PLANNING AND DESIGN OF AGRICULTURAL DRAINAGE UNDER UNCERTAINTY: A DYNAMIC MULTI-LEVEL APPROACH

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RALPH M. PARSONS
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Report Number 281

Prepared Under the Support of the
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ABSTRACT

Drainage systems are major capital investments for irrigated agriculture. Therefore,the goal is to install drainage systems that will be most beneficial to the agricultural economy as a whole. Planning for agricultural drainage has been decomposed in this thesis as a three level process. The first level is the project evaluation and scheduling phase, the second level is the planning of the collector drain network, and the third level is the design of field level lateral drains. This thesis focuses on the second and third levels of drainage planning and the interactions between these levels. Level three drainage design is based upon the physics of groundwater flow. This thesis uses the steady state Hooghoudt equation for drain design currently employed in many areas of tbe world to determine the depth and spacing of subsurface lateral drains. This thesis analyzes the uncertainty in the parameters of the Hooghoudt equation. A detailed analysis of soil permeability shows that uncertainty and spatial variability are important issues in drain design and should be incorporated into the design process. First order-second moment analysis is a methodology that provides a measure of uncertainty in a system output (mean and variance) given uncertainty in system inputs. A first order second moment analysis is performed on the Hooghoudt equation that relates uncertainty in drain performance to parameter uncertainty and system design. Two models are developed to optimally design lateral drains given uncertainty in drain performance: Chance Constraint and Stochastic Programming. The thesis shows that the Chance Constraint approach is not valid when the system response function is not monotonically non-decreasing. In drainage design the system response function is many times not monotonically non-decreasing, however, the stochastic programming approach is valid for all types of response functions. A multi-crop spatially distributed loss stochastic programming model for uniform lateral design over a collector area that accounts for uncertainty and spatial variability of soil permeability by Kriging, an optimal data interpolation technique that accounts for spatial structure, as well as accounting for economic response of multiple cropping of agricultural land.

A simulation model for level two collector drain network planning is developed. This model provides the designer with a tool for drain sizing and cost estimation of complex network alignments allowing many alternatives to be evaluated and the least cost alternative system to be selected. An analysis is performed that show that as a result of spatial variability the efficiency of the lateral field drain system. Present drainage planning is a sequential process that does not evaluate the impact of level two design upon level three. This thesis presents a dynamic multi-level planning process that incorporates feedback between level two and level three planning. A synthesis of the collector network simulation model and the multi-crop spatially distributed loss stochastic programming model witb Kriged input for the lateral drainage system is performed to provide a methodology for dynamic multi-level planning. A case study of this proposed methodology on a drainage region in the Nile Delta in Egypt is carried out. The case study shows that this methodology can provide more efficient drainage systems while providing the most economical design.

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LIST OF PRINCIPAL SYMBOLS

a	roughness coefficient in Wessling Equation for drain sizing
A	number of standard deviations for reliability α
A_i	area of each crop i in multi-crop model
AEL	annual economic loss
AYL	annual yield loss
c_1	cost function coefficient
c_2	cost function coefficient
c_3	cost function coefficient
c_4	cost function coefficient
COV(U)	covariance function
CP	crop price
d	height of drain above impervious layer
d	drivative notation
d'	Hooghoudt equivalent height above impervious layer
D	depth of drain from soil surface
D*	macimum machine digging depth
$\underline{D}$	derivative vector of a multi-input system
DWZ	dewatering zone
$\overline{DWZ}$	mean dewatering zone
DWZ*	optimal dewatering zone for crop yield
DWZ_i	midpoint of intervals of histogram of dewatering zone
E{ }	expectation operator
EL(D,L)	expected loss as a function of depth and spacing
EY_i	annual economic response for crop i in multi-crop model

$f(D,L)$	objective function in depth and spacing
$f_{DWZ}(DWZ)$	probability density function of dewatering zone
$f_{DWZ}(i)$	probability density of interval i of histogram of dewatering zone
$f_X(X)$	probability density function of random variable X
$g(X)$	function of parameter X
$G(D)$	function of depth minimized over spacing
h	height of groundwater table above drain
$\bar{h}$	mean height of groundwater table above drain
i	interest rate
i	slope of collector drain
K	soil permeability
$\bar{K}$	mean soil permeability
L	spacing between drains
LE	Egyptian pound
m	number of intervals in histogram
Mg	mean of function g
M_X	mean of random variable X
$M(\bar{X})$	mean of regionalized variable $Z(\bar{X})$
N	drainage rate or recharge rate
$\bar{N}$	mean drainage rate or mean recharge rate
nc	number of crops in multi-crop model
P_r	probability notation
PWF_i^t	present worth factor as a function of interest rate and project life
Q	discharge from collector drain
r	effective radius of drain

S	specific yield
t	project life
Y	en of K
Y(DWZ)	yield as function of dewatering zone
Y^{-1}	inverse of yield function
Y*	optimal obtainable yield
$Y(\bar{X})$	residual of regionalized variable, $Z(\bar{X})$
Z	depth of soil layer
$Z(\bar{X})$	regionalized variable over $\bar{X}$
α	reliability value
ρ_{KN}	correlation of coefficient between K and N
σ^2_{DWZ}	variance of dewatering zone
σ^2_g	variance of function g
σ^2_h	variance of groundwater level above drains
σ^2_K	variance of soil permeability
σ^2_N	variance of drainage rate
σ^2_X	variance of random variable X
$\gamma(h)$	covariogram as a function of h
$\doteq$	denotes first order approximation

CHAPTER 1

INTRODUCTION

1.1 Overview

The continued growth of the world's population has put great stress on the Agricultural community to increase production to meet the future demand for food and agricultural products. Research programs exist on means to increase production on existing lands by more efficient use of factor inputs such as: fertilizers, hybrid seeds, water application and soil conservation. At the same time, water resources and agricultural projects are being implemented to bring more land under cultivation through developing new lands and bringing old lands under multiple cropping schemes. The need for increased production makes any decline in production from present levels a serious problem. The decline in production of existing lands should be dealt with the same, if not more, urgency than bringing new (and many times less productive) lands under cultivation, since decline in production of these lands may progress to an irreversible stage and most of the present lands are the least costly for production.

One cause of reproduced production is that many agricultural lands do not have sufficient natural soil properties to drain away excess water due to rainfall or irrigation. This inability to remove excess water can lead to the problem of waterlogging. Furthermore, in arid zones, irrigated agriculture is faced with the problem of salinity build-up due to high evapotranspiration rates and poor water quality. Waterlogging and salinity results in reductions in productivity of agricultural lands and the field of agricultural drainage as a science was developed over the past 40 years to address these problems.

> ...Agricultural drainage can be summed up as the improvement of soil water conditions to enhance agricultural use of the land. Such enhancement may come about by direct efforts on crop growth, by improving the efficiency of farming operations or under irrigated conditions, by maintaining or establishing a favorable salt regime. Drainage systems are engineering structures that remove water according to the principles of soil physics and hydraulics. The consequences of drainage, however, may also include a change in the quality of the drainage water...(Van Schilfgaarde, 1974).

As mentioned in the definition above, the process of agricultural drainage is a systems problem. A system composed of many components that span many disciplines; agronomy, groundwater hydrology, economics and construction practices are a few of the major ones. However, in the development of the science of agricultural drainage almost all effort has been directed at the study of the physical components of the system; better models of the soil water movement, crop response to soil water levels, tractor digging performance. All these are important and vital contributions which put the field where it is today, but during the process, understanding the interactions of these components has been neglected. There was a need to look at the system as a whole to learn about the efficient combination of these components.

During the same period of time as agricultural drainage was developing as a science, the field of Systems Analysis was developing into a science of its own. Systems Analysis is a multidisciplinary approach that analyzes the interactions of components to provide information on the performance of a system as a whole.

The research described in this report is one of the first attempts to look at the entire agricultural drainage process from project planning to field level design from a Systems Analysis approach. It is an attempt to synthesize information about the physics, economics and social-political processes effecting agricultural drainage. This synthesis provides an analysis of the entire drainage process, thus giving decision-makers the ability to choose a system that attains the goal of enhancing agricultural land use subject to social preferences.

The objectives of this research were threefold: (1) to identify the system of the agricultural drainage process and its interactions from planning to field installation; (2) to examine the tools that presently exist for analysis at different stages of the planning/design process and propose new tools where they may be needed; (3) to develop a systems methodology for the analysis of the drainage planning/design process.

These objectives were attained and the results are presented in this report. The agricultural drainage process was identified as a multilevel process consisting of a planning level and two design levels; infrastructure level and field level. The existing analysis tools for the planning level were found adequate, but more interaction with the design levels was recommended to provide more accurate information upon which a final decision can be made. It is proposed that the planning of agricultural projects be a dynamic process which includes feedback with the design level to allow more realistic estimate of costs for field installations. The design portion of the agricultural drainage process was decomposed into two levels, the infrastructural or collector level which removes the drained water and the field level which controls the soil water conditions through field drains. These two design levels were found to form a sub-system which could be analyzed to provide the most efficient design of the total system, using a systems approach.

The design sub-system exhibited a need for tools to incorporate economics uncertainty and spatial variability in soil properties into the design process. A model for optimal design of tile drains was developed for the field level. At the collector level, a simulation model for the infrastructure design was developed using computer graphics techniques. A dynamic multilevel approach which combines these tools to provide for the optimal design of the entire drainage system under spatially varying and uncertain soil properties was developed and applied to a case study in Egypt.

This research provides a complete metholodgy for the design sub-system and an approach for the interaction of this sub-system with the planning level for an integrated approach to agricultural drainage. The application of systems analysis using simple models of the drainage process provides a foundation upon which more refined and detailed representation of the physical process can be employed.

1.2 General Approach

The focus of this report is on the design sub-system. The goal is to identify uncertainty in each of the levels of drainage design and develop tools to effectively deal with these uncertainties in an explicit manner. The first task is the definition of the uncertainties involved and the methods to quantify them. The uncertainties to be addressed in this work are physical parameter uncertainty with the emphasis upon the spatial variability and uncertainty of soil permeabilities. Due to the sparsity of and errors in data samples, the technique of "kriging" is used to provide spatial information on the mean and variance of soil permeabilities. A method is developed to provide for a measure of uncertainty in the field drainage output given uncertainty in the inputs .

The Hooghoudt equation for field drainage deaign under steady state conditions is used as a model for sub-surface drainage flow. A first order-second moment analysis is performed on the Hooghoudt equation to provide the mean and variance of the groundwater level given the mean and variance in soil permeability and drainage rate.

The optimal design of the field drains under uncertainty is addressed by two approaches. The chance-constraint approach provides for the least cost solution given a constraint upon output reliability. The stochastic programming approach uses crop loss function to define expected losses which are incorporated into the cost function and a solution is found which minimizes total costs. The approaches are compared and it is found that for certain forms of the crop loss function the chance-constraint approach is not valid and the stochastic programming approach is adopted as the model to be used in an overall planning methodology.

The level two collector problem could not be addressed by mathematical programming, so a simulation approach was adopted. A model that replicates the present manual process by computer graphical input and output devices was developed. This model allows the engineer to interact with the computer to screen a number of alternative designs to improve the efficiency of the economic performance of the collector system.

The overall planning of a drainage system is a two-level process. A dynamic multi-level planning model which incorporates uncertainty in field level design and a simulation approach to level two collector network design is developed. This combined optimization-simulation technique allows for feedback between level two and level three drainage design to incorporate the effects of spatial variability in soil properties for an efficient total system design.

1.3 Description of this Report

This report is divided into six chapters, including the present introductory chapter.

Chapter Two is a discussion of agricultural drainage; its history and present planning methodologies.

Chapter Three is a presentation of the physical processes governing drainage, and the uncertainty and spatial variability of the physical parameters of drainage. Design tools for analyzing spatial variability are presented and the sensitivity of a proposed drainage design model to spatial variability and uncertainty is tested.

Chapter Four presents two approaches to optimal design of field drains under uncertainty. The first approach is a chance-constraint mathematical programming model which is based upon reliability as the design criteria for drainage system performance. The second approach, a stochastic programming model, is based upon minimizing expected economic costs as a design criteria. The stochastic programming model is developed for both a single and a multi-crop system. Both methods are applied to a case study area in the Nile Delta and the approaches compared.

Chapter Five is divided into three sections. The first is a description of a simulation model for design of drainage collector systems. The second section is a new methodology for drainage planning, which examines the tradeoffs between capital investment of collector and field drains versus future expected losses. The third section is an application of this new methodology to a case study from the Nile Delta.

Chapter Six presents a summary of the work as well as conclusions that can be made from the research. The chapter ends with recommendations for future research.

The application of systems analysis to agricultural problems has been widely utilized. However, the application to agricultural drainage problems has been very limited and focused mainly on field level problems. There has been some work on uncertainty in water application rates applied to tile drains design. The work present in this report is the first time that uncertainty and spatial variability in soil properties have been combined with the economics of crop response in a systems approach to drain design.

The multilevel dynamic approach to total drainage system design is a new development that provides for a systematic view of the entire drainage design process. This work has also provided a first attempt at linking the planning and design process together to provide for better planning and more efficient design.

CHAPTER 2

AGRICULTURAL DRAINAGE

This chapter presents a brief overview of the field of agricultural drainage. A history of drainage practice precedes a discussion of the need for and types of drainage. Present methodologies for planning of agricultural drainageare presented. A detailed description of drainage in Egypt is given.

2.1 Background

Agricultural drainage is the modification of the groundwater regime for the purpose of improving agricultural production. The modification of the groundwater regime may be to lower the groundwater level to allow for sufficient areation of a root zone, or to provide a sufficient hydraulic gradient to allow percolation through the root zone to leach away excess salts.

The history of drainage for improved agricultural production is long. In the Fifth Century B.C. Herodotus, the Greek historial, mentioned seeing a surface drainage system in the Nile Valley. There are other accounts of the use of surface drainage for land reclamation during the ancient Greek civilization. Sub-surface drainage is believed to have begun during the Roman Age with accounts of Roman sewage being drained by sub-surface drains as early as 200 B.C. In the sixteenth and seventeenth century drainage became popular in many other places in the world (Faduka, 1976).

England is credited with being the birthplace of modern tile drainage. In 1810 James Graham drained his land by burying homemade U-shaped tile drains. In 1842 Johnritt developed a circular tile drain. In 1846 a German named Antman invented a tile-making machine and drainage prospered on the Continent. Sub-surface drainage came to America in 1835 when John Johnston drained his land with hand made tiles. In 1848 a tile machine was imported from England, and drainage use expanded due to a source of cheap drains (Faduka, 1976).

Many devices for installation and manufacturing of drains have been introduced over the past 140 years. The present state of the art is the use of corrugated polyvinyl chloride tubine (PVC) installed by highly sophisticated laser controlled machines. These products are continually improving because of the present demand for more and cheaper agricultural products. Faduka puts this in perspective,

> ....In the history of drainage, its prosperity and decay were directly related to the financial and economic situation of the country. When farm products were bringing low prices, drainage works were not practiced actively, research was neglected, and the good methods and techniques of drainage invented in prosperous times were forgotten. When drainage came to be considered important again, such methods and techniques reappeared as if newly born... (Faduka, 1976, p. 40).

The next sections will outline some of the present paths that the field of drainage is embarked upon.

2.2 Need for Drainage

When agricultural lands are faced with waterlogging due to over irrigating, poor drainage or salinity build up as a result of water quality or soil chemistry conditions, artificial drainage is one solution. The need for drainage in any agricultural situation should be coordinated with the source of water supply and the types of crops grown. The next sections will provide a brief discussion of water quality and quantity and their relationship to plant growth.

2.2.1 Waterlogging

One condition that can be alleviated by installation of a drainage system is the problem of waterlogging. Waterlogging occurs when the root zone of the plant becomes fully saturated. Water in the soils displaces air and obstructs the exchange of gases between the soil and the air. Therefore, the soil oxygen content is reduced. Due to the lack of oxygen, organic matter cannot decompose aerobically and anaerobic processes set in.

This results in a number of problems. First, anaerobic decomposition produces reduced organic compounds (such as methane, methyl, and complex aldehydes) which react with soil mineral substances and produce toxic concentrations of ferrous sulfide and manganese ions. Second, anaerobic decomposition is also much slower than aerobic decomposition and as a result, nitrogen remains bonded in organic residues, often becoming a limiting factor for plant growth. Third, the lack of oxygen and abundance of carbon dioxide in waterlogged soils cause plants to have difficulty absorbing water and nutrients, thus their growth is impaired (Luthin, 1978).

Additionally waterlogging has physical as well as biochemical effects. Excess soil moisture can damage the important top soil. Wet soils are more susceptible to compaction by animals and farm equipment. This may limit the ability to perform necessary farming operations. Also, certain plant diseases and parasites are encouraged in a waterlogged soil.

2.2.2 Salinity

A second condition that can be corrected through proper drainage is that of salinity build-up. Salinity is a severe problem in arid agricultural lands. Due to high evaporation rates, the concentration of salts in water supplies are higher in these climates than in humid climates.

When there is poor drainage and a high water table, capillary tension continually lifts groundwater to the surface, replacing the water removed from the surface by evaportranspiration. Thereby, salts in the groundwater are lifted to the surface and deposited there, sometimes forming a crusty layer. If the water table is lowered sufficiently with a drainage system, the capillary fringe will no longer reach the surface and rising of salts can be controlled.

A major advantage of a sufficient drainage system is that it permits the application of water in excess of the requirements for crop evapotranspiration. This water serves to dissolve salts and remove them from the root zone. This water is sometimes referred to as the leaching requirement (Van Schilfgaarde, 1974).

The adverse effect of salts is attributed to two processes: Osmosis and ionic toxicity. The roots of plants have a semi-permeable membrane and depend upon the osmotic pressure difference between their sap solution and relative low concentration groundwater for water uptake. As the groundwater becomes saltier the difference in osmotic pressure decreases and the plant has increasing difficulty obtaining water. Thus, a plant could suffer from lack of water, although water is available in the root zone (Luthin, 1978). The mechanism of salt toxicity on plants had not yet been adequately investigated. As a result the toxic effect of salts is generally judged on the basis of correlation between ion concentration and crop yields. There is a continuing debate over the importance of osmotic versus ion toxicity. There are three general theories as to the processes effecting plants: (1) Osmosis alone, (2) ion effect alone, (3) a combination of both osmotic and ionic effects (FAO, 1973). This presentation demonstrates the importance of groundwater level and quality upon crop production and the need to address these issues.

2.3 Drainage Alternatives

The four major types of drainage that have been developed for controlling the water table are shown in Figure 2.1. They are surface drains, sub-surface drains, mole drains, and drainage wells. In this section we will briefly describe each type and discuss the advantages and disadvantages of each.

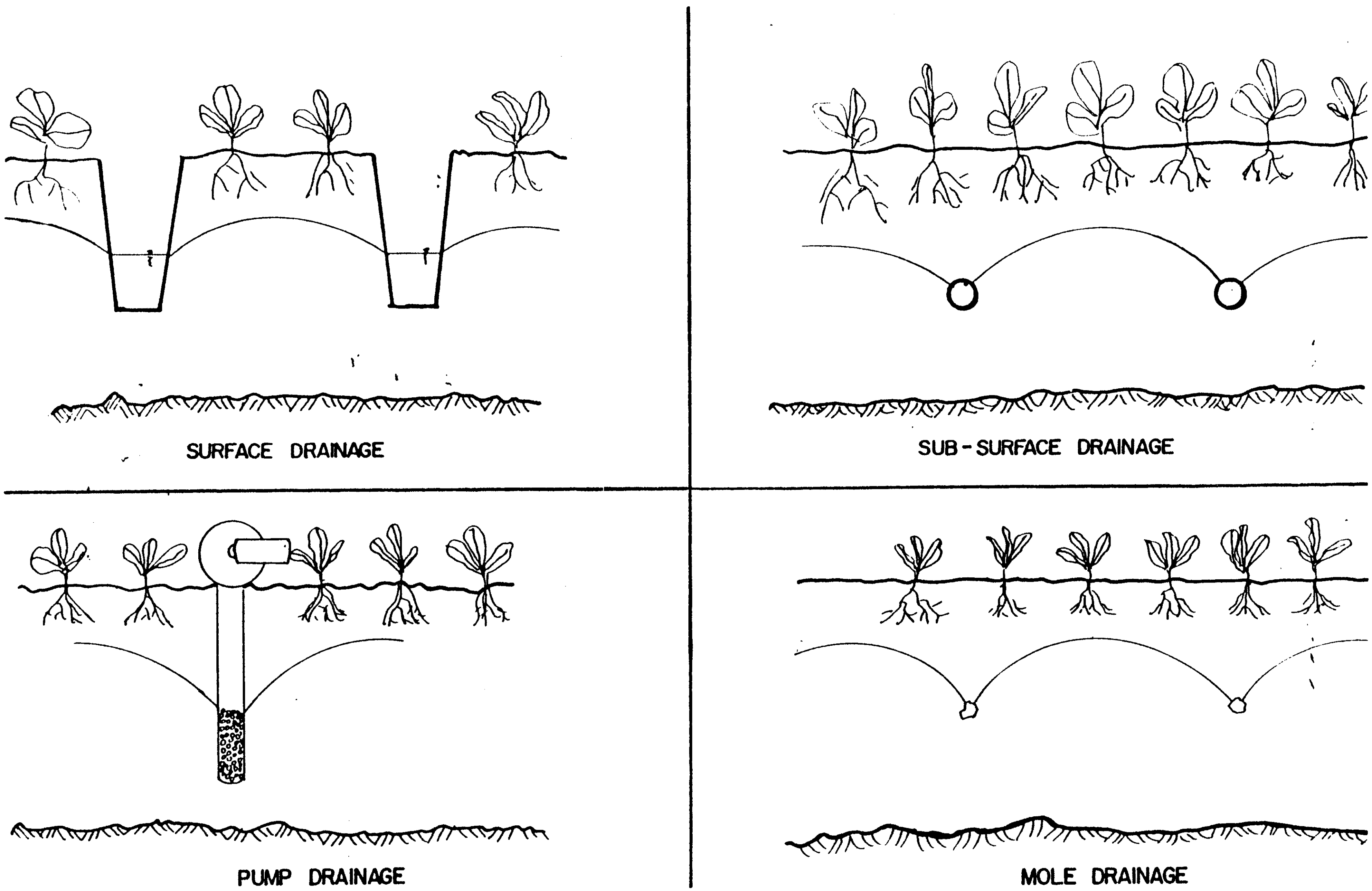

Figure 2.1 - Alternative Drainage Methods

2.3.1 Drain Types

Surface or open drains are channels constructed in the field to convey excess water away. Water enters the drain through sub-surface flow or by overland runoff. The advantages of open drains include (1) the ability to transport large quantities of water, (2) ease of construction, and (3) ease of maintenance. The disadvantages are (1) the loss of valuable farm land, (2) a constant sedimentation and weed maintenance, and (3) need for additional infra-structure.

Sub-surface drains include tile or PVC drains installed underground at varying depths and spacing. In both cases, a ditch is dug and the drain is laid and sometimes surrounded by an envelope of gravel to aid flow. The depth and spacing are based upon local hydrogeological conditions. The advantages of sub-surface drains are (1) no loss of farm land, (2) less maintenance, and (3) the ability to have some control over the water table. The disadvantages are (1) high capital cost and (2) maintenance is difficult and costly.

A mole drain system consists of a series of egg-shaped, unlined, underground conduits formed by a moling plow. This plow has a long blade with a bullet-like plug attached to the end. As this plow is drawn through the soil, it carves out the mole drain cavities. It can only be used in highly cohesive soils which will retain the shape of the conduits. These drains remain operational for 3-5 years. It has the advantage of (1) being relatively inexpensive to install and (2) construction time is short. It has a major disadvantage in that its operational life span is short, and over the long run, repeated moling can be as expensive as a permanent system.

Pump well or vertical drains consist of a newtork of wells which are used to lower the water table. This method is most effective in areas underlain by phreatic aquifers where conditions are not complicated by upward seepage from deeper lying artesian aquifers. The advantages of pump well drains are (1) its lower initial costs and (2) the possibility of using the water for irrigation or other uses. The disadvantages are (1) its high operation and maintenance cost, (2) the need for low cost power to run the pumps, and (3) the need for appropriate hydrogeologic conditions.

2.3.2 Sub-Surface Drainage Systems

This work is motivated by the drainage problems in Egypt. The conditions there warrant the use of sub-surface drainage and as such this work will focus on sub-surface or tile drains. Sub-surface drainage can be utilized in a variety of system configurations. This section will describe sub-surface drainage system components and alternative configurations.

A sub-surface drainage system can be divided into three classes of drains: field laterals, collectors and main drains. The purpose of field laterals (field drains or lateral drains) is to control the elevation of the groundwater table. The drained water in the laterals flows to the collectors which convey the water to the main drain system which conveys the water to an outlet or pumping station for disposal or reuse.

The lateral drains are constructed from clay, concrete or PVC tubing. The collectors can be either large clay or concrete pipes or open ditches and the main drains are large open channels. If the lateral drains empty into collector ditches the system is called a singular pipe drainage system as shown in Figure 2.2. If the lateral drains connect to pipe collectors, the system is called a composite pipe drainage system as shown in Figure 2.3 (Cavelaars, 1973).

To achieve a desired control of the groundwater table the lateral drains must be installed with a certain depth and spacing between drains. The depth and spacing is determined by the physics of groundwater flow and the parameters for the design area.

2.4 Multi-Level Drainage Planning

After reviewing the methods for planning of agricultural drainage worldwide, it is possible to structure drainage decision-making in a three-level hierarchy. This hierarchical approach is desirable because it allows the problem to be decomposed into segments with different problems to be solved. Each segment can then be addressed separately and the improved segments joined into a totally improved planning process.

The first level is the project evaluation level; that is the decision that decides whether the project should be undertaken or not, e.g., are the benefits of the project greater than the costs. Given that an affirmative decision is made and scheduling of drainage implementation over the region completed at the first level the next level in the process is the design of the network composed of the collectors and

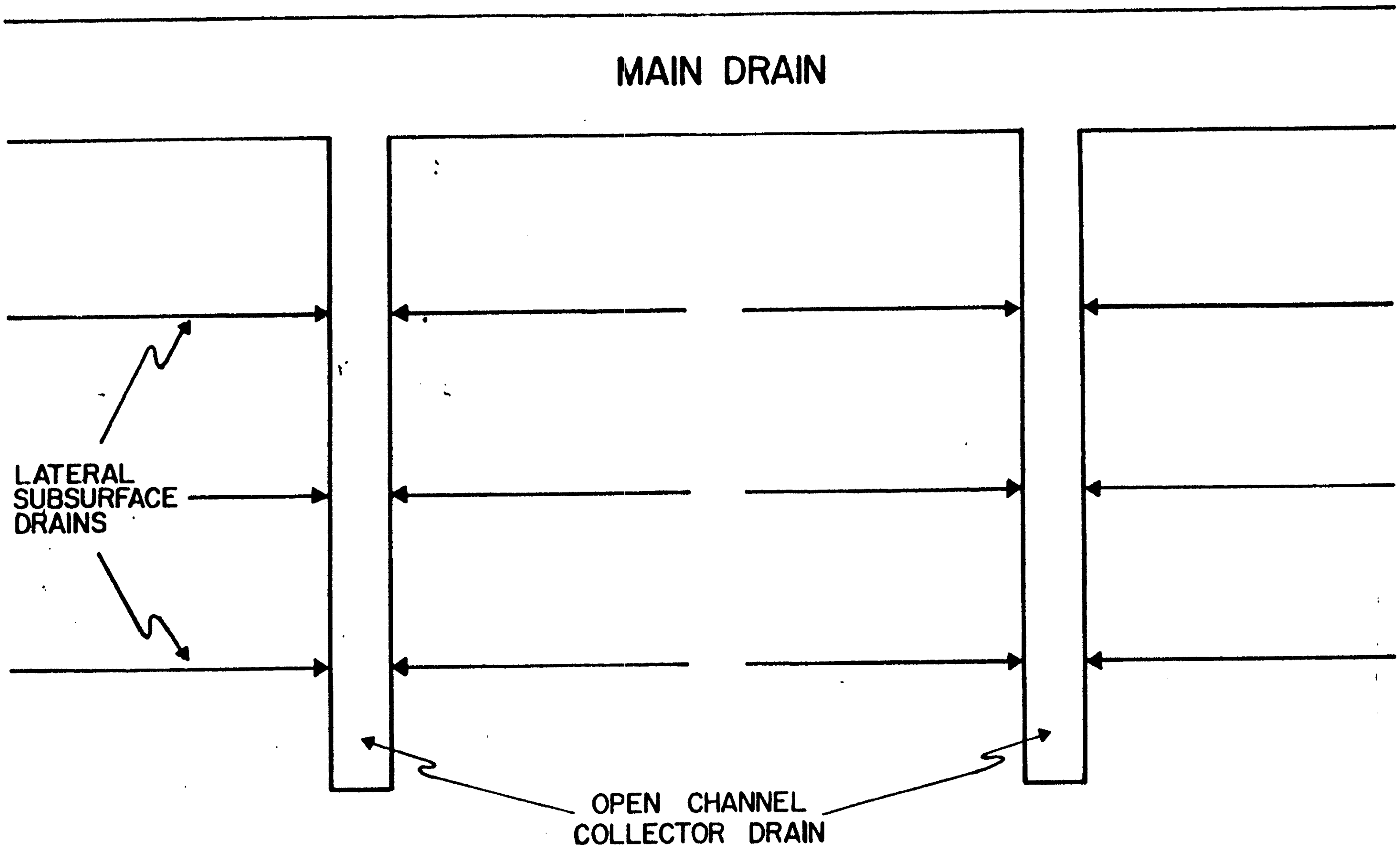

Figure 2.2 - Singular Pipe Drainage System

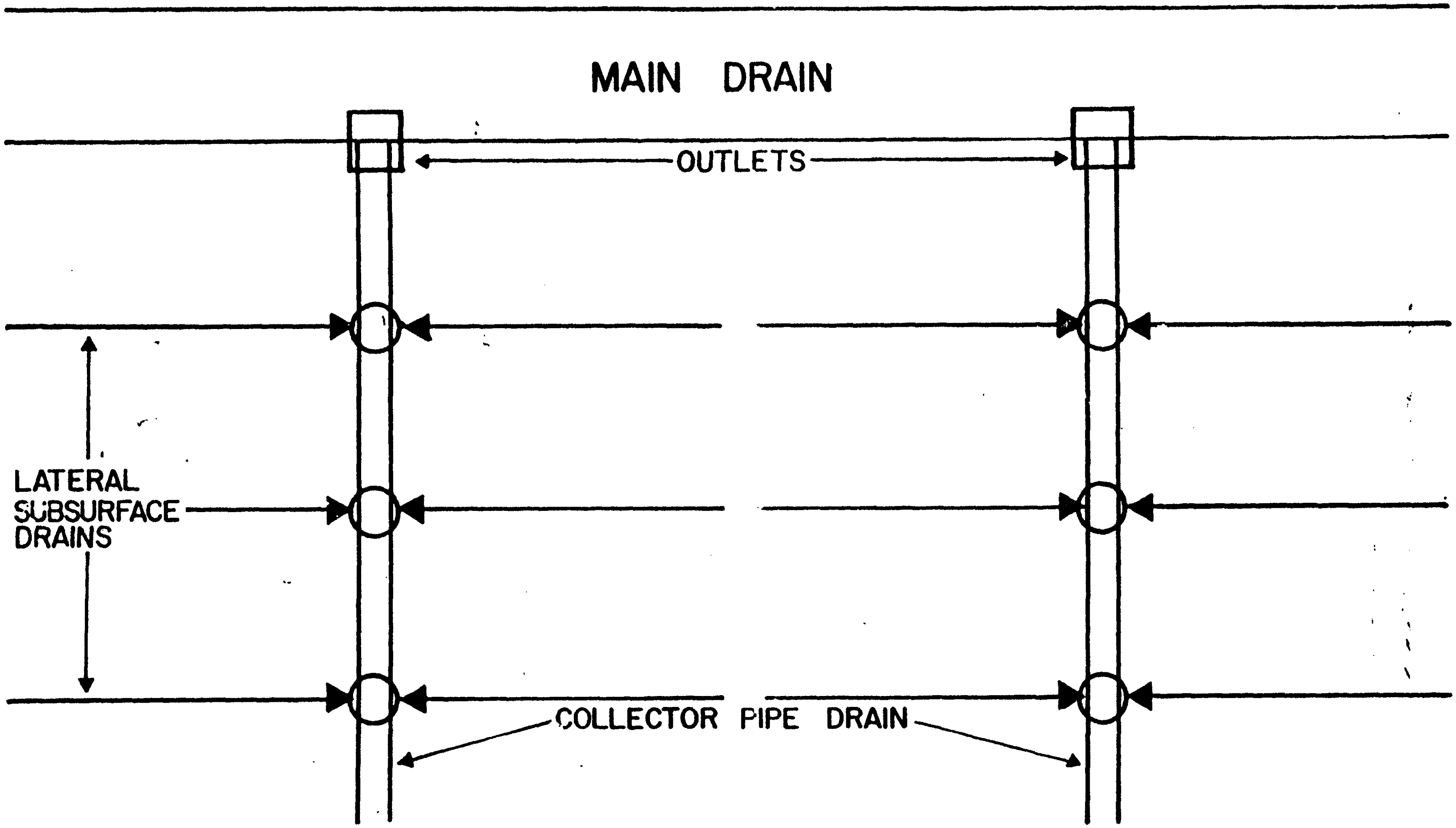

Figure 2.3 - Composite Pipe Drainage System

main drains. This will include the location, sizing and type of collector drains in the collection network. The third level is that of the design of the field drains or field level. This will include the type, material, depth and spacing of the lateral field drains. These field drains will then empty into the level-two drainage collection network. A schematic of this structure is seen in Figure 2.4.

The first level decision process is a two phase exercise in public project analysis. Here the decisions are made as to whether the drainage project is beneficial to the nation or region as a whole and how to schedule the completion of the project. A number of different techniques and schools of thought about the evaluation of public investment projects can be found in the literature. Some articles have been written which discuss the analysis of public drainage projects, (True, 1977; Trafford, 1975; Dickey, 1977; Frogge and Sanders, 1977; and Knapp, 1978). El Ghamry (1978) has analyzed the economic evaluation of drainage projects in Egypt. Very little has been written regarding the scheduling of the installation of agriculture drainage.

There is still more work to be done on the application of more advanced techniques to the first level of drainage planning. This is not the focus of this report, although a brief presentation of the hierarchial structure and multilevel interactions will be made. The main emphasis of this report is on the planning and design of the collector and lateral drains which are at the essence of the second and third level drainage planning problem.

If the first level decision concludes that the drainage project is

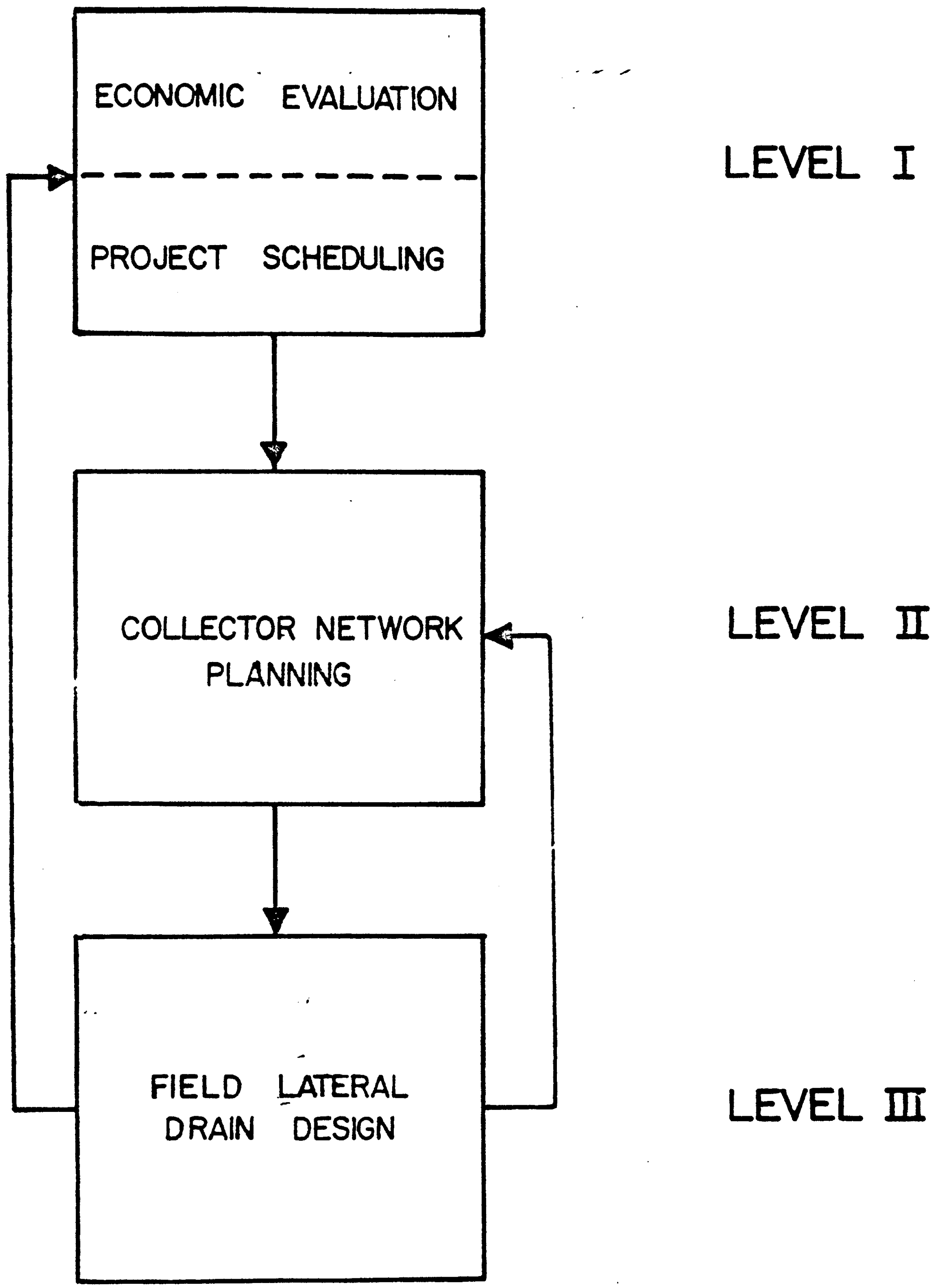

Figure 2.4 - Multi-level Drainage Planning

beneficial to the nation, it is necessary to proceed to the detailed design of the drainage system. The second level is the design of the collection and main drain network and the third level is the design of the field or lateral drains. The second level problem is presented first not suggesting that it occurs prior to the third level but for clarity of the direction of the research. It may actually take place simultaneously or following the field level process.

2.4.1 Second Level

The problem that faces the drainage system designer at the second and third levels can best be demonstrated by examining Figures 2.5 and 2.6. Figure 2.5 is the map of an agricultural area to be drained, the size of which is approximately two thousand hectares. This area has many characteristics which are functions of both space and time. Some of the more important characteristics for drainage design include: topography, soil permeability, irrigation application and crop rotation. Figure 2.6 shows one alternative drainage system for the agricultural area shown in Figure 2.5. The problem is how to generate and select from the set of feasible drainage systems the alternative which best attains the objectives for which the system is operated, subject to all the constraints of the system. As stated above, the planning of the collection network system, level-two planning, will be examined first.

The state of the art in drainage design procedure and installation practice usually requires that drain spacing and depth be constant along any collector drain. However, the characteristics that determine the depth and spacing vary in space over any drainage area and thus along any

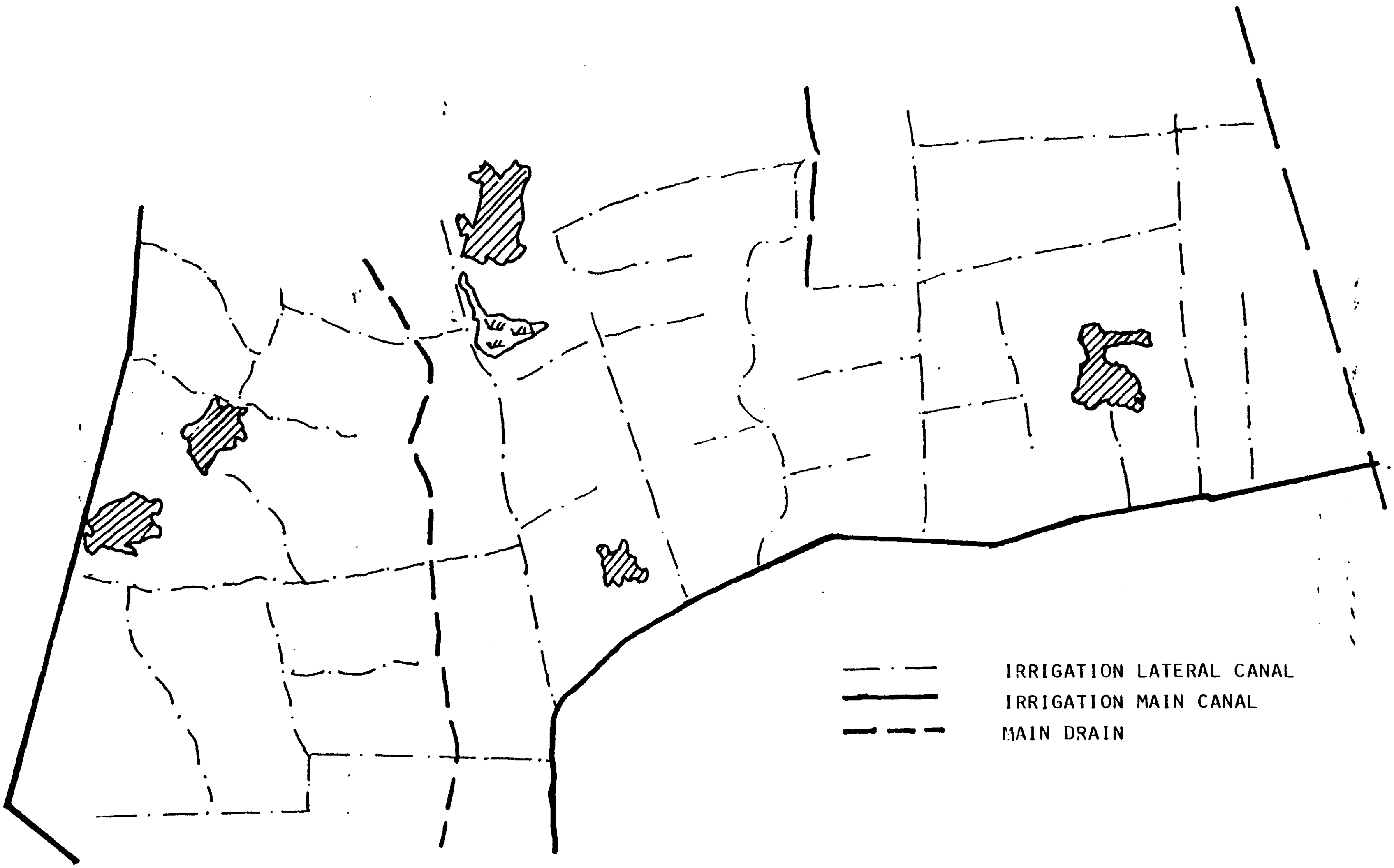

Figure 2.5 - Sample Drainage Area

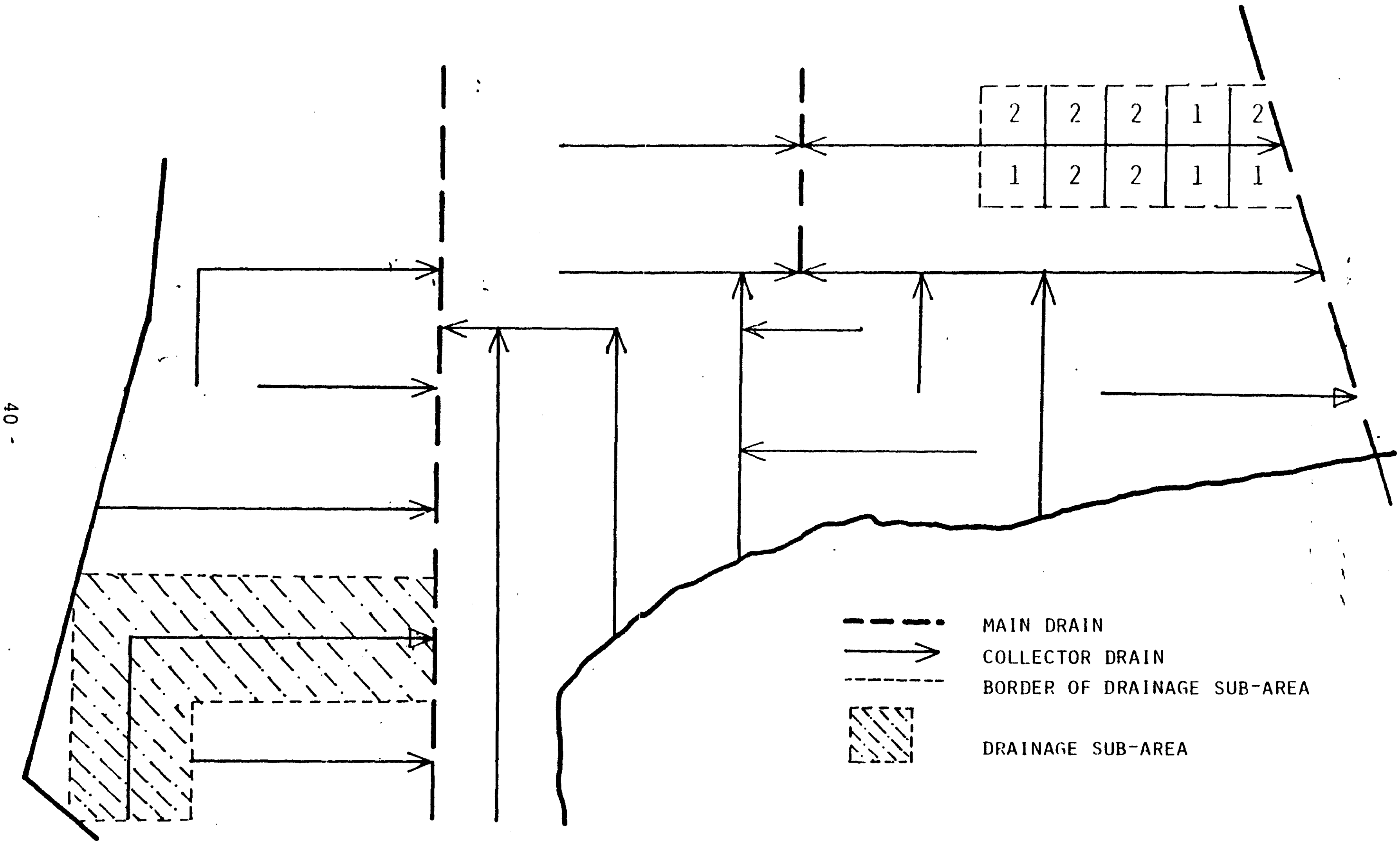

Figure 2.6 - Planning of Drainage Collection Network

collector. The placement of the collector so as to drain sub-areas with the most homogeneous set of characteristics is the objective of level-two planning. The reason for defining homogeneous sub-areas is that crop yield losses will occur when portions of sub-areas are not adequately drained, while capital resources will have been misallocated where drainage investment exceeds requirements.

Before the design process begins, data is collected over the drainage area This is done by sampling the values of pertinent drainage characteristics at locations specified by a grid network placed over the area. This data is used in the planning and design of the drainage network.

Renner and Mueller (1974), have developed a simulation model to provide the design of a system of drain laterals connected to mainline collectors assuming homogeneous characteristics. This assumption does not allow lateral spacing to vary over space. This work has expanded upon the technique by allowing lateral spacing to vary from collector to collector. This approach provides for a more realistic model which takes into account the spatial variability of soil parameters. The model is also coupled directly to a drain spacing model based upon economic criteria.

2.4.2 Third Level

The third level of drainage planning is that of field or lateral drain design. This is the most detailed of all the planning levels. Many engineering decisions must be made including the type of drains (surface or sub-surface). If sub-surface drains are chosen then a material must be selected (clay, concrete, or PVC drains) or alternatively mole drains may be selected. Once these choices are made, it is necessary to decide on the size, depth, and spacing of the drains. It is the depth and spacing of drains that will be the focus of the third level planning process. The other decisions are important but will not be addressed in this work.

In the past most drainage was designed by engineers using their empirical knowledge (True, 1977). Since 1940 many analytical equations have been developed relating depth and spacing of drains to the physics of groundwater flow and soil parameters. Equations exist for steady-state and transient conditions and many specific geometries. These have proven fairly successful in describing some of the conditions found in the real world. The government of Egypt, the Soil Conservation Service of the United States Department of Agriculture, and engineers in the Netherlands use a steady-state approach (Dickey, 1977 and Amer, 1979). The United States Bureau of Reclamation uses a transient equation (USBR, 1978). The appropriateness of either approach depends greatly upon the local climate and timing of water applications on cultivated lands. Presently all design equations assume homogeneous soil parameters. It is known that soil parameters continuously vary over space and time. However, soil parameters can only be sampled at discrete points in space. This discrete sampling leads to uncertainty in design parameters. The existing methods of design assume that the mean value of the samples represents the homogeneous soil parameters. Making this assumption can lead to either over or under design of field drains which may result in economic losses.

Van Schilfgaarde (1965), Musy and Duckstein (1976), and Wiser et al., (1974) have addressed the problem of uncertainty in water application rates for drainage design. However, no one has attacked the problem of spatial variability and uncertainty in drainage soil parameters. This work will address this problem directly and a method for analyzing uncertainty in groundwater levels due to uncertainty in soil and water parameters will be presented. The next step will be the incorporation of this uncertainty into a design criteria to allow for the development of a mathematical programming model to design "optimal" field drains under uncertainty in water application and soil parameters. Two approaches will be presented for the solution of mathematical programming problems under uncertainty, the chance-constraint approach and the stochastic programming approach. Both approaches will be solved using non-linear optimization techniques.

Christopher and Winger (1977), Renner and Mueller (1974), and Aldabagh and Beer (1974) have used simulation techniques for designing "optimal" field drains based upon deterministic design parameters. Van Shilfgaarde (1965) and Wiser, et al., (1974) have used simulation techniques to design optimal field drains given uncertainty in water application rates. Musy and Duckstein (1976) and Fogel et al., (1978) use Bayesian decision theory to optimally determine depth and spacing of field drains given uncertainty in water application rate.

The use of system analysis techniques to agricultural problems is widely accepted. They have been applied much more to irrigation problems and the concept of uncertainty has been more readily accepted. Smith (1973) presents a general review of systems application to irrigation planning. Onigkeit et al., (1969), de Lucia (1969), Cordova (1977), Dudley and Bort (1973), Howell (1974), and Yaron (1973) are a few examples of system analysis techniques applied to irrigation systems considering stochastic conditions. Anderson et al., (1976) presented the use of decision analysis as applied to agriculture. Although there are many references to the use of systems analysis techniques and stochastic process in the field of agriculture, there has been very little application of these approaches to the specific field of agricultural drainage and as such there are not many references available in the literature.

2.4.3 Interactions

The Bureau of Reclamation (1978), U.S. Department of the Interior, the Soil Conservation Service (1976), U.S. Department of Agriculture, and the International Institute for Land Reclamation and Improvement (1973) all have produced volumes on drainage planning and design. All three propose a method of sequential planning: level-one decisions lead to level-two decisions, impacting level-three decisions. There has been no discussion of feedback between the levels, or of dynamic planning.

The different levels of drainage planning will be discussed in more detail in the following chapters and a new methodology for multi-level planning will be proposed.

2.5 An Example: Drainage Planning in Egypt

2.5.1 History of Irrigated Agriculture

Several authorities (Faduka, 1976; Framji and Mahajan, 1969; and FAO, 1973) date cultivation along the Nile River as beginning about 6000 B.C. Annual floods both irrigated the Nile Valley and carried fertile sediments providing good conditions for farming. The farmer has no control of the rising or falling of the flood. The water was not always available when most needed for crops. To gain some control over the waters the farmers divided the land into sections (polders) enclosed by dikes 1-3 meters high. They were then able to direct the flood waters on the fields to a depth of 1-2 meters. The water remained for 30-60 days and rich sediments were deposited onto the fields. When the river level dropped the water remaining on the fields was drained off to the river, and wheat and barley were sown. This type of irrigation is known as "basin irrigation" and allows only one crop per year except along a narrow strip close to the river where another crop could be grown by lifting water (Feduka, 1976; Framji and Mahajan, 1969).

In ancient Egypt, much effort was put into improving irrigation practices. The central government spent time and resources on supply systems to aid in the use of the flood waters for irrigating more land under basin irrigation. These systems demonstrated a high degree of engineering sophistication (Faduka, 1976). Perennial irrigation came to Egypt in the Nile Delta in 1826. The ruler, Mohammed Ali, constructed a series of deep canals to carry the Nile's water during the summer when the river was low. The water was then lifted to the fields by human or animal power; during the remainder of the year when the river was high the system operated by gravity. This method of irrigation was very successful, but clearing silt from the canals and providing the lift during the summer required much labor.

In 1861 Mohammed Ali built the Delta Barrages on the Damietta and Rossetta branches of the Nile 23 kilometers north of Cairo. A barrage (dam) is a control structure built across a river to create a higher river level upstream of the structure to allow water to be diverted by gravity. The Delta barrages were constructed to provide water supply yearround, attenuate the silting problem, and reduce the amount of labor required. The original barrages collapsed under the increased head, but were reconstructed with the help of Indian engineers.

Before 1902 all irrigation depended on the natural flow of the Nile. With the increased development of perennial irrigation summer flows were unable to meet the demands for irrigation water. Therefore, in 1902 the Aswan Dam was completed to provide one billion cubic meters of storage (1×10^9 m^3) or about one percent of the yearly flow of the Nile. In two later stages, the Aswan Dam was raised to a total capacity of five billion cubic meters. This provided some relief, but not enough to meet the demands of a growing agricultural economy. In 1937 the Gebel Alia Dam was built on the Nile just south of Khartoum, Sudan for use by Egypt. Figure 2.7 provides a map of the entire Nile basin pointing out the major control structures. Only a small amount of within-year storage was available to redistribute some of the flood waters for summer irrigation. A side-effect of these storage dams was that much of the fertile silt and humus was deposited in the reservoirs,

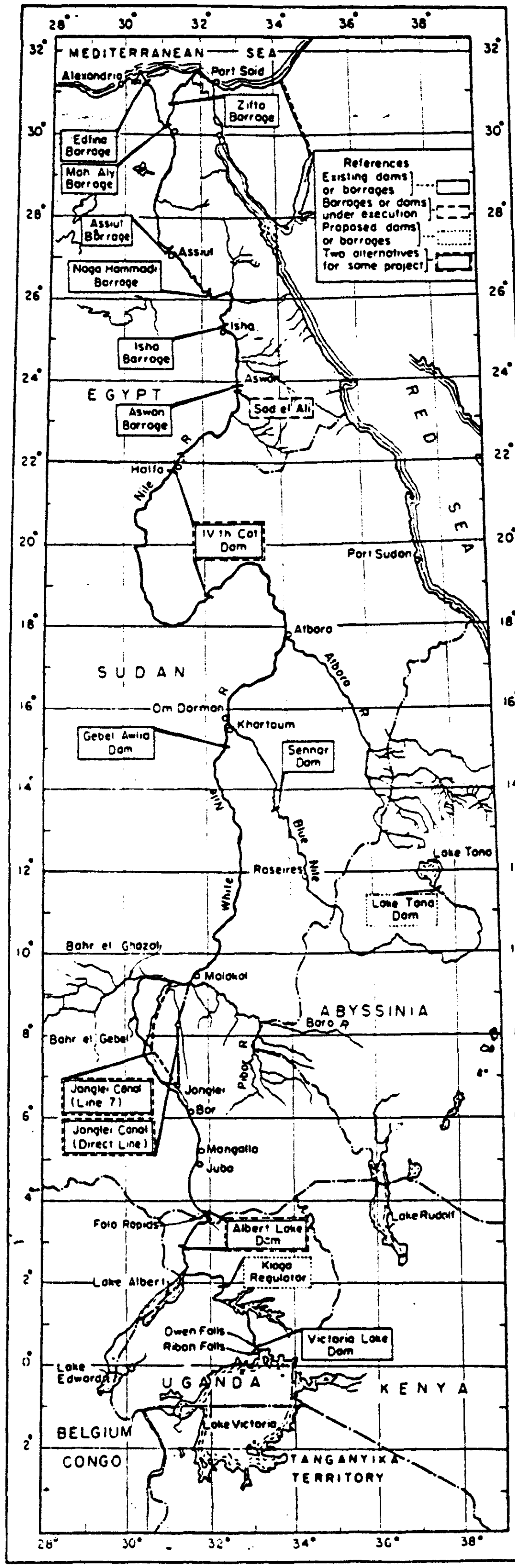

Figure 2.7 - Major Projects of Nile Basin
(from Framji and Mahajan,1969)

rather than being placed on the fields.

As agriculture grew following the Aswan Dam completion, barrages were added at Assuit, Esna, and Nag Hammadi, in upper Egypt and Zifta and Edfina in the Nile Delta. These can be seen in Figure 2.8, which is a schematic of the Egyptian irrigation network.

Even with all these projects only a small portion of the yearly flow of the Nile was able to be used for irrigation and there was a continued threat of flooding. To alleviate these problems and provide hydropower, the High Aswan Dam was constructed. The High Aswan Dam, completed in 1965, has a storage capacity of one hundred sixty four billion cubic meters, or about twice the average annual flow passing Aswan. Egypt has essentially no rainfall and must rely almost totally on the Nile flow from the upper basin. Since a majority of the water comes via Sudan a treaty has been made with Sudan for Egypt to receive an annual share of the Nile flow of fifty-five and one-half billion cubic meters. The enormous storage capacity at Aswan provides many benefits to the Egyptian people. It takes a great deal of uncertainty out of estimating the yearly water availability. It prevents damages downstream from large floods and protects agriculture from water shortages in years of drought. The hydropower from the Dam provides more than fifty percent of Egyptian electrical demand. With water available year round new lands can be brought under cultivation as well as multiple crops can be grown. Due to the High Aswan Dam, the cropping intensity in the Nile Valley is approximately one-hundred-ninty percent (World Bank, 1977). With about 30% of the Egyptian gross national product from agriculture, the High Aswan Dam has major effects upon the economy of Egypt.

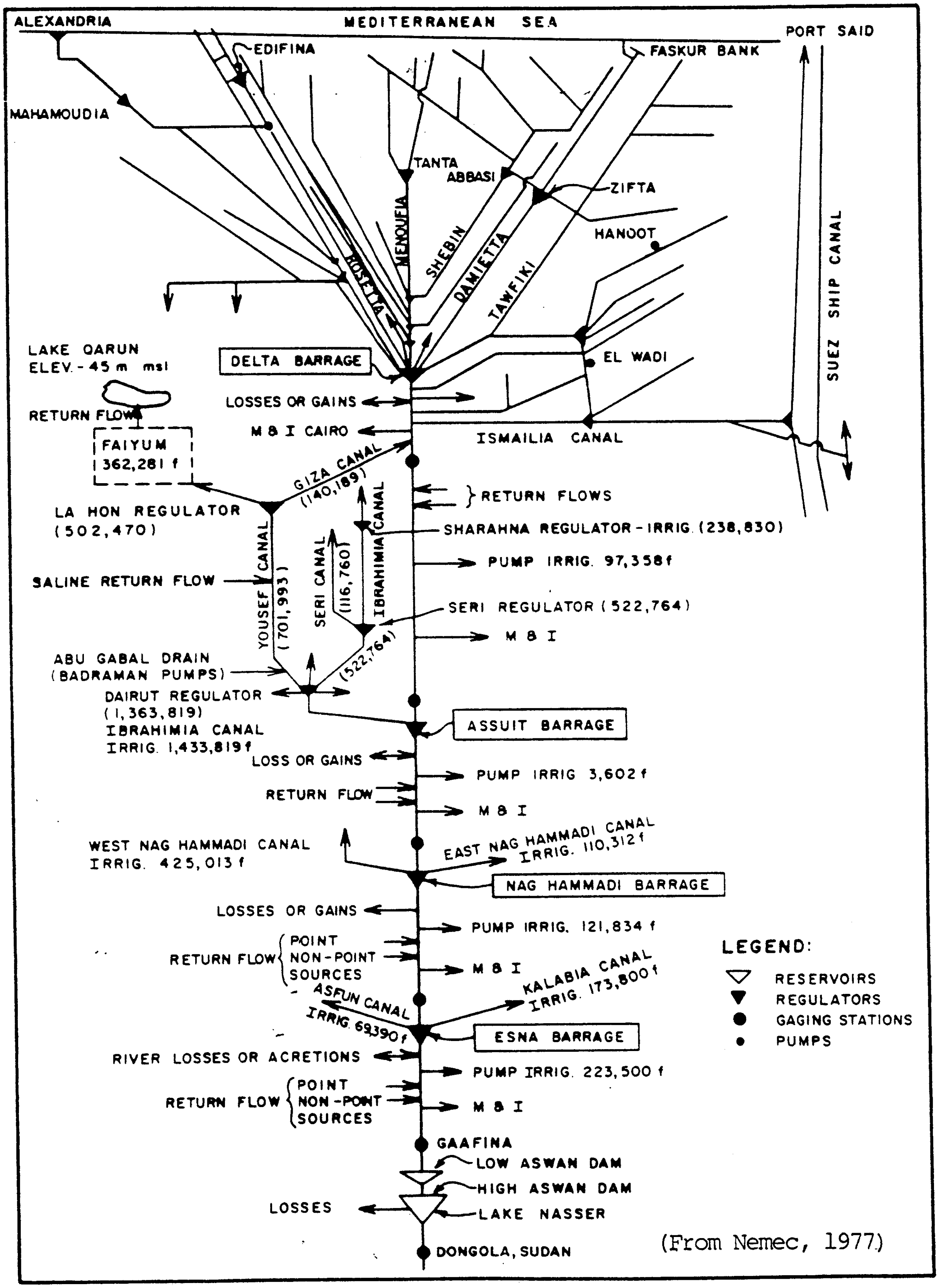

Figure 2.8 - Schematic Diagram Nile River Basin Irrigation System

2.5.2 Decline of Agricultural Production

This intensive irrigation has created a problem. In the past, with both wet and dry periods, the initial flood waters provided a mechanism for flushing away any salts that may have built up in the soil, and the groundwater table had time to slowly recede after the flood. Now due to year round irrigation the water table is constantly high and salts are not flushed. The result is that crop yields have been severely affected by waterlogging and salinity. These problems are due to poor irrigation practices and the soil properties of the Nile Valley.

It is possible to alleviate the problems of waterlogging and salinity by introducing better farm water management and agricultural drainage systems. These drainage systems allow the groundwater table to be controlled, preventing waterlogging and allowing for sufficient leaching of excess salts from the crop root zone.

With a tradition of 6000 years of basin irrigation, the poorly educated small land holding Egyptian farmer, known as "fellah", has not changed farming practices to reflect the new system of water supply. The acceptance and widespread adaptation of new farming techniques are decades away. Therefore the Egyptian government has embarked upon a monumental project of installing agriculture drains on most cultivated land in Egypt.

2.5.3 Current Planning Process in Egypt

The first-level decision of whether to install drainage has been made by the Egyptian government and the World Bank which is providing funding for the project. The Egyptian Public Authority for Drainage Projects has been formed to implement level-two and level-three planning.

The process presently employed is a sequential process of planning level-two collection networks based upon topographical criteria. Level-three field design is then based upon sampling of parameters within each collector region. A large staff exists for the investigation, planning and design of drainage systems in Egypt. Even with this large staff the task is so great that the staff is hardpressed to meet the yearly targets for drainage design. More effective and efficient methods of drainage planning and design would be very beneficial to the Egyptian government.

CHAPTER 3

DESIGN OF FIELD LEVEL LATERAL DRAINS UNDER UNCERTAINTY

This chapter is a discussion and analysis of the effects of uncertainty of drainage system performance. The physics of groundwater flow to drains is presented and the Hooghoudt model for drainage design is formulated. The uncertain input parameters to the Hooghoudt model are defined, and methods for incorporating system input uncertainty into system output uncertainty are presented. The first-order second moment approach to system uncertainty is developed. Uncertainty in the drainage design problem is focused primarily on the drainage rate and the soil permeability. The uncertainty in the soil permeability is divided into information uncertainty and large scale spatial variability. The uncertainty of soil permeability is investigated for a case study in the Nile Delta. The process of kriging, a method that represents large scale variation in soil permeability as well as information uncertainty found in sample data, is applied to the Nile Delta case study. The first order-second moment approach to the Hooghoudt model is used to define uncertainty in groundwater table elevation between two drains. The effects of large scale spatial variability upon uncertainty of water levels between drains was found to be minimal. The following sections will develop these concepts in detail.

3.1 Physics of Drainage Flow

The criteria for choosing the design groundwater levels depend on the soil, crops, climate and salinity of the irrigation water. To achieve this design water level, lateral drains must be installed at an appropriate depth and spacing. The appropriate depth and spacing is a function of mainly the irrigation water application rate, the permeability of the soil and the depth of soil layer above a possible underlying impervious layer. Figure 3.1 is a cross-sectional representation across the lateral. Several key parameters are defined:

Z = Height of ground surface above impervious surface,

D = depth of drain below ground surface,

L = spacing of drains,

d = height of drain above impervious layer,

h = height of groundwater table above drains,

DWZ = d-h = depth of unsaturated soil layer, the dewatering zone,

K = effective permeability of soil,

N = drainage or recharge rate,

r = effective radius of drains.

The typical assumption taken in drainage design is that the soil is a porous media with an impervious bottom and a variable groundwater table as a top boundary. This is known as a phreatic acquifer. The design criteria for subsurface drains is based on the groundwater table elevation between the drains. Thus, the desired model output is the phreatic surface elevation: the variation over depth of piezometric head is not in itself important. Since the piezometric head variation in the vertical direction is not important, a horizontal two dimensional

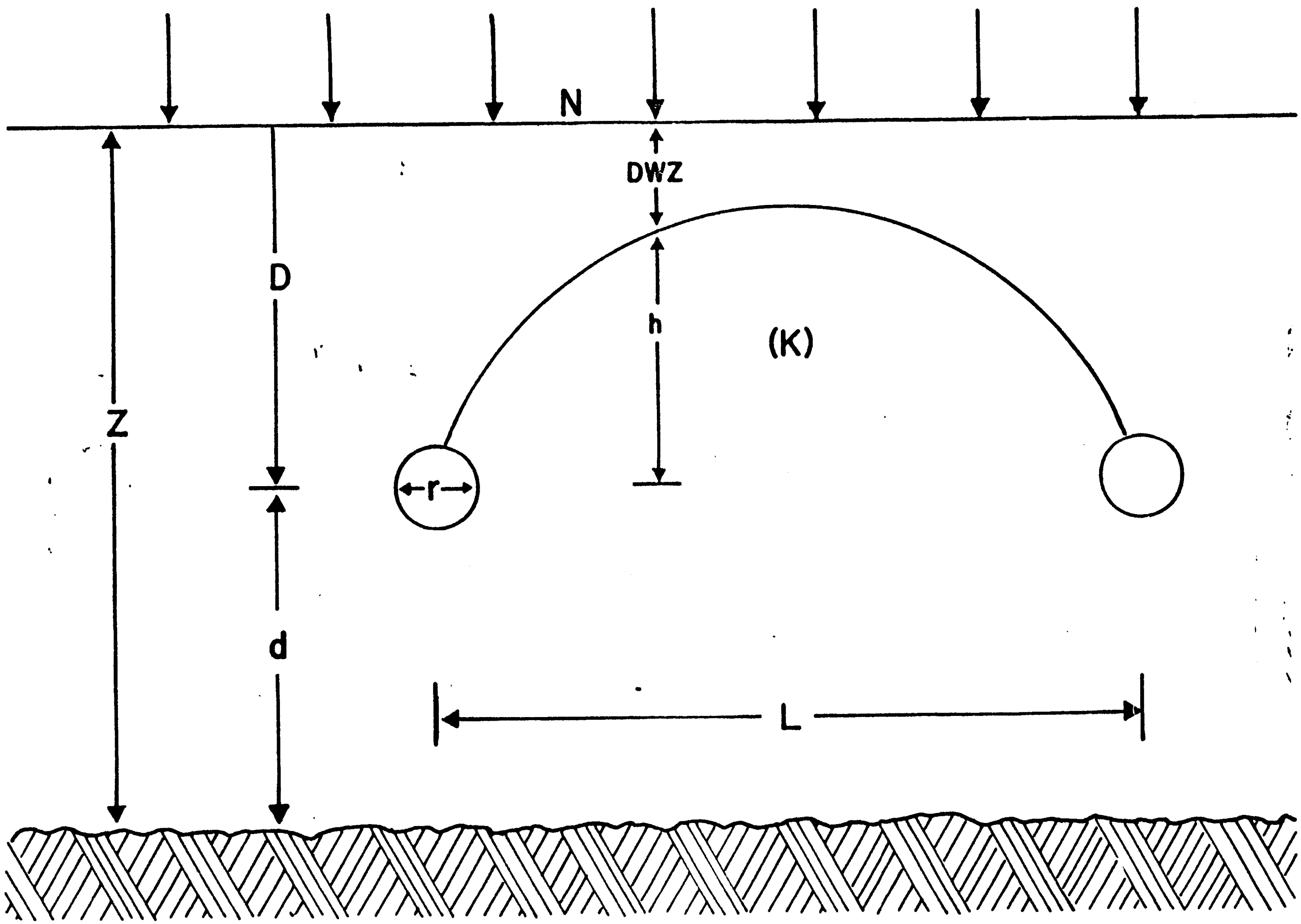

Figure 3.1 - Lateral Drain Design Problem

model of groundwater elevation is sufficient, if vertical gradients can be neglected. The next section will discuss just such a model.

3.1.1 Dupuit Equation

If the following assumptions for groundwater flow in a phreatic aquifer are made:

1. Pressure is hydrostatic, and
2. The velocity of the resulting horizontal streamlines are proportional to the slope of the free water surface but independent of depth,

the Dupuit approach can be used.

The two-dimensional Dupuit equation for flow in a phreatic horizontal bottom aquifer is (Bear, 1979)

$$\frac{\partial}{\partial x}(h(x,y)\ K\ \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(h(x,y)\ K\ \frac{\partial h}{\partial y}) + S\ \frac{\partial h}{\partial t} - N = 0 \qquad (3.1)$$

where S is the spedific yield, K is assumed to be isotropic but non-homogeneous, and x and y are horizontal Cartesian coordinates. The other terms are defined above. If the y axis is oriented parallel to the lateral drains, the water table elevation at any x between the drains will show little variability parallel to the drains due to the strong boundary effect of the drains. This means that $\frac{\partial h}{\partial y}$ will be small and $\frac{\partial}{\partial y}\ (Kh\ \frac{\partial h}{\partial y})$ will be very small and can be ignored. Applying this substitution in Equation 3.1 makes the problem one-dimensional and the variation of head between the drains over the x-axis becomes

$$\frac{\partial}{\partial x}\ (K_x h\ \frac{\partial h}{\partial x}) + S\ \frac{\partial h}{\partial t} - N = 0 \qquad (3.2)$$

In drainage design there are two different approaches to the solution of this equation; steady state and non-steady state. The steady state approach assumes that the irrigation application are uniformly distributed over the irrigation season so that conditions do not change over time and $\frac{\partial h}{\partial t} = 0$. The non-steady state assumes infrequent irrigation and the incorporation of the temporal variation of head is important. For this work the former is assumed and Equation 3.2 becomes:

$$\frac{\partial}{\partial x}\left(Kh \frac{\partial h}{\partial x}\right) = N \qquad (3.3)$$

Equation 3.3 was developed by invoking the Dupuit approximation. This horizontal flow assumption would be adequate if the drains were vertical ditches penetrating the aquifer to the impervious layer with a constant head h = 0.0 (see Figure 3.2). Given the governing equation 3.3 and this boundary condition, it is possible to describe the head or groundwater level as a function of X. Assuming K to be homogeneous Equation 3.3 becomes:

$$\frac{\partial^2 h^2}{\partial x^2} = \frac{2N}{K} \qquad (3.4)$$

which is integrated twice with boundary conditions

$h = 0.0$ at $x = 0$

$h = 0.0$ at $x = L$

to yield

$$h'^2 = d^2 - \frac{NL}{K} x \frac{Nx^2}{K} \qquad (3.5)$$

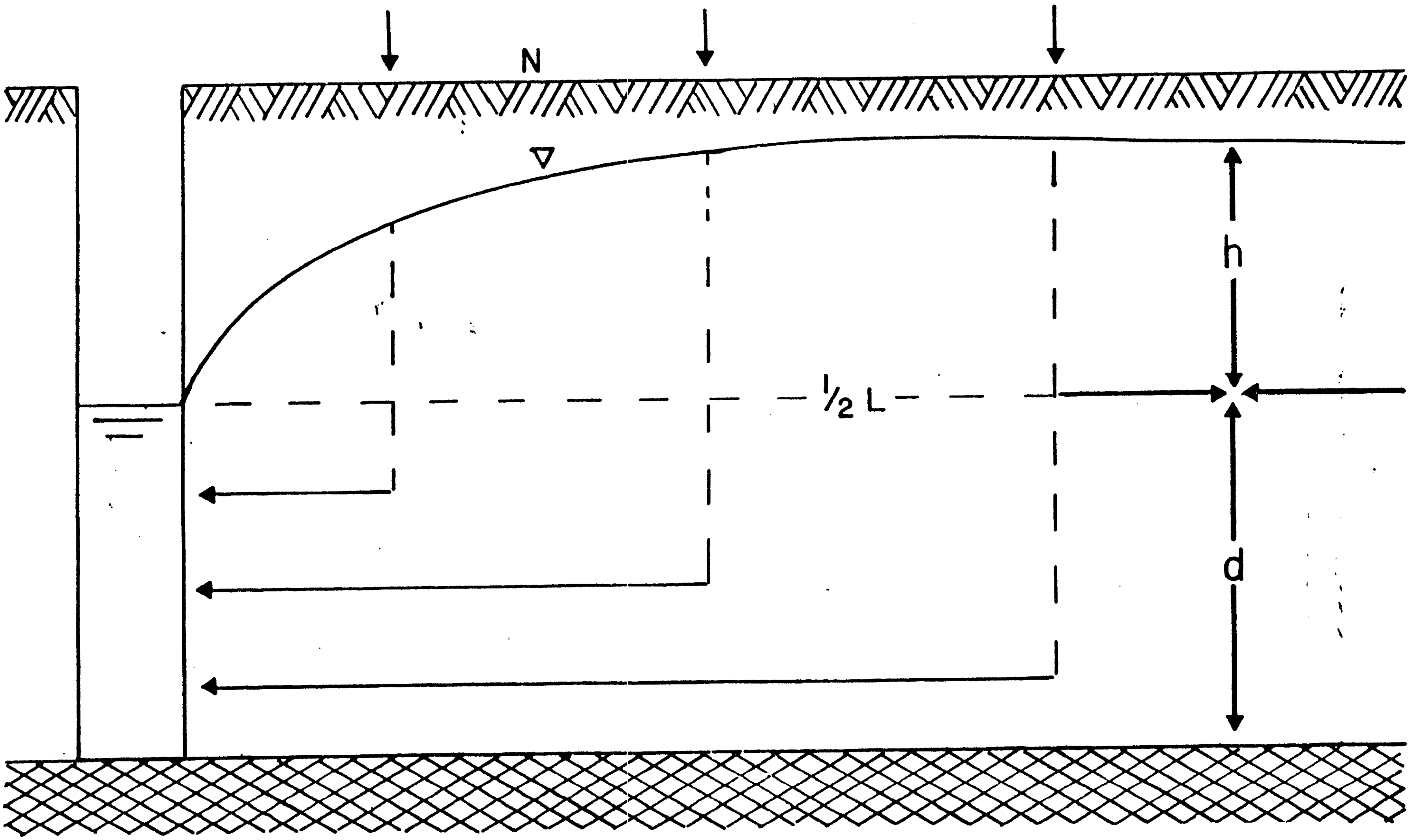

Figure 3.2 - Dupuit Model

where h' = h + d.

The usual drainage design criteria is that the maximum groundwater table elevation between the drains shall not exceed some specified level. For this case, and all cases which assume homogeneous parameters, this maximum point is located at L/2 or halfway between the drains. Replacing x by L/2 in Equation 3.5 and taking the square root produces

$$H'_{L/2} = (d^2 - \frac{NL^2}{4K})^{1/2} \tag{3.6}$$

Now substituting h = h'-d into Equation 3.6 gives the height of the water table above the drains, $h_{L/2}$

$$h_{L/2} - d = (d^2 + \frac{NL^2}{4K})^{1/2}$$

$$h_{L/2} = -d + (d^2 + \frac{NL^2}{4K})^{1/2} \tag{3.7}$$

Again, it should be emphasized that use of the Dupuit approximation works well in theory only for vertical ditch drains that penetrate to the impervious layer. When the Dupuit model is used to describe the flow to subsurface drains, it produces large errors in the estimated water levels between the drains. This is due to the unaccounted for head loss that occurs as a result of vertical upward flow to the drains (see Figure 3.3). A correction to the steady state equation to account for vertical flow is presented in the next section.

3.1.2 Hooghoudt Equation

The Dupuit approximation of horizontal streamlines fails in the case of subsurface drains as seen in Figure 3.3. The Dupuit model

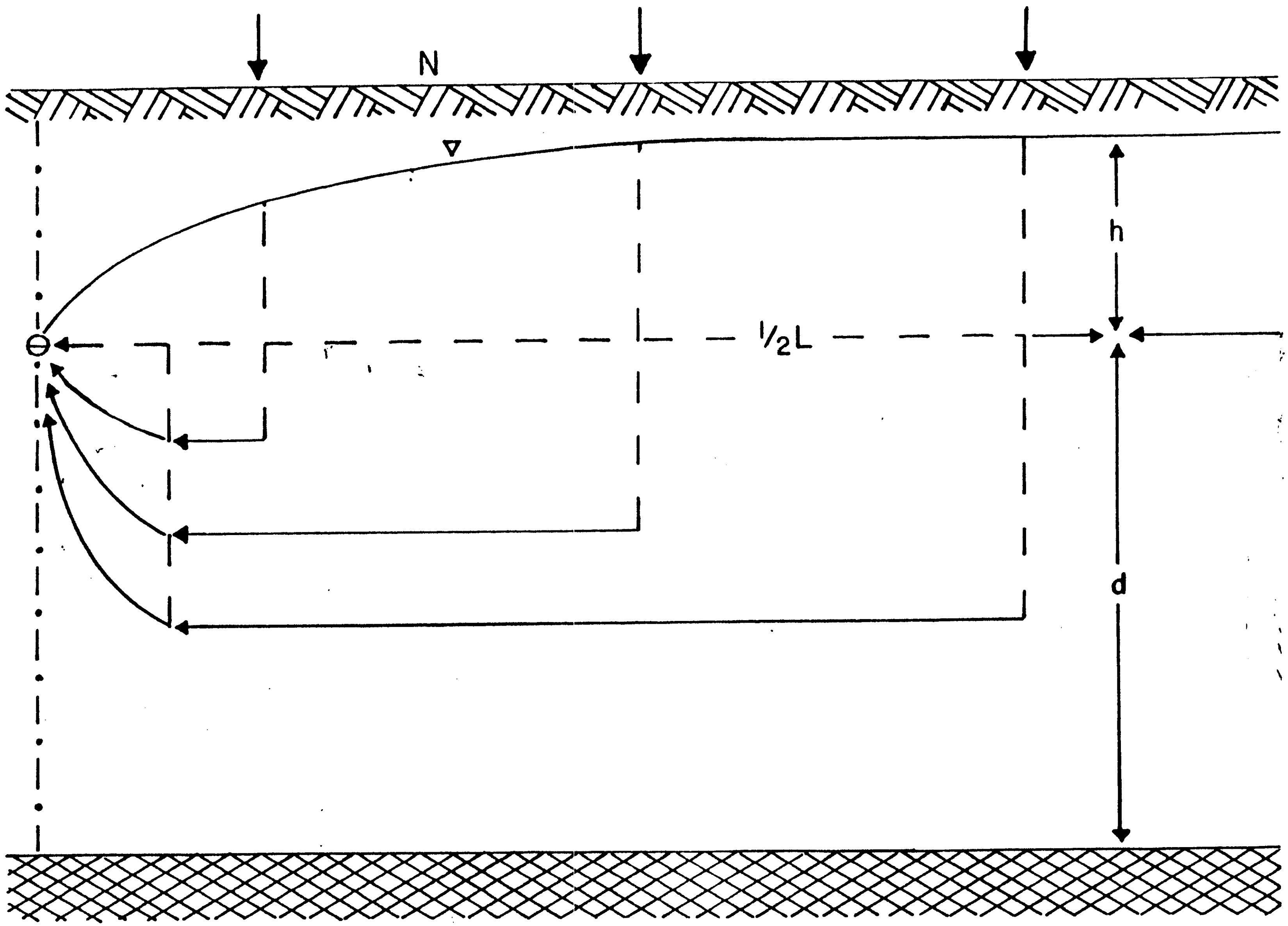

Figure 3.3 - Subsurface Drain

cannot accurately estimate water table elevations due to the vertical flow near the drains, causing substantial head loss. To overcome this shortcoming, Hooghoudt (1940) developed a correction based on radial flow assumptions. Radial flow to the drain assumes that depth to the impervious layer is very great. In most drainage design problems the depth to the impervious layer is such that neither fully horizontal flow or fully radial flow assumptions are valid. Hooghoudt's (1940) methodology incorporated both approaches. He assumed that flow through the mid-section between the drains is basically horizontal and the flow near the drains is basically radial. Then he developed a criteria to determine the mathematical transition point from horizontal to radial flow. However, the resulting equation to determine the transition point as well as the equation for the head loss is computationally tedious.

To make his work more appealing to practicing engineers, Hooghoudt developed an application procedure that is quite clever. He prepared an extensive set of tables with values of d', where d' is an "equivalent height" of the drain over the impervious layer. It is defined as the height of the ditch drain above a fictitious impervious boundary, such that if the spacing is computed using the Dupuit approximation, Equation 3.7, with d' replacing d the same answer would be obtained as when the more exact but approximate computationally tedious solution method is used (see Figure 3.4). According to Hooghoudt, the error in using the table to determine drain spacing is less than 10% (Van Schilfgaarde, 1957). USBR (1978) has provided a closed form

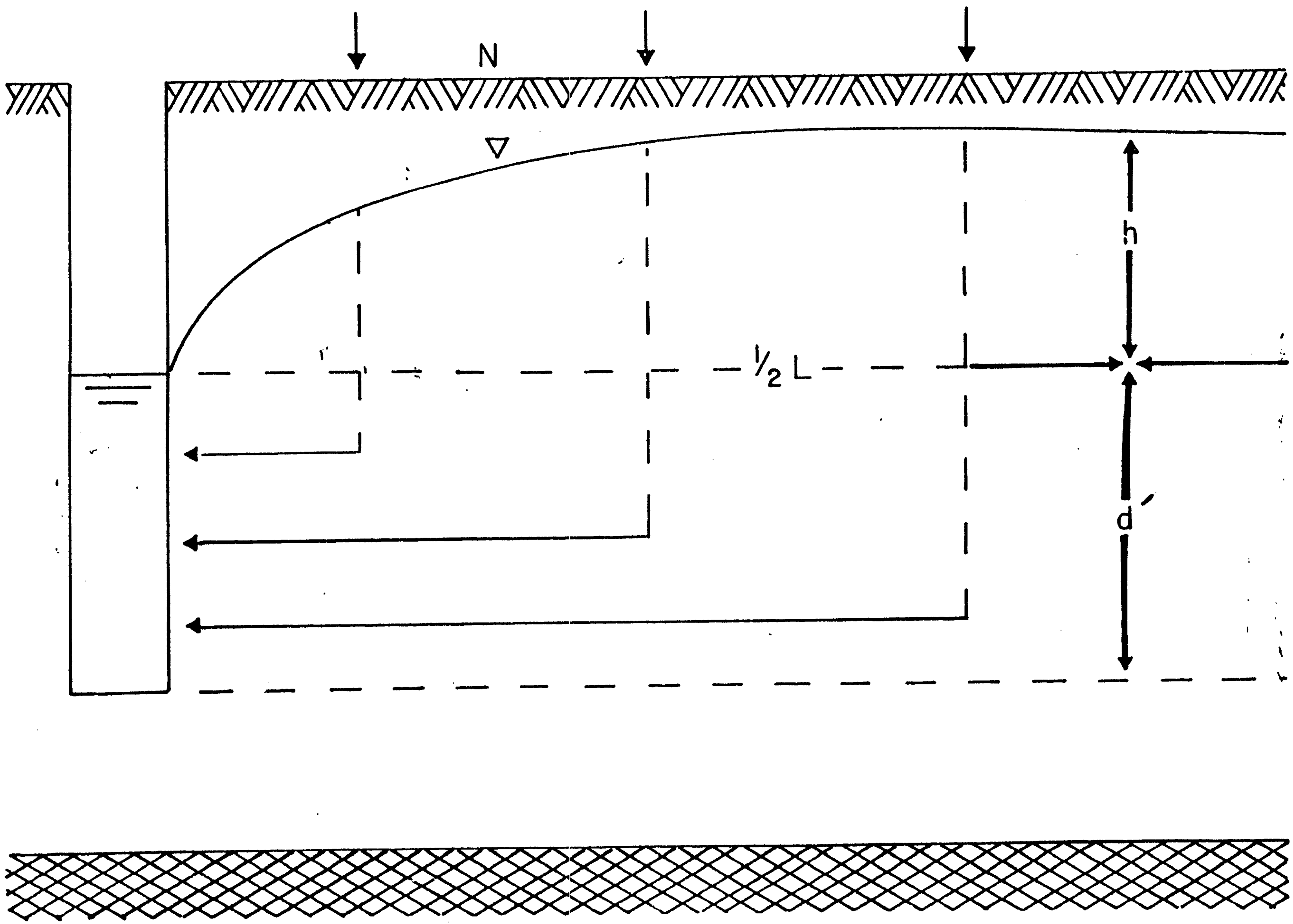

Figure 3.4 - Hooghoudt Model

formula for d', given D, L and r:

$$\text{for } 0 < \frac{d}{L} \leq 0.31 \quad d' = \frac{d}{1 + \frac{d}{L}\left(2.55 \ln \frac{d}{r} - 3.55 - 1.6 \frac{d}{L} + 2 \left(\frac{d}{L}\right)^2\right)} \tag{3.8}$$

$$\text{for } \frac{d}{L} > 0.31 \quad d' = \frac{L}{2.55 \left(\ln \frac{L}{r} - 1.15\right)} \tag{3.9}$$

Replacing d' in Equation 3.7 for water table height above the drain gives

$$h_{L/2} = -d' + (d')^2 + \frac{NL^2}{4K})^{1/2} \tag{3.10}$$

which is the Hooghoudt equation for a non-layered soil.

The Hooghoudt equation is used to find the depth and spacing of tile drains given the drainage rate N, the effective permeability K, the radius of the drains, and the design water table elevation above the drains. This approach is presently used in the Netherlands and in Egypt* as the main tool for drain design. Later in this work the methodologies developed will be applied to an Egyptian case study. For this reason the Hooghoudt equation will be the design model used throughout this work. The next sections analyze the effects of spatial variability and uncertainty upon the design of lateral drains.

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*There is little doubt that the actual drainage problem in Egypt is basically transient. However, the steady state approach is assumed acceptable for design.

3.2 Uncertainty in Drainage Design

The Hooghoudt equation for drainage design that is presently used in Egypt assumes that the drainage rate N, and the hydraulic conductivity K are constant in time and space. Assuming K to be a constant in space is stating that the aquifer is homogeneous and isotropic, which seldom is the case in nature. It follows that if K varies over the field, then N will as well due to non-homogeneous infiltration and also the non-uniform application of irrigation water. The elevation of the impervious layer varies greatly on a scale much smaller than the drain spacing but over the large scale the variation is gradual so the problem of uncertainty in the depth to impervious layer will not be addressed, (Amer, 1979).

In the design process a program of field investigation is undertaken to gather samples of the hydraulic conductivity and drainage coefficient over the field. These samples yield different values for the aquifer properties K and N. Therefore uncertainty about the true values of K and N is produced. The question of identifying this uncertainty and dealing with it in a quantifiable manner is what will be addressed in this section.

Presently all the major guidelines for drainage design recommend using some form of average value as a measure of the effective hydraulic conductivity and drainage coefficient for the one-dimensional drainage design equation. The Bureau of Reclamation (1978) in their Drainage Manual states that "the K value is obtained by averaging the results from in place hydraulic conductivity tests at different locations in the area to be drained". In Drainage of Agricultural Land, the Soil Conservation Service (1973) suggests "K = average hydraulic conductivity." Luthin (1978) recommends using an effective homogeneous hydraulic conductivity. Bower and Jackson (1974) state "The geometric mean appeared to be the best estimate of K_{Hyd} (Effective Homogeneous K)" and finally the Egyptian Public Authority for Drainage Design uses the geometric mean (Amer, 1979).

The arithmetic mean overestimates while the geometric mean underestimates the hydraulic conductivity (Bower and Jackson, 1974). The only information that either mean provides is that 50% of the time the sample value of the aquifer properties will be below the mean value and 50% of the time the sample value will be above the mean. It gives no information about how much the distribution of the true values vary from the mean. The uncertainty about the variance is very important because it means there is uncertainty about the resulting design head above the drains. This is important since the reason for installing drains is to reduce the piezometric head below a certain level as defined for the crops being grown. The crops are very sensitive to the value of this head and unsaturated zone. If the criteria is met with only 50% reliability, and no idea how much the distribution varies, the drains may not be serving their desired purpose. The types of uncertainty that exist in the aquifer properties will be addressed next.

The uncertainty that occurs in the drainage design problem can be broken down into two classes: the natural spatial variability of N and K, and information uncertainty. The natural spatial variability assumes that the groundwater system is a stochastic system with some inherent uncertainty that cannot be reduced by sampling. The information uncertainty is that due to incomplete or noisy information of the groundwater system and with sufficient time and money this uncertainty can be reduced.

The natural spatial variability of the hydraulic conductivity and drainage coefficient is a very complex process. It is composed of many scales of variation superimposed upon one another which makes any deterministic description of the process impossible. The large scale variation or trend is usually identifiable, but the small scale variations are not. Thus using the large scale trend as a description of the system by extrapolating beyond the data points will only be approximate no matter how fine a grid is used in sampling to identify the trend. The large scale process will not contain all the information about the system properties. Therefore some method for incorporating the information from the small scale level is needed. The small scale variations are a function of the development of the aquifer system. Therefore by looking at the process of aquifer formation it may be possible to infer something about the aquifer properties themselves.

One possible explanation is that in the case of alluvial valleys, where most of the world's drainage takes place, the formation of the aquifer layer was a function of the deposition of sediments transported from upstream at times of flood conditions. The process of sediment transport is governed by the velocity distributions in the river flow. This process is a turbulent flow process in which turbulence can be described statistically by a mean and a random deviation from the mean which can be described statistically. It is hypothesized that the properties of the aquifer that were formed by a process with a mean and statistically random component could also be described by a mean or trend and a random component which is described statistically.

This approach of characterizing small scale variation of a groundwater property from the trend as a probabilistic process phenomena has been followed by several authors based upon different theories for doing so. Freeze (1975), Bark et al., (1978) and Sagar (1978) as well as parallel work at MIT by Dettinger and Wilson (1981) and Wilson and Dettinger (1982) attempt to preserve the spatial statistical properties of the phenomena. This approach provides a probabilistic description of the magnitude, spatial extent and nature of the effects that the possible range of property variations can have on aquifer behavior, particularly piezometric head (Dettinger and Wilson, 1981). The description of these natural properties can be considered as a form of uncertainty that is irreducible.

On the other hand, information uncertainty represents the lack, in quantity or quality, of information concerning the aquifer system. When describing various properties of the system, inaccuracy in system parameters will be included. The inaccuracy or error results from sparse data, measurement error or model error. The errors may be due to statistical or conceptual inadequacy. This uncertainty may be reduced by increasing the size of the data samples, better measurement techniques or use of better tools. The information uncertainty can be while natural uncertainty due to spatial variability cannot be reduced.

In the case of the drainage design problem it is desired to incorporate the uncertainty in the material properties into a measure of uncertainty in the resulting piezometric head between the drains. In doing so the uncertainty due to spatial variability and information must be described.

3.3 Method for Analysis of Uncertainty

There are two basic methodologies for incorporating the uncertainty of input parameters into uncertainty of model outputs: derived distribution methods and moment methods. Figure 3.5 gives an illustration of the two methods. Both methods attempt to take information about uncertainty in input parameters (material properties, boundary condition, initial conditions) and provide information about the uncertainty of the model output (Piezometric head). This is a form of sensitivity analysis but it allows for a quantifiable measure of the expectation of occurrence for classes of events or values. Since a model has been postulated that relates input parameters to output parameters, there must be some functional relationship between them. The two methods are based upon this functional relationship.

The derived distribution method can be divided into two techniques: the analytical technique and the simulation technique. The derived distribution method uses the probability distribution of system input to derive the probability distribution of the system output. The analytical technique provides a closed-form, analytical expression of the probability distribution of the system output based upon the

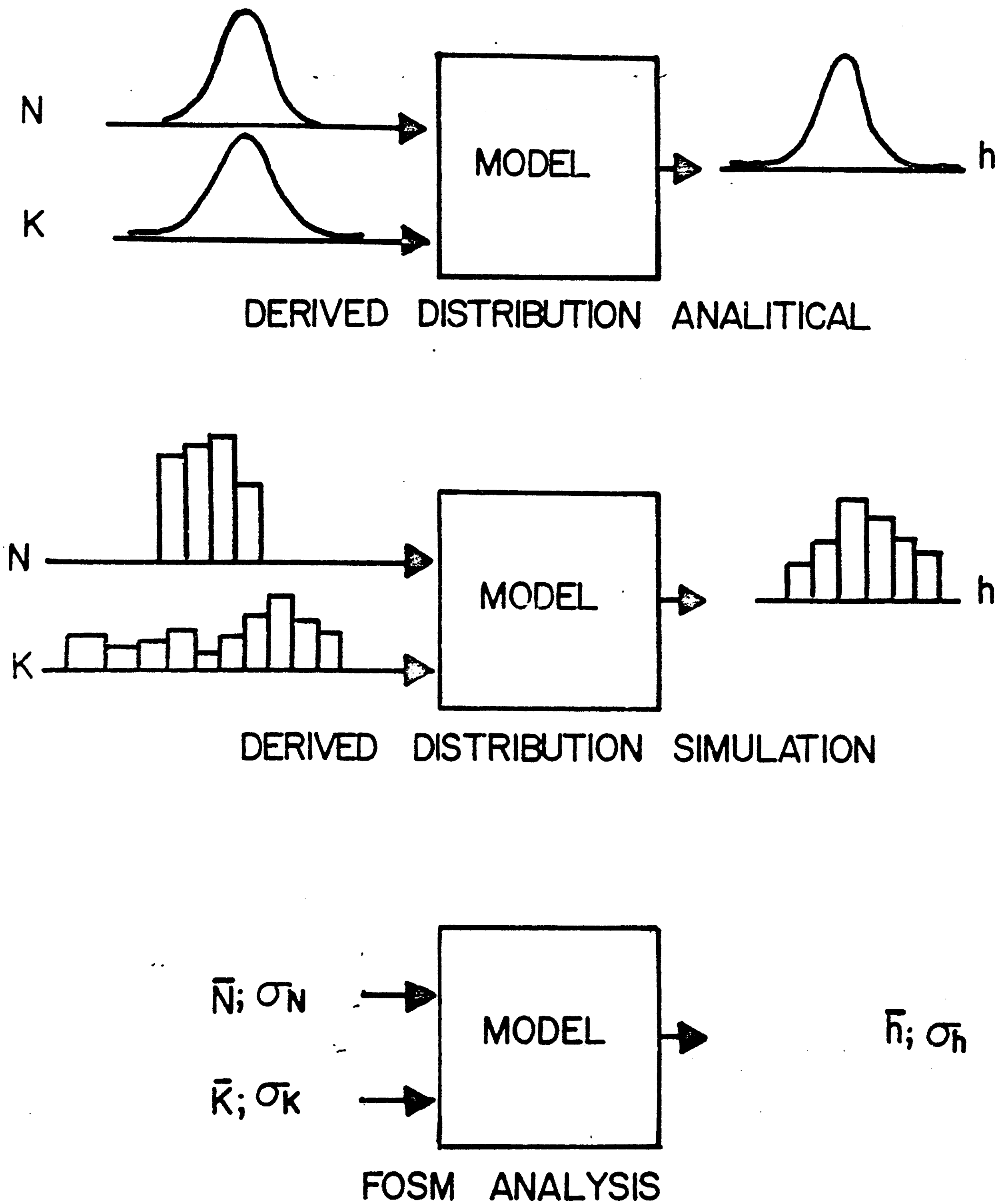


Figure 3.5 - Methods for Analyzing Uncertainty in System Outputs

probability distribution of the system input and the functional relationship between input and output using integral calculus. The analytical techniques have been applied by Eagleson (1978) for stochastic runoff due to random rainfall events, Chan and Bras (1979) for urban storm runoff; Cordova (1979) for stochastic infiltration due to random rainfall events; and Sager and Kisiel (1972) for the analysis of permeability measurements from aquifer pump tests. In practice, however, the integral analysis that produce the derived distribution results approach is often not mathematically tractable.

To overcome this barrier, but still keep information about the full probability distribution, Monte Carlo simulation is used. Monte Carlo simulation is a technique in which random inputs are generated that retain the sample statistics of the full input distribution. These discrete random inputs are then simulated to produce output values. After repeated simulation a histogram of output values is generated which will approximate the desired probability distribution. This technique has been applied to groundwater problems by Warren and Price (1961) and Freeze (1975), among others.

It is often difficult to obtain the probability distribution that is input to a derived distribution analysis and only slightly easier to estimate their moments. Thus the results obtained which depend on the exact distribution selected can be deceptive. They may reveal nothing more than an analysis conducted using only the first two moments (Dettinger and Wilson, 1981).

Alternately, the moment method for analysis of uncertainty makes the assumption that the information contained in the mean or average value and the variance-covariance which is a measure of the variation around the mean is sufficient to describe the uncertainty in the problem. The higher moments are ignored since they are either small or provide little useful information. In the case of a normal distribution, the third and all odd moments are zero and all other moments can be calculated from the variance (Benjamin and Cornell, 1970).

First and second moment methods can be applied using a perturbation and/or Taylor series expansion. The perturbation approach has been used by Tang and Pinder (1978), Bakr et al., and Gutjar et al., (1978). The Taylor series expansion approach to be followed in this work was applied by Cornell (1972) and Wilson and Dettinger (1981) to simple analytical hydrologic and groundwater problems respectively. Dettinger and Wilson (1981) and Sagar (1978) use the method with numerical models.

3.3.1 Information Uncertainty: FOSM Analysis

The following section will present the theory behind the use of the method of moments as used in this paper and by others mentioned above.

First-order second moment (FOSM) analysis requires some basic calculus and linear algebra and a limited amount of computation reducing greatly the analytical and computational burden as compared to derived distribution techniques. FOSM analysis works with the first non-zero components of any moment thereby reducing the need for information about the full probability distribution of parameters. In most cases only the first two moments, mean and variance are considered.

The disadvantages are that FOSM analysis is at best incomplete, that it may only be approximate, and that certain relationships of interest (e.g., Y = max{X}) do not lend themselves to this analysis (e.g., are not differentiable), (Cornell, 1972). In addition to previously mentioned advantages of FOSM analysis, an approach based on means and variances may be all that is justified when one appreciates (1) the data and physical arguments are often insufficient to establish the full probability law of the variable, (2) that most engineering analysis includes an important component of real, but difficult to measure, professional uncertainty (due, for example, to imperfect physical theories and to engineering approximations) and, (3) that the final output, namely the decision or design parameter is often not sensitive to moments higher than mean and variance (Cornell, 1972).

The discussion of FOSM follows that of Veneziano (1978). The first step in FOSM analysis is the linearization of the function around the point of interest. This allows higher order contributions to the mean and variance to be identified and discarded. Linearization is carried out by retaining only the first terms of a Taylor series expansion of perturbation analysis. In general, when uncertainty analysis will follow, the linearization is about the mean value of the argument. Linearization leads to the first order relationships:

$$g(x) \doteq g(M_x) + \left.\frac{dg}{dx}\right|_{x_o} (x-M_x) \qquad (3.11)$$

where g(x) is a function of the single parameter x and M_x is the mean value of the parameter x, and $\doteq$ represents first order equivalency.

For a multiple parameter function the linearization becomes

$$g(x_1,\ldots,x_n) \doteq g(M_{x_1},\ldots,M_{x_n}) + \sum_{i=1}^{n} \left.\frac{\partial g}{\partial x_n}\right|_{M_x} (x_i - M_{x_i}) \qquad (3.12)$$

where g is now a function of the vector $x_1,\ldots,x_N$, M_{x_i} is the mean of parameter x_i and M_x is the vector of mean values M_{x_1}, M_{x_N}.

In these equations, the arguments x may be interpreted as inputs or parameters. Note that this equality is exact only in the case of linear functions and includes error, in all other cases, proportional to the neglected second and higher derivatives.

Once a function has been linearized properly, finding its moments (to a first order approximation) given the moments of its arguments is trivial. The mean and variance are defined as

$$M_g = E[g] \qquad (3.13)$$

$$\sigma_g^2 = E[(g-M_g)^2] \qquad (3.14)$$

where $E[g(x)] = \int_{\infty}^{\infty} g(x)\ f_X(x)\ d_x$ and $f_X(x)$ is the probability distribution of x. Using the linearized functions, the mean and variance may be expressed.

$$M_g = E[(g(M_x) + \left.\frac{dg}{dx}\right|_{M_x} (x-M_x))]$$

$$= g(M_x) \qquad (3.15)$$

$$\sigma_g^2 = E[\{g(M_x) + \left.\frac{dg}{dx}\right|_{M_x} (x-M_x) - g(M_x)\}^2]$$

$$= e[(\left.\frac{dg}{dx}\right|_{M_x})^2 (x-M_x)^2]$$

$$= (\left.\frac{dg}{dx}\right|_{M_x})^2 \sigma_x^2 \qquad (3.16)$$

and similarly, when more than one argument is uncertain,

$$M_{g(x_1,\ldots,x_n)} = g(M_{x_1},\ldots,M_{x_n}) \tag{3.17}$$

$$\sigma_g^2 = \sum_{i,j=1}^{n} \left(\frac{\partial g}{\partial x_i}\right)^2 x_i^2 + \left(\frac{\partial g}{\partial x_j}\frac{\partial g}{\partial x_i}\right) \text{Cov } (x_i,x_j)) \tag{3.18}$$

More detail is provided by Wilson and Dettinger (1981).

An interpretation of the means and standard deviations assumed and derived in first order analysis is provided by recognition that "about 50% of the probability mass of most unimodal distribution lies within about $\pm$ 2/3 σ of the mean", (Cornell, 1972). With this in mind, estimates of the variance can be arrived at or used in much the same way as error brackets and tolerances. Only in the case of a normally distributed variable does the first order work suffice to describe the variable. In other cases the coefficient of variation $\frac{\sigma}{M}$, must be assumed to reflect an approximate or subjective (in the case of initial parameter estimates) level of uncertainty associated with the function.

The FOSM approach assumes that the input uncertainty is information uncertainty. If the uncertainty found in the system input is only information uncertainty then FOSM analysis provides a good method for describing uncertainty in system output. If the uncertainty in input parameter includes natural variability then this must be included in the analysis of output uncertainty. The next section will address the question of spatial variability and methods to categorize it.

3.3.2 Spatial Variability: Stochastic Hydrogeology

The natural uncertainty that is due to the spatial variability is a function of the structure of the phenomena being studied. The structure may have large scale trends as well as smaller scale variability. The theory of regionalized variables has been developed to describe natural phenomenon with a spatial distribution which varies from one place to another with apparent continuity (Olea, 1975). In the theory of regionalized variable the "drift" is used to denote slowly varying large scale trends while the "covarigram" describes the higher frequency variability of the structure of regionalized variable.

The drift physically represents the trend of the function over a region. It represents only the major features of large scale structure. The drift can be defined as:

$$M(\bar{X}) = E[Z(\bar{X})] \tag{3.19}$$

where the drift $M(\bar{X})$ at a point $\bar{X}$ is the expected value of the regionalized variable $Z(\bar{X})$ at a point $\bar{X}$ (Hujbregts and Matheron, 1971). The concept of drift provides a means for splitting the regionalized variable into two components, the drift which represents large scale trends, and the residual $Y(\bar{X})$ which contain information about the variability of the regionalized variable (Olea, 1975). Figure 3.6 is an illustration of a first order drift.

The residual $Y(\bar{X})$ is defined as

$$Y(\bar{X}) = Z(\bar{X}) - M(\bar{X}) \tag{3.20}$$

The residual has the property of a zero mean and can be used to calculate the covarigram, which is the measure of structural variability (Olea, 1975). The covariogram, which is calculated from the residuals, has structural information about the regionalized variable. It includes

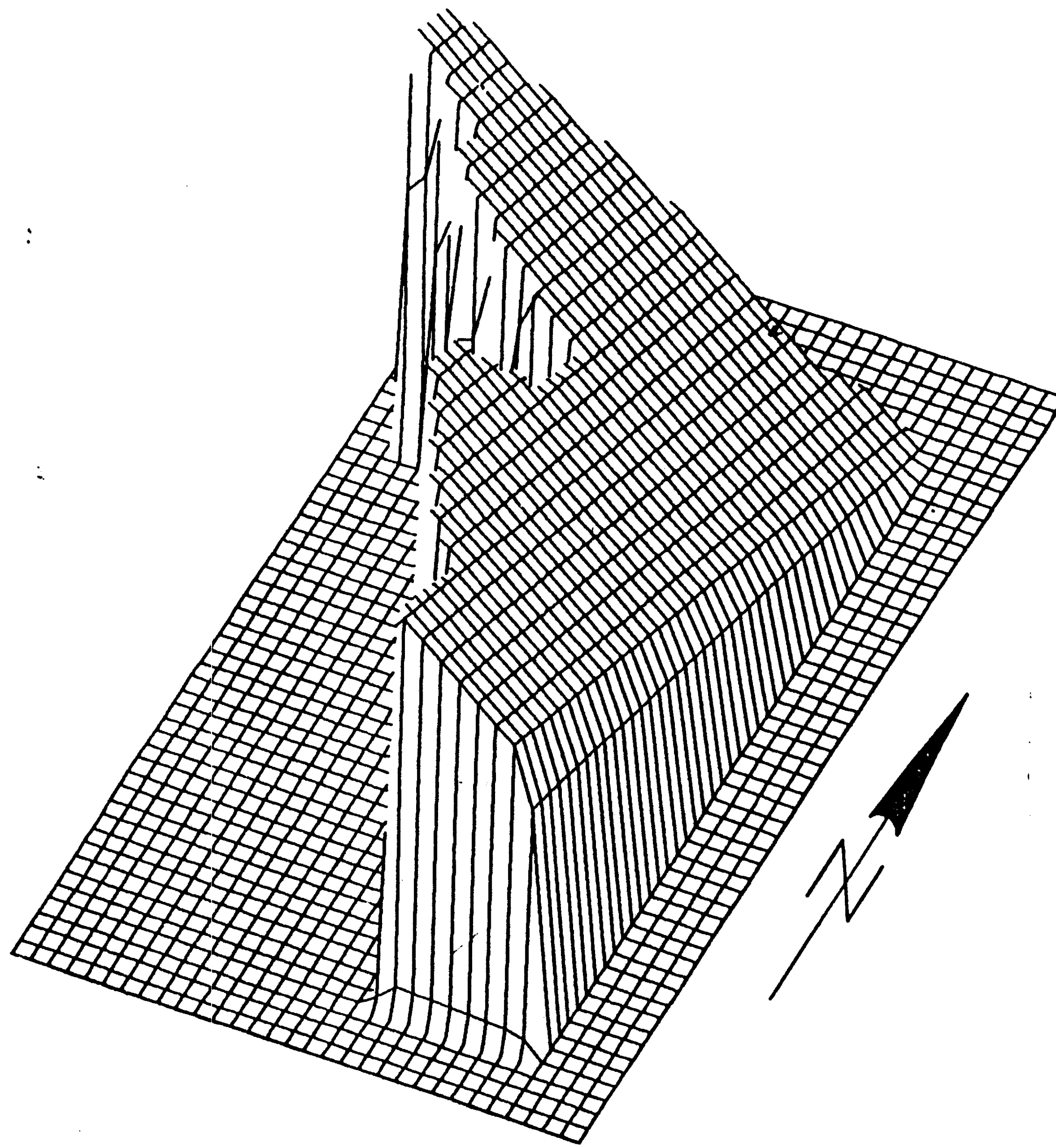


Figure 3.6 - Example First Order Drift

information about the size of the zone of influence around a sample, the isotropic nature of the variable, and the continuity of the variable through space. The covariogram, $\gamma(h)$ is defined as

$$\gamma(h) = E[\{Y(\bar{x}+h) - Y(\bar{x}) - E[Y(\bar{x}+h) - Y(\bar{x})]\}^2] \qquad (3.21)$$

where h is the distance from a sample point. An example covariogram is presented in Figure 3.7. The important parameters of the covariogram for this work is the range and the intercept at h=0. The range is the distance beyond which the influence of a sample disappears. In many cases when extrapolating the covariogram from the smallest sample distance to zero on the distance axis, the covariogram does not pass through the origin. This phenomenon is called the "nugget effect" which may occur for a number of reasons, such as poor analytical precision, poor sampling preparation (measurement error) or an even smaller scale variation occuring that cannot be detected by the large sampling interval (David, 1978). The covariogram is described in detail in Chapter 4 of David (1978). In the case where the regionalized variable has a finite variance the covariogram is related to the covariance function:

$$\gamma(h) = Var(0) - Cov(h) \qquad (3.22)$$

where Cov(h) = covariance function over h,

Var(0) = point variance,

$\gamma(h)$ = covariogram over h.

Figure 3.8 is an illustration of a covariance function.

Now that the methods to analyze both information uncertainty and spatial uncertainty have been presented the next section applies

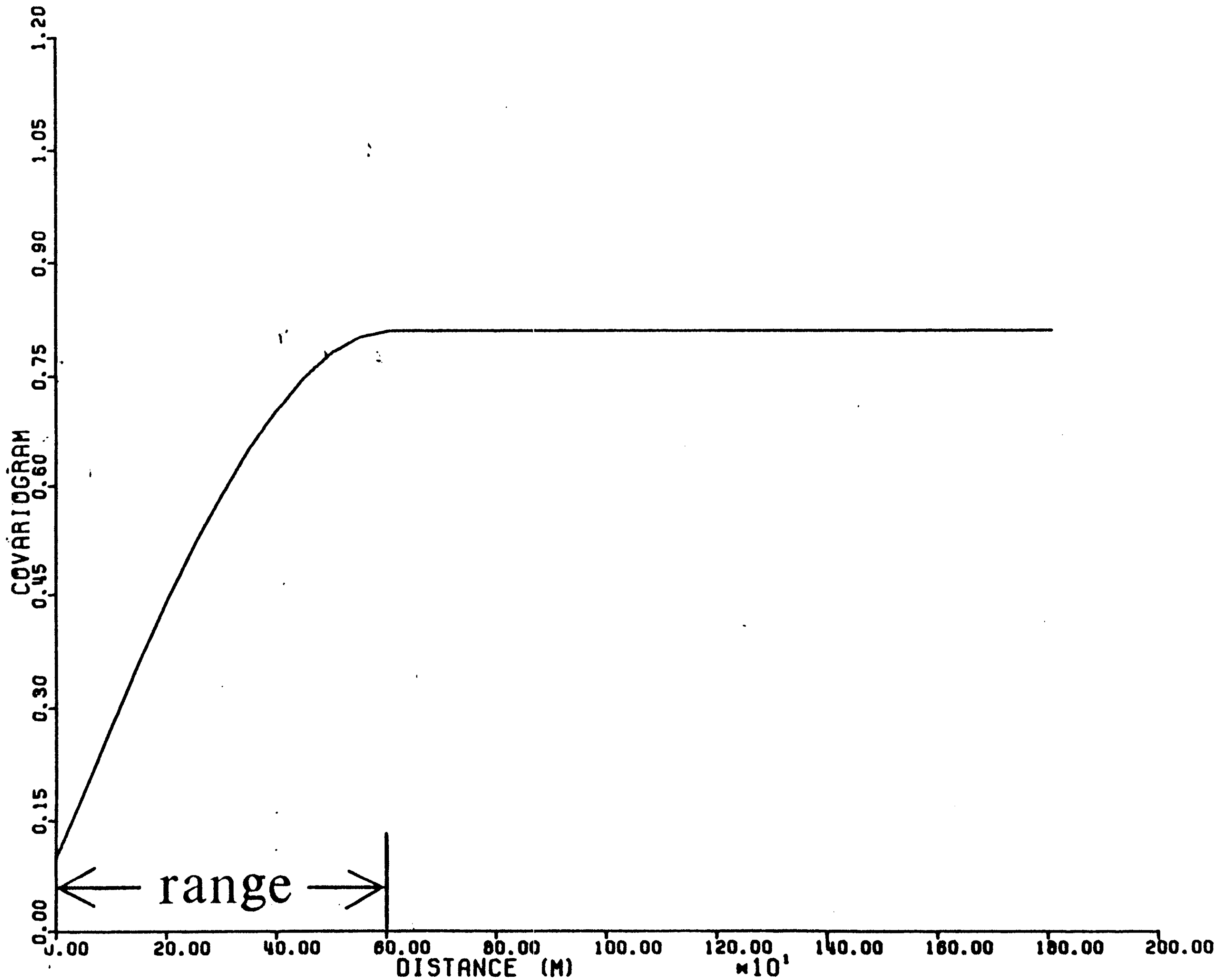


Figure 3.7 - Covariogram

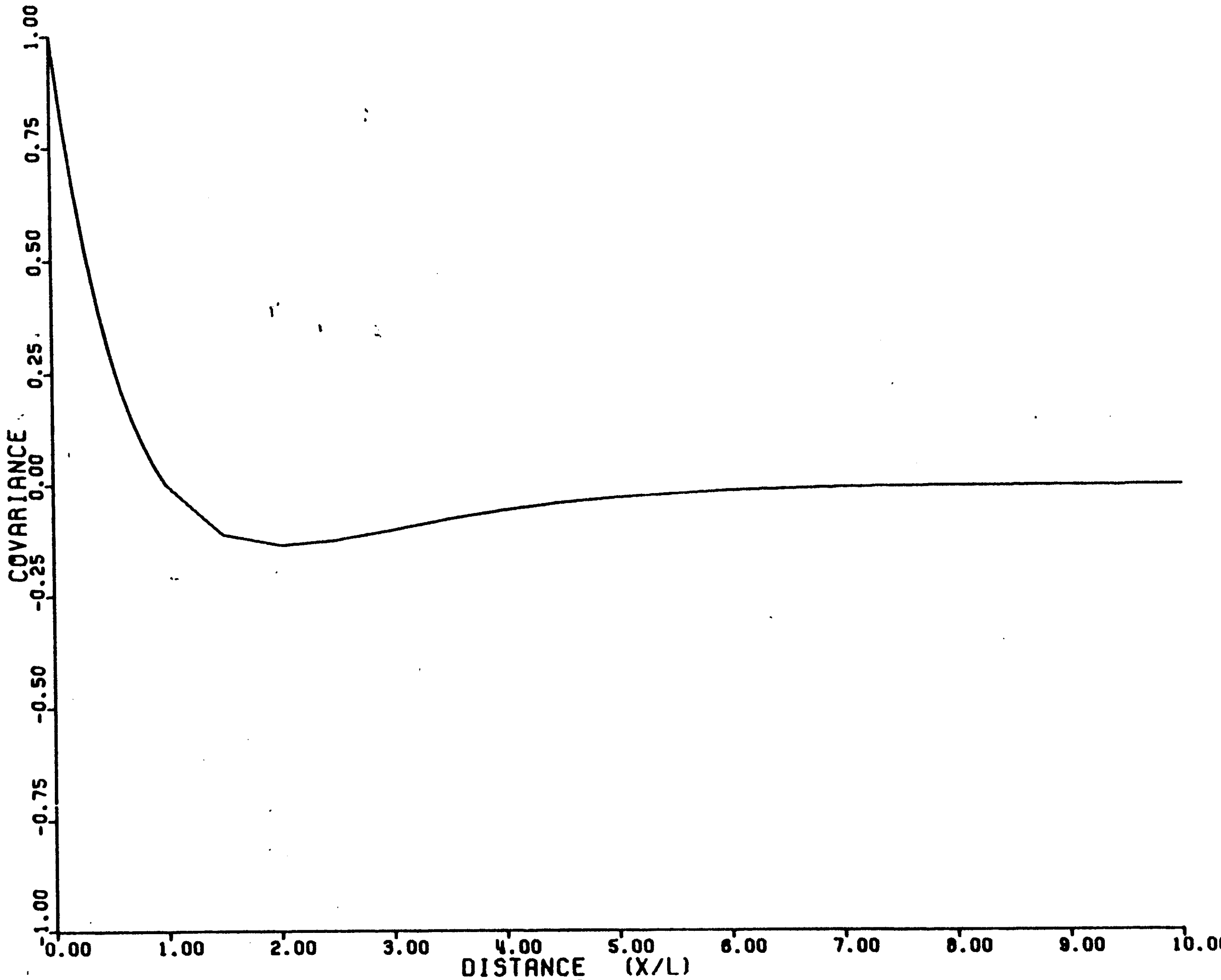


Figure 3.8 - Covariance Function

these methods to a drainage case study in Egypt.

3.4 Analysis of Uncertainty in Drainage: An Egyptian Case Study

Based upon the concepts presented above the uncertainty in soil permeability for a drainage field in Nile Delta is investigated.

The area to be studied is approximately 1500 feddan (1 feddan = .4 hectare = 1 acre) on the Embabe Drain (see Figures 3.9 and 3.10) in the Nile Delta. It is bounded on the eastern side by the Bagurize navigation canal and the western side by the Sirwasija irrigation canal. The northern border is formed by the Sabal and Shanawan drains and the southern border by the Singing road.

The Embabe drain, the downstream part of the Sabal drain and the Sabal pumping station, ensure good drainage possibilities. It can be safely assumed that with normal operation of the pumping station, a water level of approximately 2.5 meters below the surface level will be maintained in the Embabe drain. The need for drainage in this area was demonstrated by the fact that test auguring showed water table levels less than 0.5 m below ground level. In some locations the soil showed visible signs of salinity and the stand of crops was irregular. The site selected is representative of conditions found over large areas of Minoufiya Province.

The study area has 101 2-meter deep augur hole tests performed to measure hydraulic conductivity of the soil and groundwater depth. These samples were taken on a regular grid of 200 meters with some gaps. The values of the hydraulic conductivity range from .01 to .49 meters/ day. A histogram of their values is seen in Figure 3.11.

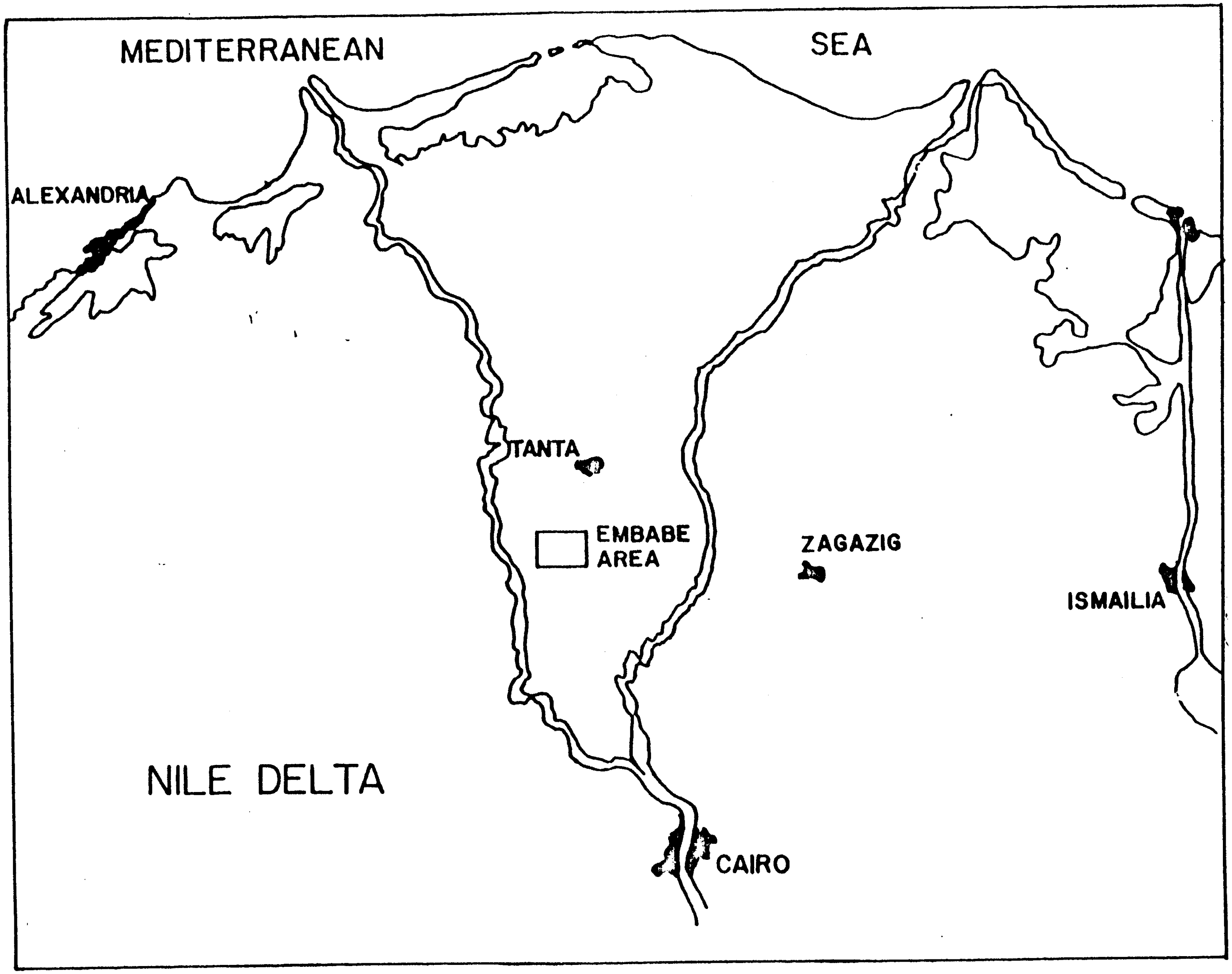


Figure 3.9 - Location of Embabe Case Study Area

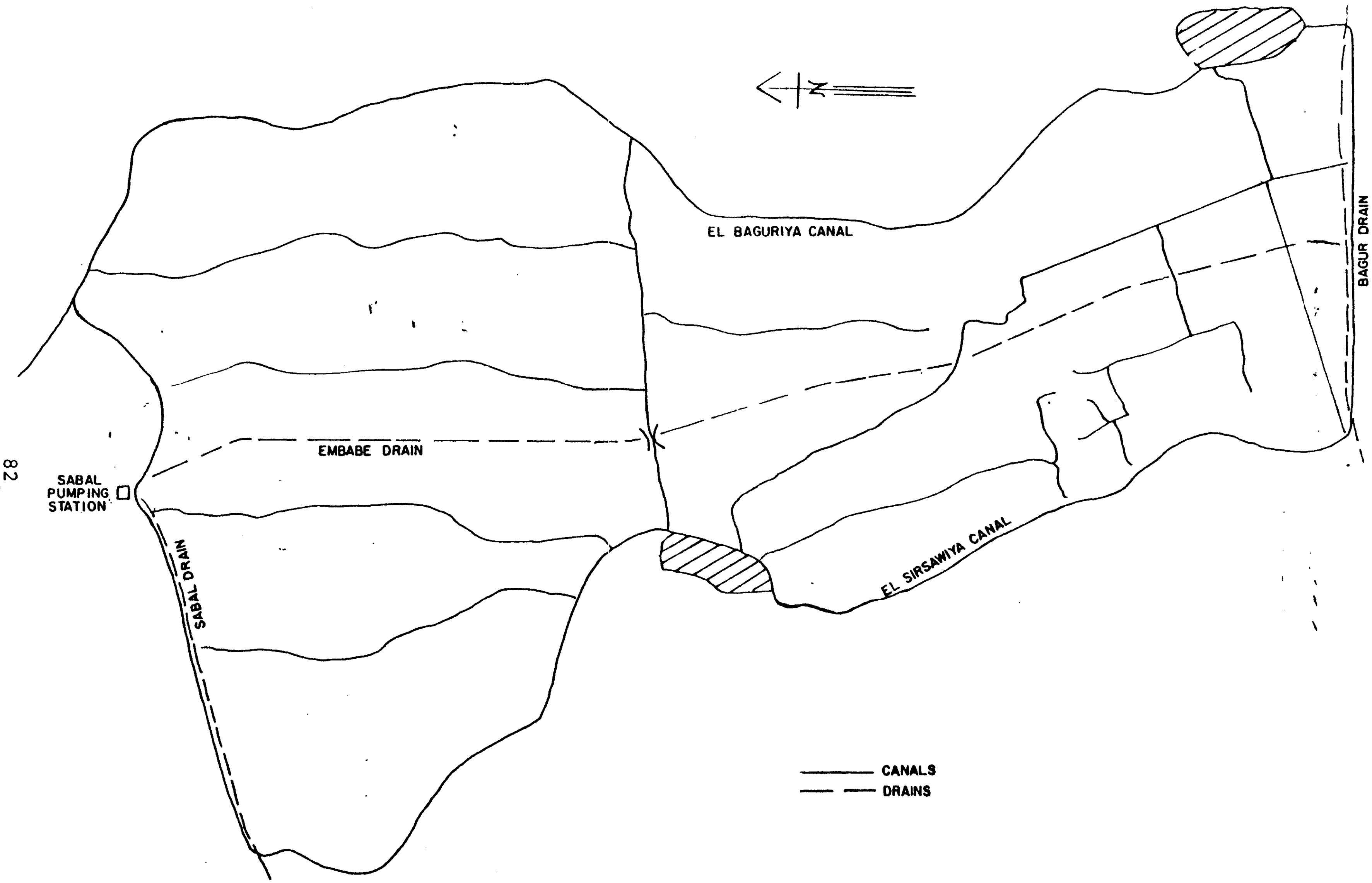


Figure 3.10 - Embabe Drain Area

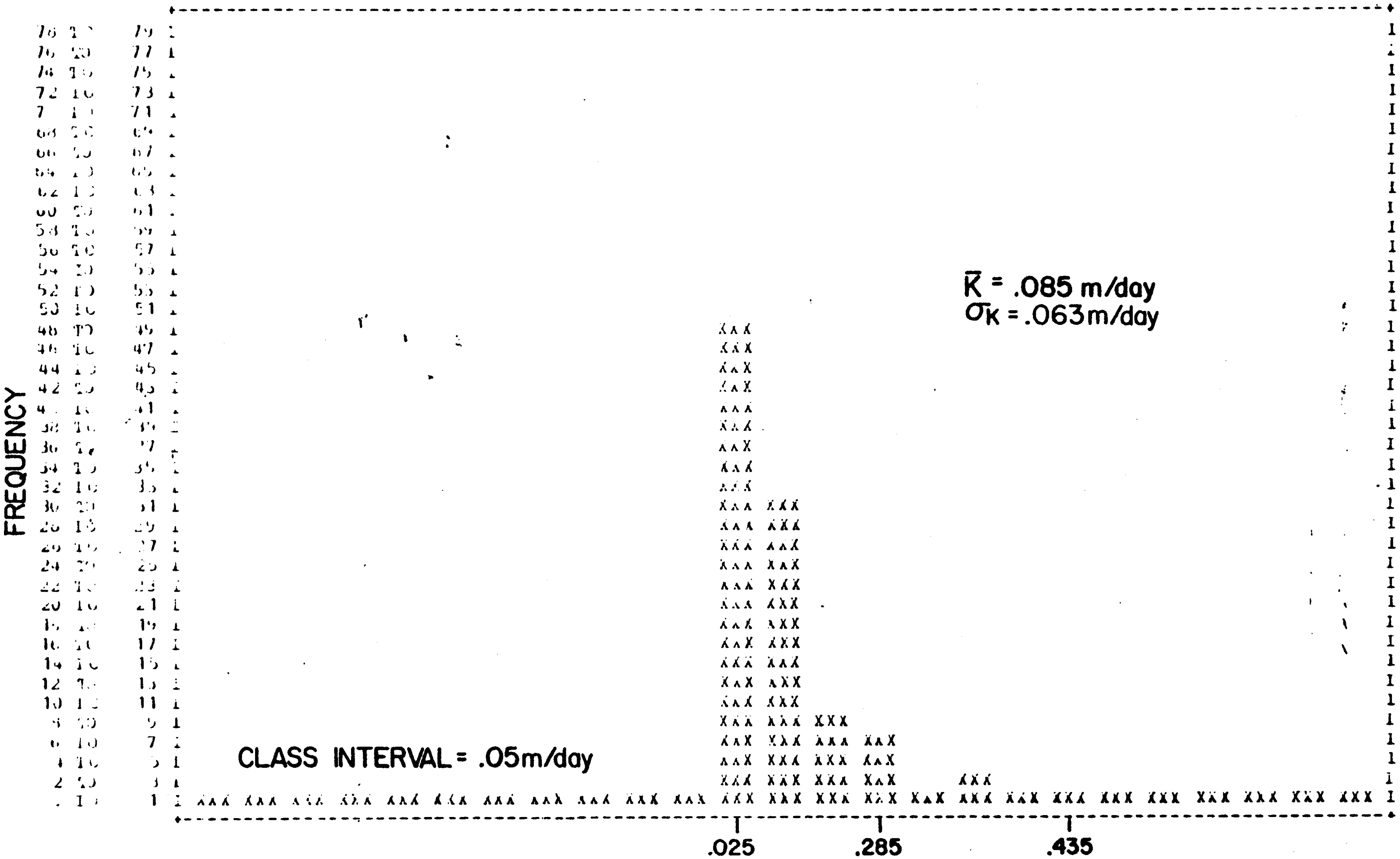


Figure 3.11 - Histogram of Hydraulic Conductivity

An analysis was performed on the sample data to identify information uncertainty and natural spatial variability. Freeze (1975) in his classic paper presents a detailed literature survey to support the assumption for log-normal distribution of hydraulic conductivity. If the hydraulic conductivity K, is log-normally distributed, a new parameter can be defined, Y = log K which is normally distributed with mean $\bar{Y}$ and variance σ_Y^2. Figure 3.12 is a histogram of the log of K for the Embabe area, where log = log base 10. The Kolmgorov-Smirnov test on this data shows that the hypothesis of log normal distribution of soil permeability can not be rejected with a significance level of 85 percent. Figure 3.13 is a contour map of the log of K over the Embabe area, also showing the sampling locations. Figure 3.14 is a three-dimensional plot of the sample data.

Using automatic, BLUEPACK, (as well as manual) drift identifiers it was found that for the Embabe area no drift can be identified. This means that the soil permeability is homogeneous in the mean. Thus subtracting a uniform mean over the field from the sample data the residuals are computed and a covariogram can be calculated. Figure 3.15 is a covariogram of residuals of the log of permeability. It can be seen that there is a "nugget effect" equal to .40. This is assumed to occur due to sample error in the auger hole tests of 25% (Amer, 1979) and smaller scale variation of permeability. The range of covariogram is found to be approximately 1000 meters. It is seen that the sill is approximately equal to the sample variance as the theory requires. This analysis provides a measure of the information uncertainty in the

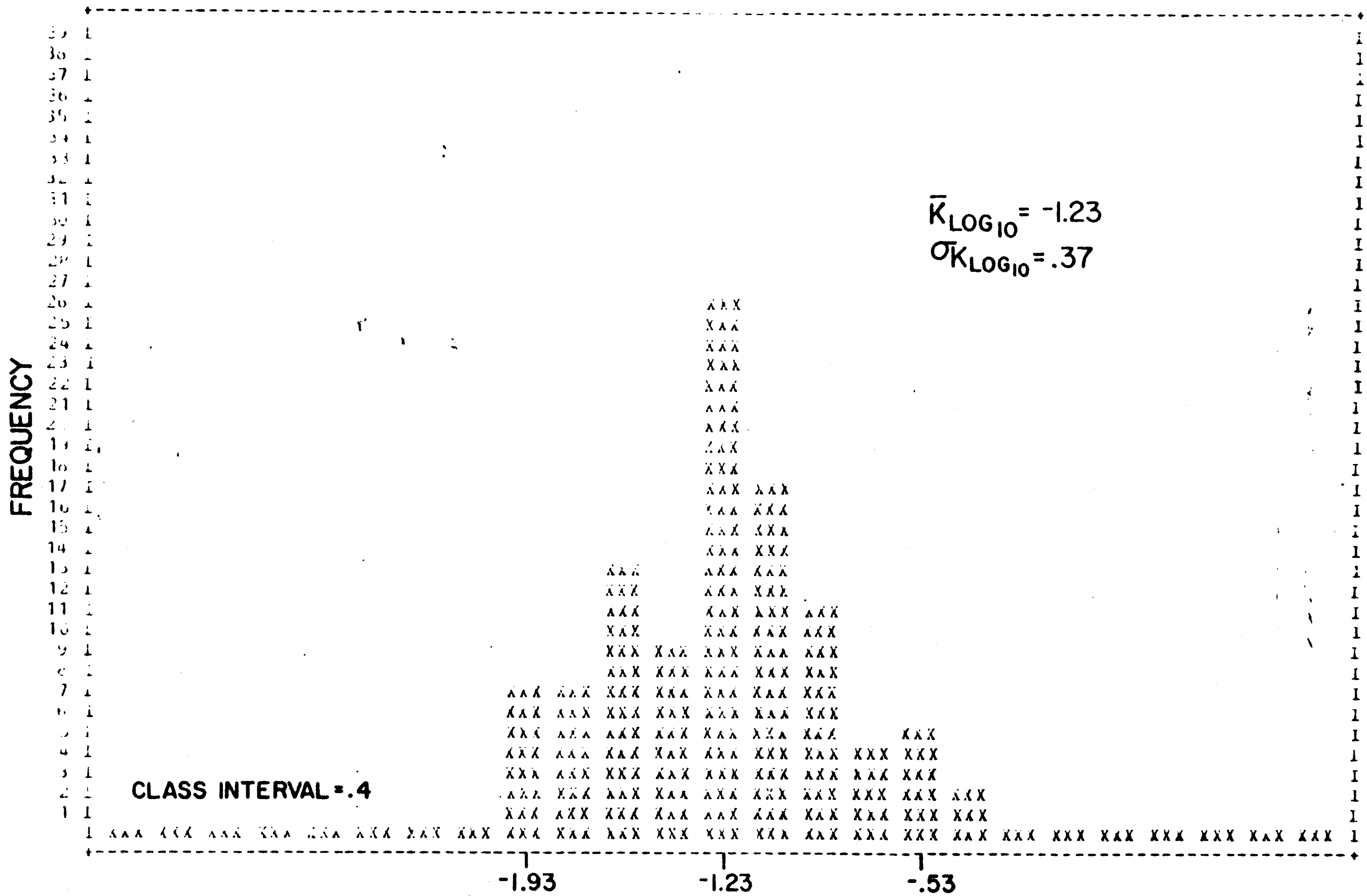


Figure 3.12 - Histogram of Log_{10} of Hydraulic Conductivity

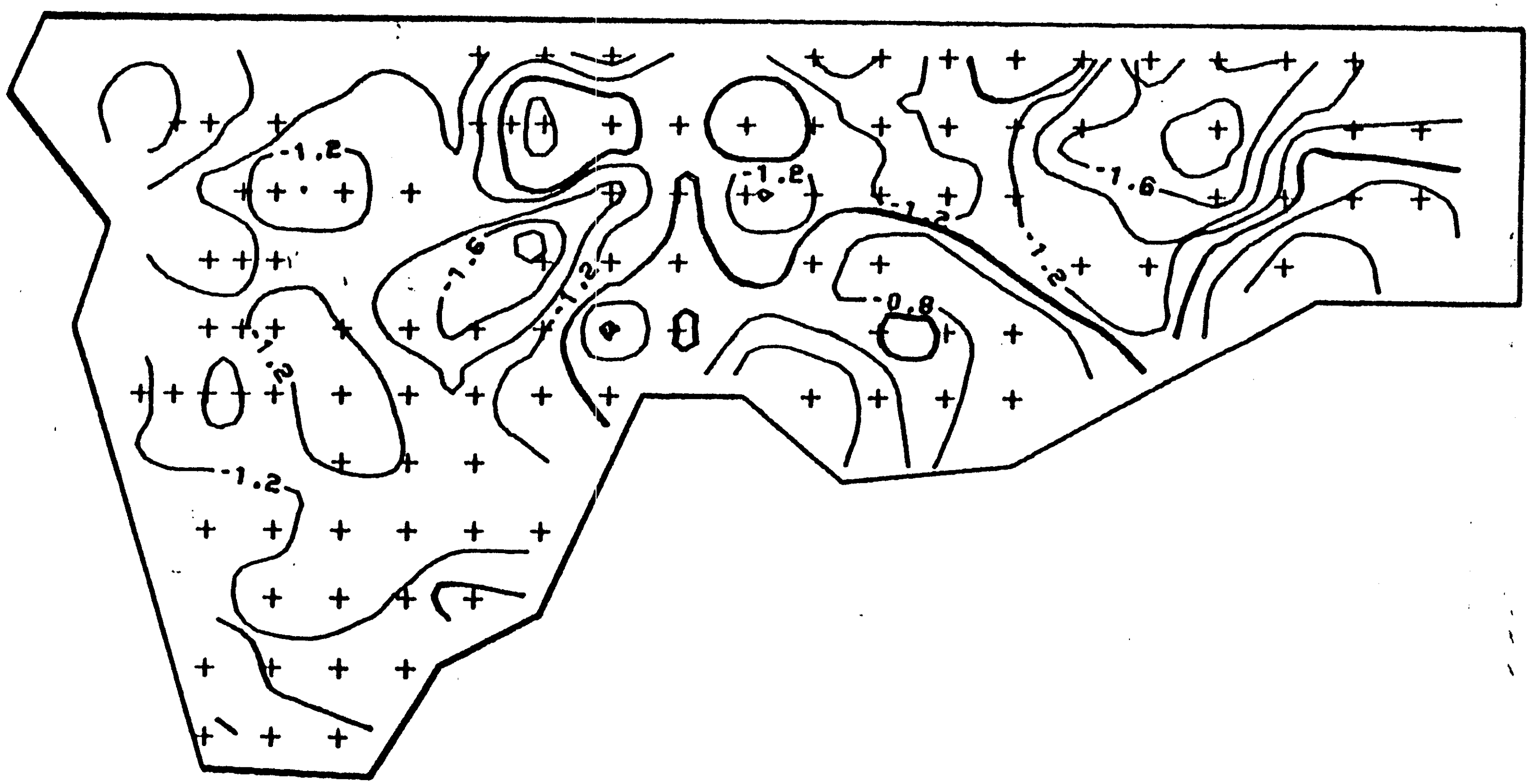


Figure 3.13 - Contour Plot of Log_{10} of Permeability

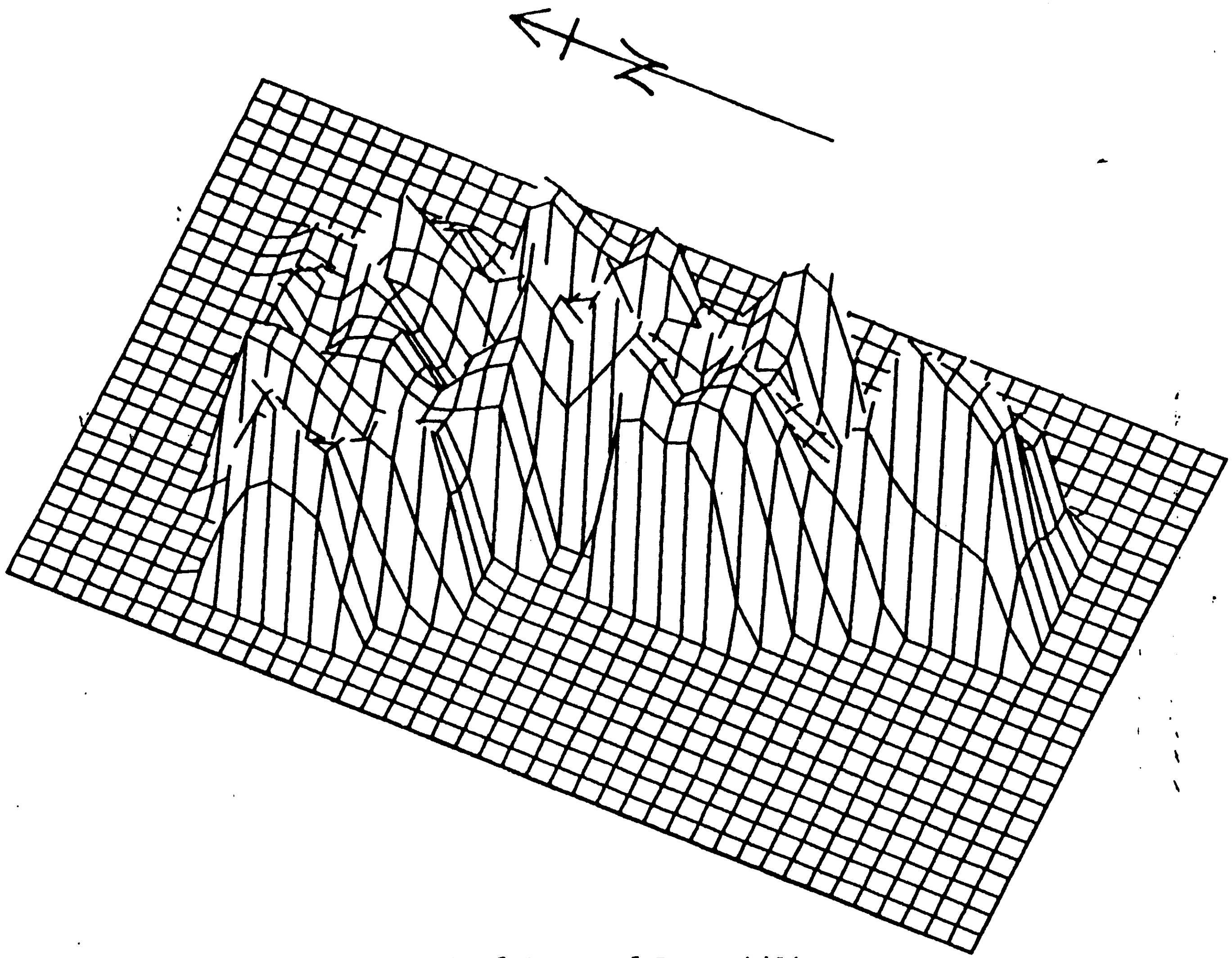


Figure 3.14 - Transact of Log_{10} of Permeability

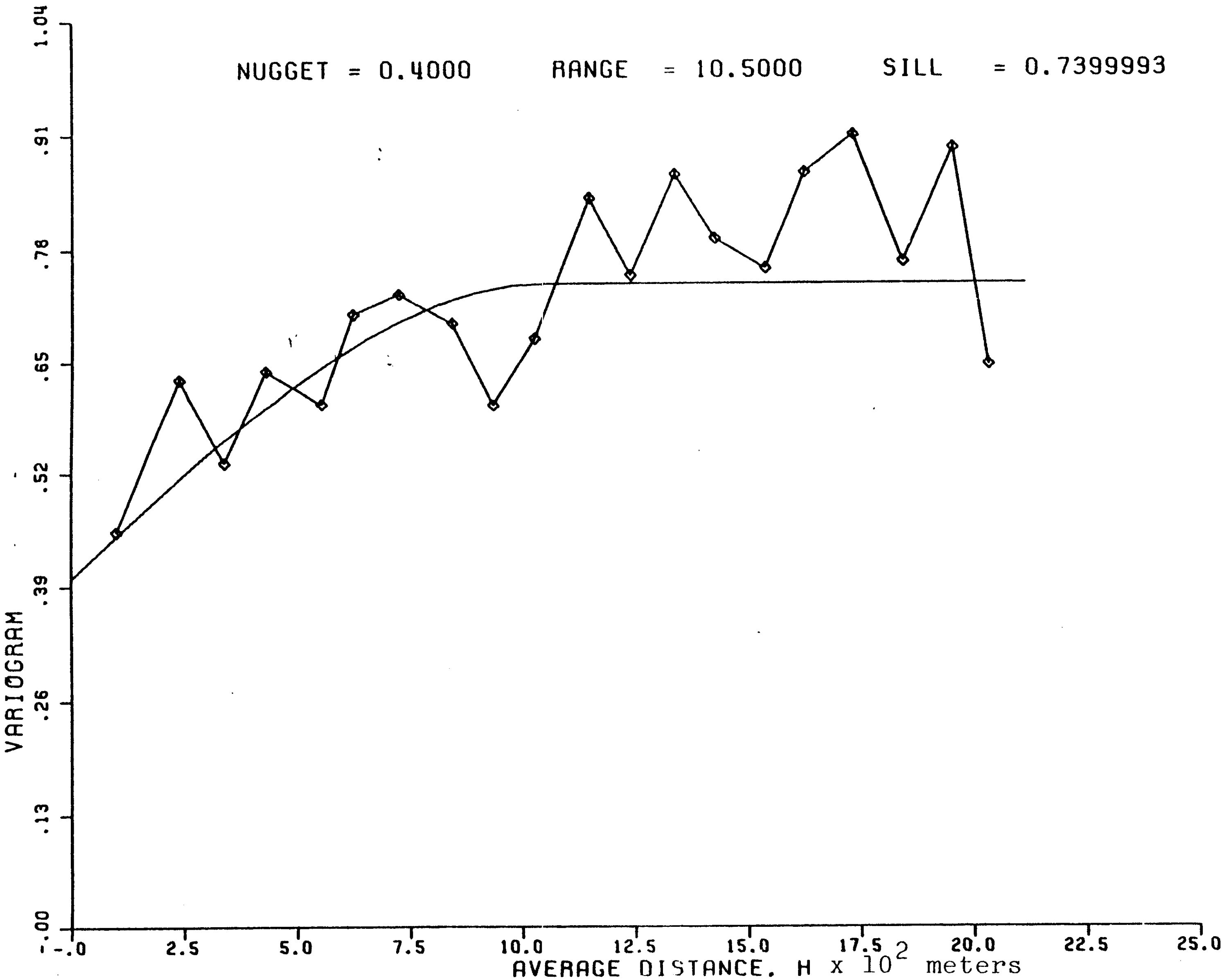


Figure 3.15 - Covariogram of Log_{10} of Permeability of Embabe Data

"nugget effect" which could be reduced with closer and more accurate sampling as well as a measure of natural spatial variability in the form of the covariogram.

3.5 Uncertainty in Prediction of the Dewatering Zone

The dewatering zone DWZ is the distance from the ground surface to the water table. The crop response to the drainage system depends on the size of this zone. A predictive model is used to estimate DWZ based on estimates of recharge rate ,N, and permeablility ,K, for each alternative design drain spacing ,L, and depth ,D. By assuming that N and K are uniform (constant) between the drains, simple analytical expressions can be used to predict DWZ. If the estimates of the uniform N and K are uncertain, then probabilistic models are employed to account for the uncertainty of the DWZ prediction, and therefore the uncertainty of the crop response. If N and K are assumed to be spatially variable, numerical models are required, with appropriate modifications to handle the stochastic nature of the variables.

The evidence collected from the Embabe area indicates that permeability is correlated over large distances, of the order of five hundred meters or so. The evidence is somewhat ambiguous because of the "nugget effect" observed in the covariogram (Figure 3.15), which may indicate sample error or simply reflect the fact that the closet data points are still a full 200 meters apart. If the correlation length of K is truly on the order of five hundred meters, then permeability fluctuations between two drains, spaced only 20 to 40 meters apart, will be relatively small. When this is the case it is possible to assume that K is uniform (constant) between the drains, but uncertain. It is uncertain because the samples contain errors and because there may be no direct measurement of K in that particular location, so that K must be inferred from measurements at nearby stations using, for example, Kriging.

No major spatial structure could be identified for the salt concentration data for the Embabe area. In addition, this is a crude indirect estimate of recharge rate N. Therefore it is assumed the recharge rate N is uniform between the drains, but uncertain.

The first model presented below is based on the assumption of constant, but uncertain N and K between the drains. However, if the correlation length of N and K is somewhat smaller, approaching in magnitude the spacing between the drains, then the spatial variation of these parameters becomes important. The second model examines stochastic spatial variation using a one-dimensional numerical discretization between the drains. A third numerical model has been formulated to examine the more realistic two-dimensional horizontal flow pattern between two drains, from the collector at which they discharge up to the edge of the field. All three models are based on the approximate probabilistic modelling approach called First Order-Second Moment (FOSM) analysis (see Benjamin and Cornell, 1970; Dettinger and Wilson, 1981, 1982, and Wilson and Dettinger, 1982). All three models focus on predicting the water table ,h, and dewatering zone, DWZ, at the midpoint between the drains, because under most conditions the water table will be a maximum at this point and DWZ a minimum. This mid-point is designated by the subscript L/2. The models are written in terms of water table height h. The statistics of the predicted dewatering zone DWZ are related to those of the water table height by the expressions in which the over bar presents the expected value.

$$DWZ = D - h \tag{3.23a}$$
$$\overline{DWZ} = D - \overline{h} \tag{3.23b}$$
$$\sigma^2_{DWZ} = \sigma^2_h \tag{3.23c}$$

3.5.1 Uniform but Uncertain Permeability and Recharge.

A model of this situation is given by the Hooghoudt equation, (Hooghoudt, 1940). From FOSM analysis, (see, for example, Wilson and Dettinger, 1981), the first order expected value of the water table elevation midway between the drains is

$$\overline{h}_{L/2} = f_1(L, d', \overline{N}, \overline{K}) = -d' + \left[d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right]^{1/2} \tag{3.24}$$

The first order estimate is identical to the deterministic estimate, with the parameters evaluated at their expected value. The vertical flow near the drains is accounted for by replacing the true depth by an equivalent depth, d', which depends on the geometry:L,d, and type and size of drain. For tile drains, the equivalent depth has been expressed in closed form (USBR, 1978).

$$d' = f_2(L, d, r)$$

$$d' = \begin{cases} \dfrac{d}{1 + \dfrac{d}{L}\left[2.55\ln(\dfrac{d}{r}) - 3.55 - 1.6(\dfrac{d}{L}) + 2(\dfrac{d}{L})^2\right]} & \text{if } 0.0 < \dfrac{d}{l} \leq 0.31 \\ \dfrac{L}{2.55\left[\ln(\dfrac{L}{r}) - 1.15\right]} & \text{if } 0.31 < \dfrac{d}{L} \end{cases} \tag{3.25}$$

It depends primarily on design parameters, and is not a function of recharge rate N or permeability K. When d'=d, the Hooghoudt model becomes a simple Dupuit model. The variance of water table estimate at the midpoint, calculated by FOSM, is

$$\sigma^2_{h_{L/2}} = f_3(L, d', \overline{N}, \sigma_N, \overline{K}, \sigma_K, \rho_{KN}) = \left[\frac{L^2}{8\overline{K}}\right]^2 \left[d'^2 + \frac{\overline{N}^2}{4\overline{K}}\right]^{-1} \left[\sigma^2_N - 2\frac{\overline{N}}{\overline{K}}\rho_{KN}\sigma_{KN} + (\frac{\overline{N}}{\overline{K}})^2\sigma^2_K\right] \tag{3.26}$$

where σ_N^2 and σ_K^2 are the variances of the estimated values of recharge N and permeability K, and ρ_{KN} is the correlation between N and K. In the Embabe case study, ρ_{KN} is almost zero $\{\rho_{KN} = -0.014\}$. When K is log normally distributed, with Y=logK normally distributed, the ratio $\frac{\sigma_K}{\bar{K}}$ in (3.25) is replaced by σ_Y and the remaining $\bar{k}$ in (3.23 and 3.24) represent geometric (logorithmic) averages of the permeability data. The correlation coefficient becomes ρ_{YN}.

Using the data from the Embabe area, (Table 3.1 with σ_N = 0.0004m/d), the predicted (3.23) water table elevation above the drains, and an estimate of its reliability (3.25) are given in Table 3.2. The drain spacing in this example is L=40m, and the depth to the impervious bottom is d=3m=d' (neglecting vertical flow head losses). The first order expected value of the water table height at the midpoint is 0.299 meters, assuming K is normally distributed. The standard deviation of this estimate is 0.396 m, neglecting the slight negative correlation between N and K, and 0.395 m accounting for it. In this example, the correlation is unimportant and is ignored below. If only the permeability is uncertain, then the estimated standard deviation drops to an almost identical value, 0.285 m. Recognizing that K is log-normally distributed hardly disturbs the first order estimate of the water table height, but it does decrease the estimated standard deviation by 6%. Because in this example the coefficients of variation of K and N are on the order of one, FOSM may be only approximate, having neglected higher order terms in the relationship between the estimate for h and the moments of K and N.

A second order estimate of the water table height can be found that depends only on the first two moments of K and N. Following the procedure in Benjamin and Cornell (1970), and Wilson and Dettinger (1981), this estimate is

Table 3.1 Field Data for the Embabe Case Study

Properties for	Mean	Standard Deviation
Sample K	0.085m/day	0.082m/day
Sample $Y=lnK$	-2.830	0.863
K calculated from Sampled Y	0.086m/day (geometric mean)	0.090m/day
Sample $W=lnS^1$	3.75	0.815
N calculated from Sampled W	0.0004m/day	0.0006m/day
Subjective Estimates for N	0.0004m/day	0.0004m/day

Sample correlation of N and K : ρ_{KN} = −0.014

[1] S = samples of salt concentration

Table 3.2 Statistics of the Water Table Elevation for Uniform, but Uncertain Parameters

Uncertain Parameters	Correlation ρ_{KN} or ρ_{YN}	$\bar{h}_{L/2}$ (m)	$\sigma_{h_{L/2}}$ (m)
K,N	0	0.299	0.396
K,N	-0.014	0.299	0.399
K	-	0.299	0.275
N	-	0.299	0.285
Y,N	0	0.299	0.374
Y,N	0	0.396*	0.374

* Second Order Estimate of Expected Value

$$\bar{h}_{L/2}\Big|_{2^{nd}order} = f_4(L, d', \bar{N}, \bar{K}, \sigma_Y)$$

$$= \bar{h}_{L/2}\Big|_{1^{st}order} + \bar{N}\frac{L^2}{8\bar{K}}\left\{d'^2 + \frac{\bar{N}^2}{4\bar{K}}\right\}^{-1/2} \times \tag{3.27}$$

$$\left[\frac{1}{2} - \bar{N}\frac{L^2}{16\bar{K}}\left\{d'^2 + \frac{\bar{N}^2}{4\bar{K}}\right\}^{-1}\right]\sigma_y^2$$

where $\bar{K}$ is log-normally distributed. The importance of this additional term for the example is shown at the bottom of Table 3.2 where it adds almost a tenth of a meter to the expected height of the water table. The log-normality of the permeability data does not change the reliability of the prediction signigicantly, but the large coefficients of variation for N and K imply that first order estimates may be non-conservative, as illustrated in this example. In the remaining analyses and designs described in this paper, K will be taken as normal, and only first order estimates of expected water tables height will be made. In practice, log-normality and second order estimates would be the rule.

3.5.2 Spatial Variation in 1-D Between the Drains.

Permeability and recharge may vary between the drains. Assume that the statistics of this stochastic spatial variation are known a priori, and are represented in terms of expected values and a covariogram or variance-covariance. If the spatial scale of the fluctuations are large compared to the distance between the drains, then the analytical Hooghoudt model based on uniform but uncertain parameters should accurately represent the uncertain physical system. If, on the other hand, the scale of fluctuation is small compared to the distance between drains, then spatial variability between the drains becomes important and a stochastic distributed parameter model for the physical response must be used. In most cases, this model will be solved numerically using Monte Carlo Simulation (see, for example, Freeze, 1975, or Smith and Freeze, 1979), or FOSM (see Dettinger and Wilson, 1981, 1982). Consider the drain design explained above with the Hooghoudt model, in which L=40m, and d=d'=3m. For spatially varying K and N, the groundwater response to this design is described by the Dupuit model

$$\frac{d}{dx}\left[K(h+d)\frac{dh}{dx}\right] = -N \qquad 0\le x \le L \tag{3.28}$$

with boundary conditions (neglecting the vertical flow under the drains, i.e., d=d'). This model can be transformed to

$$\frac{d}{dx}\left\{Kd\frac{\Phi}{dx}\right\} = -N \qquad 0\le x \le L \tag{3.29}$$

where $\Phi=\left[\frac{(h+d)^2}{2}\right]$, which has boundary conditions $\Phi=\frac{d^2}{2}$ at $x=0,L$. Solved on Dettinger and Wilson's (1981) FOSM stochastic numerical model of groundwater flow, the results, in terms of mean and standard deviation of $\Phi_{L/2}$ at the midpoint between the drains are converted to the statistics for $h_{L/2}$ via

$$\bar{h}_{L/2} = (2\bar{\Phi})^{1/2} \tag{3.30a}$$

$$\sigma_{h_{L/2}} = \frac{\sigma_{\Phi_{L/2}}}{2\bar{\Phi}_{L/2}} \tag{3.30b}$$

Spatial variation of N and K is somewhat arbitrarily represented by an exponential variogram/varianc-covariance. For example, the spatial structure of logK is described by

$$Cov_{\ln K}(U) = Var_{\ln K}(0)e^{-u/l} = \sigma^2_{\ln K}e^{-u/l} \tag{3.31a}$$

or

$$\gamma(u) = \sigma^2_{\ln K}(1 - e^{-u/l}) \tag{3.31b}$$

where l is sometimes referred to a the "correlation length".

Figures 3.16 plots dimensionless correlation length, 1/L, versus using the data of Table 3.1 (with σ_N = 0.0004 m/day), for uncertainty in K and N. In both cases, the uncertainty of the water table elevation prediction converges to the value predicted by the uniform parameter model. For $l/L \geq 1$, there is essentially no difference. The first order predicted mean is constant for all l. Thus, the uniform but uncertain model provides an accurate indication of prediction uncertainty, for spatial variation scales on the order or larger than the spacing of the drains.

3.5.3 Spatial Variation in 2-D Between the Drains and Collector.

Figure 3.17 is a plan of a section of a drainage project, bounded by drains to the left and right, by a collector at the top and the edge of the drained field below. Although it is not strictly correct for spatial stochastic systems, assume that the top and bottom boundariew are exact "no flow" boundaries of symmetry. Following the assumptions of the previous case, the groundwater flow in the field, for spatially variable K and N is described by

$$\frac{d}{dx}\left[K(h+d)\frac{dh}{dx}\right] + \frac{d}{dy}\left[K(h+d)\frac{dh}{dy}\right] = -N \qquad \begin{array}{l} 0 \leq x \leq L \\ 0 \leq y \leq B \end{array} \tag{3.32}$$

equation with boundary conditions

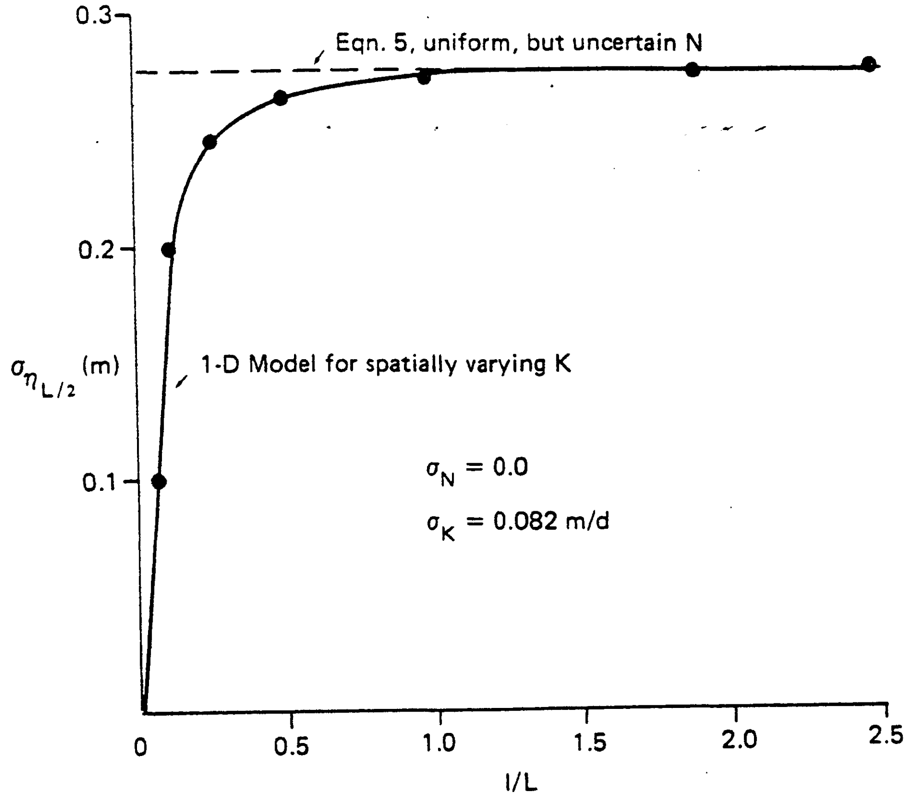
$$(h-d)\frac{dh}{dx} = 0 \qquad 0 \leq x \leq L, y = 0, B \tag{3.33a}$$

$$h = 0 \qquad 0 \leq y \leq B \;, x = 0, L \tag{3.33b}$$

In the transformed state with variable , this becomes

$$K\frac{d^2\Phi^2}{dx^2} + K\frac{d^2\Phi^2}{dy^2} = -N \tag{3.34}$$

with boundaries $\Phi = \frac{d^2}{2}$ on the drains and $\frac{d\Phi}{dy} = 0$ at the collector and at



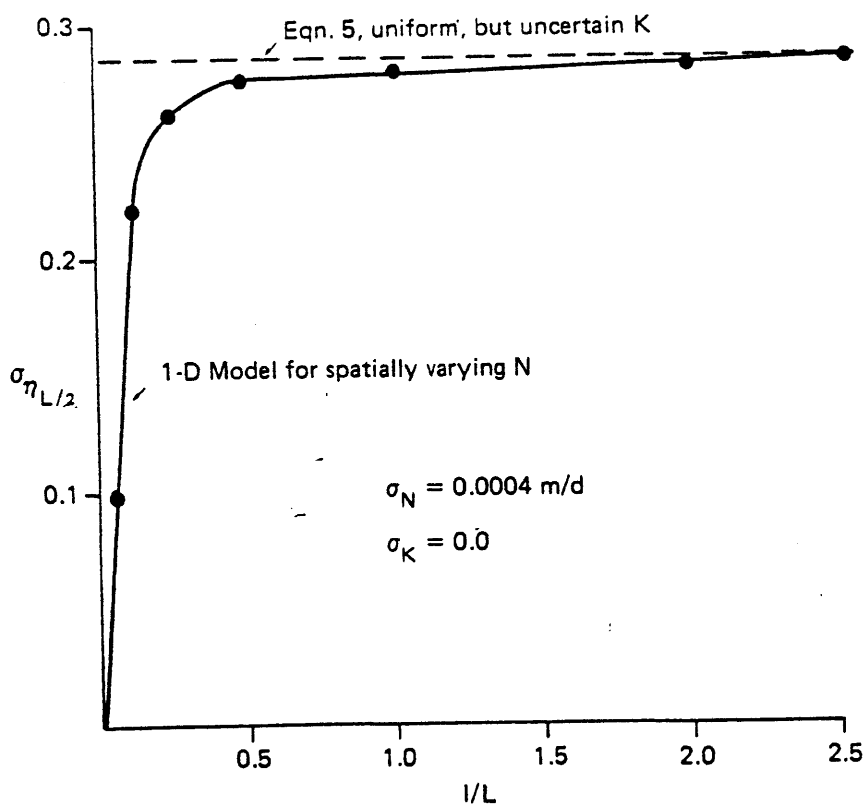


Figure 3.16 - $\sigma_{h_{L/2}}$ Versus Correlation Length of K and N

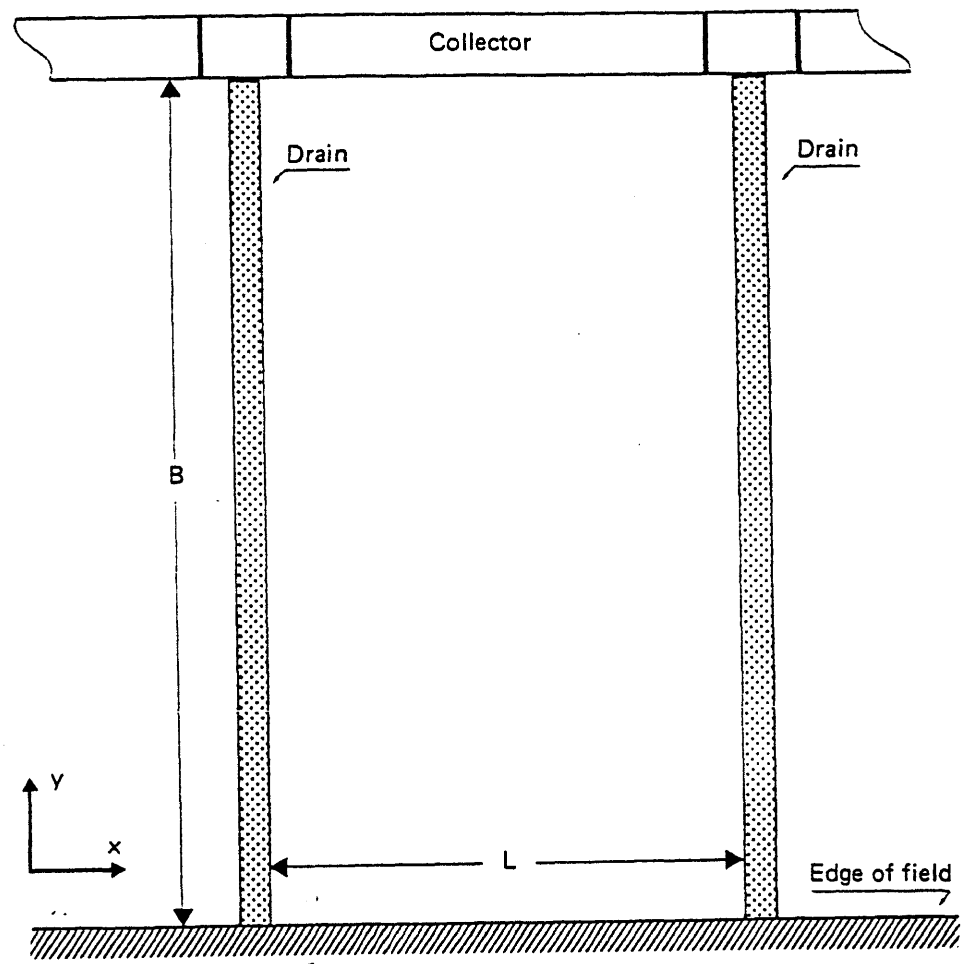


Figure 3.17 - Plan View of Drain Field

the lower edge of the drained field. Modeling this situation using Dettinger and Wilson's (1981) FOSM stochastic numerical model yields identical results to the precious models for the first order expected value of the water table. The sensitivity of the water table uncertainty in the middle of the field $[x=L/2, y=B/2]$ to permeability correlation is shown in Figure 3.18. In this multi-dimensional case, permeability variation results in a reduction of the water table uncertainty because water is now able to flow around areas of low permeability. Nevertheless, the predicted uncertainty converges to the value found for uniform by uncertain parameters for $l/L>5$, once again demonstrating that from sufficiently large correlation length the simple uniform model can be reasonably employed.

3.5.4 The PDF of h and DWZ

The FOSM models used above to examing the uncertainty of water table predictions are, by definition, second moment models. They provide estimates of the first two moments of the probability density function (PDF) of h and DWZ. However, the drainage design depends on the full PDF, not solely on its moments, when the decision is based on reliability, or expected loss. For small water table standard deviation relative to the water table height above the drains, the PDF of h or DWZ is normal. This has been demonstrated by full distributional Monte Carlo simulationw for similar problems (see, for example, Freeze, 1975; Smith and Freeze, 1979), which show that the farther from the boundaries (drains) one gets, the more normal the distribution. For larger relative variance of the water table prediction, due to increasing variance of K or N, the distribution on h or DWZ becomes skewed. Since the water table cannot rise above the groundsurface, and if we presume it will not fall below the drains (steady-state), then it is clear that the true distri-

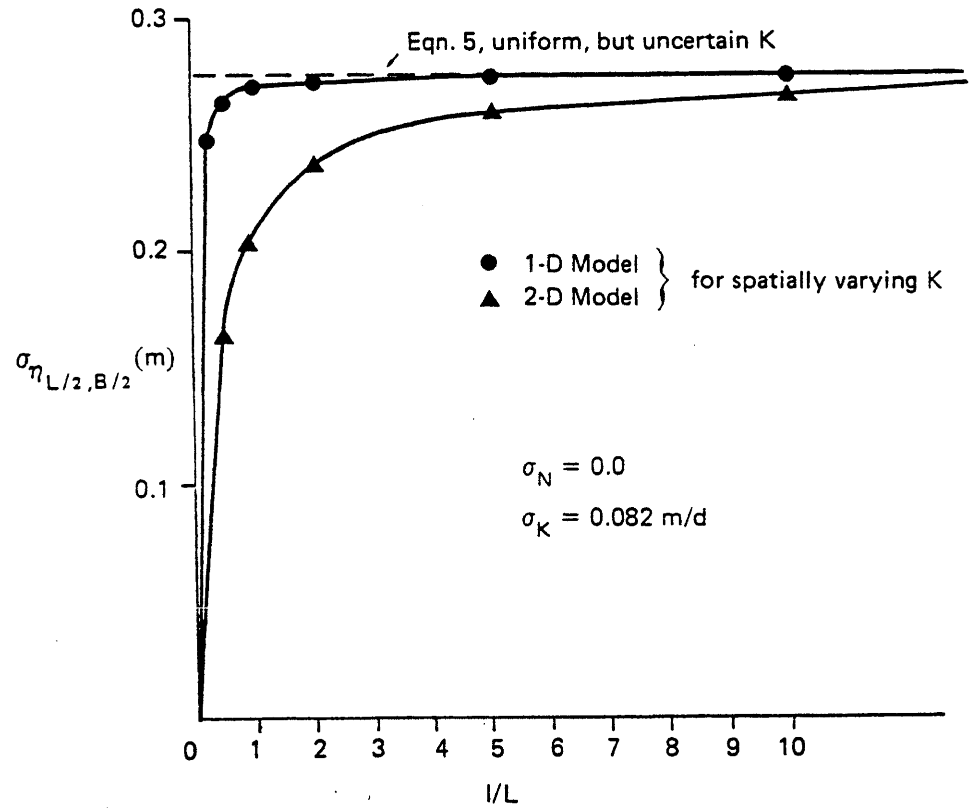


Figure 3.18 - $\sigma_{h_{L/2}}$ Versus Correlation Length K in 2-D

bution on h or DWZ if finite, $0 \leq h \leq D$,and $0 \leq DWZ \leq D$, but with various shapes depending on the position between the drains and the expected height and variance of the water table elevation.

A finite distribution that would allow for varying shapes of h would be the β distribution. Further experiments need to be performed to confirm the validity of the β distribution for the PDF of h. The results of the FOSM analysis provide $\bar{h}$ and σ_h which can be directly used to estimate the β distribution. However, in this series of papers to demonstrate the procedures, h will be assumed to be normally distributed which is true for small values of $\frac{\sigma_N}{\bar{N}}$ and $\frac{\sigma_K}{\bar{K}}$.

3.6 Optimal Interpretation of Spatial Data

Presently in Egypt the sampling interval for soil permeability is approximately 500 m. To perform a proper design of field level drains, to be discussed in the next chapter, a more dense grid of permeability values is needed. The present procedure is to perform interpolation by engineering judgment or linear (hand) interpolation. This method can be acceptable but requires a good engineer with much experience. A statistical method is trend surface analysis (i.e., least-square fitting). This method is automatic and the experience of the engineer does not matter. However, both methods do not, 1) consider the structure of the physical process or, 2) give a variance which characterizes the uncertainty about the interpolate value at each location.

As part of the theory of regionalized variables presented above, an interpolating process called kriging has been developed. Kriging is an optimal interpolation precedure, that accounts for the spatial structure of the phenomenon and provides an estimation variance at each point generated. The spatial structure is defined by the drift and the covariogram described above. The information about the spatial structure is input to an algorithm which optimally weights the contributions of the sample points at each grid point value so as to minimize the variance over the region that is kriged. There are a number of variations and extensions of the kriging process and the reader is referred to David (1978), Davis (1973), Chua (1980), and Delhomme (1979).

Kriging is a process that provides the "best linear unbiased estimator" (or B.L.U.E.) at an input. Delfiner (1976) the Centre de Morphologie de Paris, Ecole Nationale Superieure des Mines de Paria, has developed a computer code for kriging called BLUE PACK. This code will provide for automatic recognition of the structure of the phenomenon or all the information of the structure can be input.

To provide the interpolated values needed for the drainage design process the Embabe case study field, a kriged-map of the region was generated. The structure of the permeability field was identified above. Using the covariogram, (Figure 3.15), the sample data and the assumtion of no drift, the kriged realization of the log of permeability was generated on a loo meter grid using BLUE PACK. Figure 3.19 is a map of the region kriged with the sample points shown. Figure 3.20 is a map of the kriged realization and Figure 3.21 is a map of the standard deviation of the estimation error of the kriged values. Notice that the estimation error is smaller at points closest to the sample point as expected from the covariogram. The kriged values and the estimation error was generated for the log of permeability. These values of the moments of the log of permeability can be transformed back to permeability values by using the transform derived from Benjamin and Cornell (1970):

$$M_x = 10^{\left(M_{\text{Log } x} + \frac{(2.3)^2}{2} \sigma^2_{\text{Log } x}\right)} \tag{3.35}$$

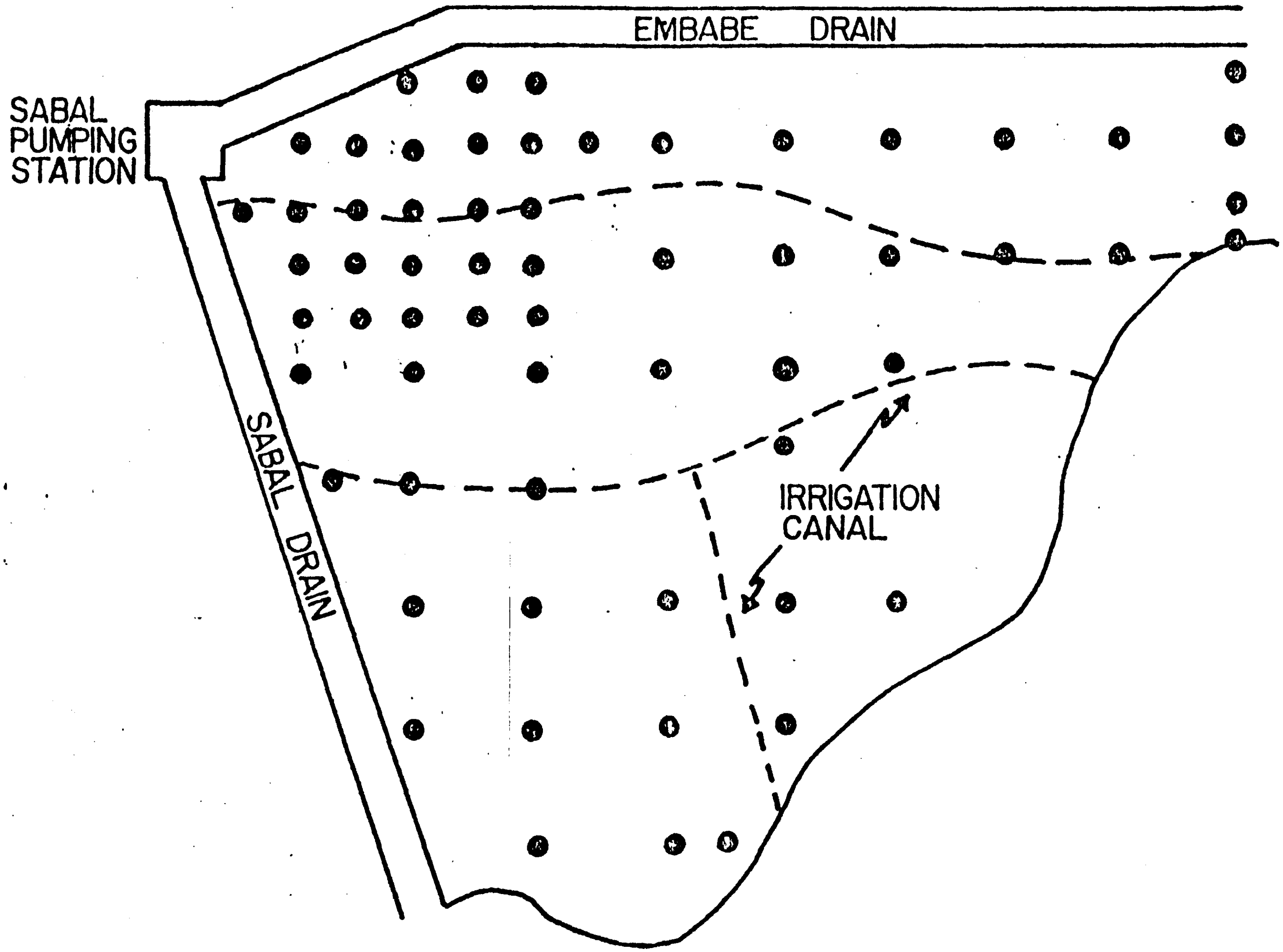


Figure 3.19 - Embabe Case Study

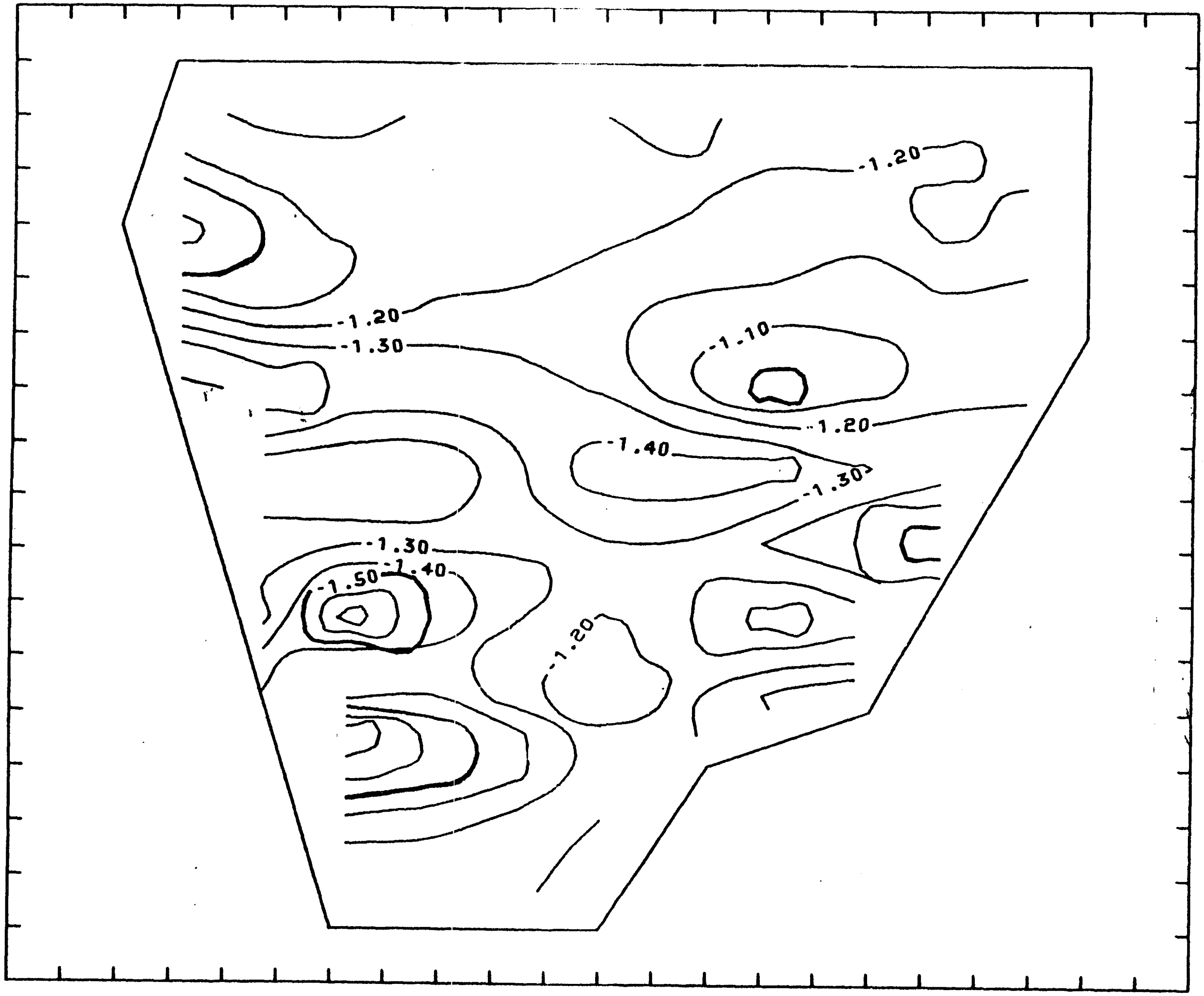


Figure 3.20 - Kriged Field of Log_{10} Permeability

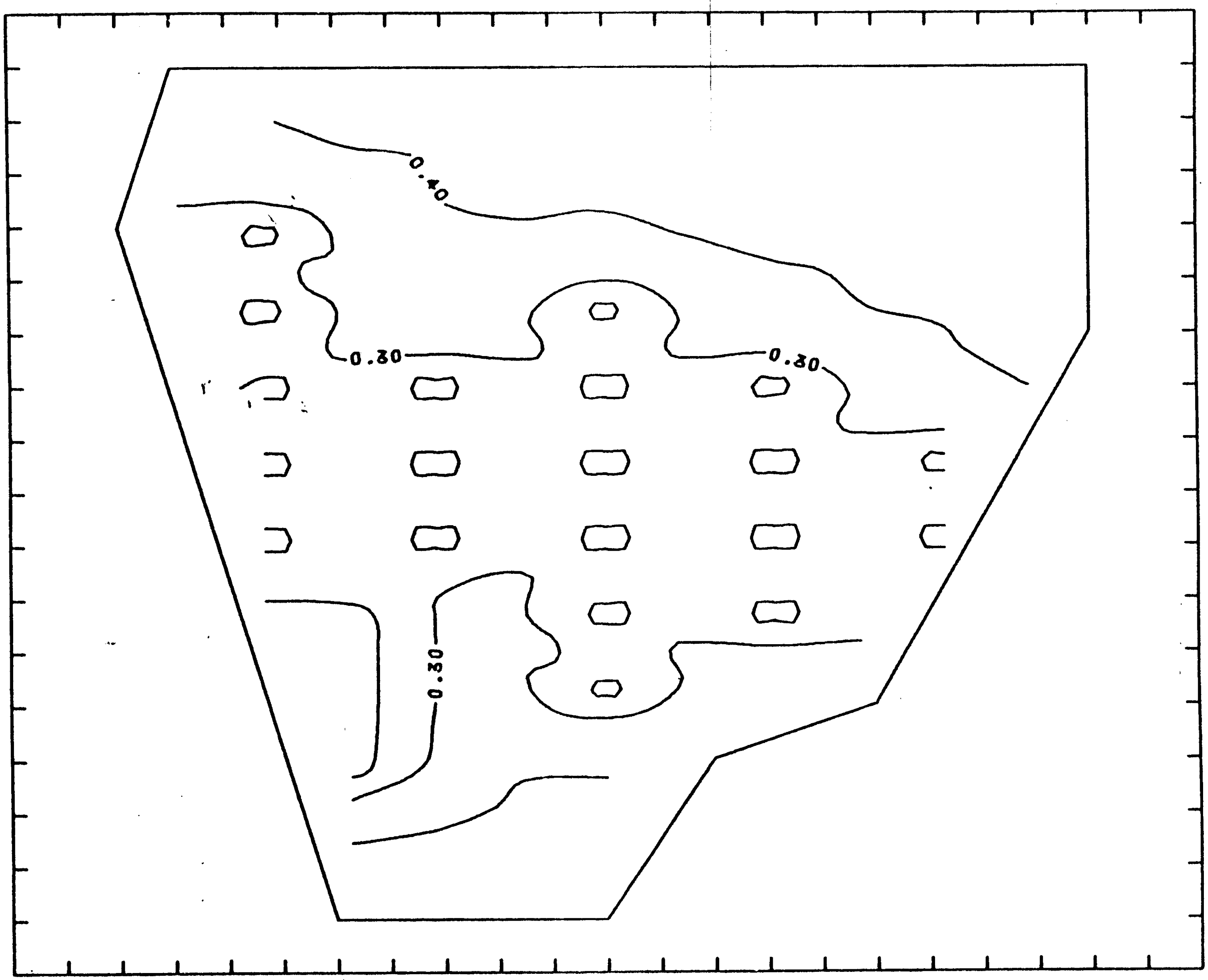


Figure 3.21 - Field of Standard Deviation of Error of Kriged Values

$$\sigma_x = M_x x \quad (10^{\sigma^2_{\mathrm{Log}\ K}} + 1)^{1/2} \qquad (3.36)$$

The transformed values can then be used in the design of agricultural drains.

CHAPTER 4

ECONOMIC DESIGN OF FIELD LEVEL LATERAL DRAINS

The goal of every design engineer is to design a system that meets the design specifications at the least cost. The same is true for drainage design. This chapter presents two mathematical programming approaches for optimal design of field level lateral drains given uncertainty in the design model. The two approaches are: 1) chance-constraint programming and 2) stochastic programming. The chapter develops both techniques for lateral drain design and analysis of the sensitivity of model parameters for each approach using the Embabe case study data. The two approaches are compared and the chance constraint approach is shown to be weaker for certain forms of the crop yield function.

4.1 Optimization Model for Drain Design

A model for steady state drainage design, the Hooghoudt equation (Eq. 3.10), has been presented in the previous chapter. The model provides a relationship for the estimation of the groundwater level mid-way between two drains as a function of depth and spacing of drains and physical parameters. Presently, it is assumed that the physical parameters are known with certainty and can be represented by their mean values. In this case, one common design specification is to provide a dewatering zone midway between the drains of a specified value. The dewatering zone is the unsaturated soil zone between the soil surface and the groundwater table. The size of the dewatering zone that is chosen is obtained from empirical data relating the dewatering zone to crop yield. The design criteria can be met by various combinations of drain depth, D, and drain spacing L, given the physical parameters. The problem then has two decision variables drain depth D and spacing L. The usual procedure is to choose a drain depth based upon some mechanical or institutional constraint, then determine the spacing that achieves the desired dewatering zone according to the Hooghoudt equation. Following this approach, there is no explicit consideration given to the economics of drain design. If there exists information relating costs to depth and spacing of drains then the design problem can be cast into a mathematical programming problem (MPP). The MPP is a procedure that will determine the maximum or minimum of an algebraic function of one or more variables restricted by algebraic equations and/or inequalities called constraints (Simmons, 1975). A MPP for drainage design would be to minimize the cost of drains subject to physical constraints governing the problem. The physical constrinats are those equations which provide for an estimated value of the dewatering zone as a function of depth and spacing of drains and physical parameters.

The cost function for drainage design does not have an absolute form. The function may vary from country to country or region to region due to the fact that labor costs, installation rates and machine types may vary, changing costs. Christopher and Winger (1975) have developed generalized cost functions for three different drain laying machines.

These functions were based upon data from the U.S. Bureau of Reclamation drainage projects. El Berry (1979) has developed very detailed cost functions for lateral drain design in the Nile Delta in Egypt. The general form of the El Berry function is

$$\text{Cost } (D,L) = \frac{c_1}{L} (c_2 \times D^{c_3} + c_4) \tag{4.1}$$

where c_1, c_2, c_3 and c_4 are coefficient specific to regional factors.

With a cost function defined, a constraint set must be defined to complete the MPP. The first constraint is to require that the dewatering zone be at least if not greater than some value DWZ*. The reader is referred to Figures 3.1 and 3.2 to help visualize the following discussion.

The dewatering zone is defined as

$$Z - (h+d) = DWZ \tag{4.2}$$

The constraint can be described as

$$Z-d-h \geq DWZ^*$$

$$D-h \geq DWZ^* \tag{4.3}$$

where Z-d is equal to the drain depth D. The Hooghoudt equation provides a functional relationship between D, L, and h at the groundwater level above the drains (Eq. 3.10). Replacing h in constraint 4.3 by Eq. 3.10 yields

$$D-(-d' + ((d')^2 + \frac{L^2\bar{N}}{4K})^{1/2}) \geq DWZ^*$$

$$D + d' -((d')^2 + \frac{L^2\bar{N}}{4K})^{1/2} \geq DWZ^* \tag{4.4}$$

where d' is the equivalent height, $\bar{N}$ is the mean drainage rate and $\bar{K}$ is the mean permeability. The equivalent height d', is a function of D and L, the decision variables and must be defined in the constraint set. The equivalent depth has a different definition over different ranges of the ratio of drain height above the impervious layer to drain spacing. These definitions comprise the following two constraints:

$$d' = \frac{d}{1 + d/L(2.55 \ln d/r - 3.55 + 1.6\, d/L - 2\, (\frac{d}{L})^2)}$$

for $d/L \leq .31$ (4.5)

and

$$d' = \frac{L}{2.55(\ln \frac{L}{r} - 1.15)} \quad \text{for } d/L > .31 \qquad (4.6)$$

where r = the effective radius of lateral pipe.

In lateral drain design the depth of the drain D may be constrainted by the maximum digging depth of drain installation equipment or because of the need to maintain gravity flow to the main drains. This condition gives rise to the following constraint

$D \leq D^*$ (4.7)

where D* is the maximum possible drain depth. And finally the depth and spacing must be no less than zero:

$L, D \geq 0$ (4.8)

Compiling the cost function and the constraint set together produces the following MPP for lateral drain design.

$$\text{Min } \frac{c_1}{L} (c_2 D^{c_3} + c_4)$$

$$\text{S.T.} \quad D + d' - ((d')^2 + \frac{L^2 \bar{N}}{4K})^{1/2} \geq DWZ^*$$

$$d' = \frac{d}{1 + d/L(2.55)\ln d/r - 3.55 + 1.6\, d/L - 2(d/L)^2)} \; ;$$

$$; \quad d/L \leq .31$$

$$d' = \frac{L}{2.55(\ln \frac{L}{r} - 1.15)} \; ; \; d/L > .31$$

$$D \leq D^*$$

$$L, D \geq 0$$

This is a mathematical programming problem with a nonlinear objective function and nonlinear constraints. It may be solved for a global optimum solution if the objective function is a quasi-convex function and the constraint set is a convex set.

Using the data from the Embabe case study area in the Nile Delta, assuming a design crop of Egyptian clover, the model was solved for the optimal drain design. The model parameters for this solution are presented in Table 4.1. Analyzing the objective function shows that it is convex and the constraint set is a convex region as well. So a global optimum can be found. Using an algorithm with enumeration over D and Newton's method to define the boundary of the constraint set, a solution was found that gives a drain depth of 2.0

Table 4.1

Parameters for Drain Design MPP

I. Physical Parameters

$\bar{N}$ = .0004 m/day

$\bar{K}$ = .085 m/day

DWZ* = 1.0 m

D* = 2.0 m

II. Objective Function Parameters

c_1 = 52.2

c_2 = 1.646

c_3 = .365

c_4 = 55.892

meters and a drain spacing of 83.45 meters. Figure 4.1 is a graphic representation of this MPP and its optimal solution of 35.16 LE per feddan.

This development assumed no uncertainty in drainage rate or permeability. The next section will address the question of optimal design given parameter uncertainty.

4.2 Chance - Constraint Programming

Uncertainty in mathematical programming can be addressed in two ways: stochastic programming and chance-constraint programming. The stochastic programming approach combines the system response function with the system output. With this approach the expected value of a system output can be obtained by the integration of the product of these two functions. This approach will not be discussed in this section but will be developed further on in this work. Chance-constraint programming, which is the technique that this section will develop, was introduced by Charnes and Cooper (1959). Chance-constraint programming is based upon the concept of system reliability where reliability is defined in terms of probabilities. The concept of reliability is introduced to the MPP by requiring that a constraint with uncertainty to be met with a certain probability

$$P_r(ax \leq b) \geq \alpha \qquad (4.9)$$

where α is the desired probability. The "chance-constraint", Equation 4.9, can be transformed into a deterministic equivalent constraint where b is a random variable. If the probability

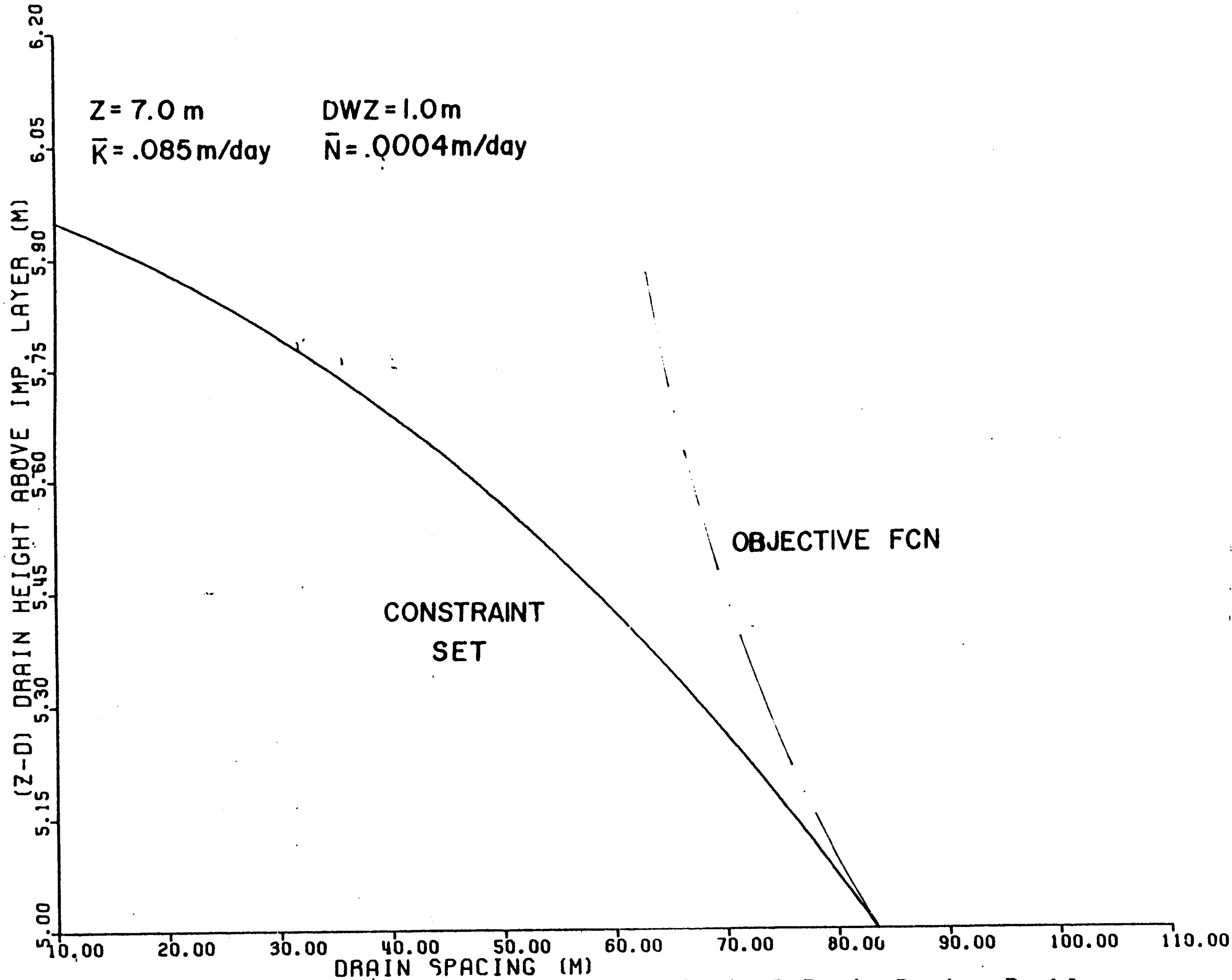


Figure 4.1 - Deterministic Optimal Drain Design Problem

distribution of b is known then the value of b that satisfies the α reliability can be found, b^{α}. Using b^{α}, the deterministic equivalent of Equation 4.9 becomes

$$ax \leq b^{\alpha} \tag{4.10}$$

The transformation of the chance-constraint into a deterministic equivalent constraint, in some cases, may be intractable. With uncertainty expressed by the FOSM analysis it is straight-forward.

4.2.1 Chance-Constraint Development

Freeze (1975) showed that the piezometric head mid-way between two boundaries in a one-dimensional groundwater model can be assumed normally distributed when the permeability is a log-normally distributed variable with a coefficient of variation of up to 0.5. The Hooghoudt equation is a one-dimensional groundwater model with fixed head boundaries. The Embabe data presented in Chapter 3 showed soil permeability to be log normally distributed with a coefficient of variation of 0.3. With these data it is possible to assume that the groundwater level mid-way between two drains is normally distributed thus DWZ is normally distributed. (See section 3.5.4.) It is now possible to incorporate uncertainty into the drain design problem.

4.2.1.1 Chance-Constraint Formulation

In the previous chapter it was demonstrated that drainage rate and permeability are not known perfectly but rather are uncertain. The deterministic approach does not account for uncertainty and must be amended. Since the depth of dewatering zone is uncertain due to input uncertainty Equation 4.4 can be converted to a "chance constraint". The "chance constraint" will require that the uncertain dewatering zone at the mid-point between drains be met with a given reliability:

$$Pr\{D + d' - ((d')^2 + \frac{\bar{N}L^2}{4K})^{1/2} \geq DWZ^*\} \geq \alpha \qquad (4.11)$$

The "chance constraint" Equation 4.11 must be converted to a deterministic equivalent constraint.

From the properties of the normal distribution, it is known that the probability that a value is exceeded can be expressed as a function of the mean and standard deviation of the distribution of the random variable.

$$P_r\{X \geq Z\} \geq \alpha \qquad (4.12)$$

is equal to

$$M_x + A\sigma_x \geq Z \qquad (4.13)$$

where A is a function of the reliability α.

For the deterministic equivalent of the chance constraint (Equation 4.11) the mean dewatering zone plus A times the standard deviation must be greater than or equal to DWZ*. From FOSM analysis of the Hooghoudt equation the mean and standard deviation of the dewatering zone can be defined. Using the concept of Equation 4.13 a deterministic equivalent for Equation 4.11 can be defined as follows:

$$D + d' - ((d')^2 + \frac{\bar{N}L^2}{4\bar{K}})^{1/2} + A \cdot \frac{L^2}{8\bar{K}} ((d')^2 + \frac{\bar{N}L^2}{4\bar{K}})^{1/2} \cdot$$

$$(\sigma_N^2 - 2 \frac{\bar{N}}{\bar{K}} \rho_{KN} \sigma_K \sigma_N - (\frac{\bar{N}}{\bar{K}})^2 \sigma_K^2)^{1/2} \geq DWZ^* \qquad (4.14)$$

By setting the reliability, α, a value for A can be found from standard normal tables. Replacing Equation 4.4 in the deterministic MPP by Equation 4.14 produces a chance constraint program for lateral drain design. The new constraint set still defines a convex region and thus can be solved for a global optimum.

4.2.1.2 Case Study Results

Each solution of the model provides the optimal depth and spacing of the drains plus the minimum cost for a desired reliability of dewatering zone. The model is then solved a number of times to develop curves to analyze the economic performance of the drainage system. For the Embabe case study the first analysis that was performed was to study the optimal design of the drainage system as a function of reliability. To achieve this goal the model was resolved with a new equivalent deterministic constraint for each level of reliability.

Figure 4.2 is a plot of cost versus reliability for variable depth and spacing. The economic interpretation of curve 1 in Figure 4.2 is that it is the variable cost function of reliability for this drainage system. If the fixed cost for the drainage system is added, the supply curve for drainage reliability is obtained.

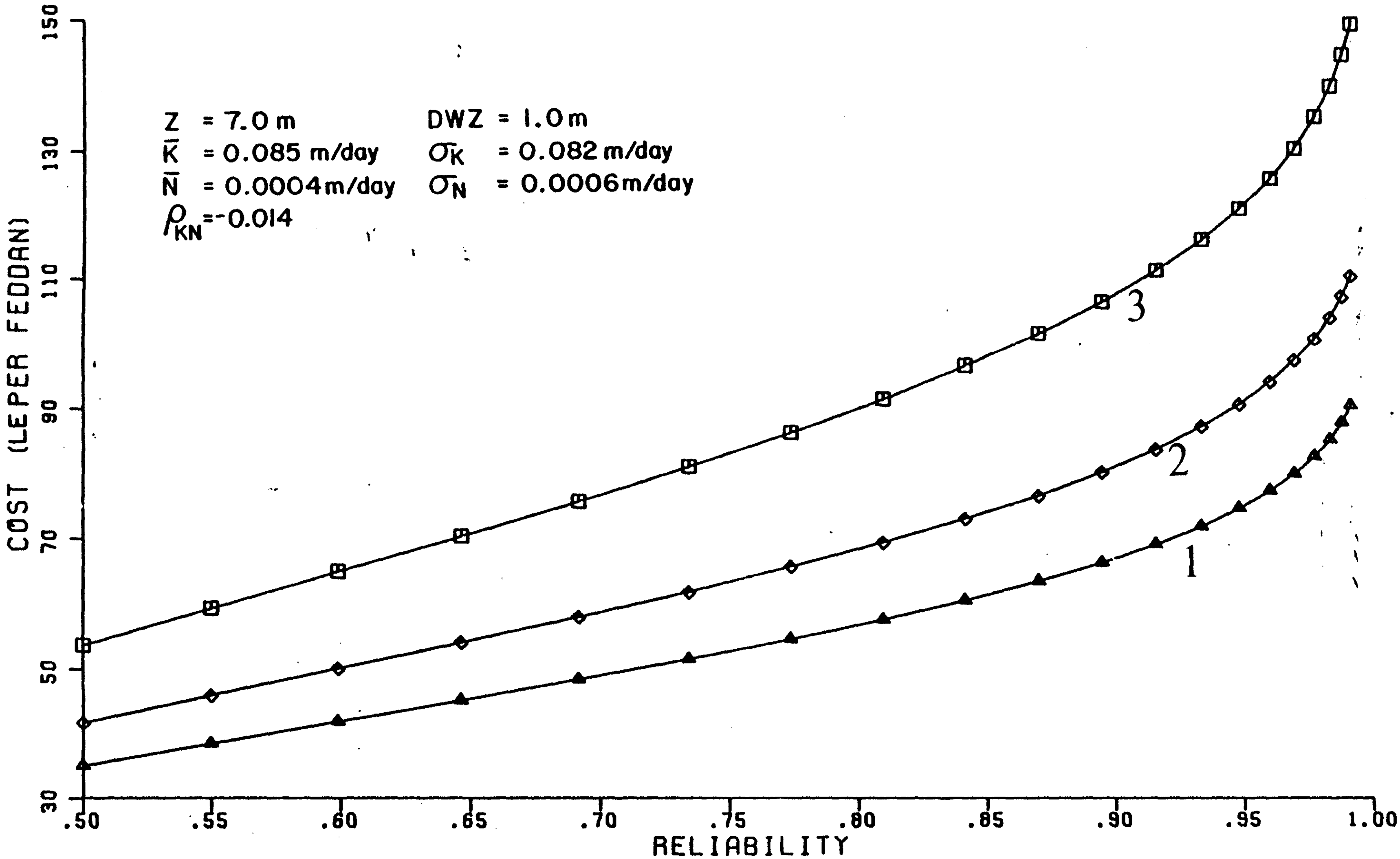


Figure 4.2 - Chance Constraint Results

The above development allowed both the depth and spacing of drains to be a design variable. However, in many situations the drainage depth is fixed due to consideration of gravity flow to the existing collector system. Curves 2 and 3 of Figure 4.2 represent solutions for fixed depths of 1.7 and 1.5 meters respectively. It can be seen that for a fixed depth the costs are higher. This is consistent with optimization theory, that as a minimization is more constrained the optimal cost will increase. The curves show that as depth decreases costs increase.

The model can also be used to aid planners in choosing the optimal depth if it must be fixed over the field. Figure 4.3 is a plot of the optimal cost per acre versus height of the drain for a reliability of 93%. The model found the optimal L to minimize costs for each depth and reliability of 93% given the input parameters and specific dewatering zone. It shows that the costs are relatively insensitive to a drainage height between 2.0 and 1.8 meters for 93% reliability.

Figures 4.4 and 4.5 are graphs of the sensitivity of the model solution to changes in the input uncertainty for permeability and drainage rate, respectively. The model was solved for a constant reliability and all other parameters held constant except the input parameters being analyzed. In both cases the figures show that for coefficients of variation less than 1.5 that the relative difference in costs are very small for most reliabilities. These results also show that the optimal solution is not very sensitive to the uncertainty

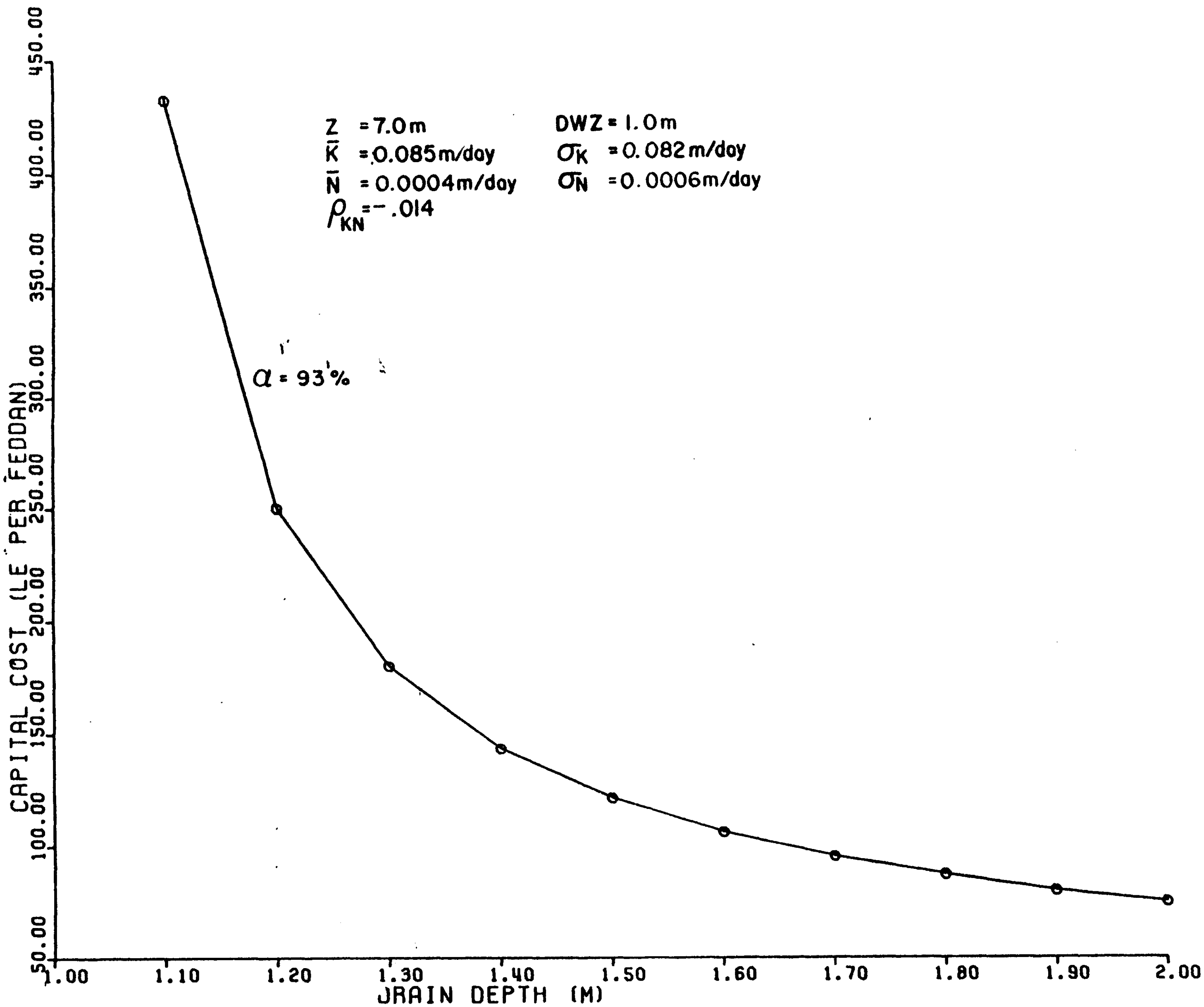


Figure 4.3 - Chance Constraint Sensitivity to Drain Depth

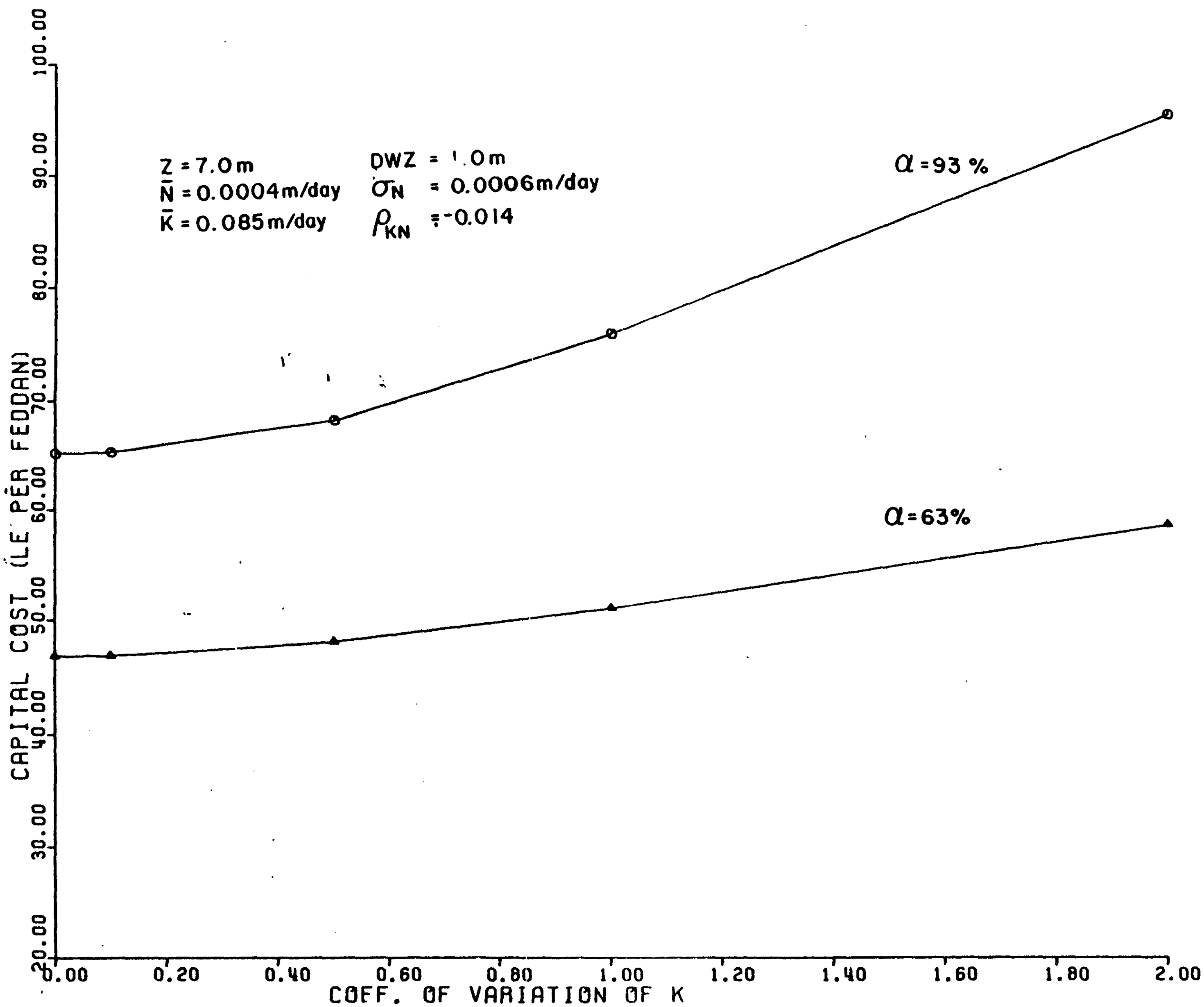


Figure 4.4 - Chance Constraint Sensitivity to Coefficient of Variation of K

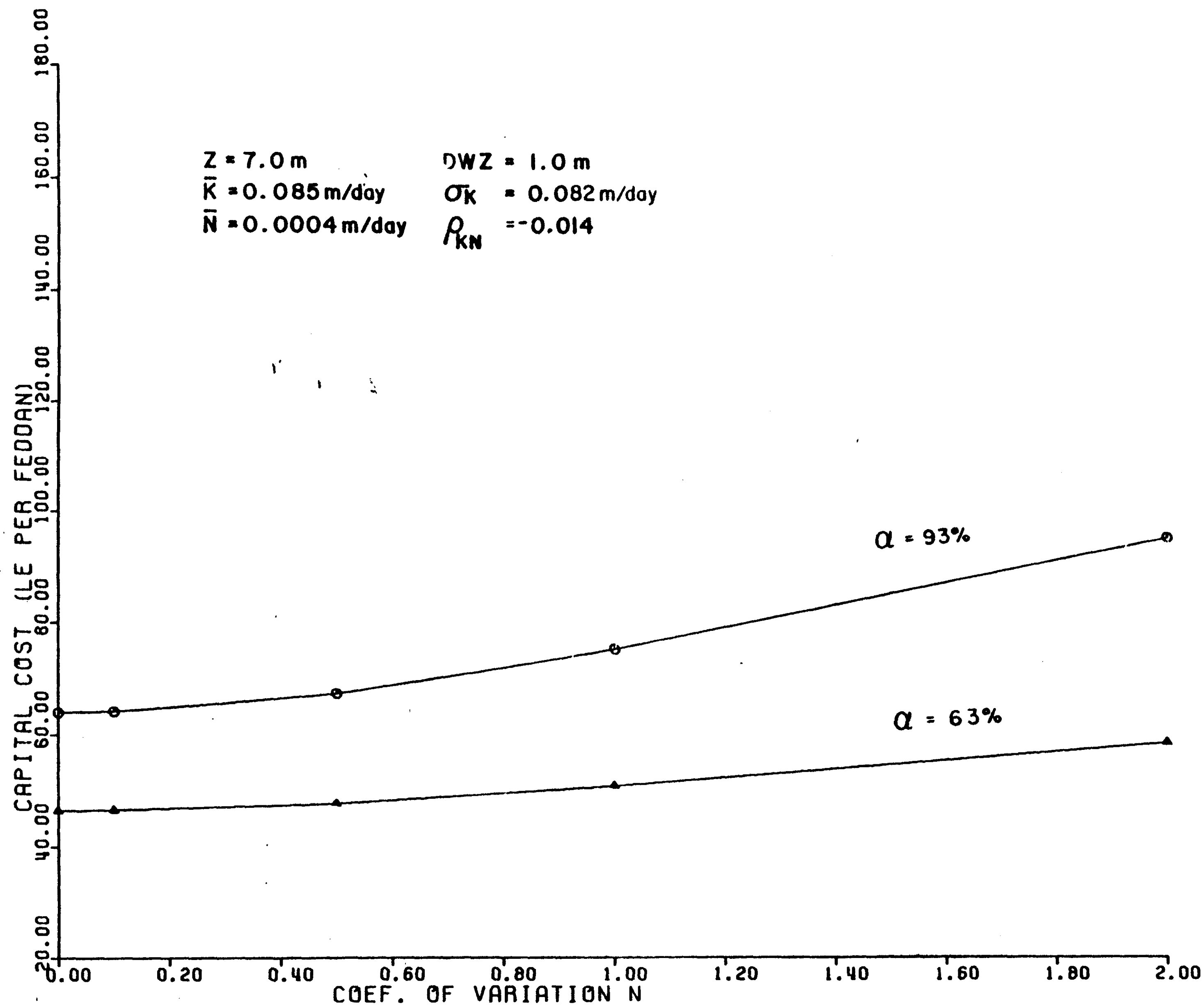


Figure 4.5 - Chance Constraint Sensitivity to Coefficient of Variation of N

in the input parameters. This takes pressure away from the engineer to precisely define the uncertainty in input parameters. In the case study data both parameters have a coefficient of variation close to 1.0 which falls on the region of the curves which are less sensitive.

Figure 4.6 is a plot of cost versus dewatering zone for a number of different reliabilities. This illustrates the main short comings of the chance constraint approach. The decision maker must choose a design dewatering zone and a design reliability. Figure 4.6 shows the tradeoffs facing the decision maker about costs of the drainage system but no information about benefits accrued. It is a design with economic benefits as an implicit factor considered, rather than as an explicit criteria of the model. The next section will address the incorporation of the explicit economic benefits in a stochastic programming approach.

4.3 Stochastic Programming Drain Design Model

An alternative to using reliability as a measure of uncertainty is to capture the entire distribution of the system output into an expected value of system performance. This alternative is the stochastic programming approach. The stochastic programming approach to uncertainty is possible if there exists a relationship between economic response and system output (Dantzig, 1955). Taking the expectation of the economic response a new economic measure of drainage system design is defined.

In agricultural drainage the objective is increased crop yield at

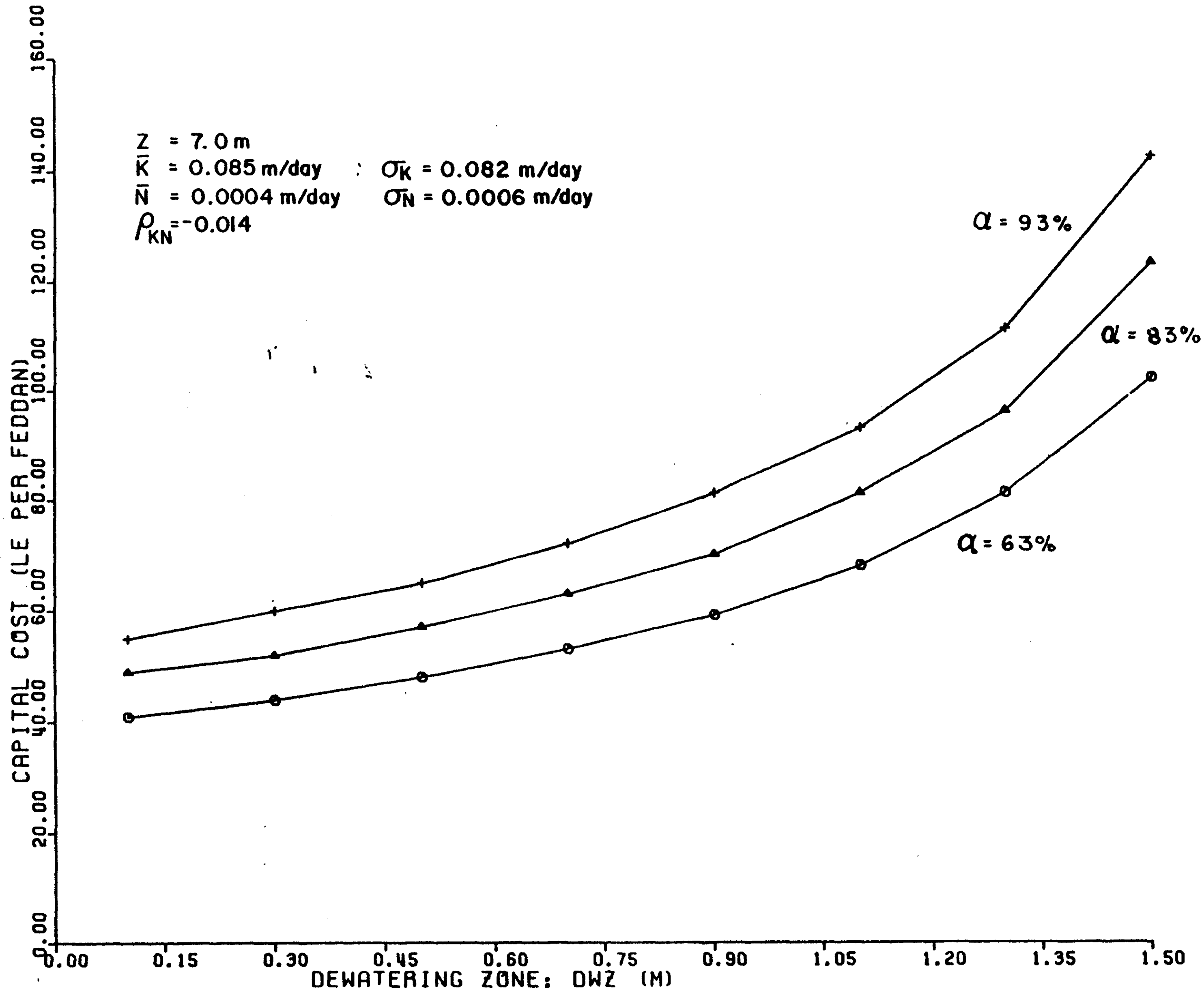


Figure 4.6 - Chance Constraint Sensitivity to Dewatering Zone

the least cost. The system output, dewatering zone, is assumed normally distributed. The FOSM analysis provides the mean and variance of the dewatering zone which fully describe the probability distribution of the dewatering zone. There exists empirical data relating crop yields to dewatering zones. With this data it is possible to calculate an expected crop yield as a function of the system design. The expected yield is subtracted from the optimal yield under perfect drainage conditions and the difference is considered the loss of benefits due to a given system design. The loss of benefits are regarded as the cost of the system in the same way as capital cost of installation. The next section will discuss the details of this approach.

4.3.1 Stochastic Programming Formulation

In the chance-constraint approach the model uncertainty was accounted for in the constraint set, while in the stochastic programming approach the uncertainty is accounted for in the objective function. The drainage design MPP becomes minimize Capital Cost plus Expected Losses subject to certain physical constraints and definitions.

The capital cost function is the same as in the previous MPP:

$$\text{Cost }(D,L) = \frac{c_1}{L}\,(c_2 D^{c_3} + c_4) \tag{4.1}$$

The expected loss function is complex and will be discussed in detail in the next section. The constraint set will now include the constraint on drain depth, the non-negativity constraints on D and L and the FOSM definition of mean dewatering zone and standard deviation of dewatering zone to be used in the objective function. The stochastic programming MPP is thus:

$$\text{Min} \quad \frac{c_1}{L} (c_2 D^{c_3} + c_4) + EL (D,L)$$

$$\text{S.T.} \quad D \leq D^*$$

$$\overline{DWZ} = D + d' - ((d')^2 + \frac{L^2\bar{N}}{4\bar{K}})^{1/2}$$

$$\sigma_{DWZ} = \frac{L^2}{8\bar{K}} \times ((d')^2 + \frac{L^2\bar{N}}{4\bar{K}})^{1/2} \cdot (\sigma^2 - 2(\frac{\bar{N}}{\bar{K}})KN \quad \sigma_N \sigma_K - (\frac{\bar{N}}{\bar{K}})^2 \sigma_K^2)^{1/2}$$

$$d' = \frac{d}{1 + d/L[2.55 \ln d/r - 3.55 + 1.6 \frac{d}{L} - 2(d/L^2)]}; \quad \frac{d}{L} \leq .31$$

$$d' = \frac{L}{2.55(\ln L/r - 1.15)} \quad \frac{d}{L} > .31$$

$$D,L \geq 0.0$$

where EL(D,L) is the expected loss function. Now that the stochastic programming model has been developed the next step is to examine the expected loss function.

4.3.2 Expected Loss Function

The model of groundwater levels between the drains that has been chosen is a steady state model. As such the predicted levels are assumed to be constant over the entire growing season and the same for each growing season over the life of the drains. The model provides the mean and variance of the groundwater head midway between the drains. With the assumption of normally distributed head the full probability distribution can be defined. The variable of interest in the loss function is not the head but rather the dewatering zone which is defined as the depth of the water table from the soil surface:

$$DWZ = D - h \qquad (4.15)$$

where

DWZ = dewatering zone

D = depth to drains

h = depth of groundwater above drains.

Using derived distribution (Benjamin and Cornell, 1970) it is possible to find the full probability distribution of the dewatering zone $f_{DWZ}(DWZ)$. Due to the linear relationship between DWZ and h, the mean of the dewatering zone is

$$\overline{DWZ} = D - \bar{h} \qquad (4.16)$$

where

$\overline{DWZ}$ = mean of dewatering zone

$\bar{h}$ = mean groundwater head

and variance of the dewatering zone is

$$\sigma^2_{DWZ} = \sigma^2_h \qquad (4.17)$$

It is clear that the dewatering zone is normally distributed with $\overline{DWZ}$ and σ_{DWZ} as parameters.

A series of general functions of yield versus dewatering zone is presented in Figure 4.7. The appropriate function must be determined specifically for each climate, soil condition, and crop. However, the general shape will fall closely to one of the seven

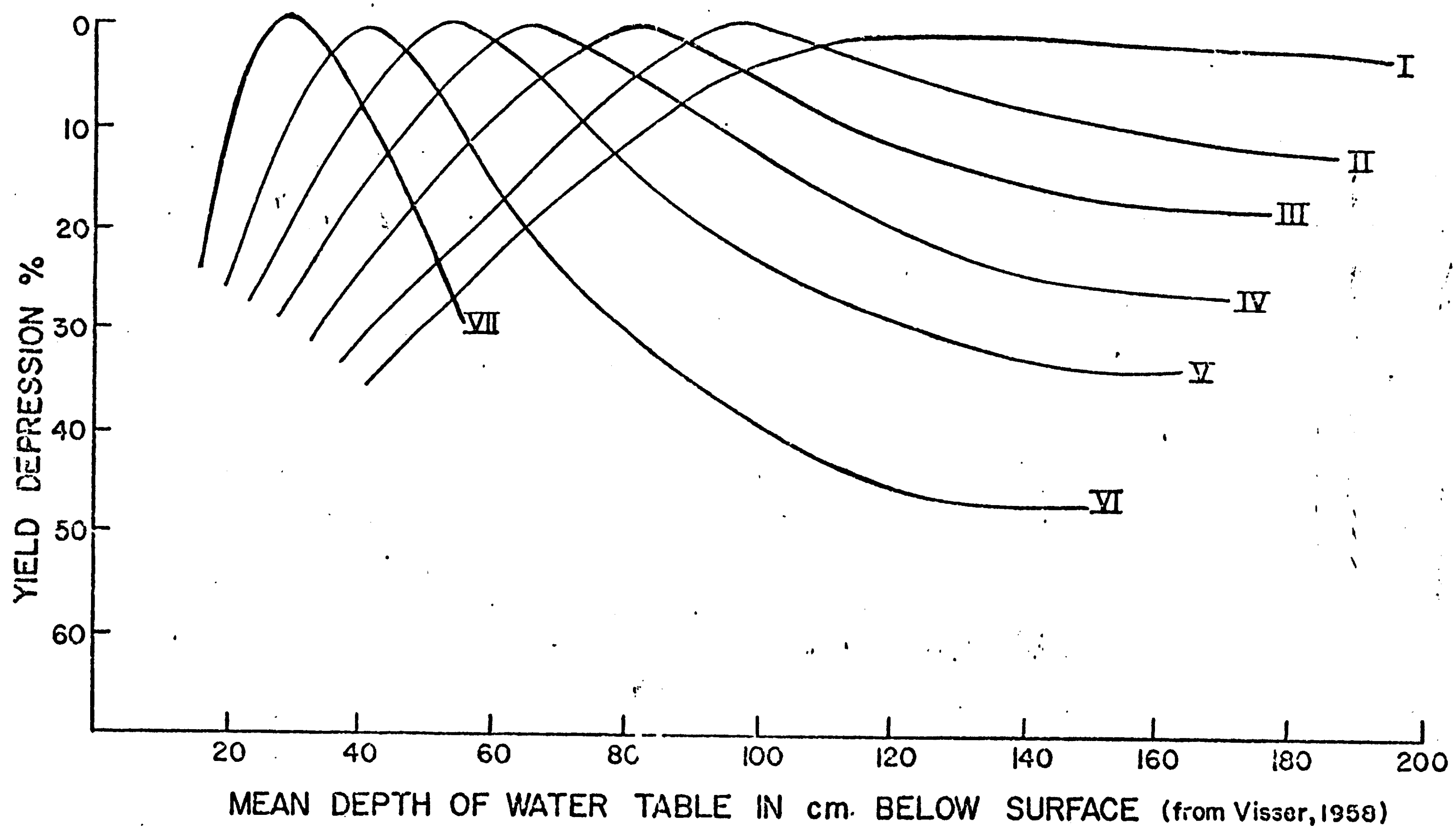


Figure 4.7 - Crop Yield Functions

presented in Figure 4.7.

The curves shown in Figure 4.7 are a measure of the crop yield as a function of the dewatering zone mid-way between the drains. These functions integrate the effects of varying DWZ between the drains. If this were not the case and the functions reflected the crop response to a uniform dewatering zone then the effect of the dewatering zone would have to be integrated over the entire drain spacing to reflect the variation in dewatering zone between drains as illustrated in Figure 3.1. If this were the case the present method could still be employed, since the Hooghoudt model provides an analytical expression for the dewatering zone as a function of x.

The expected value of yield between drains could be calculated by integrating the yields at each point x between the drain from 0 to L.

With a functional relationship between crop yield and the dewatering zone the expected yield for any crop over the growing season can be found by integrating the yield function times the probability distribution of the dewatering zone over the entire range of dewatering zones:

$$E[Y] = \int_{-\infty}^{\infty} Y(DWZ) \times f_{DWZ}(dwz)\ d_{dwz} \tag{4.18}$$

This is represented graphically in Figure 4.8. With the first and second moments and the assumption of normal distribution, the probability distribution function of DWZ has been defined. Now the appropriate yield function must be defined.

Now that an expected yield for each year has been determined,

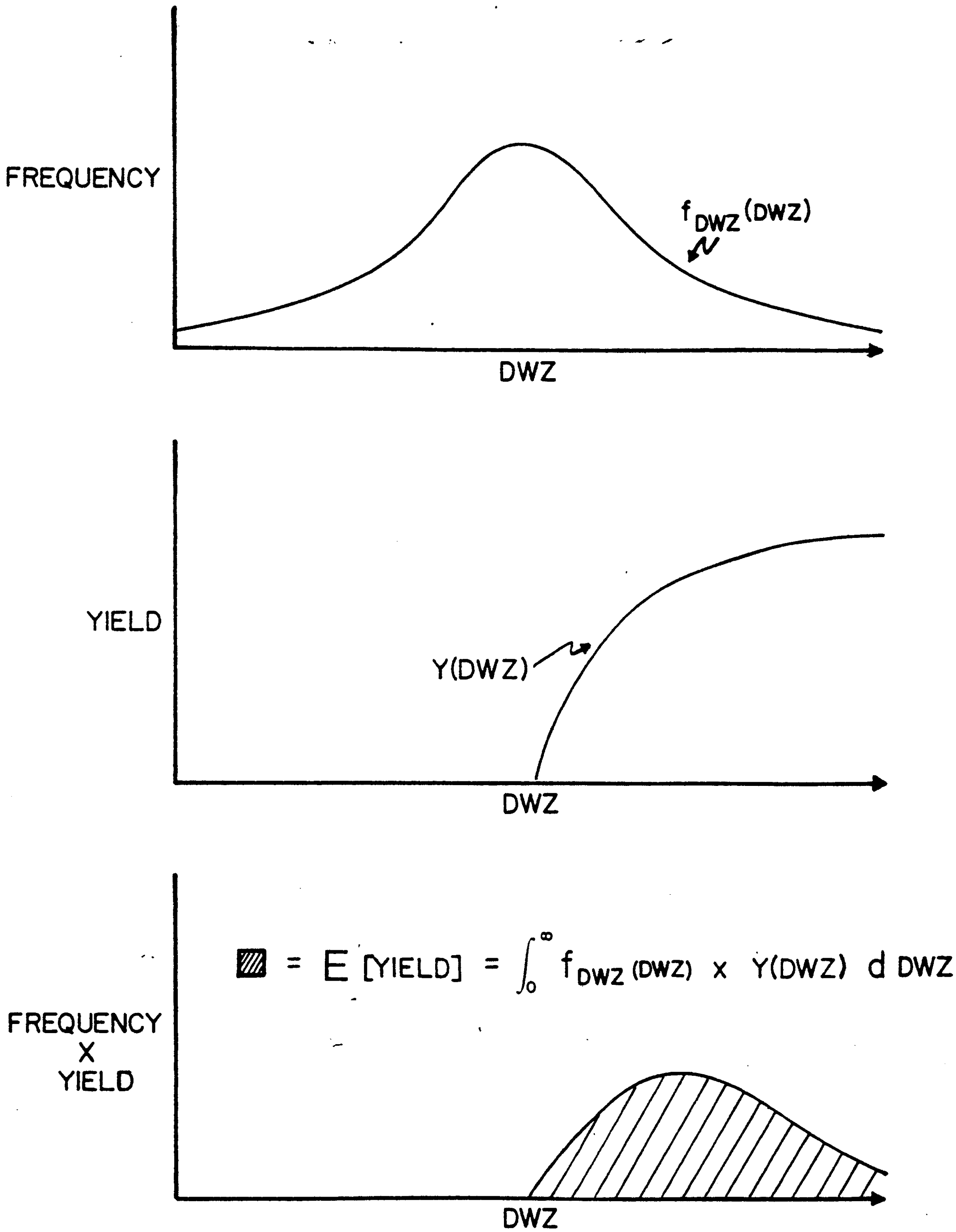


Figure 4.8 - Calculation of Expected Yield

it is possible to transfer that measure of agricultural performance into an economic measure. The difference between maximal yield possible under optimal dewatering depth and the expected yield as a result of the current system design is defined as the annual yield loss:

$$AYL = Y^* - E[Y] \tag{4.19}$$

where

AYL = annual yield loss

Y* = optimal annual yield

E[Y] = expected annual yield

This annual yield loss is then multiplied by prices for that crop and a measure of annual economic loss is defined.

$$AEL = CP \times AYL \tag{4.20}$$

where

AEL = annual economic loss

CP = price for crop

AYL = annual yield loss

It is assumed that these losses will be constant each year over the life of the project. Using standard discounting techniques the present value of the annual loss over the life of the project can be determined by multiplying the constant annual loss by the present worth factor:

$$PWF_i^t = \sum_{N=0}^{t-1} \frac{1}{(1+i)^N} \tag{4.21}$$

where

PWF_i^t = present work factor for interest rate i and project life t.

The present worth of the expected loss due to the current design is

$$EL(D,L) = AEL * PWF_i^t \tag{4.22}$$

The difficulty in this approach is that there are a number of parameters that must be specified for which it is hard to fix an exact value. They are (1) the type of loss function, (2) optimal crop yield, (3) crop prices (4) interest rate and (5) the life of the project. These parameters and the effects upon the optimal design will be investigated later in the case study.

Thus the expected loss term in the objective function is defined as

$$EL = \{Y^* - \int_{-\infty}^{\infty} Y\ (DWZ)\ .f_{DWZ}(DWZ)\ d_{DWZ}\} \times\ CP \times \sum_{N=0}^{t=1} \frac{1}{(1+i)}N \tag{4.23}$$

The question is then how to evaluate the integral in the loss function. For the functional forms presented in Figure 4.7 it is impossible to find an analytical expression for the integral in Equation 4.23. To overcome this problem the probability distribution is approximated by a histogram with small intervals. Very little probability density is found in the tails of a normal distribution and the loss functions in these areas are constant so there are no problems with approximating an infinite distribution by a finite histogram, Figure 4.9.

The integral can now be approximated by summing the product of the probability of each interval of histogram times the yield function

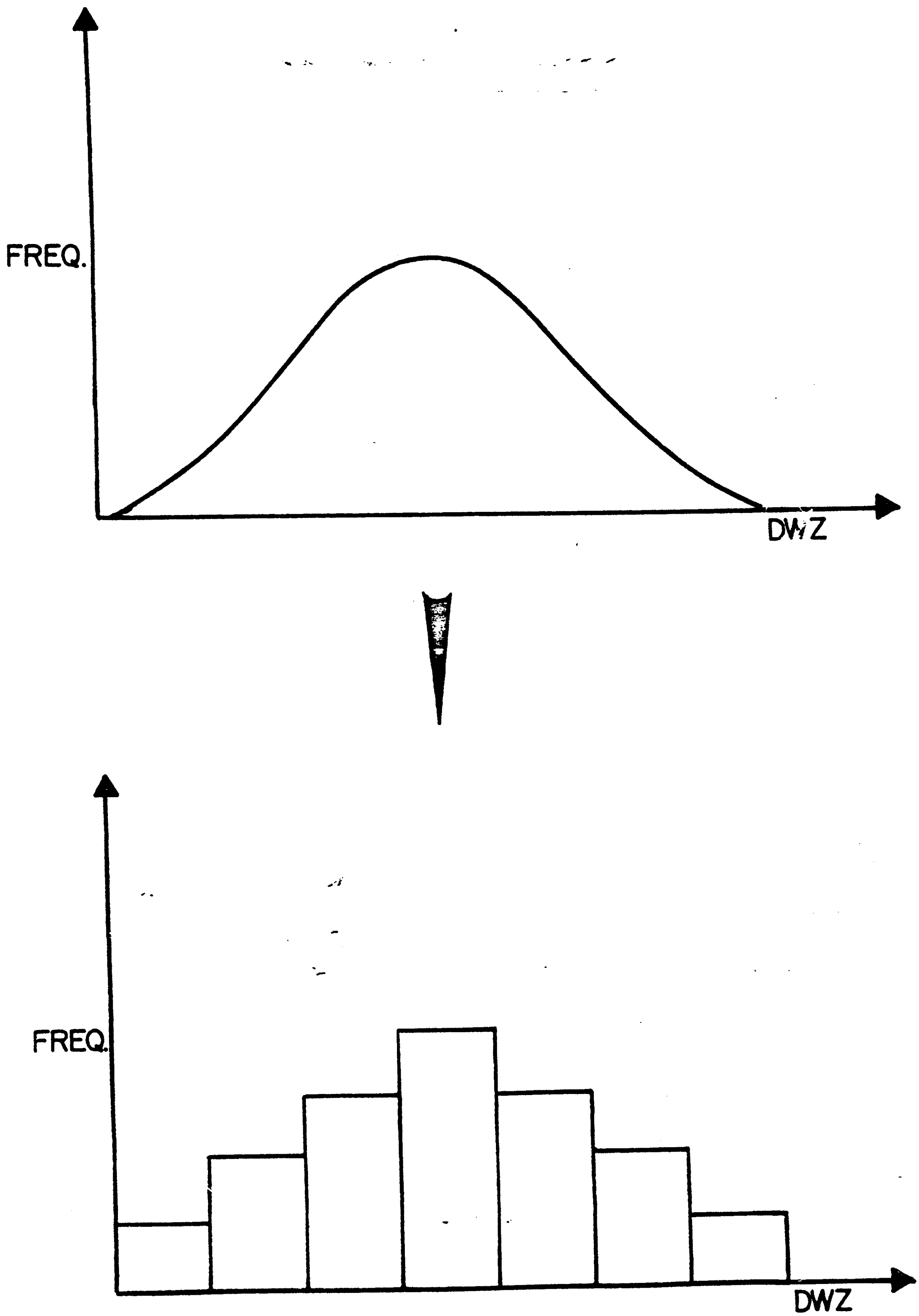


Figure 4.9 - Calculation of Histogram

evaluated at the midpoint of that interval so that

$$Y(DWZ) \cdot f(DWZ)d_{DWZ} = \sum_{n=1}^{m} f_{DWZ}^{(i)} \cdot Y(DWZ)_i \tag{4.24}$$

where

$f_{DWZ}^{(i)}$ = probability of interval i

DWZ_i = value of DWZ_i at midpoint of interval i.

Figure 4.7 illustrates basic forms of the yield function. For this work three representative forms are selected; they are labeled case I, case II and case III (Figure 4.10). In all the cases the rise to the optimal dewatering zone is parabolic. However, in case I after the optimum is reached, the yield remains at the optimum as the dewatering zone increases. This represents the condition when the supply of irrigation water to the crops is provided at frequent intervals. The function describes the process that as the dewatering zone increases more aeration is possible for the roots. But after the optimal depth is achieved the deeper depth only increases aeration and with frequent irrigation the water for the plants comes from the downward percolation and the water table does not contribute to crop water use.

Case II follows the same parabolic rise to an optimum dewatering zone. Once the optimum is reached the yield declines in a Guassian form to an assymptotic value. This case is representative of two possible conditions. The first is when there are frequent irrigations applied to a very porous soil. The water percolates very quickly to the ground-water table and the residence time of downward percolation is not

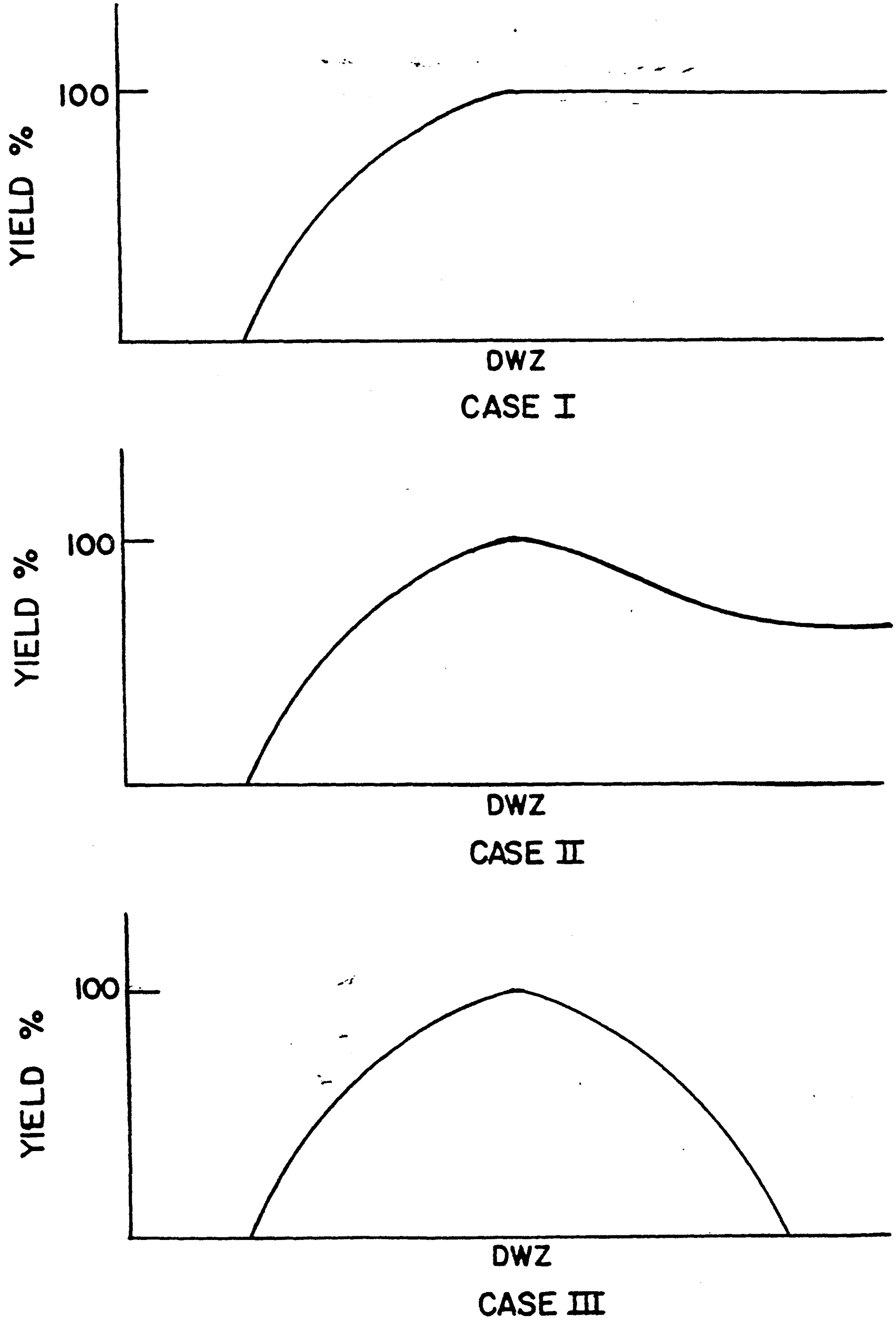


Figure 4.10 - Case Study Yield Functions

sufficient to meet the plant's needs so upward capillary movement is important. So as the dewatering zone increases the water available from the upward capillary decreases and yields decline to that supported by the downward percolation. It may also represent the condition of infrequent intensive irrigation upon a soil with high capillary, such as clay. In this condition the downward percolation takes a relatively longer time to reach the groundwater table. However, since the times between irrigation are long the groundwater becomes an important supply of water for the plants at the end of non-irrigation periods.
So that as the dewatering zone becomes larger, water is unable to reach the root zone and yields decline to that supported by downward percolation.

Finally, case III has the same parabolic rise to the optimum yield and then a parabolic decrease just as steep to zero yield. This case represents the condition of infrequent irrigation of a very porous soil where the irrigation water percolates very fast to the groundwater table and groundwater is the main source of water supply. Thus as the groundwater level decreases less and less water is available to sustain plants.

The mathematical descriptions of the three cases are as follows:

Case I

$$Y(DWZ) = 0.0 \qquad DWZ < 0$$

$$Y(DWZ) = 1-(DWZ-DWZ^*)^2 \qquad 0 \leq DWZ \leq DWZ^*$$

$$Y(DWZ) = 1.0 \qquad DWZ > DWZ^* \qquad (4.25)$$

Case II

$$Y(DWZ) = 0.0 \qquad DWZ < 0$$

$$Y(DWZ) = 1-(DWZ-DWZ^*)^2 \qquad 0 \leq DWZ \leq DWZ^*$$

$$Y(DWZ) = C_1+C_2\ e^{-A(DWZ-DWZ^*)^2} \qquad DWZ > DWZ^*$$

$$C_1+C_2 = 1 \qquad (4.26)$$

Case III

$$Y(DWZ) = 0.0 \qquad DWZ < 0.0$$

$$Y(DWZ) = 1-(DWZ-DWZ^*)^2 \qquad 0 \leq DWZ \leq Y^{-1}(0.0)$$

$$Y(DWZ) = 0.0 \qquad DWZ > Y^{-1}(0.0) \qquad (4.27)$$

Figures 4.11, 4.12 and 4.13 represent graphical representations of the loss function for Egyptian cotton for the three cases mentioned above as a function of decision variables depth D and spacing L.

The parameters used to evaluate the function are those from the Embabe case study that will be used throughout this work (Table 4.2).

Figure 4.14 is a graph of the capital cost of field drainage as functions of depth D and spacing L for this case study (ElBerry, 1979).

Combining the capital cost function with the expected loss function Figures 4.15, 4.16 and 4.17 provide a graphical representation of the stochastic programming field drainage design problem, for Cases I, II and III, repsectively. These representations include both the objective function and the constraint set.

As seen in Figures 4.15, 4.16 and 4.17 the objective function is highly nonlinear. However, this objective function is bounded by a linear

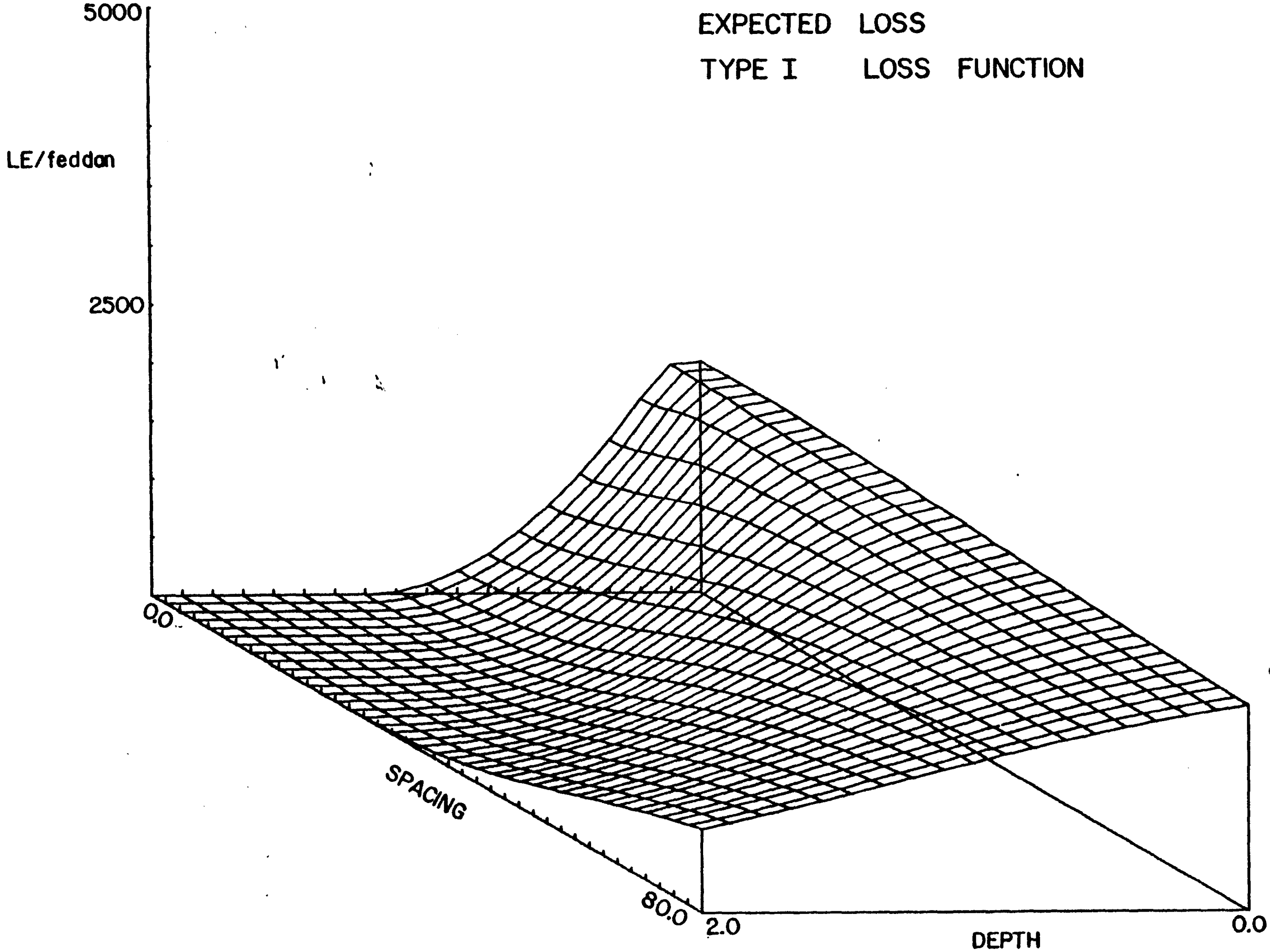


Figure 4.11 - Expected Loss: Type I Loss Function

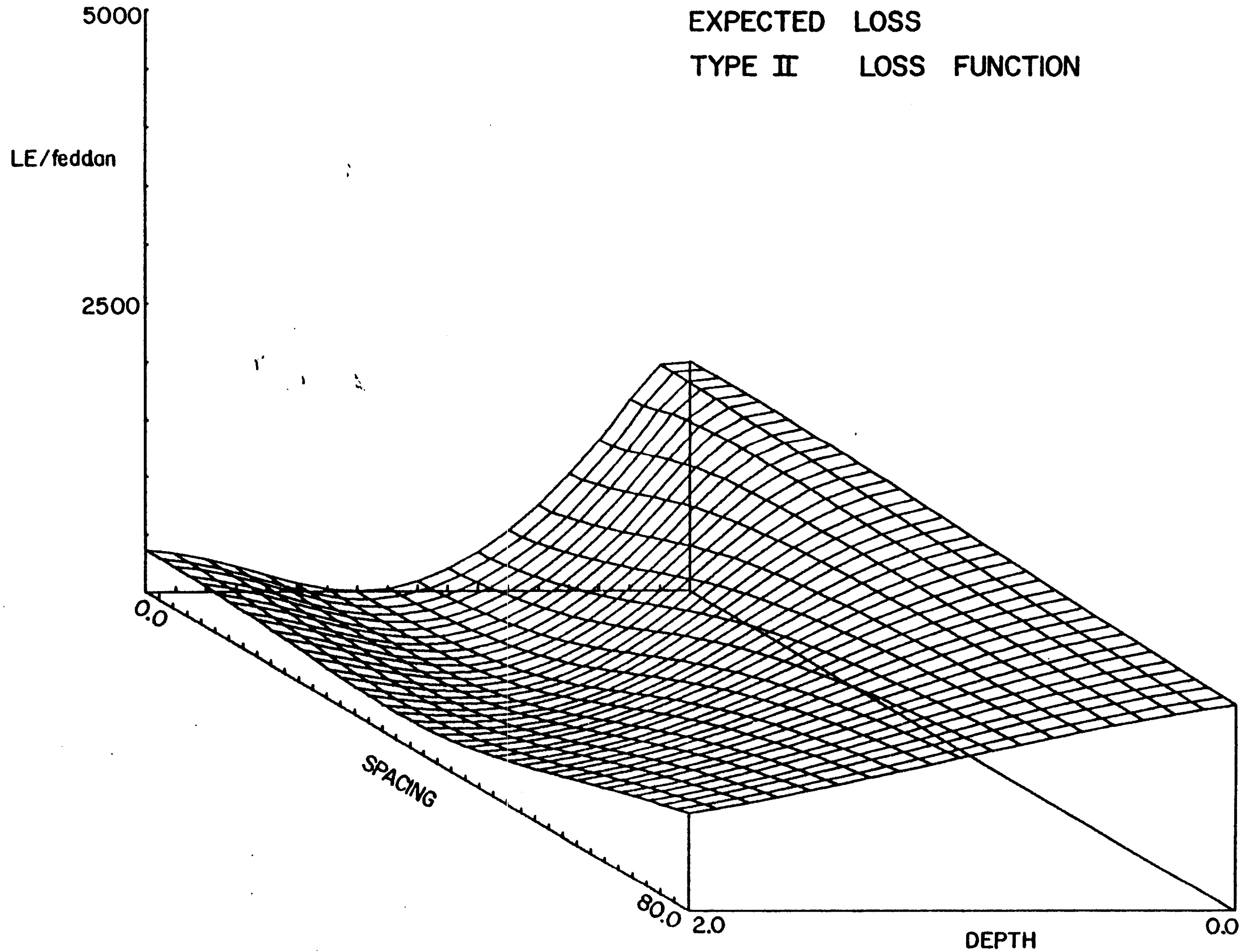


Figure 4.12 - Expected Loss: Type II Loss Function

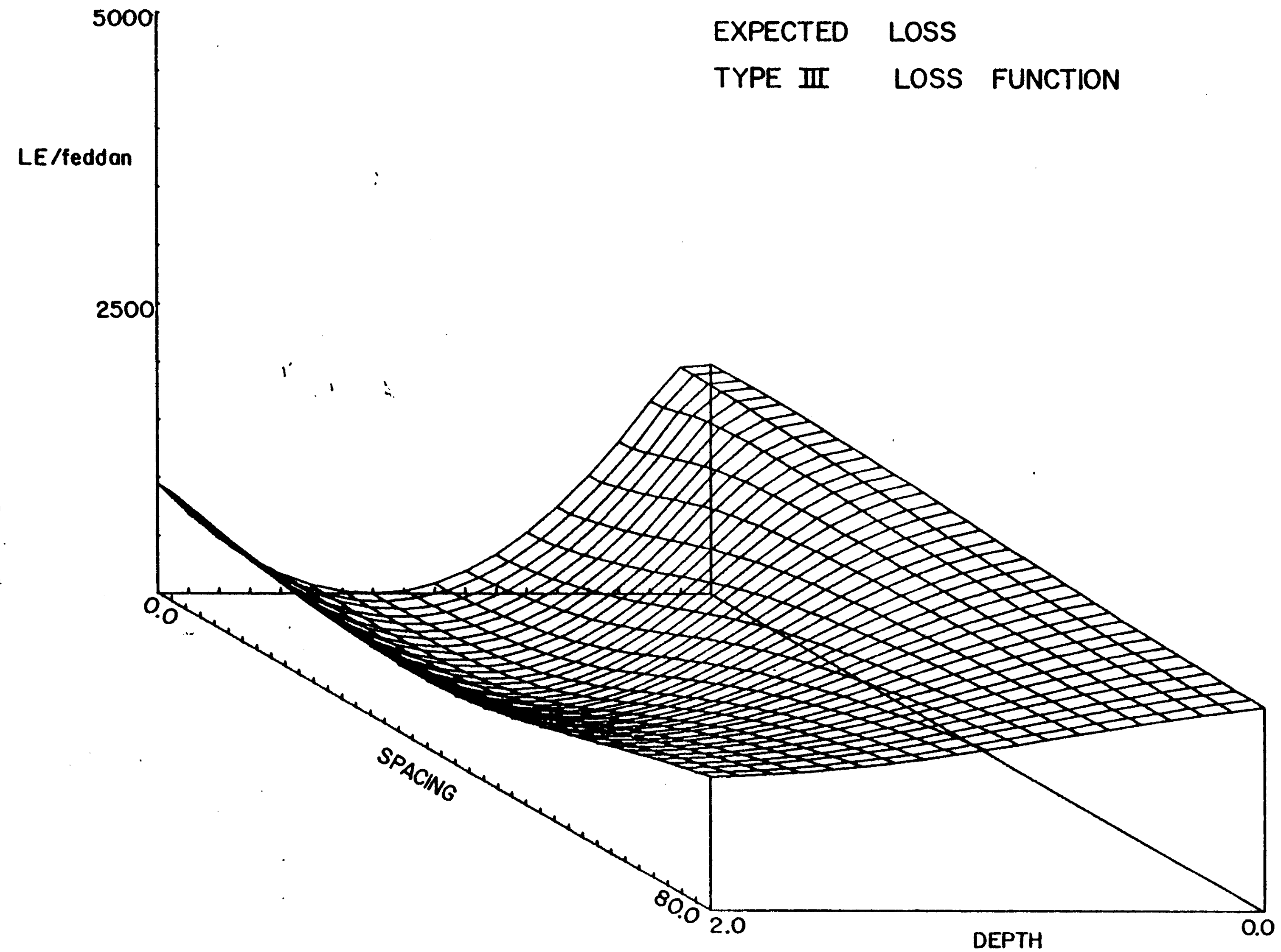


Figure 4.13 - Expected Loss: Type III Loss Function

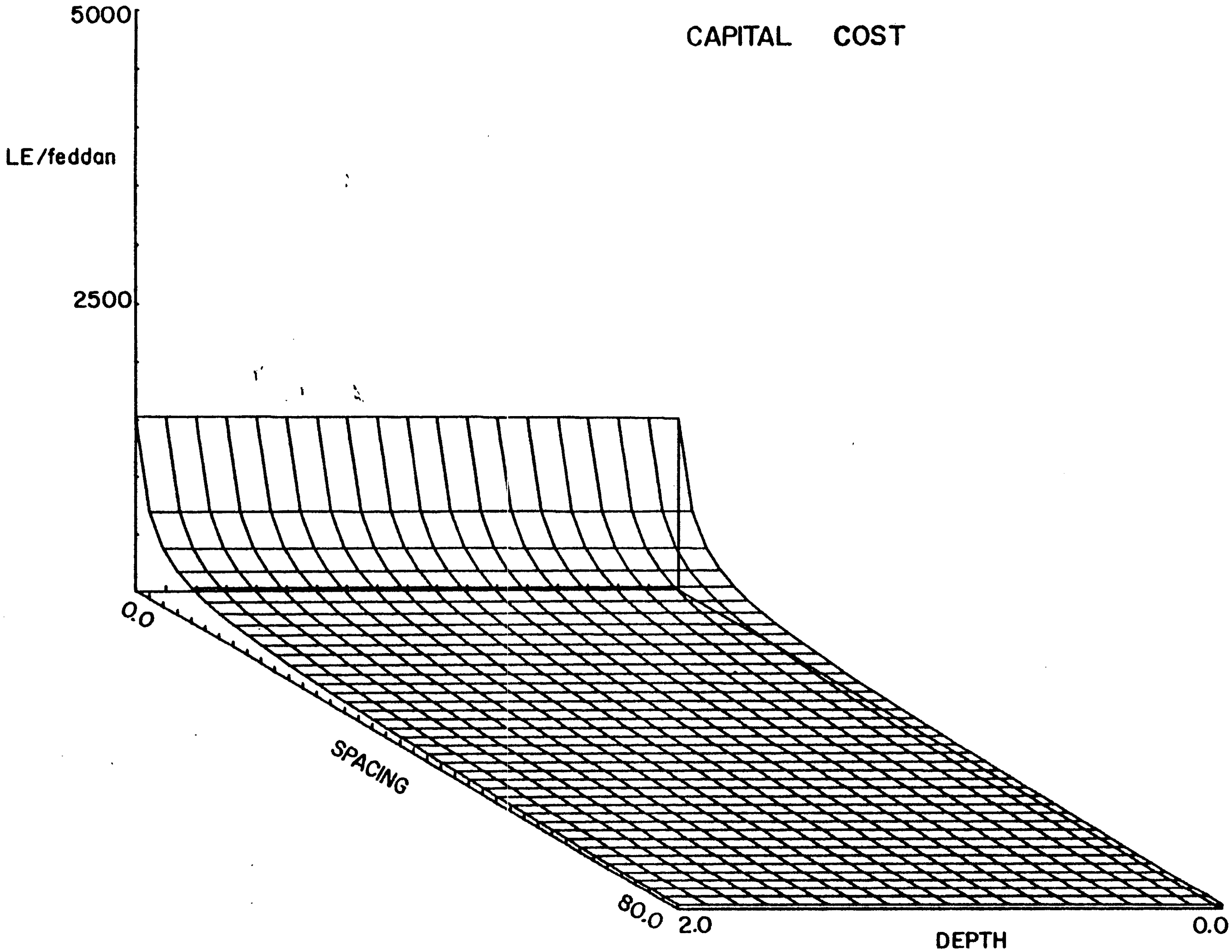


Figure 4.14 - Capital Cost

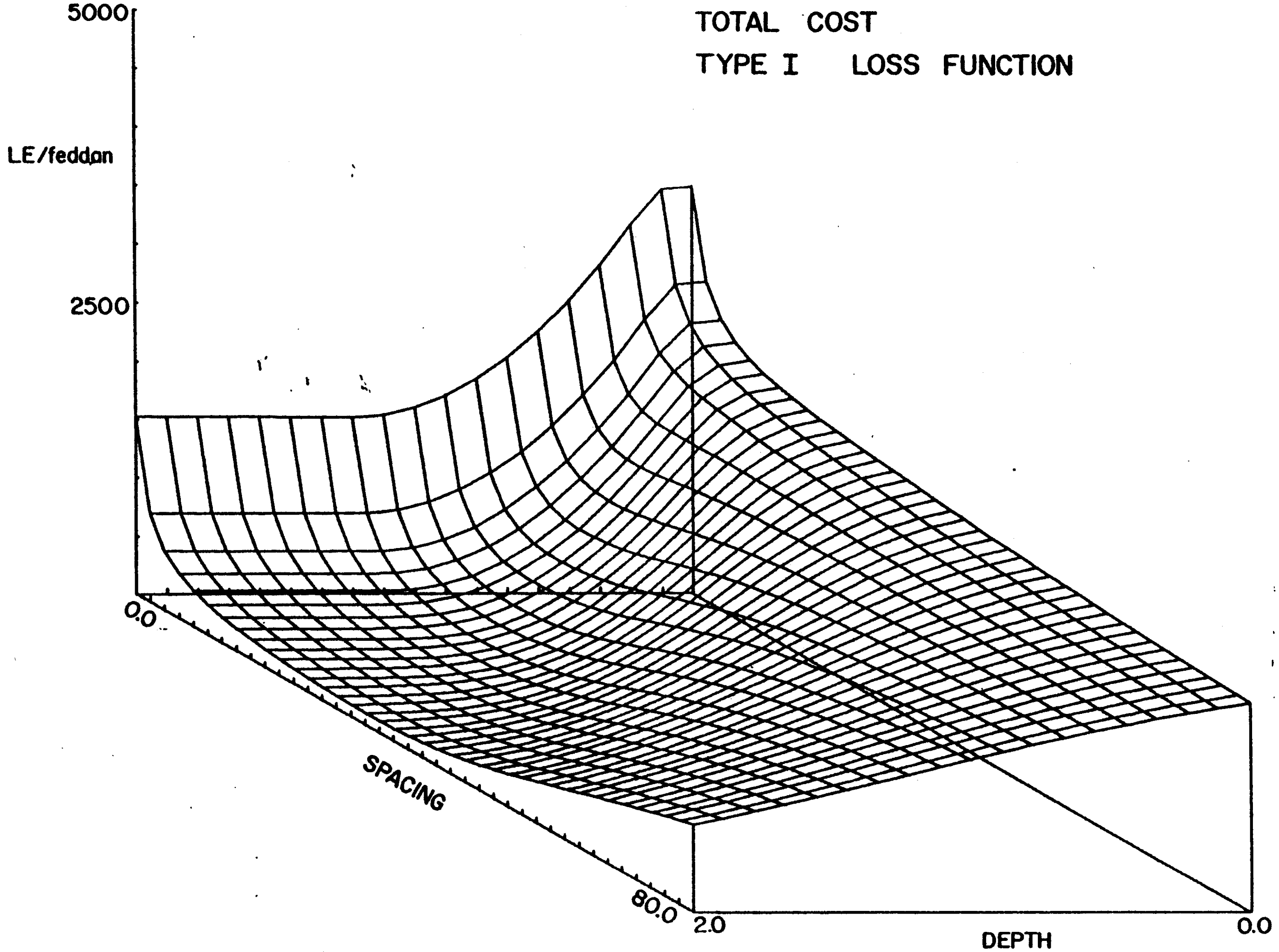


Figure 4.15 - Total Cost: Type I Loss Function

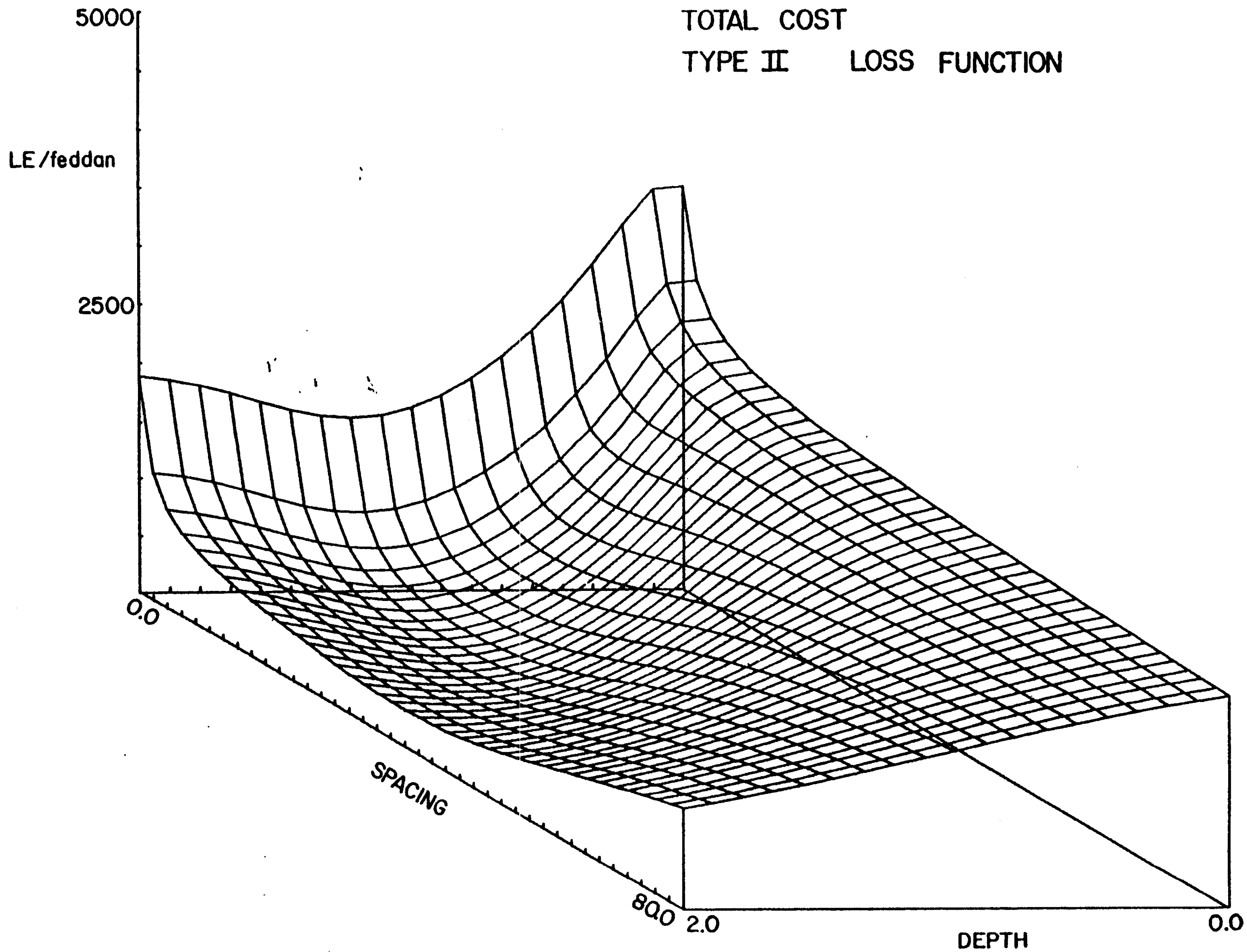


Figure 4.16 - Total Cost: Type II Loss Function

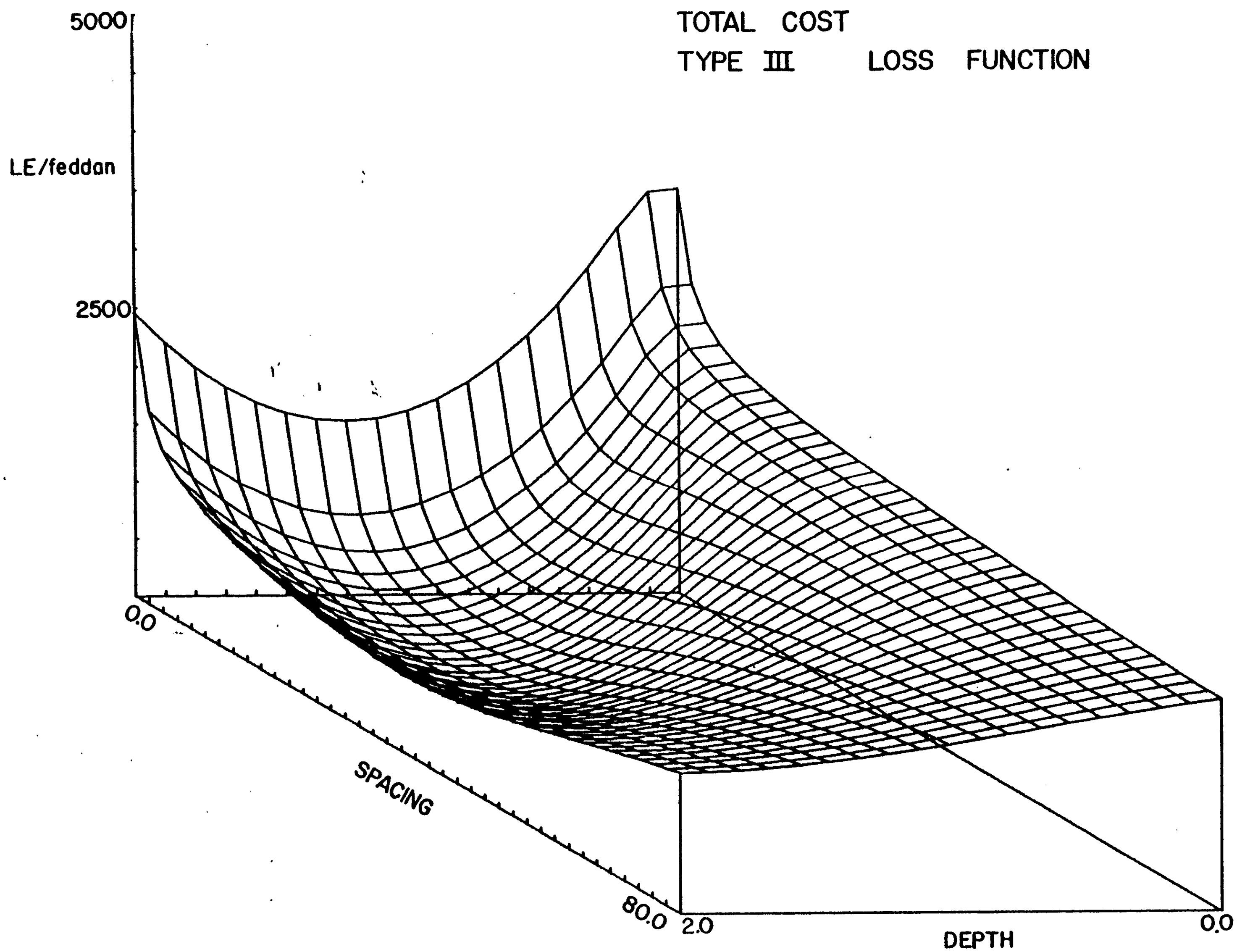


Figure 4.17 - Total Cost: Type III Loss Function

constraint set which defines a convex feasible region. The form of objective function is unimodal or quasi-convex which means that a local optimum for the minimization problem is a global optimum and standard nonlinear minimization techniques can be used. The objective function was shown to be quasi-convex over D and L respectively. This was accomplished by using a computer package that found the partial derivatives of the objective function with respect to D and L which was impractical by hand.

The package showed that the partial derivatives changed sign no more than one for each value of D with L held constant. This analysis was performed using the Embabe case study data as input parameters and over the range of D and L well beyond that expected in the case study. For all forms of the crop yield function the objective function was always quasi-convex. Although this methodology of stochastic programming is valid for any application to drainage design, the form of objective function must always be analyzed to see that it is at least quasi-convex. However, if it is not, some algorithm may be found that will allow for minimization of a non-quasi convex objective function (Lemarchal and Mifflin, 1978).

4.3.3 Solution Technique

The stochastic programming problem is again a two-dimensional nonlinear programming problem. The constraint set is linear but unbounded in L, so the solution technique proposed for the change-constraint program is not valid. The most widely used technique for this class of mathematical programming problems is the gradient search approach. However, in the stochastic programming drain design problem the objective function is so complex that the calculation of the gradient at each interaction is computationally burdensome. However, as seen in Figures 4.15, 4.16 and 4.17 the objective function is unimodal in D and L. Taking advantage of this property a recursive algorithm is developed that minimizes over D the function of the objective function minimized over L for each D.

$$\min_{D} G(D)$$

$$G(D) = \min_{L} f(D,L) \qquad (4.28)$$

where

G(D) = function of D minimized over L

F(D,L) = objective function in D and L.

Thus a two-dimensional problem is broken down into two one-dimensional problems, which is much more efficient to solve than a two-dimensional gradient search for this problem.

To solve these one-dimensional problems the golden section search method is used. This is discussed in detail in Wismer and Chattergy (1978). Briefly, the analyst chooses an interval over which the function has a minimum. Usually this interval is defined by the constraint set. If the decision variable is unbounded the analyst examines extended portions of the region if the solution converges to one of the endpoints of the original interval. The method proceeds by examining the value of the objective function at each endpoint. Then a new point is examined such that the ratio between the ending and the beginning interval of the uncertainty remain constant in each iteration. The new point is located then at the point x_1^2

$$x_1^2 = x_2^0 + r\, L_o \tag{4.29}$$

where L_o is the current interval of uncertainty and x_2^0 is one of the endpoints and r is the ratio between the interval of uncertainty. The algorithm then reduces the size of the interval by the same ratio $r = (-1 + \sqrt{5})/2$ each iteration. This ratio is known as the "golden section" from its use in ancient art and architecture of the Greeks. The procedure will continue until the interval is reduced to the desired accuracy of the analyst. In this algorithm once the desired accuracy has been reached the three points that remain are used to determine a parabola using Newton's method. The minimum of this parabola is then found and the result is the "optimal value" of the decision variable (see Figure 4.18).

The accuracy of the model is far greater than accuracy to which the drains can be installed, therefore this procedure is well suited for the problem of drainage design.

With the model now fully developed it is possible to find the optimal field level drainage design. The next section will present results for the embabe case study and sensitivity analysis of the parameters of the model.

Before presenting the results, the response of the dewatering zone as a function of depth D and spacing L, should be studied. Figure 4.19 is a graph of the mean value of the dewatering zone as a function of D and L and Figure 4.20 is a graph of the standard deviation of the dewatering zone as a function of D and L for the Embabe case study.

GOLDEN SECTION SEARCH

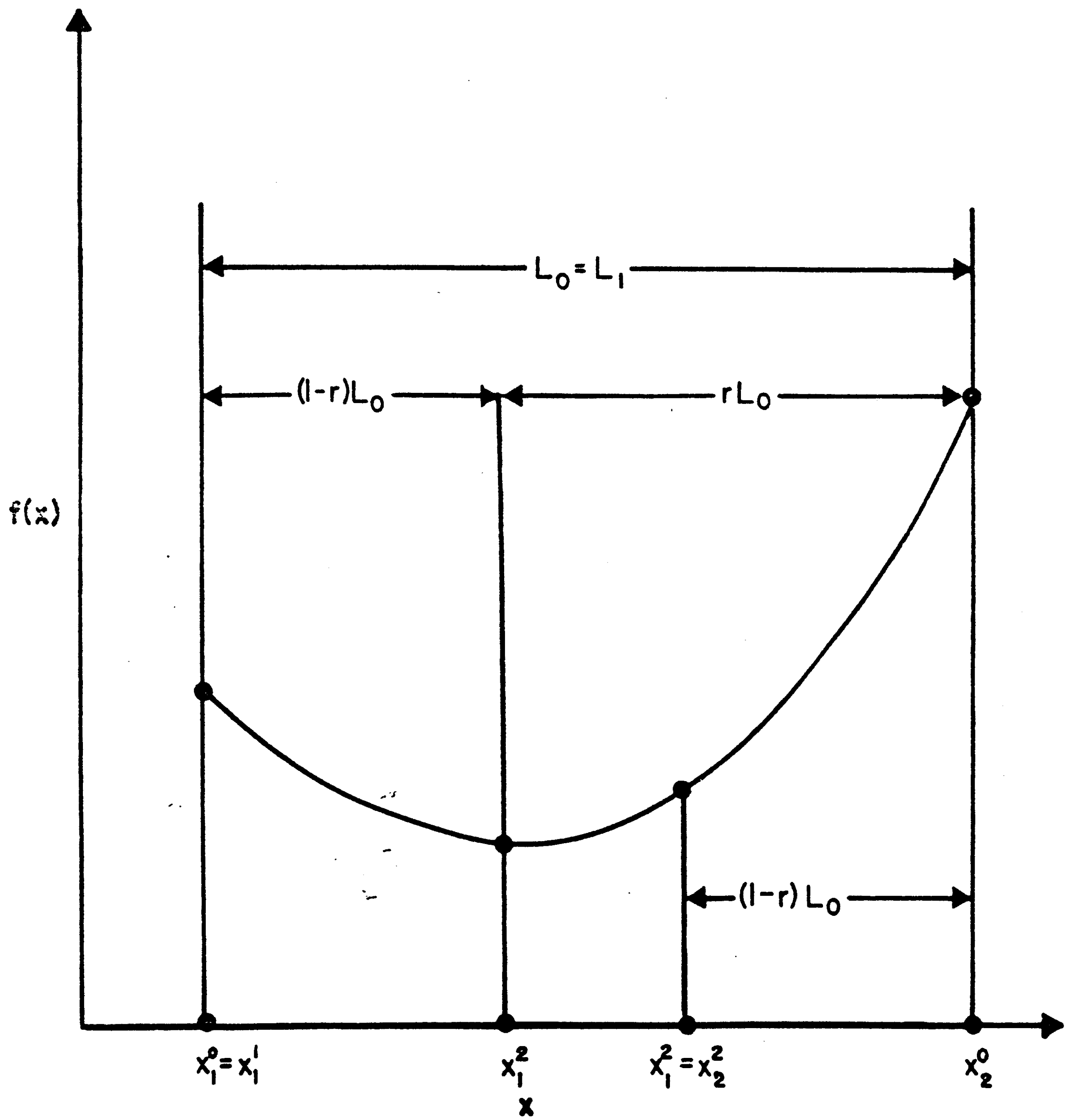


Figure 4.18 - Golden Section Search

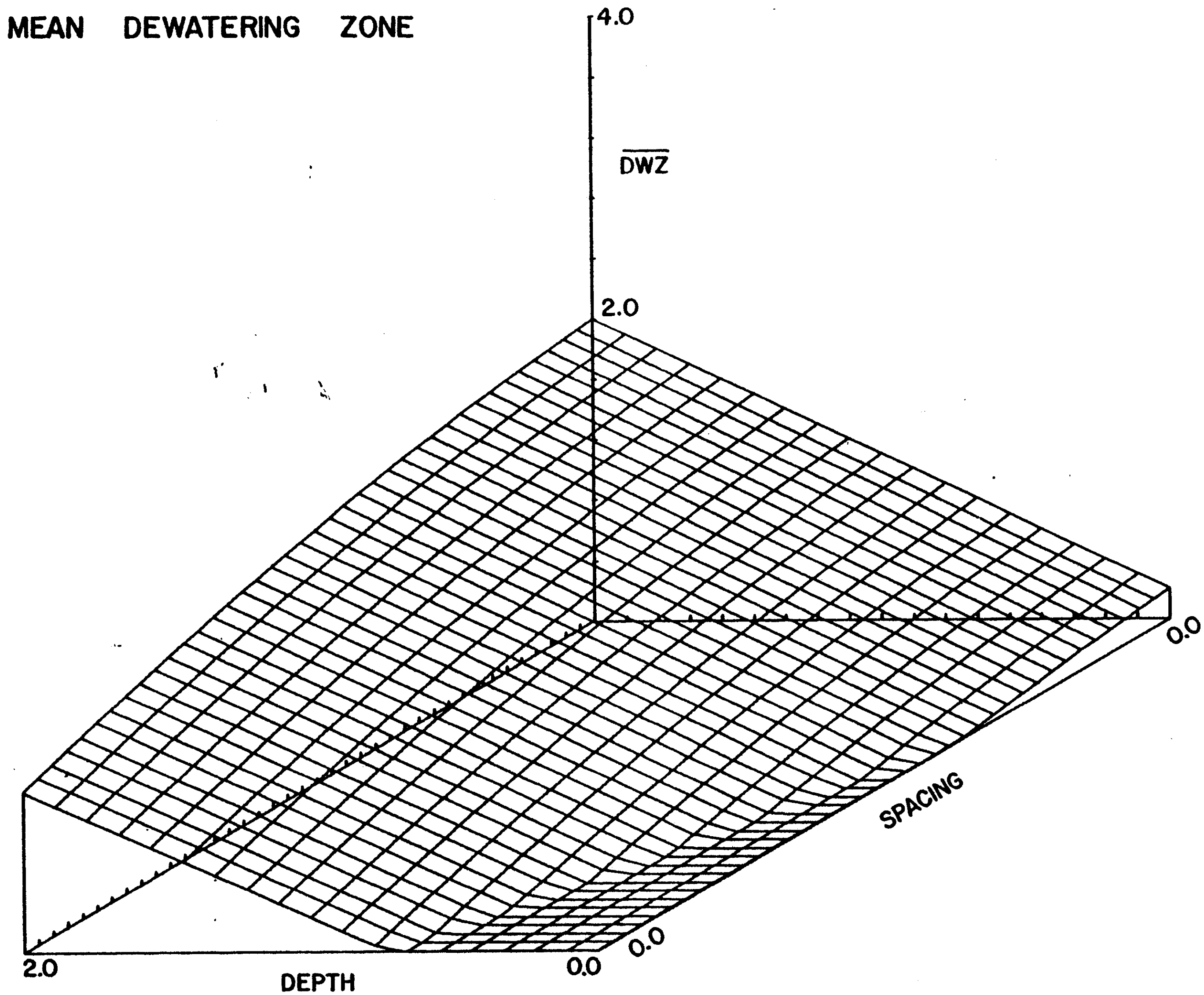


Figure 4.19 - Mean Dewatering Zone

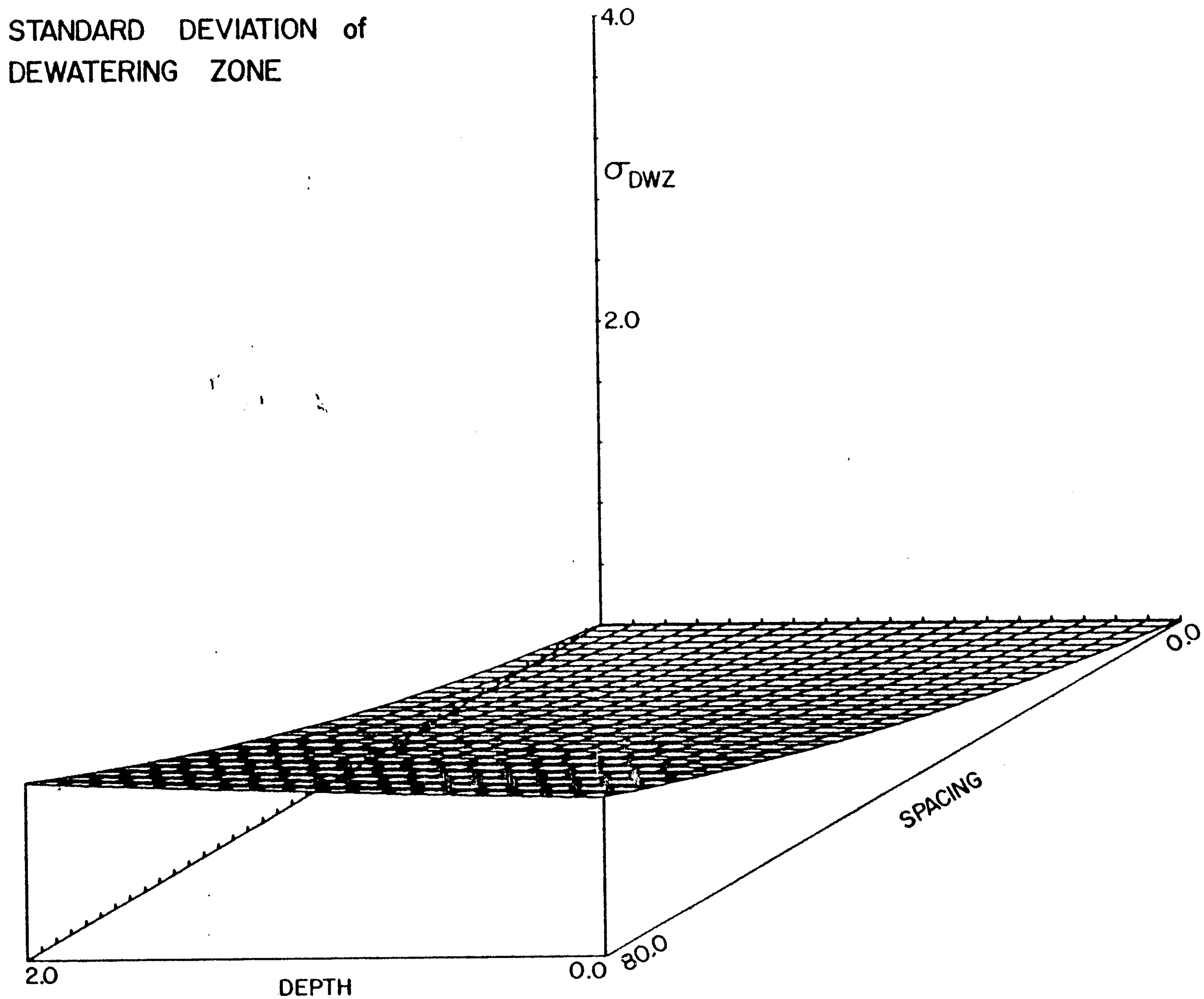


Figure 4.20 - Standard Deviation of Dewatering Zone

From Figure 4.19 it is possible to see that there are various combinations of D and L that produce contours of constant values of $\overline{DWZ}$ and these values increase as D increases and L decreases. The standard deviation σ_{DWZ} increases as L increases and decreases as D increases as shown in Figure 4.20. The model's goal is to find the design that minimizes the capital costs plus the losses due to reduced yields. Thus, the model will attempt to design a system such that $\overline{DWZ}$ will be close to DWZ optimal and σ_{DWZ} will be as small as possible to concentrate the probability density at the optimum point. This can be seen in Table 4.2 which presents the optimal drainage design for a field of Egyptian clover with three possible loss functions.

In this analysis, case I has an expected cost much less and a much different design than both cases II and III. This is due to the fact that for case I the yield is constant at the optimum value at values of DWZ beyond the optimal level. From Figure 4.19 it is seen that as the depth of the drains decreases the spacing of the drains can increase and maintain the same $\overline{DWZ}$. Thus the model will design the drains to have a large $\overline{DWZ}$ and as small a σ_{DWZ} as possible. However, the capital cost increases as $\overline{DWZ}$ increases and σ_{DWZ} decreases as $\overline{DWZ}$ increases giving a design for case I of depth, D = 2.0 meters and spacing, L = 34.7 meters.

In cases II and III the model attempts to put $\overline{DWZ}$ close to the optimum value while minimizing σ_{DWZ} as discussed above. However, a small σ_{DWZ} implies small L which implies high capital cost. The optimum design for case II, is D = 1.46 m, L = 21.95 m, and case III is D = 1.43 , L = 20.57.

Table 4.2

Optimal Drain Design Sensitivity to Crop Yield Function

I. Model Parameters

Z = 7.0m	D* = 2.0	
R = .085 m/day	σ_K = .0815 m/day	
$\overline{N}$ = .0004 m/day	σ_N = .0004 m/day	
DWZ* = 1.0 m	c_1 = .8	c_2 = .2
Crop = Clover	Yield = 200 LE/feddan	
i = 10%	Project Life = 50 years	

II. Model Results

	Loss Function		
	Type I	Type II	Type III
Drain Depth (m)	2.0	1.46	1.43
Drain Spacing (m)	34.17	21.95	20.57
Capital Cost	89.14	138.23	147.43
Expected Loss	8.93	60.09	57.21
Total Cost	98.07	198.32	204.64

NOTE: All costs in LE per feddan

There is little difference between case II and III because the range of possible probability distribution due to the constraints on D and L have little density in the area where case II approaches an asymptotic value.

For the rest of this work the assumption is to be made that for the Egyptian Delta conditions the appropriate loss function is the form of case II. This assumption is based upon the physical conditions of the Nile Delta which closely resembles those stated as a physical basis for case II. Experimental data from the Nile Delta (Ministry of Irrigation, 1965) show that for all crops of major importance to agriculture in the Nile Basin the yield function follows a case II form.

4.3.4 Sensitivity Analysis of Model Parameters

For the conditions present in the Embabe case study a detailed analysis was performed to study the change in the optimal drainage design as a function of model parameters. The analysis was performed assuming a case II type loss function for cotton with the optimal DWZ at 1.3 meters and the asymptotic value of the yield at 80% of the optimal yield.

The results are presented in two sections: physical parameters and economic parameters. The results are illustrated in two formats: tables which include the optimal design variables, the capital cost, the expected loss costs and the total costs, and graphs that plot the value of the analyzed parameters versus the total cost of the design.

Table 4.3 and Figure 4.21 illustrate the relationship of depth to the impermeable layer, D, to the optimal design. After a depth of 6 meters there is almost no effect of the depth upon the optimal design.

Table 4.3

Optimal Drain Design Sensitivity to Crop Yield Function

	Depth to Impervious Layer: Z(m)				
	3	5	7	10	15
Depth (m)	1.45	1.45	1.46	1.47	1.47
Spacing (m)	17.10	20.31	21.84	22.85	22.68
Capital Costs	177.34	149.34	138.90	132.76	133.77
Expected Loss	59.72	56.65	59.37	64.40	63.38
Total Cost	237.06	205.99	198.27	197.16	197.15

NOTE: All costs in LE per feddan

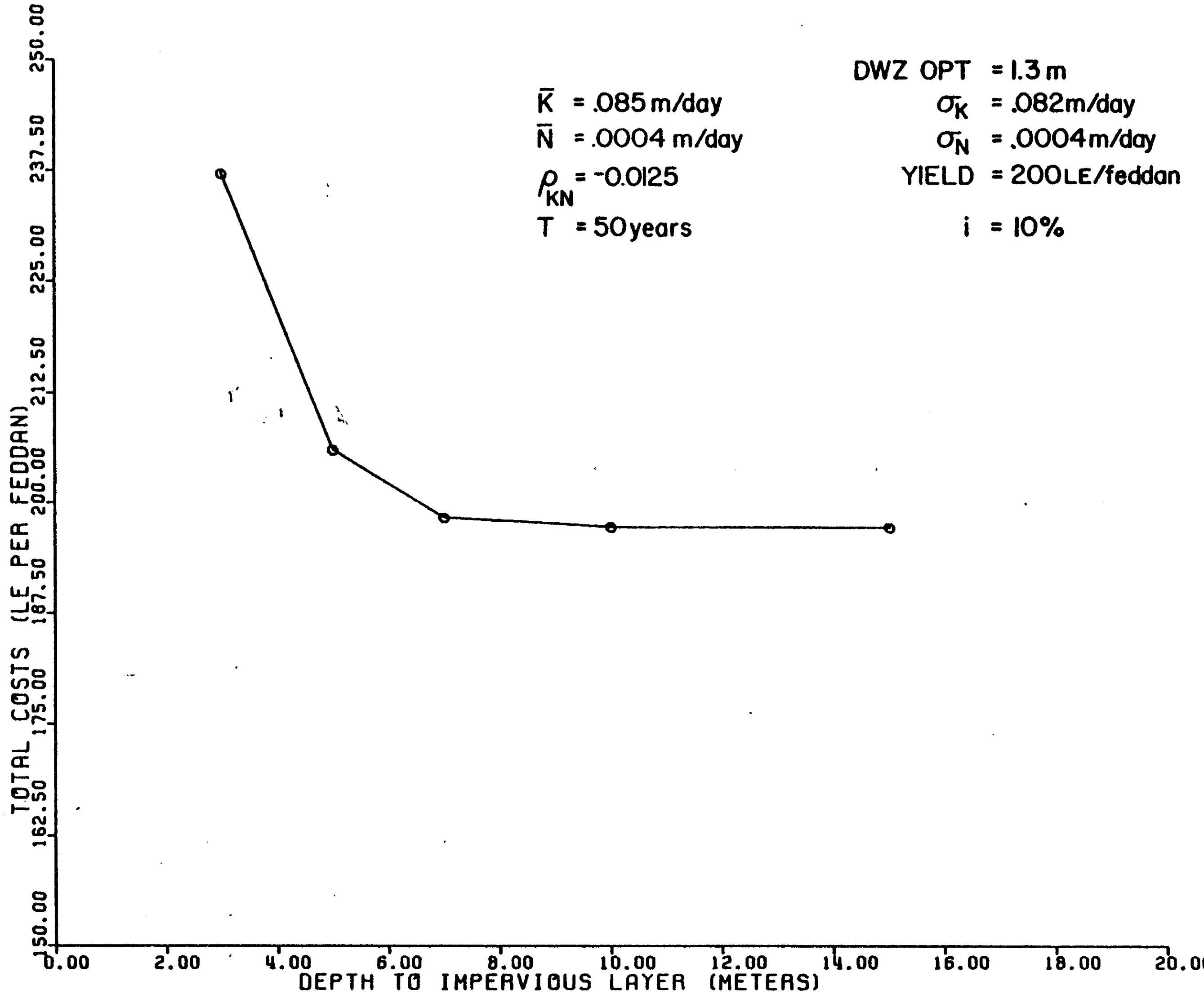


Figure 4.21 - Optimal Drain Design Sensitivity to Soil Depth

This is good because it is difficult to exactly locate the impermeable layer if it is deep. However, with all the sampling to determine permeability and groundwater elevation in the 0 to 5 meter range it will be possible to determine if the impermeable layer is in this zone.

The sensitivity of the model solution to the degree of uncertainty in the drainage coefficient N, is presented in Table 4.4 and Figure 4.22. It shows that over the range from coefficient to variation of zero to 1 there is approximately a 20% increase in total costs. Beyond 1 the cost increases greatly. In Figure 4.23 and Table 4.5 the sensitivity of the solution to the coefficient of variation of permeability, K, is present. They show a similar 20% increase of cost in the range from zero to 1 and a rapid increase beyond 1. Note that these increases are due in part to the approximations inherent in the FOSM approach.

These results show that in both cases the design depth does not change substantially, but the spacing decreases substantially. This is due to the fact that as the input variance increases the output variance increases and it increases at a greater rate for a large L as shown in Figure 4.20 and thus the model tries to reduce the output variance to increase the expected yield as described above.

The above results can be used by decision makers to aid in the design of sampling networks for K, N and Z as the tradeoff can now be made between the cost of sampling and resulting cost of system design.

The first economic parameter to be analyzed is the crop economic response which is a combination of the yield for the crop times the prices for the crop. Table 4.6 and Figure 4.24 show results for the sensitivity of the optimal design to different values for the economic yield.

Table 4.4

Optimal Drain Design Sensitivity to Coefficient of Variation of N

	Coefficient of Variation of N				
	0.0	0.1	0.5	1.0	2.0
Depth (m)	1.51	1.51	1.50	1.48	1.46
Spacing (m)	23.76	23.86	22.36	19.86	15.92
Capital Costs	127.74	127.17	135.74	152.78	190.55
Expected Loss	53.79	55.03	58.35	68.99	96.71
Total Cost	181.54	182.20	194.08	221.78	287.26

NOTE: All Costs in LE per feddan

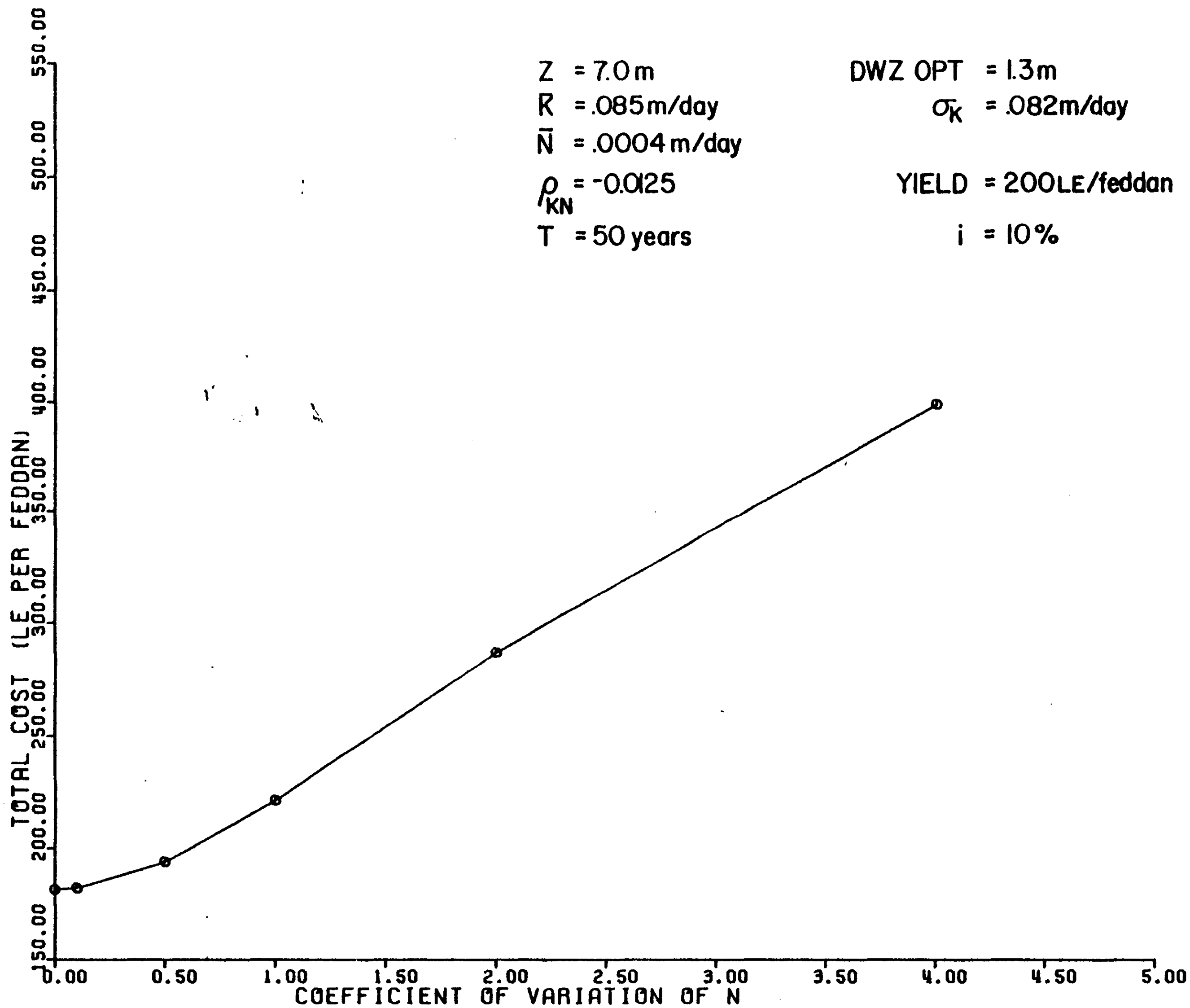


Figure 4.22 - Optimal Drain Design Sensitivity to Coefficient of Variation of N

Table 4.5

Optimal Drain Design Sensitivity to Coefficient of Variation of K

	Coefficient of Variation of K				
	0.0	0.1	0.5	1.0	2.0
Depth (m)	1.47	1.47	1.46	1.44	1.42
Spacing (m)	27.69	27.24	26.17	23.19	18.69
Capital Cost	109.55	111.36	115.91	130.77	162.28
Expected Loss	43.89	42.62	47.19	54.00	73.42
Total Cost	153.44	153.98	163.10	184.76	235.70

NOTE: All Cost in LE per feddan

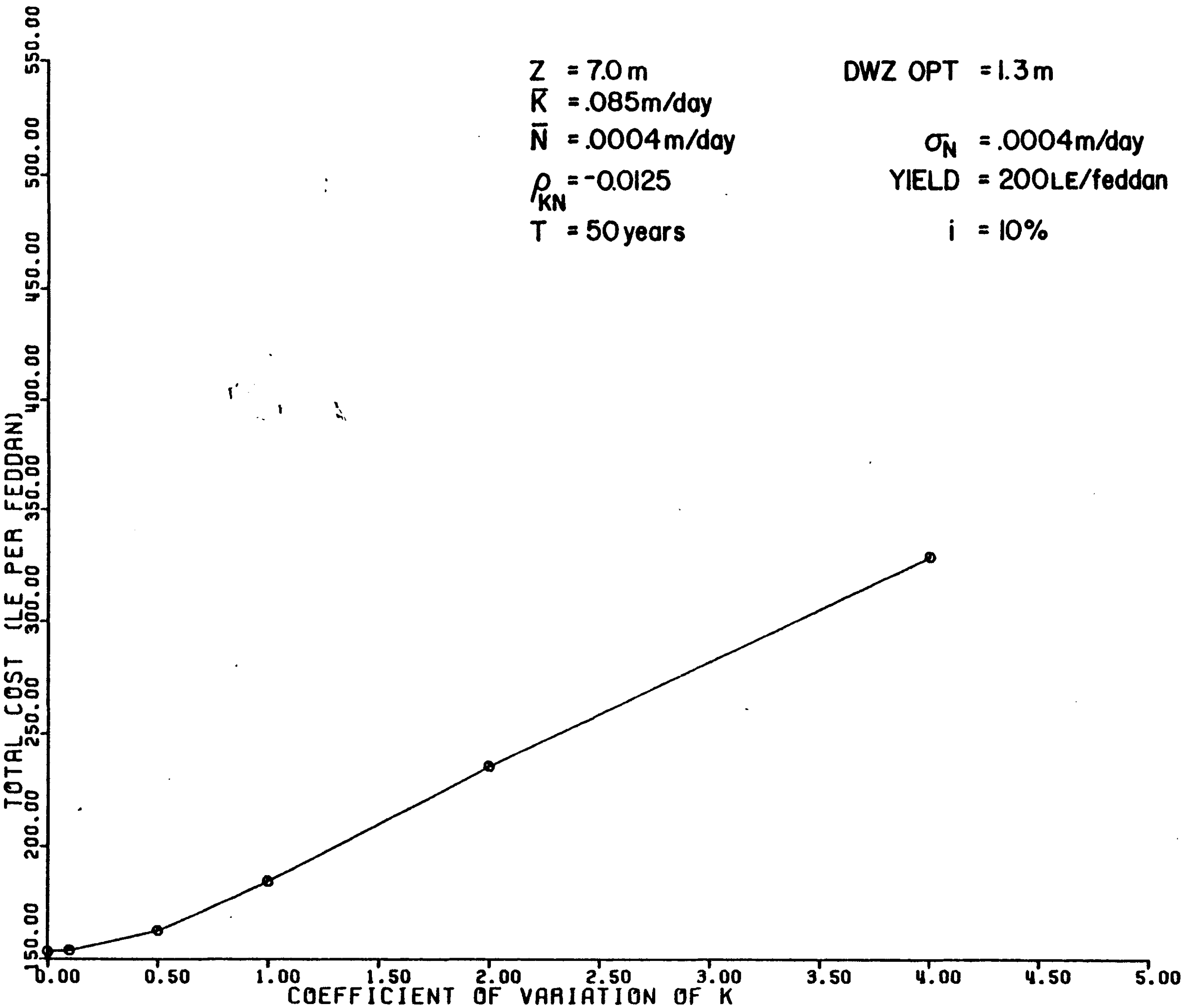


Figure 4.23 - Optimal Drain Design Sensitivity to Coefficient of Variation of K

Table 4.6

Optimal Drain Design Sensitivity to Crop Economic Yield

	Crop Economic Yield (LE per feddan)				
	10.0	50.0	100.0	350.0	500.0
Depth (m)	2.0	1.71	1.54	1.42	1.41
Spacing (m)	57.79	35.81	27.52	18.39	16.63
Capital Cost	52.70	84.88	110.29	164.86	182.30
Expected Loss	21.97	43.47	50.19	69.66	80.13
Total Cost	74.68	128.35	160.48	234.52	262.43

NOTE: All cost in LE per feddan

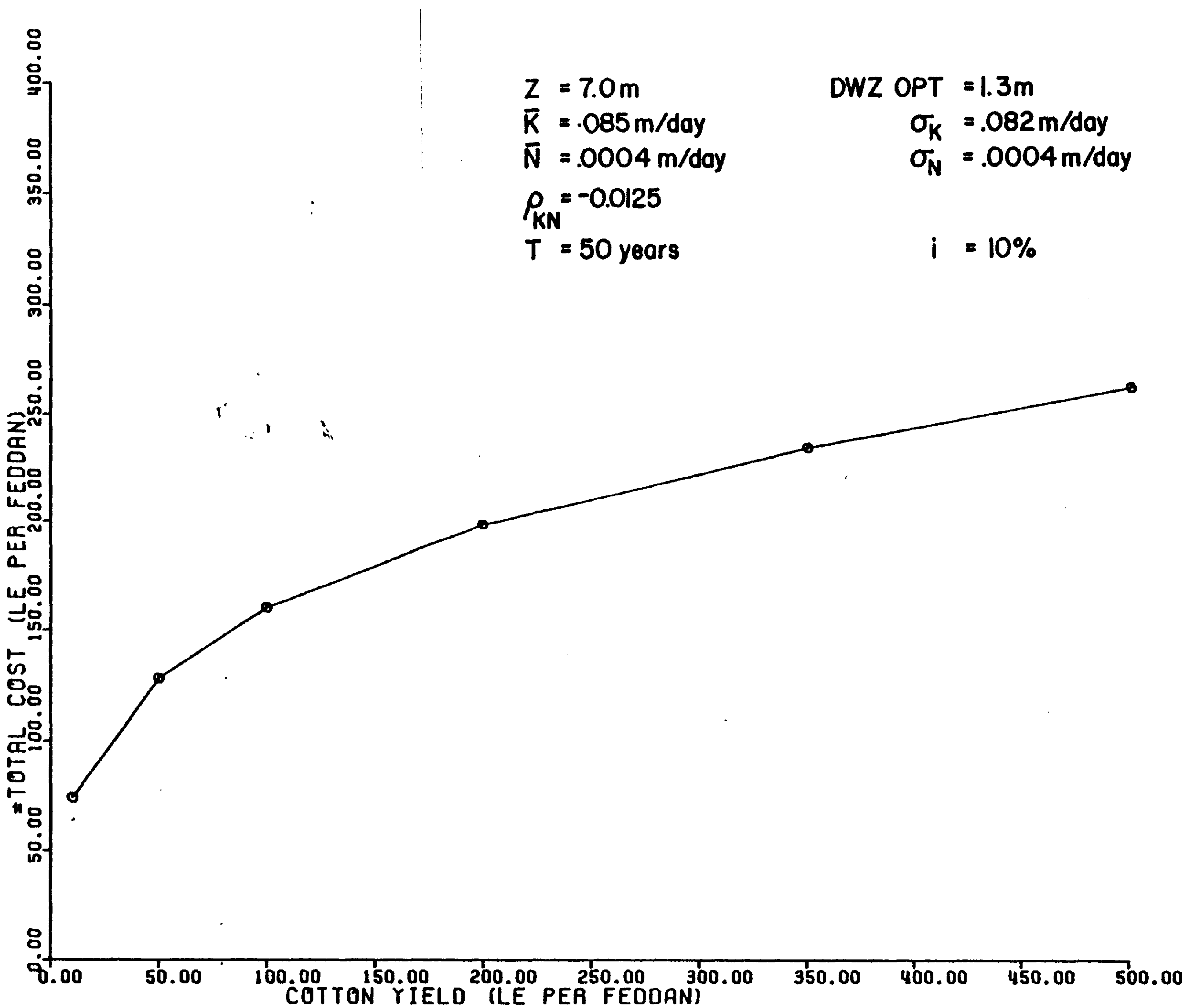


Figure 4.24 - Optimal Drain Design Sensitivity to Crop Economic Yield

The results show that costs increase as the economic yield increases. This is due to the fact that the design becomes more detailed as the possible expected loss becomes greater so that higher capital costs are paid to prevent even greater expected losses. This increased design is manifested in smaller spacing which reduces uncertainty.

Table 4.7 and Figure 4.25 illustrate the sensitivity of the optimal design to the interest rate. Low interest rates imply that the future is weighted similar to the present, while high interest rates imply the present being weighted much greater than the future. This is demonstrated well in Table 4.7. It shows that for low interest rates the design is very detailed with close spacing and large capital costs to minimize the expected loss which are weighted highly due to the low interest rate. While at the higher interest rate the capital costs are low with large spacing because the expected losses are small due to a small weighting factor. In the high interest rate case the absolute amount of crops lost over the life of the project may well be greater than the low interest rate case but the losses are weighted so little that they are not as important.

Finally, the variation of drainage design as a function of the life of the project is presented in Table 4.8 and Figure 4.26. It should be noted that the project life is not the failure time but the design life of the project. This analysis shows that after 20 years there is very little change in design due to project life. The reason for the small change is due to discounting procedures which determine the weighted value for expected loss. This value changes very little for longer time

Table 4.7

Optimal Drain Design Sensitivity to Interest Rate

	Interest Rate (%)				
	1.0	5.0	0.0	20.0	50.0
Depth (m)	1.39	1.42	1.46	1.53	1.84
Spacing (m)	14.45	18.08	21.84	27.32	40.37
Capital Cost	209.80	167.72	138.90	111.09	75.35
Expected Loss	91.69	70.40	59.37	49.81	43.63
Total Cost	301.49	238.12	198.27	160.90	118.98

NOTE: All costs in LE per feddan

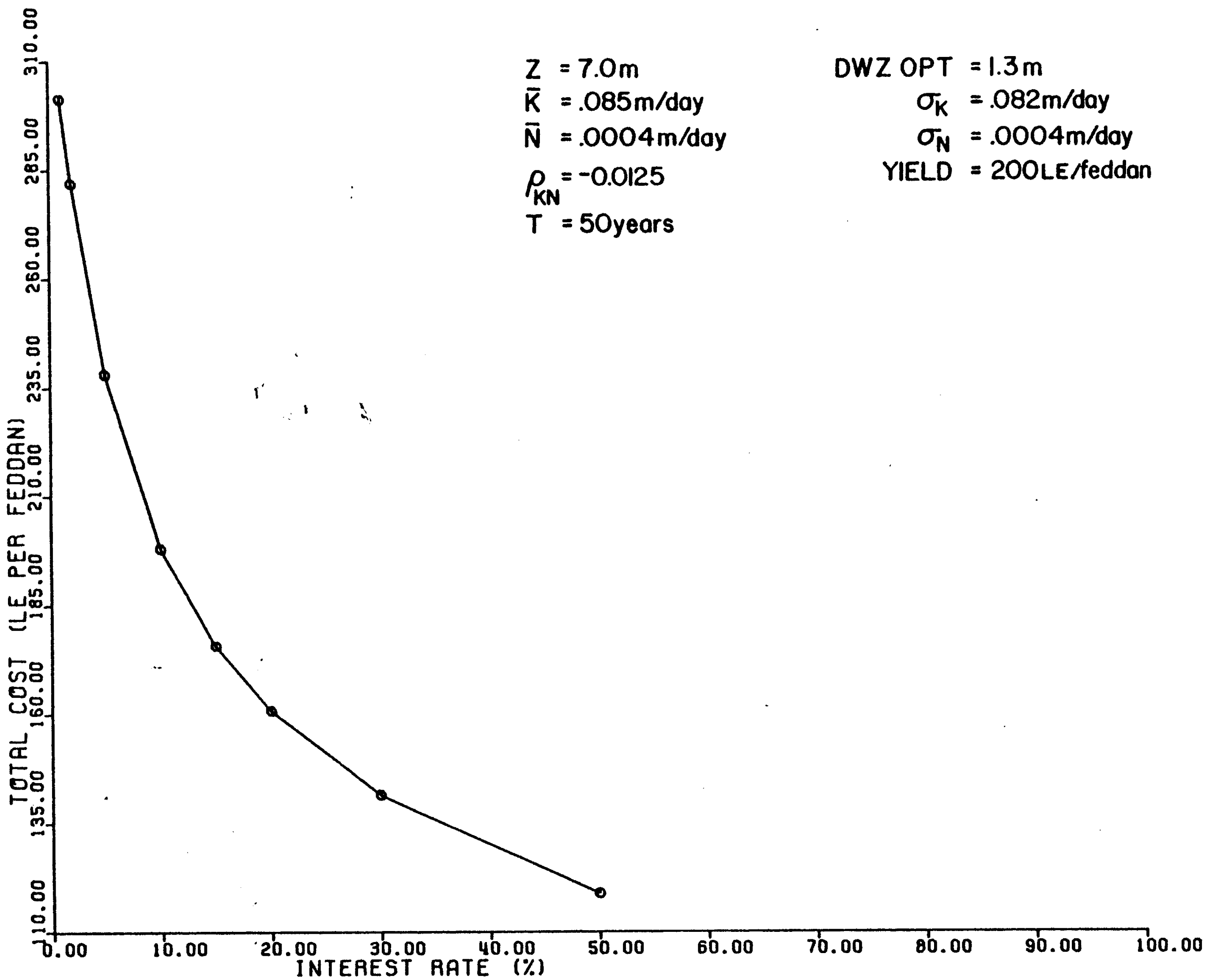


Figure 4.25 - Optimal Drain Design Sensitivity to Interest Rate

Table 4.8

Optimal Drain Design Sensitivity to Life of Project

	Life of Project				
	10.0	20.0	30.0	50.0	100.0
Depth (m)	1.50	1.47	1.46	1.46	1.45
Spacing (m)	25.33	22.85	22.23	21.84	21.77
Capital Cost	119.79	132.77	136.49	138.90	139.31
Expected Loss	51.69	56.62	58.79	59.37	59.47
Total Cost	171.48	189.38	195.28	198.27	198.78

NOTE: All costs in LE per feddan

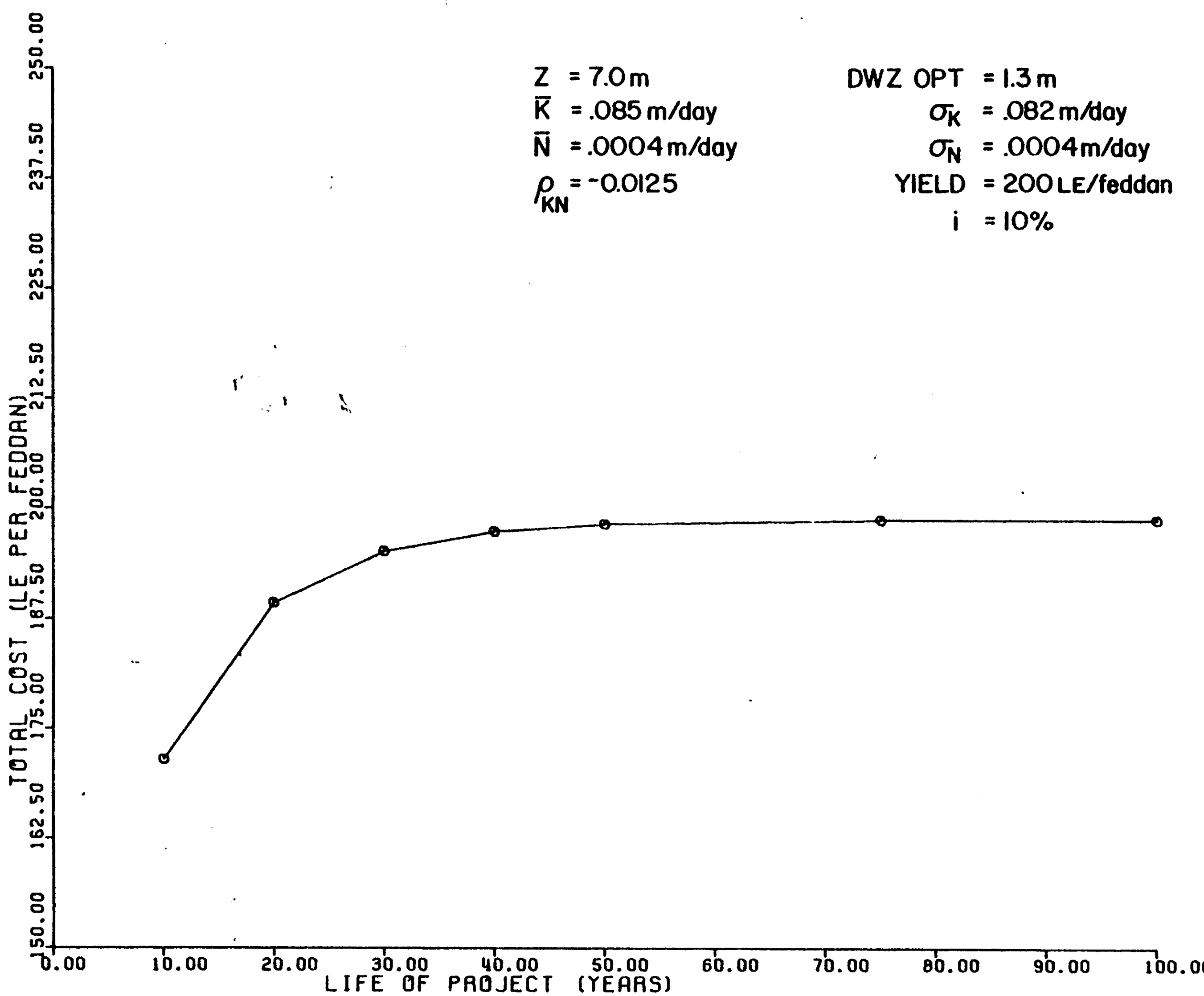


Figure 4.26 -- Optimal Drain Design Sensitivity to Life of Project

periods.

These results show the sensitivity of the optimal drainage design to all the model parameters. It shows that certain parameters are more important than others and attention must be made to define these parameters as accurately as possible. It has also been shown that more attention must be paid to the economic parameters and more research should be continued in this area.

The stochastic programming drainage design model presented above has been developed for a case with a single crop being grown over the entire design area each year. This condition may exist in some areas so the model would be applicable in these cases. It was developed in this manner for these cases as well as to be as simple as possible to allow for an understanding of processes underlying the model's operation.

However is many areas including the Egyptian Delta, there is more than one crop grown, more than one growing season each year, and crop rotation which mean yearly cycles of crops. In the next section the model is expanded to analyze this case, assuming that the same D and L is used for all crops.

4.3.5 Multiple Crop Stochastic Programming Design Model

The model will no longer have an expected value for a single crop with a single loss function but rather an expected loss for each crop with its own loss function. Each crop will then be weighted by the average area cultivated by the crop and its economic yield.

Table 4.9 is an example of the typical crop grown in the Nile Delta and the average area per feddan that is planted in that crop. The

Table 4.9

Multiple Crop Yield in Nile Delta

	Crops				
	Cotton	Maize	Wheat	Vegetables	Berseem
Area (feddan)	.25	.58	.25	.17	.62
Yield (m. ton/fed.)	.35	2.14	1.72	8.40	24.66
Price (LE/m. ton)	466.67	51.2	50.0	60.0	4.44
Total (LE/fed.)	40.83	63.55	21.5	85.68	6.78
DWZ* (m)	1.3	1.15	1.1	1.0	1.0

other row lists the economic yield.

The objective function then becomes

$$\text{Capital Cost (D.L)} + \sum_{i=1}^{nc} A_i\ EY_i[1-E(Y_i)]\ PWF_i^t \qquad (4.30)$$

where

A_i = area planted per feddan of crop i

EY_i = annual economic response for crop i

$E[Y_i]$ = expected yield crop i

PWF_i^t = present worth factor

nc = number of crops

This requires that the model find the expected yield as a function of D and L for each crop as described in Section 4.3.2, with a different loss function for each crop. These expected yields are weighted by their respective economic yields and area and the summation over the total number of crops is the new expected loss function. The last row of Table 4.9 is the weighting factor of the area per feddan times the economic yield for the crops in the Embabe case study.

Figure 4.27 is a plot of the loss as a function of D and L for the multi-crop model using the data from the Embabe case study. Figure 4.28 is a plot of the total cost for the multicrop model for the Embabe case study. It can be seen that Figure 4.28 is quasi-convex, so as global minimum can be found using the existing solution technique. The optimal solution for this case is depth D = 1.38 m, spacing L = 21.96 m, capital cost 137.9 LE/feddan, expected loss = 86.6 LE/feddan and total cost = 224.51 LE/feddan.

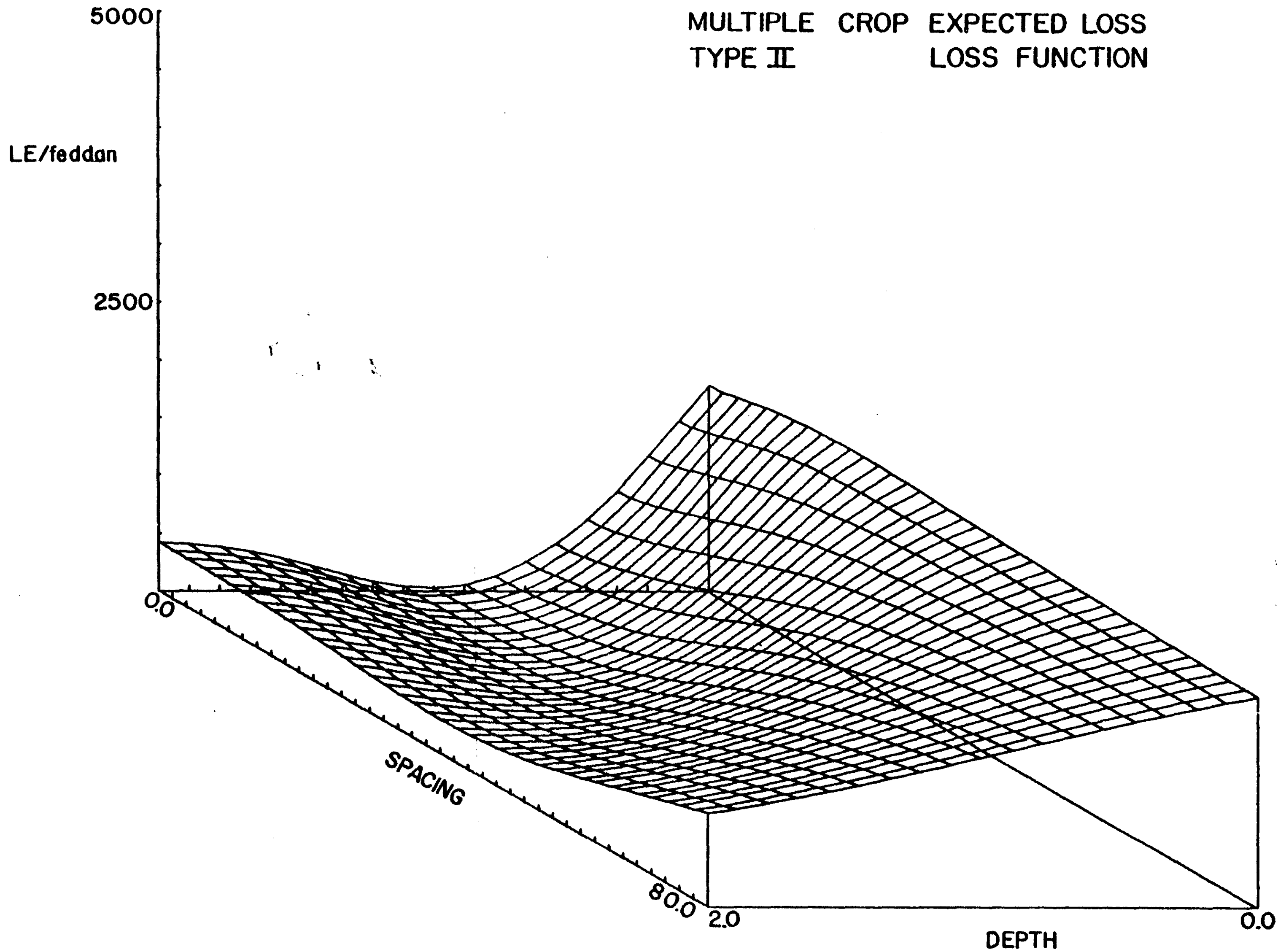


Figure 4.27 - Multiple Crop Expected Loss: Type II Loss Function

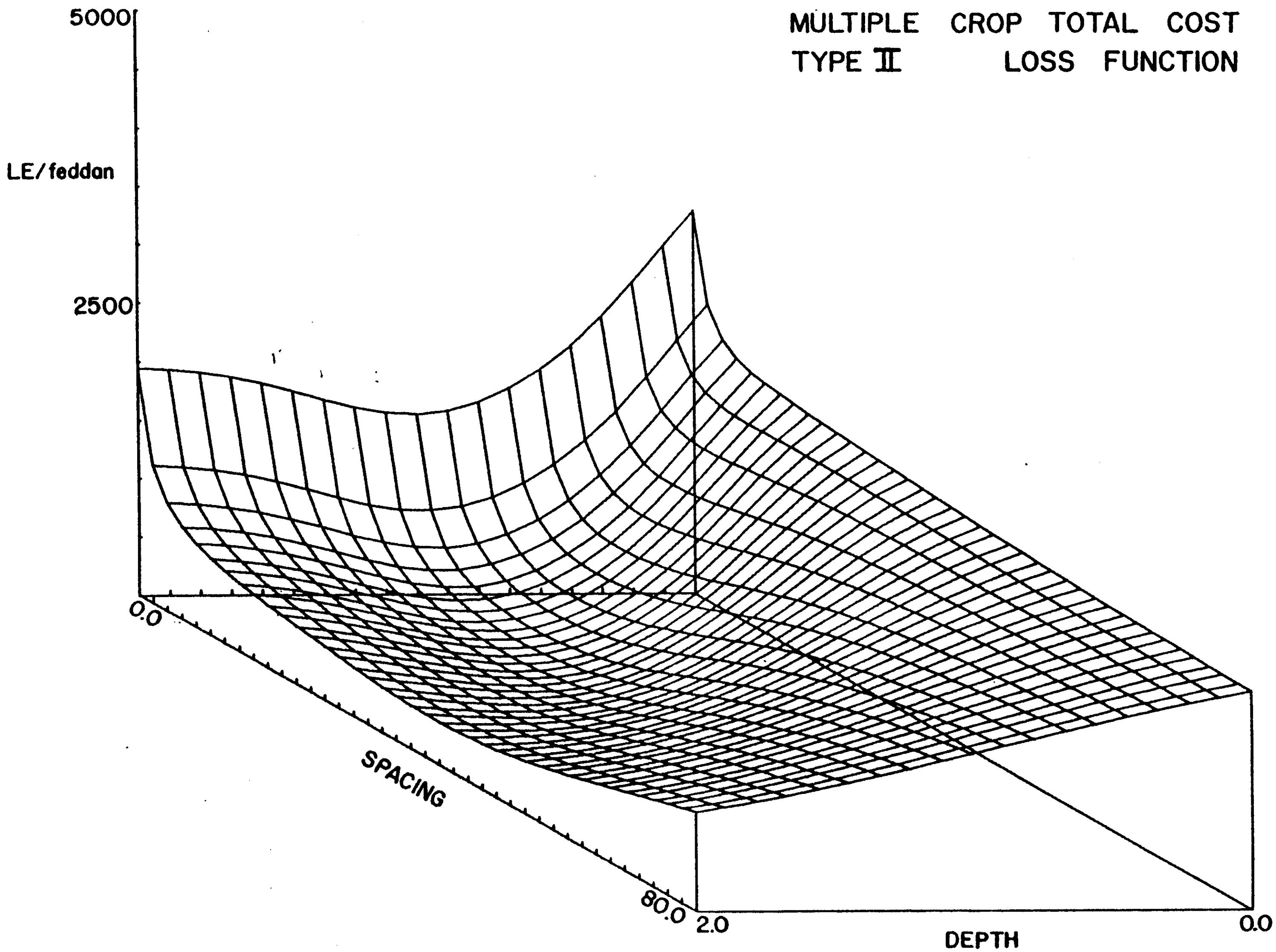


Figure 4.28 - Multiple Crop Total Cost: Type II Loss Function

4.4 Model Comparison

Thus far in the work chance-constraint and stochastic programming have been presented as alternative methods to include uncertainty in optimal drain design. This section will examine the properties of the two approaches.

The chance-constraint approach to uncertainty is a reliability approach. It requires that a system output target be met with a certain reliability. The target value for the system output is usually an optimal value for system performance. In drainage design the target value is the optimal dewatering zone for crop production. As the problem has been presented, the greater the reliability the better the system performance. This approach assumes that if the system output surpasses the target values the system performance will be as good if not better than below the target value. In other words, the system benefit function is a monotonically non-decreasing function. Figure 4.29 illustrates this argument. In case (b) the target value DWZ* is met with a reliability of 95% and the expected system benefits are greater than case (a) in which the target value is met with 80% reliability. This approach to chance-constraint programming is fine as long as the benefit function is monotonically non-decreasing. If the benefit function is not monotonically non-decreasing or the sign of the slope of the function changes over the range of possible output, the approach presented above is invalid. This will result in greater reliability of the output target producing poorer system performance than lesser reliabilities. Figure 4.30 illustrates this point. In case (a) a reliability of 50% on the

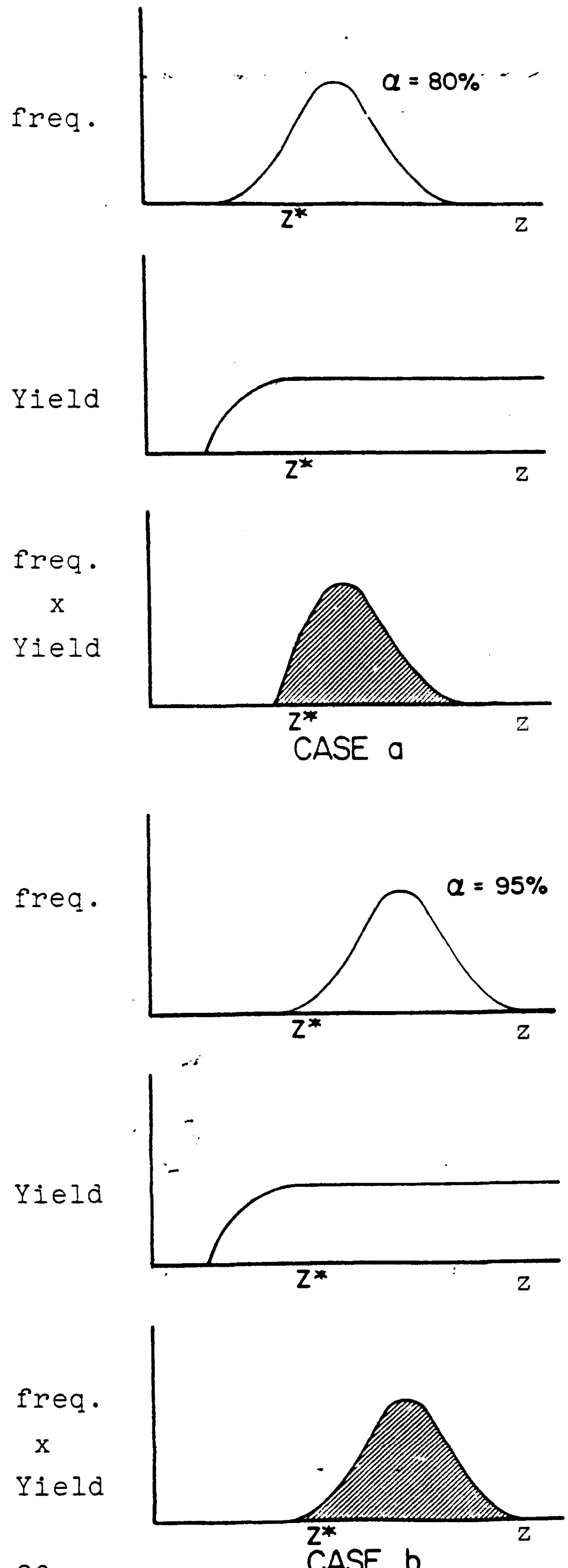


Figure 4.29 -
Reliability versus Expected Benefits: Type I Yield Function

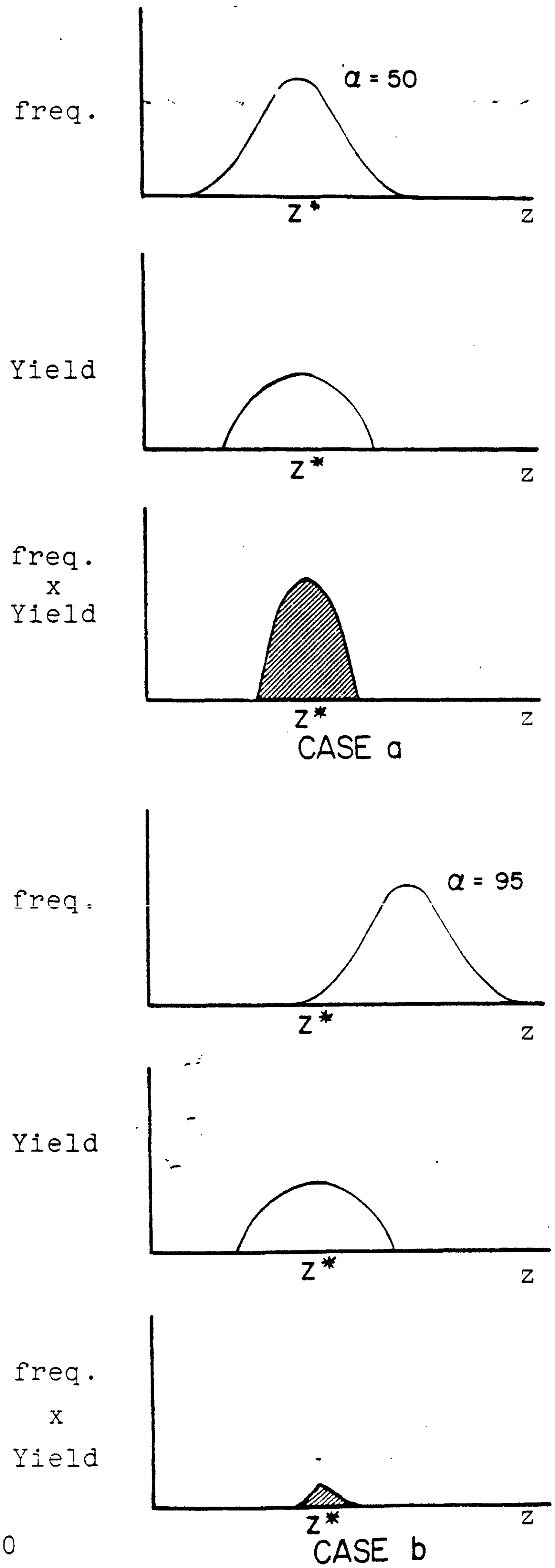


Figure 4.30
Reliability versus Expected Benefits: Type III Yield Function

target value provides substantially more expected benefits than a 95% reliability on the target value.

The implication of the above arguments for drainage design is important. In Figure 4.7 general forms of typical crop yields functions were presented. Seven functions were presented and only one was a monotonically non-decreasing function. The six others all have slopes that change sign, making the present chance-constraint approach invalid. If a monotonically non-decreasing yield function is assumed in a chance-constraint analysis and the actual yield function is not of the form, there will be a "regret". To quantify the magnitude of this regret for drainage design in the Nile Delta an experiment was performed. In Figure 4.10 three possible crop yield functions were illustrated. Type I is a monotonic non-decreasing function while type II and type III are not. An analysis was done to quantify the "regret" that would result if a drainage system was designed assuming a type I crop yield when in fact the function was actually type II or type III. The measure of regret was the difference in expected losses as described in Section 4.3.2 above. The system was designed for a 98.5% reliability of a dewatering zone of 1.0 meters for clover. Table 4.10 is a summary of the results which shows that the regret can be quite substantial. This points out the necessity to carefully define the crop yield function before proceeding with a chance-constraint approach.

The question that arises then is "what is the design reliability for a crop yield function" that has a slope that changes sign. To answer this question another experiment was performed. For each type of crop yield

Table 4.10

Economic Regret Due to Incorrect Yield Function

I. Drain Design Based Upon: Type I Yield Function

DWZ* = 1.0 m

Reliability = 98%

Depth = 2.0 m

Spacing = 34.17 m

Capital Cost = 89.1 LE per feddan

II. Economic Regret Due to Actual Yield Function Being:

Type II	218.8 LE per feddan
Type III	578.5 LE per feddan

function a system design was found using the stochastic programming model. Then the corresponding reliability of the dewatering zone on the target dewatering zone was found. Table 4.11 presents a summary of results. For type I the result is as expected, 98.5% reliability. For type III the reliability is 50%, this can be expected since the crop yield function is symmetric around DWZ*, the model will concentrate on the most dense portion of the distribution, the mean, at the optimal yield, but nothing is said about σ_{DWZ} which was shown to be important. However, for type II the reliability is 54%, and no a priori reliability could be expected or σ_{DWZ} proposed.

These results make a strong argument for the use of stochastic programming. Chance-constraint programming has been used when little or no information about the benefit function is known. This analysis has shown that this convention can lead to large losses due to the regret of assuming the wrong yield function because the chance-constraint approach does assume a form to the benefit function.

An alternative approach, but still using chance-constraints is to require the system output to be greater than a lower limit and less than an upper limit with a certain reliability. However this approach has two problems. First, to decide upon the appropriate upper and lower bounds requires almost as much information as needed to define the entire benefit function. Second, due to the irreducible uncertainty in input parameters it may be infeasible to design a system in which the output distribution can meet the desired reliability for the design interval.

Table 4.11

Stochastic Programming/Chance Constraint Results Comparison

	Crop Yield Function		
	Type I	Type II	Type III
Depth (m)	2.0	1.46	1.43
Spacing (m)	34.17	21.96	20.58
Capital Cost	89.14	138.15	147.34
Expected Loss	8.93	60.14	57.28
Total Cost	98.07	198.29	204.62
DWZ* Reliability (%)(1)	98.5	54.0	50.0

DWZ* = 1.0 for all types

1). The probability distribution of DWZ from the Stochastic Programming Solution was used to find the reliability of DWZ resulting from the optimal design.

NOTE: All costs in LE per feddan

This analysis has shown that the chance-constraint approach has problems that under certain conditions cannot be solved. The stochastic programming approach is not plagued by these problems but requires more information and additional computation. The stochastic programming approach also provides for an explicit tradeoff between economic benefits versus economic cost of drain design and allows for analysis of multiple crop areas.

The analysis has shown that chance-constraint programming is not as robust as previously perceived. For the reasons presented above the stochastic programming approach will be used as the methodology for lateral drain design in the remainder of this work.

CHAPTER 5

MULTILEVEL DRAINAGE PLANNING UNDER UNCERTAINTY

5.1 Introduction

In Chapter 2, agricultural drainage planning was described as a three level process. The level-three lateral drain design was discussed in Chapter 4, and a multi-crop stochastic programming model for optimal lateral drain design under uncertainty was presented. Level-two or drainage collector network design is addressed in the beginning of this chapter; a methodology for efficient collector network design is presented. The interactions between level-two design and level-three design are demonstrated. A dynamic planning model that synthesizes level-two planning with the stochastic programming model for level-three design is developed. This model takes into account spatial variability and uncertainty in soil parameters. Finally, a case study of the use of this model applied to a field in the Nile Delta is presented.

5.2 Drainage Collector Systems

This section discusses the planning of drainage collector systems. A review of the present practices in drainage collector planning is presented. A simulation model, to aid in more economically efficient planning is described. It is based upon present drainage practices of the International Institute of Land Reclamation and Improvement and the Egyptian Public Authority for Drainage. The model's behavior is demonstrated by a number of experimental runs.

As discussed in Chapter 2, there are many types of subsurface drainage systems. The system to be analyzed in detail in this work is the composite pipe drainage system. In the composite pipe system collector drains are subsurface pipes which collect the water drained by the lateral field drains and then empty this water to the main or measured drainage canal. The composite pipe drainage system is chosen as an example because it is the system employed in Egypt. However, the technique to be presented can be modified to represent a composite open drainage system by replacing collector pipes by open channels.

The two important design features of any collection system are network layout and pipe size. The other decisions concerning the pipe system are lateral connections or tee-sections, manholes and outlets to the main drains. The combination of all these components with the lateral field drains produce a system which provides for the removal of the excess water due to irrigation or precipitation. A poorly designed collector system may mean inadequate removal of excess water, and results in crop losses, with the lateral drains not performing as designed. The overdesign of a collector system results in the use of unnecessary resources and excess capital costs for the construction of the system. Therefore it is necessary to address the problem of collector system planning with the same emphasis as lateral field drains.

The design of drainage collector systems can be broken into two phases. The first phase is the planning of collector networks. This planning process entails the layout of the collector network, sizing of collector pipes, and location of manholes, tee-sections and outlets to the drainage canals. The second phase is the detailed design of each collector. This phase includes the design of outlet structures, modification of collector slope due to local topographic features, design of structures for passage of collector systems under or over irrigation canals, plus other site specific design features. The focus of this work is on the planning phase, because it has the greatest effect on overall performance and can be adapted to fit into a systems methodology. Many of the tasks of the detailed design phase do not lend themselves to any modelling process.

5.2.1 Present Methodology for Collector Network Planning

At the present time the process of collector layout planning by major drainage installing agencies has not been formally addressed. The procedures involved in collector layout planning are such that no standard methodology has been developed except for general guidelines set up by the major agencies for their engineers to follow. This leaves much of the planning of collector systems to the judgement of the engineer in charge. For very experienced engineers this methodology is satisfactory but for an engineer with little experience the possibility of poor judgement is significant. The present guidelines for collector planning that are available in the literature are presented below.

The Soil Conservation Service of the U.S. Department of Agriculture (1974) lists the following seven guidelines for collector layout:

1. Provide the minimum number of outlets.
2. When practical, layout the system with a short main and long laterals.

3. Orient the laterals to use the available field slope to the best advantage.

4. Follow the general direction of natural waterways with mains and sub-mains.

5. Avoid locations that result in an excessive cut.

6. Avoid crossing waterways wherever feasible. If waterways must be crossed, use as near a right-angle crossing as the situation will permit.

7. Wherever feasible, avoid soil conditions that increase installation and maintenance costs.

The Bureau of Reclamation, U.S. Department of the Interior (1978) states:

> ....There are no final rules or methods to direct the drainage engineer in locating any drain. Each location presents an individual problem which can be solved by analyzing the conditions involved. Wherever possible outlets, sub-outlets and collector drains should be located in natural drainage ways. Relief and interceptor drains should be located where they will produce the best drainage results. The location and spacing of drains require careful study and intuitive judgement on the part of the drainage engineers....

In the series Drainage Principles and Applications produced by the International Institute for Land Reclamation and Improvement, Cavelaars (1974) writes,

> ....If there is a distinct natural direction of groundwater flow, the laterals can best be laid perpendicular to the main flow as they will thus be able to intercept the flow most effectively....
>
> ....In flat or nearby areas the lateral drains are preferably installed in the direction of the main slope, if any, as this will mean an approximately equal drain depth in the entire field....

....As a general rule it is advisable to situate drains in the lowest parts of the area whenever this is feasible. This will ensure the most effective drainage at the most economical installation depth. In large drainage systems the main drains should follow the natural valleys. Quite often one is not entirely free to adopt the drain alignments to the best drainage function because of such factors as property boundaries, existing network of irrigation and drainage channels, etc.....

It is clear from these quotations that no standard method exists for planning collector network layouts, equivalent to, for example, the Hooghoudt equation for lateral drain design. However, for the sizing of the collector pipes standard methods do exist based upon the theory of flow in pipes such as Manning's equation. The parameters of interest in the pipe sizing equation are slope, material of pipe (roughness factor) and discharge or drainage water. An example of the pipe sizing equation for smooth pipes, is the Wesseling equation

$$Q = 30a^{-0.57}\ d^{2.71}\ i^{0.57} \qquad (5.1)$$

where

Q = discharge through pipe (m^3/sec)

a = roughness factor

d = diameter of pipe (m)

i = slope of collector

Other equations exist, and the choice depends upon the agency performing the analysis.

5.2.2 Drainage Collector Newtork Simulation Model

The process of planning drainage collection systems can be cast into a formal mathematical programming problem. The problem is that due to the complexities discussed above the MPP is impossible to solve using any linear or nonlinear programming package. However, it is possible to build a computer program that will simulate the procedures involved in the planning process. The simulation model does not provide an optimal solution, but rather displays the hydraulic and economic response of a given system input by the engineer to the model. The reader is referred to Ackoff (1961) for a more detailed discussion of simulation.

A simulation model is proposed that allows the engineer to make the same jusgements made with present techniques. The simulation model provides the speed of a digital computer to aid in those planning procedures that are defined mathematically or follow specific rules. For example, the pipe sizing portion of the process as well as the placement of manholes every 200 meters apart, can easily be programmed into the computer. However, the alignment of the collector network must be specified by the engineer using his judgement as well as other important parameters, because the model only simulates a system that is given and does not generate systems.

The output of the model is a complete account of the length of each pipe size, the number of manholes, the number of tee-sections, and the number of outlets needed to properly drain the area for this given system alignment. Knowing the economic costs for each of the system components that the model keeps account of, a total cost for this collector system can be obtained. With the speed that a simulation model provides, the drainage engineer is then able to analyze the cost of a number of possible alternative collector systems. Then the engineer selects the system from the various proposed alternatives which provides proper drainage at the least cost. Although this technique may not select the "global optimal solution" due to the consideration of only a few alternative systems. The use of such a model allows experienced engineers to evaluate more alternatives and provides a tool for inexperienced engineers faced with a lack of experience. While at the same time, the model will standardize many of the procedures of a drainage design office. For this work a drainage collector system simulation model was developed based upon the procedures for collector system planning utilized by the Egyptian Public Authority for Drainage Projects, as outlined by Cavelaars (1974) and Amer (1979). This model has the added advantage of being connected with a computer graphics input/output device that allows a proposed system to be input by drawing it on a graphical input device. This substantially increases the productivity of the drainage engineer. This work will focus on the use of this model in an overall methodology. A detailed discussion on the model itself is found in Alexandridis, Strzepek and Marks (1979).

5.2.3 Collector Planning Case Study

The following section is an example of the results of the model. For a relatively flat field similar to that found in the Nile Delta, a series of alternative network alignments were simulated using the model.

The example uses a rectangular field 800 meters wide by 1600 meters long, with a main open drainage canal passing along the lower boundary parallel to the X-axis (Figure 5.1). It is assumed that the entire field has lateral drains with 1.5 meter depth and 50 meter spacing. It is assumed that the average drainage rate is 3 mm/day, that the depth of the water surface in the drainage canal is 2.5 meters below ground level, and that the slope of the collector will be 3%. The collector pipes available for design will be concrete pipes which range in size from 10 cm to 50 cm in 5 cm increments. With this data and a proposed alignment the model can be run.

Figure 5.2 shows the four alternative network layouts that will be simulated. Table 5.1 presents the results of the simulation runs and presents a summary comparing the aggregated costs of each system. It can be seen from Table 5.1 that System A is the least cost system, although all the systems provide adequate drainage. System A is the least cost alternative as a result of the nonlinear costs structure of drainage pipes. This feature favors a number of short collectors of small pipe size to a few collectors with large pipe size. In System A the largest size pipe is 200 mm while in both System B and C there is a substantial length of 350 mm pipe.

Since this test case assumes a relatively flat topography the orientation of the collector to ground slope is not important. However, if topography had been a factor and drainage system A had become infeasible, and the remaining systems were feasible, System C would be the "best" alternative. This analysis was performed to demonstrate the

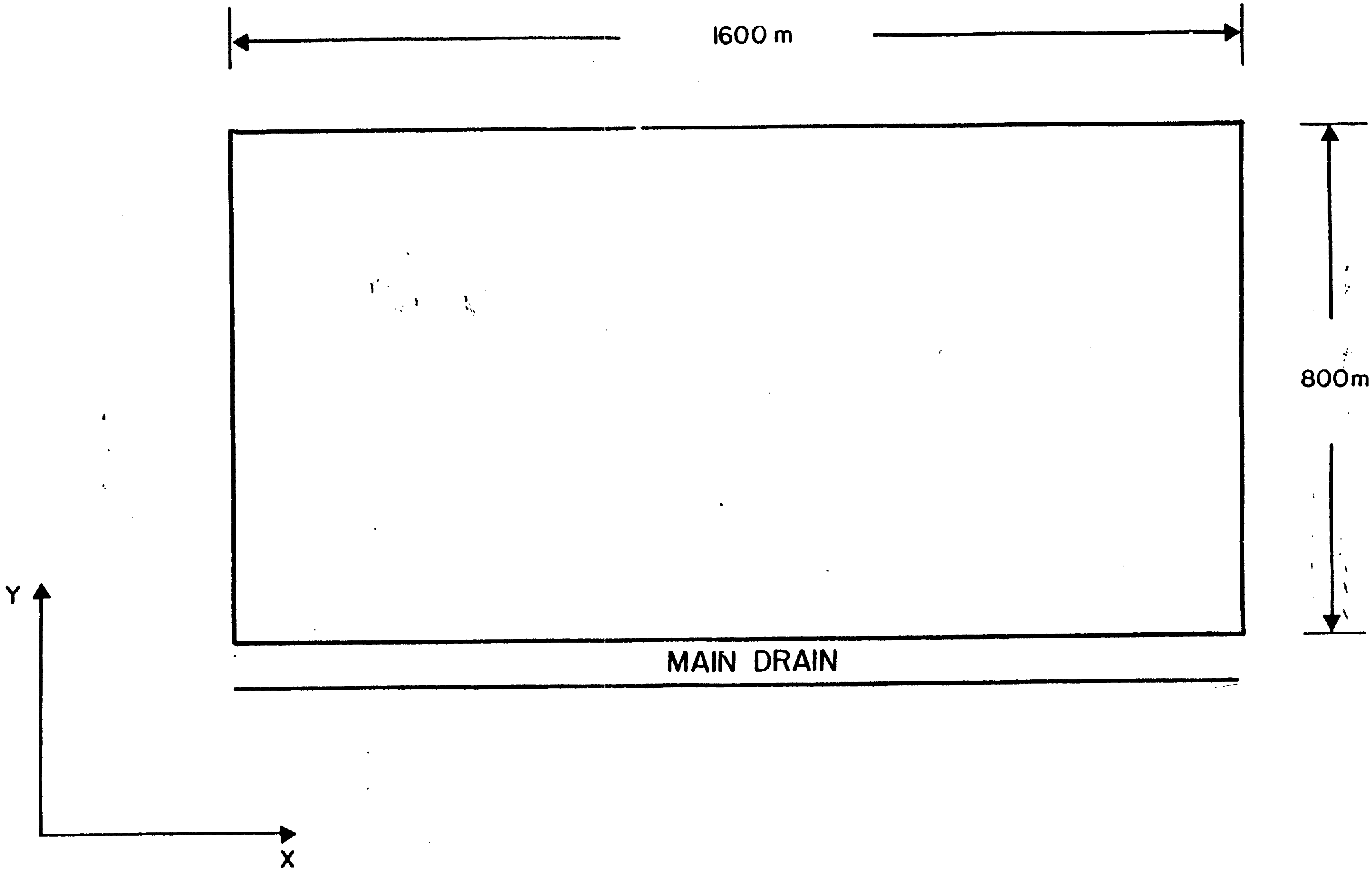


Figure 5.1 - Hypothetical Drainage Region

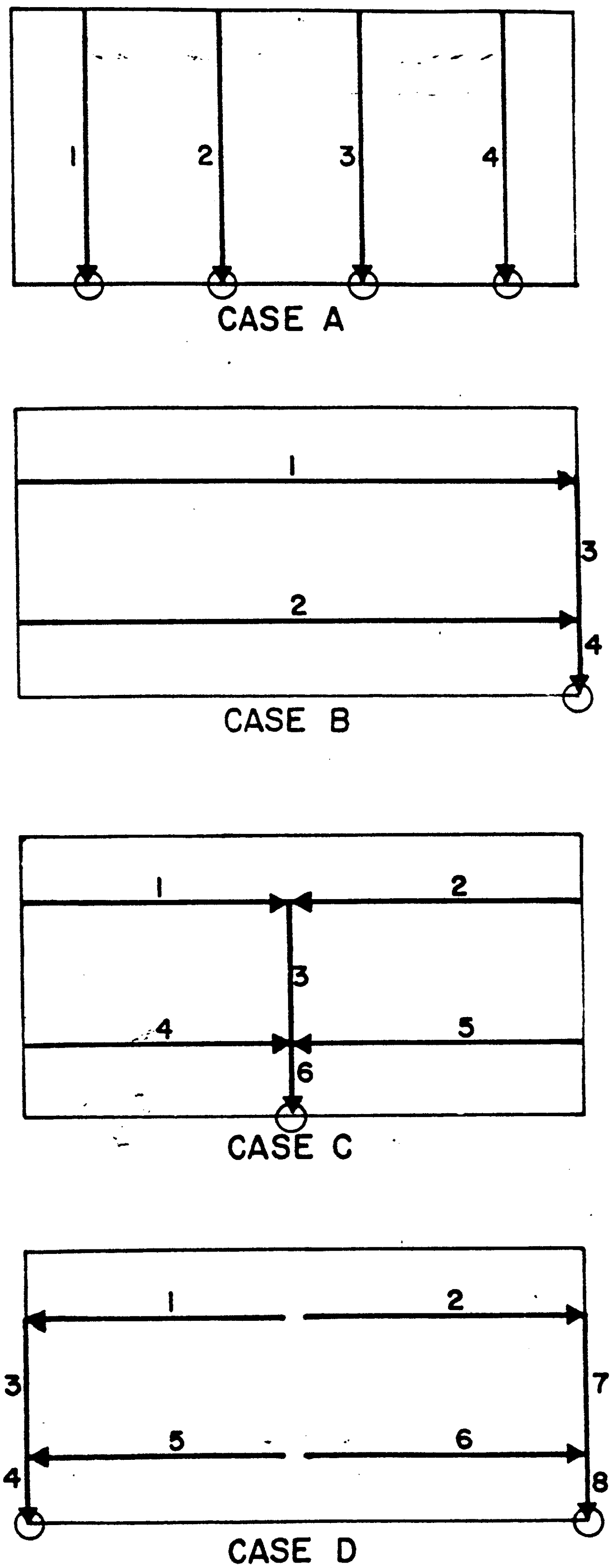


Figure 5.2 - Alternative Collector Network Layout

Table 5.1

Collector Network Simulation Model Case Study Results

Case A

Pipe Size (mm)	150.	200.
Length (m)	1200.	2000.
Cost (LE)	1320.	3386.

4 Outlets = 200 LE

Total Cost = 4706 .LE

Case B

Pipe Size (mm)	150.	200.	250.	300.	350.
Length (m)	600.	900.	1500.	600.	200.
Cost (LE)	660.	1524.	3430.	1696.	716.

1 Outlet = 50 LE

Total Cost = 8026 . LE

Case C

Pipe Size (mm)	150.	200.	250.	300.	350.
Length (m)	1200.	200.0	0.0	400.	200.
Cost (LE)	1320.	3387.	0.0	1131.	716.

1 Outlet = 50 LE

Total Cost = 6554 . LE

Case D

Pipe Size (mm)	150	200	250	300
Length (m)	1200.	2800.	0.0	400.
Cost (LE)	1320.	4741.	0.0	1131.

2 Outlets = 100 LE

Total Cost = 7182 . LE

potential of this tool to aid in selection of efficient collector systems in more complex situations.

The results presented in this section illustrate the usefulness of a simulation model for planning of drainage collector systems. The next section addresses the issue of the use of this tool in multi-level drainage planning under uncertainty.

5.3 Effect of Collector Network Design Upon Lateral Drain Design

At present the multi-level planning process proceeds sequentially from first to second to third level with no feedback between the levels. The level one decisions are made which effect level two decisions, and level one and two decisions are made with effect level three but there is no analysis of the impacts of the upper level decisions on the lower level process. It is a static planning process. This section looks at the implications of this static planning process on the efficient design of drainage systems and a methodology for a dynamic planning process using the tools developed in Chapter 4 and Section 5.2 is presented.

Chapters 3 and 4 demonstrate the importance of accounting for uncertainty and spatial variability when designing lateral field drains. Techniques for dealing with this uncertainty were evaluated and stochastic programming was selected. The remainder of this work will be based upon the stochastic programming approach for the third level.

The stochastic programming model requires statistical information about uncertain parameters as one of its inputs. The important question for lateral drain design using this model is over what area, size and location are the uncertain input parameters to be assumed constant but uncertain, with the uncertainty being presented by a probability distribution. In Chapter 3 it was illustrated that the spatial variability of the permeability of the soil is a random process and as such permeability will vary from point to point although it may possess some underlying trend. With this fact the lateral field drains ideally would be designed such that the design would differ from each set of lateral drains. However, this is quite impractical and would add unnecessary costs. Therefore a design area over which lateral spacing and depth would be kept constant must be selected. The area must be small enough to reflect local conditions but large enough to be practical. For this area the uncertain input parameters to the stochastic programming lateral drain model would be defined by statistics generated from the data samples taken from the field.

For practical reasons the minimum area over which a constant lateral drain design is feasible is that area drained by a single collector. The reason a single collector area is selected for lateral design is that each lateral must be connected to the collector. Lateral connections that are randomly placed on either side of the collector add to the number of pipe fittings and labor required. Costs would increase and the installation rate would slow down.

It then becomes clear that the alignment of the collector network will affect the lateral drain design. The present collector system planning, as discussed earlier in this chapter, does not consider the effect upon lateral spacing at all. With the added feature of examining the spatial variability and uncertainty, this present static planning process leaves much to be desired.

5.3.1 Sequential Planning Process

An example of the present sequential planning process is found in the series Drainage Principles and Application produced by the International Institute for Land Reclamation and Improvement, and Van Aart (1974). They state that;

> "... It is recalled here that drain spacing (L) varies with the square root of the hydraulic conductivity. Hence, although large differences in K values may be found, the differences in drain spacings will be much less. It is therefore better (to transform the K values into drain spacing). The values of L thus obtained are then plotted on a map on which the collector drainage system has already been delineated. It will be seen that the L-values differ less than the K values....
>
> ....The uniform drain spacing of a sub-area is found simply by calculating the arithmatic mean of the various drain spacings inside the sub-area. The drainage sub-area should, in principle, coincide with the collector block boundaries. But if two or more collector units have equal drain spacing they may be lumped together into one unit..."

An alternative method of designing field drains within a predetermined collector block is to take the geometric mean of the permeability samples as the design permeability and along with the other input parameters determine a single lateral spacing for the entire area (Amer, 1979). In some cases the collector axes are used as transects for data collection networks (El Ghamay, 1978).

These examples show that the main factor affecting lateral design is not the individual permeability measurements as much as the combining of these values into design parameters over areas defined by the collector system.

5.3.2 Spatial Variability within a Collector Area

In Chapter 3, it was shown that the uncertainty that occurs in the drainage design problem can be divided into two classes: the natural spatial variability of N and K and information uncertainty. It was demonstrated that for lateral drain design the scale of spatial variability had little effect upon the uncertainty in the groundwater level midway between two drains. This allows the assumption of uniform or homogeneous but uncertain soil permeability between drains to be made in the stochastic programming model. This assumption holds only over the range of one drain spacing. In the previous section guidelines for present drainage planning were outlined in which the assumption is made that the soil permeability is homogeneous over an entire collector region and that its effective value can be represented by a form of the mean.

Figure 5.3 presents the typical homogeneous drainage area. Its dimensions are two lateral lengths wide and the collector length long. In Egypt the average collector is 1 kilometer long and the average lateral length 200 meters. This produces a homogeneous lateral design area of 40 hectare or 95.2 feddan. It was also shown in Chapter 3 that for conditions in the Nile Delta that range or correlation distance is approximately 600 to 1000. This means that for distances between points greater than 600 meters there is little correlation in values of permeability. Therefore the assumption of homogeneity over a collector region does not hold. The failure of this assumption has two implications. First, the lateral spacing chosen for the entire collector region can not be based upon a single mean value for the region. Second, the alignment of the collector network over an agricultural field can effect the design

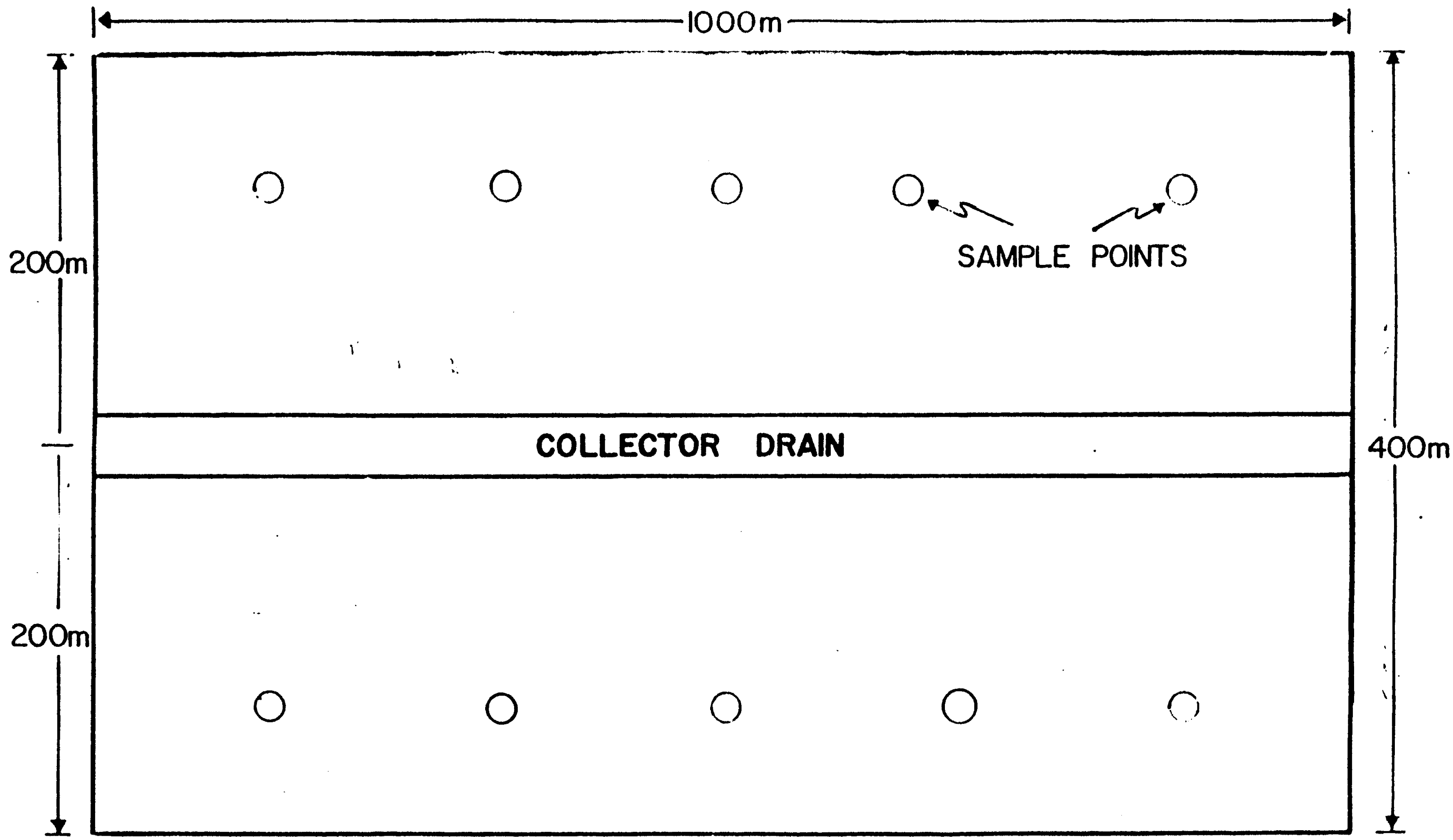


Figure 5.3 - Typical Homogeneous Drainage Area

of lateral chains. These two issues are addressed in the following sections.

5.4 Methods for Incorporating Spatial Variability in Lateral Design

In this section two approaches to account for spatial variability in lateral drain design are presented and compared.

5.4.1 Mean Areal Model

The first approach is based upon the assumption that the soil permeability is homogeneous over a collector region. This is similar to the present practice of lateral drain design. Unlike present practice, the true value of the homogeneous permeability is assumed not know with certainty, but rather the distribution of its value is known. From Chapter 3, the distribution of soil permeability can be assumed log normally distributed and defined by the mean and variance of the log of permeability. The mean and variance for each collector region is determined by the statistics of the sample permeability values that lie within each collector region. This approach accounts for spatial variability in the variance of the sample statistics. The first and second moment of the new areal permeability distribution serves as input to the multicrop stochastic programming model to obtain the optimal lateral drain design over a collector area. This approach is straight-forward; however, when the scale of variation of soil permeability is not much larger than the collector length scale the assumption of homogeneous permeability fails.

5.4.2 Spatially Distributed Loss Model

In the mean areal model the spatial variability is accounted for by the variance of sample permeability values within the collection region. In Chapter 4, it was shown that losses due to poor drainage design are nonlinear with respect to dewatering zone. Due to these nonlinearities, describing the spatial variability by a set of statistical values can give results that do not take into account the true economic costs. Ideally, a drainage system in which lateral drain design varied at each point in space would overcome this problem. It was shown that this was impractical from an installation point of view. However, from the point of view of the designer it is quite simple to develop a design for each point in space for which permeability data exists. In the same way it is possible to develop a function of the economic implications of the depth and spacing of lateral drains at each point for which permeability data is available. This function will reflect the capital cost as well as the loss due to depth of the dewatering zone, which is a function of the lateral design. In a situation where uncertainty exists this uncertainty should be included in this economic function.

Such a function has been developed for the drainage design process. It is the objective function of the stocahstic programming model for multiple crops. It gives the economic consequence of a choice of lateral drain depth D, and drain spacing L, given sufficient input parameters. So that at each point in space where data is available on permeability K, and drainage coefficient N, a cost function can be defined as a function of D and L. The values at each sample point are not without error and therefore they are described by a mean value and a measurement error or variance.

It is assumed that there must be only one design over the collector area. To determine this design a new cost function for the collector area is defined which is the summation of all the cost functions for each sample point. The "optimal"drainage design can then be found by replacing this function in the multi-crop stochastic programming model. The basis of this model is that the spatial variability of the paremeters are reflected in their true economic consequences rather than being described by a set of statistical values. This approach requires a greater computational burden due to the complexities of the objective function.

5.4.3 Model Comparison

In this section the two approaches presented above will be illustrated and a comparison between them made. A choice of the approach most appropriate to drainage planning is suggested.

To illustrate the two approaches and compare the models, a hypothetical field was considered, as presented in Figure 5.4. The field has the characteristics of an average field in the Nile Delta except that the spatial variability of the permeability values has been specified. The field is 1200 m x 1200 m square to avoid any geometric effects with a main drain bordering two sides. The proposed cropping pattern is that in Chapter 4, and the drainage rate is assumed to be uniform in space and time with a mean value of .4 mm/day and a standard deviation of .4 mm/day. All the important parameters for the field are listed in Table 5.2.

The mean of the permeability $\bar{K}$ is assumed to follow a linear trend in the Y-direction and be constant in the X-direction, this trend is illustrated in Figure 5.5 and described mathematically as follows:

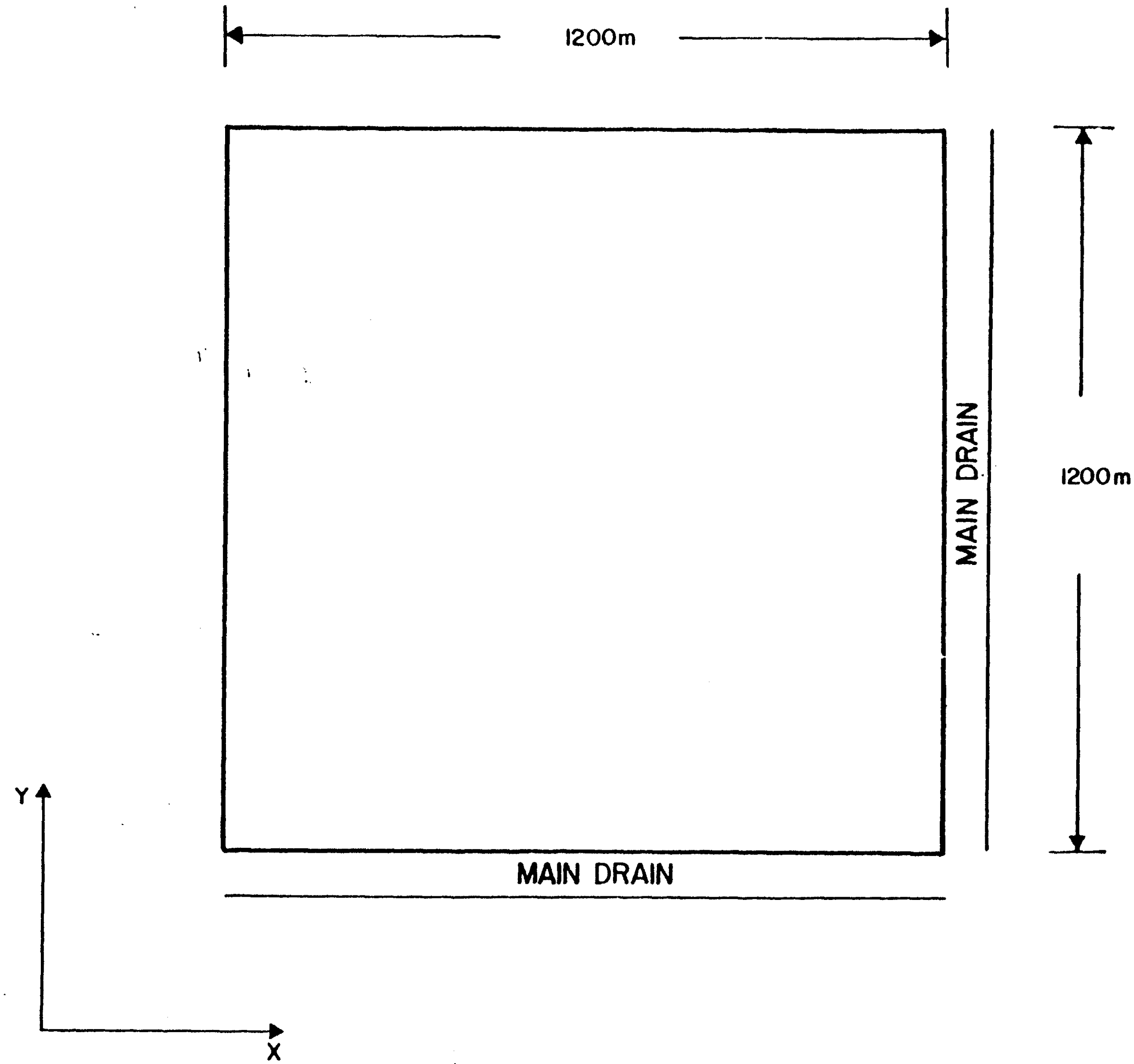


Figure 5.4 - Hypothetical Drain Field

Table 5.2

Hypothetical Field Parameters

$Z = 7$ m

$\bar{N} = .0004$ m/day

$\sigma_N = .0004$ m/day

$\rho_{KN} = -.0125$

Multiple Crop Design (Table 4.9)

Type II Crop Yield Functions

$i = 10\%$

$T = 50$ years

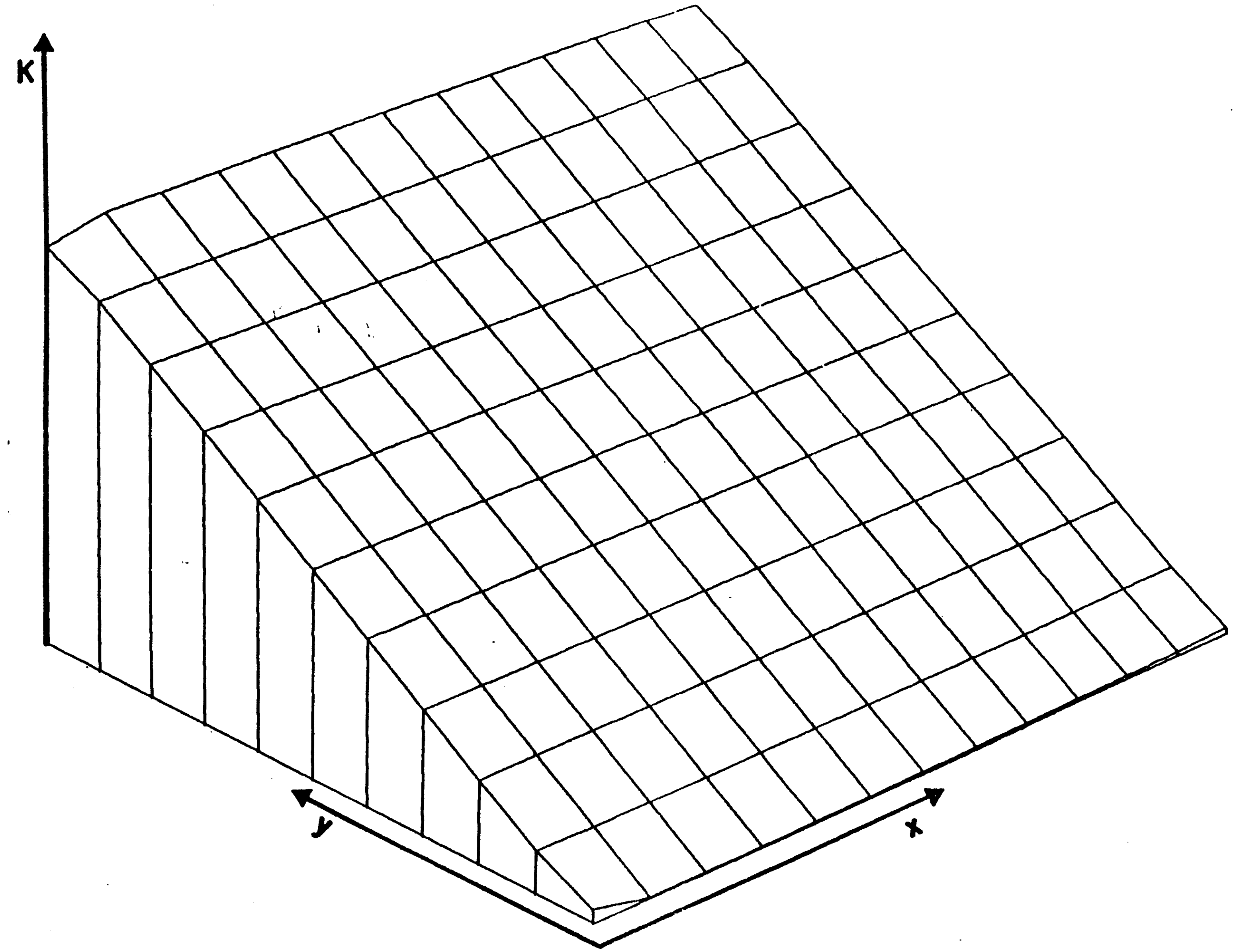


Figure 5.5 - Mean Permeability Over Hypothetical Drain Field

$$\bar{K}(X,Y) = .04 + .01 \times X \text{(m/day)}$$

where $\bar{K}(X,Y)$ is the mean. It is assumed that the variation about this mean is statistically homogeneous over the field:

$$\sigma^2_{K(X,Y)} = E[(K-\bar{K})^2] = .000936$$

To address the issues of spatial homogeneity and to demonstrate the impact of collector alignment upon total system performance two acute systems were chosen. System I, presented in Figure 5.6, aligns the collectors parallel to the Y-axis. Thus the areas defined for drainage design include the full range of possible values of the permeability; in this case the mean permeability value does not represent the true process very well. System II, presented in Figure 5.7, aligns the 3 collectors parallel to the X-axis. This allows the three drainage design areas to span only one-third of the range of permeability values and thus they better account for the true spatial process.

With the selection of drainage collection networks, lateral drain design regions are defined. Permeability statistics for each drainage area can be found. It will be seen that the alignment of the collector network effects the statistics for each drainage area. In this example, the variance of the permeability for drainage areas in System I will be greater than those in System II as well as having different mean values. The sample statistics are summarized in Table 5.3.

Using the mean areal model for lateral drain design with the data in Table 5.3, the optimal lateral drain design for each collector can be found and is given in Tables 5.4 and 5.5 for Systems I and II respectively.

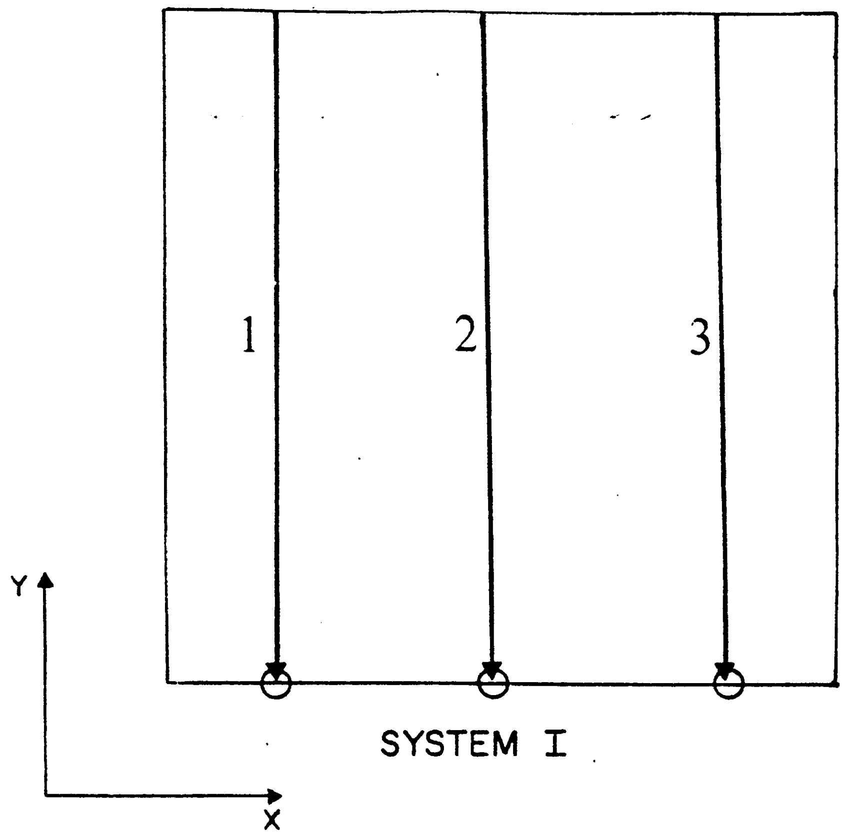


Figure 5.6 - System I Collector Alignment

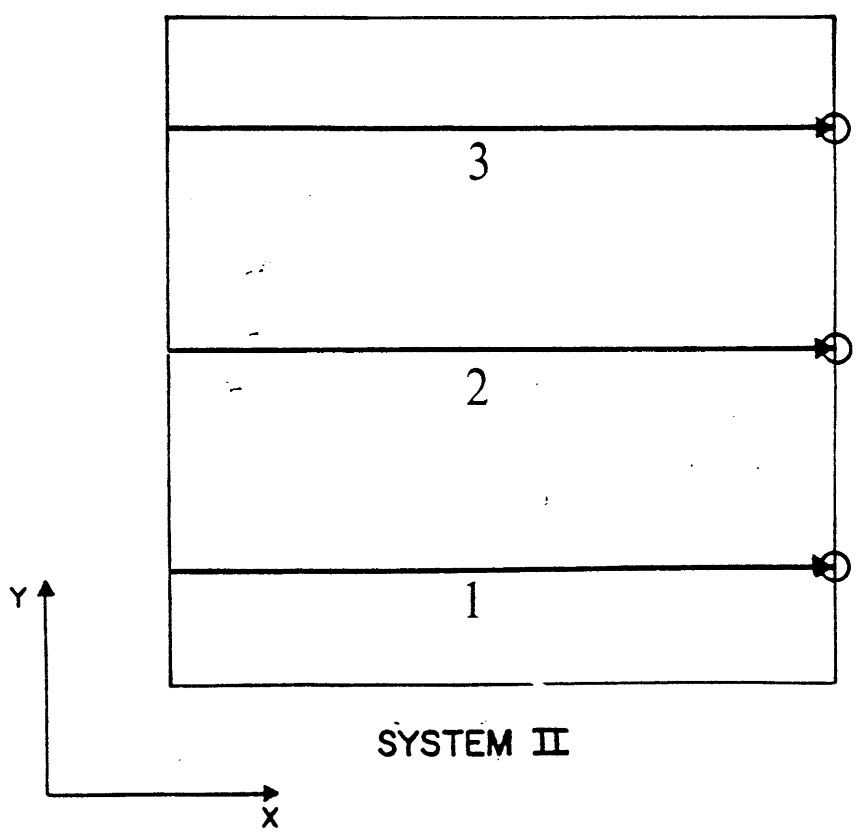


Figure 5.7 - System II Collector Alignment

Table 5.3

Permeability Statistics for Mean Areal Approach

System I

	Collector 1	Collector 2	Collector 3
$\bar{K}$ (m/day)	0.1	0.1	0.1
σ_K (m/day)	0.0506	0.0506	0.0506

System II

	Collector 1	Collector 2	Collector 3
$\bar{K}$ (m/day)	0.06	0.1	0.14
σ_K (m/day)	0.038	0.038	0.038

Table 5.4

Mean Areal Drain Design: System I

	Collector 1	Collector 2	Collector 3
Depth (m)	1.28	1.28	1.28
Spacing (m)	26.13	26.13	26.13
Capital Cost	115.93	115.93	115.93
Expected Loss	73.50	73.50	73.50
Total Cost	189.43	189.43	189.43
Area (feddan)	114.3	114.3	114.3
Collector Cost (LE)	21651.85	21651.85	21651.85

System Costs

Capital Cost	39752.4	LE
Expected Loss	25203.2	LE
Total Cost	64955.6	LE

Table 5.5

Mean Areal Drain Design: System II

	Collector		
	1	2	3
Depth (m)	1.32	1.29	1.27
Spacing (m)	20.09	26.86	31.70
Capital Cost (LE/feddan)	150.85	112.79	95.54
Expected Loss (LE/feddan)	92.35	79.59	63.13
Total Cost (LE/feddan)	243.20	185.38	158.67
Area (feddan)	114.3	114.3	114.3
Collector Cost (LE)	27797.76	21188.93	18135.98

System Costs

Capital Cost	411054.3	LE
Expected Loss	26068.4	LE
Total Cost	67122.7	LE

From these tables it can be seen that the total cost is less for System I than for System II by 2166.9 LE even though System I is a poor representation of the process. This should not suggest that System I is the better alternative but rather that there is a problem with the methodology. At close examination one can see why this result occurs and examine an alternative methodology.

In Table 5.3 the statistics of the permeability for the collector areas are shown. In System I, although the range of values spanned for each collector is three times that for the collectors in System II, the standard deviation of the permeability for System I collector areas is only 1.33 times as great. With the standard deviation only 1.33 times greater the important parameter is the mean of the permeability. For each collector area in System I the lateral drains are designed with a mean permeability of 0.1 m/day. Thus at the low permeability end of the field the lateral drains are underdesigned and yield losses will result and at the end with high permeability values the lateral drains will be overdesigned, and excess capital will be spent.

In System II the lateral drain design differs over each collector reflecting the variation of permeability over the field. The lateral design for the low permeability area requires a more detailed design to prevent yield losses, while the lateral design for the high permeability area can be a more sparse design to prevent yield losses. Due to the nonlinearities of the yield loss function the added costs incurred to prevent losses in the low permeability area are not offset by savings in costs in the high permeability area. For this reason, the experiments show that total costs for System I are less than those for System II even though the lateral drain design for System I does not account for the actual spatial variability of permeability. This example illustrates that the mean areal approach to lateral design is flawed and that the incorporation of spatial variability as the sample variance should not be used.

Returning to the hypothetical field, Figure 5.4, the two proposed collector systems, Figures 5.6 and 5.7, will again be analyzed to determine the most efficient alignment for the lateral drainage design. This time the analysis will use the spatially distributed loss model. Table 5.6 and Table 5.7 present the results for System I and System II, respectively. Table 5.8 provides a summary of the total costs for both systems. In this analysis it shows that System II is the most efficient alignment for lateral drain design. It can be seen that the expected loss for System I is greater than System II as would be expected since System II provides a design which follows true spatial process more closely.

The spatially distributed loss model accounts for the expected loss that would occur at each data point for a given lateral drain design and the sample value of the permeability at the data point. The model then sums the expected losses that would occur at all data points that lie within each collector area. The total expected losses are added to the capital costs for the given lateral drain design for the entire region. The objective is to minimize the total costs of lateral drain design for each collector. The model uses a nonlinear optimization technique (golden section search) to select the design that minimized total costs.

Table 5.6

Spatially Distributed Loss Drain Design: System I

	Collector 1	Collector 2	Collector 3
Depth (m)	1.42	1.42	1.42
Spacing (m)	29.99	29.99	29.99
Capital Cost (LE/feddan)	101.18	101.18	101.18
Expected Loss (LE/feddan)	152.55	152.55	152.55
Total Cost (LE/feddan)	253.73	253.73	253.73
Area (feddan)	114.3	114.3	114.3
Collector Cost (LE)	29001.73	29001.73	29001.73

System Costs

Capital Costs	34695.06	LE
Expected Costs	52310.13	LE
Total Costs	87005.19	LE

Table 5.7

Spatially Distributed Loss Drain Design: System II

	Collector 1	Collector 2	Collector 3
Depth (m)	1.5	1.5	1.5
Spacing (m)	24.64	34.16	43.34
Capital Cost (LE/feddan)	123.13	88.82	70.00
Expected Loss (LE/feddan)	147.36	135.13	129.96
Total Cost (LE/feddan)	270.49	223.95	199.96
Area (feddan)	114.3	114.3	114.3

System Costs

Capital Costs 32227.16 LE

Expected Costs 47142.79 LE

Total Costs 79369.95 LE

Table 5.8

Summary of Spatially Distributed Loss Approach

System I

Capital Cost	34695.06	LE
Expected Loss	52310.13	LE
Total Cost	87005.19	LE

System II

Capital Cost	32227.16	LE
Expected Loss	47142.79	LE
Total Cost	79369.95	LE

As stated, the expected loss function is nonlinear so that a collector region which spans a wide range of permeabilities will incur high total costs. These high total costs result from the need to over-design in high permeability areas to prevent severe losses in the lower permeability areas. This can be seen in Table 5.8 where System I that has collectors which span a wide range of permeability insures higher total costs than System II that has collectors which span a smaller range of permeability. System II, therefore, allows for a more efficient system of lateral drains.

In this section it has been demonstrated that the areal model fails in its attempt to account for the economic cost of spatial variability in soil permeability. However, the spatially distributed loss model has been shown to more adequately address this problem. Therefore the spatially distributed loss model is selected as the method to efficiently design lateral drains within collector areas with spatial variability in soil permeability. The next section will look at how well sample data represents spatial variability.

5.5 Representation of Spatial Variability through Kriging

The spatially distributed loss model has been chosen as the method for accounting for spatial variability in soil permeability in lateral drain design. How to best represent spatial variability of soil permeability in an agricultural field is addressed in this section. In the illustration of the spatially distributed loss model it was assumed that the sample data values were a good measure of spatial variability of soil permeability over a field. The model analyzes the expected losses associated with each data point that lies within the collector area. This approach assumes that each data point is independent of each other and that information about the soil permeability comes solely from the data points within that collector area. Information from sample points that lie in another collector area but very close to the area being analyzed as well as information about the spatial structure of permeability within a collector area are ignored.

In Chapter 3, the process of kriging was introduced. This process provides the "optimal" interpolation of spatial data by taking into account the spatial structure of a process. Kriging also provides an estimate of the reliability of the interpolated values. Given information about the spatial structure of permeability (covariogram) and sample data points, a kriged surface of permeability values of any size grid can be generated. The detail of the grid to be generated is a trade-off between the desired representative of the field and the cost of running the spatial distributed loss model for a large number of points. The use of kriging to represent the spatial variability of soil permeability allows for the most complete utilization of information about soil permeability. To illustrate the usefulness of the kriging process, a comparison was made of the design of a lateral drain system using sample data with and without measurement error and kriging data. The comparison is applied to the Egyptian case study discussed in Chapter 3.

Figure 5.8 illustrates the field to be used in the case study with the location of the data points indicated. This field which is approximately 400 feddans has 53 data points. The sampling interval for the data points is 200 meters except in one area where an intensive 100

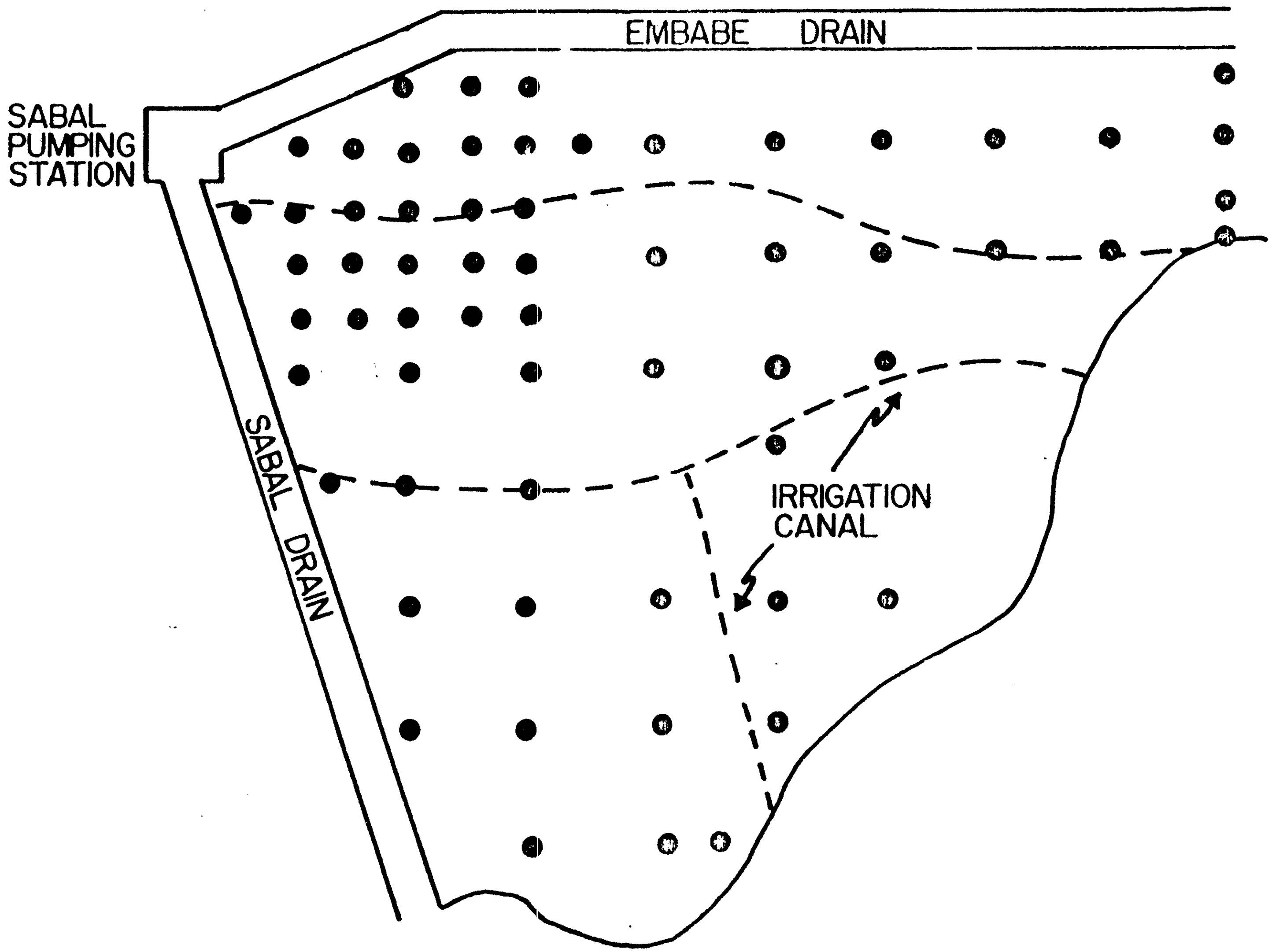


Figure 5.8 - Embabe Case Study

meter sampling took place. Presently soil sampling for drainage in Egypt is on a 500 meter sampling grid which would provide fewer data points.

The spatially distributed loss model is applied to the collector system shown in Figure 5.9 to design the lateral drain system for each of the observed values of soil permeabilities. The other important input parameters are listed in Table 5.3. Table 5.9 presents the lateral drain system design with the assumption that the sample data values have no measurement error. The total cost for this system is 126,794 LE.

It is well documented that the auger hole test for soil permeability is not free from measurement error. Amer (1979) claims that for Egyptian conditions the error is approximately twenty-five percent. If the model is rerun, this time with the assumption of an "uncorrelated" sampling error of .02 m/day, the total cost for the lateral drain system is 136,078.26 LE. The detailed results are listed in Table 5.10. The reason for the increased cost is, that due to more uncertainty in the case of measurement error, the lateral drains must be designed in more detail to prevent large expected losses and, secondly, even with the more detailed design the added uncertainty results in greater expected losses.

A kriged representation of the mean value of soil permeability and the estimation error of the mean values on a 100 meter grid was presented in Chapter 3. This provides 153 kriged data points. Using this representation of soil permeability as input to the spatially distributed loss model results in a lateral system design costing 110,800 LE. Table 5.11 lists the detailed results. This system costs 19 percent less that the total cost of the system based upon sample data with measurement

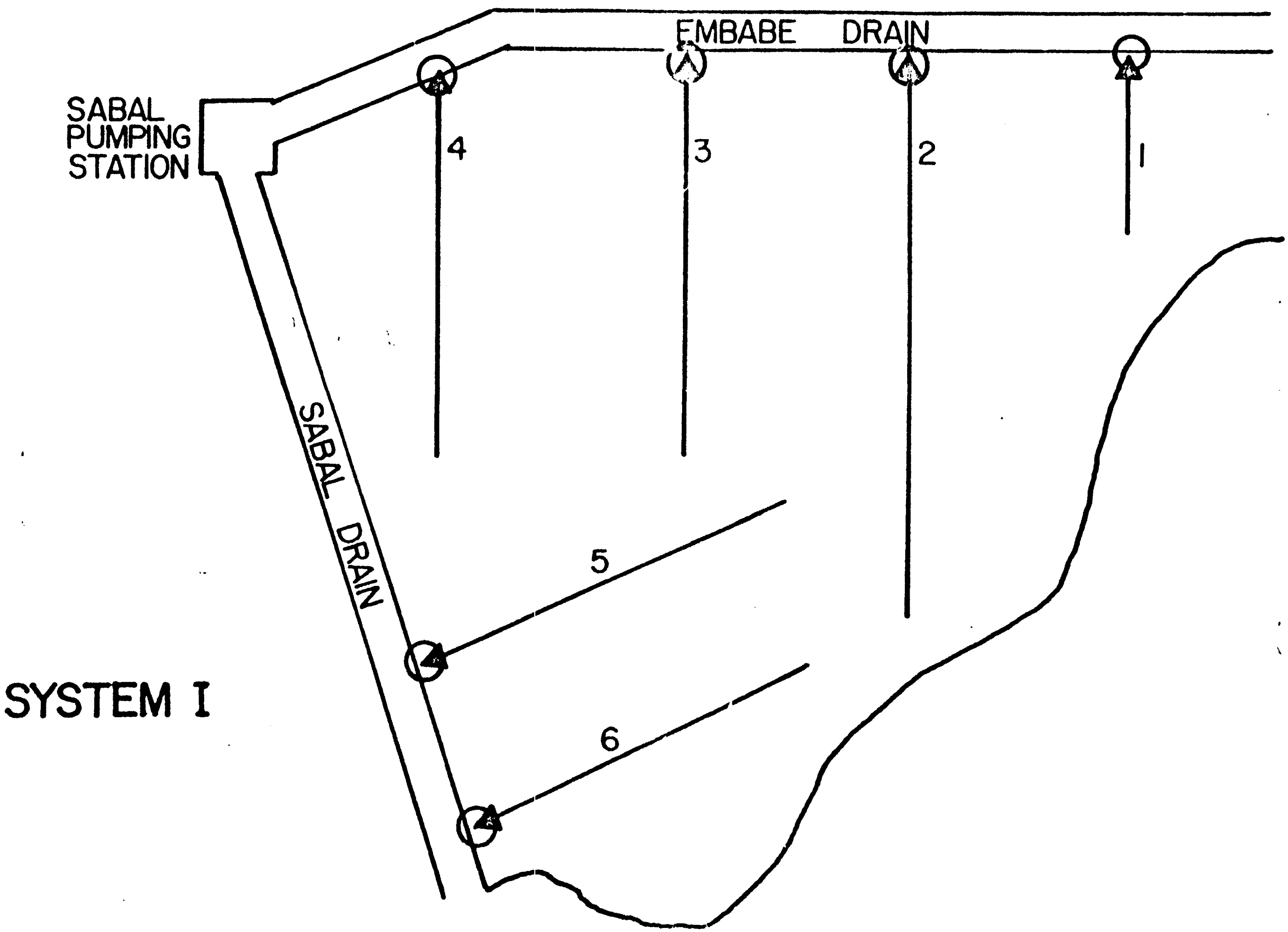


Figure 5.9 - System I

Table 5.9

Lateral Drain Design Assuming Perfect Permeability Data

	Collector					
	1	2	3	4	5	6
Depth (m)	1.5	1.5	1.5	1.5	1.5	1.5
Spacing (m)	27.30	19.10	21.30	17.77	30.68	38.81
Area (feddan)	28.6	95.2	76.2	76.2	71.4	52.4
Capital Cost (LE)	3119.	15188.	10856.	13011.	7079.	4317.
Expected Loss (LE)	4759.	15439.	17154.	16915.	11781.	7230.
Total Costs (LE)	7878.	30567.	28010.	29926.	18860.	11547.

System Costs

Capital Cost	53513.39	LE
Expected Loss	73281.12	LE
Total Cost	126794.51	LE

Table 5.10

Lateral Drain Design Assuming Permeability Data with Error

	Collector 1	2	3	4	5	6
Depth (m)	1.5	1.5	1.5	1.5	1.5	1.5
Spacing (m)	28.89	17.74	23.10	18.92	29.35	36.43
Area (feddan)	28.6	95.2	76.2	76.2	71.4	52.4
Capital Cost (LE)	3225.	16287.	10010.	12219.	7261.	4363.
Expected Loss (LE)	4950.	17413.	19599.	21175.	12235.	7340.
Total Cost (LE)	8175.	33700.	29609.	33394.	19496.	11703.

System Cost

Capital Cost	53365.85	LE
Expected Loss	85712.44	LE
Total Cost	136078.29	LE

Table 5.11

Lateral Drain Design Assuming Kriged Permeability Data

	Collector					
	1	2	3	4	5	6
Depth (m)	1.5	1.5	1.5	1.5	1.5	1.5
Spacing (m)	33.80	22.99	23.95	21.15	33.39	37.18
Area (feddan)	28.6	95.2	76.2	76.2	71.4	52.4
Capital Cost (LE)	2565.	12573.	9655.	10934.	6491.	4275.
Expected Loss (LE)	4498.	15250.	12370.	13412.	11031.	7747.
Total Cost (LE)	7063.	27823.	22025.	24346.	17522.	12022.

System Cost

Capital Cost	46492.95	LE
Expected Loss	64307.37	LE
Total Cost	110800.32	LE

error and 13 percent less than the system designed assuming perfect sample data. This result is very significant. It shows that by using information about the spatial structure of soil permeability (i.e., covariogram) it is possible to better represent the spatial variability and reduce uncertainty within each collector area. This is illustrated by the fact that in the system design using the kriged data both the capital cost and the expected losses are less than the other designs. The experiment assumes that the covariogram and the drift identify the true spatial structure of the permeability and by increasing the density of the kriged grid a more accurate representation of the ture spatial structure will be generated. However, the benefits of a more dense kriging grid to more accurately represent the permeability structure does not increase after the grid spacing reaches the scale of the drain spacing. In Chapter 3 it was demonstrated that at the range of one drain spacing the assumption of constant but unknown permeability is a valid assumption for the FOSM analysis regardless of the spatial structure.

A method now exists that not only accounts for the true economic cost of spatial variability in soil permeability but also provides a better representation of the spatial variability itself as well as analyzing the effect of collector system alignment upon lateral drainage design. The next step in the development is to combine the simulation model for collector drains with the spatial distribution loss model to measure the effect of collector drains on field lateral drains as a tool for dynamic drainage planning.

5.6 Dynamic Multi-Level Drainage Design Model

In Section 5.2 a simulation model for level two collector drain design was presented. An example illustrated that by use of the simulation model a number of alternative collector network alignments can be analyzed to find the minimum cost network. In Section 5.4 a spatially distributed loss model was presented which accounts for the true economic cost of spatial variability of soil permeability in level III lateral drain design. It was shown that the alignment of the collector system, which defines the lateral drain design areas, effects the cost of lateral system design. This demonstrates that level two design impacts the design at level three. The present sequential planning process provides no feedback between level two and level three. For example, there is no provision to analyze how the network alignment effects lateral drain design. Tools for efficient level two and level three planning do not solve the problem because in many cases the least cost collector network will not provide the most efficient lateral drain system. A methodology to provide feedback between level two and level three is developed in this section. The approach is the synthesis of the collector simulation model with the spatially distributed loss model for lateral drain design into a dynamic multi-level drainage planning model. The synthesis of these two models are needed for the design of a complete drainage system due to the fact that in many cases there may be a tradeoff between the capital costs of the collector system and the total costs of the lateral system. It is necessary then to look at the total costs of the whole drainage system; capital costs for both collector and lateral drains and the expected losses due to lateral design.

The combination of simulation and optimization techniques in a single methodology captures the true essence of the systems analysis approach. The systems analysis approach provides a tool for analyzing complex problems in a structured framework. The drainage design problem under uncertainty as outlined above is just such a complex process. By combining simulation with mathematical optimization the areas of drainage design are matched with system tools that fit the characteristics of each area.

In the area of collector network design many of the design criteria are mathematically describable, so the simulation approach fits quite well. Simulation allows the response of a system to be analyzed in an efficient manner allowing the designer to analyze many alternatives of complex systems.

The design of field drains is different. The physics and economics of the problem can be described mathematically allowing mathematical programming techniques to be used to design efficient systems. The synthesis of both approaches into a single methodology allows the complex drainage design problem to be analyzed systematically, efficiently and allows the characteristics of the system to be modeled accurately.

This combined simulation-optimization approach combines the benefits of engineering judgement and optimization techniques. It allows an engineer to choose a feasible collector alignment. The collector simulation model then provides a design for the collector system. The model sorts the permeability data and assigns values to the appropriate collector area. Then the spatially distributed loss model produces the optimal lateral drain design for each collector area. The model output is capital costs of the collector network, capital cost of lateral drain systems, expected losses for the lateral system, and the total system cost. The details of this model are described in Strzepek and Marks (1979).

With this model, the engineer can make a number of different collector alignments and find the alignment that meets the design criteria at the least cost. This tool provides the feedback link between level two and level three allowing a dynamic planning process to design the "best" drainage system. The dynamic multi-level drainage design model addresses in one model the problem of uncertainty in drainage design, the problem of spatial variability soil permeability, the economics of crop yield and drainage design, and the lack of feedback between collector and lateral designs. These are all major problems that have faced the drainage field for many years. The next section will provide an application of approach to the Egyptian case study.

5.6.1 Egyptian Case Study

To undertake the enormous task of implementing drainage over almost 5 million feddans of Egyptian agricultural land, the Egyptian Public Authority for Drainage Projects (EPADP) was established within the Ministry of Irrigation. The roles of EPADP are to perform investigations, planning, design and coordination of implementation of all drainage projects in Egypt. This is quite a large task and EPADP has done a good job considering the constraints of man-power, budget, and shortage of resources. In the process of undertaking all these tasks, certain procedures had to be adopted for drain design. Some of these procedures were adopted from different climatic and agricultural regions where they function well. Therefore, some of the procedures may not be appropriate for Egyptian conditions and should be scientifically tested in the field. This is very difficult given the tasks which lie before the EPADP, thus the use of theoretical methods to provide insight into the applicability of untested procedures is a useful experience

This section will examine two such procedures that may account for cases where tile drainage is ineffective.

1. The implementation of tile drains with a minimum design of 40 meter spacing with 1.5 meter depth.
2. The implementation of a fixed drain design over a large area.

Minimum Spacing

EPADP has adopted a policy of imposing a minimum of 40 meter spacing design for tile drains based upon an assumption that crop yield response to dewatering zone is a linear or concave function. This assumption allows one to claim that the benefits of drainage to crop production over an area of two drains providing 50% of the optimal dewatering zone is equivalent if not greater than the benefits to crop production of an area of one drain provided with optimal dewatering zone. If this assumtpion holds true, then it is quite logical to propose a design value of a water table that is less than the optimal to exploit the properties of the response function, not a minimum spacing, which is a function of soil permeability, drainage rate, and other parameters.

The reason why a minimum spacing approach fails is that dewatering zone is not linearly related to drain spacing; that is, a function of spacing and depth of drains, depth of soil layer, soil permeability, drainage rate, and irrigation practices, all of which change greatly over the region of Egypt. The goal of an efficient drain design is to find the spacing that provides the dewatering zone where the marginal cost of increased drainage equals the marginal benefits of drainage to crop production. In that way, the capital resources allocated to drainage will be utilized in an efficient manner. The other reason that a minimum spacing approach can fail in certain circumstances is that the crop yield function is not linear or concave. Figure 4.7 is a plot of the range of possible crop yield functions feasible in Egypt. One can see that although the functions rise concavely from the abssisa, there is a threshold value which does not pass through the origin which makes the function non-concave. These functions also exhibit non-concave features beyond the optimal, further complicating the analysis. In Section 4.3.5 a method to determine the optimal design of tile drains for multiple crops even under conditions where the input parameters are uncertain was developed. Based upon the above method, this section will show the cost associated with applying a minimum spacing approach under conditions where it is inappropriate.

Large Scale Areal Homogenity

The policy of EPADP in many cases is to design large areas under a single tile drain design, in effect, assuming large scale spatial homogenity of drainage properties. This policy is based on a property that as soil permeability varies in space the variation in design spacing will be less, due to the nonlinear relationship between spacing and permeability. Therefore it is easier to lump areas together since the magnitude of variation in spacings are less. Especially in the areas of greatest concern, where there are low permeabilities such as the Nile Delta Clays, the designs all show a need for spacing under 40 meters, but due to the minimum spacing policy described above, a whole region will drain at 40 meter spacing. Thus, within that region there will be many areas where the drains are totally ineffective. Even in areas where the minimum spacing policy does not come into effect, the incorporation of spatial variability of soil permeability into drain design will provide a more efficient total system.

A region along the Embabe Drain in the Nile Delta (presented above) was selected as being representative of conditions throughout the Nile Delta. Data was provided by the Ministry of Irrigation that would normally be used to design the drains. This includes soil permeability data, drainage rate, depth of soil layer and crops grown. Since the exact form of the crop yield function was not known for this region, the three that represent the range found in Egypt were used to examine the sensitivity of results to each.

The area selected for study was 2 kilometers by 1.4 kilometers and it was assumed it be drained by five parallel collectors which define the homogeneous design units.

To study the effect of a 40 meter minimum spacing a single collector area was selected. It was 400 meters wide and 1400 meters long and covers an area of 133.3 feddans. There were 11 soil permeability samples found in this collector region. For this anlaysis it was assumed that the data samples were without error as is the practice of the EPADP. However, the drainage rate was assumed to be normally distributed with a mean of .4 mm/day, with a standard deviation of .4 mm/day. This value came from the study of the Embabe region; the crops considered for this case study were wheat, maize, cotton, vegetables and berseem. A constant drain depth of 1.5 meters was assumed.

Table 5.12 provides the results of running the design model for the three types of crop yield functions shown in Figure 4.10. The results show the dramatic effect that the policy of 40 meter spacing can have under these conditions. In all three cases the drain design provided by the model gives a spacing about one-half of the 40 meters. This smaller spacing requires approximately a 100% increase in the capital costs of the tile drain installation. However, the savings in expected losses over the life of the drains, assumed to be 50 years with an interest rate of 10%, is startling for the type 1 crop function. The expected losses with a 40 meter spacing are about 14 times greater than optimal design, resulting in a total cost over the life of the drain of the 40 meter spacing being 3 times greater than optimal design provided by the model. For type 2 and 3 cases, the increased losses for 40 meter spacing were 4 and 3 times greater, respectively. This results in the total costs for both type 2 and 3 being approximately 2 times greater than the design for the Mathematical Programming Model.

Although the data used may include some assumptions and extrapolation for this region, it does show that for this data set, the adoption of a

Table 5.12 Results of analysis of effect of minimum drain spacing

Crop Yield	Type I		Type II		Type III	
Drain Design	Minimum	Optimal	Minimum	Optimal	Minimum	Optimal
Depth (meters)	1.5	1.5	1.5	1.5	1.5	1.5
Spacing (meters)	40.0	18.05	40.0	19.40	40.0	19.22
Costs:						
Capital Costs (LE)	10115.10	22419.6	10115.10	20861.0	10115.10	21047.29
Expected Losses (LE)	71241.46	4897.52	80822.42	21441.33	93422.49	30335.32
Total Costs (LE)	81356.56	27317.12	90937.52	42302.33	103539.54	51382.61

minimum spacing policy is very costly in the long run. Even if the losses are 100% over-estimated, it would not change the result. This is due to the fact that for a more costly drain design, less than 40 meters, the present values of expected losses are greater than the additional capital investment in drains. Although this is looking at losses rather than actual yields, it may explain why certain drained fields are not exhibiting any increases in yields.

Spatial Variability of Soil Permeability

The procedure of aggregating large areas of land under a single tile drain design is made for each of installation. It was proposed that a single collector area be the maximum area under which drain design is held constant. Then each independently designed collector area can be aggregated into a drainage system with little effect on the installation process.

As was demonstrated above, the ignoring of spatial variability of soil permeability within a single collector can have large costs. This section will examine the costs of ignoring spatial variability on a large field level which may span many collectors.

Table 5.13 presents results of an analysis of this problem for the case study field which is drained by 5 collectors. The analysis examines the effect of assuming a 40 meter minimum spacing over the entire field. The effect of the different crop yield functions are illustrated as well. The results provide the drain spacing for each collector designed using the Spatially Distributed Multiple Crop Stochastic Programming Approach Policy and compares this with the fixed design. The

Table 5.13 Effect of spatial variability of system design.

Crop Yield	Type 1		Type 2		Type 3	
	Min 40	Optimal	Min 40	Optimal	Min 40	Optimal
Spacing* (m) Collector						
1	40.	29.61	40	32.46	40	31.89
2	40.	18.16	40	20.47	40	20.51
3	40.	20.42	40	23.82	40	23.79
4	40.	18.05	40	19.40	40	19.22
5	40.	25.18	40	27.68	40	27.54
System Costs						
Capital (LE)	50575.48	94244.68	50575.48	84694.79	50375.48	85156.02
Expected Losses (LE)	201040.44	28041.38	256861.34	115645.30	312065.34	161366.88
Total (LE)	251615.92	122286.06	307436.82	200340.09	362640.82	246522.90

* Depth for all designs 1.5 m

capital cost, expected losses and total cost for the total 5 collector system are presented. The costs resulting from assuming a 40 meter spacing over the entire field are also shown.

In Table 5.13, the range of values for the spacing of drains for each collector can be seen. These different values are a result of the variability of the soil permeability found among the collector regions. In Section 3.4 it was shown that in this region the scale of variability of soil permeability is between 600-1000 meters. This means that little can be inferred about the value of permeability (beyond a distance of 600-1000 meters) from the value at a certain point. This is demonstrated in the results as well, since collectors 1 through 4 are continuous but show varying designs.

Table 5.13 also shows the economic results of ignoring this variability in soil permeability. For a type 1 crop yield function, the expected losses over the entire field are seven times greater with a 40 meter fixed design compared to the spatially varying design. The capital costs are 86% greater for the spatial variability case; even so, the total costs are less than 50% of those for the fixed 40 meter design. For the type 2 and 3 crop yield functions, the expected losses are approximately 2 times greater for the fixed design versus the spatially varying. This results in total costs for type 2 and 3 spatial varying design of approximately 65% of the total costs of the fixed 40 meter design.

These results are quite substantial and a spatial varying design policy should not require much more cost in installation except for the fact of the 40 meter minimum spacing that was discussed above.

Returning to the Embabe case study field, illustrated in Figure 5.8, this section outlines the use of the dynamic multi-level drainage planning model to find the "best" drainage system for this field. The example will use the kriged values of the soil permeability presented in Chapter 3 and used in Section 5.5 as input data along with the other parameters listed in Table 5.3.

There are four feasible collector networks that are analyzed. System I was the collector network used as an example in Section 5.5. Systems I, II, III and IV are illustrated in Figures 5.9-5.12. Each alternative system was analyzed using the dynamic multi-level planning model and the resulting designs are presented in Tables 5.14, 5.15, 5.16, and 5.17 for System I through System IV respectively.

Table 5.18 is a summary of total system costs for each system. From Table 5.18, it can be seen that although System I has the least cost collector network it is not the "best" system. System IV is the system with the least total cost or the "best" system. The reason that System IV is the "best" alternative is that the collector network alignment spans the field such that the homogeneous lateral drainage areas provide for the most efficient lateral drainage system of all the alternative collector alignments. The savings in lateral system costs between System IV and System I outweigh the increase in costs of the collector system between System IV and System I. Systems II and III have costs for the collector system and for the lateral system that are higher than both System I and System IV costs.

This example illustrates the importance of feedback between level

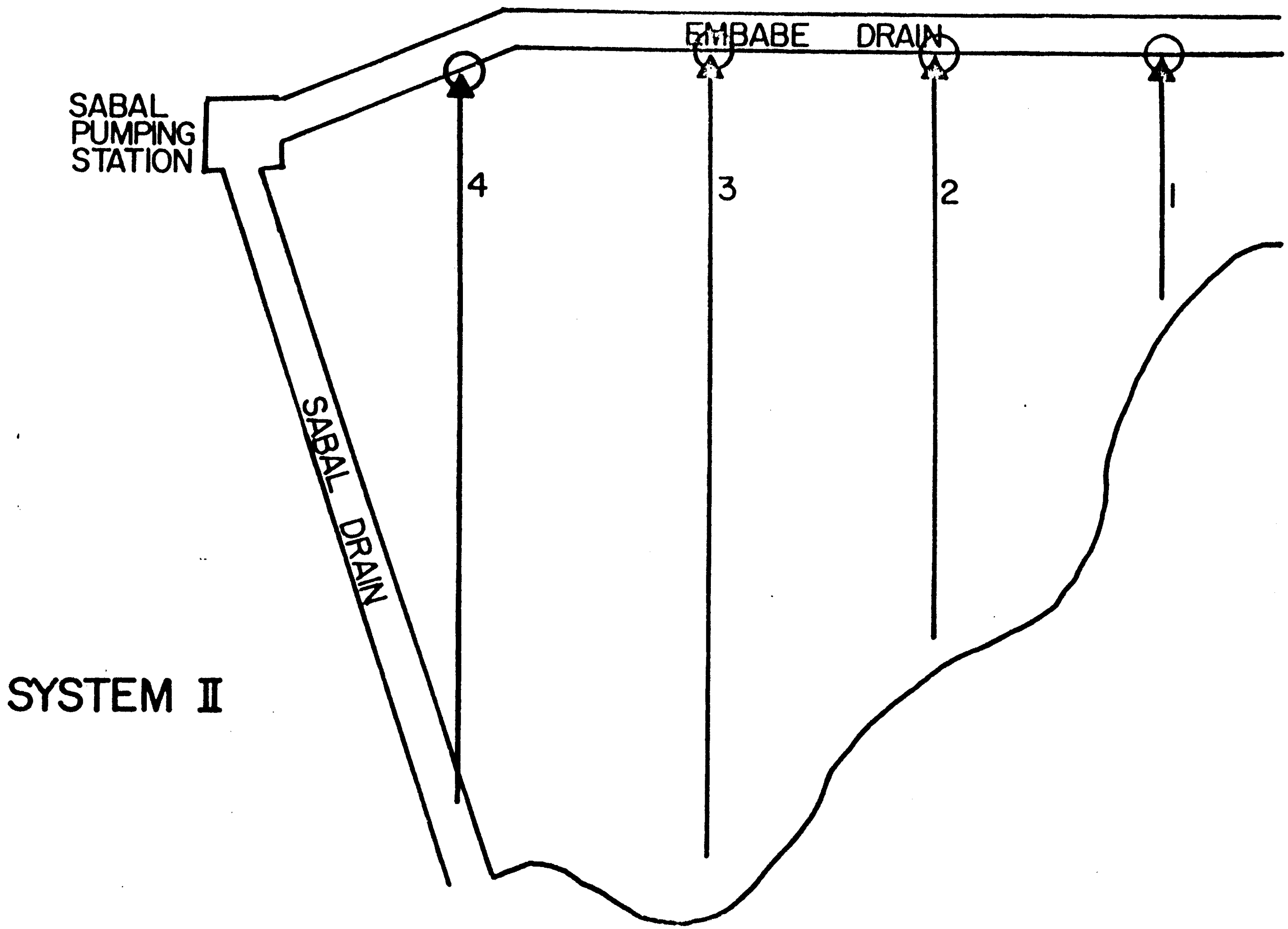


Figure 5.10 - System II

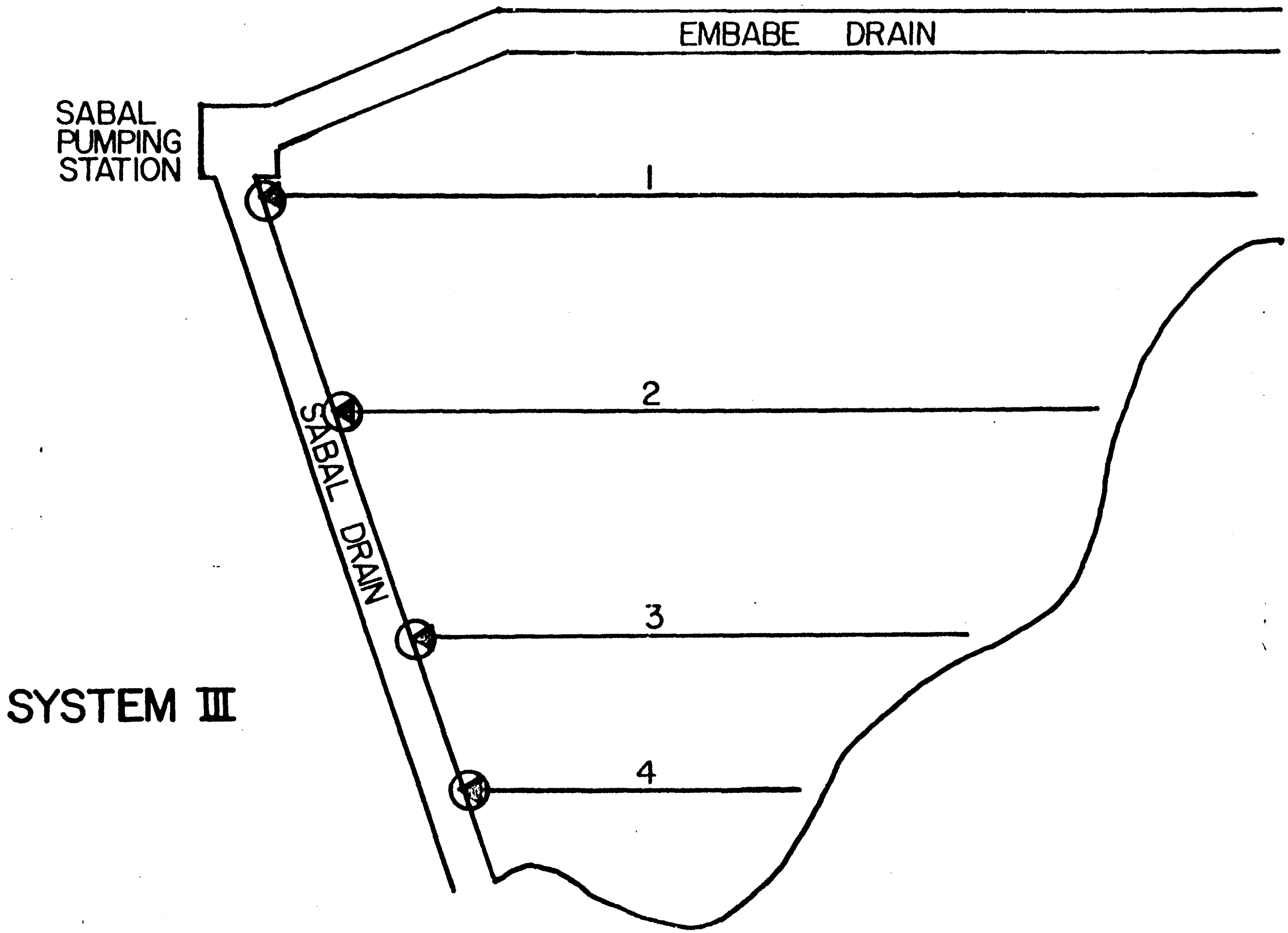


Figure 5.11 - System III

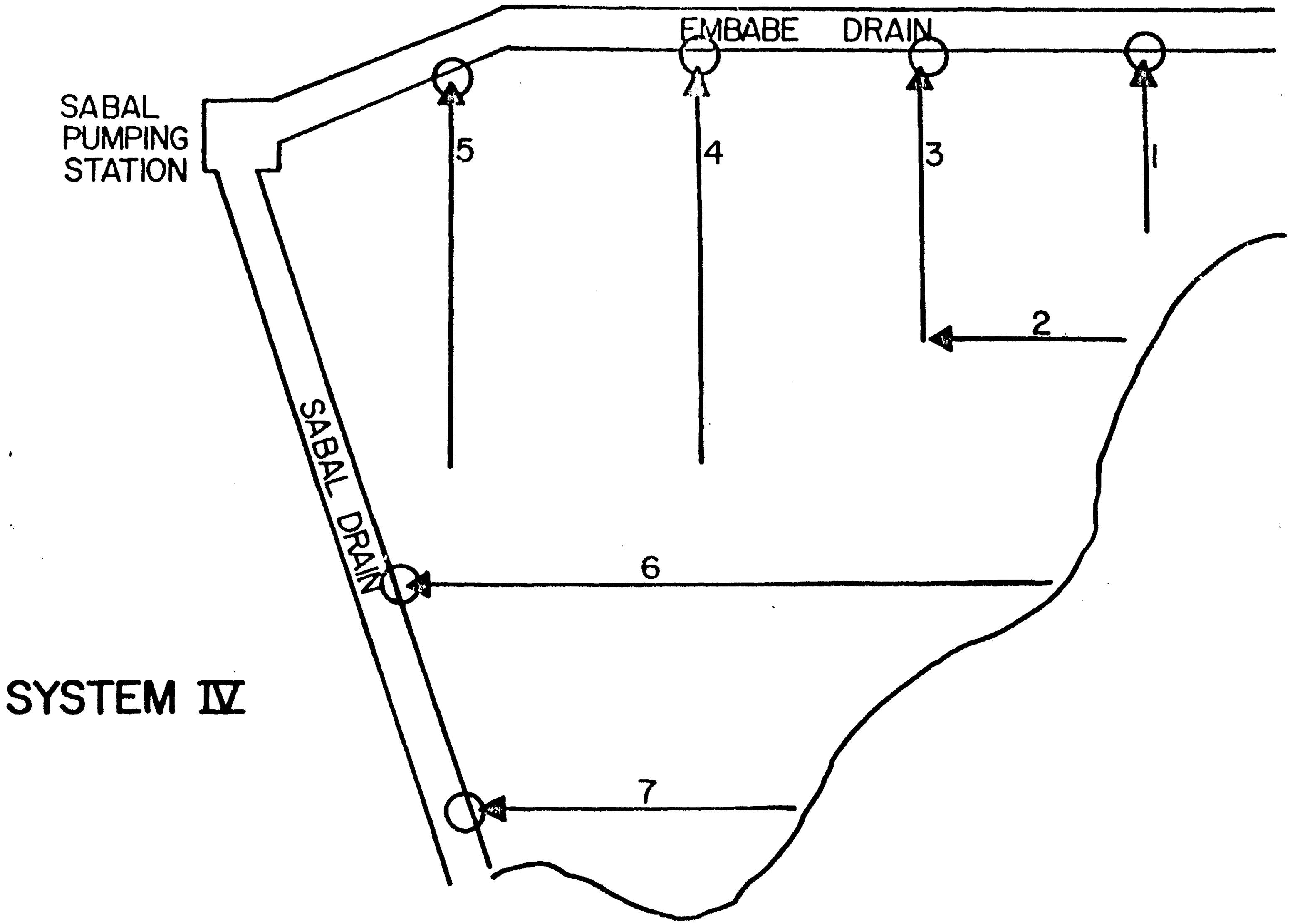


Figure 5.12 - System IV

Table 5.14

System I Drainage System Design

	Collector					
	1	2	3	4	5	6
Depth (m)	1.5	1.5	1.5	1.5	1.5	1.5
Spacing (m)	33.80	22.99	23.95	21.15	33.39	37.18
Area (feddan)	28.6	95.2	76.2	76.2	71.4	52.4
Capital Cost (LE)	2565.	12573.	9655.	10934.	6491.	4275.
Expected Loss (LE)	4498.	15250.	12370.	13412.	11030.	7747.
Collector Cost (LE)	303.	1565.	945.	945.	895.	860.
Total Cost (LE)	7366.	29388.	22970.	25291.	18416.	12882.

System Cost

Capital Cost	46492.95	LE
Expected Loss	64307.37	LE
Collector Cost	5513.15	LE
Total Cost	116313.27	LE

Table 5.15

System II Drainage System Design

	Collector 1	Collector 2	Collector 3	Collector 4
Depth (m)	1.5	1.5	1.5	1.5
Spacing (m)	32.56	22.99	26.16	25.44
Area (feddan)	47.6	95.2	133.3	123.8
Capital Cost (LE)	4437.	12573.	15469.	14769.
Expected Loss (LE)	7450.	15250.	21898.	21796.
Collector Cost (LE)	606.	1565.	2520.	2251.
Total Cost (LE)	12493.	29388.	39887.	38816.

System Cost

Capital Cost	47247.27	LE
Expected Loss	66393.44	LE
Collector Cost	6942.97	LE
Total Cost	120583.68	LE

Table 5.16

System III Drainage System Design

	Collector			
	1	2	3	4
Depth (m)	1.5	1.5	1.5	1.5
Spacing (m)	23.23	24.57	28.63	37.71
Area (feddan)	161.9	123.8	85.7	28.6
Capital Cost (LE)	21151.	15293.	9084.	2299.
Expected Loss (LE)	27644.	19333.	13617.	4167.
Collector Cost (LE)	3258.	2251.	1337.	303.
Total Cost (LE)	52053.	36877.	24038.	6769.

System Cost

Capital Cost	47827.04	LE
Expected Loss	64762.19	LE
Collector Cost	7149.20	LE
Total Cost	119738.43	LE

Table 5.17

System IV Drainage System Design

	1	2	3	4	5	6	7
Depth (m)	1.5	1.5	1.5	1.5	1.5	1.5	1.5
Spacing (m)	33.80	25.63	21.38	24.30	22.31	27.53	35.66
Area (feddan)	28.6	28.6	47.6	66.7	66.7	104.8	57.1
Capital Cost (LE)	2526.	3383.	6754.	8325.	9069.	11547.	4862.
Expected Loss (LE)	4498.	4269.	7544.	10929.	11455.	16579.	8563.
Collector Cost (LE)	303.	303.	793.	945.	945.	1794.	775.
Total Cost (LE)	7327.	7955.	15095.	20199.	21496.	29920.	14200.

Table 5.18

Summary of Case Study Results

	System I	System II	System III	System IV
Lateral Drain System Total Costs (LE)	110800.	113641.	112589.	110346.
Collector Network Total Costs (LE)	5513.	6943.	7149.	5858.
Drainage System Total Costs (LE)	116313.	120584.	119738.	116204.

two and level three. For if the level three or lateral design had been based upon the "best" level two or collector design, the resulting lateral drainage system would have been less efficient. The potential saving for Egypt can be quite substantial. If System II had been implemented rather than System IV an added cost of 4339.2 LE would have resulted which is a 3.7 percent increase in cost. This is an average cost of 1.1 LE/feddans which in the case of Egypt where the drainage installation rate is 500,000 feddan per year, results in an added cost of 500,000 LE per year. These added costs reflect only the impact of including level two - level three feedback. Using the new methodologies presented in this work the saving will be even greater. At the beginning of this section it was demonstrated the savings possible by implementation of each of the separate tools for level III and level II alone. The 3.7% saving from this latest example is only a fraction of the total savings possible if the dynamic multilevel approach is implemented.

5.7 Implication of Dynamic Multi-Level Agricultural Drainage Planning

The use of the dynamic multi-level drainage planning model in the above case study did not fully illustrate the advantage of the methods developed in this work. The case study showed that by providing feedback between level two and level three drainage planning an "optimal" total drainage system could be found (rather than combining an optimal level two decision, collector network design, with an optimal level three decision, lateral drain design based upon the level two decision).

The example did not illustrate the advantage of the dynamic multilevel drainage planning model as a synthesis of approaches that address the issues of uncertainty and spatial variability in design parameters in economic terms.

Chapter 6

SUMMARY, CONCLUSIONS AND FUTURE RESEARCH

6.1 Summary

This work has focused on the planning and design of agricultural drainage projects. It has identified the planning process as a multilevel process. The role of spatial variability and uncertainty of design parameters has been identified and a methodology for their analysis has been developed. The main contribution of this work has been the synthesis of physics, engineering, agronomy, economics and statistics into agricultural drainage planning. This synthesis has been accomplished through the development of a spatially distributed loss, multi-crop stochastic programming model for lateral drain design and a dynamic multi-level planning process that allows for the feedback of information between levels.

Chapter 2 provided an introduction to the field of agricultural drainage. Its historical development and the state of the art of drainage planning were presented. The multi-level planning process was outlined and dicussions of the procedures at each level were presented.

In Chapter 3, the physics governing groundwater flow to drains was presented. The spatial variability and uncertainty of physical drainage parameters were discussed. Methodologies for analyzing two-dimensional spatial correlation and uncertainty were presented. These methodologies were used to identify these processes in a typical drainage field in the Nile Delta. A first-order-second moment analysis to provide a measure of uncertainty in the output of the Hooghoudt drainage equation given the uncertainty in its input parameters was developed. An analysis of the effect of spatial variability upon the FOSM analysis was performed.

Chapter 4 uses the results of the first-order-second moment analysis to develop an optimization model for design of field level drains under uncertainty. Two approaches were proposed, the chance-constraint method which requires the modeller to assign a certain reliability for the system output and the stochastic programming method which minimizes the expected costs due to the uncertainty in the system output. The stochastic programming model was developed for a single crop and multi-crop situation and the two approaches were compared.

Chapter 5 presents the main theme of the work. A simulation model for drainage collector systems was discussed. A method for incorporating the spatial variability of permeability into the design of field level drains to account for the true economic costs of over and under design of field drains was developed. It was shown that the alignment of the collector system has a great effect upon the efficiency of the design of field drains. In response to this a combination optimization-simulation model for the design of total drainage systems, collector network and field laterals, was developed. This model allows tradeoffs between capital cost and expected losses to be analyzed in a systematic format. This combined design of collector and field drains and the accounting of the effects of each upon the other results in a dynamic multi-level planning process with feedback between the levels. A case study of this methodology was performed on an area in the Nile Delta.

6.2 Conclusions

The main conclusion of this work can have a major impact upon the process of agricultural drainage planning through the approaches presented and the techniques developed.

The present planning process was characterized as a sequential process where information flowed from level one to level two to level three. Figure 6.1 is an outline of the techniques that were developed to improve the efficiency of the sequential process.

At level two, a simulation model for standardized, efficient collector network planning was developed. This model provides for the cost of a proposed network design to be estimated and allows an engineer to find the least cost network. The implementation of the model on a digit computer provides for increased productivity of drainage engineers. Coupled with a graphical input device the savings due to increased productivity can be substantial in addition to the savings of designing more efficient networks.

The design of complex collector drain network systems presently is based upon engineering judgment. The use of a simulation model has provided a method for analyzing complex network alignments. This tool enhances experienced designers and greatly aids inexperienced designers.

At level three, the issue to be addressed is the optimal design of field drains. Figure 6.1 shows the development of a method to address this issue. The first step was the development of a mathematical programming model that accounts for uncertainty in the physical system. Using first - order - second moment analysis two approaches were developed: Chance-constraint programming and stochastic programming. Chance-constraint was demonstrated to be applicable only in the case of a monotonic crop field function. This is appropriate in only a small number of cases while the stochastic programming approach is applicable as long as the crop yield function can be identified.

The question of which crop to design the system for in the case of a multiple crop rotation was answered by including all the yield functions for crops

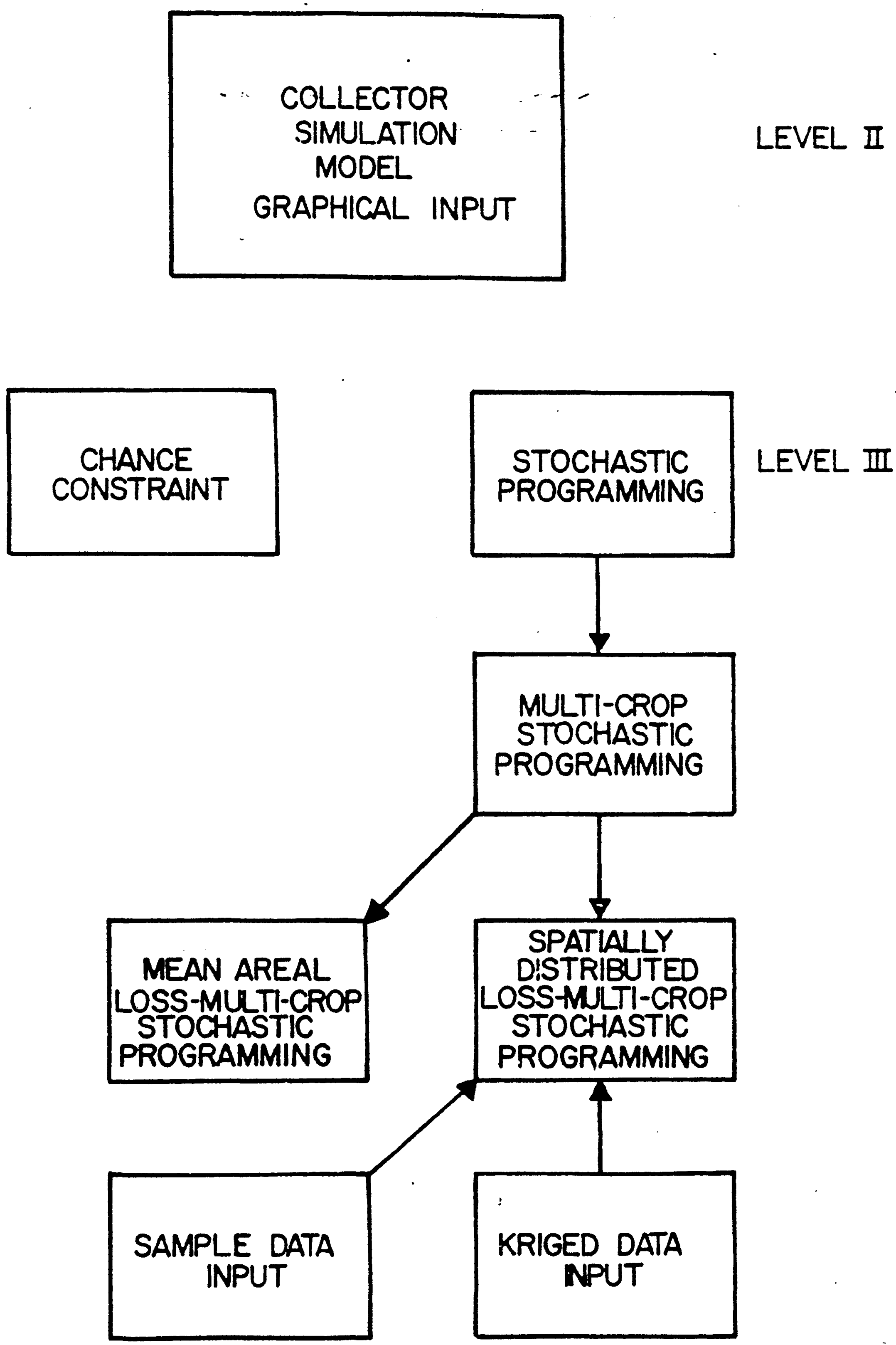


Figure 6.1 - Drainage Planning Models Developed

grown in the design area into the economic measure of the stochastic programming approach. Each crop was weighed by its cultivated area times its economic values. This allows the design of a system based upon total expected cost rather than designing for a "design" crop. The issue of spatial variability in soil permeability was addressed in two ways. One using the present concept of mean areal approach and the spatially distributed loss approach. It was shown that the mean areal approach does not account for the true economic cost of spatial variability in soil permeability. With the spatially distributed loss approach the economic cost of a drain design was analyzed at each data point and the design that minimized total economic costs was chosen. This raised the question of how to represent the spatially varying permeability data. A method of interpolating between data points accounting for their measurement error, kriging, was chosed. The use of kriging produced resulting designs with a cost nineteen percent lower than designs based upon sample data alone. Thus a model for level three design which is superior to the mean value spatial homogeneous approach presently being used was developed.

The analysis of uncertainty and spatial variability in soil permeability has not been previously applied in a design context. This analysis of uncertainty combined with the economics of crop production were incorporated into a comprehensive lateral drain design model that represents a major step forward in the use of system anaysis and economic tools in drainage design.

Although the tools developed greatly improved the efficiency of design at level two and level three, they do not address the question of the impact of level two design upon level three. This impact was studied and shown to be quite substantial under certain conditions. To address this issue a mechanism for information feedback between level two and level three was developed.

This mechanism is presented in Figure 6.2 and is called a Dynamic Multilevel Approach as compared to the present Sequential approach also shown in Figure 6.2.

This feedback mechanism was combined with the models for level two and level three to develop a dynamic multilevel drainage planning model. This approach uses simulation and optimization models together to explicity analyze the effects of collector system alignment upon lateral drain design due to spatial variability of soil permeability.

Although it is hard to quantify the advantages of the level two and level three drainage design models, it has been shown that they are more efficient and provide a more accurate design than present methods. Combining this fact with the dynamic multilevel planning approach illustrated above provides a method for agricultural drainage planning superior to the present sequential approach.

The Dynamic Multilevel Approach is a major contribution to-the agricultural drainage field for it is the first time that the problems of spatial variability as well as collector/lateral drain interaction have been addressed in a formal procedure. With the increasing acceptance of system analysis tools in the drainage planning process, this approach can have major effects upon future drainage design procedures.

In the course of this work a major finding about the use of chance-constraint programming as a method for programming under uncertainty was made. It was shown that if the response of a system being designed was not monotonically non-decreasing with respect to increased system performance then current ideas of reliability on system performance do not hold. A more detailed analysis of the response function must be made and a new measure of reliability defined.

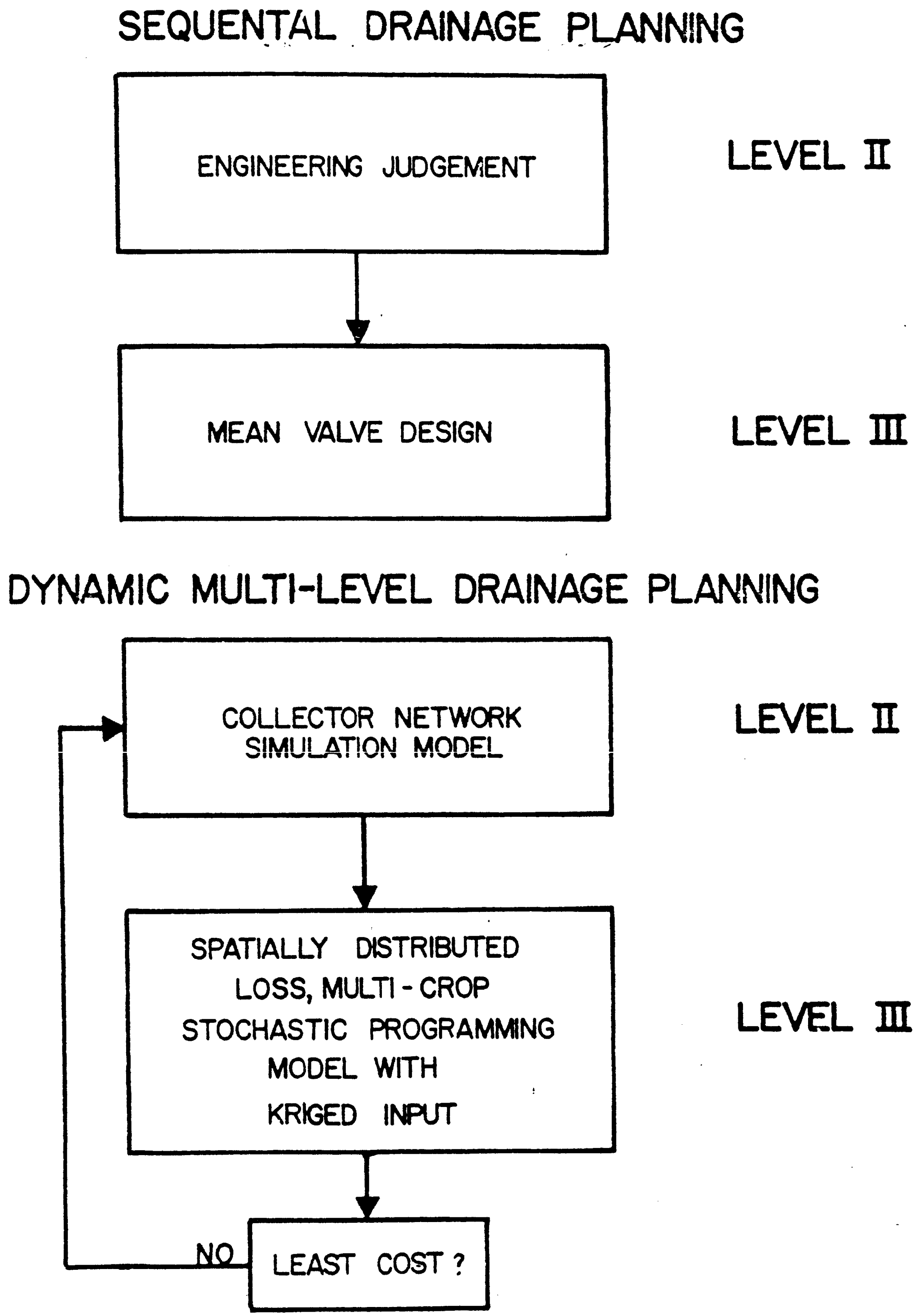


Figure 6.2 - Alternative Drainage Planning Methods

For this case of drainage design, it was shown that due to the nature of irreducible uncertainty in system inputs and yield response functions that stochastic programming was a far better technique for design under uncertainty.

6.3 Future Research

This work is a synthesis of physics, engineering, agronomy, economics and statistics as applied to agricultural drainage. Within each of these components there are areas in which further understanding would enhance the approach presented. The areas that warrant additional research are:

1. The implementation of a non-steady drainage design equation to described the groundwater level between drains.

2. The development of an agricultural yield function that is a function of non-steady state dewatering zone, the variation of the head between the drains, and the incorporation of crop response to salinity.

3. The spatial variability of soil and irrigation parameters should be studied more intensely and the design of optimal sampling network undertaken.

4. The probability and effect of drain failures due to siltation and breakage should be investigated to be incorporated in capital investment decisions

5. The incorporation of uncertainty in the economic parameters in the model presented should be developed to provide a total account of uncertainty.

6. The interaction between the first level (economic feasibility) and levels two and three must be investigated. How much level two and three analysis must be performed before an accurate level one decision can be made must be established.

7. The analysis of scheduling project complementation to provide for the most timely strain of benefits should be studied.

REFERENCES

1. Ackoff, R.L. (1961) , Progress in Operations Research, Vol. 1, John Wiley & Sons, Inc.

2. Aldabagh, A.S.Y. and C.E. Beer (1975) , Economics of Increased Mobility from Tile Drainage. Transaction of the ASAE.

3. Alexandridis, M.G., and K.M. Strzepek and D.H. Marks (1979), Computer Graphics Model for Drainage Design, Ralph M. Parsons Laboratory, MIT, TAP Working Paper.

4. Amer, M. Hassan (1979), Personal Communication.

5. Anderson, J.R., J.L. Dillion and B. Hardaker (1977) , Agricultural Decision Analysis, Iowa State University Press.

6. Bakr, A.A., L.W. Gelhar, A.L. Gutjhar and J.R. MacMillan (1978) , "Stochastic Analysis of Spatial Variability in Subsurface Flows, 1, Comparison of One- and Three-Dimensional Flows, " Water Resour. Res., 14, 263-271

7. Bear, J. (1979), Hydraulics of Groundwater, McGraw-Hill.

8. Beers, W.F.J. van (1976) , The Auger Hole Method, Bulletin No. 1, International Institute for Land Reclamation and Improvement, Wageningen, Netherlands.

9. Benjamin, J.R. and C.A. Cornell (1970) , Probability, Statistics and Decision for Civil Engineers, McGraw-Hill, Inc., New York

10. Bouwer, H. and R.D. Jackson (1974), "Determining Soil Properties," in Drainage for Agriculture, edited by Jan Van Shilfgaarde, Am. Soc. of Agronomy, Monograph No. 17.

11. Cavelaars, J.C. (1974), "Subsurface Field Drainage Systems," Vol. IV, Design and Management of Drainage Systems, Drainage Principles and Applications, International Institute for Land Reclamation and Improvement, Wageningen.

12. Chan, S. and R.L. Bras (1979) , "Urban Storm Water Management: Distribution of Flood Volumes," Water Resources Research, Vol. 15, No. 2, 371-382.

13. Charnes, A. and W.W. Cooper (1959) , "Chance Constraint Programming" Management Science, Vol. 6, No. 1, 73-79.

14. Christopher, J.W. and R.J. Winger (1975) , "Economic Drain-Depth for Irrigated Areas," paper presented at ASCE meeting Logan, Utah, August 1975.

15. Chua, Siong-Huat (1980), Optimal Estimation of Mean Areal Precipitation in Regions of Orographic Influence, S.M. Thesis, Department of Civil Engineering, Massachusetts Institute of Technology.

16. Cordova, J.R. (1977) , A Methodology to Assess the Effect of Water Supply Shortages on the Design and Operation of Irrigation Systems, M.S. Thesis, Department of Civil Engineering, Massachusetts Institute of Technology.

17. Cordova, J.R. and R.L. Bras (1979), Stochastic Control of Irrigation Systems, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Technical Report No. 239, Massachusetts Institute of Technology.

18. Cornell, C.A. (1972) , "First Order Analysis of Model and Parameter Uncertainty," International Symposium on Uncertainties in Hydrologic and Water Resource Systems, Univ. of Arizona, Tucson.

19. Dagan, G. (1979) , "Models of Groundwater Flow in Statistically Homogeneous Porous Formations," Water Resour. Res. 15, 47-63.

20. Dantzig, G.B. (1955) , "Linear Programming Under Uncertainty," Management Science, Vol. 1, 197-206.

21. David, J.C. (1973) , Geostatistical Ore Reserve Estimation, Elsevier Scientific Publishing Company.

22. Davis, J.C. (1973) , Statistics and Data Analysis in Geology, John Wiley and Sons, Inc.

23. Delfiner, P. (1976) , "Linear Estimation of Non-stationary Spatial Phenomena," in M. Guarascia, et al (ed.), Advance Geostatistics in the Mining Industry, NATO Advanced Study Series, D. Riedel Pub. Co. Boston.

24. Delhomme, J.P. (1978) , "Kriging in the Hydrosciences," Advan. Water Resour. 1, 251-266.

25. DeLucia, R. (1969) , Operating Policies for Irrigation System under Stochastic Regimes, Ph.D. Thesis, Department of Engineering and Applied Physics, Harvard University.

26. Dettinger, M.D. and J.L. Wilson (1981) , "First Order Analysis of Uncertainty in Numerical Models of Groundwater Flow, Part I: Mathematical Development," submitted to Water Resour. Res. for publication.

27. Dettinger, M.D. and J.L. Wilson (1982) , "First Order Analysis of Uncertainty in Numerical Models of Groundwater Flow",Part 2, Water Resour. Res., in press.

28. Dudley, N. and O. Burt (1973) , "Stochastic Reservoir Management and System Drainage for Irrigation," Water Resources Research 9.

29. Eagleson, P.S. (1978) , "Climate, Soil and Vegetation, 5., A Derived Distribution of Storm Surface Runoff," Water Resour. Res. 14, 741-748.

30. El Berry, A.M. (1979) , Evaluation of Current Methods for Tile Drainage Systems in Egypt, Ph.D. Thesis, Department of Agricultural Engineering, Cairo University.

31. El Ghamry, O.A. (1978), "A Stochastic Approach for the Evaluation of Drainage Projects," Paper presented at the Tenth Congress: International Commission on Irrigation and Drainage.

32. FAO (1973), Irrigation, Drainage and Salinity, Hutchinson/FAOUNESCO.

33. Fogel, M., L. Duckstein and A. Musy (1978) , "Event Based Formulation of Watershed Management," Preceedings ASCE Irrigation and Drainage Society Conference.

34. Framji, K.K. and I.K. Mahajan (1969), Irrigation and Drainage in the World, Vol. I & II, ICID, New Delhi.

35. Freeze, R.A. (1975) , "A Stochastic-conceptual Analysis of One-dimensional Groundwater Flow in Non-uniform Homogeneous Media," Water Resour. Res., 11, 725-741.

36. Frogge, R.R. and G.D. Sanders (1977), "USBR Subsurface Drainage Design Procedure," Proceedings ASCE Irrigation and Drainage Specialty Conference.

37. Fukuda, H. (1976), Irrigation in the World, University of Tokyo Press.

38. Gelhar, L.W. (1974) , "Stochastic Analysis of Phreatic Aqufiers," Water Resour. Res. 10 721-733.

39. Gelhar, L.W. (1976) , "Effects of Hydraulic Conductivity Variations on Groundwater Flows",in Proceedings Secound International IAHR Symposium on Stochastic Hydraulics IAHR, Lund, Sweden.

40. Gelhar, L.W., P.Y. Ko, H.H. Kwai, and J.L. Wilson (1974), Stochastic Modelling of Groundwater Systems, Technical Report No. 189 Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Department of Civil Engineering, Massachusetts Institute of Technology.

41. Gelhar, L.W., J.L. Wilson and A.L. Gutjhar (1979), Comment on: "Simulation of Groundwater Flow and Mass Transport under Uncertainty," Adv. Water Res., submitted for publication.

42. Gutjhar, A.L., L.W. Gelhar, A.A. Bakr and J. MacMillan (1978) , "Stochastic Analysis of Spatial Variability in Subsurface Flow 2., Evaluation and Application," Water Resour. Res., 14, 953-959.

43. Hooghoudt, S.B. (1940), Bijdragentot de Kennis van enige Natuurkundige grootheden van de grond. No. 7. Versllandbouwk, Onderz. 46:515-707.

44. Howell, T.A. (1974), Optimization of Grain Sorghum Water Use Efficiency under High Frequency Irrigation by System Simulation and Stochastic Dynamic Programming, Ph.D. Thesis, Department of Agricultural Engineering, Texas A&M University.

45. Huijbregts, C. and G. Mathern (1971), "Universal Kriging (An Optimal Method for Estimating and Contouring)," Canadian Institute of Mining and Metallurgy, Decision-Making in the Mineral Industry, Special Vol. 12, 159-169.

46. International Institute for Land Reclamation and Improvement (1973), Drainage Principles and Applications, Vols. I, II, III, IV.

47. Journel, G. and C. Huijbreghts (1978), "Mining Geostatistics" Academic Press, New York.

48. Knapp, J.W. (1978), "Economic Impacts of Delays in the Construction of Irrigation and Drainage Projects," Paper presented at the Tenth Congress: International Commission of Irrigation and Drainage.

49. Lemarchel, C. and R. Mifflin, eds. (1978), Nonsmooth Optimization, IIASA Proceedings Series No. 3, Pergamon Press.

50. Luthin, J.N. (1978), Drainage Engineering, Robert E. Krieger.

51. Luthin, J.N. (1957) Editor, Drainage of Agricultural Lands, Am. Soc. of Agronomy, Monograph No. 7.

52. Ministry of Irrigation (Arab Republic of Egypt) (1965), Crop Loss Function, M. of I. Technical Report.

53. Musy, A. and L. Duckstein (1976), "Bayesian Approach to Tile Drain Design," J. of the Irrigation and Drainage Division, ASCE, IR3.

54. Olea, R.A. (1975), Optimal Mapping Techniques Using Regionalized Variable Theory, Kansas Geological Survey.

55. Renner, D.M. and C.C. Mueller (1974) , "Drainage System Design and Analysis by Computer," Journal of Irrigation and Drainage Division, ASCE, IR3

56. Onigkeit, D., C. Kim and B. Schmid (1969), "Optimal Design of an Irrigation System," Paper presented at the 7th Congress of International Commision on Irrigation and Drainage.

57. Sagar, B. (1978), "Galerkin Finite Element Procedure for Analyzing Flow Through Random Media," Water Resour. Res., 14, 1035-1044.

58. Sagar, B. and C.C. Kisiel (1972) , "Limits of Deterministic Predictability of Saturated Flow Equations," Joint Symposium on Fundamentals of Transp. Phenomena in Porous Media, Univ. of Guelph, Canada.

59. Simmons, D.M. (1975), Nonlinear Programming for Operations Research, Prentice-Hall, Inc.

60. Smith, D.A. (1973), "Systems Analysis and Irrigation Planning," Journal of Irrigation and Drainage Division, ASCE, IR1.

61. Smith, L. and R.A. Freeze (1979), "Stochastic Analysis of Steady-State Groundwater Flow Bounded Domain, 1, One-dimensional Simulations," Water Resour. Res., forthcoming.

62. Strzepek, K.M. and D.H. Marks, (1979), A Model for Agricultural Drainage Planning Using Computer Graphic, Draft TAP Working Paper, MIT.

63. Tang, D.H. and G.F. Pinder (1977), "Simulation of Groundwater Flow and Mass Transport," Adv. Water Res., 1, 25-30.

64. Trafford, B.D. (1975), "Improving the Design of Practical Field Drainage," Soil Science, 119, No. 5

65. True, G.M. (1977), "On-farm Drainage Design Procedures used by ADS, Inc.," Paper presented at the ASCE Irrigation and Drainage Conference.

66. United States Department of Agriculture (1973), Soil Conservation Service, Drainage of Agricultural Land, Water Information Center, Inc.

67. United States Department of the Interior (1978), Bureau of Reclamation, Drainage Manual, A Water Resources Technical Publication.

68. Van Schilfgaarde, J. (1957), "Approximate Solutions to Drainage Flow Problems," Drainage of Agricultural Land, James L. Luther, editor, Monograph No. 7, American Society of Agronomy.

69. Van Schilfgaarde, J. (1965), National Symposium on Drainage, ASAE.

70. Van Schilfgaarde, J. (1974), "Introduction", Drainage for Agricultural Land, J. Van Schilfgaarde, editor, Monograph No. 17, American Society of Agronomy.

71. Visser, W.C. (1958), De Landouwwaterhuishouding in Nederland, Comm. Onderz. Land. Waterhuish. Ned. TNO. 1, p. 231.

72. Veneziano, D. (1978), Class Notes, Department of Civil Engineering, MIT.

73. Warren, J.E. and H.S. Price (1961), "Flow in Heterogeneous Porous Media," Soc. Petrol. Eng. J., 1, 153-169.

74. Wesseling, J. (1973), "Subsurface Flow into Drains," Vol. II: Theories of Field Drainage and Watershed Runoff, Drainage Principles and Applications, International Institute for Land Reclamation and Improvement, Wageningen.

75. Wilson, J.L. and M.D. Dettinger, (1982) "Elementary Applications of First Order Analysis to Groundwater Hydrology," Water Resour. Res., in press.

76. Wiser, E.H., R.C. Ward and D.A. Link (1974), "Optimized Design of a Subsurface Drainage System," Trans. of the ASAE.

77. Wismer, D.A. and R. Chattery (1978), Introduction to Nonlinear Optimization: A Problem Solving Approach, North Holland

78. World Bank (1977), Appraisal of Nile Delta Drainage II Project Arab Republic of Egypt, Report No. 1451-E6T.

79. Yaron, D. (1971), "Estimation and Use of the Water Production Function in Crops," Journal of Irrigation and Drainage Division, ASCE, IR2.