CREDIT RATIONING AND THE
COMMERCIAL LOAN MARKET

by

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ABSTRACT

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Dwight M. Jaffee

Submitted to the Department of Economics on May 10, 1968, in partial fulfillment of the requirement for the degree of Doctor of Philosophy.

This study formulates and tests a structural model of the commercial loan market with special consideration given to the question of non-price credit rationing by the commercial banks. Commercial loans are defined as short term credit extended by the commercial banks to business firms. The formal structure of the model consists of two equations on the supply side, explaining the degree of credit rationing and the commercial loan rate, and one equation on the demand side explaining the quantity of loans outstanding. The empirical tests consist primarily of analyzing ordinary least squares estimates of the model using quarterly data, both aggregated and disaggregated by asset size classes, for the post-Accord period.

Credit rationing is said to occur when there exists an excess demand for commercial loans at the ruling commercial loan rate. Three basic questions are raised with respect to credit rationing: (1) Is it rational for commercial banks to ration credit by means other than price? (2) Can credit rationing be measured, and, if so, are there significant variations in rationing over time? (3) Is the impact of credit rationing on real expenditure decisions of firms important? Affirmative answers for the first two questions are derived from the formulation of a model of commercial loan supply and credit rationing and the related empirical tests. In addition, a time series proxy variable for the degree of credit rationing is also derived. The discussion of the third question centers on the proposition that a redistribution of trade credit from large firms to small firms may reduce the direct impact of credit rationing on real expenditures. This redistribution of trade credit in periods of high credit rationing is confirmed by estimating time series equations involving trade credit and by analyzing data from a survey of the impact of the tight money conditions of 1966.

The commercial loan rate equation is based on a partial adjustment model with the timing of rate changes depending on a signal in the form of changes in the Federal Reserve discount rate. Furthermore, the desired commercial loan rate is specified as a function of the degree of credit rationing (as measured by the rationing proxy) as well as more traditional variables. The demand for loans is derived from a model containing a partial adjustment of the firm's long term liabilities and requiring that the balance sheet identity be satisfied. In this way the short run "buffer" function of loans is explicitly identified. The quantity of loans outstanding is then obtained as a function of the loan demand and the degree of credit rationing.

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PREFACE

I wish to express appreciation to the members of my thesis committee, Professor Franco Modigliani, chairman, and Professors Donald Farrar and Franklin Fisher, for their careful reading of the manuscript and many helpful suggestions. The debt of gratitude I owe to Professor Modigliani would take more than a Life Cycle to repay.

This research originated in conjunction with the Federal Reserve-M.I.T. Econometric Model project under the joint leadership of Professors Albert Ando and Franco Modigliani and Dr. Frank de Leeuw. I want to thank all the members of this research group who have helped me and, particularly, Professor Ando for his many suggestions and encouragement. I have also benefited from discussions with many fellow students at M.I.T. Phillip Cooper has been especially generous in providing help with econometric and programming problems.

Computations were performed at the computer center of the Sloan School of Management at M.I.T., primarily using the Time Series Processor ("TSP") program originally developed by Robert Hall. Data collection was financed in part by funds from a Ford Foundation grant to M.I.T. General financial aid was received in the form of a Woodrow Wilson Fellowship and a N.D.E.A. Fellowship. Miss Beatrice Rogers has done a fine job of typing the manuscript.

On the home front, my parents have provided encouragement during many years of education. Most importantly, were it not for the good humor of my son Jonathan and the many sacrifices of my wife Annette, this study could not have been completed.
"I'm sorry. We don't lend a zillion dollars to anyone."

Drawing by Mulligan;
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CHAPTER I

INTRODUCTION AND SUMMARY

A. INTRODUCTION

This study formulates and tests a structural model of the commercial loan market with special consideration given to the question of non-price credit rationing by the commercial banks. Commercial loans are defined as short term credit extended by commercial banks to business firms. This credit primarily serves to finance short term assets such as inventory stock, accounts receivable, and other liquid assets. Producers' durable equipment and structures are frequently financed by term loans, which are commercial loans with maturity exceeding one year.

It is impossible to separate true short term commercial loans and term loans on an aggregate time series basis because of data limitations. In fact, the measure of commercial loans used in this study, an unpublished Federal Reserve series, is known to contain at least some term loans. The lack of availability of separate short term commercial loan data has a conceptual basis as well as the more obvious pragmatic origin. The conceptual basis arises from bankers' reluctance to deny a request from a customer for a loan extension beyond the original maturity. To deny such a request from a financially sound company would jeopardize the bank's "customer relationship" which can be built only over a long period of time.¹ In the case of

¹Hodgman (1963) provides a complete discussion of the implications of maintaining good "customer relationships" for bank behavior.
potential bankruptcy, the large costs of collection make it expeditious to extend the loan in hope of obtaining full repayment in the future. The consequence of this "automatic" loan renewal is that, in practice at least, commercial loans and term loans are excellent substitutes for one another; and, indeed, the actual maturities in many cases will be nearly equal. Therefore, we do not feel that the data limitation is a serious problem for this study.²

The formal structure of the loan market model consists of two equations on the supply side, explaining the degree of credit rationing and the commercial loan rate, and one equation on the demand side, explaining the quantity of loans outstanding. In addition, the impact of credit rationing on the demand and supply for trade credit by business firms is also given attention as a first step in the direction of obtaining information on the relationship between credit rationing and real expenditures. The empirical tests consist primarily of analyzing ordinary least squares estimates of the basic equations. The rationing and commercial loan rate equations are fitted to aggregated data for the period 1952-II to 1965-IV. The loans outstanding equation is fitted to both aggregated and disaggregated data (by asset size) for the period 1952-III to 1965-IV; the period of fit starts one period later than the supply equations because of data limitations. Extrapolations of the fitted values into 1966 are also obtained for the three basic equations of the model. The relationship between net trade credit and credit rationing is studied by estimating time series equations and by considering survey data.

²Budzeika (1964) provides an interesting discussion of term loans and confirms the practical problems of segregating commercial loans and term loans.
The importance of commercial banks and the commercial loan market in transferring Federal Reserve monetary policy from the financial markets to the real sectors of the economy has been well recognized in discussions which take into account the institutional characteristics of the financial sector. Clearly, Federal Reserve action, whether it be a discount rate change, a reserve requirement change, or an open market security transaction, has its primary and direct impact on the commercial banks; and the effectiveness of these changes in policy parameters in influencing real expenditure decisions depends on the nature of the banks' response. In particular, the commercial loan market provides the most direct link between the banking sector and the real sector. It is for this reason that the availability doctrine, with its stress on the availability of commercial loans (and other forms of financing) rather than their cost, was considered a significant element in understanding the mechanism of monetary policy.

Until quite recently, theoretical models and econometric studies of the monetary mechanism, in contrast, have generally not given an important place to the commercial banks and the commercial loan market. The principal role assigned to banks in aggregate economic models was the gearing ratio transforming an increase in free reserves (supplied by the Federal Reserve) into an increase in the money supply. In this view, the Federal Reserve had complete and direct control over the money supply, since the commercial bank "reserve multiplier" was independent of all endogenous economic variables. Thus, while the nature of the influence of the money
supply on the real sectors differed between the Keynesian and Quantity Theory models, in either case the role of the banking system was entirely mechanical.

Recent work by many authors has suggested, however, that the determinants of the money supply are more complex.³ In their view, the banks' desired level of free reserves depends on Federal Reserve policy parameters and on interest rates, thus making the level of free reserves a function of endogenous variables. It is then but a short step to show that the money supply itself is also an endogenous variable. Stephen Goldfeld (1966) has made even further progress by incorporating the determination of the money supply into a more general theory of bank portfolio choice.⁴ His approach emphasizes the banks' demand for each asset in their portfolio, rather than only free reserves. In this way the connection between the commercial loan market and the money supply is made explicit, since commercial loans are among the most important assets held by banks.

Econometric tests of these new theories have been made concurrently with the theoretical advances and have confirmed the value of the new approach. Detailed attention has not been given to the many special "institutional" characteristics of the loan market, however, and in particular the possibility of non-price credit rationing has been slighted. These advances in the theory of money supply determination and many other recent

³For examples, see Brunner (1961), Brunner and Meltzer (1964), Meigs (1962), Orr and Mellon (1961), and Teigen (1964).

⁴Further research in this area is given in Ando and Goldfeld (forthcoming) and Modigliani and Rasche (unpublished). Goldfeld's model is based closely on the work of Tobin (unpublished manuscript) at points.
developments in the understanding of the monetary mechanism have been brought together in the Federal Reserve-M.I.T. Econometric Model project which is now being completed. Since the commercial loan rate and the volume of commercial loans are important endogenous variables in this model, a complete specification for equations explaining these variables is very important. In addition, if the model is to accurately portray the structural response of the real sectors to monetary policy, it is critical that the possibility of commercial loan credit rationing be fully tested.\textsuperscript{5}

The present study was undertaken in conjunction with the Federal Reserve-M.I.T. project with the goal of providing the necessary model and econometric estimates for the commercial loan sector. To a limited extent this has added some constraints in the choice of variables and estimation period, but, hopefully, the costs of maintaining the compatibility will be easily repaid in terms of information on the interaction of the loan market and the real sectors. This aspect of the problem is left for future work, however, pending completion of the estimation and simulation of the entire model.\textsuperscript{6}

\textsuperscript{5}See de Leeuw and Gramlich (1968) for a progress report on the Federal Reserve-M.I.T. model. The emphasis on the commercial loan market in this model can be contrasted with the Brookings Model (see de Leeuw (1965) in which commercial loans enter only as a residual.

\textsuperscript{6}It is likely that many of the special features of our model developed in detail here, particularly the sections on credit rationing, will not be included in the final version of the Federal Reserve-M.I.T. project. Thus, our study is not to be construed as an "official" report for that project.
B. SUMMARY AND CONCLUSIONS

In this section we provide a short statement of the development of the theoretical model and the results of the empirical tests, so that the harried reader may obtain at least a summary review of the work in a few pages.

1. A Sketch of the Model

In general terms the model of the commercial loan market can be viewed as the interaction of the supply of commercial loans by banks and the demand for loans by business firms. Were this a "normal" market, the variables to be determined by the supply and demand would be the quantity of loans outstanding and the price of commercial loans, namely, the commercial loan rate. The existence, or possible existence, of credit rationing, however, rules out this simple interpretation. Credit rationing is said to occur when a bank is unwilling to extend the loan demanded by a customer and supplies only a smaller amount. This can occur in the long run with the commercial loan rate at the level desired by the banks or in the short run before the rate has fully adjusted. In technical terms, credit rationing is equivalent to the existence of an excess demand for loans (by at least some customers) at the ruling commercial loan rate. It follows from this definition that even in "equilibrium" we cannot speak of "the" amount of loans, but must distinguish the banks' supply and the firms' demand, the difference being the amount of rationing.
This means there are four variables to be determined: the commercial loan rate, the loan demand, the loan supply, and the amount of rationing. Since there is an identity relating the last three variables, only three behavioral equations are necessary to provide a complete system. As already noted, the three equations actually estimated explain directly the commercial loan rate, the degree of credit rationing, and the quantity of loans outstanding. The demand for loans can then be derived by adding the amount of rationing to the quantity of loans outstanding.

In the long run, the model is simultaneous. Each of the three endogenous variables depends on its own set of exogenous variables.\(^7\) But, in addition, the commercial loan rate depends on the level of loan demand, the level of loan demand depends on the commercial loan rate, and the degree of credit rationing depends on both of these variables. In the short run, however, the model is at least "approximately" recursive. The commercial loan rate and the degree of rationing are set by the banks on the basis of exogenous variables and only the predetermined endogenous loan demand. Loan demand, in turn, is responsive to the commercial loan rate. Finally, the quantity of loans actually received by the firms, measured by loans outstanding, is a function of loan demand and the degree of credit rationing.

The reason for the approximation arises from the assumption that the pricing and rationing decisions by the banks in the short run are based on the given quantity of loans outstanding.\(^8\) The validity of this premise

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\(^7\)Almost all of the variables which are taken as exogenous for the purpose of this model -- market interest rates, business expenditures, and so on -- would of course be endogenous in any large scale econometric model.

\(^8\)The most important issue is related to the determinants of the commercial loan rate and whether bankers can be truly characterized as price-setters in this market. Goldfeld (1966), pp. 24-28, has discussed this entire question at greater length and arrives at essentially the same conclusions.
depends, of course, on the definition of short run, or, from an empirical standpoint, on the time unit used for estimation. The time unit which is actually used, a quarter, appears to be just on the margin: a longer time unit, say, of six months, would surely contradict the assumption; a shorter time unit, say, a month, would almost certainly make the assumption valid. With the technical questions of estimation procedures aside, however, it is helpful from an heuristic standpoint to identify the commercial loan rate and credit rationing with the supply side and the quantity of loans outstanding with the demand side, and we shall continue to discuss the model in this context.

2. The Supply of Commercial Loans: Credit Rationing

The possible existence of non-price credit rationing by the commercial banks has drawn considerable attention since the issue was first raised as part of the Availability Doctrine in the period immediately following World War II. Three basic questions still remain essentially unanswered, however:

(1) Is it rational for commercial banks to ration credit by means other than price?

(2) Can credit rationing be measured, and, if so, are there significant variations in rationing over time?

(3) Is the impact of credit rationing on real expenditure decisions of firms important?

We consider the first of these equations in Chapter (2) which develops a theoretical model of commercial loan supply and credit rationing. Our principal criticism of earlier attempts to answer this question is
that they focused almost solely on the supply side of the market. But credit rationing can be reasonably defined only as the difference between the quantity of loans demanded and the quantity of loans supplied at the ruling commercial loan rate. The omission of the parameters of loan demand in these earlier studies, hence, leads to only a partial solution in two senses: first, directly, information on loan supply by itself simply cannot determine the existence of credit rationing. Secondly, at least in the long run, the ruling commercial loan rate should be identified with the banks' desired loan rate and clearly this is a function of loan demand.

Our model is based on a bank which serves \( n \) customers and maximizes its expected profits, taking into account possible customer default on loans. An important distinction is drawn between the case in which the bank charges each of its customers a separate interest rate (a discriminating monopolist) and the case in which the bank classifies customers into \( m \) categories, \( m \leq n \), such that within each category a uniform rate is applied to all customers. Of course, the regime with customer classification reduces to a discriminating monopolist when \( m = n \). The bank's expected income from each loan can be formulated as an explicit function of the parameters of the demand function, the probability of repayment of any given loan, and the rate of interest charged on the loan. Thus, for both cases, we can derive the expected profit maximizing set of \( m \) rates as a function of the demand parameters, the risk parameters, the bank's opportunity cost, and the number of customer categories.
The proof of the rationality of rationing then amounts, in this view, to showing that the bank can increase its expected profits by rationing at least some customers even when it is charging the desired rate. The principal implication of the model in this connection is that the rationing of some customers may be profitable only if banks are constrained to a number of customer classes which is less than and even then only if there is a risk element present. Although this result is interesting in and of itself, it still remains to be shown that the basic assumptions of the model, that banks classify customers and that banks set the loan rate(s) at the optimal level, are at least approximately fulfilled by the United States commercial banking industry.

The basis of this correspondence can be derived from the oligopolistic structure of the commercial banking industry. At one extreme, one can imagine an industry structure in which banks participate in direct collusion, act as discriminating monopolists, and maximize industry profits. At the other extreme, a perfectly competitive solution would yield the banks zero profits. It is reasonable, however, that the true state of affairs should lie between these extremes. The oligopolistic power of the banks allows them to avoid the zero profit solution, while legal and social prohibitions against direct collusion and blatant exploitation make it impossible for the banks to maximize industry profits. This is, of course, consistent with the institutional aspect of commercial banking in which the banks do classify customers on the basis of such "objective" features as the firm's industry class, asset size, balance
sheet, and other easily recognized characteristics. In fact, to the extent that the bank's reliance on these "objective" features causes a classification of customers which differs from the optimal grouping which would be based on the risk features and demand elasticities of the customers, the expectation of rationing would become even greater.

Furthermore, in the long run one would expect the structure of rates set by the oligopolistic cartel on the m customer categories would tend to the optimal set of rates. In the short run, however, the loan rate would adjust very slowly toward the optimal level, since the time needed for the entire industry, or for at least the large banks, to agree on the desirability of a rate change would add further credence to the standard reasons for only a partial adjustment. This is of critical importance in any empirical tests of credit rationing, since it is easily shown that the amount of rationing will be increased when the banks quote a loan rate which is less than the optimal and, similarly, will be decreased when the quoted loan rate is actually above the desired level.

The empirical tests of credit rationing, referred to in the second basic question, follow the theoretical model in the sense that the amount of credit rationing is specified to be a function of the spread between the desired rate and the quoted rate in addition to the amount of rationing which occurs when the rate is in long run equilibrium level. The main problem for previous empirical studies involving credit rationing was a means of measuring the degree of rationing itself. The discussion in Chapter (3), which provides the development and results of our empirical tests of credit
rationing, shows that direct measures of rationing cannot be obtained. Fortunately, our theory of credit rationing is very suggestive about the nature of a proxy measure. The theoretical model indicates that credit rationing will take the form of limitations in the supply of loans primarily to the more risky loan customers. In fact, as already noted, risk-free firms will generally not be rationed at all. From this basis, we show in Chapter (3) that the percentage of total loans granted to risk-free firms does provide a good proxy measure for the degree of credit rationing. Furthermore, as a practical solution, a principal component of four series representing the proportion of loans granted at the prime rate and the proportions of loans which are large in size is used as the proxy measure.

The empirical experiment is based on a regression of the proxy rationing variable on the dependent variables already mentioned. The principal problem of specification is the long run desired commercial loan rate. Our formulation is based on the proposition that the marginal proceeds from all assets held by the bank must be equal in equilibrium when non-pecuniary yields are included. Thus, using Treasury bills as a convenient and important alternative bank asset, the desired loan rate is specified as a function of the Treasury bill rate with adjustment for the relative liquidity of Treasury bills, for structural changes introduced by the formation of a certificate of deposit market, and for the size of the loan portfolio.

The estimated coefficients for this regression equation have correct signs and generally are statistically significant; and the goodness of fit
is satisfactory. Three important conclusions are drawn from these results. First, the existence of rationing as an empirically important phenomenon is confirmed. Secondly, the validity of the model of credit rationing, which forms the basis of the empirical test is also supported. Thirdly, the usefulness of the rationing proxy as a measure of the degree of rationing is verified. In fact, we make direct use of this proxy in discussing the effects of credit rationing on real expenditure decisions in Chapter (5). Before summarizing these results, however, it is helpful to first consider the determinants of the commercial loan rate and the demand for commercial loans.

3. The Supply of Commercial Loans: The Commercial Loan Rate

The determinants of the desired long run commercial loan rate are developed from a theoretical viewpoint as part of the rationing model (Chapter 2), while the operational counterpart is specified in conjunction with the empirical tests of credit rationing (Chapter 3). In Chapter (4) direct tests, based on a partial adjustment model of the commercial loan rate toward the desired long run level, are provided.

A unique feature of the formulation is that the parameters of the adjustment process, which determine the timing of rate changes, are specified as functions of changes in the Federal Reserve discount rate. Changes in the discount rate, in this interpretation, serve as signals for changes in the loan rate, which allow communication between the oligopolistic set of banks without direct collusion. The empirical tests confirm the import-
ance of this mechanism and, in fact, changes in the discount rate by themselves explain a substantial percentage of the variance of the changes in the commercial loan rate variable. It should be stressed, however, that the underlying logic of this mechanism assumes that the Federal Reserve does continue to change the discount rate in a manner which allows the commercial banks to use it as an indicator. In this sense our results can be interpreted only as an explanation of past commercial bank behavior; in order to predict future changes in the commercial loan rate, one must really be able to predict "institutional" changes in Federal Reserve behavior.

The other point of interest is the specification of the desired commercial loan rate. Two possible formulations are proposed. First, one can use the specification already developed as part of the rationing tests, in which case the lagged value of the commercial loan rate must also be included. Alternatively, the rationing proxy can be used instead of the spread between the desired loan rate and the lagged value of the quoted rate, the formal equivalence of these approaches being derived in Chapter (4). Tests of both of these formulations, however, indicated collinearity between the specified variables and the changes in the discount rate. The rationing proxy, for example, enters strongly and with the correct sign only when multiplicative interaction terms are suppressed. Similarly, the direct specification of the desired commercial loan rate works well only when a simple version, based only on a long term market rate, is used.
The relative importance of the discount rate signal mechanism indicates that the commercial loan rate may deviate substantially from the desired level in the short run. This is important, since it confirms the empirical results of the existence of rationing in which short-run rationing based on a spread between the desired rate and quoted rate dominated in observable impact the rationing which occurs when this spread is zero. Furthermore, with respect to the general model of the loan market, this result is consistent with the proposition that banks act as price setters and in the short run take the level of loan demand as given.

4. The Demand for Commercial Loans and the Impact of Credit Rationing

The demand for commercial loans is developed in Chapter (5) from a formal model based on the premise that the firm's real expenditure decision is made prior to the financing decision, with the possible exception of periods of high and unexpected commercial loan credit rationing. The choice between financing the firm's net assets with commercial loans or long term liabilities ( = long term debt + equity) is emphasized together with the balance sheet identity which constrains the total of these two liabilities when all other net assets are taken as given. Furthermore, it is assumed that long term liabilities adjust only gradually to the desired level, thus creating a buffer function for commercial loans based on the need to fulfill the budget constraint in the short run. In this way the

\[9\text{Real expenditures may still depend on the cost of capital. But for our purposes we are taking this decision as given.}\]
demand for loans is derived as a function of the firm's principal assets -- inventory stock, fixed capital stock, and other net liquid assets -- as well as a function of the relative cost of commercial loan financing.

The equation which can be actually tested must have loans outstanding as the dependent variable, however, since loan demand itself is not observed if rationing occurs. At first glance it might appear that the only necessary change in the model is the addition of the amount of rationing as an independent variable with a coefficient constrained to -1. This would mean that a $1 increase in rationing would reduce both loans outstanding and loan demand by the same amount, thus leaving the coefficients of our equation unchanged. But this does not allow for the possibility that the response to rationing may involve some adjustment on the asset side of the balance sheet. Indeed, if the assets were not changed this would imply that the entire impact of credit rationing falls on long term financing, an obviously unacceptable implication. On the contrary, it is reasonable to assume that the firm does change its asset structure, in which case the direct impact of a rationing variable in the loans outstanding equation may be quite small.

This specification is tested for aggregate data, using the proxy variable for credit rationing with a free coefficient as the indicator of the additional effect of credit rationing. The results of this test confirm the general structure of the model and the small coefficient of the credit rationing variable suggests that the primary effect of credit rationing is already included in the firm's assets as specified. Additional
tests of this model were made using data disaggregated by asset size classes from the FTC-SEC Quarterly Financial Reports for Manufacturing Corporations. In general, these results confirmed the findings of the aggregate data. One important additional feature of the disaggregated data, however, is that net trade credit between business firms, which essentially cancels out in the aggregate, could be specified separately by asset class. We found that net trade credit is an important element of loan demand for all asset classes and is, in fact, more important than even inventory stock for the smaller class sizes.

This suggests a proposition, stated perhaps most strongly by Allan Meltzer (1960), that a redistribution of trade credit between large firms and small firms tends to offset the impact of credit rationing on the smaller firms. This proposition is consistent with our result that trade credit is an important element of loan demand and that almost all of the impact of credit rationing in the loans outstanding equation was included in the independent variables, among them, of course, net trade credit. As a direct test of this hypothesis, an equation explaining net trade credit was estimated for each asset class.

The dependent variable in this specification was the ratio of net trade credit extended by the class to sales (that is, essentially the average collection period) and the independent variables were the rationing proxy, the commercial loan rate, and the prime commercial paper rate. If large firms redistribute trade credit to small firms in periods of credit rationing, then one would expect a positive coefficient for the
rationing proxy in the equations for the large asset sizes and a negative coefficient in the equations for the small asset size firms. But since even the "small" firms in the sample of corporate manufacturing companies are actually relatively large compared to the rest of the business sector, it is quite possible that these "small" firms may actually extend more net trade credit in periods of rationing, although still less than the amount offered by large firms. The results of the test support this hypothesis, although the size of the coefficient does not rise monotonically with asset size.

An indirect test of the same hypothesis can be constructed by substituting the specification of net trade credit directly in the loans outstanding equation. In this way the impact of the redistribution of trade credit is explicitly included only in the rationing proxy, since the actual quantity of trade credit no longer appears in the specification. The results show that the coefficient of the rationing proxy does become more important as expected and thus this lends additional support to the hypothesis.

We conclude the study of the impact of credit rationing in Chapter (5) by analyzing data from a survey of the reactions of business firms to the tight money conditions of 1966. These data also confirm the proposition that rationed firms will substitute trade credit for bank credit. In addition, rationing causes adjustments in long term liabilities primarily for the larger firms in the sample. This suggests that the large firms which are rationed may avoid the impact of rationing by independently
obtaining other financing, while the small rationed firms must either rely on financing from large firms through trade credit or must directly reduce the level of their real assets.

This brings us back to the question raised earlier concerning the impact of credit rationing on real expenditure decisions. Preliminary and exploratory studies of the impact of credit rationing on the demand for inventory stock, fixed capital stock, and housing starts, performed as part of the Federal Reserve-M.I.T. project, could find only a small influence by credit rationing. A number of statistical problems with measuring this impact are noted in the discussion of Chapter (5). However, this result is also consistent with the existence of credit rationing, if, in fact, the redistribution of trade credit intervenes. For this to occur, the firms which supply the additional trade credit, presumably the large firms, must be able to obtain funds in periods of tight money without significantly hindering their own operations. Hopefully, empirical research on this question and the more general problem of the impact of credit rationing on real expenditures will continue.
CHAPTER 2

A THEORY OF CREDIT RATIONING*

Credit rationing has been defined as the existence of an excess demand for commercial loans at the current commercial loan rate. Furthermore, we can distinguish two forms of credit rationing, depending on the status of the commercial loan rate. The term equilibrium rationing will be used to refer to credit rationing which occurs when the commercial loan rate is set at the banks' long run desired or equilibrium level. Dynamic rationing, on the other hand, is a transient phenomenon which occurs when the commercial loan rate is only at its short run desired level.

The fundamental objective of this chapter is to provide an operational theory of credit rationing -- both equilibrium and dynamic -- consistent with (a) rational behavior by the commercial banks and (b) a realistic view of the structure and institutions of the commercial banking industry. In the literature, the theory of credit rationing has had a slow but steady development. The present work is a logical extension of this received theory in that it shares the primary goals of its predecessors while, hopefully, avoiding their failings. To clarify these goals and failings it is helpful to start with a review of the relevant literature.

*Significant portions of sections II and III of this chapter are the result of joint work with Professor Franco Modigliani. A joint publication of this material will be forthcoming. The current author retains full responsibility for any errors in the statement of the theory presented here.

1Throughout this chapter we shall refer to credit rationing in the context of commercial banks and commercial loans, although this work and the literature of which it is an extension can be interpreted within a broader context.
I. A REVIEW OF THE LITERATURE

The literature on credit rationing and the availability doctrine encompasses a collection of works which deal with a wide range of specific topics and use a variety of tools and techniques. At this point we are concerned only with the theoretical and institutional literature as it relates to the existence and mechanism of the rationing process. It is important to distinguish, at the outset, moreover, between credit rationing per se and the availability doctrine as a theory suggesting that the loan supply may be reduced by even small increases in government bond yields. As Kareken has pointed out, the failure to make this distinction has led to a substantial confusion on some important points. Thus, we will consider these arguments separately.

The two major conclusions which we wish to draw from this review can be summarized:

(1) The availability doctrine has only limited significance independent of a theory of credit rationing. For, in arguing that the banks restrict the availability of credit, the proponents of this doctrine must face the issue why the banks do not simply raise the interest rate they are charging their customers. That is, why not price ration? This, of course, is precisely the question which a theory of credit rationing must answer.

(2) Even those studies with the explicit goal of providing a theoretical justification for credit rationing fail to meet the basic issue, since they consider only the supply side of the market and neglect other very relevant questions. Only when the demand for loans and (hence) the determinants of the commercial loan rate are integrated with the supply can a complete theory be developed.

A. AVAILABILITY DOCTRINE

The availability doctrine came into prominence at the end of World War II as an alternative to the then accepted theoretical and empirical views on the monetary mechanism. The received theory indicated that for monetary policy to be effective, real expenditure decisions should be interest elastic and the monetary authority should have the ability to effect the necessary fluctuations in the relevant rates. The conditions at the end of World War II led to a rather pessimistic evaluation of monetary policy if one accepted this theory. Empirical studies available at the time indicated very little interest elasticity in any important expenditure functions. In addition, strong limitations were placed on the Federal Reserve's ability to influence the level of the interest rate on government debt due to the Treasury's desire to minimize its

3Our review of the availability doctrine is brief and highlights only the relevant analytic points. For a more complete statement of the doctrine and the events of that period, see Ellis (1951), Fforde (1951), Lindbeck (1962), Roosa (1951), Scott (1957a) and Tobin (1953).

4This received theory is discussed in the works cited above.
interest costs and the fear of disrupting this security market with widely fluctuating rates. Not surprisingly, the Federal Reserve readily endorsed the new theory.

The principal proposition of the availability doctrine was that a small variation in the rate on government securities -- achieved through open-market operations -- would indirectly influence real expenditures. Variations in government security prices, by creating uncertainty about the future course of these prices, would cause financial intermediaries -- in our case, commercial banks -- to restrict their supply of credit to the real sector. It was assumed, at least implicitly, that this reduction in the availability of credit would reduce real expenditures. The sensitivity of the financial sector to government bond yield fluctuations was greatly increased, the doctrine stressed, by the penchant of large intermediaries to hold a large share of the newly created federal debt. Several lines of argument were used to establish the proposition that the increased uncertainty following open-market sales by the Federal Reserve would lead to a restriction in the availability of credit. Ira Scott provided a convenient analytic model for deriving this result which is worthwhile considering in some detail.\(^5\)

Scott considers a financial intermediary, again we shall say a commercial bank, which holds in its portfolio two securities -- government bonds and private loans. The bank's expected return and the standard deviation of the return are given by \(e_g\) and \(\sigma_g\) and \(e_p\) and \(\sigma_p\) for the

two securities, respectively. Two different utility functions are considered. In one case, the bank maximizes its expected return constrained by some maximum level of risk (variance of portfolio). In the second case, following the work of Markowitz, a more general utility function allowing for substitution between risk and return is used. Scott considers how the portfolio composition changes when there is a \textit{ceteris paribus} increase in the risk associated with the return on the government bond. The answer for both utility functions, with Scott's assumptions, is that the bank will unambiguously increase the percentage of government bonds in its portfolio. This is easily seen in the case of the maximum variance utility function. For the initial portfolio composition, the increased risk associated with the government bond portion will increase the risk of the entire portfolio above the acceptable degree. But since the risk on government bonds is assumed, always, to be less than the risk on the private loans, the aggregate risk of the portfolio can be decreased only by transferring funds from the loans to the bonds. Similar results are also derived for the more general utility function\textsuperscript{6} Scott also notes that a \textit{ceteris paribus} increase in the yield on governments, e.g., leads, in both cases, to an unambiguous increase in the percentage of government bonds held.

This would appear to confirm the principal proposition of the availability doctrine. Both the increase in yield on government bonds and the resulting increase in uncertainty about their future prices

\textsuperscript{6} Scott, \textit{op. cit.}, pp. 46-48.
would lead to a decrease in the amount of the portfolio allocated to private loans. But there is a difficulty. The assumption of *ceteris paribus* implies that the commercial loan rate, the rate on private loans, remains fixed when the rate on government bonds changes. Scott, recognizing this point, assumes in a footnote that there is "stickiness in customer loan rates." The implication of this assumption can be easily seen in Figure 1.

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*Figure 1*

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7Scott, *op. cit.*, p. 46.
The demand curve in that figure is drawn under the assumption that there is some elasticity in the demand for credit, although we also consider the case of zero elasticity. The positive slope of the supply curve \( S_1 \) can be derived easily from Scott's smooth utility function. A *ceteris paribus* increase in the private loan rate causes a redistribution of the bank's portfolio toward private loans. First consider an initial position in which the loan rate \( r_1 \) and loan \( L_1 \) are determined by the intersection of the demand curve and initial supply curve. Now suppose the Federal Reserve undertakes an open market sale increasing the yield on government bonds and also increasing their risk. The supply curve will shift leftward, say to \( S_2 \). Now there are two cases to consider. In the first case, we shall assume that the loan rate is flexible and adjusts toward the new equilibrium value given by \( r_2 \). The amount of credit extended is now reduced to \( L_2 \). Under these conditions the availability doctrine differs very little from the earlier theory of the monetary mechanism. An increase in the government bond rate does decrease the amount of credit extended to the private sector, but the amount of reduction depends on the elasticity of the demand curve. In the polar case of a completely inelastic demand for private credit, even after the shift in the supply curve \( L_1 \) credit would still be extended.

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8 Kareken, *op. cit.*, has used a similar diagram and technique, although his object was different.

9 The availability doctrine, as evaluated here, does make one contribution. Given some elasticity in the demand for credit, the reduction in the amount of credit outstanding is achieved with (perhaps) a small change in the level of the government bond rate. The private bond rate would still fluctuate significantly as noted earlier by Kareken, *op. cit.*, pp. 300-302. This result was no doubt of much greater significance in the milieu of the pre-Accord period in which the availability doctrine first was postulated than it is today.
We can also consider the second case in which the loan rate remains at its initial value of \( r_1 \) even after the shift in the supply curve. Under this circumstance the bank would extend loans in the amount of \( L_3 \) and there would be an excess demand or credit rationing given by \( (L_1 - L_3) \). If this did occur, the availability doctrine would have made a significant contribution, for the amount of credit extended, \( L_3 \), would be independent of the shape or slope of the demand curve. But why would the banks leave the rate at \( r_1 \)? Scott's reliance on "stickiness" in this rate is more a statement of a necessary condition for his conclusion than an explanation.\(^{10}\) It is precisely this explanation, as well as conclusion, which our theory of credit rationing will attempt to clarify.\(^{11}\)

In addition to the risk-uncertainty line of argument, Robert Roosa and other early proponents of the availability doctrine stressed one other aspect of financial intermediary behavior in formulating their argument. This aspect of the theory is summarized under the heading of "locked-in" or "pinned-in" effects. Suppose, the argument runs, that the Federal Reserve makes an open market sale which causes long term bond rates to rise and causes capital losses for bankers holding these securities. Then, to

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\(^{10}\)Hodgman (1960) has also noted Scott's reliance on an unexplained stickiness in the commercial loan rate.

\(^{11}\)Before leaving Scott's model it is worth making one digression for future reference. Consider again the basic model with a maximum risk utility function, but with two forms of private loan securities, say \( P_1 \) and \( P_2 \), where the risk of \( P_1 \) exceeds the risk of \( P_2 \). Following Scott, assume that the rates on these securities are fixed at some initial values. In this case, should the rate and riskiness of the government bond increase, there would be a substitution away from the most risky security, \( P_1 \), as before. Now, however, although we know that the sum of the amount of the portfolio held in security \( P_2 \) and in government bonds must increase, the re-allocation between these securities depends on the parameter values. In particular, it is conceivable that the amount of the less risky loan, \( P_2 \), held by the bank will actually increase.
induce the bank to shift into private securities, the yield on these securities must be high enough to cover the additional risk, the transactions costs, and most importantly, the capital loss. Do private yields rise enough to induce banks to extend additional credit to the private sector? Roosa felt that they did not.\textsuperscript{12} Warren Smith and others felt that frequently they would rise sufficiently.\textsuperscript{13} As should be clear, we are again at the same impasse. If private yields rise, then the availability doctrine is shorn of much of its impact. If private yields do not rise, we are left with the need to explain why rational utility maximizing banks do not adjust the rates, but instead allow excess demand to develop in the commercial loan market.\textsuperscript{14}

Finally, at times the proponents of the availability doctrine appeared to argue that shifts in the demand for credit, influenced by the same uncertainty which affected the banks, could be relied on to improve the effectiveness of monetary policy. In terms of our earlier analysis of the Scott paper and Figure 1, this would mean that the demand for credit would shift leftward at the same time that the supply curve was shifting. Clearly, our earlier criticism still holds. If the commercial loan rate does adjust, then the availability doctrine differs little from the earlier theory. And the availability doctrine itself provides little rationale for the loan rate not to adjust.

\textsuperscript{12}Roosa (1951), pp. 286-293.

\textsuperscript{13}Smith (1956), pp. 589-593.

\textsuperscript{14}Another variant of this argument was based on the premise that the banks had a phobia against taking capital losses, even when the switch was profitable.
B. CREDIT RATIONING

References to credit rationing in England can be found as early as the period before the usury laws were repealed.\textsuperscript{15} Keynes, in his Treatise, stressed the importance of rationing and the "fringe of unsatisfied borrowers,"\textsuperscript{16} although, surprisingly, it is not in evidence in the General Theory. Ellis\textsuperscript{17} provides one of the earliest links between credit rationing and the availability doctrine. This linkage was neglected in most statements of the availability doctrine as has been stressed in the section above.

Although the existence of credit rationing was a much debated topic throughout the post-war period, the discussions of the early 1950s were primarily descriptive in nature. Three different views on credit rationing can be distinguished during this period:

1. The commercial loan rate adjusts slowly and rationing is important.

2. The commercial loan rate adjusts quickly and rationing is unimportant.

3. The commercial loan rate is sticky in the short run only and hence rationing is only a short run phenomenon.

The speed of adjustment of the rate, the element which distinguished these views, was not discussed within any analytic framework, however. Acceptance or denial of such catch-phrases as "imperfect markets," "institutional rigidities," and "administered prices" was the fashion. Furthermore, even

\textsuperscript{15} Scott (1957a), p. 538.

\textsuperscript{16} Keynes (1930), I, 212-213; II, 364-367.

\textsuperscript{17} Ellis (1951), pp. 255-256.
if we assume that these phrases did have some operational meaning, no attempt was made to show that acceptance of "administered prices," for example, would lead to credit rationing as a rational behavior pattern. Steel prices, for example, are generally believed to be administered, but it is doubtful that steel companies ration steel.\textsuperscript{18} And, on the other hand, certainly no attempt was made to give an analytic proof that the lack of administered prices would lead necessarily to the irrationality of credit rationing.

The first significant break from this view can be found in Hodgman's comments on Kareken's 1957 paper and Kareken's subsequent reply.\textsuperscript{19} The important result of this exchange was the realization that, in the absence of price rationing, some other aspect of the loan or the loan customer would be used by the bank as its rationing criterion. Potential candidates for this function would include:

1. Maturity of the loan;
2. Collateral required;
3. Length and value of the "customer relationship";
4. Amount of compensating balance;
5. Risk of partial or complete default on the loan.

A theory of credit rationing would then have as its goal to show that rational behavior by the banks would lead to their using one of these features of the loan rather than the price of the loan as the rationing device.\textsuperscript{20}

\textsuperscript{18}In the late 1940s steel was, in fact, rationed; but the issue was then centered on dynamic circumstances.

\textsuperscript{19}Hodgman (1959); Kareken (1959).

\textsuperscript{20}Guttentag (1960) appears to miss this point. His theory of credit rationing assumes that banks ration credit by altering the maturity of the loan rather than by using the rate. This begs the important question.
1. Hodgman (1960)

In his 1960 article, "Credit Risk and Credit Rationing," Donald Hodgman took up this challenge. His object was to show that credit rationing was rational even in the absence of the imperfections assumed in the earlier discussions and that the riskiness of the customer was the relevant criterion. He starts by assuming the existence of a function, \( f[x] \), which gives the bank's estimate of the probability of the firm's end of period value being \( x \).\(^{21}\) This density function is assumed to be independent of the size of the loan and is bounded in the sense that there exists a \( K \) such that

\[
f[x] = 0 \quad \text{for } x \geq K \quad \forall K \leq \infty
\]

The expected value from a given loan, \( EY \), is calculated as the sum of the expected value of repayment in full and the expected value of the available proceeds in case of partial default. The expected loss from a loan, \( EZ \), is defined as the expected value of the loss occurring when the firm fails to earn even the principal amount of the loan; hence it is independent of the interest rate charged on the loan. Utility maximization is assumed to result in the bank's requiring for each customer (i) that:

\[
(I.1) \quad \left( \frac{EY}{EZ} \right)_i \geq \left( \frac{EY}{EZ} \right)^*
\]

where \( \left( \frac{EY}{EZ} \right)^* \) is an equilibrium ratio determined by market forces.\(^{22}\)

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\(^{21}\) We have changed the notation at some points in order to retain consistency throughout our discussion.

\(^{22}\) Musgrave and Domar (1944) use a similar utility function in analyzing portfolio behavior.
To obtain a loan of given size, a customer must be able to offer an interest rate sufficiently high to satisfy (1.1). But the denominator of this ratio, EZ, does not depend on the rate of interest offered. And the numerator, though it does vary with the rate offered, has a finite bound. Thus the case may arise in which the firm cannot offer any rate which will induce the bank to grant a given loan. This is the essence of Hodgman's rationing argument.

The character of this rationing revolves around the riskiness of the firm. To see this, consider two firms similar in all respects except that the expected loss (EZ) of the first firm is higher. Then the less risky firm, with the lower EZ, can be rationed only if the bank is rationing the more risky first customer.

This also suggests a very peculiar feature of this sort of rationing. Since customers will receive their demand for some loan size whenever they can promise a sufficiently high EY, it is conceivable that the bank will be making loans to customers on the brink of bankruptcy, in the sense that the bank will take first lien on all possible returns from the investment project. This seems to describe overly risky behavior for bankers typically considered to be a conservative group. And conversely, Hodgman's model also implies that rationing occurs only when the loan demanded by the firm is so large that even the promise of the total proceeds from the project cannot induce the bank to grant the loan. Thus we find the firm willing to demand a loan for which the bank does not think

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23 Hodgman bases his argument at this point on the bounds of the density function, though this condition on EY actually holds more generally.
there is any possibility of full repayment. Such an extreme inconsistency in expectations between the bank and the firm would appear grossly unrealistic. The main point, then, is that we would never expect firms to demand loans large enough to warrant the type of rationing rationalized in Hodgman's model.24

2. Miller (1962) and Freimer and Gordon (1965)

Hodgman's model did generate a substantial amount of interest in the question of credit rationing, however, and it was the source for the models suggested by Merton Miller and Marshall Freimer and Myron Gordon. Miller's intent was to construct a model of credit rationing which would avoid the peculiar features of Hodgman's work. Instead of placing a finite bound on the density function, he explicitly acknowledges the additional costs to the bank of collecting the available proceeds in case the firm must partially default. Bankruptcy in this sense is defined whenever the gross recovery $\bar{X}$ is less than the contracted amount $(1 + r)L$. The costs of collection in this case are given as:

$$B + b \bar{X}$$

$$B > 0, \quad 1 > b > 0$$

24 The preceding argument was stressed very strongly by Sam Chase (1961). He also pointed out that Hodgman made the strong assumption that the density function, $f(x)$, and the size of the loan were independent. We discuss this aspect of the problem at greater length in section (IV.B) below. Ryder (1962) has extended Hodgman's results to the case in which the bank maximizes its long run profits on the premise that a firm once rationed will never return as a customer. See also Hodgman's replies to these comments in Hodgman (1961a) and Hodgman (1962).
Within this framework, Miller assumes that the firm maximizes its utility function.\textsuperscript{25} Attention is focused on the sign of the term \( \frac{\partial (EU)}{\partial r} \) where EU is the expected utility from the loan and r is the interest rate charged. If there are no bankruptcy costs, then for Miller's utility function at least, the sign of this partial derivative is positive. When bankruptcy costs are included, however, the expected utility reaches its maximum at a finite interest rate. These two cases are illustrated in Figure 2.

![Figure 2](image-url)

As an alternative to making the loan, the bank has the option of investing in a riskless security at an interest rate providing utility of, say, \( \bar{U} \). The firm receives a loan only when it can offer a rate with utility higher than \( \bar{U} \). For the case without bankruptcy costs, the utility

\textsuperscript{25} Miller actually uses a quadratic utility function. Very similar results can be obtained for a linear (expected value) utility function.
of the bank rises with the interest rate and thus if the firm is willing to offer a rate higher than \( r^* \) for the example in the figure, it will receive the loan. This, of course, is the traditional case of price rationing. In the case with bankruptcy costs, on the other hand, there is a limit to the utility and if, as illustrated in the diagram, the maximum utility for the loan lies below \( U^* \), the customer will not receive the loan regardless of the rate he offers. Miller calls this phenomenon credit rationing.

One drawback to this approach is the definition of bankruptcy costs. The bank must incur these costs if the firm cannot repay the contract in full. No attempt is made to distinguish between failure to repay the principal and failure to repay the interest on the principal, however. Once the loan is made, it is true that this distinction is irrelevant. In trying to explain the bank's ex ante behavior, however, it is of utmost concern. For, ex ante, there should be no cost to the bank of charging a higher interest rate; the bank gives up nothing of value. In the case used by Miller, the bank should have the ex post option of releasing the firm from the payment of the additional interest which is the cause of the bankruptcy, and thus would avoid the decline in its utility. As an alternative to Miller's assumption, we propose that bankruptcy be defined only when \( \tilde{X} < L \), that is, only when the firm cannot repay the principal. The costs of collection are still the same as in Miller's case. But the derivative \( \frac{\partial EU}{\partial r} \) is now independent of the bankruptcy costs and the marginal utility curve is the same as the case without bankruptcy costs, so that Miller's results no longer hold.
Perhaps an even more interesting point, however, is that rationing may occur in Miller's formulation even without bankruptcy costs. The reason is that the utility function has a finite upper bound as long as the expected value of the venture is finite. Under Miller's assumptions it happens that the utility level approaches this bound asymptotically. But the important point is that whenever the utility of the certain asset, $U$, exceeds this upper bound on the expected utility of the loan, in Miller's terms the customer is rationed. Consequently, there is really no qualitative difference between the cases with and without bankruptcy costs.

This is not to suggest, however, that Miller has succeeded in providing a simple general rationale for credit rationing. The problem is that he has not really asked the right question. His analysis concerns a case in which the loan request is an all-or-nothing issue; the bank's only alternative is the risk-free asset. A more realistic situation would allow the bank the alternative of granting a smaller loan to the firm, since in the general case it is certainly true that there is a loan small enough so that its expected utility exceeds $U$. This then means that the bank would make some loans but not others. Whether rationing actually occurs then depends on the interest charged by the bank and the shape of the customer's demand curve. The inadequacy of Miller's model is that neither of these elements is integrated with his formulation.

26Of course, bankruptcy costs do affect the "quantitative" character of the solution. This problem is discussed further within our own framework in section (IV.B) below.

27Freimer and Gordon (1965), p. 403, have raised a similar point, although their argument appears to stress the existence of some finite bound on the density function, an assumption which Miller explicitly disavowed.
The work of Freimer and Gordon is the most recent model of the Hodgman-Miller form. They consider the question of the optimal loan size for a given interest rate rather than the optimal rate for a given loan size, as Miller does. The locus of these optimal loans as the interest rate varies can be called "the optimal loan offer curve" and is shown in Figure 3 as the curve $\hat{L}_1 = \hat{L}_1[R]$.

The discussion of "weak credit rationing" by Freimer and Gordon is based on the fact that the size of this optimal loan reaches a finite maximum at some finite interest rate and then declines for higher interest rates. Thus, regardless of the rate the customer offers, it may not receive some large loan.

The analysis presented in this manner differs from Miller's in that the bank does consider offering the firm some smaller alternative loan. But again, the wrong question is posed. To prove that credit rationing will actually occur, it must be shown that the firm's demand exceeds the bank's optimal offer at the rate actually chosen by the bank. Thus, the existence of rationing depends on the rate chosen by the bank and the shape of the demand curve as well as the shape of the optimal loan offer curve. Since the optimal loan offer falls as the loan rate rises for sufficiently high rates, what Freimer and Gordon have shown is the existence of a "backward bending" supply curve in the loan market. But this alone is by no means sufficient to show the existence of rationing.

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28 The shape of this curve is derived rigorously in section II below.

29 Freimer and Gordon actually argue that this is valid only for the case of a "fixed size" investment project. In section IV.B below, we show that under very reasonable conditions this result can also be derived for the more general case of an "open end" investment project.
The Freimer and Gordon case of "strict credit rationing" is a much closer approach to the true problem. This form of rationing occurs when the banker chooses some interest rate and then makes loans at this rate only if the size is less than or equal to the optimal loan. To demonstrate the likely existence of strict credit rationing, they assume that the banks take as given some conventional interest rate -- 6 per cent, in fact -- and then show, using numerical examples, that it is unlikely that firms would demand credit at higher rates. Consequently, the bank has no incentive to charge a higher rate. They do not consider, however, whether the bank might do even better by charging a lower rate. In general, this amounts to asking what is the optimal loan rate. It is for this reason that we have defined our concept of "equilibrium rationing" for the case of rationing which occurs when the bank is charging its optimal rate. This, we believe, is the basic question at stake in trying to verify the rationality of credit rationing.

3. Hodgman (1961b) and Kane and Malkiel (1965)

Our review of the literature concludes with a discussion of papers by Donald Hodgman and Edward Kane and Burton Malkiel. The common ground for these two papers is that both stress the importance of the "customer relationship" in determining the characteristics of credit rationing. Hodgman's argument is based on the value of the customer's deposit to the bank. With an institutional structure which prohibits interest payment for deposits and which places a floor under the commercial loan rate in the form...
of the prime rate convention, the bank, Hodgman argues, will compete for the deposits of large corporations by providing other banking services. One of the most important of these services is the provision of bank credit in periods of tight money. Kane and Malkiel take an only slightly different tack. They argue that there is a class of customers for which the denial of a loan request increases the bank's aggregate risk. The fear of the bank is that it will lose this customer and most importantly his deposits for the long run. Thus, both theories indicate that in periods of tight money, banks will redistribute their loans toward a class of large, stable customers.

But both theories also neglect the very important question of loan rate determination. With respect to both theories, the crucial problem is why a non-prime customer cannot compete for loans by simply offering a high enough interest rate. Hodgman's argument is that the rate which the non-depositor would have to pay is so high that the firm's demand for loans at this rate is essentially zero. Rather than embarrass the firm with this very high rate, however, the bank goes through the fiction of telling the firm that there is simply no rate high enough to warrant a loan. It should be obvious that this is a classic case of price rationing. The fact that the customer's demand is zero at the relevant equilibrium rate should not cloud the issue. And the fact that the bank saves the customer's face by telling him that he is rationing is clearly irrelevant.

The Kane and Malkiel argument is equally difficult to follow. A good depositor, they argue, is more valuable to his current bank than is
his would-be value to some competing bank, since the value of a customer is only developed over time and as a result of a growing confidence in the stability of his deposits. They also note that a bank may be indifferent between a poor depositor of their own bank and a prime customer at some other bank. But this in no way implies that a non-prime customer (regardless of whether he is considered prime at some other bank) may not be able to bid loanable funds away from a bank's prime customers.

4. **Summary**

If banks do not use the interest rate as the rationing vehicle, then presumably some other characteristic of the loan contract or loan customer must be used as the criterion. The Hodgman (1960) -- Miller (1961) -- Preimer and Gordon (1965) line of argument suggested differential risk characteristics as the criterion. Hodgman (1961b) and Kane and Malkiel (1965), on the other hand, argued that banks give a preference to customers with good deposit records when it comes to allocating scarce bank funds.

To provide an explanation for credit rationing, a theory must show that at the optimal commercial loan rate it is profitable for the bank to extend a loan smaller than the amount demanded. In fact, this condition holds only if the competitive structure of the industry meets conditions which we shall develop in the remainder of this chapter. Since the studies reviewed here have generally failed to consider the demand and the determinants of the loan rate, it is not surprising that their results are of only very limited relevance to the problem.
II. SOME ANALYTIC PROPOSITIONS

The remainder of this chapter is concerned with the derivation of our theory of credit rationing. The present section contains some basic propositions concerning credit rationing by a rational banker under rather abstract conditions. The following section combines these propositions with the institutional and structural features of the commercial banking industry in order to provide the complete theory. Also in that section, the implications of the model are derived and a link between the theory and the empirical material of Chapters 3 and 4 is provided.

A. ASSUMPTIONS

The term rational behavior will be used here to mean that the bank maximizes its expected profits. The basic rationale for this assumption is that the bank services a number of customers large enough to allow us to disregard the correlations between the outcomes of sets of loans. Furthermore, since we wish to show that rationing is a rational policy, it seems fair to pick a utility function which is biased away from risk aversion. Finally, and of least importance, the expected profits formulation reduces the number of purely mathematical problems.

We want to consider a bank faced with customers. For convenience we shall write the demand function of the $i$th customer as:

$$D_i = D_i(R_i) \quad D_i \geq 0 \text{ for all } R_i$$

where $r_i$ is the interest rate charged to the $i$th customer, $R_i = 1 + r_i$, and $D_i$ is continuous and twice differentiable.

---

31 The sufficient conditions for credit rationing developed in this section remain essentially unchanged if a utility function of higher order moments is used, although the amount of credit rationing -- if it occurs at all -- will be changed. Thus the basic qualitative results hold quite generally.
We shall refer to $R_i$ as the interest factor or rate factor. We shall assume that there exists an $R^m_i < \infty$ such that:

(II.1) \[ D_i[R_i] = 0 \quad \text{for} \quad R_i \geq R^m_i \]

Also we assume that:

(II.2) \[ D_i[1] = D^m_i \quad \text{where} \quad D^m_i < \infty \]

and

(II.3) \[ \frac{dD_i}{dR_i} = D_i[R_i] < 0 \quad \text{for} \quad 1 < R_i < R_i \]

The boundary conditions for the demand curve are based on the characteristics of the underlying investment project which the firm is financing. It is assumed that each firm has a fixed size investment project which requires $D^m_i$ to finance. Thus even at a zero interest rate, the firm's demand for credit is limited to this amount. At the other extreme, we are assuming that there is a rate sufficiently high to reduce the firm's demand to zero; in this case the project is financed entirely by non-bank funds. The negative slope of the demand curve is due to the limited access of the firm to alternative means of finance. The implied absence, or at least limit, of competition between banks for the customer's business is discussed at length in section (III.A) below.

The ability of the firm to repay the loan contract depends on the outcome of the investment project. Since the project is of a fixed size, the outcome is independent of the share of the project financed by the bank.
The assumption of fixed size is clearly a rather restrictive one. However, in section (IV.B) we show that very similar results are obtained for a project in which the outcome does depend on the amount of financing provided by the bank.

The probability of the firm earning a given amount, from the bank's viewpoint, is given by a density function. This function is formulated, for ease of exposition, in terms of the end of the period value of the firm's assets. Thus, the probability of the firm's value being \( x \) at the end of the period is given by \( f_i[x] \), \( f_i \) a continuous and once differentiable function. Furthermore, we shall assume that there exist \( \underline{K}_i \) and \( \overline{K}_i \), \( 0 < \underline{K}_i < \overline{K}_i < \infty \) such that:

\[
(II.4) \quad f_i[x] = 0 \quad x \leq \underline{K}_i \quad x \geq \overline{K}_i
\]

If we define \( F_i[A] = \int_{\underline{K}_i}^{A} f_i[x] \, dx \), then it follows that

\[
(II.5) \quad F_i[x] = 0 \quad x \leq \underline{K}_i
\]

\[
F_i[x] = 1 \quad x \geq \overline{K}_i
\]

Finally, we shall assume that the end of period wealths for any two customers of the bank are uncorrelated.

With this basis we can now formulate the bank's expected profits, \( P_i \), from granting a loan to the \( i \)th customer. We want to consider the general case of a loan of size \( L_i \) at an interest rate \( r_i \), independent of whether such a loan will be demanded. To simplify the analysis, we shall
derive the expected profits as the difference between the firm's expected total repayment (interest plus principal) and the bank's total (opportunity) outlay (interest plus principal).

The bank's opportunity cost is then given by \((1 + j)L_1\) where \(j\) is the interest cost of loanable funds to the bank. This rate is assumed to be constant and independent of the size of loan granted. This is based on the premise that the bank has unlimited access to a perfect capital market at the rate \(j\).\(^{32}\) In section (IV.B) we show that a generalization to the case of an increasing opportunity cost as the loan size increases leaves the basic results unchanged.

If we now let \(I = 1 + j\) and require that \(L_1 \geq 0\) and \(R_1 \geq 1\), we have:

\[
(II.6) \quad P_i = P_i[R_1L_1] = \int_{K_1}^{R_1L_1} x f_i[x] \, dx + R_1L_1 \int_{R_1L_1}^{K_1} f_i[x] \, dx - IL_1
\]

The first term in this expression is the expected profits when the firm is not in a position to repay the agreed amount; that is, it defaults on the obligation.\(^{33}\) The second term is expected profits when the firm does repay the full amount \(R_1L_1\). The last term is, of course, the opportunity cost.

Using the fact that \(\int_{K_1}^{K_1} f_i[x] \, dx = 1\), we have:

\[
(II.6') \quad P_i = P_i[R_1L_1] = (R_1 - I)L_1 + \int_{K_1}^{R_1L_1} x f_i[x] \, dx - R_1L_1 \int_{K_1}^{R_1L_1} f_i[x] \, dx
\]

\(^{32}\)This assumption also allows us to neglect the bank's capitalization or asset base in deriving the profit function.

\(^{33}\)The case with bankruptcy costs is taken up in (IV.B) below.
Finally, integrating the middle term by parts yields:

$$(II.7) \quad P_i = P_i[R_1L_1] = (R_1 - 1)L_1 - \int_{K_i}^{R_1L_1} F_1[x] \, dx$$

The meaning of rational credit rationing can now be formalized. Suppose the bank quotes some interest rate, $R_0$. The loan demand is then given by $D_i = D_i[R_0]$. And the expected profits, $P_i = P_i(R_0D_i)$ are given by (II.7). Then rationing is rational if, and only if:

$$(II.8) \quad \frac{\partial P_i[R_1L_1]}{\partial L_1} < 0 \quad \text{for } L_1 = D_i[R_0] \text{ and } R_1 = R_0$$

Thus, rationing is rational if, and only if, expected profits can be increased by decreasing the loan size below the amount which is demanded at the desired interest factor.

B. THE OPTIMAL LOAN OFFER CURVE

Let us now ask the following question: What is the optimal loan size for the bank to grant to a customer for a given interest factor? The locus of these optimal loans as the interest factor varies will be termed the optimal loan offer curve for the firm. It is important to emphasize that the offer curve depends on the firm's ability to repay the loan but not on the firm's demand function. To find this curve, we start from the first order condition for an expected profit maximum:

$$(II.9) \quad \frac{\partial P_i[R_1L_1]}{\partial L_1} = R_i(1 - F_i[R_1L_1]) - I = 0$$
The offer curve is then defined as the implicit solution to (II.9), \( \hat{L}_i \), where \( \hat{L}_i = \hat{L}_i[R_i] \) is subject to the non-negativity condition \( \hat{L}_i \geq 0 \). (II.9) can then be rewritten as:

\[
(II.10) \quad R(1 - \hat{F}) - I = 0
\]

where the subscript \( i \) is understood and \( \hat{F}_i = \hat{F}_i[R_i \hat{L}_i[R_i]] \).

We now wish to show that the offer curve has the shape shown in Figure (3).

![Figure 3](image)

Since \( (1 - \hat{F}) \leq 1 \), and in view of the condition \( \hat{L} \geq 0 \), it is clear from (II.10) that \( \hat{L} = 0 \) when \( R < I \). Similarly, (II.10) yields a positive solution for \( \hat{L} \) when \( R = I \) only if \( (1 - \hat{F}) = 1 \).

\[34\] Formally, we require \( \hat{L}(R(1 - \hat{F}) - I) = 0 \), since if \( R(1 - \hat{F}) - I < 0 \), the bank has the option of a zero loan.
But this implies that:

$$\hat{L} \leq \frac{K}{I} \quad \text{for } R = I$$

The slope of the offer curve is derived by implicitly differentiating (II.10) with respect to $R$:

$$\frac{\partial \hat{L}}{\partial R} = \frac{1 - \hat{F} - RL\hat{f}}{R^2 \hat{F}}$$

(II.11)

where $\hat{F}$ is defined analogous to $\hat{F}$. At the point $(R = I, \hat{L} = \frac{K}{I})$ the curve must turn toward the northeast, since the derivative in (II.11) is positive. In the limit as $R$ goes to infinity, we can show that the loan size must go to zero, as follows: From (II.10) it is clear that

$$\lim_{R \to \infty} (1 - \hat{F}) = 0.$$  And from (II.4) we know that $\lim_{(1 - \hat{F}) \to 0} (RL) = \hat{F}$. Consequently, we must have $\lim_{R \to \infty} \hat{L} = 0$. The implication of this is that $\hat{L}$ must reach a maximum at some rate in the closed interval $(1, \infty)$. At the rate which yields the maximum loan, clearly $\frac{\partial \hat{L}}{\partial R} = 0$. The converse is not true, however. That is, multiple local maxima cannot be ruled out. This is discussed more formally in (IV. A) below.

In the case of certainty, of course, the offer curve reduces to a vertical line at the interest factor, $R = I$. Thus, if there is no uncertainty, the bank would choose to extend an indefinitely large loan as long as $R \geq I$.

Furthermore, we have $RL < \hat{F}$ when $R < \infty$. This implies that the optimal loan size is always finite.
Finally, we want to derive two properties of the offer curve.

First, expected profits increase (along the offer curve) as the interest rate rises. To show this, note that by substituting (II.10) into (II.6) we can derive an explicit statement for expected profits along the offer curve as:

\[ P[R\hat{L}] = \int_{K}^{RL} f[x]dx \quad \text{where} \quad \hat{L} = \hat{L}[R] \]

If we now differentiate (II.12) with respect to \( R \), we obtain:

\[ \frac{d}{dR} P[R\hat{L}] = \hat{L}(1-\hat{F}) > 0 \quad \text{as long as} \quad 0 < RL < K \]

which proves the proposition.

Secondly, we wish to show that for a given interest rate, profits decrease monotonically as we move further from the optimal loan size (in either direction).

To prove this, simply note that from (II.9) we have:

\[ \frac{\partial^2 P[RL]}{\partial L^2} = -R^2 f[RL] < 0 \quad \text{for} \quad 0 < RL < K \]

This also proves, of course, that the solution given in equation (II.10) is a maximum and not a minimum.

C. THE BANKER AS A DISCRIMINATING MONOPOLIST

We now wish to turn to a regime in which a banker has the market power of a discriminating monopolist with respect to his customers.
Consequently, the banker maximizes his expected profits with respect to each customer's demand curve individually and is free to charge each customer a different rate. Thus, we need consider only the banker's solution for the ith customer. The principal result is summarized as:

**Theorem II.1**

Let $\hat{R}_i$ be the maximum of the set of interest factors satisfying $D_i[R_i] = \hat{L}_i[R_i]$. Furthermore, let $R_i^*$ be the interest factor which maximizes the bank's expected profits from the ith customer. Then,

(a) $R_i^* \geq \hat{R}_i$

(b) Credit rationing is not profitable at $R_i^*$.

Our proof of this theorem relies on Figure (3) which shows the optimal loan offer curve and demand curve for this customer. From the construction of the offer curve and the discussion of the preceding section, we know that a loan of size $\hat{L}_i[R_i]$ at an interest factor $\hat{R}_i$ will yield greater expected profits than any loan granted at a rate equal to or less than $\hat{R}_i$. Furthermore, this loan is feasible since the customer would demand precisely this loan at $\hat{R}_i$. It is possible, indeed likely, that the bank would maximize the expected profits at some rate higher than $\hat{R}_i$. In this case the loan would equal the amount demanded by the firm at the optimal rate, but would be less than the optimal loan at the optimal rate. For loans greater than the amount demanded by the firm are clearly not feasible. And (II.14) assures us that the bank would only reduce its

---

36 There is, of course, at least one element in this set since we have $D_i[R_i] > \hat{L}_i[R_i]$ for $R_i < I$, $D_i[R_i] < \hat{L}_i[R_i]$ for $R_i > R_i^*$, and the functions have been assumed continuous.
profits by supplying a loan less than the amount demanded by the firm at the optimal rate. Thus, it is not rational for the banker to ration credit to any customer as long as it can always charge each customer the profit maximizing rate.

A more formal proof of this theorem is also instructive in that it provides a precise statement of the determinants of the optimal rate. To determine this rate we differentiate equation (II.7) with respect to $R_i$, with the demand constraint taken into account. That is:

$$\frac{\partial P_i[R_iD_i]}{\partial R_i} = D_i^*(R_i(1-F_i[R_iD_i]) - I) + D_i(1-F_i[R_iD_i])$$

where $D_i = D_i[R_i]

Setting this derivative equal to zero defines the optimal rate, $R_i^*$:37

$$\left(D_i^*\right)^* \cdot (R_i^*(1-F_i^*) - I) + D_i^*(1-F_i^*) = 0$$

where $D_i^* = D_i[R_i^*]$ and $F_i^* = F_i[R_i^*D_i^*]$. Since $D_i^*(1-F_i^*) \geq 0$ and $D_i^*[R_i^*] < 0$ for all $R_i$, we must have:

$$R_i^*(1-F_i^*) - I \geq 0$$

Equation (II.17), together with (II.9) and (II.14), assures us that Part (b) of the theorem is valid. Finally, (II.13) yields Part (a) of the theorem, as in the earlier argument.

37The second order condition for a maximum is discussed in Sections (II.D) and (IV.A) below.
Thus, for each of the bank's $n$ customers, an optimal rate, $R^*_i$, is determined and then, like the monopolist of traditional theory, the banker supplies whatever loan the firm demands at this rate. Hence, equilibrium rationing does not occur. Dynamic rationing, which results in the short run before the banks can adjust the loan rates to the long-run optimal levels, may occur, however. For whenever the banks quote a short run rate such that the demand exceeds the optimal loan offer at this rate, the banker will find rationing profitable. Thus, in Figure (3), dynamic rationing will occur whenever $R_i < R_1$ and whenever $R_2 < R_i < R_1$, where $R_i$ is the short run rate factor charged the $i$th customer.

D. A BANKER CHARGING ALL CUSTOMERS THE SAME RATE

We now consider a regime in which the banker is constrained to charge all customers the same interest rate, in direct contrast to the discriminating monopolist. We shall find under this regime that equilibrium credit rationing may be a rational policy, again in contrast to the case for the discriminating monopolist. In Section (III) below the analytic propositions developed here are merged with the institutional characteristics of the banking industry to provide our complete theory of credit rationing.

To simplify the problem, we shall assume at first that the banker faces only two customers. Our immediate goal is to find the optimal interest rate (charged both customers) which maximizes the bank's total expected profits when the demands for loans of both customers are satis-
fied. We shall refer to this aspect of the solution as Stage 1. We shall find that in some cases rationing may be profitable at the Stage 1 optimal rate. This will necessitate the solution of a second problem in which the loan to the rationed customer is constrained not by his demand curve, but by the optimal offer curve. This latter problem will be referred to as the "Stage 2" solution.

1. Stage 1

To reiterate, in Stage 1 of the solution we are concerned with the optimal rate which a banker would charge in order to maximize his expected profits from the two customers while constrained by the two demand functions. Thus, the total expected profits for the banker are derived from (II.7) as:

\[ P_t = P_1[R D_1] + P_2[R D_2] = \sum_{i=1}^{2} (R-I)D_i - \int_{K_i}^{RD_i} F_i(x)dx \]

where \( R \) is the common rate factor and the demand constraints are implicit in the notation since \( D_i = D_i[R] \). Differentiating (II.18), it is clear that

\[ \frac{\partial P_t}{\partial R} = \frac{\partial P_1[R D_1]}{\partial R} + \frac{\partial P_2[R D_2]}{\partial R} = \sum_{i=1}^{2} \left( (D_i) \ (R \ (1-F_i^D) - I) + (1-F_i^D) \ D_i \right) = 0 \]

(\text{where } F_i^D = F_i[R, D_i[R_i]])

implicitly defines the first stage optimal rate, say \( R^* \).
The second order condition for a maximum is derived by differentiating Equation (II.19):

$$
\begin{align*}
(II.20) \quad \frac{d^2 p_t}{dR^2} &= \frac{d^2 p_1[R^2]}{dR^2} + \frac{d^2 p_2[R^2]}{dR^2} < 0 \quad \text{at } R = R^* \\
\end{align*}
$$

To simplify the exposition, we shall make a stronger assumption:

$$
(II.21) \quad \frac{d^2 p_i[R^2]}{dR^2} < 0 \quad \text{for all } R \quad i = 1,2 
$$

Equation (II.21) clearly implies equation (II.20). In addition, equation (II.21) also implies that the second order conditions for the problem of the discriminating monopolist will be valid. The second order conditions for both problems are discussed further in Section (IV.A) below.

Thus, $R^*$ does provide a well defined expected profit maximization solution for the Stage 1 problem. Also $R^*_1$ and $R^*_2$ were defined above as the optimal rates the banker charged customers 1 and 2 when acting as a discriminating monopolist. We shall assume, without loss of generality, that $R^*_1 \leq R^*_2$. Under these conditions it must also be true that

$$
(II.22) \quad R^*_1 \leq R^* \leq R^*_2
$$

To demonstrate this result, we shall use a proof by contradiction. Thus, we shall assume that either $R^* > R^*_2$ or $R^* < R^*_1$. But, from the definition of $R^*_1$ given in Equation (II.16) and the second order conditions of (II.21) we have:
\[
\left\{ \frac{\partial R_1[R_1D_1]}{\partial R_1} + \frac{\partial R_2[R_2D_2]}{\partial R_2} \geq 0 \right\} \text{ as } \left\{ \begin{array}{l}
R^* < R_1^* \\
R^* > R_2^*
\end{array} \right.
\]

for \( R_1 = R_2 = R^* \), which clearly contradicts Equation (II.19). Thus, only the result given in Equation (II.22) is possible.\(^{38}\)

We can now turn to the question of credit rationing. So far we have determined the rate factor, \( R^* \), which maximizes the bank's expected profits. At this rate factor, the bank is constrained to supply the loans demanded by the firms. The question we now consider is whether under these circumstances the bank can increase its expected profits still further by rationing one or both customers. The answer is provided in the following theorem.

**Theorem II.2**

At the optimal rate factor \( R^* \) defined in equation (II.19),

(a) It is not profitable for the bank to ration customer 1.

(b) It will be profitable for the bank to ration customer 2 if, and only if

\[
(II.23) \quad R(I - F_2[R_2]) - I < 0 \quad \text{for} \quad R = R^*
\]

Before proceeding with the proof of the theorem, let us provide an intuitive explanation for the result. By virtue of (II.22) we know that customer 1 is being charged a rate at \( R^* \) which lies above the rate that a discriminating monopolist would charge. Thus the constraint of charging both customers the same rate has forced the bank to charge customer 1 a rate which is too high. Similarly, customer 2 is being charged a rate which is too low. The

\(^{38}\)The same result can be obtained using only the weaker condition that the second order conditions are valid in the neighborhood of the relevant equilibrium rates, as long as we restrict our attention to the global maximum.
condition in part (b) of the theorem means that customer 2 will be profitably rationed if the marginal expected return on his loan is less than the bank's opportunity cost. Customer 1, on the other hand, is not rationed, since the bank would prefer, if anything, to charge this customer a lower rate.  

The proof for part (a) can be seen by returning to Figure (3) and interpreting the demand curve and offer curve drawn there as belonging to customer 1. Since we know that \( R^* \geq R_1^* \), it then follows from the figure (and the properties of the offer curve) that customer 1 is not rationed. In algebraic terms, we can show that

\[
(II.24) \quad R_1 (1 - F_1[R_1D_1]) - I > 0 \quad \text{for } R_1 = R^* 
\]

But this means, using (II.10) and (II.14), that the loan demanded by customer 1 at \( R^* \) is less than the optimal loan. This, in turn, means that rationing customer 1 at \( R^* \) is not profitable.

Now let us interpret figure (3) as referring to the demand and offer curve for customer 2. Will customer 2 be rationed? The answer is yes if and only if the demand curve for this customer lies above the bank's offer curve at the optimal rate \( R^* \). But the only relevant restriction on this optimal rate is that \( R^* \leq R_2^* \). Thus, in terms of figure (3), customer 2 will be profitably rationed at \( R^* \) if (and only if) either \( R^* \leq R_1 \) or \( R_2 < R^* < R_1 \). The properties of the demand curve and density

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39 The role of customer 1 as "non-rationed" and customer 2 as rationed is of course an arbitrary assignment, depending on our assumption that \( R_1^* \leq R_2^* \).

40 This result follows from equation (II.19). That is, we must have \( R^*(1 - G[R^*D_1]) - I \) positive for at least one of the two customers. The second-order conditions and the assumption that \( R_1^* \geq R_2^* \) imply that the quantity must be positive for customer 1.
function for customer 1 determine in which region R* will actually lie. Thus, in general, we can say that the lower R+_1 the lower R*. And in the "normal" case a lower R* will increase the chances of customer 2 being rationed. The "normal" case is valid, whenever the demand curve for this customer has a slope less than the slope of the offer curve. Empirically, it is very likely that this condition will hold in the neighborhood of the relevant equilibrium rates. In fact, it is likely that the offer curve will positively slope in this region. For example, for a rectangular density function, the offer curve becomes negatively sloped only when the cumulative probability of default exceeds .50. And it should be obvious that under any "real" conditions we would never find bankers making loans of such high risk.\footnote{In the "abnormal" case, on the other hand, a lower R* may decrease the amount of rationing of customer 2 in specific regions.}

To complete the proof of part (b) of theorem (II.2) we need only show that (II.23) holds if and only if the demand curve lies above the offer curve at R*. But this follows directly from equations (II.10) and (II.14).\footnote{Since (II.24) is valid, it is clear from (II.19) that R* (1-F_2[R*D.]) - I can be either negative or positive, depending on the values of the demand curve and density function parameters for both customers.}

It is useful to consider also the case in which there is no uncertainty. In this case the condition given in part (b) of theorem (II.2) reduces to:

\[ R^* - I \leq 0 \]

But it is clear from equation (II.19) that with no uncertainty we must always have \( R^* \geq I \). Consequently, under conditions of certainty, neither customer can be profitably rationed at \( R^* \). Thus, it is the presence of uncer-
tainty, together with the requirement that the two customers be charged the same rate which distinguishes the analysis of this section from the usual theory of the monopolist and leads to equilibrium credit rationing.

This concludes the solution of the stage 1 problem. The results can be summarized by considering two cases. For the first case, let us assume that condition (b) of theorem (II.2) does not hold and that customer 2 is not rationed at \( R^* \). Under this condition it would appear that our story should end; that is, we would like to conclude that \( R^* \) is in fact the "optimum optimorum" in the sense that no interest rate can provide the bank with greater expected profits even if we do allow for the possibility of rationing. This proposition is, in fact, true, as shown in theorem (II.3) below. We shall postpone the proof, however, until we have developed some additional results. In the second case, we shall assume that the parameters are such that customer 2 can be profitably rationed at \( R^* \). Thus, as a first step the banker will restrict the loan to customer 2 to the amount indicated by the offer curve at \( R^* \). But then \( R^* \) will not, in general, be the "optimum optimorum." Instead, to determine the "optimum optimorum," we shall call it \( \tilde{R} \), we must find the solution when the bank maximizes expected profits under conditions allowing the rationing of customer 2. This means, in other words, that customer 2 is restricted to the bank's optimal loan offer curve. What about customer 1? We know that customer 1 will not be rationed. For if both customers were rationed -- that is, if both customers were granted loans
indicated by the offer curve, then the bank's optimal policy is to charge an
infinite rate and extend a zero loan. Since the customers' demands for loans
are also zero (for any rate larger than \( R^m_1 \)), technically there would be no
rationing and this contradicts the premise. Instead, the bank will charge
a rate such that customer 1 need not be rationed. To determine the level
of this rate, we turn to the second stage of our solution.

2. Stage II.

To start, let us make clear the nature of the problem. We are
assuming that \( R^* (1 - F_2[R^*D_2]) - I < 0 \), that is, it is profitable for the
bank to ration customer 2 at \( R^* \). To find the "optimum optimorum" the bank
is to maximize its total expected profits constrained by the demand curve
of the first customer and the offer curve to the second customer. Total
expected profits under this condition \( J[R, D_1, \hat{L}_2] \) can be written as:

\[
(II.25) \quad J = p_1[RD_1] + p_2[RL_2] = (R-I)D_1 + \int_{K_1}^{RD_1} F_1[x] \, dx + \int_{K_2}^{RL_2} y f_2[y] \, dy
\]

using (II.7) and (II.12).

The optimal rate can be determined by differentiating (II.25) with respect
to \( R \) and setting the result equal to zero:

\[
(II.26) \quad \frac{dJ}{dR} = \frac{dp_1[RD_1]}{dR} + \frac{dp_2[RL_2]}{dR} = 0
\]
Using (II.15) and II.13) and letting \( \hat{R} \) be the solution to this equation, yields:

\[(II.27) \quad (1 - \hat{F}_1) (\hat{R} \hat{D}_1 + \hat{D}_1) - \hat{I} \hat{D}_1 + \hat{L}_2 (1 - \hat{F}_2) = 0\]

where \( \hat{F}_1 = F_1[\hat{R} \hat{D}_1] \), \( \hat{F}_2 = F_2[\hat{R} \hat{L}_2] \), \( \hat{D}_1 = D_1[\hat{R}] \), and \( \hat{L}_2 = L_2[\hat{R}] \)

The second order condition for a profit maximum is given by:

\[(II.28) \quad \frac{d^2J}{dR^2} = \frac{d^2p_1[\hat{R} \hat{D}_1]}{dR^2} + \frac{d^2p_2[\hat{R} \hat{L}_2]}{dR^2} < 0 \quad \text{at} \quad R = \hat{R}\]

We shall assume that this condition does, in fact, hold. It is discussed further in section (IV.A) below.

Before interpreting the first order condition, it is helpful to prove the following lemma:

**Lemma II.1**

Let \( \tilde{R} \) be a (not necessarily unique) solution to \( D_2[R] = \hat{L}_2[R] \).

Then:

\[(II.29) \quad \frac{dP_2[\hat{R} \hat{L}_2]}{dR} = \frac{dP_2[\hat{R} \hat{D}_1]}{dR} \quad \text{when} \quad R = \hat{R}\]

\[(II.30) \quad \frac{d^2P_2[\hat{R} \hat{L}_2]}{dR^2} \geq \frac{d^2P_2[\hat{R} \hat{D}_1]}{dR^2} \quad \text{when} \quad R = \hat{R}\]
Part (a) of the theorem is easily proven by noting that at $\bar{R}$ we must have:

$$\bar{R} \left( 1 - \bar{F}_2 \right) - I = 0$$

where

$$\bar{F}_2 = F_2[RD_2(\bar{R})]$$

since $\hat{l}_2(\bar{R}) = D_2(\bar{R})$ by assumption. In this case, equations (II.15) and (II.13) are equivalent, which proves the result. The second part of the theorem can be proven by looking closely at the relevant second derivatives. Instead, however, we shall provide a simple intuitive explanation using figure (3). Consider a point of intersection of the demand curve and offer curve in that figure such as $\tilde{R}_1$. We have shown in part (a) of this theorem that the changes in profits along both curves, resulting from a change in the interest factor, are equal at the intersection. That is, the "marginal profits" evaluated at the intersection must be equal. But we also know that the profits along the offer curve must always equal or exceed the profits along the demand curve at the same interest factor. Thus the marginal profits along the offer curve must be rising faster, and this is exactly the meaning of (II.30).

We can now return to the interpretation of the equilibrium condition (II.27). Figure (4) summarizes the solution. $R^*_1$ and $R^*_2$ are the rates the discriminating monopolist charges the customers, respectively. $R^*$ is the equilibrium rate factor when the banker charges both customers the same rate but is not allowed to ration. $\tilde{R}$ is the rate which is the solution of the
problem of this section. Finally, $R_2$ is a rate at which the demand curve and offer curve of customer 2 intersect. We can now summarize the solution: At $R^*$ the bank found that profits could be increased further by rationing customer 2 because of condition (II.23).

Figure 4
In this case the relevant marginal profits curve is no longer \( \frac{dP_2[R^*_2]}{dR} \) but rather \( \frac{dP_2[R^*_2]}{dR} \). Comparing the later function with \( \frac{dP_1[R^*_1]}{dR} \), the bank finds that it can increase its expected profits further by reducing the rate from \( R^* \) to \( \hat{R} \). This result seems quite reasonable. For, by rationing customer 2 the bank should have more freedom in that it can now come closer to charging customer 1 the monopolist rate, \( R^*_1 \). In fact, however, this result is not generally applicable. In particular, the curves \( \frac{dP_2[R^*_2]}{dR} \) and \( \frac{dP_2[R^*_2]}{dR} \) may have multiple intersections.\(^{43}\) This is illustrated in the figure by \( \left( \frac{dP_2[R^*_2]}{dR} \right)^* \). For this case, the Stage 2 optimum, \( \hat{R}_o \), is greater than the Stage 1 optimum, \( R^* \). This result may seem at first rather strange. It indicates that the bank will raise its interest rate when given the right to ration customer 2. The reason for this behavior is that at \( R^* \) the bank finds the marginal loss of profits from customer 1 is outweighed by the marginal gain in profits from customer 2 (along his offer curve) resulting from an increase in the interest factor.\(^{44}\)

Finally, one loose end remains. The problem we have just considered occurred when at the Stage 1 optimal rate, \( R^*_1 \), it was profitable to ration customer 2. Until now we have merely assumed that if the rationing of

\(^{43}\) A sufficient, but not necessary, condition for the multiple intersection of these marginal curves is the multiple intersection of the underlying demand and offer curves.

\(^{44}\) We stress that both of these cases are equally "regular" and "normal." Expected profits from customer 1 (non-rationed) are always increased by reducing the rate (toward the monopoly rate \( R^*_1 \)) and expected profits from customer 2 are always increased by increasing the rate (until \( R_2 \) at which point rationing is no longer profitable). The result of these conflicting pressures will depend on the value of the parameters of the underlying demand functions and probability density functions.
customer 2 were not profitable at $R^*$, then $R^*$ would be the "optimum optimorum" in the sense that the maximum expected profits which could be obtained at any other rate -- and including the possibility of rationing customer 2 at these other rates -- would be less than the expected profits at $R^*$. To prove this, we must prove:

**Theorem II. 3**

Let $\tilde{n}(R, D_1, D_2)$ be the maximum expected profits which can be obtained when customer 1 is constrained to his demand curve and customer 2 is constrained to his offer curve from the bank. Let $(p_1^*[RD_1] + p_2^*[RD_2])$ be the maximum expected profits which the bank can obtain when both customers are constrained to their demand curves. Then we must have:

$$\tilde{n}(R, D_1, D_2) \leq (p_1^*[RD_1] + p_2^*[RD_2])$$

if $R(1 - p_2[RD_2]) - I > 0$ at $R = R^*$, that is, if customer 2 cannot be profitably rationed at $R^*$.

The result can be seen with the aid of figure (4). For simplicity we shall consider only the case in which the marginal profit curves,

$$\frac{dP_2[RD_2]}{dR} \quad \text{and} \quad \frac{dP_2[RL_2]}{dR}$$

intersect only once. The theorem is valid, however, for all cases. From the premise that customer 2 is not rationed at $R^*$ we know that $R^* \geq \bar{R}_2$, where it will be recalled that $\bar{R}_2$ is the rate at which the demand curve and offer curve have their (in our case, unique) intersection. If customer 2 is rationed, then the maximum expected profits, $\tilde{n}$, are determined at $\bar{R}$ as shown in the figure. Since $R^* \geq \bar{R}_2$, however, we must also
have $\hat{R} \geq \bar{R}_2$. This means that $\hat{R}$ must occur in a region in which the rationing of customer 2 is not, in fact, profitable. Thus at this $\hat{R}$ the bank could increase its profits still further by granting customer 2 the loan which he demands at this rate. But, from the definition of $R^*$ we know that the expected profits at $R^*$ must exceed the expected profits at $\hat{R}$ when customer 2 is not rationed. Hence, the expected profits at $R^*$ with customer 2 not rationed must exceed the expected profits at $\hat{R}$ when customer 2 is rationed, which proves the theorem.

3. Generalization to n Customers

We now wish to suggest how the results can be generalized to the case of n customers. First, let the bank maximize its expected profits while satisfying all n demand functions. Its expected profits would be maximized at some interest factor, call it $R^*$ as before. Also, let $R_i^*$ (for $i = 1, 2, \ldots, n$) be the rate factor that the bank would charge each customer individually when acting as a discriminating monopolist. Then it must be true that:

$$R^* > R_i^*$$

for at least some $i$

The bank would not find it profitable to ration these customers. It must also be true, however, that:

$$R^* < R_i^*$$

for at least some $i$

Among customers in this group, those whose demand function and density function are such that:

$$R (1 - \hat{F}_i[RD_i]) - I < 0 \quad \text{at} \quad R = R^*$$

will be rationed. If this last condition is not valid for any customers, our story ends; there is no rationing.

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45 This can be seen from figure 4 by drawing $\frac{\partial P_i[RD_i]}{\partial R}$ such that $R^* > \bar{R}_2$. Formally, this proposition can be derived from Lemma (II.1).
Otherwise, the bank would consider a second problem, our stage 2, in which it restricts those customers who could be profitably rationed at \( R^* \) to their respective offer curves and then maximizes expected profits. This yields a stage 2 optimal rate, \( \tilde{R} \) as before. The \( n \) customer case adds one complexity to the problem not present with only two customers. It is possible that at \( \tilde{R} \) some customers may be profitably rationed, whereas they could not be profitably rationed at \( R^* \). Should this occur, the bank would once again have to maximize expected profits, this time restricting the newly rationed customers to their respective offer curves. Presumably by successive approximations the bank would reach the point at which profits could not be increased further by rationing additional customers.

4. Generalization to \( m \) Customer Classes

So far we have been restricted in this section to the case in which the banker must charge all customers the same rate. Let us now consider a more general case in which the banker sets up a number of customer categories and charges all customers within the category the same rate. With each category is associated an interest factor \( R_{j}^{**} \) \( (j = 1, 2, \ldots, m) \), where, without loss of generality we shall assume that \( R_{j}^{**} > R_{j-1}^{**} \) \( (j = 2, 3, \ldots m) \).

To start, we shall assume that the number of categories and the rate factors associated with each of these categories are beyond the control of the bank and are taken as given. We wish to consider how the bank should classify each of its \( n \) customers, given that rationing is not allowed. If we assume that the bank's expected profits from each customer are a concave
function of the interest factor, we then obtain a particularly simple and instructive result. Under this condition the bank's expected profits from a customer reach their maximum at the monopolist rate for that customer, \( R_i^* \), and decline monotonically as the interest factor deviates from this level in either direction. Consequently, the bank will attempt to charge a customer a rate as close to the monopolist rate for this customer as possible. We can then eliminate from consideration for the classification of a particular customer all those categories but the two which bound the monopolist rate for that customer from above and below. The choice between these two categories depends on the specific shape of the profit function in this interval. If we allow for the more general case in which the profit function needs to be concave only in the neighborhood of the monopolist rate, the classification of customers becomes more complex. We need not pursue this aspect of the problem further, however, for it is clear there will always be some optimal category for each customer.

Next, we wish to consider whether there is an optimal set of interest factors, \( R_{j}^{**} \), for the (still arbitrary) \( m \) categories. The answer is obviously, yes. Among all possible sets of classification rates, there must be one set which will maximize the bank's total expected profits. (We rule out the case of ties.) Since this case is linked to the material in the following section, it is worth considering more carefully. So, let us assume that the bank has set up its \( m \) categories with the corresponding

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46 This assumption is equivalent to requiring that the second order condition hold for all values of the interest factor. See condition (II.21).
rate factors set to maximize expected profits. Rationing is still not allowed. We know that within each of these classes there must be at least one customer with a monopolist rate higher than the class rate and one customer with a monopolist rate lower than the class rate. If either of these conditions did not hold, then it is clear that the bank could increase its expected profits further by moving the class rate in the direction of the monopolist rates of all customers in the class. But this would contradict the assumption that the class rates were, in fact, the optimal rates.

Now let us ask whether it is profitable to ration any customers under this regime. Fortunately the necessary results have already been derived. Each of the categories being discussed here can be analyzed in the same manner as we discussed the single class of section (II.D.1). In particular, theorem (II.2) will be valid for each of the classes. This implies that it may be rational to ration some customers within each class. If no customers can be profitably rationed, then of course the story ends.

Otherwise, we must pursue the question of the optimal rate structure when some customers are rationed. This problem is very similar to the stage 2 solution discussed in (II.D.2) above. One further complexity arises, however. It may become profitable to move the rationed customers to categories with higher rates, or equivalently, to move the non-rationed customers into categories with lower rates. This means that the classification of
customers as well as the rate structure will change. After all the profitable shuffling of customers and rates has taken place, however, the bank will have reached its "optimum optimorum." At this point the conditions with respect to rationing for each class are identical to the case of a single class already discussed.\footnote{The possibility of credit rationing exists as long as a customer's monopolist rate exceeds the class rate for his category. Since we have already noted that this must be true for at least one customer in each class, it is clear that our procedure cannot lead to a systematic elimination of the possibility of rationing.}

Finally, to conclude, we wish to allow the number of categories, $m$, to be a bank decision variable. Fortunately, this case of an "optimal optimum optimorum" has a more simple title. It is clear that the optimal number of categories is precisely the number of customers, $n$. In this case each customer would be charged the monopolist rate. But this is exactly the case of the discriminating monopolist already discussed in section (II,c). Having come full circle, it is fitting to proceed now to the interpretation of these results.

III. A THEORY OF CREDIT RATIONING

The principal result of the previous section, summarized in (II,D.3) and (II,D.4) was that a banker charging the same rate to all customers within a rate class normally may find it profitable to ration some of these customers. In this section we wish to show first that such a regime is, in fact, a reasonable although abstract view of the commercial banking industry. Then we shall proceed to summarize the implications of the model for both dynamic and equilibrium rationing. This summary provides an important link to the empirical work reported in Chapters 3 and 4.
A. COMPETITION IN COMMERCIAL BANKING

The existence of imperfect competition in commercial banking has long been well-known and accepted. Less clear, however, is the specific form of this imperfection. For example, assume, as a polar case, that banks can and do participate in direct collusion. We can start by assuming an arbitrary distribution of customers among the various banks. Each bank is allowed to act as a perfectly discriminating monopolist toward its set of customers. The contribution of a specific customer to a bank's expected profits will, in the general case, differ between banks because of the subjective nature of the bank's evaluation of the riskiness of this customer. Thus by "buying" and "selling" customers each bank can improve its expected profits. Eventually, however, we reach the point at which no bank can improve its expected profits further without decreasing the profits of the aggregate. We will have reached a banker's Pareto Optimum. Since the banks act as discriminating monopolists toward their set of customers, Theorem (II.1) assures us that there would be no credit rationing under such a regime.

In fact, of course, direct collusion and particularly side payments are not a practical course for the banks since they are illegal. Instead, we would expect the banks to find some less explicit means to restrain competition. This problem is not, of course, unique to the banking industry. The steel industry, in particular, comes to mind. The resolution of the problem, for the steel industry at least, is described by the phrase "administered price." This "explanation" raises more questions than it answers, however. Who is the administrator? How does he select the price? We shall return to these questions in a moment.
First, however, we want to point out one important aspect of the problem which does differentiate the steel industry from the banking industry. The product sold by a steel company is typically very accurately defined in objective terms clear to all firms (and customers) in the industry. Thus, grade 5 carbon steel has objective qualities which are independent of the customer purchasing the product. Commercial loans differ from steel in this respect in that the risk of a loan may vary substantially between customers. Moreover, we have already pointed out that the evaluation of this risk may differ between banks even for the same customer. Thus there are not well defined product categories for commercial loans as there are for steel products. Yet some classification of loan customers would appear to be essential if the industry is to exert its oligopolistic power. Otherwise, in the absence of direct collusion, competition between banks will reduce the bankers' Pareto Optimum described in the beginning of this section to a borrowers' Pareto Optimum in which the profits of the banking industry are reduced to zero.

To investigate the problem of classifying customers further, it is helpful to start with the example of a single monopolist bank. In section (II.D.4) we showed that under the abstract conditions assumed there, with the number of categories a decision variable, an expected profit maximizing bank would eventually find its way to the discriminating monopolist solution of the classification problem. A more realistic appraisal of the problem suggests, however, that even such a monopolist bank would consider a more temperate classification scheme. While the bank would desire to
classify customers by their riskiness and demand elasticity as summarized by the monopolist rate, in fact, some more "objective" criteria that the firm could understand would probably be used. Thus for external purposes the bank would adopt a classification scheme using, for example, asset size and the nature of the firm's industry as well as the basic financial features of the firm. And this in turn would cause at least some compromise in the bank's internal classification of customers. In particular, there would be some tendency for the entire rate structure to be compressed toward a central value. This tendency is increased further by any usury laws which place an upper bound on the rate structure.

When we turn from the case of a single monopolist bank to the more realistic case of an oligopolist group of banks, these characteristics of the classification scheme become even more certain. We have already suggested that the industry could not reach its Pareto Optimum because of legal restrictions on collusion. In fact, even a large number of categories would be difficult to manage. For there must be agreement between the banks on the classification of customers for the system to operate successfully. But this classification scheme must not antagonize customers by creating a feeling of blatant discrimination. Potential antitrust prosecution may achieve some of the effect of the real thing. Thus banks do adopt standard categories which run along the lines of asset size, industry class, and other easily recognized features of the firm. Firms in the same class are treated equally. Furthermore, the decision-making costs of the bank are clearly reduced if the price decision is
determined by considering in which of a small number of categories a particular customer should belong.

In summary, then, the combination of legal pressure, potential legal prosecution, and perhaps even social pressure, as well as decision-making costs, all lead toward a system in which banks classify customers into a small number of fairly objective categories. Within each of these categories all firms are charged the same rate. These same factors also tend to reduce the spread between rates below that of the optimal distribution. The exact number of classes finally used cannot be determined in the abstract since it does depend on the "institutional" factors just mentioned.

We can now return to the original question of the price administrator and the level of the price. What we want to suggest is that the optimal equilibrium classification scheme determined by the oligopolistic industry as a whole, resembles very closely the problem of a single bank discussed in section (II.D.4). The number of categories, m, can be taken as a small number determined by "institutional" features already discussed. The rate structure for these classes will also resemble to a good approximation the optimal structure. The approximation arises from the fact that the bank will have to bow somewhat to social and legal pressure in classifying customers.  

In a similar vein, the discussion of rationing under the regime in section (II.D.4) must be modified slightly. In the most general case we

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48 We do not want to imply that the classification of customers by banks is a novel notion. To the contrary, it appears to be a well-accepted institutional feature of the commercial banking industry. See, for example, Alhadeff (1954), pp. 120-123.
had concluded that at least one customer in each class would not be rationed while at least one customer would be a potential candidate for rationing. The probability of rationing actually occurring increases as the customers within any category become less homogeneous, since the spreads between the (quoted) class rate and the monopolist rates for the customers would tend to increase. Thus, for the realistic case of a bank facing many customers and constrained to place customers of different risk features and demand elasticities into the same category, it seems almost certain that rationing, indeed substantial rationing, would occur. In particular, usury laws as well as general social and legal pressure tend to restrict the level of acceptable rate categories. Thus we would expect a substantial amount of rationing in the highest rate categories, since here we would find the least homogeneous group of firms. At the other extreme, firms of very low risk and very elastic demand curves would be grouped together in a low rate class. In the polar case in which these firms can be viewed as having no risk, we can be certain that no rationing of them will occur. Or, in more general terms, for the class of essentially riskless prime rate customers we should find very little rationing.

So far we have restricted our attention to the problem of the determination of the optimal or equilibrium rate structure and its implications for equilibrium rationing. It is equally important to point out that the oligopolistic structure has important implications for dynamic rationing. The basis for dynamic rationing is, of course, a "disequilibrium" interest rate structure. And, in fact, disequilibrium rates will be a common feature
if the banking industry behaves as we have just described it. Some of our reasons for expecting only a gradual adjustment in rates toward the equilibrium structure are quite standard: costs associated with changing the price; uncertainty about the reaction of the customers to the change; imperfect forecasting of the future and perhaps imperfect information of the present. To these we must add, however, the time needed for the oligopoly to come to agreement that the optimal structure has changed sufficiently to necessitate a change. In particular, this suggests the importance of some industry-wide signal when rate changes are desired. We shall leave to Chapter 4 a more detailed and empirical account of this process.

B. EQUILIBRIUM RATIONING

We are now in a position to assemble the propositions of section (II) and the discussion of (III,A) into a complete theory of credit rationing. We have agreed that banks classify customers into equivalent rate categories. The behavior with respect to rationing under these conditions has been discussed in section (II,D). To a first approximation we have assumed that the optimal rate determined in (II,D) can serve as a proxy for the rate which would actually result from the oligopolistic price setting. Our purpose now is to summarize the rationing behavior at this equilibrium rate; that is, equilibrium rationing.
1. **The Effects of Parameter Changes**

We shall describe the rationing behavior of a typical bank faced with \( n \) customers. We start, for simplicity, with the simple case in which all customers are in one class. The equilibrium rate, under these conditions, is \( \tilde{R} \) which was defined in (II.D.2). The amount of equilibrium rationing, \( E \), will be the summation of the difference between the demand and bank offer for those customers of the bank experiencing credit rationing. That is,

\[
(III.1) \quad E = \sum_{i=1}^{n} \left( \max \left( D_i(\tilde{R}) - L_i(\tilde{R}), 0 \right) \right)
\]

(The "excess supply" of loans to those customers to whom the bank would like to extend additional loans does not, of course, offset the rationing to other customers.) Since the offer curve to a firm depends on the opportunity cost of the bank, \( I \), and the firm's density function of possible outcomes, \( f_i \), in reduced form \( E \) depends on these two factors and the firms' demand functions. We wish to consider the effects of changes in these parameters on the amount of equilibrium rationing. We shall find that in most cases the effect on \( E \) of changing these parameters is ambiguous in direction. (In fact, for the purposes of the empirical work below, we shall go one step further and assume that in the aggregate these effects on equilibrium rationing are small enough to be ignored. But this gets ahead of the story.)

The evaluation of the effects of these parameter changes is quite complex. The principal difficulty is that following a change in any parameter, four possibilities must be considered:
1. A rationed customer may continue to be rationed.

2. A rationed customer may cease being rationed.

3. A non-rationed customer may start to be rationed.

4. A non-rationed customer may continue to be non-rationed.

Even for the simple case of only two customers, we must proceed through the entire analysis of section (II.D), starting with stage 1, in order to be sure that all cases are covered. We shall go through the case of a change in the opportunity cost in detail to illustrate the possible outcomes. The results for several other important changes will be outlined.

To start, we must first determine the effect of a change in \( I \) on the stage 1 optimal rate, \( R^* \). Differentiating equation (II.19) with respect to \( I \) yields:

\[
(III.2) \quad \frac{\partial R}{\partial I} = \frac{D_1' + D_2'}{\frac{\partial^2 P_1[RD_1]}{\partial R^2} + \frac{\partial^2 P_2[RD_2]}{\partial R^2}} > 0 \quad \text{for} \quad R = R^*
\]

This is positive, since the slopes of the demand curves insure the numerator is negative and the second order condition insures the denominator is negative. It is quite reasonable, of course, that an increase in the opportunity cost should lead to an increase in the equilibrium rate. Since the demand curves are negatively sloped, we know that the demand of the customers has been reduced by the increase in the opportunity cost.

\[49\] In the empirical work for the aggregate we shall assume that these rates are related linearly.
To determine changes in rationing, however, we must also consider the change in the optimal offer curve resulting from the change in the opportunity cost. Differentiating equation (II.10) totally with respect to \( I \), yields:

\[
(III,3) \quad \frac{d\hat{L}_1}{dI} = \frac{\partial \hat{L}_1}{\partial R} \frac{\partial R}{\partial I} + \frac{\partial \hat{L}_1}{\partial I} \quad \text{at } R = R^*
\]

where \( \frac{\partial \hat{L}_1}{\partial R} \) is defined in (II.11), \( \frac{\partial R}{\partial I} \) is given in (III.2), and

\[
\frac{\partial \hat{L}_1}{\partial I} = \frac{-1}{f_1 R^2}.
\]

The sign of this derivative is, in the general case, ambiguous. The second term, which represents the shift in the offer curve resulting from a ceteris paribus change in the opportunity cost, is negative. But the first term, which represents the movement along the offer curve as the equilibrium rate changes, will be positive whenever the offer curve has a positive slope. Hence it is possible that the loan offer will actually increase as the opportunity cost increases.

The consequence is that in the most general case all four possibilities listed above are possible. Thus we cannot even answer the question, which customer is rationed, without further knowledge of the values of the relevant parameters.

To illustrate the methodology, however, we shall press the issue further. Let us assume that at the original \( R^* \) it was profitable to ration customer 2 but not customer 1. Further, let us assume that after the change in \( I \) and
the resulting change in \( R^* \) it is still profitable to ration customer 2.

In this case, the equilibrium value for \( \tilde{R} \), the stage 2 optimal rate, is given in equation (II.27). Differentiating this equation with respect to \( I \) yields:

\[
(III.4) \quad \frac{\partial \tilde{R}}{\partial I} = (1 - \hat{r}_2) (\hat{L}_2' - 1) + \frac{D_1'}{\delta^2 p_1 [D_1 R]} \geq 0 \quad \text{at} \quad R = \tilde{R}
\]

since the first term may be positive or negative.\(^{50}\) Since the direction of change in the equilibrium rate is indeterminate, it should be clear that the sign for the effect of changes in \( I \) on rationing cannot be determined.

It is also interesting to consider the effects of changes in the parameters of the demand curves. For example, suppose that customer 2 is rationed in the initial equilibrium. The bank is then displaced from the equilibrium by a parallel upward shift in the demand curve. Then, even if at the new \( R^* \) it is still profitable to ration customer 2, we cannot be sure that customer 2 will be rationed more at the new equilibrium. We do know that the customer's offer curve will have remained fixed, whereas the demand curve has shifted. Since the equilibrium rate has changed, however, we must also take into account the movement along the offer curve.

\(^{50}\)It may seem strange at first that an increase in the opportunity cost may actually decrease the optimal rate. This arises because of the basic conflict between obtaining larger profits from the rationed customer and the non-rationed customer discussed earlier.
The result of these two movements will generally be ambiguous. Furthermore, the customer may not even continue to be rationed at the original rate, since if the demand curve shifts sufficiently upward, customer 1 may become the rationed customer. The ambiguity prevalent in these results should be apparent.

The possibility of shifts in the density function could also be considered. Here again, however, in the general case no definite effect can be derived. In fact, the implication of all the results noted here is that equilibrium rationing does not bear any systematic relationship to the level of the equilibrium rate, which in turn is determined by the parameters considered.

2. A Summary of Equilibrium Rationing

It is worth-while to summarize the results on equilibrium rationing before turning to dynamic rationing since the two phenomena are actually quite distinct. Our primary object with respect to equilibrium rationing was to provide a proof of the rationality of this behavior. It was shown that under a regime in which a bank classifies customers into equivalent rate categories, specific parameter values could lead to the profitability of credit rationing. In the two-customer—One-class case, it was shown that one customer would not be rationed while the other customer might be rationed. Similarly, in the n-customer—m-class case, the only certainty is that at least one customer in each class will not be rationed. Thus it is possible that none of the customers would be rationed. With banks catering to many different customers with sufficiently divergent demand functions and risk
characteristics, however, it would appear that this result would be of negligible significance and that equilibrium rationing would have practical importance.

It is also useful to point out how our theory of equilibrium credit rationing differs from other models of the phenomenon. We stressed two important points in the review of the literature. First, we pointed out that if bankers do not use the price as the rationing vehicle, then some other characteristic of the customer or loan must be used. Our model stresses the risk characteristics of the customer. Thus in the case of the single class only customers who were risk-free could be ruled out as potential rationing candidates. In the more complex case with several rate categories, a similar result follows. That is, the class with risk-free customers has no rationing. As an empirical approximation, this class of customers might be equated with those customers granted the prime rate. Within each of the other categories some rationing might occur, again depending on the risk characteristics of the customers. Finally, because of usury laws and other social pressures, substantial rationing of high-risk firms is anticipated. In the following chapter on the empirical tests of rationing, we shall use the result that the risk-free customers are not rationed as an aid in deriving an empirical measure of rationing.

While our model has stressed these risk characteristics, it would be wrong to interpret this as foreclosing any other rationale for rationing. For example, in section (IV.B) we show that bankruptcy costs change the quantitative, although not the qualitative, nature of the rationing process.
Similarly, the "customer relationship" aspect of the banking industry emphasized by Hodgman and Kane and Malkiel is not inconsistent with our model. In an heuristic sense, these characteristics of customers can be thought of as forming part of the bank's evaluation of the customer as characterized by our risk density function. It is not surprising, therefore, that their models lead to the same basic view of the rationing process as our model: that is, a tendency for rationing to result in the reduction of loans to small, risky firms rather than large established corporations. In fact, this interpretation of credit rationing is even found in the early work by Scott (1957b) and Hodgman (1960).

The second point which we stressed in our review of the literature was the absence of demand considerations and a theory of the determinants of the optimal commercial loan rate in any of the received theories of rationing. It should be clear that these elements are critical to the rationing process. To propose a theory of credit rationing without this basis is to beg the basic question. It is in this respect that we suggest our theory is unique.

Finally, we note that results of section (1) indicate that no systematic relationship exists between the direction of change in equilibrium rationing and the change in some parameter. Thus, if one aggregates all banks and all customers and views the rationing process over time, it is reasonable to consider the "micro" effects of changing parameters as essentially random disturbances which might be safely neglected in performing empirical tests. Furthermore, dynamic rationing, as we shall describe in
a moment, should be of much more importance in any aggregate, short run empirical study of credit rationing. Equilibrium rationing tends to provide the supporting cast, while dynamic rationing plays the lead.

C. DYNAMIC RATIONING

Dynamic rationing has been relatively neglected in our discussion so far and the time has come to discuss it. The basis for dynamic rationing is the imperfect, short-run adjustment of the commercial loan rate to its long run equilibrium level. Since the nature of this adjustment has already been outlined in (III.A) above and will provide the core of Chapter (4), we need not pursue this mechanism here.

To understand the modus operandi of dynamic rationing, it is most helpful to consider a shock to a system which starts in long run equilibrium. The most basic case arises when the opportunity cost of a commercial loan rises. The long run effect of this change is to increase the equilibrium level of the commercial loan rate. This mechanism has already been discussed in the previous section. In the short run, however, we are assuming that at first, at least, the quoted loan rate does not change. The optimal loan offer curve, being a function of the opportunity cost, does change, however. In fact, in Equation (III.3) we have already noted that

$$\frac{\partial \hat{L}_1}{\partial I} < 0$$

indicating that the offer curve shifts downward at every interest rate.

Since the quoted rate has not changed, the demand for loans is unchanged.
Consequently, there must be an aggregate increase in the amount of rationing.

Looking more closely at several classes of firms, moreover, we can obtain further information on the form of this rationing. First, there will be a class of firms for which the demand curve was substantially below the offer curve in the initial equilibrium. For these firms the effect of the opportunity cost change is nominal, since they will not be rationed even after the shift in the offer curve. The chief constituents of this class will be the prime customers, since they were the furthest from being rationed in the initial position. Firms in other classes may also, of course, continue to be non-rationed, even after the shift, depending on the extent of the increase. A second class consists of those firms which were not rationed before the shift but are rationed following the increase. This arises, of course, because the offer curve has moved below the demand curve for these customers. It is unlikely that there will be any riskless customers in this class, since the offer curve for these customers is a vertical line at the opportunity cost. Thus only when the opportunity cost rises above the quoted loan rate would these customers suffer even dynamic rationing. The moderately risky customers, on the other hand, will feel the effect of credit rationing more severely. Finally, there is the class of customers who were rationed even in the initial equilibrium. They will feel the effect of the increased opportunity cost the most severely since the downward shift of the offer curve to these customers results in reducing the amount of credit extended to them directly. In this class will be the most risky customers of the bank.
In the "normal" case we shall find that the amount of dynamic rationing is monotonically reduced as the loan rate adjusts toward the new equilibrium level. The "normal" case is defined to be when the slope of the demand curve is less than the slope of the offer curve. Under this condition, the reduction of loan demand must exceed the reduction in the loan offer as the interest rate rises toward its new value. Thus the amount of rationing is most severe immediately following the initial shock and then gradually returns to the much smaller equilibrium rationing level as the rate adjusts toward its equilibrium level.

In the "abnormal case", when the slope of the demand curve exceeds the slope of the offer curve, we may find that the amount of rationing fluctuates as the rate adjusts toward the new equilibrium level. A similar phenomenon was observed in the context of equilibrium rationing on page (56) below. We have termed this case "abnormal" in the sense that several different considerations all suggest that it will be of only limited empirical importance. The first point is that for simple cases, at least, the offer curve will have positive slope in the empirically relevant range. For example, for a rectangular density function, the offer curve becomes negatively sloped only when the cumulative probability of default exceeds .50. And it should be obvious that under no conditions will we find bankers making such risky loans. The positively sloped offer curve will, of course, rule out the abnormal case. Secondly, the abnormal phenomenon will be observed only if the loan rate adjusts gradually to the equilibrium level. In fact, though, the empirical evidence in Chapter (4) suggests that the adjustment
is very lumpy. That is, the loan rate adjusts from the initial disequilibrium position to the new equilibrium position in a single change. Thus, the regions between these two levels are not actually observed. Finally, we can rely, at least to a limited extent, upon the aggregation over all customers to cancel out these unlikely results.

With this basis, it is quite easy to consider the second important shock to the initial equilibrium, an increase in the demand for loans. The immediate effect of the increased demand is to raise all the demand curves, while leaving the offer curves unchanged. This is just the opposite of the effect of a change in opportunity cost. The result of this shift, on the other hand, is very similar to the case already considered. Firms not rationed in the initial equilibrium will receive larger loans, although some rationing of these firms may also occur. Firms already rationed in the initial position will find rationing even greater in the sense that the increased demand is met with an unchanged supply. Having extended a larger volume of loans, however, it is likely that the bank will find the opportunity cost of these loans rising. Consequently, superimposed upon the initial shock of an increase in demand will be the effect of an increase in opportunity cost. This effect has already been discussed, of course, and the result of the two effects will be reinforcing. Thus, for the case of a shift in demand, as well as a simple change in opportunity cost,

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51 The case of rising opportunity costs is discussed formally in section (IV,B) below.
the effect of rationing can be summarized as an "upgrading" of the bank's portfolio. On one hand, the macro effect of rationing is a redistribution of loans toward the less risky customers. On the other hand, the micro effect of reducing the loan to a customer results in a reduction of the risk of default by this customer.

In summary: Dynamic rationing occurs in the interval between a shock which disturbs the initial equilibrium and the long run adjustment of the rate. It is characterized by an increased propensity to ration all customers. Only previously rationed customers or customers on the margin of being rationed in the initial equilibrium, however, actually realize rationing.

This dynamic rationing takes the form of an upgrading in the bank's portfolio, since the rationed or near-rationed customers in the initial equilibrium are typically the bank's more risky customers. In the normal case, dynamic rationing is greatest just following the initial shock. It then declines toward the underlying equilibrium rationing level as the rate adjusts toward the system's new equilibrium position.

IV. APPENDIX TO CHAPTER 2

A. SECOND ORDER CONDITIONS INTERPRETED

A significant portion of this chapter was concerned with the solution of maximization problems. The implications of the first order conditions for these problems were discussed at length in the text. The second order
conditions were, in most cases, simply assumed to be valid. The primary purpose of this section is to provide some insight into the implications of these assumptions for the underlying density functions and demand functions.

1. Multiple (Local) Maxima for the Optimal Loan Offer Curve

The discussion in section (II,B) proved that there was a finite maximum to the optimal loan, \( \hat{L}_1 = \hat{L}_1[R_1] \). Our purpose here is to show that multiple local maxima can, in general, occur. Recall from text equation (II.11) that the slope of the offer curve is given by:

\[
\frac{\partial \hat{L}_1}{\partial R_1} = \frac{1 - \hat{F}_1 - R_1 \hat{f}_1 \hat{L}_1}{R_1^2 \hat{f}_1}
\]

This can be rewritten as

(IV.1) \[R_1 \frac{\partial \hat{L}_1}{\partial R_1} + \hat{L}_1 = \frac{(1 - \hat{F}_1)}{R_1 \hat{f}_1}\]

Differentiating (IV.1) with respect to \( R_1 \) and substituting (IV.1) yields:

(IV.2) \[\frac{R_1 \partial^2 \hat{L}_1}{\partial R_1^2} + 2 \frac{\partial \hat{L}_1}{\partial R_1} = \frac{-(1 - \hat{F}_1) \left(2 \hat{f}_1 + \hat{f}_1'(1 - \hat{F}_1)\right)}{R_1^2 \hat{f}_1^2}\]
We wish to show that \( \frac{\partial^2 L_1}{\partial R_i^2} > 0 \) can occur when \( \frac{\partial L_1}{\partial R_i} = 0 \).

And, from (IV.2) we obtain:

(IV.3) \[ \frac{\partial^2 L_1}{\partial R_i^2} = -\frac{2L_1}{R_i^2} - \frac{L_1 \hat{f}_i}{R_i \hat{r}_i} \quad \text{when} \quad \frac{\partial L_1}{\partial R_i} = 0 \]

And hence

(IV.4) \[ \frac{\partial^2 L_1}{\partial R_i^2} > 0 \quad \text{if} \quad \hat{f}_i' < \frac{-2 \hat{f}_i}{L_i \hat{r}_i} \quad \text{and} \quad \frac{\partial L_1}{\partial R_i} = 0 \]

There appears to be no economic reason for restricting \( \hat{f}_i' \); thus, multiple local maxima are possible.\(^{52}\)

\(^{52}\)The possibility of multiple (local) maxima makes the concept of "weak credit rationing" of Freimer and Gordon (1965) ambiguous, since credit rationing may occur only in specific intervals. This possibility is not mentioned, possibly because Freimer and Gordon work with a special case in which \( \hat{f}_i' = 0 \) for all \((L_i R_i)\).
2. The Discriminating Monopolist

The second order condition for this case is derived by differentiating (II,16) with respect to $R_i^*$:

\[(IV,5) \quad Z_1[R_i^*] = (D_1')^* (R_i^* (1 - F_i^*) - I) + 2 (1 - F_i^*) (D_1')^* - (R_i (D_1')^* + D_1^*)^2 f_i^* < 0\]

Thus,

\[(IV,6) \quad (D_1')^* < \frac{2(1 - F_i^*)D_i^*}{(R_i^* (1 - F_i^*) - I)^2} + \frac{(D_1')^2 r_i^2 f_i^*}{(R_i^* (1 - F_i^*) - I)^3} \quad > \quad 0\]

Thus the second order condition amounts to an upper bound on the second derivative of the demand function. The bound, in turn, depends on properties of both the demand and density functions. It is interesting to note that in the case of certainty, condition (IV.6) reduces to:

\[(IV,7) \quad \frac{\partial^2 D_i^*}{\partial (R_i^*)^2} < \frac{2D_i^*}{(R_i^* - I)^2}\]

Since we must have $R_i^* (1 - F_i^*) - I \geq 0$, it follows that condition (II.7), in fact, implies condition (II.6). Thus our solution will be a maximum as long as the corresponding solution for the case without uncertainty is a maximum.
3. **The Bank Charges Both Customers the Same Rate: Stage 1.**

The second order condition for this case is derived by differentiating equation (II.19) with respect to \( R^2 \). This can be written as:

(IV.8) \[ Z_1 [R^2] + Z_2 [R^2] < 0 \]

where \( Z_1 [R] \) is defined in equation (IV.5). Obviously (IV.8) will be valid whenever equation (IV.5) holds for \( R^2 \) as well as \( R_1^2 \) and \( R_2^2 \). Furthermore, the equivalent condition for the case of certainty will imply the above condition just as it did for the case of the discriminating monopolist.

4. **The Banker Charges Both Customers the Same Rate: Stage 2.**

The second order condition for this case is derived by differentiating equation (II.27) with respect to \( \hat{R} \). This yields:

(IV.9) \[ Z_1 [\hat{R}] + (1 - \hat{\rho}_2) (\hat{\eta}' - 1) < 0 \]

where \( Z_1 [\hat{R}] \) is defined in equation (IV.5). The first term, the slope of the marginal profit curve for the non-rationed customer, has been discussed in the previous two sections. The second term, the slope of the marginal profit curve of the rationed customer, will be negative as long as the slope of the offer curve is less than +1. In those regions in which the slope of the offer curve is greater than 1 we shall assume that the sum of the slopes is still negative.
B. THREE MORE GENERAL CASES

The analysis in the text was based on three simplifying assumptions: (1) a fixed size investment project; (2) a constant opportunity cost to the bank for loanable funds; and (3) no costs attendant with collecting the available proceeds in case of the firm's bankruptcy. In this section we will show that the conclusions of the basic model are not altered when each of these assumptions is removed individually. To start, for each case we shall derive the three critical properties of the offer curve: (1) the shape of the curve; (2) the fact that expected profits increase monotonically as we move along the curve through higher interest factors; and (3) the fact that as we move away from the offer curve by offering either smaller or larger loans for a fixed interest factor, the expected profits decrease monotonically. With this verification of the offer curve, the results in section (II.C) on the discriminating monopolist follow directly from the analysis in the text and need not be repeated here. For the case in which the banker charges all (both) customers the same rate, we shall derive the equation for the equilibrium rate of Stage 1, $R^*$. At this point it again becomes clear that the structures for each case discussed here and the structure of the basic model of the text are equivalent. Consequently, we shall not have to undertake the tedious task of rigorously proving that the results of section (II.D) hold also for each case considered here.

1. The "Open End" Investment Opportunity

The discussion in the text relied on the assumption of a "fixed size" investment opportunity for the firm. This means that the potential
end of period value of the firm was independent of the size of the loan contract. It has been suggested that a more realistic assumption would make the potential outcome a function of the size of the loan.\textsuperscript{53} Freimer and Gordon\textsuperscript{54} have also considered this problem for the special case of a rectangular density function. Our analysis is of substantially greater generality, since we do not restrict the density function. Furthermore, the emphasis of the Freimer and Gordon work appears to have been misplaced, since they conclude that the likely result is for there to be infinite optimal loan sizes for this problem. We shall show that, on the contrary, under reasonable assumptions the loan size will always be finite.

We consider a firm with a fixed equity base, \( E \), and a loan supplied by the bank, \( L \), such that its total assets are:

\begin{equation}
(IV.10) \quad A = E + L \quad \text{E > 0, } \quad L \geq 0
\end{equation}

The end of the period value of the firm is given, as before, by a random variable, \( \bar{X} \). Now, however, we shall assume that the end of period value depends on the firm's total assets and can be expressed as

\begin{equation}
(IV.11) \quad \bar{X}[A] = AP[A] \bar{Y}
\end{equation}

where \( \bar{Y} \) is a random variable with a probability distribution \( g(y)dy \) independent of the scale of the project. \( P[A] \) is a non-random function.


\textsuperscript{54}Freimer and Gordon (1965), pp. 405-408.
of total assets which adjusts the rate of return for scale. Thus, for 
\( \rho[A] = 1 \) for all \( A \) we have a constant expected rate of return to scale, 
while \( \rho'[A] < 0 \) would imply a decreasing expected rate of return to scale.\(^{55}\)

The properties of \( \rho[A] \) assumed here are, formally:

(IV.12) \[ \rho[E] = 1 \]

(IV.13) \[ \rho'[A] \leq 0 \quad \text{for all } A \]

(IV.14) \[ \gamma = \frac{d(A \rho[A])}{dA} = \rho[A] + A\rho'[A] > 0 \quad \text{for all } A \]

(IV.15) \[ \delta = \frac{d \gamma[A]}{dA} = 2\rho'[A] + A\rho''[A] < 0 \]

Condition (IV.12) provides a convenient scaling for \( \rho[A] \) in that it 
implies that \( \gamma = \gamma[E] / E \). Condition (IV.13) rules out cases of increasing 
returns to scale. Condition (IV.14) is equivalent to requiring 
d \( \gamma[A] / dA \geq 0 \); that is, the random total return is a strictly non-decreasing 
function of the size of the investment. And condition (IV.15) requires 
that the marginal total return be a decreasing function of \( A \).

The density function for \( \gamma \), the random rate of return, is given 
by \( g[y] \). We shall assume that there exist \( \underline{\gamma} \) and \( \overline{\gamma} \) such that:

(IV.16) \[ g[y] = 0 \] whenever \( y \leq \underline{\gamma} \) or \( y \geq \overline{\gamma} \) where \( 0 < \underline{\gamma} < \overline{\gamma} < \infty \)

\(^{55}\)This convenient and instructive generalization of the problem was suggested by Professor Franco Modigliani.
Letting $G[y] = \int_{y}^{Y} g[y]dy$, we also have:

$$G[y] = \begin{cases} 0 & y \leq \bar{v} \\ 1 & y \geq \bar{v} \end{cases}$$

Following the analysis of section (II.6) we start by considering the bank's expected profits from a customer as a function of the interest factor $R$ and the loan size $L$, independent of demand. Thus

$$P = P[RL] = \alpha B \int_{\bar{v}}^{Y} y g[y]dy + LR \int_{B}^{\bar{v}} g[y]dy - IL$$

where $R = i + r$, $I = i + j$, and $B = RL / \alpha \rho$

$B$ is, of course, the necessary value for $Y$ if the loan plus interest is to be fully repaid. The similarity of the structure of equation (IV.18) with that of equation (II.6) should be apparent. Thus it is clear that (IV.18) can be rewritten as:

$$P = P[RL] = L(R-I) - RL G[B] + \alpha B \int_{\bar{v}}^{Y} y g[y]dy$$

$$= L(R-I) - \alpha \rho \int_{\bar{v}}^{B} G[y]dy$$
The optimal loan size offer curve is then given by the condition:

\[(IV.20) \quad \frac{\partial P_{RL}}{\partial L} = R(1 - G[B]) + \int_0^\beta y g[y]dy - I = 0\]

The second order condition for a maximum is given by:

\[(IV.21) \quad \frac{\partial^2 P_{RL}}{\partial L^2} = R[BG[B] - \int_0^\beta g[y]dy] - \int_0^\beta g[y]dy < 0\]

This will be valid for all \(L\) and all finite \(R\) since:

\[(IV.22) \quad B = \frac{RL}{AP} = \frac{R}{\rho} = \frac{R}{\beta} \quad \text{for all } L \text{ and all finite } R\]

and

\[(IV.23) \quad \frac{\partial B}{\partial L} = \frac{R(PA - L\alpha)}{(AP)^2} > 0 \quad \text{for all } R \text{ and } L\]

The case for an infinite \(R\) is discussed below. It should also be clear that (IV.21) also insures us that as we move further from the offer curve in either direction the bank's expected profits will be decreasing monotonically.

We now can describe the shape of the offer curve. For the case \(R < I\) we must have the corner solution of a zero loan, since \(\frac{\partial P_{RL}}{\partial L}\) (from IV.20) is negative even at this point. Secondly, we must have:
since condition (IV.20) can be met for this case only if $B \leq V$. Moreover, this implies that

\[
\hat{L} < \frac{V E \rho[A]}{I - V \rho[A]} \quad \text{when } R = I \text{ and } I - V \rho[A] > 0
\]

Since $\rho$ is a decreasing function of $A$, the maximum loan size when $R = I$ will be finite as long as there exists some finite loan size, $M$ say, such that $I > V \rho[M + E]$. This means that the opportunity cost must exceed the certain rate of return from the firm's project (as adjusted by the scale factor) at some finite scale of investment. Or, conversely, if this condition does not hold, the bank can realize a certain return greater than its opportunity cost, regardless of the size of its investment in this asset. But this would imply that this certain return ($V$) should be, in fact, the bank's proper opportunity cost in evaluating the return on other bank assets, an obvious contradiction of the definition of $I$.

For $I < R < \infty$, there are two problems to be considered. The first is concerned with the possibility that at some rate factor in this range the optimal loan may be infinite. To insure that an infinite loan will not be optimal, the marginal profits, evaluated at $L = \infty$ must be negative, that is:
(IV.26) \[ \frac{\partial P[R]}{\partial L} \Bigg|_{L=\infty} = R \left( 1 - G \left[ \frac{R}{P\left(\infty\right)} \right] \right) + \int_{V}^{R} \frac{\partial P\left(\infty\right)}{\partial L} g[y] \, dy - I < 0 \]

To understand the meaning of this condition, it is helpful to consider, for the moment, the constant return to scale case in which \( \rho = y = 1 \). In general, by differentiating (IV.20) with respect to \( R \), we obtain:

(IV.27) \[ \frac{\partial^{2} P[RL]}{\partial L \partial R} = (1 - G[B]) + \frac{\partial B}{\partial R} g[B] (B - R) \text{ for all } R \text{ and } L \]

For the constant return case, this reduces to

\[ \frac{\partial^{2} P[RL]}{\partial R \partial L} = (1 - G[R]) \geq 0 \text{ as } R \leq V \text{ for } L = \infty \]

Therefore the marginal profit is increasing as \( R \) increases up to the point \( R = V \), evaluated along the locus \( L = \infty \).

Consequently, in this case the sufficient condition ruling out infinite optimal loans for \( I < R < \infty \) is the condition ruling out an infinite optimal loan at \( R = V \). This condition is easily derived from (IV.26) to be:

(IV.28) \[ I > \int_{V}^{V} g[y] \, dy \]

Thus, for this special case, as long as the opportunity cost is greater than the expected value of the rate of return on the venture, only finite

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56 The reason for choosing this special case is that this statement is not valid for the general case under consideration. That is, in the general case, expected profits may rise and fall as \( R \) rises, the loan size given.
optimal loans can occur. This condition is also very reasonable. Since the bank is an expected profit maximizer, it is assumed indifferent to risk. Thus, if there were a project with an expected return greater than the opportunity cost, it is clear that this project's rate of return would be, in fact, the relevant opportunity cost. Other assets would be evaluated using this project's rate of return as the standard. 57

In the general case under study here, an even weaker condition, already given as (IV.26), will be sufficient to rule out the possibility of infinite optimal loans. Thus, instead of the strict condition given by (II.28) we need only require that the adjustment factor for the rate of return, \( \rho \), decline sufficiently fast so as to insure that (IV.26) does hold.

With infinite optimal loans now ruled out, we can proceed with the second question concerning the shape of the offer curve. Because of the bound on the potential rate of return, we must have

\[
\lim_{R \to \infty} R(1 - G[B]) = 0
\]

Therefore, at \( R = \infty \), the marginal profit condition is:

\[
(IV.29) \quad \frac{\partial P[RL]}{\partial L} \bigg|_{R=\infty} = \gamma[A] \int_{V} \gamma g[y]dy - I = 0
\]

57 Freimer and Gordon, op. cit., pp. (406-407) implicitly argue that just the opposite is true. Their conclusion is valid only if condition (II.28) does not hold. That is, only if the expected rate of return exceeds the opportunity cost. Only then does the open end investment case lead to infinite optimal loans.
This means that \( \lim_{R \to \infty} \hat{L} = N \), where \( N \) is defined by:

\[
\gamma[N + E] \int_{V} y \ g[y] \, dy \leq I
\]

An interesting interpretation of this condition comes from the fact that at an infinite interest rate the banker is, in essence, taking over the firm. Thus, he will invest capital \( (N, \text{that is}) \) in this project up to the point at which the marginal expected return just equals the opportunity cost. The inequality arises only because of the non-negativity assumption requiring that \( \hat{L} \geq 0 \). Furthermore, in general, the optimal loan reaches its maximum at some interest rate in the closed interval \( (\nabla \gamma[N + E], \infty) \).

From this it should be clear that the offer curve just described is equivalent to the curve for the fixed investment case in its essential characteristics.\(^{58}\)

The only property of the offer curve which remains to be verified is that \( \frac{\partial P[RL]}{\partial R} \to 0 \) when evaluated along the offer curve. This result can be obtained easily by substituting (IV.20) into (IV.18) and then taking the derivative with respect to \( R \).

The final result which we wish to derive for this case is the determination of the stage 1 optimal rate, \( R^* \). This rate is defined by:

\(^{58}\) The results stated in this paragraph are equally true for the simple case \( P = \gamma = 1 \) given that condition (II.28) holds.
\[
\frac{\partial P[D_1, D_2, R]}{\partial R} = \sum_{i=1}^{2} D_i \left( R(1-G_i[B_i]) + \gamma (B_i G_i[B_i] - \int_0^{\infty} G_i[y] dy) - I \right) + D_i(1-G_i) = 0
\]

It should be clear that equations (IV.30) and (IV.18) are directly analogous to text equations (II.19) and (II.6). Thus, from equation (IV.30) theorem (II.2) can be derived as it was in the text.

2. Increasing Opportunity Costs

In the text it was assumed that the bank could invest and borrow at a fixed opportunity rate factor, I. This is reasonable since the bank has the options of lending or borrowing in the Federal Fund's Market, issuing certificates of deposit, buying or selling government securities, and finally, borrowing from the Federal Reserve. To a first approximation the yields on these alternative securities are equal when adjusted for risk and maturity. Furthermore, under moderate monetary conditions, a bank could have access to these markets at the given market price. Under more extreme conditions, however, a bank may find the relevant opportunity cost rising as its demand for credit increases. Thus, the Federal Reserve becomes reluctant to discount, the certificate of deposit rate reaches its ceiling, and so on.

Our goal here is to show that even under these conditions our basic formulation remains valid. We shall characterize these conditions by the assumption that \( I = I[L] \) where \( L \) is the total loans outstanding for the bank. The properties of this function are:
(IV.31) \[ I = I[0] = I_0 \]

(IV.32) \[ 0 < I'[L] < \infty \quad \text{for all } L, L \geq 0 \]

(IV.33) \[ \frac{\partial^2 (I[L,L])}{\partial L^2} = 2' I'[L] + LI''[L] \geq 0 \]

Condition (IV.32) implies that the bank's costs, \((LI(L))\), are increasing as the loans increase and condition (IV.33) means that the rate of increase is also rising.

We first want to consider the properties of the bank's optimal offer curve for the \(i\)th customer. Thus, we shall assume that the loans to all customers but the \(i\)th are fixed at some given value. The general formulation for the bank's expected profits from the \(i\)th customer follows directly from the text equation (II.7) with the exception that the opportunity cost is now a function of the loan granted to the \(i\)th customer:

(IV.34) \[ P[R_iL_i] = (R_i - I[L]) L_i - \int_{K_i}^{R_i} F_i[x] dx \]

Similarly the loan offer curve is derived by differentiating (IV.34) with respect to \(L_i\) and setting the result equal to zero.

(IV.35) \[ \frac{\partial P[R_iL_i]}{\partial L_i} = R_i(1 - F_i[R_iL_i]) - I[L] - L_i I'[L] = 0 \]
If we let $\hat{L}_1 = \hat{L}_1[R]$ be the implicit solution to (IV.35), we then obtain:

$$R_1(1 - \hat{F}_1) - \hat{I} - \hat{L}_1 \hat{I}' = 0$$

where $\hat{F}_1 = F_i[R_i \hat{L}_1]$ and $\hat{I} = I[\hat{L}_1]$

The second order condition for a maximum is derived by differentiating (IV.35) again with respect to $L_1$:

$$\frac{\partial^2 (P[R_1 L_1])}{\partial L_1^2} = -R_1^2 f_i[R_i L_1] - \frac{\partial^2 (I[L])L_1}{\partial L_1^2} < 0 \text{ for all } R_1$$

by virtue of assumption (IV.33).

Since $I'[L] > 0$, it is clear that $\hat{L}_1 = 0$ for $R < I_0$. In fact, even when we have $R = I_0$, $\hat{L}_1 = 0$, which is unlike the case of the text. For $I_0 < R < \infty$, the shape of the offer is also very similar to the basic case. Differentiating (II.36) with respect to $R_1$ yields:

$$\frac{\partial \hat{L}_1}{\partial R_1} = \frac{(1 - \hat{F}_1) - R_1 \hat{F}_1 \hat{L}_1}{R_1^2 \hat{F}_1 + 2I'[L] + I''[L] \hat{L}_1}$$
Comparing equation (IV.38) to equation (II.11), it is clear that the effect of introducing the rising opportunity cost is to reduce the absolute value of the slope at all interest factors. Thus, in the case in which the constant opportunity cost of the text equals the initial opportunity cost here, we must find that the optimal offer curve for rising costs always lies below the optimal offer curve with constant costs. It is equally clear that \( \lim_{R \to \infty} \frac{L_1}{R} = 0 \) is true for both cases. Thus, the shape of the offer curve is, in its essential aspects, unchanged when we introduce rising opportunity costs.

Finally, to show that profits increase along the offer curve, we differentiate (IV.34) with respect to \( R_1 \), while constraining \( L_1 = \hat{L}_1[R] \). If we then use (IV.36), the result simplifies to:

\[
(IV.39) \quad \frac{\partial P_1[R_1 \hat{L}_1]}{\partial R_1} = (1 - \hat{F}_1) \hat{L}_1 \geq 0
\]

which, of course, is the same result obtained in the text.

To find the optimal Stage 1 rate, \( R^* \) we differentiate the total expected profit function when constrained by the demand functions, with respect to \( R \):

\[
(IV.40) \quad \frac{\partial P[D_1, D_2, R]}{\partial R} = \sum_{i=1}^{2} \left\{ D_i \left( R(1-F_i[R,D_i]) - I[L_1+L_2] - \frac{\partial I}{\partial L_1} \right) \right\} + L_1(1-F_i[R,D_i]) = 0
\]
Again the similarity of the structure of this equation with its equivalent in the text is obvious.

It should be pointed out, however, that rising opportunity costs do necessitate one caveat with respect to the dynamic rationing of riskless customers. In the text it was argued that dynamic rationing of riskless customers could be ruled out, for practical purposes, since it was unlikely that the commercial loan rate would ever fall below the bank's opportunity cost. When we allow for rising opportunity costs, however, the case becomes less certain. The point to be stressed, however, is that if we do find that the riskless customers are starting to be rationed, then we can be sure that the risky customers are being rationed all the more. Thus, even under these conditions, rationing is a redistribution of loans toward the less risky.

3. Costs Attendant with Bankruptcy of the Firm

We have already discussed Merton Miller's formulation of a rationing model with bankruptcy costs in section (I.B). For Miller, bankruptcy occurs when the firm cannot repay fully the loan contract -- that is:

\[ \bar{\tilde{x}} < RL \]

in which case the bank's cost for collecting the available proceeds is:

\[ b'_0 + b'_1 \bar{\tilde{x}} \]
In the earlier section we suggested, as an alternative, that bankruptcy be defined when:

\[ \hat{X} < L \]

in which case the collection costs are:

\[ b'_o + b_1 \hat{X} \]

In this section we wish to show that including these bankruptcy costs does not affect the qualitative nature of the solution. We shall use our own definition of bankruptcy costs, although we should point out that within our model Miller's assumption leads to very similar results.

When bankruptcy costs are taken into account, the bank's expected profit function becomes:

\[
(PRL) = \int_{K}^{LR} x f[x] \, dx + RL \int_{L}^{K} f[x] \, dx - IL \int_{K}^{L} (b_o + b_1 x) f[x] \, dx
\]

where we have omitted the i subscript for the ith firm. Following the analysis of the text, we next differentiate (IV,41) with respect to L to obtain the optimal offer curve:

\[
(\text{IV,42}) \quad \frac{\partial P[RL]}{\partial L} = R (1 - F[RL]) - I - (b_o + b_1 L) f[L] = 0
\]

For \( R < I \) the optimal loan is zero. For \( R = I \), the optimal loan is defined by \( \hat{L} \leq K/I \) just as it was without bankruptcy costs. In general,
however, for \( I < R < \infty \), the effect of the existence of the bankruptcy costs is to lower the offer curve below the level for no bankruptcy costs. The \( \lim_{R \to \infty} \frac{L}{R} = 0 \), however, just as it does without bankruptcy costs. Thus the shape of the offer curve is essentially unchanged when bankruptcy costs are added. To verify that we do, indeed, have a maximum, note that:

\[
(IV.43) \quad \frac{\partial^2 P[RL]}{\partial L^2} = -R^2 f[RL] - (b_o + b_1 L) f'[L] - b_1 f[L]
\]

In general, the sign of this derivative may be either negative or positive. It will be positive if \( f'[L] \) is sufficiently negative so that the expected value of the bankruptcy costs, \( (b_o + b_1 L) f[L] \), actually declines sufficiently fast when the loan is being raised. The issue is not of any important consequence, however, and to simplify matters we shall simply assume that (IV.43) is, in fact, negative for all values of \( R \) and \( L \). Finally, note that if we differentiate (IV.41) with respect to \( R \), evaluated for \( L = \hat{L}[R] \), and then substitute (IV.42) into the result, we get:

\[
(IV.44) \quad \frac{\partial P[\hat{L}]}{\partial R} = (1 - \hat{R}) \hat{L} \geq 0
\]

Finally, to determine the stage 1 optimal rate factor, \( R^* \), we differentiate the expected profit function when constrained by both customers' demand functions:
\[ \frac{\partial P[D_1, D_2, R]}{\partial R} = \sum_{i=1}^{2} \left\{ D_i \left( R(1 - F_i[R_D]) \right) - I - (b_0 + b_1 D_1) f_i[D_1] \right\} \]

\[ + (1 - F_i[R_D]) D_i^2 = 0 \]

This result is equivalent to the corresponding equation in the basic model and, as before, we shall leave to the reader the verification that all other results follow directly.
CHAPTER 3

A MEASURE AND TEST OF CREDIT RATIONING*

The aim of this chapter is threefold. The primary object is to provide an empirical test for the existence of credit rationing. Secondly, the experiments also provide a test of our theory of credit rationing since the structure of the empirical model is based on the implications of the theoretical model. And thirdly, we wish to verify the validity of a proxy variable which can be used instead of direct measures of credit rationing in testing for the effect of rationing on the economy. We want to stress, however, that it is the existence of credit rationing as a commercial bank supply phenomenon rather than the impact of rationing on the "real" sectors, which is being studied here.

Two caveats with respect to our second objective should be noted, however. First, we use only the "aggregate" implications of the theoretical model. A more complete test of the theory per se would have to rely on microeconomic tests also. A cross-sectional study of this type should prove feasible and is planned for the future. Secondly, a related point, we do not claim that the tests have a high power in distinguishing between alternative theories of credit rationing. The similarity of the aggregate implications of most rationing theories has already been stressed in the previous chapter. Microeconomic studies would have to be used to obtain a sufficient power of discrimination.

*I am indebted to Professor Franco Modigliani for numerous suggestions on an earlier draft of this chapter -- too many to acknowledge individually -- which have improved both the value and presentation of this material.
To test the hypotheses, we rely exclusively on regression analysis of aggregate stochastic equations over the post-Accord period. In Section (I) we consider the very difficult question of finding an acceptable measure for credit rationing. Section (II) then considers the specification for the independent variables of the equation. In Section (III) the results of the tests are presented and Section (IV) provides an interpretation and summary.

I. AN EMPIRICAL MEASURE OF CREDIT RATIONING

The fact that we allocate an entire chapter to the existence of credit rationing should warn one that the measurement of credit rationing must be a severe problem. Otherwise, if data were readily available, the question of existence would be trivial. The converse, however, that difficulty in obtaining data implies the absence of credit rationing, is not valid.

A. THE MEASUREMENT PROBLEM

Credit rationing has already been defined as the existence of an excess demand for credit at the quoted rate. Excess demand is defined to be the difference between the demand and bank offer for those firms being rationed. Excess supply, defined as the case when the offer exceeds the loan demand at the quoted rate, does not of course provide an offset to rationing.
Since both the demand curve and offer curve are operational concepts, the amount of rationing is, technically, operational. Consequently, the measurement problem must arise from "practical" difficulties in data collection. These difficulties center upon the necessity of obtaining the ex ante demand curve and bank's offer curve for each individual customer.

One possible approach, for example, is to survey firms and banks, asking what is the maximum loan demand and loan offer, respectively at each interest rate. As a practical course, such questions would not elicit acceptable answers. First, the problem of communicating a hypothetical question -- the ex ante concept of a function -- would be formidable, as is common to all surveys. Even if this could be overcome, however, we face an aggregation problem. Since the excess supply of some customers does not offset the rationing of others, each case must be judged individually. The requisite data would probably not be made available, however, since, at least from the bank's position, this information could be easily used to its disadvantage.\footnote{We are not referring to surveys which test the effectiveness of either the availability or cost of credit in restricting the demand for investment goods. These studies would not distinguish between an absence of credit rationing and an absence of an effect due to rationing.}

An empirical econometric study of the demand for loans, say, is also a potential solution. The necessity of disaggregating and the fact that only the ex-post loan size is available as a dependent variable make this approach intractable for practical purposes. The discussion in Chapter 5 below emphasizes, furthermore, that even if we accept the existence
of credit rationing, the interpretation of such econometric experiments re-
mains a delicate issue.

This all suggests that an indirect approach, which uses only a proxy variable for credit rationing, may yield a more manageable solution. To construct such a test, one must set up a model of the determinants of credit rationing, on one hand, and provide a set of implications of rationing, on the other. The experiment then consists of judging whether the determinants of rationing do cause, in an empirical sense, the effects of rationing. In this way, the variable, credit rationing itself, is never needed explicitly. Of course, one must be careful to avoid correlation between the determinants and effects which is unrelated to credit rationing.

G. L. Bach and C. J. Huizenga have exploited this technique in an ingenious study, using cross-sectional data. For the period October 1955 through October 1957 a large sample of banks were classified into three groups: "tight," "medium," and "loose." A necessary condition for a bank to be labeled tight was that it have a relatively low ratio of free reserves and government bills and certificates to total deposits at the beginning of the period. In addition, however, the bank's growth in deposits over the period had to be low relative to other banks'. It was assumed that the relative demand for loans of small firms compared to large firms would be independent of the classification of the bank.

---

They used this model to test the hypothesis that tight money leads to discrimination against small firms in the availability of loan funds. The idea was that if in periods of tight money discrimination -- we might say credit rationing -- against small firms did occur, then it would be that much stronger at exceptionally "tight" banks. Since the relative demand for loans is constant across banks, discrimination or rationing would show up as a relatively fast growth in loans to small firms at "loose" banks. In fact, Bach and Huizenga find little evidence of such discrimination and reject the hypothesis. 3

Our theory of credit rationing provides a convenient framework for evaluating this experiment. The various degrees of tightness would be interpreted, in our terms, as a variation in the opportunity cost over the cross-section. Even accepting their definition of tightness, a basic question concerns the source of this variation. 4 In equilibrium, at least, one important source is the tendency for banks to specialize and service only one or a few classes of customers. This also implies, however, that the relevant "optimal" or long-run equilibrium rate on loans would differ between banks in the same way that the optimal rate differed between the various classes of customers in our theory. In the polar case in which each bank services only one class, only banks which service the

3They also consider the question of discrimination by industry class. Here too they find no evidence of credit rationing.

4The definition of tightness itself is, of course, quite arbitrary. Deane Carson (1961) has suggested several alternative definitions, although there is an obvious trade-off between feasibility and theoretical completeness. See also the reply by Bach and Huizenga (1961b).
same class would have equal equilibrium loan rates. Moreover, in our
theory we went to some length to show that there was no systematic rela-
tionship between the amount of equilibrium rationing and the level of the
opportunity cost. But this means, in terms of the Bach and Huizenga
study, that differential "equilibrium" tightness need not imply differ-
tential equilibrium rationing.5

A similar point can be made with respect to dynamic rationing.
There is no reason why the speed of adjustment of the loan rate toward
its long-run equilibrium level must be equal for all loan classes, par-
ticularly if banks do tend to service only specific classes. Dynamic
rationing depends, of course, on the spread between the desired rate and
the rate actually quoted. If the tighter banks were able to adjust this
quoted rate more quickly, then we would not expect to find a strong re-
lationship between tightness per se and dynamic rationing. In fact, more-
ever, Bach and Huizenga themselves provide very clear evidence supporting
the contention that the tight banks adjusted (raised) their rates faster
than the looser banks.6 So, in summary, what we are suggesting is that
to the extent that the classification by tightness was also a classifica-
tion by type of customer serviced, we would not expect the Bach and Huizenga
experiment to necessarily provide any evidence on credit rationing.7

5This can also be interpreted as saying that differential tightness, as de-
defined by Bach and Huizenga, has meaning only in essentially dynamic situations.

6Tussing has also noted the importance of these data, although he stressed
the possibility of shifts in relative demand. For example, since banks with
slow deposit growth may also experience low loan demand, the tightness measure
may be biased. In Bach and Huizenga (1963) the question of shifts in demand
is discussed.

7It should be pointed out that, with the exception of survey studies, the Bach
and Huizenga article is the only empirical work on credit rationing known to us.
B. A THEORETICAL PROXY FOR CREDIT RATIONING

For our empirical tests we need to construct a time series proxy for credit rationing. More accurately, we wish to construct a proxy for the degree of credit rationing. The degree of credit rationing is measured, ideally, by:

\[ H = \frac{E}{D} \]

where \( E \) is the amount of credit rationing defined in equation (II.1) of Chapter 2 and \( D \) is the aggregate demand for loans.

The basis for our proxy is the fact that credit rationing results in a redistribution of loans toward the less risky firms. It was in this sense that it was described as an upgrading of the bank's portfolio. To be complete, we should consider the distribution of credit rationing over all risk (or rate) classes. In deference to the practical problems of data availability, however, we shall distinguish only risky and risk-free firms. This can be interpreted as the simplified version of the theoretical model in which risk-free firms are not rationed and all risky firms are grouped together as potential rationing candidates. With this modification, the ideal measure for the degree of credit rationing can be rewritten as:

\[ (I.1) \quad H = \frac{D_2 - L_2}{D_1 + D_2} \]

The subscript 2 refers to risky firms. The amount of rationing is the demand of risky firms \( (D_2) \) less the loans actually supplied to these
firms \( (L_2) \). The risk-free firms (subscript 1) are not rationed and hence contribute nothing to the numerator of (I.1). The loans to these customers \( (L_1) \) just equal the demand \( (D_1) \) of these customers.

The theoretical proxy for credit rationing can be written either as:

\[
(I.2a) \quad \hat{H}_1 = \frac{L_2}{L_1 + L_2}
\]

or

\[
(I.2b) \quad \hat{H}_2 = \frac{L_1 + L_2}{L_1}
\]

The first measure, \( \hat{H}_1 \), is precisely the percentage of total loans which are granted to the risk-free customers, and hence is positively related to the degree of rationing. The second proxy, \( \hat{H}_2 \), is a measure of the proportion of loans granted to risky customers -- the inverse of \( \hat{H}_1 \) in fact -- and hence is negatively related to the degree of rationing.

The ideal measure, \( H \), and the empirical proxies, \( \hat{H}_1 \) and \( \hat{H}_2 \), are related, of course. If we let

\[
B = \frac{D_1 + D_2}{D_1}
\]

then from (I.1) we obtain:

\[
(I.3a) \quad \hat{H}_1 = \frac{1}{B(1-H)}
\]

\[
(I.3b) \quad \hat{H}_2 = B(1-H)
\]

since \( L_1 = D_1 \)
Differentiating (I.3) with respect to $H$, yields:

\[
\frac{\partial \hat{H}_1}{\partial H} = \frac{1}{B(1-H)^2} > 0
\]

\[
\frac{\partial \hat{H}_2}{\partial H} = -B < 0
\]

Equation (I.4) shows that the proxy measures are related to the ideal measure as expected. This also means that these proxies will be useful below, when we wish to study the effect of rationing on the economy. The existence of the $B$ factor in (I.3) and (I.4) introduces a possible "errors in variables" problem, however. That is, as the relative demand for loans ($B$) fluctuates, the proxy measures will deviate from the true or ideal measure of rationing. But the results presented here will not be biased by this.

This can be seen clearly, using the second proxy as the more simple example, by first rewriting (I.3b) as:

\[
\hat{H}_2 = (\bar{B} - BH) + (\varepsilon - \varepsilon H)
\]

where $\bar{B}$ is the mean of $B$ and $\varepsilon$ is the deviation from this mean. If $H$ and $\varepsilon$ are uncorrelated, then the expected value of the second term in (I.5) must be zero. And, in fact, we expect that these two variables would be uncorrelated, since our theory implies that the degree of true rationing should have no systematic relationship with the parameters of the demand functions. We also provide some empirical verification of this point when we discuss the demand for loans in Chapter (4) below.

Expression (I.4) can also be written in elasticity form as:

\[
\hat{E}H_1/EH = H/(1-H) \quad \text{and} \quad \hat{E}H_2/EH = -H/(1-H)
\]
The question at stake here, of course, is whether the use of $H_1$ or $H_2$ as the dependent variable in a linear regression will cause biased estimates for the coefficients of the independent variables. Fortunately, if the error term of the measure is uncorrelated with the true value of the variable — and it is likely this will be true in our case as just noted — the estimates are unbiased.\textsuperscript{9} On the other hand, if the amount of noise introduced by fluctuations in $B$ is very large, the efficiency of the estimates may be reduced substantially. But it is likely, and again this is confirmed in our empirical work on the demand for loans, that fluctuations in this ratio will be quite small. Finally we should note for future reference that if the proxy is used as an independent variable, for example in an equation testing the effect of rationing on the real economy, the "errors in variables" may cause biased estimates.

\textbf{C. AN EMPIRICAL PROXY MEASURE FOR CREDIT RATIONING}

The previous discussion has provided us with two theoretical measures for rationing which depend on loans granted to risk-free and risky firms. Data on total loans are readily available, of course. To obtain data on loans to risk-free customers, we must make the further assumption that risk-free customers are uniquely identified by the fact that they receive the prime rate on their bank loans. This correspondence of low risk and the lowest rate class is easily verified as both an institutional fact and an important feature of our theoretical model of credit rationing.

\textsuperscript{9}See, for example, Malinvaud (1966), chapter (10). For the case of $H_1$, this is strictly true only if a linear approximation for equation (1.36) is appropriate.
Our source of data for loans granted (and not granted) at the prime rate is the Federal Reserve's "Quarterly Interest Rate Survey," which is available from 1952. These data were used as a rationing proxy by John Hand in an unpublished paper using factor analytic techniques. The survey records the rate and volume on new loans granted during the first two weeks of the last month of the quarter. In several instances the prime rate changed during the period of the survey with the effect that one cannot distinguish between loans made at the new prime rate and loans made at this rate while the old prime rate was still in effect. To start, we smoothed and spliced these quarters as well as possible.

Preliminary experiments using these data as the source for the two proxies were satisfactory, but indicated that even further improvement could be made. As an additional safeguard, we followed Hand in using three additional series which contain information on the distribution of loans by size to further smooth the data. The premise of this method is that customers who demand and receive very large loans would also be essentially risk-free.

The actual measure of the proxy was obtained by extracting the first principal component from the following four series:

(a) The proportion of total loans granted at the prime rate.

(b) The proportion of total loans over $200,000 in size.

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10 We are grateful to John Hand for making his data readily accessible.

11 A more detailed account of the data and splicing is provided in the data appendix.
(c) The proportion of loans over $200,000 in size granted at the prime rate.

(d) The proportion of total loans $1,000 - $10,000 in size.

The first three series should enter positively into the principal component and the fourth negatively, of course. The principal component has a mean of zero and standard deviation of unity and the four factor loadings were, respectively:

(a) .988  
(b) .968  
(c) .939  
(d) -.959

This indicates that each series enters prominently and about equally into the principal component. In fact, the regression results presented below which use the principal component proxy differ primarily only in terms of goodness of fit from the results which use the simpler but presumably noisier series based on only the loans granted at the prime rate. We should also note that the principal component proxy corresponds directly to the variable $\hat{H}_1$ and not $\hat{H}_2$, since the series (a) of the principal component is, in fact, $\hat{H}_1$. Preliminary work with the simple measures indicated that it made practically no difference whether $\hat{H}_1$ or $\hat{H}_2$ was used; on this basis we chose the more straightforward measure based on $\hat{H}_1$ since it was already derived by Hand.\[12\] In Figure (1)

\[12\]The difference in the standard error of estimate between equations using the two simple proxies was less than 1 per cent.
we have plotted the principal component over the period 1952-1966. The pattern of rationing indicated by the proxy is quite reasonable. The level of the proxy in 1966 is quite encouraging, in fact, since it rises very high in the second and third quarters and then falls off some in the fourth quarter, just as would be expected, a priori.

II. THE DETERMINANTS OF CREDIT RATIONING

We can now turn to the specification for the independent variables of our regression equation. The basic structural relationship can be written as:

\[ \hat{H}_t = a_0 + a_1 (rL_t^* - rL_t) + \varepsilon_t \]

\( \hat{H} \) is, of course, the principal component proxy for credit rationing. \( rL^* \) will be called the desired commercial loan rate and is theoretically equivalent to the long run optimal rate discussed in Chapter 2. \( rL \) is the commercial loan rate actually quoted, and \( \varepsilon \) is a random error term.

The specification of (II.1) accounts for both dynamic and equilibrium rationing. When the commercial loan rate is at its long run desired level, we have only equilibrium rationing. On the microeconomic level, equilibrium rationing depends on the specific parameter values for the demand functions, density functions, and opportunity cost. But our investigation in Chapter 2 indicated that no systematic relationship existed between the degree of rationing and changes in these parameters.
Consequently, in equation (II.1) we have specified equilibrium rationing as a constant \( a_o \) plus an error term \( \epsilon \) with the latter accounting for any essentially "random" variations in the underlying parameters.

Dynamic rationing, on the other hand, occurs when the desired long run commercial loan rate differs from the rate actually quoted. The theory of Chapter 2 indicated that the degree of dynamic rationing will increase as the spread between the desired rate and actual rate increases.\(^1\) The assumption that this relationship is one of proportionality is implicit in equation (II.1).

When the quoted rate is above the equilibrium rate, the amount of dynamic rationing can be considered negative in the sense that rationing will be reduced below its equilibrium level. The total amount of rationing can never be less than zero, however, by definition. The dependent variable, as measured by our principal component proxy, will take on negative values, however. This difference in scaling will be taken up by the constant. The only point of note here is that we will not be able to unscramble the level of equilibrium rationing from the scaling effect in the constant.

While we have spoken of \( rL \) as the quoted commercial loan rate, we know that in fact, of course, there is a set of rates corresponding to the various classes of loans. Such a set of rates could be derived from

\(^{13}\)We are abstracting from the empirically implausible "abnormal" case discussed in Chapter 2, page 84.
the "Quarterly Interest Rate Survey." The collinearity between these rates, however, would clearly preclude obtaining any differential effect. Consequently we shall use only a single rate, the average rate on commercial loans, as our empirical measure.

Finally, we can turn to the important task of specifying the desired commercial loan rate. Here again, we shall assume that a single measure will be an adequate approximation for the spectrum of rates. Our theory indicated that the desired commercial loan rate would be at that level at which the marginal proceeds from a commercial loan after adjustment for risk would just equal the opportunity cost. We know also, however, that this relationship will be valid for all other assets in the bank's portfolio.

This means that the commercial loan rate will equal the market yield on any other asset after adjustment for risk, maturity, liquidity, and possibly any expectations concerning future levels of that rate. We are free, then, to choose any security as the standard of comparison, the obvious criterion being practical expedience. Our choice, on this basis, is the bank's holdings of Treasury bills. The bank's return on Treasury bills consists of essentially two components. The first component, the Treasury bill rate, rT, is straightforward. The second component, the value of the liquidity of Treasury bills, is somewhat more involved.

Our basic premise is that the liquidity yield of Treasury bills should decrease as the bank's holdings of these bills (TB) rises relative
to its deposit liabilities (DEP). Deposits are the natural scaling, since the need for liquidity arises from potential instability in these liabilities. Our _apriori_ expectation for the shape of this relationship is shown in Figure (2).

![Graph showing the relationship between Treasury bill liquidity yield and TB/DEP]

**Figure 2**

Our specification for the liquidity term takes the form $b \frac{\text{DEP}}{\text{TB}} - 1$ where $b \ (> 0)$ is an estimated parameter. This is preferable to the simple linear form, $b(1 - \frac{\text{TB}}{\text{DEP}})$, since it should provide a better approximation to the shape of the curve shown in figure (2).$^{14}$

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$^{14}$Important questions arise as to the choice of data for Treasury bills. Those of a purely technical nature, such as the source of the data, are discussed for Treasury bills as for all variables, in the data appendix. A more substantive issue, however, concerns the fact that banks are required to maintain government securities in their portfolios to cover government deposits at the bank. To the extent that this requirement (footnote cont'd)
The liquidity value of Treasury bills has been affected substantially in recent years by the existence of a well organized market for certificates of deposits (CDs). We have been able to take the banks' total deposits as given since, in the short run, the bank has little control over their level. CDs which can be purchased and sold in an active market, on the other hand, afford the banks an important means for increasing their liquidity at short notice. For this reason, the existence of CDs can be interpreted as causing a downward shift in the curve in figure (2) for the liquidity value of Treasury bills. Strictly speaking, it is only the potential of using CDs, and not the extent to which they are used, which affects the Treasury bill liquidity curve. Consequently, a simple shift parameter or dummy for the liquidity coefficient is a sufficient measure for this effect.

Practically, we shall assume that CDs became an important influence starting in 1962-I, although the arbitrary nature of this assumption is clear. Formally, we are assuming that

\[ b = b_0 - b_1 D_{62} \]

where D_{62} is a dummy variable which is zero before 1962-I and one starting in 1962-I. When the rate on CDs reaches its Regulation Q ceiling, however, one would expect that the effect of CDs on the liquidity yield of Treasury

(Footnote continued)

necessitates holding Treasury bills of some required amount, the correct variable for our analysis would be the "free bills," that is, bills above the requirement. As a practical matter, however, the total amount of government securities held by the banks exceeds by a large margin the government's bank deposits; and we know of no requirements which necessitate holding Treasury bills.
bills would largely vanish. Comparing a series of secondary market rates and
the Regulation Q ceiling on comparable CDs, indicates that the ceiling was
effective during the quarters 1962-I to 1963-II and 1965-IV of the sample
period.\footnote{15} 16 In apparent conflict with these data, however, is the fact
that for the quarters 1962-I through 1963-II the volume of CDs outstanding
was rising and, in fact, quite quickly. This would suggest that the banks
were able to extend additional CDs at the ceiling rate. One resolution of
this conflict is the possibility that in the first set of quarters the CD
market, although already very large, was still developing. In other words,
the increased demand for CDs allowed the banks to extend additional CDs at
the given ceiling rate.

If this interpretation is correct, then to specify accurately the
constraint of a ceiling rate, one would need a model of the demand for CDs.
Such a model is not now available, unfortunately, and we shall leave this
scheme for future work. To circumvent this problem, at least practically,
we have experimented with models based on "polar" assumptions about the ef-
fact of the CD market. For the first case, we assumed that the ceiling was

\footnote{15}{We are grateful to Robert Rasche for providing the secondary market rate
series, which is an average rate obtained from several published and unpub-
lished sources.}

\footnote{16}{Both 1964-IV and 1965-IV are "borderline" cases since the ceiling was
raised during the quarters with the effect that after the change the ceil-
ing was no longer binding. The dependent variable to be used in our regres-
sion tests is based on data gathered during the first two weeks of the last
month of the quarter. Since the ceiling change occurred in 1964-IV on No-
ember 24, we have assumed that the ceiling was not binding for our purposes,
while in 1965-IV the ceiling changed on December 6 and hence we assume that
the banks were still essentially constrained by the ceiling during the first
two weeks of December.}
a constraint on the banks whenever the secondary rate was above the ceiling. This hypothesis was tested by redefining the dummy (D62) so that it was zero in the quarters (noted above) when the ceiling was binding. We found, in this case, that our residuals indicated a substantial overestimate for the amount of rationing in the quarters 1962-I through 1963-II, while the residual for 1965-IV was essentially zero. This we interpreted as meaning that the CD market was, in fact, an effective liquidity source for the banks in the early set of quarters, because the demand was increasing. It also suggests that the only quarter in our sample in which the CD ceiling was actually a constraint was 1965-IV. Since this is the last quarter in our sample, we decided to simply drop it from the sample and use it, along with the 1966 quarters, for our extrapolation of the fitted values.

The second hypothesis considered assumes that the demand for CDs was the relevant constraint throughout the sample period and that the Regulation Q ceiling rate was irrelevant. Under these conditions the ratio of CDs outstanding to total deposits, CD/DEP, would be a better measure of the value of the CD market as a liquidity source than the dummy variable. The results of this test will be discussed below.

Just as we have considered the liquidity of Treasury bills, we must also account for the relative "illiquidity" of commercial loans. The illiquidity of commercial loans is due partially to the lack of a secondary market for commercial loans (short of rediscounting with the Federal Reserve) and partially because loan renewals have tended to become somewhat "automatic"
out of tradition and due to the existence of costs of collection should the firm be forced into bankruptcy. This illiquidity is then to be interpreted more as a short run constraint on the bank's control over the volume of commercial loans than as the "marketability" concept valid for Treasury bills. For this reason, the value of this illiquidity should increase as the ratio of commercial loans to the bank's total loans and investments increases. Furthermore, a priori we anticipate that this illiquidity should increase at an increasing rate as the ratio increases.

Formally, we measure this effect as \( b_2 \left( \frac{L'}{(A-L')} \right) \) where \( b_2 > 0 \) is an estimated coefficient, \( A \) is total loans and investments, and \( L' \) is the sum of commercial loans and non-farm, non-residential mortgage loans held by the banks. The mortgage loans are included since they are made to essentially the same customers as the commercial loans and hence they share the characteristic of a short run constraint on the bank's portfolio.

Finally, we should also consider changes in the liquidity ratio, since a dynamic short run increase in loans which is beyond the control of the bank would have additional (although only transitional) costs.

We have now almost completed the task of specifying the desired commercial loan rate. Three separate components have been distinguished: the Treasury bill rate itself; the liquidity value of Treasury bills, including the adjustment for the existence of CDs; and finally the illiquidity of commercial loans as measured by the ratio defined above and changes in this ratio. To obtain slightly more generality we shall formulate the desired loan rate as a linear function of the terms just summarized. The
need for the linear function arises since we have not yet formally accounted for differences in risk and maturity between commercial loans and Treasury bills. Putting all these parts together, we obtain:

\[ rL^* = c_0 + c_1 \left( rT + (b_0 - b_1D62) \frac{DEP/TB - 1}{L'/(A - L')} \right) + b_3 \Delta \frac{L'/(A - L')}{(A + L')} \]

Finally, substituting equation (II,2) into equation (II,1) yields the basic equation to be estimated:

\[ \hat{H} = (a_o + a_1c_o - d_1b_o) - a_1(rL) + d_1(rT) + d_1b_0(DEP/TB) - d_1b_1(D62) \frac{DEP/TB - 1}{L'/(A-L')} + d_1b_2 \Delta \frac{L'/(A-L')}{(A-L')} \]

where \( d_1 = a_1c_1 \)

III. ESTIMATION OF RATIONING EQUATION

In Table (A) we present the results of testing the specification contained in equation (II,3). Only ordinary least squares estimates are shown. Preliminary two-stage tests with, admittedly, a limited set of instruments indicated, however, that there is little simultaneity bias. Final two-stage tests are planned upon completion of the Federal Reserve - MIT Econometric Model.\textsuperscript{17}

\textsuperscript{17}See deLeeuw and Gramlich (1968) for a progress report on this model.
The sample period is 1952-II to 1965-III. This period was chosen in order to provide the largest feasible sample of post-Accord monetary experience. We have already noted the reason for removing the observation of 1965-IV from the sample. Interest rates are not seasonally adjusted and are measured in percentage points. All other variables are seasonally adjusted unless specifically noted and all dollar magnitudes are $ billions. A complete set of variable definitions is contained in the Appendix (Section (V)).

Equation (A-1) contains the estimates obtained by testing the complete specification contained in equation (II.3). Although the signs of all coefficients are correct, the coefficient of the interaction term, (D62) (DEP/TB - 1) is very large in absolute value relative to the coefficient of the liquidity term, DEP/TB, by itself. This suggests that the effect of the CD market may be even better specified by simply entering the dummy term linearly. The result of this test is shown as (A-2) and we do find that the standard error of estimate does fall, although very slightly. In (A-2) also, however, we still find that both the Treasury bill liquidity term and the change in the commercial loan illiquidity term are not significant. We want to suggest, though, that it is quite likely that these two variables would move together, indicating multi-collinearity. The mechanism is very simple: in the very short run with the level of both deposits and total loans and investments fixed, the bank's only response to a sudden increase in loans outstanding may be to sell off Treasury bills. This constraint is valid only in the very short run, however, which explains why the level of the bank illiquidity term is not affected by this mechanism.
<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>rL</th>
<th>rT</th>
<th>DEP</th>
<th>CD</th>
<th>D62</th>
<th>X*</th>
<th>A-L1</th>
<th>A-L1</th>
<th>DW</th>
<th>R²</th>
<th>S_e</th>
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<td>.007</td>
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<td>.83</td>
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<tr>
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<td>.009</td>
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<td>A-6**</td>
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<td>(.83)**</td>
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</tbody>
</table>

* = X = (D62)(DEP/TB - 1)
** = Dependent variable not seasonally adjusted and seasonal dummies included.
*** = Multiple correlation between fitted values and dependent variable net of seasonals.
In equations (A-3) and (A-4) we have removed from the specification the change in the loan illiquidity term and the Treasury bill liquidity term, respectively. In (A-3) it is clear that the Treasury bill rate and liquidity variable increase both in terms of significance and magnitude when the change in the commercial loan illiquidity term is dropped. And similarly, in (A-4) the Treasury bill rate becomes much less important and the change in the loan illiquidity more important, as would be expected. The magnitude of these changes is less than one would have hoped, however, and we shall accept any of the three formulations given as (A-2), (A-3), and (A-4) as equally acceptable.

In equation (A-5) we have replaced the basic CD dummy (D62) with the ratio of CDs to total deposits, for reasons discussed above. This result confirms our preference for the dummy formulation. The fit is obviously substantially poorer than the other results. Moreover, we overestimate the amount of rationing in the quarters 1962-I to 1963-II, as is implied by the low Durbin-Watson statistic. This is very important, since it is these quarters in which the existence of the CD market seems most critical for the specification. We might also note, in this equation, that the Treasury bill rate and liquidity terms become small and insignificant, again indicating a poor specification for the existence of the CD market.

Finally, in equation (A-6) we show the results of testing the specification of (A-2) with the dependent variable not seasonally adjusted, but linear seasonal dummies. It is clear that the coefficients are changed
very little in this case. In addition, in parentheses below the actual $R^2$ we have calculated the multiple correlation between the fitted values and the dependent variable net of the seasonal dummies. The small difference between the actual $R^2$ and the value net of the seasonals verifies that only a small percentage of our correlation was due to seasonal factors.

In closing this section we should also note that a number of experiments with other interest rates and other forms (linear and non-linear) for the liquidity terms have been tried. By and large, these results were similar; qualitatively, to the material presented here.\footnote{Some of these results were presented in Jaffee (1967) which was a preliminary and summary version of the material presented here. For example, the prime commercial paper rate was used instead of the Treasury bill rate and the loan illiquidity variable was specified as $L'/A$.}

IV. SUMMARY OF THE RESULTS

Our principal goal for this chapter was to provide evidence verifying the existence of credit rationing. The empirical results presented above leave little doubt, we believe, about the validity of this view. In all cases the signs of coefficients were correct; and the statistically insignificant coefficients were quite clearly related to multi-collinearity problems. The goodness of fit seems very reasonable in view of the noise, already discussed in section (I), in the dependent variable. This same evidence also substantiates, of course, the underlying theory of credit rationing and the proxy measure which was derived from it.

To provide one more experiment, it is interesting to compare the predicted values of the rationing proxy for the period 1965-IV through 1966-IV, using the estimated coefficients, with the true values for this
period. The result of this prediction for equation (A-2) is shown in Table (B).

\begin{table}
\centering
\caption{Fitted and Actual Values for Credit Rationing}
\begin{tabular}{|c|c|c|c|}
\hline
Year & Fitted Value & Fitted Value & Actual Value \\
    & Equation (A-2) & Adjusted for CDs & \\
\hline
1965-IV & .88 & 2.15 & 1.49 \\
1966-I & 1.03 & 1.03 & 1.67 \\
1966-II & 1.11 & 2.38 & 2.06 \\
1966-III & 1.34 & 2.61 & 2.41 \\
1966-IV & .98 & 2.25 & 2.37 \\
\hline
\end{tabular}
\end{table}

The predicted value from equation (A-2) is shown in the first column. The predictions for equations (A-3) and A-4, we might note, are very nearly the same as (A-2). The actual value for the rationing proxy is shown in the third column. Although the predicted series follows the actual series quite well (in fact, we catch the "turning point" in the fourth quarter), it is clear that we are substantially underpredicting the level.\footnote{The predicted values for 1967 were: 1967-I .92 \hspace{2cm} 1967-II 1.04 \hspace{2cm} 1967-III .94 \hspace{2cm} 1967-IV .97}

Because of a change in the Federal Reserve's format for the "Quarterly Interest Rate Survey," we cannot yet obtain actual values for 1967. It is reassuring, however, that the predicted value, at least, is well below the peak reached in the money squeeze of mid-1966.
It is very important, however, to note that these fitted values include the effect of the CD dummy, D62. In fact, the secondary market rate for CDs exceeded the ceiling in 1965-IV and 1966-II to 1966-IV. Furthermore, the volume of CDs was not increasing and for the most part was significantly decreasing during these quarters. This suggests that for this period the Regulation Q ceiling was a very effective constraint on the banks, and that some adjustment must be made. Because this phenomenon did not occur during the sample period, we have no structural estimates for it. As a rough measure, however, we can simply assume that the CDs ceased to provide the banks with any liquidity. We then obtain the "adjusted" predicted value for the period by adding the value of the dummy, 1.27, to the fitted values for the four quarters noted above, as shown in column 2 of Table B. This technique should, if anything, overcompensate, since the CD market still did exist and did provide at least some potential liquidity.

The values in Table B do confirm this hypothesis. In 1965-IV, 1966-II, and 1966-III, the actual value does lie between the fitted value and the adjusted fitted value as expected. In 1966-IV, on the other hand, the fitted value after adjustment is still too small, although the difference is well within the standard error of estimate. We have not corrected the value in 1966-I since the CD rate was below the ceiling in this quarter. Although we do underestimate the amount of rationing by about twice the standard error of estimate, it seems quite possible that even in this quarter some restriction in the CD market may have occurred. Although these
results are obviously not precise, they do provide hope that with a sufficient number of data observations and a more refined theory of the demand for CDs, this entire problem can be resolved.

In closing this chapter, we should like to stress two points. First, in using a proxy variable as we have done here, one always runs the risk of observing spurious correlation between the independent variables and dependent variable. The likely course for such a phenomenon in our case would be via fluctuations in the relative demand for loans of risk-free firms. We believe we have minimized the possibility of such an occurrence by carefully formulating our model. In addition, empirical evidence on this point is considered when we discuss the demand for commercial loans in Chapter 5.

Secondly, so far we have had little to say about the effectiveness of rationing, as measured by our proxy, in reducing expenditures by firms. We shall return to this topic in Chapter 5. First, however, in the following chapter, we wish to consider the determinants of the commercial loan rate.

V. DATA APPENDIX

All dollar magnitudes are measured in billions of current dollars. Interest rates are not seasonally adjusted and are measured in percentage points.

A Total loans and investments for all commercial banks, two-month average centered on end of quarter. Seasonally adjusted by ratio to moving average. FRB

CD Seasonally adjusted certificates of deposit derived by Robert Rasche from call-report data for all commercial banks.
DEP  The sum of seasonally adjusted demand deposits and time deposits.  
FRB.

D62  Dummy variable which is zero before 1962-I and one starting in 1962-I.

The rationing proxy, which is the first principal component of the four series listed above. Before taking the principal component, series (a) and (c) were spliced over quarters in which changes in the prime rate occurred during the sample period by using series (b) as a standard for an interpolation. The average value of the series interpolated from the previous quarter forward and the next quarter backward was used. The principal component was then seasonally adjusted using the Bureau of Census X-11 program.

\[ L' = L + M \]

L  Commercial and industrial loans at all commercial banks, last month of quarter, seasonally adjusted; unpublished Federal Reserve Board series.

M  Non-farm, non-residential, commercial bank mortgage loans. Seasonally adjusted by ratio to moving average.  FRB

rL  Average rate on commercial loan, nineteen large cities.  FRB

rT  Rate on three month Treasury bills, three month average.  FRB

TB  Treasury bills held by commercial banks included in the Treasury Survey of Ownership as published in the FRB. Seasonally adjusted by ratio to moving average. The principal drawback of this series is that it comes from only a partial sample of all commercial banks. In Loans and Investment by Class of Bank, FRB, data on the universe of commerce banks are available, but only on a semiannual basis. Since the totals for the smaller Treasury Survey do not appear to have drifted relative to the total for all commercial banks, the former quarterly series was used.

FRB = various issues of Federal Reserve Bulletin.
CHAPTER 4

THE DETERMINANTS OF THE COMMERCIAL LOAN RATE

The commercial loan rate is important in this study, both as a link in the overall model of the commercial loan market and as a critical element in our theory of credit rationing. The discussion of credit rationing in the previous two chapters has provided a theory of the determinants of the commercial loan rate as an important by-product. In this chapter the results of empirical tests of this theory are presented.

In section (I) we summarize the theory of rate determination as it was developed in length in Chapters 2 and 3. The theory is tested by estimating time series regression equations and in section (II) the specification of these equations is discussed. Section (III) contains the results of our experiments and section (IV) interprets and summarizes these results.

I. A REVIEW OF THE THEORY OF THE COMMERCIAL LOAN RATE

The central problem of formulating a formal theory of the commercial loan rate is the specification of the imperfectly competitive character of the industry. To start, one can consider the polar case of a single monopolist bank facing n customers. The rate charged on each loan would be such that the expected marginal income from the loan would just equal the bank's marginal cost. And, in general, the bank would charge each customer a different rate. To be more realistic, we can consider the banks as an oligopolistic set. In this case the determinants of the loan rates can be
more complex. If open collusion and side payments between banks are allowed, however, the monopolistic solution with respect to rates still persists, although the distribution of customers among the banks depends on some arbitrary initial conditions.

The degree of collusion necessary for the monopolistic solution would obviously be found illegal. On the other hand, it is well agreed that banks do have sufficient market power to avoid the perfectly competitive solution. Thus it would be expected that the banks would find some practical method for enforcing at least some of their power. The key to that method, we believe, lies in the fact that each loan customer and loan contract has unique properties. Hence banks can rationalize discriminating between customers, whereas an industry selling homogeneous products to homogeneous customers -- the steel industry, for example -- cannot. Our discussion in Chapter 2, section (III.A), indicated that this leads to a solution in which banks classify customers into categories such that within each class banks tend to charge all customers the same rate.

In Section (II.D.4) of Chapter 2, the formal properties of such a system in which the banks classify their \( n \) customers into \( m \) classes (\( m \leq n \)) were developed. The set of optimal rates, corresponding to the \( m \) rate classes, are determined by marginal profit considerations. A customer's classification depends on the parameters of its demand curve and the bank's subjective evaluation of the risk. The industry's expected profits increase as the number of allowed categories \( m \) increases
until the number of categories just equals the number of customers, in which case the monopolistic solution already discussed is attained.

Our main point is that "institutional" factors such as legal and social constraints force the banks only to approximate the optimal solution. Thus, the number of customer classes must be quite small because of both potential anti-trust prosecution and the real costs of running a complicated oligopoly. Similarly, the classification of customers is based on such "objective" factors as industry, asset size, and financial condition of the firm. This only approximates the ideal classification which is based on demand elasticity and riskiness. But, at least within the bounds of this approximation, the set of loan rates for the classes should tend in the long run toward the optimal level.

In addition to determining the desired long run rate, the oligopolistic cartel must also decide when to change the rate in the short run. It is well known that the commercial loan rate tends to lag behind other important market interest rates in adjusting to new conditions. And the reasons are clear. Imperfect forecasting, inadequate information, and similar factors all argue for at least a gradual adjustment. Most important, however, is the fact that it will take some time before the principal commercial banks, at least, will be able to agree that a change in the rates is warranted. It would be rare that a single bank would risk going it alone.¹

¹There are, of course, exceptions. For example, the Chase Manhattan Bank recently (January 26, 1967) lowered its prime rate in the absence of any industry consensus. See, for example, "Showdown in Prime Rate Looms as Chase Bank Stands Alone," Wall Street Journal, January 30, 1967.
This also suggests that some means of communication -- legal communication, that is -- between banks would be of tremendous aid in adjusting the rate. Our hypothesis is that the banks rely on a signal in determining the timing of rate changes. Thus, in periods in which the signal indicates the need to change rates, the loan rate adjusts very quickly; otherwise, the speed of adjustment is quite slow. To consider this process more closely, we now turn to the empirical counterpart of this theoretical system.

II. THE SPECIFICATION OF THE STRUCTURAL DETERMINANTS

This theory of the commercial loan rate can be stated formally as:

\[
\triangle rL = h + g(rL^* - rL_{-1})
\]

where \( rL^* \) is the long run desired rate and \( rL \) is the rate actually quoted in the short run. The desired rate can be thought of, as we have noted, as an approximation to the optimal rate discussed in Chapter 2. The adjustment process will depend on the signal; consequently, the coefficients of the process, \( h \) and \( g \), cannot be specified as constants. To make the equation testable, we must specify both the desired rate and the adjustment factors.

A. THE ADJUSTMENT PROCESS

We have already suggested the importance of a signal mechanism for the adjustment process. The plot of changes in the actual commercial loan rate, shown in Figure (1), lends additional support to this view. The changes in the rate clearly can be very large but sporadic
or lumpy. If the underlying adjustment process occurred at a constant rate, on the other hand, we would expect a much smoother cyclical pattern. This graph also suggests the nature of the signal. In Table A we have listed the quarters in which the discount rate has changed during the sample period. The high correspondence between changes in the discount rate and large changes in the commercial loan rate is clear.

**TABLE A**

<table>
<thead>
<tr>
<th>Changes in the Federal Reserve Discount Rate:</th>
<th>1952-1965^2</th>
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</thead>
<tbody>
<tr>
<td>1953-I</td>
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<tr>
<td>1954-I</td>
<td>-.25</td>
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<tr>
<td>1954-II</td>
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<td>.50</td>
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<td>1965-IV</td>
<td>.28</td>
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</table>

Furthermore, changes in the Federal Reserve's discount rate appear to fit the banks' needs extremely well. These policy-instrument changes are evident to all banks, are unambiguous in direction, and occur infrequently enough to avoid embarrassing false signals. The use of the discount rate as the signal has the further advantage that the Federal Reserve or other government agencies can scarcely criticize changes in the commercial loan rate which follow on the heels of discount rate changes. Perhaps even more important, in an environment

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^2The source and description of these data are provided in the data appendix, section (V).
in which banks treasure customer relations, timing rate changes to coincide with discount rate changes allows the banks to shift the onus onto the less personal market conditions or even better onto the Federal Reserve. If bankers actually do not use the discount rate for this purpose, it would appear that they should.

It should be emphasized, however, that the underlying logic of this formulation assumes that the Federal Reserve does continue to change the discount rate in a manner which allows the commercial banks to use it as an indicator. Clearly the Federal Reserve could easily thwart this use of one of its policy instruments by obstinately refusing to change the discount rate. More realistically, establishment of the suggestion to tie the discount rate to the Treasury bill or Federal funds rate would eliminate the discount rate as an acceptable signal. In this sense, the equation given in (II.1) should be interpreted as an explanation of past commercial bank behavior rather than as a true prediction of behavior in the future.  

This view of the adjustment process can be incorporated into the formal model by writing the coefficients of equation (II.1) as:

\[(II.2a)\quad g = g_0 + g_1(|\Delta rD|) + \text{lagged terms in } |\Delta rD|\]

\[(II.2b)\quad h = h_0(\Delta rD) + \text{lagged terms in } \Delta rD\]

\(^3\)One may also ask the question whether the discount rate can be considered an exogenous variable. For our work here, however, it seems reasonable to abstract from any feedback running from changes in the commercial loan rate to changes in the discount rate.
and by substituting (II.2) into (II.1) which yields:

\[(II.3) \quad \Delta r_L = h_0 \Delta r_D + g_0 (rL^* - rL_{-1}) + g_1 (rL^* - rL_{-1})(|\Delta r_D|) + \text{lagged terms.}\]

Equation (II.2a) means that the speed at which the banks adjust toward the desired rate increases when the discount rate changes (in either direction). One difficulty with this formulation arises if the change in the discount rate and \((rL^* - rL_{-1})\) are of opposite sign. In this case the direction indicated by the signal opposes the basic movement of the adjustment process. Such behavior by the Federal Reserve would be contrary to the basic premise of the model that the discount rate can be used as a signal. As a practical matter, moreover, it seems clear that the discount rate would not be changed in one direction (say, increased) when the banks desire a change in the opposite direction (say, a decrease).

Equation (II.2b) indicates that a change in the discount rate has an effect on the commercial loan rate independent of the basic adjustment process. The relevance of such an effect arises from the following reasoning: when the Federal Reserve raises the discount rate, it is likely that the spread between the desired rate and the rate actually being charged will vary among banks. If the signal mechanism is to function effectively, however, it is important that even those banks with a small spread raise their rates. In fact, it is the essence of the model that, in the absence of collusion, banks will rely at least in part on an exogenous
signal for the timing of rate increases. This can lead in the polar case to a situation in which all banks are in equilibrium \( (\text{\text{RL}}^* = \text{\text{RL}}) \), while the Federal Reserve still persists in changing the discount rate. In this case each individual bank, due to its ignorance of the status of other banks, would voluntarily move away from the equilibrium, although eventually the steady-state rate would be regained. An alternative formulation of this process, not pursued here, would make changes in the desired rate a function of changes in the discount rate.\(^4\)

The need for the lagged terms in equation (II.2) arises because the effect of a discount rate change on the loan rate may be distributed over time. Of course, if the signal mechanism is operating correctly, we would expect the predominant effect -- one might say the impact effect -- on the loan rate to occur almost immediately. But there are also secondary effects which may take as long as a quarter to work out. These arise because non-major banks respond more slowly to the signal and further because some loans may have been previously negotiated. Now, if the discount rate change occurs on the first day of the quarter, for example, we can safely assume that even the secondary effects would be fully developed within the quarter. In the more general case, however, the discount rate can change at any point in the quarter, which indicates that the secondary effects may be felt in the following quarter also. We

\(^4\)This section has benefited from comments on an earlier version by Professor Franklin Fisher and other participants at an M.I.T. seminar.
assume, on the other hand, that the impact effect is always in the current quarter. This suggests that the total effect of a change in the discount rate can be adequately measured by two variables. First, for the impact effect we need only the current value of the change in the discount rate, $\Delta rD$, as shown in equation (II.2). For the secondary effect, we distribute the change of the discount rate over two quarters by taking the first difference of the average rate during the quarter, the weights being the number of days the rate is in effect during the quarter. This latter variable will be noted as $\Delta rD^5$.

**B. THE SPECIFICATION OF THE DESIRED LONG RUN COMMERCIAL LOAN RATE**

For the specification of the desired long term rate, we wish to consider two alternative approaches. The most novel and perhaps attractive relies on our earlier discussion in Chapter 3 on an empirical test for credit rationing. The basic structural equation in that chapter, equation (II.1), was written:

$$\hat{H} = a_0 + a_1 (rL^* - rL)$$

where the lack of "time" subscripts indicates all variables are measured in the current quarter. $\hat{H}$ is, of course, the principal component rationing proxy. This equation can be easily rewritten as:

$$rL^* - rL = \frac{\hat{H} - a_0}{a_1}$$

---

5A further discussion of some technical problems and a formal description of this variable is provided in the data appendix.
By adding and subtracting $rL_{-1}$ to both sides and arranging terms, we obtain:

$$\begin{align*}
(II.4) \quad rL^* - rL_{-1} &= \Delta rL + \frac{\hat{H} - a_1}{a_0}
\end{align*}$$

This means that the spread between the long run desired rate and the quoted rate of the previous quarter is a linear function of the amount of rationing and the current change in the commercial loan rate. Thus, if we substitute (II.4) into (II.3), we obtain an equation in which the change in the commercial loan rate depends on the level of rationing; and most importantly, we need not specify directly either the desired loan rate nor the lagged dependent variable. If we make this substitution, we obtain an equation which is non-linear in the term $|\Delta rD|$. A linear approximation to this equation has the form:

$$\begin{align*}
(II.5) \quad \Delta rL &= v_0 + v_1 \hat{H} + v_2 \Delta rD + v_3 \hat{H} |\Delta rD| + v_4 |\Delta rD| + \text{lagged terms}
\end{align*}$$

It is important to note in this equation that the coefficient $v_4$ is not identified, since the rationing proxy, $\hat{H}$, is an index number with an arbitrarily assigned scaling. Should the mean of this proxy be changed, then the coefficient $v_4$ would also change due to the interaction term $(\hat{H} |\Delta rD|)$. The fitted values for the equation as a whole would remain unchanged, of course. The same point is also true for the constant, $v_0$, since the proxy also enters linearly. As it happens, for the scaling of $\hat{H}$ which we actually use -- mean zero -- the $v_4$ coefficient is near zero and statistically insignificant.\(^6\)

---

\(^6\)In an earlier version we were concerned that the sign of the interaction term, $\hat{H} |\Delta rD|$, would be "wrong" if the separate terms $\hat{H}$ and $\Delta rD$ differed in sign. This concern was needless, since the linear term $|\Delta rD|$ will always correct for this.
The specification of \( r_l^* \) can, of course, be handled in a more direct fashion by relying on the formulation used for the empirical tests of the rationing hypothesis (Chapter 2, section (II)). This specification, it will be recalled, derived the equilibrium rate as a function of several terms including the Treasury bill rate, Treasury bill liquidity, commercial loan illiquidity, and adjustment for the certificate of deposit market. To obtain a testable equation, a linear function of these variables is substituted in equation (II.3) and the lagged dependent variable is entered explicitly. The difficulty with this approach is that the term with the coefficient \( g_1 \) requires that we multiply the variable \( |\Delta r_D| \) by each of the terms for the equilibrium rate. As might be imagined, multicollinearity precludes obtaining reasonable structural estimates for these coefficients. Several practical solutions to this problem will be discussed with the results of the tests, to which we now turn.

III. ESTIMATION OF COMMERCIAL LOAN RATE EQUATION

The results of testing our hypotheses are shown in Table (B). The equations have been estimated by ordinary least squares. The prime source of potential bias in these results is the rationing proxy. In Chapter 3 we have already noted the "errors in variables" of this measure. In addition, there is, of course, the possibility of simultaneity bias. Preliminary two stage least squares tests using both the predicted value of the rationing proxy (from the single stage results given in Chapter 3) and limited sets of instruments, indicated little bias. This is not surprising,
<table>
<thead>
<tr>
<th>(\hat{H})</th>
<th>(\Delta r_D)</th>
<th>((\Delta r_D))</th>
<th>(\hat{H}_{-1})</th>
<th>((\Delta r_D)_{-1})</th>
<th>(r_A)</th>
<th>(r_G)</th>
<th>((r_L)_{-1})</th>
<th>DW</th>
<th>(R^2)</th>
<th>Se</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>.009</td>
<td>-.009</td>
<td>.342</td>
<td>.148</td>
<td>.147</td>
<td>.227</td>
<td>2.05</td>
<td>.76</td>
<td>.077</td>
<td></td>
</tr>
<tr>
<td>(.78)</td>
<td>(-.55)</td>
<td>(5.3)</td>
<td>(2.2)</td>
<td>(2.2)</td>
<td>(2.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-2</td>
<td>.017</td>
<td>.026</td>
<td>.284</td>
<td>.169</td>
<td></td>
<td></td>
<td>2.02</td>
<td>.67</td>
<td>.087</td>
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</tr>
<tr>
<td>(1.4)</td>
<td>(1.8)</td>
<td>(4.2)</td>
<td>(2.2)</td>
<td></td>
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<td>.012</td>
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<td>.362</td>
<td>.160</td>
<td>.295</td>
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</tr>
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<td>(5.4)</td>
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<td></td>
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<tr>
<td>B-4</td>
<td>.015</td>
<td>.021*</td>
<td>.294</td>
<td>.147</td>
<td>.159</td>
<td>2.17</td>
<td>.74</td>
<td>.079</td>
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</tr>
<tr>
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<td>(4.7)</td>
<td>(2.1)</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B-5**</td>
<td>.007</td>
<td>-.009</td>
<td>.319</td>
<td>.189</td>
<td>.132</td>
<td>.250</td>
<td>1.99</td>
<td>.80</td>
<td>.073</td>
<td></td>
</tr>
<tr>
<td>(.71)</td>
<td>(-.58)</td>
<td>(5.1)</td>
<td>(2.9)</td>
<td>(2.1)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>B-6</td>
<td>.253</td>
<td>.234</td>
<td>.155</td>
<td>.133</td>
<td>.225</td>
<td>.182</td>
<td>.210</td>
<td>1.88</td>
<td>.81</td>
<td>.068</td>
</tr>
<tr>
<td>(3.4)</td>
<td>(3.5)</td>
<td>(2.6)</td>
<td>(2.6)</td>
<td>(2.7)</td>
<td>(2.7)</td>
<td>(-3.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-7</td>
<td>.257</td>
<td>.259</td>
<td>.142</td>
<td>.148</td>
<td>.243</td>
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<td>1.97</td>
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<td>.070</td>
</tr>
<tr>
<td>(3.3)</td>
<td>(3.8)</td>
<td>(2.3)</td>
<td>(2.8)</td>
<td>(2.9)</td>
<td>(2.0)</td>
<td>(-2.7)</td>
<td></td>
<td></td>
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<tr>
<td>B-8**</td>
<td>.231</td>
<td>.235</td>
<td>.186</td>
<td>.122</td>
<td>.249</td>
<td>.146</td>
<td>-.174</td>
<td>1.89</td>
<td>.84</td>
<td>.066</td>
</tr>
<tr>
<td>(3.2)</td>
<td>(3.6)</td>
<td>(3.1)</td>
<td>(2.4)</td>
<td>(3.0)</td>
<td>(2.2)</td>
<td>(-2.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* = coefficient of \(\hat{H}_{-1}\)

** = Estimated with seasonal dummies.
since the rationing proxy, as a linear term, is not significant in the ordinary least squares tests. Further tests using a completed version of the Federal Reserve-MIT model are planned.

The estimation period is 1952-II to 1965-IV. It has been suggested that both the beginning and the end of this period are inappropriate for studies of the commercial loan rate. The excess profits taxes, it is stressed, made borrowing from the Federal Reserve unusually attractive and thus reduced the opportunity cost of commercial loans until the end of 1953, while starting in 1960 unusual government pressure made it difficult for the banks to raise their rates. But the effect of the excess profits tax, we feel, is over-rated and our model specifically specifies a signal mechanism which controls the timing of rate changes. To verify formally that no structural changes have occurred, we have also performed some Chow Tests.

Interest rates are measured in percentage points and are not seasonally adjusted, while all other variables are seasonally adjusted.

Equation (B-1) of Table (B) contains the estimates for our most complete specification of the formulation using the rationing proxy

7See, for example, Hendershott (1967) and Hendershott (1968).

8We performed two tests, both on equation (B-2), and used the period 1954-I to 1959-IV as the standard. For the excess profits tax, we could not reject the hypothesis that the coefficients for the period 1952-II to 1953-IV and the standard period were equal at the .05 level. With respect to the last part of the sample, we found we could not reject the hypotheses of equal coefficients at the .01 level for the period 1960-I to 1965-IV and the standard period, although it could be rejected at the .05 level. See Chow (1960) for a description of these F tests.
variable for the desired equilibrium rate. The specification of (B-1) differs from (II.5) in two respects. First, the lagged or secondary effect of a discount rate change is explicitly specified by $\Delta rD$, the average change discussed above, and by the interaction term $(\hat{H}_{t-1} | \Delta rD |)$. We have experimented with more complex lag structures involving other scalings and Almon lags on the change in the discount rate, but found the specification of (B-1) the most satisfactory. Secondly, in preliminary tests of all the equations shown in Table (B) we found that the term $|\Delta rD|$ was very small and statistically insignificant, for reasons already given. Consequently, we re-estimated the equations with this variable suppressed, the results of course being essentially unchanged.

In terms of goodness of fit (B-1) is quite satisfactory and the standard error of 7.7 basis points compares favorably with other formulations. The variables with the change in the discount rate enter very strongly, verifying the importance of this mechanism as a signal for the timing of rate changes. The linear rationing proxy variable is much less satisfactory, however, since it has the wrong sign and is statistically insignificant. Since this variable also enters multiplicatively in the interaction terms, it seems likely that at least part of the problem may be multicollinearity. The results of equation (B-2), in which the interaction terms are suppressed, provide evidence confirming this. Furthermore, the coefficient of $\hat{H}$ has the correct sign in (B-3) in which

---

9For example, Goldfeld (1966), p. 133, and Hendershott (1968), p. 56, obtain standard errors of 9.0 and 8.7 basis points, respectively, although the estimation periods are different.
only the interaction term with current values is suppressed, suggesting that a substantial amount of the collinearity is between the terms \( \hat{H} \) and \( \hat{H} | \Delta r_D | \) as would be expected. A very similar result is also shown in (B-4) in which we use \( \hat{H}_{-1} \) as the linear term, but include the current values of the interaction term. Finally, in equation (B-5) we have estimated the specification of (B-1) with the addition of seasonal dummies. While the seasonal factors are significant, they are small in magnitude (about 3 basis points) and do not significantly influence the other coefficients.

Instead of using the rationing proxy, it has already been suggested that equation (II.3) be estimated by directly specifying the determinants of \( rL^* \) and explicitly including the lagged dependent variable. To specify the multiplicative interaction terms, however, some summary measure of \( (rL^* - rL_{-1}) \) must be devised to prevent multicollinearity. While we had hoped to avoid using the rationing proxy for this purpose, no other satisfactory alternative could be found. In this form, equation (II.3) can be written as:

\[
(IV.1) \quad \Delta r_L = h_0 \Delta r_D + g_0 rL^* - g_0 rL_{-1} + g_1 \hat{H} | \Delta r_D | + \text{lagged terms.}
\]

For the specification of \( rL^* \) we have tried to make use of the formulation used in the empirical study of the rationing equation (see Chapter (3), p. 123 (above). Again, however, multicollinearity precludes reasonable estimates. This is not surprising, since if the rationing proxy is itself collinear with the other terms, then it is reasonable that the set
of variables used to explain rationing will have the same collinearity
tendency. In addition, it also appeared that the Treasury bill rate,
which was used in this formulation, was dominated by the change in the
discount rate variable.

This all suggests that a less complex formulation for the de-
sired long term commercial loan rate would be desirable. In particular,
a long term interest rate would be most satisfactory, since collinearity
with the change in the discount rate would be avoided. The equation to
be tested is derived by assuming that:

\[(IV.2) \quad r_L = j_0 + j_1 r_N\]

where \(r_N\) is a long term interest rate, and by substituting \((IV.2)\) into
\((IV.1)\). This specification should be adequate, since we know already
that the discount rate mechanism captures very well the short run dynamic
changes in the loan rate.

The results of testing this hypothesis are shown in Table B as
equation \((B-6)\) which uses Moody's AAA corporate bond rate \((r_A)\), and as
equation \((B-7)\) which uses the long term government bond rate \((r_G)\). In
terms of goodness of fit this formulation provides a substantial improve-
ment over that using the rationing proxy variable. In addition, the esti-
mated coefficients are very reasonable.

The basic speed of adjustment \((g_0)\) is about 21 per cent per quarter
in \((B-6)\) and about 15 per cent per quarter in \((B-7)\). This difference is
perhaps explained by a divergence in the seasonality and short run dynamics
of the rates rA and rG. In addition to the basic speed of adjustment, we must also consider the effect of a change in the discount rate. The impact of a 100 basis point increase in the discount rate, which occurs when rationing is at 1.0, is a 36 basis point increase for the commercial loan rate for (B-6) and a 41 basis point increase for (B-7). The secondary or lagged effect of this discount rate change, again when rationing is at 1.0, is a change in the commercial loan rate of .37 basis points for (B-6) and .38 basis points for (B-7). This suggests that the impact effect and the cumulative amount of lagged or secondary effect may be about equal in magnitude. The secondary effect, however, will be distributed between the current quarter and the following quarter, depending on the timing of the discount rate change in the quarter. Also, it is worth noting that in long run equilibrium, a 100 basis point increase in rA results in the loan rate rising by 87 basis points (B-6) and a 100 basis point increase in rG results in the loan rate rising by 82 basis points (B-7).

Finally, in equation (B-8) we show the effect of adding seasonal dummies to the specification of (B-6). The seasonal factors are, again, significant but only in the order of 3 basis points in size.

\footnote{We chose the value of 1.0 for $\hat{H}$ since it is one standard deviation from the mean of this variable. In 1966, when rationing was extremely high, the proxy obtained values over 2.0.}
IV. SUMMARY OF THE RESULTS

Our formulation of both the short run and long run properties of the commercial loan rate has been successful. The use of changes in the discount rate as a factor modifying the timing of loan rate changes was particularly well verified. This implies that the spread between the desired long run rate and the actual quoted rate may be substantial for even extended periods, and thus confirms the premise of Chapters 2 and 3 that dynamic rationing will dominate equilibrium rationing in observable impact.

Because of multicollinearity, it was not possible to obtain as complete a specification of the desired long term rate as wished. On the other hand, the dominance of dynamic factors in the rate equation suggests that our specification is still adequate. Furthermore, the rationing proxy proved very satisfactory when used to replace the term \((rL^* - rL_{-1})\) in the multiplicative interaction terms.

To complete this chapter, it is interesting to extrapolate the fitted values of our estimated regression into 1966. Equations (B-2) and (B-6) serve as good examples of the two approaches used. The fitted values for these two equations are shown in the first and second column, respectively, of Table (C). In the third column of this table the results of a dynamic simulation of equation (B-6), in which the lagged dependent variable is generated by the equation itself, is shown.
<table>
<thead>
<tr>
<th>Year</th>
<th>Fitted Value (B-2)</th>
<th>Fitted Value (B-6)</th>
<th>Fitted Value (B-6) (Dynamic)</th>
<th>Actual Value (rL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966-I</td>
<td>.19</td>
<td>.34</td>
<td>.34</td>
<td>.28</td>
</tr>
<tr>
<td>1966-II</td>
<td>.07</td>
<td>-.003</td>
<td>-.01</td>
<td>.27</td>
</tr>
<tr>
<td>1966-III</td>
<td>.08</td>
<td>-.001</td>
<td>.05</td>
<td>.48</td>
</tr>
<tr>
<td>1966-IV</td>
<td>.07</td>
<td>-.08</td>
<td>.06</td>
<td>.01</td>
</tr>
</tbody>
</table>

For the first and fourth quarters our predictions are uniformly acceptable, but the second and third quarters are quite poor. It is of some comfort to note that alternative specifications for the commercial loan equation also yield poor results when extrapolated through 1966.11

11For example, an equation similar to that used in Ando-Goldfeld (forthcoming) was estimated using our own data and estimation period as:

\[
\Delta r_L = .019 - .17 r_{L-1} + 2.44Q' + .19 \Delta r_T + .08 \Delta r_{T-1} - 1.22Q - \frac{10}{10} \sum_{t=1}^{10} r_{T_t-1} + .03 \frac{10}{10} .
\]

The extrapolated values for 1966 were: 1966-I .19; 1966-II .10; 1966-III .16; 1966-IV .06.

Also, in a work which has just come to our attention, Hendersott (1968), p. 57, using a slightly different estimation period, obtained somewhat better fitted values: 1966-I .19; 1966-II .21; 1966-III .27; 1966-IV .16. However, his equation implies a speed of adjustment of about 60% per quarter, which seems unrealistically high.
The principal problem with our specification centers on the fact that the Federal Reserve did not raise the discount rate in these two quarters of very tight money. A large number of ex post rationalizations for this failing of our equation can be derived from the unusual conditions in the money markets during the middle quarters of 1966. For example, on September 1, 1966, the Board of Governors felt it necessary to send letters to all member banks requesting that a special effort be made to restrict the credit expansion.\textsuperscript{12} Could this letter have served as a substitute for a discount rate change? Frankly, we are not sure; we pose this suggestion, rather, in order to emphasize the flexibility of our model in allowing the incorporation of such "institutional" developments. But while our model is valuable as a descriptive or simulation device for the commercial loan rate, obviously it will still not be able to predict "institutional" changes in Federal Reserve behavior.

V. DATA APPENDIX

A. MEASURES OF DISCOUNT RATE

Two technical problems arise in obtaining appropriate measures of the discount rate. First, the timing of discount rate changes differs between the various Federal Reserve banks, creating the problem of which bank's discount rate to use.\textsuperscript{13}

\textsuperscript{12}Federal Reserve Bulletin, September, 1966, p. 1338.

\textsuperscript{13}See Carr (1961) for an interesting study of this phenomenon.
Thus, a more extended study might attempt to explain regional differences in the adjustment of the loan rate as a function of the variations in discount rate changes among the regional Federal Reserve banks. For our purposes, however, we have used the discount rate at the New York Federal Reserve bank, since it seems likely that most banks would look to this rate for the signal.

Secondly, our measure of the commercial loan rate is obtained from the Federal Reserve's "Quarterly Interest Rate Survey" which is taken during the first two weeks of the last month of the quarter. Consequently, discount rate changes occurring during the last two weeks of the calendar quarter must be associated with the following quarter for our purposes. Furthermore, when the discount rate changes during the first two weeks of the last month of the quarter, an average rate for the period must be constructed. For the impact variable, \( rD \), in these cases (which amount to almost one-third of our observations, unfortunately) we have weighted the rate by the number of days it was in effect during the two weeks.

For the secondary effect variable, \( \bar{rD} \), we have used the average rate over the quarter, the weights being the number of days of the quarter the rate is in effect. The first difference of this variable, \( \Delta \bar{rD} \), is then equal to \( \Delta rD \) but distributed over two quarters. In summary then:

\[ rD: \text{Average discount rate at New York Federal Reserve Bank during the first two weeks of the last month of the quarter, FRB (see Table A).} \]

\[ \bar{rD}: \text{Average discount rate at New York Federal Reserve bank based on quarters ending with the 14th day of the last month of the calendar quarter, FRB.} \]
B. MEASURES OF OTHER VARIABLES

rL: Average commercial loan rate, nineteen large cities, based on
"Quarterly Interest Rate Survey," FRB

rG: Long term government bond rate, three month average, FRB

rA: Moody's AAA corporate bond rate, three month average, FRB

\( \hat{H} \): Proxy variable for credit rationing derived from principal com-
ponent of four series. (For further details, see data appendix,
Chapter 3 below.)

FRB = Source of data, Federal Reserve Bulletin (various issues).
CHAPTER 5

THE DEMAND FOR COMMERCIAL LOANS AND THE EFFECTS OF CREDIT RATIONING

Our theory of the supply side of the commercial loan market has now been fully developed. In long run equilibrium, banks quote the optimal loan rate and enforce the indicated degree of equilibrium credit rationing. In the short run, however, the quoted loan rate adjusts only imperfectly toward the long run optimal level, causing dynamic rationing which dominates equilibrium rationing in observable effect. Although formally the degree of rationing depends on loan demand, the commercial loan rate and the degree of credit rationing can be taken as given in studying the demand side of the loan market since, in the short run at least, banks will react only with a lag to changing demand.\(^1\)

The actual volume of loans outstanding is then determined by the demand for loans as influenced by the loan rate and adjusted for the degree of credit rationing. Because of the existence of credit rationing, care must be taken in defining terms. \textit{Loan demand}, \(L^d\), is defined as the firms' desired quantity of loans for a specified commercial loan rate, given that no rationing occurs. \textit{Loans outstanding}, \(L\), in contrast, refers to the quantity of loans actually received by the firms after taking account of any credit rationing. Thus, formally, we must have:

\[ E = L^d - L \]

\(^1\)The formal structure of the complete model has been discussed at greater length in section (B) of Chapter I.
where $E$ is the amount of credit rationing (measured in dollars) as defined in Chapter II, Equation (III.1).

We start in section (I) with a discussion of a theoretical model of the demand for loans under the assumption that rationing does not occur. In section (II) we allow credit rationing to enter and consider the impact of this rationing on the firms' financial and real expenditure decisions. In particular, this suggests modifications in the pure demand model, using the proxy variable for credit rationing already developed, which allow us to switch to a formulation with loans outstanding as the dependent variable. Section (III) provides estimates of the aggregate model using time series data and section (IV) tests a similar model using data disaggregated by asset-size classes of firms. One of the primary conclusions of sections (II) to (IV) is that the predominant observable effect of credit rationing is its direct impact on the firms' assets (or liabilities); this aspect of the problem is pursued further in section (V), using trade credit as an example. Finally, section (VI) summarizes the conclusions of the chapter.

I. THE DEMAND FOR COMMERCIAL LOANS

In this section we consider a firm which takes the commercial loan rate as given and formulates its demand for loans without concern for the possibility of credit rationing. The basic premise of the model, which follows the work of Locke Anderson\(^2\) and Steven Goldfeld,\(^3\) is that the

\(^2\)Anderson (1964), particularly Chapters 3 and 4.

\(^3\)Goldfeld (1966), pp. 82-87.
firm's primary concern is to meet its sales goals and opportunities, while the financial structure is given only second order priority. This means, in other words, that the financial structure can be thought of as adjusting to the firm's fixed and floating capital needs.

Table (A) contains a somewhat simplified corporate balance sheet which helps to clarify this point and also serves to define variables. The "asset" side of this balance sheet will be referred to as "net assets," since it includes accounts payable (AP), accrued tax liability (TAX), and other current liabilities (OCL) adjusted, of course, for sign. Also note that long term liabilities (T) include the equity accounts as well as long term debt.

TABLE A

<table>
<thead>
<tr>
<th>(Net) Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>M Cash</td>
<td>L Commercial loans</td>
</tr>
<tr>
<td>G Government securities</td>
<td>T Long term liabilities (= long term debt and equity)</td>
</tr>
<tr>
<td>AR Accounts receivable</td>
<td></td>
</tr>
<tr>
<td>-AP Accounts payable</td>
<td></td>
</tr>
<tr>
<td>-TAX Accrued tax liability</td>
<td></td>
</tr>
<tr>
<td>OCA Other current assets</td>
<td></td>
</tr>
<tr>
<td>OCL Other current liabilities</td>
<td></td>
</tr>
<tr>
<td>H Inventory stock (book value)</td>
<td></td>
</tr>
<tr>
<td>K Fixed assets (net)</td>
<td></td>
</tr>
</tbody>
</table>
In the long run, the structure of our model is based on the premise that commercial loans (L) and long term liabilities (T) "finance" the net assets on the left hand side of the balance sheet. The chief innovation in our approach is that we shall also explicitly account for the short run dynamic "buffer" function of commercial loans which arises from the balance sheet constraint. To understand these two functions of commercial loans more clearly, it is helpful to consider a very abstract version of the model first; the realistic details can then be filled in quite easily.

A. THE MODEL IN ABSTRACT

The balance sheet in Table (A) has been written with only long term liabilities and commercial loans on the liability side in order to emphasize that in the long run these liabilities will "finance" the net assets shown on the left hand side of the balance sheet. The division of this financing between long term liabilities and commercial loans will depend on the structure of the firm's assets with respect to maturity, liquidity, and perhaps acceptability as collateral, and on the relative cost of the two liabilities as financing.

To formalize this notion we shall separate the net assets, shown on the left hand side of the balance sheet in Table (A), into mutually exclusive and exhaustive classes with the dollar amount of net assets in each class denoted by \( P_i \) \( (i = 1, 2, \ldots, n) \). The distinguishing characteristic of the \( i \)th class is that the desired level of long term liabilities to finance this class \( \left( \bar{T}_i \right) \) can be written as:
\[ \hat{T}_i/P_i = \rho_i + f_i[r] \quad 0 \leq \rho_i \leq 1 \quad i = 1, 2, \ldots, n \]

where the coefficient for each class \( \rho_i \) is distinct and depends on the characteristics of the assets in that class. The term \( f_i[r] \), which is a function of interest rates representing the effect of the relative cost of alternative long term liability financing, has been separated from the \( \rho_i \) coefficient to provide the basis for equation (I.1) derived below.

It is also helpful to distinguish the class \( P_1 \) as the set of net assets which are financed entirely by long term liabilities, that is, \( \rho_1 = 1 \). Similarly, the class \( P_n \) will represent the set of net assets which are financed entirely by commercial loans, that is \( \rho_n = 0 \). The interest rate terms for these two classes are assumed to be identically zero. With this basis we can write the total level of desired long term liabilities (\( \hat{T} \)) as:

\[ \hat{T} = P_1 + \left( \sum_{i=2}^{n-1} \rho_i P_i \right) + \left( \sum_{i=2}^{n-1} f_i[r] P_i \right) \]

The net asset group \( P_n \) receives a zero weight in this formulation, since it is financed only by commercial loans. As a simplification of this specification we shall work with the form:

\[ (I.1) \quad \hat{T} = P_1 + \left( \sum_{i=2}^{n-1} \rho_i P_i \right) + A F[r] \]
in which the weighted interest rate terms are summarized by one function \( F[r] \) and scaled by total assets (A). This simplification reduces the model's empirical implications very little since it would be difficult to estimate these terms separately, anyway.

There is also a balance sheet identity:

\[
(I.2) \quad T + L = P_1 + \sum_{i=2}^{n-1} P_i + P_n
\]

which simply notes that the total of net assets in each group must equal the sum of long term liabilities and commercial loans. This identity is valid, of course, only for the actual quantities which exist at any time \( t \). But if we assume that the firm's desired or planned financing is consistent in the sense that even planned values for \( L \) and \( T \) satisfy the balance sheet identity, then we have:

\[
(I.3) \quad \hat{T} + \hat{L} = P_1 + \sum_{i=2}^{n-1} P_i + P_n
\]

where \( \hat{L} \) is the desired level of commercial loans. From (I.1) and (I.3) we can derive the desired level of loans as:

\[
(I.4) \quad \hat{L} = P_n + \left( \sum_{i=2}^{n-1} (1 - \rho_i P_i) \right) - A F[r]
\]

where the asset class \( P_1 \) does not enter since it is financed entirely by long term liabilities.
The principal behavioral assumption of our model is that the firm gradually adjusts its long term liabilities to the desired level given in equation (I.1). Thus, letting $\lambda$ be the speed of adjustment and using (I.2), we have:

\[ \Delta T = \lambda (T - T_{-1}) = \lambda (P_1 + \left( \sum_{i=2}^{n-1} \rho_i P_i \right) + \Lambda F[r] - P_{-1} + L_{-1}) \]

where $P = \sum_{i=1}^{n} P_i$

Since the balance sheet identity is valid also in terms of flows as:

\[ \Delta T = \Delta P - \Delta L \]

by substituting (I.6) into (I.5), solving for $L$, and arranging terms, we obtain:

\[ L^d = P_n + \sum (1-\alpha \rho_i) P_i + (1-\lambda) P_1 - \lambda \Lambda F[r] - (1-\lambda)(P_{-1} - L_{-1}) \]

where $L^d$ refers to loan demand in contrast to loans actually received.

The meaning of equation (I.7) can be stated quite easily. For a $1$ increase in net assets of class $P_i$ in the current period, long term liabilities adjust by $(\lambda \rho_i)$ and thus commercial loans must adjust by $(1-\lambda \rho_i)$ to fulfill the balance sheet identity. Similarly, for assets of the $P_n$ or $P_1$ classes commercial loans must adjust by $1$ or $(1-\lambda)$, respectively. The last term in equation (I.7), which can be written as $(1-\lambda)T_{-1}$, accounts for the replacement of commercial loans by long term liabilities as long term liabilities gradually approach the desired level.
following the initial disturbance. Finally, the term $A F[r]$ accounts for any changes in the relative cost of long term liability financing. Thus, commercial loans finance (at least some) net assets in the long run and act as a buffer which is needed because of the gradual adjustment of long term liabilities and the balance sheet identity. It is not possible to operationally distinguish these functions, however, and it should be stressed that the coefficient $A$ refers to the speed of adjustment of long term liabilities rather than commercial loans.

It should also be noted that while we have taken commercial loans as the dependent variable in our formulation, there is no reason why the demand for other short term financial assets, government securities, for example, could not be similarly treated. In this way the entire short run dynamic financial behavior of the firm could be specified consistently with respect to both the budget constraint and long run equilibrium levels. And, moreover, if an additional equation explicitly specifying the adjustment of long term liabilities were added, a complete model of corporate finance could be obtained. Such a task is planned for the future.

B. SPECIFICATION FOR THE TESTABLE EQUATION

We now wish to provide the empirical specification of the theoretical loan equation formulated as equation (I.7), while restricting our attention to the aggregate demand of all business firms. The modifications which are necessary for the model when disaggregated by asset size are discussed further in section (IV) below.
The most general formulation can be obtained by allowing each of the entries on the left hand side (net assets) of the balance sheet in Table (A) to define a unique class \( P_i \). The corresponding \( \rho_i \) coefficients would then be estimated directly by the regression. A more restricted specification is suggested, however, by our a priori knowledge, and in some cases is made necessary by data limitations. The question before us, then, is which variables should be specified individually and which can be aggregated and perhaps approximated by some summary measure.

Certainly the most important asset to specify individually is the inventory stock, since it is generally agreed to be the main determinant of loan demand.\(^4\) Or, in terms of our formal analysis, the proportion of inventory stock financed by long term liabilities should be relatively small, indicating that the coefficient for inventory stock in equation (I.1), call it \( \rho_H \), is expected to be close to zero.

Fixed capital, on the other hand, is financed primarily by long term liabilities and thus we expect that its coefficient in equation (I.1), say \( \rho_K \), will be close to unity. In the empirical work on the aggregate data we can make the further distinction between structures and producers' durable equipment. Our expectation is that loans are relatively more important in financing equipment than structures, and thus the equation (I.1) coefficient for structures (\( \rho_S \)) should be relatively smaller than the coefficient for equipment (\( \rho_E \)).

\(^4\)See Goldfeld (1966), pp. 82-86, for further evidence on this point.
We shall refer to the remaining entries listed as net assets in the balance sheet of Table (A) as net liquid assets and denote them by \( V \), where:

\[ V = M + G + AR + OCA - AP - TAX - OCL \]

Although these net liquid assets are "financial" in nature, they are directly related to the level of firm's sales, particularly when viewed as a composite, and thus can also be considered production or sales inputs to be financed by commercial loans and long term liabilities. The importance of commercial loans for financing these net assets should be bounded by the very important inventory stock and the substantially less important fixed capital. Thus, we should have:

\[ 0 \leq \rho_h \leq \rho_v \leq \rho_k \leq 1 \]

where \( \rho_v \) is the coefficient for net liquid assets \( (V) \) in equation \( (I.1) \).

Since data for the net liquid asset variable are not available on an aggregate basis, we propose to use for a proxy the level of net business sales. This should provide an adequate measure since, as we have already noted, the firm's desired level of liquid assets is directly related to its sales. To obtain an empirical measure of net sales \( (V') \), we have subtracted inventory investment and fixed capital investment from gross national product. Formally, we have:

\[ V' = Y - C - I - G - T - (X - M) \]

---

5Since we are aggregating over all business firms, accounts receivable (AR) and accounts payable (AP) will cancel except for the amount of trade credit extended to the non-business sector. This will not be true, however, when we work with the disaggregated data.

6The high correlation between the sales proxy \( (V') \) and true net liquid assets for the manufacturing sector (derived from FTC-SEC data described below), .96, provides at least a simple verification of the reliability of this proxy.
\[ V = a_1(V') \equiv a_1(GNP - \Delta H' - \Delta K_g) \]

where

- \( GNP \) = gross national product
- \( \Delta H' \) = non-farm, business inventory investment (National Accounts concept)
- \( \Delta K_g \) = gross investment (non-residential)

With these parts now in place, we can write the operational form of the aggregate equation for desired long term liabilities (equation (I,1)) as:

\[ (I,8) \quad \hat{\Gamma} = \rho_H^H + \rho_K^K + \rho_V^V a_1 V' + AF[r] \]

Similarly, the operational equivalent of the theoretical demand equation (I,7) is:

\[ (I,9) \quad L^d = (1 - \lambda \rho_H) H + (1 - \lambda \rho_K) K + (1 - \lambda \rho_V) a_1 V' - (1 - \lambda) a_1 V'_{-1} \]
\[ - \lambda AF[r] - (1 - \lambda) (H_{-1} + K_{-1} - L_{-1}) \]

Since it is difficult to obtain reliable estimates of inventory and fixed capital stock on an aggregate quarterly basis, it is necessary to take the first difference of equation (I,9):

\[ (I,10) \quad \Delta L^d = (1 - \lambda \rho_H) \Delta H + (1 - \lambda \rho_K) \Delta K + (1 - \lambda \rho_V) a_1 \Delta V' \]
\[ - (1 - \lambda) a_1 \Delta V'_{-1} - (1 - \lambda) (\Delta H_{-1} + \Delta K_{-1} - \Delta L_{-1}) \]
\[ - \lambda \Delta (AF[r]) \]
which allows us to test the hypothesis using flow data.\textsuperscript{7}

Only the interest rate term, \(-\Delta A F[r]\), remains to be given an operational interpretation. Our hypothesis is that the demand for commercial loans is negatively related to its own rate (\(r_L\)) and is positively related to a weighted average of the commercial paper rate (\(r_P\)) and Moody's AAA corporate bond rate (\(r_A\)). Thus, we have:

\[
(I.11) \quad -\Delta A F[r] = -\lambda \Delta d_0 A \left[ \frac{d_1}{d_1 + d_2} r_A + \frac{d_2}{d_1 + d_2} r_P - r_L \right]
\]

\[
= \frac{-\lambda d_0 d_1}{d_1 + d_2} A(r_A - r_L) + \frac{-\lambda d_0 d_2}{d_1 + d_2} A(r_P - r_L)
\]

\[
= w_1 \Delta A(r_A - r_L) + w_2 \Delta A(r_P - r_L)
\]

The term with Moody's AAA rate should be substantially more important since, theoretically at least, all variables with the exception of long term liabilities and commercial loans have been specified directly. Formally, the commercial paper rate enters into the formulation because our proxy for net liquid assets (including commercial paper, of course) -- that is, sales ($V'$) -- may not prove adequate in accounting for substitution between net assets arising from interest yield changes.

\textsuperscript{7}The lagged change in net sales ($\Delta V'_1$) is not constrained to a coefficient of (1- $\lambda$), in contrast to $\Delta H_{-1}$ and $\Delta K_{-1}$, because the conversion ratio of the proxy ($a_1$) is also a parameter which must be estimated.
Furthermore, it is likely that firms will substitute commercial loans for long term debt when the long term bond rate is expected to fall. Since this expectation can be measured, at least to a rough approximation, by the spread between the long term rate and the short term rate, we have additional reason to expect a relatively high weighting on the long term rate.\(^8\)

With respect to data availability, the interest rate variables provide little difficulty. The scaling variable, total assets \((A)\), however, cannot be obtained on an aggregate basis. As a proxy, we propose to use gross business product \((GBP)\) on the assumption that both variables have only small short run variability and similar trends.\(^9\) Formally, we are assuming that:

\[
A = a_2 \text{ GBP}
\]

\(^8\)More formally, the interest rate function can be written as:

\[
-F[r] = a(w_1' r_A + w_2' r_P - r_L + b(r_A - r_P))
\]

where the last term denotes the expectation of a decrease in \(r_A\). This can be simplified to:

\[
-F[r] = a [(w_1' + b) (r_A - r_L) + (w_2' - b) (r_P - r_L)]
\]

which is the form used in equation (1.11). Since \(b\) is assumed positive, it is clear that the addition of expectations increases the coefficient for \(r_A\) and decreases the coefficient for \(r_P\). In fact, if \(b\) is sufficiently large, the coefficient for \(r_P\) may even be negative.

\(^9\)The correlation between gross business product \((GBP)\) and total assets of manufacturing corporations (see section IV, below) is about .99.
Preliminary tests using the interest rate term as specified in equation (I,11) and the GBP proxy did not, however, yield significant coefficient estimates. We suspect the source of the difficulty is either the use of the proxy for total assets or the need to take the first difference of the relationship, since a similar problem does not arise for our tests of the manufacturing sector which use actual balance sheet data (see section (IV) below). To consider the problem more closely, it is helpful, using the term with Moody's AAA rate as the example, to note the following algebraic identity:

$$\Delta \begin{bmatrix} A(rA - rL) \\ (A-1) \end{bmatrix} = (A-1) \left( \Delta (rA - rL) \right) + (rA - rL)(\Delta A)$$

Thus, the interest rate term can be separated into two components: the first term represents the change in the equilibrium level of loans when the interest rate spread changes, while the second term accounts for the change in loan demand when total assets increase, the interest rate spread being given. Further preliminary tests indicated that only the second of these terms was entering significantly into regressions, suggesting that the "buffer" function of commercial loans, which is based on short-run variations in total assets is dominant. Similar results were also valid for the interest rate term with the commercial paper rate. Consequently, we shall proceed with the interest rate terms specified as:

$$-\Lambda \Delta (A F[r]) = w_1 (rA - rL)(\Delta A) + w_2 (rP - rL)(\Delta A)$$

$$= w_1 a_2 (rA - rL) (\Delta GBP) + w_2 a_2 (rP - rL) (\Delta GBP)$$
If we now substitute the interest rate relationship into equation (1.10) we obtain:

\[
(1.12) \quad \Delta L^d = (1 - \lambda \rho^e_H) \Delta H + (1 - \lambda \rho^e_K) \Delta K + (1 - \lambda \rho^e_V) a_1 \Delta V' \\
- (1 - \lambda) a_1 \Delta V'_{-1} + w_1 a_2 (rA - rL) (\Delta GBP) \\
+ w_1 a_2 (rP - rL) (\Delta GBP) - (1 - \lambda) (\Delta H_{-1} + \Delta K_{-1} - \Delta L_{-1})
\]

as the complete specification of the loan demand model. Before testing
this relationship, however, we must first turn to the issue of credit
rationing in order to transform the dependent variable from the unobserved
loan demand to the observed loans outstanding.

II. THE INTERACTION OF RATIONING AND THE DEMAND FOR LOANS

The mechanism of loan demand developed so far is quite simple. The
firm first determines the desired level of its sales and production inputs --
inventory stock, fixed capital stock, and net liquid assets. Then, on the
basis of these assets and the cost of alternative means of finance, it cal-
culates its demand for loans including any short run buffer component. And,
if there is no rationing, the firm receives exactly the loan it requests;
loan demand and loans outstanding are identical.

Now, however, we wish to explicitly include the possibility of non-
price credit rationing. In this case the firm will not be able to obtain
the loan size it requests. Any attempt by the firm to offer an interest
rate higher than the rate quoted by the bank will be to no avail, of course,
Consequently, the firm returns from the bank with a smaller loan than it requested. This might suggest, at first glance, that we need only subtract the amount of rationing (E) from the loan demand equation which has been formulated already in order to obtain an equation explaining loans outstanding as the dependent variable.

But this would be incomplete, since the response to rationing may involve also adjustment on the net asset side of the balance sheet. Furthermore, the balance sheet identity implies that for any adjustment in commercial loans there must correspond an adjustment in the net total of long term liabilities and net assets. Consequently, if net assets were not affected, this would imply that the entire impact of rationing falls on long term financing, an obviously unacceptable implication. On the contrary, it is reasonable to suppose that the firm will want to adjust its asset structure taking into account the constraint of a given loan supply. Since a reduction in net assets reduces the loan demand, at least some of the impact of rationing will be included in our specification of the net assets.\(^{10}\)

To obtain a more quantitative appraisal of the necessary changes in the demand formulation, it is helpful to start with the simple example in which the firm adjusts only the inventory stock, leaving all other net assets at the level originally planned. The amount of the "involuntary" reduction in inventory, say $E_H$, will just equal the amount of rationing (E)

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\(^{10}\) More formally, the point is that the demand for loans, as formulated here, is not independent of the amount of rationing.
in this case. As a formal property of our loan demand formulation, it can be seen from equation (I.9) that a reduction of \( E \) in inventory will reduce the demand for loans by \((1 - \lambda \rho_H)(E)\).\(^{11}\)

In the background, long term liabilities are also being reduced, by an amount \((\lambda \rho_H)(E)\), as can be seen from equation (I.5). And, in fact, this is the quantitative amount of adjustment needed for the loan demand equation, since

\[
(\lambda \rho_H)(E) = E - (1 - \lambda \rho_H)(E)
\]

is the difference between the amount of rationing and the amount of "automatic" adjustment in our formulation of loan demand. Since \( \lambda \) and \( \rho_H \) are bounded between zero and unity, the amount of additional adjustment must, in general, be less than the amount of actual rationing. Only in the case in which the adjusted asset (inventory, in our example) is financed entirely by long term liabilities \((\rho = 1)\) and the speed of adjustment of long term liabilities is unity \((\lambda = 1)\) will it be correct to reduce the demand equation by the full amount of rationing.\(^{12}\) On the other hand, if the asset is financed entirely by commercial loans \((\rho_H = 0)\), then no additional adjustment is needed (regardless of the speed of adjustment of long term liabilities).

In the general case, as we have already noted, the introduction of rationing will cause the firm to adjust all its net assets. Using the classi-

\(^{11}\)It is important to keep in mind that the term loan demand is being used in the technical sense defined at the beginning of this chapter.

\(^{12}\)The speed of adjustment, of course, depends on the unit of time used and will always equal unity for a sufficiently long period.
fications distinguished in the previous section, we would have reductions of \(E_H\) for the inventory stock, \(E_V\) for net liquid assets, and \(E_K\) for the capital stock, such that:

\[
E = E_H + E_V + E_K
\]

The reduction in loan demand "automatically" accounted for in our formulation can again be derived from equation (1.9) as:

\[
(1 - \lambda \rho_H) E_H + (1 - \lambda \rho_V) E_V + (1 - \lambda \rho_K) E_K
\]

The additional amount of adjustment necessary for an equation using loans outstanding as the dependent variable will be given, similarly, by:

\[
(\lambda \rho_H) E_H + (\lambda \rho_V) E_V + (\lambda \rho_K) E_K
\]

This means that the correction factor for rationing depends on both the firm's demand parameters (\(\rho_H\), \(\rho_V\), and \(\rho_K\)) and the impact of rationing on the firm's desired levels of the net assets to be financed. The parameters, of course, can be directly estimated by the regression itself. To determine the distribution of the "involuntary" reductions in net assets, however, additional information is needed. Although section (V) below does provide a start in this direction, the results are not sufficiently refined to be of help for the aggregate equation.\(^{13}\) It is important to keep in mind, however, that the quantitative effect at issue is probably very small, since

\(^{13}\)Special considerations relevant to the tests using disaggregated data are discussed separately in section (IV) below.
the firm would tend to reduce those assets which are most dependent on commercial loans for their financing. In fact, in the polar case in which the firm reduces only assets financed entirely by commercial loans, no additional adjustment will be necessary, as already noted.

This suggests that as an approximate solution we might safely assume the distribution of the effect of rationing on the various net assets to be constant over time. In this case the quantity of loans outstanding can be written as:

\[
L = L^d - w_3 E
\]  

(II.1)

where \(w_3\) is presumably quite small. To obtain a measure of the amount of rationing (E), we shall rely on our proxy variable for the degree of rationing \(\hat{H}\),\(^{14}\) which, it will be recalled from Chapter (3), was defined as:

\[
\hat{H} = E/L^d
\]

(II.2)

Substituting equation (II.2) into equation (II.1) yields:

\[
L = L^d - w_3 (\hat{H} L^d)
\]

(II.3)

And, upon taking first differences, we obtain:

\[
\Delta L = \Delta L^d - w_3 \Delta (\hat{H} L^d)
\]

(II.4)

---

\(^{14}\)Goldfeld (1966) accounts for rationing by using a potential deposit variable as a measure of the supply side effect. Neither Anderson (1964) nor Henderson (1968) incorporates any rationing effects in their work.
This formulation still leaves us with the practical problem of estimating the interaction terms of the rationing proxy with each component of loan demand. Our analysis of a similar problem with the interest rate variables in section (I) above suggests, however, that an acceptable equation will be:

\[(II.5) \quad \Delta L = \Delta L^d - w_3(H)(\Delta A)\]

\[= \Delta L^d - a_2w_3(H)(\Delta GBP)\]

where gross business product (GBP) is the proxy used for total assets (A).\(^{15}\) And finally, substituting the loan demand formulation given as equation (I.12) into equation (II.5), we obtain the specification to be tested:

\[(II.6) \quad \Delta L = (1 - \lambda \rho_H)\Delta H + (1 - \lambda \rho_K)\Delta K + (1 - \lambda \rho_V)a_1\Delta V' - (1 - \lambda)a_1\Delta V'_{-1} - (1 - \lambda)(\Delta H_{-1} + \Delta K_{-1} - \Delta L_{-1}) + a_2w_1(r_A - r_L)(\Delta GBP) + a_2w_2(r_P - r_L)(\Delta GBP) - a_2w_3(H)(\Delta GBP)\]

\(^{15}\)In fact, it is reasonable to require that the interest rate terms and the rationing variable have the same scaling since, from the viewpoint of the firm, every level of rationing has an analog in terms of interest rate levels. That is, the firm's unconstrained demand can be reduced to a given level (equal to the bank's supply) by charging a sufficiently high interest rate.
III. ESTIMATION OF LOAN DEMAND EQUATION: AGGREGATED DATA

Table B presents the results of testing the hypothesis summarized by equation (II.6), using aggregate data. The equations were fitted for the period 1952-III to 1965-IV, using ordinary least squares. Although all variables are defined in the data appendix, the derivation of the measures for fixed investment and inventory investment should be stressed. The proper measure for inventory investment in our model is the change in the book value of inventory stock. To obtain an aggregate measure of this variable we have subtracted the inventory valuation adjustment from the National Accounts concept of inventory investment. The proper measure for our fixed investment variable is the net investment in non-residential structures and producers' durable equipment. While we have used the National Accounts data for gross investment, it is difficult to obtain the corresponding figures for depreciation. As an approximation, we have adjusted the capital consumption allowance figures given in the National Accounts; since this measure of depreciation may be only proportional to the true figure, we have shown results with only net investment as well as with gross investment and depreciation separated.

Equation (8-1) follows the specification of equation (II.6) directly. The overall fit seems quite good; the standard error of $390 million is almost exactly 1 percent of the mean of the stock of loans outstanding. The

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16Two stage estimates provided by Robert Rasche as part of the Federal Reserve-MIT model differ very little from the results shown here.
Table B: Estimated Coefficients for Loan Demand Equation: Aggregated Data

(T statistics in parentheses)  
(ΔL Dependent Variable)

<table>
<thead>
<tr>
<th>B-1</th>
<th>.816</th>
<th>.511</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7.6)</td>
<td>(3.7)</td>
</tr>
<tr>
<td>ΔH</td>
<td>.293</td>
<td>-1.112</td>
</tr>
<tr>
<td></td>
<td>(2.6)</td>
<td>(-1.2)</td>
</tr>
<tr>
<td>ΔK_n</td>
<td>.477</td>
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<td>rPL*</td>
<td>.382</td>
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(*):  
\[ r_{AL} = (r_A - r_L)(ΔGBP) \]  
\[ r_{PL} = (r_P - r_L)(ΔGNP) \]  
\[ \hat{HG} = (\hat{H})(ΔGBP) \]  
\[ \hat{H}^{-1}G = (\hat{H}^{-1})(ΔGBP) \]  

Equation (B-1) through (B-4),  
\[ LAG = ΔH_{-1} + (ΔK_n)_{-1} - ΔL_{-1} \]

Equation (B-5)  
\[ LAG = ΔH_{-1} + (ΔK_n)_{-1} - (ΔS_g) - ΔL_{-1} \]

Equation (B-6)  
\[ LAG = ΔH_{-1} + (ΔK_n)_{-1} - (ΔK_n) - ΔL_{-1} \]
principal weakness in the equation is the rationing proxy, since its coefficient has the wrong sign and is not statistically significant. This is not too disturbing, however, as we have already indicated that the rationing effect might actually be very small. In an attempt to provide as complete a test as possible, though, we have experimented with several different specifications for the rationing variable. Only the lagged value of this variable, as shown in equation (B-2), yielded a negative coefficient with a reasonably high T statistic. But even here the coefficient is small and the impact of rationing is thus not important. For present purposes, however, we shall retain the lagged rationing variable in our formulation.

Equation (B-3) differs from (B-2) only in that net investment \((\Delta K_n)\) is now separated into two components, gross investment \((\Delta K_g)\) and depreciation \((\text{DEP})\). These results appear to confirm the accuracy of the magnitude of the depreciation measure, since the coefficients for gross investment and depreciation are essentially equal except for sign.

In our earlier discussion we had also noted the possibility of separating total investment into its components of structures \((\Delta S_g)\) and producers' durable equipment \((\Delta E_g)\). The results of this test are shown in equation (B-4). The investment variables are measured gross, with de-

---

17 There is some rationale for the use of the lagged value, since the cumulative effect of an increase in rationing will rise over time as long term liabilities decrease in response to the lower level of net assets. (See p. 177 above.)

18 This is only a partial test, however, since the lagged value of depreciation is still specified as part of net investment in the lagged variable \((\text{LAG})\). Multicollinearity precludes a complete test with the lagged value of depreciation \((\text{DEP})\) also specified separately.
preciation entered separately, since it is not possible to separate deprec-
iation for structures and equipment. These results confirm our hypothesis
that loans are more important for durable equipment than structure financing.
In fact, the coefficients indicate that in long run equilibrium, long term
liabilities finance 82 percent of producers' durable equipment but 136 per-
cent of the structures. This suggests constraining the coefficient for
structures to unity while still leaving the coefficient for equipment free.
Statistically, this is achieved by adding the variable to LAG (with sign
reversed). The results of this test are shown as equation (B-5). Al-
though the fit remains almost unchanged, these new estimates indicate that
producers' durable equipment is also financed entirely by long term liabili-
ties. As a final test, then, in equation (B-6) we have constrained the
coefficients for both components of net investment to unity. This form seems
somewhat less satisfactory than (B-5), since the estimates for the sales terms
($\Delta V'$ and $\Delta V'_{-1}$) changed significantly and the fit is not quite as good. The
principal point, however, is that the coefficients for both components ap-
pear to be very close to unity, if not actually unity, and thus we are, no
doubt, asking too much for the data to make such fine distinctions. In this
sense we view all the equations shown in Table (A) as satisfactory, although
for purposes of illustration (B-5) may be taken as typical.

---

19 This steady state value is determined, using producers' durables as the ex-
ample, by setting the coefficient of equipment equal to the value given in
equation (II.6), that is $0.589 = (1 - \lambda \rho_E)$. The value for $\lambda$ can be directly
derived from the term LAG, in this case as about 0.48. We can then
easily solve for the desired value of $\rho_E$.

20 Since the coefficient of structures in equation (II.6), $(1 - \lambda \rho_S)$, reduces
to $(1 - \lambda)$ when $\rho_S = 1$, the constraint, as a statistical matter, amounts to
including the structures variable ($\Delta S_0$) in the term with the lagged dependent
variable. The same rule is also valid for the other constraints discussed below.

21 The variable DEP accounts for depreciation from both structures and producers'
durable equipment. This differs from equation (B-4), since the coefficient for
Using (B-5) as the example, we can derive some further properties of our results. The speed of adjustment ($\lambda$) of long term liabilities is about 47 percent per quarter. While this may seem somewhat faster than would be expected, it is important to keep in mind that retained earnings, which by definition are part of long term liabilities, do "adjust" quickly. This suggests the desirability of making a further division within long term liabilities, although no attempt is made here, since it would involve an even more complex formulation. As a second result, the steady state proportion of inventory stock financed by long term liabilities can be calculated to be about 30 percent; or, conversely, commercial loans finance about 70 percent of the inventory stock. The equivalent steady state coefficients for net liquid assets and interest rate effects cannot be confidently determined because of the use of proxies. The elasticity of loan demand with respect to sales ($V'$) at the point of means, however, is approximately .50.

We can also obtain estimates of the impact effects of interest rates and rationing by evaluating the change in gross business product (GBP) at its sample mean, $1.46$ billion per quarter. In this case, an increase in the spread between the prime commercial paper rate and the commercial loan rate of one percentage point would increase the quarterly change in commercial loans by $153$ million, while a similar increase in the spread between Moody's

(footnote continued)

structures investment is now constrained and hence the coefficient for this part of depreciation should be similarly constrained. The inability to make this distinction is not too troublesome, however, since the coefficient for producers' durable equipment (.514) nearly equals the constrained coefficient for structures (.525) in (B-5).
AAA bond rate and the commercial loan rate would increase the quarterly change in commercial loans by $596 million.\textsuperscript{22} Finally, an increase in rationing by 1.0 units, a change larger than any actually observed within one quarter, would reduce the quarterly change in commercial loans by $109 million. Since the mean quarterly change in commercial loans for the sample period is $850 million and the mean of the stock of loans outstanding is over $39 billion, it is clear that the impact of rationing on commercial loans is very small. (This does not take into account, however, the impact of rationing on other net assets.)\textsuperscript{23}

\textsuperscript{22} The greater importance of the long-term rate (rA) relative to the short-term rate (rP) is consistent with the expectations hypothesis mentioned earlier.

\textsuperscript{23} We have also tested the model by explicitly specifying the Koyck lag distribution for the independent variables which is implicit in the partial adjustment model with the lagged dependent variable. The motivation for this additional test arises because serial correlation in the error term in conjunction with a lagged dependent variable leads to bias in the coefficient estimates. The "speed of adjustment" parameter, \( \lambda \), was estimated using a non-linear search routine, as .525, which is very close to the values presented for the equations in Table (B). The coefficient estimates for the other independent variables were, of course, also very similar to the results of Table (B). Without the lagged dependent variable, however, we now obtain indication of serial correlation in the residuals (DW = 1.20). To obtain more efficient estimates, we also used an autoregressive transformation in conjunction with the non-linear Koyck distribution. The estimate for the first order serial correlation coefficient was .58; the other coefficient estimates remained essentially unchanged. These tests provide important confirmation of the basic results presented in Table (B).
In total, the model and coefficient estimates appear to provide a very reasonable picture of the mechanism and structure of commercial loan demand. Perhaps the most telling criticism of the model is its complexity. For example, Hendershott (1968) has estimated a significantly more simple model as:

\[ \Delta L = .4668 \Delta H' - .450 \Delta rL + .544 \Delta L_{-1} \]

\[ (,07) \quad (,40) \quad (,08) \]

\[ R^2 = .781 \quad S_e = .406 \quad D.W. = 1.98^{24} \]

with a reduction in goodness of fit of about 10 percent compared with equation (B-5). The cost of the additional complexity in our model, however, appears to be quite small. There are ample degrees of freedom when the model is estimated with quarterly data. And furthermore, the specifications can be easily included in larger econometric models, since our dependent variables would typically be endogenous in any case. Consequently, the additional complexity easily "pays its own way."

Finally, in Table C we show the result of extending our fitted values through 1966, using equation (B-5) again as an example.\(^{25}\)

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\(^{24}\)Hendershott (1968), p. 58. His estimation period is actually 1953-I to 1965-IV and the inventory investment (which we measure at quarterly rates) was not adjusted for the inventory valuation adjustment. (Standard errors are shown in parentheses.)

\(^{25}\)Predictions for 1967 cannot be obtained, due to the unavailability of the rationing proxy.
Table C

Fitted and Actual Values for Loan Demand: Aggregated Data ($\Delta L$)

<table>
<thead>
<tr>
<th></th>
<th>Predicted Value Eq. (B-5)</th>
<th>Predicted Value Eq. (B-5) (Dynamic)</th>
<th>Predicted Value (Hendershott)</th>
<th>Predicted Value (Hendershott) (Dynamic)</th>
<th>Actual ($\Delta L$)</th>
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<td>2.2</td>
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<td>2.8</td>
<td>0.9</td>
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</table>

The first column shows the fitted values using the true lagged dependent variable, and the fifth column the actual change. Our fit is substantially too low for the first half of the year and too high for the second half, and the error is quite a bit larger than the standard error, although we do predict the turning point at mid-year and the cumulative estimate for the year is only $.1 billion too high. The dynamic prediction shown in the second column, which uses an endogenously determined lagged dependent variable, does even less well in the first half of the year, but somewhat better in the second. The principal contrast of these results with the predictions given by Hendershott's equation26 is found in the fourth quarter. Our equation forecasts a decrease in the rate of growth of loans in this quarter, whereas Hendershott's does not. This

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26Hendershott (1968), p. 58; see also p. 187 above.
difference is due principally to the presence of lagged values of inventory investment in our formulation.

IV. ESTIMATION OF LOAN DEMAND EQUATION: DISAGGREGATED DATA

To obtain further results on the demand for loans, we next turn to tests using data from the Federal Trade Commission—Securities and Exchange Commission Quarterly Financial Report for Manufacturing Corporations. The principal advantage of the FTC-SEC data is that they contain a complete and consistent balance sheet for the universe of manufacturing corporations. Furthermore, the data are disaggregated by asset-size categories. These advantages are not obtained without cost, however, since the data are quite noisy and not suitable for time-series analysis in raw form. The details of the numerous adjustments which have been made, including seasonal adjustment, are given in the data appendix.

The specification of the demand functions follows with only minor changes from the model discussed in the previous section. We shall continue to use the same notation, most of which is summarized in Table (A), although it now pertains to the FTC-SEC data, of course. We start by specifying the determinants of the desired level of long-term liabilities in a manner similar to equation (I,8):

\[ (IV.1) \quad \hat{T} = \rho_H^4 + \rho_K + \rho_{T,NAR} + \rho_{W} + A F[r] \]

where NAR = AR-AP and \( W = M + G + GCA - TAX - OCL \)
This equation differs from (1.8) only in that net trade credit (NAR) has been separated from other net liquid assets, reflecting the relative importance of trade credit, on the disaggregated level at least, as an important source of loan demand. Of course, both the trade credit and the remaining net liquid assets (W) can now be measured directly.

Preliminary tests of the loan demand equation which can be derived from equation (IV.1) indicated the estimated coefficients for fixed capital (ρK) and other net liquid assets (ρW) were both greater than unity. The result for the fixed capital coefficient is not surprising, as it is consistent with the estimates obtained for the aggregate data. Since we cannot separate producers' durable equipment from structures with the FTC-SEC data, we have simply assumed that the coefficient for total net fixed capital is unity. The result for the other net liquid assets is somewhat more surprising, however, since one would think that at least the larger firms would finance some of their net liquid assets with commercial loans. Two points should be kept in mind, however. First, the ρ coefficients are steady state parameters based, at least implicitly, on a model with static expectations. Secondly, the estimates are based on seasonally adjusted data, thus ruling out the use of loans for smoothing seasonal variations in the need for liquid assets. Consequently it may not be unreasonable for firms to finance these assets, in equilibrium, entirely through long term liabilities and we shall proceed with this coefficient also constrained to be unity.

27 The seasonal variations in loan demand are themselves of great interest, of course. We hope to incorporate this aspect of the problem into our model in future work.

28 Jaffee (1967) shows results based on a very similar model without constraining the coefficient for the liquid assets to unity.
Taking account of these constraints and following the analysis used in section (I) above, a loan demand equation of the form of equation (I.9) can be derived:

\[(IV,2) \quad L^d = (1 - \gamma \rho_H)H + (1 - \gamma \rho_T)NAR - (1 - \gamma)(H_{-1} + NAR_{-1}) - \Delta W - \Delta K - L_{-1} - A F[r]\]

Since the FTC-SEC balance sheets contain the proper stock values, there is no need to take first differences as we did for the aggregate case.

With respect to the interest rate function, \( F[r] \), the demand for loans is again assumed to be negatively related to the own rate (rL) and positively related to a weighted average of the prime commercial paper rate (rP) and the long term corporate bond rate (rA); using the formulation of equation (I.11), we have:

\[-AF[r] = w_1(rA - rL) + w_2(rP - rL)\]

To transform the loan demand formulation into a loans outstanding equation consideration must again be given to rationing. The basic argument of section (II) above, applies equally well for the disaggregated data: a substantial amount of the effect of rationing will be accounted for by changes in the firms' choice of assets, leaving only a small residual for the rationing proxy (\( \hat{H} \)) if included separately. The ability to specify the real magnitudes for trade credit and net liquid assets, however, allows us to perform additional tests which are discussed below.
One significant problem with the data is that the asset classes are adjusted for the growth (and decline) of individual firms only in the first quarter of each year. This causes discontinuities in the first quarter for all the balance sheet variables. The problem can be avoided by working only with ratios of the balance sheet variables, as the FTC-SEC does in interpreting their data. The natural scaling is total assets and fortunately our formulation is amenable to this transformation since we need only divide through by total assets in equation (IV.2).

The final equation can then be written as:

\[ L = (1 - \lambda \rho_{H})L + (1 - \lambda \rho_{T})NAR + w_1(rA - rL) + w_2(rP - rL) \]

\[ - w_3 \hat{A} - (1 - \lambda)(H_{-1} + NAR_{-1} - \Delta W - \Delta K - L_{-1}) \]

where the bars indicate variables divided by total assets, and the dependent variable is loans outstanding, since the rationing proxy (H) is included.²⁹

The estimated coefficients of this equation for six asset classes and the total are shown in Table (D). The equations were fitted for the period 1952-III to 1965-IV using ordinary least squares. In terms of goodness of fit the results are about on a par with the aggregate estimates of

²⁹Although the FTC-SEC data allow us to separate short term commercial loans from term loans, we have used the total as our dependent variable to maintain consistency with the aggregate results of the previous section. In addition, trials using only the short term loans were less successful, reflecting, perhaps, the high degree of substitution between the two forms of loans. Also, we have included "other non-current liabilities" with net fixed capital. With these two exceptions, all variables correspond directly to the FTC-SEC definitions.
TABLE D: ESTIMATED COEFFICIENTS FOR LOAN DEMAND EQUATION: DISAGGREGATED DATA
(T statistics in parentheses) (L Dependent Variable)

<table>
<thead>
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<th>Range of Assets ($ million)</th>
<th>$ \hat{H}$</th>
<th>$\text{NAR}^*$</th>
<th>$\text{LAG}^*$</th>
<th>$\text{rA-rL}$</th>
<th>$\text{rP-rL}$</th>
<th>$\hat{H}$</th>
<th>D.W.</th>
<th>$R^2$</th>
<th>$S_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1 under 1</td>
<td>.508</td>
<td>.799</td>
<td>-.437</td>
<td>.001</td>
<td>.001</td>
<td>-0.002</td>
<td>1.45</td>
<td>.82</td>
<td>.004</td>
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<tr>
<td></td>
<td>(6.1)</td>
<td>(17.4)</td>
<td>(-4.7)</td>
<td>(.32)</td>
<td>(.43)</td>
<td>(-.25)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-2 1-5</td>
<td>.566</td>
<td>.833</td>
<td>-.512</td>
<td>.005</td>
<td>.004</td>
<td>-0.003</td>
<td>1.83</td>
<td>.76</td>
<td>.004</td>
</tr>
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<td></td>
<td>(9.1)</td>
<td>(11.5)</td>
<td>(-7.2)</td>
<td>(1.0)</td>
<td>(2.6)</td>
<td>(-.31)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D-3 5-10</td>
<td>.566</td>
<td>.679</td>
<td>-.467</td>
<td>.008</td>
<td>.006</td>
<td>-0.020</td>
<td>.94</td>
<td>.72</td>
<td>.005</td>
</tr>
<tr>
<td></td>
<td>(8.7)</td>
<td>(8.3)</td>
<td>(-5.9)</td>
<td>(1.4)</td>
<td>(3.2)</td>
<td>(-1.7)</td>
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<td></td>
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<td>D-4 10-50</td>
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<td>-.598</td>
<td>.005</td>
<td>.004</td>
<td>-0.001</td>
<td>1.87</td>
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<td>.002</td>
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<tr>
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<td>(22.5)</td>
<td>(17.1)</td>
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<td>(1.9)</td>
<td>(5.5)</td>
<td>(-.22)</td>
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<tr>
<td>D-5 50-100</td>
<td>.683</td>
<td>.573</td>
<td>-.544</td>
<td>.015</td>
<td>-.002</td>
<td>.0036</td>
<td>1.73</td>
<td>.81</td>
<td>.003</td>
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<td>(16.0)</td>
<td>(7.3)</td>
<td>(-9.3)</td>
<td>(4.1)</td>
<td>(.1.2)</td>
<td>(4.0)</td>
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</tr>
<tr>
<td>D-6 over 100</td>
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<td>.559</td>
<td>-.505</td>
<td>.011</td>
<td>.002</td>
<td>.0013</td>
<td>.90</td>
<td>.81</td>
<td>.003</td>
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<tr>
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<td>(13.4)</td>
<td>(7.6)</td>
<td>(-9.2)</td>
<td>(4.0)</td>
<td>(1.8)</td>
<td>(2.1)</td>
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</tr>
<tr>
<td>D-7 Total</td>
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<td>.591</td>
<td>-.498</td>
<td>.009</td>
<td>.002</td>
<td>.0008</td>
<td>.99</td>
<td>.84</td>
<td>.002</td>
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<td>(15.6)</td>
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<td>(-10.0)</td>
<td>(3.9)</td>
<td>(2.5)</td>
<td>(1.6)</td>
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</tbody>
</table>

(*) \( \text{LAG} = H_{-1} + \text{NAR}_{-1} - \Delta \hat{W} - \Delta \hat{K} - \hat{L}_{-1} \)
the previous section after adjustment to make units equivalent. The low Durbin-Watson statistics, particularly evident in equations (D-3), (D-6), and (D-7), seem due to a sequence of quarters following the 1954 cyclical trough in which we overestimate the volume of loans and a series of quarters preceding the 1960 cyclical peak in which we underestimate the volume of loans. The FTC-SEC data are suspect in these periods, however, since they deviate from the pattern of the aggregate data in which we have more faith. Attempts to correct for the serially correlated disturbances with an autoregressive transformation were of no avail, probably because the serial correlation is centered only in the quarters just noted.

In most respects the estimated coefficients for total manufacturing, shown as equation (D-7), are consistent with the comparable estimates obtained for the aggregate in equation (B-5). The speed of adjustment of long term liabilities in (D-7) is 50 percent per quarter as compared to 47 percent per quarter for equation (B-5). The interest rate elasticities are somewhat higher in (D-7), but we again find that the long term rate is more important than the short term rate as expected. The coefficient of the rationing proxy in D-7 has the wrong sign, although the coefficient is not significant. But, as stressed earlier, we expect only a small effect from this source and hence do not find the result disturbing.30

30 We have also estimated the equations with the lagged value of the rationing proxy, and find no significant differences in coefficient estimates.
The primary difference between (D-7) and (B-5) is that the estimated coefficient for $\hat{P}_H$, the percentage of inventory stock financed by long term liabilities in equilibrium, is only .20 for (D-7) in contrast to .70 for (B-5). While it is reasonable that the manufacturing sector would rely less heavily on commercial loans for financing their inventory stock, it is unlikely that such a large difference would exist between the manufacturing sector and the aggregate. Rather, we feel the most likely explanation is that multicollinearity between the inventory stock and net trade credit has caused less weight to be placed on the inventory stock.

It is also interesting to compare the estimated coefficients across asset classes. The coefficient for the rationing proxy tends to increase in magnitude with asset size, suggesting the possibility that in periods of high rationing large firms obtain loans over and above the amount determined solely by their net assets. We shall return to this point, below. The effect of the long term rate also tends to increase with asset size, and is particularly strong for (D-5) and (D-6). This appears reasonable, since the smaller firms have less access to the corporate bond market. The results for the prime commercial paper rate are somewhat puzzling, however, since the strongest effect is found in the classes given by (D-2), (D-3), and (D-4).

Perhaps the most interesting result, however, is the relative importance of net trade credit compared to inventory stock as a determinant of loan demand, especially for the smaller firms. The impact of a net increase in trade credit extended by the smallest firms (D-1), for example, is 60 percent larger than the impact of an increase in inventory stock;
for the largest firms (D-6), on the other hand, inventory stock is slightly more important. Another view of this finding is shown in Table (E) in which the equilibrium coefficients for inventory stock and trade credit have been calculated. For the four smallest asset classes, trade credit is substantially more important than inventory stock as a determinant of equilibrium loan demand, while for the two largest classes and the total, the relationship is reversed. It is also noteworthy that no systematic relationship appears to exist between the speed of adjustment of long term liabilities (column 3 of Table E) and asset size.

The striking importance of trade credit for small firms suggests the possibility that these firms will react to credit rationing by reducing the net amount of trade credit they extend. This can occur through a reduction in the accounts receivable extended or in an increase in accounts payable, owed presumably to the larger firms. This, in turn, has implications for the impact of credit rationing on real expenditure decisions. To consider this further, we next turn to section (V) and a discussion of the effects of credit rationing.
<table>
<thead>
<tr>
<th>Range of Assets ($ million)</th>
<th>((1 - \rho_H^I)) (% of Inventory Stock Financed by Commercial Loans)</th>
<th>((1 - \rho_T^I)) (% of Net Trade Credit Financed by Commercial Loans)</th>
<th>((\lambda)) (Speed of Adjustment of Long Term Liabilities)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 1</td>
<td>.13</td>
<td>.64</td>
<td>.56</td>
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<tr>
<td>1-5</td>
<td>.10</td>
<td>.65</td>
<td>.49</td>
</tr>
<tr>
<td>5-10</td>
<td>.17</td>
<td>.40</td>
<td>.53</td>
</tr>
<tr>
<td>10-50</td>
<td>.17</td>
<td>.35</td>
<td>.40</td>
</tr>
<tr>
<td>50-100</td>
<td>.31</td>
<td>.07</td>
<td>.46</td>
</tr>
<tr>
<td>Over 100</td>
<td>.18</td>
<td>.10</td>
<td>.49</td>
</tr>
<tr>
<td>Total</td>
<td>.20</td>
<td>.18</td>
<td>.50</td>
</tr>
</tbody>
</table>

31 The arithmetic derivation of these coefficients has been given in footnote 19 above.
V. THE EFFECT OF CREDIT RATIONING

The theoretical discussion of section (II) and the empirical results of sections (III and (IV) indicate that the primary impact of credit rationing depends on the firm's revaluation and adjustment of its net asset holdings. In general, on the aggregate level rationing could affect the firm's demand for fixed capital, inventory, or net liquid assets. On the disaggregated level net trade credit may also be redistributed. One direct test for the importance of credit rationing as it affects the efficiency of monetary policy, hence, is to obtain econometric estimates of the impact of credit rationing on these expenditure decisions.

Until very recently practically no attempts had been made to obtain such econometric estimates of the effect of credit rationing. The principal drawback was the non-availability of any measure of credit rationing, an issue discussed at length in Chapter (3). A subsidiary difficulty was the inability to differentiate between the effects of tight money, that is, high interest rates, for example, and the effects of credit rationing per se.

Within the last year, however, several exploratory studies of the impact of credit rationing have been undertaken, reflecting the growing interest in obtaining quantitative estimates of the impact of monetary policy. One center of such activity has been the Federal Reserve-M.I.T. Econometric Model project where interest in credit rationing has been inspired principally by Professor Franco Modigliani. The results of
these exploratory studies have not been very revealing. Typically, the ration-
ing proxy enters with an insignificant coefficient and sometimes with an in-
correct sign. There are, of course, many problems with obtaining such esti-
mates; in addition to the two points noted above, the correct timing or lag
for the rationing variable has been particularly bothersome. These problems
serve, however, to emphasize the exploratory nature of these studies and we
still maintain hope of obtaining more encouraging results in the future.32

Of course, there is always the possibility that credit rationing may
have only a moderate or small effect on real expenditure decisions. But,
this is in apparent conflict with our findings for the existence of credit
rationing presented in Chapter (3) and the small impact of credit rationing
in directly reducing the demand for commercial loans in the previous two
sections. One potential solution of this conflict is the frequently men-
tioned view that the redistribution of trade credit between large firms
and small firms may offset the impact of rationing on the real expenditure
decisions.33

32The studies referred to in this paragraph and related to the Federal Re-
serve-M.I.T. model were by Charles Bischoff (fixed investment), Frank de
Leeuw (inventory), and Gordon Sparks (mortgage and housing market). In all
cases the research was undertaken as part of the model project, however,
and no past or continuing interest in credit rationing per se need be im-
plied. Additional study of the impact of rationing on inventory investment
and fixed capital investment has been made by John Hand (1968).

33See Meltzer (1960), Meltzer (1963), and Brechling and Lipsey (1963) for
elaborate statements of this view.
In this view the non-rationed (large) firms extend resources in the form of trade credit to the rationed (small) firms in lieu of the bank loans. This results in a reduction in total loan demand, since the reduction in small firm demand (arising from the reduction in net trade credit) exceeds the increase in large firm demand (arising from the increase in their net trade credit) as is seen in our coefficient estimates. Of course, there is also a redistribution of loans from small firms to large firms, as is consistent with the theory of credit rationing outlined in Chapter 2.

Under the proper conditions, the redistribution of trade credit may substantially dampen the effect of credit rationing on real expenditures. The smaller firms will change their expenditure decisions only to the extent that the cost of trade credit is substantially higher than the cost of the rationed commercial loans, assuming ample trade credit is provided. And even then, if the interest elasticity of investment decisions by small firms is low, and there is certainly evidence in this direction, small firms may be relatively unaffected by rationing. Similarly, large firms will change their behavior only if the cost of the bank loans, or long term liabilities, used to finance the additional trade credit exceeds the return on the trade credit. Although the implicit interest rate on trade credit is very high, the risk is also great, and thus it is difficult to determine a reasonable rate of return. 34

34 See, for example, Goldfeld (1966), p. 83.
This suggests the importance of performing tests to determine whether trade credit is redistributed in periods of high rationing. Aggregate data are of little value in this case, since trade credit sums to zero when aggregated over the entire business sector.\textsuperscript{35} On the other hand, the FTC-SEC data for the manufacturing sector with disaggregation by asset-size classes does provide a good source for such estimates, and we now turn to two experiments based on these data. Also, supporting evidence can be obtained from survey data and this is discussed in section (C) below.

A. CREDIT RATIONING AND TRADE CREDIT: A DIRECT TEST

The first test we consider consists of fitting equations which explain the ratio of net accounts receivable (NAR = AR - AP) to total sales (S), for each of the asset classes separately. Technically, the equation is in reduced form, since no attempt is made to obtain separate structural estimates for the supply (a firm's accounts receivable) or demand (accounts payable) of trade credit. The scaling by sales assumes that the net average collection period is constant, except as modified by the independent variables specified below.\textsuperscript{36}

The strongest version of the hypothesis to be tested is that the amount of net trade credit extended (relative to sales) by large firms rises and the amount extended by small firms falls in periods of tight money and

\textsuperscript{35}This abstracts from "trade credit" granted to the household sector, however. \textsuperscript{36}In equilibrium the net average collection period is \( \frac{\text{NAR}}{90} \); that is the ratio of net receivables outstanding to daily sales (assuming S is quarterly sales).
credit rationing. It is important, however, to distinguish between the redistribution of trade credit within the corporate manufacturing sector (that is, the universe for the FTC-SEC data sample) and the redistribution between this sector and the remainder of the business sector. Since the non-corporate, non-manufacturing sector consists primarily of small firms, we should find this sector receiving additional net trade credit from the manufacturing sector in periods of rationing and tight money. And even the "small" corporate manufacturers may behave like "large" firms in this respect. Thus, in periods of tight money and rationing, the small firms in the FTC-SEC universe will obtain additional credit from the large firms in this group, but also will extend further credit to the remainder of the business sector. Since the net results of these two forces cannot be determined a priori, a weaker proposition is suggested: in periods of tight money and credit rationing, large firms tend to extend relatively more trade credit than the small firms in the corporate manufacturing sector.

The specification of our equation is given by:

\[
\frac{AR - AP}{S} = \frac{NAR}{S} = f (\text{constant, } \text{RL, } H, \text{ RP})
\]

where \( f \) is a linear function. We have omitted indication of the expected signs for these variables, since they depend on the version of the hypothesis. The strong version of the proposition would suggest that the net trade credit

\[37\text{The work of Meltzer (1963) and Frazer (1965) points quite strongly in this direction.}\]
extended by large firms should rise with the commercial loan rate (rL) and rationing (H) and fall with the commercial paper rate (rP). Small firms, on the other hand, should extend less credit when rL or H rises and might be relatively indifferent to the level of rP. Part of the basis of this relationship is the institutional feature that the terms of trade credit that is, the discount and net period, tend to show very little short-run variation. Consequently, in periods of high interest rates or credit rationing, one expects firms to take more advantage of trade credit, since the relative cost has fallen. Of course, every firm is both a supplier and demander of trade credit, and the differential effect of tight money could net out. But, presumably, the larger firms are relatively insensitive to variations in interest rates, since typically they would always take the discount. And our theory of rationing indicated that the large firms would, in general, not be rationed. Hence, the net trade credit of large firms would be the resultant of additional supply (meeting the demand of small firms at the fixed terms) and relatively constant demand.

---

38 Trade credit is usually granted in the form of a discount from the quoted price if the firm pays within a stated period (ten days frequently) and otherwise the firm is expected to pay the full price within some net period (typically thirty days).

39 If the discount is 2 percent (when payment is made within ten days) and the net period is thirty days, the implicit cost of paying after thirty days and not taking the discount is an annual interest rate of 36 percent \( \left( \frac{360 \text{ days}}{20 \text{ days}} \times 2\% \right) \). Thus, large firms with access to the open market or commercial banks would have no incentive to accept trade credit beyond the discount period, even if market rates rose quite high.
The prime commercial paper rate deserves special notice, however, since it is relevant primarily only to the large firms. When it rises, the cost of extending trade credit for large firms increases, and hence we should expect some reduction in their supply of trade credit. The behavior of small firms, on the other hand, should be relatively independent of this rate.

The weaker version of our hypothesis, which takes into account net credit extended outside the corporate manufacturing sector, suggests modifications in these expectations. The small firms in the corporate manufacturing sector may actually extend more net trade credit in periods of high interest rates or credit rationing, if their supply to the rest of the business sector exceeds the additional amount obtained from the large firms. This effect should not be too pronounced with respect to rationing, however, since in this case the small firms would simply not have the resources to extend additional credit. But, in any case, the coefficients of the commercial loan rate and the rationing proxy should be at least relatively larger for the large firms, while the coefficient of the prime commercial paper rate should be relatively smaller.  

Other variables could also be added to this formulation. For example, high capacity utilization rates or low profit margins might make firms reluctant to extend additional credit, since the profit from an additional sale

---

40 While we have used the commercial loan rate as a measure of the general level of rates, a long term market rate was also tested and provided equivalent results.
will be low. Meltzer, working with a similar model, has included a liquidity variable and sales as independent variables.\textsuperscript{41} Also, the ratio of costs (of production) to sales could be included, since the level of accounts payable should be more closely related to the firm's expenditures than its sales.

We have tested all these variables as additions to the basic specification of (IV.1). In general, we found that the coefficients had the correct sign but were not significant, particularly when a time trend is added as suggested below. Consequently, we have omitted all these variables in the results presented here. This does serve to emphasize, however, the very simple structure of the model.

Before turning to the results, two technical points should be noted. First, net trade credit is obtained by the FTC-SEC from balance sheets as of the last day of the quarter. The sales variable, on the other hand, is a flow taking place during the span of the entire quarter. Since the average maturity of trade credit is quite short, well under 90 days, a considerable bias may be introduced by scaling end of quarter trade credit with the quarterly sales. As a practical and admittedly only approximate solution, we have used the ratio of average net trade credit \((NAR_\text{av} = (NAR + NAR_{-1})/2)\) to sales \((S)\) as the dependent variable. Secondly, there is a significant time trend in the ratio of net trade credit to sales, since accounts receivable have risen relative to accounts payable and both have risen relative to sales over the sample period. To some extent this may simply be the result of a similar trend in the commercial loan rate, in which case no addi-

\textsuperscript{41} Meltzer (1960), pp. 430-433.
tional specification is needed. But, other explanations also seem very likely: favorable experience with trade credit over the post-war period has made firms more willing to grant such credit; increased competition has made the terms of trade credit more favorable, resulting in increased demand for this credit; and industries with high net trade credit to sales ratios may have expanded faster than the average.\textsuperscript{42} A simple linear time trend appears to be the only reasonable proxy for these effects, although clearly it is an admission of ignorance. Consequently, we shall present results both with and without a time trend.\textsuperscript{43}

The results of these tests are presented in Table F. The estimates were obtained using ordinary least squares for the period 1952-III to 1965-IV. Although the goodness of fit for these estimates is quite high, the existence of a substantial trend should also be kept in mind. The Durbin-Watson statistics are uniformly low, for reasons which are not evident.\textsuperscript{44} It is of some consolation, however, that the estimates of coefficients are unbiased, since there is no lagged dependent variable.

The coefficient estimates for the commercial loan rate (rL) are positive for all classes and increase in magnitude with asset size until the $10-$50 million group, and decline thereafter. The coefficients for the

\textsuperscript{42}These and related issues are discussed in Federal Reserve Bank of Kansas City (1957) and (1959).

\textsuperscript{43}Since the origin of the time trend is arbitrary (for example, we use 1 for the first period of the sample), the coefficient for the constant has no significance. In fact, this is true even without the time trend, since the rationing proxy is also an index number with an arbitrary mean (in our case, zero).

\textsuperscript{44}An autoregressive estimator and first differences were also used with no success.
<table>
<thead>
<tr>
<th>Asset Range ($ million)</th>
<th>Const.</th>
<th>rL</th>
<th>rP</th>
<th>Time</th>
<th>D.W.</th>
<th>R²</th>
<th>Se</th>
<th>Mean of Dependent Variable</th>
</tr>
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<tbody>
<tr>
<td>F-1a under 1</td>
<td>-.037</td>
<td>.033</td>
<td>-.000</td>
<td>-.007</td>
<td>.41</td>
<td>.73</td>
<td>.011</td>
<td>.11</td>
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<td>(-2.6)</td>
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<td>.000</td>
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<td>-.008</td>
<td>.003</td>
<td>.58</td>
<td>.74</td>
<td>.009</td>
<td>.20</td>
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<tr>
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<tr>
<td>F-2b</td>
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<td>.017</td>
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<td>.001</td>
<td>.73</td>
<td>.82</td>
<td>.008</td>
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<td>(2.4)</td>
<td>(4.4)</td>
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<tr>
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<td>.084</td>
<td>-.030</td>
<td>.004</td>
<td>.79</td>
<td>.87</td>
<td>.013</td>
<td>.27</td>
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<td>(-.68)</td>
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<td>.80</td>
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<td>-.022</td>
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<td>.009</td>
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<tr>
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<td>.039</td>
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<td>1.09</td>
<td>.96</td>
<td>.011</td>
</tr>
<tr>
<td></td>
<td>(.89)</td>
<td>(5.8)</td>
<td>(-2.3)</td>
<td>(-.57)</td>
<td>(10.7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-6a over 100</td>
<td>-.104</td>
<td>.081</td>
<td>-.016</td>
<td>-.001</td>
<td>.45</td>
<td>.86</td>
<td>.016</td>
<td>.21</td>
</tr>
<tr>
<td></td>
<td>(-5.3)</td>
<td>(12.1)</td>
<td>(-2.8)</td>
<td>(-.27)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-6b</td>
<td>.004</td>
<td>.037</td>
<td>-.015</td>
<td>.003</td>
<td>.002</td>
<td>.92</td>
<td>.97</td>
<td>.008</td>
</tr>
<tr>
<td></td>
<td>(.27)</td>
<td>(7.7)</td>
<td>(-5.6)</td>
<td>(1.6)</td>
<td>(12.5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-7a total</td>
<td>-.060</td>
<td>.070</td>
<td>-.013</td>
<td>-.002</td>
<td>.45</td>
<td>.87</td>
<td>.013</td>
<td>.22</td>
</tr>
<tr>
<td></td>
<td>(-3.8)</td>
<td>(12.8)</td>
<td>(-2.9)</td>
<td>(-.77)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F-7b</td>
<td>.031</td>
<td>.032</td>
<td>-.013</td>
<td>.001</td>
<td>.002</td>
<td>.93</td>
<td>.98</td>
<td>.005</td>
</tr>
<tr>
<td></td>
<td>(3.4)</td>
<td>(9.7)</td>
<td>(-6.5)</td>
<td>(.80)</td>
<td>(15.0)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
two smallest classes are substantially smaller than the coefficients for the four largest groups, and in this sense at least the coefficients do rise with asset size. This is consistent with the weak hypothesis, but is inconsistent with the strong hypothesis, since even the smallest classes have positive coefficients. These results are also valid with the time trend included, although the absolute size of the coefficients diminishes as might be expected.45

The results for the prime commercial paper rate (rp) are also reasonably good. Although the coefficients do not decrease monotonically for larger asset classes, the coefficients for the smallest two groups are distinctly relatively larger than the coefficients for the four largest groups. In particular, the coefficient for the smallest class is essentially zero. The time trend has little effect on this variable, which is understandable, given its strong cyclic variation.

The results for the rationing proxy are more difficult to interpret. The coefficient for the smallest class is significantly negative, while the coefficients for the three largest classes are also negative, but smaller in absolute magnitude and not statistically significant. These results are consistent with the weak hypothesis but, again, not with the strong hypothesis. The addition of the time trend increases the coefficients for the rationing proxy, and in fact, the coefficients for the largest two classes actually become positive, although still not significant; this

45Heston (1962) shows similar results in adding a time trend to a regression for net trade credit, using only the Treasury bill rate as an independent variable.
provides at least very weak confirmation of the strong hypothesis. The disturbing aspect of these results occurs in the second and third classes where the coefficients are positive, and even significant with the time trend. This is particularly difficult to interpret, since, in terms of the commercial loan rate and commercial paper rate coefficients, the second group had the characteristics of a "small firm," while the third group behaves like "large" firms.

In general, however, we find these results quite satisfactory, given the rough nature of the data and the fairly high correlations between the independent variables. In fact, encouraged in this way, we shall now proceed with a further test of the proposition that credit rationing causes a redistribution of trade credit, which results in a decrease in the impact of credit rationing on the demand for loans by firms.

B. TRADE CREDIT AND CREDIT RATIONING: AN INDIRECT TEST

The primary hypothesis of this second test is that the importance of the rationing proxy in the loan demand formulation will be reduced substantially if trade credit adjusts to rationing, as we feel it does. The formulation given in equation (V.1) suggests, moreover, that we can replace the amount of net trade credit with a function of interest rates, rationing, and sales. Since the demand for loan equation specifies net trade credit scaled by total assets, it is convenient to rewrite (V.1) as:
(V.2) \[ \text{NAR} = \bar{S} (f \text{ [constant, } rL, \hat{H}, rP]) \]

where the bar indicates the variable is divided by total assets. Our plan is to substitute this specification for net trade credit for the actual variable (\text{NAR}) used in the tests of Table D. For this purpose it is helpful to rewrite (V.2) as an explicit linear form:

(V.3) \[ \text{NAR} = e_0 \bar{S} + e_1 rL + e_2 \hat{H} - e_3 rP \]

where the coefficient signs are consistent with our weak hypothesis for the test just performed. The tests of this new demand equation, in contrast to the results of Table D, should yield a less negative coefficient for the commercial loan rate, a more positive coefficient for the rationing proxy, and a less positive coefficient for the prime commercial paper rate. In addition, the coefficient for the long term bond rate, \text{rA}, should increase, since its effect on credit rationing is similar to that of the commercial loan rate.

This same substitution should also be made for the lagged value of net trade credit as it appears in the specification of equation (IV.3). This introduces lagged values of the variables of equation (V.3) into the new demand equation. As might be expected, tests with the current and lagged values of the interest rate terms and rationing proxy lead to small and insignificant coefficient estimates, which quite clearly are related to multicollinearity. We experimented with more restrictive assumptions, such as using only the change in these variables, but again obtained poor
results. Furthermore, the lagged value of the sales variable itself entered only with marginal significance. This, we might note, is consistent with the aggregate results in which the lagged sales proxy \( (V'_{-1}) \) did not enter strongly. In any case, if the lagged value of the sales variable is only of limited importance in the regression, then the interest rate and proxy variables corresponding to this sales variable cannot be expected to have much importance either. Consequently, we have suppressed the lagged interest rates and rationing in the results shown.

Because the coefficients of the three interest rates, \( r_L \), \( r_P \), and \( r_A \), can no longer be constrained as they were in (IV.3), they are entered in the new test separately. When this is done, the prime commercial paper rate receives a near zero coefficient in all but one of the regressions. This is consistent with our hypothesis, since we expected this coefficient to be reduced in magnitude, although we cannot rule out the possibility of simple collinearity. Consequently, this variable is also suppressed in the results shown.

The estimated coefficients of this specification are shown in Table G. The estimation period is again 1952-III to 1965-IV and ordinary least squares fits were used. The coefficients of the current sales terms are significant (except for G-5 and G-6) and are similar in order of magnitude to the results for the aggregate data shown in Table B, although there may be collinearity between the sales term and the inventory stock. The lagged value of the sales term is not significant for the three smallest classes and is only slightly stronger for the larger class sizes.
TABLE G: ESTIMATED COEFFICIENTS FOR LOAN DEMAND EQUATION, DISAGGREGATED DATA:  
FURTHER RESULTS (\( y \) Dependent Variable)

<table>
<thead>
<tr>
<th>Range of Assets ($ million)</th>
<th>H</th>
<th>S</th>
<th>S_{-1}</th>
<th>LAG*</th>
<th>rA</th>
<th>rL</th>
<th>( \hat{H} )</th>
<th>D.W.</th>
<th>( R^2 )</th>
<th>( S_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-1 under 1</td>
<td>.311</td>
<td>.104</td>
<td>.005</td>
<td>-.499</td>
<td>.008</td>
<td>-.03</td>
<td>-.001</td>
<td>1.52</td>
<td>.86</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>(3.1)</td>
<td>(2.2)</td>
<td>(.08)</td>
<td>(-7.3)</td>
<td>(2.0)</td>
<td>(-.69)</td>
<td>(-.13)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-2 1-5</td>
<td>.510</td>
<td>.151</td>
<td>-.065</td>
<td>-.583</td>
<td>.005</td>
<td>-.001</td>
<td>.001</td>
<td>1.72</td>
<td>.83</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>(7.1)</td>
<td>(2.5)</td>
<td>(-1.0)</td>
<td>(-8.3)</td>
<td>(1.0)</td>
<td>(-.20)</td>
<td>(1.0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-3 5-10</td>
<td>.314</td>
<td>.244</td>
<td>-.03</td>
<td>-.509</td>
<td>.005</td>
<td>.006</td>
<td>.002</td>
<td>1.60</td>
<td>.71</td>
<td>.005</td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(3.1)</td>
<td>(-.31)</td>
<td>(-6.1)</td>
<td>(.79)</td>
<td>(.93)</td>
<td>(1.9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-4 10-50</td>
<td>.582</td>
<td>.241</td>
<td>-.128</td>
<td>-.716</td>
<td>-.005</td>
<td>.008</td>
<td>.001</td>
<td>1.52</td>
<td>.87</td>
<td>.003</td>
</tr>
<tr>
<td></td>
<td>(10.3)</td>
<td>(3.4)</td>
<td>(-1.9)</td>
<td>(-11.9)</td>
<td>(-1.2)</td>
<td>(2.0)</td>
<td>(2.0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-5 50-100</td>
<td>.603</td>
<td>.139</td>
<td>-.119</td>
<td>-.482</td>
<td>.016</td>
<td>-.016</td>
<td>.003</td>
<td>1.77</td>
<td>.74</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>(7.4)</td>
<td>(1.6)</td>
<td>(-1.5)</td>
<td>(-7.4)</td>
<td>(3.5)</td>
<td>(-3.4)</td>
<td>(3.4)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-6 over 100</td>
<td>.565</td>
<td>.113</td>
<td>-.081</td>
<td>-.523</td>
<td>.008</td>
<td>-.007</td>
<td>.002</td>
<td>1.13</td>
<td>.80</td>
<td>.003</td>
</tr>
<tr>
<td></td>
<td>(11.1)</td>
<td>(1.7)</td>
<td>(-1.6)</td>
<td>(-6.9)</td>
<td>(2.1)</td>
<td>(-2.0)</td>
<td>(4.1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G-7 total</td>
<td>.536</td>
<td>.134</td>
<td>-.062</td>
<td>-.560</td>
<td>.005</td>
<td>-.004</td>
<td>.002</td>
<td>1.34</td>
<td>.83</td>
<td>.002</td>
</tr>
<tr>
<td></td>
<td>(11.2)</td>
<td>(2.2)</td>
<td>(-1.1)</td>
<td>(-7.7)</td>
<td>(1.7)</td>
<td>(-1.4)</td>
<td>(3.9)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) \( LAG = H_{-1} + K_{-1} - x - W_{-1} - W - T_{-1} \)
The results for the two interest rates are mixed. We anticipated that the coefficients for \( rA \) and \( rL \) should both be greater than they were in the tests shown in Table D. In general, this result is confirmed for the commercial loan rate. The coefficient for the long term rate, however, increases in only two cases, although the magnitude of the change is small in all cases. The negative coefficient in (G-4) is disturbing, of course.

Finally, the results for the rationing proxy are fairly good. With only a small exception in (G-5) the coefficient for the rationing proxy increased for the six largest classes and decreased for the smallest class. Furthermore, the \( T \) statistic for this variable also increased in every case, again with the exception of (G-5). This suggests that credit rationing does lead to an increase in the demand for loans of the larger firms and a reduction in the demand for loans of smaller firms. This result was hidden in the earlier formulation of Table D, since the redistribution of trade credit was entered explicitly. With sales used instead of trade credit, the relationship becomes much clearer, as expected.

C. TRADE CREDIT AND CREDIT RATIONING: RESULTS FROM SURVEY DATA

Surveys of the reactions of business managers to high interest rates and credit rationing provide an additional source of data for evaluating the hypothesis that net trade credit is redistributed toward smaller forms in periods of tight money. A particularly interesting survey for this purpose was undertaken jointly by the Federal Reserve-M.I.T. Econometric Model
project and Donaldson, Lufkin & Jenrette, Inc., New York investment bankers. Questionnaires were sent to a sample of 1,000 small-to-medium sized firms (assets ranging from $1 million to $50 million) in December, 1966, to determine the firms' adjustments to the tight money conditions of 1966.

The first part of the questionnaire was concerned with determining the proportion of firms in the sample which could not obtain bank credit in the desired amount. A summary of these results [on the basis of 343 acceptable responses] is shown in Table H. Of the firms in the sample, 275 actively sought bank credit during 1966, while 45 of these firms could not obtain the desired amount from the first bank which they approached. In terms of percentages (row 4 of the table), it is clear that the smallest firms experienced relatively more rationing, as would be expected. It is surprising, however, that 19 percent of the largest firms in the sample were also denied loan requests.\footnote{The survey also asked the firms to give the banks' reason for rationing. By far the most frequent response was "very little or no money available for new loans." Presumably this is merely a euphemism used by the banks and supports the contention (discussed in Chapter (3)) that survey data cannot successfully determine the existence or source of credit rationing.}

The most important and relevant portion of the survey is the response to the question, "Have you been able to secure at least part
TABLE H
PERCENTAGE OF BANK CUSTOMERS RATIONED\textsuperscript{47}

<table>
<thead>
<tr>
<th>ASSET SIZE ($ millions)</th>
<th>1 - 5.2</th>
<th>5.3-10.0</th>
<th>10.1-21.3</th>
<th>21.4-49.8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Firms in Sample</td>
<td>57</td>
<td>64</td>
<td>116</td>
<td>106</td>
<td>343</td>
</tr>
<tr>
<td>Firms Actively Seeking Credit</td>
<td>45</td>
<td>51</td>
<td>100</td>
<td>79</td>
<td>275</td>
</tr>
<tr>
<td>Firms Denied Initial Loan Request</td>
<td>12</td>
<td>7</td>
<td>11</td>
<td>15</td>
<td>45</td>
</tr>
<tr>
<td>Percentage of Firms Denied Initial Loan Request (row 3/row 2)</td>
<td>26.7%</td>
<td>13.7%</td>
<td>11.0%</td>
<td>19.0%</td>
<td>16.4%</td>
</tr>
</tbody>
</table>


\textsuperscript{47}The data for Tables (H), (I), and (J) were obtained from the tabulation of the Year End Survey in Donaldson, Infin & Jenrette, Inc. (1967), p. 44. It should be pointed out that the number of observations for data calls, particularly in Tables (I) and (J), is very small and at best these data can provide only supporting evidence. The four asset classes we have distinguished were chosen to provide a reasonable comparison with the data from the FTC-SEC Quarterly Financial Reports; a more complete classification is available in the source.
of the desired funds from other sources?" A tabulation of these results is shown in Table I. The numbers represent the percentage of firms not granted their total desired amount of bank credit at the original bank but which did succeed in obtaining at least some additional funds from other sources. "Other banks" are in general the most important of these additional sources, not too surprisingly. It is of interest, however, that small firms had about as much success as large firms in obtaining bank credit after being rationed at one bank.

Our primary interest is related to the second row of the table which shows the percentage of firms relying on trade credit as a substitute for bank credit. In terms of magnitude, it is clear from the table that trade credit does rank among the most important substitutes for bank credit. Furthermore, 33 percent of the smallest firms used this source, making it the most important substitute for this asset class. In addition, small firms relied more heavily on trade credit than any of the larger classes, although the relationship is not monotonic. The fact that 20 percent of the largest firms relied on additional long term debt offerings suggests that firms (or at least large firms with access to the open market) may be able to substitute long term debt for commercial loans in periods of credit rationing, thus not disturbing the rest of their asset holdings. Both of these results, the reliance on trade credit, especially by small firms, and the reliance on long term debt, especially

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48 A copy of the questionnaire is reproduced in Donaldson, Lufkin & Jenrette, Inc., *op. cit.*, pp. 8-9.
by large firms, is consistent with the view that the impact of credit rationing on real expenditures is reduced by an intra-business sector redistribution of funds from large firms to small firms.

### TABLE I

**ADDITIONAL SOURCES OF FINANCE FOR RATIONED FIRMS**

(Percentage of rationed firms relying on various alternative sources)

<table>
<thead>
<tr>
<th>Asset size ($ millions)</th>
<th>1-5.2</th>
<th>5.3-10.0</th>
<th>10.1-21.2</th>
<th>21.3-49.8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Other banks</td>
<td>25.0</td>
<td>28.6</td>
<td>27.3</td>
<td>26.7</td>
<td>26.7</td>
</tr>
<tr>
<td>Trade credit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(payables)</td>
<td>33.0</td>
<td>14.3</td>
<td>9.1</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Finance company</td>
<td>16.7</td>
<td>14.3</td>
<td>9.1</td>
<td>6.7</td>
<td>11.1</td>
</tr>
<tr>
<td>Insurance company</td>
<td>8.3</td>
<td>14.3</td>
<td>0.0</td>
<td>20.0</td>
<td>11.1</td>
</tr>
<tr>
<td>Long term debt</td>
<td>8.3</td>
<td>0.0</td>
<td>0.0</td>
<td>20.0</td>
<td>8.9</td>
</tr>
<tr>
<td>Other</td>
<td>25.0</td>
<td>42.9</td>
<td>0.0</td>
<td>13.3</td>
<td>17.8</td>
</tr>
</tbody>
</table>

The final set of data of interest from the survey concerns the reduced spending of business firms caused by credit rationing. The tabulation in Table J shows the percentage of rationed firms which reduced spending on the assets shown. Somewhat to our surprise, the strongest effect

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49 The question, as actually stated in the survey, did not distinguish between reductions in spending caused specifically by high interest rates and those reductions related directly to credit rationing.
TABLE J

REDUCED EXPENDITURES FOR RATIONED FIRMS

(Percentage of firms reducing expenditures on alternative assets)

<table>
<thead>
<tr>
<th>Asset size ($millions)</th>
<th>1.0-5.2</th>
<th>5.3-10.0</th>
<th>10.1-21.2</th>
<th>21.3-49.8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced expenditures on</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inventory</td>
<td>50.0</td>
<td>71.4</td>
<td>45.5</td>
<td>53.3</td>
<td>53.3</td>
</tr>
<tr>
<td>Trade credit</td>
<td>25.0</td>
<td>28.6</td>
<td>36.4</td>
<td>13.3</td>
<td>24.4</td>
</tr>
<tr>
<td>(receivables)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Modernization of</td>
<td>50.0</td>
<td>71.4</td>
<td>27.3</td>
<td>39.9</td>
<td>44.4</td>
</tr>
<tr>
<td>facilities</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additions to capacity</td>
<td>58.3</td>
<td>100.0</td>
<td>100.0</td>
<td>20.0</td>
<td>62.2</td>
</tr>
<tr>
<td>Acquisitions of other companies</td>
<td>41.7</td>
<td>85.7</td>
<td>72.3</td>
<td>46.7</td>
<td>57.8</td>
</tr>
<tr>
<td>Other</td>
<td>0.0</td>
<td>0.0</td>
<td>9.1</td>
<td>6.7</td>
<td>4.4</td>
</tr>
</tbody>
</table>

was related to expenditures associated with fixed capital, that is, modernization, additions to capacity, and acquisitions. The reduction in expenditures on accounts receivable was the least important of the five assets shown, although in terms of absolute magnitude (24 percent for the total) it still is significant. In addition, the smallest firms did reduce trade credit receivable substantially more than the largest class, although the relationship is not monotonic with asset size.

VI. SUMMARY AND CONCLUSIONS

The primary task of this chapter was the consideration of the demand for commercial loans. We have provided two tests of our model, one using aggregate and the other using disaggregated data. While the coefficient estimates for the equilibrium amount of inventory stock financed by long term liabilities differed between the two sets of data, otherwise the two estimates proved reasonable and consistent. In both cases the fits were good and almost all variables had correct signs and were statistically significant. Consequently, in general we view these tests as successful.

In order to test the demand model, consideration had to be given to the question of the impact of credit rationing. To the extent that credit rationing results in an adjustment of the firms's asset holdings, the direct explanatory power of a rationing variable, our rationing proxy in particular, should be quite small in a loan demand formulation. We found this to be the case. In addition, the estimates obtained from the
disaggregated data suggested the importance of trade credit as a source of loan demand, especially for the smaller firms. Since this is also consistent with the view that a redistribution of trade credit acts to offset the effects of credit rationing, we proceeded to provide further tests of the influence of credit rationing and high interest rates on the distribution of trade credit.

Three tests of this type were provided in section (V). In the direct test, net trade credit was regressed on independent variables consisting of interest rates, sales, and the rationing proxy. It was found that large firms definitely increase the amount of net trade credit they supply in periods of tight money. Small firms, on the other hand, reduce, or at least increase relatively little, their net supply of trade credit in these periods. In the indirect test, our specification of net trade credit, in terms of the interest rates, rationing, and sales, was used instead of the trade credit variable itself in the loan demand equation. These results were generally satisfactory and indicated that if the redistribution of trade credit is omitted, by suppressing the variable, then the impact of rationing is substantially stronger. Finally, additional supporting evidence was obtained from a survey of the effect of tight money in 1966. These data confirmed the impact of credit rationing on trade credit and were at least consistent with the redistribution toward smaller firms.

The last question to be raised, then, is whether the redistribution of net trade credit is sufficiently important to offset the possible impact
of credit rationing on real expenditure decisions. Even with a large redistribution of trade credit, presumably there still is at least some residual impact of rationing on expenditures since the redistribution itself will influence the expenditure decisions. However, if large firms can obtain credit at reasonable cost, even in periods of rationing, and if they redistribute this credit to small firms in the form of trade credit, it may be that the residual effect is very small indeed. Thus, the quantitative importance of this impact remains essentially an empirical question. Exploratory studies have already been started with the hope of measuring this direct impact of rationing on real expenditures. The lack of initial success in these tests can be attributed either to the basic statistical problems (discussed above) or to the intervention of a redistribution of trade credit. It is hoped that research on the impact of credit rationing, including its influence on both trade credit and real expenditures, will be continued.
VII. DATA APPENDIX

A. VARIABLES FIRST APPEARING IN SECTION (III)

L Commercial and industrial loans, all commercial banks, unpublished
Federal Reserve Board series

$H'$ Non-farm business inventory investment, SCB

IVA Inventory valuation adjustment, SCB

$H$ Change in book value, inventory stock ( = $H'$ - IVA)

$K_g$ Gross non-residential fixed investment, SCB

$S_g$ Gross non-residential investment in structures, SCB

$E_g$ Gross investment in producers' durable equipment, SCB

DEP Depreciation, sum of corporate and non-corporate capital consumption
allowance, less capital consumption allowance for real estate industry
(interpolated to quarterly values), SCB

GBP Gross private business product, SCB

GNP Gross national product, SCB

rA Moody's AAA corporate bond rate, 3-month average, FRB

rL Average rate on commercial loans, nineteen large cities, FRB

rP Prime commercial paper rate, 3-month average, FRB

A Rationing proxy, principal component of four series (see data appendix,
chapter (3) for further details)

Dollar values are measured in current $ billion and interest rates are measured
in percentage points. All flow variables are measured in quarterly rates.
All variables with the exception of interest rates are seasonally adjusted.

SCB = Survey of Current Business, August-September, 1965, as revised in July,
1967, Department of Commerce

FRB = Federal Reserve Bulletin, various issues
B. NOTES ON THE FEDERAL TRADE COMMISSION (FTC)-SECURITIES AND EXCHANGE COMMISSION (SEC) Quarterly Financial Reports for Manufacturing Corporations

The FTC-SEC Quarterly Financial Reports provide consistent balance sheets and income statements for the universe of manufacturing corporations derived from a selected sample.¹ The coverage is essentially complete for firms with assets over $5 million, but decreasingly smaller samples are used for the smaller firms. The official FTC-SEC view is that the data provide the best available estimate for the universe covered at the time of publication. Little attempt is made to revise the data or maintain consistency over time, although their staff is quite willing to offer what aid they can. Consequently, a significant amount of noise is found in the data and to some degree must just be accepted. Several problems are too important to ignore, however, and adjustments are noted here:

1. Splicing Sample Change Quarters

In 1951 and again in 1956 new samples were introduced (primarily for the smaller firms) to eliminate problems which had arisen with the old sample. This resulted in substantial discontinuities in the transitional quarter. Overlap observations were provided, however, for 1951-III and 1951-IV, and for 1956-II. To smooth this transition, first we calculated the ratio between the new sample and old sample for the overlap period for each series. This

¹ Anderson (1964), p. 29, and Frazer (1965), pp. 509, both provide summaries of the nature and problems of the data.
ratio was then linearly interpolated to unity at the point where the sample was first used. The interpolated ratio is then used to correct the old sample. \(^2\) The primary problem with this procedure is that the derived series need not satisfy the balance sheet identity, although in practice the inconsistency was very small.

2. **Certificates of Deposit**

Until the first quarter of 1964 a substantial percentage of the certificates of deposit were included in "other current assets" and "government securities" as well as "cash." Starting in 1964-I, all CD's are supposed to be included in cash. To make 1963-IV consistent with the new data, the FTC/SEC suggested increasing 1964-I cash by 15 percent and reducing government securities and other current assets by 5 percent and 18 percent, respectively. A telephone conversation with a SEC official indicated that it was reasonable to assume that the entire adjustment belonged to the over $100 million asset group. The official could provide no information on the amount of adjustment necessary for quarters prior to 1963-IV.

To solve this, we computed the ratio between the amount of adjustment necessary in 1963-IV and a series of total CD's outstanding (derived by Robert Rasche) for that quarter. Assuming the ratio was constant, we derived an implicit adjustment for earlier quarters by multiplying the ratio

\(^2\)Anderson, *op. cit.*, has used the same splicing method.
by the CD series for each quarter. The adjustment extends back to 1960-I, before which CD's are assumed zero.

3. "Reserves Not Reflected Elsewhere"

Starting in 1965-I, the FTC/SEC discontinued the category "reserves not reflected elsewhere" in the capital accounts. For that quarter they estimated that 10 percent of this category went to earned surplus and surplus reserves and the other 90 percent went to "other non-current liability." We assumed that these percentages were correct for the earlier quarters also and made the adjustments on this basis.

4. Accounts Receivable and Accounts Payable

Effective in 1965-I the FTC-SEC announced that certain non-trade items were being excluded from accounts payable and accounts receivable and were being shifted to other current assets and liabilities. On the basis of figures obtained from the FTC, adjustments were made for the four quarters of 1965 to maintain consistency with the earlier observations.

5. Seasonal Adjustment

The stock values of all balance sheet items and the income statement variables were individually seasonally adjusted, using the Bureau of Census X-11 program. Thanks are due to Phillip Cooper for aid in this task.
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BIографICAL NOTE

Dwight M. Jaffee was born February 7, 1943, in Chicago, Illinois, and attended public school there. He attended Oberlin College for the year 1960-1961 and Northwestern University from 1961 to 1964. He was graduated from Northwestern in June 1964, where he received a Bachelor of Arts degree with highest distinction and was elected a member of Phi Beta Kappa.

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