

DESIGN OF A PASSIVE RETARDING SYSTEM  
FOR AN ARTIFICIAL KNEE JOINT

by

JOHN A. PURBRICK

Submitted in partial fulfillment  
of the requirements for the  
Degree of Bachelor of Science  
at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1973

Signature of Author  
Department of Mechanical Engineering, May 16, 1973

Certified by . . . . . Thesis Supervisor

Accepted by . . . . .  
Chairman, Departmental Committee on Theses



ABSTRACT

When a person loses his leg above the knee, he requires a prosthesis which includes a hinge to replace the knee joint. Also included in the artificial limb should be a mechanism by means of which a braking moment can be applied to the hinge. This moment varies greatly during normal walking, and cannot be applied by a device dependent only upon the position of the limb.

It is now possible, using electromyographic (EMG) signals from the patient's remaining muscles, to determine how much force he would have put on the knee. If these signals are amplified, they can be used to control a brake.

A hydraulic braking system was devised for use in a prosthesis, employing a single-stage pressure control servo valve, driven by solenoids through a linkage designed to equalize the force-versus-stroke curve of the solenoids.

ACKNOWLEDGEMENTS

The author would like to express his deepest appreciation to his thesis advisor, Woodie Flowers, for suggesting the topic, for making his own doctoral thesis available to the author, and for his kindness and patience during the period of this project.

Thanks also to John Kassakian of the Power Systems Lab for his advice on electric motors and solenoids.

CONTENTS

ABSTRACT.....2  
ACKNOWLEDGEMENTS.....3  
CONTENTS.....4  
ILLUSTRATIONS.....5  
INTRODUCTION.....6  
PROPOSED CHANGES IN THE FLOWERS PROSTHESIS,  
    AND FEATURES RETAINED.....10  
DESIGN OF A PASSIVE RETARDING SYSTEM.....16  
PRESSURE CONTROL SERVO VALVES.....20  
APPENDIX A (SOLENOID FORCE EQUALIZER).....34  
APPENDIX B (BERNOULLI FORCE CALCULATIONS).....36  
REFERENCES.....38

LIST OF ILLUSTRATIONS

Figure no.	Title	Page
1.....	Graph of moment applied to knee joint as function of position, in ideal case.....	8
2.....	Schematic of the Flowers simulator and of the proposed prosthesis.....	13
3.....	Block diagram of servo-indexed valve with velocity feedback.....	14
4.....	Two stage pressure control servovalve.....	22
5.....	Modified valve.....	22
6.....	Valve of type proposed.....	23
7.....	Same, with pressure differential reversed.....	23
8.....	Force-equalizing linkage.....	26
9.....	Bernoulli-force-compensated valve; negative-force port....	32
10.....	End view of proposed valve; also two longitudinal sections.....	33

## INTRODUCTION

This thesis represents an attempt to continue some of the ideas suggested by the work of Woodie Flowers, as described in his doctoral thesis presented at MIT in 1972. Dr. Flowers designed a "Man-Interactive Simulator for Above-Knee Prosthetics Studies". This device consisted of a steel framework, with a hinge inside and a simulated foot, and had a leather socket at the other end into which an experimenter could put his knee. With the experimenter's knee, strapped in a fully flexed position, inserted in the prosthesis, he could hobble around using the device as a substitute knee joint. A hydraulic cylinder attached to the prosthesis could apply a chosen torque to the hinge, which torque could be enough to move the experimenter's body weight, or enough merely to put a retarding torque on the joint when it was moved by external force--as, for example, would be required when the user was walking with a normal gait.

The hydraulic cylinder was powered by a pump located externally to the prosthesis, through a servo valve attached to the limb. This device, then, was purely a captive simulator, for it was attached to a large mass of equipment by an umbilical line.

A knee joint, however, is not like, for example, the joints of the human arm. When used in a common walking gait, the muscles attached to it do not supply a net positive energy. If, therefore, an energy storage device could be attached to a prosthesis, the main requirement would become one of procuring

a mechanism to apply a braking moment of the right magnitude and at the right time. Unfortunately, the braking moment varies considerably over the time period of one step, even to the point of reversing direction (see figure 1). Devices have been developed which have a prosthetic knee joint to which a retarding moment is applied, and some are available which supply a retarding moment which varies during the time period of one step. So far however, they have all been controlled by the position of the knee--ie, the torque applied is a function of the angle assumed by the prosthesis, and is not otherwise under the patient's control. The change in applied torque is achieved by purely mechanical means, or by use of pneumatic or hydraulic devices in which a piston moving in a cylinder uncovers multiple orifices. Some incorporate energy storage mechanisms, but the closeness of the braking moment supplied by even the best of them to that observed in a real knee is not very good.

An alternative strategy, which has been used successfully in the "Boston Arm" developed by Robert W. Mann at MIT, is to use the electrical signals (electromyographic or "EMG") generated in muscles to control a prosthesis. When a limb is amputated, the muscles in the stump are tied off to the bone. The patient retains control of these muscles, and if electrodes can be attached to them (which can be done without piercing the skin surface) it is possible to determine how much force the patient would have applied to the missing joint, and to

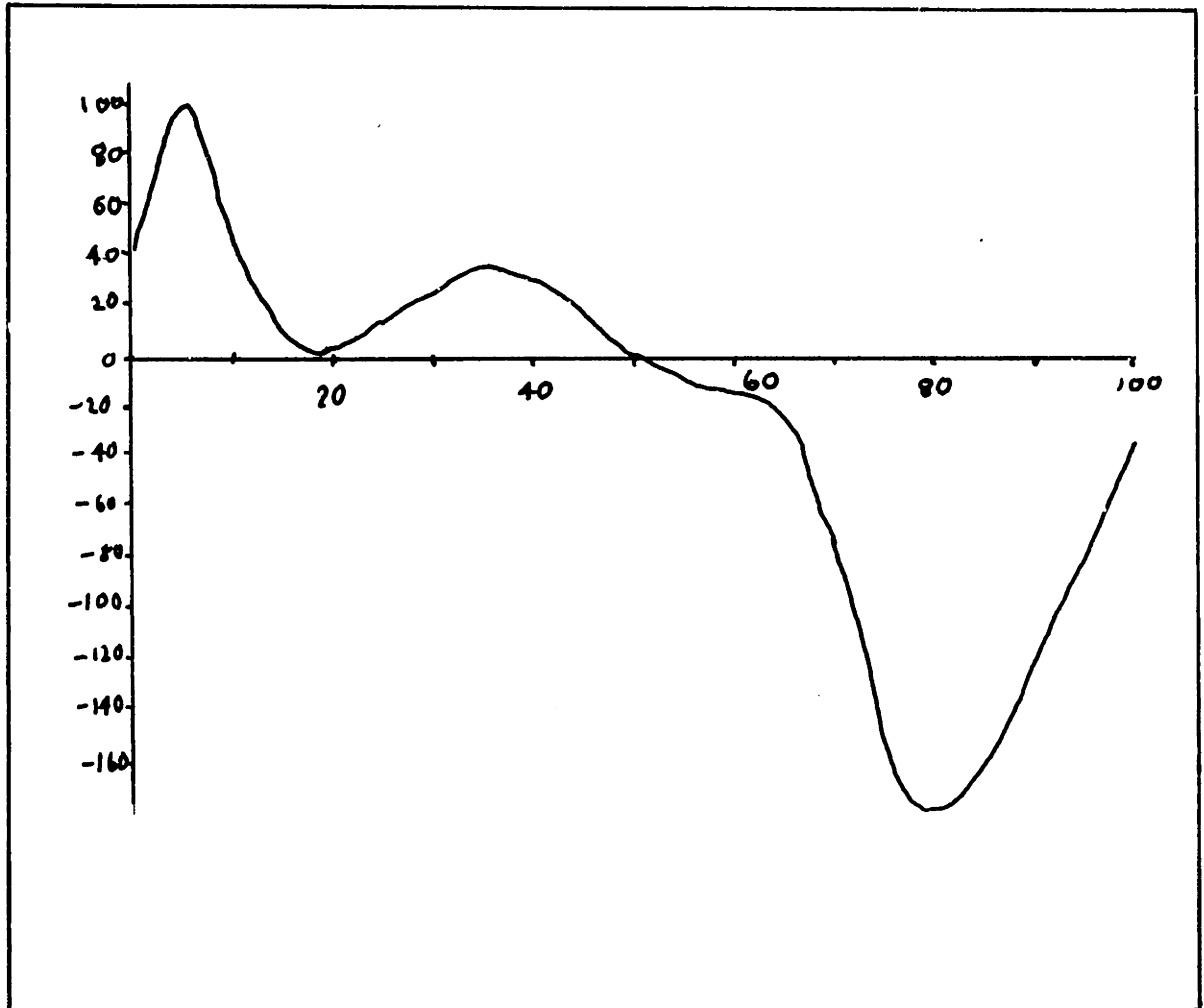


Figure 1. Graph of moment applied to knee joint as function of position, in ideal case.

Source: A. B. Wilson, "Recent Advances in Above-Knee Prosthetics", Artificial Limbs, Vol 12, no. 2, pp 1-27, 1968. Reproduced by Dr. Flowers.



operate the prosthesis accordingly. At this point, it becomes clear what is required for an EMG-controlled artificial knee joint. Most important is a braking system with variable output in either direction, having an electrical input. Then there must be an electronic circuit to pick up and amplify the EMG signals from the patient's muscles and to supply the input signal to the brake, in whatever form the signal is required. An energy storage system might be incorporated with the brake or might be separate.

What follows will be a design for the mechanical parts of the brake.

CHANGES PROPOSED IN THE FLOWERS PROSTHESIS, AND FEATURES  
RETAINED

As applied to the prosthesis simulator, Dr. Flowers' device was fairly simple. (The instrumentation backing it up was *not* simple!) A constant-pressure (1000psi) hydraulic supply fed the servo valve, and the setting of the valve was controlled by a signal derived from the EMG pickup and from the output of a load cell located on the shaft of the piston which moved the prosthesis. This arrangement was used as a servomechanism. The patient, through the EMG system, would demand that a particular force be applied to the joint; the valve would then open or close until the load cell indicated that the desired force was being applied. The fact that the controlling valve happened to be a servo valve is of no importance.

The first and most obvious step in turning Dr. Flowers' invention into a real prosthesis, capable of making life easier for a victim of accident or disease, is to do away with the umbilical line. The major content of this line is the hydraulic supply and return. Since, as explained above, there is no net power input, there should be no need for such hydraulic connections, even if they were possible. The hydraulic system, if one is to be used at all, must be entirely passive. In fact, a hydraulic system is attractive for this purpose on several accounts.

A pressure-control device using hydraulic technology would be a good choice because such systems are clean, little subject to wear, dirt or moisture, can be made so as to be unaffected by changes in temperature, have a high power to weight ratio, and are available, if commercial components are to be used, in ratings close to those required for this project. (Such ratings would be power, rate of flow, pressure differential, and speed of response.) In particular, valves are made for aerospace applications which are designed with the same objects in view as are held by the designer of a prosthesis--namely, they must be light and reliable. Unfortunately, the aerospace designer is commonly under little pressure to reduce costs, so these components are very expensive; a servo valve of the type used by Dr. Flowers costs several hundred dollars. In any event, the technology of electrically controlled hydraulic valves is highly developed. The servo valve used by Dr. Flowers would be well suited to the purpose of this project. It can be run in a purely passive mode, that is, so that it merely dissipates energy, supplying none from the hydraulic line. At no time, however, can one escape the fact that a servo valve requires a constant high-pressure hydraulic supply. A hydraulic pump, even one of only enough capacity to run the valve, would be far too heavy to use in a prosthesis. Servo valves, of the two stage type, must be ruled out.

Another feature of the Flowers prosthesis which it would be desirable to eliminate is the load cell. This component was a specially made model with a nonlinear characteristic, whose durability is open to question and which is unsuited to controlling a linear system.

It was, however, decided that most of the Flowers mechanism was a satisfactory design and should be retained. Specifically, Dr. Flowers set up a cylinder of bore 0.875ins. and stroke 2.875ins., the piston being attached to a steel cable, which, passing around a pulley, ensured a linear relationship between motion of the piston and motion of the joint. The hydraulic system was designed to operate at 1000psi maximum pressure. The maximum moment to be expected at the knee joint was 1000lb-ins. (400 lb-ins is the maximum to be expected in normal walking.) Calculations from the figures given by Dr. Flowers indicate that the pulley he used had a radius of approximately 1.8 inches (Dr. Flowers does not state the pulley radius in his thesis) and this apparatus can be retained, except that the pulley should be replaced with a double-ended one--that is, with a cylinder with a piston rod passing entirely through it. This arrangement ensures that the volume of the fluid system remains constant regardless of the position of the piston, rather than increasing as a greater length of the piston rod is forced into the cylinder.

Figures 2a and 2b show schematically the system which results from these changes. The problem is clearly one of

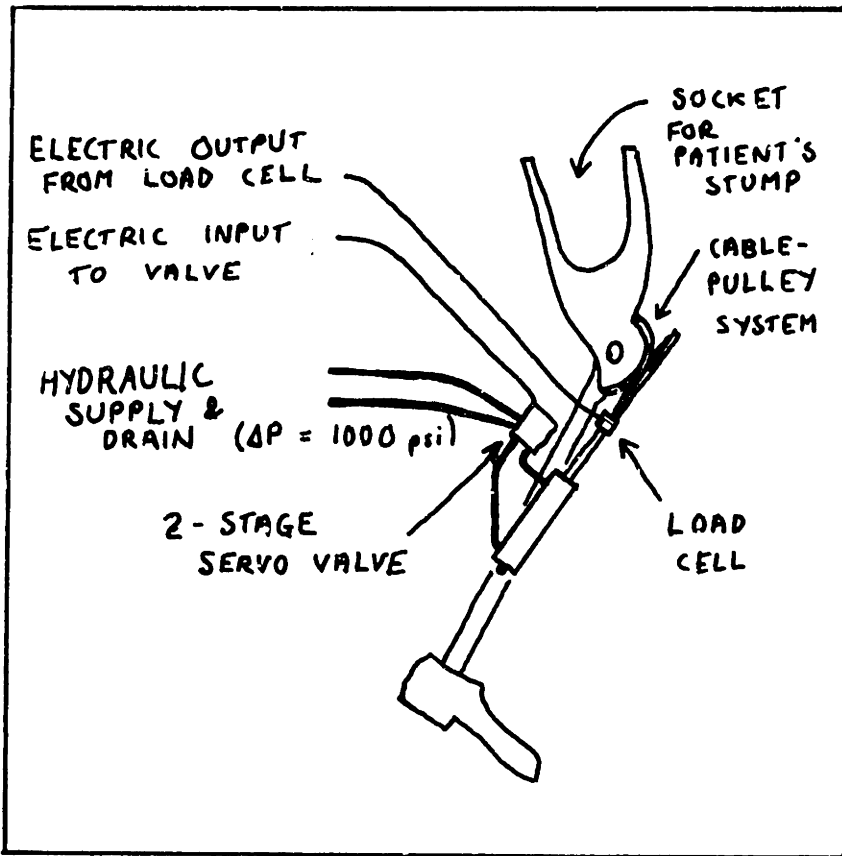


Figure 2a.  
Schematic of  
Dr. Flowers'  
simulator.

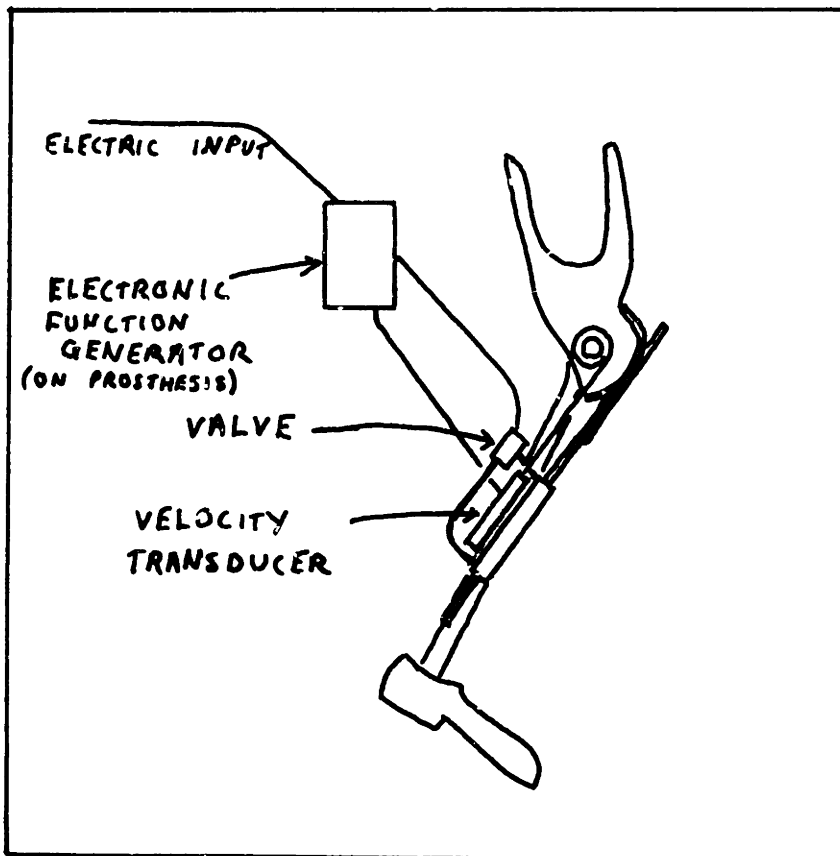


Figure 2b.  
Schematic of  
proposed  
prosthesis.

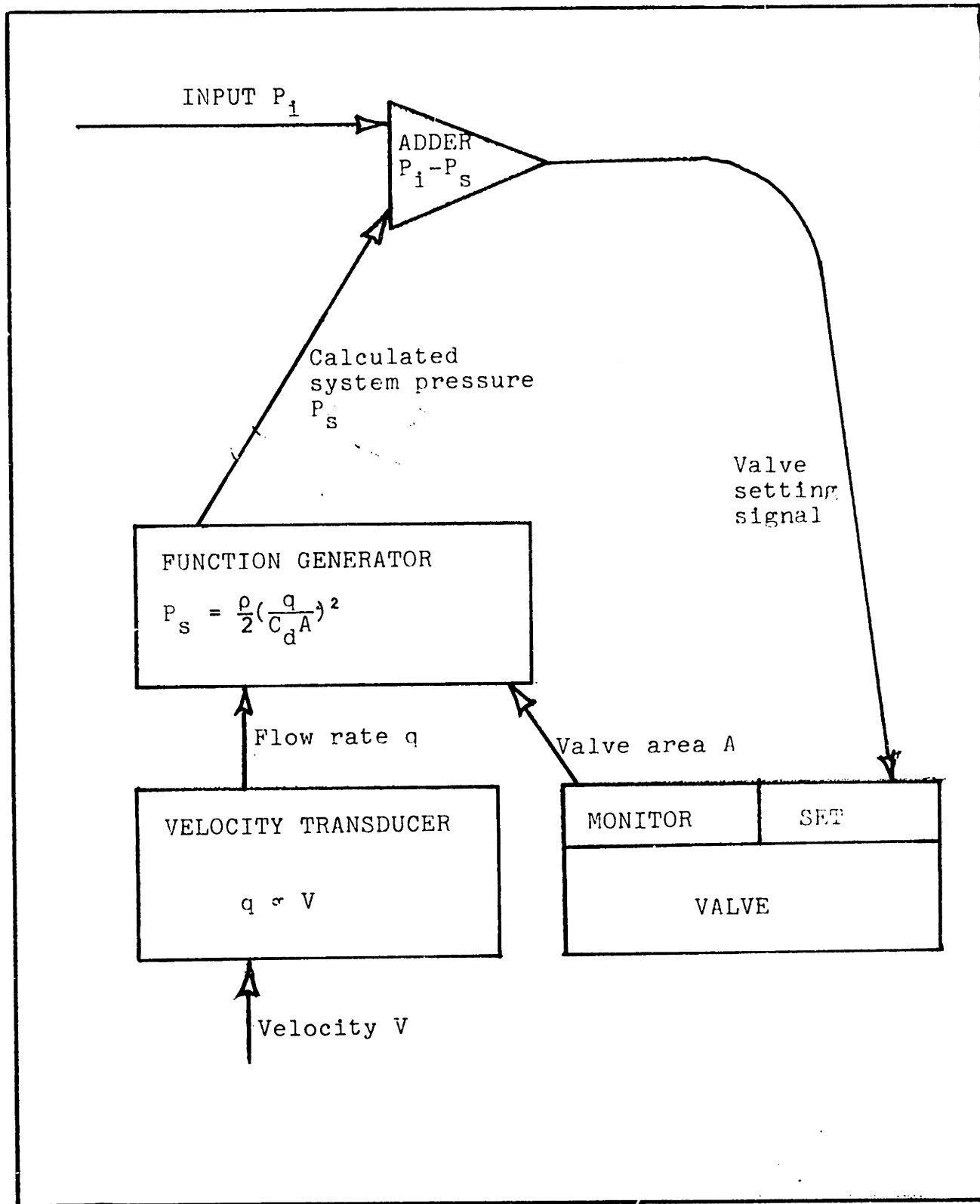


Figure 3. Block diagram of servo-indexed valve control with velocity feedback.

designing the feedback system and the control valve, as indicated in fig. 2b.

It was Dr. Flowers' feeling that the fluid lines should be kept as short as possible in order to minimize momentum and viscous losses. Momentum losses, however, are insignificant compared with the momentum of the prosthesis itself, and the viscous losses can also be minimized considerably. Viscous resistance can be predicted by the following formula:

$$Q = \frac{\pi D^4}{128\mu L} \Delta P$$

Where Q is fluid flow rate, D is the diameter of the tubing used,  $\mu$  is the viscosity of the fluid, L is the length of tubing, and  $\Delta P$  is the pressure drop along the tube.

For Dow Chemical Company's DC-200 Dimethyl Siloxane,  $\mu$  is 3.625 lb-sec/sq. in., which results, if used in Dr. Flowers' 0.15 in. inside-diameter tubing, in a fluid resistance of 0.292 lb-sec/in per linear inch. (A fluid resistance can be visualized as the pressure differential required to force one cubic inch per second of a fluid through one inch of a tube of given diameter.) This resistance is not unbearably high, but it could be reduced nearly eight times by increasing the tubing i-d to 0.25 inches. It seems that the small weight penalty caused by this would be worthwhile in terms of resistance saved. Changing to  $\frac{1}{4}$  NPT fittings on the cylinder would also be appropriate.

DESIGN OF A PASSIVE RETARDING SYSTEM

Since the load cell used by Dr. Flowers was to be avoided if possible, an attempt was made to find another parameter which could be used to provide a feedback. The most obvious alternative choice was to use a velocity transducer attached to the limb. Dr. Flowers, in fact, installed one of these on his simulator, although he used it only for the purpose of instrumentation, and not directly for control. If the pressure in the fluid system is proportional to the moment of torque applied at the joint (and it is the function of the cable and pulley to ensure this) and if rotational velocity is proportional to fluid flow rate, then a velocity transducer and a function relating fluid flow and pressure can supply a calculated figure for system pressure. It is not important that the relating function be linear--a microelectronic circuit can be designed to perform the calculation and to provide a suitable signal to the valve. The valve setting would be monitored and set by a subsidiary servomechanism, as the function relating pressure to flow rate would undoubtedly involve some parameter of the valve--probably minimum cross-sectional area of the fluid conduit. The function might be of the following type:

$$q = C_d A \frac{\sqrt{2\Delta P}}{\rho}$$

Where  $q$  is fluid flow rate,  $C_d$  is an orifice coefficient (taken as being 0.8),  $A$  is the area open to fluid flow,  $\Delta P$  is pressure drop across the device, and  $\rho$  is fluid



density.

This is, of course, the equation for fluid flow through a sharp-edged orifice. This was the first relationship examined with a view to control via velocity feedback. Various configurations were considered for implementing the valve.

At first the prime consideration was to minimize the fluid path length. The first design involved a specially-built piston-cylinder combination. Instead of fluid being forced out of the cylinder, through a valve and back into the other end of the cylinder, it would pass through an orifice in the piston itself. The piston might be made from two overlapping metal sectors, each covering slightly more than half of the cylinder's area. By rotating these relative to each other, almost half of the cylinder area could be uncovered. The piston rod would be in the form of two concentric shafts, an arrangement which would undoubtedly suffer from high resistance to changing the valve setting. Leakage between the two sections of the piston would be high, and the mechanism needed to drive the valve would have to be both strong and complex, for there would be up to 1000psi acting on the piston, and both this and the piston travel would have to be allowed for.

Another design involved surrounding the cylinder with a fluid duct. Flow in this would be controlled by a circular moving control element at one end. In such a device, however, it would be difficult to isolate the control element from the pressure drop across the piston. Such a force, if small, would

not necessarily prevent the valve from operating, since the control element would be moved into place by a position-controlled servo-motor, but the motor would have to be slower and/or heavier to overcome the extra force.

At this point, it was realized that the length of the fluid conduit was not of paramount importance. Several designs were then prepared for valves external to the cylinder, and, given the choice, it seemed preferable to use the viscous losses caused by fluid being forced through a long, constricted passage. The pressure drop is linear with flow rate, and furthermore, the equation can be expected to hold even at arbitrarily small openings, which is not the case with the sharp-edged orifice. If a servo-system is pressure- or force-controlled, this does not matter, but if one hopes to control the valve by means of some relation between  $P$ ,  $Q$  and some parameter of the valve setting, a mathematically predictable relation is vital.

Two designs came from this work. One involved a long, tapering needle which would be forced into a matching hole. Fluid would pass through the hole, and the passage through which it flowed would change in width as the needle's position in the hole was varied. The pressure of the fluid, however, would tend to force the needle into or out of the hole, depending on the direction of flow. To compensate for this, two needles would be required, working in opposite directions in different channels. A driving linkage of some kind would

be needed to force the two needles in opposite directions, and the requirements on this linkage would be considerable, for it would have to take the full pressure drop across the valve, multiplied by the cross-sectional area of the needles, in either direction. Worse, however, is the fact that as the needle advances, it must displace fluid from the conduit between itself and the wall of the hole. A standard question in fluid mechanics texts concerns how much force is needed to squeeze out all the fluid from between two plates, and this is a problem of the same order.

The other design considered involved a loosely-fitting plug which would be forced into a hole to some known depth. There would be a constricted fluid path where the liquid was forced to flow past the plug, and a relatively open area where the plug had not been inserted. There is a method of compensating this valve for the pressure difference without needing two of them working in opposite directions, by having a piston on the same shaft as the valve, exposed to the same pressure differential in reverse. When the plug is withdrawn entirely from the hole, however, an orifice is created between the edge of the plug and that of the hole. This would make the vital relationship between  $P$ ,  $Q$  and conduit area a less easy one to deal with. Perhaps this problem could have been solved by use of a tapered or trumpet-shaped hole, but before work began on this, a different approach altogether was decided upon.

### PRESSURE CONTROL SERVO VALVES

While it is the unfortunate truth that pilot-operated ("two stage") servo valves cannot be used because of the lack of a hydraulic pressure supply, there are such things as single-stage valves. They are generally available only as "flow control" models, that is, a valve where, in response to a particular current passing through a torque motor or a solenoid, the valve will open to some aperture. It is also possible to buy pressure-control servovalves. These valves maintain a specific pressure across their output ports, in response to an electrical input. They are normally sold as pilot-operated devices, but there is no reason why one should not be made in a single-stage version. The feedback which makes the valve specifically a pressure control valve (see fig. 4) is achieved by having two small piston lands attached to the main valve spool. The output pressure from the valve is passed around through ducts and is applied to these lands supplying to the spool a force proportional to system output pressure, against which a force from the pilot system acts to keep the valve stationary. If the lines from the flapper valve were cut off and joined together to equalize the volumes of the chambers into which they led, and if a control linkage could be connected to the spool, the valve would operate as a single stage servo valve. The next step is to connect together the lines which formerly ran to the hydraulic supply and return. There is no longer a hydraulic

supply, nor is there anywhere to return the fluid to; the valve is to be operated in a purely passive mode. The operating cylinder of the prosthesis takes the place of the load. (see fig. 5)

With this done, it can be seen that there are redundant parts remaining in the valve. The duct connecting the chambers whose pressure difference formerly drove the spool, and indeed the chambers themselves, are now superfluous. It is still necessary, however, to keep differential piston areas in order to have a feedback force. It was found to be possible to reduce the number of sealing lands on the piston from five to three, and eliminate the unnecessary ducting. The proposed configuration is shown in fig. 6.

The centre land on the spool is made slightly smaller than those of the two ends. The incoming and outgoing fluid conduits (which run directly to and from the operating cylinder) are connected to the chambers on the two sides of the centre land. If, in the diagram, the higher pressure is assumed to be on the right hand side, there will be a net force on the lands forming this chamber of  $P_h \Delta A$ , where  $P_h$  is the high pressure produced by the piston. On the other side of the centre land, the force produced in the opposite direction will be only  $P_l \Delta A$ , where  $P_l$  is the low pressure generated by the piston. The net force on the spool, then, is  $(P_h - P_l) \Delta A$ , or  $\Delta P \Delta A$ . If this force is applied to the end of the spool, the spool will remain stationary; otherwise

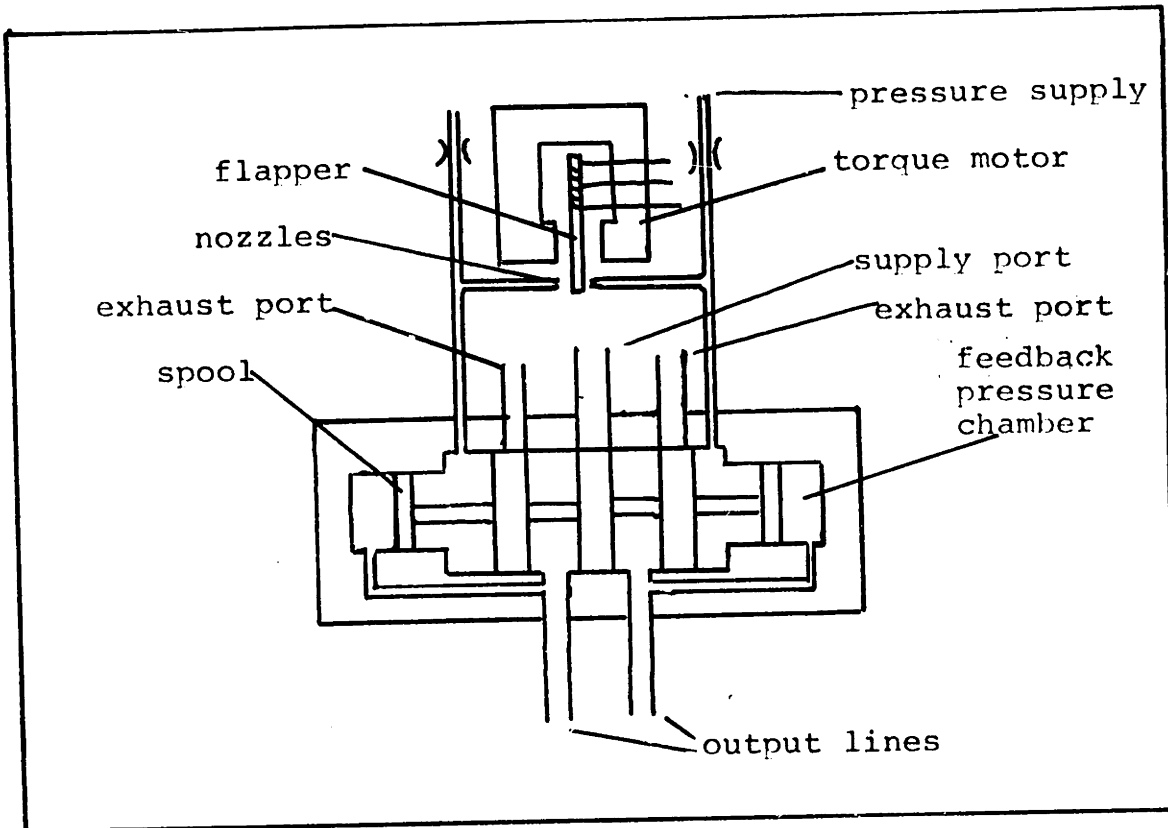
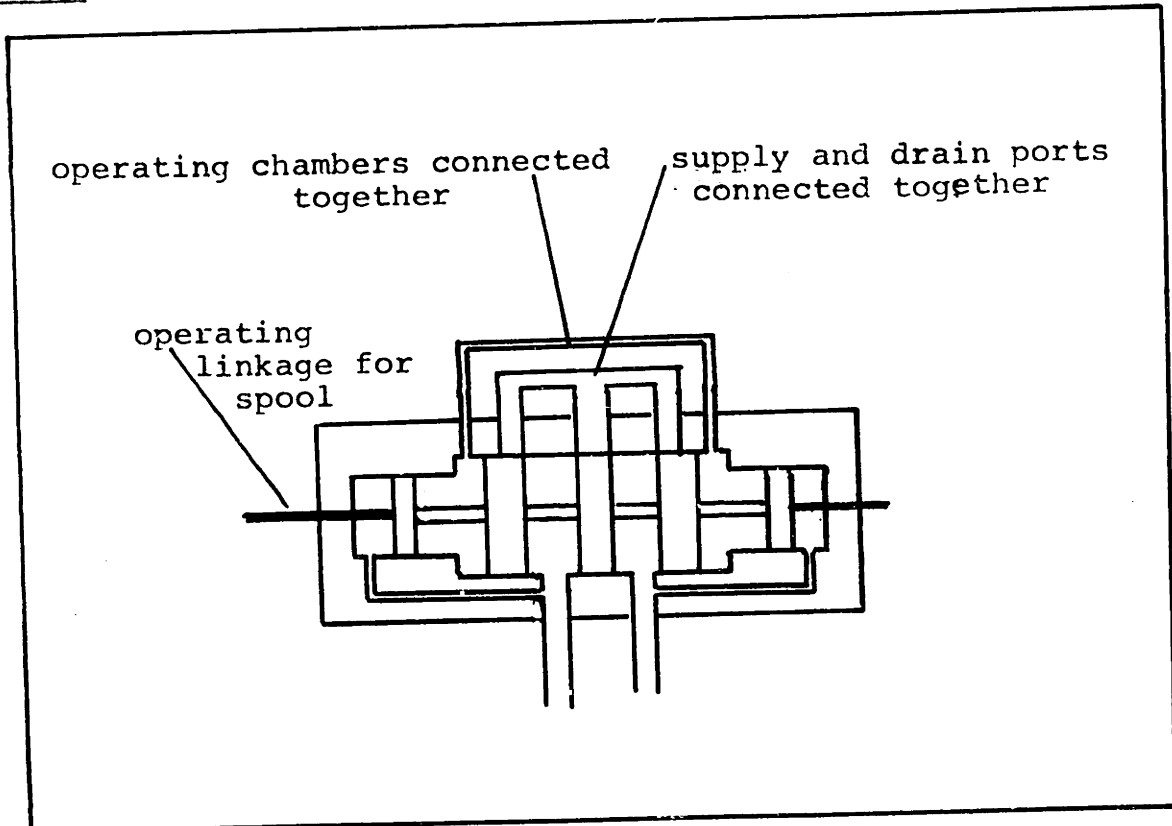


Figure 4. Two stage pressure control servovalve.

Figure 5. Two stage pressure control valve modified.



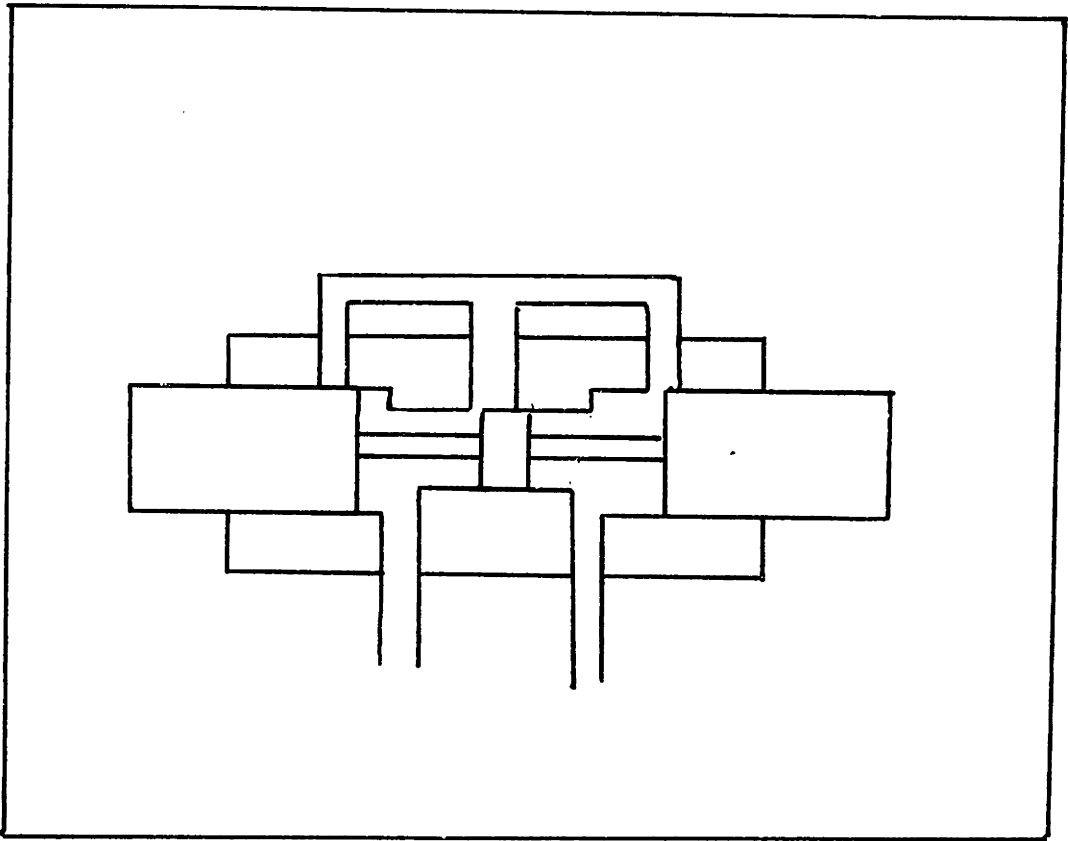
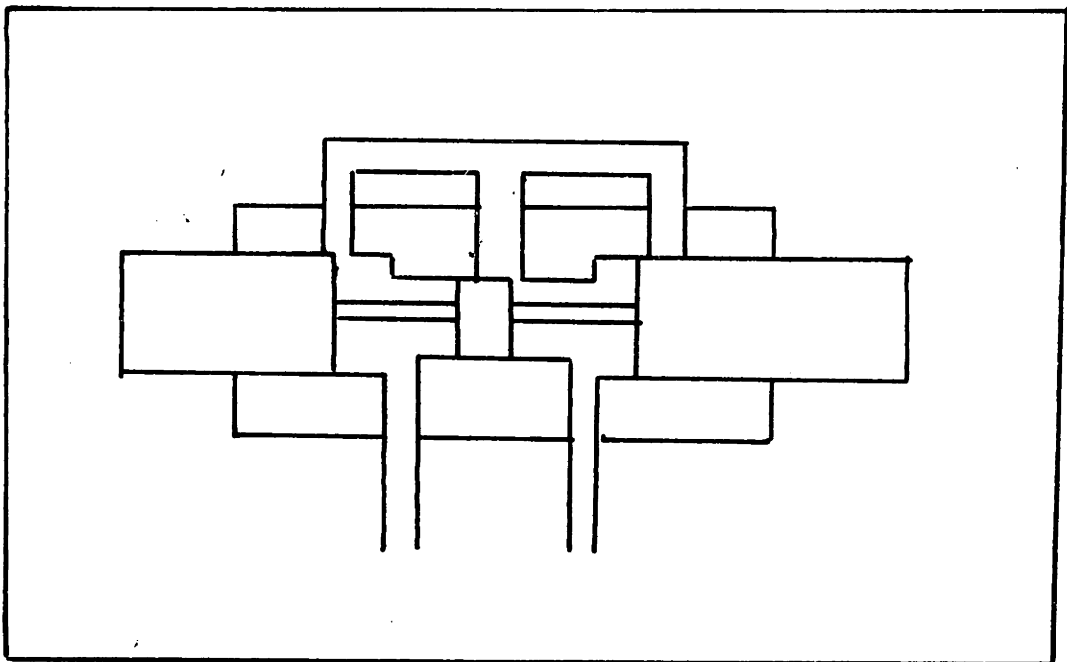


Figure 6. A valve of the type proposed. Pressure is higher in the right hand chamber than in the left.

Figure 7. The same, but with the pressure differential reversed.



it will move until it reaches the end of its travel or until the forces are in balance. The pressure differential  $\Delta P$ , which is proportional to the input and feedback forces, is caused by the pressure drop across the two orifices, one in the centre and one at one end of the spool. Note that if the valve is required to oppose a pressure drop in the other direction than the one shown in fig. 6, the spool will move in the other direction. Also, if more pressure is supplied by the piston than the valve can oppose, the spool will simply be forced to one end of its travel, without suffering any damage.

The next problem to be solved is that of driving medium and dimensions. As will be seen, these two tend to define one another, so they will be considered together. Obtaining an actual "torque motor", which would have a constant output force at any position in its operating range, is not easy, and such a machine tends to be heavy. In view of this, therefore, it was decided to use commercially available solenoids to drive the valve spool. One would be required for each direction, since they are only capable of applying a force in tension. A solenoid is light and cheap, and supplies a force which is proportional to a function of the current through its winding. The problem with most solenoids, however, is that the force supplied is highly dependent on the position of the plunger. Solenoids with a constant force output over part of the operating range are available, but



the nature of alternating magnetic fields is such that these devices are available only to run on alternating current; they are also heavy and use a lot of power. Guardian Electric Co.'s No. 11HD, for example, weighs 8 ounces and draws 48 wattts of power in order to produce somewhat less than 10 ounces of force. It is therefore proposed to use a direct-current solenoid, such as Guardian's T-6. This can be obtained for rated voltages of 6, 12 or 24 and will produce a lesser force if run at a lower voltage. It weighs 3 ounces, and its characteristic of force-versus-stroke is fairly close to a reciprocal. This information comes from a rather small and vague graph in the Guardian catalogue, but the curve is shown as smooth down to  $1/32$  inch, at which point the solenoid exerts 24 ounces of force. It is proposed to use the solenoid from this lower limit to a maximum stroke of  $3/32$  inch, at which point the force has dropped to 10 ounces. To make the solenoid act in a less position-dependent manner, a mechanical linkage between the solenoid and the valve can be used. Kinematicians claim that a linkage can be built to generate any function, and kinematic practice has reached the point where this can be done. In this case, however, a simple mechanism will be used which will, in effect, change the mechanical advantage of the solenoid relative to the valve spool. Suppose that an isosceles triangle is pivoted at its apex so that it is free to rotate in its own plane (see fig. 7). If two connecting rods are attached to the base vertices of the triangle, it is obvious that if the rods move in opposite parallel

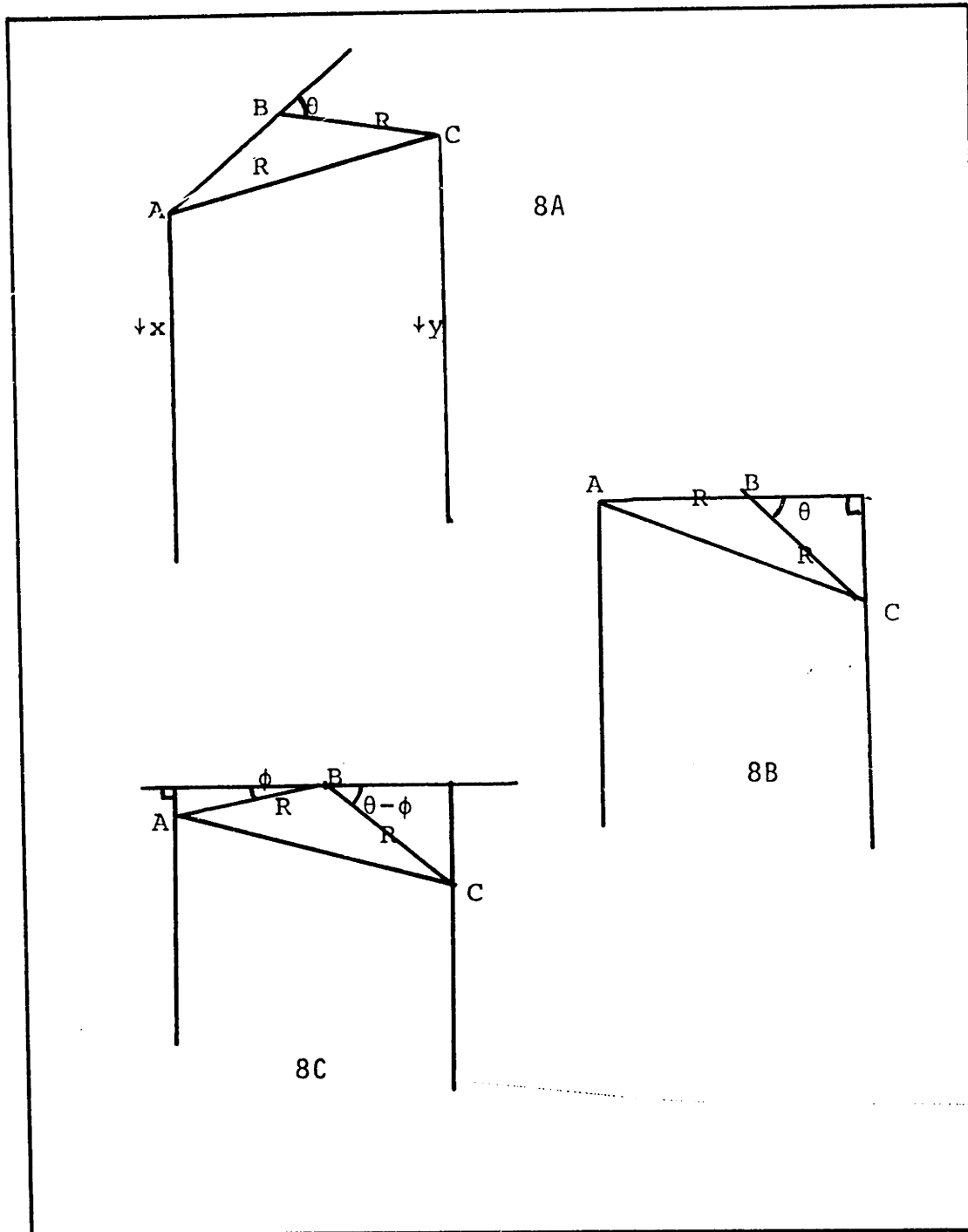


Figure 8. Linkage for equalizing position-dependent force from solenoid.

directions, one will decelerate and the other will accelerate. The size and shape of the triangle determines at what rate this will occur. A calculation for such a triangle is developed in Appendix A. This linkage will obviously not generate an arbitrary function, but it will produce a curve of the form  $y = a\sqrt{r^2 - x^2} - b$ , where  $a$ ,  $b$  and  $r$  are constants. This solenoid and linkage arrangement should be set up and tested before the valve is constructed. If it proves to be unacceptably non-linear, it might be replaced with a four bar linkage, which is considerably more versatile. If this too fails, it would be possible install a potentiometer whose resistance would change as the valve moved, or to pass an a-c signal through the solenoid to measure its changing inductance. Either of these parameters could then be used to adjust the voltage supplied to the solenoid. In either case, however, the electronics might prove rather complex and the additional potentiometer might cause mechanical complications too. Perhaps a straight potentiometer could be fitted into the pressurized valve body, and operated directly from the motion of the spool, or through a lever. This would at least not require an external mechanical connection, with all its problems of friction and leakage.

Note that in figure 8 the two connecting rods are arranged to operate in parallel but opposite directions. Remembring that there are two solenoids, this layout allows them to be placed beside the valve spool, minimizing space

requirements, and, because of closer packing of the components, allowing a valve body with less surface area and hence less weight. The solenoids should be placed adjacent to each other (see diagrams 10) rather than at opposite sides of the valve sleeve, for the same reason. Each solenoid comes with a 15/32-32 NEF-2A thread, 3/8 inch long, at the same end as the plunger, and a hole could be drilled in the valve body to fit the solenoid (body diameter is 0.76 inches maximum) with a corresponding thread at the end. It would also do no harm to drill a small hole at the closed end of each solenoid's plunger sleeve and to have this communicate with the chamber containing the operating linkage of the other solenoid. This would help prevent the solenoid from acting as a dashpot. The connecting rods from the valve spool and the solenoids could be fitted into holes drilled in the ends of the spool and the solenoid plungers. A reasonable material for these rods would be 0.0348 (20 ga.) music wire. Where it attaches to the triangular connecting links, the wire could simply be bent at right angles and pass through a hole. Such a flimsy arrangement should suffice since the forces involved are small and in tension only.

Both the solenoids and the operating linkages would be operated "wet", in an oil-filled environment. This minimizes losses due to friction on shaft seals, and prevents leakage. The coils of the solenoids could be kept outside the pressurized environment, but this adds further complication,

for then the sleeves in which the plungers move would have to be capable of withstanding system pressure. These sleeves as supplied by the manufacturer are made of phenolic plastic, and would have to be replaced. There is really no reason why the coils should be kept dry, as hydraulic fluid is non-conductive and noncorrosive, but it would be advisable to rewind the coils in an oil bath in order to eliminate air spaces between the conductors. Electrical connections can be made through the pressure chamber walls without trouble.

Finally, there is the question of the size of the valve. From the calculations for valve stroke versus solenoid stroke, the valve stroke can be calculated as 0.057 inches. Let it be required that the valve, when in its fully open position, have as large a flow area as the  $\frac{1}{4}$  inch tubing used between it and the operating cylinder. If a valve port is used which extends around the full circumference of the valve sleeve, the flow area will equal 0.057 times the circumference of the valve spool, giving a value for the spool radius of 0.1378 inches. Both this and the stroke are fairly large for a servo valve, but one of the prime requirements of the system is that the fluid resistance when the valve is wide open be as small as possible.

The area of the centre piston land is to be slightly less than that of the two outside lands. The maximum pressure differential is to be 1000psi, and the compensated solenoid can apply a maximum force of 1.5 pounds, so the area

differential is 1.5 divided by 1000, or 0.0015 square inches. The area of a piston of radius 0.1378 inches is 0.0596553 square inches, and the radius of a piston of area 0.0581553 square inches is 0.0361 inches.

With these figures available, the fluid resistance of the valve can be calculated. The average of the two radii is 0.13695 inches, and the stroke is 0.057 inches. This gives an average wide open area of 0.0488 square inches (as opposed to 0.049 for the connecting tubes). If the specific gravity of the fluid used is 0.85, the following equation, given by J. F. Blackburn in Fluid Power Control will hold:  $Q_{\max} \approx 70A_{\max} \sqrt{P}$ . The equation applies to two orifices in series, as is the case here. In this case, the product  $70A_{\max}$  equals 3.4. Applied in more real terms to the prosthesis, and with the 1.8 inch pulley mentioned earlier, a relation can be written between torque and angular velocity of the prosthesis, viz:  $\omega \approx 3.0\sqrt{\tau}$ , with the valve fully open. For a reasonable rate of swing, say 5 radians per second, the retarding force from the valve orifices alone will be 1.29 pound-inches. It is important to realize that there will be other losses than those of the orifices, but they will hopefully not be prohibitive.

One factor not yet mentioned is the so-called "Bernoulli force" on the spool, and compensation for this. When fluid flows through an orifice such as those in a spool valve, it gives rise to a longitudinal force on the valve spool. This

force does not cancel out simply because the fluid flows through different orifices in opposite directions; rather, it doubles. This force may be extremely troublesome in servo valves, for can easily swamp the force supplied by the solenoid or the pilot. The Bernoulli force for the valve proposed is calculated in Appendix B. The force is, indeed, much larger than the force supplied by the solenoid under most conditions. Blackburn, however, describes a mechanism by means of which this force may be compensated. Fluid is allowed to flow through one orifice as before. It flows through two orifices in the valve, though, and the second one is a specially-shaped negative-force port, which produces almost exactly enough force to balance the Bernoulli force from the ordinary orifice. The theory of these negative force ports, Blackburn says, is poorly understood, but the force produced by them is proportional to the stroke of the valve multiplied by the pressure drop across it, just as is the Bernoulli force. The degree of compensation can be made almost perfect, but much depends on careful machining. If the job is properly done, the remaining unbalanced force on a valve such as the one described here would be less than  $\frac{1}{4}$  ounce. Figures 9 shows how two compensated orifices would be used in the valve, and a close up section of one of them.

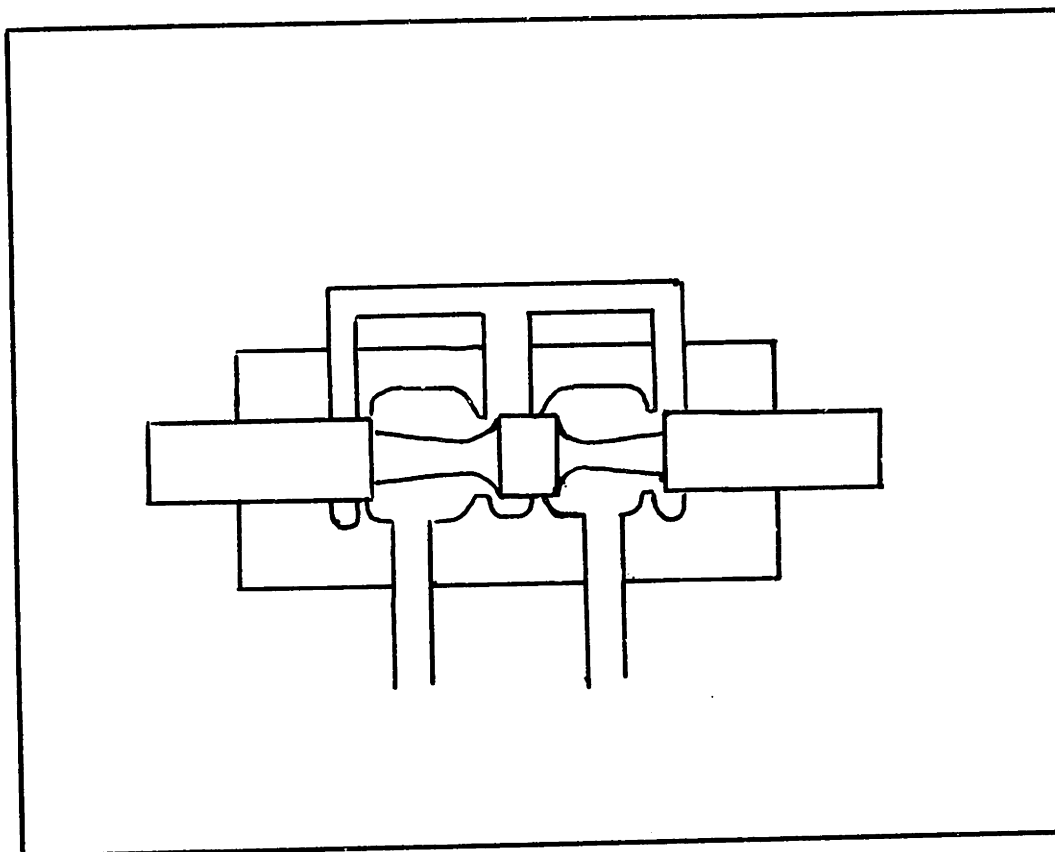
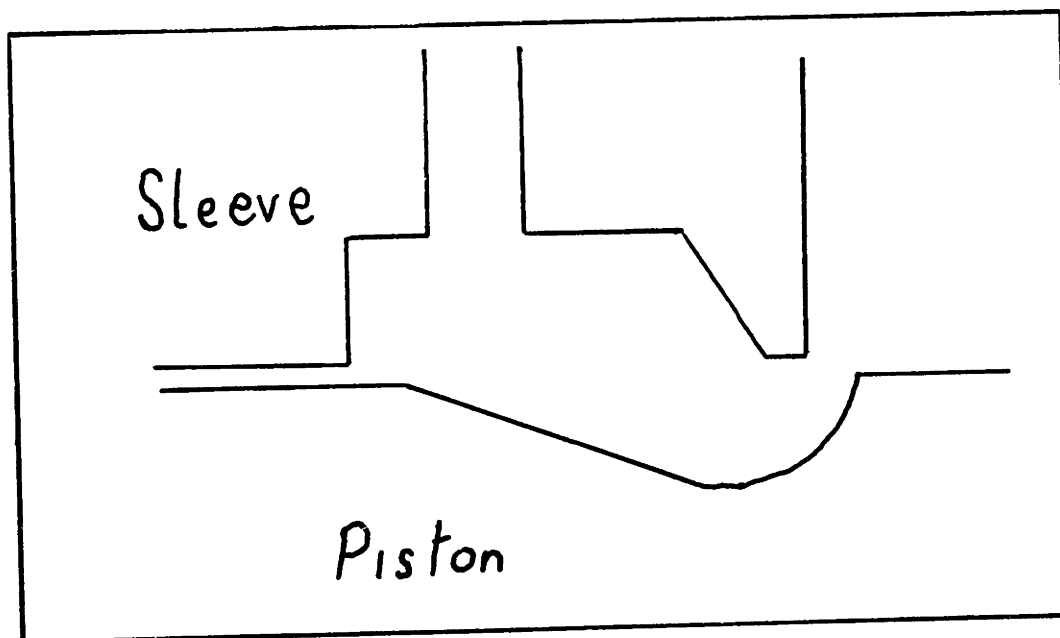


Figure 9. Valve of type proposed compensated for Bernoulli forces, above. Below, negative force port (from Blackburn).





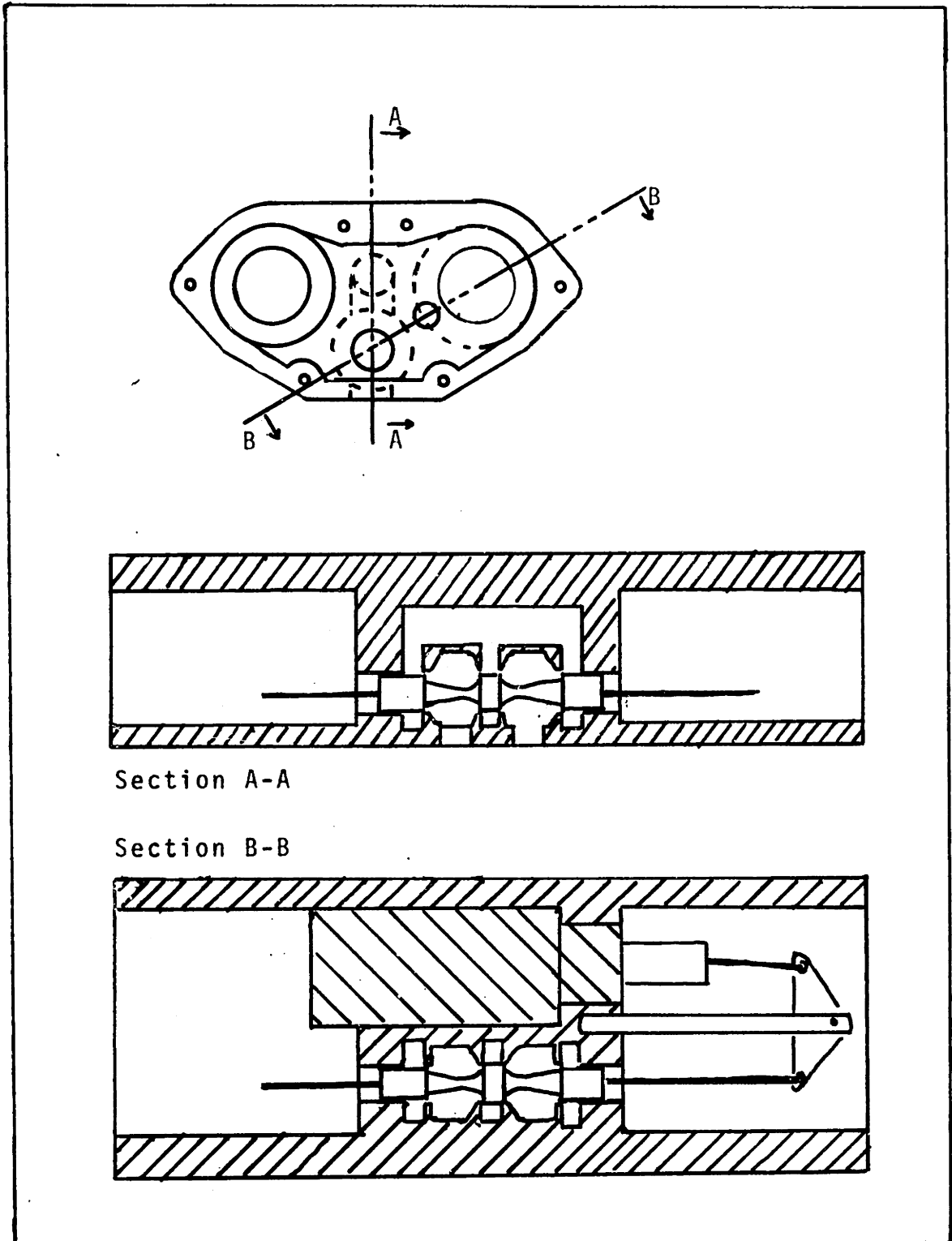


Figure 10. End view of control valve, and two longitudinal sections. Solenoid is shaded top left to bottom right.

APPENDIX A

The object of this system is to allow the solenoid to develop an increasing mechanical advantage as its stroke increases, and hence, as the force it can supply decreases. In addition, it is necessary that the mechanism to be developed will reverse the direction of applied force, thus permitting the solenoids to be placed beside the valve spool in the interest of saving space. The mechanism works through the turning of an isosceles triangle (ABC in figure 8) about its apex. The length of sides AB and BC can be specified at the designer's convenience, and in this case the length was taken as 0.4 inches, which gives a reasonable separation between the solenoid and the spool.

Suppose that the triangle is in the position shown in figure 8b. At this point, the solenoid stroke  $x$  is taken as zero, and the valve stroke  $y$  as  $R\sin\theta$ . The exterior angle of triangle ABC is to be called  $\theta$ , and it is this value which must be calculated. Clearly, the length of CD is  $R\sin\theta$ . If the triangle is now tilted through an angle  $\phi$ . The solenoid stroke  $x$  equals  $R\sin\phi$ , while the valve stroke  $y$  equals  $R\sin(\theta-\phi)$ . This latter can be expressed as  $R(\sin\theta\cos\phi-\cos\theta\sin\phi)$ , and since, by the Pythagorean formula, the length BE equals  $\sqrt{R^2-x^2}$ ,  $\cos\phi$  can be called  $\frac{\sqrt{R^2-x^2}}{R}$ , while  $\sin\phi$  simply equals  $\frac{x}{R}$ . An equation can then be written for  $y$  in

terms of  $x$  and  $\theta$  only, namely

$$y = R\left(\sin\theta\frac{\sqrt{R^2-x^2}}{R} - \cos\theta\frac{x}{R}\right)$$

Cancelling  $R$ 's, this equation can be differentiated with respect to  $x$ , viz:

$$\frac{dy}{dx} = \frac{x\sin\theta}{\sqrt{R^2-x^2}} - \cos\theta$$

The particular value in which we are interested in is that  $\frac{dy}{dx}$  equals  $\frac{5}{12}$  when  $x$  has a value  $\frac{1}{16}$  inch less than its value when the triangle is symmetrically placed relative to  $x$  and  $y$ . This latter position, it may be seen intuitively, is the point where spool and solenoid plunger would both move at the same speed, ie where  $\frac{dy}{dx}$  equals one. At this point, it is also obvious that  $x$  equals  $R\sin(\frac{\theta}{2})$ , so the value of  $x$  when  $\frac{dy}{dx} = \frac{5}{12}$  is  $R\sin(\frac{\theta}{2}) - \frac{1}{16}$ . If the values of  $x$  and  $\frac{dy}{dx}$  are plugged into the equation above,  $\theta$  can be calculated. In this case, however, this was not done. Rather, a computer iteration was performed, and  $\theta$  was found to be 1.088 radians, or  $62.33^\circ$ . The base angles of the triangle would be half this angle, or  $31.17^\circ$ .

APPENDIX B

Fluid is constrained to pass into the chamber between two lands of the valve spool in a direction perpendicular to the axis of the spool. It may pass out, however, at some angle  $\theta$  between the lands of the spool and the sleeve. Von Mises demonstrated, and it has since been proved experimentally, that this angle is in the region of  $69^\circ$ . The force on the spool is equal to the rate of change of momentum of the fluid in a direction parallel to the axis of the spool.

$$F = \frac{d(mv)}{dt}$$

$$F = \rho U Q \cos \theta$$

Where  $U$  is the velocity of the fluid at the orifice and  $Q$  is the rate of flow in cu. ins. per second

$$Q = aU$$

Where  $a$  is the area of the orifice.  $\cos 69^\circ = 0.358$ , so

$$F = .358 \rho \frac{Q^2}{a}$$

The orifice equation states

$$Q = C_d a \sqrt{\frac{2\Delta P}{\rho}}$$

Where  $\Delta P$  is the pressure drop across the orifice and

$C_d$  is the orifice coefficient of discharge.

Squaring the equation, we have

$$Q^2 = C_d^2 a^2 \frac{2\Delta P}{\rho}$$

Substituting this in the other equation, with  $C_d = 0.8$

$$\begin{aligned} F &= 0.358 \times 0.64 \times 2a\Delta P \\ &= 0.458a\Delta P \end{aligned}$$

There are, of course, two orifices, but the pressure falls equally across them. The equation must then be doubled, so an equivalent result can be obtained by treating the two equal orifices as one, with the full pressure across it.

$$F = 0.458 \times 0.049 \times 1000$$

$$= 24.44 \text{ pounds}$$

REFERENCES

The major refence for this thesis was, of course, the doctoral thesis of Woodie Flowers:

Flowers, W., "A Man-Interactive Simulator for Above-Knee Prosthetics Studies", Ph.D. Thesis, Massachusetts Institute of Technology, 1972.

Other refences were:

Yeaple, F.D., Ed., "Hydraulic and Pneumatic Power and Control", New York: McGraw-Hill Book Company, 1966.

Blackburn, J.F., et al., "Fluid Power Control", Cambridge: The M.I.T. Press, 1960.

Fitzgerald, A.E., Kingsley, C. jr., and Kusko, A., "Electric Machinery", New York: McGraw-Hill Book Company, 1961.