USING INFRARED THERMOGRAPHY TO MEASURE THE MATURITY OF CONCRETE

by

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Submitted to the Department of Civil Engineering on May 19, 1989 in partial fulfillment of the requirements for the Degree of Master of Science in Civil Engineering

ABSTRACT

Maturity, or degree of hydration, can be expressed as the product of time and temperature. According to the equation developed by Saul, maturity, M, can be represented as follows: $M = \sum (T - T_o) \Delta t$, where $T$ is the temperature in the time interval $\Delta t$. A current testing method that utilizes the maturity concept to correlate strength with maturity is maturity meters which use thermocouple probes to obtain the temperatures of the concrete. In response to the inflexibility inherent in the use of probes, this thesis examines the feasibility of using infrared thermography to measure the temperatures. After a brief synopsis of past research on the maturity concept, parametric studies are performed. Mathematical models are developed to study the effects of slab thickness (12", 24", and 36") and to investigate the influences of solar radiation, air velocity, and ambient air temperature. A heat transfer software program, ADINA-T, simulates the mathematical models and generates the resulting temperatures of the concrete.

From the results of the parametric studies, the thickness of the concrete slab largely determines the temperature, and hence, strength gradient of the concrete. Analysis of the ratio of strength at a particular depth to the average sectional strength reveals several findings: that the ratio is mainly a function of thickness, that the difference between the surface and interior strengths decreases as the curing time increases, and that the depth at which the strength is equal to the average strength is a function of thickness and independent of time. The study has shown surface temperature can used to obtain the strength of the concrete. Thus, infrared thermography is indeed feasible for the quality control of concrete.

Thesis Supervisor: Dr. Kenneth R. Maser
Title: Research Associate, Department of Civil Engineering
For my Mom whom I love so dearly
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In Kiev, capital of the industrial Ukraine, workers were in a bind to get a building up in the allotted time. The newspaper Rabochaya Gazeta said the construction crews fiddled with the architect's plans to cut the work and then produced a building in record time. When the workers eagerly swung the roof into place, the structure neatly collapsed in a heap. They had left out that part that says "allow the concrete to dry (cure)."

Source: UPI report
published in the San Francisco Sunday Examiner and Chronicle
January 4, 1976

On April 1978 a cooling tower under construction at Willow Island in West Virginia collapsed -- killing 51 workers. The contractor was using a slip-formed construction process involving a multilayer scaffold that raises itself up the wall by its own power after anchoring into the hardened concrete of the previous day's work. According to an investigation by the Office of Safety and Health Administration, the accident "could have been prevented if proper engineering practices had been followed." Investigation findings cited that one of the key factors contributing to the collapse was "a failure to make field tests to be sure that the concrete had cured sufficiently before the support form: were removed."

Source: Based of a report by Eugene Kennedy
published in the San Francisco Sunday Examiner and Chronicle
December 3, 1978
Chapter 1

Introduction

Among the oldest building materials used by man and still a most common and indispensable construction component is concrete. Concrete is a complex assemblage of relatively chemically inert materials, normally called aggregates, and cementitious binders such as lime, calcite, clay, natural cement, and artificial cement (Portland cement). Man has used concrete in his buildings for centuries, and one of the main reasons for concrete’s ubiquity is that the components of concrete occur naturally, are widespread, and are readily available. As a consequence, in places where steel and wood are largely unavailable, concrete has become the chief construction material. The lesser developed and third-world countries of the world mainly depend upon concrete for their homes and other buildings.

The popularity of concrete is readily apparent in nearly all construction projects -- from small residential homes to huge public infrastructure systems.
Concrete has almost totally replaced wooden timbers for the foundation of modern homes. Many times in large building construction, concrete with the presence of reinforcing material are a more economical yet equally adequate alternate to steel. Foundation piles are often concrete; when more floor to floor clearance is required, concrete slabs are often preferred over steel truss floors. Even when the floor is of the steel truss construction, the top layer of the floor is concrete laid over steel panels. Thus, the use of concrete seems essential to some, if not many, aspects of all construction projects.

Nature and her wonderful particularities has her own form of concrete which in many instances geologists have commonly mistaken for man-made concrete. Over the long temporal periods of earth's history, loose rocks, sand, gravel, and sea shells have combined into a sedimentary rock called sandstone. What hold these various components together in a single and solid mass are naturally occurring binders, calcium carbonate, silica, clay, etc. which with the introduction of water have seeped into the interstitial spaces and have chemically bound all the components.

Similarly, such is the basis for the formulation of man-made sandstone, i.e. concrete. Hereafter, for the sake of simplicity and clarity, concrete will refer to the product consisting of aggregates and the cementitious binder, Portland cement. Since the aggregates are chemically inert, Portland cement largely determines the strength of concrete. However, unlike steel or wood, concrete, or more specifically, portland cement requires a period of hydration to impart it the ability to bind and, thus, the ensuing strength.
This hydration, more commonly known as curing, is necessary and involves a complex chemical process in which the various constituents of the cement undergo from their dry and separate states to an eventually dried again but united structure. The hydration process is an highly exothermic reaction, thus, liberating a vast and noticeable quantity of heat. A negative consequence of this compulsory step is the time needed for the cement to hydrate long enough to gain strength sufficient to withstand the removal of forms, withstand its own load, and loads imposed by additional, external factors.

The highly competitive nature of the construction industry is widely known. Even though the profit level is low compared to many other industries, many companies are annually formed because of low entry barriers. To gain a competitive advantage over other firms, some construction companies are trying to implement "hi-tech" tools and methods in their daily construction routine. The old adage, "time is money," is never more valid than in the construction business. In the majority of large projects, some manner of financing is essential for the projects to come into fruition. Thus, interest rates and interest payments are vital factors of any construction loan. Builders may not be able to affect the interest rate, but they can and do indeed try to minimize the interest payments by completing the projects as early as possible. Owners, also benefit from early completion since they can utilize or rent out the premises sooner; therefore they frequently give contractual incentives to the construction companies to finish as soon as possible.

In order to minimize the construction time, many construction firms are utilizing CPM (Critical Path Method), new technologies, "hi-tech" equipment, etc.
As aforementioned, the use of concrete is most likely; yet compared to other building materials, concrete often has an unpretentious image and is overlooked as a source for project improvements. Unlike steel or wood, concrete requires a protracted, sometimes even substantially longer, time to place because of the needed curing process. Thus, it is not unreasonable for contractors to be eager to remove concrete forms and shores as soon as possible and to start work which depends upon the completion of the concrete activities.

However, the desire of the contractor to shorten the duration of the concrete activities conflicts with the need of the concrete to undergo a sufficient hydration time. It was inevitable that researchers attempted to predict the strength of the in-situ concrete so that the contractors could reliably predict the time when the concrete had reached a certain minimal strength so that forms and shorings could be removed. The most common test for predicting the strength of concrete is the cylinder compression test. Sample cylinders of the concrete used in the construction project are cured on-site and in the laboratory. When the job dictates early results and predictions, the cylinders are crushed and their strength measured after appropriate intervals such as 1, 3, 7, 14 days. Then the 28 day strength of the concrete is extrapolated from these laboratory results.

However, there are many reasons to question the validity of this test. A chief criticism of the methodology is the different curing environment of the sample cylinders. Even though the sample cylinders may come from the very field concrete itself, the differences of the environmental factors such as solar insolation, temperature, humidity, wind speed, etc. may significantly affect the strength of the concrete. There are other drawbacks to this method; the transfer and movement
of many cylinders can be cumbersome, a big and heavy hydraulic compression
device is necessary for the testing, and there is a possibility of a mix-up of cylinders
due to incorrect labeling or handling. Alas, because of the lack, or rather, the
paucity of advances in this field, this method remains the most commonly accepted
means of testing and predicting the strength of concrete.

Fortunately, the very idea of determining the strength of the in-situ concrete
recently spawned an interest in employing a concept that links the development of
concrete strength to the gain of strength -- the concept known as concrete
maturity. Since concrete generates heat during its curing (hydration) process, the
maturity concept attempts to correlate, due to evolution of heat, the rise in
temperature to the degree of hydration, that is, the strength of the concrete.
Several present systems utilize thermocouple probes which are imbedded into the
fresh concrete and periodically measure the temperature of the concrete. Lastly,
by means of a pre-calibrated curve, the strength of the concrete is deduced by the
maturity (temperature integrated over time) of the concrete.

Recently, there has been a growing interest in the area of non-destructive
and in-situ tests such as sound waves, ground penetrating radar, and infrared
thermography. This research investigates the feasibility of using one of these
methods, namely infrared thermography, to determine the quality and/or strength
of fresh concrete. Instead of thermocouple probes, infrared thermography is
employed to read the temperature of the curing concrete. There are several initial
advantages of infrared thermography over thermocouple probes. Infrared
thermography provides a wider coverage since it is not limited by the number of
channels available in the measuring device as in the case of probes. The output
of infrared thermography can be a 2-dimensional image, whereas, probes can only provide spot readings. And lastly, the infrared scanner is transferable from one location to another location; probes, on the other hand, are fixed in place and can only be used once because they are embedded in the concrete.

However, infrared thermography possesses the inherent disability to read temperatures other than surface temperatures. Immediate questions concerning the ability of surface temperature readings to represent the true interior temperatures of the concrete justly arise. Since the maturity concept is predicated on the temperature rise due to internal heat generation, surface temperature readings may lead to false interpretations of concrete strength.

This research explores the effects of the various environmental factors, which are present during the hydration of the concrete, upon the temperatures generated by the concrete. Parametric studies involving models with various environmental settings are used to generate the temperature gradients of the different models. From the resulting temperature readings and their implications, the ability to accurately predict interior temperatures and, hence, the feasibility of using infrared thermography are then discussed.

The research consists of several separable components. First, there is a description of the present technology involving non-destructive methods currently in use and of infrared thermography. Next, a thorough investigation of all research on the maturity concept is carried out to ascertain the validity of the concept and methodology. From the above and additional research on concrete hydration, a typical physical model is assumed. The physical model leads to the creation of a numerical model which is used in the succeeding parametric studies. Lastly, the
results of those studies are discussed, and recommendations and proposals regarding the use of infrared thermography to assist or determine quality control and, perhaps, even to predict concrete strength are given. The results will ultimately determine whether or not infrared thermography (via the maturity method) is feasible for one or both of the above stated objectives.
Chapter 2

Testing and Control of Concrete Quality

Most projects in which concrete is a major building component call for a program of quality control and assurance of the concrete. The main objective of such a program is to ensure that the finished concrete element is structurally adequate for the purpose for which it was designed so that potential disastrous structural failures (like the ones described in the epigraph) are prevented. Furthermore, a field engineer must recognize the real objectives and limitations of the various tests normally performed on concrete. For example, the slump cone test is often incorrectly used as a quality control tool. The test is meant to check the workability and consistency of the concrete mixture and not to indicate the potential strength of the concrete mixture.

The Classical Testing Approach

Up till and including today, the standard accepted method by which the quality control of concrete is obtained is the concrete cylinder compression test.
A standard procedure is described in ASTM C 39-72. 6"x12" concrete cylinder specimens are prepared from the actual concrete batch used on-site. After these cylinders are carefully labeled, half of them remain on the job site and the other half are transported to a laboratory where they are stored, cured, and then tested. The actual breaking of the specimens (field and laboratory cured cylinders) in compression occurs after 28 days after casting. However, when results are needed at earlier dates, some cylinders may be tested at 7 or 14 days in the case of concrete made with Type I (normal) cement. With Type III (high-early strength) cement, the cylinders may be tested as early as 3 days.

Abdun-Nur [1] correctly points out that this method of testing is not orginally intended to provide a measure of the strength of the concrete in the structure, but rather, the potential characteristics of the concrete being placed in the structure. He further states that experience over the years has shown that with current American design procedures and construction methods, assuming that the concrete in the structure is adequately cured and protected, these test results, if acceptable, give an indication that the concrete in the structure would serve its purpose.

A weakness of this approach is the potential situation in which some of the test cylinders fail to surpass the nominal design strength which is the minimal acceptable level. What would happen to the concrete, if, after 28 days had passed, the results of the test showed that some samples had not met the nominal design strength? As a partial solution to this possible, realistic scenario, it has been standard practice to allow the in-situ concrete to pass the quality control test if only a certain percentage of the test specimens met the nominal strength. Yet
this solution is subject to sampling errors, finite number of samples, differences in compaction and curing conditions. In many cases in which the concrete does not meet the requirements, nothing can be economically done. The cost to remove and replace the defective concrete having hardened for 28 days may just prove to be too prohibitive. Yet despite these drawbacks, this method of testing remains the most commonly used means.

The original intention for this test is solely for quality control purposes, that is, to determine if the concrete meets a minimal performance level calculated in the engineering designs and stated in the contractual agreements. If the concrete does not meet the minimal acceptance level, some remedial actions and measures are called for and taken. However, the purpose of this test has been stretched to include strength prediction. An example is to predict, for planning purposes and prior to actual construction, the time when the concrete will have attained a certain strength so that shorings can be removed. The standard compression test used for quality control has problems, therefore, the extension of the test for strength prediction is simply improper. Nonetheless, this practice exists. In addition, three of the following tests methods are similarly and improperly employed. Abun-Nur [1,p.6-7] laments the trend as revealed wonderfully by his comments on the warm-water, boiling-water, and autogenous methods:

"From the beginning the committee (ASTM Committee C-9) felt that the results of the accelerated curing tests should be used per se to evaluate the quality control of the process. This is best done through the use of a control chart, particularly one that has warning and action limits, so that corrections in the process can be made before the process gets out of control.

Unfortunately, every paper outlining or describing an accelerated curing method stresses the prediction of the 28-day strength from the accelerated test results, more than the usefulness of the methods itself for evaluating quality control. This detracts from
the early test usefulness because it keeps stressing the 28-day strengths. This predicting the 28-day strength from the accelerated tests seems to have become a fetish of a sort.

Seeing that most published papers on the subject come up with predictions reliable within ±15 percent, such predictions are no better than guessing, as an educated guess, by someone familiar with concrete in question and working conditions on the particular job, will be as close or better. In addition, all these predictions are predicated on the so-called correlations that show high coefficients of correlation. But all that this means mathematically is that the two sets of data go up and down together; it does not prove any relationship between the two. In some cases this correlation may be valid, but in more cases than not it may be illusory."

Nonclassical Testing Methods

To circumvent these limitations and drawbacks, numerous other approaches have been devised. They generally fall into two main groups, accelerated strength testing and in-situ and nondestructive testing. As the name implies, the objective of the accelerated strength testing methods is to obtain earlier predictions of the quality of the concrete by the means of accelerating the hydration process. Three such tests are the warm-water method, the boiling-water method, and the autogenous method. Reviews of these test procedures are covered by ASTM C 684. Within the in-situ and nondestructive testing category, there are two subdivisions: first, those that attempt to measure some property of concrete from which an estimate of strength, durability, and elastic behavior of the material may be obtained; and second, those that attempt to determine areas of poor consolidation, voids, and cracks. Mehta [27] summarizes the various techniques (both accelerated strength and in-situ and nondestructive testing) as the following:

**Warm-water method.** This is the simplest of the three accelerated strength tests and consists of curing standard cylinders (in their molds) in a water bath
maintained at 95°F for 24 hours. The water contributes very little to the accelerated maturity, but rather, acts as an insulator, and thus, permits the heat of hydration to provide accelerated maturity in the cylinders. Aside from the water bath, the concrete in this method is tested in the same manner as in the classical method. The advantage of this method over others which use much higher temperatures (e.g. steam curing and the boiling water) is safety, but the method is useful only where there is a laboratory on the jobsite. Another limitation of the method is that strength gain; compared to the 24-hr moist-cured concrete at normal temperature, the strength is higher but by not that much. In the mid-1970’s, the U.S. Corps of Engineers conducted an extensive study on the evaluation of the warm-water method, from which it was concluded that accelerated strength testing with this method is a reliable means of routine quality control for concrete.

**Boiling-water method.** This method consists curing normally the cylinders for 24 hours, then curing in a boiling-water bath at 212°F for 3½ hours, and testing for their compressive strength 1 hour later. The method is the most commonly used of the three procedures in this group because compared with the 24-hr warm-water method, the strength gain at 28½ hours is much higher and cylinders can be transported to a central laboratory for strength testing, thus eliminating the need for an on-site laboratory. The disadvantages are the odd hours requiring overtime (and therefore, additional costs), the danger from steam or hot cylinder burns, and the possibility of abnormal hydration products. In the early 1970’s, the method was used successfully to develop concrete mix proportions in preliminary laboratory studies and to check field concrete in the construction of a large
number of dikes, spillways, and a huge underground power station for the Churchill Falls Project in Labrador, Canada.

**Autogenous Method.** In this method, test cylinders immediately after casting are placed in insulated containers and are tested for their compressive strength 48 hours later. No external heat source is provided; subsequently, the acceleration of strength gain is achieved by the heat of hydration of the cement alone. The strength gain at end of of curing period is not high. The advantages are safe working temperatures, regular working hours, and the ability to ship the specimens to a central laboratory. Of all these three methods, this is judged to be the least accurate. Nonetheless, this method in conjunction with the maturity method was used as an integral part of the quality control program in the construction of the CN Communication Tower in Toronto, Canada [27,p.225]. Consisting of 39,800 cubic yards of slip-formed concrete to a height of 1590 feet, the CN Communication Tower is the world’s tallest free-standing structure.

**Surface hardness methods.** The surface hardness methods consist essentially of impacting the concrete surface in a standard manner, using a given energy of impact, and measuring the size of indentation or rebound. The most commonly used method employs the *Schmidt rebound hammer*, which consists of a spring-controlled hammer that slides on a plunger. A standard procedure is described in detail in ASTM C 805. The Schmidt hammer is simple and the method provides a quick and inexpensive means of checking uniformity of in-situ hardened concrete, but the results of the test are affected by smoothness, degree of carbonation and moisture condition of the surface, size and age of specimen, and type of coarse aggregate in the concrete.
Penetration resistance techniques. These techniques use power-activated devices to determine the penetration resistance of the concrete. The Windsor probe is a system currently in use. In this system, a power-activated driver fires a hardened alloy probe into the concrete. The exposed length of the probe is a measure of the penetration resistance. As in the case of the Schmidt hammer, the results of the Windsor probe vary more greatly than those of the standard compression strength tests because of the relatively small areas tested. Nonetheless, this method is excellent for measuring the relative rate of strength development of concrete at early ages, especially for the purpose of determining stripping time for formwork. A standard test procedure is described in ASTM C 803.

Pullout test. These tests consist of pulling out from concrete a specially shaped steel insert whose enlarged end has been cast into the fresh concrete. A dynamometer is used to measure the force required to pull out the insert. The inherent drawback of the method is the necessity to repair the damage done to the concrete by pulling out the insert. However, like the penetration resistance test, the pullout test is an excellent means of determining the strength development of concrete at early ages and safe form-stripping times. The technique is simple and the procedure is quick. The main advantage of the pullout test is that it attempts to measure directly the in-situ strength of concrete. The major drawback is that unlike most other in-situ test, the pullout test has to be planned in advanced. ASTM C 900 describes a suitable test procedure for employing this method.

Ultrasonic pulse velocity method. This method consists of measuring the time of travel of an ultrasonic wave passing through the concrete. The times of
travel between the initial onset and reception of the pulses are measured electronically. The path between transducers, divided by the time of travel, produces the average velocity of wave propagation. This test is recommended for the purpose of quality control only; variables such as the age of the concrete, moisture condition, aggregate / cement ratio, type of aggregate, and location of reinforcement affect the relationships pulse velocity and strength. Generally, attempts to correlate pulse velocity data with concrete strength have not been overly successful. ASTM C 597 describes a suitable apparatus and a standard procedure.

**Maturity meters.** These methods employ devices which monitor the length of curing and with thermocouples the temperatures generated by the hydrated concrete. The maturity concept, as briefly mentioned earlier, provides a means of estimating the concrete strength by the relationship among time, temperature, and maturity. Like other in-situ, nondestructive tests, this method requires a precalibrated correlation between the maturity and strength of the concrete. This precalibration must be planned and carried out prior to the actual casting of the concrete, and the calibration specimens have to be of the same concrete mix and undergo a similar curing environment so that the hydration of the specimens can represent as closely as possible the hydration of the actual concrete in place. And like the pullout methods, this test involves placing inserts into the fresh concrete during the casting.

Several systems that employ the maturity concept include the M Meter by James Instruments and the CIMS Computer Interactive Maturity System by Digital Site Systems. The basic limitations of these systems are that: they use
thermocouples which must be inserted, and therefore requiring preplanning; the thermocouple can only be used once since they will be embedded in hardened concrete; the number of thermocouples is finite, and thus, not every point of the concrete structure can be monitored accurately; the number of available channels of the systems may be too limited, therefore requiring multiple systems whose additional cost may be too prohibitive; and finally, thermo-couple probes placed near the surface do not truly represent the overall temperature variation of the concrete, thus, leading to false interpretations of the actual conditions. To obtain a more accurate picture of the overall temperature gradient, more probes are needed and must be placed at various distances from the surfaces. These steps require careful planning, meticulous placement, and considerable insertion time.

Infrared thermography faces similar problems; it is the examination of these limitations that is the inspiration for this research, which attempts to determine whether or not differences between the surface and interior temperatures of the concrete are, in fact, important at all. If the differences are very minor, then the use of infrared thermography instead of thermocouple probes is feasible and preferable in the utilization of the maturity concept. In order for infrared thermography to be effective as tool in the quality control of concrete, a basic understanding of the theory and of its limitations is needed.
Chapter 3

Infrared Thermography

Through the use of infrared sensing instruments, temperatures of a wide variety of materials (including concrete) can be measured remotely and without contact. This is accomplished by measuring the energy, in the infrared portion of the electromagnetic spectrum, radiating from the surface of the material and converting this measurement to equivalent surface temperature.

A detailed discourse on the theory of infrared radiation is readily obtainable at any library; thus, only the fundamental principles are presented here to provide to the reader a basic understanding of the subject. Every object above absolute zero radiates energy in the infrared portion of the spectrum. Figure 3.1 is a chart of the electromagnetic spectrum locating each type of radiation from radio to gamma radiation in sequence of energy wavelength. The visible portion of the spectrum subtends a narrow band with wavelengths from 0.4 microns for violet light to 0.7 microns for red light. The infrared portion, from 0.7 to about 100,
microns is directly adjacent to the visible portion. Since the human eye can only see the visible portion of the spectrum, instruments are needed to detect the radiation in the infrared range.

![Electromagnetic Spectrum](image)

**Figure 3.1: Electromagnetic Spectrum**

The intensity of thermal radiation energy as a function of wavelength and temperature is defined by the Stephan Boltzmann equation, Wein's Law, and Planck's Law. More specifically, a form of the Stephan-Boltzmann equation:

$$N_i = \varepsilon \sigma T^4$$

relates the radiant energy per unit area, $N_i$, emitted by a surface to the absolute temperature $T$. $\sigma$ is a fixed number known as the Stephan-Boltzmann constant and $\varepsilon$ is a surface characteristic known as emissivity. An emissivity of unity (1.0) indicates a perfect emitter, more commonly known as a "black body." An emissivity of zero indicates a perfect reflector. In the real world, perfect black bodies do not exist; emissivity values fall between these two limits. Most organic, painted, and non-metallic surfaces are high emitters with emissivities approaching unity. Polished metal surfaces are low emitters with emissivities approaching zero.
Accordingly, infrared instruments used to measure temperature actually measure radiance, $N$, in a portion of the infrared spectrum and convert this to the equivalent temperature through mechanization and approximation of the Stephan-Boltzmann equation.

The other area of consideration to infrared thermography is the medium through which the measurements of the sensors are made. Almost all measurements are made through the (earth's!) atmosphere. From Figure 3.2, it is apparent that the atmosphere is not uniformly transparent to the infrared as it is to the visible portion of the spectrum.

![Figure 3.2: Atmospheric transmittance across wavelengths](image)

The two spectral bands or atmospheric "windows" in which infrared sensing instruments usually operate are from 3-5 microns and 8-14 microns. For other wavelengths, the atmosphere absorbs radiant energy outside of these windows, and thereby, attenuating instrument readings.

Besides the factors concerning the target to be measured and its environment, the instrument that is used to take the measurements must also be considered. A typical system might consists of components shown in Figure 3.3.
Collecting optics and infrared lens are necessary to focus the energy radiated from the target onto an infrared detector, which in turn, converts the radiant energy to an electrical signal. The processing electronics unit amplifies this signal and performs the calculation to convert measured energy to temperature, introducing corrections for such factors as surface emissivity and ambient temperature drift. The signal is then displayed by a meter or monitor. If only spot readings are required, a meter usually suffices. However, if the area to be measured is large, two or three dimensional outputs are preferable. Adding dimensions is accomplished by adding one or more scanning elements to the optical lens system.

Infrared detectors fall into two broad categories: thermal detectors which have a broad, flat spectral response, somewhat lower peak sensitivity, and slow
time constants (on the order of milliseconds), and photon detectors which have limited spectral response, high peak sensitivity, and fast time constants (on the order of microseconds). Thermal detectors generally operate near room temperature while photon detectors almost always require refrigeration (e.g. cooled with liquified nitrogen). To select a suitable instrument, its required performance should be determined. Seven major factors to consider are as follows:

1. Temperature Range - the high and low limits over which the target may vary.

2. Accuracy - the repeatability and calibration errors as well as errors expected due to sensitivity to other stimuli.

3. Temperature Sensitivity - the smallest change that is needed to be seen.

4. Speed of Response - how fast the instruments responds to a temperature at the target.

5. Target Spot Size and Working Distance - how big and how far away the target is.

6. Output Requirements - how the output is to be displayed or recorded.

7. Usage and Ruggedness - the ruggedness of both mechanical and electrical components versus size and weight, and the people who will be handling the equipment.

Applications

The use of infrared thermography in the construction industry is presently limited but growing. A common application of the technology is in the detection of defective roofs. Leaky roofs allow water to seep into the underlying layer of insulation. Because of the presence of water, the wet insulation has different
thermal properties than the dry insulation, and as a result, the temperature of the two will be different. Infrared sensors can easily detect the temperatures of the roof surface, and different thermal readings indicate the potential defective spots of the leaky roof.

The ability to differentiate area of different temperatures is further seen in the application of infrared thermography to detect poorly insulated areas of buildings. In winter, the poorly insulated areas permit more leakage of the warm interior air to the colder exterior environment. Because of the temperature differential created, the areas with the poorer insulation show up in a different shade or color on the screen of a typical infrared camera (scanner).

Kazzi [17] discusses the relatively novel application of using infrared thermography for assessing the condition of bridge decks. Defects such as cracks, voids, and delaminations in concrete and asphalt layer of the roadways have different thermal properties because of the presence of air or water. During a sunny day, these defective areas will absorb and emit heat at a rate different from that of the sound areas. Actual field experimentation produced satisfactory results in the detection of defective areas of numerous bridges in the New England area.

As described, these three applications reveal that infrared thermography can prove to be a cost saving tool in the ever growing and increasingly expensive area of maintenance in both the private and public sectors. Likewise, the benefits of infrared thermography technology can prove to be equally useful during the construction phases -- such as for concrete quality control and form stripping times.
Chapter 4
The Concrete Maturity Concept

Even from the beginning of this century, it was known that many factors, other than curing time, govern the increase in the strength of concrete, the most important among them being the temperature of curing. The combined effects of time and temperature have been the subject of study by investigators as early as 1904, but no hypothesis, let alone any reference to the concept of maturity, was formulated until the 1950's.

During the period from 1904 to 1940, the interest in confronting the problems of winter concreting led to much research in America. Researchers, McDaniel, Wiley, and Timms & Withey [24] published the results of their investigations on the effect of strength of concrete. The purposes of their study included:

- the strength which the concrete would attain at different ages at a constant temperature.
- the age at which a particular strength could be gained at different temperature.

- the strengths which might be expected at different ages at different temperatures.

- the effects of temperatures on the hardening rate of concrete.

- the strength of concrete when exposed to the different temperatures and conditions that the concrete would experience during winter.

The significant findings of their research indicated properties that are universally known today.

- Concrete specimens cured at higher temperatures achieved high strengths than those cured at lower temperatures initially; however, the ultimate strength of the specimens cured at the higher temperatures turned out to be lower than those cured at the lower temperature.

- The concrete cured at higher temperatures reached a particular strength in less time than did the concrete cured at lower temperatures.

- For different temperatures, the hardening rate of concrete proceeded in the same manner (i.e. same hydration products) but at different rates.

- For concretes exposed to temperatures below freezing, the strength at any time after the period of initial curing depended primarily on the strength developed during the first few days of the curing period.

During the period from 1940 to 1960, the source of development and research shifted to England and Europe where there was a growing interest in the accelerated testing of concrete as opposed to the problems of winter concreting.

With their research in the steam-cured concrete, Nurse and, in particular, Saul [35] in 1951 defined the term "maturity" as follows: "The maturity of concrete may be defined as its age multiplied by the average temperature above freezing which it has maintained....Concrete of the same mix at the same maturity (reckoned in
temperature-time) has approximately the same strength whatever combination of temperature and time go to make up that maturity."

Two years later, in 1953 Bergström [5], analyzed the data of McDaniel, Wiley, Timms & Withey, and Price to check the validity of the Nurse-Saul maturity concept and the principle of superposition as suggested by Hallström [24]. By combining the Nurse-Saul and Hallström concepts, he derived the following equation which is generally known now as the Nurse-Saul maturity function.

\[ M = \sum (T - T_o) \Delta t \]  \hspace{1cm} (1)

where \( M \) = maturity in degree-time, \\
\( \Delta t \) = time interval, \\
\( T \) = temperature of concrete in time interval \( \Delta t \), \\
\( T_o \) = datum temperature below which concrete will not gain strength.

\( T_o \) has been defined by many researchers and ranges from about 10°F to 14°F for most concretes. Bergström chose 14°F for the datum based on his results.

During the next two decades, researches such as Klieger [19,32], Plowman [31,32], Goral [13], Alexander and Taplin [2], and others performed numerous studies on the validity, accuracy, and applicability of the Nurse-Saul equation. However, no general consensus concerning the validity of the maturity function developed from these studies.

In 1956, Plowman [31,32] boldly suggested the following relationship between concrete strength and maturity. The percentage of the strength obtained at a maturity of 35,600 °F-hours (i.e. the maturity obtained by a concrete cured for 28 days at 64°F with a datum temperature of 11°F) may be represented by the expression \( A + B \log_{10} \) (maturity/1000), where \( A \) and \( B \) are constants related linearly to the strength. Plowman furthered stated that the relationship was
independent of the quality of the cement, the water-cement ratio, the aggregate-cement ratio, the curing temperature below 100°F, and the shape of the test specimens.

On the other hand, Klieger’s findings [19,32] that showed concrete cured at lower temperatures reached a higher ultimate strength (a property that earlier researchers had also established) led him to conclude that there was little chance of significant correlation on the basis as simple as the product of degrees and days. Klieger also questioned the validity of Plowman’s formula over a wide variety of concretes. Plowman’s relationship was only valid if the relationship between the logarithm of maturity and strength was linear, if the initial temperature of the concrete was in the range of 60°F to 80°F, and if there was no loss of moisture by drying during the curing period.

At approximately the same time, Rastrup [24], recognizing that chemical reactions run faster at higher temperatures (rate is approximately doubled for each 10°C (18°F) the temperature is increased), published in 1954 a time-temperature function of the following form:

\[ a_1 = 2 \exp[(t_s-t_1)/10] \times a_2 \]

where \( a_1 \) = the curing time at temperature \( t_1 \) in °C,
\( a_2 \) = the curing time at temperature \( t_s \) in °C.

Two years later, the general reporter of the 1956 RILEM Symposium on Winter Concreting [25] reached the following conclusion of comparison tests between Rastrup’s and the Nurse-Saul function:

"Study of the two functions under consideration seems to indicate that Rastrup’s function may possibly describe the development of the heat of hydration better than Saul’s function, whereas Saul’s function gives a better description of the development of strength. This is an apparent contradiction, which may perhaps be due to the
fact that the conversion of the heat into strength is not exact. In the
general reporter's opinion, Saul's function is simpler in practical use
than Rastrup's."

The issue whether the reaction rate (hydration) and strength gain of concrete was
proportional was and still is unclear. In 1958, Klieger wrote in the report of his
studies [19,p.1075] what he thought was occurring within the concrete during the
early stages of curing.

"Temperature seems to affect the hydration and hence development
of strength in two ways. First, there is the known effect of
temperature on the chemical reaction, the rate of reaction increasing
as temperature increases. Since in a general sense the strength of
like concrete mixes is proportional to the amount of hydration, this
greater amount of reaction at the higher temperatures accounts for
the higher strength at early ages. However, a second factor may be
that the type of hydration product obtained or the physical make-up
of the product is influenced by the temperature during this hydration.
Lower temperatures may be conducive to a better hydration product
or better physical structure of the product."

As stated by the general reporter of the 1956 RILEM Symposium, the relatively
simple Nurse-Saul equation was easier to use, and thus, much of the subsequent
research regarding the maturity concept was based on that equation. In this
thesis, unless denoted specifically otherwise, the term "maturity" will refer to the
Nurse-Saul equation (1).

In 1970, investigations by Malhotra [26] showed that the only way to to
establish maturity-strength relationships with accelerated strength tests was by
plotting strength versus maturity because the different accelerated strength tests
produced different maturities. The results of his findings are:

- There seems to be some degree of correlation between the maturity
  obtained from various accelerated strength tests....this appears to be
  true for a wide range of water-cement ratios, even though for each
  water-cement ratio there is a different correlation.
- When the test specimens are subjected to accelerated curing using the boiling water method and subsequently moist-cured, the strength gained fails to follow the gain in maturity.

- For the same maturity, concrete made with different brands of cement gives different strength results.

In the same year, Chin Fung Kee [18] proposed another correlation between strength and maturity. After analyzing some published data, he derived the following hyperbolic relationship -- \( S = \frac{M}{CM + A} \), which can then be linearized into the form -- \( M / S = CM + A \), where \( S \) is the strength at maturity \( M \), and \( A \) and \( C \) are constants. This relationship has the advantage over Plowman's in that, the value of \( 1/C \) gives the maximum strength that the concrete will attain with age, whereas, Plowman's equation implies that any concrete will attain infinite strength with infinite time. Kee claimed his relationship is valid for concrete specimens: made with ordinary Portland and rapid hardening cements; cured at different temperatures for an initial period followed by curing at a lower or higher temperature thereafter; tested at ages from 1 day to 1 year and at maturities from 50 to 20,000°F-days; made of normal as well as of lightweight aggregates; and made with various water-cement ratios.

In the late 1970's, as a result of investigations of building failures, the Center for Building Technology of the National Bureau of Standards (NBS) began to study the application of the maturity concept as a tool for in-place strength determination of concrete at early ages [21]. Based on their findings, Lew and Reichard suggested that the strength can be linked to the maturity of concrete by the following equation [8]:

\[
S = \frac{K}{1 + K a [\log(M - 30)]^b}
\]
where $S$ is the strength, $M$ is the maturity, and $K$, $a$, and $b$ are constants obtained from non-linear least square regression analysis. In his report, Carino [8] cited that a user could determine an approximate strength-maturity relation for a concrete mix by choosing the appropriate values of $K$, $a$, and $b$ from published figures since the constants were systematic functions of cement type, and water-cement ratio.

The NBS conducted further studies [6,7,8] which revealed that a modification of a hyperbolic equation originally proposed by Kee [18] in 1971 correlated very well with the NBS's results. Thus, the strength can be related to the maturity by the three parameter formula:

$$S = \frac{(M-M_o)}{\left[ \frac{1}{A} + \frac{(M-M_o)}{Su} \right]}$$

where $S$ is the strength at maturity $M$, $M_o$ is an offset maturity, $A$ is the initial slope at $M_o$, and $Su$ is the limiting strength as maturity approaches infinity. Figure 4.1 graphically shows the above relationship.

![Figure 4.1: Assumed shape of strength-maturity relation](image)

The above relationship is not purely empirical even though the basic form of the equation was originally empirically derived from the earlier works of Kee. In his NBS report [8], Carino provides a derivation of the hyperbolic function from basic
kinetic theory with the assumptions that the rate of strength development of concrete can be represented as a second order rate equation and that the rate constant is a linear function of temperature. Carino further suggested that improvements could made if the rate constant of hydration is not taken as a linear function of temperature, but instead, is related to temperature according to Arrhenius equation: \( k = A \exp \left( -\frac{E}{T_s} \right) \), where \( k \) is the rate constant, \( A \) is a constant, \( E \) is the activation energy divided by the gas constant, and \( T_s \) is the absolute temperature. Carino did not present results using this modification; he only presented the modification as a suggestion for further research by others.

As seen from all these studies performed to test the validity of the maturity concept, there has been a gradual refinement in the theory. Researchers no longer are seeking a magical formula which can predict the strength of any sample of concrete. Instead, they have realized that the early curing temperatures and individual characteristics of different concrete mixes are major determinants of concrete strength. Thus, the maturity concept can be an effective evaluator of concrete strength if the limitations are kept in mind; the concept is valid if the concrete mix remains constant and if the curing environment and temperature of the pre-calibration specimens closely are similar to those encountered by the actual concrete cast on the job.
The very complex hydration process that takes place when water combines with the cement portion of concrete is only somewhat understood. Textbooks covering the topic of concrete usually provide some explanation of the chemical interactions and processes occurring during hydration. However, many of the explanations are either too inadequate or too confusing. A succinct and readily comprehensible description of the hydration process can be found in the publication, *Principles of Quality Concrete*, by the Portland Cement Association [33]. Below are several paragraphs from that publication.

"Cement and water combined make hydration products. Space is necessary to house these products, and since the space is that originally occupied by water, it appears that the more water that the mix contained, the more complete the hydration process would be. However, all of the cement will not hydrate even after several years in a saturated atmosphere. Strength can be attained even if all of the cement does not hydrate, and higher strength is actually attained at lower water contents.

When cement and water combine, each particle of cement is surrounded by water. Hydration begins, producing a number of complex chemical substances. The dominant product consists of tiny particles suspended in
liquid. This material is known as tobermorite gel. Although other products are formed during hydration (crystalline calcium hydroxide, for example), all these products are often lumped together and called "cement gel." This term is apt because the gel is definitely the predominant product.

As hydration proceeds, these gel particles continue to grow until those around one particle of cement become intertwined with those of other particles. At this point, the paste becomes somewhat rigid; initial set has occurred. As hydration continues, the gel particles become more firmly intertwined, and the cement gains stability. Other products of hydration begin to fill the spaces between the gel particles. If the spaces are not completely filled, as is the case when excess water is added to the mix, air spaces or capillary pores remain in the concrete. These results in concrete that is lighter, more porous, less strong, and subject to absorption of water. If the reaction products completely fill all the spaces between gel particles, the cement paste becomes strong and impermeable to water. In comparisons of specimens containing equal amounts of cement but varying amounts of water, the concretes with more water have greater volume but less strength.

The initial period of cement hydration has four stages. (1) Immediately on contact of the cement and water there is a period of about 5 min when chemical reactions occur rapidly. These cause the temperature of the mix to rise rapidly. (2) The rate of heat generation then drops and remains low for about an hour. This is called the dormant period. (3) An increase in heat generation begins, peaking about the sixth hour after mixing. (4) The fourth stage begins as the heat generation starts to drop to a very low rate, usually within 24 hr, and to still lower rates after that. Among cements of various compositions, this pattern varies...Water is essential for hydration. As long as enough of it is present in the concrete, and some cement remains, hydration continues."

Temperature is a by-product of the cement reaction and is an important determinant of the strength gain of concrete. However, environmental factors such as solar insolation, wind convection, and daily temperature fluctuation can affect the temperature of the curing concrete. These environmental factors are more often than not totally neglected in concreting.

A simple analytical model of a plain concrete slab can be used to determine and explore the influences of environmental factors upon the temperatures within the concrete slab as it cures. Since infrared thermography can only sense the temperature slab at the surface, it is necessary to determine the differences and
the extent of the differences between the surface and interior temperatures of the concrete slab. The model developed in this thesis studies horizontal concrete slabs of 12", 24" and 36" thicknesses and includes heat input from solar insolation, effects of convection, and fluctuating temperatures of the surrounding air.

Prediction of the temperature variation of the concrete is a heat transfer problem. The determination whether it is a steady state or transient analysis depends on the boundary conditions. According to Kazzi [17], a transient treatment of the model is necessary if the Biot number of the model is greater than 0.1. Kazzi gives a 8" concrete deck with typical values a Biot number of 0.32. Because the Biot number increases with greater thickness, the 12", 24", and 36" concrete slabs will naturally have Biot values greater than 0.1, thus, warranting transient analysis.

Simplification, practicality, and solvability of the analysis require a series of assumptions:

- The concrete slab will be a 1-dimensional heat transfer problem because the thickness of a typical slab is generally a lot smaller than its length or width.
- The concrete material has isotropic thermal conductivities and heat capacities both of which are also time independent.
- The concrete is placed properly so that there are no detectable air voids.
- The daily temperature fluctuation is the same day to day during the curing.
- The concrete receives adequate moisture so that hydration is not affected by lack of water.
- The number of cloudy days are factored into the solar insolation function.
The absorptivity of the concrete is equal to its emissivity.

Figure 5.1 shows the different elements of the model.

![Diagram](image)

**Figure 5.1: A sample model**

The problem that needs to be solved to determine the concrete surface temperature is basically one of conduction heat transfer through the slab with boundary conditions. It has been assumed that the heat flow is 1-dimensional, and thus, the governing equation for the slab is:

$$
\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right)
$$

where
- $\rho$ = density of concrete,
- $C$ = specific heat of concrete,
- $k$ = conductivity of concrete,
- $T = f(x,t) = \text{temperature}$,
- $t = \text{time}$,
- $x = \text{position}$. 

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It is then assumed that the conductivity does not vary with temperature, the above relationship is simplified to:
\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]
where \( \alpha = \frac{k}{\rho C} \) = thermal diffusivity of concrete.

At the surface of the slab, the heat being conducted into or out of the slab must be equal to the net heat flow to the concrete surface produced by the combined effects of solar gain, convection, and radiation exchange with the sky. That is:

\[ q_{\text{upper surface}} = -k \frac{\partial T}{\partial x} \bigg|_{\text{upper surface}} = \alpha q_{\text{solar}} - h_u(T_s - T_{\text{air}}) - \epsilon \sigma (T_s^4 - T_{\text{air}}^4) \]

where \( \alpha \) = solar absorptivity of the concrete surface,
\( q_{\text{solar}} = f(t) \) = rate of solar energy concrete slab,
\( h_u \) = convective heat transfer coefficient,
\( T_s \) = f(t) = surface temperature,
\( T_{\text{air}} \) = f(t) = outside air temperature,
\( \epsilon \) = emissivity of surface in infrared range,
\( \sigma \) = Stephan-Boltzmann constant

The parameters appearing in the model include the concrete slab properties, the external weather conditions (insolation and air temperature), and the slab's infrared emittance and absorptance. The effects of wind speed appear indirectly in the external convective heat transfer coefficient. Because of the non-linearity posed by this problem, it is not possible to obtain a general, closed-form solution. However, there are several numerical methods by which the solution for any particular case can be obtained. These include finite difference, finite element, graphical, and response factor methods. It is the finite element method that is used to solve the various analytical problems in this study.
Chapter 6

Mathematical Model

Even with the simplification of the physical model, the internal heat generation of concrete during hydration exceedingly complicates the formulation of a representative mathematical model of the heat balance. Heat generation of concrete is a function of both time and temperature (maturity), yet the temperature itself is a function of internal heat generation and external environmental factors, thus, the mutual dependency and complication. The concrete temperature mainly controls the rate of the hardening process. During hydration, the hardening concrete generates heat. The balance between this internal heat generation and the heat transmission to the surrounding environment, in turn, controls the temperature development in the hardening concrete.

From the numerous investigations cited in Chapter 4, the weakness of the Nurse-Saul time-temperature function (1) is the assumption that time and temperature equally affect the maturity of concrete. In considering the effect of
temperature on the hydration rate, Danish studies [14,22] have come up with an alternative solution which employs the Arrhenius equation to describe the hydration as a function of the temperature:

\[ H(T) = \exp\left[\frac{E}{R} \left(\frac{1}{293} - \frac{1}{2723 + T}\right)\right] \]

where

- \( H \) = hydration rate
- \( E \) = characteristic activation energy, function of temperature, cement type, fineness, w/c ration,
- \( R \) = the gas constant
- \( T \) = temperature in °C.

The integration of this function over time yields the maturity age, \( M \).

\[ M = \int_{0}^{t} H(\tau) \, d\tau \]

There currently are existing electronic instruments that automatically evaluate the maturity of concrete according to the Arrhenius equation as well as the Nurse-Saul equation.

For computational purposes, the parametric studies of this thesis will use the simpler Nurse-Saul maturity equation (1). In addition to the selection of a maturity function, the studies require the derivation of the internal heat function. The correlation between the maturity and amount of heat generated can be readily determined by adiabatic calorimetry. This measurement records the temperature increase in a concrete sample during hydration without heat transmission to the surrounding environment, resulting in the establishment of heat generation as a function of maturity. Herein lies a limiting obstacle.

The heat generation rate ideally should be a function of both time and temperature. However, the finite element program used in this research places a major constraint on the type of heat functions that can input. ADINA-T can only
accommodate heat generation curves as functions of time and position and not temperature. Thus, another assumption must be made -- all the concrete elements experience the same heat generation rate at any given time, that is, the heat generation rate is independent of temperature. Naturally, there are questions concerning the validity.

To answer those questions, we must first re-examine the objectives of our investigation; they are namely:

- to explore the effects of the various environmental factors, which are present during the hydration of the concrete, upon the temperatures generated by the concrete, specifically, the surface temperature,

- to determine the temperature gradients across concrete slabs with different thicknesses,

- and to ultimately decide the feasibility of infrared thermography in concreting.

As discussed in a previous chapter, the relationship between the hydration rate and strength gain of concrete is not totally understood. It may be true that the hydration rate may increase with an increase in temperature, but it is less certain that there is a proportionate increase in the strength gain. Studies by Klieger and results presented in the 1956 RILEM Symposium acknowledged the increase in strength, but they also added that other factors at the higher curing temperatures might mitigate the total strength gain due to higher temperatures.

To minimize the errors produced by the heat generation function proposed for the model, extreme temperatures are eliminated from the model. By keeping the air temperature and the initial temperature of the concrete mix within a small range and near those at which the concrete will likely be, the temperature difference of the various sections of the concrete slab will be minimized, and
thereby, minimizing the variation of the hydration rates. Figure 6.1 shows the temperature dependence of the reaction rate in proportion to the rate at room temperature, 68°F (20°C), which is used as the reference temperature.

![Figure 6.1: Effect of temperature on the relative hardening rate](image)

As long as the temperature range does not deviate very greatly, the difference in the relative rates of reaction for the temperatures in the range will be small. It cannot be stressed enough that higher curing temperatures do not necessarily imply greater strength, especially, in the long term, for it is well known that concrete cured under low temperatures attains a higher ultimate strength than does a concrete cured under higher temperatures.
With the above reasons as the justification for the simplification of the reaction rate, and therefore, the heat generation rate, the next task is to derive a typical rate. Orchard [34], Czernin [12], Lea [20], and Mehta [27] have published some values of the total heat of hydration over a period of time. First, the relationship between the total heat liberated and time is assumed hyperbolic of the following form:

\[ H(t) = \frac{t}{(24A + Bt)} \]  \hspace{1cm} (2)

where \( H = \) total heat of hydration of Portland cement, 
\( t = \) time in hours, 
\( A, B = \) constants to be obtained by linear regression.

Linearizing the above relationship enables the use of linear regression to obtain the values of the constants \( A \) and \( B \) which are then plugged into Equation (2). Refer to Appendix A for the respective linearization, list of values, and results of the linear regression of the resulting analysis. Differentiation of the cumulative heat function yields the desired rate of heat generation. However, the rate is for cement alone and has to be adjusted for concrete. In a concrete with a mix proportion of 0.35:1:2:4, the percentage of cement in the concrete mix is 13.6; therefore, the above rate is reduced accordingly by a factor of 0.136. For illustration purposes only, 1 pound of cement might generate 100 Btu's; but 1 pound of concrete (with cement consisting 13.6% of the total mix) generates only 13.6 Btu's, not the full 100 Btu's.

Additionally, ADINA-T, (Automatic Dynamic Incremental Nonlinear Analysis of Temperatures), the finite element program that is used to solve the models, requires the heat generation rate to have units of amount of heat per unit volume. Using the density of concrete as 150 pcf and converting all units to the English
System, the resulting rate of heat generation (as a function of hour) is:

\[ h(t) = \frac{0.011425}{(0.01015 t + 0.5304)^2} \text{ Btu/in}^3 \]  \hspace{1cm} (3)

Figure 6.2 plots the rate of heat generation over curing time. Since ADINA-T requires the function to be in tabular format, values of the function are evaluated at hourly intervals from hour 0 (0th hour of day 0) to hour 671 (23th hour of day 27), and are listed as such in the input file for the ADINA-T software.

![Rate of Heat Generation](image)

Figure 6.2: Rate of heat generation over time

Before continuing the description of the physical model, an explanation of the convention used to denote time in the study is in order. Hours are recorded as consecutive numbers from 12 noon of the first day (day 0), hour 0, to 11PM of day 27, hour 671. Thus according to this notation, the next day is day 1, noon of that day is hour 24, and midnight of that day is hour 36. For all the concrete slabs, the hydration process starts at hour 0. Hour 0 is actually not the time when the water, cement and aggregates combine, but rather, some short time afterwards. Instead, the time when the concrete begins to gain strength rapidly will be
considered the 0th hour. The time at the very beginning is not considered because the exact shape of the initial portion of the curve is not well known due to the difficulty in strength testing at such early stages. And from an engineering point of view, that early portion is not important because of the low strength. As to the extent of the length of the curing period investigated, each concrete slab will cure for 28 days (up to the 23th hour of day 27, or hour 671).

The other heat source in the model is the daily insolation. Solar tables for common latitudes and times of the year may be found in solar energy handbooks and textbooks and in astronomical charts. The magnitudes of the solar insolation are given as a function of time of day. This function is parabolic in shape; however, the model will treat the solar input as a triangular function. The actual values used in this study come from Kazzi [17]. The values of the solar radiation, as seen in Figure 6.3, are for the latitude of 42°N and for the month of August. The given values take into account the absorption in the troposphere and the absorptance coefficient of concrete.

In addition to the solar insolation, two other environmental factors that partake in heat balance between the concrete and the surrounding environment are the daily temperature swing and convection. Even though in reality, the daily temperature fluctuation varies from day to day, a simple sinusoidal function that remains the same day to day will suffice for the model. The highest air temperature will be 80°F and will occur at 12 noon (0th hour). The lowest air temperature will be 60°F and that will happen at midnight (12th hour). All the temperatures at the other hours can be obtained from the simple sinusoidal function. Figure 6.4 is the plot of the temperature swing.
Figure 6.3: Solar insolation function used in study

Figure 6.4: Daily sinusoidal temperature fluctuation
There are two basic classifications of convection -- natural and forced. Natural convection occurs when the temperature gradient between two objects of different temperatures creates a movement of air, thus transferring heat. In the case of forced convection, a moving air mass such as the wind causes heat to be transferred between the air and the object. Heat transfer is generally more considerable in forced convection, and accordingly, the convective coefficients are higher for cases of forced than for natural convection. Reference [9] gives an empirical formula for situation of forced convection:

\[ h = 0.29V + 0.95 \]

where \( h \) = convection coefficient in Btu/(hr-ft\(^2\)\(^o\)F),
\( V \) = wind velocity in ft/s.

The parametric studies in which forced convection occurs assume the presence of a 10 mph wind, and as a result, the convective coefficient is 0.0361 Btu/(hr-in\(^2\)\(^o\)F).

Natural convective coefficients are functions of temperature differentials, and typical values given by Kazzi [17] are shown in Appendix B. Kazzi's values differentiate the case whether the surface is on top of or under the air mass. This analysis does not make that distinction, and, just uses the averages of those values. The "averaged" convective coefficients are listed along Kazzi's in Appendix B, and it is the former that are used in this study.

The last heat transfer mechanism to consider is conduction. The conduction that occurs within the concrete slab is assumed to be governed by 1-dimensional Fourier's law. The two thermal properties necessary for the calculations are the specific heat capacity and the thermal conductivity. For this investigation, the
values are taken from the work of Gotfredsen & Idorn [14]. After unit conversion to conform to ADINA-T specifications, the specific heat capacity is 0.0228 Btu/(in\(^3\)°F) and the thermal conductivity is 0.106 Btu/(hrin\(^°\)F).

Since the solution method is finite element, the concrete slab is divided into a series of 1" long elements. Since we are interested in the relationship between the surface and interior temperatures and to minimize computational time, only the top half of each horizontal slab is analyzed. For the cases involving 12" thick slabs, there are 6 elements; for 24" slabs, there are 12 elements; and for 36" slabs, there are 18 elements. Nodes are points between the elements with Node 1 being the surface, Node 2 between elements 1 and 2, and so on. The temperature output of ADINA-T is at a node. Thus, the 12" slab will have 7 temperature readings at a given time; 24" will have 13; and the 36" model will have 19. In each case the following is assumed and modeled: internal heat generation is subjected by every element, whereas, convective and radiative boundary conditions are experienced only by the top, surface element.

Since no boundary conditions are specified for the highest numbered elements, ADINA-T assumes that the models are symmetric. Thus, even though the data for only the top half is given, results of the bottom half will be identical to those of the top half because of the symmetry. Indeed, simulation of the 12" slab model with the top 6 elements specified produced results identical to those of a simulation with all 12 elements specified. A benefit gained from this simplification is the enormous savings (nearly 50%) in computational time and data manipulation. The models of 12", 24", and 36" concrete slabs curing under a hypothetical environment are shown in the figure on the following page.
Figure 6.5: Model of a hypothetical case
Parametric studies are conducted to explore the effects of the various environmental factors upon the the temperatures generated by the curing concrete. These studies fall into two main categories -- the effects of varying the thickness of the concrete slab, given the same environmental conditions and the effects of varying the environmental factors, given a slab thickness of 12".

In the first group of studies, the thickness of the concrete slab varies from 12" to 24" to 36". The environmental conditions remain identical in each of the three models. The concrete slabs experience solar insolation and a wind speed of 10 miles per hour (forced convection). Comparisons are then drawn from the results of these three models.

In the second category of studies, the thickness of the concrete slabs remains constant at 12"; it is the solar insolation and type of convection that change. Results in this group are compared to the results derived from the base
model or control of a concrete slab encountering the presence of solar insolation and a 10 miles per mile wind. To study the effects of solar insolation, there is a model which incorporates no heat input from the sun (i.e. to represent concrete curing in the shade or under heavy overcast conditions). To examine how the wind speed can affect hydration, there exists a model in which the convection coefficient is based upon natural convection as a representation of little or no wind speed.

As explained earlier, the use of maturity as a representative of the strength of the curing concrete requires a calibration test to correlate the maturity values and different strength levels. This study utilizes the maturity-strength relation developed by the NBS, 

\[
S = \frac{(M-M_o)}{[1/A + (M-M_o)/S_u]}
\]

where 
\[S\] = strength in PSI,
\[M\] = maturity in °F-day,
\[M_o\] = offset maturity in °F-day (refer to Figure 4.1),
\[A\] = constant in PSI/°F-day,
\[S_u\] = limiting strength in PSI.

From NBS studies [8,p.24], for a typical concrete mix with a water-cement ratio of 0.56 and cured at 73.4°F (23°C), the value of \[M_o\] is 21, the value of \[A\] is 25, and the value of \[S_u\] is 3810. As the results of the parametric studies will show later, the 28 day strength attained by the various model is around 3.6 KSI, thus, validating the values of the above constants. Accordingly, the resulting equation from which the strength of the concrete in the models of this research is calculated becomes:

\[
S = \frac{(M-21)}{[0.04 + 0.000262(M-21)]}.
\]

where the maturity, \(M\), is calculated according Nurse-Saul Equation (1).

The temperatures which are used to derive the maturity values are recorded at four hour intervals. Taking readings at smaller intervals would provide more
accurate results but there would be numerous drawbacks. First, it would be much too costly in terms of computer time, and secondly, there would be too much data to sum up and analyze. For example, a 36" model would require the analysis of 19 nodes over a time period of 28 days. If readings were taken every hour, there would be an astounding 12,768 (19x24x28) temperature readings to sum up, and that would just be the analysis of a single model.

To make the matter worse, ADINA-T outputs the results in an format incompatible for summation. Appendix C contains a sample ADINA-T output file. The output file consist of a replication of the input data and the listing of the temperature readings generated by the program. The temperature readings are printed in a single column format. To be able to split the data according to individual nodal points requires intensive manipulation. To automate and simplify the extremely tedious but necessary procedures, the ADINA-T output file is downloaded from the Digital Vax system (on which ADINA-T runs) onto a MS-DOS compatible diskette. Next, the spreadsheet, Lotus 1-2-3, is used to import the raw data into a Lotus compatible file. Within Lotus, the long single column of data is broken down into multiple columns each representing a nodal point. In such a matrix format, the various functions of Lotus can then sum up the temperature readings, generate the maturity values, and calculate the strength. The Lotus program is also used to generate the numerous plots which appear here.

**Results & Discussion**

Before presenting the strengths results obtained by Equation 4, it might be interesting to look at the temperatures generated at the surface and at the midpoint
of the slabs during the first three days of hydration. The first five graphs plot the temperature history of the air and of the surface and interior of the concrete from Hour 0 to Hour 72.

Figures 7.1, 7.2, and 7.3 plot the temperature output from the first part of the studies in which the varying parameter is slab thickness. Predicably, the surface temperature of the concrete follows very closely the temperature fluctuation of the air in all three models. But there is a startling difference between interior temperatures of of the 12" slab model and the other two. The interior temperature curve of the 12" slab model is somewhat sinusoidal, but it is clearly higher and lags behind the air and surface curves. The interior temperature curve of the 24" slab model still exhibits sinusoidal characteristics; however, the range between the peaks and troughs have narrowed considerably. And for the first time, there is a
Figure 7.2: Temperature history of 24" concrete slab

Figure 7.3: Temperature history of 36" concrete slab
slight but noticeable trend of the interior temperature starting to decline after reaching a peak around Hour 32. Figure 7.3 shows the results for the 36" concrete. The interior temperature curve is very much smoother, and becomes more or less independent of the daily air temperature fluctuations. From a peak at Hour 36, the interior temperature starts to slowly decline. This decline is expected because of the changing (declining) rate of the internal heat generation. Overall, from these results, the thickness of the concrete slab is a strong determinant of the temperature development within the slab.

When the parameter is changed from slab thickness to environment factors, Figures 7.4 and 7.5 ensue. The results of the temperature readings of the model with solar insolation and of the model with natural convection are shown in Figures 7.4 and 7.5. The results from the no solar input model resembles those from the normal 12" slab model. The surface node seems to keep up with air temperature fluctuation time wise but not temperature wise, thus, producing a curve of smaller temperature extremes and with a time lag. As for the interior node, the temperatures fluctuate within the smallest range, and the time lag is the greatest.

The natural convection model produces some somewhat striking results. In this scenario, both the surface and interior temperatures reach significantly higher than in the normal and no solar input models. The sinusoidal and time lag characteristics are present, but the temperatures are clearly higher than the air temperatures. This feature is due to the convective coefficients; the convective coefficients of natural convection are smaller than those of forced convection; the heat liberated by the concrete during hydration does not readily dissipate out to the environment, thus, causing the temperature of the concrete to rise greatly.
Figure 7.4: Temperature history of model without solar input

Figure 7.5: Temperature history of model with natural convection
The strengths results of the first set of parametric studies are shown in Figure 7.6 to Figure 7.11. The first three figures in that set plot the strength gradients across the half section of the concrete slab modeled. The results of these three graphs are surprising. In Figure 7.6, the strength gradients at Day 1, Day 3, Day 7, Day 14, and Day 28 are all relatively flat -- from node 1, the surface node, to node 7, the interior node for the 12" slab. There is less than a 5% difference between those two nodes. And as the curing time increases, the difference becomes smaller and smaller so that at day 28 the difference is nil for all intents and purposes. In other terms, the strength across the 12" slab is very uniform, in fact, so uniform that the strength measured at the surface is a very true representation of the overall strength profile and temporal development.

Figure 7.6: Strength profile & history for 12" concrete slab
Figure 7.7: Strength profile & history for 24" concrete slab

Figure 7.8: Strength profile & history for 36" concrete slab
An added bonus is that the surface strength, representing the weakest point, is a conservative measurement; if the surface strength passes a minimal acceptable level, then naturally, the rest of the slab will also. The results for the 24" and 36" models are shown in Figure 7.7 and Figure 7.8. respectively. As in the 12" case, as the curing time increases, the strength profiles of the 24" and 36" slabs flatten out. The difference between these cases is that at the early stages, the thicker the slab is, the more pronounced the difference between the surface and interior nodes is. Yet the difference diminishes over time. In an extremely large structure of concrete (e.g. dams) in which the 1-dimensionality assumption may no longer be valid, from the trends described above it can be logically inferred that the difference in the initial strength profile should be noticeable and that surface measurements can grossly understate the overall strength of the structure.

The next three plots show the development of strength of various nodes over a 28 day curing period. All three convincingly corroborate the point that the strength profiles of the concrete slabs do not differ much. There is no discernable difference among the temporal development of strength for nodes 1, 4, and 7 in Figure 7.9. It is only in the models with the thicker slabs that any difference among the strength development curve appear. See Figures 7.10 and 7.11. In the 36" case, the surface strength is initially lower than that of the interior nodes, but as the curing time increases, it eventually catches up.

It seems that all three models would attain the same ultimate strength level since all three at day 28 have reached the same level, approximately 3.6 KSI. Again, the only difference between the results of Figures 7.9 to 7.11 is the slight difference between the surface and interior nodes in the 24" and 36" thick slabs.
Figure 7.9: Strength development history for 12" concrete slab

Figure 7.10: Strength development history for 24" concrete slab
The second half of the investigation consists of studies in which the varying parameter is not thickness but is solar insolation and convection. In this part, the thickness of the concrete slab remains identical at 12". The desired model is obtained either by setting the solar insolation function equal to zero or, as the case may be, by changing the convective coefficients from forced to natural convection-based. The model, henceforth called the "normal model," whose results serve as the basis for the following comparisons has already been solved in the earlier section. For the results of this model, please refer back to Figures 7.6 and 7.9.

The results of the strength profiles of the model without solar insolation and of the model with natural convection are shown in Figures 7.12 and 7.13. The results from the no solar input model resemble those from the normal model. The slight difference between the two is that the strength profiles of the no solar input
**Figure 7.12:** Strength profile of model without solar insolation

**Figure 7.13:** Strength profile of model with natural convection
model at Day 1 and Day 3 are not as flat as those of the normal model. This would indicate that the sun or the lack of the sun may play a role, albeit minor, in the strength development of curing concrete. Also, the 28-day strength is just slightly lower than that of the normal model.

The results of the natural convection or no wind model are shown in Figure 7.13. This model demonstrates the same systematic behavior present in the normal and no solar input models. However, upon closer examination, the strength profiles of this model differ in that they are at a higher level at any given time. This result is not unusual but should be even expected since the environmental conditions of natural convection resemble those of an adiabatic situation in which faster reaction rates and, hence, higher strengths exist. The 28-day strength profile of the natural convection model attains a level above 3.6 KSI, whereas, the gradients of the normal and the no solar input models never quite reach that high.

Figures 7.14 and 7.15 contain the strength development histories for the no solar insolation and natural convection models respectively. As in the normal case, the curves for the various nodes hardly deviate from each other. Therefore, the strength measurement taken at the surface of the concrete should present an extremely fair picture of the overall strengths in the concrete slabs in all the models containing 12" concrete slabs. These strength development curves also reveal that the 28-day strength of the no solar insolation model is slightly lower than that of the normal model, whereas, the 28-day strength of the natural convection model is slightly greater.
Figure 7.14: Strength development history for 12" no solar insolation model

Figure 7.15: Strength development history for 12" natural convection model
It is fine to plot strength versus time or versus node (i.e. distance from surface) as done heretofore to gain some ideas of the strength development, but it is somewhat lacking in the area of satisfying the objectives of this study. In the quest to ascertain the relationship between the surface and overall strength in a more quantitative sense, and perhaps in the same process, producing a better qualitative understanding -- the ratio, k, of the two strengths is obtained for the various concrete thicknesses and environmental conditions.

\[ k = \frac{S_t}{S_{ave}} \]

where \( k \) = dimensionless ratio, 
\( S_t \) = f(node) = strength at a certain distance from surface, 
\( S_{ave} \) = average strength of across the half section.

The following five figures, 7.16 to 7.20, plots the k values versus the node number which represents the distance from the surface. Node 1 is 0" away from the surface, Node 2 is 1" away, Node 3 is 2" away, and so on. Figure 7.16 contains k values derived from strength readings at Day 1; Figure 7.17 - values at Day 3; Figure 7.13 - values at Day 7; Figure 7.19 - values at Day 14; and Figure 7.20 - values at Day 28.

Even without the results of the parametric studies, it can be deduced that the k value for the surface will likely be less than 1 and that the k value for the innermost node will be greater than 1. The results of the plots do, indeed, support this presupposition. The results produce three main points of considerable interest.

The first and most noticeable is that the families of k values are highly dependent on the thickness of the concrete slabs. The k families for the three models containing 12" thick slabs are approximately the same in shape and
magnitude. Deviations among the k families of the 12" slabs become only slightly greater in the later stages of hydration as seen in Figures 7.19 and 7.20. As the thickness of the slab increases, the k families shift below and to the right, indicating that the difference between the temperatures across the half section is greater as it should be. An unexpected finding from the results is that family of k values for the 12" natural convection model has a greater dispersion than either the normal or no solar insolation model at the later stages of hydration. Figure 7.20 clearly reveals this finding.

The second interesting discovery is the decreasing range of the families of k values. From the results of all the different models, the differences between the nodal strength and the average half cross-sectional strength become smaller as the curing time increases. For example, on Day 1 for the 36" slab model, the low k

![Figure 7.16: Families of k values on Day 1](image)
**Figure 7.17:** Families of $k$ values on Day 3

**Figure 7.18:** Families of $k$ values on Day 7
Figure 7.19: Families of k values on Day 14

Figure 7.20: Families of k values on Day 28
value is 0.931 and the high k value is 1.025. But on Day 28 for the same model, the low k value becomes 0.9976 and the high k value becomes 1.0012. This temporal trend occurs in the all families of k values. This property reflects the earlier finding of the strength profile plots, namely, that the strength profile becomes flatter (horizontally) as time increases.

The last but also probably the most subtle finding to be discussed is the distance away from the surface -- the distance whose strength is the same as the average strength of the half cross-section. According to the definition of the k value, the k value of this location is 1, and thus, the location can be easily found on the plots. For each thickness, the location of this point seems to be relatively constant throughout the different stages of the curing period. For the 12” slabs, this point is about 2.25” away from the surface; for the 24” slab - about 5”; and for the 36” slab - about 7”. In all three cases, the distance of point from the surface is about 1” less than the distance of the midway point of the half cross-section. This finding may bear no relevance for infrared thermography, but it is certainly most useful for thermocouple probes. If the average strength of the concrete slab is desired, then the probes should be placed at these depths according to the sole parameter -- the thickness of the concrete slab.

Comparison Between Temperature & Strength

In all the cases modeled, the surface temperature fluctuated, following the air temperature, albeit, with a slight time lag. Doesn’t it seem very strange that the surface temperature is highly dependent upon the air temperature, whereas, the strength is not? However, there may be several logical explanations to this irony
-- the time scale involved, the "averaging" of the daily temperatures for maturity calculation, and the sensitivity of strength gain to temperature.

Initially, the temperature graphs may be misleading. There are two time scales present in the various graphs; the temperature history curves are plotted from Hour 0 to Hour 72 (3 days) and the strength history curves from Day 0 to Day 28. In order to present the extreme and intermediate points of the fluctuating air temperature, the time interval has to be small enough yet at the same time the total time spanned should be able to fit on the graph, thus, the 72 hours. On the other hand, 72 hours is just too short to represent the development of strength. Since concrete is supposed to reach its specified strength by the 28th day after pouring, the entire 28 day span is important. In short, one must be careful when comparing results plotted at 4 hour intervals versus results plotted at 1 day intervals.

The very definition of maturity causes discrepancies between temperature and strength. Maturity ideally should be calculated as the temperature integrated over time, but there are no current instruments (maturity meters) capable of performing such a task. Thus, temperatures are usually taken at hourly or greater time intervals, and the maturity is calculated as the summation of the products of temperature and time interval. But no matter how the maturity is obtained, the maturity entity is derived from a cumulation of time-temperature values. This cumulation has the effect of smoothing out sharp points in the maturity and strength curves. If the time unit for plotting strength is 1 day, the high and low temperatures for a single day will be summed up and averaged, thus, eliminating any effects of a fluctuating air temperature. Setting the time interval in maturity calculation smaller (e.g. 1 hour) is possible, but the number of temperature
readings would soon be too overwhelming to handle (however, computers might change this around in the not so distance future).

Finally, the principle reason why the difference between the surface and interior temperatures is not large despite the difference in their temperatures is that strength is not that all sensitive to small temperature changes. Remember that maturity sums up the product of time and temperature. For a time interval of 1 day, the value of the temperature used in the calculations may be 80°F or so. The difference between the surface and interior temperatures is most likely not that great, perhaps, 10-20°F. The differentiation of the strength-maturity relationship (Equation 4) used in these parametric studies yields the following:

\[
\frac{ds}{dM} = 0.04 / [0.04 + 0.000262(M-21)]^2
\]

For example on Day 2, if M has reached a value of 160°F and the difference between the surface and interior temperature is 15°F, then the difference between the resulting strengths is only 103 PSI. In fact, the differential equation (5) and any plot of Equation 4 show that, as the curing time increase (and therefore, the cumulative value of M also), differences between surface and interior temperatures increasingly diminish.

The results and the observations just made indicate that non-extreme air temperature fluctuations do not influence to any appreciable degree the development of strength within concrete even though the surface or even the inner-most point may be experiencing temperature fluctuations.
Despite the complexity of the chemical hydration process and despite the lack of total comprehension of the factors affecting concrete strength, numerous researchers have boldly attempted to carry out quality control and/or predict concrete strength with accelerated and non-destructive testing methods. The maturity method is based upon the well known relationship among time, temperature, and strength of concrete. Systems implementing thermocouple probes are currently in use but these systems have limitations. Thus, it has been proposed that infrared thermography be used in place of the thermocouple probes to measure the temperature of the curing concrete. However, infrared scanners can only detect surface temperatures. To determine whether the surface temperature are satisfactory representatives of the overall temperature gradient, and thus, the strength profile and development, a series of parametric studies have been conducted.
The ADINA-T finite element program has produced the temperature gradients across the section of concrete at different time intervals and under different conditions. Two classes of parametric studies have been conducted to explore the influences of slab thickness, solar insolation, and convection upon the temperature development of the curing concrete. Along with the maturity concept and an equation correlating maturity and strength, the temperatures have produced the strength development of the different models. The previous chapter has presented the results of such investigations.

The first set of studies in which the parameter is slab thickness shows that the strength profile is reasonably flat and becomes even flatter as the curing time increases. The overall strength development history (i.e. strength vs. time curve) for various nodes along the half cross-section is almost identical in the models with the 12" concrete slabs. However, in the models with 24" and 36" slabs, these history curves deviate a little at early curing stages, but tend to approach a common ultimate strength. This set of results has produced a finding that could have been somewhat speculated and expected or even known -- that the ultimate strength of concrete specimens of different thicknesses but made of the same mix is independent of thickness provided that the environmental conditions are the same.

The second group of studies investigates the effects of environmental conditions by varying environmental factors such as solar insolation and the wind speed. Alas, the second set of studies produces no new startling findings. The strength profiles show the same systematic behavior as in the first set of parametric studies. The only new finding, or rather different result, is that the strengths of
the no wind (natural convection) model are higher than those of the model with a 10 mph wind (normal model). Likewise but in the other direction, the no solar insolation model attains strength levels that are lower than the normal model with solar radiation.

It is with the introduction of the k value (the strength at a particular node divided by the average strength) that some interesting conclusions can be drawn from the results. It seems that the family of k values is unique for a given thickness and is more or less independent of environmental factors. Secondly, for a given concrete sample, the differences between nodes along the cross-section become smaller as the concrete cures longer. Unexpected but most intriguing is the finding that the location along the cross-section, whose strength equals the average strength of the cross-section, is relatively independent of time but largely dependent of the thickness of the concrete slab.

The validity of the above conclusion are subject to correct concreting practices. It must be restated that assumptions have been made concerning the conditions encountered during the curing process, the nature of concrete hydration, and the properties of concrete. For example, it is assumed that the concrete is properly cast and is provided with adequate water during the entire hydration process. Under most ordinary circumstances, the assumptions made in the models of this investigation are valid. Vice versa, the models are likely not valid if the concrete experiences dramatic temperature changes (e.g. from 32°F to 212°F), if the weather is stormy for the entire curing time, or if the concrete workers carelessly allow the concrete to dry up.
From the extensive research findings of the many investigators mentioned in this report, the maturity concept can be a suitable tool for concrete evaluation under general conditions. There are flaws and inconsistencies which have yet to be explained. The implementation of any testing system based on the maturity concept must heed several key points. First and foremost, the mix of the field concrete and the concrete used in the calibration tests must be identical; different mixes produce concretes with different characteristics. In deriving the strength-maturity correlation, the curing condition of the calibration specimen should closely resemble the actual environmental conditions. Concrete that is initially cured under high temperature will attain an ultimate strength that is lower than that of concrete cured under lower temperatures. And lastly, the maturity method is only useful if a calibration test has been performed prior to the method's implementation on the actual field concrete; maturity measurement systems cannot be used spontaneously.

Based upon the results of this study, the measurement of temperature taken at the surface can provide a faithful representation of the overall strength profile and development of the concrete structure. Therefore, the following recommendations can be offered as a guideline for using infrared thermography in quality assurance and strength evaluation of concrete.

Infrared thermography is useful in only those situations in which the surface of the concrete is exposed since it can only detect surface temperatures. Thus, this inherent weakness rules out the use of infrared thermography in slipforming and vertical wall because the forms holding the concrete will prevent surface readings. While it is possible to infer concrete temperature behind the forms,
properties of the form material are needed, and the resulting mathematical model becomes very complicated and extremely difficult to solve. The solution is too impractical and costly in terms of money and in time.

However, a large portion of concrete construction entails horizontal slabs where the concrete is simply poured over a metal or wooden form. The top surface of the concrete remains exposed. In this situation, the reasons for using infrared thermography are obviously not for strength prediction (i.e. no need to determine times for safe form and shoring stripping) but are for general quality control. And even in cases involving vertical wall and slipforming, infrared thermography can provide quality control services after the forms have been removed. In construction projects in which there are vast expanses of concrete work, infrared thermography can be used to detect anomalous and defective regions. Because of poor batching, casting, lack of water, missing vibration steps, etc., the concrete in those areas do not hydrate to same degree as sound concrete does, and, as a result, their temperatures will not be the same. The job at hand is to spot thermal differences, and infrared thermography is adeptly able to perform such a task. Additional quality control tests can then be applied to further evaluate the concrete areas in question.

If non-destructive tests are desired for strength evaluation as opposed to quality control, other methods such as maturity meters using thermocouple probes, the Windsor probe, or the pullout test are available. The user must be mindful that even those systems have limitations and inherent faults, thus necessitating careful planning and implementation.
The basic advantage of using infrared thermography over thermocouple probes is the portability and ability of the infrared scanners to survey large areas. Current maturity probes and other forms of non-destructive systems test only a small area of the concrete. When used in conjunction with other forms of evaluation, infrared thermography can, indeed, provide additional assurance measures in a quality control program.

This investigation has shown that infrared thermography may be use in certain applicable situations to detect the temperatures necessary in the maturity method. However, the maturity concept has not proven to be a panacea for the difficult problem of determining quality control and/or measuring concrete strength. No one testing method has emerged with great and consistent results. Each method has its own inconveniences and inherent limitations. By and largely, the dissemination of new or just different technology has remained in academia and research labs. Until very recently, the construction industry has remained immune to changes and new technology. Thus, until refined and inexpensive systems appear on the market, the cylinder compression test seems destined to remain the standard means of concrete testing and evaluation for quite some time.
List of References


Appendix A

Internal Heat Generation

Derivation of Internal Heat Generation Function

\[ H = \frac{D}{A + BD} \]

\( D = \) days
\( H = \) cumulative heat (cal/gm)

Linearization - 2 ways

\[ \frac{D}{H} = A + BD \]

\[ \frac{1}{H} = B + \frac{A}{D} \]

Regression analysis indicate that first equation is a better fit. Thus, use value of \( A \) and \( B \) from that linear regression analysis.

Convert days to hours. (t)

\[ H(t) = \frac{t/24}{A + \frac{Bt}{24}} = \frac{t}{24A + Bt} \]

\( A = 0.02235, B = 0.01015 \)

\[ b(t) = H'(t) = \frac{0.5364}{(0.5364 + 0.01015t)^2} \text{ cal/gm} \]

Need rate of heat generation

\[ \frac{d}{dt} H(t) = \frac{d}{dt} \left( \frac{t}{24A + Bt} \right) = \frac{24A}{(24A + Bt)^2} \]

Convert to Btu/cuin and factor in cement %. Multiply by factor of 0.0213

\[ b(t) = \frac{0.011425}{(0.01015t + 0.5364)^2} \]
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Regression Output for D/H = A + B * D

Constant 0.0223423167 A = 0.02235
Std Err of Y Est 0.0257444007 B = 0.01015
R Squared 0.9947458161
No. of Observations 20
Degrees of Freedom 18
X Coefficient(s) 0.0101496873
Std Err of Coef. 0.0001738653

Regression Output for 1/H = B + A * 1/D

Constant 0.0109314329 A = 0.01502
Std Err of Y Est 0.0025844009 B = 0.01093
R Squared 0.7611811696
No. of Observations 20
Degrees of Freedom 18
X Coefficient(s) 0.0150162768
Std Err of Coef. 0.0019825141
# Appendix B

## Kazzi's Convection Coefficients

**Convection Coefficients, \( H \)**

**Units:** Btu/hr-sqin-F

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Appendix C

Sample ADINAT-T Output File
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EQ.1, LUMPED EQUILIBRATION
EQ.2, CONSISTENT EQUILIBRATION

NUMBER OF CONCENTRATED MODE EQUILIBRATIONS (IP 中止) = 0
NUMBER OF PEAKS INTERPOLATED ......................... (IP 中止) = 0

CARD NUMBER 4

FREQUENCY SOLUTION CODE ........................... (IP 开始) = 0
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EQ.1, FREQUENCIES AND MODE SHAPES
AS' DETERMINED

NUMBER OF TIME STEPS BETWEEN FORMING
EQUILIBRATION FACTOR CODE ........................... (IP 中止) = 1
(ONLY APPLICABLE FOR ADINA ANALYSIS)
NUMBER OF TIME STEPS BETWEEN
EFFICIENT INTERPOLATION CODE ........................... (IP 中止) = 1
(ONLY APPLICABLE FOR ADINA ANALYSIS)
MAXIMUM NUMBER OF EQUILIBRATION
ITERATIONS PERMITTED ................................. (IP 中止) = 15
CONVERGENCE TOLERANCE ............................... (IP 中止) = 0.00000001

CARD NUMBER 5

TIME INTEGRATION CODE ............................... (IP 开始) = 1
EQ.1, NYLOR BACKWARD METHOD
EQ.2, NYLOR FORWARD METHOD
EQ.3, TRAPEZOIDAL Rule
EQ.4, AIPEL, FAMILY METHOD

TIME INTEGRATION FACTOR ............................. (IP 中止) = 0.10008401

CARD NUMBER 6

NUMBER OF BLOCKS OF INITIAL PRINTOUT .......... (IP 开始) = 1
LATEST EQUILIBRATION FACTOR ......................... (IP 中止) = 0

CARD NUMBER 7

PRINT-OUT BLOCK 1

FIRST NODE OF THIS BLOCK ........................... (IP 中止) = 1
LAST NODE OF THIS BLOCK ............................ (IP 中止) = 7

CARD NUMBER 8

PORTABLE PARAMETER ................................. (IP 中止) = 0
EQ.0, PORTABLE NOT WRITTEN
EQ.1, BINARY PORTABLE WRITTEN
EQ.2, FORMATTED PORTABLE WRITTEN

REAL RESULTS INTERVAL BETWEEN PRINTOUT .......... (IP 中止) = 1
ELEMENT RESULT INTERVAL BETWEEN PRINTOUT ........ (IP 中止) = 1
IPROGRAM ADINA-T - VERSION ADINA-T 5.0/WLS
Data Case 1 for 15" Concrete Section
LICENSED FROM ADINA R&D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

ANALYZIS TYPE (ESTABLISHED USING IP 中止, IP 中止, IP 中止)

TIME DEPENDENCY CODE ............................... (IP 中止) = 1
EQ.0, STEADY STATE ANALYSIS
EQ.1, TRANSIENT ANALYSIS

NONLINEARITY CODE ................................. (IP 中止) = 1
EQ.0, LINEAR ANALYSIS
EQ.1, NONLINEAR ANALYSIS
IPROGRAM ADINA-T - VERSION ADINA-T 5.0/WLS
Data Case 1 for 15" Concrete Section
LICENSED FROM ADINA R&D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

90
TIME STEP DATA

NUMBER OF STEPS TIME STEP INCREMENT
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PROGRAM ADINA-T - VERSION ADINA-T 3.0/W3
License from ADINA R&D Inc. for use by Massachusetts Institute of Technology

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PROGRAM ADINA-T - VERSION ADINA-T 3.0/W3
License from ADINA R&D Inc. for use by Massachusetts Institute of Technology

INITIAL CONDITIONS

INITIAL CONDITION CODE
EQ.0, REFERENCE TEMPERATURE TREF
EQ.1, DEVIATIONS FROM REFERENCE
TEMPERATURE TREF ARE READ
(PUT REFERENCE OVER-ID(R) CODE)

INITIAL CONDITION PRINT-OUT CODE
(EPRC) = 1
EQ.0, DO NOT PRINT
EQ.1, PRINT

REFERENCE TEMPERATURE (TREF) = 0.680500000000

EXTERNAL HEAT FLOW INPUT CONTROL DATA

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MAXIMUM NUMBER OF POINTS IN VOLUME FUNCTION CURVES (NVFP) = 672
LINEAR CONVECTION CODE . . . . . . . . . . . . . . . . (ELC0V) = 0
EQ.0, NO LINEAR CONVECTION BOUNDARY CONDITIONS
EQ.1, LINEAR CONVECTION BOUNDARY CONDITIONS
NUMBER OF SPECIFIED MODAL POINT TEMPERATURE ... (NTEMP) = 0
NUMBER OF BOUNDARY CONVECTION NODES . . . . . . . (NBCT) = 1
NUMBER OF BOUNDARY RADIATION NODES . . . . . . . (NBRA) = 1
NUMBER OF CONCENTRATED HEAT FLOW INPUTS . . . . . . (NHLOAD) = 1
NUMBER OF 2/D HEAT FLOW INPUTS . . . . . . . . . . . . (NHLOAD2) = 0
NUMBER OF 3/D HEAT FLOW INPUTS . . . . . . . . . . . . (NHLOAD3) = 0
NUMBER OF INTERNAL HEAT GENERATION DATA ENTS ... (NHINT) = 1
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**Modal Point Convection Temperature Data**

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**Modal Point Radiation Temperature Data**

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**Concentrated Heat Flow Input**

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**Internal Heat Generation Data**

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<table>
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<tr>
<th>ELEMENT NO.</th>
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<th>FUN. MULTIPLIER</th>
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**Element Group Data**

Conduction Element Group = 1 (Linear Conduction)

**Element Definition**

Element Type = (HMPR(1)) = 1

EQ.1, 1-DIM CONDUCTION ELEMENTS
EQ.2, 2-DIM CONDUCTION ELEMENTS
EQ.3, 3-DIM CONDUCTION ELEMENTS
EQ.4, BOUNDARY CONVECTION ELEMENTS
EQ.5, BOUNDARY RADIATION ELEMENTS

NUMBER OF ELEMENTS = (MPER(3)) = 6

**Type of Nonlinear Analysis** = (MPER(3)) = 0

EQ.1, LINEAR
EQ.2, MATERIALLY NONLINEAR ONLY

**Element Rhs and Details Options** = (MPER(4)) = 0

EQ.2, OPTION NOT ACTIVE
EQ.3, BOUNDARY OPTION ACTIVE
EQ.3, DISCRETE OPTION ACTIVE

**Material Definition**

Material Model = (MPER(15)) = 1

EQ.1, CONSTANT CONDUCTIVITY AND CONSTANT SPECIFIC HEAT
EQ.2, TEMPERATURE-DEPENDENT CONDUCTIVITY AND
COMPONENT SPECIFIC HEAT

EQ. 3. CONDUCTIVITY AND TEMPERATURE-
DEPENDENT SPECIFIC HEAT

EQ. 4. TEMPERATURE-DEPENDENT CONDUCTIVITY AND
TEMPERATURE-DEPENDENT SPECIFIC HEAT

COMPONENT SPECIFIC HEAT

NUMBER OF DIFFERENT SETS OF MATERIAL

COMPONENTS .......................... ( MPAR(16) ). = 1

NUMBER OF MATERIAL COMPONENTS PER SET. ( MPAR(17) ). = 1

( CONDUCTIVITY )

NUMBER OF MATERIAL COMPONENTS PER SET. ( MPAR(18) ). = 1

( SPECIFIC HEAT )

SET NO. ................. = 1
AREA .................. = 0.12000082401
CONDUCTIVITY ............ = 0.10000000000
SPECIFIC HEAT ............ = 0.20000000001

ELEMENT INFORMATION

n  ip  ii  jj  mm  nn  yeq  yeq

1  1  2  1  1  0.000018400  0
2  1  3  1  1  0.000018400  0
3  0  3  4  1  1  0.000018400  0
4  0  4  5  1  1  0.000018400  0
5  5  6  6  1  1  0.000018400  0
6  6  7  7  1  1  0.000018400  0

IPROGRAM ADI.H - VERSION ADI.H 5.0/6.C
Data Case 1 for 12" Concrete Section
LICENSED FROM A.D.H.A. B & D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

BOUNDARY CONVECTION ELEMENT GROUP = 2 ( LINEAR CONVECTION )

ELEMENT TYPE ................. ( WPAR(1) ). = 4
EQ. 2. LINEAR CONVECTION ELEMENTS
EQ. 2. 3-D CONVECTION ELEMENTS
EQ. 2. 3-D CONVECTION ELEMENTS
EQ. 2. 2-D CONVECTION ELEMENTS
EQ. 2. BOUNDARY CONVECTION ELEMENTS

TYPE OF NONLINEAR ANALYSIS. ............ ( WPAR(2) ). = 0
EQ. 2. NONLINEAR
EQ. 1. NONLINEAR CONVECTION COEFFICIENT

ELEMENT SIZES AND DEPTH OPTIONS .... ( WPAR(4) ). = 0
EQ. 0. OPTION NOT ACTIVE
EQ. 1. BENE OPTION ACTIVE
EQ. 2. DEPTH OPTION ACTIVE

NUMBER OF INDIVIDUAL MODES
WITH CONVECTION .......................... ( WPAR(5) ). = 1
NUMBER OF LINES WITH CONVECTION ...... ( WPAR(6) ). = 0
NUMBER OF SURFACES WITH CONVECTION ... ( WPAR(7) ). = 0

SPATIAL ISOTROPY CORRECTION INDICATOR ( WPAR(8) ). = 0
EQ. 0. NO DEGENERATION OR NO CORRECTION
FOR SPATIAL ISOTROPY
EQ. 1. SPATIAL ISOTROPY CORRECTIONS APPLIED
TO SPIRAIALLY DEGENERATED ELEMENTS

LINE TYPE CODE .......................... ( WPAR(9) ). = 0
EQ. 0. AXISTOMETRIC CONDITIONS
EQ. 1. PLANAR CONDITIONS

CONVECTION MODEL DEFINITION

TYPE OF CONVECTION BOUNDARY CONDITIONS ( WPAR(13) ). = 1
EQ. 1. CONVECTION CONVECTION CONVECTION
EQ. 2. TEMPERATURE-DEPENDENT CONVECTION COEFF.
EQ. 3. TEMPERATURE-DEPENDENT CONVECTION COEFF.

NUMBER OF DIFFERENT SETS OF
CONVECTION PROPERTIES ................ ( WPAR(14) ). = 1

NUMBER OF COMPONENTS PER CONVECTION
CONVECTION COEFFICIENT

1 0.3610092E-01

BOUNDARY RADIATION ELEMENT GROUP = 3

Emissivity Model Definition

Type of Emissivity Coefficient [ WPARE(15) ] = 1
Eq.1. Constant Emissivity Coefficients
Eq.2. Temperature Dependent Emissivity Coefficients

Number of Different Sets of Emissivity Properties [ WPARE(16) ] = 1
Number of Constants Per Emissivity Property [ WPARE(17) ] = 0

Flag Indicating Unit of Temperature [ ITEMP ] = 0
Eq.0 Temperature [ Fahrenheit ]
Eq.1 Temperature [ Centigrade ]
Eq.2 Temperature [ Rankine or Kelvin ]

Boltzmann Constant . . . . . . . . . [ SIGMA ] = 0.11875000000E-06

EMISSIVITY COEFFICIENT

1 0.9000000E+00

RADIATION NODE DATA

1 1 0 0.1000000E+01 0.1000000E+01 0.00000E+00

95
NUMBER OF MATRIX ELEMENTS .......... (NMR) = 13
MAXIMUM RANK MULTIPLE (MAX) = 2
MINIMUM RANK MULTIPLE (MIN) = 2
MAXIMUM BLOCK LENGTH ......... (EFFOR) = 13
NUMBER OF BLOCKS ............... (NBLOCK) = 1
MAXIMUM TOTAL STORAGE AVAILABLE ....... (NOUT) = 2000000

NUMBER OF COLUMNS PER BLOCK AND IUP COUPLING BLOCK
NUMBER OF BLOCKS = 1
NUMBER OF COLUMNS PER BLOCK = 7
STAGE COUPLING BLOCK = 1

PROGRAM ADINA-V - VERSION 3.8/63
Data Case 1 for 12' Concrete Section
LICENSED FROM ADINA R&D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

INITIAL CONDITIONS

MODAL POINT TEMPERATURES

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<th>MODE</th>
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DATA CHECK COMPLETED
PROGRAM ADINA-V - VERSION 3.8/63
Data Case 1 for 12' Concrete Section
LICENSED FROM ADINA R&D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

SOLUTION TIME LOG (IN SEC)

FOR PROBLEM
Data Case 1 for 12' Concrete Section

INPUT PERLS .......... 6.98
ASSEMBLEE OF LINEAR CONDUCTIVITY, HEAT CAPACITY MATRICES .......... 0.00
ASSEMBLEE OF HEAT FLOW VECTORS .......... 0.07
EQUILIBRATION ANALYSIS .......... 0.00
TRANSFORMATION OF LINEAR (EFFECTIVE) CONDUCTIVITY MATRIX .......... 0.00
STEP-BY-STEP SOLUTION ( 0 TIME STEPS )
CALCULATION OF EFFECTIVE HEAT FLOW VECTORS .......... 0.00
UPDATE EFFECTIVE CONDUCTIVITY MATRICES AND HEAT FLOW VECTORS FOR NONLINEARITIES .......... 0.00
SOLUTION OF EQUATIONS .......... 0.00
EQUILIBRATION ITERATIONS .......... 0.00
CALCULATION AND PRINTING OF TEMPERATURES .......... 0.00
STEP-BY-STEP TOTAL .......... 0.00
TOTAL SOLUTION TIME (SEC) 7.17

96
THIS PROGRAM IS THE ENTIRETY PROPRIETARY TO AND IS SUPPORTED AND MAINTAINED BY
ADINA R & D, INC

71 ELMOW AVENUE
WATERTOWN, MA 02172, U.S.A.

ADINA R & D, INC

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PROGRAM ADINA-T - VERSION ADINA-T 5.0/WLS

Data Case 1 for 12" Concrete Section
LICENSED FROM ADINA R & D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

I N I T I A L    C O N D I T I O N S

INITIAL CONDITIONS CODE

EQ.0. REFERENCE TEMPERATURE TREF
EQ.1. DEVIATIONS FROM REFERENCE TEMPERATURE TREF ARE READ
(NOT RESTART OVER-RIDES ICRM)

INITIAL CONDITIONS PRINT-OUT CODE

EQ.0. DO NOT PRINT
EQ.1. PRINT

REFERENCE TEMPERATURE

(TREF) = 58.0000004603

PROGRAM ADINA-T - VERSION ADINA-T 5.0/WLS

Data Case 1 for 12" Concrete Section
LICENSED FROM ADINA R & D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

T O T A L    S Y S T E M    D A T A

NUMBER OF EQUATIONS

(NRT) = 7

NUMBER OF MATRIX ELEMENTS

(NMR) = 13

MAXIMUM HALF BANDWIDTH

(MAXH) = 3

MINIMUM HALF BANDWIDTH

(MINH) = 2

MAXIMUM BLOCK LENGTH

(MAXB) = 13

NUMBER OF BLOCKS

(NBLOC) = 1

MAXIMUM TOTAL STORAGE AVAILABLE

(MNTO) = 2000000

NUMBER OF COVERS PER BLOCK AND 1ST COUPLED BLOCK

NUMBER OF BLOCK

1

NUMBER OF COVERS PER BLOCK

7

NUMBER OF 1ST COUPLED BLOCK

1

PROGRAM ADINA-T - VERSION ADINA-T 5.0/WLS

Data Case 1 for 12" Concrete Section
LICENSED FROM ADINA R & D INC. FOR USE BY MASSACHUSETTS INSTITUTE OF TECHNOLOGY

I N I T I A L    C O N D I T I O N S


97
### Modal Point Temperatures

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**Step Number = 1**

**Number of Equilibrium Iterations = 1**

**Step Number = 2**

**Number of Equilibrium Iterations = 1**

**Step Number = 3**

**Number of Equilibrium Iterations = 1**

**Step Number = 6**

**Number of Equilibrium Iterations = 1**

**Step Number = 7**

**Number of Equilibrium Iterations = 1**

**Step Number = 9**

**Number of Equilibrium Iterations = 1**

**Step Number = 10**

**Number of Equilibrium Iterations = 1**

**Step Number = 11**

**Number of Equilibrium Iterations = 1**

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Data Case 1 for 12" Concrete Section

Licensed from ADINA R&D Inc. for use by Massachusetts Institute of Technology.
### Modal Point Temperatures

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### Modal Point Temperatures

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STEP NUMBER = 641
NUMBER OF EQUILIBRIUM ITERATIONS = 1

STEP NUMBER = 642
NUMBER OF EQUILIBRIUM ITERATIONS = 1

STEP NUMBER = 643
NUMBER OF EQUILIBRIUM ITERATIONS = 1

STEP NUMBER = 644
NUMBER OF EQUILIBRIUM ITERATIONS = 1

STEP NUMBER = 645
NUMBER OF EQUILIBRIUM ITERATIONS = 1

STEP NUMBER = 646
NUMBER OF EQUILIBRIUM ITERATIONS = 1

STEP NUMBER = 647
NUMBER OF EQUILIBRIUM ITERATIONS = 1
### Modal Point Temperatures

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**Step Number = 469**

**Number of Equilibrium Iterations = 1**

**Step Number = 470**

**Number of Equilibrium Iterations = 1**

**Step Number = 471**

**Number of Equilibrium Iterations = 1**

---

**Solution Time Log (SEC)**

**Data Case 1 for 12’ Concrete Section**

**Problem Data**

**Calculation of Effective Heat Flow Vectors**

**Assemble of Linear Conductivity, Heat Capacity Matrices**

**Assemble of Heat Flow Vectors**

**Frequency Analysis**

**Triangulation of Linear (Effective) Conductivity Matrix**

**Step-by-Step Calculation (471 Time Steps)**

**Calculation of Effective Heat Flow Vectors**

**Updating Effective Conductivity Matrices and Heat Flow Vectors for Nonlinearities**

**Solution of Equations**

**Equilibrium Iterations**

**Calculation and Printing of Temperatures**

**Step-by-Step Total**

**Total Solution Time (SEC)**

23.69

0.20

11.97

0.00

9.00

4.81

10.67

21.22

25.72

19.34

61.96

123.25