THREE MODELS OF THE HIRING PROCESS

by

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ABSTRACT

In the three essays comprising this dissertation, I examine wage determination by constructing formal models of the hiring process. In contrast to many previous models, this analysis explicitly recognizes that firms recruit workers through multiple hiring channels: help-wanted advertising, employee referrals, etc. By focusing on these recruitment channels, I have attempted to demonstrate that observed labor-market outcomes arise because of--rather than in spite of--the institutional structure of the labor market.

In the first essay, I develop a search-theoretic explanation for the systematic differences in wages earned by observationally equivalent workers across industries. Even if all workers are fully informed of the location of and wage offered at each job opening through help-wanted advertising, coordination problems will arise as workers apply for jobs; the probability of filling a vacancy is thus plausibly an increasing function of the wage offered. In equilibrium, firms which find vacancies more costly will offer higher wages; the model thus generates persistent wage differentials between firms and predicts that these will be correlated with industry-average capital/labor ratio and profitability. Additionally, the model suggests that high-wage firms will receive more applications per job opening and that wages in the labor market will behave as "strategic complements."

In the second essay, I discuss the importance of employee referrals in the hiring process and formally examine the consequences for wage determination. Given the widespread (and apparently purposive) use of social networks, I argue that social structure must play a key role in determining labor market outcomes. Building upon the observation that workers tend to refer others like themselves, I demonstrate that in equilibrium firms employing high-ability workers will desire to hire through employee referral while firms employing low-ability workers will not. The market wage is driven down by a "lemons effect"; wages paid to referred workers are dispersed. Job seekers who are "well connected" receive (on average) more referrals and consequently higher expected wages; firms hiring referred workers face imperfect competition and thus earn positive expected profits. As the total number of social ties increases, the market "lemons effect" is exacerbated and wage dispersion

increases.

In the final essay, I extend the analysis of the second essay by adding
endogenous job assignment. If firms possess multiple technologies, job
assignment as well as compensation is influenced by social structure. If
social structure varies across demographic groups, wage determination will
also vary. Even if ability is distributed identically across groups,
females or blacks will receive lower mean wages if these groups possess
lower "network density" or "inbreeding bias" by ability. In contrast to
previous models of "statistical discrimination," variations in wages
across groups are thus generated by differences in social structure rather
than by presupposing that the productivity of workers from disadvantaged
groups is measured with (exogenously) greater error.

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INTRODUCTION

In the standard Walrasian formulation, the labor market operates frictionlessly: supply interacts with demand to determine a unique market wage, everyone desiring to work at this wage finds employment, and each firm hires workers until its marginal product equals this wage. But while this model is often a useful abstraction, it seems far removed from labor-market reality. Wage dispersion, not explained by compensating differentials nor temporary disequilibrium, exists within even the most narrowly defined occupations. Workers often experience difficulty finding jobs; firms are frequently unable to fill vacancies.

Over the last two decades, labor economists have addressed these issues by developing search-theoretic explanations for wage dispersion and unemployment. Search theory in its present form, however, constitutes only one (albeit important) step on the path toward a complete under-standing of wage and employment determination. While explicitly recognizing that firms and workers possess incomplete information, search-theoretic models have largely ignored the institutional structure of real-world labor markets. Features of the hiring process salient to labor-market participants--e.g., help-wanted advertising or employee referrals--rarely enter formal analysis.

In the three essays below, I examine wage (and to a lesser extent employment) determination by constructing formal models of the hiring process. But in contrast to many previous formal analyses, I have attempted to demonstrate that observed labor-market outcomes arise because of--rather than in spite of--the institutional structure of the labor market. By focusing on its institutions, I hope to have offered fresh

insight into the operation of the hiring process, highlighting the divergence of real-world labor markets from the standard Walrasian model and providing a foundation for future policy analysis.

In the first essay, "Equilibrium Wage Dispersion and Inter-Industry Wage Differentials," I develop a search-theoretic explanation for the systematic differences in wages earned by observationally equivalent workers across industries. Even if all workers are fully informed of the location and wage offered at each job opening through help-wanted advertising, coordination problems will arise as workers apply for jobs. Given these coordination problems, the probability of filling a vacancy is an increasing function of the wage offered. In equilibrium, firms which find vacancies more costly will offer higher wages; job seekers will be indifferent between applying to a high-wage firm (with a low acceptance probability) and a low-wage firm (with a high acceptance probability). The model thus explains the persistence of wage differentials and the correlation between these differentials and industry-average capital/labor ratio and profitability. Additionally, the model suggests that high-wage firms will receive more applications per job opening and that wages in the labor market will behave as "strategic complements."

In the second essay, "Social Networks and Labor Market Outcomes: Toward an Economic Analysis," I discuss the importance of employee referrals in the hiring process and formally examine the consequences for wage determination. Given the widespread (and apparently purposive) use of social networks, I argue that social structure must play a key role in determining labor market outcomes. Building upon the observation that

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workers tend to refer others like themselves, I demonstrate that in equilibrium firms employing high-ability workers will desire to hire through employee referral while firms employing low-ability workers will not. The market wage is driven down by a "lemons effect"; wages paid to referred workers are dispersed. Importantly, "well connected" job seekers receive (on average) more referrals and consequently higher expected wages. Firms hiring referred workers face imperfect competition and thus earn positive expected profits. As the total number of social ties increases, the market "lemons effect" is exacerbated and wage dispersion increases.

In the final essay, "Reinterpreting Models of Statistical Discrimination: Employee Referrals and the Role of Social Structure," I extend the analysis of the second essay by adding endogenous job assignment. If firms possess multiple technologies, job assignment as well as wages is influenced by social structure. By improving job matching, an increase in "network density" or "inbreeding bias" increases economic efficiency and mean wages. Thus, if social structure varies across demographic groups, wage determination will also vary. Even if ability is distributed identically across groups, females or blacks will receive lower mean wages if these groups possess lower network density or inbreeding bias. But in contrast to previous models of "statistical discrimination," variations in wages across groups are not generated by presupposing that the productivity of females or blacks is measured with (exogenously) greater error. Rather, group wage differences arise through variation in the probability that workers will be hired through employee referral and

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through variation in the expected ability of workers recruited through

each hiring channel.

Essay 1:

EQUILIBRIUM WAGE DISPERSION AND INTER-INDUSTRY WAGE DIFFERENTIALS

According to the standard competitive model of the labor market, homogeneous workers should receive equal compensation across firms. But if coordination problems in the labor market make it difficult for firms and workers to form matches, a single-wage equilibrium seems unlikely. If the probability of filling a job opening is an increasing function of the wage offered, firms which find vacancies more costly will offer higher wages. In this paper, I formally model this intuition, providing a theoretical explanation for a number of stylized facts: inter-industry wage differentials are persistent (i.e., equilibrium) phenomena and are correlated with such industry attributes as average capital/labor ratio and profitability. In addition to predicting that firms with higher valuations of output will pay higher wages, the model also suggests that these firms will receive more applications per job opening and that wages in the labor market will behave as "strategic complements".

A large number of studies have demonstrated the existence of persistent inter-industry wage differentials for observationally equivalent workers.[1] Additionally, these differentials have been shown to be positively correlated with such industry attributes as profitability and capital/labor ratio. To account for these findings, both market-clearing and non-market-clearing explanations have been offered. Market-clearing explanations include unobserved ability differences and

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[1]Recent work includes Dickens and Katz (1987a,b) and Krueger and Summers (1987a,b).

the presence of compensating differentials.[2] Non-market-clearing

explanations include a variety of efficiency wage models (in which

above-market wages are paid to reduce turnover or absenteeism, to

eliminate shirking, or for "sociological" reasons) and rent sharing

(driven by insider bargaining power).[3]

While the generic "efficiency-wage hypothesis" is often given as an

explanation of inter-industry wage differentials, different stylized facts

seem to be better explained by different versions of this hypothesis. For

instance, the correlation between industry average profitability and wage

differentials seems to fit well with the "sociological" version in which

workers' notions of a "fair" wage are influenced by firm profitability.

The correlation between industry average capital/labor ratio and wage

differentials, on the other hand, fits well with the monitoring

hypothesis: shirking becomes a bigger problem in firms utilizing expensive

machinery.

In a recent paper, Lang (1987) has suggested that search theory may

provide a more complete explanation of wage differentials, generating all

of the observed stylized facts without recourse to a wide variety of

efficiency-wage stories. The basic argument is simple: If it is somehow

difficult for firms and workers to form a match, and if a firm can

increase the probability of filling a vacancy by offering a higher wage,

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[2]See Gibbons and Katz (1987) and Murphy and Topel (1987) for analysis of
the unobserved ability hypothesis.

[3]See Katz (1986) for a survey of the efficiency wage literature.

firms for which unfilled vacancies are relatively more expensive will pay higher wages. Thus, highly profitable firms (in which valuable orders would go unfilled if a vancancy persisted) and firms with high capital/labor ratios (in which expensive machinery would set idle if a vacancy persisted) will pay higher wages. Search-theoretic models may thus represent an important contribution to the inter-industry wage differential literature.

But in applying existing search models to the issue of wage differentials, a number of problems emerge. First, as argued by Rothschild (1973), many search models are of a "partial partial-equilibrium" nature, examining behavior on only one side of the market. In the labor-market search literature, most models focus upon the decision made by job seekers facing an exogenously-given distribution of wage offers.[4] As I am attempting to explain the behavior of firms (as well as workers), such models are of limited applicability. While the industrial-organization literature on "equilibrium price dispersion" models both sides of the market, equilibrium in these models is based upon a zero-expected-profit condition for firms.[5] As I am concerned with the effect of profitability on wages, such a condition is unacceptable. Finally, both the standard labor market search models and the "equilibrium price dispersion" literature (in its labor market form) assume that jobs

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[4]See Mortensen (1986) for a survey of labor-market search literature.

[5]See, for example, Butters (1977) and Burdett and Judd (1983).

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are "hidden" from workers--job seekers are unaware of the location of any particular firm. Such an assumption seems difficult to reconcile with the existence of a nearly costless and fairly extensive listing of job openings (i.e., newspaper help-wanted advertising) found in real-world labor markets.[6]

In view of these problems, I present below an alternative model of equilibrium wage dispersion developed to examine inter-industry wage differentials. The model yields three major empirical predictions. First, firms with higher valuations (generated, for example, by higher capital/labor ratios or greater market power) will offer higher wages. Second, firms offering higher wages will attract (on average) longer queues. (More concretely, one would expect these firms to receive a larger number of applications for each job opening.) Third, wages behave as "strategic complements": wage increases at one firm (or across a subset of firms) will prompt wage increases at other firms. The model thus offers an explanation for the importance of relative wage comparisons in the labor market.

The present paper is organized as follows: In Section 1, I describe the existing "equilibrium price dispersion" literature in greater detail, further examining the difficulties in applying this literature to the issue of wage differentials, and present a verbal description of my model. In Section 2, I analyze the simplest version of this model (the case of

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[6]Lang (1987) makes a similar argument; see footnote 23 for a comparison of his analysis with my own.

two job seekers and two job openings), deriving the equilibrium vector of wages and discussing the model's empirical predictions. In Section 3, I examine the more realistic case of many applicants and many job openings, arguing that the basic results derived in Section 2 will go through. In Section 4, I examine the efficiency of the equilibria derived in the preceding two sections. I discuss possible extensions of the model in Section 5 and conclude in Section 6.

1. EXISTING SEARCH LITERATURE

As argued above, firms which find vacancies expensive are likely to pay high wages if the probability of filling an opening is an increasing function of the wage offered. In modeling this intuition, one might simply assume some functional form for this "hiring probability" and, by performing the necessary comparative statics, demonstrate that those firms with high capital/labor ratios or product market power would optimally pay higher wages. But, as Rothschild (1973) argued in an early survey of search literature, such an approach constitutes "partial partial-equilibrium" analysis: the assumed "hiring probability" function might well be inconsistent with maximizing behavior on the part of workers. Any search-theoretic model intended to explain wage differentials must incorporate both sides of the labor market, consistent with maximization by both firms and workers.

As a second modeling possibility, one might utilize the existing

"equilibrium price dispersion" literature. This literature, which developed in response to the Rothschild critique, offers a number of models in which maximizing behavior by both consumers and firms yields price dispersion in equilibrium. The earliest (and a representative) model in this literature is the Butters (1977) model of advertising and sales. In the Butters model, consumers each desire a single unit of a homogeneous good, but are uninformed about the location of the firms and the price charged by each. A firm may, at a fixed cost per advertisement, inform a random consumer of its price and location. Given this advertising technology, some consumers will received many ads (and will buy at the lowest price offered); others will received none (and will be unable to buy). In equilibrium, prices are dispersed; firms are indifferent between sending out ads offering the good at high prices (with a low probability of acceptance) or low prices (with a high probability of the offer being accepted).[7]

While such a model provides an equilibrium explanation for price dispersion, two major difficulties emerge when adapting it to a labor market setting in order to study wage differentials. First, one might argue that the advertising technology of the Butters model bears little resemblance to actual labor market institutions. In a labor market context, the Butters model implies that job openings are somehow "hidden"

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[7]One may think of the Butters model in terms of a probability theory exercise, where the consumers are represented by "urns" and the firms' advertisements are "balls" (each having a price printed on the outside of it.)

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from a worker's view--the worker learns of a particular job opening only when an offer arrives in his mailbox (or, alternately, when a friend or relative informs him of a job opening). Such a story is difficult to reconcile with the existence of newspaper help-wanted advertising which seems to provide an extensive source of current job openings at a negligible cost. (This criticism is, of course, not limited to the "equilibrium price dispersion" literature. Labor market search models typically assume that job seekers know the distribution of wage offers but are uninformed of the exact location of any offer.)

A second difficulty in adapting the "equilibrium price dispersion" models to study wage differentials is the fact that equilibrium in these models is based upon a zero-expected-profit condition. (In the Butters model, firms expect to break even on each offer sent out.) Such a condition is sensible in the industrial organization literature, as researchers are attempting to explain how equilibrium price dispersion might persist even if all firms have access to the same technology and can freely enter the market. But as I am attempting to explain how product market power (i.e., profitability differences) affect wages, a zero-expected-profit condition is unacceptable. For purposes of this paper, I am content to accept real-world product markets at face value and claim that, for whatever reason, some firms enjoy greater market power than others.

In light of these difficulties in adapting the "equilibrium price dispersion" models to study the issue at hand, I propose a new model of equilibrium wage dispersion. To motivate this model (which will be

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developed formally in following sections), I have in mind the following

story:

> The labor market consists of M job seekers and N firms (each
> having one job opening). While workers are identical, the value
> of output varies across firms. (One might imagine that some firms
> have greater product market power than others, or that some
> utilize more capital per worker and thus produce more output per
> worker.) Firms attract applicants by (costlessly) placing a help-
> wanted advertisement in the local newspaper. Thus, the night
> before the newspaper is printed, each firm (simultaneously) calls
> the newspaper and places an ad stating the wage it is offering.
> In the morning, the job seekers (costlessly) receive the newspaper
> and are thus informed of all job openings and the wage offered at
> each. Each job seeker then selects a firm at which he will apply
> and makes his way to that firm's personnel office. If a job
> seeker finds that he is the only one who has applied for a given
> job, he is hired and is paid the offered wage. If more than one
> person has applied for a job, however, the firm randomly selects a
> worker from all those who applied.[8]

Given this institutional arrangement, it seems likely that some

applicants will receive no offers and that some firms will fail to fill

their vacancies. To decrease the probability that their vacancies go

unfilled, firms with higher valuations (those with greater product market

power or higher capital/labor ratios) will offer higher wages. The model

thus predicts wage dispersion in equilibrium. (In the above story, two

assumptions were made for analytical convenience: firms have only one job

opening and workers may apply to only one firm. As discussed in Section

5, either (or both) of these assumptions may be relaxed without altering

the basic results of the model.)

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[8]Like the Butters model, this story is also analogous to a probability
theory exercise: the firms are now represented by "urns" (each having a wage
offer printed on the outside) while the applications are the "balls".

The story told above, rather than merely assuming that firms receive a stochastic number of applications, incorporates maximizing behavior on the part of both firms and workers; it thus satisfies the Rothschild critique. But in constrast to models in the "equilibrium price dispersion" literature, equilibrium is not based upon a zero-profit condition. Instead, equilibrium is derived from the requirement that the expected value of applying to firm will be equal across firms. Intuitively, a high-wage firm is likely to receive many applications. Though the worker hired at that firm receives a high wage ex-post, the ex-ante probability of receiving the job is relatively small. A low-wage firm at which an applicant is relatively likely to be hired yields the same ex-ante expected payoff.

As discussed earlier, one motivation in building this model has been to tell a more believable labor market search story in which jobs are not hidden from workers' view. Ultimately, of course, realism is an empirical issue. If workers actually tend to find jobs through friends or relatives (or, more generally, through networks) the standard labor market search model may be more appropriate. If workers tend to find jobs through answering help-wanted ads, the present model may provide a better description of the labor market. But whichever scenerio is more accurate, the present model demonstrates that a "hidden jobs assumption" is not crucial in a search model--wage dispersion may arise even in cases where job seekers are fully informed of the location and the wage offered at each job opening in the labor market.

2. THE 2 x 2 CASE

To develop the basic framework of the model, I will first examine the simplest case of two applicants and two firms. Obviously, real-world labor markets consist of many applicants and many firms. But (as will be demonstrated in the next section) the major results of the 2x2 case go through in the case of a "large" labor market. Furthermore, in this simple case, one may obtain closed for (most of) the parameters of interest, derive each firm's reaction functions (its wage as a function of the other firm's wage), and plot these reaction curves to determine the equilibrium wage vector.

It is easiest to solve this model backwards. Assuming the firms have already chosen the wages they will offer, the problem facing the applicants may be described as a (normal form) game:

Applicant 2

	Firm 1	Firm 2
Firm 1	$(1/2)w_1, (1/2)w_1$	w_1, w_2
Firm 2	w_2, w_1	$(1/2)w_2, (1/2)w_2$

Applicant 1

As discussed in the last section, each applicant may apply to only one firm. If the applicants choose different firms, both are hired and each receives the wage offered by his firm. (For example, if A1 applies to F1 and A2 applies to F2, both are hired, A1 receives w_1, and A2 receives w_2.) If both applicants apply to the same firm, however, the firm flips a coin to decide which applicant receives the job. (If both A1 and A2 apply to F1, each receives w_1 with a probability of one-half.)

Assuming $(1/2)w_2 < w_1 < 2w_2$, the game just described has three Nash equilibria--two asymmetric pure-strategy equilibria and one symmetric mixed-strategy equilibrium.[9] The strategy pairs (F1,F2) and (F2,F1) constitute the pure-strategy equilibria. In the mixed-strategy

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[9] The assumption that w_1 and w_2 are not "too far apart" is made without loss of generality--see Appendix 1 for explanation.

equilibrium, each worker applies to firm 1 with probability p (and to firm 2 with probability (1-p)). Intuitively, p will vary directly with w_1--firm 1 increases the probability each worker applies to it by increasing its wage relative to firm 2's wage.

In the present analysis, I will focus upon the mixed-strategy equilibrium. While in the simple 2x2 case presented above a pure-strategy equilibrium may seem more likely, this implies coordination on the part of the applicants. (One story might be that the applicants contact each other after reading the want ads to make sure they do not apply to the same firm.) But in a large labor market with many openings and many applicants, such coordination becomes nearly impossible. Given a listing of job openings and the wage offered at each, a participant in a large economy (in which coordination is infeasible) would likely expect a greater number of applications to be made at positions offering higher wages. In equilibrium, job seekers will then be indifferent (ex-ante) between applying for a low wage position (with a high probability of acceptance) and a high wage position (with a low probability of acceptance).[10]

To solve for the mixed-strategy equilibrium, recall the equilibrium condition that the expected value of applying to a job is constant across

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[10]While I have ruled out the pure strategy equilibria on the (intuitively plausible) grounds that coordination is difficult in a large labor market, the mixed strategy equilibrium has been justified mainly by process of elimination. A stronger argument supporting mixed-strategy Nash equilibria is that they are "reduced-form" versions of pure-strategy Bayesian equilibria where each of the players has "a little" private information. See Tirole (1986) for further explanation.

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jobs:

$$w_1 \cdot pr(\text{getting job at F1}) = w_2 \cdot pr(\text{getting job at F2}) .$$

From A1's point of view,

$$pr(\text{getting job at F1}) = pr(\text{A2 applies to F2}) + (1/2)pr(\text{A2 applies to F1}) .$$

(If A2 applies to F2, A1 will receive the job at F1 for sure; if A2
instead applies to F1, the employer flips a coin to determine which
applicant receives the job. A similar equation holds for the probability
of getting a job a firm 2.)

As A2 (in equilibrium) applies to F1 with probability p and to F2
with probability (1-p), the equilibrium condition becomes:

$$w_1 \cdot [(1/2)p + (1-p)] = w_2 \cdot [p + (1/2)(1-p)] .$$

Solving for p,

$$p = (2w_1 - w_2)/(w_1 + w_2) .$$

As predicted above, both $\partial p/\partial w_1$ and $\partial(1-p)/\partial w_2$ are greater than zero: a
firm increases the probability that each applicant applies to it (its
"application probability") by increasing its own wage.

Having examined the game played by the applicants, we now return to
the firms' problems. Each firm maximizes expected profit, which is equal

to the product of the job opening's "markup" (the difference between the opening's valuation and the wage offered) and the probability that the firm fills the vacancy. Mathematically, each firm solves:

$$\max_{w_i} \quad (v_i - w_i) \cdot pr(Fi \text{ receives at least one applicant} | w)$$

for $i \in (1,2)$ where w is the vector (w_1, w_2). As discussed in the previous section, job openings may have different valuations across firms. As wage differentials are correlated with a variety of factors, I wish to remain flexible in my interpretation of the source of these differences. Intuitively, v_i might be high because firm i possesses product market power or because it maintains a high capital/labor ratio. In either case, high valuation firms find vacancies costly.[11]

The firms' objective functions may be rewritten:

$$(v_i - w_i) \cdot [1 - pr(Fi \text{ receives no applicants} | w)] \ .$$

Defining p_i as the probability that each worker applies to firms i (firm i's "application probability"), this becomes:

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[11]One might object to this formulation, arguing that a firm's vacancy represents the absence of the "marginal" worker, whose productivity should be equal to his wage. (The firm should thus always be (almost) indifferent to filling this job as the vacancy constitutes only a second-order profit loss.) In response, I wish to argue that real-world technologies are probably not perfectly differentiable. In a plant with a Leontief production technology such that each worker operates a single machine, machines will set idle if vacancies are unfilled.

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$$(v_1 - w_1) \cdot [1 - (1 - p_1(w))^2] .$$

Substituting for p_1, firm 1 solves:

$$\max_{w_1} \; (v_1 - w_1) \cdot 3w_2 \, (2w_1 - w_2)/(w_1 + w_2)^2 .$$

The first-order condition (after much algebra) yields firm 1's "reaction function" (w_1 as a function of w_2 and v_1):

$$R_1(w_2) = w_2 \, (w_2 + 4v_1)/(5w_2 + 2v_1) .$$

Similar calculations yield firm 2's reaction function:

$$R_2(w_1) = w_1 \, (w_1 + 4v_2)/(5w_1 + 2v_2) .$$

Given v_1 and v_2, we can plot these functions to determine the equilibrium level of wages and application probabilities.[12] Figures 1A and 1B depict the cases ($v_1 = 1$, $v_2 = 1$) and ($v_1 = 2$, $v_2 = 1$). Figure 1A demonstrates (a general result) that symmetric firms (i.e., those having equal valuations) will offer the same wage in equilibrium. Given $v_1 = v_2 = 1$, both firms offer a wage of 1/2; each firm's "application probability"

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[12]Unfortunately, a closed-form solution for the equilibrium vector of wages can not be obtained in the 2x2 case. But given the concavity of the reaction functions, one can show that a unique equilibrium exists. See Appendix 2 for the proof.
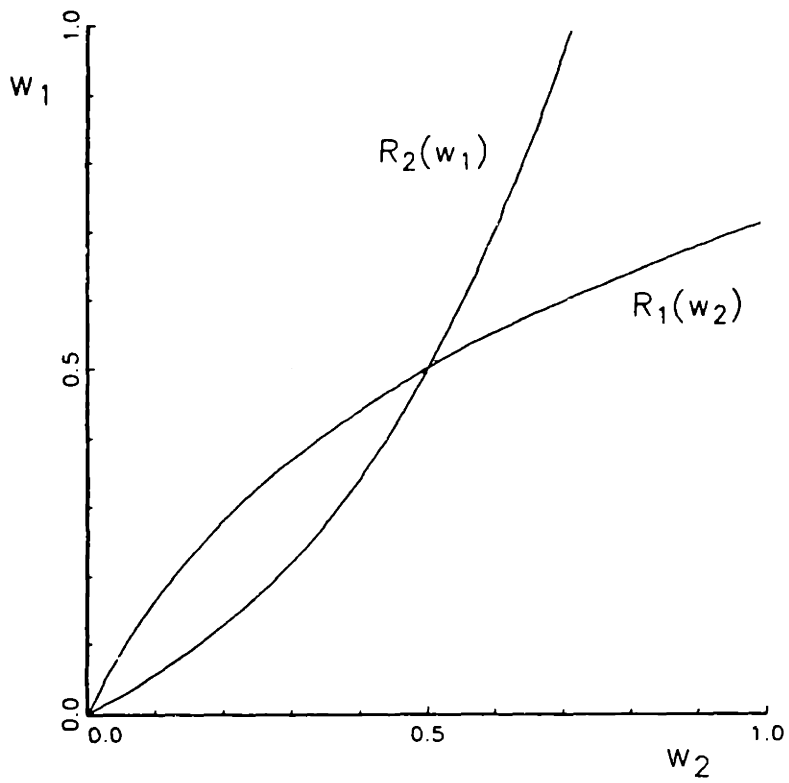
Figure 1A

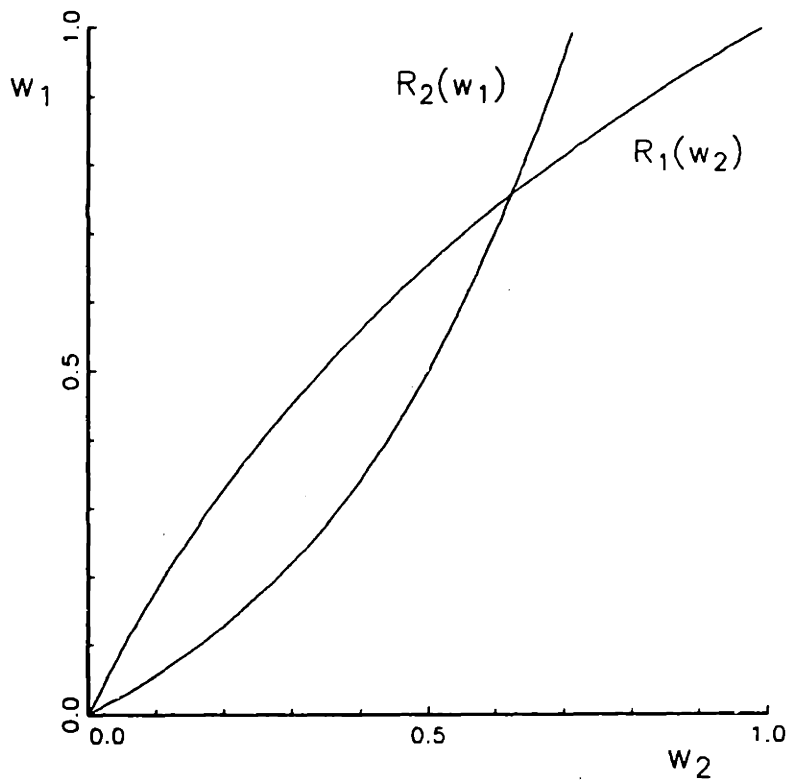Given $v_1 = v_2 = 1$:
$$w_1 = w_2 = .5$$
$$p_1 = p_2 = .5$$

Figure 1B

Given $v_1 = 2$, $v_2 = 1$:
$$w_1 = .75 \quad w_2 = .62$$
$$p_1 = .64 \quad p_2 = .36$$

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is thus equal to 1/2. If v_1 rises to 2 (holding v_2 constant), Figure 1B reveals that the equilibrium wage rises at both firms: w_1 is now (approximately) .75 while w_2 rises to .62. (Graphically, increasing firm 1's valuation causes its reaction curve to "swing upwards".) As firm 1 now offers a higher wage than firm 2, the "application probabilities" are no longer equal: p_1 (= p) rises to .64 while p_2 (= 1-p) falls to .36.

Three major results emerge from the simple 2x2 case examined above. First, the firm with the higher valuation will pay a higher wage. If the difference in the valuations were the result of differences in capital/labor ratios, for example, the more capital-intensive firm would pay a higher wage than the labor-intensive firm. Intuitively, an unfilled vancancy is more costly for the capital-intensive firm and it thus offers a higher wage in order to increase the probability it receives at least one application. (A similar story may be told about firms possessing different degrees of market power.) The model presented above thus generates the stylized facts cited earlier: inter-industry wage differentials are persistent (i.e., equilibrium phenomena) and are correlated with industry average profitability and capital/labor ratio.

Second, the model predicts varying "queue lengths" across job openings. As demonstrated in the examples above, the higher-valuation firm pays a higher wage and thereby increases the probability that each applicant applies to that firm. Defining "queue length" at firm i as the expected number of applications received ($2p_i$), the higher-valuation (and therefore higher-wage) firm expects a longer queue. Empirically, one would thus expect industries paying higher differentials to receive more

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applications per job opening. (Note that this prediction does not come out of a standard model of unobserved ability or compensating differentials.) While little direct evidence is available on application rates across industries, this prediction seems to be supported by the preliminary findings of Holzer, Katz, and Krueger (1987) who analyze Employment Opportunity Pilot Project (EOPP) data on application rates.[13]

Third, wages in the present model are "strategic complements"--an increase in one firm's wage provokes wage increases at other firms.[14] While I will not formally develop the empirical implications of this result, it seems potentially important in understanding the effect of minimum-wage laws, product-market shocks, and union power on the general level of wages in the labor market. (The standard textbook analysis of minimum-wage laws, for example, maintains that an increase in the minimum wage will cause wages to fall in the uncovered sector. The present model suggests that wages in both the covered and uncovered sectors may rise in response to such an increase.[15]) Additionally, the finding that wages are

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[13]Along similar lines, Krueger (1988) finds that the queue length for federal-government job openings is positively correlated with the ratio of government wages to private-sector wages.

[14]This terminology was introduced by Bulow, Geanakoplos, and Klemperer (1985). In oligopoly models with differentiated products where firms engage in price competition, prices are typically "strategic complements". In oligopoly models with Cournot competition, however, an increase in one firm's output prompts other firms to decrease output--quantities in these games are thus "strategic substitutes".

[15]Gramlich (1976) provides some empirical support for this hypothesis. In a related study, Grossman (1983) finds that an increase in the minimum wage compresses the overall wage distribution in the short-run; this result could also be generated from the present model.

strategic complements may provide an economic rationale for the widespread use of community wage surveys by corporate personnel departments.[16] In the present model, each firm will be (justly) concerned with its position in the community wage hierarchy, as relative wages determine its probability of attracting applicants.

3. EXTENSION TO A LARGE LABOR MARKET

Having examined the simplest case of two applicants and two firms in order to develop intuition, I now examine the more realistic case in which the labor market comprises many applicants and job openings. In the following analysis, I assume M applicants and N job openings, where M and N are "large" (in a sense to be defined below). As before, each firm solves:

$$\max_{w_i} \quad (v_i - w_i) \cdot pr(Fi \text{ receives at least one applicant} | w) \ .$$

In the large labor market case, the objective function may be rewritten:

$$(v_i - w_i) \cdot [1 - (1 - p_i(w))^M]$$

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[16]For a discussion of community wage surveys, see Doeringer and Piore (1971).

where w is the vector of wages $(w_1, w_2, \ldots, w_N)$ and p_i is again the probability that each worker applies to firm i given w.

The mixed-strategy application probabilities (i.e., the p_i's) are determined by two conditions: (1) the expected value of applying to each job opening is constant across openings and (2) the summation of these application probabilities over all job openings must equal one. Mathematically,

(1) $w_i \cdot \text{pr(getting a job at Fi)} = w_j \cdot \text{pr(getting a job at Fj)} \quad \forall \, i,j$

and (2) $\sum_{i=1}^{N} p_i = 1$.

As pr(getting a job at Fi) $= [1-(1-p_i)^M]/p_i M$, condition (1) may be rewritten[17]:

(1') $w_i \cdot [1 - (1 - p_i)^M]/p_i M = w_j \cdot [1 - (1 - p_j)^M]/p_j M \quad \forall \, i,j$.

As condition (1') represents a system of N-1 equations and condition (2) provides the Nth equation, one could conceptually solve for each of the application probabilities as a function of the vector of wages, substitute into the firm's objective function, and ultimately derive reaction functions (w_i as a function of all other wages w_{-i}). But as this approach

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[17]See Appendix 3 for a derivation of the mixed-strategy equilibrium in the large labor market case.

would prove analytically intractable, I will follow a more intuitive line of reasoning.

One can rewrite condition (1') as:

$$(1'') \quad w_i \cdot [1 - (1 - p_i)^M]/p_i M - K \qquad \forall \ i$$

where K is a direct measure of labor-market "tightness". (From condition (1), K is the expected value of applying to any job. Thus, K represents a lower bound on the set of wages a firm could offer and expect to attract workers with positive probability. Should a firm offer a wage less than K, no applicant would apply (even thought he would receive the job with probability one).)

In a small labor market (e.g., the 2x2 case analyzed above), a change in w_i has two effects--it increases both p_i and K. In words, an increase in wage i will increase the probability that each worker applies to firm i but will also "tighten" the overall labor market. As the number of firms increases, however, a change in any individual wage will have only a minor impact on market tightness--$\partial K/\partial w_i$ becomes very small. Thus, in a labor market with many firms, an increase in w_i primarily raises p_i and increases K only slightly. Given a "large enough" N, we may consider K a constant from any one firm's point of view and proceed to analyze labor-market equilibrium. (I thus define a labor market as "large" when each individual firm takes labor-market tightness (K) as given.)[18]

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[18]For purposes of tractability, I have thus slightly altered the equilibrium concept: When contemplating a deviation from its equilibrium wage, firm i

Imagine now a large labor market in which all firms have optimally set their wages and equilibrium has been attained. By the argument made in the preceding paragraph, the introduction of another firm will have little impact on labor-market tightness--from the new firm's point of view, K is a constant ($K = \bar{K}$) and the firm will maximize over the "wage-application probability" locus drawn in Figure 2. As noted above, this market equilibrium locus intersects the horizontal axis at $\bar{K}$: any wage offer below $\bar{K}$ has no chance of acceptance. Furthermore, this locus is concave due to a "crowding effect". At a firm offering a low wage, relatively few applications are expected; if such a firm increases its wage slightly, each applicant in its queue receives a relatively large increase in his expected payoff. At a firm offering a high wage, however, many applicants are expected; an increase in the firm's wage yields a smaller increase in expected earnings to each applicant in its queue.

To determine where the firm will locate on this "wage-application probability" locus, we must examine the firm's isoprofit curves. As depicted in Figure 3, these curves are convex in (w_i, p_i) space. This is easily explained by recalling that firm i's profit is equal to the product of a "markup" term $(v_i - w_i)$ and the probability it receives at least one applicant $[1-(1-p_i)^M]$. As w_i increases, the "markup" term decreases and an increase in p_i (which increases the probability that the firm fills its vacancy) becomes less valuable. Figure 3 also demonstrates that the

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assumes that labor market tightness (K)--rather than the wage vector $w_{-i} = (w_1, \ldots, w_{i-1}, w_{i+1}, \ldots, w_N)$--is fixed.
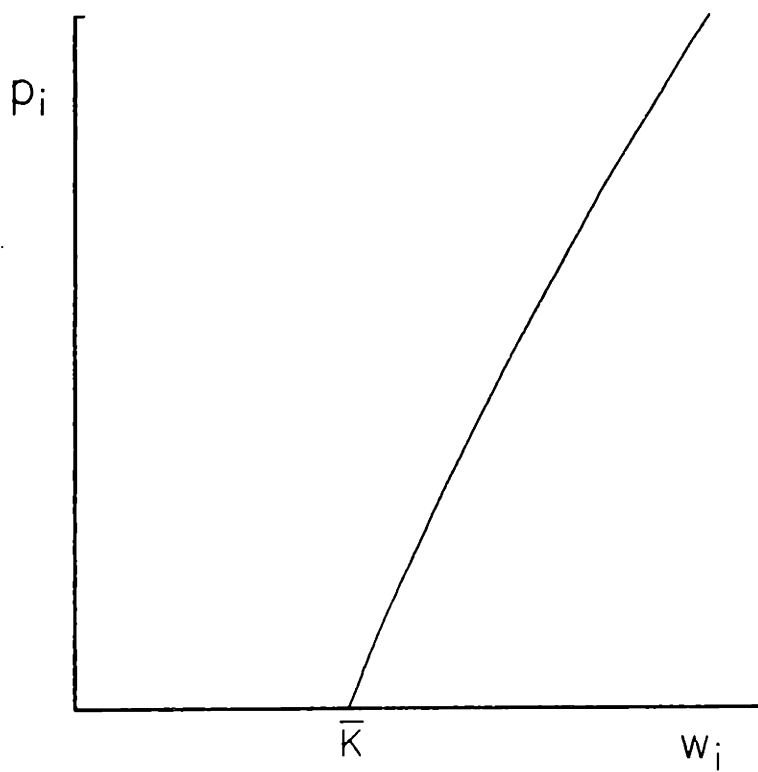
Figure 2

Market equilibrium
locus given $K = \bar{K}$
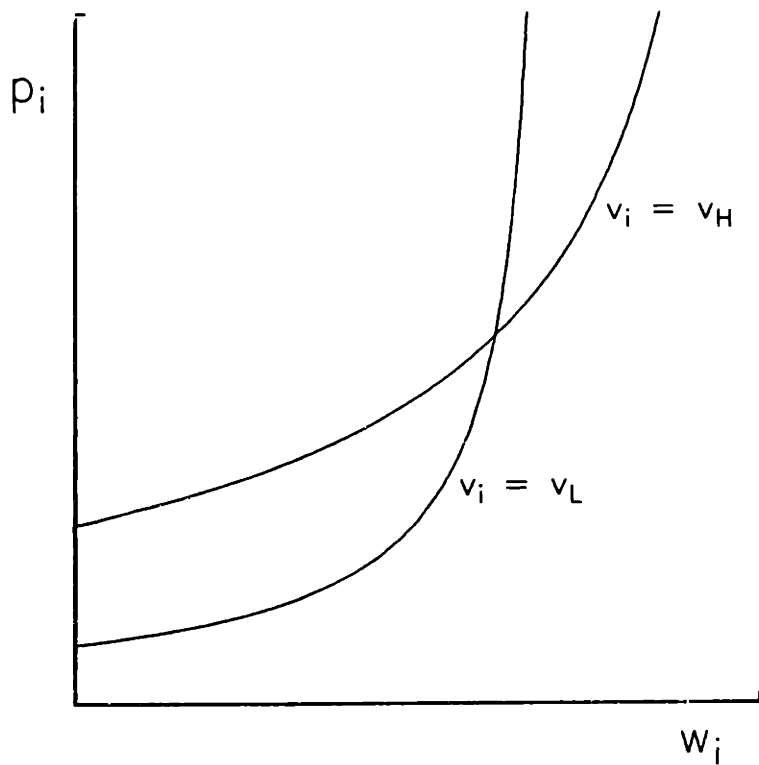
Figure 3

Firm i's isoprofit
curves given $v_H > v_L$

firm's isoprofit curves become "flatter" as v_i rises. Increases in firm i's valuation raise the "markup" term, making an increase in p_i (and thus an increase in the probability of filling the vacancy) more valuable[19].

Figure 4 combines the market equilibrium locus of Figure 2 with the isoprofit curves of Figure 3. Interpreting Figure 4 in the context of firm i's optimization problem, the wage chosen by firm i will be an increasing function of its valuation. Interpreted more broadly, Figure 4 demonstrates that firms with higher valuations will pay higher wages in equilibrium. Thus, the first major result of the 2x2 cases goes through in the more realistic case of a large number of applicants and job openings. As equilibrium application probabilities are inversely related to the wage offered, the second major result also goes through: higher valuation firms will have longer queues (where "queue length" is defined as $p_i M$, the expected number of applicants at firm i). Finally, wages remain strategic complements in the sense that an increase in labor market tightness will prompt firms to raise wages.[20] (As argued above, each individual firm has little effect on labor-market tightness and other wages. But general labor-market shocks--an increase in the minimum wage, for example--may substantially alter market tightness and prompt wage changes.)

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[19]See Appendix 4 for a mathematical statement of the slope of the firm's isoprofit curves.

[20]See Appendix 5 for a proof of this statement.
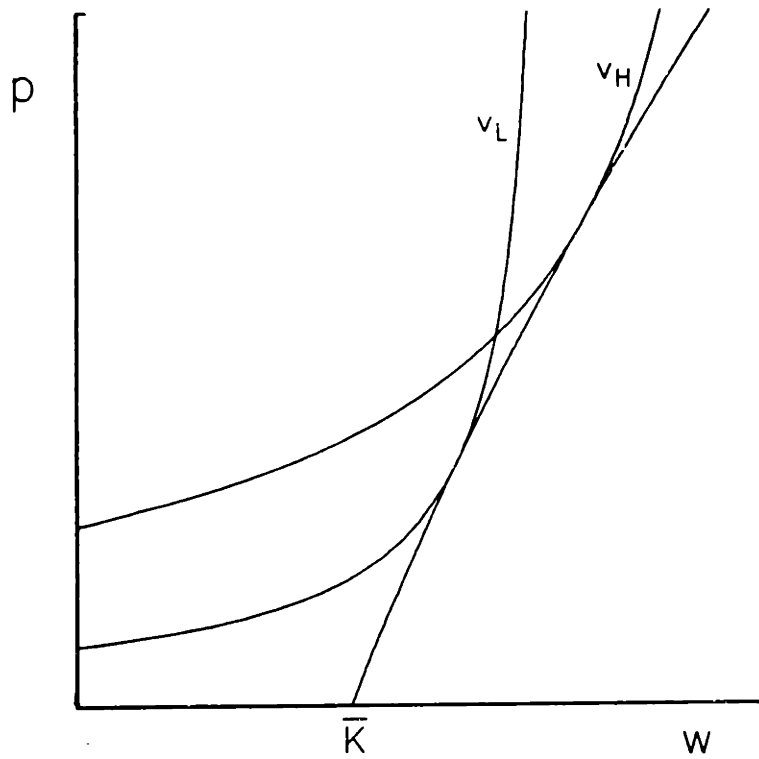
Figure 4

A high valuation ($v = v_H$) and a low valuation ($v = v_L$) firm in equilibrium

4. EFFICIENCY

In this section, I examine the efficiency of the labor market equilibrium in both the large labor market case (presented in Section 3) and the 2x2 case (presented in Section 2). Assuming that a social planner must work within the existing labor market structure, I find that the large labor market equilibrium is efficient while the equilibrium in the 2x2 case is (generally) inefficient.

As discussed above, a coordination problem in the labor market causes some firms to receive no applications and some job seekers to receive no offers. Were a social planner able to change the structure of the labor market, perhaps assigning workers to firms, the coordination problem could be overcome and, as all firms would be operating, output would be maximized.[21] But assuming the social planner must operate within the existing institutional structure, he will maximize expected output by setting wages, thus determining each worker's vector of application probabilities $(p_1,\ldots,p_N)$. (One should note that wages, representing transfers between the firms and workers, will not appear in the social objective function. All that matters from a welfare perspective is the (weighted) sum of the probabilities that each firm will operate.)

In the general case of M workers and N firms,

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[21]Note that the social planner's role is then essentially that of the Walrasian auctioneer discussed in Section 6.

$$E(\text{value of output}) = \sum_{i=1}^{N} v_i \cdot pr(\text{firm } i \text{ receives} \geq 1 \text{ applicant}) .$$

The social planner in the large labor market case thus solves:

$$\max_{p} \quad \sum_{i=1}^{N} v_i \, [1 - (1 - p_i)^M]$$

$$\text{subject to} \quad \sum_{i=1}^{N} p_i = 1$$

where $p = (p_1, p_2, \ldots, p_N)$ is the vector of application probabilities and (as before) v_i is the value of firm i's output. Defining λ as the Lagrange multiplier on the planner's constraint, the first-order conditions yield:

$$v_i \, M \, (1-p_i)^{M-1} = \lambda \quad \forall \, i .$$

The planner thus equates marginal productivities across firms; λ is determined by the requirement that the application probabilities sum to one.

In Section 3, firm i's problem was given as:

$$\max_{w_i} \quad (v_i - w_i) \, [1 - (1-p_i)^M]$$

$$\text{subject to} \quad w_i \, [1 - (1-p_i)^M]/ \, p_i M = K .$$

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Solving the firm's constraint for w_i and substituting this expression into its objective function, the firm's problem may be rewritten:

$$\max_{p_i} \quad v_i [1 - (1-p_i)^M] - Kp_iM .$$

The first-order condition on firm i's problem yields:

$$v_i M (1-p_i)^{M-1} = MK \qquad \forall i .$$

In equilibrium, marginal productivities are equated across firms; K (like λ above) is determined by the requirement that the application probabilities sum to one. Equilibrium in the large labor market is thus (constrained) efficient.

One may gain intuition for the preceding result by noting the similarity between the large labor market case and the standard competitive labor market. The first term in the firm's (rewritten) objective function is expected (total) product; the second term is the product of the expected payment to each applicant (K) and the expected number of applicants (p_iM). Given that each firm takes labor market tightness as given (i.e., assumes K is fixed), the firm's problem thus appears quite similar to the standard competitive maximization problem:

$$\max_{L} \quad Q(L) - wL$$

where Q is output (with price normalized to one), L is labor input, and w is the (fixed) market wage. Like firms in a standard competitive labor

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market, firms in the large labor market are thus essentially "price takers", as each assumes the "supply" of applicants is perfectly elastic at price K.

In the 2x2 case, the social planner's problem becomes:

$$\max_{p} \quad v_1 [1 - (1 - p)^2] + v_2 [1 - p^2]$$

where (as in Section 2) p is the probability that a worker applies to firm 1. The first-order condition yields:

$$p^* = v_1 / (v_1 + v_2)$$

where p* is the socially-optimal probability that each worker applies to firm 1. It is obvious that $p^* > 1/2$ iff $v_1 > v_2$. Intuitively, the planner wants the firm with the higher valuation to have a higher probability of operating.

As derived in Section 2, the mixed-strategy equilibrium of the game played by the job seekers implies:

$$p = (2w_1 - w_2)/(w_1 + w_2) .$$

Thus, to achieve p*, the planner must set:

$$w_1/w_2 = (2v_1 + v_2)/(v_1 + 2v_2) = r^*$$

where r* is the socially-optimal ratio of w_1 to w_2. As noted above, wages

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are merely transfers from firms to workers; the absolute level at which they are set is thus indeterminant. What matters to the social planner is the *ratio* of the wages, as this determines the vector of application probabilities.

One might now ask whether the equilibrium derived in Section 2 is efficient in the sense that $w_1/w_2 = r*$ (and thus $p = p*$). In general, the answer is negative. Assuming (w.o.l.o.g.) that $v_1 \geq v_2$, $w_1/w_2 < r*$ unless $v_1 = v_2$. In words, w_1 and w_2 are not far enough apart; too few workers are expected to apply to firm 1 relative to firm 2. (As the proof of this claim is rather round-about and yields no economic intuition, it is presented in Appendix 6.)

While firms in a large labor market are "price takers", firms in a small market are not. In the 2x2 case, each firm recognizes that an increase in its wage will not only attract more applicants but will also tighten the labor market. (This is the same argument made in Section 3.) Because each firm's wage affects the other's marginal cost, an externality exists. In general, marginal costs (and thus marginal productivities) will not be equated across firms; a social planner could increase the (expected) value of output in the labor market.

Given that the social planner desires to maximize the expected value of output, the preceding analysis has demonstrated that wage differentials are (constrained) efficient in the large labor market case and "too small" in the 2x2 case. But if the planner were instead minimizing unemployment (defined as the expected number of workers receiving no offers), he would equalize wages across the labor market, eliminating differentials

entirely.[22] (Unemployment might enter the planner's objective function if, for instance, he was unable to make transfers between employed and unemployed workers.) Intuitively, wage differentials yield varying queue lengths across firms; high wage firms will have long queues while low wage firms have short queues. As each firm hires at most one worker, many applicants who applied to high wage firms will remain unemployed. If all firms offered the same wage, queue lengths would be equalized and unemployment minimized.

5. POSSIBLE EXTENSIONS OF THE MODEL

In the model above, two assumptions were made for analytical convenience; either (or both) could be loosened without altering the basic implications of the analysis. First, I assumed that each firm has only one job opening. Multiple openings could be permitted at the cost of complicating the story told earlier: the newspaper help-wanted ads must now state not only the wage offered but also the number of vancancies to be filled. (Ceteris paribus, an applicant would apply to a firm with two openings rather than a firm with one opening, as he stands a greater chance of being hired at the first firm.)

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[22]Formally, the planner maximizes the *unweighted* sum of the probabilities that each firm will operate. The first-order conditions for this problem imply that application probabilities (and thus wages) should be equalized across firms.

Second, I assumed that each worker could apply to only one firm. Multiple applications could also be permitted at the cost of complicating the story (and the necessary math): After each firm offers its job to someone in its queue, an applicant may now hold multiple offers (and presumably accepts the one offering the highest wage). Thus, some firms may have long queues (comprised of those applicants to whom it didn't offer the job) but still have an unfilled vacancy. (While such a scenario has a realistic flavor, the best modeling strategy seems unclear: Should a firm with an unfilled vacancy be allowed to offer the job to another applicant in its queue? If so, does the firm know which of these applicants (if any) have already accepted other jobs? Or, alternately, is a second issue of the newspaper published, allowing new applicants to enter the firm's queue?) If this second assumption were relaxed, one might intuitively expect wage dispersion to decrease but not be eliminated entirely.[23]

While loosening each of these assumptions might add a measure of realism to the model, one should note that the basic equilibrium condition has not changed if this is permitted. Importantly, the expected value of applying to a firm is still constant across firms. (This condition fails

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[23]Under assumptions similar to those of the Butters model, Lang (1987) analyzes the case of multiple applications, proving that in equilibrium wages will be dispersed and some applicants will remain unemployed. Lang additionally discusses the possibility that firms require job seekers to "apply in-person"; on a formal level, this case is quite similar to the model presented above. The present model may be viewed as an extension of the Lang analysis, formally developing the "in-person application" story in the context of a finite number of applicants and job openings.

to hold only when applicants are allowed to apply to *all* firms. In that case, firms can offer any positive wage and still receive applications. If applicants can apply to only (N-1) firms, however, the equilibrium condition holds: any firm offering a wage below the labor market's "expected payoff" will receive no applications and will be forced to raise its wage.) And given this condition, one would expect the model's basic intutition to go through: firms with higher valuations will offer higher wages in order to increase to probability of filling their vacancies.

But more important than building a slightly more general model (relaxing the two assumptions discussed above) is the need for for a dynamic version of the simple model. While this static model predicts that some applicants will fail to receive offers and some firms fail to fill vancancies, it is difficult to gage the importance of this "frictional" unemployment without some measure of time. (Imagine, for instance, that a new issue of the newspaper, listing all current vacancies, could be published every ten minutes. Without inflows of new applicants and new job openings, one would expect that any unemployment would be eliminated in a relatively short time and that wage dispersion would be minimal.) In order to study policy issues, one would ultimately like to derive a steady-state solution in a dynamic model and perform the relevant comparative statics exercises.

6. CONCLUSION

Somewhere deep in the background of the competitive labor-market paradigm is a Walrasian auctioneer equating supply and demand. In this story, the auctioneer calls out a candidate market wage, learns labor supply and demand at that wage, and continues this process until he finds a wage at which the number of job openings is equal to the number of applicants. Once the market wage is determined, the auctioneer might then assign each worker to a specific firm. In such a setting, there is no reason why the value of output of a given job opening would influence the wage offered. Any firm can obtain a worker at the market wage; there is no incentive to offer a wage above it.

But in a labor market without such an auctioneer, one would expect coordination problems to develop. If the burden of deciding which firm to apply to were placed on the applicants, a candidate single-wage equilibrium would no longer guarantee each firm one worker--some firms might receive many applications while others might receive none. In such a case, wage dispersion becomes likely: firms which would find it relatively expensive to have unfilled vacancies will offer higher wages than firms which are nearly indifferent to filling a vacancy.

The model presented in this paper was developed to explain a number of stylized facts. As demonstrated by Dickens and Katz (1987) (and many others), inter-industry wage differentials for observationally equivalent workers are persistent over time and are positively correlated with such industry attributes as profitability and capital/labor ratios. As discussed above, each of these stylized facts falls easily out of a search model of the labor market. Additionally, the search story developed above

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generates other testable hypotheses: industries paying higher wages will receive (on average) more applications, and wages should behave as strategic complements.

But while the model developed in this paper has demonstrated the potential importance of search considerations for wage determination, this intuition can hardly be billed as novel--various efficiency-wage stories of turnover and absenteeism provide similar intuition.[24] This model does, however, make two contributions: First, it goes beyond a "partial partial-equilibrium" story in which an ad-hoc specification of application rates (or turnover or absenteeism) is assumed. Rather, the probability that each firm fills its vacancy is derived endogenously from the vector of wages offered. Second, the model demonstrates that a search model can be constructed without (perhaps unrealistically) assuming that jobs are hidden from workers and that firms must earn zero profits. Wage dispersion in the present model is generated by (plausible) coordination problems developing in a large labor market where all job seekers can read the want ads.

APPENDICES

1. *Proof that wages are not "too far apart" in the 2x2 case*

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[24]The non-efficiency-wage models of Lang (1987) (search considerations) and Weitzman (1987) (wage rigidity) also offer similar intuition: firms pay higher wages to increase the probability of hiring workers.

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In solving the game played by the applicants, I assumed that $(1/2)w_2 < w_1 < 2w_2$. Were this condition violated, this game has a unique pure-strategy equilibrium in which both applicants apply to the high-wage firm. (This firm thus becomes a monopsonist.) If v_1 and v_2 were far enough apart, one might expect this condition to be violated--the high-valuation firm might offer a wage sufficient to ensure that it always hires a worker. I will now prove that this is not true: It is never optimal for the high-valuation firm to completely exclude the low-valuation firm from the labor market; the assumption that w_1 and w_2 are not "too far apart" is thus made without loss of generality.

Assume (w.o.l.o.g.) that $v_1 > v_2$. Substituting firm 1's reaction function into its objective function, one obtains firm 1's value function (expected profit as a function of v_1 and w_2):

$$\Pi_1^* = (1/4)(2v_1-w_2)^2/(v_1+w_2) - (v_1^2-v_1w_2+.25w_2^2)/(v_1+w_2) .$$

This value function represents the profit earned in the "competitive" case in which both firms attract applicants positive probability. Firm 1 may, however, guarantee it receives both applications by offering $w_1 = 2w_2$. Profit in this "monopsony" case can be written:

$$\Pi_1^m = v_1 - 2w_2 .$$

(Profit is equal to the "markup" term alone, as pr{F1 receives at least one application} = 1.) Multiplying both the numerator and the denominator by (v_1+w_2), one obtains:

$$\Pi_1^m = (v_1^2-v_1w_2-2w_2^2)/(v_1+w_2) .$$

Comparing this "monopsony" profit with the "competitive" profit, it is obvious that $\Pi_1^* > \Pi_1^m$ for all v_1 and w_2; firm 1 will never completely

exclude firm 2 from the labor market.

2. *Existence of a unique equilibrium wage vector in the 2x2 case*

While a closed-form solution for the equilibrium vector of wages can not be obtained in the 2x2 case, one can prove that a unique equilibrium wage vector exists. As derived in the text, each firm's reaction function may be written:

$$R_i(w_j) - w_j (w_j + 4v_i)/(5w_j + 2v_i) .$$

Each of the reaction functions is increasing and concave; the inverse of each is thus increasing and convex:

$$\frac{\partial R_i}{\partial w_i} > 0 , \; \frac{\partial^2 R_i}{\partial w_i^2} < 0 ; \; \frac{\partial R_i^{-1}}{\partial w_j} > 0 , \; \frac{\partial^2 R_i^{-1}}{\partial w_j^2} > 0 .$$

(This is easily seen by inspection of Figure 1A or 1B.) Furthermore,

$$R_i(0) - 0 , \; R_i(2v_i) - v_i , \; \text{and} \; \frac{\partial R_i(0)}{\partial w_i} - 2 .$$

This implies:

$$R_i^{-1}(0) - 0, \; R_i^{-1}(v_i) - 2v_i , \; \text{and} \; \frac{\partial R_i^{-1}(0)}{\partial w_j} - 1/2 .$$

Given these relationships, we may now demonstrate that a unique equilibrium exists. A Nash equilibrium is obtained if:

$$\phi(w_2) = R_1(w_2) - R_2^{-1}(w_2) - 0 .$$

(Graphically, equilibrium is obtained if the reaction functions cross at some w_2.) As R_1 is strictly concave and R_2^{-1} is strictly convex, ϕ is strictly concave. Furthermore,

$$\phi(0) = 0 \quad \text{and} \quad \frac{\partial \phi(0)}{\partial w_2} = 3/2 \ .$$

Assuming (w.o.l.o.g.) that $v_1 \geq v_2$, $\phi(v_2) < 0$. Thus, ϕ is positive for small values of w_2 and negative for w_2 near v_2. As ϕ is strictly concave, this implies the existence of a unique $w_2^* > 0$ such that $\phi(w_2^*) = 0$.

3. Derivation of mixed-strategy equilibrium in the large labor market case

From an applicant's point of view, the probability of getting a job at firm i is equal to:

$$\sum_{k=0}^{M-1} \frac{1}{k+1} \cdot \text{pr}\{k \text{ other applicants apply to Fi}\} \ .$$

As the probability that firm i receives k other applicants is given by the binomial distribution with parameters p_i and $(M-1)$,

$$\text{pr}\{\text{getting a job at Fi}\} = \sum_{k=0}^{M-1} \frac{1}{k+1} \cdot \binom{M-1}{k} p_i^k (1-p_i)^{M-1-k} \ .$$

Noting that $\dfrac{1}{k+1} \binom{M-1}{k} = \dfrac{1}{M} \binom{M}{k+1}$, one can show that

pr(getting a job at Fi) $= [1-(1-p_i)^M]/p_i M$.

(This result is actually rather intuitive: the probability of being hired at firm i is equal to the expected number of workers hired at Fi ($= 1-(1-p_i)^M$) divided by the expected number of applicants at Fi ($= p_i M$).)

4. *Slope of isoprofit curves in the large labor market case*

Along the firm's isoprofit curve (with expected profits fixed at $\bar{\Pi}$),

$$\partial p_i / \partial w_i = \bar{\Pi} \cdot \left[1 - \frac{\bar{\Pi}}{v_i - w_i} \right]^{\frac{1-M}{M}} / M(v_i - w_i)^2 .$$

To obtain the result that higher valuation firms have "flatter" isoprofit curves, substitute $(v_i - w_i)[1-(1-p_i)^M]$ for $\bar{\Pi}$ and differentiate by v_i (holding p_i and w_i fixed):

$$\frac{\partial}{\partial v_i} \left(\frac{\partial p_i}{\partial w_i} \right) = \frac{- [1 - (1-p_i^M)]}{M (1-p_i)^{M-1} (v_i - w_i)^2} < 0 .$$

5. *Proof that wages increase in labor market tightness*

To prove that w_i increases in K, I will solve the firm's problem, derive w_i as a function of v_i and K, and perform the necessary comparative statics. Substituting (1″) into the firm's objective function, its problem becomes:

$$\max_{p_i} \quad v_i [1 - (1-p_i)^M] - p_i MK \quad .$$

The first-order condition yields:

$$p_i = 1 - \alpha^{1/(M-1)} \qquad \text{where} \quad \alpha = K/v_i .$$

(The firm's application probability thus decreases as K rises.)
Furthermore,

$$w_i = MK[1-\alpha^{1/(M-1)}] \, / \, [1-\alpha^{M/(M-1)}] \quad .$$

and
$$\frac{\partial w_i}{\partial K} = \frac{M \left[1 + \dfrac{1}{M-1} \alpha^{M/(M-1)} - \dfrac{M}{M-1} \alpha^{1/(M-1)} \right]}{\left[1 - \alpha^{M/(M-1)} \right]^2} \quad .$$

As $\alpha \in (0,1)$, one can show that the numerator (and thus the entire expression) is positive--the firm increases its wage as the labor market tightens.

6. *Proof of the Inefficiency of the 2x2 Equilibrium*

To demonstrate that wages in the 2x2 equilibrium are too close together, one would like to simply compare p and p*, showing that $p < p^*$ iff $v_1 > v_2$. (Or, equivalently, show that $w_1/w_2 < r^*$ iff $v_1 > v_2$.) Unfortunately, one cannot solve for w_1, w_2, nor p in closed form; a direct comparison is thus impossible. One can, however, solve for the intersection of each firm's reaction with the "socially optimal locus".

Making use of the concavity of the reaction functions, I demonstrate that the equilibrium wages w_1 and w_2 (determined by the intersection of the reaction functions) are too close together. (While the following proof is rather round-about, its graphical exposition is fairly intuitive; Figure 5 illustrates the argument.)

As derived in Section 2, the reaction functions for firm 1 and firm 2 are:

(1) $R_1(w_2) = w_2 (w_2 + 4v_1)/(5w_2 + 2v_1)$ and

(2) $R_2(w_1) = w_1 (w_1 + 4v_2)/(5w_1 + 2v_2)$.

As derived in the text, welfare is maximized when:

(3) $w_1/w_2 = (2v_1 + v_2)/(v_1 + 2v_2) = r^*$.

Now define w_2^1 as the firm 2 wage at which firm 1's reaction function (1) intersects the "socially optimal locus" (3). Similarly, define w_2^2 as the firm 2 wage at which firm 2's reaction function (2) intersects the "socially optimal locus" (3). Mathematically,

(4) $w_2^1 = \left[\dfrac{6v_1v_2}{9v_1+3v_2} \right]$

(5) $w_2^2 = \left[\dfrac{6v_1v_2}{3v_1+9v_2} \right] \left[\dfrac{v_1+2v_2}{2v_1+v_2} \right]$.

Equations (4) and (5) yield the following relationships:

(*) $w_2^1 < w_2^2$ if $v_1 > v_2$
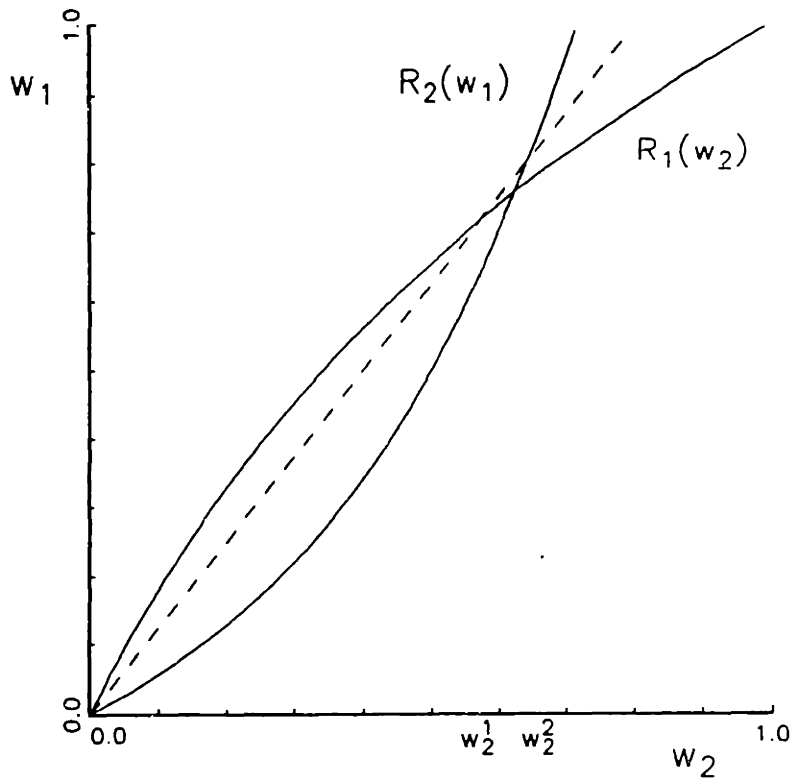
Figure 5

Figure depicts case
in which $v_1 > v_2$

Dashed line represents
socially optimal locus:
$$w_1/w_2 = r*$$

$w_2^1 > w_2^2$ if $v_2 > v_1$ and

$w_2^1 - w_2^2$ if $v_1 - v_2$.

Given the concavity of the reaction functions, one can show that (in equilibrium) $w_1/w_2 > r*$ if $w_2^1 > w_2^2$. Similarly, $w_1/w_2 < r*$ if $w_2^1 < w_2^2$. (As the validity of these claims seems obvious from inspection of Figure 5, I will not formally prove them.) The equilibrium vector of wages (w_1, w_2) is thus efficient only if $w_2^1 - w_2^2$.

Now assume (w.o.l.o.g.) that $v_1 \geq v_2$. Given (*) and the preceding claim, we have:

(**) $w_1/w_2 < r*$ if $v_1 > v_2$,

$w_1/w_2 - r*$ if $v_1 - v_2$.

The equilibrium wage vector derived in Section 2 is thus generally inefficient. Unless $v_1 - v_2$, the ratio of the higher wage to the lower wage is less than is socially optimal.

REFERENCES

Bulow, J., J. Geanakoplos, and P. Klemperer (1985), "Multimarket
    Oligopoly: Strategic Substitutes and Complements," Journal of
    Political Economy, 93, 488-511.

Burdett, K. and K. Judd (1983), "Equilibrium Price Dispersion,"
    Econometrica, 51, 955-970.

Butters, G. (1977), "Equilibrium Distributions of Sales and
    Advertising Prices," Review of Economic Studies, 44, 465-491.491.

Dickens, W. and L. Katz (1987a), "Inter-industry Wage Differences and
    Industry Characteristics," in K. Lang and J. Leonard (eds.),
    Unemployment and the Structure of Labor Markets, London: Basil
    Blackwell.

------ (1987b), "Inter-industry Wage Differences and the Theories of Wage
    Determination," NBER Working Paper # 2271, June.

Doeringer, P. and M. Piore (1971), Internal Labor Markets and Manpower
    Analysis, Lexington, MA: Heath.

Gibbons, R. and L. Katz (1987), "Learning, Mobility, and Inter-Industry
    Wage Differences," MIT mimeo, November.ber.

Gramlich, E. (1976), "Impact of Minimum Wage on Other Wages, Employment
    and Family Incomes," Brookings Papers on Economic Activity, 409-462.

Grossman, J. B. (1983), "The Impact of the Minimum-Wage on Other Wages,"
    Journal of Human Resources, 18, 359-378.

Holzer, H., L. Katz and A. Krueger (1987), "Job Queues and Wages: Some New
    Evidence on the Minimum Wage and the Industry Wage Structure," Havard
    University mimeo, November.

Katz, L. (1986), "Efficiency Wage Theories: A Partial Evaluation," is S.
    Fischer (ed.), NBER Macroeconomics Annual 1986, Cambridge, MA: MIT
    Press.

Krueger, A. (1988), "The Determinants of Queues for Federal Jobs,"
    Industrial and Labor Relations Review, forthcoming October 1988.

Kreuger, A. and L. Summers (1987a), "Efficiency Wages and the
    Inter-Industry Wage Structure," Econometrica, forthcoming.ing.

------ (1987b), "Reflections on the Inter-Industry Wage Structure," in K.
    Lang and J. Leonard (eds.), Unemployment and the Structure of Labor

Markets, London: Basil Blackwell.

Lang, K. (1987), "Persistent Wage Dispersion and Involuntary Unemployment," Boston University mimeo, December.

Mortensen, D. (1986), "Job Search and Labor Market Analysis," in O. Ashenfelter and R. Layard (eds.), Handbook of Labor Economics, Amsterdam: North-Holland.

Murphy, K. M., and R. Topel (1987), "Unemployment, Risk and Earnings," in K. Lang and J. Leonard (eds.), Unemployment and the Structure of Labor Markets, London: Basil Blackwell.

Rothschild, M. (1973), "Models of Market Organization ·_ imperfect Information: A Survey," Journal of Political Economy, 81, 1283-1308.

Tirole, J. (1986), "Non-cooperative Game Theory: A User's Manual," in The Theory of Industrial Organization, MIT mimeo, August.

Weitzman, M. (1987), "A Theory of Job Market Segmentation," MIT mimeo, August.

Essay 2:

SOCIAL NETWORKS AND LABOR MARKET OUTCOMES:
TOWARD AN ECONOMIC ANALYSIS

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Empirical research on both job search by workers and employee search by firms highlights the importance of social networks in the labor market. Workers often receive information on job openings through friends and relatives; firms often hire through employee referrals. While additional analysis may be necessary to determine precisely why workers and firms rely so heavily upon social networks, researchers have suggested a variety of plausible explanations. Friends and relatives provide accurate, low-cost information on job openings; an employee referral may implicitly (or explicitly) signal an applicant's expected productivity. But whatever the precise explanation underlying the importance of social networks, their widespread (and apparently purposeful) use raises an even larger issue: social structure must play a key role in determining labor market outcomes. Stated differently, an individual's social ties represent an important (and often neglected) constraint on his labor market success.

To explore the relationship between social networks and labor market outcomes, this paper presents a formal model of the hiring process which explicitly incorporates a simple social structure. This model serves two major purposes: First, it offers a formalization of the hiring process in which both firms and workers choose rationally between alternative hiring channels, thus moving beyond previous "partial partial-equilibrium" analyses in the economics and sociology literatures. Second, it demonstrates how social structure--perhaps a nebulous concept to many economists--may be integrated into formal labor-market analysis. From this model, an interesting (albeit tentative) picture emerges: firms' use of employee referrals may generate wage dispersion, with better

labor-market outcomes for those who are "well connected." The model

suggests that differences in average applicant ability levels across

hiring channels may arise endogenously, and additionally predicts that

firms hiring through referrals should earn higher expected profits.

The paper proceeds as follows: In Section 1, I review the

literatures on job search by workers and employee search by firms,

documenting the importance of social networks in the labor market.

Additionally, I review personnel research which suggests that workers

located through employee referrals are superior to those recruited through

other hiring channels. In Section 2, I offer a variety of explanations

why both firms and workers might rationally rely upon social networks. In

Section 3, I develop the basic premise of this paper--that social

structure constitutes an important constraint on labor-market success--and

note previous sociological research on this topic. In Section 4, I

develop a formal model of the hiring process, proving the existence of a

symmetric equilibrium and summarizing the relevant qualitative

implications. In Section 5, I conclude with a discussion of relevant

policy issues and a proposed research agenda.

SECTION 1. FACTS

In the standard Walrasian formulation, the labor market operates

frictionlessly: at the market wage, workers effortlessly find jobs and

firms effortlessly attract employees. But labor market reality is, of course, more complex. Workers entering the labor force or displaced from a job often experience considerable difficulty locating a new position. Firms, on the other hand, do not face a perfectly elastic supply of labor at some given market wage; personnel departments must actively recruit workers through a variety of hiring channels. Aware of these frictions in real-world labor markets, economists (as well as sociologists and personnel researchers) have focused considerable attention on the issue of job search by workers and (to a lesser extent) the issue of employer search for workers. I shall now briefly examine, in turn, previous research examining worker search and employer search.

Over the last two decades, economists have compiled a voluminous literature--both theoretical and empirical--on the issue of job search by workers. This literature, under the heading of search theory, focuses primarily on the duration of unemployment spells and the reservation wages chosen by job seekers.[1] But an alternative job-search literature, initiated by institutional labor economists in the 1940s and 50s, focuses instead on workers' awareness of labor market opportunities and the methods used by workers in locating jobs.[2] More specifically, these studies examine whether workers find jobs through help-wanted advertising, employment agencies, referrals through friends and relatives, direct

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[1] See Mortensen (1986) for a recent survey of this job-search literature.

[2] See, for example, Myers and Shultz (1951), Parnes (1954), Sheppard and Belitsky (1966), Rees and Shultz (1970), Granovetter (1974), Corcoran, et. al. (1980), and Wial (1988).

application, or a variety of other sources.

One major finding of this latter literature--initially suggested by relatively limited area-labor-market studies but since verified through analysis of larger data sets--is that a large percentage of workers locate jobs through friends and relatives. Table 1 reports the relevant findings of a number of these studies. While the importance of friends and relatives as a job-finding method varies somewhat across studies, the following generalization seems fair: approximately 50% of all workers currently employed found their jobs through friends and relatives. Furthermore, Table 1 reveals that social networks are more important sources of job information for blue-collar workers; white-collar workers are relatively more likely to find jobs through employment agencies or newspaper ads. While Corcoran, et. al. (1980) find that blacks rely more heavily upon referrals from friends and relatives, most studies (e.g., Rees and Shultz (1970)) report the opposite, suggesting that blacks are less likely to possess social ties to workers in high-paying positions.

While economists have devoted considerable effort to the issue of job search by workers, they have focused less attention on firms' search for workers. But researchers in several fields--economics, sociology, and human resources--have addressed this issue. Essentially, firms' search for workers is the opposite side of the coin from workers' search for jobs: firms may recruit workers through either formal hiring channels (e.g., help-wanted advertising or employment agencies) or informal hiring channels (e.g., employee referrals or direct application).[3] As shown

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[3]The distinction between formal and informal channels, as made in the job-search and employee-search literatures, rests upon the existence (or absence) of labor-market intermediaries.

TABLE 1

Job-Finding Methods Used by Workers

Source/ Data	Method				
	Friends/ Relatives	Gate Application	Employment Agency	Ads	Other
Myers and Shultz (1951)					
(Sample of displaced textile workers)					
First Job	62%	23%	6%	2%	7%
Mill Job	56	37	3	2	2
Present Job	36	14	4	0	46[1]
Rees and Shultz (1970)					
(Chicago labor-market study, 12 occuptions[2])					
Typist	37.3	5.5	34.7	16.4	6.1
Keypunch Operator	35.3	10.7	13.2	21.4	19.4
Accountant	23.5	6.4	25.9	26.4	17.8
Tab Operator	37.9	3.2	22.2	22.2	14.5
Material Handler	73.8	6.9	8.1	3.8	7.4
Janitor	65.5	13.1	7.3	4.8	9.3
Janitress	63.6	7.5	5.2	11.2	12.5
Fork Lift Operator	66.7	7.9	4.7	7.5	13.2
Punch Press Operator	65.4	5.9	7.7	15.0	6.0
Truck Driver	56.8	14.9	1.5	1.5	25.3
Maint. Electrician	57.4	17.1	3.2	11.7	10.6
Tool and Die Maker	53.6	18.2	1.5	17.3	9.4

Source/ Data	Friends/ Relatives	Gate Application	Employment Agency	Ads	Other
			Method		

Granovetter (1974)

(Sample of residents of Newton, MA)

Source/ Data	Friends/ Relatives	Gate Application	Employment Agency	Ads	Other
Professional	56.1	18.2	15.9[3]	--[3]	9.8
Technical	43.5	24.6	30.4	--	1.4
Managerial	65.4	14.8	13.6	--	6.2

Corcoran, et. al. (1980)

(PSID, 11th wave)

	Friends/ Relatives	Gate Application	Employment Agency	Ads	Other
White Males	52.0	--[4]	5.8	9.4	33.8[4]
White Females	47.1	--	5.6	14.2	33.1
Black Males	58.5	--	7.0	6.9	37.6
Black Females	43.0	--	15.2	11.0	30.8

[1]Most of these workers were rehired at previous mill job or hired at a new mill established in one of the vacated mills.

[2]In computing the cited figures, those workers rehired by previous employers and those not reporting job-finding source are excluded from the denominator.

[3]Agencies and ads are combined under the heading "Formal Means."

[4]Gate applications are included under "Other."

above, social networks play a major role in the job-search process; one might assume that firms must (almost tautologically) recruit heavily through informal hiring channels. This conjecture is substantiated by recent research: Holzer (1987), examining Equal Opportunity Pilot Project (EOPP) data, reports that 36% of firms interviewed filled their last openings with referred applicants; Campbell and Marsden (1988), analyzing data on 52 Indiana establishments, find that over 51% of jobs were filled through referral.[4]

In contrast to the research cited above, the personnel literature on employer search has attempted to evaluate employees recruited through various hiring channels.[5] The major finding of this literature, as shown in Table 2, is the lower quit rate (as measured by the one-year retention rate) of workers recruited through employee referral.[6] Additionally, Hill (1970) reports that workers recruited through referral (in each of three firms sampled) tended to have higher performance ratings. (Holzer's (1987) analysis of EOPP data provides additional (albeit weak) support for

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[4] Each of these papers examines correlations between firm characteristics and hiring channels used, finding that smaller (and perhaps less bureaucratic) firms rely more heavily upon employee referrals. Though not explicitly examining alternate hiring channels, research by Barron and Bishop (1985a, 1985b) (also utilizing EOPP data) explores the relationship between "extensive" search--locating additional applicants--and "intensive" search--screening each applicant more thoroughly. (This extensive/intensive distinction was first made by Rees (1966); it corresponds roughly to "recruitment" versus "selection" as discussed in the personnel literature.)

[5] See Ullman (1966), Hill (1970), Gannon (1971), Reid (1972), Decker and Cornelius (1979), Breaugh (1981), Schwab (1982), Taylor and Schmidt (1983) and Breaugh and Mann (1984).

[6] Datcher (1985), analyzing PSID data, finds that blacks and college graduates hired through referral tend to have lower quit rates. Surprisingly, non-college-educated white workers hired through referral tended to have higher quit rates.

this finding.) Although his sample lacks workers hired through referral, Breaugh (1981) finds that performance ratings, absenteeism, and job satisfaction are each correlated with the hiring channel through which workers were recruited. (This lends some credibility to Granovetter's (1974) finding of higher job satisfaction for workers hired through referral.)

TABLE 2

One-Year Survival Rates as a Function of Recruiting Method

Source/ Data	Method			
	Friends/ Relatives	Gate Application	Want Ads	Private Agencies
Decker and Cornelius (1979)				
Bank Employees	69%	57%	67%	52%
Insurance Agents	70	64	57	62
Abstract Service	96	90	79	94
Gannon (1971)				
Bank Employees	74	71	61	61
Reid (1972)				
Engineering and Metal Trades	39	25	16	--
Ullman (1966)				
Clerical, Company 1	25	--	12	--
Clerical, Company 2	72	--	26	38

Source: Table 5.3 from Schwab (1982), p. 113.

SECTION 2. EXPLAINING THE FACTS

As discussed in the previous section, research on both job search by workers and employee search by firms has highlighted the important role played by social networks in the hiring process. At this point, one might ask why employee referrals, an informal hiring channel, should be so important relative to more formal hiring channels such as help-wanted advertising and employment agencies. More specifically, two questions arise. First, why do workers find so many jobs through referral? Second, turning to the other side of the labor market, why do firms desire to hire through referrals? Given the evidence from the personnel literature cited above, one might answer the second question by simply noting the apparent superiority of workers recruited through referral. But given this answer, one must still explain why referred workers are superior. I shall now address each of these questions in turn.

To explain the significance of friends and relatives as sources of job information, an economist might naturally be drawn toward a search-theoretic formulation. Indeed, standard search models seem an appropriate tool: job leads received from social contacts might be aptly described as draws from an exogenously given distribution of wage offers. In a recent paper, Holzer (1988) takes this approach, extending a standard search model to permit more than one type of search method. (In addition to applying directly to firms, a worker may thus also answer want ads, visit employment agencies, query friends and relatives, etc.) One would

expect a worker's use of a given search method to increase as (1) the cost

of the method decreases and (2) the method becomes more ef.. ctive,

yielding a job offer with higher probability. As Holzer assumes that

contacting friends and relatives is relatively inexpensive and finds this

method to be highly productive in generating job offers, his model

explains workers' heavy reliance upon social contacts.

But while Holzer's finding that friends and relatives are highly

productive job-finding sources is interesting (suggesting that employers

prefer to hire referred applicants), standard search-theoretic models seem

a poor abstraction of labor market reality. Such models generally assume

that, at a cost, a worker is allowed to sample from some (usually

exogenous) wage-offer distribution; the worker accepts an offer if it

exceeds his reservation wage.[7] This implies that workers are actively

searching and that such search is costly (in terms of time and/or money).

But when jobs are found through referral, workers need not be actively

job-hunting to receive offers. Indeed, it seems more likely that job

information obtained through friends and relatives is the product of

casual social interaction not undertaken specifically for the purpose of

finding a job.[8] The number of wage offers received (and ultimately the

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[7]See Mortensen (1986) for a simple model.

[8]One can, of course, think of counter-examples: "networking" among managers
and professionals may be undertaken primarily to generate job offers. But
in blue-collar occupations (the labor-market segment in which referrals are
most significant empirically), it seems unlikely that contacts are made
specifically for the purpose of finding a job. Indeed, this conjecture held
true even for the professional, managerial and technical workers surveyed by
Granovetter (1974).

level of an accepted wage offer) are determined not by total expenditure on search, but on the number (and type) of social connections held by the worker. Thus, given the widespread use of friends and relatives as sources of job information, real-world job search may be a more passive process than suggested by standard search-theoretic models.[9]

The second question raised above--why firms hire through employee referrals--has not yet been answered definitively. But in the economics, sociology, and personnel literatures discussed above, (at least) four potential explanations emerge. In the personnel literature, the debate over the superiority of referred workers has generated two competing hypotheses: (1) alternative hiring channels draw from different underlying groups of workers, possessing inherently different abilities and (2) referred workers, being better informed of salient job characteristics by current employees, have more realistic perceptions of a given job before starting work.[10] An alternative explanation (found especially in the economics literature), holds that (3) current employees put their reputations on the line when referring a friend or relative, and will thus hesitate to refer someone unacceptable to the employer. A final explanation for firms' use of referrals (again developed most fully in the economics literature), holds that (4) whether or not referred workers are

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[9]An additional criticism of standard search models (noted explicitly by Holzer) is the "partial partial-equilibrium" nature of the analysis. No consideration is given to why wage dispersion arises (or, in Holzer's model, why firms might utilize alternate hiring channels).

[10]These two explanations are contrasted most clearly by Schwab (1982). See Taylor and Schmidt (1983) and Breaugh and Mann (1984) for efforts to distinguish empirically between the two competing hypothesis.

more productive, this hiring channel entails fewer direct hiring costs and will thus be used by some types of employers. I now elaborate on each of these four explanations in turn.

The first explanation, applied narrowly to the hiring of workers through employee referral, states that workers hired through referral have (at least on average) inherently higher ability levels. A simple argument underlies this hypothesis: Employees tend to refer others similar to themselves; job applicants referred by high-ability workers are likely to be high-ability workers themselves. The claim that employees refer other similar to themselves is cited often in the literature,[11] and one can easily imagine a variety of reasons why ability is likely to be correlated among friends and relatives. Two individuals living in the same environment (e.g., family or neighborhood) undergo a similar socialization process, ultimately developing similar traits. Alternately, friendships may often form between individuals with similar traits. Given that firms are typically unable to determine perfectly worker ability before hiring, and that other more formal hiring channels yield less information on worker ability, firms may thus prefer to hire through employee referrals.[12]

The second explanation of firms' use of employee referrals focuses on

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[11]See, for example, Rees (1966) and Doeringer and Piore (1971). This claim is also found in personnel texts, often followed by the warning that firms relying too heavily upon employee referrals may run afoul of affirmative action legislation. (See, for example, Heneman, et. al. (1980).)

[12]More formally, this explanation assumes a labor market characterized by adverse selection in which employers are unable to write (fully) output-contingent contracts. Employee referrals mitigate this adverse-selection problem by providing firms with additional information on the ability of referred applicants.

the amount (and quality) of information held by the job applicant before

the hiring decision is made. Prior to actually working for a firm, an

applicant obtained through a formal hiring channel may know little about a

job--he is uncertain of his productivity, his chances for promotion,

whether he will like his co-workers, etc. But an applicant obtained

through referral may possess considerable information on the job to be

filled, having received a credible first-hand account from his friend or

relative working in the firm.[13] As described in the personnel literature,

such information acts as a "realistic job preview" in which potential

employees are informed of all salient characteristics--both good and

bad--of the job opening.[14] Given the superior information possessed by

referred applicants, one would expect these applicants to be self-

selected: those referrals who expect to dislike a job (or see no chance

for promotion, etc.) will not bother to apply. In this way, referred

applicants may be of higher average quality than workers attracted through

more formal hiring channels who possess little information about the job.[15]

(Thus, referred applicants have higher average ability levels even in the

absence of inherent underlying differences between workers obtained

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[13]See Rees and Shultz (1970), p. 203, and Granovetter, p. 97.

[14]See Wanous (1980) for an extended discussion of realistic job previews.

[15]A formal model of this second explanation might assume the existence of a
worker-specific "match quality" for each given job. (See Jovanovic (1979)
for such an analysis.) While many workers may learn of job openings from
acquaintances (and receive a relatively precise estimate of their match
qualities), only those in the upper tail of the match-quality distribution
will actually apply for the job. Workers recruited through more formal
channels will lack a precise estimate of match quality; given the absence of
self-selection, these applicants will have lower average match qualities.

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through alternative hiring channels.)

The third explanation for firms' use of employee referrals centers upon workers' reputations; the fourth explanation highlights the importance of relative costs of alternative hiring channels. As argued by several researchers, a worker who learns of a job opening in his firm will refer only well-qualified applicants, as his reputation is at stake.[16] Implicitly, this argument assumes that an employer is willing and able to penalize workers for referring unqualified applicants (through either pecuniary or non-pecuniary means).[17] Finally, firms hiring through employee referral are likely to incur lower hiring costs--no agency or advertising fees need be paid. Indeed, one might imagine that two wage/hiring strategies might co-exist in equilibrium: some firms might pay high wages and hire through referrals while others pay low wages and recruit through more expensive formal hiring channels.[18] (Note that this explanation, in its simplest form, does not require any productivity differential between referred applicants and those recruited by other more formal means.)

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[16]See, for example, Rees and Shultz (1970), p. 203.

[17]A formal economic model of this explanation would incorporate not only adverse selection (as firms are unaware of applicants' abilities) but also moral hazard (as current workers are at least partially aware of the applicants' abilities and may hide such information from the firm). See Saloner (1985) for a similar model.

[18]This is essentially Stigler's (1962) argument that higher wages are a substitute for search. This model implicitly (and sometimes explicitly) underlies the analysis of Rees and Shultz (1970).

SECTION 3. THE ROLE OF SOCIAL STRUCTURE

Section 1 documented the empirical significance of friends and
relatives in the hiring process. To elucidate this finding, Section 2
offered several plausible explanations for the rational use of social
networks by both workers and firms. As the above discussion indicates,
more research is necessary to determine why firms use employee referrals
(or, alternatively, why referred workers are superior to those obtained
through other channels[19]). But whatever the precise explanation (and,
plausibly, each of the hypotheses offered above is of some importance),
the analysis of the previous two sections raises a larger and arguably far
more important issue. To state the argument succinctly: *Given the
widespread (and purposeful) use of employee referrals by firms and
workers, it follows that social structure--the topology of the entire web
of social connections[20]-- must play an important role in determining
labor-market outcomes.*

While practically everyone has heard the adage "It's not *what* you
know but *who* you know" (and can often supply anecdotal evidence), economic

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[19] The answer to this question is of practical importance for personnel
managers. If alternative hiring channels yield workers of inherently
different abilities, these managers should devote effort to rating the
employees obtained through each channel; these ratings will inform future
recruiting efforts. If, on the other hand, employee referrals provide
realistic job previews, personnel managers should seek to provide more
information for applicants recruited through all channels.

[20] This narrow definition of the term "social structure"--connoting a concrete
map of social networks as opposed to (more nebulous) societal "classes" or
"institutions"--was first proposed by Nadel (1957).

theorists have devoted little attention to the relationship between social structure and labor market outcomes. But a number of sociologists--often explicitly motivated by the findings discussed in the first section--have attempted to move beyond labor market formulations featuring atomistic actors and toward explanations highlighting the role of social networks. In a series of papers, Granovetter (1981, 1985, 1987) has argued that economic transactions are "embedded" in a network of social relationships; a worker's social ties constitute an important (and often neglected) constraint on his labor market success. More broadly, sociologists specializing in "social network analysis" have compiled a large literature--both theoretical and empirical--on the constraints that social structure places on individual action.[21]

While the social-structure literature contains few formal theoretical analyses of labor-market outcomes, two papers should be noted. In a model developed by Boorman (1975) and an extension of this analysis by Delany (1988), each worker may become aware of a job opportunity and (if currently employed) may allocate this job to an acquaintance. Within this framework, the welfare properties of alternative social structures may be examined.[22] From the perspective of the present paper, these models constitute an important first step toward a formal economic analysis of

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[21]For a recent overview of this subfield and examples of current research, see Wellman and Berkowitz (1988). In the final section, I elaborate further on this subfield, arguing that economists might usefully adapt some of its techniques in exploring social structure and labor-market outcomes.

[22]Boorman is concerned primarily with the "strength" of ties in a given network structure (formalizing Granovetter's (1973) "weak tie/strong tie" hypothesis) rather than alternate network topologies. But the analysis I have suggested seems (at least) conceptually similar.

social networks and labor market outcomes. But these models are only a first step, as they neglect the important role played by firms in the hiring process. While Boorman and Delany demonstrate how social ties might constrain labor-market success, they cannot explain why firms might desire to hire through employee referrals (rather than a formal hiring channel) and how wage offers will vary across hiring channels.[23] A more complete analysis of social networks must incorporate maximizing behavior by firms as well as workers.

In the next section, I present a formal model of the hiring process which I believe constitutes a second major step toward a theoretical analysis of social networks and labor market outcomes. In presenting this model, I have two major goals: A first, relatively narrow, goal is to construct a "complete" model of the hiring process--complete in the sense that both firms and workers are rationally choosing between alternative hiring channels. A second, broader, goal is to demonstrate how social structure-- perhaps a nebulous concept to many economists--might be formally integrated into labor-market analysis. In pursuing these goals, I have not sought to distinguish between the alternative explanations for firms' use of employee referrals discussed in Section 2. Rather, I have taken one plausible explanation--that workers tend to refer others similar to themselves--and explored how equilibrium wage offers differ across hiring channels (and thus how social structure may constrain labor market

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[23]The major shortcoming of the Boorman and Delany models is thus similar to that of Holzer's (1988) search-theoretic formulation: these analyses are of a "partial partial-equilibrium" nature, examining behavior on only one side of the labor market.

success).[24] Further, in formalizing social structure, I have chosen

extremely simple (and perhaps unrealistic) assumptions on the pattern of

social ties. More complex representations of social structure, building

upon work already completed by social-structure researchers, may be

necessary for policy analysis.

SECTION 4. THE MODEL

In this section, I present a formal model of the hiring process in

which firms may hire workers through employee referrals or an impersonal

"market." Workers, on the other hand, may accept the best wage-offer

received through referral (if, in fact, any are received) or wait to be

hired through the market. To explain firms' use of employee referrals, I

assume that workers tend to refer others like themselves (as discussed in

Section 2). Thus, while the market provides no worker-specific

information, a referral from a present employee does provide the firm with

some indication of an applicant's ability. As demonstrated below,

employee referrals thus allow some firms to circumvent the greater

adverse-selection problems posed by the market.[25]

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[24] I would expect that each of the other potential explanations for firms' use
of referrals would generate the same basic intuition, demonstrating that
social networks are an important determinant of labor-market success.

[25] Formally, this model is an extension of Butters' (1978) analysis of price
dispersion. (For previous extensions of this analysis to the labor market,
see Hosios (1983) and Lang (1987).) The present model differs from others
in the "equilibrium price dispersion" literature by assuming that those who
fail to receive job offers informally through referral are still able to

More concretely, I propose a two-period model[26] with the following assumptions on workers and firms:

Workers

O Each worker lives one period.

O There are 2N workers in each period; N is assumed large.

O Workers may be of two types, either high or low ability.

   To simplify the model, I further assume:

   O One-half of the workers (i.e., N) are of each type in each period;

   O High-ability workers produce one unit of output while low-ability workers produce zero units.

O Workers are observationally equivalent; employers are uncertain of the ability of any particular worker.

Firms

O Each firm may employ (at most) one worker.

O A firm's profit in each period is equal to the productivity of its employee minus the wage paid. (Product price is exogenously determined and normalized to unity.)

O Each firm must set wages before learning the productivity of its worker.

O Firms are free to enter the market in either period.

To this point, the assumptions made are standard in models of adverse

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find employment through a formal market.

[26]I further assume, merely for simplicity, no discounting between periods.

selection in the labor market.[27] Workers are observationally equivalent,
unable to signal their ability to potential employers. Each firm must set
its wage before learning the productivity of its employee; piece-rate
compensation schemes, for example, are thus prohibited. Given the
assumption of free entry of firms, expected profit (for entering firms) is
driven to zero. Thus, firms will offer wages equal to the expected
productivity of those workers on the market. Ex post, some firms will pay
a wage higher than the productivity of its employee; others will pay a
wage lower. (If the model was closed at this point, under the assumption
that all workers in each period were hired through the market, all firms
would offer a wage equal to 1/2 in each period.)

Like many models in labor theory (and economics generally), standard
adverse-selection models implicitly assume atomistic actors. Specifi-
cally, no interaction occurs between individuals looking for jobs and
workers aware of vacancies within their firms. But as such interaction is
central to any explanation of firms' use of employee referrals, it must be
explicitly modelled.[28] I thus introduce the following assumptions on
social structure:

Social Structure

O Each period-one worker "knows" at most one period-two worker,

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[27] See, for example, Greenwald (1986) and Waldman (1984).

[28] The present analysis thus attempts to formalize the "embeddedness" of labor
market outcomes in social structure described by Granovetter (1981, 1985,
1987).

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possessing such a "tie" with probability $r \in [0,1]$.

O For each period-one worker holding a "tie", the specific period-two individual "known" is selected stochastically through a two-stage process:

    O In the first stage, the period-two worker's type is chosen. With probability α, the period-one worker knows a period-two worker of his *own* type. (The period-one worker thus knows a worker of the *other* type with probability $1-\alpha$.) To formalize the notion that workers tend to know others similar to themselves, I assume $\alpha > 1/2$.

    O In the second stage, the specific period-two worker is chosen randomly from those of the appropriate type. Conditional upon holding a tie, a period-one worker thus knows any given period-two worker of his own type with probability α/N and any given worker of the other type with probability $(1-\alpha)/N$.

Given $\alpha > 1/2$, social ties typically connect workers of the same type. Further, given the stochastic nature of these ties, some period-two workers may possess several ties while others have none.[29] (Thus, while the assumed social structure is rather simplistic, it offers a plausible description of reality: some workers are "well connected" while others are not.) While convenient for the present analysis, one should note that the assumption of stochastic social ties--implying an unrealistically well-mixed and transient population--is not essential to the model. As long as firms are unaware of the precise ties held by current employees, a social structure comprising deterministic (but potentially overlapping) ties will yield the same qualitative results.

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[29]Note that the specified assignment of social ties is formally equivalent to an occupancy problem in probability theory, where period-two workers are "urns" and a tie to a given period-one worker is a "ball". As the balls are dropped into the urns stochastically, some urns will contain several balls while others contain none.

Finally, I assume the following timing:

Timing

(1) Firms hire period-one workers through the market; this clears at a wage w_{M1}.

(2) Production occurs; each firm learns the productivity of its worker.

(3) If a firm desires to hire through employee referral, it sets a "referral" wage; firm i may thus set a wage w_{Ri}.

(4) Social ties are assigned.

(5) Each period-one worker possessing a social tie relays his firm's wage offer (w_{Ri}) to his period-two acquaintance.

(6) Each period-two worker compares wage offers received, either accepting one or waiting to find employment through the market.

(7) Those period-two workers with no offers (or refusing all offers) go on the market; this clears at a wage w_{M2}.

(8) Production occurs.

More informally, the timing comprises three major stages. First, each firm hires a period-one worker through the market and learns his ability. As period-one workers are observationally equivalent--and cannot be referred for jobs by other workers--each firm hiring through the market obtains a high-ability worker with probability one-half. (As this step is relatively uninteresting, it will receive no further attention until the end of the section.) Second, knowing the ability of its current worker, each firm sets a "referral" wage; this offer is then (potentially) relayed

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to an acquaintance of the worker.[30] In order to attract this acquaintance,
the firm's offer must exceed both the market wage and any other offers
received by the acquaintance. Note that a firm not wishing to hire
through employee referral will set no referral wage. (Alternately, the
firm might offer a wage below w_{M2}--such an offer has no probability of
acceptance.) Period-two workers then compare all offers received,
accepting the highest.[31] Third, those period-two workers who receive no
offers are forced to find employment through the market, earning a wage
equal to the average expected productivity of those on the market.[32]

The solution to the model is characterized by the following three
propositions[33]:

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[30] Note that a period-one worker's only action is to inform his period-two
acquaintances of a given job opening: his role is informational rather than
strategic. (Delany (1988), making a similar assumption, labels such job
information a "donative transfer.") In a more complex model in which
period-one workers are concerned with their reputations (as suggested by
explanation 3 in Section 2), these workers would play a more strategic role:
given some penalty for referring an unqualified applicant, period-one
workers might refer only high-ability period-two acquaintances.

[31] Note that the timing in steps (3) through (5) is somewhat arbitrary. An
alternate (formally equivalent) version might run: First, social ties are
assigned (chosen by nature). Each worker then writes out the name and
address of any acquaintances and submits this list to his employer. Firms
then contact these referrals, making period-two wage offers. As before,
period-two workers compare all offers received and accept the highest. (As
will be demonstrated below, all referral wage offers will, in equilibrium,
exceed w_{M2}.)

[32] For simplicity, the equilibrium derived below assumes that firms do not
observe the actual number of workers on the period-two market before setting
the market wage. (This market wage is thus determined by the ex-ante
expected average quality of those on the market.)

[33] The following propositions are intended merely to provide the reader with a
brief--and hopefully intuitive--description of the equilibrium. Please see
the proofs for a precise statement of the period-two market wage, the
referral wage distribution, etc.

Proposition 1. The market wage, w_{M2}, is determined by the expected productivity of those workers on the market (i.e., those receiving no job offers); this wage is less than the expected (average) productivity of the population (i.e., $w_{M2} < 1/2$).

Proposition 2. Each firm employing a high-ability worker in period one will wish to hire through an employee referral. Assuming a symmetric equilibrium, each of these firms plays a mixed strategy, selecting some "referral" wage offer $w_R \in [w_{M2}, \bar{w}_R]$ from a distribution $F(w_R)$. Further, the maximum wage offered exceeds the expected (average) productivity of the population (i.e., $\bar{w}_R > 1/2$). (If a firm's offer is not accepted, either because its worker possessed no tie or because its offer was rejected, the firm is forced to hire through the period-two market.)

Proposition 3. Each firm employing a low-ability worker in period one will not desire to hire through a referral; each of these firms hires through the period-two market.

Before proving these propositions, I will briefly describe this equilibrium from the workers' perspective. Those period-two workers with ties to high-ability period-one workers receive wage offers ranging between the market wage, w_{M2}, and some upper-bound referral wage, $\bar{w}_R$. (Obviously, workers possessing more of these ties--those who are well connected--will receive more wage offers and thus higher expected maximum offers.) Those period-two workers possessing no ties to high-ability period-one workers must find employment through the market, earning w_{M2}. As workers tend to know others of the same type, high-ability period-one

workers tend to relay offers to high-ability period-two workers. Those workers remaining on the market are thus mostly low-ability: this "lemons effect" drives down the market wage.

I shall now prove each of the preceding propositions, establishing the jexistence of the specified equilibrium. (Readers interested only in the model's empirical implications may wish to skim the following proofs; relevant qualitative findings are discussed afterward.)

Proof of Proposition 1.

In the equilibrium proposed above, only those period-two workers possessing ties to high-ability period-one workers will receive referrals; period-two workers lacking such ties are forced to find employment through the market. (As all referral wage offers are greater than (or equal to) the market wage, only those workers receiving *no* offers will enter the market.) Given free entry of firms, the period-two market wage equals the expected productivity of those workers left on the market. As high-ability workers tend to refer workers of the same type, those seeking employment through the market are mostly low-ability workers; this "lemons effect" drives the market wage below the average productivity of the population.

More formally, the period-two market wage may be determined via Bayes Rule:

$$w_{M2} \; - \; Pr(H|market)$$

$$\frac{\text{Pr(market|H)} \cdot \text{Pr(H)}}{\text{Pr(market|H)} \cdot \text{Pr(H)} + \text{Pr(market|L)} \cdot \text{Pr(L)}}$$

where Pr(market|H) is the probability that a high-ability worker is on the market (i.e., receives no referral wage offers) and Pr(market|L) is defined analogously for a low-ability worker. Given the expressions for these probabilities (found in Appendix 1), the market wage may be written:

$$w_{M2}(\alpha, r) = \frac{e^{-\alpha r}}{e^{-\alpha r} + e^{-(1-\alpha)r}} \ .$$

Given the above expression, one should note that $\partial w_{M2}/\partial r < 0 \ \forall \ t \in [0,1]$. In words, the market wage falls as the (expected) number of social ties increases. Further, $w_{M2}(\alpha, 0) = 1/2$. The intuition is straightforward: In the absence of social ties, all workers are hired through market; expected productivity (and the market wage) is thus equal to 1/2. But as the number of ties increases, a larger percentage of workers--primarily of high-quality--will be hired through referrals. Thus, the market wage is driven down by this "lemons" effect.[34] Given $r > 0$, the

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[34]This terminology derives from Akerlof's (1970) model of the used-car market. Note that, contrary to Akerlof's result, adverse selection does not result in the "disappearance" of the market: because ties (and thus wage offers) are allocated stochastically, some high-ability workers will fail to receive offers, expected productivity (and thus the market wage) exceeds zero. The parameter r (the probability of possessing a social tie) thus plays much the same role as the parameter μ (the probability cf involuntary exogenous mobility) in Greenwald's (1986) model. As r falls (μ rises), a larger percentage of second-period workers are hired through the market,

market wage is pushed below the population-average productivity:

$$w_{M2}(\alpha, \tau) < 1/2. \quad Q.E.D.$$

Having determined the period-two market wage, I now examine the problem facing firms employing high-ability workers in period one.

Proof of Proposition 2.

Assuming a symmetric equilibrium, each firm employing a high-ability worker in period one plays a mixed strategy, offering each referral wage $w_R \in [w_{M2}, \bar{w}_R]$ with probability $f(w_R)$. In this section, I prove that these firms wish to hire through referrals, earning positive (expected) profits by doing so. Further, I derive an (closed-form) expression for $\bar{w}_R$ and an equation implicitly determining $f(w_R)$. Finally, I demonstrate that no firm profits by deviating from the specified strategy.

Before deriving the strategy played by each firm, it is first helpful to discuss informally the failure of (potential) pure-strategy single-wage equilibria.[35] (The discussion will proceed by contradiction.) First, assume that workers randomize over job offers if receiving more than one (best) offer. (Recall that, due to the stochastic allocation of ties, some workers receive several offers while others receive none.) Now suppose that all firms offer the same wage, w_R^*. But such an equilibrium would fail, as each firm could profit by deviating. If, by offering a wage equal to w_R^*, each firm earns a positive (expected) profit, each will

ameliorating the "lemons" effect and boosting the market wage.

[35] See Butters (1978) for a more formal analysis.

prefer to offer $w_R^* + \epsilon$. By offering an infinitesimally higher wage, a deviating firm attracts a worker *for sure*, thus increasing its profit. Even if w_R^* rose to the point where firms earn zero profit, the equilibrium would still fail: each firm would again prefer to deviate, cutting its offer to some wage (discretely) less than w_R^*. In the absence of alternative offers, a period-two worker would accept the deviating firm's (lower) offer; the firm thus earns a positive (expected) profit.

By similar reasoning, one can show that no two firms will (in equilibrium) offer the same wage with positive probability. (If this were not the case, both firms might offer the same wage to the same worker; each firm thus desires to offer a wage ϵ higher.) Furthermore, the wage offer distribution must contain no gaps (i.e., it must possess positive density over its entire range). (No firm is willing to make an offer discretely higher than any other (potential) offer: by slightly reducing such an offer, the firm would directly increase profits--by reducing wages--while the probability of attracting the referred worker remains unchanged.) In the present model, one should thus expect a mixed- (rather than pure-) strategy equilibrium.[36]

I now derive the symmetric mixed-strategy equilibrium. Conditional

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[36]Some readers may reject the notion that firms should play mixed strategies as this seems to imply that real-world firms make decisions by rolling dice or throwing darts. In defense of mixed-strategy equilibria, I would argue that such equilibria are actually reduced-form versions of pure-strategy Bayesian equilibria where each of the players has "a little" private information. See Tirole's (1988) "Non-Cooperative Game Theory: A User's Manual" for an introduction to the literature on the "purification" of mixed-strategy equilibria.

upon its high-ability period-one worker holding a social tie[37], the firm's profit may be written:

$$E\Pi(w_R) = \Pr(\text{high-ability period-two worker hired}|w_R) \cdot (1 - w_R)$$
$$+ \Pr(\text{low-ability period-two worker hired}|w_R) \cdot (-w_R) \quad .$$

Should the referred worker reject the firm's offer (having received a better one), the firm must hire through the market and receives zero expected profit. (A potential third term in the above expression, involving the probability that no worker accepts the firms offer, is thus equal to zero.) As demonstrated in Appendix 1, the firm's (conditional) expected profit may be written:

$$E\Pi(w_{R1}) = \alpha \cdot e^{-\alpha r(1-F(w_R))} \cdot (1-w_R) + (1-\alpha) \cdot e^{-(1-\alpha)r(1-F(w_R))} \cdot (-w_R)$$

where $F(\cdot)$ is the distribution of wage offers made by other firms.

In general, a player is willing to randomize over actions (i.e., play a mixed strategy) only if each potential action yields the same expected payoff. In the present model, this implies that firms must earn the same expected profit given each wage offered in equilibrium:

$$E\Pi(w_R) = c \quad \forall \ w_R \in [w_{M2}, \bar{w}_R] \quad .$$

To determine this constant (representing the expected profit for a firm

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[37]Recall that social ties are assigned after the referral wage offer is set. As an offer is potentially accepted only if the worker holds a tie, the firm determines it offer under this presumption.

whose high-ability worker possesses a tie), note that the firm could potentially deviate from its specified strategy by offering a wage of w_{M2} (for sure). In this case, the referred worker accepts the firm's offer only if no other offers were received. The firm's expected profit is given by:

$$E\Pi(w_{M2}) = \alpha e^{-\alpha r}(1-w_{M2}) + (1-\alpha)e^{-(1-\alpha)r}(-w_{M2}) - c .$$

Substituting for the period-two market wage, we obtain:

$$c = (2\alpha - 1) / (e^{\alpha r} + e^{(1-\alpha)r}) .$$

Note that (as $\alpha > 1/2$) the constant $c > 0$: firms employing high-ability workers possessing ties earn positive (expected) profits.

Given the preceding expression for c, the distribution $F(\cdot)$ may be determined by setting $E\Pi(w_R)$ equal to c for all potential wage offers w_R:

$$\alpha e^{-\alpha r(1-F(w_R))}(1-w_R) + (1-\alpha)e^{-(1-\alpha)r(1-F(w_R))}(-w_R)$$

$$= (2\alpha - 1) / (e^{\alpha r} + e^{(1-\alpha)r}) \qquad \forall\ w_R \in [w_{M2}, \bar{w}_R] .$$

Unfortunately, this expression does not yield a closed-form solution for $F(w_R)$.[38] But as $F(\bar{w}_R) = 1$ (by definition of $\bar{w}_R$), one can derive an expression for $\bar{w}_R$:

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[38] Models in the "equilibrium price dispersion" literature typically assume only one type of worker (or buyer). In this case, $\alpha = 1/2$ and the left-hand side of the equation reduces to a single term. By taking logs, one obtains a closed-form solution for $F(\cdot)$.

$$\bar{w}_R(\alpha,r) = \alpha - [(2\alpha - 1) / (e^{\alpha r} + e^{(1-\alpha)r})] = \alpha - c .$$

Intuitively, a firm offering a referral wage of $\bar{w}_R$ attracts a referred worker with (conditional) probability one--no other firm bids higher. The firm's expected profit (=c) is thus equal to the expected productivity of its referral (= α) minus the wage paid (= $\bar{w}_R$).

Furthermore, note that $\partial c/\partial r < 0$ (and thus $\partial \bar{w}_R/\partial r > 0$). In words, expected profit (of those firms employing high-ability workers possessing social ties) declines as the (expected) number of ties rises. Again the intuition seems straightforward: If only one firm were able to hire through referral, it could offer the market wage (+ ϵ) and obtain the referred worker for sure; its (expected) profit would be relatively high. But as the number of ties increases, firms face additional competition for referred workers; this competition drives down expected profit and prompts firms to raise their maximum wage offers. Finally, as $\bar{w}_R(\alpha,0) = 1/2$ and $\partial \bar{w}_R/\partial r > 0$, $\bar{w}_R > 1/2$ $\forall \alpha \in (1/2,1]$: the maximum referral wage is greater than the expected (average) productivity of the population.

To prove the second proposition, recall that $c > 0$ given $\alpha > 1/2$: the (unconditional) expected profit for a firm employing a high-ability worker in period one is thus $rc > 0$. As hiring through the market yields zero expected profit, such firms thus desire to hire through referrals. By construction, the equilibrium derived above generates constant expected profits over the range $[w_{M2}, \bar{w}_R]$. Further, no firm would wish to deviate by offering a referral wage outside this range: A lower wage will never be accepted (workers could do better on the market); higher wages directly

reduce profits but do not increase the probability of attracting a worker (as no other offer exceeds $\bar{w}_R$). Thus, no firm gains by deviating from the specified equilibrium. *Q.E.D.*

Finally, I examine the problem faced by firms employing low-ability workers in period one.

Proof of Proposition 3.

As discussed in the previous proof, firms employing high-ability workers wish to hire through referral. Further, all referral wage offers yield the same expected profit: $E\Pi(w_R) = c \quad \forall \ w_R \in [w_{M2}, \bar{w}_R]$. In the proposed equilibrium, firms employing low-ability workers in period one will not wish to hire through referral. But suppose that one such firm deviated. As I demonstrate below, the expected profit of this firm is not constant in $[w_{M2}, \bar{w}_R]$ but is decreasing in w_R. As a result, the firm maximizes expected profit by setting $w_R = w_{M2}$. Even at this wage, however, the firm's expected profit is negative; as hiring through the market guarantees zero expected profit, the firm suffers by deviating.

As shown above, the expected period-two profit of a firm employing a high-ability worker in period one may be written:

$$E\Pi(w_R) = \alpha \cdot e^{-\alpha r(1-F(w_R))} \cdot (1-w_R) + (1-\alpha) \cdot e^{-(1-\alpha)r(1-F(w_R))} \cdot (-w_R) \ .$$

Intuitively, the first half of each expression gives the expected profit

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from hiring a high-ability worker; the second half gives the expected loss from hiring a low-ability worker. Now consider $\partial E\Pi(w_R)/\partial w_R$. As $E\Pi(w_R)$ — c $\forall$ $w_R \in [w_{M2}, \bar{w}_R]$, this partial derivative must be equal to zero. Further, the partial of the second half of this equation is negative: as the referral wage rises, the firm is more likely to obtain a low-ability worker and will pay more when it does. Thus, the partial of the first half of the equation must be positive: the increased likelihood of obtaining a high-ability worker outweighs the higher wage offered.

If a firm employing a low-ability period-one worker hires through referral, its expected profit may be written:

$$E\Pi_L(w_R) = (1-\alpha) \cdot e^{-\alpha r(1-F(w_R))} \cdot (1-w_R) + \alpha \cdot e^{-(1-\alpha)r(1-F(w_R))} \cdot (-w_R) .$$

Note that $E\Pi_L$ is identical to π_1 except for the first term in each half of the expression. Intuitively, if one firm of each type offers w_R to a high-ability worker, both have the same probability of hiring the worker. The two firms differ, however, in the probability that their offers will be received by a high-ability worker. The firm employing a high-ability worker in period one will contact a high-ability period-two worker with probability $\alpha > 1/2$; the firm employing a low-ability period one worker will contact a high-ability worker with probability $1-\alpha < 1/2$.

Now consider the partial derivative $\partial E\Pi_L(w_R)/\partial w_R$. As discussed above, the partial derivative of the first half of the equation will be positive while the partial of the second half is negative. But note that firms employing low-ability period-one workers are relatively more likely

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to make offers to low-ability workers; relatively more weight is put on the second half of the expression. Thus, $\partial E\Pi_L(w_R)/\partial w_R < 0$. In words, expected profit for firms employing low-ability period-one workers is decreasing in the referral wage. As a result, such firms would maximize profits by setting the referral wage equal to the market wage.

But even setting $w_R - w_{M2}$, a firm employing a low-ability period-one worker will not profit by deviating. The expected profit for a firm offering the market wage may be written:

$$E\Pi_L(w_{M2}) = (1-\alpha) \cdot e^{-\alpha\tau} \cdot (1-w_{M2}) + \alpha \cdot e^{-(1-\alpha)\tau} \cdot (-w_{M2}) .$$

Substituting for the market wage, this becomes:

$$E\Pi_L(w_{M2}) = \frac{(1-2\alpha) \cdot e^{-\tau}}{e^{-\alpha\tau} + e^{-(1-\alpha)\tau}}$$

which, as $\alpha > 1/2$, is negative. In words, a firm employing a low-ability period-one worker earns a negative expected profit by hiring through referral; such firms will thus hire through the market. $Q.E.D.$

Having proven the existence of the specified equilibrium, I now briefly restate the model's major implications:

The lower (expected) ability of workers hired through the market is both a cause and an effect of firms' use of employee referrals.

As the market comprises primarily low-ability workers, each firm satisfied with the quality of its current workforce (i.e., each firm employing a high-ability worker) will seek to hire through employee referral. But low-ability workers are over-represented in the market precisely because many high-ability workers have already found jobs through referrals. The causation thus runs in both directions. Stated alternatively, the present analysis required no *a priori* assumption on the relative quality of workers hired through the market; the "lemons" problem arises endogenously.[39]

Wages are determined by the social ties held by workers.

Each worker's wage is determined by both the *number* and *type* of social ties he possesses: those period-two workers with more ties to high-ability period-one workers receive higher (expected) wages. Thus, wages are only *indirectly* a function of ability (as high-ability period-two workers are more likely to possess ties to high-ability period-one workers). Stated somewhat more colloquially, the present model offers some justification for the saying: "It's not *what* you know but *who* you know." Those workers with no social ties to high-ability workers (while possibly holding numerous ties to low-ability workers) are forced to find employment through the market.

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[39]This finding may prove useful to personnel researchers, who have argued that worker ability may vary across hiring channels but have not recognized the potential underlying endogeneity of such variation.

Workers finding employment through the market earn lower wages.

Because high-ability workers are more likely to find jobs through referrals, the market comprises primarily low-ability workers. This lemons effect drives the market wage below the expected (average) productivity of the population. Those without the necessary social ties to high-ability period-one workers will thus receive the (relatively low) market wage. As firms must offer wages at least as great as the market wage to attract referred workers, those hired through referral will earn higher wages. In the present analysis, the (imperfect) competition for referred workers generates wage dispersion.

Firms hiring through referrals earn higher (expected) profits.

Given the free entry of firms and the symmetric (lack of) information on the ability of workers seeking employment through the market, all firms hiring through the market earn zero (expected) profit. But each firm hiring through referral faces only *imperfect* competition for its referred worker. (Stated alternatively, the firm possesses *asymmetric* information on the ability of a worker referred by a current employee.) Given the stochastic nature of social structure (and thus wage offers), the referred worker may fail to receive any alternative offers; the firm could obtain the worker with a bid slightly greater than the market wage. In this way, firms with high-ability period-one employees--who are likely to refer high-ability period-two workers--may thus earn positive (expected) profits.

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As the number of social ties increases, the market lemons problem is
exacerbated. Specifically, as the number of ties increases:

O *the market wage decreases.*
O *the maximum (referral) wage offered increases.*
O *the expected profit of firms hiring through referral decreases.*

As the number of social ties increases (i.e., r rises), a larger
percentage of workers are hired through referral. As this group comprises
primarily high-ability workers, those seeking employment through the
market are primarily low-ability workers. This lemons effect is
exacerbated as more workers are hired through referral; the market wage
falls as the number of social ties rises. Given increased competition for
referred workers (i.e., an increase in the probability that a given worker
receives multiple offers), an increase in the number of social ties also
pushes up the maximum referral wage offered and cuts the expected profit
of firms hiring through referral.

While the last finding states that an increase in the number of
social ties exacerbates the market lemons problem, I have been unable to
analytically derive the effect on *mean* referral wages offered and
(especially) *mean* referral wage accepted. (Knowing that the maximum rises
and the minimum falls says nothing about the average.) In addressing
previous empirical work (or proposing future work), this is unfortunate:
the average wage earned is a convenient (and frequently reported) summary
statistic. But while analytical derivation of this statistic proved
infeasible, it is easily obtained through Monte Carlo simulation of the
present model. As reported in Appendix 2, both the mean referral wage

offered and the mean referral wage accepted rises as r increases. Given α

$= .9$, for example, the mean wage accepted rises from .5 at $r = 0$ to .534

at $r = 1$. (Similarly, the mean wage offered rises from .5 to .511.) To

the previous list of outcomes resulting from an increase in the number of

social ties, we may thus add that the mean wage accepted increases.

Before closing the analysis of the model, two further comments seem

warranted. First, the preceding analysis has ignored the first stage of

the model--the operation of the period-one market. As noted earlier, this

step is relatively uninteresting: as all (potential) firms have symmetric

information on the ability of each worker--knowing only the distribution

of ability in the population--the market clears at a single wage w_{M1}. It

is interesting to note, however, that this wage exceeds the average

(expected) productivity of the population (i.e., $w_{M1} > 1/2$).[40]

The intuition for this result runs as follows: Were the market wage

equal to 1/2, firms would earn zero (expected) profit during the first

period. But those firms which obtain high-ability period-one workers may

subsequently receive a positive profit in the second period. This

expected period-two profit is equal to the probability of obtaining a

high-ability period-one worker multiplied by the (unconditional) expected

profit received by such firms: $E\Pi_2 = (1/2)rc$. The period-one market wage

is thus the sum of expected period-one productivity and expected

period-two profit[41]: $w_{M1} = (1/2)(1+rc)$. Stated alternatively, firms hiring

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[40]In this respect, the first stage of the present model is very similar to
the first stage of Greenwald's (1986) model.

[41]Recall that I have assumed no discounting between periods.

through the period-one market obtain not only a first period worker but also an option to make a referral wage offer to a second period worker; the period-one market wage thus reflects both of these sources of profit.

Second, I wish to consider informally the consequences of permitting more than two worker types. For simplicity, the present analysis has assumed only two types--workers of high and low ability. As a result, all firms employing high-ability workers in period one wish to hire through referral; none of those employing low-ability workers wish to do so. But given a large number (or perhaps continuum) of types, the percentage of firms wishing to hire through referral becomes endogenous: as the number of ties increases (exacerbating the market lemons problem), a larger number of firms would prefer referred workers.

To see this, imagine that each worker's productivity is uniformly (and independently) distributed between zero and one; period-one and period-two workers could then be listed in order of ability. Again assuming that workers tend to refer others similar to themselves, those period-one workers of higher ability (those near the top of the period-one list) would typically know similar period-two workers (those near the top of the period-two list); similar ties would be held by those near the middle or bottom of each list. Given r clc e to zero, the lemons problem is minor: expected productivity of those workers on the market is close to 1/2. Only those firms employing period-one workers with ability greater than 1/2 will thus seek to hire through referral. But as r increases, a larger percentage of higher ability workers will be hired through referrals; the critical ability level determining the hiring channel

preferred by firms falls and more firms will wish to hire through referral. In this way, even "bad" jobs--those occupied by period-one workers of lower ability which subsequently offer relatively low referral wages--may be allocated through referrals rather than the market.[42]

SECTION 5. CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

Previous research examining job search by workers and employee search by firms has highlighted the importance of social networks in the hiring process. After briefly reviewing this literature, I have explored a variety of plausible explanations for the use of employee referrals by both firms and workers. Given the widespread (and apparently purposeful) use of employee referrals, I have argued that social structure must play an key role in determining labor market outcomes. To explore this issue, I have constructed a formal model of the hiring process, explicitly incorporating a simple social structure. As demonstrated above, those workers who are well connected--who possess social ties to those in high-paying jobs--will fare better than those who are poorly connected. Additionally, the model suggests that differences in average applicant ability levels across hiring channels may arise endogenously and predicts that firms hiring through referrals should earn higher profits.

But while the present model elucidates the role played by social

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[42]This result may help explain why even "secondary" jobs in a dual labor market are sometimes be allocated through referral. Note, however, that there will always be lower paying jobs allocated through the market.

networks in the labor market, it is only one step toward a full economic analysis. To indicate directions for future research, I first discuss a number of social problems--the ghetto "underclass", dual labor markets, and discrimination--that may result from the pattern of social ties held by disadvantaged workers. By explicitly incorporating assumptions on social structure, future economic models may yield fresh insights into these problems and may help evaluate alternative policy prescriptions. In discussing labor-market discrimination, I will note specifically how the present model might be extended to address the relevant policy issues. Finally, I argue that future economic analysis might profitably draw upon some of the research already undertaken by social-structure analysts.

As argued by Wilson (1987), the spread of the ghetto "underclass" and its attendant "social dislocations" during the last two decades is partly attributable to the flight of middle-class blacks from the inner city during this period.[43] This middle-class flight has had a variety of effects, placing a financial strain on neighborhood institutions and leaving ghetto youth without necessary role models who demonstrate the viability of education and hard work. But more directly, the loss of the middle-class has severed those social ties connecting ghetto residents to workers currently employed (or alternatively, workers employed in "good" jobs). Restated in the terminology of the present analysis, Wilson is claiming that black inner-city youth lack the necessary social ties to be referred for stable, high-paying jobs. These youth are thus forced to

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[43]The terms "underclass" and "social dislocations" (e.g., unemployment, crime, drug use, etc.) are Wilson's; see Lemann (1986) for a similar analysis of the ghetto underclass.

rely upon more formal hiring channels and--due to the lemons effect--will

often experience underemployment (or perhaps unemployment).[44]

In essence, Wilson's ghetto underclass hypothesis is a special case

of the dual labor markets hypothesis--a disadvantaged group is unable to

obtain "good" jobs.[45] Economists have often rejected the dual labor market

hypothesis, noting the apparent absence of "barriers to entry" prohibiting

those in the secondary sector from finding primary-sector employment;

workers in a competitive labor market are rewarded solely on the basis of

ability.[46] But given the present analysis, I would argue that the lack of

necessary social ties represents an important constraint on labor-market

mobility. Those workers without ties to primary-sector workers will not

be referred for these jobs and must accept secondary-sector jobs (or

unemployment). While this justification of the persistence of dual labor

markets is not original--a similar theme underlies Piore's (1975)

discussion of "mobility chains"[47]--previous analysis has suffered from lack

of formal modelling. Research explicitly incorporating assumptions on

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[44]See Holzer (1986) for recent empirical research on this topic.

[45]See Doeringer and Piore (1971) for a seminal discussion of dual labor
markets.

[46]See Cain (1976) for a critical review of the (early) dual labor markets
literature.

[47]As stated by Piore, "The concept of a mobility chain represents an attempt
to formalize the intuitive notion that socioeconomic movement in our society
is not random, but tends to occur in more or less regular channels. These
channels are such that any given job will tend to draw labor from a limited
and distinct number of other points.... Thus, people in a given job will
tend to be drawn from a limited range of schools, neighborhoods, and types
of family backgrounds; and conversely, people leaving the same school or
neighborhood will tend to move into one of a limited set of employment
situations." (p. 128)

social structure may not only validate previous informal explanations for dual labor markets but may also suggest new explanations for the persistence of such markets. Indeed, dual labor markets might arise from an underlying duality in social structure--a society comprising a "core" group (with many ties between members) and a "peripheral" group (whose members are poorly connected with both "core" group members and each other).

Closely related to the issues of the ghetto underclass and dual labor markets, labor-market discrimination may also result from the patterns of social ties held by various groups of workers. Economists have tradition- ally found discrimination difficult to explain, given that a competitive labor market should eliminate wage differentials not based upon produc- tivity. In Becker's (1957) classic analysis, wage differentials persist only if all employers share a preference for discrimination. But the present analysis suggests that discrimination need not depend upon "tastes"; wage differentials may instead result from the inability of firms to screen applicants perfectly. To circumvent the adverse selection problems posed by formal hiring channels, firms may hire largely through employee referrals. If a firm employs mostly white males, its referred applicants are likely also to be white males; women and minorities, lacking the necessary social ties, are thus excluded from the firm's recruitment process.[48]

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[48] In models of "statistical discrimination", the expected ability of minority workers is *assumed* to be measured with higher variance than the expected ability of majority workers. Given risk-averse firms (Aigner and Cain (1977)) or endogenous educational investment (Lundberg and Startz (1983)), this yields lower average wages for minority workers even though the

In formalizing this explanation for discrimination, the central issue

is not the random occurrence of poor labor-market outcomes (perhaps

attributable to "bad luck") but rather the persistence of these outcomes

in the long run. A dynamic extension of the present model is thus

required to address this issue. Additionally, the screening process must

be more fully developed, allowing firms to obtain more precise estimates

of applicant ability by incurring additional hiring costs. Given this

extended model, two issues may be addressed. First, starting with a

situation in which "good" jobs are held by white males, one could

determine the conditions under which discrimination will persist. Once a

small number of women or minorities are hired through formal channels, a

firm might recruit others through referral and discrimination might

subside. But such a result will inevitably depend on the assumed social

structure--the pattern of social ties held by women and minorities.[49]

Second, assuming that discrimination will persist given certain social

structures, one might then determine the optimal government intervention.

A social planner might require firms to hire through formal

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underlying ability distribution is assumed the same for both groups. One
criticism of such models is the lack of any real-world "test" which measures
minority workers' ability with higher variance. The present analysis,
however, suggests that social ties might play this role: given that workers
know others similar to themselves, employers possess a more precise estimate
of the ability of referred workers than the ability of those on the market.

[49] As noted above, the stochastic social structure of the present analysis--
implying an unrealistically well-mixed and transient population--was chosen
primarily for simplicity; one could substitute non-interacting social groups
(with varying average productivity levels across groups) or some alternative
(more complex) topological structure. Graph theory--largely an extension of
probability theory--may provide the necessary mathematical apparatus; see
Harary, et. al. (1965) and Frank (1981).

channels--thereby incurring higher screening costs than individually optimal--in order to maximize social welfare. Again, the socially-optimal affirmative-action legislation is likely to depend upon the parameters of the underlying social structure.

Given the likely importance of assumptions on social structure for the persistence of discrimination and for policy determination, the theoretical analysis proposed above must be combined with empirical investigation to determine which hypothesized social structure best represents reality. While such research seems far-removed from that typically undertaken by economists, much of the groundwork has been laid--and indeed some tentative empirical analysis performed--by social-structure analysts. In a seminal work, White (1970) argues that labor market data may be helpful in mapping social relationships. Assuming jobs are filled through employee referral, "vacancy chains"--comprising an initial worker who leaves the labor force and each subsequent worker who fills the vacancy left by the previous worker--will trace a "branch" of the overall social structure.[50]

But more generally, other network analysts have suggested alternative methods for studying social structure.[51] In social settings with a

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[50]As noted by Granovetter (1981), White's "vacancy chains" are conceptually similar to Piore's "mobility chains" discussed above; an explicit comparison (or, better, synthesis) of these ideas seems worthwhile. As a viable research strategy, the analysis of vacancy chains suffers from obvious data limitations: a single firm's personnel data would be insufficient; surveys seem problematic. White, focusing on intra-firm mobility, restricts his empirical analysis to the movement of clergy in three large denominations-- vacancy chains thus remain inside the same "firm."

[51]The discussion of this paragraph follows Wellman's (1988) informal survey; see Rapoport (1963) and Burt (1980) for more technical surveys.

relatively small number of participants (e.g., a junior high-school or a fraternity house) it is possible--using the "whole network" approach--to construct an exhaustive list of all social ties and analyze the resulting "adjacency matrices".[52] But to analyze the structure of large societies--an undertaking more relevant for our present purposes--sociologists have utilized "egocentric" approaches often incorporating "snowball" survey techniques.[53] In this type of analysis, one might ask (randomly selected) respondents to list their friends and acquaintances (or, alternatively, those they might potentially refer for employment); each of those identified would then receive a similar survey. Like White's vacancy chains, these surveys could potentially trace out "branches" of social structure and might be used to determine the topology of social networks. At present it is unclear whether the statistics currently used to summarize existing social structures--often designed to determine the "cliquishness" of a given social network[54]--are useful for elucidating the issue at hand. But future research--guided explicitly by an interest in labor market outcomes--should generate the necessary

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[52]"Adjacency matrices"--square, binary matrices recording the social ties between individuals--may be analyzed through various topological, graph-theoretic, and spatial techniques; see Marsden and Laumann (1984) for a review.

[53]See Burt (1980) for discussion and references.

[54]Much of the mathematical foundation for network analysis derives from epidemiology; a standard issue is the rate at which a disease (or alternately, rumor) will travel though a given population. As diseases spread more slowly through "cliquish" societies (consisting of densely interconnected subgroups with few ties between groups), researchers have developed a variety of statistics designed to capture this effect; see Rapoport (1963).

statistics.

APPENDIX 1: DETERMINATION OF REFERRAL WAGE OFFERS

In this appendix, I examine the problem facing firms in determining the referral wage offered (or whether to make such an offer at all). To begin, I shall examine the problem faced by some firm i whose first-period worker possesses a tie to a high-ability period-two worker. (Given the assumptions made in the text, each firm does not know (ex ante) whether its worker possesses a tie and (if so) which type of period-two worker is known; I assume the firm possesses this knowledge for expositional purposes only.) As described in the text, the (high-ability) worker accepts firm i's offer only if he has received no higher offer. Formally,

Pr(H accepts firm i's offer w_{Ri})

$$= \text{Pr(H receives no higher offer, } w_{Rj} \text{, from firm j } \forall \, j \neq i)$$

From the worker's perspective, each of the offers are allocated independently; the right-hand side of this equation may thus be rewritten:

Pr(H accepts firm i's offer w_{Ri})

$$= \prod_{j \neq i} \text{Pr(H receives no higher offer, } w_{Rj} \text{, from firm j)}$$

$$= \prod_{j \neq i} [1 - \text{Pr(H receives an offer } w_{Rj} > w_{Ri} \text{ from firm j)} \, .$$

The probability that firm j offers a wage $w_{Rj} > w_{Ri}$ to the given

high-ability worker is in turn the product of two independent
probabilities:

Pr(H receives an offer $w_{Rj} > w_{Ri}$ from firm j)

= Pr(firm j makes offer to H) $\cdot$ Pr($w_{Rj} > w_{Ri}$) .

Given the assumptions on social structure and the equilibrium strategies
of other firms, this becomes:

$$\text{Pr(H receives an offer } w_{Rj} > w_{Ri} \text{ from firm j)} = \left[\frac{\alpha r}{N}\right] \cdot (1 - F(w_{Ri}))$$

for all firms j employing a high-ability worker in period one. (Recall
that firms employing low-ability workers will make no offers through
referrals.) Given this expression (and noting the symmetry of the
equilibrium), we obtain:

$$\text{Pr(H accepts firm i's offer } w_{Ri}) = \left[1 - \left[\frac{\alpha r}{N}\right](1 - F(w_{Ri}))\right]^{(N-1)}$$

For large N, the right-hand side is well-approximated:

$$\text{Pr(H accepts firm i's offer } w_{Ri}) \approx e^{-\alpha r(1 - F(w_{Ri}))} .$$

(This approximation becomes exact as N → ∞; I thus assume it holds exactly
in further computations.) By similar calculation, one obtains the
probability that firm i's offer is accepted by a given low-ability worker:

$$\text{Pr(L accepts firm i's offer } w_{Ri}) \approx e^{-(1-\alpha)r(1 - F(w_{Ri}))} .$$

Note that, conditional upon the offer being received by a given worker, high-ability workers are less likely to accept any offer $w_{Ri} < \bar{w}_R$ as these workers tend to receive more offers. (For $w_{Ri} = \bar{w}_R$, however, note that both probabilities are equal to one--no firm offers a higher wage and the worker always accepts.)

As a period-two worker finds employment through the market only if he receives no offers, $\Pr(\text{market}|H) = \Pr(H \text{ accepts offer } w_{M2})$ and $\Pr(\text{market}|L) = \Pr(L \text{ accepts offer } w_{M2})$. As $F(w_{M2}) = 0$, we obtain:

$$\Pr(\text{market}|H) = e^{-\alpha r} \quad \text{and}$$

$$\Pr(\text{market}|L) = e^{-(1-\alpha)r} \quad .$$

To derive the (conditional) expected profit for a firm employing a high-ability worker in period one (as required for Proof 2), note that the probability of hiring a high-ability period-two worker (again conditional upon the first-period worker holding a tie) is the product of two independent probabilities:

$$\Pr(\text{high-ability period-two worker hired}|w_{Ri})$$

$$= \Pr(\text{offer made to H}) \cdot \Pr(H \text{ accepts}|\text{offer made}) \quad .$$

From the preceding analysis and the second assumption on social structure, this becomes:

$$\Pr(\text{high-ability period-two worker hired}|w_{Ri}) = \alpha \cdot e^{-\alpha r (1 - F(w_{Ri}))} \quad .$$

Similarly, the (conditional) probability of hiring low-ability worker is:

$$\Pr(\text{low-ability period-two worker hired}|w_{Ri}) = (1-\alpha) \cdot e^{-(1-\alpha)r(1 - F(w_{Ri}))} \quad .$$

In the event that no worker is hired, firm i must hire through the market and earns zero expected profit. The firm's (conditional) expected profit may thus be written:

$$\mathcal{E}\Pi(w_{Ri}) = \alpha \cdot e^{-\alpha r(1-F(w_{Ri}))} \cdot (1-w_{Ri})$$
$$+ (1-\alpha) \cdot e^{-(1-\alpha)r(1-F(w_{Ri}))} \cdot (-w_{Ri}) \; .$$

APPENDIX 2: MONTE CARLO SIMULATIONS

In this appendix, I report the results of Monte Carlo simulations of the model developed in Section 4. These simulations serve to establish the effect of an increase in the (expected) number of social ties on mean referral-wage offered and mean referral-wage accepted. Each simulation was run given 1000 workers of each type; the figures given below are the average values computed over 100 simulations. (The computer simulation program was written in BASIC and is available from the author upon request.)

		Average quality of workers on period-two market	Expected Quality[1]	Average Wage Offered	Average Wage Accepted
$\alpha = .9$	$r = 0.0$	.5000	.5000	.5000	.5000
	$r = 0.2$	.4608	.4601	.5006	.5016
	$r = 0.4$	.4204	.4207	.5023	.5064
	$r = 0.6$	.3814	.3823	.5047	.5137
	$r = 0.8$	.3447	.3452	.5084	.5242
	$r = 1.0$	.3098	.3100	.5131	.5364

$\alpha = .8$	$r = 0.0$	.5000	.5000	.5000	.5000
	$r = 0.2$	.4700	.4700	.5001	.5007
	$r = 0.4$	.4411	.4403	.5002	.5027
	$r = 0.6$	.4111	.4110	.5002	.5057
	$r = 0.8$	.3825	.3823	.5000	.5097
	$r = 1.0$	.3549	.3543	.5000	.5148

[1]Expected quality is derived via Bayes Rule as discussed in the proof of Proposition 1.

REFERENCES

Aigner, Dennis J. and Glen G. Cain. (1977) "Statistical Theories of Discrimination in Labor Markets." *Industrial and Labor Relations Review* 30: 175-187.

Akerlof, George A. (1970) "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics* 84: 488-500.

Barron, John M. and John Bishop. (1985a) "Extensive Search, Intensive Search and Hiring Costs: New Evidence on Employer Hiring Activity." *Economic Inquiry* 23: 363-382.

------ and William C. Dunkelberg. (1985b) "Employer Search: The Interviewing and Hiring of New Employees." *Review of Economics and Statistics* 67: 43-52.

Becker, Gary S. (1957) *The Economics of Discrimination.* Chicago: University of Chicago Press, 1957.

Boorman, Scott A. (1975) "A Cominatorial Optimization Model for Transmission of Job Information through Contact Networks." *Bell Journal of Economics* 6: 216-49.

Breaugh, James A. (1981) "Relationships Between Recruiting Sources and Employee Performance, Absenteeism, and Work Attitudes." *Academy of Management Journal* 24: 142-147.

------ and Rebecca B. Mann. (1984) "Recruiting Source Effects: A test of Two Alternate Explanations." *Journal of Occupational Psychology* 57: 261-267.

Butters, Gerard R. (1977) "Equilibrium Distributions of Sales and Advertising Prices." *Review of Economic Studies* 44: 465-91.

Burdett, K. and K. Judd. (1983) "Equilbrium Price Dispersion," *Econometrica* 51: 955-70.-70.

Burt, Ronald S. (1980) "Models of Network Structure" in Inkeles, et. al., eds., *Annual Review of Sociology, Vol. 6, 1980.* Palo Alto, CA: Annual Reviews, Inc. nc.

Cain, Glen C. (1976) "The Challenge of Segmented Labor Market Theories to Orthodox Theory: A Survey." *Journal of Economic Literature* 14: 1215-1257.

Campbell, Karen E. and Peter V. Marsden. (1988) "Recruitment and

Selection Processes: The Organizational Side of Job Searches" in Ronald L. Breiger, ed., *Social Mobility and Social Structure*, New York: Cambridge University Press, forthcoming.

Corcoran, Mary, et. al. (1980) "Information and Influence Networks in Labor Markets" in Greg Duncan and James Morgan, eds., *Five Thousand American Families: Patterns of Economic Progress, Vol 7.* Ann Arbor: Institute for Social Research.

Datcher, Linda. (1985) "The Impact of Informal Networks on Quit Behavior." *Review of Economics and Statistics* 65: 491-495.495.

Decker, Phillip J. and Edwin T. Cornelius III (1979) "A Note on Recruiting Sources and Job Survival Rates." *Journal of Applied Psychology* 64: 463-646.

Delany, John. (1988) "Social Networks and Efficient Resource Allocation: Computer Models of Job Vacancy Allocation through Contacts" in Wellman and Berkowitz, eds., *Social Structures: A Network Approach.* New York: Cambridge Univerity Press, 430-451.

Doeringer, Peter and Michael Piore (1971) *Internal Labor Markets and Manpower Analysis.* Lexington, MA: D. C. Heath and Company.

Frank, Ove. (1981) "A Survey of Statistical Methods for Graph Analysis" in Samuel Leinhardt, ed., *Sociological Methodology 1981*, San Francisco: Jossey-Bass.

Gannon, Martin J. (1972) "Sources of Referral and Employee Turnover." *Journal of Applied Psychology* 55: 226-228.

Granovetter, Mark S. (1973) "The Strength of Weak Ties." *American Journal of Sociology* 78: 1360-80.-80.

------ (1974) *Getting a Job: A Study of Contacts and Careers.* Cambridge: Harvard University Press.

------ (1981) "Toward a Sociological Theory of Income Differences." in Ivar Berg, ed., *Sociological Perspectives on Labor Markets.* New York: Academic Press.ess.

------ (1985) "Economic Action and Social Structure: The Problem of Embeddedness." *American Journal of Sociology* 91: 481-510.

------ (1987) "The Sociological and Economic Approaches to Labor Market Analysis: A Social Structural View." in George Farkas and Paula England, eds., *Industries, Firms, and Jobs: Sociological and Economic Approaches.* Plenum Press, forthcoming.ing.

Greenwald, Bruce C. (1986) "Adverse Selection in the Labour Market." *Review of Economics Studies* 53: 325-47.

Harary, Frank, Robert Norman and Dorwin Cartwright. (1965) *Structural Models: An Introduction to the Theory of Directed Graphs.* New York: John Wiley and Sons, Inc.

Heneman, Herbert G., et. al. (1980) *Personnel/Human Resource Management.* Homewood, IL: Richard D. Irwin, Inc.

Hill, Raymond E. (1970) "New Look at Employee Referrals." *Personnel Journal* 49: 144-148.

Holzer, Harry J. (1986) "Informal Job Search and Black Youth Unemployment." *American Economic Review* 77: 446-52.

------ (1987) "Hiring Procedures in the Firm: Their Economic Determinants and Outcomes." NBER Working Paper 2185, March.

------ (1988) "Search Method Use by Unemployed Youth." *Journal of Labor Economics* 6: 1-20.

Hosios, Arthur J. (1984) "Unemployment and Recruitment with Heterogeneous Labor." *Journal of Labor Economics* 3: 175-87.

Jovanovic, Boyan. (1979) "Job Matching and the Theory of Turnover." *Journal of Political Economy* 87: 972-990.

Katz, Lawrence. (1986) "Efficiency Wage Theories: A Partial Evaluation" *NBER Macroeconomics Annual 1986.* Cambridge, MA: MIT Press.

Lang, Kevin. (1987) "Persistent Wage Dipersion and Involutary Unemployment." Boston University, mimeo.

Lemann, N. (1986) "The Origins of the Underclass," *Atlantic* 257 (June): 31-61.

Lundberg, Shelly J. and Richard Startz. (1983) "Private Discrimination and Social Intervention in Competitive Labor Markets." *American Economic Review* 73: 340-347.

Marsden, Peter V. and Edward O. Laumann. (1984) "Mathematical Ideas in Social Structure Analysis." *Journal of Mathematical Sociology* 10: 271-294.

Mortensen, Dale. (1986) "Job Search and Labor Market Analysis" in O. Ashenfelter and R. Layard, eds., *Handbook of Labor Economics.* Amsterdam: North-Holland.

Myers, Charles A. and George P. Shultz. (1951) *The Dynamics of a Labor Market.* New York: Prentice-Hall, Inc.

Nadel, S. F. (1957) *The Theory of Social Structure*. London: Cohen and West.

Parnes, Herbert S. (1954) *Research on Labor Mobility*. New York: Social Science Research Council.

Piore, Michael J. (1975) "Notes for a Theory of Labor Market Stratification," in Edwards, Richard C., et. al., eds., *Labor Market Segmentation*. Lexington, MA: D. C. Heath and Company.

Rapoport, Anatol. (1963) "Mathematical Models of Social Interaction" in Luce, et. al., eds., *Handbook of Mathematical Psychology, Vol. 2*. New York: John Wiley and Sons, Inc.

Rees, Albert. (1966) "Information Networks in Labor Markets." *American Economic Review* 56: 559-66.

------ and George Schultz. (1970) *Workers in an Urban Labor Market*. Chicago: University of Chicago Press.

Reid, Graham L. (1972) "Job Search and the Effectiveness of Job-Finding Methods." *Industiral and Labor Relations Review* 25: 479-495.

Saloner, Garth. (1985) "The Old Boys' Network as a Screening Mechanism." *Journal of Labor Economics* 3: 255-67.

Schwab, Donald P. (1982) "Recruiting and Organizational Participation." in Kendrith M. Rowland and Gerald R. Ferris, eds., *Personnel Management*. Boston: Allyn and Bacon, Inc.Inc.

Sheppard, Harold L., and A. Harvey Belitsky. (1966) *The Job Hunt: Job-Seeking Behavior or Unemployed Workers in a Local Economy*. Baltimore: John Hopkins Press.

Stigler, George J. (1962) "Information in the Labor Market." *Journal of Political Economy* (supplement) 70: 94-105.

Taylor, M. Susan and Donald W. Schmidt. (1983) "A Process-Oriented Investigation of Recruitment Source Effectiveness." *Personnel Psychology* 36: 343-354.

Tirole, Jean. (1988) *Theory of Industrial Organization*. Cambridge, MA: MIT Press.

Ullman, J. C. (1966) "Employee Referrals: Prime Tool for Recruiting Workers." *Personnel* 43: 30-35.

Waldman, Michael. (1984) "Job Assignments, Signalling, and Efficiency." *Rand Journal of Economics* 15: 255-267.

Wanous, John P. (1980) *Organizational Entry*. Reading, MA: Addison-Wesley

Publishing Company.

Wellman, Barry. (1988) "Structural Analysis: From Method and Metaphor to
    Theory and Substance." in Wellman and Berkowitz, eds., *Social
    Structures: A Network Approach*. New York: Cambridge University
    Press.

------ and S. D. Berkowitz, eds. (1988) *Social Structures: A
    Network Approach*. New York: Cambridge University Press.

White, Harrison. (1970) *Chains of Opportunity: System Models of
    Mobility in Organizations*. Cambridge, MA: Harvard University Press.

Wial, Howard. (1988) "The Transition from Secondary to Primary
    Employment: Jobs and Workers in Ethnic Neighborhood Labor Markets."
    unpublished Ph.D. dissertation, MIT.

Wilson, William J. (1967) *The Truly Disadvantaged*. Chicago, University
    of Chicago Press.

Essay 3:

REINTERPRETING MODELS OF STATISTICAL DISCRIMINATION:
EMPLOYEE REFERRALS AND THE ROLE OF SOCIAL STRUCTURE

Attempting to explain gender and racial earnings differentials, economists have proposed various models of "statistical discrimination." But while such models may possess explanatory power, they rely upon the questionable assumption that the productivity of workers from disadvantaged groups is measured with (exogenously) greater error. In this paper, I propose a reinterpretation of this literature. In the model below, firms may hire workers through either employee referral or an impersonal market and then assign jobs endogenously. Social structure plays a key role in the analysis, determining both wages and job assignment. If social structure varies across demographic groups, mean wages will also differ even if ability is identically distributed. In this way, the differential measurement error presupposed by statistical discrimination models may be reinterpreted as variation in social structure.

In Montgomery (1988), I discussed the importance of employee referrals in the hiring process and formally examined the consequences for wage determination. Building upon the observation that workers tend to refer others like themselves, I demonstrated that in equilibrium firms employing high-ability workers prefer to hire through employee referral while those employing low-ability workers will not. Social structure determined the proportion of workers recruited through each hiring channel and the wages offered. Specifically, an increase in "network density"--the average number of social ties held by individuals--resulted in greater wage dispersion. But in that analysis, all jobs were identical; given full employment, total output and mean wages were fixed.

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In this paper, I extend my previous analysis by adding endogenous job assignment. Each firm now possesses two technologies--one more ability-sensitive than the other--and may each period choose its technology based on the expected ability of its worker. In this setting, social structure influences not only wage dispersion but also economic efficiency and mean wages. By improving job matching, increases in network density or ability "inbreeding bias"--the extent to which workers know others of the same ability level with greater-than-random probability--may increase both efficiency and mean wages. If network density or inbreeding bias varies across demographic groups, wage differentials will arise even in the absence of ability differences; some groups may thus suffer from "statistical discrimination."

The paper proceeds as follows: In Section 1, I review the statistical discrimination literature, noting the major problems associated with these models, and offer an alternative model of wage determination. In Section 2, I present the basic model in which all workers belong to the same group (i.e., are observationally equivalent). If few social ties connect members of different groups, this model can explain differences in wage determination across groups; Section 3 examines the implications for wage dispersion, economic efficiency, and mean wages. In Section 4, I discuss the effects of cross-group social ties on wage determination. Section 5 contains a brief conclusion.

SECTION 1. STATISTICAL DISCRIMINATION RECONSIDERED

As labor economists are well aware, blacks and females earn significantly less than white males. In 1981, the ratio of median wages for black males to white males was .70 while the female/male ratio was .60.[1] While these ratios have risen slightly in recent years, they remain well below unity. Further, large "wage gaps" for both blacks and females remain even after controlling for education, training, experience, turnover, and other productive characteristics. While these wage gaps might result entirely from variations in unobserved ability, many would argue that discrimination plays a major role.

Economists have generally offered two explanations for labor-market discrimination. In the first, employers, co-workers, and/or customers possess a "taste" for discrimination.[2] In the second, discrimination results not from preferences but from imperfect information on worker ability.[3] More concretely, models of statistical discrimination generally assume that a worker's expected ability (and thus wage) is determined by a pre-employment test. For some (typically unspecified) reason, this test measures the ability of workers from disadvantaged groups (e.g., women or blacks) with greater error. Consequently, employers place less weight on the "test score" obtained by these workers; wages for women and blacks are thus closer to the mean ability of the group.

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[1] See Ehrenberg and Smith (1982) and Cain (1986).

[2] See Becker's (1957) classic analysis.

[3] See Phelps (1972), Aigner and Cain (1977), Lundberg and Startz (1983), and Lang (1987).

This standard model of statistical discrimination presents two problems. First, while blacks (for instance) with above-average test scores earn less than whites with identical scores, blacks with below-average scores should earn more than similar whites. As the average wage of each group is equal to the average productivity of the group, it is questionable whether this model generates discrimination in any meaningful sense. This problem is, however, well known and has been addressed in the literature. If employers are risk-averse (Aigner and Cain (1977)) or educational investment is endogenous (Lundberg and Startz (1983)), the difference between mean wages for blacks and whites is greater than the difference in mean ability.[4]

The second (and arguably more important) problem with the standard model of statistical discrimination concerns the plausibility of its major underlying assumption: what real-world phenomenon corresponds to corresponds to the test given workers and why should it measure the ability of some workers with greater error? In defense of this assumption, Lang (1986, 1987) has argued that white male employers may have difficulty evaluating the non-verbal language used by women and blacks and thus measure the productivity of these workers with greater error.[5] But while this argument offers some justification for the existence of a differentially noisy test, it implies (as Lang explicitly

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[4]Additionally, such differences may be generated by truncating the sample below some (common) reservation wage or by assuming job matching; the model below incorporates the latter.

[5]In the latter paper, Lang also argues that firms are likely to possess more precise estimates on the ability of workers hired through referral; he thus foreshadows the present analysis.

notes) segregated firms (in which black supervisors evaluate black workers, etc.) rather than wage discrimination.

In this paper, I present an alternative model of wage determination which generates predictions similar to those of the statistical discrimination literature while free of the problems discussed above. In this model, firms may hire workers through either employee referral or an impersonal market. Wages are dispersed for workers hired through referral; workers hired through the market earn a lower wage due to a lemons effect. As demonstrated in my earlier paper, an increase in network density--the average number of social ties held by individuals--exacerbates the market lemons problem and generates greater wage dispersion. Thus, as in models of statistical discrimination, wage dispersion may differ across groups even if ability is distributed identically. But rather than presuppose a differentially noisy pre-employment test, the present analysis generates differences in wage dispersion through variations in social structure.[6]

Further, given endogenous job assignment, the mean as well as the dispersion of wages may differ across groups. In the model below, firms may assign a new employee to a task which is either more or less ability-sensitive.[7] Under perfect information, high-ability workers would be assigned to the former; low-ability workers should be assigned to the

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[6] This discussion implicitly assumes no cross-group social ties; this assumption will be discussed in Section 4.

[7] See Roy (1951) for the seminal analysis of job matching and its effect on income distribution; recent labor-market research incorporating similar assumptions includes Waldman (1984) and Gibbons and Katz (1987).

latter. But in a labor market characterized by adverse selection, workers are assigned to jobs based on their ex-ante expected ability. As demonstrated below, an increase in either network density or inbreeding bias may improve job matching, thus increasing output and the mean wage. Groups with lower network density or inbreeding bias may thus suffer from "statistical discrimination."

SECTION 2. THE MODEL

I now present a formal model of the hiring process in which firms hire workers through employee referral or an impersonal market and then assign jobs based upon workers' expected abilities. In this section, I assume that all workers belong to a single group (e.g., white males); while ability varies across workers, all are observationally equivalent to employers. Within this framework, I demonstrate how the parameters of social structure and group composition affect economic efficiency and mean wages. If the population comprises several (observationally distinct) groups which are not linked by inter-group ties, this model is sufficient to discuss differ- ences in wages across groups; implications for statistical discrimination are discussed in the next section. More generally, inter-group social ties will exist and additional analysis is required; this is discussed in the Section 4.

I propose a two-period model[8] with the following assumptions on

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[8] I further assume (merely for simplicity) no discounting between periods.

workers and firms:

Workers

O Each worker lives one period.

O There are N workers in each period; N is assumed large.

O Workers may be of two types, either high or low ability;
   a fraction $p \in [0,1]$ are of high ability in each period.
   High-ability workers possess ability $\eta = 1$ while low-ability
   workers possess ability $\eta = 0$.

O Workers are observationally equivalent; employers are
   uncertain of the ability of any particular worker.

Firms

O Each firm may employ (at most) one worker.

O Firms may choose (each period) either of two technologies:

   On job 1, output is equal to ability: $y = \eta$. Thus, high-
   ability workers produce 1 unit while low-ability workers
   produce 0 units.

   On job 2, output is less ability sensitive: $y = \eta y_H + (1-\eta)y_L$
   where $0 < y_L < y_H < 1$. Thus, high-ability workers produce y_H
   units while low-ability workers produce y_L units.

O A firm's profit in each period is equal to the output of
   its worker minus the wage paid. (Product price is
   exogenously determined and normalized to unity.)

O Each firm must set wages before learning the productivity of its
   worker.

O Firms are free to enter the market in either period.

These assumptions are identical to those made in Montgomery (1988)

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with two exceptions.[9] First, the proportion of high-ability workers in the population (i.e., p) is allowed to vary; this permits comparison of populations with different mean ability levels. Second, firms now possess two technologies, with job 1 more ability-sensitive than job 2. As discussed below, this extension implies that social structure will (in general) affect economic efficiency and mean wages.

Next, I make the following assumptions on social structure:

Social Structure

O Each period-one worker knows at most one period-two worker, possessing a social tie with probability $r \in [0,1]$.

O For each period-one worker holding a tie, the specific period-two individual known is selected stochastically through a two-stage process:

   O In the first stage, the period-two worker's type is chosen. A high-ability period-one worker knows a high-ability period-two worker with probability α_H (and thus knows a low-ability period-two worker with probability $1-\alpha_H$). A low-ability period-one worker knows a low-ability period-two worker with probability α_L.

   O In the second stage, the specific period-two worker is chosen randomly from those of the appropriate type. Conditional upon holding a tie, a high-ability period-one worker thus knows any given period-two worker of his own type with probability α_H/pN. (The remaining probabilities are derived analogously.)

The social structure is summarized by its "network density" and "inbreeding biases." Network density, r, is the average number of social ties held by period-one workers. The parameter α_H is the conditional

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[9] As noted in that paper, these assumptions resemble those of other labor-market adverse-selection models. See, for example, Greenwald (1986) and Waldman (1984).

probability of a high-ability period-one worker knowing someone of his own type. (The parameter α_L is defined analogously for a low-ability worker.) As discussed in Montgomery (1988), the pattern of social ties often displays an inbreeding bias: individuals within the same group possess a greater-than- random probability of knowing each other. More specifically, one might expect an inbreeding bias between workers of similar ability (interpreted broadly to include work habits as well as intelligence or skill).[10] Given an ability inbreeding bias, workers know others of their own type with greater-than-random probability: $\alpha_H > p$ and $\alpha_L > 1-p$.

Finally, I assume the following timing:

Timing

 (1) Firms hire period-one workers through the market; this clears at a wage w_{M1}.

 (2) Firms assign workers to jobs and production occurs; each firm learns the productivity of its worker.

 (3) If a firm desires to hire through employee referral, it sets a "referral" wage; firm i may thus set a wage w_{Ri}.

 (4) Social ties are assigned.

 (5) Each period-one worker possessing a social tie relays his firm's wage offer (w_{Ri}) to his period-two acquaintance.

 (6) Each period-two worker compares wage offers received, either accepting one or waiting to find employment through the market.

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[10]Rees and Shultz (1970), for example, note that employers satisfied with the quality of their workforce often prefer hiring through referral as workers tend to know others like themselves. (p. 203)

(7) Those period-two workers with no offers (or refusing all offers) go on the market; this clears at a wage w_{M2}.

(8) Firms assign workers to jobs and production occurs.

Except for the addition of job assignment, the timing is identical to that in Montgomery (1988). As in that paper, the timing comprises three major stages. First, each firm hires a period-one worker through the market, assigns him to a job, and learns his ability. Second, knowing the ability of its current worker, each firm determines whether it wishes to hire through referral. If so, it sets a referral wage and this offer is (potentially) relayed to an acquaintance of the worker. Finally, those period-two workers who receive no offers are forced to find employment through the period-two market.[11]

The solution to the model is characterized by the following four propositions:

Proposition 1. Each firm employing a high-ability worker in period one will wish to hire through an employee referral. Assuming a symmetric equili- brium, each of these firms plays a mixed strategy, selecting a referral wage offer $w_R \in [w_{M2}, \bar{w}_R]$ from a distribution $F(w_R)$ where w_{M2} is the period-two market wage.

Proposition 2. Each firm employing a low-ability worker in period one will not desire to hire through employee referral.

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[11]As referral wages offers always exceed the period-two market wage, no worker receiving such an offer will (in equilibrium) find employment through the market.

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Proposition 3. Expected ability varies across hiring channels. As the result of a "lemons effect", expected ability is lower for worker hired through the market than for workers hired through referral. Further, the expected ability of a referred worker is increasing in the referral wage accepted. More formally:

$$\eta_M \; < \; \eta_R(w_{M2}) \; < \; \eta_R(\bar{w}_R)$$

where η_M is the expected ability of a worker hired through the market and $\eta_R(w_R)$ is the expected ability of a referred worker hired at w_R.

Proposition 4. Each firm assigns its worker to the job which yields greater expected output. Given the critical expected ability level, $\eta^*(y_H, y_L)$, four cases may arise:

(1) If $\eta^* < \eta_M$, all workers are assigned to job 1.

(2) If $\eta_M < \eta^* < \eta_R(w_{M2})$, workers hired through the market are assigned to job 2 while referred workers are assigned to job 1.

(3) If $\eta_R(w_{M2}) < \eta^* < \eta_R(\bar{w}_R)$, only some referred workers (and no market workers) are assigned to job 2.

(4) If $\eta_R(\bar{w}_R) < \eta^*$, all workers are assigned to job 2.

Before discussing these propositions, I will briefly describe this equilibrium from the workers' perspective. Those period-two workers with ties to high-ability period-one workers receive wage offers ranging between the market wage, w_{M2}, and some upper-bound referral wage, $\bar{w}_R$. Those period-two workers possessing no ties to high-ability period-one workers must find employment through the market, earning w_{M2}. As workers

tend to know others of the same type, high-ability period-one workers tend to relay offers to high-ability period-two workers. Those workers remaining on the market are thus mostly low-ability: this "lemons effect" drives down the market wage.[12] As workers recruited through the market are of lower expected ability, they will be placed on the more ability-sensitive job (i.e., job 1) if and only if all referred workers are placed on this job. As the expected ability of referred workers varies directly with the wage offered, it is possible for only some of these workers to be assigned to job 1.

I shall now prove each of the preceding propositions, establishing the existence of the specified equilibrium. (Those readers interested only in the model's qualitative implications may wish to skip to the next section.)

Proof of Proposition 1.

Given a symmetric equilibrium, each firm employing a high-ability worker in period one plays a mixed strategy, offering each referral wage $w_R \in [w_{M2}, \bar{w}_R]$ with probability $f(w_R)$.[13] I now prove that these firms wish to hire through referrals, earning positive (expected) profits by doing so. Further, I derive an equation implicitly determining $F(w_R)$ and a (closed-form) expression for $\bar{w}_R$. Finally, I demonstrate that no firm

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[12]This terminology derives from Akerlof's (1970) adverse-selection model of the used-car market.

[13]See Montgomery (1988) for a discussion of the non-existence of pure-strategy equilibria. As noted in that paper, this portion of the model follows Butters (1977).

profits by deviating from the specified strategy.

Consider the problem faced by a firm which employs a high-ability worker in period one.[14] If the firm hires through referral and places the new worker on job one, its expected profit, $\pi_1(w_R)$, can be written:

$$\pi_1(w_R) = \alpha_H \cdot \text{Pr}(\text{accepts } w_R | H) \cdot (1-w_R) + (1-\alpha_H) \cdot \text{Pr}(\text{accepts } w_R | L) \cdot (-w_R)$$

where $\text{Pr}(\text{accepts } w_R | H)$ is the probability that a high-ability period-two worker accepts an offer of w_R conditional upon being referred and $\text{Pr}(\text{accepts } w_R | L)$ is the analogous probability for a low-ability worker. Given the assumptions on social structure, the firm's high-ability period-one worker refers a high-ability worker with probability α_H and a low-ability worker with probability $(1-\alpha_H)$. Should the referred worker reject the firm's offer (having received a better one), the firm must hire through the market and receives zero expected profit. (A potential third term in each of the above expressions, involving the probability that no worker accepts the firm's offer, is thus equal to zero.)[15] If the firm

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[14] For convenience, this proof examines the firm's problem conditional upon its first-period worker holding a social tie; the unconditional problem is exactly the same except that expected profits are multiplied by r.

[15] Perhaps more intuitively, π_1 could be written:

$$\pi_1(w_R) = \text{Pr}(\text{firm hires through referral}) \cdot (\eta_R(w_R)-w_R)$$

But as $\text{Pr}(\text{firm hires through referral})$ is equal to:

$$\alpha_H \cdot \text{Pr}(\text{accepts } w_R | H) + (1-\alpha_H) \cdot \text{Pr}(\text{accepts } w_R | L)$$

and (as shown below) $\eta(w)$ is equal to:

places the worker on job 2, its expected profit, $\pi_2(w_R)$, can be written:

$$\pi_2(w_R) = \alpha_H \cdot \Pr(\text{accepts } w_R|H) \cdot (y_H - w_R) + (1-\alpha_H) \cdot \Pr(\text{accepts } w_R|L) \cdot (y_L - w_{Ri})$$

The firm will, of course, assign the worker to the job that yields the highest expected output; expected profit can thus be written:

$$E\Pi(w_R) = \max \{ \pi_1(w_R) , \pi_2(w_R) \}$$

In general, a player is willing to randomize over actions (i.e., play a mixed strategy) only if each potential action yields the same expected payoff. In the present model, this implies that firms must earn the same expected profit given each wage offered in equilibrium:

$$E\Pi(w_R) = c \quad \forall \ w_R \in [w_{M2}, \bar{w}_R]$$

To determine this constant, note that the firm could potentially deviate from its specified strategy by offering a wage of w_{M2} (for sure); the referred worker accepts the firm's offer only if no other offers were received. In this case, $c = E\pi(w_{M2})$, which may be rewritten:

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$\alpha_H \cdot \Pr(\text{accept } w_R|H) / \Pr(\text{firm hires through referral})$,

π_1 reduces to the expression given in the text.

$$c = \max \{ \pi_1(w_{M2}) , \pi_2(w_{M2}) \} .$$

Given the free entry of firms, the period-two market wage is given by:

$$w_{M2} = \max \{ \eta_M , \eta_M \cdot y_H + (1-\eta_M) \cdot y_L \}$$

Given the expression for η_M (found below in the proof of proposition 3), one can easily show that:

$$w_{M2} = \eta_M \quad \Rightarrow \quad \pi_1(w_{M2}) > 0 \qquad \text{and}$$

$$w_{M2} = \eta_M \cdot y_H + (1-\eta_M) \cdot y_L \quad \Rightarrow \quad \pi_2(w_{M2}) > 0$$

given $\alpha_H > p$. Thus, the (conditional) expected profit of firms recruiting through referral is positive: $c > 0$.

As demonstrated in Appendix 1,

$$\Pr(\text{accepts } w_R | H) = \exp[-\alpha_H r(1-F(w_{R1}))] \qquad \text{and}$$

$$\Pr(\text{accepts } w_R | L) = \exp[-(1-\alpha_H)(p/(1-p))r(1-F(w_{R1}))]$$

where $F(\cdot)$ is the distribution of wage offers. As stated above, the distribution $F(\cdot)$ is determined implicitly by the expression: $E\Pi(w_R) = c$ $\forall$ $w_R \in [w_{M2}, \bar{w}_R]$. Writing this out,

$$\max \left\{ \alpha_H \cdot \exp[-\alpha_H r(1-F(w_{R1}))] \cdot (1-w_R) \right.$$

$$+ (1-\alpha_H) \cdot \exp[-(1-\alpha_H)(p/(1-p))r(1-F(w_{R1}))] \cdot (-w_R) \quad,$$

$$\alpha_H \cdot \exp[-\alpha_H r(1-F(w_{R1}))] \cdot (y_H - w_R)$$

$$\left. + (1-\alpha_H) \cdot \exp[-(1-\alpha_H)(p/(1-p))r(1-F(w_{R1}))] \cdot (y_L - w_R) \right\}$$

$$- c \quad \forall \; w_R \in [w_{M2}, \bar{w}_R].$$

While this expression does not yield a closed-form solution for $F(w_R)$, one may obtain such an expression for $\bar{w}_R$. As $F(\bar{w}_R) = 1$ (by definition of $\bar{w}_R$),

$$\bar{w}_R(\alpha, r) = \max \{ \alpha_H , \alpha_H y_H + (1-\alpha_H)y_L \} - c \; .$$

Intuitively, a firm offering a referral wage of $\bar{w}_R$ attracts a referred worker with probability one as no other firm bids higher. The firm's expected profit is thus equal to the expected productivity of its referral (on the preferred job) minus the wage paid.

As shown above, firms employing high-ability workers in period one earn positive expected profits by hiring through referral. As hiring through the market would generate zero expected profits (as discussed below), firms will thus wish to hire the referral. By construction, the equilibrium derived above generates constant expected profits over the

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range $[w_{M2}, \bar{w}_R]$. Further, no firm would wish to deviate by offering a referral wage outside this range: A wage below w_{M2} will never be accepted (as workers would always refuse to accept); higher wages directly reduce profits but do not increase the probability of attracting a worker (as no other offer exceeds $\bar{w}_R$). Thus, no firm gains by deviating from the specified equilibrium. *Q.E.D.*

Proof of Proposition 2.

As discussed in the previous proof, firms employing high-ability workers wish to hire through referral. Further, all referral wage offers yield the same expected profit: $E\Pi(w_R) = c \quad \forall \ w_R \in [w_{M2}, \bar{w}_R]$. In equilibrium, firms employing low-ability workers in period one will not wish to hire through referral. But suppose that one such firm deviated. As I demonstrate below, the expected profit of this firm is not constant in $[w_{M2}, \bar{w}_R]$ but is decreasing in w_R. As a result, the firm maximizes expected profit by setting $w_R = w_{M2}$. At this wage, however, its expected profit is negative; as hiring through the market guarantees zero expected profit, the firm suffers by deviating.

As shown above, the expected period-two profit of a firm employing a high-ability worker in period one may be written:

$$E\Pi(w_R) = \max\{ \ \pi_1(w_R) \ , \ \pi_2(w_R) \ \}$$

where:

$$\pi_1(w_R) = \alpha_H \cdot \Pr(\text{accepts } w_R|H) \cdot (1-w_R) + (1-\alpha_H) \cdot \Pr(\text{accepts } w_R|L) \cdot (-w_R) \quad \text{and}$$

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$$\pi_2(w_R) = \alpha_H \cdot Pr(\text{accepts } w_R|H) \cdot (y_H - w_R) + (1-\alpha_H) \cdot Pr(\text{accepts } w_R|L) \cdot (y_L - w_R) \quad .$$

Intuitively, the first half of each expression gives the expected profit from hiring a high-ability worker; the second half gives the expected loss from hiring a low-ability worker. Now consider $\partial E\Pi(w_R)/\partial w_R$. As $E\Pi(w_R) = c \ \forall \ w_R \in [w_{M2}, \bar{w}_R]$, this partial derivative must be equal to zero. Further, whether $E\Pi(w_R)$ equals $\pi_1(w_R)$ or $\pi_2(w_R)$, the partial of the second half of this equation is negative: as the referral wage rises, the firm is more likely to obtain a low-ability worker and will pay more when it does. Thus, the partial of the first half of the equation must be positive: the increased likelihood of obtaining a high-ability worker outweighs the higher wage offered.

If a firm employing a low-ability period-one worker hires through referral, its expected profit may be written:

$$E\Pi_L(w_R) = \max\{ \ \pi_{L1}(w_R) \ , \ \pi_{L2}(w_R) \ \}$$

given:

$$\pi_{L1}(w_R) = (1-\alpha_L) \cdot pr(\text{accepts } w_R|H) \cdot (1-w_R) + \alpha_L \cdot pr(\text{accepts } w_R|L) \cdot (-w_R)$$

where $\pi_{L1}(w_R)$ is the expected profit if the worker is assigned to job 1 and $\pi_{L2}(w_R)$ is the analogous profit level given the worker is assigned to job 2. Note that π_{L1} is identical to π_1 except for the first term in each half of the expression. Intuitively, if one firm of each type offers w_R

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to a high-ability worker, both have the same probability of hiring the worker. The two firms differ, however, in the probability that their offers will be received by a high-ability worker. The firm employing a high-ability worker in period one will contact a high-ability period-two worker with probability α_H; the firm employing a low-ability period one worker will contact a high-ability worker with probability $1-\alpha_L < p < \alpha_H$.

Now consider the partial derivative $\partial E\Pi_L(w_R)/\partial w_R$. Given the discussion above, whether $E\Pi_L(w_R)$ equals $\pi_{L1}(w_R)$ or $\pi_{L2}(w_R)$, the partial derivative of the first half of the equation will be positive while the partial of the second half is negative. But note that firms employing low-ability period-one workers are relatively more likely to make offers to low-ability workers; relatively more weight is put on the second half of the expression. Thus, $\partial E\Pi_L(w_R)/\partial w_R < 0$. In words, expected profit for firms employing low-ability period-one workers is decreasing in the referral wage. As a result, such firms would maximize profits by setting the referral wage equal to the market wage.

But even setting $w_R = w_{M2}$, a firm employing a low-ability period-one worker will not profit by deviating. Given the previous discussion, the expected profit for such a firm may be written:

$$E\Pi_L(w_R) \;=\; \max \{\; \pi_{L1}(w_{M2}) \;,\; \pi_{L2}(w_{M2}) \;\} \quad .$$

Given free entry of firms, the market wage is:

$$w_{M2} = \max \{ \eta_M, \eta_M \cdot y_H + (1-\eta_M) \cdot y_L \} .$$

Given the expressions for these probabilities (found below), one can easily show that:

$$E\Pi_L(w_R) = \pi_{L1}(w_{M2}) \quad \Rightarrow \quad \eta_M > \pi_{L1}(w_{M2}) \qquad \text{and}$$

$$E\Pi_L(w_R) = \pi_{L2}(w_{M2}) \quad \Rightarrow \quad \eta_M \cdot y_H + (1-\eta_M) \cdot y_L > \pi_{L2}(w_{M2})$$

given $\alpha_L > 1-p$. In words, a firm employing a low-ability period-one worker earns a negative expected profit by hiring through referral; such firms will thus hire through the market. *Q.E.D.*

Proof of Proposition 3.

In the equilibrium proposed above, only those period-two workers possessing ties to high-ability period-one workers will receive wage offers through referral. Given an ability inbreeding bias, these workers are likely to be high-ability workers themselves. Thus, a relatively large proportion of the workers hired through the market are of low ability; this "lemons effect" implies that the expected ability of workers hired through the market is lower than the expected ability of workers hired through referral. Further, the expected ability of a worker hired through referral is increasing the wage offered. I now examine these

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expected abilities more formally.

The expected ability of a worker hired through the market is simply the probability that he is of high ability; this may be obtained via Bayes Rule:

$$\eta_M = \Pr(H|market) = \frac{\Pr(market|H) \cdot p}{\Pr(market|H) \cdot p + \Pr(market|L) \cdot (1-p)} \quad .$$

In equilibrium, a worker enters the market if he receives no offers through referral. As demonstrated in Appendix 1, $\Pr(market|H) = \exp[-\alpha_H r]$ while $\Pr(market|L) = \exp[-(1-\alpha_H)(p/(1-p))r]$. Given $\alpha_H > p$, $\Pr(market|H) < \Pr(market|L)$. Thus $\eta_M < p$; a "lemons effect" drives the proportion of high-ability workers on the market below the proportion of high-ability workers in the population. Writing out the above expression,

$$\eta_M = \frac{p \cdot \exp[-\alpha_H r]}{p \cdot \exp[-\alpha_H r] + (1-p) \cdot \exp[-(1-\alpha_H)(p/(1-p))r]} \quad .$$

A firm which successfully hires a worker through referral may similarly use Bayes Rule to calculate its worker's expected ability:

$$\eta_R(w_R) = \Pr(H|\text{accepted } w_R)$$

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$$- \ \frac{\text{Pr(accepted } w_R|H) \cdot \alpha_H}{\text{Pr(accepted } w_R|H) \cdot \alpha_H + \text{Pr(accepted } w_R|L) \cdot (1-\alpha_H)} \quad .$$

Recall that Pr(accepted $w_R|H$) is the probability that a high-ability worker accepts an offer w_R conditional upon being referred; he is referred with probability α_H. As shown in Appendix 1, Pr(accepted $w_R|H$) $-$ $\exp[-\alpha_H r(1-F(w_R))]$ while Pr(accepted $w_R|L$) $-\exp[-(1-\alpha_H)(p/(1-p))r(1-F(w_R))]$. Given $\alpha_H > p$, Pr(accepted $w_R|H$) $<$ Pr(accepted $w_R|L$) . Intuitively, low-ability workers receive fewer referral offers and are thus more likely to accept at any given wage. Making the appropriate substitutions, $\eta_R(w_R)$ is equal to:

$$\frac{\alpha_H \cdot \exp[-\alpha_H r(1-F(w_R))]}{\alpha_H \cdot \exp[-\alpha_H r(1-F(w_R))] + (1-\alpha_H) \cdot \exp[-(1-\alpha_H)(p/(1-p))r(1-F(w_R))]} \quad .$$

It is easily shown that $\partial\eta_R(w_R)/\partial w_R > 0$ given $\alpha_H > p$; the expected ability of a referred worker is increasing in the referral wage. Further, $\eta_R(\bar{w}_R) - \alpha_H$. As no firm offers more than $\bar{w}_R$, the worker always accepts such an offer. Finally, consider the expected ability of a referred worker hired at the market wage. As $(1-F(w_{M2})) - 0$, we have:

$$\eta_R(w_{M2}) \ - \ \frac{\alpha_H \cdot \exp[-\alpha_H r]}{\alpha_H \cdot \exp[-\alpha_H r] + (1-\alpha_H) \cdot \exp[-(1-\alpha_H)(p/(1-p))r]} \quad .$$

Note that this expression is identical to the above expression for η_M

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except that p has been replaced by α_H. As $\alpha_H > p$, $\eta_R(w_{M2}) > \eta_M$; expected ability is higher for referred workers.   *Q.E.D.*

Proof of Proposition 4.

As described earlier, a firm may assign its worker to either of two jobs. On job 1, output is equal to η ; on job 2, output is equal to $\eta \cdot y_H + (1-\eta) \cdot y_L$. The firm is thus indifferent concerning job assignment when the worker's expected ability is:

$$\eta^*(y_H, y_L) \; - \; y_L \; / \; [1 \; - \; (y_H \; - \; y_L)] \quad .$$

If the expected ability of a new hire exceeds η^*, he is assigned to job 1; otherwise, he is assigned to job 2. Given $\eta_M < \eta_R(w_{M2}) < \eta_R(\bar{w}_R)$ as demonstrated in the preceding proof, variation in η^* yields the four cases outlined in Proposition 4.   *Q.E.D.*

SECTION 3.   IMPLICATIONS FOR WAGES AND EFFICIENCY

The previous section established the equilibrium of the basic model in which all workers belong to the same group (i.e., are observationally equivalent). I now examine how the social structure of this group affects economic efficiency and mean wages. In the absence of cross-group social ties, this analysis is sufficient for examination of wage differences

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across groups; I discuss implications for statistical discrimination below. As shown in the next section, the major qualitative predictions of this analysis are preserved when the model is generalized to permit cross-group social ties.

The present analysis is greatly simplified by the fact that total output over both periods is equal to total wages; economic efficiency, defined as total output per worker, is thus equal to the mean wage. To see this, recall that firms hiring through referral in period two earn an expected profit equal to c. (Further, given N is large, c also represents the average profit earned ex-post by such firms.) As firms hire through referral only if their high-ability workers possess social ties (and firms hiring through the market earn zero expected profit), the average profit earned by period-two firms is prc. But given the expectation of positive period-two profits (and the free entry of firms), period-one wages are bid up beyond expected period-one output: firms purchase not only output in the present period but also an "option" to make a referral wage offer. Moreover, this option value is exactly prc. Thus, period-two profits are (in the aggregate) offset by period-one losses; total output over both periods equals total wages.

As in Montgomery (1988), an increase in r increases wage dispersion by driving down the market wage and pushing up the maximum wage paid. But in that model, all jobs were identical; as all workers were employed, economic efficiency (and thus the mean wage) was fixed. In contrast, variations in social structure in the present model affect not only wage dispersion but also mean wages. Each firm in the present model possesses

two technologies; increases in r or α_H may increase efficiency (and thus mean wages) by improving job matching. More formally, I offer:

Proposition 5. In cases (1) and (4) described in Proposition 4, all workers are assigned to the same job regardless of hiring channel; small changes in social structure will have no effect on efficiency. But in cases (2) and (3), an increase in α_H increases economic efficiency by increasing the probability that a referred worker is of high ability. In case (2), an increase in r increases efficiency by increasing the absolute number of workers referred; a change in r has no effect on efficiency in case (3).

Proof of Proposition 5.

Economic efficiency, defined as total output per worker, may be written:

$$\phi(\alpha_H, r, p) = p \; [\; pr(\text{job } 1|H) \cdot 1 \; + \; pr(\text{job } 2|H) \cdot y_H \;]$$

$$+ \; (1-p) \; [\; pr(\text{job } 1|L) \cdot 0 \; + \; pr(\text{job } 2|L) \cdot y_L \;]$$

where $\Pr(\text{job } 1|H)$ is the probability that a high-ability worker is assigned to job 1 and the remaining probabilities are defined analogously. As noted in the proposition 4, there are two cases in which all workers are assigned to the same job: in case (1), $\eta^* < \eta_M$ and all workers are assigned to job 1; in case (4), $\eta_R(\bar{w}_R) < \eta^*$ and all workers are assigned to job 2. If either of these conditions holds, job assignment does not vary across hiring channels and a marginal change in either α_H or r will have no effect on efficiency.

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In case (2), however, $\eta_M < \eta^* < \eta_R(w_{M2})$; market workers are assigned to job 2, referred workers are assigned to job 1, and a marginal increase in either α_H or r will increase efficiency. To see this, note that efficiency in case (2) may be written:

$$\phi(\alpha_H, r, p) = p \ [1 - \Pr(\text{market}|H) \cdot (1-y_H)] + (1-p) \ \Pr(\text{market}|L) \cdot y_L$$

As $\partial\Pr(\text{market}|H)/\partial\alpha_H < 0$ while $\partial\Pr(\text{market}|L)/\partial\alpha_H > 0$, both halves of the expression for efficiency are increasing in α_H; thus $\partial\phi/\partial\alpha_H > 0$. To demonstrate that efficiency increases in r, note that the above expression may be rewritten:

$$\phi(\alpha_H, r, p) = p \cdot \{1 - \exp[-\alpha_H r](1-y_H)\} + (1-p) \cdot \{\exp[-(1-\alpha_H)(p/(1-p))r]y_L\} \ .$$

Taking a derivative with respect to r yields:

$$\partial\phi/\partial r = p \cdot \{\alpha_H \cdot \exp[-\alpha_H r] \cdot (1-y_H) + (1-\alpha_H) \cdot \exp[-(1-\alpha_H)(p/(1-p))r] \cdot (-y_L) \ \} \ .$$

But notice that this is simply:

$$\partial\phi/\partial r = p \cdot (\ \pi_1(w_{M2}) - \pi_2(w_{M2}) \ )$$

which, as all referred workers are assigned to job 1, must be positive.

Finally, consider case (3) in which $\eta_R(w_{M2}) < \eta^* < \eta_R(\bar{w}_R)$; only some referred workers are assigned to job 1. In this case, workers are

assigned to job 1 only if they receive a referral wage offer greater than w_R^* where w_R^* is defined by $\eta_R(w_R^*) = \eta^*$. Efficiency may be written:

$$\phi(\alpha_H, r, p) = p \cdot [1 - \Pr(\text{accept } w_R^*|H) \cdot (1-y_H)] + (1-p) \cdot \Pr(\text{accept } w_R^*|L) \cdot y_L \quad .$$

As above, $\partial\Pr(\text{accept } w_R^*|H)/\partial\alpha_H < 0$ while $\partial\Pr(\text{accept } w_R^*|L)/\partial\alpha_H > 0$; both halves of the above expression are increasing in α_H and thus $\partial\phi/\partial\alpha_H > 0$.

Economic efficiency does not, however, vary with r. To see this, note that expected ability may be written:

$$\eta_R(w_R^*) = \frac{\alpha_H \cdot \exp[-\alpha_H r(1-F(w_R^*))]}{\alpha_H \cdot \exp[-\alpha_H r(1-F(w_R^*))] + (1-\alpha_H) \cdot \exp[-(1-\alpha_H)(p/(1-p))r(1-F(w_R^*))]}$$

which, after some algebraic manipulation, becomes:

$$r(1-F(w_R^*)) = [(1-p)/(\alpha_H - p)] \cdot \ln[\alpha_H(1-\eta^*)/(1-\alpha_H)\eta^*] \quad .$$

Note that the right-hand-side of this expression does not move in r. Through substitution, we find that $\partial\Pr(\text{accepts } w_R^*|H)/\partial r = \partial\Pr(\text{accepts } w_R^*|L)/\partial r = 0$ whenever w_R^* exists (i.e., whenever only some referred workers are assigned to job 1); economic efficiency is unaffected by network density in case (3). *Q.E.D.*

Before discussing implications of this proposition for wage differences across groups, I now offer some intuition for the preceding

results. In general, increases in r or α_H increase economic efficiency by improving job matching. As all workers are assigned to the same job in cases (1) and (4), marginal changes in social structure have no effect on job matching and efficiency is thus constant. In cases (2) and (3), however, an increase in α_H implies that referred workers are more likely of high ability and those on the market are more likely of low ability. Stated differently, firms hiring through referral make fewer type II errors: such firms are less likely to accept a low-ability referred worker under the null hypothesis that he is a high-ability worker.

An increase in r increases economic efficiency in case (2) but has no effect in case (3). To gain intuition for this result, consider the following experiment: After all referral offers have been made and workers have (irrevocably) chosen employers, one additional referral wage offer is made. (Imagine that one high-ability worker "forgot" to inform his employer of a social contact until other firms had already made offers.) This additional offer will be accepted only if the worker receiving it received no offers previously (i.e., he would have been hired through the period-two market). The expected ability of this "marginal" referred worker is thus $\eta_R(w_{M2})$. In case (2), $\eta_R(w_{M2}) > \eta^*$; transferring this marginal worker from job 2 (his assignment if hired through the market) to job 1 (his assignment if referred) thus increases his expected output and economic efficiency overall. In case (3), $\eta_R(w_{M2}) < \eta^*$; the marginal worker is assigned to job 2 and economic efficiency is unchanged.

As discussed in Section 1, models of statistical discrimination generate differences in mean wages across groups by presupposing that the

productivity of some groups is measured with greater error. In contrast, the present analysis suggests that such differences result instead from variations in social structure. Given the questionable nature of the differential measurement error assumption, I propose that the standard model of statistical discrimination be reinterpreted as a reduced-form version of a more complete model in which employers utilize multiple hiring channels and social structure varies across groups. But little is gained from this reinterpretation if it simply replaces one questionable assumption with another. Thus, we must ask whether variations in social structure could plausibly account for perceived labor-market discrimination. More concretely, is network density and/or inbreeding bias in fact smaller for blacks and women?

For simplicity, the preceding model assumed complete labor-market participation. But in general, some fraction of workers will be either unemployed or out of the labor force. When searching for a job, the probability that a worker is referred depends not on his *overall* network density--i.e., the total number of individuals known--but on his *effective* network density--i.e., the number of *employed* individuals known. Thus, even if overall network density was constant across groups (as seems a sensible first approximation), effective network density will vary across groups. Moreover, as a comparatively smaller percentage of both women and blacks are employed, these groups will have lower effective network densities.[16]

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[16]One could formally obtain this result from the above model by assuming that some (exogenous) fraction of workers in each period do not work but still generate or hold social ties. Alternatively, one could simply redefine

The present analysis thus predicts that members of groups with higher unemployment and/or lower labor-market participation levels should receive fewer referrals and consequently (due to decreased job matching) lower mean wages. Empirically, many studies have found that both blacks and women are less likely to be hired through referral.[17] Further, the literature on the underclass often stresses that inner-city youth lack social ties to employed workers.[18] If effective network density is further redefined as the number of social ties held to employed workers *in the same occupation or industry*, blacks or women in occupations or industries dominated by white males will also have lower effective network densities. Finally, blacks and women might have lower absolute network densities as the result of historical barriers to association.[19]

Reasons for variation across groups in inbreeding bias--the extent to which workers know others of their own type with greater-than-random probability--are less obvious. Stated in terms of the model, it is unclear why α_H should vary holding p fixed. One should note, however, that $\partial\alpha_H/\partial p$ is likely greater than zero.[20] As a result, a decrease in the

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network density in the model to be effective network density as defined here.

[17]See, for example, Rees and Shultz (1970), Corcoran, Datcher, and Duncan (1980) and Holzer (1986,1987).

[18]See Wilson (1987).

[19]Bartlett and Miller (1985) show that female executives belonging to private clubs or on boards of directors earn higher wages.

[20]As argued by Blau (1977) and Blau and Schwartz (1984), the pattern of social relationships is determined partially by opportunities for contact. As the proportion of high-ability workers falls, *all* workers are thus less likely to know a high-ability worker.

proportion of high-ability workers increases the number of type II errors made by firms hiring through referral; job matching is reduced and mean wages fall. In this way, efficiency declines for "sociological" as well as "economic" reasons.[21] I have assumed throughout the present discussion that the proportion of high-ability workers does not vary across groups. But if fewer members of a group are of high-ability, mean wages will decline as a result of both effects discussed above. Indeed, a decline in the proportion of high-ability workers could generate an even larger decline in efficiency and mean wages. In this way, controlling for mean ability, women or blacks might earn lower wages than white males.

SECTION 4. SOCIAL TIES ACROSS GROUPS

In the previous section, I implicitly assumed that no social ties existed between members of different groups. Given this simplification, one could compare the effects of variations in social structure across groups by simply performing comparative statics on the wages of a single group. But as cross-group social ties obviously exist, I now generalize the model to permit such ties. Given the complexity of this extended

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[21]More formally, the change in efficiency due to a change in the proportion of high-ability workers may be decomposed via the chain rule:

$$d\phi/dp \;=\; \partial\phi/\partial p \;+\; (\partial\phi/\partial\alpha_H)(\partial\alpha_H/\partial p)$$

where the first term is the "economic" effect and the second is the "sociological" effect. As established above, $\partial\phi/\partial\alpha_H$ is positive; the "sociological" effect thus exacerbates the "economic" effect.

model, I shall examine a special case in which inbreeding bias is fixed across groups. In brief, the major qualitative predictions of the preceding analysis are preserved: holding inbreeding bias fixed, the group possessing higher network density has greater wage dispersion and a higher mean wage. But in contrast to the preceding analysis, the condition that total output across both periods equals total wages does not hold for each group individually; total wages paid to the group with higher network density exceeds total output. Additionally, I show that economic efficiency increases as the inbreeding bias between groups decreases.

Before generalizing the above model, one might wish to consider whether cross-group ties are empirically relevant. Indeed, if only a small percentage of all social ties are between groups, the previous analysis can be defended on these grounds. While evidence in support of an ability inbreeding bias is largely anecdotal, sociologists have collected empirical evidence on inbreeding biases between demographic groups. Marsden (1988), examining General Social Survey data on pairs of individuals who "discuss important matters", reports significant inbreeding biases between individuals of the same race/ethnicity, religion and (to a lesser extent) sex, age, and education level. Given the strength of inbreeding biases along racial lines, little is lost in the present analysis by assuming these ties away completely.[22] Inbreeding bias

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[22]Two caveats should be attached to this statement. In a dynamic setting, inter-racial ties--even if limited in number--might play a crucial role in reducing segregation and statistical discrimination. Additionally, if all social ties are bidirectional, the number of ties from blacks to whites must (tautologically) be equal to the number of ties from whites to blacks. If blacks are a minority of the population, inter-racial ties will necessarily represent a larger fraction of the social ties held by blacks; even if

by sex, however, is less strong; a more general analysis may offer additional insight. For convenience, I have thus chosen to label the two groups in the analysis below as male and female.

In contrast to the model developed in Section 2, I now assume two observationally distinct groups. While firms cannot determine the ability of workers before hiring (and setting wages), they do observe whether the worker is (for instance) male or female. As a result, wage determination will (in general) vary across groups; each group is characterized by its market wage and distribution of referral wage offers.[23] Unfortunately, by doubling the number of groups, the model becomes four times more complex. In the basic model, one must consider interaction between workers of high and low ability; permitting two observationally distinct groups implies interaction between workers in four group-ability cells. Moreover, one must consider inbreeding biases by group as well as ability. To limit the number of parameters under consideration, I thus will thus examine only one special case of the more general model.

More specifically, I assume the following changes in the basic model:

Workers

 O Workers belong to two observationally distinct groups: males and
  females. There are 2N workers in each group in each period; N is
  assumed large.

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inter-racial ties have little effect on white wages, they may have a significant impact on black wages. See Rytina and Morgan (1982) for sociological implications of this tautology.

[23] As firms employ only one worker, a public policy requiring each firm to pay all its workers the wage regardless of group would be meaningless. If firms did employ multiple workers, however, such a policy would likely create segregated firms rather than eliminate wage differences across groups.

O Workers are of two types: high-ability and low-ability. One half of each group (i.e., N workers) are of each type in each period.

Social Structure

O Each period-one worker knows at most one period-two worker. Males possess a social tie with probability r_M; females possess a tie with probability r_F where $0 \leq r_F < r_M \leq 1$.

O For each period-one worker holding a tie, the specific period-two individual known is selected stochastically through a three-stage process:

O In the first stage, the period-two worker's ability level is chosen. Conditional upon holding a tie, each worker knows another of the same ability level with probability α (and thus knows a worker of the other ability level with probability $1-\alpha$).

O In the second stage, the period-two worker's group is chosen. Conditional upon holding a tie, each worker knows another of the same group with probability β (and thus knows another from the other group with probability $1-\beta$).

O In the final stage, the specific period-two worker known is chosen randomly from among those of the appropriate ability level and group. Conditional upon holding a tie, a worker thus knows any given period-two worker of the same group and ability level with probability $\alpha\beta/N$; the remaining probabilities are derived analogously.

(The remaining assumptions on workers, firms and timing remain unchanged.) Note especially that the degree of inbreeding bias by both ability and gender is assumed constant across groups. (Given positive inbreeding biases, I assume α, $\beta > 1/2$.) Network density, however, is assumed greater for males than for females; as discussed above, one may interpret this as variation in *effective* network density.

In Section 2, I characterized the equilibrium of the model in a series of propositions and then formally proved each. While I could

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proceed similarly here, the analysis of the previous section is qualitatively unchanged by the addition of cross-group ties. In brief, period-one firms employing high-ability workers from either group prefer to hire through referrals; period-one firms employing low-ability workers do not. As network density of males exceeds that of females, the market wage for males is lower; referral wages for each group are dispersed between the group market wage and the group upper-bound referral wage; the maximum wage paid to males exceeds that paid to females. Given increased job matching, the mean wage for males is at least as great as the mean wage for females. Finally, an increase in α, r_M, or r_F may increase efficiency. (See Appendix 2 for a formal analysis of the extended model and discussion of these results.)

But the extended model serves as more than a robustness check on the previous analysis--it also offers the additional insights summarized by the following two propositions:

Proposition 6. If cross-group ties exist, total group output across the two periods is not equal to total group wages; this condition holds only for the population as a whole. Moreover, wages exceed output for the group with higher network density while output exceeds wages for the group with lower network density.

Proposition 7. Suppose all workers (regardless of group) hired through referral are assigned to job 1 while all workers hired through the market are assigned to job 2. A decrease in the group inbreeding biases, β, increases economic efficiency and thus total wages.

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Proof of Proposition 6.

As discussed in Section 3, firms hiring through the period-one market purchase not only period-one output but also an option to make a referral wage offer. The option value earned by males hired through the period-one market, OV_M, may be written:

$$OV_M \; = \; r_M \cdot [\beta c_M + (1-\beta)c_F]$$

where c_M is the expected profit earned by firms hiring a male through referral (conditional on its period-one worker holding a tie to a male) and c_F is the analogous (conditional) expected profit earned on females. The analogous option value earned females, OV_F, may be written:

$$OV_F \; = \; r_F \cdot [(1-\beta)c_M + \beta c_F] \quad .$$

Given the free entry and risk neutrality of firms, total profit across both periods is equal to zero; the profit earned on referred workers in the second period is exactly offset by the option premia paid to period-one workers. But the condition that total output equals total wages holds only for the population as a whole--not for each group individually. The average profit earned on referred males may be written:

$$\Pi_M \; = \; [\beta r_M + (1-\beta)r_F] \cdot c_M$$

while the average profit earned on referred males may be written:

$$\Pi_F \; = \; [(1-\beta)r_M + \beta r_F] \cdot c_F \quad .$$

Thus,

$$OV_M - \Pi_M - \Pi_F - OV_F - (1-\beta) [r_M c_F - r_F c_M] \ .$$

As $r_M > r_F$ and (as discussed in Appendix 2) $c_F > c_M$, this expression is greater than zero; total wages exceeds total output for men while total output exceeds total wages for women. $Q.E.D.$

Proof of Proposition 7.

Suppose that all referred workers (regardless of group) are assigned to job 1 while all workers hired through the market are assigned to job 2. (This is a generalization of case 2 in proposition 4 above.) Economic efficiency, again defined as total output per worker, may be written:

$$\phi(\alpha,\beta,r_F,r_M) - (1/4) \left\{ 1 - Pr(market|MH)\cdot(1-y_H) + Pr(market|ML)\cdot y_L \right.$$
$$\left. + 1 - Pr(market|ML)\cdot(1-y_H) + Pr(market|FL)\cdot y_L \right\}$$

where $Pr(market|MH)$ is the probability that a high-ability male worker is hired through the market and the remaining probabilities are defined analogously. Taking the derivative with respect to β,

$$d\phi/d\beta - (1/4)(r_M - r_F)\left\{ \alpha\cdot Pr(market|MH)\cdot(1-y_H) + (1-\alpha)\cdot Pr(market|ML)\cdot(-y_L) \right.$$
$$\left. - [\alpha\cdot Pr(market|FH)\cdot(1-y_H) + (1-\alpha)\cdot Pr(market|FL)\cdot(-y_L)] \right\} \ .$$

Note that the sign of this derivative is the sign of the bracketed

expression. This expression may be rewritten:

$$\left\{ \alpha \cdot \exp(-\alpha \cdot [\beta r_M + (1-\beta) r_F]) \cdot (1-y_H) + (1-\alpha) \cdot \exp(-(1-\alpha) \cdot [\beta r_M + (1-\beta) r_F]) \cdot (-y_L) \right.$$

$$\left. - \left[\alpha \cdot \exp(-\alpha \cdot [\beta r_F + (1-\beta) r_M]) \cdot (1-y_H) + (1-\alpha) \cdot \exp(-(1-\alpha) \cdot [\beta r_F + (1-\beta) r_M]) \cdot (-y_L) \right] \right\}$$

Note that both halves of this expression are of the form:

$$g(x) = \alpha \cdot \exp(-\alpha x) \cdot (1-y_H) + (1-\alpha) \cdot \exp(-(1-\alpha)x) \cdot (-y_L)$$

and that:

$$\partial g(x)/\partial x = -[\alpha^2 \cdot \exp(-\alpha x) \cdot (1-y_H) + (1-\alpha)^2 \cdot \exp(-(1-\alpha)x) \cdot (-y_L)] \ .$$

Further note that $\partial g(x)/\partial x < -(1-\alpha)g(x) < 0$; this expression is decreasing in x. As $[\beta r_M + (1-\beta) r_F] > [\beta r_F + (1-\beta) r_M]$, the bracketed expression above is thus negative; economic efficiency increases as the inbreeding bias between groups decreases.   Q.E.D.

A group's mean wage is equal to its average output across both periods plus the option value earned by period-one workers minus the profits earned by firms on referred period-two workers. Given lower female network density, average female output is lower than average male output due to decreased job matching; even in the absence of cross-group ties, females would earn lower mean wages. But given social ties between men and women, the difference in mean wages is increased even further. As

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established above, the option value earned by males exceeds the profit earned on referred males; male wages thus exceed male output. Option value earned by females is less than the profit earned on referred females; female wages thus lie below female output.

Intuitively, the option value earned by a period-one worker is increasing in the probability that (s)he holds a social tie. Given higher male network density, the probability that a male refers a female is greater than the reverse probability; men thus capture a larger share of the profits earned on referred women than vice-versa. This result is reinforced by the fact that firms tend to earn more on referred women as they face less competition from other firms.

A decrease in the group inbreeding biases causes the number of cross-group referrals to rise while the number of intra-group referrals falls. Given greater network density for males, the absolute number of referred females rises while the absolute number of referred males falls. But as the marginal female referred is of higher expected ability than the marginal male referred, there is a net improvement in job matching; economic efficiency and total wages rise.[24]

SECTION 5. CONCLUSION

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[24]Unfortunately, I have been unable to sign the effect of a change in β upon the mean wage of each group. Intuitively, as β falls, two effects pull in opposite directions: average output per female rises while the option value earned by females decreases. The reverse holds true for men. The change in the mean wage for each group depends on the relative importance of each effect.

In the present analysis, compensation varies across hiring channels; workers hired through employee referral have relatively high expected abilities and thus receive higher wages than workers hired through more formal channels. If blacks or females possess lower network densities, they will be referred less frequently and may (in the presence of job matching) receive lower mean wages. Further, if either blacks or females possess lower ability-inbreeding biases, the expected ability of referred workers declines and lower mean wages may again result. In contrast to previous models of statistical discrimination, the present analysis thus generates observed difference in wage dispersion and mean wages across groups without presupposing that the productivity of some groups is measured with exogenously greater error. These previous models may be usefully reinterpreted as reduced-form versions of the more complete model of the hiring process presented here.

In future research, I plan to examine the potential endogeneity of labor force participation and/or unemployment. In the absence of any *prima facie* evidence for differences in absolute network density across groups, I argued in Section 3 that *effective* network densities will vary directly with the proportion of the group employed. Given relatively high unemployment and low labor-market participation among women and blacks, these workers will possess lower effective network densities, receive fewer referrals, and ultimately earn lower mean wages. I have not, however, examined whether these differences in unemployment and/or labor-market participation could arise endogenously--whether these

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existing differences in employment rates are themselves generated by lower

network density. Moreover, if a worker's employment status is partially

determined by the employment status of his social contacts, job creation

may generate positive externalities; appropriate public policy (e.g., jobs

programs) might increase economic efficiency and wages.

APPENDIX 1: DETERMINATION OF REFERRAL WAGE OFFERS

In this appendix, I examine the problem facing firms in determining the referral wage offered (or whether to make such an offer at all). To begin, I examine the problem faced by some firm i whose first-period worker possesses a tie to a high-ability period-two worker. (Given the assumptions made in the text, each firm does not know (ex ante) whether its worker possesses a tie and (if so) which type of period-two worker is known; I assume the firm possesses this knowledge for expositional purposes only.) As described in the text, the (high-ability) worker accepts firm i's offer only if he has received no higher offer. Formally,

$$\Pr(H \text{ accepts firm } i\text{'s offer } w_{Ri})$$

$$= \Pr(H \text{ receives no higher offer, } w_{Rj}, \text{ from firm } j \;\; \forall \; j \neq i)$$

From the worker's perspective, each of the offers are allocated independently; the right-hand side of this equation may thus be rewritten:

$$\Pr(H \text{ accepts firm } i\text{'s offer } w_{Ri})$$

$$= \prod_{j \neq i} \Pr(H \text{ receives no higher offer, } w_{Rj}, \text{ from firm } j)$$

$$= \prod_{j \neq i} [1 - \Pr(H \text{ receives an offer } w_{Rj} > w_{Ri} \text{ from firm } j) \;.$$

The probability that firm j offers a wage $w_{Rj} > w_{Ri}$ to the given high-ability worker is in turn the product of two independent probabilities:

$$\Pr(H \text{ receives an offer } w_{Rj} > w_{Ri} \text{ from firm } j)$$

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$- \text{Pr(firm } j \text{ makes offer to H)} \cdot \text{Pr}(w_{Rj} > w_{Ri})$.

Given the assumptions on social structure and the equilibrium strategies of other firms, this becomes:

$$\text{Pr(H receives an offer } w_{Rj} > w_{Ri} \text{ from firm } j) = [\alpha_H r/pN] \cdot (1 - F(w_{Ri}))$$

for all firms j employing a high-ability worker in period one. (Recall that firms employing low-ability workers will make no offers through referrals.) Given this expression (and noting the symmetry of the equilibrium), we obtain:

$$\text{Pr(H accepts firm } i\text{'s offer } w_{Ri}) = \left[1 - [\alpha_H r/pN](1 - F(w_{Ri})) \right]^{(pN-1)}$$

For large N, the right-hand side is well-approximated:

$$\text{Pr(accepts } w_{Ri} | H) \approx \exp[-\alpha_H r(1 - F(w_{Ri}))] \quad .$$

(This approximation becomes exact as $N \to \infty$; I thus assume it holds exactly in further computations.) By similar calculation, one obtains the probability that firm i's offer is accepted by a given low-ability worker:

$$\text{Pr(accepts } w_{Ri} | L) \approx \exp[-(1-\alpha_H)(p/(1-p))r(1 - F(w_{Ri}))] \quad .$$

Note that, conditional upon the offer being received by a given worker, high-ability workers are less likely to accept any offer $w_{Ri} < \bar{w}_R$ as these workers tend to receive more offers. (For $w_{Ri} = \bar{w}_R$, however, note that both probabilities are equal to one--no firm offers a higher wage and the worker always accepts.)

As a worker finds employment through the market only if he receives no offers, $Pr(market|H) = Pr(accepts\ w_{M2}|H)$ and $Pr(market|L) = Pr(accepts\ w_{M2}|L)$. As $F(w_{M2}) = 0$, we obtain:

$$Pr(market|H) \cong exp[-\alpha_H r] \quad .$$

$$Pr(market|L) \cong exp[-(1-\alpha_H)(p/(1-p))r] \quad .$$

APPENDIX 2: WAGE DETERMINATION IN THE GENERALIZED MODEL

As in Appendix 1, I first derive the probability that each type of worker accepts w_{Ri} conditional upon this referral wage being offered. Suppose first that firm i employs a high-ability male in period one and makes an offer to a high-ability male. By following the steps taken in Appendix 1, one obtains:

$$Pr(accepts\ w_{Ri}|MH) =$$

$$\left[1 - [\alpha\beta r_M/N](1-F_{MM}(w_{Ri}))\right]^{(N-1)} \cdot \left[1 - [\alpha(1-\beta)r_F/N](1-F_{FM}(w_{Ri}))\right]^N$$

where $F_{MM}(\cdot)$ is the distribution of referral wage offers made by firms employing males in period one to period-two males and $F_{FM}(\cdot)$ is the analogous distribution of offers made by firms employing females. If α and β were allowed to vary across groups, these distributions would not (in general) be identical. But in the special case examined here, firms have exactly the same priors on the expected ability of males referred by males as they have on males referred by females. Assuming a symmetric

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equilibrium, $F_{MM}(w_R) = F_{FM}(w_R)$ $\forall$ $w_R \in [w_{M2}, \bar{w}_R]$ and I will simply write $F_M(w_R)$ for both. For large N, the above expression is well-approximated by:

$$\Pr(\text{accepts } w_{Ri} | MH) \cong \exp\{-\alpha \cdot [\beta r_M + (1-\beta) r_F] (1 - F_M(w_{Ri}))\} \quad .$$

Through similar calculation, one may obtain the analogous probabilities for low-ability males, high-ability females, and low-ability females:

$$\Pr(\text{accepts } w_{Ri} | ML) \cong \exp\{-(1-\alpha) \cdot [\beta r_M + (1-\beta) r_F] (1 - F_M(w_{Ri}))\} \quad ,$$

$$\Pr(\text{accepts } w_{Ri} | FH) \cong \exp\{-\alpha \cdot [\beta r_F + (1-\beta) r_M] (1 - F_F(w_{Ri}))\} \quad , \quad \text{and}$$

$$\Pr(\text{accepts } w_{Ri} | FL) \cong \exp\{-(1-\alpha) \cdot [\beta r_F + (1-\beta) r_M] (1 - F_F(w_{Ri}))\}$$

where $F_F(\cdot)$ is the distribution of referral wage offers made to females.

For convenience, I assume that all referred workers (regardless of group) are assigned to job 1 while all workers hired through the market are assigned to job 2. (This is a generalization of case 2 in proposition 4; it requires y_H and y_L such that $\Pr(FH | \text{market}) < \eta^*(y_H, y_L) < \Pr(FH | \text{accepts } w_{M2})$.) In this case, the market wage for males determined via Bayes Rule:

$$w_{M2}^M = \Pr(MH | \text{market}) \cdot y_H + \Pr(ML | \text{market}) \cdot y_L$$

$$= \frac{p \cdot \Pr(\text{market} | MH) \cdot y_H + (1-p) \cdot \Pr(\text{market} | ML) \cdot y_L}{p \cdot \Pr(\text{market} | MH) + (1-p) \cdot \Pr(\text{market} | ML)}$$

and the market wage for females is determined similarly:

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$$w_{M2}^{F} \quad - \quad \text{Pr(FH|market)} \cdot y_H \ + \ \text{Pr(FL|market)} \cdot y_L$$

$$- \quad \frac{p \cdot \text{Pr(market|FH)} \cdot y_H \ + \ (1-p) \cdot \text{Pr(market|FL)} \cdot y_L}{p \cdot \text{Pr(market|FH)} \ + \ (1-p) \cdot \text{Pr(market|FL)}} \quad .$$

Given $r_M > r_F$, one can easily show that the market wage for females exceeds the market wage for males. Intuitively, more males are referred and the lemons effect is stronger for this group.

The (conditional) expected profit of a firm hiring a male through referral may be written:

$$E\Pi_M(w_R) \quad - \quad \alpha \cdot \text{Pr(accepts } w_R|\text{MH)} \cdot (1-w_R) \ + \ (1-\alpha) \cdot \text{Pr(accepts } w_R|\text{ML)} \cdot (-w_R)$$

and the (conditional) expected profit of a firm hiring a female through referral may be written:

$$E\Pi_F(w_R) \quad - \quad \alpha \cdot \text{Pr(accepts } w_R|\text{FH)} \cdot (1-w_R) \ + \ (1-\alpha) \cdot \text{Pr(accepts } w_R|\text{FL)} \cdot (-w_R) \quad .$$

As in the proof of Proposition 1, the referral offer distributions are determined by setting $E\Pi_M(w_R) - c_M$ and $E\Pi_F(w_R) - c_F$ $\forall$ $w_R \in [w_{M2}, \bar{w}_R]$. Further, these constants are determined by setting $c_M - E\Pi_M(w_{M2}^{M})$ and $c_F - E\Pi_F(w_{M2}^{F})$. By substituting the appropriate market wage into each of these expressions, one can show that $c_F > c_M$. Intuitively, fewer females are referred and thus competition for referred female is less intense. Additionally, one can show that the maximum wage paid to males exceeds the maximum wage paid to females:

$$\bar{w}_R^M - \alpha - c_M > \alpha - c_F - \bar{w}_R^F \ .$$

Wage dispersion is thus greater for males than females.

Further, as both c_F and c_M are positive, period-one firms employing either high-ability males or females will wish to hire through referral. One may demonstrate the firms employing low-ability workers will not wish to hire through referral exactly as in the proof of Proposition 2: if a firm employing a low-ability worker hires through referral, its expected profit is decreasing in w_R; even at w_{M2}, however, the firm still earns a negative expected profit as $\alpha > 1/2$.

Finally, I need to show that the mean wage for males exceeds the mean wage for females and that an increase in α, r_F, or r_M increases efficiency. Average output for each group may be written:

$$\phi_M = (1/2) [1 - Pr(market|MH) \cdot (1-y_H) + Pr(market|ML) \cdot y_L]$$

$$\phi_F = (1/2) [1 - Pr(market|FH) \cdot (1-y_H) + Pr(market|FL) \cdot y_L]$$

and thus the difference in average output is:

$$\phi_M - \phi_F = (-1/2) [Pr(market|MH) \cdot (1-y_H) + Pr(market|ML) \cdot (-y_L)$$
$$- (Pr(market|FH) \cdot (1-y_H) + Pr(market|FL) \cdot (-y_L))]$$

and average output for males is higher if the bracketed expression is negative; one may obtain this result by repeating the steps taken in the proof of Proposition 7. As discussed in Section 4, the mean wage for a group is equal to its average output plus the period-one option value earned minus the profit earned on referred period-two workers. As the profit earned on referred females exceeds the option value earned by females, the mean female wage lies below average female output. But as

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the reverse is true for men, the difference in group wages is even larger
than the difference in average output.

As α increases, high-ability workers from both groups are more likely
to be hired through referral and low-ability workers are more likely to be
hired through the market; improved job matching increases the output of
both groups and thus overall efficiency. An increase in either r_M or r_F
increases the probability that both high- and low-ability workers will be
hired through referral. But as the expected ability of the marginal
worker hired through referral exceeds η^*, the efficiency gain from
additional high-ability workers outweighs the loss from additional
low-ability workers; overall efficiency thus increases.

REFERENCES

Aigner, Dennis and Glen Cain. (1977) "Statistical Theories of
    Discrimination in Labor Markets." *Industrial and Labor Relations
    Review* 30: 175-87.

Akerlof, George A. (1970) "The Market for 'Lemons'; Quality Uncertainty
    and the Market Mechanism." *Quarterly Journal of Economics* 84:
    488-500.

Bartlett, Robin L. and Timothy I. Miller. (1985) "Executive Compensation:
    Females Executives and Networking." *American Economic Review* 75:
    266-70.

Becker, Gary S. (1957) *The Economics of Discrimination.* Chicago:
    University of Chicago Press.

Blau, Peter M. (1977) *Inequality and Heterogeneity.* New York: Free Press.

----- and Joseph E. Schwartz. (1984) *Cross-Cutting Social Circles:
    Testing a Macrosociological Theory of Intergroup Relations.* New
    York: Academic Press.

Butters, Gerard R. (1977) "Equilibrium Distributions of Sales and
    Advertising Prices." *Review of Economic Studies* 44: 465-91.

Cain, Glen (1986) "Labor Market Discrimination" in O. Ashenfelter and R.
    Layard, eds., *Handbook of Labor Economics.* Amsterdam: North-Holland.

Corcoran, Mary, Linda Datcher, and Greg Duncan (1980) "Information and
    Influence Networks in Labor Markets" in Greg Duncan and James Morgan,
    eds., *Five Thousand American Families: Patterns of Economic Progress,
    Vol. 7.* Ann Arbor: Institute for Social Research.

Ehrenberg, Ronald G. and Robert S. Smith (1985) *Modern Labor Economics:
    Theory and Public Policy.* Glenview, IL: Scott, Foresman and Company.

Gibbons, Robert and Lawrence Katz. (1987) "Learning, Mobility, and
    Inter-Industry Wage Differences." MIT mimeo, November.

Greenwald, Bruce C. (1986) "Adverse Selection in the Labour Market."
    Review of Economic Studies 53: 325-47.

Holzer, Harry J. (1986) "Informal Job Search and Black Youth
    Unemployment." *American Economic Review* 77: 446-52.

----- (1986) "Hiring Procedures in the Firm: Their Economic
    Determinations and Outcomes." NBER Working Paper 2185, March.

Lang, Kevin. (1986) "A Language Thec / of Discrimination." *Quarterly Journal of Economics*, 101: 363-8?

----- (1987) "A Sorting Model of Statistical Discrimination." Boston University mimeo, October.

Lundberg, Shelly J. and Richard Startz. (1986) "Private Discrimination and Social Intervention in Competitive Labor Markets." *American Economic Review* 73: 340-7.

Marsden, Peter V. (1988) "I mogeneity in Confiding Relations." *Social Networks* 10: 57-76.

Montgomery, James. (1988) "Social Networks and Labor Market Outcomes: Toward an Economic Analysis." MIT Mimeo, November.

Phelps, Edmund S., "The Statistical Theory of Racism and Sexism." *American Economic Review*, 76: 228-34.

Rees, Albert and George Shultz. (1970) *Workers in an Urban Labor Market.* Chicago: University of Chicago Press.

Roy, A. D. (1951) "Some Thoughts on the Distribution of Earnings." *Oxford Economic Papers* 3: 135-46.

Rytina, Steve and David L. Morgan. (1982) "The Arithmetic of Social Relations: The Interplay of Category and Network." *American Journal of Sociology* 88: 88-113.

Waldman, Michael. (1984) "Job Assignments, Signalling, and Efficiency." *Rand Journal of Economics* 15: 255-67.

Wilson, William J. (1987) *The Truly Disadvantaged.* Chicago: University of Chicago Press.