SYSTEM IDENTIFICATION OF SUBMARINE HYDRODYNAMIC COEFFICIENTS FROM SIMPLE FULL SCALE TRIALS

by

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B.S. United States Coast Guard Academy (1985)

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and

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ABSTRACT

A submarine the designer has to base many major design decisions on the predictions of model tests. Presently, model tests are used to quantify the hydrodynamic and propulsive characteristics of a full size submarine. The result is the designer bases his decisions on model test data that is extrapolated from measurements taken at a Reynolds number several magnitudes lower than that of the actual submarine. This introduces "scale errors" of uncertain proportion since coefficient measurements have never been taken at such large Reynolds numbers.

This thesis uses the system identification techniques that have been developed at MIT to measure the submarine's hydrodynamic coefficients at full scale Reynolds number from several simple trial maneuvers. The sea trial data was analyzed with an Extended Kalman Filter so that the values of the resistance coefficient, thrust deduction, wake fraction, propeller thrust, and horizontal plane maneuvering coefficients could be measured. The identification produced accurate predictions of all these coefficients, proving the applicability of system identification to the submarine. These novel results have allowed the first glimpse into the Reynolds number range of actual submarines by providing a new method of accurate coefficient measurement.

Thesis Supervisor: Dr. Martin A. Abkowitz
Title: Professor of Ocean Engineering
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I thank my fiance for her support and encouragement during this long year.

This research was sponsored by Charles Stark Draper Laboratories.
Dedication

I dedicate this work to the glory of the Lord Jesus Christ in whom I can most assuredly say:

"For I am not ashamed of the gospel of Christ, for it is the power of God to salvation for everyone who believes, for the Jew first and also for the Gentile.

For in it the righteousness of God is revealed from faith to faith: as it is written, 'The just shall live by faith'."

"For whoever calls upon the name of the Lord shall be saved."

Romans 1:16-17,10:13
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1 Introduction

1.1 General Background

The development of ocean vehicles from design to construction and operation is increasingly expensive and time consuming. As a result, the ship designer must make all possible effort to reasonably assure that the vehicle, when built, can safely perform the missions it has been designed to accomplish. Paramount to both mission success and safe operation is the ability of the vehicle to achieve its design speed and maneuver adequately under mission operating conditions.

Naval Architects use much of their resources to determine, as best they can, the maneuvering and propulsive characteristics of a vehicle prior to construction. Presently the Naval Architect relies on tests which are conducted on small, geometrically similar models to determine the maneuvering and powering coefficients of a specific design. The tests conducted are resistance and self-propulsion in the towing tank, maneuvering tests on a planar motion mechanism, and model propeller tests in open water and a cavitation tunnel. Each of the experiments is designed to provide the designer information he needs to predict the maneuvering ability and power requirements of the full scale ship. However, the hydrodynamic information which the tests provide has been experimentally derived at a Reynolds number several magnitudes lower than that of the actual ship. The designer must then view the test results as either being simply qualitative or he may try to extrapolate the model data up to the higher Reynolds number of the full scale ship. The error with the extrapolation occurs when it is considered that no data has ever been accurately taken at the large Reynolds numbers which the ship operates. The designer is then simply extrapolating into a region of unknown hydrodynamic effects using assumed trends.
There remains then only one way to realistically predict the maneuvering characteristics of a ship. This is to take measurements on the full scale ship itself. This would accomplish two things; first, it would provide accurate data for the tested vessel and, second, it would provide "anchor points" in the Reynolds number region of interest that could be used to make model test extrapolations more accurate.

This thesis builds upon the work of Hwang[7] and Liu[9,10] who conducted full scale tests on the ESSO OSAKA and EXXON PHILADELPHIA, respectively. Each of these works shows that by using the process of system identification to analyze full scale sea trial data the maneuvering and powering coefficients of surface ships could be accurately measured. This work conducts the same method of identification using the Extended Kalman Filter program developed at MIT on the actual full scale trial data from a US Navy Submarine. The trial maneuvers consisted of several turns of small to full rudder deflections executed at various speeds, acceleration from stop to cruising speed, and deceleration from cruising speed. The identification process was altered to account for the different form of maneuvering data made available for the submarine.

1.2 Application to Submarines

The need for better testing information is nowhere greater than in the testing of submarines. A submarine operating below the surface, unlike a surface ship, does not create free surface waves that would alter its maneuvering and resistance characteristics. The viscous flow effects are the sole source of the hydrodynamic forces and moments that effect the submarine. Presently, it is not possible to run a model test which fully models these viscous flow effects. The following section describes the method used to determine the resistance of a submarine from model test information. This is presented here so the reader may understand the assumptions that must be made when conducting
any type of submarine model test and therefore truly see the need for better testing information.

As stated previously, the resistance of a submarine is due entirely to frictional and eddy viscous flow effects. In order to properly model these effects a model must be sized and the test designed so the Reynolds number of the model equals that of the full scale ship. Presenting this in equation form:

\[
\frac{(\rho Lu)_{\text{ship}}}{(\mu)_{\text{ship}}} = \frac{(\rho Lu)_{\text{model}}}{(\mu)_{\text{model}}}
\]  

where \( u \) is characteristic speed, \( L \) is length, \( \rho \) is water density, and \( \mu \) is water viscosity.

Obviously, this relationship causes a problem if we desire to test a model which is smaller than the actual ship. If the characteristic length of the model is smaller (as one would hope), the speed at which the test is conducted must be increased in direct proportion with the size ratio. For example, for the Reynolds numbers to be equal, a \( \frac{1}{10} \) scale model must be tested in the same fluid at 10 times the actual ship speed. This speed requirement causes the following testing problems that can not practically be resolved.

- The test mechanism must be capable of extremely high speeds.

- Model size and speed require a very deep test tank because the high speed model will have to be tested at sufficient depth to ensure that it does not induce waves at the free surface.

- The towing tank must be extremely long so that the acceleration and deceleration of the high speed carriage can be accommodated.

- No testing facility exists that can conduct tests that meet these requirements.
Today, the resistance of a submarine is predicted by running model tests at lower Reynolds numbers. This is done by testing the model resistance at ever increasing speeds until wave making effects begin to occur. The measured resistance curve is compared to a curve measured for flat planks at different Reynolds numbers. The flat plate resistance is assumed to represent the friction portion of the resistance at any given Reynolds number. The difference between the pre-wave making portion of the model resistance and the flat plate resistance is attributed to the eddy resistance caused by the hull shape. It is assumed that this difference will be constant throughout the entire Reynolds number region of interest. By then extrapolating the flat plate curve to the full scale Reynolds number and applying the eddy resistance value, the submarine resistance is predicted. The troubling issues of this process are the extrapolation of the plank resistance coefficient and the application of a constant eddy resistance. The plank resistance extrapolation is not based on any accurate information measured at the Reynolds number of the full scale submarine, but is instead an empirical extrapolation based on trends observed at Reynolds numbers which are several magnitudes lower. The eddy resistance component is assumed constant simply because no information exists to form any other method that would better predict the flow changes at the higher Reynolds numbers.

It should be noted that maneuvering data is determined using tests where the same scaling problems apply, but these tests will not be discussed here in the interest of brevity. Hopefully, this illustration has exposed the main issues that presently effect model testing efforts and why the prospect of actually measuring full scale data is so important. This thesis, for the first time, uses the Extended Kalman Filter to measure this information directly from the full scale submarine in the absence of any scale effects.
1.3 Motion Simulation

The ability to simulate the motion of an object with a computer model is a major design tool that is used today in a wide variety of engineering problems. To properly simulate the motion of an object the engineer must:

1. Obtain the system’s differential state equations in a form which fully describes the system dynamics in the desired simulation scenarios.

2. Accurately measure or calculate each term of the state equations.

3. Be able to integrate the state differential equations over the desired time interval.
Typically, to obtain the state equations of motion a Taylor expansion is used to expand the state variables about an operating condition where the system is in equilibrium, which, for the submarine is straight ahead motion at cruising speed. Chapter 3 discusses, in detail, the results of the expansion and further modifications of the submarine's state equations. Once these equations are presented in an accurate dynamic form, the partial derivative coefficients of the equations must be either calculated or measured so that they accurately reflect the system forces. Equation 1.2 is an example of some terms from the surge equation as they are derived from the Taylor expansion. To simplify the partial derivative notation the most common notational substitution is introduced here, where the equation is rewritten as 1.3.

\[ \text{Surge Force} = \frac{\partial X}{\partial u} \Delta u + \frac{\partial^2 X}{\partial v^2} \delta^2 + \frac{\partial^3 X}{\partial v \partial w \partial u} v r \Delta u + \ldots \]  \hspace{1cm} (1.2)

\[ \text{Surge Force} = X_s \Delta u + X_{ss} \delta^2 + X_{sv} \delta v r \Delta u + \ldots \] \hspace{1cm} (1.3)

Each partial derivative is substituted with a capital letter that designates to which force equation the coefficient belongs. The subscripts of the letter represent the partial derivative of the force being taken with respect to the subscript (or combination of) state variables. This notation will be used throughout this work for representation of these partial derivative coefficients.

Presently the value of these partial derivative coefficients are determined with hydrodynamic theory and scale model tests. As previously discussed, neither of these two methods provide coefficient values that can be used to accurately simulate a submarines motion. We must then look to an alternative method of determining these maneuvering coefficients so that the full scale submarine motions can be predicted more accurately, thus the need for system identification.
1.4 System Identification

System Identification is a method of solution which is useful when the coefficients of a system’s state model are not easily calculated or measured. The idea of system identification is very simple. If an accurate model which describes all of the system dynamics is known, the unknown system equation coefficients can be determined by exciting the system with known inputs and measuring the system response to those inputs. By prescribing the input and measuring the response, the unknown state equation coefficients are determined so that the response predicted by the state equations matches that which was measured on the actual system. An illustration of this method is presented in Figure 1.1.

![Diagram of System Identification Process]

Figure 1.2 General Process of System Identification

System identification does have some problems because it is essentially solving "backwards" from the input and output to the system instead of the input→system→output simulation procedure that is normally used. The most significant problem is that the system must already be built if system identification is to be used. This is not always practical, especially when the system is a multimillion dollar ship. The hope of this and other works is to provide more accurate full scale system descriptions that will allow
better pre-fabrication motion prediction in the future. Another difficulty is that the number of state equation unknowns is much greater that the number of possible response measurements. This presents a problem of parameter uniqueness and identifiability which, for the ship maneuvering case, is discussed further in Chapters 2, 3 and 4.

1.5 Submarine Sea Trial Data

Before we get further into the theory of the system identification process that was used, it is appropriate to discuss the data which was supplied for this work and how it differs from that of previous works. Hwang[7] and Liu[9,10] were supplied with full scale sea trial data which had the following properties:

- Measurements of ship motion were taken near to the center of the coordinate system as set on the ship, therefore minimizing the coupling between the states at the measurement location.

- Speed measurements were taken by a very accurate Doppler speed log relative to the water.

- Standard acceleration, deceleration, zig-zag, and hard turn maneuvers were executed specifically for use in the system identification process.

- Although it is desirable, sway velocity and yaw rate were not measured directly.

The submarine full scale data differs from this previous form of data in several very significant ways. First, the speed measurements were not taken at a location near the center of the defined submarine coordinate system but were instead taken a substantial distance forward of the coordinate center. To complicate this further, the supplied data includes only a single measurement that was taken with the electromagnetic log, shown in
Figure 1.2, which measures total speed relative to the water at this remote location. Measurement of sway velocity, $v$, and yaw rate, $r$, were not supplied. The problem is then twofold; the measurements of the electromagnetic log are less accurate than those taken by Doppler speed log and because total velocity is measured at the remote location the total velocity becomes a function of the true state velocities as represented by

$$u_{log} = \sqrt{u^2 + (v + kr)^2}$$ (1.4)

Not having the true state measurements requires many significant changes to the Extended Kalman Filter measurement model which are discussed in Chapter 2. The accuracy of the electromagnetic log will directly influence the accuracy of the final parameter identification. Use of a Doppler speed log would provide the best possible input data and therefore the best final results.

![Figure 1.3: Submarine Measurement Log Geometry](image)

Another problem is that the supplied submarine data consists only of turns which are all in the same direction. The importance of the selected maneuvers is discussed at great length is Hwang[7] where the system identification process was first used successfully. It is shown there that a zig zag maneuver is best when trying to identify the linear state equation coefficients because the maneuver provides non-constant state values
and nonzero state accelerations. With turns in only one direction it will be difficult to identify the coefficients representing the asymmetric effect of the propeller-rudder system or any coefficients describing system accelerations.

Each of these data types and forms creates a more difficult identification of the desired system parameters. Although an identification with this data is possible, especially from the acceleration and deceleration maneuvers which have no yaw or sway velocities, it would have been much better if the optimal sea trial maneuvers and measurements were supplied for this work.
2 The Kalman Filter

State equation coefficient identification in this work is accomplished with an appropriately designed Extended Kalman Filter. The following sections include a brief discussion of the basic theory of Kalman Filter operation and design. For more background in Kalman Filter optimal estimation, Gelb's work [5] is highly recommended. For more detail into the design of the specific filter used in this work consult Hwang[7] and Liu[9,10].

2.1 The Kalman Filter

The Kalman Filter is a recursive technique designed to provide an optimal estimation to a known linear system. The filter achieves this optimal result by providing variable estimators which are functions of time that minimize the error variance between the system state model and measured states. To achieve this estimation the filter must have specific a priori information such as known system qualities. First, the filter must include a system model which accurately describes the dynamics involved. Chapter 3 describes, in detail, the state equations used in this work. Secondly, to accomplish the estimation, the filter must be designed to accommodate continuous system dynamics and discrete measurements. In this continuous-discrete linear filter the system process and measurements may be corrupted by noise, for these the filter assumes the linear models:

\[ \dot{x} = Fx + w \]  
\[ z = hx + v \]

(2.1)  
(2.2)

where \( x \) is the state vector and \( z \) the measurement vector. \( w \) and \( v \) represent the process and measurement noise, respectively. These noises are assumed to be zero mean and variance, white disturbances such that
\[ E[wv'] = 0 \] (2.3)

for all time. The matrix operator F is a system particular linear function, whereas the measurement operator h varies with each time step and state vector. The states and covariance are propagated using[5]:

\[ \dot{\hat{x}} = F\hat{x} \] (2.4)

\[ \dot{P} = F\hat{x}P + PF' + Q \] (2.5)

where \( \hat{x} \) represents the expected value of the vector x. The filter updates the estimation and covariance matrix at time k with

\[ \dot{\hat{x}}_{k(+)} = \hat{x}_{k(-)} + K_k[z_k - H_k\hat{x}_{k(-)}] \] (2.6)

\[ P_{k(+)} = P_{k(-)} - P_{k(-)}H_k\hat{x}_{k(-)} \] (2.7)

\[ K_k = P_{k(-)}H_k'\hat{x}_{k(-)}\left[H_k\hat{x}_{k(-)}P_{k(-)}H_k\hat{x}_{k(-)} + R_k\right]^{-1} \] (2.8)

where Q and R are error variance matrices such that

\[ Q = E[ww'] \] (2.9)

\[ R = [vv'] \] (2.10)

This filter provides a straightforward algorithm which calculates a reasonably limited number of optimal solutions to the linear system. This algorithm is, however, inadequate for the estimation of the nonlinear systems such as the ship maneuvering problem which is considered in this work. To extend filtering applicability, the Extended Kalman Filter has been developed to provide a similar minimum error variance solution for the nonlinear problem.
2.2 The Extended Kalman Filter

The Extended Kalman Filter is of similar form to the Kalman Filter, but it allows a wider variety of solutions due to the more complicated nature of the nonlinear system. Hwang[7] and Liu[9,10] showed that this technique is applicable to and provides good results for the ship maneuvering problem. The state estimates and covariance are propagated in a fashion similar to the linear Kalman Filter in that the system is linearized about each \( \hat{x}(t) \). The result is the propagation equations are of the form:

\[
\dot{\hat{x}}(t) = f(\hat{x}(t), t) \tag{2.11}
\]

\[
\dot{P}(t) = F(\hat{x}(t))P(t) + P(t)F'(\hat{x}(t), t) + Q(t) \tag{2.12}
\]

The nonlinear filter updates the parameter estimates using

\[
\hat{\xi}_{k(\cdot)} = \hat{\xi}_{k(-)} + K_k[z_k - h_k(\hat{x}_{k(-)})] \tag{2.13}
\]

\[
P_{k(\cdot)} = [I - K_kH_k(\hat{\xi}_{k(-)})]P_{k(-)} \tag{2.14}
\]

using gain

\[
K_k = P_{k(-)}H_k'(\hat{\xi}_{k(-)})[H_k(\hat{\xi}_{k(-)})P_{k(-)}H_k'(\hat{\xi}_{k(-)}) + R_k]^{-1} \tag{2.15}
\]

with the following definitions applying to the system and measurement functions

\[
F(\hat{x}(t), t) = \frac{\partial f(x(t), t)}{\partial x(t)} \bigg|_{\hat{x}(t) = \hat{x}(t)} \tag{2.16}
\]

\[
H_k(\hat{\xi}_{k(-)}) = \frac{\partial h_k(x(t_k))}{\partial x(t_k)} \bigg|_{\hat{x}(t_k) = \hat{x}_{k(-)}} \tag{2.17}
\]
It is again worth noting that the Extended Kalman Filter admits a much larger variety of solutions than the linear filtering technique, but with careful design and preparation the Extended Kalman Filter provides the correct optimal system estimate.

2.3 State Augmentation

The Extended Kalman Filter implemented in the MIT program uses the technique of state augmentation to provide the estimates of the system states and desired coefficients. State augmentation alters the previously discussed nonlinear filter so that the state vector $\mathbf{x}$ is extended to provide augmentation with the coefficients and currents that are to be identified. The augmented state vector then becomes:

$$
\mathbf{x} = \begin{bmatrix}
    u \\
    v \\
    r \\
    \psi \\
    u_c \\
    \alpha \\
    C_1 \\
    \vdots \\
    C_k
\end{bmatrix}
$$

- **State Variables**
- **Current Variables**
- **Identified Coefficients**

Figure 2.1 Augmentation of the State Vector

The extended vector shown here is the maximum size that may presently be used in the MIT filtering program. The submarine maneuvering problem the current speed ($u_c$) and direction ($\alpha$) are not included since the submarine's velocity is measured relative to the water. The number of coefficients carried depends on the type of maneuver and the desired identification.
2.4 Measurement Model Modification for Maneuvering Trials

As discussed in Section 1.5 the sea trial data supplied for the identification is not of the optimal form that is desired for the filtering process. Because the speed was measured using the electromagnetic log, which gives the resultant speed through the water, the measurement model must be altered to properly reflect this different form of measurement during the turning maneuvers.

Recalling equation 2.2 in the continuous-discrete form used in the nonlinear filter:

$$z = h(t\hat{x}_{k|t}) + \nu$$  \hspace{1cm} (2.18)

the matrix $h$ represents the linear combination of the state variables which determine the measurement vector $z$ at each time step $k$. In the earlier work of Hwang and Liu the measurement model took the form

$$z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \nu \\ r \\ \psi \end{bmatrix} + \nu$$  \hspace{1cm} (2.19)

for the system with four measured states. In each of these works the sea trials measured the states directly. This measurement model only applies to the submarine during the acceleration and deceleration trials where $\nu$ and $r$ are negligible and equation 1.4 becomes

$$U_{log} = u$$  \hspace{1cm} (2.20)

During the turning maneuvers $\nu$ and $r$ are no longer negligible and the above simplification is not possible. The measurement model must then be changed in order to apply the Extended Kalman Filter to the maneuvering trials. The modification begins by writing equation 1.4 as
$U_{\text{log}} = \left( \sqrt{1 + \frac{(v + \kappa r)^2}{u^2}} \right) u$  

(2.21)

This allows the measurement system matrix to be written in the form

$$
\hat{h}_k = \begin{bmatrix}
\sqrt{1 + \frac{(v_k + \kappa r_k)^2}{u_k^2}} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(2.22)

where $v_k$, $r_k$, and $u_k$ represent the state values that are propagated by equation 2.13 at the time $k$. To satisfy the update equations the measurement sensitivity matrix, H, must also be evaluated at each time step $k$. Recalling the definition of H:

$$
H_k(\hat{x}_k,\hat{z}_z,\hat{z}_s) = \frac{\partial h_k(x(t_k))}{\partial x(t_k)} \bigg|_{x(t_k) = \hat{x}_k, z_z = \hat{z}_z, z_s = \hat{z}_s}
$$

(2.17)

the new H matrix becomes

$$
H_k = \begin{bmatrix}
\frac{\partial h}{\partial u} & \frac{\partial h}{\partial v} & \frac{\partial h}{\partial r} & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(2.23)

where

$$
\frac{\partial h}{\partial u} = \frac{1}{2\sqrt{1 + \frac{u_k^2}{(v_k + \kappa r_k)^2}}}
$$

$$
\frac{\partial h}{\partial v} = \frac{1}{\sqrt{1 + \frac{u_k^2}{(v_k + \kappa r_k)^2}}}
$$
\[
\frac{\partial h}{\partial r} = \frac{\kappa}{\sqrt{1 + \frac{s_i^2}{(v_s + v_{r,d})^2}}}
\]

Altering measurement and measurement sensitivity matrices in this fashion provides the necessary description of the measurement model to allow the Extended Kalman Filter to optimize the estimation of the turning circle maneuvers. Although these changes provide an accurate measurement model the more complicated form of the filter requires several more calculation steps between the propagation and update at each time step and eliminates the benefit of directly comparing the estimated states to the measured states.
3 System State Models

3.1 Derived State Model

The nonlinear state equations used in this work are the same as those used by Hwang and Liu in their work on the ESSO OSAKA and EXXON PHILADELPHIA tankers. These equations are derived using the conventional approach presented in Abkowitz[1]. The equations are found by taking a Taylor Expansion of the hydrodynamic forces and moments with respect to the forward speed ($u$) and acceleration ($\dot{u}$), transverse speed ($v$) and acceleration ($\dot{v}$), yaw rate ($r$) and rate ($\dot{r}$), and rudder deflection ($\delta$) and acceleration ($\dot{\delta}$). The basic ocean vehicle operates in a six degree of freedom dynamic environment which is illustrated in Figure 3.1.

![Figure 3.1: State Coordinate System](image-url)
The coordinate axis is fixed in the vehicle on the centerline at midships and, in the case of the submarine, along the axis of revolution of the hull. The dynamics of the system produce six coupled equations of motion. In order to simplify these equations so they may be useful for the identification process, several assumptions are made about the submarines operation during the trial maneuvers. The assumptions made in this work are listed below:

- The submarine does not significantly heave during the maneuver.

- The submarine does not significantly pitch during the maneuver.

- No Froude (surface wave) effects are present.

- Roll is not strongly coupled to the X, Y, and N equations and is therefore neglected.

The equations which result form these simplifications are presented in Figure 3.2. Given reasonable operation of the submarine during the specified maneuvers the assumptions that the heave and pitch are insignificant are more than reasonable. Examination of the maneuvering trial data confirmed the pitch and heave were negligible. The difficulty is in justifying the decoupling of the roll equation from the remaining equations of the horizontal plane. During the small rudder deflection turns and acceleration-deceleration maneuvers the submarine does not roll enough to seriously couple with the horizontal plane equations. During the hard rudder turn however, the submarine rolls significantly in the early portion of the turn due to the lift induced on the sail by the vessel’s transverse (sway) velocity. It should be noted that the initial rolling effects, although they can be violent, occur during the early portion of the trial and are followed by a steady state roll which is much less severe. In this condition the steady roll
should not strongly effect the forces and moments in the horizontal plane equations. Sea
trial data shows that the maximum roll occurs in only 15% of the time of the full
maneuver with steady state roll achieved soon after. These assumptions should be
adequate for the state model to accurately reflect the submarines maneuvers for the
specified maneuvers.

\[
\begin{align*}
\text{surge acceleration} \\ 
\dot{u} &= \frac{f_1}{(m-X_u)} \\
\text{sway acceleration} \\ 
\dot{v} &= \frac{(I_z-N_s)f_2 - (m_xG - Y_s)f_3}{f_4} \\
\text{yaw acceleration} \\ 
\dot{r} &= \frac{(m-Y_s)f_2 - (m_xG - N_s)f_3}{f_4}
\end{align*}
\]

where

\[
\begin{align*}
f_1 &= X_a \Delta u + X_{wa} (\Delta u)^2 + X_{wa} (\Delta u)^3 + X_{vr} v^2 + X_{sg} s^2 + (X_{sr} + m_xG)r^2 \\
&+ (X_{sr} + m)v_r + X_{sg} s^2 + X_{sr} v^2 + X_{sg} s^2 + X_{sr} r^2 \Delta u + X_{sr} v^2 \Delta u \\
&+ X_{sr} v_r + X_{sr} v \Delta u + X_{sr} v \Delta u + X_{sr} r \Delta u
\end{align*}
\]

\[
\begin{align*}
f_2 &= Y_o + Y_r v + (Y_r - m_xG)r + Y_s s^2 + Y_{wa} v \Delta u + Y_{sr} \Delta u + Y_{sr} \Delta u \\
&+ Y_{sr} v^3 + Y_{sr} r^2 + X_{sg} s^2 + X_{sr} v^2 + X_{sg} s^2 + X_{sr} v^2 (\Delta u)^2 + Y_{sr} v^2 \\
&+ Y_{sr} v^2 + Y_{sr} v \Delta u + Y_{sr} \Delta u + Y_{sr} \Delta u + Y_{sr} \Delta u
\end{align*}
\]

\[
\begin{align*}
f_3 &= N_o + N_r v + (N_r - m_xG)r + N_s s^2 + N_{wa} v \Delta u + N_{sr} \Delta u + N_{sr} \Delta u \\
&+ N_{sr} v^3 + N_{sr} r^2 + N_{sr} s^3 + N_{sr} v^2 + N_{sr} s^2 + N_{sr} v^2 (\Delta u)^2 + N_{sr} v^2 \\
&+ N_{sr} v^2 + N_{sr} r^2 + N_{sr} s^2 + N_{sr} s^2 + N_{sr} (\Delta u)^2 + N_{sr} v^2
\end{align*}
\]

\[
\begin{align*}
f_4 &= (m - Y_s) (I_z - N_s) - (m_xG - N_s) (m_xG - Y_s)
\end{align*}
\]

Figure 3.2: Taylor Expansion of Horizontal Plane Equations
3.2 Modifications to the State Model

With the motion of the submarine now restricted to the horizontal plane the three remaining equations of motion leave 59 coefficients which must either be calculated by valid theory or identified by the system identification process. As discussed in Hwang, the more parameters that must be identified with a given number of measurements, the less satisfactory the results become. It is obviously to our advantage to simplify the state equations as much as possible being careful not to loose any part of the system dynamics. The following sections outline the simplifications that are made to the state equations. The simplifications are based on both hydrodynamic and mathematic review of the equations and result in a simpler state model which includes all the linear and nonlinear dynamic effects of the more complicated model.

3.2.1 Elimination of Higher Order Rudder Derivatives

In order to eliminate the rudder dependent derivative it is necessary of take a step back and look at the rudder force-moment problem alone. Given the propeller-rudder system shown in Figure 3.4, the mean flow velocity into the rudder can be described as

\[
c = \sqrt{\frac{A_p}{A_R} [(1 - w)u + k u_{\Delta}]^2 + \frac{A_R - A_p}{A_R} (1 - w)^2 u^2}
\]  

(3.1)

where \( A_R \) = rudder span
\( A_p \) = rudder area covered by the propeller column
\( w \) = wake fraction
\( k \) = distance factor determined by the Law of Biot-Savart from Figure 3.3.
\( u_{\Delta} \) = far field propeller induced velocity
\[ u = \text{forward speed} \]

\[ k = \frac{u_A}{u_{A\infty}} \]

Figure 3.3 Mean Axial Velocity Induced by a Semi-infinite Tube of Ring Vorticies Determined by the Law of Biot-Savart[7]

The velocity induced by the propeller is determined using:

\[ u_{A\infty} = -(1 - w)u + \sqrt{(1 - w)^2u^2 + \frac{8}{\pi}K_i(nd)^2} \tag{3.2} \]

where \( K_i \) = propeller thrust coefficient defined as

\[ K_i = \frac{T}{\rho n^2 D^2} \tag{3.3} \]

\( n = \text{propeller RPS} \)

\( D = \text{propeller diameter} \)

\( T = \text{propeller thrust} \).
Figure 3.4: Propeller-Rudder System Flow Geometry

What has been accomplished is the rudder forces and moments can now be directly described by the actual flow over the rudder instead of the ship speed $u$. The advantage of this change is the new nondimensionalization that is possible for the higher order effects of the rudder. By nondimensionalizing the higher order cross derivatives $X_{\delta\delta}, X_{\delta\theta}, X_{\phi\phi}, Y_{\delta\delta}, Y_{\delta\theta}, Y_{\phi\phi}, N_{\delta\delta}, N_{\delta\theta}$, and $N_{\phi\phi}$ are dropped without loosing any of the information contained in the state equation.

The additional higher order $\delta$ derivatives and cross derivatives can be replaced with three force and moment terms that will account for all of the nonlinear effects which result from the deflected rudder. This simplification more realistically represents the hydrodynamic phenomena involved and also eliminates the difficulty in
differentiating the effects of each individual term during the identification process, so by combining them together a more compact and identifiable parameter is defined. To begin, the effective angle of attack of the rudder is described by the equation

\[ e = \delta - \tan^{-1} \left( \frac{v}{c} + \frac{rX_r}{c} \right) \]  

(3.4)

where \( X_r = \) distance form coordinate center to rudder

\( v = \) sway velocity

\( r = \) yaw velocity

\( c = \) mean flow velocity over rudder from equation 3.1

Using this as the descriptive variable, the lift and drag effects on the system state equations can be more simply described with the following terms:

\[ X_{\delta e} \delta^2 + X_{\delta v} \delta + X_{r \delta} \delta^2 \Delta u + X_{r \Delta u} \delta \Delta u + X_{\delta r} r \Delta u \Rightarrow X_{\delta e e} \]  

(3.5)

\[ Y_{\delta \delta \delta} \delta^3 + Y_{\delta v \delta} \delta^2 + Y_{\delta r \delta} \delta^2 + Y_{r \delta \delta} \delta^2 + Y_{\delta \delta r} \delta + Y_{r \Delta u} \delta \Delta u + Y_{\delta \Delta u} \delta \Delta u^2 + Y_{r \delta r} r \delta \Rightarrow Y_{\delta e e e} \]  

(3.6)

\[ N_{\delta v \delta} \delta^2 + N_{r \delta \delta} \delta^2 + N_{\delta r \delta} \delta^2 + N_{r \delta r} \delta + N_{r r \delta} \delta \Delta u + N_{r \Delta u} \delta \Delta u + N_{\delta \Delta u} \delta \Delta u^2 \Rightarrow N_{\delta e e e} \]  

(3.7)

3.2.2 Correction for Single Propeller Vehicles

For the vessel with a single screw there exists a hydrodynamic force and moment which is caused by the asymmetry of a single rotating propeller. This effect is commonly referred to as "propeller walk" since the ship appears to move and rotate as if the propeller were rolling along a hard surface. The force and moment terms which account for this effect, \( Y_o \) and \( N_o \), are always present when the propeller is
turning regardless of whether the ship is in motion. Thus it is more appropriate to nondimensionalize these terms with the propeller race velocity $U_{A inf}$ instead of forward speed $u$. The nondimensional terms may then be expressed as follows:

$$Y_o^- = \frac{Y_o}{\frac{1}{2} \rho L^2 \frac{U_{A inf}^2}{2}}$$  \hspace{1cm} (3.8)

$$N_o^- = \frac{N_o}{\frac{1}{2} \rho L^2 \frac{U_{A inf}^2}{2}}$$  \hspace{1cm} (3.9)

here again $U_{A inf}$ is calculated from equation 3.2.

### 3.2.3 Simplification of Surge Force Derivatives

The derivatives $X_u$, $X_{uw}$, and $X_{uwu}$ result from the speed loss in the forward motion of the ship. Following the discussions in Hwang, the surge acceleration of the ship is determined by the thrust and resistance forces giving net force as

$$(1-t)T - R$$ \hspace{1cm} (3.10)

It follows that

$$(1 - t)T - R = (1 - t)K_T \rho n^2 D^4 - \frac{\rho}{2} C_{h} S u^2$$ \hspace{1cm} (3.11)

where $t$ = thrust deduction factor

$n$ = propeller RPM

$D$ = propeller diameter

$K_T$ = propeller thrust coefficient

$\rho$ = water density
S = wetted surface area

$C_R = \text{resistance coefficient}$

$u = \text{forward speed}$

This relationship is not so helpful because $K_s$ and $C_R$ are not constant during ship operation, but are dependent on the propeller rotational speed ($n$) and ship speed ($u$), respectively. If a quadratic model of $n$ and $u$ is used to represent the propeller thrust (3.12), the speed loss derivatives can be replaced with identifiable constants.

$$(1 - r)T = \eta_1 u^2 + \eta_2 u n + \eta_3 n^2 \quad (3.12)$$

Although this parameter modification is not based on physical necessity it does not alter the application of the state model and actually enhances the parameter identification possibilities. The additional terms represent the propeller characteristics for the ship and are actually a quadratic representation of the propeller’s $K_s$-$J$ curve. Table 3.1 summarizes the physical meaning of this equation modification.[7]
Table 3.1 Physical Interpretation of Thrust Model Modifications

<table>
<thead>
<tr>
<th>$n$</th>
<th>$u$</th>
<th>Physical Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\neq 0$</td>
<td>$(1-t)T - C_R \frac{\rho}{2} S u^2$</td>
</tr>
<tr>
<td>$n_{windmilling}$</td>
<td>$\neq 0$</td>
<td>$\eta_1 u^2 - C_R \frac{\rho}{2} S u^2$</td>
</tr>
<tr>
<td>$\neq 0$</td>
<td>0</td>
<td>$\eta_3 n^2$</td>
</tr>
<tr>
<td>$n_o$</td>
<td>$u_o$</td>
<td>$\eta_1 u_o^2 + \eta_2 n_o u_o + \eta_3 n_o^2 - C_R \frac{\rho}{2} S u_o^2 = 0$</td>
</tr>
<tr>
<td>$n$</td>
<td>$u$</td>
<td>$\eta_1 u^2 + \eta_2 n u + \eta_3 n^2 - C_R \frac{\rho}{2} S u^2$</td>
</tr>
</tbody>
</table>

3.2.4 Correction For Deviation From Cruising Equilibrium

The forces on the rudder change whenever the vessel makes any significant maneuver away from straight ahead steady propeller loading equilibrium. Deviation from equilibrium causes the propeller to produce more (or less) race velocity. This in turn induces rudder forces that are not included in the $Y_v, Y_r, N_v$, and $N_r$ coefficients.
that describe the equilibrium state. The additional forces can be described using two
new coefficients $Y_8$ and $N_8$ that account for the deviation from equilibrium of the
propeller race velocity over the rudder. The additional terms take the form:

$$
\frac{Y_8}{2} (c - c_o) \frac{D}{2} L^3 r - Y_8 (c - c_o) \frac{D}{2} L^2 v
$$

(3.13)

and

$$
\frac{N_8}{2} (c - c_o) \frac{D}{2} L^3 r - N_8 (c - c_o) \frac{D}{2} L^3 v
$$

(3.14)

where $c$ is the mean flow velocity over the rudder from 3.1 and $c_o$ is the mean flow
velocity calculated at equilibrium propeller loading at ship forward speed $u$. This
modification of the state model provides the realistic description of the physical
system and the additional benefit of eliminating the terms $Y_{vu}$, $Y_{uv}$, $Y_{vuu}$, $N_{vu}$, $N_{uv}$,
$N_{vuu}$, and $N_{vuv}$ since their effects are accounted for in the new coefficients. The effects
of $X_{vu}$ and $X_{vuu}$ were found negligible by Hwang and are therefore simply dropped
from the surge equation.

3.2.5 Combination of Cubic Nonlinear Terms of $v$ and $r$

In order to further reduce the number of coefficients that must be identified and
to increase the identifiability of the third order effects of the $Y$ and $N$ equations a
combination of cubic parameters is used. Since we know that for a ship at large $v$ and
$r$:

$$
v = kr
$$

(3.15)
where \( k \) depends on the geometry, the relation of the cross derivatives is clearly seen. It is then possible to assume the model test values of the third order parameters are correct for all but one term in each of the \( Y \) and \( N \) equations. That one term can then be identified as a combination of all the terms, or more accurately, a combination of the errors that result from assuming the model test values were correct. The identified term will then become a "collector" for all the terms in its equation. Although the identified value will not be the true value of that parameter, hydrodynamically speaking, the total dynamic effect of all the cubic terms together will be correct. The simplification may be shown as follows[2]:

\[
Y_{rrr}v^2 + Y_{rrr}v^2 + Y_{rrr}r^3 + Y_{rrr}v^3
\]

\[
= Y_{rrr}v^2 + Y_{rrr}v^2 + Y_{rrr}v^2 + Y_{rrr}v^2 + Y_{rrr}v^2 + Y_{r}k^2v^2
\]

\[
= \left[ Y_{rrr} + kY_{r} + \frac{1}{k} Y_{rrr} + k^2Y_{r} \right]v^2
\]

\[
= \bar{Y}_{rrr}v^2
\]  

(3.16)

Following similar arguments for the \( X \) and \( N \) equations yields:

\[
N_{rrr}v^2 + N_{rrr}v^2 + N_{rrr}r^3 + N_{rrr}v^3 \Rightarrow \bar{N}_{rrr}v^2
\]  

(3.17)

\[
X_{r}r^2\Delta u + X_{r}v^2\Delta u + X_{r}v^2r\Delta u \Rightarrow \bar{X}_{r}v^2r^2
\]  

(3.18)

Making these assumptions provides a less complicated, more identifiable model without the loss of any hydrodynamic forces or moments and their dynamic effects.
3.2.6 Maneuvering Model - Final Form

Compiling all of the changes that have been discussed in Sections 3.2.1-3.2.5 the final version of the maneuvering state model is presented in Figure 3.4. The coefficients are presented here in nondimensional form with each individual dimensioning factor presented with the coefficient.
\[ \dot{u} = \frac{f_1}{m - X_s} \]

\[ \dot{v} = \frac{(I_x - N_s)f_2 - (mx_c - Y_s)f_3}{f_s} \]

\[ \dot{r} = \frac{(m - Y_s)f_1 - (mx_c - N_s)f_3}{f_s} \]

where

\[ f_1 = \eta_s \left[ \frac{P}{2} L^3 \right] u + \nu_s \left[ \frac{P}{2} L^3 \right] u u + \eta_N \left[ \frac{P}{2} L^3 \right] n - C_s \left[ \frac{P}{2} S u^3 \right] + X_m \left[ \frac{P}{2} L^3 \right] v \]

\[ + X_m \left[ \frac{P}{2} L^3 c^2 \right] r + (X_m + mx_c) \left[ \frac{P}{2} L^4 \right] r + (X_m + m) \left[ \frac{P}{2} L^3 \right] v \]

\[ + X_m \left[ \frac{P}{2} L^3 u^{-1} \right] v^2 r \]

\[ f_2 = Y_s \left[ \frac{P}{2} L^3 \left( \frac{u_{st}}{2} \right) \right] + Y_s \left[ \frac{P}{2} L^3 \right] u - Y_s \left[ \frac{P}{2} L^3 \right] (c - c_s) v \]

\[ + (Y_s - m_{2} u) \left[ \frac{P}{2} L^3 u \right] + Y_s \left( \frac{c - c_s}{2} \right) \left[ \frac{P}{2} L^3 \right] r + Y_s \left[ \frac{P}{2} L^3 c^2 \right] \delta \]

\[ + Y_m \left[ \frac{P}{2} L^3 u^{-1} \right] u + Y_m \left[ \frac{P}{2} L^3 u^{-1} \right] + Y_m \left( \frac{P}{2} L^3 u^{-1} \right) r^2 v \]

\[ + Y_m \left[ \frac{P}{2} L^3 u^{-1} \right] + Y_m \left[ \frac{P}{2} L^3 c^2 \right] e^3 \]

\[ f_3 = N_s \left[ \frac{P}{2} L^3 \left( \frac{u_{st}}{2} \right) \right] + N_s \left[ \frac{P}{2} L^3 u \right] - N_s \left[ \frac{P}{2} L^3 \right] (c - c_s) v \]

\[ + (N_s - mx_c u) \left[ \frac{P}{2} L^3 u \right] + N_s \left( \frac{c - c_s}{2} \right) \left[ \frac{P}{2} L^3 \right] r + N_s \left[ \frac{P}{2} L^3 c^2 \right] \delta \]

\[ + N_m \left[ \frac{P}{2} L^3 u^{-1} \right] u + N_m \left[ \frac{P}{2} L^3 u^{-1} \right] + N_m \left( \frac{P}{2} L^3 u^{-1} \right) r^2 v \]

\[ + N_m \left[ \frac{P}{2} L^3 u^{-1} \right] + N_m \left[ \frac{P}{2} L^3 c^2 \right] e^3 \]

\[ f_4 = \{(m - Y_s) \left[ \frac{P}{2} L^3 \right] (u_x - N_s) \left[ \frac{P}{2} L^3 \right] \} - \{(mx_c - N_s) \left[ \frac{P}{2} L^4 \right] (mx_c - Y_s) \left[ \frac{P}{2} L^4 \right] \} \]

Figure 3.5: Maneuvering State Model - Final Form
3.3 Resistance Model

In order to measure the resistance coefficient to a high degree of accuracy, the surge state model should be formulated to provide a model equation with the least number of unknown parameters. The state model must also be developed in light of the sea trial maneuvers which are to be executed for the identification. The chosen sea trial maneuvers are a deceleration from cruising speed to stop with windmilling propeller and acceleration from a complete stop. Both maneuvers are accomplished with minimum, preferably zero, rudder on a steady course. This serves to isolate the surge equation from any yaw and sway coupling terms that would complicate the equation and identification. By applying Newton’s law in this direction we develop the following relation:

\[(m - X_s)\ddot{u} = T - R\]  (3.19)

where \(-X_s = \) the submarines added mass in the X direction

\[m = \text{submarine mass}\]
\[T = \text{effective thrust}\]
\[R = \text{forward resistance}\]

This is the basic force balance that was developed by Liu[10] to provide the description of the system state model of surface ship surge. The activitation of the state model is outlined here with note that the final result is the same as Liu’s, but different arguments apply to the submarine hydrodynamic model than that of the surface ship.

3.3.1 Submarine Resistance

Typically the resistance of an ocean vehicle involves friction losses due to the water viscosity, wave making losses resulting from the moving vehicles induced wave
pattern, and the losses caused by turbulent eddies which are formed by the separated turbulent boundary layer that surrounds the ship. In the case of the submarine, the friction and eddy losses are always present. The wave making resistance, however, does not exist while the submarine is submerged due to the absence of the free surface. As a general rule of thumb, the free surface will not effect the resistance if the submarine operates more than three hull diameters from the surface. Properly designing the identification maneuvers can then ensure that the wave making component is not present. This simplifies the submerged submarine resistance problem to one in which the viscous flow effects around the body are the sole source of resistance to forward motion.

It is well known that the resistance on such a body increases in proportion to the square of its forward velocity. This resistance is typically presented as

\[ R = \frac{1}{2} \rho C_R(Re)S u^2 \]  

(3.20)

where \( \rho = \) density of water

\( C_R = \) nondimensional resistance coefficient which is a function of Reynolds number

\( S = \) wetted surface area

In this equation the resistance coefficient is a function of the Reynolds number and thus varies with forward speed. The identification process avoids this problem by assuming that the Reynolds number is relatively constant while the submarine operates near its cruising speed. This assumption is very reasonable in light of the high value and narrow range of Reynolds number at which the identification is made.
3.3.2 Thrust Model

As introduced in Section 3.2.3 a quadratic model in $u$ and $n$ has been introduced to model the propeller thrust characteristics.

\[(1 - \tau)T = \eta_1 u^2 + \eta_2 n u + \eta_3 n^2 \tag{3.21}\]

Each of the three terms of this equation represent various operating hydrodynamic properties of the propeller as labeled (loosely) and explained below:

- **drag-thrust coefficient, $\eta_1 u^2$:** This term represents the "locked" drag of the propeller induced by the forward velocity $u$. It is non-dimensionally represented as:

  \[\eta_1^* = \frac{\eta_1}{\rho D^2} \tag{3.22}\]

- **associated drag-thrust coefficient, $\eta_2 u n$:** This term represents the force resulting from a correlation of forward speed $u$ and the propeller RPS $n$. It is non-dimensionalized as:

  \[\eta_2^* = \frac{\eta_2}{\rho D^3} \tag{3.23}\]

- **lift-thrust coefficient, $\eta_3 n^2$:** This term describes the thrust induced by the propeller in the absence of any forward speed. It is non-dimensionalized with the equation:
\[ \eta_3 = \frac{\eta_b}{\rho D^4} \]  \hspace{1cm} (3.24)

Substituting the thrust and resistance models into equation 3.19 and dropping the nondimensional superscript yields a more descriptive form of the surge equation:

\[ \dot{u} (m - X_d) = \rho D^2 \eta_1 u^2 + \rho D^3 \eta_2 un + \rho D^4 \eta_3 n^2 - \frac{1}{2} \rho S C_k u^2 \]  \hspace{1cm} (3.25)

3.3.3 Correction for Interaction Effects

To further increase the hydrodynamic accuracy of the specialized surge equation, the interaction effects between the hull, propeller, and fluid must be accounted for. As the ship travels forward the viscous properties of the water cause the fluid to move along with the ship. The effect of this induced flow is to alter the inflow velocity to the propeller. A correction, known as the wake fraction \( w \), is used to quantify the change in follow velocity at the propeller. The wake fraction is defined by

\[ u_A = (1 - w) u \]  \hspace{1cm} (3.26)

where \( u_A \) is the propeller's speed of advance relative to the water and \( u \) is the ship speed.

The second interaction effect that must be accounted for occurs between the ship hull and the propeller. The suction of the turning propeller alters the pressure field between it and the ship. The altered pressure field, in turn, adversely affects the fluid flow around the entire hull causing an increase in the resistance. The additional resistance maybe quantified in two different ways. First, it can be modeled using the
**thrust deduction factor** \( (t) \) which represents the loss as a reduction in the supplied propeller thrust, being presented in equation 3.27. Alternatively the added resistance can be modeled with a **resistance augmentation factor** \( (a) \) which accounts for the increase in the drag term as shown in equation 3.28.

\[
R = (1 - t)T \quad (3.27)
\]

\[
T = (1 + a)R \quad (3.28)
\]

Combining the two interaction effects yields the complete uncoupled surge state model:

\[
\dot{u} (m - X_s) = (1 - t) \left[ \rho D^2 \eta_1 u_A^2 + \rho D^3 \eta_2 u_A n + \rho D^4 \eta_3 n^2 \right] - \frac{1}{2} \rho S C_R u^2 \quad (3.29)
\]

\[
\dot{u} (m - X_s) = \rho D^2 \eta_1 u_A^2 + \rho D^3 \eta_2 u_A n + \rho D^4 \eta_3 n^2 - \frac{1}{2} \rho S C_R (1 + a) u^2 \quad (3.30)
\]

or, in terms of ship forward speed;

\[
\dot{u} (m - X_s) = (1 - t) \left[ \rho D^2 \eta_1 (1 - w)^2 u^2 + \rho D^3 \eta_2 (1 - w) u n + \rho D^4 \eta_3 n^2 \right] - \frac{1}{2} \rho S C_R u^2 \quad (3.31)
\]

\[
\dot{u} (m - X_s) = \rho D^2 \eta_1 (1 - w)^2 u^2 + \rho D^3 \eta_2 (1 - w) u n + \rho D^4 \eta_3 n^2 - \frac{1}{2} \rho S C_R (1 + a) u^2 \quad (3.32)
\]

### 3.3.4 Surge Model - Final Form

The final form of the surge state model comes by defining three simplifying variables in the following manner:
\[ \eta_1^* = \eta_{1,ship}(1 - w)^2 (1 - \tau_{eq}) - \frac{1}{2} \rho C_R S \]  
(3.33)

\[ \eta_2^* = \eta_{2,ship}(1 - w)(1 - \tau_{eq}) \]  
(3.34)

\[ \eta_3^* = \eta_{3,ship}(1 - \tau_{eq}) \]  
(3.35)

Combining like terms of \( u \) and \( n \) we arrive at the most simple form of the model:

\[ \dot{u} = \frac{\eta_1^* u^2 + \eta_2^* u n + \eta_3^* n^2}{(m - X_u)} \]  
(3.36)
4 System Identification Techniques

4.1 General Identification Procedures

The general identification procedures for a single cycle of the Extended Kalman Filter process is presented in Figure 4.1. The process is very simple as shown, but may take several iterations of each loop before the proper identification is achieved.

The initial input files are formulated using the vessel's model test data and the measured sea trial data. The model test derived coefficients are used as the initial estimate of the coefficient values if these values are known. If a model test coefficient is not known, any physically reasonable approximation of the coefficient can be used. Each coefficient is identified by the filter for each maneuver in which they play a major role until the best possible estimation is achieved. The techniques that are used to locate the true coefficient estimates are discussed in Chapter 5. The sea trial data files are formulated so that each maneuver is accurately reflected in the data set. Each file starts at a point in the maneuver where the system states are known, typically, this means that the time where the yaw and sway velocities are zero becomes the starting time for the maneuvering data file. This condition is chosen because it is the only point in the maneuver where the state values are expressly known. These input files are run through the Kalman Filter program for each set of initial parameter guesses until the process noise corresponding to each measurement state is statistically correct. The statistical correctness of the process noise is evaluated using the hypothesis testing statistics discussed in Hwang[7]. The test used for evaluation is the "residual whiteness" test which provides a matrix of statistics defined by:
\[ P_{nr}(\tau) = \frac{1}{N} \sum_{n=1}^{N} r_{r}(n)r_{r}^{t}(n - \tau) \quad \tau = 0, 1, \ldots \]  

(4.1)

The hypothesis, and thus the assumed process noise, is accurate when,

\[ E\{P_{nr}(\tau)\} = \begin{cases} I & \tau = 0 \\ 0 & \tau > 0 \end{cases} \]  

(4.2)

where \( P_n \) is normally distributed as \( N \) approaches infinity.
As discussed in Chapter 2, the Extended Kalman Filtering procedure admits a wide variety of solutions due to the nonlinear properties of the ship maneuvering problem. So, in order to determine whether a statistically correct result has arrived at the true system solution, the engineer must evaluate each solution based on the following three items:
• Do all the identified coefficients achieve relatively constant value solutions within the same time period.

• Are the identified values hydrodynamically reasonable. (i.e. is a negative value identified when a negative value is physically impossible)

• Does the simulation using the identified coefficients accurately reflect the sea trial data.

If the identified coefficients do not satisfy these three criterion then the estimation is considered incorrect and the process must be repeated with new initial coefficient guesses. It is this portion of the system identification process that is so important, for as we know the filter may identify incorrect parameters. It is then up to the engineer’s knowledge of the state model and physical system to determine what part of the estimation is not correct and how it must be changed.

4.2 Surge Equation Coefficient Identification

The surge equation coefficients are identified using two maneuvers. The first is a deceleration maneuver from cruising speed with windmilling propeller. From this maneuver the first portion of the data is used to identify the total resistance coefficient $\bar{C}$, which includes the windmilling propeller. Only the first part of the maneuver is used to assure that $u$ and $\dot{u}$ are large and changing rapidly. This aids the identification in that the measurements of the higher velocity portion of the maneuver will have a much higher signal to noise ratio than the end of the maneuver when the velocity is small. Changing acceleration, $\ddot{u}$, allows the filter to evaluate new dynamic information at each time step thus insuring that the estimation is not biased to a portion of the trial where the system states are relatively constant. The identification procedures for this maneuver are
explained in detail in Section 4.2.2.

The acceleration maneuver is conducted from a complete stop where the ship is suddenly given the cruising speed RPM command. This maneuver is processed in two segments, the first being the early portion of the maneuver where \( u=0 \) and the second being the end of the maneuver where \( \dot{u}=0 \). This allows the identification of the propeller thrust coefficients at two different loading conditions. Processing this maneuver in these two segments allows the ship's propeller thrust coefficients \( \eta_1, \eta_2, \eta_3 \), to be determined along with the wake fraction, thrust deduction, and resistance coefficient. The specifics of this identification process are further discussed in Section 4.2.1.
Figure 4.2: Surge Equation Identification Procedure

4.2.1 Acceleration Maneuver

An acceleration maneuver is used to identify the ship propeller thrust coefficients, thrust deduction, wake fraction, and resistance coefficient. The measured forward speed and RPS sea trial data is presented in Figures 4.4 and 4.5. To perform the identification, the "Derived Coefficient Value" method was used. This iterative
method, developed by Liu[10], provides a logical process that allows the desired ship characteristics to be determined solely from the acceleration trial. The process begins by splitting the maneuver into two different segments. The first segment uses the portion of the trial where the submarine is near its equilibrium speed. From this portion of data the equilibrium propeller thrust coefficients $\eta_1^*, \eta_2^*$, and $\eta_3^*$ are identified. Identification of these three values defines the equilibrium operating point on the $K_t$-$J$ curve as illustrated in Figure 4.3. Remembering that these identified coefficient values implicitly include the interaction effects of the wake fraction ($w$) and thrust deduction ($t$), recall equations 3.32-3.34, we know that they uniquely define equilibrium propeller thrust and need not be identified again during this process.

Figure 4.3: Development of Ship Propeller Characteristic Curve

Now, in order to make use of this information about the ship propeller, the quadratic thrust equation is fit to the model propeller $K_t$-$J$ curve to determine the value
of the model propeller's thrust coefficients $\eta_{1\text{model}}, \eta_{2\text{model}},$ and $\eta_{3\text{model}}$. Realizing then that the thrust characteristics of the model and the full scale ship propeller depend primarily on geometry and minimally on Reynolds number, their is much less scaling error present than between the ship hull and its model. It can then be assumed that no scale error is present when the propeller is unloaded at zero thrust ($K_v=0$). This provides a point through which the ship propeller $K_v-J$ curve can be defined. The initial assumption is that the model propeller thrust coefficient, $\eta_{3\text{model}}$, is equal to that of the full size propeller installed on the ship. This allows the wake fraction and the first estimation of thrust deduction factor and resistance coefficient $C_R$ to be calculated. The Derived Coefficient iteration scheme which is presented later in this section has been developed to further refine these estimates. The procedure begins by calculating the thrust deduction with

$$t = 1 - \frac{\eta_3^*}{\eta_{3\text{ship}}} \approx 1 - \frac{\eta_3^*}{\eta_{3\text{model}}}$$  \hspace{1cm} (4.3)$$

The thrust deduction value calculated corresponds to the equilibrium thrust deduction, which is how the coefficient is defined, since $\eta_3^*$ was identified at the submarine’s operating equilibrium. The wake fraction is then calculated by dividing 3.33 by 3.34. Solving the quotient for $w$ yields:

$$w = 1 - \frac{\eta_{3\text{ship}}^* \eta_2^*}{\eta_3^* \eta_{2\text{ship}}} \approx 1 - \frac{\eta_{3\text{model}}^* \eta_2^*}{\eta_3^* \eta_{2\text{model}}}$$  \hspace{1cm} (4.4)$$

Having identified the values of the thrust deduction and wake fraction the value of $\eta_{1\alpha}$ is calculated with its definition:

$$\eta_{1\alpha} = \eta_{1\text{ship}}(1-w)^2(1-t) = \eta_{1\text{model}}(1-w)^2(1-t)$$  \hspace{1cm} (4.5)$$
Using this as an intermediate value the resistance coefficient is determined by solving equation 3.32 for $C_R$.

$$
C_R = \frac{\eta_{1a} - \eta_1^*}{\frac{s}{2D^2}} \tag{4.6}
$$

This value of $C_R$ is an estimate of the true value of submarine resistance since no additional drag effects such as the windmilling propeller remain in this value. Further refinement of this resistance coefficient is accomplished during the remainder of the iteration.

The next step of the identification process derives the $K_r$-J curve (thrust coefficients) for the ship propeller in order to determine the true equilibrium thrust deduction factor form equation 4.4. To determine the ship's $K_r$-J curve the following iterative process is used. Each cycle of this iteration requires only one additional identification of the propeller thrust coefficient $\bar{\eta}_3$. This coefficient is identified using the early portion of the acceleration trial where $\bar{\eta}_3$ dominates the propeller thrust equation since the forward speed and thrust deduction are small.

The iteration is based on the fact that the propeller-ship geometry, unlike the thrust deduction which increases with speed, is not a function of ship speed or propeller RPS but is constant throughout the entire maneuver. Abkowitz[14] solved the propeller-ship system using actuator disk theory and by applying the momentum theorem he showed that the resistance augmentation factor could be written as:

$$
a = 2k(1 - w) \left( \sqrt{1 + \frac{8K_r}{\pi D^2}} - 1 \right) \tag{4.7}
$$
where $K_\iota$ and $J$ are evaluated at the propellers operating condition and $k$ depends on the propeller-ship geometry. Liu used this equation to determine the value of $k$ and thus bridged the gap between the equilibrium and initial portions of the acceleration trial. The entire iteration is explained in detail in the following eleven items. The reader is directed to Liu's thesis for a complete explanation of the theoretical steps involved.

1. Using the model propeller characteristics determine

\[ 1 - t_{eq} \] (4.8)

\[ K_{i,eq} = \eta_1 u_{eq}^2 + \eta_2 u_{eq} n_{eq} + \eta_3 n_{eq}^2 \] (4.9)

2. Calculate the equilibrium resistance augmentation factor $a_{eq}$:

\[ a_{eq} = \frac{1}{1 - t_{eq}} - 1 \] (4.10)

3. Calculate the geometric factor $k$:

\[ k = \frac{a_{eq}}{2(1 - w) \sqrt{1 + \frac{8K_i}{\pi J_p^2} - 1}} \] (4.11)

4. Calculate the low $J$ value resistance augmentation factor $\bar{a}$:

\[ \bar{a} = 2k(1 - w) \sqrt{1 + \frac{8K_i}{\pi J_p^2} - 1} \] (4.12)

5. Calculate the value of the average thrust deduction for the initial portion of the acceleration trial:

\[ \bar{t} = \frac{SC_a J_p^2}{2D^2 K_i (1 - w)^2 \bar{a}} \] (4.13)
6. Identify $\tilde{\eta}_3$ during the initial portion of the trial after updating $\eta_1$ and $\eta_2$ with the average thrust deduction found in step 5.

7. Calculate $\eta_{3,\text{ship}}$ with:

$$\eta_{3,\text{ship}} = \frac{\eta_3}{1 - t}$$  \hspace{1cm} (4.14)

8. Calculate the new equilibrium thrust deduction $t_{eq}$:

$$t = 1 - \frac{\eta_3^*}{\eta_{3,\text{ship}}}$$  \hspace{1cm} (4.15)

9. Calculate the new value of $\eta_2$ with:

$$\eta_{2,\text{ship}} = \frac{\eta_2^*}{(1 - t_{eq})(1 - w)^2}$$  \hspace{1cm} (4.16)

10. Using the value of $\eta_{3,\text{ship}}$ which determines the $K_s$ at $J=0$ and $\eta_{2,\text{ship}}$ from step 9.

The ship's $K_s$-J quadratic curve is then fit through the newly defined $K_s$ value at $J=0$ and the assumed model $J$ value where $K_s=0$ in a proportional manner to determine a new value of $\eta_{1,\text{ship}}$.

9. Update $C_R$ using the values calculated in steps 8 and 10 with:

$$C_R = \frac{2D^2}{S} [\eta_{1,\text{ship}}(1 - t_{eq})(1 - w)^2 - \eta_3^*]$$  \hspace{1cm} (4.17)

10. Return to step 2 and iterate until $\Delta t_{eq} = 0$. 

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Forward Speed

Figure 4.4: Acceleration Trial Measured Forward Speed
Figure 4.5: Acceleration Trial Measured RPS
4.2.2 Deceleration Maneuver

Analysis of the deceleration maneuver provides an identification of the "total" resistance coefficient, $\overline{C}_r$, with good accuracy.[10] This is done by coasting the submarine to a stop from its cruising speed while allowing the propeller to freely windmill. The measured sea trial forward speed and RPS data for the specified deceleration maneuver is presented in Figures 4.6 and 4.7.

Specifying the maneuver in this way allows equation 3.30 to be further simplified so that the propeller thrust terms drop out and the deceleration state equation becomes:

$$\frac{1}{2} \rho \overline{C}_r S u^2 = (m - X_a) \dot{u}$$

(4.18)

where $\overline{C}_r$ is the total ship resistance which, in this case, includes both the ship resistance and the drag of the windmilling propeller. The beauty of this equation is that only two parameters, $\overline{C}_r$ and $X_a$, must be identified to fully describe the dynamics of this trial. The identification is again simplified since the added mass term can be calculated very accurately using the Panel Radiation Method developed at MIT. In the present work, however, the added mass was determined by applying potential theory to the submarine shape. Although this is not as accurate as the Panel method, because the added mass is only on order of 5% of the vessel's mass a 20% error in the calculation of the added mass only results in a 1% error in the entire $m - X_a$ term. This is a more than acceptable tolerance. This leaves only the resistance coefficient to be identified which is a tremendous benefit for the filtering process since the accuracy of its solution directly depends on the number of parameters that must be identified. The coefficient $\overline{C}_r$ being the total ship resistance can then be corrected for the drag of
the windmilling propeller which is of the order 5% of the true $C_R$. Here also, if the
drag of the propeller is estimated with 20% error, the true resistance coefficient will
only be off by 1%.

To determine the drag of the windmilling propeller, Liu developed the
"Recurrence Method". This method allows the null resistance coefficient to be
determined primarily from the deceleration trial. Liu showed that the true $C_R$ could be
determined with only minimal information from the acceleration trial. In this work
there is no need to prove the distinctness of each iteration scheme, so the identified
propeller thrust coefficients from the acceleration trial can be used as a starting point
for the recurrence method instead of the model test values. Using these previously
identified coefficients is in no way inconsistent and is actually beneficial in two ways;
first, it allows the Recurrence Method to converge after only one iteration saving
computation time, and secondly, convergence within a single iteration proves that the
acceleration identification did indeed produce accurate results. The Recurrence
Method consists of the following procedure:

1. Determine the propeller advance ratio for the windmilling propeller, $J_{wm}$, using
the wake fraction previously determined from equation 4.3. Also recall $J_{eq}$ and
$\bar{J}$, from the acceleration maneuver computations.

2. Calculate the windmilling thrust coefficient, $K_{wm}$, using the values of $\eta_{1\text{shp}}$,
$\eta_{2\text{shp}}$, and $\eta_{3\text{shp}}$ identified from the acceleration maneuver. Also recall the final
values of $K_{eq}$ and $\bar{K}$, from the acceleration maneuver computations.

3. Recall the geometric factor $k$ calculated from equation 4.11.
4. Calculate the windmilling resistance augmentation factor \( a_{wm} \):

\[
a_{wm} = 2k(1 - w)
\left(\sqrt{1 + \frac{8K_{nm}}{\pi J_{wm}^2}} - 1\right)
\] (4.19)

5. Calculate the resistance coefficient \( C_R \):

\[
C_R = \frac{1.0}{1 + a_{wm}} \left( C_R + \frac{2D^2K_{nm}(1 - w)^2}{SJ_{wm}^2} \right)
\] (4.20)

6. Calculate the low \( J \) value resistance augmentation factor \( a \) with equation 4.12.

7. Calculate \( \bar{t} \) with equation 4.13 using the \( C_R \) from step 5.

8. Compare \( \bar{t} \) determined here with the final value of \( \bar{t} \) determined during the acceleration maneuver computations. If they are consistent the \( \eta_{1shp}, \eta_{2shp}, \) and \( \eta_{3shp} \) are accurately identified and the \( C_R \) in step 5 is the hull \( C_R \) corrected for the drag of the windmilling propeller.

The resistance coefficient as now determined is close to the true hull resistance coefficient, but further refinement is possible since the actual wake fraction present during windmilling differs from that calculated from the acceleration maneuver. The acceleration maneuver determines the effective wake fraction which describes the wake flow in the presence of the propeller. In the absence of a propeller the wake is described with the nominal wake fraction. During windmilling, the wake fraction will be close to that of the nominal wake making it necessary to correct the calculation of \( C_R \) since the effective wake was used in previous calculations. Using the work of Huang and Groves[16] the ratio of these two wakes is determined using:
\[
\frac{1 - w_{nom}}{1 - w_{eff}} = \zeta 
\] (4.21)

The true wake is then determined by applying \( \zeta \) to the wake fraction that was identified from the acceleration maneuver:

\[
1 - w_{wm} = \zeta (1 - w_{eq})
\] (4.22)

The following steps are used to update the earlier calculations:

1a. Correct \( J_{wm} \) to reflect the new value of \( w_{wm} \).

2a. Correct \( K_{rwm} \) to reflect the new value of \( w_{wm} \).

3a. Recalculate the geometric factor with the new value of \( w_{wm} \).

4a. Recalculate \( a_{wm} \) with the updated \( K_{rwm}, J_{wm}, k, \) and \( w_{wm} \).

5a. Calculate the hull resistance with equation 4.20 and the updated values in steps 1a-4a.
Figure 4.6: Deceleration Sea Trial Forward Speed Data
Propeller RPS

Deceleration Maneuver

Figure 4.7: Deceleration Sea Trial RPS Data
4.3 Maneuvering Model Coefficient Identification

The general procedure used to identify the maneuvering model and its coefficients is illustrated in Figure 4.8. The maneuvers used for the identification are small and hard rudder turns. An overview of the sea trials that were made available is given in Section 1.5. The differences between the available and optimal type maneuvering trials and the resulting problems are discussed there. This section will discuss the identification process used on the available data which unlike the acceleration and deceleration identifications does not require any supporting calculations. In the first of the two maneuvers a small rudder deflection is used to induce a slow turn in which the linear terms dominate the forces and moments. This "linear" maneuver is further discussed in Section 4.3.1. The hard rudder maneuver is simply an execution of a full rudder command when the ship is at cruising speed. This maneuver produces a tight "nonlinear" maneuver that is further analyzed in Section 4.3.2.

The maneuvering identification uses the results of the acceleration and deceleration trials to provide the wake fraction, ship propeller characteristics, and equilibrium propeller characteristics. The introduction of these factors assures an accurate full scale representation of propeller-rudder and propeller-ship interaction.
4.3.1 Small Rudder Turn

To obtain an identification of the linear coefficients the small rudder turn sea trial data was used. The small rudder deflection induces a slow turn that is dominated by the hydrodynamic moments and forces of the linear terms of the state equations. Figures 4.9-4.11 present the speed, heading, and RPS measurement records that were provided for the trial. The yaw rate was derived from the heading angle record with a B-spline method that is fully described in Appendix C. The techniques of parallel processing and over- under estimation (Section 4.4) are used to stabilize the coefficient values during the identification.
Figure 4.9: Small Rudder TurnMeasured Forward Speed
Figure 4.10: Small Rudder Turn Measured RPS
Figure 4.11: Small Rudder Turn Measured Heading Angle
4.3.2 Hard Rudder Turn

A hard rudder turn is used to identify the nonlinear coefficients of the state equations after the initial identification of the linear coefficients is completed. The hard rudder turn provides a maneuver that is sufficiently severe to excite the nonlinear terms of the state equations. The identification process is much like that for the linear coefficients, also using the techniques of over-under estimation and parallel processing. A good identification of the coefficients, however, is more difficult to achieve than with the linear coefficients because the value of the nonlinear terms vary throughout the maneuver. The coefficients $X_{ee}$, $Y_{ee}$, and $N_{ee}$ are identified during the initial part of the turn where their contribution to the forces and moments is greatest. The remaining coefficients $X_{wr}$, $Y_{wr}$, and $N_{wr}$ are identified during the latter portion of the trial where their contribution is greatest and most stable. The measured forward speed, RPS, and heading records for the turn are presented in Figures 4.12-4.14. The yaw rate was calculated with the B-spline method described in Appendix C.
Figure 4.12: Hard Rudder Turn Measured Forward Speed
Figure 4.13: Hard Rudder Turn Measured RPS
Figure 4.14: Hard Rudder Turn Measured Heading Angle
4.4 Identification Strategies

4.4.1 Single Processing

The single processing technique is the standard method of processing in which data is processed one set at a time through the filter. The processing involves the procedures discussed in Section 4.1. Single processing is useful when the identification is very stable or when used to locate a solution before parallel processing is used to further refine the estimation.

4.4.2 Parallel Processing

Parallel processing is technique that has several uses. Primarily it is used to help eliminate the simultaneous drift phenomena that occurs while estimating the linear coefficients. The idea behind parallel processing is simple, the filter processes two (or more) data records containing different dynamic effects simultaneously producing a single optimally estimated solution. Both Gelb[5] and Hwang[7] discuss the theoretical motivation of this method in detail and the reader is directed to their works if further details of the estimation theory are desired.

During the turning circle maneuvers, the linear coefficients are subject to the "cancelation effect" which induces the simultaneous drift[7]. To reduce this effect it is beneficial to process two sets of data in which the vessel is executing maneuvers in "opposite" directions (ie. one to port and one to starboard). The design is that the hydrodynamic forces and moments that cause the drifting are acting differently in each maneuver so as to cancel each other out. Unfortunately the submarine sea trial data has been supplied so that it is not possible to take advantage of this beneficial cancellation technique since all turns were executed in the same direction.
Although this processing method was primarily developed for the drifting problem, it should be considered anytime it is beneficial to consider two portions of data simultaneously to achieve a single optimal solution. This can be very helpful when the identified coefficients model one portion of a maneuver well, while another portion is poorly predicted. Parallel processing assures that the filter simultaneously processes all relevant dynamics so that the solution to the entire maneuver is optimally estimated. Parallel processing has proven to be useful with this data in some cases, some examples are given below.

- Small rudder turns: The identification of the linear coefficients is smoothed by parallel processing the beginning and end of the maneuver simultaneously. Processing the trial in this fashion produced very smooth identifications.

- Hard rudder turn: Parallel processing was used to aid the identification of $X_{gc}$, $N_{gc}$, and $Y_{gc}$ by processing the hard rudder turn data along with a different hard turn trial, which due to data defects could not be used over the entire maneuver. Processing the two trials together doubles the amount of data that is used for the identification which was helpful because the initial portion of the data where these coefficients are identified occurs during a very short time period.

4.4.3 Over-Under Estimation

The technique of over-under estimation is used to uniquely identify any coefficients that undergo the simultaneous drift phenomena. This technique involves the following terms of the $Y$ and $N$ equations:

$$Y_y + (Y_r - mu)r + Y_\delta = Const$$

(4.23)
\( N_r v + (N_r - mx_\text{G} u)r + N_\delta \delta = \text{Const} \tag{4.24} \)

When identifying the two coefficients in these equations the combinations of false solutions are endless (hence the effect of simultaneous drift). To identify the true solution, the maneuver is processed with one coefficient being over estimated and the other under estimated. This assures that the coefficients must propagate in different directions (larger or smaller) until they converge to the true solution. Once they have converged to the true solution they remain there shortly and then commence to drift. Hwang developed this technique in his work, showing there that it is a very effective method for the estimation of \( N_r \) and \( N_\delta \) and moderately useful for \( Y_r \) and \( Y_\delta \).

The benefits of this technique are that:

- The initial over-under estimates assure that the Kalman Filter obtains the true solutions and maintains them for a period of time before simultaneous drifting occurs.

- The true solution becomes more identifiable to the user since the convergence trends are more obvious.
5 Results

5.1 Acceleration Maneuver

The iteration identification process converged to the true ship propeller characteristic values after four iterations. Calculations for each iteration are presented in Appendix E in a "relative" number format. Figures 5.1-5.3 present the identification of the equilibrium coefficients $\eta_1$, $\eta_2$, and $\eta_3$. Figure 5.4 shows the simulation of the equilibrium portion of the trial as determined with the identified coefficients. Figures 5.5-5.8 present the identification of $\bar{\eta}_3$ for each of the four iterations. The final values of thrust deduction, wake fraction, resistance coefficient, and the propeller coefficients determined with the Derived Coefficient Method are presented in Table 5.1 and the identified ship propeller $K_v$ curve is illustrated in Figure 5.9.

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<td>$w$</td>
<td>UNKNOWN</td>
<td>†</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of Identified Coefficients - Acceleration Maneuver

† these values are not publishable
Figure 5.1: Identification of $\eta_i^*$
Equilibrium $\eta_2^*$

**Equilibrium Acceleration Trial**

![Graph showing Equilibrium $\eta_2^*$ as a function of time](image)

Figure 5.2: Identification of $\eta_2^*$

---

80
Equilibrium $\eta_3^*$

Figure 5.3: Identification of $\eta_3^*$
Figure 5.4: Equilibrium Acceleration Simulation
Identification of ATA3 at Low J

Figure 5.5: Identification of $\eta_3$ - First Iteration
Identification of ATA3 at Low J

Figure 5.6: Identification of $\bar{\eta}_3$ - Second Iteration
Identification of ATA3 at Low J

Figure 5.7: Identification of $\eta_3$ - Third Iteration
Identification of ATA3 at Low J

Figure 5.8: Identification of $\eta_3$ - Fourth Iteration
5.2 Deceleration Maneuver

The deceleration maneuver provided a stable identification of the "total" resistance coefficient $C_R$ which includes parasitic propeller windmilling resistance. The Extended
Kalman Filter identification of $\overline{C}_R$ is shown in Figure 5.10. The coefficient was identified to be 104.8% of the model test value. Figure 5.11 shows the simulated deceleration maneuver produced with the identified value. The simulation has been extended beyond the actual identification time interval to illustrate the accuracy of this value over the entire maneuver.

The Recurrence Method was used to correct $\overline{C}_R$ for the parasitic drag of the propeller and to verify the ship propeller characteristics as determined from the acceleration maneuver. One iteration verified that the ship propeller coefficients had been accurately identified by the acceleration maneuver. The calculated resistance coefficient was further corrected for the difference between the windmilling and effective (cruising) wake fractions. The windmilling wake fraction was determined to be 2.3% greater than the effective wake fraction. Table 5.2 presents the value of the resistance coefficient at all stages of the correction process and Table E.2 presents a summary of calculations.

To make the corrected resistance coefficient comparable with the value determined from the acceleration maneuver, the corrected value was scaled back to the Reynolds number of the acceleration maneuver. This was done by simply determining the difference between the model resistance at the two speeds and applying the difference to the identified resistance coefficient. When corrected for the speed difference, the resistance coefficients determined from the acceleration and deceleration maneuvers were identical.
Identification of Resistance Coefficient

Deceleration Maneuver

Windmilling C_r (% of identified value)

Time (percent of reference value)

---

Identified Value + Filtered Value

Figure 5.10: Identification of $\bar{C}_r$
Figure 5.11: Simulated Deceleration Maneuver
5.3 Maneuvering Trials

The identification achieved using the Extended Kalman Filter which had been modified for use with the electromagnetic log is stable and reasonable. Figures 5.12-5.23 present the maneuver simulations that result from the identified coefficients. As expected the small rudder turn is the maneuver most accurately predicted with the Extended Kalman Filter due to the linear nature of the filtering process. Simulation of the progressively more severe turns, as one would expect, introduces more error between the measured and simulated data. Much of the error in the prediction of forward speed can, however, be attributed to the effect of submarine roll velocity on the electromagnetic log measurement. As the submarine rolls, another flow component is introduced to the resultant velocity measured by the log speed. This additional component is maximum when roll velocities are greatest and disappears when the submarine returns to a steady state. The presented curves have not been corrected for this rolling component, but it is noted that the major differences in speed prediction occur during the period of greatest roll velocity and would account for a much of the error in the forward speed simulation. The differences between the yaw rate curves is reasonable when it is considered that the trial curve has been derived, not measured, and its true value is different than the value found with the B-spline method. Additionally, the data from which the curve is derived comes from a stabilized gyrocompass that will reflect the heading change about the earth’s vertical axis which is not the axis defined in the state equations. The effect is that as the submarine rolls from the horizontal the gyrocompass under predicts the yaw rate since it is reflecting only a component of the true yaw rate that differs by the cosine of the roll angle. The end result is a derived yaw rate that is smaller than the true value that would be predicted with a simulation. Although the curves have not been corrected for
this roll angle effect, the error in yaw rate is greatest during the period of greatest roll.

The identified coefficients are presented in Table 5.2. Examples of the Extended Kalman Filter identification of these coefficients are given in Appendix F.
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model</th>
<th>Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y_v</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>Y_r</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>I_z - N_r</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>N_v</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>N_r</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>N_δ</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>Y_δ</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>N_o</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>Y_o</td>
<td>1.000</td>
<td>†</td>
</tr>
<tr>
<td>X_r + m</td>
<td>1.000</td>
<td>†</td>
</tr>
</tbody>
</table>

Table 5.2: Summary of Identified Maneuvering Model Coefficients
† This comparison was not possible because actual model test coefficients were not made available.
<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Initial Value‡</th>
<th>Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{vn}$</td>
<td>1.000</td>
<td>12.489</td>
</tr>
<tr>
<td>$X_{vvn}$</td>
<td>1.000</td>
<td>1.070</td>
</tr>
<tr>
<td>$Y_{vn}$</td>
<td>1.000</td>
<td>1.116</td>
</tr>
<tr>
<td>$X_{ve}$</td>
<td>1.000</td>
<td>1.032</td>
</tr>
<tr>
<td>$Y_{vee}$</td>
<td>1.000</td>
<td>1.088</td>
</tr>
<tr>
<td>$N_{vee}$</td>
<td>1.000</td>
<td>1.088</td>
</tr>
</tbody>
</table>

Table 5.3: Summary of Identified Nonlinear Coefficients

‡ These initial values were derived using the method discussed in Appendix D. The identified values represent combination of nonlinear effects as shown in Section 3.2.5.
Forward Speed Simulation

Figure 5.12: Small Rudder Turn Forward Speed Simulation
Figure 5.13: Small Rudder Turn Yaw Rate Simulation
Sway Speed Simulation

Final Identification: Small Rudder Turn

Figure 5.14: Small Rudder Turn Sway Speed Simulation
Figure 5.15: Small Rudder Turn Heading Angle Simulation
Forward Speed Simulation

Figure 5.16: Hard Rudder Turn Forward Speed Simulation
Figure 5.17: Hard Rudder Turn Yaw Rate Simulation
Figure 5.18: Hard Rudder Turn Sway Speed Simulation
Figure 5.19: Hard Rudder Turn Heading Angle Simulation
Figure 5.20: Mild Turn Forward Speed Simulation
Figure 5.21: Mild Turn Yaw Rate Simulation
Figure 5.22: Mild Turn Sway Speed Simulation
 Heading Angle Simulation

Mild Turning Maneuver

Figure 5.23: Mild Turn Heading Angle Simulation
6 Conclusions and Recommendations

This work implemented the Extended Kalman Filter system identification technique that has been developed at MIT on a submerged submarine for the first time. The identification was successful producing measurements of the full scale submarine's resistance coefficient, thrust deduction, wake fraction, propulsive coefficients, and maneuvering coefficients. All of the initial objectives of this research have been successfully met with excellent results. Specific conclusions are made in the following section.

6.1 Conclusions

1. Successful implementation of the Extended Kalman Filter on a submerged submarine further confirms the work of Hwang and Liu and extends the applicability of the system identification process.

2. A successful identification was achieved using maneuvers that were different than those used previously with the EXXON PHILADELPHIA and ESSO OSAKA. This proves that good results can be achieved solely from a set of turning trials, eliminating the requirement for a zig zag type maneuver.

3. The accurate simulations formed with the identified values prove that the surge state model accurately reflects the dynamics of the acceleration and deceleration of a submarine.

4. The resistance coefficient of the full scale submarine was measured by two separate maneuvers with essentially no difference between the two values. Verification of the measured value proves the accuracy and applicability of the identification method.
5. The ship propeller thrust coefficients $\eta_{1shp}$, $\eta_{2shp}$, and $\eta_{3shp}$ were successfully derived as by-products of the resistance coefficient measurement showing a 4.0% increase above the model test coefficients.

6. The ship full scale wake fraction and thrust deduction were identified, but no comparison to model test values was possible because model test values were unavailable at this time.

7. The horizontal plane maneuvering state model reasonably reflects the dynamics of a maneuvering submarine, but improvements are necessary to include roll and pitch effects.

8. The modified Extended Kalman Filter provided an accurate and stable identification of the maneuvering coefficients. This was the first time a modified filter was used to perform an identification of the ship system state model defined in this work. The result was very encouraging, proving that measurement coupled measurements can be used to produce good identification results.

6.2 Recommendations

1. To further refine the results of this work it is recommended that the maneuvering data be corrected for the errors induced by the roll angle and velocity. Correcting for each of these would lessen the error in each of the simulated maneuvers.

2. To produce more accurate results, new sea trials should be conducted that will provide uncoupled speed measurements with a Doppler speed log. Since the
accuracy of the output depends on the accuracy of the input an alternative measurement method must be found to replace the electromagnetic log measurements used in this work.

3. To improve the results of this work the added mass coefficients should be recalculated using the MIT Radiation and Refraction Program. The program would provide accurate values for the added mass terms which in turn would improve the accuracy of the identified resistance coefficient.

4. The state model should be modified to include any relevant roll and pitch dynamics. This would increase the accuracy of the state models description of the submarines motion and thus improve the application of the Extended Kalman Filter. Care must be taken however not to introduce so many new terms that any identification becomes impossible.

5. Conduct maneuvering trials on submarines of different classes so that the relationship between model and actual full scale submarines (Reynolds numbers) can be determined. This would greatly aid the design process by providing the designers with information that will allow them to predict the true performance of a full size submarine from model test data.
7 References

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17. Class Notes

MIT course 13.05 "Applied Hydrodynamics of Ship Design"

Instructed by Professor Martin Abkowitz, 1988.
8 Appendices:
A Calculation of Added Mass Coefficients

The added mass and moment coefficients \( X_v, Y_v, \) and \( N_v \) were calculated with potential flow theory using dimensions readily available in public literature.[15]. To use potential theory the submarine was approximated as an ellipse in each of two ways:

- calculate the coefficients using an ellipsoid of revolution that has the same length to diameter ratio \( \frac{L}{D} \) of the actual submarine.

- calculate the terms using an ellipse which has the same displacement and length of the actual submarine.

The additional added mass of the submarine's sail (conning tower) was determined using a flat plate approximation. The added mass of the diving planes and rudders was neglected due to the relatively small size of these appendages.

A.1 Hull Added Mass

To determine the added mass of the hull using the ellipsoid approximations previously discussed we refer to Lamb[8] who calculated the ellipse added mass coefficients \( k_1, k_2, \) and \( k_3 \) over a wide range of \( \frac{L}{D} \) ratios. The actual added mass values are determined with the following relations:

\[
X_v = k_1 \rho \nabla
\]  \hspace{1cm}  (A.1)

\[
Y_v = k_2 \rho \nabla
\]  \hspace{1cm}  (A.2)

\[
N_v = k_3 I
\]  \hspace{1cm}  (A.3)
where $\nabla =$ volume displacement of the submarine

$\rho =$ density of saltwater

$I =$ moment of inertia of the water displaced by the ellipse defined as

$$I = \frac{\pi}{120} \rho LD^4 \left[ \left( \frac{L}{D} \right)^2 + 1 \right]$$  \hspace{1cm} (A.4)

Using the first method of approximation the added mass terms were calculated using the $k$ values determined at the actual submarine $\frac{L}{D}$ of 10.97. In the second case an $\frac{L}{D}$ of 10.03 was calculated by setting the ellipse volume equal to that of the submarine while allowing the diameter to vary. The $k$ values for each case are shown in Table A.1 along with the calculated nondimensional added mass coefficients of the hull.

<table>
<thead>
<tr>
<th>$\frac{L}{D}$</th>
<th>10.97</th>
<th>10.03</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>0.0170</td>
<td>0.0200</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.9625</td>
<td>0.9620</td>
</tr>
<tr>
<td>$k_3$</td>
<td>0.8850</td>
<td>0.8800</td>
</tr>
<tr>
<td>$X_u$</td>
<td>$1.690 \times 10^4$</td>
<td>$1.988 \times 10^4$</td>
</tr>
<tr>
<td>$Y_v$</td>
<td>$9.569 \times 10^3$</td>
<td>$9.564 \times 10^3$</td>
</tr>
<tr>
<td>$N_r$</td>
<td>$3.883 \times 10^4$</td>
<td>$4.540 \times 10^4$</td>
</tr>
</tbody>
</table>

Table A.1: Hull Added Mass Coefficients
A.2 Appendage Added Mass

The added mass of the sail was calculated using equation A.5 as it is presented in [17] with the span being doubled to reflect the presence of the hull next to the sail as illustrated in Figure A.1.

![Diagram](image.png)

Figure A.1: Geometry of Submarine Hull and Sail
\[ a = \frac{\pi \rho s^2 c^2}{4 \sqrt{s^2 + c^2}} \left[ 1 - \frac{0.54}{\left( 1 + \frac{\varepsilon}{h} + \frac{1}{c} \right)} \right] \]  

(A.5)

where \( s \) = span as defined in Figure A.1
\( c \) = chord as defined in Figure A.1
\( \rho \) = density of salt water
\( a \) = added mass of the entire plate (twice that of the sail)

The additional added mass values due to the sail were found to be:

\[ X_{u,sail} = 0 \]

\[ Y_{v,sail} = 0.249 \times 10^{-3} \]

\[ N_r = 1.312 \times 10^{-7} \]

The total values of the calculated added mass are shown in Table A.2. In addition, the percent difference between each of the two cases is presented along with the percentage of ship mass that each represents.

The added mass values at the length to diameter ratio of 10.97 were selected to be used because the ellipse sizing assumptions for these values are the more standardly accepted when determining the added mass coefficients with potential theory. Assuming the same \( \frac{L}{D} \) ratio is also the more geometrically "correct" of the two methods because the ellipse and hull shapes do not differ greatly and the appendages are evaluated individually.
<table>
<thead>
<tr>
<th>( \frac{L}{D} )</th>
<th>( X_a ) (10^4)</th>
<th>( Y_v ) (10^3)</th>
<th>( N_v ) (10^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.97</td>
<td>10.03</td>
<td>10.97</td>
<td>10.03</td>
</tr>
<tr>
<td>Total Added Mass</td>
<td>1.690</td>
<td>1.988</td>
<td>9.818</td>
</tr>
<tr>
<td>% difference</td>
<td>16.48%</td>
<td>.06%</td>
<td>16.90%</td>
</tr>
<tr>
<td>% of ship mass( ^\dagger )</td>
<td>1.65%</td>
<td>1.94%</td>
<td>95.78%</td>
</tr>
</tbody>
</table>

Table A.2: Total Submarine Added Mass Coefficients

\( ^\dagger \) X\(_G\) value was removed from \( N_v \) before the percent mass was calculated.
B  Noise Analysis of Sea Trial Data

The noise level of the sea trial measurements was evaluated with the Box Jenkins method as employed by Liu[10]. Liu's noise analysis program uses a NAG library [13] subroutine that fits a nonseasonal autoregressive integrated moving average (ARIMA) model to the sea trial data with a nonlinear least squares technique that includes backforecasting. The non-seasonal model is represented by:

\[ \nabla^d x_t - c = w_t \]  \hspace{1cm} (B.1)

where  \( x_t = x_1, x_2, ..., x_N \) is the observed time series

\( \nabla^d \) is the result of applying the non-seasonal differencing of order \( d \) to the observed series \( x_t \)

\( c \) = the expected value of the differenced series

\( w_t = w_1, w_2, \text{ and } w_N \) is a stationary, zero mean ARMA model.

The model parameter estimates, in this case the sea trial measurements, and \( c \) are formed by minimizing the quadratic form of \( w_t \). The error estimate is obtained from the form of \( w_t \) which is represented as:

\[ w_t = \phi_1 w_{t-1} + \ldots + \phi_p w_{t-p} + a_t \ - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q} \]  \hspace{1cm} (B.2)

where \( \phi_1, \ldots, \phi_p \) are the autoregressive parameters, \( \theta_1, \ldots, \theta_q \) are the moving average parameters, and \( a_t \) is a zero mean white series with constant variance \( \sigma_a^2 \). The residual series, \( a_t \), provides the error estimate for the modeled time series which is the standard deviation \( \sigma_a \). This model was used to determine the measurement noise estimates for all of the sea trial maneuvers that were used by the Extended Kalman Filter. The zero mean noise residuals are presented in Figures B.1 - B.10.
Figure B.1: Noise of Forward Speed Measurement- Small Rudder Turn
Noise of Measurement - Heading Angle

Figure B.2: Noise of Heading Angle Measurement- Small Rudder Turn
Figure B.3 Noise of RPS Measurement- Small Rudder Turn
Figure B.4: Noise of Forward Speed Measurement - Hard Turn
Figure B.5: Noise of Heading Angle Measurement - Hard Turn
Figure B.6: Noise of RPS Measurement - Hard Turn
Figure B.7: Noise of Forward Speed Measurement- Acceleration Trial
Figure B.8: Noise of RPS Measurement - Acceleration Trial
Figure B.9: Noise of Forward Speed Measurement - Deceleration Trial
Noise of Measurement - RPS

Figure B.10: Noise of RPS Measurement - Deceleration Trial
C Derivation of Yaw Rate Curves

The sea trial data supplied for this work did not include the measured yaw rate for any of the maneuvering trials. To allow the identification process to continue, it was necessary to derive yaw rate curves for each of the turning maneuvers. Several methods were investigated that would allow this data to be calculated directly from the heading angle record of each trial. The methods investigated include:

- Finite difference methods applied directly to the heading angle data set.
- Polynomial curve fitting and differentiation of heading angle data.
- Polynomial data smoothing of finite differenced heading angle data.
- Cubic-Spline curve fitting and differentiation of heading angle data.
- Cubic-Spline smoothing of finite differenced heading angle data.
- Fast Fourier Transform low pass filtering of finite differenced heading angle data.

Each of these methods were evaluated for their ability to provide an accurate and smooth yaw rate curve. The finite difference method was found to be unsatisfactory because the "caging" noise induced by the gyrocompass is greatly magnified when the data set is differentiated. The noise in the differenced data proved large enough to make the data smoothing alternatives less attractive since they would admit a wider variety of solutions than if lower noise levels were present. Low pass filtering was also found not to be the best solution since the frequency difference between the true signal and the noise was not great enough to allow for accurate and clean filtering. The remaining solutions involve fitting a curve to the heading angle data before the curve is differentiated. The polynomial method was rejected due to the potential difficulty in fitting a curve of
unknown order to the data set, which in itself can be an arduous task. The cubic-spline makes curve fitting much more simple with no guess work necessary in determining the order of polynomial to be fit to the data. For these reasons the cubic-spline curve fitting method was selected for the derivation of the yaw rate curves.

A cubic-spline is a series of cubic polynomial sections that are linked together so that the first and second derivatives of the linked sections match. Linking in this manner assures link continuity and produces a full continuous curve that can be shaped easily by simply linking different sections. To perform the cubic-spline fit two NAG library[13] subroutines were used. The first routine computes the best fit cubic-spline, $S(x)$, using a weighted least squares approximation to the input data $(x_i, y_i)$. The optimal solution is the minimization of the sum of the squares of the weighted residual $R(x_i)$ as described by:

$$ R(x_i) = W(x_i) \times [S(x_i) - y_i] $$

(C.1)

where $W(x_i)$ represents the weighting factor assigned to the data point $x_i$. The optimal cubic-spline coefficients $C(1), C(2), \ldots$ are determined so that the curve can be represented with the normalized B-spline series, $N(x_i)$, such that:

$$ S(x) = C(1) \times N_1(x) + C(2) \times N_2(x) + C(3) \times N_3 + \ldots $$

(C.2)

This best fit spline is then differentiated by the second NAG routine which provides the value of the spline and its first three derivatives at each point $x_i$. This method of evaluating the derivatives provides smooth and accurate functions that are useful for the system identification process. The curves derived with this method are presented in Figures C.1 - C.3. To provide a measure of curve accuracy and smoothness each figure also contains the data obtained with the finite difference method. It is readily apparent that the cubic-spline produces reasonable and smooth yaw rate curves.
Derived Yaw Rate

Small Rudder Turn

Figure C.1: Derived Yaw Rate - Small Rudder Turn
Figure C.2: Derived Yaw Rate- Mild Turn
Figure C.3: Derived Yaw Rate- Hard Turn

D Conversion of Model Test Coefficients

There are many ways in which the state equations and their coefficients many be represented to reflect the proper dynamic relationships. In this case, the submarine model
the data was supplied by David Taylor Research Center (DTRC) which uses a different form of dynamic equation representation in their maneuvering model. Instead of using the terms of the Taylor expansion through the third derivative as has been done in this work, the DTRC model uses a modified second derivative to reflect the odd nature of the coefficient functions. The second derivative is evaluated with the state value and its absolute value so that the negative term will be carried through if the state variable changes sign. As an illustration of the form of the DTRC equations some of the lateral force equation terms are presented below:

$$\ldots + Y_{vv}v|v| + \ldots + Y_{pp}p|p| + \ldots$$

The model coefficients of the submarine were supplied by DTRC so that they fit the form of the above example equation. It then becomes necessary to convert these coefficients into the cubic form which is used in the MIT Extended Kalman Filter program. To accomplish this a chi-square curve fitting program was used to fit a cubic function through the plot of the second order absolute value function [12]. The idea is simple, the program fits the function

$$y(x) = \sum_{k=1}^{j} a_kX_k(x) \quad X_k(x) = 1, x, x^2, \ldots$$ \hspace{1cm} (D.1)

to the data derived from the DTRC supplied second order functions which have been plotted as a function of the state variable, i.e. $Y(v) = Y_v v + Y_{v|v|}v|v|$. The program fixes the linear term and fits the optimum third order coefficient by minimizing the chi-square error term:

$$\chi^2 = \sum_{i=1}^{N} \left[ \frac{y_i - \sum_{k=1}^{j} a_kX_k(x_i)}{\sigma_i} \right]^2$$ \hspace{1cm} (D.2)
where $\sigma_i$ is the standard deviation of the known data points.

The resulting fit curve and its coefficient are then of the proper form to be used in the state equation model as it is presented in Chapter 3 of this work.
Cubic Fit to Quadratic Simulation Model

Y direction - dimensionally perturbed

Figure D.1: Illustration of Coefficient Curve Fit
E  Acceleration Maneuver Iteration Calculations

Table E.1 is presented here to show the general numerical flow during the iteration process used to determine the parameters identified by the acceleration maneuver. Each number is presented as a percent of the initial calculated value or preferably the given model test value.
<table>
<thead>
<tr>
<th></th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
</tr>
<tr>
<td>$1 - t_{eq}$</td>
<td>-</td>
</tr>
<tr>
<td>$K_{Teq}$</td>
<td>-</td>
</tr>
<tr>
<td>$a_{eq}$</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{K}_r$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{a}$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{t}$</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{\eta}_3$</td>
<td>-</td>
</tr>
<tr>
<td>$\eta_{3,hp}$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\eta_{11,hp}$</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\eta_{12,hp}$</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table E.1: Summary of Calculations - Acceleration Maneuver
Table E.2 is presented here to show the general numerical flow of the Recurrence Method used to determine the parameters identified by the deceleration maneuver. Each number is presented as a percent of the initial calculated value or preferably the given model test value.

<table>
<thead>
<tr>
<th></th>
<th>By Recurrence</th>
<th>Corrected at $w_{wm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1-w$</td>
<td>1.000</td>
<td>0.970</td>
</tr>
<tr>
<td>$J_{wm}$</td>
<td>1.000</td>
<td>0.970</td>
</tr>
<tr>
<td>$K_{rwm}$</td>
<td>1.000</td>
<td>0.630</td>
</tr>
<tr>
<td>$a_{eq}$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$k$</td>
<td>1.000</td>
<td>1.030</td>
</tr>
<tr>
<td>$a_{wm}$</td>
<td>1.000</td>
<td>0.661</td>
</tr>
<tr>
<td>$C_R$</td>
<td>0.910</td>
<td>0.967</td>
</tr>
<tr>
<td>$\frac{1-w_{nom}}{1-w_{eff}}$</td>
<td>0.970</td>
<td>-</td>
</tr>
</tbody>
</table>

Table E.2: Summary of Calculations - Deceleration Maneuver
F Examples of Maneuvering Coefficient Identification Curves

The following plots are some examples of the identification output of the Extended Kalman Filter for the maneuvering trials. The remainder of the curves have been left out in the interest of brevity.
Identification of Yvrr

Figure F.1: Identification of Yvrr
Identification of Yv

Figure F.2: Identification of Yv
Identification of Ndelta

Small Rudder Turn

Coefficient Value (% of Identified)

Filtered Value
Identified Value

Figure F.3: Identification of Nδ
Identification of $X_{vr+m}$

Figure F.4: Identification of $X_{vr+m}$
u = forward speed

\[ k = \frac{u_A}{u_{A\infty}} \]

Figure 3.3 Mean Axial Velocity Induced by a Semi-infinite Tube of Ring Vorticies Determined by the Law of Biot-Savart[7]

The velocity induced by the propeller is determined using:

\[ u_{A\infty} = -(1 - w)u + \sqrt{(1 - w)^2u^2 + \frac{8}{\pi}K_r(nd)^2} \] (3.2)

where \( K_r \) = propeller thrust coefficient defined as

\[ K_r = \frac{T}{\rho n^2 D^2} \] (3.3)

\( n \) = propeller RPS

\( D \) = propeller diameter

\( T \) = propeller thrust.