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VALUE OF CATEGORICAL AND PROBABILISTIC TEMPERATURE FORECASTS FOR SCHEDULING OF POWER GENERATION

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HYDROLOGY AND WATER RESOURCE SYSTEMS

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ABSTRACT

Bayesian decision models are formulated for the use and evaluation of categorical and probabilistic forecasts of continuous variables. The models are applied to the problem of short-term scheduling of power generation in an electric system on the basis of a single-period temperature forecast. Likelihood functions are constructed using results of experiments conducted at the National Weather Service. The probabilistic forecasting scheme is of the type wherein the forecaster quantifies his degree of uncertainty in terms of variable-width, fixed-probability credible intervals. Each forecasting scheme, categorical and probabilistic, is evaluated in a coupling with two decision procedures: (1) an optimal (Bayesian) procedure which accounts for forecast uncertainty, and (2) a more conventional, nonoptimal, procedure which disregards forecast uncertainty, but which would be optimal if the forecasts were perfect. Numerical examples are presented to illustrate the economic values of both types of forecasts, gains from probabilistic forecasts, and expected opportunity losses to be incurred by decision makers who ignore forecast uncertainty.

Key words: Bayesian decision theory, value of information, meteorologic forecasts, probabilistic forecasts, electric power generation, power load forecasting.

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CHAPTER 1

INTRODUCTION

1.1 Methodological Framework

The potential value of disseminating probabilistic meteorologic forecasts has long been recognized in the literature (Thompson, 1952; Glahn, 1964; Murphy, 1966; Winkler and Murphy, 1979) as, to quote Murphy (1980, p.247),

> ...a probabilistic mode of expression provides forecasters with a means of quantitatively describing the uncertainty inherent in their forecasts... (and) ...probabilistic forecasts provide potential users of forecasts with information needed to make rational decisions in uncertain situations.

Probabilistic forecasts of binary events (such as rain/no rain tomorrow) are issued routinely by the National Weather Service, and decision models for their optimal use and evaluation have been widely investigated. We review these efforts in Chapter 2.

In this study, decision models are formulated for the use and evaluation of categorical and probabilistic forecasts of continuous variables. The models are cast in the framework of Bayesian decision theory (Raiffa and Schlaifer, 1961; DeGroot, 1970; Davis et al., 1972). The decision context is the short-term scheduling of power generation in an electric system on the basis of a single-period power load forecast. This forecast results from a deterministic transformation of a temperature forecast. Numerical examples are presented wherein the decision concerns the daily average power generation and a forecast of the daily average temperature is available with the lead time of 12-24 hours.

Forecast evaluations are often classified as either *ex post* or *ex ante* studies (Winkler and Murphy, 1979). In *ex post* studies, the value of a set of forecasts is determined from the actual losses measured after the relevant forecasts have been issued, decisions have been made, and events have been observed. In *ex ante* studies, the value of a forecast is determined from the expected losses under the joint distribution of events and their forecasts. The distribution may pertain to an existing forecast system or to a system being designed; in the former case the distribution can be estimated from a historical record of forecasts and events, whereas in the latter case it must be hypothesized. The *ex ante* perspective is employed herein.

1.2 Temperature Forecasts

The study is predicated on the results of forecasting experiments conducted by Murphy and Winkler (1974, 1975) in the operational setting of the National Weather Service Offices in Denver, Colorado, and Milwaukee, Wisconsin. These experiments were designed to test the ability of the forecasters to quantify the uncertainty in their short-term (12-36 hours) forecasts of minimum and maximum temperature. Two techniques for encoding distribution functions of continuous variates were tested: (1) fixed-width, variable-probability credible intervals technique and (2) variable-width, fixed-probability credible intervals technique.

The fixed-width, variable-probability forecasts are created by first eliciting the median and then assigning probabilities to the events that the true state of nature will fall within two specified intervals centered at the median. Given these three responses, five points on the distribution function of the state variable are obtained.

The variable-width, fixed-probability forecasts are constructed by eliciting the median and four percentiles (25, 12-1/2, 75, 87-1/2, in that order). In this way two intervals about the median are determined within which the forecaster expects the state of nature to fall with probabilities .50 and .75. Again, these five points can be used to forge a distribution function of the state variable.

A measure of performance of probabilistic forecasts is their reliability (or calibration) which is determined by the degree of correspondence betweed the relative frequencies of observed events and their forecast probabilities. Murphy and Winkler (1975, p.12) found the variable-width, fixed-probability forecasts to be more reliable than the fixed-width, variable-probability forecasts. Therefore, further references to probabilistic forecasts concern only those encoded via the variable-width, fixed-probability technique although the theoretical development is general. Categorical forecasts are defined as the medians of the probabilistic forecasts. A measure of their performance is an error defined as the difference between the forecasted and observed state values. The reliability and error statistics reported by Murphy and Winkler (1974, 1975) serve us to construct the likelihood functions for the numerical examples presented in Chapters 3 and 4.

1.3 Decision Procedures

Forecasts of hydrometeorologic phenomena are seldom perfect: categorical forecasts may be in error; probabilistic forecasts may be unreliable. Under optimal (Bayesian) decision procedures the uncertainty inherent in a forecast is explicitly accounted for. This, however, is rarely the case in practice. Behavioral studies (e.g., White, 1973;

Slovic et al., 1974; Slovic, 1981) consistently reveal that, more often than not, decision makers are unaware of the normative guides to optimal integration of probabilistic information and decision making. Typically, they exhibit strong tendencies to ignore uncertainty or to employ simple, suboptimal, heuristics to synthesize probabilistic information before it is integrated with the decision consequences. Also, more conventional decision procedures (e.g., based on mathematical programming techniques) do not take forecast uncertainty explicitly into account but utilize forecasts as if they were error-free. Therefore, a question of utmost practical relevance arises: What is the value of an uncertain forecast when the decision maker ignores the uncertainty? In this study we investigate several aspects of this question.

1.4 Forecast-Decision System

Generally speaking, the economic value of a forecast depends upon (1) the accuracy and timeliness of the forecast and (2) the degree of optimality of the decision made on the basis of the forecast. Hence, the system to be modeled must be defined so as to encompass both activities: forecasting and decision making. This issue is never brought to bear in classical applications of decision theory for it is presupposed that the decision maker unequivocally employs an optimal decision procedure. Under this premise, the *ex ante* value of a forecast is always nonnegative and maximum for a given system since the optimal decision prescribed by the theory minimizes the expected opportunity loss due to an imperfect forecast.

In this study we relax this classical assumption, and, following earlier investigations (Davis et al., 1979; Krzysztofowicz et al., 1980; Krzysztofowicz and Davis, 1982), we consider a cascaded forecast-decision system (Figure 1.1) wherein the value of a forecast explicitly depends upon

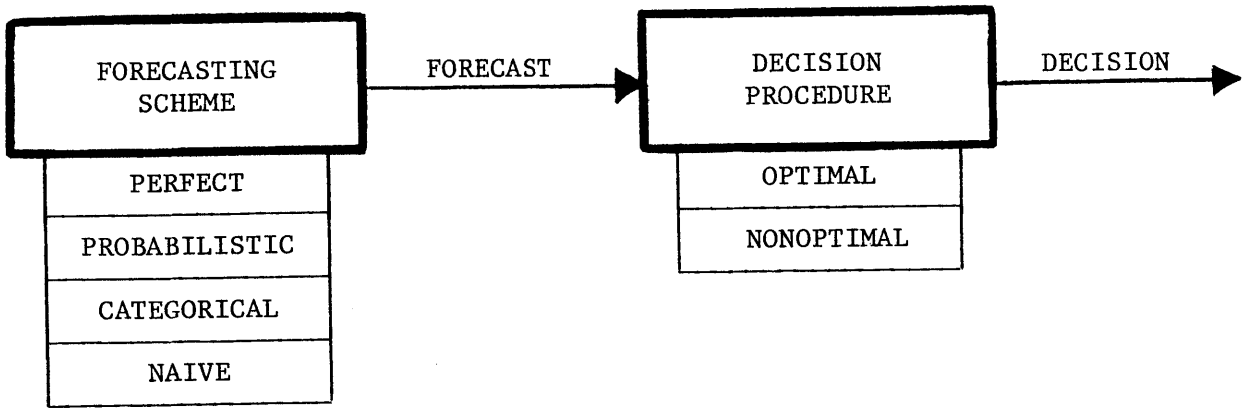

Figure 1.1 Forecast-Decision System

both the forecasting scheme (which is not necessarily perfect) and the decision procedure (which is not necessarily optimal). This decomposition affords us the ability to measure the performance of each system component as well as the total system. Thus, we can examine the effect of both the accuracy of forecasts and the optimality of decisions on the economic value of forecasts, and determine the expected opportunity losses incurred due to nonoptimal decisions.

Subject to our investigation are three types of forecasts: naive (or climatological) forecasts, categorical forecasts, and probabilistic forecasts, and two decision procedures: an optimal (Bayesian) procedure which accounts for forecast uncertainty, and a nonoptimal procedure which ignores forecast uncertainty but which would be optimal if forecasts were perfect. Therefrom, five distinct forecast-decision systems are defined:

1) categorical forecast - optimal decision,

2) categorical forecast - nonoptimal decision,

3) naive forecast - nonoptimal decision,

4) probabilistic forecast - optimal decision,

5) probabilistic forecast - nonoptimal decision.

The performance of each of these systems is evaluated in relation to the bounds established by the performance of systems composed of (a) a naive forecast - optimal decision (lower bound) and (b) a perfect forecast - optimal decision (upper bound).

1.5 Outline of the Report

In Chapter 2, the literature on the economics of weather is reviewed. In Chapter 3, the decision model for scheduling of daily average power generation in an electric system on the basis of a temperature forecast is formulated, measures of performance of a forecast-decision system are defined, and categorical temperature forecasts are evaluated for both optimal and nonoptimal decision procedures. In Chapter 4, the developments of Chapter 3 are extended to probabilistic temperature forecasts, and a comparison between the values of both types of forecasts is made. A summary of the results and conclusions comprise Chapter 5.

CHAPTER 2

REVIEW OF STUDIES ON THE ECONOMICS OF WEATHER

2.1 A Perspective

There is an ever increasing public awareness that although we may not be able to modify the weather to suit our needs, we can usually mitigate the inconvenience imposed by adverse weather events through *a priori* preparation and efficacious response to weather service forecasts. Unfortunately, we have only a partial understanding of the atmosphere and, therefore, forecasts are seldom perfect. The response to forecasts is, more often than not, determined via subjective, unaided processing of information which is known to be biased, often severely (Slovic, 1981). As a result, suboptimal decisions are frequently made by decision makers. Sometimes these decisions result in economic consequences that are less desirable than those which would result had the decision maker made his decision independent of the weather forecast. The effort, therefore, to understand and analyze the entire process of "dissemination, use and evaluation" (Nelson and Winter, 1964, p.421) of weather information has emerged as a discipline in its own right, known as the "economics of weather" (McQuigg, 1970).

According to a decision-theoretic perspective that prevails in the literature, information has value only if it improves one's ability to make decisions, the improvement being measured by a reduction of the expected loss - hence the economic connotation.

The research to date has focused mainly on static, finite state, and finite action problems, under the assumption that the forecasts are used optimally by the decision maker. We arbitrarily bifurcate the studies into those that employ a simple cost-loss ratio model and those that rely on more general statistical decision-theoretic models. These two approaches are not philosophically different - the cost-loss ratio formulation is merely a special case of a general decision-theoretic formulation.

2.2 Cost-Loss Ratio Formulations

The most prevalent decision model for the use and evaluation of weather forecasts is the cost-loss ratio formulation pioneered by Thompson (1952). The decision maker "must decide whether or not to take protective action in the face of uncertainty as to whether or not adverse weather will occur" (Murphy, 1977, p.803). Typically, a cost, c, is associated with taking protective action and an economic loss, ℓ, is incurred if no action is taken and adverse weather occurs (Table 2.1). With Pr(w) the probability of adverse weather and $Pr(\bar{w})$ the probability of favorable weather ($Pr(w) + Pr(\bar{w}) = 1.0$), the expected expenses of protection and no protection are c and $\ell Pr(w)$, respectively. Thus, the decision rule under the Bayes risk criterion is

$$\begin{aligned} &\text{to protect} && \text{if} \quad Pr(w) > c/\ell \ , \\ &\text{not to protect} && \text{if} \quad Pr(w) < c/\ell \ , \\ &\text{to protect or not to protect} && \text{if} \quad Pr(w) = c/\ell \ . \end{aligned}$$

Table 2.1 Payoff Matrix for the Cost-Loss Ratio Formulation

		ACTION	
		PROTECT	DO NOT PROTECT
STATE	ADVERSE WEATHER (w)	c	ℓ
	FAVORABLE WEATHER ($\bar{w}$)	c	0

This representation of a decision problem has been employed to study: i) the worth of precipitation forecasts to the raisin industry (Thompson and Brier, 1955; Kolb and Rapp, 1962) and the trucking, motion picture, and construction industries (Nelson and Winter, 1960; 1964), ii) the worth of tropical storm warnings to the City of Miami and the Florida Power and Light Company (Demsetz, 1962), iii) the scheduling of irrigation applications (Allen and Lambert, 1971a,b), iv) the effectiveness of air pollution forecasts in California (Kernan, 1975).

Most studies concluded that it is economically advantageous to use forecasts to make weather dependent decisions. For example, Thompson (1962, 1971) found that the average potential gains from improved atmospheric and operational forecasting constitute 5-10% of the avoidable weather losses. Murphy (1966) and Shorr (1966) reported techniques for transforming the cost-loss ratio into utilities. Murphy (1966), furthermore, derived an analytical expression for the *ex post* utility of probabilistic forecasts, in terms of the Brier probability score (Brier, 1950), under the assumption that the decision maker's utilities of outcomes are not known with certainty but conform to a uniform distribution. Murphy (1969) generalized the analysis to the evaluation of probabilistic forecasts when knowledge of the cost-loss ratio is incomplete (described by a probability density function) and developed measures for comparative forecast evaluation.

Ordinal relations between the values of probabilistic, categorical, and climatological forecasts were studied by Murphy (1977). He concluded that, in an *ex post* sense, probabilistic forecasts are unequivocally more valuable than either categorical or climatological prognostications if the forecasts are perfectly reliable (i.e., their calibration curve is a straight line). He also suggested that this relation holds for "moderately" unreliable probabilistic forecasts. Using the Brier score as a measure of probabilistic forecast accuracy, Murphy and Thompson (1977) demonstrated, again via an *ex post* analysis, that for a static, three state, three action decision problem more accurate forecasts do not necessarily ensure further mitigation of economic loss.

More recently, Winkler and Murphy (1979) indicated that, in an *ex ante* sense, perfect forecasts are more valuable than probabilistic forecasts, probabilistic forecasts more valuable than categorical, and categorical forecasts more valuable than climatological, if the categorical forecast is derived from the probabilistic forecast.

2.3 Statistical Decision-Theoretic Formulations

A general decision-theoretic treatment of categorical weather forecast evaluation was first introduced to the meteorologic literature by Gringorten (1958, 1959). In his first paper, Gringorten demonstrated that two sets of equally accurate forecasts could yield different expected gains when used operationally and suggested that users select forecasting systems that best suit their operational needs. In the second paper, he asserted that conditional forecast probabilities (conditioned on initial observations) should serve to "estimate operational gains through several courses of action with a minimum of error" and proposed an objective technique to accomplish this task.

Nelson and Winter (1960) eloquently described the two steps of decision analysis, i.e., making the decision and evaluating the decision, for categorical forecasts, yet constrained their hypothetical examples to cost-loss ratio situations. A strict mathematical exposition is presented in their later paper (Nelson and Winter, 1964). Gleeson (1960) compared two decision procedures, a classical decision-theoretic approach and a game-theoretic approach, to determine which was preferable when only discrete probability forecasts and upper and lower confidence limits on the forecast are available for an m state, n action situation. A Bayesian interpretation was brought to bear on Gleeson's problem by Epstein (1962), who considered the effect of using three different prior distributions of the forecast and evaluated three different decision procedures. This evaluation represents the first attempt to numerically compute the expected utility of the decision, as proposed by Nelson and Winter (1960).

The first author to address the value of categorical precipitation occurrence forecasts (rain/no rain) for a sequential decision problem under uncertainty was Lave (1963). He investigated the value of multiple lead time forecasts to California raisin growers, who can either pick their grapes for drying or allow them to ripen further at any 10-day interval during the months of September and October. He found that 10-day perfect forecasts were worth $18.40/acre and the worth of 30-day perfect forecasts was roughly $91/acre. Upon the premise that *all* the raisin farmers make optimal use of the rainfall forecasts, and maintain the same level of output, he concluded that weather information has negative worth since the demand for raisins is inelastic and "the inelasticity of the demand curve implies lower profit when supply increases." Hashemi and Decker (1968) reported a novel approach to the scheduling of irrigation using 12 hour probability of precipitation (POP) forecasts as the basis for decision. They developed conditional distributions of rainfall amount over the 12 hour forecast period given a set of POP forecasts. They determined, via an *ex post* analysis, that the incorporation of POP forecasts into the decision procedure substantially lessened the frequency and quantity of irrigation when compared with climatologically based decisions. A simple Markov chain (contingency matrix of the probability of a future condition given the present condition, or forecast) representation was offered by Crawford et al. (1971) to evaluate the worth of weather information, specifically wind direction change for various lag times, to a contractor who wishes to paint a house near "a factory which emits potentially damaging smoke."

Howe and Cochrane (1976) enhanced the general decision model for using categorical forecasts, as presented by Nelson and Winter (1960, 1964), to include long and short-run responses for adjusting to natural hazard events. In an application to urban snow removal, they determined that the total expected cost to the town of Rockford, Illinois, could be reduced by 50% annually if perfect forecasts were available. One further application of Nelson and Winter's (1960, 1964) model to the value of frost forecasts to pear orchardists was developed by Baquet et al. (1976). In an *ex ante* study, they estimated the sensitivity of the decision and opportunity loss to perturbations in the prior information, forecast accuracy and orchardist's utility function.

Perhaps the most complex study of the value of categorical forecasts is that by Krzysztofowicz and Davis (1982). They developed a decision-theoretic methodology for evaluation of the performance of flood forecast-response systems. During every flood event, a sequence of forecasts of the flood crest is issued. Floodplain dwellers respond to these forecasts by taking protective action such as evacuation, flood proofing, or the shutdown of a facility. This forecast-response process is modeled as a random duration Markovian decision process. The response strategy followed by floodplain dwellers is quantified via a human response model (Ferrell and Krzysztofowicz, 1982). The value of the system is measured by the expected annual reduction of the loss (cost of response plus flood damage sustained) and it depends upon both the accuracy and lead time of the forecasts and the quality of the response.

2.4 Summary

Many theoretical and applied issues pertinent to the economics of weather have been studied within the simple cost-loss ratio formulation. But there exist numerous weather sensitive decision problems that cannot readily be cast within this formulation, e.g., real-time reservoir control, irrigation scheduling, flight path determination, and power generation. More sophisticated uses of statistical decision theory in economics of weather have essentially been limited to the model of Nelson and Winter (1960, 1964), the analysis of Hashemi and Decker (1968) and the methodology of Krzysztofowicz and Davis (1982). There exist no studies on the evaluation of probabilistic forecasts of continuous predictands.

CHAPTER 3

VALUE OF CATEGORICAL FORECASTS

3.1 Decision Model

The decision problem is as follows. Every day the operator of an electric utility makes a decision as to the average power (in megawatts, MW) to be generated on the following day. The decision is made on the basis of a categorical forecast of daily average temperature. The utility suffers economic losses for either generating more or less power than that which is actually demanded. Excessive generation is undesirable because the difference between the generated and demanded power is dissipated (assuming no storage capability), while insufficient generation requires the utility to invoke more costly sources of energy to meet the excess demand on short notice.

The decision model is comprised of the following elements.

Information space (Θ,Ω). $t \in \Theta = (-\infty,\infty)$ and $v \in \Omega = [0,AM]$, where t represents the categorical forecast of the daily average temperature (in degrees Farenheit, $^{\circ}F$), and v is the corresponding categorical forecast of the daily average power load [MW]. AM is the maximum daily average power load that can occur in a given system.

State space (Θ,Ω). $\theta \in \Theta = (-\infty,\infty)$ and $\omega \in \Omega = [0,AM]$, where θ is the actual daily average temperature [$^{\circ}F$], and ω is the actual daily average power load [MW].

Decision space A. $a \in A = [0,AM]$, where a is the planned average power [MW] to be generated on the following day to meet the average load.

Load operator Ψ. The forecast temperature, t, is transformed into the forecast load, v, and the actual temperature, θ, into the actual load,

ω , by an operator:

$$v = \Psi(t) ,$$

$$\omega = \Psi(\theta) .$$

Specifically,

$$v = \begin{cases} AM & \text{if} \quad t < t_a \\ B + \alpha_h (t - t_b) & \text{if} \quad t_a \leq t < t_b \\ B & \text{if} \quad t_b \leq t \leq t_c \\ B + \alpha_c (t - t_c) & \text{if} \quad t_c < t \leq t_d \\ AM & \text{if} \quad t_d < t \end{cases}$$

where

$$\alpha_h = \frac{AM - B}{t_a - t_b} ,$$

$$\alpha_c = \frac{AM - B}{t_d - t_c} ,$$

B - weather insensitive daily base load [MW], α_h, α_c - heating and cooling coefficients [MW/^oF], t_a, t_b, t_c, t_d - reference temperatures [^oF].

Typically, B, α_h, α_c and reference temperatures t_a, t_b, t_c, t_d are dependent upon the day of the week, season, year, and whether or not the day is a holiday for a fixed location. The operator Ψ is a simple representation that has actually been used in practice (Schweppe, 1981). We harbor the assumption that the operator is exact, i.e., if we know perfectly the temperature we know the load without error. More sophisticated load forecasting models are described by Galiana (1971, 1972).

Loss function ℓ. The loss function is defined on $\Omega \times A$. $\ell(\omega,a)$ is the total loss incurred by the utility as a result of generating $a \in A$ [MW] when the actual load is $\omega \in \Omega$ [MW]. The structure of the loss is

$$\ell(\omega,a) = \begin{cases} c_o\ (\omega - a)^2 & \text{if} \quad \omega \leq a\ , \\ c_u\ (\omega - a)^2 & \text{if} \quad \omega \geq a\ , \end{cases}$$

where c_o and c_u are over and under generation cost coefficients with units [$/MW^2].

Prior information g_θ. Information about the state θ from sources other than real-time forecasts is encoded in a prior density g_θ of θ. For example, g_θ may be a climatological density estimated on the basis of observations taken over a period much longer than the period over which categorical forecasts have been provided.

Forecasting scheme Φ. A forecasting scheme is characterized by a family of densities $\Phi = \{f_t(\cdot|\theta) : \theta \in \Theta\}$, where $f_t(\cdot|\theta)$ is a density of the categorical forecast t conditional upon the actual daily average temperature $\theta \in \Theta$. For any $t \in \Theta$, $f_t(t|\cdot)$ represents the likelihood function of θ. Φ provides a probabilistic description of the errors of the categorical forecasts, and it may be viewed as a characterization of a given forecast system. Φ is to be estimated from a historical record of the forecasted and observed temperatures.

Bayesian information processor. For any categorical forecast $t \in \Theta$ produced by a known forecasting scheme Φ, the prior density g_θ of θ can be revised to obtain a posterior density $f_\theta(\cdot|t)$ of θ. The optimal revision is via Bayes theorem

$$f_\theta(\theta|t) = \frac{f_t(t|\theta)\ g_\theta(\theta)}{g_t(t)} ,$$

where

$$g_t(t) = \int_\Theta f_t(t|\theta)\ g_\theta(\theta)\ d\theta .$$

This Bayesian information processor (Sheridan and Ferrell, 1974) is shown in Figure 3.1.

3.2 Evaluation Model

3.2.1 Forecast-Decision System

The process of forecasting and decision making can be conceptualized as a system (Krzysztofowicz and Davis, 1982). The forecast-decision system is defined as a cascade coupling of two components, namely the forecast system and the decision system, as is shown in Figure 3.1. The performance of the forecast-decision system depends upon both the accuracy of the forecasts and the optimality of the decisions. This interplay between the system components is examined herein, first, theoretically and next experimentally.

Determination of the value of categorical forecasts requires meaningful measures for comparison. Perfect forecasts provide a lower bound on the Bayes risk, and naive (or climatological) forecasts provide an upper bound. The naive forecasting scheme is one which for each day specifies the same temperature equal to an estimate of θ based on the climatological (prior) density g_θ of θ. The naive forecasts are made without regard to current synoptic information. Therefore, if a real-time categorical forecasting scheme utilizing current synoptic information is to be worthwhile, it must provide forecasts whose Bayes risk is smaller than that of the naive forecasts.

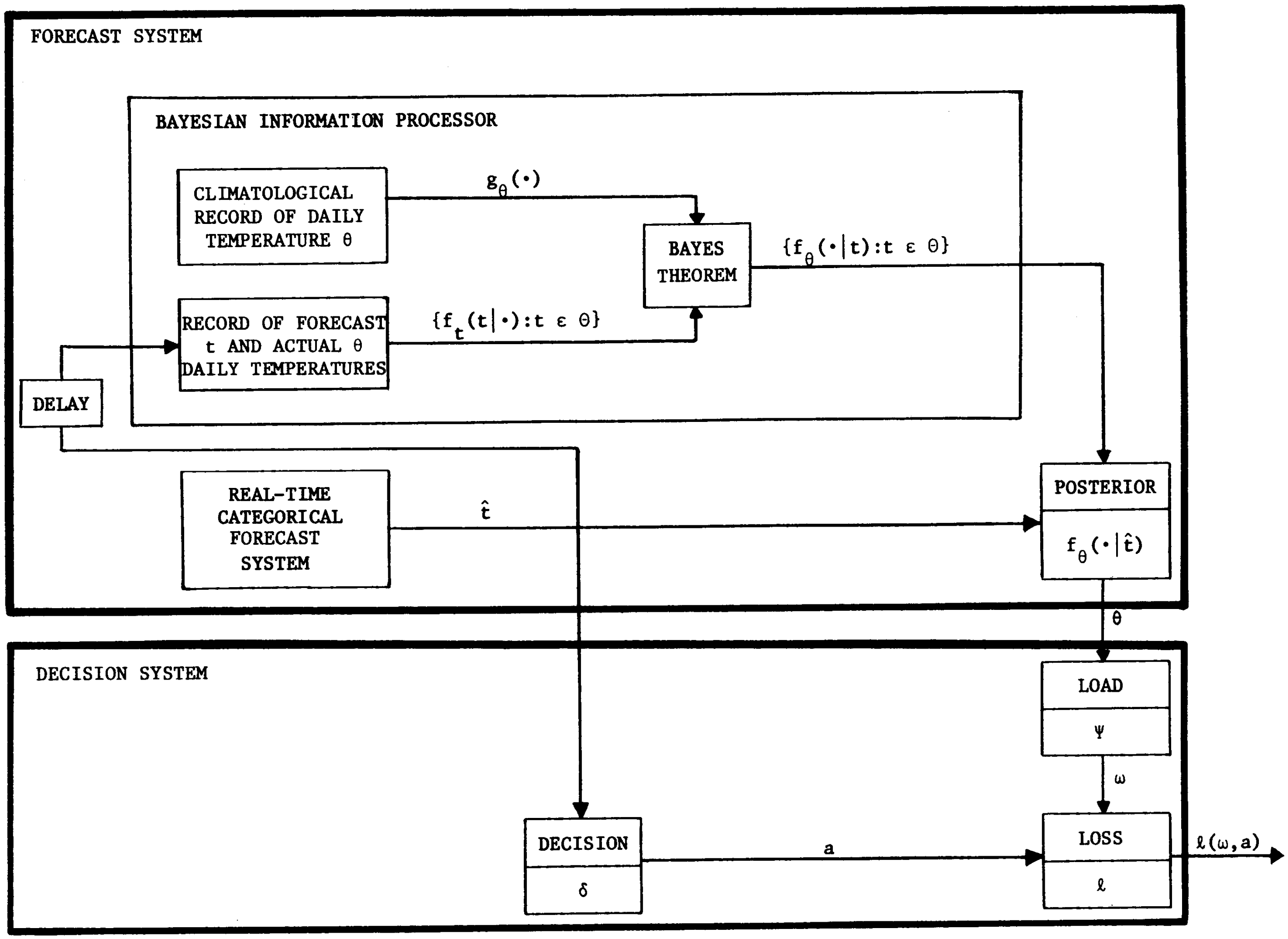

Figure 3.1 System Diagram of the Decision Model

In reality, the forecasts, both categorical and naive, are often used in a nonoptimal manner. We suggest examining such cases as well, in order to compare the value of forecasts for optimal and nonoptimal decision making. The nonoptimal decision procedure selected, and by no means the only one in popular use, is one that overlooks the intrinsic uncertainty in the forecasts, i.e., decisions are made as if the forecasts were perfect.

Table 3.1 shows the risks associated with five forecast-decision systems. To determine a risk two essential steps must be considered: making the decision and evaluating the decision. In the first step, the decision maker selects the decision or decision function that minimizes the loss or expected loss depending upon the decision procedure. In the second step, the decision maker evaluates the decision or decision function in terms of risk.

3.2.2 Nonoptimal Decision Procedures

Naive forecasting scheme. For a naive forecast of the temperature, $\hat{t} \in \Theta$, namely the climatological mean of the daily average temperature, the minimum loss, when natural uncertainty is not accounted for, is

$$\overline{LN} = \min_{a} \ell(\Psi(\hat{t}),a) .$$

Let $\hat{a} \in A$ be the decision minimizing the loss. Since $\hat{v} = \Psi(\hat{t})$ and $\hat{a}$ are elements of the same space [0,AM], $\hat{a} = \hat{v} = \psi(\hat{t})$ given the loss function delineated in Section 3.1. The risk of the decision $\hat{a}$ is specified by

Table 3.1 Risks of Forecasting Schemes and Decision Procedures

		DECISION	
		OPTIMAL	NONOPTIMAL
FORECAST	PERFECT	RF	
	CATEGORICAL	RC	$\overline{RC}$
	NAIVE	RN	$\overline{RN}$

(read $\overline{RC}$ as RC conjugate)

Notation

RF - Risk associated with a perfect forecast and an optimal decision procedure

RC - Risk associated with a categorical forecast and an optimal decision procedure

$\overline{RC}$ - Risk associated with a categorical forecast and a non-optimal decision procedure

RN - Risk associated with a naive forecast and an optimal decision procedure

$\overline{RN}$ - Risk associated with a naive forecast and a nonoptimal decision procedure

$$\overline{RN} = \int_{\Theta} \ell(\Psi(\theta),\hat{a})\ g_{\theta}(\theta)\ d\theta\ ,$$

where g_{θ} is the climatological density of the daily average temperature θ.

Categorical forecasting scheme. For categorical forecasts $t \in \Theta$, the minimum loss function is given by

$$\overline{LC}(t) = \min_{\delta} \ell(\Psi(t),\ \delta(t))\ .$$

The solution is the decision function $\hat{\delta}$ whose risk is defined by the equation

$$\overline{RC} = \iint_{\Theta\Theta} \ell(\Psi(\theta),\ \hat{\delta}(t))\ f_{\theta}(\theta|t)\ g_{t}(t)\ d\theta\ dt\ .$$

3.2.3 Optimal Decision Procedures

The following procedures are deemed optimal as the forecast uncertainty is accounted for in the decision making step.

Naive forecasting scheme. The Bayes risk associated with naive forecasts is given by

$$LN = \min_{a} \int_{\Theta} \ell(\Psi(\theta),a)\ g_{\theta}(\theta)\ d\theta\ .$$

The minimizer of this expression, $a^* \in A$, is the Bayes decision against the *natural uncertainty* in the daily average temperature. The risk of this decision is RN = LN.

Categorical forecasting scheme. In order to optimally account for uncertainty of a categorical forecast, the decision should be made against the posterior density obtained from the Bayesian information processor. Thus, the Bayes risk function for categorical forecasts, $t \in \Theta$, is

$$LC(t) = \min_{\delta} \int_{\Theta} \ell(\Psi(\theta), \delta(t)) \, f_{\theta}(\theta|t) \, d\theta \, .$$

An optimal solution to this decision problem is a decision function δ^* specifying for every forecast temperature $t \in \Theta$ the planned generation $a^* = \delta^*(t) \in A$ which minimizes the expected loss. δ^* is termed a *Bayes decision function* (DeGroot, 1970, p.138) against the *forecast uncertainty*. The risk of this decision function is given by

$$RC = \iint_{\Theta\Theta} \ell(\Psi(\theta), \delta^*(t)) \, f_{\theta}(\theta|t) \, g_t(t) \, d\theta \, dt \, ,$$

or

$$RC = \int_{\Theta} LC(t) \, g_t(t) \, dt \, .$$

It is apparent from the above development that the incorporation of a Bayesian information processor into the decision model results in an *adaptive decision procedure*. This procedure is especially attractive for real-time applications because the Bayes decision function δ^* can be found prior to decision making. Then, every day the decision maker receives the forecast $t \varepsilon \Theta$, he can make the Bayes decision $\delta^*(t) \varepsilon A$ without further calculations. If the operators Ψ, ℓ, g_θ, and Φ are not stationary, but vary periodically (e.g., from month to month), then the Bayes decision function has to be found for each period. Also, if in the course of events, the knowledge about the physical phenomenon changes and/or the forecast system undergoes a modification (e.g., by expanding the data network, introducing a novel predictive model, or hiring a new forecaster) that may affect the errors of the forecasts, then the prior distribution and/or the family of the likelihood functions should be updated and the decision function recomputed. Ultimately, the prior distribution can be updated by every new observation of the state, while the likelihood family can be updated by every new pair of the forecasted and observed state values.

Perfect forecasting scheme. The minimum loss function for perfect forecasts, $t \varepsilon \Theta$, is given by

$$LF(t) = \min_{\delta} \ell(\Psi(t), \delta(t)) .$$

The function δ^{**} that minimizes this expression is the optimal decision function against perfect forecasts. Its risk is given by

$$RF = \int_{\Theta} \ell(\Psi(\theta), \delta^{**}(\theta)) \, g_{\theta}(\theta) \, d\theta .$$

Notice that $\delta^{**} = \hat{\delta}$ and $LF = \overline{LC}$, but the difference is that the decision maker using $\hat{\delta}$ acts in a nonoptimal manner: he believes the forecasts are perfect when, in fact, they are not. This distinction is reflected in the evaluation of the decision: $RF \le \overline{RC}$. Moreover, we note that $\delta^{**}(\theta) = \Psi(\theta)$ for all $\theta \in \Theta$ and $RF = 0$ given the quadratic loss function we have chosen.

3.2.4 Ordinal Relations Between Risks

A priori, it can be shown that there exist ordinal relations among the risks for both the optimal and nonoptimal decision procedures as well as between the two sets of risks. Most obviously,

$$RF \le RC \le RN .$$

This relation is a consequence of the fact that, under optimal decision procedures, perfect forecasts are always at least as valuable as categorical or naive forecasts, and categorical forecasts are always at least as valuable as naive forecasts. These facts are demonstrated in Appendix B.

The orderings between the risks of the optimal and nonoptimal decision procedures are

$$RC \le \overline{RC} \;,\; RN \le \overline{RN} .$$

They derive from the fact that the nonoptimal decision procedures ignore the errors of the forecasts, i.e., decisions are made as if the forecasts were perfect, whereas the optimal procedures account for the forecast errors in an optimal manner, by definition. Nothing, however, can be said as to the relations between RN and $\overline{RC}$, and between $\overline{RN}$ and $\overline{RC}$. The lack of an *a priori* ordering between $\overline{RN}$ and $\overline{RC}$ is consequential. Suppose a decision maker who follows a nonoptimal decision procedure and who has been using a naive forecasting scheme ponders employing categorical forecasts. If $\overline{RC} > \overline{RN}$, the decision maker will realize negative value of forecast information. In Appendix C, we derive conditions under which this occurs.

3.2.5 Performance Measures

Krzysztofowicz et al. (1980) have developed an approach to system performance measurement. This approach will be followed herein. The sufficient statistic of system performance is a vector of values from which the efficiencies and expected opportunity losses for each of the system components as well as the total system are derived.

We propose to evaluate three forecast-decision systems:

System 1: categorical forecast - optimal decision,

System 2: categorical forecast - nonoptimal decision,

System 3: naive forecast - nonoptimal decision.

As a reference for comparing the performance of these systems, we take the case of naive forecast - optimal decision. This is the case wherein no categorical forecasts are provided and the decision maker makes the best use of the climatological record of temperature. That is, he makes a Bayes decision against the climatological distribution of the daily average temperatı and his risk is RN. The systems and their values are defined as follows

System 1. The decision maker uses the categorical forecast and accounts for the forecast uncertainty. This procedure is both adaptive and optimal. The potential value of the system is

$$PV1 = RN - RF = RN \ .$$

The actual value, AV1, which in this system equals the optimal value, OV1, since the decision procedure is optimal, is given by

$$AV1 = OV1 = RN - RC \ .$$

System 2. The decision maker uses the categorical forecast but neglects the forecast uncertainty. This procedure is adaptive but non-optimal. It would be optimal if the forecasts were perfect. The potential (PV2), optimal (OV2), and actual (AV2) values of the system are given by

$$PV2 = RN - RF = RN \ ,$$

$$OV2 = RN - RC \ ,$$

$$AV2 = RN - \overline{RC} \ .$$

The actual value, AV2, which may be either nonnegative or negative, differs from the optimal value, OV2, since a nonoptimal decision procedure is employed.

System 3. The decision maker does not use the categorical forecast, nor does he account for the natural uncertainty in the daily average temperature. Instead, he uses a naive forecast (climatological estimate) as if it were a perfect forecast. This procedure is neither adaptive nor optimal. The potential (PV3), optimal (OV3), and actual (AV3) values are given by

$$PV3 = RN - RF = RN ,$$

$$OV3 = RN - RN ,$$

$$AV3 = RN - \overline{RN} .$$

We see that the actual value, AV3, is always nonpositive since $RN \leq \overline{RN}$.

The second measure of system performance introduced by Krzysztofowicz et al. (1980) is efficiency. The generic definitions of forecast (FE), decision (DE), and total (TE) system efficiencies are given by

$$FE = \frac{OV}{PV} \; ; \; DE = \frac{AV}{OV} \; ; \; TE = \frac{AV}{PV} = FE \cdot DE .$$

For all systems FE, DE, TE $\leq$ 1. The forecast efficiency, FE, is an indicator as to how skillful the actual forecasting scheme is relative to a perfect scheme. Since the potential and optimal values are identical for Systems 1 and 2, the forecast efficiency, FE, is the same for both systems. The optimal value of System 3 is zero because if only a naive forecast is available, the optimal performance we can expect is RN. The decision efficiency, DE, is a measure of how inept is the given decision procedure when compared to the optimal (Bayesian) procedure. Analogously, the total efficiency, TE, is a composite yardstick of the actual total system performance relative to potential performance. Thus, we are now equipped with a mathematically cogent framework within which we can quantitatively examine the relative efficiency of three forecast-decision systems and their components.

The third measure of system performance is the expected opportunity loss. We define the generic forecast (FOL), decision (DOL), and total (TOL) expected opportunity losses as follows:

$$\mathrm{FOL} = \mathrm{PV} - \mathrm{OV} \ ,$$

$$\mathrm{DOL} = \mathrm{OV} - \mathrm{AV} \ ,$$

$$\mathrm{TOL} = \mathrm{PV} - \mathrm{AV} = \mathrm{FOL} + \mathrm{DOL} \ .$$

FOL represents the value of replacing the actual forecasting scheme by a perfect one. It is thus an upper bound on the daily cost of investments towards improving the actual forecasts. Similarly, DOL represents the value of replacing the actual decision procedure by an optimal one. It is thus an upper bound on the daily expenditures towards improving the actual decisions. Finally, TOL represents the value of replacing the actual forecast-decision system by a system in which forecasts are perfect and decisions optimal.

Armed with these measures, we can appraise the efficacy of an improvement in the forecast-decision system in dollars. Moreover, a preposterior analysis can be conducted to discriminate between alternative improvements of each system component.

3.3 Distribution of the Forecasting Scheme

Murphy and Winkler (1974, 1975) report statistics of the subjective temperature forecasts (categorical, variable-width, and fixed-width forecasts) from experiments conducted in Denver, Colorado, and Milwaukee, Wisconsin. The experiments were undertaken to evaluate the local forecasters' skill in constructing probabilistic forecasts of temperature for lead times from 12 to 36 hours.

Murphy and Winkler (1974, 1975) define the categorical forecast, t, to be the median of the probabilistic forecast. The statistics of the categorical forecast temperatures, t, for Denver are compared with the actual temperatures, θ, in Table 3.2. Let

$$\varepsilon = t - \theta$$

denote the forecast error and h_ε denote its density. Assuming ε is independent of θ, we have

$$f_t(t|\theta) = h_\varepsilon(t - \theta) \ .$$

Given the statistics of ε (columns 2 and 3 of Table 3.2) and two points on the distribution H_ε of ε (columns 4, 5 and 6), we can construct h_ε by postulating the underlying probability law. We postulate h_ε to be a mixed density (Benjamin and Cornell, 1970):

$$h_\varepsilon(\varepsilon) = (1 - p)\, h_n(\varepsilon) + p\, \mu(\varepsilon) \ ,$$

where $p = Pr(\varepsilon = 0)$, h_n is a normal density with parameters m_n and s_n^2 ,

Table 3.2 A Comparison of Forecast Median Temperature t and Actual Temperature θ from Experiment in Denver

SET OF FORECASTS	MEAN m_ε [^oF]	STANDARD DEVIATION s_ε [^oF]	$Pr(\theta < t)$	$Pr(\theta = t)$	$Pr(\theta > t)$	NUMBER OF FORECASTS
1	2	3	4	5	6	7
ALL	-0.5	4.9	.394	.126	.480	254
VARIABLE WIDTH	-0.1	5.2	.447	.121	.432	132
FIXED WIDTH	-0.8	4.6	.336	.131	.533	122
MAXIMUM	0.6	4.8	.472	.134	.394	127
MINIMUM	-1.5	4.8	.315	.118	.567	127
12-HOUR	-0.2	4.7	.409	.134	.457	127
24-HOUR	-0.7	5.1	.378	.118	.504	127
FORECASTER 1	0.0	5.3	.453	.109	.438	64
FORECASTER 2	-0.3	5.1	.442	.132	.426	68
FORECASTER 3	-0.1	4.6	.416	.117	.467	60
FORECASTER 4	-1.5	4.4	.258	.145	.597	62

(Source: Murphy and Winkler, Table 1, p.4, 1975) which are necessarily different from m_ε and s_ε^2, and μ is the Dirac delta function.

We shall now express the parameters of the normal density h_n in terms of the statistics of ε. By definition

$$m_\varepsilon = \int \varepsilon \, h_\varepsilon(\varepsilon) \, d\varepsilon \; ,$$

$$= (1 - p) \int \varepsilon \, h_n(\varepsilon) \, d\varepsilon + p \int \varepsilon \, \mu(\varepsilon) \, d\varepsilon \; ,$$

$$= (1 - p) \, m_n \; .$$

Thus,

$$m_n = \frac{m_\varepsilon}{1 - p} \; .$$

Similarly,

$$s_\varepsilon^2 = \int (\varepsilon - m_\varepsilon)^2 \, h_\varepsilon(\varepsilon) \, d\varepsilon \; ,$$

$$= (1 - p) \int (\varepsilon - m_\varepsilon)^2 \, h_n(\varepsilon) \, d\varepsilon + p \int (\varepsilon - m_\varepsilon)^2 \, \mu(\varepsilon) \, d\varepsilon \; ,$$

$$= (1 - p) \, [s_n^2 + m_n^2 - 2 m_\varepsilon m_n + m_\varepsilon^2] + p \, m_\varepsilon^2 \; .$$

After expressing m_n in terms of m_ε, we obtain

$$s_n^2 = \frac{1}{1 - p} \, [s_\varepsilon^2 - \frac{p}{1 - p} \, m_\varepsilon^2] \; .$$

3.4 Hypothetical Case Study

3.4.1 Input Data

Prior information g_θ. Nine years of daily average temperature data for the month of April in Boston, Massachusetts, (see Table 3.3) are used to construct the prior density g_θ. The sample data are found to be normally distributed at the 1% significance level using a Kolmogorov-Smirnov test (Benjamin and Cornell, 1970). The sample mean, variance, and precision are

$$m_\theta = 48.95\ [^oF]\ ,$$
$$s_\theta^2 = 65.29\ [^oF]^2\ ,$$
$$\tau_\theta = \frac{1}{s_\theta^2} = .015\ [^oF]^{-2}\ .$$

The resulting distribution function G_θ is shown in Figure 3.2.

Forecasting scheme Φ. The mean m_ε, variance s_ε^2, precision τ_ε of the error $\varepsilon = t - \theta$, and the reliability p of the forecasts are assumed to be equal to the values for the "ALL" set of forecasts from Table 3.2:

$$m_\varepsilon = -.5\ [^oF]\ ,$$
$$s_\varepsilon^2 = 24.01\ [^oF]^2\ ,$$
$$\tau_\varepsilon = .042\ [^oF]^{-2}\ ,$$
$$p = .126\ .$$

Using the above parameter values, the density h_ε is constructed via the method described in Section 3.3; the resulting distribution H_ε is shown in Figure 3.3. The likelihood function is then specified by $f_t(t|\theta) = h_\varepsilon(t-\theta)$ so that

Table 3.3 Daily Average Temperature in [°F] for the Month of April in Boston, Massachusetts

DAY	YEAR								
	1969	1971	1972	1973	1974	1975	1977	1978	1981
1	34	43	45	44	47	38	47	54	46
2	44	54	44	46	43	40	42	42	50
3	39	48	43	40	56	44	57	35	57
4	50	47	41	40	59	35	43	40	59
5	54	38	40	43	60	36	43	50	59
6	44	38	42	48	44	41	46	49	45
7	50	40	33	53	46	41	38	41	49
8	46	40	33	43	43	39	37	44	57
9	45	46	41	44	34	40	34	44	61
10	58	54	44	48	33	43	44	49	59
11	51	40	40	38	47	42	44	49	59
12	42	41	42	40	46	40	64	52	51
13	48	54	40	39	43	42	67	60	42
14	53	44	44	39	51	48	58	49	45
15	61	40	40	49	56	44	53	44	41
16	62	38	49	62	53	48	56	45	46
17	68	41	51	63	54	52	54	49	53
18	58	43	55	65	59	54	57	49	65
19	42	49	56	62	44	61	52	46	54
20	41	51	40	46	43	53	56	52	46
21	45	55	46	54	59	45	68	52	40
22	47	49	49	67	70	47	73	52	45
23	52	49	44	70	61	51	60	53	48
24	45	50	52	61	46	54	45	58	45
25	55	44	45	54	42	52	44	52	42
26	57	47	41	47	47	45	50	53	54
27	61	46	44	46	50	44	53	48	60
28	71	45	51	59	62	44	58	49	60
29	56	40	58	48	70	43	47	60	61
30	50	50	59	49	63	54	59	50	58

(Source: National Weather Service)

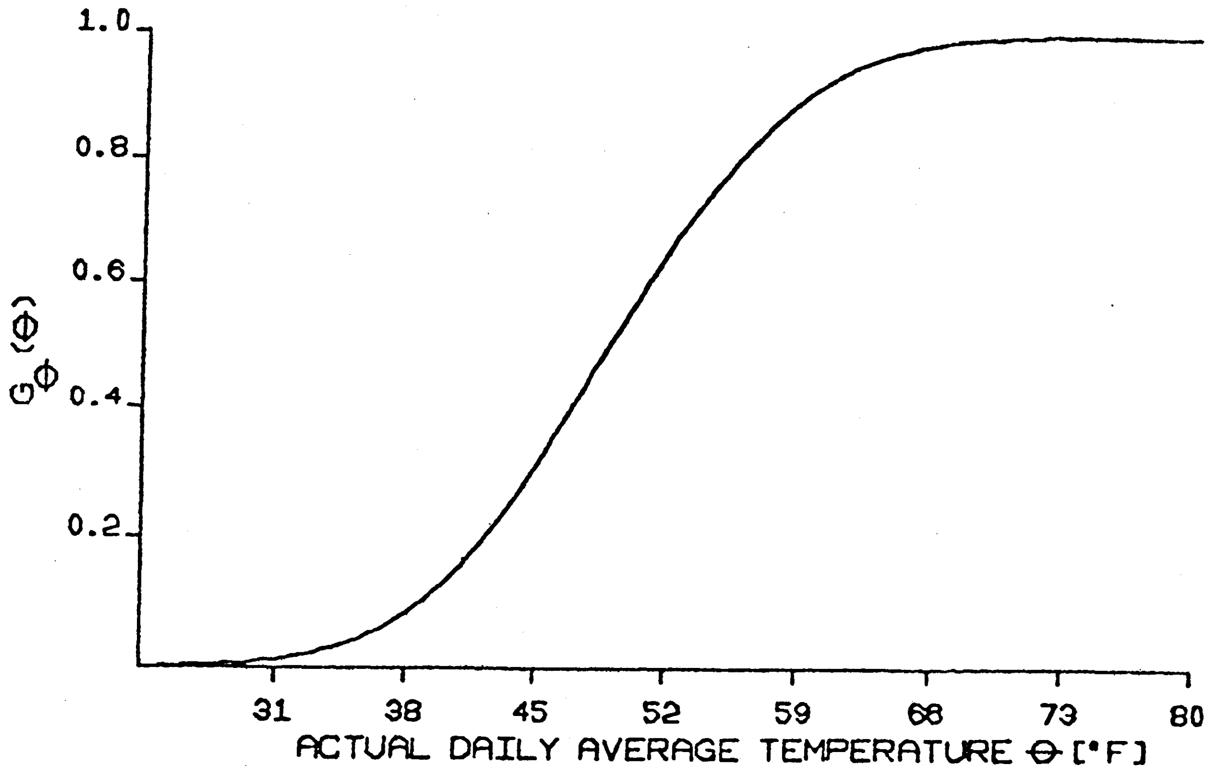

Figure 3.2 Distribution of Actual Daily Average Temperature

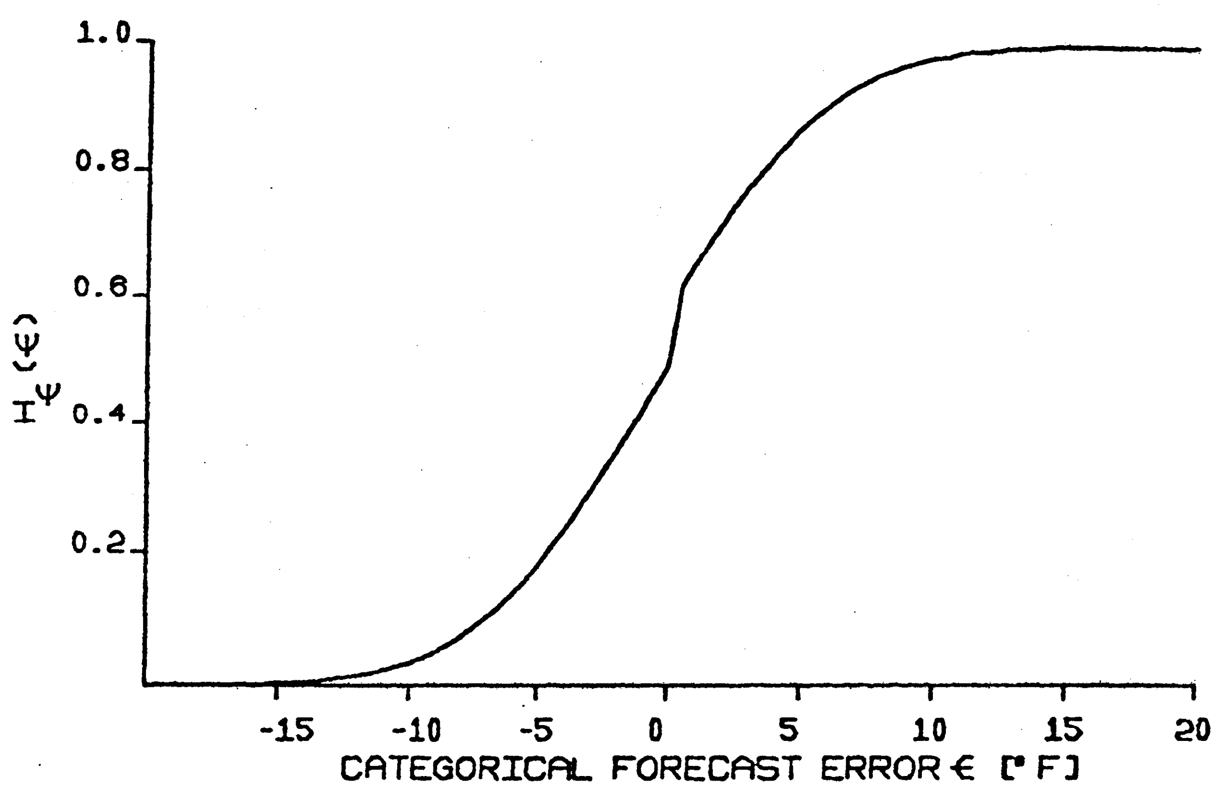

Figure 3.3 Distribution of Categorical Forecast Error

$$m_{t|\theta} = m_\varepsilon + m_\theta = 48.45 \ [^oF] ,$$

$$\tau_{t|\theta} = \tau_\varepsilon = .042 \ [^oF]^{-2} .$$

The family of posterior densities $\{f_\theta(\cdot|t) : t \in \Theta\}$ and the predictive density g_t are then computed as described in Section 3.1. The predictive distribution G_t is shown in Figure 3.4. All normal distributions are computed numerically using a polynomial approximation (Abramowitz and Stegun, 1972).

Load operator Ψ. The load operator is defined by the variables t_a, t_b, t_c, t_d and AM-B. Schweppe (1981) indicates that AM-B ranges anywhere from 5% to 30% of the base load B, and approximate values of the reference temperatures are

$$t_a = 20 \ [^oF] ,$$

$$t_b = 50 \ [^oF] ,$$

$$t_c = 70 \ [^oF] ,$$

$$t_d = 90 \ [^oF] .$$

We assume AM-B = 300 MW. The operator Ψ is displayed in Figure 3.5.

Loss function ℓ. The values of the cost coefficients are assumed to be

$$c_o = 10 \ [\$/MW^2] ,$$

$$c_u = 20 \ [\$/MW^2] .$$

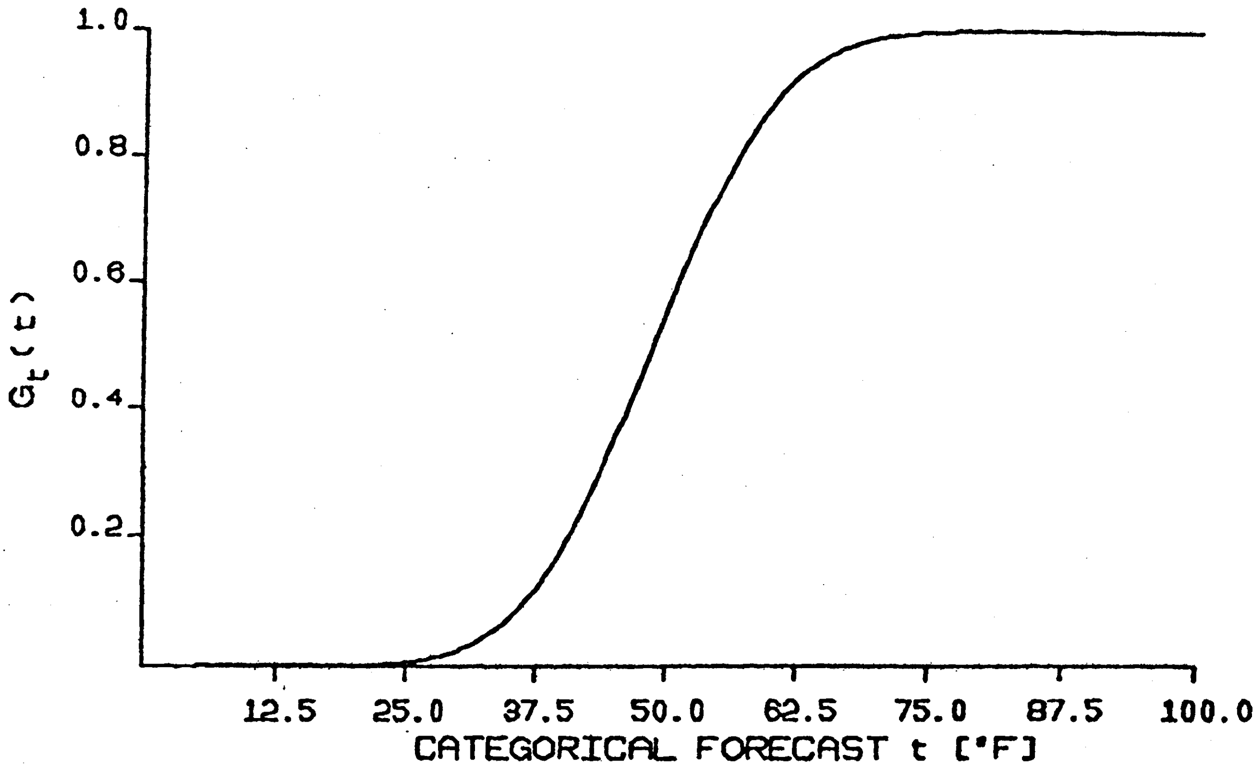

Figure 3.4 Distribution of Categorical Forecasts

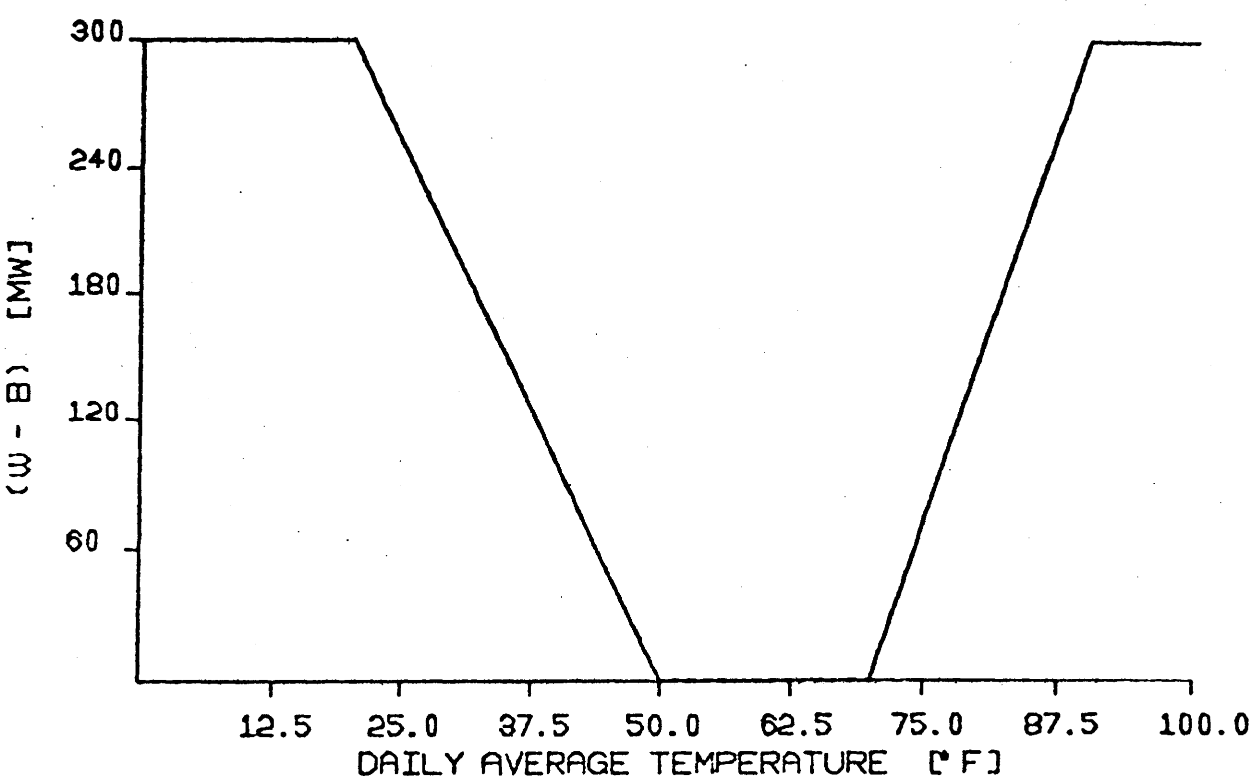

Figure 3.5 Load Operator

c_u is greater than c_o as excessive generation is wont to be less costly in comparison to insufficient generation. This supposition derives from the observation that insufficient generation would impel the utility to resort to less efficient, short-term power sources to close the gap between the actual power load and that which was planned for. Figure 3.6 illustrates the loss function. It is worth noting that the loss as a function of the state θ, $\ell(\Psi(\cdot),a)$, is not at all quadratic (see Figure 3.7).

3.4.2 Basic Evaluation

The risks for the three forecasting schemes and two decision procedures are compared in Table 3.4. Note that the risks are in \$/day. The ordinal relations between the risks suggested earlier are also confirmed in Table 3.4.

Table 3.4 Risks for the Case Study

		DECISION	
		OPTIMAL	NONOPTIMAL
FORECAST	PERFECT	RF = \$0	
	CATEGORICAL	RC = \$12327	$\overline{RC}$ = \$22318
	NAIVE	RN = \$38889	$\overline{RN}$ = \$61294

Table 3.5 presents a comparative evaluation of Systems 1, 2, and 3 with respect to all performance measures. In Systems 1 and 2, the optimal

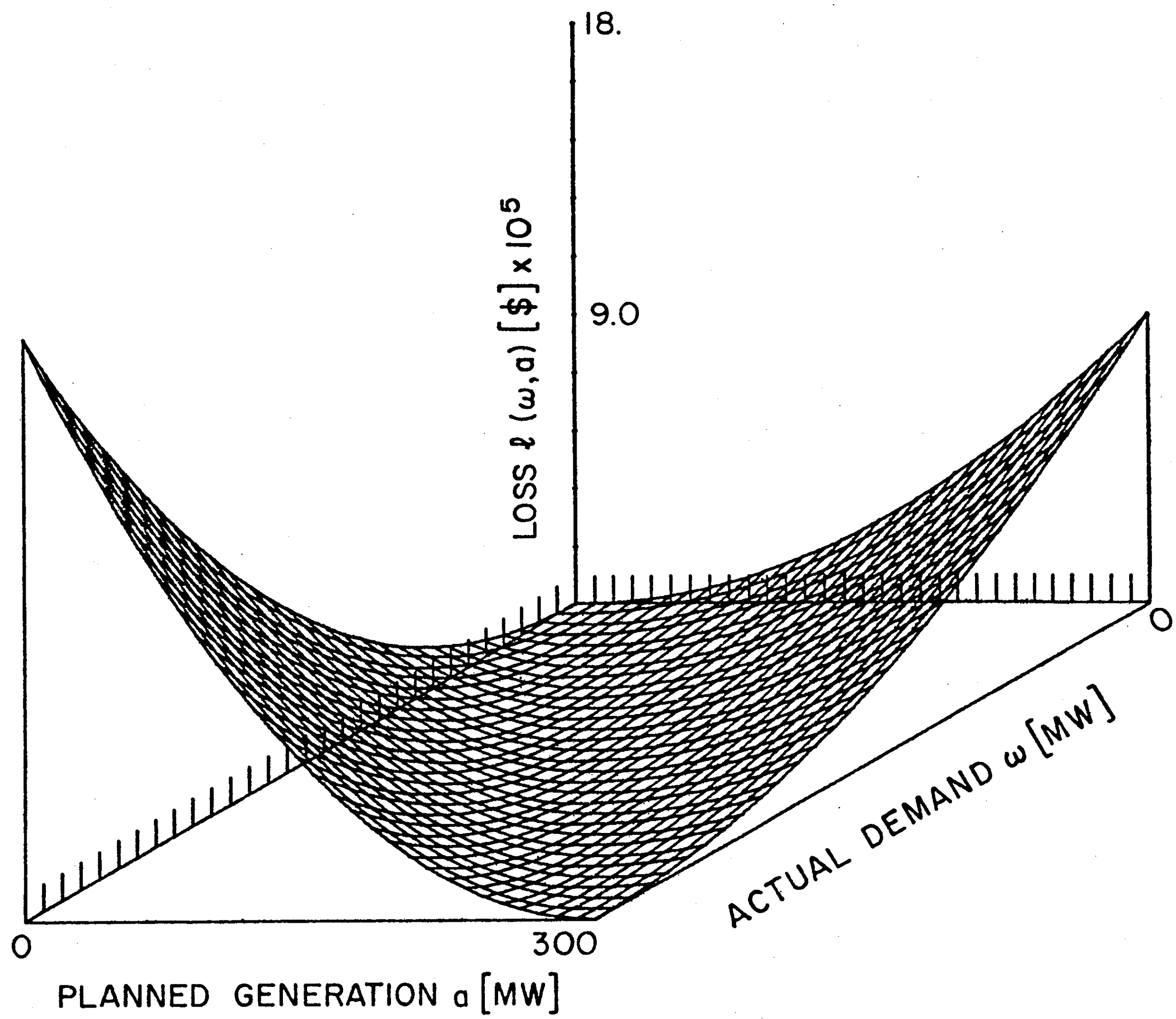

Figure 3.6 Loss Function

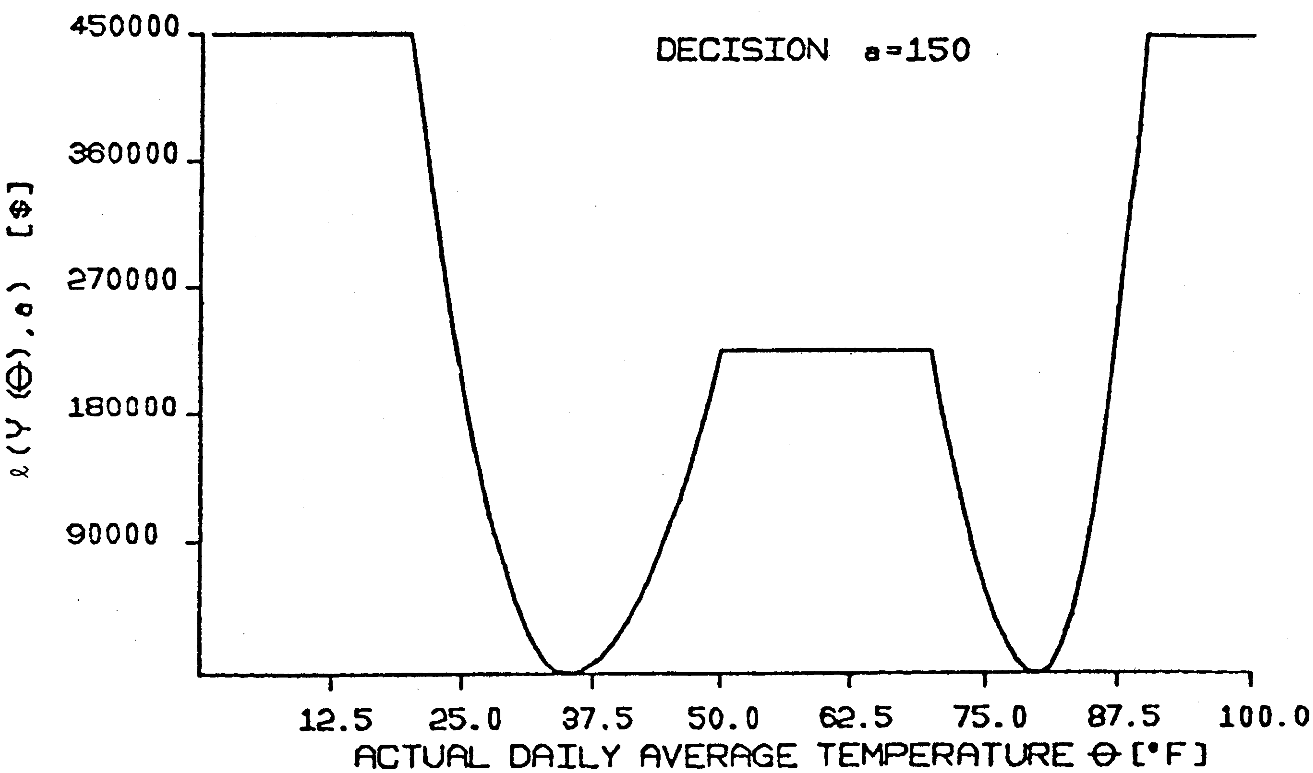

Figure 3.7 Loss as a Function of Temperature

Table 3.5 Comparative Evaluation of Three Forecast-Decision Systems

PERFORMANCE MEASURE		SYSTEM 1 CATEGORICAL FORECAST OPTIMAL DECISION	SYSTEM 2 CATEGORICAL FORECAST NONOPTIMAL DECISION	SYSTEM 3 NAIVE FORECAST NONOPTIMAL DECISION
EXPECTED DAILY LOSS [$]		RC = 12327	$\overline{RC}$ = 22318	$\overline{RN}$ = 61294
VALUES [$]				
potential value	PV	38889	38889	38889
optimal value	OV	26562	26562	0
actual value	AV	26562	16571	-22405
EFFICIENCIES				
forecast efficiency	FE	.683	.683	0.0
decision efficiency	DE	1.0	.624	$-\infty$
total efficiency	TE	.683	.426	-.576
EXPECTED OPPORTUNITY LOSSES [$]				
forecast system	FOL	12327	12327	38889
decision system	DOL	0	9991	22405
total system	TOL	12327	22318	61294

value of categorical forecasts is OV1 = 26562 \$/day, which is approximately 9.7 million dollars per year. If the forecasts were perfect, they would be worth PV1 = 38889 \$/day, or roughly 14.2 million dollars per year. The difference equal to FOL1 = 12327 \$/day, or 4.5 million dollars per year, constitutes an upper bound on the economically justified investments towards improving the quality of the forecasts, whose current efficiency FE1 = 68.3%. We can indisputably conclude that categorical temperature forecasts are of economic consequence.

Inspection of Table 3.5 reveals that System 3 is quite inferior to Systems 2 and 1, and System 2 is inferior to System 1. In System 2, the use of a nonoptimal decision procedure which neglects forecast uncertainty costs the utility, on the average, DOL2 = 9991 \$/day, which is approximately 3.6 million dollars annually. In System 3, the use of a nonoptimal decision procedure which neglects natural uncertainty in the daily average temperature costs the utility DOL3 = 22405 \$/day, or 8.1 million dollars per year. Clearly, in both systems there are expected opportunity losses due to suboptimal decisions. Categorical forecasts, however, mitigate these losses substantially. That is, if a utility employing naive forecasts and making nonoptimal decisions (System 3) were provided categorical forecasts and began to use them, although in a nonoptimal manner (System 2), then the utility would realize a benefit equal to AV2-AV3 = 16571 + 22405 = 38976 \$/day, or 14.2 million dollars annually. Interestingly, the value of categorical forecasts, in relation to naive forecasts, is higher for nonoptimal decision procedures (38976 \$/day) than for optimal decision procedures (26562 \$/day).

Analysis of the Bayes risk and decision functions for System 1 aids our understanding as to the effect each element of the decision model has upon the results. The effect of the mixed density, f_t, and the asymmetrical load operator is reflected in a plot of the Bayes risk function, LC, shown in Figure 3.8. For numerical computations, the range of categorical forecasts is constrained to be from 5 to 100 [^{o}F] since the maximum and minimum actual daily average temperatures range between 25 and 80 [^{o}F] and the error is constrained to lie within $\pm$ 20 [^{o}F]. It is easy then to intuit that local minima occur at t equal to 5 and 100 [^{o}F] as these forecasts are quite improbable. The local minimum at approximately 63.5 [^{o}F] is attributed to the fact that when $t = 63.5$ [^{o}F] the probability that the actual temperature is less than t_b or greater than t_c is quite small and thus the actual load $\omega = \Psi(\theta)$ is virtually always 0 MW; a decision a = 0 MW incurs losses only during the infrequent events $\theta < t_b$ or $\theta > t_c$. Interestingly, the local minimum is not at $(t_c - t_b)/2 = 60$. This asymmetry is due mainly to the Bayesian processing of forecasts: the likelihood function (which is only slightly skewed about t and has a spike at t) is weighted by the climatological prior whose mean is $m_\theta = 48.95$. As a result, the posterior density is decisively skewed about t in the direction of m_θ. We see this in a comparison of $f_{\theta|t=60}$ and $f_{\theta|t=65}$ (Figure 3.9) where more of the posterior density lies outside 50 to 70 [^{o}F] interval when $t = 60$ than when $t = 65$. This bears witness to the fact that $LC(t=60) > LC(t=65)$.

An asymmetry of similar nature is exhibited by the two local maxima at 31.5 and 83 [^{o}F] (the median temperatures of the sloping portions of the load operator are $(t_b - t_a)/2 = 35$ and $(t_d - t_c)/2 = 80$).

The two discontinuities at approximately 26 and 81 [^{o}F] are ascribed to the inclusion and exclusion of the spike, p, in the calculation of LC. This is purely a numerical phenomenon due to the truncation of the

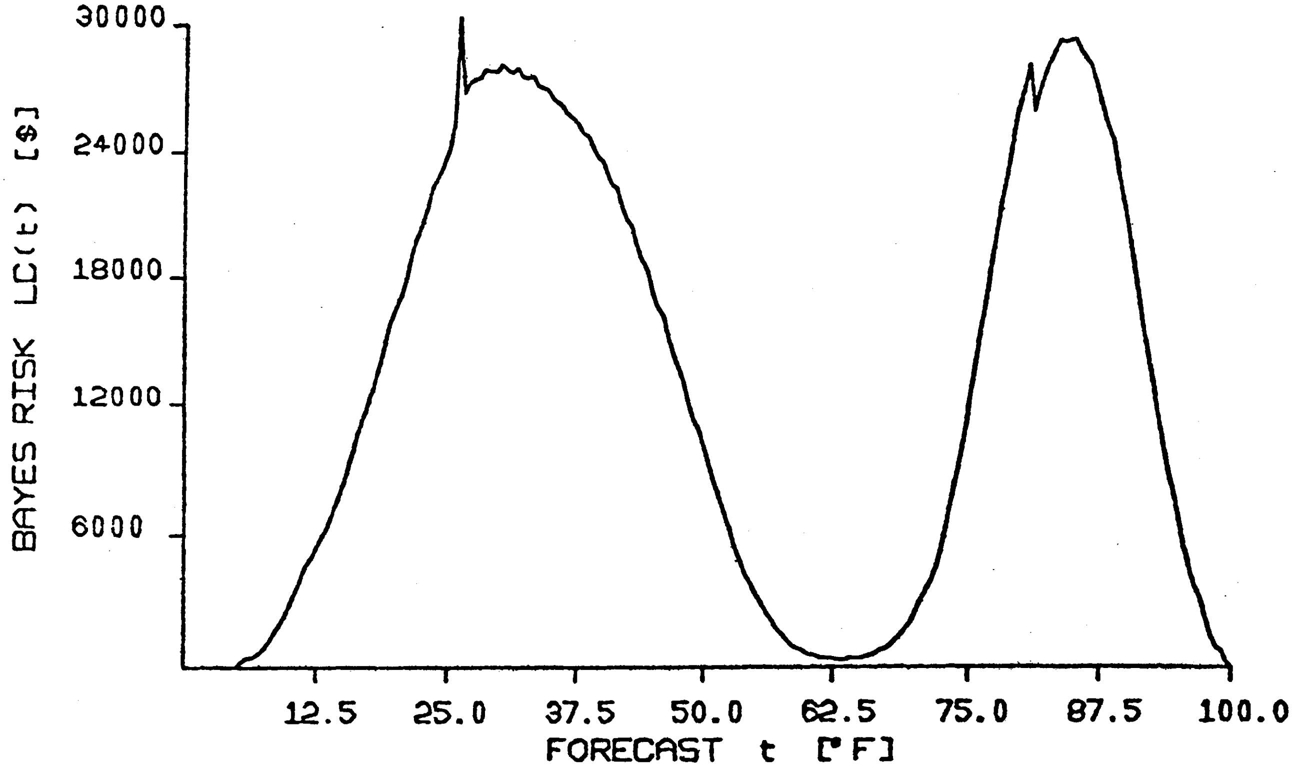

Figure 3.8 Bayes Risk Function

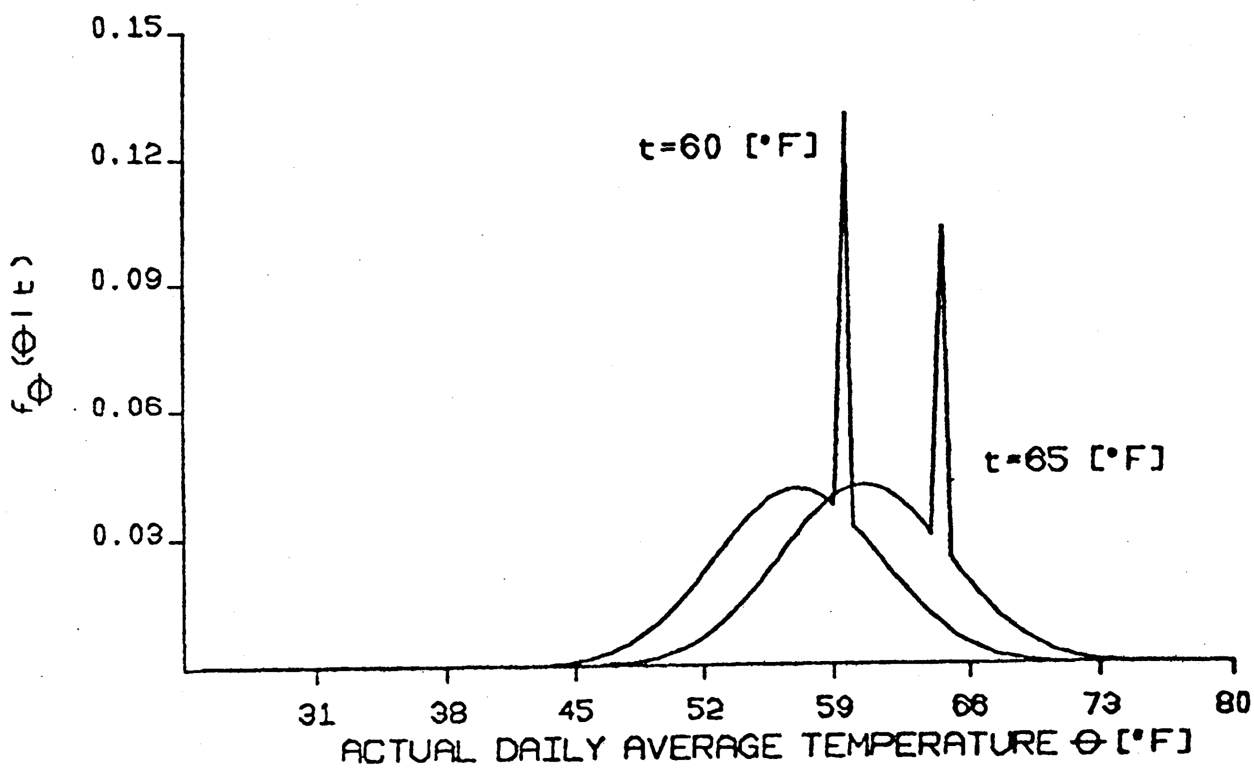

Figure 3.9 Comparative Plot of Posterior Density f_θ Given Two Categorical Forecasts

space Θ for the purpose of numerical execution of Bayes theorem and numerical integration.

Figure 3.10 illustrates the Bayes decision function δ^*; given a categorical forecast of the daily average temperature, t, $a^* = \delta^*(t)$ is the decision minimizing the risk. Although the function appears to be symmetric about 63.5 [^{o}F], careful examination reveals that the left-hand side ascends more precipitously than the right-hand side. This again is due to the fact that the climatological density is symmetric about 48.95 [^{o}F]; this causes the optimal decisions to be closer to the load operator when t is near m_θ and more gently sloped when t is much greater or less than m_θ. Figure 3.10 provides the operator with a relatively simple, yet optimal, decision rule.

3.5 Sensitivity Analyses

We seek now to perturb the precision, $\tau_{t|\theta}$, and the reliability, p, of f_t, the loss function, ℓ, and the load operator, Ψ, to obviate the need for speculation as to the system behavior under various and sundry conditions.

3.5.1 Precision of the Forecasts

Variation of the precision of f_t affords explicit quantification of the performance of the forecast-decision system as a function of the skill of the forecaster. Logically, the skill or precision should be positively correlated with the forecast reliability, but since there is no data to confirm this assertion, we vary the precision $\tau_{t|\theta}$ while holding the reliability p constant.

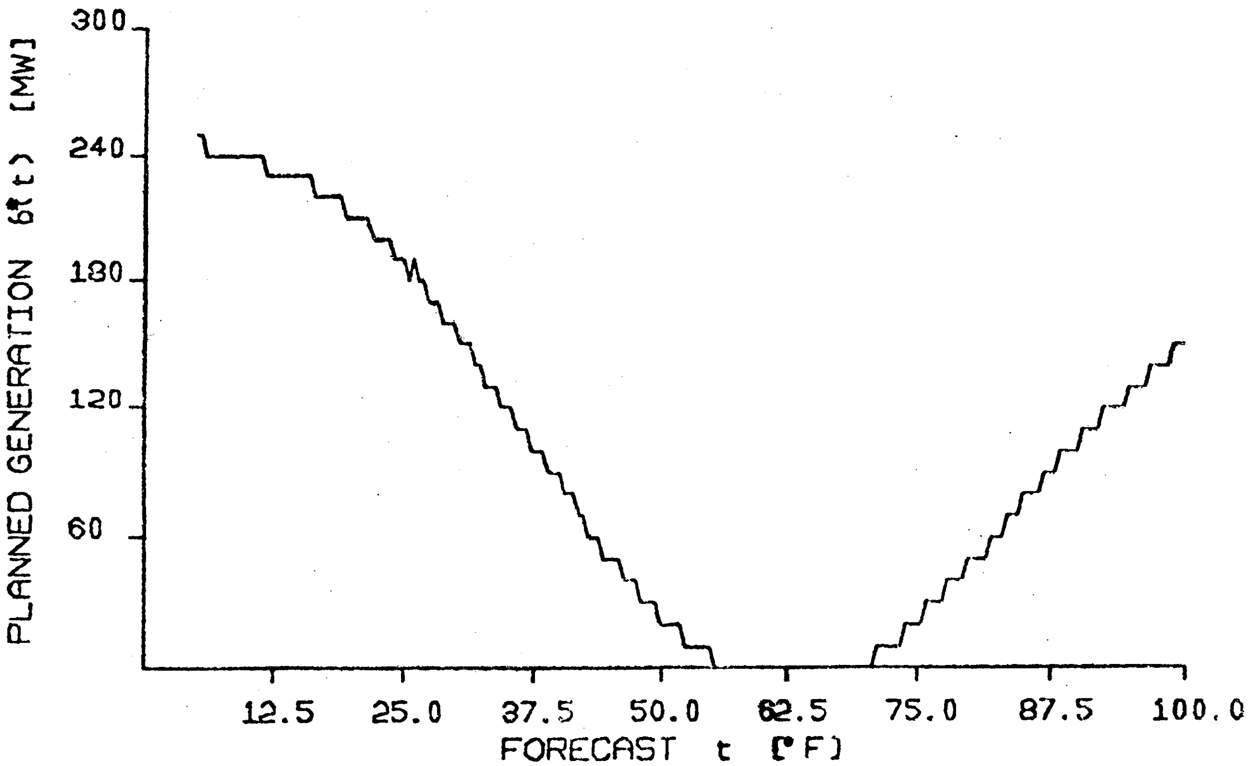

Figure 3.10 Bayes Decision Function

We expect that as $\tau_{t|\theta}$ decreases, the total efficiencies of Systems 1 and 2 will decrease via the argument expounded in Appendix D. This phenomenon is demonstrated in Figure 3.11. System 1 always outperforms Systems 2 and 3, whereas it appears that System 2 dominates System 3 for $\tau_{t|\theta} > \tau_\theta$. Figure 3.12 shows that as $\tau_{t|\theta}$ increases, it becomes less and less important to use an optimal decision procedure (DE2 approaches 1 as $\tau_{t|\theta}$ increases). The decision efficiency of System 3 is not shown as it is always negatively infinite.

Note that the forecast efficiency of Systems 1 and 2 is represented exactly by the total efficiency of System 1.

A graph of the total expected opportunity loss, TOL, versus forecast precision, $\tau_{t|\theta}$, (Figure 3.13) is informative in that the efficiencies are converted into dollars and thus proposals to increase the precision of the forecasts at a given cost, C, can be compared against the difference between the total expected opportunity losses of the *de facto* system and the improved system. If C is greater than this difference, the proposal should be rejected. We can conclude that System 3 is preferable to System 2 when $\tau_{t|\theta} < \tau_\theta$ by a wide margin. Figure 3.14 depicts the decision expected opportunity loss for all three systems.

Figure 3.15 shows a plot of the Bayes risk function LC for $\tau_{t|\theta}$ = .01, .042 $[^{o}F]^{-2}$. As $\tau_{t|\theta}$ increases, all the maxima and minima of LC, save the discontinuities at roughly 26 and 81 $[^{o}F]$, are negatively shifted when $t < m_\theta$ and positively shifted when $t > m_\theta$. Given the observation that when $t_a < t < t_b$ or $t_c < t < t_d$, it is more difficult to make decisions than when $t < t_a$ or $t > t_d$, it is apparent that for large $\tau_{t|\theta}$ the maxima lie within the sloping portions of the load operator.

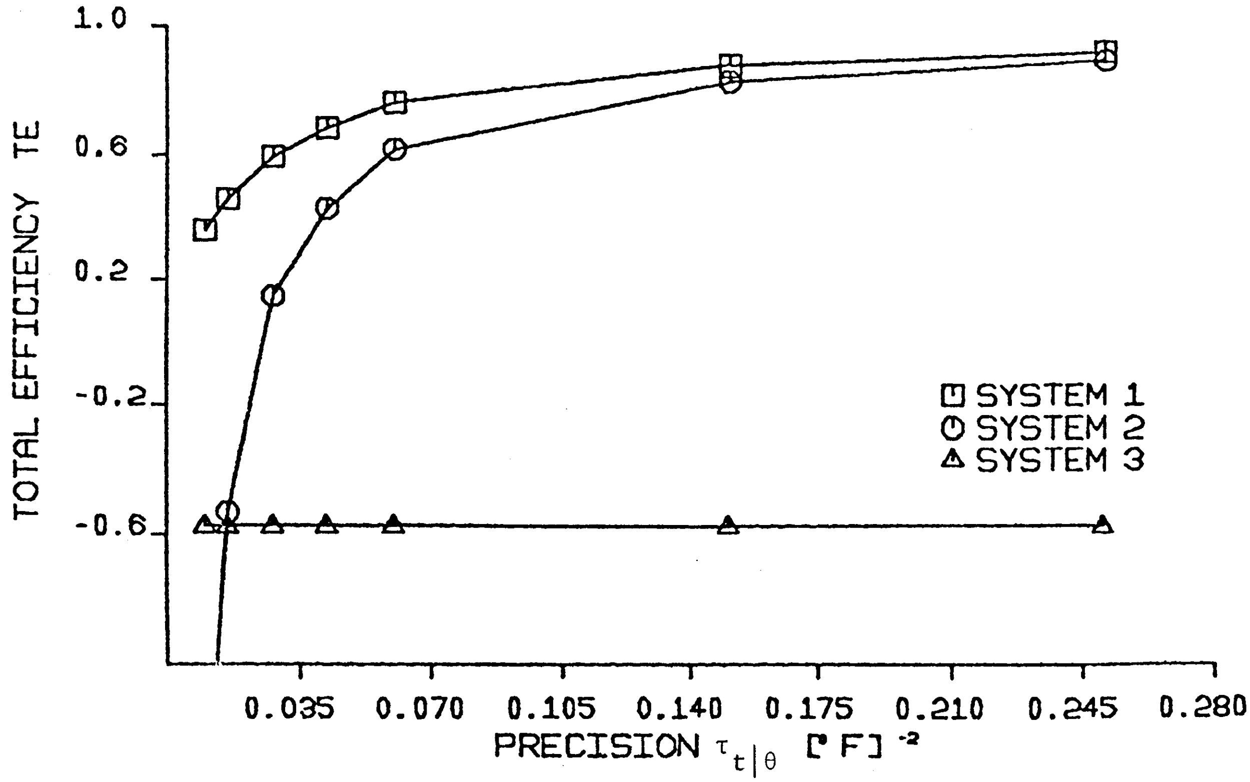

Figure 3.11 Total Efficiency versus Forecast Precision

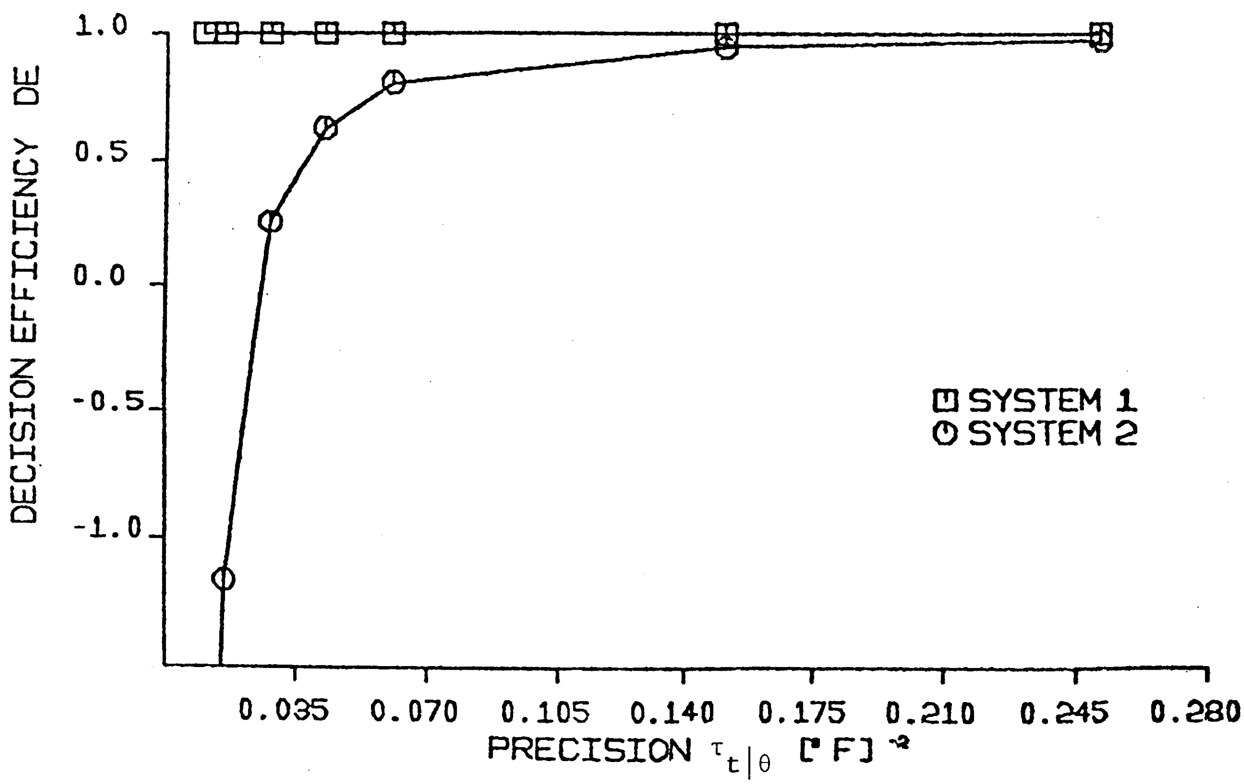

Figure 3.12 Decision Efficiency versus Forecast Precision

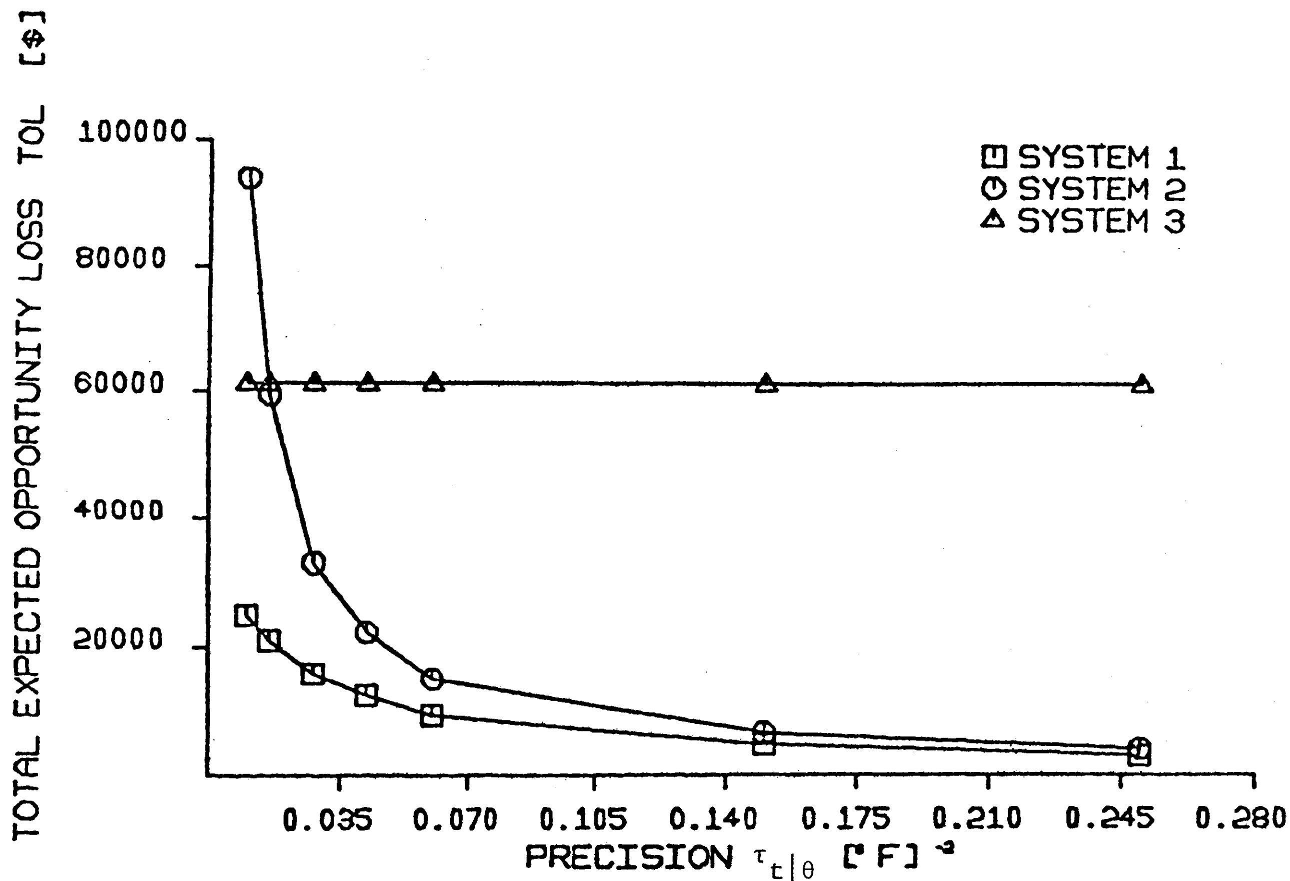

Figure 3.13 Total Expected Opportunity Loss versus Forecast Precision

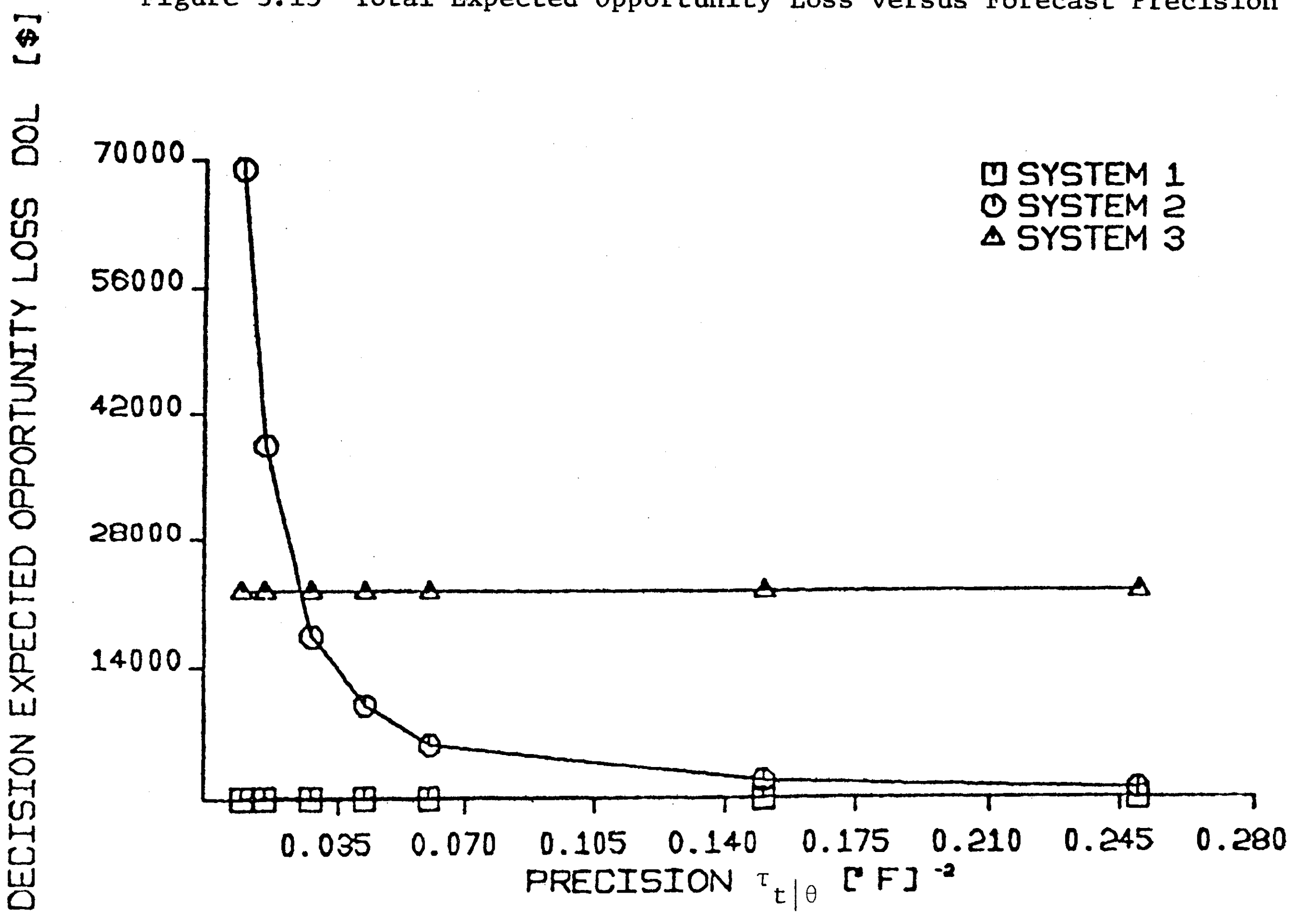

Figure 3.14 Decision Expected Opportunity Loss versus Forecast Precision

The left-hand side of the Bayes risk function when $\tau_{t|\theta} = .01$ dwarfs the Bayes risk function when $\tau_{t|\theta} = .042$ because of the subtle interaction between the load operator and the posterior density. Intuitively, one would argue that the left-hand side of LC should be less peaked than the right-hand side as $|\alpha_h| < |\alpha_c|$, but it must also be heeded that the prior density g_θ is symmetric about 48.95 [^{o}F], thereby lending more weight to forecasts from 20 to 50 [^{o}F] than from 70 to 90 [^{o}F]. This effect vanishes, however, as $\tau_{t|\theta}$ goes to infinity since perfect forecasts imply LC(t) = 0 for every $t \varepsilon \Theta$.

Figure 3.16 evinces some interesting properties of the Bayes decision function, δ^*, as a function of the forecast precision $\tau_{t|\theta}$. First, the decision function for a perfect forecast, t, is precisely the load operator $v = \Psi(t)$, which is quite logical under the assumption that the load operator, is exact. Second, the values of the optimal decisions for forecasts of extreme temperatures decrease as the precision decreases; in other words, the more uncertain the forecasts of extremes, the more "cautious" the decisions. Third, the minimum of δ^* is translated to the right as $\tau_{t|\theta}$ decreases for the same reason the maxima and minima of LC shift with decreasing $\tau_{t|\theta}$. As $\tau_{t|\theta}$ goes to zero, δ^* becomes more and more horizontal and centered about a^*, the decision that is the minimizer of LN.

3.5.2 Reliability of the Forecasts

The reliability, p, is here defined as the probability that $\theta = t$. In the basic evaluation p = .126. While perturbing the reliability we maintain a constant precision, $\tau_{t|\theta} = .042$ [^{o}F]$^{-2}$. This analysis is,

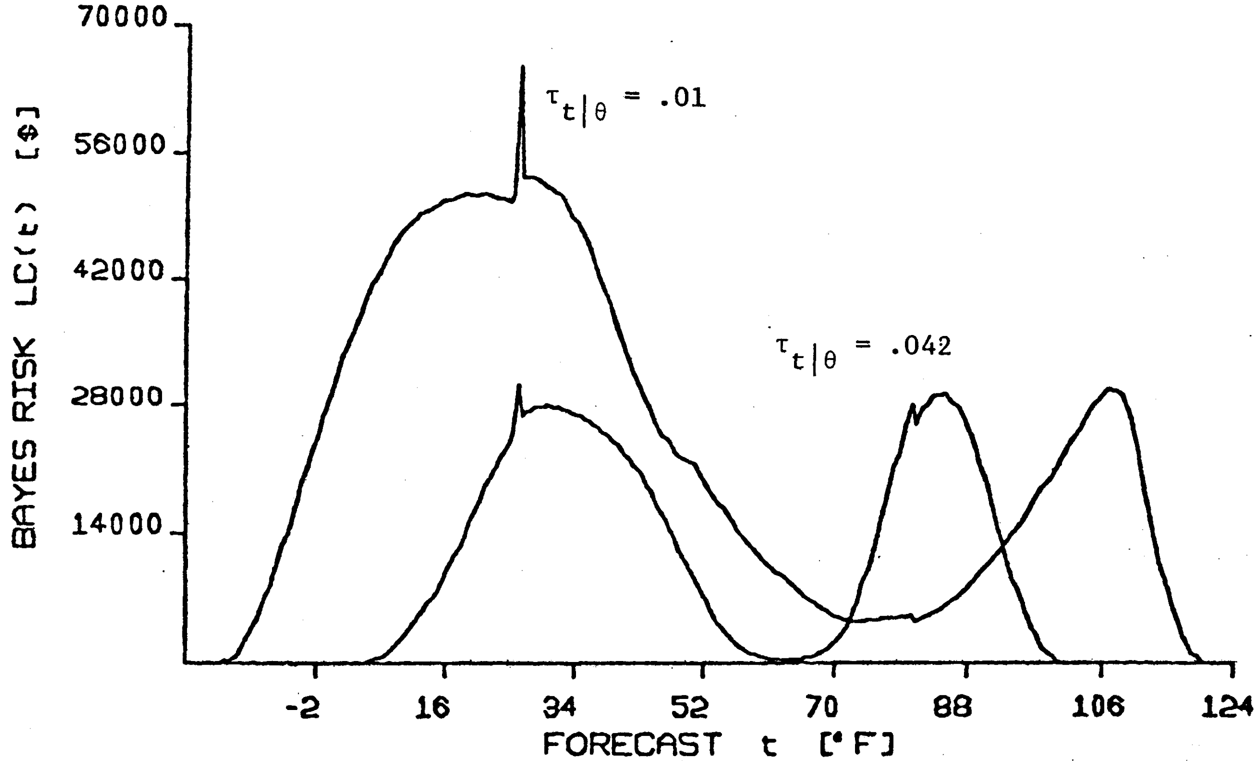

Figure 3.15 Bayes Risk Functions when $\tau_{t|\theta}$ = .01,.042 $[^{o}F]^{-2}$

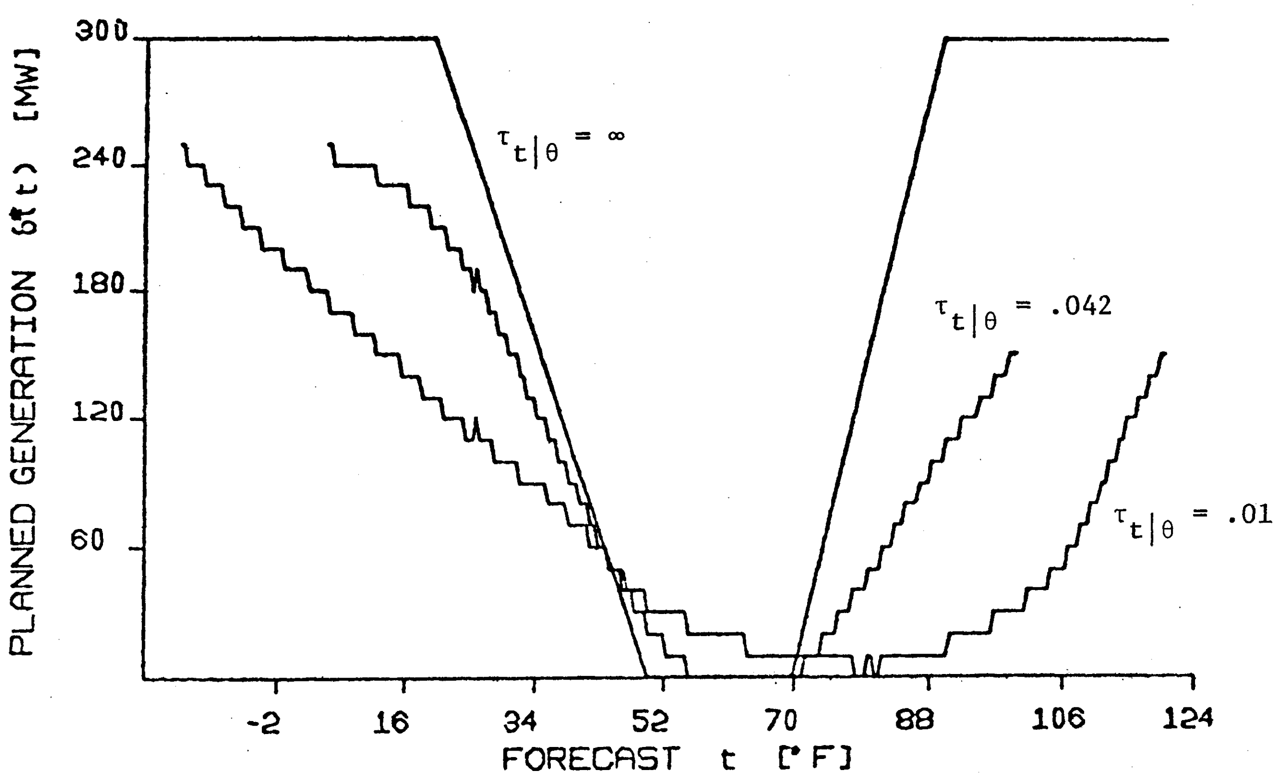

Figure 3.16 Bayes Decision Functions when $\tau_{t|\theta}$ = .01,.042,∞ $[^{o}F]^{-2}$

then, tantamount to examining the luck of a forecaster with invariant skill.

The assertion that the reliability was the cause of the discontinuity in the Bayes risk function (see Section 3.4.2) is corroborated in Figure 3.17, a plot of LC when $p = 0$, as the function is quite smooth. There are not, however, any other discernible distinctions between the two functions.

One would expect, *a priori*, that the forecast efficiency (equal to the total efficiency of System 1) would increase with p. But since $\tau_{t|\theta}$ was unaltered, s_n^2 was necessarily increased; consequently, the increase in forecast reliability was offset by a very dispersed normal density h_n. As a result, the forecast efficiency increases very slowly with p, as is shown in Figure 3.18. Not unexpectedly, the decision efficiency of System 2 monotonically decreases as p increases up to .75 (Figure 3.19). Thus, a nonoptimal user of forecasts would certainly prefer his forecaster to be unlucky but skillful rather than lucky and unskillful, unless his luck would be so great as to result in reliability exceeding .9. Similarly, for an optimal user we see that of the two properties of categorical forecasts, reliability and precision, it is far more valuable to be precise than to be reliable, up to a point.

This assessment is validated when comparing the total expected opportunity loss (Figures 3.13 and 3.20). In System 1, we see that as p increases the total expected opportunity loss decreases very slowly, whereas it summarily decreases as $\tau_{t|\theta}$ increases. Of course, in the limit, the two properties converge, i.e., a perfectly reliable forecast has

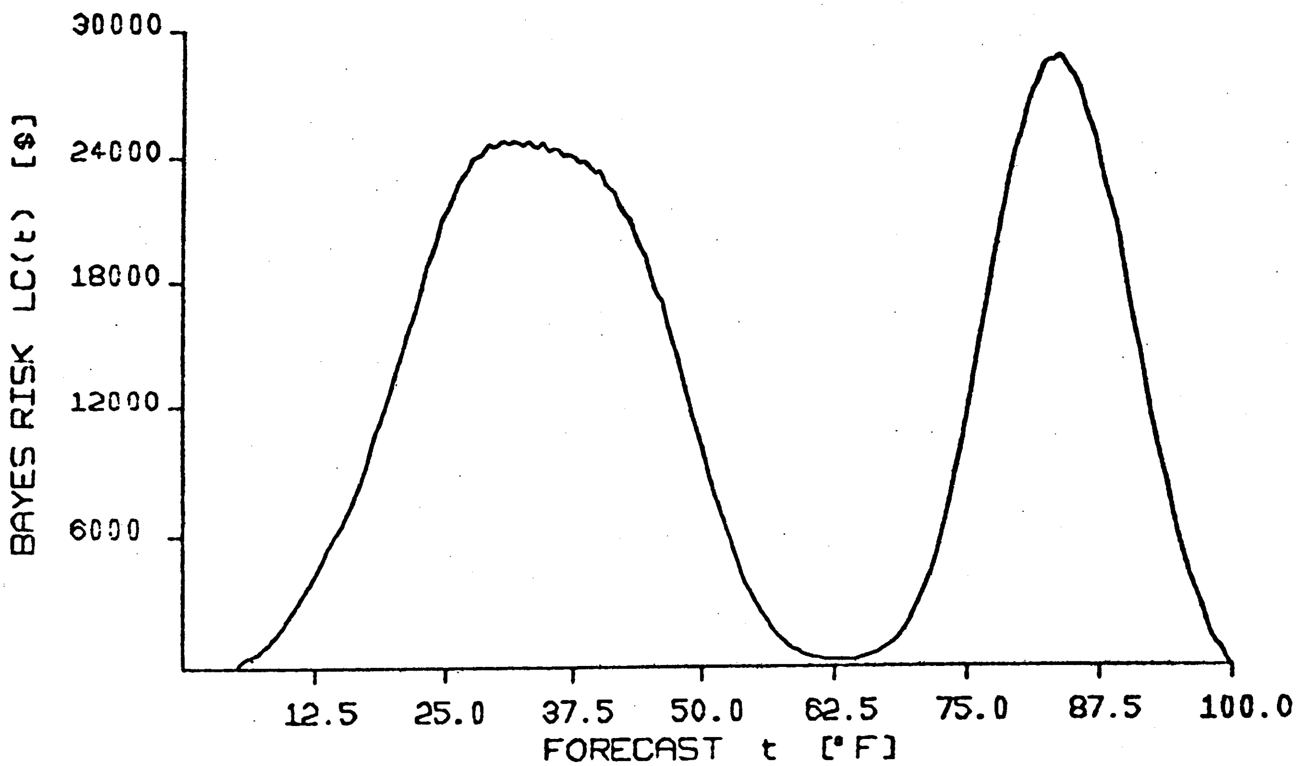

Figure 3.17 Bayes Risk Function when p = 0

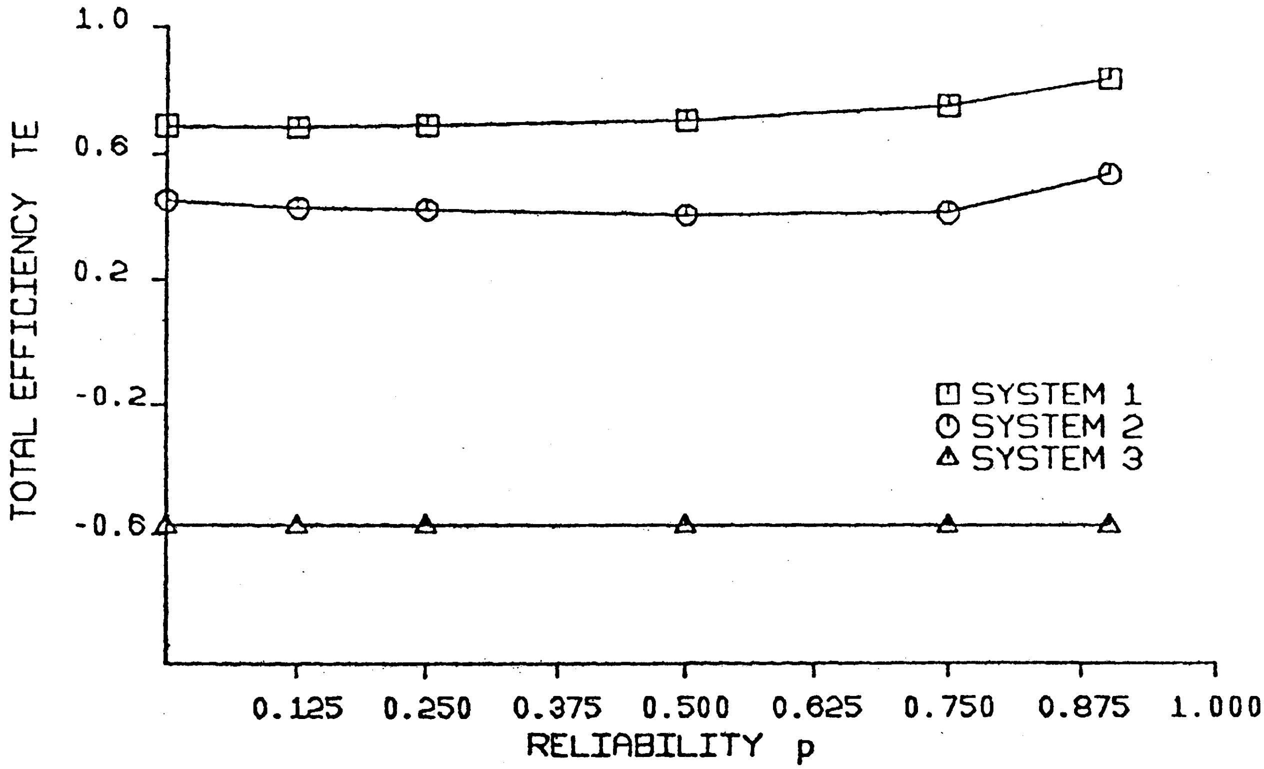

Figure 3.18 Total Efficiency versus Forecast Reliability

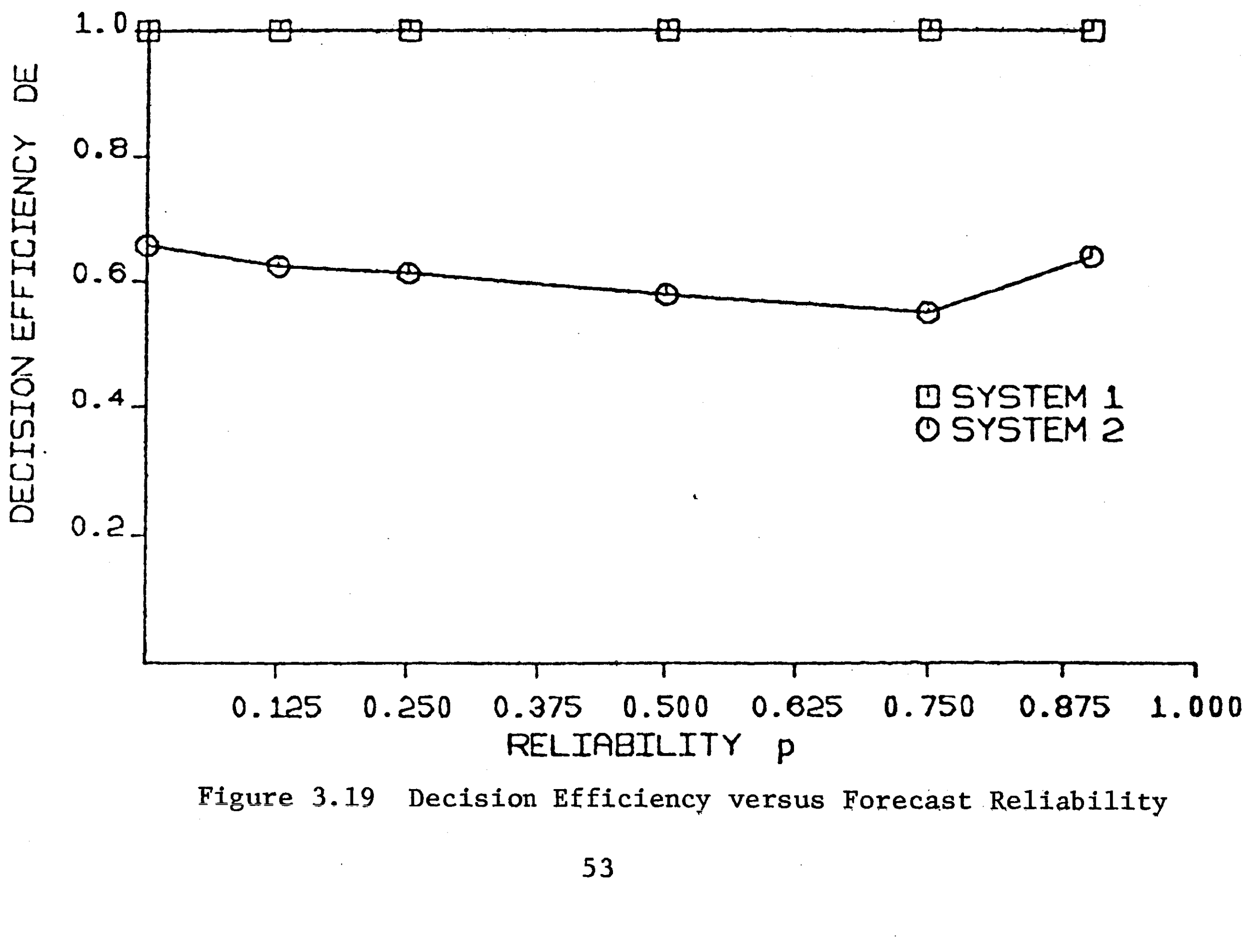

Figure 3.19 Decision Efficiency versus Forecast Reliability

infinite precision.

Examination of the decision expected opportunity loss (Figure 3.21) reveals that as p increases from 0 to .75, DOL2 increases, as it does when $\tau_{t|\theta}$ decreases. Thus given a fixed precision of the forecasts, a non-optimal user minimizes his opportunity loss when the forecasts are less reliable, i.e., when the mixed density f_t becomes more continuous.

3.5.3 Loss Ratio of Over to Under Generation

The sensitivity analysis with respect to the ratio c_o/c_u serves to elucidate the importance of exact quantification of these parameters, which is universally a difficult task. In the experiment, we fixed $c_u = 20$ [\$/MW^2] and varied c_o from 0 to 20 [\$/MW^2]. Figure 3.22 portrays the total efficiency versus c_o/c_u.

At $c_o/c_u = 0$, the total efficiency of System 1 is not defined since RN = RC = 0, and

$$TE1 = \frac{RN - RC}{RN - RF} = 1 - \frac{RC}{RN} .$$

The total efficiencies of Systems 2 and 3 go to negative infinity since

$$TE2 = \frac{RN - \overline{RC}}{RN - RF} = 1 - \frac{\overline{RC}}{RN} ,$$

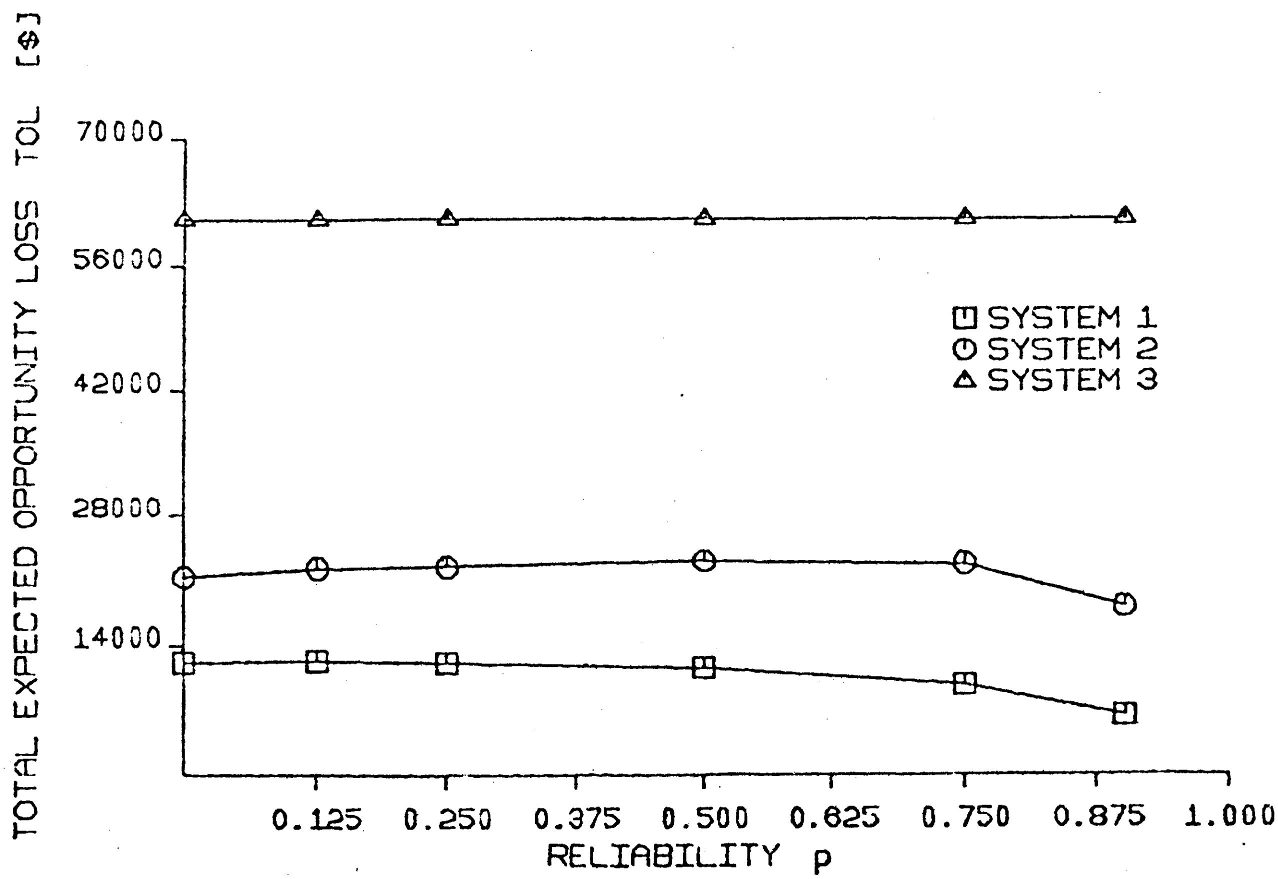

Figure 3.20 Total Expected Opportunity Loss versus Forecast Reliability

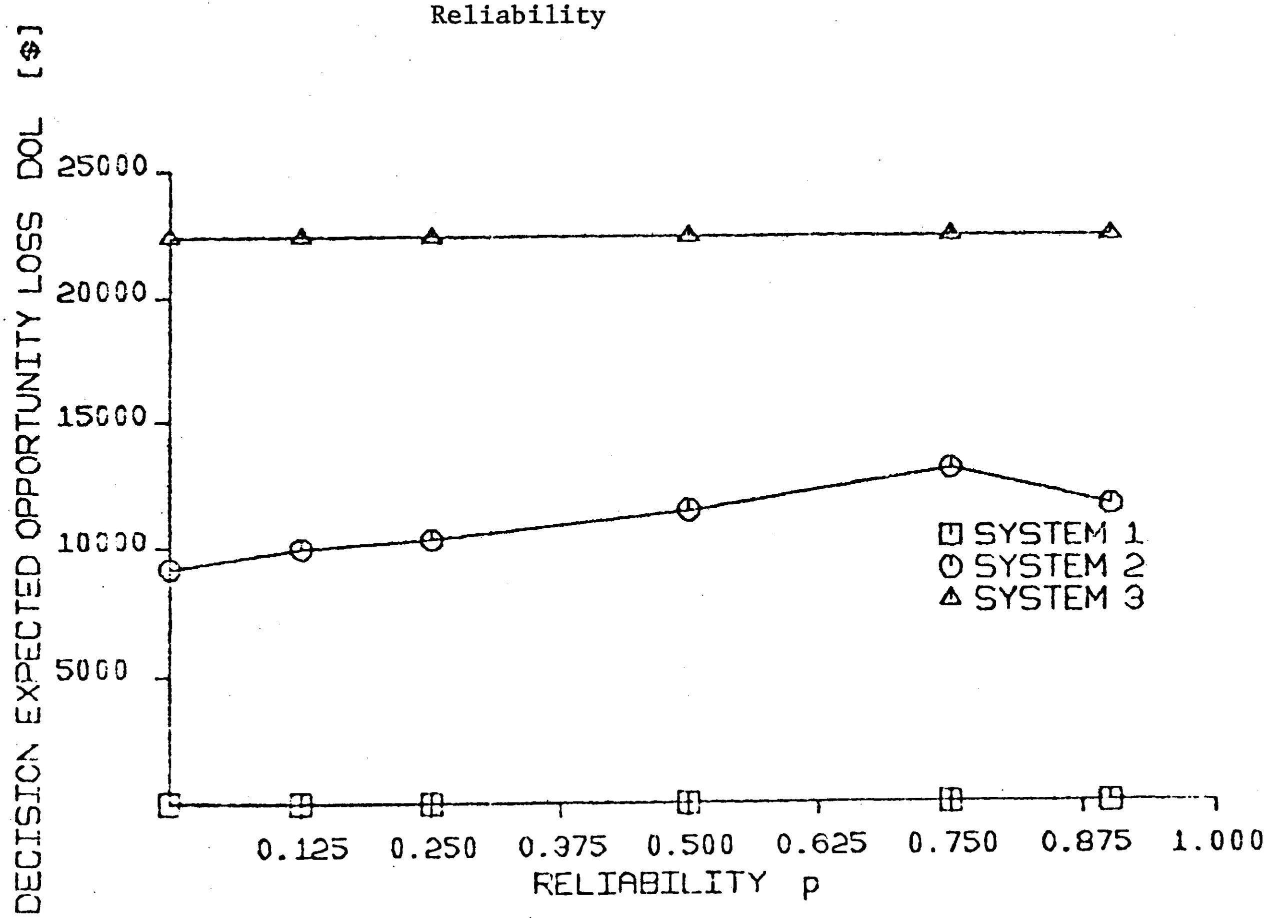

Figure 3.21 Decision Expected Opportunity Loss versus Forecast Reliability

and

$$TE3 = \frac{RN - \overline{RN}}{RN - RF} = 1 - \frac{\overline{RN}}{RN} ,$$

where $\overline{RC}$ and $\overline{RN}$ are both greater than zero. Otherwise, as c_o/c_u increases from zero to one, TE1 decreases slightly because losses are generally higher. TE2 and TE3 increase because as the loss function approaches a symmetric form, decision making becomes "easier." As c_o/c_u approaches 1 the vitality of accounting for the forecast errors diminishes (Figure 3.23).

The total expected opportunity loss of System 1, which is also the forecast expected opportunity loss for Systems 1 and 2, represents the value of a perfect forecast; we observe that as c_o/c_u approaches 1, the value of a perfect forecast increases (Figure 3.24). Improvements in the forecasting scheme should then be emphasized when ℓ is relatively symmetric in lieu of improvements in the decision procedure. This result is qualified in Figure 3.25, a plot of the decision expected opportunity loss versus c_o/c_u.

3.5.4 Heating/Cooling Gap of the Load Operator

Finally, we seek to examine the effect of lessening the difference of $(t_c - t_b)$ in the load operator, Ψ. A comparison of LC between the basic evaluation when $t_c - t_b = 20$ and the case when $t_c - t_b = 0$ is made in Figure 3.26. The total efficiencies are shown in Figure 3.27. As $(t_c - t_b)$ increases, one would expect TE1 and TE2 to increase because, simply, the range of the temperature insensitive to the load is longer. But then $|\alpha_h|$ and $|\alpha_c|$ become larger (i.e., the sloping portions are steeper) and this has a counterbalancing effect. It is this effect that

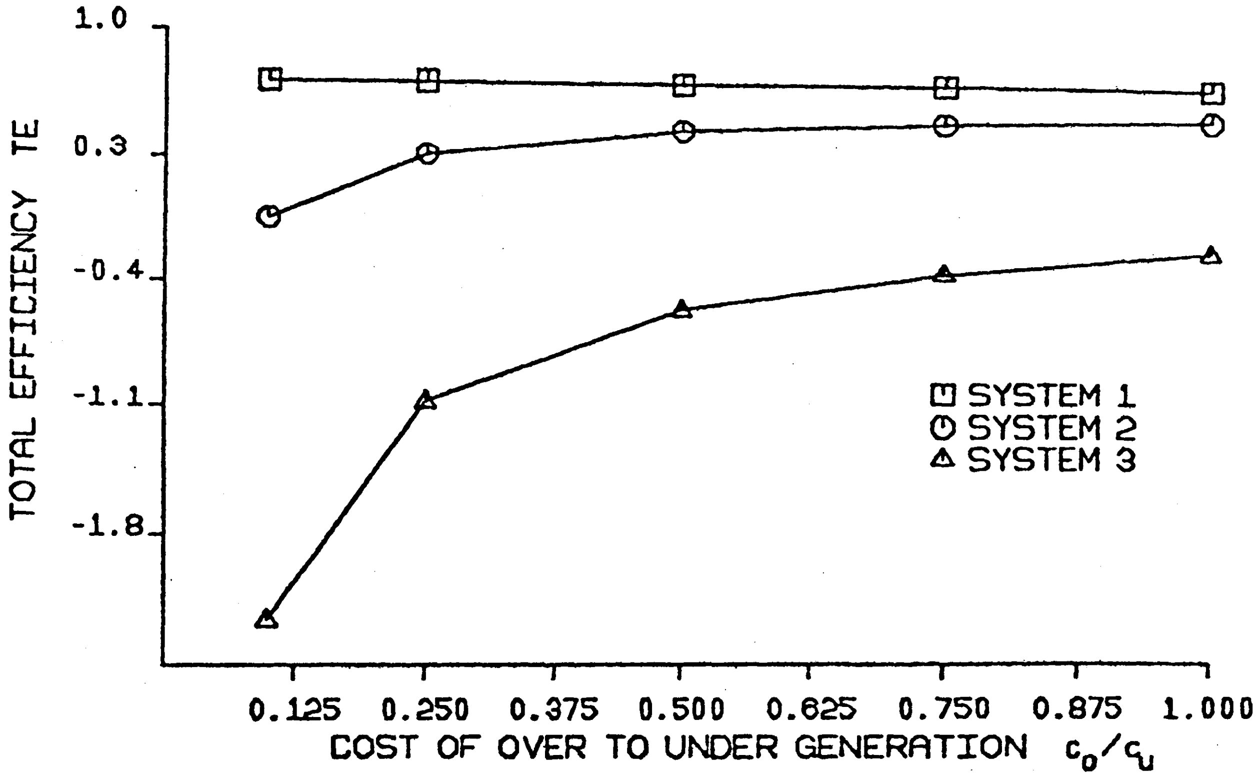

Figure 3.22 Total Efficiency versus Loss Ratio of Over to Under Generation

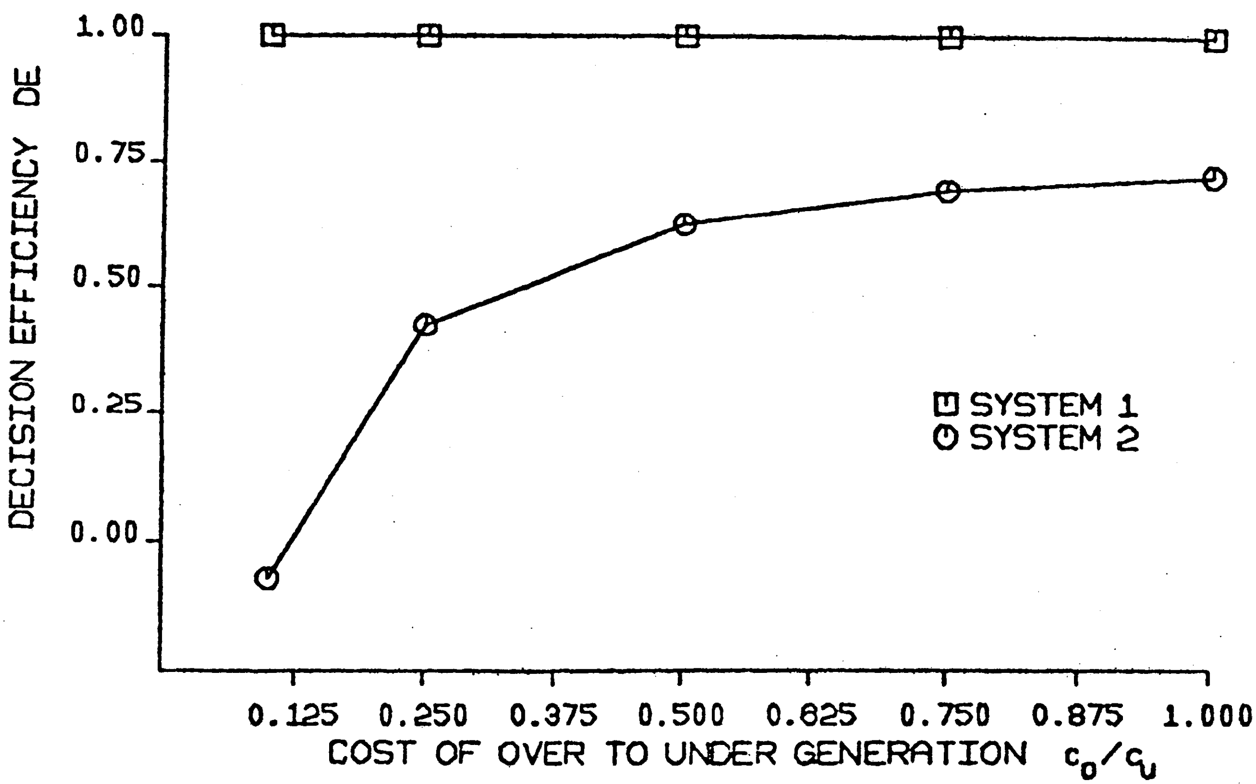

Figure 3.23 Decision Efficiency versus Loss Ratio of Over to Under Generation

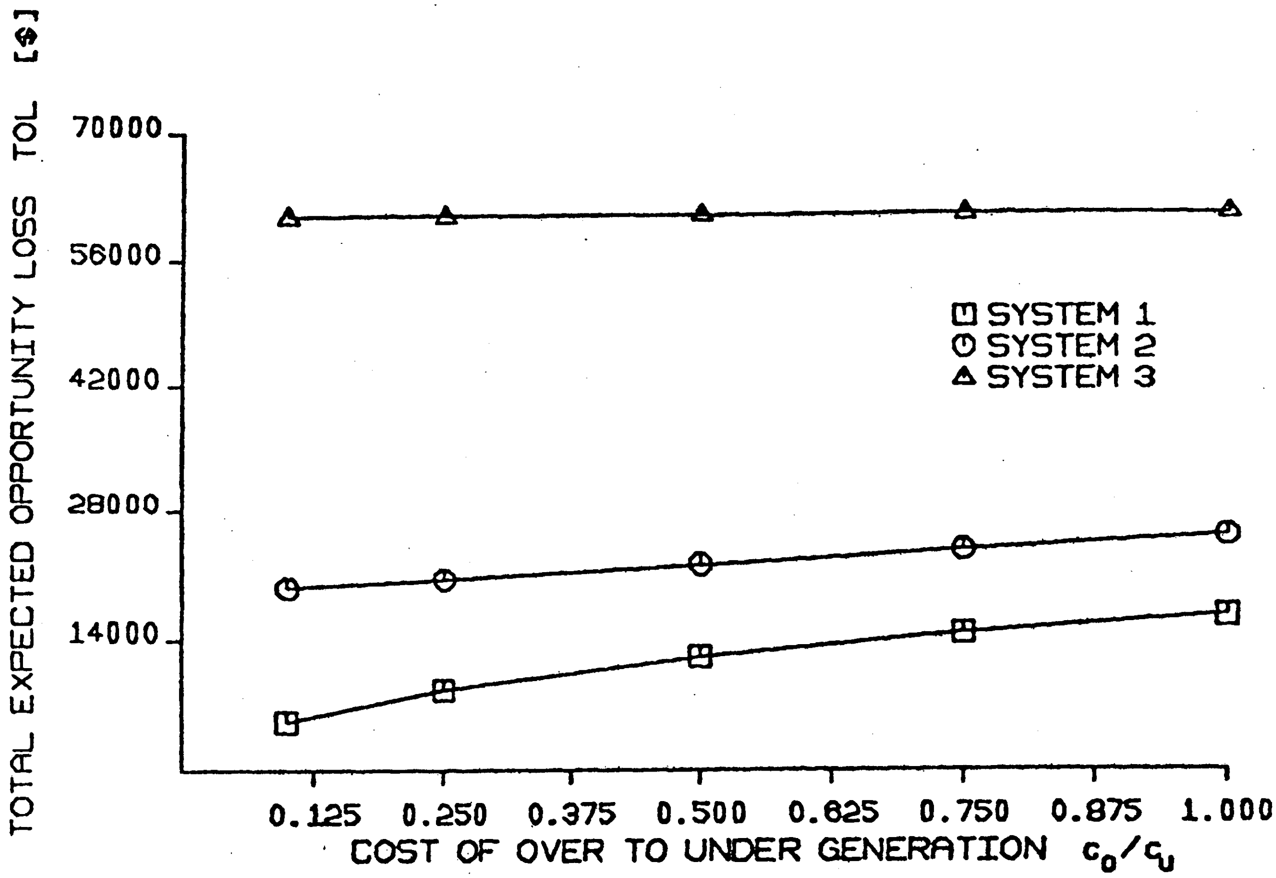

Figure 3.24 Total Expected Opportunity Loss versus Loss Ratio of Over to Under Generation

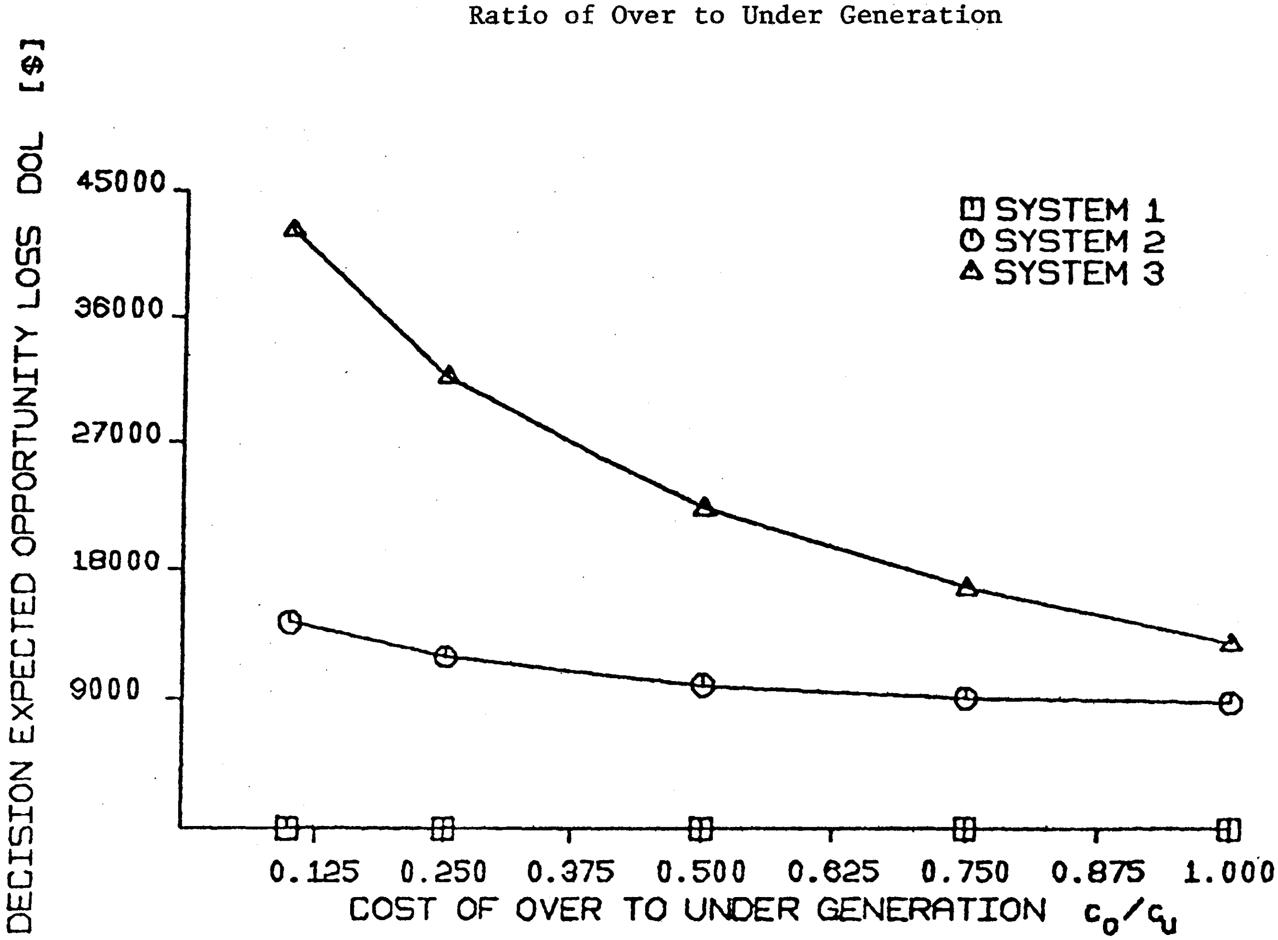

Figure 3.25 Decision Expected Opportunity Loss versus Loss Ratio of Over to Under Generation

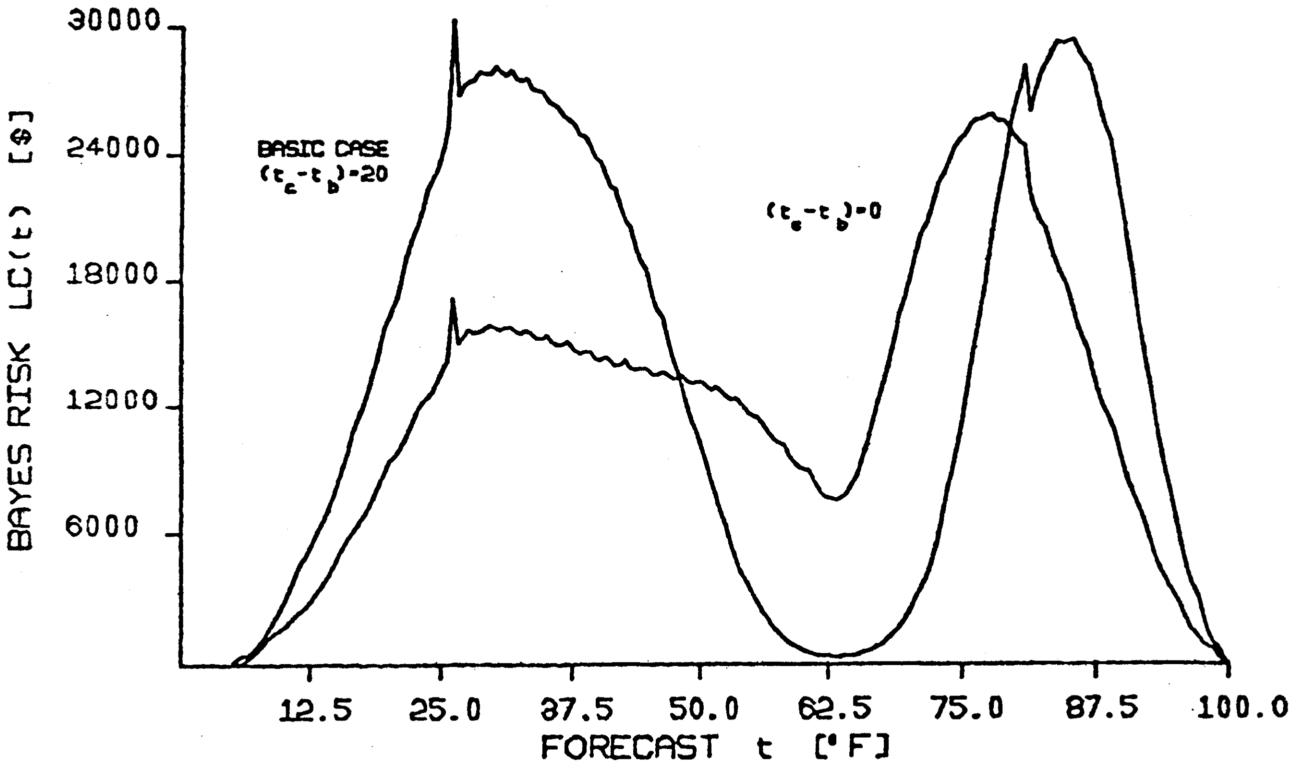

Figure 3.26 Bayes Risk Functions for the Basic Case and for Heating/Cooling Gap Equal to Zero

apparently dominates the behavior of System 3.

The value of a perfect forecast, TOL1 = FOL1, is at a minimum when the heating/cooling gap is either 0 or 20 [^{o}F] (Figure 3.28).

3.6 Summary

A simple decision model for scheduling daily average power generation on the basis of a categorical temperature forecast has been developed. A set of measures (including values, efficiencies, and expected opportunity losses) has been defined for evaluation of the performance of a forecasting scheme, a decision procedure, and a total forecast-decision system.

It has been demonstrated that categorical forecasts of the daily average temperature are, indeed, more valuable than naive (climatological) forecasts when used optimally, i.e., when the forecast uncertainty is accounted for via a Bayesian method. On the other hand, if the same categorical forecasts are used nonoptimally, i.e., under the false assumption that they are error-free, then large opportunity losses are likely to be incurred. Consequently, the value of the categorical forecasts may be negative.

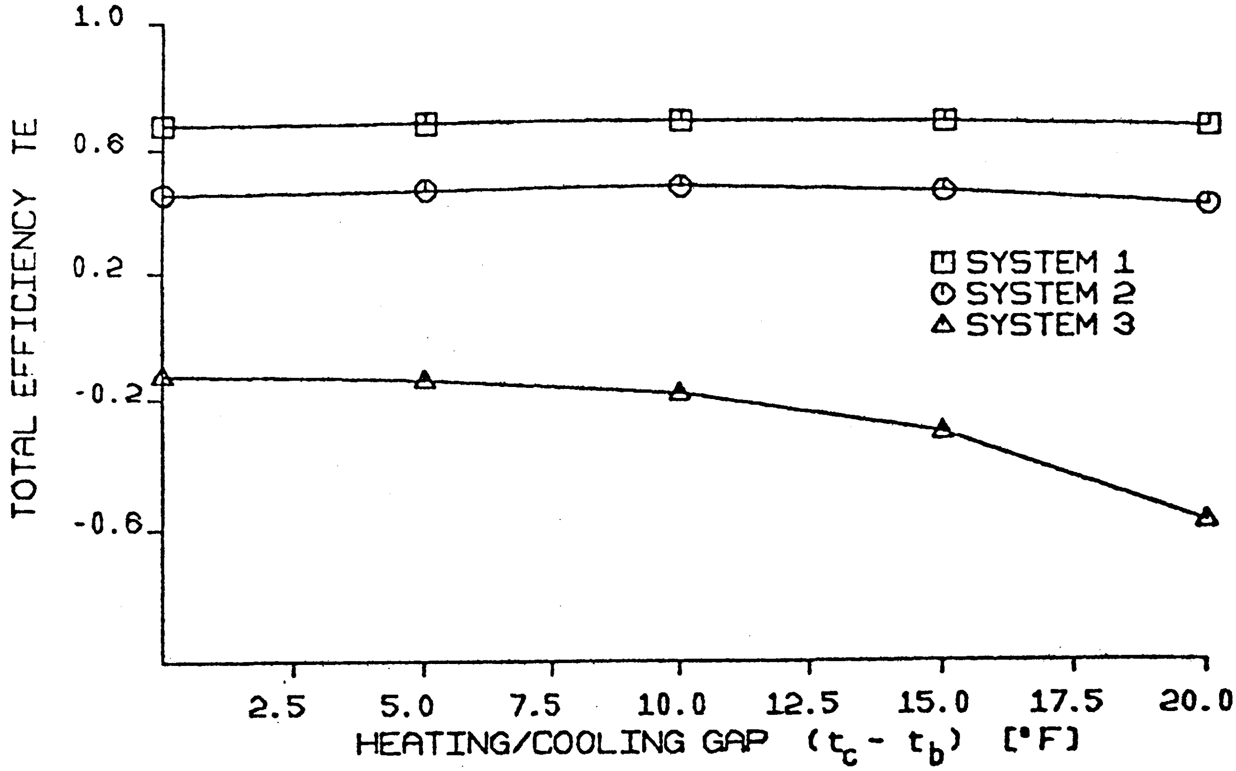

Figure 3.27 Total Efficiency versus Heating/Cooling Gap

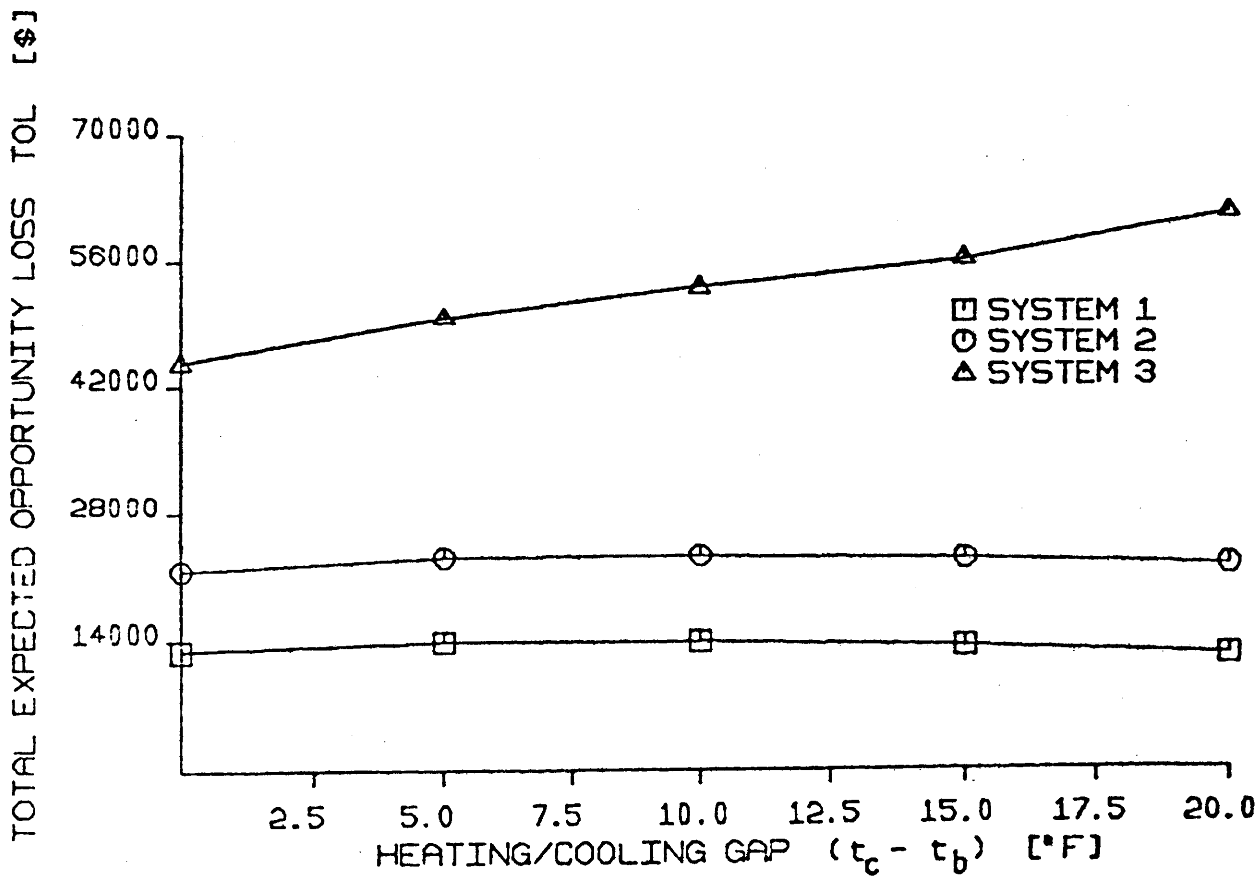

Figure 3.28 Total Expected Opportunity Loss versus Heating/ Cooling Gap

CHAPTER 4

VALUE OF PROBABILISTIC FORECASTS

4.1 Decision Model

An economic evaluation of and comparison between probabilistic, categorical, and naive forecasts will be made in the context of the electric power generation problem described in Section 3.1. Some modifications of the decision model are, however, necessary in order to supplant the categorical forecasts with the probabilistic forecasts: two elements, the parameter space, M, and the information scheme, R , need to be added to the model, and the forecasting scheme, Φ, must be redefined. The information now available to the decision maker is a subjective forecast density defined by a vector of parameters; the posterior density of the actual temperature is conditioned upon this forecast density. Formally, the changes in the decision model are as follows.

Parameter space M. $\underline{\mu} \in M$ is a vector of statistics. Typically, $\underline{\mu}$ consists of the meteorologist's responses to the queries specific to the encoding technique used to construct the subjective probabilistic forecast. $\underline{\mu}$ may also be a function of the meteorologist's responses, and, in particular, this function may constitute a sufficient statistic of fixed dimension (DeGroot, 1970, p. 155) for a family of densities R .

Information scheme R . $R = \{\rho_\theta(\cdot|\underline{\mu}) : \underline{\mu} \in M\}$, where $\rho_\theta(\cdot|\underline{\mu})$ is the subjective forecast density of the actual daily average temperature, θ, constructed on the basis of a vector of statistics $\underline{\mu} \in M$. The exact nature of this construction is dependent upon the probability elicitation technique. The procedure used to construct $\rho_\theta(\cdot|\underline{\mu})$ on the basis of the variable-width, fixed-probability elicitation technique is described in Section 4.3.

Forecasting scheme Φ. A forecasting scheme is characterized by a family of densities $\Phi = \{\phi_{\underline{\mu}}(\cdot|\theta) : \theta \varepsilon \Theta\}$, where $\phi_{\underline{\mu}}(\cdot|\theta)$ is a conditional density of the statistic $\underline{\mu}$, given the actual daily average temperature $\theta \varepsilon \Theta$. For any $\underline{\mu} \varepsilon M$, $\phi_{\underline{\mu}}(\underline{\mu}|\cdot)$ represents the likelihood function of θ. $\phi_{\underline{\mu}}$ is to be estimated from a record of probabilistic forecasts, $\rho_\theta(\cdot|\underline{\mu})$, and actual temperatures, θ. The source and construction of $\phi_{\underline{\mu}}$ is described in Section 4.3.

Bayesian information processor. For any probabilistic forecast $\rho_\theta(\cdot|\underline{\mu})$ of θ produced by a known forecasting scheme Φ, the prior density g_θ of θ can be revised to obtain a posterior density $\phi_\theta(\cdot|\underline{\mu}) = \phi_\theta(\cdot|\rho_\theta(\cdot|\underline{\mu}))$ of θ. The optimal revision is via Bayes theorem

$$\phi_\theta(\theta|\underline{\mu}) = \frac{\phi_{\underline{\mu}}(\underline{\mu}|\theta)\ g_\theta(\theta)}{g_{\underline{\mu}}(\underline{\mu})} ,$$

where

$$g_{\underline{\mu}}(\underline{\mu}) = \int_\Theta \phi_{\underline{\mu}}(\underline{\mu}|\theta)\ g_\theta(\theta)\ d\theta .$$

If the probabilistic forecasts are perfectly calibrated, then

$$\phi_\theta(\theta|\underline{\mu}) = \rho_\theta(\theta|\underline{\mu}) .$$

Given all remaining elements of the decision model unaltered, the optimal and nonoptimal decision procedures will be formulated in parallel to those for categorical forecasts.

4.2 Evaluation Model

An evaluation model for the probabilistic forecast-decision system is constructed to examine the performance of both optimal and nonoptimal decision procedures using the subjective probabilistic forecasts. For the sake of consistency, we choose a nonoptimal procedure that spurns the innate uncertainty in the forecast - i.e., the probabilistic forecast is presumed to be perfectly calibrated (reliable) and, consequently, the decision is made against ρ_θ instead of against ϕ_θ.

Table 4.1 displays the risks vital to the evaluation of the probabilistic forecast-decision systems. These risks are the expected losses of the decisions or the decision functions prescribed in the first step of the decision analysis.

4.2.1 Nonoptimal Decision Procedure

Under this procedure, the decision maker assumes, incorrectly, that the probabilistic forecast, $\rho_\theta(\cdot|\underline{\mu})$, is perfectly calibrated. The minimum expected loss function is defined as

$$\overline{\underline{LP}}(\underline{\mu}) = \min_{\delta} \int_{\Theta} \ell(\Psi(\theta),\ \delta(\underline{\mu}))\ \rho_\theta(\theta|\underline{\mu})\ d\theta\ .$$

The solution is the decision function $\hat{\delta}$ whose risk is

$$\overline{RP} = \int_M \int_\Theta \ell(\Psi(\theta),\ \hat{\delta}(\underline{\mu}))\ \phi_\theta(\theta|\underline{\mu})\ g_{\underline{\mu}}(\underline{\mu})\ d\theta\ d\underline{\mu}\ .$$

Table 4.1 Risks of Forecasting Schemes and Decision Procedures for Evaluation of Probabilistic Forecast-Decision Systems

		DECISION	
		OPTIMAL	NONOPTIMAL
FORECAST	PERFECT	RF	
	PROBABILISTIC	RP	$\overline{RP}$
	CATEGORICAL	RC	$\overline{RC}$
	NAIVE	RN	$\overline{RN}$

Notation

RP = Risk associated with a subjective probabilistic forecast and an optimal decision procedure

$\overline{RP}$ = Risk associated with a subjective probabilistic forecast and a nonoptimal decision procedure

Remaining notation is defined in Table 3.1

4.2.2 Optimal Decision Procedure

We ensure optimality by accounting, in the decision making step, for the imperfect calibration of the probabilistic forecast. The Bayes risk function is defined by the equation

$$LP(\underline{\mu}) = \min_{\delta} \int_{\Theta} \ell(\Psi(\theta), \delta(\underline{\mu})) \phi_{\theta}(\theta|\underline{\mu}) \, d\theta .$$

The function δ^* that minimizes this expression is the Bayes decision function against ϕ_θ. The risk of this decision function is defined as

$$RP = \int_{M} \int_{\Theta} \ell(\Psi(\theta), \delta^*(\underline{\mu})) \phi_{\theta}(\theta|\underline{\mu}) \, g_{\underline{\mu}}(\underline{\mu}) \, d\theta \, d\underline{\mu} ,$$

or equivalently,

$$RP = \int_{M} LP(\underline{\mu}) \, g_{\underline{\mu}}(\underline{\mu}) \, d\underline{\mu} .$$

4.2.3 Ordinal Relations Between Risks

In the order

$$RF \leq RP \leq RC \leq RN ,$$

the suborders $RF \leq RP \leq RN$ and $RF \leq RC \leq RN$ hold always; the order $RP \leq RC$ holds if the probabilistic forecast is sufficient for the categorical forecast in the sense of Blackwell's (1953) definition of sufficient experiments. A special case of sufficiency arises when the categorical forecast t appears as a parameter of the probabilistic forecast $\rho_\theta(\cdot|\underline{\mu})$, i.e., $\underline{\mu} = (t,\underline{\nu})$. For the proof in this case see Appendix B, and for more elaborate discussions of the general notion of sufficiency see DeGroot (1970, p.433), Hilton (1981), and Appendix A. The orders

$$RP \leq \overline{RP} \quad , \quad RC \leq \overline{RC} \quad , \quad RN \leq \overline{RN}$$

are obvious. No order can be established a priori among $\overline{RP}$, $\overline{RC}$, and $\overline{RN}$, between RC and $\overline{RP}$, between RN and $\overline{RC}$, and between RN and $\overline{RP}$.

4.2.4 Performance Measures

We propose to evaluate two probabilistic forecast-decision systems, in addition to the three systems evaluated in Chapter 3, one optimal and one nonoptimal. The reference system is one in which the Bayes decision is made against the climatological distribution, as in Chapter 3.

System 4. The decision maker uses the probabilistic forecast, $\rho_\theta(\cdot|\underline{\mu})$, and makes the Bayes decision against $\phi_\theta(\cdot|\underline{\mu})$. He, therefore, accounts for the imperfect calibration of the probabilistic forecast. This procedure is both adaptive and optimal. The potential value of the system is

$$PV4 = RN - RF = RN \ .$$

The actual value, AV4, which is equal to the optimal value, OV4, because of the optimality of the decision procedure, is given by the equation

$$AV4 = OV4 = RN - RP \ .$$

System 5. The decision maker uses the probabilistic forecast, $\rho_\theta(\cdot|\underline{\mu})$, as if it were perfectly calibrated. Therefore, this procedure is adaptive but nonoptimal. The system values are given by

$$PV5 = RN - RF = RN ,$$

$$OV5 = RN - RP ,$$

$$AV5 = RN - \overline{RP} .$$

The system efficiencies and expected opportunity losses are given by the equations in Section 3.2.5.

4.3 Distributions of the Information and Forecasting Schemes

Under the assumption that the variable-width, fixed-probability technique is used to encode the forecast density $\rho_\theta(\cdot|\underline{\mu})$, the vector parameter $\underline{\mu} = (t, t_{50}, t_{75})$, where t is the median, t_{50} is the width of the 50% credible interval, and t_{75} is the width of the 75% credible interval. Let

$$\varepsilon = t - \theta$$

denote the error of the median and $k_{\varepsilon|t_{50},t_{75}}$ denote its forecasted density. Assuming ε is independent of θ, we have

$$\rho_\theta(\theta|t,t_{50},t_{75}) = k_{\varepsilon|t_{50},t_{75}}(t-\theta|t_{50},t_{75}) .$$

Murphy and Winkler (1974, p.28) indicate that "... only 51% of the 50% intervals and 32% of the 75% intervals were symmetric... and the asymmetries appeared to be equally likely in either direction," but the results they report are insufficient for explicit modeling of this asymmetry. Thus, to construct the distribution $K_{\varepsilon|t_{50},t_{75}}$, it is necessary to assume that the credible intervals are symmetric about the median.[1/] Because the intervals are, by definition, symmetric about the median in terms of probability, the ordinates of the distribution $K_{\varepsilon|t_{50},t_{75}}$ are

$$\Pr(\varepsilon \leq -t_{75}/2) = .125 ,$$

$$\Pr(\varepsilon \leq -t_{50}/2) = .25 ,$$

$$\Pr(\varepsilon \leq 0) = .5 ,$$

$$\Pr(\varepsilon \leq t_{50}/2) = .75 ,$$

$$\Pr(\varepsilon \leq t_{75}/2) = .875 .$$

Let $h_{\varepsilon|t_{50},t_{75}}$ denote the conditional density of the error ε given the probabilistic forecast $\rho_\theta(\cdot|t,t_{50},t_{75})$. Again, assuming that ε is independent of θ, we have

$$f_t(t|t_{50},t_{75},\theta) = h_{\varepsilon|t_{50},t_{75}}(t-\theta|t_{50},t_{75}) .$$

[1/] This assumption will, most likely, result in the underestimation of the value of probabilistic forecasts.

By definition,

$$\phi_{\underline{\mu}}(\underline{\mu}|\theta) = \phi_{\underline{\mu}}\ (t,t_{50},t_{75}|\theta)\ ,$$

$$= f_t\ (t|t_{50},t_{75},\theta)\ f_{t_{50},t_{75}}(t_{50},t_{75}|\theta)\ .$$

Assuming (t_{50},t_{75}) to be independent of θ,

$$f_{t_{50},t_{75}}(t_{50},t_{75}|\theta) = g_{t_{50},t_{75}}(t_{50},t_{75})\ ,$$

and hence

$$\phi_{\underline{\mu}}(\underline{\mu}|\theta) = h_{\varepsilon|t_{50},t_{75}}(t-\theta|t_{50},t_{75})\ g_{t_{50},t_{75}}(t_{50},t_{75})\ .$$

Six ordinates of the distribution $H_{\varepsilon|t_{50},t_{75}}$ can be extracted from the data reported by Murphy and Winkler (1974, 1975). The "ALL" set of forecasts from Denver (Table 4.2) yields

$$Pr(\varepsilon \leq -t_{75}/2) = .159\ ,$$

$$Pr(\varepsilon \leq -t_{50}/2) = .288\ ,$$

$$Pr(\varepsilon \leq t_{50}/2) = .288 + .455 = .743\ ,$$

$$Pr(\varepsilon \leq t_{75}/2) = .159 + .735 = .894\ .$$

Table 4.2 Relative Frequency of Actual Temperature Below Interval (BI), in Interval (II), and Above Interval (AI) and Average Interval Width for Variable-Width Forecasts Issued in Denver

SET OF FORECASTS	NUMBER OF FORECASTS	PROBABILITY OF ACTUAL TEMPERATURE						AVERAGE WIDTH (STANDARD DEVIATION OF WIDTH) [°F]	
		50% INTERVALS			75% INTERVALS				
		BI	II	AI	BI	II	AI	50% INTERVALS	75% INTERVALS
1	2	3	4	5	6	7	8	9	10
ALL	132	.258	.455	.288	.106	.735	.159	6.2(1.3)	11.7(2.2)
MAXIMUM TEMPERATURE	66	.288	.515	.197	.152	.758	.090	6.3(1.2)	11.7(2.1)
MINIMUM TEMPERATURE	66	.227	.394	.379	.061	.712	.227	6.2(1.3)	11.6(2.3)
12-HOUR LEAD TIME	66	.227	.515	.258	.091	.803	.106	6.1(1.2)	11.4(2.0)
24-HOUR LEAD TIME	66	.288	.394	.318	.121	.667	.212	6.4(1.3)	11.9(2.4)
FORECASTER 1	64	.297	.375	.328	.093	.766	.141	5.8(1.3)	11.3(2.6)
FORECASTER 2	68	.221	.520	.259	.118	.706	.176	6.7(1.1)	12.0(1.7)

(Source: Murphy and Winkler, Table 3, p.7, 1975)

Moreover, from Table 3.2

$$\Pr(\varepsilon < 0) = .480 \ ,$$

$$\Pr(\varepsilon = 0) = .126 \ .$$

It is not at all apparent, though, how the users can directly transform the credible interval forecasts into the continuous distribution functions, $K_{\varepsilon|t_{50},t_{75}}$ and $H_{\varepsilon|t_{50},t_{75}}$, requisite to perform the decision analysis suggested herein. An interpolating procedure is, therefore, proposed to construct an approximation to the aforementioned distribution functions. The interpolating procedure suggested is a one-dimensional quasi-cubic hermite polynomial technique described by Akima (1970). The objective of this procedure is to mimic a curve drawn manually. The procedure, however, necessitates the specification of the 100% interval width, which renders two ordinates of $K_{\varepsilon|t_{50},t_{75}}$ and $H_{\varepsilon|t_{50},t_{75}}$. They are

$$\Pr(\varepsilon \leq -t_{100}/2) = 0 \quad ,$$

$$\Pr(\varepsilon \leq t_{100}/2) = 1.0 \ .$$

Consequently, we submit that the 100% interval width ought to be encoded when using the variable-width elicitation technique. One could obtain this parameter estimate from the forecaster by asking the following two queries:

1) What is the temperature which you are certain tomorrow's daily average will not fall below?

2) What is the temperature which you are certain tomorrow's daily average will not exceed?

Since there is no data to support other relationships, we assume that

$$t_{100} = w\, t_{75} ,$$

where $w > 1$. w is assigned a value that gives the best fit of the marginal density function

$$h_\varepsilon(\varepsilon) = \iint h_{\varepsilon|t_{50},t_{75}}(\varepsilon|t_{50},t_{75})\; g_{t_{50},t_{75}}(t_{50},t_{75})\; dt_{50}\; dt_{75}$$

to that determined for categorical forecasts in Section 3.3.

Murphy and Winkler (1974, p.26) declare that they "... expect the width of the 75% intervals to be an increasing function of the width of the 50% intervals, and the results... indicate that, on the average, such a relationship does indeed exist." Therefore, let

$$t_{75} = a\, t_{50} + b ,$$

where a and b are determined from the equations of moments:

$$m_{t_{75}} = a\, m_{t_{50}} + b ,$$
$$s^2_{t_{75}} = a^2\, s^2_{t_{50}} .$$

The simultaneous solution yields:

$$a = \frac{s_{t_{75}}}{s_{t_{50}}} ,$$

$$b = m_{t_{75}} - \frac{s_{t_{75}}}{s_{t_{50}}}\, m_{t_{50}} .$$

The mean and standard deviation of t_{50} and t_{75} are given in Columns 9 and 10 of Table 4.2.

Henceforth, t_{75} is no longer considered a random variable, and it is dropped from the distributions $K_{\varepsilon|t_{50},t_{75}}$, $H_{\varepsilon|t_{50},t_{75}}$, and $G_{t_{50},t_{75}}$ so that we are left with $K_{\varepsilon|t_{50}}$, $H_{\varepsilon|t_{50}}$, and $G_{t_{50}}$. Consequently, the density $g_{\underline{\mu}}$ is expressed as

$$g_{\underline{\mu}}(\underline{\mu}) = \int \phi_{\underline{\mu}}(\underline{\mu}|\theta)\; g_\theta(\theta)\; d\theta \;,$$

$$= \int h_{\varepsilon|t_{50}}(t-\theta|t_{50})\; g_{t_{50}}(t_{50})\; g_\theta(\theta)\; d\theta \;,$$

$$= g_{t_{50}}(t_{50}) \int h_{\varepsilon|t_{50}}(t-\theta|t_{50})\; g_\theta(\theta)\; d\theta \;.$$

Finally, the posterior density of the actual temperature given the probabilistic forecast is given by

$$\phi_\theta(\theta|\underline{\mu}) = \frac{\phi_{\underline{\mu}}(\underline{\mu}|\theta)\; g_\theta(\theta)}{g_{\underline{\mu}}(\underline{\mu})} \;,$$

$$= \frac{h_{\varepsilon|t_{50}}(t-\theta|t_{50})\; g_\theta(\theta)}{\int h_{\varepsilon|t_{50}}(t-\theta|t_{50})\; g_\theta(\theta)\; d\theta} \;.$$

The density $g_{t_{50}}$ is assumed to be Gaussian with parameter values given by the statistics in Column 9 of Table 4.2. For numerical computations all distribution functions are quantized into .5 [$^{\circ}$F] intervals.

Let $m_{\varepsilon|t_{50}}$ and $s^2_{\varepsilon|t_{50}}$ denote the mean and variance of $\varepsilon|t_{50}$ under the density $h_{\varepsilon|t_{50}}$, and $M_{\varepsilon|t_{50}}$ and $S^2_{\varepsilon|t_{50}}$ denote the mean and variance of $\varepsilon|t_{50}$ under the density $k_{\varepsilon|t_{50}}$. Under the assumption that the distributions $H_{\varepsilon|t_{50}}$ and $K_{\varepsilon|t_{50}}$ are piecewise linear, expressions for statistics of

$\varepsilon | t_{50}$ are derived. Specifically, the statistics are given by

$$m_{\varepsilon | t_{50}} = \int \varepsilon \, h_{\varepsilon | t_{50}}(\varepsilon | t_{50}) \, d\varepsilon \quad ,$$

$$s^2_{\varepsilon | t_{50}} = \int (\varepsilon - m_{\varepsilon | t_{50}})^2 \, h_{\varepsilon | t_{50}}(\varepsilon | t_{50}) \, d\varepsilon \quad .$$

To facilitate the discussion, we denote

$$-t_{100}/2 = x(1) \ ,$$

$$-t_{75}/2 = x(2) \ ,$$

$$-t_{50}/2 = x(3) \ ,$$

$$-0 = x(4) \ ,$$

$$+0 = x(5) \ ,$$

$$t_{50}/2 = x(6) \ ,$$

$$t_{75}/2 = x(7) \ ,$$

$$t_{100}/2 = x(8) \ ,$$

and $y(i) = H_{\varepsilon | t_{50}}(x(i) | t_{50})$, $i = 1, \ldots, 8$. With the differentials given by

$$dH_{\varepsilon(i) | t_{50}} = \frac{y(i+1) - y(i)}{x(i+1) - x(i)} \quad , \quad i = 1, \ldots, 7 \ ,$$

the expression for the mean is

$$m_{\varepsilon | t_{50}} = \sum_{i=1}^{7} \left(\frac{x(i+1)^2 - x(i)^2}{2} \right) dH_{\varepsilon(i) | t_{50}} \quad ,$$

and the expression for the variance is

$$s^2_{\varepsilon|t_{50}} = \sum_{i=1}^{7} \left[\frac{x(i+1)^3 - x(i)^3}{3} + (x(i)^2 - x(i+1)^2)\, m_{\varepsilon|t_{50}} + (x(i+1) - x(i))\, m^2_{\varepsilon|t_{50}} \right] dH_{\varepsilon(i)|t_{50}} .$$

A parallel set of expressions can be found for $M_{\varepsilon|t_{50}}$ and $S^2_{\varepsilon|t_{50}}$ with the proper redefinition of x(i) and y(i).

4.4 Hypothetical Case Study

A numerical example using probabilistic temperature forecasts constructed from the data reported by Murphy and Winkler (1974, 1975) is presented herein. The value of probabilistic forecasts is compared against the value of categorical and naive forecasts. Furthermore, sensitivity analyses are performed on the expected value of t_{50}, $E[t_{50}]$, the loss function, ℓ, and the load operator, Ψ.

4.4.1 Input Data

The mean and variance of the forecast parameters t_{50} and t_{75} are

$$m_{t_{50}} = 6.2 \;\; [^{o}F] ,$$

$$s^2_{t_{50}} = 1.69 \;\; [^{o}F]^2 ,$$

$$m_{t_{75}} = 11.7 \;\; [^{o}F] ,$$

$$s^2_{t_{75}} = 4.84 \;\; [^{o}F]^2 .$$

Note that these are the statistics from the "ALL" forecasts in Denver, *vide* Table 4.2. The coefficients of the equation describing the deterministic relationship between t_{50} and t_{75} are

$$a = 1.69 \ ,$$
$$b = 1.22 \ [^{o}F] \ .$$

The coefficient of the relationship between t_{75} and t_{100} is

$$w = 1.5 \ .$$

Given these parameters, we find that when $t_{50} = m_{t_{50}}$, the statistics of $\varepsilon|t_{50}$ under the distribution $H_{\varepsilon|t_{50}}$ are

$$m_{\varepsilon|t_{50}} = -.369 \ [^{o}F] \ ,$$
$$s^2_{\varepsilon|t_{50}} = 21.07 \ [^{o}F]^2 ,$$

and the statistics of $\varepsilon|t_{50}$ under the distribution $K_{\varepsilon|t_{50}}$ are

$$M_{\varepsilon|t_{50}} = 0 \ [^{o}F] \ ,$$
$$S^2_{\varepsilon|t_{50}} = 20.34 \ [^{o}F]^2 \ .$$

The load operator, Ψ, the loss function, ℓ, and the prior density, g_θ, are identical to those used in Chapter 3.

4.4.2 Test of the Numerical Models

To ensure that the decision models have been programmed correctly, the following test has been devised. The optimal decision against naive forecasts, a^*, as defined in Section 3.2.3, is employed in the probabilistic forecast-decision model, i.e., the risk is computed as

$$RP' = \int_M \int_\Theta \ell(\Psi(\theta), a^*) \; \phi_\theta(\theta|\underline{\mu}) \; g_{\underline{\mu}}(\underline{\mu}) \; d\theta \; d\underline{\mu} \; ,$$

which is equivalent to

$$RP' = \int_\Theta \ell(\Psi(\theta), a^*) \; [\int_M \phi_\theta(\theta|\underline{\mu}) \; g_{\underline{\mu}}(\underline{\mu}) \; d\underline{\mu}] \; d\theta \; ,$$

$$= \int_\Theta \ell(\Psi(\theta), a^*) \; g_\theta(\theta) \; d\theta \; .$$

Clearly, RP' is equivalent to RN.

In addition, the optimal decision function, δ^*, against the posterior density, f_θ, as defined in Section 3.2.3, is employed in the probabilistic forecast-decision model, i.e., the risk is computed as

$$RP'' = \int_M \int_\Theta \ell(\Psi(\theta), \delta^*(t)) \; \phi_\theta(\theta|\underline{\mu}) \; g_{\underline{\mu}}(\underline{\mu}) \; d\theta \; d\underline{\mu} \; ,$$

$$= \int_\Theta \int_\Theta \ell(\Psi(\theta), \delta^*(t)) \;\; [\int_\Theta \phi_\theta(\theta|t, t_{50}) \; g_{\underline{\mu}}(t, t_{50}) \; dt_{50}] \;\; d\theta \; dt \; ,$$

$$= \int_\Theta \int_\Theta \ell(\Psi(\theta), \delta^*(t)) \; f_\theta(\theta|t) \; g_t(t) \; d\theta \; dt \; .$$

Hence RP'' is identical to RC.

The results shown in Table 4.3 support the correctness of the numerical models.

Table 4.3 Accuracy of the Probabilistic Numerical Model

FORECAST	FROM CATEGORICAL MODEL	FROM PROBABILISTIC MODEL
CATEGORICAL	RC = \$12327	RP'' = \$12005
NAIVE	RN = \$38889	RP' = \$38874

4.4.3 Basic Evaluation

The risks incurred daily by the utility as a consequence of using a naive, categorical, probabilistic, or perfect forecast in an optimal or nonoptimal decision procedure are summarized in Table 4.4.

Table 4.4 Risks for the Case Study

		DECISION	
		OPTIMAL	NONOPTIMAL
FORECAST	PERFECT	RF = \$0	
	PROBABILISTIC	RP = \$11663	$\overline{RP}$ = \$16600
	CATEGORICAL	RC = \$12327	$\overline{RC}$ = \$22318
	NAIVE	RN = \$38889	$\overline{RN}$ = \$61294

The optimal use of the probabilistic forecast results in the Bayes risk RP which is lower than the Bayes risk RC associated with the categorical forecast by 5.4% on a daily basis, or approximately 240 thousand dollars per year. Furthermore, there appears to be a significant difference between the optimal (RP) and nonoptimal ($\overline{RP}$) use of the probabilistic forecast: 4937 \$/day or approximately 1.8 million dollars annually. This indicates that the probabilistic forecasts are not perfectly calibrated. In Figure 4.1 we see that while R_θ is symmetric about t, Φ_θ is skewed towards m_θ and has a spike at t, thereby lending less probability to positive errors in the median forecast than R_θ. Since most of the time the symmetry of R_θ is due to our modeling assumption (*vide* Section 4.3), the risks RP and $\overline{RP}$ are, most likely, overestimated. If the data in Table 4.2 were given a nonBayesian interpretation wherein the likelihood is interpreted as a "posterior," $\phi_{\underline{\mu}}(\theta|t,t_{50}) = h_{\varepsilon|t_{50}}(t-\theta|t_{50})$, and the decision is made against $\phi_{\underline{\mu}}$, then the closeness of $\Phi_{\underline{\mu}}$ and R_θ (Figure 4.2) implies that the risk of such a decision procedure would be similar to $\overline{RP}$.

Figure 4.3 displays the Bayes decision function, δ^*. For example, given a probabilistic forecast with median $t = 28^\circ$F and a 50% credible interval $t_{50} = 3^\circ$F, the optimal decision is to generate $\delta^*(28,3) = 210$ MW above the base load B. The influence of the credible interval t_{50} on the optimal decision $\delta^*(t,t_{50})$ is nil at about $t = 46^\circ$F and increases nonlinearly as t tends towards the extremes. In other words, the quantification of uncertainty in forecasts indicating extreme values is much more consequential for optimal decision making than the quantification of uncertainty in forecasts indicating about-average values. (Recall that the climatological mean is $m_\theta = 48.95^\circ$F.) The wide band of zero values is due to the heating/cooling gap of the load operator. The band narrows and shifts as the forecast uncertainty increases, thus reflecting the subtle interplay between the load operator Ψ, loss function ℓ, and posterior density ϕ_θ. Plotted in Figure 4.3 is also the trace of the Bayes decision function, δ^*, for categorical forecasts. For instance, given a categorical forecast $t = 28^\circ$F, the optimal decision is $\delta^*(28) = 170$ MW. The ordinate of this point is $t_{50} = 7.1^\circ$F; it may be viewed as the implied 50% credible interval of the categorical forecast. Note that the range of optimal decisions in

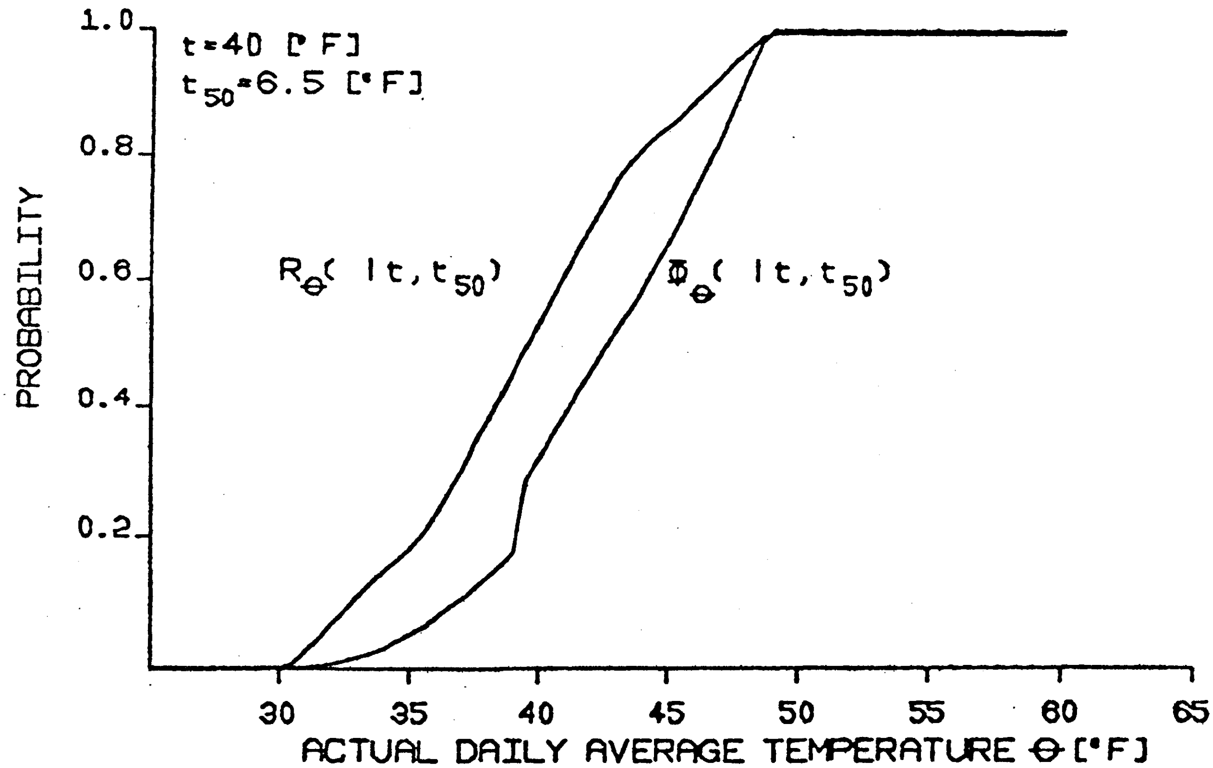

Figure 4.1 Comparison of the Forecast Distribution R_θ with the Posterior Distribution Φ_θ.

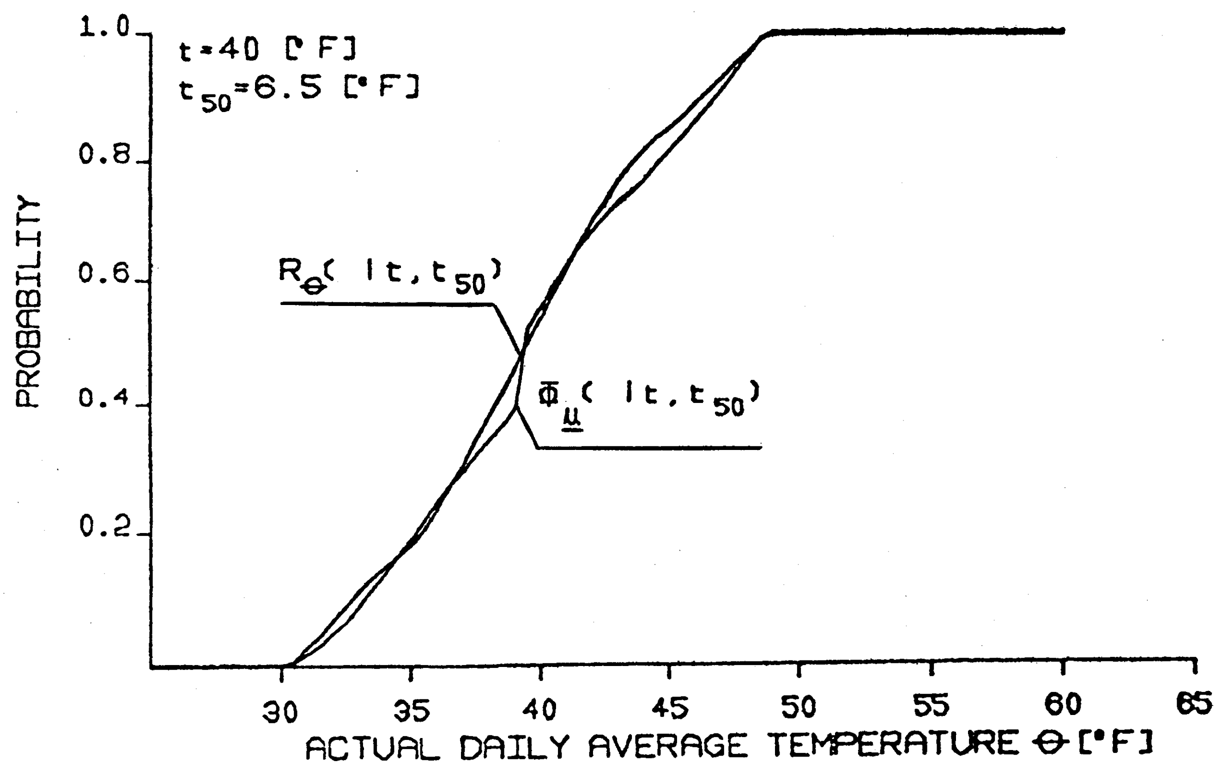

Figure 4.2 Comparison of the Forecast Distribution R_θ with the Likelihood $\Phi_{\underline{\mu}}$

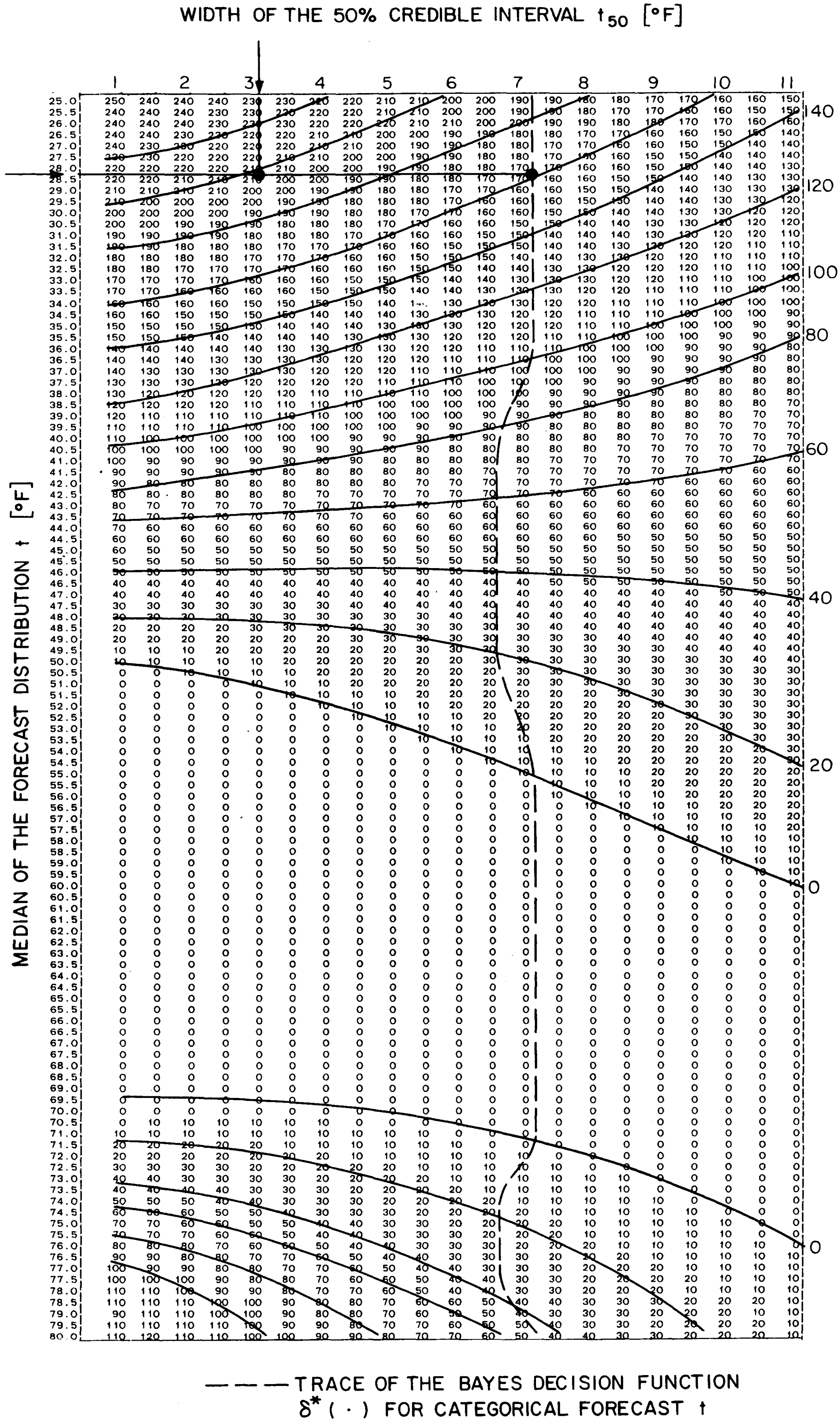

Figure 4.3 Bayes Decision Function, $\delta^*(\cdot,\cdot)$, for the Probabilistic Forecast-Decision System

response to probabilistic forecasts indicating $t = 28^{\circ}F$ is from 220 MW for very certain forecasts to 130 MW for very uncertain forecasts. This example illustrates best how much adaptivity can be built into the Bayes decision function by providing forecasts which encode the forecaster's degree of uncertainty even in such a simple form as a single credible interval.

Figure 4.4 depicts posterior distributions, Φ_{θ}, constructed for three different values of t_{50}, which result in three Bayes decision functions, $\delta^{*}(\cdot, t_{50})$, extracted from Figure 4.3 (*vide* Figure 4.6). The associated Bayes risk functions, $LP(\cdot, t_{50})$, are shown in Figure 4.5. As Figure 4.6 indicates, when the precision of the forecast increases ($t_{50} \rightarrow 0$) the Bayes decision function, δ^{*}, approaches the shape of the load operator, Ψ. The asymmetry of the Bayes decision functions and asymmetry, bimodality, and discontinuities of the Bayes risk functions are, again, attributed to the subtle interaction between the load operator Ψ, loss function ℓ, and posterior density ϕ_{θ}, as explicated in Section 3.4.2.

Table 4.5 presents values of the performance measures for Systems 1, 4, and 5. When both categorical and probabilistic forecasts are used optimally (Systems 1 and 4), their values are AV1 = 26562 and AV4 = 27226 \$/day. If the forecasts were perfect, they would be worth PV = 38889 \$/day, or roughly 14.2 million dollars per year. The forecast expected opportunity loss, FOL, constitutes an upper bound on the economically justified investments towards improving the quality of the forecasts. Thus, a decision maker already receiving probabilistic forecasts should spend no more than FOL4 = 11663 \$/day, equivalent to 4.3 million dollars annually, for a prospective forecast enhancement.

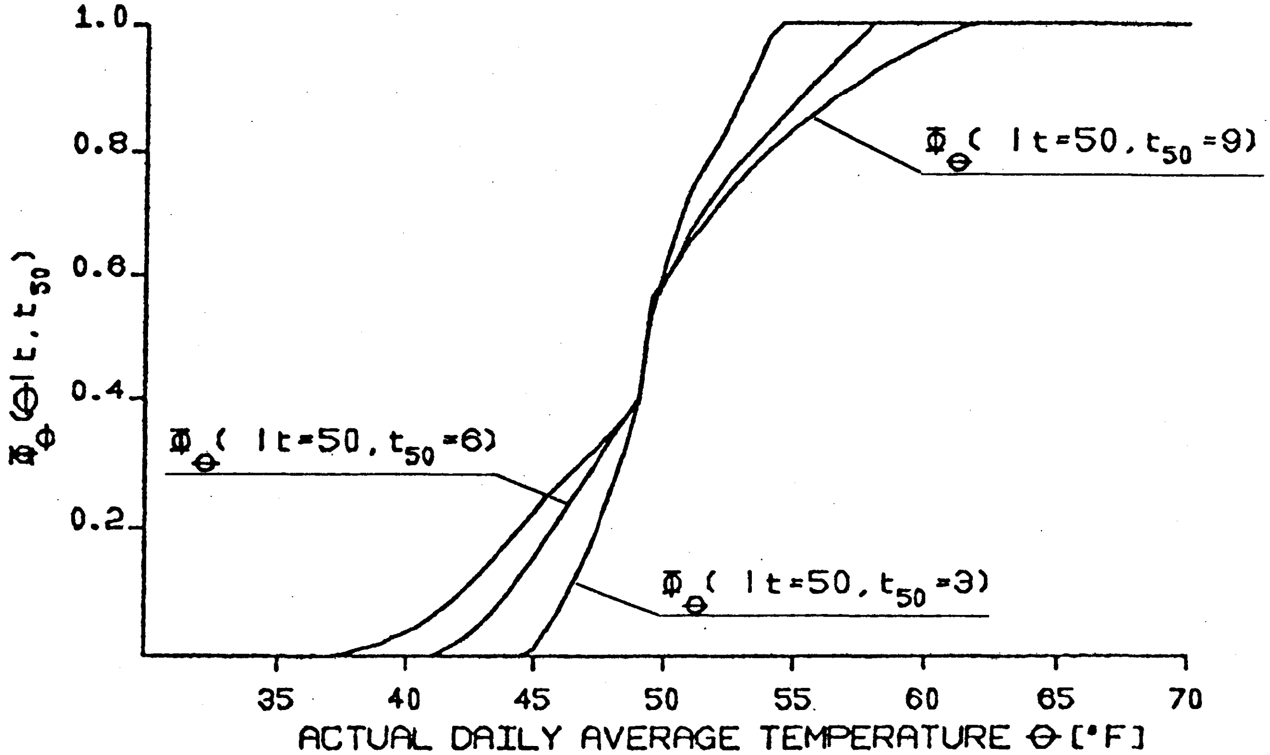

Figure 4.4 Posterior Distributions, $\Phi_\theta(\cdot|t,t_{50})$, for t = 50 and t_{50} = 3, 6, 9 [°F]

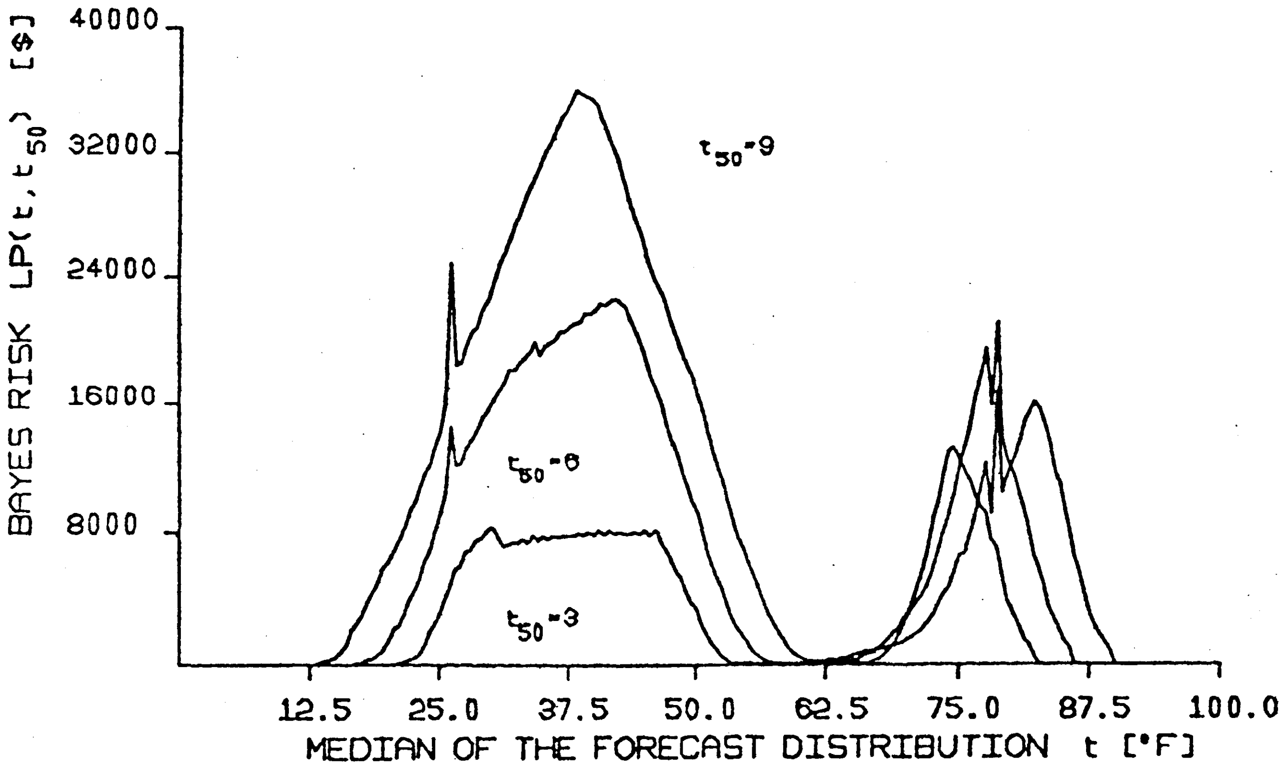

Figure 4.5 Bayes Risk Functions, $LP(\cdot,t_{50})$, for t_{50} = 3, 6, 9 [^{o}F]

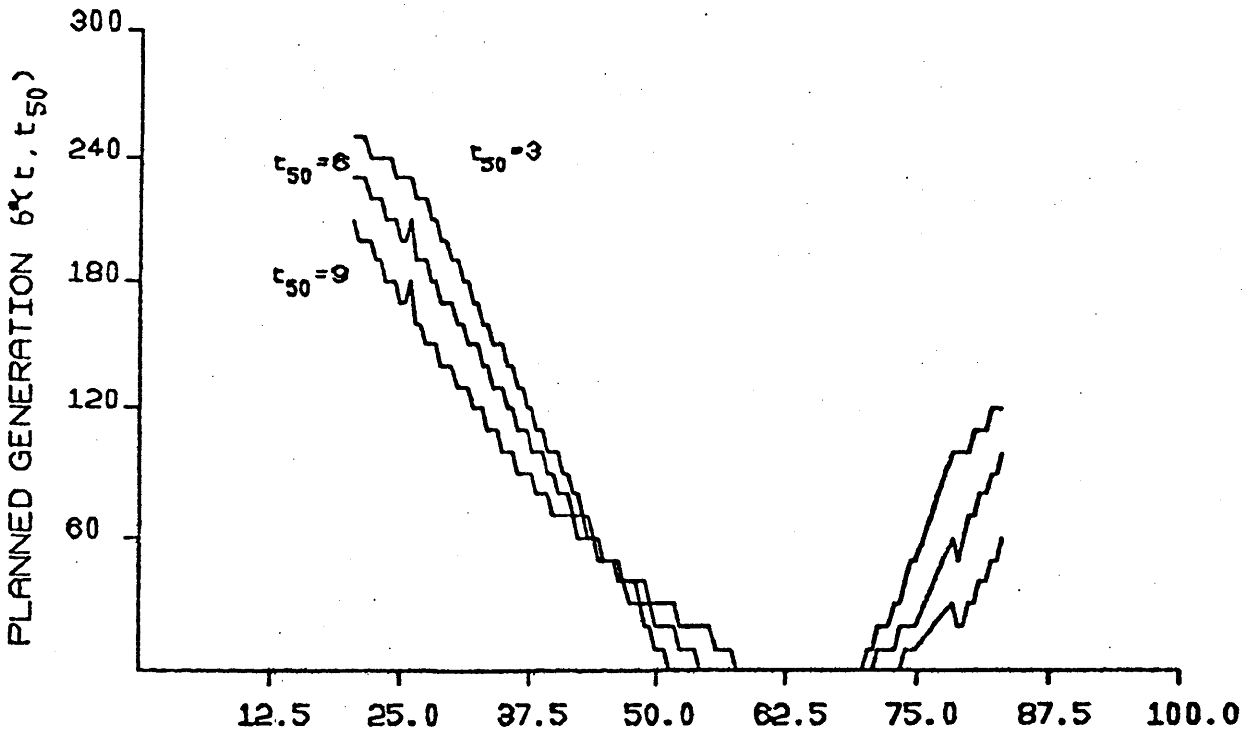

Figure 4.6 Bayes Decision Functions, $\delta^{*}(\cdot,t_{50})$, for t_{50} = 3, 6, 9 [^{o}F]

Table 4.5 Comparative Evaluation of Three Forecast-Decision Systems

PERFORMANCE MEASURE	SYSTEM 1	SYSTEM 4	SYSTEM 5
	CATEGORICAL FORECAST OPTIMAL DECISION	PROBABILISTIC FORECAST OPTIMAL DECISION	PROBABILISTIC FORECAST NONOPTIMAL DECISION
EXPECTED DAILY LOSS [$]	RC = 12327	RP = 11663	$\overline{RP}$ = 16600
VALUES [$]			
potential value PV	38889	38889	38889
optimal value OV	26562	27226	27226
actual value AV	26562	27226	22289
EFFICIENCIES			
forecast efficiency FE	.683	.700	.700
decision efficiency DE	1.0	1.0	.819
total efficiency TE	.683	.700	.573
EXPECTED OPPORTUNITY LOSSES [$]			
forecast system FOL	12327	11663	11663
decision system DOL	0	0	4937
total system TOL	12327	11663	16600

High efficiencies, FE1 = 68.3% and FE4 = 70.0%, indicate that both types of forecasts are of good quality and are much more valuable than naive forecasts which are based solely on a climatological record (efficiency of a naive forecast is zero).

Higher efficiency of probabilistic forecasts, by FE4 - FE1 = 1.7%, indicates that the forecasters were able to quantify uncertainty in their forecasts with a degree of reliability that has economic impact. The expected gain from using probabilistic forecasts instead of categorical forecasts is AV4 - AV1 = 664 $/day, or about 240 thousand dollars annually. Considering the fact that the issuance of subjective probabilistic forecasts does not require any new hardware but only additional skill and effort on the part of a forecaster, the expected net gain would be positive (even after subtracting an extra remuneration for a forecaster).

When the probabilistic forecasts are used nonoptimally (System 5), their value is AV5 = 22289 $/day. The efficiency of the nonoptimal decision procedure is DE5 = 81.9%, while the expected opportunity loss incurred by the decision maker due to the nonoptimal use of the forecast is DOL5 = 4937 $/day, or 1.8 million dollars per year. The decision maker could economically justify investments up to this amount for improving his decision procedure.

The value of the optimally used categorical forecasts, AV1, is larger than the value of the nonoptimally used probabilistic forecasts, AV5. This belies the prevailing sentiment that more information is necessarily better than less. Obviously, this is only true when the information is processed and used optimally.

When both types of forecasts are used nonoptimally, then probabilistic forecasts are TE5 - TE2 = 14.7% more valuable than categorical forecasts (compare Table 4.5 with Table 3.5). The corresponding expected gain from using probabilistic instead of categorical forecasts is AV5 - AV2 = 5718 \$/day, or roughly 2.1 million dollars annually. That is, even if the decision maker does not account for the possible imperfect calibration of probabilistic forecasts, he is much better off using probabilistic forecasts than using categorical forecasts under the (false) presumption that they are error-free.

4.5 Sensitivity Analyses

In this section, we evaluate the performance of probabilistic forecast-decision systems under conditions different than those of the basic evaluation. The consequences of modifying the expected value of the 50% credible interval, $E[t_{50}]$, the loss function, ℓ, and the load operator, Ψ, are critically examined.

4.5.1 Expected Value of the 50% Credible Interval

The expected value of the 50% credible interval, $E[t_{50}]$, is one of the parameters that reflect the forecaster's ability to construct precise forecasts. This parameter is wont to closely affect the variance $s^2_{\varepsilon|t_{50}}$ of the likelihood function $H_{\varepsilon|t_{50}}$ and the variance $S^2_{\varepsilon|t_{50}}$ of the forecast distribution $K_{\varepsilon|t_{50}}$. Using the approximations derived in Section 4.3, the variances are computed as a function of $E[t_{50}]$ in Table 4.6.

Table 4.6 Relationship Between the Expected Value of the 50% Credible Interval and the Variance of $H_{\varepsilon|t_{50}}$ and $K_{\varepsilon|t_{50}}$

$E[t_{50}]$ $[^\circ F]$	$s^2_{\varepsilon\|t_{50}}$ $[^\circ F]^2$	$S^2_{\varepsilon\|t_{50}}$ $[^\circ F]^2$
5.0	19.98	19.18
6.2	21.07	20.34
7.0	21.81	21.16
8.0	22.85	22.37

The effect of changing $E[t_{50}]$ on the posterior distribution Φ_θ is illustrated in Figure 4.7. The total efficiency of both Systems 4 and 5 decreases with increasing $E[t_{50}]$ (Figure 4.8). However, the effect of $E[t_{50}]$ is significantly more pronounced on System 5 than on System 4. It, thus, becomes more important to employ an optimal decision procedure when the precision of the forecast decreases; this observation parallels the result observed for categorical forecasts (Section 3.5.1).

Close examination of Figure 4.9 reveals that the decision expected opportunity loss of System 5, DOL5 = TOL5 - TOL4, increases from roughly 4,500 \$/day to 5,777 \$/day as $E[t_{50}]$ increases from just 5 to 8 [^{o}F].

4.5.2 Loss Ratio of Over to Under Generation

We anticipate that as the ratio c_o/c_u approaches 1, the Bayes risk functions, LP, will attain higher peaks and the Bayes decision functions, δ^*, will become broader. These properties are validated in Figures 4.10 and 4.11.

The Bayes risk function when $c_o/c_u = 0$, of course, coincides with the t axis. The associated decision function is a horizontal line at 300 MW since this structure of the loss function implies that there is no penalty for generating too much power.

The effect of the asymmetry of the loss function on the total efficiency of Systems 1, 4, and 5 is shown in Figure 4.12. As c_o/c_u approaches 1, the efficiency of System 5 declines at a faster rate than the efficiencies of Systems 1 or 4. This phenomenon is more noticeable when one considers the total expected opportunity losses (Figure 4.13). Precise knowledge, therefore, of the shape of the loss function is critical to the choice of a forecast-decision system.

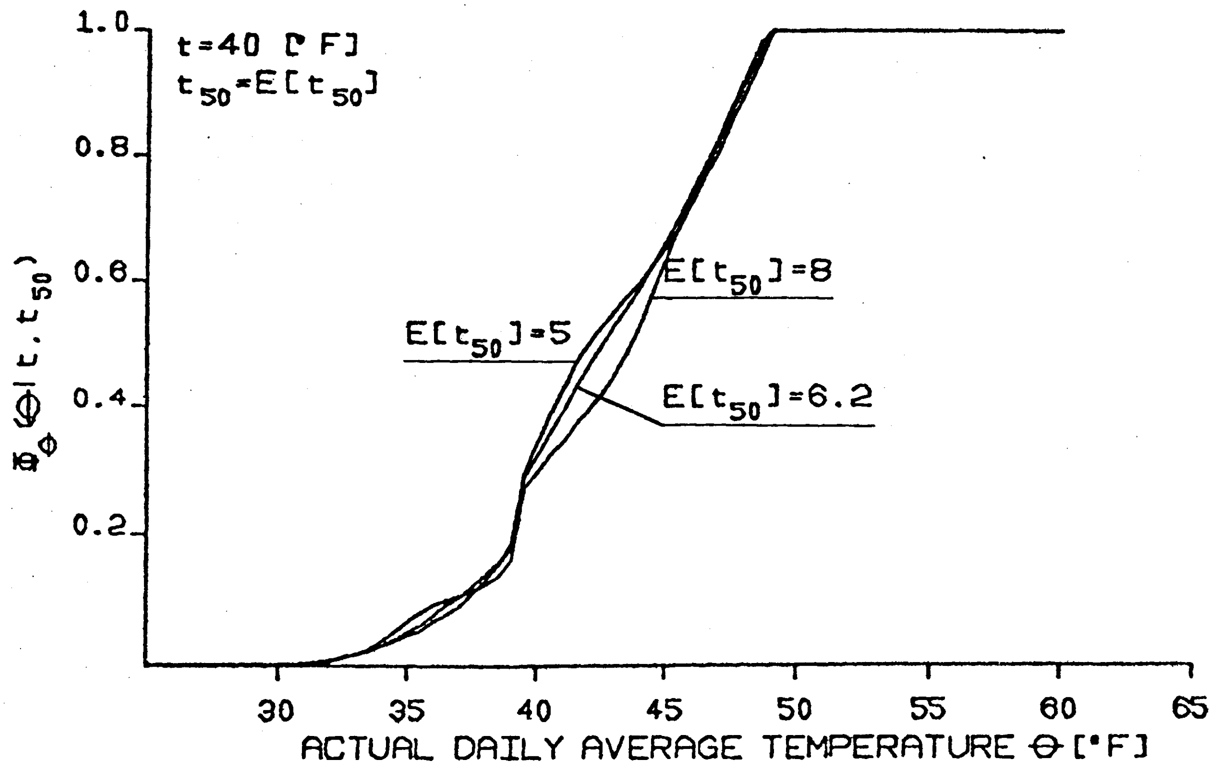

Figure 4.7 Posterior Distributions, $\Phi_\theta(\cdot|t,t_{50})$, for $E[t_{50}]$ = 5, 6.2, 8 [^oF]

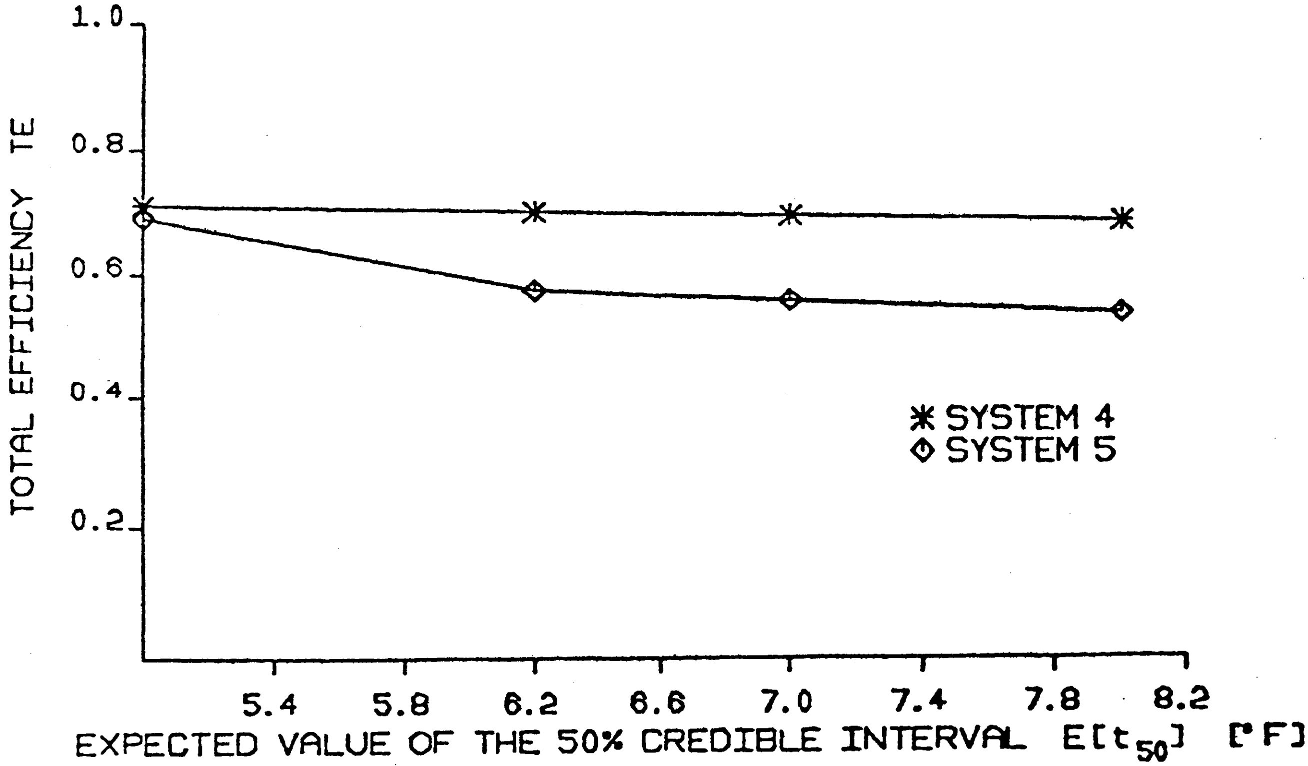

Figure 4.8 Total Efficiency versus $E[t_{50}]$

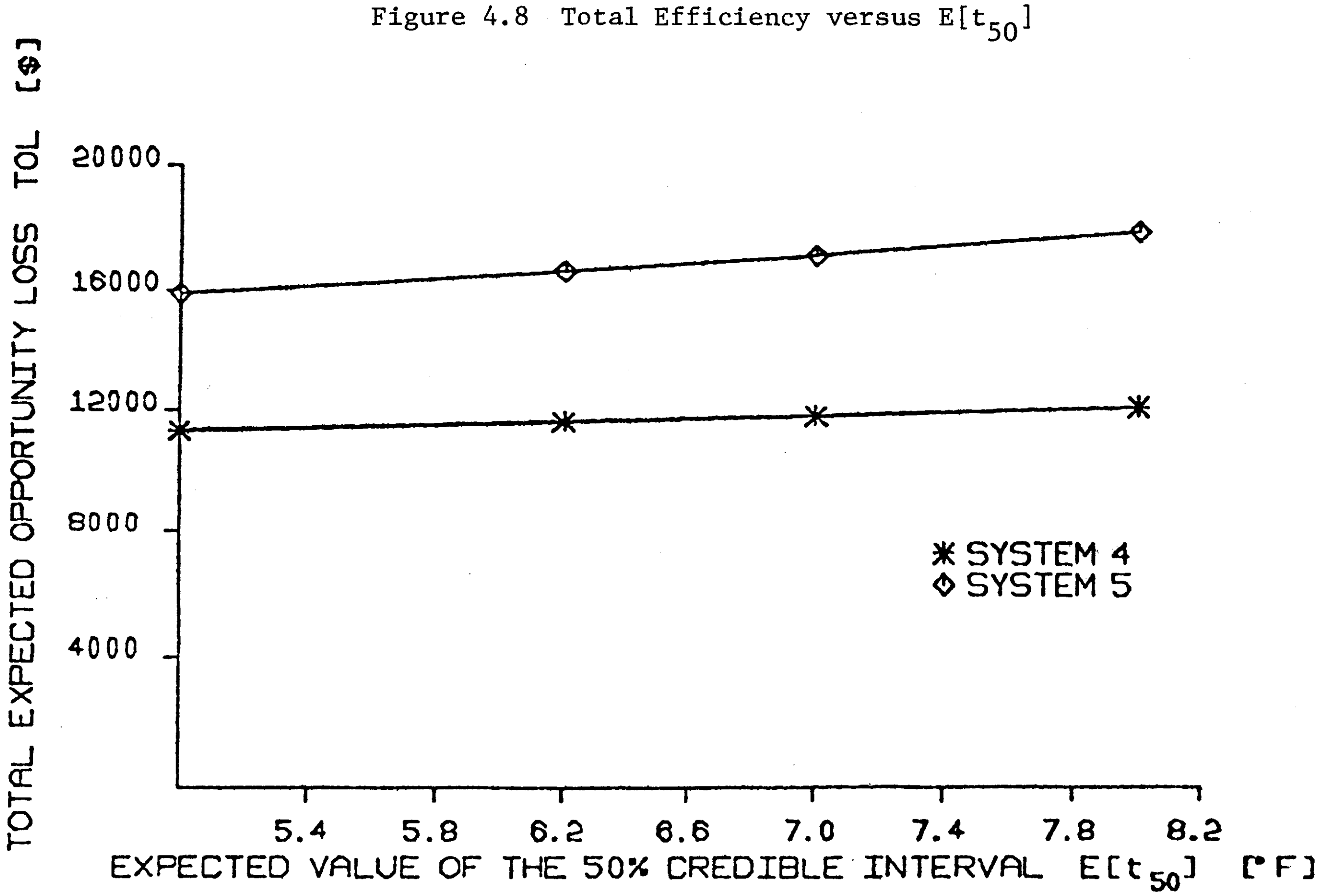

Figure 4.9 Total Expected Opportunity Loss versus $E[t_{50}]$

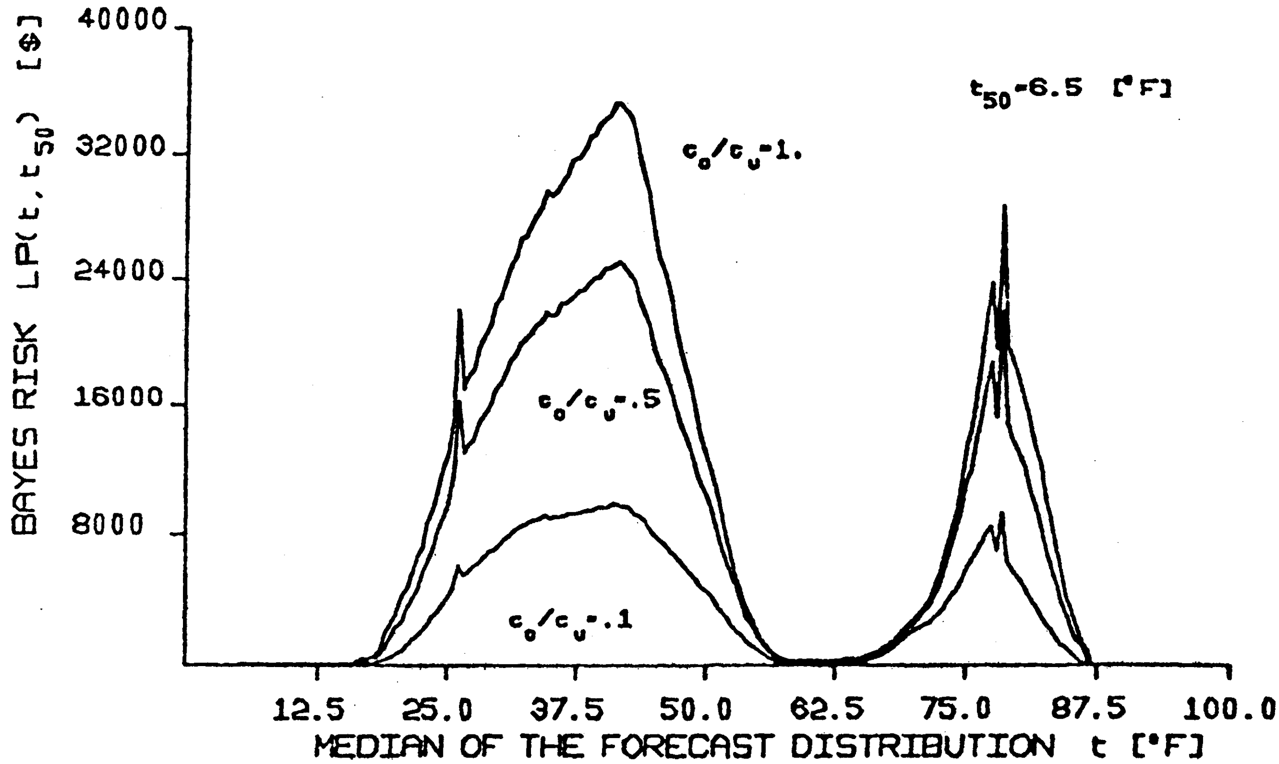

Figure 4.10 Bayes Risk Functions, $LP(\cdot,t_{50})$, when c_o/c_u = .1, .5, 1

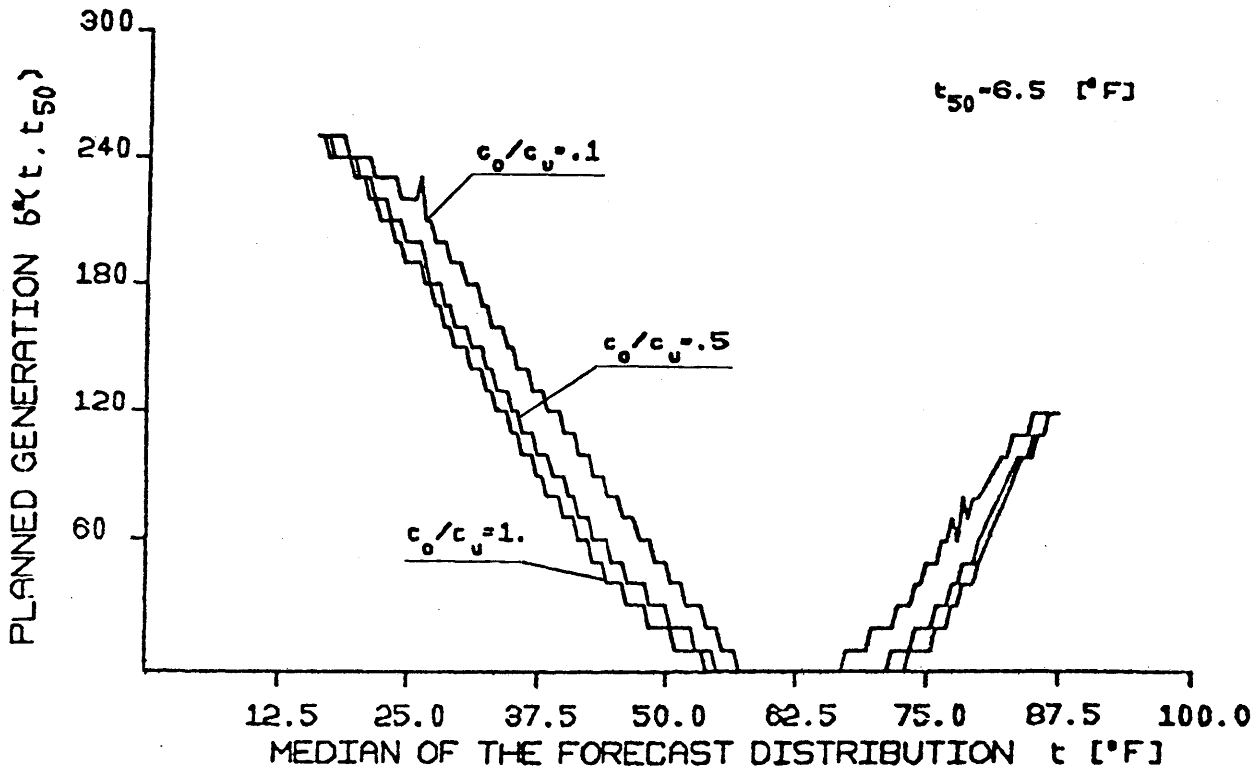

Figure 4.11 Bayes Decision Functions, $\delta^*(\cdot,t_{50})$, when c_o/c_u = .1, .5, 1

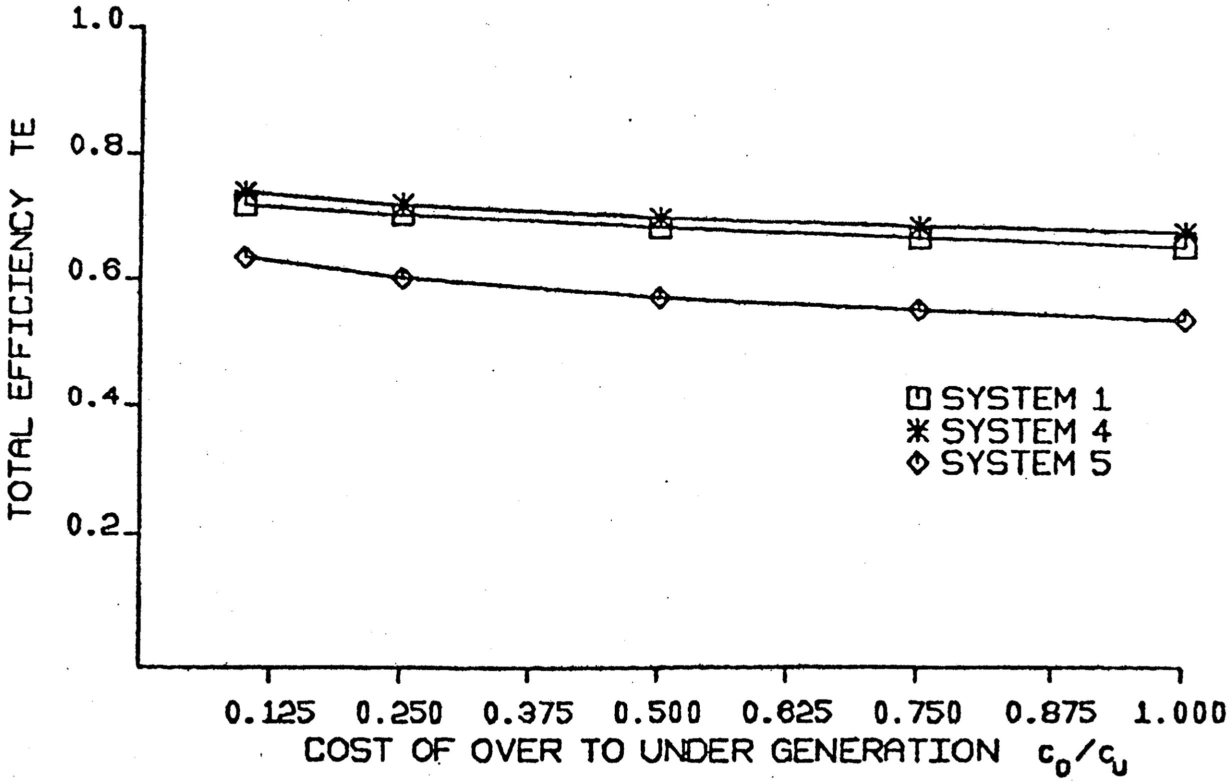

Figure 4.12 Total Efficiency versus Loss Ratio of Over to Under Generation

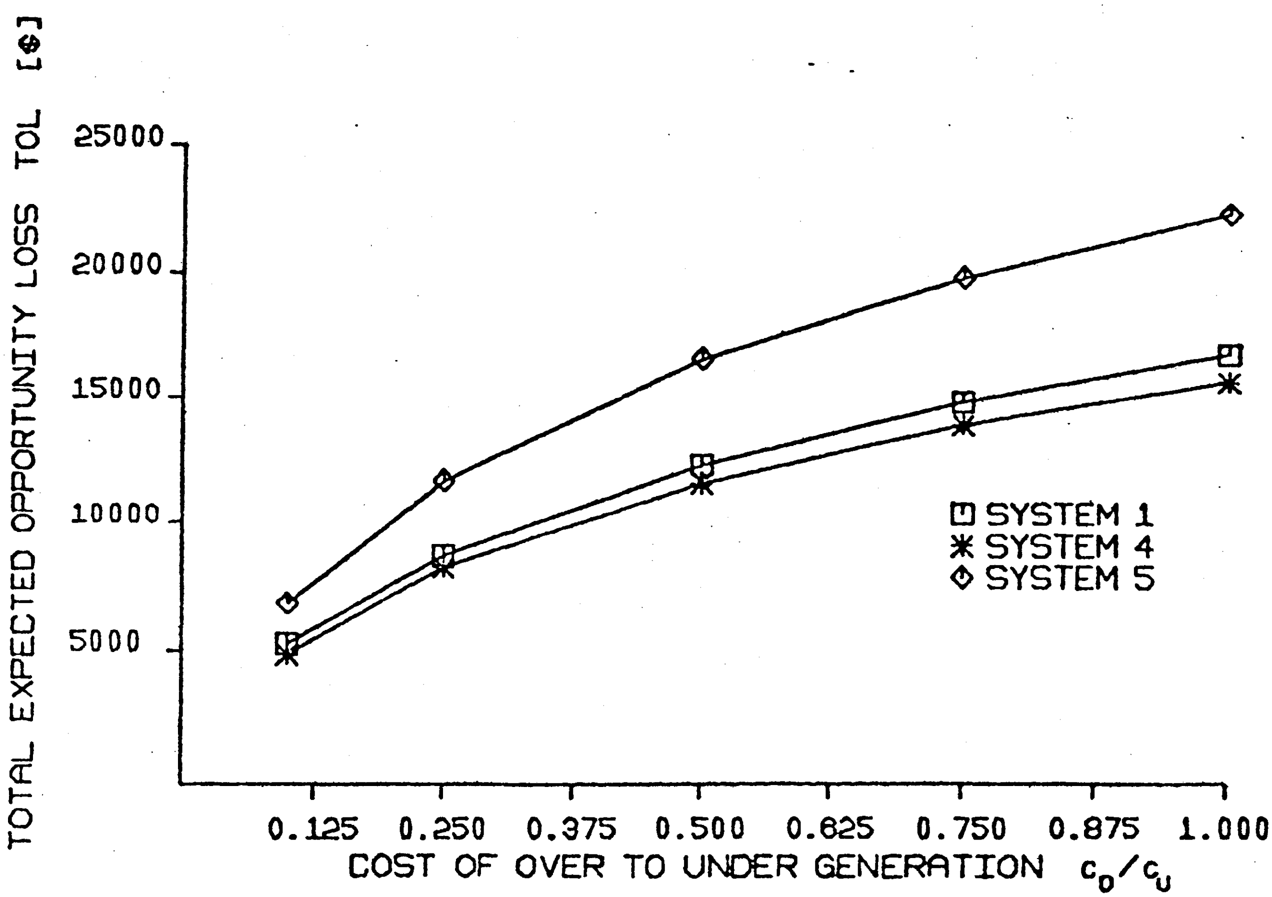

Figure 4.13 Total Expected Opportunity Loss versus Loss Ratio of Over to Under Generation

4.5.3 Heating/Cooling Gap of the Load Operator

The interplay between the heating/cooling gap and the total efficiencies of Systems 1, 4, and 5 is shown in Figure 4.14. Again, all systems have peak efficiency at $(t_c - t_b) = 10^{\circ}F$ for the reasons given in Section 3.5.4. The relative differences between the systems are almost invariant with $(t_c - t_b)$. Therefore, uncertain knowledge of the heating/cooling gap may affect ultimate system performance but will not affect the decision maker's choice as to which is the appropriate forecast to employ and should it be used optimally or not. The total expected opportunity losses are displayed in Figure 4.15.

4.6 Gains from Probabilistic Forecasts

An economic gain expected by a decision maker utilizing probabilistic forecasts instead of categorical forecasts is defined as the difference between the risks: RC - RP for optimal decision procedures, and $\overline{RC} - \overline{RP}$ for nonoptimal decision procedures. We investigate the sensitivity of these gains to the precision of the categorical forecast $\tau_{t|\theta}$. Such a sensitivity analysis is possible because the categorical forecast is defined as the median of the probabilistic forecast. Thus, the precision $\tau_{t|\theta} = \tau_{\varepsilon}$ of the likelihood function in the categorical forecasting scheme (as defined in Section 3.3) is equal to the precision τ_{ε} of the marginal distribution of errors of the medians in the probabilistic forecasting scheme (as defined in Section 4.3). The precision τ_{ε} is varied by perturbing the four ordinates of the distribution function $H_{\varepsilon|t_{50}}$ extracted from Table 4.2. We maintain the spike at $\varepsilon = 0$, however, to parallel the analysis of the categorical forecast precision presented in Section 3.5.1.

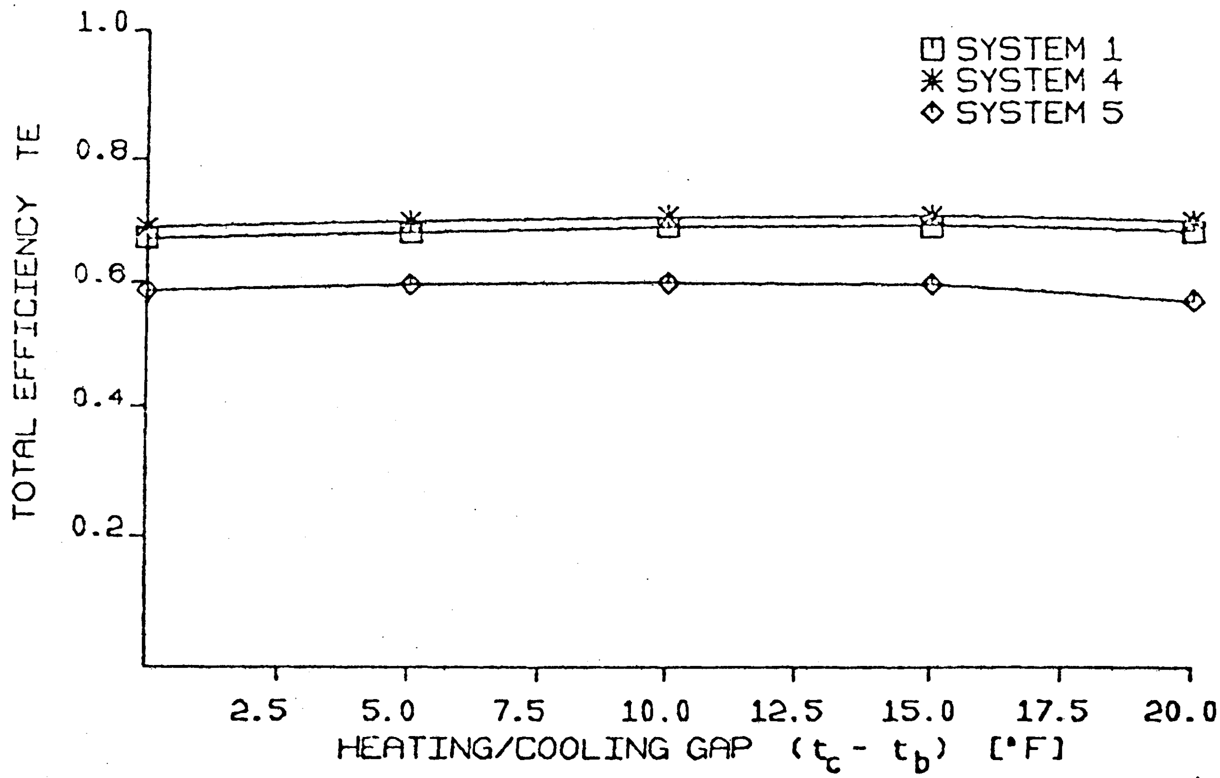

Figure 4.14 Total Efficiency versus Heating/Cooling Gap

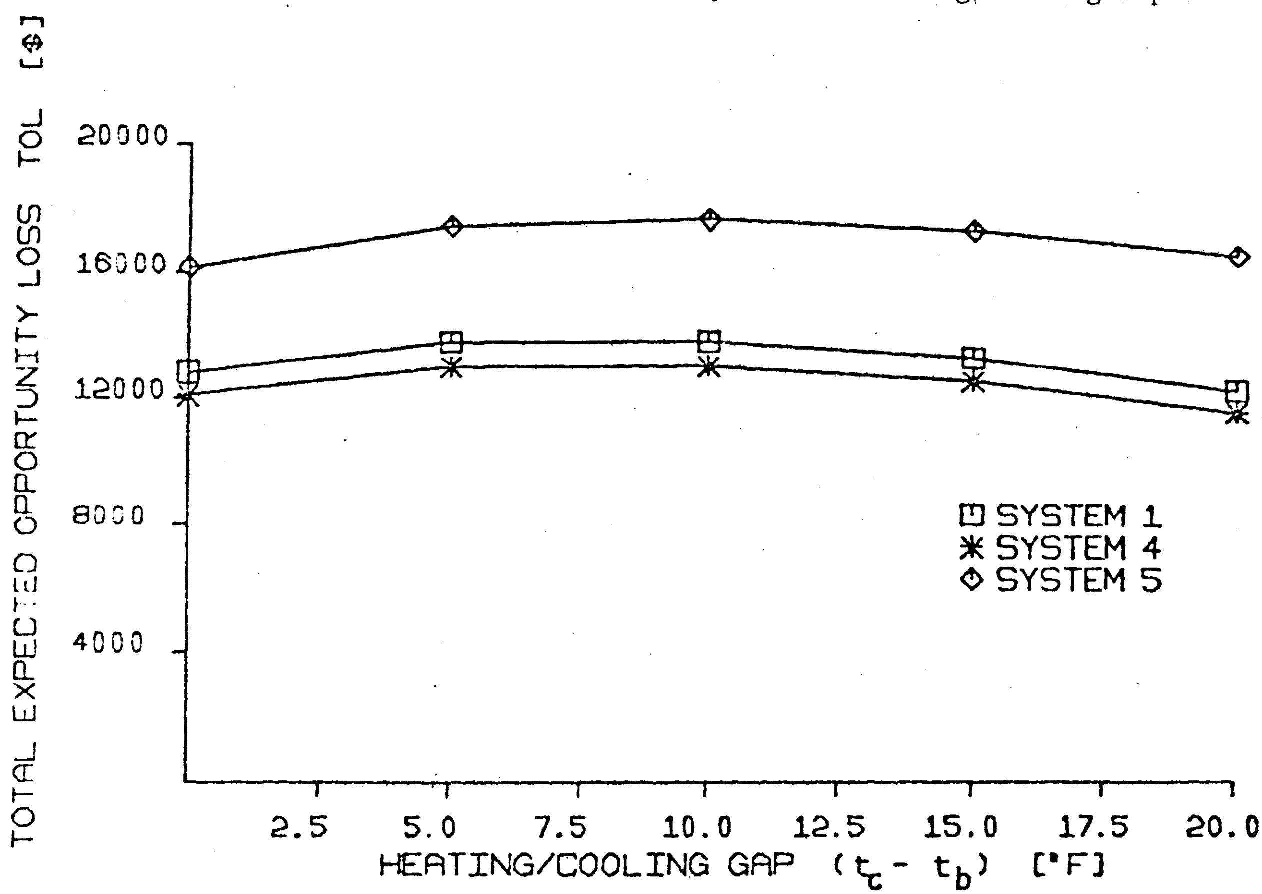

Figure 4.15 Total Expected Opportunity Loss versus Heating/Cooling Gap

The results shown in Figure 4.16 indicate that the quantification of forecast uncertainty is increasingly valuable as the precision of categorical forecasts decreases. For example, given $\tau_{t|\theta} = .016\ [^{o}F]^{-2}$, the economic gain from using probabilistic forecasts amounts to 2479 $/day, or roughly .9 million dollars per year. There is, however, little economic impetus to use probabilistic forecasts when the state variable is very predictable, i.e., as $\tau_{t|\theta} \to \infty$.

Decision makers who ignore uncertainty of categorical forecasts have much more to gain from probabilistic forecasts as is shown in Figure 4.17. The trend of gains as a function of the precision is virtually the same as in Figure 4.16, but the gains are substantially higher.

It appears, thus, that potential economic benefits to be derived from probabilistic forecasts of atmospheric variables that are less predictable than temperature (e.g., wind speed, cloud cover, precipitation) could be substantial.

4.7 Summary

A model for the use and evaluation of probabilistic forecasts has been developed. The applicability of the model has been illustrated in the context of scheduling of power generation based on daily average temperature forecasts. The probabilistic forecasts have been of the type wherein the forecaster quantifies his degree of uncertainty in terms of variable-width, fixed-probability credible intervals.

It has been found that (1) probabilistic forecasts are more valuable than categorical forecasts when used in the same (optimal or nonoptimal) decision procedure; (2) the gains from probabilistic forecasts are relatively greater for nonoptimal decision procedures (which ignore forecast

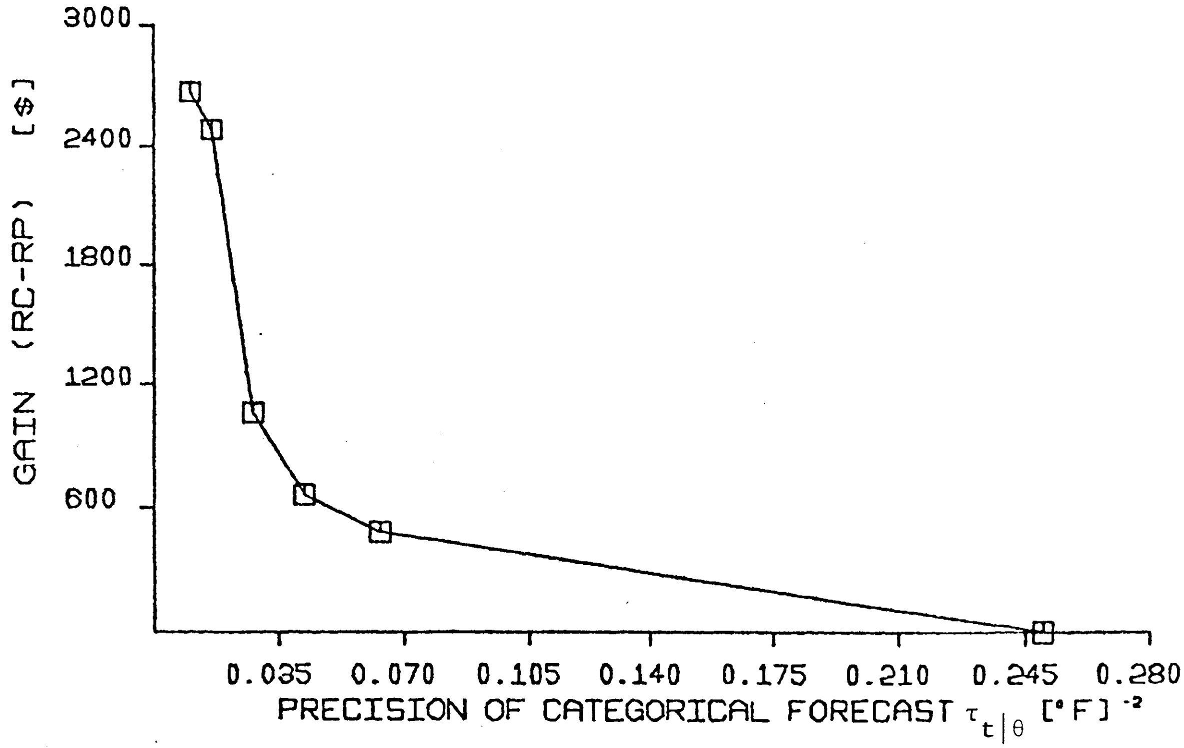

Figure 4.16 Gain from a Probabilistic Forecast as a Function of the Precision of a Categorical Forecast for an Optimal Decision Procedure

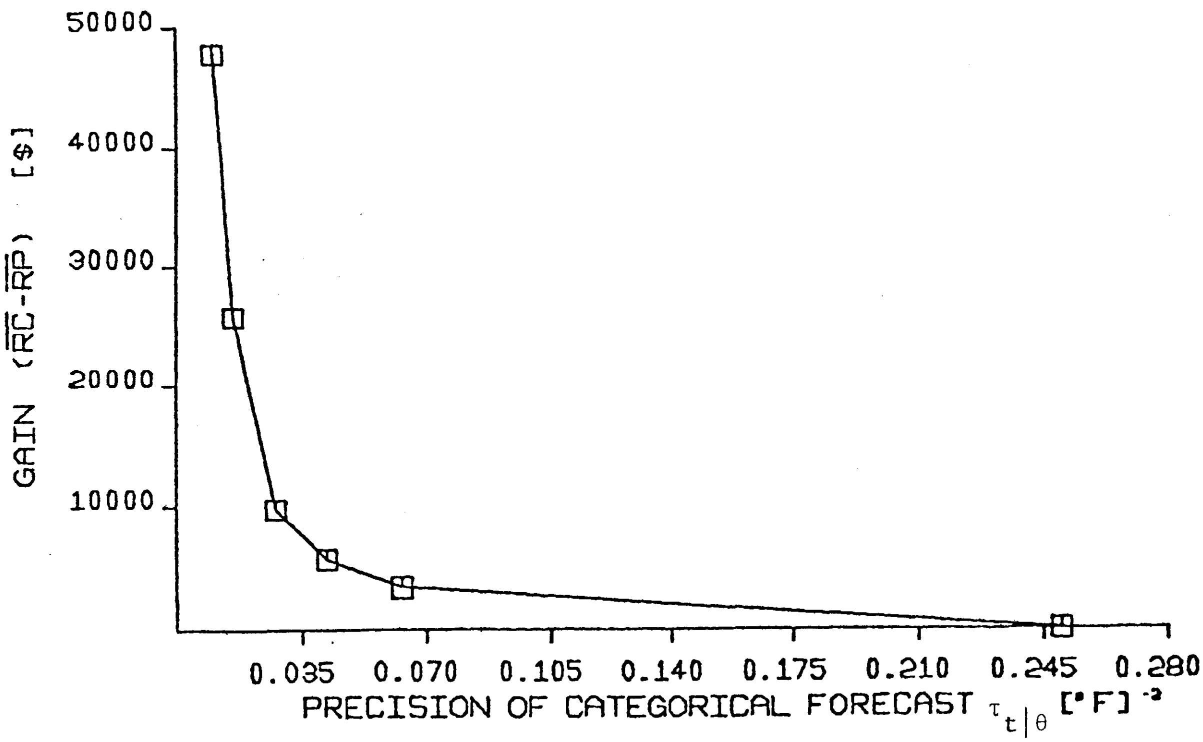

Figure 4.17 Gain from a Probabilistic Forecast as a Function of the Precision of a Categorical Forecast for a Nonoptimal Decision Procedure

uncertainty) than for the optimal ones (which account for forecast uncertainty); (3) categorical forecasts used in an optimal decision procedure are more valuable than probabilistic forecasts used in a non-optimal decision procedure; (4) the gains from probabilistic forecasts increase dramatically as the precision of categorical forecasts decreases for both optimal and nonoptimal decision procedures.

Though solution of the probabilistic forecast-decision model requires an order of magnitude more CPU time than solution of the categorical forecast-decision model, this fact is deemed irrelevant in view of the expected gains and adaptive formulation of the model due to which most users will only have to compute decision functions once a month, season, or year.

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

Decision models, cast in a Bayesian framework, have been formulated to determine optimal decisions on the basis of real-time categorical and probabilistic forecasts of a continuous state variable. The solution provided by a model is in the form of a decision function specifying for every forecast the optimal decision. For evaluation purposes, a forecast-decision system has been defined as a cascade coupling of a forecasting scheme and a decision procedure. This conceptualization enabled explicit quantitative evaluation of the performance of each system component as well as the total system using a set of decision-theoretic measures: values, efficiencies, and expected opportunity losses. Categorical and probabilistic forecasts have been evaluated in relation to naive (climatological) and perfect forecasts. For each type of forecast, two decision procedures have been evaluated: an optimal (Bayesian) procedure which accounts for forecast uncertainty and a more conventional, nonoptimal, procedure which disregards forecast uncertainty and prescribes decisions as if the forecasts were perfect.

The probabilistic forecasts have been of the type wherein the forecaster quantifies his degree of uncertainty in terms of variable-width, fixed-probability credible intervals. A technique for transforming the credible intervals into continuous forecast distributions has been suggested. The experimental results reported by Murphy and Winkler (1974, 1975), on both categorical and probabilistic subjective daily temperature forecasts, have served as a guide for constructing the likelihood functions.

A hypothetical case study concerning the short-term scheduling of power generation in an electric system on the basis of a single-period temperature forecast has been developed to illustrate the applicability of the models. Quantitative comparisons have been made between the risks incurred using the categorical and probabilistic daily average temperature forecasts in both optimal and nonoptimal decision procedures.

Ordinal relations which hold, in general, among the *ex ante* risks of perfect, probabilistic, categorical, and naive forecasts, when used in an optimal (Bayesian) decision procedure, have been confirmed experimentally.

5.2 Conclusions

- The following general facts have been demonstrated theoretically and experimentally.

1. When forecasts are used in an optimal (Bayesian) decision procedure, then, under certain well defined conditions, probabilistic forecasts are at least as valuable as categorical forecasts, and categorical forecasts are always at least as valuable as naive (climatological) forecasts.

2. When forecasts are used in a nonoptimal decision procedure which ignores forecast uncertainty, opportunity losses are always incurred; furthermore, categorical forecasts may be less valuable than naive forecasts, and probabilistic forecasts may be less valuable than either categorical or naive forecasts.

- The following conclusions are predicated on the structure of the model for short-term scheduling of power generation.

3. The gains from using probabilistic forecasts instead of categorical forecasts are greater for decision makers who employ nonoptimal decision procedures (which ignore forecast uncertainty) than for those

who already employ optimal decision procedures (which account for forecast uncertainty).

4. The gains from using probabilistic forecasts instead of categorical forecasts increase as the predictability of the state of nature diminishes; this holds for both optimal and nonoptimal decision procedures.

- The following results are specific to the hypothetical case study.

5. Optimally used probabilistic forecasts are 2% and 70% more valuable than optimally used categorical and naive forecasts, respectively.

6. Nonoptimally used probabilistic forecasts are 15% and 115% more valuable than nonoptimally used categorical and naive forecasts, respectively.

7. Optimally used categorical forecasts are 11% more valuable than nonoptimally used probabilistic forecasts.

- The above results provide strong economic arguments for the following general recommendations.

8. *Concerning forecasting.* More theoretical and experimental research should be directed towards developing probabilistic forecasting schemes, particularly for phenomena with moderate and low predictability since these are the cases wherein most economic gains from quantification of uncertainty can be accrued.

9. *Concerning decision making.* More attention should be given to Bayesian decision procedures since they are the cornerstone of optimal decision making based on uncertain forecasts.

APPENDIX A

BLACKWELL's THEOREM APPLIED TO FORECASTS

The following development is a restatement and interpretation of Blackwell's (1953) theorem in the context of a forecast-decision problem. For further discussions of the theorem see DeGroot (1970, p.433-439) and Hilton (1981).

Let $x \in X$ denote a forecast of the state $\omega \in \Omega$. In the case of a categorical forecasting scheme, $X = \Omega$ so that $x - \omega$ represents the forecast error. In the case of a probabilistic forecasting scheme, x is a parameter, or a vector of parameters, for a family of probability density functions $R = \{\rho(\cdot|x) : x \in X\}$ on the space Ω. The probabilistic forecast of ω is a density $\rho(\cdot|x) \in R$. Since the specification of $x \in X$ uniquely determines the forecast density $\rho \in R$, we shall use the term "probabilistic forecast" in reference to both x and ρ.

Suppose there are two forecasting schemes, categorical or probabilistic, issuing forecasts $x_1 \in X_1$ and $x_2 \in X_2$ of the state $\omega \in \Omega$. Let their likelihood families be $\Phi_i = \{\phi_i(\cdot|\omega) : \omega \in \Omega\}$, where $\phi_i(\cdot|\omega)$ is a probability density function on X_i $(i=1,2)$.

Definition 1. Forecast x_2 is sufficient for forecast x_1 if there exists a nonnegative function h on the product space $X_1 \times X_2$ satisfying the following three relations:

$$\phi_1(x_1|\omega) = \int_{X_2} h(x_1,x_2)\phi_2(x_2|\omega)\, dx_2 \quad \text{for} \quad \omega \in \Omega \text{ and } x_1 \in X_1\,, \tag{1}$$

$$\int_{X_1} h(x_1,x_2)\, dx_1 = 1 \quad \text{for} \quad x_2 \in X_2\,, \tag{2}$$

$$0 < \int_{X_2} h(x_1,x_2)\ dx_2 < \infty \quad \text{for} \quad x_1 \in X_1 \quad . \qquad (3)$$

A nonnegative function h satisfying (2) is called a *stochastic transformation* from X_2 to X_1. For every $x_2 \in X_2$, the function $h(\cdot,x_2)$ represents a probability density on X_1.

Insight into the notion of sufficiency can be obtained by considering the task of simulation of forecasts (DeGroot, 1970, p.434). For any fixed value of the state $\omega \in \Omega$, forecast x_2 can be generated from the density $\phi_2(\cdot|\omega)$. Forecast x_1 can be generated either in one step from the density $\phi_1(\cdot|\omega)$ or, according to the relation (1), in two steps. First, forecast x_2 is generated from $\phi_2(\cdot|\omega)$, and next, given this x_2, forecast x_1 is generated from $h(\cdot,x_2)$. Thus, in comparison to the generator of x_2, the two-step generator of x_1 involves an auxiliary randomization. An intuitively obvious implication is that having a choice between the two forecasts, the decision maker should never choose x_1 because choosing x_1 is equivalent to choosing x_2 and then subjecting it to a stochastic transformation $h(\cdot,x_2)$. Since such a transformation only adds "noise" to the information contained in x_2, it is logical to expect forecast x_1 to be no more valuable than forecast x_2. We shall now state this fact formally.

Let g be a prior density on Ω. The posterior density is defined by

$$\phi_i(\omega|x_i) = \frac{\phi_i(x_i|\omega)g(\omega)}{g_i(x_i)} \quad \text{for} \quad i=1,2 \quad ,$$

where the predictive density is

$$g_i(x_i) = \int_{\Omega} \phi_i(x_i|\omega)g(\omega)\, d\omega \quad \text{for} \quad i=1,2.$$

For any set of decisions $A = \{a\}$ and for any real-valued loss function ℓ defined on the product space $\Omega \times A$, the Bayes risk is given by

$$R_i = \int_{X_i} [\min_a \int_{\Omega} \ell(\omega,a)\phi_i(\omega|x_i)\, d\omega] g_i(x_i)\, dx_i \quad \text{for} \quad i=1,2 \quad .$$

Theorem 1 (Blackwell, 1953). If forecast x_2 is sufficient for forecast x_1, then $R_2 \leq R_1$.

The significance of Blackwell's theorem lies in that it enables one to rank two, or more, forecast systems in terms of their economic values without the necessity of performing the entire decision analysis, i.e., without explicitly specifying the set of decisions A, the loss function ℓ, and the prior density g. A straightforward implication of the theorem is that if forecast x_2 issued by one forecast system is sufficient for forecast x_1 issued by another forecast system, then the economic value of forecast x_2 is at least as great as the economic value of forecast x_1. The forecasts x_1 and x_2 may be both categorical, or both probabilistic, or one categorical and another probabilistic. The key behavioral assumption, of course, is that both forecasts are, or would be, used in an optimal (Bayesian) decision procedure. Let us now consider four examples.

Example 1. Let the state $\omega \in \{0,1\}$ describe a binary event such as the occurrence of rain $(\omega = 1)$ or no rain $(\omega = 0)$ tomorrow. A categorical forecast v of ω indicates either that rain will occur $(v = 1)$ or that it will not occur $(v = 0)$. A probabilistic forecast p $(0 \leq p \leq 1)$ of ω indicates the probability of the event $\{\omega = 1\}$, i.e., $p = Pr\{\omega = 1\}$. The probability p encodes the forecaster's degree of belief in the statement "it will rain tomorrow."

Now suppose that the forecaster prepares a probabilistic forecast p, and that a categorical forecast v is obtained from the probabilistic forecast as follows:

$$v = \begin{cases} 1 & \text{if} \quad p \geq p^* \\ 0 & \text{if} \quad p < p^* \end{cases}$$

where $p^* \varepsilon (0,1)$ is a fixed parameter. It is easy to verify that the probabilistic forecast is sufficient for the categorical forecast. Specifically, the stochastic transformation h takes the form

$$h(v=1,p) = \begin{cases} 1 & \text{if} \quad p \geq p^* \\ 0 & \text{if} \quad p < p^* \end{cases}$$

$$h(v=0,p) = 1 - h(v=1,p) \quad \text{for} \quad p \ \varepsilon \ [0,1].$$

Via Theorem 1, we conclude that no matter the decision problem (as defined by A, ℓ, and g), the decision maker is always better off by using a probabilistic forecast p rather than a categorical forecast v provided, of course, that he employs an optimal decision procedure. Decision procedures for the use of categorical and probabilistic forecasts of binary events can be found in Winkler and Murphy (1979).

Example 2 (DeGroot, 1970, p.438). As in Example 1, let the state $\omega \ \varepsilon \ \{0,1\}$ describe the occurrence of a binary event, and $y \ \varepsilon \ \{0,1\}$ and $z \ \varepsilon \ \{0,1\}$ denote two categorical forecasts of the state ω issued by two distinct forecast systems. Suppose that on the basis of records of past forecasts and actual values of the state, the likelihood functions have been estimated as follows. For System Y:

$$\phi_Y(y=1|\omega=0) = 1 - \phi_Y(y=0|\omega=0) = \frac{1}{3} \quad ,$$

$$\phi_Y(y=1|\omega=1) = 1 - \phi_Y(y=0|\omega=1) = \frac{1}{2} \quad ;$$

and for System Z:

$$\phi_Z(z=1|\omega=0) = 1 - \phi_Z(z=0|\omega=0) = \frac{3}{5} \quad ,$$

$$\phi_Z(z=1|\omega=1) = 1 - \phi_Z(z=0|\omega=1) = \frac{1}{5} \quad .$$

To demonstrate that forecast z is sufficient for forecast y, and hence System Z is superior in terms of the economic value to System Y, we must show that there exists a stochastic transformation h such that

$$\phi_Y(y|\omega=0) = h(y,z=0)\phi_Z(z=0|\omega=0) + h(y,z=1)\phi_Z(z=1|\omega=0) \quad ,$$

$$\phi_Y(y|\omega=1) = h(y,z=0)\phi_Z(z=0|\omega=1) + h(y,z=1)\phi_Z(z=1|\omega=1) \quad ,$$

for $y = 0,1$. This system of four equations has to be solved for four numbers $h(y,z)$, $y = 0,1$ and $z = 0,1$. But since relation (2) requires $h(0,0) + h(1,0) = h(0,1) + h(1,1) = 1$, it is sufficient to solve the pair of equations by letting $y = 1$. Thus we obtain

$$\frac{1}{3} = \frac{2}{5}h(1,0) + \frac{3}{5}h(1,1) \quad ,$$

$$\frac{1}{2} = \frac{4}{5}h(1,0) + \frac{1}{5}h(1,1) \quad .$$

The solution is $h(1,0) = \frac{7}{12}$ and $h(1,1) = \frac{1}{6}$. Whence, $h(0,0) = \frac{5}{12}$ and $h(0,1) = \frac{5}{6}$. Since h satisfies the properties stated in Definition 1, forecast z is sufficient for forecast y, and, by virtue of Theorem 1, the economic value of System Z is equal to or greater than the economic value of System Y.

It is possible that the reader finds the above conclusion in contradiction to his or her intuitive inference on the basis of the likelihood functions ϕ_Y and ϕ_Z. They indicate that the proportion of correct forecasts is larger in System Y than in System Z:

$$\phi_Y(y=0|\omega=0) = \frac{2}{3} > \phi_Z(z=0|\omega=0) = \frac{2}{5} \quad ,$$

$$\phi_Y(y=1|\omega=1) = \frac{1}{2} > \phi_Z(z=1|\omega=1) = \frac{1}{5} \quad .$$

But the proportion of correct forecasts is a misleading measure of the forecast value. What is important is the detectability; that is, the ability of the forecaster to discriminate between the hydrometeorologic conditions leading to rain $(\omega=1)$ or no rain $(\omega=0)$. For example, the likelihood function

$$\phi_X(x=1|\omega=0) = 1 - \phi_X(x=0|\omega=0) = 0 \quad ,$$

$$\phi_X(x=1|\omega=1) = 1 - \phi_X(x=0|\omega=1) = 1 \quad ,$$

indicates perfect detectability, and so does the likelihood function

$$\phi_X(x=1|\omega=0) = 1 - \phi_X(x=0|\omega=0) = 1 \quad ,$$

$$\phi_X(x=1|\omega=1) = 1 - \phi_X(x=0|\omega=1) = 0 \quad ,$$

although the forecasts are never "correct." The latter example is, of course, an extreme case wherein the forecaster perfectly discriminates between the two events, $\{\omega=0\}$ and $\{\omega=1\}$, but "mislabels" them. This mislabeling, or lack of correctness, is of no detrimental consequence if the decision maker interprets forecast x = 1 as being indicative of no rain while forecast $x = 0$ as being indicative of rain. In a Bayesian decision framework, the forecast is treated as an observation, or a signal, to be used for revising the decision maker's prior knowledge about the state. The "interpretation" is performed via Bayes theorem. It should be apparent, therefore, that regardless of which of the above likelihood functions represents the forecast System X , the posterior probability function of ω conditional upon x represents perfect information about ω for any prior probability function. Thus in both cases, forecast x is sufficient for, and, therefore, at least as valuable as forecast z .

Example 3. Let v be a categorical forecast of the state $\omega \in \Omega \subseteq Re$, and let $\underline{\mu} = (v, \underline{\nu})$ be a vector of parameters of a probabilistic forecast which is given in the form of a density function $\rho(\cdot|\underline{\mu})$ of ω, with ρ being a member of a family $R = \{\rho(\cdot|\underline{\mu}) : \text{all } \underline{\mu}\}$. It is easy to verify that since the parameter vector $\underline{\mu}$ includes as one of its components the categorical forecast v , the probabilistic forecast is sufficient for the categorical forecast. This is precisely the relationship assumed between categorical and probabilistic forecasts in Chapter 4 and in Appendix B. Thus by virtue of Theorem 1, the relationship between the risks is: $RP \leq RC$. In other words, probabilistic forecasts are at least as valuable as categorical forecasts. For proof of Theorem 1 in this special case see Appendix B.

Example 4 (DeGroot, 1970, p.348). Let $\omega \in \Omega \subseteq Re$ denote a state, and y and z denote two categorical forecasts of the state ω issued by two distinct forecast systems. These systems are characterized by their likelihood functions as follows. For any given value $\omega \in \Omega$, $\phi_Y(\cdot|\omega)$ is normal with mean ω and precision $\frac{1}{3}$, while $\phi_Z(\cdot|\omega)$ is also normal with mean ω and precision 1. Since the precision of forecast z is higher than the precision of forecast y , one would intuitively expect System Z to be more valuable than System Y. This is indeed the case and the formal argument proceeds as follows. Define a variate x, independent of z and ω, having a normal distribution with mean 0 and precision $\frac{1}{2}$. Then, for any given $\omega \in \Omega$, the variate $x + z$ has the same distribution as y. That is, y can be obtained from z by an auxiliary randomization. Hence, z is sufficient for y, and, consequently, the value of the forecast System Z is as least as great as the value of the forecast System Y.

APPENDIX B

ORDINAL RELATIONS BETWEEN BAYES RISKS

Consider a statistical decision problem defined by the following elements: ω - state of nature, a - decision, $\ell(\omega,a)$ - loss function, $g(\omega)$ - prior density, v - categorical forecast of ω, $f(v|\omega)$ - likelihood function of categorical forecasts, $\underline{\mu} = (v,\underline{\nu})$ - vector of statistics of a probabilistic forecast, $\rho(\omega|\underline{\mu})$ - forecast density (probabilistic forecast of ω), $\phi(\underline{\mu}|\omega)$ - likelihood function of probabilistic forecasts.

The posterior density of ω given a forecast is given by Bayes theorem. For a categorical forecast v, the posterior density

$$f(\omega|v) = \frac{f(v|\omega)\ g(\omega)}{g(v)} ,$$

where the predictive density

$$g(v) = \int f(v|\omega)\ g(\omega)\ d\omega .$$

For a probabilistic forecast $\rho(\omega|\underline{\mu})$, the posterior density

$$\phi(\omega|\underline{\mu}) = \frac{\phi(\underline{\mu}|\omega)\ g(\omega)}{g(\underline{\mu})} ,$$

where the predictive density

$$g(\underline{\mu}) = \int \phi(\underline{\mu}|\omega)\ g(\omega)\ d\omega .$$

The Bayes risks of optimal decisions or decision functions are defined as follows. For a naive forecast

$$RN = \min_a \int \ell(\omega,a)\ g(\omega)\ d\omega\ ,$$

for a categorical forecast

$$RC = \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv\ ,$$

for a probabilistic forecast

$$RP = \int [\min_a \int \ell(\omega,a)\ \phi(\omega|\underline{\mu})\ d\omega]\ g(\underline{\mu})\ d\underline{\mu}\ ,$$

and for a perfect forecast

$$RF = \int \min_a \ell(\omega,a)\ g(\omega)\ d\omega\ .$$

Theorem 1. $RF \leq RP \leq RC \leq RN$ always.

Proof. We begin by demonstrating the first inequality.

$$\begin{aligned} RF &= \int \min_a \ell(\omega,a)\ g(\omega)\ d\omega \\ &= \min_a \int \min_a \ell(\omega,a)[\int \phi(\omega|\underline{\mu})\ g(\underline{\mu})\ d\underline{\mu}]\ d\omega \\ &= \int [\min_a \int \min_a \ell(\omega,a)\ \phi(\omega|\underline{\mu})\ g(\underline{\mu})\ d\omega]\ d\underline{\mu} \end{aligned}$$

$$= \int [\min_a \int \min_a \ell(\omega,a)\ \phi(\omega|\underline{\mu})\ d\omega]\ g(\underline{\mu})\ d\underline{\mu}$$

$$\leq \int [\min_a \int \ell(\omega,a)\ \phi(\omega|\underline{\mu})\ d\omega]\ g(\underline{\mu})\ d\underline{\mu} = RP\ .$$

The second inequality is obtained as follows.

$$RP = \int [\min_a \int \ell(\omega,a)\ \phi(\omega|\underline{\mu})\ d\omega]\ g(\underline{\mu})\ d\underline{\mu}$$

$$= \int \min_a \int [\min_a \int \ell(\omega,a)\ \phi(\omega|v,\underline{\nu})\ d\omega]\ g(\underline{\nu}|v)\ g(v)\ d\underline{\nu}\ dv$$

$$\leq \int \{\min_a \int [\int \ell(\omega,a)\ \phi(\omega|v,\underline{\nu})\ d\omega]\ g(\underline{\nu}|v)\ d\underline{\nu}\}\ g(v)\ dv$$

$$\leq \int \{\min_a \int \ell(\omega,a) [\int \phi(\omega|v,\underline{\nu})\ g(\underline{\nu}|v)\ d\underline{\nu}]\ d\omega\}\ g(v)\ dv$$

$$\leq \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv = RC\ .$$

The third inequality is obtained as follows.

$$RC = \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv$$

$$= \min_a \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv$$

$$\leq \min_a \iint \ell(\omega,a)\ f(\omega|v)\ g(v)\ dv\ d\omega$$

$$\leq \min_a \int \ell(\omega,a) [\int f(\omega|v)\ g(v)\ dv]\ d\omega$$

$$\leq \min_a \int \ell(\omega,a)\ g(\omega)\ d\omega = RN\ .$$

Discussion. If probabilistic forecasts demonstrate no improvement, in the statistical sense, over categorical forecasts, i.e., the vector of statistics $\underline{\nu}$ has no predictive power whatsoever, then $\phi(\omega|v,\underline{\nu}) = f(\omega|v)$ and $g(v,\underline{\nu}) = g(v)\ g(\underline{\nu})$. Consequently RP = RC:

$$RP = \iint [\min_a \int \ell(\omega,a)\ \phi(\omega|v,\underline{\nu})\ d\omega]\ g(v,\underline{\nu})\ dv\ d\underline{\nu}$$

$$= \int \{ \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv \}\ g(\underline{\nu})\ d\underline{\nu}$$

$$= \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv = RC\ .$$

If categorical forecasts have no predictive power whatsoever, then the variates ω and v are independent so that $f(\omega|v) = g(\omega)$. In such a case RC = RN:

$$RC = \int [\min_a \int \ell(\omega,a)\ f(\omega|v)\ d\omega]\ g(v)\ dv$$

$$= \int [\min_a \int \ell(\omega,a)\ g(\omega)\ d\omega]\ g(v)\ dv$$

$$= \min_a \int \ell(\omega,a)\ g(\omega)\ d\omega = RN\ .$$

APPENDIX C

VALUE OF FORECASTS FOR A QUADRATIC DECISION PROBLEM

C1. Preliminaries

Consider a statistical decision problem defined in Appendix B wherein

$$\ell(\omega,a) = (\omega - a)^2 .$$

We shall, first, derive the expressions for all risks defined in Chapters 3 and 4, and then state the concluding theorems and discuss their implications from the viewpoint of both a forecaster and a decision maker.

The following notation will be used throughout, in addition to the notation of Appendix B:

- M_ω, S^2_ω - prior mean and variance of ω (computed from the prior density $g(\omega)$),
- $m_{\omega|v}$, $s^2_{\omega|v}$ - posterior mean and variance of ω given a categorical forecast v of ω (computed from the posterior density $f(\omega|v)$),
- $m_{\omega|\underline{\mu}}$, $s^2_{\omega|\underline{\mu}}$ - posterior mean and variance of ω given a probabilistic forecast $\rho(\omega|\underline{\mu})$ of ω (computed from the posterior density $\phi(\omega|\underline{\mu})$),
- $n_{\omega|\underline{\mu}}$, $\sigma^2_{\omega|\underline{\mu}}$ - forecast mean and variance of ω (computed from the forecast density $\rho(\omega|\underline{\mu})$),
- M_v, S^2_v - mean and variance of v (computed from the predictive density $g(v)$),
- $r_{\omega v}$ - correlation between ω and v.

C2. Risks

Naive forecast - optimal decision. For any decision a, the prior risk

$$
\begin{aligned}
LN(a) &= E[(\omega - a)^2] \\
&= S_\omega^2 + M_\omega^2 - 2a\, M_\omega + a^2 \\
&= S_\omega^2 + (M_\omega - a)^2 \; .
\end{aligned}
$$

So

$$
\frac{dLN(a)}{da} = -2 \; (M_\omega - a) \; ,
$$

wherefrom the Bayes decision minimizing LN(a) is found to be $a^* = M_\omega$, and the associated Bayes risk

$$
\begin{aligned}
RN &= LN(a^*) \\
&= S_\omega^2 \; . \qquad (1)
\end{aligned}
$$

That is, the Bayes risk is equal to the prior variance of the state of nature.

Naive forecast - nonoptimal decision. Let $\hat{v}$ denote a naive forecast of ω , so that the minimum loss

$$
\overline{LN} = \min_a \; (\hat{v} - a)^2 = 0 \; ,
$$

where the solution is $\hat{a} = \hat{v}$. The risk of the decision $\hat{a}$ is

$$
\begin{aligned}
\overline{RN} &= E[(\omega - \hat{v})^2] \\
&= S_\omega^2 + (M_\omega - \hat{v})^2 \\
&= RN + (M_\omega - \hat{v})^2 \; . \qquad (2)
\end{aligned}
$$

Clearly $\overline{RN} > RN$ unless the naive forecast is taken to be the prior mean of the state of nature, in which case $\overline{RN} = RN$.

Categorical forecast - optimal decision. For any decision function δ and a given categorical forecast v, the posterior risk function

$$LC(v,\delta) = E[(\omega - \delta(v))^2 | v]$$
$$= s^2_{\omega|v} + (m_{\omega|v} - \delta(v))^2 ,$$

and

$$\frac{dLC(v,\delta)}{d\delta(v)} = -2(m_{\omega|v} - \delta(v)) .$$

Thus, $LC(v,\delta)$ is minimized by a Bayes decision function $\delta^*(v) = m_{\omega|v}$, and the associated Bayes risk

$$\begin{aligned} RC &= E[LC(v,\delta^*)] \\ &= E[s^2_{\omega|v}] \\ &= E[E[\omega^2|v] - E^2[\omega|v]] \\ &= E[\omega^2] - E[m^2_{\omega|v}] \\ &= E[\omega^2] - E^2[\omega] - E[m^2_{\omega|v}] + E^2[m_{\omega|v}] \end{aligned}$$

since $E[\omega] = E[m_{\omega|v}]$. Finally,

$$RC = S^2_{\omega} - S^2_{m_{\omega|v}} . \tag{3}$$

Thus the Bayes risk depends on the state of nature ω only through the difference between the prior variance of ω and the predictive variance of the posterior mean of ω. Note that if the categorical forecasts are perfect, i.e., $\omega = v$, then $m_{\omega|v} = \omega$ and $S^2_{m_{\omega|v}} = S^2_{\omega}$ implying $RC = 0$, as expected.

Categorical forecast - nonoptimal decision. The minimum loss function

$$\overline{LC}(v) = \min_{\delta} (v - \delta(v))^2 = 0 ,$$

where the solution is $\hat{\delta}(v) = v$. The risk of the decision function $\hat{\delta}$ is

$$\begin{aligned}\overline{RC} &= E[(\omega - v)^2] \\ &= E[s^2_{\omega|v} + (m_{\omega|v} - v)^2] \\ &= E[s^2_{\omega|v}] + E[(m_{\omega|v} - v)^2] ,\end{aligned}$$

and recalling the expression for RC, we obtain

$$\begin{aligned}\overline{RC} &= S^2_{\omega} - S^2_{m_{\omega|v}} + E[(m_{\omega|v} - v)^2] \\ &= RC + E[(m_{\omega|v} - v)^2]. \qquad (4)\end{aligned}$$

It is apparent that $\overline{RC} > RC$ unless the categorical forecast is Bayesian, i.e., it is equal to the posterior mean of the state of nature.

For further discussion, it is convenient to derive an alternative expression for $\overline{RC}$:

$$\begin{aligned}\overline{RC} &= E[(\omega - v)^2] \\ &= E[\omega^2] - 2E[v\omega] + E[v^2] \\ &= S^2_{\omega} - 2r_{\omega v}S_{\omega}S_{v} + S^2_{v} + (M_{\omega} - M_{v})^2. \qquad (5)\end{aligned}$$

Probabilistic forecast - optimal decision. Following a development similar to that for categorical forecasts, we find a Bayes decision function

$\delta^*(\underline{\mu}) = m_{\omega|\underline{\mu}}$ whose risk (the Bayes risk) is given by

$$RP = S_{\omega}^2 - S_{m_{\omega|\underline{\mu}}}^2 \quad . \tag{6}$$

Again, the Bayes risk depends on the state of nature ω only through the prior variance of ω and the predictive variance of the posterior mean of ω . Observe that if the posterior mean $m_{\omega|\underline{\mu}}$ is a perfect prediction of ω , i.e., $\omega = m_{\omega|\underline{\mu}}$, then $S_{m_{\omega|\underline{\mu}}}^2 = S_{\omega}^2$, and consequently RP = 0.

Probabilistic forecast - nonoptimal decision. For any decision function δ and a given probabilistic forecast $\rho(\omega|\underline{\mu})$, the risk function

$$\begin{aligned}\overline{LP}(\underline{\mu},\delta) &= E[(\omega - \delta(\underline{\mu}))^2|\underline{\mu}] \\ &= \sigma_{\omega|\underline{\mu}}^2 + (n_{\omega|\underline{\mu}} - \delta(\underline{\mu}))^2\end{aligned}$$

attains its minimum at $\hat{\delta}(\underline{\mu}) = n_{\omega|\underline{\mu}}$. The risk of the decision function $\hat{\delta}$ is

$$\begin{aligned}\overline{RP} &= E[(\omega - n_{\omega|\underline{\mu}})^2] \\ &= E[s_{\omega|\underline{\mu}}^2 + (m_{\omega|\underline{\mu}} - n_{\omega|\underline{\mu}})^2] \\ &= S_{\omega}^2 - S_{m_{\omega|\underline{\mu}}}^2 + E[(m_{\omega|\underline{\mu}} - n_{\omega|\underline{\mu}})^2] \\ &= RP + E[(m_{\omega|\underline{\mu}} - n_{\omega|\underline{\mu}})^2] \quad .\end{aligned} \tag{7}$$

This result is identical to the result for categorical forecasts: $\overline{RP} > RP$ unless the probabilistic forecast is Bayesian (i.e., the mean of the forecast density is equal to the posterior mean of the state of nature), in which case $\overline{RP} = RP$.

Perfect forecast. A forecast is perfect if it indicates the actual value of the state of nature ω with probability one. Under such circumstances, the decision function minimizing the loss is clearly $\delta^*(\omega) = \omega$. and the associated risk RF = 0.

The above results are summarized in Tables 1C and 2C.

C3. Ordinal Relations Between Risks

From Appendices A and B, it is known that

$$RF \leq RP \leq RC \leq RN \ , \tag{8}$$

where the suborders $RF \leq RP \leq RN$ and $RF \leq RC \leq RN$ hold always; the order $RP \leq RC$ holds if the probabilistic forecast is sufficient for the categorical forecast. The order (8) assures a positive value of every type of forecast and the following order: the value of a perfect forecast (RN - RF) is always at least as great as the value of a probabilistic forecast (RN - RP) which, in turn, is always at least as great as the value of a categorical forecast (RN - RC).

It is advantageous to know when a similar order holds for the nonoptimal decision procedures. We shall now state the appropriate conditions.

Theorem 1. $\overline{RP} \leq \overline{RC}$ if

$$S^2_{m_{\omega|\underline{\mu}}} - E[(m_{\omega|\underline{\mu}} - n_{\omega|\underline{\mu}})^2] \geq S^2_{m_{\omega|v}} - E[(m_{\omega|v} - v)^2] \ .$$

Proof. The proof follows immediately from (4) and (7). We note that whenever the categorical forecast v is defined as the mean of the probabilistic forecast $\rho(\omega|\underline{\mu})$, that is $v = n_{\omega|\underline{\mu}}$, then the condition becomes

Table 1C Decisions and Decision Functions for a Quadratic Problem

FORECAST		DECISION: OPTIMAL	DECISION: NONOPTIMAL
	PERFECT	$\delta^{*}(\omega) = \omega$	
	PROBABILISTIC	$\delta^{*}(\underline{\mu}) = m_{\omega\|\underline{\mu}}$	$\hat{\delta}(\underline{\mu}) = n_{\omega\|\underline{\mu}}$
	CATEGORICAL	$\delta^{*}(v) = m_{\omega\|v}$	$\hat{\delta}(v) = v$
	NAIVE	$a^{*} = M_{\omega}$	$\hat{a} = \hat{v}$

Table 2C Risks of the Decisions and Decision Functions for a Quadratic Problem

FORECAST		DECISION: OPTIMAL	DECISION: NONOPTIMAL
	PERFECT	$RF = 0$	
	PROBABILISTIC	$RP = S_{\omega}^{2} - S_{m_{\omega\|\underline{\mu}}}^{2}$	$\overline{RP} = RP + E[(m_{\omega\|\underline{\mu}} - n_{\omega\|\underline{\mu}})^{2}]$
	CATEGORICAL	$RC = S_{\omega}^{2} - S_{m_{\omega\|v}}^{2}$	$\overline{RC} = RC + E[(m_{\omega\|v} - v)^{2}]$
	NAIVE	$RN = S_{\omega}^{2}$	$\overline{RN} = RN + (M_{\omega} - \hat{v})^{2}$

$S^2_{m_{\omega|\underline{\mu}}} \geq S^2_{m_{\omega|v}}$, which is the same condition that assures RP $\leq$ RC.

Theorem 2. $\overline{RC} \leq \overline{RN}$ if

(a) $r_{\omega v} \geq .5 \frac{S_v}{S_\omega}$

whenever $\hat{v} = M_v$; if

(b) $r_{\omega v} \geq .5 \frac{S_v}{S_\omega} + .5 \frac{1}{S_\omega S_v} (M_\omega - M_v)^2$

whenever $\hat{v} = M_\omega$.

Proof. From (2) and (5), $\overline{RC} \leq \overline{RN}$ if

$$-2r_{\omega v} S_\omega S_v + S_v^2 + (M_\omega - M_v)^2 \leq (M_\omega - \hat{v})^2 .$$

The final result follows upon substitution of the appropriate mean for $\hat{v}$.

Discussion. Suppose the marginal densities of ω and v are identical so that $M_\omega = M_v$ and $S_\omega = S_v$. Hence, $\overline{RC} \leq \overline{RN}$ if $r_{\omega v} \geq .5$. It is enlightening to interpret this result. Suppose a decision maker who follows a nonoptimal decision procedure and who has been using a naive forecasting scheme ponders employing categorical forecasts. If these forecasts are not informative enough so that $r_{\omega v} < .5$, then the decision maker's gain from changing the forecasting scheme is negative: $\overline{RN} - \overline{RC} < 0$. In other words, the categorical forecasts have negative value. On the contrary, if an optimal decision procedure were followed, then the categorical forecasts would never have negative value, no matter how low the correlation $r_{\omega v}$.

APPENDIX D

LIMITING BEHAVIOR OF TWO POSTERIOR DENSITIES

Case 1: uniform likelihood. Let the likelihood function be given by

$$f(v|\omega) = \begin{cases} 1/b & \text{if} \quad \omega - b/2 \le v \le \omega + b/2 \ , \\ 0 & \text{otherwise} \ , \end{cases}$$

where $b > 0$ and $-\infty < \omega < \infty$. Its mean and precision are

$$m_{v|\omega} = \omega \ ,$$

$$\tau_{v|\omega} = \frac{12}{b^2} \ .$$

The predictive density is given by

$$g(v) = \int f(v|\omega)\ g(\omega)\ d\omega = \int_{v-b/2}^{v+b/2} \frac{1}{b}\ g(\omega)\ d\omega = \frac{1}{b}\ [G(v+b/2) - G(v-b/2)] \ ,$$

where $-\infty < v < \infty$. The posterior density is given by

$$f(\omega|v) = \frac{f(v|\omega)\ g(\omega)}{g(v)} \ ,$$

$$= \begin{cases} \dfrac{g(\omega)}{G(v+b/2) - G(v-b/2)} & \text{if} \quad v - b/2 \le \omega \le v + b/2 \ , \\ 0 & \text{otherwise} \ . \end{cases}$$

Therefore as $b \to \infty$ and $\tau_{v|\omega} \to 0$, $[G(v + b/2) - G(v - b/2)] \to 1$ and consequently $f(\omega|v) \to g(\omega)$.

Case 2: normal likelihood and prior. Let $g(\omega)$ be a normal density with mean m_ω and precision τ_ω ($\tau_\omega > 0$), and $f(v|\omega)$ be a normal density with mean $m_{v|\omega}$ and precision $\tau_{v|\omega}$ ($\tau_{v|\omega} > 0$). Then the posterior density $f(\omega|v)$ is normal with statistics (*vide*, Raiffa and Schlaifer, 1961, p.54)

$$m_{\omega|v} = \frac{\tau_\omega m_\omega + \tau_{v|\omega} m_{v|\omega}}{\tau_\omega + \tau_{v|\omega}} ,$$

and

$$\tau_{\omega|v} = \tau_\omega + \tau_{v|\omega} .$$

Thus, the precision of the posterior density is always greater than or equal to the precision of the prior density. Furthermore, as the precision of the likelihood $\tau_{v|\omega} \to 0$, the precision of the posterior $\tau_{\omega|v} \to \tau_\omega$, the precision of the prior.

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