FINITE DIFFERENCE METHOD
FOR
ELECTROMAGNETIC SCATTERING PROBLEMS
by
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Submitted to the Department of Electrical Engineering and Computer Science on May 4, 1990 in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

ABSTRACT

The issues related to the application of the finite difference techniques to electromagnetic scattering and radiation problems are investigated. These issues include absorbing boundary conditions, discretization schemes on unstructured grids, treatment of frequency dispersive materials, and application to radiation by electromagnetically coupled slab structures.

A technique, which employs the pseudo-differential operator, is studied and applied to derive the absorbing boundary conditions for circular and elliptical boundaries. The pseudo-differential operator technique, based on Engquist and Majda's approach, is modified. For a circular boundary, the modified pseudo-differential operator technique leads to an absorbing boundary condition equivalent to that of Bayliss and Turkel's second-order condition. The modified pseudo-differential operator technique is then applied to derive the second-order absorbing boundary condition for an elliptical boundary. The effectiveness of the second-order absorbing boundary condition on the elliptical boundary is illustrated by calculating scattered fields from various objects. It is shown that for elongated scatterers, the application of the elliptical boundary and the corresponding absorbing boundary condition can reduce the size of the computational domain.

A finite-difference time-domain technique on triangular grid is investigated. The electric and magnetic fields are discretized on triangular grids. Maxwell's equations are approximated using the finite difference and control region techniques. The Delaunay tessellation is identified as the control region. Finite computational domain is achieved by using the circular or elliptical boundaries with appropriate boundary conditions. This algorithm provides second order accuracy in time and space when the grids are regular. For irregular grids, the spatial discretization is accurate to first order.
Abstract

Triangular grids provide the flexibility to accurately model arbitrary geometries. This algorithm is illustrated by analyzing the scattering properties of simple geometries. The accuracy of the procedure is verified by comparing results with those obtained using frequency domain techniques. Numerical results indicate that the algorithm is accurate and stable. Furthermore, the triangular grid yields more accurate geometrical modeling than rectangular grids.

A computationally efficient finite-difference time-domain technique is developed to treat frequency dispersive materials. The dispersive characteristics of the material are modeled using ordinary time differential equations which may be derived from a collection of permittivity/permability data in the frequency domain. Using the time domain models for the dispersive characteristic, the discretization, schemes is simple and the additional memory requirements is small. The proposed algorithm is numerically tested with simple one-dimensional and two dimensional configurations and compared with exact solutions in the frequency domain. The algorithm can be generalized to three-dimensional configuration with little alternation.

The finite-difference time-domain technique using rectangular grids is implemented to analyze the radiation properties of electromagnetically coupled slab structures. The slab structure consists of a conducting ground plane, a dielectric layer, and a metallic "heatsink" located above the air/dielectric interface. Radiation sources, which include various dipole configurations, are placed over the dielectric slab. These configurations simulate integrated circuit package environment. The finite-difference time-domain technique, which is primarily based on Yee's algorithm, is employed to calculate the emission levels of the slab structures. It is found, due to waveguide cutoff, that only the VED and HMD contribute to emissions. The inclusion of the dielectric material may either increase or decrease the total radiated power depending on the source. When the dimensions of the heatsink approach the critical resonance dimension, significant emission can occur. Emission level may be reduced by either grounding or shielding using metal-mixed rubber.

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To my beloved wife Cindy

To my parents and grandmother

To my grandfather
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Chapter 1

INTRODUCTION

1.1 Background

The analysis of electromagnetic scattering characteristics of arbitrary geometries is essential to numerous practical and on-going research problems [1-105]. These problems include antenna design, target detection and identification, telecommunication, analyzing microwave devices and integrated circuit package, and electromagnetic pulses. There has been a continuous demand to develop accurate and efficient techniques for analyzing these problems. In fact, this demand is even higher in the recent years due to advancements in technology and popularity of supercomputers.

There are numerous techniques developed to analyze the electromagnetic problems. These techniques are categorized into analytical techniques, asymptotic techniques, and numerical techniques. Among these categories, the analytical techniques offer elegant solutions which give insight to fundamental electromagnetic phenomena. Due to the complex nature of the problems, however, these analytical solutions are available only for few simple geometries [92]. Important asymptotic techniques
are the Geometrical Theory of Diffraction (GTD) [62-64] and the Physical Theory of Diffraction (PTD) [65]. The GTD and PTD techniques are developed to approximately analyze the problems of electromagnetic scattering. In the high frequency limit, the approximations are based on the understanding of fundamental electromagnetic phenomena provided by a few analytical solutions. To apply the asymptotic techniques to problem involving a general geometry, the geometry is assumed to be composed of a number of sections with simple shapes to which analytical solutions exist. Furthermore, by assuming the electromagnetic phenomena to be local, the appropriate analytical solutions applicable to the local geometries are used. The solution is obtained by piecing together approximate solutions corresponding to the different sections of the geometry. The asymptotic techniques are computationally efficient, but cannot satisfy growing accuracy requirements. For problems involving complex geometries, numerical techniques should be used in order to obtain accurate solutions. Numerical techniques are less efficient as compared to other techniques in terms of computational time and memory requirements. Nonetheless, because of the availability of high speed computers and demands on accurate analysis, the application of these techniques have widen rapidly.

A number of numerical techniques have been developed to solve electromagnetic scattering problems. These techniques can be applied to analyze either integral or differential equation formulations. The Method of Moments (MoM) often corresponds to numerical techniques solving integral equations; while the Finite Element Method (FEM) and finite difference method correspond to numerical techniques solving differential equations. Among these techniques, the finite difference method is
the simplest. It can be applied to frequency domain, the *Finite Difference Frequency Domain* (FD-FD), or to the time domain, the *Finite Difference Time Domain* (FD-TD).

For the electromagnetic scattering problems, although both the integral and differential equation formulations are rigorous, two techniques have different advantages. A major advantage in differential equation formulation is its simple discretization. This simplicity appears in treating scatterer geometry and dielectric/magnetic materials. The major advantage in applying the integral equation formulation is that a smaller computational domain may be used which translates to fewer discretization nodes and smaller memory requirement.

When the FD-FD or FD-TD techniques are applied to the differential formulations, the discretization procedure normally requires simple arithmetic only. The simplicity holds for inhomogeneous materials, anisotropic materials, nonlinear materials, time-varying materials [15], and frequency dispersive materials. In applying the MoM to the integral equation formulation, evaluation of multiple integrals, treatment of singular integrands, evaluation of special functions, and other intense numerical calculations are often required; this is primarily due to the complexity of the Green’s functions in the integral equation formulations. Singularities appear in the Green’s functions when the source and observation points coincide.

For scattering problem, the computational domains of the MoM are smaller than those of the FD-FD or FD-TD techniques. Due to the integral equation formulation, the computational domain of the MoM is confined to within the geometries.
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On the other hand, the theoretical computational domains of the FD-FD and FD-TD are unbounded. The unbounded computational domain is unsuitable for computations. To make the computational domain finite, an artificial boundary is placed to enclose the geometries; then, a so-called absorbing boundary condition (ABC) [28-51] is imposed to simulate the free space. As a result, the computational domain is defined by an artificial boundary. The minimum separation between the outer boundary and the scatterer is determined by the ability of the absorbing boundary condition to simulate the free space. The higher the absorbability of the absorbing boundary condition, the smaller the computational domain will be. It turns out that the artificial boundary can be located at a reasonably close distance away from the geometries. In fact, due to the high absorbability of the absorbing boundary condition and the localized nature of the finite difference techniques, the advantage of small computational domains associated with the MoM becomes less significant.
1.2 The Finite Difference Techniques

The finite difference technique has been applied to the problems of electromagnetic scattering [1-22]. In recent years, most of the research efforts have been directed at achieving more accurate discretization schemes, better absorbing boundary conditions, more efficient implementations on supercomputers, and applications to various electromagnetic problems. Examples of these problems are microwave components, wave guides, scattering problems, and problems related to medical applications. The primary concerns in applying the finite difference techniques are the discretization and absorbing boundary condition. The discretization scheme is crucial in modeling geometry of the scatterer, treating dielectric/magnetic materials, implementation, and accuracy. The absorbing boundary condition is crucial in reducing the size of the computational domain, the number of discretization nodes, and hence, the memory requirements and the computational time.

1.2.1 Discretization

The rectangular grid discretization is perhaps the most widely used discretization in computational electromagnetics and, in particular, scattering problems. In the rectangular grid, arbitrary geometries are approximated by staircases. The discretization of electric and magnetic fields follows Yee's algorithm [1]. In Yee's algorithm, the electric and magnetic fields are spatially interlaced. Within a rectangular cell, electric fields are placed along the edges, and magnetic fields are placed at the face centers. Furthermore, these electric and magnetic fields are temporally interlaced for time
domain problems. The spatial and temporal derivatives are approximated in a finite difference manner which provides accuracy to the second order (i.e., errors are proportional to the square of the grid separations or time steps). For the rectangular grid, keeping track of geometry is trivial and only requires integer arithmetic. Consequently, the implementations of FD-TD and FD-FD are straightforward with minimum storage requirements. The finite difference techniques have been applied to many problems. Yee [1] applied them to initial boundary value problems in isotropic media. Taflove [12] employed the FD-FD technique to obtain scattering characteristics for simple geometries. Taflove [13] also employed the FD-TD technique to obtain frequency domain solutions for scattering problems. By exciting with incident sinusoidal electric fields and waiting for the steady state, frequency domain solutions were extracted from the time domain technique; this investigation illustrated an alternative path in obtaining frequency domain results and the stability of the FD-TD technique. Holland [2,3] employed the FD-TD technique to analyze electromagnetic pulse problems. With the simple staircase approximation, reasonable transient responses were obtained for a few realistic but simple conducting targets. These techniques have since been thoroughly investigated and a considerable amount of effort has been expended in the computer code development.

Although the rectangular grid provides the simplest discretization scheme, the accuracy of the staircase approximation has been questioned. To improve the staircase approximation, several types of grids have been proposed. The generalized curve linear grid [4] models arbitrary scatterers by defining particular orthogonal curvilinear coordinate systems. Holland derived a finite difference formulation for these
1. Introduction

girds. The formulation was applied to spherical coordinate system [5]. For general scatterers, this finite difference scheme appears cumbersome. The distorted rectangular grid responds to the shortcoming of the staircase approximation by deforming the staircasing grids to match the scatterer. Since grids are essentially rectangular, the simplicity of the rectangular grid is maintained. The distorted rectangular grids have been successfully applied to both frequency and time domain problems [6].

The triangular grid provides accurate modeling of the scatterers. Similar to the distorted rectangular grid approach, the geometry of the interfaces is approximated by linear interpolation. Compared to the rectangular grid, more computer memory is required to keep track of the coordinates. However, the implementation of the triangular grid is simple. Also, with triangular grids, one can allocate arbitrarily dense grids at arbitrary locations. Hence, a triangular grid provides the better modeling of the scatterers and provides more flexibility. One of the discretization schemes in the triangular grid approach calls for control region approximation [9,10,11]. In this approximation, the computational domains are constructed with many Delaunay tessellations, called control regions (one for each triangular node). For these Delaunay tessellations, there exist orthogonal tessellations (Dirichlet tessellations). Because of this orthogonality property, the spatial derivatives can be easily approximated by finite differences. The control region approximation has been applied to frequency domain scattering problems where accurate numerical results have been obtained. In the frequency domain, the Helmholtz equation is integrated over each control region. The application of the divergence theorem converts the Laplace operator term into a closed loop integral of the normal derivatives. The normal derivatives are calculated
in the finite difference manner. This integral can be evaluated by summing contributions from each boundary segment of the control region. The term involving the square of the wavenumber can be approximated by the product of the value of the integrand at the center and the area of the control region. Only simple arithmetic is involved in the discretization procedure. The simplicity of the control region approximation, the flexibility and accuracy in the target modeling make the triangular grid suitable for the scattering problems.

1.2.2 Absorbing Boundary Conditions

The absorbability of the absorbing boundary conditions determines the minimum dimension of the computational domains. The absorbing boundary condition imposed on the outer boundary of the computational domain simulates the free space by permitting only outgoing waves. Methods to derive these conditions include the modal expansion [28], the integral methods [29], and the asymptotic methods [30-45]. Among them, the integral methods are most accurate but not efficient, while the asymptotic methods are least absorbing but are sufficiently accurate and efficient.

The modal expansion method generates frequency domain absorbing boundary conditions. These conditions are applicable to circular boundaries and preserve the sparse nature of finite difference techniques. Computational burdens come in when larger number of modes are needed. Fields outside the computational domain are expanded in terms of circular harmonic modes with unknown coefficients. The finite difference technique is applied to discretize the fields within the computational domain. The harmonic modes are related to the inside fields by imposing the continuity
of fields. The orthogonality between the harmonic modes allows the decomposition of the inside fields to the corresponding modes. The fields for each mode are calculated independently by solving the corresponding finite difference matrix equations. Consequently, the number of matrix equations to be solved equals the number of modes. The unimoment method [28] employs the modal expansion technique and accurate results have been obtained for simple scatterers in free space and layered media. However, for general scatterer geometry, it is shown that the number of modes required may be large [9].

The integral method generates exact absorbing boundary conditions in both frequency and time domains. These conditions are applicable to arbitrary boundaries. However, the sparsity of the finite difference matrices is destroyed in the frequency domain problem, and large computer memory is required for the time domain problem. In the integral method, Huygens' principle is applied and the fields at the outer boundaries are written in terms of fields inside. The application of these conditions to general scatterers shows accurate results in the frequency domain. Since Huygens' principle is exact, the integral method provides perfect absorbing boundary conditions. In addition, the outer boundary can be placed very close to the scatterer. However, due to the global nature of Huygens' principle, the fields at each node of the outer boundary is coupled to fields on the entire integral region. This results in very dense matrices and the computational efficiency of the finite difference technique is not realized. In the time domain, the fields at the outer boundaries are related to the known interior retardation fields and can be evaluated explicitly. Since the integrals are in the form of convolution, fields at many previous times are needed
and memory requirement is increased. Nonetheless, it is claimed [29] that the large memory requirement is compensated by the small computational domain, but no supporting results are available.

The pseudo-differential operator approach has been employed by Engquist and Majda [30] to derive asymptotic absorbing boundary conditions in both the frequency and time domains. These absorbing boundary conditions, which are applicable to rectangular and circular boundaries, preserve the sparse matrix of the finite difference technique and have sufficient absorbability. This technique factors the wave operator into a product of incoming and outgoing wave operators with respect to the computational domain. The incoming and outgoing wave operators satisfy the incoming and outgoing wave equations, respectively. Consequently, the outgoing wave operator simulates the free space and is identified to be the absorbing boundary condition. This absorbing boundary condition is in the form of a pseudo-differential operator, which is a global operator. This is approximated by differential operator and converted to local operator. Noteworthy approximations are Taylor’s expansion, Padé method, Chebyshev polynomials and least-square [36,41]. Among them the second-order Taylor expansion is the most popular and provides well-posed conditions [41]. In the second-order absorbing boundary condition, the normal derivative is expressed in two terms. The first term involves the fields at the boundary and the second term involves the second tangential derivative of the fields at the boundary. When the local wave vector is perpendicular to the boundary, these operators produce no reflected waves. The second-order absorbing boundary condition for the rectangular boundary has been successfully applied to number of problems. How-
ever, only the first-order condition can be applied at the corners of the rectangular boundary, which results in less absorbability. The absorbing boundary conditions for the circular boundary have higher absorbability than those for the rectangular boundary. It can be reduced to the Sommerfeld’s radiation condition when the radius becomes large. However, for elongated scatterers, in order to enclose the scatterer with a circular boundary, the computational domain may be unnecessarily large.

Based on Wilcox’s [51] expansion of the scattered fields, Bayliss and Turkel [33,34] derived asymptotic absorbing boundary conditions in both frequency and time domains. These absorbing boundary conditions are applicable to circular boundaries. They are local operators and have been shown to have higher absorbability than the boundary conditions derived by Engquist and Majda [30]. A circular outer boundary is chosen with the center located at the anticipated scattering center. The scattered fields are written in terms of the cylindrical coordinate variables and expanded into an inverse power series of the radius. This series converges rapidly as the radius increases, and only the leading terms are significant. The absorbing boundary conditions are constructed to absorb the leading terms. The first-order condition is identical to the Sommerfeld’s radiation condition and absorbs the first term of the series. Similarly, the second and higher order conditions absorb two or more terms of the series. Among these absorbing boundary conditions, the second-order condition is the most popular and has been applied to many problems. Accurate results have been obtained with relatively small computational domains. For elongated scatterers, the circular boundary requirement makes the computational domain unnecessarily large. Equivalent absorbing boundary conditions have been derived [42] based on similar
1. Introduction

assumptions and approaches. The absorbability is essentially identical, and circular
boundaries are required. To bypass the circular boundary requirement, numerical ex-
trapolation techniques [46] have been explored. Another way to bypass the circular
boundary requirement is to employ elliptic boundaries.

The absorbing boundary conditions on an elliptical outer boundary can be de-
erved using the pseudo-differential operator approach. While other techniques have
difficulty in handling outer boundaries other than the circular boundary, the pseudo-
differential operator approach is natural in orthogonal coordinates such as rectan-
gular, circular and elliptical coordinates. As has been pointed out, the absorbing
boundary condition for the circular boundary derived using the pseudo-differential
operator approach has less absorbability than that of [33]. Careful examination shows
a possible modification to the factorization scheme used by Engquist and Majda. This
modification will lead to an absorbing boundary condition similar to [33]. By exam-
ining the Engquist and Majda’s factorization scheme for the circular boundary, one
finds that their first-order absorbing boundary condition cannot be reduced to the
Sommerfeld’s radiation condition. The propagation term of the Sommerfeld’s radia-
tion condition is included in their first order condition; however, the decay term of
the Sommerfeld’s radiation condition appears in their second-order condition. The
fact that the terms in the Sommerfeld’s radiation condition are separated suggests
a mis-ordering of terms. A possible modification to their factorization scheme may
involve completing the square of the cylindrical coordinate wave operator. It turns
out that, after completing the square, the Sommerfeld’s radiation condition appears
naturally in the first-order absorbing boundary condition. Also, the second-order ab-
sorbing boundary condition is equivalent to that in [33]. This modified factorization scheme can be applied to derive the absorbing boundary condition for the elliptical outer boundaries. The results can be reduced to the circular case in the limit when the interfocal length vanishes.
1.3 Description of the Thesis

In this thesis, issues related to the application of the finite difference technique are discussed. In particular, the absorbing boundary condition, discretization on unstructured grids, application of the FD-TD technique to analyze problems involving frequency dispersive materials, and the application of the FD-TD technique to analyze radiation characteristics of a slab structure are studied.

In Chapter 2, the absorbing boundary conditions on the circular and elliptical artificial boundaries are derived. The derivation is based on the factorization theorem of pseudo-differential operator. The approach employed by Engquist and Majda is modified so that the first order approximation includes the Sommerfeld's radiation condition. As a result of the modification, the second order absorbing boundary condition is equivalent to that derived by Bayliss and Turkel. The modified technique is applied to derive the absorbing boundary condition on the elliptical boundary. Numerical results illustrate that the absorbing boundary conditions are effective. In the case of the elliptical boundary, the application of absorbing boundary condition can significantly reduce the number of discretization nodes.

In Chapter 3, a FD-TD technique on triangular grids is developed. The technique is the generalization of the control region approximation and the finite difference technique for rectangular grid. The discretization scheme is simple. The Delaunay tessellation are identified as the control region. The FD-TD algorithm is applicable to analyze problems involving arbitrary geometries, permeability, permittivity, and electric and magnetic losses. The circular and elliptical absorbing boundary con-
ditions are implemented. Several testcases are examined. Numerical results show that using the triangular grid provides more accurate geometry modeling than the rectangular grid. The instability criterion is similar to that of the Yee's algorithm. The application of the time domain elliptical absorbing boundary condition can offer substantial reductions in both computation time and memory.

In Chapter 4, the FD-TD technique is generalized to treat frequency dispersive materials. This FD-TD algorithm consists of four steps. Two of these four steps are direct analogs of the conventional FD-TD algorithm, in which electric fields are updated using the magnetic fields and vice versa. In the other two steps, time domain models of the dispersive characteristic are used to relate the electric field to the electric displacement and magnetic field to the magnetic flux density. The time domain models are represented by ordinary time differential equations, which may be obtained from a collection of discrete data in the frequency domain. In particular, the Debye and the molecular resonance models are studied. The algorithm is applicable to multi-dimensional problems, and is demonstrated in one and two dimensions. The algorithm is efficient in terms of computation time and memory requirement.

In Chapter 5, the radiation characteristics of an electromagnetically coupled slab structure is investigated with the FD-TD technique. The slab structure consists of a conducting ground plane, a dielectric layer, and a metallic "heatsink" located above the air/dielectric interface. The radiation source may be either a horizontal electric dipole or a horizontal magnetic dipole over the dielectric slab. The configuration simulates the integrated circuit packages with heatsink structures. The three dimensional FD-TD technique of the rectangular grid is employed to analyze
the radiation characteristic. Emission levels due to different polarizations, dielectric constants, thickness of dielectrics, and heatsink geometries are calculated.

Chapter 6 summarizes concludes the study on the application of the finite difference techniques to electromagnetic scattering and radiation problems.

The main contributions of this dissertation is summarized below. In Chapter 2, Engquist and Majda's factorization scheme for wave equation is modified to derive absorbing boundary conditions for both the circular and elliptical boundaries. For elongated scatterers, it is shown that the elliptical boundary can be used to significantly reduce the total number of discretization nodes in the computational domain. In Chapter 3, a simple time domain discretization scheme for solving electromagnetic scattering problems based on triangular grid and the control region approximation is developed. In Chapter 4, an efficient time domain model for frequency dispersive materials and a finite-difference time-domain algorithm to analyze electromagnetic scattering problems involving frequency dispersive materials are developed. Finally, in Chapter 5, the finite-difference time-domain technique is used to analyze the radiation characteristic of an electromagnetically coupled slab structures.
Chapter 2

ABSORBING BOUNDARY CONDITIONS ON CIRCULAR AND ELLIPTICAL BOUNDARIES

2.1 Introduction

Finite difference methods are becoming increasingly popular in the computational electromagnetics community. The flexibility of finite difference methods for inhomogeneous scatterers makes them more attractive than the method of moments. The sparse nature of the resulting matrices enables scientists to accurately analyze problems of electromagnetic scattering from large objects. For two-dimensional problems, typically only five to seven matrix elements per row are nonzero. Furthermore, the evaluation of the matrix elements requires only simple arithmetic, while, in the method of moments, singularities are frequently encountered and must be carefully treated. The difficulty with finite difference methods occurs when the radiation boundary conditions are considered. In the method of moments, the formulation
is within or on the scatterers. The implication is that the number of unknowns is proportional to the volume or the surface area of the scatterers. On the other hand, finite difference methods are formulated in the entire space. An outer boundary must be used to achieve a finite computational domain. The boundary condition placed on the outer boundary should simulate unbounded space and only permit outgoing scattered waves.

The boundary conditions [28-50] used on the outer boundary may be exact or approximate. The exact boundary conditions, such as the modal expansion [28] or radiation integral approaches [29], require substantial memory and computation time [30,33]. The approximate boundary conditions [30,33] have been used widely, and are known to be efficient and offer sufficient accuracy. These are local operators and are able to preserve the sparsity of finite difference methods.

Engquist and Majda [28] used the pseudo-differential operator [52,53,54] approach to obtain boundary conditions for rectangular and circular boundaries. The rectangular boundary operator has been applied to many problems [30,32], but suffers a drawback in that the normal at the corner is not defined. The boundary conditions for a circular outer boundary does not have the problem of undefined normals and has received wide attention in applications to both frequency and time domain problems.

For a circular boundary, Bayliss and Turkel [33] derived a boundary operator by assuming the Wilcox type expansion [51] for the scattered fields and then developing a series operator to eliminate the inverse power series. The most widely used
operator is the second order operator, which has been demonstrated to be effective in bringing the outer boundary closer to the scatterer, thus reducing the size of the computational domain. Also, it has been demonstrated to have higher absorbability than the corresponding Engquist and Majda condition [9].

In this chapter, we modify the pseudo-differential operator approach and obtain a circular boundary operator equivalent to Bayliss and Turkel's operator. Then, in an attempt to reduce the size of the computational domain for elongated scatterers, we employ the modified pseudo-differential operator to derive the elliptic boundary operator which reduces to the circular operator when the boundary is circular. In Section 2.2, the pseudo-differential operator approach is modified to derive the improved circular boundary condition. The modified pseudo-differential operator approach is then used in Section 2.3 to derive the second-order absorbing boundary conditions for the elliptical outer boundary. The effectiveness of the elliptical boundary in reducing the size of the computational domain for elongated scatterers is illustrated in Section 2.4 by calculating bistatic and monostatic radar cross sections of a conducting ellipse, a strip, and a hypothetical "airfoil". In the following sections, the time dependence $e^{-i\omega t}$ is assumed.
2.2 Absorbing Boundary Condition on Circular Boundary

The pseudo-differential operator approach provides a systematic way to obtain absorbing boundary conditions in orthogonal coordinate systems. This technique has been employed by Engquist and Majda [30] to derive absorbing boundary conditions for both circular and rectangular boundaries. In this section, we apply this technique to circular boundaries. We begin with an outline of [30]. Then, following a simple observation, we apply a straightforward modification to obtain an absorbing boundary condition which is equivalent to [33]. The pseudo-differential operator approach is best described in the Fourier transform domain of the transverse coordinates, but the results are identical if we treat the transverse derivative operator as an algebraic quantity.

2.2.1 Outline of Engquist and Majda's Approach

Consider a two-dimensional (2D) scattering problem where the scattered field $U_s$ is governed by the 2D Helmholtz equation:

$$
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k^2 \right) U_s = 0
$$

(2.1)

We define:

$$
L = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k^2
$$

(2.2)
Using the factorization theorem [52,53,54], the $L$ operator is factored into a product of incoming and outgoing wave operators as follows:

$$L = \left( \frac{\partial}{\partial r} + \lambda_+ \right) \left( \frac{\partial}{\partial r} - \lambda_- \right)$$ \hspace{1cm} (2.3)

where $\lambda_+$ and $\lambda_-$ are functions of $k$, $r$ and $\partial/\partial \phi$. The operator in the first parenthesis is the incoming wave operator while that in the second parenthesis is the outgoing wave operator which is an absorbing boundary operator:

$$\left( \frac{\partial}{\partial r} - \lambda_- \right) U_s = 0.$$ \hspace{1cm} (2.4)

Hence, we need to find $\lambda_-$. Following [30], we expand $\lambda_-$ into the following series:

$$\lambda_- = \lambda_-^{(1)} + \lambda_-^{(0)} + \ldots + \lambda_-^{(-n)}$$ \hspace{1cm} (2.5)

in which $\lambda_-^{(j)}$ are recursively determined. To obtain the second order boundary operator, we need only the first two terms which are given by

$$\lambda_-^{(1)} = ik \sqrt{1 + \frac{1}{k^2 r^2} \frac{\partial^2}{\partial \phi^2}} \sim ik \left(1 + \frac{1}{2k^2 r^2} \frac{\partial^2}{\partial \phi^2} \right)$$ \hspace{1cm} (2.6)

$$\lambda_-^{(0)} = -\frac{1}{2r} + \frac{1}{2r^3 k^2} \frac{\partial^2}{\partial \phi^2}$$ \hspace{1cm} (2.7)
The second-order boundary operator, obtained by combining (2.6) and (2.7), is

\[
\frac{\partial}{\partial r} = ik - \frac{1}{2r} + \frac{1}{2k^2r^2} \left( ik + \frac{1}{r} \right) \frac{\partial^2}{\partial \phi^2}
\]  

(2.8)

These are the results obtained in [30]. By ignoring terms involving \(1/r^2\) and higher-order terms from the second-order boundary operator, Sommerfeld's radiation condition can be obtained.

The first and second-order boundary operators derived by Bayliss and Turkel are given by [33]

\[
\frac{\partial}{\partial r} = ik - \frac{1}{2r}
\]  

(2.9)

\[
\frac{\partial}{\partial r} = \frac{1}{1 + \frac{i}{kr}} \left( ik - \frac{3}{2r} - \frac{3i}{8kr^2} + \frac{i}{2kr^2} \frac{\partial^2}{\partial \phi^2} \right)
\]

\[= ik - \frac{1}{2r} + \frac{ik}{2(kr)^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{4} - \frac{1}{ikr}
\]  

(2.10)

Equations (2.8) and (2.10) have been applied to many problems and it has been demonstrated [9] that (2.10) provides higher absorbability than (2.8).
2.2.2 Modified Engquist and Majda Approach

In an attempt to improve the Engquist and Majda absorbing boundary condition, we recall Sommerfeld's radiation condition:

$$\frac{\partial}{\partial r} = ik - \frac{1}{2r} \quad (2.11)$$

The above equation ensures that, in the far field, the scattered waves are cylindrical waves. We notice that the Sommerfeld radiation condition appears in the first-order condition of the Bayliss and Turkel's results. On the other hand, the $ik$ term appears in (2.6) while $1/2r$ appears in (2.7). Thus, Sommerfeld's radiation condition cannot be obtained by using (2.6) alone, and this observation motivates us to modify the approach in [30] and to group the $ik$ and $1/2r$ terms.

Considering an arbitrary function of $r$, we have the following operator relationship:

$$\left( \frac{\partial}{\partial r} + \frac{1}{2r} \right)^2 + \frac{1}{4r} = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \quad (2.12)$$

Substituting (2.12) into (2.2), we obtain

$$L = \left( \frac{\partial}{\partial r} + \frac{1}{2r} \right)^2 + k^2 + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{4r^2} \quad (2.13)$$
Now we attempt to factor \( L \) into the following form:

\[
L = \left( \frac{\partial}{\partial r} + \frac{1}{2r} + \lambda_+ \right) \left( \frac{\partial}{\partial r} + \frac{1}{2r} - \lambda_- \right) \tag{2.14}
\]

In order to find \( \lambda_- \), we multiply out (2.14) and obtain

\[
L = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{4r^2} + \frac{1}{2r} (\lambda_+ - \lambda_-) + (\lambda_+ - \lambda_-) \frac{\partial}{\partial r} - \frac{\partial \lambda_-}{\partial r} - \lambda_+ \lambda_- \tag{2.15}
\]

By comparing (2.15) with (2.1), we have

\[
\frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k^2 = -\frac{1}{4r^2} + \frac{1}{2r} (\lambda_+ - \lambda_-) + (\lambda_+ - \lambda_-) \frac{\partial}{\partial r} - \frac{\partial \lambda_-}{\partial r} - \lambda_+ \lambda_. \tag{2.16}
\]

Since the left-hand-side of the above equation does not involve \( \partial / \partial r \), we assume

\[
\lambda_+ = \lambda_- = \lambda \tag{2.17}
\]

and obtain

\[
\frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k^2 + \frac{1}{4r^2} = -\frac{\partial \lambda}{\partial r} - \lambda \lambda. \tag{2.18}
\]

To obtain the second order operator, we approximate \( \lambda \) by

\[
\lambda \sim \lambda^{(1)} + \lambda^{(0)} \tag{2.19}
\]

where the leading terms of \( \lambda^{(1)} \) and \( \lambda^{(0)} \) are \( O(1) \) and \( O(1/r^3) \), respectively. They
are determined recursively by inserting (2.19) into (2.18). For \( \lambda^{(1)} \), we equate the left-hand-side of (2.18) to the terms for which the leading terms are \( O(1) \). Assuming \( \partial \lambda^{(1)}/\partial r \) is \( O(1/r^3) \) [justified by Equation (2.21)], we have:

\[
\frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + k^2 + \frac{1}{4r^2} = -\lambda^{(1)} \lambda^{(1)},
\]

(2.20)

Therefore,

\[
\lambda^{(1)} = ik \sqrt{1 + \frac{1}{k^2 r^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{4k^2 r^2}} \sim ik \left(1 + \frac{1}{2k^2 r^2} \frac{\partial^2}{\partial \phi^2} + \frac{1}{8k^2 r^2}\right)
\]

(2.21)

For \( \lambda^{(0)} \), we set the terms for which the leading terms are \( O(1/r^3) \) to zero.

\[
- \frac{\partial \lambda^{(1)}}{\partial r} - \lambda^{(1)} \lambda^{(0)} - \lambda^{(0)} \lambda^{(1)} = 0
\]

(2.22)

We cannot, in general, assume that \( \lambda^{(1)} \) and \( \lambda^{(0)} \) commute. But keeping in mind that the transverse derivatives are algebraic quantities, since we are in the Fourier transform domain, we have:

\[
\lambda^{(0)} = - \frac{1}{2\lambda^{(1)}} \frac{\partial \lambda^{(1)}}{\partial r}
\]

(2.23)

Retaining the leading term for \( \lambda^{(1)} \) in the denominator, we obtain

\[
\lambda^{(0)} = \frac{1}{2k^2 r^3} \left( \frac{\partial^2}{\partial \phi^2} + \frac{1}{4} \right).
\]

(2.24)
2. *ABC* on Circular and Elliptical Boundaries

By adding (2.21) and (2.24), we obtain \( \lambda_- \), and consequently

\[
\frac{\partial}{\partial r} = ik - \frac{1}{2r} + \frac{ik}{2(kr)^2} \left(1 + \frac{1}{ikr}\right) \left(\frac{\partial^2}{\partial \phi^2} + \frac{1}{4}\right). \tag{2.25}
\]

The above equation is the new absorbing boundary condition, which is similar to (2.10). By substituting (2.25) into the Helmholtz equation, it can be shown that the leading residual term is proportional to \( 1/r^4 \). In fact, the simple algebraic approximation for large \( kr \)

\[
1 + \frac{1}{ikr} \sim \frac{1}{1 - \frac{1}{ikr}} \tag{2.26}
\]

converts (2.25) into (2.10).
2.3 Absorbing Boundary Condition on Elliptical Boundary

In this section we apply the pseudo-differential operator approach to find an absorbing boundary condition for elliptical boundaries. For elongated scatterers, the elliptical outer boundary may be used to reduce the size of the computational domain.

The 2D Helmholtz equation in elliptical coordinates is given by

\[
\frac{4}{d^2 \left[ \cosh^2 u - \cos^2 v \right]} \left( \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} \right) U_* + k^2 U_* = 0
\]  
(2.27)

where \( d \) is the interfocal distance and \( u, v \) are parameters of the elliptical coordinate system (Figure 2.1). The parameters \( u \) and \( v \) are related to the rectangular coordinates by

\[
x = \frac{d}{2} \cosh u \cos v, \quad y = \frac{d}{2} \sinh u \sin v
\]  
(2.28)

in which \( 0 \leq u < \infty, \ 0 \leq v < 2\pi \). Note that for a given interfocal distance \( d \), the ellipse defined by constant \( u \) approaches a circle as \( u \) is increased. This is a desired property to have for the outer boundary, since the scattered field behaves like cylindrical wave as the distance is increased. Also, when \( d = 0 \), the ellipse reduces to a circle.

By examining both the cylindrical and elliptical coordinate systems, we find in the limit as \( d \to 0 \) or \( u \to \infty \) that \( v \) becomes \( \phi \). In the cylindrical coordinate system, \( r \) is the distance along a constant \( \phi \) line. Therefore, we introduce an additional
parameter \( n \) in the elliptical coordinate system, which is defined to be the arc length along a constant \( v \) curve.

\[
n = \int_0^u \frac{d}{2} \sqrt{\cosh^2 u' - \cos^2 v} \, du'
\]  

(2.29)

Now, \( n \) becomes the radius when the ellipse approaches a circle, and \( \hat{n} \) is the unit normal vector. Partial derivatives with respect to \( n \) and \( u \) are related by

\[
\frac{\partial}{\partial n} = \hat{n} \cdot \nabla = \frac{2}{d[\cosh^2 u - \cos^2 v]^{1/2}} \frac{\partial}{\partial u'},
\]  

(2.30)

Using (2.29) and (2.30), we convert (2.27) to

\[
\frac{\partial^2}{\partial n^2} + \frac{2 \sinh u \cosh u}{d[\cosh^2 u - \cos^2 v]^{3/2}} \frac{\partial}{\partial n} + \frac{4}{d^2[\cosh^2 u - \cos^2 v]} \frac{\partial^2}{\partial v^2} + k^2 = 0
\]  

(2.31)

The above expression reduces to the cylindrical wave equation when the ellipse approaches a circle.

In order to derive the absorbing boundary conditions for the elliptical boundary, we complete the square, analogous to the circular case, and proceed to the factorization. By noticing that

\[
\frac{\partial^2}{\partial n^2} + b \frac{\partial}{\partial n} = \left( \frac{\partial}{\partial n} + \frac{b}{2} \right)^2 - \frac{b^2}{4} - \frac{1}{2} \frac{\partial b}{\partial n},
\]  

(2.32)
we rewrite (2.31) as

\[
\left( \frac{\partial}{\partial n} + \frac{\sinh u \cosh u}{d[\cosh^2 u - \cos^2 v]^{3/2}} \right)^2 + \frac{4}{d^2[\cosh^2 u - \cos^2 v]} \frac{\partial^2}{\partial v^2} + k^2 - A = 0, \quad (2.33)
\]

where

\[
A = \frac{2}{d^2[\cosh^2 u - \cos^2 v]^3} \left( \sin^2 v (\sinh^2 u + \cosh^2 u) - \sinh^2 u \left( \frac{1}{2} \cosh^2 u + 1 \right) \right)
\]

Equation (2.33) is then factored into the following form:

\[
\left( \frac{\partial}{\partial n} + \frac{\sinh u \cosh u}{d[\cosh^2 u - \cos^2 v]^{3/2}} + \lambda_+ \right) \left( \frac{\partial}{\partial n} + \frac{\sinh u \cosh u}{d[\cosh^2 u - \cos^2 v]^{3/2}} - \lambda_- \right) = 0 \quad (2.35)
\]

Then, following a similar approach as in the circular boundary case, we obtain

\[
\lambda_+^{(1)} = \lambda_-^{(1)} = \lambda^{(1)} = \frac{ik}{\sqrt{1 + \frac{4}{(kd)^2[\cosh^2 u - \cos^2 v]} \frac{\partial^2}{\partial v^2} - \frac{A}{k^2}}}
\]

\[
\simeq \frac{ik}{\sqrt{1 + \frac{2}{(kd)^2[\cosh^2 u - \cos^2 v]} \frac{\partial^2}{\partial v^2} - \frac{A}{2k^2}}} \quad (2.36)
\]

and

\[
\lambda_+^{(0)} = \lambda_-^{(0)} = \lambda^{(0)} = -\frac{1}{2\lambda^{(1)}} \frac{\partial \lambda^{(1)}}{\partial n}
\]

\[
= -\frac{1}{d\lambda^{(1)}[\cosh^2 u - \cos^2 v]^{1/2}} \frac{\partial \lambda^{(1)}}{\partial u}. \quad (2.37)
\]
Taking the leading order term from the $\lambda^{(1)}$ in the denominator, we obtain

$$
\frac{\partial}{\partial n} = ik - \frac{\sinh u \cosh u}{d[\cosh^2 u - \cos^2 v]^{3/2}} + \frac{i2k}{(kd)^2[\cosh^2 u - \cos^2 v]} \cdot \left(1 + \frac{2\sinh u \cosh u}{ikd[\cosh^2 u - \cos^2 v]^{3/2}}\right) \left(\frac{\sinh^4 u}{4[\cosh^2 u - \cos^2 v]^2} + \frac{\partial^2}{\partial v^2}\right)\quad (2.38)
$$

Then, making an approximation similar to the one used in (2.26) gives

$$
\frac{\partial}{\partial n} = ik - \frac{\sinh u \cosh u}{d[\cosh^2 u - \cos^2 v]^{3/2}} + \frac{i2k}{(kd)^2[\cosh^2 u - \cos^2 v]} \cdot \frac{\sinh^4 u}{4[\cosh^2 u - \cos^2 v]^2} + \frac{\partial^2}{\partial v^2} \quad (2.39)
$$

The above equation is the second-order absorbing boundary condition on elliptical outer boundaries. In arriving at (2.39), the higher-order terms of $A$ are ignored. The higher-order terms are determined by the power of the product of either $(d \cosh u)$ or $(d \sinh u)$, since they measure the size of the ellipse as does $r$ in the case of a circle. We note that when $d = 0$, (2.39) reduces to (2.10).

At this point, it is worthwhile to examine the “on surface radiation condition (OSRC)” [50] interpretation of (2.10). According to [50], the $r$ in (2.10) is replaced by the radius of curvature ($R$) and $\partial^2/r^2 \partial \phi^2$ by $\partial^2/\partial s^2$ where $s$ is the arc length.
2. ABC on Circular and Elliptical Boundaries

With these transformations, (2.10) becomes:

\[
\frac{\partial}{\partial n} = ik - \frac{1}{2R} + \frac{i}{2k} \frac{\partial^2}{\partial s^2} + \frac{1}{4R^2} \frac{1}{1 - \frac{1}{ikR}}. \tag{2.40}
\]

For the elliptical outer boundary,

\[
R = \frac{d \left[ \cosh^2 u - \cos^2 v \right]^{3/2}}{2 \cosh u \sinh u} \tag{2.41}
\]

\[
\frac{\partial^2}{\partial s^2} = \frac{4}{d^2 \left[ \cosh^2 u - \cos^2 v \right]} \frac{\partial^2}{\partial v^2} - \frac{2 \sin 2v}{d^2 \left[ \cosh^2 u - \cos^2 v \right]^2} \frac{\partial}{\partial v}. \tag{2.42}
\]

Then (2.40) becomes

\[
\frac{\partial}{\partial n} = ik - \frac{\sinh u \cosh u}{d \left[ \cosh^2 u - \cos^2 v \right]^{3/2}} + \frac{i2k}{(kd)^2 \left[ \cosh^2 u - \cos^2 v \right]} \cdot \left[ \frac{\sinh^2 u \cosh^2 u}{4 \left[ \cosh^2 u - \cos^2 v \right]} + \frac{\partial^2}{\partial v^2} \right] \frac{1}{1 - \frac{2 \sinh u \cosh u}{ikd \left[ \cosh^2 u - \cos^2 v \right]^{3/2}}} \tag{2.43}
\]

by ignoring the term involving \(\partial/\partial v\). Comparing (2.39) with (2.43), we note that the OSRC interpretation gives similar results with the difference appearing only in the higher order terms.
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Figure 2.1: Elliptical coordinate system.
2.4 Numerical Results and Discussions

The absorbing boundary conditions on circular outer boundaries have been studied and their effectiveness and accuracy have been demonstrated [30,33,9]. In this section, we focus on evaluating the absorbing boundary condition on elliptical outer boundaries. The control region approximation [9] with triangular grids is used to discretize Maxwell’s equations in the frequency domain (finite-difference frequency-domain, or FD-FD) and the absorbing boundary condition (2.39) is imposed on the outer boundary. The bistatic and monostatic two-dimensional radar cross sections are calculated and compared to the results obtained using the method of moments.

The three geometries considered are a conducting ellipse with a major axis of 2.5λ, an aspect ratio of 0.1, and an interfocal distance of 4.97λ, as shown in Figure 2.2, a conducting strip with 5λ width and a conducting “airfoil”. The FD-FD results will be compared with the method of moments (MoM) results. In the MoM, the pulse basis and testing functions are employed with 240 divisions in those cases, resulting in a length on the order of 0.042λ for each division. The results will be presented in terms of two-dimensional RCS which is defined as

\[ \sigma \text{ (dBm)} = 10 \log_{10} \lim_{r \to \infty} 2\pi r \left| \frac{E_x}{E_i} \right|^2 \]

and expressed as dBm (in dB after normalized by 1 meter). The frequency is assumed to be 300 MHz in all cases.

The effectiveness of the elliptical outer boundary is first demonstrated by cal-
calculating the FD-FD results with circular and elliptical outer boundaries as shown in Figure 2.2. The minimum separations from the elliptical and circular outer boundaries to the scatterer are 0.5λ and 1.2λ, respectively. Similar discretization is applied to both cases. A sample discretization for the elliptical boundary case is shown in Figure 2.3. The number of unknowns for the elliptical and circular outer boundary cases are 1632 and 4692, respectively. The bistatic RCS as a function of scattering angle for an incidence angle of 45 degrees is shown in Figure 2.4 and Figure 2.5, for the electric field and magnetic field polarizations, respectively. As can be seen from the figures, the results obtained using the elliptical outer boundary seems to compare better with the MoM results than the circular boundary results.

In order to simplify the comparison of the results, an rms error is defined as follows:

\[ \text{rmsdB} = \sqrt{\frac{\sum [\sigma(\text{dBm}) - \sigma_{\text{MoM}}(\text{dBm})]^2}{N}}. \]

For the electric field polarization, the rms errors are found to be 0.58 dB and 1.0 dB for the elliptical and circular boundaries. For the magnetic field polarization, the rms errors are found to be 1.5 dB and 2.1 dB for the elliptical and circular boundaries, respectively. This clearly demonstrates that the number of unknowns can be reduced significantly, with the same level of accuracy, for elongated scatterers by using the elliptical outer boundaries with the absorbing boundary conditions given by Equation (2.39).

In the case of the circular outer boundary, the radius of the circle is the only
parameter which needs to be determined. In the case of the elliptical outer boundary, there are two free parameters: the interfocal distance and ellipticity, or equivalently, the major and minor axes $a$ and $b$. The absorbing boundary condition is derived based on the expansion of the scattered wave about the normal of the boundary. However, in general, it is difficult to know the optimum ellipticity due to the fact that one does not know the scattered wave front prior to solving the problem. In Figure 2.6, we show the same ellipse scatterer with two elliptical outer boundaries. We argue that the choice of the flat ellipse is not as good as the other ellipse, in spite of the fact that it is bigger. The main reason is that for the flat ellipse, the scattered wave will be incident at a steep angle to a portion of the ellipse near the end. To demonstrate this point, we compare the bistatic RCS of the conducting ellipse with two elliptical outer boundaries shown in Figure 2.6. The angle of incidence is $45^\circ$ and frequency is 300 MHz. Figure 2.7 and Figure 2.8 show the bistatic RCS for electric and magnetic field polarizations, respectively. It is clear from the figures that the results for the smaller elliptical boundary compare better with the MoM results. The average $rms$ differences for the flatter elliptical outer boundary is 1.90 dB compared to 1.04 dB for the case of the smaller elliptical boundary. Thus, the outer elliptical boundary should not be chosen to be more elongated than the scatterer.

In all of the above cases, the angle of the incident wave was fixed at $45^\circ$. To further examine the absorbing boundary condition, the monostatic RCS is calculated as a function of the angle of incidence for a $5\lambda$ wide conducting strip as shown in Figure 2.9. The outer elliptical boundary, that was used, has the same shape as the one shown in Figure 2.2 ($a = 3\lambda$ and $b/a = 0.484$). The monostatic RCS at 300 MHz
is shown in Figure 2.10 and Figure 2.11 for electric field and magnetic field polarizations, respectively. Again, there is a good agreement between the MoM results and the FD-FD results obtained with the elliptical outer boundary. The average \( \text{rms} \) differences are 1.4 dB for the electric field and magnetic field polarizations.

The last geometry to be analyzed is a hypothetical "airfoil" shown in Figure 2.12. This geometry contains a portion of the ellipse analyzed previously, a half circle, and two straight sections. The same outer elliptical boundary as shown in Figure 2.9 is used. Figure 2.13 and Figure 2.14 show the monostatic RCS at 300 MHz for electric field and magnetic field polarizations, respectively. A good agreement between the MoM results and the FD-FD results is illustrated.
Figure 2.2: Circular and elliptical outer boundaries for a conducting ellipse: radius $r = 3.7\lambda$ for circular domain; $a = 3.0\lambda$ and $b/a = 0.484$ for elliptical domain.

Figure 2.3: Triangular grid discretization of an elliptical computation domain with $a = 3.0\lambda$ and $b/a = 0.484$. 
2. ABC on Circular and Elliptical Boundaries

Figure 2.4: Bistatic RCS of an conducting ellipse with $a_o = 2.5\lambda$ and $b_o/a_o = 0.1$ at 300 MHz for electric field polarization for incidence angle of $\phi_i = 45^\circ$. Solid curve: MoM; dashed curve: elliptical boundary with $a = 3.0\lambda$ and $b/a = 0.484$; circles: circular boundary with $r = 3.7\lambda$.

Figure 2.5: Bistatic RCS of an conducting ellipse with $a_o = 2.5\lambda$ and $b_o/a_o = 0.1$ at 300 MHz for magnetic field polarization for incidence angle of $\phi_i = 45^\circ$. Solid curve: MoM; dashed curve: elliptical boundary with $a = 3.0\lambda$ and $b/a = 0.484$; circles: circular boundary with $r = 3.7\lambda$. 
Figure 2.6: Two outer ellipses with different ellipticity.
Figure 2.7: Bistatic RCS of an conducting ellipse with $a_o = 2.5\lambda$ and $b_o/a_o = 0.1$ at 300 MHz for electric field polarization for incidence angle of $\phi_i = 45^\circ$. Solid curve: MoM; dashed curve: elliptical boundary with $a = 3.0\lambda$ and $b/a = 0.484$; circles: $a = 5.0\lambda$ and $b/a = 0.30$.

Figure 2.8: Bistatic RCS of an conducting ellipse with $a_o = 2.5\lambda$ and $b_o/a_o = 0.1$ at 300 MHz for magnetic field polarization for incidence angle of $\phi_i = 45^\circ$. Solid curve: MoM; dashed curve: elliptical boundary with $a = 3.0\lambda$ and $b/a = 0.484$; circles: $a = 5.0\lambda$ and $b/a = 0.30$. 
Figure 2.9: Geometrical description for FD-FD calculation of scattering by a $5\lambda$ conducting strip with an elliptical outer boundary ($a = 3.0\lambda$ and $b/a = 0.484$).
Figure 2.10: Monostatic RCS of $5\lambda$ conducting strip at 300 MHz for electrical field polarization. Solid curve: MoM; dashed curve: FD-FD with elliptical outer boundary.

Figure 2.11: Monostatic RCS of $5\lambda$ conducting strip at 300 MHz for magnetic field polarization. Solid curve: MoM; dashed curve: FD-FD with elliptical outer boundary.
Figure 2.12: Geometrical description of an hypothetical "airfoil".
Figure 2.13: Monostatic RCS of a conducting “airfoil” at 300 MHz for electrical field polarization. Solid curve: MoM; dashed curve: FD-FD with elliptical outer boundary.

Figure 2.14: Monostatic RCS of 5λ conducting “airfoil” at 300 MHz for magnetic field polarization. Solid curve: MoM; dashed curve: FD-FD with elliptical outer boundary.
2.5 Summary

We have demonstrated an effective method to modify the pseudo-differential operator approach used by Engquist and Majda to derive improved absorbing boundary conditions. The modified pseudo-differential operator approach is applied to both the circular and elliptic boundaries. The improved absorbing boundary condition for the circular boundaries is equivalent to the result obtained by Bayliss and Turkel, which has been shown to have higher absorbability. With a convenient change of variable and the modified pseudo-differential operator approach, we obtained the absorbing boundary conditions for the elliptic boundaries. This absorbing boundary condition is numerically demonstrated to be effective and can potentially offer substantial reduction in requirements of computer resources for elongated scatterers.
Chapter 3

A FINITE-DIFFERENCE TIME-DOMAIN TECHNIQUE ON TRIANGULAR GRIDS

3.1 Introduction

The Finite-Difference Time-Domain (FD-TD) techniques, derived from the differential form of Maxwell equations, have been applied to analyze various electromagnetic problems [1-22]. In analyzing the problem of electromagnetic wave scattering from arbitrarily shaped scatterers (Figure 3.1), the general procedure of the FD-TD technique involves approximating the Maxwell equations using the finite differences, imposing appropriate boundary conditions including the absorbing boundary condition (ABC) [28-51] at the outer boundary of the computational domain, and explicitly time-marching to obtain the direct time domain response. In order to apply the finite difference approximation to the Maxwell equations, the electric and magnetic fields are discretized in grids inside a finite computational domain. The discretization grids may be rectangular, spherical or other shapes. Nevertheless, the choice of the
grid should enable accurate geometry modeling of the scatterer. To achieve a finite computational domain, an artificial boundary is chosen where the ABC is imposed to simulate the unbounded space. The choice of the ABC is directly related to the shape of the artificial boundary and the size of the computational domain. The ABC should provide computational efficiency, such as small computational domain, and accurate simulation of unbounded space.

For electromagnetic problems, rectangular grids are perhaps the most commonly used grids. The Yee’s discretization scheme [1] of all field variables in space and time is employed with the rectangular grid, where the electric fields and the magnetic fields are temporally separated in half time step. In addition, they are spatially interlaced by half a grid cell. Based on this discretization scheme, a straightforward center differences in both space and time are applied to approximate the Maxwell equations. The FD-TD algorithm based on the Yee’s discretization scheme is very simple to implement. This simple implementation includes geometry modeling. Due to the fact that all grid cells are rectangular in shape, arbitrary geometries are approximated by staircases. Figure 3.2 shows a staircase approximation of a circular cylinder geometry. Usually the staircase approximation can provide reasonably accurate electromagnetic modeling ability. In some situations, e.g., when strong surface waves are present, a finer rectangular grids are needed to obtain accurate results for arbitrary geometries. The other drawback is the limitation on the shape of the outside boundary (artificial boundary). The rectangular shape outer boundary does not permit the application of second-order ABC at corners due to the fact that the normal vector is not defined at the corner.
3. A FD-TD Technique on Triangular Grids

In order to avoid the staircase approximation for the arbitrary geometry, several grids and discretization schemes have been proposed [4,5,6,7]. In [4], Holland suggested a generalized nonorthogonal grids and a correspondence discretization scheme. In his approach, a generalized coordinate system is created. The staircase approximation is eliminated by choosing the coordinate system to be naturally conform to the geometry. This approach has been applied to spherical scatterer. The application to more general geometries appears cumbersome. In [6,7], a variation of the Yee’s scheme is suggested. In this approach, the rectangular grids are used everywhere except on the surface of the geometry. The Yee’s discretization scheme and the finite difference approximation are employed away from the surface of the geometry. On the surface of the geometry, the rectangular grids are distorted to fit the geometry, and the control region approach is used to approximate Maxwell equations. This approach preserve most of the simplicity of the Yee’s algorithm, but it is not flexible enough to model complex geometry.

An alternative to avoid the staircase approximation is to use triangular grid. The triangular grids provide flexibility in the accurate electromagnetic modeling of complex geometry. In applying the triangular grids, linear interpolations are used to describe the geometry surface. Figure 3.3 shows a geometry model of a coated cylinder by using the triangular grids. The flexibility of the triangular grids can also be utilized to generate continue outer boundaries, such as circular or elliptical boundaries. There are a number of discretization scheme associated with the triangular grids. The control region [9], defined by the Delaunay tessellation, has been applied to two-dimensional (2D) frequency domain electromagnetic scattering prob-
lems. In this chapter, the control region approximation is extended to time domain electromagnetic scattering problems. The purpose is to develop a simple yet accurate algorithm for triangular grids to analyze time domain electromagnetic problems. In fact, the algorithm is basically a combination of the finite difference scheme on triangular grid and the control region approximation. The outer boundaries are either circular or elliptical.

The detail description of the algorithm and the implementation of the FD-TD technique on triangular grids are presented in this chapter. In section 3.2, the discretization schemes for all field variables and the finite difference approximation of Maxwell equations are discussed. In section 3.3, the implementation of the ABC on the circular and elliptical boundaries are discussed. In section 3.4, the FD-TD algorithm is illustrated by calculating the time domain scattering problems of simple geometries.
Figure 3.1: Configuration of electromagnetic scattering problems.
Figure 3.2: Rectangular grids and the staircase approximation of a conducting cylinder.

Figure 3.3: Triangular grids and modeling of a coated conducting cylinder.
3.2 Discretization

In the finite-difference time-domain algorithm, the discretization of the electric and magnetic fields are carried out in both time and space. The temporal discretization of fields used is similar to that of Yee scheme, where the electric and magnetic fields are interlaced in half a time step. The spatial discretization of fields are in triangular grids. Maxwell equations are approximated using the finite difference approximation in conjunction with the control region approximation. To facilitate the discussion, a local coordinate system is defined (Figure 3.4), where \( \hat{s} \) and \( \hat{n} \) are defined to be unit tangent and unit normal vectors along an arbitrary contour such that \( \hat{n} \times \hat{s} = \hat{z} \). For 2D problems, the discretization procedures can be divided into electric field polarization and the magnetic field polarization. For electric field polarization, \( \bar{E} = \hat{z} E \), Maxwell equations are written as follow:

\[
\nabla_T \times \eta_0 (\hat{n}H_n + \hat{s}H_s) = \eta_0 \sigma_e E_z + \varepsilon_e \varepsilon_0 \frac{\partial E_z}{c \partial t}
\]

(3.1)

\[
\frac{\partial E_z}{\partial n} = \sigma_m H_s + \mu_r \eta_o \frac{\partial H_s}{c \partial t}
\]

(3.2)

\[
\frac{\partial E_z}{\partial s} = -\sigma_m H_n - \mu_r \eta_o \frac{\partial H_n}{c \partial t}.
\]

(3.3)
For magnetic field polarization, $\vec{H} = \dot{z}H$, Maxwell equations are written as follow:

$$\nabla_T \times (\hat{n}E_n + \dot{z}E_s) = -\sigma_m H_z - \dot{z}\mu_r \eta_o \frac{\partial H_z}{c \partial t}$$  \hspace{1cm} (3.4)

$$\eta_o \frac{\partial H_z}{\partial n} = -\eta_o \sigma_e E_s - \epsilon_r \frac{\partial E_s}{c \partial t}$$ \hspace{1cm} (3.5)

$$\eta_o \frac{\partial H_z}{\partial \delta} = \eta_o \sigma_e E_n + \epsilon_r \frac{\partial E_n}{c \partial t}.$$ \hspace{1cm} (3.6)

In Equation (3.1) to (3.6), $E_z$, $H_z$, $E_n$, $E_s$, $H_n$, and $H_s$ are total field quantities. $\epsilon_r$, $\mu_r$, $\sigma_e (1/\Omega)$, $\sigma_m (\Omega/m)$, $\eta_o (\Omega)$, and $c (m/s)$ are relative permittivity, relative permeability, electric conductivity, magnetic conductivity, free space impedance, and speed of light in free space, respectively. The field quantities in the above equations can be broken up into the scattered and incident fields, where the incident field satisfies the free space Maxwell's equations. It turns out that FD-TD technique is simpler to implement if the total-scattered field technique based on [32] is employed.

In approximating the above equations, the control region approximation, which has been successfully applied to the frequency domain problems in the past, is used. The control region approximation calls for Delaunay and Dirichlet tessellations, which is illustrated in Figure 3.5. Dirichlet tessellations are defined by the triangular grids. The Delaunay tessellations are defined by polygons, which are orthogonal to the triangular grids. An important feature of this topology is that the edges of the Delaunay tessellations are perpendicular bi-sectors of the corresponding triangular
3.2.1 Electric Field Polarization

For electric field polarization, the FD-TD algorithm can be written in terms of $E_x$ and $H_x$. Integrating (3.1) on a finite area and applying the Green’s theorem, we obtain

$$\oint \eta_0 H_x\, ds = \iint \left( \eta_0 \epsilon_e E_x + \epsilon_r \frac{\partial E_x}{\partial t} \right) \, dA. \quad (3.7)$$

Since Equation (3.7) and (3.2) only involve $E_x$ and $H_x$ and hence $H_n$ is decoupled, we note that Equation (3.3) can be discarded in the computation. To discretize these two equations, the control region defined by the Delaunay tessellation is defined. The field discretization in a control region is shown in Figure 3.6. The $E_x$ are placed at nodes and $H_x$ are placed along the Delaunay edges. An unique orientation is assigned to each $H_x$, which is associated with two nodes. The convention of choosing the orientation is that the $H_x$ is always counter clockwise with respect to the node with larger index.

For total field node ‘j’ not lying on the interfaces and at time ‘m − 1/2’, Equations (3.7) and (3.2) are approximated by,

$$\sum_{k=1}^{M} \eta_0 H_{x[k,j]}^{m-1/2} l_{[k,j]} = \frac{\eta_0 \sigma_e A}{2} (E_{x[j]}^m + E_{x[j]}^{m-1}) + \frac{\epsilon_r A}{c \Delta t} (E_{x[j]}^m - E_{x[j]}^{m-1}) \quad (3.8)$$
\[ \frac{1}{d_{[k,j]}} (E^m_{s[k]} - E^m_{s[j]}) = \frac{\sigma_m}{2} \left( H^m_{s[k,j]} + H^m_{s[k,j]} \right) + \frac{\mu_r \eta_o}{c \Delta t} \left( H^{m+1/2}_{s[k,j]} - H^{m-1/2}_{s[k,j]} \right) \] (3.9)

where \( A \) is the area of the control region, \( l_{[k,j]} \) and \( d_{[k,j]} \) are shown in Figure 3.6. In deriving Equation (3.8), the left-hand-side of (3.7) is approximated by summing contributions of \( H^* \) along the boundary of the control region 'j', where \( j \) is assumed bigger than \( k \). If \( j \) is smaller than \( k \), the corresponding contribution is negative. For terms involving conductivities, the time average approximation is employed. Finally, spatial and temporal derivatives are approximated by center differences. If node 'j' is in the free space, the terms involved conductivities vanish and relative permittivity and permeability are unity. Equation (3.8) is used to update electric fields, and the newly updated electric fields are used in (3.9) to update magnetic fields.

For total field node 'j' lying on the interfaces and at time \( 'm - 1/2' \), the discontinuity of normal magnetic fields must be considered. Since the normal magnetic flux density continues, the normal magnetic fields on both sides of the interface are related. The field assignments for this cases are shown in Figure 3.7. In our implementation, the unknown normal magnetic fields are chosen to correspond to the field on the side with smaller relative permeability. Equations (3.7) and (3.2) are approximated using the identical expressions as in (3.8) and (3.9) except that the contributions along edges normal to the interface are split to two parts, one for each side of the interface.

For scattered field node 'j' at time \( 'm - 1/2' \), Equations (3.7) and (3.2) are
approximated as,

$$
\sum_{k=1}^{M} \eta_0 H^m_{s(z)[k,j]} \ l_{[k,j]} = \frac{\epsilon \ A}{c \Delta t} \left( E^m_{z(s)[j]} - E^{m-1}_{z(s)[j]} \right)
$$

(3.10)

$$
\frac{1}{d_{[k,j]}} \left( E^m_{z(s)[k]} - E^m_{z(s)[j]} \right) = \frac{\mu_0 \ \eta_0}{c \Delta t} \left( H^m_{s(z)[k,j]} - H^{m-1/2}_{s(z)[k,j]} \right). \quad (3.11)
$$

### 3.2.2 Magnetic Field Polarization

For magnetic field polarization, the FD-TD algorithm can be written in terms of $E_s$ and $H_s$ and is very similar to that of the electric field polarization case. Integrating (3.4) on a finite area and applying the Green's theorem, we obtain

$$
\oint E_s \ ds = - \iiint \left( \sigma_{m} H_s + \mu_0 \ \eta_0 \frac{\partial H_s}{\partial t} \right) \ dA.
$$

(3.12)

Again, since Equation (3.12) and (3.5) only involve $H_s$ and $E_s$ hence $E_n$ is decoupled, and (3.6) is discarded in the computation. The field variable assignment is shown in Figure 3.8, where $H_s$ are placed at nodes and $E_s$ are placed along the Delaunay edges (Figure 3.8). The convention used in assigning orientation to $H_s$ in the case of electric field polarization is used for assigning orientation of $E_s$.

For total field node 'j' not lying on the interfaces and at time 'm - 1/2', Equations (3.12) and (3.5) are approximated by,

$$
\sum_{k=1}^{M} E^m_{s[k,j]} l_{[k,j]} = - \frac{\sigma_{m} A}{2} \left( H^m_{s[j]} + H^{m-1}_{s[j]} \right) + \frac{\mu_0 \ \eta_0 A}{c \Delta t} \left( H^m_{s[j]} - H^{m-1}_{s[j]} \right)
$$

(3.13)
\[
\frac{\eta_0}{d_{[k,j]}} (H_{s[k]}^m - H_{s[j]}^m) = -\frac{\sigma_m \eta_0}{2} (E_{s[k,j]}^{m+1/2} + E_{s[k,j]}^{m-1/2}) - \frac{\varepsilon_r}{c \Delta t} (E_{s[k,j]}^{m+1/2} - E_{s[k,j]}^{m-1/2})
\]

(3.14)

where \( A \) is the area of the control region, \( l_{[k,j]} \) and \( d_{[k,j]} \) are shown in Figure 3.8. In deriving Equation (3.13) and (3.14), the same techniques to derive Equations (3.7) and (3.8) are used. If node ‘j’ is in the free space, the terms involved conductivities vanish and relative permittivity and permeability are unity. Equation (3.13) is used to update magnetic fields, and the newly updated magnetic fields are used in (3.14) to update electric fields.

For total field node ‘j’ lying on the interfaces and at time ‘\( m - 1/2 \)’, the discontinuity of normal electric fields must be considered. Since the normal electric displacement is continuous, the normal electric fields on both sides of the interface are related. The field assignments are shown in Figure 3.9. For the FD-TD implementation, the unknown normal electric fields are chosen to correspond to the side having smaller relative permittivity. Equations (3.12) and (3.5) are approximated using the identical expressions as in (3.13) and (3.14) except that the contributions along edges normal to the interface are split to two parts, one for each side of the interface.

For scattered field node ‘j’ at time ‘\( m - 1/2 \)’, Equations (3.12) and (3.5) are approximated by,

\[
\sum_{k=1}^{M} E_{s(k,j)}^{m-1/2} l_{[k,j]} = -\frac{u_r \eta_0 A}{c \Delta t} (H_{s(k,j)}^m - H_{s(k,j)}^{m-1})
\]

(3.15)
\[ \frac{\eta_o}{d_{[k,j]}} (H_{z(K/K)}^m - H_{z(K/K)}^m) = -\frac{\varepsilon_r}{c \Delta t} (E_{x(K/K)}^{m+1/2} - E_{x(K/K)}^{m-1/2}). \quad (3.16) \]

Equations (3.8) to (3.11) outline the two important steps of the FD-TD algorithm for the electric field polarization, and Equations (3.12) to (3.12) outline the two important steps of the FD-TD algorithm for the magnetic field polarization. The maximum dimension of a side of a triangular cell should be a small fraction of the minimum wavelength. \( \Delta t \) should be chosen such that the stability criterion is satisfied (Appendix). For equilateral triangular grids, the instability condition is

\[ \Delta t \leq \sqrt{\frac{2}{3}} \frac{d}{c}. \quad (3.17) \]

In the case of arbitrary triangular grids, a safety factor is needed.

Based on the above procedure, the evaluations of the fields at the outer boundary require field information beyond the computational domain. The fields outside the computational domain, however, are not available. An alternative to obtain the boundary fields are to impose the absorbing boundary conditions (ABC) \([28,51]\).
3. A FD-TD Technique on Triangular Grids

Figure 3.4: Local coordinate system.

Figure 3.5: Topology of the Dirichlet and Delaunay tessellation.
Figure 3.6: Field assignments on a Delaunay tessellation (electric field polarization).

Figure 3.7: Field assignments on a Delaunay tessellation along an interface (electric field polarization).
3. A FD-TD Technique on Triangular Grids

Figure 3.8: Field assignments on a Delaunay tessellation (magnetic field polarization).

Figure 3.9: Field assignments on a Delaunay tessellation along an interface (magnetic field polarization).
3.3 Implementation of Absorbing Boundary Conditions

The absorbing boundary conditions (ABC) for two types of outer boundaries are studied: circular and elliptical outer boundaries [30,33,39]. For circular boundary, ABC can be expressed in operator form [30]

\[
(\frac{\partial}{c \partial t} + \frac{1}{r}) \frac{\partial}{\partial r} = -\frac{\partial^2}{c^2 \partial t^2} - \frac{3}{2r c \partial t} - \frac{3}{8r^2} + \frac{1}{2r^2} \frac{\partial^2}{\partial \phi^2}
\] (3.18)

This operator equation applies to the scattered fields at the boundary, or \(E_z(\phi)\) for electric field polarization and \(H_z(\phi)\) for magnetic field polarization. Figure 3.10 shows the field assignments at the boundary where \(u\) are either \(E_z(\phi)\) or \(H_z(\phi)\). The implementation of the ABC is basically the same as in Reference [32]. Along the dashed line in Figure 3.10, at time step \(m\), we have

\[
(1 + \frac{\Delta_r}{\Delta_t} + \frac{3\Delta_r}{4r_i}) u_{i_0}^{m+1} = (1 - \frac{\Delta_r}{\Delta_t} - \frac{3\Delta_r}{4r_i}) u_{i_0}^{m+1} + (1 - \frac{\Delta_r}{\Delta_t} + \frac{3\Delta_r}{4r_i}) u_{i_0}^{m-1} + (-1 - \frac{\Delta_r}{\Delta_t} + \frac{3\Delta_r}{4r_i}) u_{i_0}^{m-1} - \frac{2\Delta_t}{r_i} (u_{i_1}^m - u_{i_0}^m) + \left( \frac{2\Delta_r}{\Delta_t} - \frac{3\Delta_t \Delta_r}{8r_i^2} - \frac{\Delta_t \Delta_r}{\Delta_s^2} \right) (u_{i_1}^m + u_{i_0}^m) + \frac{\Delta_t \Delta_r}{2\Delta_s^2} (u_{j1}^m + u_{jp0}^m + u_{jn1}^m + u_{jno}^m),
\] (3.19)

where \(\Delta_r\), \(\Delta_s\), and \(r_j\) are shown in Figure 3.10. \(\Delta_t\) is the time increment. In deriving (3.19), temporal and spatial center difference and time average techniques are used. In addition, the spacing between two boundary nodes are assumed to be constant.
For elliptical coordinate, the ABC in the frequency domain has been derived [39]. The frequency domain result can be converted into the time domain result by applying the following transformation:

\[ i k \rightarrow - \frac{\partial}{c \partial t}. \]

The time domain ABC of the elliptical boundary are expressed in the following operator equation:

\[
\left( \frac{\partial}{c \partial t} + \frac{b}{a^2 Z^{3/2}} \right) \frac{\partial}{\partial n} = - \frac{\partial^2}{c^2 \partial t^2} - \frac{3b}{2a^2 Z^{3/2}} \frac{\partial}{c \partial t} + \frac{1}{2a^2 Z} \frac{\partial^2}{\partial \nu^2} - \frac{\tanh^2 u}{4a^2 Z^3} \left( 2 - \frac{\tanh^2 u}{2} \right),
\]

(3.20)

where

\[ a = \frac{d}{2} \cosh u, \quad b = \frac{d}{2} \sinh u, \quad Z = 1 - \frac{\cos^2 v}{\cosh^2 u}. \]

The same technique used in implementing the circular ABC can be utilized to implement the elliptical ABC.
Figure 3.10: Field assignments on an outer boundary
3.4 Numerical Results

In this section, the FD-TD algorithm, outlined by Equations (3.8) to (3.11) for electric field polarization and Equations (3.13) to (3.16) for magnetic field polarization, and the implementation of the ABC on circular and elliptical boundaries are tested numerically. The scattering properties of two simple geometries are analyzed, circular cylinders and strips. The FD-TD algorithm is applied to analyze conducting and coated conducting cylinders. The accuracy of geometry modeling using the triangular grid is evaluated and compared to that using the staircase approximation in the rectangular grid. The capability to treat materials with electric and magnetic losses is analyzed. For strips, the focus are on time domain ABC and transient analysis. The elliptical ABC is demonstrated to reduce the computational domain in certain cases. The transient response of the strip based on Gaussian incident pulse is also illustrated.

To examine the accuracy of the FD-TD algorithm, frequency domain quantities, such as field amplitudes at a particular frequency, are calculated using the FD-TD technique. These results are then compared with those obtained using frequency domain technique. One way to obtain the frequency domain solutions via the FD-TD technique is to excite the scatterer with time dependent sinusoidal waves. The frequency domain solution is then are extracted from the steady-state or the late time responses [13].

Figure 3.11 shows a configuration used in analyzing the scattering property of a two meter diameter conducting cylinder with 0.3 meter thick coated material. This
geometry is analysis using rectangular grid FD-TD [1] and the triangular grid FD-TD. For triangular grid, the shape of the computational domain is circular with radius of 2.47 meter. For rectangular grid, a rectangular computational domain of 8 meter by 8 meter is chosen. The rectangular computational domain is considerably larger than the circular one to highlight the effect of using the rectangular grid to model the cylinder. The cylinder is excited by a 150 (MHz) sinusoidal plane wave from $\phi = 0^\circ$. The steady-state amplitudes of the scattered fields at $r = 2.47$ meter circle are calculated using three techniques: the triangular grid FD-TD, the rectangular grid FD-TD, and eigen function expansion [92].

When the relative permittivity and permeability of the coated material are unity, Figure 3.12 and Figure 3.13 show the calculated $E_z(\phi)$ for electric field polarization and $H_z(\phi)$ for magnetic field polarization, respectively. The size of the triangular and rectangular grids are about $1/20 \lambda$. The solid curves represent results calculated using the eigen function expansion. The dashed curves are calculated using the triangular grid FD-TD, and the circles represent the results obtained using the rectangular grid FD-TD. In this case, both the rectangular and triangular grids provide accurate results, though the triangular grid provides a slightly more accurate geometry model for magnetic field polarization case where the creeping waves are stronger.

The inaccuracy of the staircase geometry modeling in the rectangular grid become more noticeable when the grid size is increased with respect to the wavelength. This is illustrated by calculating the scattered field for the identical case at 300 MHz, which corresponds to the grid size of approximately $0.1 \lambda$. The results are shown in
Figure 3.14 and Figure 3.15 for electric and magnetic field polarization cases, respectively. The solid curves is calculated using the eigen function expansion. The dashed curves are calculated using the triangular grid FD-TD, and the circles represent the results obtained using the rectangular grid FD-TD. It is shown that the triangular grid provides much more accurate results than the rectangular grid. This example suggests that the FD-TD based on the triangular grid can provide accurate solution with less number of discretization nodes than the rectangular grid.

The application of FD-TD technique for the coated conducting scatterer is illustrated by calculating the scattering characteristics of a conducting cylinder coated with dielectric material having $\varepsilon_r = 3.0$ and $\sigma = 0.01/m\Omega$, as shown in Figure 3.11. The remaining parameters are the same as the previous case. For the incident plane wave at 150 MHz, the relative triangular grid size is approximately $1/14\lambda$ inside the coating material. The scattered fields along $r = 2.47$ meter radius are calculated for electric and magnetic field polarizations and shown in Figure 3.16 and Figure 3.17, respectively. The curves in Figure 3.16 represent the magnitudes of $E_z(\omega)$ along $r = 2.47$ meter for electric field polarization. The curves in Figure 3.17 represent the magnitudes of $H_z(\omega)$ along $r = 2.47$ meter for magnetic field polarization. The solid and dashed curves are the results obtained using the eigen function expansion and the FD-TD technique, respectively. As can be seen from the figures, there is an excellent agreement between two results.

In Figure 3.18 and Figure 3.19, the results for the same cylinder with different coating material ($\varepsilon_r = 3.0$ and $\sigma = 0.01/m\Omega$, and $\mu_r = 3.0$) is presented. The size of the triangular grid and other parameters are the same as the last case. Again,
there is a good agreement between the results obtained using the eigen function expansion and the triangular grid FD-TD technique. A slight discrepancy in the case of electric field polarization is primarily due to larger grid size inside the coating material (approximately $1/7 \lambda$).

The previous numerical examples demonstrated the accuracy of the triangular grid FD-TD algorithm and the implementation of the time domain ABC on circular boundaries. For the circular computational domain, very small reflections from the boundary are found when the second-order absorbing boundary condition is implemented. For elongated geometries, however, using the circular outer boundary may introduce unnecessarily large computational domains. In such cases, the elliptical outer boundary can be used to reduce the computational domain [39]. To illustrate this point, we consider a 5 meter wide, 0.13 meter thick strip with circular and elliptical computational domains, as shown in Figure 3.20. The number of nodes for the circular and elliptical computational domains are 3894 and 2075, respectively. The ellipticity and size of the elliptical computational domain are selected by keeping the minimum separation from the ellipse to the strip to be the same distance as the minimum distance from the circle to the strip and setting the interfocal distance to be approximately 5 meters. A sinusoidal plane wave at 400 MHz is incident at $\phi = 45^\circ$. The relative grid size at this frequency is approximately 0.1 wavelength. The bistatic RCS results for the electric and magnetic field polarizations are shown in Figure 3.21 and Figure 3.22, respectively. The solid curves correspond to the results calculated using the method of moments and are taken as the reference in evaluating the accuracy of the FD-TD results. The triangular grid FD-TD algorithm is used to analyze
the same problem, with dashed curves representing the results obtained using the elliptical boundary and the “o” data representing results obtained using the circular boundary. As seen from Figure 3.21 and Figure 3.22, the results obtained using the elliptical boundary appear more accurate than the results obtained using the circular boundary. It is because that the elliptical boundary captures the scattered wave front from the edges better in this configuration [39].

In all the previous examples, the steady-state (time-harmonic) results have been shown which demonstrated the stability and the accuracy of the FD-TD algorithm. In fact, the main advantage of the FD-TD technique is in its ability to handle time domain problems (e.g., transient analysis). This is demonstrated by calculating the time-domain scattered field from the strip shown in Figure 3.20. The incident wave gaussian plane wave with the pulse width of 4.5 nano-second (ns). The incident pulse is propagating along $\phi = 45^\circ$. The size of the grids and other computational domain related parameters are the same as in the last case. The scattered fields at various time steps are shown in Figure 3.23. The eight plots in Figure 3.23 represent the transient magnetic field amplitudes on the entire computational domain at different time steps. The heights of the plots indicate the field amplitudes, which have been normalized in each plots. From these plots, the reflection, the edge diffraction, and the edge coupling of two edges are clearly shown and can be traced in time.
3.5 Summary

The finite difference time domain technique on triangular grids is developed and
verified by various cases. This technique is the generalization of the control re-
gion approximation and the finite difference technique for rectangular grid. The
discretization scheme is simple, and reduces to the Yee’s algorithm when the De-
launay tessellation becomes rectangular. The discretization provides second order
accuracy in time and space when the grid is regular. For irregular grid, the spatial
discretization is accurate to first order. This algorithm can be applied to arbitrary
geometries; in addition, permeability, permittivity, electric and magnetic losses can
be included. The algorithm is tested with circular cylinder geometries. The time
harmonic results are extracted from the late time responses of a sinusoidal excitation
and compared with the eigen series solutions. Pulse responses are also calculated
for a conducting strip. The computation domains are truncated using either circular
or elliptical outer boundaries. Numerical results indicate highly absorbing nature of
the corresponding absorbing boundary conditions. An additional advantage is found
to be associated with the elliptical boundary, which provides smaller computational
domain for elongated scatterers.
3.6 Appendix: Stability Criterion

The derivation of the instability criterion for arbitrary triangular grids involve a very lengthy algebra. In this appendix, the stability condition for the equilateral triangular grids is derived. Figure 3.24 shows a Delaunay tessellation associated with the equilateral triangular grids. It should point out that the procedure to the instability condition for arbitrary triangular grids is the same. Focusing on the electric field polarization (same for magnetic field polarization), from Equations (3.1) to (3.3) and in free space

\[ \nabla^2 E_x - \frac{\partial^2 E_x}{\partial \tau^2} \]

where \( \tau = ct \) is the normalized time. Integrate this equation in the control region, we obtain

\[ \oint \frac{\partial E_x}{\partial n} \, ds - \iint \frac{\partial^2 E_x}{\partial \tau^2} \, dA = 0 \]

We apply the finite difference for normal derivatives and approximate the double integral by the value of the integrand at the node times the area of the control region.

\[ \frac{l}{h} \sum_{i=1}^{6} \left( E_{x[i]}^n - E_{x[0]}^n \right) - \frac{A}{\tau^2} \left( E_{x[0]}^{n+1} - 2E_{x[0]}^n + E_{x[0]}^{n-1} \right) = 0 \quad (3.21) \]
3. A FD-TD Technique on Triangular Grids

Assuming

\[ E_{z[0]}^n = u^n e^{i k (x_o \cos \theta + y_o \sin \theta)} , \]

and

\[ E_{z[t]}^n = u^n e^{i k (x_i \cos \theta + y_i \sin \theta)} . \]

We define the amplification factor to be

\[ z = u^{n+1}/u^n \quad (3.22) \]

Substituting into the (3.21)

\[ \frac{I}{h} z \sum_{1}^{g} \left( e^{i k ((x_i-x_o) \cos \theta + (y_i-y_o) \sin \theta)} - 1 \right) - \frac{A}{\tau^2} \left( z^2 - 2z + 1 \right) = 0. \]

In order to have a stable algorithm, the maximum magnitude of \( z \) must not exceeds unity. Based on this condition, we find

\[ t \leq \sqrt{\frac{2}{3} \frac{h}{c}} . \]
Figure 3.11: Coated conducting cylinders and computational domains.
Figure 3.12: Amplitude of $E_z(s)$ along $r = 2.47$ m for electric field polarization. Coating: $\varepsilon_r = 1.0$ and $\mu_r = 1.0$. Solid curve: eigen function expansion; dashed curve: triangular grid FD-TD; circles: rectangular grid FD-TD. Frequency: 150 MHz.

Figure 3.13: Amplitude of $H_z(s)$ along $r = 2.47$ m for magnetic field polarization. Coating: $\varepsilon_r = 1.0$ and $\mu_r = 1.0$. Solid curve: eigen function expansion; dashed curve: triangular grid FD-TD; circles: rectangular grid FD-TD. Frequency: 150 MHz.
Figure 3.14: Amplitude of $E_r(r)$ along $r = 2.47$ m for electric field polarization. Coating: $\varepsilon_r = 1.0$ and $\mu_r = 1.0$, Solid curve: eigen function expansion, dashed curve: triangular grid FD-TD; circles: rectangular grid FD-TD. Frequency: 300 MHz.

Figure 3.15: Amplitude of $H_r(r)$ along $r = 2.47$ m for magnetic field polarization. Coating: $\varepsilon_r = 1.0$ and $\mu_r = 1.0$, Solid curve: eigen function expansion; dashed curve: triangular grid FD-TD; circles: rectangular grid FD-TD. Frequency: 300 MHz.
Figure 3.16: Amplitude of $E_\phi(s)$ along $r = 2.47$ m for electric field polarization. Coating: $\varepsilon_r = 3.0$, $\mu_r = 1.0$ and $\sigma = 0.01$/m$\Omega$. Solid curve: eigen function expansion; dashed curve: triangular grid FD-TD. Frequency: 150 MHz.

Figure 3.17: Amplitude of $H_\phi(s)$ along $r = 2.47$ m for magnetic field polarization. Coating: $\varepsilon_r = 3.0$, $\mu_r = 1.0$ and $\sigma = 0.01$/m$\Omega$. Solid curve: eigen function expansion; dashed curve: triangular grid FD-TD. Frequency: 150 MHz.
Figure 3.18: Amplitude of $E_z(s)$ along $r = 2.47$ m for electric field polarization, Coating: $\varepsilon_r = 3.0$, $\mu_r = 3.0$ and $\sigma = 0.01$/mΩ. Solid curve: eigen function expansion; dashed curve: triangular grid FD-TD. Frequency: 150 MHz.

Figure 3.19: Amplitude of $H_z(s)$ along $r = 2.47$m for magnetic field polarization, Coating: $\varepsilon_r = 3.0$, $\mu_r = 3.0$ and $\sigma = 0.01$/mΩ. Solid curve: eigen function expansion, dashed curve: triangular grid FD-TD. Frequency: 150 MHz.
Figure 3.20: Conducting strip and computational domains. Grid size: 0.075 m.
Figure 3.21: Bistatic scattering width of 5 m strip for electric field polarization. Frequency: 400 MHz; incident angle: 45°. Solid curve: method of moments, dashed curve: elliptical boundary; circles: circular boundary.

Figure 3.22: Bistatic scattering width of 5 m strip for magnetic field polarization. Frequency: 400 MHz; incident angle: 45°. Solid curve: method of moments, dashed curve: elliptical boundary; circles: circular boundary.
Figure 3.23: Gaussian pulse response of 5 m conducting strip for magnetic field polarization. Pulse width: 4.5 ns; incident angle: 45°. Frame 1: 3.12ns, frame 2: 6.25ns, frame 3: 9.38ns, frame 4: 12.5ns, frame 5: 15.6ns, frame 6: 18.8ns, frame 7: 21.9ns, and frame 8: 25.0ns.
Figure 3.24: Control region for equilateral triangular grids.
Chapter 4

APPLICATION OF THE FD-TD TECHNIQUES TO DISPERSIVE MATERIALS

4.1 Introduction

The finite-difference time-domain (FD-TD) technique has been widely applied to electromagnetic problems [1-22]. In recent years, research efforts have been concentrated in more accurate discretization schemes, implementations on supercomputers, and applications to various electromagnetic problems concerning microwave components, wave guides, scattering, and medical applications. Because of the simplicity of the FD-TD algorithm, it has been applied to analyze electromagnetic problems involving complex structures, inhomogeneous materials, anisotropic materials, and time varying media. However, the application of the FD-TD technique to problems involving frequency dispersive materials has not been well explored.

Perhaps, the primary reason the FD-TD technique has not been widely applied to problems involving frequency dispersive material is the lack of an efficient time
domain model. The FD-TD technique has been applied to problems in which the frequency dispersive material has well defined time domain model, such as plasma [20]. For general frequency dispersive materials, two approaches are usually taken. In the first approach, the relative permittivity, electric conductivity, relative permeability, and magnetic conductivity are approximated to be constant in the entire frequency range. This approach is only applicable to narrow frequency band problems. In the second approach, convolution integrals are employed [16]

\[ D(t) = \varepsilon_0 \int_0^t d\tau \varepsilon(t - \tau)E(\tau) \]  \hspace{1cm} (4.1)

or

\[ B(t) = \mu_0 \int_0^t d\tau \mu(t - \tau)H(\tau) \]  \hspace{1cm} (4.2)

Since these convolutions are applied to every discretization point at every time step, this approach is time consuming and requires large memory. In addition, it is quite difficult to obtain \( \varepsilon(t) \) and \( \mu(t) \) for general dispersive materials.

In this chapter, a simple scheme to obtain efficient time domain models and a FD-TD algorithm for the frequency dispersive materials is discussed. The technique does not require time domain convolution. The time domain models of the dispersive materials are written in the form of ordinary time differential equations. The FD-TD algorithm proposed is applicable to one-dimensional (1D), two-dimensional (2D), and three dimensional (3D) problems. In Section 4.2, an introduction to general frequency dispersive materials is given. In Section 4.3, the FD-TD algorithm and the
4. Application of the FD-TD Techniques to Dispersive Materials

required time domain models of the dispersive materials are discussed. In Section 4.4, a discretization scheme for the 2D triangular grid configuration is presented. In Section 4.5, the FD-TD technique is applied to 1D and 2D problems involving frequency dispersive materials.
4.2 Frequency Domain Models

In most of the practical situations, the dispersive characteristic of a material is described by a collection of discrete data points in some frequency band. The dispersive characteristic of the material may be governed by some complex physical phenomenon. Nevertheless, the mathematical description of the complex relative permittivity and permeability should obey the causality condition and at the same time provide a reasonable match with the discrete data points.

The mathematical description of the causality condition is summarized in the Kramers-Kröning’s relationships [92]. In the case of electric dispersion, the complex relative permittivity satisfies

\[ \epsilon_R(\omega) - \epsilon_\infty = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} d\omega' \frac{\epsilon_I(\omega')}{\omega' - \omega} \]

(4.3)

\[ \epsilon_I(\omega) = -\frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} d\omega' \frac{\epsilon_R(\omega') - \epsilon_\infty}{\omega' - \omega} \]

(4.4)

where

\[ \epsilon(\omega) = \epsilon_R(\omega) + i\epsilon_I(\omega), \quad \epsilon_\infty = \epsilon(\infty) \]

(4.5)

The identical expressions are applied to the complex relative permeability in the case of magnetic dispersion.

To derive the mathematical model for the dispersive material, one may start
with a set of laws which describe the physical process within the material. For the purpose of computation, however, one can select an appropriate mathematical expression which satisfies the causality condition and reasonably describes the dispersive characteristic. There are a number of mathematical models which satisfy the causality condition. Three of such models are the Debye model [91], the molecular resonance model or the Lorentz model [93], in the case of electric dispersion, and the magnetic wall resonance model [94] in the case of magnetic dispersion. It turns out the molecular resonance model and the magnetic wall resonance model share a similar expression. For the purpose of computation, whether a model describes physically accurate electric or magnetic dispersion is not important. The important issue is to insure that the mathematical expression satisfies the causality condition. For the molecular resonance model and the magnetic wall resonance model, the following equations are used to describe the relative permittivity and permeability, respectively:

\[
\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_o^2 - \omega^2 - i\gamma_0\omega}, \tag{4.6}
\]

\[
\mu(\omega) = 1 + \frac{\omega_p^2}{\omega_o^2 - \omega^2 - i\gamma_0\omega}. \tag{4.7}
\]

Figure 4.1 shows the complex relative permeability of a sample magnetic dispersive material. The 'o' and the '+' show the experimental data of the real and imaginary parts of the complex relative permeability of the material, respectively. The solid
curves show the results calculated using (4.7) with the following parameters

$$\omega_o = 2.0\pi(200\text{GHz}), \quad \rho/\omega_o = 20.0, \quad \omega_p/\omega_o = 1.73 \quad (4.8)$$

As seen from Figure 4.1, (4.7) provides a reasonable match with the experimental data.

In the case of Debye model,

$$\epsilon(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 - i(\omega/\omega_0)}. \quad (4.9)$$

This expression has been applied to model water in which case \(\epsilon_\infty = 1.8\), \(\epsilon_s = 81.0\), and \(\omega_o = 2\pi(16.93\text{GHz})\). Figure 4.2 shows the real and imaginary parts of the relative permittivity for water.

Finally, from numerical standpoint, whether or not the mathematical model satisfies the causality condition may not be important. It is because that, in practice, only a finite number of data are given in a frequency band for the dispersive material.
4.3 FD-TD algorithm

The conventional FD-TD algorithm for electromagnetic problems consists of two major steps [1,4,7,17]. In the first step, the previous time level magnetic field is used to update the electric field. In the second step, the newly updated electric field is used to calculate the magnetic field. These two steps may be swapped. In order to account for the dispersive characteristic, the two step FD-TD algorithm is split into four steps. The additional steps are designed to relate \( \overline{E} \) to \( \overline{D} \) and \( \overline{H} \) to \( \overline{B} \).

For the reason of simplicity, the following normalized field variables are defined:

\[
\bar{d} = \overline{D}/\varepsilon_0, \quad \bar{e} = \overline{E}, \quad \bar{b} = \eta_0\overline{B}/\mu_0, \quad \bar{h} = \eta_0\overline{H}.
\]  

(4.10)

In addition, the normalized time, \( \tau = c_0 t \), is also defined. In (4.10), \( \varepsilon_0, \mu_0 \) and \( \eta_0 \) are the free space permittivity, permeability and impedance, respectively. With this normalized field quantities, the FD-TD algorithm is

\[
\nabla \times \bar{h} = \frac{\partial \bar{d}}{\partial \tau} \]

\[
\bar{d} \rightarrow \bar{e}
\]

\[
\nabla \times \bar{e} = \frac{\partial \bar{b}}{\partial \tau} \]

\[
\bar{b} \rightarrow \bar{h}
\]

(4.11)

Steps 1 and 3 are almost identical to the conventional algorithm except that the
variables \( \tilde{d} \) and \( \tilde{b} \) are introduced. To complete the algorithm, steps 2 and 4 require time domain models to describe the dispersive nature of the material.

The procedure to derive the appropriate time domain model is the same for electric and magnetic dispersive materials. Focusing on the electric dispersion,

\[
\tilde{d}(\omega) = \varepsilon(\omega) \tilde{e}(\omega),
\]

(4.12)

where \( \varepsilon(\omega) \) may be obtained by selecting the appropriate model, such as (4.6) and (4.9), with parameters obtained by curve-fitting the experimental data. As suggested by (4.12), an obvious time domain model is the convolution which is inefficient. Instead, the following substitution

\[
\frac{\partial}{\partial \tau} = -i \omega/c_0
\]

(4.13)

is made in (4.12). The substitution has been employed in many problems. Most frequently, it has been applied in designing digital filters, or in the electromagnetic community, conversion of frequency domain absorbing boundary condition to the time domain absorbing boundary condition [30,39,40]. For the frequency domain model in (4.6), the substitution leads to the following time domain model

\[
\left( \frac{\partial^2}{\partial \tau^2} + 2 \rho' \frac{\partial}{\partial \tau} + \omega_0^2 \right) d(\tau) = \left( \frac{\partial^2}{\partial \tau^2} + 2 \rho' \frac{\partial}{\partial \tau} + \omega_0^2 + \omega_p^2 \right) e(\tau)
\]

(4.14)
where

$$\rho' = \rho/c, \quad \omega_0' = \omega_0/c, \quad \omega_p' = \omega_p/c. \quad (4.15)$$

With the same substitution, the time domain model corresponding to (4.9) is

$$\left( \frac{1}{\omega_0} \frac{\partial}{\partial \tau} + 1 \right) d(\tau) = \left( \frac{\epsilon_\infty}{\omega_0} \frac{\partial}{\partial \tau} + \epsilon_s \right) e(\tau) \quad (4.16)$$

These time domain models can be easily discretized. Since the time domain models, such as (4.14) and (4.16), are written as the ordinary time differential equations, the four step algorithm and its discretization schemes are applicable to 1D, 2D and 3D problems. The major advantage of the time domain modeling are:

- avoiding convolution integral and requiring only few time levels of field information, three levels for (4.14) and two levels for (4.16) (see next section),
- simple implementation both in obtaining time domain model and its discretization.
4. Application of the FD-TD Techniques to Dispersive Materials

4.4 Discretization

There are a number of schemes available for discretizing the FD-TD algorithm for frequency dispersive material discussed in Section 4.3. In fact most of the existing schemes, for example [1, 17], can be easily integrated into the algorithm with little alteration. In this section, a discretization scheme based on [17] is discussed. The discussion is on 2D triangular grid. As pointed out in the previous section, the generalization to 3D or other discretization schemes should be straightforward.

4.4.1 Temporal Discretization

The time domain discretization of field variables is carried out in the temporal interlace manner. That is \(d(\tau)\) and \(e(\tau)\) are in integer time:

\[
\bar{d}^n, \quad e^n
\]  
(4.17)

and \(b(\tau)\) and \(h(\tau)\) are in half integer time:

\[
b^{n+1/2}, \quad h^{n+1/2}.
\]  
(4.18)

This procedure is identical to those in [1, 17], and the time marching is in the leap-frog manner. With this time arrangement the time derivatives in steps 1 and 3 of (4.11) can be discretized using the center difference.

The discretization for step 2 and 4 of Equation (4.11) may be done using various
methods available for ordinary differential equations. Using center difference in time, (4.6) can be discretized and rearranged to:

\[ e^{n+1} = d^{n+1} + \frac{\Delta^2 \omega_0'^2 - 2}{1 + \Delta \rho'} (d^n - e^n) \]

\[ - \frac{\Delta^2 \omega_p'^2}{1 + \Delta \rho'} e^n + \frac{1 - \Delta \rho'}{1 + \Delta \rho'} (d^{n-1} - e^{n-1}). \]  

(4.19)

As seen from (4.19), numerical treatment of the dispersive material modeled by (4.6) requires three levels of previous time information. Similarly, (4.16) can be discretized and rearranged to:

\[ e^{n+1} = \left( \frac{1}{\omega_0' \Delta_r} + \frac{1}{2} \right) d^{n+1} + \left( \frac{1}{2} - \frac{1}{\omega_0' \Delta_r} \right) d^n + \left( \frac{\varepsilon_\infty}{\omega_0' \Delta_r} - \frac{\varepsilon_s}{2} \right) e^n \]

\[ \frac{\varepsilon_\infty}{\omega_0' \Delta_r} + \frac{\varepsilon_s}{2}. \]  

(4.20)

Due to the fact that the time domain model involves only the first order time derivative, the corresponding numerical treatment requires two levels of previous time information.
4. Application of the FD-TD Techniques to Dispersive Materials

4.4.2 Spatial Discretization

The spatial discretization refers to the discretization in step 1 and 3 of the FD-TD algorithm in the previous section. For 2D triangular grid, the discretization is identical to the procedure described in [17], except for the interface. To facilitate the discussion, a local coordinate system (Figure 4.3) is defined. In Figure 4.3, \( \hat{s} \) and \( \hat{n} \) define the unit tangent and normal vectors along an arbitrary contour. Furthermore, \( \hat{n} \times \hat{s} = \hat{z} \). The discretization is carried out using the control regions approximation [9]. Figure 4.4 defines the discretization cell which is obtained using the Dirichlet and Delaunay tessellations.

Maxwell's equations, written in terms of the normalized variables, for electric field polarization, are

\[
\nabla_T \times (\hat{n} h_n + \hat{s} h_s) = \hat{z} \frac{\partial d_x}{\partial \tau} \tag{4.21}
\]

\[
\frac{\partial e_x}{\partial n} = \frac{\partial b_s}{\partial \tau} \tag{4.22}
\]

\[
\frac{\partial e_x}{\partial s} = - \frac{\partial b_n}{\partial \tau} \tag{4.23}
\]

The relationship from \( \vec{d} \) to \( \vec{e} \) and \( \vec{b} \) to \( \vec{h} \) are given by the time domain models of the dispersive materials. Applying an area integral and Green's theorem to (4.21), \( b_n \) and
4. Application of the FD-TD Techniques to Dispersive Materials

$h_n$ are decoupled from other field variables and the following equations are obtained:

$$\int h_s \, dl = \iint \frac{\partial d_z}{\partial \tau} \, dA \tag{4.24}$$

$$\frac{\partial e_z}{\partial n} = \frac{\partial b_z}{\partial \tau}. \tag{4.25}$$

These are the two equations corresponding to steps 1 and 3 in the FD-TD algorithm. Figure 4.5 shows the field locations in the control region. $\bar{c}$ and $\bar{d}$ are at the same location, and, similarly, $\bar{h}$ and $\bar{b}$ are at the same location. Equations (4.24) and (4.25) are discretized as follow:

$$\sum_{k=1}^{M} h_{s[k,j]}^{m-1/2} l_{[k,j]} = \frac{A}{\Delta \tau} (d_{s[j]}^{m} - d_{s[j]}^{m-1}) \tag{4.26}$$

$$\frac{1}{d_{[k,j]}} (e_{s[k]}^{m} - e_{s[j]}^{m}) = \frac{1}{\Delta \tau} (b_{s[k,j]}^{m+1/2} - b_{s[k,j]}^{m-1/2}), \tag{4.27}$$

where $A$ is the area of the control region, $\Delta \tau$ is the normalized time step, and $d$ is the separation between two nodes. The boundary conditions at perfectly conducting surfaces can be easily satisfied with this scheme. At the dispersive medium interface, however, a caution must be taken. Since both $e_z$ and $d_z$ are placed at the interface, the continuity of electric field is satisfied. The $d_z$ at the interface is split to the value just inside the interface ($d_{z+}$) and the value just outside the interface ($d_{z-}$). The continuity of normal magnetic flux density is satisfied due to the fact that $b_z$ is
normal to the interface. Different values of magnetic field are assigned at the interface; \( h_{z+} \) is the magnetic field value just inside the interface and \( h_{z-} \) is the magnetic field value just outside the interface. Figure 4.6 shows the field location assignment at the interface. With the above field location assignment, the discretization of (4.24) and (4.25) are

\[
\sum_{k=1}^{M} h_{z_{\pm}[k,j]}^m l_{[k,j] \pm} = \frac{A_+}{\Delta \tau} (d_{z[j]+}^m - d_{z[j]+}^{m-1}) + \frac{A_-}{\Delta \tau} (d_{z[j]-}^m - d_{z[j]-}^{m-1}) \tag{4.28}
\]

\[
\frac{1}{d_{[k,j]}} (e_{[k]}^m - e_{[j]}^m) = \frac{1}{\Delta \tau} (b_{[k,j]}^{m+1/2} - b_{[k,j]}^{m-1/2}) \tag{4.29}
\]

The relationship between two unknowns \( d_{z[j]+} \) and \( d_{z[j]-} \) in (4.28) can be obtained by utilizing material models on both sides of the interface and the continuity of tangential electric field. The material models on both sides of the interface are also used to calculate \( h_{z+} \) and \( h_{z-} \), once \( b_z \) is found from (4.29).

Maxwell's equations, written in terms of the normalized variables, for magnetic field polarization are:

\[
\nabla_T \times (\hat{n} e_n + \hat{e} e_z) = -\hat{z} \frac{\partial b_z}{\partial \tau} \tag{4.30}
\]

\[
\frac{\partial h_z}{\partial n} = -\frac{\partial d_z}{\partial \tau} \tag{4.31}
\]
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\[ \frac{\partial h_z}{\partial s} = \frac{\partial d_n}{\partial \tau}. \]

(4.32)

Applying an area integral and Green's theorem to (4.30), \( e_n \) and \( d_n \) are decoupled from the other field variables and the following equations are obtained:

\[ \oint e_s \, dl = - \iint \frac{\partial b_z}{\partial \tau} \, dA \]

(4.33)

\[ \frac{\partial h_z}{\partial n} = - \frac{\partial d_s}{\partial \tau} \]

(4.34)

Figure 4.7 shows the field locations in the control region. \( \bar{e} \) and \( \bar{d} \) are at the same location, and similarly \( \bar{h} \) and \( \bar{b} \) are at the same location. Equations (4.33) and (4.34) are discretized as follow:

\[ \sum_{k=1}^{M} e_{s[k,j]}^{m-1} l_{[k,j]} = \frac{A}{\Delta \tau} \left( b_{z[j]}^{m-1/2} - b_{z[j]}^{m-3/2} \right) \]

(4.35)

\[ \frac{1}{d_{[k,j]}} \left( h_{x[k]}^{m-1/2} - h_{x[j]}^{m-1/2} \right) = - \frac{1}{\Delta t} \left( d_{s[k,j]}^{m} - d_{s[k,j]}^{m-1} \right). \]

(4.36)

At the dispersive medium interface, similar to the electric field polarization case, \( b_z \) is split into \( b_{z+} \) and \( b_{z-} \) and \( e_z \) is split into \( e_{z+} \) and \( e_{z-} \). Figure 4.8 shows the field location assignment at the interface. With this field location assignment, the
discretization of (4.33) and (4.34) are

\[
\sum_{k=1}^{M} \varepsilon_{s[k,j]}^m \frac{e_{s[k,j]}}{t_{[k,j]}} \pm = - \frac{A_+}{\Delta \tau} \left( b_{s[j]}^{m-1/2} - b_{s[j]}^{m-3/2} \right) - \frac{A_-}{\Delta \tau} \left( b_{s[j]}^{m-1/2} - b_{s[j]}^{m-3/2} \right) \quad (4.37)
\]

\[
\frac{1}{d_{s[j]}} \left( \frac{h_{s[k]}^{m-1/2}}{d_{s[j]}} - \frac{h_{s[j]}^{m-1/2}}{d_{s[j]}} \right) = - \frac{1}{\Delta \tau} \left( c_{s[k,j]}^{m} - c_{s[k,j]}^{m-1} \right). \quad (4.38)
\]

The two unknowns \( b_{s[j]}^+ \) and \( b_{s[j]}^- \) in (4.37) are related by the continuity of tangential electric field. The relationship between \( b_{s[j]}^+ \) and \( b_{s[j]}^- \) are found by utilizing the materials models on both sides of the interface. The material models on both sides of the interface are also used to calculate \( e_{s}^+ \) and \( e_{s}^- \), once \( d_{s} \) is found from (4.38).
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4.5 Numerical Results

To illustrate the validity of the FD-TD algorithm for dispersive material presented in Section 4.2, three numerical examples are considered. In the first and second cases, the configurations are the half space filled with dispersive materials; the ratios of the reflected field amplitudes to the incident field amplitudes are calculated. These two cases primarily demonstrate the FD-TD algorithm and the discretization of the two models (4.14) and (4.16). In the third case, the FD-TD algorithm is applied to a 2D electromagnetic scattering problem. The configuration considered in this case is a conducting cylinder coated with dispersive material. The amplitude of the backscattered fields at a range of frequency are calculated using the FD-TD technique and compared with those using the eigen function expansion. In this case, the FD-TD algorithm and the triangular grid discretization are employed and tested for their applications to 2D problems.

In the first case, the FD-TD algorithm is applied to the problem of half space filled with water [16]. Figure 4.9 shows the configuration for the numerical simulation. The relative permittivity is modeled using the Debye formula (Figure 4.2) with the time domain model of (4.16). The incident wave is a Gaussian time pulse centered at DC having half power bandwidth of 73.4 GHz and amplitude of 1 V/m. The computational domain is located from the origin to $z = 7.5$ cm and discretized into 1000 divisions. To simulate the half space, a relatively thick medium, from 3.75 cm to 7.125 cm, is chosen so that the transmitted field is significantly attenuated before arriving at the second boundary. The computational domain is truncated
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by imposing the first order absorbing boundary condition [30,32] at \( z = 7.5 \) cm. It should be pointed out that, with this 1D configuration, the pulse reflected at the artificial boundaries is not only negligible but also separated in time with the reflected pulse at the first boundary. The incident Gaussian time pulse is launched at \( z = 0.0 \) cm. To eliminate reflection from the source, the first order absorbing boundary condition is imposed after the main portion of the Gaussian time pulse has left the source plane. The normalized time step is chosen to be half the grid size [16]. Figure 4.10 shows the incident and reflection pulse at \( z = 3 \) cm. The incident pulse is centered at about time step of 920 and the reflected pulse is at about 1325 time step. By applying Fourier transforms to the incident and the reflected pulses and taking the ratios for the corresponding frequency components, the reflection coefficient as function of frequency (from 0 to 60 GHz), is obtained. The results are presented in (Figure 4.11). where the solid curve represents the exact solution:

\[
R(\omega) = \frac{1 - \sqrt{\varepsilon(\omega) \mu(\omega)}}{1 + \sqrt{\varepsilon(\omega) \mu(\omega)}} \tag{4.39}
\]

The dashed curve shows the calculated reflection coefficient using the FD-TD technique. As seen from Figure 4.11, the calculated result matches very well with the exact result. This example demonstrates the validity and the accuracy of the FD-TD algorithm, discretization scheme, and the time domain modeling of the Debye's formula.

In the second case, the FD-TD algorithm is applied to the problem of half space filled with relative permeability of 1.0 and relative permittivity given by Figure 4.1.
This is a hypothetical material. Figure 4.12 shows the configuration for the numerical simulation. The time domain model of (4.14) is used. The incident wave is a Gaussian pulse centered at 10 GHz having half power bandwidth of 16.0 GHz and amplitude of 1 V/m. The computational domain is located from the origin to \( x = 15.0 \) cm and discretized into 1000 divisions. To simulate the half space, a relatively thick medium, from 7.5 cm to 14.25 cm, is chosen so that the transmitted field is significantly attenuated before impinging on the second boundary. The computational domain is truncated by imposing the first order absorbing boundary condition at \( x = 15.0 \) cm. The incident Gaussian time pulse is launched at \( x = 0.0 \) cm. The normalized time step is chosen to be half the grid size. Figure 4.13 shows the incident and reflection pulse at \( x = 6.0 \) cm. Figure 4.14 shows the transmitted pulse at \( x = 9.0 \) cm. As seen from Figure 4.14, the transmitted pulse is attenuated due to loss and broadened in time due to dispersion. In Figure 4.13, the incident pulse is centered at about time step of 1050 and the reflected pulse is at about 1550 time step. By applying Fourier transforms to the incident and the reflected pulses and taking the ratios for the corresponding frequency components, the reflection coefficient as function of frequency, 2 to 20 GHz, is obtained (Figure 4.15). The solid curve shown in Figure 4.15 represents the exact solution (4.39). The dashed curve shows the reflection coefficient calculated using the FD-TD technique. As seen from Figure 4.15, the calculated result matches very well with the exact result. This example again demonstrates the validity and the accuracy of the FD-TD algorithm, discretization scheme, and the time domain modeling (4.14) for (4.6).
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In the last numerical simulation, the 2D dispersive scatterer is considered. Figure 4.16 depicts the configuration. The scatterer is a conducting cylinder of 0.5 cm diameter and coated with 0.287 cm thick dispersive material. The coating has constant relative permittivity of 13.2 and relative permeability given by (4.7) with parameters given by (4.8). The relative permeability of this material as function of frequency is shown in Figure 4.1. The computational domain has radius of 1.5 cm where the time domain Bayliss and Turkel's absorbing boundary condition [39,33] is imposed. The discretization is on triangular grids. Equations (4.26) to (4.29) used for electric field polarization, and Equations (4.35) to (4.38) used for magnetic field polarization. A Gaussian pulse with center frequency of 10 GHz and half power bandwidth of 16 GHz is impinging on the coated cylinder from the positive x axis. The FD-TD algorithm (4.11) is applied to compute the transient fields in the computational domain. Figures 4.17 and 4.18 show the backscattered field amplitudes at two locations for the electric and magnetic field polarizations, respectively. The dashed curve corresponds to the field at location inside the coating ($r = 0.42$ cm). The solid curve corresponds to the field at location outside the coating ($r = 1.25$ cm). In order to evaluate the numerical scheme and isolate the effects due to geometry and dispersive material, the FD-TD algorithm with constant magnetic conductivity of $\sigma_m = 9.475 \, k\Omega/m$ is also applied. The value of the magnetic conductivity is derived from the imaginary part of the relative permeability at 10 GHz, which is the center frequency in the frequency range of interest. Figures 4.19 and 4.20 show the corresponding fields at the same two locations for the electric and magnetic field polarizations, respectively. In calculating these results, the dimension of the sides
of the triangular cells are approximately 500 microns and the size of the normalized time steps are 222.22 microns. In order to verify the accuracy, Fourier transform is applied to the fields at \( r = 1.25 \text{ cm} \). Figures 4.21 and 4.22 show the ratio of the magnitudes of the backscattered fields to the incident fields for frequencies ranging from 2 to 18 GHz for the electric and magnetic field polarizations, respectively. The 'o' represents results calculated using eigen function expansion [66]. The solid curves are results calculated using the FD-TD technique in (4.11). The dashed curves are obtained using the FD-TD technique with constant magnetic conductivity. As seen from Figure 4.21 and Figure 4.22, the FD-TD technique with proper treatment of dispersion yields much more accurate results than the constant conductivity model, as expected.
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4.6 Summary

In this chapter, the FD-TD technique for electromagnetic problems is generalized to handle frequency dispersive materials. Discretization schemes and effective time domain models are investigated and demonstrated numerically. Numerical results indicate the validity and accuracy of the algorithm. To apply the algorithm, proper frequency domain models for the dispersive materials, such as the molecule resonance model and the Debye model, are used. By making the substitution of \(-i\omega\) to \(\partial/\partial t\), the frequency domain models are transformed to the time domain model which are in the form of ordinary time differential equations relating \(\overline{D}\) to \(\overline{E}\) and \(\overline{B}\) to \(\overline{H}\). To treat the frequency dispersive materials, the conventional FD-TD algorithm for electromagnetic problems is extended by discretizing the additional equations. The algorithm is demonstrated for simple 1D and 2D problems and shown to be accurate. This algorithm can be applied to 3D problems.
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Figure 4.1: Permeability of a sample dispersive material with $\epsilon_R = 13.2$, and $\epsilon_I = 0.0$. 'o': experimental real part; '+' : experimental imaginary part; solid curve: calculated real part; dashed curve: calculated imaginary part.

Figure 4.2: Permittivity of water. Solid curve: real part; dashed curve: imaginary part.
4. Application of the FD-TD Techniques to Dispersive Materials

![Diagram of local coordinate system](image1)

Figure 4.3: Local coordinate system.

![Diagram of Dirichlet and Delaunay tessellations](image2)

--- Delaunay Tessellation  ---  Dirichlet Tessellation

Figure 4.4: Dirichlet and Delaunay tessellations.
Figure 4.5: Field locations in a control region for electric field polarization.

Figure 4.6: Field locations on the interface for electric field polarization.
Figure 4.7: Field locations in a control region for magnetic field polarization.

Figure 4.8: Field locations on the interface for magnetic field polarization.
Figure 4.9: Geometrical configuration for pulse incident on half space filled with water.
Figure 4.10: Incident and reflected electric pulses near air-water interface.

Figure 4.11: Reflection coefficient for half space filled with water.
Figure 4.12: Geometrical configuration for pulse incident on half space filled with sample dispersive material.
Figure 4.13: Incident and reflected electric pulses near air-material interface.

Figure 4.14: Transmitted electric pulse near air-material interface.
Figure 4.15: Reflection coefficient for half space filled with dispersive material.
4. Application of the FD-TD Techniques to Dispersive Materials

Figure 4.16: Geometrical configuration for coated cylinder problem.
Figure 4.17: Electric field amplitudes at $r=0.42$ cm and $r=1.25$ cm.

Figure 4.18: Magnetic field amplitudes at $r=0.42$ cm and $r=1.25$ cm.
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Figure 4.19: Electric field amplitudes at \( r=0.42 \) cm and \( r=1.25 \) cm, (constant magnetic loss).

Figure 4.20: Magnetic field amplitudes at \( r=0.42 \) cm and \( r=1.25 \) cm, (constant magnetic loss).
Figure 4.21: Ratio of backscattered electric field to incident electric field at \( r = 1.25 \) cm.

Figure 4.22: Ratio of backscattered magnetic field to incident magnetic field at \( r = 1.25 \) cm.
Chapter 5

RADIATION FROM COUPLED SLAB STRUCTURES

5.1 Introduction

The packaging of a high density integrated circuit often includes a heatsink for the purpose of cooling. As the density of the integrated circuit increases, the presence of the heatsink becomes crucial to its performance and reliability. From the electromagnetics standpoint, this type of configuration forms an electromagnetically-coupled slab structure. This structure results in an antenna. As the signal speed on the circuitry increases and with the increased restriction of emission levels by the Federal Communications Commission (FCC), the analysis of this antenna structure becomes a necessity.

The emission level from a complete system may be reduced by means of shielding, or good cabinet design. Based on current FCC limits, at the upper frequency of 1 GHz for commercial products, present cabinet designs may be acceptable. However, as the clock frequencies of computer systems increase, the edge-rates of the electronic
pulses fall, and leakage of stronger high frequency components of electromagnetic energy may occur. In practice, air-flow and cabling requirements prohibit complete shielding of structures. In addition, with the proposed raising of FCC limits to an upper frequency of approximately five times the system bandwidth, the traditional cabinetry design may be inadequate. Besides the concern for meeting FCC limits, the leakage of high frequency energy may cause interference with electronic components in close proximity. A way of avoiding such problems is through emission reduction in the vicinity of the source.

In this chapter, we will analyze the electromagnetic radiation property of the integrated circuit package structure. The package will be modeled as a layered structure which consists of a ground plane, dielectric slabs, and a heatsink. Figure 5.1 shows the geometry of this slab structure with typical dimensions. To avoid complex issues relating to the detailed current distributions within the integrated circuit, we will consider two fundamental excitations; namely the electric and magnetic dipoles. These dipoles may be placed inside or above the dielectric slab. The total radiated power will be calculated. In addition, the effects of heatsink geometry, polarization, dielectric slabs, and grounding options on the emission levels will be examined. The finite-difference time-domain (FD-TD) technique on the staggered rectangular grid, hereafter referred to as the FD-TD technique, will be applied to analyze the problem. The FD-TD technique was originally proposed by Yee [1] to solve initial value electromagnetic problems. It has been successfully demonstrated and applied to many different problems in electromagnetics, but its application to analyze integrated circuit packages has not been explored. The technique has the advantage of being
simple and flexible in treating arbitrary geometries and materials; uniform rectangular blocks are utilized to approximate geometries. This may cause inaccuracies when non-rectangular geometries are modeled. However, many integrated circuit packages are rectilinear in form, thus rendering the rectangular grid model adequate.

The FD-TD technique is a well-developed numerical tool to analyze electromagnetic problems. The applications include radar cross sections computation for arbitrary targets, medical applications such as hyperthermia, microwave and millimeterwave circuits, radiation from patch antennas, and so on. For radar cross section problems, the FD-TD technique is employed to calculate the electric and magnetic fields in and near the scatterers, leading to far field scattering patterns and radar cross sections. The FD-TD technique is employed [13] to obtain frequency domain solutions by allowing the solution to reach steady state; this implies the stability of the FD-TD technique and is particularly important for radar cross section problems. In [12], the FD-TD technique is applied to analyze the radar cross sections of relatively large targets. Using the FD-TD technique, one can study the scattering of different geometries and materials. For medical applications, the FD-TD technique has been applied to analyze the heating or temperature profiles of tissues under microwave illumination. In [14], the induced temperature on a human eye due to microwave beam was calculated by modeling the human eye as several lossy dielectric materials. The FD-TD has also been employed [23] to optimize hyperthermia treatment. In microwave and millimeter wave applications, the FD-TD technique has been utilized to calculate scattering parameters and propagation constants of microstrip lines [25]. The excitation pulse for this type of application is usually simulated by imposing
values of the electric field at the end of the line. For microwave patch antenna application, the FD-TD technique is employed to analyze the radiation pattern. In [26], the patch was excited by imposing a voltage source with respect to the ground plane, and the FD-TD technique was employed to calculate the near field; the far field pattern was obtained by a near-to-far field transformation.

In this chapter, the FD-TD technique is applied to analyze the radiation property of a simplified integrated circuit package. The effects of heatsink presence, grounding of heatsink, dielectric slabs, and dipole polarizations on the emission levels will be examined. Specifically, the heatsink geometry; the excitation with vertical and horizontal dipoles; and emission reduction options are considered.
5.2 FD-TD Algorithm on Rectangular Grid

The Maxwell's equations in a time and frequency invariant medium are written as:

\[ \nabla \times \vec{H} = \sigma \vec{E} + \varepsilon_0 \varepsilon_r \frac{\partial \vec{E}}{\partial t}, \quad \nabla \times \vec{E} = -\sigma_m \vec{H} - \mu_0 \mu_r \frac{\partial \vec{H}}{\partial t} \quad (5.1) \]

In the above equations, \( \vec{E} \) and \( \vec{H} \) are electric and magnetic fields respectively. \( \varepsilon_0 \) is the free space permittivity, \( 8.854 \times 10^{-12} \) F/m. \( \mu_0 \) is the free space permeability, \( 4\pi \times 10^{-7} \) H/m. \( \varepsilon_r \) and \( \mu_r \) are the relative permittivity and relative permeability of the medium. \( \sigma \) and \( \sigma_m \) are electric and magnetic conductivities respectively. To facilitate the discretization, the above equations are decomposed into components of the rectangular coordinates:

\[ \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial z} = \sigma \varepsilon E_x + \frac{\varepsilon_r}{\eta c_0} \frac{\partial E_x}{\partial t}, \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\sigma_m H_z - \frac{\mu_r \eta}{c_0} \frac{\partial H_z}{\partial t} \quad (5.2) \]

\[ \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} = \sigma \varepsilon E_y + \frac{\varepsilon_r}{\eta c_0} \frac{\partial E_y}{\partial t}, \quad \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\sigma_m H_y - \frac{\mu_r \eta}{c_0} \frac{\partial H_y}{\partial t} \quad (5.3) \]

\[ \frac{\partial H_y}{\partial z} - \frac{\partial H_z}{\partial y} = \sigma \varepsilon E_x + \frac{\varepsilon_r}{\eta c_0} \frac{\partial E_x}{\partial t}, \quad \frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} = -\sigma_m H_z - \frac{\mu_r \eta}{c_0} \frac{\partial H_z}{\partial t} \quad (5.4) \]

where \( c_0 = 3.0 \times 10^8 \) m/s and \( \eta_0 = 120\pi \) \( \Omega \) are the speed of light and wave impedance in free space respectively. These equations can be approximated using finite difference on a staggered rectangular grid.
In the following subsection, the implementation of the central difference scheme on the above equations, boundary conditions, and the dipole excitation will be discussed. Finally, to reduce the total number of discretization nodes a discretization scheme on expanded grids in the vertical direction will be discussed.

5.2.1 Finite Difference Approximation

Figure 5.2 shows the staggered grid cell of the electric and magnetic fields. From Figure 5.2, the electric and magnetic fields are spatially interlaced in half a unit cell; this provides convenient coupling of the electric and magnetic fields. We assign the electric fields on a time step of \( n + 1/2 \) and the magnetic fields on a time step of \( n \). By interlacing the electric fields with the magnetic fields, the first order time derivative are approximated using central difference. With this arrangement, the components form of Maxwell’s equations are converted to finite difference form:

\[
\eta_0 \sigma_e \left( \frac{e^e_{x(i,j,k)} + e^e_{x(i,j,k) \pm 1}}{2} + e^e_{x(i,j,k) - e^e_{x(i,j,k)}} \right) = \frac{h^n_{x(i,j,k)} - h^n_{x(i,j,k-1)}}{\Delta y} - \frac{h^n_{y(i,j,k)} - h^n_{y(i,j,k-1)}}{\Delta x} \tag{5.5}
\]

\[
\eta_0 \sigma_e \left( \frac{e^e_{y(i,j,k)} + e^e_{y(i,j,k) \pm 1}}{2} + e^e_{y(i,j,k) - e^e_{y(i,j,k)}} \right) = \frac{h^n_{x(i,j,k)} - h^n_{x(i,j,k-1)}}{\Delta z} - \frac{h^n_{z(i,j,k)} - h^n_{z(i-1,j,k)}}{\Delta z} \tag{5.6}
\]
5. Radiation from Coupled Slab Structures

\[
\eta_0 \sigma_e \frac{e_{z(i,j,k)}^{n+1/2} + e_{z(i,j,k)}^{n-1/2}}{2} + \varepsilon_r \frac{e_{z(i,j,k)}^{n+1/2} - e_{z(i,j,k)}^{n-1/2}}{\Delta r} = \frac{h_{y(i,j,k)}^n - h_{y(i-1,j,k)}^n}{\Delta z} - \frac{h_{z(i,j,k)}^n - h_{z(i,j-1,k)}^n}{\Delta y}
\] (5.7)

\[- \frac{\sigma_m}{\eta_0} \frac{h_{x(i,j,k)}^{n+1} + h_{x(i,j,k)}^n}{2} - \mu_r \frac{h_{x(i,j,k)}^{n+1} - h_{x(i,j,k)}^n}{\Delta r} = \frac{e_{z(i,j+1,k)}^{n+1/2} - e_{z(i,j,k)}^{n+1/2}}{\Delta y} - \frac{e_{y(i,j,k+1)}^{n+1/2} - e_{y(i,j,k)}^{n+1/2}}{\Delta z}
\] (5.8)

\[- \frac{\sigma_m}{\eta_0} \frac{h_{y(i,j,k)}^{n+1} + h_{y(i,j,k)}^n}{2} - \mu_r \frac{h_{y(i,j,k)}^{n+1} - h_{y(i,j,k)}^n}{\Delta r} = \frac{e_{z(i,j+1,k)}^{n+1/2} - e_{z(i,j,k)}^{n+1/2}}{\Delta y} - \frac{e_{z(i+1,j,k)}^{n+1/2} - e_{z(i,j,k)}^{n+1/2}}{\Delta x}
\] (5.9)

\[- \frac{\sigma_m}{\eta_0} \frac{h_{z(i,j,k)}^{n+1} + h_{z(i,j,k)}^n}{2} - \mu_r \frac{h_{z(i,j,k)}^{n+1} - h_{z(i,j,k)}^n}{\Delta r} = \frac{e_{y(i+1,j,k)}^{n+1/2} - e_{y(i,j,k)}^{n+1/2}}{\Delta x} - \frac{e_{z(i,j+1,k)}^{n+1/2} - e_{z(i,j,k)}^{n+1/2}}{\Delta y}
\] (5.10)

in which \(\Delta_r = c_o \Delta t\) is the normalized time in meters, \(\bar{h} = \eta_0 \bar{H}\) is the normalized magnetic field in V/m, and \(\bar{e} = \bar{E}\). If we assume that the electric fields at time step \(n - 1/2\) and the magnetic fields at time step \(n\) are known everywhere, Equations (5.5) through (5.10) can be solved by matching the electric field to \(n + 1/2\) time and followed by matching the magnetic field to \(n+1\) time. That is, with appropriate initial
and boundary conditions, solutions to Equations (5.5) to (5.10) can be obtained via the explicit leap-frog time-marching scheme.

5.2.2 Boundary Condition

The application of the FD-TD technique to open boundary problems, such as radiation, requires an absorbing boundary condition to simulate free space. The primary property of this boundary condition is that the boundary is transparent to the wave outgoing with respect to the computational domain. There are several ways to simulate this property. One way to obtain this property is to insert a lossy layer [26], but it has been demonstrated that the layer should be relatively thick ($\sim 1.0\lambda$) in order to absorb the outgoing waves without generating significant reflections. The second way to obtain this property is to impose an artificial boundary condition. The most common boundary condition on the rectangular grid is the Engquist and Majda's second order absorbing boundary condition [30]:

$$\frac{\partial^2}{c_0 \partial n \partial t} \bar{E} = -\frac{\partial^2}{c_0^2 \partial t^2} \bar{E} + \frac{1}{2} \nabla_T^2 \bar{E} \quad (5.11)$$

where $n$ is the normal variable with respect to the boundary, such as $\pm z$ for the $\pm z$ faces, and $T$ represents transverse coordinate with respect to the boundary. Using the idea of averaging the time derivative in space and averaging the space derivative in time [32], Equation (5.11) can be approximated by finite difference and written in
the following manner:

\[
\begin{align*}
e_{(i+1,j,k)}^{n+1/2} &= -e_{(i,j,k)}^{n-3/2} + \frac{1}{\alpha} \left(1 - \frac{\Delta_n}{\Delta_r}\right) \left(e_{(i,j,k)}^{n+1/2} + e_{(i+1,j,k)}^{n-3/2}\right) \\
&\quad + \frac{2\Delta_n}{\alpha\Delta_r} \left(e_{(i+1,j+1,k)}^{n-1/2} + e_{(i,j,k)}^{n-1/2}\right) \\
&\quad + \frac{\Delta_n\Delta_r}{2\alpha\Delta_j^2} \left(e_{(i+1,j+1,k)}^{n-1/2} + e_{(i,j+1,k)}^{n-1/2}\right) \\
&\quad -2e_{(i,j,k)}^{n-1/2} - 2e_{(i+1,j-1,k)}^{n-1/2} + e_{(i+1,j-1,k)}^{n-1/2} + e_{(i,j-1,k)}^{n-1/2} \\
&\quad + \frac{\Delta_n\Delta_r}{2\alpha\Delta_k^2} \left(e_{(i+1,j,k+1)}^{n-1/2} + e_{(i,j,k+1)}^{n-1/2}\right) \\
&\quad -2e_{(i+1,j,k)}^{n-1/2} - 2e_{(i,j,k)}^{n-1/2} + e_{(i+1,j,k-1)}^{n-1/2} + e_{(i,j,k-1)}^{n-1/2}\end{align*}
\]

(5.12)

where \(\alpha = 1 + \Delta_n/\Delta_r\). It should be pointed out that no absorbing boundary condition is necessary for magnetic field, since they are totally contained within the computational domain. Figure 5.3 depicts the necessary components needed to evaluate the \(z\) component.

### 5.2.3 Dipole Excitation

The appearance of a dipole source in the finite difference grid may be simulated by imposing field values next to the dipole location. For a dipole in an unbounded homogeneous space, the time domain electric and magnetic fields are obtained from the corresponding frequency domain solutions of the Hertzian electric or magnetic dipoles by inverse Fourier transformation. For a vertical electric dipole at the origin,
the normalized magnetic field is:

\[
\vec{h}(\vec{r}, \tau) = \frac{\eta_0}{4\pi r^2} \left( -\hat{x} y + \hat{y} x \right) \left( \frac{dIl(\tau)}{d\tau} + \frac{Il(\tau)}{r} \right) u(\tau),
\]

(5.13)

where \( \tau = t - r/c \) is the normalized time, \( r = \sqrt{x^2 + y^2 + z^2} \), \( l \) is the length of the dipole, \( Il(\tau) \) is the electric current moment on the dipole, and \( u(\tau) \) is an unit step function. Figure 5.4 shows the location of the electric dipole and the components of magnetic fields being excited. Similarly, for vertical magnetic dipole at the origin, the normalized electric field is:

\[
\vec{E}(\vec{r}, \tau) = - \frac{\eta_0}{4\pi r^2} \left( -\hat{x} y + \hat{y} x \right) \left( \frac{dml(\tau)}{d\tau} + \frac{ml(\tau)}{r} \right) u(\tau)
\]

(5.14)

where \( \tau = t - r/c \) is the normalized time, \( r = \sqrt{x^2 + y^2 + z^2} \), \( l \) is the length of the dipole, \( ml(\tau) \) is the magnetic current moment on the dipole, and \( u(\tau) \) is an unit step function. Figure 5.5 shows the location of the magnetic dipole and the components of the electric field being excited.

Imposing the electric fields for electric dipole or magnetic fields for magnetic dipole presents numerical problem. The expressions of the electric field for electric dipole and magnetic field for magnetic dipole involve integration in time. The time integration associated with these fields corresponds to the DC components. Because of initial conditions, the current cannot be turned on before \( t = 0 \). As a result, the DC component of a sinusoidal current does not vanish. In the absence of numerical errors, it will reach a steady state value. In the presence of numerical errors, however, this DC value may be slowly increasing. The problem becomes more serious when a
significant DC component exists at the absorbing boundary, since this boundary only allows outward propagating waves. To avoid such problems, these field values are not calculated using analytical expressions, but by using the finite difference technique in a self-consistent manner.

Figure 5.6 and Figure 5.7 show the radiation pattern of a vertical electric dipole on the \( xy \) and \( xz \) planes. The dipole is located in free space and is excited by a sinusoidal current at 1.0 GHz with a dipole moment of \( 10^{-6} \) A-m. The radiation pattern is calculated from the steady state field values and compared with that obtained using the exact solution [92]. Because of the finite grid size and density, the apparent dipole moment is not \( 10^{-6} \) A-m, but \( 1.45 \times 10^{-6} \) A-m. This discrepancy in the dipole moment varies with the grid ratio \( (\Delta_x/\Delta_z, \Delta_y/\Delta_z) \), but is fairly independent of the actual grid size. To resolve the ambiguity, the numerical dipole moment is calibrated. For a dipole in free space, the total radiated power is

\[
P_{ED} = \frac{\eta_0}{12\pi} \left( kI \right)^2. \tag{5.15}\]

Therefore,

\[
(Il)_{cal} = \frac{1}{k} \sqrt{\frac{12\pi P_{ED}}{\eta_0}} \tag{5.16}\]

where \( P \) is calculated numerically. The calibration technique is applied to a magnetic
dipole for which

\[ P_{MD} = \frac{\eta_0}{12\pi} \left( \frac{kml}{\eta_0} \right)^2. \] (5.17)

When \( Il = \eta_0 ml \), the radiated powers from electric and magnetic dipoles are equal. The dotted curves in Figure 5.6 and Figure 5.7 show the results after calibration.

Equations (5.15) and (5.17) can be used to calibrate dipoles when they are located at a reasonable distance away from other sources or metallic structures. For a dipole located inside slab structures, a better calibration is to consider a dipole within a parallel-plate waveguide. For a vertical electric dipole at the center of the waveguide, the total guided power in the TEM mode is

\[ P_{VED} = \frac{k\eta_0}{8d} (Il)^2. \] (5.18)

For a horizontal magnetic dipole at the center of the waveguide, the total guided power in the TEM mode is

\[ P_{HMD} = \frac{k\eta_0}{16d} \left( \frac{ml}{\eta_0} \right)^2, \] (5.19)

which is one-half of \( P_{VED} \) when \( Il = \eta_0 ml \). These results are obtained using the formulations of dipoles in layered media [92], (Appendices).
5.2.4 Expanding Grid

In a practical integrated circuit package, if we consider the critical frequency of 1 GHz, the distance between the heatsink and the ground plane is of the order of 0.01 wavelength while the side of the heatsink is about 0.1 wavelength. To resolve the field distribution in between the heatsink and the ground plane, a few grids must be placed under the heatsink. The grid size will be of the order of 0.002 wavelength. There are two obvious practical problems associated with the application of the uniform rectangular grid in this problem. The first is that with the same grid size it will require too many grids to model the heatsink in the horizontal direction. The second one is the implementation of the absorbing boundary condition. The latter usually requires that the distance from the boundary to the source must be reasonably separated, e.g., by 0.25 to 0.5 wavelength; it will, therefore, take too many grids in both vertical and horizontal directions.

The first problem can be eliminated by choosing the grid size along the horizontal direction to be several times bigger than the vertical direction. To compensate for the larger error introduced by the larger grid size, a higher order finite difference scheme is used. For example the first order derivative along $x$ at "i" is [97]:

$$
\left( \frac{\partial e}{\partial x} \right)_i \sim \frac{-e_{i+3/2} + 27e_{i+1/2} - 27e_{i-1/2} + e_{i-3/2}}{24\Delta x}
$$

$$
+ \frac{3}{640} \Delta x^4 \left( \frac{\partial^5 e}{\partial x^5} \right)_i
$$

(5.20)

The second problem can be reduced by expanding the grid size for the region
above the heatsink. The corresponding finite difference scheme should be imposed on the absorbing boundary and the connection between fine and large grid regions. The difference scheme away from the connection is unchanged except for grid size. The second order finite difference scheme at the connection can be obtained by curve fitting three points. Figure 5.8 shows the grids and field location near the connection. The derivative of magnetic field at electric field position "i + 1/2" is approximated by:

\[
\left( \frac{\partial h}{\partial x} \right)_{i+1/2} \sim \frac{8h_{i+1}}{\Delta(1+a)(3+a)} + \frac{(a-3)h_i}{\Delta(1+a)} + \frac{(1-a)h_{i-1}}{\Delta(3+a)} + \frac{4a^3 + 13a^2 - 9}{24(a+1)(a+3)} \Delta^2 \left( \frac{\partial^3 h}{\partial x^3} \right)_{i+1/2}
\]

Equation (5.21) is derived by fitting a second degree polynomial to the magnetic fields at three points. The value "a" in the expression can be as high as 10 for the problem considered. For a = 10, the coefficient in the error term is about 1.54 which may be too high for other applications, yet it is acceptable here because \( \Delta \) is very small.
5.3 Numerical Results

The basic slab structure analyzed has the following physical dimensions: The separation between the upper conductor and the ground plane is 2.54 mm (0.1 inch). The heatsink has dimensions of 48.26 mm (1.9 inches) by 48.26 mm (1.9 inches) by 2.54 mm (0.1 inch) thick. The dielectric constants for the lower and upper dielectric slabs are 4.3 and 9.0, respectively.

In the following numerical cases, the electric dipole has a calibrated magnitude of $10^{-6}$ A-m and the magnetic dipole has a calibrated magnitude of $10^{-6}\eta_0$ V-m. By choosing these magnitudes for the electric and magnetic dipoles, they will radiate the same amount of power in free space. The computational domain is chosen to be 51 by 51 by 25. The grid sizes are $\Delta_x = \Delta_y = 3.71231$ mm and $\Delta_z = 0.635$ mm. The grid expands in the vertical direction at 7.62 mm from the ground plane with expanding ratio of 10.0; these will put the top computational boundary at about 1/4 wavelength from the top of the heatsink. The dipoles are placed at about 1.9 mm above the ground plane.

The radiation property of the slab structure due to complex current sources may be understood by studying the four basic dipole configurations. In the first case, the dielectric slabs are ignored. A vertical magnetic dipole (VMD) is placed at the center of the slab structure. The dipole is driven by a sinusoidal varying current at 1.0 GHz. By employing the FD-TD technique, the steady-state solution is established after approximately 3000 time steps. The complex tangential electric field ($\mathcal{E}$) and magnetic field ($\mathcal{H}$) on a closed surface enclosing the slab structure are
extracted from the steady state solution:

\[
\mathcal{E} = \frac{2}{T} \int_{t_0}^{t_0+T} dt \, e(t) \cos(\omega t) + i \frac{2}{T} \int_{t_0}^{t_0+T} dt \, e(t) \sin(\omega t) \quad (5.22)
\]

\[
\mathcal{H} = \frac{2}{T} \int_{t_0}^{t_0+T} dt \, h(t) \cos(\omega t) + i \frac{2}{T} \int_{t_0}^{t_0+T} dt \, h(t) \sin(\omega t) \quad (5.23)
\]

where \( T \) and \( \omega \) are period and angular frequency, respectively. The power density on the surface is determined by the cross product of tangential electric and magnetic fields. The total radiated power is found to be about \( 10^{-18} \) watts. By examining the time domain electric or magnetic fields at the surface, this low power level is found to be primarily due to numerical noise.

The low level emission of VMD can be traced back to the cutoff phenomenon of a parallel plate waveguide. Assuming the dimension of the heatsink on the \( xy \) is infinite, the dipole source is inside a parallel plate waveguide. Besides the TEM mode, in the absence of the dielectric slabs, the cutoff frequencies of the TE and TM modes are

\[
f_c = \frac{mc_o}{2d} \quad (5.24)
\]

where \( m \) and \( d \) are the mode number and separation between two the plates, respectively. The first cutoff frequency corresponding to the slab dimension is about 59 GHz. The propagation constant on the horizontal plane for the TE\(_1\) and TM\(_1\) modes
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is

\[ k_\rho = k_0 \sqrt{1 - \left(\frac{\lambda}{2d}\right)^2}. \]  \hspace{1cm} (5.25)

The corresponding decay factor of the TE_1 and TM_1 modes is \( e^{-371 \rho/\lambda} \). At 1.0 GHz, the distance from the center to the edge of the heatsink is about 0.08\( \lambda \); hence the field amplitudes drop to about \( 10^{-13} \) of their values at the center. The decay of the higher order modes are even larger than the lower order modes. Thus the TEM mode is the primary source of emission. In the case of VMD, all electric fields are on the \( xy \) plane; hence the TEM mode is not excited.

The second case involves the horizontal electric dipole (HED) inside the slab structure. The dielectric slabs are ignored. Sinusoidal current at 1.0 GHz is imposed on the dipole. The total radiated power is found to be on the order of \( 10^{-18} \) watts. The situation here is very similar to that of the first case. The electric field has major component on the \( xy \) plane which does not excite TEM mode. The small vertical component of electric field is not captured by the FD-TD technique due to large contrast between the horizontal and vertical electric fields of the HED. In fact among the four fundamental dipole source configurations, only the vertical electric dipole (VED) and the horizontal magnetic dipole (HMD) generate significant vertical electric field components.

In the third case, a vertical electric dipole (VED) is placed at the center of the slab structure. The dipole is driven by a sinusoidal current at 1.0 GHz. The dielectric slabs are not present. In Figure 5.9 and Figure 5.10, the solid curves show
the radiation patterns on the \(yz\) and \(zx\) planes respectively. The radiation patterns are calculated using Huygens' principle with electric and magnetic currents derived from the tangential magnetic and electric fields on the closed surface. As seen from these figures, the radiation pattern has peaks on the horizontal plane as expected for the VED. In Figure 5.9 and Figure 5.10, the dashed curves show the radiation pattern when the heatsink is not present. The radiation patterns due to the VED with or without the heatsink differ mainly in their magnitudes; it is because of the small heatsink dimensions relative to the wavelength. The ratio of the width of the heatsink to the wavelength at 1.0 GHz is about 0.161. The total radiated power with and without the heatsink are \(12 \times 10^{-9}\) watts and \(8.8 \times 10^{-9}\) watts respectively. The presence of the heatsink induces fringe fields along the edges of heatsink which may then enhance the radiation efficiency. For an assumed current amplitude of 10 mA, the corresponding radiation resistance with and without the heatsink are 0.24 m\(\Omega\) and 0.18 m\(\Omega\) respectively.

In the fourth case, a horizontal magnetic dipole (HMD) is placed at the center of the slab structure. The dipole is driven by a sinusoidal current of 1.0 GHz. The dielectric slab is not present. In Figure 5.11 and Figure 5.12, the solid curves show the radiation patterns on the \(yz\) and \(zx\) planes respectively. As seen from Figure 5.11, the presence of the heatsink enhances the upward radiation. The enhancement of the upward radiation is due to the antisymmetric fringe fields (Figure 5.13) along the opposite sides of the heatsink. The radiation due to these fringe field adds in phase along the vertical direction. In Figure 5.11 and Figure 5.12, the dashed curves show the radiation pattern when the heatsink is not present. The radiation
The \( xy \) plane are similar for the cases with and without the heatsink; it is again because of the small heatsink dimensions relative to the wavelength. The total radiated power with and without the heatsink are \( 7.3 \times 10^{-9} \) watts and \( 8.8 \times 10^{-9} \) watts, respectively. The small reduction in emitted power with the heatsink is perhaps due to the relatively weak fringe fields. A magnetic dipole is formed by a current loop. Using the relation between magnetic current moment and electric current loop current and area, \( ml = -i k \eta_0 I A \), the current loop has a radius of approximately 2.4 mm when \( I = 10 \) mA. The corresponding radiation resistance is 0.15 m\( \Omega \).

In the fifth and sixth cases, the effects of dielectric materials on the radiation property of the slab structure are studied. As seen from Figure 5.1, the lower slab is of infinite extent on the horizontal plane. This lower layer has a dielectric constant of 4.3 while the upper slab has a dielectric constant of 9.0. For case 5, the VED is placed at the center of the slab structure and on top of the lower slab. The dipole is driven by a 1.0 GHz sinusoidal current. The total radiated power for the cases without and with the upper slabs are \( 4.4 \times 10^{-9} \) watts and \( 2.5 \times 10^{-9} \) watts, respectively. The presence of the dielectric slabs tends to trap electric energy thereby reducing the total power of radiation. The opposite trend is observed, however, for the cases of HMD. For case 6, the HMD is placed at the center of the slab structure and on top of the lower slab. The dipole is again driven by a 1.0 GHz sinusoidal current. The total radiated power for the cases without and with the upper slabs are \( 9.2 \times 10^{-9} \) watts and \( 17 \times 10^{-9} \) watts, respectively.

The magnetic cavity model may be applied to explain these feature. The magnetic cavity is formed by placing magnetic walls at the sides of the heatsink (Fig-
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Figure 5.14). Interaction or coupling between the heatsink and the ground plane occurs very strongly when the operation frequency is close to the resonance frequency of the cavity. The cavity modes are TEM, TM, and TE. Due to cutoff, the TE modes are not guided. In the absence of the slabs, the TEM mode is excited. In the presence of the slab or slabs, the quasi-TEM mode is excited. Depending on the excitation source, these TEM or quasi TEM modes may be symmetric or antisymmetric with respect to dipole position. For a VED at the center, the symmetric TEM or quasi-TEM modes may be excited. On the other hand, for a HMD at the center the antisymmetric TEM or quasi-TEM modes may be excited. Figure 5.15 shows the mode amplitude distribution. From Figure 5.15, the symmetry mode is excited when the width of the heatsink is approximately integer multiples of wavelength for which the horizontal magnetic and the vertical electric field amplitudes are minimum and maximum respectively at the edge of the heatsink. From Figure 5.15, the antisymmetric mode is excited when the width of the heatsink is approximately multiples of half wavelength for which the horizontal magnetic and the vertical electric field amplitudes are minimum and maximum respectively at the edge of the heatsink. When the slabs are present, the effective dielectric constant increases. For the lower slab, the effective dielectric constant may be approximated using the model shown in Figure 5.16. Upon solving Maxwell's equations and matching boundary conditions, the effective dielectric constant is

\[ \varepsilon_e = \sqrt{\frac{2\varepsilon_1}{\varepsilon_1 + \varepsilon_0}}. \]  

(5.26)

For \( \varepsilon_1 = 4.3, \varepsilon_0 = 1.0, \varepsilon_e = 1.27 \). In the case of two slabs, the effective dielectric
constant may be approximated by averaging over the volume to be about 3.6. The high dielectric constants reduce the effective wavelength in the horizontal direction and increase the effective width or the electrical width of the heatsink. For the lower dielectric slab, the ratio of the width of the heatsink to the effective wavelength at 1.0 GHz is approximately 0.2. For both upper and lower dielectric slabs, this ratio becomes 0.3. The inclusion of the slabs pushes the effective width of the heatsink closer to half a wavelength. Hence the radiation of the VED decreases and the radiation of HMD increases.

The frequency dependence of the radiation property is studied. Figure 5.17 and Figure 5.18 show the radiation pattern of the VED at 2.5 GHz. Figure 5.19 and Figure 5.20 show the radiation pattern of the HMD at 2.5 GHz. The features of these patterns are essentially the same as the cases at 1.0 GHz due to the fact that the heatsink is smaller than half a wavelength. Figure 5.21 and Figure 5.22 depict the radiation pattern of VED at 5.0 GHz. Figure 5.23 and Figure 5.24 depict the radiation pattern of HMD at 5.0 GHz. The minimum in Figure 5.23 is at about 34° is because of the cancellation effect due to the effective sources on the opposite sides of the heatsink (Figure 5.25). Also in Figure 5.17 to Figure 5.24, the dashed curves represent the radiation pattern due to the dipole on a ground plane. The total radiated power is summarized in Table 5.1. For VED at 5.0 GHz, the total radiated power of the slab structure increases dramatically, to about 5 times the power of dipole on ground plane. The dramatic increases in radiated power for the HMD happen at 2.5 GHz, to about 10 times the power of dipole on ground plane. This phenomenon can be explained again using the magnetic cavity model. For a magnetic
wall cavity, the lowest antisymmetric resonance may occur when the dimension of the heatsink is approximately one wavelength of the TEM mode inside the slab structure. The ratio of the width of the heatsink to the wavelength at 5.0 GHz is about 0.8 comparing to 0.4 at 2.5 GHz. Since VED corresponds to the symmetric mode, at 5.0 GHz the heatsink dimension is close to the critical dimensions for the symmetric mode. On the other hand, the HMD generates an antisymmetric mode. At 2.5 GHz the heatsink dimension is close to the critical dimension for the antisymmetric mode. The dramatic increases in radiated power for VED at 5.0 GHz and for HMD at 2.5 GHz are hence observed.

In an attempt to reduce the emissions, the heatsink is connected to the ground plane at the four corners. For the purpose of computation the grounding is simulated using rectangular columns (Figure 5.26). Table 5.2 summarizes the total radiated power before and after grounding for the slab structure at 1.0 GHz. The large emission reduction in the case HMD with upper and lower slabs is observed. Before grounding, the slab structure shows a dramatic increase in radiated power when the upper slab is included, this implies that the slab structure may in fact be close to resonance. Grounding the structure may disrupt the resonance; hence, it results in a dramatic emission reduction. On the other hand, for a VED with two slabs, the changes in total radiated power with and without heatsink are relatively small. As seen from Table 5.2, the grounding process reduces the emissions regardless of the presence of the dielectric slab. The situation may be more complicated when the frequency is higher or when the heatsink dimensions are comparable to the wavelength.
Another option for lowering the emission level is to use lossy materials for shielding. One such hypothetical material is a metal mix rubber which has a dielectric constant of about 4.0 and conductivity of 10/Ωm. The usage of a conducting rubber may provide more flexibility in assembly because of its compliancy. Figure 5.27 shows the slab geometry with lossy rubber placed beneath the heatsink perimeter. Table 5.3 summarizes the total radiated power for both a VED and a HMD at 1.0 GHz, 2.5 GHz and 5.0 GHz. The skin depth of the conducting material are 5.0 mm, 3.2 mm and 2.2 mm for 1.0 GHz, 2.5 GHz and 5.0 GHz, respectively. As a result, the radiated power is reduced in all cases.
5.4 Summary

The integrated circuit package with heatsink structure is electromagnetically modeled by a simplified electromagnetically-coupled slab structure. The radiation property of the slab structure is analyzed using the FD-TD technique. The total radiated power and the radiation pattern, which indicates where the emission energies are concentrated, are calculated. The analysis for the radiation due to complex current distributions of an integrated circuit chip is simplified by considering the four basic dipole sources, the vertical electric dipole (VED), horizontal electric dipole (HED), vertical magnetic dipole (VMD), and horizontal magnetic dipole (HMD). It is found, due to the waveguide cutoff, only the VED and HMD contribute to emission. The insertion of the dielectric material may either increase or decrease the total power of radiation. At 1.0 GHz, the insertion of dielectric slab reduces the emission for VED excitation but enhances the emission for HMD. When the operating frequency increases and the dimensions of the heatsink approach the critical resonance dimensions, significant emissions can occur. At low frequency, when the dimensions of the heatsink are small compared to the wavelength, grounding of the heatsink can significantly reduce the total radiated power. The shielding effect using lossy material such as metal mixed rubber is analyzed; the reduction in emissions appears for both low and high frequencies. The reduction is more dramatic at high frequencies due to smaller skin depths.
5.5 Appendix A: VED in Parallel-Plate Waveguide

Figure 5.28 shows the geometrical configuration of a vertical electric dipole (VED) in a parallel-plate waveguide. The solution to this problem may be derived from the general solution of a dipole in layered medium [92]. The electric field contains both the \( \hat{z} \) and \( \hat{\rho} \) components, and the magnetic field contains only the \( \hat{\phi} \) component. Assuming the TEM mode to be the only guided mode, the total propagated power is determined by \( E_z \) and \( H_\phi \). Using the notation in [92],

\[
E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left( A_0 e^{ik_0z} + B_0 e^{-ik_0z} \right) H_0^{(1)}(k_\rho \rho) \quad (5.27)
\]

\[
H_{0\phi} = -\int_{-\infty}^{\infty} dk_\rho \frac{i\omega \varepsilon_0}{k_\rho} \left( A_0 e^{ik_0z} + B_0 e^{-ik_0z} \right) H_1^{(1)}(k_\rho \rho) \quad (5.28)
\]

where \( k^2 = k_{0z}^2 + k_\rho^2 \). These expressions are evaluated separately for \( z > 0 \) and \( z < 0 \). For VED at the center of the waveguide, solutions of \( z > 0 \) and \( z < 0 \) are symmetric. For \( z > 0 \),

\[
A_0 = \frac{E_{ved}}{(1 - R)}, \quad B_0 = \frac{RE_{ved}}{(1 - R)}, \quad (5.29)
\]

where \( R = e^{ik_0z} \), and

\[
E_{ved} = -\frac{Ilk_\rho^3}{8\pi\omega \varepsilon_0 k_{0z}}. \quad (5.30)
\]
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The integrands in Equations (5.27) and (5.28) have poles at $k_0z = 2m\pi/d$ when $1-R = 0$. Equations (5.27) and (5.28) can be converted to a series. The first terms of these series ($m = 0$) represent the TEM mode. Thus

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \frac{E_{vzd}}{(1 - R)} \left( e^{ik_0z} + Re^{-ik_0z} \right) H_0^{(1)}(k_\rho \rho)$$

$$= -\frac{Ilk\eta_0}{4d} H_0^{(1)}(k_\rho) + \text{higher order modes}, \quad (5.31)$$

$$H_{0\phi} = -\int_{-\infty}^{\infty} dk_\rho \frac{i\omega\varepsilon_0}{k_\rho} \frac{E_{vzd}}{(1 - R)} \left( e^{ik_0z} + Re^{-ik_0z} \right) H_1^{(1)}(k_\rho \rho)$$

$$= i\frac{Ilk}{4d} H_1^{(1)}(k_\rho) + \text{higher order modes}. \quad (5.32)$$

The same expressions can be obtained for $z < 0$. Finally, by retaining only the TEM mode, and using the asymptotic expressions of the Hankel functions, the total power in the TEM mode is given by Equation (5.18).
5.6 Appendix B: HMD in Parallel-Plate Waveguide

Figure 5.29 shows the geometrical configuration of a horizontal magnetic dipole (HMD) in a parallel-plate waveguide. The solution to this problem may be derived from the general solution of a dipole in layered medium [92]. Unlike the case of VED, all field components exist. Nonetheless, if only the TEM mode is guided, the total propagated power is again determined by $E_z$ and $H_\phi$. Using the notation in [92],

$$E_{0z} = \int_{-\infty}^{\infty} dk_\rho \left( A_0 e^{i k_0 z} + B_0 e^{-i k_0 z} \right) H_1^{(1)}(k_\rho \rho) \sin(\phi)$$

$$H_{0\phi} = -\int_{-\infty}^{\infty} dk_\rho \frac{ik_0 z}{k_\rho^2 \rho} \left( C_0 e^{i k_0 z} - D_0 e^{-i k_0 z} \right) H_1^{(1)}(k_\rho \rho) \sin(\phi)$$

$$+ \int_{-\infty}^{\infty} dk_\rho \frac{i \omega e_0}{k_\rho} \left( A_0 e^{i k_0 z} + B_0 e^{-i k_0 z} \right) H_1^{(1)'}(k_\rho \rho) \cos(\phi)$$

where $k^2 = k_{0z}^2 + k_\rho^2$. These expressions are evaluated separately for $z > 0$ and $z < 0$. For HMD at the center of the waveguide, the solutions of $z > 0$ and $z < 0$ are symmetric. For $z > 0$,

$$A_0 = \frac{E_{hmd}}{(1 - R)}, \quad B_0 = \frac{R E_{hmd}}{(1 - R)},$$

$$C_0 = \frac{H_{hmd}}{(1 - R)}, \quad D_0 = \frac{R H_{hmd}}{(1 - R)}.$$
where \( R = e^{i k_0 z d} \),

\[
E_{hmd} = \frac{I A \omega \mu_0 k_0^2}{8\pi k_0 z}, \quad \text{and}, \quad H_{hmd} = - \frac{I A \rho^2}{8\pi}.
\] (5.37)

Hence,

\[
E_{0z} = \int_{-\infty}^{\infty} dk_0 \frac{E_{hmd}}{(1 - R)} (e^{i k_0 z z} + R e^{-i k_0 z z}) H_1^{(1)}(k_\rho \rho) \sin(\phi),
\] (5.38)

\[
H_{0\phi} = - \int_{-\infty}^{\infty} dk_0 \frac{H_{hmd}}{k_0^2 \rho (1 - R)} (e^{i k_0 z z} - R e^{-i k_0 z z}) H_1^{(1)}(k_\rho \rho) \sin(\phi)
\]

\[
+ \int_{-\infty}^{\infty} dk_0 \frac{i \omega \epsilon_0}{k_\rho} \frac{E_{hmd}}{(1 - R)} (e^{i k_0 z z} + R e^{-i k_0 z z}) H_1^{(1)'}(k_\rho \rho) \sin(\phi).
\] (5.39)

The integrands in Equations (5.38) and (5.39) have poles at \( k_0 z = 2 m \pi / d \) when \( 1 - R = 0 \). By closing the integrating path along an appropriate contour and summing up all pole contributions, Equations (5.38) and (5.39) are converted to a series. The first terms of these series (\( m = 0 \)) represent the TEM mode. Retaining only the TEM mode,

\[
E_{0z} = \frac{I A k^2 \eta_0}{4d} H_1^{(1)}(k_\rho) \sin(\phi),
\] (5.40)

\[
H_{0\phi} = i \frac{I A k^2}{4d} H_1^{(1)'}(k_\rho) \sin(\phi)
\] (5.41)
The same expressions can be obtained for \( z < 0 \). Finally using the asymptotic expressions of the Hankel functions, and \( ml = -ik\eta_0 I_A \), the total power in the TEM mode is given by Equation (5.19).
Figure 5.1: 3D Electromagnetically-coupled slab structure.

Figure 5.2: Staggered rectangular grid.
Figure 5.3: Field locations in implementing the absorbing boundary condition.
Figure 5.4: Magnetic field locations for vertical electric dipole excitation.

Figure 5.5: Electric field locations for vertical magnetic dipole excitation.
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Figure 5.6: Radiation pattern of VED on $zx$ plane.

Figure 5.7: Radiation pattern of VED on $xy$ plane.
Figure 5.8: Field locations near the fine and large grid connection.
Figure 5.9: Radiation field pattern on the $yz$ plane of the slab with VED.

Figure 5.10: Radiation field pattern on the $xy$ plane of the slab with VED.
Figure 5.11: Radiation field pattern on the $yz$ plane of the slab with HMD.

Figure 5.12: Radiation field pattern on the $xy$ plane of the slab with HMD.
Figure 5.13: Antisymmetric fringe fields along the edge of the heatsink.

Figure 5.14: Geometrical description of magnetic cavity models.
Figure 5.15: Amplitudes of symmetric and antisymmetric magnetic cavity mode.

Figure 5.16: Parallel-plate waveguide with dielectric slab.
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Figure 5.17: Radiation pattern of VED on $yz$ plane at 2.5 GHz.

Figure 5.18: Radiation pattern of VED on $xy$ plane at 2.5 GHz.
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Figure 5.19: Radiation pattern of HMD on $yz$ plane at 2.5 GHz.

Figure 5.20: Radiation pattern of HMD on $xy$ plane at 2.5 GHz.
Figure 5.21: Radiation pattern of VED on $yz$ plane at $5.0 \text{ GHz}$.

Figure 5.22: Radiation pattern of VED on $xy$ plane at $5.0 \text{ GHz}$. 
Figure 5.23: Radiation pattern of HMD on $yz$ plane at 5.0 GHz.

Figure 5.24: Radiation pattern of HMD on $xy$ plane at 5.0 GHz.
Figure 5.25: Symmetric sources along two sides of the heatsink.
Table 5.1: Summary of total power of radiation at different frequencies.

<table>
<thead>
<tr>
<th>Radiation resistance (mΩ)</th>
<th>VED</th>
<th>HMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>frequency (GHz)</td>
<td>frequency (GHz)</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>without heatsink</td>
<td>8.8</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>0.18</td>
<td>1.1</td>
</tr>
<tr>
<td>with heatsink</td>
<td>12</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 5.2: Summary of total power of radiation for heatsink grounding at four corners.

<table>
<thead>
<tr>
<th>Radiation resistance (mΩ)</th>
<th>VED</th>
<th>HMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>without dielectric slab</td>
<td>with upper dielectric slab</td>
</tr>
<tr>
<td></td>
<td>no grounding</td>
<td>grounding</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VED</td>
<td>12</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.002</td>
</tr>
<tr>
<td>HMD</td>
<td>7.3</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>0.006</td>
</tr>
</tbody>
</table>
Table 5.3: Summary of total power of radiation for heatsink shielding.

<table>
<thead>
<tr>
<th>Total power (nW)</th>
<th>VED</th>
<th>HMD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation resistance (mΩ)</td>
<td>frequency (GHz)</td>
<td>frequency (GHz)</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>without shielding</td>
<td>12</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>1.8</td>
</tr>
<tr>
<td>with shielding</td>
<td>0.1</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Figure 5.26: Geometrical description of heatsink grounding at four corners.
Figure 5.27: Geometrical description of lossy shielding.
Figure 5.28: VED in parallel plate waveguide.

Figure 5.29: HMD in parallel plate waveguide.
Chapter 6

CONCLUSIONS

In this thesis, several implementation-related issues of the FD-TD and FD-FD techniques are discussed. In particular, the absorbing boundary conditions (ABC), FD-TD technique on triangular grids, application of the FD-TD technique to problems involving frequency dispersive materials, and the application of the FD-TD technique to the radiation problem of a 3-D rectilinear configuration are presented.

The modified pseudo-differential operator technique is discussed and applied to derive the ABC on circular and elliptical boundaries. In the case of the circular boundary, the ABC derived has higher absorbability than the result obtained by Engquist and Majda, and Sommerfeld's radiation boundary condition is naturally contained in the first order term. With a convenient change of variable, the pseudo-differential operator technique is applied to obtain the ABC of the elliptic boundary. The ABC of the elliptical boundary is numerically demonstrated in the frequency domain. It is found that the application of the ABC can potentially offer substantial reduction in computer resource requirements for elongated scatterers.
6. Conclusions

The finite-difference time-domain technique on triangular grids is developed and verified by various test-cases. This technique is the generalization of the control region approximation and the finite difference technique for rectangular grid. The discretization scheme is simple, and reduces to Yee's algorithm when the Delaunay tessellation becomes rectangular. The discretization is accurate to second order in time and in space when the grid is regular. In the case of irregular grid, the first order accurate scheme is obtained. The algorithm can be applied to arbitrary geometries and can handle dielectric/magnetic materials. The algorithm is tested with circular cylinder geometries. The time harmonic results are extracted from the late time responses of a sinusoidal excitation and compared with the eigen series solutions. Pulse responses are also calculated for strip configurations. The computation domains are truncated using either circular or elliptical outer boundaries. Numerical results indicate the highly absorbing nature of corresponding absorbing boundary conditions. For elongated scatterers, the use of the elliptical boundary can reduce the computational domain.

The FD-TD technique for electromagnetic problems is generalized to handle frequency dispersive materials. Discretization schemes and effective time domain models are investigated and demonstrated numerically. Numerical results confirm the validity and accuracy of the algorithm. To apply the algorithm, proper frequency domain models for the dispersive materials, such as the molecule resonance model and the Debye model, are used. The frequency domain models are then transformed to the time domain model which are in the form of ordinary time differential equations relating $\mathbf{D}$ to $\mathbf{E}$ and $\mathbf{B}$ to $\mathbf{H}$. To treat the frequency dispersive materials, the conven-
tional FD-TD algorithms for electromagnetic problems are extended. The algorithm is efficient in terms of computation time and memory requirements. Although the algorithm is only demonstrated in 1D and 2D, it can be applied to 3D problems.

A three-dimensional integrated circuit package with heatsink structure is electromagnetically modeled by a simple electromagnetically-coupled slab structure. The radiation property of the slab structure is analyzed using the FD-TD technique. The total radiated power and the radiation pattern are calculated. The radiation due to complex current distribution of the integrated circuit package is studied by analyzing the four basic dipole sources. It is found, due to waveguide cutoff, that only the VED and HMD contribute to emissions from the overall structure. The inclusion of the dielectric material may either increase or decrease the total radiated power. At 1.0 GHz, the inclusion of a dielectric slab reduces the emission for VED excitation but enhances the emission for HMD. When the operating frequency increases and the dimensions of the heatsink approach the critical resonance dimension, significant emission can occur. For low frequencies, when the dimensions of the heatsink are small compared to the wavelength, grounding of the heatsink can significantly reduce the total radiated power. The shielding effect achieved through use of lossy material, such metal-mixed rubber beneath the heatsink perimeter is analyzed. The reduction in emission appears at both low and high frequencies. To reduce the emission from the integrated circuit package with heatsink, at low frequencies both grounding and shielding using lossy compliant material are effective; at high frequencies the shielding scheme appears more reliable.
Bibliography


Bibliography


