A PREDICTIVE MODEL CONTROL SYSTEM
FOR UNDERGROUND COAL MINING

by

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ABSTRACT

A vertical control system for underground coal mining using two different predictive models of the coal seam interface was developed. The first predictive model, based on local three dimensional Taylor series extrapolation, was found to be very sensitive to measurement noise. Several Kalman filters were developed to reduce the uncertainty of the measurement of position of the cutting drum. Only the Kalman filter for the measurement of the distance along the longwall performed better than simple averaging of multiple measurements. Thus the Taylor series model required large numbers of measurements to achieve stable and accurate predictions of the coal interface profile.

To reduce the number of required measurements, an alternate predictive model was considered which intrinsically filters the noisy measurements. The second model employs least squares estimation techniques to develop a global model of the coal interface profile. Analysis and computer simulations using a least squares filter for the measurement of the coal interface height and Kalman filters for the measurement of the distance along and perpendicular to the longwall shows that an accuracy of .015 m (.5 in) can be achieved in the predictive model with measurements of the coal interface required every .5 m (1.6 ft) along the longwall for each cut.

Thesis Supervisor: Dr. Steven Dubowsky
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NOMENCLATURE

VARIABLES

A
\[ A_e \]
parameters of coal interface profile
estimated values of parameters

a
time constant of ranging arm

b
number of independent variables

C
\[ C_0, C_1, C_2 \]
constant

E
\[ E_{Dn}, E_m, E_T, E_z, E_{zc} \]
error
error in estimate of derivative
error in measurement
truncation error
distance between cutting point and coal interface
error in estimating coal interface

q
number of parameters

h
step in X direction

K
Kalman gain matrix

k,
discrete time step

M
\[ M_{zc}, M_{Dn} \]
mean

m
number of independent variables

N
number of measurements

O(h)
order of error is h

P
covariance matrix

p
\[ P_x, P_y, P_z \]
step in Y direction

phi
angular rotation

R
Residuals

Rs
Weighted sum of squares of the residuals

S
state matrix

\[ S_e, S_{A_e} \]
standard deviation
standard deviation of estimate of parameters
\( S^n \)  
standard deviation of derivative

\( S_Y \)  
standard deviation of measurements

\( S_H \)  
standard deviation of random input

\( t \)  
time

\( T \)  
increment time

\( U \)  
deterministic input

\( U_x, U_y \)  
random measurement noise

\( V_m \)  
random input disturbance

\( W \)  

\( W_x, W_y \)  

\( X, Y, Z \)  
global cartesian coordinate system
  
  \( X \) is along the longwall face
  \( Y \) is perpendicular to the longwall face
  \( Z \) is the height

\( Z_c \)  
height of coal interface

\( Zc_m \)  
measured height of coal interface

\( Zc_e \)  
estimate of height of coal interface

\( Z_v \)  
velocity of ranging arm

**SYMBOLS**

\( A \)  
matrix

\( D^n_X(Z) = \frac{d^n Z}{dx^n} \)  
\( n \)th partial derivative with respect to \( X \)

\( e^Z \)  
base of natural logarithm

\( E[ \cdot ] \)  
expectation operator

\( f(X) \)  
function of \( X \)

\( I(k, i) = \begin{cases} 0 & \text{if } i \neq k \\ 1 & \text{if } i = k \end{cases} \)  
Kronecker delta function

\( \ln(Z) \)  
natural logarithm

\( \pi \)  
3.1415926

\( ^n \, ^\Delta \)  
\( n \)th backward difference

\( \binom{n}{k} = \frac{n!}{(n-k)!k!} \)  
binomial coefficients
CHAPTER 1

INTRODUCTION

The subject of this thesis is the design of an automatic vertical control system, or AVCS, for use in a longwall coal mining operation. The AVCS is one of three major subsystems that would comprise a fully automated, or robotic, longwall coal mining system. The AVCS that will be presented differs from previous designs in that it relies on the generation and use of a predictive model of the coal interface based on historical measurements instead of real time measurements. While this approach can overcome many of the problems encountered by current designs of AVCS's, it will be shown that care must be taken in its application.

Before beginning the discussion on the research conducted for this thesis, current coal mining practices, motivations for automation, and previous research into automation will be presented.

1.1 CURRENT COAL MINING PRACTICE

There are currently two methods of underground coal mining in use around the world; full extraction and partial extraction. The full extraction method can be further subdivided into the longwall and shortwall methods.

The partial extraction method is based on the room-and-pillar concept. Here continuous miners, basically bulldozers with front end cutting drums and some type of coal gathering system, carve out large rooms in the coal seams, leaving behind pillars of coal to support the roof. These pillars may be withdrawn upon retreat if the roof conditions allow.

The shortwall method uses techniques from both the longwall method
and the partial extraction method. A continuous miner is used, as in the partial extraction method, in conjunction with hydraulic roof support.

With the longwall method a rectangular block of coal is developed for extraction by driving a set of three or four entries ~150 m (500 ft) apart between main entries ~1200 m (4000 ft) apart into the coal seam. Figure 1-1 illustrates the plan for a longwall installation. These developed panels are then extracted by a shearer which cuts the coal and loads it into a chain conveyor which runs along the entire length of the face of a panel. A series of hydraulic, self advancing roof supports provide full support over the shearer and conveyor. As the supports advance, the roof collapse behind them. Figure 1-2 illustrates the major components of a longwall installation. The typical longwall shearer uses cutting drums which are vertically adjustable. In the U.S. two drums are typically used, one cutting to the roof with the other cutting to the floor, with the cut taken in only one direction. In the U.K. a single drum is usually used with the roof cut taken on the first pass across the longwall and the floor cut taken on the second pass. The shearer normally rides on rails mounted on the conveyor assembly at speeds of .05 to .13 m/sec (10 to 25 ft/min). In the current manual systems at least two operators are required to be with the shearer as it progresses across the face. They control the height of the drums, speed of advance, and attitude of the shearer with respect to the seam. In addition, several operators are required to advance the roof supports.

The control system of a longwall shearer, which must be supplied either in a manual or automatic mode, can be divided into three major and relatively independent, areas; the Face Advancement System which positions the conveyor and as a result, the Y motion of the shearer, the Roll Control System which controls the attitude of the shearer about the X axis, and the Vertical Control System which positions the cutting drums in the Z direction to maximize the amount of coal taken.
PLAN FOR LONGWALL INSTALLATION

figure 1-1
The face advancement system is responsible for advancing the conveyer and shield supports. The entire longwall is not indexed forward in a single step but rather a procedure called sumping is used which advances the longwall in a serpentine fashion. This is necessary since most shearer drums are not designed to cut perpendicularly into the longwall. A typical sequence of operations is shown in figure 1–3 which is an adaption of several figures from reference 3.

The roll control system provides the capability to correct for twists and modulations in the conveyor which can occur when coal or other debris becomes lodged under the conveyor. The shearer is equipped with hydraulic actuators which rotates the shearer about its longitudinal, or X, axis, thus tilting the shearing drums relative to the coal seam.

The goal of the vertical control system is to control the position of the cutting drum so as to maximize the amount of coal taken while minimizing the amount of rock cut. An equally important objective is to maintain proper step and orientation of the coal seam between successive cuts.

The longwall method of mining is used predominately in all major coal producing countries except for the U.S. and Australia. In the U.S., 5% of the underground production is by the longwall method with most longwalls averaging 810,000 kg (1,000 Tons) per 2-shift day though a high of 11,000,000 kg (120,000 Tons) a day was reached in 1977. The current longwall method is usually not considered cost effective in thick, flat-lying, easy-to-mine coalbeds since the higher initial investment required with the longwall method does not offset the possible productivity improvements. It becomes increasingly attractive in thinner, more difficult to mine coalbeds since the alternative methods become increasingly difficult to apply. This in part explains the lack of interest in the U.S. in the use of the longwall method since it generally has thicker seams than Europe.
Step 1 - start

Step 2 - advance a portion of the conveyor

Step 3 - move shearer

Step 4 - advance conveyor

FACE ADVANCEMENT SEQUENCE
figure 1-3
The automation of the longwall method has the possibility of substantially altering this view by increasing the productivity of the longwall method. The automatic, or robotic, longwall method could then become the method of choice for a much larger number of mines.

1.2 MOTIVATION FOR AUTOMATION

There have been three major driving forces behind the automation of underground coal mining; miners' health and safety, the deterioration of the mining environment, and the requirement to increase productivity.

The health and safety issue was the major catalyst for development in the U.S. The Federal Coal Mine Health and Safety Act of 1969, Public Law 91-173, began much of current federal government involvement in this area. The basic thrust of the act was to reduce miner exposure to high risk areas. The development of remotely controlled machinery was the initial response of the mining community. The subsequent research into robotics for use in coal mining was a natural outgrowth of this initial work since it offered the potential of removing the human operator totally from the dangerous areas.

The deterioration of the mining environment is especially important in Europe where thin seams deep underground are becoming increasingly common since most of the easily mined seams have already been extracted. This was the initial impetus for the longwall method and it continues to require improvements in methods and machinery. This would appear to be reason enough for the apparent British lead of two years in the automation of the longwall method.

Productivity improvements also played a part. The U.S. Bureau of Mines began the Advanced Mining Technology program in the early 70's with the intent of increasing productivity while complementing the health and safety program. This approach directly addresses a common
complaint of mine operators and machinery suppliers. That is, while mine safety has definitely improved, the effects on productivity have been ignored. They point to statistics showing that from a high of 14,000 kg (15.6 Ton) per man day in 1969, productivity has decreased to 7,700 kg (8.5 Ton) per man day in 1976. The hope is to reverse this trend while maintaining, or improving, the gains made in safety.

The longwall method in and of itself has the potential of satisfying the major objectives of increasing safety and productivity even in traditionally difficult to mine seams. The use of roof supports at the working face enhances safety. It can function in many environments which are unworkable by other methods. Automating the process would further enhance these two features while providing for potentially large productivity improvements.

It is for these reasons that the automation of the Vertical Control Subsystem of the longwall method was chosen for study in this thesis. The current manual mode of control requires two operators at the coal face where dust and rock falls not only endanger them but also make it difficult to accurately position the cutting drums. The automation of this task would remove the operator from the cutting point while allowing for the efficient positioning of the cutting drum. It is considered that of the three major subsystems in a longwall installation, the automation of the Vertical Control System would yield the greatest increase in safety and productivity for a given investment in research.

These same reasons probably account for the observation that most of the previous investigations into the automation of underground coal mining have centered on the longwall method and particularly, the vertical control system. In addition, the longwall method is usually seen as more amenable to automation since its motions are constrained to well defined paths and its basic operations are few, and generally independent, in number. This independence allows for the gradual introduction of automation since only one operation need be automated
at a time. This allows the benefits of automation to be seen more quickly than might otherwise be possible.

1.3 PREVIOUS LONGWALL AUTOMATION STUDIES

The U.S. effort in automating the longwall method has been limited to computer simulation of the overall longwall system, detailed specification of the entire system, and limited field test of subassemblies but never of the entire system. The computer simulations have shown that the overall system is capable of meeting the design goals with overall efficiencies an order of magnitude superior to current methods.\(^2\)\(^3\) However, field tests of the proposed Vertical Control system have shown that not enough attention was paid to the implementation of the theory. In all of the field tests that I am aware of, the control system was damaged to the point of being useless after less than a day of operation. The Vertical Control System that will be considered in this thesis was specifically designed so as to minimize the exposure of sensitive instruments to damage while providing equivalent of superior performance.

The U.K. effort seems to have been more complete. Several generations of computer simulations have been done, 1/4 scale models of the entire system have been built and tested, and extensive field tests have been recently done but very little detailed information has been published. It has been suggested that simulations and scale models have shown the system to be workable.\(^1\)\(^7\)

None of the previous studies propose completely automatic operation. Most assume that there will be operators to check for debris and to aid in the turn around operations at the end of each cut.

Digital controllers have been proposed in all these control subsystems mainly for reasons of flexibility in control schemes as well
as changes in equipment. The National Coal Board in the U.K. have experimented with analog controllers with decidedly poor results. The analog system proved consistently unstable mainly because of problems in predicting coal thickness at the cutter head. Digital systems are capable of minimizing this problem because of their ability to store historical data. Several investigators have proposed relatively simple systems using historical data but none have actually been field tested. This concept of using stored historical data will be more fully exploited in this thesis.

A substantial portion of the research effort of both the U.S. and the U.K. has been devoted to the development of accurate and reliable coal interface sensors. These sensors are of crucial importance since their availability and performance characteristics have in large part decided what type of vertical control strategy is required. This in fact motivated this thesis since the current vertical control strategies required the sensors be placed in excessively harsh environments where they were quickly damaged.

All the coal interface sensors to be discussed except for the thermal sensor and possibly the sensitized picks are based on measuring the thickness of coal remaining on the roof after a cut has been made. This data is then either used in real time or stored in memory for later use.

Microwave systems have been developed by the U.S. National Bureau of Mines and the U.S. National Aeronautics and Space Administration. These systems are based on differences in the dielectric constants of air, coal, and rock which effects the speed of the waves. The design of a suitable processor to detect the location of the interface has proven to be a problem. Though results of lab tests were promising, this concept has proven unworkable in a mining environment due to dust and moisture and the necessity of exposing the cone to potential rock falls.
Active nucleonic sensors have yielded the best results in both simulations and underground tests but they have been all but abandoned for fear of miner exposure to radiation.\textsuperscript{2,17} They are based on a cesium 137 source emitting gamma radiation which is backscattered by the coal and measured by a detector. The thickness of the coal controls the amount of backscatter with a greater thickness increasing the backscatter. It must ride along the surface of coal which has proven to be a problem. The range of coal thickness that can be used with good accuracy is .03-.5 m (1-20 in).\textsuperscript{9}

Natural background radiation sensors have proven to be the most accepted coal sensor detectors. Most designs are based on a sodium iodide scintillation detector which measures the amount of gamma ray radiation originating from the potassium 40 isotope content of the rock. As coal thickness increases, the scintillation drops off exponentially. The range of coal thickness that can be used with good accuracy is .03-.5 m (1-20 in).\textsuperscript{9} The signal is considerably lower than that of the nucleonic sensor and is thus noisier. It has the advantage of not being required to be in direct contact with the roof as in the nucleonic sensor. This sensor has been successfully tested underground in both the U.S. and the U.K.\textsuperscript{9,17} Its most significant shortcoming is that the rock surrounding the coal seam must contain the potassium 40 isotope. This limits its use to only certain regions in the world.

The use of instrumented cutting picks located on the cutting drums has also been explored. Generally strain gauges or accelerometers are placed on conventional cutting picks. Both force and vibration measurement techniques have been used to detect the interface. I am aware of only one field test which occurred in the U.S. There the ability to distinguish between rock and coal was not successfully proven.\textsuperscript{9} This sensor is unique in that it is applicable only in situations where the amount of coal to be left on the roof is small, usually less than .03 m (1 in), since currently only the sharp variation in hardness between coal and rock produces a signal that can be easily recognized. As such it has been used mostly as a guard
against the taking of rock. If a rock signal is sent to the controller from the picks, the main control algorithm is overridden and the cutting drums are brought away from the interface.

Recently, research has been conducted at the Massachusetts Institute of Technology to develop a more sophisticated signal recognition system so that the signal from the sensitized picks could be used to determine the strata signature of a coal seam. This would allow for the determination of both the roof and floor coal interfaces at essentially the current cutting point. Laboratory experiments have yielded encouraging results but no field tests have been carried out.\textsuperscript{10}

A thermal sensor has also been developed at the Massachusetts Institute of Technology based upon the differential heating of coal and rock.\textsuperscript{12} It differs from the other sensors, except possibly the newly developed sensitized picks, discussed above, in that it is envisioned as scanning the face of the longwall. The strata variations are used to estimate the locations of both the roof and floor coal interfaces. Field testing of the sensor has shown the concept to be workable. Several significant aspects of this sensor will affect the choice of control algorithm to be used. First, the sensor requires a substantial amount of time to make a single measurement. This leads to choosing a control algorithm that makes use of few measurements at discrete locations. Secondly, this sensor requires that the face of the coal seam be unobstructed. Thus measurements can be made only after the cut has been made. The control system will therefore be required to make use of historical data rather than real time data. Lastly, both the roof and floor coal interface can be determined whereas the previous sensors, except possibly for the advanced sensitized picks, could only determine the roof interface since debris prevents access to the floor. The control system should then be capable of utilizing this added information to improve performance.\textsuperscript{12}

The vertical control system that will be developed in this thesis
assumes that a sensor with the characteristic of the thermal sensor will be employed. It should be noted that all of the other sensors can be employed in a similar manner except that only the roof coal interface can be determined.

Two vertical control strategies have been employed by previous investigators. The first method is based on the real time measurements of the coal thickness at some distance behind the lead cutting drum. The distance between the cutting point and the sensor determines the time delay in the system. The measurement is used in a feedback loop to control the height of the cutting drum. Figure 1-4 illustrates the concept. The British have found this method to yield unstable control in their analog control system.¹⁷ Both U.S. and U.K. simulations and field tests have shown that a digital implementation of this method to be quite workable in controlled conditions.²,⁶,⁹,¹⁷ However, in actual mining environments, the coal interface sensors were quickly destroyed.⁹ Microwave, natural background radiation, and active nuleonic detectors have all been used in this approach. The thermal sensor would not be appropriate for this system since continuous real time measurements are required. If sensitized picks could be shown to be capable of accurately determining the coal interface, it could be used in this system and would, in fact, eliminate a major problem associated with this approach.

The major problem associated with this real time feedback approach is due to the coal interface detector being in a vulnerable location where it can be easily damaged by falling rock and coal. It would be desirable to have the sensor located at a greater distance behind the cutting drum. However when this is done the time delay is increased with a resulting degradation in performance and even possible instability. This problem is very real. All the field trials that I am aware of in the U.S. have ended with the sensors being damaged beyond use.⁹

The second vertical control strategy that has been employed is
PREVIOUS VERTICAL CONTROL SYSTEM

figure 1-4
based on storing sensor data on coal thickness on every pass across the longwall with this information being used on the next pass as the coal thickness at the current cutting print. A sensor which measures the location of the shearer along the length of the longwall references the data so that the present cut estimate is equal to the past cut measurement at the same distance along the face. This method assumes that the coal interface fluctuates very slowly from cut to cut and between data points. Simulation has shown this to be a stable and workable system but I am not aware of any field test conducted using this approach.6,9,2,17 Obviously the prediction will be in error by at least the amount of the change in roof height from cut to cut. For a 1 m (3.3 ft) wide cut, the roof change can be as much as .06 m (2 in) for the coal seam that was selected for use in this thesis. Measurement error will further degrade the estimate.

Reference 6 indicates that more complex coal thickness prediction methods have been considered with the conclusion that they tend to decrease performance though no details were given. Some of the methods considered included, one dimensional interpolation of future coal thickness from several past measurements, second order prediction or two-dimensional interpolation techniques which require memory of several past cuts both parallel and orthogonal to the longwall face, and a technique that makes use of local statistics of the mine such as slope, average thickness and, interface roughness.6

A predictive control method based on historical data has the possibility of eliminating the two related problems of sensor susceptibility to damage and time delay instability. Data does not have to be taken in real time. This allows for the acquisition of data at times when the instrumentation is not in danger. Continuous measurement are also not necessary if the proper prediction algorithms are used. This allows for use of sensors such as the thermal sensor which requires a significant amount of time to make each measurement as well as the other sensors already discussed. It will also be shown that there are indeed potential problems with this approach as
mentioned in reference 6 if the approach is not applied with care.

In chapter 2 an alternative vertical control system is presented that uses the predictive control method based on historical data. A model of the longwall process is developed for analysis and simulation purpose. Assumptions of the characteristics of the coal interface sensors are also presented.

Chapters 3 and 4 are concerned with the development of a predictive model of the coal seam roof profile using discrete, noisy measurements. Chapter 3 explores the use of the Taylor series while chapter 4 considers the use of polynomial least squares surface fitting techniques. In both cases the issues of sensitivity to noise, extrapolative properties, and use of Kalman filtering are explored.
CHAPTER 2

A PREDICTIVE MODEL VERTICAL CONTROL SYSTEM

It was discussed in the introduction that attempting to use continuous real time measurement of the coal thickness at some distance behind the cutting drum could lead to instabilities due to large time delays. Trying to reduce the time delay by moving the sensor closer to the cutting point places the coal interface sensor in a vulnerable situation. In addition the use of sensors that require large amounts of time to make a measurement, such as the thermal sensor, cannot be used. Use of historical coal thickness data was mentioned by several studies as being a possible solution to this problem since it would remove the requirement of making continuous real time measurements. However, no detailed studies of this concept have been carried out. This concept of using discrete historical data to predict the location of the coal interface at the current cutting point is the main issue of this thesis. Two predictive models are considered in chapters 3 and 4.

To show how such a predictive model vertical control system would be configured and to analyze the performance of the proposed techniques the following assumptions are made.

1) The position of the shearer along the longwall, \( X(t) \), is modeled as a constant velocity process with a white, zero mean, gaussianly distributed random velocity disturbance. This type of disturbance was selected for use since it can be easily handled by optimal filtering theory and it accurately represents many processes of this type. Both the nominal velocity and the standard deviation of the disturbance are assumed known. The position at \( t = (k+1)T \) is then given by:
(2-1) \[ X_{k+1} = X_k + T\omega_{k} + T\omega_{\omega_k} \]

where

\[ T = \text{time delay between measurements} \]

\[ \omega_{k} = \text{the known constant deterministic velocity} \]

\[ \omega_{\omega_k} = \text{the random velocity input} \]

2) The position of the shearer perpendicular to the longwall, \( Y(t) \), is assumed constant with a white, zero mean, gaussianly distributed random position disturbance. Sumping operations, as defined in the introduction, are not considered. It should be possible to adapt the predictive model control system at a later time to include them. The position at \( t = (k+1)T \) is then given by:

(2-2) \[ Y_{k+1} = U_y + \omega_y \]

where

\[ U_y = \text{the known constant deterministic position} \]

\[ \omega_y = \text{the random position input} \]

3) The dynamics of the cutting drum were determined experimentally for an Eickhoff shearer by Foster-Miller Associates (see reference 9). The height, \( Z \), and velocity, \( Z_v \), of the cutting drum at time \( t = (k+1)T \) is given by:

(2-3) \[
\begin{bmatrix}
Z \\
Z_v
\end{bmatrix}
_{k+1} =
\begin{bmatrix}
1 & a^{-1}(1 - e^{-aT}) \\
0 & e^{-aT}
\end{bmatrix}
\begin{bmatrix}
Z \\
Z_v
\end{bmatrix}
_k +
\begin{bmatrix}
a^{-2}(aT - 1 + e^{-aT}) \\
a^{-1}(1 - e^{-aT})
\end{bmatrix}
\begin{bmatrix}
\omega Z_k \\
\omega Z_v
\end{bmatrix}
_k
\]

where

\[ \omega Z_k = \text{the deterministic control signal} \]
If a state vector is chosen to include \( X, Y, Z, \) and \( Z_v \), the overall longwall system can then be represented by the following matrix discrete time difference equation:

\[
S_{k+1} = FS_k + LU_k + GW_k
\]

where

\[
S_k = \begin{bmatrix} X_k \\ Y_k \\ Z_k \\ Z_v_k \end{bmatrix}, \quad U_k = \begin{bmatrix} U_x_k \\ U_y_k \\ U_z_k \end{bmatrix}, \quad W_k = \begin{bmatrix} W_x_k \\ W_y_k \end{bmatrix}
\]

\[
F = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & a^{-1}x(1 - e^{-aXT}) \\ 0 & 0 & 0 & e^{-aXT} \end{bmatrix}, \quad L = \begin{bmatrix} T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a^{-2}x(aXT - 1 + e^{-aXT}) & 0 \\ 0 & 0 & a^{-1}x(1 - e^{-aXT}) & 0 \end{bmatrix}, \quad G = \begin{bmatrix} T & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\]

The output of the system, \( E_z \), will be chosen as the distance between the estimate of the height of the roof coal interface, \( Z_c \), and the top edge of the cutting drum, \( Z \).

\[
E_{z_k} = Z_{c_k} - Z_k
\]

The height of the coal interface, \( Z_c \), is obtained not from real time measurements, but from a predictive model of the roof interface profile that the control system has generated from historical measured data. Note that in general this relationship is nonlinear and time varying.
\[ Z_k = f_k(X_k, Y_k) \]

The output can be obtained from the state matrix by:

\[ Z_k = C_k (X_k, Y_k) - Z_k \]

where

\[ C_k = \begin{bmatrix} f_k' & -1 & 0 \end{bmatrix} \]

When a feedback controller is to be designed for this longwall system, the matrix will be linearized in a piecewise fashion such that linear control theory can be applied. The control gains can then be recalculated over each linear region. It will be shown in chapter 3 and 4 that the measurement of the state variables will require filtering for the final implementation of the controller. After each pass across the longwall, the predictive model of the roof is updated with the use of the new information and thus a sort of feedback is retained. Figure 2-1 shows the block diagram of the assumed system.

In the subsequent analysis and simulations, several additional assumptions will be required.

1. The shearer drum is assumed to take a 1 m (3.3 ft) cut wide on each pass along the longwall. This width includes the widest current longwall shearsers in use today.

2. The measurement of the spatial location of the shearer will be assumed to be referenced to a global coordinate system. This is a simplifying assumption that may be difficult to supply in practice.

3. The additive noise source for all the measurements will be assumed to be white (uncorrelated in time) and gaussianly distributed with zero mean and a constant known standard deviation. As with the random disturbance, the use of this
PREDICTIVE MODEL
CONTROL SYSTEM

figure 2-1
type of measurement disturbance allows for the application of optimal filtering theory. It has also been found to be valid in many applications of this sort. In general the standard deviation of the measurements will depend upon the specific sensor used and to some extent the conditions of its application. A value of .008 m (.31 in) will be used in this study as a reasonable value.

4. The required accuracy of a vertical control system will vary from one application to the next. If very high quality coal is to be mined, the accuracy of the vertical control system should be quite good and other factors such as cost of implementation and speed of operation may have lower priority than might otherwise be the case. For purposes of this thesis, a total allowable error of +.03 m (1 in) will be used. Furthermore, the total error will be evenly divided between the control system and the predictive model of the coal interface. Thus the maximum error goal in the predictive model will be set at +.015 m (.5 in).

5. For the simulations that will be performed the dynamics of the cutting drum height will not be considered. This thesis is focusing on the development of accurate and robust predictive models of the coal interface and as such the dynamic of the drum height does not need to play a significant part.

6. Only the roof coal interface is considered in the subsequent analysis. If a coal interface sensor such as the thermal sensor discussed in the introduction is used the same techniques that will be presented in chapter 3 and 4 can be used to develop a floor coal interface predictive model.
CHAPTER 3
TAYLOR SERIES PREDICTIVE MODEL
OF COAL INTERFACE PROFILE

As was discussed in the introduction, the use of real time feedback control for the vertical control system required the coal interface sensor be placed close to the current cutting point, where it could easily be damaged, or large time delays, which could lead to instability, would be introduced. Furthermore, the need for continuous real time measurement of the coal interface prevented use of some of the coal interface detectors, such as the thermal sensor, which required a substantial amount of time to make a single measurement.

The use of discrete historical data can eliminate all three of these concerns. The coal interface sensor is no longer required to make continuous real time measurements close to the current cutting point. Instead, discrete measurement of the coal interface height can be made at a safe distance behind the shearer. Figure 3-1 illustrates the resulting two dimensional grid of data points where the points are equally spaced along the longwall, X, and measurements are made after every pass of the shearer, Y. In actual practice the measurement of the height of the coal interface may be required only after a given number of cuts and not necessarily after each cut.

Chapter 2 outlined how a predictive model of the coal interface height, which would be generated from stored historical data, could be incorporated into a vertical control system. In general the predictive model should satisfy two main goals. First, it must obviously meet the accuracy requirements. In chapter 2, a maximum allowable error of 0.015 m (.5 in) for the estimates of the predictive model was selected. The second major requirement is that the predictive model should use as few data points as possible since a substantial amount of time may be required for each measurement of the coal interface. These data points will be obtained from a smooth and continuous coal interface as was shown in appendix A. A natural start
3-d GRID OF DATA POINTS

figure 3-1
to approximating this profile would be use Taylor series.

The coal interface height, \( Z_c \), can be expressed as:

\[
(3-1) \quad Z_c = Z_c(X,Y)
\]

Expanding \( Z_c \) about a point, called the pivot, in a Taylor series expansion, the following equation is obtained:

\[
(3-2) \quad Z_c(X,Y) = Z_c + \left( \frac{dZ_c}{dX} \right)_p (X-X_p) + \left( \frac{dZ_c}{dY} \right)_p (Y-Y_p) + \ldots
\]

where

\[ Z_c = \text{estimate of coal interface height} \]

The derivatives of the Taylor series can be estimated by using finite difference techniques of various orders. Equation (3-2) will be used to extrapolate from the pivot point to estimate the height of the coal interface at the current cutting point. The accuracy of this estimate is affected by several factors.

1) Truncation error – this will be shown to depend on the number of terms retained, the step size, and the function we are trying to estimate.

2) Error in approximating the derivatives – this will be shown to be a function of the number of points considered, the step size between points, and the accuracy with which these points are measured.

3) Measurement errors in \( X, Y, \) & \( Z \).

3.1 TRUNCATION ERROR

To illustrate some key points the following function will be considered:
(3-3) \[ Zc(x) = C_0 + C_1 x + C_2 x^2 \]

The Taylor Series will give an exact result if the first three terms are retained and the derivatives are assumed perfectly known since polynomials are analytic. If only the first two terms are retained and the derivatives are assumed perfectly known the following estimation to \( Zc \) is obtained:

\[
\begin{align*}
(3-4) \quad Zc_e(x) &= Zc_p + (C_1 + 2x C_2 x_p) (x - x_p) \\
&= C_0 + C_1 x + 2x C_2 x x_p - C_2 x^2_p \\
\end{align*}
\]

The error due to truncation is thus:

\[
(3-5) \quad E = Zc - Zc_e = C_2 x (x - x_p)^2
\]

Several important points are shown by this example.

1) The larger the step from the pivot, the larger the error.
2) The error is a function of the particular roof profile.
3) As more terms are truncated, the error becomes larger.

A computer simulation was performed to investigate the effect of truncation and step size on the error produced by the Taylor series when using the coal interface profile developed in appendix A. This profile, based on actual coal mine survey, was shown to be adequately modeled as an eighth order polynomial. A two dimensional Taylor series was expanded about various pivot points with the first two to nine terms retained. The derivatives were assumed to be perfectly known, the step in the \( Y \) direction was held constant at 1 m (3.3 ft) except for figure 3-2 where the step in \( Y \) is 0. The step in the \( X \) was varied continuously about the pivot. Figures 3-2 through 3-4 illustrate the percentage error in the approximation as a function of step size in \( X \) and the number of terms retained with percentage error defined as:

\[
(3-5) \quad % E = 100 \frac{(Zc_e - Zc)}{Zc}
\]
Taylor approx about X, Y=35 M with Y step = 0 Zp=1.71 m

TAYLOR SERIES APPROXIMATION TO COAL INTERFACE PROFILE

figure 3-2
Taylor approx about \( X, Y = 10 \text{ M} \) with \( Y \) step = 1 M \( Z_P = 1.45 \text{ M} \)

Number represents terms in Taylor series

Distance along longwall from pivot (m)

Taylor series approximation to coal interface profile

Figure 3-4
Figure 3-2 illustrates that an exact approximation to the coal interface profile can be achieved when the first nine terms of the Taylor series are retained. This is not surprising since the roof profile being used reduces to an eighth order polynomial in the two dimensional case. The significant point is that the lower order approximations rapidly increase in error at a step size of 15 meters (49 ft).

Figures 3-3 and 3-4 illustrate the approximation when the step in the Y direction, the distance perpendicular to the face of the longwall, is held at a constant 1 meter (3.3 ft). The significant point in this case is that an exact approximation of the coal interface profile cannot be achieved. The roof profile that is being used has only nonzero derivatives up to eighth order and yet the Taylor series which retains the first nine terms does not match the profile. This is most likely due to round off error since the magnitude of the higher order mixed derivatives is very small. As before there appears to be an upper bound of approximately 15 m (49 ft) for the step size in X at which the error begins increasing rapidly.

Figure 3-4 was chosen since it represents worst case error conditions for the roof profile that has been chosen while figure 3-3 represents more typical errors. Specific numerical results will be used from figure 3-4 when the required order and maximum allowable step size are determined. As a rough indication of what the maximum step size in X will be, the full allowable error in the predictive model will be attributed to the error resulting from the truncation of the Taylor series with the error in the derivatives and the measurements assumed to be equal to zero. From chapter two, the maximum allowable error in the estimation of the coal interface height was selected as .015 m (0.6 in).

\[(3-6) \quad \text{MAX } E_z = 0.015 \text{ m (0.6 in)} \]
\[= |Z_{e} - Z_{c}| \]
The maximum height of the coal interface in figures 3-3 and 3-4 is no greater than 1.8 m (6 ft). Thus the maximum allowable % error is:

\[
(3-7) \quad \text{MAX} \% E_z = 100 \times (0.015 \text{ m})/(1.8 \text{ m}) = 0.83
\]

Using this criteria, several of the approximations will yield conservative results if the step size in the X direction is kept below 5 meters (16.4 ft). Instead of extrapolating beyond 5 m, the pivot can be relocated and smaller step sizes can be used. With this approach accurate estimates of the coal interface height can be achieved across the entire length of the longwall. The final selection of how many terms must be kept in the Taylor series and the maximum step size in Y must await discussion of the approximation of the derivatives.

3.2 ESTIMATION OF THE DERIVATIVES

The derivatives of the the roof profile at the pivot point must be estimate from the discrete measurements if the Taylor series predictive model is to be used. A common method of obtaining these estimates is by finite backward difference techniques. Reference 14 contains a detailed derivation of the method which is outlined here.

Suppose we have the following equally spaced point.

\[
(3-8) \quad X_0, X_1, X_2, \ldots
\]

\[
Z_0(X_0), Z_1(X_1), Z_2(X_2), \ldots \quad \text{where} \quad X_i = X_0 + i \Delta x
\]

If the function Z is expanded about X in a Taylor series:

\[
(3-9) \quad Z(X-h) = Z(X) - \left( \frac{dZ}{dX} \bigg|_{X} \right) \frac{h}{1!} + \left( \frac{d^2Z}{dX^2} \bigg|_{X} \right) \frac{h^2}{2!} - \cdots
\]

Using operator notation, this can be expressed as:
(3-10) \( Z(X-h) = e^{-hxD}Z(X) \)

where

(3-11) \( e^{-hxD} = 1 - \frac{Dxh}{1!} + \frac{D^2xh^2}{2!} + \ldots \)

where

\[ D^n = \frac{d^n}{dx^n} \]

Now define the first backwards difference as:

(3-12) \( \Delta Z_i = Z(X) - Z(X-h) \)

\[ = (1 - e^{-hxD}) Z(X) \]

Now consider only the operators:

(3-13) \( \Delta = 1 - e^{-hxD} \)

(3-14) \( hxD = -ln(1 - \Delta) \)

Using the series expansion for the natural log, equation (3-14) becomes:

(3-15) \( hxD = \frac{\Delta}{1} + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \ldots \)

\[ = \sum_{i=1}^{\infty} \left( \frac{\Delta}{i} \right) \]

Taking powers of equation (3-15), the general form of the backward difference approximation of any derivative at the point it can be obtained:

(3-16) \( D^n = h^{-n}x \left( \sum_{i=1}^{\infty} \left( \frac{\Delta}{i} \right) \right)^n \)
Now the $n$th backward difference is the difference of the $(n-1)$th difference. From equation (3-12), the following equation is obtained.

\begin{equation}
\Delta^n z_i = \Delta (\Delta^{n-1} z_i)
\end{equation}

It can be shown that the coefficients of the $z_i$'s are the coefficients of the binomial expansion of $(a-b)^n$. Therefore

\begin{equation}
\Delta^n z_i = \sum_{j=0}^{n} (-1)^j \binom{n}{j} z_{i-j}
\end{equation}

where

\[
\binom{n}{j} = \frac{n!}{(n-j)!j!}
\]

is the binomial coefficient

and for example,

\begin{equation}
\Delta^1 z_i = z_i - z_{i-1}
\end{equation}

\begin{equation}
\Delta^2 z_i = z_i - 2z_{i-1} + z_{i-2}
\end{equation}

\begin{equation}
\Delta^3 z_i = z_i - 3z_{i-1} + 3z_{i-2} - z_{i-3}
\end{equation}

Using equation (3-16) the various order derivatives can now be determined. For example,

\begin{equation}
\Delta^2 + \Delta^3 + \frac{11}{12} \Delta^4 + \frac{5}{6} \Delta^5 + \ldots
\end{equation}

If only the first term in the above series is considered, then the derivative of $Z$ at $X = X_i$ is:

\begin{equation}
\Delta^1 = \frac{1}{h} (Z_i - Z_{i-1}) + O(h)
\end{equation}

where the symbol $O(h)$ stands for an error "of the order $h$" and represents the order of magnitude of the neglected terms. In general
if the first \( n \) terms of the backward difference approximation, equation (3-16), are retained, the error will be of the order \( h^n \).

The forward and central difference approximations to the derivatives can be derived in a similar fashion. The forward difference approximations are used when the derivatives are to be estimated at a point which is behind the majority of the data points in the \( X \) direction. The central difference approximations are used when the derivatives are to be estimated at a point which is close to the middle of the set of data points in the \( X \) direction. The only change is that, for the central difference, one obtains an error of the order \( h^{2n} \) when the first \( n \) terms are kept in the equation comparable to (3-16). Appendix B lists the various order approximations for the three difference techniques.

To estimate mixed derivatives, the finite difference of the single term derivatives can be taken. For example, the approximation of the first mixed partial derivative, using 1st order backward difference is given by:

\[
\frac{d^2Z}{dx dy} = \frac{d}{dx} \left( \frac{dZ}{dy} \right)
\]

\[
= \frac{1}{h} \frac{dZ}{dy_i} - \frac{dZ}{dy_{i-1}} + O(h)
\]

\[
= \frac{1}{hx_p} \left( Z_{i,j} - Z_{i-1,j} - Z_{i-1,j-1} + Z_{i-1,j-1} \right) + \frac{2}{h} xO(p) + O(h)
\]

If \( h \) and \( p \) are of the same order the resulting error is of the order \( h \). In general, though the errors of the mixed derivatives are of the same order as that of single term derivatives, they will be larger than that of the single term derivatives.

Since these approximations to the derivatives are based on Taylor series expansions, the errors in the various order approximations are
functions of number of terms retained and step size as outlined at the beginning of the chapter.

A series of computer simulations were run to determine the effect of order of approximation and step size on the approximation errors of the partial derivatives of $Z$ with respect to $X$. The actual roof profile, as given in appendix A, was used with $Y$ fixed and equally spaced deterministic measurements taken along the $X$ axis. The estimates of the derivatives are compared to the actual derivatives at the pivot point.

Figures 3-5 through 3-8 illustrate the percentage error as a function of step size and order of approximation for various order of derivatives. As expected, as the step size increases the error in the approximation increases as well. Note however that at small step sizes in the larger derivatives, round off error becomes important (the computer used 14 significant digits). The importance of this will become more apparent when noisy measurements are considered. Also significant is the observation that central difference approximations are consistently more accurate than the corresponding backward difference approximations of the same order. Numerical results from these graphs will be used for the subsequent determination of the required order of approximation of the derivatives. An attempt was made to select a series of figures that represent some worst case errors.

The maximum allowable deterministic error in the derivatives to insure that the maximum allowable error in the estimation of the height of the coal interface is not exceeded can be determined by examining the Taylor series, equation (3-2).

\begin{equation}
Z_c (X,Y) = Z_c |_{p} + \left( \frac{dZ_c}{dX} \bigg|_{p} (X-X_p) + \frac{dZ_c}{dY} \bigg|_{p} (Y-Y_p) \right) + \ldots
\end{equation}

Consider the error in each term (assuming deterministic measurements):
APPROXIMATIONS OF 2nd DERIVATIVE

numbers represents order of approx.

ACTUAL VALUE = .00375 W/M²

STEP SIZE BETWEEN DATA POINTS (m)

figure 3-6
APPROXIMATIONS OF 3rd DERIVATIVE

ACTUAL VALUE = 0.000639 m^3

STEP SIZE BETWEEN DATA POINTS (m)

PERCENTAGE ERROR

numbers represent order of approx.
\[(3-23) \quad Zc + E_{Zc} = Zc + (\frac{dZc}{dX}\bigg|_{p} E_{D_x})x(X-X_p) + (\frac{dZc}{dy}\bigg|_{p} E_{D_y})y(Y-Y_p) + \ldots + E_T \]

where

\[E_{D_x} = \text{deterministic error in partial derivative with respect to } X\]
\[E_{D_y} = \text{deterministic error in partial derivative with respect to } Y\]
\[E_T = \text{deterministic error due to truncation of the Taylor series}\]
\[E_{Zc} = \text{total error in estimate of coal interface height}\]

The total error in the estimation of the coal interface height can thus be seen to be bounded by:

\[(3-24) \quad E_{Zc} = (h^2 E_{D_x} + p^2 E_{D_y}) + \ldots + E_T \]

where

\[h = \text{extrapolation distance from pivot in } X \text{ direction}\]
\[p = \text{extrapolation distance from pivot in } Y \text{ direction}\]

Figures 3-2 through 3-8 were used to estimate each of the above deterministic errors. The step size between the data points used in the finite difference estimates of the derivatives and the extrapolation distance were assumed equal. The step size for \(Y\) was assumed to be 1 m (3.3 ft). The errors in the derivatives of the same order were assumed to be equivalent. With these assumptions equation (3-24) becomes,

\[(3-25) \quad E_{Zc} = (h+1)E_{D} + \frac{1}{2}x(h^2-2h+1)E_{D^2} + \frac{1}{6}x(h^3+3h^2+3h+1)E_{D^3} + \ldots + E_T \]

where

\[E_{D^n} = \text{error in } n^{th} \text{ order derivative}\]
Table 3-1 illustrates the total error as a function of:

1) number of terms retained in Taylor series,
2) order and type of approximation for each of the derivatives.
The first six columns are for backward and/or forward difference while the last two columns marked 2C and 4C are for central difference.
3) step size in X.

The truncation error shown is equal to $E_T$ in equation (3-25) while the error attributed to the estimation of the derivatives is the sum of the remaining terms. The table indicates that all of the estimation errors are below the maximum allowable error of .015 m (.5 in) if the step size is kept below 2 m (6.6 ft). At 3 m (9.9 ft) the Taylor series which retains only two terms must use second order central difference approximations to meet the accuracy requirement. The Taylor series which retains 6 terms can meet the accuracy requirement with all the derivative approximations considered.

Both equation (3-25) and table 3-1 show that the error attributable to the estimation of the derivatives will increase rapidly as the step size is increased. It is therefore appropriate to use as good an approximation to the derivative as possible. Use of the central difference approximations in preference to the backward or forward difference approximations would be the first logical choice. The second obvious decision of using higher order estimates of the derivatives will be shown to be not as sound as it first appears. The effect of uncertainty in the measurements will be shown to play a decisive role in the selection of the appropriate order.
## DETERMINISTIC ERROR IN ESTIMATION OF COAL INTERFACE

<table>
<thead>
<tr>
<th>Step size (m)</th>
<th>Truncation error (10^{-3} m)</th>
<th>Error due to estimation of derivative (10^{-3} m)</th>
<th>Total error (10^{-3} m)</th>
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<tr>
<td>1</td>
<td>5.8</td>
<td>0.4</td>
<td>6.2</td>
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<td>2</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Table 3-1
3.3 Effect of Measurement Noise on the Predictive Model

In the previous section several forms of the Taylor series predictive model of the coal interface were shown to meet the given accuracy goals when deterministic measurements were assumed. In the actual application of a predictive model vertical control system, measurement error will certainly be present. This is especially true when global measurements are to be used as has been assumed in this thesis. The error in the measurements can be reduced by using highly accurate, delicate and, expensive instruments. A better approach would be to design the predictive model so that it is either insensitive to noisy measurements or, preferably, inherently capable of filtering the noisy measurements.

In chapter 2 the measurement process was assumed to be modeled as a signal corrupted by an additive white, zero mean, gaussianly distributed disturbance. This was done since it accurately represents many measurement processes and it allows for the use of many powerful analysis tools by the control system designer. Figure 3-9 illustrates the process.

![Figure 3-9: Measurement Process](image)

The gaussian probability density function is given by equation 3-26.

\[
(3-26) \quad f_v(v) = (2\pi \sigma_v)^{-1} e^{-\frac{1}{2} \left(v - \mu_v \right)^2 / \sigma_v^2}
\]

where
\[ V = \text{random disturbance input} \]

\[ M_V = \text{mean of } V = E[V] = 0 \]

\[ S_V = \text{standard deviation of } V = E[V^2] \]

\[ E[q(v)] = \text{expectation operator} = \int_{-\infty}^{\infty} q(v)f_V(v) \, dv \]

\[ \pi = 3.14\ldots \]

The term white noise implies that the random disturbance is uncorrelated in time. This can be represented by equation (3-27).

\[(3-27) \quad E[V(t_1)\cdot V(t_2)] = S^2 I(t_1, t_2) \]

where

\[ I(t_1, t_2) = \text{Kronecker delta function} = \begin{cases} 0 & t_1 = t_2 \\ 1 & t_1 \neq t_2 \end{cases} \]

As mentioned in chapter 2, the standard deviation of the measurement noise will be assumed known and constant from measurement to measurement. In actual practice the standard deviation must be determined experimentally and checked periodically since the standard deviation will change with time. In addition, the assumption of zero mean must be checked as well or systematic errors will be introduced.

When measurements with the characteristics given above are used in the finite difference estimations of the derivatives, several interesting properties of the resulting estimate of the derivative can be determined. For example, the backward difference estimate of an \( n^{\text{th}} \) order derivative can be seen from appendix B to have the following form:

\[(3-28) \quad D^n = h^{-n} x \left( C_0 x^{n_1} + C_1 x^{n_1-1} + C_2 x^{n_1-2} + \ldots \right) \]
where

\[ Z_m = \text{measured value of } Z = Z + V \]

\[ V = \text{random disturbance with the above assumed properties} \]

When the measured values of \( Z \) are broken into their deterministic and random components equation (3-28) becomes:

\[
(3-29) \quad D^n = h^{-n}x( C_0 xZ_i + C_1 xZ_{i-1} + C_2 xZ_{i-2} + \ldots )
\]

\[
+ h^{-n}x( C_0 xV_i + C_1 xV_{i-1} + C_2 xV_{i-2} + \ldots )
\]

When the expected value of equation (3-29) is taken, the random component disappears because of the assumed white and zero mean properties of the random measurement disturbance.

\[
(3-30) \quad M^n = h^{-n}x( C_0 xZ_i + C_1 xZ_{i-1} + C_2 xZ_{i-2} + \ldots )
\]

Equation (3-30) shows that the mean value of the backward difference estimate of the \( n \)th derivative is equal to the value that would be obtained if no measurement error was present. It is thus possible to obtain an unbiased estimate of the uncorrupted estimate of the \( n \)th order derivative.

It can be shown from elementary probability theory that the standard deviation of the backward difference estimate of the \( n \)th order derivative is given by:

\[
(3-31) \quad S^n = h^{-n}x( C_0^2 + C_1^2 + C_2^2 + \ldots ) S_V
\]

Equation (3-31) illustrates the interesting property that the standard deviation of the estimate of the uncorrupted estimate of the \( n \)th order derivative decrease with increasing step size between data points. This is in contrast to the previous result where it was shown that the deterministic estimate of the \( n \)th order derivatives degrades
as the step size is increased. A balance must be found between the
deterministic error and the error introduced by the measurement noise.

Equation (3-31) also shows that, in general, as more data points
are considered, the standard deviation in the estimate of the $n^{th}$
order derivative increases. Since more data points are required as $n$
increases, higher order derivatives will have comparatively larger
standard deviations. The same is true of higher order backward
difference approximations since higher order estimates require
comparatively more data points.

These same results apply as well to the forward and central finite
difference estimates of the derivatives since they also have the same
general form as given in equation (3-28). Using appendix B and
equation (3-31), the values of the standard deviations of the various
finite difference techniques were determined. Table 3-2 lists the
results as a function of order and type of estimation and, order of
derivative. A significant point brought out by this table is that the
standard deviation of the central difference estimates can be as much
as several orders of magnitude lower than those of the forward or
backward difference estimates. Since the central difference estimates
were shown in figures 3-5 through 3-8 to give better deterministic
results than the comparable order backward difference estimates as
well, the use of central difference estimates is clearly to be
preferred over backward or central difference estimates.

These results were checked by running a series of computer
simulations. As before the actual coal interface profile, as given in
appendix A, was used with $Y$, the distance perpendicular to the face of
the longwall, fixed and equally spaced noise corrupted measurements of
the height of the coal interface, $Z_c$, taken along the longwall, $X$. The
estimates of the derivatives were compared to the actual derivatives.
The measurement noise for $X$ and $Z_c$ was assumed to have a standard
deviation of 0.008 meters (.3 in). Figures 3-10 through 3-12 show the
results for the first and second order backward difference
RATIO OF VARIANCE OF ESTIMATE OF DERIVATIVES TO VARIANCE OF MEASUREMENT

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backward/forward difference

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central difference

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</table>

For example, the standard deviation of the second order backward difference approximation to the first derivative is:

\[ S_D = (6.5)^5 \times S_v \times h^{-1} \]

where

\[ h = \text{step size between data points} \]
\[ S_v = \text{standard deviation of noise} \]
Figure 3.10

1ST ORDER BACKWARD DIFFERENCE APPROX TO 1ST DERIVATIVE

STEP SIZE BETWEEN DATA POINTS (m)

actual value = -0.0198 m/m

PERCENTAGE ERROR
actual value = -0.0198 m/m

2nd ORDER BACKWARD DIFFERENCE APPROX TO 1st DERIVATIVE

figure 3-11
approximations and the second order central difference approximation to the first partial derivative with respect to X. Superimposed over the program plots are the 3 times the standard deviation envelopes which use the standard deviations in table 3-2. Using 3 times the standard deviation for the envelopes will insure that there will be only a .3% probability that the actual value will exceed this error. This technique is often used to give upper and lower bounds to a value. As can be seen from the figures, the simulations match the theoretical calculations very closely.

The results presented in this section show that the finite difference estimates of the derivatives are very sensitive to noise. Several rules of thumb are clear at this point. One should try to use the lowest order derivatives possible as well as the lowest order approximations. This forces the designer to make a compromise between accuracy and susceptibility to noise. As was shown above, one choice that improves accuracy and reduces the effects of noise is to use central difference approximations wherever possible. To determine exactly which order of approximation to use for the derivatives and the step size between data points, the effects of noisy measurements and derivatives on the Taylor series predictive model must be determined. We begin by considering the Taylor series approximation of the coal interface height, equation (3-2).

\[
(3-2) \quad Z_e (X,Y) = Z_p + \left( \frac{dZ_e}{dX} \right)_p (X - X_p) + \left( \frac{dZ_e}{dY} \right)_p (Y - Y_p) + \ldots
\]

Both deterministic and random errors will be present in equation (3-2) when measurement noise is considered. The various deterministic errors were presented in equation (3-23) previously and include the error due to the truncation of equation (3-2) and the error due to the finite difference estimates of the derivatives. Figures 3-2 through 3-8 will be used to estimate these errors. The error due to measurement noise will be seen in two areas in equation (3-2). The first is in the measurement of the distance from the pivot, X and Y, and the location of the pivot, X_p, Y_p, Z_p. This error will be assumed to be three times the standard deviation of the measurement disturbance, \( \delta \). As was
mentioned previously, this is often used to give an upper bound on the error in a measurement. The second effect of measurement noise is on the estimate of the derivatives. This error will be assumed as three times the standard deviation of the derivative estimates for the same reasons given above. The standard deviations will be obtained from table 3-2. When these points are incorporated into equation (3-2), the following equation is obtained.

\[(3-32) \quad Zc + E_{Zc} = (Zc_p + 3xS_{y}) + \left( \left\{ \left( \frac{dZc}{dx} \right)_p + E_{Dx} + 3xS_{Dx} \right\} (X - X_p + 6xS_{y}) \right) \]

\[+ \left( \left\{ \left( \frac{dZc}{dy} \right)_p + E_{Dy} + 3yS_{Dy} \right\} (Y - Y_p + 6yS_{y}) \right) + \ldots + E_T \]

where

- \(E_{Dx}\) = deterministic error in partial derivative with respect to \(X\)
- \(E_{Dy}\) = deterministic error in partial derivative with respect to \(Y\)
- \(E_T\) = deterministic error due to truncation of the Taylor series,
- \(E_{Zc}\) = total error in estimate of coal interface height
- \(S_{Dx}\) = standard deviation of estimate of partial derivative with respect to \(X\)
- \(S_{Dy}\) = standard deviation of estimate of partial derivative with respect to \(Y\)
- \(S_{y}\) = standard deviation of measurement disturbance

From elementary error analysis, (see reference 16 for example), the error in equation (3-32) can be shown to be bounded by:
\[ E_{Zc} = 3S_Y + \left( \frac{2x}{D_x} + \frac{6S_Y}{h} \right)D_xh \]

\[ + \left( \frac{2y}{D_y} + \frac{6S_Y}{p} \right)D_yh \]

\[ + \ldots + E_T \]

where

\[ D_y = \frac{dZc}{dY} \] partial derivative of Zc with respect to Y at pivot

\[ h = \text{extrapolation distance from pivot in } X \text{ direction} \]

\[ p = \text{extrapolation distance from pivot in } Y \text{ direction} \]

As in the deterministic case several simplifying assumptions were made. The step size between the data points used in the finite difference estimates of the derivatives and the extrapolation distance were assumed equal. The step size for Y was assumed to be 1 m (3.3 ft). The derivatives, and their errors, of the same order were assumed to be of equal magnitude. With these assumptions equation (3-33) becomes:

\[ E_{Zc} = \left( 1 + 4D + \frac{8}{2}x(h+1)x^2D^2 + \frac{12}{6}x(h+1)^2x^3D^3 + \ldots \right)3S_Y \]

\[ + \left( (h+1)xD + \frac{1}{2}x(h+1)^2x^2E + \frac{1}{6}x(h+1)^3x^3E + \ldots \right) + E_T \]

\[ + 3x \left( (h+1)xS_D + \frac{1}{2}x(h+1)^2x^2S + \ldots \right) \]

where

\[ D^n = n^{th} \text{ order derivative} \]

Equation (3-34) can be decomposed into deterministic and random parts;

\[ E_{Zc} = 3x(h+1)xS_D + \frac{1}{2}x(h+1)^2x^2S + \frac{1}{6}x(h+1)^3x^3S + \frac{1}{24}x(h+1)^4x^4S + \ldots \]
\[ + 3(1 + 4hD + \frac{8}{2}(h+1)2D + \frac{12}{6}(h+1)^2D^3 + \frac{16}{24}(h+1)^3D^4 + \ldots ) = S_y \]

where the deterministic error is given by equation (3-25).

Equation (3-35) shows that four items will affect the total error in the estimation of the coal interface height:

1. The step size \( h \). It was shown previously that the deterministic error will increase as the step size is increased.

It was also shown that the standard deviation of the estimates of the derivatives decreases with increasing step size by various powers of the step (see table 3-2). In equation (3-35) this reduction is cancelled by the multiplication of various powers of the step size. Thus the step size will have a small effect on the error due to the standard deviation of the estimate of the derivatives.

2. The number of terms retained in the Taylor series. Figures 3-2 through 3-4 showed that the deterministic error will in general decrease as more terms are kept in the Taylor series. Equation (3-35) shows that the opposite is true for the error due to the random error. This effect is not severe for the measurement of the distance from the pivot since the higher order derivatives will in general be small. However, the effect due to the standard deviation of the estimate of the derivatives will be quite significant since the standard deviation increase quickly as higher order derivatives are considered (see table 3-2).

3. The order of approximation used for the derivatives. It was shown in figures 3-5 through 3-8 that the higher order approximations reduce the deterministic error. As in the previous point, the opposite is true for the random component and for the same reason.

4. The standard deviation in the measurements. The error due to this point will increase proportionally to the increase in the standard deviation of the measurements.

- 64 -
Equation (3-35) was used to determine the maximum allowable standard deviation in the measurements as a function of step size, number of terms is the Taylor series, and order and type of approximation to the derivatives. Numerical values were obtained from tables 3-1 and 3-2 for the deterministic error and the standard deviation of the derivatives. The assumed maximum error in the estimation of the coal interface, .015 m (.5 in), introduced in chapter 2 will be used. Table 3-3 illustrates the results.

Table 3-3 shows that, as expected, the maximum allowable standard deviation in the measurements decreases with increasing step size. At step sizes of 1 m (3.3 ft) and 2 m (6.6 ft) the Taylor series with less terms can achieve the required accuracy with comparatively more noise in the measurements. At larger step sizes more terms in the Taylor series must be used since the deterministic error becomes larger than the allowable error when only two terms are retained. Also significant is the observation that the central difference methods are in general an order of magnitude less sensitive to noise.

Table 3-3 indicates that the maximum allowable standard deviations in the measurements for all the methods is lower than the expected value of .008 m (.3 in). Some type of filtering of the measurements will be required. However, the required amount of filtering can be reduced by using as few terms in the Taylor series as possible, using the lowest order estimate of the derivatives as possible, and using central difference estimates wherever possible. These recommendations will require that the step size be kept at relatively low values.

A general method of determining the effect of design decisions on the construction of a Taylor series coal interface predictive model is now available to the designer. Before considering different filtering techniques, some simulation results will be presented to confirm the analysis presented above.
### Maximum Allowable Standard Deviation in Measurements

To assure a total error of less than .015 m in Taylor Series Predictive Model

<table>
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<th>Order of deriv</th>
<th>Maximum allowable standard deviation (10^-3 m)</th>
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<td>.008</td>
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Table 3-3
3.4 Simulation Results

To simulate the operation of coal interface predictive models, program SIMU.FTN was written. The program incorporates the model of the longwall installation developed in chapter 2 with the exception that the dynamics of the shearer's cutting drum are not considered since they will not affect the predictive model. In chapter 2 the shearer's position along the longwall, $X$, was modeled as a nominally constant velocity process with a random input velocity disturbance. The position of the shearer perpendicular to the longwall face, $Y$, was modeled as constant with a random input position disturbance. The input disturbances will initially be assumed to be zero since they will not affect the operation of the predictive model until filtering techniques are considered. The roof profile developed in appendix A was used as the actual roof profile. Measurements were assumed to be taken after every pass across the longwall at constant intervals along the longwall. All the measurements are assumed to be corrupted by a random noise source.

The Taylor series predictive model was implemented by first determining all the required derivatives of the coal interface profile at the pivot. The location of the pivot was chosen so that the distance that is required to be extrapolated is never greater than the distance between the data points. This requires that the location of the pivot change as the shearer progresses across the longwall face. This was done since it was shown earlier that the error increase rapidly as the extrapolation distance is increased. In particular, this is why measurement of the coal interface were taken after each pass across the longwall. The estimate of the height of the coal interface, which is to be used to control the cutting drum height, was determined in the middle of the cut. Backward difference estimates were used for the derivatives with respect to $Y$ since all the data points were behind the face of the longwall. For the derivatives with respect to $X$, backward, forward, and central difference were used with central difference being used whenever there were a sufficient number
of points to either side of the pivot. Appendix C contains a flowchart of SIMU,FTN and a more detailed discussion of the implementations of the longwall model and the Taylor series predictive model.

The first case that will be considered retains the first five terms in the Taylor series. Sixth order backward difference estimates are used for the derivatives in the \( \bar{Y} \) direction. Sixth order backward and forward, and fourth order central difference estimates are used in the \( X \) direction. The first series of simulations assume deterministic measurements and are shown in figures 3-13 through 3-15. The figures show the error in the Taylor series estimate of the coal interface height at the same pass of the longwall (\( \bar{Y} = 29.5 \) m) as a function of the position along the longwall face. Figure 3-13 used a step size in the distance along the longwall between the data points of 3 m (9.8 ft) while figure 3-14 used 5 m (16.4 m), and figure 3-15 used 6 m (19.7 m). Superimposed over the figures are the error bounds from table 3-1 and the maximum allowable error of .015 m (.5 in) selected in chapter 2.

Figures 3-13 agrees very well with the error bound presented in table 3-1 while figures 3-14 and 3-15 indicates that the maximum extrapolation distance is about 5 m (16.4 m) if the accuracy goal of .015 m (.5 in) from chapter 2 is used. Figures 3-14 and 3-15 clearly shows the shifting of the pivot point by the discontinuities in the curves. Also evident is the significantly better estimates of the central difference methods which are used in the middle of the curves. One can also locate the location of the pivot in each of the regions since the error is generally smaller at the pivot since there is no error due to extrapolation in the X direction. Thus for example, at the beginning of the pass the figures indicate that the pivot is located at the beginning of the interval which is indeed the case as shown in appendix C.

To illustrate the effect of measurement noise on case 1, three simulations were run with varying standard deviations in the
ERROR IN HEIGHT (M)

DISTANCE ALONG LONGWALL (M)

20:46:45 03-JUL-84
Y=29.5 M  Xstep=3 M  Ystep=1 M  SD=0

error bounds from table 3-1

maximum allowable error

DETERMINISTIC SIMULATION FOR CASE 1 WITH 3 m STEP IN X
figure 3-13
Y = 29.5 M  X step = 5 M  Y step = 1 M  SD = 0

Maximum allowable error

Deterministic simulation for Case 1 with 5 m step in X
Figure 3-14
21:02:51 03-JUL-84
Y=29.5 M Xstep=6 M Ystep=1 M SD=0

ERROR IN HEIGHT (M)

DISTANCE ALONG LONGWALL (M)

maximum allowable error

DETERMINISTIC SIMULATION FOR CASE 1 WITH 6 m STEP IN X
figure 3-15
measurement disturbance and a step size along the longwall, \( X \), of 1 m (3.3 ft) between data points. As in figures 3-13 through 3-15 the results were plotted for the pass where \( Y = 29.5 \) m (96.8 ft) and are shown in figures 3-16 through 3-18. Figure 3-16 uses a standard deviation of .008 m (.31 in) which is the standard deviation that was assumed in chapter 2 for all the sensors. Note that the scale of the error on this figure is much larger than for the other figures. These series of figures indicate that the standard deviation in the measurements must be reduced to about \( 8 \times 10^{-5} \) m (.0031 in) in order to meet the accuracy requirement of .015 m (.5 in). This is consistent with the previous analysis which resulted in a value, from table 3-3, of \( 5 \times 10^{-5} \) m (.002 in) when backward difference algorithms were used. These figures also confirm the previous observations in section 3.3 that the backward and forward difference algorithms are more sensitive than the central difference algorithms to noise. This can be seen by the larger error at the beginning and end of the longwall where backward and forward differences were used.

To contrast the performance of the first case which employed higher order derivatives and estimates of those derivatives, case 2 will use the first 2 terms of the Taylor series and 2nd order difference algorithms to estimate the derivatives. With these two cases the trade-off between accuracy, step size, and susceptibility to noise can be explored. Figures 3-19 through 3-21 illustrate the response of case 2 for the deterministic case. The error bounds from table 3-1 for the backward difference estimates were superimposed over the program plots. The figures show that the simulations are consistent with the previous analysis and in particular confirms that the maximum step size is somewhat less than 3 m (9.8 ft). In case 2 only central difference estimates of the derivatives were used in the \( X \) direction since only 1 data point is required to each side of the pivot. This is why the simulations do not degrade at the ends of the longwall as in the first case.

Figures 3-22 and 3-23 illustrate the response of case 2 for two
Stochastic simulation for Case 1 with 0.008 m standard deviation

Figure 3-16
STOCHASTIC SIMULATION FOR CASE 1 WITH $8 \times 10^{-5}$ m STANDARD DEVIATION

figure 3-18
error bounds from table 3-1

maximum allowable error

DETERMINISTIC SIMULATION FOR CASE 2 WITH 2 m STEP IN X

figure 3-20
DETERMINISTIC SIMULATION FOR CASE 2 WITH 3 m STEP IN X

figure 3-21
different measurement standard deviations. These figures indicate that this case of the Taylor series requires that the measurement standard deviation be less than or equal to .001 m (.04 in). This is consistent with the result from table 3-3 where, for the backward difference case, the maximum allowable standard deviation is .0006 m (.024 in).

The difference between cases 1 and 2 also confirm the analysis presented in section 3-3. When deterministic measurements can be assumed, case 1 will allow for step sizes of 5 m (16.4 ft) along the longwall and most probably could allow for the skipping of one or two passes across the longwall before taking measurements of the coal interface height though this was not simulated. Case 2 would allow for step sizes of about 2 m (6.6 ft) along the longwall with measurements required after every pass. When random measurements are used case 2 can use measurements 3 orders of magnitude noisier than those that can be used by case 1. Since noisy measurements will most certainly be encountered, clearly the method of choice is the Taylor series predictive model which retains only two terms, uses the second order central difference algorithm to estimate the derivative in the direction along the longwall and the second order backward difference algorithm to estimate the derivative in the Y direction. From table 3-3 the maximum step size which could be used is about 2 m (6.6 ft). An added benefit of using this case is that the resulting predictive model is a linear function of X and Y.

Even if the recommendations of the last paragraph are followed, the standard deviation of the measurements will in all likelihood have to be reduced. From the previous simulations, it was shown that the standard deviation of the measurements could be no larger than .001 m (.04 m) in order to achieve a maximum error of less than .015 m (.5 in). In chapter 2, a typical standard deviation of .008 m (.31 in) was assumed for the sensors. Thus the standard deviation must be reduced by almost an order of magnitude. How this might be achieved is the next topic to be discussed.
STOCHASTIC SIMULATION FOR CASE 2 WITH .008 m STANDARD DEVIATION

figure 3-22
3.5 FILTERING OF MEASUREMENTS

The most obvious approach to reducing the standard deviation in a measurement is to make repeated measurements of the same value. If the additive noise source is assumed gaussian and zero mean the unbiased maximum-likelihood estimate for the true value is given by the well known relationship (see ref 16):

\[ Z_e = \frac{1}{N} \sum_{i=1}^{N} Z_n \]

where

\[ N = \text{number of measurements} \]
\[ Z_e = \text{estimate of the true value of } Z \]
\[ Z_n = \text{measured value of } Z \]

If the variance of the noise source is assumed constant, the variance of \( Z_e \) about the true value of \( Z \) is given by the well known relation:

\[ S_{Z_e}^2 = N^{-1} S_Y^2 \]

where

\[ S_Y = \text{standard deviation of measurement disturbance} \]
\[ S_{Z_e} = \text{standard deviation of estimate of } Z \]

Equation (3-37) can be solved for the number of measurements required to reduce the standard deviation of the estimate given a measurement standard deviation, \( N \).

\[ N = \left( \frac{S_Y}{S_{Z_e}} \right)^2 \]

Thus knowing the standard deviation of the measurements and the
required standard deviation of the estimate, the number of measurements required can be determined. The results of typical calculations for the various Taylor series are shown in the last column of table 3-4 which is an extension of table 3-3. All the approximation methods, except possibly for the model that retains only 2 terms in the Taylor series, require unreasonably large numbers of measurements to achieve the desired accuracy.

The requirement for such large numbers of measurements is unacceptable because of the potentially large amount of time that would be required by some of the coal interface sensors. An additional limitation is that this method implicitly assumes that the value that is to be estimated is a constant. Thus it would not be possible to use averaging for determining the position of the shearer along the longwall since this is a dynamic variable. It would be possible to use averaging for the coal interface sensor since the sensor is only required to be moved in discrete steps. At each point where the coal interface is to be measured, the sensor location and the location of the coal interface would be fixed.

To improve upon averaging multiple measurements of the same value, more information about the process that generated the values to be measured must be used. A well known method that employs information about the process is Kalman filtering which is a result of optimal filtering theory. The Kalman filter equations are derived by first assuming a state-vector model of the system under consideration and then minimizing the mean squared error between the estimate of the state and the true value of the state. The resulting filter is used to determine an optimal estimate of the state vector from measurement data corrupted by noise. The method is optimal in the sense that the estimate of the state of the system is the minimum variance estimate and it is unbiased and consistent. The unbiased and consistent properties insures that the estimate of the state will converge to the true value of the state as more measurements are made. References 1 and 7 contain the derivation of the Kalman filter equations presented
# MAXIMUM ALLOWABLE STANDARD DEVIATION IN MEASUREMENTS

TO ASSURE A TOTAL ERROR OF

LESS THAN 0.015 M IN

TAYLOR SERIES PREDICTIVE MODEL

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<th>terms in Taylor Series</th>
<th>order of deriv</th>
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<td>.014</td>
<td>326531</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>4 cent</td>
<td>.41</td>
<td>381</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>6 back</td>
<td>.008</td>
<td>1000000</td>
</tr>
</tbody>
</table>

**Table 3-4**
and 7 contain the derivation of the Kalman filter equations presented below. Figure 3-24 shows the structures of the assumed state-vector model, the assumed measurement model, and the Kalman filter.

The most general form of the system model that will be considered is one which is driven by both deterministic and random inputs and is given by equation (3-39). The random input is assumed to be white (uncorrelated in time) and gaussianly distributed with zero mean and a known covariance which may be a function of time. The state vector at time $t = (k+1)T$ is given by:

\[
S_{k+1} = F_k S_k + L_k U_k + G_k \mathbf{w}_k
\]

where

$k = \text{discrete time index} \quad T = \text{time interval between measurements}$

$F_k = \text{state transition matrix} \quad \mathbf{L}_k \text{ and } \mathbf{G}_k \text{ are scaling matrices}$

$S_k = \text{state vector}$

$U_k = \text{deterministic input vector}$

$\mathbf{w}_k = \text{white gaussian random input vector with covariance } Q_k$

The measurement model that is used is the same as was assumed in chapter 2 and was further developed in section 3.3. Equation (3-40) models the measurement process as an additive random noise signal corrupting the signal that is to be measured. The random disturbance is assumed to be white (uncorrelated in time) and gaussianly distributed with zero mean and a known covariance which may be a function of time. In addition, the random measurement disturbance and the random input disturbance in equation (3-39) are assumed uncorrelated with one another. The measurement of the state of the system at $t = (k+1)T$ is given by:

\[
S_{r,k+1} = H_{k+1} S_{k+1} + \mathbf{v}_{k+1}
\]
DISCRETE KALMAN FILTER

figure 3-24
where

\[ S_{e_{k+1}} = \text{measured state vector} \quad H_{k+1} = \text{measurement matrix} \]

\[ V_{k+1} = \text{white gaussian random measurement disturbance} \quad \text{vector with covariance } R_k \]

The following equations define the Kalman filter and are the result of the optimal filtering theory.\(^1\) Equation (3-41) gives the update of the state vector estimate after the addition of a new measurement.

\[
(3-41) \quad S_{e_{k+1}} = F_k S_{e_k} + L_k U_k + K_{k+1} (S_{e_{k+1}} - H_k (F_k S_{e_k} + L_k U_k))
\]

where

\[ K_{k+1} = \text{Kalman gain matrix} \]

The Kalman gain matrix is given by equation (3-42). Note that in general it is time varying.

\[
(3-42) \quad K_{k+1} = P_{k+1} H_{k+1} R_k^{-1}
\]

where

\[ P_{k+1} = \text{error covariance matrix} \]

The error covariance provides an indication of accuracy of the estimate. The error covariance update after the addition of a new measurement is given by equation (3-43).

\[
(3-43) \quad P_{k+1} = [I - K_{k+1} H_{k+1}] F_k P_k F_k^T + G_k Q_k G_k^T
\]

Note that equations (3-42) and (3-43) will have to be solved simultaneously. Lastly, the Kalman filter must be initialized by assuming that the initial condition for the state estimate is given by the expected value of the initial condition of the state,

\[
(3-44) \quad S_{e_0} = E[ S_0 ]
\]
\[(3-45) \quad P_0 = E[ (S_0 - S_{e_0})x(S_0 - S_{e_0}) ] \]

where

\[E[ \cdot ] = \text{expectation operator} \]

In chapter 2 a model of the predictive model vertical control system was developed. Simplified models based on those developed in chapter 2 will be used as system equation (3-39) to illustrate some key aspects of both the Kalman filter and how it is applied to the longwall vertical control system.

First consider the filtering of the measurement of the position of the shear perpendicular to the longwall, \(Y(t)\). As a first approximation, it will be assumed that \(Y\) is a constant and that there is no position disturbance. The state equation (2-2) reduces to the following:

\[(3-46) \quad Y_{k+1} = Y_k \]

where the parameters from equation (3-39) are given by

\[S_k = Y_k \quad U_k = 0 \quad W_k = 0 \]
\[F_k = 1 \quad L_k = 0 \quad G_k = 0 \]

The measurement model, equation (3-40), is given by:

\[(3-47) \quad Y_{m,k+1} = Y_{k+1} + V_y \]

where

\[V_{k+1} = V_y \quad H_{k+1} = 1 \]

The state estimate update, equation (3-41), is given by:

\[(3-48) \quad Y_{e,k+1} = Y_e + K_x \times (Y_{m,k+1} - Y_{e,k}) \]
where

\( K_{k+1} = K_y y_{k+1} \) = scalar Kalman filter

The Kalman gain, equation (3-42), reduces to:

\[
(3-49) \quad K_y = P_{y_{k+1}} y_{k+1} \end{align*}
\]

where

\[ P_{y_{k+1}} = P_{y_{k+1}} \] = scalar error covariance of estimate of \( Y \)

\[ R_{k+1} = S^2_y \] = square of measurement standard deviation for \( Y \)

The error covariance, equation (3-43), reduces to:

\[
(3-50) \quad P_{y_k} = (1 - K_y) P_{y_{k+1}} y_{k+1} \end{align*}
\]

Solving equations (3-49) and (3-50) simultaneously yields the following result:

\[
(3-51) \quad P_{y_k} = (1 + P_{y_k} S^2_y y_{k+1} y_{k+1})^{-1} P_{y_k} y_{k+1} \end{align*}
\]

Solving equation (3-51) for the covariance at any time, \( k \), gives:

\[
(3-52) \quad P_{y_k} = (1 + P_{y_k} S^2_y y_{k+1} y_{k+1})^{-1} P_{y_k} y_{k+1} \end{align*}
\]

Substituting equation (3-52) into equation (3-49) gives the general solution to the Kalman gain:

\[
(3-53) \quad K_y = (1 + P_{y_k} S^2_y y_{k+1} y_{k+1})^{-1} P_{y_k} x S^2_y \end{align*}
\]

Finally the following initial condition are required:
\[ (3-54) \quad Y_e = E[Y_0 | e_0] \]

\[ (3-55) \quad P = E[(Y_0 - Y_e)^2] \]

Several interesting points can be made by considering equations (3-52) and (3-53). If the initial error covariance is equal to zero, equation (3-52) shows that the error covariance will remain zero for all time while equation (3-53) shows that the the Kalman gain will also be zero for all time. For a nonzero initial error covariance the equations show that both the error covariance and the Kalman gain go to zero over time. Figure 3-24 shows that the Kalman filter relies strictly on the model of the system to determine the estimates when the Kalman gain has gone to zero. Thus the measurements supply no new information. This points out the need for an accurate model of the system since the Kalman filter is designed to converge to the assumed model.

To compare the performance of the Kalman filter against the method of averaging, the first measurement of \( Y \) will be used as the initial state estimate and the initial error covariance will be assumed equal to the measurement variance. The number of measurement required to meet the maximum allowable standard deviation in the estimate can be determined by solving equation (3-52) for \( k \)

\[ (3-56) \quad k = \left( \frac{P_y P_{x_k}^{-1} S^2}{y_0^2} \right) \]

If the standard deviation of the measurements is assumed to be \( .008 \) m (.031 in) and the maximum allowable standard deviation in the estimate of \( Y \) is assumed to be \( .001 \) m (.04 in) as was assumed previously for the averaging procedure, \( k \) becomes:

\[ (3-57) \quad k = 63 \]

where
\[ P_{y_0} = (0.008 \, \text{m})^2 = \text{estimate of initial error covariance of } Y \]

\[ P_{y_k} = (0.001 \, \text{m})^2 = \text{maximum allowable error covariance in estimate of } Y \]

\[ S_{y} = 0.008 \, \text{m} = \text{standard deviation in measurement of } Y \]

If one is added to \( k \) to account for the first measurement, the same number of measurements are required as for simple averaging. For this case Kalman filtering and averaging are equivalent. This is not surprising since it was assumed that the position is a constant and averaging is a maximum likelihood method which can be shown to be equivalent to a minimum variance method.

It was shown above that the Kalman filter converges to the assumed model of the system. It is therefore very important that an accurate model of the position of the shearer perpendicular to the longwall face, \( Y \), be used. The first model considered, \( Y \) equal to a constant, is not an accurate representation of the actual physical system. A much more realistic model is the one developed in chapter 2 in which a random position disturbance was also present. The state equation (2-2) is given by:

\[(3-58) \quad Y_{k+1} = U_y + W_y \]

where the parameters from equation (3-39) are given by

\[ S_k = Y_k \quad U_k = U_y \quad W_k = W_y \]

\[ F_k = 0 \quad L_k = 1 \quad G_k = 1 \]

The measurement model, equation (3-40), reduces to:

\[(3-59) \quad \hat{Y}_{k+1} = \hat{Y}_{k+1} + Y \]

where
\[ V_{k+1} = V_y \quad H_{k+1} = 1 \]

The state estimate update, equation (3-41), reduces to:

\[(3-60) \quad \chi_{k+1} = U_y + K_y (Y_{k+1} - U_y) \]

where

\[ K_{k+1} = K_y = \text{scalar Kalman gain} \]

The Kalman gain, equation (3-42), reduces to:

\[(3-61) \quad K_y = P_{y_{k+1}} S_{y_y}^{-2} \]

where

\[ P_{k+1} = P_{y_{k+1}} \quad R_{k+1} = S_{y_y}^2 \]

The error covariance update, equation (3-43), reduces to:

\[(3-62) \quad P_{y_{k+1}} = (1 - K_y) S_{y_y}^{-2} \]

where

\[ Q_k = S_{W_y}^2 = \text{square of random position input standard deviation} \]

Solving equations (3-61) and (3-62) simultaneously yields:

\[(3-63) \quad P_{y_{k+1}} = (1 + S_{W_y}^2 S_{y_y}^{-2})^{-1} S_{W_y}^2 S_{y_y}^2 \]

\[(3-64) \quad K_y = (1 + S_{W_y}^2 S_{y_y}^{-2})^{-1} S_{W_y}^2 S_{y_y}^{-2} \]

This case results in a constant error covariance which can be reduced either by reducing the standard deviation of the measurements or reducing the standard deviation of the random position input. To see how well this filter functions, consider the case where the measurement standard deviation is .008 m (.31 in) as assumed in chapter
2. Furthermore the standard deviation of the control input will be assumed to be twice that of the measurements. The actual value of the control input standard deviation would have to be determined experimentally and the choice of \(0.016 \, \text{m} (0.62 \, \text{in})\) is only a reasonable value used to illustrate the operation of the Kalman filter. With these numbers equation (3-63) gives the following result.

\[
p_y^5 = 0.007 \, \text{m} (0.28 \, \text{in})
\]

where

\[
S_y = 0.008 \, \text{m} (0.31 \, \text{in}) \quad S_{\Delta y} = 0.016 \, \text{m} (0.62 \, \text{in})
\]

This result indicates that the standard deviation of the filtered estimate of \(Y\) is little better than the original standard deviation of the measurements. If this second model is believed to be accurate and the values of the standard deviations that have been chosen are reasonable, then Kalman filtering will not reduce the standard deviation to a sufficiently low value to be of use in the Taylor series predictive model. Averaging of multiple measurements of the same value will be required at every point where an estimate of \(Y\) is required.

Another important limitation of the Kalman filter for this second model of \(Y\) is that the error covariance update does not depend on the previous value of the error covariance but is a constant which is a function of the measurement and disturbance standard deviations. The outcome of this is that the Kalman filter estimate does not improve as more measurements are made. This leaves the Kalman filter susceptible to errors in the estimates of the standard deviations of the measurements and disturbance. If the standard deviations of the measurements and disturbance are sufficiently different than the values used when the filter was designed, the Kalman filter estimates may have larger standard deviations than expected and, most importantly, they would not improve with the use of more measurements.
The first two models considered for the position of the shearer perpendicular to the longwall, $Y$, represented two extremes. The first model considered $Y$ constant over the length of the longwall. The second model considered $Y$ to be bounded within a certain region but there was no relationship between the $Y$'s at two adjacent positions along the longwall due to the white property of the position disturbance. In fact there will be a relationship between both the shearer's position perpendicular to the longwall, $Y$, and its position along the longwall, $X$, since the conveyor on which the shearer rides is composed of rigid subsections which are loosely connected to one another. While the relationship between any two subsections is randomly determined to a certain degree, there is a certain deterministic relationship between $X$ and $Y$ along any one section of the conveyor. Finding both the random relationship between the conveyor sections and the deterministic relation between $X$ and $Y$ along any subsection of the conveyor will require detailed information of the conveyor machinery. Implementing a Kalman filter which used this type of detailed information was not considered in this thesis because of time constraints.

The estimation of the position of the shearer along the longwall, $X$, using Kalman filtering will now be considered. The model of the dynamics of the shearer's position along the longwall developed in chapter 2 will be used. This model, given by equation (2-1), assumed that both deterministic and random velocity inputs were present. The general form of the system model used for the Kalman filter, equation (3-39), then becomes:

\[(3-66)\]  
\[
X_{k+1} = X_k + T U_k + T W_k
\]

where

$k = \text{discrete time index}$

$T = \text{time interval between measurements}$

$S_k = X_k \quad U_k = U_k \quad W_k = W_k$
\[ F_k = 1 \quad L_k = I \quad G_k = I \]

The measurement model, equation (3-40), reduces to:

\[ x_{m_{k+1}} = x_{k+1} + v_x \]

where

\[ v_{k+1} = v_x = \text{random measurement disturbance} \]

\[ H_{k+1} = 1 \]

The state estimate update, equation (3-41), reduces to:

\[ x_{e_{k+1}} = x_{e_k} + T\mu + K_{k} ( x_{k+1} - x_{e_k} - T\mu ) \]

where

\[ K_{k+1} = K_{k} \quad \text{scalar Kalman gain} \]

The Kalman gain, equation (3-42), is given by:

\[ K_{x_{k+1}} = P_{x_{k+1}} x_{x_{k+1}}^{-2} \]

where

\[ P_{x_{k+1}} = \text{error covariance} \]

\[ R_{x_{k+1}} = S_{x_{k+1}}^2 = \text{square of the measurement standard deviation} \]

The error covariance update, equation (3-43), reduces to:

\[ P_{x_{k+1}} = (1 - K_{x_k}^2) ( P_{x_k} + T^2 S_{x_{k+1}}^2 ) \]

where

\[ Q_k = S_{x_{k+1}}^2 = \text{square of the standard deviation of the velocity input disturbance} \]

- 95 -
Substituting equation (3-69) into equation (3-70) results in the following recursive equation:

\[
(3-71) \quad P_{x_{k+1}} = \frac{P_{x_k} + T^2 S_{w}^2}{1 + \frac{P_{x_k} + T^2 S_{w}^2}{S_{v_x}^2}}
\]

Equation (3-71) is a nonlinear difference equation whose steady state solution can be found by equating the covariance at \( k \) and \( k+1 \). The steady state solution can then be found to be:

\[
(3-72) \quad P_{x_{ss}} = -0.5 T^2 S_{w}^2 + 0.5 \left[ T^4 S_{w}^4 + 4 T^2 S_{w}^2 S_{v_x}^2 S_{v_x}^2 \right]^{0.5}
\]

where

\[ P_{x_{ss}} \] is the steady state error covariance

The solution was found to converge to the steady state solution in five time steps or less in most of the cases that were considered.

Equation (3-72) shows that the steady state error covariance can be affected by the standard deviation in the measurements and the input velocity disturbance, and by the sampling rate. A smaller sampling rate will allow for larger standard deviations of both types. As an example, consider the case where the standard deviation for both measurement and input disturbance is 0.008 m (.32 in) and the maximum allowable standard deviation in the estimate of \( x \), the position of the shearer along the longwall, is 0.001 m (.04 in). Solving equation (3-72) for \( T \) yields:

\[
(3-73) \quad T = \left( S_{v_x}^2 - P_{x_{ss}} \right)^{-0.5} S_{w}^{-1} \frac{S_{v_x}^{-1}}{P_{x_{ss}}} = 0.016 \text{ seconds}
\]
where

\[ P_{x_{ss}} = (0.001 \, \text{m})^2 = (0.04 \, \text{in})^2 \]

\[ s_{y_x} = 0.008 \, \text{m} = 0.31 \, \text{in} \]

\[ s_{w_x} = 0.008 \, \text{m} = 0.31 \, \text{in} \]

The above analysis indicates that a workable filter for the position of the shearer along the longwall, \( X \), can be constructed using a realistic model. The selection of the sampling interval, \( T \), will depend on not only the estimates of the the standard deviations of the measurement and velocity disturbances, but also on the sensors available for use in measuring \( X \). Several of the references have indicated that reliable sensors are available which allow for almost instantaneous measurements.\(^3,9\) Therefore the selection of a small enough \( T \) should not prove to be a problem.

The issue of sensitivity to errors in estimating the standard deviations of the measurement and velocity disturbances must also be considered. Equation (3-73) shows that \( T \) could be selected for the worst case standard deviations. A more sophisticated approach would be to retain the recursive relation for the error covariance, equation (3-71), and monitor how far it deviates from the expected steady state error covariance. A large deviation would indicate that one of the standard deviations is larger than expected and the sampling interval could be reselected.

The reason why an effective Kalman filter could be found for the measurement of the distance along the longwall, \( X \), but not for the distance perpendicular to the longwall, \( Y \), lies in the selection of the models. For \( X \) the model assumes that the selection of a sufficiently small sampling time, \( T \), can reduce the effective random position disturbance since the velocity disturbance is multiplied by \( T \). For the measurement of \( Y \) a random position, not velocity,
disturbance was assumed and thus the sampling time did not affect the system model. There is thus no effective way of reducing the position disturbance in Y. It is the ability to reduce the position disturbance that accounts for the success of the Kalman filter in the case of measurements of X.

As was mentioned at the beginning of section 3.5 the use of averaging would not be appropriate for estimating the position of the shearer along the longwall since this is a dynamic variable. However it would be possible to use averaging to locate the position of the coal interface sensor along the longwall, X, since it has been assumed that the coal interface sensor would stay in a fixed position as it determined the location of the interface. The same form of the Kalman filter present above for the shearer could also be used for the coal interface sensor. The selection of the most appropriate estimation technique would depend on how small the sampling time, T, would have to be. If T is too small, more measurements may be required with the Kalman filter than if averaging was used. It should also be noted that since relatively accurate measurements of X can be made relatively rapidly there is no great penalty in using averaging for estimating the true value of X for the coal interface sensor.

Having determined a suitable filter for the measurement of the distance along the longwall, the construction of a Kalman filter for the measurement of the coal interface height will now be considered. In order to improve upon the averaging technique discussed at the beginning of section 3.5, the model of the coal interface measurement system must incorporate more information than simply the height of the coal interface at a given X and Y. One method of doing this is to consider how the roof height varies with X and Y.

To use information on how the roof varies with X, the distance along the longwall, and Y, the distance perpendicular to the longwall face, these variables must somehow be incorporated into the coal interface measurement system. As a first approximation, consider X to
have a constant forward velocity and $Y$ to be a constant. Knowing $X$ and $Y$, the value of the coal interface height, $Z_c$, can be determined by using the Taylor series predictive model which is nonlinear in general.

Since the Taylor series predictive model is nonlinear, the Kalman filter presented earlier is not directly applicable to the coal interface measurement system. To incorporate the present system the Extended Kalman Filter will be used.\textsuperscript{1} This filter basically linearizes the system for determination of the Kalman gain but uses the nonlinear equations for the state estimate updates. However, the filter is no longer optimal. Reference 1 contains a derivation of the equations that will be presented below.

The most general system model that will be considered for the extended Kalman filter will be the following nonlinear vector-state equation which gives the state of the system at time $t = (k+1)T$:

\begin{equation}
S_{k+1} = f_k(S_k) + g_k(S_k)\hat{W}_k + L_k^xU_k
\end{equation}

where

$k = \text{discrete time index} \quad T = \text{time interval between measurements}$

$f_k = \text{state transition matrix} \quad g_k = \text{scaling matrix}$

$S_k = \text{state vector}$

$U_k = \text{deterministic input vector}$

$\hat{W}_k = \text{white gaussian random input vector with covariance } Q_k$

The nonlinear form of the measurement model, equation (3-40), is:

\begin{equation}
S_{m_{k+1}} = h_{k+1}(S_{k+1}) + \nu_{k+1}
\end{equation}

where
\[ S_{k+1} = \text{measured state vector} \]
\[ V_{k+1} = \text{white gaussian random measurement disturbance vector with covariance} \ R_{k+1} \]

The extended Kalman filter linearizes equations (3-74) and (3-75) about the current estimate of the state of the system. The linearized version of equation (3-74) is given by:

(3-76) \[ S_{k+1} = F_k S_k + G_k W_k + L_k U_k + M_k \]

where
\[ F_k = \left. \frac{df(S)}{dS} \right|_{S=S_{e_k|k}} \]
\[ H_k = \left. \frac{dh(S)}{dS} \right|_{S=S_{e_k|k-1}} \]
\[ G_k = g_k(S_{e_k|k}) \]
\[ M_k = f_k(S_{e_k|k}) - F_k S_{e_k|k} \]

where the notation
\[ S_{e_k|k} \] refers to the estimate of \( S \) at time \( k \) given all the measurements from time 0 to \( k \)
\[ S_{e_k|k-1} \] refers to the estimate of \( S \) at time \( k \) given all the measurements from time 0 to \( k-1 \)

The linearized version of equation (3-75) is given by:

(3-77) \[ S_{k+1} = H_k S_{k+1} + V_{k+1} + N_{k+1} \]

where
\[ N_{k+1} = h_k(S_{e_{k+1|k}}) - H_k S_{e_{k+1|k}} \]

The next series of equations define the extended Kalman filter. The state estimate is first extrapolated from the beginning to the end of the current time interval by using the nonlinear equation of state. This is shown in equation (3-78):
(3-78) \[ S_{e_{k+1|k}} = f_k(S_{e_{k|k}}) + L_k u_k \]

The estimate of the state is then updated by using the current measurement:

(3-79) \[ S_{e_{k+1|k+1}} = S_{e_{k+1|k}} + K_{k+1} [ S_{e_{k+1|k}} - h_{k+1}(S_{e_{k+1|k}}) ] \]

where

\[ K_{k+1} = \text{Kalman gain matrix} \]

The Kalman gain matrix uses the linearized measurement model and is thus equivalent to equation (3-42):

(3-80) \[ K_{k+1} = P_{k+1|k}^{-1} H_{k+1}^{-1} R_{k+1} \]

where

\[ P_{k+1|k} = \text{error covariance extrapolation matrix} \]

The error covariance extrapolation also uses the linearized matrices and is given by equation (3-81):

(3-81) \[ P_{k+1|k} = F_k P_{k|k} F_k^T + G_k Q_k G_k^T \]

The error covariance update is given by equation (3-82):

(3-82) \[ P_{k+1|k+1} = [ I - K_{k+1} H_{k+1} ] P_{k+1|k} \]

The same initial conditions that were used with the linear Kalman filter will be used for the extended Kalman filter.

Now that the machinery for the Kalman filter has been extended to handle the nonlinear case, the development of an extended Kalman filter for the measurement of the coal interface height can proceed. As was mentioned previously, it will be assumed as a first approximation that the position of the coal interface sensor along the
face of the longwall, $X$, will be determined by a constant forward velocity and the sensor’s position perpendicular to the longwall face, $Y$, will be a constant. Equation (3-74) simplifies to the following for the coal interface measurement system model:

$$
(3-83) \begin{bmatrix} X \\ Y \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}_k + \begin{bmatrix} T \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}_k
$$

where

$$S_k = \begin{bmatrix} X \\ Y \end{bmatrix}_k = \text{state vector}$$

$$U_k = U_x = \text{deterministic input}$$

$$W_k = 0 = \text{random disturbance input}$$

$$F_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad L_k = \begin{bmatrix} T \\ 0 \end{bmatrix}, \quad Q_k = 0$$

As mentioned previously the height of the coal interface, which is the variable to be measured, is a nonlinear function of $X$ and $Y$. The measurement model for the extended Kalman filter, equation (3-75), reduces to the following:

$$
(3-84) \begin{bmatrix} Z_{c_{n+1}}^{k+1} \end{bmatrix} = h_{k+1}(X_{k+1}, Y_{k+1}) + \begin{bmatrix} V \end{bmatrix}
$$

where

$$S_k = \begin{bmatrix} Z_{c_{n+1}}^{k+1} \end{bmatrix} = \text{measured value of coal interface height}$$

$$U_{k+1} = \begin{bmatrix} V \end{bmatrix} = \text{white gaussian random measurement disturbance}$$

$$h_{k+1} = h_{k+1} = \text{nonlinear, time varying, and scalar function}$$

The state estimate extrapolation, equation (3-78), reduces to:

$$
(3-85) \begin{bmatrix} X \\ Y \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}_k + \begin{bmatrix} T \\ 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}_k
$$
where

\[ S_e = \begin{bmatrix} X \\ Y \end{bmatrix}_e \] = estimate of state

The state estimate update, equation (3-79), reduces to:

\[
(3-86) \quad \begin{bmatrix} X \\ Y \end{bmatrix}_{e_{k+1|k+1}} = \begin{bmatrix} X \\ Y \end{bmatrix}_{e_{k+1|k}} + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}_{k+1} \begin{bmatrix} Z_c - h_{k+1} \left( X_{e_k}, Y_{e_k} \right)_{k+1|k} \end{bmatrix}
\]

where

\[ K_{k+1} = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}_{k+1} \] = Kalman gain vector

The Kalman gain will be a vector since the state is also a vector. The Kalman gain update, equation (3-80), is given by:

\[
(3-87) \quad K_{k+1} = \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix}_{k+1|k} \begin{bmatrix} dh \\ dh \end{bmatrix}_{k+1} \begin{bmatrix} \frac{dh}{dx} \\ \frac{dh}{dY} \end{bmatrix}_{k+1} S_{v_z}^{-2}
\]

where

\[ H_{k+1} = \begin{bmatrix} \frac{dh}{dx} & \frac{dh}{dY} \end{bmatrix}_{k+1} \] = linearized measurement function

\[ P_{k+1|k} = \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix}_{k+1|k} \] = error covariance extrapolation matrix

\[ R_{k+1} = S_{v_z}^2 \] = square of measurement standard deviation

The error covariance extrapolation, equation (3-81), reduces to:

\[
(3-88) \quad P_{k+1|k} = P_{k|k}
\]

The error covariance update, equation (3-82), reduces to:
\[
\begin{bmatrix}
p_{xx} & p_{xy} \\
p_{xy} & p_{yy}
\end{bmatrix}_{k+1|k+1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}_{k+1} \times \begin{bmatrix} \frac{dh}{dx} \\ \frac{dh}{dy} \end{bmatrix}_{k+1} \times \begin{bmatrix} p_{xx} & p_{xy} \\ p_{xy} & p_{yy} \end{bmatrix}_{k+1|k}
\]

The output update is then given by equation (3-90):

\[(3-90) \quad Zc_{e_{k+1|k+1}} = h_{k+1} (X_{e_{k+1|k+1}}, Y_{e_{k+1|k+1}})\]

To apply the extended Kalman filter, the measurement function, h, relating the position of the coal interface sensor, X and Y, to the height of the coal seam, Zc, must be known. As mentioned previously, an obvious choice is to use the Taylor series predictive model that has been developed up to the point where the coal interface height is to be estimated. There are two related problems with the choice of this coal interface function.

The first problem is concerned with the observation, which was previously made when a Kalman filter was developed for the measurement of Y, that the Kalman filter will converge to the model of the system when there are no input disturbances. Accurate models of the coal interface sensor and, more significantly, the coal interface profile are thus required. It was shown in section 3.2 that accurate models of the coal interface profile require several terms in the Taylor series and higher order estimates of the derivatives. It was also shown in section 3.3 that these higher precision Taylor series models are much more sensitive to measurement noise than the lower precision Taylor series models. The final result is that the requirements on the Kalman filter are much more severe than would otherwise be required if lower precision Taylor series were used.

The second problem is due to the fact that the estimates of the coal interface height will not be error free. These faulty estimates will be used when the Taylor series predictive model is updated. The coal interface model will thus be in error by some small amount. When this faulty model is used in the Kalman filter, the estimates will degrade over the first set of estimates since the filter will converge
to a faulty model. The outcome is that the estimates of the Kalman filter will gradually diverge from the true value of the coal interface profile.

To illustrate the operation of the extended Kalman filter developed to filter the measurements of the coal interface height, a series of computer simulations, using the program SIMU.FTN presented in appendix C, were run which used the following algorithms:

1) The Kalman filters for the X and Y measurements of the shearer developed previously were used. The constant Y and constant forward velocity for X models were used.

2) The extended Kalman filter presented above was used to filter the measurement of the coal interface height. The coal interface sensor was assumed to have a constant forward velocity along the longwall and a constant distance perpendicular to the face of the longwall.

3) The coal interface predictive model used 5 terms in the Taylor series. Sixth order backward difference algorithms for the derivatives in the Y direction were used. For the derivatives in the X direction, the following algorithms were used; sixth order forward difference at the beginning of the pass, fourth order central difference in the middle of the pass, and sixth order backward difference at the end of the pass.

4) A 1 m (3.3 ft) step size between data points was used for both X and Y directions.

5) The predictive model was initialized using deterministic data. This was done to illustrate that the extended Kalman filter will function quite well when an accurate model of the coal interface profile is available.

6) The measurement standard deviation was assumed to be .008 m (.31 in).

Figures 3-25 through 3-27 illustrate the error in the filtered estimate of the coal interface height at the face of the longwall as a function of the coal interface sensor’s position along the longwall.
STOCHASTIC SIMULATION WITH EXTENDED KALMAN FILTER: 1st STEP
ERROR IN HEIGHT (M)

DISTANCE ALONG LONGWALL (M)

STOCHASTIC SIMULATION WITH EXTENDED KALMAN FILTER: 2nd STEP

Y = 20 M, Xstep = 1 M, Ystep = 1 M, SD = 0.008 M

15:20:40 09-JUL-84

figure 3-26

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STOCHASTIC SIMULATION WITH EXTENDED KALMAN FILTER: 4th STEP
Figure 3-25 uses the deterministic values of the coal interface height for the generation of the measurement model, equation (3-84), which is used in the extended Kalman filter. The subsequent figures use progressively more of the filtered estimates of the coal interface height as the coal interface sensor progresses in the Y direction. Figure 3-25 shows that the filtered estimate of the coal interface height is excellent when an accurate model of the coal interface profile is available. The subsequent figures illustrate that the extended Kalman filter quickly degrades as the extended Kalman filter tries to converge to an inaccurate model of the coal interface profile.

To overcome this divergence problem some type of real time parameter identification in the presence of noisy signals must be used to estimate the measurement model, equation (3-84). Reference 1 outlines several approaches which may be used. If parameter identification is to be used, the current state of the art imposes the following limitations on the analysis:

1) All the approaches are based on linear systems with constant or slowly varying parameters. This would require that the extrapolation distance be limited to values that would insure that a linear model is accurate.
2) Most of the algorithms that were reviewed required several hundred points to accurately estimate the parameters. This would have to be repeated over each linearized region.1

The second of these points reflects the fact that information is to be used not only to determine the true value of the measurement, but also to determine the system that generated this measurement. It is intuitive that this would require more information, or measurements, than if an accurate model of the system was available. It therefore appears that a Kalman filter which would require less measurements than averaging is unattainable.
3.6 CONCLUSIONS FOR TAYLOR SERIES

In section 3.3 it was shown that the most suitable Taylor series predictive model of the coal interface height retained 2 terms in the Taylor series and used a second order backward difference estimate of the derivative in the direction perpendicular to the longwall face and a second order central difference estimate of the derivative in the direction along the longwall. The choice of this Taylor series algorithm was mainly due to the susceptibility of the potentially more accurate Taylor series algorithm to noise.

Even if the above Taylor series algorithm was selected, it was shown in section 3.3 that the measurement standard deviation would most likely have to be reduced. In section 3.5 a Kalman filter was developed for the measurement of the distance along the longwall, \( X \), which is capable of meeting reasonable accuracy requirements of the Taylor series predictive models. A Kalman filter for the measurement of the distance perpendicular to the longwall face, \( Y \), was developed which gave only marginally better estimates of the true value of the measurement than the measurement itself. While averaging could be used for locating the position of the coal interface sensor as it makes a measurement of the coal interface height, averaging could not be used for the shearer since \( Y \) is not a constant in this case. No Kalman filter for the coal interface height, \( Z_c \), could be found which would require less measurements than using averaging. Averaging could be used for reducing the standard deviation of the estimate of the coal interface height since it was assumed that the coal interface sensor was not attached to the shearer.

To illustrate how a workable Taylor series predictive model would perform, a simulation was run using SIMU.FTN (see appendix C) and the following algorithms.

1) Two terms were retained in the Taylor series. The second order backward difference estimate of the derivative in the \( Y \) direction was used while the second order central difference estimate of
the derivative in the X direction was used.

2) The models of the shearer dynamics presented in chapter 2 were used except that the dynamics of the height of the cutting drum were not considered. The random position disturbance in Y was assumed to have a standard deviation of .016 m (.62 in) while the random velocity disturbance in the X direction was assumed to have a standard deviation of .016 m/s (.62 in/s). The forward velocity of the shearer was assumed to be .2 m/s (.6 ft/s). The shearer was assumed to make 1 m (3.3 ft) wide cuts.

3) All the measurements were assumed to have a standard deviation of .008 m (.31 in). Measurements of the position of the shearer were made every .1 seconds. Measurements of the height of the coal interface were made after every pass across the longwall and at nominal 1 m (3.3 ft) steps in the X direction.

4) The Kalman filters presented in section 3.5 were used to filter the measurement of the shearer's and the coal interface sensor's X and Y position. Averaging of the measurements of the coal interface height was used to estimate the true value of the height. 48 measurements were used at each point.

Figure 3-28 illustrates the response of a typical cut. The error in the prediction of the coal interface height in the middle of the cut is shown as a function of the shearer's position along the longwall. As before, the figure shows the discontinuities due to the shifting of the pivot. The apparent degradation in the curve as the shearer progresses across the longwall is not significant. Other simulations show that the estimates are better at the beginning of the cut. The figure also shows the gradual improvement in the prediction as the Kalman filter for the X direction gradually improves as more measurements are made. This can be seen by the decreasing randomness as the shearer progresses across the face of the longwall (many more measurements than the 5 mentioned in section 3.5 are required because the sampling time was selected as .1 seconds instead of the .016 s used previously). The response is well within the maximum error bounds of .015 m (.5 in) assumed in chapter 2.
It should be noted that the vertical control system was able to achieve the required accuracy despite the fact that the Kalman filter for the measurements of the shearer’s position perpendicular to the longwall face, \( Y \), was not able to reduce the uncertainty in the estimate to the level required by the previous analysis. This is due to the assumption that was made that all the coordinates were of equal importance. In actuality the errors in the estimates of the coal interface height, \( Z_c \), are more significant than either of the other two dimensions. This is due to the use of large numbers of measurements of \( Z_c \) by the finite difference estimates of the derivatives. In comparison, the measurements of \( X \) and \( Y \) are used only once when extrapolating.

The Taylor series predictive model used in this last simulation required about 2500 measurements of the coal interface height after each pass across the 50 m (164 ft) length of the longwall. If each measurement required only 30 seconds to make, at least twenty hours would be required to complete the required measurements after just one pass across the longwall! It is this excessive number of required measurements that motivates the consideration of the second type of predictive model. In chapter 4, a least squares estimation technique will be developed which has the potential of greatly reducing the number of required measurements.
Kalman filters for X and Y
Averaging for Z
.008 m standard deviation in all measurements
.016 m standard deviation in random disturbance in Y
.016 m/s standard deviation in random velocity disturbance in X direction

STOCHASTIC SIMULATION FOR TAYLOR SERIES PREDICTIVE MODEL
CHAPTER 4

POLYNOMIAL LEAST SQUARES PREDICTIVE MODEL

The fundamental cause for the sensitivity of the Taylor series to measurement noise is the use of point estimates of the derivatives of the coal interface generated from noisy data. This is not surprising since derivatives can be viewed as high pass filters which amplify the noise relative to the signal. To minimize the effect of noise a method that relies on global estimation instead of the point estimation will now be considered.

The method of least squares is a well known technique for fitting a curve to a set of data points so as to determine the "best" estimates of the unknown parameters of the curve. The method is "best" in the sense that the parameters determined by the least squares analysis are gaussianly distributed about the true parameters with the least possible standard deviation. In addition, the estimates will approach the true values of the parameters as the number of measurements approaches infinity. This is true only when systematic errors do not affect the measurements.

The first step in a least squares analysis is to select a functional relationship between the dependent and independent variables. The function does not have to be chosen on the basis of a physical law but only on the function's ability to accurately represent the data. Equation (4-1) shows the most general form of the relationship between Z, the dependent variable, and the X's, the independent variables. The unknown parameters which are to be estimated are the A's.

\[ Z = f(X_1, X_2, \ldots, X_b, A_1, A_2, \ldots, A_g) \]
where

\[ b \quad = \quad \text{number of independent variables} \]

\[ g \quad = \quad \text{number of unknown parameters} \]

\[ A_j \quad = \quad j^{th} \text{ unknown parameter to be determined} \quad j = 1, \ldots, g \]

\[ X_j \quad = \quad j^{th} \text{ independent variable} \quad j = 1, \ldots, b \]

\[ Z \quad = \quad \text{dependent variable} \]

Once an estimate of the parameters has been made the estimated, or calculated, relationship between the dependent variable and the independent variables is given by:

\[ (4-2) \quad Z_e = f(X_{1_e}, X_{2_e}, \ldots, X_{b_e} ; A_{1_e}, A_{2_e}, \ldots, A_{g_e}) \]

where

\[ A_{je} \quad = \quad \text{estimate of the } j^{th} \text{ parameter} \quad j = 1, \ldots, g \]

\[ X_{je} \quad = \quad \text{estimate of the } j^{th} \text{ independent variable} \quad j = 1, \ldots, b \]

\[ Z_{ie} \quad = \quad \text{estimate of dependent variable} \]

Figure 4-1 illustrates the relationship between the true, measured, and estimated variables for the case of only one independent variable, \( b = 1 \).

Next, the residuals, \( R_z \) and \( R_x \)'s, are defined as the difference between the measured and the estimated variables at \( X_i \)'s, \( Z_i \).

\[ (4-3) \quad R_{z_i} = Z_i - Z_{ie} \quad \text{for } i = 1, \ldots, N \]

\[ (4-4) \quad R_{x_{ij}} = X_{ij} - X_{ij} \quad \text{for } i = 1, \ldots, N \quad \text{for } j = 1, \ldots, b \]

where
RELATION BETWEEN TRUE, MEASURED, AND CALCULATED VARIABLES

figure 4-1
\( i \) = measurement index  
\( j \) = independent variable index  
\( N \) = number of measurements  
\( X_{ij} \) = \( i^{th} \) measured value of \( j^{th} \) independent variables \( i = 1, \ldots, N \) \( j = 1, \ldots, b \)  
\( Z_{ni} \) = \( i^{th} \) measured value of dependent variable \( i = 1, \ldots, N \)  

The weighted sum of the squares of the residuals, \( R_s \), is given by:

\[
(4-5) \quad R_s = \sum_{i=1}^{N} \left[ w_i Z_i^2 + \sum_{j=1}^{b} (w_{ij} x_{ij})^2 \right]
\]

where

\( w_i \) = weight of the \( i^{th} \) measurement of the \( j^{th} \) independent variable \( i = 1, \ldots, N \) \( j = 1, \ldots, b \)

\( w_{ij} \) = weight of the \( i^{th} \) measurement of the \( j^{th} \) dependent variables \( i = 1, \ldots, N \)

The form of weighting which is most often used, and the method which will be employed in this thesis, is called statistical weighting. The weights are defined as the reciprocals of the squares of the standard deviations of the measurements which are assumed known:

\[
(4-6) \quad w_i = S_{i}^{-2} Z_i
\]

\[
(4-6) \quad w_{ij} = S_{ij}^{-2} x_{ij}
\]

where

\( S_{i} \) = standard deviation of \( i^{th} \) measurement of \( j^{th} \) independent variable
\[ S_i \] = standard deviation of \(^{i}\)th measurement of dependent variable \( Z_i \)

Equation (4-5) is the fundamental equation for the method of least squares since the estimates of the parameters are obtained by minimizing this equation. In general the problem is highly nonlinear. The solution method that was selected linearizes the equations and locates the minimum iteratively by the method of Lagrange multipliers. Reference 16 contains a detailed derivation of the general solution method.

In elementary discussions of the method, the residuals of the independent variables are not included in equation (4-5). This is permissible only if the uncertainties in the values of the independent variables are negligible when compared to those of the dependent variable. If this is not the case the solution will not be the one which minimizes the variances of the various estimates. In addition, many of the statistical tests which can be performed on the estimates are no longer valid. Most polynomial least squares analysis makes use of this assumption since a simple closed form solution exists.

In appendix A a typical coal interface profile was shown to be adequately described by a higher order polynomial over a distance of 50 m (164 ft). Equation (4-1) will therefore be chosen to be a polynomial of initially unknown order where the independent variable will be considered the distance along the longwall, \( X \), and the dependent variable will be considered the coal interface height, \( Z_c \). Equation (4-7) shows the specification of equation (4-1):

\[
(4-7) \quad Z_c = A_1 + A_2 X + A_3 X^2 + \ldots + A_g X^{g-1}
\]

where

\[ g = \text{number of unknown parameters} \]

\[ A_i = \text{the } i^{\text{th}} \text{ unknown parameter} \quad i = 1, \ldots, g \]
\[ X = \text{the distance along the longwall,} \]
\[ \text{the independent variable} \]
\[ Zc = \text{the coal interface height,} \]
\[ \text{the dependent variable} \]

A three dimensional polynomial could also be used to generate a representation of the coal interface surface. However it was found that a simple linear extrapolation between two curves of the form of (4-7) gave good results. The additional complications of using a three dimensional polynomial were therefore not considered worthwhile. Figure 4-2 illustrates the concept of using linear extrapolation between two curves.

The order of the polynomial, \((g-1)\), will not necessarily be selected as eighth order since in general the order of the polynomial that would best represent the coal interface profile would not be known in advance. Two important points must be considered. The polynomial must be of high enough order so that the true profile can be accurately determined. Equally important is the smoothing properties of the polynomial. It would be inappropriate to select too high an order polynomial since it would begin to reflect the randomness of the data. The extreme is to select a polynomial that would interpolate all the data points. This "goodness of fit" issue will be discussed more fully in section 4.2.

Many other functions other than the polynomial could have been considered for representing the coal interface profile. Probably the most widely used is the spline approximation. Splines can be seen as a series of smooth, piecewise polynomial approximations. They were originally developed because global interpolating polynomial approximations often lead to highly inflected curves. While splines are most often used for interpolation, they have been used in least squares analysis. Splines are used widely in CAD systems because of their ability to accurately represent complicated shapes.

It is the piecewise nature of splines that render them less
LINEAR EXTRAPOLATION BETWEEN TWO CURVES

middle of current cut

figure 4-2
useful when noisy measurements are used. They tend to exhibit the randomness of the data. Splines were not considered in this thesis because of their lack of smoothing.

Any estimation technique must allow for the determination of the accuracy of its estimates for it to be useful. The accuracies of the least squares estimates will therefore be addressed in the following section.

4.1 UNCERTAINTY OF LEAST SQUARES ESTIMATES

From figure 4-1 it is seen that the relationship between the measured and true values of the variables can be expressed by:

\begin{align*}
Z_{k_i} &= Z_i + E_{Z_i} \\
X_{m_{ij}} &= X_{ij} + E_{X_{ij}}, \quad \text{where } i=1, 2, \ldots, N, \quad j=1, 2, \ldots, q
\end{align*}

where

\begin{align*}
g &= \text{number of independent variables} \\
N &= \text{number of measurements} \\
E_{X_{ij}} &= \text{error in the } i^{th} \text{ measurement of the } j^{th} \text{ independent variable} \\
E_{Z_i} &= \text{error in the } i^{th} \text{ measurement of the dependent variable}
\end{align*}

These measurement errors will cause the least squares values of the parameters to differ from the true values of the parameters. The standard deviations of the parameters are defined as:

\begin{align*}
S_{A_{e_i}} &= C \left[ \frac{\sum (A_{e_i} - A_i)^2}{N} \right]^{0.5} \quad i = 1, 2, \ldots, q
\end{align*}
where

\[ \mathbb{E} \left[ \right \] = \text{expectation operator} \]

\[ S_{\bar{A}_i} = \text{standard deviation of the estimate of the } i^{th} \text{ parameter} \]

Reference 16 outlines a procedure for estimating these standard deviations using the measured data points. The method assumes that the errors are uncorrelated. This is usually valid if systematic errors are not present. Using this assumption the resulting equation for the estimate of the standard deviation of the estimate of the parameters is given by:

\[
(4-10) \quad \left( S_{\bar{A}_i} \right)_e = \left( \frac{R_5}{N - g} \right) \left( \mathbf{C}^{-1} \right)_{ii} \]

where

\[ \mathbf{C}^{-1}_{ii} = \text{the } i^{th} \text{ row by } i^{th} \text{ column element of the inverse of the coefficient matrix which is a result of the least squares estimation of the parameters,} \]

\[ \left( S_{\bar{A}_i} \right)_e = \text{estimate of the standard deviation of the estimate of the } i^{th} \text{ parameter} \]

\( \mathbf{C}^{-1} \) is the inverse of the coefficient matrix and is a result of the general least squares solution method outlined in reference 16. It should be noted that equation (4-10) is only an unbiased estimate of the standard deviation. As the value of \( (N-g) \) increases the estimate of the standard deviation will improve. Equation (4-10) shows that the standard deviation of the estimates of the parameters can be made to approach zero by increasing the number of measurements. As one would expect, the standard deviation of the parameters will in general be larger when more parameters are to be estimated.

The estimate of the standard deviation of interpolated and/or extrapolated values can also be determined using the following equation which is derived in reference 16:
(4-11) \( \ell_{s_{f_e}} = \left( \frac{-R_s}{N_{-g}} \right) \sum_{i=1}^{9} \left[ \sum_{j=1}^{9} \frac{df_{i}\cdots df_{j}}{dA_{i} \cdots dA_{j}} (C^{-1})_{ij} \right] \ell^{5} \)

where

\( \ell_{s_{f_e}} = \) estimate or standard deviation of extrapolated value

\( \frac{df}{dA_i} = \) partial derivative of equation (4-1) with respect to \( i^{th} \) parameter

In general the standard deviation will be a function of the independent variables.

Equation (4-11) can be used to estimate the required number of measurements to meet the accuracy requirement. As was mentioned previously, the least squares model extrapolates between two curves that have been determined from data from two passes across the longwall. Assume that two curves of the roof profile have been established for the previous two cuts as illustrated in figure 4-2. The extrapolation in the direction perpendicular to the longwall face, \( Y \), at a given position along the longwall, \( X \), is given by:

(4-12) \( \ell_{c_e} = C_1 Y + C_2 \)

where

\( f = \) estimate of the height of the coal interface obtained from interpolation of the least squares curves, eq (4-2)

\( Y = \) distance perpendicular to longwall face

\( \ell_{c_e} = \) estimate of coal interface height at the current cutting point

\[ C_1 = \frac{f_2 - f_1}{Y_2 - Y_1} \quad C_2 = \frac{Y_2 f_2 - Y_1 f_1}{Y_2 - Y_1} \]

where

\( Y_1 \) and \( f_1 \) are from curve 1 \( \quad Y_2 \) and \( f_2 \) are from curve 2
The \( f \)'s are obtained not from direct roof measurements but from interpolation of the two previous curves that were determined from historical data. The \( Y \)'s will be assumed to be deterministic with \( Y_1 \) equal to zero and \( Y_2 \) equal to the width of the cut taken by the shearer which will be assumed to be 1 m (3.3 ft). This was done since it was felt that over the length of the longwall, the variation in \( Y \) will be averaged out. The error in the measurement of the distance along the longwall, \( X \), is also not considered since a realistic Kalman filter was developed in chapter 3 which can reduce this error to any arbitrary level. Considering these assumption and the error in the prediction of \( \hat{Y} \), the standard deviation of the coefficients becomes:

\[
(4-13) \quad \begin{align*}
S_{c_1}^2 &= \frac{2 \cdot S_f^2}{Y_2 - Y_1} \\
S_{c_2}^2 &= \frac{Y_2^2 + Y_1^2}{(Y_2 - Y_1)^2} \cdot S_f^2
\end{align*}
\]

\[
= 2S_f^2 \\
= S_f^2
\]

where

\[
S_{c_i} = \text{standard deviation of } i^{th} \text{ constant}
\]

\[
S_f = \text{standard deviation of least squares estimate of coal interface height given by (4-11)}
\]

The standard deviation of the estimate of the coal interface height at the current cutting point can be determined from equation (4-12) from elementary probability theory:

\[
(4-14) \quad S_{zc}^2 = Y^2 \cdot S_{c_1}^2 + S_{c_2}^2 + C_1 \cdot S_y^2
\]

where

\[
S_y = \text{standard deviation of measurement of the shearer's location in the Y coordinate}
\]

\[
S_{zc} = \text{standard deviation of the estimate of the coal height at the current cutting point}
\]

Assuming the two least squares curves are 1 m (3.3 ft) apart and
substituting in equations (4-13), equation (4-14) will yield the following:

\[(4-15) \quad s^2_{ZC} = (2xy^2 + 1)s^2_f + C_1s^2_y\]

Equation (4-15) can be solved for the standard deviation in the least squares estimates, \(s_f\).

\[(4-16) \quad s_f = \left( (2xy^2 + 1)^{-1}s^2_{ZC} - C_1s^2_y \right)^{1/2}\]

To determine the maximum allowable value of \(s_f\), the maximum allowable standard deviation in the estimate of the height of the coal interface at the current cutting point must be determined. As mentioned previously, a common approach is to assume that the expected error is equal to 3 times the maximum allowable standard deviation. In chapter 2 a maximum error of .015 m (.5 in) was selected which yields a maximum standard deviation of .005 m (.17 in). The value of \(Y\) will be 1.5 m (5 ft) if the two least squares curves are 1 m (3.3 ft) apart. The standard deviation in the measurement of \(Y\) will be assumed to be .008 m (.31 in) as was done in chapter 2. From appendix A, it can be shown that a representative value of the coal interface slope, \(C_1\), is .09 m/m (1 in/ft). Using these assumptions equation (4-16) will yield:

\[(4-17) \quad s_f = .002 \text{ m (.08 in)}\]

Equation (4-11) cannot be used directly to determine the number of points required to meet the allowable standard deviation given by equation (4-17) since the coefficient matrix is a function of the unknown parameters. In turn the number of parameters is a function of the order of the polynomial selected. Therefore the topic of goodness of fit will be discussed next.

4.2 GOODNESS OF FIT
As mentioned previously, the order of the polynomial used for the curve fit will not generally be known ahead of time. If too high an order is selected, the resulting curve will exhibit the randomness of the data. If too low an order is selected, the resulting curve will not adequately model the fluctuations of the true coal interface profile.

Several methods can be used to test the "goodness of fit". One involves determining the confidence limit of the resulting curve and then testing the plausibility of each measured data point. If a significant number of the data points lie outside the confidence interval, a different order would be called for. Another method involves testing to see if the residuals are randomly distributed. If they are not, the selected order is probably not suitable for the data in question.

An additional method involves using the fact that the sum of the squares of the residuals should be distributed according to the chi-squared distribution.\(^{16}\) This distribution is only a function of the degrees of freedom where:

\[(4-18) \quad \text{Degree of freedom} = N - q\]

where

\[q = \text{number of parameters to estimate}\]

\[N = \text{number of measurements}\]

In addition the mean of the chi-squared distribution is equal to the number of degrees of freedom. An indication of the goodness of fit is therefore available by comparing the calculated sum of the squares of the residuals to the mean of the chi-squared distribution.

To investigate the issues of "goodness of fit" and the number of required measurements a series of simulations using the program SIMU.FTN presented in appendix C were run. The following algorithms and assumptions were used in these simulations.
1) The least squares filter presented in this chapter was used to filter the measurement of the coal interface height after every pass across the longwall. The algorithms used are based on those presented in reference 16 and are given in appendix C. The Kalman filters presented in chapter 3 were used to filter the measurements of the distances parallel and perpendicular to the longwall face. All measurements were assumed to have a 0.008 m (0.31 in) standard deviation.

2) The polynomials were fit over the interval 10 to 60 m.

3) The shearer model developed in chapter 2 was used with the exception that the dynamics of the cutting drum height was not considered nor were the random disturbances. The shearer was assumed to take a 1 m (3.3 ft) wide cut.

4) The coal interface height at the current cutting point was determined from the two previous least squares curves by the extrapolation method presented earlier in the chapter. The height was determined in the middle of the cut.

Various order polynomials and step sizes between measurements of the coal interface height were used. Four consecutive cuts were taken for each combination of step size and order. An average was then calculated for the sum of the squares of the residuals, the estimated standard deviation of the curve fit, and the maximum observed error in the resulting cut. The percentage error in the residuals was then calculated using the degree of freedom as the true value. Note that an average standard deviation was calculated for each curve using the measured values of X. Table 4-1 shows the results.

As can be seen the residuals stabilize at the sixth of seventh order polynomial. This is not surprising since the roof profile was assumed to be an eighth order polynomial. More significantly, there seems to be only a slight improvement in accuracy as we increase in order. The higher order polynomials seem to be slightly better for larger step sizes.
RESIDUALS, STANDARD DEVIATION OF CURVE, AND MAXIMUM OBSERVED ERROR IN CUT

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Table 4-1
To meet the required standard deviation given by equation (4-17) requires that a step size of approximately .5 m (1.7 ft) or less be chosen. It should be noted that these maximum errors are only the averages of four cuts.

We now have a method to select the appropriate order of polynomial and number of points required to achieve a good fit. In the actual application, the sum of the squares of the residuals would be calculated for several polynomials and the lowest order polynomial that agrees with the degrees of freedom within certain limits will be chosen since the lowest order will achieve the most filtering. The standard deviation of the interpolated values of the curve would be used to decide if more points are required for the next cut.

To confirm these results several simulations were run with a sixth order polynomial filter and a step size of .5 m (1.6 ft). The same algorithms as assumed previously were used with the exception that random inputs to both the Y position and the velocity in the X direction were assumed to have a standard deviation of .016 m (.64 in) and .016 m/s (.64 in/s) respectively. The Kalman filters presented in chapter 4 are used for the X and Y directions. The resulting standard deviation for X is .001 m (.04 in). This value is used for the least squares curve fit. Figure 4-3 illustrates the filtering action of the least squares method and figure 4-4 illustrates the error in the cut.

Figure 4-3 clearly shows the filtering action of the least squares algorithm. While the fit is not exact, the estimate is both smooth and continuous. Figure 4-4 shows that indeed the resulting estimates of the coal interface height at the current cutting point is within the maximum allowable error. These estimates used 102 measurements of the coal interface height after each pass of the shearer. This is an order of magnitude reduction compared to the best Taylor series algorithm developed in chapter 3. In addition, the least squares method allows for the continual detection and adjustment to changes in the standard deviations in the measurements and coal interface profile by using the
statistical tests of the estimates of the parameters of the least squares curves.
LEAST SQUARES CURVE FIT OF MEASURED VALUES OF COAL INTERFACE HEIGHT

figure 4-3
CHAPTER 5

CONCLUSIONS

In this thesis an automatic predictive model vertical control system has been presented. In chapter 2 a method of incorporating the model of the coal interface profile into a feedback control system was presented. In chapters 3 and 4 the generation of two different predictive models of the coal interface from noisy and discrete data was presented.

The first predictive model was based on local Taylor series expansions. The derivatives of the coal interface profile were estimated at discrete points along the longwall using finite difference algorithms. The estimate of the height of the coal interface at the current cutting point could then be determined by extrapolating from the point where the derivatives have been estimated.

The estimates of the derivative proved to be very sensitive to noisy measurements so Kalman filtering of the measurement data was considered. A suitable filter was developed for the measurement of the distance along the longwall face using a reasonable model of the shearer dynamics. Kalman filters for the measurement of the height of the coal interface and the distance perpendicular to the face of the longwall were unable to yield better estimates than if averaging was employed. The recommended algorithm for the Taylor series predictive model was composed of the following:

1) Only two terms were retained in the Taylor series expansion. It was found that the susceptibility of the estimates of the derivatives increased rapidly with higher order derivatives.
2) A second order backward difference algorithm was used to estimate the derivative along the coordinate perpendicular to the longwall face while a second order central difference algorithm
was used for the coordinate parallel to the longwall face. It was found that higher order estimates were more sensitive to noise. Central difference algorithms were significantly less sensitive to noise than backward difference algorithms so they were used wherever possible.

3) Kalman filters were used for the filtering of the measurements of the distance along and perpendicular to the longwall while averaging was used for the measurement of the coal interface height.

If an accuracy goal of 0.015 m (.5 in) were used for the estimate of the coal interface height and the measurement standard deviation was 0.008 m (.31 in), the above algorithm would require that measurements of the coal interface height be made after every pass across the longwall and at 1 m (3.3) intervals. 48 measurements of the coal interface height would be required at each point to yield an acceptable estimate of the height by averaging. Therefore 2500 measurements of the coal interface height would be required after every pass of the shearer across a 50 m (164 ft) longwall.

It was the possibility of reducing this large number of required measurement that motivated the second predictive model. It was observed that the sensitivity of the Taylor series model was due to the point estimates of the derivatives. It was therefore considered that a global estimation technique would be less susceptible to noise. The method selected uses generalized least squares estimation. Higher order two dimensional polynomials were fitted to the data from one pass along the longwall. Estimates of the coal interface height at the current cutting point were determined by linear extrapolation between two of this fitted curves. Statistical tests of the resulting curves were developed that allow for the selection of the best order of the polynomial and the required number of data points. Simulations were performed with the following conditions;

1) Kalman filters for the measurements along and perpendicular to the longwall was used. The coal interface height estimates were
obtained from the least squares curve fit.

2) A maximum estimation error of .015 m (.5 m) was used. The standard deviation in the measurement was assumed to be .008 m (.31 in).

The results showed that measurements of the coal interface height would be required every .5 m (1.2 ft) for a total of 102 measurements for each pass across the 50 m (164 ft) longwall.

This significant reduction in the number of required measurements of the coal interface height makes the least squares predictive model the most appropriate of the two for the vertical control system. In addition the statistical test of the least squares estimates allows this method to easily detect and compensate for changes in the standard deviations of the measurements.

102 measurements for each pass is still a substantial number. It should be noted that the tolerance that was set for this thesis of .015 m (.5 in) is very tight. In actual application the tolerance should be substantially higher and fewer measurements of the coal interface would be required.
Appendix A

Representation of Coal Interface Profile for Simulation

For simulation purposes a mathematical representation of a typical roof profile is required. Just such a representation is presented here.

The Arthur D. Little company of Cambridge, Massachusetts was contracted by the U.S. Bureau of Mines to survey six separate longwall operations during 1977. These surveys were two dimensional plots of the altitude of the roof coal interfaces as a function of distance along the longwall. A longwall located at the Brazrah mine was selected for use in this thesis since it represented the worst case situation in that it was the most undulating of the six. The height of the coal interface, \( z \), was taken from the drawing at .76 m (30 in) intervals along the horizontal axis, \( x \), and is presented in Table A-1.\(^{13}\)

The data was first transformed to a coordinate system such that the average offset of the coal interface was parallel to the direction along the longwall, the \( x \) axis. Figure A-1 illustrate the transformation. For the Brazrah longwall the angular rotation between the two coordinate systems is:

\[
\text{(A-1)} \quad \phi = .1 \text{ radians}
\]

The transformation is then given by:

\[
\text{(A-2)} \quad x = x' \cos(\phi) - z' \sin(\phi)
\]

\[
z_c = x' \sin(\phi) + z' \cos(\phi)
\]

This data was then used to obtain a polynomial least squares curve fit. An eight order polynomial was found to give the closest fit. Figure A-2 illustrates the polynomial and the data points used for the curve fit. This figure shows that the resulting curve is slightly
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Table A-1
TRANSFORMATION USED FOR PROFILE

\[ \text{coal interface profile} \]

\[ \text{Zc} \quad \text{phi} \]

\[ x^*, zc^* : \text{original coordinate system} \]

\[ x, zc : \text{coordinate system used in thesis} \]

**figure A-1**
smoother than the actual coal interface profile such that the curve is more representative of the other five mine surveys. The coefficients of the polynomial are:

\[(2-3) \quad C_0 = -3.539988 \, \text{m} \]

\[C_1 = 1.277656 \]

\[C_2 = -1.882176 \times 10^{-1} \, \text{m}^{-1} \]

\[C_3 = 1.404704 \times 10^{-2} \, \text{m}^{-2} \]

\[C_4 = -5.943110 \times 10^{-4} \, \text{m}^{-3} \]

\[C_5 = 1.492880 \times 10^{-5} \, \text{m}^{-4} \]

\[C_6 = -2.200536 \times 10^{-7} \, \text{m}^{-5} \]

\[C_7 = 1.753255 \times 10^{-9} \, \text{m}^{-6} \]

\[C_8 = -5.815316 \times 10^{-12} \, \text{m}^{-7} \]

\[(A-4) \quad Z_c = f(X) \]

\[= C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots \]

For the simulations that will be performed, a three dimensional representation of the roof coal interface profile is required. Unfortunately, the Arthur D. Little surveys only supplied information for two dimensions. To generate a three dimensional equation, equation \(A-4\) can be modulated by an equivalent equation which is a function of the distance perpendicular to the longwall face. Equation \(A-5\) illustrates the concept.

\[(A-5) \quad Z_c(X,Y) = C_9 + C_{10} x f(X) y f(Y) \]

\[C_9 = \text{offset} = 1.3 \, \text{m} \]
\[ C_{10} = \text{amplitude} = .25 \text{ m}^{-1} \]

This form retains the basic form of the profile for a given value of \( Y \). \( C_9 \) is used to insure that the undulations of the modulated function are not too large. The offset is added to give a reasonable height to the roof coal interface. This function will be used as the actual roof coal interface in all the subsequent analysis for testing the vertical control algorithms that will be developed.
APPENDIX B

FINITE DIFFERENCE APPROXIMATIONS
OF DERIVATIVES

The following tables list the three types of finite difference approximations to various order derivatives. For example, the second order backward difference approximation to the second derivative is given by:

\[(B-1) \quad D^2 = h^{-2} \left( 2z_i - 5z_{i-1} + 4z_{i-2} - z_{i-3} \right) \]

where

\[ h = \text{step size in the independent variable, } X \]

\[ D^2 = \frac{d^2Z}{dx^2} = 2^{nd} \text{ partial derivative of } Z \text{ with respect to } X \]

The error in the estimate will be of order \(h^2\).
## Backward Difference Approximations

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**Forward Difference Approximation**

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<td>996041-1926241</td>
<td>2444981-210920</td>
<td>1233481</td>
<td>-470241</td>
<td>105791</td>
<td>-10681</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table B-2b**
## CENTRAL DIFFERENCE APPROXIMATIONS

### Second order approximations

<table>
<thead>
<tr>
<th></th>
<th>i-5</th>
<th>i-4</th>
<th>i-3</th>
<th>i-2</th>
<th>i-1</th>
<th>i</th>
<th>i+1</th>
<th>i+2</th>
<th>i+3</th>
<th>i+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2hD</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h^2 D^2</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12hD^3</td>
<td></td>
<td></td>
<td>-1</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>h^4 D^4</td>
<td></td>
<td></td>
<td>1</td>
<td>-4</td>
<td>6</td>
<td>-4</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Fourth order approximations

<table>
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<tr>
<th></th>
<th>i-5</th>
<th>i-4</th>
<th>i-3</th>
<th>i-2</th>
<th>i-1</th>
<th>i</th>
<th>i+1</th>
<th>i+2</th>
<th>i+3</th>
<th>i+4</th>
</tr>
</thead>
<tbody>
<tr>
<td>12hD</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>-8</td>
<td>0</td>
<td>8</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12h^2 D^2</td>
<td></td>
<td></td>
<td>-1</td>
<td>16</td>
<td>-30</td>
<td>16</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>18h^3 D^3</td>
<td></td>
<td></td>
<td>1</td>
<td>-8</td>
<td>13</td>
<td>0</td>
<td>-13</td>
<td>8</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>16h^4 D^4</td>
<td></td>
<td></td>
<td>-1</td>
<td>12</td>
<td>-39</td>
<td>56</td>
<td>-39</td>
<td>12</td>
<td>-1</td>
<td></td>
</tr>
</tbody>
</table>

Table B-3
APPENDIX C

PRESENTATION OF SIMULATION MODEL

To confirm the analytical results presented chapters 3 and 4 program SIMU.FTN was written. Figure C-1 contains the flowchart of the program. What follows is a brief outline of the major subroutines of the program.

Subroutine INITY initializes the program by;
1) supplying the seed for the random number generator,
2) supplying initial values to the filters if required,
3) filling in the first block of rows of stored value of the roof height with deterministic measurements. For example, if we are using a second order backward difference approximations and we retain only the first order derivatives in the Taylor Series, we will be required to measure and store the first 3 rows.

Subroutine INITX initializes the program before each cut by;
1) supplying initial positions for both the shearer and roof measurer. The cut is always assumed to progress from left to right.
2) supplying the new control input for positioning the shearer and roof measurer perpendicular to the longwall. Each cut is assumed nominal straight.
3) initialize any filters if required.

Subroutine MEASUR models the measuring process as introducing an additive zero mean gaussianly distributed disturbance. MEASUR in turn calls GRAN which is a gaussian pseudo-random number generator.

Subroutines FILTX, FILTY, FILTXZ, FILTYZ, FILTXZ model the various Kalman and least squares filters discussed in chapters 3 and 4. The first two routines filter the X and Y positions of the current cutting point of the shear. These will be used by the predictive model to
FLOWCHART OF SIMU.FTN

BEGIN
INPUT
INITIT

INITIT

MEASUR(X) MEASUR(Y)

FILTER t
FILTER(X) FILTER(Y)

filter f

NODE t
POSITION

DERIVA pivot at node forward/back

DERIVA pivot 1 ahead back/back

middle
SKIP

DERIVA pivot 1 ahead cent/back

PREDIC

SHEARER

figure C-1a
figure C-1b
determine the height of the coal seam at the cutting point. The last
three routines filter the position of the roof measuring instrument
and the actual measurement of the coal interface. The filtered
measurements will be stored for later use in updating the predictive
coal interface model.

Subroutine NODE indicates if we have reached a node. Nodes are
positions in the X, Y plane at which we measure the roof height and
possibly change to a new pivot in the predictive model.

POSITION is a section of code which indicates where the shear is
in the cut. Each cut is divided into 3 regions: start, middle, and
end. These regions refer to what type of difference algorithm is
required to estimate the derivatives in the x direction. Start
indicates that forward difference is to be used, while middle require
central, and end, backwards. Switching between methods insures that
the pivot point of the Taylor series is never more than 1 step size
away from the prediction point while also insuring that the more
accurate and stable central difference method is used wherever
possible.

To illustrate, consider figure C-2 which shows two rows of
measurements and the current cut. Say we keep only the first order
derivatives in the Taylor series and use 4th order approximations for
the derivatives. The 4th order central difference approximation
requires 5 points evenly spaced about the pivot. Thus the first pivot
that can be selected if we use central difference is node 3. The
distance to X1, the point where we want to predict Zc, from
the pivot can then be as great as 2h.

Instead we use the following strategy. Node 1 is selected as the
first pivot, with forward difference approximation used for the
derivatives. As the shearer progresses to node 2, the pivot now is
switched to node 3 and central difference is used. The process is
repeated until the end of cut is reached where we switch over to
SELECTION OF PIVOT

1 2 3 4 5 ... n-2 n-1 n

FD CD CD CD ED

X_1

last cut

current cut

figure C-2
backward difference. The strategy is to insure that we are never trying to approximate further than 1 step from the pivot, and to insure that we use central difference approximations wherever possible.

Subroutine DERIVA estimates the value of the derivatives at the pivot using backward difference approximations for the Y direction, and backward, central, or forward difference for the X direction and for the mixed derivatives. The filtered measurements are used.

Subroutine PREDIC uses the Taylor series predictive model to estimate the position of the roof in the middle of the current cut at the location of the shearer drum cutting point.

Subroutine SHEARER models the dynamics of the shearer.

Subroutine RULER models the dynamics of the roof height measuring scheme.

Some general characteristics of the program include:
1) all program decision and control rely on the filtered measurements,
2) The shearer dynamics and roof measurer dynamics have been kept independent. This was due to emphasize the non real time nature of the measurement process for control purposes,
3) Each cut is assumed nominally straight. No sumping was considered,
4) All measurements are assumed to be referenced from a glacial coordinate system. All measurements are assumed to have the same standard deviation. This last requirement can be easily changed if required. We have used discrete time models for both filtering and control.
PROGRAM SIMU

 rev date by  
 0  84 June 30 JFL First fortran 77 version,
     Uses 5 terms in Taylor Series,
     Uses (backward + central + forward)
     backward difference approximations,
     Uses Kalman filtering for all measurements
     except Z which averaging is used.
     Uses rev 1 of derived.
     0 of measur
     0 of roof
     1 of plot

 1  84 June 31 JFL
     Uses 2 terms in Taylor series,
     Uses 2nd order approx to derivatives.
     Uses rev 2 of DERIVA

 2  84 June 31 JFL
     Changed to kalman filtering for Z
     which require multiple measurements.
     Uses extended kalman filtering for Z.
     Uses 5 terms in Taylor series.
     Uses rev 1 of derived.
     Uses rev 0 of PREDIC

 3  84 July 10 JFL
     Uses random disturbance in X and Y.
     Uses kalman filter for X and Y.
     Uses rev 2 of derived
     rev 1 of predic

 4  84 July 10 JFL
     Uses random disturbance in X and Y.
     Uses Kalman filter for X and Y.
     Uses least squares filter for Z.
     Uses rev 2 of derived
     Uses rev 1 of measur
     Uses rev 1 of predic

 5  84 July 14 JFL
     Uses random disturbance in X and Y.
     Uses Kalman filter for X and Y.
     Uses least squares filter for Z fitting
     a high order polynomial to data.
     Uses rev 1 of measur
     rev 2 of predic
     rev 2 of plot

 6  84 July 24 JFL
     Uses actual least squares filter for
     filter of Z.

 7  04 Aug 84 JFL

 constants

 INTEGER N
 PARAMETER (N = 110)

 INTEGER np
 PARAMETER (np = 6)

 INTEGER Order
 PARAMETER (Order = 8)

 INTEGER Poly
 PARAMETER (Poly = 5)

 INTEGER Rev
 PARAMETER (rev = 7)

 Number of independent parameters for
 Least Squares curve fit.

 Maximum number of data points.

 Maximum number of parameters in least
 squares curve fit.

 Order of polynomial representing
 roof profile.

 Order of approximating polynomial for
 prediction.
C
INTEGER Term
PARAMETER (Term = 5)

C
DOUBLE PRECISION Xe
PARAMETER (Xe = 60.0)

C
DOUBLE PRECISION XI
PARAMETER (X1 = 10.0)

C
INTEGER Ygrid
PARAMETER (Ygrid = 2)

C
DOUBLE PRECISION Yi
PARAMETER (Y1 = 24.0)

C
DOUBLE PRECISION Ys
PARAMETER (Ys = 1.0)

C
v, t, n, m, t

C
Revision of program

C
Unit number of terminal.

C
Endings value of X.

C
Initial value of X.

C
Number of curves required to extrapolate in Y direction.

C
Initial value of Y.

C
Step between stored points. (II)

C

C
CHARACTER Ans

C
Answer to query.

C
DOUBLE PRECISION C0, Pn, Y

C
Coefficients of approximating poly.

C
Coefficients of polynomial.

C
LOGICAL Fill

C
True if filtering desired.

C
BYTE FMTX, FMTY, LABT, LABY

C
Saved for subroutine FLOT.

C
INTEGER IDEV

C
Plotting device for GEDPLT.
0 - terminal
1 - plotter

C
INTEGER Incr

C
Every Iner point will be plotted.

C
LOGICAL List

C
True if output at terminal.

C
INTEGER Npts

C
Number of points to be plotted.

C
DOUBLE PRECISION PltMin, PltMax

C
Min and max for a plot.

C
INTEGER POINT

C
Counter for points to be plotted.

C
DOUBLE PRECISION R

C
Sum of square of residuals.

C
DOUBLE PRECISION SD

C
Standard Deviation in measu.

C
INTEGER Seed(114)

C
Seed for pseudo random sequence.

C
DOUBLE PRECISION SF

C
Standard deviation of function.

C
INTEGER T

C
Time counter.

C
DOUBLE PRECISION Ux

C
Control input for X sheeter.

C
DOUBLE PRECISION Uy

C
Control input for Y for ruler.

C
DOUBLE PRECISION Xa

C
Actual value of X.

C
DOUBLE PRECISION Xf

C
Filtered value of X.
INTEGER Xindex
integer Xend
DOUBLE PRECISION Xa
REAL Xplot(400)
DOUBLE PRECISION Xs
DOUBLE PRECISION Xst(1:1)
REAL XLF(1:1)
REAL Xstr(1:1)
DOUBLE PRECISION Ya
DOUBLE PRECISION Yr
DOUBLE PRECISION Ym
DOUBLE PRECISION Yf
DOUBLE PRECISION Zm
DOUBLE PRECISION Zf
DOUBLE PRECISION Zscat(1:1)
DOUBLE PRECISION Zaf(1:1)
REAL ZstrM(1:1)
REAL ZstrA(1:1)
REAL Zerr(400)
DOUBLE PRECISION Zeta(1:1)
DOUBLE PRECISION ZetA(1:1)
DOUBLE PRECISION Zeta, Zet
DOUBLE PRECISION Zd(0:1:Order, 0:1:Min)
DOUBLE PRECISION Zf
DOUBLE PRECISION Zp
DOUBLE PRECISION Zp(1:1)
DOUBLE PRECISION ZstM(1:1)
DOUBLE PRECISION ZstA(1:1)
DOUBLE PRECISION A(1:1:1)
DOUBLE PRECISION Sx(1:1:1)
DOUBLE PRECISION X(1:1:1:1:1)
DOUBLE PRECISION Aerr(1:1:1)
DOUBLE PRECISION CC(1:1:1:1)

Index for stored values of Z.
Largest Xindex.
Measured value of X.
Array of points to be plotted.
For use with NFDPLT.
Step between stored points.
Stored values of Xf for FILTZ.
Real version of Xst for PLDT.
Stored values of Xa for PLDT.
Actual value of Y.
Filtered value of Y
Index for stored values of Z.
Measured value of Y.
Actual value of Z.
Percentage error between actual height and predicted height in middle of cut.
Array of points to be plotted.
For use with GFDPLT.
Actual and predicted value of Z in middle of cut.
Actual derivative of roof profile.
First index is for X and 2nd for Y.
Last order = 2nd partial derivative of Z with respect to Y.
Filtered value of Z.
Measured value of Z.
Predicted value of Z.
Stored values of Zm for FiltZ.
Real version of Zst for PLDT.
Stored values of Za for PLDT.

Saved for LEASOR

Unknown Parameters
Standard deviation of independent
and dependent variables. Input.
Independent variables. Input.
Difference between guess and calculated
values of unknown parameters A.

- 156 -
Coefficient matrix.

 DOUBLE PRECISION Fa(1:Np)
 Partial derivative of F with respect to A's (the parameters).

 DOUBLE PRECISION Fx(1:Np)
 Partial derivative of F with respect to X's (the independent variables).

 DOUBLE PRECISION U(1:Np)
 Solution matrix of C\*A = U.

 DOUBLE PRECISION X(m)(1:Np)
 One measured value of X's.

 The following are for GINU.

 DOUBLE PRECISION AT(1:Np)
 DOUBLE PRECISION CCC(1:Np)
 DOUBLE PRECISION CD(1:Np)
 DOUBLE PRECISION NF(1:Np)
 DOUBLE PRECISION U(1:Np)
 DOUBLE PRECISION xx(1:Np)

 COMMON area .................................................................
 COMMON (/FLINT/ IDV, FNS(1), USCALE(4), LRXX(80),
 LARY(80), LART(80), NTXX, NTXY, TH, MARK, CHW, CHHT,
 PTWD, FMTX(6), FMTY(6))
 Saved for subroutine FLOT.

 'Main GINU' .................................................................
 WRITE (Term, 5) Rev
 WRITE (Term, 10)
 FORMAT ("Program GINU revision", I2)
 WRITE (Term, 10)
 5 FORMAT ("Input step for X between stored points")
 READ (Term, X)
 TX = 0.010
 S(1) = 50
 Sy = 0.008
 WRITE (Term, 25)
 25 FORMAT ("Do you desire filtering? (Y/N)")
 READ (Term, A)
 Ans
 26 FORMAT (A1)
 IF ('A' EQ. 'Y') THEN
 Filter = .true.
 ELSE
 Filter = .false.
 END IF
 WRITE (Term, 37)
 37 FORMAT ("Do you desire output at terminal? (Y/N)")
 READ (Term, Ans)
 Ans
 38 FORMAT (A1)
 IF (Ans .EQ. 'Y') THEN
 List = .true.
 ELSE
 List = .false.
 END IF
 WRITE (Term, 39)
 39 FORMAT ('Enter number of points to skip between ones we plot.')
 READ (Term, X)

 CALL INITY(Poly, X, Y, Xs, Ys, Ygrid, Curve, Xinde, Yindex,
 Seed, Uy, Uz, y)

 CALL INITY(1, X, X, Y, Y, Xs, Ys, Ygrid, Curve, Xinde, Yindex,
 Seed, Uy, Uz, y)

 LOOP
 CONTINUE

 x in Y direction so that we are in middle of cut

 Uy = Uy + Y
 Uz = Uz + Y
Yindex = Yindex + 1

CALL INITX(Incre, SD, Seed, Uw, Xz, Nets, Yf, Xf, Xindex, Xaf, Yf)

step in X direction for shearer

REPEAT

CONTINUE

take measurements
CALL MEASUR(Xa, SD, Seed, Xm)
CALL MEASUR(Ya, SD, Seed, Ym)

IF (Filter) THEN
  filter measurements
  CALL FILTX(T, Xm, Xf)
  CALL FILTY(Uw, Ym, Yf)
ELSE
  XF = Xa
  YF = Ym
END IF

predict Z in middle of cut
Zcutx = PREDIC(Xa, Ya, fT, curve, Xf, Ys)
Zcuty = ROOF(Xa, Ya, False, Z1der)

output data
IF (Point .GE. Incre) THEN
  Point = Point + 1
  Nets = Nets + 1
  Xpoint(Nets) = Xa
  Yerr(Nets) = Zcutx - Zcuty
  IF (Yerr(Nets) .GT. FitMax) THEN
    FitMax = Yerr(Nets)
  END IF
  IF (Yerr(Nets) .LT. FitMin) THEN
    FitMin = Yerr(Nets)
  END IF
ELSE
  Point = Point + 1
END IF

END IF

IF (List) THEN
  WRITE (Term=?0) Xa, Xf, Ya, Yf, Zcutx, Zcuty
  WRITE (Term=71) Xa, F7.4, Xf, F7.4, Ya, F7.4, Yf, F7.4
  Zae = F7.4, Zea = F7.4
END IF

Step forward
T = T + 1
CALL SHEAR(SD, Seed, Uw, Xa, Ya)
UNTIL
  IF (Xf.LE. Xa) GOTO 50

CALL INITZ(SD, Seed, Uw, Xz, T, Xa, Xf, Xindex, Ya, Yf)
step in X direction for measurement of roof

REPEAT

CONTINUE

take measurements
CALL MEASUR(Xa, SD, Seed, Xm)
CALL MEASUR(Ya, SD, Seed, Ym)

IF (Filter) THEN
  filter measurements
  CALL FILTZ(T, Xm, Yf)
  CALL FILTY(Uw, Ym, Yf)

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ELSE
    XF = X
    YF = Y
END IF

IF (NGDE(XF,XINDEX+XIL+XS)) THEN
    XINDEX = XINDEX + 1
    ZA = ROOT(XBAR+YBAR+FALSE+ZDER)
    CALL REASR(ZA+SREG+SREG+2)
    XST(XINDEX) = XF
    ZST(XINDEX) = ZA
    XSTF(XINDEX) = XF
    ZSTF(XINDEX) = ZA
    XSTR(XINDEX) = ZA
    ZSTR(XINDEX) = ZA
ENDIF

CALL FULER(SD,SEED+SUN+XUN+YUN+YBAR)
UNTIL
IF (XF .LE. XE) GOTO 80
CALL Filt(ZFIL+SFIL+INDEX+XST+YSTR+ZST+CURVER
         + MS, NX+NS, RF+SF+SF+SM+XR+AM+AF+CF+FX+F+UX+TERM+AT, CR+CF+NF+UX+XX)
WRITE(1,201) N, INDEX, XS+R+SF
201 FORMAT(13Hn= 'X4', 'id= 'X4', 'Xs= 'E11.4', 'R= 'E11.4, 'SF= 'E11.4)

REPEAT
CONTINUE
WRITE(1,210) YBAR+INDEX
210 FORMAT(' Flat error in this cut? Yb= 'E11.4, ' INDEX= 'I3/
         ' Send plot to terminal or plotter or skip? (0/1/2)'
READ(TERM#) IDEV
IF ((IDEV .EQ. 0) .OR. (IDEV .EQ. 1)) THEN
    WRITE(1,20) SFTMIN, Pbars:
20 SFTMIN( 'Min for Y= E11.4, Max for Y= E11.4) CALL FLT(XSLOT+ZSRL+NETS+X1+XER+TRUE...TRUE.)
END IF
UNTIL
IF (IDEV .NE. 2) GOTO 705
DO 350 I = 3; FOLP+1
C(1) = CURVE(3+I)
CONTINUE
350 NTS = 3
DO 400 XA = X1, XE+( (X-A/10.0)
NPT = NPTS + 1
X=LOT(NTS+1) = XA
ZRR(NPTS+1) = POLYS(MC+SOLP+X)
400 CONTINUE
CONTINUE
WRITE(1,260) INDEX:
260 FORMAT(' Flat last curve fit? Yindex= 'I3/
         ' Send plot to terminal or plotter or skip? (0/1/2)'
READ(TERM#) IDEV
IF ((IDEV .EQ. 0) .OR. (IDEV .EQ. 1)) THEN
    CALL FLT(XSLOT+ZSRL+NETS+X1+XER+TRUE...TRUE.)
    CALL FLT(XST+ZST+INDEX+XI+XER+FALSE...FALSE.)
END IF
UNTIL
IF (IDEV .NE. 2) GOTO 250
WRITE(Term,270)
FORMAT(' Continue on with next cut? (Y/N)')
READ(Term,290)Ans
FORMAT(A1)
C
EXIT
IF (Ans .EQ. 'Y') GOTO 40
STOP
END

SUBROUTINE INITY(Poly, Xi, Xe, Xs, Yi, Ys, grid, Curve, Xindex, Yindex,
Seed, Uv, U2y)

C
Initialise routine.
Global constants
INTEGER Order
PARAMETER (Order = 8)

C
Order of polynomial representing
roof profile.
C
INTEGER Poly
Order of approximating polynomials. Input.
C
DOUBLE PRECISION Xi, Xe
Start and end of X. Input.
C
DOUBLE PRECISION Xs
Step between stored points. Input.
C
DOUBLE PRECISION Yi
Start value of Y. Input.
C
DOUBLE PRECISION Ys
Step between stored points. Input.
C
INTEGER grid
Number of points in Y direction
required for prediction algorithm. Input.
C
C
DOUBLE PRECISION Curve(1:grid:0:Poly)
Coefficients of polynomial. Output.
C
INTEGER Xindex
Largest value of Xindex. Output.
C
INTEGER Yindex
Index for stored points. Output.
C
INTEGER Seed(1:4)
Seed for pseudo random sequence. Output.
C
DOUBLE PRECISION Uv
Control input for Y sherrer. Output.
C
DOUBLE PRECISION U2y
Control input for Y ruler. Output.
C
C
Local variables
DOUBLE PRECISION Xs, Ya
Actual value of X and Y.
C
INTEGER Xindex
Index for stored points.
C
DOUBLE PRECISION Ys(1:110)

- 160 -
C
DOUBLE PRECISION Zder(0:Order, 0:Order)
C Actual derivative of roof profile.
C First index is for X and 2nd for Y. (ex: Zder(0, 2) = 2nd partial
C derivative of Z with respect to Y.
C
DOUBLE PRECISION Zst(1:1110)
C Stored values of Za.
C
begin INITY
C initialize random number generator
Seed(1) = 40
Seed(2) = 178
Seed(3) = 246
Seed(4) = 59
C
Xindel = IIDINT((Xr - Xi)/Xs) + 1
C
Find first Ygrid curves using deterministic inputs.
Assume deterministic input.
Ya = Yi - Ys
DO 1 DO 200 Yindex = 1, Ygrid, 1
   Ya = Ya + Ys
   Xa = Xi - Xs
   DO 100 Yindex = 1, Xindel, 1
      Ya = Xa + Xs
   100 Zst(Yindex) = Xa
   Zst(Yindex) = POOF(Xa, Ya, false, Zder)
C
100 CONTINUE
CALL LINE(Xindel, Pnv, Xst, Zst, Coeff, Error)
C
DO 110 I = 0, Pnv, 1
   Curve(Yindex(I)) = Coeff(I)
110 CONTINUE
C
200 CONTINUE
C
Place Y in middle of and Yindex at start of last cut.
Yind = Yi - Ygrid - 1
Uv = Ya - (Ys/2.0)
C
Place Y for ruler at end of last cut
Uzu = Ya
RETURN
END
C

**********************************************************************
SUBROUTINE INITX(Incr, SD, Seed(1:4), Npts, PltMin, PltMax, Point, 
T, Xa, Xf, Yindx, Ya, Yf)

Parameters
*******************************************************************************
INTSER Incre
C Increment between points that are plotted. Input.
C
DOUBLE PRECISION SD
C INTSER Seed(1:4)
C
DOUBLE PRECISION Uv
C
DOUBLE PRECISION X1
C
INTEGER Npts
C
DOUBLE PRECISION PltMax, PltMin
C
INTEGER Point
C
Max and min values for a plot.
C
- 161 -
INTEGER T
DOUBLE PRECISION Xa
DOUBLE PRECISION Xf
INTEGER Index
DOUBLE PRECISION Ya
DOUBLE PRECISION Yf

local constants

DOUBLE PRECISION Dt
PARAMETER (Dt = 1.0)
DOUBLE PRECISION Ux
PARAMETER (Ux = 0.2)

begin INIT

Xa = X1
Ya = Ux
CALL MEASU((Xa - Dt*Ux)*SD,Seed,Xf)

Index = 0
T = 0
Ntx = 0
Point = Incr

FiltMin = 1.0E+30
FiltMax = -1.0E+30
RETURN
END

SUBROUTINE INIT(Seed,Ux,X1,Xa,Yf,Xindex,Yf)

integer Seed(1:14)
DOUBLE PRECISION Ux
DOUBLE PRECISION X1
INTEGER Index
DOUBLE PRECISION Ya
DOUBLE PRECISION Yf

local constants

Counter, Output.
Time counter, output.
Actual value of X, Output.
Initial condition for FILTX, Output.
Index for X, Output.
Actual value of Y, Output.
Initial condition for filtered value of Y for FILTY, Output.

Time interval between measurements (S).
Forward speed of shearer in X (M/S).

Standard Deviation in measurement, Input.
Seed for pseudo random sequence, Input.
Control input for ruler in Y direction, Input.
Initial position of X, Input.
Time counter, Output.
Actual position of X, Output.
Filtered value of X, Output.
X index for stored values, Output.
Actual value of Y, Output.
Filtered value of Y, Output.
DOUBLE PRECISION DT
PARAMETER (DT = 1.0)  ! Time interval between measurements (S).

DOUBLE PRECISION Ux
PARAMETER (UX = 0.2)  ! Forward speed of ruler in X (M/S).

C begin INITZ ******************************************
C Place Y at end of cut from middle of cut.
Yx = 0.0
Yy = Y2
CALL MEASUR(Yx - DT*UX), SD, Seed, Y2
CALL MEASUR(Yy, SD, Seed, Y2)

C Xindex = 0
T = 0
RETURN
END

******************************************************************************
SUBROUTINE SHEAR(SD, Seed, UX, Yx, Yy)
Model of shearer. The random input is assumed to have
2 times the standard deviation of the measurements.
******************************************************************************
DOUBLE PRECISION SD  ! Standard deviation in measurements. Input.
INTEGER Seed(1:4)  ! Seed for pseudo random sequence. Input.
DOUBLE PRECISION Ux  ! Control input for Y. Input.
DOUBLE PRECISION X2, Y2  ! Actual coordinates of shearer. Input/output.

C local constants ******************************************
DOUBLE PRECISION DT
PARAMETER (DT = 1.0)  ! Time interval between measurements (S).

DOUBLE PRECISION UX
PARAMETER (UX = 0.2)  ! Forward speed of shearer in X (M/S).

C local variables ******************************************
DOUBLE PRECISION Uncer
DOUBLE PRECISION Wx, Wy  ! Random component of input.

C begin SHEAR ******************************************
Wx = 0.0
DO 100 I = 1, 4, 1
   CALL MEASUR(0.0, SD, Seed, Uncer)
   Wx = Wx + Uncer
   CONTINUE
100  CONTINUE
   Wx = 0.0
   DO 200 I = 1, 4, 1
      CALL MEASUR(0.0, SD, Seed, Uncer)
      Wx = Wx + Uncer
   CONTINUE
200  CONTINUE
   Wx = Wx + DT*UX + DT*Wx
   Yx = Yx + Wx
   RETURN
END

******************************************************************************
SUBROUTINE RULER(SD, Seed, UX, Yx, Y2)
Model of ruler. The random input is assumed to have
******************************************************************************
2 times the standard deviation of the measurements.

**Parameters**

- **DOUBLE PRECISION SD**
  - Standard deviation in measurements.
  - Input.

- **INTEGER Seed(14)**
  - Seed for pseudo random sequence.
  - Input.

- **DOUBLE PRECISION Uzw**
  - Control input for Y. Input.

- **DOUBLE PRECISION Xa, Ya**
  - Actual coordinates of ruler.
  - Input/output.

**Local constants**

- **DOUBLE PRECISION DL**
  - Time interval between measurements (S).

- **DOUBLE PRECISION Ux**
  - Forward speed of ruler in X (M/S).

- **DOUBLE PRECISION Uncer**
  - Random component of input.

**Local variables**

- **DOUBLE PRECISION Wx, Wy**

```plaintext
begin RULER

Wx = 0.0
DO 100 I = 1, 4, 1
   CALL MEASURE(0.0, SD, Seed, Uncer)
   Wx = Wx + Uncer
100 CONTINUE

Wy = 0.0
DO 200 I = 1, 4, 1
   CALL MEASURE(0.0, SD, Seed, Uncer)
   Wy = Wy + Uncer
200 CONTINUE

Xa = Ya + Ds*Ux + Ds*Wx
Ya = Uzw + Wy
RETURN
END
```

**Logical function NODE**

- **Logic if at node.**

  **Parameters**

  - **DOUBLE PRECISION X**
    - Coordinate where we currently are. Input.

  - **INTEGER Xindex**
    - Index for stored points. Input.

  - **DOUBLE PRECISION Xs**
    - Initial value of X. Input.

  - **DOUBLE PRECISION Xs**
    - Step size between stored points. Input.

  **Local constant**

  - **DOUBLE PRECISION Limit**
    - Parameter of measurements. (M)

  - **DOUBLE PRECISION Limit**
    - Parameter of measurements. (M)

```plaintext
b = in NODE
   IF (Y .GE. (X1 + Xindex*Xs) - Limit) THEN
     NODE = .true.
   END IF
RETURN
END
```
SUBROUTINE FILTX(T,Xm, XF)
The steady state Kalman filter equation for SHEAR in Z direction. The following model is used,
Xr(k+1) = Xr(k) + Dts*Ux + Dts*W
Xr(k) = Xr(k) + U
where W is zero mean and has twice the standard deviation of V.
PARAMETERS
INTEGER T
DOUBLE PRECISION Xm          
DOUBLE PRECISION XF

Local constants

DOUBLE PRECISION DT            
PARAMETER (DT = 1.0)

DOUBLE PRECISION UX
PARAMETER (UX = 0.2)

Local variables

DOUBLE PRECISION K

begin FILTX
K = 2.0**DT**2 / (DX**2 + 1)**0.5 - DT
XF = XF + K*(Xm - XF - DX*UX) + Dtx*UX
RETURN
END

SUBROUTINE FILTY(Ux,Ym, YF)
The steady state Kalman filter equations for the shearer in the Y direction. The following model is used.
Ym(k+1) = Ym(k) + W(k)
Ym(k) = Ym(k) + U(k)
where W and U are zero mean and the standard deviation of W is twice that of V.
PARAMETERS
DOUBLE PRECISION Ux

DOUBLE PRECISION Ym

Local constants

REAL K
PARAMETER (K = 0.8)

begin FILTY
YF = YF + K*(Ym - YF) + Uy
RETURN
END

SUBROUTINE FILTZ(Ux, Ym, XF)
The steady state Kalman filter equation for KULER
in Z direction, the following model is used:
\[ Xa(k+1) = Xa(k) + Dt\#Ux + Dt\#Wx \]
where Wx is zero mean and has twice the standard deviation
of Ux.

**Parameters**: Time	counter. Input.

**DOUBLE PRECISION Xa**
The measured value of the quantity
to be estimated. Input.

**DOUBLE PRECISION XF**
Filtered value of X. Input/Output.
Must be saved between calls and
initialized on the first call.

**local constants**: Time	interval between measurements (S).

**DOUBLE PRECISION DT**
PARAMETER (DT = 1.0)

**DOUBLE PRECISION UX**
PARAMETER (UX = 0.2)

**local variables**: Forward speed of RULER in X (M/S).

**DOUBLE PRECISION K**
Steady state Kalman gain.

begin FILTZ
\[ k = 2.0*DT*(DT**2 + 1)**0.5 - DT \]
end

RETURN

SUBROUTINE FILTZ(U2x, Y, XF)
The steady state Kalman filter equations for the RULER
in the Y direction. The following model is used:
\[ Ya(k+1) = Uw(k) + W(k) \]
\[ Ym(k) = Ya(k) + U(k) \]
where W and U are zero mean and the standard deviation of W
is twice that of U.

**Parameters**: Control input for Y RULER. Input.

**DOUBLE PRECISION U2x**
The measured value of the quantity
to be estimated. Input.

**DOUBLE PRECISION Y**
Predicted value of Y. Output.

**local constants**: Kalman gain.

begin FILTZ
\[ Yf = k*Ym - U2x \]
RETURN

SUBROUTINE FILZ(Fn, X1, X2, Y, X, X2t, Y2t, X2t1, Y2t1, Curve, M, N, s, Rs, S, X1, A, err, CC, FA, FX, U1, Xtemp, AL, CC, C, D, U1, Y1)

Global variables: InGxr Tr
PARAMETER (Terror = 5)
parameters Unit number of terminal.  
INTEGER Poly Order of polynomial. Input.
DOUBLE PRECISION SD Standard Deviation in measurements. Input.
INTEGER Xindex Number of data points. Input.
DOUBLE PRECISION X(I:1) Independent variable. Input.
INTEGER Yind Number of curves required for extrapolation in Y. Input.
DOUBLE PRECISION Y(I:1) Dependent variable. Input.
DOUBLE PRECISION R Sum of square of residuals.
DOUBLE PRECISION Sx(I:1) Sy Standard deviation of independent and dependent variables. Input.
DOUBLE PRECISION X(I:1:1) Independent variables. Input.
INTEGER Error Indicates error condition.  
0 - successful fit of data  
1 - solution did not converge

The following parameters are included only to allow for dimensioning by the main program. They do not need to be initialized. They are actually local variables.

DOUBLE PRECISION A(I:Np) Unknown parameters.

DOUBLE PRECISION Accr(I:Np) Difference between guess and calculated values of unknown parameters A.

DOUBLE PRECISION C(I:Np) Coefficient matrix.

DOUBLE PRECISION Fa(I:Np) Partial derivative of F with respect to A's (the parameters).

DOUBLE PRECISION Fx(I:M) Partial derivative of F with respect to X's (the independent variables).

DOUBLE PRECISION V(I:Np) Solution matrix of C*A=W.

DOUBLE PRECISION Xtemp(I:M) One measured value of X's.

The following are for GINV.

DOUBLE PRECISION AF(I:Np)  
DOUBLE PRECISION CGC(I:Np)  
DOUBLE PRECISION CG(I:Np)  
DOUBLE PRECISION NF(I:Np)  
DOUBLE PRECISION NH(I:Np)  
DOUBLE PRECISION XX(I:Np)

begin  
FILTZ Swap coefficients  
END  
100 I = 0, Poly, 1  
Curve(1:1) = Curve(2:1)  
100 CONTINUE
C
Sx(1) = SD
Sw = SD
DO 102 I = 1, Xindex+1
X(i,1) = Xst(I)
102 CONTINUE
C
For initial guess of coefficients use previous curve.
DO 105 I = 0, Poly+1
A(I+1) = Curve(2*I)
105 CONTINUE
C
Find new curve.
CALL LEASOR(M,Xindex,N+Sx+Sx2+Xtump+AT,CCC,CC,F,XX)
DO 110 I = 0, Poly
Curve(Yrid+1) = A(I+1)
110 CONTINUE
C
RETURN
END
C
******************************************************************************
DOUBLE PRECISION FUNCTION PREDIC(Poly,Yrid,Curve,X,Y)
******************************************************************************
C
rev date by
C
0 9 July 84 JPL
1 10 July 84 JPL
2 23 July 84 JPL
C
C
First five terms kept.
First two terms kept.
Using linear extrapolation of 2
points obtained from approximate
curves.
Uses rev 0 of POLYNO

C
PARAMETERS
******************************************************************************
INTEGER Poly
C
Order of approximating polynomial. Input.
C
INTEGER Yrid
C
Number of curves required. Input.
C
DOUBLE PRECISION Curve(1:Yrid+1:Poly)
C
Coefficients of curves. Input.
C
DOUBLE PRECISION X
C
Location where Z is to be predicted. Input.
C
DOUBLE PRECISION Ys
C
Step size in Y direction. Input.
C
DOUBLE PRECISION A&B
C
Coefficients of linear equation.
C
DOUBLE PRECISION C(0:19)
C
Coefficients of curves.
C
DOUBLE PRECISION SumY,SumY2,SumZ,SumZ2
C
Temporary values.
C
DOUBLE PRECISION Z1, Z2
C
Z's at the two curves.
C
BEGIN PREDIC
******************************************************************************
Find predicted heights of two points of X.
DO 10 I = 0, Poly
C(I) = Curve(1+I)
10 CONTINUE
C
Z1 = POLYNO(C[Poly+X])
DO 20 I = 0, Poly
C(I) = Curve(2+I)
20 CONTINUE
C
Z2 = POLYNO(C[Poly+X])
C
Find equation of line.
\begin{verbatim}
SUMY = Ys
SUMY2 = Ys**2
SUMZ = Ys**2
SUMZ2 = Z1 + Z2
A = (2*SUMYZ - SUMY*SUMZ)/(2*SUMY2 - (SUMY**2))
B = (SUMZ2*SUMZ - SUMY*SUMYZ)/(2*SUMY2 - (SUMY**2))

C

C Predict Z in middle of cut
PREDIC = A*(3*Ys/2) + B
RETURN
END
\end{verbatim}
DOUBLE PRECISION FUNCTION ROOF(X,Y,Evader,Zder)

Local constants

INTEGER Order
PARAMETER (Order = 8)

Order of polynomial representing roof profile.

PARAMETERS

Global constants

LOGICAL Evader

True if derivatives are to be evaluated.

DOUBLE PRECISION Zder(0:Order,0:Order)
Actual derivative of roof profile.

First index is for X and 2nd for Y.

Y, Zder(0:2) = 2nd partial derivative of Z with respect to Y.

Constants

DOUBLE PRECISION A
PARAMETER (A = 0.25)

Squared average amplitude (M^2)

DOUBLE PRECISION Factor
PARAMETER (Factor = 1.0)

DOUBLE PRECISION Offset
PARAMETER (Offset = 1.3)

DOUBLE PRECISION Offset (M)

Local variables

DOUBLE PRECISION Xfit,Yfit

DOUBLE PRECISION Xder(0:Order),Yder(0:Order)

Derivatives of roof

BEGIN

ROOF

DOUBLE PRECISION FUNCTION FIT(X,Evader,Yder)

Polynomial fit of roof data. Units are in meters.

INTEGER Order
PARAMETER (Order = 8)

Order of polynomial representing roof profile.

PARAMETERS

DOUBLE PRECISION X

Location at which Z is to be determined. Input.
LOGICAL EvalDer
C True if derivatives are to be evaluated. Input.
C DOUBLE PRECISION Fder(0:Order)
C Array of derivatives evaluated at X. Fder(0)
C is the value of the function. Output.
C local variables
C DOUBLE PRECISION C(0:Order)
C Coefficients of polynomial
C INTEGER Last
C
BEGIN FIT
C c(0) = -3.53998
C c(1) = 1.277456
C c(2) = -1.862176E-01
C c(3) = 1.404704E-02
C c(4) = -5.943110E-04
C c(5) = 1.492880E-05
C c(6) = -2.200536E-07
C c(7) = 1.732355E-09
C c(8) = -5.815316E-12
C IF (EvalDer) THEN
C Last = Order
C ELSE
C Last = 0
C END IF
C DO 200 i = 0, Last, 1
C this selects the derivative
C Fder(i) = 0.0
C DO 100 J = I, Order, 1
C this selects the coefficient
C Fder(i) = SERIES(I, J) * (X**(J-I)) + Fder(I)
C 100 CONTINUE
C 200 CONTINUE
C FIT = Fder(0)
RETURN
END
C
C ************************************************************
C DOUBLE PRECISION FUNCTION SERIES(I, J)
C Calculates J-(I-1) \* - - \* J-I \* J.
C local variables
C INTEGER I+J
C DOUBLE PRECISION R
C
BEGIN SERIES
C R = 1.0
C IF ( I .NE. 0 ) THEN
C DO 100 K = J, (J-I+1), -1
C R = R*K
C 100 CONTINUE
C END IF
C SERIES = R
RETURN
END
SUBROUTINE LEASQR(M,N,NP,SX,SY,X,Y,A,R,SF,ERR,AERR)
  !
  FarFx=VxT=xAt+CCxCG+NxU+XX
  !
  General least squares analysis program as described in
  reference 1.

  REFERENCES
  1. Wolbers, John R.I 'Prediction Analysis', D. Van
     Nostrand, 1967
     MIT CALL: SCI OA276,W848

  Global constants

  **PARAMETER (TERM = 5)**

  INTEGER Term
  
  Parameters

  **PARAMETER (TERM = 5)**

  INTEGER Term
  
  Integer N
  Number of independent variables, input.

  INTEGER N
  Number of data points, input.

  Integer Np
  Number of data points, input.

  DOUBLE PRECISION SX(1:M),Sy
  Standard deviation of independent and dependent variables, input.

  DOUBLE PRECISION X(1:M,1:N)
  Independent variable, input.

  DOUBLE PRECISION Y(1:N)
  Dependent variable, input.

  DOUBLE PRECISION A(1:Np)
  Unknown parameters. Must be initialized with a guess of the
  values. Input/output.

  DOUBLE PRECISION R
  Sum of squared residuals, output.

  DOUBLE PRECISION SF
  Average standard deviation of interpolated value, output.

  INTEGER Error
  Indicates error condition,
  0 - successful fit of data
  1 - solution did not converge

  The following parameters are included only to allow for
  dimensioning by the main program. They do not need to be
  initialized. They are actually local variables.

  DOUBLE PRECISION AERR(1:M)
  Difference between guess and calculated
  values of unknown parameters A.

  DOUBLE PRECISION C(1:M,1:Np)
  Coefficient matrix.

  DOUBLE PRECISION Fa(1:Np)
  Partial derivative of F with respect
to A's (the parameters).

  DOUBLE PRECISION Fx(1:M)
  Partial derivative of F with respect
to X'n (the independent variables).

  DOUBLE PRECISION V(1:Np)
  Solution matrix of C&N+V.

  DOUBLE PRECISION Xt=1(1:M)
The following are for GINV.
DOUBLE PRECISION AT(1:Np)
DOUBLE PRECISION CC(1:Np)
DOUBLE PRECISION CG(1:Np)
DOUBLE PRECISION NF(1:Np)
DOUBLE PRECISION U(1:Np)
DOUBLE PRECISION XX(1:Np)

Local constants
DOUBLE PRECISION Limit
PARAMETER (Limit = 0.01)

INTEGER Itt, IttMax
PARAMETER (IttMax = 50)

Maximum number of iterations allowed.

INTEGER Itt
Number of iterations that have been performed.

DOUBLE PRECISION L
Intermediate quantity.

Rescale X vector
CALL Cond(Nx*Nv*Ax+Bx)
Sx(1) = Ax*Bx

Do Itt = 0 Loop

22 Continue

Clear matrix C and vector V
Do 20 I = 1, Np, 1
   V(I) = 0.0
   Do 10 J = 1, Np, 1
      CI(I,J) = 0.0
   10 Continue
20 Continue

Itt = Itt + 1
Do 70 I = 1, Nv, 1
   Do 25 J = 1, Np, 1
      Xtemp(J) = X(J,I)
25 Continue
   Call Sfunc(Ax*Nv*Ax+Bx,Y(I),F,Fx)
60 Continue

L = Sx
Do 30 J = 1, Np, 1
   L = L + (Sx(J)*Fx(J))**2
30 Continue

Do 50 J = 1, Np, 1
   Do 40 K = 1, Nv, 1
      CI(J,K) = CI(J,K) + F(J)*Fx(K)/L
40 Continue
50 Continue
DO 60 J = 1, Nf, 1
   V(J) = V(J) + Fs(J) * F/L
60 CONTINUE
70 CONTINUE

C C C
C Fill in C below diagonal
C DO 90 J = 1, Nf, 1
   DO 80 K = J, Nf, 1
      C(K,J) = C(J,K)
80 CONTINUE
90 CONTINUE

C C C
C debus
C DO 6 II = 1, Np
   WRITE(5,5)(C(II,JJ), JJ=1,Np)
5 FORMAT(' ',6(I11,1X))
6 CONTINUE
end debus

C C C
C will now be the its inverse,
C CALL GINV(C,Nf,Nf,U,X,AT,HF,CC,XX,0,CG)
C
DO 110 K = 1, Nf, 1
   Arr(K) = 0.0
 100 DO 110 J = 1, Nf, 1
   Arr(K) = Arr(K) + C(K,J) * U(J)
110 CONTINUE

II = 1
Conv = .true.
C LOOP
310 CONTINUE
   II = II + 1
   IF (DABS(Arr(II)/A(II)) .GE. Limit) THEN
      Conv = .false.
      END IF
EXIT
   IF (Conv) .OR. (II .GE. N) GOTO 320
310 CONTINUE
GOTO 310
C END LOOP

EXIT
   IF (Conv) .OR. (II .GE. IttMax) GOTO 200
C CALL ADJUST(Np,A,Arr)
C GOTO 22
200 CONTINUE
C END LOOP
C IF (II .GE. IttMax) THEN
   Error = 1
ELS
   Error = 0
C C
C Determine sum of squares of residuals and 
C average standard deviation of function.
C R = 0.0
   SS = 0.0
   DO 400 I = 1, Nf, 1
      Xtemp(I) = X(I,II)
      CALL SFUNC(A,M,Np,Xtemp,Y(I),F,Fs,Fx)
      R = R + (F/SS)**2
   DO 400 J = 1, Nf, 1
- 174 -
DO 440 K = 1, Np, 1
  SF = SF + Fa(J)**Fa(K)**C(J,K)
  CONTINUE
440  CONTINUE
       SF = DABS(SF*R/(N*(N - Np)))
       SF = SF**0.5
C
C  Transform parameters to original system.
C  CALL TRANS(A,B,Np,AP,A)
C  Sw(i) = Sw(i)/A
C
C  Reset independent variable back to original system
DO 500 I = 1, N, 1
  X(I,1) = (X(I,1) - Bx)/Ax
500  CONTINUE
END IF
RETURN
END

SUBROUTINE COND(N,X,A,B)
C
C  The independent data points are rescaled onto the interval
C  -1 <= X <= 1.
C
INTEGER M
  Number of independent variables. Input.
INTEGER N
  Number of data points. Input.
DOUBLE PRECISION X(1:M,1:*
  Independent variable. Input/Output.
DOUBLE PRECISION A,B
  Coefficients of linear transformation.
INTEGER I
  Counter.
DOUBLE PRECISION LargeX,SmallX
  Largest and smallest value of X.
C  Begin COND ****************************************************
C  Find smallest and largest value of X.
DO 100 I = 1, N, 1
  IF (I.EQ.1) THEN
    SmallX = X(I,1)
    LargeX = X(I,1)
  ELSE
    IF (X(I,1) .LT. SmallX) THEN
      SmallX = X(I,1)
    END IF
    IF (X(I,1) .GT. LargeX) THEN
      LargeX = X(I,1)
    END IF
  END IF
100 CONTINUE
C
C  Calculate coefficients of linear transformation.
C  A = 2.0/(LargeX - SmallX)
C  B = (LargeX + SmallX)/(SmallX - LargeX)
C
C  Rescale data points.
DO 200 I = 1, N, 1
  X(I,1) = A*X(I,1) + B
200 CONTINUE
RETURN
END
SUBROUTINE ADJUST(Np,NA,Arr)
The values of A are adjusted here.
Parameters
INTEGER Np
  Number of parameters, Input.
DOUBLE PRECISION A(1:Np)
  Calculated values of unknown parameters, Input.
DOUBLE PRECISION Arr(1:Np)
  Difference between guess and calculated values of unknown parameters A,
  Input.
Local constants
DOUBLE PRECISION G
  Acceleration factor.
Begin ADJUST
  DO 10 I = 1, Np, 1
     A(I) = A(I) - Arr(I)/G
  CONTINUE
RETURN
END

SUBROUTINE SFUNC(M,Np,X,Y,F,Fa,Fx)
The constraint function and its first derivatives are evaluated here. Largest permitted polynomial is of order 10.
Parameters
INTEGER Np
  Number of independent variables, Input.
DOUBLE PRECISION A(1:Np)
  Value of parameters. Input.
DOUBLE PRECISION X(1:M)
  Independent variables. Input.
DOUBLE PRECISION Y
  Dependent variable. Input.
DOUBLE PRECISION F
  Value of constraint equation. Output.
DOUBLE PRECISION Fa(1:Np)
  Partial derivative of F with respect to A's (the parameters). Output.
DOUBLE PRECISION Fx(1:M)
  Partial derivative of F with respect to X's (the independent variables), Output.
Local variables
DOUBLE PRECISION C(0:10)
  Coefficients of polynomial.
INTEGER Order
  Order of polynomial. Max is 10.
Begin SFUNC
  Transfer parameters to C
     ORDER = Np - 1
     DO 5 I = 0, ORDER, 1
        C(I) = A(I+1)
        CONTINUE
     F = Y - POLYN(C,ORDER,X(1))

Derivatives with respect to C's
DO 10 I = 1, Np, 1
  IF (1 .le. I) THEN
    Fax(I) = -1.0
  ELSE
    Fax(I) = -X(I)**(I-1)
  END IF
  CONTINUE
10

Derivatives with respect to X's
Fax(I) = - POLY(I,Order,X(I))
RETURN
END

SUBROUTINE TRANS(A,B,Ne,C)
Transforms coefficients back to original system.
Parameters
DOUBLE PRECISION A, B
  Coefficients of linear transformation. Input.
INTEGER Ne
  Number of parameters. Input.
DOUBLE PRECISION C(1:Ne)
  Initialized as the parameters of the transformed system. Output as parameters of original system. Input/output.
Local variables
DOUBLE PRECISION Coeff(0:12)
  Actual coefficients of polynomial. Largest polynomial is 12th order.
INTEGER I, J
  Counters.
INTEGER Order
  Order of polynomial.
DOUBLE PRECISION Temp
  Temporary value.

Order = Ne - 1
DO 100 I = 0, Order, 1
  Coeff(I) = 0.0
100 CONTINUE

DO 300 I = 0, Order, 1
  DO 200 J = 0, I, 1
    Temp = BINOM(I+J)*A**(I-J)*B**J*C(I+1)
    Coeff(I-J) = Coeff(I-J) + Temp
  200 CONTINUE
300 CONTINUE

DO 400 I = 1, Np, 1
  C(I) = Coeff(I-1)
400 CONTINUE
RETURN
END

DOUBLE PRECISION FUNCTION BINOM(I,J)
Calculates binomial.
Parameters
INTEGER I, J
  Input.
Local variables
INTEGER E
   Counter.
INTEGER Temp1, Temp2, Temp3
   Temporary values.
BEGIN BINOM
   IF ((I .GE. 0) .AND. (J .GE. 0)) .AND. ((I-J) .GE. 0) THEN
       Temp1 = 1
       DO 100 E = I, 1, -1
       Temp2 = Temp2*E
       100 CONTINUE
       Temp2 = 1
       DO 200 E = J, 1, -1
       Temp3 = Temp3*E
       200 CONTINUE
       BINOM = Temp1/(Temp2*Temp3)
   ELSE
       Error
       CONTINUE
   END IF
END

******************************************************************************
SUBROUTINE NHM(A(N,H),W,N,AT,HF,C,X,CO)
   DIMENSION A(1), AT(1), HF(1), C(1), X(1), CO(1)

DOUBLE PRECISION A(W,AT,C,X,CO,BOT,DOT1,DOT2,FAC,SAV
   DOUBLE PRECISION SQRT,EPS,TOL,Z,W,TH,HF
   THIS SUBROUTINE CALCULATES THE GENERALIZED INVERSE A(H,N) OF
   THE MATRIX A(H,N), N.H.E.N, AND STORES IT IN A(H,N) SPACE.
   SOLVES A.X=B EITHER DIRECTLY OR IN A LEAST SQUARES SENSE.
   N IS THE NUMBER OF ROWS IN A(H,N),
   H IS THE NUMBER OF COLUMNS IN A(H,N),
   U(N,N) IS THE BOOK-KEEPING MATRIX.
   IX IS TO BE SET BY THE USER DEPENDING UPON THE INTENDED USE OF THE
   ALGORITHM, SEE BELOW.
   IF(IX = 0) AT2 Computes A(N+H) OR X(H) OR BOTH RESPECTIVELY.
   AT,AT+HF,AT+NCX+CO ARE ALL SINGLY DIMENSIONED ARRAYS WHICH ARE
   APPROPRIATELY DIMENSIONED IN THE CALLING PROGRAM.
   THE COEFFICIENT ARRAY IS OF DIMENSION H*N AND IS ENTERED ROW BY
   ROW IN THE MAIN PROGRAM.
   U THE BOOK-KEEPING ARRAY IS OF DIMENSION N.N.
   AT,AT+HF, AND CO ARE INCLUDED AS FORMAL PARAMETERS ONLY TO PERMIT
   THEIR DIMENSIONS TO BE SET IN THE MAIN PROGRAM. AT AND HF ARE OF
   DIMENSION H-1 AND CO IS OF DIMENSION N.
   THF VECTOR OF CONSTANT COEFFICIENTS IS OF DIMENSION H AND IS
   SUPPLIED BY THE MAIN PROGRAM. IF IX IS LESS THAN 1, C NEED NOT BE
   GIVEN BY THE MAIN PROGRAM AND MAY BE OF DIMENSION 1.
   THE VECTOR OF UNKNOWNS X IS OF DIMENSION H, BUT MAY BE OF
   DIMENSION 1, ONLY, IF IX IS LESS THAN 1.
   W = 1.0
   Z = 0.0
   TH = 200.0
   KR = 0
   DEPENDENT COLUMN TOLERANCE FOR NB BIT FLOATING FRACTION.
   NB = 55
EPS = NFB**2(NB+1)
TOL = T.H**EPS
TOL = TOL/TOL
DO 40 J = 1,N
JM1 = J-1
DO1 = DOT(A+M*N+J,J)
NFi(J) = I
DO 5 I = 1,N
IJ = I*J+N+J
C ENTER LATEST LINEARIZED A(IJ) VALUES.
C
5 U(IJ) = Z
JJ = J*N+N+J
U(JJ) = W
IF(JM1.EQ.0) GO TO 50
C
GRAM-SCHMIDT ORTHOGONALIZATION WITH CORRECTION IF REQUIRED .
C FAC = (EPS*SQRT(DO1)/FLOA(M)
LL = 0
20 LL = LL+1.
LT = 1
DO 30 K = 1,JM1
30 AT(K) = DOT(A,M*N+J,K)
DO 45 K = 1,JM1
IF(ABSM(AT(K)),G,T,FAC) LT = LL-3
DO 35 I = 1,N
IJ = I*N+N+J
IK = I+J-K
IF(NF(K).NE.0) A(IJ) = A(IJ) - AT(K)*A(IK)
IF(I.LT.EQ.K) U(IJ) = U(IJ) -AT(K)*U(IK)
35 CONTINUE
45 CONTINUE
IF(LT.LE.0) GO TO 20
50 DOT2 = DOT(U,M*N+J,K)
IF(ABSM(DOT2),LT,1.0E-37) GO TO 70
IF(DOT2/DOT1.GT.TOL) GO TO 70
C MODIFIED TREATMENT WHERE COLUMN IS NOT EFFECTIVELY INDEPENDENT.
C
DO 60 K = 1,JM1
60 AT(K) = DOT(U,K,M+N+J,K)
DO 65 I = 1,M
IJ = I*N+N+J
DO 65 K = 1,JM1
IF(NF(K).EQ.0) GO TO 65
IF(ABSM(AT(K)).LE.EPS) GO TO 65
IK = I+H-K
A(IJ) = A(IJ) - AT(K)*A(IK)
65 CONTINUE
NF(J) = 0
FAC = DOT(U+J,N+J,J)
FAC = W/SQRTM(FAC)
GO TO 75
C KR GIVES THE TOTAL OF INDEPENDENT COLUMNS, I.E. COLUMN RANK AT END
C
70 FAC = W/SQRTM(DOT2)
KR = KR+1
C COLUMN NORMALIZED.
C
75 DO 85 I = 1,N
```
IJ = IIN-N+J
A(IJ) = FAC#A(IJ)
IF(IJ.LE.I) U(IJ) = FAC#U(IJ)
85 CONTINUE
IF(IJ.LT.1) GO TO 90
FAC = Z
DO 88 I = 1,N
IJ = IIN-N+J
C
88 FAC = FAC + C(I)*A(IJ)
C0(J) = FAC
90 CONTINUE
IF(KR.LT.N) WRITE(S,800) KR,N
IF(KR.LT.N) WRITE(S,800) KR,N
800 FORMAT(1HO,10X,BHMATRIX A,BH(M,N) IS,BH OF RANK,I4,5H WITH,I3,
1 N IN COLUMNS,1H,///,1H )
IF(M.eq.N) GO TO 92
GO TO 100
92 SAV = W,
DO 95 I = 1,N
II = IIN-N+J
95 SAV = SAV#U(II)
SAV = W#SAV
WRITE(S,225) SAV
WRITE(S,225) SAV
825 FORMAT(1HO,10X,BHVALUE OF,BH DETERMI,BH NANT IS ,E16.8,///,1H )
100 IF(IJ.LT.0) RETURN
IF(IJ.EQ.0) GO TO 120
DO 115 J = 1,N
FAC = Z
DO 110 K = J,N
JK = JIN-N+K
105 FAC = FAC + C(K)*U(JK)
110 X(J) = FAC
115 CONTINUE
DO 120 J = 1,N
DO 130 I = 1,N
IJ = IIN-N+J
FAC = Z
DO 125 K = J,N
IK = IJ+K-J
JK = JIN-N+K
125 FAC = FAC + A(IK)*U(JK)
130 A(IJ) = FAC
CALL HXTRAP(A+H,N)
RETURN
END
C
SUBROUTINE HXTRAP(A,H,N)
DIMENSION A(H,H)
DOUBLE PRECISION A,S,AU
C
RETURNS THE TRANSPOSE OF MATRIX A(H,N) STORED ROW BY ROW IN A
C
SINGLE ROW ARRAY A AS A'(N,H) STORED SIMILARLY IN ARRAY A.
M = M+1
MN = H-N
MN+1 = MN-1
K = 0
I1 = 1
DO 25 I = 1,MN
IF(K.GT.MNN) GO TO 25
K = K+1
25...
```

J = K
GO TO 10
5 K = K-MHN
J = K
10 IF(J.GE.II) GO TO 15
   JM = J8N
   J = JM-MHN*(JM/MN)
   GO TO 10
15 IF(J.EQ.II) GO TO 20
   J = J+1
   SAV = A(I)
   A(I) = A(J)
   A(J) = SAV
20 II = I
25 CONTINUE
RETURN
END

FUNCTION DOT(A,M+N+J,K)
DIMENSION A(I)
DOUBLE PRECISION DS
DOUBLE PRECISION ADOT

COMPUTES THE INNER PRODUCT OF COLUMNS J AND K IN DOUBLE PRECISION
AND RETURNS THE VALUE AS FUNCTION DOT IN SINGLE PRECISION.

DS = 0.0
DO 5 I = 1,M
   J = I*M+J
   IK = I+K-J
   DS = DS + A(I)*A(IK)
5 CONTINUE
DOT = DS
RETURN
END

FUNCTION SQRTM(X)
DOUBLE PRECISION XSQRTM
S QRTM = DSQRT(X)
RETURN
END

FUNCTION ABSM(X)
DOUBLE PRECISION X,ABSM
ABSM = DABS(X)
RETURN
END

SUBROUTINE MXMLT(A,B,M,L,N,R)
DIMENSION A(I),B(I),R(K)
DOUBLE PRECISION A,AB
CALL IS FOR MXMLT(A,B,M+L,N,R)
MULTIPLY A(M,L) BY B(L,N) AND STORE RESULT OF UDBER (M,N) IN R.
R(1) = 0.0
R(2) = 0.0
CALL MLT(A,B,M,L,N,R)
RETURN
END

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SUBROUTINE MLT(A, B, H, L, N, R)
DIMENSION A(1), B(1), R(2)
DOUBLE PRECISION A, B, R
K1 = IFIX(R(1))
K2 = IFIX(R(2))
KML = K1 + (M - L)
KLH = K2 + (L - N)
DO 130 I = 1, M
   DO 140 J = 1, N
      KR = (I - 1) * N + J
      R(KR) = 0.0
   140 CONTINUE
   130 CONTINUE
RETURN
END

C

FUNCTION IFIX(X)
DOUBLE PRECISION X
IFIX = IFIX(SNGL(X))
RETURN
END
APPENDIX D

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