ESSAYS IN BEHAVIORAL ECONOMIC MODELING

by

JAMES HENRY HINES, JR.

B.A., Amherst College
(1975)

M.B.A., University of Chicago
(1978)

SUBMITTED TO THE DEPARTMENT OF MANAGEMENT
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

October 1987

© Massachusetts Institute of Technology 1987

Signature of Author __________

Department of Management, October 1987

Certified by __________

Jay W. Forrester, Thesis Supervisor

Accepted by __________

Chairman, Department Committee on
Graduate Students

Archives
ESSAYS IN BEHAVIORAL ECONOMIC MODELING

by

James H. Hines, Jr.

Submitted to the Department of Management
in October 1987 in partial fulfillment of the
requirements for the Degree of Doctor of Philosophy

ABSTRACT

This dissertation is composed of three essays which develop macro-economic formulations based on behavioral decision theory. The essays are "A Behavioral Theory of Interest Rate Mechanics," "A Disequilibrium Financial Sector," and "Present Value Analysis and Behavioral Considerations in Modeling Factor Acquisition."

A Behavioral Theory of Interest Rate Mechanics. This essay presents a behavioral model of interest rate mechanics. The theory is based upon the observation that most financial transactions go through financial intermediaries at prices set by those intermediaries. It is suggested that intermediaries raise and lower interest rates in order to adjust their financial inventories — that is, their inventories of securities and reserves. By so doing, intermediaries tend to bring the non-financial sectors' demand for and supply of securities into balance.

The interest rate that is appropriate to rebalance financial inventories is not known in advance by intermediaries. Rather, it is suggested that intermediaries use an anchoring and adjustment strategy to decide on an appropriate interest rate. In the presence of financial inventory imbalances, the intermediary will raise or lower its posted interest rates above or below an anchor here termed the "underlying interest rate." The underlying interest rate represents the interest rate environment, the expectation of what the interest rate would be now if it were not for transitory pressures encapsulated and expressed as financial inventory imbalances.

Empirical support for the behavioral theory exists at a micro and at a macro level of aggregation. Research into the nature of human decision making at the individual level indicates that anchoring and adjustment is a common strategy for making decisions like those involved in interest rate determination. Econometric estimation is used to validate the theory at a macro-economic level of aggregation.

The essay concludes with an "extended model" in which the theory of interest rate mechanics is coupled with a representation of money demand. The extended model can behave like more traditional partial equilibrium models of interest rate determination — interest rates in the the extended model can jump to new (partial) equilibria and stay there. But, with slightly altered parameters the extended model can exhibit different behavior: interest rates can remain below the equilibrium rate for an extended period of time, or interest rates can overshoot the mark.

A Disequilibrium Financial Sector. This essay presents a model of a thoroughly disequilibrium financial sector that moves funds from savers to borrowers, sets interest rates, channels interest payments and defaults from borrowers to savers, and provides a medium of exchange. The model provides sufficient scope for portraying the impact on the financial sector of non-financial, private-sector investment, and the impact on the financial sector of governmental actions including open market transactions and deficits.

Like many previous models, the model presented in this essay permits non-financial sector disequilibria to the extent that non-financial sectors adjust their actual security holdings to the desired amount over time. In addition, however, the financial sector modeled in this paper includes interest rates that do not necessarily equate desired
purchases of securities with desired sales of securities, and the model does not necessarily require that desired sales of securities equal actual sales of securities.

The key to handling financial disequilibria is the financial inventories of an aggregate financial intermediary. Intermediaries in the real world carry financial inventories because there is no assurance that the purchases of securities (deposits) will equal sales of securities (loans) from moment to moment. Commercial banks carry inventories of reserves and securities dealers carry inventories of deposits (the "reserves" of the dealer) and inventories of securities. Intermediaries' financial inventories permit the non-financial sectors' sales and purchases of securities to differ from one another for a short time.

A persistent inequality between sales and purchases or depositing and lending will show up as too high or too low an inventory of reserves—that is, as excessive or deficient liquidity. Intermediaries have two tools to control their liquidity: interest rates and credit rationing.

Simulations show that exclusive reliance on interest rates to correct quickly liquidity imbalances would likely necessitate a temporary peak of interest rates much higher than the equilibrium rate. Recent research in behavioral decision theory indicates that temporary price increases designed to "take advantage" of temporary shortages are regarded as "unfair". Accordingly, intermediaries may be reluctant to raise prices as a temporary measure as far as fast as necessary to control liquidity shortage. Credit restrictions, in place of, or along with, interest rate increases, may be attractive tools for intermediaries wishing to redress liquidity problems while continuing to treat valued customers "fairly".

Present Value Analysis and Behavioral Considerations in Modeling Factor Acquisition. This essay develops a profitability-based formulation of firms' desired acquisition of purchased factors or rented factors. Evidence from surveys and the field of behavioral decision theory are used to build an analogue to the process by which actual firms come to desire increases or decreases in factor stocks. The survey evidence suggests that discounting approaches to investment are widely used in practice and are growing in popularity. A particular discounting approach involving the benefit cost ratio is used in this essay.

The standard neoclassical approach to investment also posits a discounting approach. And, as a consequence, intermediate results in this essay are closely related to formulations developed within the neoclassical framework. The neoclassical formulation is extended, however, by explicitly including the effect of inflation on the tax depreciation shield and the effect of a time consuming production process. The essay also explores the implication of assuming that other factors will be held constant as a given factor is increased, rather than that output will be held constant as the factor increases.

The formulation enriches the neoclassical model by explicitly taking account of results from the field of behavioral decision theory. The evaluation of the benefit cost ratio is colored in the model by applying the value function from prospect theory to the benefit cost ratio, and the response to the benefit cost ratio is biased toward inaction. More fundamentally, perhaps, the present value calculation is applied to a "model within the model" corresponding to the (mental) models used by analysts and decision makers in the real world. The "analysts model", as it is called in this paper, is a simplified version of a surrounding macro-economic model, just as the models used by real-world decision makers are simplified versions of the real world. The simplification is achieved by ignoring minor structural details and by concentrating on effects that are certain and direct as opposed to uncertain or indirect.

Thesis Supervisor: Jay W. Forrester
Title: Germeshausen Professor of Management
ACKNOWLEDGEMENTS

I had a good thesis committee. Franco Modigliani made sure I was aware of relevant economic thought and focused my attention on parts of the dissertation where the argument was weak or incomplete. John Sterman encouraged me to give full play to behavioral decision theory. Over the course of many discussions he sharpened the ideas presented here. Jay Forrester, from the earliest beginning of this effort to its end, provided guidance, encouragement and, always, an uncompromising insistence on logical rigor.

Zenon Zannetos, who was also a member of my committee, passed away earlier this year. Professor Zannetos always asked questions which at first seemed to be obvious — like, "What do you mean by 'equilibrium'?” Answers to his questions turned out to be milestones toward the completion of this work.

Bob Eberlein's critiques were important, particularly to the development of the economic and statistical arguments. Alan Graham commented on early drafts of each of the essays and kept a close eye on the mathematics. Fellow students provided valuable comment and advice. Among them, Ernst Diehl, Christian Kampmann, and Gary Lovemen deserve special thanks. Margaret Kunz edited the document and packaged the whole thing up by the time it had to be in. George Richardson, in the role of counselor, drew from a very deep well of intelligence and calm good sense.

During the time I wrote this thesis, Nancy Cottrill created the world. She shaped life beyond work, invented a home, and peopled it with Hannah. To her I owe a debt which I will enjoy repaying for the rest of my life.
# TABLE OF CONTENTS

## I. A Behavioral Theory of Interest Rate Mechanics 6

1. Equilibrium Approach: Simultaneous Equations 8
2. Disequilibrium Adjustment Process 10
3. Estimation 21
4. Simulation 27
5. Summary and Conclusions 37

Appendix 1: Data Sources 39
Appendix 2: Additional Regressions 40
Appendix 3: Modification to Interest Rate Structure 41
Appendix 4: Model Documentation 42
References 47
Figures 51

## II. A Disequilibrium Financial Sector 65

1. Assets and Actors 68
2. Process and Response in the Financial Sector 82
3. The Allocation of Interest Payments and Defaults 93
4. Summary and Conclusion 98

Appendix: Model Documentation 100
References 116
Figures 120

## III. Present Value Analysis and Behavioral Considerations in Modeling Factor Acquisition 142

1. Alternative Approaches to the Investment Decision 144
2. Investment Criteria in Practice 151
3. Initial Considerations in Model Formulation 157
4. Derivation of the Benefit Cost Ratio 162
5. Desired Factor Acquisition Rate 175
6. Conclusion 184

Appendix: Benefit Cost Ratio in a Putty-Clay World 187
References 189
Figures 192
A Behavioral Theory of Interest Rate Mechanics

Introduction. In most models of the financial sector, interest rate determination is based upon an assumption that financial markets are in equilibrium in the sense that supply equals demand. However, whether financial markets should be represented as if they are in equilibrium is partially a question of the purpose of the representation and partially an empirical question.\textsuperscript{1} Equilibrium representations of the interest rate are appropriate when investigating, for example, issues in general equilibrium theory. On the other hand, investigations focused on aspects of the adjustment of interest rates to equilibrium clearly require a representation of the adjustment processes—that is, a disequilibrium representation. Studies focused on other economic processes should employ an equilibrium representation only if the mechanism that moves interest rates to equilibrium operates fast relative to the processes being studied, otherwise a disequilibrium interest rate formulation is required. In this paper I will present an explicitly disequilibrium theory of the mechanism by which interest rates move. The theory is consistent with empirical tests and consistent with the established partial-equilibrium theories. The theory of interest rate mechanics is appropriate for disequilibrium economic models and provides a starting point for investigating empirical issues such as the speed with which interest rates move to an equilibrium within the financial sector.

The paper begins with a short discussion of the neoclassical economic partial equilibrium\textsuperscript{2} theory of interest rates. That discussion concludes with the suggestion that a behaviorally-based theory of interest rate mechanics is desirable at least as a complement to the neoclassical theory. Such a behavioral theory is developed in the following sections and the theory's two parameters are estimated. Finally, the dynamics of the new theory are examined.

\textsuperscript{1} The fact that there is a question regarding equilibrium or disequilibrium formulations apparently comes as a surprise to some. Quandt (1985) reports on giving seminars on the question: "On some occasions (mostly in the U.S.) I would be interrupted by someone five minutes into the seminar with the remark, 'What you are trying to do is silly, because everybody knows that prices always clear markets...'. At other times (mostly in Europe) I would be interrupted with the remark, 'What you are trying to do is silly, because everybody knows that prices never clear markets...'."

\textsuperscript{2} The theories are "partial" in the sense that they are concerned with equilibrium in only one part of the economy, namely the financial sector.
The Interest Rate. Before proceeding it is necessary to be more specific about what is meant by "the interest rate." Figure 1 presents three different rates, the prime rate, the 20-year government bond rate, and the 90-day treasury bill rate. These rates differ with respect to both risk and maturity, and, as figure 1 shows, the different interest rates do not move in perfect lock step. However, the degree of common movement is substantial. The correlation coefficient for each rate with respect to each of the others is above .9. A great deal of the total movement in the rates is accounted for by the common movement. This paper focuses on the common movement by developing a rigorous behavioral theory as if there were only one basic rate.

In a larger macro-economic model other, "non basic" rates could be derived from the basic rate as needed by simple adjustments up or down depending upon whether the other rates are more or less risky than the basic rate.\(^3\) Whether this strategy of using a fundamental rate as a base for other rates is appropriate will depend upon the purpose of the study for which one needs to represent interest rates. The strategy would be inappropriate for a model designed to investigate how the differences between rates differ over time. On the other hand, the strategy of forming interest rates from one basic rate is probably quite appropriate for a model designed to investigate the major economic behavior modes of an industrialized economy: the business cycle, the long wave, inflation. (For discussion of the several major economic behavior modes see Schumpeter 1944; Volker 1978; and Forrester 1976, 1982). It is unlikely that the details of how various rates move against one another is important for an understanding of the major economic behavior modes. Capturing the common movement of interest rates in a basic interest rate is crucial; capturing in a detailed way the fine distinctions between rates is of less importance.

The nominal, short-term, risk-free interest rate is chosen as the basic rate in this paper. This choice is in broad keeping with economic tradition, although many theorists would focus on the real, short-term, risk-free interest rate. In the United States, however, there are no instruments carrying the real rate, and consequently representing demand and supply pressures in a behavioral model might become problematic were the real rate used. The nominal, short-term, risk-free interest rate, on the other hand, corresponds closely to

\(^3\) Translations between short and long rates would also involve expectations regarding changes in the short or long rates and, perhaps, risk premia for different maturity habitats (Modigliani and Shiller 1973, Modigliani and Sutch 1966).
the interest rate on a federal government note. The note is virtually risk-free in terms of default risk because the government has the ability to print money to pay its debt.4

1. Equilibrium Approach: Simultaneous Equations

The theory of interest rate formation developed in this essay is fundamentally a supply and demand formulation. The formulation, however, does not impose the condition that desired supply always equals desired demand, and is therefore distinguishable from supply-demand formulations in most other econometric work. In such work interest rates are chosen to equate desired supply and desired demand (see for example, Friedman 1980a, 1980b, 1977; Friedman and Roley 1980, Modigliani, Rasche, and Cooper 1970, Hendershott 1977, Bosworth and Duesenberry 1973). More specifically, the desired (primary) purchases of a security by lenders and the desired issues of that security by borrowers are specified as functions of the interest rate on that security. The condition that desired purchases (demand) equal desired issues (supply) is then imposed. With this condition the desired amount of securities and the interest rate can be determined.5

For example, if g(i,z) is a function determining desired loans DL (or securities purchases) and f(i,z) a function determining desired borrowing DB (or issues), where "i" is the interest rate and "z" represents exogenous factors, a system of three simultaneous equations may be written:

\[ \begin{align*}
DL &= g(i,z) \\
DB &= f(i,z) \\
DL &= DB
\end{align*} \]

---

4 There have been instances in which governments have defaulted on debt denominated in local currency, though such instances are not common. The default risk on the debt of the industrialized nations is extremely small.

5 Modigliani, Rasche and Cooper (1970) equate the public's demand for demand deposits to the supply offered by banks. The IS-LM approach makes use of simultaneous equalities between goods supplied and goods demanded (IS) and between money demanded and money supplied (LM) to obtain equilibrium figures for GNP and interest rates (Dornbusch and Fischer 1981, Ch. 4, eq. atonnement and 12a). Bosworth and Duesenberry (1973) use the equilibrium condition for most, but not all, rates. Hendershott (1977) uses a simultaneous equations approach that permits credit rationing via a "rationing term" in the demand equation.
Equations (1) and (2) may be substituted into (3) to yield:

\[ g(i,z) = f(i,z) \] (4)

which may be solved for a unique interest rate \( i \) under suitable restrictions on \( g \) and \( f \). The interest rate so discovered may then be substituted into either (1) or (2) to uncover the amount demanded and supplied.

The simultaneous equations approach is an equilibrium approach. In equilibrium desired quantities equal actual quantities. The desires of sellers and buyers can both be fulfilled only if the quantities they each have in mind are the same, otherwise one of the groups will find itself short of transacting partners. The simultaneous equations approach requires that sellers' desired quantities always equal buyers' desired quantities (equation (3)). Hence, the simultaneous equations approach carries the assumption that financial markets are always in equilibrium.

Theorists who use simultaneous equations do not argue that the method literally describes the way rates are set. \( F \) and \( g \) are functions that describe the preferences of suppliers and demanders of credit. In an industrial economy, the system of equations describing the preferences of everyone in an industrial economy is too large to be solved, and in any event the necessary information is unavailable. The equilibrium approach is elegant, however, even if it does not describe exactly how rates are determined. The question "What is the interest rate?" can be answered without having to deal with the potentially messy process by which interest rates adjust to the equilibrium.

---

6 More precisely, desired quantities equal actual quantities in a stress-free equilibrium. Most generally, an equilibrium might be considered as a condition in which all states (integrations) of a system are constant or are growing at a constant rate. Such a condition can result when desires are not fulfilled but where opposing forces in the system conspire to prevent further movement of actual quantities toward desired. This is termed a "stressed" equilibrium.

7 The key feature here is that markets are cleared by interest rates. Actual stocks do not have to equal the stocks that are (ultimately) desired, but desired flows have to equal actual flows. Partial adjustment models, where sectors are adjusting to desired stocks over some adjustment time, are considered equilibrium models here as long as markets are cleared by interest rates. Some simultaneous equations models are arguably disequilibrium models. For example, Hendershott (1977) includes a "credit rationing" term in some equations for desired issues of securities. This term reduces "issues relative to desired issues, [so that] ... out-of-equilibrium rates are determined" when issues are set equal to purchases (p.66).
The value of the answer produced by the equilibrium simultaneous equations approach depends upon whether the mechanisms adjusting interest rates are fast relative to the economic behavior of interest in a particular study (Cf. Eberlein 1984, p. 183 ff.). If the adjustment process operates relatively quickly, an equilibrium approach will be sound because interest rates will always be almost in equilibrium. For example, if one is interested in the long wave—a process that takes forty to sixty years to unfold—the equilibrium assumption is probably justifiable. Interest rate adjustments probably take far less than fifty years, although the adjustment speed is an empirical question which requires a disequilibrium model to investigate.

The equilibrium approach may be less justifiable if one is interested in the business cycle—a process with a three to seven year period. There is some evidence that a disequilibrium condition in the form of a credit crunch develops in the late stages of most business cycle expansions (Eckstein 1983; Sinai 1976; Wojnilower 1980, 1985; Dubofsky 1985; for more theoretical discussions see Jaffee and Modigliani 1969, Jaffee and Russel 1976, Chiang et al 1984). A credit crunch arises when borrowers would like to borrow at the going rate, but lenders refuse to lend. During a credit crunch, financial markets are not in equilibrium because demand exceeds supply. What evidence we have on the timing of credit crunches suggests that these disequilibria in the financial markets are tied to the business cycle. Consequently, if one is interested in the business cycle or in other economic processes occurring over a period equal to or shorter than the business cycle (say, four years), the equilibrium approach of simultaneous equations may be inappropriate.

2. Disequilibrium Adjustment Processes

The simultaneous equations approach, by assuming that interest rates are in equilibrium, obviates the need for an explicit consideration of the mechanism that moves interest rates to the equilibrium. When the equilibrium assumption is inappropriate however, an interest rate formulation must focus closely on the adjustment process. Walras (1954) appropriately called this process a "groping" ("tâtonnement") toward the equilibrium.

The Financial Intermediary. A disequilibrium theory is a theory of how rates are set. The answer to how may be approached by first answering who and why. In
Walras' formulation, an "auctioneer"\(^8\) acts to clear a market represented by a group of people gathered (or linked) together to buy and sell securities. The auctioneer calls out an interest rate, participants make offers to lend and borrow at this interest rate. If desired loans do not match desired borrowing, the auctioneer calls out a new interest rate. He continues in this fashion until offers to borrow equal offers to lend, at which point he permits the participants to transact.

Walras' notion of an auctioneer is considered even by those who use it as "a fairly extreme idealization of the mechanism by which prices are determined" (Malinvaud 1972, 140). Indeed, the auction is actually only a bit less unwieldy than the solution of simultaneous equations. Instead of requiring massive amounts of information and computational power, this process requires the massive and patient cooperation of institutions and the public at large. Walras' auction is little more than a hypothetical computational technique for the solution of a set of simultaneous equations (Goodwin 1951).

The lack of realism in Walras' adjustment process may be identified in at least two ways. First, transactions cannot take place before an equilibrium price is determined, whereas on real-world commodity and stock exchanges "without exception contracts are made at each of the prices called." (Malinvaud p. 139). Second, the auctioneer, although a kind of intermediary, is not permitted to take a position in the market, that is, he is not permitted to buy and sell for his own account, unlike his real-life counterparts. Realism may be improved, therefore, by permitting transactions out of equilibrium and by replacing Walras' auctioneer with an aggregated financial intermediary. The simplification involved in taking this step is quite the reverse of that made by Walras: The intermediary, rather than being a party to no transactions, can be assumed to be a party to every transaction.

The assumption that an aggregated financial intermediary is a party to every trade of financial securities closely mirrors the real world. Granted, in some transactions the price on a security is negotiated by both parties as, for example, in the case of direct placement of commercial paper by finance companies. But, in a modern economy, most financial transactions go through a dealer or a financial intermediary who stands ready to buy and sell at a price it quotes. For example, publicly traded commercial paper does go through an intermediary. Similarly, banks set the rates in the commercial loan market. It is true

\(^8\) The term "auctioneer" is common; "Master of Ceremonies" would better describe the intended function.
enough that some loan officers, more adventurous perhaps than others, will argue on behalf of their customers for special pricing. But most officers for most loans will apply standard pricing. The case is even clearer for a bank's liabilities: Banks stand ready to accept deposits at posted rates. Finally, in the largest financial market, the market for U.S. treasury securities, dealers are prepared to buy and sell at prices they quote. Financial intermediaries set prices for the bulk of financial transactions in the U.S. In other financial centers the story is much the same. Great Britain's recent "big bang" has moved virtually all financial market transactions into the hands of dealers. To a good approximation intermediaries set interest rates.

Adjusting Interest Rates to Manage "Inventories." Before moving on to discuss how intermediaries set the interest rate, it is useful to discuss why intermediaries set interest rates. It is important to realize that intermediaries make their profits primarily on the spread between rates and not on the level of rates. A dealer makes a profit on the spread between the bid and ask prices; a bank makes its profits on the difference between the cost of funds (e.g. deposits) and the rate of interest on loans. I shall argue below that the ultimate reasons that intermediaries increase or lower the interest rate also tend to keep the spreads constant. Consequently, the assumption of a constant spread is not a bad approximation. Given a constant spread, changes in the level of interest rates do not affect an intermediary's profits. The most important reason intermediaries alter the level of interest rates is not to glean directly a higher profit, but rather to adjust inventories of securities and reserves to more appropriate levels.9

Inventories of reserves and securities are necessary in order for intermediaries to permit disequilibrium trading. If the number of securities offered for sale to the intermediary at the going interest rate does not equal the number of securities desired to be bought, the intermediary can absorb the difference if it holds inventories. The intermediary must have an inventory of money or "reserves," and, depending upon the type of intermediary, it may have an inventory of securities as well. A primary market intermediary, such as a bank, only needs an inventory of reserves. When deposit inflows exceed loan outflows, reserves will increase. Conversely, when deposit inflows are less than loan outflows, reserves will decrease. The reserves are an inventory which at times accumulates excess cash inflow and which at other times permits outflows to exceed

---

9 Adjusting inventories to more appropriate levels may, of course, increase profits indirectly by balancing the costs of holding too much and too little inventory.
inflows. An intermediary working in the secondary markets, such as a government bond dealer, needs both an inventory of money and an inventory of securities. When demand from buyers for securities exceeds the supply from sellers, the dealer will dip into his inventory of securities to meet the demand while he will accumulate the excess inflow of money in his inventory of "reserves." In contrast, when the demand of security buyers is less than the supply of security sellers, the dealer will dip into his reserves to meet the excess money outflow while accumulating in his inventory of securities the excess inflow of securities. Inventories for financial intermediaries are as necessary as the inventories of manufacturers and for the same reason: The inventories decouple supply and demand.  

Intermediaries must manage their inventory positions. Inventories that are too high entail excessive carrying costs while inventories that are too low reduce the intermediaries' capacity to handle trades, thereby limiting the intermediaries' profits. Somewhere between inventories that are too low and inventories that are too high is a desired inventory level. The intermediary must control the in- and out-flows from the inventories in order to bring his actual inventories into line with his desired inventories. The intermediary can control its inventories through price and/or through rationing. When a bank's inventory of reserves is low (i.e. it is illiquid) it can reduce the amount of loans it makes below the amount demanded—that is, the bank can ration credit. On the other hand, the bank can raise the interest rate it charges on loans and the rate it offers on deposits. The result will be to encourage depositors and to discourage borrowers, thereby tending to increase

---

10 When inventories become low, intermediaries may refuse or be unable to accept certain transactions. Such a situation when economy-wide is a credit crunch. (For discussion see Hines, 1987.)

11 The desired inventory levels might be seen as the solution of a constrained optimization problem. Whether intermediaries actually approach the problem through the formal calculus of optimization is not known. Certainly they make trade-offs between the costs of high inventories and forgone profits of low trading volumes.
reserves. The spread between the cost of funds and the interest rate on loans will tend to remain constant. Conversely, if reserves are too high, that is, if the bank is excessively liquid, the bank can decrease the rate it charges on loans and decrease the rate it offers on deposits. The result will be to encourage borrowers and discourage depositors, thereby tending to decrease its reserves, and, again, tending to maintain the spread. In this case of excessive liquidity, the bank could also ration deposits, although deposit rationing seems to be rarer than credit rationing. The bank can thus use credit and deposit rationing and interest rate adjustments to control its inventories.

The methods of inventory control available to a dealer-type financial intermediary are the same as those available to a commercial bank. Figure 2 is a highly simplified sketch of a bond dealer's two inventories: bonds and money. Assume for a moment that the bond dealer begins in an equilibrium in which his two inventories are at their desired levels and that people begin selling bonds to the dealer faster than they buy bonds from him. The dealer's inventory of bonds will increase above the desired level while the dealer's inventory of money will decrease below the desired level. The dealer will be illiquid. The dealer can do two things: He can restrict his purchases of bonds and/or he can increase the

---

12 Naturally, reserves in the overall federal reserve system are fixed by the Fed. An individual bank will try to get a larger share of the reserves by raising the rate it offers on deposits.

13 This implies that financial intermediaries believe their customers' demand schedules for securities are negatively sloped and supply schedules are positively sloped. Zannetos (1966) shows how demand schedules can have positively sloped segments, and supply schedules negatively sloped segments if price expectations are elastic (i.e., if people expect future prices to change more than in proportion to the change in current prices). The beliefs of intermediaries are not the same as what the supply and demand schedules of their customers actually are. Consequently, the assumption that intermediaries believe their customer's have normally shaped demand and supply functions, says nothing about the actual shape of the supply and demand functions. If the world is characterized by oddly shaped supply and demand functions one might anticipate wide swings in interest rates and in prices of financial securities. It is possible, though controversial, that such wide swings may in fact be present (Shiller 1981, Marsh and Merton 1983). Although, the investigation of this possibility is beyond the scope of this paper, it seems that the interest rate formulation presented herein combined with Zannetos's elastic price expectations could make a contribution to the literature on large price swings in financial markets.

14 Dealers can and do hold short positions in some of the securities in which they deal. To an extent such short selling reflects speculation rather than dealing, and to that extent is beyond the scope of this paper. In addition to a speculation motivation, dealers may take short positions in order to satisfy customer demand. This does no violence to the description offered above.
interest rate (i.e. lower the price\textsuperscript{15}) at which he offers to buy and sell. Raising the interest rate, lowering price, will discourage people from selling their bonds to that particular dealer and will encourage them to buy bonds from him. Hence his inventory of bonds will decrease and his inventory of money will increase relative to leaving the price unchanged—the dealer’s illiquidity will be eased. If the dealer faces excessive liquidity (i.e. too much money and too few bonds) he can restrict his sales of bonds and/or he can lower the interest rate (raise the price). In sum, the dealer, like the commercial bank, can use restrictions on purchases and sales of bonds and/or interest rate adjustments to control his inventory positions.

The intermediary’s desire to balance inventories operates to equate supply and demand. When supply and demand are in balance, the intermediary’s inventories are unchanged. If inventories are at their desired levels and supply equals demand, the intermediary will be unmotivated to change interest rates. If supply and demand are not equal, inventories will increase above or below desired levels and the intermediary will be motivated to alter the interest rate in order to influence supply and demand to move his inventories back into line.\textsuperscript{16}

\textsuperscript{15} An increase in the interest rate is equivalent to a decrease in the price. This can be easily seen in the case of a consol such as the British government issued. A consol is a coupon bond with an infinite maturity; the issuer pays a coupon forever, but never repays the principle. The interest rate in the secondary market on a consol is: \( r = c/P \) where \( r \) is the interest rate, \( c \) is the fixed annual payment or coupon and \( P \) is the price as determined by a dealer. Since \( c \) is fixed, an increase in \( r \) means that \( P \) must decline.

\textsuperscript{16} One might argue that while intermediaries do try to balance their inventories, they also try to balance supply and demand directly to prevent their inventories from moving too far out of line. They might, for example, take (short) moving averages of their customers’ purchases and sales. When the moving averages of purchases and sales are out of kilter, the intermediary might react by altering the interest rate. This question should be resolved through observations of and discussions with intermediaries. Even in the absence of this fieldwork, however, one may examine the nature of the difference more carefully by writing the integral equations for the inventory of securities and the purchases and sales average.

\begin{align*}
\text{Securities inventory}_t &= \int_0^t (\text{Sales}_s - \text{Purchases}_s) \, ds \tag{i} \\
\text{Sales-Purchases Average} &= \frac{1}{t} \int_{t-1}^t (\text{Sales}_s - \text{Purchases}_s) \, ds \tag{ii}
\end{align*}

An exploration of the consequences of assuming intermediaries consider both (i) and (ii) (rather than only (i)) when they set rates might be worthwhile. However, adding (ii) to the intermediary’s concerns would probably have only a second order effect because the information in (ii) is contained in (i). Adding (ii) would likely result in a less oscillatory system, however, the models considered in this paper have little tendency to oscillate even without the introduction of (ii). Consequently, in this paper I
Anchoring and Adjustment. Having discussed who sets interest rates and why, it is now time to consider how interest rates are set. The approach taken here relies on behavioral decision theory (see Hogarth 1980 and Einhorn and Hogarth 1981 for surveys of the field). Behavioral decision theory is a young field. Certainly, more observation and more theory are required. Nonetheless, the body of available results is already sufficient to permit experimenting with corporate and economic models that are justified primarily by behavioral observations and generalizations. 17

A large and growing segment of the work in behavioral decision theory suggests that the process termed "adjustment and anchoring" is a common strategy for arriving at judgments like determining interest rates, where a lack of sufficient understanding, information, or time precludes solving a problem from first principles. In the adjustment and anchoring process people come to a judgment by adjusting a preexisting quantity (the anchor) by taking account of currently available information (Tversky and Kahneman 1974, Hogarth 1980 p.47, Slovic et al 1982, and Kleinmuntz 1985).

As an example consider an experiment reported by Lichtenstein et al (1978). Seventy four people were divided into two groups. One group was told that one thousand people die from electrocution every year in the United States. The people in this group were then asked to estimate how many people die each year from each of 40 other lethal events (e.g. train collisions, venomous bites, small pox, murder). The second group was told that fifty thousand people die each year from motor vehicle accidents. They, too, were given the list of other lethal events and asked to estimate how many people die each year from each cause.

The average estimate from the "1,000 electrocutions group" was lower than the average estimate from the "50,000 motor vehicle accidents group" for ninety percent of the

17 The use of behavioral decision theory in the present context involves an aggregative leap. Behavioral decision theory deals with how individuals make decisions. This paper is concerned with representing an aggregation of decisions made by many people. The leap from individual to group is well sanctioned in macro-economics. For example, the savings/consumption formulation of the life-cycle theory is developed in terms of an individual and then extended to an aggregation of individuals in an economy. The leap requires checking results at the macro-level. One way of checking is to test the resulting formulation statistically using macro-economic time series data. Initial statistical tests are reported in this paper.
other lethal events. This difference between groups seems to result from each of the two
groups using a different base, a different anchor. People in the first group used 1,000
deaths per year as an anchor; people in the other group used 50,000 deaths per year as an
anchor. Beginning from a lower base or anchor, people in the "electrocutions group"
ended up with a lower estimate than the people in the "motor vehicle accidents group." The
wide difference in the anchors allowed the results of the judgmental strategy to be
observed. In both groups people formed their judgments by anchoring and adjustment.

**Adjustment and Anchoring in an Interest Rate Formulation.** A central
hypothesis of this paper is that the adjustment and anchoring strategy undergirds the
interest rate setting process. More particularly, it is hypothesized that in the presence of a
supply and demand imbalance, summarized by the inventory positions of intermediaries,
intermediaries choose a current interest rate by adjusting up or down an anchor.

The suggestion that interest rates are set by anchoring and adjustment is not
necessarily a new one. In the "modern" interpretation of Walras (Samuelson 1947 p. 270,
Goodwin 1951, Negishi 1962), used frequently in disequilibrium models (Quandt 1985,
particularly eq 2-4), the movement in interest rate is proportional to the difference between
supply and demand of securities: \(^{18}\)

\[
(d/dt)\text{Interest rate}_t = k(\text{Supply}_t - \text{Demand}_t) 
\]

(5)

or using the infinitesimal \(\delta\)

\[
\text{Interest rate}_t = \text{Interest rate}_{t-\delta} + \delta*k(\text{Supply}_{t-\delta} - \text{Demand}_{t-\delta})
\]

(6)

Equation (6) makes clear that the "modern" interpretation of Walras is itself an anchoring
and adjustment process. The anchor is the interest rate a moment ago and the adjustment is
proportional to the difference between supply and demand.

The Walrasian formulation possesses the virtue of simplicity and may be well suited
to the analytical needs of stability theory, but the formulation suffers from several flaws.
First the state of supply and demand are incorrectly assumed to be known in the Walrasian
approach. Further, the motivation of participants is not made clear: Why do people wish
to equate supply and demand? On a more technical level, the formulation assumes the
interest rate is continuous even though it appears that interest rates can jump in response to

---

\(^{18}\) The theory is usually represented in terms of price, rather than interest rates. I have
converted to interest rates which entails a transposition of the supply and demand terms
on the right hand side.
news. Finally, the adjustment to bring supply and demand into line is the same as the movement in the anchor. Hence, the aggressiveness with which the interest rate is raised above or below the anchor is not separated from the speed with which financial-market participants move their anchor. The Walrasian formulation precludes, for example, the possibility that dealers would adjust the interest rate with respect to the anchor and then wait for the response from suppliers and demanders before adjusting the anchor.

As an alternative to the Walrasian formulation, consider the possibility that pressures causing interest rates to move are experienced by intermediaries as discrepancies between their actual and their desired inventories of securities or reserves. In response to such pressure, intermediaries immediately move their posted rates above or below their assessment of "where rates are," abstracting from transitory pressures. "Where rates are" is an anchor. The anchor is a characterization of the current rate environment and will be termed the "underlying interest rate" in this paper. A mathematical formulation suitable for use in a simulation model involves clarifying three issues. First, the manner in which the current interest rate is adjusted above or below the anchor must be considered; second, the way people form the anchor itself must be dealt with; and, finally, the manner in which inventories are translated into an adjustment term must be described.

**Multiplicative Adjustment.** Two simple basic formulations for the adjustment around the anchor suggest themselves. One is additive, the other multiplicative.

\[ RFIR_t = UIR_t + ALR_t \]  \hspace{1cm} (7)
\[ RFIR_t = UIR_t \ast ALR_t \]  \hspace{1cm} (8)
\[ UIR_t = f(\text{past interest rates}) \]
\[ ALR_t = g(\text{intermediaries' liquidity}) \]  \hspace{1cm} (9)

Where **RFIR** is the risk free interest rate, **UIR** is the underlying interest rate (the anchor), **ALR** is the adjustment from liquidity to the risk free rate, and **f** and **g** are functions discussed below.

Kahneman and Tversky (1982, p.168) report an experiment which bears on the question of additive versus multiplicative forms. Kahneman and Tversky asked one set of respondents to imagine they were buying a calculator for fifteen dollars. The salesman, however, says the identical calculator is on sale for ten dollars at the other branch, a twenty-minute drive away. A majority of respondents presented with this situation said they would make the drive. A second set of respondents were presented with an identical situation except they were told the calculator's price was one hundred and twenty five
dollars; like those in the first group, they were informed they could save five dollars by driving across town. The majority of respondents presented with the second version reported they would not make the trip to the branch store.\textsuperscript{19} The experimental results suggest that a given difference in price becomes psychologically less important as the price increases.

Kahneman and Tversky's experiment suggests that for a given state of liquidity the difference between the anchor and the risk-free rate should increase as rates increase in order for the psychological impact of the difference to remain the same. A functional form in which the effect of liquidity enters multiplicatively possesses this characteristic. Equation (8) can be rearranged in terms of the gap between the risk free interest rate (RFIR) and the anchor (UIR) to show this explicitly.

\[
\text{RFIR}_t - \text{UIR}_t = \text{UIR}_t \times (\text{ALR}_t - 1)
\]  

Clearly, for a given adjustment from liquidity, the difference between the risk free rate and the anchor increases as interest rates (represented by the anchor) increase. Hence, during periods of high interest rates, indicated by a high value of UIR, the gap between UIR and RFIR will be greater than during periods of low rates for identical values of ALR.

\textbf{The Underlying Interest Rate.} The underlying interest rate is that rate which people feel generally holds at the present time, abstracting from the effects of transitory pressures on the actual interest rate. The underlying interest rate is the reference condition to which people have become accustomed; it continually adjusts as interest rates change and people become accustomed to the changed environment. A weighted average with recent experience weighted most heavily would seem to be an appropriate mathematical representation of this concept. An exponential average possesses this property and is easy to represent in a simulation model (Forrester 1961, p. 406-411). While a small modification will be considered in the section of this essay dealing with behavior and in appendix 2, for most purposes the underlying rate may be defined as:

\[
\frac{d}{dt}\text{UIR}_t = \frac{(\text{RFIR}_t - \text{UIR}_t)}{\text{TAUIR}}
\]  

\textsuperscript{19} Kahneman and Tversky kept the total dollar purchase constant by including in the question another item (a jacket) that was not on sale.
where:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>UIR</td>
<td>Underlying Interest Rate</td>
</tr>
<tr>
<td>RFIR</td>
<td>Risk Free Interest Rate</td>
</tr>
<tr>
<td>TAUIR</td>
<td>Time to Adjust Underlying Interest Rate</td>
</tr>
</tbody>
</table>

Equation (11) represents a translation of an inherently psychological phenomenon. The underlying interest rate is not an expectation in the ordinary use of the term as meaning a forecast of what something will be. Rather, (8) states that the underlying rate is the assessment of what the short-term, risk-free interest rate (RFIR) would be in the absence of liquidity pressures, that is, if the term representing liquidity pressures (ALR) took on its neutral value of 1 (the neutral value is 0 in alternative equation (7)). Equation (11) suggests that people making this assessment form a weighted average in which recent history is weighted most heavily. It seems likely that this is, by and large, a reasonable assumption. Subsequent research, however, may suggest refinements. For example, work on the term structure (Modigliani and Sutch (1966), Modigliani and Shiller (1973)) and work on inflation and oil-price forecasts (Sterman 1986a, 1986b) suggest that trend-extrapolation may be important. Zanetos (1966) raises the further intriguing possibility that price extrapolations may be biased so that the expected relative change in prices may exceed the current perceived relative change in prices (i.e. the price elasticity of expectations may be greater than 1). Although these studies considered expectations of future quantities and are not directly applicable to the formation of the underlying rate, extrapolations, biased or not, also may be present when forming assessments of current underlying conditions. An investigation designed to explore the underlying rate more carefully would be worthwhile. Until more information is available, it is reasonable to proceed under the simpler assumption that the underlying rate is an exponential average of past rates.

An exponential average adjusts toward current conditions. If, for example, the interest rate had been at one rate for a long time and jumped suddenly to a higher rate, the underlying rate would gradually move to the higher rate. The speed of adjustment of UIR toward RFIR depends on the parameter TAUIR. Figure 3 charts the adjustment path of UIR for a step increase in RFIR for several values of TAUIR. Statistical estimation of the parameters, including TAUIR, appears below.

**Effect of Liquidity.** While the terminology of "liquidity" tends to restrict attention to money balances, it is well to note that the effect with which we are concerned can be viewed as a function of either the inventories of securities or the inventory of money. One can work with either inventory because an increase in one implies an equal
decrease in the other. Because the concern here is with movements common to rates on all securities, a potentially troublesome aggregation issue can be largely avoided by considering the inventory of money, since instruments used as money will be quite similar. Consequently, money inventories will be explicitly considered and the "liquidity" terminology is descriptive.

The simplest variable measuring the state of liquidity is the difference between actual and desired money holdings. However, this difference is measured in dollars and is sensitive to the price level and institutional size, which one might expect to be unimportant in the determination of interest rates. Inventory discrepancies, therefore, should be measured relative to some base. Either desired inventories or actual inventories are prime candidates. Choosing desired inventories as the base yields the following expression where intermediaries' money inventories are termed "reserves," meaning unborrowed reserves of the interemediaries:

\[ ALR = f(RAR) \]  
\[ RAR = \frac{(R - DR)}{DR} \]

where:

- ALR - Adjustment from Liquidity to the risk free Rate  
- RAR - Relative Available Reserves  
- R - Reserves  
- DR - Desired Reserves  
- f - a function

The function f is not completely arbitrary. It must take the neutral value of 1 when RAR takes the value of 0, indicating reserves are equal to desired. It must be negatively sloped indicating that the interest rate will increase as intermediaries become less liquid. The function cannot drop below zero, because nominal rates are never negative. A flexible function which satisfies these properties and which will prove quite convenient later is the exponential function:

\[ ALR = f(RAR) = e^{-\alpha \cdot RAR} \]

This function, for several values of \( \alpha \), appears as figure 4.

3. Estimation

This section and the next are intended to extend the understanding of and confidence in the behavioral theory of interest rates presented above. In this section the
parameters of the interest rate theory will be estimated using single equation techniques. The parameters estimated are of reasonable magnitude and of the predicted sign. These statistical results, while not conclusive, increase confidence in using the behavioral theory in a larger macro-economic model.

**Estimation.** Equations (8), (11), (13), and (14) constitute a theory of interest rate formation. The equations are reproduced below for convenience.

\[
\begin{align*}
RFIR_t &= UIR_t * ALR_t \\
(d/dt)UIR_t &= (RFIR_t - UIR_t)/TAUIR \\
ALR_t &= f(RAR_t) = e^{-\alpha RAR_t} \\
RAR_t &= (R_t - DR_t)/DR_t
\end{align*}
\]

where:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>RFIR</td>
<td>Risk Free Interest Rate</td>
</tr>
<tr>
<td>UIR</td>
<td>Underlying Interest Rate</td>
</tr>
<tr>
<td>ALR</td>
<td>Adjustment from Liquidity to the Risk Free Rate</td>
</tr>
<tr>
<td>TAUIR</td>
<td>Time to Adjust Underlying Interest Rate</td>
</tr>
<tr>
<td>R</td>
<td>Reserves</td>
</tr>
<tr>
<td>DR</td>
<td>Desired Reserves</td>
</tr>
</tbody>
</table>

The stock and flow diagram\textsuperscript{20} of the structure may be drawn as in figure 5.

The structure developed above, although based upon empirical observation at the individual level, is intended to represent macro behavior. It is necessary now to present evidence that the formulation is consistent with macro-economic observation. Toward this end, the structure will be estimated statistically, significant parameter estimates of reasonable magnitude and predicted sign will constitute evidence that the formulation is consistent with macro-economic data on interest rates and reserves.

\textsuperscript{20} Stock and flow diagrams are in common use in system dynamics. They provide a close pictorial representation of a system of integral equations. In the diagram, the box represents an integration. The heavy arrow into the box represents a rate of flow which is controlled by the "valve" symbol termed CUIR (Change in UIR). CUIR, mathematically, is the time derivative of UIR. Entities not in boxes represent constants or functions. Single-line arrows represent information connections and reveal the arguments of each function. For a good discussion of diagramming conventions in system dynamics see Richardson and Pugh (1981, chapters 1-2) and Morecroft (1982).
The first step is to convert equations (15)-(18) into a form which can be estimated. A convenient way to do this is to solve the differential equation\(^{21}\). Substituting (15) into (16) and solving for UIR yields:

\[
\ln(UIR_t) = \ln(UIR_{t_0}) + \int_{t_0}^{t} (1/\text{TAUIR}) (ALR_s - 1) \, ds
\]

From (15) we know that:

\[
UIR_t = RFIR_t / ALR_t
\]

Substituting (20) into (19) and rearranging yields:

\[
\ln(RFIR_t) = \ln(UIR_{t_0}) + \ln(ALR_t) + \int_{t_0}^{t} (1/\text{TAUIR}) (ALR_s - 1)ds
\]

Substituting for ALR using (17) and reexpressing the integral produces:

\[
\ln(RFIR_t) = \ln(UIR_{t_0}) - \alpha \cdot RAR_t + (1/\text{TAUIR}) \left\{ \int_{t_0}^{t} e^{-\alpha \cdot RAR_s}ds - (t-t_0) \right\}
\]

or for estimating purposes:

\[
\ln(RFIR_t) = k + z \cdot RAR_t + h \cdot \{ CELR_t - (t-t_0) \}
\]

where:

- \(k\) is constant for any particular regression. It is the logarithm of the underlying rate at time 0, i.e. at the beginning of the data used in the regression.
- \(z\) is \((-\alpha)\)
- \(h\) is \((1/\text{TAUIR})\)
- \(\text{CELR}\) is a month-by-month accumulation.

---

\(^{21}\) It would also be possible to approximate the differential equation with a difference equation. As will be seen, the solution of the differential equation results in a "quasi" linear equation. The difference equation would be non-linear. Further, there is some evidence that the use of difference equations as an approximation to an underlying continuous-time model can cause problems (Richardson, 1981).
The integration in equation (22) may be approximated by a month-by-month accumulation CELR. The approximation would be exact if RAR were constant during the month, changing only in the instantaneous transition from one month to the next.

The entire equation may be estimated by the following method: Start with an estimate of $\alpha$ and form the month-by-month accumulation of the exponential CELR. Next, estimate $h$ and the coefficient $z$ using ordinary least squares. Form a new month-by-month accumulation CELR by adjusting the old estimate of $\alpha$ toward $(-z)$. Continue in this manner until the $\alpha$ used to form the accumulation CELR and $(-z)$ converge.

The question of what data to use in estimating (23) remains to be discussed. Data on the risk-free nominal interest rate poses no significant problem since time series on treasury bills are readily available (see appendix 1). Observation of RAR poses a greater problem. Data relating to depository institutions can be used with the assumption that the reserve positions of depository institutions are highly correlated with the reserve positions of other intermediaries. This assumption is discussed at greater length below. In forming RAR for depository institutions, a problem exists. While information concerning the reserves of depository institutions is available (see appendix 1), data on desired reserves is not. Here, I will use required reserves (see appendix 1) as a proxy for desired reserves. This seems conscionable since required reserves are likely to be the major determinant of desired reserves. Other factors, such as changing interest rate spreads (Cf. Modigliani, Rasche and Cooper 1970, especially eq. 3.11), fluctuations in the degree of Federal Reserve displeasure at borrowing from the discount window, the covariance between deposits and withdrawals, and the degree of risk in the lending portfolio, will result in variations in desired reserves away from the level of required reserves.

It is possible to consider the distortion introduced by using RAR of depository institutions instead of RAR of all intermediaries. It seems likely that the relative inventory positions of all intermediaries are highly correlated because financial instruments may be quickly traded between intermediaries. It is difficult to imagine a pool of excess liquidity obtaining for a significant period in one set of intermediaries while another class is illiquid. It must be true that the relative available reserves of different intermediaries are correlated.

---

22 The difficulty in deriving an estimable expression for free reserves is noted by Modigliani, Rasche, and Cooper (1970).
This means that:

\[ RAR_t = j \cdot RAR_{Dt} + e_t \]  

where:

- \( RAR_{D} \) is relative available reserves for depository institutions
- \( e \) is a disturbance term
- \( j > 0 \)

This means that:

\[ ALR_t = e^{-\alpha \cdot j} \cdot RAR_t = e^{-\alpha} \cdot j \cdot RAR_{Dt} \]  

and the estimate \((-z)\), obtained from (23) is actually an estimate of \( \alpha \cdot j \). Because theory calls for positive \( j \) and \( \alpha \), a positive estimate \((-z)\) would constitute evidence in favor of the theory.

OLS estimation of equation (19), using data from January of 1959 to October of 1980, produces the results summarized on line 1 in figure 6. The low Durbin-Watson statistic in line 1 suggests the presence of an autocorrelated disturbance term. Figure 7 presents the autocorrelation and partial autocorrelation function for the errors. The gradual, though uneven, fall in the auto-correlation function and the initial spike in the partial autocorrelation function suggests a first order autocorrelated process (Box and Jenkins 1976).

---

23 No intercept constant appears in 24 because such a constant would imply that balanced liquidity among depository institutions implies imbalanced liquidity elsewhere in the economy. This seems unreasonable.

24 Bias is also introduced by the error term in (24). In the absence of an obvious instrument no correction has been made for this factor.

25 Data on bank reserves is conveniently available from January of 1959 through the present. However, beginning in October of 1980, the definition of what kinds of depository institutions maintained reserves began to change as the requirements of the Monetary Control Act of 1980 were phased in. Consequently, data from January of 1959 to October of 1980 is used in the regressions below as a consistent set of observations.
The structure of the errors prompts a reestimation using the Cochrane-Orcutt procedure to make a first order autocorrelation correction (Johnston 1972, pp. 261-263) Results are summarized in line 2 in figure 6. The time constant (TAUIR) implied by the estimate for h is a reasonable 17 months or 1.4 years. The estimate ($-z$) of $\alpha*j$ is 4. If $\alpha$ is between .5 and 2, $j$ is between 8 and 2—a reasonable, if somewhat wide, range. The autocorrelation function and partial autocorrelation functions for the errors from this regression appear as figure 8. The process is indistinguishable from white noise. As a measure of fit, $R^2$ is a bit misleading here, representing as it does the fraction of explained variance in RFIR, including the variance explained by the lagged term RFIR$_{t-1}$ necessitated by the autocorrelation correction. A better measure is the explained variance of the generalized difference (GD) of RFIR:

$$GD = RFIR_t - \rho*RFIR_{t-1}$$

The $R^2$ of the generalized difference, appearing under the heading "GD $R^2$" in figure 6, removes the effect of the autocorrelation correction from the measure of fit. Using this measure, about 19% of the variation in the risk free rate is accounted for by the regression.

In order to test the stability of these results the data has been divided in half and regressions performed on each half. The third line of figure 6 contains estimates using the first half of the data, while the fourth line contains estimates using the second half of the data. It is clear that decreasing the amount of data increases the variance of the estimates of both h (1/TAUIR) and $z$ ($-\alpha*j$). Although the increased variance of the estimates makes it difficult to observe changes in the parameters, there is no indication that the relationship tested by the regression has changed significantly over the period from the beginning of 1959 through October of 1980. It is of interest to point out that the regression for the more recent period is better, having smaller variance about the parameter estimates and having a larger "GD $R^2$" than the regression on the earlier data. Although an explanation for the shift in fit is lacking, the fact that the fit is improving should be reassuring to those who wish to use the theory presented in his paper in practical applications. Several other regressions are reported in appendix 2.
4. Simulation

Two simulations will now be considered in order to explore the dynamics and to relate the structure to partial equilibrium theories of interest rates. The first simulation will be concerned with the isolated interest rate structure derived and estimated above. The second simulation, incorporating the demand for money, will show the structure is compatible with a partial equilibrium determined by the simultaneous equations approach, and will provide an indication of how quickly interest rates adjust to a new equilibrium.

The Isolated Structure. A simulation of the structure appears in figure 9. In this simulation TAUIR is set to its estimated value of 1.4 years. $\alpha$ is set to 4 which makes the implicit assumption that $j=1$. The pattern of behavior, of course, is not sensitive to the particular parameters chosen. We will assume that initially UIR is 2% per year, and reserves are equal to desired reserves, so RAR begins at its equilibrium value of zero. The structure is disturbed from equilibrium by a 10% step decrease in reserves; after some years reserves step up to their original (equilibrium) level. Plotted in the diagram of figure 9 are the risk free interest rate (RFIR), the underlying interest rate (UIR), and relative available reserves (RAR).

Initially the structure is in equilibrium, with the risk free rate and the underlying rate equal to 2%. Because desired reserves remain unchanged, the 10% decrease in reserves translates into a step down in RAR from 0 to $-1$, as shown in the diagram. ALR immediately rises to about 1.5 and, as seen in the diagram, the risk free rate instantaneously rises to about 150% of the underlying interest rate. The underlying rate now begins adapting to the new rate environment, that is, it begins moving toward the risk free rate. As long as RAR remains at $-1$, however, the adjustment to the risk free rate remains at 150% of the underlying rate. Consequently, the underlying rate moves toward an advancing target and, as long as reserves remain low, both the risk free rate and the underlying rate grow exponentially.

Finally, reserves instantaneously step back up to their original level, returning RAR to its neutral value of 0. Pressures in the financial system are instantaneously relieved and the risk-free rate immediately returns to the underlying interest rate. With balance in supply and demand restored, there are no further pressures to generate movement in the rates.

A less technical explanation for the behavior may also be made: Because desired reserves remain unchanged, the decrease in reserves translates into a liquidity squeeze for
the intermediary. The intermediary boosts its interest rate above the prevailing rate environment in order to discourage borrowers who (it believes) would deplete its reserves and in order to attract depositors who (it believes) will increase its reserves. The assumptions are cruel though, and the increase in the interest rate does not relieve the liquidity strain. The intermediary begins to believe that the current interest rate environment is higher than it thought, and boosts its interest rate again in order to keep the pressure on lenders and to maintain its pull on deposits. This process continues until, just as suddenly as it began, the liquidity crisis ends. The intermediary, no longer wishing to pressure its borrowers nor to attract more than the usual amount of deposits, drops its rate to a level consistent with its understanding of the prevailing interest rates in the market place.

The dynamics of the isolated structure appear to be reasonable. The reverse experiment also could have been carried out: A step increase in reserves will cause rates to decay exponentially. There is a potential problem in this case, however. Equation (15) indicates that once the underlying rate reaches zero, the risk free rate will also equal zero. Equation (16) indicates that if the two rates are identical, no further movement occurs in the structure. Consequently this structure carries the implication that rates will get "stuck" if the underlying rate reaches zero. The appendix to this essay contains a small modification based on intermediation costs which corrects this problem. Because evidence of the small modification is difficult to pick up in the available time-series data, the estimation results for "normal" operating ranges still hold and we will proceed directly to consider a more important extension to the model: feedback from interest rates to intermediaries' liquidity.

The Extended Model. The above simulations explored the dynamic implications of the interest rate formulation in isolation. As discussed extensively above, financial intermediaries adjust the interest rate in order to adjust their own liquidity. Considering the interest rate formulation in isolation ignores the feedback from interest rates to the intermediary's own liquidity. Incorporating a demand for money provides a major part of the missing link. In this and the immediately following sections the simple isolated interest rate structure is extended to incorporate the demand for money. In addition to incorporating the "missing link", the extended model also demonstrates that the

26 In a truly continuous-time simulation, UIR will never reach zero. However, since actual simulations must proceed by finite steps, UIR can hit zero in an actual simulation. Furthermore, the problem of getting "stuck" does not materialize suddenly; a UIR "very close" to zero will be "very close" to being stuck at that value.
formulation for interest rates proposed in this paper is compatible with the more traditional, simultaneous equations representation of interest rates in two ways: First, the extended model has an equilibrium which can be found via simultaneous equations, and second, under reasonable parameter combinations, the dynamics of the extended model can closely approximate a "jump" to a new equilibrium.

Figure 10 presents a model in which money demand is endogenous. In the extended model the banking system's liquidity depends upon deposits held by the public and the public's deposits are a function of interest rates. The loop composed of heavy arrows is an equilibrating, negative loop as may be seen by tracing around the effects of a liquidity squeeze. Low liquidity (measured by low RAR) will cause banks to raise the interest rate. A higher interest rate implies a higher opportunity cost to holding deposits rather than interest bearing securities. Consequently, a higher interest rate will lead businesses and individuals to decrease their desired deposits. Lower desired deposits will lead to lower actual deposits. Lower deposits will relieve the liquidity squeeze. The negative loop will tend to restore the system to its desired state of liquidity.

More specifically, the the banking system's relative available reserves (RAR) is defined as above (equation (18)) as:

\[ RAR_t = (R_t - DR_t)/DR_t \]  

(26)

where:

\begin{align*}
RAR & \quad \text{Relative Available Reserves} \\
R & \quad \text{Reserves} \\
DR & \quad \text{Desired Reserves}
\end{align*}

The desired reserves of the banking system may be written as:

\[ DR_t = D_t \times RR \]  

(27)

where:

\begin{align*}
D & \quad \text{Deposits} \\
RR & \quad \text{Reserve Requirements}
\end{align*}

Hence, as deposits held by the public go down the banking system's liquidity, as measured by RAR, goes up. Deposits are a level whose rate of change depends upon interest rates. As rates go up, desired demand deposits, and hence deposits, go down because it is more expensive in terms of interest foregone to hold deposits. There is extensive empirical literature on the demand for money which can be brought to bear on the
formulation of desired deposits.  Commonly, empirical studies of the demand for money are based on a relatively simple money demand equation (Goldfeld 1973, 1976; Judd and Scadding 1982) which, in turn, is based on an inventory theory of money holdings (Baumol 1952; Tobin 1956; Miller and Orr 1966, 1968). The formulations used in econometric studies, intended for econometric models, may be manipulated to yield equivalent expressions well suited to use in system dynamics models.

The jumping off point for most econometric studies of the demand for money is an equation like (28) below.

\[ DD_t = a \cdot Y_t^{d} \cdot RFIR_t^{b} \]  \hspace{1cm} (28)

where:

- DD - Desired Deposits
- Y - Transactions
- RFIR - Interest Rate
- a, b, d - Constants

Equation (28) can be recast as:

\[ DD_t = (a \cdot RY_t^{d} \cdot RIR_t^{b}) \cdot (Y_t/RY)^{d} \cdot (RFIR_t/RIR)^{b} \]  \hspace{1cm} (29)

Defining the first term in parenthesis as Reference Deposits (RD), the second term in parenthesis as relative income (RY), and the third term in parenthesis as the relative interest rate (RI) equation (29) can be rewritten as:

\[ DD_t = RD \cdot RY_t^{d} \cdot RI_t^{b} \]  \hspace{1cm} (30)

Finally, defining the second term on the right hand side as the effect of income on deposits (EYD) and the third term as the effect of interest rates on desired deposits (EID) yields:

\[ DD_t = RD \cdot EYD_t \cdot EID_t \]  \hspace{1cm} (31)

Empirical work on the demand for money usually assumes that actual money balances adjust to desired balances with a lag. Estimation is usually carried out using logarithms (Judd and Scadding 1982, Goldfeld 1973, 1976) as in the equation below:

\[ \ln(D_t) - \ln(D_{t-1}) = c \{ \ln(DD_t) - \ln(D_{t-1}) \} \]  \hspace{1cm} (32)

---

27 In the model here there is no currency, so money and deposits are synonymous. The appropriate estimates, therefore, are those for the demand for money rather than those which deal only with the demand for demand deposits.
To see what this equation implies for the underlying adjustment process begin by taking the anti-logs. This yields:
\[ (D_t/D_{t-1}) = (DD_t/DD_{t-1})^c \]
(33)

Multiplying by \( D_{t-1} \) produces:
\[ D_t = (DD_t/DD_{t-1})^c \cdot D_{t-1} \]
(34)

To find the change in deposits each period, subtract \( D_{t-1} \) from each side:
\[ D_t - D_{t-1} = ((DD_t/DD_{t-1})^c - 1) \cdot D_{t-1} \]
(35)

or, in continuous time notation:\[28\]
\[ (d/dt)D_t = ((DD_t/DD_t)^c - 1) \cdot D_t \]
(36)

Defining the term in parenthesis as relative desired deposits (RDD) yields:
\[ (d/dt)D_t = ((RDD_t)^c - 1) \cdot D_t \]
(37)

and defining the term in curly brackets as the fractional change in deposits (FCD) yields:
\[ (d/dt)D_t = FCD_t \cdot D_t \]
(38)

Equations (29) and (36) together with equations (15)-(18) represent a model of interest rate formation in a banking system with endogenous demand for money. Equations (29) and (36) have been decomposed above to produce an intuitively appealing formulation. For convenience the equations of the model are presented together below.

\[ RFIR_t = UIR_t \cdot ALR_t \]
(39)

\[ (d/dt)UIR_t = (RFIR_t - UIR_t)/TAUIR \]
(40)

\[ ALR_t = f(RAR_t) = e^{-\alpha*RAR_t} \]
(41)

\[ RAR_t = (R_t - DR_t) / R_t \]
(42)

\[ DR_t = D_t \cdot RR \]
(43)

\[ (d/dt)D_t = FCD_t \cdot D_t \]
(44)

\[ FCD_t = (RDD_t)^c - 1 \]
(45)

\[ RDD_t = DD_t/DD_t \]
(46)

\[ DD_t = RD \cdot EYD \cdot EID_t \]
(47)

\[ EID_t = Rl^t \]
(48)

\[ RI_t = RFIR_t/RIR \]
(49)

where:

\[ RFIR \quad - \quad \text{Risk Free Interest Rate} \]
\[ UIR \quad - \quad \text{Underlying Interest Rate} \]
\[ ALR \quad - \quad \text{Adjustment from Liquidity to the risk free Rate} \]

\[28\] This is an approximation. The approximation becomes exact as the period of the discrete-time formulation goes to zero.
TAUIR - Time to Adjust Underlying Interest Rate
R - Reserves
DR - Desired Reserves
D - Deposits
RR - Reserve Requirements
FCD - Fractional Change in Deposits
RDD - Relative Desired Deposits
DD - Desired Deposits
RD - Reference Deposits
EYD - Effect of Income on Deposits
EID - Effect of Interest Rate on Deposits
RI - Relative Interest Rate
RIR - Reference Interest Rate
b,c - parameters

In the above model, EYD has been held constant and below it is set equal to its neutral value of 1. The effect of income is not considered further here, despite its importance, because such a consideration would take us well beyond the confines of the present partial disequilibrium consideration of interest rates.

Parameters in the Extended Model. The previously obtained estimates for TAUIR and \( \alpha \), 1.4 and 4 respectively, will be used initially in what follows, and their estimated standard deviations will be used in subsequent sensitivity tests. Because it is not the purpose here to add to the already voluminous literature on money demand, values for \( b \) and \( c \) will be based on estimates from the other studies. The stability of money demand represented by equations (28) and (32) has been questioned (Goldfeld 1976, Judd and Scadding 1982) and many modifications to the "conventional" equation have been suggested to correct for instability. Through all of these modifications, however, the estimates for the parameters \( b \) and \( c \) above have remained in a relatively narrow range.

The post 1973 literature on money demand equations has been reviewed by Judd and Scadding (1982). Figure 11 below shows the estimates of the parameters of concern here\(^{29}\) from the equations discussed in Judd and Scadding.\(^{30}\) Because most empirical studies use more than one interest rate, there is a potential problem in using the results from

\(^{29}\) I have shown parameter estimates rather than coefficient estimates. The translation is as follows:
Adjustment Speed Parameter = 1 - (coefficient on lagged money)
Interest Rate Parameter = (Coefficient on Interest rate)/(Adjustment Speed Parameter)

\(^{30}\) Only equations for M1 and containing transactions variables and without imposed constraints are shown in figure 6.
the studies for an estimate of the single interest rate parameter \( b \) needed in the present context. A sense for the combined impact of interest rates can be gained, however, by adding together the parameter estimates on each of the separate interest rates as shown in the table. The sum would be precisely correct if one could write

\[
\text{Rate}_{1t} = \mu \ast \text{Rate}_{2t}
\]

because in this case

\[(\text{Rate}_{1t})^w \ast (\text{Rate}_{2t})^q = \mu^q \ast \text{Rate}_{1t}^{(w+q)}\]

so that

\[b = (w+q)\]

Because equation (50) does not hold exactly, considering the sum of the parameters in estimated money demand equations to be equal to the single parameter needed for equation (48) is only an approximation. It is, however, interesting to note that the sum of the interest rate parameter estimates is less variable between equations than the parameter estimates for the individual interest rates. It is likely that the collinearity between interest rates makes the parameter estimates for a single rate less reliable than the sum of the parameter estimates.

The entries in the column labeled "Sum of Interest Rate Coefficients" are estimates of the value of the parameter \( b \) in equation (48). Thirteen of the sixteen entries are between .18 and .31. The validity of each of the three remaining equations is questionable: Equation (1.2) contains a positive estimate for the coefficient on the interest rate on time deposits and is therefore suspect (indeed Garcia and Pak argue that (1.2) is "unacceptable" (1979 p. 330)). Goldfeld considers equation (A2.4) "clearly a step in the wrong direction" (1976, p. 697). Equation (A2.5) is conceptually quite similar to (A2.6) (both include terms for debits), but (A2.6) is considered by Goldfeld to be superior to (A2.5) (Goldfeld p. 697). In brief the parameter estimates that differ markedly from the .2 to .3 range are considered by the researchers responsible to be inferior money demand equations. The difference in EID for \( b = .2 \) and for \( b = .3 \) is not large for most values of RI as can be seen from the following expression.

\[
(EID_{\text{large}} - EID_{\text{small}})/EID_{\text{small}} = (RI^{-.3} - RI^{-.2})/RI^{-.2} = RI^{-.1} - 1
\]

A value of .25 for \( b \) will be used initially.

Figure 11 also presents estimates for the "adjustment speed," parameter \( c \) in equation (45). Equation (1.2) has already been questioned. All the other parameter estimates fall between about sixteen and 30 per cent per quarter. An estimate of .2 per quarter (.8 per year) will be used initially.
In brief, the parameters in the extended model are:

\[
\begin{align*}
\alpha &= 4 \\
\text{TAR} &= 1.4 \text{ years} \\
c &= .80 \text{ per year} \\
b &= .25
\end{align*}
\]

These values fall into the mid-range of the estimates available. The sensitivity of the dynamics to the uncertainty in these estimates will be considered later.

**Comparative Statics and Dynamic Adjustment in the Extended Model.**

The extended model is compatible with more traditional simultaneous equations approaches to representing interest rates in the sense that the extended model has a unique non-zero equilibrium and the methods of simultaneous equations can be used to find it. Desired deposits are given by equation (47). After some rearranging and substitution and making use of the equilibrium assumption that EYD=1, desired deposits may be written:

\[
DD_t = (RD/RIR)^b \cdot RFIR_t
\]

The supply of deposits (SD) may be found by solving equation (42) for desired reserves, substituting using (43), and realizing that in equilibrium desired reserves will be equal to reserves. Thus:

\[
SD_t = R_t/RR
\]

The equilibrium condition is that supply equals demand simultaneously or

\[
SD_\infty = DD_\infty
\]

After making the suitable substitutions one finds that in equilibrium

\[
RFIR_\infty = RIR \cdot \left(\frac{R/(RR \cdot RD)}{1/b}\right)
\]

\[
D_\infty = R/RR
\]

The equilibrium impact of a ten per cent reduction in reserves can now be calculated. Using \(b = -.25\), \(RIR = 2\), \(RD = 10\), \(RR = .1\) as the relevant parameters, and setting \(R\) initially equal to 1, a quick calculation shows that in equilibrium \(RFIR = 2\) and \(D = 10\). If \(R\) is dropped to .9, the new equilibrium will be where \(D = 9\) and \(RFIR = 3.05\). A ten per cent reduction in reserves implies a ten per cent reduction in deposits and a fifty per cent increase in the interest rate in partial equilibrium.  

---

31 It is well to stress the discussion concerns a partial equilibrium in the financial markets. The derivation holds if transactions remain fixed. Clearly, transactions are not fixed. Assuming that transactions are fixed is reasonable only if the adjustment through deposits is fast relative to changes in real economic activity. The extended model includes the dynamic mechanisms necessary to gain a grasp of the speed of the response through deposits.
The extended model represented by equations (39) to (49) is a disequilibrium model that has an equilibrium consistent with the simultaneous equations approach. However, the extended model is a more general case than the simultaneous equations representation: It contains the dynamics by which the economy moves to equilibrium.

Consider the dynamic forces at work when the Federal Reserve reduces reserves. Intermediaries will feel a liquidity squeeze. In response, they will increase rates above the underlying interest rate in order to attract deposits and purchasers of securities, while discouraging borrowers and sellers of securities. Then, two dynamic processes will be called into play: The intermediaries will begin to reassess the underlying rate, increasing it as the interest rate remains high. This, in isolation, leads to a growing risk free rate as was seen in the simulations of the isolated structure. In the extended model, however, an additional force is called into play: Businesses and individuals begin reducing their deposits in the presence of a higher interest rate. Reduced deposits means lower desired reserves and, consequently, a lessened liquidity squeeze. In isolation the reaction of businesses and individuals would tend to push the interest rate down.

The two dynamic responses, upward revisions of the underlying interest rate and reduced deposits, work in opposite directions. The reaction of the risk free rate at any point in time will depend on the relative strengths of the two responses: On the one hand, the risk free rate will depend on how sensitive business and individual desired deposits are to the interest rate. On the other hand, the behavior of the risk free rate will depend on how quickly intermediaries reassess the underlying rate and on how far above the underlying rate they boost the risk free rate. These characteristics of businesses, individuals and intermediaries are captured in the parameters of the extended model:

\[
\begin{align*}
\text{b} & \quad \text{Relationship between desired deposits and the interest rate. Larger values indicate desired deposits are more sensitive to the interest rate.} \\
\text{c} & \quad \text{The speed with which deposits adjust to desired deposits. Larger values of } b \text{ denote a faster adjust speed.} \\
\text{TAUIR} & \quad \text{The speed with which intermediaries adjust the underlying rate toward the risk free rate.} \\
\alpha & \quad \text{The degree to which intermediaries raise the risk free rate above the underlying rate in response to liquidity pressures.}
\end{align*}
\]

Figure 12 presents a plot of the risk free interest rate and the equilibrium risk free interest rate. Surprisingly, perhaps, the risk free rate jumps immediately to its equilibrium
value and stays there. The initial increase in the risk free rate (governed by $\alpha$) moves the risk free rate to its equilibrium value. Then, the decreasing liquidity squeeze caused by depositors reducing their deposits (parameters b,c) balances the increasing underlying interest rate (parameter TAUIR). This simulation shows that the disequilibrium formulation of the extended model is consistent with the simultaneous equations approach in another important sense: The extended model can jump to a new equilibrium under suitable choices of the parameters.

It is an intriguing result suggesting, perhaps, that there may be a learning process by which financial intermediaries and businesses and individuals come to have a set of compatible responses that favor stability in interest rates. Unfortunately, this conclusion will need more substantiation than that offered by this paper. Although it is true that the jump to equilibrium holds up very well under a variety of test inputs given the parameters, these parameters are subject to uncertainty. $\alpha$ and TAUIR have been estimated in this paper and have a probability distribution about them with the standard deviations shown in figure 6. Parameters b and c are typical values from several other studies as discussed above. There is sufficient uncertainty in the parameters to produce behavior different from the jump to equilibrium. For example, figure 13 shows a far more sluggish response to a ten per cent reduction in reserves. The simulation was obtained by assuming intermediaries change rates more cautiously and more slowly ($\alpha$ reduced by one standard deviation and TAUIR increased by one standard deviation) and by assuming businesses and individuals move their deposits more slowly toward a more extreme goal (b reduced to .18 and c increased to 1.2 — the end points of their typical ranges). Under these assumptions of how people react, interest rates remain below their equilibrium values for years. The reverse assumptions, Intermediaries more aggressive and faster, depositors less extreme but faster ($\alpha$ increased and TAUIR reduced by one standard deviation; b increased and c reduced to the end points of the "typical" ranges) produces a large overshoot as shown in figure 14.

Further clouding the evidence in this paper that rates immediately jump to new equilibria is the absence of an important feedback. Interest rates affect decisions on the

---

32 The equilibrium is a partial equilibrium, because there is no feedback through income.

33 There is a slight tendency to overshoot for large (20%) reductions in reserves, a tendency to undershoot, then overshoot for reductions in reserves. The response to sinusoidally varying reserves is very good for periods of two years and four years.
"real" side of the economy—decisions to invest and to hire or fire. Those decisions will determine the need for deposits to cover transactions. Movements in the real economy, therefore, will alter the response of deposits to interest rates. An investigation into the impact of the physical responses of the economy moves us beyond the confines of this paper.

5. Summary and Conclusions

This paper has presented a behavioral model of interest rate mechanics. The theory is based upon the observation that most financial transactions go through financial intermediaries at prices set by those intermediaries. Intermediaries raise and lower interest rates in order to adjust their financial inventories (i.e. their inventories of securities and reserves). Intermediaries use an anchoring and adjustment strategy to decide where interest rates should be. In the presence of financial inventory imbalances, the intermediary will raise or lower its posted interest rates above or below an anchor here termed the underlying interest rate. The underlying interest rate represents the interest rate environment, the expectation of what the interest rate would be now if it were not for transitory pressures such as financial inventory imbalances.

The financial inventory positions of financial intermediaries accumulate, and thereby summarize, the entire history of supply and demand. For example, a low money inventory (or a high securities inventory) indicates that people in the non-financial sectors have been selling securities at a rate greater than they have been buying them. An increase in the interest rate is a response which should bring demand and supply back into line. An increase in interest rates is also the response that the illiquid intermediary will be inclined to make to adjust its own inventories. In brief the behavioral theory is a supply and demand theory.

Empirical support for the behavioral theory exists at a micro and at a macro level of aggregation. Research into the nature of human decision making at the individual level indicates that anchoring and adjustment is a common strategy for making decisions like those involved in interest rate determination. Econometric estimation was used to validate the theory at a macro-economic level of aggregation.

An "extended model" in which the interest rate theory was coupled with a representation of money demand showed that the behavioral theory can contain an equilibrium that may be determined by simultaneous equations techniques. However, the
theory developed in this paper also contains the behavioral adjustment processes that bring interest rates into equilibrium. The extended model can behave like more traditional partial equilibrium models of interest rate determination—interest rates in the extended model can jump to new (partial) equilibria and stay there. But, with slightly altered parameters the extended model can exhibit different behavior: Interest rates can dally below the equilibrium rate for an extended period time, or interest rates can overshoot the mark.

The research reported in this paper can be extended in several directions. Further work in empirical estimation of parameters, theory building, and policy analysis would all be worthwhile. Better data can be collected on the reserves and desired reserves of intermediaries. Alternatively, more sophisticated estimation techniques might be utilized to correct for biases introduced by using surrogate measures. Observation of the way interest rates are actually determined by those who actually determine them would provide an even more valuable opportunity to confirm or disconfirm the formulation suggested in this paper. In addition, such observation might reveal the ways in which the parameters of the formulation are themselves formed, allowing one to deepen the current model by transforming its parameters into variables.

The theory might also be extended through disaggregation. Although this paper dealt at a highly aggregated level, the disequilibrium process described herein could be applied to a more detailed model which included several different securities and several different intermediaries. Such a disaggregation would illuminate the processes by which the different interest rates move relative to one another.

Finally, the current work might be extended by investigating the policy implications of this theory of interest rate formation. What are the causes and consequences of credit crunches. How does the Fed influence the business cycle and other economic behavior modes? What are the impacts of targeting interest rates, reserves, money stock, or unemployment? Are there other targets or other rules for guiding open market operations which are more powerful or more reliable for fostering economic growth? Answering questions such as these will involve embedding this interest rate theory into increasingly more detailed models.
Appendix 1: Data Sources

This appendix provides information on the data, and their transformation, used to estimate equation (19). The risk-free rate is based on CITIBASE's (Citibank 1985) secondary market rate on three month treasury bills (data series FYGM3, see also Federal Reserve Bulletin, table 1.20)). This rate is calculated on a bank-discount basis.

Relative available reserves (RAR) are defined as the difference between reserves and desired reserves, divided by desired reserves. The data series used for reserves was CITIBASE's FZRN, nonborrowed reserves of depository institutions, and for desired reserves CITIBASE's FZRQ, required reserves of depository institutions. Neither FZRN nor FZRQ are seasonally adjusted, nor are they adjusted for changes in reserve requirements, in particular the change in institutions required to hold reserves associated with the Monetary Control Act of 1980. To avoid definitional problems introduced with the monetary control act of 1980, the data series used end in October of 1980. Using data adjusted for the Monetary Control Act of 1980 and using the data through 1987 materially affects the estimates. These alternative estimates would imply that the interest moves far more slowly. The estimates using these data, however, have lower $R^2$ and higher standard deviations of the estimates.
Appendix 2: Additional Regressions

Several regressions were performed in addition to those discussed in the text. One theory of the autocorrelated error term is that it arises from the month-by-month accumulation over the entire span of the regression. Lines 5 and 6 represent OLS and GLS regressions respectively in which the extended monthly accumulation is replaced by a twelve-month accumulation. The improved Durbin-Watson statistic of line 5 compared to line 1, indicates that, perhaps, some of the autocorrelation is removed by this Arlying R, although other explanations are also possible. The GLS correction yields estimates comparable to those of line 2, however the regression in terms of $R^2$ and significance of the coefficients is not quite as good. Line 7 tests whether moving to a one-month accumulation is helpful. The high standard error on H suggests that this procedure makes it difficult to obtain a good measure of the adjustment speed of the underlying rate—the underlying interest rate appears to adjust more slowly than one month.

The impact of inflation is briefly explored in lines 8 and 9. In line 8, the current period inflation rate is added to the regression equation; in line 9 the change in the inflation rate is added to the basic regression equation. In both tests, the inflation term is insignificant and carries a sign opposite from what would be expected from the Fischer theory. These results give some preliminary evidence that theory of interest rate mechanics is indeed a more proximate theory of interest rates than Irving Fischer's—that is, the influence of inflation works through liquidity, rather than directly on interest rates.
Appendix 3: Modification to Interest Rate Structure

As discussed in the text, the structure contains a risk that rates will become "stuck" at zero. A small modification removes this risk. The change involves introducing a new variable, the indicated underlying rate (IUR), toward which the underlying rate adjusts. The indicated underlying rate is identical to the risk-free rate, except the indicated underlying rate does not go to zero, but rather to some minimum value.

Behaviorally such a modification implies there is some lowest reference rate which is greater than zero. Economically this minimum rate may be determined by the costs of intermediation. The minimum rate may be interpreted as the average cost of intermediation expressed as a fraction of the amount intermediated.

A restatement of the interest rate formulation using a possible definition of the indicated underlying rate is:

\[
\begin{align*}
RFIR_t &= UIR_t \cdot ELR_t \\
(\delta_{dt})UR_t &= (IUR_t - UIR_t)/TAUR \\
IUR_t &= \max(RFIR_t, MUIR) \\
ALR_t &= e^{\alpha \cdot RAR_t}
\end{align*}
\]

(A2.1)   \hspace{1cm} (A2.2)   \hspace{1cm} (A2.3)   \hspace{1cm} (A2.4)

where:

- \text{RFIR} - Risk Free Interest Rate
- \text{UIR} - Underlying Risk Free Interest Rate
- \text{ALR} - Affect of Liquidity on the risk free Rate
- \text{IUR} - Indicated Underlying Interest Rate
- \text{MUIR} - Minimum Underlying Interest Rate, a constant
- \text{TAUR} - Time to Adjust the Underlying Interest Rate
- \text{RAR} - Relative Available Reserves
- \text{e} - The exponential function
- \text{\alpha} - a constant

The new definition of the underlying interest rate appears as equation (A2.3).

---

34 A function involving a gradual approach to MUIR, in place of the abrupt change introduced by the max function, would also be a possible assumption. The use of such a function, however, introduces additional complications and further dynamics without materially enriching the current discussion.
Appendix 4: Model Documentation

Figure 9 was generated by a model created using STELLA. Figures 12, 13, and 14 were generated by a model written in DYNAMO. Equation listings for each model follow. Note that "changes files" for the DYNAMO program are given at the end of the listing.
BASIC INTEREST RATE STRUCTURE
(Used in Figure 9)
STELLA Listing

JAMES H. HINES, JR.

□ UIR = UIR + CUIR
    INIT(UIR) = 2
□ a = 4
□ ALR = EXP(-a*RAR)
□ CUIR = (RFIR-UIR)/TAUIR
□ DR = 1
□ R = 1+(STEP(-.1,.5)-STEP(-.1,4.5))
□ RAR = (R-DR)/DR
□ RFIR = UIR*ALR
□ TAUIR = 1.4
EXTENDED INTEREST RATE MODEL
(Used in Figures 12, 13, and 14)
DYNAMO listing

JAMES H. HINES, JR.

A  RFIR.K=UIR.K*ALR.K
    RISK FREE INTEREST RATE (FRACTION/YEAR)
L  UIR.K=UIR.J+DT*CUIR.J
N  UIR=RIR*(((R/(RR*RD))**(1/B))
    UNDERLYING INTEREST RATE
A  CUIR.K=(RFIR.K-UIR.K)/TAUIR
    CHANGE IN THE UNDERLYING INTEREST RATE
    (FRACTION/YEAR/YEAR)
C  TAUIR=1.4
    TIME TO ADJUST THE UNDERLYING INTEREST RATE (YEARS)
A  ALR.K=EXP(-A*RAR.K)
    ADJUSTMENT FROM LIQUIDITY FOR THE RISK FREE RATE
    (DIMENSIONLESS)
C  A=4
    COEFFICIENT ON LIQUIDITY ADJUSTMENT
A  RAR.K=(R.K-DR.K)/DR.K
    RELATIVE AVAILABLE RESERVES (DIMENSIONLESS)
A  R.K=IR*STUP.K*WAVE.K
    RESERVES (CURRENCY UNITS)
C  IR=1
    INITIAL RESERVES
A  STUP.K=1+STEP(FSIR,TFSIR)
    STEP-UP (DIMENSIONLESS)
C  FSIR=0
    FRACTION STEP INCREASE IN RESERVES (DIMENSIONLESS)
C  TFSIR=.5
    TIME FOR FRACTION STEP INCREASE IN RESERVES (YEAR)
A  WAVE.K=1+WAVAMP*SIN(6.283*TIME.K/WAVPER)*STEP(1,TSWAVE)
    WAVE (DIMENSIONLESS)
C  TSWAVE=.5
    TIME TO START WAVE (YEAR)
C  WAVAMP=0
    WAVE FRACTIONAL AMPLITUDE
C  WAVPER=2
    WAVE PERIOD
A  DR.K=D.K*RR
    DESIRED RESERVES (CURRENCY UNITS)
C  RR=.1
    RESERVE REQUIREMENT
L  D.K=D.J+DT*CID.J
N  D=R/RR
    DEPOSITS (CURRENCY UNITS)
A  CID.K=FCD.K*D.K
    CHANGE IN DEPOSITS (CURRENCY UNITS/YEAR)
A  FCD.K=(RDD.K**C)-1
    FRACTIONAL CHANGE IN DEPOSITS (FRACTION/YEAR)
C  C=.8
COEFFICIENT ON FRACTIONAL CHANGE IN DEPOSITS
A  RDD.K=DD.K/D.K
RELATIVE DESIRED DEPOSITS (DIMENSIONLESS)
A  DD.K=RD*EYD*EID.K
DESIR ED DEPOSITS (CURRENCY UNITS)
C  RD=10
REFERENCE DEPOSITS (CURRENCY UNITS)
C  EYD=1
EFFECT OF TRANSACTIONS ON DESIRED DEPOSITS
A  EID.K=RI.K**B
EFFECT OF INTEREST RATES ON DESIRED DEPOSITS
   (DIMENSIONLESS)
C  B=.25
COEFFICIENT ON EID
A  RI.K=RFIR.K/RIR
RELATIVE INTEREST RATE (DIMENSIONLESS)
C  RIR=2
REFERENCE INTEREST RATE (FRACTION/YEAR)

A  EQUILRT.K=RIR*((R.K/(RR*RD))**(1/B))
EQUILIBRIUM RISK FREE RATE (FRACTION/YEAR)
A  EQUILD.P.K=R.K/RR
EQUILIBRIUM DEPOSITS (CURRENCY UNITS)
SPEC  LENGTH=5/SAVPER=.0625/DT=.0625/REL_ERR=0
Changes file for FIGURE 12: "Behavior of Extended Model"

---------------------------------------- Parameters

FSIR
Present  -.1
Original  0.

Changes file for FIGURE 13: "Behavior of Extended Model - Slow Response"

---------------------------------------- Parameters

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>FSIR</th>
<th>TAUIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>3.4</td>
<td>-.18</td>
<td>1.2</td>
<td>-.1</td>
</tr>
<tr>
<td>Original</td>
<td>4.</td>
<td>-.25</td>
<td>.8</td>
<td>0.</td>
</tr>
</tbody>
</table>

---------------------------------------- Run specifications

<table>
<thead>
<tr>
<th>LENGTH</th>
<th>SAVPER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>25.</td>
</tr>
<tr>
<td>Original</td>
<td>4.</td>
</tr>
</tbody>
</table>

Changes file for FIGURE 14: "Behavior of Extended Model - Overshoot Response"

---------------------------------------- Parameters

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>FSIR</th>
<th>TAUIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>4.5</td>
<td>-.31</td>
<td>.64</td>
<td>-.1</td>
</tr>
<tr>
<td>Original</td>
<td>4.</td>
<td>-.25</td>
<td>.8</td>
<td>0.</td>
</tr>
</tbody>
</table>

---------------------------------------- Run specifications

<table>
<thead>
<tr>
<th>LENGTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
</tr>
<tr>
<td>Original</td>
</tr>
</tbody>
</table>
REFERENCES


Figure 1: Selected Interest Rates

Selected interest rates do not move in precise lock-step, the degree of common movement is significant.
Figure 2: Simplified Sketch of a Securities Dealer
The Securities Dealer: As people sell their bonds to a dealer, the dealer's money balance declines while his stock of bonds increases. As people buy bonds from a dealer, the dealer's money balance increases while his stock of money decreases.
Figure 3: Exponential Adjustment Paths

Exponential Adjustment Paths: The response of the underlying interest rate to a nominal step increase in the risk free rate depends on the parameter TAUIR.
Figure 4: The Exponential Function
The adjustment of the risk free rate from liquidity (ALR) depends upon the value of relative available reserves (RAR) and on the parameter $\alpha$. 

\[ ELR = e^{-\alpha RAR} \]
Figure 5: Basic Interest Rate Structure
The risk free interest rate is the underlying interest rate adjusted for liquidity. The underlying interest rate is a weighted average of past values of the risk free rate. The adjustment from liquidity is determined by the availability of reserves relative to desired reserves.
<table>
<thead>
<tr>
<th></th>
<th>K</th>
<th>H</th>
<th>Z</th>
<th>( \rho )</th>
<th>I</th>
<th>DW</th>
<th>( R^2 )</th>
<th>( G D^2 R^2 )</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.3445</td>
<td>.014326</td>
<td>-11.2408</td>
<td>.607371</td>
<td>.174054</td>
<td>.764904</td>
<td>1959/1-1980/10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.46522</td>
<td>.059829</td>
<td>-5.536</td>
<td>.683171</td>
<td>.966161</td>
<td>1.65545</td>
<td>.975668</td>
<td>.192643</td>
<td>1959/1-1980/10</td>
</tr>
<tr>
<td>3</td>
<td>2.02442</td>
<td>.099036</td>
<td>-2.98113</td>
<td>.717602</td>
<td>.984502</td>
<td>1.72463</td>
<td>.97292</td>
<td>.066153</td>
<td>1959/1-1969/12</td>
</tr>
<tr>
<td>4</td>
<td>1.20292</td>
<td>.087458</td>
<td>-4.16393</td>
<td>.712929</td>
<td>.908224</td>
<td>1.60433</td>
<td>.948919</td>
<td>.306380</td>
<td>1970/1-1980/10</td>
</tr>
<tr>
<td>5</td>
<td>.15542</td>
<td>(.00295)</td>
<td>-9.13459</td>
<td>(.424744)</td>
<td>.368609</td>
<td>.674112</td>
<td>1960/1-1980/10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>.021767</td>
<td>(.02378)</td>
<td>-4.30546</td>
<td>(.577588)</td>
<td>.902991</td>
<td>1.8178</td>
<td>.908750</td>
<td>.186588</td>
<td>1960/2-1980/10</td>
</tr>
<tr>
<td>7</td>
<td>.0455</td>
<td>(.053072)</td>
<td>-2.13302</td>
<td>.378671</td>
<td>1.74263</td>
<td>.111788</td>
<td>1959/2-1980/10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>.46732</td>
<td>(.143931)</td>
<td>-3.98745</td>
<td>(.575691)</td>
<td>.966211</td>
<td>-.591965</td>
<td>(.149334)</td>
<td>1.65126</td>
<td>.975683</td>
</tr>
<tr>
<td>9</td>
<td>1.46622</td>
<td>(.143773)</td>
<td>-4.00471</td>
<td>(.568789)</td>
<td>.966301</td>
<td>-1.13703</td>
<td>(.854536)</td>
<td>1.64216</td>
<td>.975835</td>
</tr>
</tbody>
</table>

**Figure 6: Estimation Results**

K - Estimated value of the underlying interest at the beginning of the data range for each regression.
H - Adjustment speed parameter for the underlying interest rate. 1/H gives the adjustment speed in months.
Z - Parameter governing strength of adjustment from liquidity: The greater the magnitude of Z, the greater the response to liquidity pressures.
\( \rho \) - Autocorrelation coefficient of the errors.
I - Coefficient on inflation in line 8, coefficient on change in inflation in line 9.
\( R^2 \) - Percentage of variation of the risk free interest rate explained by the regression.
\( G D^2 R^2 \) - Percentage of variation of the generalized difference of the risk free rate explained by the regression.
RFIR\(_t\) - RFIR\(_{t-1}\) - The generalized difference is
Figure 7: Autocorrelations for OLS

Autocorrelogram and partial autocorrelogram of the OLS regression error term suggesting the presence of first order autocorrelation in the error term of the regression.
Figure 8: Autocorrelation and partial autocorrelation of the GLS regression error terms. The GLS procedure has resulted in an uncorrelated error term.
Figure 9: Behavior of Isolated Rare Structure
Intermediaries increase the risk free interest rate above the underlying rate when they are illiquid (i.e., when relative available reserves are below 0). They adjust their assessment of the underlying rate toward the risk free rate causing rates to move upward during periods of illiquidity. Intermediaries drop the risk free rate back to the level of the underlying interest rate when liquidity pressures are removed.
Figure 10: The Extended Model

Insufficient liquidity (i.e., reserves lower than desired reserves) will lead banks to increase the interest rate. The increased interest rate will cause people and businesses to reduce their deposits. The reduction in deposits will reduce the desired reserves of banks. A reduction in desired reserves will relieve the illiquidity of banks.
<table>
<thead>
<tr>
<th>Judd and Scadding's Equation</th>
<th>Source's Equation</th>
<th>Parameter Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>T-Bill or Commercial Paper Rate</td>
</tr>
<tr>
<td>1.1 Goldfeld (1976)</td>
<td>5.1</td>
<td>-.056</td>
</tr>
<tr>
<td>† 1.2 Garcia-Pak (1979)</td>
<td>1.3</td>
<td>-.957</td>
</tr>
<tr>
<td>1.4 Goldfeld (1976)</td>
<td>6.2</td>
<td>-.039</td>
</tr>
<tr>
<td>1.5 Goldfeld (1976)</td>
<td>6.1</td>
<td>-.075</td>
</tr>
<tr>
<td>A1.4 Goldfeld (1976)</td>
<td>6.3</td>
<td>-.041</td>
</tr>
<tr>
<td>A1.5 Goldfeld (1976)</td>
<td>6.4</td>
<td>-.043</td>
</tr>
<tr>
<td>A1.6 Goldfeld (1976)</td>
<td>6.5</td>
<td>-.050</td>
</tr>
<tr>
<td>A1.7 Goldfeld (1976)</td>
<td>6.6</td>
<td>-.067</td>
</tr>
<tr>
<td>† A2.2 Hafer and Hein (1979)</td>
<td>1.3</td>
<td>-.037</td>
</tr>
<tr>
<td>A2.4 Goldfeld (1976)</td>
<td>5.2</td>
<td>-.204</td>
</tr>
<tr>
<td>A2.5 Goldfeld (1976)</td>
<td>5.3</td>
<td>-.232</td>
</tr>
<tr>
<td>A2.6 Goldfeld (1976)</td>
<td>5.4</td>
<td>-.063</td>
</tr>
<tr>
<td>A2.7 Goldfeld (1976)</td>
<td>5.5</td>
<td>-.054</td>
</tr>
<tr>
<td>A2.8 Goldfeld (1976)</td>
<td>5.6</td>
<td>-.063</td>
</tr>
<tr>
<td>† * A3.1 Hafer and Hein (1979)</td>
<td>2.2</td>
<td>-.121</td>
</tr>
<tr>
<td>A3.3 Hafer and Hein (1979)</td>
<td>2.4</td>
<td>-.042</td>
</tr>
</tbody>
</table>

* The equation shown here corrects Judd and Scadding's typographical errors on the coefficients of government bonds and dividend price ratio. Judd and Scadding's rounding is preserved in the other equations.
† Equations for Hafer and Hein show the table number before the decimal. García and Pak's equation comes from the third equation of table 1.

**Figure 11: Interest Rate Coefficients and Adjustment Speed**

Judd and Scadding (1982) reviewed other economists' statistical estimates of coefficients on determinants of the money supply. The first column gives the equation number in Judd and Scadding's review; the second column gives the author and date of the original study; the third column gives the equation number in the original study.
Figure 12: Behavior of Extended Model
With $\alpha=4$, $\text{TUIR}=1.4$, $B=.25$, and $C=.8$, the risk free rate jumps to a new equilibrium value and stays there in response to a 10% decrease in reserves.
Figure 13: Behavior of Extended Model: Slow Response

With $\alpha = 3.4$, $TAUR=2.2$, $B=.18$, and $C=1.2$, the risk free rate moves slowly to a new equilibrium in response to a 10% decrease in reserves.
Figure 14: Behavior of Extended Model: Overshoot Response
With $\alpha = 4.5$, $TAUIR = 1$, $B = .31$, and $C = .64$, the risk-free rate moves and overshoots the new equilibrium in response to a 10% decrease in reserves.
II. A Disequilibrium Financial Sector

The financial sector moves funds from savers to borrowers, sets interest rates, channels interest payments and defaults from borrowers to savers, and provides a medium of exchange. The financial sector operates as a conduit: a conduit for information regarding the scarcity of financial resources and a conduit for governmental intervention and control. Macro-financial processes are important to the functioning of the larger surrounding economy as well as being of interest in their own right. Not surprisingly, economists have constructed models of the financial sector to function within larger macroeconomic models and to lay bare fundamental financial processes.

A basic choice made when modeling the financial sector is whether to employ an equilibrium representation or a disequilibrium representation. In a complete equilibrium, each sector's actual stock of securities equals its desired stock of securities; actual purchases (sales) of securities equal desired purchases (sales), and desired purchases equal desired sales. Most well known financial models (e.g., Brainard and Tobin 1968, Bosworth and Duesenberry 1973, Hendershott 1977) permit disequilibrium to the extent that actual stocks adjust toward desired stocks over time. But, the models assume that desired purchases equal actual purchases and that desired purchases equal desired sales in most markets.

To illustrate, define desired bond issues as the rate of issue required to adjust actual liabilities of a borrowing sector to desired liabilities over a certain time period. And, similarly, define desired bond purchases as the purchasing rate necessary to adjust actual financial asset holdings of a saving sector to desired:

\[
\begin{align*}
DBI &= (DL - L)/tal \\
DBP &= (DA - A)/taa
\end{align*}
\]

where:

- **DBI** Desired Bond Issues (dollars/year)
- **DL** Desired Liabilities (dollars)
- **L** Liabilities (dollars)
- **tal** Time to Adjust Liabilities (years)
- **DBP** Desired Bond Purchases (dollars/year)
- **DA** Desired financial Assets (dollars)
- **A** financial Assets (dollars)
- **taa** Time to Adjust Assets (years)

Upper case letters denote variables; lower case letters denote constants.
If desired liabilities and desired financial assets are determined by the interest rate, it is possible to determine an interest rate that equates desired purchases and desired sales. Let

\[
DL = \alpha_1 - \alpha_2 R \\
DA = \beta_1 + \beta_2 R
\]

(3) (4)

where:

\[
R \quad \text{Interest Rate (fraction/year)} \\
\alpha_1, \beta_1 \quad \text{constants (dollar-years)} \\
\alpha_2, \beta_2 \quad \text{constants (dollars)}.
\]

Substituting (3) and (4) into (1) and (2) and setting desired issues equal to desired purchases will yield an equilibrium interest rate \( R^* \) of

\[
R^* = (\text{tal}^*(A - \beta_1) - \text{taa}^*(L - \alpha_1))/(\beta_2 \cdot \text{tal} + \alpha_2 \cdot \text{taa})
\]

(5)

The desired amount transacted will equal the actual amount transacted and:

\[
\text{DBI} = \text{DBP} = (\alpha_1 - \alpha_2 R - L)/\text{tal}
\]

(6)

This is an equilibrium approach: desired bond issues always equal desired bond purchases, desired bond issues always equal actual bond issues, and desired bond purchases always equal actual bond purchases. The interest rate is determined by the mechanism of simultaneous equations representing the "market-clearing nexus of demand and supply" (Friedman and Roley 1980, p. 35).

The primary virtue of the equilibrium approach is its simplicity; the primary failing of this approach is its lack of realism. A "nexus" is a shadowy causal agent, and solving simultaneous equations is a process far removed from that by which actual people actually set interest rates. Further, in the real world the equilibrating rate \( R^* \) is unknown by any actor, so desired bond issues and desired bond purchases in the real world frequently must be unequal. Finally the regular appearance of credit crunches (Jaffee and Modigliani 1969; Jaffee and Russell 1976; Chiang, Finkelstein, Lee and Rao 1984; Sinai 1976; Wojnilower 1980, 1985; Dubofsky 1985) indicates that desired bond issues (or borrowing) are regularly not equal to actual bond issues.

Whether financial markets should be represented as if they were in equilibrium is in large part an empirical question, the answer to which will be influenced by the purposes of
the investigation at hand. The crucial questions are: (1) Is the difference between desired transactions and actual transactions or between desired purchases and desired sales large in relation to the accuracy needed to support the conclusions of a particular study? And, (2) do actual interest rates move the system to equilibrium fast relative to the speed of the economic processes that are of most concern in the study?

If the movement of interest rates is relatively fast, the equilibrium assumption may yield a model that is simpler than would otherwise be attainable without significantly corrupting the dynamics of the system. For example, the equilibrium assumption may be appropriate for investigations of the economic long wave, a process said to take about fifty years to unfold (Sterman 1984, Van Duijn 1983). In contrast, the wide spread belief that credit allocation and credit crunches occur in phase with the business cycle in many financial markets\(^1\) suggests that the equilibrium market clearing assumption may be unsatisfactory when investigating processes, such as the business cycle, that unfold over periods of four to five years.

A disequilibrium financial model is of interest for several reasons. First, the rigorous justification of the equilibrium assumption depends upon a disequilibrium representation to evaluate the time required for financial markets to move into equilibrium and to estimate the difference between disequilibrium and equilibrium quantities. Second, a disequilibrium representation will be required for studies in which the equilibrium assumption turns out to be inappropriate. Finally, a disequilibrium representation is useful in itself for understanding how disequilibria are handled in the actual economy.

In this paper, I will describe a disequilibrium financial sector that has proved useful in the context of a general disequilibrium macro-economic model.\(^2\) The issues of concern in this paper will be interest rate formation, the function of credit crunches, the impact on the financial sector of non-financial, private sector investment, and the impact on the financial sector of governmental actions including open market transactions and deficits.

---


2 The model is the System Dynamics National Model under development at M.I.T. See Forrester 1984 and 1979 for a discussion of the approach taken in the model.
Throughout, a central concern will be the role of financial intermediaries in handling financial sector disequilibria.

The financial intermediary is at the heart of the representation described in this paper. The financial intermediary is the mechanism by which funds are channeled from investors to borrowers. Further, intermediaries absorb from their own inventories much of the random inequality between non-financial sectors' desired security sales and purchases. And, when the inequality between desired sales and purchases persists, intermediaries are the mechanism by which credit is allocated in the short run and by which interest rates move to ultimately bring desired sales and desired purchases into a new balance.

The rest of this paper is organized into four sections. The next section presents the major actors and assets of the model, discusses the role of liquidity, the formation of interest rates, and their function. The second section is focused on processes and responses of the financial sector including financing the private sector and the response to deficits and open market purchases of the Federal Reserve Board. The second section also contains a discussion of credit rationing. The third section considers the allocation of interest payments and defaults. Finally, the last section presents a summary and conclusion.

1. Assets and Actors

The Financial Intermediary. The most important simplifying assumption in the financial sector described in this paper is that all financial transactions go through an aggregated financial intermediary. This simplification has two parts: (1) non-financial sectors make financial transactions only with a financial intermediary, and (2) all the various intermediaries in a real industrialized economy—commercial banks, securities dealers, insurance companies, money market funds, pension funds, and others—are aggregated together into a single actor. The assumption that some financial intermediary is a party to every trade of financial securities is hardly heroic. Although transactions in some markets, such as that for privately placed commercial paper, are made without the help of financial intermediaries, transactions in most financial markets go through some sort of intermediary. For example, publicly traded commercial paper, an increasingly significant vehicle for financing short-term credit needs of businesses, is traded by dealers. The greatest source of short-term business financing today is still commercial banks; commercial banks are intermediaries who pool deposits from savers to make loans. In the largest financial market, the market for U.S. treasury securities, government bond dealers,
who stand ready to buy and sell at a price they quote, are a party to virtually every transaction. In brief, assuming that a financial intermediary stands between the two sides of every financial transaction is a quite acceptable simplification. As will be discussed later, the assumption is not restrictive because the central intermediary in the model can mimic direct transactions between sectors.

The second part of the simplification is to aggregate all the financial intermediaries into a single one. Although this aggregation eliminates inter-intermediary dynamics, it still permits the intermediary to handle disequilibria. Because the present concern is with the disequilibrium function, and not with the inter-intermediary dynamics, the cost of the aggregation is low. The return to the dual assumption of a central, aggregated financial intermediary is high in terms of added simplicity. The assumption that all financial transactions occur via the aggregate intermediary reduces the number of links between sectors. In general, in a model with n sectors and one financial agent, the number of links is reduced by up to 100*(n-1)/(n+1) per cent if all financial transactions go through the financial agent. For example in a four sector model (e.g., government, household, consumer goods manufacturer, capital goods manufacturer) with a financial intermediary, the number of financial links is reduced by up to sixty per cent by using the financial agent as the focal point of all financial transactions. In a six-sector model the potential savings is about seventy per cent.

Financial Assets. The aggregated intermediary’s balance sheet appears below as Figure 1. The assets include loans to producers, government bonds and reserves. Loans to producers represent all business debt in the economy, net of financial assets other than transaction deposits. Government bonds represent all government debt held by the public. Reserves represent the (high-powered) “money balances” held by the financial sector, net of within-sector holding. For example, a mutual fund’s demand deposit is exactly offset by the corresponding liability at a commercial bank, and the net contribution

---


4 Runge (1976) noted the savings involved in centralization and aggregation of this type. In his representation of the labor market, he employed a centralized pool, called the "general unemployed", from which production sectors could draw. Runge noted that "while reducing significantly the complexity of the flows in the real-world labor market (where each pool of workers is potentially linked to all other pools), the structure ... represents the essentials of intersectoral mobility" (pp. 32-33).
to reserves as defined here, is zero. Hence, reserves here are principally cash in the hands of intermediaries and deposits with the Federal Reserve System—no distinction being made between them. The financial intermediary's liabilities are deposits and "bank bonds." Deposits represent transactions balances and currency. In this financial sector there is no circulating currency, so all financial transactions must be accomplished with deposits. Bank bonds represent all non-transaction-balance financial assets of the household other than equities. Bank bonds are net of financial liabilities.

Figure 2 shows how ownership of assets and liabilities are interrelated among the sectors in the economy. Note again that non-transactions savings and debt are netted against each other in the household and production sectors. As a consequence, the producing sectors, as net debtors in the economy, have a liability to the financial intermediary (loans), but no financial savings. The household, as a net saver, has no liabilities to the financial intermediary, but does have an asset: bank bonds. The desirability of netting financial assets and liabilities has nothing to do with the financial sector. The simplification achieved by this netting is achieved in the production and household sectors, not in the financial sector. Indeed, netting financial assets and liabilities necessitates a slightly more subtle interpretation of reserve requirements than would otherwise be necessary. Financial assets and liabilities are netted against each other in this paper because doing so may prove useful to those modeling household or production sectors.

Because of the assumption that all private financial transactions occur through the intermediary, all of the assets and liabilities arising in the normal course of business of the private, non-financial sectors are reflected on the financial intermediary's balance sheet. There are, however, two governmental financial accounts which appear in figure 2 but which are not reflected fully on the financial intermediary's balance sheet: Government bonds held by the Federal Reserve can be created by the direct sale of bonds to the Federal Reserve by the treasury or by the Fed purchasing government bonds from the private sector. Deposits in process at the FDIC represent insured, defaulted deposits, which have

---

5 Equity markets are not captured by the aggregate financial sector described here. Equity is potentially important in a macro-economic model. And, the pricing of equity could be handled here at the cost of defining an additional intermediary to handle equity issues. On the other hand, equity issues can be modeled as the withholding of dividends. Because the total dividend stream exceeds equity issues in the U.S. this seems like a conscionable simplification: Corporations in aggregate could meet their equity-issuing needs by withholding dividends.
not yet been paid out to depositors and, consequently, which have not yet been (re)deposited in the aggregated financial intermediary.

Figure 3 shows another view of the assets and liabilities of the financial sector emphasizing the stocks and flows. In this diagram, the heavy, straight arrows represent flows into and out of financial accumulations, represented by boxes. The lighter, curved arrows, represent information links and show how the flows and accumulations are interconnected. For example, borrowing increases the flow into loans to production sector and the flow into production sector deposits. ⁶

Let us consider figure 3 in more detail. The production sector deposits \( DEPs \) are increased by borrowing and by net cash flow \( NCFPs \), which is treated as positive when it increases deposits. Mathematically,

\[
\frac{d}{dt}DEPs = BORPs + NCFPs
\]  

Changes in the household sector's deposits are identical.

\[
\frac{d}{dt}DEPHS = BORHS + NCFHS
\]  

The production sector's borrowing increases loans as it increases deposits. Loans are decreased by maturities \( MATPs \):

\[
\frac{d}{dt}L = BORPs - MATPs
\]  

The treatment of household bank bonds is quite similar to the treatment of loans to the production sector. Indeed, despite the fact that loans are a liability to the production sector and bank bonds are an asset of the household, the two accumulations are structurally very similar. The similarity is heightened in this representation where loans and bank bonds are actually net amounts. When financial liabilities exceed financial assets (excluding deposits), the net amount is termed "loans"; when financial assets exceed liabilities, the net amount is termed "bank bonds"—loans are negative bank bonds. As a consequence, the household's borrowing decreases bank bonds—in effect, the household borrows by selling bank bonds:

\[
\frac{d}{dt}BB = -BORHS - MATHS
\]  

---

⁶ The diagram is in the format of a system dynamics stock and flow diagram. The hourglass shape represents a valve controlling the flow into or out of an accumulation. The cloud represents a sink or a source. For more information see Richardson and Pugh 1981, Ch. 1-2 and Morecroft 1982.
The net cash in-flow of the production sector is comprised of net cash flows from the other real sectors NFOS$_P$S (here, the household sector is the only other real sector and net cash flows from it are primarily sales less wages), net payments from the government TP$_P$S (receipts from the government minus taxes and purchases), less maturities of loans MAT$_P$S:

$$NCF_{PS} = NFOS_{PS} + TP_{PS} - MAT_{PS}$$  (11)

The net cash flow of the household sector is quite similar, except maturities of bank bonds increase net cash flow:

$$NCF_{HS} = NFOS_{HS} + TP_{HS} + MAT_{HS}$$  (12)

Government bonds held by bank GBHB are increased by treasury sales to the financial intermediary (TSB) and decreased by Fed open market purchases FOMP (a negative purchase is a sale):

$$(d/dt)GBHB = TSB - FOMP$$  (13)

Government bonds held by the Fed (GBHF) are increased by treasury sales to the Fed (TSF) and by Fed open market purchases (FOMP):

$$(d/dt)GBHF = TSF + FOMP$$  (14)

Because there is no circulating currency in the model, cash flows between non-financial, non-government sectors amount to the transfer of deposits between these sectors (here the household and the production sectors) and have no net impact on reserves: the financial intermediary merely notes a change in ownership of its liabilities. Similarly, borrowing has no net impact on reserves because the proceeds are immediately redeposited. (This is discussed further below). Reserves are affected only by Federal Reserve open market purchases FOMP, treasury sales to the financial intermediary TSB, and the deficit DEF:

$$(d/dt)R = DEF - TSB - FOMP$$  (15)

The deficit is the sum of the net payments to the private sectors:

$$DEF = TP_{PS} + TP_{HS}$$  (16)

**Liquidity.** One of the prime tasks in modeling a financial sector is to portray the mechanisms that permit borrowing and lending. Equilibrium is relatively easy: in equilibrium the desired lending of non-financial sectors equals their desired borrowing. In this case, the role of financial intermediaries is simply to match borrowers and lenders: Intermediaries act as brokers. Intermediaries have larger roles outside of equilibrium. When markets are in disequilibrium, desired borrowing does not equal desired lending of
non-financial sectors. Imbalances between borrowing and lending are handled in the first instance by the inventories of cash and securities held by financial intermediaries. When the supply of securities is greater than the demand for securities, intermediaries accumulate securities in their securities inventory, paying for them out of their inventory of cash. In this case intermediaries take a position in the market. In contrast, when demand is greater than supply, intermediaries sell more securities than they buy and their securities inventory declines while their cash inventory increases. Intermediaries liquidate their positions in the market (or become short sellers).  

Financial inventories provide both the information and the motivation for intermediaries to move the interest rate in a way that eventually will equilibrate supply and demand. Every transaction of a financial intermediary either adds to or depletes one of its inventories. The financial inventories accumulate and thereby summarize the entire history of realized demand and supply for financial assets of the non-financial sectors. Inventories encapsulate information about the extent and duration of past disequilibria. Importantly, the level of inventories also motivates intermediaries to use the information to respond correctly to a disequilibrium. Intermediaries need a certain amount of inventories to carry on their business; at the same time, however, inventories entail carrying costs. At some point between too little and too much inventory is the desired inventory of a financial intermediary. Consider cash inventories. When inventories of cash are above desired, intermediaries will be motivated to take actions that will encourage borrowing and discourage investing of the non-financial sectors. The situation of excessive cash inventories is one in which the supply of funds has outstripped demand, and hence the action the intermediary takes to correct its inventory imbalance is the same one it "should" take to help bring supply and demand into line. Similar comments apply to a situation of deficient cash inventories.

The types of inventories held will vary depending on the kind of intermediary. Intermediaries in the primary markets, such as commercial banks, maintain only one basic inventory: inventories of reserves (i.e., cash on hand and deposits at the Federal Reserve Banks). When the deposit inflow of a commercial bank is heavier than the loan outflow, its reserves increase. When, on the other hand, loan outflow exceeds deposit inflow,

---

7 These reasons for taking positions in the market are quite separate from speculating. Many intermediaries in addition to intermediating speculate on price movements. The "speculating" role of intermediaries will not be examined in this paper.
reserves of a commercial bank decrease. The story is similar for other intermediaries, such as dealers, in the secondary market. However, in the secondary market an inventory of securities is required in addition to an inventory of reserves (i.e., deposits at commercial banks and cash). When dealers buy more securities than they sell, their inventory of reserves declines, as in the case of banks, and their inventory of securities increases (corresponding to an increase of loans booked at a commercial bank). The converse holds when dealers sell more securities than they buy.

Banks focus on the inventory of reserves. Dealers in secondary markets can focus either on the inventory of securities or the inventory of reserves—the two will be exactly negatively correlated because every sale of a security increases reserves while it decreases securities and vice versa. Because there are several securities in the sector developed in this paper (bank bonds, loans, government bonds), a troublesome aggregation issue can be avoided by focusing on the inventory of reserves. Consequently for dealers, as well as for commercial banks, it makes sense to consider the inventory of reserves of the aggregate financial intermediary.

The relationship between the actual reserves of the financial intermediary and the desired reserves determines the liquidity of the intermediary. Commercial banks base their reserve positions largely on the amount of various types of liabilities they hold, assigning a desired fractional reserve requirement to each class. This is the approach taken in this paper. Consider first, the desired reserve fraction on deposits. Although the concept in the model relates to desired reserves, rather than the required reserves set by the Board of Governors of the Federal Reserve, the Fed requirement is probably the single most important factor determining desired reserves. Other factors, such as changing interest rate spreads (Cf. Modigliani, Rasche and Cooper 1970, especially eq. 3.11), fluctuations in the degree of Federal Reserve displeasure at borrowing from the discount window, changes in the covariance between deposits and withdrawals, and changes in the degree of risk in the

---

8 Alternatively, one could disaggregate intermediaries' actions by class of security. This might be desirable in a financial sector designed to examine financial-sector dynamics. As explained above, the current financial sector is intended as a streamlined financial mechanism in a larger, general disequilibrium macro-economic model. Consequently one wants to aggregate securities (see discussion below on interest rates). The decision to concentrate on liquidity makes this a so-called "money-market" or "bank reserves" model as distinguished from "bond-market models". Bosworth and Duesenberry (1973, pp. 42-51) and Hendershott (1977, pp. 7-21) discuss the two types of models. Their preference for bond-market models can be attributed to their interest in financial-sector dynamics as opposed to more general macro-economic dynamics.
lending portfolio, will result in variations in desired reserves away from the level of required reserves. The fractional requirements on deposits in the model is set equal to 12%, the fractional amount currently required on transactions balances at large banks.

Determining the desired fractional reserves on bank bonds and loans is complicated by our earlier assumption that bank bonds and loans represent the net amount of borrowing and lending of a sector. When a sector is a net lender, its net financial assets are termed "bank bonds." When a sector is a net borrower, its net borrowing is termed "loans." The complication arises because some of what is netted and aggregated in bank bonds or loans will carry a reserve requirement in the real world, while some will not. The financial assets of a household that are held directly by the household do not entail a reserve requirement; nor do the liabilities of the household. However, financial assets that are held by the household via intermediaries, such as savings deposits, can entail a reserve requirement. Similarly, the security holdings of the production sector held via an intermediary may entail a reserve requirement, but securities held directly by the production sector do not. Similarly, loans to the production sector do not entail a reserve requirement.

One way of setting a fractional reserve requirement on "bank bonds" and "loans" is to average the reserve requirements of all securities aggregated and netted in bank bonds and loans, weighting the average by the relative importance of each security. Figure 4 shows a breakdown of financial assets and liabilities held by the household and the production sectors with the associated approximate reserve requirements. If we assume that a net inflow or outflow of funds to bank bonds or loans will be distributed in such a way as to maintain the proportions of assets and liabilities, then the desired fractional reserve requirement can be calculated by taking a weighted average of the reserve requirement for each asset/liability. Performing this calculation results in a figure for desired fractional reserves of around .001 for both bank bonds and -.001 loans. Loans have a negative reserve requirement because non-transactions deposits are subtracted in determining net loans and, hence, an increase in net loans can be accomplished by a decrease in non-transactions deposits and thus a decrease in desired reserves.9 In sum, desired reserves of the aggregate financial intermediary (DESR) may be written:

---

9 Setting reserve requirement on bank bonds and reserve requirement on loans equal in magnitude, but opposite in sign, has a desirable property discussed below in the section dealing with "Direct Transactions."
\[ \text{DESR} = \tau_d \sum_{S=1,2} \text{DEP}_S + \tau_{bb} \text{BB} + \tau_l \text{L} \]  
\[ \tau_d = .12 \]
\[ \tau_{bb} = .001 \]
\[ \tau_l = -.001 \]

where:

- DESR: Desired Reserves
- \( \tau_d \): Fractional Reserve Requirements on Deposits
- \( \text{DEP}_S \): Deposits of sector \( S \)
- \( \tau_{bb} \): Fractional Reserve Requirements on Bank Bonds
- BB: Bank Bonds
- \( \tau_l \): Fractional Reserve Requirements on Loans
- L: Loans

(lower case letters represent constants; upper case letters represent VARIABLES).

The degree of liquidity of a financial intermediary is a measure of actual reserves (\( R \)) in relation to desired reserves (DR). It is desirable to have a measure which is dimensionless so that the measure is independent of the currency unit. The measure used here is relative available reserves (RAR), defined as the difference between reserves and desired reserves normalized by desired reserves.

\[ \text{RAR} = (R - \text{DR})/\text{DR} \]  

where:

- RAR: Relative Available Reserves
- R: Reserves
- DR: Desired Reserves

When intermediaries are exactly as liquid as the wish to be, reserves will equal desired reserves and RAR will equal zero. RAR will be below zero when intermediaries are illiquid. When intermediaries are excessively liquid, RAR will be above zero.

**Interest Rate Formation.** Intermediaries use interest rates to control their own liquidity by influencing non-financial sectors' demand for and supply of securities. The strategy used by intermediaries to set their rates is hypothesized to be one of "anchoring and adjustment" discussed in the literature on behavioral decision theory.\(^\text{10}\) In response to illiquidity, intermediaries raise their quoted rates above an anchor termed the "underlying

---

rate." The underlying interest rate is an assessment of the current rate environment; it is an assessment of where rates would be in the absence of transitory pressures, among which liquidity pressures are the most important. The underlying rate may be represented as an exponential average of past rates. Figure 5 presents these ideas graphically.

Mathematically, the risk free interest rate (RFIR) is a multiplicative adjustment of the underlying interest rate (UIR):

\[ RFIR = UIR \times ALR \quad (19) \]

where:

- RFIR: Risk Free Interest Rate
- UIR: Underlying Interest Rate
- ALR: Adjustment from Liquidity to the risk free interest Rate

The adjustment from liquidity to the risk free interest rate (ALR) is a function of liquidity (i.e., relative available reserves (RAR)). When liquidity is equal to desired, RAR will take the value of zero and ALR should take the neutral value of 1. In addition, ALR must be negatively sloped so that states of high liquidity correspond to low interest rates. Finally, ALR cannot drop below zero, because nominal interest rates cannot become negative. A flexible function estimated in the first essay is:

\[ ALR = e^{-\alpha \times RAR} \quad (20) \]
\[ \alpha = 1.65 \quad (21) \]

where:

- ALR: Adjustment from Liquidity to the risk free interest Rate
- \( \alpha \): parameter of the exponential function
- RAR: Relative Available Reserves

A mathematical representation of the underlying rate appears below where the value of the time constant is taken from the estimation performed in the first essay of this thesis (see "A Behavioral Theory of Interest Rate Mechanics"):

\[ \frac{d}{dt}(UIR) = (RFIR - UIR)/tauir \quad (22) \]
\[ tauri = 1.4 \text{ years} \quad (23) \]

where:

- RFIR: Risk Free Interest Rate
- UIR: Underlying Interest Rate
- tauri: Time to Adjust Underlying Interest Rate
Equations 34 through 38 represent a theory of how the nominal, short-term risk free interest rate is formed.\textsuperscript{11} The short-term risk free rate corresponds to the rate charged on short-term government securities. This formulation of the financial sector contains several other interest rates: the rate on deposits, the rate on bank bonds and the rate on corporate debt. As will be seen, the rate on deposits and bank bonds is determined by actual interest paid by the government and by producers. Consequently, only the rate on producer debt remains to be defined. There are several approaches to defining the rate on corporate debt. The most rigorous way of formulating the interest rate on corporate debt would be to identify another intermediary to deal in the security. Introducing another intermediary would be an appropriate strategy for use in a model designed to investigate interactions between different types of securities (and different types of intermediaries). The present purpose, to build a streamlined financial sector for a larger macro-economic model, is better served, however, by assuming that the relationship between the risk free interest rate and the rate on producer debt is an equilibrium relationship. In equilibrium, certain relationships between rates on different securities will hold. It is likely that the expected return on corporate debt would be approximately equal to the risk free rate.\textsuperscript{12}

\begin{equation}
E(R_{cd}) = RFIR
\end{equation}

where:

- $E()$ is the expectations operator
- $R_{cd}$ The rate on corporate debt
- $RFIR$ Risk free interest rate.

The return on corporate debt is the contractual rate less defaults on debt. This suggests:

\begin{equation}
E(R_{cd}) = E(C_{cd} - DR_{cd}) = RFIR
\end{equation}

\textsuperscript{11} A more complete development of the theory of interest rate mechanics is presented in the first essay of this thesis.

\textsuperscript{12} There might also be a risk premium above the expected default rate. According to the capital asset pricing model (Sharp 1964, Litner 1965, Brealey and Myers 1981, Chapter 8), there would also be a risk premium proportional to the quotient of covariance between the return on all assets and the return on corporate debt and the variance of the return on all assets [$\text{COV}(R_{\text{debt}}, R_{\text{All Assets}})/\text{VAR}(R_{\text{All Assets}})$]. Because the return on corporate debt is truncated above the contractual rate, the covariance between the return on corporate debt and the return on all assets is small. Consequently, the risk premium is small as well.
Hence,
\[ C_{cd} = RFIR + E(DR_{cd}) \]  \hspace{1cm} (26)

where:

\begin{align*}
C_{cd} & \quad \text{Contractual rate on corporate debt} \\
RFIR & \quad \text{Risk free interest rate} \\
E & \quad \text{Expectations operator} \\
DR_{cd} & \quad \text{Default rate on corporate debt.}
\end{align*}

The contractual rate, that is the rate charged on corporate debt, is the risk free interest rate plus the expected default rate.

**Borrowing to Adjust Deposits.** The interest rate controls financial sector liquidity by affecting *desired* borrowing. Borrowing here is to be understood in a broad sense where negative borrowing is saving. Desired borrowing may be thought of as being composed of two terms: Desired borrowing to fund deficits in cash-flow and desired borrowing to fund an increase in money (i.e., a portfolio effect).

\[ DBORS = BADS + BNCFS \] \hspace{1cm} (27)

where:

\begin{align*}
DBORS & \quad \text{Desired Borrowing in Sector S} \\
BADS & \quad \text{Borrowing to Adjust Deposits} \\
BNCFS & \quad \text{Borrowing for Net Cash Flow}
\end{align*}

Interest rates affect borrowing through the first term on the right hand side: borrowing to adjust deposits (BAD). Borrowing to adjust deposits represents a process loop through which sectors bring actual deposits into line with desired deposits. Specification of BAD requires knowledge of (i) desired deposits or the demand for money; and (ii) the desired rate of adjustment.

There is an extensive empirical literature on the demand for money which can be brought to bear on the formulation of desired deposits\textsuperscript{13} (see Goldfeld 1973 and Judd and Scadding 1982 for reviews). The first essay of this thesis (see "A Behavioral Theory of Interest Rate Mechanics") showed that the basic money demand equation used in most

\textsuperscript{13} In the model here there is no currency, so money and deposits are synonymous. The appropriate estimates, therefore are those for the demand for money rather than those which deal only with the demand for demand deposits.
studies of the demand for money could be put into a form suitable for system dynamics models. For convenience I will briefly repeat the money demand structure here. Borrowing to adjust deposits is the product of the desired fractional change in deposits DFCD\textsuperscript{14} and deposits D:

\[(d\textsubscript{d}/d)D\textsubscript{S} = \text{DFCD}\textsubscript{S} \times D\textsubscript{S}\]  

(28)
The desired fractional change in deposits is a function of relative desired deposits, the ratio of desired deposits to actual deposits. The function, graphed in figure 6, is

\[\text{DFCD}\textsubscript{S} = (\text{DD}\textsubscript{S}/D\textsubscript{S})^cs - 1\]  

(29)
The parameter c determines the (logarithmic) adjustment speed, expressed as a fraction per period. Goldfeld (1976) has estimated the adjustment speeds for the household and for the production sector.\textsuperscript{15} These estimates are:

\[c\textsubscript{ps} = .960 \text{ per year}\]  

(30)
\[c\textsubscript{hs} = .680 \text{ per year}\]  

(31)

Desired deposits (DD) are the product of reference deposits (rd) and two effects: one from expenditures (EED) and one from interest rates (EID). For a sector (s), desired deposits may be written:

\[DD\textsubscript{S} = \text{rd} \times \text{EED}\textsubscript{S} \times \text{EID}\textsubscript{S}\]  

(32)

Reference deposits in a model would logically be defined as the equilibrium value of deposits in order to make it simpler to put the model into equilibrium. A value for rd is necessary for some of the illustrations that follow. For these purposes, I will take reference deposits to be equal to a recent value of checkable deposits and currency (U.S. Bureau of the Census 1984, p. 487, chart no. 808):

\textsuperscript{14} The terminology here anticipates the later discussion on credit rationing. For now, however, the change in deposits is equal to the desired fraction change in deposits multiplied by deposits.

\textsuperscript{15} Estimates for the production sector money demand parameters in this equation and in (38) and (40) below are from Goldfeld 1976, Equation 10.3. Equation 10.3 performs well and the transactions variable, business product, is probably close to business expenditures as called for by theory. Parameters of the households' money demand equations (31), (37), and (39) are from Goldfeld's (1976) equation 12.3. Goldfeld 12.3 performs well and the transactions variable, consumption, is close to household expenditures. Goldfeld gives coefficients, rather than parameter estimates. The conversions are as follows:

\[C\textsubscript{S} = (1 - \text{coefficient on money lagged}) \times 4\]

\[B\textsubscript{S} = \text{coefficient on current rate}/(1 - \text{coefficient on money lagged})\]

\[D\textsubscript{S} = \text{coefficient on transactions}/(1 - \text{coefficient on money lagged})\]
\[ r_d_{hs} = \$338.7 \text{ Billion} \]  
\[ r_d_{ps} = \$103.3 \text{ Billion} \]

The effects of expenditures and interest rate on deposits are functions, graphed in figures 7 and 8, of (the relative expenditures ratio of expenditures (E) to reference expenditures (re)) on the one hand and the relative interest rate (the ratio of the risk free interest rate (RFIR) and the reference interest rate (rir)) on the other hand. For a sector (s), the effect of expenditures on deposits are:

\[ EED_s = \left( \frac{E_s}{re_s} \right)^{d_s} \]  
\[ EID_s = \left( \frac{RFIR}{rir} \right)^{b_s} \]

Goldfeld's (1976)\(^{15}\) estimates for d and b are:

\[ d_{hs} = 1.376 \]  
\[ d_{ps} = .575 \]  
\[ b_{hs} = -.388 \]  
\[ b_{ps} = -.117 \]

The estimate for \(d_{hs}\) implies decreasing returns to scale for money balances (also apparent in figure 7) and is therefore suspect. The parameters \(d_s\) will not be used in any of the following discussion, however, so there is no need to pursue the estimates here.

**Borrowing for Net Cash Flow.** An increase in cash inflow can accumulate in either deposits on the one hand or in increased bank bonds or decreased loans on the other hand. In the real world, cash flow first accumulates in cash balances or demand deposits. It might appear, therefore, that cash flow should accumulate in deposits and then be channeled into loans or bank bonds as deposits are adjusted down toward desired. This would be satisfactory if the time delay on removing the excess deposits (parameter c in equation (22)) was appropriate.

Unfortunately, as others have pointed out (e.g. Modigliani Rasche and Cooper, p. 173; Goldfeld 1973, p. 598 ff; Friedman 1977, pp. 665 ff) the estimated delay in the money demand formulation aggregates several separate delays in the real world including delays in forming perceptions of current interest rates and expectations of future interest rates and expenditure (or income), the delays necessary to collect and analyze information on alternative securities, delays in adopting mechanisms to permit a change in money's velocity, and delays in actually moving money balances toward desired. The delay in actually moving money balances toward desired is the one that is of concern when dealing with cash flows. The "action delay" is quite short compared with the other delays. The modeler has a choice: He can either disaggregate the delays, or assume that the action delay
is short enough relative to the dynamics of interest and relative to the other delays to ignore. This paper takes the second course of action: The aggregate delay will be used to represent the time necessary to form perceptions and expectations of future interest rates, to collect and analyze information on alternative securities, and to change the velocity of money. The action delay will be ignored.

Cash flow accumulates directly in bank bonds or in (reduced) loans when the action delay is ignored. This means the term BNCF (borrowing for net cash flow) in equation (20) should be set equal in magnitude and opposite in sign to the net cash flow of the sector.

\[ \text{BNCF}_S = -\text{NCF}_S \]  \hspace{1cm} (41)

where:

- \( \text{BNCF}_S \) = Borrowing for Net Cash Flow in Sector \( S \)
- \( \text{NCF}_S \) = Net Cash Flow in Sector \( S \)

2. Process and Response in the Financial Sector

**Borrowing From the Intermediary.** It is appropriate at this point to step back and compare the operation of the structure presented thus far to the way the real world works. The first issue considered is lending by the financial sector to the non-financial private sectors. In the real world a bank's reserves are reduced by the amount of any loan it makes. However, another bank's reserves are increased by the same amount when the borrower, or whomever the borrower pays, redeposits the proceeds in a transactions balance (or other account). The aggregate effect is to leave reserves unchanged. For identical reasons, when the non-financial sector reduces its non-transactions balances (i.e., "bank bonds"), the bank affected loses reserves, but another bank gains reserves, cancelling the prior reduction and leaving reserves unchanged. Equation (15) reflects these considerations: reserves are not affected by borrowing and saving of the non-financial private sectors.

Although reserves are unaffected by borrowing, desired reserves are increased. When a bank lends out a portion of its reserves to a borrower, the borrower (or whomever the borrower pays) increases his deposits when he redeposits the proceeds. Total deposits in the financial system increase (cf. equations (7) and (8)). An increase in deposits increases desired reserves of the intermediary\(^{16} \) (equation (17)). When excessively liquid, intermediaries can attempt to lend out reserves above those desired. System wide, reserves
will be unchanged, but the ensuing increase in deposits will increase desired reserves, thereby reducing the excessive liquidity (equation (17)). Liquidity, lending, and depositing are linked together in a negative equilibrating loop.

**Money Creation: Monetized Deficits and "Easy" Fed Policy.** The loop may be observed in the case of a monetized deficit. The following discussion will focus first on a deficit proper and then turn to what happens when the deficit is monetized. A deficit occurs when the treasury pays out more than it receives in taxes. In the United States the treasury funds its deficit by selling treasury bonds to bond dealers. The paying out of more money than the treasury receives would increase reserves as payees deposit their payments; however, this increase in reserves is exactly offset by the treasury's sale of bonds. On the other hand, the treasury's deficit and sale of bonds will increase desired reserves as the payments-in-excess-of-taxes are deposited. The size of the increase will depend upon whether the payment-less-taxes is deposited in bank bonds which carry in the model a .1% reserve requirement or transactions deposits which carry a 12% reserve requirement. The treasury actions result in a decrease in liquidity, which will be slight to the extent payments are accumulated in bank bonds.

The deficit is *monetized* if the Federal Reserve steps in to purchase the bonds the treasury sells to finance its deficit.16 A monetized deficit can be represented in the model's terms as the government running a deficit (TP is positive in equations (11), (12), and (16)), the treasury selling government bonds to the aggregate financial intermediary in the amount of the deficit (TSB in equations (13) and (15)) while the Federal Reserve purchases those bonds back (FOMP in equations (13), (14) and (15)).

Figure 9 shows a stock and flow diagram of the key flows and accumulations involved in a monetized deficit. Output from a simulated deficit is shown in figure 10. The system begins in equilibrium where desired reserves equal actual reserves and desired deposits equal actual deposits. The test input is a one-time "pulse" deficit equal to 10% of pre-existing reserves. The deficit is paid as a transfer payment to the household at the end of the first year. The deficit is invested and bank bonds show a small "blip" up as the

---

16 In many other countries, monetized deficits are created by the government selling bonds directly to the central bank. The end result is the same as the U.S. process described in the text.
household uses its new wealth to purchase bank bonds at the end of year one (figure 10c). The purchase of bank bonds increases the financial intermediary's reserves. Simultaneously, the treasury sells bonds to the financial intermediary which would drain reserves, except the Federal Reserve buys them back. As a result of the monetized deficit, reserves increase by the amount of the deficit.

Reserves have increased by the full amount of the accumulated deficit; desired reserves, on the other hand, have increased by a negligible .1% of the deficit because the reserve requirement on bank bonds is only .1% (figure 10a). Consequently relative available reserves, the measure of liquidity, increases from its equilibrium value of zero to just under .1, indicating that reserves are almost 10% above desired. The intermediary, excessively liquid, drops its interest rate (figure 10b) in order to attract loans and because it does not need to attract deposits. As the interest rate falls, holding money becomes relatively more attractive to the household and the production sector. The household increases its money balances by decreasing its bank bonds which fall below their previous level (figure 10c), and the production sector increases its money balances by increasing its loans (figure 10d). As the transactions deposits of the household and the production sectors increase, the desired reserves of the financial intermediary also increase, tending to relieve the state of excess liquidity in the financial intermediary. The lessening of excess liquidity decreases pressure to reduce the interest rate. In this parameterization of the model the pressure is relieved fast enough so that the interest rate actually rises. Eventually, the system settles at a point where the interest rate is sufficient to keep deposits, bank bonds, and loans at a level where the financial intermediary's desired reserves equal to

17 In the real world, the household would probably use a large part of the transfer to purchase goods from the production sector. Such a purchase would trigger further response in the production sector and feedback to the household. However, the intent of this paper is to present a formulation of the financial sector, and consequently response and feedback in the "real" sectors is ignored.

18 Note from the aggregate sectors' viewpoint the solution to excessive liquidity is the increase in desired reserves. The increase in desired reserves comes about, however, from the actions of individual financial intermediaries trying to get rid of their excess reserves directly—by lending them out or using them to purchase securities.

19 Different points in parameter space will lead to different behavior. The first essay of this thesis contains a brief discussion of the sensitivity of the interest rate dynamics to uncertainty in parameters.
reserves. At this point, there will be more demand deposits in the system than previously—money has been created.

A very similar chain of events unfolds in the absence of a deficit, when the federal reserve decides to "ease" and buys government bonds from the intermediary. In this case, there is no immediate increase in bank bonds caused by payments from the treasury as in the previous case. However, reserves still go up immediately because the federal reserve pays for its purchase of government securities with reserves. The result is that the financial intermediary is excessively liquid; it lowers its rates and the private sectors respond by increasing their transactions balances. Again, the system comes to rest at a point where the interest rate is such that the corresponding deposits, bank bonds and loans imply an amount of desired reserves equal to the actual reserves. Money has again been created.

Money Destruction: "Tight" Fed Policy. Having looked at money creation it is appropriate to examine also how reserve contraction and money destruction is handled in this representation of the financial sector. The conditions that give rise to money destruction motivate a discussion and formulation of credit rationing.

Reserves can be decreased in two ways. Either an open market sale by the Federal Reserve or budget surpluses by government will result in a decrease in reserves, a decrease in money, and at least a temporary increase in the interest rate. Because the two situations are so similar this paper will only treat a reserve decrease caused by an open market sale of government bonds to financial intermediaries.

Figure 11 shows major transmission paths. The simulation shown in figure 12 begins in equilibrium and illustrates the effect on money, interest rates, and reserves of a one-time sale of government bonds to the aggregate intermediary equal to 10% of reserves. The immediate consequence is a reduction in reserves as the financial intermediary pays for the bonds by giving its reserves to the Fed (figure 12a). Because desired reserves have not changed while actual reserves drop, the liquidity of the aggregate financial intermediary, as summarized by relative available reserves, declines from 0 to −.1. The low liquidity prompts the intermediary to raise its interest rate above the underlying interest rate (figure 12b). Higher interest rates make it more expensive to hold deposits, and producers and

20 Similar to the effects of a reserve decrease are the effects of an increase in the reserve ratio or an increase in the currency/deposit ratio. Reserve requirements are represented explicitly in the model, the currency ratio is not.
consumers begin to reduce their deposits. The household reduces its deposits by buying bank bonds (figure 12c), while producers repay their loans (figure 12d). Lower actual deposits reduce desired reserves of the intermediary, helping to alleviate the intermediary's illiquidity.

In this parameterization, the initial response of the intermediary is to increase the interest rate slightly above what eventually proves to be the equilibrium rate. As deposits come down, reducing desired reserves and thereby alleviating illiquidity, the intermediary reduces the interest rate. Eventually the system gropes its way to the new equilibrium where the interest rate and bank bonds are higher and deposits and loans are lower. The end result is dictated by the money multiplier relationship: the equation of motion in the system are such that the system finds the proper balance between interest rates reserves, deposits, bank bonds and loans.

Credit Rationing. The financial intermediary controls its liquidity by adjusting interest rates to influence the desired deposits of non-financial sectors. However, figure b shows that given the estimated parameters, the time required before liquidity (RAR) returns to normal is two or three years. During that time intermediaries may attempt to restore liquidity through non-interest rate means, that is, by restricting or rationing credit.

Credit rationing has been a source of debate among economists at least since the early fifties (see Friedman 1972 for a review). Samuelson (1952) argued that credit rationing violated principles, accepted as key by most economists, of rationality and profit maximization on the part of financial intermediaries, and therefore could only be a transitory phenomenon as interest rates adjust to their equilibrium level. Much of the economics literature since Samuelson's critique has focused on developing theories of why

---

21 Different combinations of TAUIR, α, c, and b can lead to different behavior: a slow adjustment of the interest rate to equilibrium from below is possible as is a substantial overshoot. The first essay of this thesis considered the sensitivity of the interest rate to uncertainty in these parameters.

22 Not considered in the simulation are the feedback effects flowing through the real economy. If increased interest rates act quickly to reduce economic activity, transactions deposits could be reduced faster than the simulation shows. At least some institutionally oriented observers suggest the economy does not respond quickly to increased rates (see for example, Wojnilower (1980, 1985) and Anderson and Ostas (1980, p. 533)).
credit rationing might be rational, optimizing behavior in *non-transitory* situations.\textsuperscript{23} There has, however, been another strand of literature, less theoretical, more institutional in emphasis, that has accepted credit rationing as a transitory phenomenon, but taken issue with Samuelson's implication that transitory rationing is unimportant (e.g. Anderson and Ostas 1980, Wojnilower 1980, 1985, Sinai 1976).\textsuperscript{24}

Like some of the arguments put forward in the second strand, the argument in this paper is that one cause of credit rationing is a liquidity problem that cannot be solved easily through interest rate adjustment. To assess the extent to which interest rates *could* be used to eliminate liquidity problems quickly, consider the behavior of interest rates if intermediaries attempted to reliquify over a substantially shorter period than the two to three years of figure 12b. Two parameters in the model determine how far and how quickly the intermediary moves the interest rate in response to illiquidity: $\alpha$, the parameter governing the adjustment from liquidity to the risk free rate, and TAUIR, the parameter governing the speed with which the underlying rate adjusts toward the risk free rate. The larger $\alpha$, the more the intermediary increases the risk free rate above the underlying rate for any given degree of illiquidity (see equations (19) and (20)). The smaller TAUIR, the faster the underlying interest rate increases during illiquid times (see equation (22)). Figure 13 shows output from a simulation in which $\alpha$ was doubled and TAUIR was cut in half, representing a situation in which intermediaries attempt to eliminate their illiquidity faster via the interest rate.

The simulation is interesting for two reasons. First, the simulation illustrates the difficulty of controlling liquidity through interest rates. Within a year the aggressive interest rate response turns a situation of illiquidity into one of excess liquidity which then takes two more years to rectify. Further, in order to eliminate illiquidity within a year, the

\textsuperscript{23} Jaffee and Modigliani (1969) argue that rationing will be profit maximizing behavior in a situation where the lender has some monopolistic power and must charge the same interest rates to borrowers with different default probabilities. Jaffee and Russel (1976) argue that credit rationing will be profit maximizing for both lenders and borrowers when borrowers who do not default must subsidize the defaults of those who do. Keeton (1976, chapter 1) argues that credit rationing is profit maximizing when the default rate rises with the interest rate charged. Chiang et al (1984) argue that credit rationing will be profit maximizing when a loan ceiling can be used to detect borrowers who are most likely to default.

\textsuperscript{24} Jaffee and Modigliani (1969) consider transitory or "dynamic" rationing as well as equilibrium credit rationing.
interest rate must peak at a level 70% higher than the equilibrium rate. Using rates alone to
close liquidity does not merely necessitate speeding a smooth adjustment of interest rates
toward an equilibrium, as Samuelson and others might suggest, but rather it necessitates
overshooting the equilibrium by a large margin.

Neither businesses in general nor financial intermediaries in particular are
indifferent to how the prices they charge move. At an industry level intermediaries come
under pressure from the public, and eventually congress and state legislatures, if interest
rates move too high too fast (Anderson and O'Co 1980, pp. 532-533). On a more
individual basis, intermediaries risk losing the trust of customers if they charge them more
than their competitors or if they charge them more at one time than they charge other
customers a few months later. The need to maintain good customer relationships thus
argues for caution in raising rates too fast or too far when buying securities or lending to
customers. The slow rise when buying securities or lending argues for a slow rise when
selling securities or taking deposits: Dealers will not wish to raise rates slowly when
selling to avoid having to take a larger loss than eventually proves necessary on securities
held in inventory and to maintain the spread between the rates at which securities are
bought and sold. Similarly, banks will try not to raise rates so fast or so far on deposits
that the spread between what is paid on deposits and what is received on loans disappears.

During illiquid times financial intermediaries are in a difficult situation: Illiquidity is
costly, yet so is increasing the interest rate fast and far. In such a situation, credit rationing
becomes an attractive alternative. Credit is rationed to good customers, thereby maintaining
customer relationships.

In most financial models which contain rationing terms, the amount transacted is
either the minimum of demand and supply or else the amount transacted is set equal to the
demand or supply of one sector with the other party to the transaction (usually a financial

25 The importance of customer relationships is often discussed in the context of
commercial banks (e.g., Anderson and O'Co 1980). However, relationships are
important to dealer-type intermediaries as well: Large customers who sell large blocks
of stock often call only one dealer (Smith 1985). Smaller customers too tend to deal
through one dealer (or broker). Being that one dealer for a large customer or for a large
number of small customers is very valuable.

26 For a related discussion of this affect in the context of commercial banking see Roosa
1951.
intermediary of one type or another) being purely a quantity taker. In the financial sector
developed in this paper, a more flexible approach, combining these two polar extremes is
possible because the financial intermediary is modeled as having an inventory. The
inventory permits the intermediary to act as a quantity taker for a time and then to gradually
become restrictive. To express these notions algebraically, one can write:

\[
\text{BOR} = \begin{cases} 
 DBOR*ELB & DBOR>0 \\
 DBOR*ELS & DBOR<0 
\end{cases} \tag{42}
\]

\[
\text{ELB} = f(RAR) \tag{43}
\]

\[
\text{ELS} = g(RAR) \tag{44}
\]

where:

- BOR borrowing
- DBOR desired borrowing
- ELB Effect of Liquidity on Borrowing
- ELS Effect of Liquidity on Saving
- RAR relative available reserves defined in eq()
- \( f(\cdot) \) a function; \( f>0 \)
- \( g(\cdot) \) a function; \( f<0 \)

In this formulation ELB and ELS are credit rationing terms. Two terms are necessary
because desired borrowing represents net desired borrowing in a sector. Credit will be
rationed not only in net borrowing sectors, but also in net saving sectors. Even though a
sector is a net financial saver, some people in the sector will be borrowers. Credit rationing
can limit their borrowing in which case the sector in aggregate will increase financial assets
more than desired or decrease financial assets less than desired.28

27 For examples of financial sectors in which the amount transacted is the minimum of
demand and supply see Quandt (1985, eq. 2-3) and Bosworth and Duesenberry's
(1973, pp. 73-74) treatment of the mortgage market. For examples of quantity taking
see Brainard and Tobin's (1968, pp. 109-110) treatment of demand deposits and bank
loans and Bosworth and Duesenberry's (1973, p. 67) treatment of bank loans. In
Hendershot's (1977) model, credit rationing occurs in the context of simultaneous
equations and so the amount transacted depends upon both demanders and suppliers.
Hendershot's model is an interesting use of simultaneous equations to arbitrate
between the differing desires of demanders and suppliers in the context of credit
rationing.

28 It is also possible that during illiquid times banks encourage, through advertising or
other non-price means, financial savers to save more than they otherwise would. ELB
and ELS are intended to capture this effect as well.
Estimation of \( f(\cdot) \) and \( g(\cdot) \) is not possible because good data on credit rationing are unavailable. Borrowers do not like to reveal they have been denied credit, lenders do not like to reveal they are unreliable sources of funds, and often rationing is done subtly through processing delays or through recommendations that now is not the time to come to the market with a new offering. In the absence of good data on credit rationing several proxies have been used by other investigators.\(^{29}\) Unfortunately, the relationship between these proxies and credit rationing has not been specified with a sufficient degree of precision to permit estimation of \( f(\cdot) \) and \( g(\cdot) \). Until more carefully developed proxies or more precise information from financial intermediaries themselves is available, the shapes of \( f(\cdot) \) and \( g(\cdot) \) will remain subject to significant uncertainty.

Nonetheless, credit rationing is potentially an important force and some representation of \( f(\cdot) \) and \( g(\cdot) \) will be necessary in a good model.\(^{30}\) Some aspects of \( f(\cdot) \) and \( g(\cdot) \) can be deduced a priori. Because \( f(\cdot) \) restricts lending in the face of inadequate liquidity, \( f(\cdot) \) should decrease as RAR decreases.\(^{31}\) Similarly, \( g(\cdot) \) should increase as

\(^{29}\) Such proxies include the percentage of new bond offerings rated Baa and/or below (Dubofsky 1985) and the percentage of loans granted at the prime rate (Jaffee and Modigliani 1969). Use of these proxies is based on theoretical arguments that the more risky borrowers will be denied credit first. It should be noted that these arguments are not based on liquidity. For example, Modigliani and Jaffee (1969) assume that oligopolistic banks must divide \( n \) customers with different probabilities of default into only \( m \) \((m<n)\) rate classifications; in this case the riskier customers in each class may be rationed. However, the suggestion that customers with lower credit ratings are rationed first (and most) has been durable, presumably because people believe this is how credit is actually rationed. Rationing riskier borrowers first is also consistent with liquidity-caused rationing: It is a common observation that banks' old customers are granted credit in preference to new customers when money gets tight. Presumably older, established customers have, on average, higher credit ratings. It is quite possible that other intermediaries act similarly.

\(^{30}\) A caution is ordered. Credit rationing speeds up the adjustment of actual deposits toward desired. Statistical estimation which does not take account of rationing is likely to produce estimates of the desired adjustment speed that are faster than the actual desired adjustment speed. This means that values of parameters \( c_{ps} \) and \( c_{hs} \) used in equations (30) and (31) are likely to be larger than the true values. Initial experiments with the model presented in this paper, using ELB and ELS as defined in equations (45a) and (45b), indicates that \( c_{ps} \) and \( c_{hs} \) are twice what they ought to be during times of illiquidity. Further work is needed to develop estimates of \( c_{ps} \) and \( c_{hs} \) which are compatible with a rationing formulation.

\(^{31}\) \( f(\cdot) \) is similar to the effect of inventory on shipments present in many system dynamics models (Homer undated).
RAR decreases, representing enforced savings caused by an inability to borrow. When liquidity is just adequate (i.e. RAR=0) f(*) and g(*) should be equal to 1. When liquidity is more than adequate (i.e. RAR>0), f(*) should remain at one, or perhaps rise slightly above one if intermediaries convince firms to borrow more or faster than they originally desired. Similarly, g(*) should remain at one or fall slightly below it when liquidity is more than adequate.

RAR for banks has dropped as low as −.25, and banks were still lending, so f(*) does not reach zero in the range from 0 to −.25. One possible f(*) is shown below as figure 14. A steeper slope of f(*) will intensify, and a flatter slope will lessen, the effect of liquidity on borrowing.

The relationship between f(*) and g(*) is not clear. On the one hand, net saving sectors probably have fewer borrowers, and hence, the strength of g(*) (as measured by its slope) might be expected to be less in magnitude than that of f(*). On the other hand, credit rationing seems to fall most heavily on borrowers who are poorer credit risks. It is at least possible that when a sector as a whole is adding to its financial assets, those people or companies who are still borrowing or reducing their financial assets will be the least credit worthy. This would argue for g(*) having greater strength—that is a steeper slope. In the absence of good information about the relationship between f(*) and g(*), one might as well assume a simple relationship and define g(*) in terms of f(*). One possibility is to assume that when borrowing sectors are borrowing, say 10% less than they wish, saving sectors are increasing their financial assets by 10% more than they wish. Mathematically,

\[ g(*) = 2 - f(*) \]  (45)

Although the proxies for credit rationing are insufficient for an estimation of f(*) and g(*), they do provide some confirming evidence that credit rationing may be a response to liquidity pressures. Examination of these proxies suggest that rationing occurs around the end of business cycle expansions and/or the beginning of business cycle contractions (Dubofsky 1985, p. 196; Jaffee and Modigliani p. 64, Sinai 1976, Wojnilower 1980). We know that money moves in phase with the business cycle. Desired reserves, being a function of money (see equation (17)) must also be approximately in phase with the business cycle. Because relative available reserves decline as desired reserves increase (see equation (18)); illiquidity is also in phase with the business cycle, reaching a peak about at the peak of economic activity. The peak in illiquidity produces a credit crunch. ³²

³² Naturally, liquidity is also a function of reserves. This argument says nothing about
Direct Borrowing Between the Non-financial Sectors. A key simplification in the model has been that all private sectors lend and borrow through financial intermediaries. In the real world, however, direct financing between businesses and households is possible and occurs. The purpose of this section is to show that the model can mimic direct financing even though there is no direct financial link between any two non-financial sectors. This "mimicking" of direct financing reduces the restrictiveness of the model's assumption that all transactions occur via a central intermediary.

When a business borrows directly from individuals or other businesses, it leaves the financial sector unaffected because the financial sector is not directly involved. At the most, banks record a change in ownership of deposits. An individual writes a check to the business and receives in return a promise that in the future the business will write a larger check to the individual. The check from the individual transfers the ownership of deposits to the business. The bank has precisely the same transaction deposit liability as before the transaction; from a bank's point of view, only the ownership of transactions balances has changed.

To say that the model mimics direct borrowing and lending between non-financial sectors is to say that situations which lead to such direct transactions in the real world leave the financial intermediary as unaffected in the model as its counterparts are in the real world. The modeled financial intermediary is unaffected as long as its liquidity is unaffected. Hence, the model mimics direct financing if its reserves and desired reserves are unchanged in situations which in the real world would lead to direct financing. It has previously been argued that private sector borrowing and lending (purchasing of bank bonds) leaves reserves unchanged. It remains only to show that desired reserves remain unchanged in situations which would lead to direct transactions in the real world.

reserves, which are controlled by the Federal Reserve. Nonetheless, we can still conclude that the economy tends to produce a credit crunch at the peak of the business cycle. It is unlikely that the Fed's actions prior to the peak would be such as to produce a surplus of liquidity: The Fed will be under less pressure to stimulate the economy if it has been growing recently. The Fed may also be worried about the increased money supply and rising prices caused by the accelerating demand. The only pressure causing the Fed to expand its reserves would be the illiquidity in the financial sector discussed above and the resulting rise in rates. But, because illiquidity must occur before the Fed feels pressure, the credit crunch will also occur before (or as) the Fed feels the pressure to expand reserves.
The hallmark of a direct transaction is that borrowing by one sector equals lending by another. Quite simply: What one person borrows another lends. In the model, whenever two sectors lend (purchase bank bonds) and borrow in the same amount, the dynamic impact on desired reserves is nil. A sector "lends" by purchasing bank bonds; a sector's borrowing increases loans. Bank bonds in the model carry a small positive reserve requirement, and, consequently, an increase in bank bonds in isolation would increase desired reserves. However, loans in the model carry a small negative reserve requirement equal in magnitude to the positive reserve requirement on bank bonds (see discussion above on reserve requirements). In a "direct" transaction bank bonds and loans are increased by exactly the same amount; consequently, the bank-bond-engendered increase in desired reserves is exactly offset by the corresponding decrease in desired reserves associated with the increase in loans. The net effect, therefore, is that the desired reserves of the banking system are left unchanged.

3. The Allocation of Interest Payments and Defaults

Allocation of Interest Payments. Not all processes can be captured with the structure presented thus far. In particular, interest payments and defaults on bank bonds and deposits require further consideration. Let us examine first the issue of interest payments. The financial intermediary is represented as a pure intermediary, that is, it has no equity account of its own. Without an equity account to absorb gains and losses on interest payments, all interest payments must be passed directly to the holders of deposits and bank bonds. Consequently, interest payments on bank bonds and deposits presents a problem in how to allocate interest payments between depositors and holders of bank bonds. Figure 15 presents a diagrammatic representation of the interest payment allocation discussed algebraically below.

First, we will assume that the rate on deposits and the rate on bank bonds are related linearly such that when the rate on bank bonds is zero, the rate on deposits is also zero. Mathematically:

\[ IPDEP_s = DEP_s \cdot IRDEP \]  
\[ IRDEP = IRBB \cdot \text{fird} \]

where:

- \( IPDEP_s \) Interest Payments to Deposits of sector \( s \)
- \( DEP_s \) Deposits of sector \( s \)
- \( IRDEP \) Interest rate on deposits
IRBB    Interest rate on bank bonds
fird    Fraction of interest payments allocated to deposits

The above equation indicates that the interest rate on deposits is a fraction (fird) of the interest rate paid on bank bonds. If no interest were paid on deposits (fird=0), the interest rate on bank bonds would simply be the total interest payments to the intermediary divided by the dollar amount of bank bonds. If deposits shared equally in interest payments (i.e. fird=1), the interest rate on bank bonds would be the total interest payments made to the intermediary divided by the sum of bank bonds and deposits. In the more general case, each dollar of deposits will yield a return equivalent to some fraction of a bank bond. That is, the rate paid on bonds will equal the total interest payments made to the intermediary divided by the sum of all bonds and a fraction (fird) of deposits:

\[
\begin{align*}
    \text{IPBB} &= \text{BB} \times \text{IRBB} \\
    \text{IRBB} &= \text{IPFI}/(\text{BB} + \text{fird} \times \text{TDEP}) \\
    \text{TDEP} &= \text{sum}(\text{DEP}_s)
\end{align*}
\] (48) (49) (50)

where:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPBB</td>
<td>Interest payments on bank bonds</td>
</tr>
<tr>
<td>BB</td>
<td>Bank Bonds</td>
</tr>
<tr>
<td>IRBB</td>
<td>Interest Rate on Bank Bonds</td>
</tr>
<tr>
<td>IPFI</td>
<td>Interest Payments to Financial Intermediary</td>
</tr>
<tr>
<td>fird</td>
<td>Fraction of interest payments allocated to Deposits</td>
</tr>
<tr>
<td>TDEP</td>
<td>Total Deposits</td>
</tr>
<tr>
<td>sum</td>
<td>summation operator</td>
</tr>
<tr>
<td>DEP_s</td>
<td>Deposits of sector s</td>
</tr>
</tbody>
</table>

The fraction of interest payments allocated to depositors will usually be less than one (0<=fird<=1), reflecting several factors. First, deposits are intended to represent a liability against which the banking system in the United States is required by law to keep reserves which do not bear interest. The reserve requirement is similar to a tax on deposits. The "tax" may be passed on to depositors via a reduction in the interest rate (Judd and Scadding 1982, pp. 998-999). The second factor affecting the rate on demand deposits is risk. As discussed in the next section, deposits are safer than bank bonds and, hence, depositors will accept a lower return on them. Further, deposits yield an additional return in the form of transaction services and, again, depositors will therefore be willing to accept a lower return than investors in bank bonds. Finally, fird can also be considered a government policy parameter. For example, setting fird to zero would reflect a completely effective ban on interest payments on transactions balances.
**Allocation of Defaults.** Although the technical meaning of "default" is the failure to meet any requirement in a loan agreement, in this paper the term will be used in its more popular sense: the permanent failure to repay principal or interest. Defaults in this sense occur continually in all economies, although at a greater rate during bad times than during good. During very bad times, such as the great depression in the 1930's, defaults may be a primary mechanism for clearing debt out of the system. A financial sector in a model capable of generating "very bad times" should be able to handle defaults.

Because the financial intermediary is purely an intermediary with no equity account to absorb losses, defaults must be allocated to those who invest in the financial intermediary, namely depositors and bank bond holders. Bank bonds in this portrayal of the financial sector represent investments by the household in corporate (and government) debt. Deposits represent transactions balances, primarily demand deposits. Defaults on bank bonds correspond roughly to defaults on corporate debt, and defaults on deposits correspond roughly to bank failures. A default on a bank bond is more likely than a default on a deposit except in extremely bad times. It is desirable, therefore, to allocate defaults primarily to bank bonds for moderate rates of default and to make depositors share increasingly in the default allocation as the default rate increases. A structure reflecting these comments and concerns appears diagrammatically as figure 16. Defaults on debt are split between bank bonds and deposits, based on a variable called share of defaults allocated to bank bonds SDBB. Algebraically,

\[
DBB = TDD \times SDABB \\
DDEP = TDD \times (1 - SDABB)
\]

where:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DBB</td>
<td>Defaults on Bank Bonds</td>
</tr>
<tr>
<td>TDD</td>
<td>Total Defaults on Debt</td>
</tr>
<tr>
<td>SDABB</td>
<td>Share of defaults allocated to Bank Bonds</td>
</tr>
</tbody>
</table>

A simple representation of the share of defaults allocated to bank bonds may be developed by considering the fraction of deposits which are to be shielded from default. During very good times almost all deposits will be shielded from default; but, as conditions worsen, a larger and larger fraction of deposits will be exposed to default because banks increasingly will be among those who are defaulting. At one extreme, when all deposits are shielded from default, 100 per cent of defaults will be allocated to bank bonds. At the other extreme, when no deposits are shielded from default, bank bonds will be allocated a fraction of defaults corresponding to their fractional "ownership" of the financial
intermediary, and depositors will be allocated a fraction corresponding to their fractional ownership—investors will bear the consequences of default in proportion to their holdings. This suggests:

\[ SDABB = \frac{BB + FDSD \times TDEP}{BB + TDEP} \]  \hspace{1cm} (53)

where:

- **SDABB**: Share of Defaults Allocated to Bank Bonds
- **BB**: Bank Bonds
- **FDSD**: Fraction of Deposits Shielded from Default
- **TDEP**: Total Deposits

The bond holders shield depositors from default. Consequently, the fraction of deposits shielded from default should be an increasing function of the ability of bond holders to shield deposits. When defaults are low relative to bank bonds most defaults will be absorbed by bond holders; this corresponds to a situation in which corporate default is mild. When defaults are high relative to bank bonds, bond holders will no longer be able to absorb all the defaults; this corresponds to a situation in the real world in which corporate defaults have reached a level that they increasingly threaten the viability of depository institutions. Algebraically:

\[ FDSD = h(RDBB) \]  \hspace{1cm} (54)

\[ RDBB = \frac{DEFD}{BB} \]  \hspace{1cm} (55)

where:

- **FDSD**: Fraction of Deposits Shielded from Default
- **RDBB**: Ratio of Defaults to Bank Bonds
- **h**: a function such that \( h' < 0 \)
- **DEFD**: Defaults on Debt
- **BB**: Bank Bonds

The function \( h(\bullet) \) is difficult to estimate because it depends upon aggregations (deposits and bank bonds) which are not handily available in the data; information on defaults on the aggregations would be even more difficult to obtain. Even in the absence of direct observation or statistical estimation, however, much is still known about \( h(\bullet) \). Conclusions and policy recommendations can be made as long as they are insensitive to more precise knowledge of \( h(\bullet) \). Without estimating, we know, for example, that the maximum value of \( h(\bullet) \) is one and the minimum is zero. Further, \( h(\bullet) \) must reach its maximum of one at least when RDBB is zero. The function probably decreases monotonically over its range.
Defaults on deposits differ from defaults on bank bonds in one rather crucial respect. Some deposits are insured by the Federal Deposit Insurance Corporation (FDIC) or by the Federal Savings and Loan Insurance Corporation (FSLIC). As a consequence, defaults which otherwise would reduce deposits, and therefore the money supply, are "undone" by the FDIC and the FSLIC. The FDIC and the FSLIC do not act immediately, however. There is a delay between the time a bank fails and the time the depositors receive access to their deposits again. This suggests:

\[
\begin{align*}
NDDEP &= DDEP - DRF & (56) \\
DRF &= IDIP/\text{atpd} & (57) \\
(d/dt)IDIP &= DDIF - DRF & (58) \\
DDIF &= DDEP*\text{fdi} & (59)
\end{align*}
\]

where:

- **NDDEP**: Net Defaults on Deposits
- **DDEP**: Defaults on Deposits
- **DRF**: Defaults Repaid by FDIC or FSLIC
- **IDIP**: Insured Deposits in Process
- **atpd**: Average Time to Process Deposits
- **DDIF**: Defaults on Deposits Insured by FDIC or FSLIC
- **fdi**: Fraction of Deposits Insured

According to the equations above, it is assumed that the time to process deposits is exponentially distributed with an average time of atpd. This means that most claims will be repaid more quickly than atpd but some will take substantially longer. This seems like a reasonable assumption, although one might wish to explore the impact of alternative assumptions concerning the distribution. Of somewhat greater importance is the possibility that the average processing time will increase as defaults increase because the capacity of the FDIC and FSLIC may lag the increase in volume. The above formulation treats the average processing time atpd as a constant. This must be considered an approximation which is probably adequate for most macro-economic models not focused directly on the financial sector. In a model examining the viability of the banking system, one might want to make atpd a variable instead of a constant.33

---

33 One formulation would make atpd a function of the liquid capacity of the FDIC and FSLIC relative to the backlog of claims. The liquid capacity of the FDIC or FSLIC would depend, in turn, on the speed with which loans assumed by the FDIC and FSLIC could be sold or paid off and on the willingness of congress to increase the capitalization of the two insurance funds when needed.
4. Summary and Conclusion

This paper has presented a thoroughly disequilibrium financial sector. Like many previous models, the model presented in this paper permits non-financial sectors to adjust their actual security holdings to the desired amount over time. In addition, however, the financial sector modeled in this paper includes interest rates that do not necessarily equate desire purchases of securities with desired sales of securities (borrowing), and the model does not necessarily require that desired purchases or sales of securities equal actual purchases or sales of securities. The financial sector does not assume equilibrium, but rather generates the forces that bring the financial sector into an equilibrium where all of these relationships hold—When the model comes to rest, actual security stocks equal desired security stocks, desired sales equal desired purchases, and desired sales (purchases) equal actual sales (purchases).

As a key simplification, it was assumed that all financial transactions flow through the intermediary. In the real world most financial transactions do involve intermediaries; however, some occur directly between non-financial sector actors. As was shown, however, the financial intermediary in the model by transacting simultaneously with two sectors can produce the effect of direct transactions between the sectors. Consequently, the distortion involved in making an intermediary a party to every financial transaction is slight.

Financial intermediaries have been modeled as a single aggregated intermediary. As a result, the financial model presented here cannot explore the dynamics occurring between intermediaries. On the other hand, the financial model is quite appropriate for use in a larger macro-economic model intended to investigate major macro-economic dynamics. An investigation of the major macro-economic dynamics will probably require a consideration of basic financial processes, including those of money creation and contraction and the financial sector's response to deficits and to governmental and private financing. The financial sector described in this paper is fully capable of capturing these basic processes.

The key to handling financial disequilibria is the aggregate financial intermediary, and in particular its financial inventories. Intermediaries in the real world carry financial inventories because there is no assurance that the purchases of securities (deposits) will equal sales of securities (loans) from moment to moment. Commercial banks carry inventories of reserves and securities dealers carry inventories of deposits (the 'reserves' of the dealer) and inventories of securities. Intermediaries' financial inventories permit the non-financial sectors' sales and purchases of securities to differ from one another for a
short time. In this model we focused on the inventory of reserves. A persistent inequality between sales and purchases or depositing and lending will show up as too high or too low an inventory of reserves—that is, as excessive or deficient liquidity.

Intermediaries have two tools to control their liquidity: Interest rates and credit rationing. When liquidity is too high, intermediaries will decrease the interest rate and encourage lending. When liquidity is too low, intermediaries will tend to increase the interest rate. Simulations showed that interest rates are unlikely to be the only tool used by intermediaries wishing to correct liquidity deficiencies quickly. Aggressive use of interest rates can lead to over correction where deficient liquidity is replaced by excessive liquidity. More importantly, perhaps, interest rates must peak much higher than the equilibrium rate if illiquidity is to be eliminated quickly. Intermediaries who value customer relationships—and almost all intermediaries value such relationships—will be reluctant to increase rates as fast and as far as necessary. Consequently, credit rationing, where good customers are given preferential treatment and new or poor customers may have to wait to borrow, becomes a desirable additional means of controlling reserves.

The actions taken by intermediaries to bring their inventories back into line also help bring the depositing and lending; selling and buying of financial securities back into line. Credit restrictions serve to equate the value of completed transactions while adjustments to the interest rate operate to equate the value of desired transactions. The model of the financial sector described here does not assume that equilibrium equalities hold. The model operates like the real economy: It contains the structure that produces the forces and responses which ultimately move the system to equilibrium. The model developed above, in contrast to representations which simply assume equilibrium, can be used to investigate the nature and importance of disequilibria in the financial sector.
Appendix: Model Documentation

The figures for reference deposits (RDEP) comes from 1985 Statistical Abstract (U.S. Bureau of the Census 1984 p. 487 chart no. 808). The figures are those for checkable deposits and currency for households and businesses respectively. The figure for bank bonds is total assets of the household-liabilities of households-deposits of household.

The model that generated figures 10, 12, and 13 was written in DYNAMO. A DYNAMO documenter of the model follows. Listings of relevant "changes files" are included at the end of the documenter.
Model Documentation
For Runs Shown in
Figures 10, 12, and 13

MACROS

MACRO TABTUN(TABL,IND,LOW,HIGH,INT,OZ,TUNE)
   TABTUN - PERMITS TABLES TO BE MODULATED BY
   "TUNE" PARAMETER ALLOWS BOTH ZERO-
   BASED AND ONE-BASED TABLES <2>

   TABTUN.K=$ONETAB.K*OZ+$ZERTAB.K*(1-OZ)
   TABTUN - PERMITS TABLES TO BE MODULATED BY
   "TUNE" PARAMETER ALLOWS BOTH ZERO-
   BASED AND ONE-BASED TABLES <2>

   $ONETAB.K=$TAB.K**TUNE
   $TAB - TUNE=2 IS ABOUT TWICE AS STEEP. NOTE:
   A TABLE NORMALIZED AT ONE SHOULD NOT
   REACH ZERO. <6>

   $ZERTAB.K=$TAB.K**TUNE
   $TAB - TUNE=2 IS ABOUT TWICE AS STEEP. NOTE:
   A TABLE NORMALIZED AT ONE SHOULD NOT
   REACH ZERO. <6>

   $TAB.K=TABLEL(TABL,IND,LOW,HIGH,INT)
   $TAB - TUNE=2 IS ABOUT TWICE AS STEEP. NOTE:
   A TABLE NORMALIZED AT ONE SHOULD NOT
   REACH ZERO. <6>

MEND

ARRAY INFORMATION

S=HS,PS
   S - S - SUBSCRIPT FOR SECTOR, HS=
   HOUSEHOLD, PS=PRODUCTION <8>

LIABILITIES: DEPOSITS AND BANK BONDS

DEP,K(S)=DEP,J(S)+DT*(CDEP,J(S))
DEP(S)=RDEP(S)
   DEP - TRANSACTIONS DEPOSITS (DOLLARS) <9>
   S - S - SUBSCRIPT FOR SECTOR, HS=
   HOUSEHOLD, PS=PRODUCTION <8>
   CDEP - CHANGE IN DEPOSITS (DOLLARS/YEAR)
   <10>
RDEP - REFERENCE DEPOSITS (Dollars) <11>

CDEP.K(S) = (BOR.K(S) + NCF.K(S)) * EDEPDR.K(S)
CDEP - CHANGE IN DEPOSITS (Dollars/year);
S - S - SUBSCRIPT FOR SECTOR, HS=
BOR - BORROWING (Dollars/year) <12>
NCF - NET CASH FLOW (Dollars/year) <22, 23>
EDEPDR - EFFECT OF DEPOSIT LEVEL ON DEPOSIT
REDUCTION (DIMENSIONLESS) <11>

EDEPDR.K(S) = FIFGE(1, TABLE (TEDEPDR, DEP.K(S)) /
DD.K(S), 0, 1, .2), BOR.K(S) + NCF.K(S), 0)
TEDEPDR = 0/.7/.9/1/1/1
RDEP(*) = 338.789/103.3E9

DD.K(S), 0, 1, .2), BOR.K(S) + NCF.K(S), 0)
TEDEPDR = TABLE FOR EFFECT OF DEPOSIT LEVEL ON
DEPOSIT REDUCTION <11>

DEP - TRANSACTIONS DEPOSITS (Dollars) <9>
DD - DESIRED TRANSACTIONS DEPOSITS
(Dollars) <18>
BOR - BORROWING (Dollars/year) <12>
NCF - NET CASH FLOW (Dollars/year) <22, 23>
RDEP - REFERENCE DEPOSITS (Dollars) <11>

BOR.K(S) = DBOR.K(S) * ((ELB.K) * FIFGE(1, 0, DBOR.K(S),
0) + ELS.K * FIFGE(0, 1, DBOR.K(S), 0))
BOR - BORROWING (Dollars/year) <12>
S - S - SUBSCRIPT FOR SECTOR, HS=
BOR - DESIRED BORROWING (Dollars/year) <13>
ELB - EFFECT OF LIQUIDITY ON BORROWING
(DIMENSIONLESS) <25>

DBOR.K(S) = BAD.K(S) + BNCF.K(S)
DBOR - DESIRED BORROWING (Dollars/year) <13>
S - S - SUBSCRIPT FOR SECTOR, HS=
BAD - BORROWING TO ADJUST DEPOSITS
(Dollars/year) <15>
BNCF - BORROWING FOR NET CASH FLOW (Dollars/
YEAR) <14>

BNCF.K(S) = - NCF.K(S)
BNCF - BORROWING FOR NET CASH FLOW (Dollars/
YEAR) <14>
\( S \) - \( S \) - SUBSCRIPT FOR SECTOR, HS = HOUSEHOLD, PS = PRODUCTION <8>

\( NCF \) - NET CASH FLOW (DOLLARS/YEAR) <22,23>

\( \text{BAD.K(S)} = \text{FCD.K(S)} \times \text{DEP.K(S)} \)

\( \text{BAD} \) - BORROWING TO ADJUST DEPOSITS (DOLLARS/YEAR) <15>

\( S \) - \( S \) - SUBSCRIPT FOR SECTOR, HS = HOUSEHOLD, PS = PRODUCTION <8>

\( \text{FCD} \) - FRACTIONAL CHANGE IN DEPOSITS (FRACTION/YEAR) <16>

\( \text{DEP} \) - TRANSACTIONS DEPOSITS (DOLLARS) <9>

\( \text{FCD.K(S)} = (\text{RDD.K(S)} \times \text{C(S)}) - 1 \)

\( C(*) = .680 / .960 \)

\( \text{FCD} \) - FRACTIONAL CHANGE IN DEPOSITS (FRACTION/YEAR) <16>

\( S \) - \( S \) - SUBSCRIPT FOR SECTOR, HS = HOUSEHOLD, PS = PRODUCTION <8>

\( \text{RDD} \) - RELATIVE DESIRED TRANSACTIONS DEPOSITS (DIMENSIONLESS) <17>

\( C \) - EXPONENT FOR FCD <16>

\( \text{RDD.K(S)} = \text{DD.K(S)} / \text{DEP.K(S)} \)

\( \text{RDD} \) - RELATIVE DESIRED TRANSACTIONS DEPOSITS (DIMENSIONLESS) <17>

\( S \) - \( S \) - SUBSCRIPT FOR SECTOR, HS = HOUSEHOLD, PS = PRODUCTION <8>

\( \text{DD} \) - DESIRED TRANSACTIONS DEPOSITS (DOLLARS) <18>

\( \text{DEP} \) - TRANSACTIONS DEPOSITS (DOLLARS) <9>

\( \text{DD.K(S)} = \text{RDEP(S)} \times \text{EED.K(S)} \times \text{EID.K(S)} \)

\( \text{DD} \) - DESIRED TRANSACTIONS DEPOSITS (DOLLARS) <18>

\( S \) - \( S \) - SUBSCRIPT FOR SECTOR, HS = HOUSEHOLD, PS = PRODUCTION <8>

\( \text{RDEP} \) - REFERENCE DEPOSITS (DOLLARS) <11>

\( \text{EED} \) - EFFECT OF EXPENDITURES ON DESIRED DEPOSITS (DIMENSIONLESS) <19>

\( \text{EID} \) - EFFECT OF INTEREST RATES ON DESIRED DEPOSITS (DIMENSIONLESS) <20>

\( \text{EED.K(S)} = 1 \)

\( \text{EED} \) - EFFECT OF EXPENDITURES ON DESIRED DEPOSITS (DIMENSIONLESS) <19>

\( S \) - \( S \) - SUBSCRIPT FOR SECTOR, HS = HOUSEHOLD, PS = PRODUCTION <8>

\( \text{EID.K(S)} = \text{RI.K**B(S)} \)

\( EID \) - EFFECT OF INTEREST RATES ON DESIRED DEPOSITS (DIMENSIONLESS) <20>
B(*)=-.388/-1.17
EID - EFFECT OF INTEREST RATES ON DESIRED
       DEPOSITS (DIMENSIONLESS) <20>
S    - S - SUBSCRIPT FOR SECTOR, HS=
       HOUSEHOLD, PS=PRODUCTION <8>
RI    - RELATIVE INTEREST RATE
       (DIMENSIONLESS) <21>
B    - EXPONENT FOR EID <20>

RI.K=RFIR.K/RIR
RI    - RELATIVE INTEREST RATE
       (DIMENSIONLESS) <21>
RFIR  - RISK FREE INTEREST RATE (FRACITON/
       YEAR) <40>
RIR  - REFERENCE INTEREST RATE (FRACITON/
       YEAR) <41>

NCF.K(HS)=TP.K(HS)+NFOS.K(HS)+MAT.K(HS)
NCF    - NET CASH FLOW (DOLLARS/YEAR) <22,23>
TP    - NET PAYMENT FROM GOVERNMENT (DOLLARS/
       YEAR) <48>
NFOS  - NET CASH FLOW FROM OTHER SECTOR
       (DOLLARS/YEAR) <46,47>
MAT    - MATURITIES (DOLLARS/YEAR) <27,29>

NCF.K(PS)=TP.K(PS)+NFOS.K(PS)-MAT.K(PS)
NCF    - NET CASH FLOW (DOLLARS/YEAR) <22,23>
TP    - NET PAYMENT FROM GOVERNMENT (DOLLARS/
       YEAR) <48>
NFOS  - NET CASH FLOW FROM OTHER SECTOR
       (DOLLARS/YEAR) <46,47>
MAT    - MATURITIES (DOLLARS/YEAR) <27,29>

ELS.K=(1-ELB.K)*Q+1
Q=1.084
ELB    - EFFECT OF LIQUIDITY ON BORROWING
       (DIMENSIONLESS) <25>
Q    - GRANTED ONLY 75% OF ITS NET CREDIT
       REQUESTS IS THE SAME AS THE SECTOR
       BEING FORCED TO SAVE 25%=(2-.75)
       MORE THAN IT WISHED. <24>

ELB.K=TABTUN(TELB,RAR.K,-.25,.25,.05,1,QTELB)
QTELB=0
TELB=.15/.25/.4/.7.9/1/1/1/1/1
ELB    - EFFECT OF LIQUIDITY ON BORROWING
       (DIMENSIONLESS) <25>
TABTUN - PERMITS TABLES TO BE MODULATED BY
       "TUNE" PARAMETER ALLOWS BOTH ZERO-
       BASED AND ONE-BASED TABLES <2>
This is the most complete text of the thesis available. The following page(s) were not included in the copy of the thesis deposited in the Institute Archives by the author:

pg. 105
RRD - RESERVE REQUIREMENT ON TRANSACTIONS
DEPOSITS (DIMENSIONLESS) <39>
L - LOANS (DOLLARS) <28>
RRL - RESERVE REQUIREMENT ON LOANS
(DIMENSIONLESS) <39>

TSB.K=FDFB*DEF.K
FDFB=1

TSB - TREASURY SALES OF GOVERNMENT BONDS
TO FINANCIAL INTERMEDIARY (DOLLARS/YEAR) <31>
FDFB - FRACTION OF DEFICIT FUNDED BY
FINANCIAL INTERMEDIARY
(DIMENSIONLESS) <31>
DEF - DEFICIT (DOLLARS/YEAR) <36>

GBHF.K=GBHF.J+DT*(TSF.J+FOMP.J)
GBHF=RGBHF
RGBHF=R*1.1

GBHF - GOVERNMENT BONDS HELD BY FEDERAL
RESERVES SYSTEM (DOLLARS) <32>
TSF - TREASURY SALES OF GOVERNMENT BONDS
TO FEDERAL RESERVE SYSTEM (DOLLARS/YEAR) <33>
FOMP - FEDERAL RESERVE OPEN MARKET
PURCHASES (DOLLARS/YEAR) <49>
RGBHF - REFERENCE GOVERNMENT BONDS HELD BY
FED (DOLLARS) <32>
R - RESERVES (DOLLARS) <34>

TSF.K=(1-FDFB)*DEF.K

TSF - TREASURY SALES OF GOVERNMENT BONDS
TO FEDERAL RESERVE SYSTEM (DOLLARS/YEAR) <33>
FDFB - FRACTION OF DEFICIT FUNDED BY
FINANCIAL INTERMEDIARY
(DIMENSIONLESS) <31>
DEF - DEFICIT (DOLLARS/YEAR) <36>

R.K=R.J+DT*(FOMP.J-TSB.J+DEF.J)
R=RR
RR=TD+BB-GBHB-L

R - RESERVES (DOLLARS) <34>
FOMP - FEDERAL RESERVE OPEN MARKET
PURCHASES (DOLLARS/YEAR) <49>
TSB - TREASURY SALES OF GOVERNMENT BONDS
TO FINANCIAL INTERMEDIARY (DOLLARS/YEAR) <31>
DEF - DEFICIT (DOLLARS/YEAR) <36>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>Reference Reserves (Dollars)</td>
<td>&lt;34&gt;</td>
</tr>
<tr>
<td>TD</td>
<td>Total Transactions Deposits (Dollars)</td>
<td>&lt;35&gt;</td>
</tr>
<tr>
<td>BB</td>
<td>Bank Bonds (Dollars)</td>
<td>&lt;26&gt;</td>
</tr>
<tr>
<td>GBHB</td>
<td>Government Bonds Held by Financial Intermediary (Dollars)</td>
<td>&lt;30&gt;</td>
</tr>
<tr>
<td>L</td>
<td>Loans (Dollars)</td>
<td>&lt;28&gt;</td>
</tr>
</tbody>
</table>

\[ TD.K = \text{SUM}(\text{DEP}.K) \]
\[ TD = \text{TOTAL TRANSACTIONS DEPOSITS (DOLLARS)} <35> \]
\[ DEP = \text{TRANSACTIONS DEPOSITS (DOLLARS)} <9> \]

\[ \text{DEF}.K = \text{SUM}(\text{TP}.K) \]
\[ \text{DEF} = \text{DEFICIT (DOLLARS/YEAR)} <36> \]
\[ \text{TP} = \text{NET PAYMENT FROM GOVERNMENT (DOLLARS/YEAR)} <48> \]

**LIQUIDITY**

\[ \text{RAR}.K = \text{FR}.K/\text{DR}.K \]
\[ \text{RAR} = \text{RELATIVE AVAILABLE RESERVES (DIMENSIONLESS)} <37> \]
\[ \text{FR} = \text{FREE RESERVES (DOLLARS)} <38> \]
\[ \text{DR} = \text{DESIRED RESERVES (DOLLARS)} <39> \]

\[ \text{FR}.K = \text{R}.K - \text{DR}.K \]
\[ \text{FR} = \text{FREE RESERVES (DOLLARS)} <38> \]
\[ \text{R} = \text{RESERVES (DOLLARS)} <34> \]
\[ \text{DR} = \text{DESIRED RESERVES (DOLLARS)} <39> \]

\[ \text{DR}.K = \text{TD}.K \times \text{RRD} + \text{BB}.K \times \text{RRBB} - \text{L}.K \times \text{RRL} \]

\[ \text{RRD} = .12 \]
\[ \text{RRBB} = .001 \]
\[ \text{RRL} = .001 \]
\[ \text{DR} = \text{DESIRED RESERVES (DOLLARS)} <39> \]
\[ \text{TD} = \text{TOTAL TRANSACTIONS DEPOSITS (DOLLARS)} <35> \]
\[ \text{RRD} = \text{RESERVE REQUIREMENT ON TRANSACTIONS DEPOSITS (DIMENSIONLESS)} <39> \]
\[ \text{BB} = \text{BANK BONDS (DOLLARS)} <26> \]
\[ \text{RRBB} = \text{RESERVE REQUIREMENT ON BANK BONDS (DIMENSIONLESS)} <39> \]
\[ \text{L} = \text{LOANS (DOLLARS)} <28> \]
\[ \text{RRL} = \text{RESERVE REQUIREMENT ON LOANS (DIMENSIONLESS)} <39> \]

\[ \text{RFIR}.K = \text{ALR}.K \times \text{UIR}.K \]
\[ \text{RFIR} = \text{RISK FREE INTEREST RATE (FRACTON/YEAR)} <40> \]
ALR - ADJUSTMENT FROM LIQUIDITY TO THE RISK FREE RATE (DIMENSIONLESS) \(<44>\)

UIR - UNDERLYING INTEREST RATE (FRACTION/YEAR) \(<41>\)

$$\text{UIR.K} = \text{UIR.J} + \text{DT} \times \text{CUIR.J}$$

$$\text{UIR} = \text{RIR}$$

$$\text{RIR} = .05$$

$$\text{UIR} - \text{UNDERLYING INTEREST RATE (FRACTION/YEAR)} \(<41>\)$$

$$\text{CUIR} - \text{CHANGE IN UNDERLYING INTEREST RATE (FRACTION/YEAR/YEAR)} \(<42>\)$$

$$\text{RIR} - \text{REFERENCE INTEREST RATE (FRACTON/YEAR)} \(<41>\)$$

$$\text{CUIR.K} = \left(\text{IUR.K} - \text{UIR.K}\right) / \text{TAUIR}$$

$$\text{TAUIR} = 1.4$$

$$\text{CUIR} - \text{CHANGE IN UNDERLYING INTEREST RATE (FRACTION/YEAR/YEAR)} \(<42>\)$$

$$\text{IUR} - \text{INDICATED UNDERLYING INTEREST RATE (FRACTION/YEAR)} \(<43>\)$$

$$\text{UIR} - \text{UNDERLYING INTEREST RATE (FRACTION/YEAR)} \(<41>\)$$

$$\text{TAUIR} - \text{TIME TO ADJUST THE UNDERLYING INTEREST RATE (YEARS)} \(<42>\)$$

$$\text{IUR.K} = \max(\text{RFIR.K, MUIR})$$

$$\text{MUIR} = .001$$

$$\text{IUR} - \text{INDICATED UNDERLYING INTEREST RATE (FRACTION/YEAR)} \(<43>\)$$

$$\text{RFIR} - \text{RISK FREE INTEREST RATE (FRACTON/YEAR)} \(<43>\)$$

$$\text{MUIR} - \text{MINIMUM UNDERLYING INTEREST RATE (FRACTION/YEAR)} \(<43>\)$$

$$\text{ALR.K} = \exp(-A \times \text{RAR.K})$$

$$\text{A} = 4$$

$$\text{ALR} - \text{ADJUSTMENT FROM LIQUIDITY TO THE RISK FREE RATE (DIMENSIONLESS)} \(<44>\)$$

$$\text{A} - \text{COEFFICIENT IN ALR} \(<44>\)$$

$$\text{RAR} - \text{RELATIVE AVAILABLE RESERVES (DIMENSIONLESS)} \(<37>\)$$

EXOGENOUS INPUTS

$$\text{NHP.K} = \text{PULSE}(\text{RR*FDF/DT,DT,TNHP,1E6})$$

$$\text{FDF} = 0$$

$$\text{TNHP} = 1$$
NHP  - NET PAYMENTS TO THE HOUSEHOLD FROM PRODUCERS (DOLLARS/YEAR) <45>
RR   - REFERENCE RESERVES (DOLLARS) <34>
FDF  - FRACTION OF REFERENCE RESERVES FOR NHP (DIMENSIONLESS) <45>
TNHP - TIME FOR NET PAYMENT TO HOUSEHOLD (YEAR) <45>

NFOS.K(PS)=-NHP.K
NFOS  - NET CASH FLOW FROM OTHER SECTOR (DOLLARS/YEAR) <46,47>
NHP   - NET PAYMENTS TO THE HOUSEHOLD FROM PRODUCERS (DOLLARS/YEAR) <45>

NFOS.K(HS)=NHP.K
NFOS  - NET CASH FLOW FROM OTHER SECTOR (DOLLARS/YEAR) <46,47>
NHP   - NET PAYMENTS TO THE HOUSEHOLD FROM PRODUCERS (DOLLARS/YEAR) <45>

TP.K(S)=RR*PULSE(FDTP(S)/DT,DT,TSTP,1E6)
FDTP(*)=0/0
TSTP=1

TP  - NET PAYMENT FROM GOVERNMENT (DOLLARS/YEAR) <48>
S   - S - SUBSCRIPT FOR SECTOR, HS=HOUSEHOLD, PS=PRODUCTION <8>
RR  - REFERENCE RESERVES (DOLLARS) <34>
FDTP - FRACTION OF REFERENCE RESERVES FOR TP <48>
TSTP - TIME FOR NET PAYMENT FROM GOVERNMENT (YEAR) <48>

FOMP.K=RFOMP*PULSE(1/DT,DT,TSFOMP,1E6)
RFOMP=FCRPY*RR
FCRPy=0
TSFOMP=1

FOMP  - FEDERAL RESERVE OPEN MARKET PURCHASES (DOLLARS/YEAR) <49>
RFOMP - REFERENCE FEDERAL RESERVE OPEN MARKET PURCHASES (DOLLARS) <49>
TSFOMP - TIME FOR FEDERAL RESERVE OPEN MARKET PURCHASES (YEAR) <49>
FCRPy  - FRACTION OF REFERENCE RESERVES TO PURCHASE (DIMENSIONLESS) <49>
RR   - REFERENCE RESERVES (DOLLARS) <34>

SPEC  LENGTH=20/SAVPER=.25/DT=.0625/REL_ERR=0
### LIST OF VARIABLES

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>T</th>
<th>WHR-CMP</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$$ONETAB</td>
<td>A</td>
<td>4</td>
<td>TUNE=2 IS ABOUT TWICE AS STEEP. NOTE: A TABLE-normalized AT ONE SHOULD NOT REACH ZERO. &lt;6&gt;</td>
</tr>
<tr>
<td>$$TAB</td>
<td>A</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>$$ZERTAB</td>
<td>A</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>C</td>
<td>44.1</td>
<td>COEFFICIENT IN ALR &lt;44&gt;</td>
</tr>
<tr>
<td>ALB</td>
<td>C</td>
<td>27.1</td>
<td>AVERAGE LIFE OF BANK BONDS (YEARS) &lt;27&gt;</td>
</tr>
<tr>
<td>ALL</td>
<td>C</td>
<td>29.1</td>
<td>AVERAGE LIFE OF LOANS (YEARS) &lt;29&gt;</td>
</tr>
<tr>
<td>ALR</td>
<td>A</td>
<td>44</td>
<td>ADJUSTMENT FROM LIQUIDITY TO THE RISK FREE RATE (DIMENSIONLESS) &lt;44&gt;</td>
</tr>
<tr>
<td>B(*)</td>
<td>T</td>
<td>20.1</td>
<td>EXPONENT FOR EID &lt;20&gt;</td>
</tr>
<tr>
<td>BAD(S)</td>
<td>A</td>
<td>15</td>
<td>BORROWING TO ADJUST DEPOSITS (DOLLARS/YEAR) &lt;15&gt;</td>
</tr>
<tr>
<td>BB</td>
<td>L</td>
<td>26</td>
<td>BANK BONDS (DOLLARS) &lt;26&gt;</td>
</tr>
<tr>
<td>N</td>
<td>26.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BNCF(S)</td>
<td>A</td>
<td>14</td>
<td>BORROWING FOR NET CASH FLOW (DOLLARS/YEAR) &lt;14&gt;</td>
</tr>
<tr>
<td>BOR(S)</td>
<td>A</td>
<td>12</td>
<td>BORROWING (DOLLARS/YEAR) &lt;12&gt;</td>
</tr>
<tr>
<td>C(*)</td>
<td>T</td>
<td>16.1</td>
<td>EXPONENT FOR FCD &lt;16&gt;</td>
</tr>
<tr>
<td>CDEP(S)</td>
<td>A</td>
<td>10</td>
<td>CHANGE IN DEPOSITS (DOLLARS/YEAR) &lt;10&gt;</td>
</tr>
<tr>
<td>CUIR</td>
<td>A</td>
<td>42</td>
<td>CHANGE IN UNDERLYING INTEREST RATE (FRACTION/YEAR/YEAR) &lt;42&gt;</td>
</tr>
<tr>
<td>DBOR(S)</td>
<td>A</td>
<td>13</td>
<td>DESIRED BORROWING (DOLLARS/YEAR) &lt;13&gt;</td>
</tr>
<tr>
<td>DD(S)</td>
<td>A</td>
<td>18</td>
<td>DESIRED TRANSACTIONS DEPOSITS (DOLLARS) &lt;18&gt;</td>
</tr>
<tr>
<td>DEF</td>
<td>A</td>
<td>36</td>
<td>DEFICIT (DOLLARS/YEAR) &lt;36&gt;</td>
</tr>
<tr>
<td>DEP(S)</td>
<td>L</td>
<td>9</td>
<td>TRANSACTIONS DEPOSITS (DOLLARS) &lt;9&gt;</td>
</tr>
<tr>
<td>N</td>
<td>9.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DR</td>
<td>A</td>
<td>39</td>
<td>DESIRED RESERVES (DOLLARS) &lt;39&gt;</td>
</tr>
<tr>
<td>DT</td>
<td>C</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>EDEPDR(S)</td>
<td>A</td>
<td>11</td>
<td>EFFECT OF DEPOSIT LEVEL ON DEPOSIT REDUCTION (DIMENSIONLESS) &lt;11&gt;</td>
</tr>
<tr>
<td>EED(S)</td>
<td>A</td>
<td>19</td>
<td>EFFECT OF EXPENDITURES ON DESIRED DEPOSITS (DIMENSIONLESS) &lt;19&gt;</td>
</tr>
<tr>
<td>EID(S)</td>
<td>A</td>
<td>20</td>
<td>EFFECT OF INTEREST RATES ON DESIRED DEPOSITS (DIMENSIONLESS) &lt;20&gt;</td>
</tr>
<tr>
<td>ELB</td>
<td>A</td>
<td>25</td>
<td>EFFECT OF LIQUIDITY ON BORROWING (DIMENSIONLESS) &lt;25&gt;</td>
</tr>
<tr>
<td>ELS</td>
<td>A</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>FCD(S)</td>
<td>A</td>
<td>16</td>
<td>FRACTIONAL CHANGE IN DEPOSITS (FRACTION/YEAR) &lt;16&gt;</td>
</tr>
<tr>
<td>FCRPY</td>
<td>C</td>
<td>49.2</td>
<td>FRACTION OF REFERENCE RESERVES TO PURCHASE (DIMENSIONLESS) &lt;49&gt;</td>
</tr>
<tr>
<td>FDF</td>
<td>C</td>
<td>45.1</td>
<td>FRACTION OF REFERENCE RESERVES FOR</td>
</tr>
<tr>
<td>Variable</td>
<td>Scale</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>FDFB</td>
<td>C</td>
<td>31.1 FRACTION OF DEFICIT FUNDED BY FINANCIAL INTERMEDIARY (DIMENSIONLESS) &lt;45&gt;</td>
<td></td>
</tr>
<tr>
<td>FDTP(*)</td>
<td>C</td>
<td>48.1 FRACTION OF REFERENCE RESERVES FOR TP &lt;48&gt;</td>
<td></td>
</tr>
<tr>
<td>FOMP</td>
<td>A</td>
<td>49 FEDERAL RESERVE OPEN MARKET PURCHASES (Dollars/Year) &lt;49&gt;</td>
<td></td>
</tr>
<tr>
<td>FR</td>
<td>A</td>
<td>38 FREE RESERVES (Dollars) &lt;38&gt;</td>
<td></td>
</tr>
<tr>
<td>GBHB</td>
<td>L</td>
<td>30 GOVERNMENT BONDS HELD BY FINANCIAL INTERMEDIARY (Dollars) &lt;30&gt;</td>
<td></td>
</tr>
<tr>
<td>GBHF</td>
<td>L</td>
<td>32 GOVERNMENT BONDS HELD BY FEDERAL RESERVES SYSTEM (Dollars) &lt;32&gt;</td>
<td></td>
</tr>
<tr>
<td>HS</td>
<td>e</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>IUR</td>
<td>A</td>
<td>43 INDICATED UNDERLYING INTEREST RATE (FRACTION/YEAR) &lt;43&gt;</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>28 LOANS (Dollars) &lt;28&gt;</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>28.1</td>
<td></td>
</tr>
<tr>
<td>LENGTH</td>
<td>C</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>MAT(*)</td>
<td>A</td>
<td>27 Maturities (Dollars/Year) &lt;27,29&gt;</td>
<td></td>
</tr>
<tr>
<td>MUIR</td>
<td>C</td>
<td>43.1 MINIMUM UNDERLYING INTEREST RATE (FRACTION/YEAR) &lt;43&gt;</td>
<td></td>
</tr>
<tr>
<td>NCF(*)</td>
<td>A</td>
<td>22 NET CASH FLOW (Dollars/Year) &lt;22,23&gt;</td>
<td></td>
</tr>
<tr>
<td>NFOS(*)</td>
<td>A</td>
<td>46 NET CASH FLOW FROM OTHER SECTOR (Dollars/Year) &lt;46,47&gt;</td>
<td></td>
</tr>
<tr>
<td>NHP</td>
<td>A</td>
<td>45 NET PAYMENTS TO THE HOUSEHOLD FROM PRODUCERS (Dollars/Year) &lt;45&gt;</td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>e</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>C</td>
<td>24.1 GRANTED ONLY 75% OF ITS NET CREDIT REQUESTS IS THE SAME AS THE SECTOR BEING FORCED TO SAVE 25%-=.75 MORE THAN IT WISHED. &lt;24&gt;</td>
<td></td>
</tr>
<tr>
<td>QTELB</td>
<td>C</td>
<td>25.1 TUNING PARAMETER FOR ELB &lt;25&gt;</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>L</td>
<td>34 RESERVES (Dollars) &lt;34&gt;</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>34.1</td>
<td></td>
</tr>
<tr>
<td>RAR</td>
<td>A</td>
<td>37 RELATIVE AVAILABLE RESERVES (DIMENSIONLESS) &lt;37&gt;</td>
<td></td>
</tr>
<tr>
<td>RBB</td>
<td>C</td>
<td>26.2 REFERENCE BANK BONDS (Dollars) &lt;26&gt;</td>
<td></td>
</tr>
<tr>
<td>RDD(S)</td>
<td>A</td>
<td>17 RELATIVE DESIRED TRANSACTIONS DEPOSITS (DIMENSIONLESS) &lt;17&gt;</td>
<td></td>
</tr>
<tr>
<td>RDEP(*)</td>
<td>T</td>
<td>11.2 REFERENCE DEPOSITS (Dollars) &lt;11&gt;</td>
<td></td>
</tr>
<tr>
<td>REL ERR</td>
<td>C</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>RFI†</td>
<td>A</td>
<td>40 RISK FREE INTEREST RATE (FRACITON/YEAR) &lt;40&gt;</td>
<td></td>
</tr>
<tr>
<td>RFOMP</td>
<td>N</td>
<td>49.1 REFERENCE FEDERAL RESERVE OPEN MARKET PURCHASES (Dollars) &lt;49&gt;</td>
<td></td>
</tr>
<tr>
<td>RGBH</td>
<td>N</td>
<td>32.2 REFERENCE GOVERNMENT BONDS HELD BY FED (Dollars) &lt;32&gt;</td>
<td></td>
</tr>
<tr>
<td>Code</td>
<td>Type</td>
<td>Value</td>
<td>Description</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>-------</td>
<td>-------------</td>
</tr>
<tr>
<td>RI</td>
<td>A</td>
<td>21</td>
<td>RELATIVE INTEREST RATE (DIMENSIONLESS) &lt;21&gt;</td>
</tr>
<tr>
<td>RIR</td>
<td>C</td>
<td>41.2</td>
<td>REFERENCE INTEREST RATE (FRACTON/YEAR) &lt;41&gt;</td>
</tr>
<tr>
<td>RL</td>
<td>C</td>
<td>28.2</td>
<td>REFERENCE LOANS (DOLLARS) &lt;28&gt;</td>
</tr>
<tr>
<td>RR</td>
<td>N</td>
<td>34.2</td>
<td>REFERENCE RESERVES (DOLLARS) &lt;34&gt;</td>
</tr>
<tr>
<td>RRBB</td>
<td>C</td>
<td>39.2</td>
<td>RESERVE REQUIREMENT ON BANK BONDS (DIMENSIONLESS) &lt;39&gt;</td>
</tr>
<tr>
<td>RRD</td>
<td>C</td>
<td>39.1</td>
<td>RESERVE REQUIREMENT ON TRANSACTIONS DEPOSITS (DIMENSIONLESS) &lt;39&gt;</td>
</tr>
<tr>
<td>RRL</td>
<td>C</td>
<td>39.3</td>
<td>RESERVE REQUIREMENT ON LOANS (DIMENSIONLESS) &lt;39&gt;</td>
</tr>
<tr>
<td>S</td>
<td>F</td>
<td>8</td>
<td>S - SUBSCRIPT FOR SECTOR, HS=HOUSEHOLD, PS=PRODUCTION &lt;8&gt;</td>
</tr>
<tr>
<td>SAVPER</td>
<td>C</td>
<td>50</td>
<td>PERMITS TABLES TO BE MODULATED BY &quot;TUNE&quot; PARAMETER ALLOWS BOTH ZERO-BASED AND ONE-BASED TABLES &lt;2&gt;</td>
</tr>
<tr>
<td>TABTUN</td>
<td>M</td>
<td>2</td>
<td>TOTAL TRANSACTIONS DEPOSITS (DOLLARS) &lt;35&gt;</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td></td>
<td>TOTAL TRANSACTIONS DEPOSITS (DOLLARS) &lt;35&gt;</td>
</tr>
<tr>
<td>TAUIR</td>
<td>C</td>
<td>42.1</td>
<td>TIME TO ADJUST THE UNDERLYING INTEREST RATE (YEARS) &lt;42&gt;</td>
</tr>
<tr>
<td>TD</td>
<td>A</td>
<td>35</td>
<td>TOTAL TRANSACTIONS DEPOSITS (DOLLARS) &lt;35&gt;</td>
</tr>
<tr>
<td>TEDEPDR</td>
<td>T</td>
<td>11.1</td>
<td>TABLE FOR EFFECT OF DEPOSIT LEVEL ON DEPOSIT REDUCTION &lt;11&gt;</td>
</tr>
<tr>
<td>TELB</td>
<td>T</td>
<td>25.2</td>
<td>TABLE FOR EFFECT OF LIQUIDITY ON BORROWING &lt;25&gt;</td>
</tr>
<tr>
<td>TNHP</td>
<td>C</td>
<td>45.2</td>
<td>TIME FOR NET PAYMENT TO HOUSEHOLD (YEAR) &lt;45&gt;</td>
</tr>
<tr>
<td>TP(S)</td>
<td>A</td>
<td>48</td>
<td>NET PAYMENT FROM GOVERNMENT (DOLLARS/YEAR) &lt;48&gt;</td>
</tr>
<tr>
<td>TSB</td>
<td>A</td>
<td>31</td>
<td>TREASURY SALES OF GOVERNMENT BONDS TO FINANCIAL INTERMEDIARY (DOLLARS/YEAR) &lt;31&gt;</td>
</tr>
<tr>
<td>TSF</td>
<td>A</td>
<td>33</td>
<td>TREASURY SALES OF GOVERNMENT BONDS TO FEDERAL RESERVE SYSTEM (DOLLARS/YEAR) &lt;33&gt;</td>
</tr>
<tr>
<td>TSFOMP</td>
<td>C</td>
<td>49.3</td>
<td>TIME FOR FEDERAL RESERVE OPEN MARKET PURCHASES (YEAR) &lt;49&gt;</td>
</tr>
<tr>
<td>TSTP</td>
<td>C</td>
<td>48.2</td>
<td>TIME FOR NET PAYMENT FROM GOVERNMENT (YEAR) &lt;48&gt;</td>
</tr>
<tr>
<td>UIR</td>
<td>L</td>
<td>41</td>
<td>UNDERLYING INTEREST RATE (FRACTION/YEAR) &lt;41&gt;</td>
</tr>
<tr>
<td>N</td>
<td>41.1</td>
<td></td>
<td>UNDERLYING INTEREST RATE (FRACTION/YEAR) &lt;41&gt;</td>
</tr>
<tr>
<td>SYMBOL</td>
<td>WHERE_USED</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ONETAB</td>
<td>TABTUN,A,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$TAB</td>
<td>$ONETAB,A,4/$ZERTAB,A,5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ZERTAB</td>
<td>TABTUN,A,3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>ALR,A,44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALB</td>
<td>MAT,A,27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALL</td>
<td>MAT,A,29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALR</td>
<td>RFIR,A,40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>EID,A,20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BAD</td>
<td>DBOR,A,13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BB</td>
<td>MAT,A,27/GBHB,N,30.1/RR,N,34.2/DR,A,39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BNCF</td>
<td>DBOR,A,13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BOR</td>
<td>CDEP,A,10/EDEPDR,A,11/BB,L,26/L,L,28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>FCD,A,16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDEP</td>
<td>DEP,L,9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CUIR</td>
<td>UIR,L,41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DBOR</td>
<td>BOR,A,12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td>EDEPDR,A,11/RDD,A,17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEF</td>
<td>TSB,A,31/TSF,A,33/R,L,34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEP</td>
<td>EDEPDR,A,11/BAD,A,15/RDD,A,17/TD,A,35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DR</td>
<td>RAR,A,37/FR,A,38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DT</td>
<td>NHP,A,45/TP,A,48/FOMP,A,49</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EDEPDR</td>
<td>CDEP,A,10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EED</td>
<td>DD,A,18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EID</td>
<td>DD,A,18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ELB</td>
<td>BOR,A,12/ELS,A,24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ELS</td>
<td>BOR,A,12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXP</td>
<td>ALR,A,44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FCD</td>
<td>BAD,A,15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FCRFY</td>
<td>RFOMP,N,49.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDF</td>
<td>NHP,A,45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDFB</td>
<td>TSB,A,31/TSF,A,33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FDTP</td>
<td>TP,A,48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FIFGE</td>
<td>EDEPDR,A,11/BOR,A,12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FOMP</td>
<td>GEHB,L,30/GBHF,L,32/R,L,34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FR</td>
<td>RAR,A,37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GBHB</td>
<td>RR,N,34.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HIGH</td>
<td>TABTUN,M,2/$TAB,A,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IND</td>
<td>TABTUN,M,2/$TAB,A,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>INT</td>
<td>TABTUN,M,2/$TAB,A,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IUR</td>
<td>CUIR,A,42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>MAT,A,29/GBHB,N,30.1/RR,N,34.2/DR,A,39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LOW</td>
<td>TABTUN,M,2/$TAB,A,6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAT</td>
<td>NCF,A,22/NCF,A,23/BB,L,26/L,L,28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAX</td>
<td>IUR,A,43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MUIR</td>
<td>IUR,A,43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NCF</td>
<td>CDEP,A,10/EDEPDR,A,11/BNCF,A,14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
NFOS   NCF,A,22/NCF,A,23
NHP    NFOs,A,46/NFOs,A,47
OZ     TABTUN,M,2/TABTUN,A,3
PULSe  NHP,A,45/TP,A,48/FOMP,A,49
Q      ELS,A,24
QTELb  ELB,A,25
R      RGBHF,N,32.2/FR,A,38
RAR    ELB,A,25/ALR,A,44
RBB    BB,N,26.1
RDD    FCD,A,16
RDEP   DEP,N,9.1/DD,A,18
RFIR   RI,A,21/IUR,A,43
RFOMP  FOMP,A,49
RGBHF  GBHF,N,32.1
RI     EID,A,20
RI      RI,A,21/UIR,N,41.1
RL     L,N,28.1
RR     R,N,34.1/NHP,A,45/TP,A,48/RFOMP,N,49.1
RRBB   GBHB,N,30.1/DR,A,39
RRD    GBHB,N,30.1/DR,A,39
RRL    GBHB,N,30.1/DR,A,39
SUM    TD,A,35/DEF,A,36
TABHL  $TAB,A,6
TABL   TABTUN,M,2/$TAB,A,6
TABLE  EDEPDR,A,11
TABTUN ELB,A,25
TAUIR  CUIR,A,42
TD     GBHB,N,30.1/RR,N,34.2/DR,A,39
TEDEPDR EDEPDR,A,11
TELb   ELB,A,25
TENHP  NHP,A,45
TP     NCF,A,22/NCF,A,23/DEF,A,36
TSB    GBHB,L,30/R,L,34
TSF    GBHF,L,32
TSFOMP FOMP,A,49
TSTP   TP,A,48
TUNE   TABTUN,M,2/$ONETAB,A,4/$ZERTAB,A,5
UIR    RFIR,A,40/CUIR,A,42
PARAMETER CHANGES FOR FIGURE 10

--------------------------------- Parameters
--------------------------------- FCRPY
    Present     1     2
       .1       .1     0.
Original     0.       0.
--------------------------------- Run specifications
---------------------------------

LENGTH SAVPER
    Present     8.     62.5e-3
Original     20.     .25

PARAMETER CHANGES FOR FIGURE 12

--------------------------------- Parameters
--------------------------------- FCRPY
    Present     -.1
Original     0.
--------------------------------- Run specifications
---------------------------------

LENGTH SAVPER
    Present     8.     62.5e-3
Original     20.     .25

PARAMETER CHANGES FOR FIGURE 13

--------------------------------- Parameters
--------------------------------- A   FCRPY   TAUIR
    Present     8.     -.1     .7
Original     4.       0.     1.4
--------------------------------- Run specifications
---------------------------------

LENGTH SAVPER
    Present     8.     62.5e-3
Original     20.     .25
References


Homer, Jack. "Multiplier from Inventory on Shipments." Photocopy.


_____ 1980. "The Central Role of Credit Crunches in Recent Financial History."
### FINANCIAL INTERMEDIARY

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans to Producers</td>
<td>Producers' Deposits</td>
</tr>
<tr>
<td>Gov't Bonds Held by Financial Intermediary</td>
<td>Households' Deposits</td>
</tr>
<tr>
<td>Reserves</td>
<td>Bank Bonds</td>
</tr>
</tbody>
</table>

Figure 1: Aggregated Financial Intermediary's Balance Sheet

<table>
<thead>
<tr>
<th>Liabilities of</th>
<th>Households</th>
<th>Producers</th>
<th>Government</th>
<th>Financial Intermediary</th>
<th>Monetary Authority</th>
</tr>
</thead>
<tbody>
<tr>
<td>Households</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Producers</td>
<td></td>
<td></td>
<td></td>
<td>Loans to Producers</td>
<td></td>
</tr>
<tr>
<td>Government</td>
<td></td>
<td></td>
<td></td>
<td>Gov't Bonds Held by Bank</td>
<td>Gov't Bonds Held by Fed</td>
</tr>
<tr>
<td>Financial Intermediary</td>
<td>Deposits</td>
<td>Deposits</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank Bonds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Monetary Authority</td>
<td>Deposits in Process at FDIC</td>
<td>Deposits in Process at FDIC</td>
<td></td>
<td></td>
<td>Reserves</td>
</tr>
</tbody>
</table>

Figure 2: Interrelation of Asset and Liability Ownership
Figure 3: Assets and Liabilities in the Financial Sector Stock and Flow Diagram
<table>
<thead>
<tr>
<th>Security</th>
<th>Approximate Reserve Requirement House/Prod</th>
<th>Household Holdings (per cent)</th>
<th>Production Sector Holdings (per cent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time and Savings accounts</td>
<td>0/3%</td>
<td>1673.7 (22%)</td>
<td>71.8 (02%)</td>
</tr>
<tr>
<td>Money Market Fund Shares</td>
<td>3/3</td>
<td>162.6 (02%)</td>
<td>0 (00%)</td>
</tr>
<tr>
<td>Life Insurance Reserves 2</td>
<td>.004/.004</td>
<td>262.6 (03%)</td>
<td>0 (00%)</td>
</tr>
<tr>
<td>Pension Fund Reserves 2</td>
<td>.004/.004</td>
<td>1105.6 (14%)</td>
<td>0 (00%)</td>
</tr>
<tr>
<td>Corporate Equities</td>
<td>0/0</td>
<td>1519.5 (20%)</td>
<td>0 (00%)</td>
</tr>
<tr>
<td>Credit Market Instruments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets</td>
<td>0/0</td>
<td>868.1 (11%)</td>
<td>125.7 (04%)</td>
</tr>
<tr>
<td>Liabilities</td>
<td>0/0</td>
<td>1832.2 (24%)</td>
<td>1813.3 (51%)</td>
</tr>
<tr>
<td>Security Credit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets</td>
<td>0/0</td>
<td>19.3 (00%)</td>
<td>0 (00%)</td>
</tr>
<tr>
<td>Liabilities</td>
<td>0/0</td>
<td>47.8 (01%)</td>
<td>0 (00%)</td>
</tr>
<tr>
<td>Trade Credit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets</td>
<td>0/0</td>
<td>0 (00%)</td>
<td>545.9 (15%)</td>
</tr>
<tr>
<td>Liabilities</td>
<td>0/0</td>
<td>26.5 (00%)</td>
<td>438.9 (12%)</td>
</tr>
<tr>
<td>Taxes Payable</td>
<td>0/0</td>
<td>0 (00%)</td>
<td>5.2 (00%)</td>
</tr>
<tr>
<td>Miscellaneous Claims</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assets 2</td>
<td>.004/.004</td>
<td>92.2 (01%)</td>
<td>411.7 (12%)</td>
</tr>
<tr>
<td>Liabilities</td>
<td>0/0</td>
<td>16.4 (00%)</td>
<td>110.5 (03%)</td>
</tr>
<tr>
<td><strong>Total Absolute Value</strong></td>
<td></td>
<td><strong>7626.5(100%)</strong></td>
<td><strong>3523.0(100%)</strong></td>
</tr>
</tbody>
</table>

1. Fed Funds and Security RP's are considered transactions balances and are not shown here.

2. Reserve requirement calculation: Assume non-bank intermediaries wish to hold 3% of liabilities in transactions balances. Assume banks holding these transactions balances desire to keep 12% in reserve. The net reserve fraction of the aggregate financial sector is .03*.12=.004.

Figure 5: Basic Interest Rate Structure

The risk free interest rate is the underlying interest rate adjusted for liquidity. The underlying interest rate is a weighted average of past values of the risk free rate. The adjustment from liquidity is determined by the availability of reserves relative to desired reserves.
Figure 6: Desired Fractional Change in Deposits
Line 1: Household Effect of Expenditures on Deposits
Line 2: Producer Effect of Expenditures on Deposits

Figure 7: Effect of Expenditures on Deposits
Figure 8: Effect of Interest Rate on Deposits

Line 1: Household Effect of Interest Rate on Deposits
Line 2: Producer Effect of Interest Rate on Deposits
Figure 9: The Monetized Deficit

The deficit flows to households and businesses which can use the net inflow of money to increase bankbonds (or decrease loans). The inflow of money increases reserves of the financial intermediary. The treasury funds the deficit by issuing government bonds to the intermediary. The Federal Reserve purchases those bonds from the intermediary. The financial intermediary, now excessively liquid, reduces the interest rate. The reduced interest rate causes the non-financial sectors to increase their deposits. Increased deposits increase desired reserves which reduce the excessive liquidity of the financial intermediary.
Figure 10a: Monetized Deficit
A one-time monetized deficit occurs at time 1. Reserves immediately step up. Desired reserves increase gradually as the intermediary desires to provide reserves against increasing transaction deposits from the household and production sectors (see figure 10c, d).
Figure 10b: Monetized Deficit (continued)

The increase in reserves above desired reserves (see figure 10a) translates into excessive liquidity, measured by relative available reserves. The intermediary reduces the risk free rate to encourage borrowing and discourage depositing. Liquidity declines as the intermediary needs to provide reserves against the increasing transactions deposits of the household and production sectors (see figure 10c, d).
Figure 10c: Monetized Deficit (continued)
Because the interest rate has fallen (see figure 10b), transactions deposits are relatively more attractive to the household. The household moves funds out of bank bonds into transactions deposits.
Figure 10d: Monetized Deficit (continued)

Because the interest rate has fallen (see figure 10b), transactions deposits are relatively more attractive to producers; debt is less onerous. Producers borrow to increase their transactions deposits.
**Figure 11: Open Market Sale**

The financial intermediary pays for the bonds the Fed sells with reserves. The resulting reduction in reserves will reduce the intermediary's liquidity. If it was in liquidity balance before, the intermediary will be illiquid after purchasing bonds from the Fed. In response to illiquidity, the intermediary will increase the interest rate. A higher interest rate will prompt households and businesses to reduce their deposits. A reduction in deposits implies lower desired reserves. The reduction in desired reserves alleviates the illiquidity of the financial intermediary.
Figure 12a: Open Market Purchase

The Federal Reserve purchases government bonds at time 1. Reserves immediately step down. The intermediary desires less and less reserves as the household and production sectors reduce their transactions balances (see figure 12c, d).
Figure 12b: Open Market Purchase (continued)
The fall of reserves below desired reserves (see figure 12a) translates into a drop in liquidity, measured by relative available reserves. The intermediary increases the risk free interest rate in an effort to discourage borrowers and encourage depositors. Liquidity is restored as the intermediary needs less and less reserves to cover the declining transactions deposits of the household and production sectors (see figure 12 c, d).
Figure 12c: Open Market Purchase (continued)
The increased interest rate (see figure 12b) makes transactions deposits relatively less attractive and bank bonds relatively more attractive to the household. The household moves funds from transactions accounts to bank bonds.
Figure 12d: Open Market Purchase (continued)
The increased interest rate (see figure 12b) makes transactions deposits relatively less attractive and loans more onerous to the production sector. Producers pay off loans with their transactions deposits.
Figure 13a: Aggressive Interest Rate Response to Illiquidity
An attempt by intermediaries to restore liquidity faster through interest rates alone can produce a situation of excess liquidity which takes two years to correct.
Figure 13b: Aggressive Interest Rate Response to Illiquidity (continued)

Interest rates could overshoot equilibrium by a substantial margin were intermediaries to try to use interest rates more aggressively to restore liquidity faster.
Figure 14: A Possible Shape for the Effect of Liquidity on Borrowing
As liquidity (measured by RAR) declines below 0, intermediaries will lend less than what borrowers desire. ELB measures the fraction of borrowers' requests that are granted.
Figure 15: Allocation of Interest Payments
Interest paid to the financial intermediary is allocated to holders of bank bonds and to depositors. The interest rate paid on deposits relative to that paid on bank bonds is determined by the parameter FIPD, fraction of interest rate paid to deposits.
Figure 16: Default Allocation

Defaults must be allocated to holders of deposits and bankbonds. During times of low default, almost all deposits will be "shielded" from default and almost all defaults will be allocated to holders of bonds. As defaults increase, deposits will increasingly share in the defaults. A portion of defaults on deposits are insured. After a lag, these deposits will be repaid.
III. Present Value Analysis and Behavioral Considerations in Modeling Factor Acquisition

Introduction. This essay will develop a profitability-based formulation of firms' desired acquisition of purchased factors or rented factors for use in a larger macro-economic model. The ultimate ordering decision will be determined by additional factors beyond the scope of this essay including the availability of money or credit in a sector. Further, the rate at which factors actually arrive at firms will depend on still further factors including processing and shipping delays and the availability of labor and capital. The formulation described here accounts only for the first, decision-intensive stage of the factor-ordering process. This first stage is of considerable importance as it determines how firms combine factor productivity with the interest rate, and prices of capital, labor and output goods to decide how much capital (purchased factors) or labor (rented factors) they wish to order.

Evidence from surveys and the field of behavioral decision theory will be used to build an analogue to the process by which actual firms come to desire increases or decreases in factor stocks. The formulation is intended to be used in a larger macro-economic model and, in fact, will be tailored to a particular model—the M.I.T. System Dynamics National Model. The tailoring will be evident in the choice of cash flows and in the choice of structures that determine the timing of cash flows. Tailoring ensures the decision function is consistent with the surrounding macro-economic model. Although the formulation is strictly appropriate for a particular macro model, the approach—including the approach to tailoring—can be applied to any dynamic model.²

---

1 For overviews and discussions of the approach taken in the System Dynamics National Model see Forrester 1984, 1979 and 1976.

2 System dynamics models in particular will usually employ structures that are mathematically tractable to the approach taken here.
The approach differs from the more popular neoclassical approaches in two ways. First, the approach taken here contrasts with neoclassical approaches because it does not rely on the assumption that firms attempt to maximize the utility of a stream of consumption (Jorgenson 1976, p.135). The difference is less obvious in the formulation than it might be because survey evidence indicates that decision rules used by companies are based on concepts (viz concepts of present value) which are also consistent with a utility maximizing assumption. The approach taken in this paper also differs from the neoclassical approaches in that that desired output is not determined separately from profitability considerations. In this respect, this paper's approach is closer to the "security value models" where no assumption regarding output is made (Bischoff 1971 pp. 20-21, Brainard and Tobin 1968).

Because investment criteria used in practice rely on what can be interpreted as utility maximizing concepts, the formulation in this paper paper can be viewed in part as an extension of the neoclassical formulation. The standard neoclassical formulation is extended in this work through a more detailed consideration of the proper expected inflation rates to use in forming the real interest rate by explicitly taking account of the impact of inflation on the depreciation tax shield and by considering the effect on profitability of a time-consuming production process. Finally, the neoclassical approach is extended by using the same approach to determine desired output (or changes in desired output) as is used to determine desired factor proportions.

The next section describes briefly the two basic neoclassical formulations (putty-putty and putty-clay) and the securities value model. Following this, survey evidence on the approaches actually used by U.S. companies will be reviewed. An overview of how the profitability of a factor may be determined will be given in section three. The overview will be followed by a detailed derivation. In section five, the profitability of a factor will be related to the desired factor acquisition rate. The paper ends with a summary and conclusion.

---

1. Alternative Approaches to the Investment Decision

The Standard Neo-classical Approach (Putty-Putty). A series of closely related models associated with the work of Dale Jorgenson and several coauthors are considered the "standard" neoclassical approach (Bischoff 1971, p. 21; 1967, p. 76). In this approach, a sector's desired factor acquisitions are intended to move the sector toward its desired factor stock. In defining desired factor stock, Jorgenson assumes that (i) firms maximize profits (appropriately defined), (ii) that profit is zero, and (iii) that the price of capital is equal to the present value of the rental payments. Equations (6b) and (11) below give Hall and Jorgenson's (1971) expression for desired labor and desired capital. It will prove interesting to compare these expressions to the corresponding expressions developed from different assumptions in section 4. I present a derivation of Hall and Jorgenson's desired factor stocks below.\(^4\)

Profit \(Z\) is defined as revenues less wage expense and less the rental cost of capital:

\[
Z = OP\times Q - WAGE\times L - C\times K
\]  
(1)

where:

- \(Z\): Profit (dollars/year)
- \(OP\): The price of output (dollars/output unit)
- \(Q\): The quantity of output (output units/year)
- \(WAGE\): The price of labor input (dollars/person/year)
- \(L\): The quantity of labor input (people)
- \(C\): The rental price of capital (dollars/capital unit/year)
- \(K\): The quantity of capital (capital units).

Production \(Q\) is constrained by labor and capital. Hall and Jorgenson employ a Cobb-Douglas production function with constant returns to scale:

\[
Q = a\times K^{\alpha}L^{1-\alpha}
\]  
(2)

Solving (2) for \(L\) and substituting the result into (1) yields:

\[
Z = OP\times Q - WAGE\times (Q/a)^{1/(1-\alpha)}\times K^{\alpha/(1-\alpha)} - C\times K
\]  
(3)

Taking \(Q\) as given,

---

\(^4\) The derivation which follows departs from Hall and Jorgenson (1971) in detail, but not in spirit. Hall and Jorgenson's equations 2.4 and 2.5 hold without further assumptions only if there exists a unique, finite optimal \(Q\). A constant-returns-to-scale technology, such as the Cobb-Douglas technology does not possess a unique, finite optimal \(Q\). Consequently, in order to establish their equation 2.23, they implicitly assume that profits are zero (my equation 5).
the optimal amount of capital \( K^* \) is found by setting the partial derivative of (3) with respect to \( K \) equal to zero. The result after some manipulation is:

\[
K^* = \left\{ \alpha \frac{\text{WAGE}}{(C^* \beta)} \right\} \alpha^* \{ Q/\alpha \} \tag{4a}
\]

Similar reasoning leads to an expression for desired labor:

\[
L^* = \left\{ \alpha C / (\text{WAGE}^* \alpha) \right\} \alpha^* \{ Q/\alpha \} \tag{4b}
\]

A simpler expression may be found by making the further assumption that economic profits are zero, as they will be in equilibrium:

\[
\text{OP}^*Q = C^*K + \text{WAGE}^*L \tag{5}
\]

Using (5) to express the wage in terms of capital and labor, then using (2) to express wages in terms of capital alone, and finally substituting into (4.1) yields:

\[
K^* = \alpha^* \text{OP}^*Q/C \tag{6a}
\]

Similar reasoning will lead to:

\[
L^* = (1-\alpha^*) \text{OP}^*Q/WAGE \tag{6b}
\]

It is important to note that the neoclassical desired capital stock \( K^* \) and \( L^* \) are derived assuming that expected output \( Q \) is known. The information on prices (\( \text{OP}, \text{WAGE}, \text{and C} \)) are used to determine the factor mix but not the amount of productive capacity that is needed. A more satisfactory formulation would recognize that output (capacity) is itself a function of economic profit.

The rental cost of capital represents the flow equivalent of the discounted present value of all costs. More formally, the rental cost of capital (\( C \)) can be determined as follows. In equilibrium, the price of capital \( \text{PCAP}_0, \) net of any tax credit, is equal to the present value of the rental payments a company would receive from renting the equipment and the tax benefit from depreciation:

\[
(1-\text{fict})\text{PCAP}_0 = \int_{0}^{\infty} e^{-r_t}[e^{-\text{Pdr}^*t}(1-\text{citr})C + \text{citr}\text{PCAP}_0D(t)] \, dt \tag{7}
\]

Note that this formulation for \( K^* \) and \( L^* \) requires that the constraint in equation 5 (that profits are zero) holds at all times. Consequently, it is incorrect in the neoclassical framework to use this formulation to justify a doubling of \( K^* \) and \( L^* \) if output prices change. If output price doubles and one assumes that \( c \) does not change then one must also assume that the wage does change. The change in the wage will be such that while capital doubles, labor will decline and output will remain unchanged.
where:

\[ \text{fitc} \quad \text{The proportion of the value of the assets allowable as a credit against taxes (dimensionless)} \]
\[ \text{PCAP}_0 \quad \text{The price at time 0 of a unit of capital ($/capital unit)} \]
\[ r \quad \text{The real after tax rate of return (1/years)} \]
\[ \text{pdr} \quad \text{The physical depreciation rate (1/years)} \]
\[ C \quad \text{The rental cost per unit of capital ($/capital unit/year)} \]
\[ \text{citr} \quad \text{The corporate income tax rate (dimensionless)} \]
\[ D(t) \quad \text{The fractional depreciation allowed at time t (1/years)} \]

Equation (7) assumes that a piece of equipment depreciates exponentially, that is depreciation is equal to the current capital stock multiplied by the physical depreciation rate \((\text{pdr})\). The expected interest rate is assumed constant in the future. \((1-\text{citr})C\) is the after tax rental cost of capital. \(\text{citr} \times \text{PCAP}_0 \times D(t)\) is the tax shield allowed by the taxing authority for depreciation expense.

If we assume that the taxing authority recognizes a declining balance method of depreciation, the tax depreciation \((\text{tdr})\) can be written: \(^6\)

\[ \text{tdr} = \text{cad} \times \text{pdr} \]  
\[ \text{(8)} \]

where \("\text{cad}\"\) is the coefficient of accelerated depreciation. Under double declining balances, \(\text{cad}\) would equal two. Assuming declining balance tax depreciation will result in:

\[ D(t) = (\text{cad} \times \text{pdr}) e^{-\text{(cad} \times \text{pdr})t} \]

\[ \text{(9)} \]

Substituting (10) into (8) and solving for \(C\) yields:

\[ C = \text{PCAP}_0 [r + \text{pdr}] [1 - \text{fitc} - \text{citr} (\text{cad} \times \text{pdr}) (r + \text{cad} \times \text{pdr})] / [1 - \text{citr}] \]

\[ \text{(10)} \]

Finally, substituting (11) into (6) yields:

\[ K^* = \frac{\alpha \times \text{OP} \times Q}{\text{PCAP}_0 [r + \text{pdr}] [1 - \text{fitc} - \text{citr} (\text{cad} \times \text{pdr}) (r + \text{cad} \times \text{pdr})] / [1 - \text{citr}]} \]

\[ \text{(11)} \]

It is assumed that companies order capital or labor until the amount of capital or labor on order and in place equals the desired quantity.

\(^6\) More precisely, \(\text{pdr}\) should be equal to \(1/T\) where \(T\) is the lifetime of the asset allowed by the tax authority in the straight line method. Further, Hall and Jorgenson recognize that the tax laws permit a switch over to the straight line method and derive an expression assuming the switchover is made at the optimal point (see Hall and Jorgenson's equations (2.19) and (2.21)).
Putty-Clay. In the standard neoclassical model described above, the relative amounts of factor can be increased simply by purchasing more. Capital that is already in place is like putty: A little more capital can be added, or a little taken away, so that the resulting piece of capital can work with a different amount of labor. That is, factor proportions are variable ex post. Putty-clay models in contrast assume that although companies are free to choose the amount of capital that will work with a unit of labor before the capital is put in place, once it is put into place, the clay hardens, so to speak, and one cannot simply add or remove a small piece so that the capital will work with a different amount of labor. Factor proportions are variable ex ante but fixed ex post.

Putty-clay models are developed in the work of several authors (Bischoff 1967, 1971, 1974; Ando et al 1974). A piece of equipment arriving at a company is designed to operate with a certain amount of labor. Additional labor will not increase the output of the unit of equipment and the piece of equipment cannot be operated at all with less than the required amount of labor. The amount of labor required to operate the machine cannot be changed, once the machine arrives at the company. Consequently, if factor prices change so that the company would prefer to manufacture its (given) output with, say, a more expensive machine that needs less labor to operate, the company must replace the entire existing machine with a more appropriate one. The company cannot purchase kits that will convert a machine designed to operate with one amount of labor into a machine that will work with a different amount of labor.

The capital ordering decision is not one of looking at the difference between the desired amount of capital and the actual amount of capital and ordering until the gap is eliminated. To order new machines, the company must also desire to increase capacity or else it must either throw away (or sell) existing machines or wait for the existing machines to wear out. In the putty-clay formulation investment is related to gross changes in capacity:

\[
I_k = \zeta_k \cdot \left( \frac{d}{dt}Q + D \right) \quad (12a)
\]

\[
I_i = \zeta_i \cdot \left( \frac{d}{dt}Q + D \right) \quad (12b)
\]

where:

- \( k, l \): subscripts denoting capital factor and labor factor respectively
- \( I_i \): Gross investment in the \( i \)th factor (factor units per year)
- \( \zeta_i \): Desired factor ratio of factor \( i \) to output (factor units/output unit)
- \( \frac{d}{dt}Q \): Net change in capacity (output units per year)
Depreciation (or discards) of capital capacity or layoffs/attrition of labor (output units per year)

The desired factor to output ratio $\zeta_j$ can be derived given the costs and the production function that describes how labor and capital can be combined to produce output prior to ordering capital. The desired factor/output ratios are those ratios which minimize the cost of producing a unit of output. More formally, total costs for a given quantity $K$ of capital and $L$ of labor may be written:

$$T = K*C + L*WAGE$$  \hspace{1cm} (13)

where:

- $T$ Total costs (dollars/year)
- $K$ Amount of capital (capital units)
- $C$ Rental cost of capital (dollars/capital units/year) (see equation (10) above)
- $L$ Amount of labor (labor units, i.e., people)
- $WAGE$ Cost of labor (dollars/person/year)

The cost per unit of output ($Q$) is:

$$T/Q = (K/Q)*C + (L/Q)*WAGE$$  \hspace{1cm} (14)

or, using the symbol defined above for the factor/output ratio:

$$T/Q = \zeta_k*C + \zeta_l*WAGE$$  \hspace{1cm} (15)

If one assumes a Cobb-Douglas technology with constant returns to scale, $Q$ may be written:

$$Q = a^*(\zeta_k*Q)\alpha * (\zeta_l*Q)^\beta$$  \hspace{1cm} (16)

$$\alpha = 1 - \beta$$

Solving for $\zeta_l$ yields:

$$\zeta_l = a^{-(1/\beta)}\zeta_k^{-(\alpha/\beta)}$$  \hspace{1cm} (17)

Substituting (17) into (15) gives:

$$T/Q = \zeta_k*C + a^{-(1/\beta)}\zeta_k^{-(\alpha/\beta)}*WAGE$$  \hspace{1cm} (18)

as an expression for cost per unit of output ($Q$).
The value of $\zeta_k$ which minimizes (18) may be found by setting the partial derivative of $T/Q$ with respect to $\zeta_k$ equal to zero and solving for $\zeta_k$. The result is:

$$\zeta_k^* = a^{-1} \left( \frac{(WAGE^*\alpha)}{(C^*\beta)} \right)^\beta$$

(19a)

Similarly, it may be shown that:

$$\zeta_l^* = a^{-1} \left( \frac{(C^*\beta)}{(WAGE^*\alpha)} \right)^\alpha$$

(19b)

**Putty-Putty vs. Putty-Clay.** The putty-putty and putty-clay formulations are closely related. Multiplying the optimal capital to output ratio $\zeta_k^*$ by desired output $Q$ will result in an expression for desired capital stock identical to equation (4a). And similarly, multiplying $\zeta_l^*$ by desired output results in an expression for desired labor identical to (4b).

The difference between putty-putty and putty-clay arises only when one considers how the economy moves from its current stock of, say capital, to its desired stock. And the difference is of significance principally in the case of changing relative factor prices (producing factor balancing investment), rather than in the case of investment designed to increase output. When desired output changes ($Q$ changes in equation (6)), desired investment in both models may rise above the new equilibrium replacement level until the optimal amount of capital for producing the new level of $Q$ is reached at which point investment will have returned to the replacement level. However, when there is a desire to alter factor proportions the dynamic response may differ between putty-putty and putty-clay: In a putty-putty formulation new capital can simply be added to the production process so investment can exceed the new equilibrium replacement level. In contrast, in a putty-clay formulation, which assumes factor-balancing, capital augmentation is accomplished only as capacity depreciates; desired investment will not exceed the new equilibrium replacement level.

A putty-clay formulation will be sketched toward the end of this paper. However, the primary focus of this paper will be on a formulation compatible with the putty-putty production function used in the System Dynamics National Model. A putty-putty production function is justified in models like the System Dynamics National Model by several considerations. First, capital in the National Model aggregates all capital: structures as well as equipment. As Bischoff (1971, p. 23, note 21) notes, structures "are more flexible than equipment, and...capital intensity can be varied both before and after structures are put into place." Consequently, there is some variability ex post in an aggregate function. Further, as Bischoff also notes (1967, p. 64, note 6), machines can be operated up to twenty-four hours a day. That is, a given machine can be combined with
varying amounts of labor to produce varying amounts of output in a day. Finally, although a given machine may be designed to produce at a given rate when operated by a fixed number of workers, companies can purchase different machines if the aggregate cost of equipment drops. For example, a drop in the price of capital might prompt a ship builder to automate its pipe-cutting shop, eliminating the need for most pipe cutters. In brief, it can be argued that capital in the aggregate can be combined with varying amounts of labor (in a day) and that companies can add additional capital ("realized" as additional machines) without replacing existing capital or waiting for it to wear out: Factor proportions are variable to a significant degree ex post as well as ex ante when capital represents the aggregate amount of all equipment and structures in an economy.

Securities value model. Bischoff (1971) classifies several less formally developed models into a class he describes as securities value models. The models are also termed q models after the ratio, termed "q" by Tobin (1969, p. 21), of the market value of capital to its replacement cost. These models are based on the central idea that "investment is stimulated when capital is valued more highly in the market than it costs to produce it, and discouraged when its valuation is less than its replacement cost" (Brainard and Tobin 1968). In theoretical studies, q has been considered to be the ratio of the present value of

7 The ability to operate a machine during several shifts does not precisely give a well-behaved production function if "function" is considered in its strictest meaning: a one-to-one correspondence between inputs and outputs. However, the investment that results is closer to putty-putty than putty-clay. Bischoff suggests that institutional patterns set a "normal" degree of utilization that can be altered only by incurring higher costs. In fact these costs typically increase as utilization increases: Full-time workers are more expensive than part-time workers, and over-time workers (or graveyard-shift workers) are more expensive still. Consider what happens if the price of equipment relative to labor falls and, for the sake of clarity, output is held fixed: Companies will be prompted to expand their least expensive shifts and eliminate some portion of their most expensive shift(s) by purchasing more equipment. In this case, the same amount of labor (measured as manhours per day) is used with an increased amount of capital to produce the same amount of output. However, this situation does not require the discarding of equipment currently in use, and does produce an increased likelihood that investment will rise above the long-term replacement level as in the putty-putty formulation.

8 A model which contains a more disaggregated production function or one focused on the difference between long-run and short-run responses to changes in factor prices might well employ a production function which distinguishes between an ex post and an ex ante production function. A model which focuses on a process in which retrofitting is an important dynamic might explicitly represent the possibility of altering the short-run production function. Sterman (1980) explores the dynamic consequences of successively complicating a production function in these ways.
the marginal revenue product associated with capital to the present value of the marginal
cost of capital (e.g., Abel 1982). This definition of q is quite similar to the benefit cost
ratio, the ratio of the present value of cash inflows to the present value of cash outflow,
which figures prominently in the formulation developed in this paper.

2. Investment Criteria in Practice

Firms approach the decision of acquiring additional factor in several different ways. The evidence we have deals with the acquisition of capital (or possibly projects requiring
both capital and labor) and not with the "acquisition" of people. However, it seems likely
that similar methods are employed to decide whether to take on any factor—purchased or
rented, capital or labor. Consequently, in the absence of evidence to the contrary, I will
assume that the empirical information on the capital acquisition decision is also relevant to
the hiring decision.

The problem is easily stated: Does this investment make sense now? The answer is
difficult because the consequences of the decision extend through time. Purchasing an
extra machine or hiring an extra person will generate a stream of cash inflows and cash
outflows. In the case of a capital investment, a large initial outlay may be followed by a
stream of revenues and replacement expenditures stretching into the distant future. How
can one look at the time profile of inflows and outflows and decide whether the investment
is worthwhile?

The solution used in practice is to convert the stream of payments into a single
number which can be compared against some standard. If the number is on the favorable
side of the standard, the investment is accepted, or at least has passed one hurdle on the
way to acceptance. If the number is on the unfavorable side of the standard, the investment
is not accepted (or at least requires a strong argument based on other factors). The methods
used in practice to convert the stream into a single number for comparison to a standard can
be divided into two classes: payback methods and discounting methods.

Payback Methods. The payback method proper and the simple (or accounting)
rate of return method are quite similar. In the payback method one begins by calculating
the "payback period." The payback period is the number of years until the accumulated
profit resulting from an investment equals the initial outlay for the investment.
Alternatively, the payback period may be defined as the number of years required for the
accumulated average yearly profit to equal the initial outlay. The payback period is then
compared to some standard payback period, say five years. If the investment under study has a payback period less than the standard, the investment is acceptable based on the payback criterion.

The accounting or simple rate of return is the average dollar profit divided by the initial investment. The accounting rate of return is the reciprocal of the payback period, defined as the number of years for average profit to pay back the initial outlay. The accounting rate of return is compared to a standard rate of return, say 20%. If the accounting rate of return on a particular investment is greater than the standard, the project is acceptable by the accounting rate of return method.

**Discounting Approaches.** Fundamental to all discounting approaches is the concept of present value. The present value of an investment is the amount today which could be invested in an equally risky investment, at the market discount rate, to yield the particular series of cash inflows and outflows anticipated for the investment. In this sense the present value, a single number of dollars today, is equivalent to the future series of cash inflows and outflows.

The basic present value formula is:

\[ PV(P) = \int_{0}^{\infty} Pe^{-rt} \, dt \]  

(20)

Where \( PV \) is the present value operator, \( P \) is some stream of cash flows in current dollars, and \( r \) is the nominal interest rate. Expression (20) is an accumulation. The payment at each \( dt \) is weighted by an exponential and summed (i.e., integrated). The exponential, termed the present value factor (PVF), is the solution to the differential equation and initial condition:

\[ \frac{d}{dt}PVF_t = -r*PVF_t \]  

(21a)

\[ PVF_0 = 1 \]  

(21b)

The calculation of accumulating a stream of payments, weighted by the present value factor, may also be represented using system dynamics stock and flow diagrams as in figure 2. The structure could be simulated over the future. The value of the present value accumulator at any future time is the present value of the cash stream up to that time. And the present value is the limiting value of the present value accumulator as time goes to infinity:
\[ PV(P) = \lim_{t \to \infty} (PVA) \] (22)

or,
\[ PV(P) = \int_{0}^{\infty} (1/dt)PVA \, dt \] (23)

or,
\[ PV(P) = \int_{0}^{\infty} P*PVF \, dt \] (24)

or,
\[ PV(P) = \int_{0}^{\infty} P*e^{-rt} \, dt \] (25)

which is the original equation (20).

There are several ways in which the present value calculation can be turned into an investment criteria. The three most popular, and the three which appear in the surveys with the most regularity, are net present value, internal rate of return, and the discounted benefit cost ratio (also called the profitability ratio).

The net present value criterion is the most straight forward. A negative net present value indicates that the projected series of cash inflows and outflows are equivalent to a net loss today. A negative present value means the investment is undesirable. A positive present value, on the other hand, means that the projected series of cash inflows and outflows is equivalent to a gain today. A positive present value indicates one should make the investment. The present value criterion compares the present value of a proposed investment to zero (the standard). If the present value is greater than the standard (i.e., positive), the project is acceptable on present value grounds; if the present value is less than the standard (i.e., negative) the proposed investment is unacceptable on present value grounds.

A closely related criterion relies on the internal rate of return, also called the discounted rate of return. The internal rate of return is that discount rate which, when used to discount all the cash inflows and outflows associated with a proposed investment, yields a net present value of zero. In order to decide whether an investment is worthwhile, the internal rate of return is compared to a market discount rate,\(^9\) the "standard" rate of return.

\(^9\) The market discount rate is an obvious candidate for the standard rate of return. In practice, however, a "hurdle rate," different from the market discount rate may be used by a business. An administered hurdle rate has at least one advantage: Unlike the market discount rate, an administered hurdle rate is changed infrequently. In practice, the administered hurdle rate is often set well above the market discount rate. There are several reasons for this. First, a high administered hurdle rate can counteract the bias toward a slightly unprofitable portfolio inherent in the fact that "truly" profitable projects that are mis-read as unprofitable will not be taken on, but "truly" unprofitable
If the internal rate of return is less than the market discount rate, the proposed investment offers a return less than that demanded in the market and one should not make the investment. If the internal rate of return exceeds the market discount rate, the project should be accepted.

A difficulty with using the internal rate of return is that in some cases the internal rate of return will not be unique and in other cases the internal rate of return will not exist. That is, there may be no discount rate or there may be more than one discount rate which equates a stream of cash flows to a zero present value. This problem is encountered in practice, albeit infrequently (Fremgen 1973, exhibit 3). Presumably, when the problem is encountered, the decision maker switches to another method (such as the present value method) or, when possible, eliminates clearly impossible discount rates (such as negative rates).

A third method, and the one this paper will focus on later, makes use of the discounted benefit cost ratio, also called the profitability index or the present value index. The discounted benefit cost ratio (BCR) is defined as the present value of cash inflows divided by the present value of cash outflows. The investment criterion used with the benefit cost ratio is: Invest if BCR is greater than 1, do not invest if BCR is less than 1. There is a straightforward connection between discounted benefit cost ratio and the present value of an investment opportunity. If BCR is greater than 1, the present value of the inflows exceeds the present value of outflow, and consequently the present value of the project is greater than zero. Conversely, if BCR is less than one, the present value of the proposed investment is negative.

Relationship Between Payback and Discounting. The discounting methods and the payback methods are broadly similar in intent. They attempt to translate flows of cash into a single number and compare that number to a standard. They attempt to

projects that are mis-read as profitable will be taken on. Second, the high administered rate can reflect a bias toward inaction which is discussed in the behavioral decision theory literature (Kahneman and Tversky 1982). Finally, a high administered rate can counteract an inherent tendency toward optimism on the part of supporters of a project.

For example, a project that costs $4,000 and returns $25,000 in the second period and costs $25,000 in the third period (when, perhaps, it must be dismantled and removed) has two internal rates of return: 25% and 400% (this example from Brealey and Myers 1981, pp. 71-72). As another example consider an investment that costs $1,000 returns $1,000 in the next period and costs another $1,000 in the third period; this project does not possess a (real) internal rate of return.
compress the entire financial time-profile of an investment into a single measure. That measure is intended to trade off two components: profitability and time. The more profitable and the sooner the profitability is realized, the better the investment. The discounting approaches have a more explicit theoretical grounding, the payback methods are simpler to use.

There is no exact correspondence between the payback and discounting methods. However, the methods are equivalent in the case of an investment characterized by a single initial outlay followed by a perpetual and constant stream of cash inflows. In such a case, the present value becomes:

\[ PV_0 = -I_0 + \frac{M}{r} \]  \hspace{1cm} (26)

where:
- \( PV_0 \) is the present value at time 0
- \( I_0 \) is the initial investment made at time 0
- \( M \) is the constant incoming stream of cash
- \( r \) is the discount rate.

To find the internal rate of return, one solves for an \( r \) that makes the present value zero. Making appropriate substitutions in the equation above yields:

\[ r = \frac{M}{I_0} \]  \hspace{1cm} (27)

which is also the formula for the accounting rate of return.

The benefit cost ratio of the investment is:

\[ BCR = \frac{M}{(r*I_0)} \]  \hspace{1cm} (28)

The BCR investment criteria involves determining whether BCR is greater than or less than 1:

\[ BCR = \frac{M}{(r*I_0)} < 1 \]  \hspace{1cm} (29)

or equivalently:

\[ \frac{1}{(BCR*r)} = \frac{I_0}{M} < \frac{1}{r} \]  \hspace{1cm} (30)

Hence, the inverse of the benefit cost ratio multiplied by the market discount rate is the payback period of the project. The project is acceptable if the payback period is less than the inverse of the market discount rate.

The several methods for making an investment decision converge as the project gets close to a project characterized by a single initial cash outlay followed by a constant stream
of returns forever. In such a situation, the internal rate of return will equal the accounting rate of return and the payback period of the project will equal the inverse of the benefit cost ratio multiplied by the discount rate.

Empirical Studies of Investment Criteria. A number of surveys have been conducted on capital budgeting techniques (Istvan 1961, Klammer 1972, Fremgen 1973, Schall et al 1978, and Naugle 1980). The general conclusion reached in these surveys is that discounting approaches and payback approaches are in common use. Although many companies use both approaches, companies appear to be shifting toward discounting and away from payback. Table 1 summarizes the results of the surveys.

Care must be taken in making comparisons across surveys. Klammer, Fremgen, and Naugle collected data both on which methods were in use and which methods were most important or primary. These breakdowns have been indicated. Both Istvan and Fremgen apparently insisted that companies pick only one method as the most important method. In contrast, in Klammer's and Naugle's surveys companies were free to indicate several methods as being most important.

The results reported in table 1 are probably biased away from the "other" category because questions in the survey specifically requested information on the the established methods. There may also be a bias toward reporting the use of the discounting methods since these methods may be perceived by respondents as being more sophisticated and more highly prized by the researchers. While one might mentally increase the size of the "other" category and slightly decrease the reported use of discounting methods, it is probably still correct to conclude that payback, simple rate of return, and the discounting methods are popular and that the discounting methods are growing in popularity.

Another potential objection to the results of the surveys is that people only use present values as a language to argue for gut feelings about the desirability of a proposed investment (Black 1983). While this argument may caution against a facile acceptance of corporate responses to surveys, Black's suggestion does not, in the final analysis, invalidate the conclusion that a present value calculation captures an important part of the the investment decision at many corporations. First, if people use present values as a language for disputation, it is likely that they believe this language allows them to capture the meaning of a good investment opportunity. Hence, it is likely that their "gut feeling" is the feeling that a particular investment opportunity has a positive net present value. This feeling may be colored by additional considerations such as a desire to build up one's own
department, a fear of failure, or an aura of appeal surrounding a particular product. However, these additional considerations are important in the aggregate primarily as sources of noise in the determination of present values.\textsuperscript{11} Second, if people use the terminology of present value analysis to couch their arguments, arguments showing large present values must be persuasive. Favored-by-the-gut projects which require unreasonable assumptions to show high net present values will face tough sledding in the approval process. Hence, there will be a bias toward accepting high net-present value projects. For these reasons it is likely that the survey evidence cited above actually does support the widespread use of present values to determine investments.

3. Initial Considerations in Model Formulation

Discounting vs. Payback Methods. The discounting approach is the same as the payback approach only under the rather special circumstances where a single initial outlay is followed by a perpetual and constant stream of cash inflows (see above). Outside this rather special case, the two approaches differ principally because the payback approach does not explicitly take account of interest rates.

Yet this difference is probably less salient in practice than it is in theory. Businessmen who use the payback approach almost certainly consider interest rates when they consider the cost of financing the proposed factor stock. It is likely that when financing considerations are added to the payback method, decisions based on that criteria will not be significantly different from decisions based on the discounting approach. Consequently the added complexity of explicitly representing both approaches in a macro-economic model is probably unwarranted.

If one method is to be incorporated into the model, the discounting method is preferable for two reasons. First, the discounting method provides clear guidance concerning how interest rates are folded into the factor acquisition decision. Second, the discounting methods convert cash flows into equivalent stocks. Consequently, discounting methods are as applicable to a rent (hiring) decision as they are to a purchase decision. Payback methods, on the other hand, are applicable only to a purchase decision where there is a large initial outlay. If a discounting method is used, the same criterion can be applied

\textsuperscript{11} Each individual factor may have a systematic bias. Empire building might tend to inflate present values, for example, and a fear of failure might tend to deflate present values. But, the net impact of all factors cannot be predicted from theoretical considerations alone and must await empirical investigation.
by the modeler to factor acquisition decisions whether the factor is rented or purchased. In the model formulation presented here we will assume a discounting approach.

**Desirability of BCR.** Of the discounting methods, the benefit cost ratio BCR is the most convenient for the purpose at hand. The desirability of the benefit cost ratio is less apparent at the micro level than the macro level. At a micro-decision level a potential investment is usually taken as predefined—for example, is it justifiable to buy a mainframe computer—and the decision making machinery is used to decide yes or no. In this context either the net present value or the benefit cost ratio are equally acceptable criteria.

In an aggregate model, however, a broader and more continuous view is appropriate. When speaking of the profitability of adding factor in a sector we are speaking of the profitability of the *average* opportunity involving that factor, where some opportunities will be barely profitable, some just barely unprofitable, some very profitable and some very unprofitable. A large measure of profitability in an aggregate sector means that there are many potentially profitable ways to utilize the factor in question in the sector, and those profitable opportunities are very profitable. The more profitable a factor is on average in a sector, the faster firms in the sector will add the factor.

This suggests we need not merely a measure of *whether* an investment is profitable, but a measure of *how* profitable. The benefit cost ratio is the appropriate measure here. The present value criterion, with units of dollars, is sensitive to the units of measure—the price level as well as the "unit" of capital or labor—and, therefore is not appropriate. For example a present value of a thousand dollars might indicate an extremely profitable factor if the present value of revenues is one thousand dollars and there are no costs, but a barely profitable factor, quite sensitive to assumptions, if the present value of revenues is ten million dollars and the present value of expenses is 9,999,000 dollars. The internal rate of return is unacceptable because in some situations it may not exist or it may not be unique. The benefit cost ratio, in contrast to the other alternatives, is entirely acceptable: It measures the relative margin of profitability and is not sensitive to scale or price level. A large benefit cost ratio indicates that the average investment is profitable within relatively wide ranges of assumptions regarding future prices and interest rates.

**Relevant Cash Flows, and Relevant Structure:** The "Analyst's" Model. In the real world there are many cash flows associated with an investment and a wealth of structure determining the timing of the cash flows. The various cash flows and structures are not equally salient. For example, revenue generated directly by a new
investment is clearly a significant cash inflow, but cash generated by increased economy-wide demand occasioned by hiring a worker from the pool of the unemployed is not. The delay between purchasing a new chemical factory and actually producing the first new chemicals is an important structural consideration, but the delay involved in converting an account receivable into cash may not be.

In the real world, boundedly rational analysts do not take account of every cash flow, nor are the structures they posit to determine cash-flow timing as complicated as the corresponding real-world structures. Instead, analysts use a mental model that corresponds to the actual situation but which is simpler. The factor acquisition decision in a well-formulated macro-economic model does not (in fact, should not) need to take account of every cash flow and all the structure of the macro-economic model. Rather, the factor acquisition decision should take account of a simplified version of the structure within a macro-economic model. The simplified version can be formalized as a model of the macro-economic model. The model-of-the-macro-model, termed the "analyst's model" in this paper, corresponds to the mental models of decision makers in the real world.

The factor acquisition decision derived in this paper is based on an "analyst's model" of the System Dynamics National Model (SDNM) under development at M.I.T. The cash flows represented within the analyst's model are the principle ones generated by the SDNM: revenues, purchase price and replacement expense in the case of a purchased factor, rental cost in the case of a rented factor, tax payments on income, and the investment tax credit. Similarly, the structure of the analyst's model will be composed of the major aspects of the cash-flow determining structure of the SDNM. The most relevant aspects of the SDNM include a two-stage production process, exponential real-depreciation of purchased factors, and exponential tax-depreciation (at possibly different rates). I will not consider modifications to the timing of cash flows introduced by accounts payable and receivable or by the needed accumulation of cash balances, even though these structures are included in the SDNM. These modifications are minor, and in the case of payables and receivables at least partially offsetting.

The Production Process. Stock and flow diagrams are useful for portraying the analyst's model graphically.\textsuperscript{12} Figure 3 presents a diagram of the analyst's model of

\textsuperscript{12} Stock and flow diagrams are in common use in system dynamics. They provide a close pictorial representation of a system of integral equations. In the above diagram, boxes represent integrations. Heavy arrows represent rates of flow (derivatives) which are
the production process. The figure can be viewed as part of the mental image a typical analyst has when considering the impact of increasing productive factors. Production is started, but time is required to finish it. Represented in the figure is a two stage production process where production is started and semi-finished goods accumulate in an inventory in process. After a delay, the goods are finished and put into finished goods inventory. After a further delay, the goods are sold. In the real economy, the production and distribution of manufactured goods involves more stages. Most manufacturing companies have at least these two stages of production. In addition, there are suppliers and distributors which add their own stages of production. The National Model employs two stages as an approximation.\textsuperscript{13} The impact on the BCR calculation of adding additional stages will be made clear below.

Although production starts depend upon labor and capital, the conversion of in-process inventory to finished production depends only upon the passage of time. In reality most products do require labor and capital inputs all through the production process and the assumption in the model is most accurate only for a rather small group of products, such as cheese, where aging (and bacteria) convert semi-finished products into finished products without significant human intervention. The assumption of time-dependent and factor-independent mid-stage production corresponds to the structure in the SD\textsuperscript{NM} and is intended to achieve approximately the correct delay at only a small sacrifice in terms of added complexity.

Revenues are determined by the product of shipments and the output price. The output price is a level. Price change is equal to the current price multiplied by the expected inflation rate. The analyst makes his calculations assuming that inflation will be constant in the future.

\textbf{Labor Factor.} Figure 4 combines the production process into a more complete representation of the analyst’s model of the consequences of hiring an additional unit of a rented factor such as labor. Labor is a stock, an accumulation. The diagram suggests that the "analyst in the model" visualizes an addition to labor as a stock having an attrition rate controlled by hour glass shapes representing valves. Lighter arrows represent information connections and reveal the arguments of each variable. For discussions of diagramming conventions in system dynamics see Richardson and Pugh (1981, chapters 1-2) and Morecroft (1982).

\textsuperscript{13} See Forrester 1961, appendix H, for a discussion and illustration of the impact of changing the order of the production delay.
and a replacement rate. It is assumed that the rate of replacement equals the attrition rate. Consequently, it is assumed that once a person is hired, he (or his replacement) remains.

Labor is combined with capital to produce goods. The "analyst's" perspective does not encompass the entire macro-economic model. He considers the acquisition of labor separately from the acquisition of capital. Therefore, he is concerned with how much additional production will be achieved by adding a marginal unit of additional labor to the existing capital stock. The amount of production starts generated by a marginal amount of labor will be determined by the marginal productivity of labor.

The net cash flow for labor is revenues less income tax expense less wage expense. Wage expense is the labor stock multiplied by the wage. As in the case of prices, wage inflation is assumed to be constant in the derivation. Income tax expense is assumed to be taxable net income from labor multiplied by the corporate tax rate. Taxable net income for labor is simply revenues less wage expense.

Capital Factor. Figure 5 presents an analyst's mental model of the cash flow consequences of adding additional capital (or purchased factor). Much of the analyst's model is the same as for labor: Production starts, production delays, revenues and prices are all handled similarly. The differences occur in considering replacement expense and income tax expense.

It is assumed that capital depreciates at a constant fractional rate; this corresponds to an assumption that the distribution of the time to obsolescence of the capital stock is exponential with a constant average. This assumption, for an aggregate model, is common in econometric work and in system dynamics studies \(^{14}\) and has been defended by Jorgenson (1963, p. 251) and Bischoff (1967 p. 73, note 20) among others.

We will assume that the "analyst" expects the firm to replace physically depreciating capital. The assumption is made on empirical grounds. There is a tendency in practice to exempt replacement expenditures from formal investment criteria or, at least, to consider

---

\(^{14}\) There are exceptions in both economic and system dynamics modeling: Ando, Modigliani, Rasche, and Turnovsky (1974), for example, assume that the service life of capital is itself a decision variable and an integral part of the investment decision.
replacement expenditures separately and presumably more leniently. Consequently, in most companies, analysts should assume replacements will be made.

Replacing depreciating equipment will give rise to two additional effects. First, replacements will qualify for investment tax credits. Here it is assumed that the analyst expects the investment tax credit to remain fixed. Second, replacement expense will affect the depreciation expense recognized by the taxing authorities. The value of the replacement expenditures will be capitalized on the tax books by increasing the tax book value of capital.

The tax book value of capital will be decreased by the tax depreciation expense. The taxing authority may permit the tax book value of capital to be written off at an accelerated rate. It is assumed that the analyst assumes the accelerated rate will remain constant in the future. The tax depreciation expense will affect taxable net income which determines income tax expense. Taxable net income is revenues less tax depreciation expense. Income tax expense is calculated by applying the corporate tax rate to taxable net income.

4. Derivation of the Benefit Cost Ratio

This section will move from the general discussion of how cash flow is visualized to a detailed derivation of the benefit cost ratio. The derivation is complicated by the desire to preserve the proper timing of the cash flows and to properly account for inflation. The end result will be a little more complicated than Hall and Jorgenson's equations. The value of spending time on the derivation and of the slightly more complicated expressions that result is the increased understanding of how timing and inflation affect the factor acquisition decision.

The benefit cost ratio is the ratio between the present value of cash inflows and the present value of cash outflows. The basic cash flow equations for labor (rented) factors and purchased (capital) factors are:

---

Klammer (1972) notes that replacement proposals are not subjected to estimates of profit contributions; Fremgen (1973) reports that 79% of the companies in his survey approve projects which fail to meet established financial criteria but which are necessary for maintaining existing programs or product lines. About a quarter of the companies in Naugle's (1980) sample report they apply separate formal criteria to replacement expenditures. Further, many of Naugle's companies reported they do not perform any analysis on certain projects which "were required to keep a large facility in operation."
\[ NCF_L = REV_L - WE_L - ITE_L \]
\[ NCF_C = REV_C - RE_C - ITE_C + ITC_C \]  \hspace{1cm} (31a, 31b)

where:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NCF</td>
<td>Net Cash Flow ($/factor unit/year)</td>
</tr>
<tr>
<td>C,L</td>
<td>Subscripts representing capital (purchased) and labor (rented) factors respectively</td>
</tr>
<tr>
<td>REV</td>
<td>Revenues ($/factor unit/year)</td>
</tr>
<tr>
<td>WE</td>
<td>Wage Expense ($/factor unit/year)</td>
</tr>
<tr>
<td>ITE</td>
<td>Income Tax Expense ($/factor unit/year)</td>
</tr>
<tr>
<td>RE</td>
<td>Replacement Expense ($/factor unit/year)</td>
</tr>
<tr>
<td>ITC</td>
<td>Investment Tax Credit ($/factor unit/year)</td>
</tr>
</tbody>
</table>

Making use of the linearity of the present value operator one can write:

\[ PV(NCF_L) = PV(REV_L) - PV(WE_L) - PV(ITE_L) \]  \hspace{1cm} (32a)
\[ PV(NCF_C) = PV(REV_C) - PV(RE_C) - PV(ITE_C) + PV(ITC_C) - PV(IC) \]  \hspace{1cm} (32b)

where \( PV \) is the operation of taking present values. In equation (32b) an additional term, \( PV(IC) \), has been added to represent the present value of the initial cost of an investment. Further, the investment tax credit in that equation now represents the credit resulting from the initial cost of a capital unit as well as the continuing credit resulting from replacement expenditures.

The benefit cost ratio is formed from the terms on the right hand sides of equations (32a) and (32b), each term representing the present value of a particular cash flow. We will begin the derivation by calculating the present value of each flow. These present values will then be used to construct the benefit cost ratios for rented and purchased factors.

**Present Value of Revenues Attributable to Labor.** Figure 6 presents an analyst's model for the calculation of present value of revenues. Figure 6 shows the detail of the revenue generating structure assumed in the derivation below. Attached to the revenue generating structure is the structure introduced in figure 2 for calculating the present value. (Expected) revenues are weighted by the present value factor and accumulated over future time. The structure diagrammed in figure 6 is capable of numerically generating the present value of revenues. In theory, it would be possible to place a subroutine containing the structure of figure 6 within a larger macro-economic model: at each "instant" (i.e., at each \( dt \)) in the main macro-model's simulation, the structure corresponding to figure 6 would be simulated over a sufficiently long time to produce the analyst's estimate of the present value at that instant. Because such a
simulation within the main macro-model would be very time consuming, it is preferable to use figure 6 as a guide for deriving an analytical expression for the analyst's estimate of the present value.

It is desirable to calculate the present value of revenues per unit of labor (at the margin). The structure of figure 6 will result in such a calculation. As a check, note that the marginal productivity of labor is the production per unit of labor at the margin. Consequently, the units of measure for the two inventories will be goods per unit of labor at the margin, and the units in the present value accumulator will be dollars per unit of labor at the margin as desired.

The present value of revenues \( PV(\text{REV}) \) is the value of the present value accumulator as time goes to infinity. The present value accumulator accumulates revenues weighted by the present value factor. Revenues are shipments (SHIP) multiplied by price of output (OP) and the present value factor is the exponential decay, so \( PV(\text{REV}) \) can be written as:

\[
PV(\text{REV}) = \int_{0}^{\infty} \text{SHIP} \times \text{OP} \times e^{-rt} \, dt \tag{33}
\]

Output price in this derivation is assumed to inflate or deflate at a constant price inflation rate (pir):

\[
(d/dt)\text{OP} = \text{pir} \times \text{OP} \tag{34}
\]

which may be solved to yield:

\[
\text{OP} = \text{OP}_0 \times e^{\text{pir} \times t} \tag{35}
\]

where \( \text{OP}_0 \) is the price of output currently.

The expected shipment rate in normal times will be determined by finished goods inventories (I) and by the normal inventory coverage (nic):

\[
\text{nic} = I / \text{SHIP} \tag{36}
\]

or, rearranging:

\[
\text{SHIP} = I / \text{nic} \tag{37}
\]

Substituting (35) and (37) into (38) yields:

\[
PV(\text{REV}) = \int_{0}^{\infty} \left( I / \text{nic} \right) \times \text{OP}_0 \times e^{\text{pir} \times t} \times e^{-rt} \, dt \tag{38}
\]
Finished goods inventory \((I)\) is an accumulation whose differential equation may be solved. Finished goods inventory is increased by production finishes \((\text{PRODF})\) and depleted by shipments \((\text{SHIP})\). Hence the differential equation for finished goods inventory is:

\[
\frac{d}{dt}I = \text{PRODF} - \text{SHIP}
\]  

(39)

It is assumed that production finishing is a process that empties inventory in process \((\text{IIP})\). The resident time in in-process inventory is assumed to be distributed exponentially with a constant expected resident time in in-process inventory \((\text{tiip})\). This means that \(\text{PRODF}\) is given by:

\[
\text{PRODF} = \frac{\text{IIP}}{\text{tiip}}
\]

(40)

Inventory in process is an accumulation similar to finished goods inventory. Inventory in process is increased by production starts \((\text{PRODS})\) and decreased by production finishes \((\text{PRODF})\). Mathematically,

\[
\frac{d}{dt}\text{IIP} = \text{PRODS} - \text{PRODF}
\]

(41)

Solving the above differential equation yields:

\[
\text{IIP} = \text{pmpf}^*\text{tiip}^*(1 - e^{-t/\text{tiip}})
\]

(42)

where use has been made of the fact that production starts are equal to the perceived marginal productivity of factor \((\text{pmpf})\) and the fact that inventory in process resulting from the acquisition of a new factor unit is initially zero. Substituting equation (42) into equation (40) and the result into equation (39) and solving (using equation (37)) gives:

\[
I_t = \text{pmpf}^*\text{nic}(1 - e^{-t/\text{nic}}) - \{(\text{pmpf}^*\text{ni}(1/\text{nic}) - 1/\text{tiip})\}^*(1 - e^{-t/\text{tiip}})
\]

(43)

where use has been made of the fact that the initial value of inventory \((I)\) resulting from the acquisition of a new factor unit is zero. Substituting (43) into (38) gives:

\[
\text{PV}(\text{REV}_L) = \text{OP0}^*\text{pmpf}/\{\text{rdrr}(\text{rdrr}\text{tiip} + 1)(\text{rdrr}\text{nic} + 1)\}
\]

(44)

where \(\text{rdrr}\) is the real discount rate for revenues and is equal to the nominal discount rate \((r)\) less the output price inflation rate \((orp)\).

Note that the proper real discount rate to use in calculating the present value of revenues is one corrected for inflation in output prices, not factor prices or a broader measure such as the consumer price index (CPI). However, Naugle (1975) presents evidence that inflation is not accounted for very precisely (or properly) in practice, hence,
one might want to avoid the artificial precision of using opir, using instead a broad measure such as the consumer price index. 16

Equation (44) for the present value of revenues may be decomposed into three parts: (1) the present value of revenues before delays (i.e., what the present value of sales would be were there no delays at all between manufacturing the product and selling it), (2) the effect of the delay through inventory in process, and (3) the effect of the delay through finished goods inventory. The present value of revenues before delays (PVRBD) is the well-known formula for the present value of a perpetual stream where the perpetual stream is the price (OP0) multiplied by the output flowing from a unit of labor input (pmpf):

\[ \text{PVRBD} = \text{OP0} \cdot \text{pmpf} / r \text{drr} \]  

(45)

The effect of delay through inventory in process (EDIIP) and the effect of the delay through finished inventory (EDFI) are:

\[ \text{EDIIP} = 1 / (\text{rdrr} \cdot \text{tii} + 1) \]  

(46)

\[ \text{EDFI} = 1 / (\text{rdrr} \cdot \text{nic} + 1) \]  

(47)

It is clear that assuming additional delays in the production process would involve adding additional terms of the form \(1 / (\text{rdrr} \cdot \text{average delay time} + 1)\).

The present value of revenues can now be rewritten:

\[ PV(\text{REV}_{L}) = \text{PVRBD} \cdot \text{EDIIP} \cdot \text{EDFI} \]  

(48)

Note that the effects of the delays is to decrease the present value of revenues. As either the time spent in in-process inventory (tii) or the time spend in finished goods inventory (nic) lengthens, the present value of revenues declines because the manufacturer gets his revenue later. In the limit when either tii or nic get infinitely long, the manufacturer would never sell his output and the present value would be zero.

**Present Value of Labor Costs.** There are two cost terms for labor: wage expense and tax expense. Figure 7 presents a structure capable of solving numerically the

---

16 Naugle (1975) uncovered widespread improper adjustment for inflation among the firms he surveyed. By not taking the improper adjustments into account we essentially are assuming that the improper adjustments are distributed evenly about the actual adjustment. Naugle states that "although the net bias which results from these errors is not completely clear, indications are that it would be towards overinvestment" during inflationary times (p. 2). The impacts of a systematic bias are beyond the scope of the present essay but warrant consideration in the context of a larger macro-economic model.
present value of wage expense per factor unit ($PV(\text{WE})$). The present value of wage expense $PV(\text{WE})$ is equal to the limiting value of the present value accumulator as time goes to infinity. The present value accumulator accumulates the rental cost per unit of factor, weighted by the present value factor. The present value factor is the exponential decay and the wage is an exponential growth (or decay) process completely analogous to equation (35) for the output price. Consequently,

$$PV(\text{WE}) = \text{WAGE}_0 \ast \int_{0}^{\infty} e^{(r-wir)t} \ dt$$  \hspace{1cm} (49)

or,

$$PV(\text{WE}) = \text{WAGE}_0/(r-wir)$$  \hspace{1cm} (50)

or,

$$PV(\text{WE}) = \text{WAGE}_0/rdrwe$$  \hspace{1cm} (51)

Where rdrwe is the real discount rate for wage expense and is equal to the nominal discount rate less the wage inflation rate ($rdrwe = r - wir$).

Note that the relevant real discount rate in equation (51) is the nominal rate less the inflation in wages. As before, one may wish to avoid the artificial precision inherent in deflating by the wage inflation rate and simply use a broad measure, such as the consumer price index.

The remaining present value term for labor is the present value of income tax payments. Tax payments per unit of additional labor are determined by applying the marginal tax rate to the net income recognized by the tax authority per unit of additional labor. In the model, net income per unit of additional labor is defined as revenues (REV) attributable to the additional labor unit less wage expense (WE) per additional unit of labor. Making use of the linearity of the present value operator one can write:

$$PV(\text{ITE}_L) = \text{citr}*PV(\text{REV}_L) - \text{citr}*PV(\text{WE})$$  \hspace{1cm} (52)

Where $PV(\text{REV})$ is the present value of revenues defined in equations (44) and (48) and $PV(\text{WE})$ is the present value of wage expense defined in equation (51).

Let us bring the foregoing into a formulation for the benefit cost ratio of labor $\text{BCR}_L$. First, substituting (52) into (32a) yields:

$$PV(\text{NCF}_L) = PV(\text{REV}_L) - PV(\text{WE}) - \text{citr}*PV(\text{REV}_L) + \text{citr}*PV(\text{WE})$$  \hspace{1cm} (53)

The equation above can be translated into a benefit cost ratio in several ways. I shall use a benefit cost ratio that facilitates the derivation of desired factor stocks below and also facilitates the comparison to Hall and Jorgenson's work. To wit: (46) can be rearranged
as:

\[ PV(\text{NCF}_L) = (1 - \text{citr}) * PV(\text{REV}_L) - (1 - \text{citr}) * PV(\text{WE}) \]  \hspace{1cm} (54)

Defining the first term on the right as the present value of cash inflows and the second term as the present value of cash outflows yields the following definition of BCR:

\[ \text{BCR}_L = (1 - \text{citr}) * PV(\text{REV}_L) / (1 - \text{citr}) * PV(\text{WE}) \]  \hspace{1cm} (55)

Substituting using equations (44) and (51) and cancelling the tax effect on revenues and wages yields:

\[ \text{BCR}_L = \text{OP}_0 * \text{pmpf}_f / \{ WAGE_0 (rdrr/rdrwe)(rdrr*tiip+1)(rdrr*nic+1) \} \]  \hspace{1cm} (56)

where:

- \( \text{BCR}_L \)Benefit Cost Ratio for Labor (dimensionless)
- \( \text{citr} \)Corporate Income Tax Rate (dimensionless)
- \( \text{OP}_0 \)Current Price of Output ($/Output Unit)
- \( \text{pmpf} \)Perceived Marginal Productivity of Factor (Output Units/Factor Unit/Year)
- \( \text{rdrr} \)Real Discount Rate for Revenues (rdrr = r - opir) (1/year)
- \( \text{rdrwe} \)Real Discount Rate for Wage Expense (rdrwe = r - wir) (1/year)
- \( \text{tiip} \)Time for Inventory in Process (years)
- \( \text{nic} \)Normal Inventory Coverage (years)
- \( \text{citr} \)Corporate Income Tax Rate (dimensionless)
- \( \text{WAGE}_0 \)current Wage Rate ($/Factor Unit/Year)

Note that only real interest rates affect the benefit cost ratio for labor. More particularly, note that the second term in parenthesis in the denominator shows that profitability increases as the real discount rate for wages increases relative to the real discount rate for revenues—that is, profitability increases as inflation in output prices exceeds the inflation in wages prices. This affect diminishes as the nominal interest rate increases relative to the inflation terms. The final two terms in the denominator reveal how real interest rates decrease the profitability of labor via the production delays.

**Capital (Purchased Factors).** Equation (32b), reproduced immediately below (as equation (57)) for convenience, defines the net present value of a capital factor. As before, I will first derive expressions for each of the present value terms on the right and will then combine them into a benefit cost ratio:

\[ PV(\text{NCF}_C) = PV(\text{REV}_C) - PV(\text{RE}_C) - PV(\text{ITE}_C) + PV(\text{ITC}_C) - PV(\text{IC}) \]  \hspace{1cm} (57)
where:

\[ \begin{align*}
PV & \quad \text{Present Value Operator} \\
NCF & \quad \text{Net Cash Flow ($/factor unit)} \\
C & \quad \text{Subscript representing capital (purchased) factor} \\
REV & \quad \text{Revenues ($/factor unit)} \\
RE & \quad \text{Replacement Expense ($/factor unit)} \\
ITE & \quad \text{Income Tax Expense ($/factor unit)} \\
ITC & \quad \text{Investment Tax Credit ($/factor unit)} \\
IC & \quad \text{Initial Cost ($/factor unit)}
\end{align*} \]

The calculation of the present value of revenues for a capital unit is identical to that for a labor unit; the only difference being that production starts are equal to the perceived marginal productivity of capital factor \( (\text{pmpf}_C) \), rather than labor factor. Consequently, making the appropriate subscript modifications to equation (44) gives:

\[
P_V(REV_C) = OP_0 \times \text{pmpf}_C / \{ r_{drr}(r_{drr} \times tiip + 1)(r_{drr} \times nic + 1) \}
\]

where:

\[
\begin{align*}
OP_0 & \quad \text{Current Output Price ($/output unit)} \\
\text{pmpf}_C & \quad \text{Perceived Marginal Productivity of Factor (Capital)} \\
& \quad \text{(output units/factor unit/year)} \\
r_{drr} & \quad \text{Real Discount Rate for Revenue (1/year)} \quad (r_{drr} = r - \text{opir}) \\
tiip & \quad \text{Time for in Process Inventory (years)} \\
nic & \quad \text{Normal Inventory Coverage (years)} \\
r & \quad \text{Nominal Discount Rate (1/year)} \\
\text{opir} & \quad \text{Output Price Inflation Rate (1/years)}
\end{align*}
\]

Again, the equation above can be seen as the product of the present value of revenues before delays \( (PVRBD) \) and the effect of delay in in-process inventory \( (EDII) \) and the effect of the delay in finished goods inventory \( (EDFI) \):

\[
P_V(REV_C) = PVRBD_C \times EDII \times EDFI
\]

\[
PVRBD_C = OP_0 \times \text{pmpf}_C / r_{drr}
\]

\[
EDII = 1 / (tiip \times r_{drr} + 1)
\]

\[
EDFI = 1 / (nic \times r_{drr} + 1)
\]

The present value of the initial cost per unit of capital factor is simply the current price of capital factor \( (\text{PCAP}_0) \). Because the expense is incurred immediately, there is no discounting involved. Some prefer to think of the expense of acquiring a unit of capital as the appropriate interest rate (discount rate) multiplied by the price of the capital unit. That is, one imagines having to finance the purchase through debt and equity. The cost is
constant and perpetual. Converting to present values, using the well known formula for a
perpetuity, gives the same result: the present value of the initial cost per unit is the price of
capital factor.

The present value of replacement expense is a bit more complicated. Figure 8
presents a structure, corresponding to the analyst's model, which could be used for the
numerical calculation of the present value of replacement expense. The diagram will guide
us in the derivation below. The present value calculation involves weighting the flow of
replacement expense by the present value factor and accumulating the result in the present
value accumulator. The present value of replacement expense (PV(RE)) is equal to the
limiting value of the present value accumulator as time approaches infinity:

\[ PV(RE) = \int_{t=0}^{\infty} RE \cdot PVF \, dt \]  \hspace{1cm} (63)

The present value factor is the familiar exponential decay. Replacement expense per unit of
additional factor (RE) is the physical replacement per unit of factor multiplied by the price
of capital (PCAP). Because we assume that the analyst expects physical replacement to
equal physical depreciation, the replacement per unit of factor per year is equal to the
physical depreciation rate (pdr). Consequently,

\[ PV(RE) = \int_{t=0}^{\infty} PCAP \cdot pdr \cdot e^{-rt} \, dt. \]  \hspace{1cm} (64)

The price of capital is assumed to inflate or deflate at a constant rate (cpi). The structure
determining the price of capital is identical to that determining the price of output (equation
(35)), and consequently one can immediately write:

\[ PCAP = PCAP_0 e^{cpi \cdot t} \]  \hspace{1cm} (65)

where PCAP_0 is the current price of capital. Substituting into equation (64) and solving
yields:

\[ PV(RE) = pdr \cdot PCAP_0 / (r - cpi) \]  \hspace{1cm} (66)

Or,

\[ PV(RE) = pdr \cdot PCAP_0 / r \cdot dre \]  \hspace{1cm} (67)

where rdre is the real discount rate for replacement expense.

The equation above is in the form of the formula for a perpetual cash stream. Given
the assumption that firms anticipate replacing depreciated assets, the cost of replacing those
assets per unit of capital (pdr*PCAP_0) is a perpetual stream. Note that the real discount
rate for replacement expense employs a measure of inflation different from that used in the
real discount rate for revenues and different too from that used in the real discount rate for
wage expense; but as before it may be appropriate to use a broader measure of inflation to
avoid an artificial precision in the formulation.

Income tax expense associated with the acquisition of capital (ITEC) is equal to the
tax rate (citr) multiplied by the net income attributable to acquired factor and recognized by
the tax authority. In our model taxable net income is revenue (REV) less tax depreciation
expense (TDE) which may differ from the expense of physical depreciation. The present
value of income tax expense for a capital factor may be written:

\[ PV(ITEC) = PV(citr \cdot (REV - TDE)) \quad (68) \]

Where \( PV \) is the present value operator. The linearity of \( PV \) allows one to rewrite the
above equation as:

\[ PV(ITEC) = citr \cdot PV(REV) - PV(citr \cdot TDE) \quad (69) \]

or,

\[ PV(ITEC) = citr \cdot PV(REV) - PV(DTS) \quad (70) \]

Where \( PV(REV) \) has been defined previously (equation (58) or (59)) and where the second
term on the right has been renamed the present value of the depreciation tax shield (DTS).

Figure 9 shows a structure which corresponds to a part of the analyst’s model and
which is capable of numerically calculating the present value of the depreciation tax shield.
Here, we solve the problem analytically. The present value accumulator accumulates the
depreciation tax shield (DTS), weighted by the present value factor (PVF):

\[ PV(DTS) = \int_{0}^{\infty} DTS \cdot PVF \, dt \quad (71) \]

The depreciation tax shield is the tax depreciation expense (TDE) multiplied by the
corporate income tax rate (citr):

\[ PV(DTS) = \int_{0}^{\infty} citr \cdot TDE \cdot PVF \, dt \quad (72) \]

The present value factor is the familiar exponential decay, and tax depreciation expense is
tax depreciation rate (tdr) multiplied by the tax book value of capital (TVC):

\[ PV(DTS) = \int_{0}^{\infty} citr \cdot TVC \cdot tdr \cdot e^{-rt} \, dt \quad (73) \]

Tax depreciation is a constant multiplied by the tax book value of capital. The
formulation is a declining balance method. To make this more obvious, we set the tax
depreciation rate (tdr) equal to the product of the physical depreciation rate and a coefficient
for accelerated depreciation (cad). A coefficient for accelerated depreciation equal to two (cad=2) would indicate double declining balance tax depreciation: 17

\[ PV(DTS) = \int_{0}^{\infty} c_i(t) \cdot TVC(t) \cdot (pdr \cdot cad) \cdot e^{-rt} \cdot dt \]  

(74)

The tax book value of capital (TVC) is an accumulation which is increased by replacement expense (RE) and decreased by tax depreciation expense (TDE):

\[ \frac{d}{dt}TVC = RE - TDE \]  

(75)

Replacement expense is the physical depreciation rate (pdr) multiplied by the price of capital (PCAP), and the tax depreciation expense is the tax book value multiplied by the tax depreciation rate (tdr=cad*pdr):

\[ \frac{d}{dt}TVC = pdr \cdot PCAP - TVC \cdot (pdr \cdot cad) \]  

(76)

Solving for TVC, using equation (65) to define PCAP, yields:

\[ TVC = PCAP_0 \cdot e^{-pdr \cdot cad \cdot t} + \frac{pdr \cdot PCAP_0}{(pdr \cdot cad + cpi)} \cdot (e^{cpi \cdot t} - e^{-PDR \cdot CAD \cdot t}) \]  

(77)

where use has been made of the fact that the initial value of tax book value per unit of capital is the initial value of the price of capital (PCAP_0).

Substituting the above expression for tax book value of capital into equation (74) and solving the resulting expression for the depreciation tax shield yields:

\[ PV(DTS) = \left[ \frac{pdr \cdot PCAP_0 \cdot c_i}{rdre} \right] \left[ \frac{1 + rdre/pdr}{(1 + rdre/tdr)} \right] \left[ \frac{(tdr + rdre)/(tdr + r)} \right] \]  

(78)

The above expression for the present value of the depreciation tax shield may be broken into three simpler terms: (1) the present value of physical depreciation before inflation (PVPDBI), (2) the effect of accelerated depreciation on the depreciation tax shield (EADS), and (3) the effect of inflation on the depreciation tax shield (EIS):

\[ PV(DTS) = PVPDBI \cdot EADS \cdot EIS \]  

(79)

\[ PVPDBI = c_i \cdot pdr \cdot PCAP_0 \cdot rdre \]  

(80)

\[ EADS = \frac{1 + rdre/pdr}{(1 + rdre/tdr)} = \frac{(1 + rdre/tdr)/(1 + rdre/(cad \cdot pdr))}{(1 + rdre/pdr)} \]  

(81)

\[ EIS = \frac{(tdr + rdre)/(tdr + r)} = (\frac{cad \cdot pdr + rdre}{(cad \cdot pdr + r)}) \]  

(82)

Note that as tax depreciation is increasingly accelerated (i.e., tdr gets larger relative to pdr), EADS increases, thereby increasing the present value of the depreciation tax shield. Also,

\[ 17 \text{ This is an approximation. More precisely, the physical depreciation rate (pdr) must be assumed equal to the straight line rate allowable by the FASB.} \]
as inflation increases (increasing \( r \) relative to \( r_{d irrelevant} \)), EIS decreases, reducing the present value of the depreciation shield. Inflation reduces the present value of the tax shield because the depreciation flow is determined by the historical price at which an asset was purchased. The historical price is not revalued on the books to reflect higher price levels, and consequently the depreciation flow is worth progressively less in real terms when there is inflation.

The equations above also show the interaction between the effects of accelerated depreciation and inflation. The effect of accelerated depreciation gets closer to one for any depreciation rate as the real discount rate goes to zero. At a zero real discount rate, the effect of accelerated depreciation is identically 1. The reason for this is that depreciating equipment more quickly is valuable only if receiving money faster is valuable. When the real discount rate is zero, there is no advantage to receiving money earlier: One will not receive a real increase from early receipt of money if it is invested—so there is no gain to early receipt, and one can borrow money at no real costs if receipt is delayed—so there is no loss associated with late receipt.

The effect of inflation on the depreciation shield is moderated by the coefficient of accelerated depreciation. Mathematically, as the coefficient for accelerated depreciation gets large, the effect of inflation on the depreciation shield goes to one for any inflation rate. More intuitively, the effect of inflation operates by debasing the value of the depreciation deduction as prices rise. However as depreciation is accelerated, more of the depreciation advantage is received sooner so the impact of rising prices is less significant. If one could write the entire investment off immediately, the benefit would be received immediately; the benefit would not extend into the future to be debased by rising prices.

The U.S. investment tax credit (ITC), now eliminated by tax reform, was a subtraction from taxes equal to a fraction (fitc) of capital expenditures.\(^\text{18}\) The capital expenditures associated with the purchase of a capital factor are the initial price of capital (\(PCAP_0\)) and the ongoing replacement expenditures:

\[
\begin{align*}
\text{ITC}_0 &= \text{fitc} \cdot PCAP_0 \\
\text{ITC} &= \text{fitc} \cdot RE
\end{align*}
\] (83) (84)

---

\(^{18}\) From 1962 to 1963, the time period when the Long amendment was in force, the tax credit was subtracted from allowable depreciation. The modification required to handle the Long-amendment tax credit is not considered in this paper.
where:

$$
\text{ITC}_0 \quad \text{Initial Investment Tax Credit} \\
\text{ITC} \quad \text{On-going Investment Tax Credit replacement}
$$

The present value of the investment tax credit is, then,

$$
PV(\text{ITC}) = \text{fitc} \cdot \text{PCAP}_0 + \text{fitc} \cdot PV(\text{RE}) \tag{85}
$$

where advantage has been taken of the linearity of the present value operator and of the fact that the present value of a current payment is that immediate payment.

Let us bring the equations above together into a benefit cost ratio for capital factor. Recalling that $PV(\text{IC}) = PCAP_0$ and substituting equations (70) and (85) into (57) gives:

$$
PV(NCF) = PV(\text{REV}) - PCAP_0 - PV(\text{RE}) - citr \cdot PV(\text{REV}) + PV(DTS) + \text{fitc} \cdot PCAP_0 + \text{fitc} \cdot PV(\text{RE}) \tag{86}
$$

There are several ways the terms on the right can be divided into cash inflows and outflows to form the numerator and the denominator of the benefit cost ratio. In order to facilitate the discussion of desired factor stocks and make more apparent the relationship between the work here and Hall and Jorgenson's work, I will define the benefit cost ratio for capital factor (BCR$_C$) as:

$$
\text{BCR}_C = \frac{(1 - citr)PV(\text{REV})}{(1 - \text{fitc})PCAP_0 + (1 - \text{fitc})PV(\text{RE}) - PV(DTS)} \tag{87}
$$

Substituting using equations (58), (67), and (78) yields a lengthy expression which will later serve as the basis of a comparison with Jorgenson:

$$
\text{BCR}_C = \frac{\text{OP}_0 \cdot \text{pmpf}}{\left[1/(1-citr)(\text{rdrr/rdre})(\text{rdrr*tip+1})(\text{rdrr*nic+1})\text{PCAP}_0(\text{rdre+pdr})(1-\text{fitc-citrd*pcard/pard+rd})\right]} \tag{88}
$$

where:

- **BCR$_C$**: Benefit Cost Ratio for Capital (dimensionless)
- **OP$_0$**: Current Output Price ($/capital unit)
- **citr**: Corporate Income Tax Rate (dimensionless)
- **rdrr**: Real Discount Rate for Revenues (rdr = r - oipr) (1/years)
- **rdre**: Real Discount Rate for Replacement Expense (rdre = r - cipr) (1/years)
- **tip**: Time for Inventory in Process (years)
- **nic**: Normal Inventory Coverage (years)
- **pdr**: Physical Depreciation Rate (1/years)
- **fitc**: Fractional Investment Tax Credit (dimensionless)
- **cad**: Coefficient for Accelerated Depreciation (dimensionless)
- **r**: Nominal Interest Rate (1/years)
As in the case of labor, an increase in the real discount rate for revenue, relative to the real discount rate for replacement expense will decrease the profitability of capital—that is, as the inflation rate on price of capital increases relative to the inflation of output prices, profitability of capital is impaired. As the nominal interest rate increases relative to the inflation rates, this effect diminishes. The real discount rate also decreases profitability through production delays. Further, the real discount rate determines an opportunity cost equivalent to the price of capital multiplied by the real discount rate. The opportunity cost diminishes as the fractional income tax credit increases and as the coefficient for accelerated depreciation increases. The value of the tax shield decreases as the nominal interest rate increases.

5. Desired Factor Acquisition Rate

Having defined benefit cost ratios for purchased and rented factors, we now turn to a consideration of how the ratio affects the behavior of companies in a sector. The benefit cost ratio is a measure of the average current profitability of a factor in a sector, taking account of the impact of the future cash flows associated with the factor. When a factor is profitable, firms will find profitable uses for it and will acquire the factor at a rate faster than it depreciates—the amount of factor in a sector will expand. And, conversely, when the factor is unprofitable, firms will acquire the factor at less than the rate at which it depreciates—the amount of factor will decline.

One influence on the factor acquisition rate will be the amount of factor currently in place. A manufacturer of women's clothing can more easily absorb ten additional overlock machines in a month if the company already has 1000 such machines than if the company has 5. The amount of factor currently in place provides information on the size of the company and on the relative importance of the factor in its operations. Both of these considerations will be influential in determining how fast a company can increase its factor stock.

A simple way to build the dependence on the current amount of factor into the formulation is to assume that, all else equal, desired factor acquisition is proportional to current factor stock:

\[ \text{DFAR} = \text{DFCF} \times \text{FS} \]  \hspace{1cm} (89)

\[ \text{DFCF} = f(\text{BCR}) \]  \hspace{1cm} (90)
where:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DFAR</td>
<td>Desired Factor Acquisition Rate (factor units/year)</td>
</tr>
<tr>
<td>DFCF</td>
<td>Desired Net Fractional Change in Factor (1/years)</td>
</tr>
<tr>
<td>FS</td>
<td>Factor Stock (factor units)</td>
</tr>
<tr>
<td>f(*)</td>
<td>a function</td>
</tr>
<tr>
<td>BCR</td>
<td>Benefit Cost Ratio (dimensionless)</td>
</tr>
</tbody>
</table>

A BCR of 1 indicates that for the sector overall, additional investment in factor is neither profitable nor unprofitable. Firms in the sector will be making replacement expenditures, but in aggregate will not be systematically increasing or decreasing the factor in question: $f(\text{BCR})$ will take a value of zero. As BCR increases above one, investment in the factor, on average, becomes increasingly profitable, and $f(\text{BCR})$, therefore, should become increasingly positive. Similarly, as BCR declines below 1, $f(\text{BCR})$ should become increasingly negative.

It is possible to considerably sharpen our understanding of $f(\text{BCR})$. First, we note that the ultimate shape of BCR will be determined by three sets of considerations: "rational" considerations of how BCR will change as factor is acquired, psychological or behavioral considerations relating to managers' motivation to acquire factors, and considerations relating to the companies' physical capacity to expand. We don't know, of course, the order in which the various considerations affecting $f(*)$ will be considered in practice. However, for the purpose of clarifying $f(\text{BCR})$ it is useful to consider first how BCR changes as factor is acquired. Although the argument depends on restrictive assumptions, it provides a functional form that can be used as a base for modifications suggested by additional considerations. \(^{19}\)

**The Effect on BCR of Acquiring Additional Factor.** Changing the amount of factor will affect the benefit cost ratio for the factor in several ways. As more of a factor is acquired, its marginal productivities will normally change—for example, as more and more capital is acquired, its profitability will usually decrease unless other factors (like labor) are increased at a similar pace. At a somewhat greater remove, changes in the demand for factors will affect factor prices. Further, some changes in factor quantities will be intended to change the level of output which will, in turn, change output prices. Even less directly, increased investment will increase the demand for money which will increase

---

\(^{19}\) It is interesting to note that modelers, too, resort to anchoring and adjustment when moving into unfamiliar territory. Here the anchor is the function derived from considering how BCR changes as factor is acquired. The adjustment is made on the basis of psychological and capacity considerations.
the nominal interest rate. Increased workers may result in increased income to the household which will increase final demand and, therefore, output prices. In brief, all price and productivity assumptions will be altered by changing factor stocks, and will be altered through a variety of channels.

Although price and productivity assumptions will be affected by factor acquisition decisions, it does not necessarily follow that factor acquisition decisions will be affected by the anticipation of how those decisions feed back to change all of the assumptions. Firms probably concentrate their resources on considering the largest sources of change and also tend to avoid thinking about the most uncertain changes (Cyert and March 1963, pp. 102, 118-120). These two considerations favor concentrating on productivity changes, rather than price changes. Price changes will be relatively uncertain in March and Simon's sense that the probability distribution will not be known (March and Simon 1958, p. 113) because they depend upon demand and supply schedules of other actors in the system and because the causal web through which prices are affected is not well understood. Furthermore, the effect of any one firm on prices is usually rather slight. In contrast, productivity is changed directly and significantly by the firm's decision to add more factor. Consequently, this paper will focus on effect of changes in the productivity assumption on the rate at which factor is acquired.

**Factor Stock Adjustment with Fixed Output.** To make some headway in determining how productivity changes will affect f(BCR) and to relate this discussion to the work of other investigators it is useful to convert equation (89) into an equivalent expression in terms of desired factor stocks. A common assumption is that the desired factor acquisition rate is a first order adjustment of actual stocks to desired:

\[ \text{DFAR} = \frac{(DFS - FS)}{tafs} \]  

(91)

---

20 The alternative to clarifying the uncertainty is to rely on short-run feedback (Cyert and March 1963, Hogarth 1981). Although the "analysts in the model" take prices and costs as exogenous, within the larger model prices and costs should be endogenous. The "analysts in the model," then, can rely on *seeing* the impact of their actions on prices and costs and recalculating BCR accordingly, rather than on *predicting* the impact of their decisions on prices and costs.

21 This does not mean that the *observed* investment rate is a first order adjustment. There may be delays involved in ordering and receiving equipment that intervene between the rate of desired factor acquisition and the observed investment rate. These intervening delays will produce a lag distribution different from a first order.
where:

\[
\begin{align*}
\text{DFS} & \quad \text{Desired Factor Stocks (factor units)} \\
\text{tafs} & \quad \text{time to adjust factor stocks (years)}
\end{align*}
\]

Equating equations (89) and (91), we have:

\[
\text{DFCF} = f(\text{BCR}) = \frac{[\text{(DFS/FS)}-1]}{\text{tafs}}
\]

(92)

We can now consider the subsidiary problem of defining desired factor stock (DFS). Equations (56) and (88) for the benefit cost ratio for labor and capital respectively have the same numerator and differ only in the denominator. Let us call the denominator \( V_i \), \( i = C, L \). We can then write BCR as:

\[
\text{BCR}_i = \frac{\text{OP}_0 \cdot \text{pmpf}_i}{V_i}
\]

(93)

\[
V_L = \left( \text{rdrr/tdrwe} \right) \left( \text{rdrr*tiip+1} \right) \left( \text{rdrr*nic+1} \right) \text{WAGE}_0
\]

(94)

\[
V_C = \left[ \frac{1}{(1-\text{cirr})} \right] \left( \text{rdrr/rdre} \right) \left( \text{rdrr*tiip+1} \right) \left( \text{rdrr*nic+1} \right) \text{PCAP}_0 \left( \text{rdre+pdr} \right) \left[ 1-\text{fittc-citr*tdr/(tdr+r)} \right]
\]

(95)

The productivity term is pmpf. If we assume a Cobb-Douglas production function, output (Q) can be written:

\[
Q = a \cdot FSC^\alpha \cdot FSL^\beta
\]

(96)

where \( \beta \) and \( \alpha \) are constants and FSC and FSL are the factor stocks of capital and labor respectively. If we further assume that the perceived marginal productivity of factor (pmpf) equals the marginal productivity of factor (mpf),\(^{22}\) then:

\[
\begin{align*}
\text{pmpf}_L &= \text{mpf}_L = \beta \cdot a \cdot FSC^\alpha \cdot FSL^\beta-1 \\
\text{pmpf}_C &= \text{mpf}_C = \alpha \cdot a \cdot FSC^{\alpha-1} \cdot FSL^\beta
\end{align*}
\]

(97a)

(97b)

Substituting (90) into (86) yields:

\[
\begin{align*}
\text{BCRL} &= \frac{\text{OP}_0 \cdot \beta \cdot a \cdot FSC^\alpha \cdot FSL^\beta-1}{V_L} \\
\text{BCRC} &= \frac{\text{OP}_0 \cdot \alpha \cdot a \cdot FSC^{\alpha-1} \cdot FSL^\beta}{V_C}
\end{align*}
\]

(98a)

(98b)

---

\(^{22}\) More properly, one would assume that the perceived marginal productivity of factor (PMPF) eventually will equal the marginal productivity of factor (MPF) if MPF remains unchanging or grows at a constant rate for a sufficiently long period of time. In a formulation, one would likely wish to set PMPF equal to a weighted average of past values of MPF.
In Hall and Jorgenson's neoclassical approach, output ($Q$) is assumed constant. Although I will consider a more realistic alternative assumption below, it is useful to begin with Hall and Jorgenson's assumption. Accordingly, one can rewrite the above equations as:

$$BCR_L = \beta*OP_0*Q/(FS_L*V_L) \quad (99a)$$
$$BCR_C = \alpha*OP_0*Q/(FS_C*V_C) \quad (99b)$$

It will pay firms to expand (or contract) output to the point where BCR equals one. Expanding or contracting the sector to any other level of BCR will leave at least some firms in a position where they wish to continue expanding or contracting. Consequently, if firms know their technology the sector as a whole will choose a value of desired factor stocks ($DFS_i$) such that:

$$BCR_L = 1 = \beta*OP_0*Q/DFS_L*V_L \quad (100a)$$
$$BCR_C = 1 = \alpha*OP_0*Q/DFS_C*V_C \quad (100b)$$

or, rearranging,

$$DFS_L = \beta*OP_0*Q/V_L \quad (101a)$$
$$DFS_C = \alpha*OP_0*Q/V_C \quad (101b)$$

At this point it will be instructive to substitute for $V_i$ in the equations above:

$$DFS_L = \beta*OP_0*Q/(rdrr/rdre)(rdrr*tiip+1)(rdrr*nic+1)WAGE_0 \quad (102a)$$
$$DFS_C = \alpha*OP_0*Q/[1/(1-citir)](rdrr/rdre)(rdrr*tiip+1)(rdrr*nic+1)PCAP_0(rdre+pdr)*[1-fitc-citr*tdr/(tdr+\eta)] \quad (102b)$$

The above equations are a generalization of Hall and Jorgenson's expressions for desired capital and labor (equations (6b) and (11) above). Equations (102a) and (102b) reduce to Hall and Jorgenson's expressions under the assumption of constant returns to scale ($\alpha=1-\beta$) and the more restrictive conditions that (1) there is no inflation so that:

$$rdrr=rdre=\tau \quad (103)$$

and (2) there is no delay between initiating production and selling the product so that:

$$tiip=nic=0 \quad (104)$$

---

23 This assumption corresponds to Hall and Jorgenson's assumption that profits are zero.
It is interesting that it is incorrect to account for inflation, as Hall and Jorgenson do, by simply assuming that $r$ is a real discount rate. In order to recognize inflation properly it is necessary to recognize, as many economists have (Litner 1975, p. 272), inflation's negative impact on the present value of the depreciation shield (see equation (82) and discussion).

There is, of course, an effect of inflation on the tax depreciation shield and production is a time consuming process. These two considerations work in the same direction; the standard neoclassical formulation will tend to overstate desired factor stocks. The precise amount of the overstatement will depend upon how long it takes to produce output on the tax depreciation rate, on the corporate income tax rate, on the nominal interest rate and on the inflation rate for output and replacement expense. A sense for the degree of the overstatement can be gained from figure 10 which presents the ratio of Hall and Jorgenson's formulation for desired capital stock to the formulation in equation (102b). The table gives the ratio for a variety of combinations of the nominal interest rate and the inflation rate. It is interesting to note that Hall and Jorgenson's formulation increasingly overstates desired capital as the expected inflation rate rises even when the expected real interest rate is constant and that it increasingly overstates desired capital as the expected real interest rate rises holding expected inflation constant.

Let us continue with the neoclassical assumption that output is fixed and return to the question of how the benefit cost ratio determines desired factor stocks. Equations (101a) and (101b) can be rewritten as:

\[
\begin{align*}
\text{DFS}_L &= \left( O_0^*Q/(FSL*V_L) \right) * FSL \\
\text{DFS}_C &= \left( O_0^*Q/(FSC*V_C) \right) * FSC
\end{align*}
\]

or, using equations (99a) and (99b):

\[
\begin{align*}
\text{DFS}_L &= BCR_L * FSL \\
\text{DFS}_C &= BCR_C * FSC
\end{align*}
\]

Substituting in equation (92) yields:

\[
\text{DFCF} = f(BCR_i) = (BCR_i - 1)/tafs_i
\]

The desired fractional change of factor (DFCF), as a function of BCR, is a straight line as shown in figure 11 when the sector as a whole has a Cobb-Douglas production function and output is held fixed. Note that $f(BCR)$ is an increasing function of BCR and
that when BCR takes its neutral value of 1, f(BCR) takes the value of zero indicating no net investment.

**Separability: Holding Other Factors Fixed.** Unfortunately, the assumption that output is fixed is unrealistic and is inconsistent with the analyst's model underlying the calculation of present value presented previously. A better assumption is that the other factor is held fixed. It will be recalled that the analyst acts as if the decision to acquire one factor were separable from the decision to acquire other factors. Holding output fixed would imply to the contrary that the analyst expects that an increase in one factor will be compensated for by a decrease in the other factor; holding output fixed would imply that the two decisions are closely coordinated. In contrast, assuming that the other factor is held fixed is compatible with the notion that the analyst separates the two decisions. Further, assuming that output is held fixed implies that, although the benefit cost ratio is used to balance factor proportions, it is not used to purposely increase or decrease output. But, the role of BCR should be *both* to balance the factors and to adjust output. Assuming that other factors are held fixed permits BCR to play this role and is consistent with the analyst's model.\(^{24}\)

Continuing with a Cobb-Douglas technology and again employing the assumption that firms adjust factor until BCR=1, we can write:

\[
\begin{align*}
\text{BCR}_L^* &= 1 = OP_0 \ast \beta \ast a \ast FSC_{\alpha} \ast DFL_{-1} \ast V_L (108a) \\
\text{BCR}_C^* &= 1 = OP_0 \ast \alpha \ast a \ast DFS_{-1} \ast V_C (108b)
\end{align*}
\]

Rearranging,

\[
\begin{align*}
\text{DFL}_{1-1} &= OP_0 \ast \beta \ast a \ast FSC_{\alpha} \ast V_L (109a) \\
\text{DFS}_{1-\alpha} &= OP_0 \ast \alpha \ast a \ast DFS \ast V_C (109b)
\end{align*}
\]

or,

\[
\begin{align*}
\text{DFL}_{1-1} &= (OP_0 \ast \beta \ast a \ast FSC_{\alpha} \ast FSL_{-1} \ast V_L) \ast L_{1-1} (110a) \\
\text{DFS}_{1-\alpha} &= (OP_0 \ast \alpha \ast a \ast FSC_{\alpha} \ast FSL_{-1} \ast V_C) \ast K_{1-\alpha} (110b)
\end{align*}
\]

\(^{24}\) Jorgenson in an early paper (1963) suggested "a kind of iterative process" where in one period labor and output would be determined under the assumption that capital was fixed. In the next period, capital would be determined under the assumption that labor was free to vary but that output was fixed at the previous period's value. This value of capital would provide the base for adjusting labor and output in the following period. The current formulation assumes not an iterative but a separated process where the decision to increase or decrease capital is made assuming labor is fixed and the assumption to increase or decrease labor is made assuming capital is fixed.
or,
\[ DFS_L = BCR_L^{(1/(1-\beta))} \ast FSL \]  
\[ DFS_K = BCR_K^{(1/(1-\alpha))} \ast FSC \]  

Substituting into equation (92) yields:
\[ DFCF_L = f_L(BCR_L) = (BCR_L^{(1/(1-\beta))} - 1)/\text{fs}_L \]  
\[ DFCF_C = f_C(BCR_C) = (BCR_C^{(1/(1-\alpha))} - 1)/\text{fs}_C \]

Allowing output to vary implies that the economic motivation to acquire factors increases more than proportionally with increases in BCR. This is illustrated in figure 12.

Behavioral Considerations and Constraints on the Acquisition Rate.
Figures 11 and 12 give an idea of how productivity considerations shape \( f(BCR) \). Behavioral factors and constraints on the speed of expanding or contracting a sector will further affect the the shape. There are at least two relevant factors discussed in the literature of behavioral decision theory which have not yet been incorporated into the derivation:

First, prospect theory (Kahneman and Tversky 1982, 1981, 1979) suggests that the subjective value of a gain increases less than proportionally to the gain, and conversely, the subjective value of a loss also increases less than proportionally to the absolute value of the loss.\(^{25}\) Most people, for example, feel the difference between a gain of $100 and a gain of $200 is more significant than the difference between a gain of $1000 and a gain of $1200. Similarly, the difference between a loss of $100 and a loss of $200 is subjectively greater than the difference between a loss of $1000 and a loss of $1200. This suggests that manager's motivation to increase or decrease factor stocks will increase less than proportionally with the economic justification measured by BCR. The result will be a flattening of the slope of \( f(BCR) \).

One way to incorporate the value function of prospect theory into the present framework is to assume that decision makers form the benefit cost ratio by relating the subjective value of the gain to the subjective value of the cost, rather than relating the objective gain to the objective cost as we have assumed thus far. More formally,

\(^{25}\) The value function in prospect theory is similar to the utility function in expected utility theory. The key differences are: (1) the value function is defined on changes in wealth where the utility function is usually defined on wealth and (2) the value function is concave for gains but convex for losses, where the utility function is usually assumed to be concave throughout. See Kahneman and Tversky 1979 for a discussion of the relationship between the two theories.
Kahneman and Tversky suggest that for moderate gains and losses the value function can be described by a power function:

\[ \text{value of change} = v(\text{change}) = \text{change}^\eta \]
\[ 0 \leq \eta \leq 1 \]

Hence, the subjective benefit cost ratio SBCR may be written:

\[ \text{SBCR} = \frac{v(\text{benefit})}{v(\text{cost})} = \text{benefit}^\eta / \text{cost}^\eta \]  \hspace{1cm} (113)

or,

\[ \text{SBCR} = (\text{BCR})^\eta \]  \hspace{1cm} (114)

This suggests that equations (111a) and (111b) should be rewritten so that:

\[ f_L(\text{BCR}_L) = (\text{BCR}_L^{\eta/(1-\beta)} - 1)/\text{tafs}_L \]  \hspace{1cm} (115a)
\[ f_C(\text{BCR}_C) = (\text{BCR}_C^{\eta/(1-\alpha)} - 1)/\text{tafs}_C \]  \hspace{1cm} (115b)

Figure 13 shows the impact of such a change.

Second, there is a bias toward inaction, that is, a bias toward maintaining the status quo (Kahneman and Tversky 1982). The most concrete direct evidence of this in the factor acquisition decision is the survey evidence of a willingness to make replacement expenditures even when not justifiable on purely economic grounds—that is, there is a bias against reducing the amount of factor. In addition, one might anticipate that there is a bias against increasing the factor as well. For an individual company, the bias toward inaction might be interpreted as a flat region or "dead zone" in f(BCR) around the origin representing a tendency to do nothing or little for small rewards. Once the original inertia is overcome, the bias toward inaction might have no further impact on the factor acquisition decision. For an aggregate sector, however, the impact of the bias toward inaction will likely be a smoother flattening of f(BCR) around the origin, rather than a true dead zone. Figure 14 adds such a flattening about the origin.

---

26 Kahneman and Tversky (1982) actually suggest the value function can be described by two power functions, one for gains and the other for losses. The coefficient for the power function associated with losses is typically greater than that for gains—that is, the value function is steeper for losses than it is for gains. A more rigorous formulation would involve raising the numerator of BCR to a different power than the denominator, a process that is not computationally difficult. Here I assume that the coefficients are the same in order to make clearer the effect of prospect theory on the original f(BCR). The difference between the coefficients is not huge: .95 vs. .83.
Finally, constraints on the firm's capacity to acquire or get rid of factors will tend to further flatten $f(BCR)$. The speed with which a firm can comfortably train and assign people is limited; so is the speed with which a firm can comfortably install equipment. As these limits are exceeded, the firm will find itself with an increasing number of people waiting for training or assignments and an increasing amount of equipment waiting to be installed and put to use. Similarly, there are limits to the speed with which equipment can be removed. For an aggregate sector it is difficult to sell equipment (who will buy it?). Consequently, in the aggregate, firms will find it increasingly difficult to disinvest as the net rate of disinvestment approaches the rate of physical depreciation. These considerations suggest a further flattening of $f(BCR)$ and the possibility that $f(BCR)$ approaches some limiting values. Figure 15 shows the further flattening of $f(BCR)$.

6. Conclusion

Summary. Survey evidence suggests that discounting approaches to investment are widely used in practice and are growing in popularity. In the context of macro-economic modeling, discounting approaches are superior to payback approaches, which are also widely used in practice, because discounting approaches guide the modeler in accounting for interest rates and discounting approaches are as applicable to rental decisions as they are to purchasing decisions. As a consequence, it is appropriate to base the desired factor acquisition rate in a macro-economic model on a discounting approach.

Of the discounting approaches, the most appropriate one for use in a macro-economic model is the benefit cost ratio criterion. The benefit cost ratio is a dimensionless measure of the degree to which the present value of cash inflows exceeds the present value of cash outflows associated with the acquisition of a factor. The benefit cost ratio, when applied to a sector represents the relative profitability on average of acquiring a factor. The benefit cost ratio should be closely correlated with the ease with which firms discover profitable uses of a factor. Consequently, the benefit cost ratio is useful in formulating how fast firms in a sector desire to add to or subtract from capacity. As the average profitability increases, the rate at which firms in a sector desire to acquire the factor also increases.

The calculation of the benefit cost ratio and its use in determining the desired rate of factor acquisition can be represented in a macro-economic model in a way consistent with behavioral decision theory. In particular, the present value calculation can be applied to a "model within the model" corresponding to the (mental) models used by analysts and
decision makers in the real world. The "analyst's model," as it was called in this paper, is a simplified version of the surrounding macro-economic model, just as the models used by real-world decision makers are simplified versions of the real world. The simplification is achieved by ignoring minor structural details and by concentrating on effects that are certain and direct as opposed to uncertain or indirect. The evaluation of the benefit cost ratio can be colored, as it probably is the real-world, by applying the value function from prospect theory to the benefit cost ratio, and the response to the colored benefit cost ratio can be biased toward inaction as behavioral theory suggests actual people bias their actions.

Relationship to Standard Neo-classical Formulation. The ideas, derivations, and results of this essay in part represent extensions of the standard neo-classical formulation and in part represent departures from it. The standard neo-classical formulation was extended by explicitly including the effect of inflation on the tax depreciation shield and the effect of a time consuming production process. As a result, the behavior of investment implied by the formulation developed here differs from that developed by Hall and Jorgenson: Investment is more sensitive to the real interest rate and may be expected to decline in the face of rising inflation, even if the real interest rate remains constant.

More technically, the neo-classical model was further explicated by an exploration of the implication of assuming that other factors will be held constant as a given factor is increased, rather than that output will be held constant as the factor increases. Assuming that other factors are held constant permits the formulation to account for both factor balancing and changes in output. Dynamically, holding other factors constant implies, ceteris paribus, factor acquisition increases more than proportionally as profitability (measured by the benefit cost ratio) increases; in contrast, the assumption that output will not change as factors are changed implies, ceteris paribus, that factor acquisition will increase proportionally with increases in profitability (measured by the benefit cost ratio).

Although the formulation in this paper extends the neo-classical investment function, it is important to note that the formulation results from a fundamentally different approach to formulating macro-economic decision functions. Instead of relying on a central assumption of utility maximization and rationality, the formulation begins from survey evidence of methodologies actually used by companies and then is further: articulated by the application of findings from the behavioral sciences, particularly behavioral decision theory.
It is of interest that the resulting formulation possesses a basic similarity to the standard neoclassical investment function. The most important reason for the similarity is that companies employ the machinery of present values to evaluate investment opportunities, and this machinery is consistent with utility maximization under assumptions common in neoclassical theory. If the same similarity holds for behavioral formulations of the other basic economic decision functions the foundations of neoclassical economics will be strengthened considerably. There is, of course, no special reason the similarity should continue to hold. Indeed it seems more than just a possibility that the continued application of a radically different approach will result in models that are different from those widely used today.

Whether behavioral models will truly be different and, if different, whether such models will be more useful are open but important questions. The desirability of getting answers is justification enough for further model development. This paper, by showing it is possible to ground decision functions in the theory of how people make decisions, is a link in a chain leading to such thoroughly behavioral macro-economic models.
Appendix: The Benefit Cost Ratio in a Putty-Clay World

The text considered the benefit cost ratio in the context of a putty-putty technology. In this appendix a benefit cost ratio appropriate for a putty-clay technology will be briefly sketched.

In a putty-clay world, factor proportions are fixed ex post. I will maintain the assumption that the useful life of factor is fixed. A useful way of looking at the factor acquisition problem is along the lines developed by Sterman (1980) where one factor is considered primary (the clay) and the other factors are adjusted according to factor-proportions determined by the primary factor already in place. In the current context it is reasonable to view capital as the primary factor. In this case, desired labor is defined as the labor/capital ratio of the capital stock in place now multiplied by capital stock in place now:

$$L_t^* = C L R_t^* K_t$$  \hspace{1cm} (A.1)

The capital labor ratio would be determined via a co-flow structure.

Capital acquisition would be determined by a function like that of equation (12a) in the text:

$$I_k = \zeta_k \left\{ \left( \frac{\partial}{\partial t} Q \right) + D \right\}$$  \hspace{1cm} (A.2)

Using a Cobb-Douglas production function and the analyst's model described in the text, it can be shown that the proper capital to output ratio $\zeta_k$ is:

$$\zeta_k = a^{-1} \left\{ \frac{BCR C*FS_c}{BCR_i*FS_i} \right\}^{(1-\alpha)}$$  \hspace{1cm} (A.3)

where, as before:

- $\zeta_k$ Capital-output ration (dimensionless)
- $a$ Scaling factor
- $BCR_i$ Benefit cost ratio for factor $i$

Sterman's formulation is more general than this. He recognizes the possibility of a short-term production function where factors can be combined with the primary factor in proportions different from that for which the primary factor was designed. However, the short-term production function recognizes there may be sharp reductions in productivity for using less of the non-primary factors than called for by the design specifications, and there may be only small increases in productivity for using more of the non-primary factors than the design specs would suggest. Sterman's formulation also allows for retrofits of the primary factor.
$F_{S_i}$  Factor stock of factor $i$
$c, l$  Subscripts of capital and labor respectively
$\alpha$  The coefficient on capital in the production function

The term $dQ/dt$ presents more difficulties. One approach is to set:

$$\frac{dQ}{dt} = g(\text{BCR'})Q$$  \hspace{1cm} (A.4)

Where BCR' is the benefit cost ratio of adding a marginal unit of output at the desired factor/output ratios. It may be shown that:

$$\text{BCR'} = \frac{\text{OP}}{(\zeta_k^*V_C + \zeta_i^*V_L)}$$  \hspace{1cm} (A.5)

where:

- $\text{OP}$ output price
- $V_C$ the denominator of $\text{BCR}_C$ and is defined in equation (95)
- $V_L$ the denominator of $\text{BCR}_L$ and is defined in equation (94)
- $\zeta_i$ the factor-$i$-to-output ratio calculated as in (A.3) above

$g$ would be determined by the behavioral considerations and constraints on expansion discussed in the text.
References


Table 1: Methods of Analysis

Five studies examine methods used in practice to evaluate the desirability of possible investments. The studies show that discounting approaches (net present value (NPV), present value index or benefit cost ratio (PV Index), and the internal rate of return (IRR)) are widely used and seem to be increasing in popularity.

1 Includes internal rate of return, net present value and MAPI formula
2 Includes payback reciprocal. Total figure in chart is sum of original categories and may double count
**Figure 2: Present Value Calculation**

The nominal present value of a stream of cash flows can be calculated by weighting a cash flow by the present value factor and accumulating the result. The present value factor is a stock whose initial value is one and which decays at a rate determined by the nominal discount rate.
Figure 3: The Analyst's Model of the Production Process
The "analyst" visualizes the production process as consisting of two stages: Production is begun and flows into the Inventory in Process. This Inventory is depleted by Production Finishes. As product is finished it moves into the Finished Goods Inventory. Finally, shipments are made which when multiplied by the output price yield a revenue stream. The analyst assumes that output prices inflate or deflate at a constant rate.
Figure 4: Analyst's Model: Cash Flow Attributable to Labor
Net cash flow per unit of labor is the difference between revenues and expenses per additional unit of labor. Revenues are shipments multiplied by output price. Shipments result, after a delay, from production starts. Production starts are equivalent to the perceived marginal productivity of labor. Expenses include income taxes and wage expense. The wage rate and the price of output inflate or deflate at constant rates.
Figure 5: Analyst's Model of Cash Flow Attributable to Capital

In the analyst's model, net cash flow attributable to capital is the difference between revenues and expenses per unit of additional capital. Revenues result from the sale of product from finished inventories at the output price, which inflates at a constant fractional rate. Production occurs in two stages: production starts per unit of capital (equal to the perceived marginal productivity of capital) are followed by production finishes which occur after a given time. Expenses include replacement expense and income tax expense. Replacement expense is equal to the physical replacement rate multiplied by the price of capital, which inflates at a constant fractional rate. Replacement expense gives rise to investment tax credits. Physical depreciation is equal to physical depreciation which occurs at a constant fractional rate. Income tax expense is paid on net income which is revenues less tax depreciation expense. Tax depreciation expense is calculated as a constant fraction per year of the tax book value. The tax book value accumulates the difference between replacement expense and tax depreciation expense.
Figure 6: Analyst's Structure for Calculating Present Value of Revenues
In the analyst's model, revenues per unit of labor are shipments per unit of labor multiplied by output price, which inflates at a constant fractional rate. Shipments per unit of labor are the end result of a two stage production process which begins with production starts per unit of labor, equal to the perceived marginal productivity of labor. To calculate present value, revenues per unit of labor are weighted by the present value factor and accumulated.
Figure 7: Analyst's Structure for Calculating Present Value of Wage Expense
The analyst assumes that the wage per unit of labor will inflate or deflate at a constant fractional rate. To calculate the present value, wage expense per unit of labor is weighted by the present value factor and accumulated.
Figure 8: Analyst's Structure for Calculating Present Value of Replacement Expense

In the analyst's model replacement expense is physical replacement multiplied by the price of capital. The price of capital inflates or deflates at a constant fractional rate. Physical replacement is equal to physical depreciation. Physical depreciation occurs at a constant fractional rate. To calculate the present value, replacement expense is weighted by the present value factor and accumulated.
Figure 9: Analyst's Structure for Present Value of Depreciation Tax Shield

The analyst views the tax shield as the corporate tax rate multiplied by depreciation expense. Depreciation expense is the tax book value of capital multiplied by the tax depreciation rate. The tax book value of capital accumulates the difference between replacement expense and the tax depreciation expense. Replacement expense is physical replacement multiplied by the price of capital, which inflates or deflates at a constant fractional rate. Replacement expense is equal to physical depreciation, which is assumed to occur at a constant fractional rate.
<table>
<thead>
<tr>
<th>Interest rate:</th>
<th>Inflation rate:</th>
<th>0.0%</th>
<th>5.0%</th>
<th>10.0%</th>
<th>15.0%</th>
<th>20.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0%</td>
<td>1.0000</td>
<td>1.0752</td>
<td>1.2696</td>
<td>1.9360</td>
<td>706.09</td>
<td></td>
</tr>
<tr>
<td>5.0%</td>
<td>1.0816</td>
<td>1.1111</td>
<td>1.1947</td>
<td>1.4107</td>
<td>2.15</td>
<td></td>
</tr>
<tr>
<td>10.0%</td>
<td>1.1664</td>
<td>1.1681</td>
<td>1.2000</td>
<td>1.2902</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>15.0%</td>
<td>1.2544</td>
<td>1.2371</td>
<td>1.2389</td>
<td>1.2727</td>
<td>1.37</td>
<td></td>
</tr>
<tr>
<td>20.0%</td>
<td>1.3456</td>
<td>1.3141</td>
<td>1.2960</td>
<td>1.2979</td>
<td>1.33</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 10: The Effect of Inflation and Interest Rates on Alternative Factor Stock Formulation**

This table gives the ratio of desired capital stock without, to desired factor stock with, time consuming production process and inflation impact on depreciation shield. The standard neoclassical formulation without time consuming production process and inflation impact on depreciation shield overstates desired capital as the inflation rate or the interest rate increases. Calculations in the table assume: inflation rate for revenues equals inflation rate for replacement expense, fractional investment tax credit is zero, corporate income tax is 50%, tax depreciation rate is .4, time for inprocess inventory is .8 years and normal finished inventory coverage is also .8 years.
**Figure 11: DFCF From Productivity Considerations Alone With Output Held Constant**

1. \[ DFCF = \frac{BCR - 1}{taf} \]

2. \[ DFCF = \frac{BCR (1/(1-\alpha)) - 1}{taf} \]

**Assumption:** taf=1, \( \alpha = 0.5 \)

**Figure 12: DFCF from Productivity Considerations Alone With Other Factors held Constant**

Thin Line: DFCF holding output constant

Thick Line: DFCF letting output vary, holding other factor constant
Figure 13: DFCF from Productivity Considerations Combined with Prospect Theory

Prospect theory suggests that the subjective evaluation of gains and losses increases less than proportionally with the gains and losses. The application of prospect theory produces a flattening of DFCF.

1. \[ DFCF_1 = \frac{BCR^{1/(1-\alpha)}}{taf} - 1 \]
   Assumption: taf=1, \( \alpha = .5, \eta = .88 \)

2. \[ DFCF_2 = \frac{BCR^{(\eta/(1-\alpha))}}{taf} - 1 \]
Figure 14: Modification to DFCF from a Bias Toward Inaction

Adding a bias toward inaction to DFCF. The bias toward inaction results in a flattening around the origin representing a disinclination to change the status quo.
Solid Line: Productivity Considerations alone
Grey Band: Modification from Psychological Effects Plus Constraints on Firms Capacity to Acquire and Get Rid of Factor.

Figure 15: DFCF from Productivity and Psychological Considerations Plus Constraints on Factor Acquisition
Limits on the speed with which firms can comfortably acquire factor and realistically dispose (or fire) it will result in a further flattening of DFCF.