Characteristics of Wall Pressure and Near-Wall Velocity in a Flat Plate Turbulent Boundary Layer

by

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Detailed velocity measurements of all three velocity components, taken in $U-V$ and $U-W$ pairs, and the pressure on the wall below the probe were taken in a flat plate turbulent boundary layer. Velocity measurements were taken with miniature two-wire hot-wire probes (in various configurations), and several causes of two-wire hot-wire anemometer measurement error were investigated. The most severe errors are caused by thermal interactions between the two wires and the time lag between thermal conduction to the wire supports and thermal convection to the flow. The measurement of the streamwise velocity component is generally unaffected by these effects but the measured normal and spanwise velocity components are greatly distorted. To minimize these errors a large Prandtl number working fluid, a low wire overheating ratio and a hot-wire length to diameter ratio greater than 400 are desirable.

Joint investigation of the flow velocity and the wall pressure, measured with a surface mounted 1.95mm diameter microphone, revealed that large amplitude wall pressure peaks under a turbulent boundary layer are associated with near-wall flow events. Both isolated peaks and wave trains are primarily associated with the normal component of velocity. The isolated peaks appear to be the result of rapid normal velocity fluctuations while the wave trains are associated with a flow instability in the buffer region. In either case, the flow appears to have a strong spanwise component of velocity. The wave trains appear to be similar to waves observed in the evolution of a localized disturbance in a laminar boundary layer.

Also developed were the Spectral Power Density (SPD) technique of velocity-pressure wave detection, the Polar Look-up table calibration technique for two-wire hot-wire anemometer probes and a promising parallel-wire probe.

Thesis Supervisor: Dr. Joseph H. Haritonidis, Research Engineer, Department of Aeronautics and Astronautics
To my wife and daughter for their love and support
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List of Symbols

**Roman**

$A$  
Width of hot-wire sensing box, see Figure 2.3

$a_r, a_T$  
Resistance and Temperature overtaken ratios

$B$  
Streamwire length of hot-wire sensing box

$C$  
Separation between planes of the wires (also denoted as $s$), and a constant in the Logarithmic Law, usually 5.0

$C_f$  
Coefficient of friction

$c_p$  
Specific heat at constant pressure

$d$  
Diameter of hot-wire sensing element

$E_1, E_2$  
Voltages from hot-wire anemometers

$E()$  
Spectral power of ()

$f$  
Frequency (in Hertz)

$F()$  
Flatness of variable ()

$h$  
Enthalpy or Heat transfer coefficient, $h = h_c + h_r + h_s$

$h_c, h_r, h_s$  
Heat transfer coefficient due to convection, radiation and conduction, respectively

$I$  
Current, related to voltage by $E = IR$

$K$  
Threshold level in detection techniques and height of trip

$k_f, k_w$  
Thermal conductivity of fluid and wire, respectively

$\ell_c$  
Characteristic length over which supports effect the wires' temperature

$\ell_{wire}$  
Length of hot wire sensing element

$m$  
Mass flux

$Nu$  
Nusselt number $\equiv \frac{\dot{h} d}{k_f}$

$p$  
Pressure perturbation
\( P \)  
Probability, for example \( P(u,v) \) is the joint probability of occurrence for any specific value of \( u \) and \( v \). The total area or volume under a probability curve is always 1.

Pr  
Prandtl number \( \equiv \frac{\nu}{\rho k_f} = \frac{\nu}{a} \)

\( \bar{Q}, Q \)  
Flow velocity vector and magnitude

\( \dot{q} \)  
Heat flux

\( r \)  
Radial coordinate on Polar Look-up grid

\( R_2, R_3 \)  
Resistor values on hot-wire anemometer circuit

\( R_{uv} \)  
Cross-correlation coefficient, i.e. \( \bar{uv}/(u'v') \)

\( R_{wH}, R_{wC} \)  
Resistance of wire "hot" (operating) and "cold"

\( R_w, T_w \)  
Same as \( R_{wH} \) and \( T_{wH} \)

\( Re_\delta \)  
Reynolds number based on boundary layer thickness \( \equiv \frac{U_{\infty} \delta}{\nu} \)

\( Re_\delta^* \)  
Reynolds number based on displacement thickness

\( Re_\theta \)  
Reynolds number based on momentum thickness

\( Re_z \)  
Reynolds number based on \( z \)

\( S() \)  
Skewness of variable ()

\( T^+ \)  
Time length of window used in VITA detection technique

\( T, T_{amb} \)  
Temperature, and Ambient temperature

\( T_{wH}, T_{wC} \)  
Temperature of wire "hot" (operating) and "cold"

\( t \)  
Time

\( U_\infty \)  
Free stream velocity

\( \bar{U}, U(y) \)  
Local mean velocity

\( u, v, w \)  
Perturbation velocity components

\( U, V, W \)  
Total velocity component, mean plus perturbation

\( u_r \)  
Friction velocity
\( x, y, z \) Coordinate directions, origin at leading edge, on the plate, tunnel centerline, respectively

\( X \) Measurement Position

\( y_o \) Position where Logarithmic and Arctangent Law meet

**Greek**

\( \alpha \) Included angle between hot-wires on 2 wire probes, and Temperature coefficient of resistivity

\( \alpha_1, \alpha_2 \) Angle between probe axis of symmetry and wire \#1 and wire \#2, respectively

\( \beta \) Angular coordinate on Polar Look-up grid

\( \delta \) Boundary layer thickness

\( \delta^* \) Displacement thickness

\( \varepsilon \) Error percentage

\( \gamma_{se} \) Half angular width of the thermal wake of the hot-wire at 3 standard deviations of the wake temperature distribution

\( \kappa \) Von Karman's constant, usually .41

\( \lambda \) Constant in Arctangent Law

\( \mu \) Viscosity

\( \nu \) Kinematic viscosity

\( \phi \) Angle of inclination of the probe measured between the probe axis and the streamwise axis, \( x \)

\( \psi_1, \psi_2 \) Angles between the \( x \) direction and wire \#1 and wire \#2, respectively

\( \rho \) Fluid density

\( \sigma_\gamma, \sigma_\zeta \) Angular width and thickness of the thermal wake of the hot-wire at one standard deviation of the wake temperature distribution

\( \tau_{xy}, \tau_w \) Shear stress and Shear stress at the wall
\( \theta \)  Momentum thickness and Angle of flow vector being measured, in either the \( U-V \) or \( U-W \) plane

\( \nu \)  Volume

\( \vartheta \)  Cross flow vector angle in the plane opposite \( \theta \)

\( \omega \)  Angular frequency, \( \omega = 2\pi f \)

\( \zeta \)  Direction normal to the hot-wire and the local flow direction

\( \zeta_{so} \)  Thickness of the thermal wake of the hot-wire at 3 standard deviations of the wake temperature distribution

**Superscripts**

+  Normalized by viscous wall parameters

'  Denotes RMS (square root of variance)

\( \overline{()\quad} \)  Denotes mean value

**Subscripts**

\( || \)  Parallel

\( \perp \)  Perpendicular

\( e \)  Effective, as in effective velocity, \( U_e \)

\( i, j \)  Index denoting a discrete measurement

\( \min \)  Minimum

\( \max \)  Maximum

\( \text{origin} \)  Origin of Polar Look-up Grid

\( tr \)  Transition

**Special**

\( \mid_0 \)  Evaluated at constant \( () \), where \( () \) is any variable
Chapter 1

Introduction

1.1 Motivation

The structure of wall turbulence has been investigated extensively for the past 80 years. The understanding of turbulent phenomena is of importance in a wide variety of fields: biology, meteorology, and of course aerodynamics. Boundary layer turbulence is a major source of drag on both aircraft and ships. Although it is of great importance, the behavior of turbulent boundary layers is far from fully understood, and accurate predictions are difficult except in the simplest cases. As a result, turbulence models must rely heavily on empirical theories.

Past experiments using pitot tubes, hot-wire probes, wall pressure transducers, laser doppler velocimetry, and visual methods have produced much information validating scaling laws, mixing lengths, and a number of theories describing the motion of the flow near the wall. Authoritative discussions of the classical theories of turbulence can be found in Monin and Yaglom (1971, 1975), Tennekes and Lumley (1972), Hinze (1975), and more recently Landahl and Mollo-Christensen (1986).

A different view on turbulence appeared, however, when Kline, Reynolds, Schraub and Runstadler (1967) and Corino and Brodkey (1969), presented results obtained by detailed flow visualization of all three velocity components, which showed that part of
the turbulence in a boundary layer could be described in terms of deterministic events. A series of visual studies soon followed, by Kim, Kline and Reynolds (1971), Grass (1971), Offen and Kline (1973, 1974), Falco (1977), Praturi and Brodkey (1978), and Head and Bandyopadhyay (1981) to mention a few. These investigations showed that the near-wall region of a turbulent boundary layer is characterized by regions of low speed fluid, extending in the streamwise direction, called streaks. From visual traces of hydrogen bubbles it was found that the fluid motion reveals definite sequences of ordered motions occurring randomly in space and time. Two flow features of primary importance are the fluid ejection phase and the fluid in-rush phase originally reported by Kline et al. (1967). They observed that the streaks lift from the wall and appear to oscillate. This oscillating motion increased in amplitude and scale until a breakdown occurs, at which time completely chaotic motion ensues. This outflow phase occurs on a very short timescale, and is called an ejection. Fluid in-rush phases, sweep phases as named by Corino and Brodkey (1969), are associated with the transport of high momentum fluid inward toward the wall. These are larger-scale motions originating at the outer flow field that approach the wall and clear the area of the chaotic ejection, “sweeping” it away. This inflow phase occurs on a longer timescale. Both events are usually referred to together as the “bursting” phenomenon, even though their phase is not fixed. Quantitative measurements were needed to describe the organized motions that were qualitatively evident from flow visualization. Occurring at the same time as the visual studies this need was answered by the concurrent development of both smaller and better quality velocity measurement probes as well as innovative “bursting” detection schemes. The key to using probe measurements is the availability of a reliable method for identifying bursts with velocity or pressure measurements.
1.2 Velocity Measurements

Because of their small size and high frequency response, hot-wire anemometer probes have long dominated velocity measurements in turbulence research. Single-wire probes used to measure the stream-wire component of velocity \((U)\) are the standard. But to obtain a more complete understanding of the flow structure two- and three-component probes have been built, though the latter is usually prohibitively large. The two-wire X-probe (so named because the wires appear to cross like an “X”) was first used by Reichardt (1938) to measure the simultaneous fluctuating velocities \(U\) and \(V\) (in the direction normal to the wall) in a turbulent channel flow. Also using an X-probe in channel flow, Laufer (1951, 1955) was the first to make measurements of \(U\) and \(W\) (in the spanwise direction) and to document detailed turbulence statistics for all three velocity components. His measurements were made in air with a miniature probe only five viscous units wide \((5\ell_v, \text{where } \ell_v = \nu/u_r)\) and as close as \(5\ell_v\) to the wall. At the same time, Klebanoff (1954) made \(U, V,\) and \(W\) measurements in air using a \(19\ell_v\) X-probe in a turbulent boundary layer, to within \(32\ell_v\) of the wall. Gupta and Kaplan (1972) extended the range by making the first detailed measurements of the Reynolds stress in a turbulent boundary layer, and Hussain and Reynolds (1975) performed experiments in an extremely long channel to ensure the higher order statistics reached a fully-developed state. Because of the small scales involved two approaches were taken for measurements extremely close to the wall: using specially designed sub- miniature X-probes, Willmarth and Bogar (1977), or increasing the spatial scales of the turbulence by increasing the viscosity, Eckelmann (1974). The former approach by Willmarth and Bogar (1977) using the smallest X-probe to date (.1mm) indicated that possibly turbulent eddies and shear layers smaller than their probe size \((2.5\ell_v)\) were evident in the flow, but this has not
been found by anyone else since. (Their explanation is that larger probes average out the small scales.) Eckelmann (1974) and later Kreplin and Eckelmann (1979) carried out their experiments in an oil channel with high viscosity thereby following the later approach. This allowed them to use a large probe, but due to the viscosity the sensing area width was only $1.3\zeta_*$ (the smallest non-dimensional X-probe size to date). In addition they also introduced the V-probe (so named because of the wedge shape made by the wires) to measure $U$ and $W$. This is a significant improvement over using a X-probe to measure $W$ because both wires are at the same distance from the wall thereby eliminating the effect of the velocity shear gradient in the $y$ direction. All of these experiments, except Willmarth and Bogar (1977), have produced similar velocity profile results and are referred to constantly as the definitive works. More recently, however, probably the most detailed measurement of all three velocity components in turbulent channel flow made to date were conducted by Karlsson and Johansson (1988) using Laser-Doppler Velocimetry. Their experiments used a novel technique to obtain virtually continuous time histories of the velocity components, thereby allowing calculation of higher order statistics. The advantage of the LDV is that measurements can be made without disturbing the flow, and is therefore free of most of the problems close to the wall encountered with probe measurements.

1.3 Detection Schemes

Using the smaller and more reliable hot-wire probes several techniques for detecting individual events, such as bursts or ejections have been developed. Kovasznay, Kibens, and Blackwelder (1970) were the first to introduce the concept of conditional sampling and applied it to the study of the intermittent flow in the outer part of the boundary
layer. Conditional averaging of the instantaneous velocity components yielded quantitative confirmation that ejection phases correspond with ejection of low momentum fluid outward from the boundary layer. Rao, Narasimha and Badri Narayanan (1971) used the band-pass-filtered velocity signal to detect time periods when the streamwise velocity was highly energetic. Blackwelder and Kaplan (1976) introduced the Variable-Interval Time Averaging (VITA) technique to detect large variances in the streamwise-velocity signal. Chen and Blackwelder (1978) added a slope condition to the VITA technique so that only events with rapid accelerations in the streamwise velocity component would be detected. Previously Simpson (1976) had used only a large positive slope in the velocity signal to detect events.

Other velocity-signal characteristics have also been used for burst detection. Willmarth and Lu (1972) and, Lu and Willmarth (1973) used two detection techniques: one based on the magnitude of the streamwise velocity being below a certain threshold, and the other based on the magnitude of the second quadrant $uv$ product (i.e. when $u$ is negative and $v$ is positive). Here $u$ and $v$ are the turbulent fluctuations about the mean velocity components, $\overline{U}$ and $\overline{V}$. Wallace, Brodkey and Eckelmann (1977) used a pattern-recognition technique to identify events starting with a negative gradient in the streamwise velocity, followed by a positive gradient that was greater in magnitude than the preceding negative gradient.

In spite of these differences in detection, the spatial configurations and scales of the wall structure are generally agreed upon. Conditional averaging indicates that the burst sequences correlate with an extremely high contribution to the Reynolds shear stress and hence turbulence production close to the wall. Additionally, it is suggested by the studies that turbulence production is dominated by the joint contribution from both
the sweep and ejection events. It is emphasized however, that these structural features are intermittent, forming important linked elements of a randomly repeating cycle of wall-region turbulence production. The driving force behind these features is a three-dimensional instability mechanism which is also linked to wall pressure fluctuations.

1.4 Pressure Measurements

Much of the early experimental work on wall pressure fluctuations (see the review by Willmarth, 1975) dealt with spectral and correlation measurements to determine the influence of the different regions of the boundary layer on the pressure. Of particular interest was the question of whether the linear of nonlinear mechanisms were responsible for producing the measured pressures. Most recent work, with the advent of higher quality and smaller pressure transducers, has concentrated on relating particular pressure signatures with the flow above the wall. Burton (1974), Dinkelacker and Langeheineken (1982), Thomas and Bull (1983), Her (1986) and Johansson, Her and Haritonidis (1987) used conditional sampling techniques to related flow structures with wall pressure signatures. Johansson, et al. (1987) used conditions on both the velocity and the pressure to assess if there was a bidirectional correspondence between pressure and flow signatures. It was found that this is not the case. In addition, high amplitude pressure peaks appeared to be linearly related to the velocity field through the turbulence-mean shear interaction term. Recall that the Navier-Stokes equations provide that the wall pressure can be written as

\[
p_w = \frac{\rho}{2\pi} \int_0^\infty \tilde{q} \cdot \frac{dv}{|\overline{x} - \overline{x}_w|} \quad \text{where} \quad \tilde{q} = 2 \frac{\partial U_i \partial u_j}{\partial x_j \partial x_i} + \frac{\partial^2 (u_i u_j - u_i \overline{u_j})}{\partial x_i \partial x_j}.
\]  

(1.1)
This integral expression indicates that wall pressure fluctuations beneath a turbulent boundary layer are coupled to the velocity throughout the entire boundary layer, the coupling being both through the linear interaction of fluctuations with mean shears and the nonlinear interaction of the fluctuations with themselves.

Emmerling (1973) and Dinkelacker, Hessel, Meir and Schewe (1977) found, through the use of a novel array of pressure transducers, that a variety of pressure patterns existed on the wall. In particular, large pressure fluctuations of the order of $5p'$ (where $p' = p_{rms}$) were observed as isolated peaks or parts of wave trains. Schewe (1983), using the smallest transducers to date ($d^+ = 19$, where a + superscript indicates normalization with the wall variables $v$ and $u_r$; for future reference, $\ell_* = v/u_r$ and $t_* = v/u_r^2$) found very clear evidence of wave trains in a turbulent boundary layer. The wavelength, of pressure signatures with $|p| > 3p'$, was $145\ell_*$ and the frequency $\omega^+ = 2\pi f^+ = 0.52$. The average propagation velocity was found to be $11.9u_r$, indicating that the waves are most likely associated with a flow structure located in the buffer region. It is worth noting that the measured pressure amplitudes reached values of $\pm 7p'$ and that while pressure fluctuations higher (lower) than $3p'$ occur only 1% of the time, they contribute 40% to $p'$. Thus, these energetic pressure fluctuations must be of dynamical importance.

Her (1986) and Johansson, Her and Haritonidis (1987) investigated the relation of wall pressure wave trains to flow structures in the buffer region. He found some evidence, through spectral analysis of pressure and velocity signals, that the wave trains are related to a flow instability. Breuer (1988) studied, numerically and experimentally, the evolution of a three-dimensional, localized, disturbance in a laminar boundary layer. His numerical (full Navier-Stokes) simulation shows the growth of disturbance, on a developing internal shear layer, whose initial wavelength is approximately $145\ell_*$, in
good agreement with the results of Schewe (1983) for a fully turbulent boundary layer.

1.5 Present Work

The present study addresses the origin of the pressure peaks and waves in a turbulent boundary layer. In many respects this is a repeat of the work of Her (1986) but this time using longer data sets, and new spectral wave detection criteria. Measurements of all three velocity components at $y^+ = 15$ and the pressure at the wall, below the measuring probe, are used.

Initial velocity measurements, however, showed many of the same phenomena present in measurements by Willmarth and Bogar (1977) using their sub-miniature X-probe. A more detailed look at the work of Her (1986) also showed the same errors in the velocity statistics very close to the wall, characteristic of Willmarth’s and Bogar’s findings, which until now had never been reported by anyone else. With velocity measurements in question, the present work took on the added objective to try and resolve the discrepancies between Willmarth’s and Bogar’s measurements and those of Eckelmann and Kreplin (1979) and the LDV measurements of Karlsson and Johansson (1988). Extensive testing provided evidence that the measurements and phenomena observed by Willmarth and Bogar (1977) and the initial measurements in this work are probably due to probe characteristics (wire length, and temperature overheat) and the low Prandtl number of air.
Chapter 2

Experimental Equipment and Procedures

2.1 Wind Tunnel

The experiments were conducted in the Turbulence Research Laboratory in the Department of Aeronautics and Astronautics at MIT. The tunnel and flat plate are described in detail by Mangus (1984) so only a brief description is presented here. The wind tunnel is a closed loop type with flow generated by a 50hp variable-speed DC motor driving a 1.52m diameter 5-blade fan (see Figure 2.1). The motor speed is selected manually and is maintained via feedback from a tachometer on the motor, which maintains a constant load and therefore a constant flow velocity. As reported by Mangus (1984) flow velocity is steady to within ±0.5% for velocities greater than 5m/s, and ±1% at 5m/s. Mangus reported occasional short periods of unsteadiness at flows less than 5m/s, but this is not seen in the current experiments, provided the flow is given time to stabilize. After waiting at least 5 minutes the flow is steady to within ±1% from 5m/s to 0.7m/s. Below 0.7m/s the motor cannot maintain a constant load and the flow surges approximately every 30 seconds. The useful velocity range of the tunnel is 0.7m/s - 40m/s with a free stream turbulence level \( u'/U_\infty \) (for \( f > 0.05\text{Hz} \)) less than 0.05%.

The test section is 6.1m long, 1.22m high and .6m wide, with a flat aluminum
plate mounted vertically extending the entire width and length. The plate is 1.27cm thick with an elliptical tip and flat tapered leading edge. It is mounted 10cm from the tunnel side wall at a small positive angle of attack to maintain a nominally zero pressure gradient. Typically $dc_p/dx < 10^{-4}$cm$^{-1}$. Figure 2.2 illustrates the plate and tip geometries. Nine 10cm-diameter Plexiglass plugs are flush mounted at various positions along the centerline of the plate so that instruments and transducers can be inserted at those positions. The top and bottom 10cm of the plate are porous behind which are ducts for suction of the boundary layers which grow in the corners of the test section. This capability allowed for the containment of the spreading of the corner flows for experiments in laminar boundary layers.

A right-hand coordinate system was defined with $z$ in the streamwise direction, $y$ in the direction normal to the plate, and $x$ in the spanwise direction (positive is toward the tunnel floor). Corresponding velocity components were defined as a local mean plus a perturbation: $U = \overline{U} + u$, $V = \overline{V} + v$, and $W = \overline{W} + w$, respectively, where by definition $\overline{u} = \overline{v} = \overline{w} = 0$. (The overbars denote time averages.)

The tunnel velocity was set using a centerline mounted pitot-static tube connected to a Datametrics type 570D-10T-2D1 Barocel differential pressure sensor with a range of ±10 Torr (±10mmHg). The Barocel is connected to a Datametrics type 1018C electronic manometer operable at scales of $1 \times$, $0.1 \times$, $0.01 \times$ and $0.001 \times$ of the sensor full range. Pressure is presented as an analog voltage with output range of ±10V at each scale. Resolution of the sensor-manometer system is 0.01% of the full scale range with an accuracy of ±0.02% of reading. However, when the output voltage is digitized for use in the computer the resolution was reduced to 1/2048 of 10 volts, or approximately 0.05% of the scale range. The Barocel is mounted on a 1.27cm (1/2 inch) aluminum plate.
which is in turn mounted on rubber bushings inside a sealed 1.27cm thick aluminum box. This box is mounted on 10.2cm (4 inch) rubber casters. This mounting provides isolation of the sensor from vibration transmitted through the floor, and provides a good degree of thermal stability. If the sensor was moved, it was allowed to stabilize for approximately 2 hours before use.

Free stream ambient temperature ($T_{am\dot{a}}$) was measured using a thermometer mounted on the traverse along the centerline of the test section. The resolution of the thermometer was ±.125°C, and normal operating conditions were 23°C–24°C.

2.2 Data Acquisition

Experimental data was acquired by a Phoenix Data Systems DAS 69125 Analog-to-Digital (A/D) converter. The converter was capable of digitizing up to 16 channels simultaneously at a maximum digitizing frequency per channel of 333KHz divided by the number of channels used. The voltage range accepted by the A/D was ±10V which is converted to ±2048 digital counts (≈ 5mV per count). The A/D was connected to a DEC PDP 11-55 computer which also controlled the probe positioning, timing, and all other aspects of the experimental procedure. Subsequent data reduction and graphics was performed on a DEC Microvax II computer. Most data analysis was accomplished using Fortran computer source code, though data-acquisition and hot-wire linearization were carried out by an assembly language routine. This allowed for very fast conversion of raw voltages to velocities and speeded up data collection considerably.
2.3 Hot-wire Anemometer Construction

Local flow measurements were made using constant temperature hot-wire anemometry, the theory of which is discussed in detail in Blackwelder (1981). Single component flow velocity measurements were made with a standard U-probe, while two component measurements were made with various configurations of parallel-wire, X- and V-probes. The U-probe was used to measure the $U$-component of velocity, while the parallel-wire, and X-probe measured $U-V$ or $U-W$ (depending on the orientation) and the V-probe measured $U-W$. All probes were constructed in house using Platinum–10% Rodium wire formed by the Wollaston process, nominally 1.27μm or 0.64μm in diameter ($d$), as the sensing element. Wire supports (prongs) were made from jeweler's broaches, sanded down, unless otherwise noted, to 40μm in diameter at the tips and bent to shape. Probe bodies were constructed of thermocouple ceramic tubing 1.27mm in diameter with either 2 or 4 holes (depending on probe type). The broaches were epoxied into the ceramic tubing and the hot-wire was soldered across the broach tips. Hot-wires were constructed and measured under a stereo microscope at 40× magnification (Olympus model SZ-Tr). Prong alignment and measurements are accurate to 10μm, using a ruled eyepiece.

Figure 2.3 illustrates the five general hot-wire probe types, identifying individual parts and important dimensions. In referencing X- and V-probes the sensing box size refers to the dimensions $A \times B \times C$, and the included angle is $\alpha = \alpha_1 + \alpha_2$. The wires are soldered across the broaches as straight as possible; however, a small bend is necessary to avoid having the wire destroyed by vibration of the broaches when the probe is moved from one location to another. In all cases, $\alpha_1$ and $\alpha_2$ are measured at the midpoint of the wires and care is taken in construction to insure $\alpha_1 \approx \alpha_2$. The wire
length ($\ell_{\text{wire}}$) also reflects this bend and is not simply the diagonal of the sensing box. To minimize probe blockage and to allow the sensing box to reach the wall, the probe was inclined toward the wall at an angle $\phi$. This induces a nominal flow angle of $-\phi$ which is accounted for in calibration. Figure 2.4 is a close-up of the broach tip and wire solder connection.

### 2.3.1 Measurement of $U$-$V$ components of velocity

The measurement of $U$ and $V$ was done with eight general X-probe types and a "parallel-wire" probe. The large number of probes was needed to determine the effects of probe interference and blockage. Figure 2.5 illustrates the various probe-body configurations, and naming scheme. The dimensions given are for a typical sensing box size. Specific box dimensions will only slightly change the configuration. A list of the specific probes used, their sensing box dimensions, and operating parameters are presented in Table 2.3.

The parallel-wire probe was made like an X-probe except the broaches were all the same length making the wires even and parallel. The wires were soldered on the insides of the broaches to bring the wires 25$\mu$m apart. The probe operation relies on the thermal interaction between the wires. The thermal wake of one wire interferes with the heat transfer of the other wire depending on the flow direction and vice-versa, thus making it possible to determine flow direction and velocity. This probe is illustrated in Figure 2.7.
Split-Film

In addition, a novel split-film probe was made as a next step to the parallel-wire probe. Conventional split-film probes are made using a 152μm diameter quartz rod deposited with two hemi-cylindrical coatings of Platinum with only an extremely fine gap (5μm) separating the coatings (TSI Anemometer Catalog, 1982). In contrast, the current split-film has only 25μm wide coating strips with 174μm between them (see Figure 2.8). This design was picked to minimize the thermal cross-talk problem through the rod, inherent in the conventional construction. Unfortunately the probe could not be used because the electrical resistance of the conductive silver epoxy used for the broach-film connection was too great. Welding, and soldering methods were tried but without success. A full description of the probe construction is provided here for future use.

Figure 2.8 illustrates the split-film probe made by depositing chrome in two strips, 160° apart, along a cylinder of fiber-optic glass. The chrome strips were 25μm wide by .1μm thick deposited the length of the glass rod, .76mm long by 125μm in diameter. Steps took to deposit the chrome on the glass were as follows:

1. Remove the protective jacket on a 3cm long piece of fiber-optic glass rod with a razor blade. Thoroughly clean glass rod with alcohol and cotton swab.

2. Place the glass rod in 25:1 Hydrofluoric acid : De-ionized water (HF:DI) bath for 60 seconds. The bath was agitated by rocking the dish. This bath will etch away any organic material on the rod.

3. Rinse rod in DI stop bath to remove all traces of HF.

4. Mount rod on a 25μm split mask. The mask was made by butting two razor blades, edge to edge, with a 25μm gap between the blades. The small groove
formed by the sharpened edges aligned the rod and a spring clip held the rod in
place. See Figure 2.9.

5. The mask-rod assembly was then placed in a 10KV Electron Beam Evaporative
Deposition Chamber so as to expose the underside of the mask which completely
shadows the rod except along the slit. A chromium (chrome) melt was used be-
cause of its high adhesiveness and electrical conductivity. The Vapor deposition
chamber is located in the Semi-Conductor Clean-Room Laboratory in the De-
partment of Electrical Engineering at MIT. The deposition was both timed and
measured so that exactly a .1μm ± 2Å thick layer of chrome was deposited.

6. The glass rod was then rotated 120° in the mask. However, as the spring clips
were brought into position, the rod rotated approximately 40° further, resulting
in a 160° separation between the strips.

7. The mask-rod assembly was then placed in an Oxygen plasma etch (normally used
for cleaning doped semiconductor wafers) for 90 seconds to burn off any particle
contamination picked up in the time needed to rotate the rod.

8. The mask-rod assembly was then placed back in the Vapor deposition chamber
and another chrome strip was deposited, again .1μm thick.

9. The rod was then cleaved with a razor blade to get a .76mm long piece.

The most important parameter in the construction of the split-film element is cleanliness
on a microscopic level. Failure to perform either the HF:DI bath or the oxygen plasma
etch will prevent the chrome from adhering to the glass which will result in flaking and
cracks in the strip.

The glass rod piece was then attached to a probe similar to the parallel-wire probe
with silver conductive epoxy at the four broach/chrome-strip junctions. The epoxy was chosen because it provided both a mechanical and an electrical connection. As mentioned before, the electrical resistance of the epoxy was too high to allow operation as a hot-wire. To correct this problem a separate weld or solder electrical connection in conjunction with a non-conductive epoxy mechanical connection is probably necessary, but was not tried.

2.3.2 Measurement of $U-W$ Components of Velocity

The measurement of $U$ and $W$ was done with five general V-probes and two X-probes. Again, the large number of probes was needed to check interference and blockage effects. Figure 2.10 illustrates the various V-probe configurations and naming scheme. The dimensions given are for a typical sensing box size. The two X-probes used were labeled RX1 and RX3 (where “R” denotes the rotated version of the X-probe) as seen in Figure 2.11. The X-probe was longitudinally rotated 90° so that the wires were in a plane parallel to the wall to measure $U$ and $W$. In doing this, the two wires are at different distances from the wall and, therefore, are measuring at different locations in the boundary layer. This violates the assumption that both wires “see” the same flow in order to correctly resolve the two components of velocity, but the error is minimized by keeping the separation between the wires very small. A list of the specific V-probes and X-probes used to measure $U-W$ and their dimensions is presented in Table 2.4.
2.4 Hot-wire Anemometer Operation

The hot-wire probes were mounted individually on a 30cm sting arm connected to a traversing mechanism with four degrees of freedom: $x$, $y$, $z$ and one rotational axis about the $z$ axis (used for X- and V-probe calibration). The traverse was powered by stepping motors controlled by the PDP-11 computer via a Modulynx motion control system. The spatial resolution of the traverse is 67$\mu$m in $x$, 1.8$\mu$m in $y$, 12.7$\mu$m in $z$ and .039° in rotation. The accuracy is to within one step for each motor.

The hot-wires were operated by constant temperature anemometer circuits also built in-house. The circuit schematic is shown in Figure 2.12. The frequency response of the hot-wire probes was measured using a square-wave generator. The typical response of the first stage alone was greater than 40KHz, while the through-put response of the entire circuit was 25KHz. The U-probe wire and parallel-wire probe wires were operated at a resistance overheat ratio of 30%. The X- and V-probe wires were operated at various resistance overheat ratios from 5% to 50% as listed in Table 2.3 and Table 2.4.

From Blackwelder (1981), the resistance of the sensing "hot"-wire $R_{wH}$ is related to its "hot"-temperature $T_{wH}$, and the resistance overheat ratio ($a_r$) can be shown to be proportional to the temperature overheat ratio ($a_t$) of the wire:

$$a_r = \frac{R_{wH} - R_{wO}}{R_{wO}},$$
$$a_t = \frac{T_{wH} - T_{wO}}{T_{wO}}; \quad (2.1)$$

$$R_{wH} = R_{wO} \left[ 1 + \alpha (T_{wH} - T_{wO}) \right], \quad (2.2)$$

where $R_{wO}$ and $T_{wO}$ are the "cold" resistance and temperature and $\alpha$ is the temperature coefficient of resistivity. For these experiments $T_{wO} = T_{amb} \approx 297^\circ K$ and for Pt-10%Rd wire $\alpha = .0017^\circ K^{-1}$. Substituting equation 2.1 into equation 2.2 and multiplying by
\( T_{w_G} \) gives the proportionality:

\[
a_R = \alpha T_{w_G} a_T
\]  \hspace{1cm} (2.3)

which for these experiments is \( a_R = .5a_T \).

The resistance overheat ratio was determined by the ratio of the resistor values \( R_2 \) to \( R_3 \) on the hot-wire circuit bridge as shown in Figure 2.12. The value of \( R_2 \) was preset to the approximate expected resistance of the wire, so the ratio is adjusted by varying only \( R_3 \). The bridge is balanced such that:

\[
\frac{R_{w_H}}{R_2} = \frac{R_{w_G}}{R_2 R_3 + R_3}
\]  \hspace{1cm} (2.4)

which gives a resistance overheat ratio of

\[
a_R = \frac{R_2 + R_3}{R_3} - 1.
\]  \hspace{1cm} (2.5)

For the present experiments \( R_2 \) was 47Ω and \( R_3 \) was varied to give the resistance overheat ratios shown in Table 2.1. Selecting a particular overheat ratio is a tradeoff between acceptable drift and heat input into the flow. A low overheat ratio heats the fluid less, but at the same time causes the wire to be more sensitive to variations in \( T_{a_m} \).

Operating in a constant temperature mode the electric current through the hot-wire (and voltage from Ohm's Law) responds to the cooling of the wire due to both the flow velocity as well as changes in the ambient temperature. This relation is given by Kings' Law (King, 1914),

\[
I^2 R_{w_H} = (T_{w_H} - T_{a_m}) (A + BU_e^m)
\]  \hspace{1cm} (2.6)

\[
= \left( a_R / \alpha \right) (A + BU_e^m),
\]  \hspace{1cm} (2.7)
Table 2.1: Hotwire Overheat Ratios

<table>
<thead>
<tr>
<th>$R_3$</th>
<th>$a_R$</th>
<th>$a_T$</th>
<th>$T_{wH} - T_{wC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1K\Omega$</td>
<td>4.7%</td>
<td>9.3%</td>
<td>27.6°C</td>
</tr>
<tr>
<td>$630\Omega$</td>
<td>7.5%</td>
<td>14.8%</td>
<td>44.0°C</td>
</tr>
<tr>
<td>$475\Omega$</td>
<td>9.9%</td>
<td>19.6%</td>
<td>58.0°C</td>
</tr>
<tr>
<td>$230\Omega$</td>
<td>20.4%</td>
<td>40.5%</td>
<td>120.3°C</td>
</tr>
<tr>
<td>$150\Omega$</td>
<td>31.3%</td>
<td>62.1%</td>
<td>184.4°C</td>
</tr>
<tr>
<td>$94\Omega$</td>
<td>50.0%</td>
<td>99.0%</td>
<td>294.0°C</td>
</tr>
</tbody>
</table>

where $A$ and $B$ are constants characteristic of the heat transfer of the wire, and the exponent $m$ is dependent on the wire Reynolds number (it typically has a value of $0.3−0.6$). Neglecting cross-flow, the effective velocity $U_\varepsilon$ is commonly represented as:

$$U_\varepsilon = \sqrt{U_\perp^2 + c^2U_\parallel^2}.$$ (2.8)

Here, $U_\perp$ and $U_\parallel$ represent the components of velocity normal and parallel to the sensor axis, respectively. The scaler $c$ is a measure of the wire sensitivity to parallel cooling. Usually $c \ll 1$ and is taken as zero. However, experiments by Champange, Sleicher, and Wehrmann (1967) have shown that for standard hot-wire probes $c$ can attain values of .2. From Equation 2.6 it is evident that changes in $T_{amb}$ will change the wire current.
2.5 Hot-wire Anemometer Calibration

2.5.1 U-Probe Calibration

The single wire U-probe was calibrated before each experimental run in the free stream along the centerline of the test section. All calibrations were performed directly by the computer; no linearizers or signal conditioners were used. Typically eight velocities chosen between 0 and 12m/s were used. At each speed the hot-wire voltage and tunnel velocity, via the pilot-tube, were recorded. The mean values of each were fitted to a cubic polynomial using the least-squares method giving $U$ as a function of the hotwire voltage. The error of the polynomial fit $(U_{measured} - U_{true})/U_{true}$ at all calibration velocities, except zero, was kept to ±3%. The difference in the fit error before and after an experiment was monitored. If this drift error exceeded ±5% the probe was recalibrated and the run repeated.

2.5.2 Dual-Wire Probe Calibration

For dual-wire probes ($X$, $V$, and parallel-wire) three types of calibration schemes were employed: Standard Cosine-Law, Cartesian Look-up Table, and Polar Look-up Table. Each represents a successive improvement upon the previous one.

Cosine-Law

Calibration with this method assumes forced convection cooling of a heated cylinder according to King’s (1914) law. In implementing this law the flow component perpen-
dicular to each wire of the X-probe is assumed to be the effective cooling velocity. A calibration constant is necessary to relate velocity to voltage for each wire, but flow angles not perpendicular to each wire are simply estimated by a cosine relation, and hence the name. As detailed by Bradshaw (1971) the sum of the linearized wire voltages is proportional to the $U$-component of velocity, while the difference is proportional to the $V$-component. This traditional implementation assumes that the "zero" flow angle is along the probe axis and that the wires are at exactly 45° angles to the axis.

The probes used in these experiments however, are inclined toward the wall as much as 12° to avoid probe blockage, and the wires are at angles of 30° to 60° to the probe axis. These considerations require a special, more general, derivation of the Cosine-Law calibration technique. The probe is first calibrated at the desired inclination angle in a manner similar to the single-wire probe, at different velocities but at one "zero" reference angle. Each wire voltage curve is separately fit with a polynomial expression versus the wire effective velocity.

In Figure 2.13 let $\psi_1$ and $\psi_2$ be the angles each wire makes with the calibration flow $U$, and $U_1$ and $U_2$ the effective velocity each wire "sees", and $E_1$ and $E_2$ the respective hot-wire voltages. The angles $\psi_1$ and $\psi_2$ represent the combination of the probe inclination $\phi$ and the individual wire angles. In Figure 2.13: $\psi_1 = \alpha_1 + \phi$, $\psi_2 = \alpha_2 - \phi$, and from Section 2.3: $\alpha = \alpha_1 + \alpha_2 = \psi_1 + \psi_2$. The polynomial fit of the calibration is then,

$$U_1 = U \sin \psi_1 = a_0 + a_1 E_1 + a_2 E_1^2 + a_3 E_1^3 + \cdots$$

$$U_2 = U \sin \psi_2 = b_0 + b_1 E_2 + b_2 E_2^2 + b_3 E_2^3 + \cdots$$

where the $a_i$'s and $b_i$'s are the polynomial coefficients to be solved for by a least squares fit. This gives the desired relation between the wire voltage and the sensed velocity.
In an unknown flow the sensed velocities \((U, V)\) are related to the flow velocity and angle, \(Q\) and \(\theta\), by,

\[
U_1 = Q \sin(\psi_1 + \theta) \\
U_2 = Q \sin(\psi_2 - \theta).
\]

(2.10)

Solving for \(Q\) and \(\theta\),

\[
Q = \sqrt{\left[ \frac{U_1 + U_2}{1 \sin(\alpha/2)} \right]^2 + \left[ \frac{U_1 - U_2}{2 \cos(\alpha/2)} \right]^2} \\
\theta = \tan^{-1} \left[ \frac{(U_1 - U_2)}{(U_1 + U_2)} \tan \left( \frac{\alpha}{2} \right) \right] - \frac{1}{2} (\psi_1 - \psi_2),
\]

(2.11)

(2.12)

which results in the velocity components,

\[
U = Q \cos \theta \\
V = Q \sin \theta.
\]

(2.13)

In implementing this calibration scheme, 7 calibrations velocities between .7m/s and 12m/s were used. The error between the polynomial fit derivation of \(U\) and the actual calibration velocity was \(\pm 7\%\). The drift error for runs using this calibration method could only be kept to \(\pm 3\%\). Rotating the probe through various angles away from the calibration "zero" angle gave errors as high as \(\pm 15\%\) at 1m/s and 20°, as discussed in Section 3.4.

**Cartesian Look-up Table**

The Cartesian Look-up table calibration technique involves exposing the X-probe to different velocity components by orienting the probe at several different angles with respect to the calibration flow. This method is fully described by Leuptow, Breuer,
and Haritonidis (1988), and involves recording a unique voltage pair \((E_1, E_2)\) for each
given pitch angle \((\theta)\) and calibration velocity \((Q)\). A fine cartesian grid is laid out over
this calibration with the velocity component pair \((U, V)\) stored at each intersection,
where \(U\) and \(V\) are calculated from Equation 2.13. This process is best summarized in
Figure 2.14 and the following procedure.

- Each wire voltage is measured at \(m\) velocities \((Q_{i=1\rightarrow m})\) and \(n\) angles \((\theta_{j=1\rightarrow n})\) in
  the free stream. Each voltage pair \((E_1, E_2)\) corresponds to a unique velocity–angle
  pair \((Q, \theta)\) as shown in Figure 2.14a.

- Cubic-spline curves are fit through each constant angle line, and relations for
  \(E_2 = E_2(E_1)|_{\theta_j}\) and \(Q = Q(E_1)|_{\theta_j}\) are determined, as shown in Figure 2.14b.

- At a fine grid spacing of \(E_1\), the values for \(E_2\) and \(Q\) are evaluated and recorded,
  along each of the constant angle lines.

- Cubic-spline curves are fit along each \(E_1\) grid line to give relations for \(Q = Q(E_2)|_{E_1}\) and \(\theta = \theta(E_2)|_{E_1}\) as shown in Figure 2.14c.

- At a fine grid spacing of \(E_2\), the values of \(Q\) and \(\theta\) are evaluated and recorded at
  each interval of \(E_1\). Thus, each grid point based on regular intervals of \(E_1\) and \(E_2\)
  has a corresponding \(Q\) and \(\theta\) value as shown in Figure 2.14d.

- At each grid point the velocity–angle pair \((Q, \theta)\) can be converted to the velocity
  component pair \((U, V)\) giving calibration surfaces as shown in Figure 2.14e and f.

In an unknown flow, an arbitrary \((E_1, E_2)\) pair can be converted to \((U, V)\) by finding
the closest grid points and using bilinear interpolation. Two of the advantages of
this scheme are that the probe inclination and individual wire angles are not required

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and that the X-probe is directly calibrated through the intended range of $U$- and $V$-velocities. The probe inclination is of no consequence because the flow angle ($\theta$) is defined as the angle the flow vector makes with the wall not the angle made with the probe axis (which is $\theta + \phi$). The calibration flow measured at $\theta = 0^\circ$ becomes the reference “zero” flow regardless of any constant probe inclination offset. The major problem with this look-up table is the resolution of the grid at low velocities. In Figure 2.14 near the lowest velocity $Q$, there are only 1 to 3 boxes across the calibration range using a $40 \times 40$ grid. In measuring boundary layer profiles this is precisely the region most in question and therefore in need of a better resolution. A more dense grid could be used but this leads to an enormous amount of table storage. Doubling the grid quadruples the number of grid points, which then only gives a 2–6 box resolution at low velocities.

In the present experiments a $40 \times 40$ grid was used, based on raw calibration data at 11 angles and 7 velocities (typically, $-35^\circ \leq \theta \leq +35^\circ$ and $.75 \text{m/s} \leq Q \leq 12 \text{m/s}$). As expected, the error at low velocities was greater than at higher velocities. Below 2 m/s the maximum error with respect to $Q$ was 1.7% and for $U$ and $V$ was 1.9% and 2.3% respectively over the angular calibration range. Above 2m/s these errors dropped to .3% and .6% respectively. Drift with this method could be kept to $\pm 2\%$ for both components.

Polar Look-up Table

The polar look-up table calibration method was developed to correct the inconsistent grid resolution of the Cartesian look-up table. This method consists of converting $E_1$ and $E_2$ into polar coordinates to take advantage of the fan-like shape of the raw calibration data. The procedure is identical to the Cartesian table except references to $E_1$ and $E_2$
in the Cartesian procedure are replaced with the polar equivalents \( r \) and \( \beta \) adjusted for a translation of the origin.

\[
\begin{align*}
    r &= \sqrt{(E_1 - E_{1\text{origin}})^2 + (E_2 - E_{2\text{origin}})^2} \\
    \beta &= \tan^{-1} \left[ \frac{E_2 - E_{2\text{origin}}}{E_1 - E_{1\text{origin}}} \right]
\end{align*}
\]  

(2.14)

The replacement gives the result that now at each polar grid-point \((r, \beta)\) the velocity components \((U,V)\) are known. A minor difficulty in this scheme is selecting the origin of the coordinate system, which is outlined in the following procedure. A note of caution is necessary about the distinction between lines of constant \( \beta \) and lines of constant \( \theta \). On the plots, such lines are similar, but \( \beta \) lines are straight (radial) grid lines, while \( \theta \) lines are polynomial fits to the calibration data points; they are not the same.

- On the \( E_1, E_2 \) cartesian coordinate system, straight lines are fit between the calibration point at the minimum velocity and maximum angle, pair \((Q_1, \theta_n)\) in Figure 2.15, and the calibration points at each successive velocity with maximum angle, pairs \((Q_2, \theta_n) \ldots (Q_m, \theta_n)\). The line with the minimum slope \((S_{\text{min}} = \min(S_i))\) is chosen as the lower edge of the polar coordinate system.

\[
S_i = \frac{E_2(Q_i, \theta_n) - E_2(Q_1, \theta_n)}{E_1(Q_i, \theta_n) - E_1(Q_1, \theta_n)} \quad \text{where } i = 2 \to m
\]  

(2.15)

- The upper edge of the polar coordinate system is chosen in the same way but by selecting the line with maximum slope \((S_{\text{max}})\). The lines tested in this case are between \((E_1, E_2)(Q_1, \theta_1)\) and \((E_1, E_2)(Q_i, \theta_1)\).

- The intersection of these two edge lines is the origin of the polar coordinate system.

Define the dummy variables \( B_1 \) and \( B_2 \) to be,

\[
B_1 = E_2(Q_1, \theta_n) - S_{\text{min}}E_1(Q_1, \theta_n)
\]  

(2.16)
\[ B_2 = E_{2(Q_1,s_1)} - S_{\text{max}}E_{1(Q_1,s_1)}, \]  

(2.17)

which gives the origin of the polar coordinate system at:

\[ E_{1_{\text{origin}}} = \frac{B_2 - B_1}{S_{\text{min}} - S_{\text{max}}} \]  

(2.18)

\[ E_{2_{\text{origin}}} = S_{\text{min}}E_{1_{\text{origin}}} + B_1 \]

In summary, the entire procedure is shown in Figure 2.15 and is as follows:

- Determine \((E_1, E_2)_{\text{origin}}\), from Equation 2.18.
- Convert all calibration pairs \((E_1, E_2)\) to the polar-coordinate system coordinates \((r, \theta)\), using Equation 2.14.
- Using cubic-spline fits determine \(\beta = \beta(r)|_{\theta_j}\) and \(Q = Q(r)|_{\theta_j}\).
- At regular intervals of \(r\), determine \(\beta\) and \(Q\) at constant \(\theta\).
- Using cubic-spline fits determine \(Q = Q(\beta)|_{\theta}\) and \(\theta = \theta(\beta)|_{r}\).
- At regular intervals of \(\beta\), determine \(Q\) and \(\theta\) which gives \((Q, \theta)\) at every \((r, \beta)\) grid point. Finally convert \((Q, \theta)\) to \((U, V)\) using Equation 2.13.

The advantage of this calibration table is that the grid resolution is the same for all velocities. There are as many grid boxes across the angular sweep at maximum velocity as there are at the minimum velocity. Additionally, there is no wasted storage of the table. The cartesian table has two large triangular regions of "blank" coordinates in the upper-left and lower-right portion of the table as seen in Figure 2.14e and f. In contrast, the polar table covers only the necessary calibration region.

A 40 × 40 polar grid was used based on raw calibration data at 11 angles and 7 velocities (typically, \(-35^\circ \leq \theta \leq +35^\circ\) and \(0.75\text{m/s} \leq Q \leq 12\text{m/s}\)). The error at
low velocities was still greater than at higher velocities, but only slightly. Below 2 m/s the maximum error with respect to Q was .3% and for U and V was .4% and .5% respectively over the angular calibration range. Above 2m/s both of these errors dropped to .1%. Drift with this method was kept to the same ±2% for both components.

2.5.3 Characteristics of the Calibration Plots

There are several characteristics of plotting the calibration data points on the voltage (E₁, E₂) plane that are of importance for future discussion. The general shape of the central portion of the calibration points is fan-like. Measurements at constant flow angles with variable speed yield roughly a straight line, whereas the calibration points at constant velocity and variable flow angle approximately take on the shape of a segment of an arc. This general shape was taken advantage of using a polar rather than cartesian look-up table. Calibration at large flow angles causes a folding of data points into the region of calibration already spanned by points measured at smaller angles. This behavior is discussed by Johnson and Eckelmann(1984) and can be accounted for mathematically by allowing the wire sensitivity factor in King’s law to be imaginary. The angles where the curves fold back depend mostly on the wire angles α₁ and α₂, the sensors inclination φ and the orientation of the sensor supports are discussed by Johnson and Eckelmann(1983), and can be different at different flow speeds. The value for the fold angles (or angular egde of the grid) is found for a Cartesian Look-up Table at the points where \( dE_2/dE_1 = 0 \) or \( dE_1/dE_2 = 0 \) along a constant velocity arc. The Polar Look-up Table has a slightly extended range, and the fold angles occur at \( d\beta/dr = 0 \). For both tables, the fold angles mark the range where a one-to-one correspondence between flow angles and velocity and grid coordinates exist. Flow beyond these angles
results in previously defined grid coordinates which cannot be interpreted in a unique way. In the present tests, the largest common fold angles (positive and negative) were selected as the angular calibration range.

In the ideal case, the measurements at constant flow angles and variable speed give straight lines radiating from the polar grid origin. In actuality, these lines are not straight and tend to asymmetrically "neck" together at low velocities. The asymmetry causes the lower portion of the fan to skew to one side, and is largely a result of the probe inclination ($\phi$). Setting $\phi = 0$ increases the symmetry of the lower end, but does not change the "narrowing" effect. The narrowing of the fan is linked to the change in shape of the constant velocity arcs. At high velocities, note in Figure 2.16 how the arcs are convex, approximating the Cosine law. As the speed decreases the arcs become flat (around 3 m/s in the figure), and finally become highly concave at lowest speeds. Above the "flat" arc the lines of constant angle are straight, while below, the necking effect becomes pronounced. These characteristics of the calibration plots are also found in two other works: Johnson and Eckelmann (1984) and Willmarth and Bogar (1977). But besides these two works, no other investigations using a table look-up scheme could be found where the calibration plot was actually constructed and reported. A final point to be made about the calibration plot in the location of the true ($Q = 0$, $\theta = 0^\circ$) origin. A data point measured at zero velocity, before calibration, as experienced by Johnson and Eckelmann (1984), and shown in Figure 2.17, is located below the symmetry axis corresponding to measurements at $\theta = 0^\circ$. However, starting at some finite velocity and allowing the speed to steadily decrease to zero, the measurements follow the symmetry line to almost the exact point chosen as the origin for the Polar table. But as the tunnel settles after the calibration the measurement slowly moves off of the line of symmetry, ending back at the pre-calibration point. The origins differ only in that the value of $E_1$
is too low, which implies that the difference is probably due to a convection flow induced by the hot-wires themselves. The induced flow becomes effective when no external flow is present thereby causing the drift. The origin ends up below the line of symmetry because the upper wire ($E_1$) is influenced by a convection flow which has been set up by the lower wire. Since it takes approximately a minute to observe the full change in origins as the tunnel velocity comes to rest, the origin selected for the polar grid is better representative of the true origin.

2.6 Drift Minimization

Hot-wire anemometer drift is caused by several factors: particles hitting the wires, impurities burning off, ambient temperature ($T_{amb} = T_{w_0}$) changes, poor solder connections, electronic drift in the anemometer circuit, variations in the tunnel velocity, and probe vibration (primarily during probe movement) to name a few. It is difficult to measure the effects of each of these factors independently, but combined, the common element among them is a time dependence. Therefore, to minimize the error due to drift all experiments were conducted as quickly as possible immediately following a calibration. The entire process of calibration and experiment run was typically 45 minutes. In addition, the tunnel was allowed to warm up for three hours to stabilize the temperature. Finally the hot-wire anemometer electronics rack was left on continuously and covered to avoid temperature fluctuations in circuit components. Figure 2.18 shows the drift in $Q$ over a long period of time for an X-wire, type X1.2. The drift curve was typical of all probe body configurations, wire diameter and sensing box size. As would be expected from the list of possible drift sources, the drift rate was less when the tunnel is not running as evidenced in Figure 2.18.
2.7 Boundary-layer Measurement Procedure

Boundary-layer profiles were measured moving from the wall out. This method is advantageous in that the wall region most sensitive to drift is measured first, immediately after calibration. The starting position for the hot-wire probe was chosen where the measured local mean velocity, $\bar{U}$, gave $\bar{U}/U_\infty \approx .10$. Moving any closer to the wall ran the risk of destroying the probe. The probe was moved away from the wall in exponentially increasing increments, pausing at each to take data, until the edge of the boundary layer was reached. The total sampling duration for each data set was 12.33s, or about $2100\delta/U_\infty$, where $\delta/U_\infty$ is the outer time-scale. For boundary-layer profiles, the digitizing rate was $301\mu s$ ($3.75t_\kappa$) at a sampling frequency of 3.3KHz. The edge of the boundary-layer was determined to have been reached at a point $(n - 2)$, when,

$$\frac{\bar{U}_n - \frac{1}{3} \sum_{i=0}^{2} U_{n-i}}{\frac{1}{3} \sum_{i=0}^{2} U_{n-i}} \leq .001.$$  \hspace{1cm} (2.19)

2.8 Determination of Inner Scales

Inner, otherwise known as wall, scales are determined from the friction velocity, $u_r$, and kinematic viscosity, $\nu$. The friction velocity is defined to be $u_r \equiv \sqrt{2\tau_w/\rho}$, where $\tau_w$ is the shear stress at the wall. The basic assumption is that the law of the wall (as it has evolved in the work of Prandtl (1925), Ludwig and Tillmann (1950) and Clauser (1954) is universally valid. Except in the presence of very strong pressure gradients, this is accepted as true, as shown by Coles (1964) and Coles and Hirst (1968). The procedure is to look for a $u_r$ such that the "logarithmic" region of the profile near the
wall is best fitted to the law of the wall,

$$\frac{\bar{U}}{u_r} = \frac{1}{\kappa} \ln \frac{y u_r}{\nu} + C, \quad (2.20)$$

where \( \kappa = .41 \) is Von Karman's constant and the constant \( C = 5.0 \) (Coles, 1964, Clauser, 1954, Townsend, 1976). In addition, the inner scales of length and time are given by

$$\ell_* = \frac{\nu}{u_r} \quad \text{and} \quad t_* = \frac{\nu}{u_r^2} \quad (2.21)$$

Variables non-dimensionalized by these scales are denoted with a "+" superscript. For example Equation 2.20 cast in "+" variables becomes: \( \bar{U}^+ = 1/\ln y^+ + C \).

The inner scales for these experiments are listed in Table 2.2.

### 2.9 Determination of Position

The distance moved in the \( y \)-axis relative to the starting position was kept track of by the controlling computer program; however, the starting distance from the wall was unknown beforehand forcing data point #1 initially to be at \( y = 0 \). The procedure to determine the required \( y \)-offset is to translate the non-dimensionalized profile in \( y^+ \) until the wall and buffer regions of the profile are best fit by the model proposed by Haritonidis (1987), which links the linear and logarithmic profile regions. This model, called the arctangent law for lack of any other, is

$$\bar{U}^+ = \frac{1}{\lambda} \tan^{-1}(\lambda y^+) \quad \text{over the range} \quad 0 \leq y \leq y_o^+ \quad (2.22)$$

for a zero pressure gradient flow. The constant, \( \lambda \), is determined by iteratively matching the arctangent law and logarithmic law as well as their first derivatives. This results
in \( \lambda = .0936 \), and a joining of the two laws at \( y_o^+ = 47.6 \). The use of this model may be new, but after contemplation of several hundred mean-velocity profiles there is no doubt that this law is a very good fit to the linear and buffer region profiles.

### 2.10 Calculation of Integral Profile Parameters

The displacement thickness \((\delta^*)\) and momentum thickness \((\theta)\) given by,

\[
\delta^* = \int_{y=0}^{\delta} \left(1 - \frac{U}{U_\infty}\right) dy
\]

\[(2.23)\]

\[
\theta = \int_{y=0}^{\delta} \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right) dy
\]

\[(2.24)\]

have been calculated using a modified trapezoid rule to integrate a piece-wise fit to the data from the wall to the free stream. The method combined an analytical integration of the arctangent-law from the wall to its joining the log-law at \( y_o \), together with integration of a piece-wise cubic spline fit of the data from \( y_o \) to the free stream. The integration of the piece-wise cubic spline fit is accomplished by using the analytic integral of each piece of the spline. From \( y_o \) outward, this method is functionally similar to the modified Simpson’s scheme used by Coles and Hirst (1968). Using the arctangent-law below \( y_o \) provides a more systematic approach in the important wall region.

The boundary layer thickness \((\delta)\) was determined as the total distance from the wall to the data stopping point \((n - 2)\) as determined from Equation 2.19. The thickness found by this method gives values surprisingly similar to the boundary layer thickness calculated using the \(1/7^{th}\)-power law expressions for \( \delta \) as functions of \( \delta^* \), and \( \theta \), given by Schlichting (1979).

\[
\delta = 8\delta^* \quad \text{and} \quad \delta = \frac{72}{7} \theta
\]

\[(2.25)\]
In some experiments, the original mean-velocity profiles do not level off at $\bar{U}/U_\infty = 1$ in the free stream. In such cases, $U_\infty$ is set to the maximum value of $\bar{U}$ attained ($\bar{U}_{\text{max}}$) in order to avoid anomalies in the integrals.

2.11 Calculation of Statistical Moments

Of importance is the calculations of the statistical moments of $U$, $V$, and $W$. These moments: mean, variance, skewness, and kurtosis or flatness, provide fairly established descriptive characteristics of the boundary layer profile. Calculation of these moments was done using a binomial expansion of the traditional definitions, shown in Equations 2.26–2.29, using the $U$-velocity component and $N$ data samples.

\[
\text{Mean} = \bar{U} \equiv \frac{1}{N} \sum_{i=1}^{N} U_i \tag{2.26}
\]

\[
\text{Variance} = u'^2 \equiv \frac{1}{N} \sum_{i=1}^{N} (U_i - \bar{U})^2 = \frac{1}{N} \sum_{i=1}^{N} u_i^2 - \bar{U}^2 \tag{2.27}
\]

\[
\text{Skewness} = S(u) \equiv \frac{1}{N} \sum_{i=1}^{N} \left( \frac{U_i - \bar{U}}{u'} \right)^3
\]

\[
= \frac{1}{u'^3} \left[ \frac{1}{N} \sum_{i=1}^{N} U_i^3 - \frac{3\bar{U}}{N} \sum_{i=1}^{N} U_i^2 + 2\bar{U}^3 \right] \tag{2.28}
\]

\[
\text{Flatness} = F(u) \equiv \frac{1}{N} \sum_{i=1}^{N} \left( \frac{U_i - \bar{U}}{u'} \right)^4
\]

\[
= \frac{1}{u'^4} \left[ \frac{1}{N} \sum_{i=1}^{N} U_i^4 - \frac{4\bar{U}}{N} \sum_{i=1}^{N} U_i^2 + \frac{6\bar{U}^2}{N} \sum_{i=1}^{N} N U_i^2 - 3\bar{U}^4 \right] \tag{2.29}
\]

One difference in these formulas from the exact ones is that the variance should be calculated by dividing by $N - 1$ instead of $N$, but for large data sets this point is mute.
Statistical purists (Bevington, 1969, and Press, Flannery, Teuholsky, and Vetterling, 1986) recommend avoiding the binomial expansion of the moment expressions due to accumulated round-off error and only slightly faster computing speed. The problem however, is that an excessive amount of computer memory allocation is necessary to store the entire array of data. The expanded equations were chosen because only the four sums of \( U, U^2, U^3, \) and \( U^4 \) are necessary, and these could be updated in real-time as the data was acquired. Errors were minimized by using double-precision variables and arithmetic. To test the accuracy, both methods were compared to known analytic functions as well as actual signals; no differences were found. Two other statistical quantities were kept: the Reynolds shear stress, \( \overline{u'v'} / u_r^2; \) and the cross-correlation coefficient \( R_{uv} \equiv \overline{u'v'}. \) These were calculated in real time using the expansion:

\[
\overline{uv} = \frac{1}{N} \sum_{i=1}^{N} u_i v_i - \overline{U} \overline{V}. \tag{2.30}
\]

The Reynolds stress can be estimated in the wall region using the constant stress layer assumption, which can then be used as a guide to the measurements. From Schlichting (1979) the Reynolds shear stress, obtained from the Navier Stokes equations for turbulent incompressible flow is

\[
\tau_{xy} = \mu \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right) - \rho \overline{uv}, \tag{2.31}
\]

where \( \tau_{xy} \) is the shear stress, \( \mu \) the fluid viscosity and \( \rho \) the density. Neglecting the variation of the \( V \)-velocity component \( (\partial V / \partial x \approx 0) \) and assuming \( \tau_{xy} = \tau_w \), where the viscous stress at the wall is defined by \( \tau_w \equiv \rho u_{*r}^2 \), gives

\[
\left( \frac{\mu}{\rho u_r} \right) \left( \frac{\partial U}{u_r \partial y} \right) - \frac{\overline{uv}}{u_r^2} = 1. \tag{2.32}
\]

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In viscous scales (+ units) this becomes
\[ \overline{uu}^+ = \frac{\partial U^+}{\partial y^+} - 1. \] (2.33)

Differentiating the arctangent law, Equation 2.22, and logarithmic law, Equation 2.20, the Reynolds stress can be calculated in the wall, buffer and logarithmic regions as,
\[ \overline{uu}^+ = \begin{cases} \frac{1}{\lambda^2 y^+ + 1} - 1 & 0 \leq y^+ \leq y^+_c \\ \frac{1}{\kappa y^+} - 1 & y^+_c \leq y^+ \leq 300, \end{cases} \] (2.34)

where \( y^+_c \) is the position where the arctangent and logarithmic laws meet as determined in Section 2.9. Equation 2.34 appears as a solid line in graphs of the Reynolds stress.

### 2.12 Flow Parameters

Most of the experiments were conducted at a free-stream velocity, \( U_\infty = 11 \text{m/s} \) at a location \( X = 401.5 \text{cm} \) downstream from the leading edge and on the center-line of the flat plate. This location was chosen because it corresponds to a plexiglass plug insert in the flat plate (as seen in Figure 2.2) and was similarly used by Her(1986) and Johanson, Her and Haritonidis(1987). Measurements over the plexiglass plug were desired to minimize the effects of heat conduction to the wall thereby enabling accurate measurements even in the viscous sublayer. The free-stream velocity used by Her, \( U_\infty = 10 \text{m/s} \), was initially tried in these experiments, but was changed because the boundary-layer trip used by Her proved to be inappropriate. The details of the problem of trip selection are discussed in Section 3.3. For these experiments, the flow was tripped (to artificially cause transition from a laminar to a turbulent boundary layer) 10cm downstream of the leading edge using DYMOM® brand embossing tape imprinted with
the letter "V". The strip spanned the entire test section (floor to ceiling) and was attached using the self-adhesive backing with the V's pointing downstream. The letters were 5mm (.2 inches) tall embossed onto 1.27cm wide black color-impregnated matte-finish embossing tape with a uniform embossed height of .64mm. Figure 2.19 includes all the pertinent measurement of this trip, which was formed by a DYMO® labelmaker "gun" model 1540-00 (made by Esselte Pendaflex Corp.).

Based on the measurements with the single-wire U-probe, the characteristics of the typical turbulent boundary layer and mean flow parameters used for most of these experiments are shown in Table 2.2.
Table 2.2: Mean Flow Parameters and Viscous Scales

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- $X = 401.5$cm
- $\nu = 0.1513$cm$^2$/s
- $Re_X = 2,347,321$
- $Re_{\delta^*} = 6318$
- $Re_\theta = 4589$
- $H = 1.377$
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\* \(\ell_a = 34.8 \mu m\).
### Table 2.3: Continued. X-Probe Dimensions

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<th>C (μm)</th>
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* \( \ell_s = 34.8\mu m. \)

### Table 2.4: V-Probe Dimensions

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* \( \ell_s = 34.8\mu m. \)
Figure 2.1: Wind Tunnel Diagram
Figure 2.2: (a) Test Section
Figure 2.2: (b) Flat plate leading edge geometry
Figure 2.3: Sketch of general hot-wire probe types
Figure 2.3: Continued. Sketch of general hot-wire probe types
Figure 2.4: Close-up of probe solder joint and wire junction
Figure 2.5: X-probe configurations and dimensions
Figure 2.6: Scale drawing of X-probe X3, enlarged to show detail
Figure 2.7: Parallel-wire probe
Drawn to Scale

Figure 2.8: Split-film probe
Figure 2.9: Deposition mask used for the construction of the split-film sensor
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Figure 2.12: Hot-wire anemometry circuit diagram
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Figure 2.14: Development of the Cartesian Calibration grid: (a) voltage output pairs $E_1$ and $E_2$, at $n$ angles, $\gamma$, and $m$ velocities, $Q$; (b) $n$ cubic spline fits along each angle for both $E_2$ and $Q$; (c) $E_2$ and $Q$ evaluated at regular intervals of $E_1$; (d) $Q$ and $\gamma$ evaluated at regular intervals of $E_2$; (e) X-probe calibration surface $U$-velocity plane; (f) $V$-velocity plane. From Lueptow, Breuer, and Haritonidis (1988).
Figure 2.15: Development of the Polar Calibration grid: (a) the origin of the polar grid is found from the maximum angular width of the output voltages $E_1$ and $E_2$, at $n$ angles, $\theta$, and $m$ velocities, $Q$; (b) $n$ cubic spline fits along each angle $\theta$, for both $Q$ and $\beta$, the angular polar coordinate as functions of $r$, the radial polar coordinate.
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Figure 2.16: Characteristics of the Calibration plot. A sparse grid is drawn so that the shape of the plotted voltage pairs \((E_1, E_2)\) can be easily seen. This grid, like all other polar calibration look-up tables used was actually constructed using 40 angular by 40 radial grid lines.
Figure 2.17: Location of calibration grid origin as shown by Johanson and Eckelmann (1984) on their plot of the region of calibration measured for a hot-film X-probe. Speeds given in cm/s.
- Error in $u$ at $U_{\text{inf}} = 4$ m/s
- Error in $v$ at $U_{\text{inf}} = 4$ m/s
- Error in $u$ at $U_{\text{inf}} = 11$ m/s
- Error in $v$ at $U_{\text{inf}} = 11$ m/s

Bars represent the error values at extreme flow angles.

Figure 2.18: Typical hot-wire drift error as a function of time
Figure 2.19: Diagram of DYMO brand embossing tape letter "V" trip
Chapter 3

Non Probe-Specific Errors

This chapter deals with sources of error in measuring velocity in a turbulent boundary layer profile, and the steps taken to correct them. There are three general sources of error: error in the flow, error in the measurement of that flow, and error in the reduction of those measurements. Flow error, for example, would be caused by not having a fully developed turbulent boundary layer as expected. Measurement errors are caused by how the sensors “read” the flow. And finally reduction errors are caused by improper reading of the sensors or incorrect calculations based on the measurements. Since the end result contains all three errors it is best to systematically isolate them: eliminating one before going on the next. In keeping with this approach the obvious measurement errors due to probe effects (the reason so many different probe configurations were tried) will be handled in Chapter 5, allowing this chapter to concentrate on the non probe-specific errors.

3.1 Reduction Error: Sampling Rate Selection

To process the data in real-time on the computer it was necessary to take data in blocks less than approximately 4096 points, due to the limited memory of the PDP-11/55, and then ensemble their statistics together. This results in four variables to
be selected: the total acquisition time, the digitizing rate, the number of blocks or ensembles, and the number of points per block.

Initially the total digitizing time was selected as 3000 times the outer time-scale \((3000\delta/U_\infty)\), which corresponds to the time to see 1000 outer-scale structures times a safety margin of 3. To calculate \(\delta\) a priori, the \(1/7^{th}\)-power law was used based on a fully turbulent boundary layer growing from the leading edge, given by Schlichting (1979):

\[
\delta = 0.37 \frac{x}{Re_z^{1/5}} \quad \text{where} \quad Re_z = \frac{U_\infty x}{\nu} \tag{3.1}
\]

resulting in a total acquisition time of,

\[
\text{Total time} \approx 1000 \frac{x}{U_\infty Re_z^{1/5}} \tag{3.2}
\]

Evaluating at \(x = 401.5\) cm and \(U_\infty = 11\) m/s gave a total time of 18.5s. A digitizing rate was also selected, initially chosen to be 3 times the inner time-scale \((3t_*)\). The inner-time scale \(t_*\) was calculated a priori using Equation 2.21 and the \(1/7^{th}\)-power law expression for \(u_r\).

\[
u_r = 0.172U_\infty Re_z^{-1/10} \tag{3.3}
\]

Substituting in values gives a digitizing rate of approximately 300\(\mu\)s which, over a total time of 18.5s, results in approximately 61,440 samples. This large number of samples ensures accuracy in calculation of the statistical moments to .4\% (Accuracy\(\sim 1/\sqrt{N}\)).

Optimizing for fastest computer program execution gave a computer block size of 2048 points, to be taken 30 times. The number of ensembles was later changed from 30 to 20, reducing the total number of data points, to reduce the total experiment time. This reduction did not change any of the statistics. In addition, doubling the
Table 3.1: Sampling Parameters

<table>
<thead>
<tr>
<th>Digitizing Rate</th>
<th>300,\mu s</th>
<th>Sampling Frequency</th>
<th>3.33,kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Ensembles</td>
<td>20</td>
<td>Points per Ensemble</td>
<td>2048</td>
</tr>
<tr>
<td>Total Number of Points</td>
<td>40960</td>
<td>Total Time</td>
<td>12.28s</td>
</tr>
</tbody>
</table>

The optimized sampling parameters used for turbulent boundary layer profile measurements are listed in Table 3.1.

The sampling parameters used by Her (1986) and Johansson, Her, Haritonidis (1987) were comparable, though not optimized for computer speed. The total time for their experiments was 13.4s at a rate of 101\,\mu s, taken in 65 groups of 2048 points. Any error in the statistical moments reported by Her due to his sampling parameters is therefore negligible.

3.2 Error in Calculation of Statistics

The propagation of error in the calculation of all variables was checked. Two possible categories of error were artificially introduced and allowed to contaminate the calculation procedure. The two error categories are $U_{error}$ and $(U/U_\infty)_{error}$, which seem similar but are quite different. The $U_{error}$ represents a usually constant or linear-with-time error in the measurement of $U$ by the hot-wire. Since $U_\infty$ is chosen as the maximum value of $U$, to force $\bar{U}/U_\infty$ to be unity at the boundary layer edge, $U_\infty$ is affected by the same error as $U$. The $U_{error}$ represents a joint error in $U$ and in $U_\infty$ such that $(U/U_\infty)_{measured} = (U/U_\infty)_{true}$. On the other hand, the $(U/U_\infty)_{error}$ term represents an independent
error in either $U$ or $U_\infty$ but not both. The distinction may not seem like much, but the
resulting propagation of errors is very different. In general $U_{error}$ models linear errors
such as drift over a short run, while $(U/U_\infty)_{error}$ accounts for non-linear errors such
as variations in the tunnel velocity. Unfortunately errors in the measurements contain
both categories of error, inseparable from one another.

3.2.1 Error of the Type $U_{error}$: Joint Error

A worst case error scenario was made by assuming the error to be some constant
percentage ($\varepsilon$) of the measured velocity: $U_{measured} = U_{true} + \varepsilon U_{true}$, throughout the
boundary layer. The actual error is probably a function of position and time, varying
both positively and negatively, and therefore, better than the worst-case constant model
chosen. A turbulent boundary layer profile as measured by a U-probe was used as the "true" (reference) case and known errors were artificially introduced as described
above. The error was added to all values of the reference profile, $U_\infty$ was picked, a
new $u_r$ was determined from a Clauser plot using the new data, and all other variables
recalculated in the normal way. The error was then computed, using the momentum
thickness $\theta$ as an example, as $(\theta_{new} - \theta_{true})/\theta_{true}$. Since only the change in values from
the reference case to the error case is of importance, any error in the original profile is
inconsequential. Indeed, using the $1/7^{th}$-power velocity distribution law (Schlichting,
1979) as the reference case made no difference in the resulting errors. Presented in
Figures 3.1 are the error changes resulting in several parameters based on introducing
-3% to +3% error in $U$. The error in most of the variables in the figures is as expected:
the error is approximately the same as the error in $U$. Of particular interest however,
is the insensitivity in the error of the integral moments. The error in the momentum
thickness, for example, is zero for all \( U_{\text{error}} \).

### 3.2.2 Error of the Type \((U/U_\infty)_{\text{error}}\): Independent Error

A worst case scenario for this type of error was made by using a constant percentage error in \( U \) as before, but this time fixing \( U_\infty \) to the reference case \( U_{\infty_{ref}} \), instead of letting it float to \( U_{\text{max}} \). This model then introduces a linear error in \((U/U_\infty)\) as \((U/U_\infty)_{\text{measured}} = (U/U_\infty)_{\text{true}} + \epsilon(U/U_\infty)\). Again this function is "worst-case", the actual error being non-linear and oscillating about the true value. Also, with this model an anomaly arises in that the velocity profile does not level off at \( U/U_\infty = 1 \) in the free stream, a case that is specifically avoided by renormalizing the profile by \( U_{\text{max}} \). The model is still valid because the \( U/U_\infty = 1 \) criteria could have been easily met by having a jump in \( U \) at the end of the boundary layer. Again, a measured profile was used as the reference with errors in \((U/U_\infty)\) ranging from -3\% to +3\%. The resulting errors, as seen in Figure 3.2, are not as expected. Errors in \((U/U_\infty)\) result in errors of the same order in \( u_r, \ell, \ell_*, \) and \( \Delta U^+_{\text{max}} \), but rise dramatically in the integral moments. A 1\% error in \( U/U_\infty \) results in a worst-case error in the momentum thickness of approximately 14\%, and a 2\% error in \( U/U_\infty \) gives approximately a 30\% error. The error in the shape factor \((H = \delta^*/\theta)\) is also worth mentioning. If \( U \) is less than \( U_\infty \) at the boundary layer edge the errors in \( H \) are of the same magnitude as \((U/U_\infty)\), while overshooting \( U_\infty \) results in exponentially increasing errors.

The analysis here is admittedly the worst-case, but it does indicate the extreme sensitivity of the integral moments to errors in \((U/U_\infty)\). To get accurate values of \( \delta, \delta^*, \) and \( \theta \) the necessary measurements must be ultra-precise, and conversely, large errors in the integral moments could easily result from otherwise normally acceptable error in
the mean profile.

3.3 Flow Errors: Trip Selection

Most flow peculiarities in the MIT Low Turbulence Wind Tunnel have been eliminated, or noted to be avoided, as discussed by Mangus (1984), Cohen (1987) and Breuer (1988). However, proper trip selection to cause transition from laminar to turbulent flow that will develop into a fully developed turbulent boundary layer is still an area to be investigated. The experiments performed in this section were not intended to probe the mechanics of transition, just to select a consistent tripping mechanism for the MIT wind tunnel.

A trip is necessary because normal transition occurs too far downstream on the plate. According to theory derived by Van Driest and Blumer (1963) and tests performed by Schubauer and Skramsted (1947) the Reynolds number based on the transition location downstream of the leading edge (Re_{x,tr}) is approximately 2,800,000. Based on \( U_\infty = 11\text{m/s} \) and \( \nu = .15\text{cm}^2/\text{s} \) this criterion results in a transition location of 338 cm (leaving only 13cm for the turbulent boundary layer to develop by the desired measurement location). The side wall boundary layer and free stream turbulence tend to cause transition slightly sooner, but overall the untripped boundary layer would not be sufficiently developed. Several trip designs were tested in an effort to force transition earlier and move the transition location upstream near the leading edge. The criteria sought for the trip were as follows:

- That the trip be easily removable without distorting or leaving residue on the
plate.

- That the trip was consistent: that it and the flow it produced be the same each time it was installed.

- That the turbulent boundary layer formed is fully developed at the point of measurement.

The trips attached to the plate were positioned at \( z = 10 \text{cm} \) to be able to use the seam between the leading edge and the plate as a reference for placement and to ensure the trip was attached perpendicular to the flow. The specific trips used are as follows:

1. Velcro\textsuperscript{®} strip - heavy duty type at \( z = 10 \text{cm} \) (Figure 3.3, as used by Her (1986)).

2. Dymo\textsuperscript{®} brand embossing tape with 5.1mm letters at \( z = 10 \text{cm} \) (Figure 3.4)
   - Three tape types (Black and Orange Dymo\textsuperscript{®} brand, and Red Scotch\textsuperscript{®} brand) producing different letter (symbol) thicknesses.
   - Two Orientations: symbol pointing upstream or downstream.

3. Wire trip, 0.356mm diameter, (Figure 3.5)
   - Two locations: \( z = 20 \text{cm} \), and 30cm.
   - Two heights: On the wall, and suspended at \( y = 1 \text{mm away from wall} \).

4. Rivet Trip at \( z = 10 \text{cm} \) (Figure 3.6)

5. Sand Paper, 11.4cm wide, #36 grit at \( z = 10 \text{cm} \) as used by Mangus (1884).

The Orange Dymo\textsuperscript{®} brand tape produced the thickest letter, but had to be eliminated because the orange colored glue would separate from the "clear" tape. The
residue left on the plate was very difficult to remove, so the color-impregnated tapes were used instead. Several analyses of wire trips by Gibbings (1959), Goldstein (1936), Fage and Preston (1941), sphere trips by Hall (1967) and sand-paper trips by Doenhoff and Horton (1956) and Feindt (1957) have been conducted all with mixed results. There are too many variables that define a "good" trip, and unfortunately trial and error is the best selection method. Of general consensus though, is that the average height of the trip \( K \) must be greater than a third of the displacement thickness but not taller than the boundary layer-thickness at the trip location (White, 1971). As shown in White, for a Blasius profile:

\[
\delta = 4.8 \frac{x}{\sqrt{Re_x}},
\]

\[
\delta^* = 1.743 \frac{x}{\sqrt{Re_x}},
\]

which results in \( \delta = 1.773 \text{mm} \) and \( \delta^* = .6436 \text{mm} \) at \( x = 10 \text{cm} \) and \( U_\infty = 11 \text{m/s} \). The requirement that at the trip \( \delta > K > .35^* \), was met by all trips, except the Velcro\(^\text{®} \), and rivet trip. Even though it failed this requirement, the Velcro\(^\text{®} \) was still tested because it had been used by Her(1986).

As a preliminary test, each trip was positioned at \( x = 10 \text{cm} \) and a U-probe placed at 33.6cm (just ahead of the first plexiglass plug) to measure the turbulent transition velocity. With \( U_\infty \) set at approximately 20m/s the U-probe was positioned at \( y \) where \( \overline{U}/U_\infty = .5 \). The tunnel velocity was slowly decreased until the character of the hot-wire signal abruptly changed from fully turbulent to transitioned or laminar flow. This transition velocity was noted for each trip configuration and is shown in Table 3.2. The best case(s) (lowest transition velocity) for each of the five trip types were selected for further testing. The staggered double row of V's had the lowest transition velocity but was difficult to duplicate for repeated installations. The slight decrease in \( U_\infty \) transition was
not worth the complexity of the staggered setup and was dropped in favor of the single V strip.

The next test was to measure how each of the trip generated boundary layers compared to a normal (equilibrium, fully developed) turbulent boundary layer. Two parameters are particularly sensitive to this measure of "normalcy"; the strength of the wake component $\Delta (\bar{U}/u_\tau)_{\text{max}}$ (or alternatively $\Delta \bar{U}^+_{\text{max}}$) and the shape factor ($H$). The sensitivity of $\Delta \bar{U}^+_{\text{max}}$ is extensively used by Coles (1964) to precisely categorize boundary-layer flows, while a sensitivity of $H$ to velocity profile distortions due to artificial roughness was found by Klebanoff (1966). At a free stream velocity of $U_\infty = 11\text{m/s}$, (chosen as a safety margin to insure $U_\infty > U_{\infty,\text{transition}}$ for all five trip types) the boundary layer formed by each trip (as listed in Table 3.3) was measured by a U-probe at $X = 401.5\text{cm}$. As documented in the extensive survey by Coles, the normal values for $\Delta \bar{U}^+_{\text{max}}$ and $H$ are 2.76 and 1.37 respectively, for $Re_\theta \approx 4500$. From Table 3.3, it is clear that only the Dymo® "V" trip at $x = 10\text{cm}$ produces what is termed a "normal" turbulent boundary layer. As described in Section 2.12, the Dymo® "V" trip was used for all other experiments.

The distribution of the first four moments of the streamwise velocity taken at $X = 401.5\text{cm}$ with a U-probe are shown in Figure 3.7. The statistical properties in this figure closely match those of Purtell, Klebanoff and Buckley (1981), Kreplin and Eckelmann (1979), Barlow and Johnston (1985), Laufer (1955), the numerical simulation of Spalart (1986) and Kim, Moin, and Moser (1986), and recent Laser-Doppler Velocimetry (LDV) measurements of Karlsson and Johansson (1988). The logarithmic region of the mean velocity distribution is well matched to the logarithmic law using Coles' suggested constants ($\kappa = .41, C = 5.0$), and the buffer region is well matched
to the model by Haritonidis, except below $y^+ = 1.9$ where wall affects begin to distort the measurements. The maximum turbulence intensity is $2.81u_r$ at $y^+ = 14$ which coincides with the zero crossing of the skewness, and the point of minimum flatness $(F'(u)_{\text{min}} \approx 2.5)$. Also, consistent with other measurements, the skewness rises back to zero in the log region, while the flatness remains slightly less than three, but, unlike the numerical simulations in which the skewness is negative and the flatness rises above three at $y^+ \approx 40$.

### 3.4 Calibration Error

Using X-probes, types X1.21, X2.1, and a V-probe, type W5.2, each of the three calibration techniques were tested for accuracy and drift characteristics. The probes were calibrated with each technique and then immediately used to measure known flow conditions set by adjusting $U_\infty$ and the angle the probe made with the freestream. The difference between the measured and true values for $U$ and $V$, or $U$ and $W$ were noted as calibration errors. A complete boundary layer profile was then measured in the normal way and the error checked again upon completion. The difference in errors from the start to the finish of the run was noted as drift error. Tables 3.4, 3.5, and 3.6 summarize typical errors at three velocities and three flow angles for each calibration method.

Several trials were performed and an average for each error computed. The errors in $V$ and $W$ were comparable and averaged together. At high velocities, the errors of all three techniques are similar, but at low velocities and large angles the differences are dramatic. The Polar look-up table shows a 10-20 fold decrease in calibration error over
the Cartesian look-up table and a 50-100 fold decrease over the standard Cosine-Law. The drift error also decreases with each successive calibration technique, but the percent reduction should not be emphasized because there are far too many independent factors influencing drift. What is of importance is that the Polar table calibration method has the least error. The Polar look-up table is clearly superior to the other methods and was used for all further measurements.

3.5 Drift Check

The drift of the wires was monitored over an extended period to determine how long the probes maintained their calibration. The error in total velocity ($Q$) recorded for a U-probe, X-probe and V-probe at two velocities and three overheat ratios. The error in the X- and V-probe was so similar that only the U-probe and X-probe are displayed in Figure 2.18. The drift curve is typical of all body configurations and sensing box size. The dashed lines represent drift error when the flow was at an extreme angle, $\theta = \pm 30^\circ$. As would be expected from the list of possible drift sources, the drift rate is less when the tunnel is not running as confirmed in Figure 2.18. As mentioned in Section 2.6 the effect of these errors were minimized by always calibrating before every data run and by keeping the runs as short as possible.

3.6 Cross-Talk Effects Between Wires on X-probes

To investigate the possibility of thermal interaction between the wires of two-wire probes, two tests were conducted. The first test was as follows:
1. An X-probe was calibrated in the normal way. Operating in the free stream, the calibration drift error for $U$ and $V$ was noted as well as the individual output voltages for both wires ($E_1, E_2$).

2. The hot-wire card for wire #1 was turned off, eliminating it as a heat source. As a result the voltage for wire #2 ($E_2$) dropped slightly.

3. The voltage for wire #2 ($E_2$) was then adjusted back up to its pre-wire-#1-off value. The adjustment was made by increasing the gain on the output stage of the hot-wire circuit.

4. Wire #1 was turned back on and the change in the error in $U$ and $V$ was recorded.

5. The entire procedure was repeated turning wire #2 off instead of wire #1.

Adjusting the wire voltage is a necessary step of the procedure because if no adjustment was made no error would be measured when wire #2 was turned back on. Remember both wires have to be operating in order to be able to determine $U$ and $V$ from the look-up table. Three overheat ratios and three probe box-size spacings were tested using an X-probe, type X1.5. The free stream velocity was $U_\infty = 5\text{m/s}$. Each trial was repeated at least three times to ensure an accurate assessment. Table 3.7 gives the error percentages associated with each trial for $U$ and $V$. It is possible that the errors listed in the table may be slightly too large, since adjusting the voltage to accommodate the drop seen when turning the other wire off tends to slightly amplify any errors. Wire #1-off errors are listed first followed by wire #2-off errors listed second at each entry.

From these results it is evident that there is a strong influence of one wire on the other at high overheat ratios and close wire separation, when operating at a low velocity.
in air. The velocity and working fluid, air, are stressed because at a higher velocity or in a fluid with a larger Prandtl number (ratio of viscosity to thermal diffusivity) the cross-talk error would be less. These effects are discussed in detail in Section 5.2.1. Also, since the error percentages are nearly equal for both wire #1 and #2 there does not seem to be any buoyancy (gravity) affects meaning the error is independent of probe orientation. The most important conclusion to be drawn from this test, however, is that the $U$-component of velocity is much less effected by cross-talk than is the $V$-component. The errors are approximately three times larger for $V$ than they are for $U$. Though this test indicated a strong cross-talk effect it was artificial and did not determine the actual errors to be expected in measuring the flow. To determine the actual error, a second test was conducted.

The second test was to measure the boundary layer profile using the wires of an X-probe (probe X1.10) as two independent single- wire probes, measuring with one wire while cycling the other off then on. The probe was oriented in the holder in the usual manner for an X-probe. This puts the wires in a plane perpendicular to the wall, as opposed to the usual orientation for a U-probe which has its wire in a plane parallel to the wall. Wire #1 was calibrated as a single-wire with wire #2 off, and the boundary layer profile was measured. Without recalibrating wire #1, the test was repeated again but this time with wire #2 operating. This test pair was then done for the case of wire #2 operating as a single-wire probe and wire #1 off then on. For all four tests the freestream velocity was $U_\infty = 11\text{m/s}$. The profiles for each test are superimposed on one another in Figure 3.8.

There are two key features of this figure of importance to the issue of crosstalk between the wires. First, notice that the mean velocity profile is nearly identical for all
four measurements, and second, unexpectedly, there is no difference in the moments for wire #1 with wire #2 off versus on and similarly for wire #2 with wire #1 off versus on. It was expected that the 4% error found in the first cross-talk test at 5m/s (for 30% overheat and .25mm spacing) would be evident in this test as well. This implies that the $U$-component of velocity is even less sensitive to cross-talk in an actual flow than determined in the first test. This emphasizes the conclusion that $U$ is insensitive to cross-talk errors.

As an aside, why the turbulence intensity, $u'$, and skewness, $S(u)$, are nearly 20% different in the buffer and logarithmic regions as measured by wire #1 versus wire #2 (regardless of the status of the other wire) can be explained by the differing slant angles of the wires. Each wire appeared to the flow as a slant-wire U-probe aligned perpendicular to the flow, but biased by the probe inclination ($\phi$): wire #1 was slanted to face the wall at a higher angle ($\alpha_1 + \phi = 45^\circ + 8^\circ = 53^\circ$), than wire #2 was slanted away from it ($\alpha_2 - \phi = 37^\circ$). Since the most sensitive direction of the wire is in the plane normal to it, each wire was weighted toward flow with a different $V$-component of velocity. Calibration of each wire at its respective slant insures that the mean profiles are the same, but the higher order statistics will be biased by the $V$-component, which is not present during calibration. Wire #1 is more sensitive to $V$ and therefore shows inflated values.

The conclusions drawn from these tests is that the overheat ratio and wire separation have a critical effect on the sensitivity of an X-probe to cross-talk. It is also found that these effects are very severely felt in the measurement of $V$, with $U$ appearing to be completely unaffected.
Table 3.2: Transition velocity from laminar to turbulent flow for different trip types

<table>
<thead>
<tr>
<th>Trip Type</th>
<th>$U_{\text{transition}}$ (m/s)</th>
<th>Trip Type</th>
<th>$U_{\text{transition}}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velcro® Strips, $K = 3$mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single row</td>
<td>8.1</td>
<td>Double row</td>
<td>6.8</td>
</tr>
<tr>
<td>Wire Trip, $d = .356$mm</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>On Plate, $x=20$cm</td>
<td>9.8</td>
<td>On Plate, $x=30$cm</td>
<td>11.1</td>
</tr>
<tr>
<td>$y=1$mm, $x=20$cm</td>
<td>10.3</td>
<td>$y=1$mm, $x=30$cm</td>
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<tr>
<td>Rivet Trip, $K = 4.384$mm</td>
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<tr>
<td>Spaced every $1$cm</td>
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<td></td>
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<tr>
<td>Sand Paper, $K_{\text{mean}} = 1.3$mm</td>
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<td></td>
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<tr>
<td>#36 Grit, $11.4$cm wide</td>
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</tbody>
</table>
Table 3.2: Continued. Transition velocity from laminar to turbulent flow for letter trip types

<table>
<thead>
<tr>
<th>Trip Type</th>
<th>$U_{\text{transition}}$ (m/s)</th>
<th>Trip Type</th>
<th>$U_{\text{transition}}$ (m/s)</th>
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<td>WWW</td>
<td>14.1</td>
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<td>AAA</td>
<td>13.5</td>
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<td>◢</td>
<td>///</td>
<td>13.2</td>
<td>◢</td>
</tr>
<tr>
<td>Black Dymo® letter Strips, $K = .638\text{mm}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>▼</td>
<td>VVV</td>
<td>10.3</td>
<td>▲</td>
</tr>
<tr>
<td>▼</td>
<td>WWW</td>
<td>13.7</td>
<td>▲</td>
</tr>
<tr>
<td>▼</td>
<td>AAA</td>
<td>13.2</td>
<td>▲</td>
</tr>
<tr>
<td>▼</td>
<td>TTT</td>
<td>10.4</td>
<td>▲</td>
</tr>
<tr>
<td>▼</td>
<td>VVV</td>
<td>10.2</td>
<td>▲</td>
</tr>
<tr>
<td>▼</td>
<td>TTT</td>
<td>10.3</td>
<td>▲</td>
</tr>
<tr>
<td>◢</td>
<td>XXX</td>
<td>15.6</td>
<td>◢</td>
</tr>
<tr>
<td>◢</td>
<td>///</td>
<td>12.5</td>
<td>◢</td>
</tr>
</tbody>
</table>
Table 3.3: Values of $H$ and $\Delta \bar{U}_{\text{max}}^+$ for selected trip types as measured at $X = 401.5$cm.

<table>
<thead>
<tr>
<th>Trip Type</th>
<th>XLocation</th>
<th>$U_{\infty}$</th>
<th>$H$</th>
<th>$\Delta \bar{U}_{\text{max}}^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velcro®</td>
<td>10cm</td>
<td>11m/s</td>
<td>1.383</td>
<td>3.353</td>
</tr>
<tr>
<td></td>
<td>10cm</td>
<td>15m/s</td>
<td>1.372</td>
<td>3.638</td>
</tr>
<tr>
<td>Dymo® × V®</td>
<td>10cm</td>
<td>11m/s</td>
<td>1.368</td>
<td>2.810</td>
</tr>
<tr>
<td>Wire, on plate</td>
<td>20cm</td>
<td>11m/s</td>
<td>1.381</td>
<td>3.423</td>
</tr>
<tr>
<td>Wire, on plate</td>
<td>30cm</td>
<td>11m/s</td>
<td>1.377</td>
<td>2.434</td>
</tr>
<tr>
<td>Wire, at $y=1$mm</td>
<td>20cm</td>
<td>11m/s</td>
<td>1.383</td>
<td>3.118</td>
</tr>
<tr>
<td>Wire, at $y=1$mm</td>
<td>30cm</td>
<td>11m/s</td>
<td>1.377</td>
<td>3.826</td>
</tr>
<tr>
<td>Rivet</td>
<td>10cm</td>
<td>11m/s</td>
<td>1.375</td>
<td>3.66</td>
</tr>
<tr>
<td>Sandpaper</td>
<td>10cm</td>
<td>11m/s</td>
<td>1.361</td>
<td>3.546</td>
</tr>
<tr>
<td>Desired Values, Coles (1964)</td>
<td></td>
<td></td>
<td>1.370</td>
<td>2.760</td>
</tr>
</tbody>
</table>
Table 3.4: Typical calibration and drift error when using the Cosine-Law calibration method. Calibration error is determined immediately after calibration, and drift error is determined after a typical 30 minute profile measurement. All entries are percent error.

<table>
<thead>
<tr>
<th></th>
<th>Calibration Error</th>
<th>Drift Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_{\infty}$</td>
<td>$-25^\circ$</td>
</tr>
<tr>
<td>$11 \text{m/s}$</td>
<td>.2</td>
<td>.30</td>
</tr>
<tr>
<td>$3.0 \text{m/s}$</td>
<td>4.9</td>
<td>.9</td>
</tr>
<tr>
<td>$1.0 \text{m/s}$</td>
<td>14.1</td>
<td>1.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Calibration Error</th>
<th>Drift Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_{\infty}$</td>
<td>$-25^\circ$</td>
</tr>
<tr>
<td>$11 \text{m/s}$</td>
<td>.4</td>
<td>.3</td>
</tr>
<tr>
<td>$3.0 \text{m/s}$</td>
<td>3.6</td>
<td>.7</td>
</tr>
<tr>
<td>$1.0 \text{m/s}$</td>
<td>11.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>
Table 3.5: Typical calibration and drift error when using the Cartesian Look-up Table calibration method. Calibration error is determined immediately after calibration, and drift error is determined after a typical 30 minute profile measurement. All entries are percent error.

<table>
<thead>
<tr>
<th></th>
<th>U-Velocity Component</th>
<th></th>
<th>V-, W-Velocity Component</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calibration Error</td>
<td>Drift Error</td>
<td>Calibration Error</td>
<td>Drift Error</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>$\pm 25^\circ$</td>
<td>$0$</td>
<td>$+ 25^\circ$</td>
<td>$\pm 25^\circ$</td>
</tr>
<tr>
<td>11 m/s</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.9</td>
</tr>
<tr>
<td>3.0 m/s</td>
<td>0.7</td>
<td>0.4</td>
<td>0.8</td>
<td>2.1</td>
</tr>
<tr>
<td>1.0 m/s</td>
<td>1.6</td>
<td>1.2</td>
<td>1.9</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.6: Typical calibration and drift error when using the Polar Look-up Table calibration method. Calibration error is determined immediately after calibration, and drift error is determined after a typical 30 minute profile measurement. All entries are percent error.

<table>
<thead>
<tr>
<th>U-Velocity Component</th>
<th>Calibration Error</th>
<th>Drift Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_\infty$</td>
<td>$-25^\circ$</td>
<td>0</td>
</tr>
<tr>
<td>11m/s</td>
<td>.04</td>
<td>.01</td>
</tr>
<tr>
<td>3.0m/s</td>
<td>.08</td>
<td>.03</td>
</tr>
<tr>
<td>1.0m/s</td>
<td>.1</td>
<td>.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>V-, W-Velocity Component</th>
<th>Calibration Error</th>
<th>Drift Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_\infty$</td>
<td>$-25^\circ$</td>
<td>0</td>
</tr>
<tr>
<td>11m/s</td>
<td>.04</td>
<td>.03</td>
</tr>
<tr>
<td>3.0m/s</td>
<td>.06</td>
<td>.03</td>
</tr>
<tr>
<td>1.0m/s</td>
<td>.14</td>
<td>.02</td>
</tr>
</tbody>
</table>
Table 3.7: Error in the $U$- and $V$-components of velocity due to cross-talk between the wires of an X-probe. Entries are of the form wire #1-off: wire #2-off.

**Errors in $U$**

<table>
<thead>
<tr>
<th>Spacing Between Wire Planes</th>
<th>Resistance Overheat Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30%</td>
</tr>
<tr>
<td>.13mm</td>
<td>5% : 5%</td>
</tr>
<tr>
<td>.25mm</td>
<td>3% : 4%</td>
</tr>
<tr>
<td>.38mm</td>
<td>2% : 2%</td>
</tr>
</tbody>
</table>

**Errors in $V$**

<table>
<thead>
<tr>
<th>Spacing Between Wire Planes</th>
<th>Resistance Overheat Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15% : 17%</td>
</tr>
<tr>
<td>.13mm</td>
<td>11% : 13%</td>
</tr>
<tr>
<td>.38mm</td>
<td>9% : 8%</td>
</tr>
</tbody>
</table>
Figure 3.1: Percentage error due to $U_{error}$ in: (a) $u_r$; (b) $\ell_*$; (c) $t_*$. 
Figure 3.1: Percentage error due to $U_{error}$ in: (d) $\Delta \bar{U}_{max}$; (e) $C_f$; (f) $H$
Figure 3.1: Percentage error due to $U_{error}$ in: (g) $\delta$; (h) $\delta^*$; (i) $\theta$
Figure 3.1: Percentage error due to $U_{error}$ in: (j) $Re_\delta$; (k) $Re_{\delta^*}$; (l) $Re_\xi$
Figure 3.2: Percentage error due to \((U/U_\infty)_{\text{error}}\) in: (a) \(u_r\); (b) \(\ell_s\); (c) \(\ell_s\)
Figure 3.2: Percentage error due to $(U/U_\infty)_{error}$ in: (d) $\Delta \bar{U}_{\text{max}}$; (e) $C_f$; (f) $H$
Figure 3.2: Percentage error due to \((U/U_\infty)_{\text{error}}\) in: (g) \(\delta\); (h) \(\delta^*\); (i) \(\theta\)
Figure 3.2: Percentage error due to \((U/U_\infty)_{error}\) in: (j) \(Re_\delta\); (k) \(Re_\delta^*\); (l) \(Re_\theta\)
Figure 3.3: Velcro trip
Orange 7.53 mm (3/8 in) wide DYMO® brand embossing tape

Construction: Clear tape with orange glue backing, glossy finish

Cross Section Detail

Red 7.53 mm (3/8 in) wide Scotch® brand embossing tape

Construction: Clear tape with red glue backing, glossy finish

Cross Section Detail

Black 12.7 mm (1/2 in) wide DYMO® brand embossing tape

Construction: Black color-imprinted plastic with clear adhesive backing, matte finish

Cross Section Detail

Figure 3.4: DYMO brand embossing tape trips
Figure 3.5: Suspended wire trip
Figure 3.6: Rivet trip
Figure 3.7: (a) Mean velocity profile taken with a U-probe and legend for other authors’ data. $U_\infty = 11 m/s$, $X = 401.5 cm$, and using letter “V” trip located at $x = 10 cm$. 

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Figure 3.7: (b),(c) Streamwise turbulence intensity, $u'$, and Skewness, $S(u)$, taken with a U-probe. Legend is included with Figure 3.7a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$.
Figure 3.7: (d) Streamwise velocity component Flatness, $F(u)$, taken with a U-probe. Legend is included with Figure 3.7a. $U_\infty = 11 \text{m/s}$, $X = 401.5 \text{cm}$, and using letter "V" trip located at $z = 10 \text{cm}$. 
Figure 3.8: (a) Mean velocity profile taken with X-probe, X1.10, operating as two independent wires: measurements taken using wire #1 with wire #2 off then on, and measurements taken using wire #2 with wire #1 off then on. \( U_\infty = 11 \text{m/s}, X = 401.5 \text{cm}, \) and using letter "V" trip located at \( x = 10 \text{cm} \).
Figure 3.8: (b),(c) Streamwise turbulence intensity, $u'$, and Skewness, $S(u)$, taken with X-probe, X1.10, operating as two independent wires: measurements taken using wire #1 with wire #2 off then on, and measurements taken using wire #2 with wire #1 off then on. Legend is included with Figure 3.8a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$.
Figure 3.8: (d) Streamwise velocity component Flatness, $F(u)$, taken with X-probe, X1.10, operating as two independent wires: measurements taken using wire #1 with wire #2 off then on, and measurements taken using wire #2 with wire #1 off then on. Legend is included with Figure 3.8a. $U_\infty = 11m/s$, $X = 401.5cm$, and using letter "V" trip located at $x = 10cm$. 
Chapter 4

Parallel-wire/Split-film Probes

To capitalize on the cross-talk effect between wires and turn it to an advantage, the parallel-wire and split-film probes were built. In the case of the parallel-wire probe it was hoped that by having the wires extremely close together the thermal cross-talk would make it sensitive to flow angle. The parallel-wire proved to work as expected but, unfortunately, only over a very narrow velocity range, approximately .6m/s–3.5m/s. For the present purposes of measurement of a turbulent boundary layer with $U_\infty = 11$m/s, this small range restricts measurements to $y^+ = 3–8$. Although the mean velocity may fall within the range, near the wall the maximum/minimum values are more than twice/half of the mean as shown by Johansson and Alfredsson (1982) and verified here. Displayed in Figures 4.1 – 4.3 are the calibration plots for the parallel-wire with data points taken at $y^+ = 4.3$, 10.7, 32.4 superimposed. The large velocity range mentioned above is obvious by the spread of the data, but also note how the width of the spread is funneled to a very narrow corridor at velocities greater than 3.5m/s. This funneled at high velocities is caused by the sharp angle in each constant velocity calibration arc, $E_1$ and $E_2$ are no longer unique functions of one another indicative of an insensitivity to flow angle. In contrast, at the lower velocities the calibration points are smooth and unique and measurements are possible. The flow angles present close to the wall ($y^+ = 4.3$) are within the range $-20^\circ$ to $+15^\circ$ which is in good agreement with the results of Johnson and Eckelmann (1985). This is of special importance because the measurements
Table 4.1: Parallel-wire probe $u-v$ statistics at $y^+ = 6$

<table>
<thead>
<tr>
<th></th>
<th>$\bar{U}^+, \bar{V}^+$</th>
<th>$u^{'+}, v^{'+}$</th>
<th>$S(u), S(v)$</th>
<th>$F(u), F(v)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>5.3</td>
<td>2.04</td>
<td>0.58</td>
<td>2.97</td>
</tr>
<tr>
<td>$v$</td>
<td>0.01</td>
<td>0.34</td>
<td>0.21</td>
<td>4.82</td>
</tr>
</tbody>
</table>

with most of the other probe-body types are not in agreement. It is precisely these excursions from the mean that are most interesting since their contribution to the turbulence production and to Reynolds stress is significant (Klebanoff, 1954, Johansson and Alfredsson, 1984).

The statistical moments in the near wall region are also in very good agreement with past experiments including the recent very detailed Laser-Doppler Velocimeter (LDV) measurements done by Johansson (1988). The statistics measured by the parallel-wire at $y^+ = 6$ are listed in Table 4.1, and appear in Figure 4.4. In addition, the Reynolds Stress $(\bar{u}\bar{v}/u_2^2) = -.2$ and the correlation coefficient $\bar{u}\bar{v}/(u'v') = -.37$, which is also in excellent agreement with the measurements of Kim, Moin and Moser (1987).

The initial measurements with this probe were so good that they inspired the novel split-film probe. It was hoped that the cylindrical rod would enhance the angular sensitivity of the probe at higher velocities. As mentioned before (Section 2.3.1), electrical connections between the rod and the broach could not be satisfactorily made, and the probe was never used. It is strongly recommended however that development of this probe be continued, as well as further testing of the parallel-wire probe. A modification to the parallel wire, discussed, but yet untried, is to bow the two wires toward each other. This would bring the wires closer together in the center (to say .5μm), and leave
them further apart at the ends. It is hypothesized that the close center portion will be
sensitive to angles at high velocities and the further separated wire ends sensitive at low
velocities.
Figure 4.1: Calibration plot for the parallel-wire probe, P1.1, superimposed with 2000 data samples. Samples taken at $y^* = 4.33$, $U_\infty = 11$m/s, $X = 401.5$cm, using a letter "V" trip located at $z = 10$cm.
Data taken at $y^* = 0.7$

Figure 4.2: Calibration plot for the parallel-wire probe, P1.1, superimposed with 2000 data samples. Samples taken at $y^* = 10.7$, $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, using a letter "V" trip located at $x = 10\text{cm}$. 
Figure 4.3: Calibration plot for the parallel-wire probe, P1.1, superimposed with 2000 data samples. Samples taken at $y^+ = 32.4$, $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, using a letter "V" trip located at $x = 10\text{cm}$.
Figure 4.4: (a) Mean velocity profile taken with the parallel-wire probe, P1.1 and legend for other authors' data. $U_\infty = 11\text{m/s}, X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$.
Figure 4.4: (b), (c) Turbulence intensities, $u'$ and $v'$, and Skewness, $S(u)$ and $S(v)$, taken with the parallel-wire probe, P1.1. Legend is included with Figure 4.4a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$.
Figure 4.4: (d),(e) Flatness, $F(u)$ and $F(v)$, Reynolds shear stress and cross-correlation coefficient taken with the parallel-wire probe, P1.1. Legend is included with Figure 4.4a. $U_\infty = 11\text{m/s}, X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$. 

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Chapter 5

Boundary Layer Results and Probe Specific Errors

5.1 Flow versus Measurement Errors

In implementing the polar table it was necessary to check for stray voltage pairs \((E_1, E_2)\) that were beyond the range of calibration to avoid exceeding the look-up arrays. Statistics were kept on which edge the point "fell off of" the table, so to speak. These statistics revealed that near the wall (below \(y^+ = 100\)) a significant number of data points exceeded the side boundaries of the calibration grid. Notice the voltage points that fall above and below the calibration region. This problem was first reported by Willmarth and Bogar (1977) who tracked oscilloscope traces in the \(E_1, E_2\) plane, but until now was unsubstantiated in the literature. Previous calibration techniques were blind to this problem. The Cartesian look-up table covered an excessive area in the \(E_1, E_2\) plane and the empirical expressions of the Cosine-Law could handle wire voltages of any value. Figure 5.1 shows a typical calibration grid with 2000 individual voltage points superimposed.

When points fell off the table, it was impossible to assign the correct velocity pair \((U, V)\) or \((U, W)\) to the voltage pair \((E_1, E_2)\). For these cases, the velocity pair
at the point of departure from the grid was assigned to each voltage pair thereafter
until the voltage pairs again returned to the grid. For computing purpose, assigning
values to the offensive points maintained continuity but this was just a "best" fix to
a far graver problem. Even when voltage pairs were within the calibration grid there
was no way to guarantee that the assigned velocity pair \((U, V)\) actually represents
the flow. Figure 5.2 shows values of the statistical moments for \(U-V\) and the Reynolds
stresses taken with a standard X-probe, type X1.7. The \(U\)-velocity component statistics
agree with those taken with the U-probe shown in Figure 3.7, and with those taken by
Klebanoff (1954), Laufer (1955), and Kreplin and Eckelmann (1979). The values of
the \(V\)-velocity component statistics, however, are in error for \(y^+ \leq 100\) and looking
specifically at \(y^+ = 10\): \(v'\) is 50%-60% too large, \(S(v)\) should be negative, \(F(v)\) is 28%
too small, and the correlation coefficient \(\overline{uv}/(u'v')\) is 50% too small. At this location
only 5% of the data points were off the calibration grid, see Figure 5.3. It is unlikely that
these points alone contaminated the statistics to such a high degree but rather the error
results from points that were on the table but incorrectly assigned \(U\) and \(V\) values. The
points off the plot are positive proof that something is amiss. It is emphasized again
that these extreme excursions are most likely not new, that they have been occurring
all along, and that they were just not detected using other calibration techniques.

The question at this point is whether these errors represent the physical flow near
the wall or are a manifestation of a problem due to the hot-wire probe. Hypotheses from
Willmarth and Bogar (1977) which might explain the cause of the excursions included
aerodynamic interaction between the hot-wires or their supports, the presence of the
transverse velocity component, \(W\), the presence of free convection effects and motion or
vibration of the hot-wires. A detailed analysis of the effect of each of these phenomena
is presented in Bogar (1975), with the conclusion that voltage tracers would tend to be
pulled back onto, rather than pushed off of, the grid if affected by these phenomena. The conclusion therefore drawn by Willmarth and Bogar (1977) is that it is the flow and not the hot-wire causing the excursions. They argued that one wire of the array was exposed to flow with a velocity different (in both magnitude and direction) from the velocity to which the other wire in the array was simultaneously exposed. This type of behavior would occur in severe spanwise velocity gradients or where the turbulent structure is smaller than the distance between the hot-wires. The final argument was that large probes did not experience the errors because the small scales tend to be smeared out by the excessive wire length, thereby suppressing excursions off of the calibration grid.

The initial analysis in the present work also followed along the same lines. In fact measurements made with the parallel-wire probe (Chapter 4) could be construed to support the hypothesis of Willmarth and Bogar (1977) that small scale structure and spanwise velocity gradients exist. Since the probe is used with the wires parallel to the plate it should be insensitive to spanwire gradients and since the wire spacing \( s^+ = .7 \) is 3.5 times smaller than Willmarth and Bogar's (1977), it would be expected that the probe is smaller than the proposed small scale structures, thus no excursions from the calibration grid should be seen. Indeed this was the case; the measurements did not show any excursions from the calibration grid, and the \( U, V \) and \( uv \) statistics taken very close to the wall \( y^+ \leq 10 \) were in agreement with hydrogen bubble data by Grass (1971), LDV measurements by Karlsson and Johansson (1988), and the numerical simulations by Spalart (1986) and Kim, Moin, and Moser (1987). In retrospect, however, the key point is not that this measurement supports the small scales hypothesis. The important conclusion to be drawn is that an absence of excursions implies correct measurements! No other works have presented calibration plots monitoring the voltage on the \( E_1-E_2 \) so it was not known if this was the case. The work by Willmarth and Bogar (1977)
only indicated the converse, that the presence of excursions results in errors in the determination of $U$ and $V$.

Two discrepancies arise, however, when one considers the statistical moments of $U$ and $V$ measured by Eckelmann (1974), Laufer (1955) and Klebanoff (1954) using conventional X-probes and the measurements made with probe X1.7 in the present study. First, Eckelmann's and Laufer's ($Re_D = 50,000$) probes were of comparable size ($\ell_{wire}^+ = 1.8, 5.4$ and $s^+ = 1.1, 1.8$ respectively) to Willmarth and Bogar's probe ($\ell_{wire}^+ = s^+ = 2.5$), but their $U$, $V$, and $\overline{uv}$ statistics were in agreement with the bubble data, LDV measurements and numerical simulations. If the excursions were present, yet undetected in their hot-wire measurements, there should be no agreement and serious errors should be present in $U$, $V$, and $\overline{uv}$, as was concluded using the parallel-wire probe. Second, Klebanoff's and Laufer's ($Re_D = 500,000$) probes are larger than Willmarth and Bogar's ($\ell_{wire}^+ = 19.2, 44.7$, respectively) and measure $u$ and $v$ correctly as expected, but measurements made with the relatively large X1.7 probe ($\ell_{wire}^+ = 9.1$) showed excursions and errors in the statistical moments similar to Willmarth and Bogar's (1977) measurements.

These two discrepancies indicate that the excursions may be a manifestation of the hot-wire probe or experimental set-up, plaguing measurements made with probe X1.7 in the present study and by Willmarth and Bogar (1977), but avoided by Eckelmann (1974), Laufer (1955) and Klebanoff (1954). A careful study of each of these experiments is summarized in Table 5.2, and reveals three parameters that set the experiments that have correctly measured $U$, $V$, and $\overline{uv}$ apart.

1. The Prandtl number (Pr) is extremely large.
2. The resistance overheat ratio was small, \( a_R \leq 5\% \).

3. In those experiments done in air, where the Pr is small, the wire length to diameter ratio, \( \ell_{wire}/d \), is double that of probe X1.7 and Willmarth and Bogar's probe.

These three parameters were studied in more detail and are presented in the following sections. As will be seen in Section 5.2.1 the effect of the Prandtl number and the overheat ratio are related and are handled together. In addition, the effects of two more parameters unknown in the cited experiments, were also tested: the effect of probe blockage and vibration (both prong and probe).

5.2 Measurement Errors

5.2.1 The Effect of Prandtl Number and Overheat Ratio

The Prandtl Number is the dimensionless ratio of the kinematic viscosity to the thermal diffusivity \( (Pr \equiv \nu/a \equiv c_p \mu/\rho k_f) \) and depends solely on the properties of the fluid. This ratio affects the energy equation and more specifically the heat-transfer equation which governs the operating principles of the hot-wire.

Consider the hot-wire to be a thin heated cylinder which loses heat to the cross-flow by convection. As described by Hinze (1975), the wake behind a line heat source in turbulent flow develops in three stages. Initially the wake consists of a continuous connected layer waved to and fro by large scale turbulence and deformed by small scale motions. As long as the increase in the sheet thickness due to turbulent convection is much less that the increase caused by molecular diffusion, the wake remains at stage one.
At this stage the wake is characterized by molecular diffusion only. This condition is satisfied in a region around the wire comparable in size to the smallest eddies. Distortion and break up of the hot sheet by turbulence occurs during the second and third stages at regions further away from the wire. For small probes only the first stage is important.

As derived by Batchelor and Townsend (1956), in the initial stage governed by molecular diffusion, the temperature in the wake of a heated cylinder has a gaussian cross-sectional distribution. As shown in Figure 5.4, the half-width wake thickness is usually defined as the distance from the mean to one standard deviation of the distribution, and is called \( \sigma_z \), where \( z \) is in the direction normal to the heated sheet, and hence normal to the flow–wire plane. The full half-width of the wake is actually much larger, and can be approximated as three times the standard deviation value, \( \sigma_{3\sigma} \) (thereby including 99.87% of the wake). The standard deviation of the wake thickness, governed by molecular diffusion only, was found by Batchelor and Townsend (1956) to be,

\[
\sigma_z = \sqrt{\frac{2vt}{Pr}},
\]  

(5.1)

where \( t \) is the diffusion time and \( \nu \) is the kinematic viscosity of the fluid. As a first approximation the ratio of the X-probe sensor box length \( (B) \) to the magnitude of the local instantaneous velocity vector \( (Q = |\vec{Q}|) \) can be substituted in for diffusion time \( (t = B/Q) \). The standard deviation of the wake thickness can then be expressed as,

\[
\sigma_z = \sqrt{\frac{2\nu B}{QPr}}.
\]  

(5.2)

Cast in inner scales this becomes,

\[
\sigma_z^+ = \sqrt{\frac{2B^+}{Q^+Pr}}.
\]  

(5.3)

Hence, using the simplified model of the wake illustrated in Figure 5.5, the angular
standard deviation \( (\sigma_\gamma) \) of the sheet is

\[
\sigma_\gamma = \tan^{-1} \left( \frac{\sigma_\phi}{B} \right) = \tan^{-1} \sqrt{\frac{2}{B^+Q^+Pr}},
\]  

(5.4)

where \( \sigma_\gamma \) is measured with respect to the mean velocity direction. Remember that \( \sigma_\phi \) and \( \sigma_\gamma \), in Figure 5.5, are the standard deviations of the wake thickness and angle; the full thickness and angle are approximately three times larger, \( \zeta_{3\sigma} \) and \( \gamma_{3\sigma} \), respectively.

From Equation 5.3 it can be seen that the wake thickness is a minimum when both the Prandtl number and local velocity are large, and the hot-wire box is small. Likewise, from Equation 5.4 the wake angle is minimized when the Prandtl number, the local velocity and the box size are as large as possible. Minimizing the wake dimensions is desired to prevent the thermal wakes of the two wires from interfering with one another. Unfortunately, as a probe is moved closer to the wall the local velocity decreases tending to increase the wake thickness and angle. As a result for probes where \( B \) is small \( \sigma_\gamma \) tends to be maximized near the wall. For Willmarth and Bogar's (1977) probe, the probe box size is tiny \( (B = 2.5) \), the working fluid is air which at their operating conditions gives \( Pr = .748 \), and in the regions of most interest near the wall the velocity is small \( (Q^+ \leq 10) \). Eckelmann's small probe \( (B^+ = 1.8) \) is saved by making measurements in oil where the Prandtl number is two orders of magnitude larger \( (Pr = 82) \). Values for \( \sigma_\phi^+ \) and \( \sigma_\gamma \) were calculated for Eckelmann's (1974) and Willmarth and Bogar's (1977) experiments, and compiled in Table 5.1, using \( \bar{Q}^+ \approx \bar{U}^+ = 8 \) (the middle of the buffer region, \( y^+ \approx 15 \)).

The implication of Table 5.1 is that the thermal wake of one wire on Willmarth and Bogar's probe can easily grows at an angle of 60° measured with respect to the spanwise mean flow angle. Since spanwise flow angles can typically reach ±30° near the wall (Kreplin and Eckelmann, 1979), this would easily cause the thermal wake of
Table 5.1: Thermal wake thickness and angle calculated using Equations 5.3 and 5.4

<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^+_s = .914$</td>
<td>$\sigma^+_s = .074$</td>
</tr>
<tr>
<td>$\zeta^+_s = 2.74$</td>
<td>$\zeta^+_s = .22$</td>
</tr>
<tr>
<td>$\sigma_\gamma = 20.1^\circ$</td>
<td>$\sigma_\gamma = 2.36^\circ$</td>
</tr>
<tr>
<td>$\gamma_\epsilon = 60.3^\circ$</td>
<td>$\gamma_\epsilon = 7.07^\circ$</td>
</tr>
</tbody>
</table>

one wire to completely envelope the other wire. Such a situation completely violates the assumptions made in X-probe operation, that both wires "see" the same cross-flow and that the ambient temperature doesn't change during operation. King's Law (Equation 2.6) shows that any change in the local ambient temperature around a wire will also change the hot-wire circuit current which will we interpret as a change in velocity. In fact King's Law shows that the measured current (and hence voltage) is even more sensitive to temperature fluctuations than to velocity fluctuations. Because of this heightened sensitivity, even at less extreme instantaneous spanwise flow angles, $\theta$, the problem is still present because the wires are inclined to the flow: the thermal wake of the leading edge of one wire will hit the trailing edge of the other wire when $\theta + \gamma_{thermal} = 45^\circ$ (assuming a cubical sensing box, of length $B$). An error in the determined velocity will result anytime $\zeta^+_s$ plus the spanwise flow displacement is larger than the wire spacing ($s^+$).

$$\text{Errors result when } s^+ \leq \zeta^+_s + B^+ \tan \theta.$$  \hspace{1cm} (5.5)

In contrast, Eckelmann's (1974) probe in oil is relatively immune from one wire's thermal wake contaminating the other wire's measurement. Using the criteria of Equation 5.5, and neglecting the spanwise flow angle for now, Equation 5.3 can be solved for the minimum velocity at which no thermal wake interference will occur. For Eckelmann's experiments wake interference based on this analysis would occur for $Q^+ \leq .19$. 

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Willmarth and Bogar (1977) on the other hand, will begin to have thermal wake interference when $Q^+ \leq 15$.\textsuperscript{1} Since Willmarth and Bogar used a look-up table the effect of this interference is already taken into account by the calibration since such interference would also be present at low calibration speeds in the freestream.\textsuperscript{2} What is not accounted for, however, is the spanwise flow fluctuations which will wave the wake temperature distribution back and forth across the wires. According to Krelin and Eckelmann (1979), the smallest maximum spanwise flow angle is $\vartheta = 8^\circ$ which occurs in the outer part of the turbulent boundary layer. Including this angle in the analysis results in a minimum error-free velocity of $Q_{\text{error-free}}^+ = .21$ for Eckelmann (1974), and 22 for Willmarth and Bogar (1977). Approximating $\bar{U}^+$ by $Q_{\text{error-free}}^+$, the mean velocity profile can be interpolated to determine an estimate of the minimum distance from the wall each probe can be used with "error-free" results (neglecting wall effects). From their respective mean velocity profiles $y_{\text{error-free}}^+ \approx .21$ and 500, respectively. Thus Eckelmann’s measurements are entirely unaffected by the thermal wake (even at the largest maximum $\vartheta$ of $30^\circ$ very close to the wall $y_{\text{error-free}}^+$ is only .85, which is 4 times closer than his closest data point). Willmarth and Bogar (1977), on the

\textsuperscript{1}This is a conservative estimate for Willmarth and Bogar’s probe since the wires of their probe cross more in the shape of a “V”, rather than an “X”. The streamwise distance between the centers of the wires was used for $B$, rather than the “box” length which is probably twice as long. The center spacing is used because it was the only streamwise length included in either Bogar (1975) or in Willmarth and Bogar (1977).

\textsuperscript{2}This again emphasizes the advantage of look-up table calibration methods. But it is also for this reason that Willmarth and Bogar’s attempt to check for the effect transverse velocity and heated wake interactions by calibrating a yawed probe failed to show any differences in the calibration grid. Steady-state temperature distributions and cross-flows encountered during calibration will be accounted for, leaving the calibration largely unchanged. A better test would have been to calibrate the probe un-yawed and then note the $E_1-E_3$ trace when yawing.
other hand, should see errors in their measurements for $y^+ \leq 500$, which is suspiciously similar to their reported value of 600.

This thermal wake interference model also accounts for the specific characteristics of the excursions seen on the $E_1 - E_2$ grid. As observed both by Willmarth and Bogar and in the present study with probe X1.7 the excursions *always* take the form where, say, $E_1$ is within an acceptable range but $E_2$ is too low, or vice versa. As an example, let the thermal wake of wire #1 cross wire #2. In this case wire #1 will measure the velocity in the normal way, producing a normal value for $E_1$. Wire #2, however, will be influenced by both the velocity and the increased "ambient" temperature. The current $(I)$ measured in the hot-wire is governed by the thermal equilibrium condition that the heat loss per unit time must be equal to the heat generated per unit time by the electric current through the wire. From Hinze (1975) for a wire with length $\ell$ and diameter $d$,

$$\dot{q} = I^2 R_w = h \pi d \ell (T_w - T_{amb}).$$

(5.6)

Here, $h$ is the heat-transfer coefficient; $T_{amb}$ is the ambient temperature near the wire; and $T_w$ and $R_w$ are the "hot"-wire temperature and resistance, respectively, with the H subscript dropped for simplicity. The equation assumes uniform wire heating, no end conduction (i.e. an infinite wire), and negligible diffusion beyond the end of the wire. Thus as the wake of wire #1 increase $T_{amb}$ around wire #2 the current and hence $E_2$ will decrease. For a strong thermal wake $E_2$ can appear abnormally low while $E_1$ is unaffected: the situation exactly as observed. The amount of error in the wire voltages is clearly dependent on the temperature of the wake.

The bulk temperature in the wake of the hot-wire can be estimated by balancing the energy equation through the simplified wedge shaped control volume shown in Figure 5.5. As shown in Reynolds and Perkins (1977) the heat input ($\dot{q}_{in}$) by the wire
is equated to the mass flux times the change in enthalpy \( (h) \) through the end of the thermal "wedge". At constant pressure the change in enthalpy equals the change in temperature times the specific heat of the fluid, \( c_p \). This yields

\[
\dot{q}_m = \dot{m} \Delta h = \dot{m} c_p \Delta T,
\]

where the mass flux \( (\dot{m}) \) is the fluid density times the area of the base of the wake times the instantaneous local velocity. Making the appropriate substitutions for area and mass flux this results in

\[
h \pi d \ell (T_w - T_{amb}) = \rho (2 \sigma_\gamma \ell) Q c_p (T_{wake} - T_{amb}).
\]

The heat-transfer coefficient \( (h) \) is the coefficient due to convection and radiation. It can be shown at normal operating temperatures that \( h_c \gg h_r \), thereby allowing the heat-transfer coefficient to be solely due to convection. Normally grouped in the Nusselt number (\( Nu = \frac{h_c d}{k_f} \)), \( h_c \) is a function of the Reynolds number based on the wire diameter and the Prandtl number as shown by Kreith (1965),

\[
Nu = \frac{h_c d}{k_f} = 1.1 C \left( \frac{Q d}{\nu} \right)^n Pr^{-3/4},
\]

where \( C \) and \( n \) are empirical constants whose values vary with the Reynolds number. For small wires \( Re_d \approx .4 \) which gives \( C \approx .9 \) and \( n \approx .33 \) resulting in the approximation

\[
\frac{h_c d}{k_f} \approx (Re_d Pr)^{1/4},
\]

where \( k_f \) is the thermal conductivity of the fluid. Substituting Equation 5.8 into Equation 5.10 and rearranging gives

\[
T_{wake} - T_\infty = \frac{k_f (Re_d Pr)^{1/4} \pi (T_w - T_\infty)}{2 \sigma_\gamma \rho c_p Q},
\]

Using the definition of the Prandtl number, the temperature overheat ratio Equation 2.5, and non-dimensionalizing with wall units, Equation 5.11 becomes

\[
T_{wake} = \frac{1}{2} \left( \frac{d^2}{8 Q + B^2 + 3 Pr} \right)^{1/4} \pi a r T_{amb} + T_{amb}.
\]
The influence of many different parameters is clearly seen in Equation 5.12, with the strongest influence on $T_{\text{wake}}$ coming from the overheat ratio, $a_r$ or alternately $a_R$ from Equation 2.5. The minimum wake temperature will occur, in order of influence, when $a_r$ is small, $B^+$ is large, $d^+$ is small and $Q^+$ and Pr are large. Again for Willmarth and Bogar's (1977) experiments near the wall, these parameters are the opposite of what is desired, tending to maximize $T_{\text{wake}}$. Errors caused by large values of $T_{\text{wake}}$ as a result of large overheat ratios have already been touched upon in Section 3.6: the effect of thermal cross-talk between the wires. Table 2.1 confirms the fact that large overheat ratios, $a_r$, in Equation 5.12, do indeed produce large errors. Also confirmed by Table 2.1 is that small spacing between the wires, thereby putting one wire in the thermal wake of the other as determined from Equation 5.3, also can cause large errors. However, the actual amount of error introduced by $T_{\text{wake}}$ into the measurement of $U$ and $V$ cannot be determined. The reason is that the calibration grid is non-linear and non-analytic. As seen, a small error in $E_1$ or $E_2$ in the wide part of the “fan” may result in negligible error in $U$ and $V$ while the same error at the bottom of the fan could easily produce excursions making $U$ and $V$ completely undeterminable.

There are two conclusions to be drawn from the analysis of deriving Equation 5.3 and 5.12 about the requirements of making accurate measurements with X-probe hot-wires.

1. The angular and lateral width of the thermal wake should be minimized to prevent the wake of one wire from crossing the other wire.

2. If the wake will hit the other wire, minimize the temperature of the wake to keep the induced errors small. Remember the error in $U$ and $V$ can be much worse than the error in $E_1$ and $E_2$ depending on the location on the calibration grid.
Although easily stated these minimizations are often difficult to achieve because of the conflicting requirements which dictate the measurements. The region of most interest in turbulent boundary layers is near the wall where the non-dimensional velocity is the lowest, therefore making it impossible to increase $Q^+$. To position the probe close to the wall, its size must be on the order of the closest distance required,\(^3\) thereby making $B^+$ and $a^+$ smaller rather than larger. Low overheat ratios bring with them larger drift errors, and lastly, high Reynolds number flows usually require low viscosity fluids which tend to have lower Prandtl numbers.

Laufer (1955) and Klebanoff (1954) are speculated to have achieved a good balance of these requirements, in addition to having large $l_{wire}/d$ ratios as will be discussed in Section 5.2.2. The probe sensing box sizes were larger than Willmarth and Bogar’s probe, with the appropriate sacrifice in $y^+_{min}$, and the overheat ratios, though not explicitly stated in either of the works, must have been $a_R \leq 5\%$, as opposed to Willmarth and Bogar’s of $a_R = 50\%$.

The prediction of the equations and the speculation of Laufer’s and Klebanoff’s overheat ratio were tested. Since it was unfeasible to change the working gas in the MIT wind tunnel, the Prandtl number could not be changed, but a probe 1.5 times larger, with wires 2.5 times longer, than Willmarth and Bogar’s (1977) was constructed and

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\(^3\)Actually the minimum $y^+$ position may also be dictated by either the thermal wake of the wire or the temperature and conductivity of the wall. If the wire wake is deformed by the wall then the flow direction will be altered, and if the wall thermal boundary layer crosses the wire, the local ambient temperature will be changed in Equation 5.6. A balancing of the energy equation similar to the derivation presented here with the appropriate boundary conditions could be used to determine the range of these effects.

Such an analysis was done by Walker and Bullock (1972) who reported $y^+_{min} = (32B^+/Pr)^{1/3}$. It is interesting to note that for the probes used in air mentioned here $10 \leq y^+_{min} \leq 20$, while for Eckelmann’s probe in oil $y^+_{min} = 0.8$. 

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operated at various overheat ratios from 5% to 50%, (probes X3.13–18). In addition, \( \ell_{wire}/d \) was made twice as large as Willmarth and Bogar's, as will be discussed in Section 5.2.2. Calibration plots for this probe are shown in Figures 5.6–5.9, for \( a_R = 50\%, 30\%, 10\% \) and 5% respectively. The differences in the plots are most notable in the lower region of the fan, where lower velocity increased the wake width and increased \( T_{wake} \). In Figure 5.6 with a high \( a_R \) the asymmetric "necking" effect (described in Section 2.5.3) is very pronounced, whereas in Figure 5.9 with \( a_R = 5\% \) the "necking" is absent. Additionally, looking at all the figures as the overheat ratio decreased the number of concave constant velocity arcs also decreased. The "flat" arc occurs at a lower velocity for lower values of \( a_R \); at \( a_R = 50\% \) this arc is 4.6m/s whereas at \( a_R = 5\% \) the arc is flat at approximately 3m/s. Finally, the needle like constant velocity arc at \( Q = .8m/s \) with \( a_R = 50\% \) gradually flattens out with decreasing \( a_R \) becoming smoother and "U" shaped at \( a_R = 5\% \). The overall effect of a smaller \( a_R \) is a more symmetric, fuller, and smoother varying calibration grid. The differences between the calibration plots may seem subtle but analysis of these plots is an effective way to diagnose the "health" of the X-probe. The change in the turbulent boundary layer measurements made with this probe and low overheat (probe X3.7) is dramatic as seen in Figure 5.10. The statistical moments are very similar to measurements made by Karlsson and Johansson (1988), Eckelmann (1974), Laufer (1955), and Klebanoff (1954). At \( y^+ = 10 \), the measurements had no excursions from the calibration grid (see Figure 5.3), and \( u' \) is only approximately 6% higher than \( u' \) measured by the others. However, for \( y^+ < 10 \) the number of excursions rises markedly, up to 10% of the total data record at \( y^+ \approx 4 \), bringing \( u' \) up to a value similar to that measured by probe X1.7 with a higher overheat ratio shown previously in Figure 5.2. Measurements below \( y^+ \approx 9 \) are in error and are characterized by having a dramatic increase in the amount of excursions occurring in
the data collection (approximately 10%). These points are therefore easy to pick out and eliminate. All points are left on the plots to show the effect.

Increasing the box size still further to \( \ell_{\text{wire}}^+ = 11 \) (four times larger than Willmarth and Bogar’s, 1977 probe) but still maintaining \( \ell_{\text{wire}}/d \approx 430 \) by increasing \( d \), and lowering the overheat ratio to \( a_n = 1\% \) (probe X2.2) produces a nearly ideal calibration plot, Figure 5.12. The constant angle lines are straight radiating out from the origin, and the constant velocity arcs are symmetric and “flatten” out only at the lowest velocity. The penalty paid for this ideal calibration was that the large probe could only be used accurately down to \( y^+ \approx 30 \) (like Klebanoff, 1954 who used a large probe) and that over the course of a complete turbulent boundary layer profile survey the drift error was too large because of the very low overheat. As a result only spot measurements could be made which were nearly identical with those shown in Figure 5.10 for \( y^+ \geq 40 \), with \( \pm 1\% \) variation in all moments.

In summary, it is not the fluctuations of transverse velocity component themselves that cause the excursions from the calibration grid; the assumption that the wires “see” the same cross-flow still holds. Rather the errors are caused by the fluctuations in the local ambient temperature around each wire caused by the cross-flow sweeping the thermal wake of one wire across the other. Since the hot-wire is actually more sensitive to temperature than to velocity, minor local temperature fluctuations can produce very large errors. Because the presence of cross-flow will always put one wire “upstream” of the other, one wire will be unaffected, while the other experiences an increased ambient temperature due to the wake of the first wire. This arrangement will always produce errors of the sort where one wire’s voltage is correct, while the other’s is too low, which is exactly what is observed during an excursion off of the calibration grid. The
solution to this problem as predicted by Equations 5.3–5.12 is to increase the probe box size, increase the Prandtl number (if possible), and to reduce the overheat ratio. The effectiveness of these corrections is easily detected in the calibration plots, which makes such plots very useful in analysis of hot-wire performance. The resulting measurements made by using larger box sizes and lower overheat ratios are markedly better, and could be improved even further if it was possible to increase the Prandtl number.

5.2.2 Effect of $l_{\text{wire}}/d$

As mentioned in Section 5.1 the ratio of the wire length to its diameter is also a factor in obtaining accurate measurements with a hot-wire probe. The general cause of problems is that the temperature distribution along a wire of finite length can be quite different from the constant temperature assumed previously. In Equations 5.6 – 5.12 it was assumed that the temperature of the wire was represented by a single value $T$, which corresponded to a unique resistance $R$. This idealized state corresponds to a wire of infinite length with uniform cross-sectional area. Real wires of finite length and cooler end supports have a non-uniform temperature distribution. The prongs are chosen to be much thicker than the wire, partly for reasons of strength and partly so that they shall not be heated appreciably by the electric current. Therefore the end of the wire is very nearly at the ambient temperature of the fluid, $T_{\text{amb}}$, and conductive heat transfer along the wire to the supports is appreciable. The consequence of this is to increase $h$ so that $h = h_c + h_s$. Additionally, measurements by Champagne, Sleicher, and Wehrmann (1967) indicate that for high overheat ratios the prongs may actually heat up as much as 50°C above the mean ambient temperature. This introduces an entirely new heat source, with the appropriate change in $T_{\text{amb}}$ for each wire, and the
increased complications in determining velocity.

Neglecting prong heating (assuming "cool" prongs), the increase in $h$ is derived by Blackwelder (1981). If it is assumed for simplicity that the convective heat transfer to the fluid is directly proportional to the difference between $T_{amb}$ and the temperature along the wire, $T(z)$, the steady state energy balance along the wire is a second-order differential equation. The solution, with several approximations, relates $T$ in terms of the distance $z$ from the mid-point of the wire:

$$\frac{T(z) - T_{amb}}{T_{wire,\infty}} = 1 - \frac{\cosh z/\ell_c}{\cosh \ell/2\ell_c}$$

(5.13)

where $T_{wire,\infty}$ is the temperature an infinitely long wire would reach (the value of $T_w$ used previously in Equations 5.6 and 5.11) and $\ell_c$ is a characteristic length over which the supports affect the wire's temperature. This length is defined as

$$\ell_c = \frac{d}{2} \sqrt{\frac{k_{wire}(1 + a_R)}{k_f Nu}}$$

(5.14)

where Nu is the forced-convection Nusselt number from Equation 5.9. From Bradshaw (1971) if $\ell_c$ is overestimated by using the average measured $a_R$, then the ratio of the conductive heat transfer to both supports, $h_s$, to the convective heat transfer to the fluid, $h_c$, is approximately

$$\frac{h_s}{h_c} = \frac{d}{\ell} \sqrt{\frac{k_{wire}}{k_f Nu(1 + a_R)}}$$

(5.15)

This introduces the term $h = h_c(1 + h_s/h_c)$ into Equation 5.6 for the determination of the measured wire current. For the wires on Willmarth and Bogar's probe the ratio of $h_s/h_c$ is approximately .16 which is a considerable contribution to $h$. This increase in $h$ is accounted for in the calibration of the probe, by the sensitivity constants of King's Law, or by direct calibration methods such as the Polar Look-up table. A problem arises, however, in the difference in response times of $h_c$ versus $h_s$ to fluctuations in
velocity. If they varied equally together then probe support conduction would always be compensated by the calibration, but in reality \( h_s \) lags behind \( h_c \). According to Bradshaw (1985), as the velocity fluctuates the temperature distribution along the wire is altered because \( \ell_c \) changes. This redistribution takes a finite amount of time to occur and can be obtained from dimensional analysis as \( \ell_c^2 \rho_w c_w / k_{wire} \) where \( c_w \) is the specific heat of the wire. However, the redistribution of temperature does not affect the heat transfer due to convection, \( h_c \), because the integrated wire temperature does not change. This means the main effect of the finite response time is on the conduction of heat to the prongs, \( h_s \). As a result \( h_s \) lags behind \( h_c \). The effect of this lag is to decrease \( h_c / h_s \) from the expected value during times of velocity fluctuations which causes a decrease in \( h \) and in turn a lower value of the measured voltage.

For a single-wire U-probe this decrease in \( h \) and in turn the decreased voltage will be interpreted as a lower than expected \( U \)-velocity component. For a single-wire probe Lord (1981) calculated that for constant energy per length, decreasing \( \ell_{wire} / d \) results in a lower peak time-averaged temperature at the wire center and lower mean temperatures averaged along the length of the wire. Although mean voltage variations are taken into account by calibration, Lord also found that conduction to the sensor supports also affects the dynamic behavior. From his analysis for a U-probe,

\[
\frac{u'}{U_\infty} = C d^2 \left( \frac{\ell_{wire}}{d} \right)^m ,
\]

(5.16)

where \( C \) is a constant of proportionality and \( m = 1 \) (resulting from the thermal boundary condition that there be zero temperature fluctuations at the end of the wire). This effect was first observed by Blackwelder and Haritonidis (1983) who recorded \( u' \) values that were too small when using U-probes with \( \ell_{wire} / d < 200 \). They found that \( \ell_{wire} / d \) must be at least 200 in order to accurately measure \( u' \). More recent work by
Ligrani and Bradshaw (1987) testing many hot-wire lengths and diameters found that for optimal \( u' \) measurements \( \ell_{wire}/d \) must be greater than 200, as shown in Figure 5.13. They also confirmed work by Kühn and Dressler (1985) that to avoid reductions in amplitude and phase response, specifically at lower frequencies, then \( \ell_{wire}/d \) should be greater than even 300. The necessity for these large \( \ell_{wire}/d \) ratios is corroborated by Equation 5.15 which clearly indicates that larger \( \ell_{wire}/d \) ratios will decrease \( h_s/h_c \) and thereby minimize the effect \( h_s \) has on \( h \). For example, for probe X3.13 used to compare overheat ratios in Section 5.2.1 the average \( h_s/h_c \) is only .06, 2.5 times smaller than for Willmarth and Bogar's (1977) probe; and hence 2.5 times less sensitive to the lag between \( h_s \) and \( h_c \). For X-probes, however, this lag between \( h_s \) and \( h_c \) and the resulting smaller than expected voltages are more complicated than the simple underestimate of velocity experienced by the U-probe. What results for the X-probe is independent voltage drops in \( E_1 \) and \( E_2 \). The resulting errors are therefore identical to those resulting from fluctuations in the ambient local temperature around each wire: excursions from the calibration grid.

An additional difficulty caused by the non-uniform temperature distribution is that in order to achieve an average temperature dictated by the overheat, the actual temperature in the middle of the wire is higher to make up for the cooler ends. This does not cause adverse effects in single hot-wire operation but in view of the discussion of wake temperature (Section 5.2.1) this can be an added source of error for two wire X-probes. The temperature of the wake takes on a distribution to match the wire's temperature, making the center wake band even hotter than calculated in Equation 5.12, thereby causing even larger excursions of \( E_1 \) and \( E_2 \). A large \( \ell_{wire}/d \) will flatten out the temperature distribution and lower the central wake temperature.
Both of these problems resulting from low $\ell_{\text{wire}}/d$, fluctuations in $h$ and increased wire temperature, are again enhanced at low velocities in low Prandtl number fluids because of the inverse dependence on the Nusselt number. At high velocities the effects are negligible, but close to the wall at low velocities the effects are pronounced. The effect of $\ell_{\text{wire}}/d$ can be seen clearly by comparing Figure 5.14 and Figure 5.7. Both calibration plots were made with the same probe, same overheat ratio ($a_R = 30\%$), but different wire diameters ($d = 0.63\mu m$ and $1.27\mu m$), probes X3.17 and X3.26. Figure 5.7 was made using probe X3.17 with $\ell_{\text{wire}}/d = 430$ while Figure 5.14 used probe X3.26 with $\ell_{\text{wire}}/d = 215$. Note that the lowest velocity in Figure 5.14 with the low $\ell_{\text{wire}}/d$ is only $0.9\text{m/s}$ and that the “flat” constant velocity arc occurs around $5.5\text{m/s}$. Also the lower end “necks” down considerably below about $7\text{m/s}$. Increasing the $\ell_{\text{wire}}/d$ in Figure 5.7 dramatically improves the calibration plot.

The conclusion to be drawn is that for two-wire measurements very large $\ell_{\text{wire}}/d$ ratios are necessary to make accurate measurements. The probes used by Klebanoff (1954) and Laufer (1955) had $\ell_{\text{wire}}/d$ ratios of 394 and 500 respectively, and probe X3.7 used to take the measurements appearing in Figure 5.10 had a ratio of 360. Considering these measurements plus the major differences in the calibration plots for $\ell_{\text{wire}}/d = 430$ and 215 (Figures 5.7 and 5.14) it is recommended that for two-wire probes that $\ell_{\text{wire}}/d \geq 400$. This is double the recommendation given by Blackwelder and Haritonidis (1983) and Ligrani and Bradshaw (1987) of $\ell_{\text{wire}}/d > 200$ for single-wire probes.

5.2.3 Effect of Calibration Method

In addition to $\ell_{\text{wire}}/d$ and Pr there was an additional commonality between the experiments of Klebanoff (1954), Laufer (1955), and Eckelmann (1974). They used
the cosine-law method of calibration whereas, Willmarth and Bogar (1977), and the present work used a calibration look-up table. To test the effect of the calibration method probe X3.21 (similar to probe X3.7 used in Section 5.2.1 was calibrated using the cosine-law. The turbulent boundary layer measurements are presented in Figure 5.15. The $U$ statistics and $R_{uw}$ are similar to measurements made with the Polar look-up table shown in Figure 5.10, but the $V$ statistics are somewhat different. Using the cosine-law the value of $v'$ is more in agreement with measurements of Laufer (1955) and the LDV measurements of Karlsson and Johansson (1988) with only a 1% error at $y^+ = 10$. The skewness of $V$, $S(v)$, is slightly less positive than the polar-table case, but in both cases $S(v)$ is too large for $y^+ < 40$. Below $y^+ = 40$ the skewness of $V$ rises sharply to $S(v) \approx +1.5$ at $y^+ = 6$, while the LDV measurements show $S(v)$ constant at approximately $-0.25$. The flatness, $F(v)$, is similar to both the Polar-table results and the LDV measurements. For both velocity components for all three moments, measurements below $y^+ = 7$ are incorrect which can be attributed to wall effects; however, measurements for $y^+ \leq 10$ appear to be generally better for the cosine-law case as opposed to the polar look-up table. This generalization does not apply to the Reynolds Stress measured using the cosine-law, which is 35% lower than the measurements of probe X3.7, those taken by the LDV, and the values calculated from $\overline{U}$ by differentiating the Arctangent law, as shown by Equation 2.34.

5.2.4 Probe Blockage

Probe blockage and prong interference always seem to be used as the scapegoat for measurements that are incorrect. To test the effect of blockage and interference several unique probes were constructed with varying probe geometries. The X-probes tested,
X4-X8, nicknamed the "Claw" series, are illustrated in Figure 2.5. The idea in the metamorphosis of this series was to move the prongs and probe body out and away from the wake region of the sensing box. It was thought that moving the prongs away laterally (two-dimensionally), probe X4, and, out in three-dimensions, probe X6, should show an improvement in the statistical moments from measurements of the conventional shaped X-probes X1, X2 and X3. Additionally the probe body was displaced, angled away, and shaved down, probes X5, X6, X7 and X8, with the thought that by also moving it the measurements would improve. Repeated tests with these probes did not show any definite trend, neither improvement nor deterioration of, measurements made with the conventional X-probe. A representative graph of the statistical moments taken by the "claw" probe series is shown in Figure 5.16.

Along the same lines, the V-probe went through the same metamorphosis to check for blockage effects. The probes used are illustrated in Figure 2.10. Lateral spacing of the prongs (probe V3), displacement of the sensing area (probe V2) and narrowing of the "shhaft" (probe V4) were tested against the conventional V-probe shape of probe V1. The results were similar to the tests for blockage in the X-probes: the probe configurations tried had no effect on the measurements. The V-probe shape finally used had a unique prong shape, but this was not due to blockage considerations, but rather an effort to have the prongs parallel to the wall and will be discussed fully in Section 5.3.2.

As expected, positioning very near the wall was limited by the generally larger size of these probes. However, no definitive conclusions about the effects of blockage can be made from these tests other than what was tried made no difference. The tests conducted are incomplete in that only one overheat ratio (30%) with one wire diameter
(d ≈ 1.27 μm) with an \( \ell_{\text{wire}}/d \approx 400 \) was used. Additional tests at lower overheat ratios and smaller sizes may more clearly reveal an interdependence of prong position and errors in the statistical moments. What these tests do support, however, are the conclusions of Sections 5.2.1 and 5.2.2: that both a low overheat ratio and a large \( \ell_{\text{wire}}/d \) are needed to make accurate measurements.

5.2.5 Vibration of the Hot-wire probes

Probe Body Vibration

Vibration of the probe and probe sting was checked by positioning an X-probe, types X1.8, X2.1, and X3.1, so that the probe body pressed against a small lump of modeling clay affixed to the flat plate. In addition, a small balsa wood wedge was also tried as a probe "fixing" device. Measurements were made in the normal way at \( y^+ \approx 8, 11, \) and 20 with and without probe fixing. The number of excursions from the grid did not change by more than 2% in measuring fixed versus unfixed at each location.

Prong Vibration

Vibration of the hot-wire prongs was checked by positioning a V-probe (V5.4) such that the broaches themselves (not just the probe body) pressed against the modeling clay lump used in testing probe-body vibration. The results of this test do not show any consistent change in the number of excursions using damped versus undamped prongs. The number of excursions from the grid changed at most by 2%. As a second test measurements made with an X-probe with thin broaches (25 μm diam) were compared to
those of a probe with thicker broaches (101μm diam), types X1.1, and X2.1 respectively. Measurements taken at $Y^+ \approx 8, 11, \text{ and } 20$ show a consistent 3% decrease in the number of excursions when using the thick-pronged probe versus the thin-pronged probe. This decrease, however, is probably due to the more constant thermal end conditions provided by the thicker prongs. Since the former V-probe test used the same probe for both damped and undamped cases it is probably a better test of prong vibration. The end conclusion is that effects of probe and prong vibration, if present at all, are negligible in the experiments presented here.

5.3 Problems in Measuring the $W$-component of Velocity

The measurement of $U-W$ was approached with all the lessons learned about Prandtl number, wake temperature, overheat, heat conduction to the prongs, and $\ell_{\text{wire}}/d$ firmly in mind. But it was soon discovered that there are several additional unique problems in measuring $W$ correctly.

5.3.1 Rotated X-probe Approach

Initially, in an imitation of Laufer (1955), the smallest X-probe (probe RX3.6 and RX3.8) was rotated 90° in its holder, putting the wires parallel to the plate, was used to measure $U-W$. The measurements taken by these probes (prefixed with an “R” to denote “rotated”), as seen in Figure 5.18 do not agree with measurements taken by Laufer (1976), Kreplin and Eckelmann (1979), Klebanoff (1954), the LDV measurements of Karlsson and Johansson (1988), or the numerical simulations of Spalart (1986) and Kim, Moin and Moser (1987). The key differences are that the skewness of $W$, $S(w)$,
for $y^+ \leq 70$ is positive, reaching 0.3 at $y^+ \approx 12$, when it should be zero. Also, the skewness of $U$, $S(u)$ is increasingly negative for $y^+ \leq 70$ but should turn positive at $y^+ \approx 20$. The flatness of both signals, $F(u)$ and $F(w)$ are also in error for $y^+ \leq 70$. The source of the problem is found in analyzing the cross-correlation coefficient $R_{uw}$ and the stress term $\overline{uw}^+$. The values of these two parameters should also be zero by symmetry considerations. Measured values of these parameters were typically of the order of $\pm 0.05$ in Kreplin and Eckelmann (1979). However, the correlation coefficient and Reynolds stress measured in the present study with the rotated X-probe were too large throughout the boundary layer indicating that the measured $W$-signal was probably contaminated by the $V$-component of velocity and the mean shear. In addition, both values rose sharply for $y^+ \leq 100$, indicating that the velocity shear in the $y$-direction is beginning to significantly influence each wire separately. The form of $\phi$ is caused primarily by the necessary inclination of the probe ($\phi$) to get the sensing box near the wall in the MIT wind tunnel. The angle causes the wires to have a slight angle of attack which causes them to, unintentionally, partially measure $V$, which possibly explains the erroneous skewness. The solution to these problems was the use of the V-probe. The rotated X-probes of Laufer (1955) and Klebanoff (1954) must have been used parallel to the wall to avoid these errors.

5.3.2 V-Probe Approach

As described in Section 5.2.4, several V-probes were built to test for blockage effects. When no blockage limitations were found, a small unique V-probe was built ($A^+ = 3.65$) with the prongs inclined similar to the tines of a dinner fork (probe V5), at a 6° angle. This allowed the probe body to be angled away from the wall, while the probe sensing
plane stayed parallel to the wall. Calibrations were done carefully so as to insure that the rotation plane of the probe holder was identical to the plane of the wires (not the body). This required that the body be held at a 6° angle in the holder. Small wedges of balsa wood were used to support the body and maintain the angle. As expected, measurements taken with the V-probe showed an absence of the sharp rise in all the statistics near the wall, seen with the rotated X-probes, since both wires were now at the same y position. Unexpectedly, however, initial measurements showed the same constant level of $\bar{u} w^+$ and $R_{uw}$ seen with the rotated X-probes, which the V-probe was supposed to avoid. More rigorous testing revealed that $\bar{u} w^+$ and $R_{uw}$ were very sensitive to rotation of the probe about its centerline. This rotation causes the probe to be inclined to the wall in the spanwise direction, putting one wire higher than the other. Exact rotational measurement was impossible when the probe was installed in the tunnel but initial values of $R_{uw} \approx .3$ and $\bar{u} w^+ \approx .2$ could be corrected with an approximate spanwise rotation of only $2$–spanwise angles-of-attack of approximately $5$–$6°$ were observed to cause $R_{uw}$ and $\bar{u} w^+$ to be as large as $.5$ and $.37$ respectively. Since ultra-precise rotational positioning was not available, the scheme adopted to set the probe exactly parallel to the wall was to position the probe at $y^+ \approx 100$ (the middle of the log region) and then rotate it in its holder until $\bar{u} w^+ \approx 0$. This technique finally resulted in typical $\bar{u} w^+$ values less than 0.05 throughout the boundary layer, similar to Kreplins and Eckelmann's (1979) measurement. Measurements taken with the inclined-prong V-probe, with $\bar{u} w^+$ set to zero at $y^+ \approx 100$, (probe V5.4) are shown in Figure 5.19. The statistics with this probe are greatly improved, and are in general agreement with measurements of Kreplines and Eckelmann (1979) using a V-probe, Laufer (1955), Klebanoff (1954) using X-probes, and the LDV measurements of Karlsson and Johansson (1988). Direct comparisons are difficult because of the Reynolds number
dependence of $W$ statistics. For $y^+ \leq 15$, $u'^+_{S}$ is approximately 10% too high, but all other statistics are correct. Also note that $S(u)$ is now correct very near the wall. Excursions from the calibration grid occurred at $y^+ \leq 5$ so the first point should not be trusted.

The calibration plot for the V-probe (V5.4) is shown in Figure 5.20, superimposed with 2000 date samples taken at $y^+ = 15$. The calibration plot is full, symmetric and smooth, and like the X-probe calibration plots, the constant velocity arcs change from convex to concave at $Q \approx 3$m/s. The degree of concavity, however, is much less and the bottom of the grid is much wider than for any of the X-probes, which was expected since wake effects are eliminated for the V-probe configuration. Raising the overheat ratio showed no adverse affects in the calibration plot or velocity statistics, so a higher overheat, $\sigma_R = 20\%$, was used for the final measurements. If it does not alter the measurements, a higher overheat is always desireable to minimize drift errors. The 2000 superimposed data samples confirm that there are no excursions, and that the flow angles in the $X-Z$ plane range symmetrically between $\pm 35^\circ$ with the bulk occurring between $\pm 20^\circ$. This is in agreement with the findings of Kreplin and Eckelmann (1979) who reported a symmetric distribution of angles between $\pm 30^\circ$ with the majority occurring between $\pm 17^\circ$ at $y^+ \approx 10$. 

156
Table 5.2: Comparison of Experimental Parameters for several Different Experiments

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† Speculated

\(y_{min}^+\) - Closest position to the wall data was taken

\(Re_D\) - Reynolds number based on channel depth

FP - Flat Plate

SP - Sand Paper

C- and P-table - Cartesian and Polar Look-up Tables

- Value unknown
### Table 5.2: Continued. Comparison of Experimental Parameters for several Different Experiments

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<sup>†</sup> Speculated

<sup>y<sub>min</sub></sup> – Closest position to the wall data was taken

<sup>Re<sub>D</sub></sup> – Reynolds number based on channel depth

FP – Flat Plate

SP – Sand Paper

C- and P-table – Cartesian and Polar Look-up Tables

— Value unknown

N/A – Not applicable
Figure 5.1: Calibration plot for X-probe, X1.7, superimposed with 2000 data samples. Data points taken at $y^+ = 10.3$. Notice the the points beyond the edges of the calibrated region. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using the letter "V" trip located at $x = 10\text{cm}$. 
Figure 5.2: (a) Mean velocity profile taken with X-probe, X1.7, and legend for other authors' data. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$. 

$U_+ = 2.44 \ln(y+) + 5.$ 

$U_+ = y_+$ 

Arctangent Law
Figure 5.2: (b),(c) Turbulence intensities, $u'$ and $v'$, and Skewness, $S(u)$ and $S(v)$, taken with X-probe, X1.7. Notice the error in the $V$ statistical moments. Legend is included with Figure 5.2a. $U_\infty = 11$m/s, $X = 401.5$cm, and using letter "V" trip located at $x = 10$cm.
Figure 5.2: (d), (e) Flatness, $F(u)$ and $F(v)$, Reynolds shear stress and Cross-correlation coefficient taken with X-probe, X1.7. Legend is included with Figure 5.2a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter “V” trip located at $x = 10\text{cm}$. 
Figure 5.3: Percentage of data samples that "fall off" the calibration region for X-probes, X1.7 and X3.7 as a function of distance away from the wall. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$.
Figure 5.4: Temperature distribution and characteristic widths of the wake of a heated circular cylinder
Figure 5.5: Simple wedge model of the wake of a heated wire used to balance the energy equation
Figure 5.6: Calibration plot for X-probe, X3.18, $d \approx 0.63\mu m$, $\ell_{wire}/d = 430$, with an overheat ratio of $a_R = 50\%$. 
Figure 5.7: Calibration plot for X-probe, X3.17, \(d \approx .63\mu\text{m}, \ell_{\text{wire}}/d = 430\), with an
overheat ratio of \(a_R = 30\%\).
Figure 5.8: Calibration plot for X-probe, X3.15, $d \approx 0.63\mu m$, $\ell_{wire}/d = 430$, with an overheat ratio of $a_R = 10\%$. 
Figure 5.9: Calibration plot for X-probe, X3.13, $d \approx 0.63 \mu m$, $\ell_{wire}/d = 430$, with an overheat ratio of $a_R = 5\%$. 
Figure 5.10: (a) Mean velocity profile taken with X-probe, X3.7, and legend for other authors' data. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$.
Figure 5.10: (b),(c) Turbulence intensities, $u'$ and $v'$, and Skewness, $S(u)$ and $S(v)$, taken with X-probe, X3.7. Legend is included with Figure 5.10a. $U_\infty = 11$ m/s, $X = 401.5$ cm, and using letter “V” trip located at $x = 10$ cm.
Figure 5.10: (d),(e) Flatness, $F(u)$ and $F(v)$, Reynolds shear stress and Cross-correlation coefficient taken with X-probe, X3.7. Legend is included with Figure 5.10a. $U_\infty = 11 \text{m/s}$, $X = 401.5 \text{cm}$, and using letter "V" trip located at $x = 10 \text{cm}$. 
Figure 5.11: Calibration plot for X-probe, X3.7, superimposed with 10000 data samples. Data points taken at $y^+ = 14.9$. Notice that there are no points beyond the edges of the calibrated region. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using the letter "V" trip located at $x = 10\text{cm}$. 
Figure 5.12: Nearly ideal calibration plot for X-probe, X2.2, $\ell_{\text{wire}}^+ = 11$, $\ell_{\text{wire}}/d \approx 430$, and using an overheat ratio of $a_R = 1\%$. The price to pay for this "ideal" calibration is that the probe is large, and very sensitive to ambient temperature changes.
Hot-wire response to longitudinal turbulence energy as dependent on wire length and wire diameter: each different symbol represents measurements from a different wire. \( U_\infty = 7.2 \text{ m s}^{-1} \). 

- ○, ▽, ▲, ■, \( d = 5.00 \mu\text{m} \) sensors; ○, △, ●, ▲, ▽, \( d = 1.25 \mu\text{m} \) sensors; ○, ▼, △, □, ○, ▼, △, □, ○, ▾, ○, ▾, \( d = 0.625 \mu\text{m} \) sensors.

Figure 5.13: Hotwire response to streamwise turbulence energy as dependent on wire length, \( \ell_{\text{wire}} \), wire diameter, \( d \), and wire length to diameter ratio, \( \ell_{\text{wire}}/d \). From Ligrani and Bradshaw (1987)
Figure 5.14: Calibration plot for X-probe, X3.26, $d \approx 1.27\mu m$, $\ell_{wire}/d = 215$, with an overhear ratio of $a_R = 30\%$. 
Figure 5.15: (a) Mean velocity profile taken with X-probe, X3.21, using a Cosine-law calibration and legend for other authors' data. $U_\infty = 11\text{m/s}, X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$. 
Figure 5.15: (b),(c) Turbulence intensities, $u'$ and $v'$, and Skewness, $S(u)$ and $S(v)$, taken with X-probe, X3.21, using a Cosine-law calibration. Legend is included with Figure 5.15a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$. 
Figure 5.15: (d),(e) Flatness, $F(u)$ and $F(v)$, Reynolds shear stress and Cross-correlation coefficient taken with X-probe, X3.21, using a Cosine-law calibration. Legend is included with Figure 5.15a. $U_\infty = 11$ m/s, $X = 401.5$ cm, and using letter "V" trip located at $x = 10$ cm.
Figure 5.16: (a) Mean velocity profile taken with the "claw-series" of probes, X4 – X8. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$.
Figure 5.16: (b),(c) Turbulence intensities, $u'$ and $v'$, and Skewness, $S(u)$ and $S(v)$, taken with the "claw-series" or probes, X4 – X8. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $x = 10\text{cm}$.
Figure 5.16: (d),(e) Flatness, \( F(u) \) and \( F(v) \), Reynolds shear stress and cross-correlation coefficient taken with the "claw-series" of probes, X4 - X8. \( U_\infty = 11\text{m/s}, X = 401.5\text{cm} \), and using letter "V" tripped located at \( x = 10\text{cm} \).
Figure 5.17: Calibration plot for “claw-series” of probes, X4 – X8
Figure 5.18: (a) Mean velocity profile taken with a rotated X-probe, RX3.6 and RX3.8, and legend for other authors’ data. $U_\infty = 11 \text{m/s}$, $X = 401.5 \text{cm}$, and using letter “V” trip located at $x = 10 \text{cm}$. 
Figure 5.18: (b),(c) Turbulence intensities, $u'$ and $w'$, and Skewness, S$(u)$ and S$(w)$, taken with a rotated X-probe, RX3.6 and RX3.8. Legend is included with Figure 5.18a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$. 
Figure 5.18: (d),(e) Flatness, $F(u)$ and $F(w)$, and Cross-correlation coefficient taken with a rotated X-probe, RX3.6 and RX3.8. Legend is included with Figure 5.18a. $U_\infty = 11$ m/s, $X = 401.5$ cm, and using letter "V" trip located at $x = 10$ cm.
Figure 5.19: (a) Mean velocity profile taken with V-probe, V5.4, and legend for other authors' data. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$.
Figure 5.19: (b), (c) Turbulence intensities, $u'$ and $w'$, and Skewness, $S(u)$ and $S(w)$, taken with V-probe, V5.4. Legend is included with Figure 5.19a. $U_\infty = 11$ m/s, $X = 401.5$ cm, and using letter "V" trip located at $z = 10$ cm.
Figure 5.19: (d),(e) Flatness, $F(u)$ and $F(w)$, and Cross-correlation coefficient taken with V-probe, V5.4. Legend is included with Figure 5.19a. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using letter "V" trip located at $z = 10\text{cm}$. 
Figure 5.20: Calibration plot for V-probe, V5.4, superimposed with 2000 data samples. Data points taken at $y^+ = 15.1$. $U_\infty = 11\text{m/s}$, $X = 401.5\text{cm}$, and using the letter "V" trip located at $x = 10\text{cm}$. 
Figure 5.21: (a) Probability density distribution of the angle between the instantaneous values of the $U$ and $w$ velocity components at different wall distances. (b) Distribution of the maximum values of $U-w$ angle across the channel half-width. From Kreplin and Eckelmann (1979).
Chapter 6
Pressure Measurements

The present study addresses the origin of the pressure peaks and waves in a turbulent boundary layer. Measurements of all three velocity components at $y^+ = 15$ and the pressure at the wall, below the measuring probe, are used.

6.1 Pressure Measurement

A 1/8 inch Brüel & Kjær (B&K for short), Model 4138, microphone was used to measure the wall pressure fluctuations. The frequency response is flat from 30Hz to 35KHz. Care was taken to ensure that the microphone diaphragm was flush with the wall; however, a small annular gap 0.3mm wide ($8.6 \ell_*$) and 0.05mm deep ($1.4 \ell_*$) was present between the wall and the microphone diaphragm, due to the construction of the microphone. No effort was made to eliminate it as it is believed that it should have no effect on the measurements. The actual diaphragm diameter is 1.95mm, corresponding to $56 \ell_*$.

All the measurements were taken at a free stream velocity of 11.0m/s, 401cm from the leading edge of, and on the centerline of, the plate. For these conditions, $Re_{\theta} = 4340$, $\theta = 0.79cm$, $H = 1.32$, $u_\tau = 0.43m/s$, $\ell_* = 0.035mm$ and $t_* = 81 \mu s$. Continuous data sets of 1,489,920 points were taken for two velocity components ($U, V$ or $U, W$) and wall
pressure, resulting in 496,640 data points per variable, at a sampling rate of 80μs.

Figure 6.1 shows the mean velocity distribution. Figure 6.2 shows the higher statistics of \( U \) and \( V \) and Figure 6.3 shows the higher statistics of \( U \) and \( W \). In spite of the small size of the measuring probes, the measured values appear to be in error for \( y^+ < 10 \). Figure 6.4 shows the probability density of the pressure signal. The rms value of the pressure according to Schewe (1984) should be about \(.008q_{\infty}\); no attempt was made to calibrate the output of the microphone. The maximum positive pressure observed was 8\( p' \) and the minimum \(-7.5p' \).

Data Analysis Techniques

The data was analyzed using conditional sampling and spectral techniques. The Variable-Interval Time-Averaging (VITA) method of Blackwelder and Kaplan (1976) was used to detect shear layers. The VITA algorithm was implemented by taking the local, short time, variance of the signal in question and comparing this value to a predetermined threshold level \( Ku'^2 \), for example, in the case of the \( U \)-signal, where \( K \) is the threshold constant. If the computed short time variance exceeded the threshold level, then an event had been detected. In addition to this criterion, the slope of the signal detection time was also checked so that events could be classified as accelerating or decelerating (see Johansson and Afredson, 1982).

The uv-quadrant method of Lu and Willmarth (1973) was used to detect high Reynolds (shear) stress events. Again, a threshold level was set, and events were declared for excursions of the uv signal above \( Ku'u' \). A peak detection method (Her, 1986) was used to find the high amplitude pressure peaks. A positive event was declared whenever the pressure signal exceeded \( Kp' \) and a negative peak event was declared whenever
it was less than \(-Kp'\).

A spectral peak detection (SPD) method was developed to detect waves in a particular signal. The method was applied to signals extracted from the total record using one of the above-mentioned techniques. Figure 6.5 shows such a signal, obtained using the pressure peak technique, as well as the same signal windowed using a gaussian window. This procedure enhances the activity around the detection time without sacrificing frequency resolution. Normalized power spectra, \(E(\omega^+)\), are then averaged and a threshold level is set at all frequencies. Figure 6.5b shows the power spectrum of the windowed signal in Figure 6.5a, the average of all such spectra and the threshold level. (The bumps in the averaged power spectra at \(w^+ \approx 1.0\) and 2.0 can be attributed to background noise as they are also present in the long-time power spectra of the pressure signal). If the power under any particular spectral peak was found to exceed the threshold level, then the center frequency of that spectral peak was counted as being of dynamical importance. In this example, the threshold level is the same at all frequencies. Thus a histogram of occurrences of different frequencies in the signal, weighted by the power at that frequency, verses frequency was constructed to determine the dynamically important frequencies.

### 6.2 VITA and Pressure Peak Results

The VITA method was used on the \(U\)-signal to detect high shear layer events. Figure 6.6 shows the ensemble average of accelerating events and Figure 6.7 shows the ensemble average of decelerating events. While the \(U\)-signals for the two cases are almost identical in shape, but inverted with respect to each other, the \(V\)-signals and the
p-signals are quite different. The accelerating events produce a pressure peak coincident with the middle of the event with an associated high negative $dv/dt$ which implies a positive $dv/dx$. This type of relation between $dv/dx$ and $p$ is strongly suggestive of a linear relation between the pressure and the velocity field as shown by Johansson et al. (1987). The decelerating events produce a high positive peak in the $V$-signal and a negative pressure. However, the relation between the pressure and the normal velocity component is very close to that found in the accelerating events.

Positive pressure peak detection, Figure 6.8 produces qualitatively the same results as the accelerating VITA events, indicating a qualitative reciprocity. The negative pressure peak events, Figure 6.9, are associated again directly with a positive $dv/dt$ but not with the Reynolds stress, since it is practically zero throughout. Thus, the common thread between pressure peaks and the velocity field is $dv/dt$.

All of the $uv$-quadrant results, Figures 6.10–6.13, produce pressure signatures that are again tied to $dv/dt$. However, the amplitude of the corresponding pressure peaks is much smaller than those found in the VITA events even though the $dv/dt$'s in the present case are about an order of magnitude larger for the same pressure amplitude. The conclusion to be drawn so far is that isolated $uv$ peaks are not intimately related to pressure peaks.

Ensemble averages based on accelerating VITA-on-u and pressure peak events, predictably show a signal of zero amplitude for the spanwise component $w$. However, conditional ensemble averages with $w > 0$ (or $w < 0$) at the center of the detected event, Figures 6.14 – 6.15, show that there is a well defined $w$-signature whose peak coincides with the pressure peaks. Similar results are obtained for decelerating VITA-on-u and negative pressure peak events. It therefore seems that VITA-on-u and pressure peak
events are associated with non-symmetrical structures with respect to the \( x-y \) plane (\( z \) is the coordinate in the streamwise direction and \( y \) the coordinate normal to the wall).

A different perspective of the wall pressure can be obtained from the weighted probability density \( P(u,v) \) of \( U \) and \( V \) with the pressure \( p \), \( p \cdot P(u,v) \), shown in Figure 6.16. Positive pressures are primarily associated with the fourth quadrant or sweep-type events while negative pressures are associated with both the second and third quadrants or ejections and interactions inward respectively. Positive pressures, in general, appear to be the result of fluid moving toward the wall being retarded by the presence of the wall. Negative pressures are primarily distinguished by the fact that \( u \) is negative and it is difficult on the basis of this information alone to come to any definite conclusions.

However, the conditional averages based on the VITA-on-\( u \) and pressure peak events are in accord with the long time \( p \cdot P(u,v) \) distribution. For accelerating VITA-on-\( u \) events, the pressure is low during the ejection part of the event and positive during the sweep part of the event. For decelerating events, again, the pressure is low during most of the ejection part. For negative pressure peak events, the \( U \)-velocity is almost always negative while the \( V \)-velocity is negative during the pressure fall and positive during the pressure rise, thus contributing to the symmetry of \( p \cdot P(u,v) \) for \( u > 0 \).

### 6.3 Wave Detection

Emmerling (1973) and Schewe (1983) showed clearly that both isolated pressure peaks and pressure wave trains were present in their measured wall pressures. Her (1986) found evidence that the pressure wave trains are the "footprint" of flow oscil-
lations in the buffer region as suggested by Schewe (1983) through his pressure measurements, and Kim, Kline and Reynolds (1971) through their flow visualization and hot-wire measurements in the wall region. The SPD method, described earlier, was applied to accelerating VITA-on-u events, Figure 6.17, and positive pressure peak events, Figure 6.18. Decelerating VITA events and negative pressure peak events gave essentially similar results. In both Figures 6.17 and 6.18, the power threshold level was set to twice the average power at each frequency.

It is seen that the histograms peak in the frequency range \( \omega^+ \approx .2-.4 \) for all three signals \((U, V \text{ and } p)\) is in agreement with the observations of Schewe (1983) for high amplitude pressure fluctuations and Kim et al. (1971) for the streamwise velocity component. Figure 6.19 shows the ensemble average of events corresponding to Figure 6.17 that contributed power to a frequency of .4, and likewise in Figure 6.20 for events corresponding to Figure 6.18. The velocity signals have been aligned separately from the pressure signals using the basic alignment technique of Zilberman, Wyganski and Kaplan (1977). This consists of cross-correlating the members of the ensemble with the current ensemble average and then repeating the ensemble averaging procedure by shifting the signals according to the cross-correlation time delays. In the present version, in addition to the above, the signals were windowed as described earlier in the SPD technique. After about two iterations no improvement in the ensemble average result is observed. At lower frequencies (Figures 6.21 and 6.22), the results are quantitatively similar. Close observation of the v-velocity ensemble averages around the detection time (Figure 6.22 \((V\text{-velocity})\) as well as Figure 6.6), shows a kink which may be a manifestation of a secondary wave train or instability. At higher frequencies (Figures 6.23 and 6.24), the pressure signal appears to be a single peak as opposed to a wave train. The latter result may be due to the poor resolution of the pressure transducer, whose
diameter is approximately $2/3$ of the wavelength of the pressure peak.

A comparison can be made of the above results to the computations (Navier-Stokes) of Breuer (1988) of a localized disturbance in a laminar boundary layer. The disturbance developed internal shear layers, as it progressed downstream, which then became unstable. Figure 6.25 shows the $U$ and $V$ signals at $y^+ \approx 20$ and the wall pressure, $p$. The frequency of the wave is approximately $\omega^+ \approx .42$ and the wavelength is approximately $236\ell_*$, in good agreement with the observations of Schewe (1983) and the present results.

6.4 Summary

Conditional sampling and spectral techniques applied to all three velocity signals and the wall pressure in a turbulent boundary layer were used to show the relation between the wall pressure and the near wall flow. The most prominent connection of the wall pressure and the flow field is through the normal component of velocity, $V$. Wall pressure peaks and wave trains are related to corresponding structures in the near wall flow field. The wave trains may well be the manifestation of the instability associated with the lift-up of streaks discussed by Kim et al. (1971) and simulated very convincingly by Acarlar and Smith (1987). No definite statement can be made about the formation of high amplitude pressure peaks, as opposed to waves, other than the pressure peaks may be associated with unusually rapid velocity fluctuations. The ensemble averaged $V$-signals show evidence of secondary wave trains most likely associated with a secondary instability.

It is apparent that in order to resolve small scale, but dynamically significant, pres-
sure fluctuations, that smaller pressure transducers are needed.

A comparison of the present results with computations of a localized disturbance in a laminar boundary layer indicates that both flows produce similar structures. Consequently, it may be profitable to look into the details of such simple flows in order to understand the basic physics of wall pressure as well as turbulent production.
Figure 6.1: Mean Velocity distribution measured with the X-probe. $U_\infty = 11.0\text{m/s}$, $x = 401\text{cm}$. $U_+ = 2.44\ln(Y_+) + 5$. 
Figure 6.2: $U$ and $V$ statistics.
Figure 6.3: $U$ and $W$ statistics.
Figure 6.4: Probability density distribution of pressure, \( p(p) \). \( U_\infty = 11.0 \text{m/s} \), \( x = 401 \text{cm} \).
Figure 6.5: Demonstration of the SPD technique applied to positive pressure peak events with $K = 2.0$. (a) $\cdots$, one of the events detected and $\cdots$, the same event after using a gaussian window. (b) The heavy solid line is the normalized power spectrum of the windowed signal in (a); $\cdots$, average of normalized power spectra; $\cdots$, threshold level. The detected peaks are shown by the arrows.
Figure 6.6: Accelerating VITA-on-u events. $K = 1.0$, $T^+ = 11.0$. 


Figure 6.7: Decelerating VITA-on-u events. $K = 1.0$, $T^+ = 11.0$. 
Figure 6.8: Positive pressure peak events. $K = 2.5$. 
Figure 6.9: Negative pressure peak events. $K = 2.5$. 
Figure 6.10: uv events with $K = 2.0$, quadrant I.
Figure 6.11: uv events with $K = 2.0$, quadrant II.
Figure 6.12: uv events with $K = 2.0$, quadrant III.
Figure 6.13: \(uv\) events with \(K = 2.0\), quadrant IV.
Figure 6.14: Accelerating VITA-on-u events with $w > 0$ at the time of detection. $K = 1.0$, $T^+ = 11.0$. 
Figure 6.15: Positive pressure peak events with $w > 0$ at the time of detection. $K = 2.5$. 
Figure 6.16: $pP(u,v)$, the weighted probability density of $u$ and $v$ with $p$. ---, negative pressures, —, positive pressures. Contour level interval = .001.
Figure 6.17: SPD results on accelerating VITA-on-u events with $K = 1.0$ and $T^+ = 11.0$. Power threshold level is two times the average power at each frequency.
Figure 6.18: SPD results on positive pressure peak events with $K = 2.5$. Power threshold level is two times the average power at each frequency.
Figure 6.19: Ensemble average of events from Figure 6.17 at $\omega^+ = .4$. Solid lines correspond to the aligned signals.
Figure 6.20: Ensemble average of events from Figure 6.18 at \( \omega^+ = .4 \). Solid lines correspond to the aligned signals.
Figure 6.21: Ensemble average of events from Figure 6.17 at $\omega^+ = .2$ Solid lines correspond to the aligned signals.
Figure 6.22: Ensemble average of events from Figure 6.18 at $\omega^+ = .2$. Solid lines correspond to the aligned signals.
Figure 6.23: Ensemble average of events from Figure 6.17 at $\omega^+ = .8$. Solid lines correspond to the aligned signals.
Figure 6.24: Ensemble average of events from Figure 6.18 at $\omega^+ = .8$. Solid lines correspond to aligned signals.
Figure 6.25: $u$ and $v$ velocities and wall pressure on the centerline of a localized disturbance in a laminar boundary layer. $Re_\theta \approx 950$, $y^+ \approx 20$. 
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