Evaluating Utility Investment Decisions -
An Options Approach

by

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Abstract

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In the 1970's and 80's, slower demand growth combined with constantly rising power plant construction costs has led many electric utility managers to regret commitments to construct large plants with long lead times. In retrospect, smaller more flexible alternatives would have been a better choice. Utility regulators have not been sympathetic to the industry's plight. Large nuclear and coal plant construction costs have been disallowed, which have sent many utility stocks plummeting.

Could these events been avoided had utilities adopted more flexible investment strategies? Would these more flexible strategies have been able to justify their costs?

The purpose of this thesis is to develop a methodology to answer these questions both looking to the past and to the future. In accomplishing its goal, the thesis draws upon options theory from finance. The thesis shows that methods used to value options on financial assets can also be used to value options on physical assets such as power plants.

The thesis has two main parts. Initially, an analytical model is developed which can be used to compare the economic merits of competing utility generation alternatives. The model provides a rigorous treatment of flexibility and explicitly considers its value. In the second half, the model is applied to generic utility data to demonstrate the importance of flexibility.

The main portion of the analysis compares a single 800 MW base-load coal plant to a series of five 160 MW combustion turbines. The major finding is that the flexibility associated with smaller power plants has significant monetary value. Some of the more interesting results are:

- For a reasonable set of assumptions, the value of the flexibility associated with the smaller combustion turbines ranges from 70 - 350 million dollars
This figure represents 20 to 100% of the capital costs associated with the turbines

- Decreasing the size of the combustion turbines from 160 to 40 MW increases the value of flexibility by an additional 100 - 200 million dollars.

- Building modular coal plants which are the same size as the combustion turbines, has the same effect as reducing the coal plant's capital cost by 20%.

Unfortunately, the benefits of smaller power plants are not without cost. Small scale power production offers significant challenges to utility management. These challenges can be grouped into two categories - technological and organizational. On the technology front, design and construction engineers will need to focus on developing small scale generation alternatives which do not sacrifice economies of scale in construction and thermal efficiency in operation. Organizationally, utility planners will need to adopt a "just-in-time" mentality to capacity expansion. The benefits associated with small scale generation cannot be accomplished unless utilities structure their organizations to respond quickly to the opportunities which flexibility provides.

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Of course misery loves company, and how do I love my Sloan classmates. In particular, I am grateful to Jim Elkind, Neil French, and David Orlin for reading and commenting on this thesis.

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Finally, I'd like to thank my wife, Laurel. Not only did she read this thesis, but she had to live with me during its entire composition - a heroic feat I assure you.
I. Introduction

During the mid-1970's and early 1980's many U. S. electric utilities saw their well-intentioned investment plans turn sour due to an increasingly uncertain business environment. Volatile fuel prices, declining load growth, combined with construction delays and cost overruns caught utility managers off guard. Particularly hard hit have been utilities that invested in nuclear power plants which far exceeded their initial cost estimates.

Regulators have been quick to blame utility management for not properly assessing the risks associated with the plant investments they undertook. Furthermore, utilities often committed themselves to projects which severely limited their flexibility to make future decisions. These accusations by the regulators beg the question: How does one assess risk and value flexibility? Discounted cash flow techniques do not have the capability to evaluate flexibility. As a consequence, regulators and utility analysts have debated qualitatively over these issues. However, more recently research in finance has shown that options valuation models can be used to quantitatively evaluate risk and flexibility.

Options, first popularized in financial markets, provide the owner with the right but not the obligation to buy or sell an underlying asset at a prespecified exercise price. Generally, an option can be exercised on or before a prespecified exercise date. The original research in this area focused on the valuation of options on financial instruments (Black and Scholes 1973; and Merton 1973). This research showed that option value is dependent on six fundamental variables: the current price of the underlying security, the volatility of future price movements, the expiration date of the option, the option's exercise price, the interest rate, and the dividend stream associated with the underlying security.
While the impacts of most of these variables are straightforward, it is
important to note that increased stock price volatility will enhance the value of both
calls (the option to buy) and puts (the option to sell). Intuitively, this result arises
because the worst thing that can happen to the owner of an option is that the option
expires worthless, while the upside potential is virtually limitless. Therefore,
increased volatility will result in greater upside potential without a corresponding
increase in downside risk.

Related to this point, is the fact that for non-dividend paying stocks, options
should not be exercised until the expiration date. By exercising early, the upside
potential associated with volatility is lost. The only gain from early exercise is the
dividend cash flow stream. Thus, if the dividend cash flow stream is zero, early
exercise cannot be an optimal trading policy. A more detailed discussion of the
specifics of options pricing can be found in Cox and Rubinstein (1985).

The options model is easily transferred to options on commodities or real
investment projects. The valuation of an option on a real asset sometimes called a
real option exactly parallels the valuation of financial options. For example, a utility
typically develops a number of suitable sites for constructing power stations.
Consequently, the utility has the right but not the obligation to build a plant on any
one of these sites. In this case, the real asset is the completed plant. Its value
behaves similarly to the value of a financial asset, in that it fluctuates as fuel prices
and demand fluctuate. The exercise price is the cost of constructing the plant. The
option expires when the site is no longer suitable for construction. Thus, the
economic value of a plant site is the value of an option to buy (call option) the
power plant.

A great deal of work has come out of the finance literature focusing on
options on real assets. One of the underlying themes of this resea:ch is that in
general it is best to wait as long as possible before investing in new capacity (i. e.
exercising the option). This result is a natural extension of the fact mentioned above that non-dividend paying stocks are optimally exercised on the expiration date. MacDonald and Siegel (1986) have tried to value the increased flexibility firms gain by waiting to invest in capacity. In a similar vein, Majd and Pindyck (1987) show that the value of waiting is enhanced when one considers the time it takes to complete investment projects. Myers and Majd (1985) analyze the value of the option to abandon a risky project once it's under construction.

The goal of this thesis is to apply the research on the theory of options valuation to the utility plant investment problem. This analysis can be used both in regulatory (retrospective) analysis and in future capacity planning decisions. While this analysis will focus primarily on capacity planning, there are many areas of utility management to which options theory can be applied (e.g., system operations and fuel procurement). The discussion to follow is intended to give the reader a basic understanding of the options model in a planning context and stimulate ideas for further application.

To achieve this goal, we will start in Chapter 2 with an illustrative example of the benefits of options techniques. In Chapter 3, the utility investment problem will be introduced and solved using a discounted cash flow approach. Chapter 3 will provide a useful point of reference for a later analysis of options models. The development of an options framework will be conducted in Chapter 4, while Chapter 5, will use the model to conduct several test studies which will highlight the features of the options methodology. Chapter 6 will conclude with some suggestions on how options analysis could be employed to address some of the managerial issues facing the electric utility industry.
II. **Illustrative Example**

Before developing a more detailed model to evaluate the trade-offs between a coal plant and combustion turbines, we will start with a simple example. The initial example will serve the dual purpose of introducing the options pricing methodology and demonstrating the benefits of options methods over simple discounted cash flow and decision analysis models.

Let's begin by considering the two plant types shown in Exhibit I. The plants each have a capital cost of zero. Plant type A is has 200 MW of generating capacity while plant B has a capacity of 100 MW. Demand is assumed to grow deterministically at 100 MW per year. Therefore, it will take two years for plant A to be absorbed into the system, while plant B will satisfy only a single year of demand growth.

Both plants can be constructed immediately. Plant type A has an annual fuel cost of $19 million while plant type B has a fuel cost of $20 million in the first year. The fuel cost for plant type B moves stochastically. In year 2, the cost to fuel plant B will either be 14 or 30 million dollars. Each fuel price can occur with a probability of 50%. Beyond year 2, the fuel costs will remain at year 2 levels.

Calculating the present value of costs to plant A, we take the $19 million per year fuel cost in the first period. Since fuel costs for plant A do not fluctuate, the fuel costs will be $19 million in every year beyond the current period. If the plant has a life of 30 years then, using a 10% discount rate the present value of costs can be calculated as below.

\[
\text{Cost of plant A} = 19 + \frac{19}{1.1} + \cdots + \frac{19}{1.1^{29}} = 186.7M
\]
Turning our attention to plant B, the initial fuel cost outlay in year 0 is $20 million. Next year, fuel costs will be either $14 million or $30 million. On an expected value basis, the fuel cost will be the weighted average of these two or $22 million (.5*14 + .5*30 = 22). Since fuel costs are expected to remain stable beyond year 1, we can continue to use $22 million as the fuel cost for the remainder of the 30 year plant life. The discounted value of the cash flows associated with plant B are

$$\text{Cost of plant B} = 20 + \frac{22}{1.1} + \ldots + \frac{22}{1.1^{29}} = 39.4M.$$

Comparing the present value of costs across the two plants, shows that plant A enjoys a $39.4 million advantage. This advantage is derived from the lower fuel costs attributable to plant A. Thus, the discounted cash flow analysis suggests that plant A is the proper investment choice for the utility.

Since type A’s future fuel costs are known with certainty, some people might argue that type A is less risky; consequently, its benefit is understated by the calculation above. The fallacy of this logic lies in the fact that by building plant A, the utility is committed to using fuel A for two years. Hence, the utility is giving up the flexibility to switch to plant type B if conditions warrant it (i.e. if the cost of fuel B goes down). On the other hand, if the utility builds plant B, management will have the option to build either A or B in year two.

In order to consider this option, we can use an options or decision analysis methodology. This method is superior to the naive discounted cash flow method because it considers flexibility.
Exhibit 1
Simple Example Parameters

System Data

Load Growth = 100 MW/yr
Discount Rate = 10%

Plant A

Fuel Costs (millions of dollars)

<table>
<thead>
<tr>
<th>yr 0</th>
<th>yr 1</th>
<th>yr 2</th>
<th>...</th>
<th>yr 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>19</td>
<td>19</td>
<td>...</td>
<td>19</td>
</tr>
</tbody>
</table>

Plant B

<table>
<thead>
<tr>
<th>yr 0</th>
<th>yr 1</th>
<th>yr 2</th>
<th>...</th>
<th>yr 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>30</td>
<td>30</td>
<td>...</td>
<td>30</td>
</tr>
</tbody>
</table>

20

<table>
<thead>
<tr>
<th>yr 0</th>
<th>yr 1</th>
<th>yr 2</th>
<th>...</th>
<th>yr 29</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>14</td>
<td>14</td>
<td>...</td>
<td>14</td>
</tr>
</tbody>
</table>
Exhibit 2 shows that the first period (year 0) fuel costs are $19 million and $20 million for plants A and B respectively. So far the costs are exactly the same as the discounted cash flow analysis above.

In evaluating the costs for the next year (year 1), the owner of plant A is locked into a fuel cost of $19 million; however, remember that the owner of plant B has another decision to make. By starting with plant B, the utility can either switch to plant A if fuel B increases to 30 or continue with another plant of type B if fuel B decreases to 14. By switching, the fuel cos's will be 19, while continuing will yield fuel costs of 14. Thus, if we make an investment in plant B today, our fuel costs next year will either be $19 million or $14 million. Recalculating the expected value of fuel costs in the second period (year 1) gives us $16 million (.5*19 + .5*14 = 16.5), not $22 million as in the discounted cash flow approach.

Moving to the next period (year 2), the load will now have grown sufficiently to fully utilize plant A. Thus, if our initial investment was in plant A, we will need to add some new capacity. Since fuel costs do not fluctuate beyond year 1, we will make the same decision made by the owner of plant B last period (i.e. build plant A if fuel B is 30, build plant B if fuel B is 14). Furthermore, since fuel prices are no longer stochastic, we and the owners of plant B will continue to make the same plant choice out into the future. Therefore, in year 2 and beyond the fuel cost will be the same, regardless of the plant choice today.

Thus, the only relevant costs in the options approach are the fuel costs incurred during the first two years. The present value of these costs are calculated at the bottom of Exhibit 2. The flexibility associated with plant B has tipped the scales in its favor. The net benefit to plant B is now $1.3.
Exhibit 2
Illustrative Example
Options Approach

Fuel Costs

<table>
<thead>
<tr>
<th>yr 0</th>
<th>yr 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUILD PLANT A</td>
<td>19</td>
</tr>
<tr>
<td>BUILD PLANT B</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fuel Cost Plant A = $19 + 19/1.1 +
Cost of cheaper fuel beyond year 1

Fuel Cost Plant B = $20 + 16.5/1.1 +
Cost of cheaper fuel beyond year 1

Fuel Cost A - Fuel Cost B = $1.3M,

THEREFORE BUILD PLANT B.
Note that using an options model we can calculate the value of flexibility directly. Thanks to flexibility, an initial investment plant B will result in an expected fuel cost of $16.5 million in year 1. Compared with the $19 million fuel cost we will incur by building plant A, flexibility provides a savings of $3 million. Discounting by one period, the value of flexibility today is $2.3 million (2.5/1.1 = 2.3). Thus, by using an options approach we are able to consider the flexibility associated with plant B.

The example above shows that the options model yields a better result than the discounted cash flow technique, since it explicitly considers flexibility. However, in the calculations above, it was necessary to know the probability that fuel B would move up or down. Suppose these probabilities are not known in advance as is often the case in the real world. It turns out that we can use the options methodology even if we don’t know the the true probabilities of oil price moves a priori. The fact that we don't need to know these probabilities is another attractive feature of the options method.

Demonstrating this feature requires a slightly different approach to valuing the flexibility associated with plant B. In this approach, we will first show how the flexibility associated with plant B is analogous to an option on a common stock. Once the option is identified, an arbitrage argument can be used to determine the option's value. To make the arbitrage argument, we will duplicate the benefits of plant B's flexibility with a portfolio of commodities and securities purchased today. Since the portfolio and plant B's flexibility will produce the same benefits in the future, they should have the same value today. Therefore, the value of plant B's flexibility must be equal to the value of the duplicating portfolio.

To put the problem into an options framework, recall that the fuel for plant B can move either to 30 or to 14, yet as we saw in the decision analysis example the cost that we ultimately face is either 19 or 14. Relative to plant A which costs 19 in
all cases, the payoff to flexibility is either 0 or 5. Expressed slightly differently, the payoff of flexibility is the greater of 0 or \( P_a - P_b \), where \( P_a \) is the price of fuel A and \( P_b \) is the price of fuel B. The payoff can be written more concisely as \( \max\{0, P_a - P_b\} \), where the max operator returns the greater of two values.

We can express the payoff of a put option (the option to sell a stock) in a similar form. Suppose that the exercise price of the put is \( K \) and the value of the stock at expiration is \( S^* \). If the stock price falls below \( K \), we buy the stock and sell it immediately for \( K \), netting a profit of \( K - S^* \). Otherwise, we let the option expire worthless. Thus, the payoff to a put at expiration is \( \max\{0, K - S^*\} \). This payoff is compared to the flexibility option below.

Payoff to put option at expiration: \( \max\{0, K - S^*\} \)
Payoff to flexibility: \( \max\{0, P_a - P_b\} \)

Comparing these payoffs, we see that the flexibility option is analogous to a put option on the price of fuel B with an exercise price of \( P_a \). Note that if the price of fuel B falls, we exercise our flexibility by continuing with plant B; otherwise we let the option expire.

Now that we have identified the option in the example, the next step is to value it. As mentioned above, our strategy is to duplicate the future payoffs of the option with a portfolio of commodities and securities purchased today. Since the duplicating portfolio and the option have the same future value they must also have the same value today. Therefore, the value of the option is equal to the value of the replicating portfolio. We will start by developing a general rule for stock options and later apply that rule to value the flexibility option.

To determine the composition of the duplicating portfolio, first note that the payoff of the option is closely related to the price of the stock. As owners of a put,
we are hoping that the price of the stock will fall. If the stock price does fall then the value of the put option, all else equal, will rise. Similarly, if we make a short sale in a stock, a drop in the stock price will cause the value of our position to increase. Thus, it seems reasonable that our portfolio should contain short sales of stock. Since the option does not have the same risk profile as the stock, we need to adjust the portfolio risk to the risk of the option by purchasing bonds. Given these two securities, we now need to determine the proportion of each to include in the portfolio.

\[
\begin{align*}
S & \quad \quad P_u = \max(0, K - uS) \\
\downarrow & \quad \quad P_d = \max(0, K - dS) \\
uS & \quad \quad dS
\end{align*}
\]

To solve this problem, assume that the stock price \( S \) will move either up by \((u-1)\) percent to \( uS \) or down by \((d-1)\) percent to \( dS \) (see diagram above). Given the behavior of the stock price, the value of the option, \( P \), will move to \( P_u = \max\{0, K - uS\} \) if the stock price goes up and to \( P_d = \max\{0, K - dS\} \) if the stock price goes down. If we define \( r \) as one plus the risk-free rate of interest, a \$1 \) investment in government bonds will yield \( r \) dollars at the end of one period regardless of the behavior of the stock price. In order for the portfolio to replicate the payoff of the put option, the following equalities must be satisfied:
\[ rB + nuS = P_u = \max \{0, K - uS\} \]
\[ rB + ndS = P_d = \max \{0, K - dS\} \]

where \( B \) = the dollar value of bonds purchased,
and \( n \) = the number of shares of stock in the portfolio.

Solving the above system of equations for \( B \) and \( n \) we get

\[ n = \frac{(P_u - P_d)}{S(u - d)} \]
\[ B = \frac{(uP_d - dP_u)}{r(u - d)}. \]

Thus, by purchasing \( n \) shares of stock and \( $B \) in bonds, we will exactly replicate the payoffs of the option. Therefore, the value of the option is

\[ P = nS + B. \]

Alternatively, this expression can be rewritten as follows

\[ P = \frac{(P_u - P_d)}{(u - d)} + \frac{(uP_d - dP_u)}{r(u - d)} \]
\[ P = \frac{[P_u (r - d)}{(u - d)} + \frac{P_d (u - r)}{(u - d)]/r. \]

If we let \( p = \frac{(r - d)}{(u - d)} \), then \( 1 - p = \frac{(u - r)}{(u - d)} \), and

\[ P = [pP_u + (1 - p)P_d]/r. \]

Before returning to our example, there are several features of the above formula worth noting. First, this equation is a single-period binomial option pricing formula for European puts. While it is limited to just two possible outcomes for stock
(uS and dS) and put option values (P_u and P_d), this model is easily extended to evaluate both puts and calls over many periods. Moreover, the multiperiod version of this model converges to the well-known Black-Scholes model as the time interval between stock price changes becomes increasingly small (i.e. continuous trading). Thus, the single-period model we have just derived is the basic building block for any options pricing model we choose to develop.

Second, we did not need to know the probability that the stock price would go up or down. Given that we know all possible outcomes for fuel prices, no subjectivity is introduced into the model. Therefore, as mentioned earlier, we do not need to know the probability of stock price moves to value an option.

Third, even though we have not made use of the probabilities that events will occur, the formula does show that the value of the put is the weighted average of all possible future values for the put (i.e. P_u and P_d). Nevertheless, the weights employed are not the probabilities that the stock will move up or down. Rather, these weights turn out to be those which cause the expected rate of return on the stock to equal the risk-free rate. To see this point, note that a risk-free investment of S dollars will yield rS dollars in one period. Using the same notation as previously, we see that

\[ rS = puS + (1 - p)dS, \]

and \[ p = \frac{(r - d)}{(u - d)} \] which is the same result we obtained earlier.

This point is particularly important in our valuation of the utility's flexibility option, since it clearly highlights the difference between decision analysis, which employs probability weights, and an options model.

Returning to our example, the flexibility option can be evaluated using the formula derived for stock options above. Based on our fuel price assumptions, S =
20, \( u = 1.5 \), and \( d = .7 \). Furthermore, we know that the flexibility option will either be worthless if fuel prices go up or worth $4 million if fuel prices decline. Thus, \( P_u = 0 \) and \( P_d = 5 \). Finally, we have set the discount rate at 10% so \( r = 1.1 \). Plugging into the formula:

\[
p = (1.1 - .7) / (1.5 - .7) = .5 \quad \text{and} \quad 1 - p = .5.
\]

\[
P = [.5(0) + .5(5)] / 1.1
\]

\[
P = $2.3 \text{ million}.
\]

Thus, we see that the proper economic value of flexibility is $2.3 million.

Returning to the investment problem, if plant type A is built, the flexibility option can be created by buying the replicating portfolio at a cost of $2.3 million. However, the same flexibility can be maintained by building plant B to start and incurring $1 million in added fuel costs in year 0. Therefore, starting with plant type B is the economically correct decision, since flexibility can be achieved for $1.3 million less.

Note that this result is exactly the same as the result we achieved earlier when the true probabilities of oil price moves were known. Thus, when an options model is used subjective probability weights are not required to determine the value of flexibility.

While the example is simplistic, it does show the potential for improved decision making from options theory. The options model allows for the explicit treatment of flexibility which a discounted cash flow model cannot handle. Furthermore, the options methodology does not require subjective probability weights. Using options valuation, utility planners do not need to make probability estimates of oil going up or down, rather they simply need to determine the volatility
of oil price movements over a given period of time. Thus, by using options, we are able to value flexibility more accurately with less information.

III. An Introduction to the Utility Investment Problem

The example above was intended to highlight the attractive features of the options approach. Nevertheless, the example was quite far removed from the actual utility investment problem. The purpose of this chapter is to introduce the reader to a more realistic utility investment problem. Since the utility plant investment decision is complex in its own right, this chapter will introduce the problem and solve it using a simple discounted cash flow (DCF) approach. By clearly defining the problem in the DCF framework, it will be easier to move to a more sophisticated options framework. Thus, this Chapter provides a useful stepping stone to the development of an options model in Chapter 4.

In introducing the utility investment problem and providing a transition to the options model development in Chapter 4, this Chapter will be divided into four parts. Section 3.1 will define the investment problem and outline the assumptions we will make in solving the problem. We will then define the relevant costs associated with the investment decision in section 3.2. Section 3.3 will solve the investment problem and develop a decision rule within a DCF framework. Finally, section 3.4 will summarize the DCF model developed in the first three sections.

3.1 Problem Description

Suppose that we are utility decision makers trying to choose our next generation plant. In order to keep the problem manageable, we will assume that
demand\(^1\) will grow steadily at a constant rate over the planning horizon. In addition, we will allow for power plants to be built instantaneously. Our choices for plant type are restricted to two technologies, a base load coal facility and a combustion turbine (CT) peaking facility. The capital cost for each of these technologies is known with certainty.

The fuel costs associated with each alternative are expected to grow at a constant rate. While the actual movement in fuel prices might diverge from this expected path. On average the expected growth rate provides the best estimate for future fuel prices, and thus they can be used in the analysis.

Typically, a coal plant will have a much larger megawatt capacity than a combustion turbine. This difference in size arises because there are significant economies of scale in coal plant construction. These economies of scale are not as important for combustion turbines, since CT technology does not require a boiler and other steam generation equipment. Thus, our model will assume the coal plant represents a large commitment of capacity which will satisfy several periods of demand growth. The CT’s, on the other hand, will be sized such that they satisfy a single period of demand growth.

If the coal plant is sized to meet \(\tau\) periods of load growth, then the decision the utility faces today is whether to build a single coal plant or \(\tau\) CT’s (see diagram below). To do a DCF analysis, we simply calculate the capital and operating costs of each alternative over its life and choose the alternative with the lowest cost.

\(^1\) In the electric utility industry demand is also demand is also referred to as load. Peak demand or peak load is the maximum demand which a utility serves in a given interval.
3.3 Identifying the Relevant Costs

The cost of either plant alternative can be broken down into capital and fuel components as shown below:

\[ C_{c,t+n} = k_{c,t+n} + f_{c,t+n} \quad \text{and} \quad C_{o,t+n} = k_{o,t+n} + f_{o,t+n}. \]

The variables \( k_{c,t+n} \) and \( k_{o,t+n} \) are the capital costs at time \( t+n \) for the coal plant and the CT respectively, while \( f_{c,t+n} \) and \( f_{o,t+n} \) represent the respective fuel costs at time \( t+n \).

In determining capital costs, we need to account for the difference in capacity across the two plants. To put the plants on a comparable basis, we will calculate the capital cost required for each plant type to meet \( \tau \) periods of demand growth. For the large coal plant, the capital cost, \( k_{c,t+n} \), is simply the construction cost of a coal plant at time \( t+n \), \( K_{c,t+n} \). For the CTs, we will be building a series of \( \tau \) plants, one each period. Taking the present value of the construction costs for the series of plants gives us the CT's capital cost. To calculate the present value we
can simply take a $\tau$ period annuity where the cash flow each period is equal to the construction cost of a single CT, $K_{o,t+n}$. The appropriate real (net of inflation) discount rate to use is $(r-g_c-1)$, where $g_c$ is the real escalation rate for CT construction costs. The annuity can be written as

$$k_{o,t+n} = \frac{K_{o,t+n}}{(r-g_c-1)} \left(1 + (r-g_c)^{-\tau}\right)$$

where

- $r = 1 +$ the risk-free rate for a single period
- $g_c =$ real escalation rate for CT construction costs
- $K_{o,t+n} =$ the construction cost of a single CT at time $t$.

Since we will be using annuities later in this paper, it will be helpful to simplify our notation by defining an annuity factor $A(x,y)$ as follows:

$$A(x,y) = \frac{1}{x} \left(1 + (1+x)^{-y}\right),$$

where $x$ is the real discount rate,

and $y$ is the number periods for the annuity.

For example, $A(.02,5) = 1/.02 (1 + 1.02^5)$. Using this notation to express the capital cost of the CTs over $\tau$ periods yields:

$$k_{o,t+n} = K_{o,t+n} A(r-g_c-1,\tau)$$

Focusing on fuel costs, it is important to recognize the operational impact that any new capacity will have on the entire utility system. To see why system costs are the relevant costs, consider a new CT which only operates during the
peak hour of the year. Operating for only one hour the CT will burn very little fuel and its corresponding fuel cost will be quite low. On the other hand, a new coal plant may operate in nearly every hour of the year, burning many tons of coal and racking up a very large fuel bill. Comparing these plants individually, we will find that the coal plant had a much greater fuel cost. However, if we compare the system costs in the two cases above, the coal-based system will almost surely have a lower fuel cost, since the new coal plant enables the utility to displace a significant amount of generation which was previously provided by oil plants. Ideally, we would like to have a detailed model to simulate the operation of the system under both a coal and a CT expansion scenario. Taking the difference in total system fuel costs between the two simulations, would yield the net fuel cost advantage to the coal plant. Since this type of detail is not required to demonstrate the usefulness of options methods, we will not endeavor to use a detailed fuel cost model. Nevertheless, a rough approximation for system costs will be used in calculating fuel costs associated with each plant alternative.

In developing system fuel costs, we will assume that there are two types of plants - coal and oil. All plants which burn the same fuel type get their supply from the same stockpile. In almost every conceivable case, the operating cost of a coal plant is less than that of a oil plant. Thus, if a utility builds a new coal plant, it will be able to displace some production previously supplied by oil with less expensive coal-fired production. We will define the incremental coal-fired production associated with a new coal plant at time t+n as $\Delta G_{t+n}$.

The cost savings per megawatt-hour\(^1\) (MWH) of incremental coal production is dependent on two factors. The first is the efficiency with which a power plant

\(^1\) Electrical energy production is usually measured in megawatt-hours (MWH). A one megawatt plant operating for one hour produces a MWH. A megawatt is one million watts. A 100 watt light bulb uses one MWH of energy every 10,000 hours.
converts thermal energy (measured in millions of British thermal units or mbtu’s) into electrical energy (MWH). This conversion efficiency is commonly referred to as a heat rate\(^1\). Typically, coal plants are more efficient than combustion turbines (i.e., they require fewer mbtu’s per MWH of output). We will denote the heat rates of the coal and oil plants as \(h_c\) and \(h_o\) respectively.

The second factor which determines production cost is a plant’s fuel cost. The cost of fuel used in electric utility applications is generally measured in $ per mbtu. By multiplying a plant’s fuel cost by its heat rate, we get the cost per MWH of production at the plant. We will use \(P_{o,t+n}\) and \(P_{c,t+n}\) to represent the cost per mbtu at time \(t+n\) of oil and coal respectively. Now that we have all the components, the incremental cost savings for every MWH of coal generation can be expressed as, \(h_o P_{o,t+n} - h_c P_{c,t+n}\). Thus, the total fuel cost savings, \(s_t\), attributable to a new coal plant during any period \(t+n\) are

\[
s_{t+n} = \Delta G_{t+n}(h_o P_{o,t+n} - h_c P_{c,t+n}).
\]

Note that one of the key determinants of fuel cost savings is the incremental production from the coal plant, \(\Delta G_{t+n}\). The magnitude of the incremental production will depend on a number of factors, such as the amount of existing coal capacity operated by the utility, the size of the incremental coal plant, and the number of hours during the year when the utility's demand is sufficient to effectively utilize the new coal plant. Again it would take a detailed model to properly account for all of these factors. However, for our purposes, we will assume that the new coal plant

\(^1\) A heat rate is measured in mbtu/MWH. A heat rate of 10 mbtu/MWH implies that it takes 10 million btus of fuel to generate 1 MWH of electrical energy.
operates at a known utilization or capacity factor over its entire life. From this capacity factor we can calculate the incremental coal generation, $\Delta G_{t+n}$.

The coal plant will generate fuel cost savings over its entire life of $T$ periods. To calculate these savings, we will assume that both coal and oil prices are expected to grow in real terms at a fixed percentage rate per period of $g_f$. Under this assumption, the total fuel cost savings to the coal plant is just a $T$ period annuity. The cash flow each period is the single period fuel cost savings to coal, $s_{t+n}$, which we defined above. The appropriate real discount rate is $(r-g_f-1)$. Given these definitions, the present value of the fuel cost savings to the coal plant can be written as:

$$f_{o,t+n} - f_{c,t+n} = s_{t+n} A(r-g_f-1,T),$$

where

$$s_{t+n} = \Delta G_{t+n} (h_o P_{o,t+n} - h_c P_{c,t+n}),$$

and

$$A(r-g_f-1,T) = \frac{1}{(r-g_f-1)} \left(1 + (r-g_f)^{-T}\right).$$

### 3.3 Solving the Investment Problem with a DCF Framework

Having a method to determine the costs associated with any generation alternative, we are now able to compare the costs across alternatives. In this section, we will show how discounted cash flow analysis can be used to compare these costs.

Since the coal plant is more capital-intensive than a CT, the coal plant will incur some extra construction costs ($k_{c,t+n} - k_{o,t+n}$). However, the coal plant will

---

1 The annual energy production in MWH is equal to:

Plant Capacity (MW) $\times$ Hours per year (Hrs) $\times$ Capacity Factor (%)
produce fuel cost savings \((f_{o,t+n} - f_{c,t+n})\). In order for the coal plant to be economic, the fuel cost savings must outweigh the extra capital costs. We can express the net present value of an investment in the coal plant at time \(t+n\) (NPV\(_{c,t+n}\)) as the benefits minus the costs, or

\[
\text{NPV}_{c,t+n} = (f_{o,t+n} - f_{c,t+n}) - (k_{c,t+n} - k_{o,t+n}).
\]

For positive NPV\(_{c,t}\), we will build the coal plant; otherwise, we will proceed with the series of CTs.

Expanding the NPV\(_{c,t+n}\) expression we get:

\[
\text{NPV}_{c,t+n} = s_{t+n} A(r_{gf-1},T) - (K_{c,t+n} - K_{o,t+n} A(r_{gc^{-1}},t))
\]

where \(s_{t+n} = \Delta G_{t+n} (h_o P_{o,t+n} - h_c P_{c,t+n})\).

Further expansion yields:

\[
\text{NPV}_{c,t+n} = \Delta G_{t+n} (h_o P_{o,t+n} - h_c P_{c,t+n}) A(r_{gf-1},T) - (K_{c,t+n} - K_{o,t+n} A(r_{gc^{-1}},t)).
\]

This expression is very messy indeed. In fact, it should be messy given the complexity of the utility investment problem. However, this expression can be simplified by isolating key variables of particular interest. Certainly, one of these variables is the oil price, \(P_{o,t+n}\), since it is subject to wide variability. If we isolate \(P_{o,t+n}\), we will obtain a much simpler expression which is much easier to interpret. Rewriting NPV\(_{c,t+n}\) in terms of the oil price \(P_{o,t+n}\), we get:

\[
\text{NPV}_{c,t+n} = B_1 P_{o,t+n} - B_2,
\]
where $B_1 = \Delta G_{t+n} h_o A(r-g,1,T)$, and $B_2 = K_{c,t+n} + h_c P_{c,t+n} A(r-g,1,T) - K_{o,t+n} A(r-g_c-1,T)$. 

Looking at the above expression, we see that as $P_{o,t+n}$ increases, the coal plant becomes more attractive. Alternatively, as $P_{o,t+n}$ declines, the coal plant will look worse. Note that there is a breakeven value for $P_{o,t+n}$, which will result in $NPV_{c,t+n} = 0$. In this case, the utility is indifferent between building a coal plant or a series of CTs. By setting $NPV_{c,t+n} = 0$, it is easy to see that the breakeven oil price, which we will denote as $P_{o,t+n}^*$, is equal to:

$$P_{o,t+n}^* = \frac{B_2}{B_1}.$$ 

Once we calculate $P_{o,t+n}^*$, we can determine the proper investment decision at any time $t+n$ just by looking at the oil price. Our decision rule is: if $P_{o,t+n} > P_{o,t+n}^*$, then build the coal plant; otherwise build the CTs.

### 3.4 Summary

Overall the hardest part of the utility investment problem is deciding on the cost parameters to be used in the analysis. Once these parameters have been determined it is fairly easy to compute the net present value associated with each investment alternative. The net present value of the coal plant is:

$$NPV_{c,t+n} = B_1 P_{o,t+n} - B_2,$$

where $B_1 = \Delta G_{t+n} h_o A(r-g,1,T)$, and $B_2 = K_{c,t+n} + h_c P_{c,t+n} A(r-g,1,T) - K_{o,t+n} A(r-g_c-1,T)$.
If \( \text{NPV}_{c,t+n} > 0 \), then the coal plant is the economic choice; otherwise the CT's are the economic alternative.

Alternatively we could calculate the net present value of the CT's. Since the only alternative to the CT's is the coal plant, the net present value of the CT's is always equal in magnitude and opposite in sign to the net present value of the coal plant. For example, if the net present value of the CT's is \$10\text{ million} \) then the net present value to the coal plant must be \(-\$10\text{ million}\). Using our previous notation the net present value of the CT is

\[
\text{NPV}_{o,t+n} = -\text{NPV}_{c,t+n}, \text{ or }
\text{NPV}_{o,t+n} = B_2 - B_1P_{o,t+n}.
\]

The decision rule then becomes: if \( \text{NPV}_{o,t+n} > 0 \), then the CT's are the economic choice; otherwise the coal plant is the economic alternative.

Finally, we can calculate the oil price at which the utility is indifferent between an investment in CT's or a coal plant. At this breakeven oil price the net present value of investing in either plant alternative is zero. By setting \( \text{NPV}_{c,t+n} = 0 \), it is easy to see that the breakeven oil price, which we will denote as \( P^*_{o,t+n} \), is equal to:

\[
P^*_{o,t+n} = \frac{B_2}{B_1}.
\]

Once we calculate \( P^*_{o,t+n} \), we can determine the proper investment decision at any time \( t+n \) just by looking at the oil price. Our decision rule is: if \( P_{o,t+n} > P^*_{o,t+n} \), then build the coal plant; otherwise build the CT's.

Thus, we have derived three alternative decision rules. They will all result in the same decision and can be used interchangeably. We will use these decision
rules as a starting point for the development of an options approach to the utility investment problem in Chapter 4.

IV. An Options Approach to the Utility Investment Problem

If oil prices were known with certainty, we could end the analysis here. We could just apply one of the decision rules developed above in Chapter 3 and be confident that we were making the proper investment choice. However, we know that oil prices are stochastic. Over the life of the coal plant, the actual oil price may fluctuate both above and below the breakeven oil price. Thus, the net present value from building and operating a coal plant can oscillate from positive to negative values.

This oscillation in NPV_{c,t+n} would not be a problem, if the coal plant could be added incrementally as the system demand increases. If NPV_{c,t+n} changed from positive to negative, we could readjust our investment strategy and start building CTs. However, since the coal plant represents a large commitment of capacity, we will be unable to adjust our strategy for several periods. Thus, we may be stuck with a coal plant which has a negative net present value.

On the other hand, by building CTs we will never be forced into a negative net present value situation. CTs can be added incrementally and afford the utility the flexibility to switch its investment policy each period. Thus, we must continue the analysis, by considering the value of the option to switch from CTs to coal.

The analysis in this chapter will divided into three sections. In section, 4.1 we will redefine the utility investment problem within a framework which recognizes the flexibility associated with an investment in CT's. Given this framework, we will proceed to identify the flexibility option embedded in the utility investment problem in section 4.2. Finally, in section 4.3, we will develop a procedure for valuing the
flexibility option. This procedure will lead to a new decision rule to use in choosing between investment alternatives.

4.1 Model Framework

Taking the flexibility associated with the CT's into account, we can use the following diagram to represent the decisions that the utility will face over the planning horizon.

Today at time t, we face the decision of whether or not we should build a coal plant. If we do go ahead with coal, our load will be satisfied for $\tau$ periods. At the end of time $t+\tau$, we will be in a position to make another plant investment decision. Alternatively, if we choose the CT today, we will satisfy the load for a single period.
only. As a result, we will be faced with the very same decision next period (time \( t+1 \)). As long as we continue to build CTs, we will have the option to choose between a coal plant or a CT in the very next period. However, as soon as the coal plant is built, we will be unable to add any new capacity for \( \tau \) periods. With the problem laid out in this manner, the investment in a coal plant is not a now or never proposition as in the DCF model of Chapter 3; rather this structure allows us to look at the coal plant investment as a now or later decision.

In between each of the decision nodes in the diagram above, the uncertainty regarding the next period's fuel prices is resolved. We will assume that oil prices follow a random walk about an expected rate of growth. The random walk model implies that fuel prices will tend to move up or down by a constant percentage of the current price. These changes are independent of past changes, and the movement in fuel prices over relatively small intervals also tends to be relatively small. While the random walk process does seem to fit historical movements in oil prices, the random walk assumption is not essential to the application of options theory. In fact, options models can be developed which are consistent with a variety of assumptions about the nature of fuel price movements.

We will assume that future coal prices are known with certainty. While in actuality coal prices are subject to fluctuation, the magnitude of this fluctuation is relatively small when compared with the oil price experience. Thus, a deterministic model provides a reasonable representation.

4.2 Identifying the Embedded Options

Now that we have defined the problem faced by the utility, the next task is to identify the flexibility options which are embedded in the various paths the utility might follow. To start, imagine that we, the utility management, decide to build a CT initially. After one period has passed, we look back and see how we have done. If
oil prices have gone down during the period, we will be patting ourselves on the back for the smart decision we made initially. Furthermore, we will probably stick with a winner and add another CT to the system. On the other hand, if oil prices have risen to new highs, we won’t be too disappointed, since we can immediately build a coal plant. The only thing we will have lost is the money we could have saved by owning and operating a coal plant during the first period. Note how similar this situation is to our example from Chapter 2. If fuel prices decrease, we have a payoff; while if fuel prices increase our liability is limited to nothing. Thus, like the example, we obtain a flexibility option when will build the CT. A positive payoff occurs at time t+1 whenever the expected cost of building and operating a CT over its life is less than the similar cost of a coal plant. Thus the payoff to the flexibility option is equal to max \{0, NPV_{0,t+1}\}, where NPV_{0,t+1} is the net present value of building an operating a series of CT’s at time t+1.

Now suppose that oil prices are low enough to induce us to build another CT at time t+1. So far we have built CT's in each of the first two periods. Thus, we face another capacity choice at time t+2. The fact that we have a decision to make gives us yet another flexibility option with a payoff equal to max\{0, NPV_{0,t+2}\}. Note that had a coal plant been built at time t+1, we wouldn't have a capacity choice to make at time t+2, and the flexibility option would be lost. Thus, we will receive the payoff from the second flexibility option if and only if we build CTs at each of the first two decision nodes.

Moving back to time t, if we build a CT we gain the flexibility option expiring at time t+1. In addition, we neither preclude the possibility of acquiring the second flexibility option which expires at time t+2, nor do we fully acquire it. In our state of limbo, we have an option to buy the second flexibility option. We can think of this option as a call on the second flexibility option. Thus, we have an option on an option which is often referred to as a compound option.
More generally, when a CT is built at time \( t \), we acquire the flexibility option expiring at time \( t+1 \) as well as a series of compound flexibility options expiring at time \( t+2 \) through time \( t+n-1 \). Note that each succeeding option in the series goes through another level of compounding. For example, we can define the flexibility option expiring at time \( t+n \) as \( F_{t+n} \), and define an operator \( \text{Call}_{t+n}(x) \) as the option to buy any asset \( x \) at time \( t+n \). Given these definitions, the compound flexibility option expiring in two periods is defined as \( \text{Call}_{t+1}(F_{t+2}) \). The compound flexibility option expiring in three periods is \( \text{Call}_{t+1}(\text{Call}_{t+2}(F_{t+3})) \). The compounding will continue until the end of the series is reached.

4.3 Option Valuation

In Section 4.2, we identified the flexibility options which benefit the owner of a CT. We determined that when a CT is built at time \( t \), we acquire a flexibility option expiring at time \( t+1 \) as well as a series of compound flexibility options expiring at time \( t+2 \) through time \( t+n-1 \). To tackle the problem of valuing this series of options, we have broken this section into four subsections each representing an important step in the option valuation process. The steps are:

1. Analyze the payoffs to the flexibility option as if it were a series of simple options, rather than as a series of compound options.

2. Present the model used to represent the diffusion of oil prices over time.

3. Describe the binomial option pricing model, and how it can be applied to simple options.

4. Build on steps 1-3 to develop a procedure for evaluating the compound flexibility options in the utility investment problem.
4.3.1 Simple Flexibility Option Valuation

Recall that the utility will receive a payoff from any single flexibility option whenever the CT is the economic investment alternative (i.e. the CT has a positive net present value). If the net present value of the CT is negative, then we simply exercise our flexibility to switch to coal, and the payoff is zero. Hence, if we define the net present value of the CT as \( \text{NPV}_{o,t+n} \), the payoff to a single independent flexibility option at time \( t+n \) is equal to \( \max \{0, \text{NPV}_{o,t+n} \} \). In Chapter 3 we determined that \( \text{NPV}_{o,t+n} = B_2 - B_1 P_{o,t+n} \). Substituting this result into the flexibility option payoff expression yields:

\[
\text{Payoff to flexibility option} = \max \{0, B_2 - B_1 P_{o,t+n} \}.
\]

expiring in \( n \) periods

The payoff function behaves as we might expect. At low oil prices, when the net present value of the oil plant is positive, the payoff will be positive. As the oil price rises, the payoff will decrease until \( \text{NPV}_{o,t+n} \) turns negative. At this point the owner of the CT will switch to the coal plant, and the payoff will level off at zero.

If we divide the payoff expression through by \( B_1 \), we get:

\[
\text{Payoff}/B_1 = \max \{0, B_2/B_1 - P_{o,t+n} \}.
\]

This expression is identical in form to the payoff function for a put option on a stock (i.e. \( \max \{0, K - S^* \} \)). In this case, the asset is an mbtu of oil rather than a share of stock, while the striking price \( K \) is equal \( B_2/B_1 \). Recall that in section 3.3, we showed that the striking price \( B_2/B_1 \) is equal to the breakeven oil price \( P^*_{o,t+n} \). Thus, the above payoff can be interpreted as the payoff to a put option on a single mbtu of oil with a striking price of \( P^*_{o,t+n} \).
To get back to the payoff for the flexibility option, we can multiply the above expression back through by $B_1$, which yields:

\[
\text{Payoff to flexibility option} = B_1 \max \{0, P^*_o,t+n - P_{o,t+n}\}.
\]

expiring in $n$ periods

Thus, a single independent flexibility option is equivalent to $B_1$ puts on the price of oil with a striking price equal to the breakeven oil price, $P^*_o,t+n$.

If we take a closer look at $B_1$, we see that

\[
B_1 = \Delta G_{t+n} h_o A(r-g_r-1, T),
\]

where $\Delta G_{t+n}$ = the incremental coal generation from a coal plant (MWH),

$h_o$ = the heat rate for the CT (mbtu/MWH),

and $A(r-g_r-1, T)$ = a $T$ period annuity with discount rate $r-g_r-1$.

Interpreting this expression, we see that $\Delta G_{t+n} h_o$ is the incremental number of oil mbtu's burned by the series of CT's at time $t+n$. Applying the annuity factor, $A(r-g_r-1, T)$, accounts for the incremental oil mbtu's over the entire life of the CT series. This result seems perfectly reasonable, since the payoff from flexibility should account for the operation of the all the CT's over their lives.

However, since we have a series of flexibility options, we will double count some mbtu's if we are not careful. To see why, imagine that our CT's are so small that we have 100 flexibility options in our series. We proceed to use the above expression to evaluate the payoff to the first option expiring at time $t+1$. We find that the payoff for the put option on a single mbtu of oil is positive, and we calculate the payoff to all the CT's over the entire life of the series ($i. e.$ from time $t+1$ to time $t+1+T$). At time $t+2$, we also find a positive payoff and calculate the payoff to all the CT's over the period $t+2$ to $t+2+T$. Note that we have now counted the payoff.
covering the period extending from t+2 to t+1+T twice. If we keep going through all 100 options in the series, we will count some periods 100 times!

Clearly, we only want to count each mbtu just once. To accomplish this, we need to define the payoffs for each option such that it only accounts for the incremental mbtu's up to the expiration date of the next option in the series. For example the option expiring at time t+1, should account for the incremental oil mbtu's over the interval t+1 to t+2. The option expiring at time t+2 will account for the interval t+2 to t+3. Each option will continue to cover a single interval until we reach the last option in the series at time t+\tau-1. The last option in the series will need to account for all the extra oil burned by the CT's for the remainder of their lives.

Based on this system, we can write the payoffs to all of the flexibility options in the series as follows:

Payoff at \( t+1 \) = \( \Delta G_{t+1} h_0 \) max \( \{0, \ P^{*}_{o,t+1} - P_{0,t+1}\} \),
Payoff at \( t+2 \) = \( \Delta G_{t+2} h_0 \) max \( \{0, \ P^{*}_{o,t+2} - P_{0,t+2}\} \),
\cdot
\cdot
Payoff at \( t+\tau-1 \) = \( B_1 \) max \( \{0, \ P^{*}_{o,t+\tau-1} - P_{0,t+\tau-1}\} \).

The last option in the series represents the oil burned for the entire life of the CTs beyond time \( t+\tau \), while all prior options represent a single period's fuel costs. Thus, we will not double count any fuel costs by defining the payoffs in this manner.

Now that we know the value of a simple flexibility option at any expiration date \( t+n \), we need a method to determine its value today. Since the payoff is largely dependent on the diffusion of oil prices between time \( t \) and time \( t+n \), it will be useful
to review the diffusion process we are using to represent the movement in oil prices over time.

4.3.2 Oil Price Diffusion

As stated earlier, we will assume that oil prices follow a random walk about an expected rate of change. In continuous time, the random walk representation can be described by the following differential equation:

$$\frac{dP_o}{P_o} = \alpha dt + \sigma dz$$

This equation shows that the instantaneous percentage change in oil prices is equal to the expected rate of change $\alpha$ plus a random term $\sigma dz$. The random term is often referred to as Brownian motion. $\sigma$ is the annual standard deviation of oil prices while $dz$ is a random increment. Note that the expected rate of increase in the oil price, $\alpha$, is equal to the expected rate of capital gains from holding oil, $\mu$, less the utility or convenience yield we derive from holding oil in inventory, $\delta$. Note that $\delta$ must be positive otherwise a utility would never hold any fuel in inventory. Thus, $\alpha = \mu - \delta$.

In the most general sense, the convenience yield on oil is analogous to a dividend payout on a common stock. A utility which has oil in inventory derives a benefit or convenience yield just like a stockholder collects dividends every quarter. The convenience yield is significant in option valuation, since it represents a benefit (loss) which accrues to the buyer (seller) of oil but does not accrue to the owner of an option on oil. While we will not be using a continuous time model for oil prices, we will be including the effects of the convenience yield.

Instead of the continuous time framework, we will adopt a discrete interval model to represent oil price movements. This model is depicted in in the diagram below. The basic idea is that at any time the oil price can go up or down by a fixed
percentage. For example, Exhibit 3 shows that the current oil price $P$ can either move up to $uP$ or down to $dP$ after one period. After $n$ periods, the oil price can range anywhere from $u^nP$ as the high to $d^nP$ as a low. If the convenience yield at each interval is a constant percentage, $\delta$, then for $n$ intervals we need to multiply the oil price by $(1-\delta)^n$. More generally, the oil price after $i$ upward moves in $n$ intervals will be $u^i d^{n-i}(1-\delta)^nP_{oil}$.

Random Walk Model For Oil Prices
$n = \text{number of periods}$
$i = \text{number of upmoves}$

With the discrete interval model, we only capture the movement of oil prices from interval to interval. None of the movements between intervals are recognized. This omission may seem troublesome, since we know that the movement of oil
prices is a continuous process. Hence, if we wish to fully capture all the nuances of oil price movements, we need a continuous time model. However, given that utility capacity planning decisions are generally made on an annual or even a bi-annual cycle, the discrete interval model will be sufficient for this analysis as long as the interval we choose is relatively short when compared to the length of the planning cycle.

We will assume that the interval between each decision node (planning cycle) is divided into q subintervals over which the oil price will move. In sample calculations, Cox and Rubinstein have shown that for \( q \geq 5 \), option pricing results using a discrete interval assumption very closely approximate continuous time values. This result is obtained provided that \( u \) and \( d \) are chosen such that

\[
u = e^{\sigma \sqrt{T/n}} \text{ and } d = 1/u,
\]

where \( \sigma \) is the annual standard deviation in oil prices,

and \( e \) is the base for natural logarithms.

Thus, the discrete interval model will suit our purposes in evaluating simple and then compound flexibility options in subsequent sections.

4.3.3 The Binomial Option Pricing Model

The binomial option pricing model is designed to be used in conjunction with a discrete interval diffusion model described above. Remember that the binomial model was used to calculate the value of flexibility in the simple example back in Chapter 2. In the simple example, there were two possible fuel prices and two possible values for the option. All we had to do was determine the appropriate probability weights to apply to each possible option value. Applying these weights,
we took a weighted average over the two payoffs to determine the value of the option.

In a more general setting we can apply the binomial model by following five steps. First, use the diffusion model to determine all the possible values for the underlying asset (oil prices in our case). Second, calculate the option payoffs associated with each possible value for oil prices. Third, determine the appropriate probability weights to apply to each possible terminal value or payoff of the option. Fourth, apply the weights and take the weighted average over all possible option payoffs to compute the weighted average payoff. Last, discount the weighted average payoff back to the present period.

The only step we have not discussed in detail is the weight determination problem. Fortunately, this problem is quite easy to solve, thanks to a result obtained by Cox and Ross (1976) which states that the appropriate probability weights are those which when applied to the terminal values of the stock (oil price in this case) cause the expected return on the stock to equal the risk free rate. More formally, over \( q_n \) moves in the stock price (\( n \) decision intervals) there is a vector of weights \( w_{i,q_n}, \ i = 0, \ldots, q_n \), such that the following equality is satisfied.

\[
\sum_{i=0}^{q_n} w_{i,q_n} d^{q_n-i} S = S r^{q_n}
\]

where \( w_i \) is the weight for \( i \) upward moves in \( q_n \) total moves, \( u = e^{\sigma \sqrt{r/n}} \); \( d = 1/u \), and \( S \) is the current stock price.

The right hand side of the above equation shows the value of a risk-free investment of \( S \) dollars over \( q_n \) fuel price moves. The left hand side is just a weighted sum of all possible fuel prices where the weights are the \( w_{i,q_n}, i = 0, \ldots, q_n \).
Solving for $w_{i,qn}$, we get the standard binomial probability function, which gives our model its name.

$$w_{i,qn} = \frac{qn!}{i! (qn - i)!} p^i (1 - p)^{qn - i} \quad i = 1, ..., qn$$

where $p = \frac{r - d}{u - d}$

Clearly, these weights have the characteristics of probabilities. In fact, they are the probabilities which would exist in world of risk-neutral investors. Now that we have these weights, we can just apply them to there respective option payoffs to determine the value of the option.

Given that we have solved the weight determination problem, evaluating an option is a fairly mechanical process. As an example we can evaluate the value of the simple flexibility option expiring at time $t+1$. As a first step we can enumerate all the possible oil prices at time $t+1$:

$$u^i d^{q-i}(1-\delta)^q P_o,t \quad i = 0, ..., q.$$ 

The above step comes directly from the diffusion process. This set of oil prices can then be plugged into the payoff expression we derived in Section 4.3.1. To refresh our memory the payoff was expressed as: Payoff at $t+1 = \Delta G_{t+1} h_o \max \{0, P_{o,t+1}^* - P_{o,t+1}\}$. Thus, the set of possible payoffs at time $t+1$ is

$$\Delta G_{t+1} h_o \max \{0, P_{o,t+1}^* - u^i d^{q-i}(1-\delta)^q P_o,t \} \quad i = 0, ..., q.$$ 

Note that each payoff above is just a linear function of the initial oil price. All we have done is add the striking price and multiply through by a constant $\Delta G_{t+1} h_o$. Applying the appropriate weights yields the following expression:
\[ \frac{q^i}{i! (q - i)!} p^i (1 - p)^{q-i} \Delta G_{t+1} h_0 \max \{0, \ P^*_{o,t+1} - u^i d^{q-i}(1-\delta)^q P_{o,t} \} \ i=0, \ldots q \]

Summing and discounting by the risk-free rate gives us the value of the option.

\[ \Delta G_{t+1} h_0 / r \left[ \sum_{i=0}^{q} w_{i,q} \max \{0, \ P^*_{o,t+1} - u^i d^{q-i}(1-\delta)^q P_{o,t} \} \right] \]

Note that it would be a simple procedure to calculate the remaining flexibility options as simple options. These values could all be written using the same linear form. However, since our main concern is with valuing compound options, we will save our effort for the next section.

4.3.4 Valuation of Compound Flexibility Options

Thus far, we have established that building a CT at time \( t \) will give the utility a simple flexibility option expiring at time \( t+1 \) plus a series of compound options. Unfortunately, there is no simple formula which we can use to value the compound options in this problem. Although Robert Geske (1979) has developed a compound option pricing formula, the Geske model can only handle the valuation of an option on another option. Since we are faced with many layers of compounding in the utility investment problem, his approach would be difficult to apply in this case. Nevertheless, by expanding upon the binomial option pricing method outlined above, we will proceed to develop a procedure for evaluating the compound options embedded in this problem.

To develop this procedure, it will useful to take a brief step back and review the decision faced by the utility at any of the decision nodes \( t+n \). Basically, we want
to compute the net present value of each plant type, choosing the plant with the positive NPV. Introducing the flexibility options into the problem will increase the value of the oil plant investment by the value of the option. Thus, we need to redefine the net present value of the oil plant by adding the option value, $F_{t+n}$.

Previously, we showed that when no option values are considered, $NPV_{o,t+n} = B_2 - B_1P_{o,t+n}$. Thus, the net present value of the oil plant considering options is

$$NPV_{o,t+n} = B_2 - B_1P_{o,t+n} + F_{t+n},$$

where $F_{t+n}$ = the value of the flexibility option series at time $t+n$.

Clearly, if $NPV_{o,t+n}$ is positive we will proceed with the CT.

Now suppose for a moment that we know how to evaluate the flexibility options. As we did earlier, we can set $NPV_{o,t+n} = 0$, and recalculate the breakeven oil price, $P^*_{o,t+n}$, for any period $t+n$. Furthermore, we can calculate the breakeven price at time $t+n+1$, at time $t+n+2$, ..., through $t+n+\tau-1$. This series of breakeven oil prices partitions the set of all possible future oil prices into two regions (see diagram below). The upper region includes all oil prices which exceed the breakeven oil price line, and the lower region includes all oil prices which are below the breakeven values. If the oil price ever crosses into the upper region, then we will build a coal plant, and the remaining flexibility options will be lost. The only reason we would continue to build CT's is if oil prices remain in the lower region. Thus, the following condition must be satisfied in order to receive the payoff from the compound flexibility options expiring at times $t+1$ through $t+\tau-1$:

$$P_{o,t+i} < P^*_{o,t+i} \text{ for } i = 1, ..., \tau-1.$$
This payoff condition suggests one approach for evaluating the compound options. First, we can identify all of the oil price paths which satisfy the payoff condition. Second, we can calculate the terminal value or payoff of the compound option associated with each possible path. Finally, we can appropriately weight each path to compute the current value of the compound flexibility option. Taking a weighted average of the payoffs over all paths will yield the appropriate value of the compound flexibility option.

With the discrete interval model for oil prices we described in Section 4.3.2, it is straightforward to determine the oil price paths which satisfy the payoff condition above. Likewise, from the discussion in Section 4.3.1, we know how to calculate the terminal payoffs to the flexibility options. In addition we can use a variant of the binomial option pricing model to determine the risk-free probability of each path. Unfortunately, the model cannot be solved analytically. The difficulty
arises from the fact that in order to determine the payoff condition at any time \( t+n \), we need to know \( F_{t+n} \). At the same time the calculation of \( F_{t+n} \) requires that we know the payoff condition.

For example, let's take the simple case where the coal plant satisfies the load for 3 decision intervals, while the CT meets the load for a single interval. In order to make a decision today at time \( t=0 \), we can immediately calculate the net present value of the CT investment without options (i.e. \( \text{NPV}_{0,1=0} = B_2 - B_1 P_{0,1=0} \)). Furthermore, we could use the binomial method to evaluate the flexibility option expiring next period at time \( t=1 \).

So far so good. To calculate the value of the compound flexibility option expiring at \( t=2 \), we attempt to set up a payoff condition by calculating a breakeven oil price, \( P^*_{0,1=1} \), at time \( t=1 \). We might go about this calculation as follows:

- **Step 1**: Let \( \text{NPV}_{0,1=1} = 0 = B_2 - B_1 P^*_{0,1=1} + F_{1=1} \)
- **Step 2**: Note that the flexibility option \( F_{1=1} \) includes the simple flexibility option expiring at time \( t=2 \) and a compound flexibility option expiring at time \( t=3 \).
- **Step 3**: Attempt to calculate the breakeven oil price at time \( t=2 \)
- **Step 4**: Let \( \text{NPV}_{0,1=2} = 0 = B_2 - B_1 P^*_{0,1=2} + F_{1=2} \)
- **Step 5**: Note that the flexibility option \( F_{1=2} \) includes the simple flexibility option expiring at time \( t=3 \) and a compound flexibility option expiring at time \( t=4 \).
- **Step 6**: Realize the futility of this procedure and give up

Clearly, this process sends us into a never-ending quest for a solution. The difficult feature of this problem is that every time we build another CT, we add another compound flexibility option onto the end of the series. Thus, to solve the problem, we need to devise a mechanism which prevents the perpetuation of the option series.
To get around this problem, we can choose an arbitrary decision node at
time t+a, beyond which, oil prices are no longer stochastic. Since oil prices are now
certain, the problem beyond time t+a reverts to the simple discounted cash flow
analysis that we discussed in Chapter 3. Consequently, the flexibility options will
stop regenerating and, we will be able to reach a solution.

Once we have simplified the problem by halting the diffusion process, the
entire flexibility option series can be evaluated through a recursive procedure. In
order to develop this procedure, it will be useful to define some new notation. Let
F(n, j, i) be the value of the series of flexibility options n-j periods from now, given
that the current oil price has gone through i upmoves over q(n-j) total moves.

Since oil prices no longer fluctuate beyond time t+a, there is no more
compounding, and the only option in the series is the simple option expiring at time
t+a. Thus, we can calculate the value of the flexibility option series at time t+a. By
starting at time t+a and working backwards, we can value the entire series of
flexibility options today at time t.

As a first step, we can use our new notation to express the payoffs from the
flexibility option series a periods from now. In this case, n=a and j=0, so

\[ F(a, 0, i) = B_1 \max\{0, P^*_o, t+a - u^i d^q a^i (1 - \delta)^q a^i P_o, t\} \quad i = 0, 1, \ldots, qa \]

The above expression enumerates the payoffs to flexibility at time t+a. As we have
seen earlier this expression is the payoff for $B_1$ put options on oil with a striking
price equal to the breakeven oil price at time t+a, $P^*_o, t+a$.

Moving back one period to time t+a-1, there are now two options in the
series, the simple option expiring a time t+a-1, as well as the compound option
expiring at time t+a. Thus the payoff to the option series will include the sum of
these options. The payoff to the simple option is simply:
Simple option = \[ \max\{0, \Delta G_{t+a-1}[P_{o,t+a-1}^* - u^i d^{q(a-1)i} (1-\delta)^{q(a-1)} P_{o,t}] \} \]

payoff at t+a-1 for \( i = 0, 1, ..., q(a-1) \)

While the payoff to the compound option at time t+a-1 is determined by applying the risk-free probabilities to the payoffs \( F(a, 0, i) \). The payoffs for the compound option are expressed as:

\[
\text{Compound option} = \max\{0, 1/r \sum_{h=0}^{q} \frac{q!}{h!(q-h)!} p^h (1-p)^{q-h} F(a,0,i+h) \} \quad i=0,1,\ldots,q(a-1)
\]

payoff at t+a-1

Note that if fuel prices have gone through \( i \) upmoves up to time t+a-1 then they will go through \( i+h \) upmoves by time t+a where \( h \) can range from 0 to \( q \). Thus, the expression above is just the binomial option pricing formula applied to the option expiring at time t+a.

Combining the expression for the simple and compound options we get the payoffs from the option series at time t+a-1.

\[
F(a, 1, i) = \max\{0, \Delta G_{t+a-1}[P_{o,t+a-1}^* - u^i d^{q(a-1)i} (1-\delta)^{q(a-1)} P_{o,t}] + \frac{1}{r} \sum_{h=0}^{q} \frac{q!}{h!(q-h)!} p^h (1-p)^{q-h} F(a,0,i+h) \} \quad \text{for } i = 0, 1, \ldots, q(a-1).
\]

The above expression enumerates the payoffs to the flexibility option series from t+a-1 to t+a. Stepping back to period t+a-2, the series consists of the simple option expiring at time t+a-2 as well as a compound option on the flexibility series we just enumerated (i.e., t+a-1 to t+a). Combining these options as we did above, it is easy to see that the payoffs to the flexibility option series from time t+a-2 to t+a are:
\[ F(a, 2, i) = \max\{0, \Delta G_{t+a-2}[P^*_{o,t+a-2} - u^i d^{q(a-2)-i}(1-\delta)^{q(a-2)} P_{o,t}] + \\
\frac{1}{r} \sum_{h=0}^{q} \frac{q!}{h!(q-h)!} p^h (1-p)^{q-h} F(a, 1, i+h) \} \]

for \( i = 0, 1, \ldots, q(a-2) \).

More generally at time \( t+a-j \),

\[ F(a, j, i) = \max\{0, \Delta G_{t+a-j}[P^*_{o,t+a-j} - u^i d^{q(a-j)-i}(1-\delta)^{q(a-j)} P_{o,t}] + \\
\frac{1}{r} \sum_{h=0}^{q} \frac{q!}{h!(q-h)!} p^h (1-p)^{q-h} F(a, j-1, i+h) \} \]

for \( i = 0, 1, \ldots, q(a-j) \).

Note that every time we take a step back, we use the payoffs from the previous step to calculate the payoffs at the current step. This process is illustrated in the diagram below.

Once we get back to time \( t \), then \( j=a \). At the preceding step, we calculated the payoffs from the flexibility option series at time \( t+1 \). Therefore, by bringing the payoffs back one more period, we can value the flexibility option series today.

\[ F_t = F(a, a, 0) = \frac{1}{r} \sum_{h=0}^{q} \frac{q!}{h!(q-h)!} p^h (1-p)^{q-h} F(a, a-1, i+h) \]

Now that we have a method to calculate the value of the flexibility associated with CT's we can make the economic investment choice by calculating the net present value of the CT's and applying the appropriate decision rule as shown below.

\[ \text{NPV}_{o,t} = B_2 - B_1 P_{o,t} + F_t. \]
Clearly, our decision rule is: if NPV_{o,t} > 0, then build a CT; otherwise commit to the coal plant.

The following example will help to make the recursion procedure clearer. Suppose that we let a=4, p=.5, r=1.1 and q=1. We have calculated the payoffs to each of the simple options in the flexibility options in the series without limited liability. In other words, the minimum value for the payoff can go below zero. These payoffs are shown in the tree diagram in Exhibit 3.

Starting in the last period it is easy to see that by imposing the limited liability restriction, the payoffs to the flexibility option in period 4 are as follows:

\[ F(4,0,4) = \max\{0,-100\} = 0 \]
\[ F(4,0,3) = \max\{0,-50\} = 0 \]
\[ F(4,0,2) = \max\{0\} = 0 \]
\[ F(4,0,1) = \max\{0,50\} = 50 \]
\[ F(4,0,0) = \max\{0,100\} = 100 \]

Moving back one period, we can determine the payoffs to the flexibility options series at time t+3, using the formulas above:

\[ F(4,1,3) = \max\{0, -9 + 1/1.1(0.5 \cdot F(4,0,4) + 0.5 \cdot F(4,0,3))\} = 0 \]
\[ F(4,1,2) = \max\{0, -5 + 1/1.1(0.5 \cdot F(4,0,3) + 0.5 \cdot F(4,0,2))\} = 0 \]
\[ F(4,1,1) = \max\{0, -1 + 1/1.1(0.5 \cdot F(4,0,2) + 0.5 \cdot F(4,0,1))\} = 21.7 \]
\[ F(4,1,0) = \max\{0, 3 + 1/1.1(0.5 \cdot F(4,0,1) + 0.5 \cdot F(4,0,0))\} = 71.2 \]

Note that the payoff to the simple option at time t+3 after one upmove in the oil price, would be negative (-1). The negative payoff indicates that CT's would not be a good investment at time t+3, if their flexibility were ignored. However, by considering the potential payoff to flexibility at time t+4 (the next option in the series), the CT's do have a positive net present value. Consequently, the payoff to
Exhibit 3
Payoffs to Simple Flexibility Options
Without Considering Limited Liability

\[ \begin{array}{ccc}
  t & t+1 & t+2 & t+3 & t+4 \\
  -5 & -7 & -9 & -50 & -100 \\
  -2 & -4 & -1 & 0 & 100 \\
  & -1 & 3 & 50 & & \\
 \end{array} \]
the flexibility option series at time t+3 after one upmove in the price is positive at 21.7.

Taking another step back, we can calculate the payoffs at time t+2,

\[
F(4,2,2) = \max(0, -7 + 1/1.1(0.5 \times F(4,1,3) + 0.5 \times F(4,1,2))) = 0 \\
F(4,2,1) = \max(0, -4 + 1/1.1(0.5 \times F(4,1,2) + 0.5 \times F(4,1,1))) = 5.9 \\
F(4,2,0) = \max(0, -1 + 1/1.1(0.5 \times F(4,1,1) + 0.5 \times F(4,1,0))) = 41.2.
\]

Moving back to time t+1 yields,

\[
F(4,3,1) = \max(0, -5 + 1/1.1(0.5 \times F(4,2,2) + 0.5 \times F(4,2,1))) = 0 \\
F(4,3,0) = \max(0, -2 + 1/1.1(0.5 \times F(4,2,1) + 0.5 \times F(4,2,0))) = 19.4.
\]

Finally, at time t the value of the flexibility option series is calculated as:

\[
F(4,4,0) = F_t = \max(0, 1/1.1(0.5 \times F(4,3,1) + 0.5 \times F(4,3,0))) = 8.8.
\]

This completes the development of the option valuation model. In Chapter 4, we will apply the model to some sample utility data to gain a better understanding of how flexibility considerations affect the utility investment decision.
V. Sample Study Results

5.1 Overview

In this section, the model developed in Section 3 will be run for a set of
generic utility data. In conducting these sample studies, we have three main
objectives. First, we will again demonstrate the importance of accounting for
flexibility in the investment decision, by comparing the options model to the
discounted cash flow (DCF) model. Second, the sample results throughout this
section should help to build intuition on how the options model responds to
changes in various input parameters. In particular, it will be interesting to examine
the impact of fuel price volatility on the investment decision. Finally, we will apply
the model to determine the affect of CT plant size on the investment choice.

The input parameters used as a base case for this analysis are presented in
Table 1. The data are completely fictitious, yet they are representative of a typical
utility. Our system has a peak load of 3000 MW, and demand is expected to grow at
160 MW/yr or about 5%. Fuel prices are 2 $/mbtu for coal and 3.2 $/mbtu for oil.
These assumptions translate into plant production costs of 20 $/MWH and 36.8
$/MWH for the coal and oil plants respectively. As mentioned earlier, coal prices
are known with certainty and are expected to escalate at a real escalation rate of
3%, while oil prices follow a random walk with an annual standard deviation of
20%. The capital cost for a 800 MW plant is 1300 $/kw, and a 160 MW CT can be
built for 400 $/kw. Note that all of the data and results for the sample studies are
expressed in real terms.
Table 1  
Base Case Inputs for Sample Utility

**System Characteristics**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Capacity</td>
<td>3,000 MW</td>
</tr>
<tr>
<td>Projected Growth</td>
<td>160 MW/yr</td>
</tr>
<tr>
<td>Discount Rate</td>
<td>6.0% Real</td>
</tr>
</tbody>
</table>

**Alternative Plant Characteristics**

<table>
<thead>
<tr>
<th></th>
<th>Coal Plant</th>
<th>Combustion Turbines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>800 MW</td>
<td>160 MW/ Turbine</td>
</tr>
<tr>
<td>Capital Cost</td>
<td>$1,300/kw</td>
<td>$400/kw</td>
</tr>
<tr>
<td>Fuel</td>
<td>$2/mbtu</td>
<td>$3.25/mbtu</td>
</tr>
<tr>
<td>Production Cost</td>
<td>$20/MWH</td>
<td>$36.8/MWh</td>
</tr>
<tr>
<td>Real Fuel Escalation</td>
<td>3.0 %/Year</td>
<td>3.0 %/Year</td>
</tr>
<tr>
<td>Fuel Price Uncertainty</td>
<td></td>
<td>± 20% per year</td>
</tr>
</tbody>
</table>
5.2 Comparison of Options Model to Discounted Cash Flow

In comparing the options model to DCF, it is important to recognize that DCF is just one of many planning tools used by utility decision-makers in assessing investment decisions. Several very sophisticated analytic methods, including dynamic programming and decision analysis, are currently being used in the industry. These methods possess the capability to analyze the investment problem over many periods, and to a certain extent assess flexibility. In fact, the options model shares a number of attributes with these more sophisticated models. Thus, when comparing the options model to DCF which has no provision for flexibility, we are probably overstating the the improvement in results which can be obtained if utility planning departments were to immediately adopt options techniques. Nevertheless, as demonstrated by the example in Chapter 2, the options model provides both an explicit analysis of uncertainty together with a proper economic interpretation of flexibility. Furthermore, the options approach does not require prior knowledge of the probability of future fuel price movements.

To start, we have conducted a series of runs allowing the base period oil price and the coal capital cost to vary over a reasonable range. For each case, we have calculated the net economic benefit to the CT using both DCF and the options model. The results from these runs are tabulated below. The results show that in five of nine cases, the options model calculates a positive net benefit to the CT, while DCF shows an advantage to the CT in only one case.
Net Economic Benefit to Coal Using an Options Model  
(millions of dollars)

<table>
<thead>
<tr>
<th>Coal Capital Cost ($/kw)</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>-101</td>
<td>-228</td>
<td>-358</td>
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<tr>
<td>Oil Price ($/mbtu)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25</td>
<td>111</td>
<td>-8</td>
<td>-134</td>
</tr>
<tr>
<td>3.50</td>
<td>314</td>
<td>198</td>
<td>81</td>
</tr>
</tbody>
</table>

Net Economic Benefit to Coal Using a DCF Model  
(millions of dollars)

<table>
<thead>
<tr>
<th>Coal Capital Cost ($/kw)</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
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<tbody>
<tr>
<td>3.00</td>
<td>131</td>
<td>51</td>
<td>-29</td>
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<tr>
<td>Oil Price ($/mbtu)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25</td>
<td>286</td>
<td>206</td>
<td>126</td>
</tr>
<tr>
<td>3.50</td>
<td>442</td>
<td>362</td>
<td>282</td>
</tr>
</tbody>
</table>

Clearly, the CT looks considerably more attractive when an options model is employed, but why is this? The result can be traced to two effects. First, the options model explicitly considers uncertainty, while the DCF model presumes that the conditions which exist today will continue over the entire life of the investment. For example, in the base case scenario, the coal plant starts off at a capital cost disadvantage of $732 million. Each year the coal plant provides fuel cost savings
of about $40 million, until after about 23 years the present value of the fuel cost savings offsets the initial capital cost disadvantage. Beyond year 23, the coal plant continues to provide fuel savings which have a present value of $206 million. Thus, the net present value of the coal plant alternative is $206 million. However, the coal plant's value is based on fuel price assumptions over 23 years into the future. Since these fuel price projections are probably the least reliable information we have, the results of the DCF analysis are highly suspect. In contrast, the options approach considers directly the uncertainty surrounding fuel prices.

In addition, the options model considers management's response to future events as uncertainties are resolved. Using the DCF approach, we make the implicit assumption, that if a coal plant isn't built today the opportunity to build one will never arise again. Management's role is to make a decision today and sit passively as events take their course. Of course in making real-world decisions, management has many choices and takes an active role after the initial decisions are made. We can build a coal plant today, next year, or in any other period. Furthermore, if favorable investment prospects arise, management can act quickly. Likewise, if events turn sour management has the opportunity to change course. Thus, the options approach gives management credit for doing its job.

In considering the coal vs. CT problem, building the CT today provides management with greater flexibility to respond to uncertainty. By valuing the flexibility option, the options model considers management's response; consequently, the CT appears relatively more attractive when the investment decision is evaluated within the options framework.

Another interesting feature of the options model is that it is generally more sensitive to changes in input parameters than the DCF model. For example, under DCF, changing today's oil price from 3.0 to 3.5 $/mbtu, results in an improvement in coal plant economics of $311 million, while the options model shows an
improvement of only $396 million. The greater sensitivity of the options model is highlighted by plotting the net benefit to the coal plant across a range of oil prices in Exhibit 4. The exhibit shows that the slope of the line representing the options model's results is somewhat steeper than the similar line representing the DCF results. Furthermore, this same behavior is evident when we examine the options model's response to changes in other input parameters. Exhibits 5 and 6 compare the options model to DCF over a range of coal plant capacity factors and over a range of coal unit capital costs in turn. Once again, the somewhat steeper slope of the options model line illustrates the greater sensitivity of its results.

This difference in responsiveness across the two models can also be attributed to the fact that the options model allows for management response. If an opportunity arises for a positive NPV investment, management doesn't just sit back and let it go by. Rather decision-makers will take advantage of promising investment projects. As a result, the consequences of favorable events are enhanced, and we see somewhat greater fluctuation in our results.

Overall, the comparison with DCF demonstrates the impact of both uncertainty and management's response to uncertainty. We see that the flexibility option we valued in Chapter 4 can be thought of as the value of management. If we build a coal plant, we effectively tie management's hands by limiting their flexibility in responding to future events. Therefore, if we believe that management's decisions can add value to the utility, we should use an option's model to properly reflect that value.
Exhibit 4
Net Cost Advantage to a Coal Plant by Oil Price
Discounted Cash Flow v. Options Methods

$ Millions

Oil Price ($/mbtu)

3
3.25
3.5
Exhibit 5
The Impact of Changes in Coal Plant Capacity Factor
Discounted Cash Flow v. Options Methods

$ Millions

Options
DCF

Marginal Coal Capacity Factor

1500
1000
500
0
-500
-1000

10% 20% 30% 40% 50% 60% 70%
Exhibit 6
Net Cost Advantage to a Coal Plant by CT Capital Cost Discounted Cash Flow v. Options Methods

$ Millions

300
250
200
150
100
50
0
-50
-100
-150

1200 1300 1400
Coal Capital Cost ($/kw)

- DCF
- Options
5.3 The Impact of Fuel Price Volatility

There are two reasons why a entire section is dedicated to the impact of fuel price volatility. First, volatility is one of the six fundamental determinants of option value. If fuel prices were not volatile and future fuel prices were known with certainty then option valuation would be a trivial problem. Since we know that fuel prices are not certain, we need to come up with some measure of how uncertain they are. That measure is volatility, and it will have a large impact on the results of our analysis.

Second, volatility is a subjective measure. Although it is possible to calculate past volatility, there is know way to know with certainty the volatility of future oil prices. Eventhough the options model explicitly considers all possible future oil prices, the set of oil prices which the model considers is dependent on the input for volatility. Therefore, when using an options model, it is important to look at the results over a range of volatilities, in order to see the effect of this subjective input on the results.

Increased volatility will always increase the value of an option, no matter if the option is a put or a call. This result arises because the the worst thing that can happen to the owner of an option is that the option expires worthless, while the upside potential is virtually limitless. Therefore, increased volatility will result in greater upside potential without a corresponding increase in downside risk.
A sample case will help to illustrate this point. Exhibits 7 and 8 show a 90% confidence region for future oil prices assuming an annual standard deviation or volatility of oil prices of 10% and 30% respectively. The graphs also show the breakeven oil price line. At points along the breakeven line, the utility is indifferent between building a coal plant and a CT. At points above the breakeven line the utility will choose a coal plant, while at points below the line the utility will choose a CT. We can see that there is a much greater chance that fuel prices will go above the breakeven line if the volatility is 30% rather than 10%. Therefore, one might initially conclude that the coal plant investment today would appear more favorable in the 30% case. However, in this case the options model produces exactly the opposite result. The value of coal plant is $234 million less in the 30% case when compared to the 10% case.

The cause for this result is rooted in the flexibility option. By building the CT today we later have the ability to switch to coal in the future. Thus, it doesn't matter how high the oil price goes, because before it gets there we will have built a coal plant. However, it does matter how low the oil price goes, since we stand to receive a large benefit from continuing to expand with CT's. Since the potential for low oil prices is greater under the 30% volatility case, the flexibility option will be worth more. Thus, the CT looks more attractive when oil prices are more volatile or uncertain.

Although an increase in volatility will always make the CT look more attractive, the magnitude of the volatility effect varies from case to case. In general, the impact of volatility will be largest when the total costs (both capital and fuel) of the CT and the coal plant are very close. To see this point, imagine a case where oil prices are extremely low. In order to for us to build a coal plant, oil prices will have

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1 Note that the breakeven line does not consider the effect of flexibility.
Exhibit 7
90% Confidence Interval for Oil Prices
Given an Annual Standard Deviation of 10%

Oil Price
($/mbtu)

Upper Bound
(90% Confidence Interval)

Breakeven Oil Price
(excludes options)

Lower Bound
(90% Confidence Interval)

Year

0 1 2 3 4 5

8 7 6 5 4 3 2 1 0
Exhibit 8
90% Confidence Interval for Oil Prices Given an Annual Standard Deviation of 30%
to rise substantially. Under this scenario, we are likely to build a CT today and continue to do so in the future even if volatility is high. Consequently, volatility will not have a large impact on option value. At the opposite extreme, oil prices could be so high that it is very unlikely that we would ever want to pursue a CT. Once again, an increase in volatility is not likely to improve the value of the option. Finally, in the case where the economics of the coal plant and the CT are roughly equivalent, there is good chance that we will exercise the flexibility option. An increase in volatility will make it even more likely that flexibility will be exercised. As a result, an increase in volatility will result in a relatively large increase in the value of the option. In general, volatility will have a larger impact for options where the asset value is close to the striking price. These options are often referred to as near-the-money options.

To demonstrate this point, we evaluated the flexibility options over a range of volatilities for three different scenarios. These scenarios are defined by the oil price assumption and the capital cost of the coal plant as shown below:

Scenario
Favors CT: Oil price = 2.50 $/mbtu Coal cost = 1500 $/kw
Base Case: Oil price = 3.25 $/mbtu Coal cost = 1300 $/kw
Favors Coal Oil price = 4.00$/mbtu Coal cost = 1100 $/kw

The impact of a change in volatility on the value of the flexibility option is shown in Exhibit 9 for each of the three scenarios. The results show that the base case scenario is most impacted by changes in volatility. In the base case, the value of the option increases by roughly $300 million when the volatility is increased from 10% to 30%. The increases for the "favors CT" and "favors coal" scenarios are somewhat less at $230 million and $160 million respectively.
These results are consistent with the notion that volatility has the largest impact when the relative economics of the coal plant and the CT are close. At a volatility of 20%, the base case is a toss-up with a net benefit to the CT of $8 million. Also at a 20% volatility, the "favors coal" and "favors CT" cases show large net benefits to their namesakes of $785 million and $995 million (see table at the bottom of Exhibit 9). Since these results show that the base case is "near-the-money", it is not surprising that the impact of a change in volatility is greatest in the base case.

The fact that volatility has its greatest impact for near-the-money options is quite consistent with the notion that option value reflects the value of management. If conditions are such that the investment choice is clear cut both now and in the future (i.e. very high or very low oil prices), then management could simply make the initial decision and go on vacation without having to worry about the choice again. Clearly, there is not a lot of value being added by management here. However, in the case where decision-makers are likely to have to respond to future uncertainties, then by choosing the appropriate response management will be providing significant value to the utility and its shareholders.
Exhibit 9
The Impact of a Change in Oil Price Volatility From 10% to 30% on Flexibility Option Value

Net Economic Benefit to Coal Plant
(millions of dollars)

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Favors CT</td>
<td>-994</td>
</tr>
<tr>
<td>Base Case</td>
<td>-8</td>
</tr>
<tr>
<td>Favors Coal</td>
<td>785</td>
</tr>
</tbody>
</table>
5.4 Analysis of CT Plant Size

Throughout this paper, the flexibility advantage to CT's has been discussed at length. By building a CT, we obtain the flexibility to build whichever generation alternative is cheaper in the future. The analysis thus far in this section has demonstrated that flexibility is a valuable attribute of the CT's. What is it about the CT's which makes them so flexible?

The key attribute of CT's which makes them flexible is their small size. Suppose that we could build a series of small nuclear plants that were the same size as CT's. While the nuclear plants might be more expensive to build than the CT's they would give up nothing in the way of flexibility. We could build these nuclear plants, and if another Three Mile Island or Chernobyl occurred, we could simply exercise our flexibility option and switch to CT's or coal. While this example may seem absurd, it does illustrate the point that any plant which is small in size represents less of a commitment of capacity and consequently provides the utility with flexibility.

This discussion prompts the question, if small CT's provide all this flexibility value, then maybe even smaller CT's will provide even more flexibility. This hypothesis certainly appears reasonable. By allowing management the opportunity to respond more quickly to uncertainties, management will be able to make better decisions. This argument is similar to that used to promote just-in-time and flexible manufacturing. By having the capability to switch product lines quickly, a flexible manufacturing operation is able to keep inventories low and current with consumer tastes and preferences. Similarly, by building CT's, the utility keeps its inventory of excess capacity low and is able to add new capacity "just in time" to meet load growth.
To test this hypothesis, the options model was run comparing CT plant sizes of 40, 80, and 160 MW against the coal alternative of 800 MW. These runs were made for the base case data described in section 4.1 with an oil price volatility of 20%. The values for the flexibility option for each of the CT plant sizes is summarized in Exhibit 10. The chart shows that reducing the size of the CT results in an increase in the value of the flexibility option. Going form 160 to 80 MW nets a gain of $87 million, while moving to 40 MW nets an additional $48 million. The total benefit over the 160 to 40 MW range is a hefty $135 million.

To get a feel for the magnitude of this figure, we can divide through by the 800 MW of capacity the utility is needs to build over the next five years. The 40 MW CT saves the utility 169 $/kw relative to 160 MW model. Thus, the utility can afford a capital cost which is 169 $/kw or 42% larger if it uses the smaller plants. Switching from 160 to 80MW yields a benefit of 109 $/kw or 27%.

Certainly, by building smaller CTs, the utility will incur some extra costs. A smaller plant size will probably result in a loss of some economies of scale in construction. Thus, a smaller CT may cost more on a per kw basis. In addition, a smaller plant may give up some thermal efficiency which would be reflected in the form of higher fuel costs. Furthermore, the utility will have to make decisions quickly. Instead of annually or bi-annually, planning analysis will need to be conducted quarterly or semi-annually. A shorter planning cycle will require more staff and managerial effort. Nevertheless, you can hire a lot of staff for $135 million! Consequently, provided that it is technically feasible utilities should give serious consideration to smaller plant sizes.

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1 This result is based on a capital cost of 400 $/kw for the 160 MW CT.
2 However, smaller plants can improve operational efficiency on a system-wide basis. An outage at a small plant is far less costly than an outage at a large one.
Exhibit 10
The Impact of CT Plant Size on Flexibility Option Value
Note this point is just as applicable to coal plants. Suppose we could build a coal plant which has the same 160 MW size as the CT. The CT would no longer have a flexibility advantage and the value of the flexibility option would go to 0. In the base case analysis for 160 MW CTs the flexibility option was worth $214 million or 268 $/kw. This figure represents over 20% of the capital cost of the coal plant. Thus, substantial savings can be realized from building smaller more modular coal-fired facilities.

Moreover, we don’t have to restrict the analysis to coal plants and CTs. In general any generation option, which is smaller in size provides flexibility and the option value that goes with it. Cogeneration and load management, both provide capacity options which can be added in small increments. For example, installing a direct load device on a residential air-conditioner might have the potential to reduce summer peak demand by 3 KW. Thus, installing this device is similar to installing a 3 KW power plant. Clearly the flexibility advantage of installing a plant of this size is enormous. Thus, management should look to smaller more flexible generation as a possible means to improve company performance.

Nevertheless, taking advantage of the flexibility afforded to smaller scale plants offers utility management significant challenges. These challenges can be grouped into two categories - technological and organizational. On the technology front, design and construction engineers will need to focus on developing small scale generation alternatives which do not sacrifice economies of scale in construction and thermal efficiency in operation. Organizationally, utility planners will need to adopt a "just-in-time" mentality to capacity expansion. Its not enough to talk about achieving the benefits associated with small scale generation, utilities will need to structure their organizations to respond to the opportunities which flexibility provides.
VI. Conclusion and Review

The purpose of this thesis has been to develop a methodology to assess the value of flexible generation alternatives to electric utilities. In accomplishing its goal, the thesis draws upon options theory from finance. The thesis shows that methods used to value options on financial assets can also be used to value options on physical assets such as power plants.

The options model is superior to simple discounted cash flow models in two respects. First, the options methodology allows for the explicit consideration of all possible future oil prices. Second, the options approach explicitly considers flexibility and the management's ability to benefit from flexibility in making future investment decisions.

The thesis has two main parts. Initially, an analytical model is developed which can be used to compare the economic merits of competing utility generation alternatives. The model provides a rigorous treatment of flexibility and explicitly considers its value. In the second half, the model is then applied to generic utility data to demonstrate the importance of flexibility.

In developing the model, a comparison is made between a single base-load coal facility and a series of combustion turbines as a basis for the analysis. While any two generation alternatives could have been chosen, the main feature of the two plants used here is that building a coal plant represents a significant commitment or capacity while CTs can be added incrementally to meet demand as it grows. The modularity of the CTs provides the utility with significant flexibility which can be valued using options techniques.

The analysis shows the flexibility provided by the CT can be thought of as a series of compound options. Every time a utility builds a CT, it acquires the option to choose the low cost plant type in the next period. If a utility builds a coal plant, it
gives up the option to add a new plant for several periods into the future. Based on
the binomial option pricing model, a method is developed to calculate the value of
the compound option associated with flexibility.

In the second half of the thesis, the options model is applied to some realistic
utility data. The results show that using an options model can result in significant
improvements over a naive discounted cash flow approach which does not
consider flexibility. Alternatively we can think of value associated with flexibility as
the value of management. If a firm builds a coal plant (or any other large plant),
management's hands are effectively tied. The utility has limited its flexibility in
responding to future events. Therefore, if we believe that management's decisions
can add value to the utility, we should use an options model to properly reflect that
value.

The main portion of the analysis compares a single 800 MW base-load coal
plant to a series of five 160 MW combustion turbines. The major finding is that the
flexibility associated with smaller power plants has significant monetary value.
Some of the more interesting results are:

- For a reasonable base case the value of the flexibility associated with the
  combustion turbines is roughly $200 million. This corresponds to roughly
  250 $/kw or 63% of the capital cost of the turbines.

- Decreasing the size of the combustion turbines from 160 to 40 MW
  increases the value of flexibility by an additional 135 million dollars or:
  169 $/kw.

- Building modular coal plants which are the same size as the combustion
turbines, has the same effect as reducing the coal plant's capital cost by
20%.
These results are critically dependent on the uncertainty surrounding future oil prices. In general, the greater the uncertainty or volatility the greater the value of flexibility. Unfortunately, volatility is a subjective measure. Although it is possible to calculate past volatility, there is no way to know with certainty the volatility of future oil prices. Even though the options model explicitly considers all possible future oil prices, the set of oil prices which the model considers is dependent on the input for volatility. Therefore, when using an options model, it is important to look at the results over a range of volatilities, in order to see the effect of this subjective input on the results.

Nevertheless, as long as there is some degree of uncertainty surrounding future events, flexibility will always provide value to the utility. This result has policy implications for utility management.

Any generation option, which is smaller in size provides flexibility and the option value that goes with it. Cogeneration and load management, both provide capacity options which can be added in small increments. For example, installing a direct load device on a residential air-conditioner might have the potential to reduce summer peak demand by 3 KW. Thus, installing this device is similar to installing a 3 KW power plant. Clearly the flexibility advantage of installing a plant of this size is significant. Thus, management should look to smaller more flexible generation as a possible means to improve company performance.

Taking advantage of the flexibility afforded to smaller scale plants offers utility management significant challenges. These challenges can be grouped into two categories - technological and organizational. On the technology front, design and construction engineers will need to focus on developing small scale generation options which do not sacrifice economies of scale in construction and thermal efficiency in operation. Organizationally, utility planners will need to adopt a "just-in-time" mentality to capacity expansion. The benefits associated with small
scale generation cannot be accomplished unless utilities structure their organizations to respond quickly to the opportunities which flexibility provides.
References


