW(15,K)=(BETA**4)/24.0
     AW(16,K)=(ALFA**5)/120.0
```

```
AW(17,K)=(ALFA**4)*BETA/24.0
AW(18,K)=(ALFA**3)*(BETA**2)/12.0
AW(19,K)=(ALFA**2)*(BETA**3)/12.0
AW(20,K)=ALFA*(BETA**4)/24.0
AW(21,K)=(BETA**5)/120.0
RETURN
END
```