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A SYSTEMS ANALYSIS OF THE COPPER AND ALUMINUM INDUSTRIES

AN INDUSTRIAL DYNAMICS STUDY

by

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B.S., United States Naval Academy (1951)

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1961

Signature of Author.

Certified by. . . .
2 May 1961

Mr. Kenneth J. Schlager  
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West Newton, Massachusetts  

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Massachusetts Institute of Technology
Cambridge 39, Massachusetts

May 15, 1961

Dear Professor Franklin:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "A Systems Analysis of the Copper and Aluminum Industries, An Industrial Dynamics Study."

I wish to express my gratitude to the many members of the faculty for their guidance and helpful suggestions in preparing this study. In particular, I would like to express my appreciation for the direction and assistance given by Professor Jay W. Forrester, chairman of my thesis committee. I would also like to thank Professor Morris A. Adelman, the second member of my committee, for his helpful economic insights. This work was done in part at the Massachusetts Institute of Technology Computation Center. I am grateful for the use of this facility and for the assistance of its personnel.

This thesis would not have been possible without the assistance of the Aluminum Company of America and the Reynolds Metals Company. To them, I also give my thanks.

I also want to express my appreciation to my wife, Lois, who did all of the drawings, and helped in so many other ways. My final expression of gratitude goes to Mrs. Hazel Bright, the typist, who performed far above normal standards in completing the manuscript so well and on time.

Sincerely,

Kenneth J. Schlager
A SYSTEMS ANALYSIS OF THE COPPER AND ALUMINUM INDUSTRIES
AN INDUSTRIAL DYNAMICS STUDY

by

Kenneth J. Schlager

Submitted to the School of Industrial Management on May 15, 1961 in
partial fulfillment of the requirements for the degree of Master of
Science in Industrial Management.

ABSTRACT

The prime objectives of this thesis are three in number:

1. To explain the present characteristics of the copper and
   aluminum industries by an investigation of the industrial
   structure and managerial policies of these industries.

2. To construct Industrial Dynamics models to simulate the
   operation of these industries and to use these models to
   determine improved managerial policies for individual
   companies within these industries.

3. To determine the effects on the company and the industry of a
   company pursuing these improved managerial policies in a
   competitive environment.

These two industries were selected for analysis because they
provided a contrast in past history and performance that served to ex-
pose the fundamental characteristics of Industrial Dynamic Behavior.
Copper, as an industry, has been characterized by extreme instability
of both prices and production while aluminum, in contrast, has
established a record of stability and growth unrivalled in the primary
metals group.

The mathematical models representing the dynamic operation of the
two industries are presented. These models are first described in
general outline and later in detail within the mine, concentrator,
smelter, refiner, fabricator and user sectors making up the industries.
The basic industrial structures and current and proposed policies are
outlined in each of the sectors. The results of simulating a variety
of new managerial policies on the stability and profit performance of
the industries are then described.
Specific production and pricing policies are recommended for the copper industry as a consequence of results obtained from model simulation runs. These policies are capable of being implemented by a single integrated producer such as Kennecott Copper Corporation, producing 15 per cent of the annual world supply of copper. Use of the recommended policies will serve to stabilize the industry for long-term profitable growth with no short-term profit penalties to the initiating firm. The specific parametric values to be used in the policies must be selected after a more detailed consideration of the cost structure and product-market mix of the initiating organization.

Simulation efforts in the aluminum industry were aimed at the development of production, distribution, service and pricing policies for a company in competition with other producers. The results of testing a variety of managerial policies in the model indicated that the interrelationships of the above policies are quite critical in the profitability of the company. The design of new policies must be based upon detailed consideration of the cost structure and product-market mix.

Both models are now ready for application to individual companies in the copper and aluminum industries. Although a great deal of further work will be required in adapting the model in detail to a company, results obtained in use of the models to date indicate their tremendous potential for managerial policy improvement and stable and profitable industrial growth. Interest expressed by companies in the industries indicates that such application will be an actuality in the near future.

This work was done in part at the Massachusetts Institute of Technology Computation Center, Cambridge, Massachusetts.

Thesis Advisor: Jay W. Forrester
Title: Professor of Industrial Management
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>General</td>
<td>1</td>
</tr>
<tr>
<td>Objectives and Program of Thesis</td>
<td>7</td>
</tr>
<tr>
<td>General Conclusions</td>
<td>9</td>
</tr>
<tr>
<td>II. DESCRIPTION OF THE COPPER AND ALUMINUM INDUSTRIES</td>
<td>10</td>
</tr>
<tr>
<td>General</td>
<td>10</td>
</tr>
<tr>
<td>Technology</td>
<td>10</td>
</tr>
<tr>
<td>The Market</td>
<td>13</td>
</tr>
<tr>
<td>Management</td>
<td>16</td>
</tr>
<tr>
<td>III. THE STRUCTURE AND POLICIES OF THE INDUSTRIES</td>
<td>19</td>
</tr>
<tr>
<td>Copper Industry</td>
<td>19</td>
</tr>
<tr>
<td>Aluminum Industry</td>
<td>29</td>
</tr>
<tr>
<td>IV. THE MINES SECTOR - COPPER AND ALUMINUM</td>
<td>37</td>
</tr>
<tr>
<td>General</td>
<td>37</td>
</tr>
<tr>
<td>Copper Mine</td>
<td>38</td>
</tr>
<tr>
<td>Bauxite Mine</td>
<td>51</td>
</tr>
<tr>
<td>V. THE COPPER CONCENTRATOR-SMELTER SECTORS</td>
<td>58</td>
</tr>
<tr>
<td>General</td>
<td>58</td>
</tr>
<tr>
<td>Production Policies</td>
<td>58</td>
</tr>
<tr>
<td>VI. THE ALUMINA REFINER SECTOR</td>
<td>69</td>
</tr>
<tr>
<td>General</td>
<td>69</td>
</tr>
<tr>
<td>Production - Structure and Policy</td>
<td>70</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>VII. THE REDUCTION SECTORS</td>
<td>77</td>
</tr>
<tr>
<td>General</td>
<td>77</td>
</tr>
<tr>
<td>Copper Refiner</td>
<td>78</td>
</tr>
<tr>
<td>Aluminum Reduction Smelter</td>
<td>90</td>
</tr>
<tr>
<td>VIII. THE COPPER AND ALUMINUM FABRICATOR SECTORS</td>
<td>98</td>
</tr>
<tr>
<td>General</td>
<td>98</td>
</tr>
<tr>
<td>Production Policy</td>
<td>99</td>
</tr>
<tr>
<td>Aluminum Pricing and Service Policy</td>
<td>104</td>
</tr>
<tr>
<td>Purchasing Policy</td>
<td>107</td>
</tr>
<tr>
<td>Financial Structure in Fabricator</td>
<td>108</td>
</tr>
<tr>
<td>IX. THE USER SECTOR</td>
<td>109</td>
</tr>
<tr>
<td>General</td>
<td>109</td>
</tr>
<tr>
<td>Purchasing Structure</td>
<td>110</td>
</tr>
<tr>
<td>Production Structure</td>
<td>110</td>
</tr>
<tr>
<td>Financial Structure</td>
<td>115</td>
</tr>
<tr>
<td>Future Development of the User Sector</td>
<td>117</td>
</tr>
<tr>
<td>X. SYSTEM SIMULATION AND MANAGERIAL POLICY TESTING</td>
<td>118</td>
</tr>
<tr>
<td>General</td>
<td>118</td>
</tr>
<tr>
<td>Copper Industry Simulation</td>
<td>119</td>
</tr>
<tr>
<td>Aluminum Industry Simulation</td>
<td>144</td>
</tr>
<tr>
<td>XI. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>154</td>
</tr>
<tr>
<td>General</td>
<td>154</td>
</tr>
<tr>
<td>Copper Industry</td>
<td>154</td>
</tr>
<tr>
<td>Aluminum Industry</td>
<td>157</td>
</tr>
<tr>
<td>Pricing Policy</td>
<td>161</td>
</tr>
<tr>
<td>Recommendations</td>
<td>163</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>PAGE</td>
</tr>
<tr>
<td>----------</td>
<td>------</td>
</tr>
<tr>
<td>A</td>
<td>Mathematical Description of the Copper Mine--165</td>
</tr>
<tr>
<td>B</td>
<td>Mathematical Description of the Bauxite Mine--177</td>
</tr>
<tr>
<td>C</td>
<td>Mathematical Description of the Copper Concentrator--182</td>
</tr>
<tr>
<td>D</td>
<td>Mathematical Description of the Copper Smelter--187</td>
</tr>
<tr>
<td>E</td>
<td>Mathematical Description of the Alumina Refiner--192</td>
</tr>
<tr>
<td>F</td>
<td>Mathematical Description of the Copper Refiner--198</td>
</tr>
<tr>
<td>G</td>
<td>Mathematical Description of the Aluminum Reduction Smelter--208</td>
</tr>
<tr>
<td>H</td>
<td>Mathematical Description of the Copper Fabricator--217</td>
</tr>
<tr>
<td>I</td>
<td>Mathematical Description of the Aluminum Fabricator--227</td>
</tr>
<tr>
<td>J</td>
<td>Mathematical Description of the Metal User--230</td>
</tr>
<tr>
<td>K</td>
<td>Summary of the Copper Industry Computer Simulation Runs--237</td>
</tr>
<tr>
<td>L</td>
<td>Summary of the Aluminum Industry Computer Simulation Runs--257</td>
</tr>
<tr>
<td>M</td>
<td>Distribution Policy--262</td>
</tr>
<tr>
<td></td>
<td>BIBLIOGRAPHY--264</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.</td>
<td>Flow Diagram - Copper Industry - Dual Flow</td>
<td>22</td>
</tr>
<tr>
<td>5.</td>
<td>Flow Diagram - Copper Industry - Multiple Flow</td>
<td>24</td>
</tr>
<tr>
<td>7A.</td>
<td>Flow Diagram - &quot;M&quot; Copper Mine Sector</td>
<td>39</td>
</tr>
<tr>
<td>7B.</td>
<td>Flow Diagram - &quot;Ml&quot; Copper Mine Sector</td>
<td>40</td>
</tr>
<tr>
<td>8.</td>
<td>Copper Mine Supply Curve</td>
<td>43</td>
</tr>
<tr>
<td>9A.</td>
<td>Flow Diagram - &quot;M&quot; Bauxite Mine Sector</td>
<td>52</td>
</tr>
<tr>
<td>9B.</td>
<td>Flow Diagram - &quot;Ml&quot; Bauxite Mine Sector</td>
<td>53</td>
</tr>
<tr>
<td>10A.</td>
<td>Flow Diagram - &quot;C&quot; Copper Concentrator Sector</td>
<td>59</td>
</tr>
<tr>
<td>10B.</td>
<td>Flow Diagram - &quot;Cl&quot; Copper Concentrator Sector</td>
<td>60</td>
</tr>
<tr>
<td>11A.</td>
<td>Flow Diagram - &quot;S&quot; Copper Smelter Sector</td>
<td>61</td>
</tr>
<tr>
<td>11B.</td>
<td>Flow Diagram - &quot;Sl&quot; Copper Smelter Sector</td>
<td>62</td>
</tr>
<tr>
<td>12A.</td>
<td>Flow Diagram - &quot;S&quot; Alumina Refiner Sector</td>
<td>74</td>
</tr>
<tr>
<td>12B.</td>
<td>Flow Diagram - &quot;Sl&quot; Alumina Refiner Sector</td>
<td>75</td>
</tr>
<tr>
<td>13A.</td>
<td>Flow Diagram - &quot;R&quot; Copper Refiner Sector</td>
<td>79</td>
</tr>
<tr>
<td>13B.</td>
<td>Flow Diagram - &quot;Rl&quot; Copper Refiner Sector</td>
<td>80</td>
</tr>
<tr>
<td>14.</td>
<td>Supply-Demand Pricing Curve</td>
<td>86</td>
</tr>
<tr>
<td>15.</td>
<td>History of Copper Prices and Inventories</td>
<td>88</td>
</tr>
<tr>
<td>16A.</td>
<td>Flow Diagram - &quot;R&quot; Aluminum Reduction Sector</td>
<td>91</td>
</tr>
<tr>
<td>16B.</td>
<td>Flow Diagram - &quot;Rl&quot; Aluminum Reduction Sector</td>
<td>92</td>
</tr>
<tr>
<td>17A.</td>
<td>Flow Diagram - &quot;F&quot; Copper Fabricator Sector</td>
<td>100</td>
</tr>
</tbody>
</table>
FIGURE

17B. Flow Diagram - "Fl" Copper Fabricator-----------------------------101
18A. Flow Diagram - "Fl" Aluminum Fabricator--------------------------102
18B. Flow Diagram - "Fl" Aluminum Fabricator--------------------------103
19A. Flow Diagram - Copper User Sector-------------------------------111, 112
19B. Flow Diagram - Aluminum User Sector----------------------------113, 114
20. User Metal Purchase Relationship-------------------------------116
21. Basic Response to Pulse Input----------------------------------121
22. Basic Response to Historical Consumption Inputs-----------------124
23. Response to Variation in Smelter Production Time Delay----------125
24. Response to a Decrease in Refiner's Finished Goods

Inventory----------------------------------------------------------127
25. Response to a Change in Pricing Policy - Pulse Input-----------129
26. Response to a Change in Pricing Policy - Historical Inputs-----131
27. Response to a New Smelter Production Policy--------------------132
28. Response to a New Policy Mix----------------------------------135
29. Competitive Model Response - New Policy Mix--------------------136
30. Competitive Model Response - New Policy Half-------------------137
31. Competitive Model Response - Old Policy Half--------------------138
32. Competitive Model Response - Company---------------------------140
33. Competitive Model Response - Industry---------------------------141
34. Competitive Model Response - New Mining Policy - Company-----142
35. Competitive Model Response - New Mining Policy - Industry-----143
36. Aluminum Model - Basic Response to Pulse Input----------------147
37. Aluminum Model - Quarterly Planning Period---------------------149
38. Aluminum Model - Semi-Annual Planning Period------------------150
39. Aluminum Model - Basic Response to Noise Input----------------151
Chapter I

INTRODUCTION

A. General

This thesis develops a systems analysis of managerial decision-making in the copper and aluminum industries. The prime objective of the study is to determine from an analysis of the existing industrial structure and the prevalent managerial policies the fundamental dynamic characteristics of these industries and to modify these characteristics by new policies designed to better accomplish the objectives of the companies concerned.

Every industry has certain technical and economic structural characteristics that are inherent in the processes of production, distribution, exchange and utilization.¹ At each stage of the industrial process certain operations (production processes) must be performed that require inputs of labor, materials and capital equipment. The material output of each stage must then be transported (distributed) to the next stage for further operations until it is finally transferred by means of a commercial sale (exchange) to the final user. A diagram of this single flow economic process is shown in Figure 1.

The industrial structures of the copper and aluminum industries are analogous to the process characteristics in a chemical process control system in that they involve a series of operations on materials over a period of time designed to transform the original

Figure 1 - Flow Diagram - Single Flow Economic Process
material (ore) into a more useful state (metal). As in a chemical process control system, the basic process characteristics must be well understood as a prerequisite to the design of the system as a whole. Important structural features in these industries include:

1. The number of stages of production and distribution required to provide the final product to the user
2. The time delays and inventories at each stage
3. The magnitude and nature of the value added at each stage.

With the industrial structure determined, managerial policies may be designed to achieve certain objectives desired by firms within the industry. These policies interact with the basic structural characteristics to determine the dynamic operation of the industry. The problems involved in the design of managerial policies parallel those in process controller design. In both applications, the design is directed to achieve certain system dynamic characteristics deemed desirable in view of the objectives of the system.

Most industries, however, are characterized by a number of competing companies with varying structures and policies. The dynamic operation of the whole industry will be the result of the combined interacting structures and policies of all of the firms. A realistic model of a competitive industry must consider the coupled interaction resulting from a multiple firm structure. The simple single flow

process must be altered to form a multiple parallel flow process with competitive coupling between the companies. This multiple flow structure is illustrated in Figure 2.

In order to analyze a system as complex as a competitive industry, it was necessary to develop an Industrial Dynamics computer simulation model\(^3\) for each of the metal industries. These models simulate the flow of materials, information, money and, to a limited degree, capital equipment in the two industries over a period of several years. Marketing, production and financial policies, as actually practiced by operating companies, were used initially in the model. These policies were later modified to determine the effects of alternative policies on production, prices, sales and profit fluctuations in the industries.

This thesis analyzes the operation of an industry as a total system. Copper and aluminum were selected in preference to other industries for a number of important reasons. To begin with, it should be acknowledged that this thesis could not have been pursued in its present form without the foundation provided in a previous thesis written by Ray Wayne Ballmer in 1960.\(^4\) Mr. Ballmer, a mine superintendent for the Kennecott Copper Corporation, constructed an Industrial Dynamics model of the copper industry that has been used as the

---


Figure 2. - Flow Diagram - Multiple Flow Economic Process
basis for the copper model portion of this thesis. Although changes have been made in some sectors of the copper model, especially to introduce multfirm competition, the excellence of the original work made it possible to concentrate attention in this industry on detailed analysis of structural and policy characteristics and the development of stabilization techniques to improve the system. Such concentration was vital to the objectives of this thesis.

The original copper model also served as a useful starting point for the aluminum industry model developed by this writer. Although the final models of the two industries differed to a marked degree, the original experience with the copper model was invaluable in the construction of the second model.

A subsequent but in the final analysis a more important reason for the selection of copper and aluminum rested on the fact that these two industries provided a contrast in past history that served to expose the fundamental characteristics of industrial dynamic behavior. Copper, as an industry, has been characterized by extreme instability in both price and production and has lately displayed all the indications of a "sick" industry. Aluminum, in contrast, has been the wonder of the metals group with a record of stability and growth unrivalled in its field. Of late, the lustre of aluminum has been somewhat tarnished by difficulties resulting from overexpansion, but it still retains its image of progress.

More specifically, it was felt that the structural and managerial policy differences between the industries would serve to highlight the critical characteristics determining dynamic performance. Such differences would serve to focus attention on important fundamentals at an early date, thereby conserving valuable time needed for detailed analysis.

It is the ultimate intention of the writer that this thesis be developed as a practical working tool for managerial policy formation in the industries concerned. Experience with and results from the two models to date indicate that the models with slight modification for individual company characteristics are now ready for application by companies in the industries. Interest expressed by a number of companies in both industries indicate that such application may be close to realization.

B. Objectives and Program of the Thesis

The prime objectives of this thesis are four in number:

1. To determine the technical-economic structural and managerial policy characteristics that explain the difference in dynamic behavior between the copper and aluminum industries.

2. To determine improved managerial policies to increase the efficiency of both industries as reflected in long term return on investment.

3. To determine the effect on the industry of different companies following varying policies with the goal of
discovering the critical size necessary to permit one part of the industry to improve the industry as a whole and to indicate the best policy for a single company.

4. As a special secondary result, to determine the effects of various pricing policies under the competitive conditions in the industries.

In the course of study, the basic technical-economic structures of both industries were studied to determine the production and distribution functions performed at each stage of the process, together with the value added by these functions. The nature of the exchange (sales) process was also examined to determine the factors affecting the flow of the material to the user.

With the basic structures established, policies pursued by management in typical companies were investigated to determine actual practices in the industries. In the aluminum industry, policies of the two largest companies of the "Big Four," Alcoa and Reynolds were investigated. Where the policies of these two companies differed significantly, both policies were later tested in the computer simulation. Policies in the copper industry were largely based upon the thesis of Ballmer, but critical policies were investigated and confirmed or modified through interviews with companies in the industry.

After a significant number of simulation runs designed to test the sensitivity of the model to changes in parameters, new policies
designed to improve stability and profit performance were formulated and tested. These new policies were always examined and evaluated in the light of technological and economic limitations as well as simulation results.

In order that the implications of the use of Industrial Dynamics as a top management working tool could be made clear to executive level personnel, an attempt was made to explain the final results of the thesis in terms of application to actual operations in day-to-day managerial decisions. In this light, special emphasis was placed on the results to be expected if one company with a certain share of the market were to pursue policies indicated as preferable by the model. The final policies recommended in the thesis were selected from a large number of possible policies analyzed. The final policy mix provides a balance between practicality and system improvement.

C. General Conclusions

The construction of the models of the two industries and the subsequent computer simulation testing of managerial policies in these models have allowed the development of improved production, distribution and marketing policies.

In the copper industry the new policies enable one company with a 15 per cent share of the market to stabilize the industry with a long term gain in the profit of the initiating firm.

Improved policies developed for the aluminum industry enable a company to improve its long term profit return with revised decision rules for production, distribution and marketing.
Chapter II

DESCRIPTION OF THE COPPER AND ALUMINUM INDUSTRIES

A. General

The aluminum and copper industries perform a similar function in processing a mined ore into a fabricated metal suitable for use in the metal-working industries. They even compete in some common markets where in general aluminum has replaced, or is replacing copper.

With detailed comparison, however, similarities become less and differences more sharply defined. It would seem difficult to find two other industries with such a common function and yet so unlike in their operating characteristics. The industries differ technologically in their production process, economically in both the productive cost factors and the nature of the market, and managerially as a result of historical background. An all-encompassing difference results from the fact that copper is the oldest metal used by man while aluminum is one of the newest. Copper displays all the characteristics of decline while aluminum is a young, expanding, growth industry.

B. Technology

Copper ore is extracted from the mine using conventional methods of underground or open pit excavation. It is then transported to a concentration mill where the ore is crushed and separated from the waste through a physical flotation process. The output of this process is a copper sulphide mixture known as copper concentrate which contains 16 per cent to 30 per cent copper depending on the grade of
the original ore. The concentrate is then converted at the smelter through a series of roasting, smelting and converting operations into blister copper composed of 99 per cent pure copper.\(^6\)

This blister copper is then refined in a simple but lengthy (six weeks) electrolytic process after transportation to the refiner. These refineries are usually located near the center of copper consumption far from the mines and smelting in the West. Refined copper is then sold to fabricators for use in producing brass, wire and foundry products.\(^7\)

Aluminum is developed from bauxite ore in a two stage process. In the first stage the bauxite ore, most of which is mined in South America, is converted into alumina (aluminum oxide) using the Bayer process—a chemical separation of aluminum oxide from other waste materials in the ore. This alumina is electrolytically converted into aluminum ingots through the Hall Process. The primary metal is then fabricated into sheet, extrusions and castings for final use in the many markets for the metal.\(^8\)

Technologically and economically, the interest of the copper industry has been focused on the mine. Ore yield in copper is very low, averaging around 0.8 per cent in the United States as compared to 25 per cent for bauxite (aluminum) ore and 30 per cent to 50 per cent for

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\(^7\)Ray Wayne Bellmer, op.cit.

iron ore. For this reason, the mining operation is extremely critical for slight variations in percentage yield can greatly effect final copper cost. About 70 per cent of the cost of copper is represented by the cost of the mining and concentrating operations. The smelting operation, accounting for 17 per cent of primary copper cost, and refining, 13 per cent of cost, are of secondary importance from a value-added point of view.

In the early years, prime interest in copper was centered on horizontal integration at the mine level. With a high market demand in existence, interest centered on acquiring new rich deposits. It is only since World War I that vertical integration from mine to fabricator has developed on a major scale. Major producers such as Kennecott still farm out a significant share of their smelting and refining operations.

Aluminum, in contrast, is technologically oriented to the reduction smelter. Unlike copper, integration proceeded backward from the smelter to the mine. The Aluminum Company of America, until 1940 the only producer of primary aluminum in the United States, began operations as the Pittsburgh Reduction Company. The bauxite mine and the alumina refiner are of lesser importance than the reduction facility which accounts for about 75 per cent of the mill cost of primary aluminum. Alcoa integrated backward from reduction to the mine and

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9E.B. Alderfer and H.E. Michl, op.cit., p. 89.
10Ibid., pp.95-97.
forward into aluminum fabrication. Reynolds integrated backward from an early position as a foil fabricator.\textsuperscript{11}

C. The Market

Differences in technology are matched by differences in the nature of the market. Copper is a homogeneous commodity since the refined copper of one producer differs little from that of another. Competition, therefore, turns largely on price.\textsuperscript{12} As a commodity, it is traded on a commodity exchange by brokers in a manner similar to agricultural products such as wheat and eggs. Like these products it has a futures market. Under these conditions, the prevailing practice becomes one of unrestricted production with prices fluctuating with demand. Production fluctuation results only from the fact that marginal mines cannot operate below a certain price. These mines move in and out of operation as function of the price of copper.\textsuperscript{13} The situation described has been modified in recent years by the production control policies practiced by the integrated companies. As will be explained later in the thesis, however, these policies are so affected by price that they do little to change market conditions.

The fact that such a market should exist in an industry with less than ten major producers might seem surprising. The industry is much more concentrated than steel, with three companies, Anaconda,

\textsuperscript{11} S. Malkuit, op.cit.

\textsuperscript{12} E.B. Alderfer and H.E. Michl, op.cit., p.87

\textsuperscript{13} M.R. Herman, op.cit.
Kennecott and Phelps Dodge, producing almost three quarters of the country's copper output. This concentration is modified, however, by three important factors. One is the international nature of the market. Only about one-third of copper output is produced in the United States. Chile, Central Africa and the U.S.S.R. are major producers, and international prices are greatly influenced by the London Metal Exchange. A second important market feature is the small number of large buyers who dominate the markets. Companies such as General Electric, Westinghouse and Western Electric purchase huge quantities of copper and are shrewd price-conscious buyers. A third factor preventing output restriction results from the wide differences in the nature of the cost functions of the firms in the industry. The wide variations in mine ore yields prevent a uniform decrease in output by each of the firms following a drop in demand. Rather, the high average cost—low yield—mines are forced out of operation, and the efficient mines continue at full production with a reduction in profits at the new lower price. Since high and low cost mines are not uniformly distributed among the producers, a uniform decrease in production is all but impossible.

The effects of these three factors have frustrated attempts to stabilize prices through mutual collusive agreement. Numerous valorization plans have been attempted but all have failed in times of low demand. There is recent evidence that individual firms are

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15 M.R. Herman, op.cit.
restricting production in order to maintain current copper prices. Even the Central African companies, reluctant to practice any restriction in the past because of their rich ore and low labor costs, have been putting a large quantity of smelted blister copper into temporary storage.

The aluminum market in contrast has been characterized by administered prices and flexible production control. Conditions today are greatly influenced by the fact that Alcoa was the only primary producer until 1940. Even after the entrance of Reynolds, Kaiser and the "little three,"* Alcoa retained a role of price leadership especially in primary metal. Before 1940, Alcoa maintained a policy of gradual lowering of prices through technological reduction of costs as a means of market penetration. Since World War II, the price of primary aluminum has inched upward as a result of increasing costs. The price dropped officially only in 1958 when Aluminium Limited of Canada reduced the price in response to Russian dumping of aluminum on the London Metal Market. Prices returned to their original level by 1959.

Currently, the pattern of the market is being threatened not by the price of primary aluminum but by prices of fabricated products such as sheet, extrusions and castings. Price cutting by fabricators to obtain an order is becoming quite common, and the requirement to match price is rapidly squeezing all of the profits out of the fabricators now in the business. The leading companies are still uncertain

* Anaconda, Harvey and Olin-Revere.

of the final results of this new trend, but the days of price leadership by Alcoa seem to be all but finished.

A final important difference between the two industries in the market results from their activities in the marketing function. Marketing efforts in the aluminum industry have been quite extensive with Reynolds in the lead in expanding the market through new applications. To a greater or lesser degree, all of the companies work quite closely in an applications engineering fashion with active and potential customers to expand the market through new uses for the metal. Announcements of new product applications in both the industrial and consumer markets have become a part of the normal operation of the industry.

Copper companies have been much more production-oriented, generally regarding the market as something beyond their control. Only in recent years have companies such as Anaconda made any real efforts to expand the market. This lack of promotional and product application efforts has only intensified the importance of price in this industry.

D. Management

It is extremely dangerous to generalize about management in two industries with such extreme internal and external differences as copper and aluminum. There is no typical copper management or aluminum management. Reynolds Metals in aluminum differs as much from Alcoa as Alcoa does from Phelps Dodge. Nevertheless, a few characteristics do stand out in each industry that have had a bearing on past performance.

Aluminum companies have, in general, adapted themselves, to the prevailing state of knowledge in the field of management. Reynolds,
and to a lesser extent, Kaiser, have model marketing organizations that compare with the best in other industries. Alcoa has been affected by scientific management and controllership to the extent that the organization is well directed and controlled from this point of view. And all companies in the industry have adopted the concepts of human relations and have had a peaceful industrial relations history. With the difficulties encountered in a period of very rapid expansion, it must be said that the aluminum industry has been managed well.

The copper industry has been handicapped by a wide diversity of interests and objectives by the companies within the industry. In general, vertical integration is not as well developed as in aluminum. Anaconda was the first company to develop a complete vertical integration from mine through fabrication. Kennecott, until recently, has been predominantly a mining operation. The big custom smelters, American Smelting and Refining and American Metals, have operated without mines of their own. This wide diversity of interests, and especially the peculiar economic situation of the low margin custom smelters, has tended to develop managerial policies essentially short range in nature and often not consistent with best interests of the industry as a whole.

Today, both industries exist in a condition of overcapacity and "soft prices." Copper is accustomed to instability and can view reduced production requirements with comparative equanimity because the

17 Ibid., p. 97.
larger integrated companies have low breakeven points. Aluminum, with its huge capital investment has a higher breakeven point and views current conditions with increasing concern.
Chapter III

THE STRUCTURE AND POLICIES OF THE INDUSTRIES

A. The Copper Industry

A block diagram of the copper industry viewed as a single material flow process is shown in Figure 3. Considered in its simplest form, the first four (primary) sectors of the industry—mine, concentrator, smelter and refiner—may be viewed as a single feedback loop system with a time delay. The basic feedback information loop is the price of refined copper. This price serves as an input to the supply (aggregate cost) function of the mine sector which relates a change in output to a change in price. The delay in the system—about ten weeks—is a function of the production and distribution time required to produce refined copper.

Dynamic stability of the primary metal sectors in this simplified model will be a function of

1. The mine supply (aggregate cost) function
2. The production-distribution time delay
3. The policies affecting price formation.

In such a model the first two elements—the supply function and the production-distribution time delay are structural in nature since they are part of the technical-economic characteristics of the industrial process. Price policy in contrast, since it is more subject to modification by management, is the controller to be designed to accomplish the objectives of the system.

Figure 3. Flow Diagram - Copper Industry - Single Flow
The design of this price policy will determine the stability of the system. Prices may be changed as a function of average sales, unfilled orders, production rate and refined copper inventory. The relative weighting assigned to each of these factors will greatly influence stability characteristics. If, as at present, refined copper inventory is of first importance, unstable performance is a certainty. Greater emphasis on sales and production rate will tend to produce greater stability.

This simplified model is modified in the real world by a number of factors serving to change its basic assumptions:

1. The mine supply function is made steeper by mining policies which change ore grade with price.

2. The production-distribution processes after the mine are not in fact a simple time delay, but a series of delays, inventories and production control functions. These functions can add or detract from basic stability of the system depending on their specific nature.

3. The independent mine-custom smelter processing chain introduces a parallel flow to the integrated producer material flow that has different production-inventory control and pricing policy characteristics. The simple single flow model must be replaced by a dual parallel flow model shown in Figure 4.

4. Managerial control of the process is actually vested in seven separate companies which compete in a world market. This further complicates the system with
the multiple-flow system with competitive cross-coupling influences shown in Figure 5.

Each of the above effects generally tends to increase the instability of the system and this destabilizing tendency is especially strong in the third factor above. The custom smelter—really a smelter-refiner—purchases ore or ore concentrate from the smaller independent mines. With long term contractual commitments to the mines, the custom smelter must process and sell copper as soon as possible in order not to suffer loss from a falling price. Under this pressure, the custom maintains low inventory and an unstable price policy.

Since the first two of these factors are subject to modification by the unilateral action of a single integrated producer company, changes in these policies may be used to increase the stability of the system. Such policy discussions will be postponed until the details of the problem have been brought into clearer focus.

The primary metal sectors of the industry are followed by the fabricator and user sectors. The fabricator purchases refined copper to fill orders and maintain a desired level of fabricated metal inventory. The sales orders from the user to the fabricator depend upon the sales orders of the user, his desired inventory and the price and delivery time availability of fabricated metal. Time delays and inventories, together with production and purchasing policy rules pursued introduce a dynamic effect on sales orders to the copper refiner. Some of these policies, especially the speculative purchase of copper during a period of rising prices, add to the instability of the
Figure 5 - Flow Diagram - Copper Industry - Multiple Flow
system. There is also little doubt that basic fluctuations in user industries such as construction and the utilities provide a major source of input fluctuation. National economic fluctuations aside, however, it seems apparent that both the fabricator—especially the independent fabricator—and the user are reacting to the main problem and not causing it.

Many of the user industries such as the telephone industry are characterized by long range stability. The fluctuation in the output of the wire fabricating mills, after the brass mills the major consumer of refined copper, is remarkably stable. Although the brass fabricating mills' output fluctuates to a greater extent, it appears that the primary problem lies in the primary metal sectors.

Ultimately, any improvements in this system must be the result of policy changes in the companies comprising the industry. For this reason, policy change suggestions must be capable of implementation by a single company independent of collusive agreements with other companies. Even if such collusion were not illegal, its desirability is highly questionable. The dangers of artificially high prices detrimental to copper consumers are matched by the inherent impracticability of effecting uniform equitable production or price control in an industry with such widely varying cost structures. The failures in the past when collusive action has been permitted have demonstrated the essential futility of such an approach.

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19 R.W. Ballmer, op. cit.


21 M.R. Herman, Jr. op. cit.
In order to satisfy this requirement for individual company policy implementation, it was necessary to use a multiple flow model of the industry. Ultimately, the seven channel model shown in Figure 5 could be used to demonstrate the effects of policy changes in each of the industrial organizations involved. Considering the circumstances, however, such a model is unnecessarily complicated. The integrated producers follow quite similar policies at the present time, and the tendency of the producers to follow the lead of the custom smelter in price changes leads to a system with a fairly uniform dynamic behavior. For this reason, to study the effects of the action of one company, it is only necessary to divide the industry into a dual flow model as shown in Figure 4. One flow channel will represent the company changing its policies, and the other the remainder of the industry.

Some important questions arise in the implementation of new policies by a single company. The most important of these questions would seem to be:

1. What risks does a company take in initiating such changes?

2. What are the effects of such policies on the profits of the initiating company?

3. What leverage do such policies have in stabilizing the industry as a whole?

The first two questions are related since the principal risk to a company would be loss of profits in both the short and the long run. Any analysis of such policies must include the cost and revenue
(This page has been deleted).
functions and the final results of such policies on company profits. In this study, profit effects of each policy using cost functions typical of the industry are used to determine the risks and costs of each policy. The objective, of course, is to determine a low cost policy that achieves the desired stability objective.

The influence of a single company, and its leverage effect on the industry as a whole, were analyzed with the dual flow model as will be described in a later chapter. There was reason to believe in advance, however, that leverage of a company the size of Kennecott—35 percent of the United States and 15 percent of world production—would be considerable inasmuch as the custom smelters with the same percentage of output were providing the principal destabilizing influence in the industries. Whether the size required for stabilization was greater than that for destabilization was, of course, to be determined.

The above discussion of industrial structure and pricing and production policies essentially considers the system as fixed in output capacity. Actually, of course, capital investment in mine exploration and plant and equipment results in an expanding capacity. The relation of current capacity to market demand has an important effect on the dynamics of the industry.

Undercapacity, as existed in 1955, results in a limit on production that, combined with low inventories, leads to an excessive price rise in times of increased demand. Overcapacity, in contrast, leads to depressed prices in the existing industrial structure. The drop in prices is especially sharp if competitive forces result in the least
efficient units being withdrawn from production. Such a phenomenon actually occurs in the smaller mines—custom smelter sector, but the integrated producers have recently tended to reduce output by short work weeks which would have the effect of holding price, were it not for the effects of the custom smelters' price reductions on the rest of the industry.

To incorporate the alternate effects of undercapacity or overcapacity, a growth function has been incorporated in the model to provide for varying rates of output capacity expansion. This rate of expansion is very important since it can have an important effect on short run price and production policies pursued.

B. The Aluminum Industry

At first glance, the structure of the aluminum industry, shown in Figure 6, seems quite similar to that of copper. A series of primary metal sectors are followed by the fabricator and user sectors. A feedback of information from the aluminum reduction plant provides the fundamental requirement for mine production. Although it will be noted that the concentrator sector present in copper does not exist, the basic similarity in the two systems seems apparent.

Such superficial similarities, however, seem less important than the actual differences that exist, the most important of which are the following:

1. The prime feedback is not the price of primary aluminum but a production plan based upon a sales forecast and existing inventories.
Figure 6 - Flow Diagram - Aluminum Industry - Single Flow
2. The mines are not affected by short run prices because the yield of the bauxite ore is high—about 25 per cent—and the bauxite ore value itself is a small part—less than 10 per cent of the final price of the primary metal.\(^{22}\)

3. The production of the intermediate alumina refiner-stage is coordinated into the same production plan. The presence of a significant raw materials inventory of bauxite ore at this stage permits a degree of flexibility that is impossible for the copper smelter with his minimum level of raw materials inventory.

4. The absence of independent mines and copper smelters allows for price policy in which inventory accumulation is possible with little fear of a price decline. The four major North American Producers* all have their own mines. The smaller four primary producers** do not mine bauxite. In fact, all but Ormet neither mine bauxite nor refine alumina. Instead, they purchase alumina from one of the large producers or from abroad. Though such smaller producers in a sense act as custom smelters, the lower value of the ore and the large size of the mines prevent a repetition of the copper situation.

\(^{22}\)S. Malcuit, op.cit.

* Alcoa, Reynolds, Kaiser and Aluminium Limited of Canada.

**Ormet, Harvey, Anaconda and Canadian-British.
5. The reduction smelter sector is the stage of production where the prime value is added to the product, and for this reason it is the critical part of the cost structure of the industry. Individual smelters vary in efficiency because of differences in power, labor and transportation costs.

6. Price competition is strongest at the fabricator stage where the market forces are encountered. The price of primary metal is very stable.

7. The stability of primary metal price is countered by an extremely variable price of secondary aluminum produced from scrap.

8. In general, there are significantly higher inventories in all stages of the system resulting in a greater stability of operation. Since the costs of changing production discourage frequent changes, changes in demand are absorbed by fluctuating inventories.

9. Although demand exceeded supply in the early fifties when the federal government was the most important customer, the situation in recent years has been one of significant overcapacity. 23

10. The wide geographical dispersion of mines, refiners, smelters, fabricators and users make transportation a more significant factor in the overall efficiency of the system. 24

23 Ibid.
24 Ibid.
In summation, the industrial system in aluminum is quite different from that in copper. While copper is characterized by low inventories, full capacity production—as limited by price—and a price policy that emphasizes refined copper inventory; aluminum is marked by high fluctuating inventories, reasonably coordinated production planning and a remarkably stable primary metal price.

It is instructive for clarification to examine the effects of a decrease in demand in both industries. In copper, a decrease in sales orders to the custom smelter-refiners will result in a decrease in price after a short period, as a function of order backlog and refined copper inventory. The integrated producers will follow suit and an unstable oscillatory response of the system previously described will ensue.

A similar drop in aluminum sales orders could result in a number of consequences:

1. The price could be reduced as in copper, not to increase demand—since short run demand is relatively inelastic—but to increase the market share of a particular company.

2. Production could be reduced by halting the output of high cost marginal units.

3. Production could be reduced uniformly of all units—high and low cost.

4. Production and prices may remain fixed with inventories accumulating.

5. Promotional efforts may be made to gain new sales.
In general, the first possibility—individual price reduction—is rare in the primary metal sector where only one price decrease has taken place since World War II, but it is quite common at the fabricator level where price-cutting is practiced in low sales periods. The second possibility—cutback of high cost units—has been practiced, as in the 1958 recession, to reduce excess production. The third possibility—the action of a monopolist—occurs to some extent in that not all of the high cost units are shut down. The fourth possibility—accumulation of inventories—is the principal means of absorbing short run sales fluctuations. A fifth possibility—more extensive promotional efforts—may be attempted to secure an increase share of the market or increased penetration of substitute material markets.

The final result in aluminum is a mixture of the possibilities with inventory fluctuation predominating. In comparison with copper the fluctuation in inventories is much greater than the fluctuation in price, even at the fabricator level. The possibilities listed are strongly interrelated since either a decrease in high cost unit production or an accumulation of inventory results in a downward pressure on price. At any one time, the companies in the industry will each be pursuing a policy mix of all the above possibilities in varying degrees, and the total state of the industry will be the result of the composite of these strategies.

Response of the aluminum industry to a change in demand is modified to some extent by the influence of the secondary metal market. Secondary aluminum is sold primarily to foundries and steel plants.
The former use the metal to produce aluminum castings, and the latter use aluminum shot as a deoxidizing agent. In times of reduced sales, however, the foundries of the primary producers limit their consumption of secondary to increase primary metal consumption by their own fabricators. Such a shift is profitable for the primaries as long as the price of scrap exceeds the variable costs of producing primary metal. Further depression in secondary and scrap prices, however, can make such a course unprofitable. The end effect is that variability in secondary purchases of scrap by the fabricators in integrated companies reduces the inventory accumulation of primary metal. With the growth in primary production of aluminum in recent years, however, the market share and effects of secondary aluminum have tended to decline. The effects of secondary aluminum will not be investigated in the model at this time.

Two longer range effects on the dynamics of the industry are plant and equipment investment and the market mix. The effects of undercapacity and overcapacity have already been pointed out in the previous discussion of copper. The existing overcapacity has tended to depress prices because with reduced sales and inventory accumulation, all companies reach a limit in cash resources that forces them to sell fabricated products at reduced prices. Removal of high cost units from production in periods of low demand makes such price reduction both possible and desirable.

An implicit assumption in the above discussion is a uniform market demand. In actuality a variety of markets exist and each company has its own "marketing mix." Variations in different segments of the market such as construction, automobiles or electric power transmission will introduce a unique aggregate shift in demand for each producer. The proper price, production and promotional decisions will vary with each company under a given set of market circumstances. The problem, then, is to determine the proper policies for such decisions in each company with its peculiar production cost structure and marketing mix.

The control problem in each of the industries differs although both systems must adjust to continual fluctuations in demand. In copper, price and production stability are requirements to insure the continued growth and development of the industry. The problem is one of determining stabilization policies and their costs capable of pursuance by individual companies in the industry. In aluminum, where stability already exists, the emphasis must be on the determination of short run price production and long term investment and marketing programs for most profitable operations of individual firms in the industry.
Chapter IV

THE MINES SECTOR - COPPER AND ALUMINUM

A. General

Detailed descriptions of mining operations for both copper and aluminum are available in the literature. Little purpose would be served by repeating such general descriptions here. Rather, it is the purpose of this chapter and those of the other industrial sectors that follow to describe the construction of the model and its relationship to the structural and policy characteristics of the two industries. In each sector the model will be developed in order of the time sequential flow of material and related information and money flows through the system. Though many different elements in the system will be described, most process (structural) and controller (managerial policy) elements may be classified in one of the following categories:

1. Process (Structural) Elements
   a. Storage Elements (Inventories)
   b. Time Delay Elements
   c. Output/Input Ratio Elements
   d. Financial Elements

27 M.R. Herman, Jr., op.cit.
28 S. Malcuit, op.cit.
2. Controller (Policy Elements)
   a. Production Control Policy Elements
   b. Investment Policy Elements
   c. Marketing Policy (price and delivery time) Elements.

   For each element or group of elements, the main emphasis will be on the nature of the element and its effect on the system. Only those equations necessary to describe the fundamental characteristics of the system will be listed. A complete set of the model equations together with the rationale of their formulation may be found in the appendices.

   It is important to emphasize that one of the mine sectors described (M1) is an aggregate mine for the whole industry, and for that reason is only typical and not exactly representative of any particular mine. In this chapter, the modifications to the mine sector required to represent particular companies will be described in conjunction with the dual-sector competitive model and over-all policy formulation.

B. The Copper Mine

1. Production - Structure and Policy

   A flow diagram of the copper mine sector is shown in Figures 7A and 7B. In Figure 7A the "M" copper mine sector is shown. This sector of the model was used to represent the part of the industry which was modified to test the effectiveness of new managerial policies. The flow diagram of Figure 7B represents the industry in its present
Figure 7A - Flow Diagram: "M" Copper Mine Sector
Figure 7B - Flow Diagram - "M1" Copper Mine Sector
unmodified form. Functions such as the long range growth function common to both sectors are also diagrammed in Figure 7B. In the description that follows most frequent reference will be made to Figure 7A.

The prime input variable to the mine sector is the current price of refined copper (RCP) established in the refiner sector. Since the market price established by custom smelters and the commodity markets fluctuates daily, it is necessary to time-average the price to remove high frequency random fluctuations of little interest in establishing mine production rate. This smoothed price of copper is determined in the model by the function:

\[
\text{MACP}_K = \text{MACP}_J + (DT) \left( \frac{1}{ML} \right) (\text{RCP}_J - \text{MACP}_J)
\]

MACP - Mine Average Copper Price, (dollars)
RCP - Refined Copper Price, (dollars)
ML - Smoothing period constant, (weeks).

This transformation determines the time-averaged (smoothed) price of copper by a continuous averaging of current and past price information. Selection of a smoothing time period (ML) serves to determine the amount of past information that is of interest for mine production variations. The longer the averaging time the less sensitive is mine production to rapid fluctuations in price. To realistically simulate the industry, it is important that this time not be made too long since the smaller mines are limited by financial resources to a short
term view of price to maintain profit margin and liquidity. Periods varying from twelve to twenty-four weeks were used in the model.

This time-averaged copper price (MACP) serves as an input parameter to the aggregate cost (supply) curve for the industry as illustrated in Figure 8. At any given price it will be just possible for the least efficient mine to continue operation.

In the original formulation of the model, the mine production desired (MPD) was a function solely of the average price of copper. This formulation, typical of current industrial behavior in the industry, has been maintained in the "M1" sector. To reflect improved policy changes, however, the "M" sector production policy has been modified to include variation as a function of smoothed refiner sales (RSS) as well as price (RCP).

\[ MPD.K = (M62) (M2PD.K) + (1-M62) (RSS.K) \]

**MPD** - Mine Production Desired, (lb/wk)

**M2PD** - Mine Production Desired as determined from the supply curve in Figure 8, (lb/wk)

**RSS** - Refiner Smoothed Sales, (lb/wk)

**M62** - Price-determined production weighting factor, (%).

The above policy modifies the simple production-price relationship with an anticipatory variable, refined copper sales (RSS). In the basic model M62 was one, but in the modified production policy it may be established at any value between zero and one. Increased weighting of sales tends to improve the stability of the industry. The final
Figure 8. Copper Mine Supply Curve
selection of weighting factors will be based upon both stability and profit considerations for the company formulating the policy. Recent actions of Kennecott Copper Corporation indicate that they are already initiating production policy changes based upon the above relationship.\(^{29}\) The following recent quotation from the Wall Street Journal summarizes these actions.

"Kennecott Copper Corp., largest U.S. copper miner is stepping up production by resuming seven-day operating schedules at its Western Mining division properties in Utah, Nevada, Arizona and New Mexico, O.D. Michaelson, general manager, announced in Salt Lake City. Other copper producers, surprised by the move, criticized it and said they wouldn't follow.

"'A higher copper production rate is required to meet the recent increased demand,' Mr. Michaelson said.

"Kennecott curtailed output to a six-day basis from a seven-day week February 1 'to bring domestic production in line with domestic sales.'"

Kennecott has increased its mine production at a time when other integrated and custom producers are waiting for the price rise which has not yet occurred.

The desired mine production desired (MPD) is then multiplied by the long range expansion factor (MLRSA) which reflects the results of mine expansion as a result of mine exploration and development effort. This function, which has the effect of slowly moving the supply curve

\(^{29}\)Wall Street Journal, April 17, 1961, p. 2
to the right over time, is the end effect of investment policy in the industry which will be subsequently described.

The mine adjusted production desired (MAFPD) just determined from price and sales considerations is further modified by inventory requirements of the copper concentrator, the next stage in the process. In most mine-concentrator operations, especially of integrated producers, the mine and concentrator production rates are closely coordinated, so that sizeable inventories are not required. Such ore inventories, because of their huge bulk, are very expensive and hence undesirable. Raw material inventory storage capacity at an integrated concentrator, for these reasons, is usually limited to less than one day of production. In order to make the model flexible, however, for those operations, i.e., a nonintegrated fabricator, where the mine is some distance from the concentrator, an inventory adjustment to production has been provided:

\[
\text{MAFPD}.K = \text{MAFPD}.K + \left(\frac{1}{M_{70}}\right) (\text{CRMID}.K - \text{CRMI}.K)
\]

\[
\text{MAPD}.K = (\text{MPD}.K) (\text{MLRSA}.K)
\]

MAFPD - Mine Altered and Adjusted Production Desired
for concentrator inventory requirements, (lb/wk)

MAPD - Mine Adjusted Production Desired for long range expansion of mine capacity, (lb/wk)

MPD - Mine Production Desired, (lb/wk)

CRMID - Concentrator Raw Material Inventory, Desired, (lb)

CRMI - Concentrator Raw Material Inventory, Actual, (lb)
M70 - Time to balance inventory, (weeks)

MLRSA - Long run supply adjustment ratio.

The adjustment time (M70) determines the speed of adjustment of production rate to inventory discrepancies. If inventory adjustment is not important—as in an integrated concentrator—in the operation being simulated, M70 may be made very large to make its effects on the system negligible.

Production changes at a copper mine do not occur instantaneously. To increase production, new employees and mining equipment may be needed. Adjustment time is particularly slow at an independent mine where the mine may be completely or partially deactivated. Production changes at an integrated mine are accomplished more rapidly by lengthening or shortening of the work week. The production wanted (MPW) is a time-delayed version of the mine adjusted and altered production desired (MAAPD) previously described.

The output/input ratio of the mine (MAAPD) is further modified by two other factors: ore yield and mine capacity. The latter may be described as the peak production capability of the mine as determined by the crushing and handling capacity of the mine equipment. In the usual case, the ore must be transported by train to the concentrator at least a few miles from the mine. The transport distance will vary with the layout of the mine-concentrator-smelter facility, but this handling capability in conjunction with preliminary crushing establish
a practical limit to production of the mine output. In most instances, however, this output saturation effect at the mine is not critical since it usually exceeds the capacity of the less flexible concentrator and smelter facilities. The mine capacity (MMPP), after multiplication for long range expansion (MLRSA), limits production rate to current capacity (MOPP).

The ore yield selection—one of the most important managerial policy decisions in the mine sector—deserves further explanation. In any given mine the ore yield may vary widely between 0.5 per cent and 3.0 per cent in different areas of the mine. Since the unit-pound cost of copper varies inversely with the yield, the costs of mining copper may be modified through selection of varying yields. Should the mine manager wish to maintain a constant profit margin, he will mine lower yields at higher prices according to the expression:

\[
\text{MOY}_K = \frac{\text{RCPB}}{\text{MACP}_K}
\]

\[
\text{MOY} - \text{Mine Ore Yield, } \%
\]

\[
\text{RCPB} - \text{Refined Copper Price, Base, a constant}
\]

\[
\text{MACP} - \text{Smoothed price of refined copper.}
\]

As the time-averaged price (MACP) exceeds the base price (RCPB), the yield of the ore mined will be lower. Such a policy tends to restrict output in times of rising prices which further serves to raise prices

*This equation is a simplification of equations A-258, A-259, A-288 and A-289 more completely described in Appendix A.
and amplify price-production fluctuations. From a control viewpoint such action serves to flatten out the output/input "gain" curve, inducing severe nonlinear oscillatory effects.

Alternate policies would either maintain ore yield constant on a long term economic basis or vary yield directly with price, thereby increasing output in times of shortage to act as a price-production stabilizer to the industry. The equation for the latter policy would be the reciprocal of the above equation. The relative effectiveness and costs of this means of stabilization will be compared with other approaches in a later section on policy formulation. Complete equations for alternate ore yield policies are explained in Appendix A.

The delayed production wanted function (MFW) as modified by the ore yield policy and the mine production limit just described, determines the actual production started at the mine (MPS). The ore in process is expressed by a third-order delay after which the ore is shipped (MCP) with another third-order delay to the raw material inventory of the concentrator.

The production policy of the Ml sector of Figure 7B differs from that just described for the M sector primarily in its reliance on copper price as the prime determinant of mine production rate. Similar delay, ore yield and capacity restrictions apply to that sector. The investment policy (MLRSA) is also illustrated in the Ml sector.

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30 R.W. Ballmer, op.cit.
2. Investment Policy

A most important—if not the most important—policy to be formulated in the copper industry is that of investment policy for long range expansion. In the 1955-1956 price rise, it was very evident that market demand for some time exceeded the capacity of the industry to produce. Changes in price, production and inventory policies will be to no avail if the industry is not capable of adjusting to increases in short term demand. For this reason, the rate of planned expansion is extremely critical. An investment policy providing for sufficient excess capacity to absorb short term fluctuations must be pursued. The industry must not restrict expansion to maintain full capacity operations.

In the past, undercapacity has resulted from a combination of poor forecasting and a fear of overcapacity. In the model, the expansion function (MLRSA) is based upon long term price trends which indicate the need for new capacity.\(^{31}\) In the model, expansion is based upon long term price trends. A long range upward trend in prices will lead to an expansion of mine capacity at a given price. A long term decline in price will have the opposite effect. As stability of price in the industry is achieved, this sector of the model could and should be developed to reflect growth predictions of market segments of the user sector, independent of price.

\(^{31}\)R.W. Ballmer, op.cit.
3. Financial Structure

The remaining areas of the mine sector shown in Figures 7A and 7B are financial and include the revenue, cost and profit functions. Inasmuch as a mine-concentrator-smelter unit is often treated as a profit center in an integrated copper company, revenue, cost and profit have been determined for this combination of sectors. The cost expression is the cost function of an individual firm and provides a modification of the aggregate average cost of the supply curve as a function of output volume.

Mine-concentrator-smelter costs are determined as a function of both volume and ore yield.

\[ MCOST_{KL} = \left( \frac{MUCOS.K}{MPS.JK} \right) \cdot MOY.K \]  
\[ MUCOS.K = VMCOS + FMCOS.K \]  
\[ FMCOS.K = \left( \frac{OMCOS}{MPSN} \right) \cdot MPS.JK \]

where

- \( MCOST \) - Mine-concentrator-smelter COST, total, (dollars/wk)
- \( MUCOS \) - Mine Unit COST of copper, (dollars/lb)
- \( MPS \) - Mine Production Started, (lb/wk)
- \( VMCOS \) - Variable unit COSTs of copper, (dollars/lb)
- \( OMCOS \) - Fixed COSTs of copper (dollars/lb)
- \( FMCOS.K \) - Fixed unit COSTs of copper at current volume, (dollars/lb)
- \( MPSN \) - Standard constant volume - Mine Production Started, initial, (lb/wk)
- \( MOY \) - Mine Ore Yield, (per cent)
Mine Cost (MCOST) is determined by the unit cost of copper production, as modified by the ore yield (MOY), and the production started (MPS). A higher ore yield will serve to decrease cost for a given output of copper metal. Unit cost is in turn determined by the fixed (FMCOS) and variable (VMCOS) costs of production. Unit fixed cost (FMCOS), as a function, decreases with volume as shown in equation A-267 above. The fixed and variable costs of copper are not those of any actual company but are considered to be typical of the industry.

The revenue of the mine-concentrator-smelter is determined from smelter shipments and the transfer price of blister copper which is determined by subtracting a fixed cost for the refining function from the market price of refined copper (RCP). The profit function resulting from revenue and cost variations provides a useful criterion for evaluating the desirability of stabilization policies considered later. Effects of various policies on the cost of operations in these sectors is extremely important inasmuch as over 70 per cent of the costs of production are represented in these sectors.

C. The Bauxite Mine

1. Production - Structure and Policy

Flow diagrams of the bauxite mine are shown in Figures 9A and 9B. In marked contrast to the mine in the copper industry, the bauxite mine plays a minor role in the dynamic operation of the aluminum industry. To be sure, production of primary aluminum would be impossible without bauxite, but this raw material represents less than 10
Figure 9A: Flow Diagram: "M" Bauxite Mine Sector
Figure 9B. Flow Diagram - "M1" Bauxite Mine Sector
per cent of the total value of the final primary ingot and less than 5 per cent of the fabricated product price. For that reason, production policy is less critical in the bauxite mine sector.

This lower value of the ore is matched by the absence of other characteristics important at the copper mine:

a. The aggregate (supply) cost curve variations are of little importance and have no discernible effect on over-all production policy

b. Ore yields do vary with lower yields prevailing in most domestic mines and higher yields in Jamaica and South America, but these yield cost differentials are offset by transportation costs which account for 30 to 40 per cent of the delivered cost of bauxite

c. Ore yield variation is not practiced since it is not of consequence with the low value nature of the ore.

Bauxite mines differ from copper mines in their remote locations with respect to the next stage of production. Reynolds has a mine refiner-smelter combination in close proximity in Arkansas, but most bauxite is mined in Surinam and Jamaica and is shipped to alumina refiners on the Gulf Coast.\textsuperscript{32} For this reason, transportation is a key factor in the delivered cost of the ore.

\textsuperscript{32} S. Malcuit, op.cit.
The bauxite mine production rate is based upon a production plan received from the headquarters of the company. This plan is derived from ultimate requirements for alumina at the smelter. This production requirement is supplemented by a need to maintain adequate bauxite inventories at the alumina refiner, so that the production rate is above or below long term requirements depending on the level of bauxite inventories as expressed in the equations below:

\[
\begin{align*}
MPTBS_K &= MPW_K + \left(\frac{1}{M70}\right) (SRMID_K - SRMI_K) \\
MLPD_K &= RSS_K + \left(\frac{1}{R16}\right) (RFGID_K - RFGI_K)
\end{align*}
\]

- **MPTBS** - Mine Production To Be Started, (lb/wk)
- **MLPD** - Mine Production Desired to fill reduction production rate requirements, (lb/wk)
- **SRMID** - Smelter Raw Material Inventory Desired, (lb. of bauxite)
- **SRMI** - Smelter Raw Material Inventory, Actual, (lb. of bauxite)
- **RFGID** - Refiner's Finished Goods Inventory Desired, (lb. of aluminum)
- **RFGI** - Refiner's Finished Goods Inventory, Actual, (lb. of aluminum)
- **RSS** - Smoothed refiner Sales of aluminum, (lb/wk)
- **M70** - Inventory adjustment rate, (wk)
- **R16** - Inventory adjustment time, (wk)
- **MPW** - MLPD after a time delay, (lb/wk).

This production policy provides for production of bauxite for current production rate of the system and the adjustment of bauxite inventories.
The mine production required to supply reduction production and inventory adjustment requirements (MLPD), which is in turn derived from aluminum smoothed sales and finished goods inventories requirements, is modified by the status of bauxite inventory level at the alumina refiner (SRMI). The resulting production rate (MPTBS), unlike copper, is determined independently of market prices for primary and fabricated metal.

An adjustment time lag and a production limit similar in form to those of the copper mine are incorporated. Ore yield (MOY) is a long term constant value, and long range capacity expansion is based upon market projections rather than price trends. Investment policy, unlike copper, is based upon reduction smelter and not mine capacity. The goods in process and in-transit delays are similar to those in copper except that the in-transit delay is much longer. Complete equations are described in Appendix B.

Because mining of bauxite provides ample inventories for future reduction of aluminum at a low cost, production policy at the mine will not play a major part in future recommended changes in the policies of the industry. The present production policy with slight modifications of constant is adequate support for more extensive changes in other sectors of the model.

2. Financial Structure

The format of the financial cost equations of the sector is identical to that of copper. The actual cost and price values and the
relationships between fixed and variable costs differ, of course. The transfer price out of the mine is only 2.5 cents and the standard cost but 1.5 cents, while the copper leaving the mine-concentrator-smelter complex is priced at 26 cents with a standard cost of 24 cents.\textsuperscript{33} Fixed costs are much higher percentagewise in the bauxite mine because the same material handling equipment and supervision are needed for a material of lower value.

\textsuperscript{33} Ibid.
Chapter V

THE COPPER CONCENTRATOR-SMELTER SECTORS

A. General

The copper concentrator and smelter sectors are intermediate processing stages in the production of refined copper. The flow diagrams for these sectors are shown in Figures 10A, 10B, 11A and 11B. The new policy sectors, "C" for the concentrator and "S" for the smelter are shown in Figures 10A and 11A respectively, while the sectors retaining current policies, the "Cl" and "Sl" sectors, are shown in Figures 10B and 11B.

As intermediate stages, the material flow process in these sectors has been largely regulated by the production output of the mine and the sales rate of the refiner. Inventories have been of an in-transit nature. From a control systems viewpoint, the sector itself has been of chief importance as a time lag and as a limitation of output capacity. Some of the features of prevalent production policies, however, have made these sectors an additional source of industrial instability.

B. Production Policy

Both the custom and integrated concentrator-smelters have little or no control over their raw material input. Both must accept and process all ore received. The only decisions really open to these sectors involve production for shipment versus production for inventory. In the past, all production has been programmed for immediate shipment. The basic instability in the industry resulting from a lack
Figure 10A Flow Diagram - "C" Copper Concentrate Sector
Figure 10B - Flow Diagram - "C1" Copper Concentrate Sector
Figure 11A - Flow Diagram - "S" - Copper Smelter Sector
Figure 11B - Flow Diagram - "S1" Copper Smelter Sector
of buffer inventories makes the advisability and practicality of additional inventories in these two sectors a subject of great interest.

The production rule for the "C1" concentrator and "Sl" smelter sectors, shown in Figures 10B and 11B, is one based solely upon the level of raw material inventory.34

\[
C_{1PW}.K = C_{1RMI}.K/C_4
\]

\[
S_{1PW}.K = S_{1RMI}.K/S_6
\]

**C1PW** - Concentrator Production Wanted, (lb/wk)

**S1PW** - Smelter Production Wanted, (lb/wk)

**C1RMI** - Concentrator Raw Material Inventory, (lb)

**S1RMI** - Smelter Raw Material Inventory, (lb)

* C4 - Weeks of production desired in concentrator raw material inventory, (wks)

* S6 - Weeks of production desired in smelter raw material inventory, (wks).

As shown in the flow diagrams, a raw material inventory (CRMI or C1RMI)—usually the equivalent of less than one day of production—is maintained at the input of the concentrator sector. A similar inventory of somewhat larger size (SRMI or S1RMI) is maintained at the smelter. Current production rates are determined by dividing current inventory (C1RMI or S1RMI) by the inventory desired constant (C4 or S6).

---

34 R.W. Bellmer, op.cit.
The rationale for such a production policy is understandable, especially for the smelter. Random variations in yield introduce fluctuations in the copper concentrate output of the concentrator. The copper concentrate in turn varies in copper content. For this reason, the smelter is oriented to the raw material on hand for production since the rate and grade of incoming material continuously varies. The concentrator's problems arise not so much from random inputs as from severe storage limitations. The large quantity of low grade ore required to produce a pound of concentrate—three to six times as much—make continuous production based upon inventory capacity limitations a necessity. Should the concentrator produce less than the ore mined that day, he may exceed his inventory capacity—usually less than one day of production in most cases. With too high a production rate, he may easily exceed his raw material requiring a temporary shutdown with resulting costs.

However real the reasons for this production control policy, its contribution to over-all instability of the industry make it undesirable. It is a well known phenomenon that attempts to control process flow using the integral of this flow—in this case, inventories—will produce a less stable operation than a control rule using the flow rate itself. Increased stabilization will result from supplementing this integral control with a proportional control based upon the material flow through the system. This rate measure could be the ore shipped from the mine, the concentrate shipped from the concentrator
or better yet, the sales or shipments of the refiner. Although each of
the above quantities are subject to random variations, a smoothed
quantity is entirely suitable. The revised concentrator-smelter pro-
duction policies shown in Figures 10A and 11A used in part of the new
integrated policy mix were:

\[
\begin{align*}
\text{CPD}.K &= \text{MCP}.JK + \left(\frac{1}{C3}\right) (\text{SRMID}.K - \text{SRMI}.K) \\
\text{SPD}.K &= \text{RSS}.K + \left(\frac{1}{S3}\right) (\text{RRMID}.K - \text{RRMI}.K)
\end{align*}
\]

CPD - Concentrator Production Desired, (lb/wk)  
MCP - Mine Copper Produced, (lb/wk)  
SRMID - Smelter Raw Material Inventory, Desired, (lb)  
SRMI - Smelter Raw Material Inventory, Actual, (lb)  
SPD - Smelter Production Desired, (lb/wk)  
RSS - Refiner Smoothed Sales, (lb/wk)  
RRMID - Refiner Raw Material Inventory, Desired, (lb)  
RRMI - Refiner Raw Material Inventory, Actual, (lb)  
C3, S3 - Inventory adjustment times, (wks)

In the new policy, the concentrator bases his production rate on mine
output and smelter inventory requirements. The smelter's production
rate is adapted to refiner copper sales and raw material inventory
requirements of the refiner. Both production rates in the model are
limited by capacity and raw material availability restrictions.

Limitations on inventory capacity for copper ore make it essen-
tial that concentrator production be coordinated with mine output
rather than refiner's sales. The reduction in bulk after concentration
conversion makes it possible to align smelter production with refiner sales. Both of the new policies serve to improve the stability of the industry. Effective use of such production policies is dependent upon an adequate supply of ores and concentrates in order that production not be limited by raw material inventory. Were the mine production policy not modified to include sales rate inputs, the new production policies at the concentrator and smelter would be limited by raw material inventory availability. Since there is a trade-off between larger inventories at the concentrator-smelter and the need for mine production adjustment to refiner sales, it is appropriate to examine the alternate possibilities of inventory location.

Examining the diagrams in Figures 10A and 11A, there are five possible locations for increased inventory:

1. The concentrator raw material inventory (ore)
2. The concentrator finished goods inventory (concentrate)
3. The smelter raw material inventory (concentrate)
4. The smelter finished goods inventory (blister copper)
5. The mine (ore in unmined state).

The first location would seem undesirable since so much ore would have to be stored to provide for a sizeable inventory of potential copper. The handling and storage costs of such an inventory would be unreasonable. The second and third locations may be considered together since they are usually the same inventory in an integrated operation. The storage and handling problems are considerably reduced
although here too, three to six times as much as concentrate would be required for each pound of copper. This inventory, however, would be less valuable to hold than finished blister copper. The usefulness of this inventory would also be a function of excess smelter capacity since unless the smelter could increase production to meet an increase in demand, there would be little point in having the concentrate available.

Finally, finished anode or blister copper could be inventoried after processing by the smelter. Storage and handling problems would be minimized, although the value of the material is higher at this point. This location also has the advantage of not requiring so much excess smelter capacity. The final selection is dependent on the costs of inventory, storage and handling, the costs of smelter capacity and the capacity of outgoing transportation.

In the last analysis, however, the final alternative, using unmined ore as the inventory, appears the most attractive of all in that storage and handling costs are zero. If the basic system inventory is unmined ore, only in-transit and safety stock levels of inventory need be maintained at the concentrator and smelter. The result of such an approach, of course, is that mine production rate must be flexible and at least partially independent of price according to the new mine production policy outlined in the previous chapter. The final selection of a balance between concentrator-smelter inventory levels and mine production changes will result from experience in system
simulation. Ultimately, the compromise will be based upon the alternative costs of mine production changes and inventory carrying costs.

3. Production and Distribution Structure

The actual production process at both the concentrator and the smelter are expressed by third-order delays. Material flows to finished goods inventory in the form of concentrate (concentrator) or blister copper (smelter) from which it is shipped to the next stage in the process.

Shipping delay in both sectors is represented by the sum of fixed and variable time delays. Variable delay increases with the level of the finished goods inventory since material handling and transportation personnel and facilities are limited. Such a delay function for a single material differs from the multi-product inventory, the delay for which decreases with a more extensive and varied inventory.
Chapter VI

THE ALUMINA REFINER SECTOR

A. General

Flow diagrams of the alumina refiner are shown in Figures 12A and 12B. Again, the "S" sector represents the testing area for new policies, and the "Sl" sector the remainder of the industry.

Like the copper concentrator-smelter, the alumina refiner is an intermediate process stage in the production of primary metal. Unlike the equivalent stage of copper, however, the alumina refiner is buffered by sizeable input of raw material inventory. Though the refiner's finished goods inventory is small, a sizeable inventory of alumina exists at the next stage of the system. In contrast to the in-transit type of inventories equivalent to a few days of production, prevalent in copper, raw material inventories of both bauxite and alumina of thirty to sixty days are not at all uncommon. These inventories serve to decouple the short term production rate of the refiner from that of both the mine and the reduction smelter. As a chemical process with extensive sequential operations, the in-process inventory is also large, running as high as twenty days. The presence of each of these inventories tends to make the refiner, in the short run, an essentially independent processing operation. One of the chief reasons for these inventories, of course, is the wide geographical separation between the stages of production. To insure uninterrupted production, larger inventories are a requirement.
B. Production - Structure and Policy

The production rule employed by the alumina refiner is one based upon a combination of:

1. The production rate planned for the reduction smelter (RFD).
2. The alumina inventory existing at the reduction smelter (RRMI).
3. The limit on production resulting from plant capacity and raw material inventory (bauxite) availability. In practice, excess plant capacity and large bauxite inventories make this limitation of limited significance.

To comply with these requirements the refiner production wanted (SPW) equation is as follows:

\[ \text{SPW}.K = \text{RFD}.K + \left(1/\text{S3}\right) \left(\text{RRMID}.K - \text{RRMI}.K\right) \]

\( \text{SPW} \) - Smelter Production Wanted, (lb/wk)
\( \text{RFD} \) - Refiner Production Desired, (lb/wk)
\( \text{RRMID} \) - Reduction Raw Material Inventory, Desired, (lb)
\( \text{RRMI} \) - Reduction Raw Material Inventory, actual, (lb)
\( \text{S3} \) - Inventory adjustment time, (wks).

This production policy is somewhat similar to that previously described for the copper concentrator-smelter. It differs in that production rate (RFD) rather than sales rate (RSS) at the next stage is used as the rate variable. The inventory adjustment portion of the equation is identical to that in the concentrator smelter except for the actual value of the inventory adjustment time which is somewhat larger in aluminum because of the larger inventories maintained. The
production capacity limitations, plant expansion and shipping delay
functions are similar to those in copper. Transit and goods-in-proc-
есс time delays are represented by third-order time delay equations.
A complete description of refiner structure and policies is included
in Appendix E.

The effects of this production policy on performance of the
industry will be analyzed in Chapter X.

C. Financial Structure

The fixed-variable cost and the revenue and profit functions are
identical to those in the copper mine-concentrator-smelter except for
the values of the parameters. It is of interest to note that the
delivered transfer price of an aluminum ton—equivalent of alumina (two
tons of alumina) is $0.5 cents or approximately one-third of the total
cost of the primary metal. Variable costs predominate since the major
costs are fuel, bauxite and other materials such as soda ash and lime
used in the Bayer process.

D. Transportation and the Industrial Space Network

To develop a realistic model of the aluminum industry, the key
importance of transportation costs must be recognized. Considering
the needs for bauxite transportation to the refiner, alumina trans-
portation to the smelter, aluminum transportation to the fabricator,
and finally fabricated product distribution to the user, and coupling
these movement requirements with the wide geographical separation of
stages, it is easy to understand the extent of the transportation element in final costs. In transporting alumina from the Gulf Coast to the Northwest for smelting and aluminum back to the East for fabricating, transportation accounts for over 20 per cent\textsuperscript{35,36} of the cost of the primary metal. Since the prime purpose of the model is to develop managerial policies to improve the industry and increase return on investment, it is obvious that such a predominant part of the cost structure cannot be ignored in policy decisions.

Industrial Dynamics models developed to date have simulated the flow of materials, information, personnel and, to a lesser extent, money and capital equipment, over time. A dynamic time dimension has been added to previous static concepts of industrial operation. Except insofar as it effects transit delays, however, the space dimension has been ignored. To integrate the important space factor into the aluminum model, some expressions capable of simulating the effects of alternate movement paths would be desirable.

The first stage in the system where the space factor becomes important in a dynamic sense is at the refiner. Prior to the refiner state, the predominance of Caribbean imports to the Gulf Coast make the decision a static one of initial plant location. It is here at


the refiner that dynamic choice becomes important in selection of alternate routes and destinations for the alumina produced. Because the requirements of the smelter in the form of sales and inventory variations are continually changing, the decision policy is a dynamic rather than a static one. Alcoa, for instance, has four refining plants, eight reduction smelters and seventeen fabricators scattered throughout the country. At any one time, with given inventories, sales and production at each of these locations, there is a policy which will minimize the cost of distribution. That the industry has recognized the predominance of the transportation factor is indicated by the new emphasis on location of new reduction plants in the Ohio Valley where coal-source power costs are higher than in the Northwest but where lower transportation costs override to provide a lower total cost of operation.

To provide for inclusion of this key policy area in the model, a simplified version of a two origin-two destination transportation policy is included. Although such a matrix is smaller than the actual situation of the large producers, it will illustrate the interaction between distribution, production and other managerial policy decisions. The equations used in the distribution function are described in Appendix M.
Figure 12B - Flow Diagram "S1" Alumina Refiner Sector
Chapter VII

THE REDUCTION SECTORS

COPPER REFINER AND ALUMINUM SMELTER

A. General

The reduction sectors of the industries—the refiner in copper and the smelter in aluminum—are the vital components in the dynamic performance of both systems. The underlying basis for this importance, however, differs in each industry. In copper the key element is the sales activity, while in aluminum it is the high value added to the product.

The importance of the copper refiner arises not from its increment of product value, but rather from its contact with the market and the exchange process. Here, for the first time, in the copper production and distribution process contact is made with the consumer of copper in the form of the fabricators and nonfabricating purchasers of the metal. Production and distribution processes are now supplemented by exchange processes.

The role of the refiner in pricing will be seen to be the vital controlling element in the dynamic operation of the system. Here too, for the first time, we must make a clear distinction between the characteristics of the custom and integrated producer process systems. Though the custom smelter began his operations at the concentrator, his control characteristics did not differ significantly in those sectors since, like the integrated smelter, he processed and shipped all the material he received. In the refiner sector, however, his
attitude toward inventory and price differ significantly from those of the integrated producer.

In aluminum, the importance of the sector is a direct result of the high value added—over 70 per cent of the cost of primary metal—in the electrolytic reduction process. The preeminent cost position of the aluminum smelter is not matched by importance in the market. The vast majority of metal sales are in semi-fabricated or fabricated products, and only 20 per cent to 30 per cent of total sales are in the form of the primary metal.\(^{37}\) For this reason, the price of primary has often been an internal transfer cost rather than a true market price. Market pressures on price tend to concentrate in the fabricator sector while the price of primary metal displays a remarkable resistance to change. In the aluminum model, demand effects on price are formulated in the fabricator sector.

Because the principal value added in aluminum occurs later in the production process, aluminum bears many of the characteristics of a manufacturing rather than an extractive industry. With its buffer inventories and production planning, aluminum displays little of the unstable flexibility of copper—a true example of an extracted commodity industry. For different reasons, then, the reduction sector is a vital one in the dynamic operation of both industries.

B. The Copper Refiner

1. General

The copper refiner significantly determines the dynamic operation of the industry through his actions in

a. Production policy

b. Price policy.

The first of these policies presents problems similar to those encountered in the concentrator and smelter sectors, except that closeness to the market and proximity to refined copper inventory makes refiner production of greater importance. Pricing policy and its formulation are unique to this sector but interact strongly with the supply-cost function in the mine sector to provide the basic characteristics of the system. Though these policies will at first be discussed separately, they are strongly interrelated especially in their effects on profitable results of company operations.

2. Production Policy

Flow diagrams of the copper refiner sectors are shown in Figures 13A and 13B. The "R" sector used for the formulation of new production and pricing policies is shown in Figure 13A while the "R1" sector used to represent current policies is shown in Figure 13B.

Like the concentrator and smelter, the refiner usually has no control over his incoming raw material. The integrated producer must accept the output of his smelters, and the custom refiner receives the smelted product of independent mines on the basis of long term
Figure 13A - Flow Diagram - "R" Copper Refineries Sector
Figure 13B  Flow Diagram - "R1" Copper Refiner Sector
contractual relationships. The production decision in general involves only a choice between processing the blister copper or storing it as raw material inventory. Production policy at the refiner is greatly affected by the information used in the production decision and the weighting assigned to various factors in production rate determination.

One approach to production rate selection, similar to that used in the concentrator "C1" and smelter "S1" sectors, bases rate on the level of existing raw material inventory.\(^{38}\)

\[
R_{1PW}.K = R_{1RMI}.K/R^4
\]

**R\(_{1PW}\)** - Refiner Production Wanted, (lb/wk)

**R\(_{1RMI}\)** - Refiner Raw Material Inventory (lb)

**R\(^4\)** - Constant expressing number of weeks of production desired in raw material inventory.

This policy, used in the "R1" sector of the model, has the basic destabilizing characteristic of any form of integral control as was previously explained in the concentrator and smelter sectors. In this sector, however, such a policy is unnecessary in that it ignores sales, shipment, finished goods inventory and unfilled order information immediately available to the manager. Proper use of this information permits the design of a more stable production policy.

\(^{38}\) R.W. Ballmer, op. cit.
The above production rule carries the unexpressed assumption that there is a market that will be capable of absorbing all the copper produced. This, of course, is the commodity point of view, and the real function of a commodity market is to serve as a clearing house for the exchange of the total supply on the market. Such unrestricted production, of course, leads to the very instability that all agree is not in the best interests of the industry.

If the objective of the refiner's production policy is to supply refined copper as needed by the fabricator, his production rate rule must take account of sales, order backlog and finished goods inventory. Such a rule would be similar to that applicable to most manufacturing enterprises.

$$\text{RPD}.K = (\text{RSS}.K) + \left(\frac{1}{\text{RL2}}\right) \left(\text{RFGID}.K - \text{RFGI}.K\right)$$

RPD - Refiner Production Desired, (lb/wk)

RSS - Refiner Smoothed Sales, (lb/wk)

RFGID - Refiner Finished Goods Inventory, desired, (lb)

RFGI - Refiner Finished Goods Inventory, actual, (lb)

RL2 - Inventory adjustment time, (wk)

This rate must be limited, of course, by raw material inventory availability and plant capacity. Should RPD exceed these limitations, a lower production rate must prevail.

To implement such a rule effectively, an adequate inventory of blister copper must be available to provide for short run increases in production to meet demand. In the long run the total inventory of
blister copper and refined copper inventory need be no more, in fact may be less, than the inventory resulting from an unstable production. In a later description of system simulation, the effects of this new policy on short and long term inventory levels will be discussed.

The production policy just described will be better suited to an integrated producer than a customer refiner. In Chapter X, the effects of the use of this and other policies by a company with the market share (15 per cent of world market), of Kennecott, Copper Corporation, will be described. Only after increased stability has been brought about by the action of an integrated producer will it be possible to expect the customer refiner to carry inventory of any size. The fact that the custom producer operates with a small margin for his smelting-refining services makes him particularly susceptible to falling prices, and for this reason he tries to sell as much copper as he receives in a very short period of time. The custom smelter will react in time but he cannot be the originator of policy changes in the industry.

3. Pricing Policy

Price expresses the ratio of exchange between copper material and money in the exchange process. Historically, price has been used to regulate the outflow of refined copper from the finished goods inventory. The custom refiner in particular has been prone to watch his refined copper inventory closely. With any increase in inventory level
displaying itself, prices will be reduced to insure against losing the
custom margin earned through a cyclical decline of prices. 39

In establishing the price of copper a number of pricing approaches
are open to the manager. Pricing policies considered in this study
are outlined below.

1. Demand-oriented pricing policies
   a. Sales (present or forecast)
   b. Unfilled Orders (past sales)
   c. Inventories

2. Cost-oriented Pricing policies
   a. Cost Plus Pricing
   b. Rigid Prices

3. Competitive Price Policies

Normally, the policy followed will be a mixture of the above approach-
es. Even in demand-oriented pricing, variable cost in the short run
and total costs in the long run provide a minimum below which prices
cannot be reduced. And in a cost-oriented policy, rising inventories
will eventually force some price reduction as the financial resources
of the firm become exhausted.

Of what effect is pricing policy on the stability of the industry?
How does a specific policy serve to increase or decrease the instabil-
ity existing from other causes? It would seem that pricing is really

39 M.R. Herman, op.cit.
displaying itself, prices will be reduced to insure against losing the custom margin earned through a cyclical decline of prices. 39

In establishing the price of copper a number of pricing approaches are open to the manager. Pricing policies considered in this study are outlined below.

1. Demand-oriented pricing policies
   a. Sales (present or forecast)
   b. Unfilled Orders (past sales)
   c. Inventories

2. Cost-oriented Pricing policies
   a. Cost Plus Pricing
   b. Rigid Prices

3. Competitive Price Policies

Normally, the policy followed will be a mixture of the above approaches. Even in demand-oriented pricing, variable cost in the short run and total costs in the long run provide a minimum below which prices cannot be reduced. And in a cost-oriented policy, rising inventories will eventually force some price reduction as the financial resources of the firm become exhausted.

Of what effect is pricing policy on the stability of the industry? How does a specific policy serve to increase or decrease the instability existing from other causes? It would seem that pricing is really

39 M. R. Herman, op. cit.
little different than other forms of control action such as production control. The effects of pricing as with other controllers depends upon the relationship of the parameters used in determining price and their relation to the flow of material in the system. A demand-oriented pricing policy based upon inventories is nothing more than an integral controller and like any integral control action it is destabilizing in its effects. Integral control here in this sector is much more critical than in the other sectors of the system in that price provides the prime informational feedback to the mine which actuates the initial material flow.

In the model, price formation is based upon a combined consideration of the pricing parameters above. Price (RCP) is a function of a weighted supply-demand ratio (RWRAT) shown in Figure 14. This ratio results from a selected weighting of factors expressed in the following equations:

\[
\begin{align*}
RWRAT.K &= \frac{1}{R10} \left( (R8) \left( RRATO.K \right) + (R9) \left( R11 \right) \right) \\
RRATO.K &= \frac{1}{R7} \left( (R5) \left( RSSD.K \right) + (R6) \left( RRSD.K \right) \right) \\
RSSD.K &= RUPO.K/RFGI.K \\
RRSD.K &= RSS.K/CPS.K
\end{align*}
\]

**RRATO** - Supply-demand Ratio

**RSSD** - Static pricing function

**RRSD** - Rate pricing function

**RSS** - Refiner Smoothed Sales, (lb/wk)

**RFGI** - Refiner Finished Goods Inventory, (lb)
Figure 14  Supply-Demand Pricing Curve
RUFO - Refiner UnFilled Orders, (1b)

CPS - Concentrator Production Started, (1b/wk)

R5 - Static pricing factor

R6 - Rate pricing factor

R7 = R5 + R6 - used to normalize R5 and R6 weighting

R8 - Pricing sensitivity weighting factor. This factor expresses the sensitivity of price to a change in demand (static or rate)

R9 - Price rigidity factor. This factor expresses the insensitivity (rigidity) of price to changes in demand

R10 = R8 + R9 - used to normalize R8 and R9 weighting

R11 - dummy factor used to remove all price rigidity in some runs when set equal to zero.

The pricing policy formulation involves a selection of values for R5, R6, R8 and R9. The value of R5 represents the integral or level gain in the system while R6 represents the proportional or flow gain. The R8, R9 weighting determines over-all price sensitivity. System stability is enhanced by a heavier weighting of R6 (proportional control) and a smaller value of R8.

An investigation of historical data reveals that the static aspect (RSSD) has been emphasized. An examination of Figure 15 reveals a strong inverse correlation between prices and inventories; while there appears to be no equivalent correlation between sales and prices. If a single factor explaining the instability of the copper
Figure 15  History of Refined Copper Price and Inventories
industry had to be selected, refiner price policy would be a prominent candidate.

From a practical viewpoint, changes in pricing policy must be related to the economic realities of revenue costs and profits. It seems apparent that not all firms in the industry pursue the price policy noted above, but the price leader in the form of the custom smelter does, and the others choose to follow. New pricing policy design emphasized stability, and R5/R6 ratios of 1:4 and 1:10 were used in the "R" sector of the model, while the equivalent ratios in the "R1" sectors were 4:1 and 2:1. The results of pursuing varying pricing policies will be discussed in Chapter X.

4. Financial Structure

The refiner operates on small margins and large volume. His contribution to the value of the product is small—less than 10 percent of refined copper cost. His profit is in the nature of a fixed service fee so that changes in the final price of copper are reflected back to the smelter, the concentrator and the mine. The extensive in-process time—about a month—necessary to purify the copper makes for a sizeable work-in-process inventory. The refiner purchases blister copper at the market price (refined copper price less a refining fee) and he must sell this copper four weeks later at the then current market price. His profit margin will consist of this fee plus or minus any change in price that occurs during the refining process. It is of interest to note that the custom refiners frequently emphasize that
they avoid finished goods inventory because they do not choose to speculate but rather earn their profit through a service fee. That their own lack of such inventory accounts for much of the price instability does not seem to be well recognized. The format of the refiner's financial structure is similar to that previously explained in the mine sector and is detailed in Appendix F.

C. The Aluminum Reduction Smelter

1. General

The production control and pricing problems of an aluminum smelter, shown in the flow diagrams of Figures 16A and 16B, differ greatly from those of a copper refiner. The copper refiner has no control over his incoming materials, adds only a small value to the product, maintains little raw material or finished goods inventory and prices to sell his product. The aluminum smelter, in contrast, directly controls his input of aluminum, either by direct management of his own refinery or through purchase from an integrated company. There is no requirement that all of the aluminum in the system be reduced since inventory flexibility exists.

Production flexibility in aluminum reduction is much less pronounced. Aluminum electrolytic reduction cells are connected in series to form pot lines of as many as 100 or more cells. Production changes are accomplished by starting up or shutting down complete pot
Figure 16A - Flow Diagram: "R" Aluminum Reduction Sector
Figure 16B: Flow Diagram - "RI" Aluminum Reduction Sector
lines. The cost of restarting a pot line once shut down is high,\textsuperscript{*}
and for this reason there is a tendency to resist short run production changes.\textsuperscript{41} If sales decrease temporarily, there is little sense in
decreasing production if a future upturn will permit the accumulated
inventory to be sold at a profit. The limit to such a policy is set
by the working capital available to the firm. A decision to change
production should be based upon balanced considerations of the costs
of increasing inventory and changing production.

A third degree of flexibility is open to management in its pur-
chase of old scrap for resmelting and subsequent use for fabricated
products.\textsuperscript{42} Fabricators purchase a certain percentage of scrap—
especially in periods of high demand—to supplement their virgin alu-
minum pig. With a drop in demand and a resulting accumulation of in-
ventory, curtailment of scrap purchase may become desirable even when
the above condition does not exist, since the cost disadvantage of pig
may be outweighed by the carrying costs of inventory or the costs of
decreasing production.

2. Production Policy

The present situation in primary aluminum is one of a fairly con-
stant price and production with fluctuating inventories. A desirable

\textsuperscript{*}A typical cost for a 30,000 ton pot line is $50,000.
\textsuperscript{41}S. Malcuit, \textit{op. cit.}
\textsuperscript{42}L. A. Doyle, \textit{op. cit.}
production policy would be one that maximized profit over a period of several years. Such a policy would determine at any time the proper mix of production rate variance and inventory accumulation most desirable for the company in the long run.

Any production policy must be based upon some sales forecast of the future. From such a sales forecast and a consideration of total current inventories—called the metal balance in the industry—the basic information for a production policy is at hand. The period of the sales forecast may be varied, and this period alone will influence the effectiveness of the production formulated, but with a given forecast and metal balance, the production rate (RPD) must be determined. The essential comparison is between the costs of carrying additional inventory and changing production. The following production rule encompasses such a compared selection:

\[
\text{RPD}_K = \text{SAMPLE} (\text{RLPD}_K, 26) \\
\text{RLPD}_K = \text{R2PD}_K \text{ if ICOST}_K < \text{PCOST}_K \\
\quad \quad \quad \quad = \text{R3PD}_K \text{ if ICOST}_K \geq \text{PCOST}_K \\
\quad \quad \quad \quad \quad \text{and PRGI}_K < \text{RFGIM} \\
\text{R2PD} = \text{RPS}_{JK} \\
\text{R3PD} = (\text{RSSP}_K) + (1/12) (\text{RFGID}_K - \text{RFGI}_K)
\]

RPD - The semi-annual Production Rate Desired, (lb/wk)

RLPD - The current Production rate Desired, (lb/wk)

R3PD - The new Production rate Desired if a change in rate is in order, (lb/wk)
RPS - Current Production rate, (lb/wk)

ICOST - Inventory carrying costs for the coming period, (dollars)

FCOST - Cost to change Production, (dollars)

PRFGI - Predicted total metal balance of inventory (bauxite, alumina and aluminum) in the system at the end of the forecast period, (lb)

RFGIM - Maximum metal balance inventory possible with current working capital

RFGID - Metal balance desired based on predicted sales.

The above production policy provides a rational basis for deciding whether a change in reduction production rate is desirable. The first equation for RPD indicates that a semi-annual planning period of twenty-six weeks is used in the production rate decision. If predicted sales (RPS) are less than the current production rate (RPS) a downward change in production rate will be initiated only if the carrying costs of the additional inventory that will be accumulated are greater than the costs of changing production. If predicted sales (RSSP) exceed the current production rate (RPS) an upward change in production rate will be initiated. In such a situation the ICOST function will be larger—less negative—than the PCOST function, resulting in the selection of the higher R3PD production rate. An upper limit on inventory accumulation is established by RFGIM which is based upon working capital availability.
The above decision rule provides a basis for determining production rate for the forecast period. The effectiveness of this policy will be evaluated in the system simulation discussed in Chapter X.

3. Distribution Policy

The production policy for any period will be strongly affected by the distribution policy which has as its objective the distribution of the products to the fabricators in the system to satisfy their requirements at least cost.

The distribution policy allocates the output of the reduction smelters to the fabricators so as to reduce the total sum of production, inventory and transportation costs. Limitations of time prevented incorporation of the policy in the simulation runs, but basic formulation of such a policy is outlined in Appendix M.

4. Financial Structure

With the price of primary metal stabilized by production and inventory fluctuations, profitable operation is a function of cost efficiency and the volume of operations. The same cost format is used as before, and the fixed/variable cost ratio is typical of the industry. The complete financial structure is described in Appendix G.

5. Investment Policy

Although investment policy was not explicitly studied in the model, the results of past investment are important in other policies. Plant capacity limits production rate, and cost structure in the form
of the fixed/variable cost ratio greatly affects pricing policy in the fabricator sector. Companies with a higher ratio of fixed to variable costs will find it more profitable to expand market share through reduced prices. Cost structure effects on pricing policy changes will be evaluated in system simulations.
Chapter VIII

THE COPPER AND ALUMINUM FABRICATOR

A. General

The copper and aluminum fabricators perform a similar technical function in fabricating sheet, wire, extrusions castings and other metal products suitable for further use by the metal working industry. A difference exists, however, in the economic function of each. Market pressures on price determination are much more severe for aluminum products since there is no significant price competition in the aluminum primary metal sector. Prices of copper-fabricated products are usually based upon the market price of primary metal. Because of this difference in the market location in the two industries, pricing policy has been formulated in the fabricator sector of the aluminum model. The pricing function performed by the copper refiner is the responsibility of the fabricator in the aluminum industry.

A second difference in the operation of the fabricators arises in purchasing policy. In the copper industry, the fabricator often speculates on the price of copper by excessive purchases during a price rise and heavy inventory liquidation in a falling market.\(^{43}\) Such speculation is absent in aluminum except insofar as there is a minor amount of scrap speculation by the foundries and other fabricators. Though copper price speculation is destabilizing, it is a reaction to the more basic problems existing in the industry. Speculation will decline only when other stabilizing polices make it unprofitable.

\(^{43}\)R.W. Ballmer, op. cit.
Outside of the areas of pricing and purchasing policy, the copper and aluminum fabricator sectors are identical except for values of parameters.

B. Production Policy

Flow diagrams of the copper fabricator sectors are shown in Figures 17A and 17B, and the aluminum fabricators in Figures 18A and 18B. The areas of difference as just noted occur in the purchasing and pricing policy areas. As in the previous industrial sectors, the "F" sectors are the media for experimental policy changes, while the "F1" sectors represent the remainder of the industry.

Production policy is based on exponentially smoothed sales orders and the level of finished goods inventory.

\[
FPD.K = (FSS.K) + (1/F_4) (FFGID.K - FFGI.K), \text{ (lb/wk)}
\]

\* FPD - Fabricator's Production Desired, (lb/wk)  
\* FSS - Fabricator's Smoothed Sales, (lb/wk)  
\* FFGID - Fabricator's Finished Goods Inventory, Desired, (lb)  
\* FFGI - Fabricator's Finished Goods Inventory, Actual, (lb)  
\* F_4 - Inventory rate of adjustment factor, (wk).

The principal production decision involves the selection of the rate of adjustment factor (F_4). Rapid rates of adjustment increase the instability of the system. In general, the desired finished goods inventory is higher in aluminum than copper, and the rate of adjustment less rapid. This tendency to carry larger inventories is a

* In the above equation notation the first number refers to the equation in the copper model described in Appendix H and the second, the aluminum model of Appendix I.
Figure 17A - Flow Diagram - "F" Copper Fabricator Sector
Figure 17B. Flow Diagram - "Fi" Copper Fabricator Sector
Figure 18A - Flow Diagram - "F" Aluminum Fabricator Sector
stabilizing factor in aluminum although its importance, as in copper, is less than the policies of the primary metal sector. A complete description of the production policies of the copper and aluminum fabricators will be found in Appendices H and I.

C. Aluminum Pricing and Service Policy

Pricing policy in the fabrication sector is inextricably connected with production policy in the reduction sector. The production rate in the reduction sector establishes the sales rate required in order to maintain inventories at present levels. If sales rate at current prices is below production rate, then the alternatives open to management are:

a. Increase sales

b. Decrease production

c. Accumulate additional inventory.

Sales may be increased in the short run by either reducing prices, increasing promotional efforts or providing better service. It is the function of price policy to determine if, when and by how much prices should be changed. As in the copper industry, prices are established by consideration of demand, cost and competitive factors.

There is good reason to believe that price reductions do not increase over-all demand for aluminum in the short run. To be sure, the relatively low price of aluminum has played a major role in its penetration of markets in the construction and the electrical utility industry, but this penetration has taken place over a period of years.
For shorter time intervals, the prime effect of price reduction by one company is to increase its share of the market. For this reason, pricing decisions must weigh the potential actions of competitors. In order to understand the basis for price formation, the following pricing policy equations were developed.

\[
FAP.K = F(\text{FFUN}, \text{FRSD}.K, \text{FSSD}.K) \tag{A*}
\]

\[
= \text{FUCOS}.K \text{ if } FAP.K \leq \text{FUCOS}.K
\]

\[
\text{FRSD}.K = \frac{\text{FSS}.K}{\text{RPS}.JK}
\]

\[
\text{FSSD}.K = \frac{\text{FUFO}.K}{\text{FFGI}.K}
\]

FAP - Fabricator's Aluminum Price, (dollars)

FFUN - FUNCTION relating a change in prices to a change in sales orders

FRSD - Rate factor in pricing that weighs the ratio of sales rate to reduction production rate

FSSD - Static factor in pricing that weighs the ratio of unfilled orders to finished goods inventory

FSS - Fabricator's Smoothed Sales rate, (lb/wk)

RPS - Refiner's Production Rate, (lb/wk)

FUFO - Fabricator's UnFilled Orders, (lb)

FFGI - Fabricator's Finished Goods Inventory, (lb)

FUCOS - Unit COST of fabricated aluminum, (dollars/lb).

* This function is a summary and simplification of equations F35, F36 and F39 in the aluminum model described in Appendix I.
The pricing rate factor (FRSD) directly relates pricing to reduction production and explicitly indicates the choice between a sales increase or a production decrease for a given weighting. The static factor (FSSD) introduces inventory and order levels as modifiers to rate pricing. Weighting of these two factors will depend upon considerations of internal costs and external competition. If other companies in the industry have reduced production after a drop in sales, both factors should be weighed less since there will be less pressure for the competitor to reduce prices. If, however, competitors maintain a high production rate with a falling demand, then the weightings must be increased. As was seen in copper, a more stable operation is provided by a heavier emphasis on the rate factor, though price is less important here since it does not affect production in the primary metal sectors as a prime informational feedback as in copper.

The unit cost (FUCOS) provides a lower limit to price in the model. In practice, unit cost is an elusive quantity since it is affected by a rather arbitrary approach to fixed cost allocation in the cost accounting system. The cost functions of competitors should be approximately known to establish factor weighting in price formation. In the aluminum industry, such cost estimates are not too difficult since the factor costs are generally well known in a given region.

A service function modifying sales orders based upon price has been incorporated in the model. Sales orders of the customers are modified as a function of delivery time which in turn is a function of
inventory. The principal decision for management involves the level of inventory carried. A lower level of inventory increases delivery time, leading to a reduction of sales orders to that company. This function is detailed in Appendices H and I.

D. Purchasing Policy

Since a high degree of vertical integration exists in both industries, many fabricators are "locked in" to their reduction sector suppliers. Approximately 40 per cent of all fabrication in copper, and 60 per cent in aluminum, are performed by integrated fabricators.\textsuperscript{44} Despite this lack of independence in source selection, integrated fabricators purchase orders are based on requirements for current production and inventory changes. They do not usually maintain large raw material inventories since it is more economical to retain the metal at the reduction sector and avoid carrying additional transportation costs on inventory. The purchasing policy of the integrated fabricator is designed to maintain an adequate level of primary metal at the fabricator and in the pipeline so as to insure uninterrupted production. In such a policy the only important stability consideration is the rate with which actual inventories are adjusted to desired inventories. The equations are identical in form to those used in the production policy just described except that sales rate is replaced by production rate and finished goods by raw material inventories.

\textsuperscript{44}E.B. Alderfer and H.E. Michl, op.cit.
FRMPD.K = FDPT.K + (1/F6) (FRMID.K * FRMI.K)

FRMPD - Raw Material Purchases Desired, (lb/wk)
FDPT = FPD - Fabricator Purchases for current production, (lb/wk)
FRMID - Raw Material Inventory, Desired, (lb)
F6 - Inventory adjustment time, (wk).

In copper, the desired raw material inventory (FRMID) for a non-integrated fabricator is modified by a speculation modifier (FSM). In the model such speculation is a function of the rate of change in copper prices. As prices rise purchasing is increased, and with a decline in prices, purchasing is reduced. With the stability existing in the price of primary aluminum pig, there is no incentive for such speculation by the nonintegrated aluminum fabricator.

E. Financial Structure in Fabrication

The cost structures in both industries are almost identical since the same type of equipment and personnel are required. Since fabricated products differ in cost structure, the cost structure suitable to sheet and plate in aluminum and copper was selected for the sector. Since the major part of fabricated products is in the form of sheet and plate, the cost structures are typical of the industries. The financial format, similar to that in previous sectors, is detailed in Appendix I.
Chapter IX

THE USER SECTOR

A. General

Aluminum and copper are primarily industrial products. The customer is usually an industrial enterprise that uses aluminum or copper to manufacture end products for consumer or industrial use. A notable exception to this general rule is aluminum foil which is produced by the aluminum companies for direct sale to consumers.

The markets of both metals are many and varied. The customer in the construction industry differs from the customer in the automobile industry who in turn differs from the electrical utility customer. Eventually, it would be desirable to represent the purchasing and use characteristics of each of these main classes of customers in the model. Such a representation would become especially important when the model was applied by an individual company since the model would then more closely reflect the dynamic characteristics peculiar to its "marketing mix." In this study, however, the customer was considered as an aggregate user. Varying characteristics of different users may be simulated by variations of parameters determining purchasing, production and marketing policies.

From the viewpoint of the primary metal producer, the characteristics of the user are structural in nature, since he cannot modify them significantly. The purchasing, production and marketing policies of the user, together with his basic technical and economic characteristics appear as the market structure to the copper or aluminum
producer. The primary differences between the policies and the more basic characteristics of the user lie in their susceptibility to change. Structure changes but slowly, while user policies are often modified to suit economic conditions. Policies of the producer must be developed only after a thorough consideration of those market features which are sensitive to policy changes by the users.

A final important feature of the user is the nature of his own demand inputs. Industries such as consumer durables are much more susceptible to demand fluctuations than containers and packaging products. The sales order input to the user after modification by user structure and policies is transmitted directly to the fabrication sector. The nature of the user's input as well as his structure and policies must be well understood. These inputs will be discussed more fully in the next chapter on system simulation.

B. Purchasing Structure

Flow diagrams of the user sectors are shown in Figures 19A and 19B. The essential factors incorporated in the purchasing policy of the user are:

1. Sensitivity of purchase to price
2. Sensitivity of purchase to delivery time
3. Raw material inventory policy.

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45 R.W. Ballmer, op.cit.
Figure 19A, Part 2, Flow Diagram - Copper User Sector
(Continued in Part 2)
Figure 19B Part 1 Flow Diagram: Aluminum User Sector (continued in Part 2)
Figure 19B. Part 2 Flow Diagram: Aluminum User Sector (cont. from Part 1)
In both models, two competitive producers receive their sales orders from the user as a function of the above factors. The amount of metal to be purchased is based upon raw material inventory policy which in turn is affected by sales and production rates.

With the aggregate metal requirements determined, the aluminum or copper purchases are allocated to competing companies on the basis of price, delivery time and long term service reputation. The latter factor is incorporated in the model through the use of a purchase selection function shown in Figure 20. From the curve, it can be observed that the "F" sector company will receive a certain percentage of the total business independent of price, and the other "Fl" sector companies will maintain a similar position with respect to another segment. The remainder of the market is considered to be sensitive to price, with the lower bidder receiving the larger share of the market. The purchasing percentages determined from the price ratio is then modified by the delivery time factor which is a ratio of the delivery times of the two competitors. A detailed description of the user policy will be found in Appendix J.

C. Production Structure

The production policy of the aggregate user is based upon smoothed sales orders and finished goods inventory adjustment and, except for values of parameters, is identical to that of the fabricator.
LONG RUN METAL USE - ULRCxULRAY - RATIO SCALE
"F" SECTOR

USER METAL PURCHASE RELATIONSHIP

Figure 20
D. Financial Structure

A cost structure has not been incorporated in the user sector of the model. Since the aggregate user represents such a wide variety of customers, any numbers selected would not be especially meaningful. With further development of a market segmentation in the user sector, however, a cost function for each user industry would be meaningful and helpful in explaining purchasing, production and marketing policies of the user.

E. Future Development of the User Sector

There seems little doubt that the greatest need for future development of the model exists in the user sector. Market segmentation of the user sector would be very desirable, especially in the dynamic aluminum market. Long range product and market mix policies should be formulated, along with capital investment policies. To be meaningful, however, this formulation should take place within the framework of an operating company in the industry and will require data ordinarily classified as confidential by the companies concerned.
Chapter X

SYSTEM SIMULATION AND MANAGERIAL POLICY TESTING

A. General

The preceding chapters of this report have concentrated on an explanation of the existing structure and current and proposed policies in the copper and aluminum industries. The detailed structure and policies of the industry as a whole and its successive sectors have been described in order to fulfill the first objective of the thesis, viz., the determination of the present structure and policies of the industry as a basis for the explanation of past dynamic performance and an indication of the direction for future improvement. The remaining objectives of the thesis: improved policy testing, company implementation problems and price policy effects, all require use of the Industrial Dynamics model developed for system simulation.

After an initial "debugging" and testing period, the simulation program for each model encompassed the following types of runs:

1. Runs to determine the sensitivity of the model to variations in parameters. The single flow model with one sector at each stage was used for these runs.

2. Runs to determine the influence of changes in price, production and other policies on the stability and profitability of industrial operation. The single flow model was still used for these runs.

3. Runs to determine the influence of changes in managerial policies of a single company operating in a competitive market on the company itself and on the industry as a whole. The dual flow competitive model was used for these runs.
Simulation of the aluminum and copper industries, because of the great differences in their basic problems, proceeded along quite different lines, and for this reason the test results and policies will be discussed separately.

B. The Copper Industry Simulation

1. General

The early simulation runs using the single flow model were primarily devoted to testing the sensitivity of the model to variations in parameters such as smoothing times, production time delays, the mine supply function, maximum production limitations, production control constants and other structural and policy parameters in the model. Pulse and noise inputs were used as the exogenous inputs to the model in the form of sales orders to the user sector. This sensitivity testing had the two-fold purpose of verifying the validity of the model and providing the basis for later policy investigations. Confidence in the validity of the model depended on the relative insensitivity of the model to small variations of most parameters. Excessive sensitivity to a large number of parameters would have indicated either a requirement for more exact determination and verification of the sensitive parameters or serious doubt as to validity of the model itself.

Later simulation with the single flow model emphasized changes in the decision functions used for pricing and production-inventory control. Pulse, noise and step inputs were used for some policy testing,
but the later runs were conducted with inputs similar to the historic inputs of the 1955-60 period. Consumption data for copper during this period were used for user sales orders, and production disruptions resulting from strikes were also included as exogenous inputs to mine production.

Final simulation of the copper industry was conducted with the dual flow competitive model. The policies developed with the single flow model were used by a single company competing against the remainder of the industry. The effects of the new policies on the profitability of the company and the stability of the industry were evaluated.

A complete summary of all copper simulation runs conducted is included in Appendix K. In the remainder of this section, the most important results of these runs will be presented.

2. The basic system response

The end effect of the basic structure and policies of the copper industry is displayed in its response to an input change in user sales orders (USO). In the early runs, a pulse input was used to test the stability response of the system.

A plot of the response of the system to a pulse of user sales orders is shown in Figure 21, which displays the results of RUN NUMBER 1348. With the system initially operating at steady state conditions of constant sales and production rates in all sectors, the input pulse of user sales is transmitted through the user and
Figure 21
Basic Response to Pulse Input
(RUN 1348)
fabricator sectors with a short time delay and a slight change in amplitude to the refiner sector where it both increases the average sales and order backlog and decreases the refined copper inventory (RFGI) driving up the price of copper (RCP) above the initial price of thirty cents per pound. Very quickly, the small raw material inventory of the refiner (RRMI) is depleted leading to further decrease in production (RPS) and a greater increase in price (RCP). Inventory depletion and consequent production rate decreases in the smelter and concentrator sectors further aggravate the rise in price.

To further increase the difficulties, the mine sector is influenced by the high prices to shift to a lower grade of ore which actually decreases production rate (MPS) at a time when an increase is critically required. Eventually, the mine responds slowly to the higher price with increased production (MPS) but not until average user sales orders (USS) have already begun to decline. This decline in user sales leads to a decline in user production and purchasing which decreases demand down the line leading to a rise in refined copper inventory (RFGI) and a drop in price (RCP). The system then oscillates through the remainder of the cycle to reach a state where refined inventory (RFGI) slowly accumulates and price (RCP) gradually declines.

During the initial rise, price (RCP increases from 30 to 46 cents per pound, peaking at about 36 weeks from time zero. The price of copper after seven years (360 weeks) is about 28 cents per pound.
Although the pulse input response provides some insight into the characteristics of the system, a more typical response, illustrated in Figure 22, results from using the historical consumption of copper during the 1955-1960 period as the exogenous user sales order input to the system. It is important to emphasize that no attempt was made to reproduce history with the model. Nonetheless, it is encouraging to realize that the response of the system in RUN 2053 is quite similar in basic configuration to the historical results displayed in Figure 15.

With historical consumption user sales order inputs, price (RCP) rose to about 52 cents per pound, around the end of 1955 and then declined to 27 cents per pound at the end of 1957. Inventories (RFGI) varied as the approximate mirror image of price during this period. In the latter policy-testing runs, the historical inputs were used in order to add a note of realism to the simulation results.

3. Sensitivity testing results
   a. Time delay variations

Production and transit time delay variations did not significantly affect the response of the system to a pulse input. Variations in time delays of six weeks or more, 50 per cent of the original total time delay, were tested. Price, production and inventory fluctuations were basically identical to those of the original model.

An example of this relative insensitivity is illustrated in Figure 23 which displays the results of RUN 1462 where the smelter
Figure 22

Basic Response to 1955-1960 Historical Consumption Inputs

(RUN 2053)
Response to Variation in Smelter Production Time Delay
(DEL 13)
(RUN 1462)
production time delay (DEL 13) was increased from 1.2 weeks to 4 weeks with little effect on the basic response which is almost identical to the reference response to a pulse input in Figure 21.

b. Production-inventory policy parameters variations

Variations in the amount of inventory carried and the resulting effects of these variations on production did not significantly affect the response given the same production and pricing policies, except where the desired finished goods inventory level of the refiner was changed. The desired level of the refiner's finished goods inventory (RSSC) was very critical in its effects on price formation, to be described below.

A marked increase in the instability of the system will be observed in Figure 24 illustrating the results of RUN 1569, where the desired refiner's inventory level (RSSC) was decreased from 5 weeks to 2 weeks. The refined price rose to 62 cents per pound as compared to 46 cents in the reference pulse input run of Figure 21. Parameters values were varied for other sectors as much as 100 per cent of the original values without significant effect on the response of the system.

c. Pricing policy parameters variation

The response of the system was extremely sensitive to parametric variations in the decision rule used for pricing. Variations in the sales/production rate and unfilled order/inventory weighting factors (R5 and R6) proved to be the prime sensitive areas in the primary
Figure 24

Response to a Decrease in Desired Refiner's Finished Goods Inventory (RSSC)

(RUN 1569)
metal sector of the model. Variation of the R5/R6 ratio from 2.0 in the reference run (RUN 1348) to 0.25 in RUN 1560 results in the increased system stability shown in Figure 25. The initial price rise was limited to 36 cents per pound as contrasted with 46 cents per pound in the reference run. Production and inventory were also more stable with the increase in emphasis on rate factor (R6) pricing. Such sensitivity is understandable since price is the prime information feedback loop in the system, and pricing policy is a prime element in over-all control.

4. Policy formulation and testing results

a. Design approach

New policy design efforts were concentrated in the areas of pricing and production-inventory control. The approach used was based upon fundamental design approaches used in process control system design. Although this industrial system is a high order system and very nonlinear, the intuitive concepts of linear control systems design were found very useful. In general, an attempt was made to substitute or supplement integral control policies now used in both pricing and production-inventory control with proportional and rate control information. In every case, the stability of the system was improved by these changes. A large number of policies were tested, and the most practical and least costly of these policies were selected for final use.
Figure 25

Response to a Change in Pricing Policy-Pulse Input
(RUN 1560)
b. Pricing policy

Demand and cost-plus approaches to pricing were tested. A demand pricing policy found very effective weighted the rate (sales) factor (R6) four times as heavily as the static (inventory) factor (R5).

The change in the system response to the 1955-1960 inputs with such a pricing policy (RUN 2054) is shown in Figure 26. Price, production and inventory are notably more stable. This policy was later used as part of the policy mix of the company in the competitive model.

Cost-plus pricing (RUN 2058) was found to stabilize the industry but was unprofitable in the long run and resulted in excessive variations in production.

c. Production control policy

Production control decision rules using both flow rates and stock level information were tested in each sector. The most effective of these rules involved a feedback of smoothed refiner sales to the smelter sector. The new response (RUN 1563) is illustrated in Figure 27. Initial price rise response was limited to 38 cents per pound, and over-all stability was significantly increased. It will be noted that production rates are more stable than they were with a change in price policy alone (Figure 25). Most of the new production rules (RUN 1941, RUN 2055) were able to stabilize the industry with the present price policies unchanged, but results indicated that the most profitable policy mix would combine both pricing and production control.
Figure 26
Response to a Change in Pricing Policy-Historical Inputs
(RUN 2054)
Figure 27

Response to New Smelter Production Policy

(RUN 1563)
The final production control scheme selected incorporated changes to the mine, concentrator, smelter and refiner sectors as follows:

1. The refiner's production started rule (RPS) changed from one based solely upon the level of raw material inventory (RRMI) to one using smoothed sales (RSS) and finished goods inventory level (RFGI) information. Both the old and new policies are discussed in Chapter VII.

2. The smelter's new production started rule (SPS) was also based upon refiner's smoothed sales (RSS) and inventory requirements (RRMI) as limited by his raw material inventory (SRMI) as described in Chapter V.

3. The concentrator's production started rule (CPS) was adjusted to mine copper produced (MCP) and to maintain a desired level of copper concentrate raw material inventory (SRMI) for the smelter as explained in Chapter V.

4. The mine desired production rate (MPD) was modified to incorporate refiner's smoothed sales (RSS) as well as price (RCP) as an input variable. This new policy, described in Chapter IV, was necessary to support the new production policies at the other sectors without a requirement for excessive increases in inventory. Ore yield selection was made unresponsive to price changes.

Using this production control policy mix the industry was stabilized and carrying costs on total metal inventory in the system were reduced. The response of the system using the new policy with
the 1955-1960 consumption inputs (RUN 2055) is shown in Figure 28. Both price (RCP) and inventory (RFGI) stability were improved, but production stability (RPS and FPS) were less stable than would be desirable. In a later run (RUN 2117), using the competitive model, this excessive variation in production was removed by increasing the inventory adjustment time (R12) in the refiner's production desired policy (RFD). The results of this run are shown in Figure 29.

d. Competitive model policy testing

Final model simulation runs evaluated the effects on the industry resulting from the use of the selected pricing and production policies by a single company with 50 per cent and 15 per cent of the total market. The latter share represents the position of the Kennecott Copper Corporation, the largest integrated producer of copper in the free world. These runs provided a practical and realistic test of the new policies, since it is likely that future improvements in stability must come from the action of a single company.

The first competitive model test run, splitting the industry into two halves, demonstrated that the industry could be stabilized if one-half of the industry followed the new pricing and production policies. In Figures 30 and 31 illustrating the results of RUN 2115, the marked increase in stability is emphasized by a comparison with the previous response of RUN 2053 in Figure 27.

The tabular results of the run indicated that the total profit of the new policy half exceeded that of the old policy half even though
Figure 28

Response to New Policy Mix

(RUN 2055)
Portions of the text on the following page(s) are not legible in the original.
Figure 29

Competitive Model Response to New Policy Mix
(RUN 2117)
Figure 30

Competitive Model Response—New Policy Half

(RUN 2115)
**Figure 31**

**Competitive Model Response - Old Policy Half**

(RUN 2115)
the old policy half charged higher prices. The difference resulted from higher sales volume and lower inventories in the new policy half of the industry. These results were especially significant in that no attempt had been made to "optimize" profit considerations in any way. In later runs, however, more profitable approaches to stabilization were investigated.

In the remaining competitive model runs, a firm with 15 per cent of the market (Kennecott) competed against the remainder of the industry. The firm used the new pricing and production policies. In Figures 32 and 33, the results of RUN 2116 depict the effects of the firm's actions on the industry. Both the firm (Figure 32) and the industry (Figure 33) have been increased in stability as compared to the reference response of RUN 2053 (Figure 27).

With the basic stabilization capability demonstrated, the ensuing runs emphasized the cost-profit differences of varying stabilization policies. The results (RUNS 2117, 2118, 2119, 2120, 2121) indicated that variations in the policies pursued have significant effects on both the total profit of the industry and the profit share of the stabilizing firm. The plotted results of RUN 2120, in which the mine production policy was changed to de-emphasize price (RCP) in favor of sales (RSS) in the determination of the mine production desired (MPD), is shown in Figures 34 and 35. The plot indicates increased stability for the industry, but total profits of the industry dropped 10 per cent, and the profit share of the stabilizing firm decreased by
Figure 32

Competitive Model Response - Company

(RUN 2116)
Figure 33

Competitive Model Response—Industry

(RUN 2116)
Figure 34

Competitive Model—New Mining Policy—Company

(RUN 2120)
Figure 35
Competitive Model-Industry
(RUN 2120)
2.5 per cent. The detailed results of the other runs are described in Appendix K.

The simulation runs of the copper industry have demonstrated the capability of a company with 15 per cent or more of market share to stabilize the industry with the new pricing and production policies. It has also been demonstrated that the profit results of these stabilizing policies vary significantly with changes in the new policy mix. The final determination of the policy parameters will depend upon more detailed information on the cost structure of the initiating company.

C. The Aluminum Industry Simulation

1. General

It is extremely important to understand the different objectives of the aluminum industry simulation runs. Initially, the aluminum model was developed to clarify and contrast with the structure and policies of the copper industry model. The remarkable differences between these two metal industries have been described in the previous chapters of this report. It was realized from the inception of this study that the basic problems of the aluminum industry contrasted sharply with those of the copper industry. The primary problem of the copper industry was clear—to achieve industrial stability through managerial policies capable of implementation by a single company without profit penalties to the initiating company. With stability achieved, that industry will be capable of more profitable long range growth through product and market development.
In the aluminum industry, problems are both more complex and less sharply defined. The current conditions of overcapacity and vigorous price competition in fabricated products are well known, but such conditions call for changes in policy that do more than achieve stability. Rather, a mix of short run and long run policies for profitable stability and growth are required. In other words, the aluminum industry poses problems that will exist for copper only after industrial stability is achieved.

Short run policies will include decision-making formulations for production, pricing, promotion, fabricated-product service inventories and physical distribution. The objective of these policies will be to improve return on investment on a long term basis within the constraints imposed by the effects of long term investment and product-market mix policies.

Long term policies will emphasize decision-making in property, plant and equipment investment and the development of the product and market mix. The effects of these long term policies are a cost structure and a market that provide the basic constraints for short term policy.

In the study to date, emphasis has been placed on short term policies. Long term policies have been influential only insofar as they have established the financial structure in all sectors and the purchasing and production characteristics of the sector. The primary objective of the simulation runs was to demonstrate the critical
interrelationships between production, pricing, distribution and service policies using a cost structure typical of companies in the industry. Future development of the model will emphasize the explicit incorporation of investment and product-market mix policies, together with the adaptation of short term policies to the circumstances of a particular company in the industry.

Early simulation runs were used to test the model and to determine the basic response of the system. Later runs tested the effects of changes in production, pricing and service policies. The detailed results of all of the runs are described in Appendix L. The most important of these results will be summarized in the remainder of this section.

2. The basic system response

A plot of the response of the aluminum industrial model to the same pulse input used to test the copper model is illustrated in Figure 36. This run (RUN 1937A), when compared to the equivalent copper run in Figure 21 serves to indicate the inherently greater stability of the aluminum industry. Price, production and inventories all display greater stability than their copper counterparts. These simulation results correspond to the general experience and statistical history of the industry. This run and the three following (1937B, 1937C and 2046) used the single-flow noncompetitive model. The remainder of the runs used the dual-flow competitive model.
Figure 36
Aluminum Model-Basic Response to Pulse Input
(RUN 1937A)
The first run (RUN 1937A) was based upon continuous adjustment of production rate on the basis of inventories and sales orders. Actually, the companies in the industry plan by discrete periods—months, quarters or half-years—and a more realistic model will take account of the planning period. In RUN 1937B, illustrated in Figure 38, a quarterly production planning period was used with the result that the instability of the system was noticeably increased. Further lengthening of the planning period in RUN 1937C to six months resulted in a further increase in instability as shown in Figure 38. The results of these runs demonstrate the importance of the planning period in system stability and indicate the need for considering the length of the planning period as a variable in policy formulation. The prime purpose of these initial runs, however, was to provide reference runs for later comparison with new policy test run results. Another reference run (RUN 2046) was simulated using a noise input as shown in Figure 39. This run is more typical of the history of the industry. Production and inventories fluctuate but prices are quite stable.

Previous experience gained in the early copper simulation runs made it possible to omit sensitivity testing runs in the aluminum model simulation. From the beginning, chief emphasis was placed on policy formulation by a single company under competitive conditions in the industry using the dual-flow competitive model.
Figure 37

Aluminum Model-Quarterly Planning Period

(RUN 1937B)
Figure 38

Aluminum Model-Semi-Annual Planning Period

(RUN 1937C)
3. Production policy - formulation and test

After an initial test run to check the competitive model (RUN 2050), a company with 30 per cent of the market was represented in the model and competed against the rest of the industry, using the new production policy described in Chapter VII. An earlier run (RUN 2126) contained certain errors in the model that indicated a lower profit for the company, but when these errors were corrected in a later run (RUN 2130) the company increased its profits above its normal share by 5 per cent. Later run results indicated that with further effort this profit margin could be improved by a more detailed study of policy parameters in relation to the cost structure of the company.

4. Pricing policy - formulation and test

Pricing policies tested included variations in both sales versus inventory weighting and general sensitivity to price changes with a change in demand. In RUN 2127 the company increased the weighting of the variable sales in its pricing policy with the result that profits were increased by 20 per cent and inventory carrying costs (not included in the above operating profit) were decreased by 15 per cent.

The relative sensitivity to price change as a result of sales change was increased for the company in RUN 2128. The results were unfavorable with profits declining about 8 per cent and inventory carrying costs rising by 15 per cent.

The extreme sensitivity of profits and inventories to pricing is evident. No claim is made that either of the above policies is
suitable to a particular company. It is only important to show that more extensive analysis using company cost data will enable important gains from using a rational and integrated pricing policy.

5. Service Policy - formulation and test

The service policy was modified to increase the fabricated-products inventory level (FSSC) by a factor of four to offer rapid delivery time. The results of this change in policy as tabulated in RUN 2129 indicate that the increased profits of 3 per cent were more than over-balanced by additional inventory carrying costs. A smaller increase in service inventories is more compatible with a profitable service policy.

6. Conclusions

Simulation runs of the aluminum industry have demonstrated the critical nature of short term policies and their effect on profitable operations. Application of the model to a particular company with its unique cost structure and product-market mix will permit the design of a profitable short term policy mix.
Chapter XI

CONCLUSIONS AND RECOMMENDATIONS

A. General

The conclusions of this study may be categorized into four areas which are in reference to the objectives of the thesis:

1. The explanation of present dynamic behavior
2. Policies recommended for improved dynamic performance
3. Effect of the size of the company pursuing these policies
4. General conclusions about the dynamic effects of pricing policies.

Conclusions for each of the first three categories were developed separately for the copper and aluminum industries. Pricing policy conclusions were developed from results obtained in both industries.

B. Copper Industry

1. Dynamic Characteristics

The copper industry has been characterized by a marked instability of prices, production and inventories for many years. The important structural and policy characteristics that provide for this instability are the following:

a. The varying percentages of yield obtainable from copper mines result in a steep supply (aggregate cost) curve in the region of operation. This functional relationship develops a large change in price for a small change in demand (sales orders). — Structural characteristic.
b. Two structural processing systems exist with different economic characteristics. The one system encompasses the integrated producers with common vertical ownership of the process from mine through fabrication. The other includes a large number of independent smaller mines and a few custom smelters who process and sell the output of these mines. The latter group, by reason of its essentially short range price, production and inventory control policies, provides a destabilizing element for the entire industry. — Structural characteristic.

c. The unstable pricing policy of the custom smelters' sector overemphasizes refined copper inventory level as the basis for control. — Pricing policy characteristic.

d. Production is not flexible for increases in demand. Since the industry is usually operating at full capacity, there is little or no margin for expansion with an increase in demand. — Financial investment policy characteristic.

e. Coordinated production control systems do not exist even in the integrated companies. — Production control policy characteristic.

f. There is a lack of inventory at the intermediate stages to provide for increased output on short notice when an increase in demand occurs. — Production-Inventory control policy characteristic.
g. Mine production control policy changes ore yield selection with price to maintain a constant profit margin. This policy serves to further steepen the slope of the supply curve to produce wider price swings. — Production and financial policy characteristic.

2. Policies for Improved Performance

a. The mine should base its production rate on demand in the form of average sales as well as price, and should select a long term ore yield policy that does not vary with short term price fluctuations. The details of this policy are described in the policy for mine production desired (MPD) in Chapter IV.

b. The concentrator should schedule his production rate to coordinate with the mine and should maintain concentrate inventories as required by the smelter, as described in Chapter V.

c. The smelter's production rate should correspond with refiner's average sales except as is required to maintain refiner's raw materials inventories (See Chapter V).

d. The refiner's production rate should be changed as a function of average sales and desired finished goods inventory as described in Chapter VII.
e. The refiner's pricing policy should be based more on sales rate and less on inventory levels as outlined in Chapter VII.

The recommended policies involve a combination of new pricing and production-inventory control rules at the mine, concentrator, smelter and refiner.

A company such as the Kennecott Copper Corporation, producing 15 per cent of the annual world supply of copper, may stabilize the industry by unilaterally pursuing the policies described above in a competitive market. This stabilization capability has been demonstrated by the simulation runs described in the previous chapter.

The final selection of parameters for the above policies should be accompanied by an analysis of the specific cost structure of the company initiating the policies, since variations in the cost structure can have an important effect on the profitability of the stabilization policies selected.

C. Aluminum Industry

1. Dynamic Characteristics

In marked contrast to the copper industry, production and prices of aluminum are quite stable, especially in the primary metal sector. Fluctuations in demand are largely absorbed by periodic oscillations in the level of inventories. The average levels of inventories at all stages of production are much higher than those in copper, with metal inventories as large as thirty-ninety days of production being quite
common. These inventories fluctuate over a wide range to permit stable production rates and prices.

These short term dynamic characteristics have been modified by a longer term trend toward falling prices and decreased use of existing plant facilities. Expansion in plant capacity in recent years has exceeded the growth in market demand with the result that price competition in fabricated products has developed as each firm attempts to operate at a capacity consistent with its own cost structure and market situation. The newer plants with higher fixed and lower variable costs find it desirable to reduce prices to win a larger share of the market, in order to achieve higher volume. Older plants with lower fixed and higher variable costs reduce volume but are forced to sell at the depressed prices established by their more efficient competitors.

The problem for each firm in the short run is to establish its most profitable production, service, pricing and distribution policies within the framework of its cost structure and product-market mix. In the long run, the firm must seek to design its plant facilities and product-market mix to improve the framework for more profitable operations.

A special dynamic feature of the aluminum industry derives from the nature and importance of transportation costs. The wide geographical dispersion of the successive stages of production make allocation of transportation an important and continuously changing factor in total distribution costs. Transportation allocation, as part of the
distribution policy, must be integrated with production, pricing and service in the short term policy mix.

2. Policies for Improved Performance

a. General

The policy recommendations for the aluminum industry must take a different form than those prescribed for the copper industry. Policies are more directly aimed at the interests and actions of a particular company. The model simulation runs have demonstrated the critical interrelationships between short term pricing, production and service policies and the effects of long term investment and product-market mix policies on the profitability of these short term policies. It is now necessary to apply these results to the decision-making policies of a single company within the aluminum industry. Short term and long term policy considerations will be discussed separately.

b. Short term policy design

Initially, the company concerned should determine its production, distribution, pricing and services policies as they currently exist. These policies may not be explicitly known, but the very need to clarify them will tend to expose the nature, express or implied, of current decision-making characteristics. These existing policies may then be tested in the model to determine their effectiveness as an integrated policy mix. The long term return on investment results of the current policy mix may then be compared with the profit returns of
some of the policies described within this report. Although some of
the policies tested in the model proved profitable under the assump-
tions made, these same policies may or may not prove desirable for a
particular company, given its cost structure and product-market mix.

The ultimate objective is to experimentally determine a policy
mix suitable and profitable for the company concerned. Using the
segmented-market user sector and the segmented-product fabricator
sector applicable to the company, a suitable policy mix may be deter-
mined after a reasonable number of simulation runs. Although the
basic structure of the present model will probably not change signif-
ically, a significant amount of duplication in the fabricator and
user will be necessary to characterize the multiple products and
markets. It may also be necessary to divide the industrial "1" sec-
tors of the market into segments in order to represent competitors
with varying characteristics. Such changes are not fundamental, how-
ever, and primary efforts should be directed toward the determination
of company product, market and cost parameters.

c. Long term policy Design

Short term policy design was based upon:
1. An existing product mix
2. An existing market mix
3. An existing plant and financial structure.

It is the objective of long term policy design to modify these
structural characteristics in such a way as to insure a profitable
growing enterprise. Product mix variations and their effects on the fabricator sector must be investigated in conjunction with similar analyses of markets and their effects on the user sector. Both of these policies must be integrated with investment policy which will determine the new plant and financial structure. The final integrated long term policy mix will ensure a properly balanced growth of products, market and facilities directed toward long term profitable operation.

3. Effect of Company Size on Use of Recommended Policies

Company size and its effects are not as significant as in the copper industry since over-all stability already exists, and no one company is trying to stabilize the industry. The effect of the size of the company has been included in the competitive model by appropriate initial division of sales orders according to market share. In the simulation runs to date, variation in the size of the company from 50 per cent to 30 per cent of the market produced no significant effects.

D. Pricing Policy

Pricing Policy in the copper and aluminum industrial systems was most critical in its effects on

1. Stability
2. Profitability.

Short term effects of pricing policy on stability were subject to extensive analysis in the simulation runs of both models. Stability
of pricing were greatly affected, first of all, by the structural position of price determination in the system. In the copper model, where pricing took place in the refiner sector and provided the prime informational feedback to the mine which was greatly affected by marginal cost considerations, pricing proved to be the key controlling element in the stability of the system. Pricing, in the aluminum model, in contrast, with its location in the fabricator sector where it was removed from the prime feedback loop of information to the early sectors, was not a critical factor in stability.

After structural position, the kind of information used and the weighting it received in pricing was of key importance in system stability. In general, a greater emphasis on level information such as inventories and unfilled orders as opposed to rate information such as average sales and production, tended to destabilize the system. Stability was improved through a greater weighting of the rate information.

In contrast to stability objectives in pricing, which are essentially short term in outlook and effect, profitability considerations should be based upon long term factors. Such considerations are difficult in a competitive industry plagued by overcapacity and "soft" prices. Nevertheless, it has been shown in the copper industry that a single company can determine and enforce a long term stable and profitable price policy using the proper policy mix.
E. Recommendations

It is most important that both the copper and aluminum models be developed for application to a particular company in each industry. The writer is of the opinion that the models have reached a stage of development where such particular application is both possible and necessary. Although certain additional features, as previously mentioned in this chapter, should be added to both models, the principal requirement is one of detailed application of the cost and operating characteristics of particular companies to the basic model. Only through such application can the validity and usefulness of the models be appraised.

The problems of model application in the two industries are quite different. The copper model is the more highly developed, with a specific set of policies established which are suitable after some modifications for company characteristics to actual operations. Cooperation with companies in the industry, however, has been less effective, and a much greater effort will be required to obtain application of the model in the industry. Although the achievement of stability is in the best interests of the companies in the industry, a pronounced tendency toward short range thinking in the industry must be overcome.

The situation in aluminum is a happier one. Although the model is not quite as highly developed and its orientation to individual
company characteristics makes the policy recommendations less specific, the attitude of management in the industry has been exemplary and the obstacles to future progress are few. This writer feels that in a period of about a year the model can be adapted to a company, and a set of short run and long run policies can be designed to contribute to an industry that has excellent prospects for the future. There is every indication that the use of the Industrial Dynamics model can contribute immeasurably to a greater and more profitable growth.
APPENDIX A

Mathematical Description of the Copper Mine Sectors

A. General

The "M" and "Ml" copper mine sectors (Figures 7A and 7B) will be described concurrently in order that the differences between policies in the two sectors may be clarified. The structure and policies of the mine will be described in correspondence with the time-sequenced flow of information, material and money with the following sub-divisions applying:

1. Production Policy
2. Production Structure
3. Financial Structure
4. Investment Policy
5. Initial Conditions.

Definitions of model symbology will be listed in order of appearance. A term once defined will not be redefined in the same appendix.

B. Production Policy

1. Average Copper Price (MACP and MlACP)

The prime input variable for mine production policy refined copper price (RCP) is time-averaged to develop average copper price (MACP). MACP is an exponentially-smoothed average since the current difference between RCP and MACP is divided by the smoothing time and then added to the previous MACP to form the new MACP. Identical functions exist in both mine sectors. Ml is the smoothing time.
MACP.K = MACP.J + (DT) (1/ML) (RCP.J - MACP.J), $(\$)  
MLACP.K = MLACP.J + (DT) (1/ML) (R1CP.J - MLACP.J), $(\$)  
ML = 24, (wk)

2. Mine Production Desired (MPD and MLPD)

Mine production desired (MPD and MLPD) is obtained from a function based upon the supply curve of Figure 8. Similar functions (MFUN and MLFUN) are used to determine both M2PD (M-sector) and M3PD (M1-sector).

M2PD.K = TABLE (MFUN, MACP.K, 15, 8, 0), (lb/wk)  
M3PD.K = TABLE (MFUN, MLACP.K, 15, 8, 0 ), (lb/wk)  

M2PD and M3PD are further modified to determine both MPD and MLPD.

M3PD is multiplied by a variable (M63) used to vary the production share of the sector in the division of the competitive model.

MLPD.K = (M63) (M3PD.K), (lb/wk)  
M63 = 3.4

MPD.K = (M62) (M2PD.K) + (1-M62) (RSS.K), (lb/wk)  
M62 = 0.3

3. Adjusted Desired Production (MAPD and MLAPD)

Desired Production (MPD and MLPD) is multiplied by the long run expansion factor (MLRSA) to reflect the results of new mine expansion --or retirement. The rationale for MLRSA is discussed under investment policy.
MAPD.K = (MPD.K) (MLRSA.K), (lb/wk)  
MLAPD.K = (MLPD.K) (MLRSA.K), (lb/wk)

4. Altered Adjusted Desired Production (MAAPD)

In the M sector only, desired production is further modified to allow for concentrator raw material inventory requirements. An increment of production to balance actual concentrator raw material inventory (CRMI) with desired inventory (CRMID) is added to MAPD. The adjustment time (M70) may be varied to change the speed of response. In some runs it was made very large to render the function ineffective when inventory production is not desired.

MAAPD.K = MAPD.K + (1/M70) (CRMID.K - CRMI.K), (lb/ wk)  
M70 = 8, (wk)

5. Mine Production Adjustment Time Delay

A delay in adjusting to a new desired production rate was incorporated to represent personnel and equipment adjustments to the new rate. A level (MPW) of production wanted with a feedback to a differencing function (MPCF) form a first-order time delay. The loop gain ratios (M2 and M3) determined the value of the delay. M4 is a dummy variable used to remove the delay if desired. Constants M12, M13 and M14 perform a similar function in M1.
\[
\text{MPW.K} = \text{MPW.J} + (\text{DT}) \left( \frac{1}{\text{M3}} \right) \left( \text{MPCF.J} - \text{M4} \right), \ \text{(lb/wk)} \\
\text{MLPW.K} = \text{MLPW.J} + (\text{DT}) \left( \frac{1}{\text{ML3}} \right) \left( \text{MLPCF.J} - \text{ML4} \right), \ \text{(lb/wk)} \\
\text{MPCF.K} = (\text{M2}) \left( \text{MAAPD.K} - \text{MPW.K} \right), \ \text{(lb/wk)} \\
\text{MLPCF.K} = (\text{ML2}) \left( \text{MLAPD.K} - \text{MLPW.K} \right), \ \text{(lb/wk)}
\]

\[
\text{M2} = 0.06 \\
\text{M3} = 1 \\
\text{M4} = 0 \\
\text{ML2} = 0.06 \\
\text{ML3} = 1 \\
\text{ML4} = 0
\]

6. Mine Production to be Started (MPTBS and MLPTBS)

The actual copper-content output of the mine is determined by the ore yield. Ore yield selection may be varied by the mine manager. Three basic policies are possible:

a. Long term constant yield
b. Yield increase with increase in price
c. Yield decrease with increase in price.

To follow the first policy, \text{M33} is set equal to one and \text{M31} and \text{M32} to zero in the MRATO equations below. The second policy requires an \text{M32} equal to one with the other two constants (\text{M31}, \text{M33}) both zero. In the third policy \text{M31} is one and the rest zero. Similar considerations hold for \text{M34}, \text{M35} and \text{M36} in the MIRAT equations in the M1 sector. RCPN is the initial price.
MOY.K = \((1/\text{M7}) \times (\text{M5}) \times (\text{MRAT0.K} + \text{M6})\)  
0258, A

MLOY.K = \((1/\text{ML7}) \times (\text{ML5}) \times (\text{MLRAT.K} + \text{ML6})\)  
0277, A

MRAT0.K = (\text{MPRI.K} \times \text{M31}) + (\text{MPF2.K} \times \text{M32}) + (\text{M33})  
0259, A

MLRAT.K = (\text{MPRI.K} \times \text{M34}) + (\text{MPF2.K} \times \text{M35}) + (\text{M36})  
0278, A

MPRI.K = \text{RCPN/MACP.K}  
0288, A

MPRI.K = \text{MACP.K/RCPN}  
0289, A

M31 = 0  
0518, C

M32 = 1  
0519, C

M33 = 0  
0520, C

M34 = 1  
0521, C

M35 = 1  
0522, C

M36 = 0  
0523, C

RCPN = .30, (dollars)  
0476, C

M5 = 1  
0504, C

M6 = 2  
0505, C

ML5 = 1  
0506, C

ML6 = 2  
0507, C

M7 = M5 + M6  
0387, N

ML7 = ML5 + ML6  
0401, N

MPTBS and MLPTB are then the result of a MOY (MLOY) and MFW (MLPW) multiplication. The MSTR values are external inputs used to simulate strike-induced production losses.

MPTBS.K = (\text{MPW.K} \times \text{MOY.K}) + \text{MSTR.K}, (lb/wk)  
2257, A

MLPTB.K = (\text{MLPW.K} \times \text{MLOY.K}) + (\text{MLSTR.K}), (lb/wk)  
2276, A
7. Mine Production Started (MPS and MLPS)

MPTBS (MLPTB) is the final production rate decision as long as it does not exceed mine production capacity (MOPP, MLOPP). The initial capacities, MMFP and MLMMP are changed in accordance with mine expansion (MLRSA) and ore yield (MOY, MLOY). If MPTBS (MLPTB) exceeds MOPP (MLOPP) then MOPP (MLOPP) is the rate decision.

\[ MPS.KL = \text{CLIP} (MOPP.K, MPTBS.K, MFW.K, MMFP), \ (lb/wk) \]
\[ MLPS.KL = \text{CLIP} (MLOPP.K, MLPTB.K, MLFW.K, MLMFP), \ (lb/wk) \]
\[ MOPP.K = (MAMFP.K) (MOY.K), \ (lb/wk) \]
\[ MLOPP.K = (MLAMP.K) (MLOY.K), \ (lb/wk) \]
\[ MAMFP.K = (MMFP) (MLRSA.K), \ (lb/wk) \]
\[ MLAMP.K = (MLMFP) (MLRSA.K), \ (lb/wk) \]
\[ MMFP = 45,000,000, \ (lb/wk) \]
\[ MLMFP = 255,000,000, \ (lb/wk) \]

C. Production Structure

The material production flow initiated by the production policy decision is delayed in goods in process inventory after which it is shipped to the concentrator. The third-order delay and inventory level functions used are standard Industrial Dynamics formulations.
MGIP.K = MGIP.J + (DT) (MPS.JK - MCP.JK), (lb)  
MLGIP.K = MLGIP.J + (DT) (MLPS.JK - MLCP.JK), (lb)  
MCP.KL = DELAY3 (MPS.JK, DEL18), (lb/wk)  
MLCP.KL = DELAY3 (MLPS.JK, DEL28), (lb/wk)  
DEL18 = 1.2, (wk)  
DEL28 = 1.2, (wk)  

D. Financial Structure

1. General

Costs, revenues and profits of the nine-concentrator-smelter combination are determined as a unit. The financial structure of the mine-concentrator-smelter and other sectors is a conventional fixed-variable cost formulation with unit cost decreasing with volume of production as a result of fixed cost distribution over a larger number of units.

2. Cost (MCOST and MLCONS)

Total cost (MCOST) varies with unit cost (MUCOS), production volume (MPS) and ore yield (MOY).

\[
MCOST.KL = (MUCOS.K) (MPS.JK) / (MOY.K), ($/wk) \\
MLCONS.KL = (MLUCOS.K) (MLPS.JK) / (MLOY.K), ($/wk)
\]

Ore yield effects are discussed in B6 of this appendix.

Unit cost (MUCOS) is made up of variable unit cost (VMCONS) and fixed unit cost (FMCONS).
MUCOS.K = VM COS + FMCOS.K, ($/lb)  
MLUCO.K = VLMCO + FLMCO.K, ($/lb)  

VM COS = 0.192, ($/lb)  
VLMCO = 0.192, ($/lb)  

Variable unit cost (VM COS) is constant but fixed unit cost (FMCOS)  
varies with production volume (MPS) in relation to the reference pro-
duction volume and reference overhead cost (OM COS)  

FMCOS.K = (OM COS) (MPSN)/MPS.JK, ($/lb)  
FLMCO.K = (O1MCO) (MLPSN)/MLPS.JK, ($/lb)  
OM COS = 0.048, ($/lb)  
O1MCO = 0.048, ($/lb)  

MPSN = (USON) (M60), (lb/wk)  
MLPSN = (USON) (M61), (lb/wk)  
USON = 262,000,000, (lb/wk)  
M60 = .15  
M61 = .85  

3. Revenue  

Mine-concentrator-smelter revenue (MR) varies with smelter ship-
ments (SOS) and smelter blister copper price (SCP)  

MR.KL = (SOS.JK) (SCP.K), ($/wk)  
MLR.KL = (S10S.JK) (S1CP.K), ($/wk)
Blister copper price (SCP) is obtained by subtracting the refiner toll charge from the refined market price (RCP).

\[
\text{SCP}.K = \text{RCP}.K - 0.0375, ($/\text{lb})
\]

\[
\text{SLCP}.K = \text{RLCP}.K - 0.0375, ($/\text{lb})
\]

4. Profit (MPROF and MLPRO)

Mine-concentrator-smelter profit (MPROF) is determined by the difference between revenues (MR) and cost (MCOST) as modified by corporate tax here approximated as 50 per cent of gross profits.

\[
\text{MPROF}.K = \text{MPROF}.J + (\text{DT})(1/2)(\text{MR}.JK - \text{MCOST}.JK), ($)
\]

\[
\text{MLPRO}.K = \text{MLPRO}.J + (\text{DT})(1/2)(\text{MLR} - \text{MLCOST}.JK) ($)
\]

D. Investment Policy

1. General

Industrial expansion (MLRSA) of all sectors is reflected in an investment policy that moves the supply curve of Figure 8 to the right (or left) and increases (decreases) the maximum production possible in all sectors as a function of long term price trends. A long term increase in price indicates an upward trend in demand that leads to expansion of capacity in all sectors. A long term decrease in price has the opposite effect.

2. Policy Description

Long run average price (MLRP) is calculated using a time-averaging exponential smoothing formula identical in form to that previously used to calculate average copper price (MACP).
\[
\text{MLRP}_K = \text{MLRP}_J + (\text{DT}) \left( \frac{1}{M30} \right) \left( \text{RCP}_J - \text{MLRP}_J \right), \quad (\$)
\]

\[M30 = 24, \quad (\text{wks})\]

\[\text{RCP} = \text{Refined Copper Price}\]

A long run price ratio (MLRPR) is then determined from the ratio of MLRP to a base price (RCPN)

\[\text{MLRPR}_K = \frac{\text{MLRP}_K}{\text{RCPN}}\]

The direction of change (MPDIF) of the ratio MLRPR is next determined by subtracting successive values of the ratio (MPRF*2, MPRF*3).

\[\text{MPRF*1}_K = \text{MPRF*1}_J + (\text{DT}) \left( \text{MLRPR}_J + O \right)\]

\[\text{MPRF} = \text{BOXLIN} (3,1)\]

\[\text{MPDIF}_K = \text{MPRF*2}_K - \text{MPRF*3}_K\]

The sign of the direction of change is detected by a clipping function.

\[\text{MPRAN}_K = \text{CLIP} (0, \text{MPDIF}_K, 0, \text{MPDIF}_K)\]

\[\text{MPRAN}_K = \text{CLIP} (0, \text{MPDIF}_K, \text{MPDIF}_K, 0)\]

In order to eliminate unnecessary changes in capacity for random changes in demand, threshold levels of +10 per cent and -5 per cent are established, and no change in capacity is made for any changes below the threshold level.
\[ \text{MPAF}_1.K = \text{CLIP} \left( 0, \text{MPR}_{1}.K, M41, \text{MPRF}^{*2}.K \right) \]
\[ \text{MPAF}_2.K = \text{CLIP} \left( 0, \text{MPRAN}.K, \text{MPRF}^{*2}.K, M42 \right) \]
\[ M41 = 1.10 \]
\[ M42 = 0.95 \]

MPAF1 - Production Adjustment Factor 1
MPAF2 - Production Adjustment Factor 2.

The production adjustment is then calculated from MPAF1 or MPAF2, depending upon the trend direction, as weighted by the investment constant, M40, which relates expansion desired as a function of price ratio change.

\[ \text{MPA}.K = \text{MPA}.J + (\Delta t) \left( \frac{1}{M40} \right) \left( \text{MPAF}_1.J + \text{MPAF}_2.J \right) \]
\[ M40 = 10 \]

The long range supply adjustment used in supply curve and production limit adjustment is formed from the addition of MPA and one.

\[ \text{MLRSA}.K = 1 + \text{MPA}.K \]

E. Initial Conditions

Initial conditions for mine sector level and rate equations are as follows:
MPS = RSO
MFW = MPS
MACP = RCPN
MLACP = RCPN
MPROF = 0
MR = (SOS) (SCPN)
MCOST = (MPS) (.24)
M7 = M5+M6
MLIP = (MIPS) (DEL28)
MLPS = R1SO
MLFW = MLPS
MLR0 = 0
MLR = (S1OS) (S1CPN)
MLCS = (MLPS) (.24)
M17 = M15+M16
MLRP = RCPN
M10 = M8+M9
MPA = 0
MPSN = (USN) (M50)
M1PSN = (USN) (M61)

RSO - Refiner's Sales Orders
R1SO - Refiner's Sales Orders

SCPN = 0.2625
S1CPN = 0.2625
Appendix B

MATHEMATICAL DESCRIPTION OF THE Bauxite Mine

A. General

The following structures and policies will be described in the bauxite mine sector (Figures 9A and 9B):

1. Production Policy
2. Production Structure
3. Financial Structure
4. Initial Conditions

B. Production Policy

Since the production policy of the mine is based almost completely upon the raw material requirements of the reduction smelter sector, the requirements for which are determined in that sector, the policy formulation in the mine sector is exceedingly simple and consists only of a delay, an inventory adjustment and a capacity limitation.

The delay is identical in basic formulation to that just described in the copper mine. The mine production wanted (MPW), a level, forms a first-order delay through a feedback to a differencing function (MPCF). Loop gain ratios (M3 and M4) determine the amount of delay. M4 is a dummy constant used to remove the term, if so desired.
\[ \text{MPW}.K = \text{MPW}.J + (\text{DT}) \left\{ \frac{1}{\text{M3}} \right\} (\text{MPCF}.J - \text{M4}), \ (\text{lb}/\text{wk}) \]
\[ \text{MLPW}.K = \text{MLPW}.J + (\text{DT}) \left\{ \frac{1}{\text{M3}} \right\} (\text{MLPCF}.J - \text{M4}), \ (\text{lb}/\text{wk}) \]
\[ \text{MPCF}.K = (\text{M2}) (\text{MPD}.K - \text{MPW}.K) \]
\[ \text{MLPCF}.K = (\text{ML2}) (\text{M3PD}.k - \text{MLPW}.K) \]
\[ \text{M3} = 1 \]
\[ \text{M4} = 0 \]
\[ \text{M2} = 0.06 \]
\[ \text{ML2} = 0.06 \]

The mine production wanted is then modified to fulfill the raw material inventory requirements of the alumina refiner (SRMID - SRMI) where SRMID is the desired and SRMI the actual raw material inventory. The inventory is adjusted over a time period (M70) to provide the mine production to be started, MPTBS. A similar policy formulation holds for the "ML" sector.

\[ \text{MPTBS}.K = \text{MPW}.K + (1/\text{M70}) (\text{SRMID}.K - \text{SRMI}.K), \ (\text{lb}/\text{wk}) \]
\[ \text{M1PTB}.K = \text{MLPW}.K + (1/\text{M70}) (\text{S1RMD}.K - \text{S1RMI}.K), \ (\text{lb}/\text{wk}) \]
\[ \text{M70} = 8, \ (\text{wk}) \]

The final mine production started (MPS, MLPS) is limited by the production limit (MOPP, MLOPP) based upon the initial limits (MMPP, MLMPP) and the long range supply adjustment growth (MLRSA, MLRSL) which forms the adjusted maximum production (MAMPP, MLAMP) which is then modified by the ore yield to form MOPP and MLOPP.
MAMFP.K = (MMPP) (MLRSA.K), (lb/wk)
MLAMP.K = (MLMPP) (MLLRS.K), (lb/wk)
MOPT.K = (MAMFP.K) (MOY), (lb/wk)
MLOPT.K = (MLAMP.K) (MOY), (lb/wk)
MLRSA.K = MLRSA.J + (DT) (M5+O),
MLLRS.K = MLLRS.J + (DT) (M51+O),
MPS.KL = CLIP (MQPP.K, MPTBS.K, MPW.K, MMPP), (lb/wk)
MLPS.KL = CLIP (MLQPP.K, MLP TB.K, MLFW.K, MLMP), (lb/wk)
MMPP = 80,000,000, (lb/wk)
MLMPP = 120,000,000, (lb/wk)
MOY = 1
M5 = 0.002,
M51 = 0.002,

C. Production Structure

The production structure contains a goods in process inventory (MGIP,MLGIP) and a time delay (DElli8), after which the bauxite produced (MBP) is shipped to the alumina refiner.

MGIP.K = MGIP.J + (DT) (MPS.JK - MBP.JK), (lb)
MLGIP.K = MLGIP.J + (DT) (MLPS.JK - MLBP.JK), (lb)
MBP.KL = DELAY 3 (MPS.JK,,DElli8), (lb/wk)
MLBP.KL = DELAY3 (MLPS.JK, DElli8), (lb/wk)
DElli8 = 1.2, (wk)
D. Financial Structure

The rationale for the financial structure of the bauxite mine is identical to that of the copper mine. Only the values of the constants differ.

\[
\begin{align*}
MCOST.KL &= (MUCOS.K) (MPS.JK), \text{ ($/wk)} \\
MLCOS.KL &= (MLUCO.K) (MLFS.JK), \text{ ($/wk)} \\
MUCOS.K &= VCOST + FCOST.K, \text{ ($/lb)} \\
MLUCO.K &= VLCO + FLCO.K \text{ ($/lb)} \\
FCOST.K &= (OCOST) (MPSN)/MPS.JK, \text{ ($/lb)} \\
FLCO.K &= (OLCO) (MLPSN)/MLPS.JK, \text{ ($/lb)} \\
MR.KL &= (MBF.JK) (BP), \text{ ($/wk)} \\
MLR.KL &= (MLBP.JK) (BP), \text{ ($/wk)} \\
MPROF.K &= MPROF.J + (DT) (1/2) (MR.JK - MCOST.JK, \text{ ($)} \\
MLPRO.K &= MLPRO.J + (DT) (1/2) (MLR.JK - MLCOS.JK), \text{ ($)} \\
\end{align*}
\]

MCOST, MLCOS - Mine total COST
MUCOS, MLUCO - Mine Unit COST
FCOST, FLCO - Mine Fixed unit COST
VCOST, VLCO - Mine Variable unit COST
OCOST, OLCO - Mine Overhead unit COST
BP - Bauxite Price, ($/lb)
E. Initial Conditions

The initial condition equations follow:

\[ \text{MGIP} = (\text{MBP}) (\text{DEL18}) \]
\[ \text{MPS} = \text{RSC} \]
\[ \text{MFW} = \text{MPS} \]
\[ \text{MPROF} = 0 \]
\[ \text{MR} = (\text{MBP}) (\text{BP}) \]
\[ \text{MCOST} = (\text{MPS}) (.015) \]
\[ \text{MLRSA} = 1 \]
\[ \text{MPSN} = (\text{USON}) (\text{M30}) \]
\[ \text{MLGIP} = (\text{MLBP}) (\text{DEL18}) \]
\[ \text{MLPS} = \text{R1SO} \]
\[ \text{MLPW} = \text{MLPS} \]
\[ \text{MLPRO} = 0 \]
\[ \text{MLR} = (\text{MLBP}) (\text{BP}) \]
\[ \text{MLCOS} = (\text{MLPS}) (.015) \]
\[ \text{MLRS} = 1 \]
\[ \text{MLPSN} = (\text{USON}) (\text{M31}) \]
Appendix C

MATHEMATICAL DESCRIPTION OF THE COPPER CONCENTRATOR

A. General

The "C" and "Cl" copper concentrator sectors (Figures 10A and 10B) will be described under the following structure and policy classifications:

1. Production Policy
2. Production Structure
3. Distribution Structure
4. Initial Conditions.

B. Production Policy

1. "Cl" Production Policy

The production policies in C and Cl regulating the flow of material in those sectors differ significantly. The Cl policy is considered typical of current practice in being based exclusively on raw material inventory level (ClRMI). The production wanted (ClPW) varies directly with raw material inventory level (ClRMI) and the production constant, C4, which determines the raw material inventory level desired in terms of weeks of production.

\[ \text{ClPW}.K = \text{ClRMI}.K/C4 \] (lb/wk) 
\[ C4 = 0.5, \text{ (wks)} \]

2. "C" Production Policy

A more stable production policy was formulated and tested in the C sector. This policy bases production desired (CFD) on the output
shipments of the mine (MCP) and the difference between desired raw material inventory at the smelter (SRMID) and actual raw material inventory (SRMI) with an inventory adjustment time of C3.

$$\text{CPD}_K = \text{MCP}_K + (1/C3) (\text{SRMID}_K - \text{SRMI}_K), \text{ (lb/wk)}$$

$$C3 = 0.5, \text{ (wks)}$$

This desired production rate (CPD) is limited by raw material inventory availability (CRMI) which limits production wanted (CPW) to the raw material production limit (CRMPL)

$$\text{CRMPL}_K = \text{CRMI}_K/C3, \text{ (lb/wk)}$$

$$\text{CPW}_K = \text{CLIP} (\text{CPD}_K, \text{CRMPL}_K, \text{CRMPL}_K, \text{CPD}_K), \text{ (lb/wk)}$$

Both the C1 and C final production rates (CPS, CLPS) are limited by capacity (CMPP and CLMPP) as modified by the expansion factor (MLRSA) to provide current capacity (CAMPP and CLAMP).

$$\text{CAMPP}_K = (\text{CMPP})(\text{MLRSA}_K), \text{ (lb/wk)}$$

$$\text{CLAMP}_K = (\text{CLMPP})(\text{MLRSA}_K), \text{ (lb/wk)}$$

$$\text{CPS}_KL = \text{CLIP} (\text{CAMPP}_K, \text{CPW}_K, \text{CPW}_K, \text{CAMPP}_K), \text{ (lb/wk)}$$

$$\text{CLPS}_KL = \text{CLIP} (\text{CLAMP}_K + \text{CLPW}_K, \text{CLPW}_K, \text{CLAMP}_K), \text{ (lb/wk)}$$

$$\text{CMPP} = 45,000,000, \text{ (lb/wk)}$$

$$\text{CLMPP} = 255,000,000, \text{ (lb/wk)}$$

Associated with the C policy is the desired raw material inventory which modifies mine production to maintain raw material at the concentrator. Desired inventory (CRMID) is a function of refiner
smoothed sales (RSS) and the policy constant (C3) expressing the number of production weeks level of inventory wanted.

\[ CRMID.K = (C3) (RSS.K), \quad (lb) \]

\[ C3 = 0.5, \quad (wks) \]

C. Production Structure

The basic production structure in this sector is composed of a raw material inventory (CRMI, CLRMI) and a work-in-process inventory (CGIP, CLGIP) and a third-order delay (DELL6). Material flows from raw material inventory to goods in process, and after a delay, to finished goods inventory (CFGI, CLFGI).

\[ CRMI.K = CRMI.J + (DT) (MCP.JK - CPS.JK), \quad (lb) \]

\[ CLRMI.K = CLRMI.J + (DT) (MLCP.JK - CLPS.JK), \quad (lb) \]

\[ CGIP.K = CGIP.J + (DT) (CPS.JK - CGTFI.JK), \quad (lb) \]

\[ CLGIP.K = CLGIP.J + (DT) (CLPS.JK - CLGTF.JK), \quad (lb) \]

\[ CGTFI.K = DELAY3 (CPS.JK, DELL6), \quad (lb/wk) \]

\[ CLGTF.K = DELAY3 (CLPS.JK, DELL6), \quad (lb/wk) \]

CGTFI - Goods To Finished Inventory

CLGTF - Goods To Finished Inventory

MCP, MLCP - Mine Copper Produced, (lb/wk)

D. Distribution Structure

The basic characteristic of the distribution structure is a shipping delay (COSD, CLOSD) dependent on the size of the inventory. Upon the hypothesis that delay increases with inventory level due to a
lack of material handling equipment, shipping delay increases with the level of finished goods inventory (CFG1,ClFG1). Concentrate shipped is a quotient of finished goods inventory and shipping delay (COsd,ClOsd).

The delay is explicitly determined from the relationship between normal finished goods inventory (CFGIN) and actual inventory with constants C1 and C2 with C1 expressing the fixed delay and C2 the change in delay per unit of output.

\[
\text{COsd.K} = \left(\frac{1}{\text{CFGIN}}\right) \left(\frac{(\text{C1}) \cdot \text{CFGIN} + (\text{C2}) \cdot \text{CFG1.K}}{(\text{wk})}\right),
\]

\[
\text{ClOsd.K} = \left(\frac{1}{\text{CFGIN}}\right) \left(\frac{(\text{C1}) \cdot \text{CFGIN} + (\text{C2}) \cdot \text{ClFG1.K}}{(\text{wk})}\right),
\]

\[
\text{CFG1.K} = \text{CFG1.J} + (\text{DT}) \cdot (\text{CGTF.JK - COs.JK}), (\text{lb})
\]

\[
\text{ClFG1.K} = \text{ClFG1.J} + (\text{DT}) \cdot (\text{ClGTF.JK - ClOJ.JK}), (\text{lb})
\]

C1 = 1.0
C2 = 0.1

The final shipments (COs,ClOS) from the concentrator are directly proportional to inventory (CFG1,ClFG1) and inversely proportional to the delay (COsd,ClOsd).

\[
\text{COs.KL} = \frac{\text{CFG1.K}}{\text{COsd.K}}, (\text{lb/wk})
\]

\[
\text{ClOS.KL} = \frac{\text{ClFG1.K}}{\text{ClOsd.K}}, (\text{lb/wk})
\]

E. Initial Conditions

The concentrator initial condition equations follow:
\[ \text{COS} = \text{RSO} \]
\[ \text{C1OS} = \text{R1SO} \]
\[ \text{CFG} = (\text{COS})(\text{C1+C2}) \]
\[ \text{C1FGI} = (\text{C1OS})(\text{C1+C2}) \]
\[ \text{CFGIN} = \text{CFG} \]
\[ \text{CGIP} = (\text{CGTFI})(\text{DEL16}) \]
\[ \text{C1GIP} = (\text{CIFTP})(\text{DEL16}) \]
\[ \text{CPS} = \text{RSO} \]
\[ \text{C1PS} = \text{R1SO} \]
\[ \text{CRMI} = (\text{CPS})(\text{C3}) \]
\[ \text{C1RMI} = (\text{C1PS})(\text{C4}) \]

\text{RSO, R1SO - Refiner's Sales Orders, (lb/wk)}
Appendix D

MATHEMATICAL DESCRIPTION OF THE COPPER SMELTER

A. General

The "S" and "S1" smelter sectors (Figures 11A and 11B) are described under the following structures and policies:

1. Production Policy
2. Production Structure
3. Distribution Structure
4. Initial Conditions.

B. Production Policy

The rationale for production policy in the two smelter sectors is the same as that explained in the concentrator sector in Appendix C. The S1 policy is based upon raw material inventory alone and acts as a destabilizing force on the industry. In contrast, the S policy was designed to be used by a company seeking to stabilize the industry by supplementing integral (inventory) control with proportional control. The concentrator, in order to coordinate closely with the mine to minimize raw material inventory level, used mine shipments (MCP) as the proportional variable. The S smelter uses refined smoothed sales (RSS) to integrate with requirements at the next stage of production. RSS is a rate feedback similar in principle to a rate tachometer feedback used to stabilize servomechanisms.

\[ S_{1FW.K} = S_{1RMI.K}/S_6, \quad (1b/wk) \]

\[ S_6 = 2, \quad (wk) \]

The S policy includes production for refiner raw material inventory requirements \((RRMID - RRMI)\) where \(RRMID\) is the desired and \(RRMI\)
the actual inventory. The final rates (SPS, S1PS) are limited to raw material (SRMPL) and capacity (SAMPP, SLAMP) production limits.

Initial capacities (SMPP, S1MPP) are modified by long range expansion (MLRSA) as in the concentrator sector.

\[
\begin{align*}
SPD.K &= RSS.K + \left(\frac{1}{S3}\right) (RRMID.K - RRMI.K), (\text{lb/wk}) \\
SRMPL.K &= SLRMI.K/S6, (\text{lb/wk}) \\
SPW.K &= CLIP (SPD.K, SRMPL.K, SRMPL.K, SPD.K), (\text{lb/wk}) \\
SAMPP.K &= (SMPP) (MLRSA.K), (\text{lb/wk}) \\
SLAMP.K &= (S1MPP) (MLRSA.K), (\text{lb/wk}) \\
SPS.KL &= CLIP (SAMPP.K, SPW.K, SPW.K, SAMPP.K), (\text{lb/wk}) \\
S1PS.KL &= CLIP (SLAMP.K, S1PW.K, S1PW.K, SLAMP.K), (\text{lb/wk}) \\
SMPP &= 51,000,000, (\text{lb/wk}) \\
S1MPP &= 280,000,000, (\text{lb/wk}) 
\end{align*}
\]

The desired raw material inventory (SRMID) is based upon the same rationale as the concentrator’s equivalent policy. In general, the SRMID will be larger than CRMID since the lesser bulk of concentrate as compared to ore makes such inventories practicable.

\[
\begin{align*}
SRMID.K &= (S3) (RSS.K), (\text{lb}) \\
S3 &= 2, (\text{wk}) 
\end{align*}
\]

C. Production Structure

The production structure is composed of a raw material inventory (SRMI, SLRMI), a goods-in-process-inventory, (SGIP, SLGIP) after which the goods flow to finished goods inventory (SGTFI, SLGTF) with a delay (DEL13).
SRMI.K = SRMI.J + (DT) (SCD.JK - SPS.JK), (lb) 0218, L
S1RMI.K = S1RMI.J + (DT) (S1CD.JK - S1PS.JK), (lb) 1218, L
SCD, S1CD - copper delivered, (lb/wk)
SGIP.K = SGIP.J + (DT) (SPS.JK - SGTFI.JK), (lb/wk) 0213, L
S1GIP.K = S1GIP.J + (DT) (S1PS.JK - S1GTF.JK), (lb/wk) 1213, L
SGTFI.KL = DELAY3 (SPS.JK, DEL13), (lb/wk) 0214, R
S1GTF.KL = DELAY3 (S1PS.JK, DEL13), (lb/wk) 1214, R
DEL13 = 1.5, (wk) 0425, C

D. Distribution Structure

The distribution structure includes the flow, level and delays of both incoming and outgoing material. In-transit inventory (SLCT, S1LCT) results from the time delay (DEL15) inherent in the transportation process between concentrator shipments (COS, C1OS) and smelter deliveries (SCD, S1CD).

SLCT.K = SLCT.J + (DT) (COS.JK - SCD.JK), (lb) 0219, L
S1LCT.K = S1LCT.J + (DT) (C1OS.JK - S1CD.JK), (lb) 1219, L
SCD.KL = DELAY3 (COS.JK, DEL15), (lb/wk) 0220, L
S1CD.KL = DELAY3 (C1OS.JK, DEL15), (lb/wk) 1220, L
DEL15 = 2, (wk) 0426, C

The outgoing material distribution functions are based upon the same rationale as in the concentrator sector with shipping delay (S0SD, S10SD) varying directly with the ratio of finished goods inventory (SFGI, S1FGI) to normal finished goods inventory (SFGIN).
$SFGI.K = SFGI.J + (DT) (SGTFI.JK - SOS.JK), \ (lb)$  
$S1FGI.K = S1FGI.J + (DT) (S1GTF.KL - S1OS.JK), \ (lb)$  
$SOSD.K = (1/SFGIN) \ ((S1) (SFGIN) + (S2) (SFGI.K)), \ (wk)$  
$S1OSD.K = (1/SFGIN) \ ((S1) (SFGIN) + (S2) (S1FGI.K)), \ (wk)$  
$S1 = 1.0, \ (wk)$  
$S2 = 0.1, \ (wk)$  

The final shipments from the smelter ($SOS, S1OS$) are directly proportional to inventory ($SFGI, S1FGI$) and inversely proportional to the delay ($SOSD, S1OSD$).  

$SOS.KL = SFGI.K / SOSD.K$  
$S1OS.KL = S1FGI.K / S1OSD.K$  

E. Initial conditions  

The smelter initial conditions equations follow:  

$SOS = RSO$  
$S1OS = R1SO$  
$SFGI = (SOS) (S1+S2)$  
$S1FGI = (S1OS) (S1+S2)$  
$SFGIN = SFGI$  
$S1FGN = S1FGI$  
$SGIP = (SOS) (DEll3)$  
$S1GIP = (S1OS) (DEll3)$  
$SPS = RSO$  
$S1PS = R1SO$
SRMI = (SPS)(S3) 0365, N
S1RMI = (S1PS)(S6) 1365, N
SLCT = (SCD)(DEL15) 0366, N
S1LCT = (S1CD)(DEL15) 1366, N
SPSN = (USON)(S30) 0368, N
S1PSN = (USON)(S31) 1368, N
S30 = 0.15 3494, C
S31 = 0.85 3495, C

RSO, R1SO - Refiner's Sales Orders, (lb/wk)

USON - User's Sales Orders, initial, (lb/wk)
Appendix E

MATHEMATICAL DESCRIPTION OF THE ALUMINA REFINER

A. General

The Alumina Refiner Sectors (Figures 12A and 12B) will be described under the following structures and policies:

1. Production Policy
2. Production Structure
3. Financial Structure
4. Distribution Structure
5. Initial Conditions

B. Production Policy

Production wanted (SPW, S1PW) at the alumina refiner is based primarily on the alumina requirements of the reduction smelter. The reduction production desired (RPD, R1PD) information is combined with the reduction raw material requirements (RRMID-RRMI) where RRMID is the desired and RRMI the actual raw material inventory at the reduction smelter, to provide the production rate.

\[
\text{SPW.K} = \text{RPD.K} + \left( \frac{1}{S3} \right) \left( \text{RRMID.K} - \text{RRMI.K} \right), \text{ (lb/wk)} \quad S8,A
\]

\[
\text{S1PW.K} = \text{R5PD.K} + \left( \frac{1}{S6} \right) \left( \text{R1RMD.K} - \text{R1RMI.K} \right), \text{ (lb/wk)} \quad S25,A
\]

\[
S3 = 8, \text{ (wk)} \quad C55,C
\]

\[
S6 = 8, \text{ (wk)} \quad CS11,C
\]

The production wanted (SPW, S1PW) is then limited by the maximum production possible (SAMPP, S1AMP) which is determined by the initial capacity (SMPP, S1MPP) and the long range supply adjustment expansion (SLRSA, S1LRS). The final production started is SPW (S1PW) as limited by SAMPP (S1AMP) capacity.
SPS.KL = CLIP (SAMPP.K, SPW.K, SFW.K, SAMPP.K), (lb/wk)  \[S6, R\]
SLPS.KL = CLIP (SLAMP.K, SLFW.K, SLFW.K, SLAMP.K), (lb/wk)  \[S23, R\]
SAMPP.K = (SMPP) (SLRSA.K), (lb/wk)  \[S7, A\]
SLAMP.K = (SLMPP) (SLLRS.K), (lb/wk)  \[S24, A\]
SLRSA.K = SLRSA.J + (DT) (S5)  \[S12, L\]
SLLRS.K = SLLRS.J + (DT) (S51)  \[S27, L\]
SMPP = 30,000,000, (lb/wk)  \[1CS6, C\]
SLMPP = 70,000,000, (lb/wk)  \[CS7, C\]
S5 = 0.002  \[1CS12, C\]
S51 = 0.002  \[CS13, C\]

C. Production Structure

The production structure in this sector is the conventional goods-in-process inventory (SGIP,SLGIP) and time delay (DELAY3) from which the material is shipped (SGTFI,SLGTF) to finished goods inventory (SFGI,SLFGI).

SGIP.K = SGIP.J + (DT) (SPS.JK - SGTFI.JK), (lb)  \[S4, L\]
SLGIP.K = SLGIP.J + (DT) (SLPS.JK - SLGTF.JK), (lb)  \[S21, L\]
SGTFI.KL = DELAY3 (SPS.JK, DEL3), (lb/wk)  \[S5, R\]
SLGTF.KL = DELAY3 (SLPS.JK, DEL3), (lb/wk)  \[S22, R\]
DEL3 = 3, (wk)  \[CS1, C\]

D. Distribution Structure

The distribution structure includes the functions involved in the movement of incoming and outgoing materials. Bauxite in transit from
the mine (SLET, SLLET) is delivered (SBD, SLBD) after a delay (DELI5) to
the raw material inventory of the refiner (SRMI, SLRMI).

\[
\begin{align*}
SLET.K &= SLET.J + (DT) (MBP.JK - SBD.JK), \text{ (lb)} \\
SLLET.K &= SLLET.J + (DT) (MLBP.JK - SLBD.JK), \text{ (lb)} \\
SBD.KL &= DELAY3 (MP.JK, DELI5), \text{ (lb/wk)} \\
SLBD.KL &= DELAY3 (MLBP.JK, DELI5), \text{ (lb/wk)} \\
SRMI.K &= SRMI.J + (DT) (SBD.JK - SPS.JK), \text{ (lb)} \\
SLRMI.K &= SLRMI.J + (DT) (SLBD.JK - SLPS.JK), \text{ (lb)} \\
DELI5 &= 1.2, \text{ (wk)}.
\end{align*}
\]

Desired raw material (SRMID, SLRMD), which is the basis for the
mine production control inventory adjustment is a function of reduc-
tion smoothed sales (RSS).

\[
\begin{align*}
SRMID.K &= (S3) (RSS.K), \text{ (lb)} \\
SLRMD.K &= (S6) (R1SS.K), \text{ (lb)} \\
S3 &= 8, \text{ (wk)} \\
S6 &= 8, \text{ (wk)}
\end{align*}
\]

Output material movement is governed by the same type of delay
described in the copper concentrator sector with the shipping delay
(SOSD, SL OSD) increasing with the ratio of finished goods inventory
(SFGI, SLFGI) to normal finished goods inventory (SFGIN). Shipments
(SOS, SLOS) are directly proportional to finished goods inventory
(SFGI, SLFGI) and inversely proportional to SOSD (SL OSD).

\[
\begin{align*}
SOSD.K &= \left(\frac{1}{SFGIN}\right) \left((S1) (SFGIN) + (S2) (SFGI.K)\right), \text{ (lb/wk)} \\
SL OSD.K &= \left(\frac{1}{SFGIN}\right) \left((S1) (SFGIN) + (S2) (SLFGI.K)\right), \text{ (lb/wk)} \\
S1 &= 0.3 \\
S2 &= 0.1
\end{align*}
\]
E. Financial Structure

The rationale for the alumina refiner is identical to that of the copper mine. Only the values of the constants differ.

1. Cost

\[ SCOST_\text{KL} = (\text{SUCOS}_\text{K}) (\text{SPS}_\text{JK}), \text{ ($/wk)} \]
\[ SL\text{COS}_\text{KL} = (\text{SLUCO}_\text{K}) (\text{SLPS}_\text{JK}), \text{ ($/wk)} \]
\[ \text{SUCOS}_\text{K} = \text{VSCOS} + \text{FSCOS}_\text{K}, \text{ ($/lb)} \]
\[ \text{SLUCO}_\text{K} = \text{V\text{LSCO}} + \text{F\text{LSCO}}_\text{K}, \text{ ($/lb)} \]
\[ \text{FSCOS}_\text{K} = (\text{\text{OSCO}})(\text{SPSN}/\text{SPS}_\text{JK}), \text{ ($/lb)} \]
\[ \text{FLSCO}_\text{K} = (\text{\text{OLSCO}})(\text{SLPS}_\text{JK}/\text{SLPS}_\text{JK}, \text{ ($/lb)} \]

\[ \text{VSCOS} = 0.045 \text{ - variable unit cost} \]
\[ \text{V\text{LSCO}} = 0.045 \text{ - variable unit cost} \]
\[ \text{\text{OSCO}} = 0.005 \text{ - overhead unit cost} \]
\[ \text{\text{OLSCO}} = 0.005 \text{ - overhead unit cost} \]
\[ \text{SPSN} = (\text{USON})(S30) \text{ - Initial Production Started} \]
\[ \text{SLPSN} = (\text{USON})(S31) \text{ - Initial Production started} \]
\[ \text{USON} = 80,000,000, \text{ (lb/wk)} \]
\[ S30 = 0.3 \]
\[ S31 = 0.7 \]

2. Revenue

\[ SR_\text{KL} = (S\text{OS}_\text{JK})(\text{AAP}) \]
\[ SLR_\text{KL} = (S\text{LOS}_\text{JK})(\text{AAP}) \]
\[ \text{AAP} = 0.075 \text{ - Alumina Price, ($/lb)} \]
3. Profit

\[ SPROF_{K} = SPROF_{J} + (DT) \left( \frac{1}{2} \right) (SR_{JK} - SCOST_{JK}, ($/wk) S17, L \]

\[ S1PRO.K = S1PRO.J + (DT) \left( \frac{1}{2} \right) (S1R_{JK} - S1COS_{JK}), ($/wk) S32, L \]

F. Initial Conditions

The initial condition equations follow:

\[ SOS = RSO \]

\[ SFGI = (SOS)(S1+S2) \]

\[ SFGIN = SFGI \]

\[ SGIP = (SOS)(DELI3) \]

\[ SPS = RSO \]

\[ SRMI = (SPS)(S3) \]

\[ SLBT = (SBD)(DELI5) \]

\[ SR = (SOS)(AAP) \]

\[ SCOST = (SPS)(.05) \]

\[ S1RSA = 1 \]

\[ SPROF = 0 \]

\[ SPSN = (USON)(S30) \]

\[ S1OS = R1SO \]

\[ S1FGI = (S1OS)(S1+S2) \]

\[ S1FGN = S1FGI \]

\[ S1GIP = (S1OS)(DELI3) \]

\[ S1PS = R1SO \]
SLRMI = (SLPS) (S6)
SLRBT = (SLBD) (DELI5)
SLR = (SL08) (AAP)
SLCOS = (SLPS) (.05)
SLRSH = 1
SLPRO = 0
SLPSN = (USON) (S31)
Appendix F

MATHEMATICAL DESCRIPTION OF THE COPPER REFINER

A. General

"R" and "Rl" refiner structures (Figures 13A and 13B) and policies described are as follows:

1. Production Policy
2. Production Structure
3. Pricing Policy
4. Distribution Structure
5. Financial Structure
6. Initial Conditions

B. Production Policy

Refriner production policies resemble smelter policies in both the R and the Rl sectors. The Rl policy is the familiar integral control policy:

\[ R_{LPIW.K} = R_{LRMI.K}/R_{24}, \quad (\text{lb/wk}) \]

\[ R_{24} = 3, \quad (\text{wk}) \]

The R policy uses a refined smoothed sales feedback, as at the smelter, but its inventory production is based upon its own finished goods inventory adjustment (RFGID-RFGI) rather than the raw material inventory of the next sector. The difference between the desired inventory (RFGID) and the actual inventory (RFGI) is adjusted over the adjustment time period (R12).
RPD.K = (RSS.K) + (1/R12) (RFGID.K - RFGI.K), (lb/wk) 0202,A
R12 = 52, (wk) 0484,C

The above desired production is limited by inventory (RRMI) to a maximum production limit (RRMPL).

RRMPL.K = RRMI.K/R4, (lb/wk) 0202,A
R4 = 3, (wk) 0473,C

RFW.K = CLIP (RPD.K, RRMPL.K, RRMPL.K, RFD.K), (lb/wk) 0192,A

Both the R and RL production policies are limited by capacity (RMPP, RLMP) as modified by expansion (MLRSA) to limit production to current capacity (RAMPP, RLAMP).

RAMPP.K = (RMPP) (MLRSA.K), (lb/wk) 0191,A
RLAMP.K = (RLMP) (MLRSA.K), (lb/wk) 1191,A
RMPP = 48,600,000, (lb/wk) 2478,C
RLMP = 275,400,000, (lb/wk) 3478,C

The final rates (RPS, RLPS) are the minimum of RAMPP, RLAMP and the production wanted (RFW, RLFW).

RPS.JK = CLIP (RAMPP.K, RFW.K, RFW.K, RAMPP.K), (lb/wk) 0190,R
RLPS.JK = CLIP (RLAMP.K, RLFW.K, RLFW.K, RLAMP.K), (lb/wk) 0191,R

C. Production Structure

The production structure contains the usual goods-in-process inventory (RGIP, RLGIP) from which the copper is sent (RGTFI, RLCTF) to finished goods inventory (RFGI, RLFGI) after a delay (DELI0).
\[
\begin{align*}
\text{RGIP} \cdot K &= \text{RGIP} \cdot J + (\text{DT}) \left( \text{RPS} \cdot JK - \text{RGTFI} \cdot JK \right), \quad (\text{lb}) \\
\text{RLGIP} \cdot K &= \text{RLGIP} \cdot J + (\text{DT}) \left( \text{R1PS} \cdot JK - \text{RLGTF} \cdot JK \right), \quad (\text{lb}) \\
\text{RGTFI} \cdot KL &= \text{DELAY}3 \left( \text{RPS} \cdot JK, \text{DELI}0 \right), \quad (\text{lb}/\text{wk}) \\
\text{RLGTF} \cdot KL &= \text{DELAY}3 \left( \text{R1PS} \cdot JK, \text{DELI}0 \right), \quad (\text{lb}/\text{wk}) \\
\text{DELI}0 &= 3.5, \quad (\text{wk})
\end{align*}
\]

D. Pricing Policy

The refined copper price (RCP, R2CP) is determined from a weighted supply demand ratio (RWRAT, R1WRAT) which is the input variable to the pricing function (RFUN) shown in Figure 14.

\[
\begin{align*}
\text{RCP} \cdot K &= \text{TABLE} \left( \text{RFUN}, \text{RWRAT} \cdot K, 2, 5, 0, 1 \right), \quad ($/\text{lb}) \\
\text{R2CP} \cdot K &= \text{TABLE} \left( \text{RFUN}, \text{R1WRAT} \cdot K, 2, 5, 0, 1 \right), \quad ($/\text{lb})
\end{align*}
\]

The final price quoted in the R1 sector is based upon a competitive comparison with the R2CP price and the RCP price. Should R2CP exceed RCP by a margin exceeding the accepted price margin (RPMAR) the lower of the R2CP price and another price (R3CP), which is the sum of RCP and RPMAR will be quoted. The final quoted R1 price is designated as R1CP.

\[
\begin{align*}
\text{R3CP} \cdot K &= \text{RCP} \cdot K + \text{RPMAR}, \quad ($/\text{lb}) \\
\text{R1CP} \cdot K &= \text{CLIP} \left( \text{R2CP} \cdot K, \text{R3CP} \cdot K, \text{R3CP} \cdot K, \text{R2CP} \cdot K \right), \quad ($/\text{lb}) \\
\text{RPMAR} &= 0.01, 0.02 \text{ or } 0.05, \quad ($/\text{lb})
\end{align*}
\]
RWRAT (RLWRA) is determined from a weighted consideration of a sales/production ratio (RRSD) and an unfilled order/inventory ratio (RRSD). Refiner smoothed sales (RSS) and concentrator production rate (CPS) are used as the rate variables and unfilled orders (RUFO) and refiner's finished goods inventory (RFGI) as the static variable. These ratios are weighted by constants R5 and R6 to determine the supply-demand ratio (RRATO, R1RAT).

\[
\begin{align*}
RRSD.K &= RSS.K/CPS.JK \\
R1RSD.K &= R1SS.K/C1PS.JK \\
RSSD.K &= (RUFO.K)(5)/RFGI.K(7) \\
R1SSD.K &= (R1UFO.K)(5)/(R1FGI.K)(7) \\
RRATO.K &= (1/R7)(R5)(RRSD.K) + (R6)(RRSD.K) \\
R1RAT.K &= (1/R17)(R18)(R1SSD.K) + (R19)(R1RSD.K)
\end{align*}
\]

\[
\begin{align*}
R5 &= 1 \\
R6 &= 10 \\
R18 &= 8 \\
R19 &= 2 \\
R7 &= R5 + R6 \text{ (Normalizing constant)} \\
R17 &= R18 + R19 \text{ (Normalizing constant)}
\end{align*}
\]

RRATO (R1RAT) is then modified by the pricing sensitivity constant (R8, R21) to determine RWRAT (RLWRA).
E. Distribution Structure

The distribution structure of the copper refiner is more varied and extensive than those of the previous sectors because this sector has a sales order processing function as well as a shipping and receiving function. The sales order processing function has an input of sales order (RSO, R1SO) which fills an unfilled order backlog (RUFO, R1UFO) which is depleted by shipments (ROS, R1OS). Average sales (RSS, R1SS) are determined by time-averaging RSO (R1SO) with a smoothing constant (R3).

\[
\begin{align*}
\text{RSO.KL} & = \text{DELAY3 (FCO.JK, DEI8), (lb/wk)} & 0166, R \\
\text{R1SO.KL} & = \text{DELAY3 (FLCO.JK, DEI8), (lb/wk)} & 1166, R \\
\text{RUFO.K} & = \text{RUFO.J + (DT) (RSO.JK - ROS.JK), (lb/wk)} & 0180, R \\
\text{R1UFO.K} & = \text{R1UFO.J + (DT) (R1SO.JK - R1CS.JK), (lb/wk)} & 1180, R \\
\text{RSS.K} & = \text{RSS.J + (DT) (\frac{1}{R3} (RSO.JK - RSS.J), (lb/wk)} & 0187, L \\
\text{R1SS.K} & = \text{R1SS.J + (DT) (\frac{1}{R3} (R1SO.JK - R1SS.J), (lb/wk)} & 1187, L
\end{align*}
\]
DELT = 1.2, (wk) 0421,C
R3 = 24, (wk) 0472,C

FCO - Fabricator's Copper Ordered

Incoming raw material after being in in-transit inventory (RLCT, RLCT) is delivered (RCD,RLCD) to raw material inventory (RRMI,RLRMI) after a delay (DELL2).

\[
\begin{align*}
RLCT.K &= RLCT.J + (DT) (SOS.JK - RCD.JK), (lb) \\
RLCT.L &= RLCT.J + (DT) (SLOS.JK - RLCD.JK), (lb) \\
RCD.KL &= DELAY3 (SOS.JK,DELL2), (lb/wk) \\
R1CD.KL &= DELAY3 (SLOS.JK,DELL2), (lb/wk) \\
RRMI.K &= RRMI.J + (DT) (RCD.JK - RPS.JK), (lb) \\
RLRMI.K &= RLRMI.J + (DT) (RLCD.JK - R1PS.JK), (lb) \\
DELL2 &= 4, (wk)
\end{align*}
\]

The desired raw material inventory (RRMID) for the R sector, used in the inventory adjustment of the smelter is based on smoothed sales (RSS) and a constant (R4) expressing the number of production-weeks of inventory desired.

\[
\begin{align*}
RRMID.K &= (RSS.K) (R4), (lb) \\
R4 &= 3, (wk)
\end{align*}
\]

The output shipping delay ROSD (RLOSd) is determined by a function quite different from those in the previous sectors. Since refined copper exists in various sizes and shapes, the finished goods inventory is a multi-product inventory, and delay is primarily dependent on the availability of the item in stock. Handling capacity
limitations are less significant than stock levels and delay will 
increase with decreasing inventories. The shipping delay (\(\text{ROSD,}\) 
\(\text{RLOS}\)) decreases with an increase in the ratio of actual inventory 
(\(\text{RFGI,RLFGI}\)) to desired inventory (\(\text{RFGID,RLFGD}\)).

\[
\text{ROSD.K} = \left(\frac{1}{\text{RFGI.K}}\right) \left(\frac{\text{R1} (\text{RFGI.K}) + (\text{R2}) (\text{RFGID.K})}{\text{wk}}\right), \hspace{1cm} 0185,A
\]

\[
\text{RLOS.K} = \left(\frac{1}{\text{R1FGI.K}}\right) \left(\frac{\text{R1} (\text{R1FGI.K}) + (\text{R2}) (\text{R1FGD.K})}{\text{wk}}\right), \hspace{1cm} 1185,A
\]

\[
\text{R1} = 6.7, \hspace{1cm} (\text{wk})
\]

\[
\text{R2} = .3, \hspace{1cm} (\text{wk})
\]

Material shipped (\(\text{ROS,RLOS}\)) is determined by unfilled orders 
(\(\text{RUFO,RLUFO}\)) divided by the delay (\(\text{ROSD,RLOS}\)). The material shipping 
rate, however, is limited by finished goods inventory minimum (\(\text{RFGIM}\)) 
so that if \(\text{RFGI}\) is less than \(\text{RFGIM}\) not more than one-third of the 
inventory is shipped per week (\(\text{RFGIN,RLFGN}\)) from finished goods 
inventory (\(\text{RFGI,RLFGI}\)).

\[
\text{ROST.K} = \frac{\text{RUFO.K}}{\text{ROSD.K}}, \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RLOS.T.K} = \frac{\text{RLUFO.K}}{\text{RLOS.D.K}}, \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RFGIN.K} = \text{RFGI.K}/3, \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RLFGN.K} = \text{RLFGI.K}/3, \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RFGI.K} = \text{RFGI.J} + (\text{DT}) (\text{RGTFI.JK - ROS.JK}), \hspace{1cm} (\text{lb})
\]

\[
\text{RLFGI.K} = \text{RLFGI.J} + (\text{DT}) (\text{RLGTF.JK - RLOS.JK}), \hspace{1cm} (\text{lb})
\]

\[
\text{ROS.JK} = \text{CLIP (ROST.K,RFGIN.K,RFGI.K,RFGIM)} , \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RLOS.JK} = \text{CLIP (RLOS.T.K,RLFGN.K,RLFGI.K,RFGIM)}, \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RFGIM} = 39,300,000, \hspace{1cm} (\text{lb/wk})
\]

\[
\text{RLFGM} = 222,700,000, \hspace{1cm} (\text{lb/wk})
\]
The desired finished goods inventory is a function of smoothed sales (RSS), the level desired constant in weeks of sales (RSSC) and general economic conditions (ECR).

\[
\text{RFGD.K} = (\text{ECR}) (\text{RSSC}) (\text{RSS.K}), \quad (\text{lb})
\]
\[
\text{RLFGD.K} = (\text{ECR}) (\text{RLSSC}) (\text{RLSS.K}), \quad (\text{lb})
\]

ECR = 1
RSSC = 5
RLSSC = 5

F. Financial Structure

1. General

The rationale for the financial structure of the copper refiner is identical to that of the copper mine except for the values of the parameters.

2. Cost

\[
\text{RCOST.KL} = (\text{RUCOS.K}) (\text{RPS.JK}), \quad (\$/\text{wk})
\]
\[
\text{RLUCO.K} = (\text{RLUCO.K}) (\text{RLPS.JK}), \quad (\$/\text{wk})
\]

\[
\text{RUCOS.K} = \text{VRUCOS} + \text{FRCOS.K}, \quad (\$/\text{lb})
\]

\[
\text{RLUCO.K} = \text{VLRUCO} + \text{FLRUCO.K}, \quad (\$/\text{lb})
\]

\[
\text{FRCOS.K} = (\text{ORCOS}) (\text{RPSN})/\text{RPS.JK}, \quad (\$/\text{lb})
\]

\[
\text{FLRUCO.K} = (\text{ORLUCO}) (\text{RLPSN})/\text{RLPS.JK}, \quad (\$/\text{lb})
\]

\[
\text{VRUCOS} = 0.243
\]

\[
\text{VLRUCO} = 0.243
\]

\[
\text{ORCOS} = 0.027
\]
O1RCO = 0.027
RPSN = (USON) (R30) - initial production rate, (lb/wk) 0359,N
R2PSN = (USON) (R31) - initial production rate, (lb/wk) 1359,N
R30 = 0.15 3488,C
R31 = 0.85 3489,C

3. Revenue
RR.KL = (ROS.JK) (RCP.JK), ($/wk) 0203,R
R1R.KL = (R1OS.JK) (R1CP.JK), ($/wk) 1203,R

4. Profit
RPROF.K = RPROF.J + (DT) (1/2) (RR.JK - RCOST.JK), ($/wk) 0207,L
R1PRO.K = R1PRO.J + (DT) (1/2) (R1R.JK - R1COS.JK), ($/wk) 1207,L

G. Initial Conditions

The initial condition equations follow:

ROS = RSO 0345,
R1OS = R1SO 1345,
RUF0 = (ROS) (R1+R2) 0346,
R1UF0 = (R1SO) (R1+R2) 1346,
RFGI = (ROS) (RSCC) 0347,
R1FGI = (R1OS) (R1SCC) 1347,
ECR = EC 0348,
RSS = RSO 0349,
\[ R1SS = R1SO \]
\[ RGIP = (R0S) (DELI0) \]
\[ R1GIP = (R10S) (DELI0) \]
\[ RPS = R0S \]
\[ R1PS = R10S \]
\[ RRMI = (RPS) (R4) (R14) \]
\[ R1RMI = (R1PS) (R24) (R25) \]
\[ RLC = (RCD) (DELI2) \]
\[ R1LC = (R1CD) (DELI2) \]
\[ R7 = R6+R5 \]
\[ R17 = R18+R19 \]
\[ R10 = R8+R9 \]
\[ R20 = R21+R22 \]
\[ RR = (R0S) (RCPN) \]
\[ R1R = (R10S) (R1CPN) \]
\[ RCOST = (RPS) (.27) \]
\[ R1COS = (R1PS) (.27) \]
\[ RPROF = 0 \]
\[ R1PRO = 0 \]
\[ RPSN = (USON) (R30) \]
\[ R1PSN = (USON) (R31) \]
Appendix G

MATHEMATICAL DESCRIPTION OF THE ALUMINUM REDUCTION SMELTER

A. General

The structure and policies of the aluminum reduction smelter (Figures 16A and 16B) will be described under the following subject headings:

1. Production Policy
2. Production Structure
3. Distribution Structure
4. Financial Structure
5. Initial Conditions

B. Production Policy

The production policy of the R reduction smelter attempts to achieve a balance between the costs of carrying inventory (ICOST) and the costs of changing production (PCOST). At the beginning of each half-year a decision must be made as to whether the production rate will:

1. Remain the same as in the previous period
2. Change to a higher or lower rate.

If the value of average sales or the requirements for additional service inventories necessitate a higher production rate, the production rate is increased to the value indicated by R3PD.

\[
R3PD.K = RSS.K + (1/R12) (RFGID.K - RFGI.K), \text{ (lb/wk)}
\]

RSS - Reduction Smoothed Sales, (lb/wk)
RFGID - Reduction Finished Goods Inventory, Desired, (lb)
RFGI - Reduction Finished Goods Inventory, Actual, (lb)
R12 = 12, (wk)
A different situation arises when R3PD calls for a rate lower than the present rate (RPS). A decision must then be made, either to continue at the present rate and accumulate inventory or to reduce production to the lower rate, as determined by R3PD. This decision is determined by the values of additional inventory carrying costs (ICOST) versus the costs of changing production (PCOST). The incremental inventory carrying cost (ICOST) is determined by the difference between the sales rate (RSS) and the current production rate (RPS) in the period, in this case twenty-six weeks, and the carrying costs—10 percent per annum was used.

$$\text{ICOST}.K = (26) (.20) (.05) (\text{RPS}.JK - \text{RSS}.K), (\$)$$

Production cost (PCOST) is based upon the amount of change required to produce the new rate (R3PD) and the cost per unit of rate change.

$$\text{PCOST}.K = (1.1) (\text{RPS}.JK - \text{R3PD}.K), (\$)$$

The production desired (R1PD) is then determined by the comparative magnitudes of PCOST and ICOST. If ICOST exceeds PCOST production is changed to R3PD. If PCOST exceeds ICOST the present rate (RPS) is maintained.

R1PD.K = CLIP (R3PD.K, RPS.JK, ICOST.K, PCOST.K), (lb/wk)

RFD.K = SAMPLE (R1PD.K, 26)

The planning period of one-half year (twenty-six weeks) is not necessarily typical. It was varied from one month to six months to
determine the sensitivity of the results to this parameter. Final production rate for the coming period (RPS) is limited by raw material inventory (RRMI) to maximum production rate (RRMPL) and is also limited by the initial capacity (RMPP) as modified by long range expansion (RLRSA).

\[ \text{RRMPL}_k = \frac{\text{RRMI}_k}{R4}, \text{ (lb/wk)} \]

R4 = 7, (wk)

\[ \text{RFW}_k = \text{CLIP} (\text{RPD}_k, \text{RRMPL}_k, \text{RRMPL}_k, \text{RPD}_k), \text{ (lb/wk)} \]

\[ \text{RPS}_k = \text{CLIP} (\text{RAMPP}_k, \text{RPW}_k, \text{RPW}_k, \text{RAMPP}_k), \text{ (lb/wk)} \]

\[ \text{RMPP} = 30,000,000, \text{ (lb/wk)} \]

R5 = 0.002

Production policy in the R1 sector does not explicitly consider alternative costs, but is based solely upon production desired (R4PD) as a result of predicted sales and inventory adjustment. It is limited by the inventory production limit (RLRMP) and capacity (RLAMP) as in the other sector.

\[ \text{R4PD}_k = \text{R1SS}_k + \frac{1}{\text{R18}} \left( \text{RLFGD}_k - \text{RLFGI}_k \right), \text{ (lb/wk)} \]

R18 = 12, (wk)

\[ \text{R5PD}_k = \text{SAMPLE} (\text{R4PD}_k, 26), \text{ (lb/wk)} \]

\[ \text{RLRMP}_k = \frac{\text{RLRMI}_k}{R2}, \text{ (lb/wk)} \]

R21 = 7, (wk)

\[ \text{RLPW}_k = \text{CLIP} (\text{R5PD}_k, \text{RLRMP}_k, \text{RLRMP}_k, \text{R5PD}_k), \text{ (lb/wk)} \]
RLAMP.K = (RLMPP) (RLRJS.K), (lb/wk)  
RLMPP = 70,000,000, (lb/wk)  
RLPS.KL = CLIP (RLAMP.K, RLFW.K, RLFW.K, RLAMP.K), (lb/wk)  
RLRJS.K = RLRJ.S.J + (DT) (R51)  
R51 = 0.002

Reduction production policy also determines the basic mine production desired (MPD, M3PD) based on reduction sales (RSS) and inventory (RFGID-RFGI) requirements.

MLPD.K = RSS.K + (1/R16) (RFGID.K - RFGI.K), (lb/wk)  
MPD.K = SAMPLE (MLPD.K, 26), (lb/wk)  
M2PD.K = RLSS.K + (1/R20) (R1FGD K - R1FGI.K), (lb/wk)  
M3PD.K = SAMPLE (M2PD.K, 26), (lb/wk)  
R16 = 12, (wk)  
R20 = 12, (wk)

C. Production Structure

The production structure is the conventional one with a goods-in-process inventory (RGIP, R1GIP) from which aluminum is sent (RGTFI, R1GTF) to finished goods inventory (RFGI, R1FGI) after a time delay (DELI0).

RGIP.K = RGIP.J + (DT) (RPS.JK - RGTFI.JK), (lb)  
R1GIP.K = R1GIP.J + (DT) (RLPS.JK - R1GTF.JK), (lb)  
RGTFI.KL = DELAY3 (RPS.JK, DEL10), (lb/wk)
D. Distribution Structure

Since the rationale of the distribution structure in this sector is identical to that in the copper refiner sector, only the equations and their symbolic definitions will be listed. Reference to the copper refiner description will explain fully the equations used. Symbols except as noted are defined as in the copper refiner.

\[
\begin{align*}
\text{RSO.KL} & = \text{DELAY3} (\text{FAO.JK, DEL8}), \ (\text{lb/wk}) \\
\text{RLSO.KL} & = \text{DELAY3} (\text{FLAO.JK, DEL8}), \ (\text{lb/wk}) \\
\text{RUFO.K} & = \text{RUFO.J} + (\text{DT}) \ (\text{RSO.JK - ROS.JK}), \ (\text{lb}) \\
\text{RLUFO.K} & = \text{RLUFO.J} + (\text{DT}) \ (\text{RLSO.JK - RL0S.JK}), \ (\text{lb}) \\
\text{RSS.K} & = \text{RSS.J} + (\text{DT}) \ (1/R3) \ (\text{RSO.JK - RSS.J}), \ (\text{lb/wk}) \\
\text{RLSS.K} & = \text{RLSS.J} + (\text{DT}) \ (1/R3) \ (\text{RLSO.JK - RL3S.J}), \ (\text{lb/wk}) \\
\text{DEL8} & = 1.2, \ (\text{wk}) \\
\text{R3} & = 12, \ (\text{wk}) \\
\text{FAO} & \ - \allowbreak \text{Fabricator's Aluminum Ordered, (lb/wk)} \\
\text{RLAT.K} & = \text{RLAT.J} + (\text{DT}) \ (\text{SOS.JK - RAD.JK}), \ (\text{lb}) \\
\text{RLLAT.K} & = \text{RLLAT.J} + (\text{DT}) \ (\text{SLOS.JK - R1AD.JK}), \ (\text{lb}) \\
\text{RAD.KL} & = \text{DELAY3} (\text{SOS.JK, DEL12}) \\
\text{RLAD.KL} & = \text{DELAY3} (\text{SLOS.JK, DEL12}) \\
\text{RRMI.K} & = \text{RRMI.J} + (\text{DT}) \ (\text{RAD.JK - RPS.JK}), \ (\text{lb}) \\
\text{RLRMI.K} & = \text{RLRMI.J} + (\text{DT}) \ (\text{RLAD.JK - R1PS.JK}), \ (\text{lb}) \\
\text{DEL12} & = 1.2, \ (\text{wk})
\end{align*}
\]
\[ \text{RRMID.K} = (R_4) (RSS.K) \]
\[ \text{R1RMID.K} = (R_{21}) (R1SS.K) \]
\[ R_4 = 7, \text{ (wk)} \]
\[ R_{21} = 7, \text{ (wk)} \]
\[ \text{ROSD.K} = (1/RFGI.K) \left( (R_1) (RFGI.K) + (R_2) (RFGID.K) \right) \]
\[ \text{R10SD.K} = (1/R1FGI.K) \left( (R_1) (R1FGI.K) + (R_2) (R1FGD.K) \right) \]
\[ R_1 = 6.7, \text{ (wk)} \]
\[ R_2 = 0.3, \text{ (wk)} \]
\[ \text{ROST.K} = RUFO.K/ROSD.K \]
\[ \text{R1OST.K} = R1UF0.K/R10SD.K \]
\[ \text{RFGIN.K} = RFGI.K/3 \]
\[ RFGI.K = RFGI.J + (DT) (RGTFI.JK - ROS.JK), \text{ (lb)} \]
\[ R1FGI.K = R1FGI.J + (DT) (R1GTF.JK - R1OS.JK), \text{ (lb)} \]
\[ \text{ROS.KL} = \text{CLIP} (ROST.K, RFGIN.K, RFGI.K, RFGIM), \text{ (lb/wk)} \]
\[ \text{R10S.KL} = \text{CLIP} (R10ST.K, R1FGN.K, R1FGI.K, R1FGM), \text{ (lb/wk)} \]
\[ \text{RFGIM} = 24,000,000, \text{ (lb)} \]
\[ \text{R1FGM} = 56,000,000, \text{ (lb)} \]
\[ \text{RFGID.K} = (ECR) (RSSC) (RSS.K) \]
\[ \text{R1FGD.K} = (ECR) (R1SSC) (R1SS.K) \]
\[ \text{ECR} = 1 \]
\[ \text{RSSC} = 5, \text{ (wk)} \]
\[ \text{R1SSC} = 5, \text{ (wk)} \]
E. Financial Structure

1. General

The financial structure of the previous sectors, except for values of constants, is used in this sector.

2.

\[ \text{RCOST.KL} = (\text{RUCOS.K})(\text{RPS.JK}), \text{($)}/\text{wk} \]  \hspace{1cm} \text{R22,R}

\[ \text{RLCOS.KL} = (\text{RLUCO.K})(\text{RLPS.JK}), \text{($)}/\text{wk} \]  \hspace{1cm} \text{R51,R}

\[ \text{RUCOS.K} = \text{VRUC} + \text{FRCOS.K}, \text{($)}/\text{lb} \]  \hspace{1cm} \text{R23,A}

\[ \text{RLUCO.K} = \text{VLRCO} + \text{FRUCO.K}, \text{($)}/\text{lb} \]  \hspace{1cm} \text{R52,A}

\[ \text{FRCOS.K} = (\text{ORUCOS})(\text{RPSN})/\text{RPS.JK}, \text{($)}/\text{lb} \]  \hspace{1cm} \text{R24,A}

\[ \text{FRUCO.K} = (\text{ORUCO})(\text{RLPSN})/\text{RLPS.JK}, \text{($)}/\text{lb} \]  \hspace{1cm} \text{R53,A}

\[ \text{VRUC} = 0.175, \text{($)}/\text{lb} \]  \hspace{1cm} \text{CR12,C}

\[ \text{VLRCO} = 0.175, \text{($)}/\text{lb} \]  \hspace{1cm} \text{CR25,C}

\[ \text{ORUCOS} = 0.025, \text{($)}/\text{lb} \]  \hspace{1cm} \text{CR13,C}

\[ \text{ORUCO} = 0.025, \text{($)}/\text{lb} \]  \hspace{1cm} \text{CR16,C}

\[ \text{RPSN} = (\text{USON})(\text{R30}), \text{lb}/\text{wk} \]  \hspace{1cm} \text{NR14,N}

\[ \text{R30} = .3 \]  \hspace{1cm} \text{1CR32,C}

\[ \text{R31} = .7 \]  \hspace{1cm} \text{1CR33,C}

\[ \text{USON} = 80,000,000, \text{lb}/\text{wk} \]  \hspace{1cm} \text{CV13,C}

3. Revenue

\[ \text{RR.KL} = (\text{ROS.JK})(\text{AP}), \text{($)}/\text{wk} \]  \hspace{1cm} \text{R21,R}

\[ \text{RLR.KL} = (\text{RLOS.JK})(\text{AP}), \text{($)}/\text{wk} \]  \hspace{1cm} \text{R50,R}

\[ \text{AP} = 0.26, \text{($)}/\text{lb} \]  \hspace{1cm} \text{CR7,C}
4. Profit

$$R_{PRF.K} = R_{PRF.J} + (DT) \left( \frac{1}{2} \right) (R_{R.JK} - R_{COST.JK}), \quad (\$/wk) \quad R_{27,L}$$

$$R_{1PRF.K} = R_{1PRF.J} + (DT) \left( \frac{1}{2} \right) (R_{1R.JK} - R_{1COS.JK}), \quad (\$/wk) \quad R_{56,L}$$

F. Initial Conditions

The initial condition equations follow:

$$R_{OS} = R_{SO} \quad \text{NR}_{1}, N$$

$$R_{UFO} = (R_{SO})(R_{1+R2}) \quad \text{NR}_{2}, N$$

$$R_{FGI} = (R_{OS})(R_{SSC}) \quad \text{NR}_{3}, N$$

$$E_{CR} = E_{C} \quad \text{NR}_{4}, N$$

$$R_{HSS} = R_{SO} \quad \text{NR}_{5}, N$$

$$R_{GIP} = (R_{OS})(DE_{L10}) \quad \text{NR}_{6}, N$$

$$R_{PS} = R_{OS} \quad \text{NR}_{7}, N$$

$$R_{RMI} = (R_{PS})(R_{4})(R_{14}) \quad \text{NR}_{8}, N$$

$$R_{LAT} = (R_{AD})(DE_{L12}) \quad \text{NR}_{9}, N$$

$$R_{R} = (R_{OS})(A_{P}) \quad \text{NR}_{10}, N$$

$$R_{COST} = (R_{PS})(.2) \quad \text{NR}_{11}, N$$

$$R_{LRSA} = 1 \quad \text{NR}_{12}, N$$

$$R_{PRF} = 0 \quad \text{NR}_{13}, N$$

$$R_{PSN} = (U_{SON})(R_{30}) \quad \text{NR}_{14}, N$$

$$R_{FDA} = R_{OS} \quad \text{NR}_{15}, N$$

$$M_{FD} = R_{SO} \quad \text{NR}_{16}, N$$

$$R_{1OS} = R_{1SO} \quad \text{NR}_{15}, N$$

$$R_{1UFO} = (R_{1SO})(R_{1+R2}) \quad \text{NR}_{16}, N$$
\( R_{1FGI} = (R_{1OS})(R_{1SSC}) \)
\( R_{1SS} = R_{1SO} \)
\( R_{1GIP} = (R_{1OS})(D_{EL10}) \)
\( R_{1PS} = R_{1OS} \)
\( R_{1RMI} = (R_{1PS})(R_{21})(R_{19}) \)
\( R_{1LAT} = (R_{1AD})(D_{EL12}) \)
\( R_{1R} = (R_{1OS})(A_{P}) \)
\( R_{1COS} = (R_{1PS})(.2) \)
\( R_{1LRS} = 1 \)
\( R_{1PRO} = 0 \)
\( R_{1PSN} = (US_{ON})(R_{31}) \)
\( R_{5PD} = R_{1SO} \)
\( M_{3PD} = R_{1SO} \)
Appendix H

MATHEMATICAL DESCRIPTION OF THE COPPER FABRICATOR

A. General

The description of the copper fabricator will be presented under the following structures and policies:

1. Production Policy
2. Production Structure
3. Purchasing Policy
4. Distribution Structure
5. Financial Structure
6. Initial Conditions

B. Production Policy

The production policy of the copper fabricator is similar to that of the copper refiner, with the production desired (F_{PD}, F_{LPD}) based on smoothed sales (FSS, F_{LSS}) and the finished goods inventory adjustment (F_{FGID} - F_{FGI}, F_{LFGD} - F_{LFGI}).

\[
\begin{align*}
F_{PD}.K &= FSS.K + (1/F4) (F_{FGID}.K - F_{FGI}.K), \text{ (lb/wk)} & 0152, A \\
F_{LPD}.K &= F_{LSS}.K + (1/F16) (F_{LFGD}.K - F_{LFGI}.K), \text{ (lb/wk)} & 1152, A \\
F4 &= 6, \text{ (wk)} & 1453, C \\
F16 &= 6, \text{ (wk)} & 2453, C
\end{align*}
\]

This desired production rate is limited by the raw material availability production limit (F_{PRML}, F_{LPRM}) and the production
capacity \((\text{FAMPP,FLAMP})\) as determined by the initial capacity \((\text{FMPP,FLMPP})\) and the long range supply adjustment \((\text{MLRSA})\). The final production started \((\text{FPS,FLPS})\) is delayed to account for plant adjustments needed for a change in production rate. \text{FPS} (\text{FLPS}) \text{is delayed through a level called delayed production starts} \((\text{FDPS,FLDPS})\) as regulated by the rate called production to be started \((\text{FPTBS,FLPTB})\).

\[
\begin{align*}
\text{FPRML.K} &= \text{FRMI.K}/3, \quad (\text{lb/wk}) \\
\text{FLPRM.K} &= \text{F1RMI.K}/3, \quad (\text{lb/wk}) \\
\text{FPW.K} &= \text{CLIP (FPD.K, FPRML.K, FRMI.K, FRMIM), (lb/wk)} \\
\text{FLPW.K} &= \text{CLIP (FLPD.K, FLPRM.K, F1RMI.K, FLRMM), (lb/wk)} \\
\text{FAMPP.K} &= \text{(FAMPP) (MLRSA.K), (lb/wk)} \\
\text{FLAMP.K} &= \text{(FLMPP) (MLRSA.K), (lb/wk)} \\
\text{FPTBS.K} &= \text{CLIP (FAMPP.K, FPW.K, FPW.K, FAMPP.K), (lb/wk)} \\
\text{FLPTB.K} &= \text{CLIP (FLAMP.K, FLPW.W, FLPW.K, FLAMP.K), (lb/wk)} \\
\text{FPS.KL} &= \text{DELAY3 (FPTBS.JK, DEL7), (lb/wk)} \\
\text{FLPS.KL} &= \text{DELAY3 (FPTBS.JK, DEL7), (lb/wk)} \\
\text{FDPS.K} &= \text{FDPS.J + (DT) (FPTBS.JK, FPS.JK), (lb)} \\
\text{FLDPS.K} &= \text{FLDPS.J + (DT) (FLPTB.JK, FLPS.JK), (lb)} \\
\text{FRMIM} &= 23,000,000, \quad (\text{lb}) \\
\text{FLRMM} &= 108,000,000, \quad (\text{lb}) \\
\text{FMPP} &= 54,000,000, \quad (\text{lb/wk}) \\
\text{FLMPP} &= 306,000,000, \quad (\text{lb/wk}) \\
\text{DEL7} &= 1.2, \quad (\text{wk})
\end{align*}
\]
C. Production Structure

The production structure is conventional. A goods-in-process inventory (FGIP,FLGIP) exists, from which fabricated aluminum is sent (FGTFI,FLGTF) to finished goods inventory (FFGI,FLFGI) after a third-order delay (DEI6).

\[
\begin{align*}
FGIP.K &= FGIP.J + (DT) (FPS.JK - FGTFI.JK), \quad (lb) \\
F1GIP.K &= F1GIP.J + (DT) (F1PS.JK - F1GTGF.JK), \quad (lb) \\
FGTFI.KL &= DELAY3(FPS.JK,DEI6), \quad (lb/wk) \\
F1GTGF.KL &= DELAY3 (F1PS.JK,DEI6), \quad (lb/wk) \\
DEI6 &= 1.2, \quad (wk)
\end{align*}
\]

D. Purchasing Policy

Purchasing policy is directed toward supplying current production (FDPT,FLDPT) and maintaining a desirable level of raw material inventory (FRMID,F1RMD). The desired raw material purchased (FRMPD,F1RMD) is modified by the speculation modifier (FSM) which increases the value of FRMID(F1RMD) when prices are rising and lowers it when prices are falling. Price changes (FDIFF) are determined by taking successive price differences (FDIFF*2 and FDIFF*3) and multiplying them by the speculation constant (F7) to determine the speculation modifier (FSM).

\[
\begin{align*}
FDPT.K &= CLIP (FAMPP.K,FPD.K,FPD.K,FAMPP.K), \quad (lb/wk) \\
F1DPT.K &= CLIP (F1AMP.K,F1PD.K,F1PD.K,F1AMP.K), \quad (lb/wk) \\
FSM.K &= 1 + \left(\frac{1}{F7}\right) (FDIFF*2.K - FDIFF*3.K) \\
FDIFF &= BOXLIN (3,1)
\end{align*}
\]

PRICE = BOXLIN (3,1)

PRICE*1.K = PRICE*1.J + (DT) (RCP.J)

FRMID.K = (FDPT.K) (FSM.K) (FJ), (lb)

FLRMD.K = (FLDPT.K) (FSM.K) (FJ), (lb)

FRMPD.K = FDPT.K + (1/F6) (FRMID.K - FRMI.K), (lb/wk)

FLRMP.K = FLDPT.K + (1/F17) (FLRMD.K - FLRMI.K), (lb/wk)

The actual amount of copper ordered (FCO,F1CO) is modified by the required pipeline inventory (FRPLI,F1RPL) and the actual pipeline inventory (FAPLI,F1APL), which arises from the total delay in delivery and the average copper ordered (FCOA,F1COA).

FCO.KL = FRMPD.K + (1/8) (FRPILI.K - FAPLILI.K), (lb/wk)

F1CO.KL = FLRMP.K + (1/8) (F1RPLI.K - F1APLILI.K), (lb/wk)

FRPILI.K = (FCOA.K) (DELB + DEL9 + ROSD.K), (lb)

F1RPLI.K = (F1COA.K) (DELB + DEL9 + R1OSD.K), (lb)

FAPLILI.K = FLCT.K + RUFO.K + F1CO.K, (lb)

F1APLILI.K = F1LCT.K + R1UFO.K + F1CO.K, (lb)

FCOA.K = FCOA.J + (DT) (1/F8) (F1COJ.FCOA.J), (lb)

F1COA.K = F1COA.J + (DT) (1/F18) (F1COJK - F1COA.J), (lb)

DELB = 1.2, (wk)

DELB9 = 1.2, (wk)

F8 = 12.0, (wk)

F18 = 12.0, (wk)
FLCT, FLCT - Copper in Transit, (lb)

RUFO, RLUFO - Refiner's Unfilled Orders, (lb)

Sales orders released to the refiner (RSO, R1SO) are delayed to account for clerical delays.

\[
\begin{align*}
\text{FLCO.K} &= \text{FLCO.J} + (DT) (\text{FCO.JK} - \text{RSO.JK}), \quad (lb) \\
\text{F1LCO.K} &= \text{F1LCO.J} + (DT) (\text{F1LCO.JK} - \text{R1SO.JK}), \quad (lb) \\
\text{RSO.KL} &= \text{DELAY3 (FCO.JK, DEL8), (lb/wk)} \\
\text{R1SO.KL} &= \text{DELAY3 (F1LCO.JK, DEL8), (lb/wk)}
\end{align*}
\]

0165, L  
0166, R  
1165, L  
1166, R

E. Distribution Structure

The rationale for the distribution structure of the copper fabricator is identical to that of the copper refiner. For this reason, only the equations and their symbolic definitions will be listed here.

\[
\begin{align*}
\text{FSO.KL} &= \text{DELAY3 (UCO.JK, DEL4), (lb/wk)} \\
\text{F1SO.KL} &= \text{DELAY3 (USP.JK, DEL4), (lb/wk)} \\
\text{FUFO.K} &= \text{FUFO.J} + (DT) (\text{FSO.JK} - \text{FOS.JK}), \quad (lb) \\
\text{F1FUFO.K} &= \text{F1FUFO.J} + (DT) (\text{F1SO.JK} - \text{F1OS.JK}), \quad (lb) \\
\text{FSS.K} &= \text{FSS.J} + (DT) (1/F3) (\text{FSO.JK} - \text{FSS.J}), \quad (lb/wk) \\
\text{F1SS.K} &= \text{F1SS.J} + (DT) (1/F18) (\text{F1SO.JK} - \text{F1SS.J}), \quad (lb/wk) \\
\text{DEL4} &= 1.2, \quad (wk) \\
\text{F3} &= 12, \quad (wk) \\
\text{F18} &= 12, \quad (wk)
\end{align*}
\]

0128, R  
0128, R  
0140, L  
0140, L  
0147, L  
0147, L  
0417, C  
0452, C  
3455, C
FSO,FLSO - Fabricator's Sales Orders
FUFO,FLUF0 - Fabricator's Unfilled Orders
FSS,FLSS - Fabricator's Smoothed Sales
UCO - User Copper Ordered (F sector purchases)
USP - User Substitute Copper Order (Fl sector purchases)
FOS,FLOS - Fabricator's Orders Shipped

\[ \text{FLCT.K} = \text{FLCT.J} + (\text{DT}) (\text{ROS.JK} - \text{FCD.JK}), \text{ (lb)} \]  
\[ \text{FLCT.K} = \text{FLCT.J} + (\text{DT}) (\text{R1OS.JK} - \text{FLCD.JK}), \text{ (lb)} \]  
\[ \text{FCD.KL} = \text{DELAY3 (ROS.JK,DEL9), (lb/wk)} \]  
\[ \text{FLCD.KL} = \text{DELAY3 (R1OS.JK+DEL9), (lb/wk)} \]  
\[ \text{FRMI.K} = \text{FRMI.J} + (\text{DT}) (\text{FCD.JK} - \text{FPS.JK}), \text{ (lb)} \]  
\[ \text{FLRMI.K} = \text{FLRMI.J} + (\text{DT}) (\text{FLCD.JK} - \text{FLPS.JK}), \text{ (lb)} \]  
\[ \text{DEL9} = 1.2, \text{ (wk)} \]  

FLCT,FLCT - Level of Copper in Transit
FCD,FLCD - Copper Delivered
FRMI,FLRMI - Raw Material Inventory
ROS,R1OS - Refiner's Orders Shipped

\[ \text{POS.D.K} = (1/\text{FFGI.K}) ((\text{FL}) (\text{FFGI.K}) + (\text{F2}) (\text{FFGI.D.K})), \text{ (wk)} \]  
\[ \text{FLOSD.K} = (1/\text{FLFGI.K}) ((\text{FL}) (\text{FLFGI.K}) + (\text{F2}) (\text{FLFGD.K})),\text{ (wk)} \]  
\[ \text{FL} = 6.0, \text{ (wk)} \]  
\[ \text{F2} = 0.6, \text{ (wk)} \]  

POS.D,FLOSD - Shipping Delay
FFGI,FLFGI - Finished Goods Inventory
FFGID,FLFGD - Finished Goods Inventory, Desired
\[ \text{POST.K} = \frac{FUFO.K}{FOSD.K}, \quad (\text{lb/wk}) \]
\[ \text{FLOST.K} = \frac{FLUFO.K}{FLOS.D.K}, \quad (\text{lb/wk}) \]
\[ \text{FFGIN.K} = \frac{FFGI.K}{3}, \quad (\text{lb}) \]
\[ \text{FLFGN.K} = \frac{FLFGI.K}{3}, \quad (\text{lb}) \]
\[ \text{FFGI.K} = \text{FFGI.J} + (\text{DT}) \left( \text{FGTFI.JK} - \text{FOS.JK} \right), \quad (\text{lb}) \]
\[ \text{FLFGI.K} = \text{FLFGI.J} + (\text{DT}) \left( \text{FG2GTF.JK} - \text{FLOS.JK} \right), \quad (\text{lb}) \]
\[ \text{FOS.KL} = \text{CLIP} \left( \text{POST.K}, \text{FFGIN.K}, \text{FFGI.K}, \text{FFGIM} \right), \quad (\text{lb/wk}) \]
\[ \text{FLOS.KL} = \text{CLIP} \left( \text{FLOST.K}, \text{FLFGN.K}, \text{FLFGI.K}, \text{FLFGM} \right), \quad (\text{lb/wk}) \]

- **POST, FLOST** - Order, Shipment Trial
- **FFGIN, FLFGN** - Negative Inventory Limit
- **FOS, FLOS** - Orders Shipped

\[ \text{FFGIM} = 393,000,000, \quad (\text{lb}) \]
\[ \text{FLFGM} = 222,000,000, \quad (\text{lb}) \]
\[ \text{FFGID.K} = (\text{ECL}) \left( \text{FSSC} \right) \left( \text{FSS.K} \right), \quad (\text{lb}) \]
\[ \text{FLFGD.K} = (\text{ECL}) \left( \text{FLSSC} \right) \left( \text{FLSS.K} \right), \quad (\text{lb}) \]

\[ \text{ECL} = 1 \]
\[ \text{FSSC} = 4, \quad 20, \quad (\text{wk}) \]
\[ \text{FLSSC} = 4, \quad (\text{wk}) \]

F. Financial Structure

1. General

The financial structure of the previous sectors, except for values of the parameters, applies.
2. Costs

\[ \text{FFCOS.KL} = (\text{FUCOS.K}) (\text{FPS.JK}), \text{($/wk)} \]
\[ \text{FF1CO.KL} = (\text{FLUCO.K}) (\text{FLPS.JK}), \text{($/wk)} \]
\[ \text{FUCOS.K} = \text{VFCOS} + \text{FCOS.K}, \text{($/lb)} \]
\[ \text{FLUCO.K} = \text{VLFCO} + \text{FLFCO.K}, \text{($/lb)} \]
\[ \text{FCOS.K} = (\text{OFCOS}) (\text{FPSN})/\text{FPS.JK}, \text{($/lb)} \]
\[ \text{FLFCO.K} = (\text{OLFCO}) (\text{FLPSN})/\text{FLPS.JK}, \text{($/lb)} \]
\[ \text{VFCOS} = 0.37 \]
\[ \text{VLFCO} = 0.37 \]
\[ \text{OFCOS} = 0.08 \]
\[ \text{OLFCO} = 0.08 \]
\[ \text{FPSN} = (\text{USON}) (\text{F30}) \]
\[ \text{FLPSN} = (\text{USON}) (\text{F31}) \]
\[ \text{F30} = 0.15 \]
\[ \text{F31} = 0.85 \]
\[ \text{USON} = 262,000,000, \text{(lb/wk)} \]

3. Revenue

\[ \text{FR.KL} = (\text{FCOS.JK}) (\text{FCP.K}), \text{($/wk)} \]
\[ \text{FLR.KL} = (\text{FLCD.JK}) (\text{FLCP.K}), \text{($/wk)} \]

\[ \text{FCP} \text{ K,FLCP} - \text{Fabricated Copper Price} \]
\[ \text{FCP.K} = \text{RCP.K} + 0.20 \]
\[ \text{FLCP.K} = \text{R1CP.K} + 0.20 \]
4. Profit

\[ FPROF.K = FPROF.J + (DT) (1/2) (FR.JK - FFCOS.JK), \text{($/wk$)} \]
\[ F1PRO.K = F1PRO.J + (DT) (1/2) (F1R.JK - F1LCO.JK), \text{($/wk$)} \]

G. Initial Conditions

The initial condition equations follow:

- \( FOS = FSO \)
- \( FLG = FLSO \)
- \( FUFO = (FSO) (F1+F2) \)
- \( FLUFO = (FLSO) (F1+F2) \)
- \( ECI = EC \)
- \( FLGFI = (FLOS) (FLSSC) \)
- \( FFGI = (FOS) (FSSC) \)
- \( FLGIP = (FLOS) (DEL6) \)
- \( FGIP = (FOS) (DEL6) \)
- \( FLDPS = (FLPS) (DEL7) \)
- \( FDPS = (FPS) (DEL7) \)
- \( F1PTB = F1SO \)
- \( FPTBS = FSO \)
- \( FLSS = FLSO \)
- \( FSS = FSO \)
- \( FLRMI = (FLPS) (F51) \)
- \( FRMI = (FPS) (F5) \)
- \( FCO = FSO \)
- \( F1CO = FLSO \)
\[
\begin{align*}
\text{FLCO} &= (FCO) (DEL8) & 0340, N \\
\text{FLICO} &= (FLCO) (DEL8) & 1340, N \\
\text{FLCT} &= (ROS) (DEL9) & 0341, N \\
\text{FLICT} &= (RLOS) (DEL9) & 1341, N \\
\text{FCOA} &= FCO & 0342, N \\
\text{FLCOA} &= F1CO & 1342, N \\
\text{FR} &= (FOS) (FCPN) & 0343, N \\
\text{FLR} &= (FLOS) (FLCPN) & 1343, N \\
\text{FFCOS} &= (FPS) (45) & 0344, N \\
\text{FF1CO} &= (F1PS) (45) & 1344, N \\
\text{FLPRO} &= 0 & 2344, N \\
\text{FLPSN} &= (USON) (F31) & 3344, N \\
\text{FPFROF} &= 0 & 4344, N \\
\text{FCOS} &= (FPS) (45) & 5344, N \\
\text{FPSON} &= (USON) (F30) & 7344, N
\end{align*}
\]
Appendix I

MATHEMATICAL DESCRIPTION OF THE ALUMINUM FABRICATOR

A. General

The aluminum fabricator is identical in structure and policies to the copper fabricator, except for two areas:

1. Pricing is established in the aluminum fabricator sector.

2. The speculation modifier (FSM) is absent in the aluminum fabricator.

Only the pricing policy function will be described in this appendix. The equations for all other functions are identical (omitting FSM, the speculation modifier) to those in Appendix H, except for minor modifications to symbols and equation numbers. These changes may be found in Figures 18A and 18B.

B. Pricing Policy

The basic approach to fabricated aluminum pricing is similar to that in copper refining pricing. A weighted supply-demand ratio (FWRAT, FLWRA) determines the price. The supply demand ratio is in turn a function of static (FSSD, FLSSD) and rate (FRSD, FLRSD) factors, which are based upon unfilled orders (FUFO, FUFO), finished goods inventory (FFGI, F1FGI), smoothed sales (FSS, FLSS), and reduction production rate (RPS, RLPS).

\[
\begin{align*}
FAP.K &= \text{TABLE} (\text{FFUN}, \text{FWRAT.K}, \{0.2, 3.5, 1\}, (\$/lb)) \\
F1AP.K &= \text{TABLE} (\text{FFUN}, \text{FLWRA}, \{0.2, 3.5, 1\}, (\$/lb)) \\
FWRAT.K &= (1/12) ((F13) (FRATO.K) + (F14) (F15))
\end{align*}
\]
\[ FLWRA.K = \left( \frac{1}{F22} \right) \left( F23 \right) (FLRAT.K) + (F24) (F25) \],
\[ FRATO.K = \left( \frac{1}{F9} \right) \left( F10 \right) (FSSD.K) + (F11) (FRSD.K) \]
\[ FLRAT.K = \left( \frac{1}{F19} \right) \left( F20 \right) (F1SSD.K) + (F21) (F1RSD.K) \]
\[ FSSD.K = \left( \frac{F1UFO.K \times 5}{(FFGI.K) \times 7} \right) \]
\[ FRSD.K = FSS.K / RPS.JK \]
\[ F1SSD.K = \left( \frac{F1UFO.K \times 5}{(FFGI.K) \times 7} \right) \]
\[ F1RSD.K = F1SS.K / F1PS.JK \]

\[ F10 = 1 \]
\[ F11 = 8 \]
\[ F20 = 2 \]
\[ F21 = 4 \]
\[ F9 = F10 + F11 \]
\[ F19 = F20 + F21 \]
\[ F13 = 1 \]
\[ F14 = 2 \]
\[ F15 = 1 \]
\[ F23 = 1 \]
\[ F24 = 2 \]
\[ F25 = 1 \]
\[ F12 = F13 + F14 \]
\[ F22 = F23 + F24 \]
C. Financial Structure

The structure and equations are identical to those of the copper fabricator with the following changes in the values of the constants.

\[ V_{FCOS} = 0.35, \text{ ($/lb$)} \quad \text{CF20,C} \]
\[ V_{1FCO} = 0.36, \text{ ($/lb$)} \quad \text{CF38,C} \]
\[ O_{FCOS} = 0.10, \text{ ($/lb$)} \quad \text{CF19,C} \]
\[ O_{1FCO} = 0.09, \text{ ($/lb$)} \quad \text{CF37,C} \]
\[ F_{30} = 0.3 \quad \text{LCF47,C} \]
\[ F_{31} = 0.7 \quad \text{LCF48,C} \]
Appendix J

MATHEMATICAL DESCRIPTION OF THE METAL USER

A. General

The same user sector was used for both models (aluminum and copper). The equations differ only in some symbols and in the values of the constants. Symbolic differences will be noted in the flow diagrams of Figures 19A and 19B. The sector will be described under the following structures and policies:

1. Production Policy
2. Production Structure
3. Purchasing Policy
4. Distribution Structure
5. Initial Conditions

Symbols used are from the copper model.

B. Production Policy

The production policy of the user is similar to that in the fabricator and production desired (UPD) is based upon smoothed sales (USS) and an inventory adjustment (UFGID-UFGI).

\[ \text{UPD.K} = \text{USS.K} + \left( \frac{1}{U4} \right) (\text{UFGID.K} - \text{UFGI.K}), \text{lb/wk} \]

\[ U4 = 4, \text{wk} \]

Desired production is limited by the raw material production limit (UPRML) and the plant capacity (UMPP) to determine the actual production started (UPS).
UPRML.K = URMI.K/3, (lb/wk)

UPDT.K = CLIP (UMPP, UPD.K, UD.K, UMPP), (lb/wk)

UMPP = 327,500,000, (lb/wk)

UPS.KL = CLIP (UPD.K, UPRML.K, URMI.K, URMIM), (lb/wk)

URMIM = 131,000,000, (lb/wk)

C. Production Structure

A conventional goods-in-process-inventory (UGIP) is shipped (UGTFI) after a third-order delay (DELI) to finished goods inventory (UPGI).

UGIP.K = UGIP.J + (DT) (UPS.JK - UGTFI.JK), (lb)

UGTFI.KL = DELAY3 (UPS.JK, DELI), (lb/wk)

DELI = 1.2, (wk)

D. Purchasing Policy

In the original single flow model, the industry competed in price and delivery time with a static competitor with a fixed price and delivery time. In the competitive model, however, a dynamic competition in both price and delivery time is conducted between the two parts of the industry.

The raw material purchases desired (URMPD) is based upon current production (UPDT) and raw material inventory requirements (URMID - URMI).

URMPD.KL = UPDT.K + (1/U7) (URMID.K - URMI.K), (lb/wk)

U7 = 8, (wk)
A level of desired purchases (ULRMP) is maintained with division of purchases (UCPDI) dependent on price (RCP, R1CP) and delivery time (POSD, F10SD).

The price comparison is based upon the average prices (UCPS, ULCPS) of the competing sectors of the industry.

\[
\text{UCPS}.K = \text{UCPS}.J + (DT) \left( \frac{1}{U5} \right) (\text{RCP}.J - \text{UCPS}.J), \ ($/lb) \ 0117,L
\]

\[
\text{ULCPS}.K = \text{ULCPS}.J + (DT) \left( \frac{1}{U5} \right) (\text{R1CP}.J - \text{ULCPS}.J), \ ($/lb) \ 1117,L
\]

\[
U5 = 12, \ (wk) \ 0435,C
\]

A long run price ratio is formed (ULRPR) and the trend of this ratio (UPDIF) is determined by successive differencing. The remainder of the sequence is similar to that used in the investment policy (MLRSA) in the mine sector, with the final price ratio (PRATO) similar in form to MLRSA.

\[
\text{ULRPR}.K = \frac{\text{UCPS}.K}{\text{ULCPS}.K} \ 1139,A
\]

\[
\text{UPRF*1}.K = \text{UPRF*1}.J + (DT) (\text{ULRPR}.J) \ 4138,L
\]

\[
\text{UPDIF}.K = \text{UPRF*2}.K - \text{UPRF*3}.K \ 2138,A
\]

\[
\text{UPRF} = \text{BOLSIN} (3,1) \ 3138,B
\]

\[
\text{UPRAP}.K = \text{CLIP} (0, \text{UPDIF}.K, 0, \text{UPDIF}.K) \ 0138,A
\]

\[
\text{UPRAN}.K = \text{CLIP} (0, \text{UPDIF}.K, \text{UPDIF}.K, 0) \ 1138,A
\]

\[
\text{UPAF1}.K = \text{CLIP} (0, \text{UPRAP}.K, U11, \text{UPRF*2}.K) \ 0137,A
\]

\[
\text{UPAF2}.K = \text{CLIP} (0, \text{UPRAN}.K, \text{UPRF*2}.K, U12) \ 1137,A
\]

\[
U11 = 1.03 \ 0448,C
\]

\[
U12 = 0.97 \ 0449,C
\]

\[
\text{UPA}.K = \text{UPA}.J + (DT) \left( \frac{1}{U10} \right) (\text{UPAF1} J + \text{UPAF2}.J) \ 0136,L
\]
ULRCP.K = TABLE (UFUN,PRATO.K,4,2,.1) 0115,A
PRATO.K = 1 + UPA.K 0116,A

ULRCP - Long Run Copper Purchases

The copper to be purchased from the F fabricator (ULRCP) is then modified by a short run modifier (USRCRP) based upon the ratio of the competitive delivery times to determine UPCP.

\[
UPCP.K = (ULRCP.K) (USRCRP.K) (U30)
\]

USRCRP.K = FLOSD.K/FOSD.K 1118,A

\[
U30 = 0.3 \text{ ("F" 15% of industry)}
\]

UCPDI.KL = (ULRMP.K) (UPCP.K), (lb/wk) 3449,L

UCPDI is the copper to be purchased from the F fabricator. The remainder of the purchases (USP) are ordered from the Fl fabricator.

\[
USP.KL = ULRMP.K - UCPDI.JK, \text{ (lb/wk)}
\]

0132,R

A level of F purchases (ULCPD) is maintained.

\[
ULCPD.K = ULCPD.J + (DT) (UCPDI.JK - UCO.JK)
\]

0133,A

UCO - Copper Ordered from F fabricator.

The actual copper ordered from F is affected by the pipeline inventory required (URPLI) and actual (UAPLI) and the average copper ordered (UCOA).

\[
UCO.KL = ULCPD.K + (1/8) (URPLI.K - UAPLI.K), \text{ (lb/wk)}
\]

0124,R

URPLI.K = (UCOA.K) (DEL4 + DEL5 + FOSD.K), (lb) 0125,A

UAPLI.K = ULCT.K + FUFO.K + ULCO.K, (lb) 0134,A

ULCT - Copper in Transit from F

FUFO - Fabricator F Unfilled Orders
ULCO - Delayed Copper Orders (see below)

$$\text{UCQA.K} = \text{UCQA.J} + (\text{DT})(1/\text{UL8})(\text{UCO.JK} - \text{UCQA.J}), \text{ (lb}/\text{wk})$$ 0126, A

$$\text{UL8} = 12, \text{ (wk)}$$ 0437, C

Orders are delayed in a level (ULCO) and after a third-order delay (DELI) are sent to the fabricator as sales orders (FSO).

$$\text{ULCO.K} = \text{ULCO.J} + (\text{DT})(\text{UCO.JK} - \text{FSO.JK}), \text{ (lb)}$$ 0127, L

$$\text{FSO.KL} = \text{DELAY3} (\text{UCO.JK}, \text{ DE}LI)$$

$$\text{DE}LI = 1.2, \text{ (wk)}$$ 0417, C

Orders to the substitute Fl sector (OFSM) are delayed (DELI2) and sent (DSM) as sales orders to the Fl fabricator.

$$\text{OFSM.K} = \text{OFSM.J} + (\text{DT})(\text{USP.JK} - \text{DSM.JK}), \text{ (lb)}$$ 0122, L

$$\text{DSM.KL} = \text{DELAY3} (\text{USP.JK}, \text{ DE}LI2), \text{ (lb}/\text{wk})$$ 0123, R

$$\text{DE}LI2 = 9.6, \text{ (wk)}$$ 0416, L

E. Distribution Structure

The distribution structure is similar to that of the copper refiner and has the same supporting rationale.

$$\text{UUF0.K} = \text{UUF0.J} + (\text{DT})(\text{USO.JK} - \text{UOS.JK}), \text{ (lb)}$$ 0100, L

$$\text{UFGI.K} = \text{UFGI.J} + (\text{DT})(\text{UGTPI.JK} - \text{UOS.JK}), \text{ (lb)}$$ 0101, L

$$\text{UOS.KL} = \text{CLIP} (\text{UOST.K, UFGIN.K, UFGIK, UFGIM}), \text{ (lb}/\text{wk})$$ 0102, R

$$\text{UFGIN.K} = \text{UFGI.K}/3, \text{ (lb)}$$ 0103, A

$$\text{UOST.K} = \text{UUF0.K}/\text{UOSD.K}, \text{ (lb}/\text{wk})$$ 0104, A

$$\text{UOSD.K} = (1/\text{UFGI.K}) ((\text{UL})(\text{UFGI.K}) + (\text{U2})(\text{UFGID.K}), \text{ (wk)}$$ 0105, A

$$\text{UFGID.K} = \text{(EC)} (\text{USSC})(\text{USS.K}), \text{ (lb)}$$ 0106, A

$$\text{USS.K} = \text{USS.J} + (\text{DT})(1/\text{U3})(\text{USO.JK} - \text{USS.J}), \text{ (lb}/\text{wk})$$ 0107, L
USO - User's Sales Orders

\[ UFGIM = 87,333,000, \ (lb/wk) \]
\[ U1 = 8.0, \ (wk) \]
\[ EC = 1 \]
\[ U3 = 12, \ (wk) \]
\[ U2 = 0.8, \ (wk) \]
\[ USSC = 8, \ (wk) \]

\[ URM\_K = URM\_J + (DT) \(\text{UCD}\_JK + DSM\_JK - UPS\_JK\), \ (lb) \]

\[ URMID\_K = (EC) (UPDT\_K) (UPDC) \]
\[ UPDC = 4, \ (wk) \]
\[ ULC\_K = ULC\_J + (DT) \(\text{FGS}\_JK - UCP\_JK\) \]
\[ UCD\_KL = \text{DELAY3} (\text{FGS}\_JK, \text{DEL5}) \]
\[ DEL5 = 2, \ (wk) \]

F. Initial Conditions

The initial condition equations follow:

\[ USO = USON \]
\[ UUFO = (USS) (U1+U2) \]
\[ UFGI = (EC) (USSC) (USS) \]
\[ USS = USO \]
\[ UPS = USO \]
\[ UGIP = (UPS) (DEL1) \]
\[ URMI = (EC) (UPDC) (UPS) \]
\[ URMFD = UPS \]
ULRMP = URMPD
UCPS = RCPN
UCPDI = (ULRMP) (UPCPN)
USP = ULRMP - UCPDI
ULCPD = UCPDI
UCO = UCPDI
ULCO = (UCO) (DEL4)
UCOA = UCO
OFSM = (USP) (DEL2)
ULCT = (FOS) (DEL5)
UOS = USO
UPA = 0
ULCPS = R1CPN
Appendix K

SUMMARY OF COPPER INDUSTRY
COMPUTER SIMULATION RUNS

1. RUN NUMBER 1348 KS
   a. Purpose:
      To provide a reference run of the response of the basic model to a pulse input.
   b. Changes to Model:
      None.
   c. Results:
      1) RCP rose to peak of $46\xi$ at $t=36.0$
      2) RFGI saturates at $t=252.0$
      3) Final RCP = $26\xi$.

2. RUN NUMBER 1385 KS
   a. Purpose:
      To determine the effect of a change in the mine supply curve on the response of the system to a pulse input.
   b. Changes to Model:
      Mine supply curve modified as shown in Figure 8.
   c. Results:
      1) RCP rose to peak of $38\xi$ at $t=32.0$
      2) Large increase in mine output led to excessive supply.
         RFGI saturated at $t=68.0$
      3) Final RCP=$24\xi$. 
3. **RUN NUMBER 1420**

   a. **Purpose:**
   
   To determine the effect of changing the mine price smoothing time on the response of the system.

   b. **Changes to Model:**
   
   1) M1 decreased from 24 weeks to 12 weeks

   c. **Results:**
   
   1) Initial response was essentially unchanged.

   2) Final RCP = 26f

4. **RUN NUMBER 1421**

   a. **Purpose:**
   
   To determine the effect of increasing the speed of response of the mine to a change in price.

   b. **Changes to Model:**
   
   M2 increased to 0.0375 from 0.0300.

   c. **Results:**
   
   1) Change was not large enough to affect response.

   2) Larger change was incorporated in later run.

5. **RUN NUMBER 1422**

   a. **Purpose:**
   
   To determine the effects of a decreased limit on mine production

   b. **Changes to Model:**
   
   MMPP and MIMPP decreased to 67,000,000 lb/wk from 75,000,000 lb/wk.
c. Results:

Limit not low enough to affect response.

6. RUN NUMBER 1423 KS

a. Purpose:

To determine the effect of increasing the production time delay in the mine.

b. Changes to Model:

DEL 18 increased to 4.0 weeks from 1.2 weeks

c. Results:

1) RFQI saturated earlier at t=244.0

2) No other significant changes.

7. RUN NUMBER 1460 KS

a. Purpose:

To determine the effect of a decreased limit on smelter production.

b. Changes to Model:

SMPP decreased from 170,000,000 lb/wk to 134,000,000 lb/wk.

c. Results:

Limit not low enough to affect response.

8. RUN NUMBER 1461 KS

a. Purpose:

To determine the effect of changing the smelter production rule constant.
b. Changes to Model:
   Amount of raw material inventory maintained was reduced from 4 weeks to 2 weeks (S3=2 from S3=4).

c. Results:
   No significant change.

9. RUN NUMBER 1462 KS
   a. Purpose:
      To determine the effect of changing the smelter production time delay.
   b. Changes to Model:
      DELL3 is increased from 1.5 weeks to 4 weeks.
   c. Results:
      No significant change.

10. RUN NUMBER 1462 KS
    a. Purpose:
       To determine the effect of an unfilled order feedback from the mine to the smelter production rule for SPW.
    b. Changes to Model:
       SPW.K = SRMI.K/S3 changed to
       SPW.K = (1/S3) (SRMI.K-RUFO.K)
       S3 =8.
    c. Results:
       1) Model exploded due to incorrect sign on RUFO.K.
       2) Later run used RSS.K in preference to RUFO.K.
11. RUN NUMBER 1560 KS

a. Purpose:

To determine the effects of a change in weighting of the static and rate factors to pricing.

b. Changes to Model

1) R5 (Static Weight Factor) was decreased from 4 to 2.
2) R6 (Rate Weight Factor) was increased from 2 to 8.

c. Results:

1) System stability was significantly increased!
2) RCP rose to only 36¢ at t=36.0.
3) Final RCP=30¢
4) RFGI did not saturate.

12. RUN NUMBER 1561 KS

a. Purpose

To determine the effect of a price ceiling of 30¢ on system response.

b. Changes to Model:

1) RCP.K = CLIP (.30, R1CP.K,R1CP.K, .30)
2) R1CP.K = TABLE (RFUN,RWRAT.K, .2, .3, .5, .1)

c. Results:

1) System stability was increased
2) RCP dropped to low of 28¢ at t=56.0.
3) RFGI did not saturate.
4) Final RCP=30¢.
13. RUN NUMBER 1562 KS

a. Purpose:
To determine the effect of using an average of production rates from all the sectors instead of refiner production rate alone in pricing.

b. Changes to Model:
1) \( RRSD.K = RSS.K/IAPS.JK \)
2) \( IAPS.KL = (1/4) (MTPS.JK + CPS.JK + SPS.JK + RPS.JK) \)
3) \( MTPS.KL = MPS.JK + MLP.SJK. \)

c. Results:
1) RFGI saturated later at \( t=324.0 \).
2) Initial response change was not significant.

14. RUN NUMBER 1563

a. Purpose:
To determine the effect of a refiner's smoothed sales feedback to the smelter production rule.

b. Changes to Model:
1) \( SPW.K = (1/83) (SRMI.K + RSS.K) \)
2) \( S3 = 8 \)
3) \( M2 = 0.06, M12 + 0.06 \) (mine speed of response).

15. RUN NUMBER 1564

a. Purpose
To determine the effect of cost-plus pricing.

Rescheduled in a later run.
16. RUN NUMBER 1565

a. Purpose:

To determine effect of increasing reinker production time delay.

b. Changes to Model:

1) DEL 10 increased to 10.0 weeks from 4.0 weeks
2) M2=0.06, M12=0.06.

c. Results:

RFGI saturated later as a result of better mine response.

17. RUN NUMBER 1566

a. Purpose:

To determine the effect of decreased sales smoothing time.

b. Changes to Model:

R3 = 4 weeks from R3 = 12 weeks.

c. Results:

RFGI saturated earlier at \( t=508.0 \).

18. RUN NUMBER 1567

a. Purpose:

To determine the effect of a lower reinker production limit.

b. Changes to Model:

RMFP lowered to 132,000,000 lb/wk from 162,000,000 lb/wk.

c. Results:

1) RFGI saturated earlier at \( t=300.0 \)
2) Initial response was not affected significantly.
19. **RUN NUMBER 1568 KS**

   a. **Purpose:**

      To determine the effect of a change in the refiner production factor \((R_4)\).

   b. **Changes to Model:**

      \(R_4=1\) from \(R_4=3\) (number of weeks in RRMI).

   c. **Results:**

      RFGI saturates later at \(t=316.0\).

20. **RUN NUMBER 1569 KS**

   a. **Purpose:**

      To determine the effect of a change in the level of the refiner's desired finished goods inventory.

   b. **Change to Model:**

      RSSC (sales in RFGI) decreased from 5 weeks to 2 weeks.

   c. **Results:**

      1) Marked decrease in system stability was observed!

      2) RCP rose to 62\$/ instead of 46\$/.

      3) Has effect of increasing static factor in pricing by decreasing RFGI level.

      4) RFGI saturated at \(t=128.0\).

21. **RUN NUMBER 1570 KS**

   a. **Purpose:**

      To determine the effect of a change in the pricing function curve \((RFUN)\).
b. Changes to Model:
   RFUN function modified.

c. Results:
   1) System is very sensitive to this function!
   2) Instability increased.

22. RUN NUMBER 1939 KS

   a. Purpose
      To test response of model to a noise input.

   b. Changes to Model:
      1) Noise input to USO replaced pulse
      2) rl=0/12=0/13=1.

   c. Results:
      1) Price and production stable.
      2) Inventory oscillates.
      3) Input is not typical of history of industry.

23. RUN NUMBER 1940 KS

   a. Purpose:
      To determine the effect of changing the concentrator produc-
      tion rule factor (C3).

   b. Changes to Model:
      No significant change.

25. RUN NUMBER 1941 KS

   a. Purpose:
      To test a new concentrator-smelter production policy emphasize
proportional plus integral to replace pure integral control.

b. Changes to Model:
   1) \[ CPW.K = \frac{1}{C3} (CRMI.K + MCD.JK + MLCP.JK) \]
   2) \[ SPW.K = \frac{1}{S3} (SRMI.K + SCD.JK) \]

c. Results:
   1) System stability was increased
   2) RCP rose to only \(3^{1/2}\) at \(t=32.0\).
   3) RFGI was stable.
   4) \text{Final RCP} = 29.76.

26. RUN NUMBER 1938A KS

a. Purpose:
   To determine effects of changes in the purchasing and production policies of the fabricator.

b. Changes to Model:
   1) \(FSSC = 4\) from \(FSSC = 8\)
   2) \(F6 = 4\) from \(F6 = 8\)
   3) \(F5 = 2\) from \(F5 = 5\)
   4) \(F4 = 6\) from \(F4 = 12\).

c. Results:
   1. RFGI did not saturate.
   2. No significant effects on initial response were observed.

27. RUN NUMBER 1938B KS

a. Purpose:
   To change exogenous inputs to system to include production losses from strikes 1955-1960.
b. Changes to Model

Losses in mine production put in as step inputs to mine.

c. Results:

Inputs to correctly inserted at twice correct value.

28. RUN NUMBER 2048 KS

a. Purpose:

Same as 1938B with high inventories when strikes began.

b. Changes in Model

1) UPDC = 26
2) RSSC = 8

c. Results:

No significant change from 1938B.

29. RUN NUMBER 2049 KS

a. Purpose:

To modify strike inputs to test effects and correct errors in 1938B and 2048.

b. Changes to Model:

1) 1955 strike losses in mine production were inserted.
2) Stable Price Policy R5=2; R6=8.

c. Results:

System stable even with strikes, using stable price policy.

30. RUN NUMBER 2051 KS

a. Purpose

To simulate 1955-1960 consumption of copper as inputs to the model.
b. Changes to Model


c. Results:

Higher frequency response than history of industry indicates.

31. RUN NUMBER 2052A and B

a. Purpose

Same as 2051.

b. Changes to Model:

R5=2, R6=3 (Unstable Pricing Policy)

R3=52

R8=0.25.

c. Results:

Errors in inputs discovered. Demand continued through 1955-1956.

32. RUN NUMBER 2053

a. Purpose:

To obtain inputs typical of 1955-1960 period for final evaluation of policy.

b. Changes to Model:

1) Average annual consumption of copper used for inputs 1955-1960.

2) R3=24, R4=1, F5=1.
c. Results:

1) Response of system very similar to history of 1955-1960 period.

2. System is now ready for final test of policies.

33. RUN NUMBER 2054 KS

a. Purpose:

To test the effectiveness of the new pricing policy in stabilizing the copper industry with inputs similar to those encountered in the 1955-1960 period.

b. Changes to Model:

1) R5=2; R6=8 (Stable Pricing Constants)

2) Cost-Profit functions were inserted.

c. Results:

1) System was stabilized!

2) Price rise was limited to 38¢ instead of 56¢ as in 2053.

34. RUN NUMBER 2055 KS

a. Purpose:

To test the effectiveness of the new production policy.

b. Changes to Model:

1) Unstable price policy reinstated  R5=8, R6=2.

2) \[
    \text{SPW}.K = (\text{SRMI}.K/S3) + \text{RSS}.K
\]

\[
    \text{CPW}.K = (1/C3) (\text{MCP}.JK + \text{MLCP}.JK) + (1/C3) (\text{SRMID}.K - \text{SRMI}).K
\]

\[
    \text{SRMID}.K = (S3) (\text{RSS}.K)
\]

C3 = 2
S3 = 8

\[ \text{RPD}.K = \left( \frac{1}{R13} \right) (\text{RSS}.K + 0) + \left( \frac{1}{R12} \right) (\text{RFGID}.K - \text{RFGI}.K) \]

\[ R13 = 1 \]

c. Results:

1) Industry is stabilized in price.
2) Production variations excessive.

35. RUN NUMBER 2056 KS

a. Purpose:

To test the effectiveness of a combination of the new price and production policies.

b. Changes to Model:

Combination of 2054 and 2055.

c. Results:

Improved price stability.

36. RUN NUMBER 2057A KS

a. Purpose:

To combine a mine ore yield selection policy with the price price and production policies of 2056.

b. Changes to Model:

\[ M31 = 0 \text{ from } M31 = 1 \]
\[ M32 = 1 \text{ from } M32 = 0 \]
\[ M34 = 0 \text{ from } M34 = 1 \]
\[ M35 = 1 \text{ from } M35 = 0. \]

Policy mines high grade with higher prices.
c. Results:
   1) Price less stable than 2056.
   2) Production is similar.

37. RUN NUMBER 2058
   a. Purpose:
      To test the effectiveness of cost-plus pricing.
   b. Changes to Model:
      \[ RCP.K = RUCOS.K + 0.03 \] (3¢ above unit cost at all times).
   c. Results:
      1) Price is more stable.
      2) Production is same as 2056.

38. RUN NUMBER 2059
   a. Purpose:
      To test the dual flow competitive model.
   b. Changes to Model:
      1) Model changed to have dual mine, concentrator, smelter, refiner and fabricator sector competing in price and delivery time for user sales.
      2) One-half of model is following new policies and the other half the normal policies leading to present availability.
   c. Results:
      1) Model operated satisfactorily except that price of unstable sector saturated. It will be limited on the next run.
2) Changes Indicated:

A. Mistake in CFW.K (remove MLPK)

B. Change R13 to constant less than 1.0 in R1FD.K.

C. Change Production Policies for quarterly review
   rather than continuous change.

D. Limit CPW and SFW to raw material inventory available.

39. RUN NUMBER 2115

a. Purpose:

   To test the dual flow competitive model (50-50% split)

b. Changes to Model:

   1) RFUN curve was extended to permit higher price calculations.
   2) R21 = 0.25 from R21 = 1

   (R21 price sensitivity constant).

c. Results:

   1) Stabilization capability of 50 per cent of industry was
      demonstrated! Both halves of industry more stable than
      in 1955-1960 period as previously simulated in RUN 2053.
   2) Profit of both halves about equal, with inventory carrying
      costs lower in "new policy" half.

40. RUN NUMBER 2116

a. Purpose:

   To test the dual flow competitive model with the new policy
   part holding but 15 per cent of the market.
b. Changes to Model:

1) New policy part rescaled to 15 per cent of market firm was demonstrated! Firm and industry more stable than in RUN 2053.

2) Total industry profit - 77.18

\[ TPROF = 7.61 \]
\[ T1PRO = 69.57 \]

\[ TPROF/Total = 9.9\% \]
\[ TOINV = 650.15 \]
\[ T01IN = 2885.20 \]

\[ TOINV/Total = 18.4\% \]

41. RUN 2117 KS

a. Purpose:

To test a reverse ore yield (high yield with high price) policy in the competitive model.

b. Changes to Model:

1) M31=0 M34=0

M32=1 M35=1

2) R12=52 Inventory adjustment time, R refiner.

c. Results:

1) Little change in stability noticed.

2) Total industry profit = 68.41.

\[ TPROF = 7.93 \]
\[ T1PRO = 60.48 \]
TPROF/Total = 11.6%
TOINV = 721.84
TOLIN = 4124.70.

41. RUN 2118

a. Purpose:
   To test a new pricing policy.

b. Changes to Model:
   1) R5 = 1, R6 = 10

c. Results:
   1) TPROF = 5.79
      T1PRO = 56.89
      TOINV = 701.55
      TOLIN = 3276.00.

43. RUN NUMBER 2119

a. Purpose:
   To change the competitive price margin (RPMAR) from 5¢ to 2¢.

b. Changes to Model:
   1) R3CP.K = RCP.K + RPMAR
   2) RRMAR = 0.02
   3) h.70 = 52 (Mine inventory adjustment eliminated).

c. Results:
   1) TPROF = 5.82
      T1PRO = 43.40
      TOINV = 703.15
      TOLIN = 2883.20.
44. RUN NUMBER 2120

a. Purpose:
   To test a new mining production policy.

b. Changes to Model:
   1) M62 = 0.15 was M62 = 0.30
      Less emphasis on price (RCP)

c. Results:
   1) TPROF = 5.76
      T1PRO = 56.78
      TOINV = 727.34
      TOLIN = 2022.80.

45. RUN NUMBER 2121

a. Purpose:
   To observe the effect of a fabricator sales feedback (FSS)
   and an increase in service inventory (FSSC).

b. Changes to Model:
   1) FSS.K feedback to MPD,CPW,SPW and RPW instead of RSS.K.
   2) FSSC increased from 5 to 20 weeks.
   3) RPMAR = 0.01
   4) Lower early sector inventories
      R4=3 from R4=5
      S3=2 from S3=4
      C3=0.5 from C3=4
      C4=0.5 from C4=0.1.
c. Results:

1) Price instability was increased. FLAP saturated after 100 weeks.

2) TPROF = 3.91
   TlPRO = 25.40
   TOINV = 886.03
   TOLIN = 2022.80.
Appendix L

SUMMARY OF ALUMINUM INDUSTRY

COMPUTER SIMULATION RUNS

1. RUN NUMBER 1937A

   a. Purpose:

      To test the model after initial construction and debugging
      with a pulse input.

   b. Changes to Model:

      None (original model).

   c. Results:

      1) There was indication of a system with stability in price
         and production and varying inventories corresponding with
         the actual history of the industry.

2. RUN NUMBER 1937B

   a. Purpose

      To modify the original model to reflect periodic (quarterly)
      rather than continuous control of production.

   b. Changes to Model:

      1) \[ RFD.K = \text{SAMPLE} (RLPD.K,12) \]

         \[ RLPD.K = \left(\frac{1}{R13}\right) (RSS.K+0) + \left(\frac{1}{R12}\right) (RFGID.K - RFGI.K) \]

      2) \[ MPD.K = \text{SAMPLE} (MLPD.K,12) \]

         \[ MLPD.K = RSSF.K + \left(\frac{1}{R16}\right) (RFGID.K - RFGI.K). \]

   c. Results:

      1) Greater instability in inventories and production.

      2) Average inventory level was increased.
3. RUN NUMBER 1937C
   a. Purpose:
      To test the effects of extending the production planning period from **quarterly to semi-annually**.
   b. Changes to Model:
      \[ RFD.K = SAMPLE (R1FD.K, 26). \]
   c. Results:
      1) Instability of system was further increased.
      2) Average inventory level was further increased.

4. RUN NUMBER 2046
   a. Purpose:
      To test the **response of the model to a noise input**.
   b. Changes to Model:
      1) Noise input replaced pulse input.
   c. Results:
      1) Mine production rate and inventory oscillated greatest. Price was stable.
      2) Typical input of industry.

5. RUN NUMBER 2050
   a. Purpose
      To test the **new competitive dual model**.
   b. Changes to Model:
      1) Dual model with two parallel competing material flows was constructed.
2) All time delays set at 1.2 weeks minimum.

c. Results:

1) System was less stable than in noncompetitive single
   flow model, especially in price.

6. RUN NUMBER 2125

a. Purpose:

   To determine the effectiveness of the new production policy.

b. Changes to Model:

   1) R1PD.K = CLIP (R3PD.K,R2PD.K,ICOST.K,PCOST.K)
   2) ICOST.K = (NINV.K)(.20)(.05)
   3) PCOST.K = (24)(RPS.JK - R3PD.K)
   4) NINV.K = (26)(RPR.K - RSS.K)
   5) R3PD.K = (1/13)(RSS.K) + (1/12)(RFGID.K - RFGI.K)
   6) R2PD.K = RPR.JK/1.

c. Results:

   Profits of new policy half exceeded normal share.

7. RUN NUMBER 2126

a. Purpose:

   To test the model with a 30 per cent share company following
   the production policy.

b. Changes to Model:

   Model scaled to a 30-70 split.

c. Results:

   Profits of initiating company increased when profit equation
   error was taken into account.
8. RUN NUMBER 2127
   
a. Purpose
   
   To test a new pricing policy for 30 per cent share company.
   
b. Changes to Model
   
   1) $F_{10} = 1 \quad F_{11} = 8$ (F pricing policy)

   from

   $F_{10} = 4 \quad F_{11} = 2$

   2) $F_{20} = 2 \quad F_{21} = 4$ (F1 pricing policy)

   from

   $F_{20} = 4 \quad F_{21} = 2$

   
c. Results:

   1) Profits of company rose 20 per cent above normal.

   2) Inventory costs of company decreased 15 per cent below normal.

9. RUN NUMBER 2128
   
a. Purpose
   
   To test a new pricing policy for 30 per cent share company.
   
b. Changes to Model:

   $F_{13} = 1.0$

   $F_{23} = 0.25 \quad \text{was} \quad F_{23} = 1.0.$

   
c. Results:

   1) Profits of company dropped 8 per cent below normal.

   2) Inventory costs rose 15 per cent above normal.
10. RUN NUMBER 2129

a. Purpose

To test a new service policy.

b. Changes to Model:

FSSC = 20 weeks was
FSSC = 5 weeks

Higher Fabricated Inventories.

c. Results:

1) Profits of company increased 3 per cent above normal.

2) Inventory costs increased over 100 per cent above normal, offsetting increase in operating profit.

11. RUN NUMBER 2130

a. Purpose:

To test new service policy.

b. Changes to Model:

\[ PCOST.K = (1.1) \left( RPS.JK - R3FD.K \right) \]
FSSC = 20
F23 = 0.25.

c. Results:

Similar to those of 2120.
A. General

The objective of a distribution policy is to move materials and final products at the lowest cost consistent with service. Distribution policy, however, must be consistent with production and inventory policies, so that the total costs of producing, storing and moving are minimized with a certain standard of service.

For any company, the total distribution function is a network of mines, plants, warehouses and alternate routes. At any one time, depending on sales requirements, there will be an optimal pattern of production and shipments in the network, allowing for the constraints of destination needs and source capabilities.

To demonstrate the dynamic nature of distribution policy in the system, a function encompassing the production, inventory carrying and transportation costs was developed for a simple two-source, two-destination dual channel network. The total cost (TCOST) was the sum of production costs (P1C0S, P2C0S) and inventory costs (I1C0S, I2C0S) for each of the plants, and transportation costs for each of the routes (TRC01, TRC02, TRC03, TRC04). The solution was determined in the Dynamo Compiler with a modified version of the transportation method of linear programming. With a two-by-two matrix, the solution was quite elementary. For a multiple source-destination network, more extensive programming analysis will be required.
There is little doubt that a dynamic production-distribution-inventory policy will be a key factor in the aluminum industry development.
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