THE STATICS AND DYNAMICS OF SESSILE BUBBLES
ON INCLINED SURFACES

by

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Submitted to the Department of Aeronautics and
Astronautics on May 9, 1985 in partial
fulfillment of the requirements for the
Degree of Master of Science in
Aeronautics and Astronautics

ABSTRACT

The behavior of sessile air bubbles on inclined surfaces in response to buoyancy has been studied. A model is presented for predicting the behavior of bubbles based on the following forces: adhesion force based on the anisotrophy of the contact angle, buoyancy, thermal gradient Marangoni forces, and drag if the bubbles are in motion. Bubbles were observed experimentally to exhibit three types of motion; static, where the bubbles adhere to the surface and do not move; creeping motion, where the bubbles still adhere to the surface but move in a slow motion; and separated motion, where the bubbles are separated from the surface by a thin fluid layer and move more rapidly than creeping bubbles. The model predicts the threshold between static and creeping behavior and is observed to agree with measurements for Bond numbers up to 0.68 in water. The velocity of creeping bubbles, which maintain contact with the surface, was observed often to be sporadic and jerky. From terminal velocity observations of separated bubbles it was noted that the presence of the wall increases the drag on the bubbles for Reynolds numbers greater than one, when compared to the drag on the same size bubbles freely rising in an unbounded fluid.

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ACKNOWLEDGEMENTS

This work would not have been possible without the sponsorship of The Charles Stark Draper Laboratory, Inc., and Neil Barbour. I would also like to thank John Hansman for his help and support throughout the research and the writing of this thesis. For going blind with me while recording video data, Henry Hoeh was an irreplaceable assistant whose help could not go unmentioned. Whenever research or writing just became too much to handle, I could always turn to John W. and Peter H. who were always able to supply their never ending enthusiasm, great influence, and new insights into small bubbles and their dynamic effects. This work could never have been completed without their companionship. Finally, to my parents, who, quite happily, did not have to pay for any of this, but have always been there when I've needed them.
TABLE OF CONTENTS

Abstract 2
List of Figures 7
List of Tables 9
Nomenclature 10
Introduction 12
1. Background, 18
   1.1 Overview of Bubble/Surface Dynamic Regimes 18
   1.2 Multiphase Material Interface Systems 18
2. Static Bubbles 23
   2.1 Static Bubbles - Definitions and Behavior 23
      2.1.1 Bubble Definitions 23
      2.1.2 Static Bubble Behavior 23
   2.2 Static Bubble Theory - Small Bubble Governing Equations 25
      2.2.1 Surface Tension Adhesion Force 26
         i. Basic Theory 26
         ii. Thermal Gradient Effects on Surface Tension Force 29
      2.2.2 Applied Forces 29
         i. Bouyant Force 29
         ii. Marangoni Force 30
   2.2.3 Complete Force Balance Equations 32
   2.2.4 Very Small Bubble Behavior 32
   2.2.5 Large Bubble Behavior 33
2.3 Experimental Apparatus 34
3.1 Terminal Velocity of Creeping Bubbles

3.2 Apparatus and Test Procedures

3.2.1 Procedure for Measurement of Bubble Motion

i. Experimental Setup

ii. Test Procedures

3.2.2 Procedure for Observing Base of Contact (Footprint) Behavior

i. Experimental Setup

ii. Test Procedures

3.3 Results and Discussion

4. Separated Bubbles

4.1 Governing Equations

4.1.1 Theoretical Drag Coefficients

4.1.2 Measured Drag Coefficients and the Equivolumetric Sphere

4.1.3 Wall Effects on Drag Coefficients

4.2 Apparatus and Test Procedures

4.3 Results and Discussion

4.3.1 Transition from Creeping to Separated Motion

4.3.2 Terminal Velocity - Drag Coefficients

i. Water/Glass

ii. Glycerol/All Surfaces

iii. Ethyl Alcohol/All Surfaces

iv. Water/Stainless Steel

5. Conclusions
References 93
Appendix A. Behavior of Bubbles in Gyroscopes 95
LIST OF FIGURES

1. Sessile and pendant bubbles
2. Three phase interface
3. Bubble/surface interface
4. Distorted bubble and forces - side view
5. Bubble - top and side view
6. Surface tension and buoyant forces on a bubble
7. Spherical bubble model
8. Large bubble - footprint and side view
9. Bubble test apparatus - side view
10. Bubble test apparatus - top view and test surface dimensions
11. Surface tension and buoyant force vs. volume, phi=15 degrees
12. Surface tension and buoyant force vs. volume, phi=30 degrees
13. Surface tension and buoyant force vs. volume, phi=45 degrees
14. Spherical bubble 0.5 mm diam. - side view and footprint
15. Nonspherical bubble 3.0 mm diam. - sideview and footprint
16a. Bubble prior to attachment
16b. Attached bubble
17. Bubble side view camera orientation
18. Footprint view camera orientation
19. Creeping bubble position time histories
20. Creep velocity vs. diameter, water/glass
21. Bubble motion regimes - water/plexiglass
22. Bubble motion regimes - water/stainless steel
23. Bubble motion regimes - water/glass
24. Creeping bubble footprint shape
25. Surface roughness effects on dynamic contact angle
26. Bubble footprint transition from creep to separated motion
27. Bubble transition from creep to separated motion - side view
28. Separating bubble position and velocity time histories - water/glass
29. Separating bubble position and velocity time histories - water/stainless steel
30. Drag coefficient vs. Reynolds number - water/glass
31. Drag coefficient vs. Reynolds number - glycerol/all surfaces
32. Drag coefficient vs. Reynolds number - ethyl alcohol/all surfaces
33. Drag coefficient vs. Reynolds number - water/stainless steel
34. Wall effect coefficients vs. Reynolds number
LIST OF TABLES

Table 1. Properties of test fluids at 20 C.
NOMENCLATURE

Bo - Bond number
Cd - drag coefficient
d - bubble diameter, cm or microns
deq - equivalent diameter, cm
dγ/dT - surface tension thermal variation, dynes/(cm-F)
Fb - buoyancy force, dynes
Fd - drag force, dynes
Fm - Marangoni (thermal) force, dynes
F - surface tension force, dynes
g - gravity, 981 cm/sec^2
G - number of gravities
h - bubble height, cm
H - horizontal surface tension component opposite buoyancy, dynes/cm
k - thermal gradient, F/cm
R - bubble radius, cm
Re - Reynolds number
U - bubble velocity, cm/sec
V - bubble volume, cm^3
Vc - creep velocity, cm/sec
x - bubble footprint diameter, cm
γ - surface tension, dynes/cm
Δθ - contact angle hysteresis, θ_b-θ_f, degrees
θ_a - advancing contact angle, degrees
θ_b - contact angle at the back of bubble, degrees
\( \theta_f \) - contact angle at the front of bubble, degrees
\( \theta_m \) - mean contact angle, \( \frac{\theta_b + \theta_r}{2} \)
\( \theta_r \) - receding contact angle, degrees
\( \lambda \) - wall effect coefficient
\( \mu \) - fluid viscosity, gm/(cm·sec)
\( \rho \) - fluid density, gm/(cm^3)
\( \phi \) - surface angle from horizontal, degrees
\( \psi \) - footprint azimuthal angle
Introduction

Static and dynamic behavior of bubbles on a surface have many important implications in such diverse fields as maintaining purity of samples during materials processing in space and error mechanisms in high precision mechanical, navigational gyroscopes. Little work has been done to date on the specifics of multiple phase interface phenomena associated with a bubble being in contact with a surface. In the following the static and dynamic behavior of bubbles interacting with surfaces will be modeled and tested experimentally.

Two important characteristics of bubbles in contact with a surface are the bubble shape and its contact angle. Contact angle is the physical angle the interface between the bubble and the fluid makes with the surface, as seen in the two bubbles of Figure 1. These bubbles represent the two basic types of bubbles that may adhere to a surface. A sessile bubble (Fig.1a) has the buoyant force acting towards the surface, helping keep the bubble on the surface. A pendant bubble (Fig.1b) has the buoyant force in the opposite direction as a sessile bubble, acting so as to pull the bubble away from the surface. It has been shown that the shape of bubbles in the absence of gravity must satisfy the following equation determined by Plateau,
FIGURE 1a. Sessile Bubble.

FIGURE 1b. Pendant Bubble.
\[(1/\mathcal{R}_1) + (1/\mathcal{R}_2) = \text{constant}\]

where \(\mathcal{R}_1\) and \(\mathcal{R}_2\) are the principle radii of curvature at any point on the bubble surface (1). A surface for which the Plateau relation holds is always a surface of minimum area. In the presence of gravity the equation must be modified, the effects of which are discussed in Adam (1). Using this equation and the modified versions, Hartland and Hartly (2) calculated numerically the actual shapes of many different sizes of bubbles on surfaces. The contact angle is dependant on the various surface tensions and the system geometry. It is not, in general, a quantity that can be theoretically determined. Mack (3) introduced a method for calculating the contact angle from linear measurements of small bubbles and drops.

Overall bubble motion over the surface has not been addressed directly, but motion of the interface has been studied to a great extent, primarily on an experimental basis. Rose and Heins (4) and Coney and Masica (5) both reported a velocity dependence on the interfacial contact angle. Elliott and Riddiford (6) also studied the velocity dependence and noticed an uneven, jerking motion of the interface in some of their tests, which was commented on later by Wilson (7). A compilation of studies on dynamic contact angles is included in the study by Schwartz and Tejada (8).

A very good discussion of general bubble dynamics is
included in Clift, et.al., (9). Drag coefficients for bubbles moving through infinite liquids and the effects of bubbles moving in the proximity of container surfaces are of particular interest to this research, as bubbles were observed to break away from the surface and move along, adjacent to the surface. Clift assembles many studies concerning the effects of bubble motion near surfaces and comments on their results.

Marangoni convection (10), resulting from the presence of thermal gradients, is another aspect of bubble dynamics that must be considered in the study of bubble/surface interactions. The forces are analytically described in terms of a surface energy mechanism proposed by Harper, Moore, and Pearson (11). Hardy (12) and Young, et.al., (13) both studied motion of bubbles in vertical thermal gradients, while McGrew, et.al., (14) experimented with Marangoni forces on bubbles in a horizontal thermal gradient.

Surface irregularity and roughness can influence the behavior of bubbles. Experimentally, liquid drops are easier to observe than bubbles on rough surfaces, however, the effects between the two are comparable. Oliver, Huh, and Mason (15) observed small mercury drops on idealized rough surfaces with a scanning electron microscope and observed local differences in the contact angles. This was to verify their earlier numerical computations (16). These results are relevent to the bubble problem due to the similarity of physics between bubbles and drops on surfaces.
The purpose of this work was to develop and test theoretical models for the static and dynamic behavior of bubbles on a surface. It was found experimentally that bubbles on surfaces have three basic modes of behavior: static, creeping, and separated. Static bubbles adhere to the surface and do not move under the influence of any force. Creeping bubbles also adhere, but move along the surface in a slow, sliding motion. Separated bubbles do not have a direct contact with the surface but are separated from the plate by a very thin fluid layer and characteristically move at a higher velocity.

In Chapter 1 a description of bubble interfaces is presented. Chapter 2 introduces a model for static bubble adhesion force and discusses experimental validation. The model accurately calculated the adhesion forces on small, spherical bubbles on surfaces up to a range where the Bond number (the ratio of gravity forces to surface tension forces) was near one. Above this range bubbles become significantly nonspherical and the assumptions of sphericity implicit in the model break down. Creeping bubbles are discussed in Chapter 3. This sliding motion was observed in a limited bubble size range, where the external forces (bouyancy and thermal) are on the same order as the maximum surface adhesion force that restrains the bubble from moving. Creeping motion was generally observed to be somewhat jerky in nature, particularly for the smaller bubbles, due to small irregularities in the surface. Chapter
4 discusses the transition from creeping to separated motion and considers the drag on a bubble moving near a plane surface. Transition primarily involves the area of contact between the bubble and the surface. This base of contact is observed to deform throughout the transition period, and disappears completely for separated bubbles. Once in the separated mode the bubble accelerates quickly along the surface to terminal velocity, and never reattaches to the plate. The influence of the nearby plane surface results in an increase in the drag on the bubble as compared to bubbles rising in an infinite fluid.
Chapter One - Background

1.1 Overview Of Bubble/Surface Dynamic Regimes

The behavior of sessile bubbles on a surface can be divided into three distinct regimes: static, creeping motion, and separated motion. Static bubbles adhere to the surface and do not move under the external application of buoyant or thermal forces, due to high surface adhesion forces. Creeping bubbles maintain contact with the surface but move along the plate. Separated bubbles have no direct contact with the surface, rather, they are separated by a very thin layer of fluid. Because there are no direct surface forces restricting the motion of separated bubbles they can be accelerated more rapidly by external forces than creeping bubbles. Physical models of each regime will be derived, tested, and discussed, along with the transitions between regimes, in the following.

1.2 Multiple Material Interface Systems

When multiple materials of different phases (e.g., air/water/glass) are in contact with each other their adjacent free surfaces are shaped so as to minimize the surface free energy. Surface free energy is defined as the work done on the surface of contact by a small reversible isothermal process, which could be, for example, the expansion and contraction of a bubble being raised and lowered in a column of liquid. Minimization of the free energy per unit area can be thought of in terms of a kind of force balance between the materials on opposite sides of the
interface. Figure 2 shows a schematic of the point of intersection of three different phases of different materials in contact with each other. The curved lines correspond to the surfaces (or lines) of contact. Three straight lines are drawn tangent to the lines of contact at the intersection of the phases. These straight lines each represent a surface tension force tangent to the line of contact. Surface tension is a force per unit length and is equal to the free energy per unit area for each interface. In equilibrium, the surface tension forces must balance. In Figure 2, $\gamma_{12}$ is the interfacial surface tension between phases 1 and 2 and is generally not the same value as $\gamma_{23}$ or $\gamma_{13}$. In the case of a bubble on a surface the equilibrium of the interface is simplified. Because the surface is plane, two of the surface tension forces act parallel to said surface (Fig.3). The well known equation of force equilibrium in the z direction for this case is,

\[ \gamma_{13} = \gamma_{23} + \gamma_{12} \cos \theta, \]

which defines the contact angle, $\theta$, in terms of the surface tensions. The contact angle is dependant on surface tensions, temperature, surface roughness, and other external forces on the system. It is not, in general, an absolute constant of the system, however, in some cases the contact angle is assumed to be constant for convenience.

It has been observed experimentally that there is a range
FIGURE 2. Three phase interface.

of possible contact angles for which the interface will remain motionless. The maximum value of this range is called the advancing contact angle, $\theta_a$, and the minimum is called the receding contact angle, $\theta_r$. For any angle larger than the maximum or smaller than the minimum the interface will begin to move at the point where the limit is exceeded. In a moving bubble the receding contact angle occurs at the front of the bubble while the advancing contact angle occurs at the back of the bubble, as seen in Figure 4. For a static bubble the limits of the contact angles are not reached. In this case, the contact angles at the front and the back of the bubble are defined as $\theta_f$ and $\theta_b$ respectively. These are shown in Figure 4. The difference between $\theta_b$ and $\theta_f$ is important in analyzing surface tension forces on a bubble, and will be referred to as the contact angle hysteresis, $\Delta \theta$. 
FIGURE 4. Distorted bubble and forces - side view.
Chapter Two - Static Bubbles

2.1 Static Bubbles - Definitions and Behavior

2.1.1 Bubble Definitions

In Figure 5 a schematic view of a typical static bubble is shown. The primary characteristics used in describing a bubble are bubble shape and the resulting volume, footprint shape, and the contact angle distribution. Bubble shape is determined by the geometric shape of the bubble free surface. The bubble footprint is defined as the area of contact between the bubble and a solid surface and is bounded by the line of contact (Fig. 5a). Contact angle distribution is the variation of the contact angle around the line of contact.

2.1.2 Static Bubble Behavior

In the case where there is no motion of the bubble, all forces must balance, and the bubble is said to be static. The most basic example of this is for a bubble on a level surface (Fig. 5b). No net forces exist parallel to the surface so the bubble maintains an equilibrium shape determined by the fluid density and the contact angle $\theta$, which remains constant around the base of contact. When a bubble is under the application of a force parallel to the surface on which it is attached (e.g., buoyancy due to an inclined plate, or thermal gradient), the bubble distorts slightly, resulting in a variation of the contact angle around the base of contact. This contact angle variation causes a net surface tension adhesion force at the base. In
FIGURE 5a. Bubble footprint - top view.

FIGURE 5b. Bubble side view.
the static bubble case this adhesion force exactly counteracts any externally applied forces. The magnitude of the surface tension force is directly related to the variation of the contact angle around the base of the bubble and can be characterized by the difference in the contact angles between the front and the back of the bubble, $\theta_f$ and $\theta_b$. These angles are shown in the schematic of a bubble on an inclined surface in Figure 4. Also shown in Figure 4 is the surface inclination angle convention. $\phi$ is the surface angle relative to the horizontal (i.e., $\phi = 0$ corresponds to a horizontal plate, while $\phi = 90$ corresponds to a vertical plate). The limits on the front and back contact angles, $\theta_f$ and $\theta_b$, are set by the receding and advancing contact angles, $\theta_r$ and $\theta_a$, respectively. For example, if $\theta_b > \theta_a$ the back of the bubble will begin to move. Similarly, if $\theta_f < \theta_r$ the front of the bubble will begin to move. The difference in the two angles $\theta_b$ and $\theta_f$ is defined as the contact angle hysteresis, $\Delta \theta$. The maximum hysteresis corresponds to the limits of $\theta_a$ and $\theta_r$. In physical terms this results in the greatest surface tension force restraining the bubble from moving. If the applied force is greater than that which the maximum hysteresis can counteract, the bubble will begin to move.

2.2 Static Bubble Theory – Small Bubble Governing Equations

To verify the above hypotheses concerning the balance between externally applied forces and surface tension force, equations were developed to enable the calculation of these
forces from measured bubbles.

2.2.1 Surface Tension Adhesion Force

i. Basic Theory

The surface tension force that restrains the bubble from moving in the presence of an applied force is a result of the bubble distorting at the bubble/surface interface under that applied force. This results in a variation of the contact angle around the line of contact with the surface, resulting in an asymmetric surface tension force which tends to restrict movement. In order to model the force, the footprint is assumed to be circular and the contact angle is assumed to vary sinusoidally with azimuthal angle, $\psi$. Because of the contact angle variation, there is a net horizontal component of the surface tension, $F_\gamma$, as shown in Figure 6a. The surface tension force that restrains the bubble from moving is in the opposite direction of the applied forces, as all forces must balance at the bubble/surface interface in the static case. Figure 6b shows schematically the horizontal component of surface tension force as a function of the position on the footprint boundary. Here, $\psi$ is the azimuthal angle. The horizontal component (parallel to surface) of surface tension at any point on the line of contact, opposite to the direction of the applied force $F_b$, is called $H_\gamma$, and is given by,

\begin{equation}
H_\gamma = -\gamma \cos \theta \cos \psi,
\end{equation}
FIGURE 6. Surface tension and buoyant forces on a bubble.
where \( \theta \) is a function of \( \psi \), the azimuthal angle, and \( H_y \) has the dimensions of force per unit length. Multiplying \( H_y \) by the differential length it acts over gives the surface tension force counteracting the applied force at that point on the interface. Assuming small angles, the differential length the surface tension acts over is,

\[
(2.2) \quad d\text{Length} = \frac{R}{2} \sin \theta_m d\psi,
\]

where \( R \) is the diameter of the footprint (Fig.5), and \( \theta_m \) is the mean of the front and back contact angles \( (\theta_f + \theta_b)/2 \). The sinusoidal variation of the contact angle around the footprint is given as,

\[
(2.3) \quad \Theta(\psi) = \theta_m + \frac{1}{2} \Delta \theta \cos \psi,
\]

Combining (2.1), (2.2), and (2.3) results in the differential surface tension force at any point on the footprint, \( dF_y \),

\[
(2.4) \quad dF_y = -R \sin \theta_m \cos \left[ \theta_m + \frac{1}{2} \Delta \theta \cos \psi \right] \cos \psi \, d\psi.
\]

The total surface tension force can then be found by integrating (2.4) around the line of contact,

\[
(2.5) \quad F_y = -2R \sin \theta_m \int_0^\pi \gamma \cos \left[ \theta_m + \frac{1}{2} \Delta \theta \cos \psi \right] \cos \psi \, d\psi.
\]
ii. Thermal Gradient Effects on Surface Tension Adhesion Force

A thermal gradient on the bubble results in a variation of the surface tension across the entire surface of the bubble, and thus, will change the surface tension force. This variation must be taken into account in Equation (2.5). Assuming a linear temperature gives the surface tension variation on the footprint as,

\( \gamma(\psi) = \gamma_o + \frac{d\gamma}{dT} K \frac{x}{a} \left(1 - \cos \psi\right) \) \hspace{1cm} (2.6)

where \( \gamma_o \) is the surface tension at a reference temperature, \( \frac{d\gamma}{dT} \) is the variation of the surface tension with temperature (generally a known constant of the fluid), \( K \) is the thermal gradient assumed positive in the opposite direction of bouyancy, and \( x \) is the footprint diameter. For small angles \( \frac{x}{2} = R \sin(\theta_m) \) and the surface tension variation becomes,

\( \gamma(\psi) = \gamma_o + \frac{d\gamma}{dT} K R \sin \theta_m \left(1 - \cos \psi\right) \) \hspace{1cm} (2.7)

Equation (2.7) replaces \( \gamma \) in Equation (2.5) when there is a temperature gradient.

2.2.2 Applied Forces

i. Bouyant Force

From Archimedes' principle the component of the bouyant force acting parallel to the plate surface, \( F_b \), is,
(2.8) \[ F_b = \gamma g V \sin \phi, \]

where \( \phi \) is the inclination angle of the surface with respect to horizontal. For small bubbles on a surface the volume can be approximated by that of a sphere minus a spherical cap (Fig.7),

(2.9) \[ V = \frac{1}{3} \pi R^3 \left[ 4 - (1 - \cos \Theta)^2 (2 + \cos \Theta) \right], \]

and the total buoyancy force is given by the combination of (2.8) and (2.9).

ii. Marangoni Force

As previously mentioned, a thermal gradient results in a surface tension gradient over the entire bubble surface. Not only does this change the surface tension adhesion force at the base, it also results in a bulk force that tends to move the bubble towards the higher temperature. This phenomenon is known as the Marangoni Effect and has been studied by many researchers (Refs.10-14). A model of the motion of a bubble in a thermal gradient as a solution to the Navier-Stokes equations is given in Reference 13. The result is a terminal velocity similar to Stokes flow, with the addition of a thermal gradient term. Therefore, the effective Marangoni force can be resolved from a force balance with Stokes's drag. Terminal velocity of a bubble in a thermal gradient is then given by,
\[ V_{\text{cap}} = \frac{1}{3} \pi R^3 (1 - \cos \theta)^2 (2 + \cos \theta) \]

Spherical Cap

\[ V_{\text{sphere}} = \frac{4}{3} \pi R^3 \]

\[ V_{\text{bubble}} = V_{\text{sphere}} - V_{\text{cap}} \]
\[ V_{\text{bubble}} = \frac{1}{3} \pi R^3 (4 - (1 - \cos \theta)^2 (2 + \cos \theta)) \]

FIGURE 7. Spherical bubble model.
\[ V_m = \frac{1}{a} \frac{R}{\mu} \frac{d\gamma}{dT} K, \]

where \( \mu \) is the fluid viscosity, and \( d\gamma/dt \) is the variation of surface tension with temperature. \( K \) is the thermal gradient and is defined as positive if the temperature is decreasing in the direction of buoyancy. Stokes's equation for drag on a sphere, \( D_s \), at low Reynolds numbers is,

\[ D_s = 6\pi R \mu U. \]

Combining (2.10) and (2.11) gives the Marangoni force,

\[ F_m = 3\pi R^2 \frac{d\gamma}{dT} \]

2.2.3 Complete Force Balance Equations

For equilibrium of a spherical bubble on a surface in a thermal gradient all the forces must balance (see Figure 5),

\[ F_{b} = F_{b} + F_{m}. \]

Combining equations (2.5), (2.7), (2.8), (2.9), and (2.12) gives the complete form of Equation (2.13),

\[ -2R\sin \theta \int_0^\pi \cos [\theta + \frac{1}{4} \Delta \cos \theta] \frac{d\gamma}{dT} \frac{K \sin \theta (1- \cos \theta)}{2} \cos \phi \, d\psi \]

\[ = \frac{1}{3} \rho g \pi R^2 \left[ 4 - (1 - \cos \theta)^2 (2 + \cos \theta) \right] \sin \phi + 3\pi R^2 \frac{d\gamma}{dT} K. \]

2.2.4 Very Small Bubble Behavior
For small bubbles (diameter \( d < 100 \) microns) the surface tension will dominate and the previous assumptions of circular and spherical geometry will be very good. In these cases the values of \( \theta_m \) and \( \Delta \theta \) will be determined by the materials and the integral of Equation (2.14) will be constant for the same materials. Under these assumptions, equation (2.14) simplifies to

\[
G d^2 \sin \phi = \text{constant}
\]

where \( G \) is the acceleration in earth gravities. This constant has the dimensions of length squared, and can be thought of as the square of the maximum bubble diameter which will remain static on a vertical surface.

2.2.5 Large Bubble Behavior

As bubble size increases body forces begin to dominate the surface tension forces and assumptions of spherical geometry begin to break down. If bouyancy is the primary body force this can be characterized in terms of the Bond number, \( B_0 \), which is the ratio of gravity (bouyant) forces to surface tension forces.

\[
B_0 = \frac{g d^2 \rho}{\sigma}
\]

At a Bond number of 1 the bouyant forces are of the same magnitude as the surface tension forces and the assumptions of sphericity will no longer be valid. The footprint is no
longer circular (Fig. 8a) and the bubble becomes somewhat ellipsoidal in shape (Fig. 8b) resulting from the hydrostatic pressure differences across the bubble. For buoyancy calculations, the volume of these ellipsoidal bubbles can be approximated by that of an oblate spheroid. Referring to Figure 5a,

\[ V = \frac{1}{6} \pi d^2 h. \]

In these cases, the previous static equilibrium derivation will overestimate the forces on the bubble. At high Bond numbers (Bo \( > 1 \)), small surface irregularities may influence the footprint shape due to the low surface tension forces. Discrete irregularities will result in local anomalies at the edge of the bubble footprint, causing widely varying footprint shapes. The exact equations of equilibrium for the bubbles cannot be specified on account of the uniqueness of each bubble.

2.3 Experimental Apparatus

In Figure 9 the apparatus used to observe bubbles on surfaces is shown. The apparatus consisted of a plexiglass tank which contained the test surfaces on a rotating shaft and the test fluid. The rotating shaft permitted the test surfaces to be set at any inclination between zero degrees (horizontal) and ninety degrees (vertical). The bubble was primarily viewed from the side (Fig. 5b). In order to view the bubble footprint the bottom of the bubble (Fig. 5a) was
FIGURE 8a. Nonspherical bubble - side view.

FIGURE 8b. Nonspherical bubble - footprint.

FIGURE 10. Bubble test apparatus - top view and test surface dimensions.

PIVOTING TEST SURFACE

PIVOT KNOB

6.5 cm

5.0 cm

2.5 cm

11.3 cm

7.7 cm
observed through a transparent test surface. Glass was normally chosen as the test surface for such visual observations. The dimensions of the test surface are shown in Figure 10. Before filling the tank with the test fluid, both the surface and the tank interior were cleaned thoroughly and degreased with freon to minimize any surface and fluid contaminants. Distilled water was the primary test fluid and air was used as the test gas.

Measurements of bubble shapes and limiting contact angles was accomplished by mounting the tank in an "optical comparator". The optical comparator projected a magnified (10X) shadowgraph of the side view of a bubble onto a screen, on which measurements were made. Linear dimensions could be measured to an accuracy of +/- 0.05 mm. Contact angles, however, were difficult to measure, resulting in an accuracy of +/- 3 degrees. Observation of the bubble footprint was accomplished by simply looking through the test surface from above the tank. Methods used in observing footprint behavior will be discussed in greater detail in Chapters 3 and 4 which discuss measurement of creeping and separated bubbles.

2.4 Static Bubble Test Procedures

To verify that the surface tension adhesion force model is valid, measurement of certain geometric bubble parameters was necessary. These included the bubble diameter d, the footprint diameter x, and the bubble height h. The front contact angle, $\theta_f$, and the back contact angle, $\theta_b$, were
also measured. Values were obtained directly from the optical comparator screen while a bubble was in residence on the surface. Bubbles were produced with a small (1 c.c.) syringe connected to an extremely small diameter needle (0.005 in.). Thin needles were necessary to minimize the surface at the needle's end so the bubbles would not readily adhere. This is especially critical when attempting to produce bubbles smaller than 1 mm in diameter, which tend to adhere to the needle.

To measure bubbles at various inclinations the plate was set to a specified inclination angle and bubbles then placed on the surface. Bubble diameter d, footprint diameter x, height h, and front and back contact angles \( \theta_f \) and \( \theta_b \) were then recorded. Getting bubbles to adhere to the angled plate was sometimes difficult, especially at large inclination angles. The method used was first to set the plate to the required angle. The needle was then placed on the surface and bubbles were produced. Some bubbles would move up the plate due to the bouyant force, however, some bubbles would remain stuck to the plate and were then measured. Surface inclination angles of 15, 30, and 45 degrees were studied. Above 45 degrees the bouyant force along the plate dominated, and only a few small bubbles remained static.

2.5 Results And Discussion

The surface tension adhesion force and the bouyant forces were calculated from the measured bubble dimensions and
contact angles. For these static bubbles the forces should balance. The surface adhesion force was obtained by numerically integrating the left hand side of equation (2.14) for each measured bubble. Bouyancy was calculated using equation (2.9) and the bubble volume model of equation (2.8).

The calculated surface tension adhesion force is plotted against bubble volume for three plate angles in figures 11, 12, and 13. The corresponding bouyant force is also shown by the solid line in the plots. Error bars resulting from the uncertainty in contact angle measurements are shown for a few representative bubbles. In the static bubble model, the surface tension adhesion force should exactly balance the bouyant force. For bubbles up to approximately 0.006 cc the model appears valid. Above 0.006 cc the adhesion force appears to excede the bouyant force and the model breaks down. For the larger bubbles the assumption of spherical geometry is not valid and will result in an overestimation of surface tension adhesion force. At Bond numbers of approximately 1 deviations from sphericity are expected as the hydrostatic forces begin to dominate the surface tension forces. For air bubbles in water a volume of 0.010 cc corresponds to a Bond number of 1. From the above data, the spherical model appears to be valid for volumes less than 0.006 cc, and Bond numbers less than 0.68. For Bond numbers greater than 1 the contact angle hysteresis tended to increase with bubble volume up to a maximum value of
FIGURE 11. Surface tension and buoyant force vs. volume, phi=15 degrees.
FIGURE 12. Surface tension and buoyant force vs. volume, phi=30 degrees.
FIGURE 13. Surface tension and bouyant force vs. volume, \( \phi = 45^\circ \) degrees.
hysteresis.

Figures 14 and 15 show side view and footprint photographs of two different sized bubbles, 0.5 mm and 3.0 mm in diameter, respectively. Note the degree of bubble distortion in the larger bubble side view (Fig. 15a) as compared to the more spherical smaller bubble (Fig. 14a). The contact angle hysteresis is significantly more apparent in the larger bubble. Differences in footprint shape can be seen by comparing Figures 14b and 15b. The small bubble has an approximately circular footprint although slight variations in its perimeter can be observed. These variations are probably caused by surface impurities on the plate.

For the higher Bond numbers the surface tension becomes less dominant the bubble surface becomes more susceptible to variations from external forces. For this reason, the shape of the footprint of large bubbles tends to be elongated and distorted due to buoyancy, surface irregularities, and wetting characteristics of the plate. A typical example is shown in Figure 15. Because of the irregular footprint shape, the assumption that the contact angle varies sinusoidally between the advancing and receding contact angles is no longer valid. This results in the large overestimation of the surface tension adhesion forces calculated in Figures 11, 12, and 13. These forces were calculated assuming a sinusoidal contact angle variation and a circular footprint, the diameter of which was inferred from the side measurement. Since the footprint was more
FIGURE 14a. Spherical bubble 0.5 mm diam. - side view

FIGURE 14b. Spherical bubble 0.5 mm diam. - footprint view
FIGURE 15a. Nonspherical bubble 3.0 mm diam. - side view

FIGURE 15b. Nonspherical bubble 3.0 mm diam. - footprint view
elliptical than spherical in character, the observed
diameter would correspond to the major axis of the ellipse,
and would overestimate the length of the line of contact.
This results in an overcalculation of the surface tension
adhesion force.

Another phenomenon noticed in the measurement of static
bubbles was that bubbles with the same overall diameter
dimension $d$ generally had different values of footprint
diameter $x$, height $h$, and front and back contact angles.
Differences in these measurements led to different values of
$\Delta \theta$, resulting in different values of the surface tension
adhesion force and buoyancy. This effect was observed in
both spherical and non-spherical bubbles, and is considered
a result of surface imperfections, and local variations in
the surface wetting characteristics of the plate.

Wetting characteristics of the plate surface were observed
to have important effects on the static bubble tests. The
adhering of bubbles to the test surface was found to be time
dependant, regardless as to the type of fluid used. After a
bubble is blown at the end of a needle it separates and
rises to the plate surface (Fig. 16a). When the bubble
arrives at the plate there is a thin layer of fluid between
the bubble and the plate which must be removed so the bubble
can stick. The process of removing this thin fluid layer
from the plate can be thought of as a drying of the surface.
After a short while enough fluid has been removed so that
the bubble is now attached to the surface (Fig. 16b). The
FIGURE 16a. Bubble prior to attachment

FIGURE 16b. Attached bubble
mechanism that removes the fluid layer from between the bubble and the plate is capillarity. The speed at which it occurs is governed by the fluid viscosity. For water, this time dependence is small, on the order of a second. More viscous fluids, such as glycerol which has a viscosity some three orders of magnitude greater than water, exhibited a time constant for adhesion on the order of minutes.

Long term drying effects were also noticed. The longer a bubble was allowed to remain on the surface the larger the footprint became, up to a certain limit. This implies that the capillary process of fluid removal is ongoing even after the bubble has become stuck to the plate. The long term effects of capillarity were studied qualitatively with water when a large bubble was intentionally left on the level surface for a period of approximately one hour. The plate was then tilted through many angles, and the bubble remained stuck to the surface at angles where it did not stick in the short term experiments discussed in section 2.4. It appears, therefore, that the bubble surface adhesion increases with time.
Chapter Three - Creeping Bubbles

3.1 Terminal Velocity of Creeping Bubbles

If a bubble is large enough such that the buoyant force, $F_b$, is slightly greater than the maximum retarding force due to surface tension force, $F_s$, then the bubble will move, while maintaining contact with the surface. This type of bubble motion is described as creeping. If the buoyant force is much larger than the surface tension force then the bubble is observed to separate from the plate and slide along separated from the surface by a thin fluid layer. These bubbles are referred to in the following as a separated bubbles.

The terminal velocity of a creeping bubble can be determined by balancing the drag, adhesion, buoyant, and thermal forces. It will be assumed for this theory that a creeping bubble is in the Stokes's flow regime (20), where the drag is given by equation (2.11),

\[
(2.11) \quad D_s = 6\pi R \mu U.
\]

Also, it will be assumed that once the bubble distorts to its maximum contact angle hysteresis (the difference between the contact angles at the front and the rear of the bubble) it maintains this hysteresis in steady state creeping. From Figure 4 the forces acting on a creeping bubble parallel to the plate are,
(3.1) \( \sum \text{Forces} = m \frac{dU}{dt} = F_b - F_y - F_m - F_d \),

where \( \frac{dU}{dt} \) is the bubble acceleration, \( F_b \) is the buoyant force, \( F_y \) is the surface tension adhesion force, \( F_m \) is the Marangoni force, and \( F_d \) is the drag force. Using (2.11) for the drag on the bubble (3.1) becomes,

(3.2) \( m \frac{dU}{dt} + 6 \pi R \mu \frac{U}{m} = \frac{1}{m} (F_b - F_y - F_m) \)

where \( F_y \), \( F_b \), and \( F_m \) are given by (2.5), (2.8), and (2.12), respectively.

(2.5) \( F_y = -2R \sin \Theta_m \int_0^\Pi \gamma(\psi) \cos [\Theta_m + \frac{1}{4} \Delta \Theta \cos \psi] \cos \psi d\psi \).

(2.8) \( F_b = \rho g V \sin \phi \).

(2.12) \( F_m = 3 \pi R^2 \frac{dV}{dt} K \).

For a pure fluid and an ideally flat surface the terminal creep velocity is found from the steady state solution to the differential equation (3.2),

(3.3) \( V_c = \frac{F_b - F_y - F_m}{6 \pi R \mu} \)

3.2 Apparatus and Test Procedures

Observing the behavior of creeping bubbles was made by means of a high speed video system. The system was capable
of recording sharp images of moving bubbles at frame rates of 120 or 240 frames/sec.

The plexiglass tank used in the static bubble measurements, described in Chapter 2, was also used for dynamic bubble testing. Three fluids, water, glycerol, and ethyl alcohol were tested in order to observe a wide range of viscosities and surface tensions (see Table 1). Glycerol was chosen for its high viscosity, and ethyl alcohol for its low surface tension. Three surfaces were used: glass, plexiglass, and polished stainless steel. All possible combinations of the three surfaces and three fluids were observed.

3.2.1 Procedure for Measurement of Bubble Motion

i. Experimental Setup

The apparatus used to observe bubble motion from the side is shown in Figure 17. The camera was positioned to look at the bubble through the side of the tank. A strobe, synchronized to the framing rate of the video camera, was placed on the opposite side of the tank to backlight the bubble.

ii. Test Procedures

The techniques used for recording bubble dynamics described below were used to observe both creeping and separated bubbles. The procedure was as follows. With the test surface level (φ = 0) a bubble was injected into the fluid on the underside of the surface, and the diameter, d, was measured. Then, the video camera was started and the
<table>
<thead>
<tr>
<th>FLUID</th>
<th>$\gamma$ (dyne/cm)</th>
<th>$\mu$ (gm/cm-sec)</th>
<th>$\rho$ (gm/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>72.8</td>
<td>0.010</td>
<td>1.00</td>
</tr>
<tr>
<td>Glycerol</td>
<td>63.0</td>
<td>23.30</td>
<td>1.26</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>22.0</td>
<td>0.012</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Table 1. Properties of test fluids at 20°C.
FIGURE 17. Bubble side view camera orientation.
plate surface rotated smoothly, but quickly, to a specified angle, \( \phi \). The camera continuously recorded the behavior of the bubble, be it static, creeping or separated. Plate angles of 10 to 75 degrees were studied.

The video recordings were calibrated prior to each test by recording a millimeter scale on the test surface so that during playback bubble position as a function of time could be measured directly from the video screen; the time scale was determined from the frame speed. A position could be measured every frame or every tenth frame, depending on the speed of the bubble. Velocity could be determined from the distance between position points through the frame rate.

3.2.2 Procedure for Observing Base of Contact (Footprint) Behavior

i. Experimental Setup

The shape and motion of the bubble base of contact with the surface (footprint) are very important in understanding the dynamics of a creeping bubble. Predictions of creeping velocities from the theory of this chapter are based on the assumptions of a circular line of contact and a sinusoidally varying contact angle between the minimum angle at the front of the bubble and the maximum angle at the rear of the bubble. To determine the validity of these assumptions it was necessary to view the footprint of the bubble. The test setup is shown in Figure 18. By positioning the camera above the tank, looking down on the plate surface, it was possible to locate the strobe in a position that would highlight the
footprint, as seen in Figures 14b and 15b. Because of the
different indices of refraction between the bubble,
transparent test surface, and the fluid, proper position of
the strobe would cause the footprint to become bright and
very distinct from the rest of the bubble, permitting its
shape and motion to be easily observed. There is some
distortion due to the surface being observed at an angle,
however, qualitative behavior could be observed.

ii. Test Procedures

A bubble was placed on the level plate which was then
rotated slowly until the bubble began to move, then, the
rotation was stopped and the bubble continued its motion.
The video system recorded the entire event. As the plate was
rotated it was necessary to continually move the strobe to
keep the footprint highlighted. The recordings of the
footprints were then played back and studied qualitatively.
The transition from creeping to separated flow was also
observed in this same fashion and will be discussed in the
next chapter along with separated bubbles.

3.3 Results And Discussion

Creeping bubbles occur for a small range of bubble sizes,
where the buoyant force is only slightly larger than the
maximum surface tension adhesion force. For spherical
bubbles, bouyancy goes as the cube of the radius (Eqs.2.8,
2.9), while surface tension force is linear in the radius
(Eqn.2.5). As bubble size increases, bouyancy increases at a
faster rate than the surface tension adhesion force, and
there will be only a small range of bubble sizes where the
two forces are of similar magnitude. If bouyancy is very
large a bubble will tend to separate immediately once motion
begins. This is common at large plate angles.

Creeping bubbles displayed various long term
characteristics. Some bubbles crept for a short distance
then became hung up at some point on the surface, stopping
completely. Other bubbles continued to creep or transitioned
to the separated regime. Figure 19 shows two representative
position time histories for these first two types of motion.
Figure 19a is a 2.78 mm diameter bubble in water on a glass
surface at a plate angle of 15 degrees. Figure 19b is a
14.08 mm diameter bubble in water on a plexiglass surface at
a plate angle of 10 degrees. The origin in these graphs
correspond to the time and position at which data
acquisition was initiated, and was not necessarily the point
at which the bubble began creeping. Note the constantly
changing velocity for the 2.78 mm bubble in Figure 19a, and
that the bubble actually stopped completely after
approximately 1.6 seconds. Figure 19b shows a much steadier
motion of a large 14.08 mm bubble on a plexiglass surface.
Steady motions were observed only for large bubbles with
Bond numbers greater than 1.

In the absence of thermal gradients, the terminal creep
velocity equation,

\[
V_c = \frac{F_b - F_y}{6\pi\eta R\mu}
\]
FIGURE 19a. Creeping bubble position time history.

FIGURE 19b. Creeping bubble position time history.
can be divided into two distinct parts, the buoyancy term (Eqn. 3.4) and the surface tension term (Eqn. 3.5).

\[
\frac{F_b}{6\pi R \mu} = \frac{g g R^2 [4-(1-\cos \theta_m)(2+\cos \theta_m)] \sin \phi}{18 \mu}
\]

\[
\frac{F_y}{6\pi R \mu} = \frac{-\sin \theta_m \int_0^\pi \cos \theta_m \Delta \cos \Psi \cos \Psi \, d\Psi}{3\pi \mu}
\]

The buoyancy term (3.4) is quadratic in the bubble diameter, whereas the surface tension term (3.5) is independent of bubble size. In Figure 20 measured creeping velocities as a function of bubble diameter are plotted as distinct points for bubbles on glass in water at a plate angle of 15 degrees. The curve in Figure 20 is a quadratic of the form given by equation (3.3) fitted through the data. At a diameter of 2.73 mm the buoyant force exactly balances the surface tension adhesion force, resulting in zero velocity. Below 2.73 mm, the curve implies that bubbles remains static, as negative velocities could not physically occur. A box surrounding the data points shows the narrow range in diameter in which bubbles were observed to creep.

This narrow range of creeping bubbles was apparent in all the cases that were tested. The limits of bubble behavior as a function of plate angle for the tests conducted with water are shown in Figures 21, 22, and 23. The limits of the separated and static regimes are also shown. The dots
CREEP VELOCITY
VS
DIAMETER
\( \phi = 15 \)

\[ V_C \text{ (cm/sec)} \]

\[ \text{BUBBLE DIAMETER (mm)} \]

STATIC REGION

FIGURE 20. Creep velocity vs. diameter, water/glass.
FIGURE 22. Bubble motion regimes - water/stainless steel
FIGURE 23. Bubble motion regimes - water/glass.
represent the largest static bubbles observed, and the triangles represent the smallest separated bubbles observed. In the plots each X represents individual bubbles that were observed to creep.

In all the plots there is an overlap between the static and separated regimes. There is a trend for the overlap region to become smaller with increasing plate angle. Along with the smaller overlaps were fewer creeping bubbles, and in one case, water, there were no creeping bubbles observed at plate angles over 45 degrees. The water-plexiglass combination (Fig. 21) resulted in the largest creeping bubble regions. This is thought to be due to water having a larger equilibrium contact angle on plexiglass (approximately 58 degrees) as compared to glass (approx. 40 degrees) and stainless steel (approx. 50 degrees). The larger equilibrium contact angle gives a greater adhesion force between the bubble and the surface, resulting in the larger creeping bubble regime.

Qualitatively, the transition ranges in Figures 21, 22, and 23 follow the small bubble approximation of the previous chapter, where the transition threshold has the form $gD^2\sin(\phi)$ from equation (2.15). If one looks at the trend of the largest static bubbles (the dots on the graphs) it can be seen that they follow approximately the $D^2\sin(\phi)$ dependence of equation (2.15). The scatter in the transition ranges is most likely due to the effect of surface irregularities and contaminants.
The onset of creeping appeared, visually, to occur as predicted, with the bubble distorting up to some maximum contact angle hysteresis and beginning to creep. During steady state creep the bubble approximately maintained this contact angle hysteresis on a global scale, while locally the contact angles might vary.

Creeping motion was rarely at a smooth, constant velocity. Instead, it tended to be a sporadic motion caused by local contact angle variations resulting in uneven surface tension forces around the line of contact. This allows different sections of the footprint to move at different velocities, resulting in the sporadic motion. Surface irregularities and the previous wetting history of the surface both influence the local variations in contact angle, and will be discussed below.

Footprint shape was visibly different in the creeping mode than in the static mode. Since most creeping bubbles were large and non-spherical their footprints were not circular. These bubbles show a bell shaped footprint characterized by an almost flat edge at the rear of the bubble, and a smaller radius of curvature at the leading edge. An example is shown in Figure 24. This shape appeared to correlate with the amount of bubble distortion and showed the existence of an almost two dimensional behavior (the nearly flat edge) at the rear of the bubble.

In general, the larger the bubble, the more pronounced the flat rear of the footprint became. It appeared that the
FIGURE 24. Creeping bubble footprint shape.
center of the back of the bubble, which maintains the advancing contact angle, catches up with the rest of the bubble as it moves. In the process, it entrains the rest of the rear of the bubble with it. As the line of contact moves, surface tension in the bubble tries to maintain the same contact angle across the back of the footprint, resulting in the flatness observed. The same phenomenon occurs in reverse at the front of the bubble. The front of the bubble, which is at the receding contact angle, moves ahead of the rest of the bubble, resulting in a sharper footprint shape at the front of the bubble, as compared to the rear (Fig. 24).

Previous wetting history of the surface also played an important role in the motion of the footprint. During creeping, the front of the bubble must displace the fluid in its path away from the surface, while fluid is attached back on the surface at the rear of the bubble. As creep is generally a slow, low Reynolds number flow, it was possible to distinguish the back of the bubble reaching the point where the front of the bubble began its motion. For reference, call this point P. Before reaching point P, the rear of the bubble was moving over an area of surface that was dried by capillarity when the bubble was first placed on the plate. After passing point P the rear of the bubble was then sliding over part of the surface that had been very recently voided of fluid from the passing of the front of the bubble. Because of the short time (on the order of
seconds) between the passing of the front and rear of the bubble over the same point P, the surface could not be completely dried of fluid on the microscopic level. A recently wetted surface is known to have a lower maximum contact angle hysteresis resulting in a lower surface adhesion force.

The effect of the lower adhesion force was that the velocity of the footprint, and hence, the bubble, was increased by a visible amount. Figure 19a shows an example of this effect at t=0.7 seconds. The slope of the position graph increases quickly at the mentioned time, indicating an increase in the bubble's velocity. Large bubbles (d<3.0 mm) also showed this behavior, but to a less dramatic extent. Wettability effects are also important in the transition from creeping to separated flow, and will be discussed in the following chapter on separated bubbles.

Surface irregularities also have a large effect on the footprint shape and motion. As the footprint moves at a given speed, the contact angle attempts to retain a certain value at each position around the footprint. When a small surface irregularity is encountered, the contact angle is maintained on a microscopic scale at that point. If the irregularity is assumed to be a small ridge as in Figure 25, a greater force is required at that point to move the interface over the ridge. Assuming that the remainder of the footprint is moving along without any other surface irregularities present, the existence of the ridge causes a
FIGURE 25. Surface roughness effects on dynamic contact angle.
small section of the footprint to either stop or slow down, thereby slowing the bubble and changing the shape of the footprint. If the ridge is large enough with respect to the bubble size, the entire bubble may stop moving, but usually the buoyant force of the bubble is enough to move the interface over the ridge, and results in the sporadic movement of small bubbles.

Premature onset of bubble motion may have been induced for a small number of bubbles during the process of tilting the test surface. After a bubble was placed on the surface the plate was then tilted to the required angle. Tilting was accomplished by manually rotating the plate with knob. Although the rotation was usually smooth, small fluctuations in the tilting speed may have imparted enough of an impulse to some bubbles on the verge of motion to cause them either to begin creeping or even separate.

Little evidence of creeping bubbles were observed for glycerol or ethyl alcohol. Bubbles in glycerol appeared to separate immediately from the surface as soon as it was tilted, while no bubbles in ethyl alcohol were observed to adhere to any of the surfaces, due to the low surface tension.
Chapter 4 - Separated Bubbles

4.1 Governing Equations

Separated bubble motion was observed for cases when the external forces were too great for the bubble to maintain a creeping motion. In these cases the bubble is separated from the surface by a very thin fluid layer. After separation, the bubble accelerates to a terminal velocity, which is characteristically higher than the terminal creeping velocity. In isothermal conditions, the drag coefficient of the bubble can be inferred by a force balance between the buoyant force and the drag force.

4.1.1 Theoretical Drag Coefficients

Little work has been done on the drag of bubbles moving near walls, although there should be a correlation between this drag and the drag on bubbles in an unbounded fluid. The drag on bubbles in an unbounded fluid has been studied extensively. For low speed, viscous flow ($Re < 1$) Hadamard and Rybczynski have both calculated the drag coefficient to be,

\begin{equation}
Cd = \frac{16}{Re}, \quad \text{for } Re < 1,
\end{equation}

for a spherical bubble from the stream function assuming Stokes' flow (9). For Reynolds numbers between $2 < Re < 200$ the Navier-Stokes equations have been solved numerically. Haas, et.al. (17) compiled the results from several studies and found
(4.2) \[ C_d = 14.9 \times (Re)^{(-0.78)} \] for \( 2 < Re < 200 \),

to be a good fit. For larger Reynolds numbers a solution for
the drag coefficient can be found by calculating the viscous
dissipation of energy for potential flow about a sphere
(18). In this range the drag is related to the Reynolds
number by,

(4.3) \[ C_d = 48/Re \] for \( 100 < Re < 1000 \).

Note that equation (4.2) does not have the same \( 1/Re \)
behavior as (4.1) and (4.3). This is because (4.2) covers a
range of Reynolds numbers which connects the ranges of (4.1)
and (4.3), resulting in a different Reynolds number
dependence.

4.1.2 Measured Drag Coefficients and the Equivolumetric
Sphere

In order to relate measured terminal bubble velocities to
drag coefficients some area must be assumed. In the
following the \( C_d \) will be based on the flat plate area of a
sphere with the same volume as the measured bubbles. Since
the measured bubbles were very similar in shape to oblate
spheroids, they were assumed to have the spheroidal volume
given by,

(4.4) \[ V = \frac{1}{6} \pi d^2 h, \]
where \( d \) and \( h \) are the bubble's measured diameter and height, respectively, corresponding to the major and minor axis of the spheroid. Equating this volume to that of a sphere and solving for the diameter gives the equivalent diameter, \( d_{eq} \),

\[
(4.5) \quad d_{eq} = \left[ \frac{d^2 h}{2} \right]^{\frac{1}{3}}.
\]

By equating the buoyancy to the drag force at terminal velocity, the drag coefficient can then be found. The buoyant force on the equivolumetric spherical bubble is,

\[
(4.6) \quad F_b = \frac{1}{6} \pi d_{eq}^3 \rho g \sin \phi,
\]

and the drag is given by,

\[
(4.7) \quad F_d = \frac{1}{8} \pi \rho U^2 d_{eq}^2 C_d,
\]

where \( V \) is the terminal velocity. Equating the buoyant force to the drag gives,

\[
(4.8) \quad \frac{1}{6} \pi d_{eq}^3 \rho g \sin \phi = \frac{1}{8} \pi \rho U^2 d_{eq}^2 C_d.
\]

Solving for \( C_d \) gives the final form of the drag coefficient,

\[
(4.9) \quad C_d = \frac{4}{3} \frac{g}{U^2} \frac{d_{eq}}{d} \sin \phi.
\]
4.1.3 Wall Effects on Drag Coefficients

Bubbles moving in the proximity of surfaces generally have a higher drag coefficient than those moving in an unbounded fluid. This effect has been studied primarily for bubbles rising in a cylindrical vessel and many results are gathered in Clift, et.al.(9). Most of the studies have included empirical equations to calculate the drag increase for specific flow geometries. Some theoretical derivations have been accomplished for very low Reynolds numbers (Stokes's flow). The common approach is to calculate a wall effect coefficient which is the ratio of drag with wall effects to drag on the same sized bubble in an unbounded fluid. Wall effect coefficients will be calculated for each observed Reynolds number regime using this method.

4.2 Apparatus and Test Procedures

Separated bubbles were observed by the same techniques used to observe creeping bubbles in Chapter 3. The only change in the testing procedures occurred for the tests on ethyl alcohol. Bubbles were produced in water and glycerol by blowing them onto a level surface then tilting the surface to the required angle. Because bubbles would not adhere to any surface in contact with ethyl alcohol, it was necessary to set the plate at the required angle and produce bubbles with the syringe near the lowest end of the plate. The bubbles would rise up along the surface in separated motion while being recorded with the video system. During video playback position time histories were taken from the
side view on the video screen to evaluate the terminal velocity of each bubble.

Footprint behavior during transition from creeping mode to separated mode was recorded by mounting the camera above the tank for qualitative investigation. The setup is detailed in Section 3.2 of the creeping bubble chapter.

4.3 Results and Discussion
4.3.1 Transition from Creeping to Separated Motion

The behavior of a typical bubble during transition is shown in Figure 26 for a glass surface in water. In the high speed video frames the bubble footprint is observed to undergo transition. The time between each frame is 1/240 second. In frame 1191 the creeping bubble is being accelerated by bouyancy. Fluid must be displaced ahead of the bubble as it moves. If the velocity is high enough the maximum rate of fluid removal at the surface is reached, and the bubble will begin to separate. This is evident in the footprint in frame 1192 where small rivulets of fluid trail back from the front of the bubble. The structure in the fluid trails are thought to be a result of local surface imperfections or uneven drying characteristics of the surface.

The onset of irregularity results in a slowing of the leading edge of the footprint while the rear of the footprint continues at a constant velocity, or accelerates. At this point the size of the footprint begins to decrease, as seen in frames 1193-1198. The rear of the footprint then
FIGURE 26. Bubble footprint transition from creep to separated motion.
quickly catches up with the front. When this occurs, surface tension causes the bubble to close completely and become totally separated from the surface by a thin fluid layer. Once separated, there is no longer a surface tension adhesion force restricting motion, and the bubble accelerates up to a new terminal velocity. The bubble remains near the surface due to the normal component of the buoyant force. The terminal velocity shape of the separated bubbles was somewhat ellipsoidal due to the hydrostatic pressure change across the bubble, and was also influenced by the close proximity of the wall. After separation, the bubbles were not observed to reattach themselves to the surface.

Figure 27 shows the separation phenomenon for a 6.27 mm diameter bubble in water on stainless steel as seen from the side. In this series the time between frames is 1/120 seconds. Of primary interest here are frames 1167 and 1168, where the rear of the bubble is seen just before and just after it has detached from the plate. Within four frames, or 1/30 seconds, the bubble has reached its approximate terminal velocity shape, as seen in frame 1171. After separation, some oscillation of the bubble around its final shape was commonly observed.

4.3.2 Terminal Velocity - Drag Coefficients
i. Water/Glass

Figures 28a and 29a show representative position time histories for two bubbles. Figure 28 is for a 2.93 mm
FIGURE 27. Bubble transition from creep to separated motion - side view.
FIGURE 27. (continued)
diameter bubble in water on glass with a surface incline angle of 15 degrees. Figure 29 is for a 4.8 mm diameter bubble in water on stainless steel at a surface angle of 45 degrees. The discrete data points are due to the framing rate of the camera. Separation is apparent where the slope increases dramatically. Terminal velocities were obtained for each bubble by a least squares fit to the straight section of the curve after separation. Velocities calculated by finite differencing between position data points are also shown in Figures 28b and 29b for the two representative bubbles. Due to the large amount of the noise in the velocities from finite differencing, drag coefficients were found from the least squares fit to the position data. Once bubbles separate, they are observed to accelerate to terminal velocity quickly, typically, in less than a tenth of a second.

Having calculated the terminal velocities, drag coefficients were then determined from equation (4.9) using the reference area of an equivolumetric sphere, as discussed in Section 4.1.2. Reynolds numbers were computed using the equivalent diameter of the equivolumetric sphere as the reference length. The results for water/glass are shown in the log-log plot of drag coefficient vs. Reynolds number in Figure 30. Also plotted is the theoretical drag curve for this intermediate Reynolds number regime given by equation (4.3), $Cd=48/Re$. Drag on the bubbles moving near the plate was expectedly higher than the theoretical drag on a bubble
FIGURE 28a. Separating bubble position time history - water/glass.

FIGURE 28b. Separating bubble velocity time history - water/glass.
FIGURE 29a. Separating bubble position time history - water/stainless steel.

FIGURE 29b. Separating bubble velocity time history - water/stainless steel.
FIGURE 30. Drag coefficient vs. Reynolds number – water/glass.
freely rising in an infinite fluid, as seen in Figure 30.

To calculate the wall effect coefficient a straight line with a slope parallel to the theoretical drag line was fitted by least squares to the data in the plot of Cd vs. Reynolds number. Since the plot is in log-log coordinates, the wall effect coefficient is simply the ratio of the X intercepts between the observed drag and the free bubble drag curves. For the water/glass case, the linear fit resulted in the drag given by $143/\text{Re}$, and a wall correction coefficient of $\lambda=2.98$, both of which are also shown in Figure 30.

ii. Glycerol/All Surfaces

Separated bubbles in glycerol moved very slowly, being dominated entirely by viscosity. Reynolds numbers were much less than one so the theoretical drag is calculated from (4.1), based on Stokes' flow. Experimental and theoretical results are graphed in Figure 31. Note the close agreement between theory and experiment that does not occur for the larger Reynolds number bubbles. The reason for the close agreement is that the highly viscous flow around the bubbles is not affected by the proximity of the plane surface, resulting in a wall effect coefficient of 1.

iii. Ethyl Alcohol/All Surfaces

Reynolds numbers of separated bubbles in ethyl alcohol were in a lower range than the bubbles in water, but still much greater than those in glycerol. For these bubbles the theoretical drag is given by (4.2), based on a curve fit to
DRAG COEFFICIENT VS REYNOLDS NUMBER

GLYCEROL

SURFACE
- GLASS
- PLEXIGLASS
- STAINLESS STEEL

$\frac{16}{R_E}$

$\left( \frac{C_D}{100} \right)$

$R_E \times 1000$

FIGURE 31. Drag coefficient vs. Reynolds number - glycerol/all surfaces.
numerical predictions, and the results are shown in Figure 32. The wall effect coefficient was calculated to be 2.06 for this Reynolds number range.

iv. Water/Stainless Steel

Drag coefficients for the water/stainless steel configuration are plotted against Reynolds number in Figure 33. Notice again the large deviation above the free bubble solution which, in this case, resulted in a wall effect coefficient of 14.86.

Wall effect coefficients as a function of Reynolds number range is plotted in Figure 34. The largest wall effect was for the water/stainless steel configuration, where \( \lambda = 14.86 \). Next largest was the water/glass combination, \( \lambda = 2.98 \). Ethyl alcohol exhibited a lower correction than both water configurations, \( \lambda = 2.06 \). At very small Reynolds numbers, such as the separated bubbles in glycerol, the effect of the wall becomes negligible, and the drag coefficient data agrees identically with theory. There is a clear trend for the wall effect coefficient to decrease with decreasing Reynolds number, and it appears to be linear in character, as shown in Figure 34.
FIGURE 32. Drag coefficient vs. Reynolds number - ethyl alcohol/all surfaces.
FIGURE 33. Drag coefficient vs. Reynolds number - water/stainless steel.
FIGURE 34. Wall effect coefficients vs. Reynolds number.
Chapter 5 - Conclusions

In Chapter 1 the background of bubble-surface interfaces was briefly described. In Chapter 2 a model for predicting the forces on static sessile bubbles on inclined surfaces was presented. The model assumes quasi-spherical bubble geometry. Air bubbles on a glass surface in water were tested to verify the equations of the static bubble model. The model was valid for bubbles up to a diameter of 0.22 cm or a Bond number of 0.68. For larger bubbles the model overcalculated the surface tension adhesion force due to the nonspherical shape of the bubbles. The larger the bubble the less spherical it becomes, resulting in a contact angle distribution which does not conform to the assumption of a sinusoidally varying contact angle around a circular base. For small bubbles, where the spherical assumption is valid, a simple model was developed to predict the onset of creeping motion: \( gD^2\sin(\phi) = \text{constant} \).

Creeping bubbles, which remain in contact with the surface while moving, comprise the smallest regime of observed bubble behavior in this study, and are discussed in Chapter 3. A creeping bubble terminal velocity model based on the assumptions of a spherical bubble, circular base of contact, and a sinusoidally varying contact angle was presented. Experimentally, all bubbles that were observed to creep were highly distorted nonspherical bubbles which did not fit the assumptions of the spherical model. Quite often, the
creeping bubbles would move in a sporadic motion, due to surface imperfections and irregular wetting characteristics of the test surfaces.

Chapter 4 discussed separated bubble motion and the transition between creeping and separated motion. Transition of creeping bubbles to separated motion showed many interesting aspects. In particular, the behavior of the bubble footprint as the bubble separated from the surface. The behavior of the footprint of bubbles undergoing separation was studied by means of a high speed video system. It was observed that small fluid rivulets trailed back from the front of the footprint through the area of contact as the bubble moved along the surface. When the rear of the footprint caught up to the fluid trails, it quickly caught up to the front of the footprint, at which point the bubble separated from the surface.

Most separated bubbles moving along a surface were noted to have greater drag than bubbles rising freely in an infinite fluid. Similar drag increases have been noted in the past for bubbles rising in tubes. Depending on the Reynolds number range, different wall correction coefficients were calculated from the data to give and indication of the drag increase due to the proximity of the surface. The larger the Reynolds the greater the drag increase. For very small Reynolds numbers (Re<<1), surface proximity gave no increase in drag, compared to the theoretical values.
REFERENCES


Appendix A - Behavior of Bubbles in Gyroscopes

I. Background

It has been proposed that the motion of minute bubbles, in the flotation fluid, on the surface of the gyroscope float may be a possible cause of transient float imbalances. These imbalances can limit the accuracy of high precision mechanical gyroscopes (19). The bubbles, if present, are most likely composed of hydrogen since it is used in float bearings and is known to outgas from epoxies used in the instruments. If bubbles are present in the damping fluid they will tend to migrate to the surface of the float due to its higher temperature through Marangoni convection (19). If even small (~20 microns) bubbles move, the float calibration will and the accuracy will be degraded. The goal of this appendix is to relate the results of the main body to the specific case of the high precision gyroscope.

II. Gyroscope Geometry

The inside of a high precision gyroscope consists of a sealed cylindrical instrument float suspended within the instrument case by a damping fluid. A cross-sectional view is shown in Figure A-1. The gap between the two cylinders is typically on the order of 0.005 inches. When in operation the surface of the float is maintained at a specific temperature and causes a thermal gradient of typically $236^\circ F/cm$ across the fluid gap, increasing towards the float. Any bubbles released into the gap will migrate towards the float.
FIGURE A-1. Gyroscope geometry.
surface due to Marangoni convection. Once arriving at the surface the bubbles' behavior becomes dependant on both the gravity forces and thermal gradients along the float surface.

III. Small Bubble Model

The main focus of this model is to predict the size at which bubble will begin motion along the float surface under various gravity conditions and thermal gradients. Once motion begins, the bubble will move in the direction of buoyancy and/or the Marangoni (thermal) force. The equation for static equilibrium of a small bubble on a surface was developed in Chapter 2, and is,

\[ -2R \sin \theta \int_0^\Pi \cos [\theta + \Delta \theta \cos \Psi] [\gamma_0 + \frac{d\gamma}{d\theta}] K R \sin \theta (1 - \cos \Psi) \cos \Psi d\Psi \]

\[ (A1) \]

\[ = \frac{1}{3} g R^3 \left[ 4 - (1 - \cos \theta)^2 (2 + \cos \theta) \right] \sin \phi + 3 \Pi R^2 \frac{d\gamma}{d\theta} K. \]

For small, quasi-spherical bubbles it can be assumed that both the average contact angle, \( \theta \), and the maximum contact angle hysteresis, \( \Delta \theta \), are constant. From these assumptions, the integral in (A1) is also constant. With no thermal gradients along the float surface, (A1) can then be rearranged into a simplified form as the motion threshold equation,

\[ -11760 \sin \theta \int_0^\Pi \gamma_0 \cos [\theta + \Delta \theta \cos \Psi] \cos \Psi d\Psi \]

\[ \frac{8 \Pi \left[ 4 - (1 - \cos \theta)^2 (2 + \cos \theta) \right]}{G d^3 \sin \phi} = C \]

(\( A2 \))

where \( G \) is the number of gravities, \( d \) is the bubble diameter, and \( \phi \) is the angle of the surface. Equation (A2)
gives the critical bubble diameter for the onset of creeping motion as a function of gravity and the surface angle. The constant in the equation is dependent on the gas, fluid, and surface materials, but is otherwise a constant of the system. Once the constant is known the threshold for bubble motion can be predicted for arbitrary bubble sizes, surface angles, and $G$ fields, as long as the bubbles are small and essentially spherical.

After the onset of creeping motion, as given by (A2), small bubbles will separate quickly from the surface if the average contact angle and the base of contact are small. Once separated, a bubble will remain near the float due to the large thermal gradient across the fluid gap, but will move along the surface from buoyancy and/or any thermal gradients along the float surface.

Bubbles in the damping fluid are on the order of 20 microns in diameter and are in a very low Reynolds number regime ($Re \ll 1$). Therefore, any separated bubbles at the float surface will have terminal velocities corresponding to a drag coefficient of the following form,

\[(A3) \quad Cd = \frac{16}{Re}\]

IV. Experimental Apparatus and Test Procedures

i. Apparatus

A small fluid chamber was built to measure the static and dynamic properties of hydrogen bubbles in damping fluids,
primarily the contact angle. A schematic of the contact angle apparatus (CAA) is shown in Figure A-2. It consisted of a small aluminum fluid chamber (6.67x1.25x0.95 cm) with two glass sides to permit a view of the bubbles. Inside the chamber were three 0.25 inch diameter beryllium plugs, each with a different surface roughness: $\sqrt{8}$, $\sqrt{16}$, and $\sqrt{32}$. Beryllium was used for the plugs as this is the same material used on the gyroscope float. Each plug was centrally drilled with a small hole for bubble production on the lower surface of the beryllium plug. A small tube inserted into the chamber at one end was used as a source for gas to allow pressurization of the fluid. This also allowed the damping fluid to be saturated with hydrogen. The chamber was completely sealed from the atmosphere.

ii. Test Procedures

The chamber was filled with a representative damping fluid, triazine, and sealed. Then, the entire CAA was placed on a hot plate and heated to a typical float operating temperature of 135°F. This temperature was maintained throughout the entire series of tests. Hydrogen was injected into the fluid for saturation purposes. Once saturated, small hydrogen bubbles were blown on the surface of the beryllium plugs and photographed for the contact angle measurements.

To make motion measurements in the CAA it was necessary to produce small bubbles ($d<100$ microns). This was accomplished by bubbling many large bubbles through the plugs, producing
CONTACT ANGLE APPARATUS

FIGURE A-2. Schematic of contact angle apparatus.
a multitude of small bubbles in the process. The small bubbles rose from buoyancy to the bottom surface of the plugs, where they came to rest. The CAA was then tilted to small angles (\( \theta \) = 3 to 5 degrees) and bubble motion was observed with a direct reading microscope. A calibrated scale in the microscope permitted bubbles diameters down to 0.0025 inches (63.5 microns) to be measured. Bubble diameter was recorded, as was whether or not the bubble moved over a period of five to ten minutes. From these measurements threshold conditions for motion could be determined and related to the unknown constant in equation (A2).

V. Experimental Results

i. Small Bubble Results

Small bubbles (approximately \( 60 < d < 250 \) microns) were studied at surface angles between three and five degrees on all three plugs. No measurable differences in bubble behavior between different surface roughness of \( 9 \sqrt{,16} \), and \( 32 \sqrt{} \) were observed. The breakaway constant (Eq. A2) was calculated for the smallest bubbles observed to move at each surface angle, and an average was determined, resulting in an approximate value of the constant. The average value of the constant was \( 6.5 \times 10^{-6} \) cm\(^2\). This constant corresponds to a 25 micron diameter bubble being the smallest bubble that would move on a surface at ninety degrees, in one gravity.

ii. Average Contact Angle

Accurate measurement of the average contact angle for
hydrogen bubbles in triazine on beryllium was difficult due to the small values observed. Measuring the contact angle required that a large enough bubble be produced such that the edge of the bubble was far enough away from the hole in the plug so as not to interfere with the measurement. In practice, large bubbles blown onto the beryllium plugs displayed a unique geometry shown in Figure A-3a. Because of the low contact angle in triazine, the bubble would not become attached to the bottom of the plug. A thin fluid layer remained between the bubble and the surface. The actual contact interface most likely occurred a short distance inside the hole, as depicted in Figure A-3b.

From photographs of small bubbles (Fig.A-4) together with the known breakaway constant (A2) it was possible to determine an upper limit on the average contact angle. Given the breakaway constant from above and an approximate average contact angle from the photographs, the integral term of (A2) was iteratively calculated until the breakaway constant was matched. This resulted in an upper limit of the average contact angle to be 10 degrees. The actual contact angle may be lower, but by no more than a few degrees. Therefore, the value of the contact angle will be assumed to be 10 degrees for all calculations. Effects of a slightly different average contact angle on bubble behavior will also be included in the discussion.

VI. Discussion

i. Breakaway Diameter
FIGURE A-3a. Large bubble on beryllium plug.

FIGURE A-3b. Small contact angle.
FIGURE A-4. Small bubble on beryllium plug.
With the results of the previous section, it was possible to extrapolate to bubble behavior under different gyro conditions. Given the breakaway constant of Equation (A2) and a value for the contact angle, the static force balance equation (Eq.1.15) could then be solved for \( \Delta \theta \), ignoring the thermal terms. Assuming \( \theta = 10 \) degrees, \( \Delta \theta \) is calculated to be 0.645 degrees. Fluid constants used for triazine are as follows: \( \gamma = 30.0 \) dynes/cm, \( \mu = 15.0 \) gm/(cm-sec), \( \rho = 2.4 \) gm/cm\(^3\). With these values, the model predicts the smallest size of a bubble that will break away from the surface and begin to creep under any given conditions. Once a bubble breaks free to creep it will quickly separate completely from the surface due to its small base of contact. The separated bubble will then move in the direction of the net force and will not reattach itself to the float surface.

In Figure A-5 the variation of breakaway diameter is plotted as a function of plate angle for gravity fields of 1 to 10 Gs. The small inset shows the surface angle and direction of the gravity force as related to the gyro float. For a plate angle of 90 degrees the breakaway diameter varies from about 26 microns in 1 G to 3 microns in 10 Gs.

An alternate representation of breakaway diameter is to present the calculated values for the case corresponding to a surface angle of 90 degrees, when the gravity component is a maximum. This will give the smallest breakaway diameter for a given force loading. Diameters are plotted as a function of thermal gradient along the plate in Figure A-6,
FIGURE A-5. Breakaway diameter vs. surface angle.
FIGURE A-6. Breakaway diameter vs. thermal gradient.
with each curve corresponding to a different gravity loading. A positive thermal gradient results in a thermal force opposite to the direction of gravity, and increases the size of the bubble that remains static.

The relationship between the thermal and bouyant force on bubbles is shown in Figure A-7. Each line corresponds to a different bubble diameter, while all cases are plotted for a plate angle of 90 degrees in 1 G. The important trend is that the Marangoni (thermal) force is very significant for smaller bubbles (d<20 microns).

Terminal velocities as a function of plate angle and as a function of thermal gradient are shown in Figures A-8 and A-9 for three representative bubble sizes in one gravity. Terminal velocity profiles for a 20 micron bubble as a function of plate angle are seen in Figure A-10. Note the profile is identical for each thermal gradient; the offset from the zero gradient curve is a constant proportional to the magnitude of the thermal gradient.

ii. Effect of a Smaller Average Contact Angle

As stated earlier, the average contact angle may actually be slightly less than the approximated value of 10 degrees used for calculations, and would result in a larger value of contact angle hysteresis calculated from the breakaway constant $gD^2\sin(\phi)$. This means that for a given bubble size, the bubble must distort more in order to remain static than if the contact angle were larger. The problem is that the small bubbles in gyroscopes are so entirely dominated by
FIGURE A-8. Terminal velocity vs. surface angle.
FIGURE A-10. Terminal velocity vs. surface angle for a 20 micron diameter bubble.
surface tension forces that large distortions are not likely. Due to the uncertainty of the contact angle, the error in all calculated breakaway diameters is estimated to be no more than +/- 5 microns for an error of +/- 2 degrees in the average contact angle.

iii. Surface Roughness Effects On Breakaway Diameters

The different surface roughnesses tested on the beryllium plugs played no measurable part in the determination of the breakaway behavior. This is due to the surface roughness being several magnitudes less than the size of the bubbles. A discussion of the effects of large surface imperfections and their effects on local contact angles of highly deformable bubbles was carried out in Chapter 2 Section 3.

The major difference here is that the small bubbles will remain essentially spherical no matter what the surface roughness is, and only two primary ranges of surface roughness will effect breakaway behavior.

Figure A-11a shows the perfectly flat, idealized surface assumed in the development of the included bubble theories. The first range of surface roughness to affect small bubbles is when the average irregularity height is large enough to trap small quantities of fluid between the bubble and the surface, as depicted in Figure A-11b. As the surface tension forces dominate for small bubbles, the bubble will not deform much locally at the points of contact to maintain the average contact angle, like a large bubble will. Because of this limited deformation, there is less contact with the
SURFACE ROUGHNESS EFFECTS

FIGURE A-11a. Smooth surface.

FIGURE A-11b. Slightly rough surface.

FIGURE A-11c. Very rough surface.
surface, and hence, less surface tension force to keep the bubble static. The result is that a smaller bubble will separate from the rough surface under a given force than if the surface was smooth. If the bubble is allowed to sit on the surface for a sufficiently long time some of the trapped fluid will be removed by capillarity, effectively drying part of the surface, and resulting in a greater bubble adhesion. Most surfaces, regardless of the roughness, will exhibit a similar time dependency.

The second range of surface roughness that affects bubble breakaway is when the irregularities are of the same magnitude as the bubble size, or larger. This is shown in Figure A-llc. When this occurs, the bubble can be modeled as being at some local surface orientation, as opposed to the overall surface orientation, and the above models and data on bubble breakaway hold.

iv. Concluding Remarks

It is apparent that small bubbles, on the order of less than 20 microns in diameter, will indeed breakaway from the gyro float surface under the application of gravity and moderate thermal forces. Breakaway diameters, based on a small bubble model, can be predicted for a variety of gyro conditions. Once motion begins the bubbles separate quickly, move along the surface in a predictable motion and cause an imbalance of the float which results in an error of the gyrooscope calibration. Surface roughness has a direct effect on the breakaway diameter of bubbles, but only when the
roughness is large enough to trap small quantities of fluid between the bubble and the wall. Surface roughnesses of $\sqrt[3]{8}$, $\sqrt[3]{16}$, and $\sqrt[3]{32}$ were tested and showed no measurable differences in breakaway diameters for bubbles near 20 microns in diameter.