OUT-OF-PLANE DISTORTION CAUSED BY FILLET WELDS IN ALUMINUM

by

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B.S., Escola Politécnica da Universidade de São Paulo (1961)

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Submitted to the Department of Ocean Engineering in August, 1972, in partial fulfillment of the requirements for the degrees of Master of Science and Ocean Engineer.

ABSTRACT

Out-of-plane distortion caused by angular changes at fillet welds on aluminum structural panels was analysed in two steps, namely, the experimental and the analytical.

In the experimental step, a series of data were obtained from test specimens with varying thicknesses, which were fillet welded to either a single frame or to three equally spaced frames, simulating, therefore, a free joint and a constrained joint case, respectively.

For this experimental stage, the results of the tests were analysed, considering the constrained joint case as a problem of inherent stresses originating in a statically indeterminate frame structure, and utilizing the plane stress-plane strain theory in its simplest form in order to relate the free joint angular distortion with the out-of-plane distortion of the constrained joint panels.

The results so far obtained were then compared with those available for steel panels and it was found, among other differences, that although the distortions of the aluminum panels were larger when the same amount of consumed weld metal is considered, the steel panels have shown larger distortion if the same fillet size is taken as a parameter.

In the second step, then, this behavior of the aluminum was studied, and an analytical investigation concerning the free joint angular distortion was developed, assuming that the distortion will be caused by a point heat source moving in a straight line, as the electrode tip would do.
The theoretical analysis was based on the incremental strain method theory and the thermal, elastic-plastic angular deformations were estimated for the aluminum and steel panels for the simpler case of the free-joints, because the free-joint angular distortion, as it is shown in the first part of this work, is directly related to the distortion of the constrained joint panels.

Thesis Supervisor:  Dr. Koichi Masubuchi

Title:  Professor of Naval Architecture
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1.0 - INTRODUCTION

1.1 - General

Aluminum alloys are one of the most promising and versatile engineering and construction materials due to their peculiar characteristics including light weight, corrosion resistance, electrical and thermal conductivities, workability, low temperature ductility, etc.

For the particular case of the marine application the 5XXX series of weldable magnesium-manganese aluminum alloys constitute themselves in a high corrosion resistant and high strength material, having a very good weld zone ductility and also being easily formable.

However, the use of the aluminum alloys in fabricated welded structures are by no means trouble free. One of the most important problems concerning aluminum welded structures is that related to residual stresses and thermal distortion caused by the welding processes.

It is well known that the control of the distortion in aluminum structures are usually more critical than that for the steel structures due to problems like:

a) Lower stiffness of the aluminum structures;

b) Larger plastic zones near the weld due to lower melting point and consequently lower yield strengths;

c) Larger thermal expansion coefficient and higher heat conductivity.

All these problems can come up in a combined manner so as to produce severe shrinkage and distortion in a fabricated structure, decreasing therefore its reliability, since the existence of those distortions indicates that there are highly concentrated stress zones which can cause
problems in the future, like cracks in the weldments or even the buckling of any compressed member if compressive residual stresses are present.

Due to all these factors, it is very important to conduct a rational and analytical study concerning the ways by means of which those stresses and distortions are developed, so that the results obtained from such study can serve as a basis for the establishment of methods as well as standards to be of general use by the industries.

1.2 - Scope of this study

Although many investigations have been carried out concerning the behavior of the aluminum welded structures, not so much data were available on the study of the deformation of framed aluminum panels.

By framed panels are understood such structures encountered typically in ocean as well as in aerospace vehicles, and that is composed of a flat plate to which longitudinal and transverse stiffeners are welded, as shown in Figure 1.2.1.

When those stiffeners are fillet welded to the plate, out-of-plane distortion of the mother plate occurs, which vary in both x and y directions.

Due to the mathematical difficulties in carrying a two dimensional analysis to study those deformations, most of the investigations conducted so far, the majority of them for steel, are one-dimensional, considering the deformation only in one of the directions of the plate, as is typically the case of the model shown in Figure 1.2.2.

However, at present, several two-dimensional analyses are being carried out by means of techniques such as the finite element method,
Fig. 1.2.2 Distortion due to fillet welds in two types of one dimensional models.
using high capacity computer systems.

a) Experiments

The aim of this thesis work, then, is to develop some means to predict, as accurately as possible, the out-of-plane distortion of aluminum framed panels, still using a one-dimensional approach to simplify the problem, because this kind of study using aluminum as a basic material was not yet extensively developed.

For this purpose a method similar to that developed for steel by Masubuchi et al. was followed in order that the results obtained could be compared with those available for steel on a more rational basis.

The method that will be discussed in the next chapter consists in carrying out a series of experiments with aluminum panel of different thicknesses with free and constrained joints with varying spans, and in the recording of the values of the distortion occurring during the sequential welding passes.

These out-of-plane distortions are then related to the angular changes occurring at the weld fillets, considering the problem as one of inherent stresses developed in a statically undetermined frame structure, and using the simple plane stress-strain theory for a unit width beam.

Next, by means of the principle of minimum potential energy, the free-joint angular changes are going to be related to the correspondent constrained joint angular changes, so that as a result, the so-called "coefficient of rigidity for angular changes" is obtained.

These coefficients and the values of the free-joint angular changes will then allow to estimate for each thickness and each span, the
final out-of-plane deflection of the panel structure.

b) Analysis of the results

The results thus obtained will then be compared with those available for steel panels for it was realized that there are many fundamental differences between them. Some of the differences follow:

i) Angular changes in aluminum panels are larger than those for steel if they are compared having the weight of consumed weld metal as the parameter.

ii) However, when the fillet size is taken as the parameter, generally, the steel panels show larger angular distortion for both free and constrained joints.

iii) Aluminum panels had the occurrence of the maximum angular distortions for free-joints at thicknesses varying between 6 and 8 mm (about 1/4" to 5/16"), while the steel had those maximum values between 8.5 and 9.5 mm (about 11/32" to 3/8").

iv) The occurrence of the maximum angular distortion with increasing amounts of consumed weld metal goes towards thinner plates for aluminum, while just the opposite happens for steel.

All these differences are difficult to explain using only physical reasonings since there are many variables involved in the welding process.

Therefore, in order to try to obtain some analytical means with which the above differences, or part of them, could be explained, a theoretical analysis using the incremental strain theory, taking into account the thermal elastic-plastic deformations, was developed.
The model used in this analysis, as shown later, is simply a plate strip of unit width that is heated and then cooled in order to obtain the different steps of the thermal history to carry out the computation.

The temperature profile with respect to time, then, was obtained assuming a point heat source moving in a straight line, simulating the motion of the weld tip.

The temperature distribution problem, in turn, was treated as a three-dimensional heat flow in a plate with finite thickness, in which the heat radiation from the surface was neglected.

The results of the computation actually showed that for ordinary welding conditions, the steel panels will have larger free-joint angular distortion than the aluminum panels.

It was also realized that there is a strong influence of other variables like the weld fillet shrinkage, the actual cooling rate of the weldment, the reaction of the plate against the shrinkage of the fillet, etc., as will be discussed at the final part of this thesis work.

Although the results obtained can be used only as a comparative basis between the materials considered in this study, it should be possible to develop a more elaborate and complete treatment of the subject. The results that would be obtained from this analytical investigation can actually be a very good and reliable estimation of the real deformation occurring for such a type of panel structures.
1.3 - Sequence of the study

The sequence of the steps followed in this thesis work is shown in the simplified flow chart below:

START

PREPARATION AND EXECUTION OF THE EXPERIMENTS

ANALYSIS OF RESULTS AND DETERMINATION OF THE COEFFICIENT OF RIGIDITY FOR ANGULAR CHANGE "C"

COMPARISON OF THE RESULTS OBTAINED FOR ALUMINUM WITH THOSE AVAILABLE FOR STEEL

DEVELOPMENT OF AN ANALYTICAL STUDY TO CHECK THE BEHAVIOR OF THE FILLET WELDED ALUMINUM STRUCTURAL PANEL

PREPARATION OF A COMPUTER PROGRAM TO CARRY OUT THE CALCULATIONS

ANALYSIS OF RESULTS

CONCLUSIONS

END
2.0 - EXPERIMENTAL DETERMINATION OF THE ANGULAR CHANGES
IN FILLET WELDED ALUMINUM STRUCTURAL PANELS

2.1 - General

When a one-dimensional panel structure like the one shown in the previous Figure 1.2.2 is fabricated by means of welding, a complex system of thermal stresses and consequently strains are developed, so as to produce residual distortion in the plate which can take the form of simple knuckles at the fillet in the case of free-joints (Figure 1.2.2.a) or an arc-form type deformation in the case of constrained joints (Figure 1.2.2.b).

From the engineering standpoint, what interests in the practice is a means to predict, as accurately as possible, the final maximum distortion of a fabricated structure subjected to weldments in terms of the several variables involved in the process.

These distortions, as explained earlier, are going to influence the general strength and stability of the whole structure, that can ultimately affect its reliability.

There were two options to study this problem, one of which being the development of a theoretical analysis and trying to find a suitable method of predicting the distortion of the structure, and the other carrying a set of experiments at first, and then trying to relate the different variables involved in the problem with the final values of the distortion, analysing the results obtained from the experiments.

In the present case, the second alternative was chosen, since apparently no systematic study concerning this type of investigation on
aluminum was carried out yet, and so few data are available at present to serve as a basis of comparison between the results that would be obtained from a theoretical analysis and those recorded from actual tests.

In this chapter a description of the experiments performed will be done, followed by an analysis of results trying to correlate the variables involved in the process. Finally, a brief comparative study with the data available for steel will be presented.

2.2 - Preparation of the experiments

a) Supporting jig

Figure 2.2.1 shows schematically the jig assembly used to carry out the experiments.

In order to obtain enough rigidity at the frame supports, a very stiff steel I beam was used as the supporting structure, so that the distortions recorded were actually those developed only due to the thermal strains.

Also special care was taken to avoid the relative motion between the frames and their steel supports, because this could sensibly affect the results of the experiments.

All the assembling was mounted on a steel working table and horizontally levelled so that the starting position of the test specimens could be easily settled, simply by levelling the panel plate.

The recording of the distortion was done by means of two dial gages placed underneath the previously marked centers of each panel span, since in the present case the only interest was to measure the maximum transversal deflection of the panel and relate it to the angular changes
a. GENERAL ASSEMBLING FOR THE EXPERIMENTS

b. DETAILS OF FIXATION OF THE TEST SPECIMENS

JIG ASSEMBLING FOR THE EXPERIMENTS

FIG. 2.2.1
occurring at the fillet joints.

b) Material and material properties

The material used in the experiments was the strain hardened, aluminum magnesium structural alloy 5086-H32, largely utilized in marine and general structural application.

Its characteristics and properties are as follows:

i) Chemical composition (typical):

Mg : 3.5 to 4.5%  \hspace{1cm} Cu : 0.10% max  \hspace{1cm} Zn : 0.25% max
Mn : 0.20 to 0.70%  \hspace{1cm} Si : 0.10% max  \hspace{1cm} Ti : 0.15% max
Cr : 0.05 to 0.25%  \hspace{1cm} Fe : 0.50% max  \hspace{1cm} Al : remainder

ii) Physical properties:

Density \hspace{1cm} 2.65 \text{ g/cu.cm} \ (0.096 \text{ lb/cu.in})
Liquidus Temperature \hspace{1cm} 640 \degree \text{C} \ (1184 \degree \text{F})
Coefficient of Linear Expansion \hspace{1cm} 23.9 \times 10^{-6} \degree \text{C}^{-1}
(between -60 to 300 \degree \text{C} (-76 to 572 \degree \text{F})))
Thermal Conductivity \hspace{1cm} 0.30 \text{ cal/sq.cm/cm/sec/\degree C}

iii) Mechanical properties:

Modulus of elasticity \hspace{1cm} 7,260 \text{ kg/mm}^2 \ (10.3 \times 10^6 \text{ psi})
Modulus of rigidity \hspace{1cm} 2,700 \text{ kg/mm}^2 \ (3.83 \times 10^6 \text{ psi})
Minimum tensile strength \hspace{1cm} (at room temperature) \hspace{1cm} 29.6 \text{ kg/mm}^2 \ (42,000 \text{ psi})
Minimum yield strength \hspace{1cm} (at room temperature) \hspace{1cm} 21.8 \text{ kg/mm}^2 \ (31,000 \text{ psi})
Poisson ratio \hspace{1cm} 0.334
About the filler metal, the recommended Filler - 5356 was used, which has the following chemical composition:

- Mn : 0.12 %
- Mg : 5.00 %
- Cr : 0.12 %
- Ti : 0.15 % max
- Al : remainder

c) Test specimens

Figure 2.2.2 shows the test specimens used in the experiments. The specimen shown in Figure 2.2.2.a refers to the one used in the free-joint case and the other shown in Figure 2.2.2.b was used for the constrained joint conditions.

Also shown in the Figures are the positions of the points where the distortion were recorded, as well as the welding sequence followed when the welding operations were carried out and the steps at which the recordings were taken up.

All specimens were 203.2 mm (8") wide and all the frame plates were 101.4 mm (4") high, 203.2 mm (8") wide, and 12.7 mm (1/2") thick in order to obtain enough rigidity of the frames when welding was performed.

The thicknesses and varying spans utilized in the experiments are described in the item (e) below.

d) Welding equipment

Only as a guide the specification of the welding equipment used in the experiments are presented in this item:

i) Power supply : Westinghouse Type R.S. Silicon Rectifier, Constant Voltage Power Supply for 500 Amps Max

ii) Wire Feeder : Airco Wire Feeder Model AHF-L(117 V/3A)

e) Number of tests

The tests were carried out for specimens with varying thicknesses as well as spans so that a total of 6 tests for free joint condition and 18 tests for the constrained joint case were performed.

To minimize the number of tests for the latter condition, suitable combinations of thicknesses and spans were chosen, so that such combinations considered somewhat improbable to be used in practice were neglected.

A summary of the conditions is presented in Table 2.2.1.

2.3 - Welding condition and test procedure

a) Welding condition

All tests were carried out with the same welding conditions in terms of voltage, current and wire feeding velocity. Their values were 23 volts, 155 amperes, and 26.951 g/min (given in weight of filler wire consumed per minute), respectively.

The electric arc so obtained was such that the welding could be performed smoothly and practically without noticeable spatters.

The wire feeder was set at the point E of the scale during the experiments and this position corresponded to the above value of the wire consumption rate.

However, the arc travel velocity was not constant for all tests,
READINGS CARRIED OUT AFTER PASSES 2, 4, 6, 8 AND TEN

WELDING SEQUENCE

12.7 mm (1/2")

101.6 mm (4")

609.6 mm (24")

LEFT READING STATION

152.4 mm (6")

LEFT READING STATION

RIGHT READING STATION

152.4 mm (6")

a. FREE JOINT TEST SPECIMEN

READINGS CARRIED OUT AFTER PASSES 6, 12, AND 18

12.7 mm (1/2")

101.6 mm (4")

50.8 mm (2")

LEFT SPAN 8

RIGHT SPAN 8

b. CONSTRAINED JOINT TEST SPECIMEN

ALUMINUM TEST SPECIMENS

FIG. 2.2.2
**TABLE 2.2.1**

TESTS PERFORMED WITH ALUMINUM STRUCTURAL PANEL MODELS

<table>
<thead>
<tr>
<th>Item</th>
<th>Plate thicknesses - (mm)</th>
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<tbody>
<tr>
<td></td>
<td>3.175(^{(1/8\text{&quot;})})</td>
</tr>
<tr>
<td>Free joint</td>
<td>Yes</td>
</tr>
<tr>
<td>Span (mm)</td>
<td></td>
</tr>
<tr>
<td>406.4(^{(16\text{&quot;})})</td>
<td>Yes</td>
</tr>
<tr>
<td>508.0(^{(20\text{&quot;})})</td>
<td>Yes</td>
</tr>
<tr>
<td>609.6(^{(24\text{&quot;})})</td>
<td>Yes</td>
</tr>
<tr>
<td>812.8(^{(32\text{&quot;})})</td>
<td>No</td>
</tr>
</tbody>
</table>
since a thicker plate requires more energy to be welded than a thinner one and consequently the arc velocity was reduced.

b) Test procedure

The welding sequence, as well as the steps at which the recordings were made are also shown in Figure 2.2.2. for both free and constrained joint cases.

The experiment consisted mainly in the recording of the times spent in each welding pass and in the measuring of the transversal distortion for each span (constrained joints), or each side of the panel (free-joints), at the end of the various steps of the test.

Since the wire feeding rate is constant, the time spent in each pass will give the value of the amount of metal consumed, that ultimately is related to the heat input of the welding process.

The non-availability of more sophisticated measuring equipment such as the arc-time-meter forced the use of an ordinary chronometer, so that the values recorded are relatively inaccurate, but it is believed not to substantially affect the results of the experiments.

In order that the readings were not affected by the cooling of the welded zone, a time lag of 5 minutes for the thinner plates and 10 minutes for thicker ones was left before the recording of the distortions. It was observed that those time lags were enough since when the recordings were taken the dial gages had already been still for a relatively long time.

In the free joint condition, the test specimen frame was fixed at the middle frame support of the jig. The initial as well as the
subsequent transversal distortions $\delta_1$ and $\delta_2$ were recorded for both sides of the frame at a fixed distance $l = 6$ inches from the middle line of the frame.

Figure 2.3.1.a shows schematically the configuration taken by the free-joint test specimen after the performing of the welds.

For the constrained joints the test procedure was practically the same with the main differences that now there were three fixed frames, spaced a distance $l$ apart, and the deformation of the plate was not in a polygonal shape as before, but in an arc form shape as shown in Figure 2.3.1.b.

The recording of the transversal distortion was performed after each step as indicated in Figure 2.2.2.

A small amount of back swelling was observed under the fillet bead in the thinner test specimens, but this fact, as also happened for the case of steel, was assumed to have negligible influence on the results of the tests.

2.4 - Results obtained for the free joint case

From the observed results and following the nomenclature indicated in Figure 2.3.1.a the free joint angular change $\theta_0$ and the amount of consumed weld metal are simply given by Equations (1) and (2) below.

Equation (1) was derived assuming that the panel will deform into a polygonal shape, the whole deformation being concentrated within a narrow zone near the weld fillet, so that the remaining of the panel will remain straight. This assumption is actually correct, since there
SCHEMATIC REPRESENTATION OF THE DISTORTION FOR FREE AND CONSTRAINED JOINT CASES

FIG. 2.3.1
are no external forces or moments applied on the structure, the only
stresses being those which originated from the self equilibrating resi-
dual thermal stresses at the fillet weld.

\[ \theta_0 = \frac{\delta_1 + \delta_2}{\ell} \quad (1) \quad \text{and} \quad W = \frac{r \times t}{60 \times b} \quad (2) \]

where \( \theta_0 \) = angular change at the fillet weld for each step (rad.)

(\( \theta_0 \) is assumed to be very small)

\( \delta_1 \) and \( \delta_2 \) = transversal distortion in the left and right span,
respectively (mm)

\( \ell \) = reading station coordinate from the center of the frame (mm)

\( W \) = amount of consumed weld metal per unit length of double
fillet (g/cm)

\( r \) = deposition rate (g/min) = 26.951 g/min (obtained
experimentally)

\( t \) = time spent to carry out each test step (sec)

\( b \) = test specimen width (cm)

Table 2.4.1. presents the results obtained from the experiments
for the case of the free joints, in a cumulative sequence, up to the step
No. 5.

Six plate thicknesses were tested, ranging between 3.175 mm
(1/8") and 19.050 mm (3/4"), and if a plotting of the values of \( \theta_0 \) against
\[ \log_{10} W \] is made, the graph shown in Figure 2.4.1 are obtained.

It is noticeable in Figure 2.4.1 that for a given value of
\[ \log_{10} W \], the angular change \( \theta_0 \) increases up to a point corresponding to
a thickness varying between 6 and 7 mm (about 1/4"), and then decreases
TABLE 2.4.1

RESULTS OF THE EXPERIMENTS FOR FREE JOINT CONDITION

<table>
<thead>
<tr>
<th>h (mm)</th>
<th>wt (g)</th>
<th>w (g/cm)</th>
<th>log₁₀w</th>
<th>8₀ (x10⁻³ rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.175 (1/8&quot;)</td>
<td></td>
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<tr>
<td>27.80</td>
<td>1.37</td>
<td>0.136</td>
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<td>2.47</td>
<td>0.393</td>
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<td>78.50</td>
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<td>0.586</td>
<td>70.8</td>
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<td>105.00</td>
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<td>108.2</td>
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<td>28.30</td>
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<td>0.844</td>
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<td>97.75</td>
<td>4.81</td>
<td>0.682</td>
<td>100.5</td>
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<td>130.00</td>
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<td>165.50</td>
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<td>0.911</td>
<td>143.0</td>
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</table>

Cont.
<table>
<thead>
<tr>
<th>h (mm)</th>
<th>wt (g)</th>
<th>w (g/cm)</th>
<th>( \log_{10} w )</th>
<th>( \Theta_0 \times 10^{-3} \text{rad} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.700 (1/2&quot;)</td>
<td>34.30</td>
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<td>0.890</td>
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<td>15.875 (5/8&quot;)</td>
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<td>69.50</td>
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<td>0.534</td>
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<td>5.02</td>
<td>0.701</td>
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<td>134.00</td>
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<td>0.820</td>
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<td>167.50</td>
<td>8.26</td>
<td>0.817</td>
<td>80.7</td>
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<td>5.45</td>
<td>0.736</td>
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<td></td>
<td>143.50</td>
<td>7.08</td>
<td>0.850</td>
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<td>182.20</td>
<td>8.98</td>
<td>0.954</td>
<td>51.9</td>
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</tbody>
</table>

h = plate thickness (mm)

wt = weight of consumed weld metal after each step (g)

w = weight of consumed weld metal / unit length of fillet (g/cm)

\( \Theta_0 \) = angular change at fillet (x\( 10^{-3} \text{rad} \))
VARIATION OF FREE JOINT ANGULAR CHANGE $\theta_0$ IN FUNCTION OF PLATE THICKNESS "h" AND CONSTANT $\log_{10} w$ FOR ALUMINUM

FIG. 2.4.1
for higher values of thickness.

Another point that is also noticed is the tendency of the maximum values of the angular changes to occur in the thinner side of the graph for increasing values of $\log_{10} W$.

These and other points considered of interest in this study will be discussed later, when the analysis of the results is carried out.

2.5 - Results obtained for the constrained joint case

The arc form deformation in the constrained joint case, as shown schematically in Figure 2.3.1.b, is also a consequence of the development of highly concentrated residual stresses near the weld zone. In this case, however, due to the existence of the constraints, there is in addition the presence of the so called reaction stresses which results in the constraining of the angular distortion caused by the angular change at the fillet weld.

It is interesting to recall here the difference between the terminologies "residual stress" and "reaction stress" encountered in problems of welding.

Residual stresses are those developed near the welded zone, being that their values are very high and they can originate even in the case of the free joints.

Reaction stresses, on the other hand, are originated only in the presence of some constraint, and they use to be uniformly distributed over the whole region of the welded joint, as observed for the case of the steel panels.

Therefore the arc form deformation suggests the presence of
the reaction stresses which induce bending moments at the weld joints, or in other words, at the ends of each panel span. Since there are no external forces acting on the structure and the distribution of the reaction stresses are observed to be uniformly distributed at the welded joints, the bending moment acting in each of the spans can be assumed to be constant.

Because of the fact that both spans are taken to be equal, it may be assumed for further simplification that if the weld fillets were all identical, both spans would have the same deformation, and consequently, the same angular and transversal distortion.

With these simplifications, the problem of determining the transversal distortion of this two span panel structure could be treated as one of inherent stress originating in a statically indeterminate frame structure, in which the angular changes occur at the ends of each span.

Figure 2.5.1 shows the panel distortion model and the distribution of the reaction stresses throughout the structure.

Due to the symmetry, the derivation of the equation relating the angular distortion to the transversal distortion in each span becomes very simple, considering the first order theory, in which all displacements, as well as strains, are assumed to be very small.

From the beam theory and using the nomenclature of the model shown in Figure 2.5.1.b,

\[
\frac{d^2 y}{dx^2} = \frac{d\theta}{dx} = - \frac{Mo}{EI}
\]

where

- \( Mo \) = constant bending moment acting on the beam
- \( E \) = modulus of elasticity
- \( I \) = moment of inertia of the unit width beam
a. DISTRIBUTION OF DISTORTION AND REACTION STRESSES

b. SIMPLIFIED MODEL FOR THE CALCULATIONS

PANEL DISTORTION AND REACTION STRESS DISTRIBUTION

FIG. 2.5.1
\( \theta = \) angular deformation at a point \( x \) from the origin

\( y = \) transversal deformation at a point \( x \) from the origin

Using the boundary conditions: \( y = 0 \) at \( x = 0 \) and \( x = \ell \)

and \( \theta = 0 \) at \( x = \ell/2 \),

the equations for \( \theta \) and \( y \) are obtained

\[
\theta = \frac{M \ell}{2EI} \left( 1 - \frac{2x}{\ell} \right) \quad (3)
\]

\[
y = \frac{M \ell}{2EI} \left( x - \frac{x^2}{\ell} \right) \quad (4)
\]

The maximum value of \( \theta \) is obtained at \( x = 0 \).

\[
\theta_{\text{max}} = \frac{M \ell}{2EI} \quad (5)
\]

Therefore (4) can be written in terms of (5) as:

\[
y = \theta_{\text{max}} \left( x - \frac{x^2}{\ell} \right) \quad (6)
\]

Since in the experiments the values of \( y_{\text{max}} = \delta \) for each span, at \( x = \ell/2 \), the equation relating \( \delta \) and \( \theta_{\text{max}} \) can be written

\[
\theta_{\text{max}} = \frac{4}{\ell} y_{\text{max}} = \frac{4}{\ell} \delta \quad (7)
\]

In the face of the simplifications done, equation (7) can then be used with good approximation to calculate the values of \( \theta_{\text{max}} \) that occur at each fillet joint, once \( \delta \) are available.
Tables 2.5.1 to 2.5.4 show the results of the calculations done with the values of $\delta$ recorded from the experiments, being that the nomenclature used in those tables are the same as those used in the earlier Table 2.4.1 for free joints.

For the constrained joints the left span distortion and the right span distortion were checked one against the other in order to verify if there were substantial differences between them, in which case all the assumptions done earlier would not be valid. However, it was verified that with some minor exceptions, all the results agreed quite well so that it was decided to work with the average of the recorded values in the calculations.

If now, as was done for the case of the free joints, the angular changes $\theta_{\text{max}}$ are plotted against the thicknesses $h$, for constant values of $\log_{10} W$, the graphs shown in Figure 2.5.2 are obtained.

In that Figure, it can be seen that the results obtained for spans values of 508.0 mm (20"), 609.6 mm (24"), and 812.8 mm (32") are in good agreement with what was expected from the experiments, but for the case of the span equal to 406.4 mm (16") the variation of the angular change are unexpectedly different from the other cases.

The explanation for this fact is quite difficult to be formulated since only three experimental data points were obtained for each value of the amount of consumed weld metal per unit length.

Assuming that the recording of the transversal distortion was done correctly, and that the bending moment due to the reaction stresses can be obtained from (5) as

$$M_0 = \frac{2EI}{L} \theta_{\text{max}}$$
RESULTS OF THE EXPERIMENTS

FOR THE CONSTRAINED JOINTS

SPAN = 406.4 mm (16"")

<table>
<thead>
<tr>
<th>h (mm)</th>
<th>wt (g)</th>
<th>w (g/cm)</th>
<th>log10 w</th>
<th>θ max (x10^-3 rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.175</td>
<td>27.60</td>
<td>1.36</td>
<td>0.133</td>
<td>10.3</td>
</tr>
<tr>
<td>(1/8&quot;)</td>
<td>52.20</td>
<td>2.58</td>
<td>0.411</td>
<td>16.9</td>
</tr>
<tr>
<td></td>
<td>78.50</td>
<td>3.87</td>
<td>0.588</td>
<td>31.3</td>
</tr>
<tr>
<td>6.350</td>
<td>31.20</td>
<td>1.54</td>
<td>0.186</td>
<td>12.0</td>
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<td>(1/4&quot;)</td>
<td>60.60</td>
<td>2.98</td>
<td>0.475</td>
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<td>89.60</td>
<td>4.42</td>
<td>0.645</td>
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<td>9.525</td>
<td>32.50</td>
<td>1.60</td>
<td>0.205</td>
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<td>(3/8&quot;)</td>
<td>59.90</td>
<td>2.95</td>
<td>0.470</td>
<td>23.0</td>
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<tr>
<td></td>
<td>88.70</td>
<td>4.37</td>
<td>0.641</td>
<td>26.3</td>
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</table>
### Table 2.5.2

**RESULTS OF THE EXPERIMENTS FOR THE CONSTRAINED JOINTS**

**SPAN - 508.0 mm (20")**

<table>
<thead>
<tr>
<th>h (mm)</th>
<th>wt (g)</th>
<th>w (g/cm)</th>
<th>log₁₀w</th>
<th>θ max (×10⁻³ rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.175 (1/8&quot;)</td>
<td>25.20</td>
<td>1.24</td>
<td>0.094</td>
<td>4.7</td>
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<td></td>
<td>46.80</td>
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<td>0.363</td>
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<td></td>
<td>69.80</td>
<td>3.44</td>
<td>0.536</td>
<td>12.7</td>
</tr>
<tr>
<td>6.350 (1/4&quot;)</td>
<td>29.20</td>
<td>1.44</td>
<td>0.159</td>
<td>8.8</td>
</tr>
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<td></td>
<td>55.10</td>
<td>2.71</td>
<td>0.431</td>
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<td></td>
<td>81.00</td>
<td>3.98</td>
<td>0.600</td>
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<td>55.20</td>
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<td>80.60</td>
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### TABLE 2.5.3

**RESULTS OF THE EXPERIMENTS FOR THE CONSTRAINED JOINTS**

**SPAN = 609.6 mm (24")**

<table>
<thead>
<tr>
<th>$h$ (mm)</th>
<th>wt (g)</th>
<th>$w$ (g/cm)</th>
<th>$\log_{10} w$</th>
<th>$\theta_{\text{max}}$ (x10^{-3}rad)</th>
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<tbody>
<tr>
<td>3.175 (1/8&quot;&quot;)</td>
<td>27.60</td>
<td>1.36</td>
<td>0.133</td>
<td>3.8</td>
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<td>50.20</td>
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<td>0.393</td>
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<td>76.40</td>
<td>3.86</td>
<td>0.587</td>
<td>15.8</td>
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<td>6.350 (1/4&quot;&quot;)</td>
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<td>1.59</td>
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<td>58.00</td>
<td>2.85</td>
<td>0.455</td>
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<td>87.80</td>
<td>4.32</td>
<td>0.636</td>
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## RESULTS OF THE EXPERIMENTS

FOR THE CONSTRAINED JOINTS

SPAN = 812.8 mm (32")

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<th>$h$ (mm)</th>
<th>$w$ (g)</th>
<th>$w$ (g/cm)</th>
<th>$\log_{10} w$</th>
<th>$\theta_{\text{max}}$ (x10^{-3} rad)</th>
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</thead>
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<td>0.188</td>
<td>12.4</td>
</tr>
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<td>(1/2&quot;)</td>
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<td>0.433</td>
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<td></td>
<td>81.00</td>
<td>4.00</td>
<td>0.603</td>
<td>25.6</td>
</tr>
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<td>29.10</td>
<td>1.44</td>
<td>0.159</td>
<td>6.1</td>
</tr>
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<td>(5/8&quot;)</td>
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<td>2.68</td>
<td>0.429</td>
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<td>19.050</td>
<td>27.60</td>
<td>1.36</td>
<td>0.134</td>
<td>3.9</td>
</tr>
<tr>
<td>(3/4&quot;)</td>
<td>54.60</td>
<td>2.69</td>
<td>0.430</td>
<td>8.5</td>
</tr>
<tr>
<td></td>
<td>79.90</td>
<td>3.94</td>
<td>0.596</td>
<td>8.2</td>
</tr>
</tbody>
</table>
VARIATION OF $\theta_{\text{max}}$ WITH "h" FOR CONSTANT $\log_{10}w$

FIG. 2.5.2
ANGULAR CHANGE $\theta_{\text{max}} \times 10^{-3}$ (RAD)

THICKNESS OF PLATE $h$ (mm)

c. SPAN = 609.6 mm (24"")

VARIATION OF $\theta_{\text{max}}$ WITH "$h"$ FOR CONSTANT $\log_{10} w$ (CONT.)

FIG. 2.5.2
it is noticeable from the results of the experiments, that Mo would take, in the case of the span = 406.4 mm (16"), relatively higher values than that for other spans, for thin plates up to a thickness value of 6.5 mm (about 1/4"").

It seems therefore that in the case of shorter spans (that implies in a more stiff structure), there is some kind of combination between the reaction stresses and the effect of shrinkage of the weld fillets (that is more sensitive to thinner plates), resulting in an inherent bending moment of such a magnitude so as to cause relatively larger angular distortion at the fillet welds.

The analysis of this problem, however, is out of the scope of the present thesis work, and it would demand a more careful investigation, including for instance, the above referred shrinkage effect of the weld fillet. Due to this reason, the author decided not to include the results obtained for the span of 406.4 mm (16") in the subsequent analysis that will be carried out in the following chapters.

2.6 - Derivation of a relationship between $\theta_0$ and $\theta_{\text{max}}$

The results so far obtained for free and constrained joints will be of little use unless some means to relate them be developed, so that one can predict the distortion that will occur in a panel structure, once the general conditions for the carrying out of the welding are settled.

Equations (6) and (7) of the above paragraph suggest that it is possible to estimate the distortion $\delta$ of the constrained joint panels once the value of $\theta_{\text{max}}$ are available or vice-versa. Therefore, if one could obtain $\theta_{\text{max}}$ for each welding condition, $\delta$ would be easily obtained
from (7) as:

\[ \delta = \frac{h}{4} \theta_{\text{max}} \quad (8) \]

Since \( \theta_{\text{max}} \), that from now on will be called \( \theta \) for simplicity, is a function of the plate thickness as well as \( \theta \): the panel span, for a fixed welding condition, it would be more convenient to work with a variable like \( \theta_o \), that is dependent only on the plate thickness, for the same welding condition referred to above.

The problem that remains then, is that of relating \( \theta_o \) to \( \theta \) utilizing the data available from the experiments.

As it was done in the study of steel panels, the use of the principle of minimum potential energy stored in a statically undetermined frame structure, will be made, considering the case as one of plane stress-plane strain problem, since no stresses or strains in the direction normal to the plate are taken into account in the analysis.

Considering the nomenclature of Figure 2.5.1, and assuming the first order theory, i.e., infinitesimal strains and very small displacements, the plane stress-plane strain condition for a general two dimensional problem can be expressed for the unit width strip, as:

\[ \frac{d^2y}{dx^2} = \frac{d\theta}{dx} = - \frac{Mo}{D} \quad (9) \]

where

\[ Mo = \text{constant moment assumed to be acting on the structure} \]

\[ D = \frac{Eh^3}{12(1-\nu^2)} = \text{flexural rigidity of the plate} \]

\[ E = \text{modulus of elasticity} \]

\[ h = \text{plate thickness} \]

\[ \nu = \text{Poisson ratio} \]
On the other hand the infinitesimal bending moment energy stored due to the angular change \(d\theta\) is given by:

\[
dE_b = \frac{Mo}{2} d\theta
\]  \hspace{1cm} (10)

Using (9) with the appropriate sign, since now what interests is the energy stored in the structure, Equation (11) below will be obtained, that will be used in the following developments

\[
dE_b = \frac{Mo^2}{2D} \ dx
\]  \hspace{1cm} (11)

a) Relation between \(\theta_o\) and \(\theta\)

Having already defined \(\theta\) and \(\theta_o\), and having also learned that the magnitude of \(\theta\) is always less than that of \(\theta_o\), one can assume that there must be a certain amount of energy developed in the case of constrained joints that would be necessary in order to decrease the angular change from \(\theta_o\) to \(\theta\).

If this energy can be written in a general form as

\[
\frac{dE_w}{d(\theta - \theta_o)} = C (\theta - \theta_o)
\]  \hspace{1cm} (12)

where

\(E_w\) = energy necessary to decrease the angular change of the panel structure in the case of the constrained joints from \(\theta_o\) to \(\theta\).

\(C\) = coefficient that is a function of the welding procedure, weight of deposited metal and the geometric characteristics of the structure, arbitrarily called "coefficient of rigidity for angular change", 

the total amount of energy developed at each fillet weld will be given by Equation (13) as

\[ E_w = \int_0^{(\theta_o - \theta)} C (\theta_o - \theta) \, d(\theta_o - \theta) = C \frac{(\theta_o - \theta)^2}{2} \]  

(13)

On the other hand, assuming that each weld fillet will give rise to the strain energy stored in half of each span, Equation (11) can be used to calculate the bending energy of that portion of the plate. Calling this energy as \( E_b \),

\[ E_b = \int_0^{\ell/2} \frac{M_o^2}{2D} \, dx = \frac{M_o^2 \ell}{4D} \]  

(14)

But from the previous Equation (9), the value of \( \theta \) can be derived for the plane stress-plane strain case as:

\[ \theta = \frac{M_o \ell}{2D} \quad \text{and then,} \quad M_o = \frac{2D}{\ell} \theta \]  

(15)

Substituting \( M_o \) in Equation (14) by the value given in (15)

\[ E_b = \frac{D \theta^2}{\ell} \quad \text{or} \quad E_b = \frac{E h^3}{12(1 - \nu^2)} \cdot \frac{1}{\ell} \theta^2 \]  

(16)

Therefore the total strain energy stored in the plate will be given by the sum of (13) and (16) as:

\[ E = E_w + E_b = C \frac{(\theta_o - \theta)^2}{2} + \frac{D}{\ell} \theta^2 \]  

(17)

The first term represents the energy necessary to reduce the angular distortions in a constrained panel structure from \( \theta_o \) to \( \theta \) and
the second term means the energy stored in the plate due to the bending of the panel up to an angle \( \theta \).

It is seen that one term increases when the other decreases, and the equilibrium condition, as discussed before, can be expressed by the condition that the energy stored in the structure be minimum, or in other words,

\[
\frac{dE}{d\theta} = -C(\theta_0 - \theta) + \frac{2D}{\lambda} \theta = 0 \tag{18}
\]

With (18), \( \theta \) can be obtained as:

\[
\theta = \frac{\theta_0}{1 + \frac{2D}{\lambda} \frac{1}{C}} \quad \text{or in the developed form}
\]

\[
\theta = \frac{\theta_0}{1 + \frac{2}{\lambda} \frac{Eh^3}{12(1 - \nu^2)} \cdot \frac{1}{C}} = \frac{\theta_0}{1 + \frac{Eh^3}{6(1 - \nu^2)} \cdot \frac{1}{\lambda} \cdot \frac{1}{C}} \tag{19}
\]

Equation (19) then, tells that once the value of the coefficient \( C \) is available for a given case of interest, \( \theta \) and consequently the transversal distortion \( \delta \) can be estimated by using the Equations (19) itself and (8) respectively, giving

\[
\delta = \frac{\lambda}{\lambda} \cdot \frac{\theta_0}{1 + \frac{Eh^3}{6(1 - \nu^2)} \cdot \frac{1}{\lambda} \cdot \frac{1}{C}} \tag{20}
\]
From Equation (18) one can obtain C as:

\[ C = \frac{2D}{\ell} \left( \frac{\theta}{\theta_0} - 1 \right) \quad (21) \]

so that C could be calculated for a given range of practical cases once experimental data are available. In the case of the present thesis work, C can be obtained for plate thicknesses varying between 6.35 mm (1/4") and 19.05 mm (3/4") and for spans between 508 mm (20") and 812.8 mm (32"), since the span value of 406.4 mm (16") was disregarded, as explained before.

b) Calculation of the coefficient C

Equation (21) was used to calculate the values of C for the set of experimental data recorded from the different test specimens, and the results are presented in Table 2.6.1.

In some cases, small discrepancies between the C values were found in the calculations, for a given welding condition, but in such cases their average was taken, since those discrepancies always occurred within the same range of magnitudes encountered for C.

If now a plotting of the values of C is made, as a function of \( \log_{10} W \) and the plate thicknesses, the graph shown in Figure 2.6.1 is obtained, so that interpolated values of "C" can be read in an easier manner.

c) Regressional analysis for obtaining "C" in an equation form

The tabular or the graphical form of determining the coefficient
### TABLE 2.6.1

VALUES OF "RIGIDITY" C AS A FUNCTION OF AMOUNT OF WELD METAL

CONSUMED PER UNIT WELD LENGTH AND PLATE THICKNESS

<table>
<thead>
<tr>
<th>th (mm)</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>log w</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.191</td>
<td>1.380</td>
<td>3.820</td>
<td>6.750</td>
<td>11.400</td>
</tr>
<tr>
<td>0.3</td>
<td>0.185</td>
<td>1.290</td>
<td>3.550</td>
<td>6.110</td>
<td>10.250</td>
</tr>
<tr>
<td>0.4</td>
<td>0.182</td>
<td>1.200</td>
<td>3.290</td>
<td>5.500</td>
<td>9.100</td>
</tr>
<tr>
<td>0.5</td>
<td>0.179</td>
<td>1.120</td>
<td>2.990</td>
<td>4.850</td>
<td>8.050</td>
</tr>
<tr>
<td>0.6</td>
<td>0.175</td>
<td>1.040</td>
<td>2.720</td>
<td>4.100</td>
<td>7.000</td>
</tr>
<tr>
<td>0.7</td>
<td>0.171</td>
<td>0.955</td>
<td>2.440</td>
<td>3.500</td>
<td>5.850</td>
</tr>
<tr>
<td>0.8</td>
<td>0.169</td>
<td>0.880</td>
<td>2.150</td>
<td>2.850</td>
<td>4.800</td>
</tr>
</tbody>
</table>

C \(\times 10^3 \text{ kg-mm/mm}\)
VALUES OF THE COEFFICIENT "C" PLOTTED AGAINST PLATE THICKNESS AND CONSTANT VALUES OF $\log_{10} w$

FIG. 2.6.1
"C" is not the most adequate in the computational standpoint, so that it was decided to carry out a simple but careful regresional analysis in order to obtain an equation for "C".

If values of \( \log_{10} C \), taken from Table 2.6.1, are calculated and plotted against the values of \( \log_{10} h \), for each constant \( \log_{10} W \), straight lines are obtained as shown in Figure 2.6.2.

Therefore the function that would be obtained will have a general form

\[
y = Ax^m
\]

since the plotted lines have as an equation the form

\[
\log_{10} y = \log_{10} A + m \log_{10} X
\]

or if the correct variables are taken,

\[
\log_{10} C = \log_{10} C_o + m \log_{10} h
\]

This leads to

\[
C = C_o h^m
\]

Values of \( m \) and \( C_o \) are obtained by the conventional manner by means of the equations:

\[
m = \frac{n \sum (\log_{10} C_i)(\log_{10} h_i) - \left(\sum \log_{10} h_i\right)\left(\sum \log_{10} C_i\right)}{n \sum (\log_{10} h_i)^2 - \left(\sum \log_{10} h_i\right)^2}
\]

\[
C_o = \frac{\sum \left(\log_{10} C_i\right)}{n} - m \frac{\sum \left(\log_{10} h_i\right)}{n}
\]

where \( n \) is the number of experimental points to be included in the regresional analysis.
PLOTTING OF \( \log_{10} C \) AGAINST \( \log_{10} h \) FOR CONSTANT VALUES OF \( \log_{10} w \)

FIG. 2.6.2
Carrying out the calculations for each value of $\log_{10} W$ and taking into account that all lines are converging to a unique point at $\log_{10} C$ axis, Equation (25) was obtained for the computation of $C$

$$C = 6.35 h^{(2.67 - 0.065 W)} \quad (25)$$

The range of validity of this equation, however, is limited to the following intervals of the variables involved:

i) $1.58 \leq W \leq 6.30$ (W in g/cm)

ii) $6 \text{ mm (about 1/4")} \leq h \leq 19 \text{ mm (about 3/4")}$ (h in mm)

Equation (25) seems to fit reasonably well to the experimental points obtained from the tests.

Finally, to estimate $\theta$, Equation (19) becomes:

$$\theta = \frac{\theta_o}{1 + \frac{2D}{\ell} \cdot \frac{1}{6.35 h^{(2.67 - 0.065 W)}}}$$

$$= \frac{\theta_o}{1 + \frac{2E h^3}{12(1-v^2)} \cdot \frac{1}{\ell} \cdot \frac{1}{6.35 h^{(2.67 - 0.065 W)}}}$$

$$= \frac{\theta_o}{1 + \frac{E}{38.1(1 - v^2)} \cdot \frac{h^{(0.33 + 0.065 W)}}{\ell}} \quad (26)$$

in its most reduced form.

Calculating some $\theta$ values randomly and comparing them to the corresponding experimental ones, the values in Table 2.6.2 are obtained.
**TABLE 2.6.2**

**COMPARISON BETWEEN CALCULATED AND EXPERIMENTAL VALUES OF $\theta$**

<table>
<thead>
<tr>
<th>span (mm)</th>
<th>thickness (mm)</th>
<th>$\log_{10} w$ (x$10^{-3}$ rad)</th>
<th>$\theta$ (x$10^{-3}$ rad)</th>
<th>$w$ (g/cm)</th>
<th>$\theta$ calc (x$10^{-3}$ rad)</th>
<th>$\theta$ exper. (x$10^{-3}$ rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>508.0(20&quot;)</td>
<td>6</td>
<td>0.2</td>
<td>42</td>
<td>1.581</td>
<td>22</td>
<td>12</td>
</tr>
<tr>
<td>508.0(20&quot;)</td>
<td>6</td>
<td>0.8</td>
<td>146</td>
<td>6.300</td>
<td>55</td>
<td>32</td>
</tr>
<tr>
<td>508.0(20&quot;)</td>
<td>12</td>
<td>0.6</td>
<td>71</td>
<td>3.980</td>
<td>27</td>
<td>29</td>
</tr>
<tr>
<td>508.0(20&quot;)</td>
<td>16</td>
<td>0.8</td>
<td>65</td>
<td>6.300</td>
<td>16</td>
<td>19</td>
</tr>
<tr>
<td>609.6(24&quot;)</td>
<td>8</td>
<td>0.4</td>
<td>67</td>
<td>2.510</td>
<td>33</td>
<td>22</td>
</tr>
<tr>
<td>609.6(24&quot;)</td>
<td>13</td>
<td>0.8</td>
<td>90</td>
<td>6.300</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>609.6(24&quot;)</td>
<td>17</td>
<td>0.2</td>
<td>12</td>
<td>1.581</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>812.8(32&quot;)</td>
<td>12</td>
<td>0.6</td>
<td>72</td>
<td>3.980</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>812.8(32&quot;)</td>
<td>15</td>
<td>0.8</td>
<td>73</td>
<td>6.300</td>
<td>24</td>
<td>22</td>
</tr>
<tr>
<td>812.8(32&quot;)</td>
<td>19</td>
<td>0.4</td>
<td>16</td>
<td>2.510</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>
It is realized therefore that there is a reasonably good agreement between the calculated and the experimental values with some discrepancies in the vicinity of thinner plates, but to the conservative side.

It seems, therefore, that Equation (25) can be used in general cases to predict within a reasonable accuracy the angular distortion that will be caused by a fillet weld in aluminum structural panels. Where the result seems to be inaccurate, there is always the possibility of referring to Table 2.6.1 or even to the graph in Figure 2.6.1 in order to obtain more approximate values for "C".
3.0 – COMPARISON OF THE RESULTS BETWEEN ALUMINUM AND STEEL

3.1 – General

Having obtained a way to estimate the distortion for the aluminum panels, it would be interesting now to compare the results thus far obtained with those available for steel, because there are several points that people who work with such type of structures would like to know.

As an example, one usually expects that aluminum panels, due to their lower stiffness, will show very large distortion compared to steel. This is actually what can be concluded if Figure 2.4.1 is compared with Figure 3.1.1, which shows values of $\theta_o$ for the case of steel panels.

Another point of interest is that due to faster spreading of the heat throughout the material, aluminum panel should have the occurrence of the peak distortion at a thickness larger than that for steel if only the effects of heating of the plate is taken into account. This, however, does not seem to be the case if the same figures mentioned above are compared.

Therefore, in order to check these points, it would be necessary to carry a comparative study between both materials to see which actually are the differences between them.

3.2 – Differences for free joint condition

If Figures 2.4.1 and 3.1.1 are compared, which show the test
VARIATION OF FREE JOINT ANGULAR CHANGE $\theta_0$ IN FUNCTION OF PLATE THICKNESS "h" AND CONSTANT $\log_{10}w$ FOR STEEL

FIG. 3.1.1
results obtained for $\theta_0$, the following apparent conclusions can be taken:

a) Aluminum shows quite larger angular distortion than steel throughout the thickness range covered in the experiments, when $\log_{10} W$ is taken as a parameter.

b) The occurrence of the peak distortion happens at about 6.35 mm (1/4") for aluminum, while for steel this happens in the vicinity of 9.52 mm (3/8").

c) The occurrence of the peak distortion for increasing values of $\log_{10} W$ shows a tendency to go towards lower thicknesses for aluminum, while the opposite happens for steel.

d) The variation of the angular change relative to the plate thickness for constant values of $\log_{10} W$ is more sensitive at lower thicknesses for aluminum than for steel, while just the opposite happens at higher thicknesses.

In order that this comparative study may become more specific, four representative thicknesses were chosen and the values of the free joint angular changes were calculated.

The thicknesses chosen were 6 mm (0.236"), 10 mm (0.393"), 14 mm (0.551"), and 18 mm (0.709"), respectively, which completely cover the range of validity imposed on the coefficient of rigidity "$c$".

Figure 3.2.1 presents the comparison between aluminum and steel when $\log_{10} W$ is taken as an independent variable, and one can see better the differences mentioned in items (a) through (d) above.

Now, to see if those differences prevailed even when the fillet size $Df$ is taken as the independent variable, another computation was
Comparison between values of $\theta_0$ for aluminum and steel for the same amount of the consumed weld metal.

Fig. 3.2.1
carried out, with the assumption that the deposition rate efficiency would be 95% and 65% for aluminum and steel, respectively.

The equation to calculate $D_f$, once the amount of weld metal consumed, $W$, is available, can be given by (27) below:

$$D_f = 10 \left( \frac{W \eta_d}{\gamma} \right)^{1/2}$$  \hspace{1cm} (27)

where

$D_f = \text{fillet size} \ (\text{mm})$

$W = \text{amount of consumed weld metal} / \text{unit length of fillet} \ (\text{g/cm})$

$\eta_d = \text{deposition rate efficiency} = \text{amount of deposited metal} / \text{amount of consumed metal}$

$\gamma = \text{specific weight} \ (\text{g/cm}^3) \quad \text{aluminum} = 2.65 \text{ g/cm}^3$

$\text{steel} = 7.85 \text{ g/cm}^3$

Carrying out the calculations and plotting the results, the graph shown in Figure 3.2.2 is obtained. (See Tables 3.3.1 and 3.3.2).

It is surprising to notice in this figure that when the same fillet size, $D_f$, is considered, the steel panel shows larger distortion than aluminum for all four representative thicknesses included in the study.

Roughly speaking, the fillet size actually governs the strength of the weldments. Therefore, this simply means that in practice, steel structures would have larger angular distortion and consequently transversal deflection than the aluminum.

To check the validity of the above statement, the same calculations were carried out for the constrained joints and the results
COMPARISON BETWEEN VALUES OF $\theta_0$ FOR ALUMINUM AND STEEL FOR THE SAME FILLET SIZE $D_f$

FIG. 3.2.2
are presented in the next paragraph.

3.3 - Differences for constrained joint condition

In this paragraph the calculation of the maximum transversal distortion for the case of constrained joints, considering the same representative thicknesses as before, was done for aluminum and for steel panels.

Three span values were considered, namely, 508.0 mm (20"), 609.6 mm (24") and 812.8 mm (24"), and the calculations were done with C values given by Equation (25) derived before.

Tables 3.3.1 and 3.3.2 present the results obtained for aluminum and for steel, respectively.

Plotting the results obtained, first taking values of \( \log_{10} W \) as an independent variable, the graphs shown in Figure 3.3.1 are obtained for each of the thicknesses considered.

It is noticed that in the case of the constrained joints, aluminum panels will generally show larger distortion than steel for the same values of \( \log_{10} W \), as it was the case for free joints.

If now the fillet size, \( D_f \), is taken as an independent variable, the situation is reversed, as shown in the graphs of Figure 3.3.2. Therefore, the prediction obtained when \( \theta \) was analysed is confirmed also for the case of constrained joints.

One can even notice that when \( \log_{10} W \) is taken as the independent variable, the thicker the plate, the less are the differences encountered between distortion of aluminum and of steel. For a thickness of around 18 mm (0.709"), for instance, it is visible that steel
### TABLE 3.3.1

VALUES OF THE MAXIMUM DISTORTION

FOR CONSTRAINED ALUMINUM PANELS

<table>
<thead>
<tr>
<th>pl. thick. h (mm)</th>
<th>log_{10} w</th>
<th>Df (mm)</th>
<th>\theta_0 (x10^{-3} rad)</th>
<th>\delta (mm) for spans (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>508.0 (20&quot;) 609.6 (24&quot;) 812.8 (32&quot;)</td>
</tr>
<tr>
<td>6 (0.236&quot;)</td>
<td>0.1 (estim.)</td>
<td>6.71</td>
<td>33</td>
<td>1.035</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>7.58</td>
<td>43</td>
<td>1.325</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>9.50</td>
<td>68</td>
<td>2.010</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>11.95</td>
<td>101</td>
<td>2.920</td>
</tr>
<tr>
<td>10 (0.393&quot;)</td>
<td>0.1 (estim.)</td>
<td>6.71</td>
<td>28</td>
<td>1.570</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>7.58</td>
<td>37</td>
<td>1.960</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>9.50</td>
<td>60</td>
<td>2.960</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>11.95</td>
<td>89</td>
<td>3.800</td>
</tr>
<tr>
<td>14 (0.551&quot;)</td>
<td>0.2</td>
<td>7.58</td>
<td>21</td>
<td>1.120</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>9.50</td>
<td>38</td>
<td>1.830</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>11.95</td>
<td>59</td>
<td>2.430</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>15.00</td>
<td>84</td>
<td>2.730</td>
</tr>
<tr>
<td>18 (0.709&quot;)</td>
<td>0.2</td>
<td>7.58</td>
<td>10</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>9.50</td>
<td>19</td>
<td>0.870</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>11.95</td>
<td>32</td>
<td>1.240</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>15.00</td>
<td>50</td>
<td>1.460</td>
</tr>
<tr>
<td>pl. thick. h (mm)</td>
<td>log₁₀w</td>
<td>Df (mm)</td>
<td>θ₀ (x10⁻³ rad)</td>
<td>δ (mm) for span (mm)</td>
</tr>
<tr>
<td>-------------------</td>
<td>---------</td>
<td>---------</td>
<td>----------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>6 (0.236&quot;)</td>
<td>0.4</td>
<td>4.56</td>
<td>17</td>
<td>0.794 0.924 1.540</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>5.74</td>
<td>29</td>
<td>1.100 1.490 2.460</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>7.23</td>
<td>51</td>
<td>1.640 2.240 3.700</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.12</td>
<td>77</td>
<td>2.010 2.790 4.590</td>
</tr>
<tr>
<td>10 (0.393&quot;)</td>
<td>0.4</td>
<td>4.56</td>
<td>32</td>
<td>1.725 2.300 3.500</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>5.74</td>
<td>48</td>
<td>2.460 3.270 5.050</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>7.23</td>
<td>70</td>
<td>3.200 4.310 6.640</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.12</td>
<td>95</td>
<td>3.600 4.890 7.700</td>
</tr>
<tr>
<td>14 (0.551&quot;)</td>
<td>0.4</td>
<td>4.56</td>
<td>18</td>
<td>1.105 1.438 2.120</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>5.74</td>
<td>26</td>
<td>1.541 2.030 3.100</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>7.23</td>
<td>45</td>
<td>2.350 3.120 5.080</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.12</td>
<td>64</td>
<td>2.760 3.720 6.400</td>
</tr>
<tr>
<td>18 (0.709&quot;)</td>
<td>0.4</td>
<td>4.56</td>
<td>11</td>
<td>0.835 1.072 1.430</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>5.74</td>
<td>18</td>
<td>1.203 1.550 2.310</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>7.23</td>
<td>31</td>
<td>1.965 2.565 3.870</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.12</td>
<td>45</td>
<td>2.410 3.160 4.890</td>
</tr>
<tr>
<td>SPAN (mm)</td>
<td>CASE</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------</td>
<td>------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>508.0 (20&quot;)</td>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>609.6 (24&quot;)</td>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>812.8 (32&quot;)</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

h = 6mm (0.236")

h = 10 mm (0.393")

Comparison between values of transversal distortion $\delta$ for aluminum and steel as a function of the amount of consumed weld metal (1/2)

Fig. 3.3.1
COMPARISON BETWEEN VALUES OF TRANSVERSAL DISTORTION $\delta$ FOR ALUMINUM AND STEEL AS A FUNCTION OF THE AMOUNT OF CONSUMED WELD METAL (2/2)
COMPARISON BETWEEN VALUES OF TRANSVERSAL DISTORTION \( \delta \) FOR ALUMINUM AND STEEL FOR THE SAME FILLET SIZE \( D_f \) (2/2)

FIG. 3.3.2
strain and stresses, as well as the yielding of the material.

f) The stiffness of the structure.

g) The dimension of the panel structure that will influence the capacity of the structure in absorbing and dissipating the heat supplied by the source.

h) Number of welding passes that is related to the number of thermal cycles that the structure is subjected to. Each thermal cycle is supposed to modify the state of equilibrium that was existing before the new pass, introducing new stress and strain components so that the final distortion is going to be affected.

i) Welding sequence that will affect the degree of constraining of the joints before each welding pass.

j) Welding process that will influence the net heat supplied to the weldment.

Therefore, before drawing any conclusion as a result of the present study, it is rather advisable to do an analytical study, trying to relate as many variables as possible. With the result of this analysis, then, it should be possible to obtain a more rational explanation for the behavior of the materials under discussion.
4.0 - ANALYTICAL METHOD TO PREDICT THE ANGULAR
    CHANGE IN FREE JOINTS

4.1 - General

As pointed out before, developing an analytical method to
estimate with good accuracy the transversal distortion caused by fillet
welds in panel structures is by no means an easy task.

A desirable way of analysis would be one that could include
all the variables involved in the problem so that their combined effects
could be investigated. Since this is out of the scope of the present
thesis work, this analysis will only take into account the two following
major effects, namely,

a) The effects of heating the plate by a moving point source, like the
    welding tip, considering the development of elastoplastic deformation
    in the plate.

b) The effects of the shrinkage of the weld fillet, considering only
    the elastic behavior of the weld bead during the cooling.

Also for simplicity, the method will restrict only to the case
of free joints, since it was shown that it is possible to relate the free
joint angular distortion to the constrained transversal disto. on.
4.2 - Effects of heating the plate by a moving point heat source

The procedure that will be followed to study this effect is based on a paper written by Y. Iwamura of Welding Research Laboratories of Kawasaki Heavy Industries of Kobe, Japan, with the title:


The method developed by Iwamura seems to be very consistent as far as elastoplastic analysis of models having a thermal history is concerned, and also well suited for the case of the present problem.

So, what will be done next is to adapt his study to the case of aluminum panels and develop a mean. to estimate the temperature distribution throughout the model. Once this is obtained the computer program will be rearranged due to both, the referred alterations and the introduction of a new subroutine for the establishment of the thermal history.

The general steps and the corresponding executors of this analysis is schematically presented in the flow chart below. In the flow chart is also included the part referring to the study of the effects of the shrinkage of weld fillet on the angular change.

a) The method and model for the analysis

The method of analysis is based on the incremental strain theory, with the calculation of the elastoplastic thermal deformation, using a simplified strip model, as shown in Figure 4.2.1.
FLOW CHART OF THE ANALYSIS PROCEDURE

Theoretical Estimation of Angular Change in a Two-Dimensional Model.

Basic Study for Steel - Elastoplastic Analysis to Estimate Angular Changes Caused by a Moving Point Heat Source.
- by Y. Iwamura -

Determination of Temperature Dependent Material Properties for Aluminum - by the Author

Subroutine to Calculate the Temperature Distribution of the Model - by the Author

Modification and Debugging of the Computer Program, Looking for the Best Procedure to do the Integration - by Iwamura and the Author

Simplified Analysis Using Two-Dimensional Finite Element Method to Study the Effect of the Shrinkage of the Weld Fillet - by Iwamura and the Author

Final Estimated Angular Change θ.
SIMPLIFIED STRIP MODEL FOR THE ANALYSIS

FIG. 4.2.1
The temperature distribution in the model was assumed to be two-dimensional, within each step of a thermal cycle and the plane stress condition was assumed, using therefore, the corresponding elastic-plastic stress-strain relations.

However, in this case, the calculation of the equivalent stress affecting the initiation of plastic flow becomes very complex since there are three stress components \((\sigma_x, \sigma_z, \tau_{xz})\) as well as all other temperature-dependent material properties involved in the computation.

In order to simplify the problem, Iwamura did an elastic stress analysis with an experimentally obtained temperature profile and found that actually the influence of \(\sigma_z\) and \(\tau_{xz}\) in the total strain were very small compared to that of \(\sigma_x\). Also it was realized that the strain distribution through the thickness was nearly linear so that in the analysis, a linear strain distribution was assumed for predicting the deformation of the plate.

b) **Stress strain temperature relations**

The following material properties are considered to vary with the temperature:

(i) Yield stress \((\sigma_y)\)
(ii) Strain hardening coefficient \((m)\)
(iii) Modulus of elasticity \((E)\)
(iv) Coefficient of thermal expansion \((\alpha)\)

For the case of aluminum, such material properties were taken

Those properties are presented in a graphical form in Figure 4.2.2 and will be used in the subsequent calculations.

Also to simplify the analysis, Iwamura assumed that the stress-strain curves were represented by two straight lines, considering the material as presenting an elastic, linear strain hardening behavior, and neglected also the Baushinger effects.

c) Temperature distribution

The temperature distribution of the model was calculated, assuming that the problem could be treated as a heat flow analysis in the quasi-stationary state, and that the heat is supplied by a point source moving in a straight path along the plate.

The general expression for the heat conduction is given by the Fourier heat flow equation as:

\[
\frac{\partial T}{\partial t} = x \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)
\]  \hspace{1cm} (28)

where: \( x, y, z \) are the orientation of the coordinate system shown in Figure 4.2.3. It must be pointed out that this coordinate system is different from the one shown
E: MODULUS OF ELASTICITY  
\( \sigma_y \): YIELD STRESS  
\( m \): COEFFICIENT OF STRAIN HARDENING  
\( \alpha \): COEFFICIENT OF THERMAL EXPANSION

\[ \sigma_y (\text{KG/mm}^2) \]
\[ E (10^3 \times \text{KG/mm}^2) \]
\[ m \]
\[ \alpha (10^{-6}\times\text{C}^{-1}) \]

TEMPERATURE DEPENDENT MATERIAL PROPERTIES FOR ALUMINUM

FIG. 4.2.2
SCHEMATIC TEMPERATURE DISTRIBUTION IN A PLATE DUE TO THE DEPOSITION OF A WELD BEAD

FIG. 4.2.3
in Figure 4.2.1, and that in the actual analysis, the appropriate coordinate transformation was done.

\[ T = \text{temperature at a given point (x, y, z, t) (°C)} \]
\[ \chi = \text{thermal diffusivity (cm}^2\text{•sec}^{-1}) \]
\[ t = \text{time (sec)} \]

If now a coordinate system (u, y, z) which moves at the same speed as the heat source is chosen, so that:

\[ v = \text{welding arc travelling speed = heat source travelling speed (cm} \cdot \text{sec}^{-1}) \]

Equation (28) can be written as:

\[
\frac{\partial^2 T}{\partial u^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{v}{\chi} \left( \frac{\partial T}{\partial u} \right) \tag{29}
\]

Now, the temperature \( T \) is only a function of position (u, y, z). For the case of a three-dimensional plate of finite thickness, when the radiation of the surface of the plate is neglected, the boundary condition becomes:

\[
\frac{\partial T}{\partial z} = 0 \quad \text{for } z = 0; \quad z = h
\]

where \( h \) = thickness of the plate

and the solution of this problem is given by Equation (30) below, still following the coordinate system of Figure 4.2.3:

\[
T = T_o + \frac{Q}{2\pi\chi} e^{2\chi} \left\{ -\frac{V}{2\chi} \frac{R}{R} + \sum_{n=1}^{\infty} \left\{ -\frac{V}{2\chi} \frac{R_n}{R} + \frac{V}{2\chi} \frac{R_n'}{R} \right\} \right\} \tag{30}
\]
where:

\[ T = \text{Temperature at a given point} \ (u, \ y, \ z) \]

\[ T_o = \text{Initial temperature of the plate} \ \left( ^\circ \text{C} \right) \]

\[ Q = \text{Amount of heat supplied to the work piece per unit time} \]
\[ (\text{cal} \cdot \text{sec}^{-1}) = 0.24 \ V \ I \ \eta_a \]

\[ V = \text{arc voltage} \ (\text{volts}) \]

\[ I = \text{welding current} \ (\text{Amps}) \]

\[ \eta_a = \text{arc efficiency} \]

\[ \lambda = \text{Thermal conductivity} \ (\text{cal} \cdot \text{cm}^{-1} \cdot \text{sec}^{-1} \cdot ^\circ \text{C}^{-1}) \]

\[ v = \text{arc travelling speed} \ (\text{cm} \cdot \text{sec}^{-1}) \]

\[ \chi = \text{Thermal diffusivity} \ (\text{cm}^2 \cdot \text{sec}^{-1}) \]

\[ u = x - v \cdot t \ \text{as already defined} \]

\[ R = (u^2 + y^2 + z^2)^{1/2} \ (\text{cm}) \]

\[ R_n = (u^2 + y^2 + (2nh - z)^2)^{1/2} \ (\text{cm}) \]

\[ R_n' = (u^2 + y^2 + (2nh + z)^2)^{1/2} \ (\text{cm}) \]

Equation (30), therefore, defines the temperature at any desired point, and at any given time. Once \( \chi \) is chosen, it is then possible to obtain the temperature distribution of the section, which is normal to the \( \chi \) axis at the chosen point.

In the case of this analysis, the section chosen is situated at the coordinate \( y = 0 \), following the orientation of Figure 4.2.1. This coordinate, as can be deducted, corresponds to the point \( \chi = 0 \) in the system shown in Figure 4.2.3.

The values of the time in Equation (30) are used to calculate the subsequent temperature distribution of the model, and they define the steps at which a new stress state is reached, or in other words, when a
new loading condition is imposed on the structure.

The time values are chosen in such a manner in the analysis, so that the different steps simulate a complete thermal cycle, consisting of a heating phase, followed by a cooling phase.

In order to carry out the computation for aluminum and steel strip models, the values for the different variables involved were assumed as shown in Table 4.2.1.

Those figures were taken from the experiments and references like "Welding Handbook", "Welding Data Book", and the notes of the course 13.151-J "Welding Engineering".

Just to illustrate a typical temperature profile in a complete thermal cycle, as well as the temperature distribution through a section of the strip model Figure 4.2.4 shows the results of the computation for a 19 mm (0.747") aluminum plate.

d) Solution procedure

Since the incremental form of stress-strain relations was chosen to analyse this problem, the thermal load that acts on the structure was divided into a set of load increments. This is possible by conveniently choosing the time steps taken to calculate the temperature distribution discussed in the item (c) above.

However, due to the temperature dependent stress-strain curves, the stress-strain behavior of most points in the model will transfer from one curve to another as the sequential load increments are being applied. It is necessary also to point out that the stress path followed at each load increment depends on whether the material is loading or
TABLE 4.2.1

ASSUMED VALUES OF THE VARIABLES TO

CALCULATE THE TEMPERATURE DISTRIBUTION

OF ALUMINUM AND STEEL MODELS

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Units</th>
<th>Aluminum</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_0$</td>
<td>°C</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>$V$</td>
<td>Volts</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>$I$</td>
<td>Ampe</td>
<td>155</td>
<td>230</td>
</tr>
<tr>
<td></td>
<td>$\eta_0$</td>
<td>%</td>
<td>76</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>$\lambda$</td>
<td>Cal·cm$^{-1}$·sec$^{-1}$·°C$^{-1}$</td>
<td>0.480</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>cm·sec$^{-1}$</td>
<td>0.567</td>
<td>0.130</td>
</tr>
<tr>
<td></td>
<td>$\chi$</td>
<td>cm$^2$·sec$^{-1}$</td>
<td>0.700</td>
<td>0.380</td>
</tr>
<tr>
<td></td>
<td>$T_{max}$</td>
<td>°C</td>
<td>630</td>
<td>1500</td>
</tr>
</tbody>
</table>

Remark: - See item (4.2.c) for the definition of the variables.
a. TEMPERATURE PROFILE FOR A THERMAL CYCLE

b. TEMPERATURE DISTRIBUTION AT x = 0.0

RESULTS OF THE COMPUTATION FOR A 19 mm ALUMINUM PLATE

FIG. 4.2.4
unloading corresponding to the cases of increasing or decreasing temperatures, respectively.

Figure 4.2.5 shows the assumed stress paths for loading and unloading.

Iwamura considered a total of five cases in his study for the assumed stress-strain relation due to the loading and unloading of the material. They are:

(I) Pure elastic behavior of the material during loading without yielding.

(II) Yielding of the material during loading.

(III) No yielding of the material during unloading, with no change in sign of the stress.

(IV) No yielding of the material during unloading, but with a change in sign of the stress.

(V) Yielding of the material during unloading.

These five conditions can be mathematically replaced by only three, if, as represented in Figure 2.4.5, the strain at step \( (n-1) \) is represented by the notation \( \varepsilon_{n-1}^0 \). This is possible if it is noticed that by shifting the origin of the stress-strain curve for the unloading condition, the incremental stress-strain relation for condition (V) can be described by that of condition (II), and similarly, condition (IV) can be described by condition (I).

Then, the stress conditions can conveniently be described by the equations below:
ASSUMED STRESS PATHS

FIG. 4.2.5.
\[ \sigma_n = E_n^0 (\Delta \varepsilon_n + \varepsilon_{n-1}) + \sigma_{y_n}^0 (1 - m_n) \]  

(31)

or

\[ \sigma_n = E_n^0 \left( \Delta \psi_n - \alpha_n T_n + \alpha_{n-1} T_{n-1} + \varepsilon_{n-1}^0 \right) + \sigma_{y_n}^0 (1 - m_n) \]  

(32)

\[ \sigma_n' = E_n^0 \varepsilon_{n-1}^0 + \sigma_{y_n}^0 (1 - m_n) \]  

(33)

where

\[ \varepsilon_{n-1}^0 = \varepsilon_{n-1} - \varepsilon_{n-1}^{po} \]  

(shifted strain)  

(34)

\[ \Delta \varepsilon_n = \Delta \varepsilon_n^e + \Delta \varepsilon_n^p \]  

(35)

\[ \Delta \psi_n = \Delta \varepsilon_n + \alpha_n T_n - \alpha_{n-1} T_{n-1} \]  

(36)

Subscripts \((n)\) and \((n-1)\) denote the step, subscripts \((e)\) and \((p)\) denote elastic or plastic, respectively. \(\alpha_n T_n\) denotes the thermal strain at step \((n)\) and the quantities \(\varepsilon_{n-1}^{po}, E_n^0\) and \(\sigma_{y_n}^0\) are defined in Table 4.2.2. \(\Delta \psi_n\) is called assumed strain and will be defined in the next item.

Also, the incremental strain \((\Delta \varepsilon_n)\) is available for determining loading or unloading because of the existing temperature dependent stress-strain curves.

Therefore, with the above temperature dependent stress-strain relation for the material, and the basic equations concerning the equilibrium, continuity and the boundary conditions which will be derived for the proposed model, the computation can be carried out in order to obtain the displacements and slopes of the model.
### TABLE 4.2.2

**EXPRESSIONS FOR \( \varepsilon_{n-1}^{po}, \varepsilon_n^{o}, \text{AND } \sigma_{yn}^{o} \)**

<table>
<thead>
<tr>
<th>Stress Condition</th>
<th>( \varepsilon_{n-1}^{po} )</th>
<th>( \varepsilon_n^{o} )</th>
<th>( \sigma_{yn}^{o} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I)</td>
<td>0</td>
<td>( E_n )</td>
<td>0</td>
</tr>
<tr>
<td>(II)</td>
<td>0</td>
<td>( m_n E_n )</td>
<td>( \sigma_{yn}^{*} )</td>
</tr>
<tr>
<td>(III)</td>
<td>( \varepsilon_{n-1}^{p} )</td>
<td>( E_n )</td>
<td>0</td>
</tr>
<tr>
<td>(IV)</td>
<td>( \varepsilon_{n-1}^{p} )</td>
<td>( E_n )</td>
<td>0</td>
</tr>
<tr>
<td>(V)</td>
<td>( \varepsilon_{n-1}^{p} )</td>
<td>( m_n E_n )</td>
<td>( \sigma_{yn}^{*} )</td>
</tr>
</tbody>
</table>

**Remarks:**

1. For the calculation of \( \sigma_n \), \( \sigma_{yn}^{*} = \sigma_{yn} \)
2. For the calculation of \( \sigma_n^{i} \), \( \sigma_{yn}^{*} = 0 \) in the following cases:
   a. If \( \sigma_n^{i} < \sigma_{yn} \) under the same stress condition both at step (n-1) and step (n)
   b. When any stress condition at step(n-1) changes to (V) at step (n), except the case when (IV) changes to (V), with \( \sigma_n^{i} > \sigma_{yn} \)
   c. When the stress condition of (I) at step (n-1) changes to (II) at step n, with \( \sigma_n^{i} < \sigma_{yn} \)
   d. In other cases for \( \sigma_n^{i} , \sigma_{yn}^{*} = \sigma_{yn} \)
e) Assumed strain

Since the strip model shown in Figure 4.2.1 has symmetry about the z axis, the analysis will be done only for half the model. Also, to simplify the calculations, and as explained before in (c), the strip model is assumed to be represented by the transversal section situated at the origin of the coordinate system shown in Figure 4.2.1.

Figure 4.2.6 shows the half model to be used in the computation.

As discussed earlier, the distribution of the combined strain \( \psi_n \) is linear in the z direction with some type of variation in the x direction. In order to consider this variation in the computation, the model is divided into Q segments as shown in Figure 4.2.6. Each segment end is referred to as sections. Therefore, section 1 (located at the center of the strip model) is the left of Segment 1, and so on.

For the qth segment, the combined incremental strain, including the thermal expansion increment, for the nth load increment, \( \Delta \psi_n(q) \) is assumed to have the form:

\[
\Delta \psi_n(q) = \Delta a_n(q) + \Delta b_n(q)z + \Delta c_n(q)x + \Delta d_n(q)xz
\]

(37)

where the value of x ranges from zero to the length of the qth segment. Also each segment was divided into (p-1) horizontal strips which are going to be the stations within each section to be used in the computation.

f) Displacement and slopes

Since the shear stress, as discussed earlier, is assumed to be
a. MODEL SEGMENTS AND SECTIONS

b. DIVISION OF EACH SEGMENT IN (p - 1) HORIZONTAL STRIPS

MODEL DIVIDED FOR COMPUTATION

FIG. 4.2.6
zero, and considering the material as isotropic, the shear strain must also be zero. Therefore, the shear strain

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0 \quad (38)$$

$$\frac{\partial w}{\partial x} = -\frac{\partial u}{\partial z} \quad (39)$$

where $w =$ displacement in the z direction
$u =$ displacement in the x direction

From Equation (37), the expression for the combined strain for the nth load increment and the qth segment is:

$$\psi_n(q) = \sum_{i=1}^{n} \Delta \psi_i(q) = A_n(q) + B_n(q)z + C_n(q)x + D_n(q)xz \quad (40)$$

where

$$A_n(q) = \sum_{i=1}^{n} \Delta a_i(q), \text{ etc.}$$

The incremental displacement $\Delta u_n(q)$ for the qth segment due to the nth load increment is related to the combined incremental strain by:

$$\frac{\partial \Delta u_n(q)}{\partial x} = \Delta \psi_n(q) \quad (41)$$

Recalling that x is measured by a local coordinate system, attached to each segment, the integration of Equation (41) will give the total displacement in the x direction for the qth segment, after the nth load increment.
\[ u_n(q) = \sum_{j=1}^{q-1} L(j) \left\{ A_n(j) + B_n(j)z + \frac{C_n(j)L(j)}{2} + \frac{D_n(j)L(j)z}{2} \right\} + \]

\[ A_n(q)x + B_n(q)xz + C_n(q)\frac{x^2}{2} + D_n(q)\frac{x^2z}{2} + U_o + U'_o f(z) \]  \hspace{1cm} (42)

If now (42) is differentiated with respect to \( z \) and recalling Equation (39), the slope of the \( q \)-th segment after the \( n \)-th load is given by:

\[ \frac{\partial W_n(q)}{\partial x} = - \left\{ \sum_{j=1}^{q-1} L(j) \left\{ B_n(j) + \frac{D_n(j)L(j)}{2} \right\} + B_n(q)x + D_n(q)\frac{x^2}{2} + \right\} + \]

\[ U_o \frac{\partial f(z)}{\partial z} \]  \hspace{1cm} (43)

and the total displacement in the \( z \) direction can be given by:

\[ W_n(q) = - \left\{ \sum_{j=1}^{q-1} \frac{L(j)^2}{2} \left( B_n(j) + \frac{D_n(j)L(j)}{3} \right) + B_n(q)\frac{x^2}{2} + D_n(q)\frac{x^3}{6} + \right\} \]

\[ U_o \frac{\partial f(z)}{\partial z} \left\{ \sum_{j=1}^{q-1} L(j) + x \right\} + W_o(z) \]  \hspace{1cm} (44)

\( U_o, W_o, U'_o \) in the above equations are constants of integration.

\textbf{g) Boundary conditions}

Considering the symmetry of the model, as seen in Figure 4.2.6, the boundary conditions at Section 1 are:

\[ W_n(0,z) = 0 \]  \hspace{1cm} (45)

\[ u_n(0,z) = 0 \]  \hspace{1cm} (46)
\[ \frac{\partial W}{\partial x} (o, z) = 0 \]  \hspace{1cm} (47)

Therefore the constants of integration of Equations (42), (43) and (44) become:

\[ W_o (Z) = 0 \]  \hspace{1cm} (48)
\[ U'_o = 0 \]  \hspace{1cm} (49)
\[ U_o = 0 \]  \hspace{1cm} (50)

that simplifies those equations slightly.

**h) Equilibrium conditions**

Since no external loads are acting on the structure, the following equilibrium conditions can be used:

\[ \int_o^h \sigma x_n (z) \ dz = 0 \hspace{1cm} \text{equilibrium of in plane forces} \]  \hspace{1cm} (51)

\[ \int_o^h \sigma x_n (z) z \ dz = 0 \hspace{1cm} \text{equilibrium of moments.} \]  \hspace{1cm} (52)

Therefore in terms of the assumed strains, the above two equations can be written as:

(1) For sections 1 through \( Q \)

\[ \theta_n (q) \Delta a_n (q) + \theta'_n (q) \Delta b_n (q) = \beta_n (q) \]  \hspace{1cm} (53)

\[ \theta'_n (q) \Delta a_n (q) + \theta''_n (q) \Delta b_n (q) = \beta'_n (q) \]  \hspace{1cm} (54)
(2) For section $(Q + 1)$

\[
\begin{align*}
\theta_n(Q+1)\Delta a_n(Q) + \theta'_n(Q+1)\Delta b_n(Q) + L(Q)\theta_n(Q+1)\Delta c_n(Q) + \\
+ L(Q)\theta'_n(Q+1)\Delta d_n(Q) &= \beta_n(Q+1) \\
\end{align*}
\]

(55)

\[
\begin{align*}
\theta_n(Q+1)\Delta a_n(Q) + \theta''_n(Q+1)\Delta b_n(Q) + L(Q)\theta'_n(Q+1)\Delta c_n(Q) + \\
+ L(Q)\theta''_n(Q+1)\Delta d_n(Q) &= \beta'_n(Q+1) \\
\end{align*}
\]

(56)

Functions $\theta_n, \theta'_n, \theta''_n, \beta_n$ and $\beta'_n$ are given by:

\[
\theta_n(Q) = \int_0^h E^0_n(z) \, dz
\]

\[
\theta'_n(Q) = \int_0^h E^0_n(z) \, z \, dz
\]

\[
\theta''_n(Q) = \int_0^h E^0_n(z) \, z^2 \, dz
\]

\[
\beta_n(Q) = \int_0^h E^0_n(z) \{ \alpha_n(z) T_n(z) - \alpha_{n-1}(z) T_{n-1}(z) \} \, dz - \\
-\int_0^h [E^0_n(z) \varepsilon^0_n(z) + \alpha_n(z) (1 - m_n(z))] \, dz
\]

\[
\beta'_n(Q) = \int_0^h E^0_n(z) \{ \alpha'_n(z) T_n(z) - \alpha'_{n-1}(z) T_{n-1}(z) \} \, dz - \\
-\int_0^h [E^0_n(z) \varepsilon^0_{n-1}(z) + \alpha'_n(z) (1 - m_n(z))] \, dz
\]
and \( L(q) \) is the length of segment \( q \).

(i) **Continuity condition**

Finally, within each segment continuity condition must be satisfied by the assumed deformation. So, the total combined strain distribution at the right side of the segment \( q \) must be the same as at the left side of the segment \( (q + 1) \).

This condition can be mathematically expressed in terms of the assumed strains as:

\[
\Delta a_n(q) + \Delta b_n(q)z + L(q)\Delta c_n(q) + L(q)\Delta d_n(q)z = \Delta a_n(q + 1) + \\
+\Delta b_n(q + 1)z \quad (57)
\]

So, Equation (57) is satisfied if:

\[
\Delta c_n(q) = \{\Delta a_n(q + 1) - \Delta a_n(q)\} / L(q) \quad (58)
\]

\[
\Delta d_n(q) = \{\Delta b_n(q + 1) - \Delta b_n(q)\} / L(q) \quad (59)
\]

Now all the necessary equations were derived and it will be possible to calculate the values of slopes and displacements as well as the stresses and strains where required.

The method of solution is described in the next item (j).

(j) **Method of solution**

Combining the equilibrium Equations (53), (54), (55) and (56)
and the continuity Equations (58) and (59) for each section at each load increment, a system of \(4 \times Q\) equations in terms of \(4 \times Q\) unknowns \((\Delta a_n's, \Delta b_n's, \Delta c_n's\) and \(\Delta d_n's)\) are obtained.

However, examining the set of equations it is realized that this system can be uncoupled, so that the solution is obtained only solving \(2Q + 2\) sets of equations, where each set is composed by two equations with two unknowns.

The method of solutions is described as follows:

(1) Using Equations (53) and (54) for \(q = 1\), values of \(\Delta a_1(1)\) and \(\Delta b_1(1)\) are obtained.

(2) Again using (53) and (54), values for \(\Delta a_1(q)'s\) and \(\Delta b_1(q)'s\) are obtained for all the segments 1 through Q.

(3) Then values of \(\Delta c_1(Q)\) and \(\Delta d_1(Q)\) are obtained from Equations (55) and (56). Because of the nonlinear nature of the two sets of equations, (53) - (54), and (55) - (56), an iterative solution method was used. The convergence criterion was based on changes in the values of the unknowns for consecutive iterations.

(4) For the remaining unknowns the continuity equations (58) and (59) were used.

(5) Finally, the strains, including the thermal expansion were calculated from Equation (37). The stresses were calculated from Equation (32) and the slopes and deflections from Equations (43) and (44), respectively.

(6) The load was then incremented, by increasing or reducing the temperature, depending on the step considered, and the calculation process is continued.
(k) Numerical computation

Since this calculation demands a large number of computations, and a rather cumbersome integration through the iterative converging process, Iwamura wrote a computer program that originally consisted in a main program to do all the calculations and a subroutine to solve the system of \(2Q + 2\) equations.

This original program was then modified, introducing a subroutine to calculate the temperature distribution of the model, and also an alteration was done in order to account for the temperature dependent properties of aluminum. This modification eliminated the need of feeding the computer with many data cards for each step of the computational process.

The modification also included the up-dating of the old program, since this was based on the first draft of Iwamura's paper.

The program used in the computation as well as its general flow chart is presented in the Appendix A at the end of this thesis work.

In the numerical solution, the most important aspect referred to the proper choice of the number of points taken at each section \((q)\) in order to carry out the integration.

This numerical integration was done using the Simpson rule, for no equation concerning the individual values of \(\Delta a_n\)'s and \(\Delta b_n\)'s were available. Therefore, when an insufficient number of points were taken, the convergence was not reached during the iteration process and the program entered a DO-Loop.

Therefore, after having chosen the appropriate number of sections (12 in the present analysis), so that the temperature gap
between two sections was not larger than 100 °C (180 °F), a careful choice of the number of points in each section was done.

The result was that for thinner plates up to 10 mm (0.393"), only 21 points per section were required, but for thicker plates in the order of 19 mm (0.749"), 81 points were sometimes required.

The time steps, giving the different temperature distribution through one thermal cycle, were also carefully chosen, so that temperature gaps larger than 100 °C (180 °F) between two steps were not allowed for the accuracy of the results.

(1) Results of the computation

The calculation of \( \theta_o \), only taking into account the effects of the heating of the plate, was done for aluminum and for mild steel for thicknesses of 7 mm (0.276"), 10 mm (0.393"), 13 mm (0.512"), 16 mm (0.630"), 19 mm (0.749"), and 25 mm (0.982").

For the case of steel, the data used in Iwamura's original paper was used, only modifying the value of the yield stress at room temperature, that was taken as 22 kg/mm\(^2\).

Also, only one thermal cycle for each thickness was considered and this corresponded to the fillet size of 7.52 mm (0.296") and 6.10 mm (0.240") for aluminum and for steel, respectively. The corresponding values of the amount of consumed metal are 1.58 g/cm (log \( W = 0.200 \)) for aluminum and 4.46 g/cm (log \( W = 0.650 \)) for steel.

Figure 4.2.7 shows the results of the computation for aluminum and steel. If a comparison with Figures 2.4.1 and 3.1.1 is done, it is apparent that there is a considerable lack of agreement between the
FREE JOINT ANGULAR DISTORTION $\theta_0$ (x10$^{-3}$ RAD)

PLATE THICKNESS $h$ (mm)

FREE JOINT ANGULAR DISTORTION $\theta_0$ FOR ALUMINUM AND STEEL DUE ONLY TO THE HEATING OF THE PLATE

FIG. 4.2.7
experimental results and the theoretical calculation. For aluminum, for instance, one can see by examining Figure 4.2.8 that only for thicker plates, well above 20 mm (0.787") it seems that there is some possibility of agreement between the experiments and the analysis.

When Iwamura carried out these experiments as well as the computations, using steel as a basic material, the thickness of the model had 50 mm (about 2"), precisely in the range of the thickness values where the agreement seems to exist.

However, analyzing Figure 4.2.7, the difference between steel and aluminum now appears to be confirmed, when both are compared taking as a parameter the fillet size. This was quite obvious to happen since

(1) the heat input for steel is about 2.5 times that for the aluminum, and

(2) the spreading of the heat in the aluminum plate is faster than that for steel, and so the thermal strains are more evenly distributed through the thickness of the aluminum plate.

In sight of the results obtained then, it was decided to carry out a very simple stress analysis, considering only the elastic behavior for the shrinkage of the welding fillets. This will be done in the next paragraph.
DIFFERENCE BETWEEN $\theta_0$ VALUES OBTAINED EXPERIMENTALLY AND BY DIRECT CALCULATION FOR ALUMINUM

FIG. 4.2.8
4.3 - Effects of the shrinkage of the weld fillets

The results thus far obtained in the theoretical calculation of the free joint angular distortion, when only the effects of the heating of the plate is taken into account, are not in good agreement with the experiments.

In this paragraph, therefore, a quick analysis, considering the shrinkage of the weld fillets, will be done, in order to see its influence on the overall angular distortion.

For this purpose a unit width model consisting only of two elements and with five nodes, as shown in Figure 4.3.1, will be considered.

The analysis will be done using the two-dimensional finite element method in its simplest form, and assuming that the only load on the structure considered consists of the one caused by the thermal strain due to the shrinkage of the weld fillet.

The model is assumed to be fixed at nodes 1, 4 and 5, and only the elastic behavior of the model will be considered in the analysis.

4.3.1 - Basic equations for element I

The assumed general expressions for the horizontal and vertical displacements $u$ and $w$ respectively, are:

\[ u = a_1 + a_2 x + a_3 z + a_4 x z \]  \hspace{1cm} (60)

\[ w = b_1 + b_2 x + b_3 z + b_4 x z \]  \hspace{1cm} (61)
FIGURE 4.3.1 - MODEL FOR THE ANALYSIS WITH

FIVE NODES AND TWO ELEMENTS
or in the matricial form:

\[
\begin{bmatrix}
1 & x_1 & z & x_1z
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4 \\
\end{bmatrix}
= \begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4 \\
\end{bmatrix}
\]

(62) (62')

The boundary condition at nodes 1 and 4 leads to:

(i) node 1: At \( x_1 = z = 0 \),

\[ u = w = 0 \]

So, \( a_1 = b_1 = 0 \)

(ii) node 4: At \( x_1 = 0; z = h \)

also \( u = w = 0 \)

So, \( a_3 = b_3 = 0 \)

Therefore, for element I:

\[
\begin{bmatrix}
1 & x_1 & x_1z
\end{bmatrix}
\begin{bmatrix}
a_2 \\
a_4 \\
\end{bmatrix}
= \begin{bmatrix}
b_2 \\
b_4 \\
\end{bmatrix}
\]

(63) (63')
At nodes 2 and 3, one has:

(iii) node 2: \[ x_1 = d; \quad z = 0 \]

\[ u_2 = a_2 \cdot d \quad a_2 = u_2 \cdot d \]

So that,

\[ w_2 = b_2 \cdot d \quad b_2 = w_2 \cdot d \]

(iv) node 3: \[ x_1 = d; \quad z = h \]

\[ u_3 = a_2 \cdot d + a_4 \cdot h \cdot d \quad a_4 = (u_3 - u_2) / h \cdot d \]

So that,

\[ w_3 = b_2 \cdot d + b_4 \cdot h \cdot d \quad b_4 = (w_3 - w_2) / h \cdot d \]

Substituting the values of the coefficients in the Equations (63) and (63'):

\[ u = \begin{bmatrix} x_1 & x_1z \end{bmatrix} \begin{bmatrix} h & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \left( \frac{1}{h \cdot d} \right) \quad (64) \]

\[ w = \begin{bmatrix} x_1 & x_1z \end{bmatrix} \begin{bmatrix} h & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} \left( \frac{1}{h \cdot d} \right) \quad (65) \]

b) **Strains**

Having the displacements, the strains are simply given by:

\[ \varepsilon x_1 = \frac{\partial u}{\partial x_1} = \left( \frac{1}{h \cdot d} \right) \begin{bmatrix} (h - z) & y \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \quad (66) \]
\[ \varepsilon_z = \frac{\partial w}{\partial z} = \left( \frac{1}{h \cdot d} \right) \begin{bmatrix} -x_1 & x_1 \\ x_1 & 0 \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} \] (67)

\[ \gamma_{x_1z} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x_1} = \left( \frac{1}{h \cdot d} \right) \begin{bmatrix} -x_1 & x_1 \\ x_1 & 0 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} + \begin{bmatrix} (h-z) \\ z \end{bmatrix} \begin{bmatrix} w_2 \\ w_3 \end{bmatrix} \] (68)

Therefore, the calculation of the strain energy follows:

\[ U_{x_1} = \frac{1}{2} \sigma_{x_1} \varepsilon_{x_1} = \frac{E}{2(1 + \nu^2)} (\varepsilon_{x_1}^2 - \nu \varepsilon_{x_1} \varepsilon_z) \] (69)

\[ U_z = \frac{1}{2} \sigma_z \varepsilon_z = \frac{E}{2(1 + \nu^2)} (\varepsilon_z^2 - \nu \varepsilon_{x_1} \varepsilon_z) \] (70)

\[ U_{xy} = \frac{1}{2} \gamma_{x_1z} \tau_{x_1z} = \frac{G}{2} \gamma_{x_1z} \] (71)

due to the plane stress-stain relations:

\[ \sigma_x = \frac{E}{1 + \nu^2} (\varepsilon_x - \nu \varepsilon_z) \; ; \; \sigma_z = \frac{E}{(1 + \nu^2)} (\varepsilon_z - \nu \varepsilon_{x_1}) \]

and \[ \tau_{x_1z} = \gamma_{x_1z} G. \]
Obtaining the expressions for $\varepsilon^2_{x_1}$, $\varepsilon^2_z$, $\varepsilon_{x_1} \varepsilon_z$, $\gamma^2_{x_1 z}$, and integrating within the range of $x_1$ (0 to d) and $z$ (0 to h), the following final equations are obtained for the above variables:

$$\varepsilon^2_{x_1} = \left( \frac{h}{6 \cdot d} \right) \begin{bmatrix} u \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \{u\}$$  

(72)

$$\varepsilon^2_z = \left( \frac{d}{3 \cdot h} \right) \begin{bmatrix} w \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \{w\}$$  

(73)

$$\varepsilon_{x_1} \varepsilon_z = \frac{1}{4} \begin{bmatrix} u \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \{w\}$$  

(74)

$$\gamma^2_{x_1 z} = \left( \frac{d}{3 \cdot h} \right) \begin{bmatrix} u \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \{u\} + \left( \frac{h}{6 \cdot d} \right) \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \{h\} +$$

$$+ \frac{1}{2} \begin{bmatrix} w \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \{u\}$$  

(75)

where $[ ]$ and $\{ \}$ denote line and column matrices, respectively.

Then, the total strain energy $U$ for element I can be obtained by integrating Equations (69) to (71) and using the expressions (72) to (75), so that:
\[ U_I = \int \frac{1}{2} (\sigma_{x_1} x_1 + \sigma_{z} z + \tau_{x_1} x_1 y_1 z) = \]

\[
\begin{array}{c}
\frac{E}{2(1 + v^2)} \left\{ \frac{d}{3h} \begin{bmatrix} w \\ -1 \\ 1 \end{bmatrix} \right\} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} + \frac{E}{6d} \left\{ \frac{d}{3h} \begin{bmatrix} u \\ -1 \\ 1 \end{bmatrix} \right\} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} - \\
\frac{v}{2} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \{v}\right\} + \frac{E}{(1+v)} \left\{ \frac{d}{3h} \begin{bmatrix} u \\ -1 \\ 1 \end{bmatrix} \right\} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} + \\
\frac{h}{6d} \begin{bmatrix} w \\ -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} + \frac{1}{2} \begin{bmatrix} w \\ -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \} \quad (76)
\end{array}
\]

Equation (76) will be used later in the subsequent derivations.

4.3.2 - Basic Equations for element II

(a) Displacements

Since element II consists of a triangle, as shown in Figure 4.3.1, the following assumed displacements will be considered:

\[ u = c_1 + c_2 x_2 + c_3 z \]

\[ w = d_1 + d_2 x_2 + d_3 z \]

or in a matrix notation: \( u(w) = \begin{bmatrix} 1 & x_2 & z \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \quad (77) \]

\[ (77') \]
The boundary condition at nodes 4, 5, and 3 lead to:

(i) node 4: At \( x_2 = z = 0 \)

\[
\begin{align*}
  u_4 &= w_4 = 0 \\
  \text{So,} \\
  c_1 &= d_1 = 0
\end{align*}
\]

(ii) node 5: At \( x_2 = 0; z = d \)

Also \( u_5 = w_5 = 0 \)

So, \( c_3 = d_3 = 0 \)

(iii) node 3: At \( x_2 = d; z = 0 \)

\[
\begin{align*}
  u_3 &= c_2 d \\
  c_2 &= u_3/d \\
  \text{So that,} \\
  w_3 &= d_2 d \\
  d_2 &= w_3/d
\end{align*}
\]

Then the equations for the displacements, (77) and (77') become simply:

\[
u = \frac{u_3}{d} x_2 \quad (78)
\]

\[
w = \frac{w_3}{d} x_2 \quad (79)
\]

b) Strains

For the strains the following equations are obtained:

\[
\varepsilon_{x_2} = \frac{u_3}{d} \quad (80)
\]
\[ \varepsilon_z = 0 \quad (81) \]

\[ \gamma_{x_2z} = \frac{w_3}{d} \quad (82) \]

In this case, the calculations are simpler, and the strain energy per unit volume of material are obtained for element II as:

\[ U_{x_2} = \frac{E}{2(1 + \nu^2)} \left(\frac{u_3}{d}\right)^2 \quad (83) \]

\[ U_z = 0 \quad (84) \]

\[ U_{x_2z} = \frac{E}{4(1 + \nu)} \left(\frac{w_3}{d}\right)^2 \quad (85) \]

The total strain energy for element II is, therefore,

\[ U_{II} = \frac{E}{2(1 + \nu^2)} \cdot \frac{u_3^2}{2} + \frac{E}{8(1 + \nu)} \cdot w_3^2 \quad (86) \]

4.3.3 - Energy due to the thermal strain - Initial condition

Considering the element II as contracting due to the shrinkage, the total energy due to the thermal load can be written as:

\[ U_{III} = \int \int \left( \sigma_{x_2}^0 \varepsilon_{x_2}^0 + \sigma_{z}^0 \varepsilon_{z}^0 + \tau_{x_2z}^0 \gamma_{x_2z}^0 \right) \cdot dx \cdot dz \]

The superscript \(^0\) denote the initial stress strain condition of the fillet.

Since no shear strain is supposed to exist in the thermal expansion or contraction of a structure,
\[ \gamma_{x_2 z}^o = 0, \quad \text{and due to isotropy of the material,} \quad \varepsilon_{x_2}^o = \varepsilon_z^o. \]

The total energy due to the initial condition, then, using again the plane stress-strain relations for \( \sigma_{x_2}^o \) and \( \sigma_z^o \), and Equation (81) for \( \varepsilon_z^o \),

\[
U_{III} = \frac{E}{2(1 + \nu^2)} \cdot \varepsilon_{x_2}^o (1 - \nu) u_3 \cdot d \quad (87)
\]

### 4.3.4 - Total potential energy of the structure and derivation of stiffness matrix

The total potential energy stored in the model can be mathematically represented by Equations (76), (86) and (87) as:

\[
\Pi = U_I + U_{II} - U_{III} \quad (88)
\]

Since this potential energy must be a minimum, the following conditions must be satisfied by \( \Pi \).

\[
\frac{\partial \Pi}{\partial u_2} = \frac{\partial \Pi}{\partial u_3} = \frac{\partial \Pi}{\partial w_2} = \frac{\partial \Pi}{\partial w_3} = 0 \quad (89)
\]

The application of condition (89) leads then to the system of four simultaneous equations with four unknowns, namely, the displacements \( u_2, u_3, w_2, \) and \( w_3 \).

In matricial representation, this system can be written as:
where \( D \) is the Stiffness Matrix which is a symmetrical matrix and represented by the Equation (91) below:

Solving the system of equations represented by (90), the values of the displacements are observed, so that the strains and consequently the stresses are obtained at each of the nodes of the model.

Since the present problem is concerned with the values of the slopes caused by the fillet shrinkage, only their values will be calculated, for simplicity.

Choosing node 2 as a reference point, the slope at that node will be calculated once the values of the displacements \( u_2 \) and \( w_2 \) are available.
\[
D = \begin{bmatrix}
\frac{1}{3}(\frac{2h}{d} \frac{1}{1+v^2} + \frac{d}{h} \frac{1}{1+v}) & \frac{1}{3}(\frac{h}{d} \frac{1}{1+v^2} - \frac{d}{h} \frac{1}{1+v}) & \frac{1}{4}\left(\frac{2v}{1+v^2} - \frac{1}{1+v}\right) & \frac{1}{4}\left(-\frac{2v}{1+v^2} + \frac{1}{1+v}\right) \\
\frac{1}{3}(\frac{h}{d} \frac{1}{1+v^2} - \frac{d}{h} \frac{1}{1+v}) & \frac{1}{3}(\frac{2h}{d} + \frac{3}{1+v^2} + \frac{d}{h} \frac{1}{1+v}) & \frac{1}{4}\left(\frac{2v}{1+v^2} + \frac{1}{1+v}\right) & \frac{1}{4}\left(-\frac{2v}{1+v^2} + \frac{1}{1+v}\right) \\
\frac{1}{4}\left(\frac{2v}{1+v^2} - \frac{1}{1+v}\right) & \frac{1}{4}\left(\frac{2v}{1+v^2} + \frac{1}{1+v}\right) & \frac{1}{3}(\frac{2d}{h} \frac{1}{1+v^2} + \frac{h}{d} \frac{1}{1+v}) & \frac{1}{6}\left(-\frac{4d}{h} \frac{1}{1+v^2} + \frac{h}{d} \frac{1}{1+v}\right) \\
\frac{1}{4}\left(-\frac{2v}{1+v^2} + \frac{1}{1+v}\right) & \frac{1}{4}\left(-\frac{2v}{1+v^2} + \frac{1}{1+v}\right) & \frac{1}{6}\left(-\frac{4d}{h} \frac{1}{1+v^2} + \frac{h}{d} \frac{1}{1+v}\right) & \frac{1}{3}(\frac{2d}{h} \frac{1}{1+v^2} + \frac{h}{d} \frac{1}{1+v}) + \frac{h}{d} + \frac{3}{2}\frac{1}{1+v}
\end{bmatrix}
\]
In order to conduct the calculations for different values of plate thicknesses as well as for varying fillet sizes, a computer program was written. The system of equations (90) was solved using the ordinary Gauss-Jordan reduction method. The general flow chart, as well as the program listing are presented in the Appendix B at the end of this thesis work.

4.3.4 - Results of the computation

The computation was carried out for aluminum and for steel models with thickness varying between 4.0 mm (0.157") and 20.0 mm (0.789").

The fillet sizes considered in the numerical calculation were 7.52 mm (0.296") for aluminum, and 6.10 mm (0.240") and 7.52 mm (0.296") for steel, as was done before.

In order to obtain the initial condition to calculate the energy due to the thermal strain \( \varepsilon_x^0 \), the coefficient of thermal expansion for aluminum was assumed to be \( 0.29 \times 10^{-4} \, ^\circ\text{C}^{-1} \), and for steel \( 0.13 \times 10^{-4} \, ^\circ\text{C}^{-1} \).

The temperature limits taken in the calculations were the same as assumed before for aluminum and steel, i.e., from 30 \(^\circ\text{C}\) to 630 \(^\circ\text{C}\), and from 30 \(^\circ\text{C}\) to 1500 \(^\circ\text{C}\), respectively.

The results of the computation are shown in the graph of Figure 4.3.2 (see also Table 5.1.1), and the following apparent conclusions can be drawn:

(a) The influence of the weld fillet shrinkage, as expected, is actually
FREE JOINT ANGULAR DISTORTION $\theta_0$ FOR ALUMINUM AND STEEL DUE TO SHRINKAGE OF WELD FILLET

FIG. 4.3.2
more sensitive in the range of thinner plates

(b) For the same fillet size \((D_f = 7.52 \text{ mm (0.296")})\), steel model still have larger angular distortion, but now, this difference is much smaller compared to those existent when only the effects of the heating of the plate was considered.

(c) When the fillet size of the steel model is a bit smaller than that of the aluminum, there is inclusive an inversion in the magnitudes of the angular distortion for thicker plates, although the heat input for steel is much larger than for aluminum, as seen before.

(d) The maximum values of the distortion occurred at thickness in the vicinity of 6.00 mm (about 1/4") for both aluminum and steel. This was a surprise since the temperature limits as well as the coefficients of thermal expansion were different between the two materials. However, this could be the result of considering only the elastic behavior of the material in the analysis, rather than including the plastic strains that will certainly exist in the weld fillet.

These conclusions are apparently the most important ones and now it will be possible to do a combined analysis, taking into account the two effects considered in items (4.2) and (4.3) of this chapter. This will be done next, in the final chapter, 5.0, cf this work.
5.0 - FINAL ANALYSIS OF THE RESULTS AND CONCLUSIONS

5.1 - Final conclusions

Having already in hands the experimental values of the angular distortion for aluminum as well as for steel, and also having the results of the theoretical analysis, it is necessary now to check whether there is an agreement between them, or in the negative case, to investigate why and which are the reasons of such a disagreement.

If the results of the theoretical calculation are added up and compared with those obtained experimentally, the values shown in Table 5.1.1 are obtained.

The results of combining the effects of heating the plate with that of the shrinkage of the weld fillet is also shown in Figure 5.1.1.

Table 5.1.1 shows the results of the analysis for aluminum and steel separately, comparing the experimental values of the free joint angular distortion with those obtained from theoretical calculations.

It is apparent that for both materials, the experimental values are quite above the theoretical ones, at least in the lower range of thickness covered in the analysis.

The reasons for these differences may be due to factors like:

(a) Only two elements were considered in the analysis to study the effects of the shrinkage of the weld fillets. This fact can lead to inaccuracies in the results, as it is usual when the finite element method with small number of elements is used. Since in the present case the only interest was to see how the fillet shrinkage would affect the results so far obtained, that kind of
### TABLE 5.1.1

VALUES OF $\theta_0$ FOR ALUMINUM AND STEEL OBTAINED FROM

THEORETICAL CALCULATION AND FROM EXPERIMENTS

<table>
<thead>
<tr>
<th>$h$ (mm)</th>
<th>5086-H32-Aluminum</th>
<th>Mild Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Due to plate heating</td>
<td>Due to fillet shrinkage</td>
</tr>
<tr>
<td>6.350 (1/4&quot;)</td>
<td>0.02</td>
<td>8.20</td>
</tr>
<tr>
<td>9.525 (3/8&quot;)</td>
<td>0.55</td>
<td>6.90</td>
</tr>
<tr>
<td>12.700 (1/2&quot;)</td>
<td>1.75</td>
<td>5.70</td>
</tr>
<tr>
<td>15.875 (5/8&quot;)</td>
<td>2.50</td>
<td>4.70</td>
</tr>
<tr>
<td>19.050 (3/4&quot;)</td>
<td>2.05</td>
<td>3.80</td>
</tr>
</tbody>
</table>

$D_f Al = 7.52$ mm (0.296") corresponding to $\log w = 0.200$

$D_f Steel = 6.10$ mm (0.240") corresponding to $\log w = 0.650$

Fillet sizes obtained from the recommended welding conditions for aluminum and steel.
simplification was done.

(b) Only the elastic behavior of the model was taken into account to do the analysis referred to in (a). This simplification will also probably affect the magnitude of the final angular change obtained from the calculations.

(c) A two-dimensional model was used in the analysis, since a three-dimensional model would have made the analysis quite cumbersome. This kind of approximation, although valid for the majority of cases in the structural analysis, can always introduce some kind of error that would affect the final results of the study.

(d) Only the stress in \( x \) direction was considered, to study the effect of the heating of the plate by a moving point source. When Iwamura did experiments that culminated into this approximation, he had to neglect the stress in the \( z \) direction of the plate, although its value was in the order of \( 1/10 \) of that encountered in the \( x \) direction.

(e) A linear distribution of strains was assumed through the thickness of the plate. This was also one of the simplifications done in the analysis, although in the experiments this was not precisely the case.

In spite of the discrepancies encountered in the final results, one can conclude from the theoretical analysis that the behavior of the materials investigated is in quite good agreement with that obtained in the experiments.

Comparing Figure 5.1.1 to Figures 2.4.1 and 3.1.1, the following points are readily noticeable:

(a) The maximum value of the angular change in the theoretical analysis
CALCULATED TOTAL FREE JOINT ANGULAR DISTORTION $\theta_0$ FOR ALUMINUM AND STEEL

FIG. 5.1.1
occurred at a thickness in the order of 6.35 mm (0.250"), while in the experiments, the maximum occurred at 7.25 mm (0.285").

(b) The maximum value of the angular change in the analysis for steel occurred at about 11.50 mm (0.452"), while in the experiments this happened at about 9.70 mm (0.382").

(c) The fillet size of both models analysed being about the same, steel presented larger angular changes than aluminum, therefore confirming the results obtained in the experiments, that appeared to be a strange one at first sight.

As a conclusion, it seems therefore that the simplifications introduced in the analysis, have not allowed the obtaining of more accurate results, so that theory and experiments could have agreed. However, the results showed that it is feasible to think in the development of a more advanced method of carrying out the investigation so that the above goal can be reached.

This method would be one that would take into account not only the factors which were neglected in this analysis, but also include such variables which have close relation with the thermal history of the model then chosen.

For instance, factors like the increasing effective thickness of the plate at the weld fillet, after each weld pass, should be considered, because this increase seems to highly affect the effective stiffness of the plate, as well as the degree of shrinkage of the fillet.
5.2 - Comments and suggestions

The study of the distortion in general structures, due to the welding processes to which they are submitted, have been conducted since long time ago. However, the majority of them used steel as the basic material and the methods applied were either entirely empirical or semi-empirical.

Masubuchi was one of the first investigators who tried to explain the experimental results, using the available theory of elasticity, but the lack of a more efficient means of carrying the calculation at that time did not allow the conduction of a more rational analysis on this subject.

With the advance of technology, new structural materials were developed, and also more critical structures were being designed and constructed, so that predicting the distortion became a very important problem when the designing of a new structure is concerned.

Fortunately, the advance in the technology also included improvements in the field of computer science, so that modern methods of calculation, like the finite element method, could be introduced in the structural field.

Since then, several investigations were conducted so that one can feel today that very soon, reliable estimation of the distortion, at last for the ordinary type of existing structures, will be possible to be obtained.

The simplified flow chart below shows schematically the steps followed during the progress of the studies conducted in the field under discussion.
EVOLUTION OF THE STUDIES TO PREDICT

DISTORTION OF THE STRUCTURES CAUSED

BY WELDING PROCESSES

Pure empirical procedure, using experimental results, most of them for steel - (1950)

Semi-empirical procedure, using elasticity theory to relate experimental results to the theory. Most studies conducted for steel - (Masubuchi et al. 1956 - 1959)

Development of new structural materials

Development of more critical structures

Notable advance in Computer technology (1960-1970)

Utilization of more rational methods to solve structural problems, like the Finite Element Method became possible

Great advance in the study of thermal distortion with a reasonably good agreement between theory and the experiments (e.g. the study by Iwamura - 1970-1971)

High potentiality to obtain an accurate solution for the problem, using three-dimensional models and the Finite Element Method, combined with high capacity computer systems (within this decade)
The purpose of this thesis work, therefore, was just a tentative to show that with a careful choice of the variables and an appropriate solution method, it would be possible to obtain more accurate and reliable values of the distortion.

Before finalizing, the author would like to suggest the continuation of this study by means of a more elaborate and rational method of calculation, in order to see whether the analytical results can actually be a good prediction of the distortion encountered in practice.

Once this is attained, reliable designing standards can then be settled so that this will allow the construction of safer and more economical structures, which is the ultimate goal of the present research.
APPENDIX A

This Appendix A contains:

(1) General flow chart of the computer program utilized in the analysis to study the influence of the heating of the plate in the free joint angular change.

(2) Complete listing of the computer program with the appropriate comment cards.
GENERAL FLOW CHART FOR THE CALCULATION OF FREE JOINT ANGULAR CHANGE DUE TO THE HEATING OF THE PLATE BY A POINT HEAT SOURCE

START

READ INPUT DATA

DEFINE INITIAL CONDITIONS

NUMBER OF THERMAL CYCLES

NUMBER OF STEPS PER CYCLE

NUMBER OF SECTIONS

CALL SUBROUTINE TO CALCULATE TEMPERATURE DISTRIBUTION IN THE SECTION

DETERMINE TEMPERATURE-DEPENDENT MATERIAL PROPERTIES

JUDGE THE STRESS STATE AT STEP(N-1) AND MODIFY IT IN ACCORDANCE WITH TEMPERATURE AT STEP(N)

CALL SUBROUTINE TO CALCULATE STRESS AT STEP(N) BASED ON THE ELASTIC BEHAVIOR OF ΔT(N)

1
CALL SAME SUBROUTINE AGAIN TO MODIFY PREVIOUS STRESS STATE, NOW BASED ON THE REAL CONDITION AT STEP(N) - LOADING OR UNLOADING

CHECK THE ACCURACY OF NUMERICAL CALCULATION

ITERATION PROCESS

CALCULATE THE PLASTIC STRAIN

REDEFINE MATERIAL PROPERTIES FOR THE NEXT SECTION

CALCULATE DISPLACEMENTS AND SLOPE

PRINT OUT-PUTS FOR THAT STEP

GO TO NEXT THERMAL CYCLE

END
C THIS PROGRAM WAS ORIGINALLY WRITTEN BY Y. IWAMURA TO BE USED FOR
C STEEL STRIP MODELS.
C THE PROGRAM WAS MODIFIED BY THE AUTHOR TO BECOME POSSIBLE ITS USE
C FOR ALUMINUM MODELS. A SUBROUTINE WAS ALSO ADDED IN ORDER TO
C CALCULATE THE TEMPERATURE DISTRIBUTION THROUGH THE SECTION OF
C THE MODEL.

C THIS PROGRAM IS COMPOSED BY A MAIN PROGRAM (TO CALCULATE THE
C ELASTIC PLASTIC DEFORMATION OF A PLATE WHEN THE PLATE IS HEATED
C ALONG A STRAIGHT LINE, AS IN THE CASE OF THE LINE HEATING
C METHOD OR THE BEAD ON PLATE WELDING), AND TWO SUBPROGRAMS, ONE
C OF WHICH ESTIMATES THE TEMPERATURE DISTRIBUTION ALONG A
C TRANSVERSAL SECTION OF THE PLATE AND THE OTHER TO SOLVE THE
C SIMULTANEOUS SYSTEM OF EQUATIONS AND CALCULATE THE STRESSES.
C ** ****** ****** ******
C MAIN PROGRAM
C ****** ****** ****** ******
COMMON T(15,101),TEN(15,101),TSO(101),S(101),E(101),SM(101),
ICTE(101),EE(101),SNS(101),SSY(101),H(101),TSE(101),DS(101),OA(15),
2DB(15),DC(15),DO(15),SL(15),TE(101),DST(101),DDS(101),TMSTP(25),
3Q1,EXPI,VEL,TEO
DIMENSION AD(15),BD(15),CO(15),COND(15,101),DDP(101),
1DSSN(15,101),DISP(101),DISPZ(101),DO(15),EN(15,101),PS(15,101),
2SIGN(15,101),SLOPE(15),SMN(15,101),SO(101),SP(15,101),SSYN(101),
3SY(101),SYN(15,101)
C READ MATERIAL PROPERTIES THAT ARE UNVARIABLE THROUGHOUT THE
C COMPUTATION
READ(5,900) AE2,AS2,AS3,AM2,AM3,BE2,BS2,BS3,BM2,BM3
900 FORMAT(5F12.5)
C READ NUMBER OF CASES SUBMITTED 'NC'
READ(5,902) NC
902 FORMAT(I3)
C READ PLATE THICKNESS AND VARIABLES RELATED TO WELDING PROCESS.
DC 6000 NTM=1,NC
READ(5,904) HH,TEO,VOLT,AMP,EFF,CDTY,DIFU,VEL
904 FORMAT(5F10.3)
C COMPUTE HEAT EXCHANGED AND CONSTANT TERM OF THE EXPONENTIAL FUNCT.
Q1=0.12*VOLT*AMP*EFF/(3.14156*CDTY)
EXP1=-0.5*VEL/DIFU
C READ NUMBER OF CYCLES, STEPS, SECTIONS AND POINTS THAT WILL BE
C USED IN THE COMPUTATIONS
READ(5,906) JJN,N,L,M5
906 FORMAT(4I3)
LL=L-1
MM=M-1
C READ SEGMENT LENGTHS
READ(5,908) (SL(K),K=1,LL)
908 FORMAT(10F8.3)
C READ TIME STEPS, CORRESPONDING TO DIFFERENT STEPS IN A THERMAL
C CYCLE.
READ(5,910) (TMSTP(J),J=1,N)
910 FORMAT(10F8.3)
C PRINT OUT INPUT DATA
WRITE(6,950) P4,JJN,N,L,M5
950 FORMAT(1H1,50X,'RESULTS OF THE CALCULATIONS FOR',//,54X,'PLATE THI
ICKNESS ',F5.2,' MM',///,50X,'NUMBER OF THERMAL CYCLES.....',I3,/ 2,50X,'NUMBER OF STEPS PER CYCLE.....',I3,//,50X,'NUMBER OF SECTIONS. 3.............',I3,//,10X,'NUMBER OF POINTS PER SECTION.....',I3,//)
C SEGMENT LENGTHS
WRITE(6,952)
952 FORMAT(58X,'SEGMENT LENGTHS',//,50X,'SEGMENT NO.',9X, 1'LENGTH (MM)',//)
WRITE(6,954) (K,SL(K),K=1,LL)
954 FORMAT(54X,I2,17X,F5.2)
C START THE COMPUTATION BEGINNING WITH THE DETERMINATION OF THE
C COORDINATES THROUGHOUT THE THICKNESS OF THE PLATE
H(1)=0.0
DO 200 I=2,M
H(I)=(I-1)*HH/MM

200 CONTINUE

C DEFINE INITIAL CONDITIONS
DO 300 K=1,L
DO 250 I=1,M
TEN(K,I)=0.0
DDSN(K,I)=0.0
PS(K,I)=0.0
SP(K,I)=0.0
COND(K,I)=10.0
SMN(K,I)=2.00E-2
EN(K,I)=7.26E+3
SYN(K,I)=2.18E+1

250 CONTINUE
300 CONTINUE

C INITIALIZE COEFFICIENTS OF STRAIN
DO 350 K=1,LL
AC(K)=0.0
BC(K)=0.0
CO(K)=0.0
DG(K)=0.0

350 CONTINUE

C JNJ=NUMBER OF THERMAL CYCLES
DO 5000 J=1,JNJ

C N=NUMBER OF STEPS IN EACH THERMAL CYCLE
N2=N+1
DO 4000 J=1,N2

C L=NUMBER OF SECTIONS
DO 3000 K=1,L
KK=K-1

C CALL SUBROUTINE TO CALCULATE THE TEMPERATURES FOR THE SECTION PTS.
CALL TEMP(J,K,KK,M,HH,N2)

C DETERMINE TEMPERATURE DEPENDENT MATERIAL PROPERTIES
IF(CCN(D(K,I)-10.0) 23,23,22
22 IF(CCN(D(K,I)-15.0) 24,24,25
23 TSO(I)=SP(K,I)/EN(K,I)
   SO(I)=E(I)*TSO(I)
   IF(ABS(SC(I))-SY(I)) 37,37,52
24 IF(SP(K,I)) 31,31,32
31 SSYN(I)=-1.0*SYN(K,I)
   SSY(I)=-1.0*SY(I)
   SIGN(K,I)=-1.0
   GO TC 33
32 SSYN(I)=SYN(K,I)
   SSY(I)=SY(I)
   SIGN(K,I)=1.0
33 TSO(I)=(SP(K,I)-SSYN(I)*(1.0-SMN(K,I)))/(SMN(K,I)*EN(K,I))
   SC(I)=E(I)*TSO(I)
   IF(ABS(SC(I))-SY(I)) 38,55,55
38 PS(K,I)=PS(K,I)+SP(K,I)/EN(K,I)-SO(I)/E(I)
   GO TC 34
25 TSO(I)=SP(K,I)/EN(K,I)
   SC(I)=E(I)*TSO(I)
   GO TC 37
34 CCN(D(K,I)=10.0
   GO TC 51
52 CCN(D(K,I)=15.0
53 SSY(I)=-1.0*SY(I)
   SIGN(K,I)=-1.0
   GO TC 56
54 SSY(I)=SY(I)
   SIGN(K,I)=1.0
55 SC(I)=SM(I)*E(I)*TSO(I)+SSY(I)*(1.0-SM(I))
   PS(K,I)=PS(K,I)+SP(K,I)/EN(K,I)-SO(I)/E(I)
   GO TC 51
37 IF(SC(I)) 35,36,36
35    SIGN(K,I)=-1.0
      GC TC 51
36    SIGN(K,I)=1.0
51    EE(I)=E(I)
      SNS(I)=0.0
      SSY(I)=0.0
400   CONTINUE
      NN=1
C    CALCULATION OF STRESS AT STEP 'N', BASED ON ELASTIC BEHAVIOR OF
C    INCREMENTAL THERMAL EXPANSION.
      CALL STRESS(K,M,MM,HH,NN)
      DO 450 I=1,M
      S(I)=SO(I)+DST(I)
450   CONTINUE
2001  DAN=LA(K)
      DBN=DB(K)
C    MODIFICATION OF THE STRESS STATE FOR INCREMENTAL STRAIN
      DO 550 I=1,M
      IF(CCND(K,I)-10.0) 73,73,72
      IF(CCND(K,I)-15.0) 74,74,75
      72   IF(DCS(I)*DDS(K,I)) 101,103,103
      73   IF(DDS(I)*DDS(K,I)) 92,105,105
      74   IF(S(I)*SIGN(K,I)) 93,94,94
      92   IF(ABS(S(I))-SY(I)) 96,96,95
      93   CON(I)=20.0
      GO TO 97
      72   CON(I)=10.0
      73   TSO(I)=SP(K,I)/EN(K,I)
      97   SG(I)=E(I)*TSO(I)
      74   GO TO 111
      92   TSO(I)=SP(K,I)/EN(K,I)
      93   SG(I)=E(I)*TSO(I)
      95   GO TO 105
      75   IF(DDS(I)*DDS(K,I)) 103,101,101
101 IF(S(I)*SIGN(K,I)) 103,117,117
103 IF(ABS(S(I))-SY(I)) 104,104,105
104 CON(I)=10.0
   GO TO 111
105 CON(I)=15.0
   GO TO 112
117 CON(I)=20.0
111 EE(I)=E(I)
   SSY(I)=0.0
   SNS(I)=C.0
   GO TO 550
112 IF(S(I)) 113,113,114
113 SSY(I)=-1.0*SY(I)
   GO TO 115
114 SSY(I)=SY(I)
115 EE(I)=SM(I)*E(I)
   SNS(I)=1.0-SM(I)
   GO TO 550
550 CONTINUE
C CALCULATE THE STRESS BASED ON THE MODIFIED STRESS STATE.
   CALL STRESS(K,M,MM,HH,NN)
C CHECK THE ACCURACY OF THE NUMERICAL CALCULATION
   IF(ABS(DA(K)-DAN)-1.0E-5) 122,122,2001
122 IF(ABS(DB(K)-DBN)-1.0E-6) 601,601,2001
601 CONTINUE
C CALCULATE THE PLASTIC STRAIN.
   DO 600 I=1,M
   DDP(I)=DDS(I)-(S(I)-SG(I))/E(I)
   PS(K,I)=PS(K,I)+DDP(I)
   TEN(K,I)=TE(I)
   SP(K,I)=S(I)
   DDSN(K,I)=DDS(I)
   SMN(K,I)=SM(I)
   EN(K,I)=E(I)
   DO 600
SYN(K,I)=SY(I)
COND(K,I)=CON(I)

600 CONTINUE
IF(K-1) 3000, 3000, 62
62 DC(KK)=(CA(K)-DA(KK))/SL(KK)
DD(KK)=(DB(K)-DB(KK))/SL(KK)
C GO TO THE NEXT SECTION
3000 CONTINUE
C CALCULATE THE SLOPE AND THE DISPLACEMENT.
SLOPE(1)=0.0
DISP(1)=0.0
DISPZ(1)=0.0
DC 650 K=1, LL
AO(K)=AO(K)+DA(K)
BO(K)=BO(K)+DB(K)
CO(K)=CO(K)+DC(K)
DC(K)=DO(K)+DD(K)

650 CONTINUE
DC 880 K=2, L
KK=K-1
SLOPE(K)=SLOPE(KK)+SL(KK)*(BO(KK)+SL(KK)*DO(KK)/2.0)
DISP(K)=DISP(KK)+SL(KK)*(AO(KK)+SL(KK)*CO(KK)/2.0)
DISPZ(K)=DISPZ(KK)+DISP(K)+SLOPE(K)*H(M)

880 CONTINUE
C PRINT-OUTS
WRITE(6, 960) JJ, J
960 FORMAT(1H1, 53X, 'RESULTS FOR', //, 50X, 'THERMAL CYCLE NO. ', I2, //, 50X
1, 'STEP NUMBER..... ', I2, //)
DC 860 K=1, L, 2
WRITE(6, 962)
962 FORMAT(/, 1X, 2(3X, 'SECTION', 7X, 'SLOPE', 7X, 'DISP.(Z=H)', 4X, 'DISP.(Z
1=O)', 7X)/)
KK=K+1
WRITE(6, 963) K, SLOPE(K), DISPZ(K), DISP(K)
FORMAT(6X, I2, 7X, E11.4, 3X, E11.4, 3X, E11.4)
IF(KK-L) 964, 964, 970
964  WRITE(6, 965) KK, SLOPE(KK), DISPZ(KK), DISP(KK)
965  FORMAT(1H+, 65X, I2, 7X, E11.4, 3X, E11.4, 3X, E11.4, /)
970  WRITE(6, 966)
966  FORMAT(/, 1X, 2(2X, 'THICKNESS', 5X, 'TEMP.(C)', 7X, 'STRESS', 4X, 'PLAST.
1STRAIN', 6X)/)
       WRITE(6, 967) H(1), T(K, 1), SP(K, 1), PS(K, 1)
       IF(KK-L) 893, 893, 894
893  WRITE(6, 969) H(1), T(KK, 1), SP(KK, 1), PS(KK, 1)
894  WRITE(6, 967) H(M), T(K, M), SP(K, M), PS(K, M)
       IF(KK-L) 895, 895, 890
895  WRITE(6, 969) H(M), T(KK, M), SP(KK, M), PS(KK, M)
967  FORMAT(4X, F6.2, 7X, F7.2, 5X, E11.4, 3X, E11.4)
969  FORMAT(1H+, 63X, F6.2, 7X, F7.2, 5X, E11.4, 3X, E11.4)
890  CONTINUE
800  CONTINUE
C    GO TO THE NEXT STEP
4000  CONTINUE
C    GO TO THE NEXT THERMAL CYCLE
5000  CONTINUE
C    GO TO THE NEXT THICKNESS
6000  CONTINUE
7000  CALL EXIT
STOP
END
SUBROUTINE STRESS(K,M,MM,HH,NN)
COMMON T(15,101),TEN(15,101),TSO(101),S(101),E(101),SM(101),
ICTE(101),EE(101),SNS(101),SSY(101),H(101),TSE(101),DS(101),DA(15),
2DB(15),DC(15),DD(15),SL(15),TE(101),DST(101),DDS(101),TMSTP(25),
3Q1,EXP1,VEL,TEO
C       THIS SUBROUTINE IS USED TO SOLVE THE SYSTEM OF SIMULTANEOUS EQUAT.
C       AND STRESSES.
NN=NN+1
DO 1000 I=1,M
TE(I)=ICTE(I)*T(K,I)
IF(NN-2) 81,81,82
81 TSE(I)=TE(I)-TEN(K,I)
GO TO 1000
82 TSE(I)=TE(I)-TEN(K,I)-TSO(I)
1000 CONTINUE
C       EXECUTE THE INTEGRATION.
SAE=0.0
SAEZ=0.0
SAEZZ=0.0
SAS=0.0
SASZ=0.0
SASE=0.0
SASEZ=0.0
DO 1100 I=2,MM,2
II=I+1
SAE=SAE+4.0*EE(I)+2.0*EE(II)
SAEZ=SAEZ+4.0*EE(I)*H(I)+2.0*EE(II)*H(II)
SAEZZ=SAEZZ+4.0*EE(I)*H(I)*H(I)+2.0*EE(II)*H(II)*H(II)
SAS=SAS+4.0*EE(I)*TSE(I)+2.0*EE(II)*TSE(II)
SASZ=SASZ+4.0*EE(I)*TSE(I)*H(I)+2.0*EE(II)*TSE(II)*H(II)
SASE=SASE+4.0*SSY(I)*SNS(I)+2.0*SSY(II)*SNS(II)
SASEZ=SASEZ+4.0*SSY(I)*SNS(I)*H(I)+2.0*SSY(II)*SNS(II)*H(II)
1100 CONTINUE
SIP=HH/(3.0*(M-1))
SE = SIP*(SAE + EE(1) - EE(M))
SEZ = SIP*(SAEZ - EE(M)*H(M))
SEZZ = SIP*(SAEZZ - EE(M)*H(M)*H(M))
SS = SIP*(SAS + EE(1) + TSE(1) - EE(M)*TSE(M))
SSZ = SIP*(SASZ - EE(M)*TSE(M)*H(M))
SSE = SIP*(SAE + SSY(1) + SNS(1) - SSY(M)*SNS(M))
SSEZ = SIP*(SASEZ - SSY(M)*SNS(M)*H(M))
SES = SS - SSE
SESZ = SSZ - SSEZ

C COMPUTE THE COEFFICIENT OF THE STRAIN
DT1 = SE*SEZ - SEZ*SEZ
DA(K) = (SEZZ*SES - SEZ*SESZ)/DT1
DB(K) = (SE*SESZ - SEZ*SES)/DT1
DO 1200 I = 1, M
DS(I) = DA(K) + DB(K)*H(I)
1200 CONTINUE

C CALCULATE THE STRESSES
IF(NN-2) 83, 83, 84

83 CONTINUE
DO 1400 I = 1, M
DST(I) = EE(I)*(DS(I) - TSE(I))
DDS(I) = DS(I) - TSE(I)
1400 CONTINUE
GO TO 1600

84 CONTINUE
DO 1500 I = 1, M
DDS(I) = DS(I) - TE(I) + TEN(K, I)
S(I) = EE(I)*(DS(I) - TSE(I)) + SSY(I)*SNS(I)
1500 CONTINUE

1600 CONTINUE
RETURN
END
SUBROUTINE TEMP(J,K,KK,M,HH,N2)
COMM CN T(15,101), TEN(15,101), TSO(101), S(101), E(101), SM(101),
ICTE(101), EE(101), SNS(101), SSY(101), H(101), TSE(101), DS(101), DA(15),
2DB(15), DC(15), DD(15), SL(15), TE(101), DST(101), DDS(101), TMSTP(25),
301, EXP1, VEL, TEO
C DEFINE TIME FOR COMPUTATION
IF(J=N2) 41,42,42
41 TM=TMSTP(J)
C DEFINE SECTION 'Y'
IF(K=1) 8,8,10
8 Y=0.0
GO TO 13
10 Y=0.0
DC 12 K1=1,KK
Y=Y+SL(K1)/10.0
12 CONTINUE
13 CONTINUE
C COMPUTATION OF THE TEMPERATURES FOR THE SECTION 'K'
DO 22 I=1,M
C FIRST DETERMINE COORDINATE 'Z'. HH AND H(I) ARE GIVEN IN MM.
Z=(HH-H(I))/10.0
C COMPUTE DISTANCE 'R'
R=SQR((VEL*TM)**2+Y*Y+Z*Z)
C IF 'R' IS TOO SMALL, BLOWING UP OF THE TEMPERATURE CAN HAPPEN. SO
C CHECK 'R'
IF(R<10.0E-4) 20,20,14
14 EXP3=EXP(EXP1*R)/R
C CALCULATE 'RN' AND 'RNN' CHECKING THE CONVERGENCE
NSER=0
TRN=0.0
TRNN=0.0
RNT=(VEL*TM)**2+Y*Y
DC 30 J2=1,5
NSER=J2

RN1 = (0.2 * NSER * HH - Z)**2
RNN1 = (0.2 * NSER * HH + Z)**2
RN = SQRT(RNT + RN1)
RNN = SQRT(RNT + RNN1)
EXPN = EXP(EXP1 * RN) / RN
EXPNN = EXP(EXP1 * RNN) / RNN
TRN = TRN + EXPN
TRNN = TRNN + EXPNN

30 CONTINUE
EXP2 = EXP(EXP1 * (-VEL) * TM)
T(K, I) = TEO + Q1 * EXP2 * (EXP3 + TRN + TRNN)
IF(T(K, I) - 630.0) < 22, 22, 20

20 T(K, I) = 630.0
22 CONTINUE
GO TO 45

42 CONTINUE
DO 40 I = 1, M
T(K, I) = TEO
40 CONTINUE
45 CONTINUE
RETURN
END
APPENDIX B

This Appendix B contains:

(1) General flow chart of the computer program utilized in the analysis to study the influence of the shrinkage of the weld fillet in the free joint angular change.

(2) Complete listing of the computer program, with the appropriate comment cards.
GENERAL FLOW CHART FOR THE CALCULATION OF FREE JOINT ANGULAR CHANGE DUE TO THE SHRINKAGE OF WELD FILLETS

START

READ TEMPERATURE LIMITS AND MATERIAL PROPERTIES

CALCULATE THERMAL STRAIN FOR THE INITIAL CONDITION

INITIALIZE REMAINING CONDITIONS

CALCULATE COEFFICIENTS OF THE STIFFNESS MATRIX

CALL SUBROUTINE TO CALCULATE THE VALUES OF THE ASSUMED DEFORMATIONS USING GAUSS--JORDAN REDUCTION METHOD

CALCULATE SLOPE AT THE REFERENCE NODE

PRINT OUT-PUTS

END
***ANGULAR DEFORMATION DUE TO SHRINKAGE OF THE FILET WELDMENT***

COMMON F(4),A(4,4)
DIMENSION BB(8),HH(8)

READ(5,40) MM,LL,M,RAT
FORMAT(3I3,F10.3)

*************
MM=MATERIAL
*************
DO 1600 KK=1,MM

READ(5,45) TM,TR,CTE
FORMAT(2F10.3,E11.4)

WRITE(6,48) TR,TM,CTE
FORMAT(2X,'TR=',F8.2,3X,'TM=',F7.2,3X,'CTE=',E11.4,/)  

POA1=1.0/(1.0+RAT*RAT)
POA2=1.0/(1.0+RAT)
TSRAN=CTE*(TR-TM)
F0=POA1*TSRAN*(1.0-RAT)

READ(5,50) (BB(L),L=1,LL)
READ(5,50) (HH(K),K=1,M)

FORMAT(8F10.3)

*******************************************************************************

**************
LL=NUMBER OF FILET LENGTH
*******************************************************************************

DO 560 L=1,LL
B=BB(L)
C **********************************
C M=NUMBER OF THICKNESS
C **********************************
DO 10 K=1,M
  H=HH(K)
  RHB=H/B
  RBH=B/H
C
  WRITE(6,60) H,B
  60  FORMAT(/,1X,'H=',F6.3,3X,'B=',F6.3,/)  
C
C **********************************
C INITIAL CONDITION
C **********************************
  F(1)=0.0
  F(2)=F0*B
  F(3)=0.0
  F(4)=0.0
C
C **********************************
C STIFFNESS MATRIX
C **********************************
  A(1,1)=(2.0*RHB*POA1+RBH*POA2)/3.0
  A(1,2)=(RHB*POA1-RBH*POA2)/3.0
  A(1,3)=(2.0*RAT*POA1-POA2)/4.0
  A(1,4)=-1.0*(RAT*POA1+2.0*POA2)/4.0
  A(2,1)=A(1,2)
  A(2,2)=A(1,1)+POA1
  A(2,3)=-1.0*A(1,4)
  A(2,4)=-1.0*A(1,3)
  A(3,1)=A(1,3)
  A(3,2)=A(2,3)
  A(3,3)=(2.0*RHB*POA1+RBH*POA2)/3.0
  A(3,4)=(-4.0*RHB*POA1+RBH*POA2)/6.0
  A(4,1)=A(1,4)
A(4,2)=A(2,4)
A(4,3)=A(3,4)
A(4,4)=A(3,3)+POA2/2.0

C
WRITE(6,70)
70 FORMAT(4X,'A(1,1)',7X,'A(1,2)',7X,'A(1,3)',7X,'A(1,4)',7X,
1 'A(2,2)',7X,'A(3,3)',7X,'A(3,4)',7X,'A(4,4)'//)
WRITE(6,75) A(1,1),A(1,2),A(1,3),A(1,4),A(2,2),A(3,3),
1 A(3,4),A(4,4)
75 FORMAT(1X,8(E11.4,2X)//)

C
-----------------------------------------------
CALL MATRIX(U2,U3,V2,V3)
-----------------------------------------------

C
WRITE(6,80)
80 FORMAT(6X,'U2',11X,'U3',11X,'V2',11X,'V3',//)
WRITE(6,85) U2,U3,V2,V3
85 FORMAT(1X,4(E11.4,2X)//)

C
SLOPE2=ATAN(V2/(B+U2))
SLOPE3=ATAN(V3/(B+U3))
SLOPE=(SLOPE2+SLOPE3)/2.0

C
WRITE(6,90)
90 FORMAT(4X,'SLOPE2',7X,'SLOPE3',7X,'SLOPE',//)
WRITE(6,95) SLOPE2,SLOPE3,SLOPE
95 FORMAT(1X,3(E11.4,2X)//)

C
100 CONTINUE
500 CONTINUE
1000 CONTINUE
STCP
END
SUBROUTINE MATRIX(U2,U3,V2,V3)
C ************************************************************************************************
C GAUSS-JORDAN METHOD
C ************************************************************************************************
C COMMON F(4),A(4,4)
C ************************************************************************************************
C FIRST REDUCTION
C ************************************************************************************************
F(1)=F(1)/A(1,1)
DO 5 I=2,4
F(I)=F(I)-F(1)*A(I,1)
5 CONTINUE
DO 10 J=2,4
A(1,J)=A(1,J)/A(1,1)
10 CONTINUE
DO 30 I=2,4
DO 20 J=2,4
A(I,J)=A(I,J)-A(I,1)*A(1,J)
20 CONTINUE
30 CONTINUE
C ************************************************************************************************
C SECOND REDUCTION
C ************************************************************************************************
F(2)=F(2)/A(2,2)
F(1)=F(1)-F(2)*A(1,2)
F(3)=F(3)-F(2)*A(3,2)
F(4)=F(4)-F(2)*A(4,2)
A(2,3)=A(2,3)/A(2,2)
A(2,4)=A(2,4)/A(2,2)
A(1,3)=A(1,3)-A(1,2)*A(2,3)
A(1,4)=A(1,4)-A(1,2)*A(2,4)
DO 50 I=3,4
DO 40 J=3,4
A(I,J)=A(I,J)-A(I,2)*A(2,J)
50 CONTINUE
40 CONTINUE

CONTINUE
CONTINUE

THIRD REDUCTION

F(3)=F(3)/A(3,3)
F(1)=F(1)-F(3)*A(1,3)
F(2)=F(2)-F(3)*A(2,3)
F(4)=F(4)-F(3)*A(4,3)
A(3,4)=A(3,4)/A(3,3)
A(1,4)=A(1,4)-A(1,3)*A(3,4)
A(2,4)=A(2,4)-A(2,3)*A(3,4)
A(4,4)=A(4,4)-A(4,3)*A(3,4)

FOURTH REDUCTION

F(4)=F(4)/A(4,4)
DO 60 I=1,3
  F(I)=F(I)-F(4)*A(I,4)
60 CONTINUE

DISPLACEMENT

U2=F(1)
U3=F(2)
V2=F(3)
V3=F(4)
RETURN
END
LIST OF REFERENCES


20. Masubuchi, K., Class notes of the Course 13.151J, Welding Engineering, Department of Ocean Engineering, MIT.