ESSAYS ON VERTICAL FORECLOSURE, CARTEL STABILITY AND THE STRUCTURAL DETERMINANTS OF OLIGOPOLISTIC BEHAVIOR

by

IAN AYRES

B.A. Russian and Eastern European Studies/Economics,
Yale University (1981)
J.D. Yale Law School (1986)

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                                        Department of Economics
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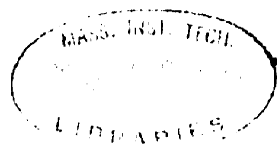
Certified by.   ......
                                    Richard L. Schmalensee
                Professor of Economics, Department of Economics
        Professor of Applied Economics, Sloan School of Management
                                                Thesis Supervisor

Accepted by...
                                        Peter Temin, Chair
                            Departmental Graduate Committee

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ABSTRACT

The first essay analyzes theories of foreclosure after vertical integration. Following the work of Steven Salop and others, a model is presented in which a duopolist is able to raise the costs of its rivals after merging upstream. Overbuying in the external input market after a vertical merger raises a rival's costs and thereby drives the rival partially from the downstream market. This foreclosure effect can occur even if the unintegrated upstream producers remain competitive after the merger. A central result is that foreclosure will turn on the change in the rivals marginal factor cost not simply the input price (the average factor cost). Indeed vertical integration can cause the marginal factor cost to rise even when the input price declines. The integrated firm's ability to strategically take account of how its overbuying effects its rivals is shown to be identical to an extensive-form game in which the integrated firm has a first-mover advantage with respect to its overbuying. Alternative strategic environments for both price and quantity games are analyzed.

The second essay develops a model of cartel stability in the presence of fringe competition. Instead of assuming a particular type of collusive behavior, the cartel's conduct is derived so as to maximize profits given a stability constraint. Under certain conditions, cartels prefer to act less collusively so as to deter defection to the competitive fringe. The model is extended to consider issues of heterogeneous costs across firms and the credibility of different collusive agreements.

The third essay examines the structural determinants of airline carrier conduct. Over 2,000 conjectural variation estimates (by carrier and route) were used as measures of

market conduct. These conjectures were then regressed against structural characteristics of the markets. The results indicate that: 1) Competition increases dramatically as the number of carriers servicing a route increases, 2) Routes with larger potential monopoly rents tend to have more collusive behavior, 3) Non-price competition substitutes for price competition, 4) Regulated conduct was both more collusive and more uniform than unregulated conduct, 5) Carriers tend to systematically expect an overly collusive response from their rivals, and 6) Tests of Bresnahan Consistency and Stackelberg leadership were rejected.

Thesis Supervisor: Richard L. Schmalensee

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INTRODUCTION

This dissertation contains three essays analyzing vertical foreclosure, cartel stability and structural tests of oligopolistic behavior. The first essay is a theoretical examination of an exclusionary practice, while the second and third essays examine, respectively, theoretical and empirical issues of collusive conduct.

In the first essay, "Vertical Integration and Overbuying: An Analysis of Foreclosure via Raised Rivals' Costs", I extend the insights of Steven Salop and others to show how the act of merging into an upstream market can make it easier for a firm to exclude its rivals from the downstream market. Vertical integration can create a marginal-factor-cost "umbrella" which an integrated firm can "raise" over itself by buying inputs from the external input market. Overbuying from the unintegrated input market raises rivals' costs and reduces their equilibrium size.

The second essay, "Deriving Cartel Behavior in a Model with Fringe Competition," is concerned with collusion instead of exclusion. Its genesis is the simple insight that cartels should modify their behavior to respond to problems of instability. In contrast to the existing literature which

has assumed a specific type of cartel behavior, the essay develops a model in which cartel behavior is derived from a class of conjectural behaviors so as to maximize cartel profits. In the model, cartels may prefer to act less collusively if they can induce a larger stable collusive group. Cartel members thus may face a trade-off between collusion and free-riding. The essay also demonstrates that when industry members face heterogeneous costs the relatively efficient firms will join the cartel, while the relatively inefficient firms will gravitate to the competitive fringe.

Finally, in the third essay, "Determinants of Airline Carrier Conduct," I construct empirical tests of predictions linking industry structure to oligopoly behavior. The essay represents a first attempt to go beyond the structure-performance regressions that have dominated the literature. By estimating conjectural variations for over 2000 routes in the airline industry, I was able to directly test structure-conduct hypotheses (without making the assumption that profits are positively correlated with collusion). The results broadly support existing theories of collusion: for example, the number of sellers has a dramatic procompetitive impact on firm behavior, and routes with larger potential monopoly rents tend to have more collusive behavior.

Finally, I would like to thank my mother and father for giving me their unconditional love throughout my life. I dedicate this dissertation to them.

CHAPTER 1:   Vertical Integration and Overbuying:

An Analysis of Foreclosure via Raised Rivals' Costs

In a series of articles, Steven Salop and several co-authors [Salop and Scheffman (1986); Krattenmaker and Salop (1986a,b); Ordover, Saloner, and Salop, (1987)] have suggested that dominant firms may engage in various forms of non-price predation by raising the costs of their rivals. Specifically, Salop and Krattenmaker argue that under certain structural conditions a firm may be able to raise its rivals' costs by 1) inducing collusion in an upstream market, or 2) by foreclosing low cost sources of its rivals' supply. Salop's second route to raising rivals' costs is identical to Salinger's definition of foreclosure [Salinger (1985)], in that both require the price of an input to increase.

Salop's notions of induced collusion have recently been formalized by Ordover, Saloner, and Salop [1987], who present a model of vertical integration in which after a merger the unintegrated upstream firms collude as a monopolist -- thus

raising the costs of the unintegrated downstream rivals.[1]

This paper, in contrast, attempts to formalize how a firm
could raise its rival's costs by overbuying in a competitive
upstream market.

Specifically, I analyze a model in which after a merger
an integrated duopolist buys some of its inputs from
unintegrated upstream producers to drive up the price that
its downstream rival must pay for the input. The model
reveals that where the downstream rival has monopsony power,
the anticompetitive effects of foreclosure will turn on
changes in marginal factor costs and not on the input price.
Indeed, in a simple model of vertical integration and over-
buying, an unintegrated rival can be partially forced out of
a market even though the spot price for its input declines.[2]
This counter-intuitive result stems from the fact that the

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[1] Salinger [1987] presents a successive Cournot oligopoly
model in which vertical integration causes the input price to
rise although there is no collusion as defined by Abreu
[1986]. The single period Cournot game with n - 1 upstream
firms simply yields the standard result of a higher price.
This result suggests that raising rival's costs (or fore-
closure) can be effected by by either an increase in the
marginal cost or by an increase in the Lerner index -- even
if firms have the same Cournot/Nash behavior.

[2] This result is a generalization of the Salinger and Salop
tests simply allowing for the fact that when downstream firms
are not price takers in the upstream market, their marginal
factor costs will be higher than the input spot price.

post-merger equilibrium may produce lower factor prices but a higher marginal factor cost for the unintegrated firms.

The interaction between vertical integration and overbuying is crucial to the analysis. Integrated downstream firms will generally buy more when they take into account how their input purchases will affect the costs of their rivals. But such overbuying will generally only move the oligopsonist level of purchases toward the competitive level, thereby increasing welfare. However, when a firm vertically integrates it stops acting as an oligopsonist with respect to its own production of the input. Vertical integration thereby creates a wedge between the marginal factor costs of the integrated and unintegrated firms buying in the unintegrated upstream market. For example, when the integrated firm buys its first unit from the unintegrated upstream producers, it must pay a slightly higher price for that unit (while the costs of its internally produced inputs remain unaffected), but it forces its unintegrated rivals to pay a slightly higher price for all their inputs. This asymmetry assures fulfillment of the Salop and Scheffman overbuying condition [1986], which requires "the vertical shift in the predator's residual demand curve [to] exceed the vertical shift in the average cost curve at the non-strategic equi-

librium."[3] Since external input purchases do not affect the cost of the internally produced inputs, an integrated firm can use external input purchases to shift its residual demand curve with only a small effect on average cost. Vertical integration can thus create an incentive for merged firms to overbuy in the upstream market, inducing both productive and allocative inefficiencies.

Modeling the specific form of strategic competition after a vertical merger, however, poses an initial problem. The analysis of Salop and Scheffman suggests that the integrated firm acts "strategically" when it chooses its external input purchases by taking account of how these purchases will affect its rivals' costs. Formally, the integrated firm makes a consistent conjecture of its unintegrated rival's reaction function with regard to the size of its external input purchases [Salop and Scheffman (1986); Salinger (1987)].

While conjectural variations models are usually subject to the criticism that they are not susceptible to formal extensive-form representation, Salop's strategic model does have an isomorphic extensive-form counterpart. Because only

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[3] Williamson made a similar average cost point in his seminal article [Williamson (1968)].

the integrated firm is modeled to make a consistent conjecture about its rival's reactions, its behavior can be identically interpreted as a first-mover advantage. Specifically, the Salopian strategic game yields the same results as a two period game in which the integrated firm moves first in choosing its <u>external</u> input purchases.

For example, consider a simple duopoly which uses a single input, A, produced in a competitive upstream market. If firm one buys some of the upstream capacity, after the merger it will have to choose how much of its inputs to produce internally, A_1^{int}, and both firms one and two will have to decide how much inputs, A_1^{ext} and A_2^{ext} respectively, to buy externally from the unintegrated upstream producers. Under these assumptions, Salop's conjectural model can be represented by the following extensive-form game: In the first period, firm one would commit itself to a certain A_1^{ext}, and in the second period firm one would choose A_1^{int} and firm two would simultaneously choose A_2^{ext}. In the second period of this game, then, firm two would chose its best A_2^{ext} given a fixed A_1^{ext}. Solving backward, firm one would then chose A_1^{ext} from the most profitable point along firm two's reaction function. Choosing this most profitable point is just the same as making a consistent conjecture with

regard to how its external purchases will effect firm two's actions.

But having given a formal game-theoretic interpretation of the strategic model only shifts the grounds for criticism. Reconceiving the model in first-mover terms, one might now question under what circumstances vertical integration would give firm one the first move in the external input market after merger. The extensive form representation, however, also leads us to consider four alternative post-merger games:

1. No First-Mover Strategic
   No First period
   Second period: firm 1 chooses A_1^{int} and A_1^{ext} and
              firm 2 chooses A_2^{ext} simultaneously.

2. Salop (External) Strategic
   First period: firm 1 chooses A_1^{ext}
   Second period: firm 1 chooses A_1^{int} and
              firm 2 chooses A_2^{ext} simultaneously.

3. Internal Strategic
   First period: firm 1 chooses A_1^{int}
   Second period: firm 1 chooses A_1^{ext} and
              firm 2 chooses A_2^{ext} simultaneously.

4. Stackelberg Strategic
   First period: firm 1 chooses A_1^{int} and A_1^{ext};
   Second period: firm 2 chooses A_2^{ext}.

While each of these strategic games generates overbuying, modelling makes a difference -- especially regarding the affect of vertical integration on welfare. Even the No First-Mover Strategic model generates overbuying in the sense that the integrated firm uses a greater proportion of

upstream inputs after integration by buying from the non-integrated upstream producers. And while in this "non-strategic" model the integrated firm does not act strategically with respect to its input supply decisions (and hence does not gain an input first-mover advantage), its decision to merge, as discussed above, is itself a cost-raising strategic behavior which forecloses its rival.[4]

The extensive form representation of the Salop strategic model -- implying first-mover advantage regarding firm one's external purchases -- naturally suggests an alternative extensive form strategic model in which firm one can commit to its internal production of the input before the external market purchases are made. Finally, for completeness, we could consider the possibility that after a merger firm one would be able to make both its internal and external input decisions before firm two got on board. By choosing both of its inputs, firm one would effectively be choosing its output and thus be acting as a Stackelberg output leader. The Stackelberg strategic model shares the common weakness of the Salop strategic model that there may exist few situations in which vertical integration should result in giving firm one a

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[4] One could also interpret firm one's opportunity to integrate as a first mover advantage.

first mover advantage in the external input market that it did not enjoy before.[5]

This criticism does not apply, however, to the Internal Strategic model and it may in fact be more reasonable to assume that firm one can precommit to _internal_ input production before the external purchasing game is played.

The first and second sections of the chapter analyze quantity and price games with no strategic first-mover effects in which vertical integration engenders overbuying in the unintegrated upstream market. In section three the foreclosure and welfare effects of both these games are examined. Section four then extends the analysis to external and internal "strategic" extensive-form games. As before, vertical integration insures overbuying and can cause welfare to decrease. The fifth section concludes.

Section 1: The Quantity Game

To investigate the interaction of vertical merger and overbuying in a simple analytic setting, consider a competi-

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[5]   Salop and Stackelberg Strategic models would be more compelling if firm one enjoyed a first-mover advantage _before_ the merger as well. However, in the current bare-bones model such an asymmetry cannot derive from the structure.

tive upstream industry producing a good, A, which is the sole input for a downstream duopoly[6] playing a Cournot game. Specifically, before the merger let total industry cost be given by the quadratic function:

$$C(A) = A^2/2k$$

Competition among the upstream firms would drive the input price to equal the marginal cost of production:

$$\alpha = A/k, \qquad \text{where } \alpha \text{ is the input price,}$$

so that the supply curve of the upstream market before the merger is:

$$A^s = \alpha k.$$

Demand for the downstream product, Q, is linear, so that the inverse industry demand curve equals:

$$P = a - bQ$$

where $Q = q_1 + q_2$ and q_i is output of ith firm.

Units of the upstream and downstream goods are chosen so that the production function for each downstream firm is simply:

$$q_i = f(A_i) = A_i \quad \text{for } i = 1, 2.$$

In this case, the pre-merger downstream reaction (best response) functions are:

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[6] The assumption of a single input in production is identical to an assumption of a fixed proportions production function with constant factor prices for other inputs.

$$q_i = \frac{ak - q_j(bk + 1)}{2(bk+1)}, \quad \text{for } i \neq j;$$

and the resulting duopoly equilibrium (denoted by d super-scripts) is:

$$q^d = \frac{ak}{3(bk + 1)}$$

$$\alpha^d = \frac{2a}{3(bk + 1)}$$

$$P^d = \frac{a(bk + 3)}{3(bk + 1)}$$

Next consider a merger in which one of the downstream duopolists buys half of the upstream capacity. The variable k in the quadratic cost function can be interpreted as total capacity [Donsimoni (1985); Perry and Porter (1985)], so that buying half of capacity would leave both the internal and external cost of production (denoted by "int" and "ext" respectively) as:

$$C(A^i) = \frac{(A^i)^2}{2(\frac{k}{2})} = \frac{(A^i)^2}{k}, \quad \text{for } i = \text{int, ext.}$$

The competitive external market price for the input will now be determined by:

$$\alpha = \frac{2A^{ext}}{k}.$$

After the merger, firm one will have to choose how much of the input to produce internally and how much to buy on the external market. Its post-merger production function will accordingly be:

$$(1) \qquad q_1 = A_1^{int} + A_1^{ext}.[7]$$

If firm one chooses to buy a positive amount on the external market, the upstream price will equal:

$$\alpha = \frac{2(A_1^{ext} + A_2^{ext})}{k} = \frac{2(A_1^{ext} + q_2)}{k}$$

The profit functions of the integrated (firm one) and unintegrated (firm 2) firms can then be written:

$$(2) \quad \pi_1 = Pq_1 - ((A_1^{int})^2/k) - 2A_1^{ext}(A_1^{ext} + q_2)/k,$$

$$(3) \quad \pi_2 = Pq_2 - 2q_1(A_1^{ext} + q_2)/k.$$

These profit equations can be simplified by substituting for q_1 from equation (1). Then differentiating π_1 with respect to A_1^{int} and A_1^{ext}, and π_2 with respect to q_2 yields three first order conditions and three unknowns:

$$(4) \quad ak - 2(bk+1)A_1^{int} - 2bkA_1^{ext} - bkq_2 = 0$$

$$(5) \quad ak - 2kA_1^{int} - 2(bk+2)A_1^{ext} - (bk+2)q_2 = 0$$

$$(6) \quad ak - bkA_1^{int} - (bk+2)A_1^{ext} - 2(bk+2)q_2 = 0$$

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[7] The Kuhn-Tucker conditions for this profit maximization problem indicate that firm one would never purchase inputs which it did not use. Thus, the expression in equation (1) will always hold with a strict equality. Throwing away inputs (or equivalently throwing away some of the outputs after using the inputs) intuitively is not a profit maximizing strategy for the following reasons: (1) Firm one would not throw away externally purchased inputs as long as it was still producing a positive quantity of inputs internally (because it would prefer to reduce its internal production); (2) but if firm one is not producing internally (i.e. is buying all its inputs from the external market), it faces the same input supply conditions as its rival and hence can obtain no advantage from overbuying.

Solving simultaneous yields the non-strategic equilibrium:

$$A_1^{int} = \frac{ak}{3bk + 2} \qquad A_1^{ext} = \frac{2ak}{3(bk+2)(3bk+2)}$$

$$q_1 = \frac{ak(3bk+8)}{3(bk+2)(3bk+2)} \qquad q_2 = \frac{ak}{3(bk+2)}$$

$$\alpha = \frac{2a(3bk+4)}{3(bk+2)(3bk+2)} \qquad P = \frac{a(3bk + 14bk + 12)}{3(bk+2)(3bk+2)}.$$

In this equilibrium, we find that even though firm one has purchased half of the upstream capacity, it still chooses to buy positive amounts from the external market ($A_1^{ext} > 0$). Not only is firm one's downstream output greater than firm two's ($q_1 > q_2$), but firm one's internal production of the input is larger than firm two's external input demands ($A_1^{int} > q_2$). Thus firm one produces more as a result of a larger internal production of the input as well as from the fact that it buys additional inputs on the external market. Finally, we find that the total amount produced internally is greater than the total amount produced in the external market ($A_1^{int} > A_1^{ext} + q_2$). This implies that there is a production inefficiency in that the same total input supply could be produced more cheaply (if the equal capacities produced equal amounts).

Comparing these results with the unintegrated duopoly equilibrium we find the surprising result that the merger causes the input price to drop:

$$\alpha^d = \frac{2a}{3(bk + 1)} \qquad > \qquad \alpha^m = \frac{2a(3bk+4)}{3(bk+2)(3bk+2)}$$

(where "m" superscripts denote the nonstrategic equilibrium values). The result is especially remarkable given that the integrated firm is buying inputs in the external market to raise the input price. Most startling of all, however, is the fact that in equilibrium the unintegrated downstream firm responds to the lower input price by buying less inputs and producing less in the downstream market:

$$q^d = \frac{ak}{3(bk + 1)} \qquad > \qquad q_2^m = \frac{ak}{3(bk + 2)}.$$

Thus, in contrast to Salopian models where rivals are forced by _higher_ input prices to recede from downstream markets, the unintegrated firm in this model is induced to partially exit even though there are _lower_ input prices.

The solution to this conundrum lies in realizing that both before and after the merger, firm 2 was not acting as a price taker in the upstream market. As a duopsonist, firm 2's input decisions are based on its marginal factor cost rather than on the average factor cost of the input, α. The

perversity of this equilibrium can then be understood by analyzing how the vertical merger has changed the marginal factor costs of both the integrated and unintegrated firms. The merger by itself has significantly changed firm 2's production incentives. For example, after the merger even if firm 1 produces the same amount using only its internal capacity ($q^d = A_1^{int}$), firm 2 would still have incentives to decrease its production below q^d as its marginal factor costs would have increased:

$$\text{pre-merger MFC} = 3q^d/k \quad < \quad \text{post-merger MFC} = 4q^d/k.$$

The change in firm two's marginal factor costs as a result of the merger are depicted in Figure 1. MFC_2' and α' represent the pre-merger marginal factor costs and input prices for different levels of firm two's input purchases (fixing firm one's input purchases constant at the duopoly level). The pre-merger input and marginal factor costs are accordingly labelled as α^d and MFC_2^d -- the levels associated with $q_2 = q^d$. MFC_2'' and α'' represent firm two's post-merger marginal factor costs and input prices assuming that firm one does not buy any inputs from the external market ($A_1^{ext} = 0$). Notice that $\alpha' = \alpha''$ at $q_2 = q^d$ because demanding half as much from an external market with half as much capacity will induce the same price. Finally, MFC_2''' and

Figure 1
2's marginal and average input price
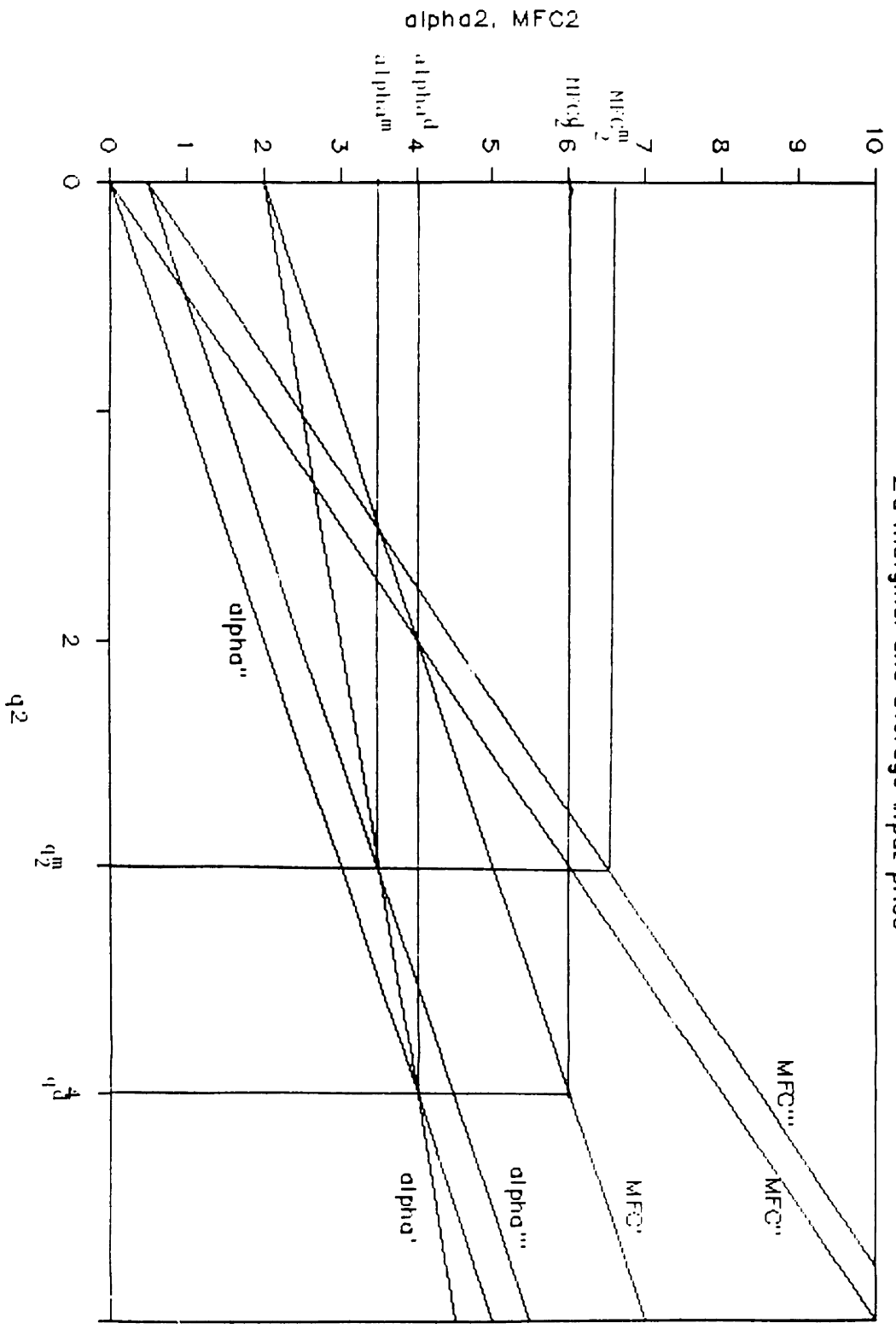
α''' depict firm two's marginal and average factor costs given firm one's equilibrium demand in the external input market $(A_1^{ext} = A_1^{ext-m})$. The non-strategic equilibrium is then illustrated by q_2^m at which the input price has fallen to α^m but at which firm two's marginal factor cost has increased to MFC_2^m.

The fact that the merger increases firm two's marginal factor cost is sufficient to cause firm two to reduce its downstream production. This implies that the act of merger itself can be thought of as "strategic" cost-raising behavior -- where, in the duopsonist context, marginal, not average, factor cost is the relevant cost concept.

In Figure 2, the reaction (best response) functions for firms one and two are shown for both the pre- and post-merger games.[8] As discussed above, firm two's reaction function shifts in because it faces higher marginal (factor) costs. This foreclosure effect is exacerbated by an outward shift in firm one's reaction (best response) function. A change in

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[8] The reactions are reduced to two-space for the post-merger three-space game $(A_1^{int}, A_2^{ext}, q_2)$ by holding A_1^{int} constant at the post-merger equilibrium value:

$$A_1^{int-m} = \frac{ak}{3bk + 2.}$$

Reactions with different slopes, but of course intersecting at the same equilibrium point could alternatively be derived holding A_1^{ext} or q_1 at their postmerger equilibrium values.

Figure 2
Pre-, Post-Merger Reaction Functions
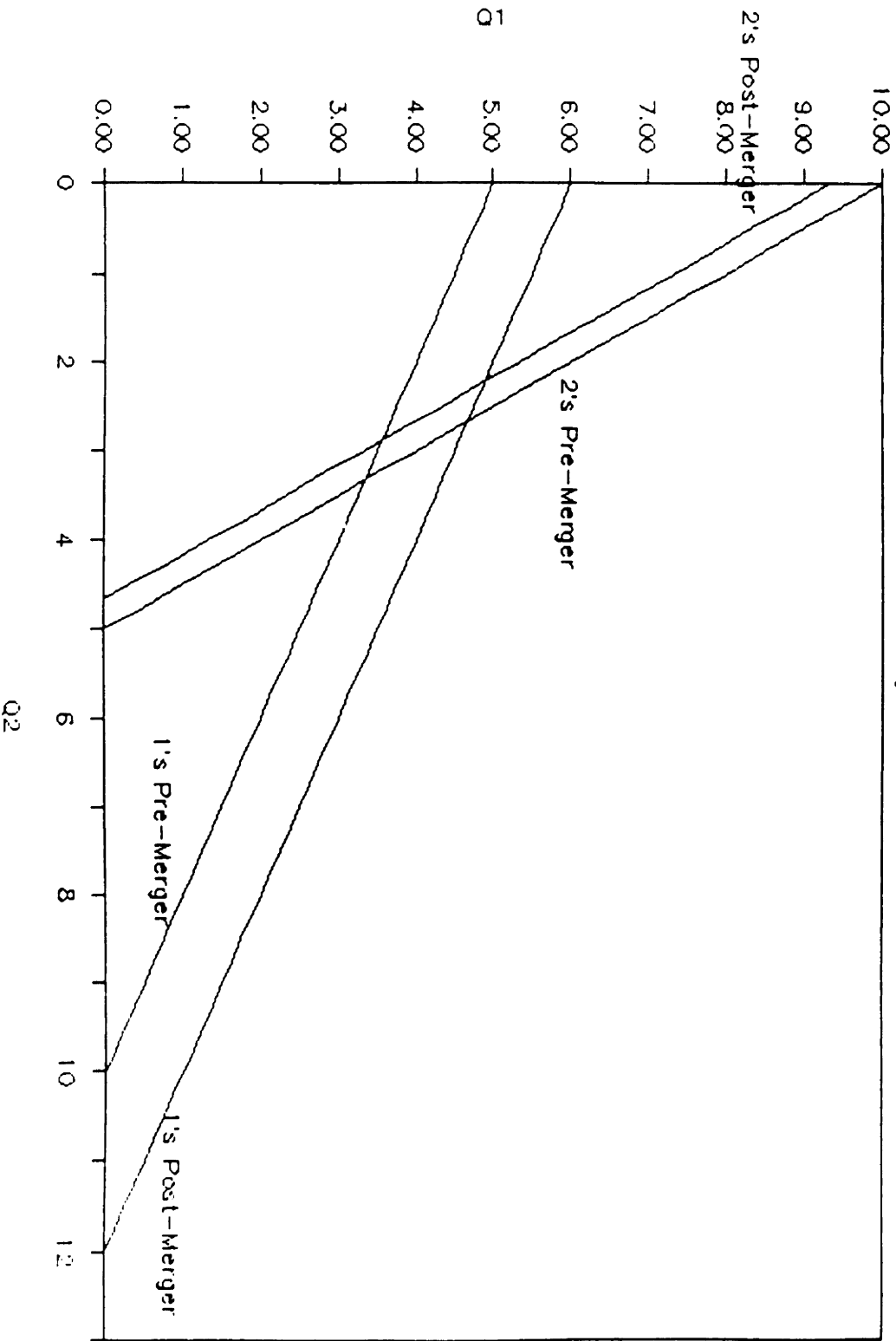
marginal costs again drives the result. Before the merger firm one faces a marginal factor cost in equilibrium of:

$$MFC_1{}^d = 3q^d/k.$$

After the merger, firm one has an incentive to grow both internally and externally beyond $A_1{}^{int} = q^d$ and $A_1{}^{ext} = 0$. At these output levels, its internal marginal cost and external costs are only:

$$MC_1 = MFC_1 = 2q^d/k.$$

After the merger firm one no longer acts as a duopsonist with respect to its own production of the input, and faces lower external marginal factor costs as well; it thus has an incentive to expand both internally and externally. Firm one's reaction function accordingly shifts out because for any value of q^2 firm one would prefer to produce more from both internal and external input sources. As one would expect, in equilibrium firm one minimizes its costs by choosing internal and external sources of supply such that the marginal cost of producing the input internally equals the marginal factor cost of buying the input externally:

$$MC_1{}^{int} = MFC_1{}^{ext} = \frac{2a}{3bk + 2}.$$

The foreclosure effect on firm two of higher marginal factor costs is therefore reinforced by firm one's cheaper internal and external sources of supply.

In Figure 2, the two less-expensive sources of supply cause firm one's reaction curve to shift out by more than firm two's reaction curve shifts in. Thus, firm one's increase in output more than crowds out firm two's reduction, causing total industry output to increase and the downstream price to drop. In equilibrium, firm two is smaller because its marginal factor cost has increased, even though its input price has fallen:

$$MFC_2{}^m = \frac{4a(bk + 1)}{(3bk+2)(bk+2)} > MFC_2{}^d = \frac{a}{bk + 1}.$$

Finally, the merger increases the profits of the merged firms. The premerger duopoly profit of each downstream firm equals:

$$\pi^d = a^2 k/9(bk+1),$$

and for the upstream industry equals:

$$\pi^{up} = 2a^2 k/9(bk+1)^2;$$

while the post-merger profits for the integrated firm are:

$$\pi_1 = \frac{a^2 k(9b^3 k^3 + 57b^2 k^2 + 100bk + 44)}{9(bk+2)^2(3bk+2)^2}.$$

Since $\pi_1 > \pi^d + \pi^{up}/2$, firm one and owners of half the upstream capacity with have a mutual incentive to come to an agreement. This is especially the case since (unlike the raising average factor cost models) there will not be a hold-

out problem by the owners of upstream capacity. Since the merger lowers the input price, the profits of unintegrated input producers are unambiguously lower. This paper does not, however, investigate the feasibility of counter-strategies that the unintegrated downstream rival might undertake to undo the effects of firm one's merger. Most notably both firm 2 and the unintegrated upstream producers would have incentives to merge to avoid their reduction in profits. Such counterstrategies are carefully analyzed in the recent work of Ordover, Saloner and Salop [1987]. For the purpose of this analysis, further vertical integration is assumed to be impossible -- either because the remaining upstream capacity is so unconcentrated that transaction costs prevent subsequent mergers or because antitrust restrictions prevent mergers that would create duopoly in the upstream market. Moreover, even if the counter-strategy of a subsequent merger were feasible, the model would still be useful as a predictor of industry movement in this area. This is an obvious area for further research but it is outside the scope of the present paper.

Section 2: The Price Game

In this section a similar vertical integration model is

analyzed in which downstream producers of differentiated products choose price instead of quantity as their decision variable. Specifically, I assume that the demands for the two downstream goods will now equal:

$$q_1 = 1 - P_1 + \gamma P_2$$
$$q_2 = 1 - P_2 + \gamma P_1,$$

where $0 < \gamma < 1$. Firms one and two thus face a symmetric cross price effect which, for stability,[9] is less than the own price effects [see Ordover, Saloner, Salop (1987)]. Assuming the same input cost and supply conditions, the Bertrand premerger reaction function equals:

$$P_i = \frac{(3 - \gamma + k) + P_j(-1 - \gamma^2 + 2\gamma + \gamma k)}{2(1 - \gamma + k)} \qquad \text{for } i \neq j,$$

and the resulting equilibrium is:

$$P^b = \frac{3 - \gamma + k}{(1 - \gamma)(3 - \gamma + k) + k}$$

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[9] For the Bertrand equilibrium to be stable, firm two's reaction function, S^2, must be larger than the slope of firm one's reaction function, S^1. This requires that:

$$S_1 = \frac{(-1 - \gamma^2 + 2\gamma + \gamma k)}{(2 - \gamma + k)}$$

< 1.

But $\frac{dS_1}{dk} > 0$ and

$$\lim_{k \to \infty} S_1 = \frac{\gamma}{2}$$

which implies that, for $0 < \gamma < 1$, the equilibrium is stable.

$$\alpha^b = \frac{2}{(1 - \gamma)(3 - \gamma + k) + k}$$

$$q^b = \frac{k}{(1 - \gamma)(3 - \gamma + k) + k}$$

$$\pi^b = \frac{k(1 - \gamma + k)}{((1 - \gamma)(3 - \gamma + k) + k)^2}.$$

If firm one buys half of the upstream capacity, as before, the post-merger equilibrium can be analogously analyzed. The three first order conditions for this price game will now equal:

$$A_1{}^{ext} = \frac{(1 + \gamma)(P_2 - P_1)}{3}$$

$$P_1 = \frac{2 + k - 2A_1{}^{ext}(1 + \gamma) + P_2\gamma(2 + k)}{2(1 + k)}$$

$$P_2 = \frac{4 + k + 2A_1{}^{ext} + P_1\gamma(4 + k)}{2(2 + k)}.$$

Solving for the three unknown variables yields the post-merger equilibrium:

$$P_1 = \frac{12 + 4\gamma + 20k + 12\gamma k + 3\gamma k^2 - 2\gamma^2 k - 8\gamma^2 + 6k^2}{12 - 8\gamma + 28k - 8\gamma k - 16\gamma^2 k - 3\gamma^2 k^2 + 2\gamma^3 k - 12\gamma^2 + 8\gamma^3 + 12k^2}$$

$$P_2 = \frac{12 + 4\gamma + 26k + 12\gamma k + 3\gamma k^2 - 2\gamma^2 k - 8\gamma^2 + 6k^2}{12 - 8\gamma + 28k - 8\gamma k - 16\gamma^2 k - 3\gamma^2 k^2 + 2\gamma^3 k - 12\gamma^2 + 8\gamma^3 + 12k^2}$$

$$A_1^{ext} = \frac{2(1+\gamma)k}{12-8\gamma+28k-8\gamma k-16\gamma^2 k-3\gamma^2 k^2+2\gamma^3 k-12\gamma^2+8\gamma^3+12k^2}$$

$$A_1^{int} = \frac{k(6+4\gamma+6k+3\gamma k-2\gamma^2)}{12-8\gamma+28k-8\gamma k-16\gamma^2 k-3\gamma^2 k^2+2\gamma^3 k-12\gamma^2+8\gamma^3+12k^2}$$

$$q_1 = \frac{k(8+6\gamma+6k+3\gamma k-2\gamma^2)}{12-8\gamma+28k-8\gamma k-16\gamma^2 k-3\gamma^2 k^2+2\gamma^3 k-12\gamma^2+8\gamma^3+12k^2}$$

$$q_2 = \frac{k(2+6k+3\gamma k-2\gamma^2)}{12-8\gamma+28k-8\gamma k-16\gamma^2 k-3\gamma^2 k^2+2\gamma^3 k-12\gamma^2+8\gamma^3+12k^2}$$

$$\alpha = \frac{2(4+2\gamma+6k+3\gamma k-2\gamma^2)}{12-8\gamma+28k-8\gamma k-16\gamma^2 k-3\gamma^2 k^2+2\gamma^3 k-12\gamma^2+8\gamma^3+12k^2}$$

At the post-merger pricing equilibrium, there is a fore-closure effect that is directly analogous to that of the quantity game. Whereas in the quantity game firm two's (one's) output was reduced (increased), now in the Bertrand equilibrium firm two's downstream price is increased by the merger and firm one's downstream price is reduced relative to the pre-merger duopoly price. The upstream price, α, unambiguously declines and for a broad range of γ and k, the vertical merger will increase the joint profits of the merging firms.

The reaction functions, depicted in Figure 3,[10] can be given a similar interpretation. Reformulating the game with

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[10] As discussed above, the Bertrand best response functions can be drawn in two space by holding A_1^{ext} fixed at its equilibrium level.

Figure 3
Bertrand Reaction Functions
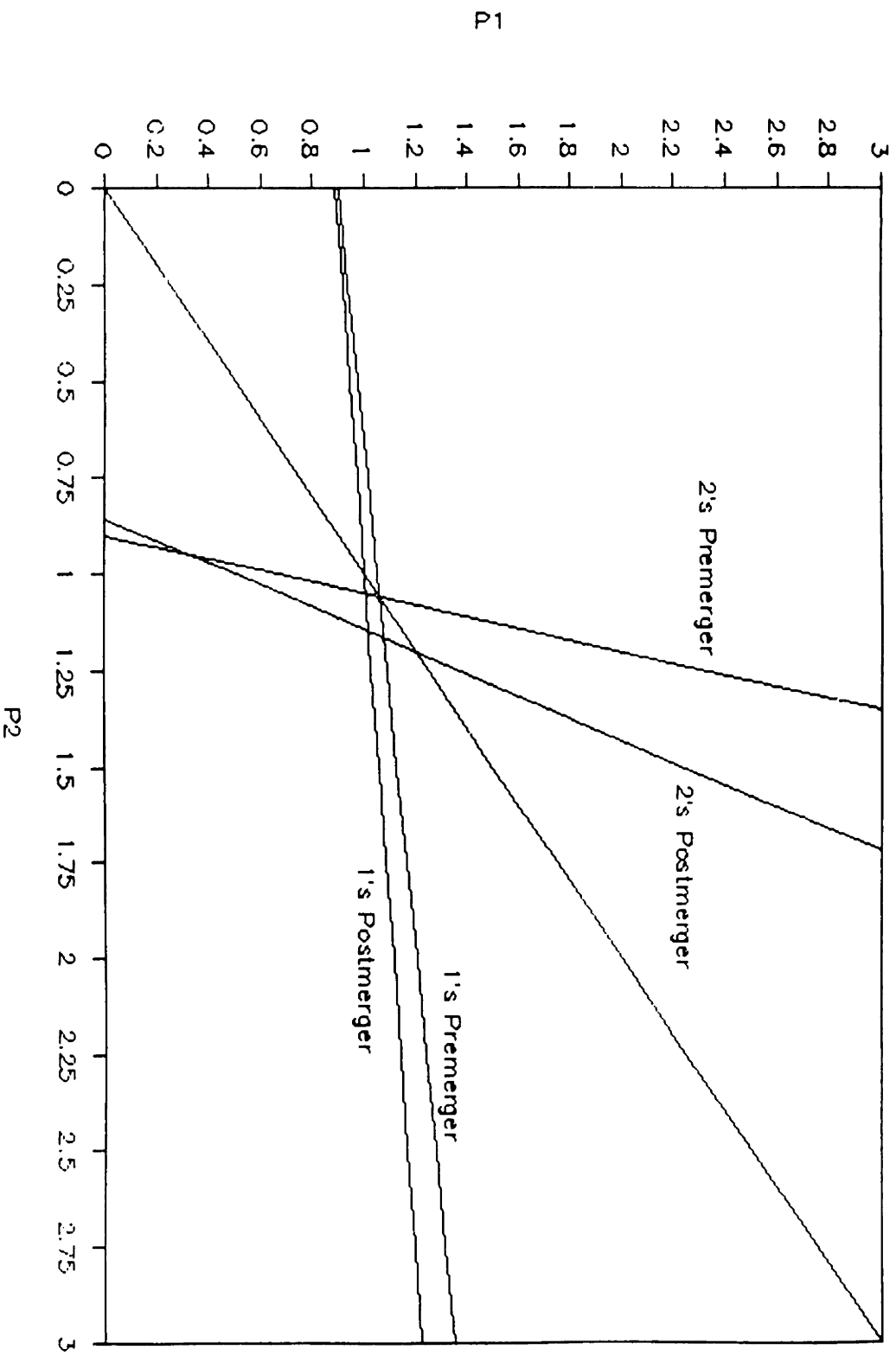
price instead of quantity as the strategic variable does not affect the cost conditions facing the firms; thus, a merger will induce shifts in the reaction curves (through the same effects on input costs). Starting again from the premerger equilibrium prices (with $A_1^{ext} = 0$), the merger will raise firm two's marginal factor costs and will lower firm one's marginal input costs (from both its internal and external sources of supply). Thus, as shown in Figure 3, firm one's postmerger reaction curve must lie below its premerger reaction curve at the pre-merger equilibrium. Similarly, firm two, facing higher marginal factor costs, must have a reaction curve that lies to the right of its premerger reaction curve at the premerger Bertrand equilibrium.

Section 3: Foreclosure and Welfare Effects

Foreclosure. In both the price and quantity games, a vertical merger makes it easier to exclude rivals. Foreclosure is achieved through the post-merger change in marginal factor costs. By manipulating the equilibrium values it is straightforward to show that for both the Cournot and Bertrand games that:

(7) $$q_1 = q_2 + 3A_1^{ext}.$$

Heuristically, this equilibrium relationship can be compared to a post-merger baseline of no overbuying and no foreclosure in which firm one produces as much inputs for itself as firm two buys on the spot market (i.e., $A_1{}^{int} = q_2 = q_1$). At the overbuying equilibrium, firm two has reduced its output by two units for every extra unit of input that firm one buys externally (beyond the no overbuying baseline). Thus, the difference between the equilibrium downstream production is equal to three times the amount that firm one purchases externally. Both the quantity and price games exhibit at equilibrium a two-for-one foreclosure rate which illustrates how vertical integration increases a merged firm's ability to exclude.

But using the post-merger no-foreclosure equilibrium ($A_1{}^{int} = q_2 = q_1$) as a baseline to measure foreclosure, ignores the fact that the merger itself, even without over-buying, engenders foreclosure. Even if firm one could not purchase inputs externally after the merger, it would produce more than firm two ($q_1 > q_2$). To capture this foreclosure effect, it is possible to recalculate the post-merger equi-librium constraining firm one's external input purchases to a given level. Using the first order conditions for the quantity game in equations (4), (5), (6), one can then calcu-

late the equilibrium values of firm one and firm two's downstream production as a function of any given external purchases:

$$q_1 = \frac{ak(bk + 4) + 2(3bk + 4)A_1^{ext}}{3b^2k^2 + 12bk + 8} \tag{7.1}$$

$$q_2 = \frac{ak(bk + 2) - 2(3bk + 2)A_1^{ext}}{3b^2k^2 + 12bk + 8} \tag{7.2}$$

In Figure 4, these functions are graphed. Even without over-buying ($A_1^{ext} = 0$), there is foreclosure as firm one's output is larger than firm two's. Overbuying creates a second source of foreclosure as the production gap widens with increasing external input purchases by firm one. These merger and over-buying foreclosure effects can be disaggregated. The over-buying foreclosure rate, F^o, is the relative effect on downstream outputs of firm one buying an extra unit of input. It can be measured by taking the ratio of derivatives of equations (7.1) and (7.2):

$$F^o = \frac{- (dq_2/dA_1^{ext})}{(dq_1/dA_1^{ext})} .[11]$$

The foreclosure effect of the merger, F^m, can be measured by comparing the equilibrium outputs of equations (7.1) and

----

[11] The denominator is not simply equal to one, because firm one's equilibrium internal input production will decrease for a given level of external input purchases.

Figure 4
Post-Merger Equilibrium Alext = .1

Output

Alext

Q1

QD

Q2
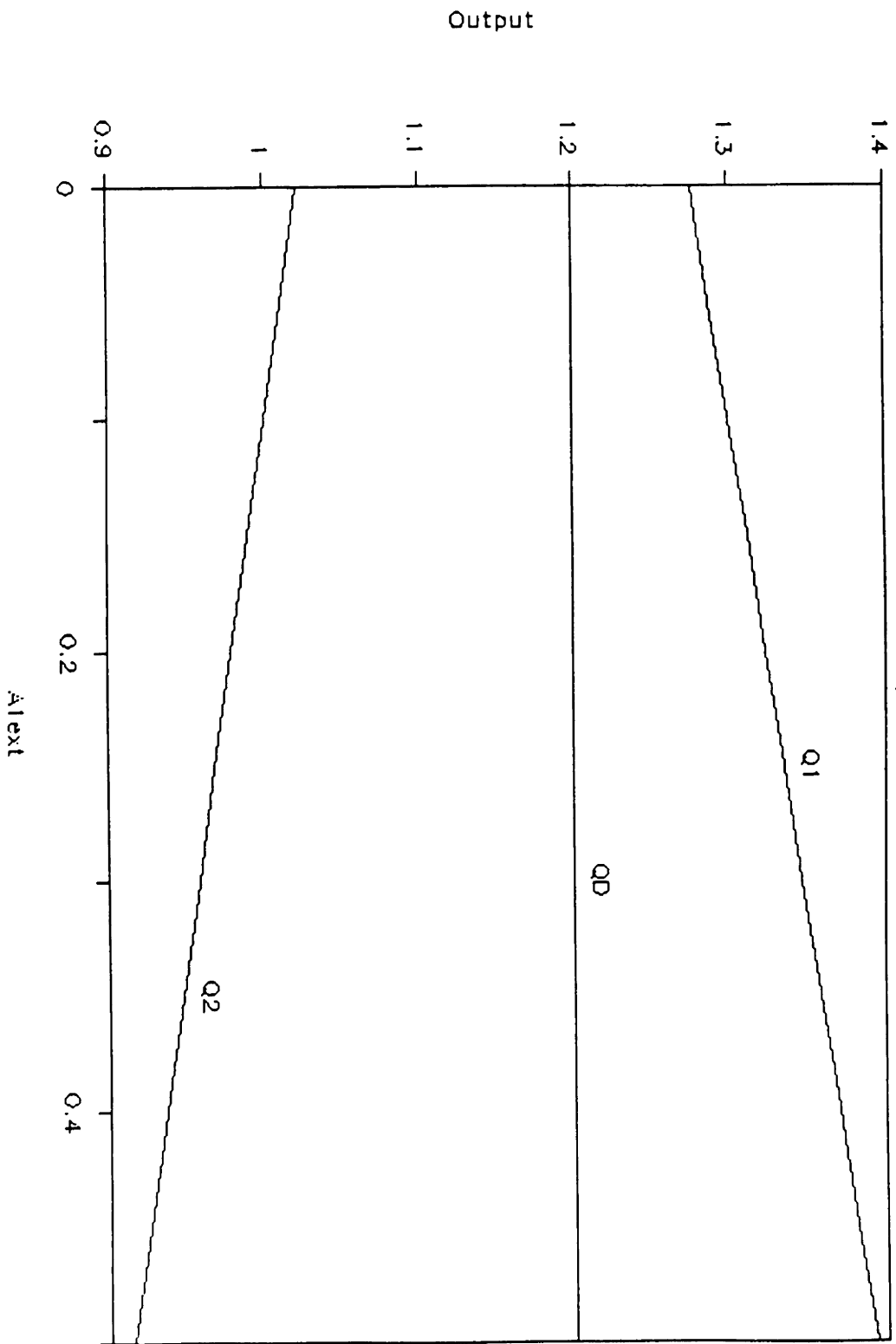
(7.2) when A_1^{ext} is fixed at zero with the pre-merger equilibrium output, q^d:

$$F^m = \frac{(q^d - q_2(A_1^{ext}=0))}{(q_1(A_1^{ext}=0) - q^d)}.$$

For the linear quantity game, both foreclosure rates equal:

$$F^o = F^m = \frac{(3bk + 2)}{(3bk + 4)}.$$

Thus, the total foreclosure effect, F^n, of both merger and overbuying is:

$$F^n = \frac{(q^d - q_2)}{(q_1 - q^d)} = \frac{(3bk + 2)}{(3bk + 4)}.$$

This total foreclosure rate will always be less than one but greater than one-half. The increase of firm one's downstream production is therefore less than the decrease of firm two's production, but the merger has made it easier to foreclose. In the pre-merger equilibrium, increased input purchases by firm one only would cause firm two to reduce its output by half as much [as seen in the best response function of equation (1a)]; in the post-merger equilibrium, however, the rate of foreclosure, F^n, is strictly larger.

Welfare Effects. As Ordover and Saloner [1987] have stressed, however, not all behavior which puts rivals at a competitive disadvantage should give rise to antitrust con-

cern. This <u>caveat</u> has been equally reflected in the long line of antitrust case law which has reaffirmed Chief Justice Warren's holding that the antitrust laws were enacted for "the protection of competition, not competitors" [Brown Shoe Co. v. United States (1962)]. Thus, cost-raising strategies of vertical integration should ultimately be judged by their effect on social welfare.

In the simple linear model analyzed here, the welfare effects of a vertical merger may be disaggregated into two components: affecting production and allocative efficiency, respectively. In both the price and quantity models, firm one produces more goods internally than the external market produces:

$$A_1^{int} > A_1^{ext} + q_2.$$

Because both integrated and unintegrated input producers have equal amounts of capacity, producing different amounts internally and externally creates an inefficiency in production. This inefficiency is the loss, L^p, attributable to the extra costs of producing an equal amount of inputs before the merger:

$$L^p = \frac{(A_1^{int})^2}{k} + \frac{(q_2 + A_1^{ext})^2}{k} - \frac{(A_1^{int} + q_2 + A_1^{ext})^2}{2k},$$

which simplifies to:

$$L^p = \frac{(A_1^{int} - (q_2 + A_1^{ext}))^2}{2k.}$$

But from equation (7) we know that:

$$q_1 = q_2 + 3A_1^{ext}$$

$$A_1^{int} = q_2 + 2A_1^{ext}$$

$$A_1^{ext} = A_1^{int} - q_2 - A_1^{ext}$$

so that the productive inefficiency may be expressed as:

$$L^p = \frac{(A_1^{ext})^2}{2k.}$$

The welfare loss due to inefficient production is thus directly a function of the amount of firm one's overbuying -- its external input purchases.

Vertical integration also affects welfare by changing the downstream price or prices. In the quantity game an implication of the two-for-one foreclosure is that firm one's output increases more than firm two's output contracts, thus lowering the downstream price. Consumers in the quantity game thus unambiguously gain and allocative efficiency is improved.

Aggregating the allocative and production effects together, one finds for the quantity game that the premerger welfare equals:

$$W^d = \frac{4a^2k}{9(bk + 1),}$$

and that the postmerger welfare is:

$$W^m = \frac{2a^2k(15b^2k^2 + 42bk + 17)}{9(3bk + 2)^2(bk + 2).}$$

The net change in welfare then becomes

$$W^d - W^m = \frac{- 2a^2k(3bk + 1)}{9(3bk + 2)^2(bk + 2)(bk + 1).} < 0$$

Because W^m is less than W^d, in this simple linear Cournot model at least, the downstream price effect dominates -- meaning that the merger on net increases welfare.

For the price game, however, the effect on welfare is not so straightforward. The two-for-one foreclosure now leads firm two to increase its price by more than firm one reduces its price, given stable (i.e., upward-sloping) reaction functions. A vertical merger in the pricing game can induce allocative inefficiencies both because the average downstream price increases and because downstream price dispersion, ceteris paribus, lowers welfare.

The latter point can be seen by analyzing the combined area under the two firm's demand curves:

$$AREA = \int_0^{q_1^B} (1 - q_1 + \gamma P_2^B)\, dq_1 + \int_0^{q_2^B} (1 - q_2 + \gamma P_1^B)\, dq_2$$

$$= (1+\gamma P_2^B)q_1^B - (q_1^B)^2/2 + (1+\gamma P_1^B)q_2^B - (q_2^B)^2/2$$

$$= [(1+\gamma P_1^B)^2 - (P_1^B)^2 + (1+\gamma P_2^B)^2 - (P_2^B)^2]/2.$$

Then for any given fixed average downstream price $(P_1^B + P_2^B = X)$, we know (by substituting for P_2^B and then taking the derivative with respect to P_1^B) that the area under the demand curves is maximized by setting $P_1^B = P_2^B$.

The first point -- that allocative efficiency decreases as the average downstream price increases -- stems directly from the fact that the combined areas under the demand curves generally increase as the as a common downstream price falls.[12]

Thus, when the average downstream price rises allocative inefficiency is established in two steps:

1) post-merger welfare with price dispersion is less than the same average price without price dispersion, and

2) post-merger welfare without price dispersion is less premerger welfare (without price dispersion) because the average downstream price has increased.

----

[12] For relatively large values of γ such that
$$\gamma/(1-\gamma^2) > P$$
the area under the curves increases as $P_1 = P_2 = P$ rises. But this result that welfare can increase indefinitely as prices rise is an artifact of the simple linear differentiated product demands that, while tractable, are not derived from proper constrained utility functions. Indeed for $\gamma = 1$, the linear demand curves imply paradoxically that consumers' expenditures, E (and, if the budget constraint is binding, consumers' income) increase as the price rises:
$$E = P_1 q_1 + P_1 q_1 = P_1(1 - P_1 + \gamma P_2) + P_2(1 - P_2 + \gamma P_1)$$
$$dE/dP_1 = 1 + 2(\gamma P_2 - P_1)$$
which for $\gamma = 0$ and $P_1 = P_2$ is:
$$dE/dP_1 = 1.$$

For example, then, when $\gamma = 1/2$, $k = 2$, firm one's price increases more than firm two's price decreases so that the average downstream price increases with merger,[13] and, from the foregoing argument, allocative efficiency must decrease. And since a merger induces productive inefficiencies as well, net social welfare must decline.

In sum, while the quantity and price games yield similar amounts of foreclosure and productive inefficiency, vertical integration in the linear quantity game increases welfare (as the lower downstream price outweighs the inefficient production); in the price game, however, vertical integration decreases welfare for a broad range of parameter values by increasing 1) the average downstream price, 2) the downstream price dispersion and 3) inefficient production.

Section 4: Strategic Behavior

The equilibrium changes significantly if, following Salop and Scheffman [1986], we change the model so that the merged firm takes account of how its external input purchases increase its rival's costs (and consequently decrease its

----

[13] For $\gamma = 1/2$ and $k = 2$: $P^d = 1.058$; $P_1^B = 1.016$; $P_2^B = 1.147$.

rival's downstream production). As discussed in the intro-
duction, Salop strategies are identical to an extensive form
game in which the integrated firm enjoys a first-mover
advantage with respect to its external input purchases. This
section analyzes both Salop's external strategic and the
internal strategic games.

Salop's External Strategic Game. Under the external
strategic assumption, the second first order condition,
$d\pi_1/dA_1{}^{ext} = 0$ (equation (5)), no longer obtains because firm
one now makes a consistent conjecture about how its external
input purchases effect its rival's output. Rearranging equa-
tion (6), firm two's reaction (best response) function
equals:

$$(7) \quad q_2 = \frac{ak - bkA_1{}^{int} - (bk + 2)A_1{}^{ext}}{2(bk + 2)}.$$

The derivative of q_2 with respect to $A_1{}^{ext}$ is $- 1/2$. This
can be substituted, as firm one's consistent conjecture for
$dq_2/dA_1{}^{ext}$, into a new first order equation:

$d\pi_1/dA_1{}^{ext} = P - \alpha - [(2/k)A_1{}^{ext} + bq_1](1 + dq_2/dA_1{}^{ext}) =$

$(8) \quad 2ak - 2q_2(bk + 2) - 3A_1{}^{ext}(bk + 2) - 3bkA_1{}^{int} = 0.$

Solving first order equations (4),(6) and (8), yields the new
strategic equilibrium:

$$A_1{}^{int} = \frac{ak(8-bk)}{8(3bk+2)} \qquad A_1{}^{ext} = \frac{ak(b_2k_2+4bk+8)}{8(bk+2)(3bk+2)}$$

$$q_1 = \frac{ak(5bk+12)}{4(bk+2)(3bk+2)} \qquad q_2 = \frac{ak}{4(bk+2)}$$

$$\alpha = \frac{a(b_2k_2+10bk+12)}{4(bk+2)(3bk+2)} \qquad P = \frac{a(2b_2k_2+9bk+8)}{4(bk+2)(3bk+2)}.$$

As the slope of either the demand curve or the upstream supply curve become more elastic (via increases in b or k), firm one chooses to obtain a larger proportion of its input from the external input market. Indeed for bk > 8, firm one would prefer to produce negative amounts of the input for itself -- an impossible result. At bk = 8, firm one has purchased half the capacity in the industry but refuses to use it, choosing instead to buy all of its inputs on the external market. Accordingly, if bk = 8 a merger will cause the joint profits of the merging firms to decrease. Indeed, the change in profits from changing equals:

$$\pi_1 - (\pi^d + \pi^{up}/2)$$

$$= \frac{a^2k[-9b^4k^4 + 6b^3k^3 + 247b^2k^2 + 376bk + 208]}{576(bk+2)(3bk+2)(bk+1)^2},$$

which as shown in Figure 5 is only positive for bk less than approximately 6.3. Accordingly, the following analysis is restricted to these values of bk such that there are positive profits to be earned from merging. In other instances (for higher values of bk), firms will have no incentive to merge

CHANGE IN PROFITS AFTER MERGER
(Thousands)
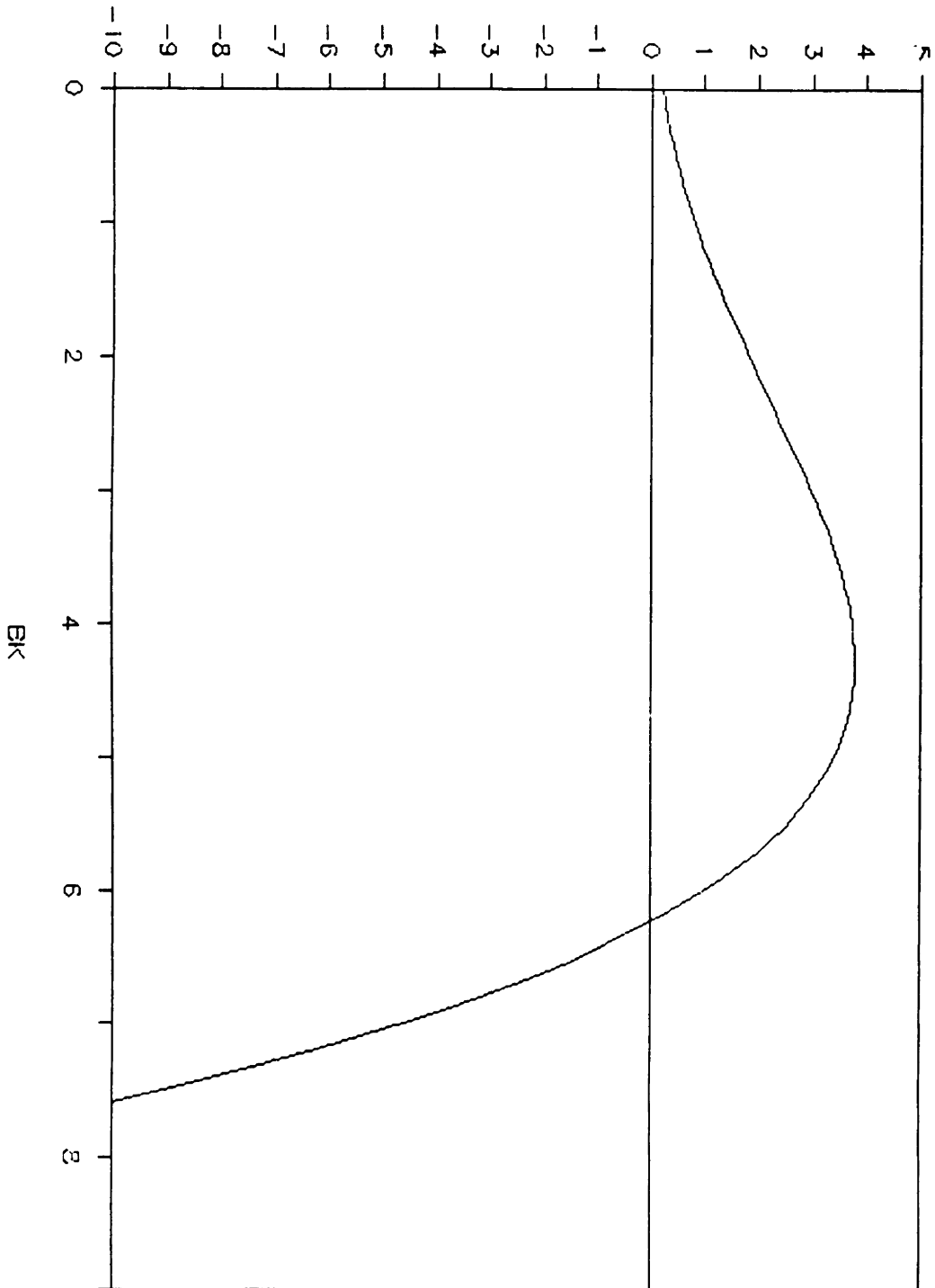
FIGURE 5
PROFITABILITY OF STRATEGIC INTEGRATION

and the unintegrated duopoly/duopsony equilibrium will remain.

For the merger equilibria of interest, however, we find that strategic behavior has significantly increased firm one's incentives to purchase inputs from the external market. Whereas in the nonstrategic equilibrium, firm one produced more input than the external producers, now firm one may produce less than the unintegrated external supply. The difference between internal and external production ($q_1 - q_2 + A_1^{ext}$) will have the same sign as:

$$b^2k^2 + 2bk - 2$$

which has as its root of interest

$$bk = (3)^{1/2} - 1$$

which is approximately .73. Thus for bk larger than .73, firm one will produce more of the input than the unintegrated input producers. This implies that the marginal costs of internal production will be lower than those of the unintegrated upstream firms, so that the strategic equilibrium can generate an analog to the nonstrategic productive inefficiency -- except that firm one now produces internally too little instead of too much.

Moreover, firm one's increased purchases in the external input market more than crowd out the premerger demand of its

unintegrated downstream rival, thus unambiguously driving up the post-merger input price:

$$\alpha^{strat} - \alpha^d = \frac{a(3b^3k^3 + 9b^2k^2 + 2bk + 4)}{12(bk+2)(3bk+2)(bk+1)} > 0$$

In the original quantity game, firm one purchased external inputs up to the point where its marginal factor cost equaled its marginal revenue product:

$$MFC_1^{ext} = MRP_1 = \frac{2a}{3bk + 2}$$

Firm one's strategic behavior, however, now causes it to raise its marginal factor cost above its marginal revenue product, as Salop and Scheffman [1986] have demonstrated in a different context:

$$MFC_1^{ext,strat} - MRP_1^{strat} = a/4 > 0$$

The intuition here is that firm one is willing to purchase external inputs whose marginal cost is greater than their marginal revenue, because by doing so it will favorably shift both itsresidual downstream demand and residual upstream supply curves. By the envelope theorem, at equilibrium the marginal increase in profits will equal the markup, $P - \alpha$, because increasing A_1^{int} causes firm one's profit rectangle to increase by this amount as it shifts to the right. Thus, firm one purchases external inputs up to the point where the

marginal loss on the input equals the increase in profits due to the shifts of the residual curves:

$$MFC_1{}^{ext} - MRP_1 = a/4 = P^{strat} - {}_\alpha{}^{strat}$$

The relevant residual demand and supply curves for the strategic equilibrium are shown in Figure 6 for $b = 2$, $k = 2$. At the equilibrium value of $A_1{}^{ext}$, firm one's marginal factor cost exceeds its marginal revenue product by the same interval as its markup.

That foreclosure in the strategic equilibrium is more severe than foreclosure in the first quantity game is also evinced by the fact that even after buying half of the upstream capacity, firm one buys more on the external market than its unintegrated rival ($A_1{}^{ext} > q_2$). Moreover, since firm two's foreclosure is again caused by its increased marginal factor costs, the pre-merger and post-merger equilibria have the relation:

$$MFC_1{}^{ext} > MFC_2 > MFC^d.$$

Indeed, as depicted in Figure 7, for some parameter values, firm one will even purchase external inputs whose marginal factor cost is greater than the downstream price, demonstrating a strong form of price predation -- but one which is instantaneously profitable.

Finally, total industry output increases with the merger (because firm one increases its downstream production by more

Figure 6
Strategic Equilibrium
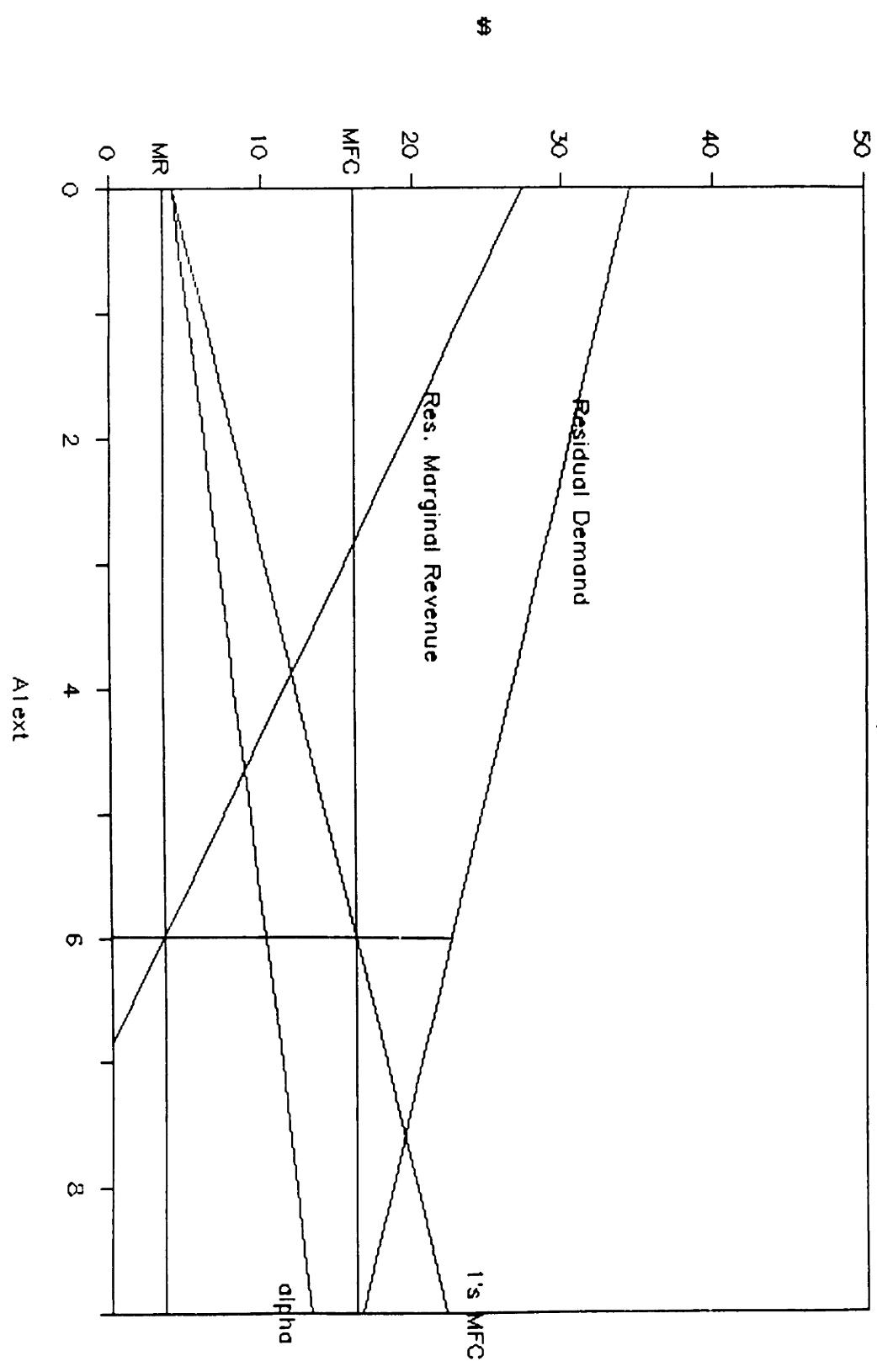
Figure 7
Strateg. Equilibrium
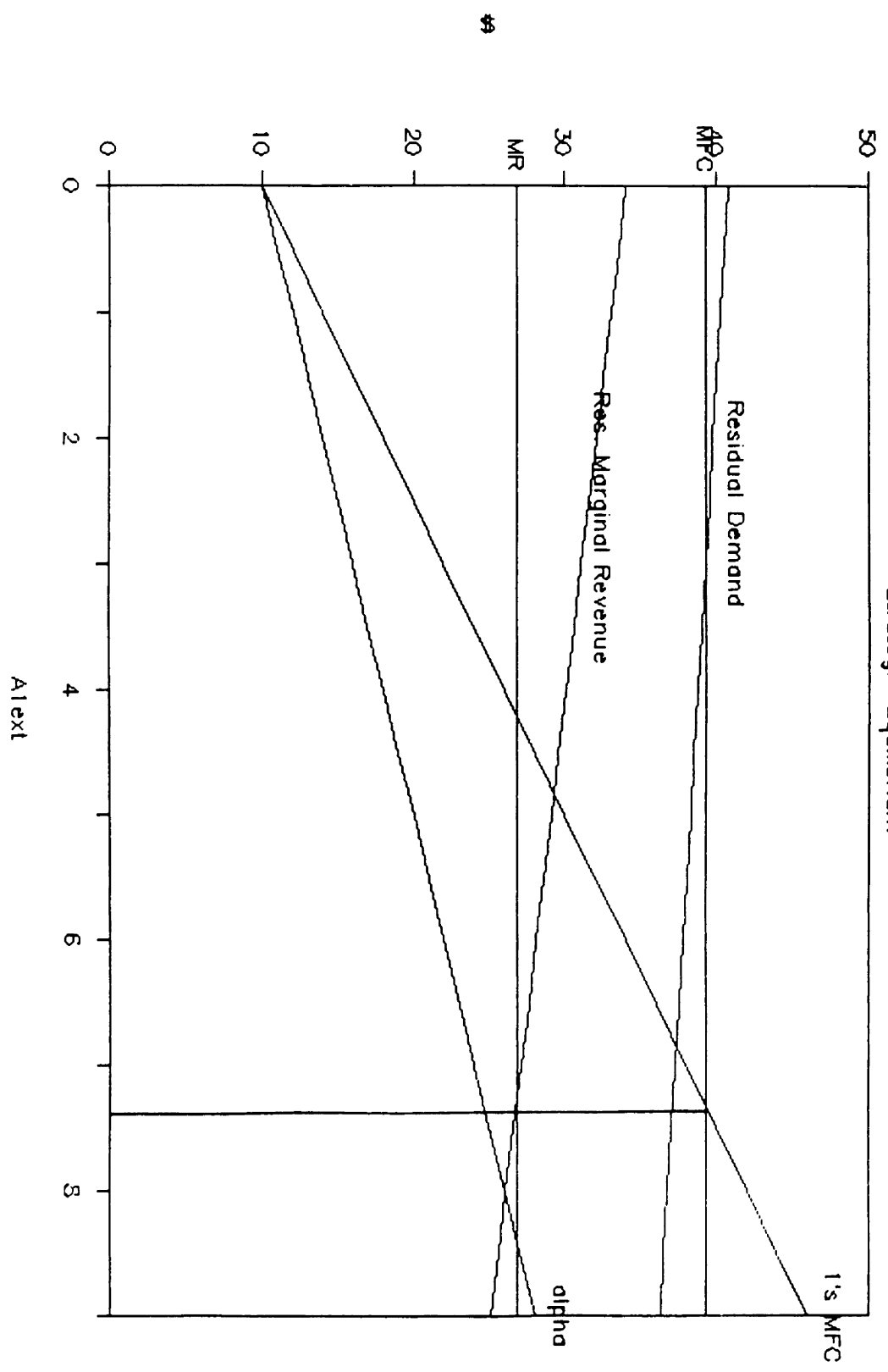
than the drop in firm two's output). Therefore, the downstream price falls and consumer welfare increases:

$$P^d - P^{strat} = \frac{abk(bk + 5)}{6(bk+2)(3bk+2)(bk+1)} > 0.$$

But as with the Section One's quantity game, the welfare-increasing allocative effect of a lower downstream price is opposed by the welfare reducing effect of production inefficiency. On net, the change in welfare from the merger equals:

$$W^d - W^{strat} = \frac{a^2 k[9b^4 k^4 + 27b^3 k^3 - 66b^2 k^2 - 304bk - 92]}{6(bk+2)(3bk+2)(bk+1)},$$

which as shown in Figure 8 is positive approximately for 3.2 < bk. Vertical merger and strategic overbuying in a quantity game, as with the nonstrategic price game, can thus lead to a reduction in welfare, even without inducing collusion among unintegrated upstream producers. Although the highly artificial assumptions of the model limit its application, this possible reduction in welfare should be of special antitrust concern because it indicates a mechanism through which competition writ large can be damaged by the interaction of vertical merger and overbuying.

The Internal Strategic Game. As indicated in the introduction, there may be situations in which it might be more

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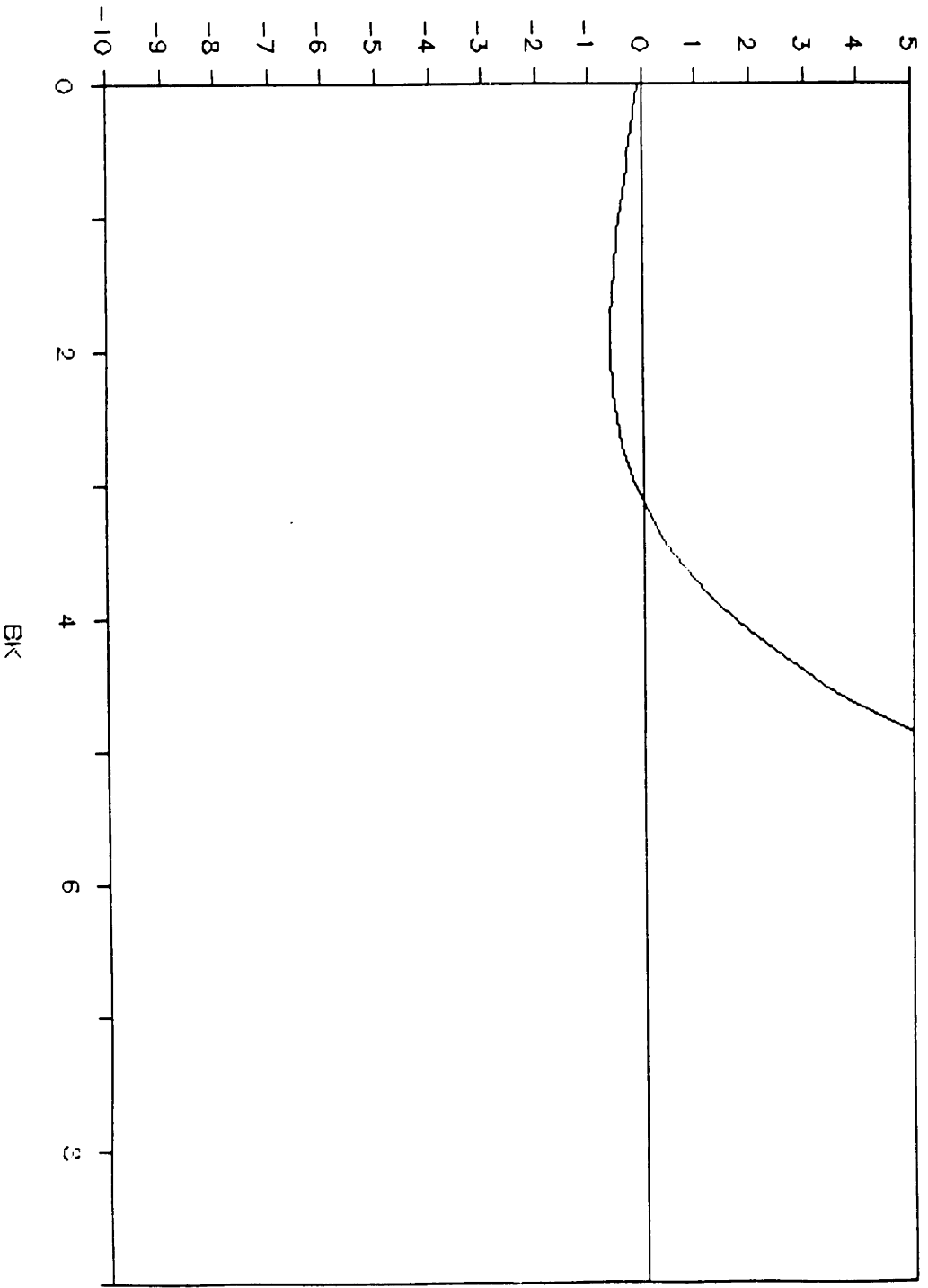
FIGURE 8
WELFARE CHANGE AFTER INTEGRATION

BK

reasonable to model a vertical merger as giving the integrated firm a first-mover advantage with respect to its internal input production rather than its external input purchases. For example, in a slightly more complicated setting, we might be able to conceive of the integrated firm being able to commit to internal input production before its rivals purchase inputs externally, but it is harder to envision how a merger would give the integrated firm an ability to precommit to its external input purchases.

The Internal Strategic model may be solved analogously to the Salop Strategic model by allowing firm one to make a consistent conjecture with respect to the effect that its first-move variable will have on firm two. Here, then firm one makes a consistent conjecture about how its internal input production effects its rivals behavior. From equation (7) firm one would conjecture that:

$$dq_2/dA_1^{int} = \frac{-bk}{2(bk + 2)}.$$

Substituting this conjecture into a new first order condition $(d\pi_1/dA_1^{int} = 0)$, we can then solve the other first order conditions $(d\pi_1/dA_1^{ext} = 0, \ d\pi_2/dq_2 = 0)$ for the Internal Strategic Equilibrium:

$$A_1^{int} = \frac{ak(12 + bk)}{12(3bk + 2)}$$

$$A_1^{ext} = \frac{ak(8 - b^2k^2)}{12(3bk + 2)(bk + 2)}$$

$$q_1 = \frac{ak(7bk + 16)}{6(3bk + 2)(bk + 2)}$$

$$q_2 = \frac{ak}{3(bk + 2)}$$

$$P = \frac{a(5b_2k_2 + 28bk + 24)}{6(3bk + 2)(bk + 2)}$$

$$\alpha = \frac{a(-b_2k_2 + 12bk + 16)}{6(3bk + 2)(bk + 2)}.$$

For $(bk)^2 < 8$ this internal strategic equilibrium is like the other models of this paper is characterized by overbuying $(A_1^{int} > 0)$. And as in the quantity model of Section one, vertical integration causes both the upstream and downstream prices to fall. As with the simple quantity model, firm one's output falls and firm two's input increases, with internal input production outstripping unintegrated production. As above, the welfare enhancing post-merger downstream price decreases will work against a production inefficiency induced by the merger. The net effect on welfare can be derived as:

$$W^{delta} = \frac{a^2k(32 + 112bk + 70b^2k^2 + 31b^3k^3 + 8b^4k^4 - b^5k^5)}{72(bk + 1)(3bk + 2)(bk + 2)^2},$$

which as shown in Figure 9 is positive for all relevant parameter values, so that internal strategic behavior is on net welfare enhancing.

Section 5: Conclusion

The models in this essay demonstrate that vertical integration can make it easier to exclude rivals. A vertical merger can create a marginal-factor-cost "umbrella" which an integrated firm can "raise" over itself by buying inputs from the external input market. Such integration can be a profitable strategy which drives rivals from the market and reduces welfare and thus is an appropriate concern of antitrust.

The essay also shows that Salopian notions of strategic overbuying can be given an extensive-form game theory inter-pretation that imply that an integrated firm has a first-mover advantage with regard to at least one input choice. Strategic overbuying can lead as expected to more drastic forms of foreclosure, including situations in which integrated firms purchase inputs whose marginal factor costs are greater than the downstream price. Whether a specific type of strategic overbuying is appropriate in a particular

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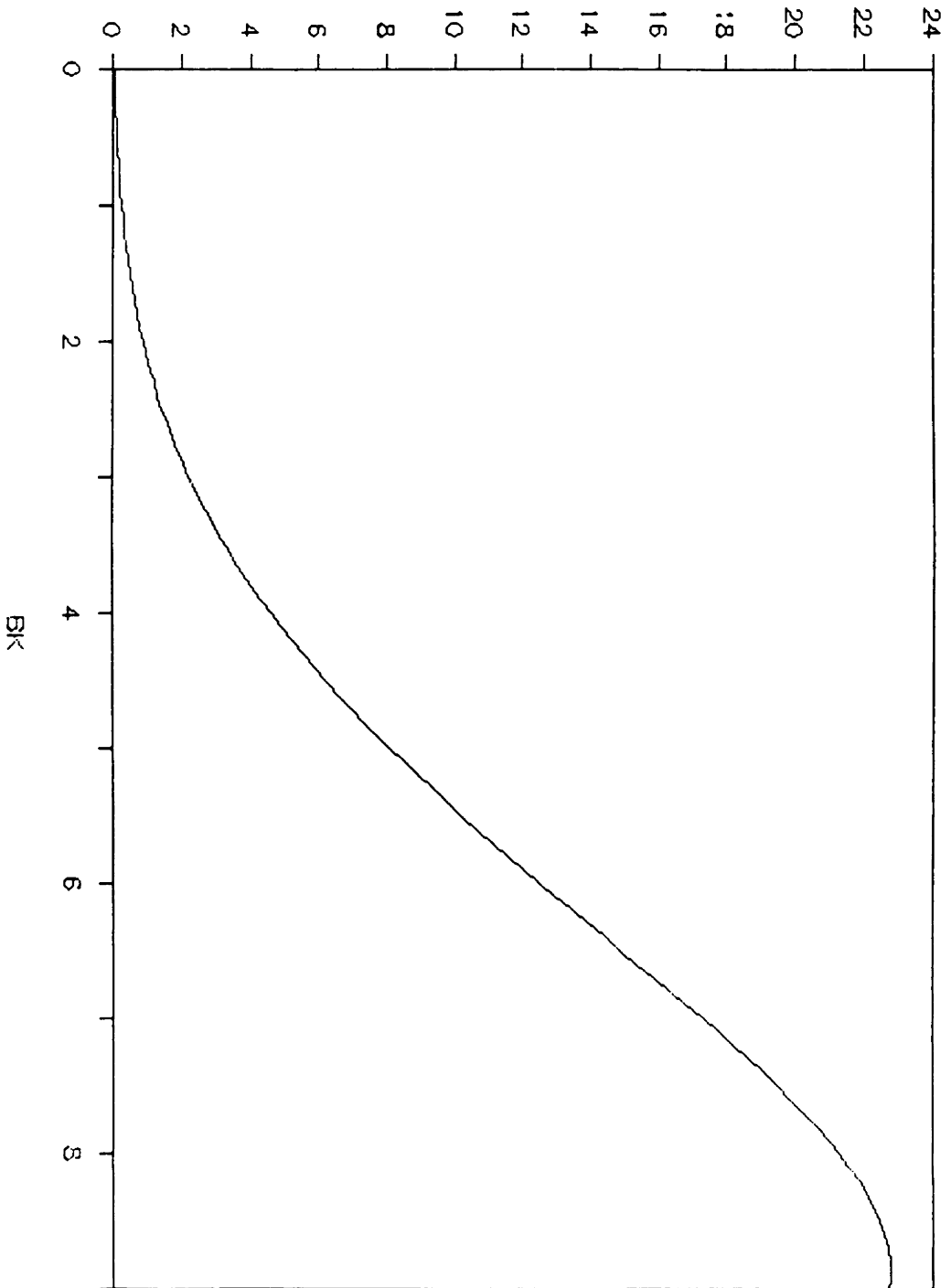
FIGURE 9
INTERNAL STRATEGIC WELFARE CHANGE

setting will turn, however, on whether the act of vertical merger is thought to give the integrated firm a first-mover advantage with regard to its strategic choice variable. Even without a first-mover advantage, vertical merger can reduce welfare (as was seen in the non-strategic pricing game). But for the simple quantity models analyzed, the possibly more reasonable internal strategies were welfare enhancing while Salop's external strategies caused more extreme and socially deleterious forms of overbuying and foreclosure. As Schmalensee [1979] has noted, the varying results of different models may not reflect the poverty of theory but the richness of the realty, so that solid antitrust analysis will necessarily involve picking the right model.

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CHAPTER 2: Deriving Cartel Behavior
in a Model with Fringe Competition

In many oligopoly models, a specific form of interdependent behavior is assumed from the outset. But as Stigler [1968] and others [Friedman (1972)] have pointed out, equilibrium behavior should derive from the structural characteristics of a market and not be merely assumed. This paper attempts to derive equilibrium behavior in a collusive price leadership model with fringe competition. The ability of firms to join or defect from the dominant cartel limits the degrees of collusive behavior that will be stable for a given cartel size. In general, larger cartels can only be stable if they act less collusively, because more collusion will increase free-riding by the competitive fringe. Cartel members thus face a trade-off between having more collusion or less free-riding (that is a larger stable cartel). At least in some situations cartel members will profit more from a larger cartel with less collusive behavior. This implies that from a cartel member's perspective less collusive strategies can Pareto dominate what is often considered "perfect" collusion.

Consider an industry with n firms[14] (producing a homogeneous product with identical cost functions, such that $c'(q) > 0$), k firms decide to combine and form a cartel, while the remaining n-k constitute the competitive fringe. Fringe competitors are price takers that produce up to the point that marginal cost equals price.

One can think of actions taking place in three stages. In the first stage a cartel strategy is determined.[15]   In the second stage firms commit themselves to joining the cartel or not, and in the third stage production takes place.

The specification of cartel behavior is central to my model.   For tractability, I only consider sy..metric cartel behavior.   Cartel behavior is assumed to be explicitly

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[14]   As D'Aspremont et al. [1983] stressed the discreteness of firms is critical for a cartel of any size to be stable.   The assumption of a discrete firms seems especially appropriate in the oligopoly context.

[15]   This cartel strategy could be determined in two different ways: 1) a pre-determined cartel member or small core of cartel firms could choose a strategy that would maximize the expected profits of each firm in the cartel; or 2) a trade association could pick a strategy that would maximize total industry profits (including the profits of competitive fringe).   This latter mechanism might imply a Rawlsian veil of ignorance -- where firms ex ante don't know whether or not they will be cartel member.   Since fringe firms always benefit from formation of a cartel, there will always be an incentive for such a trade association to form.   The issue of how a trade association might behave is addressed below.

cooperative and the cartel members can costlessly reach an agreement and deter cheating (by detecting and punishing deviations from the agreement). The cartel cannot, however, force firms to join or exit the cartel[16], but instead can influence the equilibrium cartel size by choosing a particular strategy.

The fundamental difference between earlier work and the model in this piece is that the former assumes behavior ex ante. For example, D'Aspremont et al [1983] assumes "perfect collusion"[17] instead of deriving behavior from the model. I instead allow the cartel to choose from a broad family of symmetric collusive behaviors.

Because the firms in the competitive fringe are price takers, a cartel strategy can be completely described by a price schedule that sets the market price as a function of

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[16] The possibility of a cartel to exclude members is considered separately below.

[17] In the D'Aspremont model, the cartel calculates a residual demand curve by subtracting the competitive fringe supply and then chooses the price that will set marginal revenue equal to marginal cost. This behavior is only "perfect" given a particular cartel size, in that the cartel profits for a cartel of that size cannot be higher. But as will be demonstrated below, once behaviors' effect on cartel size is taken into account, other forms of less collusive behavior can yield higher cartel profits, by reducing the free-riding.

the cartel size.[18] Although behavior is most directly measured by the cartel's price choice, using a conjectural measure better illustrates the degree of collusion as compared to standard benchmarks of competitive, Cournot, or "perfectly" collusive behavior. In trying to choose an appropriate conjectural parametrization for behavior, I immediately face two distinct approaches in the literature on cartel stability and incentives to merger[19]. One branch, represented by works of Perry and Porter [1985] and Salant, Switzer, and Reynolds [1983], has modeled cartel behavior as having constant conjectural variations for different cartel sizes. The other branch, represented by works of D'Aspremont et al. [1983] and Donsimoni [1985], has instead chosen a cartel behavior which implies that conjectures increase with cartel size. In the following sections I explore the implications of both conjectural assumptions and will return again to this issue of conjectural parametrization.

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[18] Given the price and cartel size, the cartel firm output can be derived so that market demand and supply will be equal.

[19] The incentives for merger literature of Salant, Switzer and Reynolds [1983] and Perry and Porter [1985] is closely related to questions of cartel stability, as these authors ask under what conditions cartel will be externally unstable, as defined below.

The essential insight of D'Aspremont et al. [1983] concerned the ability of different sized cartels to keep individual firms from entering or exiting the cartel. These authors developed two conditions of stability. A cartel with k firms will be internally stable when a cartel member will not profit more from exiting the cartel (joining the competitive fringe) than from staying in the cartel. Thus, a cartel with k firms is <u>internally stable</u> if:

$$\pi_c(k-1) < \pi_d(k),$$

where "π" represents the profit; and the subscripts "c" and "d," here and throughout the paper, refer to competitive fringe and ("dominant") cartel firms respectively. A cartel with k firms is <u>externally stable</u> when no fringe firm has an incentive to join the cartel, that is, if:

$$\pi_d(k+1) < \pi_c(k).$$

These two conditions can be unified with the following definition.

<u>Definition 1</u>: Define a function S(k), such that

$$S(k) = \pi_c(k-1) - \pi_d(k). \qquad (1)$$

A cartel of size k^S is <u>locally stable</u> (that is, both internally and externally stable) if and only if: 1) $S(k^S) < 0$

(internal stability), and 2) $S(k^S+1) > 0$ (external stability).[20]

This definition, because of its local quality, does not assure that these stable points will be reached. A cartel size would be globally stable if the instability of larger or smaller sizes would move the cartel toward the stable size. Thus, a notion of global stability should insure both that all cartel sizes greater than the stable size are internally unstable, and that all cartel sizes less than the stable size are externally unstable.

Definition 2: A cartel size of k^S is globally stable if and only if: 1) $S(k^X) < 0$, for all $0 \le k^X \le k^S$; and 2) $S(k^Z) > 0$, for all $k^S \le k^Z \le n$.

Lemma 1: For a given behavior, a globally stable cartel size will be a uniquely stable equilibrium.

Proof of Lemma 1: By definition 2, all k greater than a globally stable cartel size must be internally unstable, and

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[20] To be locally stable, a cartel of all the firms in the industry, k = n, need only be internally stable, and a null cartel, k = 0, need only be externally stable.

all k less than a globally stable cartel size must be
externally unstable. Hence, for a given behavior, a globally
stable cartel will be the unique equilibrium that is both
internally and externally stable.
Q.E.D.

This paper explores how these stability conditions can
limit equilibrium cartel behavior. The following two sec-
tions examine two different families of cartel behavior and
derive optimal cartel behavior within each family.

Section 1: Constant Conjectures

For tractability I assume linear demand and marginal
costs of the form:

$$Q_t = a - bP$$

$$C(q) = (1/2)q^2 \quad MC = q$$

where P is price; $C(q)$ and MC are the firms' total and
marginal costs respectively; and Q_t equals total industry
output such that $Q_t = kq_d + (n-k)q_c$ with q_d, q_c being the out-
put of the individual dominant cartel firms and competitive
fringe firms respectively. The dominant cartel faces a
residual demand curve equalling total demand minus the fringe
supply:

$$Q_r = a - (b + n - k)P$$

where Q_r is the residual demand. The standard first order condition for the ith cartel member is then:

$$MR_i = A - BQ_r - Bq_{di}\gamma_i = q_{di}$$

where $A = \dfrac{a}{(b+n-k)}$; $B = \dfrac{1}{(b+n-k)}$; and $\gamma^i = 1 + \sum_{i \neq j} \dfrac{\partial q_{dj}}{\partial q_{di}}$.

"γ_i" is the conjectural variation measure; it is the ith firms conjecture about how the cartel's output will be affected by a change in its production.

Because of the symmetry assumption, all γ_i can be denoted simply by "γ." Under the constant conjecture parametrization, this conjectural measure can take on any value, but once chosen is constant in the sense that it does not vary with cartel size (i.e., $d\gamma/dk = 0$).[21] Given a specific conjecture, the equilibrium price, quantities and profits can be easily calculated for different cartel sizes:

$$q_d = a/(h + \gamma) \qquad\qquad (2.1)$$

$$q_c = P = a(h - k + \gamma)/(h - k)(h + \gamma) \qquad (2.2)$$

$$\pi_d = a^2(h - k + 2\gamma)/(2(h - k)(h + \gamma)^2) \qquad (2.3)$$

$$\pi_c = a^2(h - k + \gamma)^2/(2(h - k)^2(h + \gamma)^2), \qquad (2.4)$$

where h=b+n.

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[21] Cournot behavior corresponds to $\gamma = 1$; Bertrand competition corresponds to $\gamma = 0$; and "perfect" collusion corresponds to $\gamma = k$.

Proposition 1: With constant conjectures, the stable cartel's size, k*, will either consist of all the firms in the industry or no firms (i.e, k* = n, or 0):

(1) for $\gamma < 2h/(h-1)$,   k* = n;

(2) for $2h/(h-1) \leq \gamma \leq 2(b+1)/b$,   k* = n,0; and

(3) for $\gamma > 2(b+1)/b$,   k* = 0.

Proof of Proposition 1:[22] The stability measure, S, for constant conjectures, γ, reduces to:

$$S(k) = \frac{(a^2/2)\gamma[k(2-\gamma) - h(2-\gamma) - 2]}{(h+\gamma)^2(h-k+1)^2(h-k)} \qquad (3.1)$$

To prove Part (3): Notice that dS/dk < 0, for all $\gamma > 2$, and that:

$$S(k=n) = \frac{(a^2/2)\gamma(\gamma b - 2(b+1))}{b(h+\gamma)^2(b+1)^2} \qquad (3.2)$$

Because S(k=n) > 0 for all $\gamma > 2(b+1)/b > 2$, and because dS/dk < 0 for all such γ, then S(k) > 0 for all k when $\gamma > 2(b+1)/b$. For $\gamma > 2(b+1)/b$, therefore, no cartel will form.

To prove Part (1): (A.) Notice that from equation (3.1):

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[22] For a similar, demonstration see Perry and Porter [1985].

$$S(k=1) = \frac{(a^2/2)\gamma(\gamma(h-1) - 2h)}{(h+\gamma)^2(h)^2(h-1).} \qquad (3.3)$$

Because $S(k=1) < 0$ for all $\gamma < 2h/(h-1)$, and because $dS/dk < 0$ for all $2 < \gamma < 2h/(h-1)$, then $S(k) < 0$ for all $2 < \gamma < 2h/(h-1)$.

(B.) From equation (3.2), notice also that:

$$S(k=n,\gamma=2) = -2a^2/(b(h+\gamma)^2(b+1)^2) < 0, (3.4)$$

and that $dS/dk > 0$ for all $\gamma < 2$, so that $S(k) < 0$ for all $\gamma < 2$

(A.) and (B.) together then show that for $\gamma < 2h/(h-1)$, $S(k) < 0$ for all k, and that therefore all firms will join the cartel.

To prove Part (2): Notice that, as shown in equation (3.3), $S(k=1) \geq 0$ for all $\gamma \geq 2h/(h-1) > 2$, and that, as shown in equation (3.2), $S(k=n) \leq 0$ for all $\gamma < 2(b+1)/b$. Since S is monotonically increasing in k for $2h/(h-1) \leq \gamma \leq 2(b+1)/b$, and since $S(k=1) \leq 0$ and $S(k=n) \geq 0$ for these values of γ, there must be one (usually non-integer) value, k^0, which will make $S(k) = 0$. Solving $S(k) = 0$ yields this value:

$$k^0 = (2 + h(2-\gamma))/(2-\gamma).$$

For $k < k^0$, $S(k) > 0$ and for $k > k^0$, $S(k) < 0$, so that for $2h/(h-1) \leq \gamma \leq 2(b+1)/b$ both $k = 0$ and $k = n$ are locally stable cartel sizes.

Q.E.D.

Proposition 1 reflects how the competitive fringe can free ride on the output reductions of the cartel. For relatively collusive behavior (where $\gamma \geq 2(b+1)/b$), the implied output reduction of cartel firms is so severe that individual firms have an incentive, for any k, to join the competitive fringe. Analogously, for relatively competitive behavior ($\gamma \leq 2h/(h-1)$), the output reductions of the cartel are moderate enough to deter any free-riding so that all firms will join the cartel. For the intermediate range of behavior, however, there will be incentives for individual firms to free ride unless there are enough cartel firms to share in the output reduction. If this critical mass is reached, a firm will gain more from joining the cartel and increasing the collusion than from defecting to the competitive fringe. For these intermediate behaviors, both the smallest and the largest cartel sizes ($k = 0$, n) are stable. An example of this dual equilibrium is shown in Figure 10. (for a = $1000, b = 10, n = 10, and γ = 2.15): cartel sizes less than or
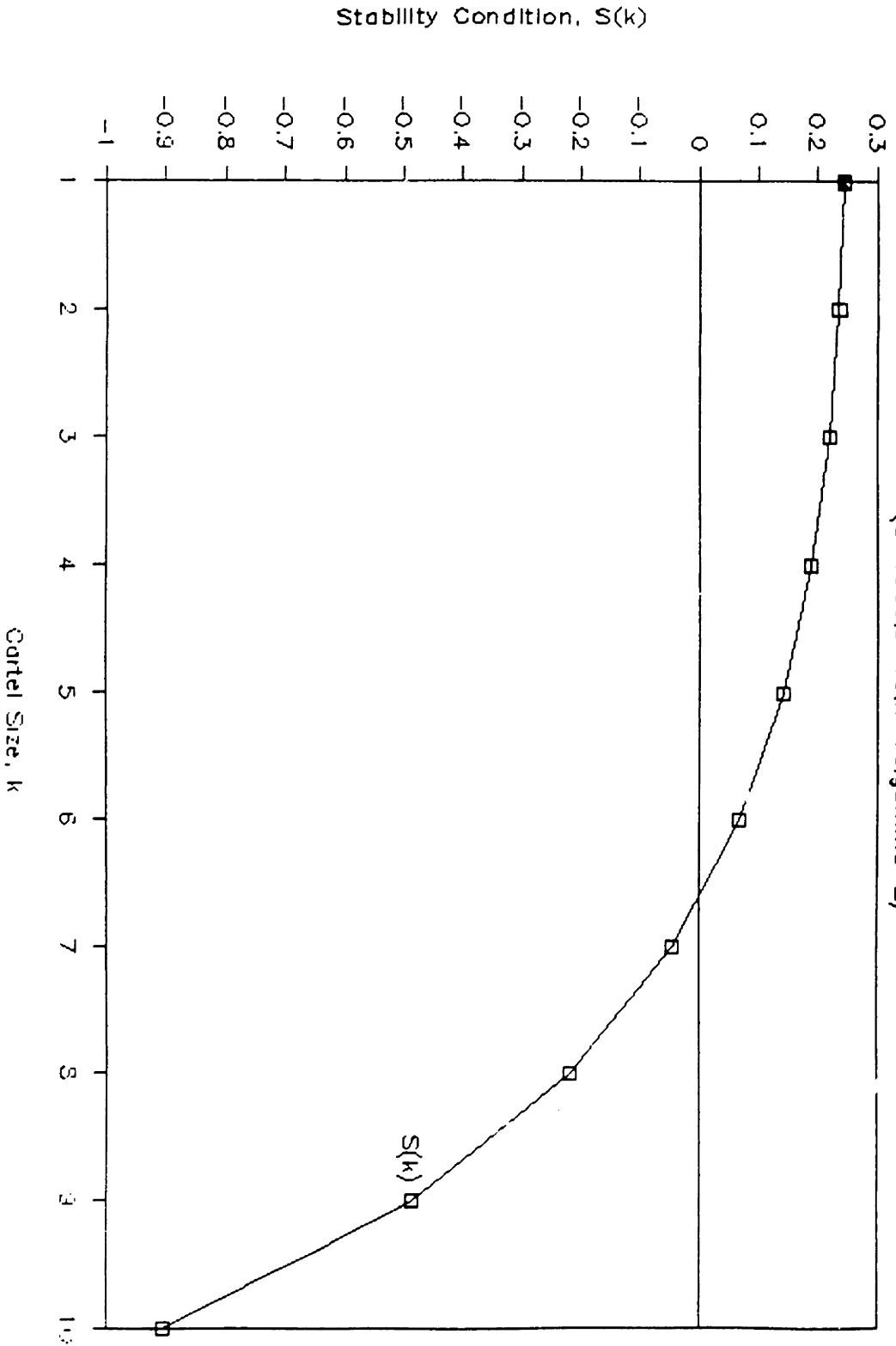
Figure 10
(a=1000,b=10,n=10,gamma=2)

equal to six are internally unstable, cartel sizes greater than or equal to seven are externally unstable ($k^0 = 6.66$).

Characterizing the stable equilibrium for any given behavior, allows us to examine which behavior within the constant conjecture family will be profit maximizing. This is accomplished in the following two propositions.

Proposition 2: The most profitable cartel behavior which yields a locally stable cartel size is $\gamma = 2(b+1)/b$. This behavior can support a stable cartel which includes all the firms in the industry, $k = n$, and yields per firm profits of:

$$\pi_d(k=n, \gamma=2(b+1)/b) =$$
$$(a^2/2)(b^2 + 4(b+1))/(2(1+b) + bh)^2.$$

Proof of Proposition 2: The partial derivative of cartel profits with respect to γ equals:

$$\partial \pi_d/\partial \gamma = a^2(k-\gamma)/(h+\gamma)^3(h-k).$$

For a given cartel size k, the cartel profits are therefore an increasing function of the conjectural variation size, γ, as long as $\gamma < k$. Since by Proposition 1, the only stable cartel sizes are 0 or n, the optimal behavior is the most collusive (i.e. largest γ) that will still support a stable cartel of $k = n$. By Proposition 1, the largest γ, to support

a stable cartel size of $k = n$ is $\gamma = 2(b+1)/b$. Since the only other stable cartels are either 1) cartels with no members and only competitive profits or 2) cartels with the entire industry but less collusive behavior, $\gamma = 2(b+1)/b$ is the optimum. The cartel profits for this local optimum are directly computed from equation (2.3).
Q.E.D.

Proposition 3: The optimal cartel behavior which yields a globally stable and unique cartel size is $\gamma = 2h/(h-1)$. For this value of γ, the stable cartel will include all firms in the industry and cartel profits per firm will be:

$$\pi_d(k=n, \gamma=2h/(h-1)) = \frac{a^2(bh - b + 4h)(h-1)}{2bh^2(h+1)^2}.$$

Proof of Proposition 3: By Proposition 1, the only values of γ for which there is a unique and globally stable cartel size are $\gamma \leq 2h/(h-1)$, and the stable cartel size for all such γ is $k = n$. As shown in the proof to Proposition 2, since $\partial\pi_d/\partial\gamma > 0$ for a given cartel size and for these values of γ, then the most profitable cartel behavior is simply the most collusive (largest γ) that still supports the globally stable equilibrium. $\gamma = 2h/(h-1)$ is therefore the optimum

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behavior. The cartel firm profits for this behavior are computed directly from equation (2.3).

Q.E.D.

It is straightforward to show that the locally stable equilibrium, associated with $\gamma = 2(b+1)/b$, generates higher cartel profits than the globally stable equilibrium, associated with $\gamma = 2h/(h-1)$. Intuitively, since both behaviors support a cartel size including all n firms but the former does so with more collusion, it is the more profitable. A profit maximizing cartel will therefore choose the former if we assume that the cartel can get to the most profitable locally stable cartel size.

This analysis suggests that fringe competition can dramatically effect cartel behavior. A cartel of a given size, k^{fix}, which maximized profits without fringe competition would simply set $\gamma = k^{fix}$. But once fringe competition opens the possibility of internal instability, a profit maximizing cartel will take into account the effect its behavior has on cartel size. For example, Table I compares the price and profits in a 10 firm industry (where a=$1000,b=10) when there is and isn't a fixed cartel size. When the cartel size is fixed at five, the profit maximizing price (corresponding

to $\gamma = 5$) is \$53.33 and cartel members earn \$1333.33 in profits. At this price however, a cartel member would be better off joining the competitive fringe and would thereby increase its profits to \$1378.13. Therefore, if cartel defection is possible, $\gamma = k = 5$ is internally unstable. When defection is possible, a maximizing cartel would behave more competitively (setting $\gamma = 2(b+1)/b = 2.2$) not only to make the cartel size of five stable but to induce all ten firms to participate. As shown in Table 1, allowing entry and exit actually benefits the cartel as the more-competitive, but larger cartel has much higher profits of \$1460.92.

Table 1
(a = \$1000, b = 10, n = 10)

	Fixed Cartel Size	Varying Cartel Size
γ	5	2.2
k	5	10.
P	\$53.33	\$54.95
π_d	\$1333.33	\$1460.92

Having derived the optimal cartel behavior for constant conjectures in Proposition 2, it is possible to evaluate the welfare effects of fringe competition. For constant conjectures, the Lerner index of market power, L, is:

$$L(k,\gamma) = \gamma/(h - k + \gamma). \tag{4.1}$$

At the optimal cartel behavior and locally stable cartel size the equilibrium Lerner index is:

$$L(k=n,\gamma=2(b+1)/b) = 2(1+b) / (2(1+b) + b^2). \tag{4.2}$$

Following Schmalensee [1982], the dead-weight loss, DW, for this fringe competition model can also be calculated for constant conjectures:

$$DW(k,\gamma) = (a\gamma)^2 k / 2h(h-k)(h+\gamma)^2, \tag{4.3}$$

which for the optimizing cartel equilibrium equals:

$$DW(k=n,\gamma=2(b+1)/b) = 2na^2(1+b)^2 / hb(2(1+b) + bh)2. \tag{4.4}$$

The comparative statics for the optimizing cartel equilibrium are given in the following Lemma:

<u>Lemma 2</u>: The dead-weight loss evaluated for the optimal behavior, $\gamma = 2(b+1)/b$, and cartel size, $k = n$, is (1) a positive function of consumers' willingness to pay, a, and (2) a decreasing function of b, the slope of the demand curve (and a monotonic transformation of demand elasticity). Finally, (3) dead-weight loss is a positive function of the industry size, n, when b is large relative to n. Specifically, dDW/dn is greater or less than zero as $(b+1)^2 - n(2n+b)$ is greater or less than zero.

Proof of Lemma 2: From equation (4.4):

$$dDW/da = 4na(1+b)^2 / hb(2(1+b) + bh)^2 > 0$$

which proves (1).

By analyzing partial derivatives of equation (4.4) we see:

$$dDW/db = \partial DW/\partial b + (\partial DW/\partial\gamma)(d\gamma/db).$$

$$\partial DW/\partial b = - \frac{(a^2\gamma^2 k(2h(h-k) + (h+\gamma)(2h-k)))}{2(h^2(h+\gamma)^3(h-k)^2)} \leq 0$$

$$d\gamma/db = - 2/b^2$$

$$\partial DW/\partial\gamma = a^2 hk\gamma/h(h-k)(h+\gamma)^3 \leq 0$$

which is sufficient to show that $dDW/db \leq 0$, which proves (2).

Similarly by taking partial derivatives of equation (4.4), it can be shown that:

$$dDW/dn = \partial DW/\partial n + (\partial DW/\partial k)(dk/dn)$$

$$= \frac{a^2\gamma^2[(b+1)^2 - n(2n+b)]}{2(h+\gamma)^3(h-k)}.$$

The sign of this expression will be the same as the expression in the brackets, which completes the proof. Q.E.D.

When $b > n$, $dDW/dn > 0$, even though the partial, $\partial DW/\partial n$, is always negative. This seemingly counter-intuitive result

may be explained by the fact that increasing the number of firms in the industry will also increase the number of firms in the cartel (dk/dn = 1). For sufficiently elastic demand the harm of increasing the cartel size can outweigh the good of increasing the number of firms in the industry ($\partial DW/\partial k$ > - $\partial DW/\partial n$).

Social welfare is affected by both pricing and production inefficiencies. The usual pricing inefficiency is caused by the wedge between price and marginal cost. But there is a productive inefficiency as well stemming from the fact that fringe firms produce more than cartel firms. Because there are increasing marginal costs, the same aggregate output could be produced more cheaply by symmetric production. Returning to the example in Table 1, changing the cartel size from five to ten firms eliminates this production inefficiency at the same time that it increases the pricing inefficiency. In fact, society as well as the cartel members gain from the shift to the larger cartel--as the elimination of production inefficiencies more than offset the larger price wedge. Plugging in the parameters of the example into equation (4.3) shows that the dead-weight loss falls from $333 to $245.[23]. In fact, efficient production only occurs

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[23] This result is analogous to Williamson's insight [1968] that the production economies from merger could outweigh the social harm of post-merger market power.

when k = 0 or n, and this may in part explain the model's preference for corner solutions.

Restricting cartel behavior to the family of constant conjectures has allowed us to derive the optimal behavior within this set of behaviors, but raises some questions. The all-or-nothing stable cartel size using constant conjectures is, however, unappealing. Our intuitions about stable cartels question such razor-edged results, as we expect instead to find stable cartel sizes with some but not all firms in the industry -- this after all is the starting point of a dominant cartel/competitive fringe model.

Section 2: Conjectures that Vary with Cartel Size

To overcome this weakness, it is worthwhile to consider other types of cartel behavior. The major alternative to constant conjectures is "perfect collusion." Perfect collusion in conjectural terms would set γ_i equal to the reciprocal of the ith firm's share of the cartel output (SH_i). Assuming symmetry:

$$\gamma = (1/SH) = k.$$

A major difference between this type of behavior and the constant conjecture specification is that the conjecture varies

with the cartel size -- $d\gamma/dk$ is not equal to zero. One reason that a constant conjecture specification of behavior, like the ubiquitous Cournot assumption, may be biased toward corner stability solutions is that as cartel size increases, these constant conjectures become relatively more and more competitive in that there is a greater and greater short-fall between the constant conjecture and the profit maximizing conjecture for larger cartel sizes.[24]

I have chosen the simplest linear parametrization that allows the conjecture to vary with cartel size. The conjectural measure can be conveniently rewritten with $\gamma = \sigma k$,[25]

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[24] As noted above, behavior can also be measured as a price schedule for different cartel sizes, $P(k)$. Even under constant conjectures the cartel price will vary with the cartel size:

$$P(k) = a(h-k+\gamma) \; / \; (h-k)(h+\gamma).$$

But varying conjectures amplifies the positive price response to changes in cartel size:

$$dP/dk = \partial P/\partial k + (\partial P/\partial \gamma)(d\gamma/dk)$$
$$(ak)/((h+\gamma)^2(h-k)) \; + \; a\gamma/(h+\gamma)(h-k)^2(d\gamma/dk)$$

The added collusive response to cartel sizes seems to be necessary to generate interior cartel sizes that are stable. Increasing this response effectively increases the punishment for defection and thus implicates notions of credibility to which we will return.

[25] The specification $\gamma = \sigma_1 + \sigma_2 k$ would include both the constant conjectures and the text's linear specification as special cases, $\sigma_2 = 0$ and $\sigma_1 = 0$ respectively. The market structure could then determine optimal cartel behavior. This specification is similar to Boyer and Moreaux [1983] linear modelling of conjectures as a function of output. Unfortunately, analytic characterization of stability over two parameters has proven intractable.

so that σ taking on values between zero and one describes a range of behaviors between perfect competition and perfect collusion.[26] Like D'Aspremont et al. [1983], this parametrization lets behavior vary with cartel size, but unlike earlier efforts, it lets structural characteristics of the markets determine the specific type of behavior.

Using varying conjectures, the first order equation for the identical cartel firms becomes:

$$MR = A - BQ_r - Bq_d\sigma k = q_d$$

and the equilibrium price, quantities, and profit are easily calculated:

$$q_d = a/(h + \sigma k) \qquad (5.1)$$

$$q_c = P = a(h + (\sigma - 1)k)/(h - k)(h + \sigma k) \qquad (5.2)$$

$$\pi_d = a^2((2\sigma - 1)k + h)/(2(h - k)(h + \sigma k)^2) \qquad (5.3)$$

$$\pi_c = a^2(h + (\sigma - 1)k)^2/(2(h - k)^2(h + \sigma k)^2). \qquad (5.4)$$

Substituting equations (5.3) and (5.4) into equation (1) yields an expression stability measure S(k):

$$S(k,\sigma) = (a^2/2)[(h+(\sigma-1)(k-1))^2(h-k)(h+\sigma k)^2 -$$
$$((2\sigma-1)k+h)(h-k+1)^2(h+\sigma(k-1))^2] /$$
$$(h-k+1)^2(h+\sigma(k-1))^2(h-k)(h+\sigma k)^2. \qquad (5.5)$$

D'Aspremont et al. [1983] prove that when the cartel colludes "perfectly" ($\sigma = 1$), there will exist at least one stable

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[26] $\sigma = 1/k$ similarly describes Cournot behavior ($\gamma = 1$).

cartel size. For example, in Figure 11 for n = 10, a = $1000, and b = 10, the stable cartel size is 3 as S(k=3) is negative and S(k=4) is positive. In fact, for this linear model, Donsimoni et al. [1985] have shown that under certain conditions[27] there is a unique stable cartel size for perfectly colluding cartels.

Allowing the cartel members to choose other forms of symmetric behavior, however, can dramatically increase the stable cartel size. For example in Figure 12, by lowering σ from 1 to .5, the size of a stable cartel is increased from k = 3 to k = 6.
The following propositions characterize the possible equilibria.

Proposition 4: For any $\sigma > 0$, there will always exist a locally stable and positive cartel size.

Proof of Proposition 4: From equations (5.5) it can be shown that:

$$S(k=1) = (a^2/2)[\sigma(h(\sigma-2) - \sigma)] / h^2(h-1)(h+\sigma)^2 \leq 0.$$

The latter inequality implies that for $\sigma > 0$, there will always be an incentive for one firm to join a cartel (because

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[27] These conditions are roughly equivalent to b > .302(n).

Figure 11 — Perfect Collusion Stability
(a=1000,b=10,n=10)
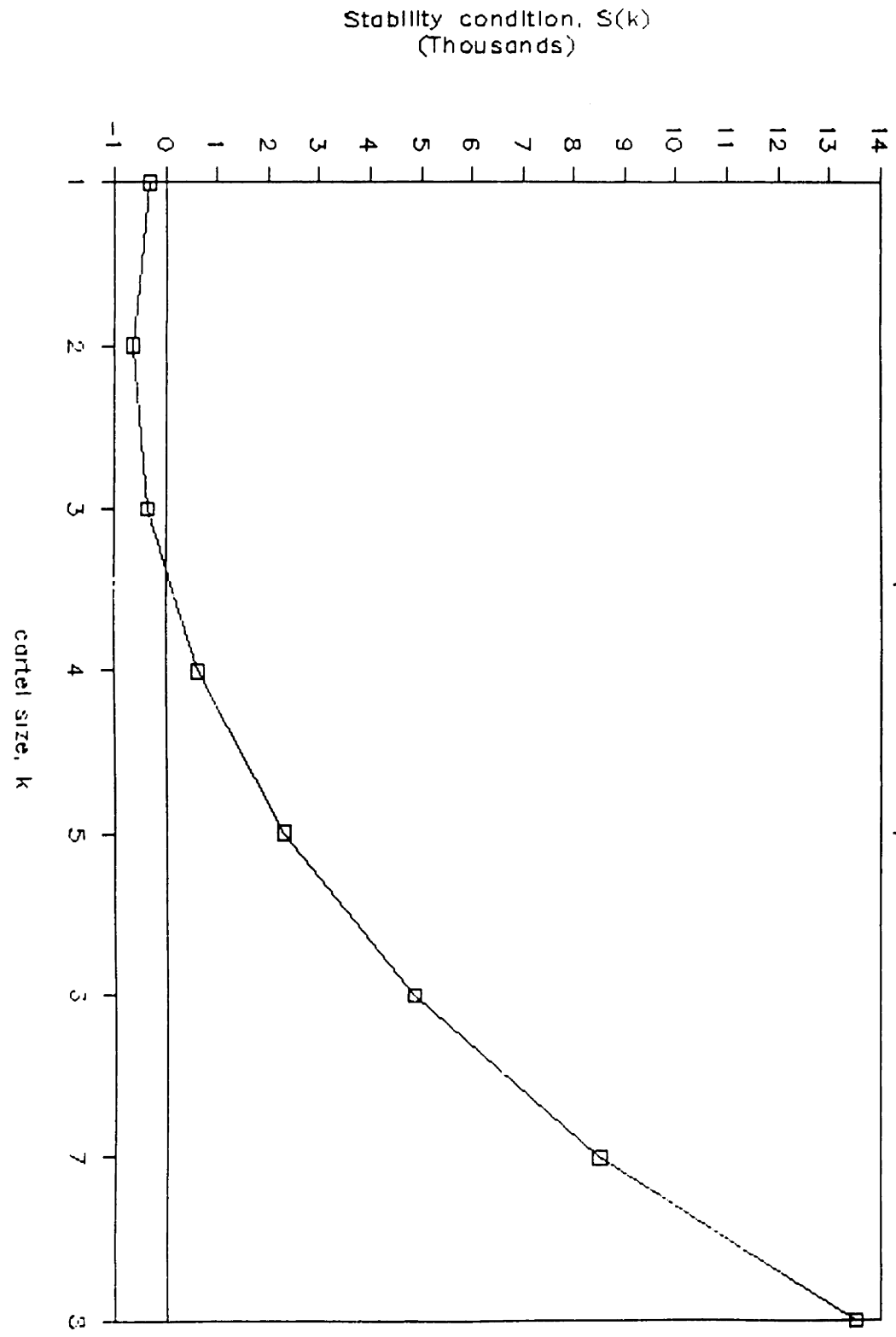
Figure 12
Varying Behavior
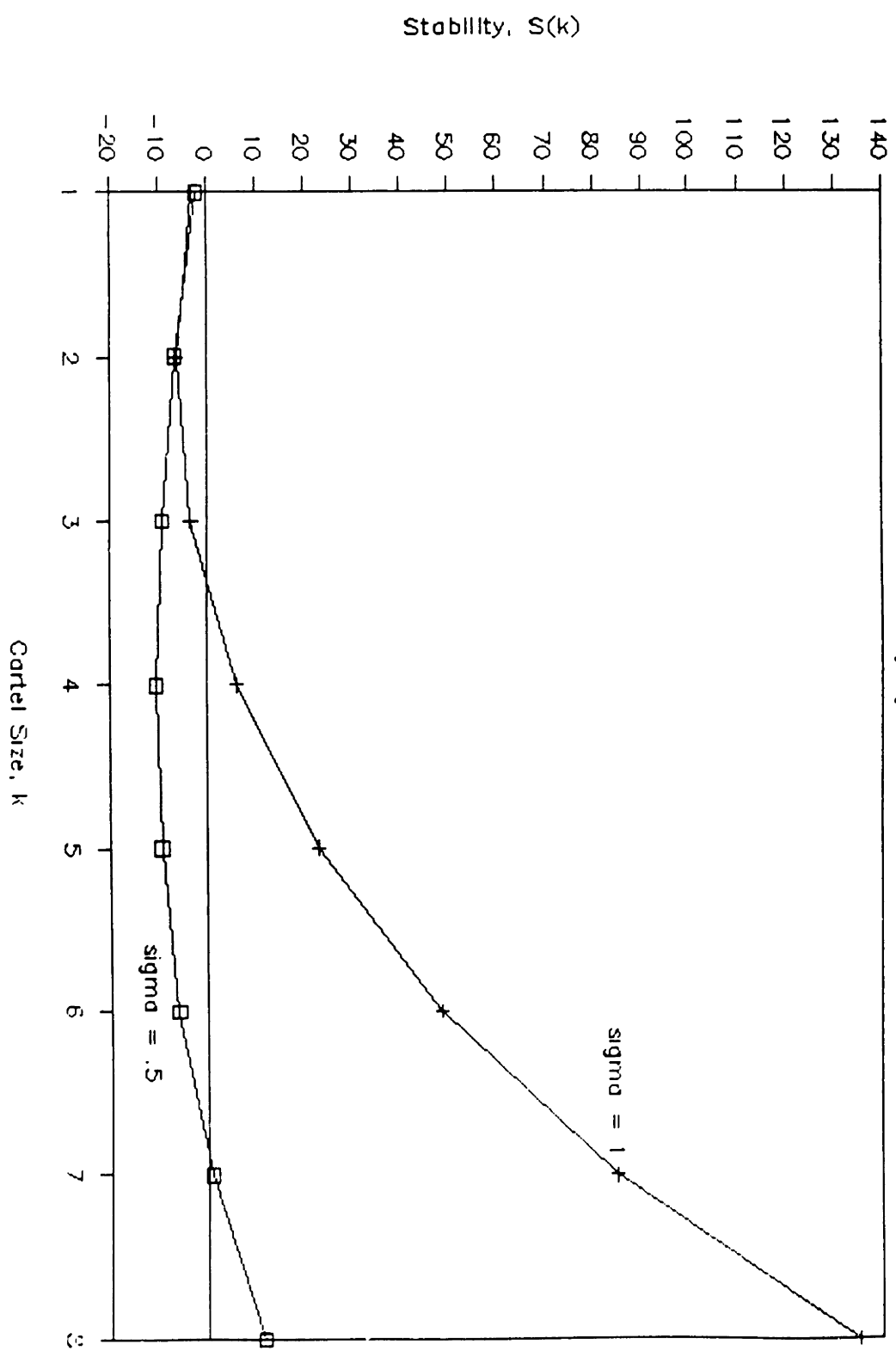
S(k=1) < 0). Following the proof of D'Aspremont et al. [1983], we know that if S(k=2) $\geq$ 0, k = 1 is a locally stable cartel, if not we consider S(k=3) and so on. In general, if S(1+i) remains negative for all i = [1,n-1], then a cartel that includes all firms, k = n, is locally (and in this case globally) stable. If, as i increases, S(1+i*) turns positive before i = n-1, then k = i* is a locally stable cartel. Q.E.D.

The next two propositions characterize the equilibrium for σ=1.

Proposition 5: For σ = 1, there exists a unique equilibrium cartel size for any b > b* = n[(v/(v^2-1)$^{(1/2)}$) - 1], where v = 8/(1 + (17)$^{(1/2)}$). When n $\geq$ 4, this equilibrium will be in the interior (k = 3).

Proof of Proposition 5: The complicated proof of uniqueness is given in given in Donsimoni et al [1985] and entails a demonstration that for these values of b and n, the horizontal distance between the profit functions montonically increases in k.

From equation (5.5), the stability condition for σ=1 reduces to:

$$S(k,\sigma=1) = \frac{(a^2/2)[h^2(k^2-4k+2) - (k-1)^4]}{(h-k)(h-k+1)^2(h-k-1)^2(h+k)}.$$

This expression is negative for $k \leq 3$ for any values of b or n, and positive for $k \geq 4$ for all $b > b*$. Thus, (probably as an artifact of the linearity assumptions) 3 will be the unique stable cartel size when $b > b*$.

Q.E.D.

For the remainder of the paper, I restrict attention to this unique equilibrium region of $b > b*$. I have been unable to find sufficient conditions to prove such uniqueness, due to the fact that equation (5.5) is a fourth-order polynomial in σ, and a fifth-order polynomial in k. A grid simulation indicates that for many reasonable parameters such uniqueness hold.[28]

Proposition 6: For $b < ((n-1)^2 / (n^2-4n+2)^{(1/2)}) - n$, $\sigma = 1$ will be the optimum behavior, and will create a locally stable cartel size of $k = n$.

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[28] The results of the grid simulation are described in an appendix.

Proof of Proposition 6: Again from equation (5.5) it can
be shown that:

$$S(k=n,\sigma=1) = (a^2/2)[h^2(n^2-4n+2)-(n-1)^4]/b(b+1)^2(b-1)^2(h+n).$$

Analyzing this equation, one can see that the sign of
$S(k=n,\sigma=1)$ will be the same as the sign of the bracketed
expression. Moreover, the bracketed expression will be less
than zero whenever:

$$b < ((n-1)^2 / (n^2-4n+2)^{(1/2)}) - n.$$

$S(k=n,\sigma=1)$ is less than zero for these values of b and n, and
k = n is therefore a stable cartel size. That this equi-
librium is optimal follows directly from the fact that for k
= n, and $\sigma = 1$, this cartel of all the industry's firms is
able to act like a perfect monopolist.
Q.E.D.

Now through several propositions, we investigate the
properties of equilibria for σ not equal to 1.

Proposition 7: To support a cartel size larg r than the
unique size (of k = 3) supported by $\sigma = 1$, the cartel must
behave more competitively.

Proof of Proposition 7:   In order for k = 3 to be a unique stable cartel size, we know (from Proposition 5 and the definition of stability) that:

1) there exists a k^1 implicitly defined by the equation, $S(\sigma=1,k^1) = 0$,   where $3 < k^1 < 4$,   and

2) $\partial S(\sigma=1,k^1)/\partial k > 0$.

We know from the proof of Proposition 4 that $S(\sigma,k=1) < 0$. Intuitively then, (as shown in Figure 11), $S(\sigma=1,k) < 0$ for k < 3 and $S(\sigma=1,k) > 0$ for k > 4, so there must be some k between 3 and 4 which intersects the x-axis and the slope at that point must be positive.

To support the larger cartel size of k = 4, $S(\sigma,k)$ must be shifted to cut the x-axis at k > 4. Taking the total derivative of $S(\sigma=1,k^1) = 0$ and applying the implicit function theorem we know that at $\sigma = 1$ and $k = k^1$:

$$dk/d\sigma = -\ (\partial S/\partial \sigma)\ /\ (\partial S/\partial k) \qquad (5.6)$$

From 2) we know that $\partial S/\partial k > 0$. We now evaluate $\partial S/\partial \sigma$.

The expression for $S(k,\sigma)$ in equation (5.5) can be set equal to zero and, for a fixed k, solved for its conjectural roots. $S(k,\sigma)$ is a fourth order polynomial in σ. Solving for these roots, one finds that there are two real and two complex roots. One real root is $\sigma = 0$, which follows from the fact that $\pi_d(\sigma=0) = \pi_c(\sigma=0)$ for any k (Intuitively, since $\sigma=0$

corresponds to perfectly competitive behavior, under this behavior there is never an incentive to enter or exit a cartel of any size because cartel members and fringe firms both act the same.). The real root of interest, σ^O, after many manipulations, can be shown to equal:

$$\sigma^O(k) = AA + (X^1/AA) + X^2, \qquad (6)$$

where:

$AA = ((b(9k^7 - 192k^6 + 831k^5 - 1644k^4 + 1776k^3 - 1086k^2 + 354k - 48) + 36^7 - 208k^6 + b^3(- k^6 + 63k^5 - 363k^4 + 808k^3 - 882k^2 + 474k - 100) + b^2(- 39k^6 + 387k^5 - 1230k^4 + 1875k^3 - 1515k^2 + 627k - 105) + b^4(3k^5 - 45k^4 + 153k^3 - 210k^2 + 136k - 36) + 513k^5 - 699k^4 + b^5(- 3k^4 + 12k^3 - 21k^2 + 9k) + b^6k^3 + 567k^3 - 273k^2 + 72k - 8) / (-27k^9 + b(81k^8 - 243k^7 + 243k^6 - 81k^5) + 81k^8 - 81k^7 + b^2(-81k^7 + 243k^6 - 243k^5 + 81k^4) + b^3(27k^6 - 81k^5 + 81k^4 - 27k^3) + 27k^6) + (k-b-1)((8k^7 + (-b^2 - 16b - 36)k^6 + (b^3 - b^2 + 52b + 66)k^5 + (b^4 + 17b^3 + 22b^2 - 68b - 63)k^4 + (-b^5 - 7b^4 - 48b^3 - 26b^2 + 49b + 33)k^3 + (-b^5 + 10b^4 + 20b^3 - 6b^2 - 23b - 9)k^2 + (8b^4 + 20b^3 + 18b^2 + 7b + 1)k - 8b^4 - 12b^3 - 6b^2 - b)/(k^2 + (-b-1)k + b))^{(1/2)}/(3(3^{(1/2)})k^3(k^2 + -b-1)k + b))^{(1/3)}$;

$X^1 = (-6k^5 + b(20k^4 - 80k^3 + 114k^2 - 70k + 16) + b^2(k^4 - 22k^3 + 63k^2 - 60k + 19) + 31k^4 - 60k^3 + b^3(- 2k^3 + 8k^2 - 14k$

+ 6) + b^4k^2 + $55k^2$ $-24k$ + 4) / ($9k^6$ - $18k^5$ + b(- $18k^5$ + $36k^4$ - $18k^3$) + b^2($9k^4$ - $18k^3$ + $9k^2$) + $9k^4$);

X^2 = - (($4k^2$ + b($-2k^2$ - 4k + 4) + $2b^2k$ - 6k + 2) / (3($-k^3$ + b(k^2 -k) + k^2))).

Taking the partial derivative of equation (5.5) with respect to σ and evaluating it at σ = 0 yields:

$$\partial S(k,\sigma=0)/\partial\sigma = a^2((h-k) - k(2h-k)) / n^3(h-k) < 0.$$

Since equation (5.5) has two real roots in σ and at the 0-root $\partial S/\partial\sigma$ < 0, we know that $\partial S(k,\sigma^o(k))/\partial\sigma$ must be greater than zero. As shown in Figure 13, the stability condition, S, as a function of σ is zero and has a negative slope at σ = 0 so that in order for it to reach the x-axis again at its other real root it must take on a positive slope.

We can now sign $dk/d\sigma$. Since by definition $\sigma^o(k^1)$ = 1 [k^1 is that k which makes $\sigma^o(k=k^1)$ = 1], we know, from the proceeding paragraph, that $\partial S(k^1,\sigma=1)/\partial\sigma$ > 0 so that from equation (5.6) $dk/d\sigma$ < 0. Therefore to encourage new firms to join an σ=1 cartel, the cartel must behave more competitively.

Lemma 3: If there is a unique stable cartel size for any given σ, then $d\sigma^o/dk$ < 0.

Figure 13
(a=1000,b=10,n=10,k=5)

Stability, S(sigma)

Conjectures, sigma
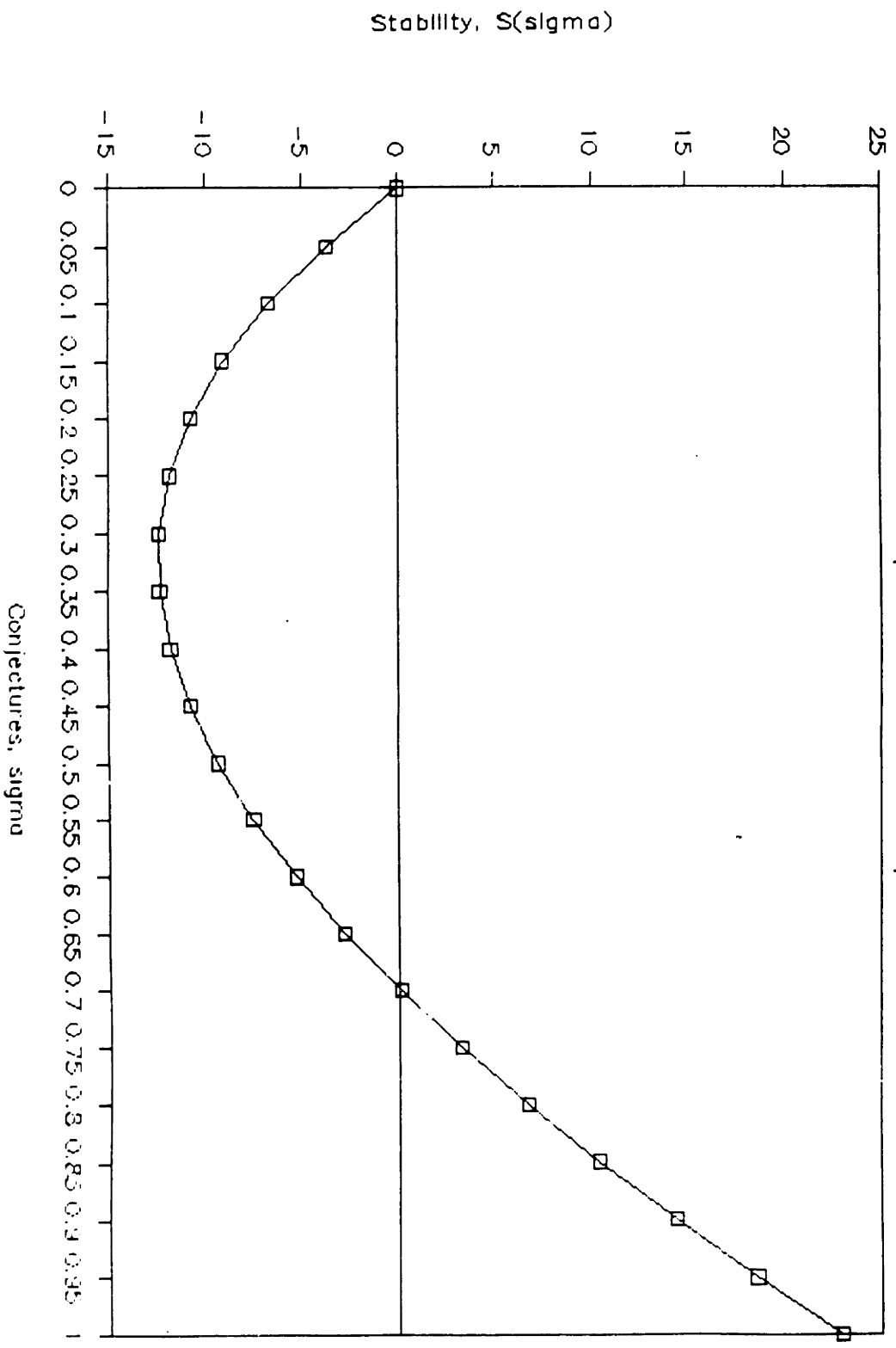
Proof of Lemma 3: If an equilibrium remains unique for any σ, then for any interior stable cartel size $(k < n)$, $\partial S(k,\sigma^O(k))/\partial k > 0$. Again using the implicit derivative in equation (5.6) and the fact that $(\partial S/\partial \sigma) > 0$, we know that $d\sigma^O/dk < 0$.

Q.E.D.

Proposition 8: If there is a unique stable cartel size for any given σ, then there exists an $\sigma > 0$ that will make any cartel size stable.

Proof of Proposition 8: From the definition of unique-ness, we know that for any given σ, if $S(k^Z,\sigma) = 0$, then $S(k) < 0$ for all $k < k^Z$, and $S(k) > 0$ for all $k > k^Z$. By defini-tion, substituting the conjectural root, $\sigma^O(k)$, into the stability measure yields

$$S(k,\sigma^O(k)) = 0$$

so that choosing $\sigma = \sigma^O(k)$, will make any k a stable cartel size.

Q.E.D.

Proposition 9: For a given k^Z, there will be a well defined range of σ that will yield a locally stable cartel

configuration. In particular, a cartel size, k^Z, will be stable for all $\sigma^O(k^Z+1) < \sigma < \sigma^O(k^Z)$.

$\underline{\text{Proof of Proposition 9}}$: Evaluating equation (5.5) and its derivative with respect to σ at $\sigma = 0$ yields:

$$S(k,\sigma=0) = 0$$

$$dS(k,\sigma=0)/dk = a^2((h-k) - k(2h-k)) / h^3(h-k) < 0.$$

Because (as we proved in Proposition 7) $S(k,\sigma)$ has only two real roots in σ, 0 and σ^O, this indicates that $S(k^Z,\sigma) > 0$ for $\sigma > \sigma^C(k^Z)$ (In Figure 13, for example, the stability measure is graphed as a function of behavior, $S(\sigma)$, for a five firm cartel. $\sigma^O(k=5)$ is approximately 0.71.). By definition 1, this implies that $\sigma > \sigma^O(k^Z)$ will never make k^Z an internally stable cartel size.

By analogy we know that $S(k^Z,\sigma) < 0$ for $\sigma < \sigma^O(k^Z)$, which implies that all $\sigma < \sigma^O(k^Z)$ will be internally stable. But to be externally stable as well means that $\pi_c(k^Z) > \pi_d(k^Z+1)$, which implies that $S(k^Z+1,\sigma) > 0$. From Lemma 3 we know that:

$$\sigma^O(k^Z+1) < \sigma^O(k^Z).$$

Following the argument in the preceding paragraph, k^Z will be externally stable (that is, $S(k^Z+1,\sigma) > 0$) only for $\sigma > \sigma^O(k^Z+1)$. Therefore only $\sigma^O(k^Z+1) < \sigma < \sigma^O(k^Z)$ will make k^Z both internally and externally stable.

Q.E.D.

Armed with this last proposition, an initial result of this model is that by knowing a given cartel structure (that is a value for k), we should be able to bound equilibrium stable behavior.

But we can go further. Profit maximization should tell us what σ the cartel will choose. To understand how a profit maximizing σ will be chosen requires an intuition about how fringe competition interacts with cartel profits. The fringe competitors free ride on the quantity reductions of the cartel. The cartel can, thus, collude more effectively by either behaving more collusively or by increasing the size of the cartel to reduce this free-riding ($\partial \pi_d / \partial \sigma > 0$, for $\sigma \leq 1$; and $\partial \pi_d / \partial k > 0$). Since each cartel size is only stable for certain values of σ, cartels will maximize profits by taking into account not only the direct effect of σ on π_d but also the indirect effect of σ on π_d through k. Optimal behavior is characterized in the following propositions.

Proposition 10: $\sigma > 1$ will never be a profit-maximizing cartel behavior.

<u>Proof of Proposition 10</u>:   From Lemma 3, $d\sigma^o/dk < 0$
implies that for large enough increases of σ the stable cartel
size will decrease (For example, in Figure 14, $\sigma^o(k=7)$ is only
approximately 0.49 as opposed to about 0.71 for $\sigma^o(k=5)$.).
From equation (5.3) we know also that:

$$d\pi_d/d\sigma = a^2k^2(1-\sigma) \ / \ (h-k)(h+\sigma k)^3 > 0 \text{ for } \sigma < 1$$

$$< 0 \text{ for } \sigma > 1$$

$$d\pi_d/dk = a^2\sigma k(h(2-\sigma) + k(2\sigma-1)) \ / \ (h-k)^2(h+\sigma k)^3 > 0$$

These derivatives imply that if a cartel started with $\sigma > 1$
(In Figures 1 and 2, for example, a cartel would need $\sigma > 1$ to
make cartels of 1 or 2 firms (k=1 or 2) stable.), it could
increase its profitability by decreasing σ to 1 . A lower σ
would increase profits through both the direct effect of σ on
π_d ($d\pi_d/d\sigma < 0$, for $\sigma > 1$) and the possible indirect effect of
σ on π_d of increasing the stable cartel size.   Therefore, all
equilibrium σ should be 1 or less.
Q.E.D.

This reverse argument, however, does not hold: lowering σ
below 1 (to make a larger cartel size stable, as in Figure
12) can increase the profits of individual cartel firms.   It
is possible that cartel firms will gain more from decreasing
the competitive fringe's free ride than they will lose from

Figure 14
(a=1000,b=10,n=10,)
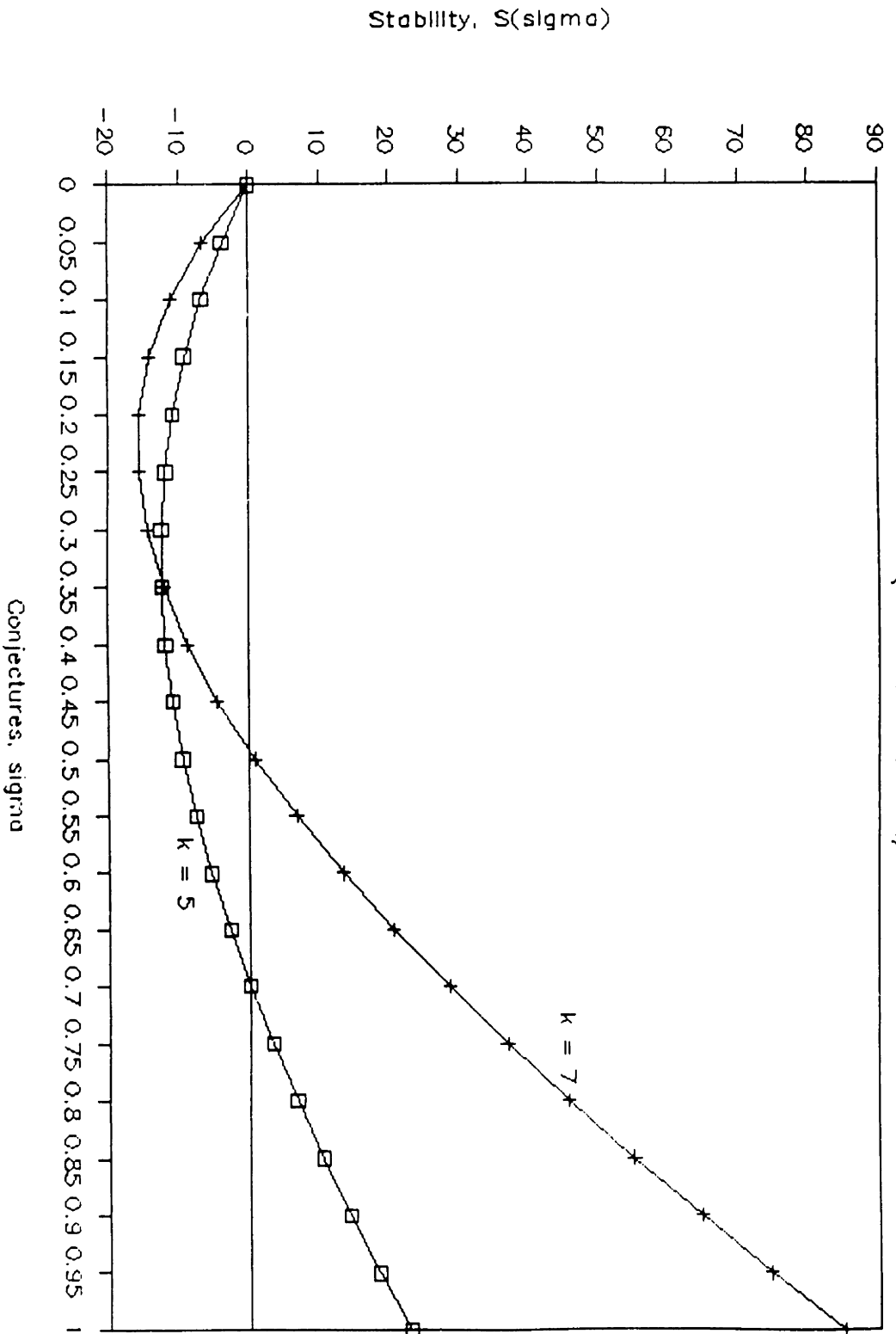
acting less collusively (lowering σ), heuristically the partial derivatives might be:

$$\partial\pi_d/\partial\sigma + (\partial\pi_d/\partial k^*)(dk^*/d\sigma) < 0, \qquad (7)$$

where $k^*(\sigma)$ is the stable cartel size supported for any given σ. In fact, from Proposition 9, we know that $k^*(\sigma)$ is a discontinuous function (so that $dk^*/d\sigma$ is undefined), remaining a constant k between $\sigma^O(k+1) < \sigma < \sigma^O(k)$ and jumping at each critical $\sigma^O(k)$ to $k + 1$. This description of $k^*(\sigma)$ also allows us to prove the following proposition.

Proposition 11: For a given cartel size, k^z, $\sigma^O(k^z)$ will be the profit-maximizing behavior[29] within the range of all possible behaviors that make k^z stable ($\sigma^O(k^z+1) < \sigma < \sigma^O(k^z)$).

Proof of Proposition 11: Because the stable cartel size $k^*(\sigma)$ remains constant within the range $\sigma^O(k+1) < \sigma < \sigma^O(k)$, there will not be an indirect effect of σ on π_d through a change in the stable cartel size. Since the direct effect of σ on π_d is positive for all σ less than one ($d\pi_d/d\sigma > 0$, $\sigma < 1$), profits will be maximized by the largest stable σ, $\sigma^O(k)$.

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[29] The optimal cartel behavior will actually be σ^* - epsilon, or in other words the largest σ for which $S(\sigma)$ is still negative.

Q.E.D.

Combining Propositions 10 and 11 drastically reduces the set of possible globally optimal behaviors. Proposition 11 limits the set to n possible behaviors, defined by $\sigma^O(k)$ for k = int[1,n], and Proposition 10 lets us exclude all $\sigma^O(k) > 1$. Hence, to determine the optimal behavior, σ^{opt}, we need only find which $\sigma^O(k) \leq 1$ yields the highest profits:[30]

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[30] If a trade association determines behavior in a Rawlsian fashion, as described above, it would not maximize simply the cartel members profits, π_d, but instead total industry profits, π_t:

$$\pi_t = k\pi_d + (n-k)\pi_c.$$

Proposition 9 would not necessarily still be valid under such a regime. Super-collusive behavior of $\sigma > 1$ might be optimal because:

$$d\pi^c/d\sigma = a^2 k^2 (h + (\sigma-1)k) \,/\, (h-k)^2 (h+\sigma k)^2 > 0$$

which implies that the fringe firms might gain more from super-collusive cartel behavior than the cartel firms would loose. In fact, it can be shown that the expression for $d\pi_t/d\sigma$ has the same sign as:

$$(n(h-k)/kb) - \sigma,$$

so that for $(n(h-k)/kb) > \sigma > 1$, $d\pi_t/d\sigma > 0$. Larger values of σ, thus, directly reduce the industry's profits. The possible indirect effect of reducing the stable cartel size also

$$\sigma^{opt} = \sigma^O(\text{argmax}_{k} \; \pi_d(k,\sigma^O(k))) \quad \text{for } k = \text{int}[1,n]; \text{ redefining all } \sigma^O(k) \geq 1 \text{ to equal } 1.$$

Stepping back for a moment, we see that $\sigma^O(k)$, as defined in equation (6), is a function of the cartel size, k, the slope of the demand curve, b, and the number of firms in the industry, n. The optimal behavior is thus derived from and

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reduces both cartel and frirge member's profits, as:

$$\partial\pi_c/\partial k = a^2\sigma k(h+(\sigma-1)k)(2b + (\sigma-1)k) \; / \; (h+\sigma k)^3(h-k)^3 > 0,$$

$$\partial\pi_d/\partial k = a^2\sigma k(h(2-\sigma) + k(2\sigma-1)) \; / \; (h-k)^2(h+\sigma k)^3 > 0.$$

For a trade association, Proposition 9 could be modified to say that $\sigma > (n(h-k)/kb)$ will never be an optimal behavior.

Proposition 10, however, is still valid for these remaining values of σ because for a given k, both cartel and fringe firms profit from having the highest stable σ $(d\pi_t/d\sigma > 0)$. This implies that the set of optimal behaviors will again be restricted to the set of $\sigma^O(k)$.

This issue of maximizing industry profits as opposed to cartel profits did not affect the analysis for constant conjecture behavior, because the only positive stable cartel sizes included all firms in the industries so that the maximands were always identical.

depends on the structural characteristics of the market and is not assumed.

Proposition 12: "Perfect" collusion, as traditionally defined ($\sigma = 1$), need not be the optimal cartel behavior.

Proof of Proposition 12: Proof is by counter example. Using the same parameter values as in Figures 1-4 (a=$1000,b=10,n=10), the "perfect" collusion of the D'Aspremont model ($\sigma = 1$) yields cartel firm profits of 1278 for a three firm cartel. When the cartel members can choose a better-than-"perfect" strategy, however, they choose $\sigma^{opt} = \sigma^{o}(k=10) = .33$, which induces a stable cartel of all 10 firms in the industry (k=n=10) and cartel firm profits of 1528. Q.E.D.

This striking result -- that it can be more profitable to choose $\sigma < 1$ -- indicates that the free-riding on the cartel's output reduction can be substantial. That structural constraints can cause dominant cartels to act more competitively is not new in the industrial organization literature. For example, Gaskins [1971] has shown that when faced with the threat of entry a dominant cartel (or monopolist) might act

more competitively. In Gaskins' model, the more-competitive limit pricing was employed to discourage entry; in this model, entry is given, and the more competitive behavior is to encourage cartel membership. Similarly, in Rotemberg and Saloner [1986], varying future demand (and therefore credible punishment) reduces the self-enforcing collusive price below the monopoly level.

Knowing the optimal behavior, σ^{opt}, also allows us to assess the social welfare consequences of collusion. The Lerner index, $L(\sigma)$, as a measure of market power will equal

$$L(\sigma) = \sigma k^*(\sigma)/(h + k^*(\sigma)(\sigma - 1).$$

And following Schmalensee [1982], the total deadweight loss, $DW(\sigma)$, from the market will equal:

$$DW(\sigma) = a^2\sigma^2 k^*(\sigma)^3/(2h(h - k^*(\sigma))(h + \sigma k^*(\sigma))^2).$$

For the parameter values used in the proof of Proposition 11 and the optimal behavior, $\sigma^{opt} = \sigma^o(k=10) = .33$, we find that:

$L(a=\$1000,b=n=k=10,\sigma=.33) = .25$

$DW(a=\$1000,b=n=k=10,\sigma=.33) = 501.48.$

Comparing these estimates to those with $\sigma = 1$, and $k^*(\sigma=1) = 3$, we see that society can be better off with perfectly col-lusive behavior if the resulting cartel defection is large enough:

$L(a=\$1000,b=n=10,k=3,\sigma=1) = .15$

$$DW(a=\$1000,b=n=10,k=3,\sigma=1) = 127.60.$$

These figures sharply contrast with Table 1's constant conjecture example. Although in both cases behaving more competitively eliminates the production inefficiency (since all firms in the industry produce the same quantity), with varying conjectures the added pricing inefficiency (seen through the rising Lerner index) dominates.

The structural determination of behavior in this model sharply reduces the varying kinds of equilibria predicted by Ordover, Sykes, and Willig [1982, p. 1862]. These authors use virtually the same dominant cartel/competitive fringe model but suggest a much larger range of possible deadweight losses. Because only certain types of behavior are stable, the degree of collusion and of collusive harm is, however, much more determinate.

Although the foregoing analysis proceeded as though internal and external stability were qualitatively the similar, intuition suggests that external stability might be easier to impose directly than internal stability. After all, a cartel can keep a firm's representative from entering

a room, but cannot keep her from leaving.[31] Therefore, it seems appropriate to analyze the two stability constraints separately to determine when each is binding. As proven in the following proposition, recognizing that external stability is a weaker constraint does not change the above analysis because external stability is never binding at equilibrium. Ironically, the incentive for merger literature of Perry and Porter [1985] and Salant, et al. [1983] focuses entirely on the lack of external stability.

Proposition 13: External stability will never be a binding constraint, as long as $\partial \pi_d / \partial k > 0$.

Proof of Proposition 13: When $\partial \pi_d / \partial k \geq 0$ (as it is for both the constant and varying conjectural parameterizations), current cartel members would profit from an increase in cartel size. Thus, external instability, which causes cartel size to increase, would never harm individual cartel firms because each of them would welcome (profit from) the new members.

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[31] A trickier question, however, is whether a cartel can more easily keep a firm from "acting" like a cartel member, even if the cartel can physically keep the firm from attending meetings.

Q.E.D.

Internal stability, however, will reduce the cartel's potential profits, because for any chosen behavior cartel members' profits will fall as cartel size falls (for $d\pi_d/dk >$ 0). Thus a perfectly-colluding cartel's willingness to change their behavior (act more competitively) will always be aimed at increasing cartel size and will always be in response to the internal stability constraint. Thus, perfectly-colluding cartels in this model will never worry about having to exclude members but may well have to create incentives to join.

Section 3: Comparing Different Families of Behavior

Having analyzed the optimal equilibria for the constant and varying conjectural families, it is useful to examine which type of optimal behavior will profit the cartel most. Returning to the parameter values a=$1000,b=n=10, we find that the optimal varying conjecture of σ = .33 yields higher cartel profits ($1528) than those created by the optimal constant conjecture of γ = 2.19 ($1460).[32] But we find that the

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[32] Since both behaviors cause the entire industry to cartelize, the higher profitability of the varying conjectures behavior can be see from the fact that σk = γ = 3.3 > 2.19.

optimum constant conjecture of $\gamma = 2.19$ still produces higher cartel profits than D'Aspremont perfect collusion of $\sigma = 1$, which yields cartel profits of only $1278. This example underscores the reasonableness of considering wider varieties of cartel behavior. Moreover, the superiority (from the cartel's perspective) of the varying conjectures behavior is encouraging. Given our uneasiness with the all-or-nothing quality of the constant conjecture equilibrium, the possibility of optimal intermediate cartel sizes with varying conjectures reconfirms our intuition about dominant cartel/competitive fringe market structures.

Yet having analyzed these two families of cartel behavior still leaves open the question of whether there are other types of behavior a perfectly colluding cartel would prefer. This question is answered in the final Proposition:

Proposition 14: Letting $\gamma^* = 1$ for $k \in [1,n-1]$ and $\gamma^* = n$ for $k = n$ supports a globally stable cartel of all the firms in the industry and yields the largest possible cartel profits.

Proof of Proposition 14: Proposition 1 proves that $S(k,\gamma=1) < 0$, so $S(k,\gamma^*) < 0$, for $k \in [1,n-1]$. By direct computation it is easy to show that:

$$\pi_c(k=n-1,\gamma=1) - \pi_d(k=n,\gamma=n) =$$

$$- (a^2/2b)[n(n+2) + 2b(b+2) + 2bn(n-1) + nb^2(n-2) + 1]$$

$$/ (1+b+n)^2(b+1)^2(b+2n) < 0,$$

so that $S(k=n,\gamma^*) < 0$. Thus, by definitions 1 and 2, $k = n$ is both a locally and global stable cartel size. That γ^* yields maximum cartel profits comes immediately from the fact that $\gamma=k$ is the maximum profits for any given k and that $d\pi_d/dk$ equals:

$$a^2\gamma/(h+\gamma)^2(h-k) > 0$$

Q.E.D.

Proposition 14 pushes the limits of this analysis. Agreeing to Cournot behavior when any firm deserts the cartel, is analogous to penalizing firms for leaving the cartel. Serious questions remains whether this type of behavior would be credible.

Figure 15 highlights the issues of credibility and again allows us to compare the constant and varying conjectural parameterizations. As stated above, cartel behavior could be completely characterized by a price schedule for different sized cartels, $P(k)$. But since $P(k)$ is a monotonically increasing function of the conjectural variation, γ, we can reparameterize behavior in terms of γ without loss of generality. Moreover, as illustrated in Figure 15, recasting

Figure 15
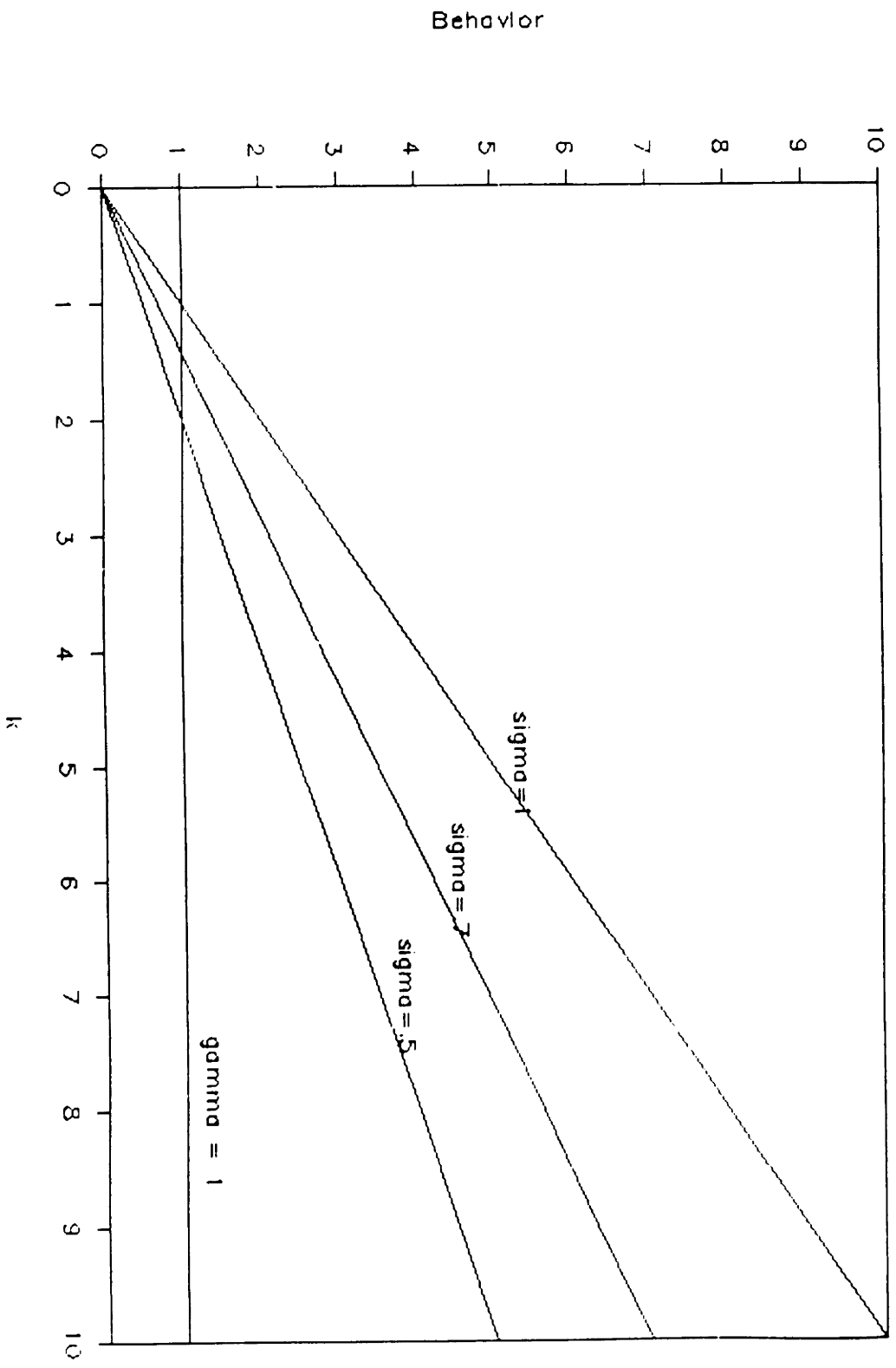
behavior in terms of γ more clearly illustrates the traditional strategic benchmarks:

$\gamma = k$ corresponds to "perfect collusion,"

$\gamma = 1$ corresponds to Cournot behavior, and

$\gamma = 0$ corresponds to perfect competition.

A minimum restriction on credibility might restrict possible cartel behaviors to the area below the $\gamma = k$ line. Since $\gamma > k$ would never be an equilibrium for a fixed k, we might expect that it would not be credible when cartel stability was at issue. This credibility restriction on cartel behavior may not be sufficient, however, as the behavior envisioned in Proposition 14 meets this requirement while still offending our intuitions about credibility. Further research seems appropriate.

From Figure 15, we can also see the dual flaw of the constant conjectural specification, $\gamma = \gamma^{fix}$. For small cartels ($k < \gamma^{fix}$), the behavior is overly collusive and possibly not credible. Moreover, as the cartel size increases ($k > \gamma^{fix}$), the behavior becomes increasingly competitive -- as the shortfall between γ^{fix} and "perfect collusion" monotonically increases. By way of comparison, the varying conjectural specification remains a constant percentage of the perfect conjecture. At a minimum, it seems clear that if "perfect

collusion" is credible for different cartel sizes (as D'Aspremont et al have modeled it), less collusive varying conjectures of the type $\gamma = \sigma k$ should also be credible.

Section 4:   Heterogeneous Costs

One of the problems with the foregoing cartel/competitive fringe model is that for any stable cartel size and any given behavior, the fringe firms produce more and are more profitable than the cartel firms. Both of these results collide with the commonly accepted stylized facts about fringe competition. We usually think of fringe firms as being small and relatively inefficient compared to the dominant firms (like, for example, American Motors relative to General Motors). There is some intuition that it is the size and possible efficiency of large firms that make them the more natural price leaders in the industry.

In this section, I try to formalize some of these intuitions by allowing for differences in size and efficiency (with heterogeneous cost functions). I extend Donsimoni's model [1985] to show that even within a broad class of varying conjectures the efficient firms in the industry will be the first to join a cartel. Moreover, I show that it is pos-

sible that the relatively efficient firms within the cartel may both produce and profit more than the free-riding inefficient firms in the competitive fringe. Finally, I show that when the competitive fringe does profit more there can be bizarre, socially inefficient strategies for cartel firms, which include throwing away part of their capital stock.

To model cost heterogeneity simply, assume now that each firm in the industry has a different cost function:

$$C(q_i) = (q_i)^2 / (2\beta_i), \quad \text{for } i = 1, n,$$

$$\text{where } \sum_{i=1}^{n} \beta_i = 1.$$

As β_i increases a firm becomes more efficient. Following Perry and Porter [1985], the β_i may be thought of as each firm's capital stock, which is a necessary input in the production process, and whose total supply is fixed for the industry (and normalized to unity).

Again, we must choose a paremeterization of cartel behavior. Ignoring the destabilizing effects of fringe competition, perfect collusion implies that cartel firm's conjectures would equal:

$$\gamma_i = \beta_k/\beta_i$$
$$\text{where } \beta_k = \sum_{i=1}^{k} \beta_i$$

Such behavior maximizes total cartel profits by equalizing each firm's marginal costs with the cartel's residual marginal revenue. A natural analog to setting:

$$\gamma_i = \gamma = \sigma k,$$

in the symmetric cost case above, is to set:

$$\gamma_i = \sigma\beta_k/\beta_i$$

when cost functions are heterogeneous. Letting σ vary between 0 and 1 spans the perfect competition and perfect collusion benchmarks. Moreover, for any σ, the marginal costs of producing the last unit will be the same for each cartel firm. This parametrization therefore eliminates any production inefficiencies within the cartel.[33] Using this cost and behavioral paremeterization, the equilibrium for any cartel group with k firms (and owning β_k capital) can be characterized:

$$P = a(h' + (\sigma-1)\beta_k) \,/\, (h' - \beta_k)(h' + \sigma\beta_k) \quad (20.1)$$

$$q_{di} = a\beta_i \,/\, (h' + \sigma\beta_k) \quad\quad\quad (20.2)$$

$$q_{ci} = a\beta_i(h' + (\sigma-1)\beta_k) \,/\, (h' - \beta_k)(h' + \sigma\beta_k) \quad (20.3)$$

$$\pi_{di} = (a^2\beta_i/2)(h' + (2\sigma-1)\beta_k) \,/\, (h' - \beta_k)(h' + \sigma\beta_k) \quad (20.4)$$

$$\pi_{ci} = (a^2\beta_i/2)(h' + (\sigma-1)\beta_k) \,/\, (h'-\beta_k)^2(h' + \sigma\beta_k)^2 \quad (20.5)$$

where h' equals b + 1.

Donsimoni [1985] showed for $\sigma = 1$, that for any stable cartel configuration, the relatively efficient firms would join the cartel and the relatively inefficient firms would

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[33] There will remain, however, the production inefficiency between cartel and non-cartel members referred to above.

remain in the competitive fringe. This result is generalized to the case of any σ in the following proposition.

Proposition 15: For any given cartel configuration:

(1) if the cartel is externally stable with respect to a firm of type β^X, then it will also be externally stable with respect to any less efficient firm $(\beta < \beta^X)$;

(2) if the cartel is internally stable with respect to a firm of type β^X, then it will also be internally stable with respect to any more efficient firm $(\beta > \beta^X)$.

Proof of Proposition 15: To prove Part (1) notice that if a cartel (with $\beta_k = \underline{\beta}_k$) is externally stable with respect to a firm of type β^X, this implies that:

$$\pi_{ci}(\beta_i = \beta^X, \ \beta_k = \underline{\beta}_k) > \pi_{di}(\beta_i = \beta^X, \ \beta_k = \underline{\beta}_k + \beta^X). \quad (21.1)$$

From equations (20.4) and (20.5), we know that for any j:

$$\pi_{cj}(\beta_j = \beta_j, \ \beta_k = \underline{\beta}_k) \ / \ \pi_{ci}(\beta_i = \beta^X, \ \beta_k = \underline{\beta}_k) =$$

$$\pi_{dj}(\beta_j = \beta_j, \ \beta_k = \underline{\beta}_k + \beta^X) \ / \ \pi_{di}(\beta_i = \beta^X, \ \beta_k = \underline{\beta}_k + \beta^X)$$

$$= \beta_j \ / \ \beta^X. \quad (21.2)$$

Together inequality (21.1) and equations (21.2) imply that for any j:

$$\pi_{cj}(\beta_j = \beta_j, \ \beta_k = \underline{\beta}_k) > \pi_{dj}(\beta_j = \beta_j, \ \beta_k = \underline{\beta}_k + \beta^X). \quad (21.3)$$

Taking the derivative of equation (20.4) with respect to β_k yields:

$$d\pi_{di}/d\beta_k = a^2\beta_i\sigma\beta_k[h'((1-\sigma)h'+(h'-\beta_k))+\sigma\beta_k(h'\sigma+2\sigma\beta_k+4h'-\beta_k)]$$
$$/ \quad (h' + \sigma\beta_k)^3(h'-\beta_k)^3 \geq 0. \qquad (21.4)$$

Because this derivative is positive for all $\sigma > 0$, we know that:

$$\pi_{dj}(\beta_j=\beta_j, \ \beta_k=\underline{\beta}_k+\beta^X) > \pi_{dj}(\beta_j=\beta_j, \ \beta_k=\underline{\beta}_k+\beta_j) \quad (21.5)$$

for all $\beta_j < \beta^X$.

Finally, combining the inequalities (21.3) and (21.5) gives us that:

$$\pi_{cj}(\beta_j = \beta_j, \ \beta_k = \underline{\beta}_k) > \pi_{dj}(\beta_j = \beta_j, \ \beta_k = \underline{\beta}_k + \beta_j) \quad (21.6)$$

for all $\beta_j < \beta^X$. This inequality is the condition for external cartel stability -- which proves that if β^X is externally stable, any $\beta < \beta^X$ will also be.

A symmetric argument can establish Part (2). Q.E.D.

Intuitively, this result -- that the relatively efficient firms will join the cartel -- is caused by a supply effect.[34] If relatively efficient firms are in the competitive fringe, the inefficient firms' residual demand is relatively small and the cartel price becomes so low that the efficient firms profit more from leading (and restricting output) than from following (and free-riding). The inefficient firms also

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[34] This analysis is taken from Donsimoni [1985].

profit from this arrangement because they prefer to free-ride on a higher cartel price (than to restrict output with a lower price).

This efficient price leadership result[35] creates the possibility that some of the cartel firms will produce more and be more profitable than some of the fringe firms. This possibility is caused by the fact that the relative efficiency could make cartel members more profitable (and more productive) than the free-riding fringe. In other words the fringes relative inefficiency could keep them from cashing in on the cartel's output restrictions.

Proposition 16: A cartel firm with capital stock β_d will be more profitable than a fringe firm with capital stock β_c, iff:

$$\beta_d/\beta_c > R_p = (h' + (\sigma-1)\beta_k)^2 \ / \ (h' + (2\sigma-1)\beta_k)(h' - \beta_k), \quad (22.1)$$

and will produce more iff:

$$\beta_d/\beta_c > R_q = (h' + (\sigma-1)\beta_k) \ / \ (h' - \beta_k). \quad (22.2)$$

----

[35] In a supergame model, Rotemberg and Saloner [1986] have similarly derived a collusive equilibrium where the firm most efficient at acquiring demand information becomes the price leader.

<u>Proof of Proposition 16</u>: The inequalities (22.1) and (22.2) can be directly calculated from equations (20.2) - (20.5).

Q.E.D.

Since $R_p < R_q$, any cartel firm that produces more than a fringe firm will be more profitable (but greater profitability does not imply greater production). From Proposition X, we know that $\beta_d > \beta_c$, and the larger this efficiency disparity the more likely that the cartel firm will be more profitable or produce more. It is possible that each fringe firm would be more profitable than each cartel firm.[36]

<u>Proposition 17</u>: Each firm in a cartel will be more profitable than each firm in the competitive fringe iff:

$$\beta_{marg-d}/\beta_{marg-c} > R_y \qquad (23)$$

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[36] For example, imagine a cartel configuration where all the cartel firms were only slightly more efficient than all the fringe firms -- so that the average cartel efficiency, β_d, was only slightly larger than the average fringe firm efficiency, $\beta_c = \beta_d$ + epsilon. Under this situation the free-riding effect would certainly dominate the efficiency effect -- thus making the fringe firms more productive and profitable.

where $\beta_{marg\text{-}d}$ and $\beta_{marg\text{-}c}$ are the respective efficiencies of the least efficient cartel firm and the most efficient fringe firm.

Proof of Proposition 17: From Proposition 17 we know that inequality (23) guarantees that the infra-marginal cartel firm is more profitable than the infra-marginal fringe firm. Since all the other cartel firms have, by definition, $\beta > \beta_{marg\text{-}d}$, all the cartel firms are more profitable that the infra-marginal fringe firm. By analogy we know that all the other fringe firms are less profitable than the infra-marginal cartel firm.

Q.E.D.

Focusing on the possibility of having the cartel firms be more profitable than fringe firms does more than salve our casual empiricism. For if fringe firms are more profitable than cartel firms, competition for these profits can take a bizarre form.

Anytime a less efficient fringe firm is more profitable than a cartel firm, the cartel firm would be willing to pay the fringe firm to take some of its capital. This is because if the cartel firm shifted the difference in capital to the

fringe firm, it would trade places with that firm and begin to make the higher fringe profits.

Moreover, even if the fringe firm refused such an offer (or if such offers were outlawed), there will be situations where efficient cartel firms will have incentives to destroy some of their capital so that they can become relatively inefficient and thus join the more profitable ranks of the competitive fringe. Assume, for example, that the infra-marginal cartel firm (i.e. the one with the smallest amount of capital) has just slightly more than the infra-marginal fringe firm. By throwing away some capital and thereby committing to inefficiency and smallness, the infra-marginal cartel firm could trade places with the marginal fringe firm and increase its profits!

This result raises the possibility that only skewed distributions of firm sizes (for example, as implied by inequality (23)) will be stable. In other words, a new notion of cartel stability might require that cartel members be sufficiently big relative to the fringe and so that there are not incentives to throw away capital. Such a skewedness requirement is strongly corroborative of Gelman and Salop [1983] conception of judo economics.[37]

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[37] It also is analogous to the result in Quasi-Monopolies, where small defectors were not worth punishing.'

Section 5: Conclusion

In this paper I have analyzed the optimal behavior patterns within each of two symmetric families of cartel behavior. Although other parameterizations of behavior might exist which generate higher cartel profits, the results of this paper at least suggest that restricting cartel behavior to what has been traditionally called perfect collusion is not necessarily optimal. Moreover, in looking at equilibrium cartel sizes we should be able to draw inferences about the types of behavior that would be stable.

Finally, I have examined heterogeneous costs structures. Under the varying conjectures specification, the relatively efficient firms join the equilibrium cartel. That the efficient are the natural price leaders not only confirms our intuitions, but suggests another type of stability condition under which each cartel firm must be more profitable than each fringe firm. Such a restriction would imply a discontinuous jump in market share between cartel and competitive fringe firms -- a skewedness which in itself comports with casual observation.

Appendix

This appendix describes the results of a grid search to evaluate whether stable cartel sizes were unique for different industry sizes, demand slopes and cartel behavior. The stability condition of equation (5.5):

$$S(k,e) = (a^2/2)[(h+(e-1)(k-1))^2(h-k)(h+ek)^2 -$$
$$((2e-1)k+h)(h-k+1)^2(h+e(k-1))^2] /$$
$$(h-k+1)^2(h+e(k-1))^2(h-k)(h+ek)^2,$$

was evaluated for:

industry sizes, n, ranging between 1 and 10

(varying by 1);

demand slopes, b, ranging between 0 and 10

(varying by .5);

cartel behavior, e, ranging between 0 and 1

(varying by .05).

For each permutation of these values the stability condition for each possible cartel size (k varying by 1 between 0 and n) was calculated. By comparing the signs of the stability conditions for the different cartel sizes, the equilibrium stable cartel sizes for each permutation (fixed n, b, and e) was determined.

Of the 2000 industry permutations that were evaluated (10 n's x 20 b's x 20 e's) only 27 exhibited non-unique cartel size equilibrium. The non-unique equilibria were:

```
n =  6; b = .5; e = .90, .95, 1.00
n =  7; b = .5; e = .85, .90, .95, 1.00
n =  8; b = .5; e = .65, .70, .75, .80, .85
n =  8; b =  1; e = .60
n =  9; b = .5; e = .55, .60, .65, .70, .75
n =  9; b =  1; e = .50
n = 10; b = .5; e = .50, .55, .60, .65, .70
n = 10; b =  1; e = .45
```

In each instance in which the equilibrium cartel size was not unique, there were two stable cartel sizes: one in the interior (that is less than the number of firms in the industry) and a second that comprised all the firms in the industry.

Each instance in which uniqueness failed corresponded to instances in which b was small relative to n. Donsimoni et al [1985] proved that for e = 1, the stable cartel size will be unique for all $b > b*(n) = n[(v/(v^2-1)^{(1/2)}) - 1]$, where $v = 8/(1 + (17)^{(1/2)})$. b* is approximately equal to .302(n). All instances of non-uniqueness occurred well within the Donsimoni condition of b < b*(n).

The results of the grid search confirm that the equilibrium cartel size will be unique for a broad range of reasonable parameter values. Only when the slope of the demand curve is so flat relative to size of the industry is there a

danger of non-uniqueness and in those instances there will be an interior and an industry stable cartel.

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CHAPTER 3:   Determinants of Airline Carrier Conduct

Section 1: Introduction

Since Joe Bain's seminal work [Bain (1959)], industrial organization research has focused on structure, conduct, and performance. Bain thought that, aside from feedbacks of second order, structural variables determined industry conduct and that in turn structure and conduct determined industry performance. Dozens of empirical articles have examined the structure-performance relationship [Phillips (1976)] by regressing structural variables (such as concentration) on performance variables (such as profits). The underlying purpose of such regressions, however, was to test the relationship between structure and conduct -- for example, whether concentration facilitated collusion. Demsetz [1973] noted, however, that often such tests of conduct are unidentified -- since firm-specific efficiency as well as collusion could induce a positive correlation between profits and concentration. This chapter represents a first attempt in trying to overcomes the inherent problems of inferring conduct from performance by regressing estimates of conduct, itself, on the structural variables that theory suggests

induce collusive behavior. Such a regression can then be used to test more directly, for example, Stigler's hypothesis [1964] that the number of sellers influence the degree of collusive behavior and Posner's hypothesis [1975] that collusion is likely to take place in markets in which the gains from collusion are the greatest.

Calculating quantitative estimates of behavior is the crucial starting point. Following Iwata [1974], marginal cost and elasticity estimates were combined with price and market share data to estimate conjectural variations in the airline industry both before and after deregulation. Collapsing an airline's behavior into a scalar strategy variable places restrictive assumptions on the model. Moreover, as discussed below, this chapter's calculation of Iwata's conjectural measure is closely related to the Lerner index of market power. As such, many of the regressions can be given a traditional performance-on-structure interpretation [see, for example, Collins and Preston (1969); Weiss (1974)].

Studying the airline industry, however, allowed the estimation of over two thousand conjectural variations (by carrier and route) using similar marginal cost data. And the conjectural variation of a firm retains some appeal as an appropriate measure of market conduct because it not only

represents the firm's expectations of other firms' conduct, but by feeding into its own reaction function determines the firm's own conduct (its non-cooperative strategy).^[1]

Section two of the chapter describes the calculation of the conjectural variations, and tests whether carriers displayed competitive, collusive or Cournot behavior. Section three forms the central part of the chapter. There, I describe the estimation of the structure/conduct regressions, test whether specific structural variables influence conduct and examine the robustness of my regressions. In section four, slopes of the firms' reaction curves are estimated. Tests for the equality of the conjectured and actual reaction curve slopes are made. Such tests are shown to test not only for the presence of Bresnahan consistency [1981] but also for a generalized form of Stackelberg leadership.

Section 2: Conjectural Variation Estimates

As Iwata showed in his 1978 <u>Econometrica</u> article [1974], estimates of conjectural variations can be derived from firms' profit maximizing first-order conditions. Marginal revenues equals:

(1) $MR_i = P + q_i(dP/dq_i) = P + q_i(dP/dQ)(dQ/dq_i)$

$\qquad = P + q_i(dP/dQ)(1 + (dq_j/dq_i))$

(1)' $\qquad = P + q_i(dP/dQ)(1 + k_i)$

$\qquad = P - P(S_i/e)(1 + k_i)$

where P = price, q_i = firm i's output, Q = total market output, k_i = dq_j/dq_i = firm i's conjectural variation, e = market price elasticity and S_i = firm i's market share. By setting marginal revenue equal to marginal cost, MC_i, we can derive a measure of conjectural variations in terms of the Lerner index (price and marginal cost), market share and elasticity of demand:

(2) $\qquad MC_i = P(1 - (S_i/e)(1 + k_i))$

$\qquad (P - MC_i)/P = L_i = (S_i(1 + k_i))/e$

(3) $\qquad k_i = ((eL_i)/S_i) - 1$

The conjectural variation measure ($k_i = dq_j/dq_i$) is an expectation of how competitively a firm's rivals will react to a change in the firm's output. A higher value of k_i indicates that 1) a firm expects its competition to act more collusively, and 2) the firm, itself will act more col-

lusively (the slope of its reaction function changes). Under the competitive (or Bertrand) assumption $k_i = -1$; under Cournot $k_i = 0$; and under perfect collusion $k_i = (1/S_i) - 1$[2]. More generally, since positive conjectures reflect the expectation that output restrictions will be matched by competitors, Anderson [1977] has suggested that such "matching" conjectures imply at least an attempt at collusion.

Conjectural variations were calculated for the regulated year 1975 and the deregulated year 1982.[3] The conjectural variation estimates for 1975, a year in which CAB set fares, are used as a regulatory benchmark in analyzing the effect of deregulation and should be interpreted "as if" firms had these conjectures. This derivation of the conjectures implies a single price and a single quality. In the airline industry such assumptions are suspect. Fares vary not only between firms but within firms [see Borenstein (1983)]. Competition by offering more frequent flights or through other and other types of quality characteristics increase the dimensionality of the strategy space. Moreover, conjectural variations are assumed to be exogenous to the determination of price and market share.

Finally, as noted in the introduction, the conjectural measure of conduct is closely related to the Lerner measure

of performance. From equation (3), a firm's conjectural variation estimate is a function of the market demand elasticity, the firm's Lerner index and the firm's market share. Since the estimation of equation (3) assumed a constant elasticity across routes,^[3.5] the deregulated conjecture for a firm can be reinterpreted as a monotonic transformation of a firm's Lerner index weighted by its market share. The correspondence between the conjectural estimate and the Lerner index is even closer for the conjectural estimates of the regulated era -- as the lack of market share data necessitated the calculation of average route conjectures that are solely a function of the Lerner indicies on a particular route.^[3.6]

The variance of the conjectures was also estimated. Using a first-degree Taylor expansion, equation (3) becomes:

$$(\hat{k}_i - k_i) = (dk_i/de)(\hat{e} - e) + (dk_i/dMC_i)(\hat{MC}_i - MC_i)$$

where hatted and unhatted symbols represent estimated and true values, respectively. If we assume that the marginal costs and elasticity estimates are unrelated, squaring and taking the expectation of this equation yields an expression for the variance of the conjectural variation estimate.

(4) $\text{Var}(k_i) = (dk_i/de)^2 \text{Var}(e) + (dk_i/dMC)^2 \text{Var}(MC).$^[4]

Results. The estimates of conjectural variation in 1982 were distributed:

k<-4	-4<k<-2	-2<k<-1	-1<k<0	0<k<1	1<k<2	2<k<4	4<k
7	29	103	518	682	307	306	137

The mean conjecture for 1982 was .885, roughly halfway between Cournot conduct and the average perfectly collusive conjecture of 1.86 (derived from the average carrier share of .34). 24 percent of the carriers had relatively competitive conjectures between Cournot and Competitive conduct(-1<k<0). While the conjectures less than -1 imply, by equation (3), that price was below the estimate for marginal cost, such observed shortfalls (6 percent of the sample) were never sig- nificantly different from zero.

T-tests (reported in Table 2) clearly rejected (at 1 per- cent significance) the extreme behaviors of perfect competi- tion or collusion for any carrier. The average conduct for each carrier was more collusive than Cournot and for 11 of the 16 carriers significantly so (at 5 percent level). Trunk carriers appeared to act more collusively than the local car-

riers, whose average conjectures were .969 and .610 respec-
tively. Indeed, the equality of the trunk and local means was
rejected $(F(1,2087) = 31.5)$. This result is not unexpected
given that the trunks are larger and more firmly established.

In Table 3 the results of individual t-tests reinforce
this picture of a matching behavior more collusive than
Cournot but less than perfect collusion. In only 14 percent
of the sample can matching conduct be rejected.

Table 2

Average Conjectural variations and T-Tests for Competitive (-1), Cournot (0), and Perfectly Collusive ((1/S) - 1) Behavior

Carrier	Average Conjecture	Perfectly Competitive	Cournot	Perfectly Collusive
All Carriers	.885	5.927*	2.782*	-6.899*
Trunk	.969	6.194*	3.048*	-5.993*
United	.719	5.406*	2.260**	-5.791*
Eastern	1.306	7.254*	4.109*	-5.472*
Delta	.867	5.872*	2.726*	-6.129*
American	1.138	6.723*	3.578*	-7.448*
Trans World	.777	5.588*	2.443**	-8.109*
Braniff	2.855	12.109*	8.968*	-6.951*
Northwest	1.526	7.944*	4.799*	-6.039*
Western	.354	4.260*	1.116	-9.079*
Continental	.472	4.627*	1.483	-10.826*
Local	.610	5.074*	1.922**	-6.584*
US Air	.258	3.959*	.811	-5.766*
Ozark	.287	4.051*	.905	-4.988*
Piedmont	.369	4.307*	1.162	-7.379*
Republic	1.001	6.294*	3.149*	-5.776*
Texas Int.	.631	5.126*	1.983**	-7.824*
Frontier	.797	5.651*	2.507**	-9.887*

 * 1 percent significance level
 ** 5 percent significance level

Table 3

The Number of Carrier-Routes for which the Estimated Conjectures Were Consistent With Competitive (-1), Cournot (0), Matching (>0) and Perfectly Collusive ((1/S) - 1) Behavior*

Carrier	Perfectly Competitive	Cournot	Matching	Perfectly Collusive
All Carriers	916(1773)	891(1198)	1791(298)	434(1655)
Trunk	273(1350)	650(943)	1364(229)	308(1285)
United	65(224)	137(152)	231(58)	57(232)
Eastern	15(235)	91(159)	233(17)	65(185)
Delta	52(259)	142(169)	269(42)	81(230)
American	32(191)	90(133)	198(25)	29(194)
Trans World	28(123)	61(90)	122(29)	26(125)
Braniff	0(41)	4(37)	41(0)	4(37)
Northwest	5(60)	28(37)	62(3)	21(44)
Western	17(73)	30(60)	67(23)	13(77)
Continental	27(112)	55(84)	108(31)	9(130)
Local	75(423)	243(255)	429(69)	128(370)
US Air	25(92)	61(56)	92(25)	42(75)
Ozark	2(34)	19(17)	32(4)	12(24)
Piedmont	9(73)	51(31)	76(6)	22(60)
Republic	23(139)	73(89)	145(17)	40(122)
Texas Int.	7(42)	23(26)	41(8)	6(43)
Frontier	7(43)	14(36)	41(9)	4(46)

*A conjecture is considered consistent with a given behavior if its value is not significantly different from the postulated behavior at a 5 percent significance level. The number of observation in which the behavior is not consistent is in parentheses.

The estimates of 1822 conjectures in 1975 were distrib-uted:

k<-4	-4<k<-2	-2<k<-1	-1<k<0	0<k<1	1<k<2	2<k<4	4<k
0	0	0	183	902	554	157	26.

A comparison of the regulated and unregulated conjectures support common theories about deregulation. The mean conjec-ture was higher under regulation (.982) than on the same routes in 1982 (.680) indicating that as we might expect there was more competition after deregulation.[5] An F-test of each year's mean strongly rejected their equality $(F(1,3652) = 74.13)$. The variance of the conjectures, more-over, increased with deregulation (from .896 in 1975 to 1.35 in 1982). Thus, regulated conduct was less competitive but more uniform. This result suggests the possibility that regulation, while in general impeding competition, may have improved conduct on certain routes which were particularly susceptible to collusion (for example, because of high bar-riers to entry or concentration).

Section 3: Determination of Conjectural Variations

The central analysis of this chapter is an attempt to estimate how conjectural variations are determined -- that is, how firms form expectations about how their rivals will act. Within the model used in this chapter, the concentration of an industry cannot determine the conjecture. In this model the first-order equations of the n firms in the market determine the price and the n-1 market shares:

$$MC_i = P(1 - (S_i/e)(1 + k_i)) \text{ for } i = 1, n.$$

The conjectural variations are assumed to be exogenous to these equations. The conjectures, then, determine the price and market shares; the market shares (and combinations of them like the Herfindahl index) do not determine the conjectural variation. The determination of conjectural variations may be part of a larger simultaneous system in which market share and conjectures are jointly determined. This would especially make sense in a dynamic system. For the purposes of this chapter, however, I assume that the variations are determined by a set of variables that are exogenous to the firm.

In searching for the appropriate set of exogenous variables, it is important to realize that the firm in forming its expectation is analytically in the same position as the

economist in forming her expectation about which structural characteristics will lead to collusive behavior. Armed with this insight, we can then look to see if theories of how structure effects conduct coincide with the expectations of the firms themselves.

A) From Stigler's "A Theory of Oligopoly," [1964] firms should expect more collusive behavior from their rivals when the number of sellers is small. The greater the number of sellers, the greater the gains from deviating from collusive behavior, and the harder it is to detect cheating. Like Stigler, I have assumed the number of sellers to be exogenous.

B) Following Posner [1975], firms should expect more collusion on routes in which the gains from collusion are greater. The routes with large potential monopoly rents are those with relatively inelastic demand. Tourist and long-haul demand has been found to be more elastic than business and short-haul demand.^[6] Accordingly, route distance and a tourism measure were hypothesized to influence firm behavior.

C) Because excluding new entry is necessary for successful collusion, I included a dummy variable for slot constrained airports and the number of newly certified carriers. In 1982 the runways at four airports (New York's Kennedy and

Laguardia, Chicago's O'Hare and Washington's National) were so congested that the F.A.A. limited the number of takeoffs and landings (constrained the number of slots) [Creager (1983)]. Slot constraints, by excluding willing competitors, should have allowed carriers to collect scarcity rents. More generally, barriers to entry were proxied by the number of newly certified carriers serving a route under the theory that the absence of new competition should have allowed more collusive conduct.[7]

D) Both empirical and theoretical studies of the airline industry have concluded that non-price competition tended to replace price competition during the regulated era [for example, see Douglas and Miller (1974)]. To test whether non-price competition continued to be a substitute for price competition, the number of routes flights per week where included in the regression.[8]

E) Finally, the conjectured response of rivals may hinge on the identity of either the carrier making the expectation or the rivals whose response is being predicted. For example, as noted by Gollop and Roberts [1979], different rivals may have different capacities to respond to changes in output. Conversely, different firms might correctly expect different responses from a given rival.[9] The identities of the

firms making the expectations and the identities of their route-specific rivals also might capture inter-route reputational effects. Carriers may systematically misestimate their rivals' behavior or attempt to establish "tough" reputations themselves. For these reasons, dummy variables for both the carrier forming the expectation (the ith carrier) and for its route-specific rivals were included in the regression. The final specification of the conduct equation was:

$$k = a_c + a_r + a_{slot} + b_1 \#CARRIERS + b_2 \#NEWCERT +$$
$$b_3 TOURISM + b_4 DIST + b_5 FLIGHTS,$$

where: a_c = carrier dummies
a_r = rival dummies
a_{slot} = slot dummies
#CARRIERS = number of carriers serving route
#NEWCERT = number of newly certified carriers serving route
TOURISM = index of tourism
DIST = route distance
FLIGHTS = number of non-stop flights per week.[10]

Results. The slope coefficients (excluding carrier and rival dummies which are reported in Table 4) for the 1982 structural regression were:

Regressor	Coefficient
a_{slot}	.1074 (1.2693)
#CARRIERS	- .4655 (-4.2267)

#NEWCERT	- .0170
	(- .1652)
TOURISM	-2.4654
	(-1.2439)
DIST	- .0013
	(-15.5170)
FLIGHTS	.0020
	(1.9890)

All estimates from both regressions are of the expected sign. The number of carriers is not only significant but large. The addition of two carriers would cause virtually competitive conduct on a route that otherwise would be Cournot.

The competitive impact of the number of firms on carrier conduct is direct evidence to support Stigler's collusion and refute Demsetz's efficiency hypotheses. Its size is also a rejection of Baumol, Panzar and Willig's concept of contestability [1982]. While Bailey [1981] has used airlines as an illustration of a contestable industry, contestability should lead not only to more competitive conduct but to conduct which is insensitive to the number of carriers actually serving a route. In a contestable market those waiting in the wings should exert as great a pro-competitive force as actual rivals. This lack of contestability in 1982 may, however,

reflect a transition to a deregulated equilibrium (Bailey and Baumol [1984]).

While the presence of newly certified carriers has the expected sign, it is neither large nor significant, implying that their competitive influence is largely captured through the #CARRIERS variable. Although casual empiricism might suggest that certain new carriers (for example, People's Express) behave quite competitively, this tendency has a large variance. The slope estimates of the elasticity variables (tourism and distance) seem to indicate that, as Posner predicted, carriers collude more on less elastic routes. Route distance, however, is by far more important than tourism in influencing behavior. Not only is the distance coefficient estimated more accurately, but it contributes much more to changes in route conduct. For example, the conjecture for the Denver-Phoenix route (589 miles) should, ceteris paribus, be 1.3 less than Denver-Philadelphia (1569 miles). Tourism's impact, however, even on relatively different routes is negligible. For example, the conjecture for the Detroit-Dayton route (a low tourism route) should be, focusing only on tourism, only .05 less than Detroit-Fort Lauderdale (a high tourism route). The number of non-stop flights per week, a proxy for non-price competition, was

found to depress price competition significantly. A route with many non-stop flights (300 per week) should have a conjecture .5 higher than a route with relatively few non-stop flights (50 per week). Finally, an F-test (F(29,2053) = 9.75) decisively rejected equality of the carrier and rival dummies. The trunk carriers have systematically less competitive conjectures (and therefore behavior) than those of the locals.

The regression results were robust to the use of alternative elasticity and marginal cost estimates. The signs, magnitudes and significance of all the structural coefficients were robust to the use of other elasticity measures ranging from Borenstein's estimate [1983] of -2 to Devany's estimate [1975] of -1.07. The conduct regression was similarly robust to two alternative estimates of marginal cost which controlled for quality^[11] and endogeneity of output^[12] respectively.

Table 4

Regressor	Coefficient
a_{slot}	.1074 (1.2693)
#CARRIERS	− .4655 (−4.2267)
#NEWCERT	− .0170 (− .1652)
TOURISM	−2.4654 (−1.2439)
DIST	− .0013 (−15.5170)
FLIGHTS	.0020 (1.9890)

Trunks	As Carrier Forming Expectation	As Rival
United	2.7955 (16.7360)	.5226 (3.7044)
Eastern	2.7148 (14.5370)	.3967 (2.8729)
Delta	2.5049 (14.6922)	.6444 (4.7281)
American	3.3636 (17.5310)	.6947 (4.8718)
T.W.A.	2.6067 (14.2470)	.4476 (3.0163)
Braniff	3.8087 (13.7190)	.3171 (2.1422)
Northwest	3.0468 (12.1630)	.5749 (4.0559)

	As Carrier Forming Expectation	As Rival
Western	2.3262 (11.1780)	.4573 (3.3651)
Continental	2.4402 (13.2500)	.3103 (2.1353)

Locals
U.S. Air	1.3252 (8.1903)	.7093 (4.7182)
Ozark	1.3495 (4.9029)	.6938 (3.9930)
Piedmont	1.3883 (8.0216)	.5180 (3.3158)
Republic	2.2449 (14.1370)	.5489 (3.9369)
Texas Int.	1.7949 (1.1976)	.4452 (2.3792)
Frontier	2.4187 (10.7830)	.3489 (2.3181)

R-squared 0.3334
Number of Observations 2089
Heteroskedastic-Consistent t-Statistics in parentheses

Section 4: The Slope of the Reaction Curve and Consistency

If we maintain our implicit assumption of constant demand elasticity, it is relatively straight-forward to calculate the slope of the reaction function. Let the inverse demand function be:

$$P = aQ^{-1/e}$$

so that the elasticity of demand is:

$$e = -(dQ/dP)(P/Q)$$

Marginal cost and marginal revenue from equations (1) and (3A) equal:

$$MR_i = P + q_i(dP/dQ)(1 + k_i)$$

$$MC_i = (CT_i/q_i)(a_q + d_{qq}\ln q_i + \Sigma_i\, r_{qi}\ln W_i + \Sigma_i\, m_{qi}\ln Z_i)$$

By setting marginal cost equal to marginal revenue and using the implicit function theorem, it can be shown that the slope of firm i's reaction curve equals:

(4) $dq_i/dq_{-i} = -\,P'(1-B)/(P'(2 + k_i - B) - MC')$

where $q_{-i} = \sum_{j \neq i} q_j$,

$P' = (dP/dQ) = -(1/e)(P/Q)$

$B = S_i(1 + k_i)(1 + e)/e$

$$MC' = (AC_i d_{qq} + (MC_i/AC_i)(MC_i - AC_i))/q_i$$

The slope of the reaction function is the actual response of a firm to its competitors' actions. The reaction function expresses a firm's strategy, and the firm's conjectures of rivals' behavior (k_i) directly effect this strategy.

Bresnahan [1981] has suggested that the multiple equilibria of non-cooperative games be restricted to those that are consistent conjectural variation equilibria. In a consistent equilibrium, the slope of the reaction curve is locally the same as the conjectural variation. Using equation (4), it is possible to compare the conjectural variation of a firm to the actual slope of the rivals' aggregate reaction curve. If the markets are consistent, then:

$$k_i = \sum_{j \neq i} \frac{dq_j}{dq_i} = \sum_{j \neq i} \left(\frac{dq_j}{dq_{-j}}\right) \left(\frac{dq_{-j}}{dq_i}\right) = \sum_{j \neq i} \left(\frac{dq_j}{dq_{-j}}\right),$$

because $\dfrac{dq_{-j}}{dq_i} = 1.$

In other words the firms expectation about how its rivals will react to a change in its output must equal the rivals actual reaction, the sum of the slopes of the rivals' actual reaction curves. For example, as Bresnahan showed [1981], Bertrand conjectures (equaling -1) with constant returns to scale are consistent. This c⌐n be seen as equation (4) equals

-1 when k_i equals -1.

The concept of consistency can also be used heuristically to shed light on Stackelberg leadership. To be consistent is, in a sense, to know your rival's reaction function. This in essence is what a Stackelberg leader knows.^[13]

In a Cournot-duopoly world with linear demand and constant marginal cost, Figure 16 shows that if firm A changes to a consistent conjecture (from k=0 to k=-1/2,entailing a shift of firm A's reaction curve from AA to AA'), the Stackelberg outputs (and price) are duplicated. If rivals expectations are inconsistently fixed, a firm has a powerful incentive to make a consistent conjecture; the incentive is the profits of a Stackelberg leader. The striving of all firms to be consistent may be interpreted as the desire to be a Stackelberg leader. Consistency, however, generalizes the Stackelberg concept because it allows "leadership" even when rivals act more or less collusively than Cournot.

Results. The mean of the estimated slope parameter for 1982 was .119, while the mean conjecture was .885. This result (that the conjectured reaction of rivals was systematically more collusive than the rivals' actual reactions) parallels the linear Cournot model in which firms expect relatively collusive conducts (k = 0) when in fact the rivals

Figure 16
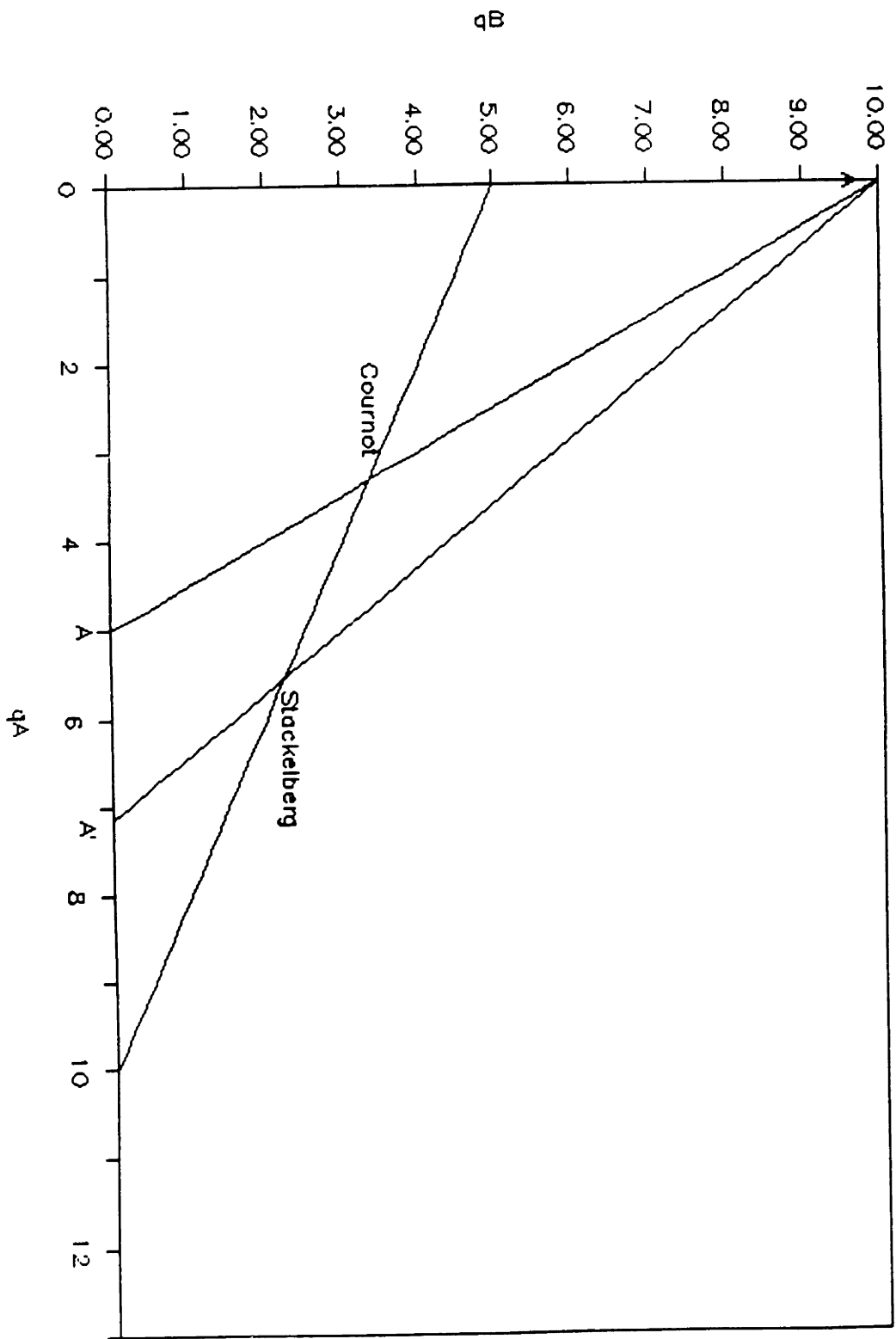
respond more competitively (slope = -.5). This tendency to systematically expect overly collusive behavior might be explained as a temporary effect of deregulation and is an area for further research.

The variance of the slope was approximated using the same method used to estimate the variance of k_i in equation (4)[14] and heuristic tests for the difference between the actual and conjectured slope were made.[15]

The null hypothesis of conjectural consistency is strongly rejected. As reported in Table 5, the difference between the mean conjecture and the mean slope was significantly different from zero at more than 1 percent confidence level (t = 20.32). More generally, the local carriers came closer to being consistent -- with 4 of the 6 failing to reject the possibility, while all the trunks rejected the null at a 1 percent significance level.

TABLE 5

Test for Consistent Conjectures and Stackelberg Leadership

Carrier	t-Statistic
All Carriers	20.3*
Trunk	
United	5.3*
Easter	20.0*
Delta	9.2*
American	8.4*
Trans World	5.3*
Braniff	30.9*
Northwest	6.3*
Western	2.3*
Continental	2.3*
Local	
US Air	.3
Ozark	1.3
Piedmont	1.1
Republic	6.2*
Texas Int.	1.4
Frontier	6.7*

* 1 percent significance level.

The size and competitive reputation of the trunk carriers may make it easier to predict their responses and partly explain why local carriers more consistently estimate their rivals' reactions. This result also illuminates the relationship between consistency and Stackelberg leadership. Under the conjectural reformulation, large and established firms like the trunk carriers have no natural advantage in being Stackelberg leaders. All firms have incentives to make consistent conjectures.

Section 5: Conclusion

This chapter has attempted to test structure-conduct relationships. Reducing the conduct space to scalar strategies requires extreme assumptions and the conjectural measures of conduct, themselves, are closely related to performance measures they were intended to supplant. Even interpreted as modified Lerner indices, however, the conjectural measures can claim to be superior to raw price-cost margins as they control for differences in market shares.

Given these qualifications, this chapter has examined specific determinants of firm conduct. Despite the extreme assumptions of the model, the regressions confirm our intuitions that:

1) the number of sellers has a dramatic procompetitive impact on firm behavior, and

2) routes with larger potential monopoly rents tend to have more collusive behavior.

Moreover, as theories of regulation might predict, deregulated conduct is more competitive but less uniform than regulated conduct. Finally, measures of the firms' reaction functions indicates that conjectures are not consistent but overly collusive.

Footnotes

^[1] These conjectures, however, like the non-cooperative models they come from, are static measures of firm conduct. The impossibility of reactions in one-period games makes these models internally inconsistent. Not only are the assumptions of the such models literally false, but given the assumptions no normal form representation of the game exists.

^[2] Such collusive conjectures maximize total industry profit. For example, in a duopoly the first order conditions for joint profit maximization is:

$$P + q_i(dP/dQ) + q_j(dP/dQ) = MC_i$$

setting this equal to equation (1)' yields

$$(dP/dQ)q_i(1 + k_i) = (dP/dQ)(q_i + q_j)$$

which when solved for k_i equals:

$$k_i = (1/S_i) - 1.$$

However, if carrier's marginal costs, as described below in note 3 are not equal, perfect collusion would entail output only from the low-cost producer and side payments.

^[3] DESCRIPTION OF DATA. All route-specific data come from CAB _Origin and Destination Survey_ (_O+D Survey_), a 10 percent sample of all airline tickets. This data set (kindly provided by Severin Borenstein) is on a CAB computer tape referred to as the O+D Dollar Amount of Fares, DB1 Summary Computer Tape.

Routes. The 1073 routes represented in the sample were city-pairs in the _O+D Survey_ which averaged at least 50 passengers a day.

Carrier/Routes. The 2089 Carrier/Routes (for example, United serving Boston-Washington) were in the sample. Observations in which carriers had less than 10 percent of a route market were discarded under the assumption that such fringe competitors were price takers. Observations for carriers whose marginal cost was unavailable (because they were excluded

from Caves, Christensen and Tretheway [1984] data set, see below) were also not included.

Fares. Fares for 1975 and 1982 were estimated as the average fare in the different passenger classes (Y, K, etc.) weighted by the proportion of passengers flying in each class. Both the proportions of passengers and the class fares were taken from the CAB, DB1 Summary Computer Tape.

Route Market Share and Distance. Both route distances and 1982 market shares were taken from CAB, DB1 Summary Computer Tape. Market shares were not available for 1975. Using the fact, however, that shares add to one, an average route conjectural variation for the 1073 routes in the sample was derived. From equation (3) we have:

$$S_i = (eL_i)/(k + 1)$$

$$\Sigma_i \ S_i = 1 = e\Sigma_i \quad (L_i/k_i + 1)$$

Assuming $k_i = k_j = k$:

$$k = (e\Sigma_i \ L_i) - 1$$

Elasticity of Demand. An elasticity of 1.3 was taken from an estimate of Brown and Watkins [1971]. This measure was bounded by several airline demand studies. Devany [1975] estimated an elasticity of demand of 1.07; Borenstein [1983] 2. My estimate of the Panzar-Rosse test statistic [1977] similarly could not reject the null hypothesis of elastic demand. The test is based on the reduced form estimation of the log of revenues on all demand and supply side exogenous variables, including the logs of input prices. A negative sum of the input coefficients indicates that the firm is facing inelastic demand.
The reduced form equation was specified:

$$\ln REV = a_T + a_F + \Sigma_i \ b_i \ln W_i + \Sigma_i \ f_i \ln Z_i$$

$$+ .5\Sigma_i \ \Sigma_j \ t_{ij} \ln Z_i \ln Z_j + \Sigma_i \ d_i DEMAND$$

where REV = total annual revenues per carrier
   $DEMAND_i$ = demand side variables (per capita disposable

income, unemployment rate, train/bus/car
price indices, frequency delay).

The results of the test for pre- and post deregulation
were:

	P/R Statistic	Standard Error
1970 - 1978	-1.38442	45.81
1979 - 1981	-0.00004	1459.27

While the point estimates of the Panzar-Rosse statistic were
all negative, the null hypothesis of elastic demand could not
be rejected because of the imprecision of the estimates.
The elasticity measurement suffers from its invariance
between routes. Several studies (Abrahams [1983], Morrison
and Winston [1983], and Anderson and Kraus [1981]) have indi-
cated that 1)tourist routes have more elastic demand than
business routes and 2) long-hauls have more elastic demand
then short-haul. The robustness of its use is discussed
below.

Marginal Cost. Estimates of marginal cost were derived from
the work of Caves, Christensen, and Tretheway [1984]. Those
authors used a panel data set of 27 carriers (trunk and
local) from 1970-81 to estimate a translog cost function.
This translog model related airline costs to annual output
(passenger miles), three input prices (labor,fuel and capi-
tal) and four measures of airline characteristics (average
stage length, load factor, points served, and frequency
delay). The final translog specification was:

(1A) $\ln CT = \Sigma_T a_T + \Sigma_F a_F + a_q \ln q + \Sigma_i b_i \ln W_i$

$+ \Sigma_i f_i \ln Z_i + .5 d_{qq}(\ln q)^2 + .5\Sigma_i \Sigma_j g_{ij}\ln W_i \ln W[j$

$+ .5\Sigma_i \Sigma_j t_{ij}\ln Z_i \ln Z_j + \Sigma_i r_{qi}\ln q \ln W_i$

$+ \Sigma_i m_{qi}\ln q \ln Z_i + \Sigma_i \Sigma_j l_{ij}\ln W_i \ln Z_j$

where $g_{ij} = g_{ji}$, $t_{ij} = t_{ji}$

CT = total annual costs of carrier
a_T = year effects
a_F = firm effects

q = total passenger-miles of carrier
W_i = input prices (fuel,labor and capital)
Z_i = airline characteristic control variable
(load factor, average route distance, number
of points served, and frequency delay)

Using the Caves, Christensen and Tretheway parameter
estimates (shown in Table 1), I was able to calculate
marginal costs per passenger mile from the analytic deriva-
tives of cost:

(3A) $MC = (dlnCT/dlnq)(CT/q) = (CT/q)(a_q + d_{qq}lnq$

$+ \Sigma_i r_{qi}lnW_i + \Sigma_i m_{qi}lnZ_i)$

TABLE 1

Variable tion	Coefficient Estimate	Standard Deviation
a_q	.804	.034
d_{qq}	.034	.054
r_{labor}	-.009	.005
r_{fuel}	.018	.003
$r_{capital}$	-.010	.004
$m_{route\ distance}$	-.050	.085
$m_{load\ factor}$	.036	.119
$m_{points\ served}$	-.123	.064

The 1982 marginal costs (by carrier and route) were then
estimated by plugging in the individual carrier character-
istics and the individual route distances.

The estimates of marginal cost are reasonable. The
marginal cost of the airlines estimated at their means is 8.4
cents per passenger-mile (with a standard error of .13 cents)
which was less than the 10.5 cents average cost per
passenger-mile (as expected because of increasing returns to
scale).

^[3.5] See description of conjectural calculation in
footnote 3.

^[3.6] As derived in footnote 3, the estimates of route
conjectures for the regulated 1975 data equaled:

$$k = (e\Sigma_i L_i) - 1.$$

^[4] The variance of the demand elasticity, also taken from the Brown and Watkins study [1971] was .10. The variance of marginal cost (which accounted for the estimated covariance between the coefficients in the Caves Christensen and Tretheway translog regression) was, evaluated at the sample mean, .13.

^[5] The airline cost data also indicates that under regulation CAB did not set fares competitively. Following Friedlaender and Spady [1981], the derivative of the cost function with respect to output equals:

$$(dlnCT/dlnq) = (dCT/dq)(q/CT) = a_q + d_{qq}lnq$$

$$+ \Sigma_i r_{qi}lnW_i + \Sigma_i m_{qi}lnZ_i$$

If CAB set fares equal to marginal cost this derivative is:

$$Pq/CT = REV/CT = a_q + d_{qq}lnq$$

$$+ \Sigma_i r_{qi}lnW_i + \Sigma_i m_{qi}lnZ_i$$

The estimates of these slope coefficients can be compared to those of Caves, Christensen and Tretheway [1984]:

	Regulation Estimate	Cost Function Estimate
a_q	0.96	0.804
d_{qq}	0.01	0.034
r_{labor}	0.04	-0.009
r_{fuel}	-0.05	0.018
$r_{capital}$	0.01	-0.010
m_{load}	0.06	0.036
m_{stage}	-0.06	-0.050
m_{points}	-0.03	-0.123

The coefficients in equation (5) varied greatly from those when estimated jointly with the cost function. A sum of

squared residuals F-test with 7, and 57 degrees of freedom equalling 11.35 easily rejected the null of competitive marginal cost pricing at more than a 1 percent significance level.

^[6] See Anderson and Kraus [1981], Morrison and Winston [1983], and Abrahams [1977].

^[7] The number of newly certified carriers is an imperfect proxy for barriers to entry. Non-newly certified carriers may have entered some of the sampled routes. More-over, the absence of new entry does not entail the absence of potential competition and does not theoretically explain why barriers differed between routes.

^[8] Because frequency competition is likely to be endogenously determined, the number of route flights may induce simultaneity bias. It is assumed to be exogenous.

^[9] For example, a given rival might respond less aggressively to increased output from a fringe competitor.

^[10] DESCRIPTION OF STRUCTURAL DATA.

Carrier Dummy = 1 for carrier forming conjecture, 0 other-wise. Source: CAB, DB1 Computer Tape.

Rival Dummy = 1 if rival carrier serves route, 0 otherwise. Source: CAB, DB1 Computer Tape.

Slot Dummy = 1 if one endpoint is slot constrained, 2 if both endpoints are slot constrained, 0 otherwise. In 1982 the FAA had imposed slot constraints (limited the number of takeoffs and landings) at New York's Kennedy and Laguardia, Washing-ton's National and Chicago's O'Hare airports. Source: 14 C.F.R. 93.123 (1976) also see Graham, Kaplan and Sibley [1983].

#CARRIERS. Source: CAB, DB1 Computer Tape.

#NEWCERT = the number of carriers serving the route who were certified for interstate service after passage of the Airline Deregulation Act of 1978. Source: DB1 Computer Tape.

TOURISM. Following Borenstein [1983], This index is a weighted average of 1977 tourist hotel revenues as a percent-

age of total business revenues in each end-point city. The weight for each endpoint is the proportion of trips on the route that originate at the other endpoint. For instance, since most trips on the Albany-Miami route originate in Albany, the tourist attractiveness of Miami gets great weight than that of Albany. Source, U.S. Census of Service Industries, 1977.

FLIGHTS = number of non-stop flights per week. Source, Official Airline Guide, North American Edition, May 1, 1982.

^[11] Recent studies of airline demand [see, e.g., Abrahams (1983), Anderson and Kraus (1981), Ippolito (1981)] have controlled for airline quality with frequency delay measures. The average number of flights per city pair was included in the translog specification (equation x) as a measure of the frequency delay quality. The SUR estimate of the frequency delay coefficient was negative (-.118), indicating economies to increasing the number of flights per route) and significant (t = -2.6).

^[12] Output was instrumented with pric? indices for other modes of transportation (car, bus, train) as well as general macro activity aggregates (per capita disposable income, unemployment rate) and transportation complements (hotel revenues). Instrument data was taken from the Statistical Abstracts of the United States. All data are national aggregates. The transportation indices were measured in price per passenger mile. A Hausman test, however, could not reject a null hypothesis of output exogeneity (chi-squared (60 d.f.) = 0.63).

^[13] This connection between consistency and Stackelberg leadership is suggestive at best. The concepts relate to different games (one and two period respectively) with different strategies. But as noted in footnote 1, conjectural variation models are static models striving to be dynamic. These repressed dynamics bolsters the comparison with the Stackelberg model.

^[14] The slope variance was estimated under the assumption that the covariance of k_i with MC_i and e was zero.

^[15] The form of the t-test:

$$(k - slope)/(Var(k) + Var(slope))^{.5}$$

assumes that the slope and k estimates are independently distributed. Because a positive covariance is more likely, this makes my estimate of the denominator too large and biases the test toward accepting the null of consistency (equality).

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