Demand Analysis using Strategic Reports: An application to a school choice mechanism*

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Abstract

Several school districts use assignment systems that give students an incentive to misrepresent their preferences. We find evidence consistent with strategic behavior in Cambridge. Such strategizing can complicate preference analysis. This paper develops empirical methods for studying random utility models in a new and large class of school choice mechanisms. We show that preferences are non-parametrically identified under either sufficient variation in choice environments or a preference shifter. We then develop a tractable estimation procedure and apply it to Cambridge. Estimates suggest that while 82% of students are assigned to their stated first choice, only 72% are assigned to their true first choice because students avoid ranking competitive schools. Assuming that students behave optimally, the Immediate Acceptance mechanism is preferred by the average student to the Deferred Acceptance mechanism by an equivalent of 0.08 miles. The estimated difference is smaller if beliefs are biased, and reversed if students report truthfully.

JEL: C14, C57, C78, D47, D82

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1 Introduction

Admissions to public schools throughout the world is commonly based on assignment mechanisms that use reported rankings of various schooling options (Abdulkadiroglu and Sonmez, 2003). The design of such school choice mechanisms has garnered significant attention in the theoretical literature (Abdulkadiroglu, 2013). Although mechanisms that incentivize truthful revelation of preferences have been strongly advocated for in the theoretical literature (see Pathak and Sonmez, 2008; Azevedo and Budish, 2017, for example), with rare exceptions, real-world school choice systems are susceptible to gaming. Table I presents a partial list of mechanisms in use at school districts around the world. To our knowledge, only Boston and Amsterdam currently employ mechanisms that are not manipulable.

The widespread use of manipulable school choice systems poses two important questions. First, what are the costs and benefits of manipulable mechanisms from the student assignment perspective? Theoretical results on this question yield ambiguous answers. Abdulkadiroglu et al. (2011) use a stylized model to show that strategic choice in the Immediate Acceptance mechanism, also known as the (old) Boston mechanism, can effectively elicit cardinal information on preferences and can improve average student welfare. However, this potential benefit comes at a cost of violating notions of fairness and stability of the final assignments. Second, how does one interpret and analyze administrative data on reported rankings generated by these mechanisms if the reports cannot be taken as truthful? Information on preferences can be useful for academic research on the effects of school choice on student welfare (Abdulkadiroglu et al., 2017a), student achievement (Hastings et al., 2009), and school competition (Nielson, 2013). Additionally, student preference information can also be useful for directing school reforms by identifying which schools are more desirable than others.[1] However, ignoring strategic incentives for such purposes can lead to incorrect conclusions and mis-directed policies.[2]

This paper addresses these two questions by developing a general method for estimating the underlying distribution of student preferences for schools using data from manipulable mechanisms and by applying these techniques to data from elementary school admissions

[1] School accountability and improvement programs and district-wide reforms often use stated rank-order lists as direct indicators of school desirability or student preferences. Boston's Controlled Choice Plan formally used the number of applications to a school as an indicator of school performance in an improvement program. Similarly, Glenn (1991) argues that school choice caused improvements in the Boston school system based on observing an increase in the number of students who were assigned to their top choice.

[2] Previous empirical work has typically assumed that observed rank-order lists are a truthful representation of the students' preferences (Hastings et al., 2009; Abdulkadiroglu et al., 2017a; Ayaji, 2017), allowing a direct extension of discrete choice demand methods. The assumption is usually motivated by arguing that strategic behavior may be limited in the specific environment. A handful of contemporaneous papers discussed below allow for agents to be strategic (He, 2014; Calsamiglia et al., 2017; Hwang, 2016).

in Cambridge, MA. We make several methodological and empirical contributions. First, we document strategic behavior to show that student reports respond to the incentives present in the mechanism. Second, we propose a new revealed preference method for analyzing the reported rank-order lists of students. We use this technique to propose a new estimator for the distribution of student preferences and to derive its limit properties. These technical results are applicable to a broad class of school choice mechanisms that includes the systems listed on Table I, except for the Top Trading Cycles mechanism. Third, we derive conditions under which the distribution of preferences is non-parametrically identified. Finally, we apply these methods to estimate the distribution of preferences in Cambridge, which uses an Immediate Acceptance mechanism. These preference estimates can then be used to analyze how often students are assigned to their true first choice school and to compare the outcomes under the current mechanism to an alternative that uses the student-proposing Deferred Acceptance mechanism.

Interpreting observed rank-order lists requires a model of agent behavior. Anecdotal evidence from Boston (Pathak and Sonmez, 2008) and laboratory experiments (Chen and Sonmez, 2006; Calsamiglia et al., 2010) suggest that strategic behavior may be widespread in manipulable school choice systems. Indeed, our analysis of ranking behavior for admissions into public elementary schools in Cambridge indicates significant gaming. There are strong incentives for behaving strategically in Cambridge. Because Cambridge uses an Immediate Acceptance Mechanism, some schools are rarely assigned to students who rank it second, while others have spare capacity after all students have been considered. Students therefore may lose their priority at a competitive school if they do not rank it first. We investigate whether students appear to respond to these incentives by using a regression discontinuity design based on the fact that students receive proximity priority at the two closest schools. We find that student ranking behavior changes discontinuously with the change in priority. This result is not consistent with a model in which students rank schools in order of their true preferences if residential decisions are not made in consideration of proximity priority. Reassuringly, we do not find evidence that aggregate residential decisions or house prices are affected by this priority.

Therefore, instead of interpreting reported rank-order lists as true preferences, we assume that the report corresponds to an optimal choice of a lottery over assignments to various schools. The lottery implied by a rank-order list consists of the probabilities of getting assigned to each of the schools on that list. These probabilities depend on the student's priority type and report, a randomly generated tie-breaker, and the reports and priorities of the other students. Given a belief for the assignment probabilities corresponding to each rank-order list, the expected utility from the chosen lottery must be greater than other lotteries the agent could have chosen. We begin by assuming that students best respond to the strategies of other students. This rational expectations assumption is an important baseline that accounts for strategic behavior. However, we also consider models in which students have less information or have biased beliefs. First, we consider a model in which agents are unaware of the fine distinctions in the mechanism between various priority and student types. Second, we consider a model with adaptive expectations in which beliefs are based on the previous year. In an extension, we also estimate a mixture model with both naïve and sophisticated players.

Once a model of strategic behavior has been assumed, it is natural to ask whether it can be used to learn about the distribution of preferences using a typical dataset. To address this question, we study identification of a flexible random utility model that allows for student and school unobservables (see Block and Marshak, 1960; McFadden, 1973; Manski, 1977). Under the models of agent beliefs discussed above, estimates of assignment probabilities obtained from the data can be substituted for the students' beliefs. As we discuss later, consistent estimates of the assignment probabilities (as a function of reports and priority types) can be obtained using the data and the knowledge of the mechanism. Our results show that, given estimated assignment probabilities, two types of variation can be used to learn about the distribution of preferences. The first is variation in choice environments that may arise when two identical populations of students face different mechanisms or a different number of school seats. We characterize the identified set of preference distributions under such variation. The second form of variation assumes the availability of a special regressor that is additively separable in the indirect utility function. Such a regressor can be used to "trace-out" the distribution of preferences (Manski, 1985; Matzkin, 1992; Lewbel, 2000). Similar assumptions are commonly made to identify preferences in discrete choice models. In our application, we use distance to school as a shifter of preferences. Our empirical specification therefore rules out within-district residential sorting based on unobserved determinants of school preferences. This assumption is commonly made in the existing empirical work on school choice and models of joint schooling and residential decisions is left for future work.[3]

This analysis naturally suggests a two-step estimation procedure for estimating the distribution of preferences. In a first step, we estimate the lottery over school assignments associated with each report and priority type. In a second step, we estimate the parameters

[3] We investigated whether house prices or the fraction of residential units occupied by Cambridge elementary school students is higher on the side of a priority zone boundary where priority is accorded at a better performing school. Our results suggest that priorities are not a strong enough driver of residential decisions to generate differences in house prices or the aggregate number of families locating in an area. These findings are consistent with families not paying attention to the details of the admissions system at the time of choosing where to live. Details available upon request.

governing the distribution of preferences using a likelihood based method. Specifically, we implement a Gibbs' sampler adapted from McCulloch and Rossi (1994). This procedure is convenient in our setting because the set of utility vectors for which a given report is optimal can be expressed in terms of linear inequalities, and it allows us to avoid computing or simulating the likelihood that a report is optimal given a parameter vector. In an appendix, we prove that our estimator is consistent and asymptotically normal. The primary technical contribution is a limit theorem for the estimated lotteries. This result requires a consideration of dependent data because assignments depend on the reports of all students in the market. That school choice mechanisms are usually described in terms of algorithms rather than functions with well-known properties further complicates the analysis. We solve this problem for a new and large class of school choice mechanisms that includes all mechanisms in table I, except the Top Trading Cycles mechanism.

We then apply our methods to estimate student preferences in Cambridge in order to address a wide range of issues. First, we investigate the extent to which students avoid ranking competitive schools in order to increase their chances of assignment at less competitive options. Prevalence of such behavior can result in mis-estimating the attractiveness of certain schools if stated ranks are interpreted at face value. Ignoring strategic behavior may therefore result in inefficient allocation of public resources for improving school quality. Further, a large number of students assigned to their first choice may not be an indication of student satisfaction or heterogeneity in preferences. We therefore also report on whether strategic behavior results in fewer students being assigned to their true first choice as compared to their stated first choice.

Second, we study the welfare effects of switching to the student proposing Deferred Acceptance mechanism. The theoretical literature supports strategy-proof mechanisms on the basis of their simplicity, robustness to information available to participants, and fairness (see Azevedo and Budish, 2017, and references therein). However, it is possible that ordinal strategy-proof mechanisms compromise student welfare by not screening students based on the intensity of their preferences (Miralles, 2009; Abdulkadiroglu et al., 2011). We quantify student welfare from the assignment under these two mechanisms under alternative models of agent beliefs and behavior. This approach abstracts away from potential costs of strategizing and acquiring information, which are difficult to quantify given the available data. Nonetheless, allocative efficiency is a central consideration in mechanism choice, along with other criteria such as differential costs of participating, fairness, and strategy-proofness (Abdulkadiroglu et al., 2009).

Our baseline results, which assume equilibrium behavior, indicate that the average student prefers the assignments under the Cambridge mechanism to the Deferred Acceptance mechanism. Interestingly, this difference is driven by paid-lunch students, who face stronger strategic incentives than free-lunch students because of quotas based on free-lunch eligibility. A cost of improved assignments in Cambridge is that some students (2-10% depending on the specification and the student group) have justified envy.[4] We then evaluate the mechanisms assuming that agents have biased beliefs about assignment probabilities. These estimates suggest that biased beliefs may mitigate the screening benefits of the Cambridge mechanism because mistakes can be costly in some cases.

Finally, we evaluate a mixture model with naïve and sophisticated agents to assess the distributional consequences across agents who vary in their ability to game the mechanism. We estimate that about a third of paid-lunch and free-lunch students report their preferences sincerely even if it may not be optimal to do so. Although naïve agents behave suboptimally, we find that the average naïve student prefers the assignments under the Cambridge mechanism. This occurs because naïve students rank their most preferred school first and gain priority at this school at the cost of sophisticates who avoid ranking these schools. The cost of not receiving their true second or third choices turns out to be smaller than this benefit.

Related Literature

These empirical contributions are closely related to a handful of recent papers that estimate preferences for schools using manipulable mechanisms (He, 2014; Calsamiglia et al., 2017; Hwang, 2016). He (2014) proposes an estimator based on theoretically deriving properties of undominated reports using specifics of the school choice implementation in Beijing.[5] Hwang (2016) proposes a subset of restrictions on agent behavior based on simple rules to derive a bounds-based estimation approach. Compared to our procedure, these approaches avoid using restrictions that are implied if agents have information about which schools are more competitive than others. Calsamiglia et al. (2017) estimates a mixture model in which strategic agents solve for the optimal report in an Immediate Acceptance mechanism using backwards-induction from lower- to higher-ranked choices.

There are a few other general distinguishing features from the aforementioned papers worth noting. First, most papers mentioned above use approaches that are specifically tailored to the school choice mechanism analyzed, and it may be necessary for a researcher to modify the ideas before applying them elsewhere. In contrast, we allow analysis for a more general class of mechanisms, including mechanisms with student priority groups. Second,

[4]Student i has justified envy if another student i' is assigned to a school j that student i prefers to her assignment and student i has (strictly) higher priority at j than student i'.

[5]The estimators proposed in He (2014) that do not assume optimal play are based on a limited number of restrictions implied by rationality, the specific number of schools and ranks that can be submitted in Beijing based on the fact that the school district treats all agents symmetrically.

results on identification and large market properties of the estimator are not considered in the papers mentioned above. Finally, our empirical exercise investigates the consequences of specific forms of subjective beliefs on the comparison between mechanisms.

Our technical results on the large sample properties of our estimator use results from the work on large matching markets by Azevedo and Leshno (2016) and Azevedo and Budish (2017). The results on identification build on work on discrete choice demand (Manski, 1985; Matzkin, 1992; Lewbel, 2000; Berry and Haile, 2010). While the primitives are similar, unlike discrete choice demand, the probability of assignment to a schools may not be 0 or 1. This feature is similar to estimation of preference models under risk and uncertainty (Cardon and Hendel, 2001; Cohen and Einav, 2007; Chiappori et al., 2012). Cardon and Hendel (2001) and Cohen and Einav (2007) model uncertainty in outcomes within each insurance contract rather than uncertainty over which option is ultimately allocated. Chiappori et al. (2012) focuses on risk attitudes rather than the value of underlying prizes.

Our paper provides an empirical complement to the large theoretical literature that has taken a mechanism design approach to the student assignment problem (Gale and Shapley, 1962; Shapley and Scarf, 1974; Abdulkadiroglu and Sonmez, 2003). A significant literature debates the trade-offs between manipulable and non-manipulable mechanisms (Ergin and Sonmez, 2006; Pathak and Sonmez, 2008; Miralles, 2009; Abdulkadiroglu et al., 2011; Featherstone and Niederle, 2016; Troyan, 2012; Pathak and Sonmez, 2013). Theoretical results from this literature have been used to guide redesigns of matching markets (Roth and Peranson, 1999; Abdulkadiroglu et al., 2006, 2009).

A growing literature is interested in methods of analyzing preferences in matching markets, usually using pairwise stability (Choo and Siow, 2006; Fox, 2010, 2017; Chiappori et al., 2017; Agarwal, 2015; Diamond and Agarwal, 2016). In some cases, estimates are based on the strategic decision to engage in costly courting decisions (Hitsch et al., 2010). Similar considerations are important when applying to colleges (Chade and Smith, 2006).

The proposed two-step estimator uses insights from the industrial organization literature, specifically the estimation of empirical auctions (Guerre et al., 2000; Cassola et al., 2013), single agent dynamic models (Hotz and Miller, 1993; Hotz et al., 1994), and dynamic games (Bajari et al., 2007; Pakes et al., 2007; Aguirregabiria and Mira, 2007). As in the methods used in those contexts, we use a two-step estimation procedure where the distribution of actions from other agents is used in a first step estimator.

Overview

Section 2 describes the Cambridge Controlled Choice Plan and presents evidence that students are responding to strategic incentives provided by the mechanism. Sections 3 and 4 present the model and the main insight on how to use submitted rank-order lists. Section 5 and Section 6 discuss identification and estimation. A reader solely interested in the empirical application instead of the econometric techniques may skip these sections. Section 7 applies our techniques to the dataset from Cambridge, MA.

2 Evidence on Strategic Behavior

2.1 The Controlled Choice Plan in Cambridge, MA

We use data from the Cambridge Public Schools' (CPS) Controlled Choice Plan for the academic years 2004-2005 to 2008-2009. Elementary schools in the CPS system assigns about 41% of the seats through partnerships with pre-schools or an appeals process for special needs students. We focus on the remaining seats that are assigned through a school choice system that takes place in January for students entering kindergarten.

Table II summarizes the students and schools. The system has 13 schools and about 400 students participating per year. One of the schools, Amigos, was divided into bilingual Spanish and regular programs in 2005. Bilingual Spanish speaking students are considered only for the bilingual program, and non-bilingual students are considered only for the regular program.[6] King Open OLA is a Portuguese immersion school open to all students. Tobin, a Montessori school, divided admissions for four- and five-year olds starting in 2007.

An explicit goal of the Controlled Choice Plan is to achieve socio-economic diversity by maintaining the proportion of students who qualify for the federal free/reduced lunch program in each school close to the district-wide average. Only for the purposes of the assignment mechanism, all schools except Amigos are divided into paid-lunch and free/reduced lunch programs. Students eligible for federal free or reduced lunch are only considered for the corresponding program.[7] About 34% of the students are on free/reduced lunch. Each program has a maximum number of seats, and the overall school capacity may be lower than the sum of the seats in the two programs. Our dataset contains both the total number of seats in the school as well as the seats available in each of the programs.

The Cambridge Controlled Choice Mechanism

We now describe the process used to place students at schools. It prioritizes students based on two criteria:

[6] A student voluntarily declares whether she is bilingual on the application form.

[7] Households with income below 130% (185%) of the Federal Poverty line are eligible for free (reduced) lunch programs. For a household size of 4, the annual income threshold was approximately \$27,500 (\$39,000) in 2008-2009.

(i) Students with siblings who are attending that school get the highest priority.

(ii) Students receive priority at the two schools closest to their residence.

Students can submit a ranking of up to three programs at which they are eligible. Cambridge uses an Immediate Acceptance mechanism and assigns students as follows:

Step 0: Draw a single tie-breaker for each student

Step k = 1,2,3: Each school considers all students who have not been previously assigned and have listed it in the k-th position. Students are sorted in order of priority, breaking ties using a random tie-breaker. Each student is considered sequentially for the paid-lunch program if she is not eligible for a federal lunch subsidy and for the free/subsidized lunch program otherwise. She is assigned to the corresponding program unless,

(a) There are no seats available in the program, or

(b) There are no seats available in the school.

If either of the conditions above are satisfied, the student is rejected.

There are a few notable features of this mechanism. First, the mechanism prioritizes students at higher-ranked options. The effective priority therefore depends on the report of the student. Second, there is a cutoff for each program/school, and all students with an effective priority below that cutoff are rejected. This cutoff is set so that the number of students assigned to the program/school does not exceed its capacity. Finally, students are assigned to the highest-ranked option for which their effective priority is above the cutoff.

These features of the Cambridge Controlled Choice Plan are shared with a large class of mechanisms that we will formally introduce below. There are two clear reasons why there may be strategic incentives in such mechanisms. First, the dependence of the effective priority on the report provides incentives to skew ranking towards options where priority is most valuable. Second, if the length of the list is limited, students should avoid ranking too many schools where their priority is likely to be below the cutoff. We now describe the ranking behavior and document strategic decision-making in response to these incentives in Cambridge.

2.2 Evidence on Ranking Behavior and Strategic Incentives

Panels A and D in table III show that over 80% of the students rank the maximum allowed number of schools and over 80% of the students are assigned to their top-ranked choice in a typical year. Researchers in education have interpreted similar statistics in school districts as indicators of student satisfaction and heterogeneity in student preferences. For instance, Glenn (1991) argues that school choice caused improvements in the Boston school system based on an observed increase in the number of students who were assigned to their top choice.[8] Similarly, Glazerman and Meyer (1994) interpret a high fraction of students getting assigned to their top choice in Minneapolis as indicative of heterogeneous student preferences.

Conclusions based on interpreting stated preferences as truthful are suspect when a mechanism provides strategic incentives for students. For example, it is tempting to conclude from ranking patterns in panels E and F that students have extremely strong preferences for attending a nearby school or the same school as their sibling. However, such behavior can also be driven by strategic incentives if a student "loses her priority" when a school is not ranked at the top of the list in an Immediate Acceptance Mechanism (Ergin and Sonmez, 2006).

Indeed, table IV shows that the incentive to "cash the priority" is strong. While there is significant heterogeneity in their competitiveness, panel A shows that Baldwin, Haggerty, Amigos, Morse, Tobin, Graham & Parks, and Cambridgeport have many more students ranking them than there is capacity. A typical student would be rejected in these schools if she does not rank it as her top choice. Indeed, Graham & Parks rejected all non-priority paid lunch students even if they had ranked it first in each of the five years. Additionally, panels B and C show that the competitiveness of schools differs by paid-lunch status. Graham & Parks, for instance, did not reject any free/reduced lunch students who applied to it in a typical year. Overall, a larger number of schools are competitive for paid-lunch students than for free-lunch students.

There are two other features that are worth highlighting. First, most schools either reject all students who did not rank them first or do not reject any students. Therefore, students must rank competitive school first in order to gain admission but may rank non-competitive schools at any position. This suggests that, at least in Cambridge, considering which school to rank first is important. Second, table IV shows that several paid-lunch students rank competitive schools as their second or third choice. This may appear hard to rationalize as optimal behavior. However, it should be noted that these choices are often not pivotal because an extremely large number of students are assigned to their top choice. Another possibility is that these students count on back-up schools, either at the third-ranked school or at a private or charter school, in case they remain unassigned. Finally, students may simply believe that there is a small chance of assignment even at competitive schools. We further discuss these issues when we present our estimates.

[8]The argument is based on ranking and assignment data generated when Boston used a manipulable assignment system.

2.3 Strategic Behavior: A Regression Discontinuity Approach

We now study whether students are ranking schools strategically. Our empirical strategy is based on the assignment of proximity priority in Cambridge. A student receives priority at the two closest schools to her residence. We can therefore compare the ranking behavior of students who are on either side of a geographical boundary where the proximity priority changes. If students are not behaving strategically and the distribution of preferences are continuous in distance to school, we would not expect the ranking behavior to change discretely at this boundary. On the other hand, the results in table IV indicate that a student may lose her proximity priority at competitive schools if she does not rank it first. Therefore, some students may find it optimal to manipulate their reports in order to avoid losing proximity priority. Strategic students may rank a competitive school at which they have priority as their first choice instead of their most preferred school.

Figure I and table V present the results of this discontinuity design. The figure plots the probability of ranking a school in a particular position against the distance from a proximity priority boundary. Specifically, let d_{i2} and d_{i3} be the second and third closest schools to student i. For any school j, the horizontal axis is the difference $\Delta d_{ij} = d_{ij} - \frac{1}{2}(d_{i2} + d_{i3})$. Because Cambridge assigns a student priority at the two closest schools, Δd_{ij} is negative if student i has priority at school j and positive otherwise. The vertical line represents this boundary of interest where we assess ranking behavior. The black dashed lines are generated from a local linear regression of the ranking outcome y_{ij} on the distance from this boundary, Δd_{ij}, estimated separately using data on either side of the boundary. The black points represent a bin-scatter plot of these data, with a 95% confidence interval depicted with the bars. The gray points control for school fixed effects. Table V presents the estimated size of the discontinuity using the procedure recommended by Imbens and Kalyanaraman (2011) and their test of whether the outcome studied changes discontinuously at a priority boundary.

Figure I(i) shows that the probability that a student ranks a school first decreases discontinuously at the proximity boundary. Further, the response to distance to school is also higher to the left of the boundary, probably reflecting the preference to attend a school closer to home. The jump at the boundary may be attenuated because a student can rank only one of the two schools where she has priority as her top choice.[9] In contrast to figure I(i), I(ii) and I(iii) do not show a large jump in the probability that a school is ranked second or third. This should be expected because we saw earlier that one's priority is unlikely to be pivotal in the second or third choices. Further, the probability of ranking a school that is extremely

[9] Figure **??**(i) in the appendix focuses on the second and third closest schools and shows that the discontinuity is still discernible.

close to a student's home in the second or third position is low because students tend to rank nearby schools first. Table V presents the estimated size of this discontinuity. The first column shows that the probability that a school is ranked first drops by 5.75% at the boundary where the student loses proximity priority. This effect is statistically significant at the 1% level. Further, panels B and C of the table show that this change is larger for paid lunch students than for free lunch students. This is consistent with the theory that paid lunch students are responding to the stronger strategic incentives as compared to free lunch students. The next two columns present these estimates for the second- and third-ranked choices. As expected, the estimated effects here are smaller and often not statistically significant.

Strategic pressures to rank a school first may be particularly important if the school is competitive. Figures I(iv) and I(v) investigate the differential response to proximity priority by school competitiveness. We split the schools based on whether they rejected some students in a typical year or not as delineated in table IV. Consistent with strategic behavior, figure I(iv) shows that the probability of ranking a competitive school first falls discontinuously at the boundary but less so in figure I(v), which focuses on non-competitive schools. Indeed, the fourth and fifth columns of table V confirm that the estimated drop in the probability of ranking a competitive schools first is 7.27%, which is larger than the overall estimate. Additionally, panels B and C of table V shows that the estimated response to proximity priority is larger for paid-lunch students at 11.07% as compared to 1.47% for free-lunch students.[10] Non-competitive schools, in stark contrast, have an statistically insignificant estimated drop of only 2.06%, which is consistent with strategic pressures being less stringent. However, we view the estimated effects at non-competitive schools for free-lunch students as inconclusive because the point estimates are large but imprecise. The findings are therefore consistent with paid-lunch students responding to significant strategic pressures in the Cambridge mechanism, but not free-lunch students because of the lower strategic pressure they face.

Finally, we consider a placebo test in which we repeat the analysis assuming that proximity priority is only given at the closest school. Figure I(vi) shows no discernible difference in the ranking probability at this placebo boundary. The estimated size of the discontinuity, presented in the last column of table V, is only 0.07% and statistically indistinguishable from zero. Figure **??**(iv) in the appendix presents a second placebo boundary, dropping the two closest schools and constructing priorities at the two closest remaining schools. As expected, we do not find a discontinuous response at this placebo boundary.

Taken together, these results strongly suggest that ranking behavior is discontinuous at

[10] Figures **??** (ii) and **??**(iii) in the appendix show the corresponding plots by free-lunch status.

the boundary where proximity priority changes. However, there are two important caveats that must be noted before concluding that agents in Cambridge are behaving strategically. First, the results do not show that all students are responding to strategic incentives in the mechanism or that their expectations are correct. We therefore consider models with biases in beliefs and non-optimal behavior in addition to a rational expectations model. Second, it is possible that the response is driven in part by residential choices, with parents picking a home so that the student receives priority at a more preferred school. While the previous literature has found evidence of residential sorting across school districts (Black, 1999; Bayer et al., 2007), we are not aware of any research on the effects of priorities on the housing market within a unified school district. A boundary discontinuity design similar to Black (1999) suggests that neither house prices nor the aggregate number of families in an area are related to priorities.[11] A more thorough analysis of this issue or a full model that considers the joint residential and school choice decision is left for future research.

These results contrast with Hastings et al. (2009), who find that the average quality of schools ranked did not respond to a change in the neighborhood boundaries in the year the change took place. As suggested by Hastings et al. (2009), strategic behavior may not be widespread if the details of the mechanism and the change in neighborhood priorities are not well advertised. Note that the Charlotte-Mecklenburg school district did not make the precise mechanism clear. In contrast, Cambridge's Controlled Choice Plan is published on the school district's website and has been in place for several years. These institutional features may account for the differences in the student behavior.

3 Model

We consider school choice mechanisms in which students are indexed by $i \in \{1, \ldots, n\}$, programs indexed by $j \in \{0, 1, \ldots, J\} = \mathcal{J}$, and schools are indexed by $s \in \mathcal{S}$. Program 0 denotes being unmatched. Each program or school, $k \in \mathcal{J} \cup \mathcal{S}$, has $n \times q_k^n$ seats, with $q_k^n \in (0, 1)$ and $q_0 = 1$.[12] The school capacities are such that $q_s^n \leq \sum_{\{j:s_j=s\}} q_j^n$, where $s \in \mathcal{S}$ and s_j is the school corresponding to program j. We now describe how students are assigned to these seats, their preferences over assignments, and assumptions on behavior.

[11] Details available upon request. Note that the proximity priority system for kindergarten admission is not used for higher grades in Cambridge.

[12] This convention will be convenient because we will be considering limits as $n \to \infty$, in which $q_k^n \to q_k \in (0, 1)$.

3.1 Utilities and Preferences

We assume that student i's utility from assignment into program j is given by $V(z_{ij}, \xi_j, \epsilon_i)$, where z_{ij} is a vector of observable characteristics that may vary by program or student or both, and ξ_j and ϵ_i are (vector-valued) unobserved characteristics. Let $v_i = (v_{i1}, \dots, v_{iJ})$ be the random vector of indirect utilities for student i with conditional joint density $f_V(v_{i1}, \dots, v_{iJ}|\xi, z_i)$, where $\xi = (\xi_1, \dots, \xi_J)$ and $z_i = (z_{i1}, \dots, z_{iJ})$. We normalize the utility of not being assigned through the assignment process, v_{i0}, to zero. Therefore, v_{i0} is best interpreted as the inclusive value of remaining unassigned and participating in the post-assignment wait-list.

This formulation allows for heterogeneous and non-additive preferences conditional on observables. The primary restriction thus far is that a student's indirect utility depends only on her own assignment and not on the assignments of other students. This rules out preferences for peer groups or for conveniences that carpool arrangements may afford.

Except when explicitly noted, we assume that the set of observables $z_{ij} \in \mathbb{R}^{K_z}$ can be partitioned into $z_{ij}^2 \in \mathbb{R}^{K_z-1}$ and $z_{ij}^1 \in \mathbb{R}$. The indirect utility function is additively separable in z_{ij}^1:

$$V(z_{ij}, \xi_j, \epsilon_i) = U(z_{ij}^2, \xi_j, \epsilon_i) - z_{ij}^1. \tag{1}$$

We assume that $\epsilon_i \perp (z_{i1}^1, \dots, z_{iJ}^1)$, which implies that any unobserved characteristics that affect the taste for schools is independent of z^1. This representation normalizes the scale of utilities by setting the coefficient on z_{ij}^1 to -1. The model is observationally equivalent to one with student-specific tastes, α_i, for z_{ij}^1 as long as it is negative for all i. The term z_{ij}^1 in this representation is sometimes referred to as a special regressor (Manski, 1985; Matzkin, 1992; Lewbel, 2000). The combination of the additively separable form and the independence of ϵ_i is the main restriction in this formulation.

In the school choice context, this assumption needs to be made on a characteristic that varies by student and school. We follow a common approach in the school choice literature by assuming that distance to school is additively separable and independent of unobserved student preferences (see Abdulkadiroglu et al., 2017a, for instance). The independence assumption is violated if unobserved determinants of student preferences simultaneously determine residential choices. As mentioned earlier, we do not find evidence that school choice incentives in Cambridge affect house prices or aggregate residential decisions. Nonetheless, the potential for interactions between school and residential choice warrants further research. Our empirical approach will include fixed-effects for whether a student has priority at a school as a determinant of preference to partially control for residential choice.

While our identification results do not make parametric assumptions on utilities, we specify student i's indirect utility for school j in the empirical application as

$$\begin{aligned} v_{ij} &= \sum_{k=1}^{K} \beta_{kj} z_{ijk} - d_{ij} + \varepsilon_{ij} \\ v_{i0} &= 0, \end{aligned} \tag{2}$$

where d_{ij} is the road distance between student i's home and school j; z_{ijk} are student-school specific covariates; β_{kj} are school-specific parameters to be estimated; and $\varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iJ}) \sim \mathcal{N}(0, \Sigma)$ independently of z, d.[13] The scale normalization is embedded in the assumption that the coefficient on d_{ij} is -1. Our estimated specification constructs z_{ijk} by interacting indicators of student paid-lunch status, sibling priority, proximity priority, ethnicity, home-language, and a constant with school-specific dummies.[14]

3.2 Assignment Mechanisms

School choice mechanisms typically use submitted rank-order lists and student priority types to determine final assignments. Let $R_i \in \mathcal{R}_i$ be a rank-order list, where jR_ij' indicates that j is ranked above j'.[15] Students, if they so choose, may not submit a report in which the most preferred schools are ranked in order of true preferences. Let student i's priority type be denoted $t_i \in T$. In Cambridge, t_i is defined by the free-lunch type, the set of schools where the student has proximity priority, and whether or not the student has a sibling in the school.[16]

A mechanism is usually described as an outcome of an algorithm that takes these rank-order lists and priorities as inputs. It will be convenient for our analysis to define a mechanism as a function that depends on the number of students n.

Definition 1. *A **mechanism** is a function $\Phi^n : \mathcal{R}^n \times T^n \to \left(\Delta^J\right)^n$ such that for all (R_i, t_i), $R_{-i} = (R_1, \ldots, R_{i-1}, R_{i+1}, \ldots, R_n)$ and $T_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n)$, $\Phi^n_{ij}((R_i, t_i), (R_{-i}, T_{-i}))$ denotes the probability that i is assigned to program j.*

Therefore, the assignment probabilities for each student depends on her report, her pri-

[13] Our specification allows for heteroskedastic errors ε_{ij} and arbitrary correlation between ε_{ij} and $\varepsilon_{ij'}$. This specification relaxes homoskedastic and independent preference shocks commonly used in logit specifications.

[14] The school-specific dummies interacted with the constant subsume the unobservable ξ_j.

[15] The set $\mathcal{R}_i$ may depend on the student's priority type t_i and may be constrained. For example, students in Cambridge can rank up to three schools, and programs are distinguished by paid-lunch status of the student.

[16] We assume that students take their priority type as given. Cambridge verifies residence and free-lunch eligibility using documentary evidence. Because most schools are less competitive for free-lunch students and the classes of instruction are not split by free-lunch status, it is unlikely that not declaring free-lunch eligibility is beneficial for an eligible student.

ority type as well as the reports and priority types of the other students. In addition, the final outcome depends on a random number used to break ties between students. As we demonstrate below, a student can face uncertainty about both the reports of other students as well as the tie-breaker.

In Cambridge, $\Phi^n((R_i, t_i), (R_{-i}, T_{-i}))$ is determined by a cutoff rule and the effective priority of the student, which depends on her report. Specifically, the priority of student i at school j is

$$e_{ij} = f_j(R_i, t_i, \nu_i) = \frac{3 - R_i(j) + \frac{t_{ij} + \nu_i}{4}}{3},$$

where $\nu_i \in [0, 1]$ is student i's draw of the random tie-breaker, $R_i(j)$ is the position of school j in the rank-order list R_i,[17] and t_{ij} is respectively 0, 1, 2 or 3 if the student has no priority, proximity priority only, sibling priority only, or both proximity and sibling priority. The function f chosen for Cambridge ensures that students who rank a school higher than other students are given precedence, with ties broken first by proximity and/or sibling priority, and then by the random tie-breaker.

Given the priorities e_{ij} above, let p_j be the highest priority student who was rejected at program j. The algorithm described in Section 2.1 places student i in program j if $e_{ij} > p_j$ and student i ranks program j above any other program j', with $e_{ij'} > p_{j'}$. Hence, the algorithm assigns student i to program j if $D_j^{(R_i,t_i,\nu_i)}(p) = 1$, where

$$D_j^{(R_i,t_i,\nu_i)}(p) = 1\{e_{ij} > p_j, jR_i0\} \prod_{j' \neq j} 1\{jR_ij' \text{ or } e_{ij'} \leq p_{j'}\}$$

and $e_{ij} = f_j(R_i, t_i, \nu_i)$.

The total fraction of students who would be assigned to program j if the cutoffs were p is therefore given by $D_j(p) = \frac{1}{n} \sum_i D_j^{(R_i,t_i,\nu_i)}(p)$. Because the number of students assigned to program j (likewise school s) cannot exceed the number of available seats $n \times q_j^n$ (likewise $n \times q_s^n$), it is easy to verify that the cutoffs determined by the algorithm in Section 2.1 have the following property.

Definition 2. *The vector of cutoffs $p \in [0, 1]^J$ is* ***market clearing*** *for $(D(p), q^n)$, if, for each program $j \in \mathcal{J}$,*

$$D_j(p) - q_j^n \leq 0,$$

[17] $R_i(j)$ is set to 4 if school j is not ranked.

with equality if $p_j > \min\{p_{j'} : j' \neq j, s_{j'} = s_j\}$,[18] *and for each school* $s \in \mathcal{S}$,

$$\sum_{\{j:s_j=s\}} D_j(p) - q_s^n \leq 0,$$

with equality if $\min\{p_j : s_j = s\} > 0$.

The first constraint ensures that program j has a cutoff larger than the other programs in the same school only if a student was rejected because the program has exhausted its capacity. The second constraint ensures that the lowest cutoff in the school is strictly positive only if the school has exhausted its total capacity.

In fact, this representation of the assignments in terms of the market clearing cutoffs and a priority that depends on a student's report is not unique to Cambridge. Many school choice mechanisms prioritize students based on their rank-order list and implement a cutoff rule that rejects all students below a program/school specific threshold. We now formally define a large class of mechanisms for which such a representation is valid.

Definition 3. *The mechanism* Φ^n *is a* ***Report-Specific Priority + Cutoff*** *mechanism if there exists a function* $f : \mathcal{R} \times T \times [0,1]^J \to [0,1]^J$ *and a measure* γ_ν *over* $[0,1]^J$ *such that*

(i) $\Phi_j^n((R_i, t_i), (R_{-i}, t_{-i}))$ *is given by*

$$\int \ldots \int D^{(R_i,t_i,\nu_i)}(p^n) \mathrm{d}\gamma_{\nu_1} \ldots \mathrm{d}\gamma_{\nu_n},$$

where $f(R_i, t_i, \nu_i)$ *is the eligibility score vector for student* i,

(ii) p^n *are market clearing cutoffs for* $(D(p), q^n)$, *where* $D(p) = \dfrac{1}{n} \sum_i D_j^{(R_i,t_i,\nu_i)}(p)$.

(iii) $f_j(R_i, t_i, \nu_i)$ *is strictly increasing in* ν_{ij} *and does not depend on* $\nu_{ij'}$ *for* $j' \neq j$.[19]

The representation highlights two ways in which these mechanisms can be manipulable. First, the report of an agent can affect her eligibility score. Even with a fixed cutoff, agents may have the direct incentive to make reports that may not be truthful. Second, even if eligibility does not depend on the report, an agent may (correctly) believe that the cutoff for a school will be high, making it unlikely that she will be eligible. If the rank-order list is constrained in length, she may choose to omit certain competitive schools.[20]

[18] We use the convention that $\min\{p_{j'} : j' \neq j, s_{j'} = s_j\} = 0$ if $\{j' \neq j, s_{j'} = s_j\} = \emptyset$.

[19] We can allow for a single tie-breaker (as in Cambridge) by ensuring that ν_{ij} and $\nu_{ij'}$ are perfectly correlated.

[20] The representation extends the characterization of stable matchings by Azevedo and Leshno (2016) in

Table I presents a partial list of school choice mechanisms currently used around the world. As we show in Appendix **??**, all mechanisms in table I, except for the Top Trading Cycles mechanism, belong to this class of cutoff-based mechanisms.[21] A remarkable feature is that these mechanisms differ essentially by the choice of f. The techniques we develop below are applicable to this entire class.

3.3 Agent Beliefs and Best Responses

Evidence presented in Section 2 suggests that agents are responding to strategic incentives in the Cambridge mechanism. To model this strategic behavior, we assume that agents submit rank-order lists that are optimal given a set of beliefs about the probability of assignment at various schools. We assume that agents have private information about their preferences, v_i. For an agent with priority type t_i, let the lottery $L^{\sigma}_{R_i,t_i} \in \Delta^J$ consist of the believed assignment probabilities at various schools when other agents in the market follow the strategy σ and she reports R_i. Because agents do not know the preferences of others in the market, these beliefs do not depend directly on the preferences of others, v_{-i}, or on the (realized) reports of others, R_{-i}. Instead, we specify these beliefs as a function of the mechanism and the strategies of the other agents in the mechanism but will drop the dependence on these objects for notational simplicity.

Given a preference vector $v_i \in \mathbb{R}^J$ and priority type t_i, the agent's expected utility from reporting R_i is therefore $v_i \cdot L_{R_i,t_i}$.[22] By choosing different rank-order lists, this agent can choose lotteries in the set $\mathcal{L}_{t_i} = \{L_{R_i,t_i} : R_i \in \mathcal{R}_i\}$. Therefore, this agent reports R_i only if

$$v_i \cdot L_{R_i,t_i} \geq v_i \cdot L_{R'_i,t_i}, \text{ for all } R'_i \in \mathcal{R}_i. \tag{3}$$

It is important to emphasize that optimality of R_i is with respect to the agent's belief about her assignment probabilities, which may or may not be the true assignment probabilities generated by the mechanism. In our application, we consider specifications with three alter-

terms of demand-supply and market clearing to discuss mechanisms. Particularly, we can use the framework to consider mechanisms that produce matchings that are not stable. The representation may therefore be of independent theoretical and empirical interest. For example, Abdulkadiroglu et al. (2017b) use a related cutoff-based approach for evaluating achievement gains from attending various types of schools for the case of the Deferred Acceptance Algorithm. Our representation suggests that it may be possible to extend their techniques to the entire class of RSP+C mechanisms.

[21] Leshno and Lo (2017) derive a cutoff representation for the Top Trading Cycles mechanism that does not belong to the class of RSP+C mechanisms.

[22] Although our model incorporates social interactions through L, it differs from a multinomial version of Brock and Durlauf (2001) because we do not specify an idiosyncratic payoff shock for each possible report R_i. Instead, we micro-found the expected utility of a report R_i through preferences for schools incorporated in v_i.

native forms of beliefs below.

3.3.1 Rational Expectations

Our baseline model assumes that agents have correct beliefs about the probabilities of assignments given their own type (v, t), the population distribution of types and the ranking strategies used in the district. Specifically, let $\sigma : \mathbb{R}^J \times T \to \Delta^{|\mathcal{R}|}$ be a (symmetric) mixed strategy used by the students in the district.[23] The first argument of σ is the vector of utilities over the various schools, and the second argument is the priority type of the student. Each student believes that the lottery when she reports $R_i \in \mathcal{R}_i$ is given by

$$\begin{aligned} L_{R_i,t_i} &= \mathbb{E}_\sigma[\Phi^n((R_i,t_i),(R_{-i},T_{-i}))|R_i,t_i] \\ &= \sum_{(R_{-i},T_{-i})} \Phi^n((R_i,t_i),(R_{-i},T_{-i})) \prod_{k\neq i} f_\sigma(R_k,t_k) \end{aligned} \tag{4}$$

where $\mathbb{E}_\sigma[\cdot]$ denotes expectations taken over the random variable $(R_{i'}, t_{i'})$ for $i' \neq i$ drawn iid with probability $f_\sigma(R,t) = f_T(t) \int \sigma_R(V,t) \mathrm{d}F_{V|T=t}$, and $F_{V,T}$ is the joint distribution of preference and priority types in the population. As a reminder, the dependence of L on the mechanism, Φ^n and the strategy, σ, have been suppressed for notational simplicity.

In this model of beliefs, the perceived probability of assignment depends on both the tie-breaker in the mechanism and the distribution of reports by the other students in the district. Therefore, a student perceives uncertainty due to both the reports submitted by other students as well the uncertainty in the lotteries within in the mechanism. This contrasts with models of complete information about the reports submitted by the other students, where the latter form of uncertainty is not present.[24] We believe that this model is more realistic than a complete information model because students are unlikely to be aware of all the reports that will be submitted by others. This assumption is commonly made in the analysis of other non-dominant strategy mechanisms, for example, in empirical analysis of auctions (Guerre et al., 2000; Cassola et al., 2013, among others).[25]

[23]Note that when the distribution of utilities admits a density, a unique pure strategy is optimal except for a measure zero set of types.

[24]Complete information Nash Equilibrium models are common in the literature on assignment mechanisms (see Ergin and Sonmez, 2006, for example). Results based on assuming that agents have knowledge of R_{-i} are both quantitatively and qualitatively similar to the ones presented here and are available on request.

[25]Indeed, when all agents optimally respond to such beliefs, the behavior is consistent with a Bayesian Nash Equilibrium:

Definition 4. *The strategy σ^* is a **type-symmetric Bayesian Nash Equilibrium** if $v_i \cdot L^{\sigma^*}_{R_i,t_i} \geq v_i \cdot L^{\sigma^*}_{R'_i,t_i}$ for all $R'_i \in \mathcal{R}_i$ whenever $\sigma^*_{R_i}(v_i,t_i) > 0$, where $L^{\sigma^*}_{R_i,t_i}$ and $L^{\sigma^*}_{R'_i,t_i}$ are given by equation (4).*

It is important to note that agents need not have beliefs over a very high dimensional object in Report-Specific Priority + Cutoff Mechanisms in order to compute the assignment probabilities and best responses. It is sufficient for agents to form beliefs over the distribution of cutoffs for the various programs. Our empirical approach will use this feature to reduce the dimension of the problem.[26]

3.3.2 Adaptive Expectations

Assuming rational expectations implies a strong degree of knowledge and sophistication. One may reasonably argue that the primary source of information for parents may be based on prior year information. We address this concern by considering the following alternative model of agent beliefs:

$$L_{R_i,t_i} = \mathbb{E}_{\sigma^{-1}}[\Phi^{-1,n}((R_i,t_i),(R_{-i},T_{-i}))|R_i,t_i] \tag{5}$$

where the notation -1 indicates the use of previous year quantities rather than the current year. Specifically, we assume that the agents have knowledge about the previous year strategy σ^{-1}, believe the school/program capacities are the same as the previous year, and that the distribution of other student types are the same as the previous year as well.

Such beliefs may arise if agents form expectations about the competitiveness of various schools based on the experiences of parents that participated in the previous year. Agents, in this case, may be systematically mis-informed, for example, about increases or decreases in capacity. Estimates from this model will be used to investigate whether the potential screening benefits of manipulable mechanisms hinge crucially on agents forming rational expectations.

3.3.3 Coarse Expectations

Another form of mis-information in the mechanism may be about the specific use of priorities and the program quotas based on free/reduced lunch status. If agents are not informed about these details of the mechanism, their expected probability of assignment given report R_i will not depend on their priority type, t_i. For example, beliefs that are based only on aggregate information about the number of applicants and capacities at each school would have this property.

[26] Kapor et al. (2017) use a survey of students in New Haven to construct estimates of students' beliefs over these cutoffs. They then extend our methods to estimate the preference distribution using these estimated beliefs.

To address the possibility that beliefs are coarse, we consider a model in which an agent believes that the lotteries are given by

$$L_{R_i} = \sum_{t_i \in T} \mathbb{E}_\sigma[\Phi^n((R_i, t_i), (R_{-i}, T_{-i}))|R_i, t_i] f(t_i), \tag{6}$$

where σ is the strategy used by the students in the district. Such beliefs may have distributional consequences and may undo some of the goals of the Controlled Choice Plan of maintaining a diverse student mix within programs. It is possible that schools that are popular among paid-lunch students, such as Graham & Parks, may be under-subscribed by free/reduced lunch students because of such coarse beliefs.

4 A Revealed Preference Approach

This section illustrates the key insight that allows us to learn about the preferences of students from their (potentially manipulated) report and presents an overview of our method for estimating preferences. Equation (3) reveals that a student's optimal choice, given her priority type, depends on the assignment probabilities a student believes she can achieve by altering her report. The choice of a report by a student is, therefore, a choice from the set of lotteries,

$$\mathcal{L} = \{L_{R_i} : R_i \in \mathcal{R}_i\},$$

where the dependence on t_i is dropped for simplicity. The various forms of beliefs described in Section 3.3 specify particular values for L_{R_i}.

Assume, for the moment, that a student's belief for the assignment probabilities is known to the analyst, and consider her decision problem. Figure II illustrates an example with two schools and an outside option. Each possible report corresponds to a probability of assignment into each of the schools and a probability of remaining unassigned. Figure II(i) depicts three lotteries $L_R, L_{R'}, L_{R''}$ corresponding to the reports R, R' and R'' respectively on a unit simplex.[27] The dashed lines show the linear indifference curves over the lotteries for an agent with a utility vector parallel to the vector a. A student with a utility vector parallel to a_R will therefore find L_R optimal (figure II(ii)). A student who is indifferent between L_R and $L_{R'}$ must have indifference curves that are parallel to the line segment connecting the two points and, therefore, a utility vector that is parallel to $a_{R,R'}$ (figure II(ii)). Likewise, students with a utility vector parallel to $a_{R,R''}$ are indifferent between L_R and $L_{R''}$. In fact, L_R is optimal for all students with utility vectors that are linear combinations of $a_{R,R'}$ and

[27] The simplex is often referred to as the Marschak-Machina triangle.

$a_{R,R''}$ with positive coefficients. A similar reasoning can be applied to $L_{R'}$ and $L_{R''}$, resulting in the vector $a_{R',R''}$ depicted in figure II(iii). We now turn our attention to utility space in figure II(iv). The rays starting from the origin and parallel to each of these vectors partition this space. As argued above, L_R is optimal for students with utility vectors $v \in C_R$, for example if v is parallel to a_R. In symbols, for any J and set of lotteries $\mathcal{L}$, choosing L_R is optimal if and only if the utility vector belongs to the cone:

$$C_R = \{v \in \mathbb{R}^J : v \cdot (L_R - L_{R'}) \geq 0 \text{ for all } R' \in \mathcal{R}\}. \tag{7}$$

For all values of v in this cone, the expected utility from choosing R is at least as large as choosing any other report.[28] Similarly, reports R, R', and R'' are only optimal for students with utility vectors in the regions C_R, $C_{R'}$, and $C_{R''}$ respectively. Further, these regions may intersect only at their boundaries and together cover the utility space. The choice of report therefore reveals the region in utility space to which a student's preference vector belongs.

Remarkably, a partition of this form is implied for all school choice mechanisms that use tie-breakers if a student's belief for assignment probabilities is identified. The argument does not rely on uniqueness of equilibria as long as agent beliefs are identified and can be estimated using the data. Further, these inequalities use all the restrictions on preferences given the beliefs of the agents and the mechanism. We can use this insight to construct the likelihood of observing a given choice as a function of the distribution of utilities:

$$\mathbb{P}(R|z,\xi) \quad = \quad \mathbb{P}(R = \arg\max_{R' \in \mathcal{R}} v \cdot L_R | z, \xi; f) = \int 1\{v \in C_R\} f_{V|z,\xi}(v|z,\xi) \mathrm{d}v. \tag{8}$$

This expression presents a link between the observed choices of the students in the market and the distribution of the underlying preferences, and it is the basis of our empirical approach. It presents rich information about utilities because the number of regions is equal to the number of reports that may be submitted to a mechanism.[29]

There are three remaining issues to consider. First, we need to show that agent beliefs over assignment probabilities can be consistently estimated so that the regions C_R used to construct the likelihood can be determined. The objective is to justify an approximation to ex-ante student beliefs using a realized sample of reports. Section 6.1 shows that assignment probabilities for the strategies used by students observed in the data can be consistently estimated in all RSP+C mechanisms. Therefore, our procedure is robust to the

[28]We allow for the possibility that $L_R = L_{R'}$ for two reports R and R'. This may occur if the student lists a school she is sure to be assigned to as her first choice. In such cases, our revealed preference method does not deduce any preference information from later ranked choices.

[29]There are a total of 1,885 elements in $\mathcal{R}$ because students in Cambridge can rank up to three programs from 13.

possibility of multiple equilibria in case one wishes to assume that students play equilibrium strategies.[30] Second, Section 5 provides conditions under which the distribution of utilities is non-parametrically identified. We can obtain point identification by "tracing out" the distribution of utilities with either variation in lottery sets faced by students or by using an additively separable student-school specific observable characteristic. Third, Section 6.2 proposes a computationally tractable estimator that can be used to estimate the parameters of the preference distribution. Here, we use a first step estimate of assignment probabilities.

5 Identification

Section 4 showed that the choice of report by a student allows us to determine the cone, $C_R \subseteq \mathbb{R}^J$ for $R \in \mathcal{R}$, that contains her utility vector v. The argument required knowledge of student's beliefs for assignment probabilities that constitute $L_{R,t}$. Knowledge of the mechanism and the joint distribution of reports and types directly identifies the forms of beliefs specified in equations (4)-(6).[31] This section presents our results on identification of the distribution of indirect utilities. Knowledge of this distribution is sufficient for positive analysis of various types. For example, it allows for analysis of assignments under counterfactual mechanisms and a determination of the fraction of students who are assigned to their true first choice. Additionally, certain forms of normative analysis that involve comparing the proportion of students who prefer one mechanism over another can also be conducted.[32]

We now show what one can learn about the distribution of utilities using equation (8):

$$\mathbb{P}(R \in \mathcal{R}|z, t, \xi, b) = \int 1\{v \in C_{b,R,t}\} f_{V|z,t,\xi}(v|z, t, \xi)\mathrm{d}v,$$

where b is a market subscript and the dependence on t has been reintroduced for notational clarity. This expression shows that two potential sources of variation that can be used to "trace out" the densities $f_{V|T,z,\xi}(v|z, t, \xi)$. The first results from choice environments with different values of $C_{b,R,t}$. The second results from variation in the observable characteristic z. We consider each of these below. As is standard in the identification literature, we treat the assignment probabilities and the fraction of students who choose any report as observed.

[30] We avoid common issues in deriving a likelihood in games with multiple equilibria (Ciliberto and Tamer, 2009; Galichon and Henry, 2011, for example) because we are able to identify and estimate agent beliefs.

[31] Section 6.1 presents a consistent estimator for $L_{R,t}$ as defined by each of the forms of beliefs.

[32] Solving the social planner's problem or comparing mechanisms using a Kaldor-Hicks criterion requires additional assumptions on the transferability of utility or a choice of Pareto weights.

5.1 Identification Under Varying Choice Environments

In some cases, researchers may be willing to exclude certain elements of the priority structure, t, from preferences, or they may observe data from multiple years in which the set of schools are the same but capacities vary across years. Such variation in choice environments can result in rich information about preferences. Our arguments in this sub-section will therefore relax the additive separability assumption made in equation (1) by allowing $V(\cdot)$ to have a general functional form. Because utilities can be determined only up to positive affine transformations, we instead normalize the scale as $\|v_i\| = 1$ for each student i. This normalization is without loss of generality. Hence, it is sufficient to consider the case when v_i has support only on the unit circle.

To gain intuition, assume that a researcher has data from two years with one school adding a classroom in the second year. If we assume that school capacity can be excluded from the distribution of preferences, i.e. $v|z, \xi, t, b \sim v|z, \xi, t, \tilde{b}$ for the years indexed by b and $\tilde{b}$, we effectively observe students with the same distribution of preferences facing two different lottery choice sets. For example, let the choice sets faced by students be $\mathcal{L} = \{L_R, L_{R'}, L_{R''}\}$ and $\tilde{\mathcal{L}} = \{L_R, \tilde{L}_{R'}, L_{R''}\}$ respectively. Figure III(i) illustrates these choice sets. The change from $L_{R'}$ to $\tilde{L}_{R'}$ affects the set of utilities for which the various choices are optimal. The set of types for which L_R is optimal now also includes the dotted cone. The utilities in this cone can be written as linear combinations of $\tilde{a}_{R,R'}$ and $a_{R,R'}$ with positive coefficients. Observing the difference in likelihood of reporting R for students with the two types allows us to determine the weight on this region:

$$\mathbb{P}(R|\tilde{b}) - \mathbb{P}(R|b) = \int (1\{v \in \tilde{C}_R\} - 1\{v \in C_R\}) f_V(v) dv,$$

where explicit conditioning on the vector (z, t, ξ) is dropped because it is held fixed. Figure III(ii) illustrates that this variation allows us to determine the weight on the arc $\tilde{h}_R - h_R$. Carvalho et al. (2017) independently developed a similar identification argument in the case where there are only two programs (i.e. $J = 2$) and there is rich variation in the choice environment. Appendix **??** proves a result for the general case and characterizes the identified set even when the variation in choice environments is limited.

The discussion suggests that enough variation in the set of lotteries faced by individuals with the same distribution of utilities can be used to identify the preference distribution. The arc above traces the density of utilities along the circle when such variation is available. However, typical school choice systems have only a small number of priority types and datasets typically cover a small number of years. Therefore, due to limited support, we will typically partially identify f_V. While this source of variation may not be rich enough for a basis for non-parametric identification, it makes minimal restrictions on the distribution of utilities. In particular, the result allows for the preference distribution to depend arbitrarily on residential locations through z. Although beyond the scope of this paper, this framework may be a useful building block for a model that incorporates both residential and schooling choices.

5.2 Identification With Preference Shifters

This subsection drops the scale normalization, $\|v_i\| = 1$, made in the previous section and reverts back to the additive separable form with the scale normalization in equation (1). We describe how variation in z^1 within a market, fixing ξ and z^2, can be used to learn about the distribution of indirect utilities. The objective is to identify the joint distribution of $u_{ij} = U(z^2_{ij}, \xi_j, \epsilon_i)$ given (ξ, z^2), where we drop this conditioning for simplicity of notation. Because ϵ_i is independent of z^1_{ij}, we have that

$$f_{V|Z^1}(v|z^1) = g(v + z^1)$$

where g is the density of u and z^1 is observed on a set ζ. Therefore, our goal is to identify g.[33]

Consider the lottery set faced by a set of students and the corresponding region, C_R, of the utility space that rationalizes choice R (figure II). A student with $z^1_i = z$ chooses R if $v = u - z \in C_R$. The values of u that rationalize this choice are given by $z + \alpha_1 a_{R,R'} + \alpha_2 a_{R,R''}$ for any two positive coefficients α_1 and α_2. Figure IV illustrates the values of u that make R optimal. As discussed in Section 4, observing the choices of individuals allows us to determine the fraction of students with utilities in this set. Similarly, by focusing on the set of students with $z^1_i \in \{z', z'', z'''\}$, we can determine the fraction of students with utilities in the corresponding regions (see Figure IV). By appropriately adding and subtracting these fractions, we can learn the fraction of students with utilities in the parallelogram defined by (z, z', z''', z''). This allows us to learn the total weight placed by the distribution g on such parallelograms of arbitrarily small size. It turns out that we can learn the density of g around any point z in the interior of ζ by focusing on variation in the neighborhood of z. The next result formalizes this intuition.

[33] In recent work, Allen and Rehbeck (2017) study identification in a general class of demand models and consider the case where $v_{ij} = u_{ij} - h(z^1_{ij})$. Specifically, they study the identification of $h(\cdot)$ and show that it can be identified up to scale and location without knowledge of the distribution of u_{ij}. Our goal is to identify the distribution of indirect utilities, which requires knowledge of u_{ij}. It may be possible to combine these results to relax the restriction that (indirect) utilities are linear in z^1_{ij}.

Theorem 1. *If C_R is spanned by J linearly independent vectors $\{a_{R,R^1},\ldots,a_{R,R^J}\}$, where $R^1,\ldots,R^J$ belong to $\mathcal{R}\backslash\{R\}$, then $g(\cdot)$ is identified in the interior of ζ. Hence, $f_V(v|z^1)=g(v+z^1)$ is identified for all v such that $v+z^1$ is in the interior of ζ.*

Proof. Let $A_R=[a_{R,R^1},\ldots,a_{R,R^J}]$ and note that $C_R=\{v:v=A_Rx \text{ for some } x\geq 0\}$. Assume, wlog, $|\det A_R|=1$. Evaluate $h_{C_R}(z)=\mathbb{P}(v\in C_R|z)$ at A_Rx:

$$\begin{aligned} h_{C_R}(A_Rx) &= \int_{\mathbb{R}^J} 1\{u-A_Rx\in C_R\}g(u)\,\mathrm{d}u \\ &= \int_{\mathbb{R}^J} 1\{A_R(y-x)\in C_R\}g(A_Ry)dy = \int_{x_1}^{\infty}\ldots\int_{x_J}^{\infty} g(A_Ry)\,dy, \end{aligned}$$

where the second equality follows from a change of variables, $u=A_Ry$, and the third equality follows because $A_R(y-x)\in C_R \iff y\geq x$. Therefore,

$$\frac{\partial^J h_{C_R}(A_Rx)}{\partial x_1\ldots\partial x_J}=g(A_Rx).$$

In particular, for any z^1 in the interior of ζ, $g(z^1)$ is given by $\frac{\partial^J h_{C_R}(A_Rx)}{\partial x_1\ldots\partial x_J}$ evaluated at $x=A_R^{-1}z^1$. The derivative is identified on the interior of ζ because $h_{C_R}(\cdot)$ is observed on ζ. $\square$

Intuitively, z_i^1 shifts the distribution of indirect utilities. For example, when z_{ij}^1 denotes the distance to school j, then all else equal, students closer to a particular school should have stronger preferences for attending that school. These students should be more likely to rank it on their list. The extent to which students who are closer to a given school are more likely to rank it is indicative of the importance of distance relative to other factors that affect preferences that are captured by $U(\cdot)$.[34]

One drawback of the formal result above is that it places a restriction on C_R, which is a non-primitive object. However, the condition can be verified in the data because C_R is identified. Moreover, Theorem A.2 in the Appendix shows that we can identify g under weaker conditions on C_R if g has exponentially decreasing tails and $\zeta=\mathbb{R}^J$. The proof is based on Fourier-deconvolution techniques because the distribution of v is given by a location family parametrized by z^1. The conditions on g are quite weak and are satisfied for commonly used distributions with additive errors such as normal distributions, generalized

[34]The nature of this identification result articulates the fact that identification of the density at a point does not rely on observing extreme values of z^1. Of course, identification of the tails of the distribution of u will rely on support of extreme values of z^1. Also note that our identification result requires only one convex cone generated by a lottery and, therefore, observing additional lotteries with simplicial cones generates testable restrictions on the assumption that z^1 is a special regressor.

extreme value distributions, or if u has bounded support.[35]

When z^1_{ij} is assumed to be the road or walking distance from student i's residence to school j, then the support of z^1 will be limited by geographical constraints. In this case, such variation provides partial information on f_V. Our estimator, which is described in the next section, will use variation from this source in addition to variation in choice environments in a parametric specification.

6 Estimating Assignment Probabilities and Preference Parameters

We estimate our preference parameters, $\theta = (\beta, \Sigma)$, using a two-step estimator. In the first step, we estimate the assignment probabilities for each lottery, $L_{R,t}$. The second step estimates θ taking the estimate from the first step $\hat{L}$, as given. Although it may be possible to estimate these two sets of parameters jointly, the two-step procedure is computationally tractable, albeit at potential efficiency costs. Theorems 2.1 and 6.1 of Newey and McFadden (1994) show conditions under which a two-step estimator is consistent and asymptotically normal. These results require that the first-step is consistent and asymptotically normal, and the second-step is reasonably well-behaved. Our second-step estimator will be equivalent to a maximum likelihood estimator. The main challenge, therefore, is to show that our first-step is consistent and asymptotically normal.

The asymptotic framework assumes that the number of students n grows large in a single year and the number of programs is held fixed. The capacity of the programs, $n \times q^n_j$, increases proportionally to the number of students, i.e. $q^n_j \to q_j \in (0, 1)$. These limits are meant to capture an environment, such as the one in Cambridge, where the number of students is large relative to the number of schools.[36] It is sufficient for the researcher to observe data from a single year of the mechanism with many students for consistency and asymptotic normality of the estimator. In our application, the parameters governing the distribution of preferences given the observables, θ, is held constant across years and multiple years of data to improve the precision of $\hat{\theta}$. For simplicity of notation, we omit the dependence of L on the application year.

[35] We do not require that g has a non-vanishing characteristic function. Further, when u has bounded support, we can allow for ζ to be a corresponding bounded set.

[36] Azevedo and Leshno (2016) use similar limits to analyze properties of stable matchings in a large market.

6.1 First Step: Estimating Assignment Probabilities

The first step requires us to estimate the probabilities in equation (4) for each value of (R_i, t_i). The equation highlights two sources of uncertainty facing the student when forming this expectation. First, at the time of submitting the report, the student does not know the realization of (R_{-i}, T_{-i}). The agents form expectations for this realization based on population distribution of types and the forecast strategy σ. The second source of uncertainty is within Φ^n because of the random tie-breaker used to determine assignments. Even with rational expectations, these two sources of uncertainty imply that the realized empirical assignment probabilities differ from the agent's expectations.

We approximate the first source of uncertainty using a resampling procedure because the data consists of a large sample of reports and priorities drawn from the population distribution. For the second source of uncertainty, we can use the fact that Cambridge uses a mechanism with a Report-Specific Priority + Cutoff (RSP+C) representation. For each draw of $(R_{-i}, T_{-i})_b$ and tie-breakers $\nu_{-i,b}$, we compute a market clearing cutoff p_b^{n-1} by simulating the mechanism.[37] The assignment for a student with priority type t_i, report R_i, and a draw of the random tie-breaker ν_i is given by $D^{(R_i,t_i,\nu_i)}(p_b^{n-1})$, where $e_i = f(R_i, t_i, \nu_i)$. This reasoning suggests the following estimator, $\hat{L}$, for the rational expectations case:

$$
\begin{aligned}
L_{R_i,t_i} &\approx \frac{1}{B}\sum_{b=1}^{B} \Phi^n((R_i, t_i), (R_{-i}, T_{-i})_b), \\
&\approx \frac{1}{B}\sum_{b=1}^{B} \int D^{(R_i,t_i,e_i)}(p_b^{n-1})\mathrm{d}\gamma_{\nu_i} \\
&= \hat{L}_{R_i,t_i}, \qquad (9)
\end{aligned}
$$

where $(R_{-i}, T_{-i})_b$ is the b-th sample of $n-1$ reports and priority types and γ_{ν_i} is the CDF of the tie-breaker. The estimator therefore incorporates information on all the submitted applications, data on school capacities in each year, and knowledge of the assignment mechanism. Further, it uses a single set of draws for p_b^{n-1} to compute assignment probabilities for all values of (R_i, t_i). This makes the computation tractable even with a large number of possible rank-order lists.[38]

[37] Our estimator approximates the cutoffs by ignoring the report of agent i because, in a large market, any single agent has a negligible impact on the cutoffs. That the resulting approximation error is negligible is formalized in Appendix A.3, where we show that the approximation error in using only the other $n-1$ agents is of order $1/n$.

[38] One may instead use observed assignment frequencies conditional on priorities and the rank-order lists in this step. However, this approach is likely to yield poor estimates due to the curse of dimensionality unless there are many students relative to the number of rank-order list and priority combinations. In Cambridge, there are just under 2,000 feasible rank-order lists for each priority type. Our sample consists of 2,129

Our estimator for the adaptive expectations case is analogous. We draw $(R_{-i}, T_{-i})_b$ from the observed reports in the previous year and calculate p_b^{n-1} using the previous year's capacity. The estimate for L_{R_i} in the coarse expectations case uses the estimate $\hat{L}_{R_i,t_i}$ for the rational expectations case and averages it for each R_i using the empirical distribution of t_i.

The equation above highlights why the RSP+C representation is useful for estimating assignment probabilities, a potentially complicated task for general mechanisms. Because mechanisms are usually described in terms of algorithms that use the reports and priority types of all the students in the district, there are few a priori restrictions that prevent them from being ill-behaved. A small change in students' reports could potentially have large effects on the assignment probabilities.[39] Moreover, our objective is to estimate assignment probabilities simultaneously for all priority-types and each possible rank-order list that can be submitted by a student. The RSP+C representation allows us to obtain results on the limiting distribution of $\hat{L}$ by examining the limit behavior of the cutoffs, p_b^{n-1}.

Theorem 2. *Suppose that Φ^n is an RSP+C mechanism with a random tie-breaker $\nu_{ij} = \nu_{ij'} = \nu_i$ that is drawn from the uniform distribution on $[0,1]$.*

(i) If p^* is the unique market clearing cutoff for $(E[D(p)], q)$, where $q = \lim q^n$, then for each (R,t), $|\hat{L}_{R,t} - L_{R,t}| \xrightarrow{p} 0$.

(ii) Further, if $p_j^* > 0$ for all j, $\|q^n - q\| = o_p(n^{-1/2})$ and $\nabla_p E[D(p^*)]$ is invertible, then for each each (R,t),

$$\sqrt{n}(\hat{L}_{R,t} - L_{R,t}) \xrightarrow{d} N(0, \Omega^*)$$

where Ω^* is given in Theorem A.3.

Proof. The result is a special case of Theorem A.3 in the Appendix, which relies on less restrictive assumptions on γ_ν and does not assume that $p_j^* > 0$. □

This result is based on showing that the cutoffs determining the eligibility thresholds are close to p^* if the number of students is large and then analyzing the limit distribution of $\sqrt{n}(p_b^{n-1} - p^*)$. As discussed in Appendix **??**, the uniqueness of the cutoff p^* is a generic

students. The knowledge of the mechanism and administrative data on capacities and submitted reports is helpful in reducing the dimension of the problem to the number of cutoffs, which is equal to the number of programs.

[39] Two pathological examples allowed by Definition 1 are instructive. The first example is one in which the assignment of all students depends only on student 1's report. The second is an algorithm that depends on whether an odd or even number of students apply to schools. It is easy to see why the conclusions of Theorem 2 will not apply in these cases.

property.[40] Uniqueness of p^* in the Cambridge mechanism follows if for all j, $D_j(\cdot)$ is strictly decreasing in p_j (see Proposition **??** in the Appendix). Results on p_b^{n-1} can be translated to $\hat{L}$ using smoothness imparted by the non-degenerate distribution of ν. Finally, as formalized in Proposition **??**, our results also imply convergence of finite market equilibria to large-market limits.

6.2 Second Step: Preference Estimates

The second step is defined as a maximum likelihood estimator and it takes the estimate $\hat{L}$ from the first step as given. Specifically, equation (8) implies that the maximum likelihood estimator is given by

$$\hat{\theta} = \arg\max_{\theta \in \Theta} \quad \textstyle\sum_{i=1}^{n} \log \mathbb{P}(R_i = \arg\max_{R \in \mathcal{R}_i} v_i \cdot \hat{L}_{R,t_i} | z_i, t_i; \theta), \tag{10}$$

where R_i is the report submitted by student i, z_i is the vector of observables that the distribution of v_i depends on, t_i is the priority type of agent i, and θ parametrizes the distribution of v as given in Section 3.1.

Unfortunately, our model does not yield a simple closed-form solution for this likelihood. Further, the relatively large number of potential rank-order lists implies that a simulated maximum likelihood with enough draws to avoid bias is computationally burdensome.[41] To solve this problem, we adapt the Gibbs' sampler used by McCulloch and Rossi (1994) to estimate a discrete choice model. It offers a computationally convenient likelihood-based method for estimating parameters in some cases when an analytic form for the likelihood function is not available. The Gibbs' sampler obtains draws of β and Σ from the posterior distribution by constructing a Markov chain of draws from any initial set of parameters $\theta^0 = (\beta^0, \Sigma^0)$. The invariant distribution of the resulting Markov chain is the posterior given the prior and the data. The posterior mean of this sampler is asymptotically equivalent to the maximum likelihood estimator (see van der Vaart, 2000, Theorem 10.1 (Bernstein-von-Mises)).

As in the discrete choice case, we first use data augmentation to pick a utility vector for each agent consistent with their choice. Here, we initialize $v_i^0 \in C_{R_i}$ for each student i, where R_i is the report chosen by agent i. We set v_i^0 by using a linear programming solver to find

[40] Specifically, we use results from Azevedo and Leshno (2016) to show that market clearing cutoffs are generically unique. Using techniques from Berry et al. (2013) and Berry and Haile (2010), we can also derive stronger conditions for global uniqueness of the market clearing cutoffs.

[41] There are 1,885 possible ways to rank up to 3 schools from 13. This is the potential number of rank-order list for each student priority type.

a solution to the constraints $v_i^0 \cdot (\hat{L}_{R_i,t_i} - \hat{L}_{R',t_i}) \geq 0$ for all $R' \in \mathcal{R}$.[42]

The chain is then constructed by sampling from the conditional posteriors of the parameters and the utility vectors given the previous draws. The sampler iterates through the following sequence of conditional posteriors:

$$\begin{aligned} \beta^{s+1} \quad &| \quad v_i^s, \Sigma^s \\ \Sigma^{s+1} \quad &| \quad v_i^s, \beta^{s+1} \\ v_i^{s+1} \quad &| \quad v_i^s, C_{R_i}, \beta^{s+1}, \Sigma^{s+1} \end{aligned}$$

The first two steps update the parameters β and Σ in equation (2) using standard procedures (see McCulloch and Rossi, 1994, and Appendix **??** for details). The last step, which draws v_i^{s+1} for each student, differs slightly from McCulloch and Rossi (1994). Specifically, we need to condition on the regions C_{R_i} and sample from the following conditional posteriors:

$$v_{ij}^{s+1} | v_{i1}^{s+1}, \ldots, v_{ij-1}^{s+1}, v_{ij+1}^{s}, \ldots, v_{iJ}^{s}, \beta^{s+1}, C_{R_i}, \Sigma^{s+1}.$$

This requires us to draw from a (potentially two-sided) truncated normal distribution with mean, variance, and truncation points determined by β^{s+1}, Σ^{s+1}, C_{R_i} and $v_{i,-j}$.[43] We can ensure that $v_i^{s+1} \in C_{R_i}$ for every student i in every step by calculating the bounds on v_{ij}^{s+1} conditional on $v_{i,-j}$ defined by the restriction $v_i^{s+1} \cdot (L_{R_i} - L_R) \geq 0$ for all $R \in \mathcal{R}$.

We specify independent and diffuse conjugate prior distributions according to standard practice. These details and other implementation issues are described in Appendix **??**. Standard errors in our case also need to account for estimation error in the first step. We do this using the bootstrap procedure described in Appendix **??**.

7 Application to Cambridge

7.1 Assignment Probabilities and Preference Parameters

Table VI presents the assignment probabilities for various schools averaged over various student subgroups.[44] As in table IV, the estimates indicate considerable heterogeneity in school competitiveness. The typical student isn't guaranteed assignment at one of the more competitive schools even if she ranks it first. On the other hand, several schools are sure

[42]A linear programming solver can be used to eliminate linearly dependent constraints with positive coefficients in order to further simplify the later stages of the Gibbs' sampler.

[43]The standard discrete choice model only involves sampling from one-sided truncated normal distributions.

[44]Table **??** provides an estimate for standard errors of $\hat{L}$ constructed by bootstrapping the estimator.

shots for students who rank them first. The probability of not getting assigned to a school also differs with paid-lunch status. A comparison of estimates in panel A with those in panels D and E indicates that having priority at a school significantly improves the chances of assignment. The differential is larger if the school is ranked first.

Panel A of table VII presents the (normalized) mean utility for various schools net of distance by student group for four specifications. The first specification treats the agent reports as truthful, while the second, third, and fourth specifications assume that students best respond to beliefs given by rational expectations, adaptive expectations, and coarse beliefs respectively.[45] In each of these specifications, we find significant heterogeneity in willingness to travel for the various schooling options. Paid-lunch students, for instance, place a higher value on the competitive schools as compared to the non-competitive schools. Although not presented in the mean utilities, Spanish- and Portuguese-speaking students disproportionately value schools with bilingual and immersion programs in their home language. Students also place a large premium on going to school with their siblings.

A comparison between the first column and the others suggests that treating stated preferences as truthful may lead to underestimates of the value of competitive schools relative to non-competitive schools. This differential is best illustrated using Graham & Parks as an example. Treating stated preferences as truthful, we estimate that paid-lunch students have an estimated mean utility that is an equivalent of 1.29 miles higher than the average public school option. This is an underestimate relative to the models that assume students correctly believe that Graham & Parks is a competitive school. In contrast, the value of Graham & Parks for free-lunch students is over-estimated by the truthful model relative to both the rational expectations and adaptive expectations model. The difference is due to the fact that Graham & Parks is not competitive for free-lunch students and therefore, the low number of applications it receives indicates particular dislike for the school.

Overall, estimates based on modeling expectations as adaptive are strikingly similar to those from assuming rational expectations. In part, this occurs because the relative competitiveness of the various schooling options in Cambridge is fairly stable even though there is some annual variation in assignment probabilities across schools. This result is comforting and suggestive of the robustness of our estimates to small mis-specifications of agent beliefs. Estimates that endow agents with coarse beliefs continue to indicate that treating reports as truthful underestimates the relative preference for the most competitive schools such as Graham & Parks, Haggerty, Baldwin, and Morse. The results are more mixed for the less desirable schools. As in the models that treat preferences as truthfully

[45] The underlying parameter estimates for the two baseline specifications, rational expectations and truthful reporting, are presented in tables **??** and **??**.

reported, free-lunch and paid-lunch students are in broad agreement on the relative ranking of the various schools.

Another significant difference between the estimates that treat agents as truthful and those that do not is in the number of schools students find preferable to the outside option. Panel B shows that estimates that treat stated preferences as truthful suggest that about half the students have five or more schools where assignment is preferable to the outside option. On the other hand, treating agents as best responding to one of the three forms of beliefs studied here suggests that about half the students find at most two schools in the system preferable to the outside option. To understand these results, note that treating preferences as truthful extrapolates from the few students (about 13%) that do not have complete rank-order lists. On the other hand, the model that treats students as being strategic interprets the decision to rank long-shots in the second and third choices as evidence of dislike for the remaining schools relative to the outside option.

These results should be viewed in light of Cambridge's thick after-market. About 92% of the students who are not assigned through the school choice process are assigned to one of the schools in the system. Only a quarter of unassigned students are placed at their top-ranked school through a wait-list that is processed after the assignment process in Cambridge. Most of the remaining unassigned students are placed in an unranked school. Cambridge also has charter and private school options that unassigned students may enroll in. The value of the outside option is therefore best interpreted in terms of the inclusive value of participating in this after-market.[46]

These specifications estimated the preference parameters using the set of students who submitted a rank-order list consistent with optimal play (i.e. submitted a list corresponding to an extremal lottery). For the rational expectations model, a total of 2,071 students (97.3% of the sample) submitted a rationalizable list.[47] The large fraction of students with rationalizable lists may initially appear surprising. However, theorem A.1 in the appendix indicates that the lists that are not rationalized are likely the ones where assignment probabilities for one of the choices is zero. Our estimates in table VI suggest that this is rare, except for a few schools. Most of the students with lists that cannot be rationalized listed Graham & Parks

[46]Students who are assigned through the process can later enroll in other schools with open seats, approximately 91% of the students register at their assigned school. Some differences between assignments and registrations can be caused by changed in student preferences or the arrival of new information (Narita, 2016). Because the wait-list process in Cambridge allows students to choose a set of schools at which to apply, we explored whether this feature results in significant bias. Specifically, we estimated the probability that a student is able to ultimately register at a school where she was rejected during the main application process. Beliefs based on these probabilities resulted in quantitatively similar results to our baseline specifications. We therefore avoid modeling the after-market.

[47]One student was dropped because the recorded home address data could not be matched with a valid Cambridge street address.

as their second choice. Indeed, the reports can be rationalized as optimal if agents believe that there is a small but non-zero chance of assignment at these competitive schools. One concern with dropping students with lists that cannot be rationalized is that we are liable to underestimate the desirability of competitive schools. Although not reported, estimates that add a small probability of assignment to each of the ranked options yield very similar results.

7.2 Ranking Behavior, Out-of-Equilibrium Truthtelling and Assignment to Top Choice

In this section, we investigate the ranking strategy of agents, whether they would suffer large losses from out-of-equilibrium truth-telling, and how strategic manipulation may affect student welfare.

Table VIII presents the fraction of students who find truthful reporting optimal and losses from truthful behavior relative to optimal play as estimated using the two polar assumptions on student behavior and beliefs. The first three columns are based on the assumption that the observed reports are truthful and analyze the losses as a result of such naïvete. These estimates can be interpreted as analyzing the true loss to students from not behaving strategically if they are indeed out-of-equilibrium truth-tellers. The estimates suggest that the truthful report is optimal for 57% of the students. The average student suffers a loss equal to 0.18 miles by making a truthful report, or 0.42 miles conditional on regretting truthful behavior. We also estimate heterogeneous losses across student groups. Free-lunch students, for instance, suffer losses from truthful play less often and suffer lower losses conditional on any losses. This reflects the fact that the Cambridge school system is not competitive for these students because of the seats specifically reserved for this group.

The last three columns use estimates based on rational expectations and tabulate losses from non-strategic behavior.[48] Again, these estimates suggest that about half the students, and disproportionately paid-lunch students, have strategic incentives to manipulate their reports. Together, the observations suggest that markets where students face large competitive pressures are precisely the markets where treating preferences as truthful may lead to biased assessments of how desirable various schools are.

The estimated losses using both specifications may seem small on first glance, but can be explained by noting that whenever a student has a strong preference for a school, she will rank it as her first choice in her optimal report (and potentially manipulate lower-ranked

[48]These estimates differ from the ones based of truthful reporting only because of differences in preference parameters.

choices). The priority given to the first-ranked choice results in a low chance that the student is not assigned to this highly desired school. This fact significantly lowers the potential of large losses from truthful reporting.

Our estimates that about half the students find it optimal to behave truthfully is likely to affect our assessment of how many students are assigned to their top choice. Table IX presents this fraction by student paid-lunch status. The last column indicates that 85.2% of the students rank their top choice first. This occurs because many students avoid ranking competitive schools as their top rank in favor of increasing the odds of assignment to a less preferred option. As a result, fewer students rank Graham & Parks as their top choice, instead favoring Haggerty or Baldwin. We therefore see over-subscription to Haggerty and Baldwin by paid-lunch students relative to the true first choice. The last column indicates that while 83.4% were assigned to their stated first choice, only 72.3% were assigned to their true first choice. This pattern is particularly stark for paid-lunch students, who are assigned to their true first choice only 64.6% of the time. Table VI indicates that assignment to competitive schools is less likely for paid-lunch students. Together, these results suggest that calculations of whether students are assigned to their preferred options based on stated preferences may be misleading and differentially so by student demographics.

7.3 Evaluating Assignments under Alternative Mechanisms

A central question in the mechanism design literature is whether an Immediate Acceptance mechanism is worse for student welfare compared to strategy-proof mechanisms such as the Deferred Acceptance mechanism. This question has been debated in the theoretical literature with stylized assumptions on the preference distribution (see Miralles, 2009; Abdulkadiroglu et al., 2011; Featherstone and Niederle, 2016). The Immediate Acceptance mechanism exposes students to the possibility that they are not assigned to their top listed choices, which can harm welfare when they strategically choose not to report their most preferred schools. However, this possibility has a countervailing force that agents with particularly high valuations for their top choice will find it worthwhile listing competitive schools on top. Hence, the mechanism screens agents for cardinal preferences and can result in assignments with higher aggregate student welfare. Additionally, assignments under an Immediate Acceptance mechanism may be preferable under a utilitarian criterion because they need not eliminate justified envy (equivalently, they may not be stable). These are situations in which a student envies the assignment of another student even though the envied student has lower priority at that school.

Table X presents a quantitative comparison between the Cambridge mechanism and the Student Proposing Deferred Acceptance mechanism[49] using the preference estimates presented earlier. Because the Deferred Acceptance mechanism is strategy-proof, evaluating the counterfactual market with this mechanism is relatively straightforward because it does not involve hurdles in computing an equilibrium.[50]

An approach that treats agents' stated preferences as truthful finds that the average welfare is higher in the Deferred Acceptance mechanism. Panels A and B show that although Cambridge assigns more students to their top choice due to the additional priority awarded to students at schools that are ranked first, Deferred Acceptance does better at assigning students to less preferable options, including fourth and fifth choices. Recall that estimates from this specification indicated that many students prefer these options to remaining unassigned (see table VII). However, estimates assuming optimal behavior showed that the vast majority find less than three schools in Cambridge preferable to remaining unassigned. Therefore, the conclusion that Deferred Acceptance improves on average student welfare may be incorrect if strategic behavior is widespread.

In contrast to estimates assuming truthful behavior, results that treat agents as responding to strategic incentives indicate that the assignments produced by the Cambridge mechanism are preferable to those produced by the Deferred Acceptance mechanism. The fraction of students assigned to their true first choice remains higher under the Cambridge mechanism but, interestingly, the mechanism also places students at their true second choices with high probability if agents are strategic. This occurs because some students report their true second choice as their top choice. Indeed, panel C shows that more students prefer the Cambridge mechanism's assignments to the Deferred Acceptance mechanism's assignments than the other way around. Although the mechanism is effectively screening based on cardinal utilities, the average student prefers the assignments under the Cambridge mechanism by only an equivalent of 0.08 miles. The table also illustrates differences across student groups. Paid-lunch students prefer the Cambridge assignments more than free/reduced lunch students, perhaps due to strategic pressures. The estimated effects are of similar magnitude to the difference between Deferred Acceptance and Student Optimal Stable Matching (SOSM) as measured in New York City High Schools by Abdulkadiroglu et al. (2017a). Unlike SOSM,

[49] We construct a Deferred Acceptance mechanism by adapting the Cambridge Controlled Choice Plan. Schools consider students according to their priority + tie-breaking number. A paid-lunch student's application is held if the total number of applications in the paid-lunch category is less than the number of available seats and if the total number of applications at the school is less than the total number of seats. Free-lunch student applications are held in a similar manner. We allow students to rank all available choices.

[50] To evaluate the Cambridge mechanism, we use the beliefs estimated for each model to determine choices and compute outcomes using the estimated assignment probabilities. Alternatively, it may be possible to solve for the equilibria of manipulable Report-Specific Priority + Cutoff mechanisms because only equilibrium cutoffs need to be obtained. These cutoffs can then be used to compute assignment probabilities and beliefs.

however, the Cambridge mechanism need not result in a Pareto improvement relative to the Deferred Acceptance mechanism.

Specifications with biased beliefs indicate that the cardinal screening benefits of an Immediate Acceptance mechanism may be further diminished and instances of justified envy may be larger if beliefs are not well aligned with true assignment probabilities. In the models with biased beliefs, free-lunch students tend to prefer the assignment produced by the Deferred Acceptance mechanism relative to the one produced by the Cambridge mechanism. Further, the benefits to paid-lunch students are lower than the model that treats agents as having rational expectations. The significant aggregate benefits to free-lunch students under the Deferred Acceptance mechanism is driven, in part, by the large fraction of students assigned to their top two choices. Paid-lunch students continue to prefer assignments in the Cambridge mechanism to the strategy-proof counterpart.

Finally, a potential undesirable feature of the Cambridge mechanism is that it may result in instances of justified envy. Panel C shows that there are few instances of justified envy if agents have rational expectations, only about 2.5%, because students are often assigned at one of the top two choices. Under truthful reporting the estimated instances of justified envy is higher, just under 8%. The other specifications yielded intermediate results.

Our quantitative results contribute to the debate in the theoretical literature about the welfare properties of an Immediate Acceptance mechanism, which is similar to the Cambridge mechanism. The results are different in spirit from Ergin and Sonmez (2006), which suggests that full-information Nash equilibria of an Immediate Acceptance mechanism are Pareto inferior to outcomes under Deferred Acceptance. This difference stems from our focus on beliefs that account for ex-ante uncertainty faced by the students. Our results provide a quantitative counterpart to the theoretical claims in Abdulkadiroglu et al. (2011). They argue that an Immediate Acceptance mechanism can effectively screen for the intensity of preferences and can have better welfare properties than the Deferred Acceptance mechanism. Troyan (2012) shows that the theoretical results in this literature that are based on notions of interim efficiency are not robust to students having priorities, and he advocates for an ex-ante comparison such as the one performed in this paper.

Given the small benefits of the Cambridge mechanism, it is important to note that agents may face costs of strategizing because students may need to gather additional information about the competitiveness of various schools before formulating ranking strategies. These costs may weigh against using an Immediate Acceptance mechanism for school assignment. Additionally, there may be distributional consequences if agents vary in their ability to strategize (Pathak and Sonmez, 2008). While we cannot quantify the direct costs of strategizing and gathering information with our data, we extend our model to address distributional consequences of heterogeneous sophistication and biased beliefs in the next section.

7.4 Extension: Heterogeneous Agent Sophistication

The specifications presented above have modeled a homogeneous population of agents who make optimal reports given beliefs consistent with the data. However, agents may differ in their information about the competitiveness of various schools or may vary in their understanding of the mechanism. The difficulty in empirically analyzing extremely flexible models of heterogeneously sophisticated agents stems from the fact that a researcher has to disentangle heterogeneity in sophistication from preference heterogeneity while only observing the actions of the agents. Theorem A.1 in the appendix shows it is typically possible to rationalize any rank order list as optimal for some vector of utilities. Simultaneously identifying preferences and heterogeneity in sophistication will therefore require restricting behavioral rules and parametric assumptions.

We estimate a stylized model with heterogeneous agent sophistication based on Pathak and Sonmez (2008).[51] They theoretically compare the Deferred Acceptance mechanism to the Immediate Acceptance mechanism using a model with two types of agents: naïve and sophisticated. Naïve agents report their preferences sincerely by ranking the schools in order of their true preferences. Sophisticated agents, on the other hand, recognize that truthful reporting is not optimal because schools differ in the extent to which they are competitive and because of the details of the mechanism. Reports made by sophisticated agents are optimal given the reports of the other agents.

We model a population with a mixture of sophisticated and naïve agents who have the same distribution of preferences but differ in their behavior. Naifs report their preferences truthfully while sophisticated agents report optimally given their (correct) beliefs about the probability of assignment at each option given their report. The distribution of preferences is parametrized as in equation (2). In addition to parametric assumptions, the model embeds two strong restrictions. First, it is a mixture of two extreme forms of agent behavior: perfect sophistication and complete naïvete. Second, the distribution of preferences does not depend on whether the agent is sophisticated. These simplifications allow us to keep the estimation procedure tractable. Appendix **??** details the Gibbs' sampler for this model, which needed to be modified. We use estimates of the beliefs in the rational expectations model for the sophisticated agents.

Table XI presents the estimated mean utilities and the fraction of agents who are naïve. The estimated mean utilities are similar to the estimates in the other specifications, and

[51] See Calsamiglia et al. (2017) for another empirical model of agents that are heterogeneous in their sophistication.

usually in between the specifications treating agents as either truthful or fully sophisticated (table VII). Panel B shows that about a third of paid-lunch and free-lunch students are estimated to be naïve. These results contrast from estimates obtained by Calsamiglia et al. (2017) in Barcelona, where they estimate that over 94% of households are strategic in their decisions. One potential reason driving this difference is that the 93% of students in Barcelona are assigned to their top-ranked choice, while in Cambridge, this number is only 84%.

Table XII describes the differences between outcomes in the Cambridge and the Deferred Acceptance mechanisms. Because Deferred Acceptance is strategy-proof, both naïves and sophisticates report their preferences truthfully. Therefore, their outcomes are identical. The fractions of students assigned to their first, second, and third choices are similar to the results presented previously. We also see a similar overall increase in the fraction of students assigned to their top choice school in the Cambridge mechanism and a decrease in fractions assigned at lower-ranked choices. Interestingly, the probability of a student assigned to their top choice under the Cambridge mechanism is larger for naïve agents than for sophisticated agents even though they have identical preferences (78.4% vs 76.2%). This relatively larger probability of assignment at the top choice is at the cost of a significantly lower probability of assignment at the second choice, which is 6.5% for naifs and 12.3% for sophisticates. These differences are particularly stark for the paid-lunch students, who face a more demanding strategic environment. Our estimates suggest that, relative to sophisticates, naïve students effectively increase their chances of placement at their top choice school at the cost of losing out at less preferred choices.

These results can be explained by the difference between the propensity of naifs and sophisticates for ranking popular schools. While naïve students disregard a school's competitiveness, sophisticates are likely to avoid ranking competitive schools. Therefore, naifs effectively gain priority at their first choice school relative to sophisticated students with the same true first choice if the school is competitive. For example, Graham & Parks is estimated to be the top choice for 17.7% of students, but about a third of the sophisticated students for whom it is the top choice avoid ranking it first. Consequently, naive students are more likely to be assigned to Graham & Parks if it is their first choice. Qualitatively similar patterns hold for the other competitive schools such as Haggerty, Baldwin, and Morse. This increase in assignment probability at the top choice comes at the cost of a reduction in the probability of assignment at the second choice. For example, while 14.7% of sophisticated paid-lunch students are assigned to their second choice school, only 6.6% of naïve paid-lunch students get placed at their second choice. As Pathak and Sonmez (2008) pointed out, naïve students effectively "lose priority" at their second and lower choice schools to sophisticated students who rank the school first. It is therefore not surprising that the instances of justified envy are largest amongst naïve students, and particularly paid-lunch naifs. About 17% of paid-lunch naifs remain unassigned while about 6% of paid-lunch sophisticates are unassigned. Further, of the 27.6% paid-lunch naifs who are not assigned to their top choice, just under a third have justified envy for another student's assignment.

The aggregate welfare effects for naïve students therefore depend on whether the benefits of an increased likelihood of assignment at the top choice outweighs the lost priority at less preferred options. Although the naïve agents are making mistakes in the Cambridge mechanism, our comparison of assignments under the Deferred Acceptance mechanism to those under the Cambridge mechanism in panel B of table X shows that only 23.2% of the naïve paid-lunch students prefer the Deferred Acceptance mechanism to the Cambridge mechanism. This compares with 9.4% for paid-lunch sophisticates and less than 22% for free-lunch naifs and free-lunch sophisticates. Overall, we find that the average naïve student prefers assignments under the Cambridge mechanism by an equivalent of 0.024 miles. Because sophisticates are optimally responding to incentives in their environment, their estimated value for the assignments in the Cambridge mechanism is larger, at an equivalent of 0.094 miles.

8 Conclusion

We show that students in Cambridge respond to the strategic incentives in the mechanism. Specifically, students who reside on either side of the boundary where proximity priority changes have observably different ranking behavior. This finding weighs against the assumption that agents are ranking schools in order of true preferences if proximity priority is not a significant driver of residential choice.

Motivated by these results, we develop a general method for analyzing preferences from reports made to a single unit assignment mechanism that may not be truthfully implementable. The approach views the choice of report as an optimal choice from available assignment probabilities. We show that these probabilities can be consistently estimated for a broad class of school choice mechanisms, including the Immediate Acceptance and the Deferred Acceptance mechanisms. We consider models in which agents have biased beliefs in addition to a rational expectations model. Further, we characterize the identified set of preference distributions and propose a computationally tractable estimator.

Our empirical results indicate that treating preferences as truthful is likely to result in biased estimates in markets where students face stiff competition for their preferred schools. The stated preferences therefore exaggerate the fraction of students assigned to their true top choice. We then evaluate changes in the design of the market, where we find that the typical student prefers the Cambridge mechanism's assignment to the Deferred Acceptance mechanism's assignment by an equivalent of 0.08 miles. These losses are concentrated for the paid-lunch students, for whom the scarcity of seats at desirable programs results in the highest advantage from screening based on intensity of preferences. Free-lunch students, on the other hand, face a less complex strategic environment in the Cambridge mechanism, and the average student is close to indifferent between the two mechanisms.

Estimates from models in which agents have biased beliefs about assignment probabilities have a less optimistic view on the cardinal screening benefits of the Cambridge mechanism. A model with heterogeneously sophisticated agents finds that the Cambridge mechanism is preferable to naïve students because they gain priority at their top choice. Across specifications, we find relatively few instances of justified envy in the Cambridge mechanism due to the significant majority of students who are assigned to their top choice in this school district.

The relatively small welfare advantage of the Cambridge mechanism should be weighed against potential costs and distributional consequences of strategizing. Quantifying these effects may be difficult without directly observing differences in information acquisition activities across mechanisms. More broadly, our results motivate further research on mechanisms that use the intensity of student preferences in allocation more directly without some of the potential costs of strategic behavior.

Our methods can be extended to many other settings. Within school choice systems, the study of mechanisms that use finer priorities or exams is left for future research. Another important setting where agents make similar trade-offs is when they apply to college. A challenge in directly extending our approach is that colleges' admission decisions may be based on unobserved factors. Also closely related are multi-unit assignment mechanisms such as course allocation mechanisms. These settings, however, will require a richer space of preferences with complementarities over objects.

It is worth re-emphasizing that our approach to school assignment is predicated on two important assumptions that deserve further research. First, we assume that families do not make residential decisions in response to latent tastes for schools. Future research that investigates potential sources of residential sorting in unified school districts and jointly models residential and school choice would be valuable. Second, we infer beliefs based on observed ranking behavior and assume optimal responses. Further work that directly measures beliefs and ranking behavior in the field and develops appropriate models can help us better understand how agents interact with assignment mechanisms.

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Table I: School Choice Mechanisms

Mechanism	Manipulable	Examples
Immediate Acceptance	Y	Barcelona[1], Beijing[2], Boston (pre 2005), Charlotte-Mecklenberg[3], Chicago (pre 2009), Denver, Miami-Dade, Minneapolis, Seattle (pre 1999 and post 2009), Tampa-St. Petersburg.
Deferred Acceptance		
w/ Truncated Lists	Y	New York City[4], Ghanian Schools, various districts in England (since mid '00s)
w/ Unrestricted Lists	N	Boston (post 2005), Seattle (1999-2008), Amsterdam (post 2015)[5]
Serial Dictatorships		
w/ Truncated Lists	Y	Chicago (2009 onwards)
First Preferences First	Y	various districts in England (before mid '00s)
Chinese Parallel	Y	Shanghai and several other Chinese provinces[6]
Cambridge	Y	Cambridge[7]
Pan London Admissions	Y	London[8]
Top Trading Cycles		
w/ Truncated Lists	Y	New Orleans[9]
New Haven Mechanism	Y	New Haven[10]

Notes: Source Table 1, Pathak and Sonmez (2008) unless otherwise stated. See several references therein for details. Other sources: [1] Calsamiglia and Guell (2017); [2]He (2014); [3]Hastings et al. (2009); [4]Abdulkadiroglu et al. (2009); [5] de Haan et al. (2016); [6]Chen and Kesten (2013); [7] "Controlled Choice Plan" CPS, December 18, 2001; [8]Pennell et al. (2006); [9]`http://www.nola.com/education/index.ssf/2012/05/new_orleans_schools_say_new_pu.html` accessed May 20, 2014. [10]Kapor et al. (2017)

Table II: Cambridge Elementary Schools and Students

Year	2004	2005	2006	2007	2008	Average
		Panel A: District Characteristics				
Schools	13	13	13	13	13	13
Programs	24	25	25	27	27	25.6
Seats	473	456	476	508	438	470
Students	412	432	397	457	431	426
Free/Reduced Lunch	32%	38%	37%	29%	32%	34%
Paid Lunch	68%	62%	63%	71%	68%	66%
		Panel B: Student's Ethnicity				
White	47%	47%	45%	49%	49%	47%
Black	27%	22%	24%	22%	23%	24%
Asian	17%	18%	15%	13%	18%	16%
Hispanic	9%	11%	10%	9%	9%	10%
		Panel C: Language spoken at home				
English	72%	73%	73%	78%	81%	76%
Spanish	3%	4%	4%	4%	3%	3%
Portuguese	0%	1%	1%	1%	1%	1%
		Panel D: Distances (miles)				
Closest School	0.43	0.67	0.43	0.47	0.45	0.49
Average School	1.91	1.93	1.93	1.93	1.89	1.92

Notes: Students participating in the January Kindergarten Lottery. Free/Reduced lunch based on student's application for Federal lunch subsidy.

Table III: Cambridge Elementary Schools and Students

Year	2004	2005	2006	2007	2008	Average
	Panel A: Round of assignment					
First	81%	84%	85%	83%	75%	82%
Second	8%	3%	4%	7%	5%	5%
Third	5%	2%	2%	2%	4%	3%
Unassigned	6%	11%	9%	8%	16%	10%
	Panel B: Round of assignment: Paid Lunch Students					
First	80%	77%	78%	79%	68%	76%
Second	5%	4%	5%	8%	5%	5%
Third	6%	3%	4%	2%	3%	4%
Unassigned	9%	16%	14%	11%	24%	15%
	Panel C: Round of assignment: Free Lunch Students					
First	85%	95%	98%	94%	89%	92%
Second	14%	1%	2%	4%	6%	5%
Third	2%	1%	0%	1%	4%	1%
Unassigned	0%	4%	0%	2%	1%	1%
	Panel D: Number of Programs Ranked					
One	2%	6%	9%	5%	12%	7%
Two	5%	6%	9%	7%	7%	7%
Three	93%	89%	82%	88%	81%	87%
	Panel E: Students with Priority at Ranked Schools					
Sibling Priority at 1st Choice	38%	34%	32%	24%	34%	32%
Sibling Priority at 2nd Choice	4%	3%	1%	2%	2%	2%
Sibling Priority at 3rd Choice	0%	2%	1%	1%	0%	1%
Proximity at 1st Choice	53%	52%	50%	51%	52%	51%
Proximity at 2nd Choice	42%	34%	37%	33%	37%	36%
Proximity at 3rd Choice	22%	24%	24%	25%	21%	23%
	Panel F: Mean Distance (miles)					
Ranked first	1.19	1.18	1.24	1.29	1.19	1.22
All ranked schools	1.37	1.41	1.38	1.40	1.34	1.38
Assigned School	1.10	1.01	1.07	1.12	0.92	1.04

Notes: Sibling and proximity priority as reported in the Cambridge Public School assignment files. Students with older siblings enrolled in CPS receive priority at their sibling's school. Students also receive proximity priority at their two closest schools. Percentages, where reported, are based on the total number of applicants each year.

Table IV: School Popularity and Competitiveness

	Graham Parks	Haggerty	Baldwin	Morse	Amigos	Cambridgeport	King Open	Peabody	Tobin	Flet Mayn	Kenn Long	MLK	King Open Ola
						Panel A: All Students							
Ranked First	60	56	53	47	37	34	33	31	25	18	16	12	5
Ranked Second	72	37	66	25	18	44	39	38	17	10	18	20	0
Ranked Third	56	33	46	31	19	44	37	32	20	15	16	15	0
Ranked Anywhere	192	120	166	102	75	113	114	105	64	48	54	51	6
Capacity	41	41	41	42	41	27	51	48	35	38	41	37	15
First Rejected	1-P	1-R	1-R	1-R	1-R	1-R	NR	NR	1-R	NR	NR	NR	NR
						Panel B: Paid Lunch Students							
Ranked First	49	45	40	29	25	24	25	17	13	4	7	4	2
Ranked Second	60	28	56	14	12	29	23	27	10	3	6	6	0
Ranked Third	47	29	33	19	15	34	24	18	11	4	8	10	0
Ranked Anywhere	152	95	128	60	51	87	70	65	33	9	21	20	3
Capacity	29	27	27	29	41	18	36	34	29	35	34	27	15
First Rejected	1-P	1-R	1-R	1-R	1-R	1-R	NR	NR	3-R	NR	NR	NR	NR
						Panel C: Free Lunch Students							
Ranked First	9	12	12	17	12	11	13	10	12	16	10	9	2
Ranked Second	13	8	7	11	5	12	17	12	8	8	14	11	0
Ranked Third	10	4	9	10	4	12	13	13	9	10	11	4	0
Ranked Anywhere	29	24	25	40	20	36	44	38	31	36	34	25	2
Capacity	25	23	26	26	41	17	33	31	19	18	26	24	15
First Rejected	NR	NR	NR	1-R	1-R	2-P	NR	NR	1-R	NR	NR	NR	NR

Notes: Median number of applicants and seats over the years 2004-2008. First rejected is the round and priority of the first rejected student, e.g., 1-P indicates that a student with proximity priority was rejected in the first round. S: Sibling priority, PS: both proximity and sibling priority, R: regular/no prioirity, and NR: no student was rejected in any round. Free/Reduced lunch based on student's application for Federal lunch subsidy.

Table V: Regression Discontinuity Estimates

	Baseline			Competitive School	Non-Competitive School	Placebo Boundary
	Rank First	Rank Second	Rank Third	Rank First	Rank First	Rank First
			Panel A: All Students			
Estimate	-5.75%	-2.38%	-0.86%	-7.27%	-2.06%	0.07%
	(0.013)	(0.012)	(0.011)	(0.018)	(0.019)	(0.024)
t-statistic	-4.54	-2.02	-0.80	-3.96	-1.10	0.03
			Panel B: Paid Lunch Students			
Estimate	-7.44%	-2.65%	-0.68%	-11.07%	-1.22%	1.88%
	(0.016)	(0.014)	(0.015)	(0.025)	(0.018)	(0.031)
t-statistic	-4.64	-1.90	-0.46	-4.45	-0.67	0.61
			Panel C: Free Lunch Students			
Estimate	-3.55%	-2.59%	-3.15%	-1.47%	-5.23%	-3.55%
	(0.022)	(0.021)	(0.022)	(0.031)	(0.031)	(0.033)
t-statistic	-1.60	-1.22	-1.43	-0.47	-1.67	-1.06

Notes: Regression discontinuity estimates based bandwidth selection rule proposed by Imbens and Kalyaraman (2011). All estimates use rankings by 2,128 students. Competitive schools are Graham & Parks, Haggerty, Baldwin, Morse, Amigos, Cambridgeport and Tobin. Placebo boundary at the mid-point of the two-closest schools. Standard errors clustered at the student level in parenthesis.

Table VI: Estimated Assignment Probabilities

Ranked	Graham Parks	Haggerty	Baldwin	Morse	Amigos	Amigos Bilingual	Cambridgeport	King Open	Peabody	Tobin	Tobin K4	Tobin K5	Fletcher Maynard	Kenn Long	MLK	King Open Ola
Panel A: All Students																
First	0.43	0.59	0.63	0.57	0.73	0.98	0.60	1.00	0.94	0.85	0.31	0.34	0.92	1.00	1.00	1.00
Second	0.24	0.25	0.23	0.20	0.35	0.94	0.18	0.92	0.83	0.74	0.04	0.14	0.86	1.00	0.99	1.00
Third	0.21	0.19	0.18	0.10	0.25	0.83	0.10	0.67	0.61	0.66	0.02	0.08	0.77	0.90	0.90	0.89
Panel B: Paid Lunch																
First	0.22	0.45	0.49	0.54	0.73	1.00	0.51	1.00	0.94	0.93	0.32	0.36	1.00	1.00	1.00	1.00
Second	0.00	0.05	0.03	0.16	0.35	1.00	0.08	0.89	0.82	0.76	0.03	0.16	1.00	1.00	1.00	1.00
Third	0.00	0.01	0.00	0.06	0.24	0.85	0.01	0.56	0.56	0.64	0.01	0.09	0.89	0.89	0.89	0.87
Panel C: Free/Reduced Lunch																
First	0.82	0.87	0.90	0.64	0.74	0.97	0.77	1.00	0.94	0.72	0.31	0.29	0.76	1.00	1.00	1.00
Second	0.71	0.65	0.61	0.26	0.35	0.90	0.39	0.98	0.86	0.71	0.07	0.08	0.59	0.99	0.99	1.00
Third	0.62	0.56	0.52	0.18	0.27	0.83	0.28	0.87	0.70	0.68	0.03	0.05	0.53	0.92	0.92	0.94
Panel D: Proximity Priority																
First	0.63	0.97	0.95	0.89	0.95	0.99	0.94	1.00	1.00	0.92	0.55	0.54	0.95	1.00	1.00	1.00
Second	0.12	0.21	0.15	0.24	0.38	0.97	0.30	0.97	0.84	0.76	0.07	0.16	0.80	1.00	1.00	1.00
Third	0.10	0.11	0.06	0.12	0.28	0.84	0.16	0.76	0.66	0.67	0.02	0.12	0.72	0.89	0.90	0.88
Panel E: No Priority																
First	0.37	0.55	0.60	0.52	0.71	0.97	0.55	1.00	0.94	0.85	0.28	0.33	0.92	1.00	1.00	1.00
Second	0.28	0.25	0.23	0.19	0.34	0.96	0.17	0.92	0.83	0.75	0.03	0.14	0.88	1.00	1.00	1.00
Third	0.25	0.20	0.18	0.10	0.24	0.85	0.10	0.66	0.60	0.66	0.02	0.08	0.78	0.90	0.90	0.89

Note: Average estimates weighted by number of students of each type. Probabilities estimated using B=1,000. Ranks and priority types of opposing students are drawn with replacement from the observed data. Second and third rank assignment probabilities are conditional on no assignment to higher ranked choices, averaged across feasible rank order lists.

Table VII: Estimated Mean Utilities

	Truthful		Rational Expectations		Adaptive Expectations		Coarse Beliefs	
	Paid Lunch	Free Lunch	Paid Lunch	Free Lunch	Paid Lunch	Free Lunch	Paid Lunch	Free Lunch
Panel A: Mean Utility								
Graham Parks	1.29	0.40	1.93	-0.20	1.89	-0.19	1.85	0.47
	[0.06]	[0.08]	[0.27]	[0.33]	[0.36]	[0.52]	[0.12]	[0.16]
Haggerty	1.39	0.72	1.45	0.69	1.41	0.89	1.56	0.68
	[0.07]	[0.11]	[0.16]	[0.24]	[0.20]	[0.31]	[0.13]	[0.18]
Baldwin	1.26	0.50	1.36	0.70	1.05	-0.14	1.57	0.72
	[0.05]	[0.09]	[0.14]	[0.16]	[0.17]	[0.37]	[0.10]	[0.15]
Morse	0.66	0.70	0.77	0.97	0.61	0.78	0.82	0.8
	[0.07]	[0.08]	[0.12]	[0.17]	[0.18]	[0.33]	[0.14]	[0.16]
Amigos	-0.01	-0.38	0.13	-0.29	0.05	-0.43	-0.11	-0.64
	[0.13]	[0.15]	[0.18]	[0.27]	[0.22]	[0.32]	[0.19]	[0.27]
Cambridgeport	0.77	0.18	0.60	0.29	0.41	0.25	0.91	0.21
	[0.06]	[0.08]	[0.14]	[0.18]	[0.21]	[0.26]	[0.12]	[0.16]
King Open	0.65	0.40	0.58	0.27	0.44	0.37	0.52	0.24
	[0.06]	[0.07]	[0.10]	[0.13]	[0.13]	[0.18]	[0.10]	[0.12]
Peabody	0.22	0.48	0.06	0.30	0.03	0.52	0.05	0.31
	[0.08]	[0.09]	[0.12]	[0.15]	[0.16]	[0.20]	[0.11]	[0.15]
Tobin	-0.49	0.64	-0.92	0.38	-0.83	0.39	-0.74	0.28
	[0.11]	[0.12]	[0.21]	[0.26]	[0.25]	[0.30]	[0.18]	[0.21]
Fletcher Maynard	-1.30	-0.05	-2.03	-0.14	-1.67	0.12	-2.21	-0.3
	[0.14]	[0.10]	[0.42]	[0.23]	[0.32]	[0.22]	[0.30]	[0.18]
Kenn Long	-0.19	0.47	-0.40	0.11	-0.54	0.08	-0.51	0.25
	[0.09]	[0.07]	[0.20]	[0.17]	[0.27]	[0.21]	[0.18]	[0.15]
MLK	-0.66	0.08	-1.14	-0.28	-0.82	0.16	-1.24	-0.27
	[0.10]	[0.09]	[0.23]	[0.19]	[0.22]	[0.22]	[0.18]	[0.17]
King Open Ola	-3.60	-4.13	-2.39	-2.79	-2.01	-2.79	-2.47	-2.75
	[0.35]	[0.39]	[0.53]	[0.67]	[0.62]	[0.93]	[0.47]	[0.63]
Outside Option	-2.08	-1.44	-0.55	-0.92	-0.64	-0.74	-0.49	-0.73
	[0.10]	[0.09]	[0.08]	[0.08]	[0.09]	[0.13]	[0.06]	[0.07]
Panel B: Percentage of Acceptable Schools								
up to 1	10%	20%	24%	14%	23%	19%	16%	21%
up to 2	16%	30%	61%	40%	61%	45%	58%	51%
up to 3	23%	40%	85%	62%	86%	69%	84%	74%
up to 4	34%	51%	95%	79%	95%	85%	95%	89%
up to 5	44%	61%	98%	91%	99%	95%	99%	96%

Notes: Average estimated utility for each school, normalizing the mean utility of the inside options to zero. Utilities calculated by averaging the predicted utility given the non-distance covariates. Bootstrap standard errors in brackets, except for Truthful reporting where we present the standard deviation of the posterior. Adaptive Expectations based on reported lists from 2005, 2006 and 2008 with assignment probabilites estimated using data from 2004, 2005 and 2007 respectively. This specification drops data from 2007 in preference estimates since Tobin split by entering age in that year. The fraction of students with rationalizable lists is 97.3%, 96.8%, and 99.3% for the Rational Expectations, Adaptive Expectations and Coarse Beliefs specifications respectively.

Table VIII: Losses from Truthful Reports

	Truthful						Rational Expectations					
	No Loss		Mean Loss		Std Loss		No Loss		Mean Loss		Std Loss	
	mean	s.e.	mean	s.e.	mean	s.e.	mean	s.e.	mean	s.e.	mean	s.e.
All	57%	0.01	0.18	0.02	0.53	0.05	46%	0.01	0.07	0.01	0.26	0.03
Free Lunch	68%	0.02	0.01	0.00	0.09	0.03	62%	0.02	0.01	0.00	0.07	0.03
Paid Lunch	51%	0.01	0.26	0.03	0.64	0.06	38%	0.01	0.10	0.01	0.31	0.04
Black	65%	0.02	0.06	0.02	0.30	0.07	56%	0.02	0.04	0.01	0.19	0.05
Asian	56%	0.03	0.19	0.04	0.56	0.09	46%	0.03	0.07	0.02	0.25	0.06
Hispanic	60%	0.04	0.10	0.03	0.36	0.09	51%	0.03	0.04	0.01	0.18	0.06
White	52%	0.01	0.24	0.03	0.62	0.06	40%	0.02	0.09	0.01	0.30	0.04
Other Race	47%	0.06	0.20	0.07	0.51	0.15	39%	0.05	0.08	0.04	0.24	0.09

Notes: Estimated loss from reporting preferences truthfully, relative to optimal report in distance units (miles).

Table IX: Ranking and Assignment of Top Choice

	Graham Parks	Haggerty	Baldwin	Morse	Amigos	Cambridgeport	King Open	Peabody	Tobin	Fletcher Maynard	Kenn Long	MLK	King Open Ola	Total
Panel A: All Students														
Preferred School	24.8	11.6	9.4	10.0	7.9	6.1	7.2	6.1	5.4	4.0	3.6	2.2	1.0	99.3
Ranked #1 (simul)	16.2	12.6	11.6	11.2	8.4	8.0	8.6	7.0	4.9	3.9	3.6	2.3	1.0	99.3
Ranked #1 (data)	14.3	12.6	11.9	11.0	8.8	7.7	8.2	7.8	5.7	4.4	3.8	2.7	1.2	100.0
Preferred and Ranked #1	15.3	10.4	8.2	9.6	7.5	5.8	7.2	6.1	4.6	3.9	3.6	2.2	1.0	85.2
Preferred and Assigned	9.9	8.5	6.6	8.2	6.8	5.1	7.2	6.0	3.8	3.5	3.6	2.2	1.0	72.3
Ranked #1 and Assigned	10.4	10.1	9.0	9.5	7.6	6.8	8.6	6.9	4.2	3.6	3.6	2.3	1.0	83.4
Panel B: Free Lunch Students														
Preferred School	9.0	8.1	6.5	12.5	6.9	6.8	7.2	7.9	10.7	10.8	7.1	4.6	1.6	99.5
Ranked #1 (simul)	8.9	8.5	6.8	12.7	6.9	7.1	7.2	8.2	9.0	10.6	7.2	4.7	1.6	99.5
Ranked #1 (data)	6.7	8.3	8.0	12.2	7.8	6.4	7.7	9.0	8.5	10.8	7.1	5.5	2.0	100.0
Preferred and Ranked #1	8.6	8.0	6.4	12.2	6.7	6.6	7.2	7.8	8.9	10.3	7.1	4.6	1.6	96.1
Preferred and Assigned	7.9	7.5	6.0	10.7	6.0	6.1	7.2	7.8	7.1	9.3	7.1	4.6	1.6	88.8
Ranked #1 and Assigned	8.3	7.9	6.4	11.1	6.2	6.5	7.2	8.1	7.2	9.5	7.2	4.7	1.6	91.9
Panel C: Paid Lunch Students														
Preferred School	32.6	13.3	10.9	8.7	8.6	5.7	7.3	5.3	2.7	0.6	1.8	1.1	0.7	99.3
Ranked #1 (simul)	20.0	14.6	13.9	10.4	9.3	8.3	9.3	6.5	2.9	0.6	1.8	1.1	0.7	99.3
Ranked #1 (data)	18.8	14.6	13.4	10.6	9.5	7.9	8.7	7.2	3.7	1.2	2.2	1.3	0.9	100.0
Preferred and Ranked #1	18.9	11.5	9.2	8.2	8.1	5.3	7.3	5.3	2.4	0.6	1.8	1.1	0.7	80.2
Preferred and Assigned	11.0	8.9	7.0	6.9	7.3	4.6	7.3	5.2	2.2	0.6	1.8	1.1	0.7	64.6
Ranked #1 and Assigned	11.7	11.1	10.2	8.6	8.3	6.9	9.3	6.4	2.7	0.6	1.8	1.1	0.7	79.4

Notes: Unless otherwise noted, table presents averages over 1,000 simulations from the posterior mean of the parameters estimated from the rational expectations model.

Table X: Deferred Acceptance vs Cambridge

	Truthful			Rational Expectations			Coarse Beliefs			Adaptive Expectations		
	All Students	Paid Lunch	Free Lunch	All Students	Paid Lunch	Free Lunch	All Students	Paid Lunch	Free Lunch	All Students	Paid Lunch	Free Lunch
Panel A: Deferred Acceptance												
Percent Assigned to First Choice	71.0	64.0	84.8	67.8	58.4	86.4	70.4	62.0	86.9	68.9	58.0	88.4
Percent Assigned to Second Choice	12.0	13.6	8.7	15.8	18.7	10.0	12.3	14.1	8.8	13.8	17.3	7.5
Percent Assigned to Third Choice	5.1	7.0	1.4	5.2	7.1	1.5	4.9	6.8	1.1	5.1	7.2	1.3
Percent Assigned to Fourth Choice	3.0	4.4	0.3	1.3	1.9	0.2	1.8	2.6	0.1	1.5	2.3	0.2
Percent Assigned to Fifth Choice	1.7	2.5	0.1	0.2	0.3	0.0	0.4	0.6	0.0	0.2	0.4	0.0
Panel B: Cambridge Mechanism												
Percent Assigned to First Choice	78.9	74.5	87.7	72.3	63.9	88.8	73.9	67.3	86.9	72.2	63.0	88.9
Percent Assigned to Second Choice	6.5	6.8	5.9	14.7	18.1	7.9	10.2	11.1	8.3	11.9	15.1	6.1
Percent Assigned to Third Choice	3.1	4.0	1.3	3.9	5.1	1.3	3.5	4.6	1.5	3.6	4.7	1.5
Percent Assigned to Fourth Choice	0.0	0.0	0.0	1.0	1.4	0.3	1.5	2.1	0.3	1.2	1.7	0.3
Percent Assigned to Fifth Choice	0.0	0.0	0.0	0.2	0.2	0.0	0.4	0.5	0.0	0.2	0.3	0.1
Panel C: Deferred Acceptance vs Cambridge												
Mean Utility DA - Cambridge	0.065	0.102	-0.008	-0.078	-0.107	-0.021	-0.035	-0.052	-0.001	-0.032	-0.069	0.033
	(0.019)	(0.027)	(0.008)	(0.009)	(0.012)	(0.009)	(0.009)	(0.012)	(0.011)	(0.025)	(0.033)	(0.032)
Std. Utility DA - Cambridge	0.149	0.168	0.044	0.109	0.120	0.046	0.095	0.104	0.061	0.178	0.124	0.233
Percent DA > Cambridge	26.1	28.0	22.2	17.3	15.6	20.6	25.2	24.6	26.3	25.0	23.4	27.8
	(1.6)	(1.9)	(2.4)	(1.8)	(2.2)	(2.4)	(1.5)	(1.8)	(2.2)	(2.7)	(3.5)	(3.4)
Percent DA ≈ Cambridge	32.4	27.4	42.4	31.2	28.0	37.5	32.1	28.2	39.7	31.6	26.3	41.0
	(1.2)	(1.5)	(2.0)	(1.2)	(1.5)	(2.0)	(1.2)	(1.4)	(1.9)	(1.7)	(1.9)	(2.8)
Percent DA < Cambridge	41.5	44.6	35.4	51.5	56.4	41.9	42.8	47.2	34.0	43.5	50.3	31.2
	(1.8)	(2.1)	(2.5)	(2.0)	(2.4)	(2.7)	(1.7)	(2.0)	(2.4)	(2.8)	(3.7)	(3.5)
Percent with Justified Envy	7.7	10.4	2.4	2.5	2.7	2.3	5.0	5.8	3.4	3.8	4.2	2.9
	(0.5)	(0.7)	(0.4)	(0.2)	(0.3)	(0.4)	(0.3)	(0.4)	(0.4)	(0.7)	(1.0)	(0.6)

Notes: Panels A and B present percentages of students assigned to true k-th choice. Panel C compares the expected utility difference between Deferred Acceptance and Cambridge Mechanism. Simulations of the Deferred Acceptance mechanism draw other student reports using the estimated utility distribution. We say DA ≈ Cambridge if the expected utility is within 10^{-5} miles. Point estimates use the estimated parameters and 1,000 simulations of the mechanisms. Bootstrap standard errors using 250 draws of the parameters in parentheses, except for Truthful reporting, where we use the 1,000 draws from the posterior distribution.

Table XI: Estimated Mean Utilities using a Mixture Model

	Mixture Model		
	All Students	Paid Lunch	Free Lunch
	Panel A: Mean Utility		
Graham Parks	1.19	1.53	0.52
	[0.11]	[0.12]	[0.15]
Haggerty	1.27	1.53	0.76
	[0.14]	[0.13]	[0.22]
Baldwin	1.25	1.45	0.84
	[0.10]	[0.10]	[0.13]
Morse	0.74	0.68	0.86
	[0.11]	[0.11]	[0.13]
Amigos	-0.12	0.00	-0.38
	[0.21]	[0.19]	[0.30]
Cambridgeport	0.56	0.68	0.31
	[0.11]	[0.11]	[0.15]
King Open	0.48	0.57	0.32
	[0.09]	[0.10]	[0.12]
Peabody	0.08	0.03	0.16
	[0.13]	[0.13]	[0.16]
Tobin	-0.45	-0.81	0.26
	[0.16]	[0.18]	[0.23]
Flet Mayn	-1.02	-1.52	-0.04
	[0.24]	[0.30]	[0.18]
Kenn Long	-0.07	-0.23	0.24
	[0.14]	[0.16]	[0.15]
MLK	-0.68	-0.97	-0.10
	[0.14]	[0.17]	[0.15]
King Open Ola	-3.23	-2.96	-3.77
	[0.40]	[0.43]	[0.46]
Outside Option	-1.11	-1.03	-1.26
	[0.09]	[0.09]	[0.11]
	Panel B: Agent Behavior		
Fraction Naïve		0.378	0.316
		[0.0079]	[0.0079]

Notes: Panel A presents the average estimated utility for each school, normalizing the mean utility of the inside options to zero. Utilities calculated by averaging the predicted utility given the non-distance covariates. Panel B reports the estimated fraction of naive agents by free-lunch status. Bootstrap standard errors in brackets.

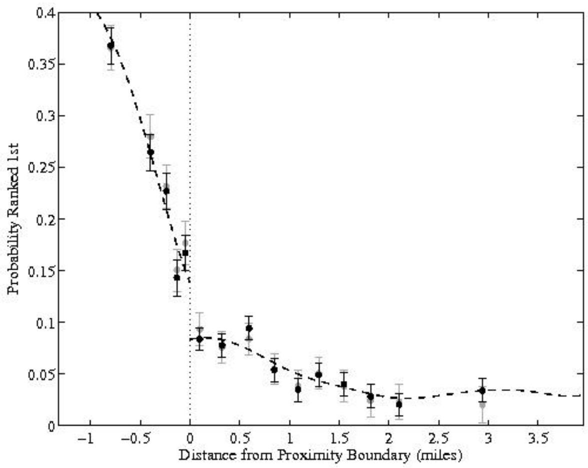
Table XII: Deferred Acceptance vs Cambridge using a Mixture Model

	All Students		Paid Lunch		Free Lunch	
			Panel A: Deferred Acceptance			
Percent Assigned to First Choice	70.0		61.8		86.3	
Percent Assigned to Second Choice	13.1		14.5		10.3	
Percent Assigned to Third Choice	5.8		7.8		1.7	
Percent Assigned to Fourth Choice	2.8		4.0		0.3	
Percent Assigned to Fifth Choice	0.9		1.4		0.1	
			Panel B: Cambridge Mechanism			
	Naïve	Sophisticated	Naïve	Sophisticated	Naïve	Sophisticated
Percent of Students	35.7	64.3	37.8	62.2	31.6	68.4
Percent Assigned to First Choice	78.4	76.2	72.4	69.5	90.2	89.6
Percent Assigned to Second Choice	6.5	12.3	6.6	14.7	6.3	7.6
Percent Assigned to Third Choice	3.3	4.3	4.1	5.9	1.7	1.3
Percent Assigned to Fourth Choice	0.0	1.8	0.0	2.6	0.0	0.3
Percent Assigned to Fifth Choice	0.0	0.5	0.0	0.8	0.0	0.1

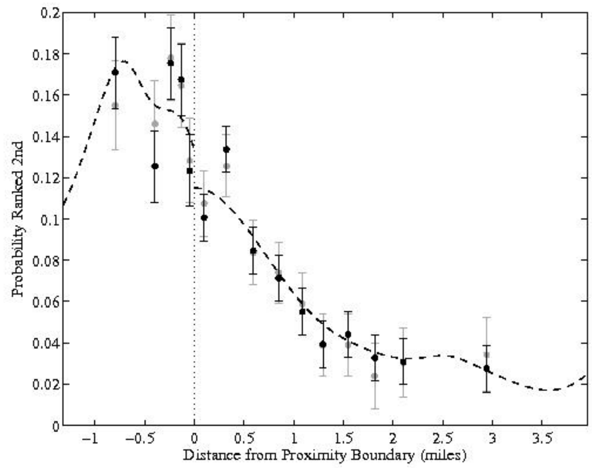
Panel C: Deferred Acceptance vs Cambridge	Naïve	Sophisticated	Naïve	Sophisticated	Naïve	Sophisticated
Mean Utility DA - Cambridge	-0.024	-0.094	-0.029	-0.13	-0.014	-0.023
	(0.010)	(0.010)	(0.014)	(0.013)	(0.008)	(0.007)
Std. Utility DA - Cambridge	0.096	0.125	0.112	0.137	0.047	0.046
Percent DA > Cambridge	22.5	12.8	23.2	9.4	21.0	19.4
	(1.6)	(1.6)	(2.0)	(1.8)	(2.2)	(2.3)
Percent DA Cambridge	30.4	30.6	26.6	26.7	38.1	38.2
	(1.2)	(1.2)	(1.4)	(1.4)	(2.0)	(2.0)
Percent DA < Cambridge	47.1	56.7	50.2	63.9	40.9	42.4
	(1.6)	(1.6)	(1.9)	(1.9)	(2.5)	(2.6)
Percent with Justified Envy	6.4	2.5	8.3	2.7	2.6	2.3
	(0.4)	(0.2)	(0.6)	(0.3)	(0.4)	(0.4)

Notes: Panels A and B present percentages of students assigned to true k-th choice. Panel C compares the expected utility difference between Deferred Acceptance and Cambridge Mechanism. Simulations of the Deferred Acceptance mechanism draw other student reports using the estimated utility distribution. We say DA ≈ Cambridge if the expected utility is within 10^{-5} miles. Bootstrap standard errors in brackets.

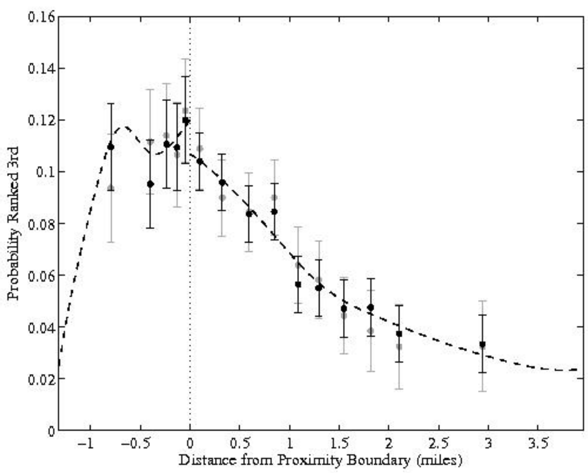
Figure I: Effect of Proximity Priority on Ranking Behavior



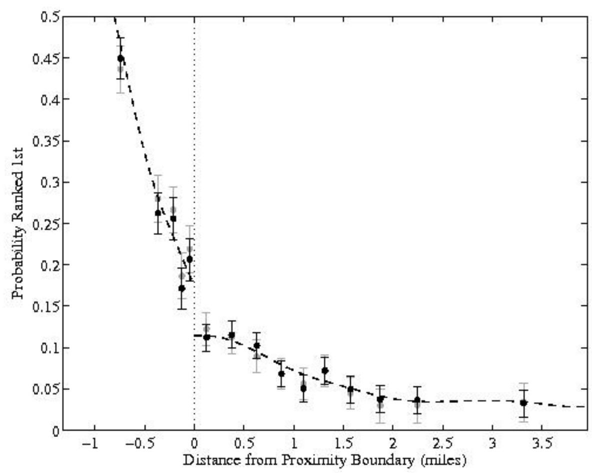
(i) First Rank



(ii) Second Rank



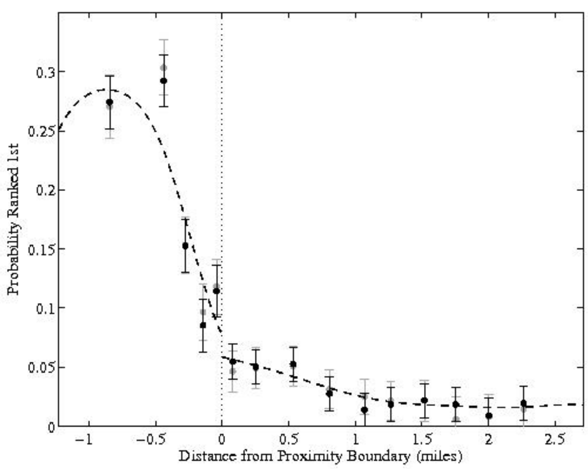
(iii) Third Rank



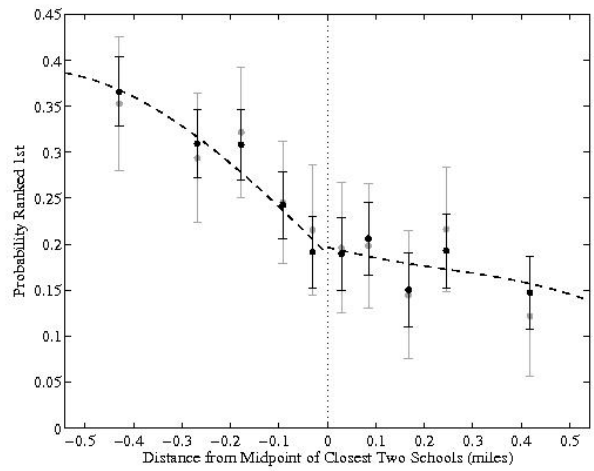
(iv) Competitive schools

Cont'd...

Figure I: Effect of Proximity Priority on Ranking Behavior (cont'd)



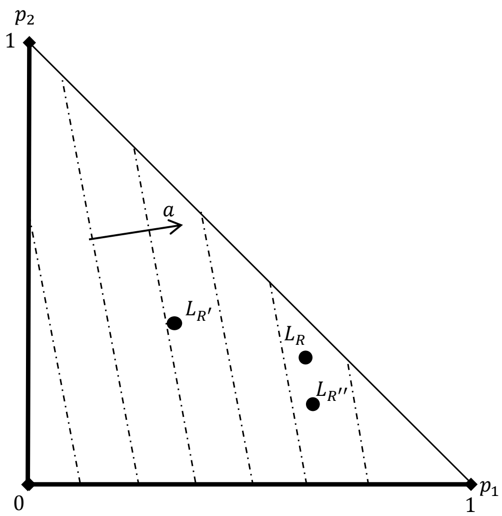
(v) Non-competitive schools



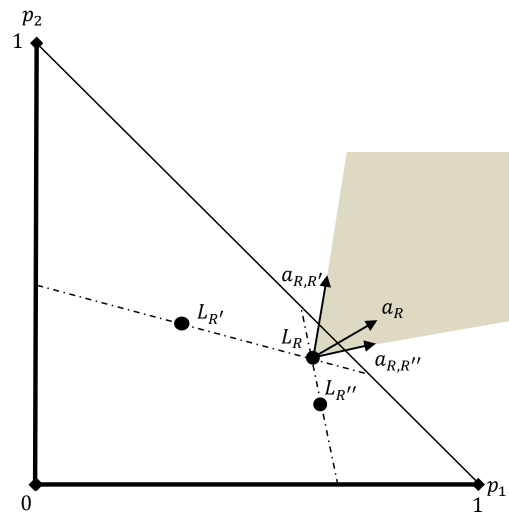
(vi) Placebo at prioritized schools

Notes: The graphs are bin-scatter plots (based on distance) with equally sized bins on either side of the boundary. For each student, we construct a boundary distance, $\bar{d}_i$, based on her distance to the schooling options. For a given school-student pair, the horizontal axis represents $d_{ij} - \bar{d}_i$. The vertical axis is the probability that a student ranks the school in the relevant distance bin. Range plots depict 95% confidence intervals. Black plot points are based on the raw data, while the grey points control for school fixed effects. Dashed lines represent local linear fits estimated on either side of the boundary based on bandwidth selection rules recommended in Bowman and Azzalini (1997) (page 50). Panels (a) through (e) use the average distance between the second and third closest schools as the boundary. A student is given proximity priority at the schools to the left of the boundary and does not receive priority at schools to the right. Competitive schools considered in panel (d) are Graham & Parks, Haggerty, Baldwin, Morse, Amigos, Cambridgeport and Tobin. The remaining schools are considered non-competitive in panel (e). Panel (f) considers only the two closest schools and uses the average distance between the closest and second closest schools. Only the two schools where students have proximity priority are considered. Panels (a), (d), (e) and (f) plot the probability that a school is ranked first. Panels (b) and (c) plot the probability that a school is ranked second and third respectively. Distances as calculated using ArcGIS. Graphs are qualitatively similar when using only students with consistent calculated and recorded priorities. Details in data appendix.

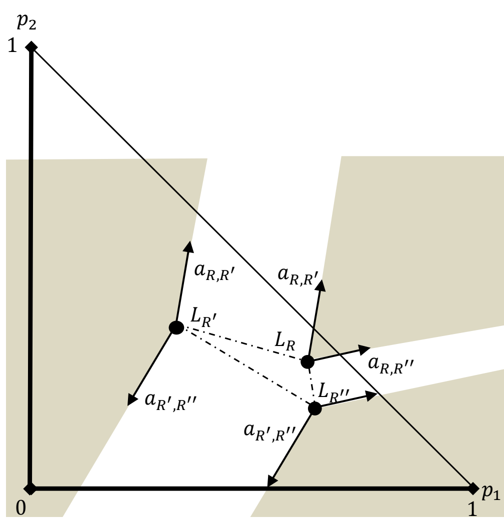
Figure II: A Revealed Preference Argument



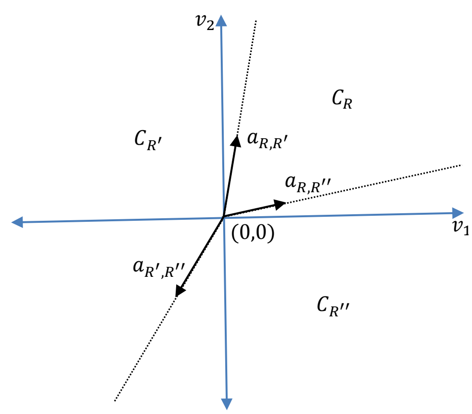
(i) Indifference curves for utility vectors parallel to a, and choices over three lotteries



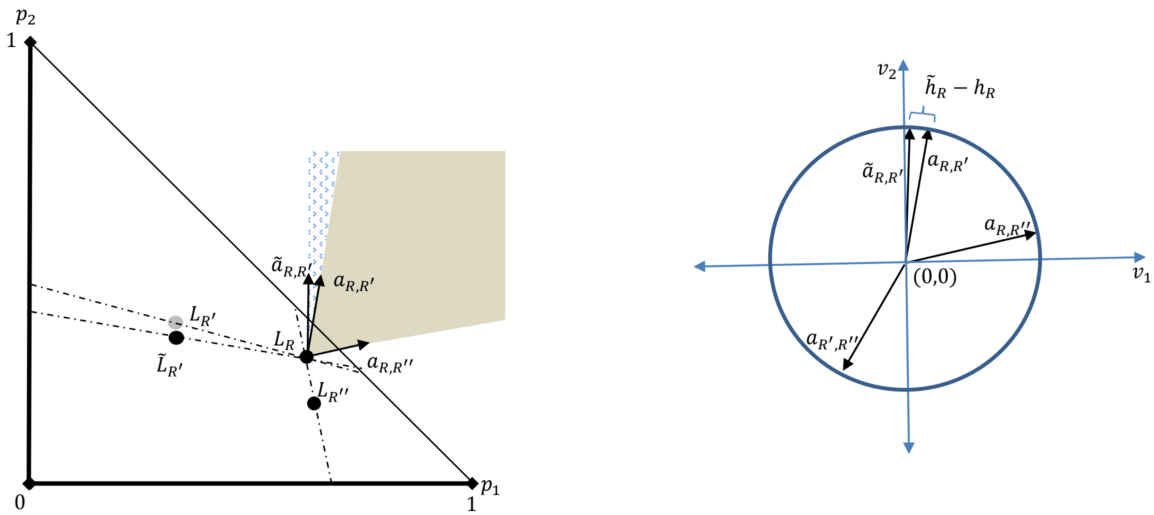
(ii) L_R is optimal for a_R if $a_R = \alpha_1 a_{R,R'} + \alpha_2 a_{R',R''}$ for $\alpha_1, \alpha_2 > 0$



(iii) Normal cones for each lottery



(iv) Lottery choice reveals utility region



(i) Variation in the assignment probabilities induces variation in the normal cones

(ii) Allows identifying the density of utility distribution on the unit circe

Figure III: Variation in Lotteries

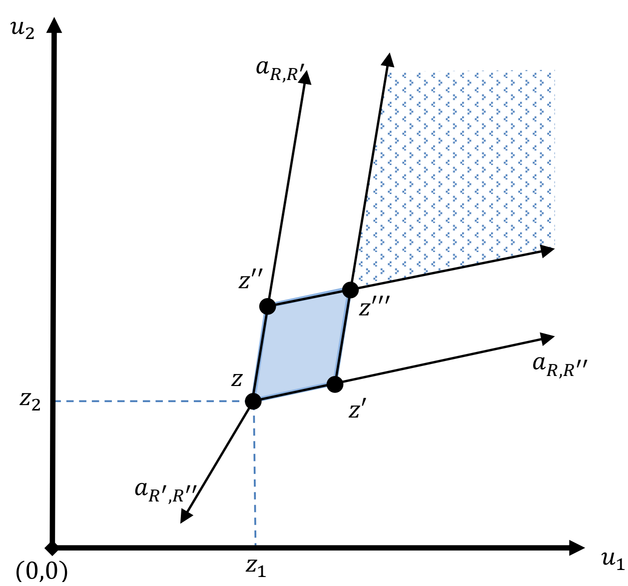


Figure IV: Local variation in z identifies the density of u

A Appendix

This appendix provides the main results for the paper. Appendices **??** to **??** appear online and contain additional results and preliminaries cited here.

A.1 Testable Restrictions of Optimal Behavior

Our empirical methods are based on the assumption that agent behavior is optimal. Therefore, if agents maximize their utility, they must pick lotteries that are extremal in the set of lotteries with probability one because ties in expected utilities are non-generic:

Proposition A.1. *Let the distribution of indirect utilities admit a density. If L_R is not an extreme point of the convex hull of $\mathcal{L}$, the set of utilities v such that $v \cdot L_R \geq v \cdot L_{R'}$ for all $L_{R'} \in \mathcal{L}$ has measure zero.*

Proof. If L_R is not an extreme point of the convex hull of $\mathcal{L}$, then $C_R = \{v \in \mathbb{R}^J : \forall L_{R'} \in \mathcal{L}, v \cdot (L_R - L'_R) \geq 0\}$ has Lebesgue-measure zero. Since v admits a density, $\int 1\{v \in C_R\} \mathrm{d}F_V = 0$. □

The result also indicates that the fraction of students with behavior that is not consistent with optimal play can be identified. This suggests that the assumption that agents behave optimally is testable. However, as we will show below, we should expect that observed behavior can be rationalized as optimal in most assignment mechanisms.

Consider a mechanism in which reports correspond to rank-orders over the available options. Therefore, a report is a function $R : \{1, \ldots, K\} \to \mathcal{J}$ such that (i) for all $k, k' \in \{1, \ldots, K\}$, $R(k) = R(k') \neq 0 \Rightarrow k = k'$ and (ii) $R(k) = 0 \implies R(k') = 0$ if $k' > k$. Let $\mathcal{R}$ be the space of such functions. Therefore, R is a (partial) rank-order list and $R(k)$ denotes the identity of the k-th ranked school. As discussed earlier, the mechanism produces lotteries $L_{R,t}$ for each report submitted by an agent with priority type t. Let $L_{R,j}$ be the probability that a student with priority type t is assigned to program j when submitting R, where we suppress the dependence on t for notational simplicity.

Definition 5. *The set of lotteries $\mathcal{L} = \{L_R \in \Delta^J : R \in \mathcal{R}\}$ is **rank-monotonic** for priority type t, if for all $R, R' \in \mathcal{R}$, $R_{-i} \in \mathcal{R}_{-i}$ and $k \leq K$ we have that $(R(1), \ldots, R(k-1)) = (R'(1), \ldots, R'(k-1))$ implies*

$$L_{R,R(k)} \geq L_{R',R(k)}.$$

*Further, $\mathcal{L}_t$ is **strictly rank-monotonic** for priority-type t if the inequality above is strict if $R(k) \neq R'(k)$, and $L_{R,R(k)} > 0$*

Rank-monotonicity is a natural condition that should be satisfied by many single-unit assignment mechanisms. Specifically, it requires that the assignment probability at the k-th ranked school does not depend on schools ranked below it, and that ranking a school higher weakly increases a student's chances of getting assigned to it. Under strict rank-monotonicity, ranking a school higher strictly increases the assignment probability unless this probability is zero.

We now show that in all strictly rank-monotonic ranking mechanisms, all agents that pick a report that gives them a positive probability of assignment at each of their options are behaving in a manner consistent with equilibrium play.[52]

Theorem A.1. *Assume that $\mathcal{L}$ is strictly rank-monotonic. The report $R \in \mathcal{R}$ corresponds to an extremal lottery $L_R \in \mathcal{L}$ if $L_{R,R(k)} > 0$ for all k such that $\sum_{k'<k} L_{R,R(k')} < 1$.*

Proof. Consider a report $R \in \mathcal{R}$ such that for any $k = 1, 2, .., K$, $\sum_{k'<k} L_{R,R(k')} < 1$ and $L_{R,R(k)} > 0$. Take any vector of coefficients λ such that:

$$\begin{aligned} \lambda_{\tilde{R}} &\geq 0 \text{ for every } \tilde{R} \in \mathcal{R} \\ \sum_{\tilde{R} \in \mathcal{R}} \lambda_{\tilde{R}} &= 1 \\ \sum_{\tilde{R} \in \mathcal{R}} \lambda_{\tilde{R}} L_{\tilde{R}} &= L_R \end{aligned}$$

We will show that $\lambda_R = 1$. The proof follows by induction. Consider some report $\tilde{R}$ where $R(1) \neq \tilde{R}(1)$. Strict rank-monotonicity and our assumption on R imply $\lambda_{\tilde{R}} = 0$. We have shown that for $k = 1$, $R(k') \neq \tilde{R}(k')$ for any $k' \leq k \implies \lambda_{\tilde{R}} = 0$. Suppose that this statement is true for all $l \leq k-1$ and that $\sum_{l<k} L_{R,R(l)} < 1$. Take any report $\tilde{R}$ where $R(l) \neq \tilde{R}(l)$ for some $l \leq k$. If $l < k$, $\lambda_{\tilde{R}} = 0$ by the inductive hypothesis. If $l = k$, Strict rank-monotonicity and our assumption on R imply $\lambda_{\tilde{R}} = 0$. By induction, $R(l) \neq \tilde{R}(l)$ and $\sum_{l<k} L_{R,R(l)} < 1 \implies \lambda_{\tilde{R}} = 0$.

Suppose that there is a $j \in S$ and $\tilde{R} \in \mathcal{R}$ such that $L_{R,j} \neq L_{\tilde{R},j}$; we will show that $\lambda_{\tilde{R}} = 0$. Let $\tilde{k}$ be the minimum k such that $R(k) \neq \tilde{R}(k)$. Rank-monotonicity and the fact that either $L_{R,j} > 0$ or $L_{\tilde{R},j} > 0$ imply that

$$\sum_{l<\tilde{k}} L_{R(l),\tilde{R}} = \sum_{l<\tilde{k}} L_{R,R(l)} < 1.$$

[52]Strict-rank monotonicity does not rule out that two different reports result in the same lottery, e.g., if $R_1 = (A, B, C)$ and $R_2 = (A, B, D)$ both result in assignment probabilities for A,B,C and D equal to $[\phi_A, 1 - \phi_A, 0, 0]$.

Thus, our previous results imply that $\lambda_{\tilde{R}} = 0$.

$\square$

The result implies that every report with non-zero assignment probabilities is rationalizable as an optimal report for a priority type if the mechanism is strictly rank-monotonic. Intuitively, this is the case because upgrading any school in the reported rank-order list strictly increases the probability of assignment and there exists a utility vector for which such a report is optimal.

Although optimal play is testable predictions, we do not develop a statistical test for the null hypothesis that play is consistent with optimal behavior. The technical challenge arises because failing to reject that the probability of ranking a sub-optimal report is zero is not enough. Rejecting the null of optimal behavior amounts to showing that the probability is indeed equal to zero.

A.2 Identification with Non-Simplicial Cones

In this section, we consider identification for the case when the cone C_R is not spanned by linearly independent vectors. We need that there exists a report for which the normal cone satisfies the following property:

Definition 6. *A cone C is **salient** if $v \in C \implies -v \notin C$ for all $v \neq 0$.*

Our results require that the tails of the distribution of utilities are light. Formally, assume that for some $c > 0$, the density of u belongs to the set

$$\mathcal{G}_c \equiv \{g \in \mathbb{L}^1(\mathbb{R}^J) : e^{c|u|} g(u) \in \mathbb{L}^1(\mathbb{R}^J)\},$$

where $\mathbb{L}^1$ is the space of Lebesgue integrable functions.

Theorem A.2. *Assume that $g \in \mathcal{G}_c$ and there is a lottery L_R such that C_R is a salient convex cone with a non-empty interior. If $\zeta = \mathbb{R}^J$, then the distribution of utilities $F_V(v|z^1)$ is identified from*

$$h_{C_R}\left(z^1\right) = P(L_R \in \mathcal{L}|z^1).$$

Proof. For a fixed lottery L_R such that C_R is salient, define the linear operator A:

$$Ag(z) = \int_{C_R} g\left(v + z\right) \mathrm{d}v.$$

We need to show that if $A(g' - g'') = 0$ a.e. then $g' - g'' = 0$ a.e. The proof is by contradiction.

Since the cone C_R is salient, its dual T_R has a nonempty interior. Let $\varepsilon \in \text{int}(T_R)$, with $|\varepsilon|$ sufficiently small so that $g_\varepsilon(u) = g(u)e^{2\pi\langle\varepsilon,u\rangle} \in \mathbb{L}^1$. Note that $1\{u \in C_R\}e^{-2\pi\langle\varepsilon,u\rangle} \in \mathbb{L}^1$ for every $\varepsilon \in \text{int}(T_R)$ because $\langle\varepsilon, u\rangle > 0$.

Towards a contradiction, suppose that $A(g' - g'') = 0$ a.e. but $|g' - g''|_1 > 0$. Since $\zeta = \mathbb{R}^J$, we have that for almost all $z \in \mathbb{R}^J$,

$$Ag(z) \quad = \quad e^{-2\pi\langle\varepsilon,z\rangle}\int 1\{v \in C_R\}\, e^{-2\pi\langle\varepsilon,v\rangle}e^{2\pi\langle\varepsilon,v+z\rangle}g(v+z)dv = 0.$$

Since $e^{-2\pi\langle\varepsilon,z\rangle} > 0$, $Ag = 0$ for almost all $z \iff \hat{f}_{\varepsilon,C_R}(\xi) \cdot \overline{\hat{g}}_\varepsilon(\xi) = 0$, where $\hat{f}_{\varepsilon,C_R}$ is the Fourier Transform of $f_{\varepsilon,C_R}(x) = 1\{x \in C_R\}e^{-2\pi\langle\varepsilon,x\rangle}$ and $\overline{\hat{g}}_\varepsilon$ is the conjugate of the Fourier Transform of $g_\varepsilon(x)$, both continuous functions in $\mathbb{L}^1$. Since $\overline{\hat{g}}_\varepsilon$ is continuous, the set where $\overline{\hat{g}}_\varepsilon \neq 0$ is open. Further, since $|g|_1 > 0$, the support of $\overline{\hat{g}}_\varepsilon$ is non-empty. It follows that there is an open Z_ϵ where $\overline{\hat{g}}_\varepsilon$ is different from zero, and therefore, $\hat{f}_{\varepsilon,C_R}(\xi) = 0$ for all $\xi \in Z_\epsilon$. This contradicts the fact that $\hat{f}_{\varepsilon,C_R}$ is an entire function, as shown in Lemma **??**.

Finally, since $g(u)$ is known for almost all u, we have that $F_V(v|z^1) = \int_{-\infty}^{v-z^1} g(u)\mathrm{d}u$ is identified. □

The condition that there exists a lottery L_R such that C_R is salient and has a non-empty interior is satisfied for all school choice mechanisms in which (i) singleton rank-order lists are allowed, (ii) the probability of assignment into the top-ranked school is non-zero and (iii) the probability of assignment into unranked schools is zero. A rank-order list in which only one school is ranked will then yield a salient cone with a non-empty interior.[53]

The key insight is that Fourier transform of an exponential density restricted to any salient cone is non-zero on any open set. We first show a preliminary which specializes results in De Carli (1992, 2012).

A.3 Asymptotic Theory for RSP+C Mechanisms

Our main results in this section derive the properties of our estimator $\hat{L}$ for $L^n_{R,t}$ defined in equation (9) in the main text where the dependence of L on n is re-introduced in the notation for clarity. We hold σ unless explicitly conditioned on and treat the rational expectations case. Results for the other forms of beliefs follow as a consequence. We start by introducing some notation and definitions.

[53] It is easy to see that the two conditions imply that the convex hull of $\mathcal{L}$ has a non-empty interior. Let L_R be an extremal point in $\mathcal{L}$. Because the convex hull of $\mathcal{L}$ has a non-empty interior, there exists $\mathcal{L}' \subseteq \mathcal{L}$ such that the matrix $A = [L_R - L_{R_1}, \ldots, L_R - L_{R_J}]$ where each $L_{R_k} \in \mathcal{L}'$ has full rank. Consider $v \neq 0$. Because A is full rank, $A'v \neq 0$. Therefore, if $v \in C_R$, then it must be that $A'v$ has a strictly positive component. It follows that $-v \notin C_R$. Hence, C_R is salient.

Although the text stated our result for the uniform distribution, in our main results, we will assume that the mechanism uses a general /non-degenerate distribution of tie-breakers.

Definition 7 (Non-degenerate tie-breakers)**.** *Fix a function $f(R,t,\nu)$. The tie-breaker is non-degenerate if there exists some $\kappa > 0$, such that for each p, $p' \in [0,1]^J$, $j \in \{1,\ldots,J\}$, and $(R,t) \in \mathcal{R} \times T$,*

$$\gamma_\nu(\{\nu : p_j \wedge p'_j \leq f_j(R,t,\nu) \leq p_j \vee p'_j\}) \leq \kappa |p_j - p'_j|.$$

Non-degenerate tie-breakers is a strengthening of strict preferences in Azevedo and Leshno (2016). The assumption is straightforward to verify with knowledge of the mechanism. For example, it is satisfied if a random number is used to break ties between multiple students with the same priority type. It also allows for a situation in which a single tie-breaking number that is used by all schools to break ties.

Given a sample (R_i, t_i, ν_i), for $i \in \{1,\ldots,n\}$, we can obtain an empirical measure $\eta^n = \dfrac{1}{n}\sum_{i=1}^n \delta_{(R_i,t_i,\nu_i)}$, where $\delta_{(R_i,t_i,\nu_i)}$ is the Dirac-delta measure on (R_i,t_i,ν_i). Given η^n and a cutoff vector p, we can define the fraction of students that would be assigned to each program j as follows:

$$D_j(p|\eta^n) = \eta^n \left(\{f_j(R_i,t_i,\nu_i) > p_j,\ jR_i0\} \bigcap_{j' \neq j} (\{jR_ij'\} \cup \{f_{j'}(R_i,t_i,\nu_i) \leq p_{j'}\}) \right).$$

As a proof device, we will use a continuum economy. Let η be a probability measure over Borel sets in $\mathcal{R} \times T \times [0,1]^J$. If agents in the economy are using strategy σ, then $\eta = m^\sigma \times \gamma_\nu$, where $m^\sigma((R,t)) = f_T(t) \int \sigma_R(v,t) \mathrm{d}F_{V|T=t}$. Analogously, define the fraction of students that would be assigned to each program j in the continuum economy:

$$D_j(p|\eta) = \eta \left(\{f_j(R_i,t_i,\nu_i) > p_j,\ jR_i0\} \bigcap_{j' \neq j} (\{jR_ij'\} \cup \{f_{j'}(R_i,t_i,\nu_i) \leq p_{j'}\}) \right). \tag{11}$$

It is straightforward to see that $D_j(p|\eta)$ is a continuum analog of $D_j(p|\eta^n)$ because if (R_i,t_i,ν_i) are drawn i.i.d. from η, then $E[D_j(p|\eta^n)] = D_j(p|\eta)$.

Market clearing cutoffs (Definition 2) embody two sets of constraints, one set for the programs and another for schools. It will be useful to combine them in a single set. Define a $J \times S$ matrix A with entries $a_{js} = 1$ if $s_j = s$, i.e., if program j belongs to school s, and 0 otherwise. Here, S is the total number of schools. Let $\tilde{A} = [I_J\ A]$, where I_J is the

J-dimensional identity matrix, and

$$\tilde{D}(\tilde{p}|\eta) = \tilde{A}' D(\tilde{A}\tilde{p}|\eta) \in [0,1]^{J+S}, \tag{12}$$

where $\tilde{p} \in [0,1]^{J+S}$. The function $\tilde{D}$ stacks the program and school aggregates of the number of students demanding assignment given the cutoffs $p = \tilde{A}\tilde{p}$. In this notation, we have an equivalent definition of market clearing cutoffs in terms of $\tilde{p}$ and $\tilde{D}$:

Proposition A.2. *The cutoffs $p \in [0,1]^J$ are market clearing cutoffs for $D(p|\eta) \in [0,1]^J$ and $q \in [0,1]^{J+S}$ if and only if for each $k \in \mathcal{J} \cup \mathcal{S}$,*

$$\tilde{D}_k(\tilde{p}|\eta) - q_k \leq 0, \text{ with equality if } \tilde{p}_k > 0, \tag{13}$$

where $p = \tilde{A}\tilde{p}$ and $\tilde{p} = [\tilde{p}_{\mathcal{J}}, \tilde{p}_{\mathcal{S}}]$ with $\tilde{p}_{\mathcal{S},s} = \min\{p_j : s_j = s\}$ for $s \in \{1, \ldots, S\}$ and $\tilde{p}_{\mathcal{J}} = p - A\tilde{p}_{\mathcal{S}}$.

Proof. It's easy to verify that the inequalities $\tilde{D}_k(\tilde{p}|\eta) - q_k \leq 0$ are equivalent to those in the definition for market clearing cutoffs. Therefore, we only need to verify that the set of restrictions satisfied with equality coincide. For every $j \in \mathcal{J}$, $\tilde{p}_j > 0$ if and only if $p_j > \min\{p_{j'} : j' \neq j, s_{j'} = s_j\}$. Similarly, for every school $s \in \mathcal{S}$, $\tilde{p}_{\mathcal{S},s} > 0$ if and only if $\min\{p_j : s_j = s\} > 0$. $\square$

In what follows, we will therefore work with $\tilde{p}$ instead of p. Finally, let p_+ be the subvector of p with strictly positive elements and $D_+(p|\eta)$ be the corresponding subvector of $D(p|\eta)$.

We are now ready to state the main result of this section.

Theorem A.3. *Suppose that Φ^n is a RSP+C mechanism that uses non-degenerate tie-breakers, and for each $k \in \mathcal{J} \cup \mathcal{S}$, $q_k^n - q_k = o(1/\sqrt{n})$. For strategy σ, consider $\eta = m^\sigma \times \gamma_\nu$. If $\tilde{p}^*$ is the unique solution to equation (13), then for each each (R,t),*

$$|\hat{L}_{R,t} - L^n_{R,t}| \xrightarrow{p} 0.$$

If, additionally, $\nabla_{\tilde{p}^*_+} \tilde{D}_+(\tilde{p}^*|\eta)$ is invertible, then

$$\sqrt{n}(\hat{L}_{R,t} - L^n_{R,t}) \xrightarrow{d} \Gamma \tilde{A} \nabla \tilde{D} \tilde{A}' Z$$

where $Z \sim \mathcal{N}(0,\Omega)$, $\Gamma = \nabla_p \int D^{(R,t,\nu)}(\tilde{A}\tilde{p}^*)\mathrm{d}\gamma_\nu$,

$$\nabla\tilde{D} = \begin{bmatrix} (\nabla_{\tilde{p}^*_+}\tilde{D}_+(\tilde{p}^*|\eta))^{-1} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Omega = \left(1+\frac{1}{B}\right) V\left(\int D^{(R,t,\nu)}(\tilde{A}\tilde{p}^*)\mathrm{d}\gamma_\nu\right) + \frac{\mathbb{E}_\sigma\left[V\left(D^{(R,t,\nu)}(\tilde{A}\tilde{p}^*)\,\middle|\, R,t\right)\right]}{B}.$$

The first part of the result shows that if a RSP+C mechanism uses non-degenerate tie-breakers and the market-clearing cutoff is unique in the continuum economy, then $\hat{L}$ is a consistent estimator for L^n. Non-degeneracy of the tie-breaker is straightforward to verify with knowledge of the mechanism. Appendix **??** derives conditions on $D(p)$ and q under which uniqueness is guaranteed, and weaker conditions under which uniqueness is generically guaranteed using results from Azevedo and Leshno (2016) and Berry et al. (2013).

Under additional smoothness conditions, the result also provides a limit distribution for $\hat{L}$. The expression shows that the variance of the estimator depends on the inherent sampling variation in the observed reports and priority types. In addition, the estimator also has an additional independent source of variance due to resampling. This variance decreases with the number of resamples B used to construct the estimator.

Proof. We first define market clearing cutoffs p^n given that an agent of type t reports R. Let

$$\eta^n = \frac{1}{n}\delta_{(R,t,\nu)} + \frac{n-1}{n}\eta^{n-1},$$

and $\eta^{n-1} = \dfrac{1}{n-1}\sum_{i=1}^{n-1}\delta_{(R_i,t_i,\nu_i)}$ with (R_i,t_i,ν_i) drawn from η. Define $\tilde{p}^n_k$ such that $\tilde{D}_k(\tilde{p}|\eta^n) - q^n_k \le 0$ with equality only if $\tilde{p}^n_k > 0$. Note that $\tilde{p}^n$ exists by assumption since Φ^n is an RSP+C mechanism.

We define similar objects for a bootstrap sample. Index a draw in the b-th bootstrap sample from the empirical sample $(R_1,t_1),\ldots,(R_n,t_n)$ with i_b, and denote the bootstrap empirical measure $m^{n-1}_b = \dfrac{1}{n-1}\sum_{i_b=1}^{n-1}\delta_{(R_{i_b},t_{i_b})}$. Since the distribution of ν is known, we can draw ν_{i_b} directly from γ_ν for each i_b. Therefore, ignoring the report of one agent, we can define

$$\eta^{n-1}_b = \frac{1}{n}\sum_{i_b=1}^{n-1}\delta_{(R_{i_b},t_{i_b},\nu_{i_b})},$$

where ν_{i_b} is a draw from γ_ν, independently of all other random variables. Let $\tilde{p}^{n-1}_b$ be such that $\tilde{D}_k(\tilde{p}|\eta^{n-1}_b) - q^n_k \le 0$ with equality only if $\tilde{p}^{n-1}_{b,k} > 0$.

For each (R,t), consider the difference $\hat{L}_{R,t} - L^n_{R,t}$. Since Φ^n is and RSP+C mechanism, we have that

$$\hat{L}_{R,t} - L^n_{R,t} \;\; = \;\; \frac{1}{B}\sum_b \int D^{(R,t,\nu)}(p_b^{n-1})\mathrm{d}\gamma_\nu - E\left[\int D^{(R,t,\nu)}(p^n)\mathrm{d}\gamma_\nu \middle| R,t\right],$$

where $p_b^{n-1} = \tilde{A}\tilde{p}_b^{n-1}$, and $p^n = \tilde{A}\tilde{p}^n$.

We will derive the limit properties of the difference in the equation above using the limit distributions of p_b^{n-1} and p^n and smoothness of the integals in the expressions.

By definition of $D(p|\eta^n)$, we have that $\sup_p \|D(p|\eta^n)-D(p|\eta^{n-1})\| = O(1/n)$ and $\sup_p \|D(p|\eta_b^{n-1})-D(p|\frac{n}{n-1}\eta_b^{n-1})\| = O(1/n)$. The definition of $\tilde{D}(\tilde{p}|\eta)$ and Lemma **??** implies that

(i) for each $k \in \mathcal{J}\cup\mathcal{S}$, $\sup_{\tilde{p}} |\tilde{D}_k(\tilde{p}|\eta) - \tilde{D}_k(\tilde{p}|\eta^n)|$ converges in probability to 0,

(ii) $\sqrt{n}\left(\dfrac{1}{B}\sum_b D(\tilde{A}\tilde{p}^*|\eta_b^{n-1}) - D(\tilde{A}\tilde{p}^*|\eta)\right)$ converges in distribution to Z, and therefore,

$$\sqrt{n}\left(\frac{1}{B}\sum_b \tilde{D}(\tilde{p}_0^*|\eta_b^{n-1}) - \tilde{D}(\tilde{p}^*|\eta)\right) \xrightarrow{d} \tilde{A}'Z,$$

(iii) For any $\tilde{p}^*$ and any sequence of δ_n decreasing to 0,

$$\sup_{\|\tilde{p}-\tilde{p}^*\|\le\delta_n} \sqrt{n}\|\tilde{D}(\tilde{p}|\eta^n) - \tilde{D}(\tilde{p}|\eta) + \tilde{D}(\tilde{p}^*|\eta) - \tilde{D}(\tilde{p}^*|\eta^n)\| = o_p(1),$$

and likewise

$$\sup_{\|\tilde{p}-\tilde{p}^*\|\le\delta_n} \sqrt{n}\|\tilde{D}(\tilde{p}|\eta_b^{n-1}) - \tilde{D}(\tilde{p}|\eta) + \tilde{D}(\tilde{p}^*|\eta) - \tilde{D}(\tilde{p}^*|\eta_b^{n-1})\| = o_p(1).$$

Since $E[\tilde{p}^n] = E[\tilde{p}^n|m^\sigma]$ by definition and $E[\tilde{D}(\tilde{p}|\eta^n)] = \tilde{D}(\tilde{p}|\eta)$, Lemma **??** applied to $\tilde{D}(\tilde{p}|\eta)$ and $\tilde{p}^*$ implies that

$$\left\|\frac{1}{B}\sum_b \tilde{p}_b^{n-1} - \tilde{p}^*\right\| \xrightarrow{p} 0, \|\tilde{p}^n - \tilde{p}^*\| \xrightarrow{p} 0$$

and

$$\sqrt{n}\left(\frac{1}{B}\sum_b \tilde{p}_b^{n-1} - E[\tilde{p}^n]\right) \xrightarrow{d} \nabla\tilde{D}\tilde{A}'Z,$$

where $\tilde{p}^n$ and $\tilde{p}_b^{n-1}$ are respectively market clearing cutoffs for $(\tilde{D}(\tilde{p}|\eta), q^n)$ and $(\tilde{D}(\tilde{p}|\eta_b^{n-1}), q^n)$.

Pre-multiplying by $\tilde{A}$, we have that

$$\left\| \frac{1}{B} \sum_b p_b^{n-1} - E[p^n] \right\| \xrightarrow{p} 0$$

by the triangle inequality, and because p^n is bounded. Further, by Slutsky's theorem,

$$\sqrt{n} \left(\frac{1}{B} \sum_b p_b^{n-1} - E[p^n] \right) \xrightarrow{d} \tilde{A} \nabla \tilde{D} \tilde{A}' Z.$$

Since the tie-breaker ν is non-degenerate, γ_ν admits a density. Therefore, $\int D^{(R,t,\nu)}(p) \mathrm{d}\gamma_\nu$ is differentiable at every p since $D^{(R,t,\nu)}(p)$ is an indicator for $f(R,t,\nu)$ belonging to a hypercube:

$$D_j^{(R,t,\nu)}(p) = 1\{f_j(R,t,\nu) > p_j, jR0\} \prod 1\{f_{j'}(R,t,\nu) \leq p_{j'} \text{ or } jRj'\}.$$

Hence, $\hat{L}_{R,t}$ is a differentiable function of $\frac{1}{B} \sum_b p_b^{n-1}$. Therefore, by the Continuous Mapping Theorem,

$$\sup_{R,t} |\hat{L}_{R,t} - L_{R,t}^{n,\sigma}| \xrightarrow{p} 0$$

and by the Delta Method

$$\sqrt{n} \left(\hat{L}_{R,t} - L_{R,t}^{n,\sigma} \right) \xrightarrow{d} \Gamma \tilde{A} \nabla \tilde{D} \tilde{A}' Z.$$

□