This technical memorandum consists of the unaltered thesis of James J. Beville, Jr., submitted in partial fulfillment of the requirements for the degree of Master of Science at the Massachusetts Institute of Technology in June, 1968. The preparation and publication of this report, including the research on which it is based, was supported in part under a grant to the Massachusetts Institute of Technology by the General Motors Corporation for highway transportation and safety research. This study was conducted at the Electronic Systems Laboratory under MIT DSR Project Number 79723.
MERGING AND MAINSTREAM CONTROL TECHNIQUES
FOR AN AUTOMATED HIGHWAY SYSTEM

by

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MERGING AND MAINSTREAM CONTROL TECHNIQUES FOR AN AUTOMATED HIGHWAY SYSTEM

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Submitted to the Department of Electrical Engineering on May 17, 1968 in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

It is desired to develop methods and criteria for evaluating sensing and control devices for vehicle traffic flow during normal mainstream driving and merging type situations before the expensive phases of breadboard experimentation are carried out. A secondary goal is to determine an optimal manner in which the transition from present non-aided to fully automated driving might best be made.

To accomplish these objectives, a model based on car-following theory was developed to simulate mainstream and merging type driving situations. By the inclusion and variation of a number of parameters in the model, the evolutionary sequence from purely human control to fully automatic control of vehicles could be simulated. This allowed the study to encompass normal driver-vehicle-roadway interactions, driving with the introduction of sensory and control aids, and travel on a fully automated system.

Initial test runs of the model, using the MIT CTSS (compatible time sharing system) 7094, determined that the model very accurately simulated observed human-vehicle-roadway interactions. Further runs in which parameter values were varied, showed that the key elements in stable traffic flow for a given set of conditions are the vehicle operator's reaction time and sensitivity, (i.e., the severity with which the vehicle controller makes acceleration and deceleration corrections). Through additional runs, general stability relationships were verified and optimal roadway usage criteria set.


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CHAPTER I
INTRODUCTION

A growing population, coupled with the increasing affluence of our society, is placing ever increasing strains on our transportation systems especially in the area of automotive applications. Thus far, our only solution has been to build more and more highway systems of questionable overall efficiency. Space limitations along with soaring costs will make this approach impractical in the not too distant future. Necessity dictates that we must increase markedly the quality and not just the quantity of our roadway networks, especially in and around our major metropolitan areas. It is for these reasons that we must now start planning and developing the highway systems of the future, which, by necessity, will have an ever increasing degree of automation.

Development of quantitative simulation methods for studying traffic flow is then of considerable interest. The most immediate need for this type of analysis is in the development of driver aid type devices for improving safety and roadway usage. By developing reliable quantitative methods for representing vehicle traffic flow in various types of driving situations (i.e., mainstream traffic driving, merging, exits and intersections), much initial evaluational work can be carried out on a computer before costly research is necessary.

What we are now about to enter into is an evolutionary, or more appropriately, revolutionary, phase in the development of automotive
transportation. Automated travel will not come about in a one step process from the solely human control form, which we now know, to some highly sophisticated fully automated system. Instead, a transient process, now in its initial phases, will occur in which first attempts at adding devices to insure safety will give way to more and more sophisticated items, not only to improve safety, but also to improve roadway usage.

The purpose of this investigation then is to develop tools to study this evolution and basic criteria on which to guide its development. Accordingly, the investigation has been broken into four phases described in the next four chapters. Chapter II gives a brief review of the presently available methods and theories for quantitatively representing and analyzing traffic flow. The next portion, covered in Chapter III, provides a survey of proposals and experiments thus far suggested or carried out in the development of automated travel. Basically the purpose of this chapter is to acquire some feel for the hardware and general system policies which can be expected to be found in an eventual automated system. Chapter IV attempts to tie together from these earlier chapters those facts which seem pertinent to the formulation of a system model. The final phase established the authenticity of the model and then utilized it to develop criteria for guiding the automating process.
CHAPTER II
REVIEW OF TRAFFIC FLOW THEORY

INTRODUCTION

Numerous mathematical theories have been developed in recent years in an attempt to describe the flow of vehicles in different traffic situations. Basically these theories can be categorized into three groups: hydrodynamic theory, car following, and queueing theory.

HYDRODYNAMIC THEORY

Hydrodynamic flow theory views traffic as a compressible fluid having density, concentration and velocity. This type of approach is based on a partial differential equation expressing the conservation of matter and an assumed empirical relationship between flow and concentration. Discontinuities in traffic flow are propagated like shock waves in a compressible liquid. Basic to this approach are the following definitions which are related by the flow equation:

\[ Q = U \cdot K \]

where

\( Q = \) Flow, which is a term representing the number of vehicles passing a point of the road per unit time. It is then a measure of the volume of traffic being handled by a particular roadway with units of vehicles per hour.
U = Speed, which as the name implies, is the distance traveled per unit time.

K = Concentration or density, which is the number of vehicles per mile of road.

According to Haight, the basic relationship between flow and concentration for a single lane of traffic is the "fundamental diagram of traffic flow," an example being the function shown in Fig. 1. Intuitively one should be able to imagine the shape of this curve. The density of traffic flow can vary from zero when there are no cars to a maximum \( K_j \) (jam density). With the density equal to zero, the flow should also equal zero, since there are no vehicles on the road. With the density equal to jam conditions, the flow again is equal to zero because there is no room for vehicles to move. With an increase in density from \( K = 0 \), or decrease from \( K = K_j \), the flow increases from zero. Referring to Fig. 1, it can be seen that when traffic is sparse and interactions between vehicles small, the flow increases almost linearly with concentration. This is generally referred to as the "stable region" of traffic flow. As concentration increases, the rate of increase tapers off until a maximum flow rate is reached. This maximum flow rate is called "road capacity". Once density exceeds the value corresponding to maximum flow rate, vehicle interactions increase markedly, resulting in a reduction in flow. This region of post maximum flow is then called the "unstable region", or region of "saturated flow". The density flow diagram also allows us to calculate mean speed at any operating point by simply connecting that point on the curve with the origin by a straight line. The slope of this line is equal to the mean speed of the system.
Flow (Veh/Hr.)

\[ Q \]

-1500

-1000

-500

Stable Region

Unstable Region

Concentration (Veh/mile)

Fig. 1 Q-K Diagram of Traffic Flow for a Single One Directional Lane

(Redrawn from Introduction to Traffic Flow and Theory, 1964, Highway Research Board)
Lighthill and Whitham\textsuperscript{17} did most of the pioneering work in the area of shock wave propagation in traffic flow caused by vehicles slowing down to avoid hitting the vehicle in front. This theory predicts that a shock wave is propagated at a velocity given by:

\[ U_s = \frac{\Delta Q}{\Delta K} = \frac{Q_2 - Q_1}{K_2 - K_1} \]

where
\[ \Delta Q = \text{Difference in flow rates between two points on the } Q-K \text{ curve.} \]
\[ \Delta K = \text{Difference in traffic density between two points on the } Q-K \text{ curve.} \]

Thus the speed of shock wave propagation for any operational point on the Q-K curve is simply the slope of the curve at that point. This type of theory has found its primary applications in investigating long stretches of roads and flow near junctions.

While many mathematical models have been proposed for Q-K functions, the two most noted are those of Greenshields\textsuperscript{10} and Greenberg\textsuperscript{11}.

Greenshield's model: \[ Q = 2C*K*(1-K/K_j) \]

Greenberg's model: \[ Q = C*K*ln(K_j/K) \]

where \[ Q = \text{Flow rate} \]
\[ K = \text{Traffic density or concentration} \]
\[ C = \text{Constant} \]

Through use of the Q-K relationship, traffic speed can easily be determined.

Greenshield's model: \[ U = 2C*(1-K/K_j) \]

Greenberg's model: \[ U = C*ln(K_j/K) \]
CAR FOLLOWING THEORY

Car following theory is the study of stimulus-response type interactions in a single lane of traffic caused by various acceleration and deceleration patterns induced in vehicles. Thus, car following theory is used to study acceleration and deceleration patterns in traffic and the flows resulting when traffic is regulated in various ways. Alternately one could envision it as the study of the aggregate effect of the behavior of individual drivers. Car following attempts to describe how the average driver would react to various velocity and acceleration patterns of the vehicle in front of him.

Models developed using this type of theory are deterministic in nature with the basic premise being that once the interactions of individual vehicles are understood, the overall characteristics of the stream can be predicted. In the car following laws, the responses of the driver is assumed to be proportional to both his sensitivity and the magnitude of the stimulus.

While many mathematical descriptions of car-following theory have been proposed, the most commonly accepted representation is found in the work of Gazis, Herman and Potts, they postulate the equation:

\[ \ddot{X}_{n+1} = \frac{C_s(X_n - \dot{X}_{n+1})}{(X_n - \dot{X}_{n+1})^2} \]

where

\[ X_n = \text{Position of vehicle } n \]
\[ \dot{X}_n = \text{Velocity of vehicle } n \]
\[ \ddot{X}_n = \text{Acceleration of vehicle } n \]
\[ C = \text{Variable which is a function of driver sensitivity and vehicle velocity} \]
The link between the macroscopic approach of hydrodynamic theory and the microscopic approach of car following theory is illustrated in the following derivation. This derivation of Greenberg's flow-concentration curve from the above car following law was done originally by Gazis, Herman and Potts.\(^1\)

Integrating the general car following equation to obtain \(X_{n+1}\) = velocity of vehicle \(n+1\).

\[
\dot{X}_{n+1} = C \times \ln(X_n - X_{n+1}) + C_1
\]

Spacing is inversely proportional to density.

\[
\dot{X}_{n+1} = C \times \ln(1/K) + C_1
\]

\[
U = C \times \ln(1/K) + C_1
\]

\[
Q = U \times K = C \times K \times \ln(1/K) + C_2
\]

\[
Q = C \times K \times (\ln(1/K) + \ln(C_3))
\]

\[
Q = C \times K \times \ln(C_3/K)
\]

Applying boundary conditions at jam conditions.

\[
Q = 0, \text{ when } K = K_j
\]

\[
C_3 = K_j
\]

\[
Q = C \times K \times \ln(K_j/K) \text{ Greenberg's Law}
\]

Thus we can clearly see a definite link between car-following theory and hydrodynamic theory. Car following theory, however, has the advantage over a hydrodynamic study in that the dynamics and control response of the individual vehicle can be investigated.

**Queueing Theory**

The third major approach to vehicular flow is that of queueing theory. In a typical highway network situation, vehicles of different types, operated by drivers with different desires and characteristics
are involved. Variable phenomena of this type are referred to as "stochastic" and the methods of probability and statistics provide a means by which it is possible to predict certain characteristics. In this type of theory, the function that characterizes traffic is the probability distribution for velocity. At low concentration, this function tends to be a free distribution representing the probability density of velocities at which drivers would operate vehicles if nothing impeded them. At higher concentrations, the velocities are lower due to interactions.

In the stochastic approach, vehicles are not randomly distributed on the roadway, but instead appear in clusters called queues. These queues have different velocities and lengths, the theory being that faster vehicles cluster up behind slower vehicles while waiting for chances to pass. Queueing theory then provides information such as the average number of vehicles in a queue, the average delay for a vehicle in a queue, and the probability of n vehicles being found in a queue. Queueing theory is especially applicable to the study of delays, such as occur at signalized intersections where queue length is directly proportional to the length of a red or green light. Queueing theory has also been extensively used in studying the problem of traffic merging. In applying queueing theory to the study of the merging problem, whether it be the forced merge, intersection or ramp, it must be realized that a complete mathematical model for merging has not yet been developed. Thus most of these studies, depend heavily on computer simulation to obtain results. Typical of most work done on the merging problem using queueing theory is the work of Oliver and Bisbee. They have postulated that minor stream
queue lengths (i.e., merging lane queue lengths) are a function of the major stream flow rates. Basically, this can be summed up in the following equation for the average number of vehicles in the merging lane or minor queue.

\[ E(n) = \left( \frac{Q_a}{Q_b} \right)^2 \frac{(1 - Q_b \cdot T \cdot \exp(-T \cdot Q_b))}{(\exp(-T \cdot Q_b) - (\frac{Q_a}{Q_b}) \cdot \exp(-T \cdot Q_b))} \]

where

\( Q_a \) = Minor stream flow
\( Q_b \) = Major stream flow
\( T \) = Minimum acceptable headway distance between vehicles to insure safe driving. This is usually a function of the velocity and the controller's reaction time of the vehicle.

CONCLUSION

Hydrodynamic theory and queueing theory while providing excellent information as to overall system dynamics, have the major disadvantage of not providing much information with respect to the dynamics of the individual vehicle. Car following theory does provide a means for studying both the dynamics of the individual vehicle and gives a great deal of insight into what is happening throughout the entire traffic system. Thus in attempting to develop a model to study main line vehicle control and merging for highway systems ranging from normal human driving to fully automated situations, car following theory appears to be the most applicable.
CHAPTER III
SYSTEM HARDWARE AND POLICY CONSIDERATIONS

INTRODUCTION

Before attempting to develop a model to investigate control and safety requirements for main line and merging type driving situations, certain design and component proposals which appear especially applicable to automated and semi-automated driving situations are investigated. The general philosophy carried throughout this study is that the development of automated travel will be an evolutionary transformation, as sensing and control devices gradually take over more and more of the driving functions now performed solely by the human driver.

FUNDAMENTAL CONSIDERATIONS OF SYSTEM EVOLUTION

Evolutionary development of automated travel will simply be a process of public demand stimulating governmental and industrial organizations to produce operational systems fulfilling specific public requirements. While the eventual cost of a fully automated system will be sizable, it can be effectively argued that this should be more than made up by increased safety, reliability, and convenience. Economic considerations will also play a major role in determining when such a system will become a reality and also place additional constraints on it.

Meyer, Kain and Wohl in a recently published book on urban transportation problems foresee three significant methods for reducing costs while increasing service in an automobile transportation system.
These are the large-scale development and usage of small vehicles, leasing or rental arrangements and "electronic highways". Arguing for the reduction in the size of vehicles, they feel that significant cost reductions could be realized, not only in actual unit construction costs, but also in reduced parking facilities and greater roadway flow capabilities. Thus it appears that one of the major restrictions placed on system development by economics will be the physical size of its vehicles.

A second approach, the adoption of a widely accepted policy of renting or leasing vehicles, will almost have to follow due to the probable high capital investment associated with the individual ownership of vehicles. Leasing vehicles will provide a service with most of the attractions of private ownership, such as privacy and scheduling flexibility, while reducing the cost to the user since each vehicle can be leased many times during a day and not simply occupy a parking area. The final area for improvement is the roadway system itself. Only through some form of a system coordination and control can optimal usage and safety be attained.

The first and primary consideration throughout the automating process will be safety. Thus we can expect the first steps taken toward an eventual fully automated system to be in the area of incorporating more safety devices in vehicles, especially in the form of informational display units.

The second step in this probable progression of automation is likely to be attempts at optimizing road usage by routing vehicles and controlling their speeds in a manner to cut down on traffic congestion. Typical of the work being done in this area is General Motors' "Driver Aid Information and Routing System". Features of this system include
a visual signal reminder (a panel display in the vehicle triggered by roadway signals from magnets or low frequency transmitters). These signals are used as route reminders to guide drivers to destinations via directional displays on the instrumentation panel. In this particular approach, as the driver enters the system through some sort of a toll booth, he would dial the final destination he desires and the system would provide him with a punched card containing routing information which is then used to program the individual vehicle's routing equipment.

In addition to routing directions, information as to prescribed driving speeds to alleviate congestion or to facilitate merging might be flashed to the driver through computer-controlled roadway signs. First steps have already been taken in this direction in Toronto's use of a computer system to coordinate traffic signals.

Probably the most startling difference between fully automated travel versus normal driving will be in the use of automatic devices to control and coordinate all aspects of the trip. While some studies in this area tend to feel that each vehicle on the system at all times should be controlled by one central control system, it seems more reasonable to provide each vehicle, once it has entered the system, with its own self-contained control unit based on some sort of special-purpose computer. This would then alleviate the necessity to provide some central computer with prohibitively large quantities of information. On the other hand, it seems reasonable to expect that such aspects of the trip as merging or exiting the system will be coordinated by some overall controller, since it is desirable to optimize such operations in a manner as to minimize delays to the system. This overall system controller could also perform most of the bookkeeping functions associated
with a leasing system. By the time public opinion and technology are ready for the implementation of such a system, cost factors should no longer provide a major stumbling block to the mass use of such equipment as small special-purpose digital computers. This is in large part due to the rapid advances made in microcircuit technology both in regards to reliability and cost.

Mr. C. Hogan of Motorola, Inc. in the June 1964 issue of the IEEE Spectrum confirms this in the following statement:

"Very complex silicon circuits can now be constructed on a chip approximately 50X50 mils. The cost to carry a wafer through all steps of diffusion and metalization, provided it is run at moderately high volume and at 100 percent yield before packaging, is about $10 per wafer including normal overhead. Assuming that a one-inch square wafer can be processed at 100 percent yield, each wafer will contain 400 individual circuits which cost 2 1/2 cents apiece. Even if the ultimate yield is as low as 50 percent, the cost of the finished silicon monolithic integrated circuit before packaging will be less than 5 cents.

Over the next three years, the resolution with which silicon circuits can be built will steadily improve. In the research and development laboratories, one bit of a two place shift register has been placed successfully on a 70 mil square die. Again assuming a 50 percent yield, this entire circuit consisting of 33 transistors, 27 resistors and 2 diodes could be built for less than 10 cents."

Project Metran, which was a special M.I.T. study on urban transportation, forecasts that a special-purpose digital computer with roughly the specifications required by an automated system might cost by 1990 in the region of $500.

In addition to controlling merges and exits, the overall system controller could also perform the function of testing the vehicle for mechanical or electronic deficiencies before allowing it to enter the system. Testing such as this should considerably reduce slowdown and accidents due to vehicle failures.
VEHICLE CONTROL

In keeping with the philosophy of investigating not only the final automated system, but also the evolutionary process through which automation might best be attained, the first question to be asked is, "What can be done and by what degree to increase a driver's ability to handle high speed dense traffic situations?". The fact that safety can be improved through the incorporation of driver informational aids has already been established by Bierley in his experiments studying the effect of providing the average driver with various types of information such as relative spacing and velocity. By providing the individual driver with this type of information, marked differences were observed in his ability to follow other vehicles in queues, keeping some desired spacing and velocity. Figure 2 illustrates how information from sensing devices fits into the basic interactions of vehicle control.

Sensing devices then appear to be the starting point for the growth of an automated system. Dr. Joseph Treiter of Ohio State University has experimented with inter-vehicle infrared sensing devices to provide relative vehicle spacing information. In such a system, each vehicle carries an infrared light source on its rear bumper which is pulsed in proportion to the vehicle's speed by a bladed disk. An infrared sensor on the following vehicle's front bumper detects this pulsating infrared source and the received signal's intensity is used in determining relative distances.

Some interesting control and sensing concepts developed during project Metran include a very accurate speed measuring device. This proposal makes use of light reflectors mounted on the roadway surface, and a light source mounted on each vehicle. By spacing the
Fig. 2 Basic Sensing and Control Interactions
light reflectors at intervals along the road, light from the vehicle is then periodically reflected and received. A photocell could be used to detect the reflected light, producing pulses whose frequency is dependent upon vehicle speed.

In addition to devices carried on vehicles, equipment incorporated in the roadway system itself can provide a great deal of information to the vehicle controller. Robert, Spangler, and Snell have suggested a vehicle guidance and control system using only passive roadbed equipment. In this approach, the control decisions are based on the detection of speed and position information by means of radio-frequency magnetic fields induced in roadbed loops by a vehicle-borne generator. By suitable coupling of the guidance detector to the vehicle's steering mechanism, the guidance system is made null-seeking. The pulse frequency derived from the spacing of roadbed loops is then used to operate the speed control system.

Experiments in actually instrumenting vehicles with sensing devices and using their informational output to control the vehicles have already been carried out by Oshima, Kikuchi, Kimua, and Matsumoto. Their control system performs three functions: speed control, steering and collision prevention. In the operation of the speed control, the actual speed is detected by a tachometer generator coupled to the output shaft of the transmission. The tachometer output is compared with the desired speed to furnish the control signal. Steering control is performed by a guidance cable in the center of the road. A signal with a frequency of 4.3 KHZ is transmitted along this cable while two pickup coils are mounted on the front bumper to detect deviations of the vehicle from the guidance cable. For stability purposes, receiver coils
were also mounted on the rear bumper. Similar automated car studies were conducted by General Motors in Warren, Michigan around 1958.

**VEHICLE PROPULSION**

While initial automatic devices will undoubtedly be applied to internal combustion powered vehicles, the final fully-automated system, in all probability, will rely on electric power for its primary propulsion source. Electric motors will not only prove more controllable, but also reduce air pollution in urban areas. The advent of the electric car as a practical means of transportation is approaching with the rapid development of fuel cells. Up to now, the primary reason for not making more extensive use of electric power for vehicles has been the need to rely on large heavy storage batteries, greatly limiting a vehicle's range. Fuel cells, however, should alter this situation, utilizing fuels that are easily handled, obtained at low cost and generally available commercially. As now conceived in the "Commucar" and other similar proposals, once the vehicle is off the system it will have the capability to proceed to its desired destination under its own power. On the system, however, the highway network itself supplies the vehicle with power.

Most studies on the general requirements for an automated system's propulsion tend to stress two possible approaches, both involving electric drives. One approach makes use of synchronous linear motors and the other, speed-controller linear motors. An excellent reference in this general area is contained in the *Survey of High Speed Ground Transport*, the report of a study conducted at MIT for the U.S. Department of Commerce.
Basically in a synchronous linear drive motor system, all vehicles would be interconnected causing them to operate in strict synchronism with no net slip. An example of this type of device is the polyphase synchronous linear motor in which either the armature or the field is developed linearly along the extent of the roadway. This type of approach is often likened to a conveyor belt with links. As long as two vehicles are not forced into the same link, there will be safe, precise control on all vehicles.

Another possibility is the use of speed-controlled linear motors. With the advent of relatively low cost semiconductors, it is now possible to develop efficient solid state frequency converters. This has paved the way for the so-called "brushless" D.C. motors, that have solid state switches to perform commutation. The speed-controlled linear motor can be controlled from a fixed frequency reference, providing "apparent synchronous" operation. An added advantage over synchronous motors is the capability for external control of operational speed at acceleration and deceleration ramps and control of individual vehicle maneuvers within a string of vehicles.

Electrically powered vehicles then have the advantage over other forms in that they are easily controlled, produce no toxic exhaust fumes, and have high tractive power. Additionally, during periods of high sustained speeds, the electric motor is more efficient than conventional motors. Figure 3 shows one possible control scheme making use of an electric motor with power supplied by the system.

**VEHICLE DESIGN**

Several factors will dictate the design features of commuting type vehicles for an automated system. First, as mentioned, will be
Fig. 3  Block Diagram of Vehicle Control System

(Re drawn from Project METRAN, An Integrated, Evolutionary Transportation System for Urban Areas, p. 111)
the requirement for smaller vehicles. Studies have already been insti-
gated on such vehicles and some concrete design proposals have been
made such as the "Commucar". As envisioned in most of these stud-
ies, the vehicle will be electrically driven, carry up to four passengers
and have the capability of being manually-driven and self powered when
off the system, and automatically-controlled and powered when on the
system.

Such a vehicle then must provide means for power and control
pickup from the system. The "Commucar" study suggests a two arm
control and pickup device in which the vehicle itself performs switching
at merging and exit points, rather than the system. During normal
driving, both arms remain engaged. However, before each intersec-
tion, one of the arms is detached from the system power and control
rails. If routing information dictates that the vehicle not turn off, the
arm connected to the rail in the mainline direction is left connected.
If routing information commands a turnoff, the arm connected to the
rail which follows the exit ramp is left connected. Once the intersec-
tion has been passed, both arms are again engaged.

CONCLUSION

The function of this chapter has not been to give a complete
report on all proposals thus far submitted or even to discuss the de-
tailed aspects of a practical automated system. Instead, what has
been given is a general foreshadowing of what to expect in the areas of
automatic hardware and their possible evolution. Basically, we can
conclude that the two major requirements for the system will be safety
and efficient roadway usage. The vehicles should be easily controlled
through regulation of their propulsion units, probably some form of a
linear electric motor. Sensing devices will be available with accuracy levels necessary to insure safety. As a main controller element, each vehicle will have some form of a logic device with provision for memory storage to perform those functions now carried out by the human driver, only in a faster and more exacting manner. These requirements and constraints on the system, coupled with some of the fundamentals in mathematically representing vehicle traffic flow discussed in Chapter II, are combined in the next chapter in the development of a simulation model to study the design of the automated system.
the requirement for smaller vehicles. Studies have already been insti-
gated on such vehicles and some concrete design proposals have been
made such as the "Commucar". As envisioned in most of these stud-
ies, the vehicle will be electrically driven, carry up to four passengers
and have the capability of being manually-driven and self powered when
off the system, and automatically-controlled and powered when on the
system.

Such a vehicle then must provide means for power and control
pickup from the system. The "Commucar" study suggests a two arm
control and pickup device in which the vehicle itself performs switching
at merging and exit points, rather than the system. During normal
driving, both arms remain engaged. However, before each intersec-
tion, one of the arms is detached from the system power and control
rails. If routing information dictates that the vehicle not turn off, the
arm connected to the rail in the mainline direction is left connected.
If routing information commands a turnoff, the arm connected to the
rail which follows the exit ramp is left connected. Once the intersec-
tion has been passed, both arms are again engaged.

CONCLUSION

The function of this chapter has not been to give a complete
report on all proposals thus far submitted or even to discuss the de-
tailed aspects of a practical automated system. Instead, what has
been given is a general foreshadowing of what to expect in the areas of
automatic hardware and their possible evolution. Basically, we can
conclude that the two major requirements for the system will be safety
and efficient roadway usage. The vehicles should be easily controlled
through regulation of their propulsion units, probably some form of a
CHAPTER IV
SYSTEM MODEL

MODEL CONSIDERATIONS

In attempting to study the progression from present human-controlled driving to the sophisticated computer-controlled systems envisioned by some traffic engineers, a model had to be developed that could simulate this transition without losing its authenticity. The model then must contain an accurate description of driver response, coupled with means for modifying this type of response to accurately depict the automatic vehicle control situation.

It turns out that the human driver is a fairly effective servo-mechanism device in vehicle-road-type interactions. His main shortcomings lie in his sensing abilities and reaction or delay time in carrying out corrective actions. The strategy decided upon in developing this model was to mimic the human driver and his stimuli-response actions to as large an extent as possible. This was accomplished by including a number of variable parameters in the model. By properly varying these parameters and studying their subsequent effect on the behavior of the system, questions as to how this evolutionary process might most efficiently take place can be answered. Information can likewise be provided as to what human traits and characteristics should be left in the system and which need improvement and by what degree. The model then has no random characteristics, but is based on a deterministic, rather than Monte Carlo approach.

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In attempting to develop as general a model as possible to describe as many system interactions as possible, the merging situation was picked, allowing study of a main flow of traffic and also the influence of a secondary flow interacting with it. The model is broken up into two areas. The main traffic stream in the evolutionary process will either be controlled by the driver in each vehicle with certain informational aids or automatically controlled by special-purpose computers. Control of vehicles on the merging lane in the automated situation, however, can be expected to be performed by some system coordinator to insure optimal stream interactions.

**VEHICLE RESPONSE**

To develop the relations to describe the vehicle's dynamical behavior to various stimuli, the basic relations of car-following theory were applied for the reasons outlined in Chapter II. As a basis on which to build, the car-following relations found in the works of Gazis, Herman, and Potts \(^{12}\) and Fox and Lehman \(^{9}\) were used. Gazis, Herman, and Potts car-following expression relates acceleration response to changes in relative velocity, relative spacing and a driver or system sensitivity factor as described by the following equation:

\[
\frac{d^2X_{n+1}(t)}{dt^2} = \frac{\text{ALPHA} \cdot V_{n+1}(t) \cdot (V_n(t-T) - V_{n+1}(t-T))}{(X_n(t-T) - X_{n+1}(t-T))^2}
\]

where

- \(X_n\) = Position of vehicle \(n\)
- \(V_n\) = Velocity of vehicle \(n\)
- \(\text{ALPHA}\) = Sensitivity of driver or system response
- \(T\) = Time delay in vehicle and driver response
Coupled to this equation is a factor which Fox and Lehman\(^9\) call the "threshold of awareness". It has been observed from human driving (and in all probability would be true for vehicle sensing devices) that at large vehicle spacing intervals, relative velocity changes are very difficult to detect. For the human driver, this ability to detect changes in relative velocity, according to Fox and Lehman\(^9\), depends on the "rate of change of angular motion of an image across the retina of the eye". A threshold equation is therefore introduced into the model whose value depends on relative vehicle velocities and spacing.

\[
\text{Threshold} = W^* \left( \frac{V_n - V_{n+1}}{(X_n - X_{n+1})^2} \right)
\]

Where \(W\) is a sensitivity factor, which can be varied to simulate the various degrees of accuracy of system sensing devices.

During mainstream flow interactions, the individual vehicles will be assumed to be driving in one of two modes. "Distance detection mode" refers to driving situations in which the vehicle's threshold value is below some specified level. In this mode, vehicle controllers are aware of only changes in relative spacing between vehicles. This type of driving situation is encountered when traffic conditions are light and vehicles have large spacing intervals.

During conditions in which the threshold has been exceeded, vehicles are controlled by a "velocity detection mode" control scheme. This mode, which is of greatest interest to this study, governs following dynamics when vehicles are closely spaced. Each vehicle controller must be provided not only with relative spacing, but also relative velocity information.
In attempting to ascertain what information is essential to safe, high speed, small-gap driving, it was decided to provide the vehicle controller not only with information relative to the vehicle directly ahead, but also information about the dynamics of the vehicle in front of the vehicle ahead. By weighing the information provided from both leading vehicles, the importance of each could be determined. Thus a final threshold equation was arrived at in the form:

\[
\text{Threshold} = \frac{W_1 \times \text{VREL}_1}{(\text{XREL}_1)^2} + \frac{W_2 \times \text{VREL}_2}{(\text{XREL}_2)^2}
\]

where

- \(\text{VREL}_1\) = Relative velocity with respect to the vehicle just ahead.
- \(\text{VREL}_2\) = Relative velocity with respect to the velocity two ahead.
- \(\text{XREL}_1\) = Relative spacing with respect to the vehicle just ahead.
- \(\text{XREL}_2\) = Relative spacing with respect to the vehicle two ahead.
- \(W_1\) and \(W_2\) = Weighing factors which can be varied to determine the relative importance of information about each vehicle.

**GAP DISTANCES**

Probably the most important consideration to be dealt with is that of gap distances, (i.e., distance from the front bumper of one vehicle to the rear bumper of the vehicle just ahead). For any given set of conditions this factor determines the capacity of the system. Gap requirements must be considered for two situations. First, what is the smallest allowable gap distance required to insure mainstream flow safety. Secondly, what gap distances are required for efficient and safe merges. The basic consideration must, of course, be safety,
which for non-automated driving has been considered a distance roughly equivalent to the minimum stopping distance of a vehicle. Haight, Wojcik and Bisbee in a mathematical analysis of this problem arrived at the following result for minimum gap distances for main stream driving:

\[ Y = V_n T_n + \frac{(V_n)^2}{2X_n} - \frac{(V_{n+1})^2}{2X_{n+1}} \]

where

- \( Y \) = Required gap distance
- \( T_n \) = Reaction time of vehicle \( n \)
- \( X_n \) = Maximum deceleration possible for vehicle \( n \)
- \( X_{n+1} \) = Maximum deceleration possible for vehicle \( n+1 \)
- \( V_n \) = Velocity of vehicle \( n \)
- \( V_{n+1} \) = Velocity of vehicle \( n+1 \)

The most commonly accepted rule of thumb is the California Driving Law, which states that there should be at least one car length for each 10 mph of speed. In both cases, the required gap distance is dependent on vehicle velocity and in the former reaction time. It was decided in the model to use a rather general equation which takes into account all the important parameters involved.

\[ Y = SPDES - CL \]

\[ SPDES = B + T V_{n+1} \]

where

- \( Y \) = Required gap distance
- \( SPDES \) = Desired spacing measured from the front bumper of one vehicle to the front bumper of the vehicle ahead
- \( CL \) = Vehicle length
B = Safety factor, which is independent of speed and has a minimum value equal to the vehicle's length.

T = Controller reaction time.

By varying B and T, the effect of improving the sensitivity and reaction time of the system can be studied as the transition from the human driver to a computer-controller system evolves. The equation, however, is based on the assumption that both vehicles decelerate at the same rate.

A safe gap distance cannot be a constant value, however, but instead should be dependent on the actions of the vehicle ahead. This model takes into account such circumstances, by requiring a greater gap distance if the vehicle ahead is decelerating rapidly. This refinement was attained by modifying the spacing equation just discussed by introducing a variable which Lehman and Fox\(^9\) refer to as CONSA.

CONSA, shown in Fig. 4, has a normal value of one, which causes no change in the requirement for gap distance. However, as the deceleration of the vehicle ahead exceeds a certain critical level, CONSA begins to rise in a linear fashion until it reaches a value of two when the vehicle ahead is using maximum braking. Quantitatively, CONSA is described by the following equation:

\[
\text{CONSA} = \frac{\text{DAC}_n + \text{DCMAX} - 2 \times \text{CRITWV}}{\text{DCMAX} - \text{CRITWV}}
\]

where

- \(\text{DAC}_n\) = Deceleration of the vehicle ahead.
- \(\text{DCMAX}\) = Maximum deceleration possible.
- \(\text{CRITWV}\) = Critical value of deceleration above which the gap requirement begins to grow.
Fig. 4  Effect on CONSA of Deceleration by the Vehicle Ahead

(Redrawn from Safety In Car Following, p. 17, by: P. Foy and F. Lehman)
The final model spacing equation taking CONSA into account was:

\[ SPDES = B + T \times CONSA \times V_n \]

Thus in this modified equation for desired spacing, variations in the deceleration of the vehicle ahead are reflected in CONSA.

Another question to be considered is what amount of deviation from the desired velocity (i.e., each vehicle has some speed which it wishes to maintain) will be allowed before corrective actions are taken. Two factors must be considered, the accuracy of the system sensing devices in determining speed and relative spacing and also the desire to avoid continual corrective action and thus smooth the ride to as large an extent as possible. Two variables were introduced in the model to account for this, FVL and FVH. FVL dictates the lowest acceptable velocity before corrective action is initiated and FVH gives the highest velocity acceptable. These values will tend to approach the desired velocity as more sophisticated systems are simulated.

**REACTION TIME**

Reaction time, like spacing requirements is not a constant value, but varies depending on the particular driving conditions. For example, human driver reaction time varies over a wide range depending on whether the driver is under some form of stress, such as in an area where vehicles are merging. This idea of a variable action time for an automated system seems practical as well. When traffic is light and spacing gaps large, a long reaction time will in effect smooth out the ride while still maintaining a high safety level. In closely spaced driving, however, this period of delay must be minimized to as great an extent as possible. The model handles this problem of
variable reaction time by use of step and ramp type functions, shown in Fig. 5. In the distance detection mode of driving, where headways are large and there is little to worry about, the reaction time is set as some maximum value. However, when threshold is exceeded, (i.e., the vehicle's control transfers from a "distance detection mode" to a "velocity detection mode" scheme) the reaction time is stepped down to some minimum level, from which it gradually increases, barring an unforeseen crises, until it again attains a maximum value. Reaction time will then stay at this level until a new crisis appears or a merging or exit point is reached, when it again drops to a minimum value. As mentioned, the value of this minimum level of reaction time will have a strong effect on the maximum capacity of the system. Complementing the actual reaction time are parameters dictating how quickly the vehicle controller can detect the deceleration of the vehicle ahead and how long it is required to maintain an enlarged desired spacing distance when CONSA exceeds one.

**VEHICLE ACCELERATION DETERMINATION**

The variable to be controlled by the system is the acceleration pattern of each vehicle and to this end the following criteria has been established. In the distance detection mode of driving, the vehicle controller attempts to keep each vehicle's acceleration equal to zero, once the desired velocity has been attained. Over threshold, the general car-following equation described earlier in the chapter is used to determine acceleration with a slight refinement to enable studying the effect of providing information about the vehicle two ahead. Thus acceleration commands in the "velocity detection mode" are determined by the following equation:
Fig. 5 Vehicle Controller Reaction Time
\[ A_c = (\text{ALPHA} \cdot V_{n+1}) \cdot \left( \frac{W_1 \cdot V_{\text{REL1}}}{(X_{\text{REL1}})^2} + \frac{W_2 \cdot V_{\text{REL2}}}{(X_{\text{REL2}})^2} \right) \]

where

- \( A_c \) = Acceleration required
- \( \text{ALPHA} \) = Intensity of driver or system response
- \( V_{\text{REL}} \) = Relative velocity
- \( X_{\text{REL}} \) = Relative spacing

\( W_1 \) and \( W_2 \) = Weighing factors representing the relative importance of information about the vehicle just ahead, versus the one two ahead. \((W_1 + W_2 = 1)\)

This equation applies directly to situations where headways are greater than required. In cases with spacing less than desired, the equation is further modified by the addition of an acceleration response factor, \( \text{ACF} \) (Fig. 6). \( \text{ACF} \) is equal to zero for satisfactory relative spacing and increases in a linear fashion to value of one at zero vehicle separation. The complete model equation for acceleration is then given by:

\[ A = \text{ACF} \cdot \text{DCMAX} + (1.0 - \text{ACF}) \cdot A_c \]

where

- \( \text{DCMAX} \) = Maximum deceleration possible
- \( A_c \) = Acceleration term previously described
- \( \text{ACF} \) = Response factor

An additional variable in the form of a warning signal has been programmed into the model, denoted by the term BLT (brake light). As in normal driving, when a vehicle is braked, a brake light is turned on to warn the following vehicle. This usually has the effect of decreasing the following driver's reaction times. In the model, if the vehicle controller finds that deceleration is greater than the value
Fig. 6 Variation of Acceleration Factor with Various Vehicle Relative Spacings
associated with normal vehicle drag, the vehicle's brake lights are
turned on, which in turn decreases the reaction time of vehicles fol-
lowing.

With the vehicle's acceleration known, the velocity and distance
traversed are computed by simple integrations:

\[
V(n, 1) = V(n, 2) + \frac{\Delta t}{2} \times (A(n, 1) + A(n, 2))
\]
\[
X(n, 1) = X(n, 2) + \frac{\Delta t}{2} \times (V(n, 1) + V(n, 2))
\]

where

\[V(n, I) = \text{Velocity of vehicle } n \text{ at time } I,\]
where \(I = 1\) indicates the present
time interval

\[X(n, I) = \text{Distance of vehicle } n \text{ at time } I\]

\[A(n, I) = \text{Acceleration of vehicle } n \text{ at time } I\]

\[\Delta t = \text{Time duration between computations}\]

Figures 7 and 8 depict in flow chart form the logic performed by
the controller of each vehicle. Varying the parameters allows the sys-
tem to simulate the entire spectrum from human control, through
human-aided control by sensing devices to a fully automated system.

**MERGING LANE**

The same basic approach used in determining policies and con-
trol criteria for the mainstream has been applied to the merging lane
and the subsequent interactions between streams as vehicles transfer
from the merging to the mainstream lane. Refer to Figs. 10 and 11 for
a flow chart description of the following discussion.

Basically, the merging process is quite simple. The vehicle
controller, whether it be a human driver or system controller,
searches downstream for an acceptable gap and a determination is
made whether it is possible for the merging vehicle and gap opening to
Compute: XREL1
      XREL2
      XREL1
      XREL2

Check For Brake Lights

Distance Mode

Determine Mode:
1. Distance Detection
2. Velocity Detection

Velocity Mode

Is It Time To Check Threshold

no

Is XREL1 Greater Than 10 Car Lengths

yes

Reset Counter

no

Is Vehicle Over Threshold

yes

Fig. 7 Vehicle Mainstream Control Logic
Compute New Desired Velocity
\[ V(i, 1) = V(i, 2) + \Delta t/2*(A(i, 1) + A(i, 2)) \]

Compute Drag, If Deceleration Is Greater Than Drag Turn On Break Lights

Return

Has There Been A Recent Large Greater Than Drag Turn On Deceleration In The Vehicle Ahead

yes

Solve For CONSA

no

Solve For Desired Spacing
\[ SPDES = B + HDES*V(i, 2) \]

Solve For Desired Spacing
\[ SPDES = B + CONSA*HDES*V(i, 2) \]

Has Threshold Just Been Exceeded

yes

Reaction Time Set To Minimum Level

no

Solve For Acceleration
\[ A_c = ALPH*V(i, 2)*EQ \]

Is Relative Spacing Less Than Desired Level

yes

Recalculate A With The Addition Of The Acceleration Factor
\[ A = ACF*ACMIN + (1-ACF)*A_c \]

no

Fig. 8 Vehicle Mainstream Control Logic Continued
intercept the merging point at the same instant. While the process seems easy enough, this is probably one of the most complex interactions carried out in highway driving, and it is most inefficient when performed by the unaided driver. Human drivers, in almost all cases, tend to underestimate the speed of the mainstream and, thus, after entering the mainstream must accelerate up to speed causing drivers behind the merging gap to slow down. These fluctuations lower the capacity of the roadway. In the envisioned fully automated system, this operation becomes even more critical, since gap distances will be small with dense traffic at high speeds. To study the merging process in as great detail as possible, the entire process has been broken into three phases. One distinction should be noted in the merging control process versus the main flow control scheme. In the main flow control, the assumed fully automated system differs from normal driving by substituting in each vehicle sensing and logic devices in place of a human, while in the assumed automated merging process, the merging vehicle is controlled by an overall system coordinator during the merging process and not returned to its internal control until it has been absorbed into the main flow stream.

In the first phase of the merging process, the overall system controller, which can still represent the human driver, searches the mainstream for an acceptable gap. The basic criteria for an acceptable gap is given in the following equation:

\[
\text{GREQ} = 2\times \text{GAPMAIN} + \text{CL} + \text{SF}
\]

\[
\text{GAPMAIN} = \text{SPDES} - \text{CL}
\]
where $GREQ = \text{Gap distance required for merging}$

$\text{GAPMAIN} = \text{Required mainstream normal gap distance}$

$\text{CL} = \text{Vehicle length}$

$\text{SF} = \text{Safety factor}$

$\text{SPDES} = \text{Desired mainstream spacing}$

The requirement for a suitable merging gap is that it be twice the normal main flow gap distance, plus the length of the vehicle, plus some safety factor to be determined by simulation runs.

Upon ascertaining the existence of an acceptable gap, the controller must decide whether it is possible to intercept this gap without exceeding acceleration limitations. If this can be achieved, an appropriate acceleration trajectory must be determined.

The model provides for two types of acceleration patterns for control of the merging vehicle, a bang-bang type in which the merging vehicle is subjected to a constant acceleration while in the merging lane. This type of control assures that the vehicle will intercept the gap at the proper time. Once in the mainstream another acceleration value is applied to bring the vehicle up to proper mainstream speed. The advantage of this type of merge is that it provides for a relatively smooth entrance, since acceleration is kept constant and entrance speed can be lower than mainstream speed, resulting in an additional safety factor. Its main drawback is that a larger gap distance is required to accommodate the accelerating process of bringing the vehicle up to mainline speed. To a very high degree, this simulates the way in which a human driver traverses the acceleration ramp.

The second type of acceleration pattern programmed into the model is a scheme for gradually increasing acceleration that not only
assures intercepting the gap at the merging point, but also assures that it will be reached at mainstream speed, thus eliminating the need to accelerate in the mainstream. This is then an attempt to model an optimal type controller; where optimality is defined as intercepting the mainstream merging slot at the proper instant with a velocity relative to the mainline flow which is equal to mainline speed. To accomplish this, the system controller solves a two-point boundary equation to obtain an acceleration pattern similar to the one shown in Fig. 9 where the acceleration is given by the following equation:

\[ A_n(t) = A^*(1 - \exp(-t/R)) \]

where \( A_n \) = Acceleration of the merging vehicle as a function of time,

\( A \) = Constant to be determined by the controller from initial and final conditions.

\( R \) = Time constant, which determines the rate at which the acceleration increases, and is determined from initial and final conditions.

These merging acceleration schemes then determine the behavior of the merging vehicle during the first phase of the process, that is, while the vehicle is entirely on the acceleration ramp. By varying the controller parameters, most types of merges can be simulated, from human type actions up to highly efficient system controlled merges.

In the second phase of the merging process, the model turns the vehicle onto the mainstream lane at a determined turning rate. This is the most critical phase of the entire operation for, if the vehicle is not adjusted to mainstream speed very quickly, a marked slowdown in the system will occur. A variable denoted by THETA was introduced into the model to represent the relative angle between mainstream and
Merging Vehicle Acceleration

\[ A_n(t) = A \left( 1 - \exp\left( -\frac{t}{R} \right) \right) \]

Fig. 9 Acceleration Pattern for Merging Vehicle on the Acceleration Ramp
merging vehicles. Once the vehicle has had its direction completely adjusted to that of the mainstream and its velocity brought up to a desired level the vehicle is returned to its own internal headway control and the mechanics of the merging process are complete. As discussed earlier, the model also reduces the vehicle's reaction times to a minimum level during merging processes. In the fully automated type system, the system controller will have the added capability for merging the vehicle in such a way as to form gaps for additional merges further down stream.

The final phase of the merging process is the propagation of shock waves in the mainstream, induced as mainline vehicles adjust to the addition of the merging vehicle into the mainline flow.

Chapter V discusses simulation runs made with this model, concluding with a discussion of criteria and parameter values which tend to stabilize and optimize the system. A complete program listing is given at the end of the paper.
Merging Phase Zero

Check Mainstream Headways Between Vehicles K and K+1

Is The Headway Greater Than Required

yes

Calculate Acceleration Required To Intercept Gap

Is The Required Acceleration Within Bounds

no

yes

Initiate System Control Over Vehicle To Intercept Gap

Merge Phase = 1

Merging Phase One

Maintain Same Control

Is It Time To Sample To See If Gap Is Still Available

no

yes

Check To See If Intercept Gap Is Still Available

no

yes

Check For Alternate Gap Merge = 0

Has The Merging Point Been Reached

no

yes

Maintain Control Scheme

Set Reaction Time To Minimum

Merge = 2

THETA = Angle Of Merge

Fig. 10 Merging Controller Logic
Merging Phase Two

Has Vehicle Completely Turned On To Main Line
Does THETA = 0

no
Steer Vehicle In Main Line Direction

THETA = THETA - (Turning Rate) * Δt

Is Vehicle Up To Main Line Speed

no
Accelerate Vehicle

yes
Set Acceleration Equal To 0

If THETA = 0
V = Main Stream Speed

no
yes
Set Merge = 3

Return Control To Vehicle

Fig. 11 Merging Controller Logic Continued
CHAPTER V
ANALYSIS OF MODEL RUNS

INTRODUCTION

The model formulated in the last chapter was implemented using FORTRAN IV on the MIT IBM CTSS 7094. FORTRAN IV was chosen for a model language for two reasons. First, the model required a language oriented toward a continuously running time base type problem, which FORTRAN IV is well suited for. Second, FORTRAN IV is the most widely known and available language, and thus the model could be transferred from one computer to another with little difficulty. In final form, the program consisted of approximately 350 instructions, the complete program listing being given in Appendix A. Debugging difficulties were greatly reduced by use of the time sharing system, however, several initial test runs were made with the model and then verified by hand calculations before the program was pronounced ready for use. A typical run corresponded to a real time driving situation of 20 seconds for which variables were updated for every .1 second of real time. Actual computing time for such a run using the MIT CTSS 7094 was 80 seconds. During normal runs, output consisted of each vehicle's velocity, position and the acceleration of the merging vehicle.

Model studies were divided into three phases. In the initial phase, a set of runs were conducted to establish the validity of the model and gain some understanding of parameter magnitudes.
Accordingly, just the mainstream portion of the model was run, with a comparison made between computed results and data available from car following experiments conducted by the New York Port Authority. The second phase of the study involved runs made with the addition of the merging lane under normal driving conditions. During the last phase of the studies, the model was modified through parameter optimization to simulate a possible fully automated highway configuration.

**MAIN-LINE TRAFFIC RUNS**

To establish those parameter values which cause the model to operate in a manner consistent with observed normal driver-vehicle type interactions, a process was used in which parameter values from the literature and previous car-following computer experiments were tried and then modified until the results correlated with available test data. Work previously done by Fox and Lehman proved especially helpful in this regard. While the experimental data from the Port Authority experiments in car-following was not as detailed as would be desired (i.e., data was sampled only every five seconds), it does provide a means for quasi-verification of the model. Figures 12 and 13 show by means of a plot of relative spacing versus time, a comparison between actual test data and the computed results furnished by the model. In this particular run, vehicle operators wish to maintain a velocity of 65 ft/sec (44.2 mph.) and a spacing interval of 100 feet. The results then show the interactions of vehicles as the operators try to adjust to these desired levels from some initial set of conditions. By adjusting the model parameters to the values given in Table 1, the computed results were made to
VEHICLE POSITIONS (spacing feet)

TIME (seconds)

DESIRED VELOCITY = 65 ft/sec

Fig. 12 New York Port Authority Data
(Redrawn from Safety in Car-Following, page 80 by Fox and Lehman)
Fig. 13 Computed Car Following Data
Comprising Just Main Stream

VEHICLE POSITIONS (spacing feet)

TIME (seconds)

DESIRED SPACING = 125 ft
DESIRED VELOCITY = 65 ft
Table 1
Parameter Values Matching Port Authority Data

<table>
<thead>
<tr>
<th>Program Notation</th>
<th>Value</th>
<th>Parameter Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACELB (ft/sec/sec)</td>
<td>1.5</td>
<td>Acceleration increment used in adjusting vehicle velocity</td>
</tr>
<tr>
<td>ACFMIN (ft/sec/sec)</td>
<td>-20.0</td>
<td>Maximum Deceleration possible</td>
</tr>
<tr>
<td>ACZERO (ft/sec/sec)</td>
<td>15.0</td>
<td>Maximum acceleration from standing start</td>
</tr>
<tr>
<td>ALPHA</td>
<td>100.0</td>
<td>Sensitivity Factor</td>
</tr>
<tr>
<td>BTT (sec)</td>
<td>0.5</td>
<td>Reaction time in detecting a brake light</td>
</tr>
<tr>
<td>CRITWV (ft/sec/sec)</td>
<td>-11.0</td>
<td>Critical value of deceleration</td>
</tr>
<tr>
<td>DRINC (sec/sec)</td>
<td>0.2</td>
<td>Rate of increase of reaction time once it has been reduced</td>
</tr>
<tr>
<td>FVH</td>
<td>1.08</td>
<td>Highest acceptable speed factor</td>
</tr>
<tr>
<td>FVL</td>
<td>0.85</td>
<td>Lowest acceptable speed factor</td>
</tr>
<tr>
<td>HDES (sec)</td>
<td>1.4</td>
<td>Reaction time factor used in spacing equation</td>
</tr>
<tr>
<td>THRSH</td>
<td>0.0001</td>
<td>Threshold value</td>
</tr>
<tr>
<td>TKKK (sec)</td>
<td>0.6</td>
<td>Time to react to vehicle ahead decelerating</td>
</tr>
<tr>
<td>THMED (sec)</td>
<td>3.5</td>
<td>Period larger distance spacing is required, after vehicle ahead has rapidly decelerated</td>
</tr>
<tr>
<td>VDESR (ft/sec)</td>
<td>65.0</td>
<td>Desired velocity</td>
</tr>
<tr>
<td>VMAX (ft/sec)</td>
<td>150.0</td>
<td>Maximum velocity possible</td>
</tr>
<tr>
<td>B (ft)</td>
<td>25.0</td>
<td>Vehicle Length, plus safety factor</td>
</tr>
<tr>
<td>$W_1$</td>
<td>0.75</td>
<td>Weighting Factor in regards to relative importance of information from vehicle just ahead</td>
</tr>
<tr>
<td>$W_2$</td>
<td>0.25</td>
<td>Weighting factor in regards to relative importance of information from vehicle two ahead</td>
</tr>
</tbody>
</table>
approximate the actual test data as can be seen in comparing Figs. 12 and 13.

Those differences which do exist between the relative spacing curves from the experimental data and computer runs can be traced to two factors. First, the accuracy of the data is very much in question, as data was sampled only every five seconds. Secondly, drivers in each vehicle possessed different characteristics (i.e., reaction time, ability to judge relative spacing and velocity changes, etc.), while the model assumed identical characteristics for each vehicle operator. As with any case of a model such as this, by performing enough parameter adjustments, any set of curves could be closely approximated, however; expenditure of that much computer time was judged not merited since it was only the purpose of this set of runs to verify general parameter magnitudes and the ability of the model to regulate vehicles in a manner consistent with observed behavior.

VEHICLE MERGING UNDER NORMAL DRIVING CONDITIONS

With the establishment of parameter values consistent with human type response, runs were made with the addition of a merging lane, thus allowing the study of the interaction of two traffic lanes and any disturbances then resulting. For this set of runs, normal mainstream speed was set at 88 ft/sec. (60 mph.), while merging speeds of vehicles entering the mainstream from the acceleration ramp were set at values considerably lower, simulating a normal merging situation. Once in the mainstream flow, the merging vehicle was accelerated to a velocity consistent with mainstream desired speed and then the vehicle's control reverted to the mainstream car-following equation.
Figure 14 represents the results from such a run with the output showing changes in vehicle velocities with respect to time as cars following the merging vehicle attempt to adjust their dynamics to compensate for the merging vehicle under conditions of relatively high speed and dense traffic. For this particular run, those model parameters established during the initial mainline flow runs, which caused the model to approximate the Port Authority data, were used. The merging vehicle's entrance speed was 67 ft./sec. with a desired mainline gap spacing of 108 feet and a merging gap of 260 feet. From the graph it can be seen that the system is not overly unstable, but there are considerable oscillations and over-shoots.

In attempting to reduce these oscillations and improve the response of the system, various combinations of $W_1$ and $W_2$, (i.e., the weighing information factors used in determining the relative importance of information concerning the dynamics of the vehicle just ahead versus two ahead in computing vehicle acceleration using the car following equation) were tried. The results showed very little difference in relative stability as more weight was attached to information concerning the vehicle two ahead. This lack of direct results can partially be explained by the fact that information was still provided concerning the state of the brake lights of the vehicle two ahead (i.e., the application of brake lights by a vehicle has the effect of reducing the reaction time of the vehicles following).

The next attempt at trying to improve the stability of the system was carried out by adjusting "THRESH", the threshold value of awareness, (i.e., the value determining whether the vehicle controller is aware of changes in relative velocity and distance or only
Fig. 14 Merging Under Normal Driving Conditions

DESIRED MAIN STREAM VELOCITY = 88 FT SEC
DESIRED SPACING = 100 FT
ALPHA = 100
0.3 ≤ REACTION TIME ≤ 1.5

MERGE TAKES PLACE AT TIME = 0
Again negligible effects were observed with regards to improving stability by raising or lowering the original value. The most probable explanation for this is that the vehicles are already so tightly spaced and the merging vehicle induces such a large disturbance that the system is always in a velocity detection driving mode.

Spacing requirements were then studied to fix their relative importance in achieving stability during vehicle merging. As formulated in the model, mainline required spacing is given by:

\[
SPDES = B + CONSA \times HDES \times VELOCITY
\]

Where as mentioned, "B" represents an interval comprising vehicle length and some safety factor, while HDES*VELOCITY is roughly equivalent to the distance traveled before a vehicle is aware of changes in the driving behavior of the vehicle just ahead. From runs made in attempting to adapt the model to Port Authority data, it was found that for stable mainstream flow, "HDES" had to exceed a value of one, while changes in "B" led to small differences in relative stability.

With the incorporation of a merging lane, the merging gap distance required to maintain mainstream stability was found to be directly dependent on the difference in velocities between mainstream speed and merging entrance speed. Quantitatively the gap distance required to insure stable merging can be represented by the following equations:

\[
GAPMAIN = SPDES - CL
\]

\[
GREQ = 2 \times GAPMAIN + CL + SF
\]

where

\[
GAPMAIN = \text{Normal mainstream headway distance required for stable flow.}
\]

\[
SPDES = \text{Desired spacing as measured from the front bumper of one vehicle to the front bumper of the vehicle ahead.}
\]
CL = Vehicle length
GREQ = Mainstream gap distance required for stable merging.
SF = Safety factor which is a function of relative velocity differences.

In reality, this safety factor must include the distance lost while the merging vehicle is accelerating up to mainstream speed. Thus for stable merging, this safety factor is given by the following equation under the assumption that the merging vehicle attains mainstream speed in a very nearly linear manner.

\[ SF \geq \frac{V_{\text{MAIN}}(V_{\text{MAIN}} - V_{\text{MERGE}})}{A} - \frac{(V_{\text{MAIN}}^2 - V_{\text{MERGE}}^2)}{2A} \]

where

- \( V_{\text{MAIN}} \) = Mainstream desired velocity
- \( V_{\text{MERGE}} \) = Merging vehicle's entrance speed on to the mainstream relative to the mainstream direction.
- \( A \) = Average acceleration value, used by the merging vehicle in attaining mainline speed.

Thus as one would intuitively reason, merging gap requirements are dependent on the difference between injection speed and mainline velocity. It is also interesting to note from the runs made, that decreasing spacing below specified requirements, tends to quickly force the system into a much more unstable mode of operation, increasing spacing beyond that required has little effect on stability. This appears reasonable, however, for as vehicles cluster into queues with relative spacings smaller than required, they constantly interact, while attempting to regulate their spacing to desired levels. With headway distances larger than required, this added instability factor of vehicles attempting to attain some desired spacing is eliminated and thus
all further instabilities are generated purely by changes in the dynamics of one or a set of vehicles in the flow.

Reaction time, however, turns out to be one of the most important variables in determining system stability. As discussed in Chapter IV, a variable reaction time was incorporated into the model to simulate changes in driver's reaction time induced by various driving situations. The range over which reaction time can vary is illustrated by the results recorded in Table 2 of tests made in 1934 on brake reaction time. \(^1\) Basically they show that an individual driver's reaction time during normal driving can fluctuate from a value of less than .3 seconds to over 1.6 seconds. If anything, this is probably a conservative estimate. As expected, the system proved to be more stable for smaller reaction times, while more oscillatory for larger values.

In addition to normal reaction time, other reaction variables included were BTT (the reaction time in sensing a brake-light) which was set at .5 seconds, TKK (the reaction time for observing a rapid deceleration in the vehicle ahead) which was placed at .6 seconds and THMED (the time interval for increased desired spacing) set at 5 seconds. Again in reducing these values, as in the case of general reaction time, increased stability was observed.

ALPHA, the sensitivity or gain factor for the model is, in effect, a measure of the degree of the driver's response to various actions in the applying of the brake or the gas pedal. In the model itself, this factor shows up in the basic relation for acceleration determination given by the car-following equation:

\[ A_c = \text{ALPHA} \cdot \text{VELOCITY} \cdot \left( w_1 \cdot \frac{\text{VREL}_1}{\text{XREL}_1} + w_2 \cdot \frac{\text{VREL}_2}{\text{XREL}_2} \right) \]
### Table 2

<table>
<thead>
<tr>
<th>Vehicle Dynamics</th>
<th>Stimulus</th>
<th>Starting Foot Position</th>
<th>Reaction Time (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standing</td>
<td>Audible</td>
<td>Brake Pedal</td>
<td>0.24</td>
</tr>
<tr>
<td>Standing</td>
<td>Bright Light</td>
<td>Brake Pedal</td>
<td>0.26</td>
</tr>
<tr>
<td>Standing</td>
<td>Stop Light</td>
<td>Brake Pedal</td>
<td>0.36</td>
</tr>
<tr>
<td>Standing</td>
<td>Audible</td>
<td>Accelerator</td>
<td>0.42</td>
</tr>
<tr>
<td>Standing</td>
<td>Bright Light</td>
<td>Accelerator</td>
<td>0.44</td>
</tr>
<tr>
<td>Moving-normal road</td>
<td>Audible</td>
<td>Accelerator</td>
<td>0.46</td>
</tr>
<tr>
<td>conditions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standing</td>
<td>Stop Light</td>
<td>Accelerator</td>
<td>0.52</td>
</tr>
<tr>
<td>Moving-test conditions</td>
<td>Stop Light</td>
<td>Accelerator</td>
<td>0.68</td>
</tr>
<tr>
<td>Moving-normal road</td>
<td>Stop Light</td>
<td>Accelerator</td>
<td>0.82</td>
</tr>
<tr>
<td>conditions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moving-test conditions</td>
<td>None-Stop</td>
<td>Accelerator</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td>light hidden</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moving-normal road</td>
<td>None-Stop</td>
<td>Accelerator</td>
<td>1.65</td>
</tr>
<tr>
<td>conditions</td>
<td>light hidden</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*(Source: Traffic Engineering Handbook, (Baerwald, 1965) p.82)*

Both results from computer runs and mathematical stability analyses point to this sensitivity factor, coupled with reaction time, as being the prime determinants of system stability. Under normal driving conditions, the human driver gain factor, \( \text{ALPHA} \), according to Edie\(^7\) (1961), is around 92. Fox and Lehman\(^9\) in their study provide for two values of \( \text{ALPHA} \) in the following manner:

\[
\text{ALPHA} = \begin{cases} 
92 & \text{for acceleration response} \\
106 & \text{for deceleration response}
\end{cases}
\]
During initial runs of this investigation, in attempting to correlate the model with the Port Authority data, a value of 100 was used. The improvement in system stability by the reduction in the value of ALPHA can readily be seen in Fig. 15 which contains the same parameter values as that of Fig. 14, but with ALPHA reduced from 100 to 20. Thus it appears from model runs that decreasing ALPHA has a stabilizing effect on the system while increasing it tends to amplify any oscillations and increase their frequency leading to unstable conditions. This conclusion will later be collaborated by mathematical analysis.

Finally, this phase of the investigation was concluded by investigating the effect that vehicle performance characteristics play in determining system stability. Typical vehicle parameters were taken as follows: ACZERO, the maximum possible acceleration from a standing start, 15 ft/sec/sec, VMAX, the maximum possible vehicle velocity, 140 ft/sec and ACMIN, the maximum possible deceleration 20 ft/sec/sec. Owing to the fact that a vehicle's acceleration capabilities are a function of its velocity, the following relation was used in determining maximum acceleration:

\[
ACMAX = (VMAX - VELOCITY)*(ACZERO/VMAX)
\]

where

- **ACMAX** = Maximum acceleration
- **VMAX** = Maximum vehicle velocity possible
- **VELOCITY** = Current velocity of the vehicle
- **ACZERO** = Maximum acceleration possible from a standing start

As one would guess, vehicle parameters have no effect on the operation of the system as long as mainstream cruising speed is sufficiently lower than vehicle velocity limitations to allow for those
Fig. 15 Merging Under Modified Conditions

VELOCITY (feet/second)

DESIRED MAIN STREAM VELOCITY = 88 FT/SEC
DESIRED SPACING = 108 FT
\[ \alpha = 20 \]
\[ 0.3 \leq \text{REACTION TIME} \leq 1.5 \]

MERGE TAKES PLACE AT TIME = 0

MERGING VEHICLE

ENTRANCE ONTO MAIN RAMP
accelerations which might be required to compensate the dynamics of the vehicle for mainstream disturbances.

**VEHICLE MERGING IN AN AUTOMATED SYSTEM**

With the determination of those parameters which play a dominate role in determining stability, an optimal road configuration was modeled in which vehicles were spaced at gap intervals as small as 8.8 feet. Basic to this analysis was the assumption that since all vehicles are under the same form of control, a vehicle is limited in its maximum decelerating capabilities to that of every other vehicle in the system (i.e., we are discounting the possibility of hitting something in the middle of the roadway and coming to an instantaneous stop). Under this assumption, the minimum mainstream headway requirement is given by:

\[
\text{MAINGAP} = (\text{Reaction time}) \times (\text{Velocity})
\]

where \( \text{MAINGAP} \) = Minimum mainstream gap requirement

This distance then allows a vehicle to stop in time to avoid hitting the vehicle in front, if it suddenly applies maximum braking power. The runs now described were made for a system with a mainline speed of 88 ft/sec (60 mph.) and a headway distance for mainstream flow of 9 feet. This closely approximates an optimal automated system.

Assuming a vehicle length of 17 feet, a flow rate of 13,000 vehicles per hour is possible for a single lane of automated traffic. This compares with 6700 vehicles per hour for the average eight lane grade-separated (four lanes in each direction) highway of today, which only attains this maximum flow rate at vehicle speeds in the 35 to 45 mph. range.
For such an automated system as outlined, an initial run (refer to Fig. 16) was made using those parameters which match human driver response. As can be observed, the system is very unstable during the merging process with the vehicle speed deviation being greatly amplified by each vehicle as it propagates down the chain.

As noted in discussing normal merging, ALPHA, the sensitivity factor, and the reaction time played the dominant role in determining system stability. Thus, as a first attempt to stabilize the system, it seemed reasonable to reduce reaction time to a fixed level of .1 second. As can be readily observed from Figs. 16, 17, and 18 this greatly improves the stability of the system. Next the effect of varying ALPHA was studied, with the result that decreasing ALPHA has a stabilizing effect. Complete stability for the merging process was observed for a reaction time of .1 seconds and an ALPHA of 20. This indicates then, that the main disadvantages of the human operator as the main component in the vehicle control system are his slow reaction time and his overly-sensitive corrective actions.

Vehicle parameter values were left largely the same as those for normal vehicles as described in the last section.

\[
\begin{align*}
VMAX &= 140 \text{ ft/sec/sec} \\
ACZERO &= 15 \text{ ft/sec/sec} \\
ACMIN &= -20 \text{ ft/sec/sec}
\end{align*}
\]

The last portion of investigating this automated highway configuration centered around the dynamics of the merging vehicle. In studying the effect of various entrance techniques, a semistable main flow line was assumed with a reaction time of .1 second but a sensitivity factor of 100. Figure 19 shows the resultant velocity
2nd VEHICLE BEHIND MERGING VEHICLE

MAIN STREAM DESIRED VELOCITY = 88 FT/SEC

DESIRED SPACING = 9 FT

ALPHA = 100

.3 ≤ REACTION TIME ≤ 1.5

3rd VEHICLE BEHIND MERGING VEHICLE

MERGE TAKES PLACE
AT TIME = 0

Fig. 16 Merging in Automated Configuration Under Human Control Parameters
Fig. 17  Velocity Trajectories of Vehicles Following Merging Vehicle in Automated Condition
Fig. 18 Velocity Trajectory of the 4th Vehicle Behind the Merging Vehicle in an Automated Configuration
patterns of the vehicle directly behind the entering vehicle caused by disturbances induced by various entrance velocities. It was found that if the merging vehicle enters the mainstream with a relative velocity very close to that of the mainstream flow, no disturbances are induced. For this particular type of driving situation, an acceptable velocity range for a mainstream velocity of 88 ft/sec turns out to be:

$$87 \text{ ft/sec} \leq VMERG \leq 89 \text{ ft/sec}$$

If, however, entrance speeds exceed or are less than these values, oscillations result. The amplitude and frequency of these oscillations are determined by the stability of the mainstream flow. This indicates that there are two methods for stabilizing the merging process, either to stabilize the mainstream so that disturbances will be damped out, or to control the merging entrance dynamics so that no disturbances are induced. This last method means that speeds on the acceleration ramp will have to exceed that of the main flow if velocity relative to the mainstream is to be kept in bounds. Obviously the more important aspect is the stabilization of the mainstream. By stabilizing both, optimal operation results.

Merging gap requirements for the automated system coincides very closely with that observed for the nonautomated situation, that is, gap requirements are a function of reaction time and velocity differences between mainline and merging speeds. This requirement is represented in the following equations:

$$GREQ = 2*GAPMAIN + CL + SF$$

$$SF \geq \frac{(VMAIN* (VMAIN-VMERGE))}{A} - \frac{(VMAIN^2 - VMERGE^2)}{2*A}$$
where

\[ \text{GREQ} = \text{Required gap distance} \]
\[ \text{GAPMAIN} = \text{Normal mainstream gap required} \]
\[ \text{CL} = \text{Vehicle length} \]
\[ \text{SF} = \text{Safety factor representing the distance loss by the merging vehicle in attempting to attain mainstream speed} \]

As the merging process is optimized, this safety factor will approach zero.

**STABILITY ANALYSIS**

In attempting to verify the results obtained from runs made with the model, several quantitative methods of stability analysis for car-following theory were investigated. Basically, the type of stability we are most interested in is asymptotic stability; where asymptotic instability is defined as the case when each vehicle amplifies the signal it receives from the vehicle in front so that disturbances tend to grow.

Gazis, Herman and Rothery \(^{13}\) did an analysis for asymptotic instability caused by small perturbations in a string of vehicles. As a basis for their work, they used the car-following equation used in this model. While Appendix B contains the complete derivation, the results in the form of requirements for asymptotical stability can be summarized in the following equation:

\[
\alpha \cdot \text{VELOCITY} \cdot \frac{T}{Y^2} < \frac{1}{2}
\]

where

\[ \alpha = \text{Sensitivity factor} \]
\[ \text{VELOCITY} = \text{Mainstream velocity} \]
\[ T = \text{Reaction time} \]
\[ Y = \text{Vehicle spacing, measured from front bumper to front bumper} \]
This result affirms what had earlier been learned from model runs, that the main determinant for stability, given a particular driving situation, are the controller's reaction time and sensitivity. For example, in the automated driving situation with human parameters which proved so unstable:

\[
\frac{(\alpha \cdot v \cdot t)}{y^2} = 18
\]

While for the completely stable configuration in which \(\alpha\) was set equal to 20 and reaction time was a fixed constant at .1 second:

\[
\frac{(\alpha \cdot v \cdot t)}{y^2} = .37
\]

The simulated results then tend to reinforce this stability relationship, which appears to be critical in the design of any automated system. It is worth noting that another stability analysis was done by Prof. Meyer (1966) for stability with respect to large perturbations. His stability requirements are given in the following relationship:

\[
\frac{(\alpha \cdot v \cdot t)}{y^2} < \frac{1}{2} + \frac{\alpha}{y \cdot (e^{\alpha/y} - 1)}
\]

While this result differs from the work of Gazis, Herman and Rothery by the \(\frac{\alpha}{y \cdot (e^{\alpha/y} - 1)}\) term, it appears that they can be related in the following manner:

Defining: \(F(\alpha, y) = \frac{1}{2} + \frac{\alpha}{y \cdot (e^{\alpha/y} - 1)}\)

Taking the limit as \(\alpha\) approaches zero:

\[
F(\alpha, y) = \frac{1}{2} + \frac{1}{y} \cdot \left(\frac{1}{y} \cdot (e^{\alpha/y}) - 1\right) = \frac{3}{2}
\]

Taking the limit as \(\alpha\) approaches infinity:

\[
F(\alpha, y) = \frac{1}{2}
\]
Plotting $F(\text{ALPHA}, Y)$ as a function of $\text{ALPHA}$, a curve similar to the one shown in Fig. 20 is realized. Hence for large values of $\text{ALPHA}$, both stability relations coincide.
Fig. 20 Plot of Factor Found in Meyer's Stability Relationship
CHAPTER VI
CONCLUSIONS

From the results of the model test runs and mathematical analysis described in the last chapter, it is clear that the basic criteria in judging the merit of adding specific sensory or other driver aid devices to vehicles is the degree by which it will improve the stability of the system, thus reducing gap requirements and optimizing road usage.

For sensory aided driving there appears to be two directions in which improvements can be made; in reducing reaction time and driver sensitivity, (i.e., the amount of accelerating or braking a driver uses in making corrections). Probably the most plausible is an attempt to reduce driver sensitivity. This could be attained by furnishing the vehicle operator with some form of a null-meter display, indicating to the driver when he is accelerating or braking too hard. Information for such a meter control device could be attained by sensor devices detecting relative spacing and velocity or both as in the work of Bierley\textsuperscript{3} which was discussed in Chapter III. Another factor in the stabilization of a flow of vehicles is maintaining at least a specified minimum gap distance. This could be accomplished by placing lines across the road, separated by required gap distances for the particular velocity at which the roadway is normally operated. The vehicle operator would then be instructed to maintain a distance behind the vehicle in front so that as his front bumper crosses one line, the front bumper of the vehicle ahead is crossing the next line.
As mentioned, the other major factor in stabilizing vehicle flow is the reduction of driver reaction time. The ability of devices to decrease a driver's reaction time and then hold it at this lower level for any sizable amount of time is very questionable. Devices in this area will in all probability have to be restricted to purely warning types.

There appears to be little doubt that fully automated travel, especially in and around our urban areas, will become a reality. As discussed in the last chapter, a two lane automated system (one lane for travel in each direction) could handle approximately what now requires four eight lane (four lanes in each direction) systems. For such an automatic system, flow rates in excess of 13,000 vehicles per hour for a single lane are not at all out of the question since gap requirements are limited only by the reaction time of the system. In fact, the optimal flow rate for an automated system from the Q-K relationship is found to be:

\[
Q = \frac{V \times 3600}{CL + V \times T}
\]

where

- **Q** = Vehicle flow rate in terms of vehicles per hour
- **V** = Average velocity of mainstream vehicles (ft/sec)
- **T** = Reaction time

Automated systems provide for two methods of stabilization. First is stabilization of mainstream flow dictated by the following relationship:

\[
\frac{\text{ALPHA} \times \text{VELOCITY} \times T}{Y^2} < \frac{1}{2}
\]
where \( \text{ALPHA} = \text{Sensitivity factor} \)

\[ Y = \text{Mainstream vehicle spacing distance} \]

\[ \text{VELOCITY} = \text{Average mainstream velocity (ft/sec)} \]

\[ T = \text{Reaction time} \]

Second is stabilization attained by optimizing the merging process; where optimal merging is defined as merging at speeds very close to those of the mainstream.

What then has been developed is a tool in the form of a computer program with which control and sensory devices can be evaluated from the aspect of their effect on stabilizing the system. Secondly, criteria in the form of particular performance requirements which must be met to insure safety for given desired driving conditions have been set.
CHAPTER VII
RECOMMENDATIONS FOR FUTURE WORK

It has been the goal of this investigation to formulate a model encompassing those driver-vehicle interactions which appeared necessary to study mainstream driving and vehicle merging for the evolutionary process from normal driving through fully automated travel. From this point there are many interesting and challenging directions in which to go.

One major disadvantage of this model as far as investigating normal driving interactions has been its lack of provision for multi-lane mainstream flow with vehicles interacting between lanes. Thus it should prove interesting to enlarge the model to a two or three lane type system.

In regards to the problem of optimal scheduling, (i.e., merging vehicles into the system in such a manner as to optimize roadway usage), the model could be enlarged to include several entrances and exits in overlapping succession.

By far the most intriguing possibility is the application of this model to a driving simulator. Essentially as the model now exists, computer logic is substituted for the reactions of the human driver. By allowing the human to interact directly with the simulation by actually controlling one of the vehicles various driving situations could be more effectively studied, especially in the area of sensory aided driving.
Appendix A

Program Listing

C PROGRAM FOR MERGING AND MAIN STREAM STUDY

DIMENSION BLT(8,20),VLP(8),XP(8)
DIMENSION OLCON(8),CONTIM(8),ACELBT(8),ACMIN(8),SAMPS(8),SAMPT(8)
DIMENSION VDES(8)
DIMENSION BT(8),VMAX(8),ACZERO(8),NINC(8),DRINC(8)
DIMENSION SAMPV(8),DSMPV(8),DSMPS(8),DSMPT(8),WAVE(1)=A(8,20)
DIMENSION FVL(8),FVH(8),AK(8),KKK(8),THRESH(8)
DIMENSION DK(8),HDES(8),FCTR1(8),FCTR2(8),T(8)
DIMENSION TIMAX(8),ZEROD(8),WAVE(8),A(8,20)

162 FORMAT(20X,27HCONTROL RETURNED TO VEHICLE//)
159 FORMAT(10X,46HMERGE IS TAKING PLACE MERGING VEHICLE NUMBER =,1X,F3)
160 FORMAT(2X,4HTIME,,4HVEH1,,4HVEH2,,4HVEH3,,4HVEH4,,4HVEH5)
161 FORMAT(2X,4HTIME,,8X,,4HVEH1,,8X,,4HVEH2,,8X,,4HVEH3,,8X,,4HVEH4,,8X,,4HVEH5)

PRINT 161

C INITIAL SPACING OF VEHICLES SPECIFIED

SPAB(1)=200.
SPAB(2)=112.
SPAB(3)=200.
SPAB(4)=112.
SPAB(5)=112.
SPAB(6)=112.

C READ IN SYSTEM VEHICLE PARAMETERS

D) 41 1=1,6
VBL(1)=88.

41 CONTINUE

VBL(7)=88.

DO 1 BLT(I)=X(8,20),VLP(I)=X(8,20),XP(I)=X(8,20)
OLCON(I)=10.0
CONTIM(I)=10.0
ACELBT(I)=1.5
ACMIN(I)=20.0
SAMPS(I)=40
SAMPT(I)=40
VDES(I)=88.
SAMPV(I)=40
DSMPV(I)=40
DSMPS(I)=40
DSMPT(I)=40
WAVE(I)=10.0
BIT(I)=5.

-74-
A(I) = 0.0
V(I) = 10.0
D(I) = 2
F(I) = 95
T(I) = 1.0
A(I) = 2.0
T(I) = 60
T(I) = 0.0
D(I) = 105
F(I) = 90
T(I) = 1.0
A(I) = 0.0
T(I) = 0.0
D(I) = 106
H(I) = -100
F(I) = 40
F(I) = 3
F(I) = 1
MODE(I) = 1
FOR NEXT RUN INITIATE CAR POSITIONS AND VELOCITIES
A(I) = 0.0
X(I) = 0.0
B(1) = 20.0
W(1) = 0.0003
W(1) = 0.0003
I(I) = 1
MODE(I) = 1
CONTINUE
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
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X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
A(I,2) = 0.0
X(I,2) = X(I-1,2) - SPAB(I)
18 V(I,1)=V(I,2)+5*DELT*(A(I,1)+A(I,2)) 01230
X(I,1)=X(I,2)+5*DELT*(V(I,1)+V(I,2)) 01240
THETA=THETA-(PI/44) 01250
IF (THETA*GE.0) GO TO 19 01260
THETA=0 01270
19 IF (S=NE.7) GO TO 28 01280
MERGE=3 01290
PRINT 162 01300
C CAR HAS BEEN FULLY ABSORBED INTO THE MAIN FLOW, CONTROL RETURNED 01310
GO TO 28 01320
14 CONTINUE 01330
KK=ITKXI+1+0001)/DELT+1*0 01340
JJ=(BT(I)+0001)/DELT+1*0 01350
C INTERPOLATE FOR XREL 1*2 01360
J=(TI+1+0001)/DELT+1*0 01370
VREL1=V(I-1,J)-V(I,J) 01380
XREL1=X(I-1,J)-X(I,J) 01390
BLT1=BLT(I-1,J) 01400
IF (I*EQ.2) GO TO 933 01410
932 VREL2=V(I-2,J)-V(I,J) 01420
XREL2=X(I-2,J)-X(I,J) 01430
BLT2=BLT(I-2,J) 01440
933 A(SW*MODE(I)) 01450
IF (A(SW*EQ.2)) GO TO 922 01460
C MODE(I)=1 IS DISTANCE DETECTION MODE 01470
C MODE I=2 IS VELOCITY DETECTION MODE 01480
C THIS IS DISTANCE DETECTION MODE 01490
BKAI=TIME-SAMPT(I) 01500
I=(BKAI*GE.0) GO TO 902 01510
901 MODE(I)=1 01520
IT(I)=1 01530
A(I,1)=A(I,2) 01540
PATH=0 01550
IF (T(I)TIMAX(I)+GE.0) GO TO 904 01560
T(I)=T(I)+DRINC(I)*DELT 01570
904 CONTINUE 01580
IF ((TIME-SAMPS(I))*LT.0)) GO TO 905 01590
SAMPS(I)=TIME+DSAMPS(I) 01600
IF ((TIME+CONTIM(I)+TMED)) +GE.0) GO TO 908 01610
CONS=0 01620
GO TO 909 01630
908 CONS=1 01640
909 SPDDES=B(I)+CONS*HDES(N)(I)*V(I,2) 01650
IF (XREL1=SPDES*LT.0) GO TO 660 01660
910 CONTINUE 01670
905 IF ((TIME-SAMPV(I))*LT.0) GO TO 680 01680
SAMPV(I)=TIME+DSAMPV(I) 01690
GO TO 665 01700
902 SAMPT(I)=TIME+DAMPT(I) 01710
922 W(I)=5 01720
W2=5 01730
IF (10)*XREL1/CL)*LT.0) GO TO 936 01740
W3=0 01750
GO TO 912 01760
936 W1=(10-XREL1/CL)*R.0)*W3A(I) 01770
IF (1*EQ.2) GO TO 912 01780
C IF DECELERATING RAPIDLY, DRIVER WONT WORRY ABOUT SECOND CAR 01790
IF (W1*EQ.1) GO TO 912 01800
IF (W2*EQ.2) GO TO 912 01810
IF (W3*EQ.3) GO TO 912 01820
919 E=W1*VREL1/XREL1**2+W2*VREL2/XREL2**2 01830
TH=SORT(EQ.2)+W3*BLT1+W4*BLT? 01840
GO TO 920 01850
912 E=W1*VREL1/XREL1**2 01860
TH=SQRT((EQ**2)+3*B`)\[1\]
920 IF (THTH**2)-THRESHHERE**2)+43*BLT1) GO TO 901
925 MDF1=2
C OVER THRESHOLD VELOCITY
1. (TIME~WAVEI)+TMED) LE.0.0) GO TO 923
C IF (COL=NEJ1) G0 TO 925
C KI ALL EAVALS 0 UNTIL A COLLISION HAS OCCURRED
929 CONSA=2.0*CRITWV-(VI-1,KK)-VI-1,KK+1)/DELT-ACMIN(I)
CONSA=CONSA/(CRITWV-ACMIN(I))
930 (CONSA-ACM1(I)) LE.0.0) GO TO 927
C M1=1.0
927 IF (CONSA-OLCON(I)) LE.0.0) GO TO 928
CONTIM(I)=TIME
OLCON(I)=CONSA
GO TO 600
928 CONTINUE
IF (TIME-(CONTIM(I)+TMED)) LE.0.0) GO TO 931
OLCON(I)=1.0
GO TO 600
931 CONSA=OLCON(I)
600 SDFS=B(I)+CONSA*HDFSN(I)*V(I,2)
ZAP=IT(I)
C IF (ZAP*EQ.O.O) GO TO 610
1.0=IF THRESHOLD EXCEEDED
T(I)=0
T(I)=FCTR2*TIME
IF ((T(I)-.1).GT.0.O) GO TO 600
T(I)=1
T(I)=EMAX(I)
IF (T(I)-.1).GT.0.O) GO TO 600
T(I)=1
0.0) GO TO 705
IF (V(I,2)+DELT*5*(A(I,1)+A(I,2))) GO TO 660
IF (V(I,1)+EQ.0.0) GO TO 705
IF (V(I,1)+GT.0.0) GO TO 640
V(I,1)=0.0
GO TO 705
IF \((V(I,2) - FVH(I)*VDES(I)) \leq 0.0\) GO TO 705

\[ A(I,1) = A(I,1) - 1.3 \]

\[ V(t,1) = V(I,2) + 0.5 + \Delta T \times (A(I,1) + A(I,2)) \]

\[ X(I,1) = X(I,2) + 0.5 \times \Delta T \times (V(SI) + V(I,1)) \]

\[ DRAG = -0.000068212343 \times V(I,1)^2 + 0.034842529 \times V(I,1) - 0.3036002 \]

DECELERATION

\[ D = \text{DRAG} \]

IF \((A(I,1) - \text{DRAG}) > 0.0\) GO TO 28

\[ B(I,1) = 1.0 \]

CONTINUE

BEGINNING MERGE SUBROUTINE

IF \((\text{MERGE} \cdot 0)\) GO TO 512

IF \((\text{MERGE} \neq 0)\) GO TO 520

512 \(K = 2\)

517 \(\text{AGAP} = X(K,2) - X(K+1,2)\)

\[ \text{SPDES} = 8(I) + \text{HDES}(I) \times 88 \times \text{RGAP} = CL + (2 \times \text{SPDES}) + 10 \times \]

IF \((\text{AGAP} \geq 0)\) GO TO 513

IF \((K \leq 5)\) GO TO 515

\[ A(8,1) = ACMN(I) \]

\[ V(8,1) = V(8,2) + 3 \times \Delta T \times (A(8,1) + A(8,2)) \]

IF \((V(8,1) \geq 0.0)\) GO TO 516

\[ V(8,1) = 0.0 \]

516 \(X(8,1) = X(8,2) + 0.5 \times \Delta T \times (V(8,1) + V(8,2)) \)

GO TO 21

515 \(K = K + 1\)

GO TO 517

513 \(\text{INCEPT} = K + 1\)

SOLVE FOR TIME TO INTERCEPT

\[ \text{CEPTIM} = (-X(K,2) + \text{SPDES} + 0.1) / V(K,2) \]

IS VELOCITY AND ACCELERATION WITHIN BOUNDS

\[ \text{ACREQ} = 2 \times (-X(8,2) - V(8,2) \times \text{CEPTIM}) / \text{CEPTIM}^2 \]

\[ \text{VERE6} = \sqrt{V(8,2) + 2 \times \text{ACREQ} \times X(8,2)} \]

IF \((\text{VERE6} - 110) \times \text{GE} \times 0.0)\) GO TO 515

GAP HAS BEEN FOUND

\[ \text{MERGE} = 1 \]

\[ V(8,1) = \text{ACREQ} \]

\[ V(8,1) = V(8,2) + 3 \times \Delta T \times (A(8,1) + A(8,2)) \]

\[ X(8,1) = X(8,2) + 3 \times \Delta T \times (V(8,1) + V(8,2)) \]

IF \((X(8,1) \leq 0.0)\) GO TO 21

MERGE IS TAKING PLACE

\[ \text{MERGE} = 2 \]

523 \(DO 521 L = 1, 20\)

\[ V(MCAR \cdot L) = V(INCEPT \cdot L) \]

\[ M\cdot CAR = M\cdot CAR + 1 \]

\[ A(MCAR \cdot L) = A(INCEPT \cdot L) \]

\[ B(LT(MCAR \cdot L)) = B(LT(INCEPT \cdot L)) \]

\[ X(MCAR \cdot L) = X(INCEPT \cdot L) \]

CONTINUE

\[ IT(MCAR) = IT(INCEPT) \]

\[ MODE(MCAR) = MODE(INCEPT) \]

\[ T(MCAR) = 1 \]

IF \((\text{INCEPT} \cdot LE \cdot \text{INCEPT})\) GO TO 522

\[ M\cdot CAR = M\cdot CAR + 1 \]

GO TO 523

522 \(DO 524 J = 1, 20\)

\[ V(INCEPT \cdot J) = V(8, J) \]

\[ X(INCEPT \cdot J) = X(8, J) \]

\[ A(INCEPT \cdot J) = A(8, J) \]

\[ BLT(INCEPT \cdot J) = B(LT(8, J)) \]

CONTINUE

\[ IT(INCEPT) = 1 \]

\[ T(INCEPT) = 1 \]

\[ MODE(INCEPT) = 2 \]

\[ PI = 3.141592 \]

\[ \text{THETA} = PI / 8 \times 0 \]
NCAR=7
S=1
X(INCEPT,1)=0.0
V(INCEPT,1)=V(INCEPT,1)*COS(THETA)
A(INCEPT,1)=A(INCEPT,1)*COS(THETA)
V(INCEPT,2)=V(INCEPT,2)*COS(THETA)
V(INCEPT,3)=V(INCEPT,3)*COS(THETA)
PRINT 159,INCEPT
PRINT 160
21 IF (MERGE-GE= 2) GO TO 22
PRINT 157,TIME,V(1,1),V(2,1),V(3,1),V(4,1),V(5,1),V(6,1),V(7,1)
1=1(8:1)
PRINT 158,X(1,1),X(2,1),X(3,1),X(4,1),X(5,1),X(6,1),X(8,1)
G) TO 23
22 PRINT 157,TIME,V(1,1),V(2,1),V(3,1),V(4,1),V(5,1),V(6,1),V(7,1)
1=1(INCEPT)
PRINT 158,X(1,1),X(2,1),X(3,1),X(4,1),X(5,1),X(6,1),X(7,1)
23 CONTINUE
DO 34 J=2,20
K=22-J
D3 34 1=1.8
BLT(I,K)=BLT(I,K-1)
Y(I+K)=X(I+K-1)
V(I+K)=V(I+K-1)
34 CONTINUE
DO 16 I=2,8
A(I+2)=A(I+1)
16 CONTINUE
END
APPENDIX B

STABILITY ANALYSIS FOR CAR FOLLOWING

The following analysis was originally done by Gazis, Herman and Rothery. Considering the general car-following law described by the following equation:

\[
\frac{d^2 X_{n+1}(t)}{dt^2} = \frac{\text{ALPHA} \cdot V_n}{(X_n - X_{n+1})^2} \cdot \frac{dX_n(t)}{dt} - \frac{dX_{n+1}(t)}{dt}
\]

where

- \( X_n \) = Position of vehicle \( n \)
- \( V_n \) = Velocity of vehicle \( n \)
- \( \text{ALPHA} \) = Sensitivity factor

Now consider the Fourier component of the fluctuation of the lead vehicle, (i.e., the 0th vehicle in the chain).

\[
\frac{dX_n(t)}{dt} = F_n \exp(j\omega t)
\]

where: \( F_0 = 1 \)

Letting:

\[
L = \frac{\text{ALPHA} \cdot V_n}{(X_n - X_{n+1})^2}
\]

Substituting into the general car-following equation:

\[
(j\omega \exp(j\omega t)) \cdot F_{n+1} = L \cdot (F_n - F_{n+1})
\]

\[
F_{n+1} = \frac{(1 + j\omega \exp(j\omega t))^{-1} \cdot F_n}{L}
\]
Solving for $F_n$ in terms of $F_0$:

$$F_n = (1 + j\omega/L*\exp(-jwt))^{-n}F_0$$

Integrating to solve for $X_n(t)$:

$$X_n(t) = (1 + \omega^2/L^2 - (2\omega/L)*\sin(\omega t))^{-n/2} *$$

$$\exp(i(\omega t - \tan^{-1}(\omega/L)*\cos(\omega t)/(1-(\omega/L)*\sin(\omega t))))$$

If $\omega^2/L^2$ is greater than $2(\omega/L)*\sin(\omega t)$; where $\omega$ is equal to the frequency of oscillation, the amplitude decreases with increasing values of $n$; thus as $\omega$ approaches zero:

$$L*T < 1/2$$

or substituting in the value of $L$:

$$(\text{ALPHA} * V_0 * T)/Y^2 < 1/2$$

where

- $\text{ALPHA} = \text{Sensitivity factor}$
- $V_0 = \text{Velocity of the mainstream}$
- $T = \text{Reaction time}$
- $Y = \text{Vehicle spacing}$


