INVESTIGATION OF CURRENT FLOW IN THE INNER EAR 
DURING ELECTRICAL STIMULATION 
OF INTRACOCHLEAR ELECTRODES 

by 

Gary Girzon 

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Signature of Author ________________________ 
Department of Electrical Engineering and Computer Science 
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Certified by ________________________________ 
Donald K. Eddington, Ph.D. 
Thesis Supervisor 

Accepted by ________________________________ 
Professor Arthur C. Smith 
Chairman, Departmental Committee on Graduate Students 

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Investigation of Current Flow in the Inner Ear During Electrical Stimulation of Intracochlear Electrodes

by

Gary Girzon

Submitted to the Department of Electrical Engineering and Computer Science on January 30, 1987 in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering and Computer Science

Abstract

A fundamental problem in cochlear implant research is understanding the spatial and temporal characteristics of the electric field generated by electrical stimulation of intracochlear electrodes. It is shown in this thesis that an anatomically accurate, electrical model provides a feasible method of investigating the generated electric field. The three dimensional, electro-anatomical model developed in this thesis is based on human temporal bone sections, and encompasses the cochlea, portions of the nerve trunk and surrounding temporal bone. To define the discrete model, the structures of the inner ear are sampled with elements of uniform dimensions. The electrical boundary value problem is formulated using a three dimensional, finite difference method and solved using an iterative method. Sufficient anatomical and electrical data are available to define a first order model and a three dimensional version (with over 500,000 elements) can be formulated and solved on a scientific minicomputer (VAX 11/750) within practical computing constraints (40 Mbytes of secondary storage and 40 hours required for solution.)

The results of the electro-anatomical model are validated in part by experimental measurements of potential in implanted cochleas. The experiments support the hypothesis that the electrical pathways during stimulation of scala tympani electrodes are primarily resistive. Both the electro-anatomical model and the experimental data show an asymmetric distribution of potential along the length of the scala tympani. For example, during monopolar stimulation, steep potential gradients occur at the basal end of the scala tympani, while the apical potentials reach a plateau. The robustness of the electrical model can be attributed to the preservation of cochlear anatomy and boundary conditions in the electro-anatomical model.
Given sufficient resolution, the three dimensional, electro-anatomical model can be used to ascertain the magnitude and direction of current density in any structure of interest. In this fashion, the present model can compute the current density distribution in the vicinity of neural tissue. Further validation of the model is required using other experimental data (e.g., evoked response and psychophysical experiments.) Combined with experimental data, the numerical model provides a basis for the design of more efficient and more powerful cochlear implant systems.

Thesis Supervisor: Donald K. Eddington, Ph.D.
Title: Research Scientist, Research Laboratory of Electronics, Massachusetts Institute of Technology

and

Director, Cochlear Implant Research Laboratory, Massachusetts Eye and Ear Infirmary.
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It is hard to summarize my past 2½ years at M.I.T in a few sentences. At times, things were very difficult, lonely, scary. At times, it was a blast. The past few years, however, have never been boring. The institute is a special place with very special people. Ultimately, it’s the individuals of M.I.T. who have made my educational stay a very rewarding experience. I would like to express my gratitude to those who have helped me survive the “experience”. Thanks to Charlie Marge for showing me that there is humanity in the Sloan School (and also for being a great roommate all these years.) Xiao-Dong Pang, I thank you for showing me the ropes at M.I.T. (Where else should two individuals from opposite ends of the world get an education?) I am very grateful to Daisy and Robert Kunstaetter for improving my standard of eating and skiing, and for demonstrating a lifestyle I shall try hard to emulate (work hard and vacation long!) Last but not least, I would like to thank Bonnie and Bob Kidd for opera and squash, and just for being good friends.
This work is dedicated to my parents who have sacrificed much to provide our family with a future and a chance to live free.

The mean man thinks: "I am so generous".
The shallow man: "I am profound".
Sometimes God will sigh: "I am a worm".
The worm hisses: "I am God!"

The worms climb arrogantly upwards.
The coward rejoices to be in the clouds.
Only the free man
Thinks:
"I am a slave".

Yevgeny Yevtushenko†

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Chapter I

Introduction

Cochlear implants are devices that are used to restore limited hearing to individuals with sensorineural deafness. Most of these individuals retain a significant complement of excitable auditory nerve fibers but have lost the ability to transduce acoustic input into electrical spike activity on these fibers. The function of the implant is to elicit activity on these remaining fibers by electrically stimulating electrodes placed within the inner ear. A typical prosthesis system consists of a microphone that presents an electrical analogue of the acoustic signal to the processing electronics which then produce the electrical stimuli that are delivered to the implanted electrodes. Currently, there are about a dozen different cochlear implant systems being investigated by research groups around the world [Parkins, 1983]. Implanted individuals report that the implants improve communication, especially when combined with lip reading. However, while subjects describe the percepts elicited by electrical stimulation as "sound like", the goal of providing fluent speech recognition without lip reading has not been attained.

The stimulation of intracochlear electrodes produces an electric field in the inner ear. A fundamental problem in cochlear implant research is understanding the spatial and temporal characteristics of the generated field. Being able to predict and thus control the field is essential to developing more effective cochlear implants because the neural response is related to the potential distribution in the vicinity of nerve tissue. If the electric field can be controlled, localized stimuli can be delivered to specific nerve sites. Furthermore, neural activity can be predicted given the potential distribution along peripheral nerve fibers. The aim of a cochlear prosthesis is to replicate the neural activity produced in the normal ear during acoustic stimulation. Hence, when combined with other measures (e.g., psychophysical, brain stem evoked response, etc.), a working model for the electric field distribution may lead to a better understanding and the development of more successful cochlear implant systems.

The framework of the thesis is to study the feasibility of developing anatomically accurate, electrical models of the implanted cochlea. Such electro-anatomical models can be used for predicting the electric field distribution in the inner ear during electrical stimulation of intracochlear electrodes. By preserving cochlear anatomy in a three dimensional model, the effects of the spiraling, heterogeneous structure of the cochlea on the potential distribution can be investigated. Placing an electrode in the
environment of the cochlea is unlike placing it in an infinite homogeneous medium; the potential distribution is certainly affected by the structure and physical boundary conditions of the cochlea.

One is limited experimentally in measuring the potential or current distribution in the implanted cochlea because of the small size and complex structure of the inner ear. However, potential measurements can be made using the implanted electrode array. A series of such measurements was performed as a part of this thesis and is used to test the predictions of the electro-anatomical models. Furthermore, the measurements provide significant insight into the potential distribution across a range of subjects and stimulus conditions.

In this thesis, the feasibility of electro-anatomical models is examined in the contexts of the available electro-anatomical data, model formulation and solution, model testing, and numerical error analysis. The thesis begins with a brief introduction to the peripheral auditory system and to a typical cochlear prosthesis. Several issues important to cochlear implant performance and design are discussed in Chapter II, leading to the electro-anatomical questions addressed in this thesis. Chapter III includes a literature survey of past work in the electro-anatomy and electrical modeling of the cochlea. The measurements and models described in Chapters II and III provide some insight into the major electrical pathways of the inner ear. The modeling of implanted cochleas, however, requires a different type of electrical model.

The development of a three dimensional, electro-anatomical model begins in Chapter IV. The mathematics governing the electric field, modeling assumptions and the choice of anatomical and electrical data are discussed in Chapter IV. One numerical approach to modeling the electro-anatomy of the implanted cochlea is presented next. Chapter V continues with a description of how such a model is defined and formulated as a set of simultaneous linear equations using a finite difference method. In the three dimensional model, the structures of the inner ear (e.g., the cochlea, temporal bone, nerve trunk) are sampled and represented in the discrete domain by elements of uniform dimensions. Chapter V includes sections on anatomical data entry, finite difference formulation, and an iterative method of solution. Chapter VI presents several results from different electro-anatomical models. Here, the effects of model resolution, electrode representation, and electrical composition are examined.

The next two chapters concern the intracochlear potential measurements. Chapter VII describes the experiments and presents data for several subjects and stimulation conditions. In Chapter VIII, the experimental and computed results are compared.
Finally, Chapter IX presents an analysis of numerical error in the model developed. The chapter includes an analytical development of model truncation error. Furthermore, the rate of convergence of the electro-anatomical model is estimated by comparing the solutions of models of different resolution. The chapter concludes with a brief discussion of methods of increasing the model resolution without incurring penalties in computation time and storage required for model solution.

A more complete description of the modeling system developed is contained in the Appendix. The model data structures, the numerical method of solution and programs for anatomical data entry, manipulation and display are described.
Chapter II

Background

2.1 A Brief Introduction to the Peripheral Auditory System

The auditory process consists of several steps during which airborne sound waves are transformed from pressure waves to neural signals traveling toward the higher brain centers. The peripheral auditory system can be divided into three anatomical regions: the external ear, the middle ear and the inner ear (Fig. 2.1). The external ear receives the sound waves at the pinna. The waves then travel through the external auditory meatus before displacing the tympanic membrane. The tympanic membrane, situated across the medial end of the external auditory meatus, forms the boundary between the external and middle ears. The middle ear couples sound energy from the outer ear to the fluid filled environment of the inner ear. The energy is transmitted by the action of three small bones (ossicles): the malleus, incus and stapes. The malleus is connected to the tympanic membrane, while the stapes communicates directly with the oval window of the inner ear. An inward displacement of the tympanic membrane causes rotation of the malleus and incus. The momentum is then transferred to the stapes, which pushes inward, displacing cochlear fluid and triggering the next chain of events in the inner ear.

The inner ear is embedded in the temporal bone and contains the cochlea, a spiraling, multiple cavity tube. The axial length of the human cochlea is about 5 mm. The cochlea winds 2½ turns around the modiolus and measures about 31 to 33 mm in length. The cochlear duct is a membranous tube suspended within the bony cochlea (Fig. 2.2). The chamber within the cochlear duct is known as the scala media and contains a fluid called endolymph. Attached to the osseous spiral lamina and the external bony wall, the cochlear duct subdivides the bony cochlea into the scala tympani and scala vestibuli. Scala tympani and scala vestibuli communicate at the apex of the cochlear spiral (the helicotrema) and contain a fluid known as perilymph. The perilymphatic spaces do not communicate with the endolymphatic spaces.

The stapes footplate is held in the oval window (the basal end of scala vestibuli) by the annular ligament. (The round window membrane separates the perilymph at the basal end of the scala tympani from the middle ear cavity.) The motion of the stapes during acoustic stimulation causes the displacement of cochlear fluids and the motion of the basilar membrane, known as the traveling wave. The traveling wave spreads
Fig. 2.1: A drawing of the cross section of the human ear. The "ear canal" is referred to as the "external auditory meatus"; the "ear drum" is referred to as the "tympanic membrane" in the text. (from [Brödel, 1946]).
Fig. 2.2: Drawing of a cross section of one turn of the cochlea, prepared from a midmodiolar section of a cat’s cochlea. (from [Davis, 1962]).
from the basal end to the apex. The point of maximum displacement along the basilar membrane depends on the frequency of the stimulus. For example, high frequency stimuli move the point of maximum displacement toward the base, while at low frequencies, the maximum point moves toward the apex.

The tuned, mechanical motion of the basilar membrane triggers the auditory transducer, the hair cell, found in the organ of Corti. The complex transduction process is initiated by the bending of the stereociliary tufts of the hair cells. There are about 25,000 hair cells in the human cochlea. Hair cells, like the basilar membrane, respond best to acoustic inputs of particular frequency. For example, hair cells at the base respond maximally to high frequency stimuli, while hair cells at the apex respond maximally to low frequency stimuli. This spectral response is known as the tonotopic organization of the cochlea.

The link between the peripheral auditory system and the brain is the auditory nerve. The human auditory nerve contains about 30,000 fibers. Together with the cochlear blood vessels, the auditory nerve is located in the modiolus, the central cavity of the cochlea.

2.2 A Brief Introduction to Cochlear Implants

Individuals with sensorineural deafness retain a significant complement of auditory nerve fibers but have lost the ability to transduce acoustic input to electrical spike activity on these fibers. Such loss can be caused by noise trauma, drugs, disease, or the aging process. The function of the cochlear implant is to elicit activity on the remaining fibers by electrically stimulating electrodes placed within the inner ear. Thus the outer, middle and inner ear pathways are bypassed.

A typical prosthesis system consists of two general components: an implanted electrode array and an external signal processor. A microphone presents an electrical analogue of the acoustic signal to the signal processing electronics which then produce the electrical stimuli that are delivered to the electrode array. The electrode array is implanted by surgically exposing the round window and inserting the electrode array through the window into scala tympani.

Several stimulation schemes and electrode configurations are being investigated by research groups around the world today†. The SYMBION prosthesis (INERAID®

† Recent surveys of cochlear implant research programs can be found in [Parkins, 1983] and [Feigenbaum, 1986].
cochlear implant, SYMBION INC., Salt Lake City, Utah) [Eddington, 1980] has been implanted in over 80 human subjects (including five at the Massachusetts Eye and Ear Infirmary, Boston) at the time of writing. The SYMBION electrode array (Fig. 2.3) contains eight platinum ball electrodes. Teflon insulated leads join the electrodes to a percutaneous connector which is located behind the pinna. Six of the electrodes, spaced uniformly 4 mm apart, are placed inside the cochlea. The two remaining electrodes are placed in the bony promontory in the middle ear and under the temporalis muscle. The muscle electrode, which has a slightly greater radius, functions as the common return or ground electrode for the intracochlear electrodes.

Stimulation schemes based on a common return electrode are called monopolar. Each of the electrodes can be thought of as a monopole current source with respect to the far field ground electrode located in the temporalis muscle (Fig. 2.4). This is the normal stimulation scheme used by the INERAID cochlear prosthesis. Bipolar stimulation schemes include a separate return electrode for each stimulus electrode. The spacing of these electrode pairs is kept small compared with the active/return pairs used in the monopolar stimulus configurations. The electric field produced by bipolar electrodes has the characteristics of an electric dipole. The advantage of using bipolar schemes is a more spatially localized electric field. Bipolar electrodes have been shown to excite a smaller nerve population than do monopolar electrodes [Merzenich et al., 1979]. The bipolar schemes, however, require twice as many electrodes as the monopolar schemes to deliver the same number of stimulus channels. Furthermore, because the current density is concentrated between the active pair of electrodes, bipolar schemes can require higher stimulus strengths than monopolar schemes when the excitable tissue is not located between the electrode pair.

The INERAID prosthesis is called a multichannel device since each of the electrodes has the ability to deliver a distinct stimulus. Single channel devices present a common stimulus to one or more electrode pairs. In the INERAID portable processor, the input signal is first passed through an automatic gain stage after which the signal is split into four separate channels. Each of these channels is composed of a bandpass filter and voltage-to-current converter. The output of each channel is presented to a different intracochlear electrode. (The two most basal electrodes, as well as the promontory electrode, are unused in the portable processor.) The four channels are stimulated simultaneously with respect to the return (temporalis muscle) electrode.

One strategy for designing speech processing algorithms for cochlear implants is to generate electrical stimuli that produce patterns of electrical activity similar to those elicited by acoustic signals in the normal ear. If such patterns could be sufficiently
Fig. 2.3: The SYMBION electrode array. Six of the electrodes are bundled and placed inside the inner ear. Electrode 6 is the most basal electrode while electrode 1 is the most apical. The spacing between the intracochlear electrodes is fixed at 4 mm. The Return electrode is placed under the temporalis muscle. The Promontory electrode placement varies across subjects but is typically located near the round window in the middle ear.

Fig. 2.4: Monopolar multichannel cochlear implant. In this type of implant, each electrode is stimulated by a distinct stimulus with respect to a common (G) electrode.
duplicated, an implanted subject with post-lingual deafness would be able to understand speech spontaneously. However, while subjects do report the percepts elicited by electrical stimulation as “sound like” and the majority of users lip read significantly better with their devices than without, the goal of providing fluent speech recognition, without lip reading, has not yet been attained.

There are several aspects of current systems that may limit cochlear implant performance. For example, all implants rely on a limited number (2–22) of electrodes to deliver stimuli to the sensorineurally deaf ear while the normal ear functions with thousands of receptor cells and nerve fibers. Also, limited and non-uniform nerve survival, as well as other pathological conditions, certainly influence the performance of implants across subjects. Even if arrays with thousands of electrodes were available in ears with normal nerve fiber populations, questions regarding the optimal electrical stimulus (e.g., which acoustic cues are necessary for speech recognition and how to reproduce these cues in the neural activity patterns) remain largely unanswered.

2.3 Thesis Goals

Finally, there is a set of electro-anatomical questions which this thesis addresses directly. Analogous to the chain of events that occur in the normal peripheral auditory system, a similar transformation of sound energy occurs during electrical stimulation of intracochlear electrodes. The sound energy is first transformed into the electrical stimuli (stimulus function) at the implanted electrodes. Electrical stimulation produces an electric field (potential or current density function) in the cochlea and its surroundings. The firing of neural fibers (neural excitation function) occurs once the current density reaches some threshold value. The neural excitation function depends on the electrode configuration, stimulus waveform, cochlear electro-anatomy and the surviving nerve population. Ultimately, one wants to be able to predict and thus control the neural excitation function given any stimulus function.

The electro-anatomical questions deal with the problem of understanding how the anatomical structure of the inner ear and the geometry of the electrodes determine the patterns of spatial and temporal current distribution (the potential or current density functions) and eventually, the patterns of neural excitation (the neural excitation functions.) The cochlea and the implanted electrodes constitute an electrical system that is irregular in geometry and heterogeneous in composition. Understanding the electrical characteristics of this system is crucial to the development of higher performance prosthetic systems. This thesis is a first step in the development of a theoretical model of the implanted cochlea that is electro-anatomically accurate. Ultimately, such a
model would predict the current density function at the site of neural tissue, and thus the neural excitation function, given an arbitrary stimulus function. The current density function is difficult to predict because of the complex heterogeneous structure and geometry of the inner ear. Furthermore, due to its small size, heterogeneous structure and bony wall, the cochlea does not lend itself to direct measurements of the electric field distribution. Any such measurement (e.g., of current density at peripheral nerve fibers) is likely to be altered by the presence of the recording apparatus. Measurements made during electrical stimulation such as nerve fiber recordings [Moxon, 1971], brainstem recordings [Merzenich et al., 1979; O’Leary et al., 1985] or psychophysical experiments [Shannon, 1983; Cotter, 1986] provide only an indirect measure of the current density function.

The nature of the current spread in the implanted ear can have important implications in the design and performance of cochlear implant systems. The aim of a multichannel prosthesis is to excite specific areas of nerve tissue by stimulating individual electrodes. Taking advantage of the tonotopic organization of the cochlea, basal electrodes should excite nerve fibers with high characteristic frequencies, and apical electrodes should excite nerve fibers with low characteristic frequencies. In other words, stimulus functions should ideally generate independent, non-overlapping (spatial) neural functions. This will be true if the current density function is “narrow” or localized to a short length of the cochlea. However, for extended or “broad” current spread, it may not be feasible to deliver localized stimuli because of the overlap in the current density functions from adjacent electrodes. The effects of electrode interaction are most severe in the case where all the electrodes are stimulated simultaneously (as in the INERAID portable processor). Such electrode interaction may reduce the number of independent stimulation sites.

It may be possible, however, by mapping the current density functions and by carefully manipulating the stimuli, to actually “narrow” the generated electric field. For example, if a target electrode is made to deliver a stimulus function out of phase with its neighbors, the resultant current density function may be narrower than produced by the target electrode alone. However, the ability to “deconvolve” the desired neural function into individual components of the stimulus function depends in part on the extent of current spread in the cochlea [Van Compernolle, 1985].

The electric properties of the implanted cochlea most likely fall somewhere between the two hypothetical models shown in Fig. 2.5. The first electrical model (used with certain modifications in [Van Compernolle, 1985] and [Cotter, 1986]) is a homogeneous volume conductor. The conductor extends to infinity in all directions. In
\[ V_{\text{target}} = k \frac{I_{\text{source}}}{r} \]

**Infinite Homogeneous Volume Conductor**

\[ V_{\text{target}} = V_{\text{source}} e^{-x/\lambda} \]

**Infinite Uniform Transmission Line**

*Fig. 2.5:* Two hypothetical electrical models for the potential distribution in the cochlea. The transmission line is assumed to be resistive.
such a model, for a point source, the current will spread equally in all directions. For the cochlea, such an approximation would be valid if biological tissue conductances were all similar and if boundary conditions (i.e., the finite size of the biological system) could be ignored.

The opposite extreme is described by a uniform transmission line model [Black and Clark, 1972; O'Leary et al., 1985]. In such a model, current spread is longitudinal. For the cochlea, such an approximation assumes that current flows primarily along the length of the scala tympani. The approximation is typically justified by the insulative properties of temporal bone and the higher conductivity of the cochlear fluids. The three dimensional structure of the cochlear spiral is thus simplified to a two dimensional transmission line. This assumes little or no electrical communication between adjacent turns of the cochlea. The electrical "distance" between the site of current injection ("source") and the final destination ("target") is measured along the length of the unwound cochlea. The transmission line analogy may not be a bad one for short, uniform lengths of the cochlea. It does not account, however, for the different physical boundary conditions at apical and basal ends of the cochlea. Nor does it account for the differences in the grounding pathways (through the auditory nerve and blood vessels) along the length of the cochlea.

This thesis investigates the feasibility of developing a theoretical model that can predict the current distribution during electrical stimulation. Assumptions and approximations about the electrical properties of the cochlea apropos cochlear implants have been made in the past. The framework of this thesis, however, is to develop an electro-anatomical model that preserves the cochlear anatomy and makes no a priori assumptions (with the exception of fundamental mathematical equations governing current flow and biological material properties) about how current should flow in such a system. In this fashion, the effects of cochlear anatomy and electrical heterogeneity can be examined. The next chapter presents a brief literature survey in cochlear electro-anatomy and how this research relates to the present thesis.
Chapter III

Electro-Anatomical Studies of the Cochlea

3.1 Introduction

Electro-anatomy of the guinea pig cochlea has been studied in the past by several research groups. Concurrently, electrical models were proposed to describe the generation and spread of cochlear potentials. Both standing and evoked (by acoustical stimulation) potentials can be recorded in the normal cochlea. These potentials are thought to be a manifestation of the active cochlear transduction process. The experimental investigations described in this chapter represent the most direct characterizations of the electrical properties of a mammalian cochlea. This chapter presents a brief literature survey of relevant electro-anatomical results and models.

3.2 Cochlear Potentials

The electro-anatomical models discussed in this chapter represent sections of the cochlea by electrical networks. In the electrical networks, the electrical pathways between major cochlear structures (e.g., scala tympani (ST) and scala media (SM), scala vestibuli (SV) and ST) are represented by passive elements. Cochlear potentials, which play an active role in the transduction process [Davis, 1965], are modeled by current or voltage sources. Both standing (e.g., the d.c. endolymphatic potential) and evoked (e.g., d.c. summing potential and the a.c. cochlear microphonic) potentials occur in the normal cochlea. The evoked potentials most likely do not occur in the sensorineurally deaf ear. Furthermore, the d.c. potentials are of no consequence during electrical stimulation since neural excitation is related to the potential gradient [Rank, 1975].

Nevertheless, the recorded cochlear potentials suggest the relative importance of the electrical pathways present. For example, scala vestibuli and scala tympani are both only slightly positive with respect to an indifferent muscle electrode. The spiral ligament (SL) is also close to zero potential. Therefore, one expects a pathway of low resistance between the spiral ligament, scala tympani and scala vestibuli. Scala media, on the other hand, maintains a high positive (80–100 mv) endocochlear potential. Hence, one expects little communication between the endolymph and the perilymph, and other tissues surrounding the scala media.
3.3 The Experiments of Von Békésy†

Von Békésy was among the first to investigate the electro-anatomy of the guinea pig cochlea. His experiments showed that the cochlea is well insulated by its bony walls from the rest of the body. Furthermore, von Békésy demonstrated that the grounding resistances between the cochlear scalae and the body lie in the auditory nerve and the blood vessels. Because the nerve and blood vessels enter the cochlear duct from within the modiolus all along the length of the cochlea, von Békésy speculated that “there must exist considerable cross conduction between the different turns of the cochlea” [1951, p. 661].

Von Békésy performed a series of experiments measuring the potential distribution across the cochlear partition (ST–SV) for different sites of current injection [1951; see also Dallos, 1973]. Based on these experiments, von Békésy proposed a lumped parameter, electro-anatomical model for small longitudinal sections of the cochlea. The model (Fig. 3.1) includes longitudinal resistances representing SV, ST and the grounding pathway G (the internal resistance of the nerve and blood vessels in the modiolus). Within a transverse section, the structures are assumed to be at an equipotential and are therefore modeled as single nodes. The cochlear partition is modeled as the SV–ST transverse resistance. SV and ST also join through the SV–G and ST–G resistances. Resistances linking non-adjacent transverse sections (the Ω’s) are also included in the model. These resistances represent electrical coupling between the different turns of the cochlea. Only if the coupling between turns is low (i.e., the Ω’s are large compared with the resistance of the other pathways) can the cochlea be modeled as a transmission line where the three dimensional structure of the coiled cochlea is ignored. However, von Békésy states that “there is no complete insulation between the different parts of the cochlear winding” [p. 666, 1951]. The major grounding pathway, through the auditory nerve and blood vessel system, forms a low resistance path along the whole length of the cochlea.

One of von Békésy’s experiments was a basic calculation testing the validity of the transmission line model [1952]. He estimated the hypothetical loop resistance (from the stapes footplate to the helicotrema to the round window) of scala tympani and scala vestibuli for well insulated canals. Such a calculation can be performed given perilymph conductivity, and the cross sectional areas and lengths of the two scalae which he determined experimentally. The calculated loop resistance turned out

† All references in this section are from [von Békésy, 1951 and 1952].
Fig. 3.1: Von Bekesy's transmission line model for short lengths of the cochlea. Note the resistors representing the electrical coupling between different turns of the cochlea. See text for details. (Adapted from Fig. 14–25 in von Bekesy, 1951).
to be roughly four times greater than that measured experimentally. Therefore, the presence of grounding pathways in the model (which include the SV–ST pathway through the spiral ligament (SL)) and pathways linking the different turns of the cochlea can be quite significant in reducing the effective loop resistance.

Von Békésy’s results have been questioned based on his use of 1000 Hz a.c. stimuli which may have increased current shunting because of membrane capacitances. Von Békésy also failed to report any capacitive effects in his measurements, unlike other investigators [Sitko, 1976; Cannon, 1976]. Von Békésy’s model ignored certain current pathways, such as along the endolymph in SM, between ST–SL and SV–SL. Other research groups have continued the development of lumped parameter models by performing further measurements in the cochlea.

3.4 Other Electro-Anatomical Investigations and Models

Misrahy et al. [1958] improved on von Békésy’s technique by using a four electrode technique to measure the distribution of d.c. potentials and the resistance of the cochlear partition (SM–ST). K–Cl electrodes were placed in ST, SM and SV at different turns of the guinea pig cochlea. Current pulses were injected through one pair of electrodes, while an adjacent pair of electrodes was used to measure the source potential difference. Potentials were also recorded at different positions along the cochlea. Each turn of the cochlear spiral was then modeled as a uniform transmission line, similar to von Békésy’s analysis. The bulk resistance, or what the authors believed was the characteristic impedance of the transmission line, was determined directly from the voltage-current relationship for a particular pair of electrodes. The average resistance of the cochlear partition (SM–ST) was shown to decrease from 4700 ohms at the round window to 600 ohms at the third turn. Misrahy et al. then proceeded to calculate the length constant, a characteristic transmission line parameter. The calculation was made assuming that the effective resistance of the two canals could be determined given the conductivity of the scalar fluid and geometry of the scalae. However, this assumption is contradicted by von Békésy’s experiment described above. Thus it is not surprising that the Misrahy et al. length constants are rather low when compared with more recent measurements since the length constant is inversely proportional to longitudinal resistance of the transmission line. Nevertheless,

† Von Békésy measured the longitudinal resistance of ST and SV using a Wheatstone bridge to eliminate current flow through the cochlear partition (ST–SV).

†† Misrahy et al. measured it directly while von Békésy predicted the characteristic impedance using longitudinal resistance and attenuation measurements.
the reported length constants and resistance measurements suggest the electrical non-uniformity of the "unwound" cochlea.

A fundamental problem with the Misrahy et al. analysis is their simple transmission line model. They consider ST and SV to be the outer core of their transmission line. SM is the inner conductor. The cochlea, however, is better represented as two transmission lines with one (inner) line embedded in another (outer) line [Cannon, 1976]. The inner line is composed of the endolymph (SM) as the inner core and scala media boundary as its outer core. The outer transmission line is composed of the perilymph (ST, SV) and SL as the inner core and the bony wall of the cochlea as the outer core. Lumping the multiple cavity cochlea into a transmission line consisting of only two longitudinal sections most certainly ignores some of the major current pathways.

A study of the resistive pathways in the first turn of the guinea pig cochlea was performed by Johnstone et al. [1966]. A four electrode scheme was used to perform the measurements. Pairs of electrodes were placed in the three scalae (SM, ST and SV) and an indifferent electrode was located in the neck of the animal. The stimulus consisted of a 1 or 2 µA current step, with a duration of up to 5 seconds. The results from the three stimulus/recording sites were modeled as an electrical network with four nodes, plus ground (Fig. 3.2).

Honrubia and his co-workers made a similar set of measurements in the first turn [Honrubia et al., 1976] and then extended these measures to turns two and three [Honrubia, 1974; Sitko, 1976]. The measurements show that the SM–ST and SM–SV resistances decrease progressively in the higher turns. The grounding resistance (modeled as SL–NE, where NE is the neutral temporalis electrode) increases in the higher turns. Thus the pathway of least resistance to the body is located at the base of the cochlea.

Sitko [1976] also measured the potential difference across the cochlear partition along the length of the cochlea when current was injected into turn one and turn four of the cochlea. For current injected into turn one, the potential declined only up to turn two, after which the potential remained constant. For current injected into turn four, the potential decreased exponentially with distance from source. Both the resistance measures and the potential distribution experiment suggest that the grounding pathway is rather limited in the higher turns. A significant portion of current injected in the higher turns, therefore, flows into the basal turn of the cochlear spiral.
Fig. 3.2: Electrical model for the first turn of the cochlea. The model represents a cochlear section of 2 mm length. Abbreviations: SV: scala vestibuli; SM: scala media; ST: scala tympani; SL-MOD: the spiral ligament (blood vessels) and modiolus (nerve tissue) pathways to ground. (Adapted from Fig. 2 in Johnstone et al., 1966).
3.5 Three Dimensional Models

A three dimensional lumped parameter model for the guinea pig cochlea was developed by Strelloff [1971, 1973] to simulate the generation and distribution of cochlear potentials. Fig. 3.3 shows his model of the "unwound" cochlea. No electrical coupling is assumed to be present across different turns. The full model consists of 90 sections of varying thickness. Each section contains five nodes representing the scala media, scala tympani, scala vestibuli, the organ of Corti and the spiral ligament. The passive elements in the model are resistive, while active elements represent the standing cochlear potentials. Strelloff obtained the passive element values from a variety of sources in literature. Most of the literature data, however, describe the basal turn of the cochlea and hence elements for the rest of the model were adjusted in proportion to the geometrical dimensions of the pertinent structures.

A somewhat different approach to modeling the guinea pig cochlea was undertaken by Cannon [1976]. Instead of attempting to build a physical model to predict the phenomenon of cochlear potentials, Cannon relied on the electrophysiological data to build a network model. Similar to Strelloff's model, Cannon's lumped element model is composed of transverse sections joined by longitudinal elements. The model, however, includes two grounding pathways leading to the modiolus — one through the stria vascularis/spiral ligament and the blood vessels in the bony wall, and another through the blood vessels and nerve tissue leaving the organ of Corti (Fig. 3.4). Cannon did not attempt to model the whole length of the cochlea with a single model. Instead, he modeled each turn of the cochlea as a short transmission line.

The difficulty in using a network model to describe cochlear electro-anatomy is demonstrated by the assignment of element values in Cannon's model. For example, Cannon decided to assign equal values for the three effective resistance branches between spiral ligament and scala media (i.e., SL-SM; SL-SV, SV-SM; SL-MOD, MOD-OC, SL-ST, SM-OC, OC-ST). Such a choice may seem puzzling at first. However, with variations as large as 50% in the electro-anatomical measurements [Johnstone et al., 1966; Honrubia et al., 1976], Cannon's decision is as good as one can make for this type of model.

3.6 Capacitive Effects

To account for the evoked intracochlear potentials, such as the cochlear microphonic, network models of the cochlea have included capacitive and/or non-linear elements. Examples of network non-linearities and capacitances can be found in [Weiss et al., 1971] and [Haas, 1973; Cannon, 1976], respectively. Capacitive measurements
Fig. 3.3: Three dimensional, network model of the electrical properties of the cochlea. (from [Strelioff, 1971]).
Fig. 3.4: Resistance values for the first section of Cannon's cochlear model. The complete model is composed of six sections, each of which represents 1 mm of the cochlea. All resistances, except the one representing the basilar membrane (ST–OC), are constant for all sections of the model. Abbreviations: SV: scala vestibuli; SM: scala media; ST: scala tympani; SL: the spiral ligament and stria vascularis; MOD: the modiolus; OC: organ of Corti. (Adapted from Fig. 30 in Cannon, 1976).
have been reported in the more recent electro-anatomical investigations. Johnstone et al. [1966] described two time constants for the transient response of scala media to a 11 sec current step: a short constant (20 msec) and a much longer time constant. Sitko [1976] observed the longer time constant as well and speculated that it may be caused by an electrodiffusion process.

The shorter time constant has been attributed to capacitances in the endolymphatic wall of the scala media. Cannon [1976] modeled the stria vascularis, a highly folded cell structure, as the primary capacitance (SM–SL model element in Fig. 3.4) although the SM–OC and SM–SV model elements were also assigned capacitive components. Both Cannon [1976] and Sitko [1976] measured the impedance of the cochlear partition using sinusoidal stimuli. Cannon's SM–ST measurements in the first turn show that the magnitude of the impedance remains constant at 4000 ohms for frequencies up to 100 Hz. The impedance magnitude decreases to 2500 ohms (−4 dB) at 1000 Hz and to 1500 ohms (−9 dB) at 4000 Hz. The phase lag of the impedance is equal to 10° at 100 Hz and is less than 30° at 1000 Hz. Similar measurements of the ST–SL impedance show no change in magnitude for frequencies up to 4000 Hz. Sitko's data show a slightly greater SM–ST impedance attenuation. The magnitude decreases by 12 dB at 1000 Hz and by 17 dB at 4000 Hz (re 0 dB at 100 Hz). Thus the capacitances associated with the scala media boundaries can effectively act as a current shunt at high frequencies.

3.7 Discussion

The lumped element, electro-anatomical models described in this chapter do provide certain insight about the nature of the current pathways in the cochlea. However, these models have been developed with the assumption that the cochlea can be “unwound” by electrically uncoupling the different turns. Furthermore, current is assumed to flow largely longitudinally, along the length of the cochlea. Of course, the models are based on the bulk impedance measurements. The bulk measurements supply the effective impedance between any two regions but not the pathway of the current flow.

Nevertheless, certain electrical properties of the cochlea do emerge. The scala media is well insulated, with modeled resistive pathways (20–40 kohms) of order 10–20 times greater than pathways linking the other canals. Scala vestibuli, scala tympani and the spiral ligament seem to be linked by a relatively low resistance. The neural body is connected by the blood vessel system to the spiral ligament. Because the nerve has a larger volume than the blood vessels, the major grounding pathway to the
body is through the auditory nerve. The auditory nerve and blood vessels reach all
regions of the cochlea. But because of the spiraling nature of the cochlea and the con-
centrations of nerve tissue at the base of the modiolus, the grounding pathway is rather
limited in the higher turns.

Black and Clark [1980] used a three dimensional, transmission line model to
compute a measure of current spread due to electrical stimulation by intracochlear
electrodes. Separate models were developed for each of the three turns. The model
did not include capacitive elements, nor did it reflect the spiraling geometry of the
cochlea. The electrodes were modeled as ideal, point current (or voltage) sources. The
purpose of the model was to determine the current distribution in the vicinity of excita-
able (neural) tissue for different electrode configurations. However, because of the
gross quantization effects in such a model, the neural tissue could be lumped into
several possible model elements (e.g., ST-SM, SM-G). The model was found inade-
quate because of discrepancies in length constants calculated for elements of the model
assumed to contain neural tissue. Furthermore, certain stimulus configurations, such as
ST–ST electrodes, could not be represented due to the low resolution of the model.

When modeling the current density function during electrical stimulation of
intracochlear electrodes, there is a need to know the current distribution more precisely
than can be deduced from a discrete transmission line model. For instance, predicting
the magnitude and direction of the current density along the path of nerve tissue in the
osseous spiral lamina and in the modiolus, requires a model with greater resolution
than provided by transmission line models. The required anatomical detail is not
available when the whole of scala tympani (or any other structure) of a transverse sec-
tion is modeled by a single node. In addition, the transmission line models do not take
into account the three dimensional structure of the inner ear since they ignore possible
current paths between turns and simplify the grounding pathways. The transmission
line approximation also fails at the apex of the cochlea, the helicotrema, where scala
tympani and scala vestibuli join. Finally, an implanted electrode changes the electrical
environment by occupying a significant volume of the cochlear fluids. The implant
electrodes should not be approximated as point sources without investigating the
effects of such modeling.

Hence, a need for a different type of model which can overcome the inadequacies
of the transmission line model arises when modeling cochlear implant systems. A high
resolution model was developed by Wilson and Finley [1984a-d] to investigate current
spread due to a bipolar electrode. However, the model assumed both a homogeneous
medium and collapsed the three dimensional structure of the cochlea into a two
dimensional plane. Even though the authors claim that the model is consistent with results obtained during bipolar electrical stimulation, it is hard to believe that an adequate model can be developed with such severe simplifications.

The model proposed in this thesis is a first step in preserving the three dimensional, heterogeneous structure of the cochlea. Such a model can be used to gain a better understanding of how the geometry of the cochlea and the implanted electrodes effect the current spread. The development of the electro-anatomical model begins in the next chapter.
4.1 Introduction

A goal of this thesis is to investigate the feasibility of developing a three dimensional, electro-anatomical model of the implanted cochlea. The inadequacies of low resolution, transmission line models in predicting current spread due to intracochlear stimulation were discussed in the previous chapter. A three dimensional, electro-anatomical model can be developed by increasing the number of lumped parameters and configuring them in an anatomically correct fashion. Ideally, such a model should include enough resolution to represent the smallest structure of interest. When properly formulated, such a model could address questions not answered by the simpler models:

- As a consequence of the anatomy of the inner ear, are there any current pathways in the cochlea which are not represented in "unwound" models?
- What is the current density in more specific anatomical locations (e.g., the nerve tissue in the spiral lamina)?
- What impact does the electrode array have on the resultant electric field?

This thesis addresses the feasibility of developing such a model in several contexts. The governing mathematics and assumptions in electrically modeling the implanted cochlea are discussed in this chapter. This chapter also discusses the choice of the electro-anatomical data required to describe the model. The topics discussed in this chapter apply to any electrical model of the cochlea. Subsequent chapters describe the formulation and solution of the particular modeling scheme implemented as part of this thesis and compare the model results with potential measurements made in the scala tympani of subjects with implanted intracochlear electrodes.
4.2 Governing Mathematical Equations

In general, physiological electric field problems can be formulated quasi-statically [Plonsey, 1969]. The electromagnetic effects are considered to be negligible and only the quasi-static terms in Maxwell's equations are used in such a formulation.

The current density vector \( \mathbf{J} \) (A/m\(^2\)) obeys:

\[
\nabla \cdot \mathbf{J} = I, \tag{4.1}
\]

where \( I \) is the internal volume current source (A/m\(^3\)). The following relationship relates the current density to the electric field \( \mathbf{E} \) (V/m):

\[
\mathbf{J} = \mathbf{k} \, \mathbf{E} \tag{4.2}
\]

where \( \mathbf{k} = (\sigma + j\omega\varepsilon) \) is the general conductivity tensor, \( \sigma \) is conductivity (S/m)\(^†\), \( \varepsilon \) is permittivity (F/m), and \( \omega \) is frequency in radians per second. The electric field is related to the electric potential \( \Phi \) (V) by:

\[
\mathbf{E} = -\nabla \Phi. \tag{4.3}
\]

Thus by combining Eqs. (4.1) - (4.3), one gets:

\[
\nabla \cdot \mathbf{J} = \nabla \cdot (\mathbf{k} \, (-\nabla \Phi)) = I. \tag{4.4}
\]

Under the quasi-static formulation, the governing equation for a volume conductor with internal sources is a general Poisson's equation.

Note that if:

\[
\frac{\omega \varepsilon}{\sigma} \equiv \omega \tau \ll 1, \tag{4.5}
\]

the general conductivity tensor \( \mathbf{k} \) becomes just the conductivity tensor \( \sigma \). For biological materials, \( \tau \) or the dielectric relaxation time is much shorter than the excitation period \( \left( \frac{1}{2\pi \omega} \right) \) [Plonsey, 1969]. The condition is equivalent to having ionic drift dominate diffusion in the electrolyte current flow equations [Grodzinsky, 1985]. The electrolyte can then be modeled as an ohmic conductor:

\[
\mathbf{J} = \sigma \mathbf{E} \tag{4.6}
\]

and

\[
\nabla \cdot \mathbf{J} = \nabla \cdot (\sigma \, (-\nabla \Phi)) = I. \tag{4.7}
\]

The use of Eqs. (4.6) and (4.7) assumes that the conductor is purely resistive and

\[\nonumber \]

\( \dagger \) conductivity \( \sigma \) (S/m) is the reciprocal of resistivity \( \rho \) (ohms-m).
capacitive effects are negligible. Thus \( J, E \) and \( \Phi \) become purely spatial variables, with no time or frequency dependence.

If \( \mathbf{\sigma} = [\sigma_x, \sigma_y, \sigma_z] \) varies both along the principal axes \((x,y,z)\) and in space, the volume conductor is described as heterogeneous and anisotropic. If \( \mathbf{\sigma} \) has the same value along axes in all directions \((\sigma_x = \sigma_y = \sigma_z)\), the conductor is described as heterogeneous and isotropic. Furthermore, if there is no spatial variation \((\mathbf{\sigma} \) becomes a scalar constant), the conductor is described as homogeneous. In a homogeneous volume conductor with no internal current sources \((l = 0)\), Eq. (4.4) reduces to Laplace’s equation:

\[
\nabla^2 \Phi = 0. \tag{4.8}
\]

The complete formulation of the boundary value problem requires the specification of source and boundary conditions. The active sources during electrical stimulation are the electrodes which deliver a controlled current stimulus. The potential at the electrodes, which can also be used to specify a source condition, is difficult to specify because of the potential due to the electrode polarization impedance [Schwan, 1963]. A potential measured at the stimulus electrode may not reflect the potential in the adjacent cochlear fluid. The current, however, must be conserved across the electrode–electrolyte interface.

For the general class of elliptic equations such as the Poisson’s equation in Eq. (4.7), one of several types of boundary conditions must be specified. Along a closed boundary of the problem region one may specify a constant potential \( \Phi \), or the normal derivative \( \frac{\partial \Phi}{\partial n} \) (a quantity proportional to the current flow across the boundary), or a combination of both conditions [Hildebrand, 1976, p. 491].

In a heterogeneous volume conductor, material interface conditions must also be conserved. At the junction of two ohmic conductors 1 and 2, the interface condition is:

\[
n \cdot \mathbf{\sigma}_1 \mathbf{E}_1 = n \cdot \mathbf{\sigma}_2 \mathbf{E}_2. \tag{4.9}
\]

Thus the normal \((n)\) component of the current density vector must be conserved across the material interface.

4.3 Model Data

The definition of the three dimensional model requires the specification of two distinct yet interrelated parameters. The anatomical data are defined by the three dimensional structure of the cochlea and the implanted electrodes. The electrical data consist of the specific electrical conductivity of materials such as the soft tissue, bone
and fluid of the inner ear, and the conductors and insulators of the electrode array.

4.3.1 Anatomical Data

The anatomy of the inner ear is well known and data suitable for model development are available as temporal bone sections. Of course, the precise anatomy of a particular individual implanted with a particular electrode array cannot be ascertained. However, using computer aided tomography (CAT) techniques, it may be possible to extract the location of the implanted electrodes and the shape and path of the cochlear spiral [Ketten, 1986]. These measures could then be used to adjust the “generic” three dimensional model to individual cases. The choice and application of the anatomical data are discussed in the next chapter.

4.3.2 Electrical Data

Of the two parameters defining the model, the specification of the cochlear electrical characteristics is the more difficult task. Both the small size and complex structure of the inner ear have made the measurement of tissue conductivities a rather difficult task. In fact, aside from the bulk impedance measurements described in Chapter III, only the resistivities of the perilymph and endolymph have been measured. The determination of conductivity from a bulk resistance measurement requires that all the injected current pass through a volume whose dimensions can be measured or are known a priori. Because the measurements are done in vivo, one cannot be sure of the exact dimensions encompassing the area of current spread. Furthermore, a typical bulk resistance measurement usually includes several types of biological materials. For example, consider the bulk resistance of the scala media – scala tympani partition. Within the measurement, conductivities from fluid, bone, nerve and supporting tissue are all lumped together. All these materials contribute to the total resistance. Furthermore, the measurement may include other electrical pathways, such as through the spiral ligament to the scala vestibuli. It has not been possible to establish individual tissue conductivities from such bulk impedance measurements.

There is another alternative for specifying the electrical properties of the cochlear materials. Data for the impedance of various soft tissue (e.g., nerve), bone and fluids exist in the literature [Geddes and Baker, 1967; Schwan and Kay, 1957a, 1957b]. Although such measures are not made using the cochlear tissues themselves, they do provide a first order estimate of electrical conductivity of cochlear materials.
4.3.3 Model Electrical Assumptions

Several assumptions are made in formulating the governing equations in the model. These are:

- Quasi-static formulation
- Resistive media
- Isotropic media

The governing mathematical equations are formulated under the quasi-static assumptions, as described previously. The other two assumptions place restrictions on the electrical properties of model materials. The assumption of a resistive media makes the conductivity (a real quantity) the sole electrical property of the biological materials. Capacitive effects are thus considered negligible and all electrical variables have no time or frequency dependence. Measurements described in Chapter III, however, suggest that the scala media walls are both resistive and capacitive [Sitko, 1976; Cannon, 1976]. The magnitude of the impedance of the cochlear partition can be attenuated by as much as as 17 dB at 4 kHz (re 100 Hz). The phase of the impedance is on the order of 30° at 1 kHz.

It is likely, however, that this capacitance will have little effect on the distribution of potential in the scala tympani or the nerve tissue during electrical stimulation of scala tympani electrodes. In the present model, the capacitive tissues encompass a very small portion (by volume) of the whole model domain. Furthermore, the capacitances are concentrated at the boundaries of the scala media which are well insulated. For example, even when the effective † SM–ST resistance is reduced by 17 dB to represent the change in magnitude at 4 kHz, it remains several times greater than the longitudinal resistance of the perilymph for similar lengths of the cochlea (using Strelivoff’s [1973] model). This means that the principal current pathway from a scala tympani electrode to a far field ground electrode located outside the cochlea will be along the perilymph and through the relatively low impedance pathways provided by the nerve and blood vessels. Since these materials have been shown to be primarily resistive up to frequencies of 10 kHz ([Geddes and Baker, 1967; Schwan and Kay, 1957b]), it is not surprising that bulk impedance measurements in an implanted cochlea using scala tympani electrodes show that the magnitude of the impedance

† effective resistance refers to the value assigned to the SM-ST partition in transmission line models. This value is higher than the bulk resistance of this partition because the effects of other cochlear pathways are included in the models.
varies by less than 10% while the maximum phase lag reaches 6° at 5 kHz [Spelman et al., 1983]. Also, no capacitive effects are seen in scala tympani - spiral ligament measurements [Cannon, 1976] — a current pathway that most likely excludes the scala media. These results are also supported by the experiments described in Chapter VII of this thesis. For the purpose of this thesis, since the primary concern is the potential distribution in the scala tympani, the cochlea is assumed to be resistive.

The second assumption states that the model media are isotropic in conductivity. Tissues such as nerve, and perhaps the osseous spiral lamina (due to its perforated structure), are anisotropic. For example, conductivity of nerve in the longitudinal and transverse directions can differ by a factor 2-10 [Geddes and Baker]. Other biological tissues and fluids are largely isotropic [Geddes and Baker]. The anisotropic tissues, like the capacitive tissues, encompass a very small portion of the model domain. For the present model, the effect of anisotropy on the global results is likely to be minimal. However, as stated previously, the full material properties should be considered in any future model of greater resolution.

### 4.3.4 Specific Biological Material Resistivities

In the model, the resistivity of several types of biological materials must be specified. These are the cochlear fluids (endolymph and perilymph), the various bone (spiral lamina, temporal bone), the nerve and other soft tissue.

The resistivity of the cochlear fluids was first measured *in vitro* by von Békésy [1952]. The ohmic resistivity of perilymph and endolymph can also be calculated from chemical analysis [Peterson et al., 1978]. While the chemical composition of perilymph and endolymph differ, their resistivities are quite similar. From these two sources, the resistivity of cochlear fluids lies between 50 and 100 ohm-cm.

The values for resistivity of bone and nerve tissue are taken from Geddes and Baker [1967]. Bone resistivity typically ranges from 1000 to 5000 ohm-cm. The resistivity of cochlear bone tissue most likely falls in upper half of this range since the otic capsule is exceptionally hard and avascular as compared to other bone tissues [Bergerson et al., 1984]. The perfusion of fluid canals and blood vessels in parts of bone, however, may lower its resistivity. Nerve tissue has a resistivity between 100 and 400 ohms-cm, depending on the type of matter (gray-white) and orientation (longitudinal-transverse). Soft tissues, such as found in the spiral ligament, are assigned resistivities similar to that of nerve tissue. (A value for spiral ligament resistivity can be derived from Cannon [1976] who estimated the resistance from experimental measures. His value for spiral ligament is 5 kohms/mm. Assuming a cross
<table>
<thead>
<tr>
<th>biological material</th>
<th>resistivity (ohms-cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>fluid</td>
<td>50 – 100</td>
</tr>
<tr>
<td>bone</td>
<td>2000 – 5000</td>
</tr>
<tr>
<td>nerve</td>
<td>300</td>
</tr>
<tr>
<td>soft tissue</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 4.1: Assignment of resistivities to biological materials in the electro-anatomical model. The resistivities of fluid and bone vary as indicated; the effects of these resistivities are examined in Chapter VI of this thesis.

sectional area of 0.5 mm$^2$ in the first turn [Wever, 1949], the resistivity of the spiral ligament is equal to 250 ohm-cm.) Typical material resistivities used in the electro-anatomical model are shown in Table 4.1.
Chapter V

The Three Dimensional Electro-Anatomical Model: Definition, Formulation and Solution Methods

5.1 Introduction

Once the mathematical equations governing the potential distribution in the implanted cochlea have been defined, a method for formulating and solving the equivalent boundary value problem (Eq. (4.7)) must be chosen. An analytical method of solution is not feasible due to the complex geometry and non-homogeneity of the structure modeled. Therefore, a numerical method is used to transform the boundary value problem from a continuous into a discrete or spatially sampled domain. The electrostatic formulation and numerical solution of physiological boundary value problems has been treated by several researchers in the past [Plonsey, 1969; Witwer et al., 1972; Coburn, 1980; Heringa et al., 1981; Kim, 1982; Sepulveda et al., 1983; Coburn and Sin, 1985]. A finite difference method is used to divide the problem domain into a finite number of uniform hexahedral or "brick" elements. Within each brick element, the electrical conductivity of the biological material is assumed to be homogeneous and isotropic. Each brick is bounded by eight vertices or nodes, and each adjacent brick shares four of these nodes. The solution of the model yields the potentials for all the nodes in the problem domain, from which the current density distribution can then be obtained. This chapter begins with a description of the anatomical data entry, continues with the formulation of an equivalent electrical network, and ends with a description of the method used to compute the solution. The results from several types of electro-anatomical models which vary in resolution, source representation and electrical composition are presented in the next chapter.

5.2 Anatomical Data Entry

The anatomical data for the three dimensional model described in this thesis were derived from a set of serial sections of a human temporal bone. The temporal bone set†, from the right ear of a 12 year old male, was obtained from the Temporal Bone Library of the Massachusetts Eye and Ear Infirmary. The sections are cut at a plane parallel to the axis of the modiolus. The sections are 20 μm thick and at least every

† a complete description of temporal bone preparation is contained in [Schuknecht, 1974].
tenth section is stained and mounted on a standard microscope slide. A total of 37 sections are used from the temporal bone set to describe the implanted cochlea. Because only every tenth section was mounted and available, the discrete model resolution in the sectioning direction is fixed at 200 \( \mu \text{m} \). Fig. 5.1 shows a micrograph of a typical midmodiolar section.

Entry of anatomical data was accomplished in several stages which successively transformed each temporal bone section from a continuous into a discrete domain. The data transformation stages are described below, and summarized in Table 5.1.

<table>
<thead>
<tr>
<th>stage</th>
<th>name</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>temporal bone sections</td>
<td>the original anatomical data</td>
</tr>
<tr>
<td>2</td>
<td>pencil tracings</td>
<td>hand tracings of the main cochlear structures</td>
</tr>
<tr>
<td>3</td>
<td>CAA (vector) format</td>
<td>aligned, digitized vector representation of the pencil drawings</td>
</tr>
<tr>
<td>4</td>
<td>RASTER (mesh) format</td>
<td>the final, sampled model format.</td>
</tr>
</tbody>
</table>

Table 5.1: The main stages of the data entry process.

Each microscope slide was magnified and projected on a sheet of paper. Outlines of the major cochlear structures — the three major scalae (media, tympani and vestibuli), the stria vascularis and spiral ligament, and the nerve trunk — were traced by hand. Transparencies of the tracings were made and the cochlear sections aligned manually. Once aligned and registered, the tracings were digitized as a series of two dimensional vectors using the Computer Aided Anatomy (CAA) hardware and software at the Eaton Peabody Laboratory, Massachusetts Eye and Ear Infirmary. The digitization procedure involved retracing each of the paper drawings along specific anatomical structures (e.g., the scala media outline) using a graphics tablet and digitizing puck. Each pencil drawing was thus transformed into a set of two dimensional vectors. The vectors representing each structure were tagged with different names and identification numbers. For example, all of the scala tympani outlines were tagged as ‘1’. Fig. 5.2 shows a representation of a typical midmodiolar section in the CAA data format.

The data were then converted from the vector (CAA) format to the model domain format. The discrete model domain (hereafter referred to as the RASTER format) forms a three dimensional Cartesian space, divided into uniform hexahedral elements or
Fig. 5.1: A photomicrograph of a representative midmodiolar section. The anatomical data for the three dimensional model are based on a series of such sections. Only the cochlea, the surrounding temporal bone and a portion of the nerve trunk (i.e., the lower left quadrant) are included in the model. (Note that this section is not the same one used to obtain the CAA and RASTER model sections shown in Figs. 5.2 and 5.3, respectively.)
bricks. Each brick represents a homogeneous volume of biological material such as soft issue, bone, fluid, or electrode. Alternately, the element may be part of a structural outline (e.g., the boundaries of scala media or scala tympani.) Once the model boundaries (e.g., the midmodiolar window in Figs. 5.2 and 5.3) and the model resolution (i.e., the number of elements) are specified, a set of RASTER meshes is automatically produced from the CAA data. Note that only a two dimensional window is required to specify the model domain since the third model dimension (in the direction of temporal bone sectioning) is automatically fixed by the number of sections used. Each RASTER mesh represents a single section that has been spatially sampled (Fig. 5.3). The mesh consists of a single ‘‘wall’’ of bricks which is 200 μm thick (assuming no interpolation is performed between sections.) Thus the original temporal bone sections, which are 20 μm thick, are assumed to extend for 200 μm — the distance between available temporal bone sections. It is possible to change the inter-mesh resolution by geometrically interpolating between adjacent meshes or by simply including duplicate meshes in the model.

For simplicity, assume that a series of 40 cochlear sections containing all the structures of interest within an 8 mm by 8 mm square region (in the sectioning plane) have been digitized. This defines the model spatial domain — a cube with an 8 mm edge length. A cube element with edge length of 200 μm is then used to quantize the model domain. Thus each RASTER mesh contains 1600 (40 × 40) bricks, and the whole model is composed of 64 000 (40 × 40 × 40) bricks. Note that each RASTER mesh is bounded by 3362 (2 × 41 × 41) nodes, while the model described includes a total of 67 240 (40 × 41 × 41) nodes. Fig. 5.4 shows a typical model element and the complete RASTER model.

The anatomical data were initially quantized and stored as 512 × 512 element RASTER meshes. The model size (i.e., the number of model elements) and thus the model resolution is primarily dictated by the time allowed for model solution. Another limitation lies in the memory required for model storage and solution. Given the computer system (VAX 11/750 (Digital Equipment Corp., Maynard, MA) minicomputer and IRIS 1200 Graphics Terminal (Silicon Graphics Inc., Mountain View, CA)) and the method of solution used, the dimensions of the largest solvable model are set at 128 × 128 × 40 (over 655 000 model elements.) Certainly, larger models can be handled by the present computer system but not without expanding the solution time to unacceptable limits. For example, the solution time for the 128 × 128 × 40 model is approximately 40 hours. Expanding the model dimensions to 512 × 512 × 40 would increase the solution time to at best 640 hours. The memory and time required for
Fig. 5.2: The CAA format representation of a typical midmodiolar cochlear section. The anatomical outlines shown were first traced from a magnified projection of the temporal bone section, then digitized and tagged using a graphics tablet and puck. The three '+' are registration marks used to align the modeled sections. The rectangular border represents the approximate midmodiolar window that defines the model boundaries for this histological section.
model solution are described in the thesis Appendix.

The anatomical data, however, have been preserved at the higher resolution for future modeling. For example, once a model is solved at low resolution, the solution can be upsampled and the model solved again — this time at higher resolution but within a smaller model domain. Using such a technique, it may be feasible to determine the electric field distribution in select anatomical locations with increased resolution but without the overhead required by a full three dimensional model. The feasibility of interpolating model solutions across different resolutions is discussed in Chapter IX of this thesis.

The model domain for the electro-anatomical model developed in the thesis consists of a region 9.6 mm by 5.76 mm by 7.4 mm, spatially quantized by model elements with dimensions 18.75 μm by 11.25 μm by 200 μm. The element dimensions described are for the highest resolution (512 × 512 mesh) model (Table 5.2).

<table>
<thead>
<tr>
<th>element mesh size</th>
<th>element (μm)</th>
<th>Δx</th>
<th>Δy</th>
</tr>
</thead>
<tbody>
<tr>
<td>512 × 512</td>
<td></td>
<td>18.75</td>
<td>11.25</td>
</tr>
<tr>
<td>256 × 256</td>
<td></td>
<td>37.50</td>
<td>22.50</td>
</tr>
<tr>
<td>128 × 128</td>
<td></td>
<td>75.00</td>
<td>45.00</td>
</tr>
<tr>
<td>64 × 64</td>
<td></td>
<td>150.00</td>
<td>90.00</td>
</tr>
<tr>
<td>32 × 32</td>
<td></td>
<td>300.00</td>
<td>180.00</td>
</tr>
<tr>
<td>16 × 16</td>
<td></td>
<td>600.00</td>
<td>360.00</td>
</tr>
</tbody>
</table>

Table 5.2: Changes in element size with mesh resolution.

In the 512 × 512 mesh model, each element is tagged with a distinct name representing the soft tissue, bone, fluid, or material interface/boundary that the element encompasses. The tags are used during the formulation of the electrical network when resistivity values are assigned to the individual model elements (Table 5.3).

Some of the anatomical data were tagged manually after the data were converted to the RASTER format. For example, while the outline of the scala tympani (ST) had been originally digitized as one complete vector, portions of the ST outline (e.g., ST – spiral ligament, ST – osseous spiral lamina) were tagged separately so that it would be possible to manipulate the electrical characteristics independently.
<table>
<thead>
<tr>
<th>tag</th>
<th>(ohms-cm)</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5000</td>
<td>Temporal bone</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>ST (Perilymph)</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>SM (Endolymph)</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>SV (Perilymph)</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>Stria vascularis, SL</td>
</tr>
<tr>
<td>6</td>
<td>200</td>
<td>Interior of Semicircular canals</td>
</tr>
<tr>
<td>7</td>
<td>300</td>
<td>Nerve tissue</td>
</tr>
<tr>
<td>10</td>
<td>500</td>
<td>ST–Bony wall of the cochlea, ST–Spiral lamina boundaries</td>
</tr>
<tr>
<td>11</td>
<td>50</td>
<td>ST–SL boundary</td>
</tr>
<tr>
<td>12</td>
<td>50</td>
<td>ST–Organ of Corti, ST–Habenula perforata boundaries</td>
</tr>
<tr>
<td>20</td>
<td>20000</td>
<td>SM boundaries</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
<td>SV–Bony wall boundaries</td>
</tr>
<tr>
<td>31</td>
<td>50</td>
<td>SV–Stria vascularis</td>
</tr>
<tr>
<td>34</td>
<td>20000</td>
<td>SV–Limbus/Spiral lamina boundary</td>
</tr>
<tr>
<td>40</td>
<td>300</td>
<td>SL–Bony wall boundary</td>
</tr>
<tr>
<td>60</td>
<td>300</td>
<td>Boundaries of Semicircular canals</td>
</tr>
</tbody>
</table>

Table 5.3: A list of model element tags and the representative resistivities. The scala media and scala vestibuli–limbus boundaries (tags 20 and 34) have super high resistivities which are eventually lowered to the temporal bone resistivity during the downsampling process. Certain boundaries (e.g., SV–Bony wall) are assigned relatively low resistance since they are surrounded by high resistance temporal bone. Abbreviations: SV: scala vestibuli; SM: scala media; ST: scala tympani; SL: spiral ligament.

Other anatomical data that were tagged manually consisted of the modiolar boundaries and nerve tissue, both in the modiolar region and in the spiral lamina. The modiolus and the nerve trunk (Fig. 5.3) were “filled” with nerve tissue. The nerve fibers passing through the osseous spiral lamina originally were not digitized as a distinct (“nerve tissue”) structure but left as the “gap” between scala tympani and scala vestibuli and “filled in” manually. The boundaries of the osseous spiral lamina were tagged independently for scala tympani and scala vestibuli. Furthermore, a short segment of the spiral lamina – scala tympani boundary was tagged differently from the rest of the scala tympani boundary. This segment, typically encompassing several
model elements, is intended to represent the habenula perforata region, a point where the nerve fibers exit the spiral lamina and travel to the hair cells in the organ of Corti.

5.3 Decreasing the Model Resolution (Downsampling)

In order to formulate a model that was solvable given practical time and storage constraints, the model resolution was decreased from the $512 \times 512$ (mesh) model originally defined. As mentioned, given the computational software and hardware used for the model solution, the largest model used consisted of $128 \times 128$ element meshes. The quantized anatomical data, defined in the RASTER format as $512 \times 512$ element meshes, were converted by regular decimation (downsampled) to $128 \times 128$ (or lower) meshes using a hierarchical, two dimensional downsampling process. The data were first downsampled from a $512 \times 512$ mesh to a $256 \times 256$ mesh, then from a $256 \times 256$ mesh to the next level (a $128 \times 128$ mesh) and so on for all the model meshes (Fig. 5.5). A typical $512 \times 512$ mesh is shown in Fig. 5.3; Figs. 5.6, 5.7, and 5.8 show the $256 \times 256$, $128 \times 128$, and $64 \times 64$ meshes, respectively. Thus at each downsampling step, the element volume increased by a factor of four, or alternatively, the total number of elements in the destination (downsampled) model decreased by a factor of four with respect to the source (high resolution) model. The same algorithm was used at each step; the downsampling procedure is summarized below:

1. Convert numerical tags into material resistivity. This step is performed only for the initial $512 \times 512$ mesh. See Table 5.3 for element resistivities.

2. Assign super high resistivity values to certain boundaries (e.g., scala media outline) which otherwise may get blurred out in the downsampling process. Again, this step is done only for the $512 \times 512$ mesh.

3. For each element in the low resolution (destination) mesh, perform a local (9 element) weighted average in the higher resolution (source) mesh. The average is performed at spatially equivalent elements in the source mesh. Thus, for a 4:1 downsampling process, every second element in every second row in the high resolution mesh is convolved with the filter to produce a low resolution model element. The averaging filter, chosen ad hoc using the
Fig. 5.3: RASTER representation of a typical midmodiolar section. This mesh contains 65,556 (256 × 256) elements. The mesh is obtained by sampling the CAA forma data (shown in Fig. 5.2). Elements are tagged according to the biological material or boundary they represent. For example, the complete interior of the modiolus and the nerve trunk are tagged as "nerve tissue". (The portion of the modiolus that was defined manually can be seen by comparing this figure with Fig. 5.2). The boundaries of the scala media can be seen as the light outline of the scala.
criterion to be described in Chapter VII, is shown in Fig. 5.9.

4. Repeat step 3 for additional levels of downsampling.

5. Clip element resistivities to a set maximum (typically equal to the resistivity of bone) to eliminate super high resistivities. This step is performed only for the final destination mesh.

The averaging was two dimensional (i.e., performed only in the RASTER mesh plane) based on the assumption that there is less variation in cochlear anatomy between adjacent meshes than in the mesh plane. Models with mesh dimensions 64 x 64 or higher contain elements with dimensions (in the mesh plane) which are less than the 200 μm distance between adjacent cochlear meshes (Table 5.2). The inter-mesh resolution is fixed at 200 μm because additional anatomical sections were not available.

Note that steps 2 and 5 maintain the high resistivity of anatomical structures such as Reissner’s membrane and the rest of scala media wall. Certain boundary structures may not be preserved in the lower resolution models due to anatomical undersampling in the high resolution model. In other words, if a structure is represented by only a single row of elements at high resolution, there is no spatial equivalent (i.e., half an element) at low resolution. Thus while the fine detail of an anatomical structure (e.g., the boundaries of scala media) may not be preserved when downsampled, the structure’s electrical property of higher resistivity is maintained by the assignment of the super high resistivity. The averaging filter acts as a low pass spatial filter, distributing the boundary structure (high resolution) resistance among the downsampled (low resolution) elements.

5.4 Finite Difference Formulation

The next stage in the modeling process is the formulation of the numerical equations governing the potential distribution in the discrete model domain. The solution of the discrete model consists of the solution of a set of simultaneous linear equations (one equation per model node.) The node equations are derived using a finite difference method which treats each brick in the model as a homogeneous and isotropic volume conductor.
Fig. 5.4a: An example of a model element (or brick). For a 128 by 128 by 37 model, \( \Delta x = 75 \, \mu m \), \( \Delta y = 90 \, \mu m \), and \( \Delta z = 200 \, \mu m \).

Fig. 5.4b: The complete three-dimensional model. The model is composed of \( N \) meshes (\( N = 37 \), typically). Each mesh is uniformly divided into model elements shown in (a). The model dimensions are 9.6 mm by 5.7 mm by 7.4 mm in the \( x \), \( y \) and \( z \) axes, respectively. Note that the anatomical outline in mesh 1 is drawn for illustrative purposes only.
Fig. 5.5: A schematic representation of the down-sampling procedure. For each section of the model, the high resolution (source) mesh is downsampled to the low resolution (destination) mesh. Hence, one element of the destination mesh represents four elements of the source mesh.
Fig. 5.6: RASTER (256 × 256) representation of a typical midmodiolar section. This mesh was obtained by downsampling the 512 × 512 mesh shown in Fig. 5.3 and assigning resistivities to the tagged elements. Note that variation in the gray-scale shade does not correspond to a similar variation in resistivity, but elements with the same gray-scale shade have identical resistivities.
Fig. 5.7: RASTER (128 × 128) representation of a typical midmodiolar section. This mesh was obtained by down sampling the 256 × 256 mesh shown in Fig. 5.6. Note that variation in gray-scale shade does not correspond to a similar variation in resistivity, but elements with the same gray-scale have identical resistivities.
Fig. 5.8: RASTER (64 × 64) representation of a typical midmodiolar section. This mesh was obtained by downsampling the 128 × 128 mesh shown in Fig. 5.7. Note that variation in gray-scale shade does not correspond to a similar variation in resistivity, but elements with the same gray-scale shade have identical resistivities.
Fig. 5.9: The downsampling filter. The nine element, two dimensional spatial filter is convolved with the source mesh to produce the destination mesh. The filter is applied at every second element of every second row in the source mesh (for a 4:1 element downsampling ratio). The greatest weight (30.0) is assigned to the central element while the 8 neighbors are weighted according to distance from the central element. The filter was chosen using a lowest average percentage difference (APD) criteria for the source and destination meshes (see Chapter IX for details.)
5.4.1 Derivation of Node Potential Equations

To derive the node equations, first consider a brick element of constant conductivity $\sigma$. Scalar values for the current $I$ and potential $\Phi$ can be defined for each side of the brick. Scalars $I_l, I_r, I_d, I_u, I_f, I_b$ refer to the current flowing through the left, right, bottom, top, front and back surfaces, respectively (Fig. 5.10). The current density is defined by:

$$\mathbf{J} = -\sigma \nabla \Phi,$$  \hspace{1cm} (5.1)

and the current by:

$$I = \int \mathbf{J} \ ds,$$  \hspace{1cm} (5.2a)

thus yielding:

$$I = \int -\sigma \nabla \Phi \ ds$$ \hspace{1cm} (5.2b)

$$= -\sigma \Phi \times \text{surface area},$$  \hspace{1cm} (5.2c)

where \text{surface area} refers to the brick surface through which a particular current $I$ flows. Therefore, each of the surface currents can be expressed as a function of the electric field in the given direction and the brick conductivity:

$$I_l = -\sigma \left( \frac{\partial \Phi}{\partial x} \right)_l \Delta z \Delta y $$ \hspace{1cm} (5.3a)

$$I_r = -\sigma \left( \frac{\partial \Phi}{\partial x} \right)_r \Delta z \Delta y $$ \hspace{1cm} (5.3b)

$$I_d = -\sigma \left( \frac{\partial \Phi}{\partial y} \right)_d \Delta z \Delta x $$ \hspace{1cm} (5.3c)

$$I_u = -\sigma \left( \frac{\partial \Phi}{\partial y} \right)_u \Delta z \Delta x $$ \hspace{1cm} (5.3d)

$$I_f = -\sigma \left( \frac{\partial \Phi}{\partial z} \right)_f \Delta x \Delta y $$ \hspace{1cm} (5.3e)

$$I_b = -\sigma \left( \frac{\partial \Phi}{\partial z} \right)_b \Delta x \Delta y $$ \hspace{1cm} (5.3f)

According to Kirchhoff's current law, the net current flowing through the element (a closed surface) is equal to zero. If there are no current sources inside the element, the total current entering and leaving the element is zero. Thus:

$$I_l - I_r + I_d - I_u + I_f - I_b = 0.$$  \hspace{1cm} (5.4)
Fig. 5.10: Currents in a solid (homogeneous) element used to derive the finite difference set of equations.

Fig. 5.11: A composite element. The element is composed of 8 bricks (brick 7 is hidden.) The center nodes of the front (f), top (u) and right (r) surfaces are shown, while the nodes of the left (l), bottom (d) and back (b) are hidden. A center (c) node is located at the junction of the 8 bricks inside the composite element.
Upon rearrangement of the terms in Eq. (5.3) and using Eq. (5.4) with the limit as \( \Delta x, \Delta y, \) and \( \Delta z \) go to zero, Laplace’s equation \( (\nabla^2 \Phi = 0) \) is obtained [Vemuri and Karplus, 1981].

Consider the same element but now composed of eight smaller bricks, each with a distinct conductivity \( \sigma_1 \) through \( \sigma_8 \) (Fig. 5.11). At the center of each exterior surface, a node is placed. These nodes have potentials \( \Phi_l, \Phi_r, \Phi_d, \Phi_u, \Phi_f, \) and \( \Phi_b \) at the left, right, bottom, top, front, and back surfaces, respectively. Furthermore, there is a node located in the center of the composite element which has potential \( \Phi_c \). Each of the six exterior surfaces is assumed to be at an equipotential which is equal to the potential of the central node for the surface. For example, the potential of the right surface is equal to \( \Phi_r \), while the potential of the top surface is equal to \( \Phi_u \). Furthermore, the potential of the three interior surfaces obtained by slicing the composite element across the brick boundaries is equal to \( \Phi_c \), the center node potential.

For any exterior surface in the composite element, the total current density is equal to the sum of the current densities flowing across the surfaces of the bricks which form that element surface. Thus for the front surface, the total current density is:

\[
J_f = J_1^f + J_2^f + J_3^f + J_4^f ,
\]  

(5.5)

where \( J_1^f \) through \( J_4^f \) are the “front” direction components of the current densities at the four front surface bricks. The current density is calculated using Eq. (5.1a) with the potential gradient between the front and central surfaces being the same for all the front bricks:

\[
\left( \frac{\partial \Phi}{\partial x} \right)_f = \frac{\Phi_f - \Phi_c}{\Delta z} .
\]  

(5.6)

Thus the brick current densities are equal to the product of their individual conductivities and the common potential gradient:

\[
J_1 = \frac{\Phi_f - \Phi_c}{\Delta z} \sigma_1 ,
\]  

(5.7a)

\[
J_2 = \frac{\Phi_f - \Phi_c}{\Delta z} \sigma_2 ,
\]  

(5.7b)

\[
J_3 = \frac{\Phi_f - \Phi_c}{\Delta z} \sigma_3 ,
\]  

(5.7c)

\[
J_4 = \frac{\Phi_f - \Phi_c}{\Delta z} \sigma_4 .
\]  

(5.7d)
The total current density at the front surface is therefore:

\[ J_f = \frac{\Phi_f - \Phi_c}{\Delta z} (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) . \] (5.8)

The "front" current is obtained by multiplying the current density by the brick surface area:

\[ I_f = \frac{\Phi_f - \Phi_c}{\Delta z} \frac{\Delta x \Delta y}{4} (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) . \] (5.9a)

Similarly, the other five surface currents are:

\[ I_b = \frac{\Phi_c - \Phi_b}{\Delta z} \frac{\Delta x \Delta y}{4} (\sigma_5 + \sigma_6 + \sigma_7 + \sigma_8) , \] (5.9b)

\[ I_l = \frac{\Phi_c - \Phi_l}{\Delta x} \frac{\Delta z \Delta y}{4} (\sigma_1 + \sigma_3 + \sigma_5 + \sigma_7) , \] (5.9c)

\[ I_r = \frac{\Phi_c - \Phi_r}{\Delta x} \frac{\Delta z \Delta y}{4} (\sigma_2 + \sigma_4 + \sigma_6 + \sigma_8) , \] (5.9d)

\[ I_d = \frac{\Phi_c - \Phi_d}{\Delta y} \frac{\Delta x \Delta z}{4} (\sigma_3 + \sigma_4 + \sigma_7 + \sigma_8) , \] (5.9e)

\[ I_u = \frac{\Phi_c - \Phi_u}{\Delta y} \frac{\Delta x \Delta z}{4} (\sigma_1 + \sigma_2 + \sigma_5 + \sigma_6) . \] (5.9f)

Using Eq. (5.4), the nodal equations become:

\[ \Phi_I \beta_I + \Phi_R \beta_R + \Phi_U \beta_U + \Phi_D \beta_D + \Phi_B \beta_B + \Phi_F \beta_F = \Phi_C \beta_C \] (5.10)

where

\[ \beta_I = \frac{(\sigma_1 + \sigma_3 + \sigma_5 + \sigma_7)}{4\Delta x} \Delta y \Delta z , \] (5.11a)

\[ \beta_R = \frac{(\sigma_2 + \sigma_4 + \sigma_6 + \sigma_8)}{4\Delta x} \Delta y \Delta z , \] (5.11b)

\[ \beta_F = \frac{(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)}{4\Delta z} \Delta x \Delta y , \] (5.11c)

\[ \beta_B = \frac{(\sigma_5 + \sigma_6 + \sigma_7 + \sigma_8)}{4\Delta z} \Delta x \Delta y , \] (5.11d)

\[ \beta_U = \frac{(\sigma_1 + \sigma_2 + \sigma_5 + \sigma_6)}{4\Delta y} \Delta x \Delta z , \] (5.11e)

\[ \beta_I = \frac{(\sigma_3 + \sigma_4 + \sigma_7 + \sigma_8)}{4\Delta y} \Delta x \Delta z \] (5.11f)
and

\[ \beta_c = \beta_l + \beta_r + \beta_u + \beta_d + \beta_f + \beta_b. \]  

(5.11g)

In the homogeneous case and with \( \Delta x = \Delta y = \Delta z \), Eq. (5.10) becomes:

\[ \Phi_c = \frac{\Phi_l + \Phi_r + \Phi_u + \Phi_d + \Phi_b + \Phi_f}{6} \]  

(5.12)

which is a well known six neighbor approximation of the three dimensional Laplace equation [Vemuri and Karplus, 1981].

### 5.4.2 Calculation of Current Density from Model Solution

The solution of the model yields the potential at each node in the model domain. To calculate the current density \( J = (J_x, J_y, J_z) \), consider once more a typical model element (Fig. 5.12) of constant conductivity \( \sigma_e \). Each surface of the model element comprises four nodes. Here, the brick surface potential is approximated as the average of the four node potentials. Using Eq. (5.1), the current density components are:

\[ J_x = \sigma_e \frac{(\Phi_a + \Phi_c + \Phi_e + \Phi_g) - (\Phi_d + \Phi_b + \Phi_f + \Phi_h)}{4 \Delta x}, \]  

(5.13a)

\[ J_y = \sigma_e \frac{(\Phi_c + \Phi_d + \Phi_h + \Phi_g) - (\Phi_a + \Phi_b + \Phi_f + \Phi_e)}{4 \Delta y}, \]  

(5.13b)

\[ J_z = \sigma_e \frac{(\Phi_a + \Phi_b + \Phi_c + \Phi_d) - (\Phi_e + \Phi_f + \Phi_g + \Phi_h)}{4 \Delta z}. \]  

(5.13c)

The magnitude of the current density is equal to:

\[ J = \sqrt{J_x^2 + J_y^2 + J_z^2}. \]  

(5.13d)

### 5.4.3 Alternative Boundary Value Problem Formulations

An alternate view of the finite difference formulation is to consider the discrete boundary value problem as an electrical network problem. The network consists of resistors joining neighboring nodes. Each resistor is defined according to element geometry and the composite conductance between nodes. In the finite difference formulation, with the exception of the boundary nodes, adjacent nodes share four model elements. Thus the effective resistance between any two nodes can be expressed as four resistors in parallel. Each of the resistors can be determined from the formula for the conductance of a volume conductor with conductivity \( \sigma \), length \( \Delta l \), and cross-
Fig. 5.12: The homogeneous element used in the calculation of current density.
sectional area $A$:

$$G = \frac{1}{\sigma} \frac{A}{\Delta l}.$$  \hspace{1cm} (5.14)

Note the similarity between Eq. (5.14) and Eqs. (5.11a-f), if a particular $\beta$ is considered as a conductance $G$.

The composite element in the derivation of the finite difference method (Fig. 5.11) encloses a total of 27 nodes. However, only seven nodes — the central node and its six neighbors — are used in the numerical method. A better approximation would incorporate all the node potentials into the second order derivative approximation (Eq. (5.10)), or use a greater number of nodes within the same composite element. Such a formulation, however, would increase the bandwidth of resultant system of equations. For example, the present formulation results in only seven non-zero entries for each node equation (Eq. (5.10)). Using all the node potentials would increase the number of non-zero entries from seven to as high as twenty seven. Due to the large number of nodes in the model, increases in memory overhead and/or solution time would be significant.

The present method of formulation is slightly different from other finite difference formulations. For example, a method described in [Witwer et al., 1972] places nodes in the center of each brick or model element. Using this method, the effective resistance between nodes is equal to two resistors in series (i.e., sum of resistances). However, the present parallel resistance formulation is more suitable to downsampling of anatomical data than the alternate series resistance formulation. This is because the anatomical data (i.e., the model elements) are stored in meshes with even rank, while electrical data (i.e., the model nodes) are stored in meshes with odd rank. Hence, the anatomical data is easily downsamped and upsamped by factors which are powers of 4 (e.g., 4, 16, 64 ...). Furthermore, the present formulation is closer to a three dimensional finite element method based on tetrahedral [Silvester and Ferrari, 1983] and hexahedral [Kim, 1982] elements. In the finite difference formulation, as described, the second order derivative is estimated using a six neighbor approximation. For the same composite element, as many as twenty seven nodes are used by the finite element method. The finite element formulation may yield more accurate solutions [Silvester and Ferrari, 1983] but with the added overhead in the memory and/or solution time mentioned above.
5.5 Solution of Model†

Once all the anatomical data have been entered, the conductivities assigned, and boundary and source conditions defined, the model can be solved. (The choice of boundary and source conditions will be discussed in the next chapter.) For every node in the model domain, with the exception of boundary nodes, an equation such as Eq. (5.10) is written. Essentially, the solution of the model involves solving a set of \( N \) simultaneous linear equations in the form \( Hx = b \), where \( N \) is the number of unknown nodes in the three dimensional model, \( H \) is the \( N \times N \) conductance matrix built using the finite difference method, \( b \) is the \( N \) element source vector, and \( x \) is the \( N \) element potential vector to be calculated. The \( H \) matrix is symmetric and very sparse, having at most seven non-zero entries per row. For the model developed, the number of unknowns is approximately 655,000 (128 \( \times \) 128 mesh model). An iterative method of solution becomes a very attractive and perhaps the only means of solution since direct methods (e.g., Gaussian elimination) require much greater storage overhead during execution. The sparse nature of the resulting set of simultaneous equations makes an iterative method even more attractive. Furthermore, for certain types of problems, the iterative method to be described is more accurate (in terms of machine round-off error) and faster than a direct one. This section presents a brief overview of the solution algorithm. The software implementation as well as the data structures involved in the formulation and solution of the model are described in the Appendix of this thesis.

5.5.1 Iterative Methods

A class of iterative methods used to solve the set of linear equations in the form \( Hx = b \) can be derived by solving an equivalent problem of minimizing the quadratic functional \( f(x) \),

\[
f(x) = \frac{1}{2} x^T H x - b^T x + c, \quad x \in \mathbb{R}^N, \tag{5.15}
\]

where \( H \) is an \( N \times N \) symmetric matrix, \( b \in \mathbb{R}^N \), and \( c \in \mathbb{R} \).

Most iterative methods converge towards a final solution by taking steps in the form

\[
x^{k+1} = x^k + \tau_k d^k, \tag{5.16}
\]

where each step is based on two parameters: \( d^k \), (the "search direction"), and \( \tau_k \) (the

† Note that all the algorithms described in this section have been taken from [Axelsson and Barker, 1985].
"step size"). A scalar chosen to minimize or reduce the functional \( f(x) \) passing through \( x^k \) in the direction \( d^k \). The speed or number of iterations required for convergence as well as the efficiency of iterative methods depend on how these two parameters are calculated. In developing iterative algorithms, there is always a trade off between the speed of the method and the overhead (in both time and storage) required for the calculation of the stepping parameters. A faster method will require greater overhead. For example, a very simple but slow iterative algorithm is the Method of Steepest Descent:

\[
g^k = Hx^k - b,
\]

\[
\tau_k = g^k T g^k / d^k T H d^k,
\]

\[
x^{k+1} = x^k - \tau_k g^k,
\]

where \( k = 0, 1, \ldots \), and \( g^k = g(x^k) \). Here, the search direction is chosen to be the gradient \(-g(x)\). The step size \( \tau_k \) can be shown to uniquely minimize the functional \( f(x^{k+1}) \). The method requires very little storage and the computational effort can be further reduced by performing the matrix multiplication recursively.

In the PreConditioned Conjugate Gradient method (PCCG) (the algorithm used in solving the model), the search direction \( d^k \) is also updated at every iteration from the gradient \( g^k \) by \( \beta^k \) (an additional "step size variable.") Essentially, for every iteration, a search direction \( d^k \) is chosen such that it is orthogonal to the previous search direction. The complete algorithm is as follows:

\[
\tau_k = g^k T h^k / d^k T H d^k,
\]

\[
x^{k+1} = x^k + \tau_k d^k,
\]

\[
g^{k+1} = g^k + \tau_k H d^k,
\]

\[
h^{k+1} = C^{-1} g^{k+1},
\]

\[
\beta_k = g^{k+1} T h^{k+1} / g^k T d^k,
\]

\[
d^{k+1} = -h^{k+1} + \beta_k d^k,
\]

where \( k = 0, 1, \ldots \). To begin, \( x^0 \) is chosen and \( g^0 = Hx^0 - b \), \( h^0 = C^{-1} g^0 \), and \( d^0 = -h^0 \) are set. Given infinite precision, such a method must end with a zero search direction. At this point the exact solution is obtained since all search vectors are exhausted. Thus
the Conjugate Gradient method must converge to the exact solution after at most \( N \) iterations. Such an upper bound does not exist for the Method of Steepest Descent. In the PCCG method, the linear system of equations is preconditioned by multiplying the system by a conditioning matrix \( C \) (described in the Appendix) and then the new system of linear equations is solved. At convergence, the conditioned solution must be multiplied by the inverse of the conditioning matrix to obtain the correct solution. Preconditioning can be shown to significantly decrease the number of iterations required for convergence.

5.6 Discussion

To summarize, the anatomical data in the form of anatomical sections were first transformed into the vector (CAA) format and then spatially sampled into the model (RASTER) format. The full three dimensional, electro-anatomical model consists of 37 cochlear sections. At the highest resolution, each cochlear section is represented by a mesh or a two dimensional array with \( 512 \times 512 \) elements. Each model element is assigned a numerical value according to the type of biological material the element represents. The model domain has dimensions of \( 9.6 \) mm by \( 5.76 \) mm by \( 7.4 \) mm which corresponds to a model element with dimensions of \( 18.75 \) \( \mu \)m by \( 11.25 \) \( \mu \)m by \( 200 \) \( \mu \)m (for the \( 512 \times 512 \) model.)

The electro-anatomical model includes sufficient resolution to overcome some of the inadequacies of the transmission line models. For example, given the solution of such a model, the magnitude and direction of the current density in any model element can be computed. Hence, with sufficient resolution, current density in any structure of interest can be computed. One electrical pathway that is not included in the model is the blood vessel system in the cochlea (see Chapter III.) The effects of this pathway can be investigated by grounding elements in the spiral ligament (a region with a large blood supply [Schuknecht, 1974].) The major grounding pathway, however, remains the auditory nerve due to its large volume.

Using the methods described in this chapter, it is both feasible and practical to formulate and solve an electrical model of the implanted cochlea. The model, based on anatomical sections of a human temporal bone, represents the cochlea as a sampled three dimensional structure. Even though the data entry procedure requires manual tracing and digitizing of each temporal bone section, the complete model can be defined (from cochlear sections to the RASTER format) in the time span of 2–4 weeks. Furthermore, once the data are digitized and tagged, RASTER format models of any resolution can be derived with very little manual supervision. The data structures used
allow for flexible variation of the model electrical parameters. Changing the bone resistivity, for example, requires a change in the resistivity entry table (Table 5.3) and the re-downsampling of the model meshes, starting from the $512 \times 512$ model. Some of the manual aspects of data entry could be improved. If, for example, all anatomical boundaries of interest are tagged at the digitizing stage (rather than tagging some manually in the \textit{RASTER} format), conversion from the \textit{CAA} to the \textit{RASTER} format could be completely automated.

Uniform quantization was used in the present model because it facilitates the transformation of anatomical data from the continuous to the discrete model (\textit{RASTER}) domain. Once the data are in the \textit{RASTER} format, applying the finite difference method becomes a relatively straightforward task. Other numerical methods, most notably the finite element method, are more flexible in allowing for variations in model element dimensions. For example, it would be advantageous to use smaller elements to represent the bone and nerve tissue of the osseous spiral lamina than those used to represent the perilymphatic spaces, or the surrounding temporal bone. However, if the size of the model elements is allowed to vary, each element must be defined individually. The uniformly sampled model contains over 500,000 elements; even if the number of elements could be reduced by half using the finite element method, the data entry stage would remain an enormous task. Presently, the model is quantized by elements with fixed dimensions, a procedure that is easy to automate. There is a direct correspondence between any physical location and a model element due to the uniform quantization of the model domain. Thus to represent the model domain, the anatomical tags for the model elements are stored sequentially. If the quantization were based on irregular elements, both the element tag and the physical dimensions of the element (e.g., the element vertices) would have to be stored — a significant increase in storage required to describe the model. The primary disadvantage of a uniform model is that the number of model elements increases sharply with increasing resolution. For example, doubling the resolution in all model dimensions would increase the total number of elements by a factor of eight.

The model resolution is limited by two practical factors: the memory requirements and time to compute the model solution. Typically, a faster method of solution will require more storage overhead. For example, using the \textit{PCGG} method, the solution converges in about 40 hours for a typical heterogeneous ($128 \times 128 \times 37$) model. The \textit{PCGG} algorithm requires about 40 megabytes of secondary storage during the solution procedure. Using the simpler \textit{CG} algorithm requires only one quarter the storage. However, a typical two dimensional (single mesh) problem (only 16 000
nodes vs 600 000 nodes for the three dimensional problem) requires over 20 hours of computation using the CG algorithm.
Chapter VI

The Three Dimensional Electro-Anatomical Model: Results

6.1 Introduction

This chapter presents the solutions for various three dimensional electro-anatomical models. The complete three dimensional model consists of 37 uniform meshes. The model as a whole (the model domain) may be viewed as a hexahedron which can be divided into smaller hexahedrons, the model elements or bricks. All the model elements have uniform dimensions. The model domain has dimensions of 9.6 mm by 5.76 mm by 7.4 mm, along the x, y, and z axes, respectively (Fig. 5.4). The model domain includes the cochlea and a portion of the surrounding temporal bone. As it leaves the modiolus, the auditory nerve trunk is truncated by the bottom plane of the model. Therefore, five of the six model domain surfaces border temporal bone, while the sixth (bottom) surface borders both bone and nerve tissue.

In the model, the electrodes are referred to as “source” and “sink” electrodes. Current is injected at the “source” electrode and must return to the “sink electrode, which is fixed at zero potential. All model results can be linearly scaled to stimulus intensity (current strength) and polarity because the model is a linear, passive electrical network. Hence, for a positive source, the model predicts a positive potential distribution. To model physical stimuli (typically, biphasic waveforms), the model results should be interpreted as representing the magnitude of the potential (or current density, if computed) distribution during one phase of the stimulus waveform. Only the polarity (or direction, for current density) of the model results changes for a “source” electrode of negative amplitude.

6.2 Model Boundary Conditions

The complete formulation of the boundary value problem requires the specification of electrical boundary conditions at the exterior surfaces of the model domain. The boundary conditions must specify a constant potential at a model surface, a constant current flow across a model surface, or a combination of both conditions. Typically, the boundary potential is set to zero, indicating a current sink. Similarly, forcing the current flow to be zero is equivalent to insulating the model surface.
Fig. 6.1: Monopolar boundary conditions for a mesh representing a midmodiolar histological section. The nerve trunk at the bottom boundary is grounded while the remainder of the mesh is insulated.
Due to the finite size of the model domain, one can only approximate the physical boundary conditions. First, consider a monopolar electrode configuration that includes a single intracochlear electrode stimulated with respect to a ground electrode located in the temporalis muscle. Because the nerve and blood vessels provide the major (low resistivity) pathways out of the cochlea while the bony walls and the temporal bone insulate the intracochlear spaces (see Chapter III), all the surfaces of this model are insulated with the exception of the bottom surface that represents nerve. (Fig. 6.1) The neural tissue is grounded to represent its electrical proximity to the temporalis muscle electrode.

Alternatively, the physical boundary conditions during monopolar stimulation could be modeled by grounding all the model surfaces. Such conditions would force the potential to drop to zero at the model boundaries. However, while the potential outside the cochlea may approach zero, the grounding of the surfaces would also create a large sink electrode all around the model domain. Based on informal comparisons, solutions of such models do not predict the asymmetrical distribution of potential seen in the insulated models and the experimental results (Chapter VII). Furthermore, the potential attenuates more rapidly away from site of current injection in these models than observed in insulated models and experimental data because of the larger sink electrode.

A model that is completely insulated at all surfaces is used to represent the physical boundary conditions for pseudo-bipolar electrode configurations. For these configurations, the stimulus electrode pair resides inside the scala tympani. Thus most of the current pathways are contained within the model domain.

6.3 Typical Potential Distribution

A typical potential distribution for the complete three dimensional model is shown in Fig. 6.2. The results are for a model composed of 37, 64 × 64 meshes. The model has a heterogeneous composition: temporal bone, nerve tissue and cochlear fluid resistivities are 5000 ohms-cm, 300 ohms-cm and 50 ohms-cm, respectively. (All tissue resistivities are as described in Table 5.3). The model simulates a monopolar stimulus configuration with a “source” electrode located in the scala tympani of mesh 12 (approximately 15 mm from the round window.)

The model results are displayed in the form of equipotential contour lines, superimposed on the outline of the bony wall of the cochlea, the modiolus and the nerve trunk of each cochlear section. The potential difference between the equipotential contour lines is constant in all the model meshes. The absolute potential values are
Fig. 6.2: Equipotential contour plots for a monopolar configuration of the electro-anatomical model. The potential difference between the contour lines is constant across panels (0.1). Each panel represents a single model mesh (indicated below the lower right corner of each panel.) The full model includes 37 meshes (4–41); only meshes 5-37 are shown in the figure. Outlines of the cochlea, the modiolus, the nerve trunk and a portion of a semicircular canal can be seen in each panel. For example, mesh 16 is also represented in Fig. 5.3. The arrow indicates the maximum potential within a model section (except for mesh 12, the site of current injection, where the potential gradients are very steep). The table on the next page lists the maximum potential of each section (peak); the value of the maximum equipotential contour (max contour); the value of the minimum equipotential contour (min contour); and the potential difference between contours expressed as a percentage of the peak potential in the section (% of peak in each section).
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</tbody>
</table>

Fig. 6.2 (cont'd)
arbitrary since the data have not been scaled with respect to the actual stimulus intensity. However, due to the linear nature of the model, the spacing of the contour lines yields a relative measure of the potential gradient (which is a function of current density and conductivity) for particular model locations.

The model boundary conditions are responsible for several aspects of the contour plots. For example, equipotential lines are normal to the model's insulated boundaries because the normal component of the potential gradient is zero when the current flow is zero. Equipotential lines are parallel to the grounded boundaries containing nerve tissue (see the bottom surfaces of meshes 10–29) because the potential gradient or the direction of current flow is toward the grounded boundary.

The steepest potential gradients occur at the site of the current source (mesh 12), at the bony wall separating the scala tympani from the modiolus (meshes 6–13) and throughout the nerve trunk and the modiolus (meshes 13–27). The large gradients in the bony wall are due to the potential difference maintained between the scala tympani and the nerve tissue of the modiolus by the thin segment of bone of high resistivity. Hence the largest gradients (and the highest current densities in bone) occur at the thinnest boundary between the scala tympani and the modiolus. Note, however, that the absolute current flow across the bony wall is rather small when compared to the current flow inside the scala tympani or nerve tissue because of the difference in their conductivities.

The upper left corner of meshes 5, 6 and 18–36 encompass a broadly distributed region of relatively high equipotential. The size of the equipotential region within each mesh increases with the distance of the mesh from the site of current injection. Furthermore, as the distance from the mesh to the site of current injection increases, the maximum potential within each mesh shifts toward the upper left corner. This distribution of potential is due to the model boundary conditions. For example, if the model domain were completely insulated, a potential source would force all the nodes in the model to assume the source potential. (There is no equivalent scenario for a current source because such a combination of source and boundary conditions would force the model domain potential to infinity.) Therefore, it is not surprising that the well insulated upper left half corner is at an equipotential. (The upper right corner is equally well insulated but since the current source is located in the left half of the model and more of the grounded nerve tissue is located in the right half of the model, the left corner assumes a higher equipotential.)

The current density in a particular element is equal to a product of the potential gradient across the element and the element conductivity. The model predicts less
than a 10:1 ratio in the maximum to minimum potential within the cochlea, and a similar ratio for the complete model if the grounded elements representing the nerve tissue are excluded. For example, within the cochlea, the maximum (source) potential is 1.57 (mesh 12) while the minimum potential is 0.2 (mesh 33, near the round window)†. The ratio of element conductivities in the model is approximately 100:1 (fluid to bone). Thus, because of the larger conductivity ratio, the current density will emphasize the conductivity rather than the potential ratio. The current density contours, if plotted, would assume the shape of the major anatomical structures (e.g., scala tympani, modiolus and nerve trunk) since most of the current flows through the fluid filled spaces, the auditory nerve and other soft tissue.

6.4 The Potential Distribution Along the Length of the Scala Tympani

The comparison of computed results with experimental measurements (Chapter VII) requires the distribution of potential computed by the electro-anatomical model along the scala tympani. Hence, to extract the potentials along the length of the scala tympani (the path of the implanted electrode array), the path of the scala tympani was mapped in the anatomical model. Restricting the analysis to a limited set of locations facilitates the analysis and comparison of the different model solutions. For example, the model solution (over 150 000 node potentials) is represented by 109 data points in the scala tympani.

To map the path of the scala tympani, the centers or midpoints of each scala tympani cross section were estimated and recorded. Because a single section of the temporal bone may cut the cochlear duct several times as it spirals from base to apex, several midpoints were recorded for some model sections. The assembly of all the scalar midpoints has the cumulative effect of unwinding the scala tympani. A potential distribution can now be described as a function of distance from the beginning of the scala tympani (the round window) to its end (the helicotrema.) The potential distribution along the length of the scala tympani as computed by the model is referred to as a trace in this thesis.

Fig. 6.3 shows the sampled version of scala tympani projected on two of the model surfaces. A total of 109 nodes represent the length of the scala. The length of scala tympani, if calculated as the sum of distances between the individual midpoints, is equal to 35 mm. The calculated length is within the range reported in literature for

† All units are arbitrary since the potential data have not been adjusted to actual stimulus intensity.
Fig. 6.3: The projection of the map of scala tympani on two of the model planes. The map is derived by recording the midpoints (n=109) of the scala in each cochlear cross section. Refer to Fig. 5.4 for orientation.
the length of the basilar membrane [Wever, 1949].

Since potential varies within any particular scala tympani cross section, the choice of the section center as the potential to represent the whole cross section may seem rather arbitrary. Large potential gradients will occur near the site of current injection. The spatial variation of the scalar potential, however, decreases quickly with distance away from the site of current injection. For example, as seen in equipotential contour plots (Fig. 6.2), scala tympani potentials for distances 400 µm from the current source (meshes 10 and 14) vary by less than 12% from their local maximum potentials. For other meshes located further from the current source, the scala tympani typically lies in a single equipotential region. In the worst case, the potential variation in these meshes may be as high as 20% (calculated by expressing the contour spacing as a percentage of scalar maximum potential.) However, the maximum potential at these meshes is small with respect to the maximum potential in the model domain, making the potential deviation at the scalar midpoints of these meshes not as significant.

6.5 Modeling of Electrodes

During the electrical stimulation of implanted subjects, the injected current strength is controlled. Thus for a fixed stimulus intensity, a constant current flows between the “source” and “sink” electrode pair. The current is independent of the electrode impedance which may vary across subjects and electrode pairs due to manufacturing differences and electro-anatomical changes. In the numerical model, it is therefore desirable to specify the current intensity at the modeled electrodes. However, only two types of source conditions can be specified at the nodes of the model domain: voltage (V) or volume current density (A/m^3). Neither of these two source conditions corresponds exactly with measurable quantities. For example, the voltage measured at the stimulus electrode pair includes the potential due to the electrode polarization impedance [Schwan, 1963.] What can be calculated is the approximate current density (if one assumes the ball electrodes are perfect spheres) at the site of the electrodes. But the current density is described for a surface (A/m^2) and cannot be directly specified in the three dimensional model. The current density, however, can be computed from the model potential distribution. Methods of adjusting the model results to physical stimulation conditions are described in Chapter VIII of this thesis.

Another aspect of electrode modeling is the geometrical representation of the electrode array. The actual electrode array occupies a significant volume of the scala tympani. In the numerical model, electrodes can be modeled as point current or voltage sources. The potential distribution due to a point source and a charged sphere may be
Fig. 6.4: The effects of electrode representation on the distribution of potential along the length of the scala tympani.

Fig. 6.5: The effects of model resolution on the distribution of potential along the length of the scala tympani.
very different near the active source. Furthermore, the presence of the implant array (the electrodes and insulated leads) could also alter the potential distribution in the cochlea due to the displacement of cochlear fluids.

To compare the effects of electrode geometries, two models with different electrode representations were formulated. The electro-anatomical models consisted of 37, 128 × 128 element meshes. Boundary conditions were representative of monopolar stimulation. The two models were:

- **Point source** model:
  The electrodes were modeled as ideal current sources. Current was injected at a single node in the model.

- **Solid (volume) source** model:
  The electrodes (both active and non-active) were modeled as volumes approximating a sphere containing elements of high conductivity. The implant array leads were modeled with elements of zero conductivity. Current was injected at a single node enclosed within a region of high conductivity, resulting in an equipotential within the region. The equipotential represents the potential at the surface of the "source" electrode.

Note that only current injection models are considered in subsequent analysis. It was found that the solutions of high resolution models (e.g., 32 × 32 or greater mesh resolution) with both current and voltage point sources were equivalent if normalized to a non-source potential. The source potential at the site of current injection ranges with model resolution although it appears to converge to a constant value as the resolution is increased; theoretically, the potential at a point current source is undefined.

The potential traces derived from the **point source** and **volume source** models are shown in Fig. 6.4. The potentials have been normalized to the maximum potential in each trace. The two models show very close agreement for potential distributions along the length of the scala tympani. The average potential difference between the two traces (defined in Chapter IX) is equal to 3.11% (sd = 1.05%, n0 = 109). The gaps in the **volume source** model trace are due to the zero potentials for the nodes inside the (insulated) implant leads.

Hence, because there is no significant difference between the two solutions, the simpler **point source** electrode representation is used for all subsequent models.
6.6 Model Resolution

The effects of changing the model resolution (from $128 \times 128$ to $64 \times 64$ mesh models) are shown in Fig. 6.5. The anatomical data have been downsampled from the high resolution $128 \times 128$ mesh model to the lower resolution $64 \times 64$ model. The two models share boundary and source conditions representing monopolar stimulation. The two potential distributions are very similar. The average potential difference is equal to 6.63% (sd = 6.95%, n = 109). Note that the greatest difference in potential occurs at the basal end. The average percentage difference reduces to 4.54% (sd = 1.83, n = 88) for the final 20 mm of scala tympani. Comparison of all the node potentials (based on the analysis performed in Chapter IX) yields an average difference of 4.3% for the change in resolution described.

Hence, $64 \times 64$ (mesh) models are used for subsequent analysis. The lower resolution models offer the convenience of a faster solution which is well within 10% of the $128 \times 128$ model solution for potentials along the scala tympani.

6.7 Model Electro-Anatomical Composition

The composition of a three dimensional electro-anatomical model is defined by the assignment of electrical resistivity to model elements. In this section, the potential distributions for three models which differ in electro-anatomical composition are compared. The models share boundary and source conditions representing monopolar stimulation. The models are:

- **Homogeneous:**
  
  In this model, all the elements have equal resistivity.

- **Heterogeneous:**
  
  This model has biological material resistivities assigned to the various soft tissue, bones and fluids in the model (Table 5.3).

- **Insulated Tube:**
  
  This model has the same geometry as the heterogeneous model but the ratio of the resistivity of elements representing bone and nerve tissue, and elements representing the cochlear fluids, is very high (5000:1). The potential distribution in such a model should be characteristic of a well insulated tube (with the appropriate boundary conditions.)

The resistivities of elements representing the major cochlear tissues and fluids in the three models are given in Table 6.1. Therefore, the ratio of bone to fluid
<table>
<thead>
<tr>
<th>model type</th>
<th>resistivity (ohms-cm)</th>
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<tr>
<td></td>
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</tr>
<tr>
<td>homogeneous</td>
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<tr>
<td>heterogeneous</td>
<td>50</td>
</tr>
<tr>
<td>insulated tube</td>
<td>1</td>
</tr>
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Table 6.1: The resistivities of the elements representing the major tissues and fluids in the three models.

It is 1:1, 100:1, and 5000:1, in the homogeneous, heterogeneous and tube models, respectively. Informal comparison of solution of an insulated tube model with nerve tissue resistivity equal to 300 ohms-cm with the present insulated tube model (other elements and boundary conditions being equivalent) did not reveal any significant differences between the two models. This result suggests that the high conductivity of the cochlear fluids and the low conductivity of the surrounding temporal bone, as well as the insulated boundary conditions, have the greatest effect on the potential distribution in the scala tympani in insulated tube models.

Other biological tissues in the three models have resistivities in the range 200–400 ohms-cm (Table 5.3). However, since the volume occupied by these tissues with respect to the full model domain is small, the global results (potentials along the scala tympani) of models of monopolar and pseudo-bipolar electrode configurations are not very sensitive to changes in the resistivities of these tissues.

### 6.7.1 Potential and Current Density Traces: Changes in Models

The potential and current density traces for three scala sites (apical, central, basal)† of current injection are shown in Figs. 6.6, 6.7 and 6.8, for the homogeneous, heterogeneous and tube models, respectively. The potentials are extracted from the nodes representing the midpoints of the scala tympani. The current density, however, represents an average of the current density magnitudes (Eq. (5.13)) in the 4 model

† The sites of current injection relate directly to the SYMBION electrode array. Thus the apical site corresponds to electrode 1, the central site corresponds to electrode 3 and the basal site corresponds to electrode 6. The distance between the apical and basal sites is equal to 20 mm. Similarly, the other inter-site distances correspond to the spacing of electrodes in the SYMBION array (4 mm). Electrode 6 is located 2.4 mm from the round window along the length of the scala tympani.
Fig. 6.6: Potential and current density distributions along the length of the scala tympani (ST) for three current injection sites in the homogeneous model.
Fig. 6.7: Potential and current density distributions along the length of the scala tympani (ST) for three current injection sites in the heterogeneous model.
Fig. 6.8: Potential and current density distributions along the length of the scala tympani (ST) for three current injection sites in the insulated tube model.
elements surrounding the scalar midpoint.

The *homogeneous* model predicts the greatest decrease in potential with distance from the site of current injection. The smallest rate of attenuation is predicted by the *tube* model. For example, the typical potential attenuation at a distance 1 mm away from the site of injection is 11.0 dB, 2.5 dB, and 0.5 dB, (all with respect to the maximum or "source potential") in the *homogeneous*, *heterogeneous*, and *tube* models, respectively. The potential attenuates rapidly in the *homogeneous* model because there is no insulation of the cochlea. The current flows almost exclusively along the scala tympani in the *tube* model because of the high bone to fluid resistivity ratio. Toward the apex, which is very well insulated from the “sink” electrode (the grounded nerve tissue located near the base), there is very little change in potential. The scalar current, manifested by the scalar potential gradient, flows toward the base of the scala tympani in the *tube* model.

The distribution of potential in the *heterogeneous* model falls somewhere between the *tube* and *homogeneous* models. For example, the *homogeneous* model predicts very large "bumps" in the potential distribution. The "bumps" are due to the spiraling geometry of the cochlea and the homogeneous nature of the model. The *homogeneous* model predicts that potential will depend in part on the direct (straight-line) distance between the source and target, as opposed to the distance measured along the scala tympani. For example, examination of the basal stimulus trace (Fig. 6.6) reveals a "bump" in the vicinity of 22 mm and a "depression" at 17 mm. Both of these distances are measured along the scala tympani. However, if measured by the straight-line distance, the model node at 22 mm is closer to the "source" than the node at 17 mm, thus creating a "bump" in the potential distribution. The presence of "bumps" suggests that current is flowing between cochlear turns, either across the bony wall separating adjacent turns, or through nerve tissue in the modiolus. If the cochlea is well insulated (i.e., all the current flows longitudinally along the scala tympani), the "bumps" should not exist. In a model of a well insulated cochlea, the potential distribution decreases monotonically with distance (along the scala) away from site of current injection. For example, there are no "bumps" present in the potential trace of the *tube* model.

The potential trace of the *heterogeneous* model exhibits the characteristics of both the *homogeneous* and *tube* models. While the high bone resistivity does provide some insulation, the *heterogeneous* trace is not without "bumps". The "bumps", however, are not as large as predicted by the *homogeneous* model. Some of the "bumps", such as the "bump" at 8 mm during apical stimulation in the *homogeneous* model, do not
appear in the heterogeneous model. The model boundary conditions are also demonstrated in the potential trace of the heterogeneous model. For example, relatively little current flows along the scala tympani toward the apex as manifested by a flat potential distribution or plateau at the apex. As in the tube model, the potential continues to decrease toward the base of scala tympani, while the potential tends to plateau toward the apex. The highest plateau in the homogeneous and tube models is achieved during apical current injection.

While the potentials across the three current injection sites remain constant in the homogeneous and heterogeneous models, the tube model predicts a marked change in the source potential. The source potential in the tube model decreases as current is injected more basally. The decrease is probably due to the fact that the basal electrode is located closer (distance measured along the length of the scala tympani) to the “sink” electrode than either the central or apical electrodes. Apparently, the composition of either the heterogeneous or the homogeneous model does not seem to affect the magnitude of the source potentials. None of the models, however, predict any variation in the magnitude of current density near the stimulus electrodes. The peak current densities are very similar in the heterogeneous and tube models even though these models have a very different fluid to bone resistivity ratio. The current density magnitude is smaller in the homogeneous model due to its uniform conductivity.

The current density traces in general reflect the characteristics observed in the potential traces. There are “bumps” in the homogeneous and heterogeneous current density traces, although the current density at the “bumps” is well attenuated with respect to the peak current densities. The range of the current density is several orders of magnitude greater than the range of potential for the equivalent electrical configuration. These comparisons are best seen in the normalized potential and current density traces.

6.7.2 Potential and Current Density Traces: Changes in Stimulus Site

Figs. 6.9, 6.10, and 6.11 show the potential and current density traces in the three models for basal, central, and apical electrodes, respectively. The data in each trace have been normalized and expressed as attenuation in dB with respect to the maximum potential or current density for each trace. The maxima occur near the “source” electrode, with the exception of the tube model where the current density at the basal end of scala tympani sometimes exceeds the “source” density (but by no more than 1 dB.)

During basal stimulation, the potential trace of the tube model decreases by less than 0.5 dB toward the apex, while the attenuation is equal to 10.0 dB and 20.0 dB for
Fig. 6.9: Potential and current density distributions along the length of the scala tympani (ST) for apical stimulation in the three models. Each trace is normalized by its maximum.
Fig. 6.10: Potential and current density distributions along the length of the scala tympani (ST) for central stimulation in the three models. Each trace is normalized by its maximum.
Fig. 6.11: Potential and current density distributions along the length of the scala tympani (ST) for basal stimulation in the three models. Each trace is normalized by its maximum.
the heterogeneous and homogeneous models, respectively. The relative positions of these apical potential plateaus are similar for central and apical current stimulation. All models predict a decreasing potential at the basal end of scala tympani when current is injected more apically.

The differences in the range between potential and current density can be seen in the normalized plots. For example, the dynamic range for the scalar potential for all configurations is less than 30 dB, while the current density for the equivalent configuration varies by as much as 90 dB. This is as expected since the current density is a function of the potential gradient and should therefore decrease more rapidly with distance from the excitation site. For example, while the potential is attenuated by only 0.5–11.0 dB/mm at the site of current injection, the magnitude of the current density is attenuated by 15.0–38.0 dB/mm (variation due to models and site of current injection.) As expected from the model boundary conditions, the greatest attenuation of current density occurs at the apical end of scala tympani during basal stimulation.

Current flow in the scala tympani from the site of stimulation toward the base is smallest in the homogeneous model and greatest in the tube model. In the tube model, where low resistivity elements (the cochlear fluids) are completely enclosed by high resistivity elements (bone and nerve tissues), current flows to the base along the path of least resistance (the fluids). At the base, the current returns to the "sink" along the shortest path through through elements of higher resistivity. In the homogeneous model, where there is no electrical distinction between model elements, current flow is not restricted to the scalae. Hence, the current density in the path plotted (the scala tympani) will be smaller. The heterogeneous model, which falls between the tube and homogeneous models, predicts a moderate current density spread toward the basal end of scala tympani.

Current flow in the direction of the apex is not as easily generalized. In general, for all stimulus sites, there is a downward trend of the current density from site of current injection toward the apex. This trend becomes more accentuated as the cochlea is better insulated. For apical current injection, the current density for the three models is similarly ranked in magnitude for both the basal and apical directions. For basal and central current injection, current flow is greatest in the heterogeneous model. Near the basal and apical electrode sites, the tube model predicts a higher current density than the homogeneous model. At the apical end of the scala tympani, however, the greatest attenuation of the normalized current density is predicted by the tube model. As described previously, current flow is restricted along the well insulated, low resistivity, cochlear fluid in the tube model. Hence, it's not surprising that
during basal and central current injection, the current density is lower at the apex than at the base of the scala tympani in the tube model. A similar distribution of current density is seen in the heterogeneous trace during basal and central current injection, although the attenuation of current density at the apex is not as high as in the tube model.

While the current density typically peaks at the site of the "source" electrode, there is a corresponding increase in current density near the "sink" electrode. The "sink" electrode is represented in the monopolar models as the grounded nerve tissue region at the bottom plane of the model domain. Hence, the electrical pathways to the "sink" electrode are more significant at the base of the cochlea and thus the current density increases at the base of scala tympani, independent of stimulus site. The greatest increase is predicted by the tube model, where the current density at the base of the scala tympani sometimes exceeds the current density near the source electrode.

6.7.3 Change in Bone Resistivity

Fig. 6.12 shows the potential and current density traces for two heterogeneous models with varying bone resistivity (5000 and 2000 ohms-cm). The rest of the elements in both models are assigned resistivities shown in Table 5.3. Raising the bone resistivity increases the electrical insulation of the cochlea and thus implies a less rapid attenuation of potential away from source of injection. For example, at the stimulus site, the potential gradient is somewhat steeper in the model with lower bone resistivity. Higher bone resistivity also raises the potential in the scala tympani. For example, the peak ("source") potentials are on the average 18% higher in the 5000 ohms-cm (bone) model. The difference between normalized potentials along the length of the scala tympani, however, is not large (typically 1–2 dB). There is virtually no difference in the current densities calculated at the sites of current injection, and except for the 10–15 mm region, little variation in the distribution of current density along the length of scala tympani for the two models.

The differences in the potential and current density distributions are not as significant as one might expect from the ratio of bone resistivity (2.5:1) for the two models. Although the temporal bone acts to insulate the cochlear spiral, the high conductance of the cochlear fluids and the insulation of the model boundaries have a greater influence on the potential distribution along the length of the scala tympani.
Fig. 6.12: Potential and current density distributions along the length of the scala tympani for basal and apical stimulation of the scala tympani in two heterogeneous models with varying bone resistivity. Elements representing the other biological materials are assigned similarly in the two models.
Fig. 6.13: Potential and current density distributions along the length of the scala tympani for basal and apical stimulation of the scala tympani in two heterogeneous models with varying fluid resistivity. Elements representing the other biological materials are assigned similarly in the two models.
6.7.4 Change in Fluid Resistivity

Fig. 6.13 shows the potential and current density traces for two heterogeneous models with varying fluid resistivity (50 and 100 ohms-cm). The rest of the elements in both models are assigned resistivities shown in Table 5.3. The model with the higher fluid resistivity predicts a higher potential along the length of the scala tympani. The difference in potential is greatest at the sites of current injection. For example, the peak potentials are on the average 37% higher in the 100 ohms-cm (fluid) model. The differences in the potential beyond the site of current injection are not as significant for the two models.

Raising the fluid resistivity increases the resistance of the major current pathway (the fluid filled cochlea). The 100 ohms-cm (fluid) model predicts a higher attenuation of potential (re source potential) in the vicinity (+/- 2 mm) of the site of current injection. Hence, for normalized data, the potentials predicted by 50 ohms-cm (fluid) model exceed the potentials predicted by the 100 ohms-cm (fluid) model. The two models predict a similar current density distribution with the exception of the 10–15 mm region noted in the bone resistivity analysis.

6.7.5 Pseudo-Bipolar Stimulation Models

The potential and current density distributions for two pseudo-bipolar stimulus configurations are shown in Fig. 6.14. The pseudo-bipolar models are similar in composition to the monopolar heterogeneous model with element resistivities assigned as in Table 5.3. In these models, the current “source” and “sink” electrodes are located in the scala tympani. Both electrodes are modeled as point sources: current is injected into the “source” node while the “sink” node is kept grounded (zero potential.) Two stimulus configurations are modeled: current passed between the most apical and most basal electrodes (stimulus 1-6), and current passed between two centrally located electrodes (stimulus 3-4).

The potential trace is biphasic, with a peak in potential near the “source” electrode and a depression near the “sink” electrode. There is relatively little change in potential beyond the “source” or “sink” electrodes. Most of the current, therefore, flows in the region between the two electrodes. The current density is equal in magnitude but opposite in direction at the source and sink electrodes. As the distance between the “source” and “sink” electrodes is decreased, the electric field produced by such a configuration should approach that of a dipole source. The dipole field is characterized by a much more rapid attenuation of potential and current density with distance from site of stimulation. However, the attenuation rates near the electrode
Fig. 6.14: Potential and current density distributions along the length of the scala tympani for two pseudo-bipolar electrode configurations in a heterogeneous model.
sites are not significantly greater in the pseudo-bipolar models than predicted by the monopolar models — perhaps because the spacing between the stimulus pair is relatively large (equal to 4 mm for stimulus 3-4). For example, the pseudo-bipolar models predict an attenuation of 2.0 and 1.2 dB/mm for potential, and 22.0 dB/mm and 26.0 dB/mm for current density magnitude, during stimulus of electrodes 3-4 and 1-6, respectively (calculations for distances 1 mm from "source" electrodes.)

6.8 Model Results: Discussion

In this section, results from several types of models have been presented. The majority of the results have been taken from $64 \times 64$ mesh models based on the small average percentage difference (less than 5%) in potential between a typical $64 \times 64$ and a $128 \times 128$ model. Furthermore, no significant difference was found in representing the electrodes as point sources or as volume sources represented by a number of model elements. It is not clear, however, whether this result is a model artifact or is a function of the heterogeneous structure of the implanted cochlea.

Even in the highest resolution model, the electrode representation is rather coarse due to the dimensions of the model elements (75 µm by 45 µm by 200 µm). Because the ball electrodes have a radius of approximately 250 µm, each electrode can at best be represented as a sphere enclosed in a region of $8 \times 11 \times 3$ model elements. Furthermore, the finite difference formulation has the effect of smoothing out the variations in biological and electrode material resistivities. Therefore, the relatively high resistivity of the electrode array leads or the low resistance of the metal electrodes may not significantly alter the potential distribution in the scala tympani if the electrode array can only be represented by a small number of elements.

On the other hand, the electrode result could be explained by the electrical properties of the modeled system. The distribution of potential along the length of the scala tympani may indeed be unaltered by the presence of the electrode array. The low resistivity of the cochlear fluids and the insulation of the cochlea may contribute to the result obtained. In other words, during monopolar stimulation the potential spread may be so great in the scala tympani as to not be affected by the presence of the electrode array. Further analysis of implications of electrode representation is required using higher resolution models.

The potential distribution in the model domain is largely a function of the model boundary conditions. Because most of the model boundaries are insulated, there is relatively little variation of potential with distance away from the site of current injection. This is especially true near the insulated boundaries. The largest potential gradients
occur near the site of current injection (in the scala tympani), across the thin portions of the bony wall (scala tympani – modiolus), in the modiolus and near the grounded portion of the nerve trunk toward the bottom surface of the model. Because of the small variation in potential, however, current densities are primarily functions of the material conductivity and not the potential gradient. Thus in a heterogeneous model, most of the current flows in the cochlear fluids, followed by nerve and bone tissue (in order of decreasing conductivity.)

The heterogeneous model, a first order approximation of the anatomical and electrical properties of the implanted cochlea, falls somewhere between the homogeneous and tube models. The homogeneous model, in which there is no electrical distinction between the various biological materials, predicts "narrow" potential and current density distributions with "bumps" showing up in the potential and current density traces. The "bumps" occur because both potential and current are functions of the straight-line distance between "source" and "target" nodes (direct distance) in the "homogeneous model". The potential and current density traces, however, are plotted as a function of distance measured along the length of the scala tympani (scalar distance.) The scalar distance often exceeds the direct distance because of the spiraling, three dimensional geometry of the cochlea. The "bumps" in the heterogeneous model, however, are not as great as in the homogeneous model due to the insulation provided the model elements enclosing the cochlea. As predicted by the tube model, higher insulation (as characterized by the ratio of the bone/nerve to fluid resistivities) implies increased current flow in the scala tympani. The are no "bumps" in the tube model because virtually all the current flows longitudinally in the cochlea. However, the "bumps" in the potential and current density traces of the heterogeneous model suggest the presence of indirect current paths linking the different cochlear turns.

The model boundary conditions are reflected the potential and current density traces of the heterogeneous model. In the basal regions of the scala tympani, the current density is equal to or greater than the current density in the apical regions of the scala tympani. Current densities increase at the base of the scala due to the proximity of the grounded nerve tissue (representing the "sink" electrode) to the base of the cochlea. In contrast, the current density at the apex is attenuated because of the electrical insulation of the apex. The magnitude of the current density attenuates rapidly with distance from the current injection site. For example, the current density magnitude in the scala tympani decreases by at least 20 dB at distances of 5 mm or greater, for all stimulus sites.
The increase in the current density magnitude† at the basal end of the scala tympani is due to the grounding of the nerve tissue at the bottom surface of the model domain. This assignment of boundary conditions is made necessary by the finite size of the model domain. Therefore, the increase in the magnitude of the current density predicted by the model may not occur in the implanted cochlea. For example, the resistance of the current pathway to the “sink” electrode may be under-represented by the model, thus creating steeper potential gradients at the base. Nevertheless, the monopolar boundary conditions force a significant portion of the current to flow toward the base, even during more apical stimulation.

It is interesting to compare the basal spread of current with an analogous spread of energy during the acoustic excitation in the normal ear. Electrical excitation at the apex may result in current spread toward the base, resulting in a stimulus intended to excite low frequency fibers reaching nerve tissue with high characteristic frequency. Similar spread of energy occurs during acoustic stimulation. Tuning curves (frequency threshold curves) for nerve units of all characteristic frequencies have very sharp high frequency slopes, while tuning curves for high frequency fibers often exhibit low frequency tails [Kiang and Moxon, 1974.] Thus it is unlikely that a nerve unit will be excited by a tone above its characteristic frequency but the same unit may respond to a tone below its characteristic frequency. The implications for cochlear implants, especially for devices with extended current spread, are obvious. For example, apical stimuli may interact more with basal stimuli during multiple electrode stimulation than in the reverse case. However, it remains to be seen if the predicted asymmetries in the scala tympani potential and current density distributions will be manifested in the implanted subject’s evoked responses. The hypothesis may be tested, for example, by psychophysical experiments that measure interaction between simultaneously stimulated electrodes.

† Note that only the magnitude and not the direction of the current density vector is considered here. However, if the stimulus waveform is biphasic, the current density will reverse in polarity.
Chapter VII

Potential Measurements During Electrical Stimulation

7.1 Introduction

To test the theoretical predictions of the three dimensional, electro-anatomical model developed in this thesis and to better characterize the electrical properties of the implanted cochlea, a series of electrical measurements was performed in five subjects. In the experiments, a sinusoidal stimulus was presented to one pair of implanted electrodes and potential differences for all the other combinations of electrodes were measured. Such measurements can be directly compared with the predictions of the model developed in this thesis as well as with other computed results. In this chapter, the results of the potential measurements in five subjects are presented. In the next chapter, the experimental data are compared with computed results from the three dimensional, electro-anatomical models and the hypothetical electrical models discussed previously.

7.2 Experiment Description

The five subjects participating in the experiments were implanted with the SYMBION electrode array at the Massachusetts Eye and Ear Infirmary (Boston, MA.) The SYMBION array, first described in Chapter II, consists of eight platinum ball electrodes, six of which are implanted in the scala tympani. One electrode is placed at the promontory, and another in the temporalis muscle and serves as the return electrode during monopolar stimulation. An additional electrode was placed on the subject’s ipsilateral arm to serve as an additional far field recording site. A constant current stimulus was presented to one pair of electrodes while the potential differences between all the remaining electrode pairs were recorded (Fig. 7.1). The potential measurements did not include the stimulating electrodes since the effects of the electrode polarization impedance [Schwan, 1963] would make such recordings difficult to interpret.

The hardware setup for the potential measurements is shown in Fig. 7.2. The 1270 Hz (typical) a.c. stimulus was generated by a function generator (LFG-1300S, Leader Electronics Corp., Plainview, NY). The generator output was then passed through an optical isolation stage, voltage-to-current converter, and finally, a capacitor that assured a.c.-coupling with the electrodes. The stimulus current strength was set to 20 $\mu$A RMS. At this strength, the 1270 Hz stimulus was inaudible for all subjects.
Fig. 7.1: Schematic diagram illustrating the measurement of intracochlear potential. The SYMBION electrode array is shown as implanted in an unwound cochlea (represented by the trapezoid). Electrode 6 is located nearest the base of the scala tympani, while electrode 1 is located most apically. The experiments consist of presenting a controlled current stimulus to one pair of electrodes while simultaneously recording the potential differences between all the remaining pairs of electrodes.

Fig. 7.2: A diagram of the stimulation and recording hardware for measuring the intracochlear potentials. The stimulus current was monitored across a 1 kohm resistor. See text for description of equipment.
On the recording side, the signal from the recording pair of electrodes was first presented to a programmable gain instrumentation amplifier (AD524C, Analog Devices Inc., Norwood, MA). The amplified signal was then passed through a high-Q passive bandpass filter (Model 2-B, Allison Laboratories, La Puente, CA). The filter was tuned to the stimulus frequency to eliminate line frequency and other noise or interference signals. Finally, the potential difference (RMS) was measured using a digital voltmeter (Model 8060, John Fluke Mfg. Co., Inc, Everett, WA). The phase of the potential (recording) waveform was measured relative to the current (stimulus) waveform on an oscilloscope (Model 7633 with 7A22 differential amplifiers and 7B53A time base, Tektronix, Inc., Beaverton, OR.) The complete system was calibrated for both gain and phase measurements.

7.3 Data Description

The first set of measurements were conducted to determine if the cochlea/electrode system behaved linearly for a wide range of input current strengths. Fig. 7.3 is a plot of the magnitude of the potential difference as a function of input current. Since the potential is proportional to the current strength (r = .99998), the system behaves linearly for the range of currents presented. Thus all potential data presented in this chapter can be normalized to the input current strength.

The measurements that follow were recorded as a data matrix, as shown in Fig. 7.4. The columns of the matrix represent the "positive" recording electrode, while the rows represent the "negative" electrode. (Stimulus configurations are labeled as "positive"—"negative". For example, during monopolar stimulation, the "positive" stimulus was always presented to the intracochlear or "source" electrode, while the "negative" stimulus was presented to the return or "sink" electrode. See Fig. 7.2 for reference.) Each entry represents the potential difference (in mV RMS) for a 20 μA RMS stimulus. For a particular electrode configuration, entries in either the row or column representing the stimulus electrodes were left blank. Also, the potential difference for diagonal entries was not recorded but set to zero after ascertaining that it was equal to the noise floor (less than 0.5 mV RMS) of the system. For most of the measurements, only one half of the data matrix was recorded because in a linear, passive and bilateral system, symmetrical off-diagonal entries should be equal in magnitude but 180 degrees out of phase. This hypothesis was tested for two data matrices and could not be rejected even at the 0.6 significance level.
Fig. 7.3: A plot of stimulus current vs intracochlear potential difference. The current was injected into electrode pair 1–8; potential was measured between electrodes 2–6. Stimulus frequency was 1270 Hz; all electrical units are in RMS. The plot suggests a linear relationship between the stimulus current and the measured potential ($r = 0.99998$).
Fig. 7.4: The data matrix used for intracochlear measurements. For each stimulus configuration, the recorded potential differences were entered in such a matrix. The matrix shown represents an experiment where electrode pair 1–G (1–8) is stimulated since entries in rows (and columns) 1 and G are not recorded. Each entry represents the potential difference between the source (+) and sink (−) electrodes.

![Data Matrix](image)

<table>
<thead>
<tr>
<th>Source (+) Electrode</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
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<td>V_{53}</td>
<td>V_{63}</td>
<td>V_{73}</td>
<td>V_{83}</td>
<td>V_{93}</td>
</tr>
<tr>
<td>3</td>
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<td>V_{34}</td>
<td>V_{44}</td>
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<td>V_{75}</td>
<td>V_{85}</td>
<td>V_{95}</td>
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<td>V_{36}</td>
<td>V_{46}</td>
<td>V_{56}</td>
<td>V_{66}</td>
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<td>V_{96}</td>
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<td>V_{3P}</td>
<td>V_{4P}</td>
<td>V_{5P}</td>
<td>V_{6P}</td>
<td>V_{7P}</td>
<td>V_{8P}</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>V_{2A}</td>
<td>V_{3A}</td>
<td>V_{4A}</td>
<td>V_{5A}</td>
<td>V_{6A}</td>
<td>V_{7A}</td>
<td>V_{8A}</td>
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<tr>
<td>8</td>
<td>A</td>
<td>V_{2A}</td>
<td>V_{3A}</td>
<td>V_{4A}</td>
<td>V_{5A}</td>
<td>V_{6A}</td>
<td>V_{7A}</td>
<td>V_{8A}</td>
</tr>
</tbody>
</table>

Fig. 7.5: Family of curves obtained by plotting the intracochlear measurement potential matrix. The family of curves is symmetrical about the zero potential line (indicated as dashed) due to duplication of lower-half matrix entries in the upper-half of the matrix. See text for details.

![Potential Curves](image)
Absolute phase measurements were performed in several trials and the phase of the potential never exceeded 16°.† This is consistent with results reported [Spelman et al., 1983] in the monkey cochlea and with data for specific biological materials [Schwan, 1963]. For most of the experiments, the phase of the potential difference was simply recorded as being in or out of phase with respect to the current waveform. For example, if the potential measurement lagged the current stimulus waveform by less than 20°, the phase was recorded as in. If the lag was as great as 160–200°, the phase was recorded as out. The assumption underlying these phase assignments is that during electrical stimulation of scala tympani electrodes (with respect to a temporalis muscle electrode, for monopolar stimulation), most of the current flows through pathways that are primarily resistive. Capacitive effects, therefore, can be neglected. This is not surprising, since most of the current flows in the cochlear fluids and the nerve tissue. Bulk measurements suggest that the fluids and tissues are primarily resistive [Geddes and Baker, 1967]

Further support for the system being largely resistive was provided by measuring the potential for stimuli of different frequency. In a resistive system, potential measurements should be independent of frequency. Measurements were performed for one electrical stimulus configuration (1-8) in one subject (MLP) using stimuli of three different frequencies: 161 Hz, 1270 Hz and 4800 Hz. The potentials were first divided by the stimulus current strength to normalize the data. (The current strength was adjusted for each stimulus frequency such that the stimulus remained inaudible to the subject.) Then, the normalized potential measurements at equivalent electrodes but measured at different frequencies were used to calculate changes in potential with frequency. The results are shown in Table 6.1. With the exception of 6 measures that included electrode 2, the potential changes by less than 1 dB at 1270 and 4800 Hz with respect to 161 Hz. For all data, there is less than 0.5 dB difference between 1270 and 4800 Hz. The large deviation occurs for measurements including electrode 2 at 161 Hz.

Another test of the resistive assumption was to examine the matrix data for element additivity. In a stationary and linear system, the potential measurement between any two points is independent of the path by which it is measured. For example,

† In one trial, a larger (30°) phase shift was measured. However, during this trial, the hardware was functioning erratically and the stimulus switching box was found to be at fault.
Table 7.1: Frequency response at 161 Hz, 1270 Hz and 4800 Hz for stimulus 1-8. A total of 21 potential measurements were performed at each frequency at electrodes 2 through 7. These measurements were normalized by the respective stimulus current strengths before the attenuation statistics were calculated.

The potential \( \Phi_{a,c} \) can be expressed as the sum \( \Phi_{a,b} + \Phi_{b,c} \). Furthermore, in a resistive system, the phase of all the measures is equal to zero and the summation process can be performed using only the magnitude and sign of the potential. This hypothesis was tested using analysis of variance of the data matrices. Because only one half of a data matrix was typically recorded, the matrices were “filled” by duplicating each measurement. Symmetrical off-diagonal matrix entries were set to be equal in magnitude but opposite in polarity. The variance of the matrix rows and columns accounted for over 99% of the total variance in all subjects (25 data matrices). Such a result, together with the phase measurements, is consistent with the hypothesis that the significant electrical pathways during intracochlear stimulation are primarily resistive.
not plotted. The vertical distance between any two curves is equal to the potential difference at the column electrodes. If these measurements had been made in a noiseless system, the curves would be "parallel" in the sense that the distance between all the points for any pair of curves would be constant.

A representative curve for any particular stimulus pair of electrodes can be derived by averaging the family of curves shown in Fig. 7.5. This is equivalent to finding the column average of the data matrix and plotting it as a function of the row (the row average of the data matrix is equal in magnitude but opposite in sign to the column average.) The matrix average, or the average of all the matrix entries, is equal to zero. This is because of the symmetrical nature of the matrix and the duplication of entries in the lower and upper half-matrices. Therefore, the average curve represents an average potential distribution taken from all the measurements. Again, if the measurements were noiseless, the curves would all have identical shapes but with varying "d.c." offsets. The averaging eliminates the "d.c." offset present in each measurement reference point, producing a characteristic "a.c." (or zero mean) potential distribution†.

7.4 Display of Data and Calculation of Attenuation Statistics

7.4.1 Data Plots

For each electrode configuration, the characteristic potential distributions for all subjects are plotted in two ways: as raw (averaged) and normalized curves. The raw data have been scaled to correspond to a 20 μA stimulus current strength; otherwise, the measurements have not been altered in any fashion. All potential differences are expressed in mV RMS.

The normalization procedure for monopolar measurements is described by:

$$\Phi_i^n = \frac{\Phi_i - \Phi_{\text{min}}}{\Phi_{\text{max}} - \Phi_{\text{min}}} \times 100.0$$  \hspace{1cm} (7.1)

where $\Phi_i$ is the potential at electrode $i$, and $\Phi_{\text{min}}$ and $\Phi_{\text{max}}$ are the minimum and maximum potentials for the curve, respectively. During monopolar stimulation, the minimum potential always occurred at the arm electrode (except for subject MTH; see

† The characteristic potential distribution for a particular pair of stimulating electrodes is referred to as a characteristic potential curve, or just curve.
below) reflecting the electrical proximity of the arm and return electrodes. For all configurations, the subject curves coincide at the sites of minimum (0%) and maximum (100%) potential.

The normalization procedure is somewhat different for the pseudo-bipolar measurements. To be consistent with the monopolar calculations, for the attenuation statistics near the site of the "sink" or "negative" electrode, a different offset is used in the normalization process. Thus Eq. (7.1) becomes:

\[ \Phi_i^n = \frac{\Phi_i - \Phi_{\text{max}}}{\Phi_{\text{min}} - \Phi_{\text{max}}} \times 100.0 , \]  

The pseudo-bipolar data are normalized using either Eq. (7.1) or Eq. (7.2), depending on the location of the recording electrode. If the recording electrode is closer to the "sink" electrode, Eq. (7.2) is used instead of Eq. (7.1). For example, the normalized potentials for the 3-4 stimulus configuration are calculated using Eq. (7.1) for electrodes 1 and 2, and using Eq. (7.2) for electrodes 5 and 6.

**7.4.2 Attenuation Statistics**

Two interrelated statistics are calculated for each characteristic potential curve. For adjacent pairs of electrodes, the potential attenuation in dB is calculated as:

\[ K_{i, i+1} = 20 \log \left( \frac{\Phi_i^n}{\Phi_{i+1}^n} \right) \]  

where \( \Phi_i^n \) is the normalized potential at electrode \( i \), using Eq. (7.1) or Eq. (7.2). (Actually, the denominator of Eqs. (7.1) and (7.2) has no effect on the attenuation statistics.) The inter-electrode length constant in mm is calculated using transmission line theory as:

\[ \lambda_{i, i+1} = -\frac{\Delta x}{\ln \frac{\Phi_i^n}{\Phi_{i+1}^n}} , \]  

where \( \Delta x \) is equal to the distance between electrodes (4 mm). Note that

\[ \lambda_{i, i+1} = \frac{20 \Delta x}{\ln (10) K_{i, i+1}} . \]  

Inter-electrode attenuation and length constants are shown in Tables 7.2 and 7.3,
respectively. Note that statistics for sites near the "sink" electrode during pseudobipolar stimulation have been calculated using the normalization procedure described in Eq. (7.2).

<table>
<thead>
<tr>
<th>Electrode Pair</th>
<th>JFA</th>
<th>RLB</th>
<th>MTH</th>
<th>TWM</th>
<th>MLP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stimulus: 1-8</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-3</td>
<td>-1.86</td>
<td>-1.48</td>
<td>-0.92</td>
<td>-4.63</td>
<td>-2.50</td>
</tr>
<tr>
<td>3-4</td>
<td>-1.01</td>
<td>-1.84</td>
<td>-0.95</td>
<td>-2.47</td>
<td>-1.32</td>
</tr>
<tr>
<td>4-5</td>
<td>-2.05</td>
<td>-1.34</td>
<td>-</td>
<td>-2.74</td>
<td>-0.93</td>
</tr>
<tr>
<td>5-6</td>
<td>-2.71</td>
<td>-1.38</td>
<td>-</td>
<td>-3.55</td>
<td>-3.77</td>
</tr>
<tr>
<td><strong>Stimulus: 3-8</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>0.35</td>
<td>0.36</td>
<td>1.00</td>
<td>2.21</td>
<td>0.48</td>
</tr>
<tr>
<td>4-5</td>
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<td>-</td>
<td>-4.94</td>
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</tr>
<tr>
<td>5-6</td>
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<td>-1.82</td>
<td>-</td>
<td>-4.94</td>
<td>-4.35</td>
</tr>
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<td>-0.01</td>
<td>0.80</td>
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</tr>
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<tr>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td>-1.40</td>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
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<td>-1.45</td>
<td>-0.85</td>
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<td>-3.46</td>
<td>-3.65</td>
<td>-4.23</td>
</tr>
</tbody>
</table>

Table 7.2: Inter-electrode attenuation in dB. The potentials have been offset by the potential of the electrode nearest (electrically) to the "sink" (or "source", where indicated *) electrode for each stimulus configuration. MTH monopolar data are offset by the potential of electrode 5 due to the apparent short between electrodes 5 and 8. Statistics including electrode 5, therefore, cannot be computed. See text for details.
Fig. 7.6: Raw (upper panel) and normalized (lower panel) characteristic potential curves for stimulation of electrode pair 1–8. (8 is the return electrode.) The data in this figure have been normalized using Eq. (7.1). Note that for subject MTH, the minimum potential occurs at electrode 5, which is not plotted in the normalized panel for clarity. See text for details.
Fig. 7.7: Raw (upper panel) and normalized (lower panel) characteristic potential curves for stimulation of electrode pair 6–8. (8 is the return electrode.) The data in this figure have been normalized using Eq. (7.1). Note that for subject MTH, the minimum potential occurs at electrode 5, which is not plotted in the normalized panel for clarity. MTH data are normalized with respect to the promontory electrode potential. See text for details.
Fig. 7.8: Raw (upper panel) and normalized (lower panel) characteristic potential curves for stimulation of electrode pair 3–8. (8 is the return electrode.) The data in this figure have been normalized using Eq. (7.1). Note that for subject MTH, the minimum potential occurs at electrode 5, which is not plotted in the normalized panel for clarity. See text for details.
Table 7.3: Inter-electrode length constants in mm. The potentials have been offset by the potential of the electrode nearest (electrically) to the “sink” (or “source”, where indicated *) electrode for each stimulus configuration. MTH monopolar data are offset by the potential of electrode 5 due to the apparent short between electrodes 5 and 8. Statistics including electrode 5, therefore, cannot be computed. See text for details.

7.5 Characteristic Potential Curves: Monopolar Stimulation

For current injection at the most apical electrode (configuration 1-8, Fig. 7.6), the potential decreases monotonically from apex to base for all subjects. The notable exception is subject MTH whose electrode 5 was found to be shorted to electrode 8 during the monopolar measuring session. (Electrode 5 behaved similar to other subjects electrodes during the pseudo-bipolar measures, which were performed a month earlier. Thus it is likely that a short circuit between electrodes 5 and 8 developed
between these two sessions. Such a change in electrode impedance has not been observed in any other subject.) The potential curves are similar in their general shape and range of values. The raw data agree especially well for electrodes 3 through 6. However, the inter-electrode potentials do not behave in any systematic fashion across subjects. For example, for subject MLP, the potential attenuation changes from 0.93 dB at electrode pair 4-5 to 3.77 at electrode pair 5-6 (Table 7.2). The inter-electrode attenuation is more uniform for the other subjects, most notably RLB. The normalized plots (Fig. 7.6) show the best agreement between the curves for subjects JFA, RLB and MLP. The offset in subject TWMS's curve is due to the large raw potential at electrode 2. The length constants for electrode pair 2-3 range from 7.5 to 23.5 mm (Table 7.3), suggesting an extended potential distribution during monopolar stimulation.

The potential distribution for current injected at the base of the cochlea (configuration 6-8, Fig. 7.7) is very different from the distribution when current is injected at the apex. The greatest change in potential occurs at the electrode pair situated nearest to the site of current injection (5-4). With respect to electrode 5, the attenuation at electrode 4 ranges from 3.9 to 4.7 dB, or about 60% of the electrode 5 potential. The variation in potential between electrodes 4 and 1 is less than 10% in subjects JFA, RLB, MLP and MTH, and less than 20% in subject TWMS. Thus, judging from the potential gradient, most current flows toward the base of the cochlea, while a relatively small amount flows toward the apex.

An interesting phenomena occurs for some subjects at the apical (1, 2, 3) electrodes. For subjects RLB and JFA, the potential at electrode 1 is greater than the potential at electrode 2 or 3. For subject MLP, the potential at electrode 2 is less than at electrode 1 or 3. Subjects TWMS's and MTH's electrode 2 potentials are greater than the potentials at electrodes 1 and 3. Although the the deviations are small (typically, less than 0.5 dB or about 2% of normalized maximum potential,) they can be observed in all the individual subject measurements (i.e., this is not just a result of averaging the potential differences.) This non-monotonic behavior, first described in Chapter VI as "bumps" in the potential and current density distributions, suggests that a portion of the current is flowing outside the scala tympani. For example, the current may be passing directly across the modiolus, the bony wall or through some other indirect pathway from the "source" to "target" electrodes.

The potential curves for the final monopolar configuration (3-8, Fig. 7.8) combines aspects of the two previous curves. By injecting current in the middle of the cochlear duct, the potential at both the basal and apical ends of the scala tympani can
be measured. Judging from the previous measurements, it is not surprising that changes between the more apical pair of electrodes (1-2) are smaller than changes between the basal pairs (4-5 and 5-6). Thus based on the potential gradient, more current appears to be flowing toward the base of the scala tympani than toward the apex. Near the "source" electrode (3), however, the current appears to be iso-directional based on the projected gradients for electrode pairs 3-2 and 3-4. For all subjects, potentials at electrodes 2 and 4 vary by less than 13% (normalized data.)

7.6 Characteristic Potential Curves: Pseudo-Bipolar Stimulation

The characteristic potential curves for two pseudo-bipolar stimulus configurations are shown in Figs. 7.9 and 7.10. All the measures were performed as previously described, with the exception of subject MTH. The stimulus waveform during the MTH measurement was a biphasic square wave. The potential recording (peak-to-peak) was performed manually using an oscilloscope. The potentials have been scaled to a 20 μA source strength to correspond with the other subjects.

Current injected across the length of the cochlea (configuration 1-6, Fig. 7.9) has a potential distribution similar to current injected at the apex (configuration 1-8). The attenuation statistics are very similar to those obtained for apical (1-8) and basal (6-8) monopolar stimuli (Tables 6.2 and 6.3). For example, the attenuation statistics near the current "source" (electrode pair 2-3) are very much like the statistics for the same pair of electrodes during stimulus 1-8. Similarly, statistics near the current "sink" (electrode pair 4-5) correspond well with same site statistics during stimulus 6-8.

Finally, the last bipolar configuration introduces current in the middle of the cochlear spiral (electrodes 3-4, Fig. 7.10). As in the monopolar mid-cochlear stimulus (3-8), there is a larger potential difference between the basal pair (5-6) than the apical pair (1-2) of electrodes. It is therefore not surprising that the attenuation statistics for stimulus 3-4 correspond with those for stimulus 3-8. A notable difference in this pseudo-bipolar configuration is the smaller range of potentials when compared with other electrode configurations (e.g., -7 to +7 vs -16 to +18 mV RMS). For all stimulus configurations, the peak potential occurs near the "source" electrode while the minimum occurs near the "sink" electrode. When the stimulus electrodes are particularly close together, as in the case of configuration 3-4, it is not surprising that a low range of potentials can be measured on the fringes of the electrode array (i.e., electrodes 1, 2 and 5, 6.) During the stimulation of electrode pair 3-4, most of the current flows in the region between the two active electrodes. Hence, the peak potentials occur in the same region.
**Fig. 7.9:** Raw (upper panel) and normalized (lower panel) characteristic potential curves for stimulation of electrode pair 1–6. The data in this figure have been normalized using Eq. (7.1). See text for details.
Fig. 7.10: Raw (upper panel) and normalized (lower panel) characteristic potential curves for stimulation of electrode pair 3–4. The data in this figure have been normalized using Eq. (7.1) for electrodes 1 and 2 and using Eq. (7.2) for electrodes 5 and 6. See text for details.
7.7 Characteristic Potential Curves: Discussion

In general, the characteristic potential curves for a particular stimulus configuration agree qualitatively (i.e., in the shape of the curves and in local trends, such as potential plateaus) across subjects. It is very difficult to find a means of normalizing these curves. Due to the nature of the SYMBION array, which has only six monopolar electrodes, it impossible to ascertain the exact potential at the site of current injection. The potentials cannot be measured with reference to either of the stimulating electrodes without introducing potentials due to electrode polarization impedances — although the arm electrode, as seen in both measurements, can be used as the return reference. However, while there are deviations between individual electrode potentials, the agreement in the shape of the curves and in the attenuation statistics supports the notion of similarities in cochlear anatomy and site of implantation for the five subjects.

For most stimulus configurations, there is a monotonic decrease of potential magnitude away from the site of current injection. The notable exception occurs in the potential distribution for current injected at the base (6-8). The (6-8) curves display non-monotonic behavior at the apical end of the scala tympani. Such potential "bumps", although small in magnitude, suggest the presence of current pathways outside the scalar fluids. The "bumps" can be explained by considering the difference between direct and scalar distance† in the cochlea. For example, consider a scenario in the implanted cochlea where the direct distance between electrode 6 (most basal) and electrode 1 (most apical) is less than the direct distance between electrodes 6 and 2. The scalar distance between electrodes 6 and 1, however, is greater than between electrodes 6 and 2. Thus if the current flow is contained along the cochlear canals, the potential distribution along the scala tympani should monotonically decrease as one travels away from the "source" of current injection. Hence, the potential at electrode 2 should be greater than at electrode 1 for stimulation of electrodes 6-8 in the "well insulated" cochlea. As the resistivity of the materials bounding the scala tympani decreases, the potential becomes more a function of the direct distance between "source" and "target" electrodes. Hence, the potential at electrode 1 may exceed the

† Direct distance refers to the straight-line distance between two measurement points; Scalar distance refers to distance measured along the length of the scala tympani. The scalar distance is fixed by the spacing of the electrode array.
potential at electrode 2. The measured potential distribution along the length of the scala tympani is a function of both the direct and scalar distances. Furthermore, when current is injected at the base of the cochlea, the current flow outside the scala tympani is accentuated because there is little current flowing in the apical region in the first place.

It is clear from the measurements that the implanted cochlea cannot be simply viewed as a uniform transmission line. For example, comparing the potential distributions for stimulation at the basal (6-8) and apical (1-8) ends reveals some of the special boundary conditions in the cochlea. The measured potential distribution is consistent with the view that the grounding pathway is located primarily at the base of the cochlea, while the apex remains well insulated. In such a scenario, most current flow will be directed toward the base of the cochlea. A similar potential distribution was reported by Sitko [1976] in his guinea pig measurements.

The extent of potential spread is characterized by the length constant, a measure of distance at which the potential attenuates to roughly 38% of the reference potential. The computed monopolar length constants for electrode pairs nearest the ‘‘source’’ electrode range from 7 to 25 mm.†† During basal (6-8) stimulation, the average length constant is equal to 8.13 mm (sd = 0.6, n = 4), close to the range (8–16 mm) computed by Black and Clark [1980] based on electric threshold data for neurons in the central nucleus of the inferior colliculus. Care must be taken, however, in interpreting the length constant and attenuation figures. Because a potential reference is not available, the raw potential data (which are bipolar and have zero mean) have been offset by the potential of the electrode closest to the current ‘‘sink’’ (or ‘‘source’’ for certain pseudo-bipolar calculations). The attenuation calculations are very sensitive to this offset. The offset used in the monopolar calculations, the arm electrode, is probably a reliable reference for the ‘‘sink’’ electrode. The reference is harder to set for the pseudo-bipolar calculations since no similarly situated electrode exists for such electrode configurations. The choice of an extreme (maximum or minimum) potential represents the best estimate one can make for the ‘‘source’’ or ‘‘sink’’ potential. If the offset is increased in magnitude, the attenuation decreases and the length constants increases. Thus it is likely that the pseudo-bipolar length constants (Table 7.3) are too low and attenuation figures (Table 6.3) too high since the magnitudes of the ‘‘sink’’

†† Except for the length constants for electrode pair 1–2 during stimulation of 3–4. These curves have a clearly asymmetric potential distribution, resulting in large length constants toward the apex. The length constants near the site of current injection cannot be established due to lack of resolution in the measurements.
(or "source") potentials probably exceed those of their neighboring electrodes.

A further inadequacy of the attenuation calculations has to do with the resolution of the SYMBION electrode array. For example, the potentials can only be measured at distances 4 mm apart (or greater), and for at most five intracochlear sites. Furthermore, the length constants represent the electrical characteristics at least 6 mm (between the closest non-active electrode pair) away from the site of current injection. At this distance, as well as for distances between measurements as large as the electrode spacing, the transmission line analogy (for which the length constant is a characteristic measure) may not hold since cochlear geometry and electrical properties vary for these lengths. The potential gradients at the active electrodes most likely exceed the gradients predicted by projecting the experimental data. These gradients cannot be measured with the SYMBION electrode array.

In spite of the coarse nature of these measurements, the experiments demonstrate that there is an extended spread of potential during monopolar stimulation and pseudo-bipolar stimulation. The connection between potential and current spread cannot be made directly because of the coarse nature of these measurements. A large spread in potential may not necessarily imply an equally extended current spread. Thus while the measurements are useful in predicting global trends (i.e., the flow of current toward the base and the potential plateau at the apex), it is not possible to ascertain anything about the local current distribution (e.g., at the nerve tissue in the osseous spiral lamina.) The next chapter compares these experimental results with the predictions of electro-anatomical models. Such models can be used in ascertaining the current density at more specific locations in the implanted cochlea.
Chapter VIII

Comparison of Experimental Data with Computed Results

8.1 Introduction

In this chapter, the characteristic potential curves (described in Chapter VII) are compared with the potential distributions computed by electro-anatomical models (described in Chapters V and VI) and by models for two electrical extremes (described in Chapter II). First, the results of all the models are normalized using a linear procedure. In this fashion, the model results can be compared qualitatively with the experimental data (i.e., how well the computed potential distributions match the measured potential distributions.) Finally, the computed current densities (using the normalized results of the electro-anatomical model) are compared with estimates of current density at the "source" electrodes during electrical stimulation. In this fashion, the results of the electro-anatomical models can be compared quantitatively with the actual stimulus intensities.

8.2 Electro-Anatomical Models

To simulate the pseudo-bipolar and monopolar stimulation experiments, two electro-anatomical models have been formulated. The three dimensional models have identical electro-anatomical composition and differ only in their source and boundary conditions. The models are described in Chapters V and VI of this thesis. Both models are heterogeneous with bone, fluid and nerve tissue resistivities set at 5000 ohms-cm, 50 ohms-cm, and 300 ohms-cm, respectively (Table 5.3).

8.3 Models for the Hypothetical Electrical Extremes

Together with the electro-anatomical model results, the monopolar experimental measurements are also compared with the two models for the electrical extremes described in Chapter II (Fig. 2.5) and with the transmission line model described below. The potential distribution in the infinite homogeneous volume conductor and the infinite uniform transmission line models can be calculated analytically. The third model, the terminated uniform transmission line, is formulated and solved numerically. The analytical models predict potential distributions that are functions of distance from the source to the measurement site or target. The potential distribution in the homogeneous volume conductor is described by:
\[ \Phi_{\text{target}} = \frac{k \sigma l_{\text{source}}}{r}, \quad (8.1) \]

where \( l_{\text{source}} \) is the source current strength, \( k \) is a constant due to the source geometry, \( \sigma \) is the volume conductivity, and \( r \) is the direct distance between the source and target sites. For a uniform transmission line the potential is described by:

\[ \Phi_{\text{target}} = \Phi_{\text{source}} e^{-\frac{d}{\lambda}}, \quad (8.2) \]

where \( \Phi_{\text{source}} \) is the source (or other reference) potential and \( d \) is the distance between the source and target as measured along the length of the transmission line. The length constant \( \lambda \) determines the rate of potential attenuation. (The choice of \( \lambda \) was dictated by a fitting criterion explained below.) The two distance parameters, \( d \) and \( r \), are extracted from the three dimensional model. The comparison of potentials between models is thus made at identical sites along the length of the scala tympani.

The additional terminated transmission line model is a more realistic representation of the physical boundary conditions (i.e., grounded base and insulated apex in the cochlear spiral.) The simulation was performed by formulating a three dimensional model of a transmission line and specifying the appropriate boundary conditions (Fig. 8.1). The same software tools used in the development of the three dimensional electro-anatomical model were also used to define and solve the terminated transmission line model. Special boundary conditions consisted of insulating the back surface of the model domain (representing the apex) while the remainder of the model domain exterior was grounded.

### 8.4 Placement of Electrodes

The electrode array was placed in the electro-anatomical model with the most basal electrode (6) located 2.4 mm from the round window. The rest of the intracochlear electrodes (5 through 1) were placed at 4 mm intervals along the length of the scala tympani. Although analysis of CAT scans of implanted cochleas (which may be useful in determining the position of the electrode array) had not been performed at the time of writing, operative notes for the five subjects studied indicate that the placement of electrode 6 ranges from the round window to approximately 4 mm inside the scala tympani. Only the position of electrode 6 is required to ascertain the positions of the remainder of the electrodes because the electrodes are spaced uniformly along the implanted array (assuming no deformation has occurred during array insertion.)
Fig. 8.1: The terminated transmission line model. In this model, only the back surface is insulated while all the other surfaces are grounded. The numerical model, formulated similarly to the electro-anatomical models described in Chapter V, consists of 37 meshes. Each mesh is composed of two homogeneous regions: the circular region in the center (which becomes a cylinder in the three dimensional model) and the surrounding region. The dimensions of the model correspond to the dimensions of the 'unwound' scala tympani (diameter: 0.5 mm, length: 35.0 mm). With a source placed in the center of the numerical model and all the external surfaces grounded, the conductivities of the two regions were adjusted to provide a potential distribution along the length of the numerical model within 6 percent of the distribution obtained with the continuous transmission line model (solved analytically).
8.5 Normalization of Data

Because it is not feasible to measure the potential at the site of current injection, a direct comparison (e.g., by normalizing data with respect to the source potential) between the experimental measurements and the model results is not possible. The experimental data (i.e., the characteristic potential curves) consist of both positive and negative potentials for fixed sites along the cochlear spiral. The model results, on the other hand, are distributed between the "sink" (zero) and "source" potentials. During the experiments, a controlled current stimulus is presented to the active pair of electrodes. In the electro-anatomical model, a current volume density is specified at the "source" nodes. (The model nodes can also be constrained to a fixed potential.) Thus without a common reference, quantitative comparison of data is difficult.

The computed potential distributions are compared with the experimental measurements using an ad hoc linear normalization procedure. From each model, a potential distribution along the length of the scala tympani ("trace") is computed for each electrode configuration. The traces are then normalized by a scale factor $k$ and an offset constant $c$:

$$
\Phi^N(d) = k \cdot \Phi(d) + c,
$$

where $\Phi(d)$ is the trace potential at distance $d$; $d$ is measured along the length of the scala tympani. For each of the three model types, a $k$ and a $c$ have been chosen to best fit the curve for stimulus configuration 1-8. The same two parameters are then used in normalizing the traces of each model for other configurations. The transmission line parameter $\lambda$ is also chosen to best fit the 1-8 stimulus configuration. Table 8.1 lists the fitting parameters for all models, together with the free parameters for the alternate models.

The physical significance of adjusting the scale factor $k$ is analogous to changing the strength of the injected current. As seen earlier, the measured potentials scale linearly with input stimulus strength (Fig. 7.3). The models, which differ in composition and source conditions (e.g., certain models may specify a current source while others a potential source), are scaled so that the computed potentials fall within the range of the measured potentials. Because the measured potentials were normalized to a stimulus intensity of 20 $\mu$A for all stimulus configurations, a single value of $k$ is maintained across all stimulus configurations for a specific model type.

The parameter $c$ shifts the (scaled by $k$) trace so that it corresponds with the measured potential distribution. As mentioned previously, because the two data lack a common potential reference, it is not feasible to offset both sets of data to make it
<table>
<thead>
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<th>parameter</th>
</tr>
</thead>
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<tr>
<td>infinite volume conductor model</td>
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<td>transmission line models</td>
<td></td>
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<tr>
<td>electro-anatomical (pseudo-bipolar) models</td>
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</tr>
<tr>
<td></td>
<td>c: -5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>k: .55</td>
</tr>
<tr>
<td></td>
<td>c: -73 / -55</td>
</tr>
</tbody>
</table>

Table 8.1: The fitting parameters for the various models. The other parameters in the infinite volume conductor and the two transmission line models \( \Phi_{source}, I_{source} \) were set to 1. All electrical units in the model traces are arbitrary; the model traces were normalized to correspond with the measured characteristic potential curves (in mV RMS). In the transmission line models, \( \lambda = 10 \) mm. The offset c varies in the two pseudo-bipolar models (1–6 / 3–4) as indicated. See text for details.

Positive with respect to a common "sink" potential. The offset, however, does not affect the calculation of potential gradient (and current density) which is related to neural activity.

8.6 Comparison of Model Results with Monopolar Stimulus Experiments

Figs. 8.2 - 8.4 plot the normalized results of the various computed models (traces) together with the characteristic potential distributions (curves) for the subjects. The top panel in each figure shows the electro-anatomical model results, while the lower panel shows the results from the three alternative models: the infinite homogeneous volume conductor, the infinite uniform transmission line and the numerical simulation of a terminated transmission line.

As mentioned, stimulus configuration 1-8 was used for the choice of the fitting parameters, k and c (and \( \lambda \) for the transmission line models). The electro-anatomical model fits the subject potential distribution best at the central and basal regions of scala tympani. At the apical end, the model predicts a sharper attenuation of potential than seen in the experimental measures. The electro-anatomical trace can be divided into three segments: the sharp decrease in potential from site of current injection, followed by a much slower decrease between electrodes 2 and 5, and finally a larger attenuation for electrodes 5 and 6 than seen in the previous segment. Compared with
Fig. 6.2: Comparison of experimental results with the electro-anatomical model (upper panel) and the three alternate models (lower panel) for stimulus configuration 1–8. Subject MTH is not plotted; see text for details.
Fig. 8.3: Comparison of experimental results with the electro-anatomical model (upper panel) and the three alternate models (lower panel) for stimulus configuration 6–8. Subject MTH is not plotted; see text for details.
Fig. 8.4: Comparison of experimental results with the electro-anatomical model (upper panel) and the three alternate models (lower panel) for stimulus configuration 3–8. Subject MTH is not plotted; see text for details.
the subject attenuation statistics (Table 7.1), the model trend at the base agrees with most of the experimental data. For example, in subjects JFA, TWM and MLP, the attenuation is greater between electrode pair 5-6 than 2-3.

Both transmission line models agree well with the experimental data. For current injected at the apex, the trace for the terminated transmission line model does not differ a great deal from the trace for the infinite transmission line model. The variations in the two traces are due to the discrete nature and finite boundary conditions of the numerical model. While in general the fit is good, the transmission line models cannot account for the minor local variations in potential, such as the three segments in the potential distribution described above. The transmission line models predict a uniform attenuation of potential along the length of the scala tympani. For example, the attenuation between the most basal pair of electrodes is the same as between the most apical pair — a result which contrasts sharply with the electro-anatomical model predictions and the measurements.

Of all the models, the infinite homogeneous volume conductor predicts the highest attenuation of potential (with respect to the source potential) with distance away from the site of current injection. This means that the trace of the homogeneous volume conductor must be scaled significantly to fall within range of the experimental measurements, causing the appearance of large "bumps". For example, the homogeneous model predicts a net decrease in potential from electrode 6 to electrode 3. No matter how the fitting parameters are chosen, this model does not fit any of the stimulus configurations.

For stimulus configuration 6-8, the electro-anatomical trace agrees well with the curve between electrodes 4 and 5. This region is also the place of best agreement across subjects for normalized data (Fig. 7.7). The electro-anatomical trace also displays the potential plateau at the apical end, albeit the potential continues to decrease past electrode 3 and undershoots the measured potential by 4 mV. The model predicts a "bump" at electrode 1. The potential difference between electrodes 1-2 is about 2 mV for the model, which is greater than the 0.1–0.8 mV "bumps" observed for the subjects (Fig. 7.8).

The transmission line models fail to characterize the potential gradient near the site of current injection. Although a model with a shorter length constant (e.g., 8 mm instead of 10 mm) could better account for the potentials between the source and electrode 4, such models cannot account for the potential plateau at the apical end of the scala. The contrasting potential distributions for stimulus configurations 1-8 and 6-8 imply that special boundary conditions are necessary to model the cochlear tube. A
uniform transmission line model cannot predict the potential distribution for both apical and basal current injection. Neither can a transmission line model that is insulated at one end but still has a uniform grounding pathway along the rest of its length. Development of a model with non-uniform boundary conditions and grounding pathways is not unlike the definition of a full electro-anatomical model. Hence, the simplicity of the transmission line models vanishes once special boundary conditions are imposed.

For stimulus configuration 6-8, the homogeneous volume conductor fails again. After showing some agreement with the experimental data between electrodes 5 and 3, the model clearly overestimates the “bump” near electrode 1 due to the large k required to scale the model trace.

The robustness of the electro-anatomical model can be seen in the final monopolar stimulus configuration, 3-8. The characteristics of both the apical and basal potential distributions are preserved in the model trace. The model does predict a net increase in potential directed apically from electrode 1. This trend does not show up in the experimental data due to the spacing of the electrode array. For example, while both the electro-anatomical model and experiments suggest a net decrease in potential from electrode 2 to 1, only the model has sufficient resolution to predict a local “depression” between these two electrodes.

The alternate models display the same deficiencies seen in the previous stimulus configurations. The uniform transmission line cannot account for the differences between the basal and apical potential gradients. The terminated transmission line does a slightly better job although it does not adequately represent the plateau at the apical end. The homogeneous volume conductor represents the apical plateau but fails to represent the attenuation between electrodes 4 and 5, and the basal side of the source.

8.7 Comparison of Model Results with Pseudo-Bipolar Stimulus Experiments

Traces from the electro-anatomical model are compared with the subjects’ characteristic potential curves for pseudo-bipolar configurations in Figs. 8.5 and 8.6. The pseudo-bipolar configuration cannot be modeled using the alternate models due to electrode placement (the electrode pair resides in the scala tympani) and physical boundary conditions (the grounding pathway through the nerve and blood vessels no longer exists and the whole of the cochlea is well insulated).
Fig. 8.5: Comparison of experimental results with the electro-anatomical model for stimulus configuration 1–6.

Fig. 8.6: Comparison of experimental results with the electro-anatomical model for stimulus configuration 3–4.
As in the monopolar normalization procedure, the pseudo-bipolar model traces have been scaled by the same factor \( k \) that was used for the monopolar electro-anatomical traces. A new offset parameter \( c \) was chosen to bring the scaled potentials within the same range as the experimental measurements (Table 8.1). The change in offset is likely due to the change in boundary conditions for the two types of models. For an insulated boundary condition (used in the complete pseudo-bipolar model domain), the potential can only be determined within an arbitrary additive constant [Hildebrand, 1976, p. 491].

For stimulus configuration 1-6, the electro-anatomical model predicts a three part potential distribution, as was also seen in the 1-8 measurements and model results. The model fits the small potential difference between electrodes 2 and 3 as compared with the difference between electrodes 3 and 5 (subjects JFA, RLB and MTH.) The potential gradient between electrodes 1 and 2 is as high as predicted by extrapolating the curves of subjects TWM and MLP. This behavior is also observed in the electro-anatomical trace for configuration 1-8.

For stimulus configuration 3-4, the model fits the basal part of the curve extremely well. The apical potential plateau, however, is offset by 4 mV. The choice of fitting parameter \( c \) dictates the which of the regions (apical or basal) are best fit. Nevertheless, the potential gradient is well fitted in the basal region regardless of parameter \( c \). The apical gradient is greater than estimated from the measurements.

### 8.8 Comparison of Computed Current Density (Electro-Anatomical Models) with Current Density at the Stimulus Electrodes

The normalized results of the electro-anatomical models can be used to compute the current density in the vicinity of the active electrodes. The computed current density can then be compared with an estimate of the current density at the stimulus electrodes during electrical stimulation. The experimental current density (\( \mu A/mm^2 \)) can be calculated given the stimulus current strength (\( \mu A \)) and the surface area of the electrodes (\( mm^2 \)). If the SYMBION electrodes are modeled as perfect spheres, the current density at the surface of the electrodes is described by:

\[
J_s = \frac{I}{4\pi r^2},
\]

where \( I \) is the source current strength and \( r \) is the radius of the electrodes. The magnitude of the current density is equal to \( J_s \) and is directed radially out of the source electrode and radially into the sink electrode. For \( I = 20 \mu A \) and \( r = 250 \mu m \),
\[ J_s = 25.5 \, \mu A/mm^2 \]. The surface area is equal to 0.785 mm².

For the three dimensional, electro-anatomical models, the current density at the site of current injection is calculated using Eq. (5.13). In the model, the electrode is modeled as a point source. The current density in the model corresponding to the surface current density \( J_s \) should be calculated at a distance \( r \) from the point source. However, because of the "brick" structure of the electro-anatomical model, the spherical surface shape must be approximated by the model elements. Hence three current density computations — one for a hexahedron contained inside the electrode sphere \( (r < 250 \, \mu m) \), one for a hexahedron which is a geometrically closest to the electrode sphere, and one for a hexahedron completely outside the electrode sphere \( (r > 250 \, \mu m) \) — are calculated from the electro-anatomical model results.

The three current density computations are thus: an "8 neighbor" element average (inside sphere); a "32 neighbor" element average (nearest to sphere); and a "56 neighbor" element average (outside sphere). The "8 neighbor" computation is an average of the current density magnitude for the 8 elements surrounding the point current source. The "56 neighbor" computation is derived from the 56 elements that surround the former 8 elements. The "32 neighbor" average is derived from the same 8 elements plus an additional 24 elements surrounding the source node. The "8 neighbor" average thus encompasses a surface described by a 300 \( \mu m \) by 180 \( \mu m \) by 400 \( \mu m \) hexahedron (surface area = 0.492 \( mm^2 \)), the "32 neighbor" average is for a 600 \( \mu m \) by 360 \( \mu m \) by 400 \( \mu m \) surface (surface area = 1.20 \( mm^2 \)), while "32 neighbor" average is for a 600 \( \mu m \) by 360 \( \mu m \) by 800 \( \mu m \) surface (surface area = 1.97 \( mm^2 \)). The current densities, after being scaled by \( k \) (the electro-anatomical model scale factor), can then be compared to the current density computed analytically for the spherical electrodes. Table 8.2 shows the model predictions.

The predicted current densities are 2-10 times as great as estimated analytically based on stimulus current strength. The models predict nearly identical current densities in the vicinity of all sources, regardless of electrode configuration or stimulus type. Furthermore, the current density magnitude in the vicinity of the current source does not vary with changes in bone or fluid resistivities (Chapter VI.) The potential distribution, however, is more dependent on the electrical composition of the electrical model (Figs. 6.12 and 6.13.) This may have an effect on the scale factor \( k \) used to fit the model data. For example, the 100 ohms-cm (fluid) model best fits the experimental data with \( k = 45 \), as opposed to \( k = 55 \) in the 50 ohms-cm (fluid) model. Hence, the predicted current densities in Table 8.2 decrease by approximately 20% for the 100 ohms-cm (fluid) model. This model, however, does not provide as good a fit for
### Current Density Magnitude (μA/mm²)

<table>
<thead>
<tr>
<th>stimulus electrode pair</th>
<th>8 neighbor</th>
<th>32 neighbor</th>
<th>56 neighbor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ave.</td>
<td>s.d.</td>
<td>ave.</td>
</tr>
<tr>
<td>1-8</td>
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<td>3-4</td>
<td>269</td>
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</tr>
</tbody>
</table>

Table 8.4: Average of the current density (magnitude) in the vicinity of the point source electrode. The "8 neighbor" figures refer to the 8 elements in the two meshes surrounding the point source. All these elements lie inside the electrode sphere. Similarly, the "32 neighbor" ("56 neighbor") figures refer to the 32 (56) elements in two (four) meshes surrounding the immediate source node (8 elements surrounding the source node.) The exterior surface of the 32 (56) elements approximates (encloses) the electrode sphere. The standard deviation (s.d.) is greater for the "56 neighbor" averages since elements are located at varying distances from the point source. The estimated current density for a spherical electrode is equal to 25.5 μA.

...all the stimulus configurations as the 5000 ohms-cm (bone), 50 ohms-cm (fluid) electro-anatomical model.

The current density is most likely underestimated in the analytical calculation because the effective surface area of the electrodes is probably overestimated. For example, much of the electrode surface is covered by array leads and insulation. Decreasing the electrode surface area increases the current density (Eq. (8.6)). For example, if 25% of the ball electrode is covered by leads and insulation, the surface current density increases by 33%.
8.9 Discussion

In this chapter, the results from several types of electrical models were compared with the experimental measurements. Of all the models, the three dimensional electro-anatomical model is superior in accurately characterizing the potential distribution along the scala tympani. The robustness of the anatomical model is demonstrated by its ability to capture the salient features of the potential distribution during the simulation of three monopolar and two pseudo-bipolar stimulus configurations. The homogeneous volume conductor model fails due to excessive attenuation of potential (with respect to the source potential) with distance from the site of current injection. The large attenuation requires excessive scaling to bring the model results within the range of the potentials measured in the human subjects. The transmission line models fail to account for differences between the basal and apical potential distributions. Furthermore, the local variations in the potential distribution (e.g., the three part potential distribution for configurations 1-8 and 6-8) cannot be predicted by models that predict uniform attenuation of potential along the length of the cochlea.

The characteristic potential curves illustrate the electrical non-uniformity of the cochlea and exhibit the same overall shape of the traces produced from the electro-anatomical models. For example, both the electro-anatomical models and the characteristic potential curves predict an asymmetric potential distribution in the scala tympani. The good correlation between the experimental and electro-anatomical model results are likely due to the representation of physical boundary conditions and conservation of cochlear anatomy in such models. Typically, the potential decreases monotonically with distance away from site of current injection. Such measurements are consistent with the notion that most of the current flows in the high conductivity fluids of the cochlear duct. However, the presence of non-monotonic behavior in at least one potential curve (configuration 6-8) suggests the presence of indirect or non-scalar current pathways. These deviations in potential are significantly smaller than the equivalent potential "bumps" predicted by electro-anatomical models. The potential measurements, however, suffer from lack of resolution in the electrode array. Thus a possible "bump" or "depression" may be missed if it occurs in the region between electrodes.

The usefulness of any theoretical result lies in its physical significance. The electro-anatomical model best represents the qualitative characteristics of the characteristic potential distributions (e.g., the shape of the curves and the local deviations in potential). Also, the quantitative correlation between model and subject data (i.e., comparing the current density as computed from the model and estimated from the
electrode surface area and stimulus strength) is reasonable considering that the *electro-anatomical* model is only a first order approximation of the implanted cochlea. The model predicts current density at the stimulus electrode surface that is 2-10 times greater than calculated analytically. However, the difference may be reduced (perhaps by as much as a factor of 2) because the *effective* electrode surface area is smaller than estimated due to array leads. Furthermore, the electrode model, an ideal point source, may overestimate the current density at the site of current injection. The representation of electrodes needs to be investigated further to establish the most appropriate electrode model.

Given the verification of model results for potentials along the length of the scala tympani, the *electro-anatomical* models should be extended to provide potentials at the sites of possible nerve excitation, such as the nerve tissue in the spiral lamina. With the present model resolution, however, these structures cannot be adequately represented. The next chapter includes error analysis of model resolution and presents methods of increasing model resolution.
Chapter IX

Model Error Analysis

9.1 Introduction

In the numerical simulation of current flow in the implanted ear, a fundamental assumption is that a spatially continuous and electrically non-uniform system, the inner ear, can be made discrete (quantized) and modeled as a distributed, lumped element electrical network. During the modeling procedure, the model domain is quantized into a set of uniformly spaced meshes. As the discrete model resolution is increased (by decreasing the spatial sampling interval), the finite difference solution should approach the continuous or exact solution. This chapter examines the errors introduced in the context of changing the model resolution.

The present three dimensional model is limited in resolution by the uniform quantization scheme. Certain structures, in particular nerve tissue in the spiral lamina bone — a location of primary interest since neural excitation may occur there — are undersampled in the present model. The finer anatomical details which occupy a small percentage of the total model domain (by volume) can be omitted without significantly influencing the global solutions (e.g., the current distribution along the scala tympani) if the bulk electrical properties are preserved. When the current distribution in the vicinity of these structures is required, the global results can be used as initial or starting conditions in a model that specifies a smaller segment of the global model in greater detail. In this fashion, the model resolution can be increased without an increase in modeling overhead (time and storage.) Methods for developing higher resolution models are also discussed in this chapter.

9.2 Types of Errors

Errors in numerical models such as the one developed in this thesis can be grouped into several classes†. Modeling Error and Measurement Error are two classes of error that occur during the formulation and definition of the numerical model. The Modeling Error results from basic properties of the model selected (e.g., using a discrete system to model a continuous system, approximating smooth boundaries by

† The error classification and analytical development of truncation error is from [Vemuri and Karplus, pp. 88 - 92].
piecewise elements, and assuming isotropic and resistive media.) The \textit{Measurement Error} is due to error in the model data used to specify the parameters of the model (e.g., inaccuracies in the anatomical and electrical parameters.) These two error classes are often unavoidable because of practical constraints. For example, all discrete models have finite spatial resolution. Consequently, certain anatomical structures may not be adequately represented. Increasing the model resolution results in penalties of increasing both the storage and time required for a model solution. Other modeling assumptions (e.g., the quasi-static formulation and resistive media) facilitate model formulation and solution. The choice of electro-anatomical data in the present model is dictated by the unavailability of more specific cochlear measurements. Until more data are available for the electrical properties of specific cochlear tissues (e.g., temporal and spiral lamina bone, nerve, stria vascularis), the conductivities are approximated from data available for similar biological materials.

The next two classes of error occur once the boundary value problem is formulated. \textit{Roundoff Error} occurs because of the finite precision of digital computers. For example, all numerical quantities must be represented by a finite number of bits. During calculations, numbers must be either rounded up or down; the cumulative effect is known as the \textit{Roundoff Error}. The dominant error in computer calculations, however, is the \textit{Truncation Error}. \textit{Truncation Error} is a consequence of representing an infinite continuum by a quantized model of finite dimensions. Most solutions of boundary value problems are analogous to \ldots infinite (spatial) series. In a discrete simulation, the solution is truncated at a limited number of terms. As the number of terms in the model solution is increased (analogous to increasing the model resolution), the discrete solution should approach the infinite series solution. The \textit{Truncation Error} for simple problems can be analyzed using two methods, one based on Taylor series and the other on Fourier series. These methods are discussed below.

\subsection*{9.3 Analytical Considerations of Truncation Error}

To analyze the truncation error, consider a homogeneous medium. The potential distribution satisfies Laplace's equation:

\begin{equation}
\nabla^2 \Phi = 0 .
\end{equation}

In the three dimensional model, Eq. (9.1) becomes:

\begin{equation}
\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0
\end{equation}

To analyze the truncation error it is sufficient to consider only a single term of
Eq. (9.1). Hence, assuming a one dimensional potential distribution (i.e., \( \Phi = \Phi(x) \)), Eq. (9.1) becomes:

\[
\frac{d^2\Phi}{dx^2} = 0.
\]  

(9.3)

9.3.1 Taylor Series Method

The finite difference approximation of Eq. (9.3) can be derived by considering a Taylor series expansion along a uniformly spaced line. The potential at node \( i+1 \) is described by:

\[
\Phi_{i+1} = \Phi_i + h \left[ \frac{d\Phi}{dx} \right]_i + \frac{h^2}{2} \left[ \frac{d^2\Phi}{dx^2} \right]_i + \alpha h^3 + \beta h^4 + \cdots,
\]

(9.4)

where \( \Phi_i \) represents the potential at node \( i \), \( h \) is the spacing between adjacent nodes (the spatial sampling interval), and \( \alpha, \beta, \cdots \) are functions of \( h \) and derivatives (second order or higher) of \( \Phi \) evaluated at node \( i \). Similarly, the potential at node \( i-1 \) is described by:

\[
\Phi_{i-1} = \Phi_i - h \left[ \frac{d\Phi}{dx} \right]_i + \frac{h^2}{2} \left[ \frac{d^2\Phi}{dx^2} \right]_i - \alpha h^3 + \beta h^4 + \cdots.
\]

(9.5)

Adding Eqs. (9.4) and (9.5):

\[
\Phi_{i+1} + \Phi_{i-1} = 2\Phi_i + h^2 \left[ \frac{d^2\Phi}{dx^2} \right]_i + \beta h^4.
\]

(9.6)

Hence:

\[
\left[ \frac{d^2\Phi}{dx^2} \right]_i = \frac{\Phi_{i+1} - 2\Phi_i + \Phi_{i-1}}{h^2} + O(h^2).
\]

(9.7)

In other words, the error in the finite difference approximation of the second order derivative is proportional to \( h^2 \) and higher powers of \( h \). (Eq. (9.7) is better known as the central difference approximation.) If \( h \) is small, the truncation error is dominated by the \( h^2 \) term. To reduce the truncation error, \( h \) must be decreased. This is accomplished by reducing the mesh spacing, i.e., refining the mesh or increasing mesh resolution. There are other means of reducing the order of the truncation error such as using a different approximation of the derivatives. However, both the alternative
approximations and mesh refinement can be very expensive in terms of computing requirements. Note that Taylor series analysis does not specify the size of the truncation error. Rather, it places a theoretical bound on the relative truncation error as a function of mesh spacing, h.

9.3.2 Fourier Series Method

Consider a continuous function $\Phi(x)$ defined in the interval $[0,1]$. A quantized or sampled representation of the function is:

$$\Gamma_i = \Phi(x_i), \quad i = 0, 1, 2, \ldots, L-1$$

(9.8)

Such a representation of $\Phi$ may be incomplete. For example, information between samples may be lost if $\Phi$ is undersampled. However, in certain instances, the continuous function can be recovered from the sampled version. If $\Phi$ changes rapidly between samples $i$ and $i+1$, then $\Gamma$ is not likely to be a good representative of $\Phi$. In terms of spatial frequency, slowly varying functions can be thought of having “low-frequency” or “long-wavelength” components. Conversely, fast varying functions can be thought of having “high-frequency” or “short-wavelength” components. Finite difference methods typically do a better job of representing the “long-wavelength” components and the approximation of the “short-wavelength”, “high-frequency” components gets better with mesh refinement.

The sampled version of $\Phi$ can be expressed as a discrete Fourier series. Thus:

$$\Gamma_i = \sum_{k=0}^{L-1} g_k e^{\frac{j2\pi ki}{L}},$$

(9.9)

where $g_k$, the amplitude of each spatial frequency mode, is equal to:

$$g_k = \frac{1}{L} \sum_{i=0}^{L-1} \Gamma_i e^{-\frac{j2\pi ki}{L}}.$$  

(9.10)

Note that phenomena with spatial wavelengths less than $h$, the mesh spacing ($h = \frac{1}{L-1}$), cannot be described by $\Gamma$.

The accuracy of the central difference approximation of the second order derivative can be estimated by considering a function having a single frequency mode:

$$\Phi(x) = g e^{jkx}.$$  

(9.11)

Substituting $\Phi(x)$ for the left hand side of Eq. (9.7) yields the exact second order
derivative:
\[
\frac{d^2 \Phi}{dx^2}
\bigg|_{x=m} = -k^2 \Phi(m) .
\] (9.12)

The right hand side of Eq. (9.7) yields the central difference approximation at node \(i\):
\[
\frac{\Gamma_{i+1} - 2\Gamma_i + \Gamma_{i-1}}{h^2} = \frac{\Phi(m+h) - 2\Phi(m) + \Phi(m-h)}{h^2}
\] (9.13a)
\[
= g e^{j(k(m+h))} - 2g e^{jkm} + g e^{j(k(m-h))}
\] (9.13b)
\[
= \frac{g e^{jkm}}{h^2} \left[ e^{jkh} + e^{-jkh} - 2 \right]
\] (9.13c)
\[
= \frac{2g e^{jkm}}{h^2} \left[ \cos(kh) - 1 \right]
\] (9.13d)
\[
= \frac{2g e^{jkm}}{h^2} \left[ \cos^2(kh/2) - \sin^2(kh/2) - 1 \right]
\] (9.13e)
\[
= -k^2 g e^{jkm} \left[ \frac{\sin(kh/2)}{(kh/2)} \right]^2
\] (9.13f)
\[
= -k^2 \sin^2(\tau) \Phi(m), \quad \tau = \frac{kh}{2} .
\] (9.13g)

Hence, the central difference approximation differs from the exact second order derivative by the \(\text{sinc}^2(\tau)\) function, which varies from 0 to 1. For small \(\tau\), the central difference approximation approaches the exact value of the second order derivative. Hence, for low spatial frequency modes \(k\) (or long spatial wavelengths \(\lambda\)) and high resolution meshes (small \(h\), Eq. (9.7) is a good approximation because the truncation error remains small.

The deviation from the exact second order derivative in the central difference approximation can be computed as a function of mesh resolution. If \(N\) represents the minimum samples of spatial interval \(h\) allotted to represent the shortest wavelength \(\lambda\), then:
\[
\lambda = \frac{2\pi}{k} \geq Nh .
\] (9.14)
Hence:

\[ \left( \frac{kh}{2} \right) = \tau \leq \frac{\pi}{N}. \quad (9.15) \]

Eq. (9.15) yields a lower bound on \( \text{sinc}(\tau) \) as a function of \( N \). The values of the \( \text{sinc}^2(\tau) \) function are computed for representative values of \( N \) in Table 9.1.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \text{sinc}^2(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>.992</td>
</tr>
<tr>
<td>10</td>
<td>.967</td>
</tr>
<tr>
<td>7</td>
<td>.935</td>
</tr>
<tr>
<td>5</td>
<td>.875</td>
</tr>
<tr>
<td>3</td>
<td>.684</td>
</tr>
<tr>
<td>2</td>
<td>.405</td>
</tr>
</tbody>
</table>

Table 9.1: The deviation in the central difference approximation of the second order derivative from the exact derivative as a function of mesh resolution. \( N \) represents the number of spatial intervals allotted to represent the lowest spatial wavelength.

The Fourier method places a lower bound on the deviation due to truncation error between the exact and central difference approximation of the second order derivative. For example, consider a function whose shortest spatial wavelength is equal to \( \lambda \). If mesh spacing \( h \) is equal to or less than \( \frac{\lambda}{2} \), Fourier analysis predicts that the discrete approximation is within 90\% of the exact derivative at wavelength \( \lambda \). If \( h \) is greater than \( \frac{\lambda}{2} \), the deviation can be as high as 60\%. Conversely, a model with mesh resolution of 100 \( \mu \text{m} \) can accurately represent (within 90\%) potential solutions with spatial wavelengths 700 \( \mu \text{m} \) or greater.

**9.4 Error Simulations**

The analytical methods described above yield lower bounds for the behavior of the truncation error for geometrically regular and homogeneous problems. Heterogeneous problems of irregular geometry cannot be treated in the same, straightforward manner. One cannot simply compare discrete and exact solutions because the exact solution cannot be obtained. The truncation error in practical models depends on the electrical composition, the boundary and source conditions, and the geometry of the
model. To gain a measure of the truncation error in such models, however, solutions from models of varying resolution can be compared. In this fashion, the rate of convergence can be compared against the theoretical bounds.

9.4.1 Model Scenarios

To observe the effects of resolution on model solutions, as well as the effects of downsampling, several types of numerical models were formulated. Both two dimensional (single mesh) and three dimensional (37 mesh) models were used in the error analysis. Three types of numerical models were formulated: a completely homogeneous model, and two heterogeneous, electro-anatomical models. The electro-anatomical models are similar in electrical composition, and in boundary and source conditions, to the models described previously in this thesis. The model specifics are described below:

- **Homogeneous:**
  No spatial variation in conductance; only mesh refinement performed here. The truncation error should vary with $h^2$, as predicted by Eq. (9.7).

- **Heterogeneous Upsampled:**
  The model was initially defined at low resolution (eg., $16 \times 16$ element mesh model) and hierarchically upsampled to higher resolution models. For example, upsampling a $16 \times 16$ element mesh to a $32 \times 32$ mesh involves replicating each low resolution element with four elements, each one with one quarter the volume of the original low resolution element. The four elements share the conductance of the original element. The geometry of the model is preserved in the upsampling procedure while the model is simultaneously refined. Again, since the model composition remains constant while the resolution is increased, the truncation error should vary with $h^2$ (at least within homogeneous regions).

- **Heterogeneous Downsampled:**
  The model was initially defined at high resolution (eg., $128 \times 128$ element mesh model) and hierarchically downsampled to lower resolution models. The downsampling process does not preserve the model geometry but attempts to best represent the electrical properties of the model at lower resolution. Downsampling, unlike upsampling, results
in a slightly different boundary value problem being solved because the model geometry isn’t necessarily preserved. (Upsampling results in the same problem but in a refined model.) Some aspects of the high resolution model are lost in the downsampling procedure. Hence, the truncation error may be of smaller order (suggesting lower convergence) than \(O(h^2)\) predicted for refined models.

Expanded sections of high and low resolution meshes are shown in Fig. 9.1. High resolution mesh \(H\) contains nodes \(H_1\) through \(H_{25}\) and elements \(a\) through \(p\). Similarly, low resolution mesh \(L\) contains nodes \(L_1\) through \(L_9\) and elements \(A\) through \(D\). For example, the conductivity of element \(A\) in mesh \(L\) is a downsampled representation of the conductivities of elements \(a, b, e\) and \(f\) in mesh \(H\). As mesh resolution is increased, potentials at spatially equivalent nodes should converge towards the same (exact or continuous) solution. Thus the potentials at the low resolution nodes \(L_1, L_2, L_4\) and \(L_5\) should approach the solutions at the higher resolution nodes \(H_1, H_3, H_{11}\) and \(H_{13}\), respectively.

### 9.4.2 Statistical Measures of Error

Two statistical measures are used to examine the differences in the solutions. The first calculates the *average percentage difference* (APD) between discrete model solutions and exact solutions and is defined by:

\[
APD = \frac{1}{N} \sum_{i=1}^{N} \frac{\Phi_i - \Phi_i^h}{\Phi_i} \times 100.0 ,
\]

where \(\Phi\) is the exact solution, \(\Phi^h\) is the discrete model solution, and \(N\) is the number of nodes in the numerical model. The second statistic, the \(L_2\) norm, is defined in the continuous domain \(\Omega = (a,b)\) by

\[
L_2 = \| \Phi - \Phi^h \|_0 = \left( \int_a^b |\Phi - \Phi^h|^2 \, ds \right)^{1/2} .
\]

and in the discrete domain as

\[
= \left( \sum_{i=1}^{N} |\Phi_i - \Phi_i^h|^2 ds \right)^{1/2} ,
\]

where \(ds\) is the surface area of a mesh element for a two dimensional analysis, or
Fig. 9.1: Schematic representation of high (H) and low (L) resolution meshes. In the low resolution mesh, each element corresponds to four elements of the high resolution mesh. The nodes in the low resolution mesh should converge to the same potentials as the spatially equivalent nodes in the high resolution mesh.
volume for a three dimensional analysis.

An upper bound can be placed for the $L_2$ norm,

$$L_2 \leq ch^2$$  

(9.18)

where $c$ is a constant due to model geometry and $h$ is the model resolution [Ready, 1984, p. 138]. In Eq. (9.17b), $ds$ becomes $h$. Similar to the result obtained using Taylor series analysis, Eq. (9.18) only yields information on how rapidly the error decreases with resolution and nothing about the size of the actual error incurred. Nevertheless, comparison of $L_2$ norms from models of different resolution yields the model rate of convergence. For example, if the logarithm of the $L_2$ norm is plotted as a function of the logarithm of $h$, the resultant graph should be a straight line with slope equal to 2.

The solution software has been tested by comparing the model solution with an exact solution of a two dimensional boundary value problem (a rectangular region with one side maintained at a fixed potential and the other sides set to zero [Spiegel, 1974].) The APD between the model and the analytical solution was found to be less than 0.06% for models ranging in resolution from $64 \times 64$ to $256 \times 256$.

Note that Eqs. (9.16) and (9.17) require $\Phi$, the exact solution. Since the exact solution cannot be obtained for the heterogeneous models, the highest resolution solution from each model type is chosen as the comparison solution. The comparison solution, which is closest to the exact solution, is then used as the basis for comparison with lower resolution models. For example, for each three dimensional model scenario, a $128 \times 128$ element mesh model is solved and then compared with $64 \times 64$, $32 \times 32$ and $16 \times 16$ mesh models. For two dimensional analysis, the comparison solution is obtained from the $256 \times 256$ model.

### 9.4.3 Results for Two Dimensional Models

Fig. 9.2a plots the average percentage difference (Eq. 9.16) as a function of mesh resolution for the three single mesh models. The smallest APD is observed between the homogeneous solutions — from 0.04% for the $128 \times 128$ model to 0.5% for the $16 \times 16$ model. The APD for the heterogeneous models ranges from 0.1% to 2.0%, and 1.1% to 16.0%, between the upsamped and downsamped model solutions, respectively. As predicted, raising the model resolution decreases the percentage deviation from the comparison solution. All the models suggest convergence towards the comparison solution with increasing resolution.
Fig. 9.2: Two dimensional error analysis for models of varying resolution. The upper panel (a) displays the APD as a function of model resolution, while the lower panel (b) displays the L2 norm as a function of model resolution. The homogeneous L2 curve has been shifted upward for display purposes.
The greatest APD is observed for the heterogeneous downsampled models. This result is due to the change in the model resistivities as the model is downsampled. In other words, a slightly different boundary value problem is solved at each model resolution. However, the difference between the two highest resolution models and the comparison model (less than 5%) suggests that low spatial frequency components are converging. Changes between downsampled models are therefore not significant enough to lead to drastically different solutions. The heterogeneous downsampled models converge (in the APD sense) at rates similar to the other two models.

The $L_2$ error norm is shown plotted as a function of model resolution in Fig. 9.2b. Theory predicts that a log-log plot of $h$ vs $L_2$ norm will have a slope of 2 or less (Eq. (9.18)). The greatest rate of convergence, as suggested by the slope of the $L_2$, is observed for the homogeneous and upsampled models. The heterogeneous downsampled models do not converge as quickly as the other two models because of the downsampling procedure. Between the two highest resolution models, the incremental $L_2$ slope (Table 9.2) is equal to 1.88 and 1.78, for the homogeneous and upsampled models, respectively. The equivalent slope in the downsampled model decreases to 1.49. The incremental slope values are somewhat smaller for the lower resolutions of all model types. This is perhaps due to the small number of values used in the discrete $L_2$ norm calculation to approximate the integral.

9.4.4 Results for Three Dimensional Models

Fig. 9.3a shows the APD plotted as a function of model resolution for the three dimensional models. Again, the homogeneous model has the smallest deviation from the comparison solution. The APD for the homogeneous model ranges from .06 to .93%. The APD ranges from .25 to 2.3% and 4.3 to 13.1%, for the heterogeneous upsampled and downsampled models, respectively. In general, the APD range is similar to the APD seen in the two dimensional analysis. The APD changes are likely due to the comparison solution used in the three dimensional analysis (128 x 128 as opposed to 256 x 256.)

The $L_2$ error norms for three dimensional models are plotted in Fig. 9.3b. The homogeneous and heterogeneous upsampled models have nearly identical incremental slopes, which corresponds well with the analytical predictions. For example, between the two highest resolutions, the incremental $L_2$ slope is equal to 1.97 — almost the theoretical limit of 2. The downsampled model has an incremental slope of 1.45 for the same change of resolution. As observed in the two dimensional analysis, the three
Fig. 9.3: Three dimensional error analysis for models of varying resolution. The upper panel (a) displays the APD as a function of model resolution, while the lower panel (b) displays the L2 norm as a function of model resolution. The homogeneous L2 curve has been shifted upward for display purposes.
Table 9.2: Incremental $L_2$ slopes. The slopes are based on the log-log plots of the $L_2$ norm versus the model resolution (Figs. 9.2b and 9.3b). For three and two dimensional analysis, the norm calculation is based on the $128 \times 128$ element and $256 \times 256$ models, respectively. Each increment between successive models changes $h$ by a factor of four, the element surface area. This is also true for three dimensional models because the depth resolution (i.e., number of meshes in the complete model) is kept constant across models. Abbreviations: HOM: homogeneous; HET UP: heterogeneous upsampled; HET DOWN: heterogeneous downsampled.

Dimensional models are converging towards the comparison solutions as the resolution is raised.

Analysis of three dimensional model behavior reveals the same trends first observed in the two dimensional analysis. In a sense, the analysis remains two dimensional because the number of meshes is kept constant across models of different resolution. Because the model refinement is two dimensional, the ratio of element volume (for three dimensional models) is equal to the ratio of element surface area (for two dimensional models). However, the methods of formulation and solution are different for three dimensional models. The similar range of results suggests not only model convergence but the correctness of model formulation and solution software for both the two and three dimensional models.

9.5 General Conclusions and Further Directions

The software for model solution has been validated by comparing the model and analytical solutions for a two dimensional boundary value problem. Thus based on the analytical and numerical error analysis, the heterogeneous downsampled model is converging towards a comparison solution and hence the exact solution. Although the rate of convergence (1.45) is not as high as predicted by theory (2.0), the theoretical result
is based on a homogeneous model that conserves the problem geometry. Considering the model heterogeneity, the numerical results are rather good. For example, the global APD measure suggests less than a 5% differential between the \(128 \times 128\) and \(64 \times 64\) three dimensional models. Assuming that the APD error between successively refined models continues to decrease by a factor of 2 and that this trend can be extrapolated toward higher resolution models, the \(128 \times 128\) model is less than 5% from the exact solution.

Note that mesh refinement does not significantly change the global model solution. For example, between the \(16 \times 16\) and \(128 \times 128\) heterogeneous upsampled models there is less than a 3% change. The mesh is refined, however, by a factor of 8 (in each dimension) between these models. Hence, the drastic results predicted by Fourier analysis (Table 9.1) are not observed in the global potential results. In terms of its harmonic content, the model solution is primarily "low-frequency".

The results of the error analysis suggest that the three dimensional model results are adequate for the prediction of some global results, such as the potentials in the scala tympani. The scala tympani is defined by an adequate number of model elements. Even though the results in this thesis are derived from \(64 \times 64\) mesh models, it is likely that \(32 \times 32\) models would yield adequate results. However, the long term purpose of the model is the extraction of potentials along the length of peripheral nerve fibers. In the present model, portions of the osseus spiral lamina, cochlear fluid and nerve tissue are sometimes lumped together. Hence, while the joint electrical properties of the biological materials are conserved in the model, the potentials near or at the neural tissue cannot be ascertained correctly due to lack of model resolution.

What is required is a model with higher resolution. For example, the present model is limited to elements with dimensions 75 \(\mu\)m by 45 \(\mu\)m by 200 \(\mu\)m. To correctly model the potentials near or at the nerve fibers in the osseous spiral lamina, the resolution should be increased at least 2-5 times the resolution of the smallest structure. For example, if the typical width of the spiral lamina bone is taken as 20 \(\mu\)m, the model mesh dimensions need be increased from \(128 \times 128\) to \(2048 \times 2048\)! The model can be redesigned to handle more elements but such a "brute force" approach will result in large penalties in the storage and time required for solution. For example, doubling the resolution in all model dimensions increases the time and memory requirements by a factor of 8. Hence even with more computing power (e.g., a faster central processor, an array processor, or even several parallel processors), the overhead in a "brute force" approach quickly swamps all available resources.
Instead of building larger and larger models, the present model should be used as the "long wavelength" solution. Only the region of interest should then be refined, resulting in a model with a higher resolution but with the same physical mesh dimensions and modeling overhead. For example, assume that one is interested in the potential distribution along the peripheral nerve track in a particular model section. Once the global, 37 mesh model is solved, the region of interest can be refined resulting in a new, local model of the region of interest (e.g., the osseous spiral lamina). Hence, model resolution is increased but the size of the model (i.e., the number of model elements) remains constant or can even be reduced (Fig. 9.4). The "long wavelength" or global solution is used as the starting solution in the new model.

Similar approaches to solving the high resolution boundary value problem are known as Multigrid Methods [Axelson and Barker, 1985]. Multigrid methods operate on several mesh levels. Each level corresponds to a mesh of different resolution. Initially, the finest mesh is operated upon for several iterations — called smoothing because it tends to damp out the "high-frequency" components but leave the smoother components. The fine mesh solution is updated by a correction calculated on a coarser mesh, i.e. at the next mesh level. The smoothing — correction cycle continues until the correction factor is less than some criterion. However, the calculation in the coarse mesh proceeds in the same fashion as in the fine mesh. Hence, by recursion, the whole sequence of meshes is used until a mesh that is coarse enough to be solved directly is reached. Typically, the multigrid cycle proceeds uniformly from fine to coarse meshes or vice-versa, but it can also operate locally between adjacent mesh levels. Surprisingly, very simple smoothing algorithms (Gauss-Seidel relaxation or Chebyshev iteration) work best in multigrid methods. This is because at each level, only a few iterations are required to generate the smoother components of the solution. The total computation time and computation complexity (at each level) can be reduced by the use of multigrid methods.
Fig. 9.4: Schematic illustration of global and local models. The global model encompasses the complete cochlear section shown. The local model encompasses only the region outlined in the rectangle in the global model. The local model represents the osseous spiral lamina and portions of scala media and scala tympani. Both models have the same number of elements but the resolution increases in the local model. Note that the grid spacing in actual models is denser than illustrated in the figure.
Chapter X

Conclusions

In order to investigate the current flow in the inner ear during electrical stimulation of intracochlear electrodes, a three dimensional, electro-anatomical model of the implanted cochlea has been developed. Such a model is a first order approximation of the anatomy and electrical properties of the inner ear. This type of model offers several advantages over simpler, distributed parameter (e.g., transmission line) models. The anatomy of the implanted cochlea is preserved in the three dimensional model. Hence, the non-uniform physical boundary conditions and current pathways that are simplified or ignored in transmission line models are preserved in the electro-anatomical model. Because transmission line models typically incorporate only a limited number of elements, only the bulk current pathways can be ascertained in such models. With the electro-anatomical model, the direction and magnitude of the current density in any structure of interest can be computed, given sufficient model resolution. Furthermore, the three dimensional, electro-anatomical model can be used to investigate the effects of different electrode geometries and configurations.

The definition of a three dimensional model requires the specification of the electrical and anatomical properties of the inner ear. The specification of the electrical properties remains a difficult task because the specific electrical properties of many cochlear tissues have not been measured. However, a first order model can be defined based on the conductivities of similar tissues found in the literature. Although the present model assumes that the biological materials in the cochlea are electrically resistive and isotropic, element capacitances and anisotropies could be incorporated into a three dimensional model. The measurements, however, support the hypothesis that the major current pathways during stimulation of scala tympani electrodes are resistive.

Anatomical data suitable for model development are available as temporal bone sections. Using CAT scan images of implanted cochleas, it may be feasible in the future to establish the site of implantation for the electrode array. Hence, if differences exist in sites of implantation across subjects, the model could be fit individually for each implanted subject.

A three dimensional, electro-anatomical model can be defined, formulated and solved on a typical scientific minicomputer. For example, a $64 \times 64 \times 37$ element model (with element dimensions of 150 \(\mu\text{m}\) by 180 \(\mu\text{m}\) by 200 \(\mu\text{m}\)), requires approximately 17 Mbytes of secondary storage and 6 hours for solution on a VAX 11/750.
The storage required (as well as the solution time, based on the solution of \(64 \times 64\) and \(128 \times 128\) models) increases linearly with the number of elements in the model.

Higher resolution models are required to ascertain the current densities in the vicinity of excitable tissue. Using the results of the low resolution models (based on methods described in Chapter IX), high resolution solutions for restricted regions of special interest can be obtained within practical computational constraints.

The data entry procedure (during which the model geometry is defined) takes approximately 3 weeks. Once the anatomical data are entered, the rest of the modeling procedure (formulation and solution) is automated.

Electro-anatomical models for monopolar stimulus configurations predict an asymmetric distribution of potential along the length of the scala tympani. The potential gradients are highest at the basal end of the scala and reach a plateau at the apical end. The magnitude of the current density is similarly elevated at the base of the scala. These results are due to the boundary conditions imposed in the monopolar models. The boundaries of the monopolar models are insulated with the exception of the nerve trunk at one model surface (base), which is grounded. Hence, the ground is situated physically and electrically (because of the high conductance pathway through the auditory nerve) closer to the basal end of the scala tympani. Current injected more apically flows to the base of the model, taking the path of least resistance through the cochlear fluids. Therefore, it is not surprising to see elevated current densities at the base of the scala tympani.

The distribution of potential in the model is largely a function of the imposed boundary conditions and the high conductivity of the cochlear fluids with respect to the conductivities of other tissues. Changing the bone or fluid conductivities by a factor of two does not significantly alter the distribution of potential and current density along the length of the scala tympani. As long as the conductivity of the cochlear fluids remains high with respect to the surrounding bone conductivity, the potential and current density distributions retain their characteristic shapes.

Potential measurements made in the implanted cochleas of 5 human subjects agree qualitatively (e.g., in the shape of the characteristic potential curves) and quantitatively (e.g., range of inter-electrode length constants) across subjects for similar stimulus configurations. This suggests similarities in cochlear anatomy and site of implantation across subjects.

The monopolar curves derived from the measurements are asymmetric for current injected centrally (3-8) and at the base (6-8). For example, potential gradients are
greatest at the base, while potential does not differ significantly the apex.

The normalized results of the electro-anatomical model compare well with the characteristic potential curves derived from the potential measurements made in the 5 subjects. Of the four models considered (the homogeneous volume conductor, uniform and terminated transmission lines, and the electro-anatomical model), only the electro-anatomical model predicts the asymmetric distribution of potential along the scala tympani for the three monopolar configurations considered. Also, the electro-anatomical model predicts the two pseudo-bipolar configurations. Hence, the monopolar measurements are consistent with an electrical model of the cochlea with the ground located electrically and physically closer to the base than the apex.

The electro-anatomical model predicts current densities at the source electrodes that are 2–10 times higher than those computed analytically based on stimulus intensities and surface area of the electrodes. However, it is likely that the current densities are overestimated in the model due to the type of electrode representation used (i.e., a point source.) Furthermore, it is likely that the current density is underestimated in the analytical calculations because the effective surface area of an electrode is smaller (e.g., due to obstruction by electrode leads) than computed from the diameter of a ball electrode. Given these factors and the nature of the electro-anatomical model (e.g., a first order approximation), the current densities predicted by the model and by computations based on electrode surface area seem to be in reasonable agreement.

In addition to cross-turn current pathways, the "bumps" in the model traces (Chapter VI) and the deviations in the characteristic potential curves for stimulus 6-8 (Chapter VII) suggest the presence of indirect current pathways linking the different cochlear turns.

Several aspects of the present model should be investigated further. For example, it is not clear from the present analysis whether point source electrodes form an adequate representation of the full electrode array. Also, the effects of different electrode geometries and configurations (e.g., closely spaced, bipolar electrodes) should be investigated.

A possible current pathway that is not included in the present electro-anatomical model is the blood vessel system of the cochlea. The magnitude of this pathway is likely to be small when compared to the large (in volume) auditory nerve pathway. Furthermore, the blood vessel system in the modiolus is probably represented adequately in the electro-anatomical model by lumping the nerve tissue and blood vessels together. However, the effect of the blood vessel pathway from the spiral ligament to ground should be investigated. This can be accomplished by grounding nodes in the
spiral ligament.

Other aspects of the model that should be investigated include minimizing the overhead in model definition and solution that will result from incorporating element capacitances and anisotropies. Numerical methods which may decrease the computational requirements (e.g., the multigrid methods mentioned in Chapter IX) should also be examined.

While the present model has sufficient resolution to compute the current or potential distribution along the length of the scala tympani, higher resolution models should be computed to ascertain the potential and current density distributions in the vicinity of excitable tissue (e.g., nerve tissue in the osseus spiral lamina or the modiolus.) Such models could be formulated using the methods discussed in Chapter IX. In this fashion, the potential distribution along the peripheral nerve fibers could be used to model the nerve excitation function. Psychophysical experiments should then be used to validate the results of the high resolution models. For example, masking experiments could be used to derive the effective fields of each of the implanted electrodes. These fields could then be compared directly with the current density functions predicted by the model. Different current density functions could then be produced by combining the functions of several electrodes. Similarly, a desired neural excitation function could be “deconvolved” into individual electrode components.

In this fashion, the electro-anatomical model can become a powerful tool in cochlear implant research. Combined with electrical and psychophysical measurements from implanted subjects, the numerical model can provide a clearer picture of the electric field produced during stimulation of intracochlear electrodes. The electric field is the first step in the chain of events leading to neural excitation, and eventually, the subject response. A better understanding of this important step can only lead to more effective and more efficient cochlear implant systems.
Appendix

Model Data Structures and Software

A.1 Introduction

This appendix describes in greater detail the data structures used in manipulating the anatomical and electrical data, and the software (the PreConditioned Conjugate Gradient (PCCG)) algorithm used to solve the electro-anatomical model. All the software was developed on a VAX 11/750 minicomputer under the 4.2 BSD UNIX operating system in the C programming language. Some of the data entry and display software requires the use of the IRIS graphics display terminal.

A.2 Data Structures

The anatomical and electrical data for the complete three-dimensional model are stored as a set of meshes or two-dimensional data arrays, with two data arrays (resistivity and potential data) per anatomical section. Each model is assigned a name and the complete set of data resides in a directory (called \textit{name}). Each directory contains a header file (\textit{name.head}) which holds specific information about the model. The header file is 1024 bytes long and has the following fields (all data are stored as short integers in binary representation):
<table>
<thead>
<tr>
<th>location (bytes)</th>
<th>data</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>top plane</td>
<td>currently not used</td>
</tr>
<tr>
<td>2</td>
<td>rank x</td>
<td>number of nodes in x dimension</td>
</tr>
<tr>
<td>4</td>
<td>rank y</td>
<td>number of nodes in y dimension</td>
</tr>
<tr>
<td>6</td>
<td>default resistivity</td>
<td></td>
</tr>
<tr>
<td>8 - 127</td>
<td>currently not used</td>
<td></td>
</tr>
<tr>
<td>128 - 383</td>
<td>distance to next plane</td>
<td>fixed at 200 μm in the present model</td>
</tr>
<tr>
<td>384 - 639</td>
<td>currently not used</td>
<td></td>
</tr>
<tr>
<td>640 - 895</td>
<td>model section† number</td>
<td>(for up to 128 planes)</td>
</tr>
<tr>
<td>896 - 1023</td>
<td>currently not used</td>
<td></td>
</tr>
</tbody>
</table>

For every model section (or plane)† there exists two files: one containing the resistivity data and the other the potential data. The potential array has rank $M$ and contains the potentials computed for every node in that particular model section; the resistivity array has rank $M-1$ and contains the resistivity values (or resistivity index values — see pccgdf program description) for model elements between plane numbers $n$ and $n+1$. The elements in the last column of every row as well as the last row of the resistivity array are not used. Note that in the present model, all data arrays are square.

The data arrays are called planen.IMP for the resistivity data and planen.POT for the potential data, where $n$ is the model plane number. The resistivity data are stored as short integers (16 bits per value in the present system) while the potential data are stored as single precision floating point numbers or floats (32 bits per value) – even though the solution may be carried out in double precision. The anatomical data defined at the highest RASTER resolution ($512 \times 512$ meshes) are stored as bytes (8 bits per value). Each byte specifies one of 256 anatomical tags.

† Section refers to the temporal bone section. These are numbered as in the temporal bone description (see Chapter V). Thus section numbers differ by 10 (i.e. 200 μm.) Plane refers to the model mesh number. Plane numbers differ by 1.
A plane may also be assigned a source file called planen.b. The source file contains an ASCII text (unlike the other files, which contain binary data) description of the location and strength of active sources. Each line in the planen.b file describes a single source described by the following fields (fields are separated by blanks or tabs):

<table>
<thead>
<tr>
<th>field name</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>column of source</td>
</tr>
<tr>
<td>Y</td>
<td>row of source</td>
</tr>
<tr>
<td>strength</td>
<td>source amplitude</td>
</tr>
<tr>
<td>type</td>
<td>type of source (see below)</td>
</tr>
</tbody>
</table>

Currently, two types of sources are supported by the pccgdf program: a fixed potential at node (X,Y) and a current source at node (X,Y). If the type field is left blank, this signifies current injection at node (X,Y) while a potential source is indicated by a "p" in the type field. Note that in the model, a current sink is specified as a source of zero potential.

The model file types are summarized below:

<table>
<thead>
<tr>
<th>file name</th>
<th>description</th>
<th>data type</th>
</tr>
</thead>
<tbody>
<tr>
<td>name.head</td>
<td>single header file per model</td>
<td>1024 bytes</td>
</tr>
<tr>
<td>planen.pot</td>
<td>potential data for plane n</td>
<td>floating point</td>
</tr>
<tr>
<td>planen.imp</td>
<td>resistivity data for plane n</td>
<td>short integers</td>
</tr>
<tr>
<td>planen.b</td>
<td>source file for plane n</td>
<td>ASCII text</td>
</tr>
</tbody>
</table>

A.3 Data Display and Manipulation Utilities

This section contains short descriptions of the programs used to manipulate and display the anatomical and electrical data. The name of the program is specified in **bold** and the arguments in *italics*. (Additional options or arguments are also indicated in **bold**.)
addone [–options] < source_file > out_file

Increase the rank of the source_file (resistivity only) by 1. This is done by padding the last column and rows with zero elements. Command line options are:

- b       the source_file contains byte data (default is short integers).
- rnnn    set rank of source_file to nnn.

clearp [–options] name

Clear the specified plane in directory name, setting all the potential nodes to zero (or all the resistivity elements to 1.) By default, the potential data file is cleared. Command line options are:

- pnnn    clear plane number nnn.
- rnnn    set rank of matrix to nnn (must be specified).
- z       clear resistivity data

current [–options] name > out_file

Compute the current density from the potential solution. This program can be used in two ways. The first (default) mode calculates the current density for the complete model section. In this mode, the section is taken from model directory name, and the computed current density array is written to file out_file. The command line options are:

- pnnn    set plane number to nnn
- xnnn    set the mesh x sampling interval to nnn
- ynnn    set the mesh y sampling interval to nnn
The second **current** mode allows the calculation of current densities in individual model elements. It is invoked using the single `-v` option, i.e.:

```
current -v name < in_file
```

Here, `in_file` contains the position(s) (model section, `x`, `y`) of model element(s) (one per line), while `name` specifies the model directory.
**differ** [--options] *file1* *file2* > *out_file*

Compare the potentials from two arrays, stored in *file1* and *file2*. Several comparison modes are available. These are:

<table>
<thead>
<tr>
<th>mode</th>
<th>calculation performed</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFFER</td>
<td>$\left[ \text{file1} - \text{file2} \right]$</td>
</tr>
<tr>
<td>DIVIDE</td>
<td>$\left[ \frac{\text{file1}}{\text{file2}} \right]$</td>
</tr>
<tr>
<td>PERCENT</td>
<td>$\left[ \frac{\text{file1} - \text{file2}}{\text{file1}} \right] \times 100.0$</td>
</tr>
</tbody>
</table>

The calculations specified are performed for every pair of nodes in the two arrays. All calculations yield the absolute value of the operation performed. If the two arrays differ in size, the calculations stop at the smaller node count. When all the node pairs are processed, **differ** generates the following statistics:

- the number of node pairs compared
- square root of sum of squares of all calculations
- the average of all calculations
- standard deviation of all calculations
- the maximum of all calculations
- the sum of all values in *file1*
The command line options for **differ** are:

- `-v` set verbose mode (useful for debugging)
- `-d` set DIVIDE mode of calculation
- `-p` set PERCENT mode of calculation
- `-o` generate only the statistics
- `-r` raw format statistics (suppress headers and tabulate)

**down** [–options] < input_file > output_file

Convert by regular decimation ("downsample") the resistivity plane files. The data are downsampled from the high resolution *input_file* to the low resolution *output_file* by a factor of 2 in each dimension. The following downsampling modes are available:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL</td>
<td>choose a maximum from 4 adjacent elements</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>average of 4 adjacent elements</td>
</tr>
<tr>
<td>DOWN</td>
<td>downsample by taking every second element in every second row.</td>
</tr>
<tr>
<td>FILTER</td>
<td>a weighted average of 9 adjacent elements. Central element weighting is set by the -m option, and averaged values are clipped to the limit set by the -l option.</td>
</tr>
</tbody>
</table>
The default downsampling mode is NORMAL. If a look-up file is specified, the default is set to AVERAGE mode. The command line options for down are:

- **-cnnn** specify the resistivity look-up file
- **-lnnn** set the clipping limit to nnn. This option is used to implement the super-high resistivity clipping discussed in the text (Chapter V)
- **-mnnn** set the averaging filter central value to nnn/100
- **-rnnn** set input matrix rank to nnn
- **-v** set the verbose mode (useful for debugging)
- **-a** set AVERAGE mode
- **-o** set FILTER mode
- **-d** set DOWN mode

**downp** [–options] < input_file > output_file

Downsample potential plane files. The data are downsamplied from the high resolution input file to the low resolution output file by a factor of 2 in each dimension. Hence, a 65 × 65 potential array is downsamplied into a 33 × 33 potential array. Two downsampling modes are available:

<table>
<thead>
<tr>
<th>mode</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NORMAL</td>
<td>decimate the input file by taking every second sample in every second row. The boundary nodes are conserved. This is the default mode.</td>
</tr>
<tr>
<td>AVERAGE</td>
<td>average four node potentials in the input file to produce a single output node.</td>
</tr>
</tbody>
</table>

The command line options are:

- **-a** set AVERAGE mode
- **-rnnn** specify the array rank
- **-v** set verbose mode (useful for debugging)
getpot [–options] name < input_file > output_file

Extract potentials from model data arrays. The model directory is specified by name. The input_file specifies the desired nodal positions (model section, x, y), one node per line. If the section number is specified in the command line (using the -p option), the potential for the single node (location specified by the -x and -y options) is extracted. The command line options are:

-a average the four nodes to generate the final result. This option is used when the potential file actually contains current density data (see current).

-dxnnn specify the model x spatial sampling interval (default is 75 μm)

-dynnn specify the model y spatial sampling interval (default is 45 μm)

-dznnn specify the section thickness (default is 20 μm)

-m generate maximum statistics (see below)

-pnnn single node plane number

-xnnn single node x position

-ynnn single node y position

-v verbose mode

The following statistics are generated for each node (on a single line of output, separated by tabs):

• section number
• plane number
• incremental count of nodes
• node y location
• node x location
• distance from first node
• the node potential
If the maximum mode is set (-m), additional statistics are generated on the same line:

- x location of maximum potential in the nodal plane
- y location of maximum potential in the nodal plane
- the maximum node potential
- percentage deviation from maximum at the current node

**header name**

Edit a model header file. Run-time commands are:

- r change rank
- h help
- q quit
- a add to header
- d delete from header
- u update header
- n new header
- i print information

**info name**

Print out information (rank, plane, section numbers) about model *name*. 
**make_res** < source_file > out_file

Generate a binary resistance file (to be used in generating model resistivity meshes from anatomical data – see **down**) from an ASCII text source_file. Up to 256 different resistivities can be specified in the source_file. Each line of the file contains two fields: the index [0→255], specifying the anatomical tag, and a numerical value for the material resistivity. A negative index specifies the default (typically, bone) resistivity. The default resistivity is then assigned to all non-indexed entries in the binary resistivity file. A resistivity value of zero implies that the element is insulated.

**maximum** name

Display the maximum potentials and their locations in model name.
show [-options] name

Display plane data on standard output. Default display file is plane1.pot in
the named directory. The command line options are:

-d assume potential data stored using double precision (64 bits per value)
-e convert 0 potentials to 1E-20 (for log plotting)
-g calculate and display the magnitude of the potential gradient from
    potential data
-j calculate and display the magnitude of the current density from po-
    tential and resistivity data
-l LOCAL display mode; file name is assumed to be in the current
directory
-m display the middle row (unless otherwise specified by -x or -y)
-pnnn show plane number nnn
-rnnn set the rank of array (only needed for LOCAL mode)
-v take a vertical slice from the whole model
-yynnn set the row to be displayed
-xnnn set the column to be displayed
-t tabulate data
-n if midline, print the row or column index
-z display resistivity data (default is potential)
make_plane [−options] name < source_file

Create files for a new model plane. If directory name exists, update header file. Otherwise, create a new directory and header file. Source_file may contain the following (field name followed by the numerical value, nnn):

<table>
<thead>
<tr>
<th>field</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>plane nnn</td>
<td>plane number</td>
</tr>
<tr>
<td>size_x nnn</td>
<td>number of columns</td>
</tr>
<tr>
<td>size_y nnn</td>
<td>number of rows</td>
</tr>
<tr>
<td>neighbor nnn</td>
<td>distance to next plane</td>
</tr>
<tr>
<td>default_imp nnn</td>
<td>default resistivity</td>
</tr>
</tbody>
</table>

If the source_file is not specified, distance to next plane and the default resistivity are both set to 1. Some parameters may be described by the following command line options:

- pnnn set plane number to nnn
- rnnn set rank of data matrices to nnn

upsamplez -rnnn < input_file > output_file

Interpolate ("upsample") the resistivity data by a factor of 4. Each element in the input_file is quadruplicated in the output_file. The -r option specifies the rank of the input_file.

A.4 dsp: Software to Display and Edit Model Results and Data
Usage: dsp [−options] name

dsp is a general purpose program used to display two dimensional data arrays on the IRIS graphics terminal. Its primary purpose is to edit and display electro-anatomical model data and results. dsp can also be used to graphically display any other data organized as two dimensional arrays. The data are displayed as two dimensional grids, with each data point represented by a rectangular element. A gray scale shade or a pseudo-color is assigned to the element, based on the value of that element represents. dsp can display data arrays ranging in rank from 1 to 1024, which
can be in byte, short integer or floating point formats. A version that can display double precision, floating point data is also available. Once the data is displayed, it can be graphically edited, normalized, and displayed with a different colormap for contrast enhancement. Other dsp features include the ability to draw lines and circles (used to generate arbitrary geometrical models), a “zoom” mode which is used to expand specific array regions and a graphical display of (equipotential) contours and (current) gradients.

dsp uses the complete IRIS screen for display and the IRIS keyboard and mouse for input. The display is organized into 7 major areas. Text is displayed in the textport, located in the lower part of the screen. A narrow mouseport, located near the top of the screen, presents information about the mouse buttons and active viewport. There are two data viewports: potential data (typically, floats) are displayed in the potential viewport while resistivity or impedance data (typically, shorts) are displayed in the impedance viewport. By default, the potential viewport occupies the largest part of the screen and is located in the left half of the screen. The impedance viewport is located in the upper left half corner of the screen. The roles and sizes of the viewports can be changed during execution. The three final areas of the screen are the header viewport which is displays information about model sections (if a model directory is being displayed — otherwise, it is blanked), the menuport, which provides a user interface to several dsp functions, and the colorbar, which displays the present colormap and number of colors being used. The menuport is located on the right hand side of the screen. The header is located in the upper left corner of the IRIS screen.

The optional argument name in the dsp command line specifies either the model directory (MODEL mode) or the file (LOCAL mode) to be displayed, depending on other command line options. If the directory is specified, the user is prompted for the model plane number. The plane potential and resistivity files are then read in, and the user can display them by selecting the appropriate menu functions. The dsp command line options are:

- **b**
  Sets the binary flag and turns on the LOCAL mode. Also, the potential and impedance viewports are swapped. Hence, the file name contains a byte format array which is displayed in the “old” potential viewport.

- **d**
  Expands the textport to several text lines. The default is a single line of text.
-i  Turn on LOCAL mode. File name contains a short integer format array which is displayed in the potential viewport.

-nnnn  Sets the number of colors to nnn. The default is 256 and the limit is 4096. Furthermore, nnn must be a power of 2.

-p  Turn on LOCAL mode. File name contains a float format array which is displayed in the potential viewport.

-rnnn  Sets the rank of the arrays to nnn. The arrays are assumed to be square. Note that the rank need not be specified to display the model directory (in MODEL mode.)

-rxnnn  Sets the x dimension of the array to nnn.

-rynnn  Sets the y dimension of the array to nnn.

-s  Swap the potential and impedance viewports.

-z  Same as the -i option.

Note that if one of the array dimensions is specified (i.e., either -rx or -ry) but not the other, the two dimensions are equated. Unspecified dimensions in the LOCAL mode will result in error and program exit. The rank, however, need not be specified in the MODEL mode since it’s extracted from the model header file.

Once the data are read in and the color map generated (this may take a few seconds), a variety of functions may be specified by clicking the mouse buttons in a viewport or on a specific menu item. Initially, the data are not displayed (the viewports are blank.) Hence, a display item must be selected from the menu. The functions that can be performed by clicking any mouse button while on the menu item are:
Zoom

Toggle ZOOM mode. In the ZOOM mode, a small portion of the impedance array is displayed in the full potential viewport. When ZOOM mode is toggled on, the user is required to specify the zoom window. This is done by positioning the cursor by moving the mouse into the impedance viewport. Clicking the LEFT mouse button sets the origin of the zoom window. Clicking the MIDDLE mouse button sets the zoom window endpoint. The window can be expanded or shrunk by moving the mouse and clicking on the MIDDLE mouse button to select a new endpoint. Similarly, the origin and thus the whole window can be redefined by clicking the LEFT mouse button. Clicking on the RIGHT mouse button freezes the selected window and displays the zoomed-in data in the potential viewport. The definition procedure can be aborted at any time by pressing the BREAK key. If the ZOOM mode is on, choosing the ZOOM menu item turns it off. The original function of the potential viewport is restored.

Potential

Display the potential data array. In ZOOM mode, the windowed impedance array is displayed.

Impedance

Display the impedance data array.

Update

Update the potential and impedance data files. Any changes, such as those due to viewport editing and gradient calculation, will be saved. Hence the original data array may be changed.

C-Shell

Escape to a C-shell (csh) in the textport.

Quit

Exit the dsp program. NOTE THAT THIS IS THE ONLY CLEAN WAY TO EXIT.

ReadPotential

Read the potential data array into memory (in MODEL mode). In the LOCAL mode, file name is read. This file must contain floating point data!

ReadImpedance

Read the impedance data array into memory (in MODEL mode). In the LOCAL mode, file name is read. This file must contain short integer data!

ClearPotential.

Erase the Potential viewport.
ClearImped.
Erase the Impedance viewport.

NewPlane Request a new plane to be read in (MODEL mode.)

Contour Generate a log scale contour of the potential data in the Impedance viewport.

ColorMap Generate a pseudo-color map

GrayScaleMap
Generate a gray scale color map

MapEntries Enter the number of color map entries. This number must be a power of 2 and less than 4096. Note that the first eight colors [0-7] are left as originally defined in the IRIS colormap.

Overlap Overlap the impedance data on top of the potential display. SPECIAL must be OFF.

Def.Pot.View
Define a new Potential viewport. The windowing procedure is as in zoom. Note that the previous viewport is not erased.

Def.Imp.View
Define a new Potential viewport. The windowing procedure is as in zoom. Note that the previous viewport is not erased.

NormalizePot.
Normalize the potential data. If SPECIAL is ON, the data are normalized by the maximum potential value; if SPECIAL is OFF, the data are normalized by the average value. The maximum is printed in the textport.

NormalizeImp.
Normalize the impedance data. If SPECIAL is ON, the data are normalized by the maximum impedance value; if SPECIAL is OFF, the data are normalized by the average value. The maximum is printed in the textport.

Gradient If SPECIAL is ON, calculate the potential gradient; else, calculate the two dimensional current density using both the potential and impedance data. The direction of calculated vector is displayed. The magnitude of the calculated vector is now stored in the potential data array. (Hence, choosing item Update will
overwrite the potential file.) The magnitude can be displayed by choosing item Potential.

Newdir  Change to a new directory (MODEL mode). The user will be prompted for a new plane number.

Newfile Read in a new file (LOCAL mode). There is no way to switch dimensions once dsp is executed, hence the new file must be of the same dimensions (or larger, with side effects.)

ScaleImp Divide by 10 all impedance values which are greater than 100 and re-display the impedance data. In this fashion, the contrast of the impedance array is enhanced. This operation alters the impedance data array.

EraseToggle Toggle ERASE. If ERASE is ON, every new re-display will clear the viewport.

NextPlane Step to the next model section and read it (MODEL mode).

BackPlane Step to the previous model section and read it (MODEL mode).

DrawCircle Draw a circle in the impedance data array. The user must specify the color (optional), origin x and y positions and the radius. If the color is left unspecified, the previously defined color is used.

DrawLine Draw a line in the impedance data array. The user must specify the color (optional), starting and ending x and y positions. If the color is left unspecified, the previously defined color is used.

Special Toggle the SPECIAL flag.

Reverse Toggle the REVERSE flag. If REVERSE is ON, the viewport origin is moved to the upper left half corner. The default for the origin is in the lower left half corner.

Several functions can be performed while the cursor is in one of the data viewports. These are accomplished by clicking the appropriate mouse buttons. These functions always operate on the array element pointed to by the cursor. The button functions are:
<table>
<thead>
<tr>
<th>button (combination)</th>
<th>function description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEFT</td>
<td>display the element location and value</td>
</tr>
<tr>
<td>MIDDLE</td>
<td>replace the element with SET value (set by RIGHT button)</td>
</tr>
<tr>
<td>RIGHT</td>
<td>define a new SET value</td>
</tr>
<tr>
<td>RIGHT&amp;MIDDLE</td>
<td>&quot;Flood fill&quot; an enclosed (impedance) area with a SET value. Note that the area must be enclosed by boundaries which have higher values than the SET value. The fill can also be used in a zoom window.</td>
</tr>
<tr>
<td>LEFT&amp;MIDDLE</td>
<td>Substitute (impedance) elements specified with a new value. The specified elements are blinked. This command may be aborted by specifying a negative new value.</td>
</tr>
</tbody>
</table>

A.5 pcegdf: Software for Formulation and Solution of Model

Usage: `pcegdf [\--options] name`

Program pcegdf is used to formulate (using a finite difference scheme) and solve the three dimensional, electro-anatomical model. The resultant system of linear equations is solved iteratively using using the PCCG (PreConditioned Conjugate Gradient) algorithm [Axelson and Barker, 1985]. The main stages in the pcegdf program are as follows:
<table>
<thead>
<tr>
<th>stage</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialization</td>
<td>Initialize (via command line arguments) various model parameters. These are documented below.</td>
</tr>
<tr>
<td>Build $H$</td>
<td>Assemble the conductivity matrix ($H$), using the model resistivity data files. Here, the finite difference scheme is applied.</td>
</tr>
<tr>
<td>Solve Model</td>
<td>The longest part of the program. Here, the $PCCG$ algorithm is used to iteratively solve the electro-anatomical model equations.</td>
</tr>
<tr>
<td>Update Model</td>
<td>Once the model solved, the data may be written back to the model directory.</td>
</tr>
</tbody>
</table>

The standard $pccgdf$ program assumes the following defaults\(^\dagger\): the potential data are stored using floating point precision and the resistivity data are stored as short integers. All internal calculations, including temporary data storage, are done in double precision. The command line options are:

- **-bnnn** Sets the size of the memory block allocated for any single data array to $nnn$. See below for memory requirements. The default is likely to be too small for most electro-anatomical models.

- **-ennn** Sets stopping criteria ($\varepsilon$) to $nnn$. $pccgdf$ stops when $\delta I \leq \varepsilon$. See the $PPCG$ algorithm description. Default $\varepsilon$ is equal to 0.001.

- **-i** Turn on insulation flag. For each model mesh, the boundaries are insulated. Note that this does not mean that the whole model is insulated since the first and last planes specify the model end-plane potentials. To insulate the whole model, the resistivity of the first and last planes should be set to 0 (see $make\_res$).

- **-pnnn** Sets the starting plane number to $nnn$. The previous plane ($nnn-1$) must exist. The default starting plane is plane 1 (plane 0

\(^\dagger\) Other format versions are also available. For example, $pccg3ff$ performs all calculations in floating point. Also, a two dimensional version is available ($pconj$) with identical arguments. $pconj$ does not require secondary (disk) storage.
must exist in this case.)

-rn
 Sets the number of planes, exclusive of first and last planes, to be included in the model to nnn. Default is 1.

-s
 Turn on statistics flag. At every iteration, generate the following statistics: iteration number, square root of preconditioned gradient dot product ($\sqrt{\delta I}$), the greatest element in the gradient vector (g) and the step size scalar $\tau$. See the PCCG algorithm description for details. This option is very useful for tracking long jobs by redirecting output to an error file. Default is off.

-v
 Set the write flag to write potentials back into the model directory upon the solution of the model. Default is off — if flag is not set, the model will not be updated.

The PCCG Algorithm

The PCCG algorithm is as follows:

1. $x := x^0$
2. $g := b$
3. $g := F^{-1}(Hx - g)$
4. $x := F^Tx$
5. $h := Gg$
6. $d := -h$
7. $\delta 0 := g^Th$
8. IF $\delta 0 \leq \varepsilon$ THEN GOTO 23.
9. $e := F^{-T}d$
10. $h := Ee + d$
11. $h := F^{-T}h$
12. $h := h + e$
13. $\tau := \delta 0 / d^Th$
14. $x := x + \tau d$
15. $g := g + \tau h$
16. $h := Gg$
17. $\delta 1 := g^Th$
18. IF $\delta 1 \leq \varepsilon$ THEN GOTO 23.
19. \( \beta := \delta_1 / \delta_0 \)
20. \( \delta_0 := \delta_1 \)
21. \( d := -h + \beta d \)
22. GOTO 9.
23. \( x := F^{-T}x \)

STOP

\( x^0 \) and \( b \) are the input vectors (the potential and source data, respectively), and \( x^0 = F^{-T}G(F^{-1}b) \). Matrices \( G, F \) and \( E \) are derived from the model conductivity matrix, \( H \). (\( H \) is of rank \( N \times N \), where \( N \) is the total number of nodes in the model. Each row of the \( H \) matrix has at most seven non-zero entries, representing the conductances of the central node and its six neighbors. See Chapter V for details.) For SSOR (Symmetric Successive Over Relaxation) preconditioning, the conductivity matrix can be decomposed as follows:

\[ H = D + L + L_T, \]

where \( D \) and \( L \) are the diagonal and lower triangular parts of \( H \), respectively. Then:

\[ F = (1/\omega)D + L, \quad G = (1/\omega)D, \quad E = [1 - (2/\omega)]D, \]

where \( \omega \) is the SSOR relaxation factor, ranging from 1 to 2. The solution \( (x) \) and auxiliary vectors \( (e, d, g, h) \) are all of dimension \( N \).

**Modeling Overhead: Memory and Time Required for Solution**

The present version of \texttt{pcegdf} requires temporary storage during model solution for the solution and auxiliary vectors, and the conductance matrix. If \( N \) is the total number of nodes in the model, each vector has dimension \( N \) and the conductance matrix is dimension \( N \times 7 \) (only the non-zero entries are stored.) Using floating point (4 bytes per value) representation for the conductance matrix \( H \) and double precision (8 bytes per value) for the solution and auxiliary vectors, the secondary storage requirements are:

\[ \text{total bytes} = 5 \times N \times 8 + 7 \times N \times 4. \]

\[ = 68 \times N \]

This space must be available on disk. The program \texttt{pcegdf} also requires a smaller segment of primary memory during execution. The minimum segment allocated (set by -b
command line option) must be equal to the memory required for a conductance matrix of a single mesh, or $7 \times M \times 5$ ($M$ is the number of nodes in a single mesh.) The memory required and typical times for model solution are shown below for two electro-anatomical models. Both models are composed of 37 meshes.

<table>
<thead>
<tr>
<th>Model (mesh)† Resolution</th>
<th>Disk Space</th>
<th>Memory Allocated</th>
<th>Time of Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 × 64</td>
<td>10 630 kbytes</td>
<td>118 300 bytes</td>
<td>≈ 6 hours</td>
</tr>
<tr>
<td>128 × 128</td>
<td>41 869 kbytes</td>
<td>465 948 bytes</td>
<td>≈ 40 hours</td>
</tr>
</tbody>
</table>

The solution times are for error criterion ($e$) equal to $10^{-10}$, which is greater than the precision (floating point) used for the stored data. The computations, however, occur in double precision.

A.6 A Typical Modeling Session

The main stages in a typical modeling session are described in the table below.

† The resolution stated is for model elements. All data, however, are stored in arrays appropriate for model nodes. Hence, the real size of these meshes (to be used in calculating the memory requirements) is 65 × 65 and 129 × 129.
<table>
<thead>
<tr>
<th>Stage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>DATA ENTRY</td>
<td>Define the anatomical data of the model. Includes the generation of CAA format data which must be then transformed to the model RASTER format.</td>
</tr>
<tr>
<td>MODEL DEFINITION</td>
<td>The anatomical data is downsampled into a model suitable for solution. The resistivity of the model elements and active sources in the model are defined.</td>
</tr>
<tr>
<td>MODEL SOLUTION</td>
<td>The model is formulated and solved during this stage</td>
</tr>
<tr>
<td>ANALYSIS OF RESULTS</td>
<td>The model results are displayed, or re-calculated (e.g., for current density) for analysis.</td>
</tr>
</tbody>
</table>

**Data Entry and Model Definition**

The anatomical data are resident in the highest resolution RASTER meshes (512 × 512) as byte format, two dimensional arrays. Each RASTER array represents a single cochlear section. The array elements are tagged by name, representing the biological materials of the cochlea. The anatomical data must be converted from this format into a format suitable for electro-anatomical model solution. This involves transforming the anatomical data (byte) into resistivity data (short integers). Furthermore, the meshes must be downsampled to a size that can be handled by the pccgdf program.

The resistivity file is built from a source file using the program make_res. (A typical source file is shown in Table 5.3). The resistivity file (res6) is generated by:

```
make_res < res6.data > res6
```

where res6.data is the source file.

The 512 × 512 anatomical meshes may now be downsampled. Assume that a 64 × 64 model is desired. The following shell script will downsample the anatomical data from the anatomical data mesh (section171) into the model (plane17.imp) mesh of desired resolution.

```
addone -r512 -b < section171 > temp.513
down -r513 -o -m3000 -cres6 < temp.513 > temp.257
down -r257 -l32000 -o -m3000 < temp.257 > temp.129
```
down -r129 -l5000 -o -m3000 < temp.129 > g_65/plane17.imp
rm temp.257 temp.129

The destination model name is g_65. Note that this script must be executed once for every model section.

Model Solution

Once the model is downsampled, and the sources defined, it can be solved. This is accomplished by:

\[ pccgdf \ -s \ -v \ -i \ \text{-ttempx} \ -p4 \ -r37 \ -b500000 \ -e10e-10 \ -w1.6 \ -x150 \ -y90 \ g_65 \ > \ stat\_file \]

Analysis of Results

Once the model is solved, the results can be displayed using programs dsp or show. For example, getpot can be used to extract a more specific potential distribution (e.g., along the length of the scala tympani):

\[ \text{getpot} \ -m \ -dx150 \ -dy90 \ g_65 \ < \ \text{spiral}_65 \ > \ g65\_pot \]

File spiral_65 contains a set of nodes describing the length of the scala tympani.

Adjusting Model Results to Physical Stimuli

The spatial units in the electro-anatomical are specified in \( \mu \text{m} \) (10\(^{-6}\) m). Resistivity values are always specified in ohms-cm. The electrical units, however, are arbitrary, because the model solutions can be scaled to the source strength. For example, the solution (i.e., all the node potentials) of a model with a “source” current strength set to 10 is equal to 10 times the solution of a model with a “source” current set to 1.

The current densities (as computed by current, for example) need be adjusted in order to correspond to the scaled model potentials. For example, assume the model solution yields potentials in mV. The current density is equal to:

\[ J_x = \frac{1}{\rho} \frac{\Delta \Phi}{\Delta x} \]

where \( \rho \) is the resistivity (ohms-cm), \( \Phi \) is the potential (mV), and \( \Delta x \) is a distance.
parameter (μm). Then, given the model parameters, the units of current density are:

\[
\text{current} = \frac{1}{10^{-2} \text{ ohms-m} \times 10^{-6} \text{ m}} \times 10^{-3} \text{ V} = 10^5 \frac{A}{m^2} = 10^5 \frac{\mu A}{mm^2}
\]

Hence, the results computed by current need to be multiplied by 10^5 to correspond with the model potentials being expressed as mV and current densities being expressed as μA/mm^2.
References


Wilson, B. S. and Finley, C.C. (1984 a,b,c,d) Speech Processors For Auditory Prosthesis, Second-Fourth Quarterly Progress Reports, NINCDS, NIH.