COPPER MARKET FLUCTUATIONS
AN INDUSTRIAL DYNAMICS STUDY

by

Ray Wayne Ballmer
B.S., New Mexico School of Mines
1949

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

at the

Massachusetts Institute of Technology
1960

Signature of Author

School of Industrial Management

Certified by

Faculty Advisor of the Thesis
Mr. Ray W. Balmer
Room 52-455
M. I. T.

Dear Mr. Ballmer:

This letter is to give you permission to print additional copies of your thesis by the Multilith process, and to submit to the School of Industrial Management copies thus produced, in lieu of the typed copies normally required.

A copy of this letter is to be reproduced by the same process and is to be placed in each copy of the thesis immediately following its title page.

Sincerely yours,

Richard B. Maffei
Thesis Supervisor
School of Industrial Management
May 16, 1960

Professor Philip Franklin
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Franklin:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "Copper Market Fluctuations An Industrial Dynamics Study".

I wish to express my gratitude to the many members of the faculty for their guidance and helpful suggestions in preparing this study. In particular I would like to express my appreciation for the council and assistance rendered by Professor Jay W. Forrester, chairman of my thesis committee. This work was done in part at the M.I.T. Computation Center. I am grateful for the use of this facility and for the assistance of its personnel.

I should also like to thank the men associated with the copper industry who so generously contributed their time, knowledge and experience. Without the assistance of these gentlemen, many of whom preferred not to be identified, the study could not have been effectively pursued.

I also want to express my appreciation for the help and encouragement provided in so many countless ways by my wife Doris.

Sincerely,

Ray Wayne Ballmer
COPPER MARKET FLUCTUATIONS: AN INDUSTRIAL DYNAMICS STUDY

by

Ray Wayne Ballmer

Submitted to the School of Industrial Management on May 16, 1960 in partial fulfillment of the requirements for the degree of Master of Science in Industrial Management.

ABSTRACT

This study was undertaken with two objectives in mind: the desire to study the behavior of the copper market, and the desire to evaluate Industrial Dynamics as a method for studying this type of problem.

Small deficiencies or excesses in supply historically have resulted in wide fluctuations in copper price. These wide fluctuations are believed to be detrimental to the long-term growth of the copper industry.

Industrial dynamics is a dynamic system study method being developed at the School of Industrial Management of the Massachusetts Institute of Technology by the staff of the Industrial Dynamics Department under the direction of Professor Jay W. Forrester. Industry simulation is obtained through the use of an IBM 704 digital computer and a mathematical model constructed to represent the system under study.

A mathematical model representing the general behavior of the aggregate primary copper industry is presented. This model contains six major sectors one each representing the aggregate copper user, the aggregate fabricator, the aggregate smelter, the aggregate concentrator, and the aggregate miner. The model contains systems representing the flow of orders, materials and information.

A brief description of model construction is provided to show the adoption of study method and industry characteristics.

Conclusions are reached in two areas, first the study method and second, the industry based on simulation.
Industrial Dynamics as such is believed to be a powerful tool for examining the alternative risks and expectations of business enterprise. Much can be learned from model construction in itself, while simulation provides an abundance of information about the simulated system.

Conclusions based on simulation are valid with respect to the model, but it would not be correct to project these conclusions directly to the industry. While an attempt was made to construct the model in the image of the aggregate industry, more work is necessary to verify or to achieve adequate representation.

Two methods for achieving some degree of price stabilization are demonstrated. The first method is to change the mining policy for part of the industry. The changed policy required the mining of higher grade ore during periods of high price and mining of lower grade ore during periods of low price.

Time was not available to explore the second method except in the crudest fashion. Here refiner production was planned in an attempt to maintain a constant relation between refiners' finished goods inventory and refiners' unfilled orders. While there are many obvious deterrents to such a policy, the strength of the stabilizing force achieved was so strong that further investigation is indicated.

Further study of copper market fluctuations with Industrial Dynamics as the study method is recommended.

This work was done in part at the M.I.T. Computation Center, Cambridge, Massachusetts.

Thesis Advisor: Jay W. Forrester
Title: Professor of Industrial Management
TABLE OF CONTENTS

Letter of Transmittal ........................................ iii
List of Tables .................................................. ix
List of Illustrations ........................................... ix

CHAPTER I

INTRODUCTION .................................................. 1
  Purpose ....................................................... 2
  Study Method ................................................ 3
  Major Conclusions ......................................... 3

CHAPTER II

THE PRIMARY COPPER INDUSTRY ............................... 6
  Structure of the Primary Industry ....................... 7
  Marketing of Copper ........................................ 12
  Competition .................................................. 14

CHAPTER III

INDUSTRIAL DYNAMICS ........................................ 16
  Industrial Dynamics ......................................... 16

CHAPTER IV

THE COPPER USER ............................................. 21
  Sales, Shipments, and Finished Goods Inventories .......... 22
  Production ................................................... 27
  Raw Material Inventory and Purchase Decision ............... 28
  The Where to Purchase Decision ............................ 29

CHAPTER V

THE COPPER FABRICATOR ..................................... 34
  Sales, Shipments, and Finished Goods Inventory ............. 35
  Production .................................................... 36
  Raw Material Inventory and Purchase Decision ............... 38
CHAPTER XII

CONCLUSIONS ................................................. 86
Industrial Dynamics as a Study Method ................. 86
Industry Conclusions ..................................... 87
Recommendations for Further Study ..................... 91

Appendix A. Mathematical Representation of the Primary Copper User ................................. 93
Appendix B. Mathematical Representation of the Aggregate Fabricator .................................. 101
Appendix C. Mathematical Representation of the Aggregate Refiner ...................................... 106
Appendix D. Mathematical Representation of the Aggregate Smelter ................................... 109
Appendix E. Mathematical Representation of the Aggregate Concentrator ............................. 111
Appendix F. Mathematical Representation of the "M" Mine Sector ........................................ 113
Appendix G. Mathematical Representation of the "M1" Mine Sector ...................................... 116
Appendix H. Changes to the Refiners Sector ................................................................. 121

Bibliography ..................................................... 123

viii
List of Tables

1. Annual Copper Production by Countries of Origin . . . 11
2. World Copper Consumption . . . . . . . . . . . . . . . 12

List of Illustrations

Figure 1. Comparison of Refined Copper Prices . . . . 13
Figure 2. Flow Diagram of the Copper User . . . . 23
Figure 3. Variable Shipping Delay . . . . . . . . . . 26
Figure 4. Users Purchase Relationship . . . . . . . . . 30
Figure 5. Flow Diagram of the Aggregate Fabricator . . . 37
Figure 6. Flow Diagram of the Aggregate Refiner . . . 44
Figure 7. Copper Price and Supply Demand Relationship . 46
Figure 8. Variable Shipping Delay . . . . . . . . . . . . 50
Figure 9. Flow Diagram of the Aggregate Smelter . . . 51
Figure 10. Flow Diagram of the Aggregate Concentrator . 52
Figure 11. Mine Supply Curve . . . . . . . . . . . . . . 61
Figure 12. Flow Diagram of the "M" Mine Sector . . . 63
Figure 13. Flow Diagram of the "M1" Mine Sector . . . 65
Figure 14. Basic Model Response to Pulse Input . . . 80
Figure 15. Modified Model Response to Pulse Input
          New Mining Policy for "M1" Mine Sector . . . 82
Figure 16. Price Stabilization by Planned Refiner Pro-
          duction . . . . . . . . . . . . . . . . . . . . . . . 84
CHAPTER I

INTRODUCTION

The wide fluctuation in world copper price is the central problem for this study. Copper historically has passed from periods of short supply and relatively high price to periods of long supply and relatively low price. The nature of this problem is indicated by the following quotation from the U.S. National Resources Board.

Economic stability is of peculiar importance to the non-ferrous metals. They are especially subject to and they suffer from wide variations of price. They need, above all things, to balance supply and demand, to avoid needless expansion of capacity, and to temper the extremes of price fluctuations, whether sudden advances or violent decline.

The above statement made in 1935 could well have been made in the year 1960 for industry leaders are still faced with the problem of avoiding "boom or bust" gyrations.

Mr. Jean Vuillequez, Vice President of American Metal Climax, Inc., in a recent article, said:

Much has been said about the volatility of the price of copper. In the past it appears that relatively small excesses of production or deficiencies in supply resulted in wide price swings. Undeniably such price fluctuations have been a deterrent to the growth of consumption. However, some of the past price fluctuations were due to causes beyond the control of the industry.

---


Chapter I

The problem of minimizing price fluctuation is an important one for the industry. Mr. Vuillequez, in stressing its importance, said:

The essential ingredients to continued growth in copper consumption are there, but we must find the right recipe if we are to produce a palatable future. Above all, we must try to avoid exaggerated prices, ..., which inevitably produce depression prices. Copper must minimize shortages that stunt its growth and overproduction that discourages the capital needed for long-term development.

Purpose

The industry has in the past been subject to periods of heavy demand that could not be anticipated. These demands (for example, World War II and the Korean War) resulted in shortages that ultimately encouraged copper consumers to find substitute materials. Mr. Vuillequez's article suggests however, that perhaps some of the past price fluctuations may have been subject to control by the industry if the right control method(s) had been known.

The central purpose of this study is to examine industry behavior in an attempt to find some of the internal behavioral mechanisms that tend to cause wide price swings from relatively small excesses in supply and demand. One area to be explored is the selection of basic mining policy. For example, should the long-term economic definition of ore be modified by short-run economic conditions? A secondary purpose of the study is to evaluate Industrial Dynamics as a study method.

3Ibid.
Chapter I

Study Method

Industrial Dynamics is a dynamic systems study method being developed by the Industrial Dynamics Department of the School of Industrial Management at the Massachusetts Institute of Technology. In brief, Industrial Dynamics provides a method of obtaining simulation for determining industry behavior under various conditions. Information describing industry behavior is generated by a digital computer. Computer speed is such that simulation can readily be obtained for different conditions imposed on the mathematical model used to represent the industry.

Industrial Dynamics appeared to be a useful method for examining some of the decision-making functions of the copper industry and was thus chosen as the study method. Although time would not permit detailed consideration of all the factors bearing on the copper industry, I felt the use of this method would provide me with a better understanding of industry behavior and perhaps provide useful information concerning possible alternative industry policies.

Major Conclusions

Conclusions to be drawn from this study fall into two categories, first those concerning the study method, and second those concerning the primary copper industry. The study method will be discussed first since the study method used bears heavily on the results achieved.
Chapter I

In my judgment, Industrial Dynamics can be a powerful management tool. Model construction, in itself, sharpens the awareness of the dynamic characteristics of business activity. Much can be learned about the nature of the industry through model construction. Simulation enables the researcher to evaluate the effect of present policy and the impact of proposed changes on the industrial system under study. With proper application, Industrial Dynamics can, I believe, greatly assist in the management of risk.

Major conclusions concerning the copper industry were reached on the basis of model construction and industry simulation. Conclusions are valid to the extent that the model represents the industry. No attempt was made to accurately portray all possible aspects of the industry, rather the attempt was to represent, in an aggregate model, those characteristics believed to be most important for defining industry behavior.

The introduction of a flexible mining policy, one that would permit the production of higher grade ore during periods of relatively high price and the mining of a lower grade ore during periods of relatively low price, succeeded in reducing price fluctuation to some extent.

A marked degree of price stabilization was achieved in the model by providing an industry stockpile of blister
CHAPTER II

THE PRIMARY COPPER INDUSTRY

The following information is presented to provide the reader with a general background of copper and the copper industry.

Man's first use of copper is not recorded. Copper occurred as a native metal in many parts of the world and could be used as such without metallurgical treatment. Though one of the earliest metals used by man, copper is still generally regarded as the most important of the non-ferrous metals.¹

Copper is virtually indestructible, a given mass of copper may be used over and over again without suffering chemical or physical deterioration. A recent estimate is that approximately 60 per cent of copper entering the economy will ultimately be recovered and used again. The cycle of copper's usefulness has been estimated to be approximately 35 years.² The indestructible properties of copper give rise to two general industries, the primary copper industry and the secondary copper industry.

The secondary industry employs scrap copper and copper bearing materials as a major source of raw materials


²Copper the Cornerstone of Civilization, Number 15 of the Copper and Brass Research Association, 420 Lexington Avenue, New York 17, New York, 1957, p. 18.
Chapter II

to produce castings, powdered metals, chemicals and other products that do not require refined copper.\textsuperscript{3}

The primary industry produces fabricated products from brass mills, copper wire from copper wire mills and other products requiring the use of refined copper. The major source of raw materials for the primary industry is from copper bearing ores.\textsuperscript{4}

A small amount of scrap copper is used as feed for certain segments of the primary industry. There are two main types of scrap material, new scrap from the processing of materials within the primary industry, and old scrap from discarded copper and copper bearing products. New scrap, or "run-around copper," is not credited as contributing to the total new supply of copper and is generally consumed within the primary industry. Old scrap on the other hand does add to the new supply of copper and is the main source of copper for the secondary industry.\textsuperscript{5}

Structure of the Primary Industry

The primary copper industry as an integrated system consists of various sectors classified by type of operation. These sectors include mining, concentrating, smelting, refining and fabricating.

\textsuperscript{3}Ibid., p. 19.
\textsuperscript{4}Ibid.,
\textsuperscript{5}National Security Resources Board, op. cit., p. S-5.
Chapter II

Mining is the process of removing copper bearing ores from the earth. This is generally accomplished by either underground methods or by open pit methods. The mining method, within these broad classifications, is usually dependent upon the nature and composition of the ore body and the state of technological development. Open pit methods continue to become more prominent with the continual development of techniques for moving large quantities of material at a low cost.⁶

Copper is so widespread in nature that almost every country has some ore deposits. In 1950, for example, 36 countries recorded some output, of these, 18 countries produced over 10,000 tons of recoverable copper. In spite of the wide dispersion of copper ores, the major part of the world's known copper is concentrated in a few places. About 90 percent of the world's unmined copper resources are located in four regions, south-central Africa, Chile, western United States, and Kazakhstan, U.S.S.R.⁷ In the long-run, however, discovery of new ore deposits in various parts of the world has caused a continual shifting in dominance among countries and deposits. Table 1 shows the annual production by countries for several selected years.

---

⁶ See, National Security Resources Board, op. cit., pp. II-1 through II-164. This work gives a good description of the primary copper industry.

Chapter II

Ore reserve figures are generally considered to be economically defined, that is, only that copper bearing material which can be mined and processed at a profit is considered ore. Informed guesses of the potential copper resources of the world show over 200 million tons of copper in 2 percent ore or better. In spite of the sporadic "finds" and the wide variations in ore bodies, one writer concludes that the long-run response of supply to demand is relatively uniform.  

Concentrating is a physical process for removing some of the non-copper bearing material from copper ore. Normally this function is performed at or near the mine to reduce later handling and transportation costs. Concentrating processes can vary widely, but in general the chemical composition of the ore is not changed in the process.

Smelting is a pyrometallurgical process for removing non-metallics from the ore concentrates. The final product of a smelter is known as blister copper because of the rough upper surface of the solidified product. Smelting, as the term is used here, embraces the three major steps of roasting,
smelting, and converting. If possible, smelters are situated near mines and concentrators to reduce transportation charges on non-copper material. The capital investment required for a smelter is so large that many small producers can not afford the expenditure. This has resulted in creation of custom smelters in various parts of the world. A custom smelter normally buys ores or concentrates from many small producers and thus fills the gap in the chain from mine to consumer. 10

Blister copper contains gases and other metals contained in the smelter charge, and must therefore be further purified or refined. Refining can be accomplished by several methods, but the electrolytic process is by far the most predominant method in use today. Refineries are usually situated near or in copper consumption centers. Refined copper is sold to fabricators in many different shapes and sizes as required to suit their needs. Refiners generally maintain the major industry inventory of refined copper. 11

Fabricators are the major customer for refined copper. The term fabricator, as applied to the metal industries, refers to the shaping and finishing of metals and alloys from the bulk forms of refined metal or scrap into standard shapes,

11 Ibid., p. 40.
Chapter II

sizes, temper, and finishes as are required by manufacturers of finished articles or by the construction industries. In the primary copper industry fabricators generally fall into one of two groups, (1) copper and brass mills who produce copper and copper-base alloy products, and (2) copper wire mills who produce various types and grades of copper wire.

Fabricators sell their products to various manufacturers, assemblers, and installers for use in final end products. The extensive industrial use of copper depends chiefly on its: (1) electrical conductivity, (2) corrosion resistance, (3) ductility, and (4) heat conductivity. The commercial importance of copper is also closely related to its alloying properties. The ultimate end uses for copper are represented in almost every activity of man. Table 2 shows world copper consumption, by regions, for several selected years. A comparison of Table 1 and Table 2 shows the wide geographical separation between some of the consumption centers and the production regions.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANNUAL COPPER PRODUCTION BY COUNTRIES OF ORIGIN</td>
</tr>
<tr>
<td>(Thousands of short tons)</td>
</tr>
<tr>
<td>1951</td>
</tr>
<tr>
<td>United States</td>
</tr>
<tr>
<td>Chile</td>
</tr>
<tr>
<td>No. Rhodesia</td>
</tr>
<tr>
<td>U.S.S.R.*</td>
</tr>
<tr>
<td>Canada</td>
</tr>
<tr>
<td>Belgian Congo</td>
</tr>
<tr>
<td>Japan</td>
</tr>
<tr>
<td>Mexico</td>
</tr>
<tr>
<td>Un. South Africa</td>
</tr>
<tr>
<td>Totals</td>
</tr>
</tbody>
</table>

*A.B.M.S. (estimated in part)*
Table 2\textsuperscript{13}

\begin{center}
\begin{tabular}{lccccc}
 & 1951 & 1953 & 1955 & 1957 \\
\hline
Americas & 1,613 & 1,614 & 1,664 & 1,483 \\
Europe & 1,012 & 990 & 1,582 & 1,687 \\
Asia & 142 & 133 & 147 & 246 \\
Africa & 25 & 13 & 23 & 34 \\
Australia & 36 & 15 & 50 & 41 \\
U.S.S.R.* and \\
Satellites & 300 & 360 & 408 & 509 \\
Totals & 3,131 & 3,125 & 3,873 & 4,001 \\
United States & 1,368 & 1,444 & 1,446 & 1,278 \\
\end{tabular}
\end{center}

Marketing of Copper

The marketing of copper is in several respects different from commodity marketing as it is generally thought of. Copper is sold as an ore, as a concentrate, and as a metal in many forms and states of purity. Many of the large producers are vertically integrated, to varying degrees, from mining through fabricating. These integrated producers often sell a great portion of their copper as a finished product rather than as a commodity.

The major portion of raw copper is sold in two broad markets, (1) the European market, and (2) various domestic markets. Price is based on chemical purity; at any one time there may exist several prices for identical copper.\textsuperscript{14}

\textsuperscript{12} Copper the Cornerstone of Civilization, op. cit., p. 30. The totals may not agree due to rounding.
\textsuperscript{13} Ibid.
\textsuperscript{14} For a good description of the various markets see, M. R. Herman, Jr., op. cit., pp. 89-94
COMPARISON OF REFINED COPPER PRICES

Figure 1

Source: Copper Industry Report (quarterly), United States Department of Commerce
Chapter II

Figure 1 illustrates the variability of these prices. While the individual prices at times vary considerably, they generally reflect the same state of supply and demand within the industry.

**Competition**

Although copper deposits are widespread throughout the world the dominant primary production is controlled by a relatively few companies. Ten companies in 4 foreign countries produced approximately 38 per cent of the world output of copper in 1950. If the United States and the U.S.S.R. are excluded, the percentage of world mine production supplied by these companies is raised to 65 per cent. In 1950 there were approximately 300 active copper producing mines in the United States. Most of them were small operations. The 25 largest mines accounted for 98 per cent of the total, and the 10 leading mines furnished 85 per cent of the aggregate.

The relative concentration of production among a few large producers has led to many organized attempts to secure market and production control as a means for reducing wide price swings. Few of these attempts have been successful for any length of time. In 1952 there did not seem to be sufficient concentration or unanimity of circumstance among producers to effectively achieve control.15 A more recent

---

Chapter II

study of the industry has concluded that the structure is sufficiently concentrated to permit collusive activity to take place, but co-operative action on an extensive scale for any period of time has not been successful because of the force for change arising out of the discovery and development of new deposits.\footnote{Orris C. Herfendahl, \textit{op. cit.}, p. 197.}
CHAPTER III

INDUSTRIAL DYNAMICS

It has been suggested that industry fluctuations in price and production rates are amplified by, or indeed may be caused by, various characteristics within the industry. If this is true, the possible correlative relationship is immediately suggested. Can these responsible practices or policies be discovered and neutralized? Or better yet, can these practices or policies be changed so as to have a stabilizing effect on the industry from external cyclic forces? Industrial Dynamics is a study method developed to analyze this type of problem.

The purpose of this chapter is to explain briefly the study method used in examining the various possibilities posed by the problem. The chapters following will present the adaption of problem and method.

Industrial Dynamics

Industrial dynamics as a study method is essentially the study of industry behavior by simulation. The conditions which describe industry operations are set up in a digital computer which then generates information describing industry

---

behavior under the imposed conditions.  

In essence a mathematical model is a description of the industry for simulation. If the model is to provide a basis for answering questions about the industry or system it represents, it must be an adequate description of the system.  

When completed such a model will be too complex to yield an analytical solution to the problem, if indeed such a solution exists. The completed model will be used, with a digital computer, to trace through time a particular course of action that results from the dynamic interaction of the various factors contained in and imposed on the model. Different factors or different conditions may well yield different time patterns or responses. This is an experimental, empirical approach to a better understanding of the industry system under study. Better understanding may well lead to better control.  

To successfully construct a meaningful model, the industry under study must be considered as an integrated feedback system. Only in this way can we properly deal with the way information is used for the purpose of control.  

---

2Ibid., p. 39.  
3Jay W. Forrester, "Memorandum D-46," a copyrighted, privately published paper which describes Models of Dynamic Behavior of Industrial and Economic Systems—a Section of Industrial Dynamics Class Notes. Written at the Massachusetts Institute of Technology, April 1, 1959. Much of the material in this chapter is based on various sections of this memorandum.
Chapter III

Without an awareness and representation of information flow principals we cannot find internal causal mechanisms, if they exist.

I do not propose to describe model requirements in any detail, since this information is available to the interested reader. In general a model may well include various broad classifications of flow systems such as: material, orders, money, capital equipment, men, and information. In constructing a dynamic simulation model, the factors and flow systems to be included arise directly from the questions that are to be answered. Another important controlling determinant of model content is the information feedback loop. An information-feedback loop exists when an environment leads to decisions, the decisions lead to action, and the action alters the original environment. If part of the feedback loop is omitted from the model, the simulated system will not function as does the real system.

Another comment on Industrial Dynamics as a study method; it is believed model construction should not wait for a complete gathering of all pertinent statistical information. If we are looking for internal causal mechanisms,

---

Ibid. Also a book on the subject is soon to be published by the same author.
Chapter III

it is doubtful that these can be found by simulation with a model built from external observation. It is believed more important to build the model on the basis of the general nature of the inter-relationships within the system. Once built, plausible values can then be assigned to the various coefficients. By simulation the validity of the assumptions or the importance of the item in question can be judged. It may be found that extensive data collection has been wasted in some areas, while there is a need for careful data collection in some unsuspected area.

One final note, Industrial Dynamics in its present state of development is not a tool for forecasting the future. A model that accurately represents an industry today will accurately forecast the future only if all relationships remain as described. What reason have we to expect that these relationships will remain unchanged over time?

Within the limits of available time, this model of the primary copper industry was constructed in conformance with the principals of Industrial Dynamics and the general principal of aggregation to achieve cumulative policy and behavior relationships.

Essentially the model consists of an inter-relationship between order, material, and information flows septems. Sufficient time was not available to separately incorporate labor, capital, and other flows present within the industry.
Chapter III

The presence of the omitted flow system is imp
cumulative behavioral aspects of the flow syst
The flow of money is incorporated in an abbrev
show the effect of different policies on profi

The customer, or user, of copper and cc
products is included as a major sector of the
dictated by the feedback loop between the cons
producer. The producer (fabricator, refiner,
miner) acts in response to the consumers actic
tent that the consumer changes his action base
ducers response, the producer is in turn affec
change in consumer action) by his own action,
loop is present.

A similar evaluation led to the provis:
model, for competition from other industries.
Since the main focus is on the copper industr
is represented in a much abbreviated form.
CHAPTER IV

THE COPPER USER

The purpose of this chapter is to briefly explain the users' sector of the primary copper industry model. In this chapter I will discuss in general terms the mathematical representation of the various decision functions contained in the sector. The equations for the user are presented in Appendix A.

The end uses of copper are many and varied. Consequently we should not expect all copper users to behave in the same manner. Time does not permit separate customer identification, or later representation within the model, however, I do not feel this poses any great threat to model validity.

As economists construct aggregate supply and demand curves, I have constructed an aggregate copper user. Our copper user as such probably does not exist, but if his behavior under any given conditions reasonably represents the aggregate response of the many diverse behavioral aspects of the real world user, we can be satisfied to let him stand for all users. For example, assume that under a certain condition some users wanted to increase inventories, other users wanted to decrease inventories, and still others wanted to make no change in inventories; we could represent separately these various actions (there would undoubtedly be differences
in rate of inventory accumulation and liquidation), but the net effect can just as well be represented by one action that is the aggregate of the separate actions. This principal, or concept, of aggregation is used as a cornerstone for constructing this model and therefore it is necessary that it be well understood. From this point on, aggregate behavior is implied in reference to users, competitors, fabricators, etc. The aggregate copper user is probably a typical customer for most raw material industries since the use of copper is represented in almost every activity of man.

Sales, Shipments, and Finished Goods Inventories

Users' sales are taken as given in this model, no provision for change in user sales by action of the user or copper industry is provided. If this study was focused on the user, such an assumption would not be valid. The validity here is contingent on another assumption; users can obtain (through time) all needed raw material. Those requirements not provided by the primary copper industry will be obtained from its competitors. User sales will be used as the main external input to the model.

New sales enter a backlog of orders. Shipments to fill orders are made on the basis of a variable shipping delay and the level of unfilled orders. Shipments reduce the order backlog and the finished goods inventory. As the unfilled order backlog increases, shipments from inventory increase. An increase in the order shipping delay will decrease
Figure 2 - Part 1

Flow Diagram - Copper User (continued in Part 2)
Figure 2 - Part 2
Flow Diagram - Copper User (continued from Part 1)
Chapter IV

the order shipping rate. This decreased shipping rate will tend to increase the order backlog; the increased backlog will tend to increase the shipping rate, but the increased shipping rate may reduce the finished goods inventory so much that the shipping delay is increased. The increased shipping delay will tend to reduce the order shipping rate; reduction of the unfilled order backlog by increased order shipments will also tend to reduce the order shipping rate. We have here a good example of a feedback loop, the net response to some change in the loop can not be determined without considering the total effect and response over time.

Figure 2 is a flow diagram of the copper user. Material flows are represented by a solid line, order flows are represented by solid line containing small circles, and the flow of information is shown by a dotted line. Reference to Figure 2 will assist the reader in visualizing the representation of the copper user.

The variable shipping delay is based on the relation of actual to desired finished goods inventory. The method of formulating this delay will be explained since similar formulations are used in other sectors. In Figure 3 the horizontal axis represents the ratio between actual and desired inventory, the vertical axis represents the average shipping delay. The average shipping delay is composed of two parts, (1) a minimum fixed delay (U1) which includes
Chapter IV

processing time and the average lead time of order received in April may specify July delivery. An order received in April may request immediate delivery.

(2) the average additional variable delay (U2) can be computed only when the actual level of inventory is equal to the desired level of inventory. This delay is a function of the ratio of actual inventory to desired inventory.

\[
\text{RATIO} = \frac{\text{ACTUAL INVENTORY - LBS.}}{\text{DESIRED INVENTORY - LBS.}}
\]

Figure 3

Shipping Delay

The total delay would then be equal to the sum of the product of U2 and the ratio of actual inventory to desired inventory.

The rationale for the existence of U2 under certain conditions of balanced inventory may not at first be obvious. If the inventory under discussion consisted of only one product with no different sizes, color, etc., then there would be no delay caused by out-of-stock items when the desired inventories were equal. This, however, is not the case for user finished goods inventory consists of all

\[\text{Assuming, of course, that the inventory is equal to the order.}\]
made from copper and competing materials. While the actual and desired inventories may well be equal in terms of dollar value, weight, number, or some other measure, it is apparent that the actual inventory will not contain every article (in the proper number, size, color, etc.) that may be ordered. This out-of-stock condition gives rise to the use of the above concept.

The desired inventory is determined from an average sales rate and the number of weeks of sales required or wanted in inventory. As formulated, the average sales will change in response to changes in sales, but no provisions have been made to allow for a change in weeks of sales desired in inventory.

Production

The production function is represented as a third order exponential delay. The output from production goes to and tends to increase finished goods inventory. The input to production comes from and tends to reduce raw material inventory.

The production decision consists of two parts, (1) that production necessary to supply average sales, and (2) that production necessary to bring desired and actual finished

---

goods inventory into balance. The second part of the production decision may well be negative when actual inventory exceeds desired inventory and thus the production rate would be less than the sales rate. It is recognized that the correction of an imbalance between actual and desired inventory is not attempted at once, but over a period of time. Therefore, only a fraction of the second part of the production decision is added to the production for average sales.

To insure that model performance represents actual performance, an inventory production limit has been provided. This prevents production beyond the ability of raw material inventory to supply the necessary raw materials, and also prevents production beyond physical capabilities (manpower, capital, etc.)

**Raw Material Inventory and Purchase Decision**

Raw material inventory is decreased by material going into production and is increased by receipt of material from the copper industry and the competing industries. The raw material purchasing decision, similar in construction to production decision, consists of two parts, (1) the purchase of materials for production and (2) the purchase of materials necessary to balance desired and actual raw materials inventory. The desired raw material inventory is dependent upon the number of weeks of production wanted in raw materials inventory and the production rate desired.
Chapter IV

The Where to Purchase Decision

The decision of how much raw material to purchase has been represented. This decision is based on the inter-action of sales, production rates, and the receipt of raw material. Since the model provided two main sources of supply (copper and competing materials) a means must be provided for deciding where the material is to be purchased.

One underlying assumption is that user sales represent all the recognized uses for refined copper. Some of the uses are filled by copper while others are filled by competing materials. Figure 4 is a chart that shows the various relationships of this assumption. It has been assumed that 20 per cent of all possible uses for copper will always be filled by copper due to its special properties, while 20 per cent of possible copper uses will never be filled by copper due to the special properties of competing materials. Between the upper and lower limits, the use of copper is dependent on the relation of the price of copper to the price of competing materials. To simplify this study, the price of competing materials has been assumed to remain constant.

The relationship between the use of copper and the use of competing materials has not been derived from

---

LONG RUN COPPER USE - ULRCP - RATIO SCALE

Figure 4

User Copper Purchase Relationship
statistical data, yet I believe that in general the relationship is correct. The use of any material is dependent not only on its properties, but also on the relative cost of use in comparison with the cost of use of substitute materials. Figure 4 shows two curves that represent the above general assumptions. It is planned to make computer runs with each curve to determine how important the details of the relationship are to general industry behavior. Another simplifying assumption used is that of a "normal" price for copper with the price for substitute materials in equivalent terms being equal to this normal copper price.

The normal price concept is used in several places throughout the model. For this normal price I have chosen 30 cents per pound. This is no special significance to this figure other than it is in the range of experts' estimates of a "proper" price. 4

Returning to the users' sector, copper is assumed to supply 50 per cent of the users needs (for recognized possible copper uses) when the price for copper is "normal". For any given purchase the percentage of the requirements to be filled by the copper industry is determined from the relationship shown in Figure 4, and the remaining amount is filled by the competitor. The decision of where to purchase becomes

4Orris C. Herfindahl, op. cit., p. 145.
Chapter IV

a decision of how much (or what percentage) to purchase from the copper industry (the balance to be supplied by competitors).

This decision is based on two items, (1) the long-run consideration (dependent upon copper price) and (2), the short-run consideration dependent upon availability of copper. The long-run decision is a function of average copper price and the relation shown by Figure 4. This ratio (PRATO) used to enter the table for the long-run decision is modified by a mechanism to represent "stickiness" on the part of the user in changing production process for different raw material. This mechanism is discussed in more detail in Appendix A for the interested reader. The short-run consideration, based on the availability of copper, is the ratio of the normal order filling time (fabricators' normal shipping delay) to actual order filling time (actual fabricators' shipping delay). The decision for determining copper's share of the total raw material purchases is achieved by modifying the long-run consideration by the short-run consideration.

The actual amount of copper to be purchased (in lbs.) is the result of two functions, the percentage of total requirements times the amount of raw material desired, plus any amount necessary to balance actual and normal (or required) pipeline inventories. The normal pipeline inventory is derived by multiplying the average copper order rate time the normal delays (clerical, fabricator shipping, transportation,
Chapter IV

etc.), while the actual pipeline inventory is derived by measuring the amount of undelivered copper that has been ordered. The resultant orders for copper serve as the input to the fabricators sector of the model.

This model does not directly provide for expansion or contraction of copper sales by promotional activities. This factor, if representation were desired, could be presented by modifying the users' sales orders. By assumption, the users' sector represents all the recognized possible uses of copper, if promotional activity were to find new uses for copper the orders entering the users sector would be increased.
CHAPTER V

THE COPPER FABRICATOR

This chapter briefly describes the aggregate fabricator and the representation of this fabricator in the model of the primary copper industry.

The fabricator buys refined copper for use as a raw material in fabricating copper wire, brass, and other copper-base products. These products are then sold to the various copper users. Generally domestic fabricators buy their copper from domestic sources while foreign fabricators buy their copper from foreign sources. There is, however, considerable movement of refined copper and fabricated copper-base products from one market to another.¹

Prior to the second world war the United States was a net exporter of refined copper, since then it has been a net importer in most years. In spite of the present tariff, the domestic consumption of foreign copper-base fabricated products has increased in recent years. My purpose is not to discuss the pros and cons of this issue, but to illustrate the world flow of copper and copper-base products. As price and/or availability dictates, copper or copper-base products will flow from one region to another.²

Chapter V

In manufacturing goods from fabricated copper products, there is a certain amount of new scrap created. Usually this scrap is purchased from the users by the fabricator and re-enters the fabrication process. A certain amount of scrap is also generated in the fabrication process. This material also generally re-enters the manufacturing cycle of the fabricator. To simplify model construction, I have not specifically represented the circular flow of "run around" scrap. The equations used to represent the fabricator are contained in Appendix B.

Sales, Shipment, and Finished Goods Inventory

This part of the fabricators' sector is represented in much the same manner as is the similar segment of the users' sector. Fabricator sales orders (user purchase orders) enter a backlog of unfilled orders. Shipments from finished goods inventory are based on the size of the order backlog and a variable shipping delay. Desired finished goods inventory is a function of average sales and the number of weeks of sales wanted in inventory.

Fabricator's finished goods consist of a wide variety of copper and copper-base products, therefore, the variable

---

3 During an interview with one industry leader in the fabricators' sector (January 1960) he stated that user purchase contracts normally require the fabricator to purchase all new scrap from the user.
shipping delay is formulated along the same line of reasoning used in representing the user variable shipping delay. Within this circuit of orders and shipments the various interactions described in Chapter 4 take place, but here the input (sales orders) is the resultant of the dynamic interaction of flow systems in the user sector. Fabricator's shipments to the user has an effect on the user's purchases of fabricator's products; the user's purchases have an effect on the flow of goods to the user. Sales and shipments serve to create a feedback loop tying the two sectors together. The flow diagram of the aggregate fabricator is presented in Figure 5.

Production

As in the user's sector, fabricator's desired production is based on average sales and the necessity for balancing desired and actual finished goods inventory. The model has been provided with a mechanism to prevent actual production from exceeding possible production. Maximum production is limited by physical capabilities of the facility (manpower, plant, etc.) as well as by the availability of raw materials.

The actual fabrication process is represented by a third order delay. Fabricated products flow from production into finished goods inventory. Raw material for production flows from the fabricator raw material inventory. Production flow tends to increase finished goods inventory and to reduce raw material inventory.
Figure 5

Flow Diagram of the Aggregate Fabricator
Chapter V

Raw Material Inventory and Purchase Decisions

The decision of how much raw material to purchase is based on two items, (1) sufficient material to replace that being used in the production process, and (2) any amount necessary to adjust actual and desired inventories.

Many observers feel that a certain amount of inventory speculation is practiced by some fabricators. This speculation could exist for two main purposes, (1) to insure an adequate supply of raw materials in times of short supply, and (2) to profit on price changes in copper.

Regardless of the motive, speculation would tend to increase the purchase rate (over normal) of copper during a period of rapidly rising prices and would tend to decrease the purchase rate of copper during a period of rapidly falling prices. This would result in increased demand when the availability of copper is relatively low and a decreased demand when the availability is relatively high.

Speculation as represented in the model effects the desired raw material in entory, and thus the ultimate purchase

---

Thomas J. Keller, op. cit., February 15, 1960, p. 2; Interviews with industry leaders during the first week of February 1960 also touched on this point. Persons interviewed felt that fabricator speculation existed among some fabricators, but not to the same extent as previously. Domestic copper was in short supply due to industry strikes at the time of these interviews.
Chapter V

of refined copper. The actual purchase orders for refined copper are determined from two items, first the desired purchases, and second any purchases necessary to balance pipeline inventories.

The desired purchases are the resultant of speculation, production, and raw material inventory imbalance. The purchases necessary to balance pipeline inventories are the result of comparing actual and required pipeline inventories. The resultant purchase orders for refined copper serve as the sale orders for the refiners and order input to that sector.
CHAPTER VI

THE COPPER REFINER

This chapter describes the aggregate copper refiner and the representation of that refiner in the primary copper industry model. The equations used to represent the aggregate refiner in the model are presented in Appendix C.

The refiner uses blister copper as a raw material to produce refined copper in many shapes and sizes for sale to the fabricators. As with the user and fabricator, there are many refiners with different operating policies etc. Some copper refiners act as a part of an integrated company processing blister copper from their own sources, others act as toll refiners and/or as custom refiners.

In a toll contract the refiner performs a service for the owner of the copper processed. A custom refiner buys blister copper and sells the refined product. Custom refiners for the most part attempt to make their money on the treatment charges and not on changes in refined copper price. In general they enter into long-term purchase agreements and pay for the blister copper (or ores and concentrates) on the basis of the price realized from the sale of refined copper.¹

¹M. R. Herman, Jr., op. cit., p. 22-24.
This refiner for the most part is "locked in" with his source of supply in so far as receipt of raw material is concerned. In the short-term he has no control over the amount of raw material he receives. In the long-term a refiner can increase or decrease his source of supply (within limits of available copper) by entering into or by canceling contracts. The custom refiner is under pressure to sell each period as much refined copper as he receives in blister copper during the period, otherwise he could suffer a loss from a drop in copper price.\(^2\)

Most authorities agree that the price for refined copper is the result of the interaction between demand and supply.\(^3\) Various pressures to modify the "free" price of copper undoubtedly exist, but these pressures have not been particularly effective. The existence of several prices in the domestic market illustrates the existence of some of these pressures.\(^4\)

Integrated domestic producers generally attempt to maintain a stable price. In period of decreased availability the custom price will generally exceed that quoted by the integrated producer. In periods of increasing availability the custom price will generally be lower than that of

---

\(^2\)Ibid., pp. 100-103.

\(^3\)Robert G. Page, op. cit., p. 44; This view was also expressed in many of the February interviews.

\(^4\)See Figure 1 of Chapter 2.
Chapter VI

the integrated producer. The integrated producer's ability to sell refined copper under these conditions depends in part on his fairness in allocating available copper during periods of short supply. Fabricators will continue to purchase from the integrated producer during periods of excess supply to insure their "fair" share of copper during periods of tight supply. 5

Sales, Shipments, and Finished Goods Inventory

This segment of the refiner sector of the primary copper industry is represented in much the same manner as are the similar segments in the user and fabricator sectors of the model.

Fabricators' purchase orders serve as refiners' sales orders and as the main order input to the sector. Sales orders enter a backlog of unfilled orders. The size of the unfilled order backlog and the magnitude of the variable shipping delay determine the rate of flow of refined copper to the fabricator.

The variable shipping delay is represented, as in the user and fabricator sectors, as the sum of a fixed delay and a variable delay dependent upon the ratio of actual to desired finished goods inventory. This method of representations is possible since the finished goods inventory consists of various sizes and shapes of refined copper.

5 This point was stressed heavily during the February interviews.
The finished goods inventory is a function of average sales and the number of weeks of average sales desired in inventory. The actual inventory is increased by the flow of material from production and decreased by the flow of materials to the fabricator. Figure 6 shows the flow diagram of the aggregate copper refiner.

Production and Raw Material Inventory

As previously noted, refiners are committed to take smelter production. The smelter production or shipments enter raw material inventory, so the refiners' production is a function of their raw material inventory. The production rate is limited by two considerations, first, the physical capabilities of the refiners (plant, capital, etc.), second, the necessity or desire for maintaining a minimum raw material inventory to insure continuous production.

The raw material inventory is increased by the receipt of smelter shipments and decreased by the production rate of refiners. The absolute size of the inventory is a function of the refiners' production rate and the smelters' shipping rate.

Price Determination

Price is a function of the demand for, and the supply of, refined copper. Prices are quoted by the refiner so

---

6M. R. Herman, Jr., op. cit., p. 79.
Figure 6

Flow Diagram of the Refiner Sector
this is the appropriate place to represent the price determining function.

As represented, the copper price is determined by two aspects, (1) a so-called static aspect, and (2) a so-called rate aspect. The static aspect is the result of comparing unfilled orders for refined copper with refined copper inventory. When this relationship is normal, it would tend to create the normal ($0.30 per pound) price. The rate aspect is a comparison of the average sales rate with the total mine production rate of new copper. When these rates are equal, the rate aspect would tend to provide a normal price.

The actual relationship, employed in the model, for determining copper price is shown in Figure 7. The supply-demand ratio used for entering this chart is the weighted combined results of the static comparison and the rate comparison.

The rationale for using both a static and rate comparison may not immediately be apparent. The static aspect shows the present relationship between supply and demand. The rate aspect shows the expected future relationship between supply and demand. Both the present relationship and the expected future relationship between supply and demand are important determinants of copper price. For example, if supply and demand relationships are normal, a major curtailment of mine
Figure 7

Copper Price and Supply - Demand Relationship
production (assume a strike) rapidly affects copper prices. The effect will take place before its impact in the stocks of refined copper can be felt. In the model this behavior is represented by the rate aspect. If a more detailed model were to be constructed, the rate aspect should provide a comparison of average sales with production rates along the entire pipeline from mine to refiners finished goods inventory.

As represented, the pricing mechanism provides a copper price based on the relationship between supply and demand. For the purpose of this study it was not felt necessary nor desirable to generate separate prices for custom refiners, integrated producers, the London Metal Exchange, etc., therefore, only one price mechanism is provided.

---

7Copper the Cornerstone of Civilization, op. cit., p. 31; Robert G. Page, op. cit; Thomas J. Keller, op. cit., February 1, 1960, p. 2.
Chapter VII

The custom smelter also purchases some secondary scrap for smelter feed. This source of supply is not separately represented in the model. The availability of secondary scrap in general responds to price movements in a fashion similar to the availability of copper concentrates or ore,² therefore, scrap is assumed to be an integral part of the mines sector.

The aggregate primary copper smelter is represented in an extremely simple fashion in this model. Raw materials from the concentrator sector are received in the raw material inventory. Production is a function of the size of this inventory, but it is limited to the maximum production capabilities of plant facilities etc. Goods from the production process enter the finished goods inventory for eventual shipment to the refiner. Shipment to the refiner and based on the size of the finished goods inventory and a variable shipping delay.

The variable shipping delay consists of two parts, (1) an average fixed minimum delay and (2) an additional variable delay that is a function of actual inventory to normal inventory. Figure 8 represents the general relationship

for determining the shipping delay. The fixed minimum delay is shown as S1

\[ \text{RATIO} = \frac{\text{Actual Inventory - Lbs.}}{\text{Normal Inventory - Lbs.}} \]

**Figure 8**

Shipping Delay

and the variable additional delay is shown as S2. The sum of S1 and S2 is the total shipping delay.

A list of equations showing the mathematical representation of the primary copper smelter is shown in Appendix D, while a flow diagram of this sector is shown in Figure 9.

**The Copper Concentrator**

The copper concentrator receives mine ore as a raw material for producing copper concentrates. Nearly all concentrators operate in conjunction with some specific mine or groups of mines. From the standpoint of this study their function is quite simply that of receiving ore, processing it and shipping the concentrates to the smelter. In practice, new techniques in this area have contributed greatly to the mining and processing on lower grade ores.
Figure 9
Flow Diagram of the Aggregate Smelter
Figure 10
Flow Diagram of the Aggregate Concentrator
Chapter VII

The representation of the concentrator is similar except for parameter values to that used for representing the aggregate smelter. The list of equations that describe the concentrator are shown in Appendix E and a flow diagram of this representation is shown in Figure 10.
CHAPTER VIII

THE MINES SECTOR

The aggregate mine behavior and its representation in the primary copper industry model is described in this chapter.

Mines can be extremely different from one another in many respects. Mining methods vary widely, ore grade and ore reserves are seldom the same, but in spite of these differences all mines have the common function of mining ore that can be processed into refined copper at a profit.

The control of total mine production has been the subject of much discussion by persons connected with the primary copper industry. The cost and time required for developing new mines have prevented the rapid expansion of mine output during periods of short supply. The shut down and maintenance costs of idle mines have on the other hand tended to prevent a rapid contraction of mine production in time of long supply. The inability of the mining sector to rapidly adjust production to changes in demand has been cited as one of the major problems facing the industry. ¹

The problem of increasing facilities to provide for sufficient sources of supply in periods of high demand is

¹Dr. J. Zimmerman, Editorial, Daily Metal Reporter, Annual Metal and Steel Number, Volume 60, Number 10, Part 2 (New York, January 1960); Robert G. Page, op. cit., Jean Vuillequez, op. cit.
one that has faced the industry from the second world war until just recently. At present and for the next few years, it appears that available supply will exceed expected demand by a comfortable margin. This condition brings into focus the problem of how to best restrict production to demand.

Mr. Robert G. Page, President of the Phelps Dodge Corporation, in discussing this problem states that it is a historical fact that high cost mines do not shut down to any important degree when copper is in excess supply and prices are low. He further argues that in spite of economic "laws," it would not be desirable for the low cost producer to drive the high cost producer out of business. Therefore, he feels that the low cost producer should curtail production sufficiently in times of excess supply to permit the high-cost operator to stay in business.

I do not propose to discuss the merits of this or other suggested production curtailments methods from the point of economic good. The adjustment of production to meet changing conditions of demand is a real problem of the industry. My interest in this study is to find some of the causes and possible solutions for the short-term cyclic nature of copper production and price.

---

3 Robert G. Page, op. cit.
Chapter VIII

I have not attempted to construct a growth model of the primary copper industry for two reasons. 1) I feel that partial stabilization of copper price and availability is a prerequisite of rapid growth, and 2) time available for any study is limited. I feel the available time will best be used in studying the immediate problems and possible solutions to price stabilization. If this study is at all successful, it will provide a firmer foundation for studying growth as such.

General Data

As previously noted, mining methods vary considerably, but to provide a better basis for understanding the model construction (particularly the mines sector) I will describe an open pit mine operation and certain later processes only in enough detail to point out necessary concepts.

The first assumption necessary is that we are discussing an operating mine. Exploration and development work will have shown us that we have a minable ore body that will yield an assumed average ore grade of 1 per cent. To mine each ton of ore it will be necessary to remove two tons (assumed) of barren and waste material. For simplicity's

---

4Jean Vuillequez, op. cit., Mr. Vuillequez asserts that the essential ingredients for growth are present, but the industry must find the right recipe.
Chapter VIII

sake another assumption is that a long range mining program shows this ore body can be mined in a manner to maintain the average ore grade for the life of the property. This does not mean, however, that each ton of ore will average 1 percent. In most ore deposits the copper content can and does vary widely. A cut-off grade is usually established for that material which contains the minimum amount of copper that can be mined at a profit. Other ores in the mine may contain copper far in excess of the average ore. Like all averages, the average ore consists of material below and above the average.

Since it is necessary to remove barren and waste material to remove the ore, mine capacity is generally limited by the capacity to remove material, be it ore or waste. This capacity is not necessarily a one for one exchange ratio since waste material will probably not have to be moved the same distance as will ore. For example, a mine may have the capacity for removing 1,000 tons of ore per hour and 2,000 tons of waste per hour. If all mining equipment is committed to waste removal, we can't state in certainty that the mine can remove 3,000 tons of waste per hour. Nevertheless, mine capacity is in general limited by the capabilities for removing material be it ore or waste.

After removal, ore is next crushed and finely ground to permit separation of the ore mineral from the barren host
rock. This process (concentration) will result in two products, (1) an ore concentrate, and (2) a tailing or waste material. The concentrate will contain most, but not all of the copper contained in the ore. The measure of the total copper recovered from the ore in the concentrate is commonly referred to as recovery and is stated as a percentage. Recoveries usually range from 80 per cent to over 90 per cent.

The copper content of the concentrate will depend upon the amount of non-copper minerals contained in the concentrate. The ratio of concentration and recovery are, to a great extent, determined by the characteristics of the ore. Different ores from the same ore body often yield markedly different results in the concentrating process. Generally speaking, concentrator production is limited by the physical capacity for handling and processing ore material, although copper content and ore characteristics can often be important determinants of capacity.

The concentrates are then smelted to remove the non-metallic content of the concentrate. A smelter is usually designed to process a given amount of concentrate per period of time. This rate, the amount of concentrate handled, generally limits smelter capacity. The limit on copper produced by the smelter is therefore a function of concentrate grade and capacity for processing concentrate.
Chapter VIII

In our imaginary situation we see that the mine has handled three tons of material to supply the concentrator with one ton of ore containing twenty pounds of copper. If we assume the concentrator produces a product containing 20% copper with a recovery of 90 per cent, we will find the smelter feed from our original ton of ore will consist of ninety pounds of material containing eighteen pounds of copper. The smelter product from this feed will consist of approximately eighteen pounds of material containing something less than pure copper.

To simplify the model construction, all material is represented in terms of equivalent pounds of copper. In each sector the production limit has been stated (or has been implied) in terms of pounds of copper per week. The reader must recognize that variable conversion ratios can exist. For example, if the aggregate concentrator were to suddenly produce a product containing much less copper than before, the smelters production limit (as stated in pounds of copper) would have to drop since the smelter can only process a given amount of material.

Mine Production

Mine production is represented by a third order delay. Produced ore goes into the concentrator's raw material inventory. The mine production decision for the aggregate
Chapter VIII

miner is in general based on copper price. Figure 11 shows a typical supply curve that is used for determining desired mine production. This chart is entered with an average copper price which yields desired production in terms of pounds of copper per week.

The rationale for determining production in this manner should be apparent. As previously noted, mines vary considerably in terms of size, copper content, and other characteristics. Because of these variances, the costs of producing copper also vary quite widely. Some mines are only able to produce copper at a profit under conditions of a relatively high price, while others can profitably produce at a lower price. Thus, as the price of copper increases additional ore bodies will become profitable and additional supply is in effect "called out". The relationship shown in Figure 11 is not the result of extensive investigation, and while no claim is made for the accuracy of the curve, I believe it adequately represents the general nature of the price-supply relationship.

Since aggregate mine production is slow in changing to meet new conditions (barring strikes, etc.) a mechanism is provided in the model to represent this condition. This mechanism is called the "production wanted function" for identification purposes.
Figure 11

Mine Supply Curve
It was also necessary to provide a means for representing a change in conversion factors to represent a change in grade of ore mined. This mechanism is called "ore yield," and is a weighted comparison of normal and actual copper price. This function, "ore yield," is used to modify "production wanted" to reflect any changes caused by temporary changes in grade of ore mined. Mine production is also limited by physical plant capacity. This limit is adjusted, as necessary, by the "ore yield" factor to provide a correct limit for that grade of ore mined.

The necessity for providing a method for supply growth is represented by an auxiliary mechanism called "adjusted production desired." This auxiliary achieves the effect of slowly moving the supply curve over time. (Figure 11).

The rational for this concept is based on the belief that the finding and development of new sources of supply is an orderly function dependent upon demand. This auxiliary tends to shift the supply curve to the right when the long-term average price exceeds the normal price by a significant amount. In a similar fashion it will shift the supply curve to the left where the long-term average price falls below normal price by a significant amount. Short-run price changes will not cause a shift in the supply curve. The "adjusted production desired" auxiliary also acts on the various maximum

---

5 Orris C. Herfendahl, op. cit., p. 63
Figure 12

Flow Diagram of the "M" Mine Sector
production limits to reflect industry-wide the changes in the supply curve.

Profits

To provide a rapid means for evaluating industry effectiveness and the desirability of any changes made to the basic model, two simple money flow systems were incorporated in the model to measure profits. To simplify this representation, the concept of normal profits was used, and only those profits in excess of (or less than) normal are measured.

Profits are measured in the mine sector and for the industry (mine through refiner) as a whole. Mine profits are defined as being equal to one-half the difference between revenue and cost. Mine revenue is defined as being equal to the product of mine production (in pounds) and current copper price. Mine cost is defined as being equal to the product of the normal copper price ($0.30 per pound) and the mine production divided by "ore yield". This latter function reflects the fact that mining costs in general are dependent on material handled, be it ore or waste. Industry profits are defined as one-half the difference between total industry revenue and total mine costs. Industry revenue is defined as the product of refiners' shipments and current copper price. Under conditions of normal production and normal copper price, both profit functions would be zero.
Figure 13

Flow Diagram of the "M1" Mine Sector
Chapter VIII

Two Mine Sectors

The mine sector consists of two mines. The purpose for providing two sectors was to enable the researcher to experiment with different policies to study the effect on the total system. Both mine sectors are constructed in an identical fashion, changes in policy are achieved, in the model, by using different constants to change the effect of decision functions in each sector. These changes will be discussed in a later chapter.

Appendix F contains a list of the equation that represent the "M" mine sector, while the flow diagram for this sector is presented in Figure 12. Equations for the "M1" mine sector are presented in Appendix G. Figure 13 is a flow diagram for this sector. Equations for the industry profit center and its flow diagram are presented as a part of the "M1" mine sector.
CHAPTER IX

SIMULATION REQUIREMENTS

This chapter will briefly discuss the prerequisites for simulation and the basis for determining system constants.

The different equations used for describing this model of the primary copper industry were written to conform with the specifications for the Dynamo II A program. Dynamo II A is a code for generating a running IBM 704 program from a set of equations key-punched, according to the specified format, into special cards for later "reading in" to the IBM 704 electronic computer.¹

Simulation, as provided by Dynamo II A, is accomplished by solving all model equations over consecutive time periods. The solution of equations in 5th period, for example, is based on the solution results of the 4th period. This calculation process can be carried out as long as necessary to provide simulation over any period of time desired. Calculations of values for the first time period require that values for certain types of equations be initially specified.

¹P. Fox and others, "Memorandum D-47," a privately published paper which describes the Dynamo II program. Prepared at the Massachusetts Institute of Technology, July 24, 1959; P. Fox and others, "Memorandum D-60", a privately published paper that describes changes in the Dynamo II program and results in the Dynamo II A program. This memo also modifies certain parts of memorandum D-47. Prepared at the M.I.T., December 1959.
Chapter IX

These values, called initial conditions, are specified to represent the industry at some point in time.

To insure against mathematical errors, I chose to initially represent the industry in a steady state and check the model over a short period of time. If all flows remained constant at the initially specified values, I could be certain that no mathematical errors existed. This would not, however, insure correct formulation. Several attempts were required to achieve this steady state.

The next step, following stabilization, is that of model verification. Here the system is simulated under appropriate conditions and the results over time are evaluated to insure that the model response, in general, represents the industry. For example, if the time response should show copper price to always move with the movement of stocks of refined copper, we would conclude that the model did not represent the primary copper industry.

If the model clearly does not represent the industry, this can easily be shown, but model validity is more difficult to prove, if indeed it can be. We can only judge validity on the basis of our knowledge of the industry. Detailed accurate data necessary to insure perfect correlation is not available, but I seriously doubt the necessity for insuring such high correlation. If the model responds in a way that reasonably represents industry response, we can learn much from it.
Chapter IX

This model of the primary copper industry was built with that in mind. I was more interested in constructing a model that would represent present industry behavior than I was in constructing a model that would trace out past industry history. For that reason the model was constructed more on the basis of belief of industry behavior than on the basis of statistical data.

Determination of Constant Values

The constants in the model equations in some cases represent measurable items while in other places they are only useful concepts used to provide mathematical representation. For example, Del 1, USSC, and Del 5 (Appendix A) are constants that represent measurable items. Del 1 is the average time required by the user to produce an item. If we knew exactly which product was to be made and knew which factory was to make it, we could definitely establish the average production time. Since we are dealing with the aggregate rather than the specific, determination of the actual values were made on the basis of judgment rather than on the basis of measurement.

The constant F7 (Appendix B) is representative of those constants which are used to achieve mathematical representation. F7 is used to weight the "speculation modifier" in the fabricators sector. Values for constants of this type are, in the final analysis, determined experimentally.
Where possible, values for constants representing measurable items were based on available statistical data and the information secured during interviews with industry representatives.

The normal user order rate is one of the basic determinants of other functions in the model. This order rate was derived by doubling the assumed normal primary copper production rate. After consulting several statistical sources, the normal industry production rate was assumed to be 131,000,000 pounds of refined copper per week. This rate has been used as the "normal" production rate throughout the model. As with the normal "copper" price, no attempt will be made to justify the accuracy of this figure, but I doubt that the use of a slightly different figure would yield significantly different results.

---

2 Year Book of the American Bureau of Metal Statistics, Thirty-eighth Annual Issue for the Year 1958, (New York, 1959); George H. Cleaver, "Copper", Engineering and Mining Journal, 91st Annual Survey and Outlook, (New York, February 1960); Jean Vuillequez, "Copper," Mining Congress Journal, Volume 46, Number 2, Annual Review (Washington, February 1960); Copper the Cornerstone of Civilization, op. cit.; Various issues of Copper Industry Report, issued quarterly and annually by the U.S. Business and Defense Service Administration, U.S. Department of Commerce (Washington; Government Printing Office); Daily Metal Reporter, Volume 60, Number 10, Part No. 2, Annual Metal and Steel Number (New York, January 15, 1960), this contains abundant statistical data compiled from several sources such as the Copper Institute and others.
Chapter IX

Various statistical sources were reviewed as a guide in establishing other physical constants such as maximum refiner capacity etc. Here again an attempt was made to establish realistic and correct value, but no attempt will be made to prove their accuracy. They are believed to be reasonably correct for use in the model.

---

3 Same source as listed in footnote 2, page 70.
CHAPTER X

PROPOSALS FOR ACHIEVING SOME DEGREE OF MARKET STABILIZATION

This chapter will discuss several proposed methods for achieving some degree of market stability.

The problem of reducing price and production fluctuation brings to mind several possible solutions. The changes indicated for these solutions can be made to the model, and the effectiveness and desirability of these solutions can then be studied by simulation. How such changes are to be made in the industry is another question, but perhaps model results will serve as a guide for what to attempt and what not to attempt.

Mine Production Control

At this writing I have been unable to secure data that will either prove or disprove a suspicion of mine. That suspicion is that the average mine tends to operate in such a fashion as to yield regular profits. More specifically, I believe the average mine tends to produce a lower grade ore in periods of high price and higher grade ore in periods of low price.

---

1 The Bureau of Mines, in response to my request, is assembling data on the monthly grade of copper ore mined for the states of Arizona, New Mexico, and Utah. This data is kept for only the two most recent years. At the time of this writing this data has not yet been received. It was felt that a comparison, on a short-term basis, of copper price and grade of ore mined may provide an insight to this question.
Chapter X

Copper mining, unlike many industries, results in the depletion of the industries most valuable asset. For any given ore body, as each ton of ore is mined, there remains one less ton of ore to be mined. Handling costs are such that once a ton of copper bearing material is moved from the mine to a waste dump, the chances are that it will never be moved again. Thus, the copper contained in this waste material (copper content below the "cut-off") is lost for human consumption. (Continual developments in methods for recovering copper from waste dumps modifies this statement.)

The conservationist's tendencies of man are such that he will attempt to insure that all material containing sufficient copper to yield a profit will be treated as ore. This tendency could result in the mining of a lower grade ore during periods of high price and the mining of a higher grade ore during periods of low prices.

Such a policy, if followed, could tend to reinforce or amplify market fluctuations. One cause for price increases is a relative shortage of supply. Conservationism would tend to increase this relative shortage and perhaps give rise to further price increases. A similar reinforcement would take place under conditions of low price. Here, there would be

---

Chapter X

a relative oversupply, conservationism would tend to further increase the supply and perhaps result in even lower prices.

From the standpoint of production control, perhaps the easiest method for determining daily ore grade and ore "cut-off's" is to establish a grade "cut-off" based on long-range expected costs and prices. Once this "cut-off" is established it dictates the average grade of the ore body. From this point an engineering study would be used to develop a mining program for the most efficient ore removal. This program would dictate the average ore grade for each phase of exploitation. Depending upon the physical characteristics of the ore body, the average grade could remain uniform throughout the life of the mine or vary from one stage of development to another.

Mining programs are developed as described above, but in practice the short-term profit and loss statement tends to be a pressure that modifies the indicated practice. This pressure is probably most effective during periods of low price. If the program has been developed on the basis of a normal profit at a normal price, it can easily be seen that adherence to the program at a price below normal will result in short-term profits below normal. Such results, particularly if the low price should prevail for an extended period, are not desirable from the operators viewpoint. In a period of high prices, adherence to the mining program could yield
short-term profits high enough to permit conservationism to modify the program by mining a lower grade of ore than specified.

In summary, both pressure from conservationism (as defined) and pressure from short-term profit and loss statements would tend to result in a higher than normal ore grade under conditions of lower than normal price. Under conditions of higher than normal price, conservationism would tend to result in a lower than normal grade, while pressures from the short-term profit and loss statement would perhaps permit a lower grade ore to be mined.

Various computer runs to show the effect of different mining policies are planned. The results of these runs and the indicated conclusions will be discussed in a later chapter.

Fabricators Speculation

Fabricator speculation can also be accused of increasing price swings in copper industry. Some fabricators are believed to increase their purchase rates above normal purchase requirements as price tends to increase. The rationale for this, as far as the fabricator is concerned, is based on the past history of the industry. Many times copper has been in extremely short supply, those fabricators without sufficient raw material have been unable to produce at the desired rate and may well have had to reduce production rates. As possible

---

3 M. R. Herman, Jr. op. cit., p. 71 and 171.
Chapter X

shortages began to appear, the fabricator would increase his copper order rate in an attempt to provide sufficient raw material for his future operation. As the price levels off he would reduce excess buying, and even reduce his buying rate below normal as the price would start to fall. This reduction of purchase rates is based on the belief that he can purchase the necessary copper at some later date at a lower price.\textsuperscript{4}

If speculation is viewed from the price aspect alone, the same results emerge. The speculator attempts to buy at a low price so he can later sell at a higher price. As prices fall he postpones purchases so he can buy raw material at a lower price for goods he knows he will have to sell at a lower price. As long as the speculator must only compete with people facing the same markets (raw material and product) he does, he may improve his results through speculation. If his share of the market is too small for his speculation to have any effect on prices or supply, he can still improve his results, with respect to the industry, in competition with the industry and other industries.\textsuperscript{5}

\textsuperscript{4} M. R. Herman, Jr., \textit{op. cit.}, p. 100.
\textsuperscript{5} Ibid., pp. 75-76.
Speculation if present to a significant degree, can affect the industry. Raw material purchase rates would be above required at a time of relatively short supply. This excess demand would further increase the imbalance between supply and demand and tend to result in increased price and perhaps further excessive buying. On the downswing, purchase rates would be below required at a time of relatively excess supply. The reduced purchase rates would tend to increase the imbalance between supply and demand and thus tend to further depress prices. Speculation can, therefore, accentuate both price rise and fall. To the extent that it does so, speculation can handicap the industries competitive position with respect to other industries.

It is planned to make some computer runs to show the effect of various degrees of speculation. A later chapter will discuss these results.

**Larger Stocks of Refined Copper**

Increasing stocks of refined copper as a method for market stabilization was discussed during some of the February interviews. On the surface this would appear to be an attractive proposal for providing sufficient refined copper in periods of excessive demand. Aside from the cost of implementing such a proposal, there are several aspects to this proposal that tend to render such a method ineffective.⁶

⁶Ibid., p. 80.
The size of refined copper inventories are an important determinant of price. If refiners were to substantially increase the size of their finished goods inventory, this in itself would create a tremendous pressure for lower prices. In some way refiners would have to convince fabricators that the new inventories were "normal," such a feat is not easy to accomplish.

Refined copper is sold to fabricators in so many sizes and shapes that it would be difficult to determine composition of proper inventory. In periods of high demand, some of the inventory would possibly need to be reworked to provide the required products. Aside from the additional cost, re-handling capacity would be limited. This was the basis for one criticism of the government stockpiling program by one of the industry representatives interviewed.

Computer runs to show the effect of increased stocks of refined copper are not planned at this time because of the above criticisms and the definite shortage of research time. If time to make computer runs necessary to explore this possibility does present itself, the results will be discussed in a later chapter.
CHAPTER XI

SIMULATION RESULTS

The preceding chapters have discussed the industry and its representation by the primary copper industry model. This chapter will describe various computer runs and their results.

Model Verification

This aggregate model of the primary copper industry was not constructed with the idea that it would accurately represent all aspects of the primary copper industry. It was hoped, however, that the model response would represent aggregate industry response in a general way. If this representation could be achieved, then simulation studies could provide useful information about industry behavior.

Several computer runs were made to achieve model verification. The results of each run were analyzed. Particular attention was paid to the behavior of the price structure in evaluating the adequacy of industry representation. Indicated changes in model construction and/or the value of system constants were made prior to new computer runs.

The lack of research time did not permit model verification attempts to be made under all the environmental conditions that should be studied. Verification runs were only attempted under one set of conditions as follows:
Chapter XI

the runs were started in the steady state with all conditions normal, after a short period the copper user received in one calculation period an order pulse equal to one month's normal orders. The effect of this severe disturbance as it passed through the system was evaluated to judge model validity. Figure 14 shows model response under these conditions.  

The sudden increase in orders resulted in a sharp rise in copper price that was soon followed by a rapid fall. The sharp price rise resulted in decreased copper purchases by the user as indicated by refiners production, and in an immediate decrease in mine production later followed by an eventual increase in mine capacity. The decreased user purchases combined with the increase in mine capacity resulted in constantly increasing the stocks of refined copper and constantly decreasing copper price. This run also shows that more work is necessary to make the model mine time response faster to more accurately represent the industry.

Mine Production Policies

Several computer runs were made to determine the effect of different mining policies. The "M1" mine sector was made to operate with the policy of mining lower grade ore under conditions of lower than normal price. Figure

1 COMPUTER RUN No. 0638RB. A copy of these runs is on file in the Industrial Dynamics Department of the School of Industrial Management at Massachusetts Institute of Technology.
Figure 15

Modified Model Response to Pulse Input
New Mining Policy For "M1" Mine Sector
Chapter XI

15 shows the model response to the pulse input where the "M1" mine sector has half of the industry capacity and is operating under the new policy. A comparison of Figure 14 and Figure 15 shows that this policy used by half of the industry does result in reducing the severity of price fluctuations. Under the new policy users do not decrease copper purchases to the same extent they did previously, nor is mine capacity increased to as great an extent as before. Here again results indicate that additional work is necessary to achieve more realistic results. It is believed that the constants M2 and M12 should be increased.

Fabricator Speculation

One computer run was made with fabricator speculation removed. These results showed that fabricator speculation did tend to increase the magnitude of price fluctuation.

Planned Production for Refiners

The study of model performance indicated a possible means for achieving stabilization that had not previously been considered. Figure 14 shows that the refiner is the slowest sector to respond to indicated changes in production schedules, yet refiner production is perhaps the most critical in the determination of copper price. It was reasoned that if the refiners response could be achieved sooner, price fluctuation could be minimized.

2 COMPUTER RUN No. 0640RB
3 COMPUTER RUN No. 0642RB
Chapter XI

The discovery of this possibility came so late in the research period that it could not be examined properly. Substantial model modification would be required and time to make these changes was not available. In a crude attempt to provide a rough evaluation of this possibility, a mechanism for providing refiner production planned on the basis of maintaining a constant relationship between their finished goods inventory and their backlog of orders was devised. Appendix H contains a description of the changes made to the refiners sector to achieve this crude production control. Figure 16 shows the marked effect of the new production rule. Refiners increased production soon enough to prevent the copper price from reaching the high level experienced in other runs. This resulted in a smaller decrease of user orders for copper and minimized the increase of excess mine capacity. The significance of results obtained by industry simulation will be discussed in Chapter XII.

---

4 COMPUTER RUN NO. 0643RB
CHAPTER XII

CONCLUSIONS

Conclusions from this study are in two major areas; the study method and the primary copper industry. The study method will be evaluated first since it bears heavily on the validity of industry conclusions.

Industrial Dynamics as a Study Method

Industrial Dynamics can be a powerful management tool. It provides a method for carefully examining system behavior and the impact of policy decisions.

The use of Industrial Dynamics provides benefits in two stages, first in system investigation and model construction, and second in system simulation.

Perhaps the greatest benefit to be derived from the construction of an industry model is the general sharpening of the thinking process. It becomes necessary to "think through" the total effect of all things to be represented in the model. Decision function must be analyzed in their total relation to the rest of the industry or system under study before they can be adequately represented. This examination process necessary for model construction in itself creates a new awareness of the dynamic characteristics of business endeavor.

Industry or system simulation provides an abundance of information for analysis. In the process of constructing
Chapter XII

a model one gets the general "feel" for the relation of
various decision functions. Simulation provides quantitative
information concerning the relation of the various factors.
Management can study the need for or the effect of a change
in policy in various areas.

Industrial Dynamics does not by itself provide solu-
tions to business problems, it does provide a means for
generating information and knowledge as a foundation for
business decisions.

In discussing the goal of management science, Peter F.
Drucker said:

"The main goal of a management science must be to
enable business to take the right risk. Indeed, it must
be to enable business to take greater risks—by providing
knowledge and understanding of alternative risks and
alternative expectation:...

Industrial Dynamics is one method for providing information
and understanding necessary for risk taking.

Industry Conclusions

Conclusions about the primary copper industry reached
as a result of this study are of general nature. Conclusions
are based on information resulting from simulation. Detailed
conclusions concerning the model would be valid, but without

\footnotetext{1Peter F. Drucker, "Thinking Ahead: Potentials of
Management Science," Harvard Business Review, Volume 37,
Number 1 (Boston, January/February 1960).}
further work it would not be correct to directly project these conclusions to the industry.

An attempt was made to construct an aggregate model that would represent the primary copper industry. In view of time requirements many aspects of the industry were not separately represented. Aggregation in itself tends to hide some behavioral aspects that may be highly important to various organizations within the industry. As one example, government pressures in some parts of the world for continued production despite falling prices are of great importance to the companies concerned. This pressure is not separately represented in the model but becomes a part of the "stickiness" in changing mine production. It is believed, however, that model results can well indicate fruitful areas for further study.

Computer runs showed that use of a mining policy that permitted the mining of higher grades of ore during periods of relative high price and the mining of a lower grade of ore during periods of relatively low price tended to reduce price fluctuation. Under conditions of the test input the sudden increase of user orders and the resultant increased demand for copper caused first a sharp rise and then a rapid decline in copper price. This price disturbance resulted in a decreased use of copper by the user and an increase in mine capacity. This imbalance between supply and demand resulted in excess stocks of refined copper. When one half of the
Chapter XII

mines, in the model, followed the new policy the imbalance between supply and demand was less than that created when all mines operated on the same policy of mining lower grade ore under conditions of higher price.

Results clearly indicated that the model mine did not respond soon enough to accurately represent the industry. It is believed that the mine production change factors M2 and M12 should be increased. With a more rapid mine response it is believed that the proposed stabilization method would be more effective.

Fabricator speculation as such will not be discussed in any detail because of the difficulty for the industry to directly control this. Simulation studies showed that speculation did tend to exaggerate price fluctuation. Perhaps speculation is best eliminated by removing the need or reasons for its existence.

The evaluation of simulation results did reveal one possible method for achieving a marked degree of price stabilization. Time was not available to study this possibility in detail. Reference to Figure 16 in comparison with Figure 14 will show the marked degree of stabilization achieved by introducing a policy of planning refiner production for finished goods inventory. In view of time considerations, this concept was represented in a very crude fashion. The
Chapter XII

refiner's raw material inventory was arbitrarily initially increased three times and then allowed to "float". Refiner production was planned to maintain a constant relationship with refiners unfilled orders. There are several objections that can be raised to the production method as presented in the model. First, if this practice were to be followed by all refiners, it would not be long before the existence of the raw material inventory would be recognized by fabricators and thus come to play an important part of price determination. This would of course tend to defeat the stabilization control achieved. Second, the investment necessary to maintain this raw material inventory is in itself a valid objection.

On the other hand, such a proposal eliminates one of the major objections voiced against increasing stocks of refined copper. Reprocessing into the desired shapes would not be necessary, thus reprocessing costs that might be necessary for maintaining larger stocks of refined copper would be eliminated by this proposal. In view of the simulation results, I believe the adoption of such a policy by only a small part of the industry could possibly result in marked degree of price stabilization for the entire industry. Adoption by a small part of the industry might reduce or eliminate the tendency for refiner raw material inventory to become an important part of the price determining mechanism.
Chapter XII

Refiner production for the part of the industry adopting this policy could be made a function of desired finished goods inventory modified by the size of raw material inventory on hand. If properly conceived some control on the investment necessary for instituting such a policy would then be achieved.

Recommendations for Further Study

The primary copper industry model in its present form provides only a rough representation of the industry. Additional work necessary for this model to more closely represent the industry is indicated. For example, the size of some inventories is strongly influenced by tax considerations at year's end.\(^2\) This factor is not represented in the model, and therefore the effect of short-term accounting periods is not shown. Perhaps this influence plays an important part in creating or sustaining price fluctuation. This aspect of industry behavior, and others not included, could well be made a part of the model structure through additional work.

Additional investigation of the proposed stabilization methods is certainly warranted in view of the results obtained to date.

\(^2\) An industry representative in the fabricators sector emphasized the importance of LIFO in the determination of inventory size.
Chapter XII

Additional studies could show the industry effect if a small portion of the industry were to adopt a combination of both proposed methods. This work could also show the effect of such action on the segment of the industry adopting these policies. Further work may well result in the discovery of more desirable methods for obtaining stabilization and growth.

The model as constructed was intended to represent the industry only under a limited number of circumstances. For example, if price restrictions were to be imposed, the model as presently constructed could not properly represent the industry. The limitations of time and not the framework of Industrial Dynamics were responsible for the many simplifying assumptions used. Additional work could well result in a more meaningful model that would provide more valuable information about industry behavior.
the reader in visualizing the following equations in the proper relationships. The letter "U" at the beginning of a symbol serves to identify the symbol with the user's sector.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>( \text{UUFO.} K = \text{UUFO.} J \div (\text{DT})(\text{USO.} JK - \text{UOS.} JK) )</td>
<td>( \text{UFGI.} K = \text{UFGI.} J \div (\text{DT})(\text{UGTFI.} JK - \text{UOS.} JK) )</td>
<td>( \text{UOS.} KL = \text{UOST.} K \text{ if } \text{UFGI.} K \geq \text{UFGIM} ) ( = \text{UFGIN.} K \text{ if } \text{UFGI.} K &lt; \text{UFGIM} )</td>
<td>( \text{UFGIN.} K = \text{UFGI.} K/3 )</td>
<td>( \text{UOST.} K = \text{UUFO.} K/\text{UOSD.} K )</td>
<td>( \text{UOSD.} K = (1/\text{UFGI.} K)[(U1)(\text{UFGI.} K) + (U2)(\text{UFGID.} K)] )</td>
<td>( \text{UFGID.} K = (\text{EC})(\text{USSC})(\text{USS.} K) )</td>
<td>( \text{USS.} K = \text{USS.} J \div (\text{DT})(1/\text{U3})(\text{USO.} JK - \text{USS.} J) )</td>
<td>( \text{UPD.} K = \text{USS.} K \div (1/\text{U4})(\text{UFGID.} K - \text{UFGI.} K) )</td>
<td>( \text{UGIP.} K = \text{UGIP.} J \div (\text{DT})(\text{UPS.} JK - \text{UGTFI.} JK) )</td>
<td>( \text{UGTFI.} KL = \text{DELAY} 3 (\text{UPS.} JK, \text{Del} 1) )</td>
<td>( \text{UPS.} KL = \text{UPDT.} K \text{ if } \text{URMI.} K \geq \text{URMIM} ) ( = \text{UPRML.} K \text{ if } \text{URMI.} K &lt; \text{URMIM} )</td>
</tr>
</tbody>
</table>
Negative production limit, raw material inventory - lbs/wk

\[ \text{UPRML.K} = \frac{\text{URMI.K}}{3} \]

Raw material inventory - lbs

\[ \text{URMI.K} = \text{URMI.J} + (\text{DT})(\text{UCD.JK} + \text{DSM.JK} - \text{UPS.JK}) \]

Desired raw material inventory - lbs

\[ \text{URMID.K} = (\text{EC})(\text{UPDT.K})(\text{UPDC}) \]

Long range copper use - (percentage of total purchases) - \%

\[ \text{ULRCP.K} = \text{TABLE(UFUN, PRATG.K)} \]

ULRCP.K is found by entering a table (UFUN) with
the argument PRATO.K. This table was derived from

Figure 4, Chapter 4.

Price ratio used to enter table UFUN - no units

\[ \text{PRATO.K} = 1 + \text{UPA.K} \]

Copper price - smoothed - \$/lb

\[ \text{UCPS.K} = \text{UCPS.J} + (\text{DT})(1/U5)(\text{RCP.J} - \text{UCPS.J}) \]

User, percent of copper to be purchased - \%

\[ \text{UPCP.K} = \frac{(\text{ULRCP.K})(\text{USRCP.K})}{\text{USRCP.K}} \]

Short-run copper purchases modifier - no units

\[ \text{USRCP.K} = \frac{\text{FOSDN/FOSD.K}}{\text{USRCP.K}} \]

Raw material purchases desired - lbs/wk

\[ \text{URMPD.KL} = \text{UPDT.K} + (1/U7)(\text{URMID.K} - \text{URMI.K}) \]

Level of raw material to be purchased - lbs

\[ \text{ULRMP.K} = \text{ULRMP.J} + (\text{DT})(\text{URMPD.JK} - \text{UCPDI.JK} - \text{USP.JK}) \]

Level of orders for substitute material - lbs

\[ \text{OFSM.K} = \text{OFSM.J} + (\text{DT})(\text{USP.JK} - \text{DSM.JK}) \]

Delivery of substitute materials - lbs/wk

\[ \text{DSM.KL} = \text{DELAY 3 (USP.JK, DEL2)} \]

Copper ordered - lbs/wk

\[ \text{UCO.KL} = \text{ULCPD.K} + (1/8)(\text{URPLI.K} - \text{UAPLI.K}) \]
Required pipeline inventory - lbs

\[ URPLI.K = (UCOA.K)(DEL4 + DEL5 + FOSD.K) \]

Copper ordered - average - lbs/wk

\[ UCOA.K = UCOA.J + (DT)(1/U8)(UCO.JK - UCOA.J) \]

Level of copper ordered - lbs

\[ ULCO.K = ULCO.J + (DT)(UCO.JK - FSO.JK) \]

Level of copper in transit - lbs

\[ ULCT.K = ULCT.J + (DT)(FOS.JK - UCD.JK) \]

Copper delivered - lbs/wk

\[ UCD.KL = DELAY 3(FOS.JK, DEL5) \]

Copper purchases desired - lbs/wk

\[ UCPDI.KL = (ULRMP.K)(UPCP.K) \]

Substitute purchases - lbs/wk

\[ USP.KL = ULRMP.K - UCPDI.JK \]

Level of copper purchases desired - lbs

\[ ULCPD.K = ULCPD.J + (DT)(UCPDI.JK - UCO.JK) \]

Actual pipeline inventory - lbs

\[ UAPLI.K = ULCT.K + FUFO.K + ULCO.K \]

Purchases desired - information - lbs/wk

\[ UPDT.K = UMPP \quad \text{if} \quad UPD.K \geq UMPP \]
\[ = UPD.K \quad \text{if} \quad UPD.K < UMPP \]

Purchase adjustment - no units


The purpose of this function is to shift the purchase relation curve Figure 4, Chapter 4, to represent "stickiness". This function is described in more detail at the end of this appendix.

Purchase adjustment factor 1 - no units

\[ UPAF1.K = 0 \quad \text{if} \quad U11 \geq UPRF*2.K \]
\[ = UPRAP.K \quad \text{if} \quad U11 < UPRF*2.K \]
Purchase adjustment factor 2 - no units

:\text{UPAF2.K} = 0 \text{ if } \text{UPRF*2.K} \geq \text{U12} \quad 1137
\text{=} \text{UPRAN.K} \text{ if } \text{UPRF*2.K} < \text{U12}

Price ratio adjustment - positive - no units
\text{UPRAP.K} = 0 \quad \text{if } 0 \geq \text{UPDIF.K} \quad 138
\text{=} \text{UPDIF.K} \text{ if } 0 < \text{UPDIF.K}

Price ratio adjustment - negative - no units
\text{UPRAN.K} = 0 \quad \text{if } \text{UPDIF.K} \geq 0 \quad 1138
\text{=} \text{UPDIF.K} \text{ if } \text{UPDIF.K} < 0

Price ratio difference - no units
\text{UPDIF.K} = \text{UPRF*2.K} - \text{UPRF*3.K} \quad 2138

Price ratio file - no units
\text{UPRF} = \text{BOXLIN}(3.1) \quad 3138

This is a device for storing information beyond the normal calculation periods. Here the file keeps records for 3 periods—each file is changed each week.

Price ratio file - drawer 1 - no units
\text{UPRF*1.K} = \text{UPRF*1.J} \div (\text{DT}(\text{ULRPR},J \div 0)) \quad 4138

This is the first drawer of the file—each week its contents will be moved to the second drawer, and the second drawer's contents will be moved to the third drawer. The contents of the third drawer will be discarded. Any drawer contents are available for use during model simulation.

Long run price ratio - no units
\text{ULRPR.K} = \text{UCPS.K}/\text{USPS} \quad 139
The following are constants used in this sector.

**Production delay - wks**

\[ \text{DEL } 1 = 1.2 \]

**Competition delay in filling orders - wks**

\[ \text{DEL } 2 = 9.6 \]

**Order placing delay - wks**

\[ \text{DEL } 4 = 1.2 \]

**Transit delay to user - wks**

\[ \text{DEL } 5 = 2 \]

**Economic conditions - no units**

\[ \text{EC } = 1 \]

**Fixed shipping delay - wks**

\[ \text{U1} = 8 \]

**Additional shipping delay with balanced inventory - wks**

\[ \text{U2} = .8 \]

**Time over which sales are averaged - wks**

\[ \text{U3} = 12 \]

**Time to balance finished inventory - wks**

\[ \text{U4} = 12 \]

**Time over which copper price is averaged - wks**

\[ \text{U5} = 12 \]

**Time to balance raw material inventory - wks**

\[ \text{U7} = 8 \]

**Time over which copper purchases are averaged - wks**

\[ \text{U8} = 12 \]

**Sales in finished inventory - wks**

\[ \text{USSC} = 8 \]

**Substitute price - $/lb**

\[ \text{USPS} = .3 \]
Minimum desired finished inventory - lbs
UFGIM = 873,330,000

Production in raw material inventory - wks
UPDC = 4

Minimum desired raw material inventory - lbs
URMIM = 131,000,000

Maximum production possible - lbs/wk
UMPP = 327,500,000

Weighting factor in UPA - no units
U10 = 100

Price ratio threshold for increasing PRATO - no units
U11 = 1.03

Price ratio threshold for increasing PRATO - no units
U12 = .97

The equations used to determine the value of PRATO essentially accomplish the following: PRATO is increased, and copper purchases decreased, when the ratio (ULRPR) of long-run average copper price to substitute price is above 1.03 and the ratio is increasing. PRATO will remain constant at this point until the ratio ULRPR falls below .97, at this point PRATO will begin to decrease and will continue to decrease so long as the ratio is falling. If the ratio should remain constant or start to rise, PRATO will remain constant until ULRPR exceeds 1.03 and is rising. This mechanism represents the "stickiness" of the user in selecting his raw material.

DELAY 3 refers to a third order delay. For example, the equation UGTFI.KL = DELAY 3(UPS.JK, DEL 1) states that
UGTFI, KL is the delayed flow out of a level or reservoir filled by UPS, JK. The average delay is equal to DEL1.¹

Appendix B

Mathematical Representation of the Aggregate Fabricator

The equations for representing the behavior of the aggregate primary copper fabricator are contained in the following list. The use of the letter F as the first letter of various symbols serves to identify the fabricator. Reference to Figure 5, Chapter 5, will assist the reader in visualizing the relationships of the following equations.

Equation No.

Fabricator, sales orders - lbs/wk
\[ \text{FSO}_K = \text{DELAY} 3 (\text{UCO}_{JK}, \text{DEL4}) \] 128

Fabricator sales orders, the order input to this sector, is the output of a third order delay in the users sector. The input to this delay is user copper ordered.

Unfilled orders backlog - lbs
\[ \text{FUFO}_K = \text{FUFO}_J + (\text{DT})(\text{FSO}_{JK} - \text{FOS}_{JK}) \] 140

Finished good inventory - lbs
\[ \text{FFGI}_K = \text{FFGI}_J + (\text{DT})(\text{FGTFI}_{JK} - \text{FOS}_K) \] 141

Orders shipped - lbs/wk
\[ \text{FOS}_{KL} = \text{FOS}_{JK} \text{ if } \text{FFGI}_K \geq \text{FFGIM} \]
\[ = \text{FFGIN}_K \text{ if } \text{FFGI}_K < \text{FFGIM} \] 142

Negative Limit - finished inventory - lbs/wk
\[ \text{FFGIN}_K = \text{FFGI}_K/3 \] 143

Order shipping trial - lbs/wk
\[ \text{FOST}_K = \text{FUFO}_K/\text{FOSD}_K \] 144

Order shipping delay - wks
\[ \text{FOSU}_K = (1/\text{FFGI}_K)[(F1)(\text{FFGI}_K) + (F2)(\text{FFGID}_K)] \] 145
Desired finished goods inventory - lbs
\[ FFGID.K = (EC1)(FSSC)(FSS.K) \]

Smoothed sales - lbs/wk
\[ FSS.K = FSS.J + (DT)(1/F3)(FSO.JK - FSS.J) \]

Goods in process - lbs
\[ FGIP.K = FGIP.J + (DT)(FPS.JK - FGTFI.JK) \]

Goods to finished inventory - lbs/wk
\[ FGTFI.KL = DELAY 3(FPS.JK, DEL6) \]

Delayed production started - lbs
\[ FDPS.K = FDPS.J + (DT)(FPTBS.JK - FPS.JK) \]

Production started - lbs/wk
\[ FPS.KL = DELAY 3(FPTBS.JK, DEL7) \]

Production desired - lbs/wk
\[ FPD.K = FSS.K + (1/F4)(FFGID.K - FFGI.K) \]

Raw material inventory - lbs
\[ FRMI.K = FRMI.J + (DT)(FCD.JK - FPS.JK) \]

Raw material purchases desired - lbs/wk
\[ FRMPD.K = FDPT.K + (1/F6)(FRMID.K - FRMI.K) \]

Desired raw material inventory - lbs
\[ FRMID.K = (FDPT.K)(FSM.K)(FS) \]

Speculation modifier - no units
\[ FSM.K = 1 + (1/F7)(FDIFF*2.K - FDIFF*3.K) \]

Price difference file - no units
\[ FDIFF = BOXLIN (3,1) \]
This file has 3 drawers and contents are changed each week.

First drawer of price difference file - no units

Price file - no units
\[ PRICE = BOXLIN (3,1) \]
This week's average price - $/lbs

\[
\text{PRICE}^{*1.1}_K = \text{PRICE}^{*1.1}_J + (\text{DT}) (\text{RCP}^{.1}_J + 0) 
\]

Copper ordered - lbs/wk

\[
\text{FCO}^{.1}_K = \text{FRMPD}^{.1}_K + (1/12) (\text{FRPL}^{.1}_K - \text{FAPL}^{.1}_K) 
\]

Required pipeline inventory - lbs

\[
\text{FRPL}^{.1}_K = (\text{FCOA}_K)(\text{DEL}^{.1}_8 + \text{DEL}^{.1}_9 + \text{ROSD}_K) 
\]

Actual pipeline inventory - lbs

\[
\text{FAPL}^{.1}_K = \text{FLCT}_K + \text{RUFO}_K + \text{FLCO}_K 
\]

Copper ordered - average - lbs/wk

\[
\text{FCOA}_K = \text{FCOA}_J + (\text{DT})(1/8) (\text{FCOJ}_K - \text{FCOA}_J) 
\]

Level of copper ordered - lbs

\[
\text{FLCO}_K = \text{FLCO}_J + (\text{DT})(\text{FCOJ}_K - \text{RSOJ}_K) 
\]

Level of copper in transit - lbs

\[
\text{FLCT}_K = \text{FLCT}_J + (\text{DT})(\text{ROSO}_K - \text{FCD}_K) 
\]

Copper delivered - lbs/wk

\[
\text{FCD}_K = \text{DELAY}^{3}_3 (\text{ROSO}_K, \text{DEL}^{.1}_9) 
\]

Production to be started - lbs/wk

\[
\text{FPTBS}_K = \text{FAMPP}_K \text{ if } \text{FPW}_K \geq \text{FAMPP}_K \\
= \text{FPW}_K \text{ if } \text{FPW}_K \leq \text{FAMPP}_K 
\]

Adjusted maximum production possible - lbs/wk

\[
\text{FAMPP}_K = (\text{FMPP})(\text{MLRSA}_K) 
\]

Production limit, raw material - lbs/wk

\[
\text{FPRML}_K = \text{FRMI}_K/3 
\]

Production wanted - lbs/wk

\[
\text{FPW}_K = \text{FDP}_K \text{ if } \text{FRMI}_K \geq \text{FRMM}_K \\
= \text{FPRML}_K \text{ if } \text{FRMI}_K < \text{FRMM}_K 
\]

Purchases Desired, information - lbs/wk

\[
\text{FDPT}_K = \text{FAMPP}_K \text{ if } \text{FPD}_K \geq \text{FAMPP}_K \\
= \text{FPD}_K \text{ if } \text{FPD}_K < \text{FAMPP}_K 
\]
The following are constants used in the fabricators sector.

Production delay - wks
   DEL6 = 1.2

Delay in changing production - wks
   DEL7 = 1.2

Order delay - wks
   DEL8 = 1.2

Transit delay - wks
   DEL9 = 1.2

Fixed shipping delay - wks
   F1 = 6

Additional shipping delay with balance inventory - wks
   F2 = .6

Time over which sales are averaged - wks
   F3 = 12

Time to balance finished inventory - wks
   F4 = 12

Production time in raw material inventory - wks
   F5 = 5

Time to balance raw material inventory - wks
   F6 = 8

Speculation weight - no units
   F7 = .05

Time to average orders - wks
   F8 = 12

Normal shipping delay - wks
   FOSDN = 6.6

Sales in finished inventory - wks
   FSSC = 8
Minimum desired finished inventory - lbs

FFGIM = 131,000,000

Maximum production possible - lbs/wk

FMPP = 180,000,000

Minimum desired raw material inventory - lbs

FRMIM = 65,500,000
Appendix C

Mathematical Representation of the Aggregate Refiner

The following equations are used to represent the behavior of the aggregate primary copper refiner. The price generating mechanism for the model is a part of the refiners sector and is represented by the appropriate equations in the following list. The first letter R identifies the symbols as a part of the refiners sector reference to Figure 6, Chapter 6.

Refiners sales orders - lbs/wk

\[ RSO_{KL} = \text{DELAY} \ 3(FCO\_JK, \text{DEL}8) \]

These orders serve as the order input to this sector.
The input to this third order delay is fabricators copper ordered.

Unfilled order backlog - lbs

\[ RUFO\_K = RUFO\_J \times (DT)(RSO\_JK - ROS\_JK) \]

Finished goods inventory - lbs

\[ RFGI\_K = RFGI\_J + (DT)(RGTFI\_JK - ROS\_JK) \]

Orders shipped - lbs/wk

\[ ROS\_KL = ROST\_K \text{ if } RFGI\_K \geq RFGIM \]
\[ = RFGIN\_K \text{ if } RFGI\_K < RFGIM \]

Negative limit - finished goods inventory - lbs/wk

\[ RFGIN\_K = RFGI\_K/3 \]

Order shipping trial - lb/wk

\[ ROST\_K = RUFO\_K/ROSD\_K \]

Order shipping delay - wks

\[ ROSD\_K = (1/RFGI\_K)\left[(R1)(RFGI\_K) + (R2)(RFGID\_K)\right] \]
Finished goods inventory desired - lbs  
\[ \text{RFGID.K} = (\text{ECR})(\text{RSSC})(\text{RSS.K}) \]

Smoothed sales - lbs/wk  
\[ \text{RSS.K} = \text{RSS.J} + (\text{DT})(1/R3)(\text{RSO.JK} - \text{RSS.J}) \]

Goods in process - lbs  
\[ \text{RGIP.K} = \text{RGIP.J} + (\text{DT})(\text{RPS.JK} - \text{RGTFI.JK}) \]

Goods to finished goods inventory - lbs/wk  
\[ \text{RGTFI.KL} = \text{DELAY 3} (\text{RPS.JK, DEL10}) \]

Production started - lbs/wk  
\[ \text{RPS.KL} = \text{RAMPP.K} \text{ if } \text{RPW.K} \geq \text{RAMPP.K} \]
\[ = \text{RPW.K} \text{ if } \text{RPW.K} < \text{RAMPP.K} \]

Adjusted maximum production possible - lbs/wk  
\[ \text{RAMPP.K} = (\text{RMPP})(\text{MLRSA.K}) \]

Production wanted - lbs/wk  
\[ \text{RPW.K} = \text{RRMI.K/R4} \]

Raw material inventory - lbs  
\[ \text{RRMI.K} = \text{RRMI.J} + (\text{DT})(\text{RCD.JK} - \text{RPS.JK}) \]

Level of copper in transit - lbs  
\[ \text{RLCT.K} = \text{RLCT.J} + (\text{DT})(\text{SOS.JK} - \text{RCD.JK}) \]

Copper delivered - lbs/wk  
\[ \text{RCD.KL} = \text{DELAY 3(SOS.JK,DEL12)} \]

Copper price - $/lb  
\[ \text{RCP.K} = \text{TABLE(RFUN,RWRAT.K,.2,3.5,.1)} \]

Supply, demand ratio - no units  
\[ \text{RRATO.K} = (1/R7)[(R5)(\text{RSSD.K}) + (R6)(\text{RRSD.K})] \]

Static - supply, demand - no units  
\[ \text{RSSD.K} = (\text{RUFO.K})(5)(1)/[(\text{RFGI.K})(7)(1)] \]

Rate - supply, demand - no units  
\[ \text{RRSD.K} = \text{RSS.K/CPS.JK} \]

Weighted - supply, demand ratio - no units  
\[ \text{RWRAT.K} = (1/R10)[(R8)(\text{RRATO.K}) + (R9)(R11)] \]
The following are the constants used in the refiner's sector.

Production delay - wks
   DEL 10 = 3.5

Transit delay - wks
   DEL 12 = 4

Fixed shipping delay - wks
   R 1 = 6.7

Additional delay with balanced inventory - wks
   R2 = .3

Time to average sales - wks
   R 3 = 12

Production in raw material inventory - wks
   R 4 = 3

Weighting factor in price mechanism - no units
   R 5 = 4

Weighting factor in price mechanism - no units
   R 6 = 2

Normal copper price - $/lb
   RCPN = .3

Sales in finished goods inventory - wks
   RSSC = 5

Maximum production possible - lbs/wk
   RMPP = 162,000,000

Minimum finished goods inventory - lbs
   RFGIM = 131,000,000

Weighting factor in price mechanism - no units
   R 8 = 1

Weighting factor in price mechanism - no units
   R 9 = 2

Weighting factor in price mechanism - no units
   R 11 = 1
Appendix D

Mathematical Representation of the Aggregate Smelter

The aggregate smelter, as represented in the model is defined by the following list of equations. The letter "\( \mathbb{K} \)" used as the first letter of a symbol serves to identify it with the smelter sector. Reference to Figure 9, Chapter 7, will enable the reader to visualize the various equations in the proper relationship.

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>( \text{Orders shipped - lbs/wk} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SOS.KL} = \frac{SFGI.K}{SOSD.K} )</td>
</tr>
<tr>
<td>211</td>
<td>( \text{Order shipping delay - wks} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SOSD.K} = \frac{1}{SFGIN}[(S1)(SFGIN) + (S2)(SFGI.K)] )</td>
</tr>
<tr>
<td>212</td>
<td>( \text{Finished goods inventory - lbs} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SFGI.K} = SFGI.J + (DT)(SGTFI.JK - SOS.JK) )</td>
</tr>
<tr>
<td>213</td>
<td>( \text{Goods in process - lbs} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SGIP.K} = SGIP.J + (DT)(SPS.JK - SGTFI.JK) )</td>
</tr>
<tr>
<td>214</td>
<td>( \text{Goods to finished goods inventory - lbs/wk} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SGTFI.KL} = \text{DELAY 3 (SPS.JK, DEL 13)} )</td>
</tr>
<tr>
<td>215</td>
<td>( \text{Production started - lbs/wk} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SPS.KL} = \text{SAMPP.K if SPW.K} \succ \text{SAMPP.K} )</td>
</tr>
<tr>
<td></td>
<td>( = \text{SPW.K if SPW.K} \prec \text{SAMPP.K} )</td>
</tr>
<tr>
<td>216</td>
<td>( \text{Adjusted maximum production possible - lbs/wk} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SAMPP.K} = (SMPP)(MLRSA.K) )</td>
</tr>
<tr>
<td>217</td>
<td>( \text{Production wanted - lbs/wk} )</td>
</tr>
<tr>
<td></td>
<td>( \text{SPW.K} = \frac{SRMI.K}{S3} )</td>
</tr>
</tbody>
</table>
Raw material inventory - lbs
\[ SRMI.K = SRMI.J + (DT)(SCD.JK - SPS.JK) \]

Level of copper in transit - lbs
\[ SLCT.K = SLCT.J + (DT)(COS.JK - SCD.JK) \]

Copper delivered - lbs/wk
\[ SCD.KL = DELAY 3 (CGS.JK, DEL15) \]

The following are the constants for the smelters sector.

Production delay - wks
\[ DEL\ 13 = 1.5 \]

Transit delay - wks
\[ DEL\ 15 = 2 \]

Fixed shipping delay - wks
\[ S\ 1 = 1 \]

Additional delay with normal inventory - wks
\[ S\ 2 = .1 \]

Production in raw material inventory - wks
\[ S\ 3 = 4 \]

Maximum production possible - lbs/wk
\[ SMPP = 170,000,000 \]
Appendix E

Mathematical Representation of the Aggregate Concentrator

The following equations define the aggregate concentrator as represented in the model of the primary copper industry. The letter "C" used as the first letter of a symbol serves to identify the symbol with the concentrators sector. Reference to Figure 10, Chapter 7, will enable the reader to visualize the relationships between the following equations.

<table>
<thead>
<tr>
<th>Equation No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>230</td>
</tr>
<tr>
<td>231</td>
</tr>
<tr>
<td>232</td>
</tr>
<tr>
<td>233</td>
</tr>
<tr>
<td>234</td>
</tr>
<tr>
<td>235</td>
</tr>
<tr>
<td>236</td>
</tr>
<tr>
<td>237</td>
</tr>
</tbody>
</table>

Orders shipped - lbs/wk

\[
COS,KL = CFGI,K/COSD,K
\]

Order shipping delay - wks

\[
COSD,K = (1/CFGIN)[C1(CFGIN) + (C2(CFGI,K))]
\]

Finished goods inventory - lbs

\[
CFGI,K = CFGI,J + (DT)(CGTFI,JK - COS,JK)
\]

Goods in process - lbs

\[
CGIP,K = CGIP,J + (DT)(CPS,JK - CGTFI,JK)
\]

Goods to finished goods inventory - lbs/wk

\[
CGTFI,KL = \text{DELAY 3 (CPS,JK,DEL 16)}
\]

Production started - lbs/wk

\[
\begin{align*}
CPS,KL &= \text{CAMPP,K if CPW,P} > \text{CAMPP,K} \\
&= \text{CPW,K if CPW,P} < \text{CAMPP,K}
\end{align*}
\]

Adjusted maximum production possible - lbs/wk

\[
\text{CAMPP,K = (CMPP)(MLRSA,K)}
\]

Production wanted - lbs/wk

\[
\text{CPW,K = CRMI,K/C3}
\]
Raw material inventory - lbs

\[ CRMI.K = CRMI.J \div (DT)(MCP,JK + M1CP.JK - CPS.JK+O+O+O) \]

The constants used in the concentrator sector follow.

Production delay - wks

\[ \text{DEL 16} = 1.2 \]

Fixed shipping delay - wks

\[ C_1 = 1 \]

Additional delay with normal inventory - wks

\[ C_2 = 0.1 \]

Production in raw material inventory - wks

\[ C_3 = 0.1 \]

Maximum production possible - lbs/wks

\[ \text{CMPP} = 150,000,000 \]
Appendix F

Mathematical Representation of the "M" Mine Sector

The behavior of the "M" mine sector is defined by the following equations. The use of the letter "M" as the first letter of a symbol serves to identify the symbol with the "M" mine sector. Reference to Figure 12, Chapter 8, will assist the reader in visualizing the proper relationship of the following equations.

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Goods in process - lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>MGIP.K = MGIP.J + (DT)(MPS.JK - MCP.JK)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Copper produced - lbs/wk</th>
</tr>
</thead>
<tbody>
<tr>
<td>251</td>
<td>MCP.KL = DELAY 3 (MPS.JK, DEL18)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Production started - lbs/wk</th>
</tr>
</thead>
<tbody>
<tr>
<td>252</td>
<td>MPS.KL = MOPP.K if MPW.K \geq MMPP</td>
</tr>
<tr>
<td></td>
<td>MPTBS.K if MPW.K \leq MMPP</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Production wanted - lbs/wk</th>
</tr>
</thead>
<tbody>
<tr>
<td>253</td>
<td>MPW.K = MPW,J + (DT)(1/M3)(MPCF.J - M4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Average copper price - $/lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>254</td>
<td>MACP.K = MACP.J + (DT)(1/M1)(RCP.J - MACP,J)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Production desired - lbs/wk</th>
</tr>
</thead>
<tbody>
<tr>
<td>255</td>
<td>MPD.K = TABLE(MFUN, MACP.K, .15, .8, .05)</td>
</tr>
</tbody>
</table>

This equation says that MPD.K is found by entering a table (MFUN) with the argument MACP.K. This table is derived from Figure 11, Chapter 8.

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Production change factor - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>256</td>
<td>MPCF.K = (M2)(MAPD.K - MPW.K)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation No.</th>
<th>Production to be started - lbs/wk</th>
</tr>
</thead>
<tbody>
<tr>
<td>257</td>
<td>MPTBS.K = (MPW.K)(MOY.K)</td>
</tr>
</tbody>
</table>
Ore yield - no units
\[ \text{MOY.K} = (1/M7)[(M5)(MRATO.K) + (M6)(1)] \]

Price ratio - no units
\[ (M33) \]

This device is provided to change the behavior of the "M" mine sector in the model. Two of the constants M31, M32, M33 are always zero, while the other is set equal to one. With proper use of this function the mine can have different response to price changes.

One yield production limit - lbs/wk
\[ \text{MOPP.K} = (MAMPP.K)(\text{MOY.K}) \]

Adjusted maximum production - lbs/wk
\[ \text{MAMPP.K} = (\ast \text{ PP})(\text{MLRSA.K}) \]

Profit - $
\[ \text{MPROF.K} = \text{MPROF.J} + (DT)(1/2)(\text{MR.JK} - \text{MCOST.JK}) \]

Revenue - $
\[ \text{MR.KL} = (\text{MCP.JK})(\text{RCP.K}) \]

Cost - $
\[ \text{MCOST.KL} = (\text{RCPN})(\text{MPS.JK})/\text{MOY.K} \]

Adjusted production desired - lbs/wk
\[ \text{MAPD.K} = (\text{MPD.K})(\text{MLRSA.K}) \]

The constants for this sector follow.

Production delay - wks
\[ \text{DEL} = 1, 2 \]

Time to average price - wks
\[ \text{M} = 24 \]

Weighting factor in production change - no units
\[ \text{M} = .0125 \]
Weighting factor for production wanted - no units

\[ M_{3} = 0.1 \] 502

Weighting factor for production wanted - no units

\[ M_{4} = 0 \] 503

Weighting factor for ore yield

\[ M_{5} = 1 \] 504

Weighting factor for ore yield - no units

\[ M_{6} = 2 \] 505

Maximum production possible - lbs/wk

\[ \text{MMPP} = 75,000,000 \] 511

Weighting factor for price ratio - no units

\[ M_{31} = 1 \] 518

Weighting factor for price ratio - no units

\[ M_{32} = 0 \] 519

Weighting factor for price ratio - no units

\[ M_{33} = 0 \] 520
Mathematical Representation of the "M1" Mine Sector

The following equations describe the behavior of the "M1" mine sector of the primary copper industry model. The use of "M1" as the first part of various symbols serves to identify the symbol with the "M1" mine sector. The mechanism used to provide for shifting of the mine supply curves for both mine sectors is contained in this list, as is the profit function for the industry. Reference to Figure 13, Chapter 8, will assist the reader in visualizing the proper relationships of the various equations.

**Goods in process - lbs**

\[ M_{1GIP,K} = M_{1GIP,J} + (DT)(M_{1PS,JK} - M_{1CP,JK}) \]  

Equation No. 270

**Copper produced - lbs/wk**

\[ M_{1CP,KL} = \text{DELAY 2} (M_{1PS,JK}, \text{DEL } 28) \]  

Equation No. 271

**Production started - lbs/wk**

\[ M_{1PS, KL} = M_{1OPP,K} \text{ if } M_{1PW,K} \geq M_{1MPP} \]
\[ = M_{1PTB,K} \text{ if } M_{1PW,K} < M_{1MPP} \]  

Equation No. 272

**Production wanted - lbs/wk**

\[ M_{1PW,K} = M_{1PW,J} + (DT)(1/M_{13})(M_{1PCF,J} - M_{14}) \]  

Equation No. 273

**Production desired - lbs/wk**

\[ M_{1PD,K} = \text{TABLE}(M_{1FUN}, M_{ACP,K}, .15, .8, .05) \]  

Equation No. 274

This says that \( M_{1PD,F} \) is found by entering table \( M_{1FUN} \) with the argument \( M_{ACP,K} \). This table is derived from the supply curve in Figure 11, Chapter 8.

**Production change factor - no units**

\[ M_{1PCF,K} = (M_{12})(M_{1APD,K} - M_{1PW,K}) \]  

Equation No. 275

**Production to be started - lbs/wk**

\[ M_{1PTB,K} = (M_{1PW,K})(M_{10Y,K}) \]  

Equation No. 276
Ore yield - no units
\[ M_{10Y.K} = \left(\frac{1}{M_{17}}\right)(M_{15})(M_{1RAT.K}) + (M_{16})(1) \] 277

Price ratio - no units
\[ M_{1RAT.K} = \frac{(M_{PR1.K})(m_{34})}{(M_{PR2.K})(M_{35})} + (1)(M_{36}) \] 278

This function is provided to enable the researcher to change mine behavior by changing the value of the constants M_{34}, M_{35}, and M_{36}.

Ore yield production limit - lbs/wk
\[ M_{10PP.K} = (M_{1AMP.K})(M_{10Y.K}) \] 279

Adjusted maximum production - lbs/wk
\[ M_{1AMP.K} = (M_{1MPP})(M_{LRSA.K}) \] 280

Profit - $
\[ M_{1PRO.K} = M_{1PRO.J} + (DT)(1/2)(M_{1R.JK} - M_{1COS.JK}) \] 281

Revenue - $
\[ M_{1R.KL} = (M_{1CP.JK})(R_{CP.K}) \] 282

Cost - $
\[ M_{1COS.KL} = (R_{CPN})(M_{1PS.JK})/M_{10Y.K} \] 283

Adjusted production desired - lbs/wk
\[ M_{1APD.K} = (M_{1PD.K})(M_{LRSA.K}) \] 284

Industry profit - $
\[ C_{IPRF.K} = C_{IPRF.J} + (DT)(1/2)(C_{IR.JK} - C_{C.JK}) \] 285

Industry cost - $
\[ C_{IC.KL} = M_{COST.JK} + M_{1COS.JK} \] 286

Industry revenue - $
\[ C_{IR.KL} = (R_{OS.JK})(R_{CP.K}) \] 287

Price ratio 1 - no units
\[ M_{PR1.K} = R_{CPN}/R_{CP.K} \] 288

This function may be used in both the "M" and the "Ml" mine sectors in equations No. 259 and No. 274.
As price (RCP,K) changes the result would be to have the mine produce a lower grade ore at a high price and a higher grade ore at a low price.

**Price ratio 2 - no units**

\[
\text{MPR2.K} = \frac{\text{MACP.K}}{\text{RCPN}}
\]

This function is used in a similar manner to that of MPR1.K (No.288). Here, however, a higher price will cause the mine to produce a higher grade ore, while a lower price will cause the mine to produce a lower grade ore.

**Long run supply adjustment - no units**

\[
\text{MLRSA.K} = 1 + \text{MPA.K}
\]

**Long run price ratio - no units**

\[
\text{MLRPR.K} = \frac{\text{MLRP.K}}{\text{RCPN}}
\]

**Long run price - $/lb**

\[
\text{MLRP.K} = \text{MLRP.J} + (\text{DT})(1/\text{M30})(\text{RCP.J} - \text{MLRP.J})
\]

**Price adjustment - supply - no units**

\[
\text{MPA.K} = \text{MPA.J} + (\text{DT})(1/\text{M40})(\text{MPAF1.J} + \text{MPAF2.J})
\]

This equation and those that describe it are used to obtain a "stickiness" in shifting of the supply curve Figure 11, Chapter 8.

**Price adjustment factor 1 - no units**

\[
\begin{align*}
\text{MPAF1.K} &= 0 \quad \text{if M41} \geq \text{MPRF*2.K} \\
&= \text{MPRAP.K} \quad \text{if M41} < \text{MPRF*2.K}
\end{align*}
\]

**Price adjustment factor 2 - no units**

\[
\begin{align*}
\text{MPAT2.K} &= 0 \quad \text{if MPRF*2,K} \geq \text{M42} \\
&= \text{MPRAN.K} \quad \text{if MPRF*2,K} < \text{M42}
\end{align*}
\]

**Price ratio adjustment, positive - no units**

\[
\begin{align*}
\text{MPRAP.K} &= 0 \quad \text{if 0} \geq \text{MPDIF,K} \\
&= \text{MPDIF,K} \quad \text{if 0} < \text{MPDIF.K}
\end{align*}
\]
Price ratio adjustment negative - no units

\[ \text{MPRAN}.K = 0 \quad \text{if} \quad \text{MPDIF}.K \geq 0 \]
\[ = \text{MPDIF}.K \quad \text{if} \quad \text{MPDIF}.K < 0 \]

Average price ratio difference - no units

\[ \text{MPDIF}.K = \text{MPRF}^*2.K - \text{UPRF}^*3.K \]

Price ratio file - no units

\[ \text{MPRF} = \text{BOXLIN} (3,1) \]

First drawer in price ratio file - no units

\[ \text{MPRF}^*1.K = \text{MPRF}^*1.J + (\text{DT})(\text{MLRPR}.J + 0) \]

The constants for this sector follow:

Production delay

\[ \text{DEL28} = 1.2 \]

Weighting factor in production change - no units

\[ \text{M12} = .0125 \]

Weighting factor for production wanted - no units

\[ \text{M13} = .1 \]

Weighting factor for production wanted - no units

\[ \text{M14} = 0 \]

Weighting factor for ore yield - no units

\[ \text{M15} = 1 \]

Weighting factor for ore yield - no units

\[ \text{M16} = 2 \]

Maximum production possible - lbs/wk

\[ \text{M1MPP} = 75,000,000 \]

Weighting factor for long range supply adjustment - no units

\[ \text{M8} = 1 \]

Weighting factor for long range supply adjustment - no units

\[ \text{M9} = 2 \]
<table>
<thead>
<tr>
<th>Time to average long run price - wks</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 30 = 24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weighting factor for price ratio - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 34 = 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weighting factor for price ratio - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 35 = 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weighting factor for price ratio - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 36 = 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price adjustment weighting factor - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 40 = 10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price adjustment weighting factor - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 41 = 1.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price adjustment weighting factor - no units</th>
</tr>
</thead>
<tbody>
<tr>
<td>M 42 = .5</td>
</tr>
</tbody>
</table>

Page 120
APPENDIX H

CHANGES TO THE REFINERS SECTOR

The desire to explore the results of refiner production based on maintaining a constant relation between refiners finished goods inventory and refiners unfilled orders resulted in the following changes to the refiners sector of the model.

RPW, equation No. 192 was changed to be a function of RPD (Production desired), the new function added.

New equations are as follows:

\[
\begin{align*}
RPW.K &= RPD.K \text{ if } RPD.K \geq RRMLP.K \\
       &\quad RRMLP.K \text{ if } RPD.K < RRMLP.K \\
RRMLP.K &= RRMI.K/R14 \\
RPD.K &= (1/R13)(RSS.K) + (1/R12)(RFGID.K - RFGI.K)
\end{align*}
\]

where:

- RPW = Production Wanted - lbs/wk.
- RPP = Production Desired - lbs/wk.
- RRMLP = Raw Material Production Limit - lbs/wk.
- RRMI = Raw Material Inventory - lbs.
- RSS = Average Sales - lbs/wk.
- RFGID = Desired Finished Goods Inventory - lbs.
- RFGI = Actual Finished Goods Inventory - lbs.
- R14 = Minimum Wanted Raw Material Inventory - wks = 3.
- R12 = Time To Balance Finished Goods Inventory - wks = 26
- R13 = Programming Device - no units.

The above equations say in effect that refiners will produce to maintain desired finished goods inventory within the limit of available raw materials. The programming constant R13 was provided as a means for short-circuiting the
Appendix H

desired production function when the model was run to represent the industry as it is. By making R13 a small number the desired production would be beyond the limits established by the raw material production limit and therefore make this function inoperative. For runs where the desired production function was to operate, R13 was set equal to 1 and initial raw material inventories were arbitrarily increased three times.
BIBLIOGRAPHY OF SELECTED SOURCES

1. Books


2. Documents and Pamphlets


Copper the Cornerstone of Civilization; Number 15 of the Copper and Brass Research Association, 420 Lexington Avenue, New York, 1959.


"Description of the Model of Production Scheduling and Customer Delay as Used in the 1959 Summer Session," Memorandum D-51, an unpublished paper of the Industrial Dynamics Research Group, School of Industrial Management, Massachusetts Institute of Technology, 1959.

3. Articles


4. Statistical Sources


