COMPUTATIONAL MODELING OF THE GAS-PHASE TRANSPORT PHENOMENA AND EXPERIMENTAL INVESTIGATION OF SURFACE TEMPERATURES DURING FLAME-JET THERMAL SPALLATION DRILLING

by

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ABSTRACT

Thermal spallation is a method whereby the surface of a rock is rapidly heated causing small (100-500 μm) flakes, or spalls, to form. When applicable to drilling, a supersonic, high temperature (2600 K) gas jet is directed at the rock to provide the heat source and sweep away the spalls.

Previous studies of thermal spallation drilling indicate that penetration rates of up to 30 m/hr (100 ft/hr), or greater than ten times those commonly obtained using conventional rotary mechanical methods, can be achieved in coherent, hard rock such as granite. These results suggest that substantial cost reductions could be possible in the construction of hot dry rock geothermal power plants where rotary mechanical methods are used for deep well drilling and account for an estimated 50% of the initial capital cost.

The current study has focussed on gaining a better understanding of both the rock failure mechanism that occurs during thermal spallation, and the heat transfer from the gas jet to the rock surface. Rock mechanics modeling leads to an expression for the surface temperature during spallation as a function of rock physical properties and the incident heat flux. Surface temperature and heat flux determination during laser and flame-jet induced thermal spallation are used to provide appropriate values of the "Weibull parameters" that describe the size-strength relationship in granite. Use of these parameters allows accurate estimation of surface temperatures in the numerical simulation model developed to calculate the transport phenomena occurring in the flow-field above the rock surface.

Gas side heat transfer predictions at the rock/gas interface have been made by numerically solving the Reynolds averaged form of the mass, momentum, and energy conservation equations in an axisymmetric coordinate system. The solution scheme utilizes a k-ε turbulence model for estimating turbulent transport coefficients, and a second order accurate finite volume method, stabilized by a four-stage Runge-Kutta time stepping scheme and by added "artificial dissipation", for determination of the steady-state flow-field and hole geometry. Model estimates of penetration rates and hole diameters are within ± 5% of those found experimentally.

Thesis Supervisor: Dr. Jefferson W. Tester
Title: Professor of Chemical Engineering
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1 Thesis Digest

1.1 Introduction

Thermal spallation can be broadly defined as fragmentation of the surface of a brittle solid into small disk-like flakes, called spalls, by rapidly heating a relatively small fraction of the solid (e.g. less than 10% of the exposed area). Thermal stresses cause failure and arise from the tendency of the material to expand as temperature is increased. When thermal spallation is used for drilling a "flame-jet" like the one shown in Figure 1.1 is commonly used to impart the high heat fluxes (typically greater than 1.0 MW/m²) required and to sweep away the spalls.

Recent interest in the use of naturally occurring underground regions of hot dry rock (HDR) for power generation has resulted in a need for inexpensive, reliable drilling methods in hard, usually granitic, crystalline basement rock that is typically encountered (Armstead and Tester, 1987). The second law of thermodynamics limits the amount of useful work (electric power) that can be produced in such a heat mining process. The amount of electric power produced per unit of heat removed from the earth increases as the geothermal fluid (water) temperature increases. Since the temperature of the geothermal fluid is governed by the temperature of the hot underground reservoir and the earth's temperature usually increases with depth, the power generating capability of a HDR energy extraction system can be increased only by drilling deeper.

The high sensitivity of total geothermal power plant cost to drilling costs, combined with the inherent difficulties encountered when
Figure 1.1 Schematic of Thermal Spallation Drilling (Williams et al., 1988).
using conventional methods for these applications provides a strong economic incentive to explore alternative drilling techniques. As HDR well costs decrease, a larger fraction of the earth's available resource becomes commercially viable. In addition, other fields of interest including tunneling (for transportation and waste disposal applications), mining, deep-well waste storage, and oil and gas drilling in the overthrust belt would benefit from a rapid, inexpensive method for penetrating through crystalline rock.

In general, spallation drilling works best in rocks with high quartz ($SiO_2$) content, such as granite, quartzite, and some sandstones. These rock types are apparently able to build up the required compressive stresses, under rapid heating conditions, before stress relieving mechanisms such as softening or melting occur. A further requirement for good spallation drilling characteristics is that the rock mass being drilled be relatively free of macroscopic defects such as faults that would enable stress relief to occur.

These requirements are met by most crystalline basement rocks that are encountered during deep (>1 km) well drilling. However, more typically encountered rocks such as limestone and shale, which are found in oil and gas well drilling, either spall very poorly, or do not spall at all. Research aimed at extending the use of thermal spallation to these "soft" rocks is currently being conducted jointly by LANL and the New Mexico Institute of Mining and Technology (Potter, 1988).
1.2 Modeling of Thermal Spallation Drilling

Knowledge of the heat flux and surface temperature at any point on the spalling surface allows determination of the local drilling velocity through a heat balance on the control volume surrounding the rock-gas interface. The heat balance can be written:

\[ Q = (\rho C_p)_r U_{dr} (T_s - T_{r0}) + \Delta H_{misc} \]  \hspace{1cm} (1.1)

where

- \((\rho C_p)_r\) = rock thermal density (J/m\(^3\) K),
- \(Q\) = local heat flux to the rock (W/m\(^2\)),
- \(U_{dr}\) = local penetration rate normal to the surface (m/s),
- \(\Delta H_{misc}\) = energy losses due to miscellaneous phase transitions and crack formation (W/m\(^2\)),
- \(T_s\) = local surface temperature (K), and
- \(T_{r0}\) = initial rock temperature (K).

The second term on the right-hand side of equation (1.1) is assumed to be negligible during thermal spallation drilling.

In a somewhat idealized case of steady-state drilling, the average hole shape in the actively spalling region must remain constant and satisfy the condition that the forward component of the local penetration rate is the same everywhere. This is illustrated in Figure 1.2 and can be expressed mathematically as:

\[ V_{dr} = \frac{U_{dr}}{\cos(\theta)} \]  \hspace{1cm} (1.2)

where \(V_{dr}\) = forward advance rate of drill head (m/s), and \(\theta\) = angle of the tangent of the rock boundary to the horizontal plane.

In the present study and in a study conducted by Rauenzahn (1986), computer simulations have been developed for predicting the heat fluxes along the rock surface during thermal spallation drilling. Comparison
Figure 1.2 Illustration of modeled region and self-consistency condition (equation 1.2).
of heat flux and hole diameter predictions with experimental results is accomplished by forming non-dimensional Stanton numbers and hole radii. In this case, the Stanton number (St) is defined as the ratio of the rock surface heat flux ($Q_r$) to the total jet inlet heat flux ($Q_{jet}$), or:

$$\text{St} = Q_r / Q_{jet}$$  \hspace{1cm} (1.3),

and

$$Q_{jet} = (\rho C_p U)_{jet} (T_{jet} - T_s)$$  \hspace{1cm} (1.4)

where $T_{jet}$ is the temperature at the nozzle outlet. In order to determine the Stanton number from an experimentally measured penetration rate ($V_{dr}$), $Q_r$ is calculated from equation (1.1) giving:

$$\text{St}_{exp} = \frac{(\rho C_p)_r V_{dr} (T_s - T_{r0})}{(\rho C_p U)_{jet} (T_{jet} - T_s)}$$  \hspace{1cm} (1.5)

Computer simulation results directly predict the rock surface heat flux so that predicted Stanton Numbers are given by

$$\text{St}_p = \frac{Q_p}{(\rho C_p U)_{jet} (T_{jet} - T_s)}$$  \hspace{1cm} (1.6)

where $Q_p$ = predicted heat flux.

Figure 1.3 shows Rauenzahn's results for both Stanton numbers as a function of the non-dimensional drill head stand-off distance (SOD) defined as:

$$\text{SOD} = \frac{Z_d}{R_{noz}}$$  \hspace{1cm} (1.7)

where $Z_d$ = distance from bottom of drill to bottom of hole, and $R_{noz}$ = nozzle inside radius. Predicted Stanton numbers ($\text{St}_p$) were from three to five times greater than the experimental ones ($\text{St}_{exp}$). This large discrepancy provides the major motivation for the current study.
\( \triangle = \text{Prediction (St}_p\text{).} \)

\( \bullet = \text{Barre Granite Small-Scale Drilling Experiments (St}_{\text{exp}}\text{).} \)

\( \square = \text{Westerly Granite Small-Scale Drilling Experiments (St}_{\text{exp}}\text{).} \)

**Figure 1.3** Stanton number versus stand-off distance (Rauenzahn, 1986).
1.3 Thesis Objectives

The overall goal of this work is to be able to accurately predict performance characteristics during spallation drilling. Forward drilling rate and hole radius are both determined by local drilling velocities, which in turn are determined from predicted values of local heat fluxes (Q) and rock surface temperatures (T_s) through equation (1.1). The specific objectives of this thesis are:

1. to develop an accurate method of predicting the heat flux to a spalling surface, and
2. to measure the surface temperatures during spallation for several spallable rocks being subjected to known values of heat flux.

Section 1.4 contains a description of the rock mechanics modeling work performed by Dey (1984). Section 1.5 describes experiments conducted in the present study aimed at verifying Dey’s modeling. Section 1.6 outlines the gas-side heat transfer and fluid mechanics modeling and presents results leading to overall predictions of spallation drilling performance. Finally, Section 1.7 contains a summary of the conclusions and recommendations.

1.4 Analysis of Rock Failure During Thermal Spallation Drilling

Preston (1934) was the first worker to propose the currently accepted mechanism for spall formation. Preston's mechanism, shown in Figure 1.4 (Rauenzahn, 1986), is as follows:

1. A pre-existing flaw, near a surface, is acted upon by a compressive stress (thermally induced in the case of thermal spallation).
2. The flaw propagates in the direction of the applied stress, and parallel to the free surface.
3. The high aspect ratio (diameter:thickness) of the resulting plate and the high compressive stress cause buckling to occur, forming a "spall".
Figure 1.4 Simplified Chain of Events Leading to Spall Formation on a Surface of a Semi-Infinite Solid (Preston, 1934).
Although the mechanism described above is useful for providing a qualitative image of the processes occurring during thermal spallation, it does not describe the real situation where two or more flaws of flaws various orientations and lengths may interact and when certain components of the rock do not undergo brittle failure at the conditions of interest.

With the above mentioned uncertainties in mind, Dey (1984) developed a mathematical description of the initiation of spallation based upon four underlying assumptions:

1) compressive failure occurs during spallation,
2) the stress level at failure is governed by the orientation, size, and number density of pre-existing flaws,
3) the Weibull Statistical failure theory describes this stress level distribution, and
4) the temperature field can be approximated as being one-dimensional and treated as that at a subliming solid/gas interface.

Weibull theory considers that there is a probability of fracture associated with a given sample under stress such that the cumulative probability of failure can be given by (Weibull, 1939):

\[ G(\sigma) = 1.0 - \exp\left[\int_0^V (\sigma/\sigma_0)^m dV\right] \]  

(1.8)

where

- \( \sigma_0 \) = compressive strength of rock per unit volume (MPa m³/m),
- \( V \) = sample volume under stress (m³), and
- \( m \) = "homogeneity factor".

Integration of the right-hand side of equation (1.8) requires knowledge of the stress distribution within the sample volume. Assuming elastic behavior of the rock, constant surface temperature \( T_s \) and heat flux \( Q \), and that each spall is shaped like a cylinder with diameter to thickness ratio \( C_L \) of about 15 to 1 leads to the following
expression for the median surface spallation temperature (where \( G(\sigma) = 0.5 \)) (Dey, 1984):

\[
T_s = T_{r0} + \left[ \frac{3}{\rho C_p} \left( \frac{1 - \nu}{\beta_r E} \sigma_0 \right)^m \frac{2(0.693)}{\pi C_L^2} \left( \frac{m}{\alpha_r} \right)^{1/(m+3)} \right]
\]  

(1.9)

\( E \) = Young's modulus,
\( \nu \) = Poisson's ratio, and
\( \alpha_r \) = thermal diffusivity of the rock.

All physical properties and the Weibull strength parameters, \( m \) and \( \sigma_0 \), were considered to be temperature independent when deriving equation (1.9). Therefore, the parameter values that are currently used for making calculations with equation (1.9) must be considered as temperature-averaged in some sense.

1.5 Experimental Determination of Spallation Surface Temperatures

In order to use the theory described in the preceding section to estimate the spalling surface temperature, values for the Weibull parameters, \( m \) and \( \sigma_0 \), are needed. Dey and Kranz (1985) performed mechanical tests at ambient temperature on Berkeley "blue" granite. Rauenzahn (1986) studied thermal spallation on Westerly and Barre granite at low heat fluxes (<1.0 MW/m²) using a 500 W CO₂ laser and a calibrated welding torch as heat sources. Both experimenters estimated values of \( m \approx 20 \) and \( \sigma_0 \approx 70 \text{ MPa-m}^3/20 \). Rauenzahn estimated spallation temperatures by recording heating tests on high speed videotape at 2000 frames per second and reviewing the tapes to determine the time taken for the first point on the surface to spall. The rock was assumed to behave as a one-dimensional, semi-infinite solid subjected to a constant heat flux (Q in W/m²) as described by (Carslaw and Jaeger, 1959):
\[ T_s = T_{r0} + 2 \frac{Q}{k_r} \left( \frac{a_r t}{\pi} \right)^{1/2} \]  

where \( t = \) time (s). Therefore, knowing the heat flux and the time taken for the first spall to appear, the surface temperature (in K) can be estimated from equation (1.10). However, using the first spall as the criterion for determining the spallation temperature complicates interpretation. In the context of the theory described earlier, spallation is expected to occur at a distribution of stress levels and the spallation temperature is defined as that occurring at the median cumulative probability of failure. The first spall to appear is in the low-end tail of the failure stress distribution where spalling temperatures are expected to be below the median spallation temperature.

Additional difficulties arise from the small beam diameters (0.5 and 2.0 mm) used during Rauenzahn's laser heating tests. The one-dimensional heating assumption used in deriving equation (1.9) is violated by both of these beam radii because the depth of the heated region is on the order of 1 mm which is close to the beam diameter. Furthermore, the small volumes that are heated probably do not include enough flaws and grains for the Weibull-based theory developed by Dey (1984) to adequately describe the statistics of the failure processes occurring.

Experiments conducted in this study were aimed at avoiding the difficulties described above. The specific experimental objectives were:

1. to extend the range of heat fluxes at which spallation temperatures in common granite rocks are known,
2. to have direct surface temperature measurements during thermal spallation,
3. to have accurate values for heat fluxes at these temperatures.

**Flame-jet Spallation Experiments:** Figure 1.5 is a schematic diagram of the experimental set-up that was used during flame-jet spallation tests. An infrared (IR) scanner was used for surface temperature measurement. A Unistrut beam supported the torch and acted as a hinge to the support frame at point "A" so that the torch could be quickly moved to and from the sample during operation. Guides at point "B" assured that the torch was correctly positioned when it was lowered, as shown in Figure 1.5b. Parts of the rack exposed to hot gases were wrapped in 2.54 cm (1 inch) thick refractory ceramic fiber in order to protect them from the intense heat. A one-inch thick refractory board protected the rock samples while the torch was being lowered into position. The torch was ignited in the "upright" position, shown in Figure 1.5a, by opening the propane flow control valve on the small-scale torch slightly and passing the flame issuing from a hand-held propane torch underneath the nozzle outlet.

Barre and Westerly granite were used in these tests. All Barre samples were 0.3 m x 0.3 m x 0.3 m (1x1x1 ft³) and Westerly samples varied in cross section and thickness from about 0.3 m x 0.3 m x 0.1 m (0.3 ft) to 0.9 m (3 ft) x 0.3 m x 0.15 m (0.5 ft).

**Laser-heating Tests:** The experimental laser set-up consists of a 25 kW continuous CO₂ laser, associated optics, an optical beam spinner, and a focusing head as shown in Figure 1.6. These experiments were conducted at the United Technologies Industrial Lasers facility in East Hartford, CT. The infrared scanner and a Kodak Ektapro 1000 high speed
a. Torch in Raised Position.

b. Torch in Lowered Position.

Figure 1.5 Small-scale flame-jet spallation experimental set-up.
United Technologies 25 kW Laser.

Figure 1.6 Schematic of experimental set-up for laser-induced spallation. (Not to scale.)
videotaping unit, capable of recording at 1000 frames/s, were used for data acquisition. The operating procedure was as follows:

1) enter laser power level and time of irradiation into numerical controller,
2) start infrared scanner and high speed videotaping unit, and
3) open shutter.

Temperature measurements were made on samples of Barre and Westerly granite, Sioux Quartzite, and Webatuck Dolomite during the laser spallation experiments.

**Methods of Heat Flux Calculation:** Validation of equation (1.9) and determination of the Weibull parameters requires that $T_s$ be known as a function of the heat flux ($Q$). Determination of $T_s$ was described above. Two methods have been used to determine the heat fluxes in this study. In the first method the rock was again modeled as a one-dimensional, semi-infinite solid subjected to a constant heat flux at the free surface (see equation (1.10)). A least-squares line-fitting technique was used to match experimental curves of $T_s$ versus $t^{1/2}$ to equation (1.10). The heat flux is related to the slope of this line as follows:

$$Q = \frac{Ak_r}{2(\alpha_r/\pi)^{1/2}}$$

(1.11)

where $A = \text{slope of "best-fit" line}$. An example of the results of a calculation to determine the best-fit line through the $T_s$ versus $t^{1/2}$ data and the heat flux for a laser heating test is illustrated in Figure 1.7.

The second heat flux calculation method has been used for analyzing the flame-jet spallation data from measured drilling rates ($V_d$) and steady-state stagnation-point surface temperatures ($T_{s, st}$). The
Figure 1.7 Example of transient temperature-rise curve for laser heating.
fundamental energy balance at the rock surface, given previously in
equation (1.1) relates $T_{s, st}$ and $V_{dr}$ to the heat flux. Values of $V_{dr}$
were measured by Rauenzahn (1986) during small-scale field drilling
experiments using the same torch as used in the present study.

**Experimental Results:** The results of laser and flame-jet spallation
tests are illustrated in Figures 1.8 and 1.9 for Barre and Westerly
granite, respectively. The curve representing the Weibull theory
spallation temperature predictions (equation (1.9)) is obtained by using
both mechanically-determined Weibull parameters (Dey and Kranz, 1985)
and by fitting values to the data obtained in this study. In addition,
the theoretical 90% probability interval, defined as the region bounded
by the temperature values at which 5% and 95% cumulative probabilities
of spalling are predicted to occur, is shown for the fitted Weibull
curve.

At heat fluxes above about 1 MW/m$^2$ the surface temperatures on Barre
granite are higher and more sensitive to heat flux than predicted using
mechanically determined Weibull parameters, implying that a smaller
value of the homogeneity parameter, $m$, should be used. At 2.9 MW/m$^2$ the
measured surface temperature is about a factor of two higher than the
Weibull prediction.

Although considerable scatter exists in the data, especially for the
experiments performed using Westerly granite, the laser results follow
the general trends mapped out by the flame-jet data. This strongly
implies that the rock failure processes occurring in both cases are
similar and independent of the type of heat source. Furthermore, the
Figure 1.8 Surface temperature versus heat flux for Barre Granite -- comparison between laser and flame-jet results indicating steady-state spallation.
Figure 1.9 Surface temperature versus heat flux for Westerly Granite -- comparison between laser and flame-jet results indicating steady-state spallation.
agreement between the results calculated using the transient heating method for calculating the heat flux with those calculated using the heat balance and independently obtained drilling rates implies that the assumptions inherent in both methods are most probably valid under the conditions of these experiments.

Comparison of the results illustrated in Figures 1.8 and 1.9 reveals that spallation temperatures for Barre granite are higher than those for Westerly granite at heat fluxes above approximately 1 MW/m². In addition, spallation temperatures for Westerly granite are not as strongly dependent on heat flux as they are for Barre granite, indicating that Westerly granite has a higher homogeneity parameter.

Two potential causes for the large amount of data scatter in the results presented in Figures 1.8 and 1.9 are identified in this thesis. First, because spallation failure is governed by the stress field and the inherent flaw structure near the surface, and because locations for surface temperature measurements have been chosen arbitrarily in this study, spallation temperatures should be expected to form a distribution governed by the Weibull theory. As expected for statistical phenomena, the predicted 90 % probability envelope contains most of the data scatter (Figures 1.8 and 1.9).

The second factor identified as contributing to the data scatter is due to experimental error caused by the relatively coarse time and temperature resolution capabilities of the IR scanner. Each of the data points presented in Figures 1.8 and 1.9 with heat flux calculated by the transient analysis (equation (1.10)) is estimated to have a ± 35 % uncertainty region with respect to both heat flux and temperature due
solely to experimental procedures. Future studies should employ high speed, high resolution IR scanning for surface temperature and heat flux determination.

The laser spallation temperature results for Sioux Quartzite and Webatuck Dolomite are presented in Chapter 4 of this thesis. The most important result from these experiments is that Sioux quartzite was found to produce spalls at a constant surface temperature of approximately 320 °C regardless of heat flux.

Effect of Differences in Surface Temperatures on Stanton Numbers:
The value of the surface temperature strongly affects calculated values of experimental Stanton numbers because $T_s$ appears both in the numerator and the denominator of equation (1.6). For example, a temperature change from 700 K to 1000 K with a jet temperature of 2830 K causes more than a two-fold increase in Stanton number.

The newly calculated Stanton numbers obtained using values for the penetration rates measured during Rauenzahn's field drilling experiments are plotted along with the old ones in Figure 1.10. Fitted values of $m$ and $\sigma_0$, listed on Figures 1.8 and 1.9 were used for temperature determination. Figure 1.10 shows that Stanton numbers calculated using the fitted Weibull parameters are approximately a factor of two greater than those calculated using mechanically determined Weibull parameters.

The major conclusions and recommendations that are drawn from the discussion presented above are summarized in Section 1.7.
Figure 1.10  Comparison of curves of Stanton number versus SOD using Weibull parameters obtained in the present study and by mechanical testing procedures.
1.6 Prediction of Thermal Spallation Drilling Performance

**Characteristics: Model Development and Simulation Results**

Prediction of thermal spallation drilling performance characteristics requires knowledge of the spatial distribution of rock surface temperature and heat flux. Determination of the heat flux requires solution of the mass, momentum, and energy conservation equations for the flowing gas stream that impinges on the rock surface. The purpose of this section is to describe the modeling and the solution method for the conservation equations that were used to calculate penetration rates and hole diameters during flame-jet induced thermal spallation drilling.

**Governing Equations:** The governing conservation equations for mass, momentum, and energy are solved subject to the appropriate boundary conditions and constitutive relationships. The equations are approximated on a finite grid of points. Estimated values of the surface heat flux are used to move the rock boundary until the hole shape conforms to the consistency criterion given by equation (1.2).

The governing equations and boundary conditions are expressed in axisymmetric, vector form as:

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{1}{r} \frac{\partial \mathbf{E}}{\partial r} + \frac{\partial \mathbf{G}}{\partial z} = -\frac{\partial \mathbf{R}}{\partial r} + \frac{\partial \mathbf{S}}{\partial z} + \mathbf{M}
\]  

(1.12)

where:

\[
\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix} ; \quad \mathbf{E} = \begin{pmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho u(E + P) \end{pmatrix} ; \quad \mathbf{G} = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho v(E + P) \end{pmatrix} ; \\
\mathbf{R} = \begin{pmatrix} 0 \\ \tau_{rr} \\ \tau_{rz} \\ u\tau_{rr} + v\tau_{rz} - q_r \end{pmatrix} ;
\]
\[ S = \begin{bmatrix} 0 \\ \tau_{rz} \\ \tau_{zz} \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ \sigma_{r\theta}/r \\ 0 \end{bmatrix}; \]

\[ \sigma_{r\theta} = P - \left( 2 \mu \frac{u}{r} - \frac{2}{3} \mu \text{div}(\mathbf{V}) \right); \]

\[ \text{div}(\mathbf{V}) = \frac{1}{r} \frac{\partial ru}{\partial r} + \frac{\partial v}{\partial z}; \]

\( \tau_{i,j} \) = stress tensor component i,j,
\( u,v \) = radial and axial velocity components, and
\( r,z \) = radial and axial coordinates.

The set of equations employed in this study and expressed by equation (1.12) are called the "Reynolds averaged" conservation equations (Reynolds, 1841). These equations include the effects of turbulent momentum and heat transport and therefore contain additional components in the stress tensor known as the "Reynolds stresses":

\[ \tau_{i,j} = -P\delta_{i,j} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{i,j} \frac{\partial u_k}{\partial x_k} - \rho u'_i u'_j \quad (1.13) \]

"Reynolds stresses"

where \( u'_i \) = the fluctuating component of velocity in direction i, and \( u_i \) = the mean component of velocity in direction i. The heat flux vector also has an added term due to the splitting of the temperature into mean and fluctuating parts:

\[ q_i = -\lambda \frac{\partial T}{\partial x_i} + \rho c_p \overline{t' u'_i} \quad (1.14) \]

where \( \lambda \) is the gas thermal conductivity (W/m K).

Calculation of the stress tensor and heat flux vector requires evaluation of the added terms due to turbulence on the right hand sides.
of equations (1.13) and (1.14). The most widely used method, and the one adopted for this work, is to relate the turbulent stresses and heat fluxes to mean gradients in the flow field through turbulent transport coefficients (Boussinesq, 1877). This is stated mathematically as:

\[
-\rho u_i u_j = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{\delta_{ij}}{3} \left( 2 \mu_t \frac{\partial u_k}{\partial x_k} + \rho u_k u_k \right)
\]

(1.15)

\[-\rho C_p \left( \frac{T'}{u_i} \right) = \lambda_t \frac{\partial T}{\partial x_i}\]

(1.16)

where \( \mu_t \) = turbulent "viscosity", and \( \lambda_t \) = turbulent "thermal conductivity".

In this study the flow-field is conceptually divided into a near wall region, where the behavior is similar to that in a turbulent flat-plate boundary layer, and a far-field, where simple models based upon well-known velocity and temperature profiles cannot be used. In the far-field region, the k-\( \epsilon \) turbulence model described by Launder and Spaulding (1974) is used to calculate the turbulent viscosity coefficient according to the following relationships, obtained from scaling arguments applied to the transport equation for turbulent kinetic energy:

\[\mu_t = \frac{\rho C_{\mu} k^2}{\epsilon}\]

(1.17)

where \( k \) is the turbulent kinetic energy per unit mass, and \( \epsilon \) is the isotropic dissipation rate of turbulent kinetic energy per unit mass. The value of \( C_{\mu} \) has been experimentally determined to equal 0.09 (Jones and Launder, 1972).

When using the k-\( \epsilon \) turbulence model, transport equations are written
for $k$, and $\epsilon$ as a function of mean flow-field properties. These equations have been derived from the turbulent form of the conservation of momentum equations. The final modeled turbulent kinetic energy and dissipation rate equations used in this study are given by Viegas and Rubeson (1983) as:

$$
\frac{\partial \rho k}{\partial t} = -\left( \frac{1}{r} \frac{\partial (\rho u k)}{\partial r} + \frac{\partial (\rho v k)}{\partial z} \right) + \left( \frac{1}{r} \frac{\partial (r \mu_k \partial k/\partial r)}{\partial r} + \frac{\partial (\mu_k \partial k/\partial z)}{\partial z} \right) + P_k - \rho \epsilon
$$

(1.18)

$$
\frac{\partial \rho \epsilon}{\partial t} = -\left( \frac{1}{r} \frac{\partial (\rho u \epsilon)}{\partial r} + \frac{\partial (\rho v \epsilon)}{\partial z} \right) + \left( \frac{1}{r} \frac{\partial (r \mu_\epsilon \partial \epsilon/\partial r)}{\partial r} + \frac{\partial (\mu_\epsilon \partial \epsilon/\partial z)}{\partial z} \right) + \frac{\epsilon}{k} (C_1(P_\epsilon) - C_2(\rho \epsilon))
$$

(1.18)

where $P_k$ and $P_\epsilon$ are the rates of production of turbulent kinetic energy, and the rate of production of dissipation of turbulent kinetic energy, respectively,

$$
\mu_k, \mu_\epsilon = \text{turbulent diffusion coefficients for } k, \text{ and } \epsilon,
$$

$C_1 = 1.55,$
$C_2 = 2.0,$ and
$C_3 = 1.0.$

These equations are solved throughout the designated "far-field" domain in order to be able to ultimately calculate the wall-region heat transfer. A modified form of the semi-theoretical Prandtl (1925) mixing length model is used in the near-wall region for predicting velocity and temperature profiles. The final expressions for the wall shear stress ($\tau_w$) and heat flux ($Q_w$) are:
\[ \tau_w = \frac{\rho_w u_p \Theta C_\mu^{1/4} \sqrt{\kappa}}{1/\kappa \ln(32.6 \gamma^*/h^*) + 1} \]  
(1.19)

\[ Q_w = \frac{(C_p(T_p - T_w) + 1/2 \ u^2) \rho_w C_\mu^{1/4} \sqrt{\kappa} \Theta}{\Pr_t/\kappa \ln(32.6 \gamma^*/h^*) + 1} + St_{rh}^{-1} \]  
(1.20)

where

\[ \Theta = \int_0^1 \frac{1}{(\rho/\rho_w)^{1/2}} \ d(u^*/u_p^*) \]

\[ St_{rh}^{-1} = \frac{(T_{rh} - T_w)}{(Q_w/(\rho_w C_p))} \left( \frac{\tau_w}{\rho_w} \right)^{1/2}, \]  
and

\[ T_{rh} \text{ = temperature at the outer edge of the surface roughness,} \]
\[ h^* = \frac{h u_t}{\nu}, \]
\[ h \text{ = r.m.s. surface roughness height,} \]
\[ \kappa \text{ = von Karman constant (0.41),} \]
\[ \Pr_t \text{ = turbulent Prandtl number (0.86),} \]
\[ u_t = \frac{(\tau_w/\rho_w)^{1/2}}, \]  
and

\[ St_{rh}^{-1} = 5.19 \ h^{+0.20} \ Pr^{0.44} \]  
(Dipprey and Sabersky, 1963)  
(1.21)

**Boundary Conditions:** The boundary conditions that are used to solve the conservation equations (1.12) are depicted on Figure 1.11. The centerline boundary is assumed to be an axis of symmetry and therefore represents a zero flux condition. Solid walls, for example the spalling rock surface, are treated as no slip boundaries with zero pressure gradients normal to the surface. Temperature is specified on the rock surface from equation (1.9), whereas the drill housing was assumed to be adiabatic. Physical properties and Weibull parameters, m and \( \sigma_0 \), used are averages of Barre and Westerly granite values. The turbulent kinetic energy is set equal to zero on solid surfaces, and the dissipation rate is specified at the computational node nearest to the wall by the following relationship (Launder and Spaulding, 1974):
**Figure 1.11** Illustration of boundary conditions used for thermal spallation drilling simulation.
\[ \epsilon = \frac{C_\mu k^{3/2}}{y} \] (1.22)

At the nozzle outlet, the turbulent kinetic energy is specified as 5% of the mean flow kinetic energy and the turbulent length scale is specified as one-third of the nozzle radius. At the outlet from the cavity, zero streamwise gradients are assumed for all variables except pressure. Pressure is set from the user-defined under-expansion ratio \( \left( \frac{P_{noz}}{P_{amb}} \right) \) which equals 4.4 for all cases tested in this study.

**Solution of Governing Equations:** Discretization techniques are adopted whereby the property conservation laws are expressed for small (but finite) volume elements that are formed by breaking up the flow-field domain into a grid of points. The net flux of mass, momentum, and energy into each volume element (or "cell") is approximated and numerical time integration is performed until all cells are at steady-state conditions (no accumulation).

A central differencing scheme is used in the present study to approximate the derivatives appearing in the equation set represented by equation (1.12). Since central spatial differencing can lead to instabilities in the solution algorithm if used in combination with forward time stepping, a four-stage Runge-Kutta time-stepping scheme is used and "artificial dissipation" terms are added.

The artificial dissipation has two components: a second-difference term that is turned on only at shocks, and a fourth difference term that is turned on where the turbulent viscosity is not high enough to damp odd-even decoupling. The fourth-difference term is \( O(\Delta x^3) \) and therefore does not affect the formal solution accuracy. However, the second
difference term degrades the solution accuracy to first order in shocks.

**Grid Generation:** The desirable features of a computational grid are:

(a) minimum number of grid points,
(b) sufficient resolution of gradients,
(c) orthogonality of cell edges, and
(d) equal cell spacings.

The goal of (a) is to enhance the convergence rate whereas the others focus on obtaining accurate solutions. In practice, (a) - (d) are impossible to satisfy simultaneously. Instead, grid generation is done so that each is approached as closely as possible.

Grid generation is broken down into two steps in this study: determination of an initial grid with appropriate point clustering in high gradient regions (features (a) and (b)) such as near the nozzle outlet and the rock/gas interface, and smoothing of the grid using an elliptic equation set to equalize grid spacings and force the cell edges closer to orthogonality with one another (features (c) and (d)). A typical example of a final grid is illustrated in Figure 1.12.

**Simulation Results:** Prediction of penetration rates and hole radii depends upon attaining a steady-state flow-field for each hole geometry, and a steady-state hole geometry for each specified stand-off distance. Each flow-field was assumed to have converged when an averaged value of the Stanton number (St) varied by less than 0.5 % per 1000 iterations. Predicted hole shape usually oscillated toward a steady-state value and convergence was assumed when the amplitude of the oscillations in the hole radius was less than 0.25 nozzle radii.

Computer simulation runs were conducted for three sets of modeling
Figure 1.12 Smoothed computational grid
assumptions: the basic solver, as outlined above; the basic solver with a heat capacity that varies with gas temperature, and the basic solver with a variable heat capacity and mass injection at the spalling surface to represent the momentum deficit introduced into the boundary layer by spall liberation. The effect of each of these modeling methods is illustrated in the plot of St versus non-dimensional hole radius in Figure 1.13.

The most important improvements in the basic solver algorithm used in this study over that developed by Rauenzahn (1985) are second order accurate advection and node point clustering in high gradient regions. These improvements should result in less numerically-induced smearing of flow-field gradients and correspondingly higher values for wall heat fluxes and shear stresses. In Figure 1.13, the curve generated in the present study is seen to be higher than the results predicted by Rauenzahn (1986), and both predicted curves are higher than the experimental results.

In the basic solver, the value of the heat capacity is set equal to the inlet value throughout the flow-field. In reality, the heat capacity decreases at lower gas temperatures resulting in lower heat capacities near the rock wall than at the nozzle. A lower heat capacity results in a proportionally lower turbulent thermal conductivity ($\lambda_t$) because the turbulent Prandtl number ($Pr_t = \mu_t C_p / \lambda_t$) is assumed constant. The results for the variable heat capacity case illustrated in Figure 1.13 indicate that the predicted Stanton numbers decrease by approximately 30% compared to the values predicted using the basic solver. However, the results are still about 1.5 times higher than
Figure 1.13 Stanton number versus hole radius – composite plot illustrating effects of changes in modeling assumptions on prediction accuracy.
experimentally observed Stanton numbers.

The liberation of solids at a rock surface results in a change of the turbulent boundary layer structure and a corresponding decrease in the heat transfer to the rock according to Roberts et al. (1983). In the present study, the influence of entrained spalls on the transport phenomena is, perhaps crudely, approximated by assuming that it is the same as that of transpiration blowing. The modeling is implemented by applying blowing boundary conditions to only the convective terms of the conservation equations. Heat fluxes, and viscous stresses are calculated from wall functions (equations 1.19 and 1.20) for a single point inside the log-law region of the boundary layer, without including terms for the cross-stream variations in shear stress and heat flux that would occur during true transpiration blowing.

The simulation results including variable heat capacity and mass injection plotted in Figure 1.13 are within 5% of experimental Stanton numbers and hole radii. The use of variable heat capacity and mass injection make comparable contributions toward diminishing the discrepancy between experimental data and results predicted by the basic solver.

Despite the high level of agreement between predicted and experimental results achieved using the model developed in the present study, it should be emphasized that detailed agreement with boundary layer velocity and temperature profiles is necessary to validate the modeling of the erosion of the boundary layer as described above. The present approach has been taken only for purposes of approximating and placing bounds on the potential influence of spall liberation on the
transport phenomena occurring during spallation drilling. Future experimental studies of the physical interactions between gases and solids within turbulent boundary layers are necessary before more detailed modeling can be justified for solution of the flow-field that exists during thermal spallation drilling.

1.7 Summary of Conclusions and Recommendations

The conclusions and recommendations of this thesis are summarized below.

Conclusions and Recommendations I: Experimental Investigation of Spallation Surface Temperatures

The major conclusions that are drawn from the results and discussion presented in Section 1.5 are:

1. Spallation temperatures at heat fluxes above 1 MW/m² are up to a factor of two greater than predicted using Weibull parameters obtained by mechanical or low heat flux testing procedures.
2. Stanton numbers calculated from field drilling data obtained by Rauenzahn (1986) are significantly closer to those predicted by Rauenzahn’s simulator than initially thought.
3. Surface temperatures obtained during laser and flame-jet spallation under the same heat fluxes result in surface temperatures that are within the data scatter of one another.
4. Westerly granite spalls at lower temperatures than Barre granite at the same heat fluxes (for example, approximately 200 K lower at 3 MW/m²).

The recommendations for future investigations of the rock mechanics governing thermal spallation are:

1. **Objective**: Improve ability to measure heat fluxes and surface temperatures during thermal spallation. **Approach**: Use high speed (>50 Hz) IR scanning with broad temperature range capabilities, as described by Morgan and Schultz (1985).
in a flawed, but otherwise homogeneous, material and compare results of varying material properties and heat flux with those predicted by Weibull theory (Eq. 1.9).

3. **Objective:** Determine Weibull parameters for a large number of rock types.
   **Approach:** Use high powered laser spallation techniques as described above with the improved IR scanner technology developed in accordance with recommendation # 1.

**Conclusions and Recommendations II: Prediction of Performance Characteristics**

The following major conclusions are drawn based on the results and discussion concerning the modeling of the flow-field that exists during thermal spallation drilling:

1. The general trend of the predicted Stanton number versus hole radius curve is well predicted using either the upwind differencing method of Rauenzahn (1986), or the second order accurate method employed in the present study.
2. The use of the second order numerical method described above results in about 30% higher values of predicted Stanton numbers for the same hole radii (or stand-off distance) than those predicted with Rauenzahn’s method.
3. Including the effect of the temperature variation on the gas heat capacity decreases predicted Stanton numbers by approximately 30%.
4. The use of a continuous transpiration blowing model to simulate the effect of spall entrainment results in a 35% decrease in predicted Stanton numbers and excellent agreement (within 5%) with experimental results, suggesting that the mass-injection effect upon the boundary layer should be investigated further.

Recommendations aimed at improving the flow-field modeling and the ability to verify such modeling are:

1. **Objective:** Improve ability to validate simulation model.
   **Approach 1:** Perform additional small-scale field drilling experiments with varying $P_{jet}$, $T_{jet}$, $R_{jet}$, $M_{jet}$.
   **Approach 2:** Diminish coupling of rock mechanics with heat transfer by performing drilling experiments in Quartzite.
2. **Objective:** Improve flexibility of computational model for prediction of alternate geometries (e.g. cavity formation).
   **Approach:** Set up unstructured methodology for grid generation and flow model implementation.
3. **Objective:** Improve understanding of the influence of spall entrainment on near-wall transport phenomena.
   **Approach:** Perform heat flux and turbulence structure measurements on a simplified flow geometry (e.g. a flat plate)
with intermittent solids injection.

4. **Objective**: Develop ability to rapidly estimate penetration rate and hole geometry trends during thermal spallation drilling. **Approach**: Develop lumped parameter model(s) using insight gained from numerical modeling and field experiments.
2 Introduction

2.1 Motivation for a Rapid Drilling Technique in Hard Rock

Recent interest in the use of naturally occurring underground regions of hot dry rock for power generation has resulted in a need for inexpensive, reliable drilling methods in hard, usually granitic, crystalline basement rock that is typically encountered. The schematic of a hot dry rock (HDR) reservoir system is shown in Figure 2.1. In the present concept, water is pumped down an injection well and heated by being forced through a hot, fractured volume of rock at depths ranging from three to six kilometers below the earth’s surface. The hot water returns to the surface through a recovery well and may be used directly, for example to provide domestic or process heat, or indirectly to generate electricity in a power cycle. The second law of thermodynamics limits the amount of work that can be extracted in such a process according to:

\[
\frac{\delta W_{\text{max}}}{\delta Q_{gf}} = -\frac{(T_{gf} - T_0)}{T_{gf}} \quad (2.1)
\]

where \( \delta W_{\text{max}} \) is the maximum differential quantity of work that can be generated from the transfer of a differential amount of heat \( \delta Q_{gf} \) from the geothermal fluid at a temperature \( T_{gf} \) to the atmosphere at temperature \( T_0 \). A thermodynamic first law analysis of the idealized process whereby the geothermal fluid is cooled isobarically to ambient conditions; transferring heat to the surroundings using Carnot engines leads to:

\[
\delta Q_{gf} = C_p dT_{gf} \quad (2.2)
\]
Figure 2.1 Hot dry rock (HDR) geothermal system concept (Tester et al., 1989).
where $C_p$ is the fluid heat capacity at constant pressure (J/kg K). For $C_p = \text{constant}$, combination of equations (2.1) and (2.2) followed by integration from the geothermal fluid temperature at the wellbore ($T_{gf,w}$) to atmospheric temperature leads to:

$$P_{\text{max}} = -\dot{m}_{gf}(T_0 - T_{gf,w}) - T_0 \ln(T_0/T_{gf,w})$$ \hspace{1cm} (2.3)

where $\dot{m}_{gf}$ = the mass flow rate of geothermal fluid (kg/s) and $P_{\text{max}}$ = the maximum power generating capacity of the system.

Equation (2.3) implies that for a given atmospheric temperature and geothermal fluid mass flow rate, the amount of power that can be produced increases as the geothermal fluid (water) temperature increases. Since the temperature of the geothermal fluid is governed by the temperature of the hot underground reservoir and the earth's temperature usually increase with depth (Armstead and Tester, 1987), the previous discussion leads to the conclusion that the power generating capability of a HDR power process can be increased only by drilling deeper.

However, drilling to gain access to hot dry rock (HDR) at depths in excess of 3 km. is a difficult and costly process when using current rotary-mechanical techniques. Drilling-related costs are almost always the single largest capital component of any geothermal power plant (Armstead and Tester, 1987). Instantaneous penetration rates are low, typically 1-7 m/hr., because it is difficult to crush the hard,
polycrystalline rock encountered in HDR systems. Furthermore, average penetration rates, which include the time taken for drilling as well as operations requiring shut down of the drive system, are less than the already low instantaneous penetration rates cited above because of the extra time required for dealing with problems associated with failure or wear of drilling hardware, such as drill bits, and when twisted-off sections of drill pipe become lodged in a deep hole. The available data indicates that the penetration distance per rotary mechanical bit is typically between 30 and 110 m (100 and 350 ft) when drilling for HDR applications (Armstead and Tester, 1987). Moreover, the time required for the "tripping" process wherein the drill string is removed from the hole, the bit is replaced, and the drill string is returned to the hole takes about 8 hours at 3 km (10,000 ft) and is estimated to require more than 24 hours at 10 km (30,000 ft) (Armstead and Tester, 1987).

"Twist-off's" occur more frequently when drilling at depth in hard crystalline rock than, for example, drilling for gas or oil in softer sedimentary rocks, such as limestone or sandstone, because of the significantly higher torques that are required to turn rotary drill bits. Figure 2.2 illustrates the distribution of activities during the drilling of the Los Alamos National Laboratory (LANL) HDR project hole EE-3 to a total depth of 4.2 km (13,800 ft) in granite. "Fishing-out" and side-tracking around broken drill string amounted to almost 44% of the total time whereas actual drilling took only 12.1%.

Figure 2.3 compares estimated well drilling costs for HDR wells in
Figure 2.2 Distribution of activities during the drilling of EE-3 at Fenton Hill (Armstead and Tester, 1987).
Figure 2.3  HDR well drilling costs in crystalline, granite formation at Fenton Hill (Tester et al., 1989). (symbols denote wells drilled)
granite formations to costs for oil and gas wells drilled to the same depths (Tester et al. 1989). The HDR well cost trend is generally higher than that for oil and gas. However, all of the HDR data is from experimental holes at the Fenton Hill site and much of the methodology used there had not been previously tested under such adverse high temperature and hard rock conditions encountered. Many technical problems that occurred while drilling these initial, "first generation" holes have already been solved by workers at LANL in the US and at the Camborne School of Mines in the UK. Therefore, cost improvements are expected if rotary drilling is used in the future. For example, drilling costs for the newest hole, EE-2A, lie on the average oil and gas well cost line (Figure 2.3), reflecting the improvements that have been made in HDR drilling technology and procedures.

Nevertheless, even if well drilling costs become identical to those for oil and gas, exponential increases with depth are still expected. Furthermore, the sensitivity of total geothermal power plant cost to drilling costs, combined with the inherent difficulties encountered when using conventional methods for these applications provides a strong economic incentive to explore alternative drilling techniques. As HDR well costs decrease, a larger fraction of the earths available resource becomes commercially viable. In addition, other fields of interest including tunneling (for transportation and waste disposal applications), mining, deep-well waste storage, and oil and gas drilling in the overthrust belt would benefit from a rapid, inexpensive method for penetrating through crystalline rock. Several novel drilling
concepts have been developed and were reviewed by Maurer (1975; 1980). The most promising of these "advanced" methods avoid the inherent need for direct-contact fracturing, crushing, and grinding that are normally responsible for mechanical inefficiencies and limitations during the comminution process occurring at the rock surface. Two of these are discussed in the following section.

2.2 Thermal Drilling Techniques

Two types of thermal hard rock drilling techniques have been investigated. The first type is the "subterrene" drill (Fogelson, 1974) which melts through the rock and either removes the molten rock through the drill housing or extrudes it into pores or fractures in the hole wall. Subterrene drilling was determined to be uneconomical by LANL after extensive research in the mid 1970's (Hanold and Altseimer, 1977) and was abandoned in 1976. A major technical difficulty with the subterrene is the very low drilling velocity (< 1 m/hr.) due to the buildup of high viscosity, low thermal conductivity molten rock between the penetrator and the solid rock surface.

An alternative to Subterrene fusion drilling is flame-jet induced thermal spallation drilling, shown schematically in Figure 2.4. Thermal spallation drilling is carried out at relatively low surface temperatures (≈ 500 °C) where, ideally, no melting occurs. Instead, compressive stresses, induced by rapid localized heating of the confined rock, fracture the surface into small chips that are then swept away by
the high speed gas stream. Penetration rates of 8 to 30 m/hr. and a maximum depth of 330 m have been achieved in granite using a drill head similar to the one shown schematically in Figure 2.5 (Browning, 1981). The total direct operating cost of drilling to this depth is estimated
**Figure 2.4** Schematic of supersonic flame-jet issuing from the combustion chamber nozzle, expanding, and impinging on the underlying rock (Rauenzahn, 1986).
Figure 2.5 Browning Engineering flame-jet drill schematic (Browning, 1981).
by Browning to be about $6.60/m ($2.00/ft) (1981 dollars) using his flame-jet whereas estimated "trouble-free" well drilling costs at Fenton Hill for holes ranging from 3.0 - 4.7 km are $300.00/m ($100/ft) (1981 dollars).

In a typical spallation drilling rig, fuel oil and air are delivered to the combustion chamber through hoses that are also used to support the drill head assembly (Figure 2.6). Combustion takes place inside the burner at a pressure of about 0.93 MPa (120 psig) and the high temperature products are passed through a converging/diverging nozzle, exiting the tool at over four times atmospheric pressure and slightly above Mach 1. The turbulent jet formed near the nozzle outlet impinges on the underlying rock surface imparting stagnation point heat fluxes in the range of 1-10 MW/m².

In Browning's burner (Figure 2.5), cooling water is circulated between the combustion chamber and drill housing and is then injected radially into the hot exhaust gases several feet above the actively spalling region. Quenching provided by vaporization of the cold water spray protects the utility hoses from suffering thermal damage.

Field tests have produced impressive results but they have also indicated some limitations on deep well drilling with flame-jets. The inability to penetrate non-spallable zones of rock is the most important limitation. Conventional methods are required to continue drilling in these zones and an on-site, stand-by rotary rig would
Figure 2.6 Schematic of thermal spallation drilling (Williams et al., 1988).
probably be necessary, thereby potentially destroying some of the economic advantages of using thermal spallation. This and other problems, such as elongation of the support/utilities delivery hoses caused by the heavy loads and high downhole temperatures, may be solved by designing a tool that can be integrated with standard drill pipe. Preliminary tests at LANL have demonstrated the feasibility of such a system including the use of a spark plug for downhole re-ignition after the torch has been extinguished (Williams et al, 1988).

Research aimed at extending the use of thermal spallation to "soft" rocks encountered in oil mining, such as limestone, shale, and sandstone, is being conducted jointly by LANL and the New Mexico Institute of Mining and Technology (Potter, 1988). With the exception of certain very hard sandstones, these "softer" rocks have been found to be non-spallable, presumably because at the temperatures encountered during thermal spallation they do not undergo brittle failure, which is necessary for spallation to occur (see Chapter 3).

2.3 Modeling of Thermal Spallation Drilling

Early studies of thermal spallation have not provided quantitative insight into either the rock failure mechanism or the heat transfer phenomena occurring during thermal spallation drilling (see Chapter 3 for details). Recently, however, extensive experimental and theoretical studies of thermal spallation drilling have been carried out at LANL (Dey, 1984; Williams, 1988) and at the Massachusetts Institute of
Technology (MIT) (Rauenzahn, 1986; Rauenzahn and Tester, 1985). The major result of Dey's work is a method of predicting the pseudo-steady state temperature of the spalling rock surface as a function of heat flux, rock physical properties, and two empirical factors related to the size dependent compressive strength properties of the rock. Rauenzahn's major contribution was a predictive code for simulating the flow-field in the wellbore and the resulting heat flux to the rock.

Knowledge of the heat flux and surface temperature at any point on the spalling surface allows determination of the local drilling velocity through a heat balance on the control volume surrounding the rock-gas interface. The heat balance can be written:

\[ Q = (\rho C_p)_r U_{dr} (T_s - T_{r0}) + \Delta H_{misc} \]  \hspace{1cm} (2.4a)

where

\((\rho C_p)_r\) = rock thermal density \((\text{J/m}^3 \text{ K})\),

\(Q\) = local heat flux to the rock \((\text{MW/m}^2)\),

\(U_{dr}\) = local penetration rate normal to the surface \((\text{m/s})\),

\(\Delta H_{misc}\) = losses of heat due to miscellaneous phase transitions and crack formation \((\text{W/m}^2)\),

\(T_s\) = local surface temperature \((\text{K})\), and

\(T_{r0}\) = initial rock temperature \((\text{K})\).

The second term on the right-hand side of equation (2.4a) is neglected in this thesis on the grounds that the processes at the spalling surface occur at temperatures too low for significant amounts of melting to occur and at rates too high for the \(\alpha-\beta\) quartz transition to occur. Furthermore, crack formation and phase changes such as the vaporization
of entrapped water (≈ 0.1% by volume) require negligible amounts of energy relative to the sensible heat required to raise the rock surface temperature to spalling conditions.

Equation (2.4a) can be rearranged to solve for penetration rate:

\[ U_{d_r} = \frac{Q}{(\rho C_p) r (T_s - T_r0)} \]  (2.4b)

In the somewhat idealized case of steady-state drilling that was modeled by Rauenzahn (1986) and is modeled in this work, the average hole shape in the actively spalling region is assumed to remain constant and satisfy the condition that the forward component of the local penetration rate \( U_{d_r} \) is the same everywhere. This is illustrated in Figure 2.7 and can be expressed mathematically as:

\[ V_{d_r} = \frac{U_{d_r}}{\cos(\theta)} \]  (2.5)

where \( V_{d_r} \) = forward advance rate of drill head (m/s), and \( \theta \) = angle of the tangent of the rock boundary to the horizontal plane.

The computer codes developed by Rauenzahn (1986) and in this study predict local values of the heat flux along the spalling rock surface by solving the mass, momentum, and energy conservation equations in the gas. The boundary shape is then forced to conform to the consistency condition given by equation (2.5).
Figure 2.7 Illustration of modeled region and self-consistency condition (Eq. 2.5).
2.4 Presentation of Thermal Spallation Drilling Performance Characteristics

Comparison of heat flux and hole diameter predictions with experimental results is accomplished by forming non-dimensional Stanton numbers and hole radii. In this case, the Stanton number \( \text{St} \) is defined as the ratio of the rock surface heat flux \( Q_r \) to the total jet inlet heat flux \( Q_{\text{jet}} \), or:

\[
\text{St} = \frac{Q_r}{Q_{\text{jet}}} \quad (2.6)
\]

and

\[
Q_{\text{jet}} = (\rho C_p U)_{\text{jet}} (T_{\text{jet}} - T_s) \quad (2.7)
\]

Where \( T_{\text{jet}} \) is the temperature at nozzle outlet. \( Q_r \) is calculated from equation (2.4a) for experimental Stanton numbers giving

\[
\text{St}_{\text{exp}} = \frac{(\rho C_p)_{\text{r}} U_{\text{dr}} (T_s - T_{r0})}{(\rho C_p U)_{\text{jet}} (T_{\text{jet}} - T_s)} \quad (2.8)
\]

Computer simulation results directly predict the rock surface heat flux so that predicted Stanton Numbers are given by

\[
\text{St}_p = \frac{Q_p}{(\rho C_p U)_{\text{jet}} (T_{\text{jet}} - T_s)} \quad (2.9)
\]

where \( Q_p \) = predicted heat flux.

Experimental Stanton numbers \( \text{St}_{\text{exp}} \) given by equation (2.8) contain some inherent uncertainty because they utilize an assumed relationship between surface temperature and heat flux in order to convert measured drilling rates into heat fluxes. In Chapter 4, this is shown to have a pronounced effect because surface temperature appears in both the...
numerator and denominator of equations (2.8) and (2.9). Predicted drilling rates are significantly less influenced by the assumed temperature/heat-flux dependency for reasons that are discussed in chapter 6.

Figure 2.8 is a plot of Rauenzahn's results for both Stanton numbers as a function of the non-dimensional drill head stand-off distance (SOD) defined as:

\[
\text{SOD} = \frac{Z_{dr}}{R_{noz}},
\]

(2.10)

where \(Z_{dr}\) = distance from the bottom of the drill to the bottom of hole, and \(R_{noz}\) = nozzle inside radius. Predicted Stanton numbers (\(St_p\)) are from three to five times greater than the experimental ones (\(St_{exp}\)). This large discrepancy provides the major motivation for the current study.
Figure 2.8 Stanton number versus stand-off distance (Rauenzahn, 1986).
2.5 Thesis Objectives and Approach

The overall goal of the current study work has been to develop the ability to accurately predict performance characteristics, for example hole radius and drill penetration rate, during spallation drilling. However, drill penetration rate and hole radius are both determined by local drilling velocities, which in turn are related to predicted values of local heat fluxes (Q) and rock surface temperatures (T_s) through equation (2.4b). Therefore the specific objectives of this thesis were:

1) to develop an accurate method of predicting the heat flux to a spalling surface for a given average rock surface temperature distribution, and

2) to have accurate values of the surface temperatures that occur during thermal spallation for rocks being subjected to known values of heat flux.

The first objective is met by implementing improvements in the physical and numerical modeling of the flow-field and heat transfer phenomena. For example, the introduction of mass injection at the rock/gas interface to represent the eroding rock surface, and a variable gas heat capacity are examined to see if they substantially improve the predictive capability of the model. Further possible improvements, including enhanced grid resolution and higher order numerical modeling of the convective terms in the conservation equations, are also examined.
The second objective is met by performing direct measurements of the infrared radiation leaving the rock surface during spallation and back calculating the surface temperature by using the modified Stefan-Boltzmann law, given by Kreith and Black (1980) as:

\[ E = \epsilon \sigma T_s^4 \]  
(2.11)

where \( E \) = total emissive heat flux incident on the infrared detector, \( \epsilon \) = emissivity of the rock surface (\( \approx 0.9 \) for granite), and \( \sigma \) = Stefan-Boltzmann constant = \( 5.67 \times 10^{-8} \) (W/m\(^2\) K\(^4\)).

Chapter 3 contains a discussion of the fundamental relationship between the rock surface temperature during thermal spallation and the failure mechanism that occurs. A brief review of early studies is followed by the current mathematical description of the rock mechanics of thermal spallation, leading to an expression for the rock surface temperature as a function of the applied heat flux. Chapter 4 presents results and discussion of experiments aimed at verifying the theoretical modeling of spallation from a rock mechanics perspective. Chapters 5 and 6 detail the gas-side heat transfer and fluid mechanics modeling and present results leading to overall predictions of spallation drilling performance. Finally, Chapter 7 contains the conclusions and recommendations for future work.
3 Previous Studies of Thermal Spallation

3.1 Relationship Between Rock Failure and Surface Temperature at Spallation

Spallation failure occurs when the local rock strength \((\sigma_c, I_{oc})\) is exceeded by the local stress \((\sigma)\). The local stress in the heated portion of a confined rock mass is induced by the tendency of the rock to expand as temperature increases. Therefore, prediction of the temperature at a spalling surface requires knowledge of the rock failure mechanism governing the local rock strength.

The thermal expansion behavior for an unconfined, uniformly heated sample is expressed by the defining equation for the linear thermal expansion coefficient \((\beta)\):

\[
\epsilon = \frac{l - l_0}{l_0} = \beta(T - T_0)
\]

(3.1)

where

- \(\epsilon\) = linear strain of the sample,
- \(l\) = heated sample length,
- \(l_0\) = length of sample at \(T_0\),
- \(T_0\) = initial sample temperature, and
- \(T\) = final sample temperature.

An approximate equation for the stress induced by heating a confined rock sample is derived by assuming elastic behavior:

\[
\sigma = \frac{E\epsilon}{1 - \nu} = \frac{E\beta\Delta T}{1 - \nu}
\]

(3.2)

where
E = Young's modulus (Pa),

ν = Poisson's ratio, and

ΔT = temperature rise (K).

Failure will occur when the induced thermal stress exceeds the local compressive strength (σc,loc). Therefore, manipulation of equation (3.2) results in an expression for the predicted local surface temperature rise in terms of the intrinsic compressive strength:

\[ ΔT_{sp} = \frac{(1 - ν)σ_{c,loc}}{βE} \]  \hspace{1cm} (3.3)

where ΔT_{sp} = temperature rise at the point of rock failure.

In reality, the local compressive strength of a polycrystalline rock undergoing brittle failure is governed by the intrinsic material strength properties and the distribution of pre-existing flaws. A large spectrum of local failure strengths and sample size dependent global compressive and tensile strengths are observed for these materials. A plot of the global compressive strength of granite as a function of sample volume is shown in Figure 3.1 (Lundborg, 1967). Analytical modeling work performed by Dey (1984) (see Section 3.3 for details) provides a method of statistically relating the measured size-strength behavior of a rock sample to local surface temperature values.
Figure 3.1 Compressive strength of granite versus test sample volume (Lundborg, 1967).
3.2 General Survey of Early Studies of Thermal Spallation

3.2.1 Rock Failure Mechanisms

This section provides a brief historical summary of the various mechanisms and theories that have been proposed to explain spallation. A more extensive survey may be found in Rauenzahn (1986).

Norton (1925) claimed that spallation in a semi-infinite medium is the result of shearing between isotherms parallel to a free surface. However, this is impossible for true thermal spallation because the heated area is small compared to the total sample size and the confinement provided by the surrounding rock prevents shearing from occurring in the absence of flaws. Norton performed experiments that appeared to verify his hypothesis; but, in fact, his experimental set-up did not provide enough confinement. Therefore, tensile failure in a direction normal to the free surface being heated was actually observed by Norton because of the stress exerted by the expanding outer layer on the sample core.

Preston (1934) recognized that a spall is formed when the compressive stress extends a pre-existing material flaw parallel to the heated surface (Figure 3.2). The flaw propagates because of global compression which causes shear failure at its tips. Buckling of the rock plate that is formed results in a rapid release of the elastic energy contained in the compressed spall, explaining the observed tendency of spalls to eject violently from a rock surface. Preston
Figure 3.2 Simplified Chain of Events Leading to Spall Formation on a Surface of a Semi-Infinite Solid (Preston, 1934).

a. HEAT PENETRATES TO DOTTED LINE

b. CRITICAL FLAW LOCATED AT THIS POINT

c. SURFACE LAYER ARCHES AND BUCKLES AS FLAW LENGTHENS

d. SPALL FORMATION COMPLETES AS SIDES SEPARATE
heated spheres of clay at varying rates in a radiation furnace and caused both spallation and macroscopic tensile failure to occur. Spallation occurs at high heat fluxes where the surface temperature becomes high before an appreciable amount of the sample volume is heated. Tensile failure occurs at low heat fluxes because the heated layer is thick enough relative to the cold core so that the thermally induced stresses pull the inner sphere apart.

Marovelli et al. (1965) first proposed the use of the Weibull (1939) failure theory to explain crack initiation and formation of spalls. Weibull theory (described in more detail in Section 3.3.3) is a statistically-based method to account for the number, orientation, and size distribution of cracks per unit volume. Ultimately, the theory leads to a semi-empirical prediction of the distribution of failure strengths for a given sample under compression. The probability of failure is highest when the product of the stress and the sample volume stressed is maximized. Weibull theory is used as a basis for our current modeling of rock failure during spallation (Rauenzahn, 1986; Rauenzahn and Tester, 1989).

Gray (1965) proposed a mechanism similar to Norton's whereby shear failure occurs at an angle to the free rock surface followed by shear fracturing along a plane parallel to that surface. He approximates the rock as a subliming solid that is uniformly heated at a constant temperature surface moving forward at a rate $U_{dr}$. The temperature profile is governed by the following one-dimensional, steady-state form
of the energy conservation equation in the rock:
\[
\frac{d^2 T}{dz^2} - \alpha_r \frac{dT}{dz} = U_{dr}(\rho C_p)_r \frac{dT}{dz}
\] (3.4)

where
\[
\alpha_r = \text{rock thermal diffusivity (m}^2/\text{s}), \text{ and}
\]
\[
z = \text{distance measured from the rock surface inward (m)}.
\]

The solution to this equation close to the surface is:
\[
T = T_s \exp\left[-\frac{U_{dr}}{\alpha_r} z\right]
\] (3.5)

where \(T_s\) is the rock surface temperature.

Lauriello (1971) studied the stress field developed in a semi-infinite solid subjected to laser heating. The rock weakening that resulted was analyzed by applying an integral transform technique to determine the temperature and stress fields from the governing equations of thermo-elasticity. Laser induced cracking experiments were performed in granite to validate the theoretical analysis. Nelson (1969), in an earlier study of the same heating configuration as Lauriello, assumed that spallation occurs when the tensile stress in the surface-normal direction exceeds the rock tensile strength.

The results of Lauriello and Nelson indicate that the time required for the locus of maximum tension to reach the necessary depth to form a spall is several seconds at relevant heat fluxes. In contrast, high speed videotaping of thermal spallation performed by Rauenzahn (1986) indicate that spalls are liberated within about 0.1 to 0.5 seconds.
Furthermore, Rauenzahn (1986) points out that beam diameters used by Lauriello and Nelson for experimental verification of their modeling are too small to represent classical thermal spallation because they are close to the rock grain size (about 0.5 mm) and the thermal penetration depth. Additional evidence that true spallation is not obtained by Nelson is that his "spalls" have thickness-to-diameter ratios of about 0.40 compared to about 0.07 found by most investigators.

3.2.2 Field Studies

Union Carbide's Linde Division developed a thermal spallation burner head (Figure 3.3) for primary blast hole production in hard rock in 1947. A fuel oil/oxygen mixture is combusted and expanded to produce a supersonic jet. This "flame-jet" technique has been used extensively for making blast holes in taconite mining (Calaman and Rolseth, 1961) and for creating slots around granite blocks during quarrying operations. Concentric hole reamers are used in taconite drilling because appreciable amounts of molten rock are formed and must be removed. The angled port in the burner helps spall the side walls of the hole as the drill pipe is rotated. More than 140 million tons of crude taconite were produced through the use of this burner head, and its subsequent modifications, between 1947 and 1961 alone (Calaman and Rolseth, 1961).
Figure 3.3 Union Carbide burner head used for blast-hole production by spalling and melting of Taconite (Calaman and Rolseth, 1961).
Extensive spallation drilling experience using the Union Carbide torch is summarized by Calaman and Rolseth (1961) who found that the "spallability index" (SI) of a rock relates to the thermal diffusivity, grain size, thermal expansion coefficient, and rock strength according to:

$$\text{SI} \propto \frac{\text{(Thermal Diffusivity)} \times \text{(% expansion at } T_s)}{\text{(Grain Size)} \times \text{Rock Strength at } T_s}$$

where $T_s$ is the surface temperature at spallation. The higher the spallability index, the higher the penetration rate.

Above a rock-specific temperature, spallation ceases to occur. Calaman and Rolseth attributed this to the rock becoming plastic at high temperatures and thereby relieving the stress before it can build up to cause brittle failure.

Penetration rates observed by Calaman and Rolseth in various rock and iron ore formations are given in Table 3.1, although without heat fluxes and hole diameters the data are only qualitatively useful. Drilling rates reportedly diminish in weathered and/or oxidized rock. However, flame-jet drilling is less controlled by these effects than mechanical drilling.
Table 3.1  Thermal Spallation Rates Obtained by Linde for Various Rock and Iron Ore Formations (Calaman and Rolseth, 1961).

<table>
<thead>
<tr>
<th>Rock or Iron Formation</th>
<th>Penetration rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m/hr</td>
</tr>
<tr>
<td>Oxidized and Altered Taconite</td>
<td>4.3</td>
</tr>
<tr>
<td>Lower Cherty Magnetic Taconite,</td>
<td></td>
</tr>
<tr>
<td>Area A</td>
<td>7.6</td>
</tr>
<tr>
<td>Area B</td>
<td>3.7</td>
</tr>
<tr>
<td>Area C</td>
<td>7.0</td>
</tr>
<tr>
<td>Lower Slately Taconite</td>
<td>3.7</td>
</tr>
<tr>
<td>Michigan Quartzite</td>
<td>9.1</td>
</tr>
<tr>
<td>Michigan Jasper</td>
<td>5.8 - 12.0</td>
</tr>
<tr>
<td>Labrador Specularite</td>
<td>7.0 - 12.0</td>
</tr>
<tr>
<td>Michigan Conglomerate</td>
<td>6.1</td>
</tr>
<tr>
<td>Michigan Diorite</td>
<td>3.7</td>
</tr>
<tr>
<td>Michigan Seretic Dike</td>
<td>2.7</td>
</tr>
<tr>
<td>Michigan Argillaceous Iron Formation</td>
<td>4.3</td>
</tr>
<tr>
<td>Arkansas Syenite</td>
<td>5.5</td>
</tr>
<tr>
<td>Massachusetts Granite</td>
<td>6.1</td>
</tr>
<tr>
<td>Potsdam Sandstone</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Browning Engineering Corporation measured the ratio of the volumetric rock removal rate during spallation to the volumetric oxygen feed rate for fuel-oil or kerosene being burned in varying mixtures of nitrogen and oxygen (Browning et al., 1965). Their results, illustrated in Figure 3.4, reveal that the optimum rock removal rate from a granite surface that is initially nominally flat is achieved by using a 50:50 oxygen:nitrogen ratio. Apparently the high momentum flux for a given heat flux of this mixture, compared to that resulting from burning pure oxygen, leads to an increased ability to clear debris from the rock/fluid interface thereby increasing the effective heat transfer area.
Figure 3.4 Rock removal efficiency of various burners using three different oxidants to burn kerosene or fuel-oil (Browning et al., 1965).
More recently, Browning (1981) has drilled holes in granite to 335 m (1100 ft.) at Conway, N.H., and 130 m (430 ft.) at Barre Vt. (Table 3.2). These holes were drilled using fuel-oil and air for combustion at 3.6 MPa and 0.86 MPa, respectively. Hoses were used to deliver fuel, oxidant, and cooling water, and to support the burner head (Figure 2.5). The average drilling rates were 15.8 m/hr at Conway, and 7.6 m/hr at Barre. Near the end of the Conway hole, the drilling rate increased to greater than 30 m/hr, presumably because the drill encountered a zone containing more competent, less weathered rock. Browning proposed that these higher rates may be more representative when drilling deeper holes in this type of rock.

Table 3.2 Summary of Full-Scale Spallation Drilling Exercises in Crystalline Rock (adapted from Rauenzahn, 1986).

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Location</th>
<th>Depth m (ft.)</th>
<th>Avg. Rate m/hr. (ft./hr.)</th>
<th>Diameter m (in.)</th>
<th>Pressure MPa (psia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Browning</td>
<td>Conway, N.H.</td>
<td>335 (1100)</td>
<td>15.8 (52)</td>
<td>0.2-0.25 (8-10)</td>
<td>3.4 (500)</td>
</tr>
<tr>
<td>Browning</td>
<td>Barre, VT.</td>
<td>130 (430)</td>
<td>7.6 (25)</td>
<td>0.35-0.40 (14-16)</td>
<td>0.86 (125)</td>
</tr>
<tr>
<td>LANL*</td>
<td>Pedernal Hills, N.M.</td>
<td>30 (100)</td>
<td>6-7 (22)</td>
<td>0.35-0.45 (14-18)</td>
<td>0.76 (110)</td>
</tr>
</tbody>
</table>

*LANL - Los Alamos National Laboratory

Field spallation drilling experiments were conducted by LANL in granitic rock with the following objectives (Williams et al., 1988):
1) to verify the accuracy of analytical work conducted by Dey (1984) and Rauenzahn (1986),
2) to develop and test equipment and methods for obtaining downhole ignition of spallation burners,
3) to integrate the capability to perform thermal spallation drilling with standard oil-field drilling equipment, and
4) to investigate the capabilities of spallation for large cavity formation and tunneling.

A Browning flame-jet Burner was adapted to fit 2 7/8 inch drill pipe and a downhole ignition system, consisting of a spark plug, high voltage transformer, and start-up oxygen passages, was added (Figure 2.5). Although drilling was stopped at a depth of 20 m below the surface, LANL successfully demonstrated the ability to perform downhole ignition and the use of drill pipe as a burner support and conduit for combustion air. Furthermore, a cavity with a maximum diameter of 1.6 m was formed by leaving the burner head stationary in a pre-drilled 0.6 m diameter hole (Figure 3.5). Later, a second hole was drilled and expanded until it contacted the first, whereupon the first was used to hold the spalls formed during further expansion of the new cavity. Williams et al. (1988) concluded that the rapid penetration rates, low equipment wear, rates and less trip time associated with thermal spallation drilling should result in a method that is more economical than standard rotary mechanical drilling. This is in agreement with the conclusions of a conceptual economic design study conducted for Sandia National Laboratories by Rinaldi (1984).
Figure 3.5 Schematic of cavity formation experiment performed by LANL (Williams et al., 1988)
Rauenzahn (1986) conducted small-scale field drilling experiments in Barre, VT and Westerly, RI using a custom built torch (Figure 3.6) made by Browning Engineering. In this design, propane and oxygen are fed to the combustion chamber in the torch at a pressure of approximately 0.90 MPa (120 psig). The burned gases issue from the sonic nozzle at 4.4 times atmospheric pressure. Advance rate is varied by adjusting the rotation speed of a motor driven screw that supports the torch (Figure 3.7). Measured values of stand-off distance and hole diameter for each penetration rate setting used by Rauenzahn are listed in Table 3.3.
Figure 3.6 Internals of Rautenstahm's custom-built flame-jet spallation drill constructed by Browning (Rautenstahm, 1986).
Figure 3.7  Schematic of Rauenzahn’s small-scale burner rack and advancing mechanism for piercing horizontal or angled holes at known rates (Rauenzahn, 1986).
Table 3.3 Results of Small-Scale Quarry Drilling Performed by Rauenzahn (1986).

**Barre Granite**

<table>
<thead>
<tr>
<th>Drilling Velocity $V_{dr}$ (m/hr)</th>
<th>Stand-off Distance $Z_{dr}$ (cm.)</th>
<th>Hole Radius $R_h$ (cm.)</th>
<th>Measured</th>
<th>Non-Dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.77</td>
<td>6.4</td>
<td>3.2-3.5</td>
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<tr>
<td>7.31</td>
<td>7.0</td>
<td>3.5</td>
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<td>3.8</td>
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</tr>
<tr>
<td>5.49</td>
<td>7.6</td>
<td>3.8-4.4</td>
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<tr>
<td>4.57</td>
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</tr>
<tr>
<td>4.11</td>
<td>10.2</td>
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<tr>
<td>2.74</td>
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<table>
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<tr>
<td>R_{noz}</td>
<td>R_{noz}</td>
<td></td>
</tr>
<tr>
<td>22</td>
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<td>24</td>
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</tr>
<tr>
<td>70</td>
<td>28</td>
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</tr>
</tbody>
</table>

**Westerly Granite**

<table>
<thead>
<tr>
<th>Drilling Velocity $V_{dr}$ (m/hr)</th>
<th>Stand-off Distance $Z_{dr}$ (cm.)</th>
<th>Hole Radius $R_h$ (cm.)</th>
<th>Measured</th>
<th>Non-Dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.31</td>
<td>8.9</td>
<td>4.4-5.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.62</td>
<td>10.2</td>
<td>5.7-6.4</td>
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<tr>
<td>4.57</td>
<td>15.2</td>
<td>6.4-7.0</td>
<td></td>
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<tr>
<td>3.39</td>
<td>17.1</td>
<td>7.6-8.3</td>
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</table>

<table>
<thead>
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<th>SOD $Z_{dr}$</th>
<th>Hole Radius $R_h$</th>
<th>Non-Dimensional</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_{noz}</td>
<td>R_{noz}</td>
<td></td>
</tr>
<tr>
<td>30</td>
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<td>20-22</td>
<td></td>
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<td>56</td>
<td>24-26</td>
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</tr>
</tbody>
</table>

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3.3 Rock Failure During Thermal Spallation: Current Theory

3.3.1 Key Assumptions

A mathematical description of the initiation of spallation was developed by Dey (1984) based upon four underlying assumptions:

1) compressive failure occurs during spallation,
2) the stress level at failure is governed by the orientation, size, and number density of pre-existing flaws,
3) the Weibull Statistical failure theory describes this stress level, and
4) the temperature field can be approximated as being one-dimensional and treated as that at a subliming solid/gas interface.

A brief discussion of the reasoning behind each of these assumptions is given below. Reference is made to several of the studies that were described in Section 3.2. Further details can be found there.

Assumption # 1 is based upon Preston's (1934) experiments that distinguished between true spallation, which was shown to be caused by compression, versus splitting in a direction normal to the heated surface, caused by gross tensile failure. Furthermore, Lauriello (1971) calculated the stress field under a rapidly heated spot in a brittle material and determined that a state of compression exists in the rock at depths where spalling is expected to occur.
The effect of the size, orientation, and number of pre-existing flaws on rock failure strength (assumption #2) has been pointed out by numerous researches (Wong and Brace, 1979; Nematt-Nasser and Horii, 1982; Fairhurst and Cook, 1966). The use of Weibull theory to describe this phenomenon during spallation (assumption #3) was first suggested by Marovelli et al. (1966) and was also successfully used by Lundborg (1967) to describe the size-strength relationships of granite under both compressive and tensile stresses.

Assumption #4 implies that a pseudosteady state temperature and heat flux occur at the rock surface during thermal spallation. The validity of this is investigated by calculating the discrete drop in temperature that occurs when a spall is liberated. The temperature profile is given by equation (3.5):

$$T = \exp\left(-\frac{U_{dr}}{\alpha_r} z\right)$$

(3.5).

For thermal spallation drilling, representative values of the variable appearing in equation (3.5) are:

- $U_{dr} = 6.0 \times 10^{-4}$ m/s,
- $\alpha_r = 1 \times 10^{-6}$ m$^2$/s, and
- $z =$ spall thickness $= 4 \times 10^{-4}$ m.

Therefore, using equation (3.5) gives:

$$\frac{\Delta T_s}{T_s} \approx 21\%$$

where $\Delta T_s$ is the difference between the frontside and backside surface
temperatures on the spall. Furthermore, one can easily show that
\[
\frac{\delta Q}{Q} \approx \frac{\delta T_s}{T_{\text{jet}} - T_s} \approx 6.5\% 
\]
where \( T_{\text{jet}} \) and \( T_s \) are assumed to be 2830 K and 900 K, respectively.

Rauenzahn (1986) has demonstrated that errors in the physical property values that are necessary in the analysis that follows are probably comparable to the 21% error shown above to be caused by assuming a steady-state surface temperature. Furthermore, the alternative to making assumption # 4 is numerical solution of the 1-D energy conduction equation in the rock for transient temperature and heat flux boundary conditions. Considering the uncertainty inherent in such an analysis and recognizing that the theory that will be described in Section 3.3.3 is only an approximation, the added computational effort is not warranted.

3.3.2 Compressive Rock Failure Mechanism

The currently accepted mechanism of crack extension under the influence of a compressive stress is illustrated in Figure 3.8 (Rauenzahn, 1986). A single flaw that is, in general, oriented neither parallel nor perpendicular to the stress vector is shown in compression.
a) Flawed Rock - Unheated

b) Heat Applied - Flawed Region is Loaded Compressively ($\sigma > 0$)

c) Flaw Begins to Extend in Direction of Stress

d) Fractured Region Enters Buckling Mode and Begins to Separate

e) Spall is Formed and Separated from Surface

Figure 3.8 Spallation failure process (Rauenzahn, 1986).
The events leading to crack extension are:

1) buildup of tensile (or shear) stress at both ends of the flaw,
2) initial crack propagation at an angle of about 70 degrees to the flaw (Nematt-Nasser and Horii, 1982), and
3) extension of the crack to align itself with the stress field.

During spallation the crack under consideration is near a free surface. Therefore, buckling will occur and a "spall" will be ejected from the rock if the following approximate criterion is met (Holzhauser and Johnson, 1979):

$$\frac{d}{\delta} \geq \left[ \frac{E}{6(1 - \nu^2)\sigma} \right]$$  \hspace{1cm} (3.6)

where

- $\sigma$ = stress level in the rock (MPa),
- $\delta$ = spall thickness (m), and
- $d$ = spall diameter (m).

The variation of local rock strength throughout the sample can be understood by noticing that, for a given spall thickness, a smaller diameter or, equivalently, a smaller crack length is required for failure at high stress than at low stress. Therefore, the higher the stress level, the more likely that the first flaw encountered by the thermal penetration wave will elongate to (or already be at) the critical buckling length.

Unfortunately, understanding the compressive failure near a single
flaw does not lead directly to quantitative predictions of failure in a rock sample because no account is made for the size, orientation, and density distributions of flaws. Therefore, a statistical method is commonly used to capture this composite aspect of failure in a microscopically heterogeneous material such as granite.

3.3.3 Weibull Statistical Failure Theory

The true intrinsic strength of a material under stress cannot, in general, be achieved because of the presence of flaws. Weibull theory considers that there is a probability of fracture associated with a given sample under stress such that the cumulative probability of failure can be given by (Weibull, 1939):

\[ G(\sigma) = 1.0 - \exp \left[ \int_{0}^{V} \left( \frac{\sigma}{\sigma_0} \right)^m dV \right] \]  

(3.7)

where

\( \sigma_0 = \) compressive strength of rock per unit volume (MPa-m^3/m^3),

\( V = \) volume of rock containing flaws capable of removing a given point on the surface (m^3), and

\( m = \) "homogeneity factor".

This expression has been shown to describe the relationship between sample size and measured strength of granite and other flawed materials undergoing brittle failure (Lundborg, 1967). Substitution of expressions giving the variation of \( \sigma \) and \( V \) with \( z \) into equation (3.7) results in a tractable, closed form expression for the spallation
surface temperature as a function of rock physical properties, the empirical Weibull parameters (m and \( \sigma_0 \)), and the applied heat flux (see Section 3.3.4).

Weibull parameters can be obtained independently from mechanical tests conducted at room temperature. The first parameter, m, is interpreted as a measure of the structural homogeneity of the material being stressed. In the limiting case when m tends toward infinity, the cumulative probability of failure tends toward zero for all values of \( \sigma \) less than \( \sigma_0 \), and to one for all \( \sigma \) greater than \( \sigma_0 \), reflecting the fact that in a perfectly homogeneous material the intrinsic strength measured for any sample size is the same. Values of m and \( \sigma_0 \) have been found to lie in the ranges of 5 - 25 and 50 - 70 MPa-m\(^{3}/m\), respectively, for granite at room temperature (Dey and Kranz, 1985; Lundborg, 1967).

3.3.4 Estimation of Spallation Temperature

Integration of the right-hand side of equation (3.7) requires knowledge of the stress distribution within the sample volume. The thermally induced stress is expressed as a function of the local temperature rise in the rock by using equation (3.2). Local temperature rise as a function of depth is determined according to the method proposed by Gray (1965) whereby the rock surface is assumed to be at a constant temperature and the thermal field is approximately the same as that for a subliming solid. Combination of equations (3.2) and (3.5) leads to the variation of stress with depth into the rock:
\[ \sigma = \Delta T_s \exp \left( - \frac{U_d r}{\alpha_r} z \right) \frac{\beta E}{(1 - \nu)} \] (3.8).

The volume \( V \) that contains a critical flaw capable of removing the surface point in question is the final term necessary for integration of the right-hand side of equation (3.7). Since many spalls have been found experimentally to be shaped like cylinders with diameter to thickness ratios of about 15:1, \( V \) can be approximated by:

\[ V = z^3 (\pi C_L^2 / 4) \] (3.9),

and

\[ dV = 3z^2 (\pi C_L^2 / 4) \, dz \] (3.10),

where \( C_L \) is the spall aspect ratio (≈ 15 for granite).

At median spalling conditions, by definition, the cumulative probability of failure \( G(\sigma) \) is 0.5. Integrating equation (3.7) and solving for the surface temperature results in the following expression (Rauenzahn, 1986):

\[ T_s = T_{r0} + \left[ \left( \frac{Q}{\rho C_p} \right)^3 \left( \frac{1 - \nu}{\beta_r E} \sigma_0 \right)^m \left( \frac{2(0.693)}{\pi C_L^2} \right) \left( \frac{m}{\alpha_r} \right)^3 \right]^{1/(m+3)} \] (3.11)

The most important contribution to the spallation surface temperature is due to the biaxial stress term (2). Term (1) accounts for the relatively weak dependence of the surface temperature at failure on applied heat flux. For a typical value of \( m = 20 \),

\[ T_s \propto Q^{0.13}. \]

As \( m \) approaches infinity the heat flux dependence tends towards zero and the surface temperature rise becomes only a function of physical properties and a single, size-independent, strength parameter \( (\sigma_0) \) as
expected.

It is important to note that all physical properties and the Weibull strength parameters \( m \) and \( \sigma_0 \) were considered to be temperature independent when deriving equation (3.11). However, Rauenzahn (1986) has shown that additional microcracks form as a polycrystalline material is heated because of the different thermal expansion characteristics of mineral inclusions. This is expected to change the homogeneity parameter, although insufficient data exist to quantify the effect. Furthermore, physical properties can vary widely over the range of rock temperatures typically encountered during thermal spallation (Heuze, 1983). Therefore, the parameter values that are currently used for making calculations with equation (3.11) must be considered as temperature averaged in some sense, particularly when deriving Weibull parameters directly from surface temperature versus heat flux data obtained during thermal spallation.

The major advantages of using the Wiebull statistical theory to derive equation (3.11) rather than solving the full set of thermoelasticity equations for the rock (even assuming that the initial flaw distribution was known) are the explicit closed form and the transparency of the physical insight gained from equation (3.11). No numerical integration is required and the probabilistic nature of brittle compressive failure is accounted for without the high costs typical of Monte-Carlo methods that would be necessary otherwise.
In the past, the validity of equation (3.11) has been tested by comparing predicted surface temperatures using Weibull parameters determined from mechanical testing procedures to experimentally determined spallation onset temperatures. At heat fluxes from about 0.6-1.0 MW/m² the results agreed to within the uncertainty in physical parameters. Experiments, described in the next section, have been conducted during this study to test the validity of the model over a wide range of heat fluxes under steady-state spalling conditions.
4 Experimental Results

4.1 Introduction

Use of the theory described in the previous chapter to estimate the rock surface temperature during thermal spallation drilling requires Weibull parameters - m and \( \sigma_0 \) values. Dey and Kranz (1985) performed mechanical tests at ambient temperature on Berkeley "blue" Granite. Rauenzahn (1986) studied thermal spallation on Westerly and Barre Granite using a 500 W CO\(_2\) laser and a calibrated welding torch as heat sources. Both experimenters estimated values of \( m \approx 20 \) and \( \sigma_0 \approx 70 \) MPa-m\(^{3/20}\). Rauenzahn's welding torch results are displayed in Figure 4.1 and compared to those obtained using equation (3.11) with the fitted Weibull parameters. Rauenzahn estimated spallation temperatures by recording heating tests on high speed videotape at 2000 frames per second and reviewing the tapes to determine the time taken for the first point on the surface to spall. The rock was assumed to behave as a one-dimensional, semi-infinite solid subjected to a constant heat flux (Q) as described by Carslaw and Jaeger (1959):

\[
T_s = T_{r0} + 2\frac{Q}{k_r} \left( \frac{\alpha_r t}{\pi} \right)^{1/2}
\]  
(4.1)

where \( k_r \) = rock thermal conductivity (W/m\(^2\) K), \( \alpha_r \) = rock thermal diffusivity (m\(^2\)/s), and \( t \) = time (s).
Figure 4.1 Calculated spallation onset temperature versus calibrated heat flux during welding-torch heating (Rauenzahn, 1986).
Therefore, knowing the heat flux and the time taken for the onset of spallation, the surface temperature (in K) was estimated from equation (4.1). However, using the first spall as the criterion for determining the "spallation temperature" complicates interpretation of temperatures so-obtained. In the context of the theory described in the previous chapter, spallation occurs at a distribution of stress levels and the spallation temperature is defined as that occurring at the median cumulative probability of failure. The first spall to appear is in the low-end tail of the failure stress distribution where spalling temperatures are expected to be below the median spallation temperature. Therefore, ideally, random points on a surface at a spatially uniform heat flux should be chosen and the surface temperatures at spallation averaged for determination of the median spalling temperature.

Rauenzahn points out that difficulties also arise from the small beam diameters used during his laser heating tests. A typical beam heat flux profile obtained from the burn pattern produced in a plexiglass sample reveals an inner constant heat flux region surrounded by an outer linearly decreasing flux (Figure 4.2 (Rauenzahn, 1985)). The two values of inner radius (r') obtained by Rauenzahn were 0.5 and 2.0 mm. The one-dimensional heating assumption used in deriving equation (3.11) is violated by both of these beam radii because the depth of the heated region is on the order of 1 mm which is close to the beam diameter. Furthermore, the small volumes that are heated may not include enough flaws and grains for the Weibull-based theory developed by Dey (1984) to adequately describe the statistics of the failure
Figure 4.2 Laser heat flux pattern from plexiglass burn experiments (Rauenzahn, 1986).
processes occurring.

Experiments performed in this study were aimed at avoiding the difficulties described above. Laser and flame-jet induced thermal spallation have been performed in several well-characterized rock types while monitoring surface temperatures with a commercially available infrared (IR) scanner (described in Section 4.3). A high powered (25 kW) laser firing an annular beam with a 4:1 outside-to-inside diameter ratio was used so that large overall diameters of 5 and 15.8 cm. could be achieved while maintaining heat fluxes in the important range from about 0.1 - 3.0 MW/m². In addition, larger beam diameters provided more representative samples of rock surface, comparable to the those encountered in the torch experiments. The heat source for flame-jet spallation tests was the torch used by Rauenzahn during his small-scale field drilling experiments.

The specific experimental objectives were:

1) to extend the range of heat fluxes at which spallation temperatures in common granite rocks are known,

2) to have direct surface temperature measurements during thermal spallation,

3) to have accurate values for heat fluxes at these temperatures, and

4) to verify that the rock failure mechanism occurring during flame-jet and laser induced thermal spallation are similar.
4.2 Details of Experiments

4.2.1 Flame-jet Spallation

Figure 4.3 is a schematic diagram of the experimental set-up that was used during flame-jet spallation tests. An infrared (IR) scanner, which is described later in Section 4.3, was used for non-contact surface temperature measurement. A Unistrut beam supported the torch and acted as a hinge to the table at point "A" so that the torch could be quickly moved to and from the sample during operation. Guides at point "B" assured that the torch was correctly positioned when it was lowered, as shown in Figure 4.3b. Parts of the rack exposed to hot gases were wrapped in 2.54 cm (1 inch) thick refractory ceramic fiber in order to protect them from the intense heat. A one-inch thick refractory board protected the sample while the torch was being lowered into position.

The torch was ignited in the "upright" position, shown in Figure 4.3a, by opening the propane flow control valve slightly and passing a flame underneath the nozzle outlet. Once lit, the oxygen flow rate was slowly increased until a characteristic "pop" was heard, indicating that the flame had flashed-back to the internal flame-holder, whereupon both flow control valves were completely opened.

After several minutes of operation the torch was stabilized and the infrared scanner was started. The torch was lowered into position by one operator, while another held the protective board in place. The
a. Torch in Raised Position.

b. Torch in Lowered Position.

**Figure 4.3** Small-scale flame-jet spallation experimental set-up.
protective board was pulled away in about 0.1 seconds and spalling occurred within 0.1 to 20 seconds depending on the stand-off distance of the burner head from the rock sample. Rock samples were either turned to expose a new face, or removed after each experiment.

Barre and Westerly Granite were used in these tests. All Barre samples were 0.3 m x 0.3 m x 0.3 m (1x1x1 ft³) and Westerly samples varied in cross section and thickness from about 0.3 m x 0.3 m x 0.1 m (0.3 ft) to 0.9 m (3 ft) x 0.3 m x 0.15 m (0.5 ft). Table 4.1 lists the petrographic mineral compositions of Barre and Westerly Granite (Krech et al., 1974).

**Table 4.1** Mineral Compositions of Barre and Westerly Granite.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Barre Granite</th>
<th>Westerly Granite</th>
</tr>
</thead>
<tbody>
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<td>Plagioclase</td>
<td>50.0</td>
<td>43.0</td>
</tr>
<tr>
<td>Quartz</td>
<td>22.0</td>
<td>24.6</td>
</tr>
<tr>
<td>Microcline</td>
<td>10.0</td>
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</tr>
<tr>
<td>Biotite</td>
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</tr>
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<td>Muscovite</td>
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</tr>
<tr>
<td>other</td>
<td>4.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**4.2.2 Laser-heating Tests**

The experimental laser set-up consisted of the laser, associated optics, an optical beam spinner, and a focusing head as illustrated in Figure 4.4. The infrared scanner and a Kodak Ektapro 1000 high speed videotaping unit, capable of recording at 1000 frames/s, were used for data acquisition.
The laser and associated hardware were set-up by personnel employed by United Technologies Industrial Lasers in East Hartford, Connecticut, from whom time on the machine was rented. An optical beam spinner was used to smooth azimuthal non-uniformities in the raw laser beam by turning the beam at 4000 - 6000 rpm. The focusing head put a slight angle of divergence on the beam so that beam impingement diameters could be easily varied by changing the distance from the sample to the head. The pattern created by the laser beam was an annular ring with an outside-to-inside diameter ratio of four-to-one. The beam impingement
Figure 4.4 Schematic of experimental set-up for laser-induced spallation. (Not to scale.)
time was controlled by user-defined settings in a numerical controller machine that sent electrical impulses to open or close the external shutter that was positioned in the path of the laser beam. High speed videotapes reveal that the shutter took approximately 50 ms to fully open.

The operating procedure was as follows:

1) Position and label rock sample.
2) Enter laser power level and time of irradiation into numerical controller,
3) Start infrared scanner and high speed videotaping unit, and
4) Open shutter.

All laser settings were recorded, and before and after photographs of samples were taken.

4.3 Surface Temperature Measurements: Using IR scanning

The radiation emitted from the surface of a body is a function of its temperature. When an infrared scanner device is used, this radiation is measured and the surface temperature determined without the normal requirement of physical contact between the temperature sensor and the object being measured. Several researchers have attempted to measure spallation temperatures by implanting thermocouples in the rock and monitoring the transient temperature rise up to the time when the thermocouple fails due to contact with the flame (Thurumalai,
1969). The major problems associated with this technique are: inaccuracies because thermocouple bead diameters are typically about the same size as the heated layer thickness (≈ 1 mm), local rock weakening resulting from drilling the holes that contain the thermocouples, and poor thermal contact between the thermocouples and the rock samples due to the different thermal expansion characteristics of the thermocouple bead and the rock. Furthermore, a large number of thermocouples must be used to obtain lateral surface temperature distributions.

The IR scanner provides direct surface temperature measurements at a sampling frequency of 20 Hz. and a spatial resolution of 240 horizontal x 512 vertical zones. The temperature information is displayed as colors on a video screen and a color key allows conversion back into temperatures. In addition, a screen cursor displays a direct digital temperature value at a user-chosen point of interest. The principles of operation of the IR scanner are now described.

4.3.1 Principles of Operation

The infrared scanning system is a Hughes Probeye model 3100 consisting of three basic units (Figure 4.5): a set of infrared detectors that transmit voltage signals related in a highly non-linear pattern to incident radiation; an optical system for scanning the surface and focusing the radiation from the points being measured onto the detectors, and a data logging and processing unit for conversion of detector signals into temperature values.
Figure 4.5 Block diagram of infrared scanner.
The detectors are constructed from mercury cadmium telluride and respond to IR radiation in the 3.2 to 5.6 μm portion of the electromagnetic wave-length spectrum. The atmospheric attenuation of IR radiation in this range is minimal (Morgan and Schultz, 1985). Therefore, most of the radiation leaving the observed part of the rock surface is received at the detectors. A focusing lens is used so that the only radiation measured is that from the image of the detectors in the focal plane, which in this case is adjusted until it coincides with the rock surface.

The actual amount of radiation received by the detectors depends upon the detector area, optical magnification, the solid angle from the target subtended by the lens, the emissivity and temperature of the rock, transmission and reflection losses in the optical system and surrounding environment, and reflected radiation from other high temperature objects. Optical system interference is accounted for in each sensor during the short time between temperature measurement by viewing a temperature reference inside the scanner but outside the first lens. The algorithm used for conversion of sensor signals into temperatures accounts for any inaccuracies during the subsequent measurement. Other potential error sources have been checked in this study by performing calibration experiments (described in Section 4.3.2).

Conversion of the voltage signal into a temperature is done using a calibration table for that particular sensor. The processing unit in
the IR scanner detects a voltage, corrects the voltage based on the indicated temperature reference mentioned above, and finds the corresponding temperature value in the stored calibration table.

Scanning of the entire surface is accomplished by a set of eight mirror facets mounted on a vertical spinning axis. The mirrors scan across the field of view and re-direct the incoming radiation into the optical system. Thirty detectors are used and each mirror is at a different angle relative to the axis of rotation resulting in 240 horizontal measurement lines. The mirrors spin at a frequency of 20 Hz. so that each point is updated every 50 ms.

4.3.2 Independent Calibration

The accuracy of the IR scanner for this application is dependent upon using the correct surface emissivity for the rock and minimizing atmospheric attenuation between the rock and the scanner lens. Furthermore, when the flame-jet is being used as a heat source, reflection of the radiation transmitted from the hot gases to the surface must be accounted for. Reflected radiation should be unimportant during laser heating because the laser emits monochromatic light at 10.6 μm which is well above the 5.6 μm upper detection limit for the IR scanner.

The first calibration experiment was aimed at verifying the ability of the scanner to accurately measure granite surface temperatures in the
range expected during spallation. A 1.9 cm. diameter x 4.5 cm. long cylinder of granite was heated in a tube furnace (Figure 4.6) and the rock surface temperature was measured using a type K thermocouple positioned at the center of the tube. The rock surface temperature was approximately equal to the air temperature in the tube after one hour of heating. Unfortunately, only two high temperature measurements were recorded because of time and budget limitations. The results are listed in Table 4.2 where it is seen that the errors are 1.2 % and 0.42 % at 563 °C and 592 °C respectively. These errors are of the same order of magnitude as the uncertainty associated with the thermocouple measurements.

Table 4.2 High Temperature Infrared Scanner Calibration Results.

<table>
<thead>
<tr>
<th>Thermocouple Reading (°C)</th>
<th>IR Scanner Reading (°C)</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>563.0</td>
<td>569.5</td>
<td>1.2</td>
</tr>
<tr>
<td>592.0</td>
<td>594.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

rock emissivity = 0.90

In addition to these experiments, crude calibration tests were performed after some of the flame-jet and laser heating tests on Barre and Westerly Granite, Sioux Quartzite, and Webatuck Dolomite by placing a thermocouple against the rock surface when it had cooled to between 40 - 100 °C. In all cases the thermocouple and IR scanner results were within 10 % of each other and temperatures indicated by the thermocouple were always lower.
Figure 4.6 IR scanner calibration set-up.
A final set of calibration tests were aimed at quantifying the effect of radiation from primarily CO$_2$ and H$_2$O in the gas stream leaving the flame-jet burner nozzle at about 2830 K. Monochromatic gas emissivities for CO$_2$ and H$_2$O vary widely over the electromagnetic wavelength spectrum and with the mean beam length of the gas making a-priori determination of the cumulative interference over all wavelengths from 3.2 to 5.6 μm difficult. However, the black-body radiation at 2830 K is calculated from equation (2.3) as 3.6 MW/m$^2$ compared to about 50 kW/m$^2$ at typical rock surface temperatures. Therefore, the potential for "flame" interference cannot be ignored.

The influence of the propane/oxygen torch used in this study on the accuracy of surface temperatures measured during flame-jet tests has been estimated by allowing steady-state spallation to develop and then rapidly removing the torch from the rock sample while simultaneously monitoring the temperature-time history. The results of one such test are plotted in Figure 4.7. Spallation temperatures measured by the IR scanner varied between about 710 °C and 730 °C while the torch was impinging upon it. When the torch was removed the indicated temperature dropped by 10 - 20 °C within 50 ms and then continued dropping at a similar but declining rate until observations were stopped approximately 1 s later. As described above, the IR scanner was calibrated at approximately 600 °C so that high accuracy is expected at these temperatures. Therefore, since some cooling occurred between measurements, the 3 % temperature drop indicated immediately after torch
Figure 4.7 Effect of flame radiation on IR scanner determined surface temperatures.
removal is a conservative upper limit estimate for the effect of flame interference on the indicated steady-state surface temperatures.

Similar results have been obtained for all flame-jet spallation experiments regardless of the torch stand-off distance or rock type. Therefore, the effect of flame interference on the results that are presented in Section 4.5 of this thesis is assumed to be negligible.

4.4 Heat Flux Calculations

Validation of equation (3.11) and determination of the Weibull parameters requires that $T_s$ be known as a function of the heat flux ($Q$). The most common methods of determining the heat flux to a surface are: measurement of the steady-state cooling requirement for a sensor of known surface area and measurement of the temperature gradient across a substance with a known thermal conductivity and a one-dimensional temperature profile. Both of these methods are impractical for conditions on the rock surface during flame-jet or laser induced spallation because they require that sensors be mounted in the hostile environment existing at the rock surface during heating. Furthermore, the heat flux to the sensors is likely to be substantially different from that to the rough rock surface. Finally, before steady-state spalling conditions could be set-up in the rock the surface would probably have receded past the sensors.

With the above limitations in mind, several methods have been used
to determine the heat fluxes in this study. In the first method the rock is again modeled as a one-dimensional, semi-infinite solid subjected to a constant heat flux at the free surface. The relationship between surface temperature and heating time is given by equation (4.1) as:

\[ T_s = T_{f0} + \frac{Q}{2k_r} \left( \frac{\alpha_r t}{\pi} \right)^{1/2} \]  \hspace{1cm} (4.1)

where \( \alpha_r \) = rock thermal diffusivity (\( m^2/s \)), and \( t \) = time (s). Transient temperature rise data have been recorded for some of the flame-jet, and all of the laser induced spallation experiments. Therefore, a least-squares line-fitting technique is used to match experimental curves of \( T_s \) versus \( t^{1/2} \) to equation (4.1). The heat flux is related to the slope of this line as follows:

\[ Q = \frac{A k_r}{2(\alpha_r / \pi)^{1/2}} \]  \hspace{1cm} (4.2)

where \( A \) = slope of "best-fit" line. All temperature values used are taken from the digital cursor read-out on the IR scanner to avoid the subjectivity that would be inherently introduced otherwise when matching displayed colors to the color key. An example of the results of a calculation to determine the best-fit line through the \( T_s \) versus \( t^{1/2} \) data and the heat flux for a laser heating test is illustrated in Figure 4.8. The good fit of the data verifies the validity of the one-dimensional temperature profile assumption made in deriving equation (4.1).
Figure 4.8 Example of a transient temperature-rise curve for laser heating.
Another heat flux calculation method has been used for analyzing the flame-jet spallation data from measured drilling rates \( V_{dr} \) and steady-state stagnation-point surface temperatures \( T_{s, st} \). The fundamental energy balance at the rock surface, given previously in equation (2.4) relates \( T_{s, st} \) and \( V_{dr} \) to the heat flux:

\[
Q_{st} = (\rho C_p) V_{dr} (T_{s, st} - T_{r0})
\]  

(2.4),

where \( Q_{st} \) = stagnation point heat flux, \( V_{dr} \) = forward drilling rate, and \( T_{s, st} \) = stagnation point surface temperature. Values of \( V_{dr} \) were obtained from Rauenzahn's small-scale field drilling data, listed in Table 3.2, and \( T_{s, st} \) values have been measured in the present study. The stagnation-point heat flux obtained at a given stand-off distance during steady-state drilling experiments conducted by Rauenzahn (1986) is assumed to be equal to that at the same stand-off distance on a flat granite surface. Furthermore, surface phase transformations and crack propagation energies are assumed negligible for equation (2.4) to be true.

A second method for calculation of the laser-induced heat flux was also attempted. The power delivered from the laser beam to an approximately 10 cm. diameter, commercially available "hockey-puck" calorimeter located at the sample position nearest to the focusing head (Figure 4.4) was measured at each narrow-beam power level setting that was used during the spallation experiments. The ratio of power delivered to power setting for the large beam impingement diameter was
assumed to equal the average of the narrow beam results. The measured power divided by the beam area equals the average heat flux. Therefore, test burns to determine beam diameters and radial power variations were performed in 1.3 cm. (1/2 inch) thick plexiglass squares. The results of these experiments are illustrated in Figure 4.9 where overall beam diameters are shown to increase with power level setting, especially for the wide diameter beams. Temperature profiles obtained from the IR scanner indicate that surface temperature, and therefore heat flux varies by less than 5% over 95% of the beam thickness so that with overall beam radii, calibrated power levels, and the measured monochromatic rock emissivity for granite at 10.6 μm of 0.90 (Hovis and Callahan, 1966) heat fluxes can be calculated.

Unfortunately, inconsistencies were found between spallation onset times obtained using supposedly identical heat fluxes at the large and small beam diameters (Figure 4.10). Comparison with the transient heat-up curves from infrared scanning indicates that the heat fluxes calculated as described above are incorrect. One possible explanation for this discrepancy is that atmospheric absorption of the laser beam occurs, leading to lower than estimated heat fluxes at the large beam diameter sample location, which is farther away from the focusing head than the narrow beam location. Therefore, since transient heat-up curves were measured for all laser tests, values calculated from equation (4.1) as described above are used for all laser-induced heat fluxes reported in the remainder of this thesis.
Figure 4.9 Laser beam diameters measured on plexiglass blocks.
Figure 4.10 Calculated global onset temperature versus calibrated laser heat flux.
4.5 Results

4.5.1 Flame-Jet Tests

The results of the flame-jet spallation tests are listed in tables 4.3 and 4.4, and illustrated in Figures 4.11 and 4.12 for Barre and Westerly Granite, respectively. The curve representing the Weibull theory spallation temperature predictions (equation (3.11)) is obtained by using mechanically-determined Weibull parameters (Dey and Kranz, 1985) and averaged physical properties for Barre and Westerly Granite (Table 4.3). In addition, the theoretical 90 % probability interval, defined as the region bounded by the temperature values at which 5 % and 95 % cumulative probabilities of spalling are predicted to occur, is shown for reference.

Table 4.3 Flame-Jet Spallation Surface Temperature Values -- Heat Flux From Energy Balance.

<table>
<thead>
<tr>
<th>Barre Granite</th>
<th>Stand-Off Distance</th>
<th>( V_{d_r}^* ) (est.)</th>
<th>( T_{s, st} ) °C</th>
<th>( Q_{s, st} ) MW/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{Z_{d_r}}{R_{noz}} ) in.</td>
<td>cm.</td>
<td>m/hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>4.0</td>
<td>10.2</td>
<td>4.3</td>
<td>925</td>
</tr>
<tr>
<td>40</td>
<td>5.0</td>
<td>12.7</td>
<td>3.3</td>
<td>937, 937, 837</td>
</tr>
<tr>
<td>47</td>
<td>5.9</td>
<td>14.9</td>
<td>3.1</td>
<td>879, 850, 930, 890, 785</td>
</tr>
<tr>
<td>63</td>
<td>7.9</td>
<td>20.0</td>
<td>2.8</td>
<td>710, 710</td>
</tr>
<tr>
<td>80</td>
<td>10.0</td>
<td>25.4</td>
<td>2.5</td>
<td>520, 520, 508, 520</td>
</tr>
<tr>
<td>96</td>
<td>12.0</td>
<td>30.5</td>
<td>2.2</td>
<td>475, 520</td>
</tr>
</tbody>
</table>
Figure 4.11 Surface temperature versus heat flux -- flame-jet spallation in Barre Granite.
Figure 4.12 Surface temperature versus heat flux -- flame-jet spallation in Westerly Granite.
Table 4.3  Flame-Jet Spallation Surface Temperature Values -- Heat Flux From Energy Balance (Continued).

Westerly Granite

<table>
<thead>
<tr>
<th>Stand-Off Distance $Z_{dr}$</th>
<th>$V_{dr}^*$ (est.)</th>
<th>$T_{s, st}$</th>
<th>$Q_{st}^\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{noz}$</td>
<td>in.</td>
<td>cm.</td>
<td>m/hr</td>
</tr>
<tr>
<td>32</td>
<td>4.0</td>
<td>10.2</td>
<td>5.6</td>
</tr>
<tr>
<td>80</td>
<td>10.0</td>
<td>25.4</td>
<td>2.9</td>
</tr>
<tr>
<td>96</td>
<td>12.0</td>
<td>30.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

* Rauenzahn (1986),

$Q_{st}^\dagger = (\rho C_p)_{r} V_{dr} (T_{s, st} - T_{r0})$,

$(\rho C_p)_{r} = 2.64 \times 10^6$ J/m³ K, and

$T_{r0} = 300$ K.
Table 4.4 Flame-Jet Spallation Surface Temperature Values -- Heat Flux From Transient Analysis.

**Barre Granite**

<table>
<thead>
<tr>
<th>$\frac{Z_{dr}}{R_{dr}}$</th>
<th>in.</th>
<th>cm.</th>
<th>$V_{dr}^*$ (est.)</th>
<th>$T_s$</th>
<th>$Q^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>4.0</td>
<td>10.2</td>
<td>4.3</td>
<td>785</td>
<td>1.8</td>
</tr>
<tr>
<td>47</td>
<td>5.9</td>
<td>14.9</td>
<td>3.1</td>
<td>925</td>
<td>2.0</td>
</tr>
<tr>
<td>47</td>
<td>5.9</td>
<td>14.9</td>
<td>3.1</td>
<td>785</td>
<td>1.3</td>
</tr>
<tr>
<td>47</td>
<td>5.9</td>
<td>14.9</td>
<td>3.1</td>
<td>808</td>
<td>1.1</td>
</tr>
<tr>
<td>47</td>
<td>5.9</td>
<td>14.9</td>
<td>3.1</td>
<td>832</td>
<td>1.4</td>
</tr>
<tr>
<td>63</td>
<td>7.9</td>
<td>20.0</td>
<td>2.8</td>
<td>638</td>
<td>0.6</td>
</tr>
<tr>
<td>80</td>
<td>10.0</td>
<td>25.4</td>
<td>2.5</td>
<td>588</td>
<td>0.5</td>
</tr>
</tbody>
</table>

$^*$ Rauenzahn (1986)

$^+$ Heat flux from Transient Analysis,

\[ k_r = 2.14 \text{ W/m K}, \text{ and} \]

\[ \alpha_r = 0.87 \times 10^{-6} \text{ m}^2/\text{s}. \]

Table 4.5 Averaged Barre and Westerly Granite Physical Property Values used for Prediction of Weibull Temperature Curves in Figures 4.11 and 4.12.

\[ (\rho C_p)_r = 2.64 \times 10^6 \text{ (J/m}^3 \text{ K}) \]

\[ \beta_r = 9 \times 10^{-6} \text{ (°C}^{-1}) \]

\[ E = 52.5 \text{ (GPa)} \]

\[ \nu = 0.25 \]

\[ \alpha_r = 0.9 \times 10^{-6} \text{ (m}^2/\text{s}) \]

\[ m = 20 \]

\[ \sigma_0 = 70 \text{ MPa-m}^3/20 \]

(Properties reported in Rauenzahn, 1986)
At heat fluxes above about 1 MW/m² the surface temperatures on Barre Granite are higher and more sensitive to heat flux than predicted, implying that a smaller value of the homogeneity parameter, m, should be used. At 2.9 MW/m² the measured surface temperature is about a factor of two higher than the Weibull prediction.

Surface temperatures measured on Barre Granite at low heat fluxes (≈ 1 MW/m²) are within experimental scatter of onset temperatures measured by Rauenzahn using a calibrated welding torch to induce spallation. The consistency of the two sets of results between about 0.6 - 1.2 MW/m² confirms that the IR scanner is not grossly inaccurate and indicates that at low heat fluxes the spallation onset temperatures are representative of those achieved during steady spalling. Therefore, one might expect that at low enough heat flux (and surface temperature rise) the mechanically obtained Weibull parameters should accurately represent the rock failure that occurs. This point will be investigated further in the discussion of laser induced spallation (Section 4.5.2).

Comparison of the results illustrated in Figures 4.11 and 4.12 reveals that spallation temperatures for Barre Granite are higher than those for Westerly Granite at heat fluxes above approximately 1 MW/m². In addition, spallation temperatures for Westerly Granite are not as strongly dependent on heat flux as they are for Barre Granite, indicating that Westerly Granite has a higher homogeneity parameter. This is consistent with the observation that the average grain size in Westerly Granite is about one-half of that in Barre Granite. For a
given thermal penetration depth, more grain boundaries will be encountered in Westerly Granite. Therefore, if flaws extend preferentially between grains, more potentially critical flaws will be encountered at a given surface temperature, so a lower spallation temperature should be expected to cause failure. However, the different spallation temperatures at high heat fluxes for Barre and Westerly Granite could also potentially be caused by the different mineral compositions and physical properties. Further study is required before firm conclusions can be drawn to explain the cause(s) of the different spallation temperatures that are measured.

4.5.2 Laser Spallation Tests

Laser spallation tests have been conducted on Barre and Westerly Granite, Sioux Quartzite, and Webatuck Dolomite (Table 4.6). Measured surface temperatures as a function of heat flux are plotted along with 90 % probability envelopes for Barre and Westerly Granite in Figures 4.13 and 4.14, respectively. Considerable data scatter exists especially for the Westerly Granite, but the laser results follow the general trends mapped out by the flame-jet data. The consistency in these trends is a further indication that infrared interference effects did not substantially influence temperature measurements made during flame-jet spallation.
Figure 4.13 Surface temperature versus heat flux for Barre granite -- comparison between laser and flame-jet results indicating steady-state spallation.
Figure 4.14 Surface temperature versus heat flux for Westerly granite -- comparison between laser and flame-jet results indicating steady-state spallation.
Table 4.6  Measured Surface Temperatures and Calculated Heat Fluxes from Laser Spallation Experiments.

**Barre Granite**

<table>
<thead>
<tr>
<th>Q (MW/m²)</th>
<th>Tₛ (℃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>481</td>
</tr>
<tr>
<td>0.35</td>
<td>495</td>
</tr>
<tr>
<td>0.38</td>
<td>520</td>
</tr>
<tr>
<td>0.43</td>
<td>495</td>
</tr>
<tr>
<td>0.47</td>
<td>457</td>
</tr>
<tr>
<td>0.53</td>
<td>497</td>
</tr>
<tr>
<td>0.54</td>
<td>497</td>
</tr>
<tr>
<td>0.55</td>
<td>423</td>
</tr>
<tr>
<td>0.58</td>
<td>423</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q (MW/m²)</th>
<th>Tₛ (℃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.59</td>
<td>630</td>
</tr>
<tr>
<td>1.47</td>
<td>604</td>
</tr>
<tr>
<td>1.55</td>
<td>870</td>
</tr>
<tr>
<td>1.78</td>
<td>690</td>
</tr>
<tr>
<td>1.92</td>
<td>685</td>
</tr>
<tr>
<td>1.95</td>
<td>808</td>
</tr>
<tr>
<td>2.28</td>
<td>686</td>
</tr>
<tr>
<td>2.39</td>
<td>930</td>
</tr>
<tr>
<td>2.41</td>
<td>890</td>
</tr>
</tbody>
</table>

**Westerly Granite**

<table>
<thead>
<tr>
<th>Q (MW/m²)</th>
<th>Tₛ (℃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23</td>
<td>457</td>
</tr>
<tr>
<td>0.40</td>
<td>524</td>
</tr>
<tr>
<td>0.45</td>
<td>457</td>
</tr>
<tr>
<td>0.52</td>
<td>423</td>
</tr>
<tr>
<td>0.56</td>
<td>423</td>
</tr>
<tr>
<td>1.07</td>
<td>442</td>
</tr>
<tr>
<td>1.34</td>
<td>645</td>
</tr>
<tr>
<td>1.48</td>
<td>598</td>
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<tr>
<td>1.54</td>
<td>646</td>
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<td>1.64</td>
<td>645</td>
</tr>
<tr>
<td>1.77</td>
<td>710</td>
</tr>
<tr>
<td>1.95</td>
<td>767</td>
</tr>
<tr>
<td>2.14</td>
<td>483</td>
</tr>
<tr>
<td>2.50</td>
<td>551</td>
</tr>
</tbody>
</table>

**Sioux Quartzite**

<table>
<thead>
<tr>
<th>Q (MW/m²)</th>
<th>Tₛ (℃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>361</td>
</tr>
<tr>
<td>0.83</td>
<td>364</td>
</tr>
<tr>
<td>1.62</td>
<td>331</td>
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<td>1.86</td>
<td>350</td>
</tr>
<tr>
<td>2.13</td>
<td>364</td>
</tr>
<tr>
<td>2.16</td>
<td>364</td>
</tr>
<tr>
<td>2.29</td>
<td>350</td>
</tr>
</tbody>
</table>

\[ k_r = 3.0 \, \text{W/m K} \]
\[ \alpha_r = 0.9 \times 10^{-6} \, \text{m}^2/\text{s} \]
(estimated)

**Webatuck Dolomite**

<table>
<thead>
<tr>
<th>Q (MW/m²)</th>
<th>Tₛ (℃)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.24</td>
<td>504</td>
</tr>
<tr>
<td>1.32</td>
<td>457</td>
</tr>
<tr>
<td>2.79</td>
<td>598</td>
</tr>
<tr>
<td>2.86</td>
<td>551</td>
</tr>
<tr>
<td>3.92</td>
<td>645</td>
</tr>
</tbody>
</table>

\[ k_r = 3.33 \, \text{W/m K} \]
\[ \alpha_r = 0.93 \times 10^{-6} \, \text{m}^2/\text{s} \]
(Birch et al., 1942)
Included in Figures 4.13 and 4.14 are Weibull theory predictions using \( m \) and \( \sigma_0 \) parameters fit to the laser data along with the physical properties listed in Table 4.7. The predicted 90% probability envelopes contain most of the scatter. Specifically, spallation failure at the surface is governed by the stress field and the inherent flaw structure near the surface. The locations for measurements have been chosen arbitrarily in this study so that temperatures should be expected to form a distribution governed by the Weibull statistics rather than conform to a median condition. This is in contrast to the relatively small amount of scatter in the flame-jet stagnation-point temperatures reported in this study where the temperatures are area-weighted averages in the stagnation region of the flow-field.

Table 4.7 Weibull Parameters Fit to Laser Spallation Data and Physical Property Values used for Barre and Westerly Granite.

<table>
<thead>
<tr>
<th></th>
<th>Fitted Weibull Parameters</th>
<th>Physical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m = 7.7 )</td>
<td>( (\rho C_p)_r = 2.64 \times 10^6 ) (J/m³ K)</td>
</tr>
<tr>
<td></td>
<td>( \sigma_0 = 42 ) MPa-m³/7.7</td>
<td>( \beta_r = 11 \times 10^{-6} ) (°C⁻¹)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E = 58 ) (GPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \nu = 0.225 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha_r = 0.9 \times 10^{-6} ) (m²/s)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Fitted Weibull Parameters</th>
<th>Physical Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( m = 12.8 )</td>
<td>( (\rho C_p)_r = 2.64 \times 10^6 ) (J/m³ K)</td>
</tr>
<tr>
<td></td>
<td>( \sigma_0 = 51 ) MPa-m³/12.8</td>
<td>( \beta_r = 8 \times 10^{-6} ) (°C⁻¹)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( E = 47 ) (GPa)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \nu = 0.275 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \alpha_r = 0.9 \times 10^{-6} ) (m²/s)</td>
</tr>
</tbody>
</table>

(Physical properties reported in Rauenzahn, 1986)
Another factor contributing to the data scatter of the laser results is the relatively coarse time and temperature resolution capabilities of the IR scanner. At a heat flux of 3.0 MW/m², spallation times are typically on the order of 150 - 200 milliseconds and surface temperatures of about 900 °C are achieved. However, the infrared scanner samples at a rate of once every 50 milliseconds and has a temperature resolution capability (for the range required at this heat flux) of about 90 °C. The consequences of this relatively coarse discretization of the temperature history have been investigated by successively removing single data points from some of the transient temperature rise curves and re-calcultating the heat fluxes. Variations of up to 35 % in the calculated heat flux were found. Therefore, each of the data points presented in Figures 4.11-4.14 with heat flux calculated by the transient analysis (equations (4.1) and (4.2)) contains an inherent ± 35 % uncertainty region with respect to both heat flux and temperature due solely to experimental procedures. This probably contributes substantially to the observed data scatter.

Sioux Quartzite and Webatuck Dolomite surface temperature data are also included in Table 4.6. Sioux Quartzite spalls at an average surface temperature of approximately 355 °C and has no measurable dependence on applied heat flux over the range tested (0.7 - 2.3 MW/m²). Sioux Quartzite is approximately 93 % pure crystalline quartz and is fine grained (0.2 - 0.5 mm diameter). Therefore, if the distribution of flaws capable of initiating spallation is related to composition and grain size, the Weibul-based theory described in Chapter 3 dictates that
spallation temperatures should be relatively independent of heat flux. Unfortunately, Weibull parameters obtained from mechanical testing procedures are not available for Sioux Quartzite so comparison of experimental surface temperature data with predictions made using equation (3.11) is not possible. In the future, Weibull strength parameters for Sioux Quartzite should be experimentally determined.

The spallation surface temperatures of Webatuck Dolomite vary from 457 °C to 645 °C for heat fluxes ranging from 1.24 MW/m² to 3.92 MW/m². However, it is unclear whether true spallation was occurring during these tests since rock chips were not observed to violently eject from the surface, as they characteristically do during spallation with granite and quartzite. Instead, the rock surface appeared to disintegrate and rock particles dropped off the surface forming a pile a hot material at the base of the sample. The observed behavior indicates that the Dolomite, which is approximately 80 % MgCO₃, may have been reacting to form MgO and CO₂ through a calcination type of mechanism. However, no independent analysis of the residual material has been conducted to confirm this possibility. Since mechanical Weibull parameters are not available for estimation of surface temperatures using equation (3.11), and since the present study was aimed toward drilling in crystalline rocks, no further investigation of Dolomite was performed.
4.5.3 Global Onset Temperatures

The global onset temperature is defined as the temperature of the rock surface when the first spall is removed from any point on the surface. Values of spallation global onset temperature are seen in Figure 4.15 to be relatively insensitive to the applied heat flux level. Furthermore, most of the measured values are within about 30% of those predicted using mechanical Weibull parameters and those obtained experimentally by Rauenzahn. However, above about 1.0 MW/m², global spallation onset temperatures are significantly lower than continuous flame-jet temperatures and local laser spallation temperatures. This is again explained by recognizing the stochastic nature of spallation. Since the global onset temperature is that at which the first spall is ejected from any point on the surface, it represents the low temperature limit of the statistical distribution. Values of surface temperature at random, or at least arbitrarily selected, locations are more useful in determining average surface conditions during spallation drilling.

4.5.4 Experimental Stanton Numbers

The value of the surface temperature strongly affects calculated values of experimental Stanton numbers because $T_s$ appears both in the numerator and the denominator of equation (2.8):

$$St_{exp} = \frac{(\rho C_p)_r U_d (T_s - T_{r0})}{(\rho C_p U)_{jet} (T_{jet} - T_s)} \quad (2.8)$$

For example, a temperature change from 700 K to 1000 K with a jet
Figure 4.15 Global spallation onset temperature versus heat flux.

Assuming Physical Properties are Those at 573 K.

Legend:
- = Laser Induced Spallation -- Barre Granite (this study).
- = Laser Induced Spallation -- Westerly Granite (this study).
○ = Welding Torch Spallation -- Barre Granite (Rauenzahn, 1986).
□ = Welding Torch Spallation -- Westerly Granite (Rauenzahn, 1986).

- = Weibull Theory -- Parameters from Mechanical Tests.
- - - = Weibull Theory -- 90% Probability Envelope.
temperature of 2830 K causes more than a two-fold increase in Stanton number if all other variables remain constant.

The newly calculated Stanton numbers obtained using values for the penetration rate obtained from Rauenzahn's field drilling experiments and surface temperatures estimated in the present study are plotted along with old Stanton numbers in Figure 4.16. Fitted values of m and \( \sigma_0 \), along with physical property values listed in Table 4.7 are used for temperature determination.

The results shown in Figure 4.16 for non-dimensional SOD values below 26 are of unknown accuracy because the heat fluxes and surface temperatures predicted by equation (3.11) are higher than any that were measurable with the infrared scanner. Surface temperatures calculated for SOD values of 22, 24, and 26 are close to the rock melting temperature (\( \approx 1050 \) °C) and may lead to higher calculated Stanton numbers than are actually achieved.
Figure 4.16 Comparison of curves of Stanton number versus SOD using Weibull parameters obtained in the present study and by mechanical testing procedures.
4.6 Summary

The results displayed in Figure 4.16 show that experimental Stanton numbers for Rauenzahn's field drilling experiments are substantially closer to his simulation results than those originally reported (see Figure 2.8). Unfortunately, however, this improvement does not necessarily substantiate the use of a Weibull-based theory to describe the rock failure mechanism that occurs during thermal spallation. This point is illustrated by considering a possible alternative mechanism to account for the discrepancy between results obtained using mechanical Weibull parameters and those obtained during thermal spallation.

The sequence of events leading to failure by the alternative mechanism shown in Figure 4.17 is as follows:

1) crack extension and alignment with the compressive stress field,
2) partial buckling,
3) "overheating" of the spall during the time taken while competition between stress build-up, due to thermal expansion of the "spall", and stress relief, due to deformations in adjacent "soft" materials, occurs, and
4) spall ejection.

Mechanical Weibull parameters are expected to accurately describe the statistics of steps 1 and 2, but the overheating is a higher-order phenomena that becomes important at high heat fluxes and is not included
Figure 4.17 Illustration of possible mechanism for "overheating" at high heat fluxes.
in mechanical testing procedures used to validate the Weibull theory
description of rock failure.

The above mechanism is consistent with the few values of spallation
temperature obtained in this study for Quartzite (Table 4.6) where it
is observed that onset and steady state spallation temperatures are
identical. Quartzite is approximately 93% pure quartz so no
appreciable softening, or overheating, would be expected at the measured
surface temperatures. Unfortunately, mechanical Weibull parameters for
Sioux Quartzite have not been found in the literature for comparison.

Future work should be directed at increasing the data base for
granites and other spallable rocks. Laser heating tests like those
performed in this study should be used to investigate the statistical
spread of spallation temperatures at a given heat flux. However, high
speed IR scanning equipment with a broader temperature measurement
range, and higher precision, will be necessary in order to perform
meaningful analysis at heat fluxes in excess of about 2 MW/m².

Further study of thermal spallation at low heat fluxes will
eliminate potential "overheating" effects described above. This could
be performed using a subsonic jet, such as a welding torch, or a large
diameter laser. Some researchers theorize that "nonspallable" rocks
such as limestone could be spalled if the surface temperatures were
limited to below a point at which ductile, instead of brittle, failure
occurs in these rocks (Potter, 1988). Low heat flux spallation using
lasers should be used as a method to provide insight into this potential phenomenon. The use of lasers will decrease the coupling that occurs between the heat transfer mechanism and the rock mechanics during flame-jet thermal spallation.

The flow field simulation model developed in this study is described in the following chapter. For the purposes of this modeling, the surface temperature is assumed to be accurately described by equation (3.11) which is based upon Weibull theory. The physical properties and Weibull parameters, $m$ and $\sigma_0$, used are the averages of Barre and Westerly granite values that were listed in Table 4.7. These values should provide reasonable surface temperature estimates for the simulation regardless of whether Weibull theory is strictly a correct description of the rock failure criteria, since the Weibull parameters have been fitted to match experimental data obtained in this study.
5 Prediction of Thermal Spallation Drilling Performance

Characteristics: Model Development

5.1 Introduction

5.1.1 Background and Motivation

Prediction of thermal spallation drilling performance characteristics requires knowledge of the spatial distribution of rock surface temperature and heat flux. A quantitative relationship between the steady-state spallation surface temperature and applied heat flux was developed by using the Weibull-based statistical method (Rauenzahn (1986); Dey (1984)). Weibull parameters were fit to temperature/heat flux data obtained during small-scale thermal spallation experiments (Chapter 4). Determination of the heat flux requires solution of the mass, momentum, and energy conservation equations for the flowing gas stream that impinges on the rock surface.

Rauenzahn (1986) predicted Stanton numbers that were three to four times those that he had calculated from experimental drilling rates. In the present study, approximately one-half of this discrepancy has been attributed to experimental error (see Chapter 4). Additional sources of error are dealt with by improving the prediction of the transport phenomena.

The purpose of this chapter is to describe the modeling and the solution method for the conservation equations that were used to
calculate penetration rates and hole diameters during flame-jet induced thermal spallation drilling. Most of the flow-field modeling is similar or identical to that used by Rauenzahn (1986). However, the numerical method for solution of the governing equations, the grid generation procedure, and the rock boundary movement algorithm are different. Table 5.1 compares the important features of the approach taken in this study to that used by Rauenzahn (1986). The various entries in Table 5.1 are described in detail in this chapter.

**Table 5.1** Comparison of methodology employed in the present study with that of Rauenzahn (1986).

<table>
<thead>
<tr>
<th>Feature</th>
<th>Rauenzahn (1986)</th>
<th>This study</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordinate System</td>
<td>Axisymmetric</td>
<td>Axisymmetric</td>
</tr>
<tr>
<td>Turbulence Model</td>
<td>k-ε with wall functions</td>
<td>k-ε with wall functions</td>
</tr>
<tr>
<td>Spatial Discretization</td>
<td>Upwind differencing (accuracy $\propto \Delta x$)</td>
<td>Centered differencing (accuracy $\propto \Delta x^2$)</td>
</tr>
<tr>
<td>Temporal Discretization</td>
<td>Forward time</td>
<td>4-stage Runge-Kutta</td>
</tr>
<tr>
<td>Artificial Dissipation</td>
<td>None (inherent 2nd difference)</td>
<td>2nd and 4th difference blend</td>
</tr>
<tr>
<td>Computational Grid</td>
<td>Aligned with coordinate axes</td>
<td>Fit to hole shape</td>
</tr>
<tr>
<td>Hole Shape Adjustment</td>
<td>Based on local boundary angle</td>
<td>Based on local boundary angle and penetration rate</td>
</tr>
</tbody>
</table>
5.1.2 General Description of Solution Method

Prediction of the transport phenomena is accomplished by solving the governing conservation equations for mass, momentum, and energy subject to the appropriate boundary conditions and constitutive relationships. The equations are approximated on a finite grid of points. Estimated values of the surface heat flux are used to move the rock boundary until the hole shape conforms at every point to the consistency criterion given by equation (2.5):

\[
V_{dr} = \frac{U_{dr}}{\cos(\theta)} \quad (2.5)
\]

where \( V_{dr} \) = forward advance rate of drill head (m/s), and \( \theta \) = angle of the tangent of the rock boundary to the horizontal plane.

The modeling developed in this chapter assumes a single-phase flow of an ideal gas with a constant heat capacity and no chemical reactions. A single-phase flow is assumed throughout the flow-field since the high-speed wall-jet along the rock surface is expected to rapidly remove most of the solid particles. However, in the near wall region, the local volume fraction of solids in the flow is expected to be higher than the overall average volume fraction. Therefore, the influence on the transport phenomena is expected to be greatest near the wall as relatively cold particles ejected from the rock surface interact with the hot, turbulent gases in the boundary layer. In Chapter 6, a simple method aimed at assessing the near wall influence of spall entrainment by the fluid is presented.
A constant heat capacity is used in the "basic solver" developed in this chapter so that comparisons made with Rauenzahn's simulation results, which assumed \( C_p \) was constant, will highlight differences due to changes in the numerical methods used. In Chapter 6, results generated in the present study using a variable heat capacity are presented.

The assumption that no chemical reactions occur is made in order to simplify the computations. If reactions were included, important chemical species and reactions would have to be identified, and a complete set of species conservation equations would be necessary. The additional computational requirements and modeling efforts necessary are beyond the budget and scope of the present study. Nevertheless, future work should be directed at assessing the influence of reactions on the transport phenomena occurring at the rock surface during thermal spallation drilling.

5.2 Governing Equations and Boundary Conditions

5.2.1 Conservation Equations

The equations expressing conservation of mass, momentum, and energy for the flow of a continuous, single-phase fluid are given in vector form as (Anderson et al., 1984):
continuity:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{v} \rho = 0 \]  \hspace{1cm} (5.1)

momentum:

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \rho \mathbf{f} + \nabla \cdot \mathbf{\Pi}_{ij} \]  \hspace{1cm} (5.2)

energy:

\[ \frac{\partial (\rho E)}{\partial t} + \nabla \cdot (\rho E) = \frac{\partial Q}{\partial t} - \nabla \cdot \mathbf{q} + \rho \mathbf{f} \cdot \mathbf{v} + \nabla \cdot \left( \mathbf{\Pi}_{ij} \cdot \mathbf{v} \right) \]  \hspace{1cm} (5.3)

where,

\( \rho = \) fluid density (kg/m²),
\( \mathbf{v} = \) velocity vector \((u_1, u_2, u_3)\),
\( \mathbf{f} = \) body force vector, including gravity \((f_1, f_2, f_3)\),
\( E = \) total energy of the fluid (internal \(I\) + kinetic \(KE\) + potential \(PE\) +...) \((J/kg)\),
\( \mathbf{q} = \) heat flux vector \((q_1, q_2, q_3)\),
\( Q = \) external heat addition, and
\( \mathbf{\Pi}_{ij} = \) fluid stress tensor.

The thermodynamic equation of state is given by the ideal gas law in terms of absolute temperature as:

\[ P = \rho RT \]  \hspace{1cm} (5.4).

If potential energy effects are negligible, then the ideal gas law is also expressible as:

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\[ P = (\gamma - 1)(\rho E - \rho/2 \mathbf{V} \cdot \mathbf{V}) \tag{5.5} \]

where the internal energy (\(I\)) reference state is \(I = 0\) at \(T = 0\) K and

\(P\) = pressure (Pa),
\n\(\gamma\) = ratio of constant pressure to constant volume heat capacity
\n\((C_p/C_v \approx 1.26\) for most hydrocarbon/oxygen combustion product mixtures),

\(R\) = specific ideal gas constant (\(\approx 286\) J/kg K for propane/oxygen mixtures, and
\n\(T\) = temperature (K).

Fourier’s law is used to relate the components of the heat flux vector to the local temperature gradient:

\[ q_i = -\lambda \frac{\partial T}{\partial x_i} \quad (i = 1,2,3) \tag{5.6} \]

The thermal conductivity of the fluid (\(\lambda\)) is assumed to be related to the fluid dynamic viscosity (\(\mu\)) by a constant value of the Prandtl number (\(Pr\)):

\[ \lambda = \frac{\mu C_p}{Pr} \tag{5.7} \]

where \(C_p\) = gas heat capacity (J/kg K), and \(Pr \approx 0.71\) for most gases.

Finally, the stress tensor for a Newtonian fluid is written in indicial notation as:

\[ \Pi_{i,j} = -\rho \delta_{i,j} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{i,j} \frac{\partial u_k}{\partial x_k} \tag{5.8} \]

where

\[ u_i = i\text{'th component of the velocity vector, and} \]

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\( \mu = \text{dynamic fluid viscosity, and} \)
\[ \delta_{ij} = 0 \ (i \neq j); \ 1 \ (i = j). \]

Stokes' hypothesis (Schlichting, 1979) was used to relate the dynamic and bulk viscosities in deriving equation (5.8).

The effects of body forces, such as gravity, and external heat addition, for example radiation heat transfer, have been neglected in this model. Furthermore, a major simplification of the governing equations is made by assuming that the flow-field can be modeled as axisymmetric. Thus, equations (5.1) - (5.3) are rewritten in axisymmetric vector form as:

\[
\frac{\partial \mathbf{U}}{\partial t} + \frac{1}{r} \frac{\partial \mathbf{F}}{\partial r} + \frac{\partial \mathbf{G}}{\partial z} = -\frac{1}{r} \frac{\partial \mathbf{B}}{\partial r} + \frac{\partial \mathbf{S}}{\partial z} + \mathbf{M} \quad (5.9)
\]

where

\[
\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix} ; \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho u v \\ \rho u (E + P) \end{bmatrix} ; \quad \mathbf{G} = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + P \\ \rho v (E + P) \end{bmatrix} ;
\]

\[
\mathbf{R} = \frac{1}{Re_r} \begin{bmatrix} 0 \\ \tau_{rr} \\ \tau_{rz} \\ \rho u \tau_{rr} + \nu \tau_{rz} - q_r \end{bmatrix} ;
\]

\[
\mathbf{S} = \frac{1}{Re_r} \begin{bmatrix} 0 \\ \tau_{rz} \\ \tau_{zz} \\ \rho u \tau_{rz} + \nu \tau_{zz} - q_z \end{bmatrix} ; \quad \mathbf{M} = \frac{1}{Re_r} \begin{bmatrix} 0 \\ \sigma_{\theta \theta} / r \end{bmatrix} ;
\]

\[
\sigma_{\theta \theta} = P \left[ 2 \mu \frac{u}{r} - \frac{2}{3} \mu \text{div}(\mathbf{V}) \right];
\]

\[
\text{div}(\mathbf{V}) = \frac{1}{r} \frac{\partial ru}{\partial r} + \frac{\partial v}{\partial z};
\]

\[
Re_r = U_{\text{jet}} R_{\text{noz}} / \nu_{\text{jet}},
\]

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\( R_{noz} = \) nozzle outlet radius,
\( M_{jet} = \) jet Mach number = \( U_{jet}/a_{jet} \),
\( a_{jet} = \) speed of sound at nozzle outlet,
\( \tau_{i,j} = \) non-dimensional stress tensor component \( i,j \),
\( q^{*}_r = \) non-dimensional radial heat flux = \( q_r/\rho U^2_{jet}(1/(\gamma-1)M_{jet}^2Re_r) \),
\( q^{*}_z = \) non-dimensional axial heat flux = \( q_z/\rho U^2_{jet}(1/(\gamma-1)M_{jet}^2Re_r) \),
\( u,v = \) non-dimensional radial and axial velocity components (\( u_r/U_{jet} \) and \( u_z/U_{jet} \)), and
\( r,z = \) non-dimensional radial and axial coordinates (\( x_1/R_{noz} \) and \( x_2/R_{noz} \)).

Note that in equations (5.9), the auxiliary relationships presented above, and throughout the computer code written in the present study, non dimensional variables are used. Velocities are non-dimensionalized by dividing by \( U_{jet} \), lengths are divided by \( R_{noz} \), \( \rho E \) and \( P \) are divided by \( \rho_{jet} U^2_{jet} \), and \( \mu \) is divided by \( \mu_{jet} \).

5.2.2 Mean Flow Equations

In principle, equations (5.9) could be used directly to solve for the turbulent flow-field that occurs during spallation drilling. However, a numerical solution is not feasible using current modeling techniques because of the wide range of time and length scales that must be taken into account (White, 1974). A more practical method is to consider turbulence as being made up of randomly fluctuating flow components superimposed on a mean flow field. To this end, density, velocities, pressure, and temperature are expressed as:

\[ \rho = \bar{\rho} + \rho' \]
\[ u = \bar{u} + u' \]
\[ v = \bar{v} + v' \]
\[ p = \bar{p} + p' \]
\[ T = \bar{T} + t' \]

where the overbar (\( \bar{\cdot} \)) denotes ensemble averaging, and the prime (\( ' \)) denotes a fluctuating value. Unless otherwise noted, the overbar is not used in the remainder of this thesis, but all quantities except those primed represent temporal averages.

The resulting set of governing equations are called the "Reynolds averaged" conservation equations (Reynolds, 1841), and contain additional components in the stress tensor known as the "Reynolds stresses":

\[
\Pi_{i,j} = -p\delta_{i,j} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu \delta_{i,j} \frac{\partial u_k}{\partial x_k} - \rho u'_i u'_j \tag{5.10}
\]

"Reynolds stresses"

where \( u'_i \) = the fluctuating component of velocity in direction \( i \), and \( u_i \) = the mean component of velocity in direction \( i \). The heat flux vector has an added term due to the splitting of the temperature into mean and fluctuating parts:

\[
q_i = -\lambda \frac{\partial T}{\partial x_i} + \rho c_p \bar{t}' u'_i \tag{5.11}
\]

The method of determining the Reynolds stresses and turbulent heat fluxes is described next.
5.2.3 Closure of the Mean Flow Equations

Calculation of the stress tensor and heat flux vector requires evaluation of the added terms due to turbulence on the right hand sides of equations (5.10) and (5.11). The most widely used method, and the one adopted for this work, is to relate the turbulent stresses and heat fluxes to mean gradients in the flow field through turbulent transport coefficients (Boussinesq, 1877). This is stated mathematically as:

\[-\rho u_i u_j = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{\delta_{ij}}{3} \left[ 2 \mu_t \frac{\partial u_k}{\partial x_k} + \rho u_k u_k \right] \] (5.12)

\[-\rho C_p \, t' u_i' = \lambda_t \frac{\partial T}{\partial x_i} \] (5.13)

where \( \mu_t \) = turbulent "viscosity", and \( \lambda_t \) = turbulent "thermal conductivity".

The heat and momentum transport coefficients (\( \lambda_t \) and \( \mu_t \)) bear resemblance to the molecular transport coefficients in equations (5.6) and (5.8). However, unlike those for laminar flow, the turbulent thermal conductivity and viscosity depend upon both local mean conditions and the flow history. Therefore, "turbulence models" are employed to relate the turbulent transport coefficients to known average quantities.

In this study the flow-field is conceptually divided into an inner (near wall) region, where the behavior is assumed to be similar to that in a pure-fluid, turbulent flat-plate boundary layer, and an outer (far-
field) region, where simple models based upon well-known velocity and temperature profiles cannot be used. In the modeling described below, the effect of entrained solids on the turbulence structure is neglected on the basis that accurate methods for treating this situation have not yet been developed.

5.2.4 Far-Field Turbulence Model

In order to obtain solutions of the Reynolds averaged conservation equations, it is necessary to have a method for calculating the turbulent, or "eddy" viscosity throughout the flow field. By dimensional arguments, and by analogy with the kinetic-molecular gas theory, the turbulent viscosity can be modeled as:

\[ \mu_t = \rho v_t l \]  \hspace{1cm} (5.14)

where \( v_t \) and \( l \) are characteristic turbulent velocity and length scales. The distinctions between most practical turbulence models arise from the methods employed to predict \( v_t \) and \( l \). Three classes of widely used turbulence models are described briefly in this section: "zero-equation", "one-equation", and "two-equation" models.

The well known zero-equation model of Prandtl (1925) reduces the task of solving equation (5.14) to the prediction of a "mixing-length" \( (l_m) \) through an algebraic, empirical relationship. He set \( l = l_m \) in equation (5.14) and related the turbulent velocity \( (v_t) \) to the mixing length according to:

\[ v_t = l_m |du_t/dy| \]  \hspace{1cm} (5.15)
where \( u_t \) is the velocity component parallel to the wall, and \( y \) is a coordinate measured in a direction normal to the wall. In a region sufficiently close to the wall, the length over which a fluid particle can maintain its identity is limited by the distance to the wall \( (y) \). Therefore, Prandtl reasoned that the mixing length \( (l_m) \) is proportional to this distance, giving:

\[
l_m = \kappa y = 0.41 y \tag{5.16}
\]

where \( \kappa \) is a universal constant that has been determined experimentally by numerous researchers to equal approximately 0.41 (Karman (1930); Laufer (1951)).

A comparison between calculated and measured mixing lengths, shown in Figure 5.1, illustrates the good agreement for \( y \) values less than about 5% of the total boundary layer thickness \( (\delta) \) and outside of the so-called "viscous sublayer". Figure 5.1 also illustrates the division of a turbulent boundary layer into two regions: the "wake", where the velocity and temperature profiles resemble those occurring in the wake that forms behind a stationary bluff body immersed in a moving fluid, and the "log-law" region, where equation (5.16) applies. There is an additional region that is still closer to the wall known as the viscous sublayer, where viscous, rather than inertial (turbulent) stresses dominate. No data are shown for this region and calculations performed by Rauenzahn (1986) indicate that during spallation drilling the viscous sublayer does not appear because the roughness of the rock surface protrudes well into the fully turbulent log-law region of the boundary layer.
Figure 5.1 Illustration of the good fit between Prandtl's mixing length expression (Eq.'n (5.16)) and experimental data for flow over a flat plate (Anderson et al., 1975).
Prandtl's model gives results accurate to within 1-2 % for boundary layer flows over flat plates and in pipes. A variation of Prandtl's model is used in the present study for prediction of the wall heat flux and shear stress, therefore further discussion is presented later in Section 5.2.5.

The most important shortcoming of the algebraic, zero-equation models is the need to specify the length scale $l$ at every point in the flow field. In most of these models the length scale is specified in terms of local flow-field characteristics, such as distance from the wall for Prandtl's model, without taking into account any upstream flow field "history" effects on the turbulence structure. Although for certain types of flows the local nature of the turbulence is an adequate approximation, many complex flows of engineering interest, particularly those with pressure gradients and/or separation, are affected by upstream conditions. Furthermore, even for those flows that are governed by local flow field characteristics, algebraic models for specifying the length scale have been experimentally determined for only the most common and simple cases.

In an attempt to incorporate flow-field "history" effects into the turbulence modeling, Prandtl (1945) and Kolmogorov (1942) proposed that the turbulent velocity should be approximated as being proportional to the square root of the turbulent kinetic energy per unit mass given by:
\[ k = 1/2 \left( u_i' u_i' \right) \]  \hspace{1cm} (5.17)

and \[ v_t \propto k^{1/2} \]  \hspace{1cm} (5.18)

A transport equation, analogous to the conservation equations expressed by equation (5.9), can be derived from the momentum conservation equations so that \( k \) can be solved for throughout the flow field and upstream effects are accounted for by the convective and diffusive terms. The specific details of the \( k \)-transport equation are described below during the discussion of two-equation turbulence models.

Unfortunately, the length scale in equation (5.14) again remains to be specified by auxiliary, empirically determined relationships when the one-equation approach is employed. Furthermore, the \( k \)-transport equation that is usually employed has several terms that must be modeled based on the small amounts of experimental data that are available for complex flows of interest. Therefore, one-equation models are less general, and computationally more expensive than the already restrictive zero-equation models. Moreover, even where experimental data have been available, the predictions that have been made with one-equation models have not been consistently superior to those made with zero-equation models (Anderson et al., 1984).

In order to correct these deficiencies, two equation models have been developed to calculate the value of \( l \) throughout the flow field using an additional PDE derived from the momentum equations, thereby incorporating "history" into values of both \( v_t \) and \( l \). Although a transport equation can be developed directly for \( l \), the most successful
approaches have used modeled forms of an equation for the isotropic
dissipation rate of turbulent kinetic energy ($\epsilon$) (Harlow and Nakayama,
1968; Jones and Launder, 1972; Launder and Spalding, 1974). The length
scale (1) is related to $\epsilon$ by scaling arguments:

$$l = C_\mu k^{3/2}/\epsilon$$  \hspace{1cm} (5.19)

where $C_\mu$ is a universal proportionality constant. Combination of
equations (5.14), (5.18), and (5.19) results in the expression used for
determination of the turbulent viscosity ($\mu_t$) in these so-called $k-\epsilon$
models

$$\mu_t = \frac{\rho C_\mu k^2}{\epsilon}$$  \hspace{1cm} (5.20)

The value of $C_\mu$ has been experimentally determined to equal 0.09 (Jones
and Launder, 1972).

Hinze (1975) derives a transport equation for the turbulent kinetic
energy ($k$) of an incompressible fluid from the momentum conservation
equations, giving:

$$\frac{\partial k}{\partial t} + u \frac{\partial k}{\partial x_k} + u_i' u_k' \frac{\partial u_i}{\partial x_k} = - \frac{\partial}{\partial x_k} \left( \frac{p'}{\rho} + k \right) u_k' + \nu \frac{\partial^2 k}{\partial x_1 \partial x_1} - \nu \left( \frac{\partial u_i'}{\partial x_1} \frac{\partial u_i'}{\partial x_1} \right)$$  \hspace{1cm} (I)  \hspace{1cm} (II)  \hspace{1cm} (III)  \hspace{1cm} (IV)  \hspace{1cm} (V)  \hspace{1cm} (VI)

(5.21)

where terms (I) - (VI) are interpreted as follows:

(I) time rate of change of $k$,

(II) advection of $k$,

(III) production of $k$ (henceforth denoted $P_k$),
(IV) turbulent diffusion of \( k \),

(V) viscous diffusion of \( k \), and

(VI) dissipation of \( k \).

Terms (III), (IV), and (VI) contain unknown quantities that are modeled as functions of known average flow properties. Term (III) is simply modeled by replacing the Reynolds stresses with the corresponding terms from the Boussinesq approximation (eq. 5.12). The second term (IV) is modeled by a gradient-diffusion expression:

\[
\left( \frac{\rho'}{\rho} + k \right) u_k' = -\nu_k \frac{\partial k}{\partial x_k} \tag{5.22}
\]

where \( \nu_k \) = turbulent kinetic energy diffusion coefficient. The turbulent kinetic energy diffusion coefficient, \( \nu_k \), is assumed to be proportional to the turbulent viscosity.

As mentioned previously, the dissipation of turbulent kinetic energy (term VI) is solved for by deriving another transport equation from the momentum conservation equations. Terms resembling (III), (IV), and (V) appear and are modeled as described above for the corresponding terms in equation (5.21). The final modeled turbulent kinetic energy and dissipation rate equations used in this study are the non-dimensionalized forms of those given by Viegas and Rubesin (1983):

\[
\frac{\partial \rho k}{\partial t} = \left( -\frac{1}{r} \frac{\partial (\rho u_k)}{\partial r} + \frac{\partial (\rho v_k)}{\partial z} \right) + \left( \frac{1}{r} \frac{\partial (\mu_k \partial k/\partial r)}{\partial r} + \frac{\partial (\mu_k \partial k/\partial z)}{\partial z} \right) \frac{1}{Re_r} \\
+ (Re_r/Re_k^2) p_k - N_\epsilon \rho \epsilon \tag{5.23}
\]
\[
\frac{\partial \rho \varepsilon}{\partial t} = -\frac{1}{r} \frac{\partial (\rho \varepsilon)}{\partial r} + \frac{\partial (\rho \varepsilon)}{\partial z} + \left( \frac{1}{r} \frac{\partial (\mu_\varepsilon \partial \varepsilon / \partial r)}{\partial r} + \frac{\partial (\mu_\varepsilon \partial \varepsilon / \partial z)}{\partial z} \right) \\
+ \frac{\varepsilon}{k} \left(C_1 ((\text{Re}_r/\text{Re}_k^2) P_\varepsilon) - C_2 N_\varepsilon (\rho \varepsilon) \right) \\
\text{(5.24)}
\]

where
\[
P_k = \frac{\mu_t}{\mu_t} \left[ \tau_{\text{rr}} \left( \frac{\partial u}{\partial r} \right) + \tau_{\text{rz}} \left( \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \right) + \tau_{\text{zz}} \left( \frac{\partial v}{\partial z} \right) - \frac{\tau_{\theta \theta} u}{r} \right] \\
- \frac{2}{3} \rho k (\text{Re}_k^2/\text{Re}_r) \text{div}(V), \tag{5.25a}
\]

\[
P_\varepsilon = \frac{\mu_t}{\mu_t} \left[ C_3 \tau_{\text{rr}} \left( \frac{\partial u}{\partial r} \right) + \tau_{\text{rz}} \left( \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \right) + C_3 \tau_{\text{zz}} \left( \frac{\partial v}{\partial z} \right) - C_3 \frac{\tau_{\theta \theta} u}{r} \right] \\
- \frac{2}{3} C_3 \rho k (\text{Re}_k^2/\text{Re}_r) \text{div}(V), \tag{5.25b}
\]

\[
\text{Re}_k = k^{1/2} \text{j}_\text{jet} \text{R}_{\text{noz}} / \nu_{\text{jet}},
\]

\[
N_\varepsilon = \varepsilon_{\text{jet}} R_{\text{noz}} / k_{\text{jet}} U_{\text{jet}} = C_\mu / l_{\text{jet}}^* (\text{Re}_k / \text{Re}_r),
\]

\[
l_{\text{jet}}^* = l_{\text{jet}} / R_{\text{noz}},
\]

\[
l_{\text{jet}} = \text{turbulence length scale at inlet conditions},
\]

\[
\mu_t = \mu_k + \mu,
\]

\[
\mu_k, \mu_\varepsilon = \text{turbulent diffusion coefficients for k, and } \varepsilon,
\]

\[
C_1 = 1.55, C_2 = 2.0, C_3 = 1.0, \text{ and } k \text{ and } \varepsilon \text{ have been non-dimensionalized by their inlet values, } k_{\text{jet}} \text{ and } \varepsilon_{\text{jet}} \text{ respectively.}
\]

These equations must be solved throughout the designated "far-field" domain in order to be able to ultimately calculate the wall-region heat transfer.
5.2.5 Wall-Region Turbulence Model: Wall Functions

The high temperature, high speed gas that flows along the solid-rock boundary results in large velocity and temperature gradients. Accurate representation of the high gradients would require a large number of computational grid points if the k-ε model were used in this region. Furthermore, the k-ε model described in Section 5.2.4 cannot be directly applied to wall flows. Instead, ad-hoc terms must be added to each transport equation to allow the correct variations in k and ԑ to occur (Jones and Launder, 1972; Lam and Bremhorst, 1981; Chien, 1982). Therefore, a modified form of the semi-theoretical Prandtl (1925) mixing length model described in the previous section is used in the near-wall region for predicting velocity and temperature profiles.

From equations (5.14) and (5.15), Prandtl's final expression for the turbulent "eddy viscosity" is:

\[ \mu_t = \rho l_n^2 \left| \frac{\partial u_t}{\partial y} \right| \]  

(5.26)

where \( l_n = \kappa y \). However, during spallation drilling, added drag beyond that due directly to turbulent motions is caused by pressure forces acting on the sides of roughness elements that protrude from the rock surface into the turbulent boundary layer. This situation is modeled by assuming that an added length scale (\( \delta y_0 \)) exists at the tips of the roughness giving (Kays and Crawford, 1980):
\[ l_m = \kappa (y + \delta y_0) \]  
(5.27)

where \( y \) is now measured from the tips of the roughness elements.

Pimenta et al. (1975) conducted an experimental study of rough-walled boundary layers and found that \( \delta y_0 \) is proportional to the rms roughness height. The proportionality constant is 0.031.

Solution of the conservation equations for the special case when there is no pressure gradient, and no normal velocity component at the wall leads to the conclusion that a Couette flow exists, implying a constant shear stress across the boundary layer. Combining equations (5.12) and (5.27), neglecting the effects of molecular viscosity, and using standard boundary layer assumptions (Schlichting, 1979) leads to the following equation for the shear stress:

\[ \frac{\tau_w}{\rho} = \kappa^2 (y + \delta y_0)^2 \left( \frac{du_t}{dy} \right)^2 \]  
(5.28)

Defining "wall variables" as \( u_r = (\tau_w/\rho)^{1/2} \); \( u^+ = u_t/u_r \); \( y^+ = y \nu/u_r \), and re-writing equation (5.28) leads to:

\[ \left( \frac{\rho_w}{\rho} \right) = \kappa^2 (y^+ + \delta y_0^+)^2 \left( \frac{du^+}{dy^+} \right)^2 \]  
(5.29)

where "+" denotes scaling with wall variables, as described above. This expression is manipulated to yield:

\[ \left( \frac{\rho}{\rho_w} \right)^{1/2} \frac{du^+}{dy^+} = \frac{dy^+}{\kappa (y^+ + \delta y_0^+)} \]  
(5.30)
Figure 5.2 Illustration of the coupling between inner and outer region modeling.
Integration from the wall to an arbitrary point "p", illustrated in Figure 5.2, within the log-law region of the boundary layer leads to:

\[
\left(\frac{T_w}{\rho}\right)^{1/2} = \frac{u_p \theta}{1/\kappa \ln(32.6 y^+/h^*) + 1} \tag{5.31}
\]

where \(u_p\) is the tangential velocity at the point "p", \(h^+ = u_r/\nu\), \(\rho\) = r.m.s. surface roughness height, and

\[
\theta = \int_0^1 \left(\frac{\rho}{\rho_w}\right)^{1/2} d(u^+/u_{p^+}).
\]

Van Driest (1951) has calculated \(\theta\) as:

\[
\theta = A \left[ \sin^{-1} \left( \frac{2A^2 - B}{(B^2 + 4A^2)^{1/2}} \right) + \sin^{-1} \left( \frac{B}{(B^2 + 4A^2)^{1/2}} \right) \right] \tag{5.32}
\]

where

\[
A^2 = \frac{(\gamma - 1)M_a^2}{2(T_w/T)}
\]

\[
B = \frac{1 + (\gamma - 1)M_a^2}{(T_w/T)} - 1
\]

\(M_a = \) Mach number = \(u_p/a\), and
\(a = \) speed of sound.

Equation (5.31) is matched to the far-field turbulence model at point "p" as illustrated in Figure 5.2.. Experimental measurements in turbulent boundary layers formed near pipe walls and on flat plates prove that, if the pressure gradient is zero and no mass injection occurs, the production of turbulent kinetic energy \(P_k\) is approximately equal to the rate of dissipation \(\rho \epsilon\) (Klebanoff, 1954; Laufer, 1955). In this case, equation (5.23) leads to:

\[
\frac{\tau_w}{\rho} \frac{du_t}{dy} = \epsilon \tag{5.33}
\]
Combination of equations (5.12) and (5.20) leads to:

\[ \rho \varepsilon = \frac{\rho^2 C_\mu k^2}{\tau_w} \left( \frac{du_t}{dy} \right) \]  

(5.34)

Equation (5.33) and (5.34) are combined to yield:

\[ \frac{\tau_w}{\rho} = \frac{1}{2} C_\mu k \]  

(5.35)

Equation (5.35) is substituted into equation (5.31) giving the final expression for the wall stress:

\[ \tau_w = \frac{\rho_w u_p \Theta C_\mu^{1/4} \sqrt{k}}{1/\kappa \ln(32.6y^+/h^+ + 1)} \]  

(5.36)

Similar reasoning and calculations lead to an expression for the wall heat flux (Viegas et al., 1985; Rauenzahn, 1986; Amano and Jensen, 1982) presented here without derivation as:

\[ Q_w = \frac{(C_p (T_p - T_w) + 1/2 \ u^2) \rho_w C_\mu^{1/4} \sqrt{k} \Theta}{Pr_t/\kappa \ln(32.6y^+/h^+ + 1) + St_{rh}^{-1}} \]  

(5.37)

where

\[ St_{rh}^{-1} = \frac{(T_{rh} - T_w)}{(Q_w/(\rho_w C_p))^{1/2}} \left( \frac{\tau_w}{\rho_w} \right)^{1/2}, \]

\[ T_{rh} = \text{temperature at the outer edge of the surface roughness,} \]

\[ \kappa = \text{von Karman constant (0.41),} \]

\[ Pr_t = \text{turbulent Prandtl number (0.96),} \]

\[ u_r = (\tau_w/\rho_w)^{1/2}, \text{ and} \]

\[ St_{rh}^{-1} \text{ represents the non-dimensional temperature change from the wall temperature, } T_w, \text{ to the temperature at the edge of the roughness, } T_{rh}. \]

Dipprey and Sabersky (1963) developed a semi-empirical expression for \( St_{rh}^{-1} \) based on dimensional analysis and comparison with data:

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\[ S_{r,h}^{-1} = 5.19 \ h^*-20 \ \Pr^{-0.44} \]  \hspace{1cm} (5.38)

where \( C = 5.19 \). Figure 5.3 illustrates the excellent correlation of the data obtained by Dipprey and Sabersky (1963) using equation (5.38) from the upper portion of the transitionally rough into the fully rough region where \( h^* \) is greater than approximately 50. Note that the abscissa in Figure 5.3 is plotted as \( S_{r,h}^{-1}(\Pr^{-0.44}) \) in order to collapse the data for different Prandtl number fluids onto a single curve. The coefficient \( C \) is a function of the roughness pattern on the surface whereas the exponents in equation (5.38) are universally valid. Dipprey and Sabersky (1963) performed their experiments in tubes with close-packed "granular" type surface roughness with average roughness diameter-to-thickness ratios ranging from 20 to 200 and found that \( C \) was identical for all cases tested. Since spall liberation is responsible for producing the roughness that occurs on the rock surface during spallation drilling, the surface roughness aspect ratio should be approximately 10-20 (see Chapter 4). Therefore, the value of \( C \) used in this study was assumed to be the same as that found by Dipprey and Sabersky (1963).

However, the roughness pattern that exists on the rock surface during thermal spallation drilling is highly irregular. Observation of the surface as spallation occurs reveals that relatively large (1-2 cm) pieces of rock often protrude into the flow-field, probably beyond the boundary layer, for several seconds before being removed by the gas.
Figure 5.3 Inverse Stanton Number Function from Equation (5.38) versus Roughness Height Scaled in Wall Variables (Dipprey and Sabersky, 1963).
These adhered "spalls" are substantially larger than the rms roughness height on the surface and are not adequately accounted for by the current modeling. Future efforts should focus on understanding the effects of such a highly irregular roughness pattern on turbulent boundary layer transport phenomena.

5.2.6 Boundary Conditions

The boundary conditions that are used to solve the conservation equations (5.9) are depicted on Figure 5.4. The centerline boundary is assumed to be an axis of symmetry and therefore represents a zero flux condition. Solid walls, for example the spalling rock surface, are treated as no slip boundaries with zero pressure gradients normal to the surface. Temperature is specified on the rock surface from equation (3.11), whereas the drill housing was assumed to be adiabatic after several test runs had been performed and indicated that penetration rate predictions were unchanged by using adiabatic versus wall function boundary conditions. The turbulent kinetic energy is set equal to zero on solid surfaces, and the dissipation rate is specified at the computational node nearest to the wall by the following relationship (Launder and Spalding, 1974):

\[ \epsilon = \frac{C_{\mu} k^{3/2}}{y} \]  

(5.39)

This expression is derived by forcing the k-\( \epsilon \) model to match Prandtl mixing length theory in the log-law region.
**Figure 5.4** Illustration of boundary conditions used for thermal spallation drilling simulation.
At the nozzle outlet, the turbulent kinetic energy is specified as 5% of the mean flow kinetic energy and the turbulent length scale is specified as one-third of the nozzle radius. Preliminary simulation runs indicated that the effects of these parameters on the results are less than 1% when they are varied within the range of commonly reported nozzle turbulence parameters.

At the outlet from the cavity, zero streamwise gradients are assumed for all variables except pressure. Pressure is set from the user-defined under-expansion ratio \( (P_{\text{noz}}/P_{\text{amb}}) \). The under-expansion ratio used in this study equals 4.4, corresponding to the conditions during small-scale field drilling experiments performed by Rauenzahn (1986), and during full scale tests performed by Browning (1981).

5.3 Solution of Governing Equations

5.3.1 Fundamentals of Discretization Techniques

Equations (5.9), (5.23), and (5.24) have not been solved analytically for the boundary conditions and geometry that are being investigated in this study. Therefore, discretization techniques are adopted whereby the property conservation laws are expressed for small (but finite) volume elements that are formed by breaking up the flow-field domain into a grid of points. The net flux of mass, momentum, and energy into each volume element (or "cell") is approximated and numerical time integration is performed until all cells are at steady-state conditions (no accumulation).
Numerical methods are distinguished from one another by the particular procedures used for the cell flux calculations and time integration. Anderson et al. (1984) describe a large number of commonly used numerical methods. The most important factors governing the use of a particular method are accuracy and stability. Time accuracy is not important in this study because only the final steady-state is required and any suitable path can be used for time stepping. However, high spatial accuracy is desirable so that gradients and heat fluxes are well represented. A stable algorithm damps out transient disturbances introduced by round-off and truncation errors.

For most methods, including the one used here, spatial accuracy is governed by the size of the computational cell whereas stability is controlled by both the cell size and the time step size. Accuracy and stability are demonstrated below for a one-dimensional wave equation using central differencing. The characteristics of this example problem are similar to those for the equations and discretization method employed in this study.

**Accuracy**

The model wave equation is given by:

$$\frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} = 0 \quad (5.40)$$

where $a =$ wave velocity, $y =$ wave amplitude, $x =$ distance from the origin, and $t =$ time. Centered differencing is used and the distance between all node points (Figure 5.5) is assumed constant, resulting in

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\[ \begin{align*}
\Delta x & \quad \Delta x \\
\hline \\
i - 1 & \quad i & \quad i + 1 \\
\hline \\
\frac{\partial y}{\partial x} & \approx \frac{y_{i+1} - y_{i-1}}{2\Delta x}
\end{align*} \]

**Figure 5.5** Discretization stencil and centered difference approximation for model wave equation (5.40).
the following expression for the spatial derivative:

$$\frac{\partial y}{\partial x} \approx \frac{y_{i+1} - y_{i-1}}{2\Delta x} \quad (5.41)$$

Substitution into equation (5.41) leads to the following semi-discrete form of equation (5.40):

$$\frac{dy}{dt} + a \frac{y_{i+1} - y_{i-1}}{2\Delta x} = 0 \quad (5.42)$$

The spatial accuracy of equation (5.42) is examined by expanding $y$ in a Taylor series in each direction about point "i" and calculating the $y$ values at $i + 1$, and $i - 1$. These expressions are substituted back into equation (5.42) giving:

$$\frac{\partial y}{\partial t} + a \frac{\partial y}{\partial x} = \frac{\Delta x^2}{6} \frac{\partial^3 y}{\partial x^3} + O(\Delta x^4) \quad (5.43)$$

Equation (5.43) is the so-called "modified" differential equation that the numerical method solves instead of the exact equation given by equation (5.40). The numerical method is said to be second order accurate because the leading order error term on the right-hand side of equation (5.43) is proportional to $\Delta x^2$.

Rauenzahn (1986) used "upwind differencing" rather than central differencing in his simulation of thermal spallation drilling. This means that the spatial derivative in equation (5.40) was expressed as:

$$\frac{\partial y}{\partial x} = \frac{y_i - y_{i-1}}{\Delta x} \quad (5.44)$$
The order of accuracy for the upwind differencing method is proportional to $\Delta x$. Analysis of the modified equation reveals that diffusion-like terms are added by the truncation error. A solution calculated using upwind differencing is theoretically less accurate than one obtained using centered differencing on the same grid. However, as will be shown next, centered differencing results in an unstable algorithm if equation (5.40) is solved by discrete forward time stepping.

**Stability**

A "Fourier/von Neumann" stability analysis of equation (5.42) is described in this section. The numerical solution at any time is expressed as the sum of the exact value ($y_e$) and an error ($\epsilon$) as:

$$y^n = y_e^n + \epsilon^n \quad (5.45)$$

where the superscript "n" is a discrete time level index. Substitution of equation (5.45) into (5.42) followed by subtraction of the exact equation given by equation (5.40) leads to the conclusion that the errors must satisfy the original equation:

$$\frac{d\epsilon^n_j}{dt} \approx -a \frac{\epsilon^n_{j+1} - \epsilon^n_{j-1}}{2\Delta x} \quad (5.46)$$

The stability of equation (5.46) is investigated by substituting the general form of a solution to a linear differential equation with constant coefficients into (5.46) and determining whether errors grow or decay in time. A general solution is given by (Wylie, 1979):

$$\epsilon(x,t) = \epsilon_0(t)\exp(ikx) \quad (5.47)$$

where $\epsilon_0$ denotes the value of $\epsilon(x,t)$ at $x=0$, $i = \sqrt{-1}$, and $k$ is the wave number of the particular error mode. In discrete form this becomes (Anderson et al., 1984):
\( \varepsilon_j(t) = \varepsilon_0(t) \exp(ik(j\Delta x)) \) \hspace{1cm} (5.48)

Substitution of (5.48) into (5.46) and the use of Euler's identity results in the following:

\[
\frac{d\varepsilon_j}{dt} = -a \frac{\text{isin}(\theta)}{\Delta x} \varepsilon_j
\]\hspace{1cm} (5.49)

where \( \theta = k\Delta x \). The left hand side of equation (5.49) can be approximated with a forward time difference to give:

\[
\frac{\varepsilon_j^{n+1} - \varepsilon_j^n}{\Delta t} = -a \frac{\text{isin}(\theta)}{\Delta x} \varepsilon_j^n
\]\hspace{1cm} (5.50)

The amplitude ratio describing the transient error growth is given by:

\[
\left| \frac{\varepsilon_j^{n+1}}{\varepsilon_j^n} \right| = 1 + \left( a \frac{\Delta t}{\Delta x} \right)^2 \sin^2(\theta)
\]\hspace{1cm} (5.51)

Since this ratio is always greater than or equal to one, errors grow in time and the solution is unconditionally unstable.

The central differencing scheme used in this study is stabilized by using a four-stage Runge-Kutta time-stepping scheme and by the addition of "artificial dissipation". These techniques are explained in Sections 5.3.4, and 5.3.5 following the description of the finite volume spatial discretization methodology.
5.3.2 Finite Volume Formulation

The discrete set of conservation equations are formulated by integrating equations (5.9) over a cell volume giving:

\[
\begin{align*}
\iiint_V \frac{\partial \mathbf{U}}{\partial t} r dr dz d\theta &+ \iiint_V \left( \frac{1}{r} \frac{\partial (rE_2)}{\partial r} + \frac{\partial G}{\partial z} \right) r dr dz d\theta - \iiint_V \left( \frac{1}{r} \frac{\partial (rR)}{\partial r} + \frac{\partial S}{\partial z} \right) r dr dz d\theta \\
&= \iiint_V \nabla \cdot \mathbf{F}_2 r dr dz d\theta \quad \text{(I)} \quad \text{(II)} \quad \text{(III)}
\end{align*}
\]

\[
\iiint_V \frac{\partial P}{\partial r} r dr dz d\theta = \iiint_V \mathbf{M}_2 r dr dz d\theta \quad \text{(V)}
\]

where

\[
E_2 = E - P,
\]

\[
P = \begin{pmatrix} 0 \\ -P \\ P \\ 0 \end{pmatrix},
\]

\[
\mathbf{M}_2 = \frac{1}{Re_r} \begin{pmatrix} 0 \\ -\tau_{\theta\theta}/r \\ 0 \\ 0 \end{pmatrix},
\]

\[
\tau_{\theta\theta} = 2 \mu_t \frac{u}{r} - \frac{2}{3} \mu_t \text{div} \mathbf{V}, \quad \text{and}
\]

\[
V = \text{cell volume}.
\]

The dependence on \( \theta \) is integrated out of (5.52) based on the assumption of axisymmetry. A volume averaged state vector is defined as:

\[
\overline{\mathbf{U}} = \frac{\iiint \mathbf{U} r dr dz}{\iiint r dr dz} \quad \text{(5.53)}
\]

Terms (II) and (III) in equation (5.52) are rewritten by applying...
Green's theorem resulting in the following expressions:

\[
\text{Term (II)} = \oint [(rE_2)az - (rG)ar] \tag{5.54}
\]

\[
\text{Term (III)} = \oint [(rB)az - (rS)ar] \tag{5.55}
\]

Term (V) is averaged by the same method as that used in equation (5.53). Thus, with these simplifications, the final integral form of the conservation equations is:

\[
V \frac{\partial U}{\partial t} + \oint [(rE_2)az - (rG)ar] - \oint [(rB)az - (rS)ar]
- \iint \frac{\partial P}{\partial r} rdrdz = V \overline{M}_2 \tag{5.56}
\]

Equation (5.56) serves as a starting point for discretizing the flow-field and approximating the flux balances. The first term on the left hand side of equation (5.56) represents the rate of change of the state vector inside a given spatial region having a volume \(V\). The second and third terms are convective and diffusive flux balances that effectively sum all inflows and outflows of conserved quantities from the volume. The pressure integral to the left of the "=" sign and the source term appearing on the right hand side of the equation arise from the use of an axisymmetric co-ordinate system. The following sections give details of the method used for solving (5.56) in the physical domain applicable to spallation drilling.

5.3.3 Flux Calculations

The line integrals appearing in equations (5.56) are rewritten in terms of face integrals on each edge of the computational cell and then
the fluxes are approximated. For example, the convective flux integral is written (see Figure 5.6):

\[
\phi^B \left[ (rE_2) \partial z - (rG) \partial r \right] + \phi^C \left[ (rE_2) \partial z - (rG) \partial r \right] + \phi^D \left[ (rE_2) \partial z - (rG) \partial r \right] + \phi^A \left[ (rE_2) \partial z - (rG) \partial r \right]
\]

These integrals are approximated by assuming that \( r, z, E_2, \) and \( G \) vary linearly along the faces. For face A-B this gives:

\[
r = r_A (1 - \xi) + r_B \xi,
\]

\[
E_2 = E_{2A}(1 - \xi) + E_{2B} \xi, \text{ etc...}
\]

where \( \xi \) varies linearly from zero to one with distance along the face. The integrated expression for the convective fluxes on face A-B is given by (Roe, 1987):

\[
\phi^B \left[ (rE_2) \partial z - (rG) \partial r \right] = \left[ \frac{r_B - r_A}{6} \right] [r_A (2E_{2A} + E_{2B}) + r_B (2E_{2B} + E_{2A})] + \left[ \frac{Z_B - Z_A}{6} \right] [z_A (2G_A + G_B) + r_B (2G_B + G_A)]
\]

where \( E_{2A} \), and \( G_A \) are flux vectors evaluated at node point A.

This formulation results in leading error magnitudes of \( O(\Delta x^2) \) everywhere in the flow-field whereas more commonly used and straightforward method of neglecting the variations of \( r \) in the face integrals result in \( O(\Delta x) \) or less accuracy near the axis of symmetry where \( r \to 0 \).

Term (IV) in equation (5.52) is discretized using a pseudo finite-
Figure 5.6 Example of a finite cell used for convective flux balance integration.
element technique in which the pressure derivative is expanded using the chain rule:

\[ \frac{\partial P}{\partial r} = \frac{1}{|J|} \begin{vmatrix} \partial P/\partial \xi & \partial P/\partial \eta \\ \partial z/\partial \xi & \partial z/\partial \eta \end{vmatrix} \]

where

\[ |J| = \begin{vmatrix} \partial r/\partial \xi & \partial r/\partial \eta \\ \partial z/\partial \xi & \partial z/\partial \eta \end{vmatrix} \]

This leads to the following

\[ \frac{\partial P}{\partial r} = \frac{1}{|J|} \left( \frac{\partial P}{\partial \xi} \frac{\partial z}{\partial \eta} - \frac{\partial P}{\partial \eta} \frac{\partial z}{\partial \xi} \right) \quad (5.59) \]

where

\[ |J| = \text{Jacobian determinant, and} \]

\[ \xi, \eta = \text{local coordinates in computational space} \]

\[ (0 \leq \xi, \eta \leq 1). \]

Radius and pressure variations on a cell are given by:

\[ r = r_A(1 - \xi)(1 - \eta) + r_B(\xi)(1 - \eta) + r_C(\xi)(\eta) + r_D(1 - \xi)(\eta) \quad (5.60) \]

\[ P = P_A(1 - \xi)(1 - \eta) + P_B(\xi)(1 - \eta) + P_C(\xi)(\eta) + P_D(1 - \xi)(\eta) \quad (5.61) \]

Substitution of equations (5.60), (5.61), and (5.59) into term (IV) of equation (5.52) leads to (Roe, 1987):

\[ \int \int r \frac{\partial P}{\partial r} \, dr \, dz = \]

\[ \frac{(z_B - z_D)}{24} \left[ P_A(4r_A + 3r_B + 2r_C + 3r_D) - P_A(2r_A + 3r_B + 4r_C + 3r_D) \right] \]

\[ - \frac{(z_A - z_C)}{24} \left[ P_B(3r_A + 4r_B + 3r_C + 2r_D) - P_B(3r_A + 2r_B + 3r_C + 4r_D) \right] \]
\[
\frac{(r_B - r_D)}{24} \left[ P_A(x_B + x_D - 2x_C) - P_C(x_B + x_D - 2x_A) \right] \\
- \frac{(r_A - r_C)}{24} \left[ P_B(x_A + x_C - 2x_D) - P_D(x_A + x_C - 2x_B) \right]
\]  
(5.62)

Velocity and temperature derivatives are required for evaluation of the viscous stresses and heat fluxes ($\mathbf{R}$ and $\mathbf{S}$) appearing in equation (5.56). Green's theorem is used to approximate each derivative as an integral around a cell, resulting in cell-centered viscous fluxes. The derivatives are calculated according to the method described by Peyret and Taylor (1983):

\[
\frac{\partial u}{\partial r} = - \frac{1}{A} \int u \, dz
\]  
(5.63)

\[
\frac{\partial u}{\partial z} = - \frac{1}{A} \int u \, dr
\]  
(5.64)

where $A$ is the cell area.

The integrals in equations (5.63) and (5.64) are approximated as finite sums giving:

\[
\int u \, dy = \sum_{f=1}^{4} u_f \Delta y_f
\]  
(5.65)

where "$f$" is a cell face index. This method of calculating derivatives automatically accounts for grid lines that are not parallel to the coordinate axes.

The integration regions used for evaluation of the convective and
viscous fluxes are different (Figure 5.7). Convective fluxes are calculated at nodes, resulting in cell centered flux balances (Figure 5.7a), whereas viscous fluxes are defined at cell centers, resulting in node-centered flux balances (Figure 5.7b). The entire scheme becomes node-centered by summing the convective flux balances from the four cells making up the "node super-cell" (Figure 5.7c).

The methodology described above defines a scheme that is formally second-order accurate. However, for forward time differencing, the scheme is unstable when viscous terms are small because the convective flux integration is equivalent to a centered difference approximation of the spatial derivatives appearing in equations (5.9). Artificial dissipation and a four-stage time-stepping methods described in the following sections are used to stabilize the numerical algorithm.

5.3.4 Artificial Dissipation

The flux integration method described above is equivalent to a centered difference approximation of the flux vector derivatives in equation (5.56). As discussed in Section 5.3.1, centered differencing of convective derivatives is unconditionally unstable when used with forward time stepping. Furthermore, equation (5.43) proves that the leading order error term of the modified differential equation that is actually being solved by the numerical method is proportional to the third derivative of the dependent variable. Odd order derivatives lead to wave dispersion phenomena resulting in oscillations near
a) Convective Flux Cell (shaded).

b) Node Cell -- Viscous Flux Balance.

c) Node Super Cell -- Total Flux Balance.

**Figure 5.7** Integration areas for convective, diffusive, and total flux balances.
discontinuities, such as shock waves, and "odd-even" decoupling of the numerical solution at neighboring node points. Odd-even decoupling means that disturbances with a wavelength equal to twice the grid spacing are not damped and therefore lead to aphysical wiggles and/or instabilities in the numerical solution for the flow field. Stable numerical methods usually add "artificial viscosity" to overcome odd-even decoupling. This can be done either explicitly, by adding dissipative terms to the equations of motion, or implicitly, by formulating a numerical method that has a dissipative leading order truncation error.

The major advantage of explicitly adding dissipative terms is that their magnitude can be controlled to minimize the attenuation caused in the numerical solution. Implicit methods apply smoothing globally and this smoothing cannot be turned off. The major disadvantage of explicit numerical smoothing is that ad-hoc rules must be formulated to specify where, and how to adjust smoothing parameters.

The method of introducing artificial dissipation into the numerical solution algorithm used in this study is an extension and modification of explicit methods described by Ni (1981), Rizzi and Eriksson (1984), and Powell (1988). The artificial dissipation has two components: a second-difference term that is turned on only at shocks, and a fourth difference term that is turned on where the turbulent viscosity is not high enough to damp odd-even decoupling. The fourth-difference term is $O(\Delta x^3)$ and therefore does not affect the formal solution accuracy.
However, the second difference term degrades the solution accuracy to first order in shocks. This is typical of all "shock-capturing" numerical schemes because the shock structures must be smeared out to at least the width of one grid cell for these numerical methods to be stable.

The fourth-difference operator is given by:

\[ D_4 (u) = \epsilon_4 L^2 (u) \]  

(5.66)

where \( \epsilon_4 \) is the fourth-difference artificial viscosity coefficient, \( L \) is an unweighted Laplacian operator, and \( L^2 \) is the (unweighted) biharmonic operator. Away from the boundaries, the Laplacian is expressed as (see Figure 5.8):

\[ L (u) = u_{SW} + u_{SE} + u_{NE} + u_{NW} + 2(u_S + u_E) + u_N + u_W - 12u \]  

(5.67)

The biharmonic operator appearing in equation (5.66) is simply calculated by applying the Laplacian operator to the Laplacian of the state vector.

The second difference operator takes the form:

\[ D_2 (u) = \epsilon_2 \Gamma (u, \delta P) \]  

(5.68)

where \( \epsilon_2 \) is the second difference artificial viscosity coefficient (\( \approx 0.01-0.05 \)), and \( \Gamma \) is a modified Laplacian operator that uses a weighting function, \( \delta P \), as follows:
Figure 5.8 Laplacian operator stencil (adapted from Powell, 1988).
\[ L(\mathbf{u}, \delta P) = (u_{SW} - \mathbf{u})\delta P_{SW} + (u_{SE} - \mathbf{u})\delta P_{SE} + (u_{NE} - \mathbf{u})\delta P_{NE} + (u_{NW} - \mathbf{u})\delta P_{NW} + 2(u_{S} - \mathbf{u})\delta P_{S} + 2(u_{E} - \mathbf{u})\delta P_{E} + 2(u_{N} - \mathbf{u})\delta P_{N} + 2(u_{W} - \mathbf{u})\delta P_{W} \] (5.69)

The weighting function is defined at node points by:

\[ \delta P = \frac{|L(P)/P|}{\max|L(P)/P|} \] (5.70)

where \( \max \) denotes the maximum of the enclosed quantity over the entire solution domain. Values of \( \delta P \) between nodes are obtained by taking the maximum, for example:

\[ \delta P_{SW} = \max[\delta P, \delta P_{SW}] \] (5.71)

Boundary values of \( \delta P \) are set equal to zero. The use of the pressure switch given by equation (5.70) as a weighting function allows the algorithm to detect when shocks occur and automatically add second difference smoothing terms only in these regions. This is done in an attempt to use the minimum amount of smoothing possible for stable shock capturing.

Numerical experiments conducted during this study indicate that the fourth-difference artificial dissipation expressed by equation (5.66) must be turned off in shocks, high turbulent viscosity regions, and in the boundary layer along the spalling rock surface in order to obtain solutions that are independent of the fourth difference artificial dissipation coefficient. The method used to automatically switch off the fourth difference artificial dissipation in the regions described above is expressed by:
\[ \varepsilon_4 = \max[\varepsilon_4^0 - \varepsilon_2 \delta \rho - 3\left(\frac{\mu_t}{Re_r}\right), 0] \] (5.72)

where \( \varepsilon_4^0 = \) user specified fourth difference damping coefficient (\( \approx 0.001 \)). In addition, the total artificial dissipation was linearly reduced to zero across the boundary layer by setting:

\[ D_{b1}(\mathbf{U}) = D(\mathbf{U})(y/R_{\infty}) \] (5.73)

where \( D_{b1}(\mathbf{U}) = \) total (second plus fourth difference) artificial dissipation in the boundary layer.

The second and fourth difference contributions are combined in the following way:

\[ D(\mathbf{U}) = D_2(\mathbf{U}) - D_4(\mathbf{U}) \] (5.74)

and then added to the right-hand side of (5.56).

The next section describes the numerical time integration method used to achieve the steady-state solution of equations (5.56).

5.3.5 Temporal Discretization

Equations (5.56) are a coupled set of semi-discrete, non-linear equations describing the time rate of change of the state vector, \( \mathbf{U} \). A multistage scheme is used to integrate these equations to steady state. The method is given by Jameson (1986) as:

\[ U^1 = U^n - \alpha_1(\Delta t)(R^n - \mathbf{D}^n) \]
\[ U^2 = U^n - \alpha_2(\Delta t)(R^1 - \mathbf{D}^n) \]
\[ U^3 = U^n - \alpha_3(\Delta t)(R^2 - \mathbf{D}^n) \]
\[ U^4 = U^n - \alpha_1 (\Delta t) (R^k - \mathcal{D}^n) \]
\[ U^{n+1} = U^4 \]  
(5.75)

where

\[ \alpha_1 = 0.25; \alpha_2 = 0.33; \alpha_3 = 0.50; \alpha_4 = 1.0, \]

\( R^k \) = the residual representing the spatially discretized form of equations (5.56) evaluated after stage "k",

\( \mathcal{D}^n \) = artificial dissipation evaluated after stage \( n \), and

\( \Delta t \) = time step.

Computational efficiency is achieved by evaluating the physical and artificial dissipation terms during only the first stage.

The stability of the discretization scheme is examined by performing a Fourier/von Neumann analysis on a two-dimensional model equation. Since the second difference damping terms are turned on only in shocks, they are neglected in this analysis. The model equation is the finite-volume form of a two-dimensional convective diffusion equation derived by using the discretization techniques described in Section 5.3. The expansion of the spatial discretization terms is done according to the method explained in Section 5.3.1. For clarity, the terms are not written out explicitly, but the final form of the equation governing the transient error propagation is written:

\[ \frac{\partial \varepsilon_0}{\partial t} = z \varepsilon_0 \]  
(5.76)

where \( z \) is the Fourier-space representation of the spatial discretization operator. Assuming that \( z \) is, in general, a complex number with complex and real parts denoted by \( z_i \) and \( z_r \), respectively,
the multistage time-integration given by equation (5.75) is written:

$$\frac{\epsilon_0^{n+1}}{\epsilon_0^n} = 1 + \alpha_4 z + \alpha_4\alpha_3 z(z_1) + \alpha_4\alpha_3\alpha_2 z(z_1)^2 + \alpha_4\alpha_3\alpha_2\alpha_1 z(z_1)^3$$  \hspace{1cm} (5.77)

where $\Delta t$ has been absorbed into the expression for $z$. The magnitude of the left-hand side of equation (5.77) is defined as the error amplification factor. The stability region for the multistage time-stepping method, plotted as a function of $z_1$ and $z_r$, is illustrated in Figure 5.9. The outermost line is for an amplification factor of one and therefore represents the stability threshold. All of the contours inside of the stability threshold represent amplification factors less than one. Note that the components of $z$ shown in Figure 5.9 are independent of the spatial discretization technique. The CFL number and $\epsilon_4$ must be chosen so that $z_1$ and $z_r$ calculated for the spatial operator attain values only within the stable region of the time-stepping scheme.

The expressions for $z_1$ and $z_r$ for the complete spatial operator are presented here without derivation as:

$$z_1 = \frac{i \text{CFL}}{2}[(\cos(\theta)\sin(\phi) + \sin(\phi)) + (\sin(\theta)\cos(\phi) + \sin(\theta))\text{AR}]$$

$$z_r = D_2[(1 - \text{AR}^{-1})(\cos(\theta) - \cos(\phi)) + (1 + \text{AR}^{-1})(\cos(\theta)\cos(\phi) - 1)] - \epsilon_4 \text{CFL}[164 + 12 \cos(2\theta) - 80 \cos(\theta) + 16 \cos(2\theta)\cos(\phi) - 64 \cos(\theta)\cos(\phi) + 4 \cos(2\theta)\cos(2\phi) + 16 \cos(\theta)\cos(2\theta) - 80 \cos(\phi) + 12 \cos(2\phi)]/1 + \text{AR}^2$$  \hspace{1cm} (5.79)
Figure 5.9 Amplification factor contours illustrating the temporal stability region.
where
\[ \text{CFL} = \frac{a \Delta t \Delta r}{A_{nc}}, \]
\[ a = \text{maximum local wave speed}, \]
\[ D_2 = \frac{\mu_t \Delta t}{\text{Re } A_{nc}}, \]
\[ A_{nc} = \text{node cell area}, \]
\[ AR = \text{cell aspect ratio} = \frac{\Delta z}{\Delta r}, \]
\[ \theta = k_r \Delta r, \]
\[ \phi = k_z \Delta z, \]
\[ k_r = r\text{-direction wavenumber}, \text{ and} \]
\[ k_z = z\text{-direction wavenumber}. \]

Substitution of equations (5.78) and (5.79) into (5.77) allows determination of the specific amplification factors for the combined spatial and temporal discretization methods employed in this study. The family of curves of values of \( z_i \) and \( z_r \) for values of \( \theta \) and \( \phi \) between 0 and \( 2\pi \) are referred to as wave number locii and are plotted along with the temporal stability limit curve in Figure 5.10. The parameters used to generate the wave-number locii are representative of those calculated during simulation runs and are given in Table 5.2.
Figure 5.10 Superimposed temporal stability region and spatial wave number locii with artificial dissipation evaluated at the first stage of the multistage time integration only.
Table 5.2 Parameters used for Stability Analysis (Figure 5.10).

AR = 1.0  
\( \mu_t/\mu_{n0z} = 500 \)  
Re_r = 20,000  
\( \epsilon_4 = 0.001 \)  
\( a = 2.0 \) (corresponds to flow at Ma = 1.0)  
CFL_{max} = 2.15

Note: Artificial and physical diffusion terms are evaluated only at first stage.

The maximum stable CFL number is found to equal 2.16 since at this value the outermost wave-number locus meets the temporal stability line. Increasing the CFL number causes the amplification factor to become larger than one and therefore the method becomes unstable.

The influence of evaluating artificial and physical diffusion terms during only the first stage of the time stepping update is to decrease the operation count of the method at the cost of a slightly lower maximum stable time step. Figure 5.11 is a plot of the wave-number locii and stability interval for the case when all diffusion terms are updated at every stage of the time-stepping scheme. The efficiency of freezing diffusive terms after the first stage is demonstrated by noting that the time step size, as indicated by CFL_{max}, decreases by only 4.5 % whereas the computational work required drops by greater than a factor of 2.

In practice, AR, D_z, and \( \Delta z \) vary dramatically throughout the flow field. The most efficient and reliable input parameter values have been found by trial-and-error to be:
Figure 5.11 Superimposed temporal stability region and spatial wave number locii with artificial dissipation evaluated at every stage of the multistage time integration.
CFL = 0.8, and 
\( \varepsilon_4 = 0.001. \)

5.3.6 Solution of Turbulence Transport Equations

The k-\( \varepsilon \) model equations given by equations (5.23) and (5.24) are of the same form as those in equations (5.9). Therefore, the convective, diffusive, and artificial dissipation terms are discretized as detailed in Sections 5.3.2 - 5.3.5. However, the turbulent kinetic energy production and dissipation terms must be treated implicitly during each stage of the time integration procedure in order to ensure numerical stability. The discrete, implicit forms of the turbulence transport equations are given by:

\[
\begin{align*}
\frac{(\rho k)}{k+1/2} &= \left( \frac{(\rho k)}{k+1} \right) - \alpha_k \text{CFL}(\rho \varepsilon) + \alpha_k \text{CFL}(P_k) \\
(\rho \varepsilon) &= (\rho \varepsilon) + \alpha_k \text{CFL}(C_1) P_k \left( \frac{\rho \varepsilon}{\rho_k} \right)^{k+1} - \alpha_k \text{CFL}C_2 \left( \frac{(\rho \varepsilon)^2}{\rho_k} \right)^{k+1} 
\end{align*}
\]

(5.80)  
(5.81)

where superscript "k+1/2" implies that the values have already been updated to account for the convective, diffusive, and artificial dissipation flux balances calculated at stage k.

Equations (5.80) and (5.81) are manipulated to yield an expression for the value of (\( \rho \varepsilon \)) at stage k+1:

\[
(\rho \varepsilon) = \frac{k+1 - A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1} 
\]

(5.82)

where

\[ A_1 = \alpha_k \text{CFL}(C_2 - 1), \]

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\[ A_2 = (\rho k)^{1/2} + \alpha_k \text{CFL}(\rho \varepsilon) - \alpha_k (\text{CFL} \, C_\varepsilon^2) P_k, \text{ and} \]

\[ A_3 = -[(\rho \varepsilon)(\rho k)]. \]

The solution to equation (5.82) is substituted back into equation (5.80) to determine the updated value of \((\rho k)\). After solution of equations (5.81) and (5.82), values of the turbulent viscosity are updated at all node points using equation (5.20):

\[ \mu_t = \frac{\rho C_\mu k^2}{\varepsilon} \quad (5.20) \]

As discussed earlier, \(\mu_t\) relates the strain rate to the apparent viscous stresses throughout the turbulent flow field.

5.3.7 Solver Validation

The purpose of this section is to illustrate the accuracy of the code used in this study for predicting flow-fields similar to the one that occurs during thermal spallation drilling. This is accomplished by comparing simulation results for subsonic and supersonic free-jets with experimental data obtained by Snedeker and Donaldson (1964), Wyganski and Fiedler (1969), and Rodi (1972). In addition, comparison is made with predictions of other researchers using \(k-\varepsilon\) turbulence models for predicting subsonic jets.

The rate of spread of a turbulent jet is governed by the rate of entrainment of surrounding fluid through the action of turbulent
diffusion at the jet boundary (Schlichting, 1979). Therefore, comparison of predicted and experimental jet spreading rates provides a test of the combined ability of the turbulence model and numerical method to accurately represent flow fields similar to those that occur near the nozzle outlet during spallation drilling.

**Subsonic Free Jet**

Plots of simulated and experimental (Snedeker and Donaldson, 1964) jet half width versus downstream distance for a subsonic turbulent free jet exhausting into stagnant surroundings are shown in Figure 5.12. Jet half-width is defined for a given downstream distance as the radius at which the axial velocity equals one-half of the maximum axial jet velocity. Both jet half-width and downstream distance are non-dimensionalized with nozzle radius. The nozzle outlet conditions and numerical parameters used for modeling this flow-field are listed in Table 5.3.

**Table 5.3** Nozzle Outlet Conditions and Modeling Parameters for Subsonic Free-Jet.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle Mach Number (Ma)</td>
<td>0.52</td>
</tr>
<tr>
<td>Nozzle Reynolds Number (Re_D)</td>
<td>129,200</td>
</tr>
<tr>
<td>Inlet Turbulence Intensity C_1, C_2, C_3</td>
<td>.5 % of mean kinetic energy</td>
</tr>
<tr>
<td>CFL</td>
<td>1.42, 1.92, 3.13</td>
</tr>
<tr>
<td>ε_0, ε_4</td>
<td>1.0</td>
</tr>
<tr>
<td>ε_0, ε_4</td>
<td>0.0, 0.001</td>
</tr>
</tbody>
</table>

The half width of a free-jet is known to grow linearly with
Figure 5.12  Radial spread of a subsonic jet -- comparison between experimental data and model predictions.
downstream distance from the nozzle once the potential core has disappeared and self similarity of cross-stream velocity profiles has occurred (Hinze, 1975). In this case, self similarity is said to occur when plots of axial velocity, turbulence intensity, temperature, etc., non-dimensionalized with their respective centerline values versus radius, non-dimensionalized with the jet half-width, fall on the same curves regardless of the axial location. The slope of the line joining the jet half widths in the self similar region of the flow-field is defined as the spreading rate.

Table 5.4 is a comparison of spreading rates obtained by several experimenters (Rodi, 1972; Wyganski & Fiedler, 1969; Snedeker & Donaldson, 1964) with predictions of other workers and with predictions made during this study. The level of error in the experimental results is unknown but the good agreement obtained between the results for the three studies cited indicates that the error is probably less than 1%. The present prediction (shown as "this study" in Table 5.4) uses the same method as the "modified" form of Hanjalic and Launder (1981) and is about 15% too high when compared to the experiments. In the "modified" k-\(\epsilon\) model, a separate constant (\(C_3\)) is used to multiply the terms responsible for production by normal stress components in the transport equation for the dissipation rate of turbulent kinetic energy. This modified form contrasts to the "standard" k-\(\epsilon\) model which does not properly account for the abnormally high normal stresses that occur in round turbulent jets and, therefore, overpredicts the spreading rate by about 30-35% (Table 5.4). Both the standard and the modified k-\(\epsilon\) models
predict spreading rates for planar jets to within approximately 2% of experimentally measured values.

Table 5.4 Comparison of Predicted Round-Jet Spreading Rates with Measurements and Predictions of Other Investigators.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.086</td>
<td>0.087</td>
<td>0.136</td>
<td>0.115</td>
<td>0.098</td>
<td>0.100</td>
</tr>
</tbody>
</table>

*Note: Spreading rate is defined as $\partial r_{1/2}/\partial x$ where $r_{1/2}$ is the radius at which the jet velocity equals one-half of its centerline value and $x$ is axial distance measured from the nozzle outlet. In the self similar jet region, the spreading rate is constant.

Deficiencies still exist in the turbulence modeling for round-jets given that the high accuracy results mentioned above for plane jets are possible using the standard k-ε model whereas even with the modified model round jet spreading rates are off by about 15%. Numerous workers have attempted to eliminate these deficiencies through more elaborate modifications of the ε-transport equation than those mentioned above but have achieved similar levels of agreement with experimental data. In addition, Launder & Morse (1979) used a full Reynolds stress closure scheme which resulted in an even greater overprediction of the round-jet
spreading rate by more than 50% (see Table 5.4). For comparison with previous predictions and because of the straight-forward implementation of the Hanjalic-Launder method, it was chosen for use in this code-validation study.

A comparison between predicted and experimental (Rodi, 1972) non-dimensional turbulent kinetic energy profiles in the self similar region of the jet is shown in Figure 5.13. In this case, the turbulent kinetic energy \( k \) is scaled with centerline velocity \( U_c \) squared and radial position is scaled with jet half width \( r_{1/2} \). The 8-10% underprediction of centerline turbulence levels is comparable with the results of Hanjalic and Launder (1981) and indicates that the turbulent flow field is being well represented using the current methodology.

**Supersonic Free-Jet**

Figure 5.14 is a comparison of predicted jet half-widths with experimental values (Snedeker & Donaldson, 1964) for a sonic \( (Ma = 1) \), highly underexpanded \( (P_{jet}/P_{amb} = 3.57) \) jet exhausting into stagnant surroundings. Other parameters are listed in Table 5.5.
Figure 5.13 Predicted and experimental turbulent kinetic energy profiles (specifications given in Table 5.2).
Figure 5.14  Radial spread of a supersonic jet -- comparison between experimental data and model predictions.
Table 5.5 Nozzle Outlet Conditions and Modeling Parameters for Supersonic Free-Jet.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nozzle Mach Number (Ma)</td>
<td>1.0</td>
</tr>
<tr>
<td>Nozzle Reynolds Number (Re₀)</td>
<td>600,000</td>
</tr>
<tr>
<td>Inlet Turbulence Intensity</td>
<td>5% of mean kinetic energy</td>
</tr>
<tr>
<td>C₁, C₂, C₃</td>
<td>1.42, 1.92, 3.13</td>
</tr>
<tr>
<td>CFL</td>
<td>1.0</td>
</tr>
<tr>
<td>ε₂₀, ε₄₀</td>
<td>0.01, 0.001</td>
</tr>
<tr>
<td>P_{jet}/P_{amb}</td>
<td>3.57</td>
</tr>
</tbody>
</table>

The disparity in the results for this case may be somewhat exaggerated by the lack of detailed experimental resolution in the initial jet-expansion region. Experimental errors are estimated as ± 5%. However, the terminal spreading rate is overpredicted by about the same percentage in this case as for the subsonic case. Furthermore, the predicted supersonic core length (the last downstream point at which the centerline Ma ≥ 1) is 36 nozzle radii (Figure 5.15), as compared to experimental values of about 40-44 nozzle radii (Snedeker and Donaldson, 1964). Correct prediction of the supersonic core length is important because it determines whether or not a normal stand-off shock occurs in
Figure 5.15 Centerline Mach number as a function of distance from the nozzle outlet.
the impinging flow-field during thermal spallation drilling.

Figure 5.15 also shows that approximately three shock diamonds, corresponding to pairs of extrema in the Ma versus Y curve, are predicted at the nozzle outlet as the high inlet pressure adjusts to the ambient pressure. The first of these shocks results in a Mach number less than one, implying that a "Mach disk" forms normal to the mean flow direction. The second and third shocks are predicted to be oblique, although the data of Snedeker and Donaldson indicate that the second one is also a Mach disk. Lowering the artificial dissipation coefficient in the present scheme results in correct prediction of the second normal Mach disk but destabilizes the code by allowing small oscillations of the shock structure. Supersonic core length and jet-spreading rate were affected by less than 4% by changing the artificial dissipation level.

Free-jet predictions using the "standard" form of the k-ε model have also been conducted during this study. Although not shown, the resulting sub-sonic jet spreading rates are about 2% lower than those obtained by Hanjalic and Launder (1981) using the same model (see Table 5.4). Calculation of the super-sonic case resulted in a core length of 32 nozzle radii and a terminal spreading rate approximately equal to that given by Hanjalic and Launder's subsonic prediction (standard model).

The modified value of C₃ used in the validation work described above was to illustrate the variations in prediction accuracy possible for
relatively minor changes to the modeling constants and to confirm that similar levels of accuracy could be obtained using the flow-field solver implemented in this study as those developed previously by other workers. Preliminary simulation runs were conducted using standard and modified models for predicting impinging jet flow fields similar to those occurring during thermal spallation drilling and revealed that the increased sensitivity to normal strain rates of the modified model was leading to nonphysical results near the stagnation point. Predicted turbulent viscosity values approached zero because of the higher predicted levels of the turbulence dissipation rate. This caused a large flow recirculation zone to form starting at the stagnation point and having a radius of approximately 6 nozzle radii. These results conflict with experimental observations made by Snedeker and Donaldson (1964) for impinging supersonic jets. Moreover, predicted penetration rates were two orders of magnitude less than those routinely observed using thermal spallation drilling.

The standard k-ε model did not predict the anomalous recirculation zone described above, and predicted heat transfer rates and spatial heat transfer distributions were consistent with those observed in practice for both subsonic and supersonic jets at moderate stand-off distances and pressures. Furthermore, this model has been extensively tested by other researchers and the various modeling parameters have been more firmly established than the modified constants of Hanjalic and Launder (1981). Therefore, the standard k-ε model was used for all full-scale thermal spallation simulations conducted in this study. The maximum
accuracy that is expected to be achievable for jet-spreading rate and 
super-sonic core length is approximately ± 35 %, corresponding to the 
errors for the "standard" model results given in Table 5.4. Since the 
behavior of the wall-jet and corresponding heat transfer to the rock 
surface is heavily influenced by the impinging jet flow-field, similar 
accuracy should be expected of the overall simulation results.

5.4 Boundary Movement Algorithm

The component of the penetration rate that is parallel to the drill 
axis ($V_{dr}$) must be the same at steady-state conditions for all points 
along the spalling rock surface. This is expressed by:

$$V_{dr} = \frac{U_{dr}}{\cos(\theta)}$$  \hspace{1cm} (2.5)

$V_{dr}$ is calculated as an average of predicted penetration rates for all 
points from a radius of one-half nozzle diameters to the point where $\theta$ = 
45°.

The average of the drilling velocities predicted for the previous 
and current boundary heat flux profiles is used to determine the 
movement necessary to force self-consistency. The equation used is:

$$U_{dr, i} = \frac{1}{2} \left( U_{dr, i}^{old} + \frac{Q_{i}}{(\rho C_{p})_r (T_{s, i} - T_{r, 0})} \right)$$  \hspace{1cm} (5.83)

where

old
$U_{dr, i} = $ previous predicted value of penetration rate at location 
"i".
The updated boundary angle at each location is determined by averaging the current value with that predicted by the local penetration rate:

$$\cos(\theta_i) = 0.5 (\cos(\theta_i)^{old} + \frac{U_{dr_i}}{V_{dr}})$$  \hspace{1cm} (5.84)

The non-dimensional ratio of the local excess to average penetration rate ($R_{pr,i}$) at each point on the surface is given by:

$$R_{pr,i} = \frac{U_{dr,i}}{V_{dr}} - 1$$  \hspace{1cm} (5.85)

These values are smoothed by averaging each with the values from the two nearest neighboring boundary points so that "wiggles" do not appear. The boundary movement is finally performed by using the smoothed values of $R_{pr,i}$ giving:

$$r_i^{new} = r_i^{old} + \omega R_{pr,i} \sin(\theta)$$  \hspace{1cm} (5.86)

$$z_i^{new} = z_i^{old} - \omega R_{pr,i} \cos(\theta)$$  \hspace{1cm} (5.87)

where $r_i$ and $z_i$ are non-dimensional boundary coordinates, and $\omega$ is a user-defined acceleration parameter ($\approx 2.0$). $R_{pr,i}$ is always set equal to zero at the centerline and the boundary is not allowed to move below $z = 0$ so that the stand-off distance cannot change during a simulation.

During most runs, the starting boundary shape was elliptical and the first steady-state solution was achieved after 1000 - 1500 iterations. Typical requirements for the remainder of the simulation were: less than 1000 iterations to achieve a steady-state flow-field at each fixed

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boundary position, and 50 - 70 boundary movements to reach the final
hole-shape. Convergence of the hole shape was assumed to have occurred
when the predicted variations were less than 0.5 nozzle radii.
Typically, about one hour of cpu time on the Cray-XMP computer at Los
Alamos National Laboratory was required for convergence at each stand-
off distance and set of conditions.

5.5 Grid Generation

5.5.1 Introduction

The desirable features of a computational grid are:

(a) minimum number of grid points,
(b) sufficient resolution of gradients,
(c) orthogonality of cell edges, and
(d) equal cell spacings.

The goal of (a) is to enhance the convergence rate whereas the others
focus on obtaining accurate solutions. In practice, (a) - (d) are
impossible to satisfy simultaneously. Instead, grid generation is done
so that each is approached as closely as possible.

Grid generation is broken down into two steps in this study:
determination of an initial grid with appropriate point clustering in
high gradient regions (features (a) and (b)), and smoothing of the grid
using an elliptic equation set to equalize grid spacings and force the
cell edges closer to orthogonality with one another (features (c) and (d)).

5.5.2 Initial Grid Generation Procedure

The clustered regions in the initial grid are generated by using "stretching" (Figure 5.16) whereby consecutive values of grid spacing are related according to:

$$\Delta y_{j+1} = (1 + \alpha)\Delta y_j$$

where $\alpha = "stretching\ parameter" \approx 0.1$, and $\Delta y =$ node point spacing for cell j.

Grid stretching was applied in two regions where flow-field gradients are known to be high: near the lip of the nozzle outlet and along the rock boundary. Stretching near the rock boundary occurred from the wall out to 0.8 - 2 nozzle radii normal to the surface. The stretching parameters used were 8 - 15 % depending on the simulation run. Except for stretched regions, all grid lines were spaced evenly. A typical initial grid generated for an ellipse with a drill head stand-off distance of 20 nozzle radii is shown in Figure 5.17.

5.5.3 Elliptic Grid Smoothing Procedure

The elliptical grid generation procedure proposed by Steger and Sorenson (1979) was used to smooth the initial grid. This method was chosen because of the straightforward implementation, flexibility, and
\[ \Delta z_2 = (1 + \alpha) \Delta z_1 \]

\[ \alpha \equiv \text{"stretching parameter"} = 0.10 \]

**Figure 5.16** Grid Stretching.
Figure 5.17 Initial computational grid.
small number of user specified parameters necessary to achieve satisfactory grids. General details of the method are presented in this section.

The Steger and Sorenson method maps the coordinate space described by \( r \) and \( z \) into a rectangular computational coordinate space defined by two new coordinates, \( \xi \) and \( \eta \) (Figure 5.18). Lines of constant \( \xi \) are analogous to stream lines in irrotational, potential flow, and lines of constant \( \eta \) are analogous to velocity potential lines. With this in mind, Steger and Sorenson propose that equations analogous to those governing irrotational potential flow be solved for \( \xi \) and \( \eta \) giving:

\[
\xi_{rr} + \xi_{zz} = P(\xi, \eta) \tag{5.88}
\]

\[
\eta_{rr} + \eta_{zz} = Q(\xi, \eta) \tag{5.89}
\]

where

\[
\xi_{rr} \equiv \frac{\partial^2 \xi}{\partial r^2} \quad \text{(likewise for similar terms), and}
\]

\( P, Q \) = source terms used for grid clustering.

Although \( P \) and \( Q \) are present in the grid generation routine written for this study, they were found to be unnecessary and were set equal to zero. Equations (5.88) and (5.89) are transformed so that \( \xi \) and \( \eta \) become the independent variables, leading to the following equations:

\[
\alpha r_{\xi \xi} - 2\beta r_{\xi \eta} + \gamma r_{\eta \eta} = 0 \tag{5.90}
\]

\[
\alpha z_{\xi \xi} - 2\beta z_{\xi \eta} + \gamma z_{\eta \eta} = 0 \tag{5.91}
\]

where

\[
\alpha = r_{\eta}^2 + z_{\eta}^2, \\
\beta = r_{\xi} r_{\eta} + z_{\xi} z_{\eta}, \text{ and} \\
\gamma = r_{\xi}^2 + z_{\xi}^2.
\]

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Figure 5.18 Coordinate mapping from physical into computational space.
An alternating-direction implicit algorithm was used to solve equations (5.90), and (5.91). However, for this study, only four sweeps of the solver through the flow field were used, rather than the 10 - 20 required for full convergence because otherwise the desired clustering introduced with the initial grid would be smoothed out completely. Figure 5.19 illustrates the final grid that resulted from the initial one given in Figure 5.17. As expected, the grid lines are closer to being orthogonal, and the stretched regions are somewhat thicker than for the initial grid.
Figure 5.19 Smoothed computational grid.
6 Simulation Results

In this chapter, the results of applying the model and numerical methods described in Chapter 5 are presented and compared to Rauenzahn's (1986) experimental and simulation results. In addition, modifications to account for the effects of the variation of gas heat capacity with temperature and to account for the injection of rock chips into the flow-field are proposed and tested. The final results show good agreement with experiments but further work is required to individually validate the modeling assumptions.

6.1 Convergence Criteria and Details of Simulation

Two important assumptions made in the spallation simulator described in Chapter 5 are:

1. a steady-state flow-field exists for a given hole geometry and set of conditions, and
2. a steady-state hole geometry exists for a given stand-off distance.

These idealizations neglect the effects of rock heterogeneities and discrete spall formation on the flow field. Detailed transient phenomena, for example changes in the rock boundary shape when a spall is ejected, are not captured using the current modeling approach. Therefore, the simulation results obtained in the present study must be viewed as time smoothed.
Convergence of each flow-field was determined by monitoring changes in the surface averaged Stanton number \(<St>\), defined as:

\[
<St> = \frac{1}{i_{45}} \sum_{i=1}^{i_{45}} \left( \frac{St_i}{\cos \theta_i} \right)
\]

where \(St_i = \frac{Q}{(\rho C_p U)_{jet}(T_{jet} - T_{s,i})}\),

\(T_{s,i}\) = rock surface temperature at node point "i", and \(i_{45}\) = index of the boundary node-point nearest to the centerline having a tangent angle to the horizontal \(\theta_i\) greater than 45 degrees. Flow-field convergence was assumed when \(<St>\) varied by less than 0.5 % per 1000 iterations. This usually required from 300 to 1200 iterations to achieve and occurred when the maximum calculated change per iteration of any state vector component at any point throughout the flow-field, non-dimensionalized by its value at the nozzle outlet, was less than \(2 \times 10^{-4}\). Predicted hole shape usually oscillated toward a steady-state value and convergence was assumed when the amplitude of the oscillations in the hole radius was less than 3 % of the hole radius.

The first simulation performed was started by setting the initial conditions as: an ellipsoidal cavity; a SOD equal to 20, and no flow inside the cavity. The turbulent viscosity was held constant and the \(k-\epsilon\) turbulence model was not used until steady-state conditions had occurred. Computational efficiency was achieved in all subsequent runs by using the final flow-field calculated from one run as the initial flow-field for the next, in which the geometry and/or flow conditions were altered.
Solution accuracy depends upon the level of grid resolution. The required level of grid resolution was determined by comparing radial Stanton number distributions computed for different numbers of grid points in a fixed hole geometry. The effect of increasing the global grid resolution from 2701 to 5225 node points is illustrated in Figure 6.1 for a SOD of 60 and a prescribed ellipsoidal boundary shape. Since the Stanton numbers vary by less than 2%, the lower density grid is sufficient for this and any runs conducted at smaller values of SOD where the grid density becomes higher.

6.2 Modeling Considerations and Results

Computer simulation runs have been conducted for three sets of modeling assumptions: the basic solver, as outlined in Chapter 5; the basic solver with a heat capacity that varies with gas temperature, and the basic solver with a variable heat capacity and mass injection at the spalling surface to represent the momentum deficit introduced into the boundary layer by spall liberation.

6.2.1 Basic Solver

The most important improvements in the numerical algorithm used in this study over that developed by Rauenzahn (1986) are second order accurate advection and node point clustering in high gradient regions. These improvements should result in less numerically-induced smearing of
Figure 6.1 Effect of grid resolution on Stanton number distribution.
flow-field gradients (Anderson et al., 1984; Peyret and Taylor, 1983) and correspondingly higher values for wall heat fluxes and shear stresses. Figure 6.2 compares the curve of predicted Stanton number versus non-dimensional hole radius from this study to predictions made by Rauenzahn (1986), and to the curve calculated from experimental small-scale field drilling data obtained by Rauenzahn. The curve generated in the present study is seen to be higher than the results predicted by Rauenzahn (1986), and both predicted curves are higher than the experimental results. The higher Stanton numbers predicted in this study, corresponding to higher heat fluxes, are consistent with the assertion made above that steeper flow-field gradients are captured by the higher order numerics being employed.

The high heat fluxes predicted by the present method decrease the level of agreement between predicted and experimental results compared to predictions obtained by Rauenzahn (1986). Therefore several potential improvements to the physical modeling have been investigated and are discussed in the following section.

6.2.2 Effect of Variable Heat Capacity

In the basic solver, the value of the heat capacity is set equal to the inlet value throughout the flow-field. In reality, the heat capacity decreases at lower gas temperatures resulting in lower heat capacities near the rock wall, where the temperature is typically about 900 K, than at the nozzle, where the temperature is approximately 2830
Figure 6.2 Stanton number versus hole radius -- illustration of basic solver accuracy.
K. A lower heat capacity results in a proportionally lower turbulent thermal conductivity ($\lambda_t$) according to the following relationship:

$$\lambda_t = \frac{\mu_t C_p(T)}{Pr_t}$$  \hspace{1cm} (6.1)

where $Pr_t$ is assumed constant (0.86). The heat capacity ($C_{p,m}$) for the CO$_2$ and H$_2$O gas mixture formed from the combustion of propane and oxygen is estimated by summing the mole fraction ($y_i$) weighted values for the pure ideal gas heat capacities ($C_{p,i}$) according to $C_{p,m} = \sum y_i C_{p,i}$ (ideal mixing). The expression for the molar heat capacity of the mixture, using component heat capacities from Kelley (1960) is:

$$C_p = 36.4 + 9.65 \times 10^{-3}T - 3.7 \times 10^5T^{-2} \text{ (J/mol K)}$$  \hspace{1cm} (6.2)

where $y_{CO2} = 3/7$, and $y_{H2O} = 4/7$. For typical wall temperatures, the ratio of $\lambda_t$ when $C_p = C_{p,wall}$ to $\lambda_t$ when $C_p = C_{p,jet}$ is approximately:

$$\frac{\lambda_{t,wall}}{\lambda_{t,jet}} = \frac{C_p(T_{wall})}{C_p(T_{jet})} = 70\%$$  \hspace{1cm} (6.3)

Figure 6.3 illustrates that the predicted Stanton numbers decrease by approximately 30% compared to the values predicted using the basic solver. However, the results are still about a factor of two higher than experimentally observed Stanton numbers. It is interesting to note that the predicted hole radii are unchanged when the effect of a variable heat capacity is included. This is because the use of a variable heat capacity lowers the heat fluxes everywhere along the rock boundary by approximately the same amount. Therefore, the variation of heat flux with radius, and the resulting predicted hole shape, should not be substantially different from the base case.
Figure 6.3 Stanton number versus hole radius -- illustration of effects of variable heat capacity on simulation accuracy.
6.2.3 Effect of Mass Injection

The liberation of solids at a rock surface results in a change of the turbulent boundary layer structure and a corresponding decrease in the heat transfer to the rock according to Roberts et al. (1983) who have conducted experiments during which sand particles were ingested into a hot turbulent boundary layer and observed that heat transfer rates decrease by greater than 75% for mass loadings (solid mass:fluid mass) of order one. They cited calculations and performed measurements proving that a combination of boundary layer thickening and the thermal insulating effect of the relatively cold, entrained solids are responsible for the lower heat transfer rates. However, no method for modeling this phenomena was presented.

Mirels (1984) assumed that the momentum deficit introduced by sand particles being entrained into a turbulent boundary layer is the same as that for an equivalent mass flux of gas. Therefore, he modeled entrainment of solids as transpiration blowing at the surface. During spallation drilling, however, the portion of the boundary layer very near the wall should not be strongly influenced by the process of mass injection because of the relatively low average frequency at which spalls are liberated. This is demonstrated by examining the non-dimensional frequency (Strouhal number, Sr) of spall liberation, defined as:

\[
Sr = \frac{f_s \delta_b}{U_{*,b1}}
\]  

(6.4)
where
\[ f_s = \text{frequency of spall liberation} \approx 13 \text{ Hz. near stagnation point}, \]
\[ \delta_{b1} = \text{representative boundary layer thickness} \approx R_{noz}, \]
\[ U_{\infty, b1} = \text{representative boundary layer velocity} \approx 0.5 U_{jet}, \text{ and} \]
\[ \text{therefore}, \]
\[ Sr \approx 10^{-4} \ll 1. \]

In this case, the Strouhal number equals the number of particles "seen" leaving the surface per boundary layer thickness traveled in the streamwise direction by the flowing gas.

The low value of the Strouhal number indicates that locally, very near the rock surface, there is effectively no blowing. The only influence of the rock chips is due to the momentum and temperature deficits, relative to the solids-free flow, advecting downstream in the outer region of the flow-field.

In this study, the influence of entrained spalls on the transport phenomena is, perhaps crudely, approximated by assuming that it is the same as that of transpiration blowing. The modeling is implemented by applying blowing boundary conditions to only the convective terms of the conservation equations. Heat fluxes, and viscous stresses are calculated from pure-fluid wall functions (equations 5.36 and 5.37) without including terms for the cross-stream variations in shear stress and heat flux that would occur during true transpiration blowing.

The simulation results including variable heat capacity and mass
injection are plotted in Figure 6.4 and agreement to within 5% of experimental Stanton numbers and hole radii is achieved. Figure 6.5 is a composite plot of the results from Figures 6.2 through 6.4 illustrating the relative effects of each change in the modeling assumptions on the simulation accuracy. The use of variable heat capacity and mass injection make comparable contributions toward diminishing the discrepancy between predicted and experimental results. The mass injection model changes the predicted hole radii and Stanton numbers because most of the effect of mass injection occurs near the stagnation point where the penetration rate, and therefore the calculated blowing rate, is highest. This changes the heat transfer distribution along the rock surface and results in a change in the predicted hole shape.

The decrease in heat transfer that would occur if the full effect of continuous transpiration blowing were present, including the direct effect on the inner region shear stress and heat flux, is investigated below as a limiting case for the predicted results. Moffat and Kays (1984) present a well known theoretical based relationship for estimating the ratio of the Stanton number with blowing to that without blowing as a function of the "blowing ratio" ($B_h$):

$$\frac{St}{St_0} = \frac{\ln(1 + B_h)}{B_h}$$  \hspace{1cm} (6.5),

where $St = $ Stanton number with blowing, $St_0 = $ Stanton number without blowing,

$$B_h = \frac{\left(\rho u_h\right)_{\text{wall}}}{\left(\rho u_t\right)_{\text{St}}}. $$
Figure 6.4 Stanton number versus hole radius -- comparison of experimental results with predictions made using variable heat capacity and mass injection.
Figure 6.5 Stanton number versus hole radius -- composite plot illustrating effects of changes in modeling assumptions on prediction accuracy.
\( u_n \) = the velocity component in a direction normal to the wall, \( u_t \) = the velocity component tangent to the wall, and the subscript "\( \omega \)" denotes conditions at the boundary layer edge. Typical parameter values calculated from field drilling rate data for a stand-off distance of 20 nozzle radii are \( (\rho u_n)_{wall}/(\rho u_t)_{\omega} = 0.02 \),

\[
B_n \approx \frac{0.02}{St}, \text{ and}
\]

\[
St_0 \approx 6 \times 10^{-3}.
\]

Therefore

\[
\frac{St}{St_0} \approx 12 \%.
\]

This analysis demonstrates that the Stanton number can decrease by over a factor of eight if the full effect of blowing is present at every point along the rock boundary. Since spallation is modeled in the present study by including the effects of blowing on the convective terms only, the predicted drop in heat transfer is only about 35-40 \% (see Figure 6.5).

In addition to matching predicted and experimental curves of Stanton number versus hole radius, matching the curve of hole radius versus SOD is important since it is SOD that determines both penetration rates and hole radius for a given set of nozzle conditions. Figure 6.6 illustrates that good agreement is obtained between predicted and experimental non-dimensional hole radii as a function of non-dimensional stand-off distances from 20 to 70. Also shown are predictions made by
Figure 6.6 Comparison of predicted and experimental hole radii versus SOD.
Rauenzahn (1986). These results act as a secondary verification of the accuracy of the modeling techniques that have been employed since in this case the hole radii are parameterized by stand-off distance instead of Stanton number.

However, it should be understood that detailed agreement with boundary layer velocity and temperature profiles is necessary to validate the modeling of the erosion of the boundary layer as described above. The present approach has been taken only for purposes of approximating and placing bounds on the potential influence of spall liberation on the transport phenomena occurring during spallation drilling. Future experimental studies of the physical interactions between gases and solids within turbulent boundary layers are necessary before more detailed modeling can be justified for solution of the flow-field that exists during thermal spallation drilling. Therefore, the results presented above should not be viewed as unequivocal proof that the modeling technique is correct, despite the excellent agreement that has been achieved. However, they suggest a plausible mechanistic interpretation of how solids ejection during thermal spallation may influence drilling or penetration rates.

6.3 Simulation of Full-Scale Drilling

The simulator accuracy was assessed in the previous section by comparing curves of predicted Stanton number versus non-dimensional hole radius with hole radii and Stanton numbers calculated from measured
drilling rates (Rauenzahn, 1986) and surface temperatures (Chapter 4). Unfortunately, full scale penetration rate and hole radius measurements are not well suited for model validation because of the erratic mode of operation typically employed by commercial operators. Usually, holes are drilled in an oscillatory, uncontrolled manner in which the burner head is lowered until it hits the hole bottom whereupon it is raised and held for a short time and then lowered again. Hole shapes and penetration rates during this type of operation are unlikely to be the same as those obtained during the steady-state drilling predicted by the simulator developed in the present study. However, in order to assess the effects of several operating parameters on drilling performance, steady-state predictions have been made for the flame-jet nozzle type used by Browning (1981) at Barre, VT. The operating conditions and parameters used are listed below in Table 6.1.


\[
\begin{align*}
\rho_{\text{jet}} & = 0.8 \text{ kg/m}^2 \\
C_{p,\text{jet}} & = 2366 \text{ J/kg K} \\
U_{\text{jet}} & = 837 \text{ m/s} \\
T_{\text{jet}} & = 1940 \text{ K} \\
Re_{\text{r}} & = 350,000 \\
R_{\text{noz}} & = 3.81 \text{ cm} \\
P_{\text{jet}}/P_{\text{amb}} & = 4.4 \\
\gamma & = C_p/C_v = 1.26
\end{align*}
\]

Figure 6.7 compares the relationship between Stanton number and non-dimensional hole radius predicted in the present study with predictions made by Rauenzahn (1986) and experimental results calculated from the overall average penetration rates reported by Browning (1981) in Barre granite, and Williams et al. (1988) at Pedernal Hills, N.M. In both
Jet Temperature = 1940 K.
Fuel-Oil and Air.
Re = 3.5 \times 10^5.

Figure 6.7 Variation of Stanton number with hole radius -- comparison of predictions with full-scale field drilling results.
cases, estimated errors in "experimental" Stanton numbers are approximately ± 35% due to uncertainties in the measured penetration rates, and uncertainties in the calculated rock spallation temperatures.

The most important uncertainty associated with the rock surface temperature used for calculation of the Stanton number achieved in Barre granite occurs because the heat flux is estimated using Weibull parameters determined from small-scale drilling experiments (Chapter 4). The estimated value of the full-scale heat flux is 5.8 MW/m², compared to the maximum small-scale measured heat flux of approximately 3.0 MW/m². The predicted surface temperature rise at 5.8 MW/m² is 1330 K, which is above the granite melting temperature of approximately 1320 K (Carmichael, 1982). The Weibull theory presented in Chapter 4 is based on a brittle failure mechanism and is therefore unlikely to be valid if temperatures approach the rock melting point where softening occurs. Near the rock melting point, the brittle failure mechanism probably becomes influenced by inelastic, or ductile deformations that occur in some of the component minerals in the granite. Therefore, it is doubtful that the predicted surface temperature, and therefore the calculated Stanton number, is accurate at such high heat fluxes.

The estimated heat flux for the Pedernal Hills drilling data is 3.2 MW/m² leading to a calculated surface temperature of 1128 K which is below the melting point of most granites. However, no experiments were performed in the type of granite found at Pedernal Hills to determine
the correct Weibull parameters. Therefore, for estimating purposes, the arithmetic average of the Barre and Westerly granite values determined in the present study (Chapter 4) were used.

The comparison between predictions and experiments shown in Figure 6.7 illustrates that the values predicted by Rauenzahn (1986) and in the present study are within the estimated ± 35 % experimental uncertainty interval. Unfortunately, insufficient data has been obtained to test the modeling assumptions in detail.

Figure 6.8 illustrates the expected variation of Stanton number with nozzle Reynolds number \((Re_n)\) at a fixed SOD of 20. Most Subsonic stagnation point correlations predict that \(St \propto Re_n^{-0.2}\) (Hrycak, 1981). However, the predicted trend illustrated in Figure 6.8 indicates that an approximately linear decay of Stanton number with increasing Reynolds number occurs. This is presumably due to a superposition of multiple phenomena related to the supersonic, impinging flow-field. Nevertheless, the absolute variation of \(St\) with \(Re_n\) is small. For \(Re_n\) ranging from 80,000 - 700,000, the predicted Stanton number changes by only 20 %. This relatively minor variation is overshadowed by the uncertainties inherent in the flow-field modeling.

Figure 6.9 illustrates the predicted variation of \(St\) with jet temperature, non-dimensionalized by the initial rock temperature (300 K), at SOD = 20. Stanton number increases by approximately 20 % as the jet temperature ratio is increased from 5.0 to 9.4. The actual
Figure 6.8 Predicted variation of Stanton number with Reynolds number for a full-scale fuel-oil/air torch.
temperature ratio used by Browning was approximately 6.5. Although the maximum attainable jet temperature ratio for fuel-oil and air is approximately 7.3, corresponding to stoichiometric proportions and no nozzle cooling, higher values are presented in Figure 6.9 for illustration of predicted trends. Other fuel-oil/oxidant combinations and/or preheating of feed streams would be necessary to achieve the higher temperatures in practice.

Figure 6.10 illustrates the predicted variations of penetration rate (m/s) with nozzle volumetric flow rate (m³/s) for three non-dimensional stand-off distances (SOD). The curve for SOD = 20 was calculated directly from the results for St versus Reₚ shown in Figure 6.8. The curves for SOD = 15 and SOD = 30 were estimated from the simulation results at Reₚ = 350,000 by assuming that each ratio of U₃ₚ at a given value of SOD to that at SOD = 20 remains constant, for all Reynolds numbers, at ratio values presented in the form of Stanton numbers in Figure 6.7. Figure 6.10 provides additional physical insight from the non-dimensional forms of the same results given earlier in Figures 6.7 and 6.8. The results shown in Figure 6.10 indicate that doubling the volumetric flow rate through the nozzle will approximately double forward penetration rates at volumetric flow rates less than about 3 m³/s. At volumetric flow rates approaching 6 m³/s, the increase of penetration rate with volumetric flow rate has leveled off to less than a factor of 0.5.
SOD = 20.
Fuel-Oil and Air.
Re₉ = 3.5 x 10⁵.

Figure 6.9 Predicted variation of Stanton number with \( T_{\text{jet}} / T_{r,0} \) using a fuel-oil/Air torch.
Figure 6.10 Estimated variation of drilling rate with volumetric flow rate through nozzle at three different stand-off distances.
The results presented in Figures 6.7 - 6.10 indicate that the highest drilling rate for a given hole diameter is achieved by maintaining the shortest possible SOD (within the range tested), the highest jet temperature ratio, and the highest possible volumetric flow rate (jet Reynolds number). The net trade-off of operating conditions will be limited in practice by economic considerations. For example, operation at smaller SOD's results in higher drilling rates but increases the likelihood of damage to the end of the drill head caused by flying spalls, as observed by Browning (1981). In addition, higher operating temperatures, which result in faster drilling, require more cooling of the drill head and increase the possibility of damage if cooling water supply interruption occurs. Therefore, data on drill head lifetimes and operating costs must be obtained and incorporated before economically optimum drilling strategies can be devised. It is therefore recommended that future studies be performed aimed at experimentally determining drill head lifetimes under various operating conditions and using various materials of construction and drill head designs.

6.4 Summary

Improvements in the basic simulator accuracy have been made by incorporating the effects of the variation of the gas heat capacity with temperature, and by accounting for the injection of rock chips (spalls) in the turbulent boundary layer. The latter effect is modeled in a similar fashion to continuous transpiration blowing along the spalling
rock surface. The combination of these modeling improvements, and the use of a numerical solution algorithm with higher order accuracy has lead to a level of agreement between predictions and small-scale experiments of \( \pm 5\% \). Comparison of predictions and experiments for full-scale drilling lead to worse agreement. Results obtained from both the simulator developed in the current study, and the one developed by Rauenzahn (1986) were within \( \pm 35\% \) of results measured during full-scale field experiments.

The most important uncertainty in the modeling used in the present study is the approximation of the effects of rock chips on the flow-field by using continuous mass injection boundary conditions. Fundamental experiments aimed at assessing the influence of intermittent mass injection and the presence of solid particles on turbulent boundary layer structure are necessary before more detailed and more rigorous modeling can be developed. However, the analysis of the potential effects of transpiration blowing and the good agreement achieved using the present method indicates that a substantial decrease in heat transfer due to spall entrainment along the rock surface may be occurring. Future modeling refinements will necessarily require more fundamental experiments aimed at detailed determination of gas-solid momentum and energy interactions that occur in a turbulent boundary layer.
7 Conclusions and Recommendations

This chapter contains a summary of the conclusions and recommendations that have been made in Chapters 4-6. Section 7.1 refers to the work detailed in Chapter 4. This work was aimed at developing an improved understanding of the interaction between the rock mechanics and heat transfer occurring during thermal spallation drilling. Section 7.2 summarizes the results of the flow-field modeling described in Chapters 5 and 6. The objective of this work was to be able to accurately predict the heat transfer rate to the rock surface under a variety of flow-field conditions that occur during flame-jet thermal spallation drilling.

7.1 Relationship Between Spallation Surface Temperature and Applied Heat Flux

The theoretical modeling work of Dey (1984) and Rauenzahn (1986) led to an expression for the average surface temperature during spallation as a function of applied heat flux, rock physical and thermal properties, and two strength related parameters:

\[ T_s = T_{r,0} + \left[ \left( \frac{Q}{\rho C_p} \right)^3 \left( \frac{1 - \nu}{\beta r E} \right)^m \left( \frac{2(0.693)}{\pi C_L^2} \right) \left( \frac{m - 3}{\alpha_r} \right)^3 \right]^{1/(m+3)} \]  \hspace{1cm} (3.11)

Prior to this thesis, equation (3.11) was used with Weibull strength parameters determined from mechanical tests conducted at room temperature (Dey and Kranz, 1985). Experimental difficulties and the
inability to directly measure surface temperatures and heat fluxes hindered earlier attempts to validate equation (3.11) during steady-state thermal spallation (Rauenzahn, 1986). Furthermore, the inconsistency between spallation temperatures obtained by Rauenzahn (1986) using a small beam diameter (≈ 5.0 mm) laser and those obtained under a welding torch flame with a 2.54 cm spouting diameter suggested a possible dependency of the spallation characteristics on the mode of heat transfer. Specifically, the laser beam diameters used in previous studies were probably too small to uniformly activate a statistically significant number of flaws within the heated sample volume.

The current study has been aimed at understanding the variation of spallation surface temperature with applied heat flux and extending the range of the experimental data. Surface temperatures were measured for high powered (25 kW) laser and supersonic flame-jet induced heat fluxes ranging from 0.3 - 3.0 MW/m². The major conclusions are:

1) At heat fluxes greater than 1 MW/m², rock surface temperature rises are up to a factor of two higher than those estimated using mechanically determined Weibull parameters.

2) The dominant processes occurring during thermal spallation are independent of whether a laser beam or a flame-jet is used as the heat source.

3) Temperatures measured at the point on the rock surface where the first spall appears are more than a factor of two less than median temperatures on a spalling surface at typical values of
incident heat flux.

4) Stanton numbers calculated from field drilling data obtained by Rauenzahn (1986) are significantly closer to those predicted by Rauenzahn's simulator than originally thought.

5) Westerly granite spalls at lower temperatures than Barre granite at the same heat fluxes (for example, approximately 200 K lower at 3 MW/m²).

Analysis of the measured temperature rise versus heat flux data presented in Figures 4.13 and 4.14 leads directly to conclusions 1 and 2. Errors in the estimated heat flux values are expected to be in the range of approximately ± 35 % because of the limited time (50 ms) and temperature resolution capabilities (typically ± 70 °C) of the I.R. scanner used for measuring transient surface temperature rise profiles. Additional scatter is attributed to the naturally occurring statistical distribution of failure strengths in a given rock sample. Nevertheless, the general trends mapped out for surface temperature as a function of heat flux are similar regardless of whether laser or flame jet induced thermal spallation tests are being conducted.

A practical implication of conclusion 1 leads to conclusion 4 as illustrated in Figure 4.16 where Stanton numbers derived from measured drilling rates and surface temperatures estimated from equation (3.11) approximately double as a result of using the newly determined Weibull parameters. As stated in conclusion 4, this accounts for a substantial portion of the factor of four discrepancy between Rauenzahn's (1986)
predicted and experimental results. Factors contributing to better predictive capabilities have also been identified in this thesis and are summarized in Section 7.2.

Conclusion 3 was reached by comparing steady flame-jet spallation data shown in Figures 4.13 and 4.14 with global spallation onset temperatures illustrated in Figure 4.15. The global onset temperature, defined as the temperature achieved when the first spall leaves a heated surface, is seen to be more than a factor of two less than that achieved under true steady-state conditions except at relatively low heat fluxes (< 1 MW/m²). This phenomenon occurs presumably because the global onset temperature is representative of the weakest point on a spalling surface and therefore is in the low temperature end of the spallation surface temperature probability distribution curve.

A possible alternative to the rock failure mechanism described by Rauezahn (1986) is presented in chapter 4 for thermal spallation at high heat fluxes. In the proposed mechanism (Figure 4.17), certain components of the rock yield in a ductile, stress-relieving mode thereby inhibiting brittle failure until the stress level is increased by additional heating. At high heat fluxes, this may result in a substantial fraction of rock surface overheating.

Unfortunately, insufficient data exist for determining the mechanism occurring during thermal spallation. Future studies should focus on theoretical modeling of this phenomena, if only in an
approximate sense, to determine whether the observed factor of two or greater overheating temperature rise that occurs at heat fluxes of 2.5-3.0 MW/m² is consistent with the proposed mechanism.

Specific recommendations for further study of the relationship between the surface temperature and applied heat flux during thermal spallation are listed below.

1) **Objective:** Improve the accuracy of measured heat fluxes and surface temperatures during thermal spallation.

   **Recommended approach:** Develop a high speed IR scanner capable of data acquisition rates of at least 50 Hz, higher temperature resolution, and a broader temperature range (up to 1200 °C instead of 1000 °C). Morgan and Schultz (1985) describe the necessary components and considerations for constructing such a device.

2) **Objective:** Improve the theoretical understanding of rock failure mechanism occurring during thermal spallation at high heat fluxes to assess the validity of the Weibull model developed by Dey (1984).

   **Recommended approach:** Numerically solve one-dimensional stress and temperature fields in rock and apply local failure criteria based upon initial crack length distribution and requirements for crack growth as outlined by Wong and Brace (1979) and Ashby et al. (1986). Stress relief by inelastic deformations
of non-spallable inclusions could be incorporated by randomly placing zones that deform under a load. An ideal test case could be simulated in which the initial fractures are distributed about an average length and have randomly distributed orientations. Simulated mechanical testing procedures could be used to determine the Weibull parameters. Then, stress fields corresponding to conditions occurring during thermal spallation could be simulated to determine the variations of surface temperatures and Weibull parameters with heat flux. Such a model would enable one to alter individual parameters and compare the effects to those predicted by equation (3.11). For example, Young’s modulus could be changed for an otherwise identical material and one could test to see if both models predict that surface temperatures vary with an exponent of $1/(m+3)$ on $E$, as given by equation (3.11).

3) **Objective:** Expand data base to include additional $m$ and $\sigma_0$ Weibull parameter values obtained during thermal spallation for a large number of commonly encountered spallable rocks. These rocks should include commercially important ones, for example granites, taconite, and sandstone, and rock types that have been experimentally studied, such as the Pedernal Hills granite pierced by workers at LANL (Williams et al., 1989), Sioux quartzite, and Webatuck Dolomite. **Recommended approach:** Perform thermal spallation tests using a high powered laser while monitoring the surface temperature
with high speed IR scanning and fit the predicted Weibull curve to the data by adjusting the Weibull parameters (see Section 4.5.2).

7.2 Gas Dynamics and Heat Transfer Modeling

The penetration rate during thermal spallation drilling is related to the heat flux and surface temperature according to equation (2.4b):

\[ U_{dr} = \frac{Q}{(\rho C_p)_r(T_s - T_{r,0})} \]  (2.4b)

The heat flux is determined primarily by the gas-side transport phenomena and to a lesser degree by the rock surface temperature. In this study, the Reynolds-averaged mass, momentum, and energy equations have been numerically solved resulting in steady-state heat flux predictions along the rock/gas interface formed during spallation drilling. Local heat fluxes were used to calculate an estimate for the boundary shape by forcing the local consistency criterion to apply:

\[ U_{dr} = V_{dr} \cos(\theta) \]  (2.5)

A k-ε turbulence model was used for evaluation of the turbulent eddy viscosity throughout the flow field except near solid walls where wall functions based on Prandtl’s mixing length theory were used to calculate the heat flux and wall shear stress from mean flow properties.

The numerical method used in this study is a second order accurate, centered differencing scheme that is stabilized by the combined use of a
four-stage, modified Runge-Kutta technique and artificial dissipation terms that suppress odd-even decoupling and allow shock discontinuities to be captured. The computational grid was formed by clustering node points in high gradient regions along the rock boundary and near the nozzle lip, followed by smoothing of the grid with an elliptic equation solver similar to the one proposed by Steger and Sorenson (1979).

The simulation accuracy has been tested by comparing predictions to small-scale field drilling data obtained by Rauenzahn (1986) in Barre VT. and Westerly RI. granite (Figure 6.5). Three cases were numerically simulated: the basic solver, analogous to that developed by Rauenzahn (1986) but with second order accurate advection and node clustering in high gradient regions, the basic solver with the temperature dependence of the gas heat capacity included, and the basic solver with the heat capacity dependence and modeling to account for the entrainment of spalls into the flow-field. The major conclusions are:

1) The use of second-order accurate numerical advection increases the predicted heat fluxes by about 30% over those predicted by Rauenzahn (1986) using first-order accurate advection.

2) The inclusion of a temperature dependent gas heat capacity lowers predicted heat fluxes by about 30% relative to the constant heat capacity model. The heat fluxes are lowered uniformly along the spalling rock surface.

3) The use of a simple "blowing" boundary layer model to simulate the effects of spall entrainment lowers predicted heat fluxes
by about 35%.

4) The combined use of second order accurate numerics, variable heat capacity, and the mass injection model leads to agreement between experimental and predicted Stanton numbers and hole radii to within about ± 5%.

5) The general trend of predicted Stanton number versus hole radius is qualitatively correct whether upwind methods employed by Rauenzahn (1986), or the second order methods described in the present study are used.

The basic solver predicts Stanton numbers (St) that are about a factor of three higher than those measured. These Stanton numbers are higher than those predicted by Rauenzahn (1986). However, this increase is consistent with the well-known tendency for upwind differencing methods, like the one used by Rauenzahn, to "smear" out flow-field gradients (Anderson et al, 1984) thereby lowering predicted stress and heat flux levels.

As stated in conclusion 2, the most important effect of including the temperature dependence of the gas heat capacity was to lower predicted heat transfer rates by about 30% (Figure 6.5). This is consistent with the expected drop in the turbulent thermal conductivity with temperature from the nozzle exhaust condition where \( T_{\text{jet}} \approx 2830 \) K to the relatively cold wall temperature of about 900 K. The turbulent thermal conductivity coefficient \( (\Lambda_t) \) is related to the turbulent viscosity according to:
\[ \lambda_t = \mu_t C_p(T)/Pr_t \] (6.1)

Therefore, since the turbulent Prandtl number and viscosity do not vary with temperature, the turbulent thermal conductivity decreases proportionally to the heat capacity.

Conclusion 3 refers to the modeling of spall entrainment as continuous mass injection along the rock boundary, similar to the situation that exists in a boundary layer with transpiration blowing (see Section 6.2.3). However, unlike blowing, in this study spalls are assumed to affect only the outer region of the turbulent boundary layer. Unfortunately, insufficient data exist to thoroughly test the spallation simulator over a wide range of nozzle pressures, temperatures, and stand-off distances.

The recommendations are:

1) **Objective:** Improve the ability to validate the simulation model.

**Recommended Approach 1:** Perform additional small-scale field drilling experiments with varying \( P_{jet}, T_{jet}, Re_{jet}, Ma_{jet} \).

**Recommended Approach 2:** Diminish coupling of rock mechanics with heat transfer by performing drilling experiments in Quartzite. Quartzite is recommended instead of granite because granite, although important from a practical standpoint, is not ideally suited for simulator verification experiments because of the wide distribution of spalling surface temperatures (Figures 4.13 and 4.14) and uncertainties concerning the influence of non-spallable components on surface
characteristics (e.g. roughness) during spallation drilling. For example, protrusions of non-spallable materials may occur because these materials must be eroded away by a combination of spalling in adjacent regions and the sweeping action of the gas flow over the rock. Quartzite is a better material for use in conducting thermal spallation experiments aimed at simulator validation because of the virtually constant surface temperature over a wide range of heat fluxes, the narrow distribution of spallation temperatures at a given heat flux (Table 4.5), and the low concentration of non-spallable components (< 3%, Thurumalai (1969)).

2) **Objective:** Improve flexibility of computational model for prediction of alternate geometries (e.g. cavity formation).

**Recommended Approach:** Set up unstructured methodology for grid generation and flow model implementation.

3) **Objective:** Improve understanding of the influence of spall entrainment on near-wall transport phenomena.

**Recommended Approach:** Perform heat flux and turbulence structure measurements on a simplified flow geometry (e.g. a flat plate) with intermittent solids injection.

4) **Objective:** Develop the ability to rapidly estimate penetration rate and hole geometry trends during thermal spallation drilling.
Recommended approach: Develop lumped parameter model correlations similar to those commonly used for predicting heat transfer in subsonic flows through pipes and over flat plates using non-dimensional parameters to describe flow-field and geometrical characteristics. These models generally relate a dimensionless heat transfer coefficient or Nusselt number (Nu) to the local Reynolds and Prandtl numbers. For the case of thermal spallation where a supersonic expanding jet impinges on a concave surface, Nu is a function of Re, Pr, SOD, $P_{jet}/P_0$, $T_{jet}/T_{r0}$, and inlet turbulence characteristics. Furthermore, suitable relationships must be devised for estimating the variation of heat transfer with distance along the rock boundary from the stagnation point, since most correlations proposed for subsonic impinging jets predict only stagnation point Nusselt number values (Kataoka et. al, 1985; Anderson and Stresino, 1963; Popeil at al., 1980; Hrycak, 1981; Perry, 1955). Insight into the functional relationships required for lumped parameter modeling could be gained from the combined use of the predictive simulation developed in this study and further experimental data. The major advantage of adopting a lumped parameter approach for estimation of performance characteristics would be that reasonable accuracy might be possible at orders of magnitude less computational time. As long as the lumped parameter model is capable of correctly predicting trends, some loss of accuracy would be more than compensated for by the increased computational speed.
Appendix

Computer Program Listing

SPSIM -- Spallation Simulator

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\textbf{Nomenclature}

\begin{itemize}
  \item \text{} a \quad \text{Speed of sound (m/s).}
  \item \text{} \text{A} \quad \text{Slope of best-fit line.}
  \item \text{} B_{th} \quad \text{Blowing parameter } ((\rho u_n)_{th}/St_0(\rho u_c)_{th}).
  \item \text{} CFL \quad \text{Non-dimensional timestep (a\Delta t/\Delta x).}
  \item \text{} C_i \quad \text{Spall diameter:thickness ratio.}
  \item \text{} C_p \quad \text{Constant pressure heat capacity (J/kg K).}
  \item \text{} C_v \quad \text{Constant volume heat capacity (J/kg K).}
  \item \text{} C_{\mu} \quad \text{Universal turbulent viscosity constant (0.09).}
  \item \text{} C_{1,2,3} \quad \text{Parameters for k-\varepsilon model.}
  \item \text{} D_2 \quad \text{Second difference contribution to artificial dissipation.}
  \item \text{} D_4 \quad \text{Fourth difference contribution to artificial dissipation.}
  \item \text{} E \quad \text{Young's modulus (GPa); total energy.}
  \item \text{} f \quad \text{Frequency of spall liberation (s^{-1}).}
  \item \text{} F \quad \text{Body force vector.}
  \item \text{} G(\sigma) \quad \text{Cumulative probability of failure.}
  \item \text{} G \quad \text{Convective radial flux vector.}
  \item \text{} h \quad \text{rms roughness height (m).}
  \item \text{} H \quad \text{Enthalpy (J/kg K).}
  \item \text{} I \quad \text{Internal energy (J).}
  \item \text{} k \quad \text{Turbulent kinetic energy.}
  \item \text{} k_r \quad \text{Rock thermal conductivity (W/m K).}
  \item \text{} L \quad \text{Unidiff Laplacian operator.}
  \item \text{} l \quad \text{Sample length (m); turbulent length scale (m).}
  \item \text{} l^* \quad \text{Non-dimensional turbulent length scale (l/R_{noz}).}
  \item \text{} m \quad \text{Homogeneity parameter.}
  \item \text{} M \quad \text{Source term vector.}
  \item \text{} M_{jet} \quad \text{Jet Mach number (U_{jet}/a_{jet}).}
  \item \text{} N_\varepsilon \quad \text{Non-dimensional scaling parameter (\varepsilon_{jet}R_{noz}/k_{jet}U_{jet}).}
  \item \text{} P \quad \text{Pressure (Pa).}
  \item \text{} P_k \quad \text{Production rate of turbulent kinetic energy.}
  \item \text{} P_{max} \quad \text{Ideal maximum achievable power output (W).}
  \item \text{} P_\varepsilon \quad \text{Production rate of dissipation of turbulent kinetic energy.}
  \item \text{} Pr_i \quad \text{Turbulent Prandtl number (0.86).}
  \item \text{} Q_i \quad \text{Heat flux in direction i.}
  \item \text{} Q \quad \text{Local heat flux to rock (W/m^2); source term in grid smoothing routine.}
  \item \text{} Q_{max} \quad \text{Heat transfer for producing W_{max} (Q).}
  \item \text{} r \quad \text{Radial coordinate direction.}
  \item \text{} R \quad \text{Ideal gas constant (8.314 J/mol K).}
  \item \text{} R_{Re_k} \quad \text{Turbulent Reynolds number (k_{jet}^{1/2}R_{noz}/\nu_{jet}).}
  \item \text{} R_{noz} \quad \text{Nozzle radius (m).}
  \item \text{} R_{jet} \quad \text{Jet Reynolds number (U_{jet}R_{noz}/\nu_{jet}).}
  \item \text{} R_h \quad \text{Hole radius (m).}
  \item \text{} R_{pr} \quad \text{Ratio of local to average penetration rates.}
  \item \text{} \text{B} \quad \text{Diffusive radial flux vector.}
  \item \text{} SI \quad \text{Spallability index.}
\end{itemize}
SOD  Dimensionless stand-off distance (≡ Z_{dr}/R_{dr}).
St  Stanton number.
St_{rh}  Stanton number based on heat flux from roughness layer edge to the wall.
Sr  Strouhal number (f_{x}δ_{b1}/U_{m,b1}).
S  Diffusive axial flux vector.
t  Time (s).
t'  Fluctuating component of temperature (K).
T  Temperature (K).
u  Radial velocity component.
u_i  Velocity component in coordinate direction "i" (i=1,2,3).
u_f  Friction velocity (m/s) = (τ_w/ρ_w)^{1/2}.
U_{dr}  Local drilling velocity normal to rock surface (m/s).
U_{jet}  Jet velocity (m/s).
U_{e,b1}  Tangential velocity at boundary layer edge (m/s).
U  State vector.
v  Axial velocity component.
V  Volume (m^3).
V  Velocity vector.
V_{dr}  Forward penetration rate of drill head (m/s).
W  Work (J).
x_i  Coordinate in direction i.
y  Distance measured in a direction perpendicular to the rock wall.
z  Axial coordinate direction; distance measured normal to rock surface (toward rock) (m); Fourier space representation of spatial discretization operator.
Z_{dr}  Stand-off distance (m).
Greek Letters

\( \alpha \)
Thermal diffusivity \((m^2/s)\); multistage time integration parameter; grid stretching parameter.

\( \alpha_0 \)
Weibull parameter \((\text{MPA-m}^3/\text{m})\).

\( \delta \)
Boundary layer thickness \((\text{m})\).

\( \delta_{ij} \)
Kronecker delta \((1 \text{ if } i=j; \ 0 \text{ if } i\neq j)\).

\( \epsilon \)
Turbulent dissipation rate of turbulent kinetic energy.

\( \gamma \)
Heat capacity ratio \((C_p/C_v)\).

\( \kappa \)
von Karman turbulence constant \((0.41)\).

\( \lambda \)
Thermal conductivity \((\text{W/m K})\).

\( \mu \)
Dynamic viscosity \((\text{kg/m s})\).

\( \nu \)
Kinematic viscosity \((m^2/s)\); Poisson’s ratio.

\( \rho \)
Density \((\text{kg/m}^3)\).

\( \sigma \)
Rock stress; deviatoric component of normal fluid stress \((\text{MPa})\).

\( \tau \)
Deviatoric stress \((\text{MPa})\).

\( \eta \)
Vertical axis coordinate in 2-D computational space.

\( \theta \)
Angle between tangent to the rock surface and the horizontal axis; coordinate direction.

\( \xi \)
Horizontal axis coordinate in 2-D computational space.

\( \omega \)
Boundary movement acceleration parameter.
Subscripts

amb    Property at ambient conditions.
c      Compressive.
dr     Drill.
e      Exact solution value.
exp    Experimental value.
f      Cell face property.
gf     Geothermal fluid property.
jet    Property at nozzle outlet conditions.
k      Relevant to turbulent kinetic energy equation.
e      Relevant to turbulent dissipation rate equation.
loc    Local conditions.
max    Maximum.
c      Node-cell property.
noz    Property at nozzle outlet conditions.
0      Initial; inherent property (Weibull theory); without blowing.
∞      Boundary layer edge property.
p      Conditions at the first computational point away from the wall; predicted value.
r      Rock property; radial component.
rh     Conditions at edge of surface roughness.
s      Rock surface property.
sp     At spallation.
st     Stagnation point property.
t      Turbulent
T      Total.
w      Wall conditions.
z      Axial direction.

Superscripts

'      Fluctuating component.
-      Cell averaged; Reynolds averaged.
+      Scaled with wall variables.
References


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