THE EFFECTS OF SURFACE CONDITIONS ON BOILING CHARACTERISTICS

by

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ABSTRACT

A unified model relating surface variables to the nucleate pool
boiling characteristics was developed. A simple trapping mechanism was
postulated and a geometrical model developed for idealized conical cavities
relating the effective radius for nucleation to cavity radius, cone angle,
and contact angle. The model predicts that fluids with small contact
angles (organics or liquid metals) should nucleate at a smaller effective
radius than fluids with relatively large contact angles (water) for a
given cavity. Experimental results for water and organics were shown to
be consistent with the theory.

A gas diffusion technique was developed to measure the effective
radius for natural cavities and was used successfully to predict the
minimum \( \Delta T \) necessary to sustain bubble production from individual
cavities. In a modified form the capability of gas diffusion technique
can be extended to provide the cumulative distribution of active cavities
for any given surface. Utilizing gas diffusion and boiling experiments,
the nucleation criteria \( \Delta T = \frac{2 \delta T_s \nu_2}{h_i g^2} \) was verified for small natural
cavities, in which case the effect of the thermal layer is insignificant.
It was shown analytically that dynamic effects in nucleation can be
neglected for most cavities of practical interest.

Photo and electron micrographs of active sites were obtained, pro-
viding information on cavity geometry. A surface model was developed for
the cumulative site density curve in terms of geometrical parameters. For
mechanically ground surfaces it was shown sufficient to assume that the
population of cavities possesses a single equivalent cone angle and that
the distribution of mouth radii follows the power law. The model was used
to explain experimental data from this and other studies. Methods were
discussed for measuring the important geometric surface parameters in
advance.

A transient heat conduction model was experimentally verified for
water and organics at atmospheric pressure. The model, even though strictly
valid for only the isolated bubble region, was extrapolated to predict per-
formance at higher heat flux. By relating surface variables to the boiling
heat flux, a unified expression was developed showing explicitly the role
of surface geometry and contact angle.

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NOMENCLATURE

A  surface area  
A1  area of influence  
A2  dimensional parameter, eqn. (15d)  
B2  dimensional parameter, eqn. (15e)  
C2  dimensional parameter, eqn. (15f)  
C  specific heat and dissolved gas concentration  
Db  bubble departure diameter  
f(e,φ)  dimensionless function, eqn. (10)  
g  acceleration of gravity  
g0  gravitational conversion coefficient  
h  heat transfer coefficient  
h  boiling heat transfer coefficient  
H/ro  dimensionless parameter, eqn. (15i)  
hflu  latent heat of vaporization  
K  thermal conductivity  
M/ro  dimensionless penetration distance, eqn. (15h)  
n/A  number of sites/unit area  
Ps  system pressure  
Pv  vapor pressure  
Pv0  Ps + 2δρ  initial vapor pressure in cavity  
(q/A)  heat flux  
(q/A)b  boiling component of the heat flux  
R  cavity radius
\[ r_0 \quad \rho_0 \cos(\theta - \phi \pi) \quad \text{local cavity radius at} \ M \]

distance co-ordinates

\( T_s \quad \text{saturation temperature corresponding to system pressure} \)

\[ T_2 + \frac{2 \frac{\partial T_s}{\partial t} u_{fg}}{h_{fg} p} \quad t \quad \text{time} \]

\( U_{fg} \quad \text{specific volume change of evaporation} \)

\( \Delta T \quad T - T_s \)

\( W_3 \quad \text{dimensionless parameter, eqn. (15b)} \)

\( W_2 \quad \text{dimensionless parameter, eqn. (15c)} \)

\( W_{g_0} \quad \text{initial mass of trapped vapor} \)

\[ \Delta T_{2\beta_0} \quad \frac{2 \frac{\partial T_s}{\partial t} u_{fg}}{h_{fg} p} \quad \text{excess superheat} \]

\( \alpha \quad \text{thermal diffusivity} \)

\( \delta \quad \text{thermal layer thickness} \)

\( \mu \quad \text{chemical potential and viscosity} \)

\( \rho \quad \text{radius of curvature} \)

\( \rho_0 \quad \text{initial value of radius of curvature} \)

\( \rho_r \quad \text{vapor density} \)

\( \rho_l \quad \text{liquid density} \)

\( \theta \quad \text{contact angle} \)

\( \phi \quad \text{cone angle} \)

Subscripts

\( \text{a} \quad \text{air} \)

\( \text{b} \quad \text{boiling} \)

\( \text{l} \quad \text{liquid} \)
\( \text{O}_2 \) oxygen

\( i \) initial condition

\( s \) saturation condition

\( v \) vapor

\( w \) wall
I. INTRODUCTION

I.1 Motivation For This Study

In heat transfer work involving nucleate pool boiling, one of the primary considerations is the \( q_A \) vs \( \Delta T \) behavior. The most direct way of obtaining the nucleate boiling characteristic of a fluid surface combination is by an actual boiling experiment. It is however of interest from both a practical and analytical point of view to develop methods for predicting the nucleate boiling curve in advance without the necessity of a boiling experiment. Non boiling methods are shown in the flow chart of Figure 1, where the direct and indirect methods refer to the manner in which the nucleation site density curve, \( n_A \) vs \( \rho \), is obtained. Using a direct method \( n_A \) vs \( \rho \) is immediately obtained but with the indirect method a surface measurement and model are required before \( n_A \) vs \( \rho \) can be determined. In either case subsequent nucleation and heat flux models must be utilized for predicting \( q_A \) vs \( \Delta T \). It is apparent that the information required to develop the preceding model involves nearly every aspect of boiling. In spite of the vast amount of boiling research performed over the years many fundamental questions raised by the model relating to nucleation, cavity distribution and heat flux models still remain unanswered.

Griffith and Wallis [1] proposed a criteria for predicting nucleation from a cavity with a given mouth radius using thermodynamic equilibrium concepts. This model was verified in Reference [1] for uniformly superheated systems, and in
[2,3,12] for liquid metals. However, when applied to the general problem of boiling in a saturated pool, the predicted superheat was consistently too small. Other models incorporating dynamic effects [4] and the thermal layer [5,6,7] were also unable to explain experimental results conclusively.

The first heat flux models were based on bubble agitation but later Moore and Messler [8] contended that microlayer evaporation was the dominant mechanism. Han and Griffith [5,6] formulated the problem as a transient conduction process. Then Graham and Hendricks [8] and Cooper [10,11] theoretically determined the conditions under which the microlayer becomes important. Deane [10] was the first to incorporate all the effects into a single model and experimentally predict the boiling characteristics. His observations were limited to temperature fluctuations under a single nucleation site. Mikic and Rohsenow [13] proposed a transient conduction model of the process (based on the arguments by Han and Griffith [5,6]) for conditions under which the microlayer effect should be insignificant; but more data is needed to substantiate this model.

Less analytical work has been done on characterizing boiling surfaces than on any other aspect of boiling. Most of the work has been primarily experimental. The influence of surface conditions was first investigated by Corty and Foust [14]. Later Westwater [15,16] photographed and counted nucleation sites, but did not attempt to model the surface.
Many experimenters tried to characterize surfaces using roughness but this parameter proved inadequate [17].

I.2 Scope

Both direct and indirect non-boiling methods of predicting the $\frac{Q}{A} \text{ vs } \Delta T$ behavior in advance will be considered, with a primary emphasis on investigating the effects of surface conditions on boiling characteristics. In order to achieve these objectives every aspect of boiling mentioned in Section I.1 must be considered—nucleation phenomena, nucleation site distribution and heat flux model.

This work begins in Chapter II with a theoretical and experimental analysis of nucleation. This includes a study of the role of contact angle in determining the effective radius for nucleation. Also an analytical study is made to determine inertial and transient heat transfer effects upon nucleation. A gas diffusion experiment is described which is used to perform nucleation experiments and verify the theory. Further supporting experimental evidence from the literature is then given.

In Chapter III a surface model is developed relating surface parameters to the nucleation site distribution. Also supporting experimental evidence is presented for water and several organics. A discussion of possible techniques for measuring the cavity distribution is presented.

In Chapter IV a transient heat conduction model is
verified experimentally. The surface model is then incorporated with the heat flux model providing a unified description of the process going from the surface to the boiling heat flux.
II. EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental equipment necessary for this study consisted of a boiling and gas diffusion apparatus. The boiling apparatus was used to obtain $\varphi_A$ vs $\Delta T$ curves, cumulate nucleation site distributions, and for studying nucleation from individual sites. The effective radius for natural cavities was determined by the gas diffusion experiments. A very brief description of these experiments is given in this chapter and a more detailed description in Appendices A and B.

II.1 Boiling Apparatus

Boiling occurred from the end of a $\frac{1}{8}$" diameter copper cylinder mounted almost flush with the base of the teflon coated brass enclosure. A photograph and sketch of the apparatus are shown in Figures 31 and 32. Heat was supplied to the straight conductor section by a copper heater section containing a high density cartridge heater. The heater and conductor sections are not permanently fastened permitting the test surface to be easily dismantled for refinishing. Guard heaters on each side maintain the bulk at saturation while two reflux condensers return the condensate to the system. Visual observation of the boiling process was possible through pyrex windows on the front and back of the enclosure. Four thermocouples along the axis of the straight section permitted
determination of wall superheat and heat flux while the bulk temperature was measured with a thermocouple placed about 1" above the boiling surface.

Before each set of runs the test section was prepared in the Metallurgy Laboratory of M.I.T. by grinding with a specified grit paper. A set of runs consisted generally of boiling water and organics on the same surface. Since oxidation of the boiling surface occurred less with organics than with water, organics were always boiled first. Before taking data the liquid was degassed at \( \frac{3}{4} \) of the burnout heat flux for one hour. Data was taken traversing the boiling curve from high to low heat flux. Incrementally the heat flux was lowered and the system allowed to reach equilibrium at which points the thermocouple readings were recorded. When the isolated bubble region was encountered the number of sites/in\(^2\) was determined by visual counting. The test was terminated after a couple of data points were obtained in the natural convection region.

II.2 Gas Diffusion Apparatus and Procedure

A photograph and sketch of the apparatus are shown in Figures 34 and 35. The apparatus consists essentially of a high pressure tank, plexiglass tray, electric stirring device, oxygen probe and test surface. A supersaturated solution of water with dissolved air is allowed to flow from the tank into
the tray containing a test surface. Different concentrations of dissolved air (as measured by the probe) can be maintained by adjusting the inflow rate of supersaturated water. Action of the electric stirrer aids in eliminating concentration gradients in the neighborhood of the test surface. Corresponding to a given concentration a unique equilibrium interface radius exists for the system. If the equilibrium radius is less than or equal to the effective radius for nucleation a bubble will grow from an active site on the test surface. By lowering the concentration of dissolved gas to the point at which a bubble no longer appears from a site the effective radius of that cavity can be determined. It is easy to show that the radius obtained by this procedure is the same radius that would be obtained in a boiling experiment with a uniformly superheated liquid.
III. NUCLEATION

III.1 General Considerations

For thermodynamic equilibrium across a spherical vapor-liquid interface the following relations must be satisfied

$$P_V - P_L = \frac{2\phi}{\rho}$$

$$\mu_L = \mu_V$$

$$T_V = T_L$$

(1)  
(2)  
(3)

By writing a force balance across the interface relation (1) is easily verified. It can be shown [18] that relation (2) implies that the vapor states are essentially saturation states at the system temperature. By approximating the saturated vapor states by the Claperon equation, expressions (1), (2) and (3) can be restated as

$$\rho = \frac{2\phi T_s u_{fg}}{h_{fg} \Delta T}$$

(4)

Expression (4) will often be referred to in this work as simply $\rho = \frac{2\phi}{\Delta \rho}$. Using this expression Griffith and Wallis [1] were able to predict the $\Delta T$ required for incipient nucleation from artificial cavities in a uniformly superheated system. Later this expression was shown valid for incipience in liquid metals [2,3,12], which have sufficiently high conductivity to behave as uniformly superheated systems. Expression (4) failed, however, to predict the incipient $\Delta T$ when applied to a system in which the bulk was saturated. Modifications of (4) were suggested by several experiments [5,6,7,19,20] to
account for the presence of a significant thermal gradient in the liquid close to the heating surface.

The nucleation criteria of Hsu [7] and Bergles and Rohsenow [19] was simply that the temperature at the tip of the bubble at the hemispherical state, shown in Figure 2 must be greater than or equal to the equilibrium superheat required at that radius from equation (4). Assuming a linear temperature gradient in the thermal layer yields

$$ T_x = T_w - \chi \delta $$

where $\delta$ is the approximate limiting thermal layer thickness given by

$$ \delta = \frac{K \Delta T}{\chi A} \text{n.c.} $$

for nucleate pool boiling.

Solving (4) and (5) in accordance with the nucleation criteria yields the following

$$ \rho = \frac{\delta}{2M} \left[ 1 \pm \sqrt{1 - \frac{8M \delta T_s \sqrt{\gamma g}}{h \gamma g \Delta T \delta}} \right] $$

where $M = 1$ for Hsu and Bergles and Rohsenow and $M = \frac{3}{2}$ for Han and Griffith.

Expression (7) sets limits upon the maximum and minimum radius cavity that can be active at a given $\Delta T$. Several experimental studies [7,20,21,22] with artificial cavities have revealed only fair agreement with equation (7). Figure 3
shows experimental results from [21], in which water and organics were tested using artificial cylindrical cavities. Except for the very largest cavity it is seen that the prediction from equation (7) is generally too low. For water the prediction is approximately 100% low while for organics the prediction ranges from 200% to almost 1000% too small. It is quite probable that the large deviation for the organics, which have good wetting characteristics, is partly due to the fact that they nucleate at an effective radius much smaller than the mouth radius. The dependence of the effective radius of nucleation on wetting characteristics will be discussed in Section III.3.

In Figure 3 it is seen that expression (4) and (7) merge as the cavity size decreases, revealing that for small cavities the effect of the temperature gradient is insignificant. The smallest artificial cavities generally used to check expression (7) were about $7 \times 10^{-4}$" in radius which is about the size of the very largest natural cavities created by mechanical grinding. Since this work is primarily concerned with small natural cavities on mechanically finished metal surfaces, equation (4) will be assumed valid as a working model subject to later experimental verification.

III.2 Growth and Stability

The equilibrium positions that an interface may attain within a cavity can be mapped on a $\frac{1}{\rho}$ versus Volume curve.
The shape of this curve is a function of cavity geometry and contact angle. Photographic studies of active cavities, discussed in Chapter IV and shown in Figures 17, 18 and 19, suggest a general conical geometry with a superimposed roughness scale on the walls. Figure 4 shows a comparison between the \( \frac{1}{\rho} \) versus Volume curves for smooth and rough walled conical cavities, for a contact angle between 0 and \( \pi/2 \). The shape of these curves is important in determining the equilibrium radius and cavity stability.

Suppose that an interface is located at position 1 of Figure 4a and the system is held at a uniform superheat corresponding to this position. If a small decrease in temperature occurs, causing condensation, the interface recedes to point 2 at which condition the equilibrium requirement is for an even higher \( \Delta T \). Since the system is held fixed at condition 1 further condensation results, causing the interface to eventually collapse deactivating the cavity. If the opposite occurred and the perturbation (increase in temperature) were to produce evaporation the interface would grow producing a bubble. In a similar manner it can be argued that a positive sloping portion of a \( \frac{1}{\rho} \) versus Volume curve is stable for small perturbations. If the initial interfacial position is located at \( \frac{1}{\rho} > \frac{1}{R} \) this initial radius is then equal to the effective radius for nucleation. If the interface is formed between \( \frac{1}{\rho_{\text{max}}} < \frac{1}{\rho} < \frac{1}{R} \) then \( R \) determines the superheat
required to produce a bubble.

The shape of the $f$ versus volume curve for the cavity in Figure 4b is generally similar to that in Figure 4a except many extrema exist providing positions at which the interface is stable for small perturbations. If the roughness scale is sufficiently small the effective nucleation radius for smooth and rough cavities may be nearly equal at the same initial volume except the rough cavity will be stable for very small perturbations. The next section develops a quantitative model for predicting the effective radius when the cavity geometry is specified.

III.3 Effective Radius for Nucleation

Bankoff [23] first described the necessary conditions for vapor to be trapped in surface grooves. The relationship between contact angle and cone angle determines whether or not vapor will be trapped. In wedge shaped grooves for which $\theta > \phi$ vapor will not be completely displaced by the advancing liquid front and trapping occurs. If $\theta < \phi$ the vapor will be completely displaced by liquid and the cavity deactivated. Since contact angle plays a major role in vapor trapping, Appendix C provides a detailed discussion of that phenomena. Crude contact angle measurements revealed that the advancing contact angle for water and organics are $35^\circ$ and $7^\circ$ respectively. Intuitively it is expected that organics will penetrate farther
into cavities than does water during the trapping process. This reasoning is now extended to provide a means of calculating the effective radius.

Assume that a bubble is departing from a conical cavity of radius \( R \) and cone angle \( \phi \). Due to a slight asymmetry in the bubble departure, liquid with contact angle \( \theta \) will advance into the cavity from a favored direction as shown in Figure 5a. The shape of the liquid surface is assumed nominally flat as the front penetrates into the cavity. Trapping is completed when the liquid surface intersects the opposite edge of the cavity at point A of Figure 5b. A flat liquid surface is certainly an idealization of reality since the fluid surface combination must satisfy the boundary condition of essentially equal contact angle at each point along the cavity walls. Also hydrodynamic effects should be considered in a model that would more accurately describe the liquid surface shape. A qualitative sketch showing this more complicated trapping mechanism is shown in Figure 11.

By geometrical arguments the volume of trapped vapor, Figure 5a, can be related to the cavity radius, cone angle, and contact angle by the following expression

\[
\frac{V_v}{R^3} = g_1(\theta, \phi) = \frac{\pi \sin(\theta-\phi) \tan^3 \psi (\tan \gamma \cot \psi - 1)^2}{3 \sin \frac{\phi}{2} \cos \gamma (\tan^2 \psi - \tan^2 \gamma)^{\frac{3}{2}}}
\]

(8)

where
\[
\psi = 90 - \phi_2 \quad \gamma = 90 - (\theta - \phi_2)
\]
and $V_i$ is the initial trapped volume of vapor.

Now assume that this mass of vapor takes the shape shown in Figure 5b. In going from the state shown in Figure 5a to that of 5b the vapor is compressed by the action of surface tension. The vapor volume in Figure 5b determines the resulting interfacial curvature, which is given by another geometric relation as follows:

$$\frac{V_2}{\rho_2^3} = g_2(\theta, \phi) = \frac{\pi}{3} \left[ \frac{\cos^3(\theta - \phi_2)}{\tan \phi_2/2} + S^2(3 - S) \right] \tag{9}$$

where $S = 1 - \sin(\theta - \phi_2)$

and $V_2$ is the final volume of vapor in Figure 5b.

Combining these expressions the following is obtained relating the effective radius to cavity geometry:

$$\frac{\rho}{R} = (\frac{V_2}{V_i})^{1/3} f(\theta, \phi) \approx f(\theta, \phi) \tag{10}$$

where $f(\theta, \phi) = (\frac{V_2}{V_i})^{1/3}$ and $(\frac{V_2}{V_i})^{1/3} \approx 1$.

In relation (10) it was assumed that $V_2 = V_i$. If instead an adiabatic or isothermal vapor compression from state 5a to 5b was assumed a $\frac{\rho}{R}$ value about 10% smaller is obtained.

Expression (10) is shown in Figure 6 where $\frac{\rho}{R} vs \phi$ is plotted for various values of $\theta$. Consider a single curve of constant $\phi$. Below $\theta = \phi$ no vapor is trapped which is consistent with Bankoff's argument. For small cone angles $\frac{\rho}{R}$ is very sensitive to contact angle for values of $\theta$ slightly greater than $\phi$. As $\theta$ increases the rate of change of $\frac{\rho}{R}$ with $\theta$ decreases up to $\frac{\rho}{R} = 1$ at which point the cavity radius
determines the required superheat. For larger cone angles, \( \frac{\theta}{R} \) increases smoothly with \( \theta \). It is interesting to note that for \( 0 < \theta < 90° \) the interface will be formed on the left leg of the \( \theta \) versus Volume curve of Figure 4a.

This model is based strictly on geometrical arguments. If all dynamic effects were absent the \( \Delta T \) required for nucleation could be predicted from equation (4) for \( \frac{\theta}{R} \leq 1 \) calculated from equation (10) or Figure 6. The following section consideres dynamic effects in detail.

III.4 Dynamic Effects on Nucleation

III.4.1 General Considerations

During the vapor trapping process described in the previous section, liquid at some initial temperature enters the cavity and establishes an initial position at some depth \( M \) as shown in Figure 7. Initially the liquid possesses some velocity \( V_0 \) and the cavity wall is held at some temperature \( T_w \). If the temperature at the liquid vapor interface is above the initial temperature of the liquid within the cavity, heat will be conducted into the liquid causing condensation to occur. Compression of the vapor resulting from inertial effect further increases the interfacial temperature thus enhancing the driving force for condensation. It is necessary to determine the conditions under which the interface will not completely collapse, but remain stable eventually growing and
producing a bubble.

Bankoff [4] first considered the problem by modelling the process as viscous controlled capillary penetration due to condensation using a one dimensional model neglecting radial heat flux and inertial effects. Later Marto and Rohsenow [3] and Shai and Rohsenow [2] considered the problem for liquid metals, using a one dimensional heat flux model neglecting radial heat flux, inertial, and viscous effects. The present model is essentially a reformulation of Bankoff's model incorporating radial heat flux and inertial effects.

III.4.2 Formulation of the Problem

A co-ordinate system moving with the fluid is chosen as shown in Figure 7 for determination of the temperature field within the liquid. Convective effects are assumed negligible and two dimensional transient conduction is the governing heat transfer process within the liquid. For momentum considerations the control volume shown in Figure 7 is employed with the interfacial boundary moving with the fluid. The external forces acting on the control volume are the shear force on the wall and the unbalanced pressure force on the vertical boundaries. The vapor pressure exerted at the interfacial boundary of the control volume is determined by the net effect of condensation and vapor compression. The penetration velocity in this formulation is determined not only by the
rate of condensation or evaporation but also by the vapor compression resulting from inertial forces. Details of the formulation of the problem are given in Appendix D, with the following assumptions

1. Three dimensional effects are negligible in the neighborhood of the interface.

2. Convection effects are negligible.

3. Temperature at cavity mouth is constant at saturation temperature corresponding to system pressure.

4. Conductivity of vapor neglected.

5. Velocity field is parabolic being a function of \( r \) and time only.

6. Pressure at cavity mouth is constant at system pressure.

7. Momentum change of the vapor at interface is negligible.

8. The initial liquid temperature is equal to the mean temperature of the thermal layer, i.e. \( \frac{Tw + Ts}{2} \).

In this analysis \( z \) will represent the penetration coordinate fixed to the moving fluid and measured from position \( M \). The initial radius of curvature of the interface at position \( M \) is denoted as \( \rho_0 \). For simplicity the analysis will be performed using the cylindrical coordinate system of Figure 7b, but maintaining the essential conical relations involving \( M, \beta, \) and \( \rho \). In dimensionless form the governing equations, boundary conditions and initial conditions are as follows:
Momentum Equation

\[ \frac{d^2 z'}{d \tau'^2} + A_2 \frac{d z'}{d \tau'} + B_2 z' = - c_2 \int_{0}^{\tau'} \int_{0}^{r'} \frac{\partial T'}{\partial \beta'} r' dr' \]  \hspace{1cm} (11)

Energy Equation

\[ \frac{d^2 T'}{d \tau'^2} + \frac{1}{r'} \frac{\partial T'}{\partial r'} + \frac{d^2 T'}{d \beta'^2} = \frac{\partial T'}{\partial \tau'} \]  \hspace{1cm} (12)

Boundary Conditions

\[ r' = 1 \quad , \quad T_w = 1 + \delta \]  \hspace{1cm} (13a)

\[ r' = 0 \quad , \quad \frac{\partial T'}{\partial r'} = 0 \]  \hspace{1cm} (13b)

\[ \beta' = \frac{M}{r_0} + \beta' \quad , \quad T' = 0 \]  \hspace{1cm} (13c)

\[ \beta' = 0 \quad , \quad T_i' = 1 + w_3 \int_{0}^{\tau'} \int_{0}^{r'} \frac{\partial T'}{\partial \beta'} r' dr' + w_2 z' \]  \hspace{1cm} (13d)

Initial Conditions

\[ \tau = 0 \]  \hspace{1cm} (14a)

\[ T' = 0.5 \]  \hspace{1cm} (14b)

\[ z' = 0 \]  \hspace{1cm} (14c)

\[ \frac{d z'}{d \tau'} = V_2 \]

where

\[ T' \equiv \frac{T - T_s}{T_2 - T_s} \quad , \quad \tau \equiv \frac{t}{r_0^2} \quad , \quad \beta' , r' , z' \equiv \frac{\beta , r , z}{r_0} \]  \hspace{1cm} (15a)

\[ w_3 \equiv \frac{\pi \Delta T r_0^4 k P_v^0}{h_{fg} \alpha \delta w g_0 \cos (\theta - \phi_2)} \]  \hspace{1cm} (15b)

\[ w_2 \equiv \frac{3 r_0^2 P_v^0}{2 \delta h \cos (\theta - \phi_2)} \]  \hspace{1cm} (15c)

\[ A_2 \equiv \frac{8 \mu}{\delta H^2} \]  \hspace{1cm} (15d)
\[ B_2 \equiv \left( \frac{3 P_v}{\rho_e M H} - \frac{2 \rho T \tan \Phi_2 \cos(\theta - \Phi_2)}{\rho_e M R_0^4} \right) \frac{R_0^4}{\alpha^2} \] (15e)

\[ C_2 \equiv \frac{2 \pi \Delta T R_0^6 \kappa P_v}{\rho_e M w_0 \frac{\alpha}{\alpha_0}^3} \] (15f)

\[ V_2 \equiv \frac{V_0 r_0}{\alpha} \] (15g)

\[ \frac{M}{R_0} \equiv \frac{1}{\tan \Phi_2} \left( \frac{1}{f(\theta, \phi) \cos(\theta - \Phi_2)} - 1 \right) \] (15h)

\[ \frac{H}{R_0} \equiv \frac{1}{\tan \Phi_2} \] (15i)

Essentially six dimensionless groups are required for a solution. In the momentum equation \( A_2 \frac{dz}{dt} \) is a viscous term and \( B_2 z' \) and \( C_2 \left( \int_{\Omega_0}^{\Omega_1} r' dr' \right) \) are pressure terms representing the change in pressure within the vapor space due to compression and condensation respectively. In boundary condition (13d), \( W_3 \left( \int_{\Omega_0}^{\Omega_1} r' dr' + w_2 z' \right) \) represents the net effect of vapor compression and condensation upon the vapor pressure and hence interfacial saturation temperature corresponding to that pressure. When compression and condensation occur simultaneously \( w_2 z' \) is positive tending to increase \( T_i' \) while \( W_3 \left( \int_{\Omega_0}^{\Omega_1} r' dr' \right) \) is negative tending to decrease \( T_i' \). In boundary condition (13a), \( f \) will be given small incremental values such as .1, .2, and .3 which means that the cavity wall is either 10%, 20% or 30% higher than \( \Delta T \frac{\alpha}{\rho_0} = \frac{2 \rho T s}{h f_2} \).
III.4.3 Numerical Results and Discussion

The solution to the preceding problem was obtained by solving equation (12) with boundary conditions (13a-d) and initial condition (14a) using an implicit finite difference scheme and equation (11) with initial conditions (14b,c) using a second order Runge Kutta technique. A listing of the computer program is given in Appendix E. Solutions were obtained for cavities in which vapor trapping is possible as determined from the criteria $\phi \leq \Theta$. For water and organics this requires that $\phi$ is less than $35^\circ$ and $7^\circ$ respectively. Table 1 shows values of the dimensionless groups (15a-$\xi$) along with $M/r_o$ and $H/r_o$ obtained for water and organics for appropriate choices of the input variables. From high speed photographs of bubble departure from Reference [5,6] an estimated value of $V_o = 10$ ft/sec was obtained for water. Essentially the "necking" velocity at the base of the bubble was determined at the point of departure. This velocity which is measured parallel to the boiling surface overestimates the velocity with which fluid enters the cavity since the fluid must change direction upon entering. In the following numerical solutions values of $V_o$ ranging from 1 ft/sec to 10 ft/sec will be used.

Figure 8 shows a plot of penetration depth versus time for water at various values of $\xi$ resulting from solution of the governing equations (11,12) using $\phi = 10^\circ$, $\Theta = 35^\circ$, $r_o = 1 \times 10^{-4}$" and $V_o = 1$ ft/sec (see table 1). For $\xi = .1$
**TABLE 1**

**LIST OF PARAMETERS USED IN THE ANALYSIS OF DYNAMIC EFFECTS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WATER</th>
<th>METHANOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>$10^\circ$</td>
<td>$5^\circ$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$35^\circ$</td>
<td>$7^\circ$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>$1 \times 10^{-4}$</td>
<td>$.5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>$1 - 10$ ft/sec</td>
<td>$1 - 10$ ft/sec</td>
</tr>
<tr>
<td>$W_3$, eqn.(15b)</td>
<td>25.11</td>
<td>11.11</td>
</tr>
<tr>
<td>$W_2$</td>
<td>.68</td>
<td>.55</td>
</tr>
<tr>
<td>$A_2$</td>
<td>15.1</td>
<td>52.7</td>
</tr>
<tr>
<td>$B_2$</td>
<td>2227.4</td>
<td>157.9</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$95 \times 10^3$</td>
<td>$3.5 \times 10^3$</td>
</tr>
<tr>
<td>$V_2$</td>
<td>4 - 40</td>
<td>4 - 40</td>
</tr>
<tr>
<td>$M/r_0$</td>
<td>1.85</td>
<td>25</td>
</tr>
<tr>
<td>$H/r_0$</td>
<td>11.5</td>
<td>25</td>
</tr>
</tbody>
</table>
the interface is unstable, collapsing at an increasing rate with time. Eventually the interface will reach the bottom of the cavity when $Z' = H/r_0 \approx 10$ for this case. When the wall temperature is increased to $\chi = 0.3$, the interface penetrates to a maximum of about $Z' = 0.8$ and subsequently grows by evaporation. For this cavity it appears that nucleation should occur at some value of $\chi$ between 0.2 and 0.3, say about 0.25.

For water the effect of initial velocity, $V_0$, is shown in Figure 9 for the same cavity as before with $T_w = 0.3$. As $V_0$ increases from 1 to 10 ft/sec the interface penetrates deeper into the cavity before subsequent growth occurs. As $V_0$ is increased greater penetration results not only from inertial effects but also due to additional condensation resulting from internal compression which tends to increase the vapor pressure and hence interfacial saturation temperature corresponding to that pressure. Solutions beyond $V_0 = 10$ ft/sec were not obtained but it appears as though a stability limit is rapidly being approached beyond which point a further increase in velocity will cause the interface to completely collapse.

Solutions (not plotted) for increasing values of cone angle from $\phi = 10^\circ$ to $\phi = 30^\circ$ at $V_0 = 1$ ft/sec for water indicated that the required superheat for nucleation increases from about 25% to slightly over 30%. This increase can be attributed mainly to the fact that a smaller volume of vapor
is trapped when $\phi = 30^\circ$ resulting in a relatively greater incremental increase in interfacial temperature for a given incremental penetration depth. Hence the condensation driving force is enhanced causing further penetration.

Solutions for organics using $\phi = 5^\circ, \theta = 7^\circ$, $r_0 = .5 \times 10^{-4}$" and Vo values from 1 ft/sec to 10 ft/sec revealed that a value of $C = .15$ is sufficient to cause nucleation. For organics, which penetrate deep into the cavities, the increased viscous force upon the cavity wall tends to dampen inertial effects permitting nucleation at smaller values of $C$ than for water.

The predicted correction in the required superheat which is about 30% for water and 15% for organics is not considered to be sufficiently large to justify incorporating dynamic effects into the nucleation model. Instead the equilibrium model, equation(4), will be assumed valid for predicting nucleation for cavities with effective cone angles $\phi < 35^\circ$ for water and $\phi < 7^\circ$ for organics. Natural cavities should require less additional superheat than idealized smooth walled cavities since (see Figure 4b) regions exist in which the radius of curvature actually increases upon decreasing volume. This enhances the stability situation considerably since $\Delta T$ is lowered at the interface tending to suppress condensation.
III.5 Experimental Results

Since the previously described model for nucleation indicates that wall superheat for nucleation can be predicted from $\Delta \theta = \frac{2 \delta}{\rho}$ even in a saturated system, an experiment was devised to test that assertion.

III.5.1 Boiling Experiment

A copper test section boiled water and several organic liquids with the apparatus and procedure described in Appendix A. The heat flux was lowered to a point at which only about 8 cavities were active. Subsequently the heat flux was gradually decreased and the $\Delta T$ recorded when bubble production from each cavity ceased. For each cavity a tentative value of $\theta$ was calculated from (4).

These cavities were distinguishable from run to run by circular scale spots formed around each cavity when water was boiled from the surface. Analysis of the scale revealed that it was a negligibly thin deposit consisting of minute traces of silica derived from the water. Such a deposit was also encountered by other experimenters [24] who boiled distilled and deionized water on stainless steel.

Another independent determination of $\rho$ for the same 8 cavities is needed for comparison. Such a determination could be obtained in a nucleation experiment under uniformly superheated conditions, similar to the Griffith-Wallis experiment[1].
Unfortunately the very high uniform superheat required for nucleation of small natural cavities is hard to obtain in practice, precluding the use of this method. Therefore an alternative gas diffusion technique was devised.

III.5.2 Gas Diffusion Experiment

This experiment, described in Appendix B, was designed to measure the effective radius for natural cavities. Since for a given cavity the effective radius depends upon the wetting properties, a separate gas diffusion should be performed for water and organics. Due to experimental difficulty in maintaining a sufficiently high dissolved gas concentration for organics, only water was used. In this experiment a uniformly supersaturated system of water with dissolved air is maintained at various concentration levels. Corresponding to a given concentration and system temperature a unique radius exists for a curved interface at equilibrium and is given by

\[ \rho = \frac{2 \phi}{P_a + P_v - P_l} \]  

(16)

Where \( P_a, P_v \) and \( P_l \) are all measurable quantities. When the test surface is immersed in the liquid having a high gas concentration, bubbles may emerge from the active sites. Incrementally the concentration is lowered and the bubbles mechanically wiped from the surface. This is necessary due to the extremely slow bubble growth rates. A point will be
reached where upon wiping a bubble from a cavity a new one does not soon appear. An effective radius is then calculated from (16), which should be the same as that obtained in a uniformly superheated nucleation experiment. Since dynamic effects should be negligible in the gas diffusion experiment the value of $\rho$ from (16) should be precisely $\rho_0$ shown in Figure 7.

III.5.3 Results and Discussion

III.5.3.1 General

Figure 10 shows a comparison of the effective radius as calculated in the boiling and gas diffusion experiments. The good agreement for water suggests that $\rho = \frac{2 \rho}{\Delta P}$ properly predicts the effective radius for saturated systems provided that $\rho/\delta$ is sufficiently small. A value of $\rho/\delta = .02$ was obtained from the boiling experiment where $\delta$ was determined from expression (6) and $\rho$ from (4). As expected from the analysis of dynamic effects, the values of $\rho$ predicted from the boiling experiment are slightly less (5%-25%) than the equilibrium values of $\rho$, i.e. $\rho_0$, as determined from the gas diffusion experiment.

The effective radii for organics is seen to be about one half that of water. As in the case of water a value of $\rho/\delta \approx .02$ was obtained which is sufficiently small to insure that the thermal gradient effect is insignificant. Using
the arguments of Section III.3 the difference between the predicted $\rho$ for water and organics should be expected since these organics have very small contact angles compared with water; hence $\rho/\rho_{H_2O}$ is smaller for organics on the same cavity, Figure 6. The difference in the calculated radii of curvature cannot be attributed to the dynamic effects previously discussed because water should be affected more than organics tending to lessen the discrepancy.

A more quantitative description of the decrease in effective radius can be obtained by considering a particular cavity, say #1, for which $\rho_{org}/\rho_{H_2O} \approx \frac{1}{2}$. Advancing contact angles of $\theta = 7^\circ$ and $35^\circ$ will be used for organics and water respectively as determined from contact angle measurements. A unique value of $\phi$ can be obtained by trial and error from the graph of Figure 6 in accordance with the following

$$\frac{\rho_{org}}{\rho_{H_2O}} = \frac{\rho_{org}}{\rho_{H_2O}} = \frac{f(\theta_1, \phi)}{\frac{f(\theta_2, \phi)}{\rho}} = \frac{1}{2}$$

(17)

at $\theta_1 = 7^\circ$, $\theta_2 = 35^\circ$.

For this cavity a cone angle of about $\phi = 5^\circ$ is obtained.

Note also that for this cavity water has a sufficiently high contact angle to be limited by the cavity radius, i.e. $\rho = \rho$ for water. Looking at cavities #3, 4, 5, and 7 approximately the same $\phi$ value of $5^\circ$ is obtained. Apparently cavities 8 and 9 have $\phi$ values greater than $7^\circ$ and hence are washed out by the organics. Cavity 10 has a $\phi$ value somewhat greater
than 5°, say 6°.

It is interesting to compare the cavity photographs and sketches in Figures 17, 18, and 19 (discussed in Section IV.3) with the experimental results. According to the model the photographically measured cavity radius should agree with that predicted from $\rho = \frac{2\delta}{\Delta \rho}$ for the water data. Indeed the majority of the photographs support the model, as shown in Table 2. For cavities 1, 7, 9 and 10 the agreement is excellent while for cavities 3 and 5 the radii are larger than predicted from equation (4).

The cavity shapes as sketched in Figures 17, 18, and 19 are not ideal straight walled cones as modelled in Figure 6 so the deduced $\delta$ values for these cavities should be regarded as effective values. Due to the micro roughness along the cavity wall coupled with hydrodynamic effects, trapping is possible at larger cone angles than predicted by the idealized model of Figure 6. In actual trapping process, which is sketched qualitatively in Figure 11, the interface is retarded at the wall as it negotiates the roughened surface contour, while the fluid near the cavity mouth tends to advance rapidly across the cavity. These effects tend to permit trapping at relatively large cone angles. It is then more realistic to consider $\phi$ as an effect cone angle which accounts not only for deviations from conical geometry but also for hydrodynamic and roughness effects.
III.5.3.2 Cavity Stability

When the $\Delta T$ was lowered just enough to extinguish an active cavity and then increased, the cavity would often become reactivated at nearly the same $\Delta T$. However, when the $\Delta T$ was decreased to near zero and then increased no activity occurred in the case of organics until the superheat was sufficiently high to cause the entire surface to explode into bubble production. With water some cavities remained active and reappeared with virtually no hysteresis. Others came into activity only after a neighboring bubble spread over the site.

The preceding reveals that a stable region often exists near the effective radius of nucleation and that water is more stable than organics. This can be explained by considering the $1/\rho$ vs Volume curve for a cavity possessing some internal micro roughness such as that shown in Figure 12a. This "idealized" rough cavity possesses a single saw tooth element of roughness. The accompanying $1/\rho$ vs Volume curve shows the existence of a relative maximum at point A and a relative minimum at A$^-$ in the neighborhood of the roughness element. For this cavity assume that point A represents the effective radius for nucleation. When the superheat is decreased slightly below point A, bubble production ceases and the nucleus remains on the stable leg A$^-$A$. When the surface is reheated to the $\Delta T$ corresponding to A, the cavity goes again with no hysteresis. However, if the superheat were reduced below point A$^-$ the cavity
would be deactivated. The "length" of the stable leg $A-A^-$ is greater for water than for organics. In fact for water point $A^-$ is in some cases very close to zero as inferred from the lack of hysteresis when the superheat was lowered to near saturation. To show that extrema $A$ and $A^-$ can be caused by a saw tooth roughness element as shown, refer to Figure 12c. At point $A^-$ which is a small distance below the apex, the radius of curvature $\rho_A$ can be related to the local radius of the cavity $r_A \approx r_A$ by geometrical considerations as follows:

$$\frac{r_A}{\rho_A} = \cos(\theta - \phi + \alpha)$$

(18)

where $\alpha$ is the pitch angle as shown in Figure 12c. At the apex where a range of contact angles can be attained, it can be seen by geometry that the radius of curvature $\rho$ passes through a minimum of $\theta - \phi \leq \alpha$. The value of $\rho$ at this minimum point is $\rho_{A_{\min}} = r_A$. Assuming $\phi = 30^\circ$, $\alpha = 60^\circ$ the inequality $\theta - \phi \leq \alpha$ holds for water and organics showing that a minimum exists at point $A$. Furthermore substituting $\phi$ and $\alpha$ into equation (18) yields $\frac{\rho_A}{r_A} = 5.7$ and $1.6$ for water and organics respectively showing that the "length" of the stable leg $A-A^-$ is greater for water than for organics in agreement with experimental results. For practically all choices of $\phi$ and $\alpha$ stability tends to increase with increasing contact angle. In general a cavity possesses many extrema in the $1/\rho$ vs Volume curve permitting a stable region to exist near the initial trapped volume.
III.5.4 Comparison With Other Data

III.5.4.1 Comparison With Griffith and Wallis

The model and experimental data from this study directly contradict the results of Griffith and Wallis [1] who claimed that the single dimension is sufficient to characterize a cavity. They boiled organics and water on the same surface obtaining the cumulative site density curve, i.e., $\eta_A$ vs $\Delta T$ for each. Then converting $\Delta T$ to $\rho$ from equation (4) their data for water, methanol, and ethanol fell nearly on a single curve. According to the model of Section III.3 and the experimental results of this study shown in Figure 10 the data for organics should fall to the left of the water since organics have a smaller contact angle.

Griffith and Wallis employed a subcooled system in order to facilitate bubble counting. Since their system was opened at the top a partial pressure of air equal to the atmospheric pressure minus the saturation pressure corresponding to the subcooled bulk temperature is present at the upper liquid surface. The presence of the gas can significantly alter the required superheat for nucleation. It can easily be shown that the presence of gas in a cavity will diminish the required superheat for nucleation and is given by

$$\Delta T = \frac{T_s - \nu_{fg}}{h_{fg}} \left( \frac{2 \phi}{\rho} - \rho_g \right)$$

(19)
where \( \text{Pa} \) is the partial pressure of \text{gas} in the cavity. Even if a liquid contains dissolved \text{gas}, a sufficiently high \text{bubble departure frequency} tends to maintain \( \text{Pa} \) negligibly small by dilution; but if the frequency is diminished \( \text{Pa} \) may become significantly large.

Experiments were run in this study to determine the effect of subcooling. For \text{water} and \text{methanol} the effect was to decrease the \( \Delta T \) (or increase the apparent \( \rho \)) by nearly 100\% in some cases. Furthermore, the departure frequency near the point at which an active site was extinguished was significantly reduced, more in the case of \text{water} than \text{organics}. Thus the data of \text{Griffith and Wallis} is questionable for two reasons. First, their predicted cavity radii are probably too large caused by the presence of \text{gas} in the cavities. Secondly, their photographic technique used for \text{bubble counting}, which consisted of an instantaneous picture, probably missed counting more \text{water} bubbles than \text{methanol}. These two effects added in such a way as to fortuitously produce a unique curve.

\text{III.5.4.2 Comparison With Others}

In reference [25] the cumulative site density-versus \text{excess wall superheat}, i.e., \( n_A \) vs \( \Delta T \), was obtained for several \text{organics} boiled on the same surface. In Figure 13 the data is replotted by converting \( \Delta T \) to \( \rho \) using \( \rho = \frac{\Delta \phi}{\Delta \rho} \). The
data falls nearly on a single curve as expected from the
model of Section III.3 since the fluids have similar wetting
behavior.

From the work of Hatton and Hall [22], who boiled water
at different pressures on similar surfaces, data for $\eta_A/\Delta T$
can be obtained. Figures 14 and 15 show some of this data
replotted on $\eta_A/\rho$ coordinates using the steam tables. The
data falls nearly on a single curve for each surface as shown,
revealing that expression (4) can properly predict the pressure
dependence. Data for a 30/μ″ surface, which is not plotted,
showed scatter of ±50%.

Figures 22-27 which will be discussed in detail in the
next chapter show data of this work and reference [26] for
water and organics boiled on the same surface. The data for
organics is consistently to the left of the water data as
expected.

The nucleation data from reference [21] for water and
organics with artificial cavities, discussed in Section III.1
is replotted in Figure 16 using equation (4) to predict a new
value of $\rho$ for each cavity. The cavities are numbered in order
of increasing actual mouth radius. The replotted data is
apparently consistent with the model of this chapter in that
$\rho_{\text{water}} > \rho_{\text{organics}}$ and all the organics exhibit approximately
the same $\rho$ for a given cavity. However, for water the pre-
dicted radius is not equal to the cavity radius which implies
the following: either the interface is formed at some position in the cavity where the radius of curvature is indeed equal to \( \rho = \frac{2 \delta}{\Delta \rho} \) as predicted from equation (4), or else the cavity is nucleating at \( \rho = r_c \) but the nucleation criteria (4) is unable to predict the value. If equation (7) were used with say \( M = 1 \) the minimum \( \rho \) predicted for each cavity would be approximately the same as that obtained from equation (4) and the maximum \( \rho \) would be approximately equal to the thermal layer thickness \( \delta = 44 \times 10^{-4} \) . It is true that these values for the minimum and maximum radii bound the actual mouth radii but neither can predict the mouth radius exactly. If high speed photographs were taken of bubble departure from these cavities and it was determined that the interface is indeed formed near the cavity mouth, at which point \( r_c \) should be the radius which determines nucleation, it could be reasoned that equation (7) is inaccurate since it does not predict \( r_c \) at either limit. For cavities #1-4 the value of \( \frac{r_c}{\delta} \) is about .16, .23, .41, and .75 respectively indicating that the cavity size is comparable to the thermal layer thickness. For such large values of \( \frac{r_c}{\delta} \) it is possible that equation (7) cannot properly model the effect of the significant temperature gradient in the neighborhood of the heating surface. However without photographic evidence it is not clear where the interface is formed and hence whether or not equation (7) is correct. Due to the consistency of the data for organics it is quite
probable that the $\rho$ predicted from equation (4) is correct for these fluids which can penetrate deep into the cavities to positions of small $\rho$.

III.6 Summary and Conclusions

Using a boiling and gas diffusion experiment the criteria $\Delta P = \frac{2\gamma}{\rho}$ was verified for small natural cavities. Analytically it was shown that dynamic effects due to inertia and condensation have a relatively small effect upon nucleation. A simple trapping mechanism was postulated and a geometrical model was developed relating the effective radius to the cavity radius, cone angle, and contact angle. The model predicts that fluids with small contact angles (organics) should nucleate at a smaller effective radius than fluids with relatively large contact angles (water) for a given cavity. Experimental results for organics and water were shown to be consistent with the theory. To incorporate hydrodynamic and roughness effects into vapor trapping, the cone angle as deduced from the trapping model should be interpreted as an effective cone angle. Cavity stability and trapping potential are enhanced by roughness beyond that expected for smooth walled conical cavities. Data from the literature generally substantiates the experimental and analytical work presented in this chapter.
IV. CHARACTERIZATION OF BOILING SURFACES

IV.1 General

For a given surface the heat transfer characteristics can always be obtained by actually boiling the surface. However it is of interest both from a practical and theoretical point of view to develop a non boiling measurement technique with which the performance of a given surface can be assessed in advance. The flow chart of Figure 1 traces the different alternatives. Using the gas diffusion technique \( \eta_A vs \rho \) is directly obtained; then, converting \( \rho \) to \( \Delta T \) from the nucleation criteria, i.e. \( \Delta \phi_0 \rho \), gives \( \eta_A vs \Delta T \) from which the heat flux model predicts \( q_A vs \Delta T \). If instead it is decided to perform a geometrical surface measurement it is necessary to develop a surface model with which it is possible to go from a surface measurement to a prediction of \( \eta_A vs \rho \). This chapter is primarily devoted to developing a surface model incorporating the important surface variables. Also, methods of measuring these variables will be discussed.

IV.2 Previous Investigations

Corty and Foust [14] were among the first to study surface variables in nucleate boiling. They argued in a qualitative way that surface microroughness and contact angle determine cavity stability. Experimentally they showed that upon increasing roughness more cavities were active at a given \( \Delta T \) and suggested that rms roughness may be enough to correlate
surface effects. Vachon et al. [17] produced equivalent \( \text{rms} \) roughnesses on separate surfaces using different preparation techniques. Different \( \frac{\varphi}{A} \) vs \( \Delta T \) curves resulted revealing that \( \text{rms} \) alone is insufficient to correlate the data. Other experimental results from [17] indicated that the boiling curve does not shift monotonically to the left upon increasing roughness (with a given finishing technique) but reaches an optimum point beyond which the heat flux actually decreases for a given \( \Delta T \). Westwater et al. [15] photographed active cavities and in a later work [16] counted nucleation sites on metallic surfaces. Various experimenters [13,26,27,28] studied surface variables and related them to heat transfer characteristics.

IV.3 Nature and Origin of Cavities

Figures 17, 18, and 19 show photo and electron micrographs of cavities along with rough sketches of the probable internal geometry. The location of these cavities was identified by small circular scale spots formed when water boiled from the surface. When the central area of the scale spot was examined with a microscope a cavity was invariably detected. The internal geometry was studied by focussing a microscope at different levels within the cavity. It was very surprising to find such large cavities on this surface which originally had a bright shiny mirror finish to the naked eye. The surface was finished by grinding with progressively finer silicon
carbide grit paper and then polished with a .5μ alumina slurry on a polishing cloth.

Apparently the cavities were caused by inclusions or pieces of grit imbedding into the surface at some earlier stage in the polishing process and subsequently popping out leaving a hole. Figure 20 shows electron micrographs of two pieces of silicon carbide grit paper. Note the sharp points and cutting edges which are ideal for creating cavities. As long as the grit remains bound to the backing only scratches result on the surface, but when the grit becomes dislodged it has the potential of creating a cavity in the metal.

Westwater [16] determined that the spacial arrangement of cavities on mechanically polished copper was random following a Poisson distribution. Very careful mechanical polishing can minimize the number of such cavities but only by electropolishing of a pure metal can surface cavities be virtually eliminated.

In general the cavity geometry does not appear to be related to shape of the surrounding surface. For this reason it might be expected that average statistics such as $\text{rms}$ roughness alone would be insufficient to correlate surface effects. The next section is devoted to formulating a surface model using appropriate surface statistics. This idealized model will assume that cavities are smooth walled conical cavities. From the sketches in Figures 17, 18, and 19 it was observed that natural cavities possess a roughness scale and
are in general tapered but are not truly conical. The cone angle $\phi$ will then be regarded as an effective cone angle accounting for deviations from conical geometry, roughness effects, and hydrodynamic effects encountered in the trapping process.

IV.4 Formulation of Surface Model

Before a surface model can be formulated it is first necessary to describe the effective radius for nucleation analytically in terms of cavity geometry and contact angle. Assuming all cavities are conical with cone angle $\phi$ and mouth radius $R$, the definition of $\rho$ was given by expression (10) of Section III.3 as

$$\frac{\rho}{R} = f(\theta, \phi) \quad (10)$$

The next consideration is to specify a joint probability density function $\rho(\phi, R)$ for a cavity having a cone angle $\phi$ between $\phi$ and $\phi + d\phi$ and a mouth radius $R$ between $R$ and $R + dR$ along with the total number of cavities $\eta_A^2$, $\eta_{(\phi)}$:

$$\rho = \frac{\eta_A}{\eta_{(\phi)}} = \rho(\phi, R) \quad (20)$$

In this expression $\eta_A$ is the number of cavities having a cone angle between $\phi$ and $\phi + d\phi$ and mouth radius $R$ between $R$ and $R + dR$, i.e., $\eta_A = \eta_{(\phi)} \rho(\phi, R)$.

Utilizing expressions (10) and (20) it is possible to calculate the cumulative distribution $\eta_A$ vs $\rho$ using the math-
emathematical algorithm utilized in Appendix F. However, a simple cumulative distribution can be obtained in a straightforward manner by assuming after Mikic and Rohsenow \[11\] that the cumulative distribution for $R$ is given as

$$\frac{n}{A} = \left(\frac{R_s}{R}\right)^m$$

(21)

where the dimensional coefficient (1/unit area) of $\left(\frac{R_s}{R}\right)^m$ is understood and $m$ and $R_s$ depend upon surface geometry. Assuming furthermore that the distribution of $\phi$ is a sufficiently narrow band so that a single equivalent value, i.e. $\bar{\phi}$, can be specified for all the cavities, expression (10) becomes

$$\frac{\rho}{R} = f(\theta, \bar{\phi})$$

(22)

Combining (21) and (22) the cumulative distribution is

$$\frac{n}{A} = \left(\frac{\int f(\theta, \bar{\phi}) R_s}{\rho}\right)^m$$

(23)

For $\bar{\phi} = 5^\circ$, $R_s = 2 \times 10^{-4}$, $m = 5$, $\eta_A vs \rho$ is plotted in Figure 21 for various values of $\theta$. It can be seen that at values of $\theta$ slightly greater than $\bar{\phi}$, $\eta_A vs \rho$ curve is very sensitive to contact angle. This is a consequence of the fact that $\frac{\rho}{R} = f(\theta, \bar{\phi})$ is sensitive to contact angle at small cone angles as revealed in Figure 6. Upon increasing $\theta$ the $\eta_A vs \rho$ curve shifts to the right until $\theta \geq 30^\circ$ at which point $\eta_A vs \rho$ is unique. From Figure 6 it is seen that beyond $\theta = 30^\circ$, $\frac{\rho}{R} = 1$ and from equation (23) it is evident that $\eta_A vs \rho$ is equivalent to $\eta_A vs R$. For larger equivalent cone angles, $\bar{\phi}$, the rate at which $\eta_A vs \rho$ shifts to the right with $\theta$
decreases in accordance with the behavior of \( \frac{\rho}{R} = f(\theta, \Phi) \).

The three unknown parameters in expression (23) can be determined experimentally for any given surface as follows: boiling experiments must be performed for two fluids with different wetting characteristics yielding two distinct \( \eta_A \) vs \( \rho \) curves which are parallel when plotted on log-log coordinates. The slope of either curve immediately yields \( m \). By taking the ratio of \( \rho \) values from the two curves at the same value of \( \eta_A \), the following is obtained which can be solved by trial-and-error from Figure 6 for \( \Phi \):

\[
\frac{\rho_1}{\rho_2} = \frac{\rho_{1/R}}{\rho_{2/R}} = \frac{f(\theta_1, \Phi)}{f(\theta_2, \Phi)}
\]

Then \( R_S \) can be obtained from either \( \eta_A \) vs \( \rho \) curve by determining the \( \rho \) intercept at \( \eta_A = 1/\text{in}^2 \) where \( \rho = \rho_S \), then

\[
R_S = \frac{\rho_S}{f(\theta, \Phi)}
\]

With knowledge of these three parameters the nucleation site distribution is completely specified and can be used for predicting the performance of a system possessing some arbitrary contact angle. Other possible methods for determining these three parameters without resorting to a boiling experiment will be discussed in Section IV,6.

Inherently this model assumes that if \( \theta > \Phi \) all the cavities will be active at some value of \( \rho \). If it were desired to build into the model the possibility of having a portion of the cavities deactivated, a different assumption
for the distribution of cone angles must be made. In Appendix F an \( n_A \nu s \phi \) distribution is derived assuming that \( \phi \) is uniformly distributed between 0 and some \( \phi_M \). In that case only the fraction of cavities with cone angles less than \( \Theta \) will be active.

IV.5 Experimental Results and Discussion

Boiling experiments were conducted with water and several organic fluids boiled on the same surface. The cumulative site density \( n_A \nu s \Delta T \) was obtained by visual counting and \( \Delta T \) converted to \( \rho \) from \( \rho = \frac{2\phi}{\Delta T} \). Figures 22-27 show results from this study and reference [26] for different surface finishes. Notice that the \( n_A \nu s \phi \) distributions are approximately straight parallel lines on log-log which is consistent with the surface model. Furthermore since the organics have nearly the same contact angle they should indeed yield similar nucleation site densities. Water has a higher contact angle and hence lie farther to the right. The larger slopes of these curves suggest a narrow band distribution of mouth radii resulting as a consequence of the very uniform surface finishing procedure that was employed.

This experimental data of Figures 22-27 for organics and water can be interpreted quantitatively utilizing the model of the previous section. Referring to say Figure 26, consider an average line through the organic data parallel
to the water data. The ratio of $\frac{\sigma_{\text{org}}}{\sigma_{\text{H}_2\text{O}}}$ at any given $n/A$ is a constant which in this case is about $\frac{1}{3}$. Referring to Figure 6 and taking $\theta = 7^\circ$ and $35^\circ$ as contact angles for organics and water respectively a unique value of $\Phi$ can be obtained by trial-and-error and satisfies equation (24). For this surface a $\Phi$ value of $6^\circ$ is obtained and like the example of Section III.5 water has sufficiently high contact angle that $\rho = R$. From $n/A vs \rho$ for water $R_s$ and $m$ can be determined immediately yielding $R_s = 2 \times 10^{-4}$" and $m = 7$. The $\Phi$ and $R_s$ values on the surfaces tested in this work and Reference[26] were all very nearly $6^\circ$ and $2 \times 10^{-4}$" respectively while $m$ varied from 5 to 20. More data is needed using other surface finishing procedures such as lapping, bead blasting, etching, and the like before a detailed study of the variations of $\Phi$, $R_s$, and $m$ can be made. Even more fundamentally it must be determined whether or not a straight line on lox-loc can describe $n/A vs \rho$ for arbitrary surface finishes. There is good experimental evidence that this assumption is valid for a wide range of surface finishes since $\frac{\rho}{A}$ vs $\Delta T$ curves are often straight lines on lox-loc. It will be shown in Chapter V that the shape of the $\frac{\rho}{A}$ vs $\Delta T$ curve indeed implies the shape of $n/A vs \rho$.

For large values of $n/A$ experimental data appears to bend slightly upwards as seen in Figures 14 and 15. This trend was also observed by Mikic [13] who replotted the data
of Westwater [16] on $\eta_A vs \phi$ coordinates. When the data bends upwards at large $\eta_A$, i.e. $\eta_A > 40$ in \(^2\) the expression for from a curve fit at lower $\eta_A$ will be assumed to characterize the distribution. Using this distribution it will be seen in Section V, that not only the isolated bubble region but also higher heat flux regions of the boiling curve can be predicted.

Generally the spread in $\eta_A vs \phi$ curves for organics boiled on the same surfaces was very small as was seen in Figures 13 and 22-27. In one series of runs however the methanol and benzene curves were initially significantly different shown in Figure 28 as run 1 and 2. Not only were the slopes different but there was about a 100% separation in the curves. Following these runs several other runs were taken alternately boiling methanol and benzene on the same surface. Curves for both fluids essentially shifted towards each other until they finally stabilized in runs 6 and 7 with a 20% spread. Apparently small contact angle change occured until the surface conditions become constant. Separations in the nucleation site density curves occured in other runs but usually amounted to less than about 20%. It is quite remarkable that the data for organics is generally so consistent in view of Figure 6 which shows that the effective radius is very sensitive to contact angle (for small contact angles). A more complete description of the trapping process would probably reveal that $\phi$
has a somewhat weaker dependence on contact angle.

IV.6 Measurement of Cavity Distribution

IV.6.1 Mechanical and Optical Methods

The unknown parameters describing the nucleation site distribution have been reduced to $R_s$, $m$, and $\Phi$. If all the idealizations built into the model of Section IV.4 were valid then the following procedure would provide the three parameters: $\Phi$ could be determined either optically or by tracing a Talysurf Profilometer stylus carefully across several cavities measuring their cone angles and taking the average value. If assumptions were made concerning the spacial distribution of cavities it is possible that $m$ and $R_s$ could be deduced statistically from a set of traces. It is possible that the complete distribution of mouth radii could also be determined optically by going through the tedious process of counting the number and sizes of all cavity radii.

In reality neither $\Phi$ nor the mouth radii can be easily determined. If hydrodynamic and roughness effects are included, the precise definition of $\phi$ and hence $\Phi$ loses some of its geometrical significance as was noted in Section III.5.3. In addition the location of cavities on a real surface is
often obscured by the presence of microscale surface roughness. Aside from this difficulty a talysurf trace will hit so few cavities that the meaningful statistics may not be obtained. The problems involved in such a measurement are indeed formidable and as of yet have not been solved.

**IV.6.2 Gas Diffusion Experiment**

If an experiment such as the gas diffusion experiment could be performed on a test surface the $\eta_A^{VS} P$ distribution could directly be determined. Of course this is only true provided the fluid used has similar wetting properties to the fluid to be boiled. If results of a gas diffusion experiment using water are applied to organics the effective $\Phi$ would have to be specified.

In Section III.5.3 it was noted that the gas diffusion experiment was used successfully for measuring individual cavities that were known to be stable from a previous boiling experiment. This technique yielded cavity distributions that were too large when applied to an entire arbitrary surface. Apparently this was due to metastable cavities remaining active which would not be stable in nucleate boiling. Many departures are needed from cavities before they prove themselves to be good vapor traps and this is not attained using an air-water system. This might be remedied by using a gas with higher solubility such as $CO_2$. Preliminary tests
showed some potential problems that may be encountered such as maintaining the pool at a sufficiently high concentration. However, it is hoped that with some refinement this experiment will be able to predict the cavity distribution in advance.

IV.7 Summary and Conclusions

Photo and electron micrographs of active cavities were obtained providing insight into the nature and origin of cavities. A model was developed for the cumulative site density curve in terms of geometrical parameters. Essentially the model assumes that the population of surface cavities possess a single equivalent cone angle $\Phi$ with the distribution of mouth radii following the power law as follows.

$$n_A = \left( \frac{f(\theta, \Phi) R_s}{\rho} \right)^m$$

Experimental data from this study and other studies was successfully explained with the model. Further data for surface finishes produced by means other than mechanical grinding must be obtained before generalizations of the surface model are made. Possible methods for measuring the pertinent surface variables were discussed along with the gas diffusion experiment.
V. PREDICTION OF THE BOILING HEAT FLUX

The flow chart of Figure 1 shows that in order to predict the boiling curve from knowledge of $\eta/A$ vs $\Delta T$ data it is necessary to utilize a heat flux model. This chapter will deal with choosing an acceptable heat flux model and incorporating the surface variables into a unified heat transfer model.

V.1 Previous Investigations

The first heat flux models were based solely on bubble agitation but later Moore and Messler [8] contended that microlayer evaporation was the dominant mechanism. Han and Griffith [5,6] formulated the problem as a transient conduction process involving repeated removal of the thermal layer. An evaluation of the various heat transfer mechanisms was performed by Graham and Hendricks [9] showing that no one process dominates over all ranges. Cooper and Lloyd [11] studied the microlayer evaporation in detail stating the conditions under which the effect is significant. An overall model including microlayer and transient conduction effects was developed by Deane [12] for liquid metals.

Kurihara and Myers [26] were among the first to analytically incorporate surface effects into a heat transfer model. For a limited range of surface finishes Kuhloor and Radhakrishnan [27] correlated surface effects by defining a roughness parameter. Mikic and Rohsenow [13] following
the arguments of [28] developed a transient conduction heat transfer model including the effects of surface conditions.

V.2 Heat Flux Model

For cases in which the microlayer is insignificant, i.e., for relatively high pressure, low wall superheat, or subcooled bulk temperature [11], Mikic and Rohsenow [13] derived an expression for the boiling heat flux. Their analysis is stated briefly as follows:

When a bubble departs from an active site the portion of the thermal layer within the area of influence, taken to be twice the bubble departure diameter \( D_b \), is removed and replaced by saturated liquid at \( T_s \) from the bulk. This process occurs with a frequency \( f \) equal to the average bubble departure frequency. During one cycle the heat removed \( \frac{q}{A} \) at an active site per cycle on a surface at \( T_w \) is

\[
\frac{q}{A} \bigg|_{Ai} = \int_0^f \left( \frac{q}{A} \right) \, dt = \frac{2K\Delta T}{\sqrt{\pi\alpha f}}
\]

(26)

where

\[
\frac{q}{A} = \frac{K\Delta T}{\sqrt{\pi\alpha t}}
\]

and \( A_i \) is the area of influence.

Then over a unit time the average heat flux per site is

\[
\frac{q}{A} \bigg|_{av, Ai} = \int \frac{q}{A} \bigg|_{Ai} \, dt = \frac{2K\Delta T \sqrt{f}}{\sqrt{\pi\alpha}}
\]

(27)

Assuming that the area of influence is equal to \( \pi D_b^2 \) and
that no overlapping of active sites occur the average boiling heat flux over the entire surface is

$$\frac{q}{A} = \frac{n_A}{A} \left( \frac{q}{A} \right)_{av, A} \pi D_b^2 = 2\pi \sqrt{\frac{k}{\rho c_p l}} \sqrt{f} D_b^2 \frac{n_A}{A} \Delta T$$  \hspace{1cm} (28)$$

where $n_A$ is the number of sites per unit area. In this expression $D_b$, and $f$ can be determined by correlations such as those of Cole and Rohsenow [29] and Cole [30] as follows:

$$D_b = C_2 \left[ \frac{\delta q_o}{g(\rho_e - \rho_v)} \right]^{\frac{1}{2}} \left( J_a^* \right)^{\frac{5}{4}}$$  \hspace{1cm} (29)$$

where $C_2 = 1.5 \times 10^{-4}$ for water and $4.65 \times 10^{-4}$ for other liquids and

$$J_a^* \equiv \frac{\rho_e C_e T_s}{h_f h_g}$$

and

$$f D_b = C_3 \left( \frac{\delta q_o g(\rho_e - \rho_v)}{\rho_e^2} \right)^{\frac{1}{4}}$$  \hspace{1cm} (30)$$

where $C_3 = .6$

The total heat flux can be obtained by adding on the contribution of natural convection

$$\frac{q}{A} = \frac{q}{A} + \frac{A_{n.c.}}{A_{tot}} \left( \frac{q}{A} \right)_{n.c.}$$  \hspace{1cm} (31)$$

where

$$\frac{A_{n.c.}}{A_{tot}} = 1 - \frac{n_A}{A_{tot}} \pi D_b^2$$

and $A_{tot}$ is the total area of heating surface. $\frac{q}{A}_{n.c.}$ can be
determined by the following:

For Laminar Range, \(10^4 < Ra < 10^9\)

where

\[
Ra = \frac{\gamma g \Delta T A^{3/2}}{\alpha \sqrt{\nu}}
\]

and \(A\) = area of heating surface

\[
\left(\frac{Q}{A}\right)_{n.c.} = 0.54 \rho_c \left(\frac{\gamma g \Delta T A^{5/4}}{\alpha \sqrt{A}}\right)
\]

(32)

For Turbulent Range: \(10^9 < Ra < 10^{12}\)

\[
\left(\frac{Q}{A}\right)_{n.c.} = 0.14 \rho_c \left(\frac{\gamma g \Delta T A^{4/3}}{\alpha \sqrt{\nu}}\right)^{1/3}
\]

(33)

In expression (28) the only missing information is \(\eta_A\)

which will be provided shortly.

When the system pressure is low or at high wall superheats the contribution of the microlayer, which must be added to (28), can be expressed simplistically as

\[
\left(\frac{Q}{A}\right)_{b,mic} = \int tgr \ h_{av}(C Db^2) \eta_A (Tw - Ts)
\]

(34)

where \(tgr\) is the bubble growth time, \(h_{av}\) is an average heat transfer coefficient over the bubble base and \((C Db^2)\) is the average base area over which evaporation occurs.

The term \((Tw - Ts)\) in expression (34) would have to be modified for high conductivity liquids or low conductivity surfaces since strong variations of surface temperature occur during evaporation. For the case of liquid sodium Deane [12] combined transient conduction and microlayer effects into an overall heat transfer model.
In the present work only data at one atmosphere was obtained (and moderate wall temperatures) in which cases expression (28) should be valid since microlayer effects are small. The next section presents data verifying expression (28) for water and organics.

V.3 Experimental Results and Discussion

Water and methyl alcohol were boiled on copper with different surface finishes. The number of active sites was visually determined in the isolated bubble region at each heat flux \( q_A \) and \( \Delta T \). When the heat flux was lowered to the point at which no bubbles appeared, natural convection data was obtained. Using the above data \( q_A \) is easily determined from equation (31). Experimental results are plotted in Figure 29 using the following form of equation (28):

\[
\frac{q_A}{q_A}^* = \frac{q_A}{\sqrt{\pi K \rho c}} \frac{1}{f} \frac{1}{D_b^2 \Delta T} = \eta_A
\]

(28a)

Values for \( f \) and \( D_b \) were taken from Reference [31]. The excellent agreement indicates that the process is indeed one of transient conduction and predicted accurately by equation (28).

Expression (28) and the experimental results of Figure 29 show that \( h_b = \frac{q_A}{\Delta T} \sim \frac{\eta_A}{A} \) whereas the data of [25] reveals that \( h \sim (\eta_A)^{1/2} \). The reason for the difference is that the natural convection component was subtracted from the total heat flux in the present work yielding only the boiling component. At very low heat fluxes the natural convection component dominates since few sites are active. At
higher fluxes natural convection effects become insignificant.

Knowledge of \( q_A \) vs \( \Delta T \) behavior for low heat fluxes is sufficient to predict well into the higher heat flux regions as shown in Figure 30. The good agreement is fortuitous, however, since expression (28) is strictly valid only in the isolated bubble region. As stated by Mikic [13] the underestimation of \( q_A \) at high heat flux is balanced by the decrease in departure diameter and the two effects cancel.

V.4 Unification of the Model

The results of Chapters III and IV can now be incorporated into an overall model for boiling heat flux. By substituting expression (23) for \( q_A \) into (28) and letting

\[
\varphi = \frac{2}{h_f g} \Delta T \frac{v_f g}{h_f g} \Delta T
\]

the following is obtained

\[
q_A = \frac{\sqrt{\pi} \left( R_f \Phi(\theta) \right)^m}{2^{m-1}} \left( \frac{h_f g \rho_v}{T_s \delta} \right)^m \sqrt{\tau} D_b^2 \Delta T^{m+\frac{1}{2}} \tag{35}
\]

Equation (35) is an expression for the boiling heat flux showing explicitly the role of surface conditions and contact angle. It is seen that surface variables affect both the slope and intercept of the \( q_A \) vs \( \Delta T \) curve. The slope depends only on the surface parameter \( m \), while the intercept is function \( m, \Phi, \) and \( R_f \). In principle then by employing the proper finishing procedure it is possible to obtain any desired slope intercept combination.
V.5 Summary and Conclusions

A transient conduction heat flux model for the isolated bubble region was experimentally verified and incorporated into the surface model. The model can be extrapolated to predict the boiling heat fluxes at much higher $\Delta T$'s. The unified expression relates surfaces variables to the boiling heat flux explicitly showing the role of surface geometry and contact angle. It was seen that surface variables effect both the slope and intercept of the $\frac{\varphi}{A}$ vs $\Delta T$ curve.
VI. CONCLUSIONS AND RECOMMENDATIONS

VI.1 Conclusions

Using boiling and gas diffusion experiments the nucleation criteria \( \rho = \frac{2 \Delta T_s \nu_{fg}}{h_{fg} \Delta T} \) was verified for small natural cavities in which case the thermal gradient near the heating surface is negligible. A gas diffusion experiment was developed and employed successfully for predicting the effective radius of nucleation for certain natural cavities. In a modified form it should be capable of providing the entire cumulative nucleation site density for a given fluid surface combination. Dynamic effects in nucleation are relatively small and can be neglected for most cavities of practical interest. A simple trapping mechanism can be postulated and a geometrical model developed for idealized conical cavities relating the effective radius to the cavity radius, cone angle, and contact angle. The model predicts that for a given cavity, organics due to their excellent wetting problems should nucleate at a smaller effective radius than water. Experimental results from this and other studies are consistent with the model. To incorporate hydrodynamic and roughness effects the cone angle must be interpreted as an effective cone angle. Cavity stability and trapping potential are enhanced by roughness beyond that expected for smooth wall conical cavities.

A surface model can be developed for the cumulative site density curve \( n_A vs \rho \) in terms of the geometrical parameters
\( \Phi, \Theta, R_s, \) and \( m, \) equation (23). For mechanically ground surfaces it is sufficient to assume that the population of cavities possesses a single equivalent cone angle and that the distribution of mouth radii follows the power law. The model can explain experimental data from this and other studies. However, further data for a wider range of surfaces must be obtained before generalizations of the surface model are made. Possible methods for measuring the cavity distribution consist of the gas diffusion experiment and Taly-surf Profilometer.

A transient heat conduction model can accurately describe the boiling curve for water and organics at atmospheric pressure. The model, even though strictly valid for only the isolated bubble region, can be extrapolated to predict performance at higher heat fluxes. By incorporating the surface and heat flux models a unified expression relating surface variables to the boiling heat flux was developed, showing explicitly the role of surface geometry and contact angle.

VI.2 Recommendations

This work attempted to provide a framework for predicting the boiling curve in advance by defining the important geometric parameters that effect boiling. Some of the idealizations such as the postulated trapping mechanism must be modified to account for the roughness and hydrodynamic effects.
Until this is done the cone angle $\phi$ is not completely a geometric parameter. A wider range of surface finishes must be investigated to provide feedback information for modifying the surface model.

Utilizing the Talysurf Profilometer as a surface measuring device should be further investigated. Refining the gas diffusion experiment should be pursued since this device gives direct input to the heat flux model eliminating the necessity of a surface model and geometrical measuring technique. Systems using $CO_2$-water and air-organics should be developed which would hopefully establish a sufficiently high bubble departure frequency to insure cavity stability.
REFERENCES


APPENDIX A

BOILING APPARATUS AND PROCEDURE

A.1 Apparatus

The main body of the apparatus consisted of a rectangular brass enclosure 4x4x6" which was coated inside and out with FEP teflon providing excellent corrosion resistance. The main heater was a 940 watt Hot Watt high density cartridge heater 5/8" diameter, 2" long, fit into a copper cylinder 1 1/8" diameter, 3" long. Two 400 watt Chomaloxy annular shaped guard heaters fastened to the sides of the enclosure maintained the bulk at saturation. Power as regulated to the main and guard heaters by 220 volt and 110 volt General Radio type variacs respectively. Four 30 gauge copper-constantan thermocouples were placed in .04" diameter thermocouple holes located along the axis of the straight conductor section at positions .173", .675", 1.176", and 1.676" from the boiling surface. Temperatures were measured with a Leeds and Northrup Series #262237 potentiometer. The heating section was attached to the conductor section with machine screws located in brass flanges silver soldered to the copper sections. Before fastening, "Silver Goop" was applied to the mating surfaces to insure good thermal contact. A thin 304 stainless steel flange .04" thick was silver soldered around the periphery of the boiling surface to provide a means of fastening the conductor section to the enclosure. A teflon gasket was used for proper sealing.
The vapor was returned to the system by 2 reflux condensers located at the top of the enclosure. Since boiling runs were taken with benzene and methyl alcohol, whose vapors are toxic and flammable, it was necessary to place a safety hood above the condensers removing any vapor which did not condense. Two 2x5" windows on the front and back of the enclosure provided a clear view of the boiling process.

A.2 Procedure

A.2.1 Surface Preparation

The copper test section was finished before each set of runs in the Metallurgical Laboratory of M.I.T. Wet grinding was employed using silicon carbide grit paper with grit sizes ranging from #180 to #600. The test surface was held flush with light pressure against a rotating grinding disk continually rinsed with water. In order to remove the circular grinding pattern resulting from this rotation the surface was subsequently ground on a non-rotating grinding disk. About 50 light grinding strokes in one direction on a fresh continuously washed grinding disk completed the process. To the naked eye the surface appeared to have uniform parallel scratches. After being rinsed with a jet of alcohol the surface was ready for boiling.

A.2.2 Boiling Procedure

A set of runs generally consisted of boiling water and
organics on the same surface. When organics are boiled on a metal surface little oxidation occurs; however, water tends to cause visual oxidation of the boiling surface unless special precautions are taken. Since this study requires that boiling data be taken on surfaces that initially possess nearly the same surface chemistry, organics were boiled before water on a given surface. The organics were degassed by boiling at about $\frac{2}{3}$ burnout heat flux for about $\frac{1}{2}$ hour. Partial degassing of distilled water to eliminate oxidation was accomplished initially in a separate glass beaker before the liquid was admitted into the enclosure. Again the water was boiled for about one hour at $\frac{1}{2}$ the burnout heat flux. Data was taken as the boiling curve was traversed from high to low heat flux. Incrementally the heat flux was lowered and equilibrium established at which conditions the thermocouple readings were recorded. Usually about 20 minutes was sufficient time for the system to reach equilibrium at each heat flux level. When the isolated bubble region was encountered not only thermocouple readings but also visual counts of active sites were obtained. For water counting first became possible at about $30/\text{in}^2$ and organics at about $40/\text{in}^2$. The test was terminated after a couple of data points were obtained in the natural convection region.

A.2.3 Systematic Errors

Using an error analysis scheme developed by Wilcox[41]
the maximum error in temperature $\Delta T$ measurement was estimated to be only about 2% at the highest heat flux. Heat loss from the sides of the straight conduction section was negligible as liberal amounts of fiberglass insulations were wrapped around the section and entrie apparatus. The heat loss from the flange at the boiling surface was approximated by a simple fin analysis. The fin loss, which was roughly one half of the natural convection heat transfer component, was then subtracted from the heat flux determined from the temperature gradient, yielding the total heat flux at the copper boiling surface.
APPENDIX B

GAS DIFFUSION EXPERIMENT

B.1 General

Using gas diffusion experiments to study nucleate boiling is not an entirely new idea. Westwater and Buel [36] studied growth of CO2 bubbles in supersaturated water, looking at the influence of contact angle. By the analogy between heat and mass transfer certain generalizations can be made regarding boiling systems. They concluded, among other things, that the effect of contact angle on bubble growth is insignificant a fact that was merely assumed by previous investigators. One of the most novel uses of a gas diffusion experiment was devised by Brown [37] who claimed it was possible to measure the cumulative nucleation site density for a fluid surface combination with a device called the "Bubble Meter". The alternative indirect method of obtaining $\eta_A^{vs} \varphi$ (discussed in Chapter IV) requires the development of a surface model and geometric measuring technique. The difficulties involved therein are circumvented by using the gas diffusion experiment. Besides providing gross information on the nucleation site density, the gas diffusion experiment could be used as a tool for studying nucleation from individual cavities. This Appendix begins with a brief discussion of equilibrium concepts and then describes a gas diffusion experiment which is essentially an improvement of Brown's technique.
B.2 **Equilibrium Radius**

For thermodynamic equilibrium of an air water system across a spherical interface (shown in Figure 33) the following relations must be satisfied:

\[
\begin{align*}
\mu_{H_2O}\,^\ell &= \mu_{H_2O}\,^g \\
\mu_{AIR}\,^\ell &= \mu_{AIR}\,^g \\
T\,^\ell &= T\,^g \\
(P_a + P_v) - P\,^\ell &= 2\phi \frac{\sigma}{\rho}
\end{align*}
\]  

(B1) \hspace{1cm} (B2) \hspace{1cm} (B3) \hspace{1cm} (B4)

where \(P_a\) and \(P_v\) are the partial pressures of air and water in the gas phase, \(P_l\) is the pressure of the liquid, and \(\mu\) is the chemical potential of each component in each phase. As a consequence of condition (B1) it can be shown [18] that the vapor states are nearly saturation states at the system temperature. Condition (B2) stated in terms of Henry’s Law is

\[
P_a = m \, C_a
\]

(B5)

where \(P_a\) is the partial pressure of air in gas phase, \(C_a\) is the concentration of dissolved air in the liquid phase and \(m\) is Henry’s Law constant which is a function of temperature only (for partial pressures less than 1 atmosphere). Condition (B4) is simply a force balance across the interface. From relations (B1 - B4) it can be seen that for a given system total pressure temperature, and dissolved air concentration (fixing \(P_a\), \(P_v\), \(P_l\)) a unique radius of curvature exists as
specified by relation (B4) rewritten as follows:

\[ \rho = \frac{2 \delta}{(P_a + P_v) - P_\ell} \quad (B4) \]

In this study the value of \( \rho \) will be varied by regulating the concentration \( C_a \) and hence \( P_a \) in relation (B4). Relation (B4) will often be referred to as \( \rho = \frac{2 \delta}{\Delta P} \)

where \( \Delta P = (P_a + P_v) - P_\ell \).

B.3 Nucleation and Growth of Gas Bubbles

The equilibrium positions that an interface can attain within a cavity are mapped on a \( \frac{1}{\rho} \) vs Volume curve in Figure 33b for a smooth walled conical cavity. If the system is held at a concentration level corresponding to point 1 and the interface occupies a position in the cavity corresponding to point 3, diffusion will occur into the bubble causing growth. Similarly if the interface occupies position 2 diffusion will occur into the liquid and collapse will occur. In general an interfacial position on a negative sloping portion of the \( \frac{1}{\rho} \) vs Volume curve is unstable while that on a positive sloping portion is stable. If the initial interfacial position is located at \( \frac{1}{\rho} > \frac{1}{R} \), then this initial radius is equal to the effective radius for determining bubble growth. If the interface is formed between \( \frac{1}{\rho_{\text{MAX}}} \leq \frac{1}{\rho} \leq \frac{1}{R} \) then \( R \) determines the necessary concentration. These conditions are precisely those for bubble growth in a uniformly superheated degassed boiling system. Thus the effective radius obtained
from a gas diffusion experiment should be able to predict the $\Delta T$ necessary for bubble growth in a boiling system from the nucleation criteria

$$\Delta T = \frac{2 \alpha T_s U_{fg}}{h_{fg} \rho}$$

**B.4 Apparatus and Procedure**

In order to obtain effective radii for various cavity sizes it is necessary to develop a means of obtaining, controlling, and measuring different supersaturation levels. To accomplish this the apparatus shown in Figures 34 and 35 was designed, consisting of a 15 gallon glass lined high pressure tank, 10x8x4" plexiglass tray, an electric stirrer, and an oxygen probe.

Air is vigorously bubbled at about 50 psig for 5 hours up through the bottom of the tank containing ten gallons of filtered deionized water. This operation increases the dissolved air concentration in the water to nearly that corresponding to the tank pressure. This water is then admitted into the opened plexiglass tray containing the test surface. With respect to this new condition the liquid is supersaturated and bubble growth is possible. When the water level reaches a depth of about 2" it is drained off and thereafter the inlet and outlet flow rates are equal. An electric stirrer placed near the test surface insures that no significant concentration
gradients develop. An equilibrium bulk concentration will be established between the amount of dissolved gas being supplied at the inlet, and that being lost at the drain, free surface, and consumed in bubble growth within the system. By varying the inlet flow rate the concentration can be regulated. An oxygen probe, described in Section B.5.3 placed near the test surface continuously monitors this concentration.

If the concentration is sufficiently high a large population of bubbles will emerge from the test surface. All bubbles that appear must have initiated from cavities with effective radii greater than or equal to the equilibrium radius. At a given system condition the concentration is recorded and a photograph taken from which bubbles are counted. Subsequently the concentration is lowered by lowering the flow rate. When a steady concentration prevails the old bubbles are wiped from the surface with a teflon strip. At the rate of about once every five minutes. Generally the departure frequency of bubbles was about 1/15 minutes which made physical wiping of the bubbles necessary. The slow growth rates (and departure frequencies) are primarily due to the low solubility of air in water. After wiping the surface another population soon will appear consisting of fewer bubbles and the process repeated at progressively lower concentrations until no bubbles reappear. In this way the
cumulative nucleation site density $\eta_A$ can be obtained. If it is desired individual sites can be singled out for study.

Since contact angle plays an important role in determining the effective radius, a gas diffusion experiment must be performed with the same fluid that will be eventually boiled from the surface.

B.5 Measurement of Air Partial Pressure

B.5.1 General

In order to determine $\rho$ in expression (B4), $P_a$ must be determined. Air consists mainly of the following gases

- $N_2$: 78.09%
- $O_2$: 20.95%
- $A$: .93%
- $CO_2$: .03%

Dissolved air in solution is difficult to measure if each component is to be measured separately. Nitrogen and Argon are relatively inert and difficult to measure at least by chemical methods. A possible method of measuring the combined partial pressure of all components is with a device used by Murphy [38] called an Aerometer. With this apparatus a sample of liquid is extracted from the system and evaporated by depressurization releasing the dissolved air. Upon repressurization the pure vapor condenses trapping the gas above the liquid where it can be measured. Other possible methods will be discussed in the following sections.
B.5.2 **Bubble Meter**

Brown [27] suggested that $P_a$ could be measured by inserting a device called the "Bubble Meter" as shown in Figure 33 into the solution. The disk is porous (made of fritted glass) and should allow equilibrium to be established between the gas and liquid. Then $P_a + P_v$ could be measured simply by the height of mercury in the manometer.

The following experiments were performed to test this device. The bubble meter was immersed into a supersaturated solution of water with air at room temperature. Even though the supersaturation level should have been sufficiently high to raise $P_a + P_v$ about 5 psi above atmospheric pressure the manometer indicated $P_a + P_v = 14.7$ psi. Another experiment consisted of placing the bubble meter into a beaker of water into which air was vigorously bubbled maintaining saturation at $P_a + P_v = 14.7$ psi. At the same time the liquid was slowly heated. The bubble meter should have indicated a constant pressure of 14.7 psi but instead the mercury level kept rising with increasing temperature.

In effect the bubble meter does not appear to come into equilibrium with the air liquid system, i.e. insufficient diffusion occurs. When the bubble meter is immersed in the water a fixed mass of air is trapped. The total pressure $P_T = P_a + P_v$ can be expressed as $P_T = \frac{w_g RT}{V} + P_v$ where $P_a = \frac{w_g RT}{V}$ by the perfect gas law. Assuming $\frac{w_g R}{V} = \text{const} = C$ the
total pressure is \( P_T = cT + P_v \). The constant \( c \) is determined by one experimental point (measuring the pressure \( P_T \) at a given temperature). This relation predicted the response of the bubble meter to within 20% for values of temperature from \( 75^\circ F < T < 140^\circ F \). The preceding arguments indicate that no significant diffusion occurs as \( P_T \) merely responds to temperature. The obtained value of \( P_a + P_v \) is not the true equilibrium value and can not be used in expression (24) for predicting \( \rho \). Brown employed heating of the liquid to obtain different supersaturation levels. Using the bubble meter he was able to predict nucleation with good accuracy from artificial cylindrical cavities of known sizes. The agreement was only fortuitous however since the true variation of \( P_a + P_v \) with temperature is not significantly different from \( P_T = cT + P_v \). If the technique of increasing concentration at a constant temperature were used then no such agreement would have resulted.

The diffusional resistance of the porous disk and the large gaseous space are most likely the reasons for not reaching equilibrium. Other bubble meters were made using silastic membranes (0.001" thick) having a low diffusional resistance. Even when \( CO_2 \) was used as the diffusing gas results were basically the same as before. This perhaps was due to the large gas space or elasticity of the silastic membrane.
B.5.3 **Probe Method**

For all bubbles in equilibrium in the air water system it is assumed the ratio of the partial pressure of all other gases to that of oxygen is a fixed constant, $K$. Thus

$$P_a = (1 + K) P_{o_2}$$

where $P_{o_2}$ and $P_a$ are the partial pressures of oxygen and air respectively in a bubble. It should be expected that $K$ is approximately $79 \div 21 = 3.76$, i.e. the ratio of nitrogen to oxygen in air. Expression (B4) can be restated as

$$\rho = \frac{2 \sigma}{(1 + K) P_{o_2} + P_v - P_e}$$  \hspace{1cm} \text{(B6)}

The partial pressure of oxygen, $P_{o_2}$, can be measured with an oxygen probe which will be described shortly. The procedure of Section B.4 was used with a test surface containing artificial cylindrical cavities with known mouth radii ranging from $9 \times 10^{-4}$" to $2 \times 10^{-4}$". For cylindrical cavities the mouth radius is indeed the effective radius which determines bubble growth. Using equation (B6) the constant $K$ was determined to be about $3.3$. Once $K$ is determined the expression can then be used to determine unknown effective radii.

The oxygen probe used in this study is similar to those commercially available. Details of the construction are given in References [39,40]. The purpose of the probe is to provide a continuous measurement of oxygen partial pressure $P_{o_2}$. The probe as shown in Figure 37 consists of a lead
anode, silver cathode, and a suitable electrolyte. The silver cathode is covered by a thin (.002) teflon membrane through which oxygen diffuses. The oxygen is consumed at the silver and a current is produced. At the silver cathode essentially zero concentration of oxygen is maintained. The concentration gradient of dissolved oxygen is linear across membrane and some oxygen concentration \( C'_{O_2} \) exists at the outer membrane boundary. Note \( C'_{O_2} = C_{O_2} \) (where \( C_{O_2} \) is the bulk concentration of \( O_2 \) to be measured) if sufficient agitation is present in the outside fluid. The rate of oxygen diffusion, \( \dot{m}_{O_2} \), across the membrane can be expressed as

\[
\dot{m}_{O_2} = a_1 C_{O_2} \approx a_1 C_0 \tag{B7}
\]

where \( a_1 \) is some constant which is a function of the membrane properties and temperature. The current, \( I \), is directly proportional to the rate of \( O_2 \) consumption yielding

\[
\dot{m}_{O_2} = a_2 I \tag{B8}
\]

Combining (B7) and (B8) the following is obtained:

\[
C_{O_2} = a_3 I \tag{B9}
\]

Since \( P_{O_2} = m C_{O_2} \) and \( V = I R \), where \( R \) is an external load resistor across the probe leads, relation (B9) can be written as

\[
P_{O_2} = a_4 V \tag{B10}
\]
where \( q_4 = \frac{q_3}{m_R} \) is some function of temperature for a given system. This relation predicts that the probe output voltage should be directly proportional to the oxygen partial pressure. Figure 38a shows a typical calibration curve for a probe tested with three gaseous mixture of oxygen with \( P_{O_2} \) values of 0, 21% and 50%. It is seen that the probes have good linearity but do not pass exactly through the origin. This small residual voltage at \( P_{O_2} \) is caused by diffusion of oxygen from the atmosphere at the top of the probe. The sensitivity of the probe to temperatures is shown in Figure 38b where temperature is plotted against voltage at a given \( P_{O_2} \). Corrections for temperature were taken into account when necessary. When used in a liquid containing dissolved oxygen the calibration curve of Figure 38a is valid if adequate agitation of the fluid is maintained. In the present system only moderate agitation is provided by the electric stirrer so a new calibration curve is necessary. Keeping the stirring speed constant one calibration point was taken before each run by vigorously bubbling air at atmospheric pressure into water which fixes \( P_{O_2} \) = .2095 (14.7 - \( P_r \)) and recording the probe output. The calibration curve was constructed by passing a straight line through that calibration point and the origin. Of course, this calibration curve is valid only if all subsequent readings are made at the same stirring speed.
Experimental Results and Discussion

Experimental results for individual cavities discussed in Section III.5.3 and shown in Figure10 revealed excellent agreement with boiling experiment. As expected the data using water was not valid for organics due to the differences in contact angle. A separate experiment would have to be performed for organics. Even though the gas diffusion experiment was valid for studying individual sites that were known in advance to be stable, the experiment failed to predict $\rho$ for arbitrary cavities. The reason for this is most likely due to an insufficient number of departures from sites, permitting even metastable cavities to remain active. In nucleate boiling of water about 50 departures/sec. occur throughout the course of boiling. Only cavities which are excellent vapor traps can survive so many departure cycles without being deactivated. If all the vapor in a cavity is displaced by liquid in any one cycle the cavity is permanently lost (unless it is reactivated by a neighbor or homogenous nucleation occurs). However in the gas diffusion experiment only one departure per five minutes is usually produced by mechanical wiping. In a given test of about an hour duration about 12 departures occur which is a mere fraction of that occurring in nucleate boiling. A cavity is simply not given a sufficient test of stability.

An improved gas diffusion experiment would employ a carbon dioxide water system because the solubility of $\text{CO}_2$ in water is
100 times greater than that of air in water. A departure frequency of about 1/sec should be expected eliminating the need of mechanical wiping. Preliminary runs using CO$_2$-water revealed potential difficulties in maintaining and measuring high concentrations of CO$_2$. The high solubility certainly causes bubbles to grow faster but unfortunately promotes considerable CO$_2$ loss by diffusion at the free surface. A system using air-methanol was also considered since air has a high solubility in alcohol with the potential of producing rapid bubble growth. As with the CO$_2$-water system difficulty was encountered in maintaining high concentrations. It is believed that some modifications in the system will eliminate this problem. At the present time such modifications are being studied in the Heat Transfer Laboratory of M.I.T.

B.7 Summary and Conclusion

A gas diffusion experiment was developed for measuring in advance the cumulative nucleation site distribution for a surface. Good experimental agreement resulted for individual sites that were known in advance to be stable from a previous boiling experiment. The experiment was not able to predict the total cumulative site density for an arbitrary surface due to the existence of metastable cavities. In a modified form it is believed that this device will be successful in providing $n_A$ vs $\rho$ for arbitrary surfaces.
APPENDIX C

CONTACT ANGLE

C.1 Equilibrium Contact Angle

Surface tension or surface free energy, is a function of the chemical state of an interface. When a drop of liquid is placed on a smooth surface, as shown in Figure 39a, a contact angle is established which can be described by a force balance of surface tensions at the triple interface yielding

\[ \frac{\gamma_{sl}}{\gamma_{sv}} + \frac{\gamma_{lv}}{\gamma_{lv}} \cos \theta = 0 \]  \hspace{1cm} (C1)

or

\[ \theta = \cos^{-1}\left(\frac{\gamma_{sv} - \gamma_{sl}}{\gamma_{lv}}\right) \]  \hspace{1cm} (C2)

where \( \gamma_{sv} \), \( \gamma_{sl} \), and \( \gamma_{lv} \) are the interfacial tensions of the solid vapor, solid liquid, and liquid vapor interfaces respectively.

If a pure liquid is placed on a clean metallic surface spreading should occur, i.e. \( \theta = 0 \) \[32\]. If however the surface possesses an oxide layer, absorbed gas, or other contamination a finite contact angle results. The greater the contamination of the system the greater the contact angle. Since a meticulously clean surface is untypical in engineering practice the contact angle for most fluid-surface combinations is greater than zero. If the surface possesses a microscale roughness equation (C2) still is valid except that the contact angle is measured with respect to the undulations,
shown in Figure 39b. The apparent contact angle as measured from the nominal horizontal surface is then different from the actual contact angle.

C.2 Dynamic Contact Angles

As noted by Huh and Scriven [33], dynamic contact angles can be classified in two ways: First, is a contact angle measured in a fluid having a moving triple interface. Second, is a contact angle measured in a fluid that has recently come to rest. It is generally true that the advancing contact angle is greater than the receding angle, a phenomenon known as hysteresis. Intuitively a difference is expected since the advancing portion sees a dry surface while the receding portion sees a surface emerging from the liquid.

Contact angles of the second type depend upon surface roughness, contamination, surface heterogeneity, and the time between movement and measurement [34]. If the surface is not homogeneous the surface tensions (and hence contact angles) may vary over the surface. If the time interval between movement and measurement is short, equilibrium values of the surface tensions are not attained, thus yielding non equilibrium contact angles. The truly dynamic contact angles of the first type possess all the difficulties of the second type but are also coupled with hydrodynamic phenomena. The nature of the triple interface movement is essentially
irregular, unsteady, stick and slip motion, moving rapidly over short distances at a time [33]. Further physical and mathematical difficulties involved with this problem are discussed in Reference [33].

C.3 Experimental Measurement of Contact Angle

It is obvious that contact angles, especially dynamic ones, are difficult to predict in advance for most engineering systems. Hence an experiment was performed to estimate the magnitudes of equilibrium and dynamic contact angle.

C.3.1 Apparatus and Procedure

Figure 39c shows the apparatus in which an air bubble is produced on the bottom side of polished copper surface immersed in the test fluid at room temperature. By adjusting the light properly and viewing the surface from about 1° below the horizontal, a good reflection of the bubble is produced in the copper. A photograph is taken through a microscope at about 8x and the included angle measured at the point of contact between the bubble and its reflection (shown in Figure 40). Half of this value then is the contact angle. By expanding and contracting the bubble an estimate of the dynamic receding and advancing contact angle can be obtained.
C.3.2 Results

The equilibrium contact angle for methanol is shown to be about $7^\circ$ in Figure 40c. Unfortunately the accuracy of this measurement is extremely low for such small angles (about $\pm 30\%$), the accuracy increasing for larger contact angles. Deciding where contact with the surface first occurs is the main source of error. When the bubbles were made to slowly grow or collapse no change in the contact angle could be observed.

The contact angle of water varied from $10^\circ$ to $90^\circ$ depending on surface cleanliness and whether the interface was advancing or receding. Figures 40a, b show the hysteresis between advancing and receding contact angles for water on a surface which was exposed to the atmosphere prior to testing. When the surface was polished under water and maintained under water, eliminating exposure to organic vapors or dust from the air, the equilibrium angle was about $10^\circ$ and little hysteresis was observed. When meticulous care was not taken to clean the surface the average advancing contact angle was approximately $35^\circ$. These contact angle measurements are generally consistent with the recent measurements made in Reference [35] for organics and water.

Probably the most convincing experiment for observing the difference in contact angle between organics and water is merely to place a drop of each fluid on a engineering metal
surface. Invariably the water will show a contact angle and the organic will almost appear to spread on the surface.

For quantitative purposes the contact angle of organics is taken to be about 7° while that of water about 35°.
APPENDIX D

DERIVATION OF GOVERNING EQUATIONS FOR DYNAMIC EFFECTS

D.1 Derivation of Momentum Equation

The governing equation of motion for the penetration depth is derived by considering a control volume as shown in Figure 7b, with the interfacial boundary located on the liquid side of the interface moving with the fluid. The following expression relates the external forces to the momentum flux across the boundary and the rate of change of momentum within the control volume.

\[ \sum F_z = \iiint_{C.S.} V_z \left( \rho \mathbf{v} \cdot d\mathbf{A} \right) + \frac{\partial}{\partial t} \iint_{C.V.} \rho v_z \, d\mathbf{v} \]  \hspace{1cm} (D1)

where \( z \) is positive to the left.

In this formulation the velocity profile is assumed to be a parabolic function of \( r \) as follows

\[ V_z = 2V \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right] \]  \hspace{1cm} (D2)

where \( V \) is the average velocity which will be a function of time only, defined as

\[ V \equiv \frac{d\bar{z}}{dt} \]

Substituting (D2) into the momentum terms of equation (D1) and performing the integration yields,

\[ \iint_{C.S.} = - \rho V^2 \left( \pi r_o^2 \right) \]  \hspace{1cm} (D3)

\[ \frac{\partial}{\partial t} \iint_{C.V.} \pi r_o^2 \rho (M+z) \frac{dv}{dt} + \rho V^2 \left( \pi r_o^2 \right) \]  \hspace{1cm} (D4)
The external forces acting on the control volume, i.e. $\Sigma F_z$, consist of the unbalance pressure and shear forces. The shear force acting to the left is determined from the derivative of the velocity profile at the wall yielding

$$-rac{4 \mu V}{r_0} \frac{2 \pi r_0}{(M+z)}$$

(D5)

The unbalanced pressure force acting on the vertical boundaries is given as

$$-(P_l - P_s) \pi r_0^2$$

(D6)

where $P_s$ is the pressure at the cavity mouth and $P_l$ is the pressure on the liquid side of the interface.

The liquid pressure $P_l$ can be related to vapor pressure across the curved interface by the following

$$P_v - P_l = 2 \frac{\partial \rho}{\partial \rho} + \frac{m^2}{P_v}$$

(D7)

where the momentum change of the vapor, i.e. $\frac{m^2}{P_v}$, was estimated to be small compared to $2 \frac{\partial \rho}{\partial \rho}$.

From the perfect gas law, i.e. $P_v = \frac{W_g R T}{\nu}$, a small change in pressure can be expressed as

$$\frac{P_v - P_v^0}{P_v^0} = \frac{\Delta W_g}{W_g^0} - \frac{\Delta \nu}{\nu^0} + \frac{\Delta T}{T^0}$$

where changes in pressure due to temperature changes $\frac{\Delta T}{T^0}$ were estimated to be small compared to $\frac{\Delta W_g}{W_g^0}$ and $\frac{\Delta \nu}{\nu^0}$

where $P_v^0$, $\nu^0$, and $W_g^0$ are initial values of these parameters.
at time \( t = 0 \). The initial pressure is simply

\[
P_v^o = P_T + 2 \rho/ho_0
\]  
(D9)

where \( \rho_0 \) is the initial radius of curvature.

Combining (D7) and (D8) and substituting into (D6) the unbalanced pressure force is given as

\[
-(P_T-P_T) \pi r_0^2 = - \pi r_0^2 \left[ \frac{P_v^o \Delta \omega_g}{\omega_g^o} - \frac{P_v^o \Delta \nu}{\nu_v^o} - \frac{2 \rho_0}{\rho_0} + \frac{2 \rho}{\rho_0} \right]
\]  
(D10)

The radius of curvature \( \rho \) at some position \( z \) can be related to \( \rho_0 \) and \( z \) from geometrical considerations as follows

\[
\rho = \rho_0 - \Delta \rho
\]  
(D11)

where

\[
\Delta \rho = \frac{T \tan \Theta_2}{\cos(\Theta - \Theta_2)} z
\]  
(D12)

From geometry considerations \( \Delta \nu/\nu_v^o \) can be expressed as follows for small \( z \)

\[
\frac{\Delta \nu}{\nu_v^o} \approx \frac{3z}{H}
\]  
(D13)

The value of \( \Delta \omega_g \) must be determined by the net condensation that occurs from \( t = 0 \) to \( t \) and is determined from the net heat transfer at the interface, i.e.

\[
\Delta \omega_g = \frac{1}{h \Delta g} \int_0^T \int_0^{r_0} \frac{K \Delta T}{\Delta \rho} 2 \pi r dr
dr
\]  
(D14)
Substituting (D11)-(D14) into (D10) and plugging (D10), (D5), (D4) and (D3) into (D1) the following momentum equation results upon rearrangement.

$$\frac{d^2 z}{dt^2} + \frac{8 \mu}{r_0^2 P_k} \frac{dz}{dt} + \left( \frac{3 P_v^o}{M H P_k} - \frac{2 \Delta \tan \phi^a}{P_k M P_0^2 \cos (\theta - \phi)} \right) z = -P_v^o \left( \int k d\theta \right) 2 \pi r dr$$

(D15)

with initial conditions

$$t = 0$$

$$z = 0$$

$$\frac{dz}{dt} = V_0$$

(D16a) (D16b)

Of course this equation cannot be solved separately without knowledge of the temperature field.

D.2 Derivation of Energy Equation

The governing energy equation under the assumptions of negligible convective effects is simply the following two dimensional transient conduction equation:

$$\frac{\partial^2 T}{\partial z^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

(D17)

with boundary conditions

$$\frac{\partial T}{\partial r} = 0 \ , \ r = 0$$

$$T = T_w \ , \ r = r_o$$

$$T = T_s \ , \ z = M + z$$

$$T = T_i \ , \ z = 0$$

(D18a) (D18b) (D18c) (D18d)

where $T_i$ is to be derived.
and initial conditions

\[ t = 0 \]
\[ T = \frac{Tw + Ts}{2} \]  \hspace{1cm} (D18e)

Boundary condition (D18d) is derived as follows.

From the previous arguments in Section D.1 the increase in vapor pressure is given as

\[ P_v - P_s = \frac{2 \Delta}{P_0} + P_v^0 \frac{\Delta \omega_g}{\omega_g^0} + P_v^0 \frac{3z}{H} \]  \hspace{1cm} (D19)

At the interface \( P_v - P_s \) can be related to \( T_i - T_s \) by the Clausius-Clapeyron equation

\[ P_v - P_s = \frac{J \ h_f g}{V_f g \ T_s} (T_i - T_s) \]  \hspace{1cm} (D20)

Dividing expression (D19) by \( \frac{2 \Delta}{P_0} \), substituting (D14) for \( \Delta \omega_g \), and noting that

\[ \frac{P_v - P_s}{2 \Delta P_0} = \frac{P_v - P_s}{P_v^0 - P_s} = \frac{T_i - T_s}{T_s \Delta P_0 - T_s} \]  \hspace{1cm} (D21)

expression (D19) can be rewritten as

\[ \frac{T_i - T_s}{T_s \Delta P_0 - T_s} = 1 + \frac{P_0 \ P_v^0}{2 \ \delta \omega_g \ h_f g} \int_{0}^{\ini} k^{\theta} I \ 2\pi r \ dr + \frac{3 \ \rho_0 \ P_v^0 \ z}{2 \ \delta \ H} \]  \hspace{1cm} (D22)

This expression for \( T_i \) is then the boundary condition at \( z = 0 \).

Upon non-dimensionalizing the preceding equations using

\[ T = \frac{T - T_s}{T_s \Delta P_0 - T_s} \]  \hspace{1cm} (D23)
\[ \delta' = \frac{\delta}{r_0} \quad r' = \frac{r}{r_0} \quad z' = \frac{z}{r_0} \quad (D24) \]

\[ \tau = \frac{t \alpha}{r_0^2} \quad (D25) \]

the governing equations, boundary conditions and initial conditions (11)-(15) are obtained.
APPENDIX E

COMPUTER PROGRAM LISTING
THIS IS A COMPUTER PROGRAM WHICH SOLVES A TWO DIMENSIONAL TRANSIENT CONDUCTION EQUATION BY THE IMPLICIT FINITE DIFFERENCE SCHEME COUPLED WITH A SECOND ORDER DIFFERENTIAL EQUATION OF MOTION SOLVED BY A SECOND ORDER RUNGE-KUTTA TECHNIQUE

IMPLICIT REAL*8 (A-H,C-Z)
REAL*8 LAM22, LAM33, LAM44, LAM1, LAM2, LAM5, LAM6, IFLUX, MU, MTR, KON
DIMENSION A(91,91), T(11,11), G(11), B(1900), BDUM(1900), R(91),
1 HR(3), RIM(3), WG(3), RCS(3), PH(3)

MMX=11
M1=MMX-1
M2=MMX-2
KK=6
MUD=M1
MLD=M1
DTAU=.008
RHO=.5D-4
MU=1.08
RL=46.8
PVS=20.
SIG=1.3D-3
THET=7.*3.14/180.
ALPH=3.5D-3
DELT=13.
KON=.111
HFG=473.
HR(1)=12.5D-4
HR(2)=3.7D-4
HR(3)=1.D-4
RIM(1)=12.5D-4
RIM(2)=6.3D-4
RIM(3)=.5D-4
WG(1)=16.3D-17
WG(2)=19.D-17
WG(3)=10.D-17
ROS(1)=1.
ROS(2)=1.
ROS(3) = .8
PH(1) = 5.*3.14/180.
PH(2) = 30.*3.14/180.
PH(3) = 70.*3.14/180.
DO 130 IJN=1,1
HTR=HR(IJN)
MTR=RIM(IJN)
WGO=WG(IJN)
PHI=PH(IJN)
ANG2=PHI/2.
ANG1=THET-PHI/2.
RO=RHO*DCOS(ANG1)
Q=(8.*MU/(RO**2.*RL))*(144./3600.)
V=(3.*PVS/(RL*MTR*HTR))*(12.**4.)
V=V*32.
BB=2.*SIG*D*SIN(ANG2)/DCOS(ANG2)
BB=BB/(RL*MTR*RHO**2.*DCOS(ANG1))
BB=BB*32.*1728.
B1=V-BB
A2=(Q*RO**2./*ALPH)*3600./144.
B2=(B1*RO**4./*ALPH**2.)
B2=B2*3600.**2./(12.**4.)
CC=PVS/(RL*MTR*WGO)
CC=CC*1728.*32.
C2=(C2/(HFG*ALPH**3.))*3600.**2./12.**6.
C2=-C2
WW=2.*3.14*DELT*RO**3.*KON/(HFG*ALPH)
WW=WW/1728.
W1=RHO*PVS/(2.*SIG*WGO)
W1=W1*12.
W3=WW*W1
W2=(1.5.)*(RO*RHO*PVS/(SIG*HTR))
W2=W2*12.
YY=0.
DZ=1./(MMX-1.)
DR=DZ
LAM1=DTAU/DZ**2.
LAM2=LAM1
MAX=(MMX-1)*(MMX-2)+1
TU=0
LAM22=LAM1*(1.+1./(2.*M2))
LIM=MAX-M1
LL=MAX-M2
MC=1+MUD+MLD
MA=MAX*MC-((MC-MLD-1)*(MC-MLD)/2)
LAM33=1.+2.*LAM1+LAM2)
LAM44=1.+2.*LAM2+4.*LAM1
C DEFINE ELEMENTS OF MATRIX A
C SET A COEFF=0
   DO 2 I=1,MAX
   DO 2 J=1,MAX
2   A(I,J)=0
C SET NON ZERO ELEMENTS IN FIRST COLUMN EXCEPT ROW 1
   DO 4 I=2,MMX
4   A(I,1)=-LAM2
C SET ALL LAMS AT UPPER
   DO 6 I=2,LIM
5   II=M1+I
6   A(I,II)=-LAM2
C SET ALL LAMS AT LOWER
   DO 8 I=2,LIM
7   II=I+M1
8   A(II,I)=-LAM2
C SET ALL DIAG ELEMENTS = LAM33
   N=MMX
   M=3
   DO 15 J=1,M2
   DO 14 I=M,N
14   A(I,I)=LAM33
15   N=N+M1
C SET ALL DIAG ELEMENTS = LAM44
  DO 17 I=2,LL,M1
  17 A(I,I)=LAM44
C SET ELEMENTS WITH COEF=LAM11
    N=MMX
    NN=1
    M=3
    DO 19 J=1,M2
    DO 18 I=M,N,1
  18 A(I,I-1)=-LAM1*(1.-1./(2.*N-1.))
    M=M+M1
    N=N+M1
  19 NN=NN+M1
C SET ELEMENTS WITH COEFF=LAM22
    N=M1
    M=3
    NN=1
    DO 21 J=1,M2
    DO 20 I=M,N
  20 A(I,I+1)=-LAM1*(1.+1./(2.*N-1.))
    M=M+M1
    N=N+M1
  21 NN=NN+M1
C SET COEF= 4*LAM4
  DO 25 I=2,LL,M1
  25 A(I,I+1)=-4.*LAM1
  DO 110 IIN=1,1
    V2=4.
C SET UPPER ROW
    SUM=0
    DO 30 I=1,M2
  30 SUM=SUM+I
    SUM=SUM+1.*8.*((M2+3.)/4.)/2.
    LAM5=(1.*((W3*DTAU*DR**2./DZ)*SUM)
    LAM6=W3*DTAU*DR**2./DZ
    A(1,1)=LAM5
A(1,2)=-LAM6/8.
DO 32 I=3,MMX
32 A(I,1)=-LAM6*(I-2)
C CONVERT A TO GELB MATRIX B
L=0
DO 40 I=1,MAX
IRH=MUD+I
IF(IRH.GT.MAX) IRH=MAX
ILH=I-MLD
IF(ILH.LT.1) ILH=1
DO 40 J=ILH,IRH
L=L+1
40 B(L)=A(I,J)
DO 110 IN=1,11,5
V2=4.*IN
TW=1.3
DO 1 J=2,M1
DO 1 I=1,M1
1 T(I,J)=.5
WRITE(KK,60) TW,YY,MA
60 FORMAT(1H1,3X,'TW=',F4.1,2X,'YY=',F6.3,2X,'MA=',I6)
WRITE(KK,62)
62 FORMAT(2X,'TIME',4X,'TV',3X,'DIST',4X,'FLUX',
12X,'DWG0/WGO',2X,'PL-FS',20X,'DT/DZ')
IFLUX=0
Z2OLD=0
Z1OLD=V2
DO 100 L1=1,50
C SET DUMMY MATRIX
DO 50 I=1,MA
50 BDUM(I)=B(I)
C SET MATRIX OF CONSTANTS AND BEGIN RUNGE-KUTTA
F1OLD=-A2*Z1OLD-B2*Z2OLD+IFLUX*C2
AZ1=Z1OLD+DTAU*F1OLD
F2OLD=Z1OLD
AF2N=A*1
$Z2NEW = Z2OLD + (DTAU/2) * (F2OLD + AF2N)$
$R(1) = ROS(1) + NAM6 * TW^2 (M2 + 3) / 4 / 2.$
$R(1) = R(1) + W3 * IFLUX + W2 * Z2NEW$

A = M1
M = 2
NN = 1
DO 34 J = 2, M2
DO 33 I = M, N
II = I - NN
33 R(I) = T(II, J)
   NN = NN + M1
   M = M + M1
34 N = N + M1
   NN = M2
   DO 36 I = MX, LIM, M1
   II = I - NN
   R(I) = T(M1, II) + LAM22 * TW
36 NN = NN + M2
C SET FINAL ROW
DO 38 J = 1, M2
   JJ = LIM + J
38 R(JJ) = T(J, M1) + LAM2 * TU
C SET LAST NODE
R(MAX) = T(M1, M1) + LAM2 * TU + LAM22 * TW
   CALL DGELB(R, BDUM, MAX, 1, MUD, MLD, 1.0, 16, 1, IER)
C DGELB IS A DOUBLE PRECISION IBM SCIENTIFIC SUBROUTINE PACKAGE VERSION
C III WHICH SOLVES A BANDED MATRIX BY GAUSSIAN ELIMINATION
C DOCUMENTED IN IBM MANUAL H 20-0205
C RENAME TEMPS
TV = R(1)
N = 0
DO 44 J = 2, M1
C 42 I = 1, M1
   II = I + N + 1
42 T(I, J) = R(II)
44 N = N + M1
C CALCULATE HEAT FLUX

DO 46 I=1,M1
46 G(I)=(T(I,2)-TV)/DZ
G(MMX)=(TW-TV)/DZ
FLUX=0
DO 48 I=2,M1
48 FLUX=FLUX+G(I)*((I-1)*DR**2.0)*DZ
IFLUX=IFLUX+FLUX*DTAU
AZ2=Z2OLD+DTAU*F2OLD
AF1N=-A2*AZ1-B2*AZ2+IFLUX*C2
Z1NEW=Z1OLD+(DTAU/2.0)*(F1OLD+AF1N)
TAU=L1*DTAU
FDWG=WW*IFLUX/WGO
DPL=B2*Z2NEW-C2*IFLUX
WRITE(KK,101) TAU,TV,Z2NEW,FLUX,FDWG,DPL,(G(I),I=1,11)
Z1OLD=Z1NEW
100 Z2OLD=Z2NEW
101 FORMAT(F7.4,F5.2,F8.4,F9.4,F6.2,F9.2,3X,11F5.1)
WRITE(KK,150) V2
150 FORMAT(//,F5.1)
110 WRITE(KK,7C) IER
70 FORMAT(//,3X,'IER=',I3,//)
WRITE(KK,123)
123 FORMAT(7X,'A',11X,'R',10X,'BB',6X,'A2',4X,'B2',
15X,'C2',8X,'W2',7X,'W3',//)
130 WRITE(KK,122) Q,V,BB,A2,B2,C2,W2,W3
CALL EXIT
END
APPENDIX F

DERIVATION OF CUMULATION SITE DISTRIBUTION

As stated in Chapter IV knowledge of the joint probability density function for \( R \) and \( \phi \) and a definition of \( \varphi \) in terms of cavity geometry is sufficient information to predict \( \eta_A \) vs \( \varphi \). In the surface model of Chapter IV the cavities were assumed to have an equivalent cone angle \( \bar{\Phi} \) and a distribution of mouth radii following the power law \( \eta_A = \left( \frac{R_s}{R} \right)^m \). This assumption implies that all cavities are active if \( \theta > \bar{\Phi} \). The following example will allow for the possibility of a portion of the cavities being deactivated when \( \theta < \phi \) for individual cavities. As before it will be assumed that the distribution of mouth radii is \( \eta_A = \left( \frac{R_s}{R} \right)^m \); however, the cone angles are uniformly distributed between 0 and some maximum value \( \phi_M \). The following expressions sketched in Figures 41a,b,c essentially define the problem

\[
\frac{\eta}{A} = \left( \frac{R_s}{R} \right)^m
\]

\[
\rho(\phi) = \frac{1}{\phi_M}
\]

\[
\frac{\rho}{R} = f(\theta, \phi)
\]

where \( \rho(\phi) \) is the probability that \( \phi \) is between \( \phi \) and \( \phi + d\phi \). To simplify the problem expression (F3) is crudely approximated by a series of straight parallel lines shown in Figure 41a and given as
\[
\frac{\rho}{R} = \beta (\theta - \phi)
\]  \hspace{1cm} (F4)

where \(\beta\) is an average slope approximately equal to \(\frac{1}{2} \theta\). A single value of \(\varphi\) (for a given \(\theta\)) can be obtained from an entire set of cavities which have the proper combination of mouth radii \(R\) and cone angles \(\phi\). If \(\theta - \frac{1}{\beta} > 0\) for a given \(\theta\) and \(\varphi\), Figure 41d shows the locus of mouth radii \(R\) and cone angles \(\phi\) in the \(R-\phi\) plane of the corresponding cavities. This curve essentially is a replot of Expression (F4) in which \(\rho\) and \(\theta\) are constant and \(R\) and \(\phi\) are variables.

Portion #1 is the region in which \(R = \rho\) and #2 in which \(R = \frac{\rho}{\beta (\theta - \phi)}\). Note the upper cut off for cavities is at \(R = R_s\). As shown in Figure 41b the maximum value of \(R\) is \(R_s\) at which point \(\varphi_a = 1\). For \(\phi\) values greater than \(\theta\) no active cavities exist since they are completely deactivated.

The shaded region bounded by the curves #1, #2, \(R = R_s\) and \(\phi = \phi_m\) represents all active cavities with effective radii between \(\rho\) and \(\rho = \rho_s = R_s\). When \(\theta - \frac{1}{\beta} < 0\) Figure 41e shows that \(R = \frac{\rho}{\beta (\theta - \phi)}\), \(R = R_s\), and \(\phi = \phi_m\) bound the entire region since \(R \neq \rho\) for any cavities.

Referring to Figures 41d and 41e the following cases must be considered in this analysis.

case 1 \(\theta - \frac{1}{\beta} > 0\)

1a \(\phi_m \geq \theta - \frac{\rho}{\beta} R_s\)

1b \(\theta - \frac{1}{\beta} < \phi_m < \theta - \frac{\rho}{\beta} R_s\)

1c \(0 \leq \phi_m \leq \theta - \frac{1}{\beta}\)
case 2 \[ \theta - \frac{1}{\theta} < 0 \]

2a \[ \phi_m \geq \theta - \frac{2}{\theta R_s} \]

2b \[ 0 \leq \phi_m \leq \theta - \frac{2}{\theta R_s} \]

Case 1a is for cavities with a relatively wide band distribution of cavities in which the maximum cone angle \( \phi_m \) is greater than the contact angle. Case 1c is the opposite extreme for a very narrow distribution of cone angles in which event it is obvious that \( n_A v_s \rho \) is equivalent to \( n_A v_s R \). Case 1b is somewhere between 1a and 1c. For all cases an analytical expression will be derived for \( n_A v_s \rho \).

Before proceeding expression (F1) is converted into a probability density \( p(R) \) by differentiation and normalization by some total number of \( n^2 A \), \( n\rho_A \), where \( n\rho_A = \left( \frac{R_s}{R_k} \right)^m \)

\[ p(R) = -\frac{m R_s m R^{-m-1}}{n\rho_A}, \quad R_k < R < R_s \] \hspace{1cm} (F5)

where \( R_k \) is some arbitrary lower cut off. Expression (F5) is negative since the cumulative density is monotonically decreasing with \( R \). It will be treated, however, in the usual way as if it were positive.

The joint probability density \( p(\phi, R) \) is given as

\[ p(\phi, R) = p(\phi) p(R) \] \hspace{1cm} (F6)

where \( p(\phi) \) and \( p(R) \) are assumed independent. Expression (F6) can be considered as representing a surface above the \( \phi - R \) plane of Figures 41d,e. The volume bounded by this surface
and the proper shaded region represents the fraction of the total number of cavities with effective radius $\rho$ between $\rho$ and $\rho_s$. To convert this fraction to an actual number it is necessary to multiply by the total number of cavities/in$^2$, $n_A$. Stated mathematically the general expression for $n_A$ vs $\rho$ is

$$n_A - 1 = \iint_{\text{SHADED AREA}} n_A p(\phi) p(R) dR d\phi$$

(F7)

The number one appears on the left side of Expression (F7) as a consequence of the fact that at $R = R_s$ the value of $n_A$ is $1$/in$^2$ (See Figure 41b)

For case 1a in which $\theta - \frac{l}{\phi} > 0$, $\phi_M \geq \theta - \frac{\rho}{\phi} R_s$ expression (F7) becomes

$$n_A - 1 = -\frac{m R_s}{\phi_M} \left( R_s^m \left( \frac{\theta - \frac{l}{\phi}}{R} \right) - 1 \right) - \frac{m R_s}{\phi_M} \left( \frac{\theta - \frac{\rho R_s}{\phi}}{R} \right) - \frac{m R_s}{\phi_M} \left( \frac{\theta - \frac{\rho}{\phi} R_s}{R} \right)$$

performing this calculation yields

$$\frac{n}{A} = \frac{1}{\phi_M} \left( \theta - \frac{l}{\phi} \right) \left( \frac{R_s^m}{\phi} - 1 \right) + \frac{\left( \frac{\rho R_s}{\phi} \right)^m}{\phi_M} \left[ \frac{m+1}{\phi} \left( \frac{R_s}{\phi} \right)^m \right] - \frac{1}{\phi_M} \left( \frac{l}{\phi} - \frac{\rho}{\phi R_s} \right) + 1$$

where $(\theta - \phi_M) \beta R_s \leq \rho \leq R_s$

In a similar manner the other cases can be derived by integrating over the proper regions yielding the following: case 1b

$$\frac{n}{A} = \frac{1}{\phi_M} \left( \theta - \frac{l}{\phi} \right) \left( \frac{R_s^m}{\phi} - 1 \right) + \frac{\left( \frac{\rho R_s}{\phi} \right)^m}{\phi_M} \left[ \frac{m+1}{\phi} \left( \frac{R_s}{\phi} \right)^m \right] - \frac{1}{\phi_M} \left( \phi_M - \theta + \frac{l}{\phi} \right) + 1$$
where \( \rho \leq (\theta - \phi_m) \beta R_s \)

**case 1c** \( \theta - \frac{1}{\beta} > 0, 0 < \phi_m < \theta - \frac{1}{\beta} \)

\[
\eta_A = \left( \frac{R_s}{\rho} \right)^m
\]

where \( \rho \leq R_s \)

**case 2a** \( \theta - \frac{1}{\beta} < 0, \phi_m \geq \theta - \frac{\rho}{\beta R_s} \)

\[
\eta_A = \frac{(\theta R_s/\rho)^m}{\phi_m m!} \left[ \theta^{m+1} \left( \frac{\rho}{\rho R_s} \right)^m \right] - \frac{1}{\phi_m} \left( \theta - \frac{\rho}{\rho R_s} \right) + 1
\]

where \( (\theta - \phi_m) R_s \leq \rho \leq \theta R_s \)

**case 2b** \( \theta - \frac{1}{\beta} < 0, 0 \leq \phi_m \leq \theta - \frac{\rho}{\beta R_s} \)

\[
\eta_A = \frac{(\theta R_s/\rho)^m}{\phi_m m!} \left[ \theta^{m+1} (\theta - \phi_m)^m \right]
\]

where \( (\theta - \phi_m) \beta R_s \geq \rho \)

Like the surface model of Chapter IV three parameters are needed in order to describe \( \eta_A \nu S \rho \). As before \( m \) and \( R_s \) are required but instead of \( \Phi \), the parameter \( \phi_m \) now appears. It is seen that assuming a uniform distribution for \( \phi \) results in a more complex form of \( \eta_A \nu S \rho \). Essentially five different cases must be considered to describe all of the possibilities. Various other distributions could be assumed for \( \phi \) and \( R \). In fact it is also possible to derive an expression for \( \eta_A \nu S \rho \) if a distribution for \( \theta \), i.e. \( \rho(\theta) \) is also assumed. In view of the simplicity of the model of Chapter IV and its
consistency with experimental results, it will be adopted as a working surface model. If future inadequacies result in that model more attention should be focussed on developing surface models by the procedures used in this appendix.
Figure 1. Flow Chart Tracing Alternative Routes to the Boiling Curve
Figure 2. Sketch of Nucleation Conditions from References [7,19]
KEY

i = eqn. (4)

j = eqn. (7) M = 1

k = eqn. (7) M = 3/2

δ = 34 x 10^-5 ft

for all fluids

![Graphs showing nucleation data for different fluids.](image)

a. WATER

b. ETHANOL

c. n-HEPTANE

d. BENZENE

Figure 3. Nucleation Data for Organics and Water from Reference [21]
a. IDEALIZED CONICAL CAVITY

b. NATURAL CAVITY WITH ROUGHNESS

Figure 4. Sketch of $\frac{1}{\rho}$ vs Volume Curves for Idealized and Rough Cavities
Figure 5. Sketch of Idealized Vapor Trapping Process
Figure 6. Graph of Expression (10) Relating Effective Radius to Cavity Geometry and Contact Angle

\( \rho_r = f(e, \phi) \), Eqn. (10)
a. CONICAL CAVITY

b. SIMPLIFIED CYLINDRICAL CAVITY

Figure 7. Sketch Showing Solution Domain for Dynamic Effects
Figure 8. Effect of Wall Superheat on Penetration Depth for Water
Figure 9. Effect of Initial Velocity on Penetration Depth for Water
Figure 10. Results of Gas Diffusion and Boiling Experiments for Individual Cavities
Figure 10. Results of Gas Diffusion and Boiling Experiments for Individual Cavities

**Note:** Each point is an average of several runs.

**KEY**
- ▼ GAS DIFFUSION (WATER)
- ○ WATER
- □ METHANOL
- △ BENZENE
- ○ FREON-113

**Chart Details:**
- **Y-axis:** $\rho \times 10^4$ (in)
- **X-axis:** ACTIVE SITE NUMBER
- **Legend:** BOILING DATA
TABLE 2
COMPARISON OF PHOTOGRAPHICALLY DETERMINED CAVITY RADIUS WITH EFFECTIVE RADIUS FROM BOILING AND GAS DIFFUSION EXPERIMENTS FOR WATER

<table>
<thead>
<tr>
<th>CAVITY #</th>
<th>EFFECTIVE RADIUS $\times 10^4$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BOILING</td>
</tr>
<tr>
<td>#1</td>
<td>1.05</td>
</tr>
<tr>
<td>#3</td>
<td>.98</td>
</tr>
<tr>
<td>#5</td>
<td>1.15</td>
</tr>
<tr>
<td>#7</td>
<td>1.12</td>
</tr>
<tr>
<td>#9</td>
<td>1.45</td>
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<tr>
<td>#10</td>
<td>1.65</td>
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KEY
- 14.7
- 6.26
- 4.47
- 1.87
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Run number refers to the order in which runs were taken.

KEY
○ METHANOL
□ BENZENE

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PLUNGE FOR EXPANDING AND CONTRACTING THE BUBBLE

FLEXIBLE TUBE

SMALL HOLE DRILLED THROUGH SURFACE

COPPER SURFACE

MICROSCOPE AND CAMERA

VIEWING FROM ABOUT 1° BELOW THE HORIZONTAL

GLASS CUBE

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BIOGRAPHICAL NOTE

James J. Lorenz was born January 22, 1944, in Philadelphia, Pennsylvania, and was raised in the suburbs where he attended Haverford High School in Havertown. After graduation in June of 1962, he entered the Mechanical Engineering Department of Drexel Institute of Technology. While at Drexel he co-oped at the Chrysler Corporation Space Division in New Orleans, Louisiana.

Upon receiving his B.S.M.E. from Drexel in June 1967, he was admitted to the Mechanical Engineering Graduate School at M.I.T. where he received his S.M. in Mechanical Engineering in February 1969.* From June to September of 1969 he worked on the Summer Staff at the M.I.T. Instrumentation Laboratory in the Naval Mechanical Design Group.

In February 1969, he entered the Ph.D. program in Mechanical Engineering at M.I.T. working on the effects of surface conditions on boiling characteristics under the supervision of Professor W.M. Rohsenow.

August 6, 1966, he was married to the former Virginia Elaine Buck. The couple has a three year old daughter, Nancy.

He has accepted a position at Argonne National Laboratory in a Thermal and Hydraulics Group doing research on Liquid Metal Fast Breeder Reactors.