THREE ESSAYS ON EXCHANGE RATE DETERMINATION

by

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B.S. U.C. BERKELEY
(1982)

Submitted to the Department of Economics
in Partial Fulfillment of the Requirements
of the Degree of

DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

August 1987

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ABSTRACT

The determination of flexible exchange rates is an issue of manifold proportions that continues to incite further investigation. As usual, the further the investigation proceeds the more new puzzles seem to arise. This thesis addresses a select few of these puzzles.

First, we consider the apparently excessive movements in exchange rates in response to disturbances. The key revelation of this chapter is that money stock innovations have two principal effects on the exchange rate. The first effect is the rational adjustment of the exchange rate to the changed stock as captured by the Overshooting Hypothesis. The second, and potentially much more important effect is the result of the change in money growth rate expectations induced by the stock innovation. The change in the growth rate expectation, which is tantamount to an added stream of future expected changes in the stock, must be fully adjusted for at the time of the expectation change. Simulations verify that this front-end loading of the growth rate effect can produce quite extreme exchange rate volatility using very reasonable parameter values.

Next, we confront the issue of whether or not time-varying risk premia are built into the structure of returns across similar assets denominated in different currencies. We do so by utilizing the second moment expectations implied by currency option pricing to demonstrate that these expectations are in fact systematically related to expected return differentials. Because the measured deviations from uncovered interest rate parity are tied to variables which theory links to the risk premium, the results provide substantial evidence that a risk premium does indeed exist, as opposed to the alternative of a violation of rational expectations. However, like previous attempts, the data do not support an explicit mean-variance formulation of the risk premium.

Finally, we investigate the behavior of floating rates in the developing country context. Focusing on the Peruvian experience with floating rates from 1950-1954, we find that, as a model of the underlying fundamentals, a purchasing power parity model fits quite well whereas the monetary models and a model that emphasizes the world price of Peru's exports fare much less successfully. All told, Peru's experience with floating rates is promising: both the real and nominal exchange rates remained relatively stable. Nonetheless, we call attention to some important caveats. First, the constellation of policies and institutions in Peru at the time were consistent with the relative success of the experiment. These conditions are less likely to be present in most developing countries today. And second, after the experience with generalized floating among industrial countries, it is less likely that speculators in a developing country today would be using a model of fundamentals that would generate the same degree of real exchange rate stability.

Thesis Supervisor: Rudiger Dornbusch
Title: Professor of Economics
ACKNOWLEDGEMENTS

It is a pleasure to acknowledge those people who have helped me in myriad ways through my years here at M.I.T. They have made the experience enjoyable as well as challenging and productive.

In particular, I would like to thank Rudiger Dornbusch and Stanley Fischer for their support and extremely insightful advice at every juncture. The depth and breadth of their understanding provides much inspiration.

I would also like to thank:

Paul Krugman, for very helpful suggestions, and for his perspective, which I greatly admire;

Danny Quah, for numerous useful discussions when unshaped thoughts needed tooling;

Frances Van Loo, who planted the seed of Economics in me at Berkeley;

and numerous fellow graduate students, without whose friendship and collegial support I would still be a Thesis Writer.

I would also like to express my appreciation of the very generous financial support from the National Science Foundation, and my internship at the Board of Governors of the Federal Reserve System.

Most important, I would like to express my gratitude to my parents for their never-ending love and confidence. I am indebted to my mother, who taught me to enjoy, and to my father, who taught me how to set my sights. This dissertation is dedicated to them both.
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Chapter I: Introduction
The determination of flexible exchange rates is an issue of manifold proportions that continues to incite further investigation. As usual, the further the investigation proceeds the more new puzzles seem to arise. The chapters that follow address three of these puzzles. While all three are within the same domain, the analyses involved project a certain independence, so we choose not to force a unification.

First, in chapter II we consider the issue of exchange rate volatility. Why do rates appear to move excessively in response to new information? Dornbusch(1976) provides one very clean rationalization with his Overshooting Hypothesis. And yet, his set-up of a one-time unanticipated shock leaves an important element out of the story: individuals are better characterized by partial knowledge of certain key variables, updating their beliefs over time as new information arises. We build on this extension within a sticky-price monetary model by recognizing that individuals are uncertain of the underlying money growth rate and form their expectations optimally using the path of the observable stock. Thus, if money stock innovations induce changes in money growth rate expectations then a potentially very important additional channel is opened up through which stock innovations can affect exchange rates in an overshooting-type fashion. In effect, a change in the growth rate expectation is tantamount to an added stream of future expected changes in the stock, and must be fully adjusted for at the time of the expectation change. Using a number of different price adjustment rules, we verify that this indirect effect of stock innovations can produce quite extreme exchange rate volatility, and for most parameter values is much more significant than the direct effect captured by Dornbusch(1976).

In contrast to the analysis in chapter II, which abstracts from risk considerations by assuming risk neutrality, the analysis in chapter III takes direct aim at the foreign exchange risk premium. The emphasis is on the potential role of time-varying second moment expectations. Despite their potential importance in assessing risk-return tradeoffs, past work often assumes that exchange rate second moment expectations are constant. The major
contribution of this chapter is the use of the market's second moment expectation as implied by currency option pricing. Using an appropriate option pricing model together with realized market prices, we solve for the unique implied expectation. Then, we use these series to test some common propositions regarding the risk premium. The upshot is that these expectations are in fact systematically related to expected return differentials across similar assets denominated in different currencies. Moreover, because the measured differentials are tied to variables which theory links to the risk premium, the results provide substantial evidence that a risk premium does indeed exist, as opposed to the alternative of a violation of rational expectations. However, when we attempt to fit an explicit mean-variance formulation of the risk premium, we find it at odds with the data. The poor performance of the model here is in keeping with previous attempts in the litterature.

Finally, in chapter IV we leave the industrialized world to investigate the behavior of floating rates in the developing country context. Focusing on the Peruvian experience with floating rates from 1950-1954, we ask and answer two key questions. First, what fundamentals underlay the market's determination of the exchange rate? In the end, we find that an expectational purchasing power parity model fits quite well whereas the monetary models and a model that emphasizes the world price of Peru's exports fare much less successfully. And second, is a floating rate system a viable choice for developing countries such as Peru? All told, Peru's experience with floating rates is promising: both the real and nominal exchange rates remained relatively stable. Nonetheless, we call attention to some important caveats. First, the constellation of policies and institutions in Peru at the time were consistent with the relative success of the experiment. These conditions are less likely to be present in most developing countries today. And second, after the experience with generalized floating among industrial countries, it is less likely that speculators in a developing country today would be using a model of fundamentals that would generate the same degree of real exchange rate stability.
Chapter II: Exchange Rate Dynamics With Monetary Policy Uncertainty
I. Introduction

This chapter addresses an important and yet surprisingly neglected question: how do exchange rates behave if individuals are uncertain of the true money growth rate, forming their expectations optimally using the path of the observable stock? The question certainly seems realistic enough. Some reflection on the issue should help with a simple realization: if money stock innovations induce changes in money growth rate expectations then a potentially very important additional channel is opened up through which stock innovations can affect exchange rates in an overshooting type fashion. The thesis of this paper is that in fact this additional channel is quite significant, probably more so than the direct effect captured by Dornbusch(1976). As a framework for the analysis we use a simple sticky-price monetary model of exchange rate determination. Simulations of the model using various price adjustment rules verify that the indirect effect through growth rate expectations can induce extreme exchange rate volatility.

The primary issue the sticky-price models set out to address is the volatility of exchange rates, especially with respect to the relative stability of national price levels. The seminal contribution along these lines is Dornbusch's(1976) overshooting hypothesis. According to this hypothesis, when (for example) the money stock unexpectedly increases, the nominal exchange rate will first depreciate by more than its long-run depreciation, and the appreciate back to the steady state. This "volatility" effected by the discrete change in the money stock is due to the different speeds of adjustment of asset and goods markets, which magnifies the impact effect on the exchange rate.

Variations on the original theme are numerous. Wilson(1979) shows that while anticipated money stock changes still result in overshooting, the impact effect of the announcement is less than the impact effect in the unanticipated case. Other work demonstrates that undershooting is also a possibility. In the original article, Dornbusch points out that overshooting is not necessary when output is variable. Frenkel and
Rodriguez (1982) show that sufficiently imperfect capital mobility can cause undershooting. Similarly, Papell (1984a) demonstrates that undershooting can result if monetary policy accommodates prices. In a later paper, Papell (1984b) investigates the overshooting hypothesis empirically. Defining all money supply deviations from a time trend as unanticipated changes, he finds evidence that the mark exhibits overshooting while the yen generally undershoots.\(^1\)

Extensions into the analysis of money growth rates and secular inflation include Mussa (1977, 1982), Frankel (1979), Buiter and Miller (1981, 1982), and Flood (1982a). These analyses consider growth rates which are known with certainty except at moments of discrete change. For example, Buiter and Miller (1982) analyze the effects of a one-time unexpected drop in money growth on the real exchange rate and real output. In the model below, rather than adjusting to shocks in a world of perfect certainty, agents act using their best (Bayesian) expectation of money growth given all available information. A Bayesian approach is also used by Lewis (1986). In her model, agents learn about the true level of the money stock over time and this leads to on-average mistaken exchange rate forecasts. And finally, empirical work by Engel and Frankel (1984) is relevant to the issues at hand. They attempt to disentangle the puzzle of why money supply announcements move interest rates by focusing attention on the exchange rate: if a positive innovation in the money supply is associated with depreciation of the currency then this suggests increased inflationary expectations; whereas if it is associated with appreciation then this suggests anticipation of future tightening. They find more evidence of the latter.

We solve and simulate the sticky price model under each of the three appropriate price adjustment rules that appear in the literature: (1) the Barro and Grossman (1976) rule, (2) the Mussa (1981) rule, and (3) the Buiter and Miller (1982) rule. While it is true that for

\(^{1}\) These few examples of analysis of adjustment to stock changes are meant to illustrate the spirit of past work rather than provide a complete survey.
the right parameter choice the Barro-Grossman and Mussa rules are structurally equivalent [see Obstfeld and Rogoff(1984)], we use the same parameter values for all simulations, which produces slightly different results. Unlike the other two rules, the Buiter-Miller rule includes a sluggish core inflation in the specification.

The chapter is organized as follows. Section II describes the various versions of the model in detail, including the equilibrium sources of exchange rate and price level dynamics. Section III presents the results of the simulated exchange rate behavior and compares the volatility with the empirical record. Conclusions are in section IV.

II. The Model

The model is the very simple rational expectations sticky-price monetary model used by Buiter and Miller (1982) and Flood(1982a) to analyze unexpected one-time permanent changes in the rate of money growth:

\[
\begin{align*}
(1) \quad m - p &= ky - \lambda i \\
(2) \quad i &= i^* + \hat{c} \\
(3) \quad p &= \alpha w + (1-\alpha)e \\
(4) \quad y &= \delta(e - p)
\end{align*}
\]

Here, \( w \) is the logarithm of the money price of domestically-produced goods; \( e \) is the logarithm of the exchange rate (defined as the domestic currency price of foreign
currency); $i$ is the domestic nominal interest rate; and $p$ is the domestic consumer price index. Dots over variables indicate rates of change.\textsuperscript{2} The remaining variables are exogenous: $m$ is the logarithm of the nominal money supply; $\alpha$ is the share of home goods in the price index; and $i^*$ is the nominal interest rate on foreign currency bonds. $\alpha$ is assumed fixed and $i^*$ is assumed fixed and equal to zero.

To complete the model, a specification for the gradual adjustment of sticky prices must be added. Equations (5a)–(5c) describe the three appropriate price adjustment rules which appear in the literature:

(5a) \[ \dot{w} = \phi y + \dot{w} \]
(5b) \[ \dot{w} = \phi y + \dot{w} \]
(5c.1) \[ \dot{w} = \phi y + \pi \]
(5c.2) \[ \dot{\pi} = \xi (\dot{p} - \pi) \]

Each of the three specifications is appropriate for moving long-run equilibria and anticipated changes in exogenous variables. Equation (5a) is the sticky-price adjustment rule suggested by Barro and Grossman(1976). The variable $\tilde{w}$ denotes the output price that would prevail in a hypothetical Walrasian equilibrium with fully flexible prices:

(6) \[ \tilde{w}_t = (1/\lambda) \int_t^\infty \exp[(t-\tau)/\lambda] m_{t\tau} d\tau \]

\textsuperscript{2} These rates of change are right-hand derivatives.
where in a stochastic setting the integral would apply to expected future money supplies. It is important to note, however, that this added term which affects the rate of change of prices does not alter the fact that \( \bar{w} \) is a predetermined variable. While unexpected changes in exogenous variables can cause \( \bar{w} \) to jump, \( w \) is precluded from doing so; it is only the right-hand derivative of \( \bar{w} \) which influences \( \dot{w} \). This added term makes the rule appropriate in settings of anticipated future changes in exogenous variables: without it, rational adjustment of \( w \) to anticipated changes would necessitate current disequilibrium in the goods market, and this is inappropriate (asymptotically) in the case where the changes are "infinitely" far in the future.

Equation (5b) is the rule derived by Mussa(1981). \( \bar{w} \) is defined as the domestic output price that would clear the goods market given the actual (possibly disequilibrium) values of endogenous variables \( e, i, \dot{e}, \) and \( \dot{w} \). With the simple specification of the goods market in equation (4), \( \bar{w} \) is equal to the exchange rate \( e \). Alternative microeconomic rationales for the Mussa rule are presented in McCallum(1980), Mussa(1981), and Flood(1982b). Mussa's derivation, for example, assumes monopolistic firms for whom price changes are costly. Flood's inventory-adjustment story assumes that firms set their prices a period in advance of market transactions.

Equations (5c.1) and (5c.2) describe the sluggish core inflation specification originally proposed by Buiter and Miller(1982) who use it to investigate an unexpected permanent change in the money growth rate. \( \pi \) is defined as a backward-looking weighted average of past inflation rates with exponentially declining weights:
(7) \[ \pi_t = \xi \int_{-\infty}^t \exp[-\xi(t-\tau)] \dot{p}_t \, d\tau \]

While this specification maintains the assumption that \( w \) is predetermined, it does not rule out discrete jumps in \( \pi \): as per equation (7), \( \pi_t \) also depends on current \( \dot{p}_t \); if \( p_t \) makes a discontinuous jump due to a jump in \( e_t \), \( \dot{p}_t \) becomes unbounded and so does \( \dot{\pi}_t \). Thus, \( \pi \) must jump. Using the parameter values of the simulations below, it is easily checked that the following relation holds:

(8) \[ \pi_t - \pi_t^- = \xi(1-\alpha)(e_t - e_t^-) = 0.125(e_t - e_t^-) \]

where

\[ \pi_t^- = \lim_{t \to t^-} \pi_t \]

Thus, a 10 percent jump in the exchange rate will induce a 1.25 percent jump in the core inflation rate.

Before moving on to more complicated monetary dynamics, it is helpful to review the response of the model to an unanticipated permanent change in the rate of money growth as analyzed by Flood(1982a) and Buiter and Miller(1982). Using the Barro–Grossman price adjustment rule together with the parameter values used by both Flood and Buiter–Miller, \( k=1, \lambda=2, \alpha=3/4, \delta=\phi=1/2 \), the paths of the exchange rate and price level are described by:
\[
\begin{bmatrix}
  e_t \\
  w_t
\end{bmatrix} = C_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-(1/4)t} + C_2 \begin{bmatrix} 1 \\ 1/3 \end{bmatrix} e^{(3/8)t} + \begin{bmatrix} m_t + 2\mu \\ m_t + 2\mu \end{bmatrix}
\]

Thus, an increase in \( \mu \) to \( \mu + \Delta \mu \) shifts the saddle path out to SP1 as shown in Figure 1. Given the level of \( w \) there exists a unique exchange rate consistent with both rational expectations and stability of the system. The exchange rate must jump from point A to point B, after which the system gradually converges to the new long-run equilibrium at C with the appropriate lower level of real money balances.

**FIGURE 1**

The spirit of the more complicated analysis that follows is the same as that of the analysis above as well as the original overshooting analysis of Dornbusch(1976). We assume that expectations are rational and that the economy is stable. As in Dornbusch(1976), the money supply process outlined below includes unexpected stock changes; as in Wilson(1979) agents must respond to expected future changes in the stock;
as in Buiter and Miller (1982) agents must respond to unexpected changes in the money growth rate. And, unlike previous work, agents must also respond to expected growth rate changes.\(^3\)

The monetary block of the model is described by the following three equations:

\[
\begin{align*}
(10) & \quad \text{dm}_t = \mu_t dt + \sigma dz_1 \\
(11) & \quad \text{d}\mu_t = \theta(\mu_{ss} - \mu_t) dt + b dz_2 \\
(12) & \quad \text{Corr}(dz_1, dz_2) = \rho > 0
\end{align*}
\]

Here, \(m_t\) is the log of the nominal money stock at time \(t\); \(\mu_t\) and \(\sigma^2\) the instantaneous mean and variance of log changes per unit time; \(\mu_{ss}\) the steady-state growth rate; \(dz_1\) and \(dz_2\) increments of standard Wiener processes; and \(\theta, b,\) and \(\rho\) constants. The parameters \(\sigma, \theta, \)
\(b,\) and \(\rho\) are assumed known. Using the observable money stock process, agents form their conditional expectation \((\mu_t^e)\) and conditional variance \((v_t)\) of \(\mu_t\) using Bayesian dynamic optimization.\(^4\) The evolution of these expectations is described by equations (13) and (14):\(^5\)

\[
\begin{align*}
(13) & \quad \text{d}\mu_t^e = \theta(\mu_{ss} - \mu_t^e) dt + [(v_t + \rho \sigma b) / \sigma^2] (\text{dm}_t - \mu_t^e dt) \\
(14) & \quad \text{dv}_t = [(b^2 - 2\theta v_t) - (v_t + \rho \sigma b)^2 / \sigma^2] dt
\end{align*}
\]

\(^3\) In effect, growth rate changes are tantamount to changes in the future stock.

\(^4\) See Appendix 1 for a more detailed description of the optimal inference procedure.

\(^5\) We are assuming that \(\mu_{ss}\) is known with certainty.
It deserves mention at this point that these expectations are optimal given all information available, which includes the values of all endogenous variables of the full system described above.

As is clear from equation (11), deviations of the growth rate from the steady-state level regress back to \( \mu_{ss} \) at a rate determined by \( \Theta \). Rational adjustments to current innovations in \( \mu \) must take this fact into account. It is assumed throughout the analysis that the value of the parameter \( b \) is negative. That is, positive innovations in the money stock are associated with reductions in the underlying rate of growth. This assumption is both a priori much more plausible than its alternative and technically much more useful (see Appendix 1). Because innovations in the money stock induce revisions in growth rate expectations, the impact effect on the exchange rate of the stock innovations works through a second channel on top of the direct channel captured by the overshooting hypothesis. It is important to note, however, that the negative sign of \( b \) does not imply that individuals will adjust downward their growth rate expectation following a positive stock innovation, causing the impact effect through this second channel to go in the direction opposite that of the direct channel. The sign of the revision depends upon the sign of the "covariance" term \((v_t + \rho \sigma b)\) in equation (13). For reasonable parameter values this term has a positive sign, implying that the two effects work in the same direction.

Appendix 2 derives the solution paths for the systems that individuals face under the different price adjustment rules. (The derivations treat only the deterministic solutions, which is appropriate given that agents are assumed risk neutral; they must recalibrate the
system with each innovation.) Consider, as an example, the solution paths for \( e_t \) and \( w_t \) under the Barro–Grossman rule:

\[
\begin{bmatrix}
  e_t \\
  w_t
\end{bmatrix} = C_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-(1/4)t} + C_2 \begin{bmatrix} 1 \\ 1/3 \end{bmatrix} e^{(3/8)t} + \begin{bmatrix} e_p \\ w_p \end{bmatrix}
\]

where

\[
e_p = w_p = m_0 + t\mu_{ss} + 2\mu_{ss} + (1/\theta)(\mu_0^e - \mu_{ss})(1 - e^{-\theta t}) + K(\mu_0^e - \mu_{ss})e^{-\theta t}
\]

and

\[
K \equiv (1/\theta)[1 - (1 + \lambda\theta)^{-1}]
\]

\[
= (1/\theta)[1 - (1 + 2\theta)^{-1}]
\]

The variable \( K \) should be interpreted as a coefficient of adjustment to an unexpected change in the money growth rate. As one would expect, as the money growth rate regression parameter \( \theta \to \infty \) \( K \to 0 \) and as \( \theta \to 0^+ \) \( K \to \lambda \) which is the full adjustment corresponding to the permanent change case of Buiter–Miller and Flood. Note, however, that for this rule as well as the Mussa rule, whatever the value of \( K \) the impact effect of innovations in money growth rates are taken fully by the exchange rate due to predetermined goods prices. Under the Buiter–Miller rule, the core inflation rate is also immediately affected. For all three versions, the dynamics are defined in terms of \( \mu_t^e \) since individuals are acting upon their best expectation of what the current \( \mu \) is. Note also that \( (1/\theta)(\mu_0^e - \mu_{ss}) \) equals the integral from \( t=0 \) to \( t=\infty \) of all expected added money due
to the increase in $\mu_0 e$ above $\mu_{ss}$.

In any interval from $t$ to $t+\Delta t$ where $\Delta t$ is "small" the endogenous variables respond to six different exogenous sources of motion. The first four, which affect both the exchange rate and the price level, would be present in the corresponding deterministic system. These sources are: (1) expected money growth corresponding to $\mu_{ss}$, (2) expected money growth corresponding to $(\mu_0 e - \mu_{ss})$, (3) expected regression of $\mu_0$ toward $\mu_{ss}$, and (4) movement along the saddle path. The remaining two sources of motion arise from (1) innovations in the money stock, $m$, and (2) "innovations" in money growth rate expectations. Since both of these sources are unexpected by definition, their impact effect is wholly upon the exchange rate with the Barro–Grossman and Mussa rules and primarily upon the exchange rate with the Buiter–Miller rule.

III. Simulated Exchange Rate Behavior

The simulations correspond to a period of five years. The "small" interval referred to in section II as $\Delta t$ is set equal to one day. Consequently, the results of the weekly sampling presented below cover seven independent adjustment intervals. Once again, the parameter values are set equal to those used in Buiter and Miller(1982) and Flood(1982a): $k=1$, $\lambda=2$, $\alpha=3/4$, and $\delta=\phi=\xi=1/2$. The parameter $\rho$ of the money supply process is set equal to 0.8.\(^6\)

\(^6\) This parameter has very little effect on the results. For example, in a simulation using the Barro–Grossman rule with $\sigma=0.01$, $b=-0.10$, and $\theta=0.5$, the mean squared change in the log exchange rate for $\rho=0.4$ and $\rho=0.95$ are both 0.020, exactly the same as the $\rho=0.8$ case reported in Table 1A. This somewhat surprising result is due to offsetting effects of changes in $\rho$ in the inference equations (13) and (14).
Tables 1A–1C present results for the mean squared annual changes in the log of the exchange rate for various money supply parameters and price adjustment rules. The corresponding empirical values are provided for comparison. Some comments on the selection of parameter values are in order. First, the time interval to which \( \theta \) applies is one year. That is, a given deviation from \( \mu_{ss} \) regresses at a rate of 100\% percent per year. Second, we choose \( \sigma \) to roughly match the standard error (annual basis) of Barro and Rush's (1981) unanticipated money regressions which range from 0.012 to 0.014. While this is admittedly a rough proxy for \( \sigma \), the tables demonstrate clearly that the results are not sensitive to \( \sigma \). And finally, we choose the most important parameter \( b \) in order to generate money growth variations in realistic ranges. The results of the extreme values for any single week over the five year period are presented in Table 2. Only the various values of the parameter \( b \) are included since the evolution of the growth rate is independent of \( \sigma \).

Pronounced volatility comes out clearly from the tables. Perhaps more important, the 100\% increase in the magnitude of the stock innovations that results when \( \sigma \) is adjusted from 0.01 to 0.02 has an almost imperceptible effect on exchange rate volatility under any of the price adjustment rules. The exchange rate shifts implied by stock innovations of these plausible magnitudes are actually quite small, even though the model still has the overshooting property. The great bulk of the saddle-path adjustment is driven by the updating of growth rate expectations. Additionally, comparing the results

---

7 Since we solve the model under the Buiter–Miller rule using four dimensions (see Appendix II), the solutions we obtain are not appropriate for the limiting case of \( \theta = 0 \). In doing the simulations we set \( \theta = 0.001 \).

8 These values are from Krugman (1981).
TABLE 1A
B–G RULE: SIMULATED MEAN SQUARED CHANGE IN THE LOG EXCHANGE RATE (ANNUAL BASIS)

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<thead>
<tr>
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<th>b=−.05</th>
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θ=0

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θ=0.5

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θ=1

EMPIRICAL VALUES:

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<td>$/BP</td>
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(1973–1980)
TABLE 1B
MUSSA RULE: SIMULATED MEAN SQUARED CHANGE IN THE LOG EXCHANGE RATE (ANNUAL BASIS)

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<tr>
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θ=1

EMPIRICAL VALUES:
$/$DM = .011
$/$Y = .019
$/$BP = .019

(1973–1980)
TABLE 1C
B–M RULE: SIMULATED MEAN SQUARED CHANGE
IN THE LOG EXCHANGE RATE (ANNUAL BASIS)

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<td>.092</td>
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\(\theta=0\)

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EMPIRICAL VALUES:

\(\$/DM = 0.011\)

(1973–1980)

\(\$/Y = 0.019\)

\(\$/BP = 0.019\)
TABLE 2
CORRESPONDING MONEY GROWTH EXTREMES
PERCENT PER WEEK

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<td>0.0055</td>
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<td>0</td>
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<tr>
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<td>-0.0041</td>
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$\theta = 0$

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<td>0.0035</td>
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<td>0</td>
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<tr>
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<td>-0.0034</td>
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<tr>
<td>MIN</td>
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$\theta = 1$
across the different rules, we find considerable similarity. The Mussa rule tends to generate the most volatility, but differences are slight and depend upon the pattern of parameter settings.

Tables 3A–3C bring very strong evidence to the question of the relative importance of the direct and indirect (growth rate) effects of stock innovations on the nominal exchange rate. These values represent the average ratio of the impact effect of growth rate innovations to that of stock innovations over the five year simulation period. As we would expect, the results for the Barro–Grossman and Mussa rules are precisely the same owing to the fact that effectively the only difference between the two is the slope of the saddle path and this slope cancels out when the impact ratios are calculated. The results for the Buiter–Miller rule are quite similar to those for the other two rules. In sum, the results stress the relative importance of growth rate revisions as an explanation of exchange rate volatility.

Tables 4A–4C provide the magnitudes of deviations from purchasing power parity that result under the different parameter configurations. On the following page, Plots 1 and 2 of the real exchange rate over a five year period give an idea of the degree of persistence involved for the Barro–Grossman and Buiter–Miller rules; deviations are considerably more persistent under the core inflation specification.

Finally, using the Barro–Grossman rule, Plot 3 nets weekly nominal exchange rate depreciation of its expected value (the value with which it would have ended the week if no innovations had shocked the system). It should be pointed out, however, that expected depreciation is a very small fraction of the typical weekly changes so that the plot looks very similar to the series without expectations netted out. Additionally, the stock
change series provides a view of the relative importance of the driving forces. That is, given the slope of the saddle-path implied by the parameters (−3), the direct effect of money stock innovations should cause a 33% overshooting of the exchange rate. If one interprets each "blip" in the stock series as an innovation, and considers only the direct effect, then the exchange rate path would only be about a third again more extreme. It appears that the indirect effect of the growth rate updating is doing most all of the work.
**TABLE 3A**

B–G RULE: RATIO OF EXCHANGE RATE ADJUSTMENT INDUCED BY GROWTH RATE INNOVATIONS TO THAT INDUCED BY STOCK INNOVATIONS

<table>
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<td>10.0</td>
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</tr>
<tr>
<td>σ=.015</td>
<td>6.7</td>
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<tr>
<td>σ=.02</td>
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<td>10.0</td>
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\( \theta=0 \)

<table>
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<tr>
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\( \theta=1 \)
**TABLE 3B**

MUSSA RULE: RATIO OF EXCHANGE RATE ADJUSTMENT INDUCED BY GROWTH RATE INNOVATIONS TO THAT INDUCED BY STOCK INNOVATIONS

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$\theta = 1$
# TABLE 3C

**B–M RULE: RATIO OF EXCHANGE RATE ADJUSTMENT INDUCED BY GROWTH RATE INNOVATIONS TO THAT INDUCED BY STOCK INNOVATIONS**

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TABLE 4A
B–G RULE: SIMULATED MAXIMUM % DEVIATIONS FROM PPP

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TABLE 4B
MUSSA RULE: SIMULATED MAXIMUM % DEVIATIONS FROM PPP

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TABLE 4C
B–M RULE: SIMULATED MAXIMUM % DEVIATIONS FROM PPP

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\[ \theta = 0 \]

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\[ \theta = 0.5 \]

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</tr>
<tr>
<td>.02</td>
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<td>19</td>
</tr>
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\[ \theta = 1 \]
PLOT #1: REAL EXCHANGE RATE (B-G MODEL)

PLOT #2: REAL EXCHANGE RATE (B-M MODEL)

* Parameter values for plots are: $b = -0.10$, $\sigma = 0.01$, $\theta = 0.5$, and $\rho = 0.8$. 
PLOT #3: NOMINAL EXCHANGE RATE VOLATILITY

![Graph showing percent change in week versus week for depreciation in excess of expected and money stock change.](image-url)
IV. Conclusion

The purpose of this chapter is not to push a purely monetary explanation of exchange rate variability. Most would agree that some combination of fiscal and asset-market/psychological factors also play a role. The purpose is rather to highlight a neglected and very significant channel through which money stock innovations can affect the exchange rate. Namely, if individuals form their money growth rate expectations using the path of the observable stock, then stock innovations will produce changes in growth rate expectations which independently induce an impact effect on the exchange rate. The simulation results suggest that this indirect effect is probably stronger than the direct effect as captured by Dornbusch (1976).

A number of directions for further work are apparent. First, one could consider the efficiency in terms of lost output of different disinflation policies tied to various values of $\theta$, the adjustment parameter of the growth rate. Would some type of Laffer curve result? Second, what if the steady-state growth rate were to change? What would the updating procedure look like? This is a much more difficult question technically. And third, what if $\theta$ were made stochastic?
APPENDIX 1

This appendix describes the Bayesian updating procedure for the conditional expectation and the conditional variance of the money growth rate. These results are due to Liptser and Shiryaev (1978) and are generalizations of continuous-time Kalman filters. A more detailed description appears in Gennotte (1984).

We begin with the case where there is no mean reversion in $\mu$. The money stock is determined by a single state variable $z$. The instantaneous change in the log of the money stock and money growth rate are given by:

\begin{align*}
\text{(A1.1)} & \quad dm_t = \mu_t dt + \sigma dz \\
\text{(A1.2)} & \quad d\mu_t = bdz
\end{align*}

where $b$ is a constant scalar and $\sigma$ is a positive constant. Denote the expectation and variance of $\mu_t$ conditional on observing realized money supplies as $\mu_t^e$ and $\nu_t$ respectively. The distribution of $\mu$ at time zero is assumed to be Gaussian with mean $\mu_0^e$ and variance $\nu_0$. The changes in the conditional expectation are given by:

\begin{equation}
\text{(A1.3)} \quad d\mu_t^e = \frac{[(\nu_t + b\sigma)/\sigma^2]}{2} (dm - \mu_t^e dt)
\end{equation}

---

9 The dimension one example in Gennotte (1984), page 17, is a diffusion process in proportional terms. The translation to the logarithm involves an application of Ito’s lemma to arrive at the correct mean growth rate. If the mean growth rate in proportional terms is $\zeta$ then the mean growth rate of the log is $\mu = \zeta - \sigma^2/2$.

10 The equation II–2.3 on page 17 of Gennotte (1984) is incorrect; the factor "$dz$" should not be included.
Equation (A1.3) can be interpreted as:

\[(A1.4) \quad d\mu_t^e = \left[ \text{Cov}(dm_t, \mu_{t+dt}) / \text{Var}(dm) \right] (dm - \mu_t^e dt) \]

Equation A1.4 has the same form as the usual inference equation for Gaussian variables and \( \mu_t^e \) is also the least squares estimator. The change in the expectation is the innovation in the instantaneous change in the log money stock multiplied by the ratio of the covariance of the two variables and the variance of the signal \( dm \). If the innovation is positive and if the correlation between \( \mu \) and \( dm \) is positive, agents will adjust their expectation upward.

The conditional variance \( \nu_t \) is determined by \( \nu_0 \) and \( dv \):

\[(A1.5) \quad dv_t = \left[ b^2 - (\nu_t + b\sigma)^2/\sigma^2 \right] dt \]

\( \nu_t \) is thus a predictable function of time. Equation A1.5 can be interpreted as:

\[(A1.6) \quad dv_t = d[\text{Var}(\mu_t)] - \text{Cov}(dm_t, \mu_{t+dt})^2/\text{Var}(dm) \]

This equation is also similar to the usual inference equation for the conditional variance. The first term is the ex-ante increment in the conditional variance corresponding to the unobservable variation of \( \mu \) from \( t \) to \( t+dt \). The second term is always negative and
represents the reduction in variance due to the information dm. The variance decreases if the new information is precise enough to outweigh the additional noise due to the variation of $\mu$.

It can be shown that the conditional variance of $\mu^e_t$ can be expressed as:

\begin{equation}
(A1.7) \quad v_t = -2b\sigma[1 - \{1 - (v_0 + 2b\sigma)^{-1}v_0\exp(-2bt/\sigma)\}^{-1}]
\end{equation}

Thus, if $b$ is positive the limit of $v_t$ as $t$ tends to infinity is zero. If $b$ is negative the limit is $-2b\sigma$ and the estimation error persists through time. If $v_0=0$ then $v_t=0$ for all $t$.

Extension of the above to the case where $\mu$ regresses back toward a steady-state $\mu_{ss}$ is straightforward as long as $\mu_{ss}$ is known with certainty. Otherwise, the updating process must adjust both $\mu^{e, ss}$ and $\mu^{e, t}$. This problem is considerably more complicated. Technically, however, in order to allow for non-zero conditional variance after time $t=0$ in the case where $\mu_{ss}$ is known with certainty it is necessary to introduce a second Wiener increment so that money stock innovations and growth rate innovations are not perfectly correlated. The money process used in the paper includes this change:

\begin{equation}
(A1.8) \quad dm_t = \mu_t dt + \sigma dz_1
\end{equation}

\begin{equation}
(A1.9) \quad d\mu_t = \theta(\mu_{ss} - \mu_t)dt + bdz_2
\end{equation}

\begin{equation}
(A1.10) \quad \text{Corr}(dz_1, dz_2) = \rho > 0
\end{equation}
Under this specification the evolution of the optimal expectations is described by:

\begin{align*}
(A1.11) & \quad d\mu_t^e = \theta(\mu_{ss} - \mu_t^e)dt + \left[(v_t + \rho \sigma b) / \sigma^2\right] (dm_t - \mu_t^edt) \\
(A1.12) & \quad dv_t = [(b^2 - 2\theta v_t) - (v_t + \rho \sigma b)^2 / \sigma^2] dt
\end{align*}

Interpretation of these updating procedures is the same as that in the single Wiener increment case with no regression back to a mean.
APPENDIX 2

This appendix solves the model under four different price adjustment scenarios: (1) the case of perfect price flexibility, (2) the Barro-Grossman rule, (3) the Mussa rule, and (4) the core inflation rule of Buiter-Miller.

Adjustment With Perfect Goods Price Flexibility

We solve the flexible price model in order to determine the unique coefficient of adjustment to a money growth change such that price level and exchange rate expectations will be fully consistent with non-explosive evolution of the system. Letting $K$ denote this coefficient, in the flexible price model $K = d\tilde{p}/d\mu = d\tilde{w}/d\mu = de/d\mu$, where $\tilde{p}$ and $\tilde{w}$ denote the consumer price index and the domestic output price under perfect flexibility. It is assumed that deviations of $\mu$ from the steady-state $\mu_{ss}$ regress back to $\mu_{ss}$ at rate $\theta$. The system is described by the following five equations:

(A2.1) \[ m - \tilde{p} = ky - \lambda i \]

(A2.2) \[ i = i^* + \dot{e} \]

(A2.3) \[ y = \delta(e - \tilde{p}) \]

(A2.4) \[ \tilde{p} = \alpha \tilde{w} + (1-\alpha)e \]

(A2.5) \[ \dot{\mu} = \theta(\mu_{ss} - \mu) \]
Normalizing $i^* = y = 0$, and recognizing that with perfect price flexibility $e = \bar{w}$ and $\dot{e} = \dot{\bar{w}}$ we obtain the system:

(A2.6) \[ \dot{e}_t = (1/\lambda)e_t - (1/\lambda)m_t \]

where

(A2.7) \[ m_t = m_0 + t\mu_{ss} + (\mu_0 - \mu_{ss}) \int_0^t e^{-\theta\tau} d\tau \]

\[ = m_0 + t\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{-\theta t}) \]

Note that $(1/\theta)(\mu_0 - \mu_{ss})$ is the integral from $t=0$ to $t=\infty$ of all added money due to the increase in $\mu_0$ above $\mu_{ss}$. The term $e^{-\theta t}$ captures that fraction which has not yet occurred at time $t$.

The general solution to the nonhomogeneous differential equation A2.6 has the form:

(A2.8) \[ e_t = C_1 e^{(1/\lambda)t} + e_p \]

Where $e_p$ is any particular solution to A2.6. We will use the method of undetermined coefficients to solve for $e_p$. Given the form of $m_t$ it is appropriate to postulate that the particular solution will have the form:

(A2.9) \[ e_p = d_1 + d_2 t + d_3 e^{-\theta t} \]
Computing $\dot{e}_p$ and substituting into A2.6 yields:

\[(A2.10) \quad d_2 - \theta d_3 e^{-\theta t} = (1/\lambda)(d_1 + d_2 t + d_3 e^{-\theta t}) - (1/\lambda)[m_0 + \mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{-\theta t})] \]

Equating coefficients of like powers of $t$ generates three equations:

\[(A2.11) \quad d_2 = (1/\lambda)d_1 - (1/\lambda)m_0 - (1/\lambda\theta)(\mu_0 - \mu_{ss}) \]

\[(A2.12) \quad 0 = (1/\lambda)d_2 - (1/\lambda)\mu_{ss} \]

\[(A2.13) \quad - \theta d_3 = (1/\lambda)d_3 + (1/\lambda\theta)(\mu_0 - \mu_{ss}) \]

which leads to the following particular solution:

\[(A2.14) \quad e_p = m_0 + t\mu_{ss} + \lambda\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss}) - (1/\theta)(\mu_0 - \mu_{ss})(1 + \lambda\theta)^{-1} e^{-\theta t} \]

Given that the eigenvalue of the system, $1/\lambda$, is positive, $e_0$ must be such that $C_1$ in equation A2.8 equals 0. This yields:

\[(A2.15) \quad e_0 = m_0 + \lambda\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - (1 + \lambda\theta)^{-1}) \]

Defining $K \equiv (1/\theta)[1 - (1 + \lambda\theta)^{-1}]$ we can then write:
\[ e_0 = m_0 + \lambda \mu_{ss} + K(\mu_0 - \mu_{ss}) . \]

Note that:
\[ \lim_{\theta \to \infty} K = 0 \]

and
\[ \lim_{\theta \to 0^+} K = \lambda. \]

Thus, in the case where there is no regression of \( \mu \) back to a \( \mu_{ss} \), i.e. \( \theta=0 \), we are left with the unexpected one-time permanent \( \Delta \mu \) case. When \( \theta \) is very large, a change in \( \mu \) scarcely moves \( e \) and \( \bar{w} \).

**Adjustment With Sticky Goods Prices: The Barro-Grossman Rule**

In this case the model described by equations A2.1 through A2.5 is augmented by the following relationship describing the gradual adjustment of goods prices:

\[ (A2.16) \quad \dot{\bar{w}} = \varphi y + \dot{\bar{w}} \]

where \( \bar{w} \) is the flexible price equilibrium domestic output price. Recognizing that in this setting:

\[ (A2.17) \quad \frac{d\bar{w}}{dt} = (d\bar{w}/dm)(dm/dt) + (d\bar{w}/d\mu)(d\mu/dt) \]

\[ (A2.18) \quad (d\bar{w}/dm)(dm/dt) = (1)(\mu_t) = \mu_t \]
(A2.19) \( (d\bar{w}/d\mu)(d\mu/dt) = (K)\left[-\theta \left(\mu_0 - \mu_{ss}\right) e^{-\theta t}\right] \)

and normalizing \( i^* = \bar{y} = 0 \), we can then write the system as:

\[
\begin{bmatrix}
\dot{e}_t \\
\dot{w}_t
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
e_t \\
w_t
\end{bmatrix} +
\begin{bmatrix}
-(1/\lambda)m_t \\
\mu_t - K\theta(\mu_0 - \mu_{ss})e^{-\theta t}
\end{bmatrix}
\]

where \( m_t \) is as in equation A2.7 and:

(A2.21) \( a_{11} = (1/\lambda)[(1 - \alpha) + k\delta - k\delta(1 - \alpha)] \)

(A2.22) \( a_{12} = (1/\lambda)(\alpha - k\delta\alpha) \)

(A2.23) \( a_{21} = \varphi\delta - \varphi\delta(1 - \alpha) \)

(A2.24) \( a_{22} = -\varphi\delta\alpha \)

(A2.25) \( \mu_t = \mu_{ss} + (\mu_0 - \mu_{ss})e^{-\theta t} \)

Substituting in the parameter values used in Buitter and Miller(1982) and Flood(1982),\(^{11}\) the general solution to the system in equations A2.20 can be written in the form:

---

\(^{11}\) These values are \( k=1, \varphi=\delta=1/2, \lambda=2, \) and \( \alpha=3/4. \)
\[
\begin{bmatrix}
e_t \\
w_t
\end{bmatrix} = C_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-(1/4)t} + C_2 \begin{bmatrix} 1 \\ 1/3 \end{bmatrix} e^{(3/8)t} + \begin{bmatrix} e_p \\ w_p \end{bmatrix}
\]

where \(e_p\) and \(w_p\) are corresponding particular solutions. As before, we use the method of undetermined coefficients to solve for \(e_p\) and \(w_p\). Given the form of the nonhomogeneous terms in equation A2.20 it is appropriate to postulate that the particular solutions will have the form:

(A2.27)
\[
\begin{align*}
e_p &= d_1 + d_2 t + d_3 e^{-\theta t} \\
w_p &= d_4 + d_5 t + d_6 e^{-\theta t}
\end{align*}
\]

Computing \(\dot{e}_p\) and \(\dot{w}_p\) and substituting into equation A2.20 yields the equations:

(A2.28)
\[
d_2 - \theta d_3 e^{-\theta t} = (5/16)(d_1 + d_2 t + d_3 e^{\theta t}) + (3/16)(d_4 + d_5 t + d_6 e^{-\theta t})
\]
\[
- (1/2)[\mu_0 + \mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{\theta t})]
\]

(A2.29)
\[
d_5 - \theta d_6 e^{-\theta t} = (3/16)(d_1 + d_2 t + d_3 e^{\theta t}) - (3/16)(d_4 + d_5 t + d_6 e^{-\theta t})
\]
\[
+ [\mu_{ss} + (\mu_0 - \mu_{ss})e^{\theta t} - K\theta(\mu_0 - \mu_{ss})e^{-\theta t}]
\]
Equating coefficients of like powers of $t$ generates three pairs of equations:

\[(A2.30) \quad d_2 = (5/16)d_1 + (3/16)d_4 - (1/2)m_0 - (1/2\theta)(\mu_0 - \mu_{ss})\]

\[(A2.31) \quad d_5 = (3/16)d_1 - (3/16)d_4 + \mu_{ss}\]

\[(A2.32) \quad 0 = (5/16)d_2 + (3/16)d_5 - (1/2)\mu_{ss}\]

\[(A2.33) \quad 0 = (3/16)d_2 - (3/16)d_5\]

\[(A2.34) \quad -\theta d_3 = (5/16)d_3 + (3/16)d_6 + (1/2\theta)(\mu_0 - \mu_{ss})\]

\[(A2.35) \quad -\theta d_6 = (3/16)d_3 - (3/16)d_6 + (1 - K\theta)(\mu_0 - \mu_{ss})\]

The resulting coefficient solutions are:

\[(A2.36) \quad d_1 = d_4 = m_0 + 2\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})\]

\[(A2.37) \quad d_2 = d_5 = \mu_{ss}\]

\[(A2.38) \quad d_3 = d_6 = -(1/\theta)(1 - K\theta)(\mu_0 - \mu_{ss})\]

Which yields the particular solutions of equation A2.26:

\[(A2.39) \quad e_p = m_0 + t\mu_{ss} + 2\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{-\theta t}) + K(\mu_0 - \mu_{ss})e^{-\theta t}\]

\[(A2.40) \quad w_p = m_0 + t\mu_{ss} + 2\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{-\theta t}) + K(\mu_0 - \mu_{ss})e^{-\theta t}\]

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Note again that \( (1/\theta)(\mu_0 - \mu_{ss}) \) is the integral from \( t=0 \) to \( t=\infty \) of all added money due to the increase in \( \mu_0 \) above \( \mu_{ss} \). The term \( (1 - e^{-\theta t}) \) captures that fraction that has not yet occurred at time \( t \). The long run equilibrium will have \( e \) and \( w \) increasing at rate \( \mu_{ss} \) with the level fully adjusted for the added money stock. The coefficient of adjustment \( K \) ensures the proper realignment of the system to a deviation of \( \mu \) from \( \mu_{ss} \). The initial realignment decays over time as \( \mu_t \to \mu_{ss} \). In the case of no mean reversion in \( \mu \), \( K \) equals 2 (=\( \lambda \)) and the system is similar to that analyzed by Buiter and Miller(1982) and Flood(1982) as discussed in the text.

**Adjustment With Sticky Goods Prices: The Mussa Rule**

In this case the price adjustment rule is:

\( \dot{w} = \phi y + \dot{w} \) \hspace{1cm} (A2.41)

where \( \dot{w} \) is defined as the domestic output price that would clear the goods market given the actual (possibly disequilibrium) values of endogenous variables. For the version of the model described by equations A2.1 through A2.5, \( \bar{w} = e \). Normalizing \( i^* = \bar{y} = 0 \), we can write the system as:
(A2.42) \[
\begin{bmatrix}
\dot{e}_t \\
\dot{w}_t
\end{bmatrix}
= \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
e_t \\
w_t
\end{bmatrix}
+ \begin{bmatrix}
-(1/\lambda)m_t \\
-(1/\lambda)m_t
\end{bmatrix}
\]

where \( m_t \) is as in equation A2.7 and:

(A2.43) \[ a_{11} = (1/\lambda)((1 - \alpha) + k\delta \alpha) \]

(A2.44) \[ a_{12} = (1/\lambda)(\alpha - k\delta \alpha) \]

(A2.45) \[ a_{21} = \varphi \delta \alpha + (1/\lambda)((1 - \alpha) + k\delta \alpha) \]

(A2.46) \[ a_{22} = (1/\lambda)(\alpha - k\delta \alpha) - \varphi \delta \alpha \]

(A2.47) \[ \mu_t = \mu_{ss} + (\mu_0 - \mu_{ss})e^{-\theta t} \]

Substituting in the parameter values, the general solution to the system in equations A2.42 can be written in the form:

(A2.48) \[
\begin{bmatrix}
e_t \\
w_t
\end{bmatrix}
= C_1 \begin{bmatrix} 1 \\ -8/3 \end{bmatrix} e^{-(3/16)t} + C_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{(1/2)t} + \begin{bmatrix} e_p \\ w_p \end{bmatrix}
\]
where \( e_p \) and \( w_p \) are corresponding particular solutions. Using the method of undetermined coefficients, the particular solutions can be shown to equal those for the Barro-Grossman rule case:

\[
\begin{align*}
(A2.49) \quad e_p &= m_0 + t\mu_{ss} + 2\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{-\theta t}) + K(\mu_0 - \mu_{ss})e^{-\theta t} \\
(A2.50) \quad w_p &= m_0 + t\mu_{ss} + 2\mu_{ss} + (1/\theta)(\mu_0 - \mu_{ss})(1 - e^{-\theta t}) + K(\mu_0 - \mu_{ss})e^{-\theta t}
\end{align*}
\]

**Adjustment With Sticky Goods Prices and Core Inflation**

The core inflation specification includes the price adjustment rule of Buiter and Miller(1982):

\[
\begin{align*}
(A2.51) \quad \dot{w} &= \phi y + \pi \\
(A2.52) \quad \dot{\pi} &= \xi(\dot{p} - \pi)
\end{align*}
\]

Equation A2.52 defines \( \pi \) as a backward-looking weighted average of past rates of inflation with exponentially declining weights.

It is inappropriate to treat this model as a three-dimensional linear system with time-dependent nonhomogeneous terms analogous to the two-dimensional systems above: given the price-adjustment rule, \( w \) is no longer free to immediately adjust to anticipated continuous money growth as it is with the Barro–Grossman and Mussa rules. Thus, the
economics necessitate a higher-dimensional system which incorporates this important
difference. To facilitate the analysis, we assume that $\mu_{ss}=0$ so that four dimensions will
be sufficient. Normalizing $i^*=\bar{y}=0$ and substituting in all parameter values save $\theta$, the
system can be written as:

\[
(A.253) \quad \begin{bmatrix}
\dot{e} \\
\dot{w} \\
\dot{\pi} \\
\dot{m}
\end{bmatrix} = \begin{bmatrix}
5/16 & 3/16 & 0 & -1/2 \\
3/16 & -3/16 & 1 & 0 \\
7/64 & -3/64 & -1/8 & -1/16 \\
0 & 0 & 0 & -\theta
\end{bmatrix} \begin{bmatrix}
e \\
w \\
\pi \\
m
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
\theta m_0 + \mu_0
\end{bmatrix}
\]

The general solution to the system in A2.53 can be written in the form:

\[
(A.254) \quad \begin{bmatrix}
e_t \\
w_t \\
\pi_t \\
m_t
\end{bmatrix} = C_1 Z_1 e^{\eta_1 t} + C_2 Z_2 e^{\eta_2 t} + C_3 Z_3 e^{\eta_3 t} + C_4 Z_4 e^{\eta_4 t} + \begin{bmatrix}
m_0 + (1/\theta)\mu_0 \\
m_0 + (1/\theta)\mu_0 \\
m_0 + (1/\theta)\mu_0 \\
m_0 + (1/\theta)\mu_0
\end{bmatrix}
\]

where $\eta_1 = 0.4179$
$\eta_2 = -\theta$
$\eta_3 = -0.2090 + 0.2617i$
$\eta_4 = -0.2090 - 0.2617i$

\[
Z_1 = \begin{bmatrix}
1 \\
0.562 \\
0.153 \\
0
\end{bmatrix} \quad Z_3 = \begin{bmatrix}
1 \\
-2.781 + 1.396i \\
-0.493 - 0.758i \\
0
\end{bmatrix} \quad Z_4 = \begin{bmatrix}
1 \\
-2.781 - 1.396i \\
-0.493 + 0.758i \\
0
\end{bmatrix}
\]
and \[ Z_2 = \begin{bmatrix} Z_{21} \\ Z_{22} \\ Z_{23} \\ Z_{24} \end{bmatrix} \]

where \[ Z_{21} = 1 \]
\[ Z_{22} = (3-10\theta)/(32\theta^2-10\theta+3) \]
\[ Z_{23} = (3/16 - \theta)Z_{22} - 3/16 \]
\[ Z_{24} = (-3/4)Z_{22} + (16\theta-2)Z_{23} + 7/4 \]

Transforming the terms that correspond to complex roots yields the equivalent system:\(^{12}\)

\[
\begin{bmatrix} e_t \\ w_t \\ \pi_t \\ m_t \end{bmatrix} = C_1 \begin{bmatrix} Z_1 \\ \end{bmatrix} e^\eta_{1t} + C_2 \begin{bmatrix} Z_2 \\ \end{bmatrix} e^\eta_{2t} + C_3 \begin{bmatrix} Z_5 \\ \end{bmatrix} e^{-0.2090t} + C_4 \begin{bmatrix} Z_6 \\ \end{bmatrix} e^{-0.2090t} + \begin{bmatrix} m_0 + (1/\theta)\mu_0 \\ m_0 + (1/\theta)\mu_0 \\ 0 \\ m_0 + (1/\theta)\mu_0 \end{bmatrix}
\]

where \( Z_1 \) and \( Z_2 \) are as before and:

\[ Z_5 = \begin{bmatrix} \cos(\nu t) \\ -2.781\cos(\nu t) - 1.396\sin(\nu t) \\ -0.493\cos(\nu t) + 0.758\sin(\nu t) \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(\nu t) \\ -3.112\cos(\nu t - 0.465) \\ -0.904\cos(\nu t + 0.994) \\ 0 \end{bmatrix} \]

\[ Z_6 = \begin{bmatrix} \sin(\nu t) \\ -2.781\sin(\nu t) + 1.396\cos(\nu t) \\ -0.493\sin(\nu t) - 0.758\cos(\nu t) \\ 0 \end{bmatrix} = \begin{bmatrix} \sin(\nu t) \\ -3.112\cos(\nu t + 4.247) \\ -0.904\cos(\nu t - 0.577) \\ 0 \end{bmatrix} \]

and \( \nu = 0.2617 \).

\(^{12}\) See theorem 8.4-2, page 392 in Martin(1983).
Chapter III: Tests of the Foreign Exchange Risk Premium Using the
Expected Second Moments Implied by Option Pricing
I. Introduction

The search for the foreign exchange risk premium and the variables which drive it has produced a vast empirical literature, with very limited success. However, there is reason to be cautious when interpreting many of the results, for they are generally based on an extremely tenuous assumption: the tests generally assume that conditional second moments are constant [e.g., Frankel(1982b) and Hodrick and Srivastava(1984)]. Recent work by Cumby and Obstfeld(1985) and Giovannini and Jorion(1987) strongly rejects the hypothesis that the conditional variance-covariance matrix of rates of return is constant. Not only do these results suggest that many of the previous tests involve serious misspecification errors, they also have important policy implications (e.g., for the reliability of the portfolio balance effects of sterilized intervention).

This chapter applies a new method for investigating the risk premium which explicitly recognizes this previously-neglected dimension of the risk-return tradeoff. The approach exploits information revealed in the foreign currency options market—a market whose principal objective is the prediction of future exchange rate second moments. Using a currency option pricing model, together with realized market option prices, we solve for the market's conditional second moment expectation. Then, we use these time-varying expectations to test some hypotheses commonly tested in the literature to determine whether they provide any new insights.

Using the options market to measure time-varying expectations offers three key benefits. First, this measure of the second moment expectation is rooted in actual market behavior. It therefore reflects the basis on which investors are assessing risk-return tradeoffs, and is properly forward-looking. Second, it avoids the difficult question of whether to measure variance as dispersion about a conditional mean or a sample mean. Past

---

\(^1\) Examples of work on implied volatility include Schmalensee and Trippi(1978), Beckers(1981), and Shastri and Tandon(1986b).
work on asset market volatility often measures time-varying second moments using either moving variance about moving mean (MVAMM) estimators, or the ARCH model developed by Engle(1982). The option market approach differs from these in the sense that it implicitly measures the market's method of conditioning. And third, this approach allows sudden jumps in the conditional variance, which are ruled out with other methods (e.g., ARCH). With these three benefits, however, must be accepted a drawback: the approach assumes that the implied volatilities map into second moments. Yet, although the sufficient conditions for an exact correspondence do not hold, the resulting series show a striking similarity to realized second moments.

We first use the implied expectations to test the hypothesis of unbiasedness of forward exchange rates (which implies jointly that traders have rational expectations and that there is no foreign exchange risk premium). We do so by testing for significant correlation between expected variances/covariances and net-of-depreciation interest differentials, where expected covariances are constructed using sample correlations together with expected standard deviations. The results of these tests allow us to answer two key questions. First, can we reject unbiasedness using a variable upon which risk premia theoretically depend? Although the joint unbiasedness hypothesis has been strongly rejected in the literature [e.g., Hsieh(1984)], the rejections are not due to factors that theory links to the risk premium. Consequently, they provide little evidence for the alternative hypothesis that a risk premium does indeed exist. Because theory establishes a relation between second moment changes and the risk premium, this test provides a stronger basis than previous tests for interpreting rejection of the joint hypothesis as evidence for the existence of a risk premium, as opposed to a violation of rational expectations.

The second key question the unbiasedness tests will answer concerns a proposition asserted by Krugman(1981). He makes the case that the role of Jensen's inequality in international portfolio decisions is theoretically similar in magnitude to that of the risk premium itself. Since the effects of Jensen's inequality necessarily arise as long as investors
maximize real returns, a not uncommon assumption in finance theory, we net the dependent variable of these terms, which produces an estimate of the additional effect of expected variances/covariances on the equilibrium relationship.\footnote{Thus, we do not test for the influence of Jensen's inequality; the data do not allow us to discriminate between the role of Jensen's inequality and that of the risk premium. Rather, since the existence of terms arising from Jensen's inequality involve such a commonly assumed sufficient condition, we impose their presence throughout the analysis. Because their influence turns out to be quite small relative to that of the measured risk premium, netting them from the dependent variable has no substantive effect on any of the results presented.} To the extent that these additional second moment effects are large relative to the Jensen's terms, this constitutes evidence against the proposition of Krugman.

We then apply some "structure" to help interpret the unbiasedness results by applying a portfolio balance model developed by Kouri(1976,1977) and Dornbusch(1983). This intermediate step naturally leads into a test of the Kouri-Dornbusch model itself. While this model has previously been tested by Frankel(1982b), his test assumes a constant return variance-covariance matrix. Our test replicates Frankel's except for the fact that it explicitly includes a time-varying matrix. This is an important consideration; while there is a tradition in the finance literature of assuming a constant variance-covariance matrix of stock returns, the measured expectations suggest that this assumption is far less justifiable when applied to the foreign exchange market.

The chapter is organized as follows. Section II outlines the hypothesis of forward rate unbiasedness as well as the Kouri-Dornbusch model, and discusses some previous empirical results. Section III describes the binomial option pricing model and presents plots of the resulting series. Section IV presents the results of the various tests. Finally, conclusions are in section V.
II. RISK PREMIUM THEORY AND PAST EMPIRICAL RESULTS

II.A The Hypothesis of Forward Rate Unbiasedness

The hypothesis that the forward rate is an unbiased predictor of the corresponding future spot rate is typically tested as a joint hypothesis which contends that (1) traders have rational expectations and (2) there is no risk premium. If we define the forward rate prediction error $\varepsilon_{t+1,1}$ as:

(1) $\varepsilon_{t+1,1} = f_{t,1} - s_{t+1} - \mu_t$

where $f_{t,1}$ is the natural logarithm of the forward rate contracted at date $t$ for delivery at date $t+1$, $s_{t+1}$ is the log of the realized spot rate at date $t+1$, and $\mu_t$ are terms independent of risk considerations that are due to Jensen's inequality, then the unbiasedness hypothesis implies that $\varepsilon_{t+1,1}$ has zero mean and is uncorrelated with any information available at date $t$. Generally, unbiasedness is tested without regard to the terms arising due to Jensen's inequality [e.g., Hsieh(1984)]. However, these terms are functions of exchange rate second moments, which we explicitly recognize as time-varying, so they should be taken into consideration.\footnote{We refer the reader to the exposition in Hsieh(1984) for a more detailed treatment.}

\footnote{See Krugman(1981) and Frankel (1982b), Appendix 1, for derivations of Jensen's inequality terms. The expression in equation (2) applies in the case where consumption patterns differ across countries.}
\[ \mu_t = [\omega_t'Qw_t - \sigma^2_t / 2] \n\]

where \( \omega_t \) is an \( n \)-element column vector containing the variance, \( \sigma^2_t \), of the dollar exchange rate being tested bilaterally plus \( n-1 \) covariances with the other dollar exchange rates in the investment opportunity set, \( Q \) is a matrix whose \( n+1 \) columns indicate the consumption preferences of residents of the \( n+1 \) countries (the \( n+1 \)st being the rest of the world), and \( w_t \) is a vector of the \( n+1 \) wealth shares. In the event no risk premium were present, if one were to neglect these terms and find significant coefficients on the measured second moment expectations due to the role of Jensen's inequality, then the joint null hypothesis would be spuriously rejected.

Substantial empirical evidence has been built up against the unbiasedness of the forward rate. Typically, researchers interpret rejection of this joint hypothesis as evidence that a risk premium does indeed exist. However, none of the strong rejections is due to factors that theory links to the risk premium:5 evidence for rejection is generally either (1) serial correlation of forward rate prediction errors6 or (2) correlation of forward rate prediction errors with known information that is not linked theoretically to the risk premium.7 Consequently, past rejections of unbiasedness do not offer clear evidence of a risk premium, per se.8

7 Hsieh(1984) provides the strongest evidence along these lines. His tests find a significant correlation between forward rate prediction errors and both holding period yields and forward discounts.
8 Evidence is now appearing which points to the role of expectational errors. For example, working with survey data on exchange rate expectations, Frankel and Froot(1987) suggest that systematic prediction errors are significant in explaining forward discount bias.
In section IV we test whether there exists a significant correlation between expected variances/covariances and forward rate prediction errors. This test differs in an important way from previous tests because theory establishes a relation between second moment changes and the risk premium (outlined below). A rejection, then, would provide stronger evidence than previous tests for the alternative hypothesis that a risk premium exists, as opposed to the alternative of a violation of rational expectations.

II.B A Portfolio Balance Model of the Risk Premium

The Kouri(1977)-Dornbusch(1983) model of the risk premium makes explicit a relation between the risk premium and the second moments of exchange rates. Because the derivation of the model appears elsewhere in the literature,\(^9\) we present only a very brief description here.

Investors are assumed to maximize a function of the mean and variance of end-of-period real wealth, where bonds of different currency denominations are the only available assets. The choice problem over the vector of portfolio shares generates the equilibrium relationship:\(^{10}\)

\[
(3) \quad i_t - i^S_t - E\Delta s_{t+1} - \mu_t = \theta \Omega_t (X_t - Qw_t)
\]

\(^9\) See Frankel(1982b) for a clear exposition.

\(^{10}\) The result presented in equation (3) requires the additional assumption that goods prices are nonstochastic when expressed in the currency of the producing country. That is, the only uncertainty is exchange rate uncertainty. This assumption is made by Krugman(1981), but is considered only one special case by Dornbusch(1983). Frankel(1982b) makes this assumption in his tests of the model.
\[ \mu_t = [\Omega_t Q w_t - \sigma_t^2/2] \]

where: \( i_t' = [i_{t}^m \ i_{t}^p \ i_{t}^y] \) is a vector of nominal returns on the bonds denominated in marks, pounds, and yen;\(^{11}\) \( l \) is a column vector of three ones; \( E \) is the expectation operator; \( \Delta \) denotes "change in"; \( s_t \) is now a three-element vector of the logs of the exchange rates; \( \mu_t \) is a vector of terms arising from Jensen's inequality; \( \theta \) is a coefficient of relative risk aversion; \( \Omega_t \) is the variance-covariance matrix of currency depreciation; \( X_t \) is the three-element vector of asset supply shares; and \( Q \) and \( w_t \) are the consumption share matrix and wealth vector referred to above.

Equation (3) specifies the risk premium as a function of three different factors: the coefficient of relative risk aversion \( \theta \), the exchange rate variance-covariance matrix \( \Omega_t \), and the difference between the vector of asset supply shares and the vector of wealth-weighted sums of consumption shares. Intuitively, wealth plays a role because the demand for a given country's asset depends positively not only on its expected relative return, but also on the wealth of those investors who have a relatively greater preference for that country's goods and thus for its assets.

Rational expectations implies that \( E \Delta s_{t+1} = \Delta s_{t+1} + \epsilon_{t+1} \), yielding the regression equation:

\[ i_t - l i s_t' - \Delta s_{t+1} - \mu_t = \theta \Omega_t (X_t - Q w_t) + \epsilon_{t+1} \]

\(^{11}\) The choice of currencies for the empirical analysis is constrained by data availability; only the mark, pound, and yen rate options provide sufficient observations.
This is the equation that Frankel(1982b) estimates under the assumption of a constant variance-covariance matrix $\Omega_t$. The null hypothesis of his test is that the coefficient of relative risk aversion is zero, i.e., that there exists no risk premium of the form predicted by the model. He is unable to reject the null. In section III we replicate Frankel's test in the sense that we include the same assets and use the same consumption shares that he does. However, this test differs in one very important way in that it does not constrain the variance-covariance matrix to be constant. This introduces a new, very volatile driving force as a complement to the much slower-moving variable $(X_t - Qw_t)$; the results should shed some light on their relative importance.

III. THE OPTION VALUATION MODEL AND IMPLIED EXPECTATIONS

III.A The Binomial Pricing Model

The Black-Scholes(1973) option pricing model is not appropriate for valuing currency options because the underlying asset is an amount of foreign currency, not a share of stock. Biger and Hull(1983), Garman and Kohlhagen(1983), and Grabbe(1983) develop a model for valuing a European foreign exchange call;\footnote{A call option gives the owner the right to purchase the underlying asset at a predetermined price--the exercise price. A European option can be exercised only on the expiration date, while an American option can be exercised at any time up to and including the expiration date.} however, the currency options that have been available in this country until very recently have been American options,\footnote{The Chicago Board Options Exchange began trading European foreign exchange options in late 1985.}
which permit exercise at any time up to and including the expiration date. One could use the European valuation formula to solve for the implied standard deviations as is done in Shastri and Tandon (1986b), but this approximation can be quite rough for call options that are likely to be exercised early.\(^{14}\) In order to generate a more precise measure from the American options, we use a more general model from which the Black-Scholes model can be derived as a limiting case: the Binomial Option Pricing Method of Cox, Ross, and Rubinstein (1979). The adaptation to currency options appears in Cox (1985) and is extended by Bodurtha and Courtadon (1987). This method allows one to both (1) modify the valuation procedure to take into account the different nature of the underlying asset and (2) properly include the value of early exercise.

III.B Inputs For The Valuation Model

American spot currency options began trading on the Philadelphia Exchange (PHLX) in early 1983. Beginning in September 1985, the Chicago Board Options Exchange (CBOE) began trading European currency options. In order to generate a sufficiently long time series, we use the PHLX option prices to recover the second moment expectations. Additionally, while the PHLX now trades options on six major currencies, the data are too sparse on all but the three most heavily traded currency options: the British Pound, the West German Mark, and the Japanese Yen.

A very important consideration when attempting to back implied variances out of an option pricing model is the synchronous data problem. A spot price that does not correspond

\(^{14}\) On a stock that pays no dividends, the values of American and European call options are identical; since no "rents" (dividends) are being forgone by not exercising an American call, it will always be worth more alive than exercised. However, in the case of foreign exchange options, rents are being forgone: the interest rate applying to the foreign currency if the option were exercised. Thus, it's the differentials between domestic and foreign interest rates that make the optimality of early exercise a possibility.
to the timing of the option trade will distort the variances if the spot rate differs from the rate prevailing at the time of the trade. For this reason, we use a transactions database which provides the underlying spot rate at the time of each option trade (the PHLX-OSU Currency Options Data Base, compiled with the support of the Philadelphia Stock Exchange by James N. Bodurtha Jr.). The data base runs from February 28th, 1983 through June 27, 1985.

The weekly data used in the simple efficiency regressions covers a sample period from July, 1983 through May, 1986, for a total of 151 observations (Wednesday closing prices). The first four months of the transactions data are not included due to the thinness of trading; there were eight trading days between March and June, 1983, on which no call option traded for at least one of the three currencies, while this occurred only once after June, 1983. (The closing trade of the previous day is used). In order to lengthen the sample beyond June 27, 1985 we use the Philadelphia Exchange closing option and spot prices as reported in the New York Times (46 of the 151 observations). The spot rate that is printed in the Times is the Philadelphia Exchange spot price at the close of the options market (2:30 Eastern Time).

Since trading volume is quite high through the latter third of the sample, the 2:30 spot price is likely to be a very good approximation to the spot rate prevailing at the time of the final option trade. The resulting series do not appear to be affected by the change.

For each observation we choose the call option that is closest to being at-the-money, with a time to maturity between three and six months.\(^{15}\) There are a number of justifications for focusing on the at-the-money options.\(^{16}\) First, the Black-Scholes model has a tendency to work best for options nearest the money. Second, the partial derivative of the call price with respect to the future volatility is higher the closer the call is to being at-the-money,

\(^{15}\) Hsieh and Manas-Anton(1986) demonstrate that maturity and exercise price biases are present in implied volatilities calculated from Deutsche Mark futures options. Our choice of maturity and exercise price parameters help to minimize these biases.

\(^{16}\) Stan Beckers(1981) demonstrates that, with implicit variances computed from daily closing option and stock prices, using only the implicit variance of the option nearest the money produces as good a prediction of future variance as other more elaborate selection/weighting schemes.
suggesting that the prices of these options are more sensitively linked to the underlying volatility. And third, these options are more heavily traded. The choice of the time to maturity window between three and six months is motivated by a balancing of the increasing significance of the method's approximations at shorter maturities against the increasing thinness of trading at longer maturities.\(^\text{17}\)

The binomial pricing model establishes a foreign exchange call option's value, \(C\), as a numerical approximation using seven variables:

\[
(6) \quad C = F(S, K, t, i, i^*, \sigma, n)
\]

For each calculation of the implied volatility \(\sigma\), \(S\) is the spot price of the corresponding currency from the transactions data base (PHLX spot rate at option market close after July, 1985), \(K\) is the exercise price of the option, \(t\) is the number of calendar days until the last trading day prior to expiration, and the number of subintervals \(n\) is set at 50 (a number of subintervals that provides a "sufficiently" close approximation to the Black-Scholes value when valuing a stock call option).\(^\text{18}\)

We calculate the domestic interest rate using the average of the Bid and Ask prices for the U. S. Treasury Bill expiring on the date closest to the expiration of the option. To calculate the ratio \(i/i^*\), we use the currency futures contract expiring at the same time as the option, together with the covered interest parity relation.\(^\text{19}\) The Treasury Bill discounts and futures prices are from the New York Times (Wednesday closing prices).

\(^{17}\) Since currency options mature on only four dates per year during most of the sample, it is necessary to use a different time to maturity from week to week. However, maturity bias tends not to be a problem across call options with different expiration dates when the option nearest to expiration has more than six weeks remaining [see Hsieh and Manas-Anton (1986)].

\(^{18}\) See Cox and Rubinstein, Options Markets, p. 243, for examples of the accuracy of the Binomial Approximation to the Black-Scholes values for different values of \(n\).

\(^{19}\) Violations of covered interest parity represent riskless arbitrage opportunities; the condition holds very well across assets within the same jurisdiction. For evidence see Frenkel and Levich (1979).
III.C Generating The Expected Variance Series

The binomial valuation model requires working through a binomial "valuation tree". The numerical approximation procedure used starts with an initial value for the implicit volatility and combines it with the other determining variables in the model. After going through the tree completely, the procedure generates a call value. This call value is then compared to the actual market price. At this point an adjustment is made to the initial volatility value according to the Newton-Raphson numerical search procedure. The process then iterates until a change in the annual spot volatility (standard deviation) of less than .0001 is encountered. Like previous work with implied volatilities, we calculate annual volatilities by linearly scaling the three to six month horizons (e.g., a volatility expected over six months is multiplied by two).

The resulting three series of the expected variance (of changes in the log of the exchange rate on an annual basis) are plotted below in Figure 1 together with a measure of the historical mean squared change in the log of the exchange rate calculated from the previous month's daily spot movements (20 business days) according to:

\[
RVAR_t = \frac{1}{18} \sum_{j=t-19}^{t-1} (R_j - \bar{R})^2
\]

20 Cox and Rubinstein(1985), page 242, show that the choice of the binomial factors \(u = \exp(\sigma \sqrt{t/n})\) and \(d = 1/u\) provide the model with the proper limiting properties when pricing a stock option. When pricing a currency option, Bodurtha and Courtadon(1987) demonstrate that theoretical consistency requires that \(u = \exp((i - r) \sqrt{t/n} + \sigma \sqrt{t/n})\) and \(d = 1/u\). However, for empirical implementation they also point out that for values of \(n\) that are practical computationally, the stock option factors are usually preferable since smaller values of \(n\) induce biases in the currency option formula. With a value of \(n = 50\), we use the Cox and Rubinstein factors.

21 See Manaster and Koehler(1982) for further information regarding the search procedure.

22 Since the derivative of a call value with respect to the volatility is always positive, if an implied volatility exists then it is unique.
where \[ R_j = \ln(S_j) - \ln(S_{j-1}) \]

and \[ \bar{R} = \sum_{j=t-19}^{t-1} \frac{R_j}{19} \]

Since \( \text{RVAR}_t \) in (7) is a measure of the variance of daily changes in the log of the exchange rate, we use an assumption of independence to arrive at the variance on an annual basis by multiplying by 262.\(^{23}\) This is the series that appears as the REALIZED VARIANCE in the plots.\(^{24}\) On the whole, the two series correspond quite well.

Two additional points deserve attention. First, if the assumptions underlying the option pricing model were an exact description of reality, then we would expect an exact mapping between an implied volatility and the market's second moment expectation.\(^{25}\) However, the assumptions are not an exact description of reality. The issue, then, is the validity of the assumptions for the purpose at hand. On this point, perhaps the best evidence is Figure 1, which suggests that the implied expectations correspond well with realized variances. There is some tendency for the implied variance expectations to be slightly higher, particularly in the case of the yen; this is most likely attributable to the facts that empirical exchange rate distributions show more kurtosis than the log-normal and that expected variances change over time, both of which tend to increase the value of an

\(^{23}\) This multiple corresponds to the number of business days in a year. While the annualization is admittedly rough, it does not alter the basic shape of the series included in figure 1.

\(^{24}\) The horizon over which the REALIZED VARIANCE is calculated is chosen so as to generate similar volatility in the two series; a longer horizon smooths the series while a shorter one increases the volatility.

\(^{25}\) The assumptions underlying the model are: (1) the spot rate is log-normally distributed with constant variance, (2) the risk-free interest rates are known and constant, and (3) capital markets are frictionless.
FIGURE #1

VARIANCES OF ANNUAL LOG SPOT RATE CHANGES

---

- = IMPLIED BP VARIANCE
- - - - = REALIZED BP VARIANCE

---

- = IMPLIED DM VARIANCE
- - - - = REALIZED DM VARIANCE

---

- = IMPLIED JY VARIANCE
- - - - = REALIZED JY VARIANCE

1984 1985
Second, it is likely that errors in variables would be a problem if we were to use the calculated expectations as explanatory variables. Moreover, generating covariance measures using products of these variables could magnify any imprecision. For this reason, we choose to instrument for the implied variances in all the tests that follow, calculating the covariances from the projections.

IV. EMPIRICAL RESULTS

Forward rate unbiasedness implies that forward rate prediction errors have zero mean and are uncorrelated with any information available to agents at the time the forward rate is determined. In subsection I.A, we define the one-period forward rate prediction error as

$$\varepsilon_{t+1,1} \equiv f_{t+1} - s_{t+1} - \mu_t$$

where $f_{t+1}$ is the log of the one-period forward rate set at time $t$, $s_{t+1}$ is the log of the realized spot rate at $t+1$, and $\mu_t$ are the appropriate terms arising from Jensen's inequality. As is common in the literature [e.g., Hsieh(1984)], we assume that covered interest parity holds, which implies that:

$$i^*_t - i_t - \Delta s_{t+1} - \mu_t = \varepsilon_{t+1}$$

---

26 Clearly, the way in which the underlying variance is changing over time is a key factor in determining the sign of this partial derivative. Work along these lines, however, is still in its infancy.

27 For the unbiasedness tests in Tables 1-3, we calculate the Jensen's adjustment on the basis that consumption shares differ across countries.
where $\varepsilon_{t+1}$ is the prediction error realized at $t+1$ (we will suppress the one-period subscript from now on). Notice that the left-hand-side is the same as that which appears in the Kouri-Dornbusch model. That is, the test of the Kouri-Dornbusch model is really a test of forward rate unbiasedness in which the exact form of the bias—the risk premium—is specified. This fact will be helpful for interpreting the coefficients in the unbiasedness tests.

First, we test to see if expected exchange rate variances and covariances are correlated with the prediction errors. We generate the covariance terms by assuming that the correlations are constant, and then using the relation $\text{Cov}(X,Y) = \rho_{X,Y} \sigma_X \sigma_Y$ where $\rho$ is taken to be the sample correlations, and the standard deviations are the square roots of the implied variances. The Two-Stage Least Squares results for the single equation unbiasedness test appear in Table 1 below (weekly data). These regressions include the full sample period from July, 1983 through May, 1986 (149 weekly observations); the interest rates are 7-day Eurocurrency deposit rates expressed on a weekly basis.

The results provide important evidence on two fronts. First, the significance of individual coefficients provides substantial evidence against the orthogonality conditions implied by the joint unbiasedness hypothesis, particularly with the mark and yen. Moreover, this evidence is qualitatively very different in that it is due to variables which theory links to the risk premium. In this sense, it is better evidence for the alternative hypothesis that a risk

---

28 The sample correlations are: $\rho_{pm} = 0.736$, $\rho_{py} = 0.378$, and $\rho_{my} = 0.550$.

29 Throughout the empirical analysis, the instruments for each of the three implied variance series are the first two lags of the series plus a measure of historical variance calculated from daily data over the previous 100 business days.

30 A White test for heteroskedasticity in the Mark equation is not rejected at the 5% level. For the Pound equation, the test is rejected at the 5% level but not at the 1% level; we calculate the standard errors using the White correction. The Yen equation shows evidence of both heteroskedasticity (rejection at the 1% level) and serial correlation (Durbin-Watson statistic below the lower bound at the 5% level but not at the 1% level); we calculate the standard errors using Hansen's(1982) asymptotically appropriate method of moments estimator (with two lags imposed).
TABLE 1

\[ i^*_t - i_t - \Delta s_{t+1} - \mu_t = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon_{t+1} \]

<table>
<thead>
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<th></th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>R2</th>
<th>DW</th>
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<td>MARK</td>
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<td>(-3.27)</td>
<td>(1.26)</td>
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<tr>
<td>POUND</td>
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<td>107</td>
<td>-89.5</td>
<td>-2.03</td>
<td>0.08</td>
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<td>(-2.07)</td>
<td>(-0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YEN</td>
<td>-0.004</td>
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<td>-190</td>
<td>21.8</td>
<td>0.06</td>
<td>1.51</td>
</tr>
<tr>
<td></td>
<td>(-0.81)</td>
<td>(1.75)</td>
<td>(-2.30)</td>
<td>(0.77)</td>
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</tr>
</tbody>
</table>

* T-statistics in parentheses.

Mark equation: \( X_1 = \) variance of log changes in the dollar/mark rate (weekly basis) \( X_2 = \) covariance of log changes in the mark rate and pound rate \( X_3 = \) covariance of log changes in the mark rate and yen rate

Pound equation: \( X_1 = \) covariance of log changes in the pound rate and mark rate \( X_2 = \) variance of log changes in the pound rate \( X_3 = \) covariance of log changes in the pound rate and yen rate

Yen equation: \( X_1 = \) covariance of log changes in the yen rate and mark rate \( X_2 = \) covariance of log changes in the yen rate and pound rate \( X_3 = \) variance of log changes in the yen rate

... premium does indeed exist, as opposed to the alternative of a violation of rational expectations.

The second point to be gleaned from Table 1 concerns the relative magnitude of the Jensen's inequality terms and the risk premium. Krugman(1981) makes the case that in theory they are of approximately the same magnitude. The results, however, demonstrate that the coefficients \( \beta_1, \beta_2, \) and \( \beta_3 \) (which measure the influence of second moments over
and above their Jensen’s inequality effect) are generally much larger than the fractional coefficients (consumption shares) imputed to Jensen’s inequality per equation (2). For example, in the case of the pound under the simplifying assumption that consumption shares are equal across countries, we have:

\[
\mu^P_t = [ (Q_p - 0.5) \sigma_{p,t}^2 + Q_m \sigma_{pm,t} + Q_y \sigma_{py,t} ]
\]

Thus, the results suggest a much larger role for the risk premium. Additionally, however, the results do not bode well for the theory we test below, in that it is essentially the same as that of Krugman.

Consider now an interpretation of the coefficients which derives from the portfolio balance model of Kouri and Dornbusch. The model, described by equation (3), specifies the risk premium (RP) on currency \( i \) as:

\[
RP_{i,t} = \theta \Omega_{i,t} (X_t - Qw_t) = \theta (X_{i,t} - Q_i w_t) \sigma_{i,t}^2 + \theta (X_{j,t} - Q_j w_t) \sigma_{ij,t} + \theta (X_{k,t} - Q_k w_t) \sigma_{ik,t}
\]

where \( \theta \) is the coefficient of relative risk aversion, \( \Omega_i \) is the \( i \)th row of the variance-covariance matrix of exchange rate changes, \( X_t \) is the vector of asset supply shares denominated in the different currencies, and \( Qw_t \) is the vector of consumption shares in the different currencies. If one assumes that the difference between the asset supply shares and the consumption shares \( (X_t - Qw_t) \) is constant, then each of the coefficients in Table 1 should be the same across the three equations. That is, \( \beta_1 = \theta (X_m - Q_m w) \), \( \beta_2 = \theta (X_p - Q_p w) \), and \( \beta_3 = \theta (X_y - Q_y w) \). The first row of Table 2 presents the 2SLS results for the pooled regression:

70
The significance of the coefficients $\beta_2$ and $\beta_3$ is further evidence against the null hypothesis that all information is orthogonal to the left-hand-side. Additionally, assuming risk averse agents, the negative sign of $\beta_2$ and $\beta_3$ suggests that investors perceive the existing share of pound- and yen-denominated assets to be lower than that share which minimizes consumption risk. The insignificance of $\beta_1$ is consistent with the perception that the mark asset share is more closely in balance with the minimum risk share. The implication for $\theta$ of the magnitudes of the coefficients is puzzling, however; since $(X - Qw)$ is the difference of two fractions (shares), the magnitudes suggest that investors are "extremely" averse to exchange rate risk, i.e. that $\theta$ is much larger than is commonly believed.

The second row of Table 2 presents results for the same specification except for the inclusion of the contemporaneous forward discount (measured using the interest differential).
Since the forward discount has been shown to cause rejection of the joint unbiasedness hypothesis, it is interesting to test whether or not its significance dominates that of the expected second moments or vice-versa. The results suggest that in fact the significance of the expected second moments is the dominant of the two: the forward discount is not even close to entering significantly while the coefficient $\beta_3$ remains significant at about the 1% level. Nonetheless, the coefficient $\beta_2$ does become insignificant due to both a decrease in its estimated value and a decrease in the precision of its estimation.

Heteroskedasticity plagues most all of the empirical work on forward rate unbiasedness and most of the equations estimated thus far are no exception. Typically, researchers either use heteroskedasticity-consistent estimators or use OLS with appropriately corrected standard errors. However, since under the null hypothesis the variance of the prediction error is equal to the variance of the change in the log exchange rate [see equation (8)], in our case we can use Weighted Least Squares with the implied standard deviations as weights. Table 3 presents the results for the pooled equation:

\[
(i^* - i_t - \Delta s_{t+1} - \mu_t)/\sigma_t = \beta_0 + \beta_1 X_1/\sigma_t + \beta_2 X_2/\sigma_t + \beta_3 X_3/\sigma_t + \epsilon_{t+1}/\sigma_t
\]

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>R2</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>WLS</td>
<td>.005</td>
<td>24.6</td>
<td>-30.0</td>
<td>-32.3</td>
<td>.02</td>
<td>1.81</td>
</tr>
<tr>
<td></td>
<td>(2.12)</td>
<td>(1.94)</td>
<td>(-2.58)</td>
<td>(-2.65)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* T-statistics in parentheses.
As expected, the coefficients do not change much in magnitude and are slightly more significant.

The analysis above naturally leads us to a test of the Kouri-Dornbusch model itself. Once we have a measure of the asset supply shares and the consumption shares, we can estimate the single coefficient $\theta$ in (5):

\[
(5) \quad i_t - \delta_t - \Delta s_{t+1} - \mu_t = \theta \Omega_t (X_t - Q w_t) + e_{t+1}
\]

As previous results point out, however, given the correlations of the variables in $\Omega_t$ with the left-hand-side, the estimated coefficient $\theta$ will be very sensitive both in sign and magnitude to the sign and magnitude of the relatively stable $(X_t - Q w_t)$. This said, we choose to measure the asset shares as is done previously in the literature by Frankel(1982b), so that the effect of changing second moments can more easily be discerned.\(^{31}\)

Frankel estimates $\theta$ under the strong assumption of a constant variance-covariance matrix $\Omega_t$. The null hypothesis of the test is that the coefficient of relative risk aversion is zero, i.e., that there is no portfolio balance risk premium of the form the model predicts. He is unable to reject. The test below also tests for the null hypothesis of a zero coefficient of risk aversion. However, this test permits the covariance matrix to vary. The elements of each month’s covariance matrix are determined by the (instrumented) implied expected variances from the options market (monthly basis), with the covariances determined as above using the assumption of constant correlations.

\(^{31}\) The world portfolio which he considers includes only outside government debt. See Frankel (1982b), Appendix 4, for definitions.
Table 4 presents the results of the 2SLS regressions using the two consumption specifications considered by Frankel. They cover monthly observations from April, 1983 through December, 1985, for a total of 33 months (making 99 observations). The EQUAL CSN SHARES regression corresponds to the case in which all countries consume the same basket of goods (see appendix); the interest rates are those for one month Eurocurrency deposits (monthly basis); the spot rate is that used to generate the implied expected variances; the coefficient $\beta_1$ corresponds to the coefficient of relative risk aversion.$^{32}$

\[
i^*_t - i_t - \Delta s_{t+1} - \mu_t = \beta_0 + \beta_1 * \Omega_t (X_t - Qw_t) + \epsilon_{t+1}
\]

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>R2</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQUAL CSN SHARES</td>
<td>-0.008</td>
<td>-3.05</td>
<td>0.08</td>
<td>2.19</td>
</tr>
<tr>
<td></td>
<td>(-1.74)</td>
<td>(-2.98)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSN SHARES VARY</td>
<td>0.010</td>
<td>-2.83</td>
<td>0.04</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>(1.60)</td>
<td>(-1.93)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* T-statistics in parentheses.

The results are both discouraging and interesting. The coefficient of relative risk aversion ($\beta_1$) is negative and much larger in absolute value than one would expect. Thus, the test results provide no support for the model. The existence of a "pole" at the point $(X_t - Qw_t) = 0$ is a possible explanation of the wrong sign, however. That is, given the demon-

$^{32}$ With both equations, a White test for heteroskedasticity shows no sign of it at conventional significance levels.
strated relationship between the relatively volatile \( \Omega_t \) and the left-hand-side, measuring the relatively stable \( (X_t - Qw_t) \) with a sign that is opposite that perceived by the market would generate a negative coefficient rather than a positive one if the model were in fact true. The extremely significant coefficient in the equal-consumption-shares case suggests that a relationship exists. At the very least, it constitutes a very strong rejection of the unbiasedness hypothesis.

In order to gauge the sensitivity of the results to the consumption shares, we examine two alternative cases: (1) investors measure consumption risk against dollar-denominated consumption, so that all elements of the consumption share vector are zero and (2) investors measure consumption risk wholly against home-denominated consumption, so that the home share equals one. Table 5 presents the results of the 2SLS regressions:33

\[
i^*_t - i_t - \Delta s_{t+1} - \mu_t = \beta_0 + \beta_1 \Omega_t (X_t - Qw_t) + \varepsilon_{t+1}
\]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>R2</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOLLAR SHARE = 1</td>
<td>-.037 (-2.84)</td>
<td>112 (2.99)</td>
<td>.08</td>
<td>2.36</td>
</tr>
<tr>
<td>HOME SHARE = 1</td>
<td>-.015 (-1.64)</td>
<td>-340 (-1.81)</td>
<td>.03</td>
<td>2.19</td>
</tr>
</tbody>
</table>

* T-statistics in parentheses.

33 With both equations, a White test for heteroskedasticity shows no sign of it at conventional significance levels.
In the dollar-share-equals-one regression the coefficient of relative risk aversion now has the correct sign. While this case is perhaps less plausible than the others, the result underscores the sensitivity to the measurement of the difference \(X_t - Q_{W_t}\).

As a final measure to help identify the role of various factors in producing the estimates above, we regress the left-hand-side on the asset supply share series alone in bilateral equations. Theory predicts that the coefficients on asset supplies should be positive: a higher asset supply requires a higher expected return to induce investors to hold them. Previous work with this equation [e.g., Rogoff(1984), Frankel(1982a), Dooley and Isard(1983), and Danker et al(1985)] finds little or no evidence of this prediction in the data. Table 6 presents the results of the OLS regressions:\(^{34}\)

\[
i_t^* - i_t - \Delta s_{t+1} - \mu_t = \beta_0 + \beta_1(\text{ASSET SHARE})_t + \epsilon_{t+1}
\]

<table>
<thead>
<tr>
<th></th>
<th>(\beta_0)</th>
<th>(\beta_1)</th>
<th>R2</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARK</td>
<td>.112</td>
<td>-1.86</td>
<td>.14</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>(2.23)</td>
<td>(-2.29)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POUND</td>
<td>.020</td>
<td>-1.20</td>
<td>.08</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>(1.21)</td>
<td>(-1.28)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YEN</td>
<td>.326</td>
<td>-1.40</td>
<td>.21</td>
<td>1.96</td>
</tr>
<tr>
<td></td>
<td>(2.92)</td>
<td>(-2.88)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* T-statistics in parentheses.

\(^{34}\) Only the pound equation shows evidence of heteroskedasticity using the White test; we calculate the standard errors in this case using the White correction.
All of the coefficients on the asset supply shares are of the wrong sign, significantly so in the case of the yen and mark. To the extent that the measured shares represent the relevant assets, this is further evidence against the theory. Additionally, these results shed some light on why the measured coefficients of risk aversion are typically negative.

In addition to the above analysis, the volatility of expectations in itself has potential policy implications, e.g., for the reliability of the portfolio balance effects of sterilized intervention. Consider for illustration a two-country model of the risk premium:

\[
i^*_t - i_t - E\Delta s_{t+1} = \theta \sigma^2_t (X_t - Qw_t)
\]

A common view is that sterilized intervention (instrument) affects the exchange rate (target) according to the above relationship with a stable, if not constant, exchange rate variance. However, the pronounced volatility of market's second moment expectation seriously undermines this view. The extreme noise introduced would swamp any policy induced variation in \( X_t \) due to sterilized intervention, with the sign of the effects of changes in \( \sigma^2_t \) depending on the sign of \( (X_t - Qw_t) \). Additionally, to the extent that current account imbalances affect exchange rates through the above relationship (by reallocating \( w \)), their role as a driving force pales in comparison to the role of \( \sigma^2_t \).
V. CONCLUSIONS

The foreign currency option market provides a unique method for measuring ex-ante variance expectations: beliefs embedded in option prices are rooted in actual market behavior and reflect the "true" information set available to investors. Moreover, the model used to recover the implied expectations is preference-free, relying wholly upon arbitrage pricing methods. The result is a series which corresponds quite closely to actual variances.

Overall, the results of the tests support one conclusion above all others: changes in the market's second moment expectations are systematically related to expected return differentials. The joint unbiasedness hypothesis is strongly rejected in a number of different tests. Rejection of this joint hypothesis is not new, however. What is new is that the rejections are due to variables which theory ties to the risk premium. In this sense they provide much better evidence in favor of the alternative hypothesis that a risk premium does indeed exist, as opposed to the alternative of a violation of rational expectations.

The test of the Kouri-Dornbusch model of the risk premium, however, finds little support in the data. The absolute size of the measured coefficients of relative risk aversion are broadly consistent with the magnitudes of the coefficients in the unbiasedness regressions, but the sign is typically wrong. There are at least two explanations for this result which are consistent with the validity of the theory. First, as explained in section III, the sign and magnitude of the measured coefficient are very sensitive to the asset and consumption share definitions, given the correlation between the elements of the variance-covariance matrix and the dependent variable (as demonstrated in the unbiasedness tests). This suggests the possibility of experimenting with broader asset definitions than those used in Frankel(1982b). A second explanation works from the other side of the equation. That is, perhaps replacing expected depreciation with realized depreciation plus a random error term is off the mark, especially during the sample period considered (1983-1986). Working with survey data on exchange rate expectations, Frankel and
Froot(1987) find evidence that systematic prediction errors are playing a significant role. A possible remedy for this would be to replace the realized depreciation with the median survey expectation on the left-hand-side. The results in this case are likely to be quite different from the current results in light of the fact that most all of the variability in the dependent variable is due to this component.

A final note concerns the more general implications of rapidly varying second moment expectations. First, they call into question the reliability of the portfolio balance effects of sterilized intervention. Feasible policy-induced changes in relative asset supplies are dwarfed by autonomous movements in investors' second moment expectations. And second, theoretical and empirical work that neglects the perceived nonconstancy of exchange rate second moments may be overlooking much of what is fundamental to agents' behavior.
DATA APPENDIX

Rates of Return

The rates of return used for the regressions reported are the seven-day and one month Eurocurrency interest rates as reported in the London Financial Times. Wednesday closing rates are used for the weekly data, while rates for the last business day of the month are used for the monthly data. The depreciation of the spot rate is measured as the change in the logarithm of the spot rate that is used to calculate the implied option volatility each week/month (Philadelphia Exchange). This corresponds to the transactions data described in the text from July, 1983 through June, 1985, and to the Philadelphia option market closing spot reported in the New York Times for July, 1985 through May, 1986.

Asset and Wealth Data

We calculate the asset and wealth data as described in Frankel(1982b).

Consumption Shares

When residents of all countries are assumed to consume the same basket of goods, the share of world consumption allocated to each country's goods is computed as its 1983 GNP divided by the total GNP of the four countries in dollars. The shares are: U.S. 0.596, U.K. 0.079, Germany 0.111, and Japan 0.214. When residents of different countries are allowed to consume different baskets of goods, we use the consumption shares computed and reported in Frankel(1982b, p. 272), which are derived from import and export to GNP ratios in 1973.
Chapter IV: Floating Exchange Rates in Developing Countries: The Case of Peru. 1950-54
I. Introduction

In November 1949, the Peruvian government adopted a system of dual floating exchange rates which remained in force through 1954. The duration of the episode coupled with the richness of its context of macro policy and world events make it a valuable opportunity for analyzing the behavior of floating rate systems in developing countries. Although a number of issues are worthy of attention, the analysis below proceeds along two fronts. First, what fundamentals underlay the market's determination of the exchange rate? For instance, is the monetary approach to exchange rate determination which is so often applied to industrialized countries' floating rate experience the best approach in the less-developed Peruvian case? An alternative model suggests that it is not. And second, was the floating rate system a viable choice for Peru? What was its role in Peru's adjustment to the large terms of trade shocks that occurred in the period? Here, we consider some key dimensions of the adjustment, such as the behavior of the real exchange rate, to determine in what ways it was alleviated or aggravated. We then consider the implications of the experience for the viability of flexible rates in developing countries today.

The economic and institutional characteristics of Peru at the time were similar to those in most developing countries: exports depended heavily on a small number of commodities, the domestic capital market was repressed, and there were no organized forward markets for the Peruvian sol. In August 1948 an important structural shift had occurred--a military coup from the right—which ushered in a period of economic liberalization and orthodoxy lasting nearly two decades. Only a year after the coup, the government adopted the new exchange rate system, largely on the recommendation of a private U.S. consulting firm. The system established two different floating rates: the 'certificate' rate for trade transactions and the so-called 'draft' rate for financial transactions. Although it was known that, from time to time, the central bank intervened in order to smooth out short-term fluctuations in the
certificate rate, it was claimed that no attempt was made during this period to prevent its long-term adjustments.\(^1\) The draft market, on the other hand, was free of intervention. Not surprisingly, the two floating rates tended to move in tandem, with little difference between them. In addition, the policy shift of November 1949 also eliminated virtually all trade and capital movement restrictions. Thus, individuals had a great deal of freedom to act in their best interest vis-a-vis the floating exchange rates.

The episode has certainly not gone without interpretation. Tsiang(1957) and Schott(1959) provide earlier, more qualitative analyses. It is not until much later that Edwards(1983) provides the first, and to this author's knowledge only, econometric appraisal of events. He uses a short-run monetary model and concludes that, "the results obtained are quite satisfactory, indicating that the monetary view of exchange rate determination provides a useful benchmark for analyzing the behavior of floating exchange rates in developing countries." In contrast, Tsiang(1957) suggests a very different view that emphasizes world prices for Peruvian export goods as a fundamental determinant. He asserts in his first paragraph:

From the outbreak of the [Korean] war to the end of 1952, when export prices were very favorable, the exchange rates were remarkably stable, despite considerable monetary expansion. In 1953, however, the certificate rate depreciated by 28 percent, because world market prices of Peruvian exports fell sharply while domestic costs and prices continued to rise on account of internal inflation.

Below, we propose a model that admits the dollar price of exports as a proximate determinant of the exchange rate. We then test this 'Tsiang model' against some competing alternatives.

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\(^1\) This capacity to manage the short term fluctuations in the trade transactions market was one of the primary reasons for establishing two floating rates instead of a unified rate.
The principal alternative model against which we test the Tsiang and monetary models is a purchasing power parity (PPP) model. This model is, in fact, a very sensible point of departure given that PPP is the foundation stone of the monetary model. We find that, contrary to the very poor performance of PPP models for the industrial countries since generalized floating in 1973, the PPP model provides a very good description of the floating rate's behavior in Peru.\(^2\) This is true in spite of the fact that inflation over the period was never greater than 10 percent per year. Moreover, this is important evidence vis-a-vis whether a floating rate system was a viable choice for Peru. Indeed, real exchange rate variability is often singled out as the biggest disappointment of floating rates among the industrial countries.

The chapter is organized as follows. Section II provides a more detailed description of the Peruvian setting including plots of key variables and other vital statistics. Section III outlines the competing models: a PPP model, various monetary models, and a model formulated along the lines of the views put forth by Tsiang(1957). Section IV presents the results obtained for the different alternatives. Section V considers whether or not the floating rate system was a viable choice for Peru. Conclusions are in section VI.

II. The Peruvian Setting

As suggested above, the military coup of August 1948 was an economic turning point of the first order. For most of the 1940's Peru was ruled by governments more or less committed to capturing and maintaining the support of the middle classes and, for the first

\(^2\) See Frenkel(1981) for evidence on the collapse of purchasing power parities.
time in Peruvian history, policies of state economic intervention coupled with exchange and import controls made their appearance, aimed at extracting surplus from export sectors for the benefit of urban groups. By 1948, however, the governing coalition was in disarray: the experiment with economic control was generally recognized to have proved a failure, the economy faced an increasingly-acute balance-of-payments crisis, and rapid inflation had alienated much of the previous middle-class support for the new policies. In August 1948 General Manuel Odria, backed by the traditional ruling class, seized power and swung the country firmly back toward the traditional policies of liberalism and outward-orientation. As Hunt(1974) states, from 1948 on: In a continent that was witnessing ever-increasing state intervention in economic life in country after country, Peru . . . turned around to begin a march in the other direction that continued for the next 18 years.

The interventionist policies had come to a definitive end and there then followed a period of nearly two decades of total integration into the international system, with complete commitment to the rules of the game. Entry of foreign capital and the repatriation of profits were virtually unrestricted, government intervention and participation were kept to a minimum, and the export sectors continued as the mainspring of the economy. Table #1 presents data on the key export sectors at that time; notice that throughout the floating rate period cotton was by far the dominant export commodity, with sugar a strong second. It was not until 1960 that copper and fish derivatives began to dominate export receipts.

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3 See Thorp and Bertram(1978), pages 147-201.


5 To help put these dollar figures in perspective, Peru's 1950 GNP in dollars was $1032 million.
Table #1
Peruvian Composition of Exports, 1950 and 1954

<table>
<thead>
<tr>
<th></th>
<th>1950</th>
<th></th>
<th>1954</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value($ millions)</td>
<td>Share(percent)</td>
<td>Value($ millions)</td>
<td>Share(percent)</td>
</tr>
<tr>
<td>Cotton</td>
<td>68.0</td>
<td>35.1</td>
<td>64.8</td>
<td>26.2</td>
</tr>
<tr>
<td>Sugar</td>
<td>29.7</td>
<td>15.3</td>
<td>33.0</td>
<td>13.3</td>
</tr>
<tr>
<td>Petroleum Products</td>
<td>25.3</td>
<td>13.1</td>
<td>17.1</td>
<td>6.9</td>
</tr>
<tr>
<td>Lead</td>
<td>12.3</td>
<td>6.4</td>
<td>23.7</td>
<td>9.6</td>
</tr>
<tr>
<td>Zinc</td>
<td>10.3</td>
<td>5.3</td>
<td>9.1</td>
<td>3.7</td>
</tr>
<tr>
<td>Copper</td>
<td>10.2</td>
<td>5.3</td>
<td>20.0</td>
<td>8.1</td>
</tr>
<tr>
<td>Others</td>
<td>37.8</td>
<td>19.5</td>
<td>79.9</td>
<td>32.3</td>
</tr>
<tr>
<td>Total</td>
<td>193.6</td>
<td></td>
<td>247.6</td>
<td></td>
</tr>
</tbody>
</table>

Source: Cuentas Nacionales del Peru (1966), pages 52-3.

We turn now to the 1949 exchange reform. The establishment of dual markets was intended to prevent speculative capital movements from disturbing the certificate rate which applied to trade transactions, while giving full freedom to capital movements regarded as necessary to attract foreign investment. Theoretically, the rates in the two markets were free to move divergently, but there were implicit links between the demands and supplies in the two markets. The draft rate clearly could not fall below the certificate rate or demand for foreign exchange in the certificate market would simply be shifted to the draft market, thereby exerting downward pressure on the certificate rate and upward pressure on the draft rate. However, because the certificate rate could only be applied to trade transactions, the draft rate was not prevented from rising above the certificate rate. Nevertheless, except during the periods November 1949-November 1950 and February-June 1954, the draft rate seldom exceeded the certificate rate by more than 2 percent. Thus, on the whole the behavior of the dual rate system was not substantively different from that of a unified floating rate.

The persistent narrowness of the spread might be attributed to the under-invoicing of

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6 Thus, it is incorrect to think of the certificate rate as a floor which the draft rate could bump into but not go below.
exports and over-invoicing of imports. However, as Tsiang points out, it is unlikely that such illegal measures took place to any important extent since the spread between the rates was seldom large enough incentive to take the risk and trouble involved. Rather, Tsiang suggests that a more important link is probably found in the potential for foreign direct investors to arbitrage the spread between the two markets. That is, if the spread tended to widen too much it may have been worth while for some of the investing firms to sell part of their foreign exchange funds on the draft market and use the sol proceeds to buy dollars in the certificate market for their imports of equipment and materials, rather than use their foreign exchange resources directly.

Some comments on central bank intervention in the certificate market are in order since the linkages between the two markets provide a channel through which intervention might have affected the determination of the draft rate. The authorities are known to have intervened from time to time to smooth out short-term fluctuations in the certificate rate, but to have left the draft rate entirely free to find its level. Moreover, as a matter of policy no attempt was to be made to prevent long-term adjustments of the certificate rate. Referring to the period from 1950 through 1952, Tsiang writes: 7,8

Except in 1951 when the Central Bank absorbed in reserves some $9.5 million of foreign exchange, in order to stabilize the rate, the authorities appear not to have intervened persistently in either direction.

Nonetheless, the central bank does not appear to have been fighting the fundamentals reflected in the draft market in 1951. We offer two pieces of evidence of this. First, the absorption of reserves appears to have been partially the result of the rules governing the

7 Page 460.

8 To put the $9.5 million in perspective, Peru's total reserves at the end of January 1951 stood at $58.2 million and her January 1951 imports were $19.6 million. Source: International Financial Statistics.
supply side of the certificate market. That is, the floating rate system was established with the requirement that 100 percent of dollar export receipts had to be surrendered to the central bank for conversion into 'certificates' which could then be sold in the certificate market. These certificates were valid for 60 days, after which they had to be sold back to the central bank at the current certificate rate less 2 percent. Due to the flow controls, there appears to have been an artificially-inflated supply of dollars to the certificate market arising from extremely favorable terms of trade shocks. In an attempt to reduce the flow, the central bank reduced the surrender requirement to 75 percent on March 21, 1951 and further to 50 percent on April 23 of that same year. The surrender requirement returned to 100 percent later that year.

Second, if fundamentals had pointed to a more appreciated level of the exchange rate, the pressures for the draft rate to fall would have sucked the demand side out of the certificate market either causing a fall in the certificate rate or causing the central bank to assume the whole demand side. Neither of these scenarios seems to have been the case, which suggests that the central bank was not in fact battling the fundamentals in the draft market.

The only period through which central bank intervention appears to have delayed adjustment of the certificate rate is the first half of 1953. At the beginning of the year, there was pressure toward depreciation coming from both the demand and supply sides. Reluctant to allow too rapid a rise, the monetary authorities carried out net sales of some $12.5 million. During the latter half of 1953, however, the authorities seem to have given up major attempts to intervene and to have resigned themselves to letting the rate find its own level. The net sales of foreign exchange to the private sector during the second half of the year amounted to no more than a fraction of a million dollars. In these circumstances, the certificate rate rose rapidly from 16.13 soles/dollar at the end of June 1953 to 19.89 soles/dollar at the end of

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9 See Tsiang (1957) for details.
December 1954.

Previous analysis has considered the behavior of each of the rates: Tsiang couches his analysis in terms of the certificate rate, whereas Edwards directs his attention to the draft rate. Like Edwards, we restrict our attention to the draft rate since it is more purely market-determined in the sense that the central bank made no attempt to manage it.\textsuperscript{10} Plot #1 below illustrates the behavior of the nominal and real exchange rates over the five year period.

\textbf{PLOT #1: REAL AND NOMINAL EXCHANGE RATES (S/$)}

Note that the nominal rate remained quite stable until well into 1953 while the real exchange rate appreciated slightly (less than 10%). In contrast, late 1953 brought a considerable depreciation of both the nominal and real rates. From August 1953 to its peak in January 1954 the nominal sol/dollar rate went from 16.25 to 21.00. In February 1954, the President
of Peru announced a stabilization program which included *inter alia* (1) the reduction of
government expenditures to the level of current receipts, particularly by slowing down
public works; (2) the checking of domestic credit expansion; and (3) more active regulatory
intervention by the central bank in the certificate market. The announcement brought about
an almost immediate improvement in the exchange markets. Following the impact effect,
the rate continued to drift downward toward the 19.00 soles/dollar rate attained at year-end
1954. Additionally, it is noteworthy that the real rate was never more than 10 percent from
its initial value.

Table #2 presents some key figures on the Peruvian macroeconomy. Money growth
was quite rapid through 1952, peaking at greater than 20 percent growth in 1951. In
contrast, money growth was relatively low through 1953 and 1954—the years of a relatively
depreciated nominal and real exchange rate. Inflation, as measured by the cost of living in
Lima, remained modest and relatively stable throughout. Real GNP growth peaks in 1950
with a resumption of higher growth at the end of the period.

<table>
<thead>
<tr>
<th></th>
<th>1950</th>
<th>1951</th>
<th>1952</th>
<th>1953</th>
<th>1954</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 Growth</td>
<td>16.7</td>
<td>23.5</td>
<td>16.8</td>
<td>10.5</td>
<td>7.0</td>
</tr>
<tr>
<td>Change in CPI (Lima)</td>
<td>9.3</td>
<td>7.4</td>
<td>6.8</td>
<td>9.0</td>
<td>4.7</td>
</tr>
<tr>
<td>Real GNP Growth</td>
<td>13.1</td>
<td>1.3</td>
<td>1.1</td>
<td>6.8</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Source: International Financial Statistics (M1 and CPI) and Cuentas Nacionales del Peru (GNP).

Plots #2 and #3 provide an illustration of the correspondent behavior of the nominal
rate and Peru's terms-of-trade and export-cost-of-dollars respectively. In both plots, a priori
we might expect a positive association between the two series; however, neither one exhibits
what could be considered a clearly evident relationship. We construct the terms-of-trade
index using the indices of import and export prices in *International Financial Statistics*. The export-price-of-dollars variable appearing in Plot #3 figures prominently in the analysis of Tsing; he defines it as the cost of living index divided by the unit dollar value of exports. These data are also from *IFS*.

**PLOT #2: NOMINAL EXCHANGE RATE AND TOT**

![Graph showing nominal exchange rate and import prices/export prices](image)

**PLOT #3: NOMINAL RATE AND EXPORT COST OF DOLLARS**

![Graph showing nominal exchange rate and export cost of dollars](image)
We turn our attention now to key balance of payments data. As pointed out above, Tsiang describes the potential linkage between the certificate and draft markets arising from direct foreign investment. And given the extremely hospitable orientation of the government, it is not surprising that direct investment was substantial through the period:

Table #3
Foreign Direct Investment (millions of $)

<table>
<thead>
<tr>
<th></th>
<th>1950</th>
<th>1951</th>
<th>1952</th>
<th>1953</th>
<th>1954</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign Direct Investment</td>
<td>-8.2</td>
<td>30.2</td>
<td>50.3</td>
<td>37.4</td>
<td>19.7</td>
</tr>
</tbody>
</table>

Source: International Monetary Fund, *Balance of Payments Yearbook*.

Only in 1950 was there a net disinvestment. From 1951 to 1954 direct investment was heavy, and was concentrated in the extractive export industries.

The behavior of the trade balance and reserve position are illustrated in Plots #4 and #5.

**PLOT #4: TRADE BALANCE AS PERCENT 1952 GNP**

![Graph of trade balance as a percent of 1952 GNP from 1950 to 1955.](image-url)
The trade balance is rather volatile, with surplus periods tending to correspond to periods of improved terms-of-trade. Additionally, corresponding roughly to the improved terms-of-trade in the first half of 1951, the level of reserves climbs markedly and then, broadly speaking, tapers downward to the end of period level.

Finally, we consider some of the short-run asset market conditions. Table #4 provides data on private short-term capital exports by Peruvian residents.\textsuperscript{11} Note that there is no indication of speculative capital movement in 1953--the year of pronounced depreciation:

\textsuperscript{11} We remind the reader that data of this type is generally not of the highest quality due to incentives for underreporting.
Table #4
Private Short-Term Capital Exports (millions of $)

<table>
<thead>
<tr>
<th></th>
<th>1950</th>
<th>1951</th>
<th>1952</th>
<th>1953</th>
<th>1954</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in private foreign assets held through Peruvian banks</td>
<td>10.6</td>
<td>-11.0</td>
<td>0.2</td>
<td>2.0</td>
<td>1.6</td>
</tr>
<tr>
<td>Change in private dollar balances held in U.S.</td>
<td>-3.3</td>
<td>0.7</td>
<td>13.9</td>
<td>3.3</td>
<td>9.5</td>
</tr>
<tr>
<td>Total</td>
<td>7.3</td>
<td>-10.3</td>
<td>14.1</td>
<td>5.3</td>
<td>11.1</td>
</tr>
</tbody>
</table>

Source: International Monetary Fund, *Balance of Payments Yearbook*.

An important related variable is the interest differential across assets denominated in the different currencies. Bearing in mind that the market for the Peruvian government’s long-term debt was far less deep and well-developed than that for U.S. long-term government debt, Plot #6 provides a measure of the nominal return differential across the two different assets:

**PLOT #6: LONG-TERM NOMINAL INTEREST RATES**

![Graph showing interest rate comparison between Peruvian and U.S.](image-url)
The differential is quite stable but for the period from mid-1953 to mid-1954. For all intents and purposes, variation in the differential is due to variation in the Peruvian long-term rate.

III. The Models

To proceed along the first of the two fronts of the investigation described in section I--on what basis was the market determining the equilibrium exchange rate--we first outline some of the relevant theory and the models which have grown out of it. Our point of departure, purchasing power parity (PPP), is motivated by the central importance of the real exchange rate in outward-directed developing countries. In its absolute version, PPP theory states that the equilibrium rate between two currencies equals the ratio of the two price levels. While it is quite well established that PPP models provide a very poor description of the industrial countries' floating rate experience since generalized floating [see, for example, Frenkel (1981)], the case of Peru is quite different in two key respects: (1) Peru is not an industrialized country and (2) the thinking in the 1950's and 1960's was far more accepting of the idea that real exchange rates would remain constant or nearly constant in the context of a freely floating exchange rate.\footnote{Indicative of the era, Friedman's (1953) classic defense of floating rates made strongly the argument that exchange rates would follow PPP, suggesting that it was far easier to allow exchange rates to adjust to differing rates of inflation than vice-versa.} Among other reasons, the first difference is important because speculators realize that pronounced deviations from PPP are likely to be much less sustainable for developing countries; in the face of pronounced deviations, floating rate experiments are more likely to be abandoned with a concomitant policy-induced adjustment of the rate back toward the PPP level. The second difference is quite important in that, as a result, we would expect speculators in the 1950's to have put a much higher weight on a PPP model of exchange rate fundamentals.
Indeed, supporters of flexible exchange rates at this time generally expected exchange rates to follow PPP. Theoretically, this would be the likely outcome if any one of the following three conditions are met. First, PPP would prevail if differing rates of inflation were the only source of shocks to the balance of payments. Second, if shocks from other sources are present, PPP would prevail if import and export demand elasticities were so high that adjustment would require only very small exchange rate changes. And third, PPP would prevail if domestic wages and prices were completely flexible because any shock to the exchange rate would first produce a change in the price of tradables and then rapid and parallel changes in wages and all other prices in the economy, assuming an accommodative monetary policy. Clearly, the first condition is unlikely to be met in any real world economy. However, even if the second and third conditions are only true over the longer run, PPP might be saved if rational speculators conclude that the long-run exchange rate path is determined by relative price levels.

An Expectational Purchasing Power Parity Model

We consider a model which allows short-run deviations from PPP.\textsuperscript{13}

\begin{equation}
  s_t = E[p_t | \Omega_{t-1}] - p_t^* + \gamma d_{t-1} + \epsilon_t \quad 0 < \gamma < 1
\end{equation}

\textsuperscript{13} We might ask whether the direction of 'causality' from prices to the exchange rate is consistent with the data. Frenkel(1978) investigates the same question regarding the industrialized countries' floating rates in the 1920's. His Sims(1972) test for Granger causality yields evidence that, in fact, the proper specification for the countries he considers should have exchange rates on the right hand side. He then goes on to estimate 'price equations.' For the Peruvian episode, however, results of a Sims test between the nominal exchange rate and the Peruvian price index yield no evidence that the exchange rate 'causes' prices but prices do not 'cause' the exchange rate: the null hypothesis of econometric exogeneity can not be rejected in either direction.
where

\[ s_t = \log \text{ of the spot draft exchange rate (soles/dollar)} \]

\[ p_t, p_t^* = \log \text{ of Peruvian and U.S. price levels, respectively} \]

\[ E[ . | \Omega_{t-1}] = \text{expectation conditional (linearly) on information at time } t-1 \]

\[ d_t = \text{deviations from PPP in period } t \]

\[ \epsilon_t = \text{white noise random disturbance} \]

Equation (1) postulates that deviations from PPP can be represented as a stationary first-order autoregressive process. This means that even though there are short-run deviations from PPP, in the long-run these deviations tend to disappear. The speed at which deviations from PPP are eliminated depends on the value of \( \gamma \). For estimation, presumed joint endogeneity precludes the use of OLS when the contemporaneous price level \( p_t \) is included as a regressor. Under these circumstances, we prefer to eliminate possible inconsistency in the estimate of this parameter by including only lagged values of the instruments used in the first stage regression of 2SLS. This has the natural interpretation of a linear projection onto the previous period's information set.

**The Short-Run Monetary Model**

We now consider the short-run monetary model employed by Edwards (1983). In essence, the model begins with the PPP relation above, and then substitutes in expressions for the determination of the price levels in the money markets:

\[ d_t \equiv s_t - p_t + p_t^* = \gamma d_{t-1} + \epsilon_t \quad 0 < \gamma < 1 \]
(3) \((M_t - p_t^d) = ay_t - bi_t\) ; \((M_t^* - p_t^*)^d = ay_t^* - bi_t^*\)

(4) \(m_t - m_{t-1} = \theta(m_t^d - m_{t-1})\) ; \(m_t^* - m_{t-1}^* = \theta(m_t^{*d} - m_{t-1}^{*})\)

where

\(M_t, M_t^* = \text{log of domestic and foreign nominal money stock}\)

\(m_t, m_t^* = \text{log of domestic and foreign real money stock}\)

\(y_t, y_t^* = \text{log of domestic and foreign real income}\)

\(i_t, i_t^* = \text{domestic and foreign nominal interest rates}\)

The model also includes the simplifying assumption that the coefficients a, b, and \(\theta\) are equal across the two countries. Equation (3) presents the demand for money functions in the domestic and foreign countries. The assumption of slow adjustment in the money markets is captured in equation (4), where \(\theta\) is the speed of adjustment coefficient.

The equilibrium price levels at home and abroad can be found by replacing \(m_t^d\) and \(m_t^{*d}\) [eq. (3)] into the partial adjustment equation (4). The resulting expressions for \(p_t\) and \(p_t^*\) can then be used in (2) to find the following equation for the equilibrium exchange rate in the short-run:

\( (5) s_t = (M_t - M_t^*) - \left[\theta a / (1 - (1-\theta)L)\right](y_t - y_t^*)\)

\(+ \left[\theta b / (1 - (1-\theta)L)\right](i_t - i_t^*) + \gamma d_{t-1} + \varepsilon_t\)

where \(L\) is the lag operator.
Equation (5) suggests that in the short-run the log of the exchange rate is related with a unitary coefficient to the difference in the log of the nominal money stocks at home and abroad, and that current and lagged values of $y, y^*$, $i$, and $i^*$ affect the current value of the exchange rate. The presence of lagged $y$, $y^*$, $i$, and $i^*$ and of $\gamma d_{t-1}$ are the main differences between the short-run formulation of exchange rate determination presented in (5) and the most simple versions of the monetary approach. As may be seen, the coefficients of current and lagged values of $y_t$, $y^*_t$, $i_t$, and $i^*_t$ are related to the parameters of the money demand equations and to the speed of adjustment coefficient $\theta$. If, for example, it only takes one period for the money markets to clear (i.e., $\theta=1$), then the coefficients of the income and interest rate terms will be equal to their respective coefficients in the money demand equations.

In his OLS estimation of the coefficients on the income and interest rate differentials Edwards uses a third-order polynomial lag over eighteen months with a zero end constraint. According to his model, the coefficient on the money stock differential should not be significantly different from one, the sum of the coefficients on the income differential should be negative, the sum of the coefficients on the interest rate differential should be positive, and the coefficient $\gamma$ on the PPP deviation should be positive. Table #5 presents his results for the draft sol/dollar rate: 14

14 Edwards uses consumer price indices to calculate $d_t$.  

99
Table #5
Edwards' Results for Short-run Monetary Model

\[ s_t = \beta_0 + \beta_1(M_t-M_t^*) + \beta_2(y_{t-j}-y_{t-j}^*) + \beta_3(i_{t-j}-i_{t-j}^*) + \beta_4 d_{t-1} \]

<table>
<thead>
<tr>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$R^2$</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>-17.063</td>
<td>1.053</td>
<td>-2.901</td>
<td>0.267</td>
<td>0.243</td>
<td>0.98</td>
<td>1.68</td>
</tr>
<tr>
<td>(3.083)</td>
<td>(0.101)</td>
<td>(0.532)</td>
<td>(0.040)</td>
<td>(0.141)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

From this he concludes that, "generally, these results support the short-run monetary model of exchange rate determination for the Peruvian experience with a floating rate." Although on the surface the results do indeed appear supportive, the model and its estimation are not without shortcomings. We consider two here, the second of which is more important. First, the imposed lag structure is quite long and its estimated shape is very different from that implied by the model. The partial adjustment in the money market implies that the coefficients should be geometrically declining. However, a replication of Edwards' results verifies that the largest coefficients in absolute value occur at the twelve month lag for both the income and interest rate differentials. Plots #7 and #8 make it clear that the measured influence of these variables is certainly not geometrically declining.
More important, Edwards' estimation begins with July, 1951, a full eighteen months into the floating rate period—a length of time which corresponds to his choice of
lag length. In so doing, he neglects nearly a third of the available monthly data. Moreover, it is this third which captures the United States' deepest involvement in the Korean War, the influence of which is highlighted by Tsiang (1957) for reasons quite apart from purely monetary considerations. There is no mention of this sampling choice in the Edwards (1983) paper. One might argue that it would be inappropriate to use data for the lagged regressors from a period in which a different exchange rate system is operative. However, in his monetary model the lagged regressors are only relevant insofar as they determine the current price level within the money market. The exchange rate is then determined by substituting these price level determinants into the PPP relationship described in equation (1). Since his model of the money market is independent of the exchange rate, there is little reason to exclude such a large portion of the data. Results for Edwards' model estimated over the full sample appear in section III.

The Tsiang Model

The views expressed by Tsiang (1957) suggest an alternative model that emphasizes the role of world prices for Peruvian exports as a proximate determinant of the exchange rate. On page 460 he asserts that:

On the assumption that no very great changes in the volume of exports are required to keep the balance of payments in equilibrium, the sol price of dollars . . . could be expected to tend to adapt itself to the changes in the domestic cost of producing a dollar's worth of exports.
Citing only annual figures, he demonstrates a rough correspondence between the certificate rate and the estimated average cost of a dollar's worth of exports measured as the ratio of the Peruvian cost of living to unit export prices in dollars. This is the extent to which Tsiang might be said to have 'modeled' the exchange rate in his analysis. For an econometric test of the role of export prices, one possible specification of Tsiang's view includes the Peruvian price level and dollar export prices as the proximate determinants of the spot rate, with the PPP adjustment term of the previous models to assure appropriate long–run behavior:

\[ s_t = \beta_0 + \beta_1 E[p_t|\Omega_{t-1}] + \beta_2 DPE_t + \gamma d_{t-1} + \varepsilon_t \quad 0<\gamma<1 \]

where \( DPE_t \) equals the log of the dollar price of exports and the predicted sign of the coefficient \( \beta_2 \) is negative.

IV. Results Over The Full Sample

We begin with estimation of the core expectational PPP model described in equation (1). The model predicts coefficients on the domestic and U.S. price levels of one and negative one respectively. Table #6 presents the results using Two–Stage Least Squares: 15

---

15 The instruments for the domestic price level are a constant, a time trend, and the first three lags of the price level, the money supply, the interest rate differential, and the real income differential.
Table #6
Results for 'Expectational' Purchasing Power Parity Model

\[ s_t = \beta_0 + \beta_1 E_{t-1}[p_t] + \beta_2 p_t^* + \beta_3 d_{t-1} \]

<table>
<thead>
<tr>
<th></th>
<th>(\beta_0)</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>(R^2)</th>
<th>DW</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>0.561</td>
<td>1.062</td>
<td>-1.083</td>
<td>0.854</td>
<td>0.94</td>
<td>1.50</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(0.709)</td>
<td>(0.068)</td>
<td>(0.171)</td>
<td>(0.066)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WPI</td>
<td>-0.033</td>
<td>0.992</td>
<td>-0.923</td>
<td>0.869</td>
<td>0.92</td>
<td>1.61</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(0.536)</td>
<td>(0.043)</td>
<td>(0.115)</td>
<td>(0.066)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

These equations fit quite well, particularly in light of the dismal performance of PPP models for the industrial countries since generalized floating. Neither of the price level coefficients are significantly different from their predicted values, the coefficient on \(d_{t-1}\) is positive and less than one, and the \(R^2\)'s are high. Although both Durbin–Watson statistics are in the inconclusive range at the 5% level, the Ljung–Box Q-statistics show no evidence of autocorrelation. (The Q values reported are the marginal significance levels of the tests and are nowhere near the conventional 5% level.) All told, the model works quite well. Moreover, it provides a nice touchstone for evaluating the performance of the alternative models.

We turn now to the results for the monetary model. In effect, the monetary model simply couples PPP with a money market description of price level determination, as laid out in equations (2) through (4). Table #7 presents the OLS results for two alternative specifications of the monetary model. Edwards' short-run model appears first and
includes the 18-month lags of the real income and interest differentials.\textsuperscript{16} The second model could be referred to as a 'long-run' formulation in which lagged values of income and interest rate differentials do not appear.\textsuperscript{17} In terms of Edwards' model this corresponds to a money market that fully adjusts every period; i.e., the value of θ is one in equation (4).

\textbf{Table \#7}  
Results for Monetary Models Over the Full Sample

\[ s_t = \beta_0 + \beta_1(M_{t-1} - M_{t-1}^*) + \beta_2\Sigma(y_{t-j} - y_{t-j}^*) + \beta_3\Sigma(i_{t-j} - i_{t-j}^*) + \beta_4d_{t-1} \]

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$R^2$</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>WITH LAGS</td>
<td>1.127</td>
<td>0.318</td>
<td>0.072</td>
<td>0.134</td>
<td>0.209</td>
<td>0.91</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(1.991)</td>
<td>(0.038)</td>
<td>(0.344)</td>
<td>(0.051)</td>
<td>(0.155)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO LAGS</td>
<td>3.087</td>
<td>0.309</td>
<td>0.465</td>
<td>0.072</td>
<td>0.473</td>
<td>0.89</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(0.817)</td>
<td>(0.032)</td>
<td>(0.115)</td>
<td>(0.014)</td>
<td>(0.137)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

While the coefficient on the log money stock differential is significantly different from zero for both models, it is now very significantly different from its predicted value of one under the monetary model. In the short-run model, the coefficients on the log income differential and the lagged PPP deviation are no longer significant. In the long-run

\textsuperscript{16} Except for the real income series, the data are the same as those used by Edwards as verified by the replication of Edwards' results in equation (5) over the shorter period. Since the source of the real income series Edwards uses does not include the 1948–1949 period, we use nominal GNP figures deflated by the consumer price index for the period 1948:7–1949:12.
\textsuperscript{17} This is not strictly the long-run formulation of the monetary model in that it still admits transitory deviations from purchasing power parity.
model, both the log income and interest differential coefficients are significant but the former is now of the wrong sign. Additionally, the dismal Durbin–Watson statistics signal very strongly the likelihood of omitted variables. In sum, the models are a relatively poor description of the full five–year floating rate period.

Before turning to the Tsiang model, we re–estimate the two different formulations of the monetary model treating the foreign (U.S.) price level as exogenous. This specification has the advantage that it helps to further clarify the role of exogenous determinants by separating the U.S. price level from the domestic money market: \( p_t^* \) is included as a separate regressor and the money supply, real income, and interest rate terms in the equation now reflect only the conditions in the Peruvian money market. Table #8 presents the OLS results. In both equations, \( p_t^* \) is the U.S. consumer price index:

**Table #8**
Results for Monetary Models with U.S. Price Level Treated as Exogenous

\[
s_t = \beta_0 + \beta_1 M_t + \beta_2 y_{t-j} + \beta_3 i_{t-j} + \beta_4 d_{t-1} + \beta_5 p_t^* 
\]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( R^2 )</th>
<th>( DW )</th>
</tr>
</thead>
<tbody>
<tr>
<td>WITH LAGS</td>
<td>8.563</td>
<td>0.210</td>
<td>0.572</td>
<td>0.155</td>
<td>0.442</td>
<td>−1.802</td>
<td>0.96</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>(2.919)</td>
<td>(0.098)</td>
<td>(0.395)</td>
<td>(0.047)</td>
<td>(0.093)</td>
<td>(0.571)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO LAGS</td>
<td>8.087</td>
<td>0.370</td>
<td>0.976</td>
<td>0.051</td>
<td>0.634</td>
<td>−1.883</td>
<td>0.94</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>(1.221)</td>
<td>(0.090)</td>
<td>(0.194)</td>
<td>(0.009)</td>
<td>(0.081)</td>
<td>(0.309)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.
The U.S. price level is very significant in both equations but its magnitude is much greater than the theoretical value of negative one, significantly so in the equation with no real income or interest rate lags included. The coefficients on the log money supply are still very significantly less than one and the coefficients on the log real income are of the wrong sign. While the $R^2$s are now higher, the very low Durbin–Watson statistics continue to signal strongly the likelihood of omitted variables. In short, the monetary models are not supported by the data.

We turn now to the results for the Tsiang model described in equation (6). Again, the linchpin of his view is that the dollar price of Peruvian exports was a fundamental exogenous determinant of the exchange (certificate) rate. Table #9 presents the Two–Stage Least–Squares results using the Peruvian consumer price index and wholesale price index. Recall that the variable DPE denotes the log of the dollar price of exports:18

Table #9
Results for 'Tsiang' Model

\[ s_t = \beta_0 + \beta_1 E_{t-1}[p_t] + \beta_2 \text{DPE}_t + \beta_3 d_{t-1} \]

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$R^2$</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>-2.749</td>
<td>0.689</td>
<td>-0.102</td>
<td>0.996</td>
<td>0.93</td>
<td>1.22</td>
</tr>
<tr>
<td></td>
<td>(0.260)</td>
<td>(0.033)</td>
<td>(0.026)</td>
<td>(0.066)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>WPI</td>
<td>-2.930</td>
<td>0.898</td>
<td>-0.217</td>
<td>0.941</td>
<td>0.91</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>(0.313)</td>
<td>(0.040)</td>
<td>(0.031)</td>
<td>(0.067)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

18 As noted in the data appendix, the dollar export price index we use is built from the sol export price index.
These results for the Tsiang model are mixed. While the coefficients appear well determined with the appropriate sign and the $R^2$s are only slightly lower than those for the core expectational PPP model, the Durbin–Watson statistics continue to signal the likelihood of omitted variables. In an effort to distinguish the better model, we estimate the core PPP model with the dollar–price–of–exports included as an explanatory variable. Is there information in the export price variable that is not captured by the U.S. price level variable? Table #10 presents the Two–Stage Least–Squares results:

<table>
<thead>
<tr>
<th>Table #10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results for 'Expectational' Purchasing Power Parity Model With Dollar Export Prices</td>
</tr>
<tr>
<td>$s_t = \beta_0 + \beta_1E_{t-1}[p_t] + \beta_2p_t^* + \beta_3d_{t-1} + \beta_4DPE_t$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$R^2$</th>
<th>DW</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>0.153</td>
<td>1.047</td>
<td>-1.030</td>
<td>0.855</td>
<td>-0.018</td>
<td>0.94</td>
<td>1.49</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(0.741)</td>
<td>(0.087)</td>
<td>(0.235)</td>
<td>(0.068)</td>
<td>(0.031)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WPI</td>
<td>-0.748</td>
<td>0.974</td>
<td>-0.703</td>
<td>0.864</td>
<td>-0.076</td>
<td>0.93</td>
<td>1.60</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(0.636)</td>
<td>(0.041)</td>
<td>(0.161)</td>
<td>(0.061)</td>
<td>(0.041)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

In neither equation is the dollar–price–of–exports variable significant at conventional significance levels. Moreover, none of the price level coefficients are significantly different in 
  International Financial Statistics deflated by the current draft exchange rate. We attempted to improve the fit of the Tsiang model by constructing four different export price indices from U.S. wholesale prices in the Survey of Current Business; the results, however, were not substantively different from those reported above.
ferent from their predicted values, although the U.S. WPI coefficient does come down a bit. Thus, these results establish rather conclusively the dominance of the core expectational PPP model.

As a last avenue of inquiry we look into the role of inflationary expectations in the determination of the exchange rate. Following the method used by other authors [e.g. Frankel(1979)], we use the long-term interest differential as a proxy for the expected long-run inflation differential. Admittedly, the market for long-term government debt in Peru was nowhere near as deep as that for U.S. long-term government debt; however, it seems reasonable that much of the long-term rate's movement was directly or indirectly (through policy induced adjustments in the rate) the result of changes in inflationary expectations. Two-Stage Least Squares results for the core PPP specification including the (instrumented) contemporaneous long-run interest differential appear in Table #11: 19

Table #11
Results for Expectational PPP Model With Expected Inflation Differential

\[ s_t = \beta_0 + \beta_1 E_{t-1}[p_t] + \beta_2 p_t^* + \beta_3 d_{t-1} + \beta_4 (i_t - i_t^*) \]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( R^2 )</th>
<th>( DW )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>0.244</td>
<td>0.998</td>
<td>-0.995</td>
<td>0.773</td>
<td>0.025</td>
<td>0.95</td>
<td>1.63</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>(0.564)</td>
<td>(0.073)</td>
<td>(0.172)</td>
<td>(0.072)</td>
<td>(0.012)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WPI</td>
<td>0.875</td>
<td>0.931</td>
<td>-0.986</td>
<td>0.640</td>
<td>0.041</td>
<td>0.94</td>
<td>1.64</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.486)</td>
<td>(0.047)</td>
<td>(0.104)</td>
<td>(0.085)</td>
<td>(0.014)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses.

19 The instruments for the interest differential are the same as those used for the price level. See note 15.
The expected inflation differential, as proxied by the interest differential, enters significantly in both equations. None of the coefficients on the price levels are significantly different from their theoretical values and the R2's are both higher. The coefficient on the 'inflation differential' in the CPI equation implies that a one percent increase (annual basis) induces a 2.5 percent depreciation of the sol. Thus, the sign is as we would expect a priori, and the magnitude is quite reasonable.

Since the Peruvian capital market was indeed repressed through this period, one possible alternative interpretation of the interest rate–exchange rate relation is that expectations of higher inflation, and therefore depreciation through PPP, in the first half of 1953 were not met with increases in the nominal long–term interest rate, causing depreciation of the sol. The depreciation was then met in late 1953 and early 1954 (see Plot #6) with policy induced adjustments in the interest rate which created pressure for appreciation. If this story were true, then we would expect that the exchange rate series would be Granger–causally prior to the interest rate series. We use Sim's(1972) procedure to test whether the interest rate differential is in fact 'prior' to the nominal exchange rate or vice–versa. As Table #12 indicates, the null hypothesis that the interest differential does not 'cause' the exchange rate is rejected whereas the null that the exchange rate does not 'cause' the interest differential is not. Thus, there does not appear to be any evidence for this alternative interpretation.
**Table #12**

Granger Causality Tests Between the Long-Run Interest Differential and the Exchange Rate

<table>
<thead>
<tr>
<th>Differential causes s</th>
<th>s causes Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>F Statistic</td>
<td>3.27</td>
</tr>
</tbody>
</table>

Critical values are 5%: 2.67 and 1%: 3.97.

In the end, the core expectational PPP model emerges a clear winner. This stands in sharp contrast to its performance vis-à-vis industrial countries' exchange rate behavior since 1973. When the full sample is considered, the added step taken by the monetary models of replacing the (endogenous) price level with the money stock and (endogenous) interest rate and real income is not supported empirically. Perhaps more surprising, when the information embedded in the U.S. price level is included, world dollar prices of Peruvian export goods do not help to explain the exchange rate's behavior. Judging from Plot #3, it appears possible that the market was reacting asymmetrically to changes in world export prices: favorable prices did not generate significant appreciation whereas unfavorable prices seem to have been associated with marked depreciation. In fact, Tsiang's analysis implicitly points to this asymmetry, as evinced by the quote on page two of this paper. The puzzle is in the year 1951. One might argue that the central bank's absorption of reserves in that year is what prevented the adjustment. However, this is very unlikely since after May of that year reserves were actually falling but no adjustment occurred. Perhaps a more significant explanation can be found in either or both of the following two factors. First, the increase in reserves may have indirectly precluded
appreciation by generating expectations of future price level increases, and therefore
depreciation, due to the very rapid money growth in 1951 (see Table #2) that resulted
from not sterilizing the reserve inflow. And second, to a large extent, the markets may
have looked through the transitory favorable export prices arising out of the U.S.'s
involvement in the Korean War, particularly since President Truman's decision in April
1951 to relieve General MacArthur of his commands signalled an end to American
insistence on total victory.

V. The Viability of Flexible Rates

Whether or not the flexible rate system was a viable choice for Peru is a question of
many dimensions with no clear answer. In light of this, we choose to examine five
important aspects of the floating rate experience and their implications: (1) the relative
volatility of export prices and the exchange rate, (2) the changing relative price of traded
and non-traded goods, (3) the possibility of destabilizing speculation, (4) the exchange
rate regime as part of a constellation of policies, and (5) the stability of the model
underlying the market's behavior. Since the behavior of the dual rate system was on the
whole not substantively different from that of a unified rate, we make no distinction
between the two in what follows.

The Relative Volatility of Dollar Export Prices and the Exchange Rate

Were exchange rate movements large relative to export price movements? One
might expect that developing countries' dependence upon the vagaries of world commodity
prices would cause a magnification effect of export price changes on the flexible exchange rate. Table #13 presents the mean squared change in the log of dollar export prices and the log of the nominal exchange rate from January 1950 to December 1954:

Table #13
Mean Squared Change from 1950 to 1954 (Annual basis)

<table>
<thead>
<tr>
<th>Log of Dollar Export Prices</th>
<th>Log of Nominal Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.064</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Sources: *International Financial Statistics* (export prices) and *Pick’s Currency Yearbook* (nominal rate).

Thus, dollar export prices were in fact much more volatile over the period, by a factor of more than 10. Is this because export prices were exceptionally volatile or because the nominal rate was quite stable? The answer is both. It is true that export prices, owing to the Korean War and other factors, were exceptionally volatile. Yet, in comparison to the industrial countries' experience since generalized floating, the nominal rate was actually surprisingly stable. Table #14 presents the same statistic for the dollar/mark, the dollar/yen, and the dollar/pound exchange rates from 1973–1980:

Table #14
Mean Squared Change from 1973 to 1980 (Annual basis)

<table>
<thead>
<tr>
<th>Log Dollar/DM rate</th>
<th>Log Dollar/Y rate</th>
<th>Log Dollar/GBP rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.011</td>
<td>0.019</td>
<td>0.019</td>
</tr>
</tbody>
</table>

In sum, the expectation that for a developing country export price volatility would generate pronounced nominal exchange rate volatility is not borne out in the case of Peru. This finding is consistent with the result that dollar export prices did not figure prominently in the success of the models in the previous section. As pointed out there, part of this may have been due to the perceived transitoriness of the price changes. Another factor was probably the high positive correlation between import prices and export prices over the period (correlation coefficient = 0.78).

The Changing Relative Price of Traded and Non–Traded Goods

An essential element in any adjustment to changing external conditions is the relative price of traded and non–traded goods.\(^{20}\) Plot #9 illustrates the behavior and degree of relative price changes.

PLOT #9: RELATIVE PRICE OF TRADED AND NON-TRADED GOODS

\(^{20}\) A more salient concept is the relative price of tradables and non–tradables. Since the data used accord better with the use of the terms 'traded' and 'non–traded', we use the latter.
The traded goods price is calculated as an equal weighted average of import prices and export prices.

Two points warrant comment. First, this key relative price exhibits pronounced deviations from its initial value. This is not surprising, however, given the facts that the nominal exchange rate tends to adjust very little in the face of changes in dollar export prices and that import and export prices are highly positively correlated. These movements in the relative price resulting from the real shock induce the resource shift into the traded goods sectors where they can be more efficiently utilized.

This leads to the second point: disturbances to the relative price are not only large, they are also persistent. This would not have been the case if Peru's internal price structure had been characterized by considerable flexibility. The resource transfer would have bid up wages and prices relatively quickly until the initial relative price was restored. Perhaps more important, this has a key implication regarding interpretation of the empirical success of the expectational PPP model. In effect, it undercuts the interpretation that PPP is a good model because shocks to the exchange rate, whatever the source, are being transmitted quickly and widely into the domestic price structure, rather than the opposite story, as effected through the asset markets, being more nearly the truth.

The Possibility of Destabilizing Speculation

The issue of destabilizing speculation is certainly of central importance vis-a-vis the viability of floating rates in developing countries.\(^{21}\) For the Peruvian experience, what evidence is available suggests that speculative behavior was not in fact destabilizing. The key period upon which this judgement is based, the year 1953, includes several months previous to the only significant depreciation, as well as the lion's share of the depreciation

\(^{21}\) The arguments in this subsection are due to Tsiang(1953). We include them here because the analysis would be incomplete without them. See his paper for more details.
itself (see Plot #1). There are three principal pieces of evidence. First, Table #4 establishes that short-term capital exports were lower in 1953 than in 1952 or 1954, the former being a year of considerable exchange rate stability and the latter a year of appreciation. Second, the outstanding amount of import credits received from foreign suppliers continued to increase in 1953, which indicates that importers as a whole did not attempt to reduce significantly their future foreign exchange commitments because of expected depreciation. And finally, Tsiang(1953) finds insufficient evidence to justify the belief that imports in 1953 were artificially inflated by speculative anticipation of further depreciation. Thus, the key indicators show no sign of destabilizing behavior.

The Exchange Rate Regime as Part of a Constellation of Policies

Any choice of exchange rate system must necessarily fit into an existing institutional structure and coexist with a general policy orientation. This idea is not new but deserves emphasis. The Peruvian government, in most all respects, seems to have created an environment in which a floating rate system could operate effectively, with relative stability. However, the extreme economic liberalism which characterized the government's orientation is not so common, nor is it welcome in much of the developing world. Differences in orientation, producing differences in policy objectives, might very well compromise the potential stability of a floating rate. For example, in a repressed capital market in which interest rates are largely administered, non-adjustment or slow adjustment of rates in the face of inflationary expectations, in pursuit of a particular policy objective, might induce considerable exchange rate instability. The point is, going part of the way may be worse than not going at all.
The Stability of the Model Underlying the Market's Behavior

Finally, in order to consider floating rates in the current developing country context it is important to ask whether or not the market-perceived model of fundamentals might have changed over the last three decades. We make the point in section III that because thinking in the 1950's was far more accepting of the idea that real exchange rates would remain constant or nearly so under floating rates, we would expect speculators to have put a much higher weight on a PPP model of fundamentals than they would today. Indeed, experience since 1973 has taught that real exchange rates are not even close to remaining constant in the face of floating rates, at least for the industrial countries. Ceterus paribus, it stands to reason that other driving forces would most likely be afforded more weight in today's asset markets.

VI. Conclusions

In our efforts to model the fundamentals which underlay the market's determination of Peru's floating exchange rate, we find that the expectational purchasing power parity model not only dominates the others, but also fits quite well. Even though the monetary models begin with the assumption that PPP holds, their substitution of the price levels with their money market determinants is not congruent with the data. This stands in sharp contrast to the results and conclusions in Edwards(1983). Additionally, the emphasis that Tsiang(1953) puts on the role of the dollar price of Peru's exports in the determination of the exchange rate is also not supported: when the dollar price of exports is added to the expectational PPP model its coefficient is insignificant at conventional levels, and the other coefficients remain at their theoretically predicted levels.

All told, Peru's experience with floating exchange rates is promising: the real exchange rate never ventured from its initial value more than 10 percent in either direction; there does not appear to be any evidence of destabilizing speculation; and the nominal
exchange rate remained relatively stable in the face of quite volatile world prices for Peru's exports. Nonetheless, some important caveats are in order. A floating rate system that is not supported by a consistent constellation of policies and institutions is likely to fare much worse. For example, one might expect considerable volatility in the context of a repressed capital market in which nominal interest rates are slow in or prevented from adjusting to market conditions. And finally, after the experience with generalized floating among the industrial countries, in a developing country today it is unlikely that speculators would be using a model of fundamentals that would generate the same degree of real exchange rate stability.
DATA APPENDIX

PERU

Money Supply: Seasonally adjusted M1. The unadjusted figures are from Boletin del Banco de la Reserva del Peru.

Interest Rate: Yield of long-term government bonds from International Financial Statistics.

Price Indices: CPI is from Boletín del Banco de la Reserva del Peru. Wholesale, export, import, and home goods price indices are from International Financial Statistics.

Exchange Rate: The end of period draft rate in the free market from Pick's Currency Yearbook.

Real Income: Constructed interpolating annual data on real GNP from Cuentas Nacionales del Peru.

USA


Interest Rate: Yield on long-term government bonds from Supplement to Banking and Monetary Statistics.


Real Income: Seasonally adjusted index of industrial production from Survey of Current Business.
REFERENCES


------- "Real Exchange Rate Overshooting and the Output Cost of Bringing Down Inflation." European Economic Review. 18, no. 1/2 (May/June 1982): 85-123.


_____ , "Sticky Prices and Inventory Adjustment." Board of Governors of the Federal Reserve System, mimeo (February, 1982b).


Tsiang, Sho C. "An Experiment with Flexible Exchange Rate Systems: The Case of Peru, 1950-54." International Monetary Fund Staff Papers. 5 (November 1957): 449-76.