A STUDY OF ACOUSTIC RADIATION FROM AN
ELECTRICAL SPARK DISCHARGE IN AIR

by

ROBERT EDWARD KLINKOWSTEIN

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Certified by...........

Thesis Supervisor

Accepted by.........................
Chairman, Department Committee on Graduate Students
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Submitted to the Department of Mechanical Engineering on July 12, 1974 in partial fulfillment of the requirements for the Degrees of Bachelor of Science and Master of Science.

ABSTRACT

The electrical spark discharge is studied as a source of acoustic energy. A theoretical explanation of the formation and propagation of N waves produced by sparks is presented, and experimental studies are given to support this theory. The effect of changing parameters of the discharge circuit on the spark as an acoustic source is also discussed. Finally, the theory is presented as a tool for predicting the characteristics and realistic limitations of sparks as sound sources.

Thesis Supervisor: Richard H. Lyon
Title: Professor, Department of Mechanical Engineering
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>2</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>3</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>4</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>5</td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>7</td>
</tr>
<tr>
<td>II. N WAVE THEORY</td>
<td>9</td>
</tr>
<tr>
<td>1. Formation of N Wave</td>
<td>9</td>
</tr>
<tr>
<td>2. N Wave Propagation Distortion</td>
<td>16</td>
</tr>
<tr>
<td>3. Hopkinson Blast Scaling</td>
<td>23</td>
</tr>
<tr>
<td>4. Mathematics of the N Wave</td>
<td>25</td>
</tr>
<tr>
<td>III. THE SPARK AS AN ACOUSTIC SOURCE</td>
<td>30</td>
</tr>
<tr>
<td>1. The Spark Circuit</td>
<td>30</td>
</tr>
<tr>
<td>2. Triggering Methods</td>
<td>33</td>
</tr>
<tr>
<td>IV. EXPERIMENTAL INVESTIGATION</td>
<td>39</td>
</tr>
<tr>
<td>1. Signal Processing</td>
<td>39</td>
</tr>
<tr>
<td>2. Analysis and Experimental Results</td>
<td>41</td>
</tr>
<tr>
<td>V. CONCLUSIONS</td>
<td>55</td>
</tr>
<tr>
<td>APPENDIX A - EFFECTS OF NONLINEAR PROPAGATION ON ACOUSTIC MEASUREMENTS</td>
<td>57</td>
</tr>
<tr>
<td>APPENDIX B - AN EXAMPLE CACULATION</td>
<td>67</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>74</td>
</tr>
</tbody>
</table>

-4-
LIST OF FIGURES

<table>
<thead>
<tr>
<th>No.</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Finite Amplitude Distortion</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>Spherical Propagation of a Pulse</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>Ideal N Wave</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>Hopkinson Scaling</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>Energy Spectrum of an N Wave</td>
<td>28</td>
</tr>
<tr>
<td>6</td>
<td>Spark Discharge Current</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>a. Current Measuring Circuit</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>b. Current Waveform</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>Spark Discharge Triggering Circuits</td>
<td>34</td>
</tr>
<tr>
<td>8</td>
<td>Signal Processing Instrumentation</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>N Wave From a 4.5J Spark at 1.5 Meters</td>
<td>42</td>
</tr>
<tr>
<td>10</td>
<td>Energy Spectrum for a 4.5J Spark at 1.5 Meters</td>
<td>43</td>
</tr>
<tr>
<td>11</td>
<td>Energy Versus Spark Gap Dimension</td>
<td>46</td>
</tr>
<tr>
<td>12</td>
<td>N Wave Propagation Distortion</td>
<td>48</td>
</tr>
<tr>
<td>13</td>
<td>N Wave Half Period Versus Propagation Distance</td>
<td>49</td>
</tr>
<tr>
<td>14</td>
<td>Peak Overpressure Versus Propagation Distance</td>
<td>51</td>
</tr>
<tr>
<td>15</td>
<td>Hopkinson Scaling Data</td>
<td>53</td>
</tr>
</tbody>
</table>

TABLE 1  Maximums and Zeros of Spherical Bessel Functions.  29
LIST OF FIGURES (CONTINUED)

APPENDIX A

EFFECTS OF NONLINEAR PROPAGATION ON ACOUSTIC MEASUREMENTS

1a Excess Attenuation from Nonlinear Propagation......60
2a Nonlinear Propagation Effects on a 4.5J Spark Measured in 2kHz Bandwidths.........................63
3a Energy Spectrum of a 4.5J Spark at 1.5 Meters......65

APPENDIX B

AN EXAMPLE CALCULATION

1b N Wave and Energy Spectrum for a 4.5J Spark at 1m..68
2b N Wave and Energy Spectrum for a 12.5J Spark at 2m.71
I. INTRODUCTION

The study of the electric spark as a source of acoustic energy presents an unusual problem to the acoustician accustomed to the linear mathematical descriptions of acoustical phenomena. The spark discharge is inherently a nonlinear transduction, converting electrical energy into acoustic energy. Its nonlinear nature becomes evident when examining the characteristics of a typical spark channel where temperatures often reach several thousand degrees Kelvin, pressures may reach several atmospheres and the speed of acoustic propagation is much faster than the ambient sound velocity.

Increasing interests in acoustically modeling urban environments [1], airports and other situations where difficulties arise in controlling full size model parameters [2], has provided the motivation for an intense study of the spark as an acoustic source. In reduced scale models such as these an acoustic source with frequency content up to 200 kHz is desireable. The electric spark provides such a source. In addition the directivity pattern is essentially omnidirectional in the equatorial plane, and the acoustic characteristics have proven to be repeatable for successive sparks.

The spark has also gained acceptance as an impulse
source for testing microphones and loudspeakers [3]. This testing procedure enables measurements to be made without the need for an anechoic test facility.

Very little investigation has been done of the electric spark as an acoustic source, however, the theory of shock wave propagation has been well developed and applied to acoustic radiation from explosions [4]. Shock wave production from sparks is a similar phenomena and should obey similar behavior. The purpose of this thesis is first, to present this shock wave theory as it applies to acoustic radiation produced by electrical spark discharges. Secondly, it will utilize this theory to gain insight into the usefulness and limitations of the resulting acoustic pulse, and provide a technique for obtaining predictable and reliable data. And finally, this thesis will provide experimental justification of this theory.
II. N WAVE THEORY

The theory of nonlinear acoustics essential to understanding the production and propagation of N waves produced by electric sparks is presented here. Acoustic radiation from sparks undergoes three regions of transformation as it propagates from its point of origin. In the first, there is little effect due to dissipation and nonlinear mechanisms lead to the formation of an N shaped wave terminated on both ends by shock discontinuities. In the second region the effects of dissipation become important. As the amplitude decreases the effects of finite amplitude slowly give way to dissipation by viscosity, heat conduction, and molecular relaxation. Equations of linear acoustics apply in this third region where the wave geometry is altered only by spherical spreading and frequency dependent attenuation.

II.1 Formation of N Wave

In the development of one dimensional linear acoustics, amplitudes are assumed to be small, and the resultant linear equations are easily solved. Solutions of the form $f(x \pm ct)$ result, which correspond to traveling waves whose profile does not change with distance or time. The term "profile" will be defined here as the distribution of
density, velocity, etc. along the direction of propagation. Each point of the profile propagates with velocity $c$. Velocity, density, pressure and other characteristic quantities are functions only dependent on the quantity $(x \pm ct)$. Since this is the case, each of these quantities may be expressed as functions of each other, where time and coordinates do not appear explicitly, (e.g. $p = \rho \langle \rho \rangle$, $v = v(p)$).

When finite amplitude effects become important, solutions of the form $f(x \pm ct)$ are no longer possible. It has been found, however, that a general solution of the exact equations of fluid motion is possible for a one-dimensional wave. This solution is a generalized form of the approximate solutions valid for small amplitudes. It is known as the Riemann solution of the equations of motion. To obtain this solution the assumption is made that, for a wave of any amplitude the velocity can be expressed as a function of the density and that the flow is adiabatic in the absence of any shock waves. Therefore, $s$, the entropy per unit mass is assumed constant.

A summary of the development presented by Landau [5] will be given here. Begin with the equation of continuity and Euler's equation, for plane wave propagation in the $x$ direction

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0 \quad ,$$

(1)
\[
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0 \quad .
\]  

(2)

Where \(\rho\) is the density, \(p\) is pressure, and \(v\) is fluid velocity. These equations can be solved [5] to give the general relation between the velocity and density or pressure

\[
v = \pm \int \frac{c}{\rho} \, dp = \pm \int \frac{dp}{\rho c} \quad ,
\]

(3)

and a solution of \(x\) as a function of \(v\) in the form

\[
x = t[v \pm c(v)] + f(v) \quad ,
\]

(4)

where \(f(v)\) is an arbitrary function of \(v\) and \(c(v)\) is given by (3).

Relationships (3) and (4) give the required general solution attributed to Riemann. These formulae determine the velocity and therefore all other quantities as implicit functions of \(x,\) and \(t.\) Fluid flow which behaves in the manner described by (3) and (4) is called a simple wave, and remains simple until shocks are formed.

Explicit relations for other flow variables can also be obtained from Riemann's solution.

\[
c = c_o \pm \frac{1}{2}(\gamma - 1)v
\]

(5)
\[ \rho = \rho_0 \left(1 \pm \frac{1}{2}(\gamma - 1)v/c_0\right)^{2/(\gamma - 1)} \]  

(6)

\[ p = p_0 \left(1 \pm \frac{1}{2}(\gamma - 1)v/c_0\right)^{2\gamma/(\gamma - 1)} \]  

(7)

Where \( \rho_0, p_0 \) and \( c_0 \) are the equilibrium values of density, pressure, and velocity of sound respectively, and \( \gamma \) is the ratio of the specific heats at constant pressure and constant volume, \( c_p/c_v \).

The velocity of a point of the wave profile is given by

\[ u = v(\rho) \pm c(v) \]  

(8)

which is essentially different from linearized theory since \( u \) is now a function of density and therefore different for different points of the wave profile. Thus, in generalized theory of plane wave propagation there is no well defined wave velocity.

Consider a wave propagating in the positive direction (i.e. \( u = v + c \)). From (3) we obtain \( du/d\rho > 0 \). In compressions \( \rho > \rho_0 \) and \( c > c_0 \) and similarly in rarefactions \( \rho < \rho_0, c < c_0 \). The changing nature of the velocity for different points of the wave profile causes its shape to change with time. Points of the compression move forward while points of rarefaction move backward, as shown in Fig-
ure 1. Eventually the profile may become multivalued (i.e. \(\rho(x)\) is triple valued at some point as shown in Figure lc.) at this point the wave ceases to be a simple wave. In reality this is not physically possible and shock fronts are formed which result in dissipation of energy.

The results from Riemann's solution can be used to obtain results of some properties, in the second order, for finite amplitude waves. Linear theory being first order solutions. Distortion of the wave profile results from second order effects. If the wave profile is purely harmonic at some instant, it will not be so at some later time. If we think of expanding the wave by Fourier theory, propagation results in introducing higher order harmonics.

The velocity, \(u\), of points of the wave profile is obtained by considering Equation (8). In the first approximation \(v = 0\) and therefore \(u = c_0\), which is consistent with linear acoustic theory. In the second order,

\[
\frac{\partial u}{\partial \rho_o} = c_o + \rho_o \left( \frac{\partial u}{\partial \rho_o} \right)_o \frac{\rho_o}{c_o} v
\]

\(\frac{\partial u}{\partial \rho}\) can be solved from Riemann's results to give

\[
u = c_o + \alpha v
\]

where \(\alpha = \frac{1}{2}(\gamma + 1)\) for a perfect gas.
Figure 1.

Finite Amplitude Distortion
In the general case the wave is no longer simple after the discontinuity has formed. However, Landau [5] has shown that in the second order the wave remains simple since the change in entropy across a discontinuity is of third order. Therefore, we may consider these equations to be correct solutions for plane wave propagation to second order after the formation of a shock. If a spherical outgoing wave is considered far from its origin it may be regarded as being plane over a short distance. If the wave is to be studied for long intervals of time, however, the $1/r$ dependence of velocity must be taken into account. $v$ may be expressed as

$$v = v_0 \frac{r_0}{r},$$

(11)

where $v_0$ is the value of $v$ at some reference position $r_0$, and the velocity $u$ of points of the profile is given by

$$u = c_0 + \alpha \frac{v_0 r_0}{r}.$$  

(12)

The first term is the ordinary velocity of sound and the second term results in distortion of the profile. The additional movement to points of the profile afforded to this second term during a time $t = (r - r_0)/c$ is then given by
\[ \Delta r = (a v_o r_o / c_o) \log(r/r_o) \]  \hspace{1cm} (13)

Therefore the distortion of a propagating spherical wave increases with the logarithm of the distance.

Now consider the shock wave formed by a single pulse from a point source. The spherical case is distinguished from the plane wave case by one important feature. The region of compression formed by the pulse must be followed by a region of rarefaction, in the spherical case, but this is not necessary in the plane wave case. This can be argued as follows: before the spherical wave has arrived the velocity potential, \( \phi \), must be constant in a medium that is at rest. After the wave has passed, the potential must again be constant in the region. For spherical waves, however, \( \phi \) is of the form

\[ \phi = f(ct - r)/r \]  \hspace{1cm} (14)

Such a function may be constant only if \( f \) tends to zero. Therefore the potential must be zero both before and after passage of the wave.

The variation of pressure is related to \( \phi \) by

\[ p_1 = -\rho \frac{\partial \phi}{\partial t} \]  \hspace{1cm} (15)

From these observations it is clear that
\[ \int_{-\infty}^{\infty} \text{pdt} = 0, \quad (16) \]

and therefore both compressions and rarefactions must be observed at any point as the wave passes.

At this point we may explain the formation of an \( N \) wave from a single compression pulse produced by a point source. The positive pulse viewed at any point away from the origin must be followed by a region of rarefaction. Distortion of this wave form proceeds at a rate proportional to the logarithm of distance and ultimately results in the formation of two shock fronts. One in the region of compression and one in the region of rarefaction. This process is shown graphically in Figure 2a.

One further observation may be made from this development of \( N \) formation. Figure 2b shows how \( N \) wave formation may be independent of the initial compression wave form.

II.2 N Wave Propagation Distortion

The theory of \( N \) wave propagation distortion has been worked out in detail [6] and the results will be presented here. In the derivation, the effects of finite amplitude waves are retained to second order. That is corrections to linearized theory necessary from the result of \( p_s/p_o \) to the first power. Where \( p_s \) is the peak over pressure
Figure 2.
Spherical Propagation of a Pulse
and $p_0$ is the ambient pressure. It is suggested that these results are applicable where $p_s$ is 1% or less of $p_0$. Energy is assumed to be lost only at the wave fronts. The derivation is also limited to distances where $r$ is large compared to a wavelength. For our work this indicates validity for distances greater than approximately 10cm.

Figure 3 shows the pressure, time profile of an ideal N wave. The N wave is characterized by two quantities $p_s$, and $T$ as they are defined by this figure. The theoretical results are presented below in terms of these parameters

$$\delta = \frac{c}{r[\ln(r/a_1)]^{1/2}} ,$$

$$T = \frac{1}{2c_o} \frac{\gamma + 1}{\gamma} c[\ln(r/a_1)]^{1/2} ;$$

$$\frac{dT}{dr} = \frac{\gamma + 1}{4\gamma c_o} \delta .$$

Where $\delta$ is defined as $p_s/p_0$ and $c$ and $a_1$ are constants.

These results may be expressed independent of $c$ and $a_1$ [7], making them useful for comparison of experimental results.

$$\delta = r_o \delta_0 / \{r[1 + k\ln(r/r_o)]^{1/2}\}$$

-19-
Figure 3.
Ideal N Wave
\[ T = T_0 \left[ 1 + k \ln \left( \frac{r}{r_o} \right) \right] \]  

(21)

where \( k \) is defined by

\[ k = \frac{(1 + \gamma) r_o \delta_o}{2 \gamma c_o T_o} \]  

(22)

and \( r_o \) and \( T_o \) are constants given by a reference position and \( T \) at that position, respectively.

From these equations a number of results can be obtained. First, the period of the N wave, \( 2T \), will increase as the wave travels outward from the source, and the most rapid increase will take place close to the source. The amplitude decay will be greater than \( r^{-1} \), associated with spherical spreading. The propagation laws depend only upon \( \gamma \), the ratio of the specific heat, and \( c_o \) the unperturbed speed of sound in the gas.

As the N wave amplitude decreases, the finite amplitude effects will become of less importance and the propagation laws presented here are of little consequence. Lighthill[16] has defined a Reynolds for the N wave and suggests that when \( R \) becomes comparable to unity, finite amplitude effects should cease to be important.

\( R \) is defined as,
\[ R = 1450\delta T \]  \hspace{1cm} (23)

for typical laboratory conditions and \( T \) is measured in microseconds.

In the region where this Reynolds number is much greater than one, energy dissipation of the \( N \) wave is dominated by an entropy change across the wave front rather than air absorption tending to change the frequency spectrum of the wave. Lighthill [16] and other researchers [6] have shown that this is the case. Energy dissipation by this mechanism is accounted for in Equations (17) through (21). For Reynolds numbers of order 1 or less, frequency dependent air attenuation theory applies. The reasoning of these results is as follows: frequency dependent attenuation tends to increase the rise time of wave front and round the sharp corners of the \( N \) wave. If the amplitude of the resulting wave is large enough, however, it will restore itself (as explained previously) to the ideal \( N \) wave, maintaining sharp corners and a fast rise time. The \( N \) wave will continue to preserve its geometry until the amplitude is no longer large enough to distort at a rate great enough to overcome the tendencies of air absorption by relaxation mechanisms. This transition region is defined by the conditions given above.
II.3 Hopkinson Blast Scaling

Several schemes have been developed to predict the characteristics of blast waves resulting from large explosions. Once the characteristics from a given explosion resulting from a specific explosive have been measured, information about blast waves resulting from different amounts of this explosive can be obtained. This is known as blast scaling, and finds wide use in the study of large explosive discharges where expense and difficulties in measurements prohibit repeated explosions.

The most common form of blast scaling is known as Hopkinson scaling, and has been shown to apply over very wide ranges of distances and explosive energies [4]. The results of Hopkinson scaling that will be of particular use in the study of spark discharges, is best illustrated by Figure 4. If a microphone located at a distance $r_1$ from the spark experiences an N wave with peak overpressure $p_s$, half period $T$ and some characteristic time history, $p(t)$, then a microphone positioned at a distance $r_1(E_2/E_1)^{1/3}$ from a spark of "similar explosive characteristics" will experience an N wave with peak overpressure $p_s$, half period $T(E_2/E_1)^{1/3}$ and time history $p((E_2/E_1)^{1/3}t)$.

The phrase "similar explosive characteristics" used
Figure 4.
Hopkinson Scaling
here suggests that the efficiency of energy conversion for the two sparks is the same. It will be shown later that this is primarily dependent on the spark gap.

II.4. Mathematics of the N Wave

The ideal N wave was shown in Figure 3. Fourier analysis of this N wave has been recently worked out [8-9] for studies of sonic booms. The amplitude spectrum is denoted as \( P(f) \) and the pressure time history is denoted as \( p(t) \). The Fourier transform pair is then given by

\[
p(t) = \int_{-\infty}^{\infty} P(f) e^{i2\pi ft} df , \quad (24)
\]

\[
P(f) = \int_{-\infty}^{\infty} p(t)e^{-i2\pi ft} dt , \quad (25)
\]

and the energy spectrum is defined as, \( |P(f)|^2 \).

The results are obtained in the form

\[
P(f) = ip_{s}2T \text{sinc}2Tf\{(1/\pi 2Tf) - \cot \pi 2Tf\} \quad (26)
\]

where \( \text{sinc}f = \frac{\sin \pi f}{\pi f} \).

This result can be simplified in terms of spherical Bessel functions using the identity,
\[
\text{sinc}2Tf\{(1/2Tf) - \cot \pi 2Tf\} = j_1(\pi 2Tf), \quad (27)
\]

where \(j_1\) is the spherical Bessel function of the first kind. Therefore,

\[
P(f) = iP_s 2T(j_1(\pi 2Tf)) \quad (28)
\]

\[
|P(f)|^2 = 4P_s^2 T^2 (j_1(\pi 2Tf))^2 \quad (29)
\]

The asymptotic behavior at high and low frequencies can also be found. For low frequencies,

\[
j_1(\pi 2Tf) \rightarrow \frac{\pi 2Tf}{3}, \quad (30)
\]

which gives,

\[
|P(f)|^2 = \frac{16}{9} P_s^2 T^4 \pi^2 f^2 \quad (31)
\]

This result indicates a 6db per octave roll off for low frequencies (i.e. frequencies less than the first maximum of \(|P(f)|^2\)).

For high frequencies,

\[
j_1(\pi 2Tf) \rightarrow \frac{-1}{\pi 2Tf} \cos(2T \pi f) \quad . \quad (32)
\]
This gives,

\[ |P(f)|^2 = \frac{p_s^2}{\pi^2 f^2} [\cos(2\pi Tf)]^2. \]  \hspace{1cm} (33)

The peaks of the high frequency magnitude also decline at a rate of 6 db per octave, and this asymptotic magnitude is independent of the half period T. The maximum, and minimum points are determined by the \cos term, in the high frequency limit.

The above results are shown in Figure 5, where the energy spectrum is shown for an N wave with T = 25\mu sec. To facilitate these results, Table 1 gives the first four maximums and zeros of \( j_1(x) \). For frequencies higher than these the high frequency approximation is suitable.
Figure 5.
Energy Spectral Density of N Wave
<table>
<thead>
<tr>
<th>x</th>
<th>$j_4(x)$</th>
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<tr>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2.1</td>
<td>0.4361</td>
</tr>
<tr>
<td>4.5</td>
<td>0.0</td>
</tr>
<tr>
<td>5.9</td>
<td>-0.1679</td>
</tr>
<tr>
<td>7.7</td>
<td>0.0</td>
</tr>
<tr>
<td>9.2</td>
<td>1.086</td>
</tr>
<tr>
<td>11.0</td>
<td>0.0</td>
</tr>
<tr>
<td>12.4</td>
<td>-0.082</td>
</tr>
</tbody>
</table>

Table 1.
Maximuns And Zeros
of Spherical Bessel Functions
III. THE SPARK AS AN ACOUSTIC SOURCE

The process of electrical breakdown has received extensive consideration [10] and no attempt will be made to present its theoretical development. The process may be outlined as follows. When the voltage across the gap reaches a sufficiently high potential (breakdown voltage), causing ionization in the air around the gap, a very narrow cylindrical region between the gap becomes a good conductor. The energy stored in the circuit surges through this region, often raising the temperature to several thousand degrees Kelvin. This results in the rapid expansion of the spark channel, forming a cylindrical shock ahead of it. The details of the spark channel expansion are not known. The spark channel may eventually contract, thus forming a rarefaction, as was indicated by some researchers [11]. The initial shock usually pulls away from the spark channel within 1μsec, and the shock front is first observed to be ellipsoidal, with its major axis along the axis of the spark. Within 10μsec, however, it assumes a nearly perfect spherical shape [12].

III.1 The Spark Circuit

The essential elements of the spark discharge circuit are shown in Figure 7a. A high voltage power supply
charges a storage capacitor through a current limiting resistor, and the discharge of the capacitor occurs through the spark gap which may reach one ohm of resistance or less during discharge. The means of triggering the discharge can take several forms, and will be discussed later.

The circuit used for the experimental work of this thesis utilized a capacitor designed and constructed by the author. Capacitance could be varied from .04\(\mu\)fd to 2\(\mu\)fd. It was of the parallel plate design, using 10 mil myler for dielectric, and aluminum foil for plate conductors. The breakdown voltage was estimated to be greater than 6KV. A 0 - 5KV power supply was used to charge the capacitor through a 10 meg ohm resistor. Aluminum plates (cross section 15cm x .2cm) with a minimum of spacing between them, to minimize inductance, were used for conductors between the capacitor and the spark gap. The electrode wires for the gap were 10 mil tungsten wire, and the gap could be varied from 0 - 1.5cm. Provisions were made for monitoring voltage and current during discharge. The current was measured using a toroid through which the discharge current passed, and the aid of a simple integrating circuit shown in Figure 6, that was effective down to 100 Hz.

A typical discharge current is shown in Figure 6b. It is characterized by an underdamped oscillation with
(a) Current Measurement Circuit

(b) Current Waveform

Figure 6. Spark Discharge Current
a frequency of approximately 300kHz. The self inductance of the circuit was determined to be approximately .35μh and the spark gap resistance was in the order of 1Ω.

The efficiency of conversion from electrical energy to acoustic energy has been found [7] to be 1.2% for sparks where discharge was initiated by reaching the breakdown voltage of the gap. Higher, or lower efficiencies are possible if different triggering methods are used. Efficiencies of energy conversion for the spark are comparable to those of common explosions. Typical values for explosions are given as .85 - 2.85% [13].

III.2 Triggering Methods

There are basically four methods used for triggering the electrical discharge of the spark circuit. These four methods are shown schematically in Figure 7. The first method, 7a, used by Wright [7], is the simplest of the four schemes. The voltage across the capacitor, c, is simply increased until the breakdown voltage of the gap is reached.

The method shown in Figure 7b was used for earlier work at MIT [1],[2]. The voltage across the capacitor is set to a higher value than the breakdown voltage and a trigger module is used to complete the circuit. The trigger module is operated by a second high voltage pulse supply.
Figure 7.
Spark Discharge Triggering Circuits
Figure 7. (cont.)
Spark Discharge Triggering Circuits
This trigger module adds considerable resistance to the discharge circuit, however, and therefore, decreases the efficiency of energy conversion. The discharge current in this circuit will generally not be of an oscillatory nature, but is characterized by rise time associated with inductance and capacitance of the circuit and a current decay related to RC.

The third circuit shown in Figure 7c, was used by the author for the experimental results presented in this thesis. It offers several advantages over the other methods. A third electrode is placed near the center of the gap to provide an initial ionization of the air in the gap. A low energy high voltage pulse source provides the energy to create a very weak spark from the trigger electrode to one of the discharge electrodes, thus ionizing a region of air in the gap. The storage capacitor is kept charged at a voltage lower than the breakdown voltage of the gap. All of the energy in the capacitor is dissipated in the gap since no additional resistance is added. This triggering method offers the distinct advantage of allowing the discharge voltage to be varied over a considerable range without altering the geometry of the gap. It was found that discharge voltages in the range 530 - 6000 volts could be used with a gap of .4cm.
The energy stored in the circuit is given by

\[ E = \frac{1}{2} CV^2, \]  

(36)

where \( V \) is the voltage measured across the capacitor. This implies that the voltage would be a most useful parameter for controlling the stored energy since \( E \) is proportional to the first power of capacitance. The voltage range indicated above corresponds to a change in energy by a magnitude greater than 100. This circuit also facilitated the study of the effect of changing the gap dimension while holding the voltage, capacitance and thus the energy constant.

The final method of triggering was used in work by von Békésy [14]. It is simply a mechanical means of triggering, and should provide similar experimental results as the method illustrates in 7a. This method is shown in Figure 7d.

III.3 Acoustic Characteristics

The spark discharge has proven to be an acoustic impulse source with repeatable properties. For a circuit of specified capacitance, voltage, and spark gap, the waveform geometry and energy spectrum at a given distance can
be repeated with a very high degree of accuracy. Experimental results have shown that standard deviations of less than one dB in frequency bands of 2kHz from 4kHz to 160kHz can be expected for the energy spectrum.

The spark provides a nearly perfect onmidirectional directivity pattern in the equatorial plane of the spark. This topic will not be studied in detail here, however. Directional characteristics of the spark have been studied by other researchers [15].
IV. EXPERIMENTAL INVESTIGATION

IV.1 Signal Processing

The signal processing system used for the experimental investigation of the generated pressure pulse is shown schematically in Figure 8. The pressure pulse is initially detected by a 1/10" BBN piezoelectric crystal microphone which has a flat frequency response from 100Hz to 150kHz, and a sensitivity of -114dB re 1 volt/μbar. The signal from the microphone is stored digitally by a transient recorder which has interfacing capabilities for computer processing of the stored signal. Triggering of the transient recorder is achieved by detecting the electromagnetic radiation from the spark. The stored pressure signal is simultaneously displayed on an oscilloscope.

To obtain the energy spectrum of the pressure signal the interfacing capabilities of the transient recorder are utilized. The signal is read by an Interdata 70 computer and operated on by a Fast Fourier Transform. The results of the F.F.T. are then converted into an energy spectrum in 2kHz frequency bands. The output of the computer is given by a teletype where the energy levels for frequencies between 2kHz and 160kHz are given in dB re .0002μbars. Several spectra may be averaged by the com-
Figure 8.
Signal Processing Instrumentation

-40-
puter and standard deviations for each frequency band calculated.

IV.2 Analysis and Experimental Results

A theoretical study of ideal acoustic N waves has been presented previously. In this section the justification for applying this theory to the acoustic radiation from electrical sparks will be presented, utilizing the results of an experimental study of the spark. Each of the theoretical observations made previously will be studied individually and as they affect each other. Some modifications to the theory will be necessary to account for the difference between the pressure pulse actually produced by the spark and the ideal N wave used in the development of the theory.

The pressure signature produced by an electrical spark discharge is characterized by a pulse closely resembling the ideal N wave of Figure 3, and a resulting energy spectrum of Figure 5. A typical pulse and its spectrum are shown in Figures 9 and 10, respectively. An accurate record of the initial shock front was not possible using the equipment available. The rise time of the shock is generally shorter than the mechanical rise time of the microphone. Also, low damping, characteristic of crystal
Figure 9.
N Wave From A 4.5 J Spark At 1.5 Meters
microphones, resulted in overshoot and ringing at the natural frequency of the microphone (200kHz). Therefore, this portion of the waveform must be observed with caution. A critical difference in this waveform and the ideal N wave is that it is not characterized by a zero crossing point exactly midway between the initial shock front and second shock front. This time, $T_1$, is critical since the peak frequencies of the spectrum are directly related to this time. Small changes in $T$ can make substantial changes in the resulting energy spectrum. It is convenient to define a new time $T_1$ using the actual spectrum resulting from the pulse. This time is best determined by noting the location of the first minimum or second maximum of the spectrum. For example, if the first minimum occurs at a frequency, $f_1$, then by Equation (21) and Table 1

$$T_1 = \frac{4.5}{2\pi f_2}.$$  

Using this time, it was found that the entire geometry of the spectrum could be calculated quite accurately. Experimental results show that generally good results for $T_1$ can be obtained using the empirical relationship $T_1 = 1.14T$. Other differences in the pulse appear to have had little effect on the spectrum geometry. Only this modification in
the characteristic time, T, was necessary to achieve good agreement with theory.

The 6dB per octave lower frequency asymptote and the -6dB per octave upper bound for the high frequencies, agreed well with the ideal N wave theory. This is indicated in Figure 10.

Experimental results support the statement that the acoustic properties of a spark produced N wave are dependent only on the energy released by the spark and the efficiency of converting this energy to acoustic energy. That is, any combination of capacitance and voltage resulting in the same value of the product, CV^2, will yield the same N wave. The efficiency of conversion from electrical energy to acoustic energy is governed by the size of the spark gap. The efficiency of conversion increases with gap dimension. This efficiency would be expected to vary linearly with the doubling of spark gap dimension, d, (i.e. with the log(d)). Experimental results agreed with this observation as is shown in Figure 11.

From the results of theory presented earlier, the effects of finite amplitude to second order, predict that the N wave will elongate as it propagates and its peak overpressure will decrease with distance. More specifically, the points of the profile will distort proportional to the log
Figure 11
Energy versus Spark Gap Dimension
-46-
of distance and the amplitude will decrease at a rate somewhat faster than $1/r$ normally associated with spherical spreading. Figure 12 illustrates the results of recording the pressure profile of a 4.5 joule spark at distances from .5m to 2.5m from the spark. The elongation process is evident from these traces, and they show that the shock of the rarefaction phase does not completely form. The initial shock front, commonly called the head shock does not appear to change its rise time as propagation proceeds. This observation is consistent with theory since effects of high frequency attenuation by the air become important only at greater distances than those recorded here. Examination of N waves recorded at greater distances indicate the expected increased rise time.

From Equation (18) we see that, if the square of the half period, $T$, is plotted versus the $\ln(r)$, where $r$ is the distance from the spark, then a straight line would be expected. This has been done in Figure 13 for 4.5J and 12.5J sparks, and a least squares curve fit for the data has been performed. By finding the least squares fit for this data, an experimental means of determining the constant, $k$, in Equations (20), (21), and (22), has been found without the need to measure $p_S$ directly. Therefore, information
Figure 13.
N Wave Half Period Versus Propagation Distance

-49-
about the half period, $T$, as the wave propagates is sufficient to uniquely determine $p_s$ as a function of $r$.

Combining Equations (20) and (22) gives the relationship between, $\delta = \frac{p_s}{p_o}$, and the propagation distance $r$.

$$\delta = \frac{2\gamma k C_o T_o}{\{ (\gamma + 1)r[1 + k\ln\left(\frac{r}{r_o}\right)]^{1/2} \}} \quad (38)$$

The constant $k$ is found by comparing Equation (21) with the least squared fit to the data.

Figure 14a shows how data for $p_s$ was taken form the oscilloscope traces, to account for the mechanical response of the microphone. Data taken in this manner showed excellent agreement with the values predicted by Equation (38). Equation (38) and the data points are plotted in Figure 14b. Such close agreement between theory and data suggests strongly that our previous suspicion of an underdamped impulse response of the microphone was correct. These results have also been verified for weaker sparks, .006 - .4J [7].

Hopkinson blast scaling has been widely accepted as an accurate means of predicting $N$ wave characteristics from explosive discharges much greater in magnitude than the sparks considered here. Its proven validity over large
Figure 14.
Peak Overpressure Versus Propagation Distance
ranges of explosive energy [4], suggests that it would also prove valuable in the study of the spark. Hopkinson blast scaling was verified for sparks with energies between 2J and 12.5J. A reference distance of \( r_1 = 1 \) meter was chosen for a spark of 4.5J of energy. The energy of the spark was changed and the resulting pressure pulse was recorded at a distance of \( r_1 (E_2/E_1)^{1/3} \). According to theory, the recorded pulse should have an overpressure, \( p_s \), the same as the 4.5J spark recorded at 1 meter and a time, \( T \), given by \( T (E_2/E_1)^{1/3} \). The results illustrated in Figure 15, show good agreement with this theory.

The theory that has been verified for sparks in this section provides some very useful tools for predicting the acoustic properties of spark produced N waves. Propagation distortion describes how the geometry of the N wave changes as it propagates. Hopkinson scaling provides a link between sparks of different energies, and Fourier theory relates the N wave geometry to frequency composition and energy content.

To the acoustician interested in using the spark as an acoustic source, the major concern is with the energy spectrum produced and not the geometrical properties of the N wave. It is most convenient, however, to describe the production and propagation of sound from the spark in terms of the profile, rather than the energy spectrum. The nece-
Figure 15
Hopkinson Scaling

-53-
sity of the link between these two descriptions, is of great importance to the success of this investigation. The ultimate goal, of achieving an accurate means of predicting the energy spectrum at a location produced by a spark of arbitrary energy, relies on coordinating all of these verified theories. Experimental results have demonstrated that this is possible, and an example is included in Appendix B. Nonlinear propagation of the N wave can have serious effects on common acoustical measurements of the spark, and this problem will be considered in Appendix A.
V. CONCLUSIONS

The formation of an acoustic N wave from an electrical spark is explained by wave steepening, described by non-linear acoustic theory, and the effects of spherical spreading on an impulsive compressional source. The second order effects of acoustic theory predict, that the formation and distortion of the N wave will proceed at a rate proportional to the logarithm of the propagation distance. This distortion results in an energy spectrum that shifts toward lower frequencies as the wave propagates.

Fourier theory shows that the high frequency energy of the N wave will observe an upper bound of a -6dB per octave slope and low frequencies will tend to a 6dB per octave asymptote. The low frequency energy magnitude is proportional to the product of the overpressure squared, $p_s^2$ and the half period to the fourth power, $T^4$. The magnitude of the high frequency energy bound is proportional to $p_s^2$ and independent of the half period.

The spark provides a source of acoustic energy that is repeatable over large ranges of energy that are most easily selected by adjustment of the voltage. The spark gap dimension, d, can also be used as a means to vary the acoustic energy from the spark since the efficiency of
electrical to acoustical energy conversion increases at a rate proportional to \( \log(d) \). In the equatorial plane of the spark the directivity characteristics have shown to be omnidirectional.

Experimental results have shown that N shaped pulses produced by the spark behave as predicted by theory. The ability to use this theory in its entirety, as a tool for predicting the energy spectrum from sparks of different energies and over large ranges of propagation distance, is of primary importance. Experimental data indicate that this is possible.
APPENDIX A

EFFECTS OF NONLINEAR PROPAGATION ON ACOUSTIC MEASUREMENTS

This appendix will be devoted to examining the effects of nonlinearity on some common acoustic measurements of spark produced acoustic pulses.

The spherical spreading N wave produced by a spark distorts as it propagates due to finite amplitude effects normally omitted in linear acoustic theory. This theory has been worked out in detail [6] and the results are presented here to second order.

\[
\delta = r_o \delta_o / \left[ r \left( 1 + k \ln (r/r_o) \right) \right]^{1/2}
\]

\[
T = T_o \left( 1 + k \ln (r/r_o) \right)^{1/2}
\]

\( \delta \) is defined by the ratio \( p_s/p_o \), where \( p_s \) is the peak overpressure of the N wave and \( p_o \) is the ambient pressure. \( T \) is the time from the initial pressure front of the N wave to the zero crossing of the wave near its midpoint. The constant, \( k \), is a characteristic of the spark source being used and can be calculated experimentally. It is defined by the expression,

\[
k = \frac{(\gamma + 1) r_o \delta_o}{2 \gamma c_o T_o},
\]
where $\delta_o$ is the value of $\delta$ measured at a reference position, $r_o$, and $T_o$ is the value of $T$ measured at this position. $c_o$ is the ambient sound velocity. $\delta_o$ and $T_o$ may also be obtained from the energy spectrum. This will be shown shortly.

The above equations show how the geometry of the N wave distorts as it propagates. By relating this geometrical distortion to the energy spectrum, some insight can be gained about acoustic measurements of the N wave.

The energy spectrum is defined as $|P(f)|^2$ where $p(f)$ is the Fourier transform of $p(t)$. That is,

$$P(f) = \int_{-\infty}^{\infty} p(t)e^{-i2\pi ft}dt \quad (4a)$$

This gives the result

$$|P(f)|^2 = 4P_s^2T^2(j_1(2\pi Tf))^2 \quad (5a)$$

which simplifies, for higher frequencies to,

$$|P(f)|^2 = \frac{P_s^2}{\pi^2f^2} \cos^2(2\pi Tf) \quad (6a)$$

$j_1$ in Equation (5a) is the spherical Bessel function of the first kind. For high frequencies the energy spectrum is
characterized by an envelope that declines at a rate of
6dB. per octave. Equation (6a) can also be expressed as a
function of propagation distance using Equation (1a) to give
the result,

$$|P(f)|^2 = \frac{r_o^2 \delta_o^2}{\pi f^2 r^2 [1 + k \ln\left(\frac{r}{r_o}\right)]} \cos(2\pi Tf) \quad (7a)$$

From this we see that the envelope of the energy spectrum
decreases in magnitude as a function of distance, $r$, given
by

$$\Delta dB = -20 \log\left(\frac{r}{r_o}\right) - 10 \log[1 + k \ln\left(\frac{r}{r_o}\right)] \quad (8a)$$

The first term is associated with spherical spreading,
while the second term is an added excess attenuation due
to nonlinear effects. This term can be plotted versus
distance for different $k$ values to obtain a graph of excess
attenuation from nonlinear effects. This has been done in
Figure 1a for a reference position, $r_o$, of 1 meter, and
several $k$ values.

The value of $k$ can be obtained experimentally in
several ways. The simplest way, mentioned previously,
is to record the N wave at a distance $r_o$ from Equation (3a).
Figure 1a

Excess Attenuation from Nonlinear Propagation
The simplicity of this method is appealing, but it can often lead to inaccurate results. Some difficulty in obtaining an accurate value of $p_s$ can be experienced because of inadequate high frequency response or problems related to the transient response of the microphone (e.g. piezoelectric microphones). The second method that may be used for determining $k$, utilizes Equation (2a). If the value of $T$ is recorded at several distances from the spark and $T^2$ is plotted versus $\ln(r/r_o)$ a straight line should result. A least squares straight line can be fitted to this data and then compared with Equation (2a) to obtain $k$. Finally, $k$ may be found by examining the energy spectrum from the spark recorded at $r_o$. $T_o$ is simply obtained by observing that the first minimum of the spectrum occurs when

$$j_1(2\pi T f) = 0$$

or

$$T = \frac{4.5}{2\pi f}$$

or

(10a)

The value $p_s$ and hence $\delta_o$ is obtained from the spectrum by using the magnitude of the -6dB per octave slope for high frequencies and applying this to Equation (6a).

The one-third octave band envelope spectrum can be expressed as,
\begin{align}
\frac{ps^2}{\pi^2f} \cdot 0.23 \\
\frac{ps^2}{\pi^2f} \cdot 0.71
\end{align}

(11a)

and the octave band envelope spectrum can be expressed as,

\begin{align}
\frac{ps^2}{\pi^2f} \\
\frac{ps^2}{\pi^2f}
\end{align}

(12a)

for high frequencies. Each of these is characterized by a slope of -3dB per octave.

Exact determination of the constant \( k \) has been shown to be unnecessary for measurements taken from 1 to 10 meters from the spark since nearly all spark sources used for model studies have \( k \) values of 0.2 to 0.3 for \( r_o = 1 \) meter. A maximum error of approximately 1/4dB could result by choosing \( k = 0.25 \).

The basis for determining excess attenuation due to nonlinear effects has been founded entirely on the high frequency envelope of the spectrum rather than the spectrum itself. This assumption places some limitations on the size of the bandwidth that may be used for measurements. Figure 2a shows the energy spectrum of a 4.5J spark measured in 2kHz bandwidths from 8 to 50kHz. The detail of the spectrum revealed by taking measurements in such narrow bandwidths prohibits the use of this theory. Notice the first minimum of the spectrum located at approximately 25kHz. In this
Figure 2a.
Nonlinear Propagation Effects On A 4.5J Spark
Measured In 2kHz Bandwidths

-63-
2 meter propagation distance it shifts 4kHz down to 21kHz and a maximum point now occupies the 25kHz region. This could obviously lead to invalid measurements.

Figure 3a shows the energy spectrum of a 4.5J spark recorded at 1.5 meters. It is measured in 2kHz constant bandwidths, one-third octave bands, and octave bands. The usefulness of considering the envelope is evident when examining the one-third octave and octave band measurements. Each exhibits the -3dB per octave slope predicted by theory, and any deviations from this line indicate the maximum uncertainty that could be expected when taking measurements in constant percentage bandwidths.

When dealing with the measurement of frequencies that are of interest in model studies and propagation distances as large as 10 meters, the effect of air absorption may appear to play an important role. It has been shown, however, [16] that typical spark energies used for model studies produce N waves of sufficient amplitude that all dissipated energy is essentially accounted for at the wave front and the geometry of the N wave is preserved. This dissipation at the wave front is accounted for in the propagation Equations (1a, 2a) presented previously. Therefore, the resulting spectrum always obeys the -6dB per
Figure 3a
Energy Spectrum for a 4.5J Spark at 1.5 M
octave slope in the high frequencies. Attenuation due to nonlinear propagation effects need only be considered. As is shown in Figure la this is nearly a constant attenuation per doubling of distance for typical k values. For k = .25 there is an excess attenuation of approximately .6dB per doubling or a total attenuation of 6.6dB per doubling in the high frequencies.
APPENDIX B
AN EXAMPLE CALCULATION

A sensitive test of the theory that has been presented is to apply it in its entirety. That is, if the N wave, or energy spectrum produced by a single spark is recorded at a known distance, then the N wave and energy spectrum produced by a spark circuit of any voltage and capacitance, and recorded at any distance can be determined. As an illustration of how this calculation is performed, and as a final test of the theory, a reference spark of 4.5J recorded at 1 meter will be used to predict the acoustic properties of a 12.5J spark at 2 meters.

In Figure 1b the N wave and resulting energy spectrum are shown for a 4.5J spark at 1 meter. From this we obtain \( P_{s1} = 300 \, \text{nt/m}^2 \) and \( T_{o1} = 28\mu\text{sec} \). Since the energy spectrum is of particular interest in this case, \( T_{11} \), the half period of the N wave associated with the spectrum must be obtained. To calculate \( T_{11} \) from the spectrum, it is best to choose the first minimum or second peak since the first peak is too broad to choose a definitive maximum. The first minimum results when

\[
2\pi f T_{11} = 4.5
\]

\( T_{11} \) is therefore 31.8\( \mu\text{sec} \)
Figure 1b.
N Wave And Energy Spectrum For A 4.5J Spark At 1m
If Hopkinson scaling is now applied to the problem, we see that at a distance given by

\[ 1 \text{ meter} \left(\frac{12.5}{4.5}\right)^{1/3} = 1.406 \text{ meter.} \]

an N wave will be observed with the following features.

\[ P_{s2} = 300 \text{nt/m}^2 \]

\[ T_{o2} = \left(\frac{12.5}{4.5}\right)^{1/3} 27.5 \mu\text{sec} = 38.7 \mu\text{sec} \]

\[ T_{12} = \left(\frac{12.5}{4.5}\right)^{1/3} 31.8 \mu\text{sec} = 44.7 \mu\text{sec} \]

This information gives a reference N wave for the 12.5J spark.

From Equation (22), \( k = .269 \) and \( \delta, T_{o2} \) and \( T_{12} \) are given by

\[ \delta_2(r) = \frac{.422}{r \left[1 + .269 \ln\left(\frac{r}{140.6}\right)\right]^{1/2}} \]

\[ T_{o2}(r) = 38.7 \left[1 + .269 \ln\left(\frac{r}{140.6}\right)\right]^{1/2} \]
\[ T_{12}(r) = 44.7\left[1 + 0.269\ln\left(\frac{r}{140.6}\right)\right]^{1/2}. \]

Where \( r \) is measured in cm. At 200 cm these become

\[ \delta_2 = 0.002 \]

\[ T_{o2} = 40.5\mu\text{sec} \]

\[ T_{12} = 46.8\mu\text{sec} \]

The N wave and energy spectrum from a 12.7 J spark measured at 200 cm are shown in Figure 2b. The actual results give

\[ \delta_2 = 0.0018 \]

\[ T_{o2} = 39\mu\text{sec} \]

and the spectrum calculated from \( T_{12} \) is shown in Figure 2b.

Some simple observations about the spectrum can be made without plotting the entire spectrum. Using Equations (31) and (33) to predict the high and low frequency behavior gives the following results. For low frequencies

\[
\frac{|P_2(f)|^2}{|P_1(f)|^2} = \frac{(0.002)^2(40.5)^4}{(0.003)^2(28)^4} = 1.95
\]
Figure 2b

N Wave And Energy Spectrum For A 12.5 J Spark At 2 m
\[ 10 \log\left(\frac{|P_2(f)|^2}{|P_1(f)|^2}\right) = 2.9 \text{ dB} \]

Therefore, the low frequencies should be 2.9 dB above the reference spectrum of the 4.5J spark.

For high frequencies we have the high frequency upper bound given by

\[ \frac{|P_2(f)|^2}{|P_1(f)|} = \left(\frac{0.02}{0.03}\right)^2 = .444 \]

\[ 10 \log\left(\frac{|P_2(f)|^2}{|P_1(f)|^2}\right) = -3.5 \text{ dB} \]

The change in level of the first peak is given by Equation (29).

\[ 10 \log\left(\frac{\delta_2^2}{\delta_1^2}\right) = -.2 \text{ dB} \]

The ability to accurately predict the amplitude is hampered somewhat by the second and fourth power dependency on \(T\). Trends in high, low, and middle frequencies behave as they are predicted, but the absolute change in levels are difficult to predict within an accuracy of 1 dB. Accuracy in
predicting locations of frequency maximums and minimums is quite good, however.
REFERENCES


