EFFECTIVE STRATEGIES FOR
OLIGOPOLISTIC GAMES

by

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S.B., Massachusetts Institute of Technology
(1983)

S.M., Massachusetts Institute of Technology
(1985)

Submitted to the Sloan School of Management
in partial fulfillment of the degree of

Doctor of Philosophy in Management

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
15 May 1987

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ABSTRACT

Traditional wisdom, as expressed in the marketing literature and by managers, suggests
that firms often have strong incentives to take actions (e.g., price cuts or promotional
campaigns) to give themselves short-term advantages over their rivals. However, many
models and actual cases also suggest that these actions can become self-defeating,
particularly when other firms in the same market respond with similar actions (perhaps
leading to a price or promotion war).

This thesis addresses this important managerial problem, known in the social sciences as
the Prisoner's Dilemma. Our primary objectives are (1) to understand the origins and
consequences of this type of competitive behavior, and (2) to learn how to analyze and
formulate effective strategies for firms involved in these situations.

The problem is approached in two ways, (1) using mathematical theory and (2) through
simulated competitive strategy tournaments. We generalize the classic Prisoner's Dilemma
to allow for more than two players and continuous alternatives instead of the two-player, two-
alternative structure normally used. Each of these extensions is relevant when trying to
accurately model oligopolistic competition. The N-player generalization leads to the notion of
implicit coalitions among subgroups (less than N) firms. The continuous alternatives
generalization allows players to respond with different degrees of aggressiveness to price cuts
by competing firms.

Strategies that recognize either or both of these concepts are shown to perform quite well
in a variety of competitive environments. Implicit coalitions respond well to non-cooperative
behavior, thereby earning relatively high profits for member firms. We demonstrate that
coalition-seeking behavior is extremely robust, even when very few firms are aware of
coalitions.

Strategies that use continuous alternatives reap many benefits, including the ability to
avoid large, unprofitable oscillations between high and low prices. We specify the framework
for a generic strategy that can take advantage of both coalitions and continuous action.

The competitive strategy tournaments allow us to examine the importance of these
extensions and to judge the relative effectiveness of a large set of proposed strategies. Our
first tournament, involving a simple model of price competition, supports the ideas generated
by our early analytical work and suggests additional properties, such as self-awareness and
boundedness, which help distinguish successful strategies from weaker ones. The second
tournament provides further insight by allowing us to study the applicability of the implicit
cohesion concept in an environment where coalitions are harder to form and maintain.

Beyond our analysis of the tournaments, we discuss a class of relatively complex
strategies that can yield excellent long-run profits against many different competitors. These
strategies, known as LOOK-AHEAD strategies, first try to model each opponents' previous
prices with a linear reaction function and then choose prices to optimize any particular
objective, such as long-run profits. We discuss the advantages and disadvantages of this idea,
and examine the performance of one prototype against the full set of entries in our second
tournament.

Thesis Committee:

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ACKNOWLEDGEMENTS

It's been eight very long but very short years here at MIT. Now as I am about to leave this comfy, cozy nest, I would like to acknowledge the efforts of all those people who helped give me just the push I've needed.

First, I would like to thank my thesis committee: John Little, Richard Schmalensee, and my advisor and mentor, John Hauser. John encouraged me to get involved with this thesis in the first place and then gave me the freedom to develop it on my own. Fortunately, he has always been there to steer me in the right direction after hitting one of my many dead ends. I appreciate all the time and effort that John has devoted to me, and I hope to remember all the valuable lessons I have learned from him.

I am also thankful for the kind souls who have wandered into or out of the MIT Marketing group during my four years, including Jim Lattin, Lizz Restuccia, Deborah Marlino, Annie Cooper, Cheryl Mottley, and two of my favorite marketing buddies, Leigh McAlister and Fred Feinberg. All these folks contributed to my cause in numerous ways and helped me keep a proper perspective on this thesis and everything else.

Finally, I would like to express sincere gratitude to my family, my in-laws, and especially my wife Mina, who has worked incredibly hard to keep me afloat. You've made a lot of sacrifices for me; now it's my turn to live up to my potential.
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CHAPTER i:
INTRODUCTION

The essence of competitive strategy consists of not only making the right strategic decisions, but also reacting to competitors' decisions in an appropriate, timely fashion. The success (or failure) of a firm can often be linked to its ability (or inability) to anticipate, preempt, or cope with other firms' maneuvers.

The incentives to increase profits (e.g., by cutting price, increasing advertising expenditures, or running promotional campaigns) usually exist for most firms in a particular market; however, one firm's actions, such as price cuts, often provoke similar or stronger responses from other firms. A small gesture by one firm may lead to aggressive price wars -- changing a stable cooperative market into a chaotic competitive one where all firms are worse off than before.

Specific examples of this type of competitive behavior are common in today's business environment. Consider price competition in industries such as the domestic airlines or steel production: frequently, when one firm announces new low rates, one or more of its rivals is likely to match the new rates or announce their own new pricing policies. Matching rivals' moves is a vital aspect of competition; substantial profits can be lost by firms which stand pat.

Perhaps the most dramatic contemporary example is OPEC. After a decade of highly profitable collusion the cartel collapsed, partially because of cheating by member nations. Saudi Arabia, which for years staunchly abided by formal price and output guidelines, was no longer willing to be the sole cooperator. Early in 1986, the Saudis finally gave up their attempts to maintain cooperation and began increasing output and offering price discounts to try to regain lost sales. This policy proved to be unsuccessful; oil prices dropped as low as $8/barrel. In
recent months, the Saudis and OPEC have changed their tactics once again. Member nations have recently reached agreement to try to stabilize prices at about $18/barrel, but industry experts are unsure as to how long worldwide oil prices will stay at that level.

The OPEC example suggests that output quantities can also be viewed as a strategic variable that can lead to self-defeating competition within an industry. Packaged-goods firms often rely on three additional strategic weapons: advertising, promotion, and product differentiation. The well-documented battles among soft drink producers and fast food firms provide ample evidence for the ubiquity of each of these types of competition.

The purpose of this thesis to address these types of competitive phenomena. There are no easy solutions to the scenarios mentioned above and no single research project can hope to solve all of them. But we can make some important steps forward by addressing several of the theoretical and managerial issues that arise in this area of marketing, such as describing several prominent features of profitable strategies.

Central to the model and theory developed in this paper is a popular game-theoretic construct, the Prisoner's Dilemma. The basic idea behind the Prisoner's Dilemma is simple: individual firms often have short-term incentives to outmaneuver their rivals, but when all firms engage in such activities, the resulting industry profits are lower than if everybody had resisted the temptation to outmaneuver each other.

The Prisoner's Dilemma is a common feature in many models of competition, yet most models imbed it without explicitly acknowledging its presence. We intend to highlight and extend the Prisoner's Dilemma. The resulting
understanding of how the Prisoner's Dilemma (PD) affects competition in a complex environment should help managers make better use of existing models of competition and should help researchers create better models.

When considering the PD as more than just a conceptual model of price competition (or any other type of interfirm rivalry), several of its shortcomings become particularly evident and troublesome. One of the objectives of this thesis, therefore, is to generalize the classic Prisoner's Dilemma paradigm to make it a more realistic model of interfirm competition. We show that there are a variety of possible definitions for a Generalized Prisoner's Dilemma (GPD); we then choose one viable generalization and explore it in some detail.

The GPD models discussed here are relatively uncomplicated, especially when compared to other popular models of competition. For example, we consider only one strategic variable (price) and leave to future elaboration any cost or demand dynamics which may be important components of many other models. We believe that our principal focus -- the Prisoner's Dilemma -- is by itself enough to keep our analysis interesting and relevant. We gain valuable benefits from our parsimonious assumptions: the ability to trace and understand the behavioral dynamics of a competitive market.

Despite the simplicity of the models we use, we are able to identify several features of the GPD that deserve special consideration when formulating strategies. First is the notion of an implicit coalition, in which subgroups of firms may be able to maintain some degree of cooperation even when one or more rivals are totally uncooperative. Another important idea is continuous action, refering to the infinite number of strategic and tactical choices that are available to
players in GPD games. We discuss some of the analytic implications of these concepts, and suggest ways to implement them in strategies for GPD games.

We operationalize the models and test our analytic insights via computer simulation, using strategies submitted by participants around the world. These strategies cover a broad range of complexity and cooperativeness. Some are simple heuristics that require very little information about competitors, and some represent realistic decision rules used by managers in actual competitive situations. We examine these algorithms in two separate competitive strategy tournaments.

Both tournaments strongly support the key properties of the GPD. However, the very best strategies in each tournament are somewhat surprising. We explain the reasons for their success and propose alternative strategies which improve upon them.

The two tournaments provide a good idea of what it takes to succeed in different types of competitive environments. After discussing both tournaments, we turn to a class of complex strategies that are designed to apply and take advantage of the most important lessons from the tournaments. The main idea behind these strategies is to understand competitors' behavior, anticipate their future prices, and then choose prices to optimize long-run profits. We develop these strategies for a general class of games and test one specific version within the context of our second tournament.

Finally, we conclude with a brief discussion about how or whether all these theoretical ideas can be used to improve the decision-making ability of human players involved in similar games. Some early empirical evidence is presented and future experiments are suggested.
CHAPTER 2:
THE PRISONER'S DILEMMA

For nearly three decades, social scientists have devoted considerable effort towards studying a simple two-player two-alternative game known as the Prisoner's Dilemma (PD). The inception of the game is often attributed to Merrill Flood and Melvin Dresher, or Albert W. Tucker, but Luce and Raiffa (1957) are usually given the most credit for its formal development. The title is not highly relevant to the game; a simple pricing example serves equally well to illustrate the basic dilemma.

Suppose two firms, A and B, have identical cost and demand functions and each has only two possible prices for their product: $10 or $15. Each firm seeks to choose the price level which will maximize its own profits. If both firms choose the higher price, each makes a (one period) profit of $3 million. However, each has a strong incentive to cheat -- if one firm undercuts the other by pricing at $10 per unit, that firm (the price cutter) stands to earn $5 million, while its competitor makes no profit at all. If both firms cut price to $10, then each earns $1 million. The symmetric payoff matrix shown in Table 2.1 summarizes the possibilities:

<table>
<thead>
<tr>
<th>Firm A's price</th>
<th>$15</th>
<th>$10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15</td>
<td>(3,3)</td>
<td>(0,5)</td>
</tr>
<tr>
<td>$10</td>
<td>(5,0)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>

Table 2.1. Sample payoff matrix
The first number in parentheses refers to Firm A's one-period profit (in millions of dollars), while the second number is Firm B's profit. A quick scan of this matrix reveals the fact that each firm has incentive to choose the lower price regardless of its opponent's decision. Thus, the $10 price level is said to be a dominant alternative for each player. If each firm chooses its dominant alternative, then the (1,1) payoff results. Note that in a one-period game, neither firm has any incentive to unilaterally deviate from this outcome; therefore the $10 price is an equilibrium strategy for each player. Therein lies the dilemma: if each firm were willing to use its dominated alternative ($15), both would be better off. Some sort of binding agreement to maintain the higher price would be to the mutual benefit of both parties, but in a game such as this, with no binding agreements allowed (a non-cooperative game), it is often difficult to attain (or maintain) the cooperative payoff.

Of course, this 2x2 game is an extreme oversimplification of actual price competition. But the essence of the PD -- firms being unable to achieve high, cooperative payoffs due to incentives to cheat -- is a common theme in the Marketing and Economics literature. Dozens of researchers have cited the PD as a prevalent feature of oligopolistic competition; no textbook on industrial economics can be complete without at least a token reference to the PD. For instance, Scherer (1980) mentions the PD not only as a pervasive element in price competition, but also notes its presence in advertising expenditures and new product development.

Different strategic variables which lead to PD situations have also been modeled, including output quantities (Rao and Bass 1985), consumer promotions (Dickson 1985), product positioning (Hauser 1985), and channels of distribution
(McGuire and Staelin 1983, Jeuland and Shugan 1983). Many other studies do not mention the PD by name, but allow PD-like behavior to occur under certain conditions. Furthermore, texts on competitive strategy (e.g. Porter 1980, 1985) devote chapters to the analysis of PD situations, but without the catchword of "Prisoner's Dilemma." In his most recent book, Michael Porter makes the following observation:

Often firms make strategic choices without considering the long-term consequences for industry structure. They see a gain in their competitive position if a move is successful, but they fail to anticipate the consequences of competitive reaction. If imitation of a move by major competitors has the effect of wrecking industry structure, then everyone is worse off. (Porter 1985, p. 8)

The PD concept is widespread in contemporary business literature, but is not limited to business situations alone. Indeed, over its 30-year lifespan, the PD has been one of the most frequently studied phenomena in economics, political science, sociology, and psychology. See Axelrod (1981, 1984) for a review of these and other applications of the PD.

The classic 2x2 PD allows each player to either cooperate (C) or defect (D). Cooperation is analogous to choosing a high price in the aforementioned pricing game, while defecting is equivalent to cutting price. If both players cooperate in a given period, then each is rewarded with a payoff of R points (or dollars). If one player defects from mutual cooperation, he receives the temptation payoff of T, while the cooperating player gets the "sucker's payoff" of S. If both choose to defect, then each receives the punishment payoff of P.¹

¹The assumption that the payoff matrix is symmetric is not necessary in defining a Prisoner's Dilemma, but is commonly used for convenience. We will maintain this assumption for all games and models presented in this paper.
Table 2.2. Canonical Payoff Matrix for the 2x2 Prisoner's Dilemma

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Cooperate (C)</th>
<th>Defect (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate (C)</td>
<td>(R,R)</td>
<td>(S,T)</td>
</tr>
<tr>
<td>Defect (D)</td>
<td>(T,S)</td>
<td>(P,P)</td>
</tr>
</tbody>
</table>

In the pricing game of Table 2.1, the reward for cooperation is $R = 3$ million, the temptation to defect is $T = 5$ million, the sucker's payoff is $S = 0$, and the punishment for mutual defection is $P = 1$ million.

In order to make sure that the PD property holds, the payoffs $R$, $T$, $S$, and $P$ must meet certain constraints. The essential property, once again, is that each player has a dominant alternative (to defect), but if both defect, the resulting payoff $(P)$ is less than the payoff for mutual cooperation $(R)$. Three general types of conditions emerge from this property:

1) Regardless of what our opponent does, we are best off defecting. If he cooperates we prefer to defect (i.e., $T > R$), and if he defects we still favor defection ($P > S$).

2) Regardless of what option we choose, we are better off if our opponent is lenient and chooses his dominated alternative (cooperation). Thus, $R > S$ for when we cooperate, and $T > P$ for when we defect.

3) Mutual cooperation is always preferred to mutual defection: $R > P$. 
These three sets of inequalities can be combined into one compound inequality: $T > R > P > S$. This is the heart of the PD.†

Most researchers include an additional constraint in defining a PD. Imagine an iterated PD game, in which both players repeatedly make choices from the same payoff matrix. In this dynamic game, there may be forms of cooperation more complex than repeated choices of C by both players. For instance, the two players may agree (implicitly) to alternate choices of C and D on each move, resulting in alternating payoffs of T and S for each player. Since the $T > R > P > S$ constraint says nothing about this type of collusion, an additional constraint, $T + S < 2R$, is normally introduced to eliminate the incentives to profit by alternating C's and D's.

The Repeated Prisoner's Dilemma Game

Oligopolistic competition is often a dynamic process, and therefore should be modelled as such. Few markets exist for only one period: firms are likely to interact with one another (and with sellers) numerous time in the future. The recent growth of dynamic game theory has brought forth a variety of new modelling tools; researchers can finally go beyond the static models which have persisted for so many years (Friedman 1977).

† Although the PD and its $T > R > P > S$ condition are well accepted for their accurate representation of many different phenomena, slight variations of this condition have also been analyzed often. For example, the game with $T > R > S > P$ is commonly known as "Chicken," and is a popular alternative to the PD (Hamburger 1979). Other variations of these ordinal conditions are examined by Rapoport, Guyer, and Gordon (1976).
The easiest way to turn a static PD game into a dynamic one is to repeat the game a large (possibly infinite) number of times. Several standard assumptions are made about this dynamic game. Two have already been mentioned: both players seek to maximize their own profits, and no communication or binding agreements are allowed. It is further assumed that time is divided into discrete periods, and the payoff matrix does not change as the game progresses.†

Furthermore, each player is assumed to have complete knowledge about all previous moves (by both players). When a particular round ends, both choices (C or D) and the corresponding payoffs are revealed to both players with no uncertainty. The external validity of this assumption depends on the strategic variable(s) being modelled. Firms can often observe competitors' prices without much difficulty, but promotion expenditures or other less tangible variables might be harder to measure precisely.

Before any moves are actually made in a PD game, each player must choose a strategy. In game theory, a strategy is defined as a "plan which dictates the play of a player as a function of his information. A simple way of viewing a strategy is that it is the book of instructions that you might leave with a representative who is going to play the game for you," (Shubik 1970, p. 183). In other words, a player's strategy specifies how he will react under any possible contingency which may arise during the course of the game. A strategy may specify radical changes in behavior given a certain series of competitors' moves, but the strategy itself can not change once the game begins.

†Game theorists have devised PD-like models which relax both of these latter assumptions. Non-zero sum differential games, which incorporate time as a continuous variable, are frequently applied to marketing models, e.g., Wernerfelt 1985, Rao and Bass 1985. Both of these cited models also have time-dependent payoffs, otherwise known as stochastic games (Friedman 1977).
In the iterated 2x2 Prisoner's Dilemma one of the most basic strategies is constant defection (known as ALL-D). Although this strategy operates independently of the other player's behavior, it should always provoke the other player to act the same way. If player 1 is using ALL-D and player 2 is fully aware of this fact (perhaps through past experience), then player 2 never has incentive to cooperate; player 1 would be entirely insensitive to any cooperative gestures. Hence, the best response to ALL-D is ALL-D. This example demonstrates the multi-period analogue to the equilibrium strategy presented earlier. In any 2-player game, static or dynamic, a strategy \( S_i^* \) is a best response for player \( i \) if it yields the highest possible payoff against a given opposing strategy, \( S_j \):

\[
\Pi_i(S_i^*, S_j) = \max_{S_i} \{ \Pi_i(S_i, S_j) \} \text{ for all possible } S_i
\]

where \( \Pi_i(S_i, S_j) \) is the profit to player \( i \) when he chooses \( S_i \) against player \( j \)'s \( S_j \).

If both strategies, \( S_i \) and \( S_j \), represent best responses against each other, a Nash equilibrium results. As player 1 tries to anticipate the strategy of player 2, he (as a rational player) chooses the strategy that leaves himself best off. Similarly, if player 2 is rational, he will also make assumptions about player 1's behavior and choose his best strategy. If each player makes correct assumptions about his opponent's behavior and chooses his best response, then both strategies will be equilibrium strategies and neither player will have incentive to unilaterally deviate. Moorthy (1985) carefully develops the equilibrium concept and discusses its relevance in models of marketing and economic behavior.¹

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¹In general, neither best responses nor equilibria need exist or be unique. Such properties are subject to investigation.
A more specific type of equilibrium, known as collective stability (Axelrod 1984), occurs when a particular strategy is the best response to itself:

$$\Pi_i(S^*, S^*) = \max\{\Pi_i(S_i, S^*)\} \text{ for all possible } S_i$$

Thus, ALL-D is an example of a collectively stable strategy. Like Axelrod, we focus almost exclusively on collective stability when discussing equilibrium strategies.

Many game theorists who study the iterated PD game concentrate on creating equilibrium strategies for the game. Perhaps the most notable example is a strategy first popularized by Friedman (1971): Start cooperatively and continue cooperating until the other player defects for the first time. At that point switch to defection and play ALL-D for the remainder of the game. This model is often posed as a simplified but intuitive model of oligopolistic competition. It has been cited and extended many times in the marketing and economics literature (Moorthy 1985, p. 274). Friedman (1971) demonstrates that under certain conditions, this strategy will indeed be an equilibrium strategy.†

Because it specifies such an extreme change in behavior after a single defection, this strategy will be called XTRM for the remainder of this paper.

Other game theorists have chosen to eschew the equilibrium concept and instead have tried to create strategies that either describe human behavior in experimental PD games (Downing 1975) or encourage cooperation (Rappoport and Chammah 1965). A prominent example of the latter type of strategy is TIT FOR TAT. Like XTRM, this strategy starts cooperatively, but in all subsequent

†These conditions include uncertain game lengths or sufficiently low discounting of future profits. See Axelrod (1984) for additional details.
rounds it merely mimics what its opponent did in the previous round. TIT FOR TAT is the game theory equivalent of traditional wisdom such as "an eye for an eye." It is also a concise model of leader-follower behavior in oligopolies. Axelrod (1984) scrutinizes TIT FOR TAT in great depth, providing a host of cites and examples to show why this strategy is a simple but effective model of many different competitive situations.†

† Although TIT FOR TAT was not intentionally designed with equilibria in mind, it often is a collectively stable equilibrium strategy, particularly in situations where some players believe that other players are using TIT FOR TAT. See the discussions by Axelrod (1984) and Kreps, Milgrom, Roberts, and Wilson (1982)
CHAPTER 3:
THE AXELROD TOURNAMENTS

Creating strategies for games such as the repeated Prisoner's Dilemma is relatively easy; there are an infinite number of possible strategies. The hard part is trying to create effective strategies.

The effectiveness of a particular strategy can vary considerably, depending on what opposing strategies are to be faced. "Nice" strategies such as XTRM and TIT FOR TAT (i.e., those that never defect first) will fare very well against other nice strategies: each will receive the reward payoff every period. On the other hand, one does not want to be too nice against an opponent like ALL-D. The best response against ALL-D is to match its extreme uncooperativeness each period and accept the Punishment payoff each period for the remainder of the game.

Because the competitive environment plays such a large role in determining the success of each strategy, it is impossible to claim that any single strategy is "optimal" for the repeated PD. Nevertheless, in 1979 a political scientist named Robert Axelrod began a series of studies aimed at identifying the best strategy (or set of strategies) under a broad set of possible circumstances.

Axelrod first proposed a tournament in which fourteen professional game theorists submitted strategies in the form of computer subroutines, and all possible pairs of strategies played five 200 round PD games.† The entries included XTRM, TIT FOR TAT, and twelve others. In addition, RANDOM, which randomly chooses C or D on each move was also included.

† Axelrod's tournaments are not the first applications of programmed strategies in PD games. Previously, programmed strategies were used to induce and measure cooperation in games with human opponents (Oskamp 1971). However, Axelrod is believed to be the first to run PD games exclusively with programmed strategies in an attempt to determine a "winning" strategy.
The winner was the simplest entry of all, TIT FOR TAT, submitted by Anatol Rapoport. The top eight entries were nice, but none of the bottom seven finishers was nice. Each of these results came as quite a surprise. All of the entrants were aware of TIT FOR TAT (Axelrod included it as a sample strategy in the original proposal for the tournament), and several participants entered modified versions of it, but none could outscore it overall.

The strong performance of TIT FOR TAT is even more surprising in light of the fact that TIT FOR TAT can never win a single game. In many cases, TIT FOR TAT will tie its opponent (e.g., when the opponent is nice), but no strategy, even RANDOM, can ever lose to TIT FOR TAT.

One unexpected reason for the success of TIT FOR TAT is the influence of echo effects. Several strategies consist of basic TIT FOR TAT with occasional defections thrown in, hoping to catch cooperative entries off guard. Indeed, such strategies would earn the Temptation payoff for a single period against TIT FOR TAT, but frequently the rest of the game would be mutual defection. The only way a sneaky strategy could escape this rut is to absorb a Sucker's payoff for one round against TIT FOR TAT's defection. The net result is that both strategies earn an S, a T, and one P for each period of mutual defection. The nice alternative, i.e. always cooperating and receiving R's is generally more profitable.†‡

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† Two of the conditions defining the 2x2 PD, namely R > P and 2R > T + S, can be combined to form a compound condition 3R > P + T + S, so the result follows.

‡ The only time echo effects do not apply is at the end of the game. If a player knows when the last period will occur, it pays for him to defect in the final period against TIT FOR TAT's anticipated cooperation, since TIT FOR TAT will be unable to strike back. Several strategies took advantage of the fixed game length in Axelrod's first tournament, but these end-game effects had a minimal impact on the overall tournament standings.
Another key property which helped make TIT FOR TAT so successful is forgiveness. Consider XTRM: this strategy is nice and doesn't fall prey to echo effects. However, its punishment for an opponent's defection is perhaps too severe. Unlike TIT FOR TAT, XTRM never gives a defecting strategy an opportunity to atone for its transgressions. For this reason, XTRM came in seventh in the tournament, second worst among the nice strategies.

TIT FOR TAT is the most forgiving strategy among the fourteen entrants; it is always willing to start with a clean slate, as long as its opponent first makes a conciliatory gesture.

The Second Tournament

TIT FOR TAT's victory in Axelrod's first tournament, while highly informative, is not proof that TIT FOR TAT is "optimal." In fact, Axelrod identifies three strategies which would have won the tourney if they had been submitted. One is TIT FOR TWO TATS which defects only if its opponent defected on the previous two moves. Axelrod suggests that this finding highlights the importance of forgiveness, which apparently was largely ignored by the expert strategists.

A second tournament was run soon after the first tournament's results were tallied. This time Axelrod received 62 entries from participants representing a wide range of ages, disciplines, and geographic origins. Although end-game effects were negligible in the first round, Axelrod chose to run five games with different (a priori unknown) lengths for each of the 2016 possible pairs (including RANDOM again). The game lengths were chosen at random from a distribution with a median of 200 rounds.†

† A game-ending probability of 0.99654 was used at each round since it implies an expected median game length of 200 rounds. The game lengths actually chosen were 63, 77, 151, 156, and 308 rounds; the average and median length (151 rounds) were shorter than expected.
Most of the contestants were aware of the principal results from the first round, so many of the new entries were designed to cooperate with or take advantage of strategies which were nice and forgiving. Several attempts were made to modify TIT FOR TAT to be able to identify RANDOM or highly uncooperative opponents.

Despite all these efforts to unseat the defending champion, Anatol Rapoport's TIT FOR TAT won again.†

Niceness proved once again to be the most influential property. Fourteen of the top fifteen strategies would never defect first, while fourteen of the fifteen worst entries lacked this property. Forgiveness was still important, although the different blend of entered strategies led to different conclusions. For example, TIT FOR TWO TATS, which would have won the first round, would have placed 24th in the second tournament. However, the least forgiving entry, XTRM still performed poorly -- 52nd overall and last among nice entries.

Provocability is another fundamental feature that helps explain the success of TIT FOR TAT in the second tournament. According to Axelrod, a rule is provicable if it defects immediately after an "uncalled for" defection. A precise definition of "uncalled for" is not given, but, intuitively, any defection which destroys a stable, mutually cooperative relationship is always uncalled for. A strategy such as TIT FOR TWO TATS is not highly provicable, since it only defects after the second of two consecutive defections. Several of the entries in the second tournament defect frequently but never consecutively against TIT FOR TWO TATS, and therefore badly exploit its low provocability.

† Anyone was eligible to enter TIT FOR TAT or any other rule, but only Rapoport chose to do so.
Provocability and forgiveness are both commonsense properties for firms engaged in oligopolistic games. Provocability suggests that each firm should recognize and appropriately punish any price cuts by its competitors. Firms often engage in price wars to assert their provocability: witness the countless retailers who boldly claim, "We will not be undersold!" Eventually a price war will end when one or more firms raise prices and others follow suit. This is equivalent to forgiveness.

Axelrod identifies two additional properties which further help distinguish between good and bad strategies: envy and clarity.

Envy refers to strategies which not only try to maximize their own profits, but also try to reduce competitors' profits at the same time. It is an undesirable trait for players in PD games. Beating an opponent by a wide margin does not guarantee a high score for the winning strategy. TIT FOR TAT is the best example: as noted earlier, TIT FOR TAT never wins a game, but it succeeds by consistently scoring well, even when it loses.

Behr (1981) reanalyzed Axelrod's first tournament, examining number of wins and winning margins as the primary objectives. Not surprisingly, the nasty strategies were more successful at winning games than they were at earning high scores. TIT FOR TAT, on the other hand, placed last among the fifteen first round strategies when ranked according to victories (with zero), and ranked twelfth when ties were counted also.

Envy is an important aspect of actual oligopolistic competition and experimental PD games. A short-sighted manager/player may gain satisfaction (but not necessarily profits) in trying to top a competitor's earnings. This is
particularly true if salaries and bonuses are based on comparisons between a manager's firm and the other firms in the industry. Such a player often ends up with a bigger slice of a smaller pie. Shubik (1959, 1982) has given the name "difference maximizing" to this type of behavior and has studied other types of envious behavior as well (Shubik 1971). Axelrod (1984, p. 110-111) discusses experimental games with students given explicit instructions to try to score well for themselves instead of playing to win:

> These instructions simply do not work. The students look for a standard of comparison to see if they are doing well or poorly. The standard, which is readily available to them, is the comparison of their score with the score of the other player. Sooner or later, one student defects to get ahead, or at least to see what will happen. Then the other usually defects so as not to get behind. Then the situation is likely to deteriorate with mutual recriminations. Soon the players realize that they are not doing as well as they might have, and one of them tries to restore mutual cooperation. But the other is not sure whether this is a ploy that will lead to being exploited again as soon as cooperation begins once more.

Human players often pay dearly before they fully understand the ramifications of envy. Unfortunately, many players (and programmed strategies) never rid themselves of their envious yearnings. The few who do learn the lesson are usually glad they did.

Clarity is another uncomplicated but surprising property: do not be too clever. In both tournaments, there was no overall association between each strategy's success and its complexity, but one observation held true in both cases -- the simplest strategy (TIT FOR TAT) was the most successful. This was no fluke, according to Axelrod. TIT FOR TAT is easy to recognize as an opponent, and once it is recognized, the optimal response is readily determined: play TIT FOR TAT (since it is collectively stable).'

* Other nice strategies such as ALICE and XTRM are also best responses to TIT FOR TAT. In this case, therefore, the best response exists but is not unique.
Several entries relied on complex probabilistic processes to choose C or D. An opponent of such a rule might not be able to infer the exact nature of the process, and sometimes games would degenerate into mutual defection as a result of this misunderstanding. The last-place rule in the first tournament is a fine example: it usually cooperated between 30 and 70 percent of the time, leading some strategies to believe that it was RANDOM.

The decision processes of human players and managers are far more complex than any programmed strategy, so even in a simple 2x2 game, the observed behavior of these players is often impossible to decipher. Perhaps if players tried to clarify their motives and objectives, there might be less confusion and misunderstandings in actual games. Nobel laureate Herbert Simon invokes Occam’s Razor as guidance for behavior in business organizations: “Accept the simplest theory that works ... [and] make no more assumptions than necessary to account for the phenomena,” (Simon 1979, p. 495).

The Robustness of TIT FOR TAT in Different Environments

The sharp decline of TIT FOR TWO TATS from the first tournament to the second highlights the pivotal role of the competitive environment. The performance of any strategy greatly depends on the nature of the opponents to be faced. Before we have complete faith in any one strategy, we demand evidence that it will succeed in any environment which might arise.

To pursue this idea, Axelrod analyzed several hypothetical tournaments, creating each new tournament by varying the competitive environment in sensible ways. First he increased the presence of five representative strategies (as determined by stepwise multiple regression). Five hypothetical tournaments
were created in which the size of one representative strategy was quintupled; that is, all games involving the representative strategy were weighted five times as highly as other games. A sixth tournament was created by similarly weighting the residuals from the multiple regression. See Axelrod (1984, pps. 203-205) for more details on this procedure.

Since the five representative rules and the residuals cover a broad spectrum of possible strategies, each of the six hypothetical tournaments includes a competitive environment substantially different from that of the actual tournament. For instance, two of the representatives are nasty rules which do not fare well in the actual tourney; the hypothetical tournaments which emphasize each of these two rules are therefore characterized by extremely competitive behavior. If a particular strategy can perform well in each of these hostile environments as well as in the nicer environments, we can validly claim that the strategy is robust.

TIT FOR TAT did not disappoint us. It prevailed again, winning five of the six hypothetical tournaments, including both which featured a nasty representative, and it placed second in the only hypothetical tournament it failed to capture. Looking at all the strategies, the scores in the hypothetical tournaments correlated fairly well with the scores of the actual tournament (the correlations ranged from 0.82 to 0.96), so the overall results of the second tournament seem quite robust.

Despite this convincing evidence, Axelrod chose to run one more hypothetical tournament, an ecological tournament based on the notion of survival of the fittest. Imagine that each entry in the tournament could clone itself and create offspring in proportion to its total score. Thus, TIT FOR TAT would spawn the
most clones, while a less successful rule, such as RANDOM, would have a relatively small representation in the second generation. It would then be possible to run a tournament among this group of offspring, and again, each second-generation entry would clone itself in proportion to its second-generation score. This process could be repeated over and over, until the relative proportion of each entry in the population becomes stable.

The original motivation for this type of tournament stems from evolutionary biology. Each strategy can be considered as a different animal within a given species. When two animals meet they can try to cooperate with or exploit each other. The success or failure of each animal's endeavors will dictate whether or not it survives until the next round of interactions.

A similar story can be told for firms competing in the same market. If one firm is frequently exploited by its competitors, it will lose money and eventually drop out. Conversely, a consistently successful firm will enjoy substantial profits and will become an even greater presence in the future.

Axelrod ran over 1000 of these simulated generations. The environment at stage $t$ consisted of the 63 strategies weighted by their scores from stage $t-1$. Figure 3.1 shows that by the 200th generation, most of the original entries were negligible in the environment. Rule 8, the only nasty rule to finish in the top 15 in the actual tournament, thrived for a while on the growing proportion of nice strategies. It eventually foundered, however, when easily-exploited strategies began to disappear. With a few exceptions (e.g. rules 5 and 8), the best strategies from the first generation tend to stay on top. By generation 1000, TIT FOR TAT is clearly in the lead, and is widening its margin after each generation.
After all these impressive analyses by Axelrod, and all these equally impressive performances by TIT FOR TAT, it may be tempting to claim that TIT FOR TAT truly is the best strategy. Axelrod cautions against making such an inference. Although TIT FOR TAT is doubtlessly robust and effective, it can be beaten in certain environments. Three examples have been shown so far: TIT FOR TWO TATS would have won the first tournament; in one hypothetical tournament, TIT FOR TAT came in second; and in a world consisting only of ALL-D players, ALL-D is the best response. Furthermore, it is theoretically possible to create a modified TIT FOR TAT which can successfully identify and defect against RANDOM, but play like regular TIT FOR TAT at all other times. Such a strategy could do no worse than TIT FOR TAT, and would beat TIT FOR TAT whenever RANDOM is present. (Axelrod reports several attempts to create such a strategy, none successful.)
To summarize, Axelrod’s tournaments firmly establish the importance of five pivotal properties in repeated Prisoner's Dilemma games. First and foremost is niceness (i.e. always cooperate if your opponent has not yet defected). Forgiveness (i.e. give your opponent another chance to cooperate after he defects) was surprisingly influential, especially in the first tournament. In the second tournament, however, the role of forgiveness was tempered somewhat by provocability (i.e. always punish “uncalled for” defections immediately). Successful strategies should also strive for clarity, but should avoid envy.

Any strategy that abides by all five of these properties will generally perform well for itself, but the clearest, simplest manifestation of these properties, TIT FOR TAT, proved to be the most prosperous strategy of all.

Beyond the 2x2 World

Less than one-quarter of Axelrod's book is devoted to the simulated PD games described above. After making his observations about the success of TIT FOR TAT in computer tournaments, the author begins to apply his findings. He describes a number of situations, including trench warfare, legislative logrolling, and buyer-seller relationships which can be interpreted within the 2x2 framework. Axelrod especially concentrates on how TIT FOR TAT can operate in such systems, and more generally, how cooperation can begin and flourish in a world of "meanies."

Axelrod's work has been highly praised, both for its insightful contributions to a well-known discipline (non-cooperative game theory) and for its social and political relevance. Some reviewers, however, take issue with some of the assumptions made in the book. For example, Axelrod only analyzes a two-player
game, but draws implications for several multi-player affairs. One critic, while favorably inclined towards the book as a whole, does not accept Axelrod's N-person generalizations: "To these situations, Axelrod's simulation and formalization have, as far as I can tell, no applicability," (Bonacich 1985, p. 10).

Throughout the history of the Prisoner's Dilemma, cautious researchers have warned against overapplying the 2x2 game. Martin Shubik, one of the most prolific contributors to non-cooperative game theory, is especially wary of real-world generalizations drawn from the PD:

The very simplicity of this game is a danger. Analogies between it and human affairs are best employed to study their inadequacies and to pinpoint what has been left out rather than to claim how much of the world can be packed into a 2x2 matrix. (Shubik 1970, p. 181)

Although Axelrod's liberal use of the PD as a model of different phenomena might go against Shubik's advice, Axelrod does recognize that various extensions to the 2x2 PD might provide better representations of these same events. In a footnote to his book (p. 221), Axelrod lists some interesting possibilities which await examination, including:

1. The length of the game might depend on the success or failure of its players. For instance, firms may go bankrupt and drop out of an industry. Shubik (1959) suggests a class of games known as "games of economic survival" which incorporate this concept. More recently, Majeski (1984) analyzes an iterated PD game with different subjective termination probabilities for each player.

2. The constituent (i.e. one-period) game may not be the simple 2-alternative PD game that Axelrod analyzes. The game may not be a PD at all, or the payoff matrix may change over time. A third possibility, which we will focus on, is that each player may have more than two alternatives at each round.
3. The game may involve more than two players at a time. The N-person PD, as we will discuss in the next chapter, can be far more complicated than the two-person game. Since each player must worry about at least two opponents, it is much harder to maintain cooperation.

4. Each player may have incomplete information about his opponent's past behavior. As previously mentioned, different strategic variables may be associated with different levels of certainty. To examine the effects of uncertainty, Axelrod reran his second tournament with a one percent chance that each player misperceives his opponent's move at each round. TIT FOR TAT won again.

Each of these extensions can help improve the descriptive power of the basic PD, especially for models of oligopolistic competition. Incorporating these changes is not a difficult task; many existing models routinely include all or some of these extensions. However, models that are more realistic are also more complicated; the focus strays from the PD structure, which becomes hidden in a complex model structure.

The models to be developed in this paper employ two extensions -- multiple players and multiple alternatives -- while still concentrating on the PD concept. In the chapters that follow, we introduce each of these extensions slowly, thereby generalizing the Prisoner's Dilemma without masking its importance. Although the models presented here are extremely simple in comparison to other contemporary models of oligopolistic competition, the types of behavior generated by these models, as will be shown, are far from trivial. We will demonstrate that the PD, even in simple models of oligopoly is truly a force to be reckoned with.

After introducing these extensions and discussing some of the mathematical properties of the Generalized Prisoner's Dilemma (GPD), we will motivate two specific GPD models and discuss two Axelrod-like tournaments that we have
completed. The intuitive appeal of Axelrod's methodology carries over nicely to a GPD model; although our first tournament is considerably more complicated than Axelrod's (in terms of size and observed behavior) and our second tournament is even more complicated than the first, we still find a manageable set of properties such as niceness that help explain the success or failure of most strategies.
CHAPTER 4:
THE GENERALIZED PRISONER'S DILEMMA

When we first specified the constraints on the payoffs in a 2x2 PD, four types of conditions were listed for the one-shot PD game: (1) Each player prefers to defect, regardless of the other player's decision; (2) Each player is better off if his opponent becomes more cooperative; (3) Both players prefer mutual cooperation to mutual defection; (4) Constant cooperation is more preferable than alternating cooperation.

These conditions can be restated in the context of a multi-firm continuous-alternative pricing game essentially by substituting the words "cut price" for "defect," and "raise price" for "cooperate":

(1) Each firm can increase his profits by cutting his own price, regardless of the prices chosen by his opponents

(2) Each firm's profits increase as one or more competitors raise their prices.

(3) All firms are better off if all prices increase by a uniform amount.

(4) All firms are better off maintaining constant prices rather than periodically alternating between high and low prices.

These four propositions qualitatively define the Prisoner's Dilemma for any strategic game with an arbitrary number of players and alternatives. More importantly, each of these ideas is consistent with basic economic theory and will likely hold true for a large number of oligopolistic markets. Isolated exceptions can be identified (e.g., status goods with strong price-quality relationships), but by and large, these conditions are highly realistic.
Previous work on PD games beyond the 2x2 case has led to similar conditions on the payoffs to each player. Two different approaches have been taken to extend the 2x2 game:

**N-Player 2-Alternative Games:** Many sociologists have suggested that each individual in a large group can become entangled in a PD when facing a societally relevant action such as polluting, contributing to a charitable cause, or using an exhaustible resource.

Littering on public streets is a common example. An individual may drop a wrapper or empty can in the gutter and rationalize his action by thinking, "It's far easier for me to drop it rather than carry it to a wastebasket. Besides, it's only one small piece of garbage." But suppose that all people in one city made the same decision: the city would become a dirty, unhealthy place. All residents would suffer.

The central dilemma is in accordance with the generalized PD just presented. Littering is a dominant alternative; each person is better off littering regardless of what others do. But each person prefers a cleaner city, and if everyone chooses to litter, the total "payoff" to society (and to each individual) is worse than if everyone cooperated.

Luce and Raiffa (1957) provided an early economic example of the N-player PD, although they did not formally develop the model:

As an n-person analogy to the prisoner's dilemma, consider the case of many wheat farmers where each farmer has, as an idealization, two strategies: "restricted production" and "full production." If all farmers use restricted production the price is high and individually they fare rather well; if all use full production the price is low and individually they fare rather poorly. The strategy of a given farmer, however, does not significantly affect the price level - this is the assumption of a competitive market - so that regardless of the strategies of the other
farmers, he is better off in all circumstances with full production; yet if each acts rationally they all fare poorly. (Luce and Raiffa 1957, p. 97).

The N-player PD, also known as "The Tragedy of the Commons," (Hardin 1968) was formally developed by several authors in the mid 1970's (Hamberger 1973, Schelling 1973, Goehring and Kahan 1976, Taylor 1976), and was recently reviewed by Dawes (1980) and Hardin (1982).

**2-Player Multiple-Alternative Games:** Relatively little work has focused on the multiple alternative PD game. Most applications have been models of disarmament. In the mid 1960's, Marc Pilisuk performed several experimental studies with an "extended" PD (e.g. Pilisuk and Rapoport 1964; Pilisuk, Potter, Rapoport, and Winter 1965; Pilisuk and Skolnick 1968) to study the relationship between personality characteristics ("Peace Doves" vs. "War Hawks") and subjects' willingness to cooperate. The primary motivation for the use of multiple alternatives, according to Pilisuk, et al (1965), is that the added choice flexibility permits a more refined study of gestures and responses which may lead players to suspicion or trust.

The payoff matrices used in these extended PD games usually obey the three generalized PD propositions stated above. For example, Pilisuk and Skolnick (1968) used the following payoff matrix where each player chooses a discrete action between 0 and 5 (shown along the outside of the matrix). The payoffs to the row player are shown above the diagonal in each entry, and the column player's payoffs are shown below the diagonal.
Notice that each 2x2 submatrix is by itself a legitimate 2x2 PD. Each interior entry is the (R,R) payoff for one 2x2 game, the (P,P) payoff for another, and the (T,S) and (S,T) payoffs for two other 2x2 games. The unique equilibrium for a one-shot game with this payoff matrix is the lower right corner, in this case entry (5,5) with a payoff of 0 to each player. If either player falls prey to his own greedy self-interest, then both players might be trapped in this corner for the duration of the game.

The models for the proposed tournaments go a step beyond these multiple-alternative games: we assume that each player faces a continuous range of alternatives (prices). Continuous alternative PD games are not entirely new; most models of oligopoly use continuous profit functions, but few explicitly acknowledge the PD structure (if present), and even fewer explicitly define the conditions which imply the presence of a Prisoner's Dilemma. We will undertake that task as part of this thesis. We now present one possible definition and interpretation of the generalized PD (GPD).

**Formalizing the Generalized Prisoner's Dilemma**

We continue to use an oligopolistic market (with price as the sole strategic variable) as our prototypical GPD model. However, the relevance of our work is
not limited to this type of model. We can easily apply our GPD framework to
different oligopolistic markets (e.g., with output quantities as the strategic
variable), or to other common PD scenarios, such as disarmament. We leave
these different applications to the reader.

Let $\Pi_i(P_1, P_2, \ldots, P_i, \ldots, P_N)$ represent the payoffs (profit) to player (firm) $i$
where $P_1$ through $P_N$ are the prices set by each of the $N$ firms. For the remainder
of this paper we call $\Pi_i$ profit and $P_i$ price with the understanding that they can
be general payoffs and actions. (Since profit normally declines in price, the
reader will find he must reverse the signs of our conditions when analyzing
actions in which more is better, such as output quantities.)

First we define two key prices: the short-term, non-cooperative, profit-
maximizing price, $P^*$, and the joint-profit maximizing (cooperative) price, $P^0$,
which are equivalent to $D$ and $C$, respectively, in the 2x2 game. Each firm can
maximize its one-period profits by choosing $P^*$, regardless of other players'
prices.† At the other extreme, if all firms are totally cooperative, they will all
choose $P^0$ to maximize joint profits.

For simplicity and consistency with the 2x2 game, we first consider the class
of games in which $P^*$ and $P^0$ are fixed and invariant with respect to competitors' 
prices.‡ Thus, each firm has the short-run incentive to set its price as low as $P^*$,

---

† $P^*$ is also known as the Cournot or Cournot-Nash price. The Cournot model was originally
designed to model oligopolistic markets with output as the strategic variable, but has been
frequently used to model price competition as well. See Friedman (1977) for additional
discussion on the Nash and Cournot equilibrium concepts and their applications.

‡ In general, the Cournot price will not be unique and invariant, as we assume in this first
model. This property, also known as separability (Friedman 1977), is a convenient feature of the
demand function of this initial GPD game. Our first tournament will be consistent with this
assumption, but in the second tournament $P^*$ will be a function of competitors' prices. More
generally, $P^*$ can be affected by external factors, such as time and random demand shocks.
but not any lower. If all competitors' prices are held constant, each player will reduce his profits if he cuts his price below $P^*$. Similarly, no group of cooperating firms should ever jointly raise their prices above $P^0$. Joint profits can only decrease as any firm's price goes higher than $P^0$.

With these two important prices defined, we can now specify the GPD conditions using the derivatives of the profit function. Since we are assuming a symmetric game, we arbitrarily choose firm 1 as our focal player and state all conditions based only on firm 1's profit function $\Pi_1(P_1, P_2, \ldots, P_N)$. The same conditions must hold for all other firms.

*Condition 1*

First we must ensure that firm 1 can always increase its short-run profit by lowering its price, as long as $P_1 > P^*$:

$$\frac{\partial \Pi_1}{\partial P_1} < 0 \quad \text{for} \quad P_1 > P^* \quad (4.1)$$

An alternative interpretation of this constraint is that firm 1 always decreases its profit by unilaterally increasing its price (when $P_1 > P^*$). Condition 1 is the generalized version of $T > R$ and $P > S$ in the 2x2 Prisoner's Dilemma. In a broader sense, this condition states that each firm faces a downward-sloping demand curve with respect to its own prices, a standard feature in virtually all models of price competition.

*Condition 2*

The next condition says that any price increase (decrease) by any one competitor will always increase (decrease) firm 1's profit:
\[
\frac{\partial \Pi_1}{\partial P_j} > 0 \quad \text{for } j = 2, 3, \ldots, N \quad (4.2)
\]

Note that this condition holds for the full range of competitor's prices. That is, even if a competitor is outside of the \((P^*, P^0)\) range, its price increases still aid firm 1. Condition 2 is the generalized version of \(R > S\) and \(T > P\) in the 2x2 Prisoner's Dilemma.

**Condition 3**

We must guarantee that mutual cooperation is profitable. If all firms were willing to increase their prices by a uniform amount, then firm 1 should be better off (as long as no prices exceed \(P^0\)):

\[
\frac{\partial \Pi_1}{\partial P_1} + \frac{\partial \Pi_1}{\partial P_2} + \ldots + \frac{\partial \Pi_1}{\partial P_N} > 0 \quad P_j < P^0 \quad \text{for } j = 1, 2, \ldots, N \quad (4.3.1)
\]

Consider (4.3.1) for the 2-player game:

\[
\frac{\partial \Pi_1}{\partial P_1} + \frac{\partial \Pi_1}{\partial P_2} > 0 \quad \text{or} \quad \frac{\partial \Pi_1}{\partial P_2} > -\frac{\partial \Pi_1}{\partial P_1} \quad \text{for } P_1, P_2 < P^0 \quad (4.3.2)
\]

This condition has a particularly interesting interpretation: firm 2's price increase apparently helps firm 1 more than firm 1's own price decrease. Fortunately, there is a sound economic argument behind this surprising result. Either price change (increasing \(P_2\) or decreasing \(P_1\)) will boost firm 1's demand, but only under the latter change can firm 1 increase its sales without cutting its margin. Whether or not this condition holds in actual markets is an empirical question, but if a competitive pricing game is to be interpreted as a Prisoner's Dilemma, this condition must hold true. Condition 3 is the generalized version of \(R > P\) in the 2x2 Prisoner's Dilemma.
Condition 4

Our final condition eliminates possible incentives for profitable oscillations and is an analogy to the $2R > T + S$ condition in the $2 \times 2$ game. We state it first for the two-firm game. For that game we wish to discourage oscillations in which the firms periodically alternate taking price cuts of $d$ dollars. In particular, we want profits to decrease for deeper and deeper price cut oscillations. One way to model this condition, for prices between $P^*$ and $P^0$, is:

$$\frac{\partial}{\partial d} [\Pi_i(P_1 - d, P_2) + \Pi_i(P_1, P_2 - d)] < 0 \quad \text{for } P_i < P^0 \quad (4.4.1)$$

Note the similarity of (4.3.2) and (4.4.1). The two conditions have inequalities in opposite directions because (4.3.2) refers to price increases whereas (4.4.1) refers to price decreases. More generally, condition (4.4.1) can be extended to rule out oscillations among three or more firms:

$$\frac{\partial}{\partial d} [\Pi_i(P_1 - d, P_2, ..., P_N) + \Pi_i(P_1, P_2 - d, ..., P_N) + ... + \Pi_i(P_1, P_2, ..., P_N - d)] < 0 \quad (4.4.2)$$

for $P_i < P^0$.

However, there are other ways to specify this condition that are not merely restatements of (4.3.1). For example, if we assume that all $N$ firms have identical prices ($P_1 = P_2 = ... = P_N = \hat{P}$) then condition 4 can also be stated as:

$$\frac{\partial \Pi_1}{\partial \hat{P}} + \frac{\partial \Pi_2}{\partial \hat{P}} + ... + \frac{\partial \Pi_N}{\partial \hat{P}} > 0 \quad \text{for } \hat{P} < P^0 \quad (4.4.3)$$

Although this condition still looks like (4.3.1), it is different because it explicitly accounts for each firm's profit function.

---

*There may be times when firms do have strong incentives to engage in coordinated oscillations. One common example is that of Coke and Pepsi offering substantial price promotions on alternate weeks within a given store. Lal (1986) develops a model of price promotions which features coordinated oscillations as part of an equilibrium cooperative strategy.*
We close this section by acknowledging that there are a number of different ways to represent condition 4, but all suggest the same basic idea: avoid oscillations. And finally, we emphasize that we have included condition 4 not for its structural realism, but because it generalizes a condition that is commonly included in the 2x2 PD game. Pricing games that obey conditions 1-3 but not condition 4 are still relevant and interesting and can be analyzed in other contexts.

Conditions 1-3, as stated, define the GPD. However, each condition only indicates the profitability of unilateral or uniform price changes. In defining a GPD, we also wish to know whether or not firm 1 can profit by a particular non-uniform price change by two or more firms. We can begin to analyze such changes by viewing conditions 1-3 as *directional derivatives* of firm 1's profit function. Consider a three firm game, for example. Condition 1 suggests that the gradient of $\Pi$, is positive everywhere along the vector $(-k_1, 0, 0)$ for any positive value of $k_1$. Condition 2 says the same about the vectors $(0, k_2, 0)$ and $(0, 0, k_3)$. Finally, condition 3 generalizes into the vector $(k_4, k_4, k_4)$, where $k_4$ represents the uniform price increase for all three firms.

The major benefit of this directional derivative interpretation is that we can take convex combinations of these vectors and still maintain a strictly positive gradient. Thus any simultaneous price change that is associated with a vector within the space

$$(-k_1, 0, 0) + (0, k_2, 0) + (0, 0, k_3) + (k_4, k_4, k_4)$$

where $k_1-k_4$ are non-negative (and at least one element is strictly positive) is a profitable change for firm 1.
Having expressed our initial assumptions and definitions for the GPD and the profit function, we now present the specific model used in our first tournament.

**An Example GPD Model**

For the first competitive strategy tournament (MITCS1), we assume a symmetric three-firm market with a time-invariant constant elasticity demand curve:

\[
Q_1 = KP_1^{-a}P_2^{\beta_2}P_3^{\beta_3}
\]  

(4.5)

The price elasticity of demand for firm 1 is assumed to be \( a = 3.5 \), while the cross-price elasticities for player 1 with respect to firms 2 and 3 are set at \( \beta_2 = \beta_3 = 0.25 \). We also set the scaling constant, \( K \), equal to 3375. These elasticities are reasonable and consistent with previously estimated values for a variety of markets (Telser 1962; Lambert, Naert, and Bultez 1975; Simon 1979).

Furthermore, we believe that the game is not particularly sensitive to the actual values chosen; our empirical analyses suggest that many values obeying conditions 1-4 will lead to equivalent types of behavior, given the constant elasticity demand curve in equation 4.5.

To properly scale each firm's profits, we assume without loss of generality that each firm faces fixed costs of 480 currency units per period and that variable costs are constant and equal to one currency unit per item sold. Thus, the profit function faced by each firm at each period is:

\[
\Pi_1 = 3375P_1^{-3.5}P_2^{2.5}P_3^{2.5}(P_1 - 1) - 480
\]  

(4.6)

This profit function is a very simple model of oligopolistic pricing, but it does

---

*Since the game is symmetric and there is no preferred ordering among the players, we continue to state demand and profit functions, etc., only for player 1.*
contain the fundamental elements of price competition. More importantly, it is a legitimate generalization of the PD, as the reader can verify.

Having specified the profit the profit function (4.6), we are now able to calculate the two key prices, $P^*$ and $P^0$. The non-cooperative price, $P^*$, is calculated by maximizing (4.6) with respect to firm 1's price. If we take the derivative of (4.6) and set it equal to zero, this price emerges as:

$$ P^* = \frac{a}{a-1} = \frac{3.5}{2.5} = 1.4 $$

Since the profit function is completely symmetric, $P^*$ for firms 2 and 3 is also 1.4.

The Pareto optimal price, $P^0$, is obtained by maximizing the total profit to all three firms if they are perfectly cooperative. Joint profits are maximized at:

$$ P^0 = \frac{a - \beta_1 - \beta_2}{a - \beta_1 - \beta_2 - 1} = \frac{3}{2} = 1.5 $$

The close analogy between $(P^0, P^*)$ and (C, D) is evidence that our game, when restricted to these two prices, meets the requirements of a 2x2 Prisoner's Dilemma. Suppose, for a moment, that firms 2 and 3 were choosing the same price as one another (although any explicit agreements are still illegal in our game). Table 4.1 illustrates the approximate payoffs under this two alternative game:

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1 = P^0$</td>
<td>$P_2 = P_3 = P^0$</td>
</tr>
<tr>
<td>(20,20)</td>
<td>(3,21)</td>
</tr>
<tr>
<td>$P_1 = P^*$</td>
<td></td>
</tr>
<tr>
<td>(29,11)</td>
<td>(12,12)</td>
</tr>
</tbody>
</table>

Table 4.1. Example Payoffs from One GPD Game.
The first number in parentheses is the profit to firm 1 and the second number is the profit earned (separately) by firm2 and firm 3. These payoffs, although asymmetric [because $\Pi_1(P^*, P^0, P^0) > \Pi_1(P^*, P^*, P^c)$], clearly obey the two constraints for a 2x2 PD.

Of course, these restrictions ($P_2 = P_3 = P^* or P^0$) are not realistic, nor are they actually imposed on the tournaments. But this exercise does help to show the close relationship between the 2x2 PD and our more complex model of price competition.

**Envy**

Before proceeding to our theoretical analysis, we note one more important feature of the GPD model, the envious price. When discussing Axelrod’s tournaments, we noted that some players often defect in an attempt to beat their rivals rather than to score well for themselves. In the GPD, a distinct price, $P^e$, is associated with this type of behavior.

The envious price is defined as the price that maximizes the difference between one player's profits and his opponents' profits. It is consistent with the notion of *difference maximization* mentioned earlier. Any player who misses the main point of the game (i.e. maximize profits) and instead plays to win each game will frequently choose the envious price:

$$P^e = \frac{a + \beta}{a + \beta - 1} = \frac{15}{11} \approx 1.36$$

where $\beta = \beta_2 = \beta_3$.

In our game, each firm can assure itself of getting more profit (or at worst equal profit) than its competitors by constantly choosing the envious price. Note,
however, that a profit-maximizing player should never use $P^*$, except in cases that require severe punishments for non-cooperative behavior.¹

¹Abreu (1986a,b) has recently proposed a class of strategies known as "carrot and stick" strategies which use severe punishments (as low as $P^*$ and even lower) as a credible threat to enforce maximally collusive behavior.
CHAPTER 5: IMPLICIT COALITIONS

The combination of multiple players and continuous alternatives provides for intermediate forms of cooperation somewhere between mutual defection and optimal cooperation. In this chapter, we discuss the tactics that firms can (and should) use if they wish to keep their profits as high as possible, even after a cooperative market begins to disintegrate towards a competitive one. Firms can signal and respond to other firms' cooperative gestures in two ways, by engaging in implicit coalitions with some (but not all) of their competitors, and by taking full advantage of the continuity of the strategic variable in the GPD. We introduce each of these concepts and pursue several theoretical issues that demonstrate their importance in the next two chapters. This theoretical work should help us gain some understanding about why these tactics are so important; it should also illustrate how smart oligopolists should behave to perform most effectively in repeated GPD games. Our GPD tournaments, discussed in the next two chapters, provide empirical evidence that will allow us to test the validity of our theoretical propositions.

Implicit Coalitions: Overview

Suppose firm 3 in our hypothetical three firm market is consistently choosing \( P^* \) (the GPD analogue to ALL-D). How should the firms 1 and 2 react? One option is to punish the defector by also playing ALL-\( P^* \), since that strategy is a best response to itself. However, if firms 1 and 2 are still willing to cooperate with each other, they might prefer a price higher than \( P^* \) that could earn them greater profits than three-way mutual defection. What then is the optimal choice for the two cooperative firms? The answer is the implicit coalition price, \( P^c \),
which maximizes the joint profits of two firms when they account for the uncooperative behavior of the third firm. In other words, if firm 3 is uncooperative, firms 1 and 2 seek to set their prices at \( P^c \) such that:

\[
\Pi_i(P^c, P^c, P^*) = \max \{\Pi_i(P_i, P_i, P^*)\} \quad \text{for all possible} \quad P_i \quad (i = 1, 2).
\]

For our first GPD game, \( P^c \) is calculated as:

\[
P^c = \frac{a - \beta}{a - \beta - 1} = \frac{13}{9} \approx 1.44.
\]

This price, a natural compromise between \( P^0 \) and \( P^* \), is fixed at 1.44 regardless of the third player's price, and applies to any of the three possible implicit coalitions which may emerge in our game. In an \( N \) player game there are \( N - 2 \) implicit coalition prices corresponding to implicit coalitions of 2, 3, \ldots, \( N - 1 \) players. (One might also wish to consistently describe \( P^* \) and \( P^0 \) as coalition prices for coalitions of 1 and \( N \) players, respectively.)

Table 5.1 presents the possible payoffs in a 3-player 3-alternative game limited to \((P^*, P^c, P^0)\).

**Several Potential Applications**

It is possible to identify a number of oligopolistic markets which may involve implicit coalitions. The domestic airlines often provide a good example. When major carriers such as United, Delta, and American are not trying to beat each other up with their own fare wars, they often maintain relatively high prices while low-price airlines such as Peoples Express (or in its latest incarnation, Peoples Express).
Table 5.1. Payoffs in a 3x3 Prisoner's Dilemma, where $P^0 = 1.50$, $P^c = 1.44$, and $P^* = 1.40$. The first, second, and third lines in each entry refer to the payoff to player 1, 2, and 3, respectively.

Continental) are flying the same routes at much lower rates. Thus, United, et al, may be purposely choosing some sort of coalition price ($P^c$) to maximize their profits, given the uncooperative ($P^*$) price of Peoples Express.

Sometimes these coalitions may be clearly explicit instead of implicit, as in the case of oil-producing nations. In late 1985, OPEC recognized that they could no longer sustain prices at the collusive levels that had persisted for the previous decade, due to a worldwide oil glut and the larger role being played by Britain, Mexico, and other non-OPEC nations. Saudi Arabia led a series of price cuts and production increases in the hope that Britain, et al, would begin to act more cooperatively. However, Britain continued to refuse, and OPEC members gave up their attempts to bully Britain into cooperating. Now OPEC is trying to reestablish a new cartel price, at a level far lower than the old collusive prices. These tactics can be viewed as OPEC's attempt to establish some sort of $P^c$ as the best cartel response to the continuing non-cooperative behavior of Britain.
Cooperative R&D ventures and some of the protectionist policies now being implemented in various U.S. industries are other examples of coalitions. American semiconductor manufacturers, for instance, see the harm in Japanese "dumping" practices, but also realize that price wars are not the best solution. With the approval of the U.S. government, several American firms are now seeking more cooperative ways to meet the Japanese challenge.

Although the GPD is still a simplification of these international competitive scenarios, we can see that it allows for some of the key phenomena that the 2x2 game misses entirely.

**Theoretical Background**

The economics literature on cartels and coalitions dates back many years, but until recently, relatively few papers have examined the benefits gained by firms which engage in implicit coalitions. One of the more famous early studies is that of Orr and MacAvoy (1965) who illustrated the punitive value of implicit coalitions. Orr and MacAvoy showed that a potential cheating firm often lacks the incentive to cut price if the remaining firms respond by choosing the coalition price, $P^c$. In our notation, this idea can be expressed as:

$$\Pi_i(P^s, P^c, P^c) \leq \Pi_i(P^0, P^0, P^0)$$  \hspace{1cm} (5.1)

Indeed, this inequality holds true not only for our simple model, but for a broad class of pricing games, as discussed below.

More recently, game theorists have analyzed implicit coalitions using two alternative notions of stability, internal stability (i.e., coalition members having no incentive to leave the coalition) and external stability (i.e., outside firms having no incentive to join the coalition). The term stable is reserved only for coalitions of any size that satisfy internal and external stability simultaneously.
D'Aspremont, Jacquemin, Jaskold Gabszewicz, and Weymark (1983) first introduced this definition of stability and proved that at least one stable coalition will always exist (under very general conditions). The original work of d'Aspremont et al has been applied and further extended by Donsimoni, Economides, and Polemarchakis (1986); in addition, Donsimoni (1985) has applied the d'Aspremont et al framework to a more generalized model by allowing demand and cost conditions to vary across firms.

Before we use these studies as proof of the stability of implicit coalitions in our model, a comment is in order: each of these three papers assumes von Stackelberg behavior between a dominant coalition (or cartel) and a price-taking "competitive fringe." That is, members of the coalition are assumed to set a single coalition price with full knowledge that the fringe will take that price as given and choose profit-maximizing output quantities. Thus, much of the competitive behavior and analytical results generated by these models may not be directly applicable to a model of simultaneous price competition among equivalent (or nearly equivalent) firms.

Several recent papers, particularly one by Deneckere and Davidson (1985), have examined similar types of stability under the same structural assumptions that we make in our model. Without making explicit references to internal or external stability, Deneckere and Davidson prove an important result: under most models of price competition, all firms have incentives to form implicit coalitions; i.e., all implicit coalitions are internally stable.1 In our model, for

---

1 Deneckere and Davidson show (in the appendix to their 1985 paper) that virtually all profit functions obeying GDP-like conditions will lead to global internal stability. However, realistic exceptions do exist; for example, if marginal costs decrease rapidly enough for one or both firms, it may be possible to violate the necessary conditions described by Deneckere and Davidson. Donsimoni et al (1986) were among the first to recognize this potential effect of decreasing marginal costs.
example, the two-firm implicit coalition is more profitable to its members than no coalition at all:

$$\Pi_i(P^c, P^c; P^*) \geq \Pi_i(P^*, P^*, P^*) \quad \text{for } i = 1, 2 \quad (5.2)$$

Furthermore, Deneckere and Davidson show that the profitability of an implicit coalition improves as more firms cooperate. Thus, the three-firm implicit coalition leaves all three players better off than the two-firm coalition:

$$\Pi_i(P^0, P^0, P^0) \geq \Pi_i(P^c, P^c, P^*) \quad \text{for } i = 1, 2, 3 \quad (5.3)$$

Note the similarity of (5.3) to (5.1) above. A non-cooperative firm will generally earn greater profits than each member of a two-firm implicit coalition (i.e., $$\Pi_i(P^*, P^c, P^c) \geq \Pi_i(P^*, P^*, P^c)$$ for $$i = 2, 3$$), but the "free-rider" is still better off by cooperating at $$P^0$$. Thus, Orr and MacAvey's findings are reconfirmed.

In this section, we have only discussed the role of implicit coalitions in pricing games. It is enlightening to apply the same analysis to a different strategic variable, namely output quantities. Several researchers (e.g., Salant, Switzer, and Reynolds 1983) have studied the incentives to cooperate under the assumption that firms only set quantities. The conclusions of these studies are generally opposite to the ideas mentioned above: when subsets of firms cooperate on output against non-cooperative rivals, they earn lower profits than they would earn from simple non-cooperative behavior. Virtually all of the incentives to form implicit coalitions seem to disappear.

Deneckere and Davidson (among others) attribute this startling difference to the opposite slopes of the reaction functions in these different games: pricing games generally have upward-sloped reaction functions while reaction functions in quantity games usually slope downwards. See Deneckere and Davidson for a deeper analysis of this difference. Future research in this area might examine
coalition formation in more complicated games, e.g., situations in which firms must choose prices and quantities. Such work would yield a much more realistic picture of how implicit coalitions emerge and prosper in real-world competition.

"Macro-stability" of Implicit Coalitions: Introduction

The previous section reviewed various game-theoretic approaches used to analyze implicit coalitions. The studies cited rely on different behavioral or structural assumptions, but all attempt to explain the endogenous incentives (or disincentives) to form (or avoid) implicit coalitions in a single market with repeated interactions.

In this section, we broaden our focus and return to the Axelrod paradigm to demonstrate how profitable implicit coalitions can be when each firm can engage in separate oligopolistic markets with all possible pairings of opponents. Thus, we are not only interested in studying the impact of coalitions at the micro level of one market, but also at the macro level of an entire economy of oligopolistic firms.

In Axelrod's extensive post-tournament analysis, he showed how cooperative strategies can grow and prosper in a world of "meanies." Imagine a 2x2 PD world populated only by totally non-cooperative players, i.e., everyone chooses ALL-D. Then because ALL-D is a collectively stable strategy (as defined in chapter 2), no other strategy can be introduced this world singly and earn greater profits than ALL-D.

But suppose small clusters of one or more other strategies are allowed to invade this cruel world: in this case, players choosing cooperative strategies, such as TIT FOR TAT, can cooperate with one another and earn relatively high
scores. Of course, interactions between these cooperative invaders will be quite rare (due to the small cluster size), but in some cases, the big gains earned from (infrequent) cooperative interactions will offset the small but frequent losses earned from games with ALL-D.

If the proportion of cooperative players reaches a certain critical mass, and if the interactions (i.e., game lengths) are sufficiently long, then the few cooperative players will receive higher scores (on average) than their non-cooperative counterparts. Axelrod demonstrated this idea in a variety of ways, and showed why TIT FOR TAT is particularly well-suited to invade nasty worlds.

This same line of reasoning can be applied to our GPD game. In a nasty world (or multi-market industry) filled with ALL-\(P^*\) firms, small groups of coalition-seeking cooperative firms can successfully establish a foothold and earn greater profits than their greedy rivals. Furthermore, like TIT FOR TAT, coalition-seeking firms are generally better at this invasion process than other cooperative strategies are. Many different cooperative strategies may eventually overtake ALL-\(P^*\), but the coalition strategies will usually perform best of all.

In the next section we introduce a simple coalition strategy and then demonstrate how small clusters of it can successfully invade a world of ALL-\(P^*\) players, alone and in tandem with other cooperative strategies such as XTRM.¹

¹We retain Axelrod's story that a small cluster of a particular strategy is invading a world consisting of other strategies in known proportions. An alternative view is that the entering strategy will play in only one repeated game against two opponents chosen with probabilities equal to those same proportions. The analysis is identical in both cases.
Example Coalition Strategy

The simplest possible coalition strategy, named COALITION, relies only on the three focal prices, $P^*$, $P^c$, and $P^0$. COALITION is a nice strategy: it starts at $P^0$ and in subsequent rounds it chooses:

- $P^0$ if both opponents chose $P^0$ last round.
- $P^c$ if one opponent was at or above $P^c$ last round.
- $P^*$ if both opponents were below $P^c$ last round.

We can also define the obvious generalization of COALITION for $N$-player games.†

How well will COALITION do? In general we can not say. In a single three-firm market, the best response to two firms playing ALL-$P^*$ is still to play ALL-$P^*$. However, in multi-market environments where some cooperative firms exist, we expect COALITION to perform well, even if many of the cooperative firms do not recognize implicit coalitions.

Proposition 1 illustrates the power of even the tiniest cluster of COALITION players:

**Proposition 1:** Consider a multi-market world made up only of myopic Nash optimizers (ALL-$P^*$) and COALITION firms in proportions $\rho_A$ and $\rho_C$, respectively. Assume an infinite period game with discount factor $w^t$ in period $t$. Suppose that the first GPD condition (4.1) and (5.1–5.3) hold true. Then for any non-zero combination of $\rho_A$ and $\rho_C$, there exists a critical discount factor, $w^*_{CA} \in (0,1)$, such that for all $w \geq w^*_{CA}$, each of the COALITION firms will earn higher average profits than each of the ALL-$P^*$ firms.

---

†The coalition strategy, as defined here and applied in Proposition 1 depends on the uniqueness of $P^*$, $P^c$, and $P^0$. However, there are a variety of possible analogues for COALITION for more complicated games; hence the generality of the the proposition is not limited to our specific GPD formulation.
Proof: Each player can face three possible sets of competitors in each repeated game: (ALL-$P^*$, ALL-$P^*$), (ALL-$P^*$, COALITION), and (COALITION, COALITION). Table 5.2 displays the prices for each round in each of these games. (For the moment, ignore the rows involving XTRM.) Note that prices do not vary after round 3; each game will be characterized by total cooperation ($P^0$, $P^0$, $P^0$), total non-cooperation ($P^*$, $P^*$, $P^*$), or a two-firm implicit coalition ($P^c$, $P^c$, $P^c$). The average profit earned by ALL-$P^*$, denoted as $\Pi_{ALL-P^*}$, is:

$$\Pi_{ALL-P^*} = \rho_A^2[\Pi_i(P^*, P^*, P^*)/(1-\nu)] + 2\rho_A\rho_C[\Pi_i(P^*, P^*, P^0) + w\Pi_i(P^*, P^*, P^*)/(1-w)] + \rho_C^2[\Pi_i(P^*, P^0, P^0) + w\Pi_i(P^*, P^*, P^c)/(1-w)]$$

(5.4)

Similarly, $\Pi_{COAL}$, the average profit for COALITION is:

$$\Pi_{COAL} = \rho_A^2[\Pi_i(P^0, P^*, P^*) + w\Pi_i(P^*, P^*, P^*)/(1-w)] + 2\rho_A\rho_C[\Pi_i(P^0, P^0, P^*) + w\Pi_i(P^c, P^*, P^*)/(1-w)] + \rho_C^2[\Pi_i(P^0, P^0, P^0)/(1-w)]$$

(5.5)

For simplicity, we shall use the following notation: let $\Pi^{000}$ represent $\Pi_i(P^0, P^0, P^0)$, $\Pi^{***}$ represent $\Pi_i(P^*, P^*, P^*)$, and so on for each of the profit expressions in (5.4) and (5.5).

The difference between $\Pi_{COAL}$ and $\Pi_{ALL-P^*}$ can then be expressed as:

$$\Pi_{COAL} - \Pi_{ALL-P^*} = \rho_A^2[\Pi^{0**} - \Pi^{***}] + 2\rho_A\rho_C[\Pi^{00*} - \Pi^{*0*}] + \rho_C^2[\Pi^{000} - \Pi^{*00}] + (w/(1-w))\{\rho_C^2[\Pi^{000} - \Pi^{**c}] + 2\rho_A\rho_C[\Pi^{ccc} - \Pi^{***}]\}$$

(5.6)

We wish to determine $w^*_{CA}$, the critical value of $w$ at which $\Pi_{COAL} = \Pi_{ALL-P^*}$.

After simple algebra, we find:
<table>
<thead>
<tr>
<th>FIRM 1'S STRATEGY</th>
<th>COMPETITORS</th>
<th>PROPORTION</th>
<th>ROUND 1</th>
<th>ROUND 2</th>
<th>ROUNDS 3+</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALL-P*</td>
<td>(ALL-P*, ALL-P*)</td>
<td>$\rho_A^2$</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(ALL-P*, XTRM)</td>
<td>$2\rho_A \rho_X$</td>
<td>($P^<em>, P^</em>, P_0$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(ALL-P*, COAL)</td>
<td>$2\rho_A \rho_c$</td>
<td>($P^<em>, P^</em>, P_0$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(XTRM, COAL)</td>
<td>$2\rho_A \rho_X$</td>
<td>($P^*, P_0, P_0$)</td>
<td>($P^<em>, P^</em>, P_c$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(XTRM, XTRM)</td>
<td>$\rho_X^2$</td>
<td>($P^*, P_0, P_0$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(COAL, COAL)</td>
<td>$\rho_c^2$</td>
<td>($P^*, P_0, P_0$)</td>
<td>($P^<em>, P^</em>, P_c$)</td>
<td>($P^<em>, P^</em>, P_c$)</td>
</tr>
<tr>
<td>XTRM</td>
<td>(ALL-P*, ALL-P*)</td>
<td>$\rho_A^2$</td>
<td>($P_0, P^<em>, P^</em>$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(ALL-P*, XTRM)</td>
<td>$2\rho_A \rho_X$</td>
<td>($P_0, P^*, P_0$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
<td>($P^<em>, P^</em>, P^*$)</td>
</tr>
<tr>
<td></td>
<td>(ALL-P*, COAL)</td>
<td>$2\rho_A \rho_c$</td>
<td>($P_0, P^*, P_0$)</td>
<td>($P^<em>, P^</em>, P_c$)</td>
<td>($P^<em>, P^</em>, P_c$)</td>
</tr>
<tr>
<td></td>
<td>(XTRM, COAL)</td>
<td>$2\rho_A \rho_X$</td>
<td>($P_0, P_0, P_0$)</td>
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<tr>
<td></td>
<td>(XTRM, XTRM)</td>
<td>$\rho_X^2$</td>
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</tr>
<tr>
<td></td>
<td>(COAL, COAL)</td>
<td>$\rho_c^2$</td>
<td>($P_0, P_0, P_0$)</td>
<td>($P_0, P_0, P_0$)</td>
<td>($P_0, P_0, P_0$)</td>
</tr>
</tbody>
</table>

Table 5.2: COMPARISON OF OUTCOMES IN A WORLD OF ALL-P*, XTRM, AND COALITION. (Competitors column lists strategies of firms 2 and 3 respectively. By symmetry the payoffs for firm 1 are the same if we switch firms 2 and 3. The columns under round n list the play in that round for firms 1, 2, and 3, respectively.)
\[ w_{CA}^* = \left[ 1 + \frac{\rho_C^2(\Pi^{000} - \Pi^{cc}) + 2\rho_A^2\rho_C(\Pi^{cc} - \Pi^{**})}{\rho_A^2(\Pi^{**} - \Pi^{0**}) + 2\rho_A^2\rho_C(\Pi^{0*} - \Pi^{000}) + \rho_C^2(\Pi^{000} - \Pi^{000})} \right]^{-1} \] (5.7)

Consider the fraction in brackets above. The two terms in the numerator are positive (or at least non-negative) due to (5.2) and (5.3), while each of the terms in the denominator are positive by (4.1) (and by the very definition of \( P^* \)). Thus, \( w_{CA}^* \) must lie in the interval (0,1). As an illustration, if we set \( \rho_A = \rho_C \) for our GPD model, \( w_{CA}^* = 0.901 \). With a smaller cluster of COALITION firms (i.e., \( \rho_A > \rho_C \)) \( w_{CA}^* \) increases. For example, if \( \rho_A = 0.90 \) and \( \rho_C = 0.10 \), \( w_{CA}^* = 0.9636 \).

Note how rapidly the "critical mass" for COALITION falls as \( w \) increases.

When \( w = 0.99654 \) (the value used in our tournaments), fewer than 1% (actually 0.84%) of the firms in the economy must be COALITION in order for COALITION to beat ALL-\( P^* \) overall. These numbers compare favorably with Axelrod's equivalent analysis for ALL-D and TIT FOR TAT.

Now suppose we change the cooperative invader from COALITION to XTRM. Let \( \rho_X \) represent the proportion of XTRM firms in the economy. Through the same type of analysis as above, we can determine a new critical discount factor, \( w_{XA}^* \), above which XTRM is more profitable overall than ALL-\( P^* \):

\[ w_{XA}^* = \left[ 1 + \frac{\rho_X^2(\Pi^{000} - \Pi^{***})}{\rho_A^2(\Pi^{**} - \Pi^{0**}) + 2\rho_A\rho_X(\Pi^{0*} - \Pi^{000}) + \rho_X^2(\Pi^{000} - \Pi^{000})} \right]^{-1} \] (5.8)

In this case, equal-sized clusters (\( \rho_A = \rho_X \)) lead to \( w_{XA}^* = 0.819 \). It may seem somewhat surprising that XTRM requires a lower discount factor than COALITION to beat ALL-\( P^* \) in the equal-sized cluster situation, but as the invading cluster gets smaller, XTRM quickly loses this advantage. When \( \rho_A = 0.90 \) and \( \rho_X = 0.10 \), \( w_{XA}^* = 0.9951 \). At \( w_{XA}^* = 0.99654 \), XTRM must make up over 6% of the total population in order to succeed.
Let us now go one step further. Consider a world filled with all three strategies (ALL-P*, COALITION, and XTRM) in proportions \( \rho_A, \rho_C, \) and \( \rho_X. \) Again using the prices in Table 5.2, we can derive two new critical discount factors, \( w_{cax}^* \) and \( w_{xac}^*. \) The subscript "CAX," for example, indicates that the profits earned by COALITION are being compared to those earned by ALL-P* while we ignore the earnings (but not the presence) of XTRM. Similarly, "XAC" indicates that the comparison is between XTRM and ALL-P* in the presence of COALITION. Thus, for \( w > w_{cax}^* \), COALITION earns greater profits than ALL-P*, and for \( w > w_{xac}^* \), XTRM beats ALL-P*.

If both of these discount factors can be shown to exist in the range \((0,1)\), then we have further evidence about the robustness of nice strategies in a complex environment. Furthermore, we would like to compare these two values to determine the ultimate winner in the limit as \( w \) approaches 1.

Because we must compare three strategies simultaneously to get \( w_{cax}^* \) or \( w_{xac}^* \), we must solve quadratic equations instead of a linear equation like (5.6). Thus, the derived values cannot be expressed in a form like (5.7) and (5.8). The solutions to the quadratic equation can be expressed in the following form:

\[
w^* = \frac{a + b - \sqrt{(a-b)^2 + 4ac}}{2(b-c)}
\]

where:

\[
a_{cax} = \rho_A^2 [\Pi^{***} - \Pi^{0*}] + 2\rho_A (\rho_C + \rho_X)[\Pi^{***} - \Pi^{0*}] + (\rho_C + \rho_X)^2 [\Pi^{*00} - \Pi^{000}]
\]

\[
b_{cax} = \rho_C^2 [\Pi^{000} - \Pi^{**}] + 2\rho_C \rho_X [\Pi^{**} - \Pi^{**}] + \rho_X^2 [\Pi^{000} - \Pi^{**}] + 2\rho_C \rho_X [\Pi^{000} - \Pi^{**}] - 2\rho_C \rho_X [\Pi^{***} - \Pi^{**}]
\]

\[
c_{cax} = \rho_C^2 [\Pi^{000} - \Pi^{***}] + 2\rho_C \rho_X [\Pi^{***} - \Pi^{***}] + (2\rho_C \rho_X + \rho_X)^2 [\Pi^{000} - \Pi^{***}]
\]
and:

\[ a_{XAC} = \rho_A^2(\Pi^{***} - \Pi^{*0}) + 2\rho_A(\rho_C + \rho_X)(\Pi^{*0} - \Pi^{00}) + (\rho_C + \rho_X)^2(\Pi^{00} - \Pi^{000}) \]

\[ b_{XAC} = \rho_C^2(\Pi^{00} - \Pi^{*0}) + 2\rho_A\rho_C(\Pi^{*0} - \Pi^{**}) + \rho_X^2(\Pi^{00} - \Pi^{***}) + 2\rho_A\rho_X(\Pi^{00} - \Pi^{*0}) \]

\[ b_{XAC} = \rho_C^2(\Pi^{00} - \Pi^{*0}) + (2\rho_C\rho_X + \rho_X^2)(\Pi^{00} - \Pi^{***}) \] (5.9.2)

Although these expressions seem fairly cumbersome, we can still make a few observations about them. First of all, both \( w^{*\text{CAX}} \) and \( w^{*\text{XAC}} \) lie between 0 and 1. This might not be obvious from (5.9.1) and (5.9.2), but it follows from the quadratic equation which yields (5.9). Secondly, the two solutions are remarkably similar: the "a" values are identical, while the "b" and "c" values differ only by one term each. It comes as no surprise that \( w^{*\text{CAX}} \) will be approximately equal to \( w^{*\text{XAC}} \). In the equal-proportions case \( (\rho_A = \rho_C = \rho_X) \), \( w^{*\text{CAX}} = 0.765 \) and \( w^{*\text{XAC}} = 0.772 \). Thus, both COALITION and XTRM will eventually pass ALL-\(P^*\), but COALITION will pass ALL-\(P^*\) slightly earlier than XTRM will.

We still must address an important question: under what conditions will COALITION beat out XTRM as \( w \) approaches 1? From the equal-proportions values listed in the previous paragraph, COALITION appears to have an edge, since it beats out ALL-\(P^*\) slightly faster; however, it is possible to choose values of \( \rho_A, \rho_C, \) and \( \rho_X \) such that \( w^{*\text{CAX}} > w^{*\text{XAC}} \).†

Although there are cases when XTRM passes ALL-\(P^*\) faster than COALITION does, we can show that COALITION will always beat out XTRM as \( w \) continues to increase. Any possible ambiguity is resolved by yet another critical discount factor, \( w^{*\text{CXA}} \). Using the same analytical techniques as above,

† This latter inequality can be obtained by choosing \( \rho_X \) much larger than \( \rho_A \) and \( \rho_C \).
we can derive:

\[
    w_{CXA}^* = \frac{\rho_A[\Pi^{*\star} - \Pi^{ce*}] + \rho_X[\Pi^{***} - \Pi^{c**}]}{\rho_A[\Pi^{*\star} - \Pi^{ce*}] + \rho_X[\Pi^{***} - \Pi^{c**}] + \rho_A[\Pi^{ce*} - \Pi^{***}]} 
\]  

(5.11)

Each of the bracketed quantities is positive, so this discount factor is clearly bounded by (0, 1). Hence, COALITION will always beat out XTRM for sufficiently large \( w \).

Our general hypothesis appears to be supported: nice guys don't finish last; in fact, as the discount factor (or game length) becomes sufficiently large, only the nasty player is a sure loser. Our argument in favor of COALITION (or more general coalition-seeking strategies) is supported by the existence of two critical discount factors, \( w_{CAX}^* \) and \( w_{CXA}^* \). For all values of \( w \geq \max(w_{CAX}^*, w_{CXA}^*) \), all COALITION players will earn the highest average profits.

Although we have presented convincing analytical evidence concerning the robustness of COALITION, we have done so only for a world with two opposing strategies (XTRM and ALL-\( P^* \)). Nevertheless, it seems plausible that COALITION will continue to do well in worlds with additional strategies (cooperative and non-cooperative). For example, consider a general class of “TIT FOR TAT followers” that start cooperatively and then choose a weighted average of their competitors’ previous prices:

\[
P_1 = \alpha\{\max(P_2, P_3)\} + (1-\alpha)\{\min(P_2, P_3)\}
\]

where \( P_2 \) and \( P_3 \) are the opponents’ previous prices, and \( 0 < \alpha < 1 \). Although the analytics become quite complex, it is still possible to show that COALITION still emerges as the most profitable strategy (for sufficiently large \( w \)). As the game length gets longer and longer, the averaging process will always drive the TIT FOR TAT players’ prices down to \( P^* \) (whenever the strategy is matched against
one or more ALL-P* players). In the limit, therefore, this strategy will behave just like XTRM; the qualitative differences will be minimal. (Note that setting \( \alpha = 0 \) yields a strategy that will behave exactly like XTRM in any match involving COALITION and/or ALL-P*.)

But just as we should acknowledge the robustness of COALITION, we should also recognize that it will not win \textit{all} the time, even for arbitrarily large \( w \). Suppose, for example, that we have a world with three strategies: COALITION, ALL-P*, and MAX, a generous TIT FOR TAT follower that always mimics the previous maximum opposing price (i.e., \( \alpha = 1 \)). If the proportion of MAX players is large enough, then ALL-P* will beat COALITION for all \( w \). This is due to the fact that ALL-P* will earn higher profits than COALITION when either is matched against two MAX players: \( \Pi_1(P^*, P^0, P^0) > \Pi_1(P^0, P^0, P^0) \). As the game progresses, ALL-P* will continually abuse MAX to an extent that COALITION can never overcome.
CHAPTER 6: CONTINUOUS ALTERNATIVES: DEGREES OF NICE AND NASTY

Implicit coalitions address the problem of how we should behave when some (but not all) of our rivals are uncooperative. But by no means do implicit coalitions "solve" the GPD. Other difficult issues remain, such as what to do when one or more of our competitors cuts price by a small amount.

If a price cut is very small, we may choose to ignore it, but over the long term our passivity may lead to exploitation. Axelrod's principle of provocability suggests that we must respond to any price cut, but with a continuous range of prices available, there is no obvious optimal response. We could punish defectors by matching their price cuts, amplifying them, or going all the way to $P^*$. Or we could try to ignore the price-cutting firms by choosing the appropriate $P^c$ (as the COALITION strategy would do).

Literature Review

Virtually all models of price competition include price as a continuous variable. However, most game-theoretic pricing strategies avoid the strategic dilemmas referred to in the previous section by specifying a discrete set of prices to use in any possible situation. XTRM is a common example: this strategy will only use $P^0$ or $P^*$, and nothing in between. Strategies like XTRM, sometimes known as trigger-price strategies (Porter 1983b, Green and Porter 1984), use a limited set of prices for analytic convenience. Most models involving simple trigger-price strategies consider two different behavioral phases: a cooperation phase ($P^0$) and a punishment phase ($P^*$). In general, these strategies need not be as restrictive as XTRM. Unlike XTRM, some allow a return to cooperation after
a sufficiently long punishment period. Some also allow for additional phases in which cooperating firms can reward themselves after carrying out the appropriate punishments, or resort to stiffer punishments to penalize firms that did not comply with the specified punishment policy.

But despite these relatively minor differences among trigger strategies, they are all still discrete strategies operating in a continuous world. Fortunately, there has been some work done in formulating continuous strategies. James Friedman (the acknowledged father of XTRM) analyzed a class of reaction function strategies in 1968 and demonstrated that Nash equilibria can exist for such strategies. Friedman considered only a two-firm market and restricted each firm's reaction function to be a function of the opponent's previous price alone.

In recent years, several game theorists have reexamined the equilibria from Friedman's model. Stanford (1986a) showed that these reaction functions fail to attain subgame perfection, a common and desirable feature of most discrete models. Nevertheless, Stanford has pursued continuous reaction function models in other papers (Stanford 1986b, Kalai and Stanford 1985) to investigate the strategic and analytical implications of continuous reaction functions when the model assumptions are modified slightly.

Another direction that game theorists have taken in trying to formulate continuous-alternative strategies is through models of price-matching guarantees. Several researchers, including Kalai and Satterthwaite (1986), have created and analyzed games in which each firm is committed to match the lowest price of any of its opponents. If this price-matching behavior is enforced, then no firm has an incentive to cut its price below the optimal collusive price. We have briefly examined this type of strategy in the previous chapter (it is a "TIT FOR
TAT follower" with $a = 0$), and we will see it again when we discuss the competitive strategy tournaments.

Because models with continuous reaction functions or price-matching guarantees can often be more realistic and behaviorally richer than most discrete models, we are encouraged by the ongoing work of researchers like William Stanford, who states, "continuous reaction function models continue to hold an appeal which is hard to resist. They also continue to harbor significant questions" (Stanford 1986a, p. 231).

We are intrigued by several relevant questions as well. For example, how do the prices and profits of continuous reaction function strategies compare to those of discrete strategies? We know of only one study (Slade 1986a) that develops both types of strategies for a given profit function, but the author stops short of making any direct comparisons of the two. Furthermore, we know of no studies that allow a mixture of continuous and discrete strategies within a single market. This is a promising area for future work.

Two-Firm Games

We now make a first attempt at comparing continuous and discrete strategies. In order to make the problem more realistic (and more complex), we take our first CPD profit function (4.5) and add a bit of uncertainty. When uncertainty of some kind is present in the environment, firms generally find it more difficult to accurately convey their intentions. A firm may be unable to distinguish a truly cooperative competitor from one which makes small price cuts. If one firm overreacts to these apparent defections, it might bring on the decline of industry profitability.
To help understand the implications of continuous action we introduce a hypothetical environment called a $d$-$T$ world. In such a market, firms make random price cuts (of $d$ cents) in period $T$, where $d$ and $T$ follow arbitrary probability distribution functions that may vary from firm to firm. In a $d$-$T$ world we can examine any set of strategies (including nice ones) to determine how well each one copes with defections of different sizes.\(^\dagger\)

We first illustrate degrees of nice and nasty in a two-firm continuous action game and then discuss a generalization of the concept to $N$-firm games.

Let $P_i(t)$ represent firm $i$'s price in period $t$, and let $\epsilon$ be a number that is small in comparison to the difference between $P^0$ and $P^*$. The following strategies are three ways to generalize TIT FOR TAT to two-player continuous alternative games. All three strategies start at $P^0$, but in later rounds their responses differ:

**Discrete TFT:**

$$P(t) = \begin{cases} 
  P^0 & \text{if } P_{2(t-1)} = P^0 \\
  P^* & \text{otherwise}
\end{cases}$$

**Robust TFT:**

$$P(t) = \begin{cases} 
  P^0 & \text{if } P_{2(t-1)} \geq P^0 - \epsilon \\
  P^* & \text{otherwise}
\end{cases}$$

**Continuous TFT:**

$$P(t) = P_{2(t-1)}$$

Discrete TFT is a close analogue to ordinary TIT FOR TAT in the 2x2 game. It does not consider the intermediate range of prices between $P^*$ and $P^0$ at all.

\(^\dagger\)To keep the size and number of defections within reasonable limits, we assume that the distribution of $d$ is bounded within the range $(0, P^0-P^*)$ and we limit our attention to the behavioral dynamics caused by the first defection alone.
Robust TFT forgives slight defections, but will not use any intermediate prices itself. Continuous TFT provides a more even balance of forgiveness and provocability.

Each of the three pairwise comparisons (in a $d \cdot T$ world) yields insights about the importance of recognizing continuous action. We conjecture that Robust TFT and Continuous TFT will each perform better than Discrete TFT for virtually any distributions of $d$ and $T$, but the comparison between Robust and Continuous TFT will be dependent on these distributions.

Each strategy, when matched against itself in a $d \cdot T$ world, tends to lead to uniquely different behavior. Consider first Discrete TFT. Mutual cooperation begins in round 1 and continues until round $T$, when the first random price cut occurs: in that round, one firm (say firm 2) will defect to $P^0 - d$. In the next round firm 1 will respond by choosing $P^*$, but firm 2 will be back at $P^0$. Thereafter, play oscillates between $(P^0, P^*)$ and $(P^*, P^0)$. This alternating will continue until the game ends. By condition 4 of the GPD, this oscillation is less profitable than sustained mutual cooperation.

Now consider Robust TFT. Firm 2 defects in round $T$ again, and if the size of the defection is greater than $\varepsilon$, the game is identical to the Discrete TFT behavior just described. On the other hand, if $d < \varepsilon$, firm 1 will forgive the small defection and continue cooperating. If the game is sufficiently long, the profit from mutual cooperation will outweigh the one-time cost of forgiving the small defection.

Finally, consider Continuous TFT. As in the case of Discrete TFT, Continuous TFT will begin to oscillate forever after the initial defection. However, instead of oscillating between $P^0$ and $P^*$, this strategy will oscillate
only between \( P^0 \) and \( P^0 - d \). Thus, by condition 4 of the GPD, the oscillations of Continuous TFT will be more profitable than those of Discrete TFT.

We now compare the strategies to one another in a more formal manner.

**Discrete TFT vs. Robust TFT:** Robust TFT outperforms Discrete TFT because it reduces the likelihood of oscillation. Suppose each strategy is matched against itself in comparable \( d \cdot T \) worlds, where the constituent GPD game is repeated \( M \) times and profits are discounted each round by discount factor \( w \). Since both strategies are nice, their prices and profits are identical and equal to \( \Pi_1(P^0, P^0) \) until round \( T \), when the first defection occurs. If \( M - T \) is even, the payoffs from subsequent rounds are:

\[
(M - T)\left[\Pi_1(P^*, P^0) + \Pi_1(P^0, P^*)\right]/2 \quad \text{for Discrete TFT}
\]

\[
(M - T)\left[\Pi_1(P^*, P^0) + \Pi_1(P^0, P^*)\right]/2 \quad \text{for Robust TFT, } d > \varepsilon
\]

\[
(M - T)\left[\Pi_1(P^0, P^0) + \Pi_1(P^0, P^0)\right]/2 \quad \text{for Robust TFT, } d \leq \varepsilon
\]

Since \( 2 \Pi_1(P^0, P^0) > \Pi_1(P^*, P^0) + \Pi_1(P^0, P^*) \) by condition 4, Robust TFT will outperform Discrete TFT as long as \( \text{Prob}\{d \leq \varepsilon\} = 0 \).

If \( M - T \) is odd, Discrete TFT gains \( \Pi_1(P^*, P^0) - \Pi_1(P^0, P^0) \) on the last round, hence the condition on \( w \) to assure that the rewards from the \( M - T - 1 \) rounds of cooperation outweigh the last round penalty.

Note that the same ideas will apply if both parties can defect or if either can defect more than once. They are also independent of the distributions of \( d \) and \( T \), as long as we maintain our assumption that \( \text{Prob}\{d > P^0 - P^*\} = 0 \).
Discrete TFT vs. Continuous TFT: When describing Continuous TFT above, we noted that it will handle oscillations better than Discrete TFT. The proof parallels that for Robust TFT. The strategies cooperate for the first $T-1$ rounds, then differ after the defection in round $T$. For $M-T$ even, the payoffs are:

$$(M-T)[\Pi_1(P^*, P^0) + \Pi_1(P^0, P^*)]/2 \quad \text{for Discrete TFT}$$

$$(M-T)[\Pi_1(P^0 - d, P^0) + \Pi_1(P^0, P^0 - d)]/2 \quad \text{for Continuous TFT}$$

Since $P^0 - d \leq P^*$ by the definition of a $d$-$T$ world and since $\delta[\Pi_1(P^0 - d, P^0) + \Pi_1(P^0, P^0 - d)]/\delta d < 0$ by condition 4, continuous TFT will outperform Discrete TFT.

The requirement that $w$ be sufficiently large comes from the recognition that for $M-T$ odd, Discrete TFT gains $\Pi_1(P^*, P^0) - \Pi_1(P^0 - d, P^0)$ on the last round.

Robust TFT vs. Continuous TFT: This comparison is more complex than the previous two. Comparing the two proofs sketched above, we see that the advantage of Robust TFT comes from the fact that unproductive oscillations are avoided with probability $\text{Prob}\{d \leq \varepsilon\}$. On the other hand, the advantage of continuous TFT comes from the fact that the oscillations are dampened to a smaller magnitude. Thus there is a tradeoff between the eternal but small oscillations of Continuous TFT and the more limited but larger oscillations of Robust TFT. The relative advantage of these strategies will depend upon the specific payoff function and the specific probabilities on $d$ and $T$.

In particular, Robust TFT will be superior if it is better to eliminate oscillations begun by small deviations whereas Continuous TFT will be superior if it is better to dampen their magnitude. Of course $d$-$T$ worlds are only part of
the story since they do not account explicitly for exploitive behavior. Continuous 
TFT punishes such exploiters; Robust TFT does not. On more qualitative 
grounds, Axelrod’s principle of provocability favors Continuous TFT, since it 
responds to any price cut. In either case, it is clear that potential advantages 
exist for strategies that can exploit the continuous nature of the GPD.

N-Firm Games

Throughout this chapter we have indicated different analytical approaches to 
show that effective strategies in GPD games should be able to (1) recognize and 
use the implicit coalition price(s), and (2) take advantage of continuous pricing. 
However, we have not given any suggestions about how these two tactics can be 
integrated with one another. Thus, we propose a generic strategy that can 
achieve both of these ideals, but is still flexible enough to allow for different types 
of competitive behavior.

Let \( f_1(P_2, P_3) \) be a general function mapping our competitors’ previous prices 
onto the range \((P^c, P^o)\) and let \( f_2(P_2, P_3) \) be a general function mapping our 
competitors’ previous prices onto the range \((P^*, P^c)\). Then, suppressing the \( t-1 \) 
argument on the right hand side, we state GENERIC as:

\[
\text{GENERIC: } P_1(t) = \begin{cases} 
\begin{align*} 
& f_1(P_2, P_3) \text{ if } P_2, P_3 > P^c \\
& f_2(P_2, P_3) \text{ if } P_2, P_3 < P^c \\
& P^c \text{ otherwise} 
\end{align*} 
\end{cases}
\]

The idea behind GENERIC is that it is willing to choose a high price \((>P^c)\) 
only if both competitors are also fairly cooperative, and it will give up on 
coalitions if both competitors chose low prices \((<P^c)\) in the previous period.
Furthermore, it restricts its prices to the range \((P^*, P^0)\), as was suggested in our discussion about the GPD in chapter four.

We can make the obvious generalization of GENERIC to more than three firms.

For later analysis, we define one version of GENERIC, named GENERIC1, where \(f_1(P_2, P_3) = \min(P_2, P_3, P^0)\) and \(f_2(P_2, P_3) = \max(P_2, P_3, P^*)\). We turn now to our first tournament to examine the validity of our proposals.
CHAPTER 7:
MITCS1: THE FIRST GPD TOURNAMENT

In November 1984 we announced our first tournament (named MITCS1), similar in design to Axelrod's second tournament, but featuring the Generalized Prisoner's Dilemma. The game was posed as a managerial problem with price as the sole strategic variable. Each game in the tourney was the repeated GPD game defined in equation 4.6 with three programmed strategies choosing prices each period from a continuous range of prices. As in Axelrod's tournament, each possible grouping of entries engaged in five repeated games, and the overall winner was the strategy that amassed the highest total profits across all games in which it participated. Contestants were given full information about the profit function used, and in every game, each player had access to the past prices set by all three players in that game. Entries were submitted in the form of FORTRAN IV subroutines.

By July 1985, we had received over 40 algorithms (including several duplicates) from a diverse group of participants around the world. The field of entrants included economists, political scientists, game theorists, marketing academics, and managers. Several universities and major corporations submitted the best and most creative entries they had found after running their own mini-tournaments. Thus, the pool of algorithms available for this first empirical analysis of the GPD contains a wide variety of creative efforts from some very strategically-minded people.

Description of Entries

Many entrants, having learned Axelrod's lessons, attempted to generalize TIT FOR TAT. Six strategies recognized implicit coalitions and incorporated the
implicit coalition price, $P^c$ into their algorithms. We label these algorithms IC for implicit coalition. Several IC entries fit into the GENERIC framework, including some that used very complex functions for $f_1$ and $f_2$ that involved many of the previous decisions of each competitor, not just their most recent prices.

Most algorithms tried to incorporate continuous alternatives, but participants did so in a variety of ways, including:

- **MIN**: Start at $P^0$. In all subsequent rounds, choose the minimum of your opponents' prices from the previous round.
- **MAX**: Start at $P^0$. In all subsequent rounds, choose the maximum of your opponents' prices from the previous round.
- **AVG**: Start at $P^0$. In all subsequent rounds, choose the average of your opponents' prices from the previous round.

Each of these strategies will lead to very different types of behavior. MAX is extremely forgiving but not highly provokable. Two MAXs, playing together against a nasty opponent, will remain at $P^0$, ignoring the exploitative moves by the third player. On the other hand, MIN is extremely competitive, and will raise its price only if both opponents do so first. AVG is the most moderate of the three, trying to balance forgiveness and provocation at the same time. (Axelrod himself entered AVG into the tournament.)

A slightly more complicated generalization of TIT FOR TAT is MXCM (pronounced MAXCUM) which also starts at $P^0$, but in later rounds it mimics the previous price of the opposing strategy with the greatest cumulative profits at that point in the game. Thus, MXCM does not attempt to use the previous prices of both competitors -- it only focuses on the more profitable of the two and adopts the passive mimicking strategy, just like TIT FOR TAT in the 2-player game. Like AVG, MXCM is both provokable and forgiving since it follows any price.
changes made by the leading strategy. However, MXCM is distinguished from AVG because it is able to ignore ineffective strategies. In contrast, AVG will always give equal weight to the prices of both opponents, regardless of how well they perform.

Other entrants made no attempt to generalize TIT FOR TAT. Some were content with the basic XTRM strategy or slight variations of it. A few participants chose constant price strategies (e.g., always cooperate (ALL-\(P^0\)), always defect (ALL-\(P^*\)), or always be envious (ALL-\(P^*\)), or random (RND) strategies which chose prices randomly from the range (\(P^*, P^0\)) or used a random walk technique. Hence most of the algorithms could be classified into one of eight broad categories: MIN, MAX, AVG, MXCM, IC, XTRM, constant price, or RND.

Beyond these general descriptions of strategy types, the entries differed due to specific tactics or features that were frequently employed. For example:

Following Axelrod (1984), strategies which start the game cooperatively and are never the first to cut price below \(P^0\) are termed nice, as opposed to nasty strategies, which can be the first to defect.

Self-awareness allows strategies to consider the previous decisions of all three firms (not just the two opponents) when choosing prices. This feature tended to reduce cycling and echo effects.

Many strategies restricted their prices to the range (\(P^*, P^0\)), since there is no way to increase profit by choosing prices outside of this range. Strategies that were willing to go below \(P^*\) or above \(P^0\) are known as unbounded.

Some strategies tried to induce cooperation by raising prices slightly above the level specified by a general strategy type (e.g. play MIN but add a few cents to the minimum price). These strategies have price-raising initiative.
On the other hand, some strategies have price-cutting initiative; that is, they were willing to cut price below the specified level, usually in an attempt to punish an earlier price cut made by another firm.

Finally, several strategies occasionally used the envious price $P^*$ to try to outscore their opponents (rather than maximizing their own profit).

These descriptions are admittedly vague. The actual implementation of some of these features can vary greatly from strategy to strategy. For instance, price-raising algorithms can vary the magnitude and frequency of price increases. One algorithm added five cents to the minimum opposing price every round, while another only added two cents to the minimum price at random times. For this analysis, however, we shall ignore such fine-grain differences, since the mere presence of a particular feature was generally more important than the manner in which it was implemented.

**MITCS1 Results and Interpretation**

Table 7.1 presents a summary of the strategies and their performances. The strategies are ranked by their average profit per round. For comparison, mutual cooperation pays 20 currency units per period to each firm, while each period of mutual defection (i.e. all three firms choosing $P^*$) yields approximately 12 currency units to each firm. (See Table 5.1 for additional payoff comparisons.)

The winning algorithm, entered by Terry Elrod of Vanderbilt University, was the simplest possible IC strategy, COALITION. The top four algorithms in the tournament recognized the coalition property, and all six IC strategies finished in the top ten overall.
Table 7.1. Official MITCS1 Results.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Entrant</th>
<th>Strategy Type</th>
<th>Features(^{(1)})</th>
<th>Average Profits per Round</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Terry Elrod</td>
<td>IC</td>
<td></td>
<td>17.182</td>
</tr>
<tr>
<td>2</td>
<td>(anonymous)</td>
<td>IC</td>
<td>U</td>
<td>17.172</td>
</tr>
<tr>
<td>3</td>
<td>Avraham Beja &amp; Shlomo Kalish</td>
<td>IC</td>
<td>S</td>
<td>17.172</td>
</tr>
<tr>
<td>4</td>
<td>Steve Shugan</td>
<td>IC</td>
<td></td>
<td>17.157</td>
</tr>
<tr>
<td>5</td>
<td>(MIT)(^2)</td>
<td>AVG(^{10})</td>
<td>S</td>
<td>17.157</td>
</tr>
<tr>
<td>6</td>
<td>Gary A. Lines</td>
<td>AVG</td>
<td></td>
<td>17.104</td>
</tr>
<tr>
<td>7</td>
<td>(MIT)</td>
<td>MIN</td>
<td>S</td>
<td>17.063</td>
</tr>
<tr>
<td>8</td>
<td>Steve Shugan</td>
<td>IC</td>
<td>S</td>
<td>17.014</td>
</tr>
<tr>
<td>9</td>
<td>Beja &amp; Kalish</td>
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<td>S</td>
<td>16.927</td>
</tr>
<tr>
<td>10</td>
<td>(MIT)</td>
<td>MXCM</td>
<td>S</td>
<td>16.914</td>
</tr>
<tr>
<td>11</td>
<td>Terry Elrod; John Roberts</td>
<td>MAX</td>
<td></td>
<td>16.879</td>
</tr>
<tr>
<td>12</td>
<td>Gary Gaeth &amp; Gerard Tellis;</td>
<td>MIN</td>
<td></td>
<td>16.851</td>
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<tr>
<td></td>
<td>Terry Elrod; Gary A. Lines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>James M. Lattin</td>
<td>MAX</td>
<td>S</td>
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<td>14</td>
<td>John A. Cadley</td>
<td>XTRM</td>
<td>R</td>
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<td>Steve Borgatti</td>
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<td>(MIT)(^4)</td>
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<td>John Roberts</td>
<td>AVG(^{10})</td>
<td>U</td>
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<td>21</td>
<td>Barbara Bruner &amp; James Olver</td>
<td>AVG</td>
<td>N</td>
<td>16.127</td>
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<td>22</td>
<td>Robert F. Bordley</td>
<td>MIN</td>
<td>U</td>
<td>15.976</td>
</tr>
<tr>
<td>23</td>
<td>Robert E. Marks</td>
<td>XTRM(^{6})</td>
<td>U</td>
<td>14.535</td>
</tr>
<tr>
<td>24</td>
<td>Robert E. Marks</td>
<td>XTRM</td>
<td>E</td>
<td>14.497</td>
</tr>
<tr>
<td>25</td>
<td>James M. Lattin</td>
<td>ALL (P^0)</td>
<td></td>
<td>14.351</td>
</tr>
<tr>
<td>26</td>
<td>Shlomo Maital</td>
<td>MXCM</td>
<td>N</td>
<td>13.944</td>
</tr>
<tr>
<td>27</td>
<td>Beja &amp; Kalish</td>
<td>RND(^{7})</td>
<td>N</td>
<td>13.809</td>
</tr>
<tr>
<td>28</td>
<td>Robert E. Marks</td>
<td>(8)</td>
<td>N</td>
<td>13.763</td>
</tr>
<tr>
<td>29</td>
<td>Steve Borgatti</td>
<td>MIN</td>
<td>N</td>
<td>13.740</td>
</tr>
<tr>
<td>30</td>
<td>Roland Rust; Robert F. Bordley;</td>
<td>ALL (P^*)</td>
<td>N</td>
<td>13.637</td>
</tr>
<tr>
<td></td>
<td>John Roberts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Beja &amp; Kalish</td>
<td>RND(^{7})</td>
<td>N</td>
<td>13.575</td>
</tr>
<tr>
<td>32</td>
<td>(MIT)</td>
<td>RND(^{2})</td>
<td>N</td>
<td>13.447</td>
</tr>
<tr>
<td>33</td>
<td>Shlomo Maital</td>
<td>MXCM</td>
<td>N</td>
<td>13.384</td>
</tr>
<tr>
<td>34</td>
<td>Robert F. Bordley</td>
<td>MIN</td>
<td>N</td>
<td>12.918</td>
</tr>
<tr>
<td>35</td>
<td>Kenneth L. Stott, Jr.,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Francis J. Vasko, &amp; Floyd F. Wolf</td>
<td>ALL (P^o)</td>
<td>N</td>
<td>12.151</td>
</tr>
<tr>
<td>36</td>
<td>Robert E. Marks</td>
<td>ALL (P^*)</td>
<td>N</td>
<td>9.909</td>
</tr>
<tr>
<td>37</td>
<td>(anonymous)</td>
<td>MIN(^{10})</td>
<td>N</td>
<td>9.684</td>
</tr>
<tr>
<td>38</td>
<td>(anonymous)</td>
<td>AVG(^{11})</td>
<td>U</td>
<td>9.643</td>
</tr>
</tbody>
</table>

(see Notes to Table 7.1, next page)
Recognizing implicit coalitions proved to be the single most important factor in the tournament, but another crucial factor was how the strategies dealt with the continuous nature of the prices. Most entrants used simple heuristics (e.g. MIN, MAX, AVG) to address this problem with varying degrees of success. The standard averaging strategy (i.e. nice, bounded AVG with no self-awareness, envy, or price-raising/cutting initiative) finished in 6th place, easily beating standard MAX (ranked 11th) and standard MIN (12th).

Several of the descriptive features were highly influential. First and foremost is niceness, reconfirming the findings of Axelrod (1984). Nice algorithms were able to reap great benefits by avoiding the short-term temptation to cut price. The best nasty strategy played standard AVG most of the time, but would occasionally make small price cuts as long as both competitors remained at $P^0$. If either opponent responded to these cuts, this strategy would return to standard AVG for the remainder of the game. This clever form of exploitation helped make
this algorithm far more profitable than other nasty entries, but still could not provide any better than a 21st place finish.

Other important features were bounded prices and lack of envy. Only one successful algorithm ever exhibited envious behavior, but that strategy (ranked 2nd) would only go to $P^e$ if both opponents priced at or below $P^e$ in the previous round, a fairly rare occurrence. Perhaps if this second-ranked strategy did not try to battle envious opponents on their terms, it might have been able to win the tournament.

The value of bounded prices can be seen by comparing standard AVG and MIN (ranked 6th and 12th, respectively) to their equivalent but unbounded counterparts (ranked 19th and 22nd, respectively). Boundedness was worth nearly 70 cents per round to AVG and nearly 90 cents per round to MIN.

Several entrants found self-awareness to be a blessing. For example, some strategies gave all three firms a fair say in the pricing process, e.g. averaging prices across all three firms (ranked 5th), and 3-firm MXCM (10th place). But self-awareness was a curse to others, including those who used it as a ratchet on prices. The algorithms ranked 8th and 15th, for example, only let their prices move downwards, regardless of the cooperative gestures made by their opponents.

Little can be said about the effectiveness of price-raising and price-cutting initiative. Some of the nice, bounded entries were able to encourage cooperation and discourage cheating with appropriate rewards and penalties, but these successes were counterbalanced by the unsuccessful strategies that brought on their own demise by raising or cutting prices too much at the wrong times.
Table 7.1 seems to depict a tight three-way battle for first place. However, it should be noted that each algorithm played in nearly 1000 three-firm matches in each of the five games in the tournament. This information, combined with the fact that each match lasted approximately 200 rounds, implies that each strategy chose prices in nearly 1 million total rounds. Thus, a difference of .01 currency units on a profit-per-round basis is equivalent to a 10,000 currency unit difference in total profit.

An Alternative Champion

The winning algorithm, COALITION, was the only highly-ranked strategy which did not acknowledge the continuity of prices. Apparently, none of its top rivals could use “continuous pricing” effectively enough to overcome the winner’s discrete simplicity. However, this does not imply that the task is impossible; another algorithm, GENERIC1 would have easily won the tournament had it been entered. The pricing rule for this strategy is only slightly more complicated than that of COALITION, but it can take better advantage of continuous prices:

\[
P_1(t) = \begin{cases} 
\min\{P_2, P_3, P^0\} & \text{if } P_2, P_3 \geq P^c \\
P^c & \text{if } \max\{P_2, P_3\} \geq P^c \text{ and } \min\{P_2, P_3\} \leq P^c \\
\max\{P_2, P_3, P^*\} & \text{if } P_2, P_3 \leq P^c
\end{cases}
\]

where \( P_2 \) and \( P_3 \) represent firm 1’s opponents’ prices in the previous round.

Table 7.2 shows the top ten entries in the revised tournament with GENERIC1 included. Note that the relative rankings of the original strategies are unchanged, although average profits have increased because of the presence of the cooperative newcomer. The margin of victory for the new algorithm is
quite significant; the gap between first and second place is larger than the margin between second and seventh place.

Table 7.2. Revised MITCS1 Results.

<table>
<thead>
<tr>
<th>New Rank</th>
<th>Entrant</th>
<th>Old Rank</th>
<th>Strategy Type</th>
<th>Features</th>
<th>Average Profits per Round</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GENERIC1</td>
<td>--</td>
<td>IC</td>
<td></td>
<td>17.353</td>
</tr>
<tr>
<td>2</td>
<td>Terry Elrod</td>
<td>1</td>
<td>IC</td>
<td></td>
<td>17.257</td>
</tr>
<tr>
<td>3</td>
<td>(anonymous)</td>
<td>2</td>
<td>IC</td>
<td>S U R E</td>
<td>17.249</td>
</tr>
<tr>
<td>4</td>
<td>Avraham Beja &amp; Shlomo Kalish</td>
<td>3</td>
<td>IC</td>
<td>S C</td>
<td>17.245</td>
</tr>
<tr>
<td>5</td>
<td>Steve Shugan</td>
<td>4</td>
<td>IC</td>
<td></td>
<td>17.235</td>
</tr>
<tr>
<td>6</td>
<td>(MIT)²</td>
<td>5</td>
<td>AVG³</td>
<td>S</td>
<td>17.225</td>
</tr>
<tr>
<td>7</td>
<td>Gary A. Lines</td>
<td>6</td>
<td>AVG</td>
<td></td>
<td>17.175</td>
</tr>
<tr>
<td>8</td>
<td>(MIT)</td>
<td>7</td>
<td>MIN</td>
<td>S R</td>
<td>17.136</td>
</tr>
<tr>
<td>9</td>
<td>Steve Shugan</td>
<td>8</td>
<td>IC</td>
<td>S</td>
<td>17.101</td>
</tr>
<tr>
<td>10</td>
<td>Beja &amp; Kalish</td>
<td>9</td>
<td>IC</td>
<td>S C</td>
<td>17.000</td>
</tr>
</tbody>
</table>

NOTES:
(1) Default features include niceness, no self-awareness, bounded prices, no price-raising or -cutting initiative, and no envy. Exceptions are noted as N for nastiness, S for self-awareness, U for unbounded prices, R for price-raising initiative, C for price-cutting initiative, and E for envy.
(2) (MIT) denotes an algorithm entered by a member of the MIT community.
(3) Weighted average of all three players' prices, using cumulative profits as weights.

More importantly, the success of this algorithm is not very sensitive to variations in the competitive environment. Extreme changes, such as doubling the presence of all nasty entries, usually can not unseat this new winner. Many of the aforementioned procedures that Axelrod used to demonstrate the robustness of TIT FOR TAT have been applied to this tournament, with strong results favoring the proposed generic strategy.

Where does this powerful new strategy come from? The idea of GENERIC follows from our theoretical analysis, but to be honest, the key factor behind our discovery of the specifics of GENERIC1 is hindsight. We benefitted from the luxury of being able to rerun the tournament at will, adding and deleting algorithms as we pleased. It did not take us too long to notice that MAX algorithms tended to perform well when competitors were tough, while MIN
seemed to do better in a more cooperative setting. In the next section, we give a robust theoretical justification for the success of GENERIC1.

Surprisingly enough, this strategy has been cited at least once in the pricing literature. In a comprehensive book titled, "Pricing in the Electrical Oligopoly, Volume 1," Ralph Sultan noted that General Electric exhibited precisely this type of pricing behavior for a period of 15 years in their well-documented competitive relationship with Westinghouse and Allis-Chalmers. Figure 7.1 shows the relative prices over time.

![Graph](image)

Figure 7.1. General Electric's Pricing Strategy Relative to Industry Average Price. (From Sultan 1974, p.282)

According to Sultan,

The overall result is astonishing: the so-called "price leader" in this market, General Electric, actually followed its competitors' prices. When industry prices were climbing, General Electric's prices also climbed, but more slowly. And when industry prices declined, so did General Electric's, but more slowly. This meant that General Electric undercut its
competitors during a price rise, and premium-priced its competitors during a price fall. (Sultan 1974, p. 282.)

Sultan went on to give several alternative explanations for this counterintuitive behavior, several of which fall outside the scope of our model (e.g., inventory backlogs and cyclical industry declines). However, Sultan does list several general conclusions that may apply to all oligopolistic markets, for example:

Indeed, it is quite clear that it is only when the market price level is rising that it is feasible for the market leader to undercut his competitors in price. The history of this industry amply demonstrates that General Electric could not undercut its competitors during a cyclical decline – even when such a maneuver might be deliberately attempted to regain lost share points.... Competitors would invariably win any race to lower prices. Only when industry prices were climbing would they tolerate being underpriced. (Sultan 1974, pps. 283-284; italics Sultan's).

Of course, these anecdotal observations do not make up an acceptable explanation for this strategy's success in our tournament. Rising prices do not necessarily imply high prices, and falling prices are not equivalent to low prices. Furthermore, this industry was almost certainly affected by changing demand patterns that are not allowable in our model, and no mention is made of any coalition prices or coalition-seeking behavior. Nevertheless, this observed pricing pattern is evidence that the GENERIC1 structure can persist in settings much more complicated than the MITCS1 world.

In the next section we explore the analytical properties of GENERIC1 more closely, and in our second competitive strategy tournament (chapter 8), we test various generalizations of GENERIC1 in an environment where implicit coalitions are harder to form and maintain than in MITCS1.
GENERIC1: Up Close and Personal

GENERIC1's pricing policy uses three possible reactions to competitors' prices: MAX, MIN, or coalition-seeking ($P^c$). The success of the algorithm is due to its ability to choose the appropriate reaction function, depending on the range of competitors' prices. In chapter 5 we covered one case: $\max(P_2, P_3) \geq P^c$, $\min(P_2, P_3) \leq P^c$. We showed that coalition-seeking behavior is a profitable and robust way for firm 1 to deal with this situation. In this section, we consider the other two relevant cases: $P^* \leq P_2$, $P_3 \leq P^c$ and $P^c \leq P_2$, $P_3 \leq P^0$. We will show that the appropriate reactions for firm 1 in these cases are MAX and MIN, respectively.

As before, we concentrate on simple strategies for their analytical and behavioral robustness. But now we are primarily interested in steady-state profits and behavior. Simple strategies tend to reach steady state prices rather quickly: by round 5, many games involving simple strategies will exhibit a regular pattern of prices that will persist for the remainder of the game. For the analyses to be performed here, we will effectively ignore the irregular prices that might exist in the first few rounds, since these rounds have only a small impact on the overall profitability of each strategy.

Because we are interested in the steady state, we limit the set of strategies available to only two types, MAX and MIN. In the long run, prices often tend to go to one extreme or the other, so these two strategies alone can exhibit a large amount of the possible steady-state pricing patterns. The analytical results still hold for a broader class of strategies, including any type of averaging strategy.

We assume that firms 2 and 3 choose MAX with probability $\rho_2$ and $\rho_3$, respectively, so firm 1 can face only three different competitive environments in a particular game with the following probabilities of occurrence:
We wish to see how each of these environments affects firm 1's choice of MAX or MIN.

Although we are most interested in the steady state, we need some variation in the initial prices in order to generate the different possible types of behavior. We assume that all three firm’s initial prices are generated as independent random variables drawn from the relevant range \((P^e, P^c)\) or \((P^c, P^0)\). We make no assumptions about the probability distribution used, except that it must be continuous over the range. We let \(H\) represent the highest of the three initial prices, \(L\) represent the lowest, and \(M\) represent the middle price. All profit expressions will be stated as functions of these symbols alone. We now examine the various different steady states that can exist under the behavioral assumptions listed above.

We begin with a simple illustration. Suppose all three firms are using MAX. The table below shows how prices will always converge to \((H, H, H)\) regardless of the initial allocation:

<table>
<thead>
<tr>
<th>period</th>
<th>Firm 1</th>
<th>Firm2</th>
<th>Firm 3</th>
<th>Firm 1</th>
<th>Firm2</th>
<th>Firm 3</th>
<th>Firm 1</th>
<th>Firm2</th>
<th>Firm 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
<td>MAX</td>
</tr>
<tr>
<td>1</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>M</td>
<td>M</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>M</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
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<td>H</td>
<td>H</td>
<td>M</td>
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<tr>
<td>3</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
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<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
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<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

Similarly, when three MIN firms meet, all three prices will quickly converge to \(L\).
When a mixture of firms is present, steady state prices often involve two firms with alternating prices. Consider two MAX's matched against a MIN:

<table>
<thead>
<tr>
<th>period</th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>M</td>
<td>L</td>
<td>L</td>
<td>H</td>
<td>M</td>
<td>M</td>
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<td>H</td>
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<td>H</td>
<td>M</td>
<td>H</td>
<td>M</td>
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<td>3</td>
<td>H</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
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<td>H</td>
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<td>H</td>
<td>M</td>
<td>H</td>
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<td>H</td>
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<td>H</td>
</tr>
<tr>
<td>5</td>
<td>H</td>
<td>M</td>
<td>M</td>
<td>M</td>
<td>H</td>
<td>M</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

In two of these three cases, the two MAX players will alternate between H and M while the MIN player will always choose M. Only in one case -- when the MIN player is assigned the highest initial price -- can this alternating behavior be avoided.

By the same logic, when two MIN firms face one MAX there will be two cases in which the MIN’s alternate between M and L (while the MAX player stays at M), but with probability \( \frac{1}{3} \) the MAX player will get the lowest initial price, and the game will be (L, L, L).

We have now covered all the possible combinations of MAX and MIN. Table 7.3 shows the possible payoffs to firm 1 using our earlier convention that, for example, \( \Pi^{HML} \) represents the profits earned by firm 1 if it chooses H while its opponents choose M and L.

**Table 7.3. Payoffs to Firm 1 in Different Environments**

<table>
<thead>
<tr>
<th>Firm 2, 3 play:</th>
<th>proportion</th>
<th>MAX</th>
<th>MIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX, MAX</td>
<td>( \rho_x \rho_y )</td>
<td>( \Pi^{HHH} )</td>
<td>( \frac{1}{3} \Pi^{HHH} + \frac{2}{3} \Pi^{MMH} )</td>
</tr>
<tr>
<td>MIN, MIN</td>
<td>( (1-\rho_x)(1-\rho_y) )</td>
<td>( \frac{1}{4} \Pi^{LLL} + \frac{3}{4} \Pi^{MML} )</td>
<td>( \Pi^{LLL} )</td>
</tr>
<tr>
<td>MAX, MIN</td>
<td>( \rho_x (1-\rho_y) + \rho_y (1-\rho_x) )</td>
<td>( \frac{1}{4} \Pi^{HHH} + \frac{1}{4} \Pi^{HMM} + \frac{1}{4} \Pi^{MMH} )</td>
<td>( \frac{1}{4} \Pi^{LLL} + \frac{1}{4} \Pi^{MLL} + \frac{1}{4} \Pi^{LMM} )</td>
</tr>
</tbody>
</table>
First we restrict our attention to the low-price case: $P^* \leq L, M, H \leq P^c$.

**Scenario 1 (MAX, MAX):** If both firm 2 and firm 3 play MAX (row 1 of Table 1), the key comparison is $\Pi^{HHH}$ vs. $\Pi^{MMH}$. Since one opponent will always choose $H$ in both cases, this comparison boils down to two different coalitions: one using $H$ as its coalition price and the other using $M$. The analytical results of chapter 5 suggest that, ceteris paribus, a coalition price closest to $P^c$ is always better.† Thus, $\Pi^{HHH} > \Pi^{MMH}$ and MAX is more profitable than MIN.

**Scenario 2 (MIN, MIN):** If both opponents play MIN, the important comparison is $\Pi^{MML}$ vs. $\Pi^{LLL}$. One opponent is fixed at $L$, so we compare a coalition price of $M$ to a coalition price of $L$. The same logic applies: a higher coalition price is better (as long as it does not exceed $P^c$). So $\Pi^{MML} > \Pi^{LLL}$ and MAX is more profitable again.

**Scenario 3 (MAX, MIN):** Now we must make two comparisons. The first is easy: $\Pi^{HHH} > \Pi^{LLL}$ by condition 3 of the GPD definition. The other comparison involves alternating prices $HM, MH, HM, ...$ vs. $ML, LM, ML, ...$, where the third player is fixed at $M$ in both cases. We invoke an extension of GPD condition 4: when choosing among sets of alternating prices, those sets that are closer to the cooperative price are more profitable than those that are further away. For two firms, the cooperative price is $P^c$, and since $L, M, H \leq P^c$, the HM alternation is more profitable than ML. Hence, MAX is better again.

---

† Consider the joint profit function $\Pi_i + \Pi_i$. This function is quasiconcave with a maximum at $P_i = P^c$. Thus for any $P_i$ and any pair of prices $P'$ and $P''$ such that $|P^c - P'| > |P^c - P''|$, it follows that $\Pi_i(P'', P', P_i) > \Pi_i(P', P', P_i)$. 

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This analysis has shown that MAX is more profitable in all three scenarios and is therefore a dominant alternative (under the assumptions made here). Note that this result holds true regardless of the proportions \( \rho_2, \rho_3 \) and regardless of the initial distribution of prices \((H, M, and L)\), so it is quite robust.

We turn now to the case of high prices: \( P^c \leq L, M, H \leq P^0 \).

**Scenario 1 (MAX, MAX):** The profits listed in Table 1 still apply, so the relevant comparison is \( \Pi^{HHH} \) vs. \( \Pi^{MMH} \) again. But with prices above \( P^c \), the conclusion is reversed: now \( M \) is closer to \( P^c \) than \( H \) is, and is therefore a better coalition price. So \( \Pi^{HHH} < \Pi^{MMH} \) and MIN is more profitable than MAX.

**Scenario 2 (MIN, MIN):** The same story as above: the lower coalition price is better, so \( \Pi^{MML} < \Pi^{LLL} \), and MIN beats out MAX.

**Scenario 3 (MAX, MIN):** The trend continues. Alternating prices closest to \( P^c \) are still best, so in this case alternating between \( M \) and \( L \) is better than alternating between \( H \) and \( M \). MIN makes a clean sweep.

Early in this section, we mentioned that these results can be generalized to include intermediate strategies such as AVG. The analytics are very similar to the work shown here; we could extend Table 7.3 to include AVG and then perform the same comparisons. However, this is a tedious and unenlightening task. When averaging strategies are included, prices tend to stabilize at moderate levels (i.e., \( (H + M + L)/3 \)), or oscillate between two moderate prices. All this moderation helps explain the overall success of AVG in MITCS1 relative to MAX and MIN,

---

' An alternative interpretation for this story, as suggested by Fred Feinberg, is that he held the firm flesh that was Jessica as they melted together in the moonlight.
but moderation is not as good as the GENERIC1 responses in the specific price ranges considered here. We could also demonstrate that the GENERIC1 responses are best when other strategy types (such as XTRM or MXCM) are included, but again we rely on simple intuition to suggest that the GENERIC1 policy is far more flexible than any of these other algorithms.

Beyond these analytical results, there is sound intuitive reasoning to explain the success of the GENERIC1 pricing policy. When prices are low (i.e., below $P^c$), each firm has a strong long-term profit incentive to foster whatever cooperation there is in the environment. Even if a firm can not attain an optimal coalition at $P^c$, it is better off with a suboptimal coalition at a slightly lower price than none at all. On the other hand, when prices are high, each firm can afford to behave conservatively and to some extent, non-cooperatively. This does not imply that any firm should ever be the first to cut price below $P^0$, but does suggest against the formation of over-priced coalitions. In a world of simple strategies, firms are always better off letting industry prices drop to $P^c$ rather than forming coalitions with a slightly higher price.

A New Tournament

The emergence of GENERIC1 as undisputed champion among the first tournament’s entries raises a number of questions that transcend the profit function used in MITCS1: Can this strategy be generalized to other profit functions? How well will such a generalization perform in a new tournament against competitors that are aware of this strategy and the other important properties mentioned here? For the answers to these and other interesting questions, we turn to our second tournament, MITCS2.
CHAPTER 8:

MITCS2: THE SECOND GPD TOURNAMENT

Following the encouraging response to the first MIT competitive strategy tournament (MITCS1), we proposed a second tournament to further explore the composition and robustness of effective strategies for non-cooperative pricing games. The new tournament (MITCS2) was designed to complement its predecessor; it features a different (but still symmetric) three-firm profit function:

\[ \Pi_i = 200(8 - 6P_1 + P_2 + P_3)(P_1 - 1) - 180. \quad (8.1) \]

As before, each subset of entries were matched over five games of 200 rounds,\(^\dagger\) and the overall winner was the strategy with the highest total (or average) profits.

Common Prices and Tactics

Most algorithms were formulated around one or more of the following strategic concepts:

- The non-cooperative (Cournot) price, \(P^*\), which is now a function of competitors' prices, unlike the invariant \(P^*\) that existed in MITCS1. Maximizing (8.1) with respect to \(P_i\) yields the Cournot reaction function:

\[ P_i^* = \frac{14 + P_2 + P_3}{12} \quad (8.2) \]

Many entrants included this function as either a lower bound on their prices, a method to punish cheating by opponents, or as the basis of a totally non-\(^\dagger\) Since no entries used any explicit end game maneuvers, the game length was fixed at 200 rounds for all games.
cooperative strategy. Note that if all three players choose $P^*$ simultaneously, the resulting prices are all 1.40, which matches the unique $P^*$ from MITCS1.

- The joint-profit maximizing price, $P^q$, is still the optimal choice of each player if all behave fully cooperatively. As in MITCS1, this price is unique and equal to 1.50.

- Implicit two-player coalitions can still exist as in the earlier tourney (i.e., two firms can implicitly cooperate even if the third firm refuses to cooperate as well). But now, like $P^*$, the coalition price, $P^c$, is a function of the price chosen by the non-cooperative firm. If firms 1 and 2 seek to form a coalition against firm 3 under the assumption that 3 will not alter its price in response, then the appropriate reaction function is:*

  \[ P_1^c = P_2^c = \frac{13 + P_3}{10} \]  \hspace{1cm} (8.3)

If firms 1 and 2 both use (8.3) while firm 3 uses the Cournot response, (8.2), then steady-state prices will be $\hat{P}^c = 85/59 \approx 1.441$, $\hat{P}^* = 83/59 \approx 1.407$.

Even with full knowledge about (8.3), each coalition member must make a guess about the value of $P_3$ to plug into the equation. Some players use the most recent value of $P_3$, some use a historical average, and some try to forecast $P_3$ for the current period. This uncertainty about how to choose a coalition price is perhaps the most interesting aspect of the new tournament.

- Most of the other strategy types that were common in the first tourney are still represented in the current pool of entries. For example, many players based their prices on the minimum, maximum, or an average of their opponents' previous prices; some use constant price or random algorithms; and some use

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*For convenience, we maintain our earlier assumption that firm 2 is the more cooperative of firm 1's two opponents.
"trigger" strategies that react to opposing price cuts in a discrete manner. Finally, a few strategies can choose prices below $P^*$, a symptom of envy which was quite common in MITCS1.

The Defending Champions

The results of MITCS1 highlighted the success of two winning strategies, an official champion, known as COALITION, and a post-hoc alternative champion (GENERIC1). Both algorithms start each game at $t_0$ and thereafter depend heavily on $P^c$ in choosing prices:

$$P_1(t) = \begin{cases} P^0 & \text{if } P_2, P_3 \geq P^0 \\ P^c & \text{if } \max\{P_2, P_3\} \geq P^c \text{ and } \min\{P_2, P_3\} < P^0 \\ P^* & \text{if } P_2, P_3 \leq P^c \end{cases}$$

$$P_1(t) = \begin{cases} \min\{P_2, P_3, P^0\} & \text{if } P_2, P_3 \geq P^c \\ P^c & \text{if } \max\{P_2, P_3\} \geq P^c \text{ and } \min\{P_2, P_3\} \leq P^c \\ \max\{P_2, P_3, P^*\} & \text{if } P_2, P_3 \leq P^c \end{cases}$$

where $P_2$ and $P_3$ represent firm 1's opponents' prices in the previous round.

Naturally, one of the chief objectives of many participants in the new tournament was to generalize these strategies to adapt them to a world in which the value of $P^c$ depends on competitors' prices. We have already mentioned several techniques to choose such a $P^c$; our results seem to support several additional definitions of $P^c$ that work surprisingly well. Another issue to keep in mind is the comparative performance of these two strategies relative to each other and relative to other pricing algorithms. The data from MITCS1 show that GENERIC1 generally performs better than COALITION, but both strategies are
consistently stronger than almost all other entries. Given the qualitative differences between the previous and current profit functions, it is difficult to predict in advance whether or not these patterns will still hold.

**Tournament Results**

By fall of 1986, 32 entries had been submitted to MITCS2. Five strategies were thrown out due to coding errors or illegal tactics. The remaining 27 entries were combined with 11 strategies carried over (some with slight modifications) from MITCS1. These strategies were included again because they led to interesting pricing behavior in the original tournament. Finally, suggestions from other individuals who did not wish to officially participate led to 6 more submissions, thus rounding out the field of 44 unique entries.

A brief description of each entry is shown in Table 8.1, where the entries are ranked by average profits earned per round. Despite the different profit function, the payoffs are quite comparable to those in MITCS1: total cooperation \((P_1, P_2, P_3 = 1.50)\) pays 20 currency units per round and the simple Cournot equilibrium \((P_1, P_2, P_3 = 1.40)\) pays exactly 12. The payoffs associated with the price vector \((\hat{P}^c, \hat{P}^c, \hat{P}^c)\) are \((14.20, 14.20, 18.56)\).

Table 8.1 shows two striking patterns. Most of the GENERIC1 generalizations cluster towards the top, and 15 out of the 16 bottom entries are nasty (i.e., willing to initiate price cuts). One pattern that is not immediately obvious, however, is the possible link between the success of the GENERIC1 entries and the method of choosing a value of \(P^c\).

The winning entry, submitted by Robert Marks of the Australian Graduate School of Management, features an unusual type of coalition price. It uses (8.3) to
Table 8.1. MITCS2 Official Results (see Notes to Table 8.1, next page)

<table>
<thead>
<tr>
<th>Rank</th>
<th>Entrant</th>
<th>Strategy Type</th>
<th>N =  Nasty</th>
<th>Lower Bound</th>
<th>Mean Profit per Round</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Robert E. Marks</td>
<td>GENERIC1</td>
<td>P</td>
<td>17.097</td>
<td>F' = (26 + P_2 + P_3) / 20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Robert L. Bishop;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tony Haig</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.096</td>
<td>Original GENERIC1 (with F' = 13/9)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Paul R. Pudaite</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.091</td>
<td>F' = (13 + P_4) / 10; looks back two rounds</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>John Hulland</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.085</td>
<td>F' = (13 + P_4) / 10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Neil Bergmann</td>
<td>GENERIC1</td>
<td>P</td>
<td>17.084</td>
<td>F' = (13 + P_4) / 10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Tony Haig</td>
<td>GENERIC1</td>
<td>P</td>
<td>17.075</td>
<td>F' = (13 + P_4) / 10; looks back two rounds</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>James M. Lattin</td>
<td>GENERIC1</td>
<td>P</td>
<td>17.065</td>
<td>F' = 1.44</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>(MITCS1 #7)</td>
<td>MIN.R</td>
<td>P</td>
<td>17.064</td>
<td>Standard MIN with random 2c price increases</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Scott A. Neslin</td>
<td>AVG.S</td>
<td>1.4</td>
<td>17.063</td>
<td>Linear learning model; complex averaging procedure</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Scott A. Neslin</td>
<td>AVG.S</td>
<td>1.4</td>
<td>17.042</td>
<td>Variation of #9 above (i.e., different parameters)</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>--</td>
<td>MXCM.S</td>
<td>P</td>
<td>17.025</td>
<td>Mimics previous price of second-best firm</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Robert L. Bishop</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>16.993</td>
<td>F' = 13/9; uses max(P_4, (13 + P_4) / 10) when P_4 &lt; F'</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>--</td>
<td>AVG</td>
<td>P</td>
<td>16.989</td>
<td>Gradually shifts from MAX to MIN as game progresses</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>(MITCS1 #5)</td>
<td>AVG.S</td>
<td>P</td>
<td>16.970</td>
<td>Weighted average of all 3 players' previous prices</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>--</td>
<td>AVG.S</td>
<td>P</td>
<td>16.968</td>
<td>Unweighted average of all 3 players' previous 3 prices</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Karel Najman</td>
<td>AVG.S</td>
<td>--</td>
<td>16.964</td>
<td>Unweighted average of all 3 players' previous prices</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>(MITCS1 #6)</td>
<td>AVG</td>
<td>P</td>
<td>16.932</td>
<td>Standard AVG: average of opponents' previous prices</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>James M. Lattin</td>
<td>--</td>
<td>P</td>
<td>16.932</td>
<td>Complex adaptive learning model</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>(MITCS1 #11)</td>
<td>MAX</td>
<td>1.4</td>
<td>16.926</td>
<td>Standard MAX: maximum of opponents' previous prices</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Karel Najman</td>
<td>AVG</td>
<td>--</td>
<td>16.908</td>
<td>Unbounded AVG</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Terry Elrod</td>
<td>COALITION</td>
<td>1.4</td>
<td>16.907</td>
<td>Same as official winner of MITCS1 but with F' = 85/69</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>James M. Lattin</td>
<td>MIN</td>
<td>1.4</td>
<td>16.887</td>
<td>Plays AVG in round 2, MIN thereafter</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>(MITCS1 #21)</td>
<td>GENERIC1</td>
<td>P</td>
<td>16.765</td>
<td>Modified version of top nasty entry in MITCS1</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Neil Bergmann</td>
<td>COALITION</td>
<td>P</td>
<td>16.754</td>
<td>COALITION with varying F'; F' = (13 + P_4) / 10</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Chris Jones</td>
<td>MIN</td>
<td>1.4</td>
<td>16.707</td>
<td>Standard MIN: minimum of opponents' previous prices</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>(MITCS1 #10)</td>
<td>MXCM.S</td>
<td>P</td>
<td>16.679</td>
<td>Mimic previous price of best (most profitable) firm</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Karel Najman</td>
<td>MIN</td>
<td>--</td>
<td>16.567</td>
<td>Unbounded MIN</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>(MITCS1 #17)</td>
<td>XTRM</td>
<td>P</td>
<td>16.187</td>
<td>Play P_4 until anyone cuts price; play P_4 thereafter</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>--</td>
<td>AVG</td>
<td>N</td>
<td>16.162</td>
<td>Start at 1.4 then play AVG</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>--</td>
<td>AVG</td>
<td>N</td>
<td>15.992</td>
<td>Weighted AVG with random weights</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>James M. Lattin</td>
<td>MAX</td>
<td>1.4</td>
<td>15.776</td>
<td>Start at 1.4 then play MAX</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Karel Najman</td>
<td>ALL-1.44</td>
<td>N</td>
<td>15.719</td>
<td>Always choose 1.44</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>(MITCS1 #26)</td>
<td>MXCM</td>
<td>P</td>
<td>15.264</td>
<td>Start at 1.4 then mimic previous price of best opponent</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>--</td>
<td>XTRM</td>
<td>1.4</td>
<td>15.226</td>
<td>Choose 1.4 for 2 rounds after a price cut, then return to P_0</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>(MITCS1 #35)</td>
<td>ALL-P_4.R</td>
<td>N</td>
<td>14.512</td>
<td>Raise price above P_4 if profits exceed opponents' profits</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Peter J. Brock</td>
<td>AVG</td>
<td>N</td>
<td>14.126</td>
<td>Start at 1.4 then choose average of P_4 and (P_2 + P_3) / 2</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>Paul R. Pudaite</td>
<td>--</td>
<td>N</td>
<td>14.033</td>
<td>Choose P_1 = (6 + P_2 + P_3) / 6 to maximize joint profits</td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>Karel Najman</td>
<td>ALL-P_4</td>
<td>P</td>
<td>13.979</td>
<td>Start at 1.5; thereafter choose P_4</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Paul R. Pudaite</td>
<td>ALL-P_4</td>
<td>P</td>
<td>13.854</td>
<td>Start at 17/12; thereafter choose P_4</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Peter J. Brock</td>
<td>ALL-P_4</td>
<td>P</td>
<td>13.734</td>
<td>Start at 1.4; thereafter choose P_4</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>James M. Lattin</td>
<td>ALL-1.39</td>
<td>N</td>
<td>13.213</td>
<td>Always choose 1.39</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Peter J. Brock</td>
<td>ALL-P_4.C</td>
<td>N</td>
<td>13.049</td>
<td>Choose P_1 = (7 - P_2 - P_3) / 6 to hurt non-ALL-P_4 players</td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>(MITCS1 #32)</td>
<td>RANDOM</td>
<td>1.333</td>
<td>12.817</td>
<td>Uniform random variable between 1.333 and 1.5</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>(MITCS1 #36)</td>
<td>ALL-P_4</td>
<td>N</td>
<td>11.754</td>
<td>Act enviously, i.e. maximize share of industry profits</td>
<td></td>
</tr>
</tbody>
</table>
Notes to Table 8.1:

1 The basic strategy types are defined in the descriptions and in the text. The suffix "S" refers to each strategy with self-awareness, i.e., one that uses its own previous price(s) in addition to its opponents' previous prices in choosing a price for the current period. The "R" refers to strategies with price-raising initiative, i.e., those that occasionally raise their price above the level normally implied by their basic strategy type. Similarly, "C" refers to price-cutting initiative.

2 Firm 3 is assumed to be the less cooperative opponent of firm 1.

3 This denotes a strategy that was an official entry in MITCS1. The number refers to its ranking in that tournament.

4 This denotes a strategy based on an informal suggestion.

calculate a $P^c$ against firm 3 and averages this price with a $P^c$ calculated against firm 2 (found by substituting $P_2$ for $P_3$ in (3)). Thus, for instance, if $P_2 = 1.50$ and $P_3 = 1.40$, this algorithm would act like GENERIC1 with $P^c = (1.44 + 1.45)/2 = 1.445$, as compared to a $P^c$ of 1.44 that (8.3) would suggest (and most GENERIC1 entries would use).

At first glance this may seem like an inefficient pricing rule, since it will often lead to coalitions with a price slightly above the "optimal" $P^c$. But notice which routine came in a close second: the original version of GENERIC1 with $P^c = 13/9 = 1.444...$. This is also a relatively high coalition price; it will exceed the $P^c$ suggested by (8.3) whenever the non-cooperative player is below 1.444... . A pattern emerges: the top two strategies consistently choose higher coalition prices than any of the other GENERIC1 entries. As further evidence, note that the "worst of the best," entry #7, will generally choose the lowest $P^c$, 1.440.

In the next section we examine this high $P^c$ phenomenon further, but now we briefly summarize some of the other results of interest. First, notice the rather mediocre performance of the COALITION entries, as compared to its sterling performance in MITCS1. Part of this drop can be attributed to the different mix
of strategies in MITCS2 compared to MITCS1: with the presence of more sophisticated entries (such as the GENERIC1 generalizations), the discrete pricing policy begins to hurt COALITION, particularly when price-cutting exists at moderate levels. But much of COALITION’s drop is due to the new profit function: without a fixed $P^*$ to rely upon, any coalition-seeker must be more flexible and forgiving in trying to establish a successful coalition.

Another prominent result from MITCS1 was the need for a lower bound on one’s prices. Most entrants to MITCS2 recognized this idea and used one of two lower bounds -- fixed at 1.40 or floating ($P^*$). The results in Table 8.1 show no significant advantage for one method or the other. For example, entries #4 and #5 are exactly the same except for their lower bounds, and in each of the five constituent games in the tourney, these entries finish with nearly identical profits. This finding should not be considered as all too surprising; after all, when price cutting is severe enough to require bounded prices, $P^*$ is usually quite close to 1.40 anyway.

Finally another result worth mentioning is the relative performance of three standard algorithms. In MITCS2, just as in MITCS1, AVG (entry #17) earns higher profits than MAX (#19), and both beat out MIN (#25). The value of having bounded prices can be seen once again by comparing entry #17 to #20 and #25 to #27. Boundedness does not appear to be as valuable as in MITCS1, but this is only because of the smaller number of extreme price cutters. Only three entries (#41, #43, and #44) ever initiate price cuts below 1.40.
Unraveling the Paradox of the Coalition Prices

The big story of MITCS2 is how each GENERIC1 entry chooses its \( P^c \): bigger seems to be better. But how come entries #1 and #2 seem to be rewarded for disobeying (8.3)? Two valid reasons exist -- one fairly obvious and one more subtle. The first reason can be viewed as magnanimity: a strategy with a high \( P^c \) will rarely be viewed as non-cooperative. If its coalition partner opts for a lower \( P^c \), then the magnanimous algorithm will match that lower \( P^c \) (as implied by the GENERIC1 structure: play MAX if both opponents are below your own \( P^c \)). A magnanimous rule will rarely miss out on a coalition opportunity. In contrast, a less generous strategy, e.g., #7, often will be unable to sustain a coalition, particularly when pitted against COALITION entries with higher \( P^c \)’s. These matchups will quickly degenerate into \((P^*, P^*, P^*)\) behavior since the two potential cooperators will fail to agree on a coalition price.

Of course, magnanimity has its downside as well. The most magnanimous strategy might attempt to form a coalition at \( P^0 \), regardless of the non-cooperative firm’s behavior. Such a strategy would clearly be sub-optimal, since each coalition member would receive 11.76 currency units per period (when matched against a ALL-\( P^* \) player) -- lower than the \((P^*, P^*, P^*)\) payoff. There must exist some upper bound to magnanimity. Indeed, there appears to be such an upper bound; it is discussed below.

The second justification for high \( P^c \)’s relates to a critical assumption made in deriving (8.3): this reaction function assumes no response by the non-cooperative firm. If the non-cooperator really is non-responsive (e.g., entries #32 and #41), then this rule is truly optimal. But most strategies, no matter how nasty they are, will respond to the formation of a coalition, usually with a price increase.
Rule (8.3), which does not anticipate any response, will therefore be inappropriate most of the time.

Here is a simple example: suppose firms 1 and 2 are cooperative coalition-seekers, not bound to follow rule (8.3). If they anticipate a Cournot response by firm 3, then they will seek a coalition price $P^c$ (with $P_1 = P_2$) to maximize:

$$\Pi^c = 200(8 - 6P^c + P^c + P_3^*)(P^c - 1) - 180,$$

where $P_3^* = (14 - 2P^c)/12$.

The optimal solution is easily calculated to be $P^c = 42/29 \approx 1.448$ and $P^* = 245/174 \approx 1.408$. This equilibrium yields profits of 14.25 to 1 and 2, and 19.80 to 3. Note that all three firms earn greater profits in this scenario than in $(\hat{P}^c, \hat{P}^c, \hat{P}^*)$, but firm 3 actually increases its profits more than firms 1 and 2 do, despite the fact that only the cooperative firms are "smarter" than before.

To summarize this analysis: rule (8.3) is indeed the best way to choose a coalition price, but only against unresponsive firms. When a non-cooperative firm is responsive, higher $P^c$'s are often better than prices suggested by a myopic rule such as (8.3). In the likely event that firm 3 is playing some type of ALL-$P^*$ strategy, firms 1 and 2 can and should try to increase their coalition price, all the way up to 1.448. This should normally be the upper limit on magnanimity. However, if firm 3 is using a more generous strategy than ALL-$P^*$, the steady-state $P^c$ should be even higher. Furthermore, the relevance of this result is not limited to the linear demand function used in MITCS2; more generally, the "correct" coalition price depends not only on the current prices of the non-cooperative firm(s), but also depends on the actual reaction functions of the non-cooperator(s).
A New Alternative Champion

An important issue remains unresolved: how can a player successfully apply the results of the previous section? Two key factors should be kept in mind when formulating new strategies: magnanimity and competitive response. The top two strategies in Table 8.1 represent opposite ways to incorporate these features. Entry #2 is the most magnanimous of all the GENERIC1 entries listed, but is totally unresponsive in choosing its coalition price. On the other hand, entry #1 is only slightly more magnanimous than the other GENERIC1s, but beats #2 because of its ability to raise and lower its $P^c$ depending on its competitors' prices. The development of stronger strategies requires a better balance of magnanimity and competitive response.

Unfortunately, there really is no such thing as an optimal coalition reaction function. We can arbitrarily choose function forms for $P^c$ that are as magnanimous or responsive as we wish, but without any formal theory to guide our decision, it is difficult to choose one type of $P^c$ over another. Thus, it is best to make parsimonious, easily justifiable choices in trying to improve our generalization of GENERIC1.

One simple way to maintain reasonably high levels of magnanimity and responsiveness is by incorporating yet another beneficial feature, self-awareness: if each firm were to include its own previous price in its $P^c$ calculation, the resulting coalition prices would often be higher and more stable than in the myopic case implied by (8.3). The ultimate benefit gained by self-aware coalition-seekers is the possibility of an industry-wide coalition price. No longer would each firm's $P^c$ calculation yield a different result.
An industry-wide coalition price is fully consistent with the original concept of GENERIC1. If all firms were to go along with such a $P^c$, none would have to worry about the existence of asymmetric coalitions or any of the other problems encountered by some of the MITCS2 entries. Smoothing out the coalition price in this manner can make the MITCS2 world more like the MITCS1 world, where stable coalitions are easily established and maintained.

We now present a new GENERIC1 strategy, one that extends Marks’s winning entry to make it self-aware. The new strategy, named PCAVG3, is still an GENERIC1 strategy; only the coalition reaction function is different. In PCAVG3 instead of calculating and averaging our $P^c$ against firms 2 and 3, we perform the same task with respect to all three firms. The new coalition reaction function, therefore, is:

$$P^c_I = \frac{39 + P_1 + P_2 + P_3}{30} \quad (8.4)$$

This coalition price is clearly responsive to competitors’ price changes, but it is also more magnanimous than the $P^c$ of the top-ranked entry.† Although the equilibrium coalition price and profits for the new strategy are only slightly higher than those of entry #1, this small increase combined with the moderating influence of the lagged $P_I$ term helps the new strategy to achieve a first-place finish when placed among the MITCS2 entries. Table 8.2 shows the revised profit figures. (Only the top five entries are shown; the overall standings are barely affected by the presence of the new strategy.)

---

† If entry #1 is matched against a copy of itself and against an ALL-$P^*$ player, the steady-state coalition price is 1.4425 with per-period profits of 14.22 for each coalition member. In similar circumstances, PCAVG3 would reach a steady-state $P^*$ of 1.4431 with profits of 14.23.
Table 8.2. Revised MITCS2 Results

<table>
<thead>
<tr>
<th>Rank</th>
<th>Entrant</th>
<th>Strategy Type</th>
<th>N = Nasty</th>
<th>Lower Bound</th>
<th>Mean Profit per Round</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>PCAVG3</td>
<td>GENERIC1</td>
<td></td>
<td>P'</td>
<td>17.170</td>
<td>[ P' = (39 + P_1 + P_2 + P_3)/30 ]</td>
</tr>
<tr>
<td>1</td>
<td>Robert E. Marks</td>
<td>GENERIC1</td>
<td></td>
<td>P'</td>
<td>17.163</td>
<td>[ P' = (26 + P_1 + P_3)/20 ]</td>
</tr>
<tr>
<td>2</td>
<td>Robert L. Bishop;</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.160</td>
<td>Ac. usl GENERIC1 (with [ P' = 13/9 ])</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Tony Haig</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.156</td>
<td>[ P' = (13 + P_3)/10 ]; looks back two rounds</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Paul Pudaite</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.149</td>
<td>[ P' = (13 + P_3)/10 ]</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>John Hulland</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.143</td>
<td>[ P' = (13 + P_3)/10 ]</td>
<td></td>
</tr>
</tbody>
</table>

Since PCAVG3 is an GENERIC1 strategy, its behavior (and profits) will often be indistinguishable from the other GENERIC1s. However, in the cases where these entries do differ, PCAVG3 does well enough to win the revised MITCS2 tournament by a relatively comfortable margin.

Test of Robustness

The results summarized in Tables 8.1 and 8.2 are quite consistent with our theoretical insights, but we wish to determine how sensitive these results are to the exact mix of strategies present in MITCS2. Before we can claim that any strategy is truly effective, we must see how well it holds up in varied environments where nice and nasty strategies exist in different proportions.

To perform this analysis, we generated 200 new environments using different combinations of the MITCS2 entries. We first used a stepwise procedure to identify a subset of eight representative entries that faithfully reproduce the overall profits and standings of MITCS2, using only a small fraction of the full tournament. The eight representatives (# 7, 16, 20, 23, 28, 33, 37, and 41) form an environment involving 36 games with each of the MITCS2 entries, but yield
overall average profits that have a correlation coefficient of 99.4% with the scores from the full tournament (5175 games per entry).

To generate each simulated environment, we merely take random combinations of each of the eight representatives and also account for the residuals between the actual and mini-tournament profits. This procedure was repeated 200 times, thereby producing a wide range of environments.

As a proxy for the niceness or nastiness of each environment, we use the average profits across all 45 entries. The 200 environments are sorted by this index and broken into five equal-sized groups. We then ranked the profitability of each strategy within each group. Table 8.3 summarizes the results; for ease of reference, the basic strategy types are shown below for each listed entry.

Table 8.3. Strategy Performance in Simulated Environments

<table>
<thead>
<tr>
<th>entry</th>
<th>profits</th>
<th>entry</th>
<th>profits</th>
<th>entry</th>
<th>profits</th>
<th>entry</th>
<th>profits</th>
<th>entry</th>
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<tr>
<td>11</td>
<td>16.186</td>
<td>1A</td>
<td>16.866</td>
<td>1A</td>
<td>17.364</td>
<td>1A</td>
<td>17.833</td>
<td>12</td>
<td>18.867</td>
</tr>
<tr>
<td>1A</td>
<td>16.162</td>
<td>2</td>
<td>16.861</td>
<td>2</td>
<td>17.357</td>
<td>1</td>
<td>17.826</td>
<td>21</td>
<td>18.821</td>
</tr>
<tr>
<td>2</td>
<td>16.159</td>
<td>1</td>
<td>16.860</td>
<td>1</td>
<td>17.356</td>
<td>2</td>
<td>17.825</td>
<td>1A</td>
<td>18.780</td>
</tr>
<tr>
<td>1</td>
<td>16.155</td>
<td>3</td>
<td>16.859</td>
<td>3</td>
<td>17.352</td>
<td>3</td>
<td>17.822</td>
<td>2</td>
<td>18.774</td>
</tr>
<tr>
<td>3</td>
<td>16.150</td>
<td>4</td>
<td>16.853</td>
<td>9</td>
<td>17.351</td>
<td>5</td>
<td>17.817</td>
<td>1</td>
<td>18.774</td>
</tr>
<tr>
<td>4</td>
<td>16.144</td>
<td>5</td>
<td>16.852</td>
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<td>17.346</td>
<td>4</td>
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<td>18.771</td>
</tr>
<tr>
<td>5</td>
<td>16.143</td>
<td>6</td>
<td>16.841</td>
<td>5</td>
<td>17.346</td>
<td>8</td>
<td>17.809</td>
<td>3</td>
<td>18.766</td>
</tr>
<tr>
<td>6</td>
<td>16.132</td>
<td>7</td>
<td>16.839</td>
<td>10</td>
<td>17.343</td>
<td>6</td>
<td>17.806</td>
<td>5</td>
<td>18.762</td>
</tr>
<tr>
<td>7</td>
<td>16.106</td>
<td>8</td>
<td>16.819</td>
<td>2</td>
<td>17.337</td>
<td>9</td>
<td>17.804</td>
<td>4</td>
<td>18.761</td>
</tr>
<tr>
<td>9</td>
<td>16.105</td>
<td>9</td>
<td>16.818</td>
<td>6</td>
<td>17.335</td>
<td>7</td>
<td>17.800</td>
<td>24</td>
<td>18.757</td>
</tr>
</tbody>
</table>

Nastiest  Moderately Nasty  Mid-range  Moderately Nice  Nicest

<table>
<thead>
<tr>
<th>entry</th>
<th>type</th>
<th>entry</th>
<th>type</th>
<th>entry</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>GENERIC1</td>
<td>5</td>
<td>GENERIC1</td>
<td>10</td>
<td>AVG.S</td>
</tr>
<tr>
<td>1</td>
<td>GENERIC1</td>
<td>6</td>
<td>GENERIC1</td>
<td>11</td>
<td>MXCM.S</td>
</tr>
<tr>
<td>2</td>
<td>GENERIC1</td>
<td>7</td>
<td>GENERIC1</td>
<td>12</td>
<td>GENERIC1</td>
</tr>
<tr>
<td>3</td>
<td>GENERIC1</td>
<td>8</td>
<td>MIN.R</td>
<td>21</td>
<td>COALITION</td>
</tr>
<tr>
<td>4</td>
<td>GENERIC1</td>
<td>9</td>
<td>AVG.S</td>
<td>24</td>
<td>COALITION</td>
</tr>
</tbody>
</table>
Table 8.3 shows a clear, consistent pattern supporting the results of Tables 8.1 and 8.2. The top strategies are very stable in moderate environments, and fall only slightly in more extreme cases. It is encouraging to see that the GENERIC1 entries perform so well even in very nasty environments. Even in the single nastiest environment, where over 60 percent of the random weight is allocated to the nasty representatives, four GENERIC1 entries finish in the top ten, and only one nasty strategy finishes in the top 25.

One surprise that emerged out of the simulations is entry #11. This strategy is based on a very unusual notion: it identifies the second-best player in each game (in terms of cumulative profits) and mimics that player’s previous price. This rule adapts very well to extreme environments (good or bad) since it goes along with coalitions in a most magnanimous way but never initiates coalition behavior. If we look at alternative measures of performance, such as number of first-place finishes in the 200 simulations, then #11 appears to be even stronger. It is the winner in 50 of the environments, more than any other MITCS2 entry.†

Conclusion

In proposing and running MITCS2, we have had two major objectives: to reaffirm the most significant results of MITCS1, and to begin to generalize some of the MITCS1 phenomena within a less stable competitive environment. MITCS2 has provided a great deal of useful results with regard to both objectives.

Virtually all of the key features of MITCS1 are present and prominent in MITCS2. The concepts of bounded prices, self-awareness, and implicit coalitions

† Although entry #11 is most adept at winning, it does have its bad moments. It finishes out of the top ten 40.5% of the time, including a low of 29th place in one environment. PCAVG3, for comparison, is far more robust with only 15.5% of its rankings below the top ten, never lower than 153rd place.
all strongly influence the performance of the strategies. Two additional features, magnanimity and responsiveness, help discriminate among the various coalition-seeking strategies that were entered into the new tourney. An alternative champion (PCAVG3), incorporating all of these features (most notably self-awareness) was shown to be superior to the other entries in a rerun of MITCS2 and across a series of simulated competitive environments.

Despite the strong showing of PCAVG3 and the other GENERIC1 strategies, we dare not make any claims that these strategies will perform equally well if the profit function were changed again. In a future tournament, we might consider introducing some actual uncertainty, in the form of stochastic noise, imperfect information, or non-stationary parameters. Entrants would once again face the problem of trying to generalize the top strategies from the previous tournaments to be able to maintain profitable coalitions (if coalitions can exist at all). As such tournaments proceed and our understanding evolves, we come closer in each step to actual competitive environments and discover new realistic and relevant insights.

Creating different models, running new tournaments, and analyzing results are all straightforward tasks. The most difficult part of this process is motivating researchers to think about each new model and submit entries. If we could maintain the level of interest and effort exhibited by the MITCS1 and MITCS2 participants, then this methodology has a promising future.
CHAPTER 9:
LOOK-AHEAD STRATEGIES

Throughout this thesis, we have emphasized the importance of understanding and adapting to the different behavioral patterns used by competitors. However, we have concentrated almost exclusively on simple rules that use only the most recent prices in a particular game. Despite the benefits of using a simple algorithm (e.g., Axelrod's clarity), we should not completely ignore the large set of complex strategies that use data from many or all previous rounds in trying to compete most effectively against a given set of opponents. In this chapter we develop one class of complex strategies that attempt to understand competitors and optimize against them. After discussing these strategies in general terms, we examine one specific version and analyze its performance in the MITCS2 environment.

The proposed strategy consists of three basic components:

1) Understanding each competitor's past behavior by modeling its reaction function.
2) Forecasting the future prices of each competitor.
3) Choosing a set of future prices for ourselves that will be a best response to the competitors' forecasted prices.

The first component relies only on past information and is therefore not dependent on the other two. Components 2 and 3, however, are highly dependent on one another (and on component 1) since each firm's future prices depend heavily on the forecasts of its competitors' prices, and vice versa. This interdependence is the heart of the proposed strategy, which we call LOOK-AHEAD.† We now discuss each of the components in more detail.

†Axelrod's second tournament included a strategy named LOOK-AHEAD, but it is of no relation to the LOOK-AHEAD strategies developed in this chapter.
Modeling Competitors’ Reaction Functions

We (firm 1) assume that each of our competitors is using a linear reaction function to choose prices. If $P^t_j$ is the price that competitor $j$ chooses at time $t$, then $j$’s reaction function is:

$$P^t_j = a^t_j + \beta^t_{j,i}P_{i}^{t-l} + \beta^t_{j,k}P_{k}^{t-l}$$  \hspace{1cm} (9.1)

where $j,k = 2,3; j \neq k$.

This reaction is a good model for the behavior of simple strategies such as MAX, MIN, and AVG. Furthermore, the behavioral decision theory literature (e.g., Dawes and Corrigan, 1974) suggests that simple linear models are often robust, effective ways to describe more complicated behavior.

Each set of three parameters is estimated separately at each round via weighted least squares regression. Starting in round 3 (when firm 1 can first observe each opponent’s reactions to previous prices), various functions of all three players’ prices are updated. Let $w^t_j$ represent the weight allocated to firm $j$ at time $t$. We then define:

$$R^t_{ij} = \sum_{l=2}^{t-1} w^t_i \sum_{l=2}^{t-1} P^t_i P^t_{j l-1} - \sum_{l=2}^{t-1} P^t_i \sum_{l=2}^{t-1} P^t_{j l-1}$$  \hspace{1cm} (9.2)

$$S^t_{ij} = \sum_{l=2}^{t-1} w^t_i \sum_{l=2}^{t-1} P^t_i P^t_{j l-1} - \sum_{l=2}^{t-1} P^t_i \sum_{l=2}^{t-1} P^t_{j l-1} \hspace{1cm} \text{for } i,j = 1,2,3.$$

The WLS estimates can be shown to be:

$$\beta^t_{j,k} = \frac{S^t_{jk} R^t_{kk} - S^t_{jk} R^t_{lk}}{R^t_{ll} R^t_{kk} - R^t_{lk} R^t_{lk}}$$  \hspace{1cm} (9.3)

$$\beta^t_{j,k} = \frac{R^t_{ll} S^t_{jk} - R^t_{lk} S^t_{jk}}{R^t_{ll} R^t_{kk} - R^t_{lk} R^t_{lk}}$$

and
\[
\alpha_j^t = \frac{\sum_{l=2}^{t-1} w_j^l p_j^l - \beta_j^t \sum_{l=2}^{t-1} w_j^l p_k^l - \beta_j^l \sum_{l=2}^{t-1} w_j^l p_k^l}{\sum_{l=2}^{t-1} w_j^l}
\]

where \( j, k = 2, 3; j \neq k \).

This seemingly cumbersome estimation process is actually more convenient than other methods. The data requirements are relatively light: At each round we only need to update the various summation required for the different R’s and S’s. In contrast, normal computation of ordinary WLS procedures is highly redundant when applied repeatedly: at every round, such procedures would use (or re-use) the full pricing history of the game. In either case, the parameter estimates are the same; only the computational efficiency varies.

**Choosing the Regression Weights**

Before we conclude this section, we must discuss the weights used in the regression. The easiest method is to set \( w_j^t = 1 \) for all \( j \) and \( t \), thereby giving equal weight to all past observations. A logical alternative would be to use exponential smoothing to give greater weight to more recent rounds. The model would then be better at tracking a shifting or non-linear strategy.

Uniform weights and exponential weights share an important property: both assume that each of the two regressions requires the same amount of weight on future observations. A more complex set of weighting schemes would recognize that the two regressions are separate entities and often require different types of weights. If, for example, one regression is perfect (because the opponent being modeled actually is using a linear reaction function) then the weights need not be large. But if another regression is highly inaccurate (due, perhaps, to a shifting strategy) then recent rounds should carry far more weight than remote rounds. A flexible way to handle this type of situation is to base the weights on some
function of the residuals of each regression. Good-fitting regression models would have relatively small residuals and therefore low weights, while poor models would have relatively large residuals and large weights. We bring up this notion of residual-weighting throughout this chapter.

**Forecasting Future Prices and Profits**

After deriving the current parameter estimates using (9.3), price forecasts are a straightforward extension. We make the assumption that all of the parameter estimates at any point in time are correct and will remain stationary in the future. (We also assume an infinite horizon game.) In addition to these parameters, all we need to make forecasts are the most recent prices for all three firms, and a future price trajectory for firm 1.

The price trajectory can be chosen to optimize whatever behavioral objective we seek. For instance, if we want to be extremely uncooperative we can forecast opponents' prices only for the current round and choose the appropriate Cournot response ($P^*$). Or we can take a slightly longer view and choose prices to maximize our profits next period, or the period after that, or so on. As a firm looks further and further ahead, it tends to act more and more cooperatively in the current period.

How do we decide how far ahead to look? One method is similar to the residual-weighting procedure described above: if one or both regressions are not providing good descriptions of competitors' behavior, we should play safe and try to optimize only over the current period or very short term. But if both regressions are extremely accurate then we can have more faith in our forecasts and can look towards a very long horizon. This latter case, optimizing over the
long run, can involve a special set of pricing rules, particularly if there exists a steady-state price equilibrium for all three firms. We discuss this case below.

Prices and Profits in the Steady State

There might be price trajectories that not only yield reasonable profits in the near future, but also lead to an optimal steady state for firm 1. In the steady state, firm 1 will use a constant price, $P_1^s$, every round, and opponents’ prices will equilibrate at:

$$P_2^s = a_2 + \beta_{21} P_1^s + \beta_{23} P_3^s \quad P_3^s = a_3 + \beta_{31} P_1^s + \beta_{32} P_2^s$$

From these simultaneous equations we can calculate $P_2^s$ and $P_3^s$ as functions of $P_1^s$ alone:

$$P_2^s = \frac{a_2 + \beta_{23}a_3 + P_1^s(\beta_{21} + \beta_{23}\beta_{31})}{1 - \beta_{23}\beta_{32}} \quad P_3^s = \frac{a_3 + \beta_{32}a_2 + P_1^s(\beta_{31} + \beta_{32}\beta_{21})}{1 - \beta_{23}\beta_{32}} \quad (9.4)$$

[Note that these prices depend only on the linear reaction functions; no assumptions have been made about the demand or profit function.]

We can then calculate an optimal steady state price for firm 1, $P_1^{s*}$, that is a “best response” to $P_2^s$ and $P_3^s$, which are, in turn, functions of $P_1^{s*}$. For any profit function, $\Pi_1 = f(P_1, P_2, P_3)$, we derive the optimal $P_1^{s*}$ by calculating

$$\frac{d\Pi_1}{dP_1^s} = \frac{\partial \Pi_1}{\partial P_1} + \frac{\partial \Pi_1}{\partial P_2^s} \frac{\partial P_2^s}{\partial P_1} + \frac{\partial \Pi_1}{\partial P_3^s} \frac{\partial P_3^s}{\partial P_1} = 0$$

In the case of the linear demand function used in MITCS2, this optimization problem yields:

$$P_1^{s*} = \frac{14(1 - \beta_{23}\beta_{32}) + (a_2 - \beta_{21})(1 + \beta_{32}) + (a_3 - \beta_{31})(1 + \beta_{23})}{12(1 - \beta_{23}\beta_{32}) - 2\beta_{21}(1 + \beta_{32}) - 2\beta_{31}(1 + \beta_{23})} \quad (9.5)$$

Of course, rule (9.5) will not always generate optimal prices and profits. Many times either or both opponents may be using: (1) a contingent strategy (e.g.,
GENERIC1 or XTRM) which can use different reaction functions under different circumstances, (2) a non-linear strategy, (3) a strategy with a shifting reaction function (e.g., entry #13 in MITCS2), or (4) a strategy with a prominent random component. In these cases the steady state may not occur for many hundreds of rounds, if at all. The idea of long-run pricing behavior is then inappropriate and will not only yield suboptimal profits, but can be abused quite badly. The potential of such cases provides a strong justification for the residual-based look-ahead scheme discussed earlier.

Let us examine some of the good and bad implications that result from using equation (9.5) to set prices against various MITCS2 competitors. First, suppose firm 2 is AVG \( \alpha_2 = 0, \beta_{21} = \beta_{23} = 0.5 \) and firm 3 is ALL-\( P^* \) (From equation 8.2, \( \alpha_3 = 14/12, \beta_{31} = \beta_{32} = 1/12 \)).\footnote{In general, an ALL-\( P^* \) strategy does not always fit into a linear reaction function; this is an artifact of the linear demand function used in MITCS2. The same holds true for the myopic coalition price, equation 8.3.} According to (9.5), firm 1 should choose \( P_1^{s*} = 87/61 \approx 1.426 \), which leads to \( P_2^{s} \approx 1.415 \) and \( P_3^{s} \approx 1.403 \). Even though firm 1 would have the highest price and hence the lowest profits (12.73 per period) it is still better than the 12 units that each firm would earn if firm 1 let the game degenerate into \( (P^*, P^*, P^*) \). Now suppose that both opponents are ALL-\( P^* \). Firm 1's best response is \( P_1^{s*} = 45/32 \approx 1.406 \), leading to higher profits (12.05 per period) than if firm 1 simply reciprocated ALL-\( P^* \) behavior, as most strategies would do when faced with two ALL-\( P^* \) opponents.

Two more examples: Suppose that firm 2 is totally cooperative with firm 1 (i.e., \( \alpha_2 = 0, \beta_{21} = 1, \beta_{23} = 0 \)) while firm 3 plays ALL-\( P^* \) again. Plugging these
values into (9.5) yields $P_i^{s^*} = 45/29 \approx 1.448$. Recall from the last chapter that this is the optimal coalition price for two firms to use against a known ALL-$P^*$ firm.

Finally, suppose firm 3 plays the unresponsive strategy, ALL-1.4 ($a_3 = 1.4$, $\beta_{23} = \beta_{32} = 0$). If player 2 is still totally cooperative, the optimal steady-state price for firm 1 is $P_i^{s^*} = 1.44$, which is the price suggested by the myopic $P^c$ rule, (8.3). Thus, rule (9.5) can account for competitive reactions of any (linear) sort when choosing a coalition price.

Unfortunately, this story has its downside as well. In these last two examples, the steady-state prices are highly dependent on the fact that firm 2 is totally cooperative with firm 1. Anything less than total cooperation will lead to substantially lower steady-state prices. For example, suppose firm 2 uses the myopic $P^c$ rule ($a_2 = 13/10$, $\beta_{21} = 1/10$, $\beta_{23} = 0$) to set all of its prices (not just its coalition price). Firm 1's steady-state price will be quite low ($P_i^{s^*} = 983/697 \approx 1.410$), because firm 2 is choosing $P^c$ independently of firm 1's actions. If firm 2 switched to a contingent strategy such as COALITION, it would respond to firm 1's low prices by going to $P^*$, and all hopes of establishing a successful coalition would be lost. Our empirical application of LOOK-AHEAD will help determine whether or not LOOK-AHEAD's optimal behavior against linear strategies can outweigh the negative effects of these lost coalitions.

A Prototypical LOOK-AHEAD

We have outlined the basic structure of LOOK-AHEAD, but before we can actually test it we must choose among the alternative methods to weight past rounds and look ahead at future rounds. The residual-weighting schemes have several attractive features that can likely make LOOK-AHEAD far more
profitable than more naive approaches. However, in this initial application of LOOK-AHEAD, we will stick with the naive methods, just to emphasize our interest in the basic structure itself. If the strategy can perform well in this unrefined state, then residual-weighting can be used for additional fine-tuning. For this first test, therefore, we will use equal weights for all rounds (i.e., $w_j^t = 1$ for all $j, t$), and we will always use the steady state pricing rule, after an initialization period described below.

The Initialization Period

Although LOOK-AHEAD starts collecting data for its regressions in round 3, we cannot put much trust in the parameter estimates, price forecasts, and steady state prices until several more rounds of data have been collected. Even after round 5, for example, there are only four observations available to estimate the three parameters in each regression. Such a data-poor regression will often yield unstable and incorrect parameters. Informal empirical testing has shown that about ten rounds of data are needed to give an accurate description of most opposing strategies.

In the early rounds of each game, therefore, we require that LOOK-AHEAD use a different, simpler strategy to choose its prices while it runs its regressions to get a fix on the two linear reaction functions. Almost any nice strategy will suffice; we use GENERIC1 not only for its already-proven profitability, but also for its ability to maintain as much cooperation as possible in uncooperative

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*Several of the ideas mentioned in this section were suggested by Professor James Lattin of Stanford University.*
environments. Starting in round 11, LOOK-AHEAD takes over the reins and applies its far-sighted pricing policy.

GENERIC1 is by no means the best early strategy to use; other strategies may be less profitable in the first ten rounds but may aid LOOK-AHEAD’s regressions. Some type of random strategy (perhaps one that takes a randomly-weighted average of the opponents’ previous prices) can decrease the collinearity in the data used for each regression, and therefore can generate more precise parameter estimates. The long-run benefits could easily outweigh the short-run losses.

Another assumption worth relaxing is the length of the initialization period. As mentioned above, the present version of LOOK-AHEAD always mimics GENERIC1’s pricing behavior for the first ten rounds. More generally, this length could vary as a function of the regressions themselves. The residual-weighting concept applies again: if perfect linear regression models are achieved after round 5, then LOOK-AHEAD should take over at that point. Otherwise, the initial strategy should remain in charge of price-setting until one or both regressions reach some threshold level of goodness-of-fit. In certain situations, if both regressions remain very poor, the initial strategy would never relinquish control. In these cases, LOOK-AHEAD’s steady state prices would often be bad choices anyway. We leave this variable-length initialization period as a feature for future versions of LOOK-AHEAD.

We now review the structure of our prototypical LOOK-AHEAD strategy before discussing its actual performance. LOOK-AHEAD plays exactly like GENERIC1 from rounds 1 to 10. During that time it updates the summations that it needs for its ordinary (unweighted) least squares regressions. Starting in
round 11, LOOK-AHEAD uses each of these two regressions as a model of each opponent's reaction function, and predicts future prices. Under the assumption that a steady state will exist, LOOK-AHEAD chooses a price $P_1^{s'}$ that will yield optimal long-run profits for firm 1 if its current parameter estimates are indeed correct. When these estimates are not correct, LOOK-AHEAD must suffer the consequences, since it is totally committed to long-term behavior in its current form.

An Empirical Test of LOOK-AHEAD

We have now depicted one version of LOOK-AHEAD and have described how its pricing policy will operate when faced with the MITCS2 profit function. To see how well it performs, we translated it into FORTRAN code and included it as yet another alternative entry in MITCS2. The adjusted results (for the top seven entries) are shown in Table 9.1.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Entrant</th>
<th>Strategy Type</th>
<th>Nasty</th>
<th>Lower Round</th>
<th>Mean Profit per Round</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>LOOK-AHEAD</td>
<td>--</td>
<td>--</td>
<td>17.370</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>1A</td>
<td>PCAVG3</td>
<td>GENERIC1</td>
<td>$P'$</td>
<td>17.281</td>
<td>$P' = (39 + P + P + P_{1/30}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Robert E. Marks</td>
<td>GENERIC1</td>
<td>$P'$</td>
<td>17.274</td>
<td>$P' = (26 + P + P_{1/20}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Robert L. Bishop; Tony Haig</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.273</td>
<td>Original GENERIC1 (with $P' = 13/9$)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Paul Pudaite</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.270</td>
<td>$P' = (13 + P_{1/10};$ looks back two rounds</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>John Hulland</td>
<td>GENERIC1</td>
<td>1.4</td>
<td>17.263</td>
<td>$P' = (13 + P_{1/10}$</td>
<td></td>
</tr>
</tbody>
</table>

LOOK-AHEAD comes through with an impressive win. Its average profit per round is far higher than any other entry; in fact, its margin of victory is greater
than the margin separating entries #1 and #13 in the original MITCS2 tourney (Table 8.1).

Naturally, LOOK-AHEAD does best when matched against any pair of linear reaction function strategies. Only nine of the MITCS2 entries actually use linear reaction functions all the time. Seven other entries have linear reaction functions with a lower bound. Another half-dozen entries are not inherently linear, but will exhibit linear (or constant price) behavior throughout many games. Thus, over half of LOOK-AHEAD's games involve at least one strategy which will not be modeled perfectly by LOOK-AHEAD's linear reaction function models.

As mentioned earlier, LOOK-AHEAD does has a difficult time when matched against an COALITION or GENERIC1 strategy and a nasty entry. Even after ten rounds of initialization, LOOK-AHEAD can still be confused by the coalition seekers, since they often use several different reaction functions (MIN, MAX, \(P\)) in the early part of the game. When LOOK-AHEAD finally gets some idea of what is going on, prices have usually eroded towards \(P^*\). After that, LOOK-AHEAD might begin to raise its prices towards the appropriate coalition price, but this process is slow: most games end before the steady state arrives. LOOK-AHEAD therefore misses out on much of the profits earned by standard coalition strategies.

For the same basic reasons, LOOK-AHEAD does not do well against itself (when the third player is nasty). The changes in \(P_i^{**}\) from one period to another are generally not a linear function of past prices, so LOOK-AHEAD has a hard time modeling and optimizing against a clone of itself.
The overall picture of the prototypical LOOK-AHEAD shows a strategy that is highly variable. When used in the right situation (i.e., against two linear competitors) it does extremely well, often attaining a true optimum for most of the game. But at other times, LOOK-AHEAD's strict reliance on its linear models and steady state assumptions can lead to a mediocre performance.

Because LOOK-AHEAD is so highly dependent on the composition of its competitive environment, it is a prime candidate to be included in the simulated MITCS2 environments discussed in the last chapter. We ran LOOK-AHEAD against the same 200 combinations of the eight representatives with the following results (see Table 8.3 for comparison).

<table>
<thead>
<tr>
<th>environment</th>
<th>average profits</th>
<th>rank</th>
<th>(winning margin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>nastiest</td>
<td>16.019</td>
<td>16</td>
<td>-0.167</td>
</tr>
<tr>
<td>moderately nasty</td>
<td>16.926</td>
<td>1</td>
<td>+0.060</td>
</tr>
<tr>
<td>mid-range</td>
<td>17.453</td>
<td>1</td>
<td>+0.089</td>
</tr>
<tr>
<td>moderately nice</td>
<td>18.041</td>
<td>1</td>
<td>+0.208</td>
</tr>
<tr>
<td>nicest</td>
<td>19.079</td>
<td>1</td>
<td>+1.212</td>
</tr>
</tbody>
</table>

The evidence is clear: LOOK-AHEAD requires some threshold amount of niceness to do well, and its performance improves dramatically as niceness increases. These results may seem somewhat surprising given LOOK-AHEAD's difficulties in dealing with most coalition strategies. Perhaps coalitions per se are relatively unimportant in very nice environments; getting industry prices to reach $P^c$ is often easy when times are good -- the hard part is to get prices all the way back to $P^0$ after a price war. LOOK-AHEAD is quite good at initiating price increases as part of its long-term outlook.
In the nastiest environments, LOOK-AHEAD is hurt by its farsightedness. All too often, LOOK-AHEAD is trying to raise industry prices above $P^*$, but these efforts are not worth the trouble and expense when followers are scarce.

LOOK-AHEAD also does very well in terms of total wins; in 86 of the 200 simulations, it achieves the best score. Its average winning margin of 14.7 cents per round is more than double the margin of 6.2 cents per round for the other 114 winners. However, in 77 environments, LOOK-AHEAD does not even make the top ten, reaching a low of 29th place. Interestingly enough, only two of LOOK-AHEAD's 86 wins supplant the previous win champion, entry #11. Thus in 134 of the 200 simulations (67%), one of these two entries is the winner.

Future Directions for LOOK-AHEAD

LOOK-AHEAD's impressive but erratic initial performance leaves great promise for future work. A primary task would be to incorporate one or more of the residual-weighting schemes described earlier. Although these procedures will further complicate an already complex algorithm, they can reduce the dependence on the strict assumptions that significantly influence LOOK-AHEAD's performance in MITCS2. Another test for LOOK-AHEAD can be found back in MITCS1: the algorithm could be included in its present form, or with its steady state pricing rule adjusted for the constant elasticity profit function used in that tourney. Finally, one might try to analytically investigate LOOK-AHEAD's equilibrium properties (in a game-theoretic sense). The complexity of the strategy may inhibit such attempts, but any analytic insights would provide a nice balance for the empirical work done here.
CHAPTER 10:
HUMAN PERFORMANCE IN THE GPD AND SOME COMMENTS ON CLASSICAL GAME THEORY

In addition to the analytical work and simulations presented in this thesis, we have also done some informal work involving human players in GPD games. Over the past two years we have run several hundred games matching two human teams (of one to four players each) against one programmed strategy in a repeated game with the MITCS1 profit function. The programmed strategies include several MITCS1 entries and some other simple variations.

Because the literature on experimental gaming is large, we did not begin to replicate studies that have tested the success and/or cooperativeness of human subjects in non-cooperative settings. Instead, we concentrated on the key GPD properties, such as implicit coalitions, bounded prices, and envy; we wanted to determine whether or not humans are capable of identifying and obeying (or avoiding) these properties.

Although we have run our game with many groups of students and executives, we have not yet performed any formal experiments or tests. However, across all these games we have noticed a number of consistent patterns. Hopefully, our observations, discussed below, will help us design future experiments that will accurately test and measure the phenomena we seek.

**Cooperation:** In very few games have we seen sustained periods of mutual cooperation. A primary reason, naturally, is the basic PD property: people find it hard to resist the short-term gains earned from a unilateral price cut. But another reason may be subjects' desire to experiment. Even though they were given an example payoff matrix for the game, most people wanted to see what
would happen if they cheated a bit. Thus *niceness* was rarely observed. Equally important is the fact that few subjects were able to raise industry prices after a price war. Some of the programmed strategies, such as MITCS1 #7, initiated price increases to encourage cooperation, but most people were content to take advantage of these higher prices without reciprocating.

*Envy:* As we noted in chapter 3, human players frequently play enviously in the 2x2 Prisoner's Dilemma, even when given explicit instructions not to do so. In the more complicated environment of the GPD, we expected to see a great deal of envious behavior, and indeed we did. The envious price, $P^e$, proved to be more of a focal point than any other single price. In some games, the human teams were so competitive that prices would occasionally go well below $P^e$ and stay there for several rounds. Most players eventually learned not to go so low, but few could ever rise above $P^e$ once they found it. Occasionally, we would talk to the players as they played the game. We would casually remind them about the stated objective (maximize profits), and point out the payoff matrix that we prepared for them. More often than not, this only served to confuse the players; they found it difficult to justify an ALL-$P^e$ type of strategy especially when both opponents were earning greater profits by playing something like ALL-$P^c$.

*Implicit Coalitions:* Two coalition-seeking algorithms were included as programmed strategies: COALITION and GENERIC1. In none of our sessions did we explain implicit coalitions to the participants. Unfortunately, they did not discover the coalition property spontaneously. We have yet to observe one instance where a human team would go along with the optimal coalition price, $P^c$, for any period of time.

As soon as price-cutting began, most players would go well below $P^e$ and remain there for the duration of the game. Thus, the coalition strategies would
usually end up playing like ALL-P* for most of the game. However, we have seen cases in which both human teams try to gang up on the programmed strategy. But their intentions were usual envious and context-specific, e.g., "let's outsmart the computer." Nevertheless, these episodes show that at least some people thought about cooperating with one opponent and tried to implement the appropriate tactics for their cause.

*Bounded Prices:* The paragraph on envy suggests that humans had little regard for the "Cournot price," P*; most players kept cutting until they noticed a drop in their own profitability. Even when the programmed strategy was bounded at P*, few players would take the hint; many believed that the computer was just using a bad or "wimpy" strategy. In general, only the unbounded algorithms, such as unbounded MIN (MITCS1 # 22), were viewed by the human teams as difficult opponents.

Although these comments are anecdotal, they are strongly consistent with our intuition and the earlier literature on experimental gaming. In the future, we might consider testing them rigorously. One direction would be to choose a specific set of programmed strategies that can account for the different properties we wish to examine. Thus, we might include standard and unbounded versions of MIN, MAX, and AVG to properly test the effects of boundedness. In order to test the coalition strategies, we might vary the structure of the market, perhaps by matching one human team against two programmed opponents. We can also change the game itself by altering the profit function or adding more players. Finally, we might pursue a creative and ambitious use of human players. For example, we could devise a scheme in which the human teams play the basic game for a number of rounds and then devise their own explicit strategy to actually make their moves for them in future rounds. We would then run
additional games and mini-tournaments to compare the effectiveness of our own programmed strategies and those generated by our subjects. This would be a fine way to integrate our past simulation work into our future experimental work.

Some Comments on Classical Game Theory

This thesis has an unusual position with respect to the standard game-theoretic paradigm. This document does contain some of the essential elements for a typical game-theoretic study, such as a discussion of the Nash equilibrium, various references to equilibrium behavior and profits, and a host of cites to other recent game-theoretic work. But any reader well-versed in contemporary game theory will quickly recognize that our intents, methods, and conclusions are quite different from those of most game-theoretic papers.

One big difference between our work and the usual game theoretic approach is the role of equilibria. As a broad (and perhaps unfair) generalization, game theory papers treat equilibria as the key concept: many papers do no more than prove the existence, uniqueness, subgame perfection, and in some cases, optimallity of various equilibria. Furthermore, in deriving these equilibria, two heroic assumptions are often made: (1) all players are rational profit-maximizers (and know that everyone else is a rational profit-maximizer), and (2) all firms choose the same strategy.

We agree that equilibrium behavior and profits deserve special attention when studying the effectiveness of any strategy in a repeated game. However, we find it difficult to go along with these two assumptions. The first is widely challenged and rarely trusted (especially in actual oligopolistic games), and the second is highly unrealistic. This is not meant to imply that all of our models and strategies are perfect; all methods have their weaknesses. Many of the simple
strategies that we have examined may fail to be good descriptive models of actual behavior, but they provide an initial step towards recognizing that a mix of strategies may be present in any market. Our emphasis on the tournament approach and robustness reflects our belief that a truly effective strategy must be able to perform well in a variety of competitive environments.

In the future, we expect to see studies featuring the best of both worlds: equilibrium-based models that allow firms to have different strategies and objectives. We hope that game theorists find our results provocative.

Despite these differences, this thesis still has much in common with game theory. The most important similarity is the notion of best response. Game theorists start with this idea, add their behavioral assumptions, and derive their equilibria. We have taken the same essential idea and used it in a different context by demonstrating and discussing various best-response schemes throughout this thesis. For example, implicit coalitions are a best response for a subgroup of cooperating firms; the GENERIC1 strategy (i.e., play MIN in good times, MAX in bad) is a best response against a fairly large class of competitors; and the LOOK-AHEAD strategies of the last chapter try to choose a best (long-run) response against any set of opponents.

Thus, depending on the reader's viewpoint, this thesis may or may not fall into the realm of game theory. But regardless of how this work is classified, we feel that we have achieved a certain symbiosis with other studies in game theory: much of this thesis extends and applies various game-theoretic contributions, and future game-theorists will hopefully pick up on some of the ideas presented here and extend them even further.
CHAPTER 11:
SUMMARY AND FUTURE DIRECTIONS

This thesis has examined effective strategies in a Prisoner's Dilemma game generalized to more than two players and to continuous alternatives. Both generalizations are relevant and interesting. Throughout the history of the PD, people have claimed it to be an intuitive model of interfirm rivalry (and other social phenomena), but few researchers have attempted to formally link it to the more complicated models that pervade the marketing and economics literature. We feel that the generalized Prisoner's Dilemma, as presented here, is a major step towards bridging the gap between the intuitive simplicity of the 2x2 PD and the complexity of "mainstream" models of competition.

We reviewed the PD literature, emphasizing Axelrod's innovative tournaments, which for the first time identified effective strategies for the 2x2 PD game. In chapter 4, we introduced the generalized Prisoner's Dilemma by extending the conditions used to define the 2x2 game. Each of these conditions has an intuitive interpretation behind it, but at least one of them is not unique. After defining the GPD, we presented our first model, a pricing game with a constant-elasticity demand function. In chapters 5 and 6, we used this model as a simple example of the key GPD phenomena, including implicit coalitions and continuous alternatives.

Implicit coalitions have existed for a long time in many industries, but game theorists have only recently begun to explore the numerous theoretical insights that relate to coalition behavior. We used two approaches to demonstrate the value of coalition-based strategies: first we discussed micro-stability, i.e., the fact that coalition-seeking behavior can lead to stability within a given market; next
we looked at *macro-stability*, where we showed that a coalition strategy can earn greater profits than a broad range of opponents under fairly general and realistic conditions.

The generalization to continuous alternatives is equally provocative. We motivated models that allow a comparison of continuous and discrete strategies, and then discussed one simple example (the \(d-T\) world) to demonstrate some of the benefits gained by recognizing continuous alternatives. In the \(d-T\) world, we observed that a strategy's performance is directly linked to its ability to cope with oscillations. We then set up a framework for a generic strategy that can take full advantage of coalitions and continuous alternatives in any \(N\)-player game.

Chapters 7 and 8 were devoted to our two competitive strategy tournaments, MITCS1 and MITCS2, where we tested our theoretical propositions and sought to compare the profitability and robustness of various different strategy types. In MITCS1, many entrants tried to generalize TIT FOR TAT, and indeed, the winning entry exhibited several TIT FOR TAT features such as niceness, forgiveness, and provocability. This entry was a simple coalition strategy, but did not use continuous alternatives. In our post-tournament analysis, we constructed another strategy that recognized both coalitions and continuous action; this alternative champion, named GENERIC1, was consistently more profitable than any other algorithm examined in the MITCS1 world. We then theoretically justified the pricing policy of GENERIC1, i.e. play MIN when both opponents' prices are high, MAX when both opponents' prices are low, and go for a coalition if only one price is reasonably high.

In MITCS2, where the coalition price was no longer unique, several entrants turned their attention from generalizing TIT FOR TAT to generalizing
Most of these generalizations performed no better than the original GENERIC1 itself, despite the fact that GENERIC1 is not able to go along with varying coalition prices. The only entry to earn higher profits used the basic GENERIC1 structure with an unusual coalition pricing rule that takes account of the previous prices of both opponents, not just the non-cooperative firm's. We extended this concept to involve all three firms' prices and showed that such a strategy would outperform all of the MITCS2 entries. Further evidence came from the results of a robustness test showing that the top MITCS2 entries tend to perform quite well in a wide range of competitive environments, from very nasty to very nice.

After concentrating our analysis almost exclusively on simple strategies in MITCS1 and MITCS2, we presented a complex class of LOOK-AHEAD strategies in chapter 9. These strategies were designed to model competitors' behavior as linear reaction functions and then choose current prices to maximize long-run profits. We discussed some of the advantages and disadvantages of this scheme: it has the ability to earn excellent profits if both opponents are using simple strategies, but against anything fairly complicated (including the basic coalition strategies), profits and behavior can be quite erratic. We tested an initial version of a LOOK-AHEAD strategy by including it in a rerun of MITCS2. Despite its potential flaws, LOOK-AHEAD proved to be a very strong winner in MITCS2 and in all but one of the robustness-test environments. We have high hopes for refined versions of LOOK-AHEAD; we discussed various ways to make it better able to cope with a wider range of opponents.

**Future Directions**

This thesis has sought to obtain synergies from both theoretical and empirical work. A fundamental idea (the Generalized Prisoner's Dilemma) led to MITCS1.
Some of the tournament results, particularly the importance of implicit coalitions and GENERIC1, led us to investigate the coalition concept further in the second tourney. Two of the major themes in MITCS2, magnanimity and responsiveness, were the inspiration for the LOOK-AHEAD strategies that attempt to incorporate both properties in long-run pricing behavior. In the future, we could pursue this stream of research in the same manner.

Every time we run a tournament and perform the analytical work that surrounds it, we can relax some of the more restrictive and unrealistic assumptions that we relied on before. Probably the most important first step would be to incorporate some type of uncertainty, as discussed throughout the thesis. Despite the robustness of implicit coalitions in MITCS1 and MITCS2, we have no assurances that coalitions will hold up in a world where the model parameters change over time, or where information about past prices is imperfect. We might find the basic LOOK-AHEAD strategy of chapter 9 to be very successful in such an environment, since it can "smooth out" some of the noise that might be encountered. Other model changes to consider include asymmetric demand functions, differing (non-linear) cost functions, and markets with more than three firms (perhaps with entry and exit). But before we jump wildly into a new tournament, we must identify our priorities.

Besides all these tournament possibilities, there are several worthwhile analytical questions that have not yet been answered. We might choose to examine the equilibrium properties of some of the strategies we have considered here. We have briefly mentioned two equilibrium situations of interest, one involving a mix of continuous and discrete strategies, and one involving the LOOK-AHEAD strategies. We might also devote some attention to the steady-state properties of AVG and GENERIC1.
Back on the empirical front, we might find some interesting stories still lurking in the MITCS1 and MITCS2 data. In this thesis, we have considered overall average profit as the sole criterion when comparing strategies. We might find different results if we look at median or minimum profits. Both of these statistics are good indicators of robustness, maybe more reliable than the average. Any major discrepancies among these different ranking methods might lead to further analytical work.

We might also look to the real world to ask more questions and seek more answers. Perhaps we can perform econometric work to confirm Sultan's suspicions about the electrical oligopoly and find other oligopolies with similar behavior. Some game theorists, e.g. Porter (1983a) and Slade (1986b), have used real data to test the validity of different strategies, and we could do the same. Finally, we might focus more on experimental work with human subjects, as described in chapter 10. We've come a long way in our ability to model and understand oligopolistic competition, but there is still a long future ahead.
REFERENCES


