DESIGN PROCEDURES FOR SUBMARINE-DEPLOYABLE
BISTATIC SONAR SYSTEMS

by

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(1981)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF

MASTERS OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1984

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Philippe Porter Dondey

Submitted to the department of Ocean Engineering on 21 May 1984 in partial fulfillment of the requirements for the degree of Master of Science in Ocean Engineering

ABSTRACT

Bistatic submarine-deployable sonar systems consist of a small underwater sound source and a typically larger submarine which receives the active sonar signal. In this context, the object of this thesis is to develop a computer-based procedure allowing the exploration of a variety of issues in the design of the bistatic system.

To approach a more precise specification of the design procedure, the bistatic sonar equation is introduced and each of its terms is analyzed and illustrated. Possible interrelationships between the main system parameters are shown, and the distinction between particular and standard parameters is drawn; a particular parameter is the input of only one term of the sonar equation, whereas a standard parameter is used in more than one term.

The design procedure is intended to solve any possible design problem specified by the user. Solvability of the sonar equation for each term, and of each term for its particular parameters, are thus considered. Cases where solution of the sonar equation is not relevant arise. For instance, there is no sense in trying to solve for the reverberation strength when the background is dominated by ambient noise rather than reverberation. Standard parameters cannot be solved for directly, and an input characteristic is defined between the specified
and the unknown: scanned parameters, associated with graphic possibilities, allow the designer to study their effect on any other (solvable) system parameter. As a result, a practical input format is developed where the user defines inputs as specified, scanned or unknown.

In other respects, the structure of the design procedure is based on the bistatic sonar equation and is kept modular. It is the frame in which the subsequent system designer should provide his/her own prediction models for each term of the sonar equation. Applications in this thesis using very approximate models illustrate what type of problem can be specified, and the corresponding graphical possibilities. The results obtained show a reasonable qualitative behavior, if not verifiable.

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ACKNOWLEDGMENTS

I would like to express my sincere gratitude to Ira Dyer, my thesis advisor, for his unfailing and patient guidance in the progress of this thesis. His advice in the definition of the scope of my research helped me to make my way through the broad implications of this subject. I consider myself very fortunate to have worked with him.

I am equally indebted to John Purze for his friendly guidance and supervision and for arranging my financial support in the Draper Fellows program. He is responsible for making my stay at CSDL a rewarding and enjoyable experience I shall not forget soon.

Thanks are also due to Steven Sifferlen and James Veale for their constant assistance in the specification and the implementation of the computer procedure. I am truly grateful for the dedicated help of James Veale, who developed the graphical output software. I also want to thank John Polcari who provided me with his original ray-tracing program.

Finally I would like to thank the Technical Publications Department who most kindly and patiently typed the manuscript and produced an expertly edited thesis.

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CHAPTER 1

INTRODUCTION

1.1 Physical Description

This thesis presents a frame for the design of a bistatic sonar system. References are made to abundant literature in underwater acoustics research, mainly the Journal of the Acoustical Society of America, as well as handbooks for engineers, among which are R. J. Urick's Principles of Underwater Sound [1].

Most active sonars, naval or commercial, are monostatic, i.e., the source and the receiver are located at the same place. But all the design rules and traditional "trade-offs" and compromises between performance and achievable values do not apply or must be adapted to the particular and relatively unexplored subject of bistatic sonar configuration, where the source and receiver are separated. A sketch of this configuration is given in Figure 1-1.

![Diagram of bistatic sonar configuration]

**Figure 1-1. Bistatic configuration.**
The basic assumptions on which the following work is based are:

- The capabilities of the sonar are detection and estimation of range, bearing, and speed.
- The ranges $R_1$ and $R_2$ are large with respect to the signal wavelength, the source array, receive array, and target dimension, and, eventually, the depth of the ocean.
- Source and receiver are both submarines navigating in a pre-planned coordinated manner.
- The source array is omnidirectional in the horizontal plane.
- The ocean is typically the deep ocean and may be ice-covered.

1.2 Design Parameters

The assumptions stated above do not translate directly into system parameters; rather, several models based on the sonar equation as developed in Chapter 2 should allow one to determine the effect of environmental assumptions and system parameters on the performance of the system.

The environment descriptors are:

- The sound velocity profile and ocean depth, and its dependence on range when a range dependent transmission loss model is used (see Section 2.4).
- The surface and bottom roughness to allow the computation of reflection loss and reverberation. Ice-covered oceans lead to specialized transmission loss, reverberation, and noise models.
- The noise generating environment: wind, distant shipping density, biological activity, etc.

The system parameters are:

- The geometrical configuration: the relative ranges $R_1$ and $R_2$, the respective depths of source target and receiver, the angle defining the target aspect as viewed from the receiver, the bistatic angle, $\psi$.  

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- The central frequency $f$, the signal bandwidth $W$, the source acoustic power $P$, the receiver beam pattern $B^2(\theta, \phi)$.

These parameters should allow calculation of a signal-to-noise ratio for further design of the signal processor. The receiver bandwidth is needed to compute the post-array noise level (see Section 2.5), since available noise data are given in a spectral value, usually in a 1 Hz band.

1.3 **Objectives**

The objective of this thesis is to build the framework for a computer-based design procedure. The basic assumption stated above implies the basis of the design procedure, namely, the bistatic sonar equation. The algorithm of the procedure will also have to provide sufficient flexibility to allow subsequent users to address a variety of issues in the system design.

A principal characteristic of this procedure is that it shall be as independent as possible of the specific data and/or prediction model provided by the system designer. The only constraint is an agreement on the definitions of the terms of the sonar equation, and on the system parameters that interrelate these terms.

In Chapter 2, the sonar equation is presented for the bistatic case, and its various terms are analyzed, with reference given to existing models in the nonclassified literature.

In Chapter 3, a design procedure is presented, which allows the insertion of any relevant computer model. First-order models are used to give an example of signal to noise ratio calculations as a function of frequency and geometrical configuration.
CHAPTER 2

THE SONAR EQUATION AND ITS MODELING

2.1 Sonar Equation for the Bistatic Case

2.1.1 Introduction

The sonar equation is a formal way of linking the system and environmental parameters with the purpose of either performance prediction or system design. The basic procedure of system design is to assess all but one of the terms in the sonar equation, to solve for that term, and hence to determine the corresponding system parameters. The various terms of the sonar equation represent how the sound signal intensity is affected as it travels from the source to the receiver, via the target. They are expressed in decibels with reference to the mean square pressure of a one micro-Pascal wave (dB re 1 μPa). Following loosely the notation conventions in Urick [1], they are as follows:

Sonar System Terms:
- Projector Source Level: SL
- Receiving Array Gain: AG
- Self Noise Level: NLS
- Detection Threshold: DT

Environment Terms:
- Source-Target Transmission Loss: TL₁
- Target Receiver Transmission Loss: TL₂
- Reverberation Level: RL
- Ambient Noise Level: NLA

Target Dependent Terms:
- Target Strength: TS

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Referring to Figure 2-1, we can lay down the sonar equation, looking backwards at the process of sound transmission.

![Diagram](image)

Figure 2-1. Illustration of the sonar equation.

The detection threshold \( DT \) is the signal-to-noise ratio when the sonar function is just performed. It is understood that functions other than detection, e.g., target range, bearing or velocity estimations, and, moreover, identification or classification require a higher signal to noise ratio, and the distinction is deferred to Section 2.8, with the convention that \( DT \) is to be replaced by a parameter estimation threshold (PET), when necessary.

The signal-to-noise ratio \( SNR \) is the ratio between the echo level and the background level at the terminals of the receiver array. The echo level is the source level minus both transmission losses plus the target strength. The background level is the unwanted part of the signal at the receiver, and is the incoherent sum of the post array noise level and the reverberation level. In turn, the post array noise level \( NLD \) is the incoherent sum of the self noise and the ambient noise levels (see Section 2.5)
Hence the signal to noise ratio, in decibels

\[ \text{SNR} = \text{SL} - \text{TL}_1 - \text{TL}_2 + \text{TS} - \text{BG}, \quad \text{dB} \]

with

\[ \text{BG} = \frac{\text{NLD} + \text{RL}}{10} = 10 \log \left( 10^{\frac{\text{NLD}}{10}} + 10^{\frac{\text{RL}}{10}} \right), \text{dB re} \, 1 \mu \text{Pa} \]

and

\[ \text{NLD} = \text{NLS} + \text{NLA}, \quad \text{dB re} \, 1 \mu \text{Pa} \]

In the next section, a transient form of the sonar equation is introduced, and issues of time and frequency characteristics of the signal are explored. The following sections present an analysis of each of the sonar equation terms SL, TL, NLD, TS, RL, and DT.

2.1.2 Purpose of a Modeling Analysis

This analysis is intended to list the main effects of system parameters on each term, and to suggest the degree of complexity required by possible models if some degree of precision is to be respected. It is not a description of the state-of-the-art in sonar design, although references are made to existing prediction models. All the equations stated are to be considered as illustrations rather than exact references for the system designer.

It should be noted here that the reliability of the results depends directly on the models provided by the system designer. The best models possible will be experimental measurements, done in conditions as close as possible to the operational conditions, with equipment equivalent to the specified sonar, or parts thereof.

For instance, if the receiver part of the system is an already existing monostatic sonar, the best model for the post array noise level is field measurements; if part of the receiver signal processor is used, its structure will imply a relevant model for the detection and parameter estimation thresholds.
2.2 The Ocean as a Communication Channel

2.2.1 Transient Form of the Sonar Equation

The above equations have been written in terms of intensity, i.e., of the time average of the acoustic power per unit area. They apply for continuous waves or long pulse signals when medium interactions such as multipath time spreading and fluctuation remain negligible.

For active short pulse sonar the echo duration $\tau_e$ is increased over the emitted pulse duration $\tau_o$ by the time spreading in the channel $\tau_m$ and an additional duration $\tau_t$ imposed by the extension in range of the target. Strictly $\tau_e$ can be found by convolving each of the time-spread functions, but often only one or two dominate.

When the averaging window $T$ of the receiver signal processor does not take the time spread $L$ into account, then a correction term $10 \log \frac{T}{\tau_e}$ should be added to the source level to represent an equivalent pulse, longer and weaker, in a non-time stretching medium (see Section 2.3). It may alternatively be considered as an additional term in transmission loss (see Section 2.4).

2.2.2 Fluctuation of the Sound Signal

Other features in the sound propagation impose limits on the precision with which time and frequency characteristics of the signal can be measured. W. S. Burdick [2], mentions the thermal microstructure in the top layers of the ocean as a principal cause of inhomogeneities of the sound speed and subsequently of the sound fluctuation. S. M. Flatté devotes an entire book [3] to the effects of fluctuating inhomogeneities of sound speed in the volume of the ocean, including thermal finestuctures and microstructures, eddies, and internal waves.

Other sources of fluctuation are the scattering from the moving sea surface, modeled as a random filter in Fortuin [4]. Another form of frequency spreading is due to the relative motions of the source target and receiver (see Section 2.2.5).
2.2.3 The Ocean Channel as a Communication Filter

A very appealing description of the ocean channel is introduced in Knight [5]. Because the governing equations of sound propagation are linear, the ocean channel may be modeled as a linear filter

\[ h(t, \tau, \vec{r}, \vec{r}_s) \] relating the observed field \( p(t, \vec{r}) \) to the input \( g(t, \vec{r}) \) by

\[ p(t, \vec{r}) = \int h(t, \tau, \vec{r}, \vec{r}_s) \ g(t - \tau, \vec{r}_s) \ d\tau \]

\( h(t, \tau, \vec{r}, \vec{r}_s) \) accounts for the effects on the sound signal stated previously; namely, the time spread \( L \) which is the duration of the filter's impulse response, the frequency spread \( B \) in hertz, which results in the time domain in a time varying frequency modulation of the sinusoidal carrier.

In a doubly spread channel, to avoid frequency selective fading, a signal should have its bandwidth \( W \) less than \( L^{-1} \) Hz. On the other hand, to avoid temporal fading, the signal duration \( T \) must be much less than \( B^{-1} \) seconds.

2.2.4 Time Spread

An analysis of the time spread due to multipath propagation is carried in Smith [6]. The depth-averaged impulse response of a channel is estimated by a theory based on ray acoustics, assuming lossy specular reflection from the boundaries (see Section 2.4 for the introduction of ray theory and Appendix A for an example of ray tracing program). Transmission losses are computed based on angular spreading, and the results are presented in Figure 2-2. The pulse duration \( \tau \) turns out to be a function of the source position relative to the sound speed profile, as is shown further in the simple case of the isogradient channel. The rays included in the front pulse have followed various refracted-surface reflected (RSR) paths. The reverberant tail includes bottom reflected rays, and has a rate of decay which is a function of the volumetric and reflection absorption coefficients.
Figure 2-2. Theoretical averaged impulse response.

With a typical sound speed profile as in Figure 2-3, RSR paths going deep into the ocean are faster than the totally refracted or ducted paths which remain in the vicinity of the channel axis where the sound speed is minimum.

Figure 2-3. A typical sound speed profile.

The pulse duration $\tau$ will not be affected if the sound speed profile is substituted by a linear profile with the same source and bottom speeds $C_s$ and $C_b$, the distinction between RSR and bottom reflected paths depending only on $C_s$ and $C_b$. 17
In that case the pulse duration is, from [6]

\[ \tau = R^2 \frac{\theta}{6} C_s \]

where \( \theta \) is the limiting angle \( \cos \theta = \frac{C_s}{C_b} \). The time spread is thus proportional to the range. A numerical example is carried in Appendix A, and confirms the above approximation.

2.2.5 Frequency Spread

Frequency or Doppler spread can be associated with the shorter of the decorrelation times of relative motions of the source, target and receiver, \( \tau_m \), and of internal waves, \( \tau_i \).

If \( \ell \) is the horizontal correlation scale of the medium and \( \bar{v} \) is the mean source-target-receiver (horizontal) motion, then

\[ \tau_m = \frac{\ell}{\bar{v}} \]

In Smith and Stern [7], an expression for the horizontal correlation range is given

\[ \ell = \frac{52 \times 10^3 \lambda}{\Delta} \]

where \( \Delta \) is the depth excess, as illustrated in Figure 2-3.

Models on the effect of internal waves on the decorrelation time \( \tau_i \) are discussed in Psaraftis [8]. An expression of \( \tau_i \), fitted to an experiment presented in Dyson [9] could be

\[ \tau_i \approx \frac{1}{2\bar{v}} \left( \frac{406}{f} \right) \left( \frac{300}{R} \right)^{1/2} \]

where

\[ R = \text{range in km} \]
\[ f = \text{frequency in Hz} \]
\[ \bar{v} = \text{root mean square single path phase rate (} 3.1 \times 10^{-3} \text{ Hz from Porter [10])} \]
2.3 **Source Level**

The source level is the mean square pressure of the sound wave leaving the source, on axis in the case of a directional source, measured in decibels with reference to a 1 µPa plane wave, at 1 meter from the source acoustic center. It is related to the acoustic power by the relation

\[ SL = L_p + DI + 171 \text{ dB re 1 µPa, 1 m} \]

where

\[ L_p = 10 \log P = \text{acoustic power level (dB re 1W)} \]

\[ DI = \text{projector directivity index (dB)} \]

The directivity index is the main design parameter for the source. DI results from the integration of the source beam pattern \( B^2(\theta, \phi) \) which weights the radiated sound off the specified radiation axis

\[ DI = -10 \log \frac{1}{4\pi} \int B^2(\theta, \phi) \cos \theta \, d\theta \, d\phi \]

where \( \theta \) is the depression angle, measured from the horizontal, and \( \phi \) the azimuth angle. Since \( B^2 \) is less than 1 off axis, DI is a positive number, ranging from 0 dB for omnidirectional sources to typical values of 30 dB for large projectors. DI depends mainly on the ratio of the array size to the sound wavelength. Directivity indices of simple transducers are listed in [1], Table 3.2. The directivity index may also be expressed in terms of the equivalent solid angle \( \Omega_e \)

\[ DI = -10 \log \frac{\Omega_e}{4\pi} \]

When the source has a rectangular beam pattern, \( \Omega_e \) is a function of the horizontal equivalent angle \( \phi_e \) and the vertical equivalent angle \( \theta_e \)
\[ \Omega_e = \phi_e \int_{-\theta_e/2}^{\theta_e/2} \cos \theta \, d\theta = 2\phi_e \sin \frac{\theta_e}{2} \]

The cost of increasing both dimensions of the source array and hence of the source itself has to be compared to the advantages of a directional source. Clearly, the bistatic configuration doesn't require a horizontally directional source. Assuming also that the electrical power devoted to the projector array is only a fraction of that needed for propulsion, power will be more efficiently used if a small high-level projector is powered by a small navigation-efficient submarine than if a larger projector requiring less electrical power to perform the same SL, has to be carried by a bigger, more power consuming submarine.

This trade-off between array size, acoustic power, and overall propulsion power may also be formulated with respect to the vertical dimension and vertical beamwidth of the array.

Except for size considerations the vertical beamwidth \( \theta_e \) of the source need not be larger than the long-range limiting angle \( \theta' \) determined by the source and bottom sound speeds \( C_s \) and \( C_b \)

\[ \theta_e \leq 2\theta' \]

where

\[ \cos \theta' = \frac{C_s}{C_b} \]

2.4 Transmission Loss

2.4.1 Introduction

Signal transmission loss is among the most comprehensively studied fields of underwater acoustics. Moreover, the application to the bistatic system is straightforward: the total transmission loss, \( TL \), is the sum of the transmission loss from the source to the target, \( TL_1 \), and from the target to the receiver, \( TL_2 \)

\[ TL = TL_1 + TL_2 \]
From here on, any standard model of transmission loss, based on all available assumptions on the operating conditions, may be used. After a brief introduction to the principal features of sound propagation and transmission loss, several analytical methods and computational models will be referenced. The particular case of under-ice sound propagation is considered. Finally a computational model based on standard graphs is introduced. A summary of transmission loss models can be found in Urick [11], Chapter 3.

2.4.2 Basic Assumptions

The source-target and target-receiver ranges $R_1$ and $R_2$ should be allowed to vary between 20 miles and 200 miles. The ocean in which the sonar would operate will be typically deep, between 1000 and 5000 meters, although source, target, and receiver depths are relatively smaller, between 0 and 500 meters. These assumptions imply long range, deep ocean propagation modeling.

The operating frequency of the sonar, between 500 and 5000 Hz, allows ray theory to give a good qualitative description of the sound propagation, except in the neighborhood of caustics and convergence zones (see further, ray theory and Appendix A).

2.4.3 Principal Features of Transmission Loss

As introduced in Urick [1], Chapter 5, transmission loss, which is a measure of the loss in mean square pressure between a point at one unit distance from the source and a point at a distance in the sea, may be considered to be the sum of a loss due to spreading and a loss due to attenuation.

Spreading loss may be spherical, when the model is of a point source in a homogeneous, unbounded, lossless medium. From conservation of power across concentric spheres

$$P = \int \hat{I} d \hat{s} = 4\pi r^2 \hat{I}_1 = 4\pi r^2 \hat{I}$$
the transmission loss is

$$\text{TL} = 10 \log \frac{I_1}{I} = 20 \log r \text{ dB re } 1 \text{ m}$$

when \( r_1 = 1 \) meter, and \( r \) is measured in meters.

In long range propagation, if the ocean is assumed lossless and bounded by a horizontal and perfectly reflecting surface and bottom, the spreading loss is cylindrical

$$P = 2\pi r_1 I_1 \int \cos \theta \, dz = 2\pi r I \int \cos \theta \, dz$$

where \( \theta \) is the ray grazing angle, of which the integral over depth is constant in an isovelocity medium. This yields

$$\text{TL} = 10 \log r \text{ dB re } 1 \text{ m}$$

The following table summarizes various spreading laws.

Table 2-1. Spreading laws (from Urick [1]).

<table>
<thead>
<tr>
<th>Type</th>
<th>Intensity Varies As</th>
<th>Transmission Loss, dB re 1 m</th>
<th>Propagation In</th>
</tr>
</thead>
<tbody>
<tr>
<td>No spreading*</td>
<td>( r^0 )</td>
<td>0</td>
<td>Tube</td>
</tr>
<tr>
<td>Cylindrical</td>
<td>( r^{-1} )</td>
<td>10 log ( r )</td>
<td>long-range sound channel</td>
</tr>
<tr>
<td>Spherical</td>
<td>( r^{-2} )</td>
<td>20 log ( r )</td>
<td>free field</td>
</tr>
</tbody>
</table>

*Hypothetical for sonar.

In the long-range propagation of a short sonar waveform, multi-path propagation causes the signal to be stretched in time proportionally to the distance travelled (see Section 2.2.4). The mean square pressure falls then again as the inverse square of the range instead of as the inverse first power.
Volumetric absorption is due to the conversion of acoustic energy into heat; its law is exponential with range since the rate of change of the intensity with range is proportional to the intensity

\[ \frac{dI}{dx} = -nI \]

Hence the volumetric absorption

\[ VA = 10 \log \frac{I_1}{I_2} = \alpha(r_2 - r_1) \]

where \( \alpha \) is usually expressed in dB/km, and has been extensively studied, as shown in Urick [11], Chapter 5. A semi-empirical model presented in Fisher [12] shows that \( \alpha \) varies roughly as the square of the frequency, and is, to a lesser extent, a function of temperature, pressure, and salinity. In the ice-covered Arctic, because of surface scattering, \( \alpha \) varies as the first power of the frequency (see Figure 2-4).

Surface and bottom attenuation occur when the reflection of the sound on the surface and the bottom is not perfect. The surface attenuation, due to scattering, and the bottom attenuation, due to refraction into the bottom and scattering, may be predicted and measured in decibels per bounce.

2.4.4 Ray Theory

The wave equation relating the pressure or the velocity potential to time and space coordinates is

\[ \nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0 \]

It describes the propagation of sound waves in a continuous medium. The harmonic time separation yields the Helmholtz equation
Figure 2-4. Volumetric absorption as a function of frequency. From Urick [12].
\[ \nabla^2 \phi + k^2(\hat{r}) \phi = 0 \]

with \( \phi(r, t) = \phi(\hat{r}, \omega) e^{-i\omega t} \) for a monochromatic wave.

Ray theory describes wave fronts, in a medium which is slowly varying in time and space

\[ \frac{1}{c} |\nabla \phi| \ll |k(\hat{r})| \]

where \( k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \) is the wave number.

Separating the amplitude and the phase in the plane wave velocity potential

\[ \phi = Ae^{iS} \]

leads to the Eikonal equation

\[ |\nabla S|^2 = k^2(\hat{r}) \]

The most practical application of the equation is Snell's Law. Assume that \( k \) is locally a function of depth \( z \) only. Introducing the inclination, \( \theta \), and the path, \( s \), of a single ray, as in the following figure.

\[ \frac{dx}{ds} = \cos \theta, \frac{dz}{ds} = \sin \theta \]
one can separate range and depth dependence in the Eikonal equation

\[ \frac{\partial s}{\partial x} = k \frac{\partial x}{\partial s}, \quad \frac{\partial s}{\partial z} = k \frac{\partial z}{\partial s} \]

differentiating the first equation with respect to \( s \)

\[ \frac{d}{ds} \left( \frac{\partial s}{\partial x} \right) = \frac{\partial}{\partial x} \left( \frac{ds}{ds} \right) = \frac{\partial}{\partial x} k(z) = 0 \]

on the other hand

\[ \frac{d}{ds} \left( k \frac{\partial x}{\partial s} \right) = \frac{\partial}{\partial s} (k \cos \theta) \]

The result that \( \frac{c}{\cos \theta} \) is a constant along any one ray, called the ray parameter, is one application of Snell's Law and is used in all ray tracing programs to find the shape of ray paths, given a continuous sound velocity profile \( c(z) \). See Appendix A for an example of a simple ray tracing program.

Ray theory programs can also handle horizontal variation in the velocity profile or the water depth relatively easily. Snell's law is also applied, but \( \theta \) becomes the angle of the ray with the plane normal to the sound speed gradient. Ray theory is exact within the limit of short wavelength or high frequency. This limitation excludes intensity calculations in the shadow zones or near caustics (a caustic is the envelope of the intersection of adjacent rays in a ray diagram). Propagation models use modified ray theory to calculate the transmission loss in the proximity of shadow zones and caustics (see Blatstein [13]).

2.4.5 Normal Mode Theory

In a horizontally stratified medium, with an omnidirectional source of unit amplitude located at \( r_0 = 0, z_0 \) in cylindrical coordinates, the Helmholtz equation becomes
\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial z^2} + k_n^2(z) \phi = -2 \frac{\delta(r)}{r} \delta(z - z_0) \]

The potential can be expanded in terms of eigenfunctions

\[ \phi = \sum_n u_n^*(z_0) u_n(z) R_n(r) \]

where the normal modes satisfy the equations

\[ \frac{d^2 u}{dz^2} r(z) + \left[ k_n^2(z) - k_n^2 \right] u_n(z) = 0 \]

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial R_n(r)}{\partial r} \right] + k_n^2 R_n(r) = -2 \frac{\delta(r)}{r} \]

According to the boundary conditions on \( \phi \), Bessel and Hankel transforms are used. An indication on the convergence of the eigenfunction expansion is obtained by considering an isovelocity ocean bounded by a pressure release surface and rigid bottom. The boundary conditions are

\[ \phi = 0 \quad \text{at} \quad z = 0 \]

\[ \frac{\partial \phi}{\partial z} = 0 \quad \text{at} \quad z = h \]

The radial components are then Hankel functions

\[ R_n(r) = i \pi H_n^{(1)}(k_n r) \]

\[ = \sqrt{\frac{2}{\pi k_n r}} e^{i(k_n r - \pi/4)} \quad \text{at large ranges} \]

and the vertical components are

\[ u_n(z) = \sqrt{\frac{2}{h}} \sin \gamma_n z \]

with

\[ \gamma_n = \frac{(n - 1/2) \pi}{h} \]
define the vertical phase velocity

\[ c_n = \frac{\omega}{k_n} = \frac{k c}{(k^2 - \gamma_n^2)^{1/2}} \]

Propagating modes are limited by requiring the square root to remain real

\[ n < n_{\text{max}} = \frac{2k}{\lambda} + \frac{1}{2} \]

For a 100 Hz signal in 75 meters of water, 10 modes are enough, but theoretically, 2000 modes are needed to describe a 1 kHz signal in 1500 meters of water. Normal mode theory is hence best suited for low frequency and shallow waters. Nevertheless, it is exact and is used in some long range, deep ocean models (DiNapoli [14]).

2.4.6 Summary

Most propagation models are based on modified ray theory (to take caustics and convergence zones correctly into account), or modified normal modes (to reduce the amount of computation). As introduced in Appendix A, a simple ray tracing program based on Snell's law is enough to give some insight into the propagation modes, and to compute travel times.

Transmission loss tables as a function of range and frequency could be considered as an input to the design package, as the result of a separate transmission loss model, where the user has specified other environmental parameters (sound speed profile, bottom reflection, source and receiver depths).

In Appendix B, a way to input these tables is proposed, and is a set of curves for various environmental situations, is presented for sake of illustration.

2.5 Noise Level

2.5.1 Noise Rejection

The term NLD in the sonar equation

\[ DT = SL - TL + TS - NLD \quad (2.5-1) \]
represents the post-array noise level, in decibels with reference to 1 micro pascal. The noise is the part of the overall sound signal arriving at the receiver, which the receiving array is designed to minimize through its spatial selectivity. Noise rejection considerations also impact another area of sonar design, namely, signal waveform design.

The center frequency and the bandwidth of the active sound waveform are two parameters essential to the design of the signal processor, but also dictate what part of the noise spectra affects the receiving array. However, when the noise spectra are sufficiently smooth within the signal bandwidth—which is typically true for narrow bandwidth ranging signals—then the bandwidth calculation of noise levels is simplified. If the noise level spectrum level is $S_N(f)$, then the mean square pressure received in a bandwidth $W$ around a center frequency $f_0$ is

$$P_N = \frac{1}{W} \int_{f_0-W/2}^{f_0+W/2} S_N(f) \, df$$

(2.5-2)

When $S_N(f)$ is flat, one may approximate this integral by

$$P_N = W S_N(f_0)$$

(2.5-3)

Specification of the bandwidth, which is needed at that point, may be postponed until later, by setting here $W = 1$ Hertz, thus calculating the post array noise spectral density.
2.5.2 Array Gain

In the spatial domain, detection and ranging echoes typically occupy a sector much smaller than the incoming noise. The array gain is a measure of how well the array rejects noise.

The overall noise spectrum level is that measured by an omnidirectional receiver. It is an average of the directional noise spectrum level

\[
S_{N}(f_0) = \frac{1}{4\pi} \int S(f_0, \Omega) \, d\Omega \tag{2.5-4}
\]

where \( \Omega \) is the solid angle: \( d\Omega = \cos \theta \, d\theta \, d\phi \).

A directional array receives the signal \( S_{ND}(f_0) \) in a bandwidth \( W \)

\[
S_{ND}(f_0) = \frac{W}{4\pi} \int S(f_0, \Omega) \, B^{2}(\Omega) \, d\Omega \tag{2.5-5}
\]

where \( B^{2} \) is the beam pattern of the array.

The array gain is defined by

\[
AG = 10 \log_{10} \frac{\int S(f_0, \Omega) \, d\Omega}{\int S(f_0, \Omega) \, B^{2}(\Omega) \, d\Omega} \tag{2.5-6}
\]

In decibels, the post array noise level in a bandwidth \( W \) is thus related to the omnidirectionally measured spectrum level by

\[
NLD = NL + 10 \log W - AG \quad \text{dB re } 1 \mu Pa \tag{2.5-7}
\]

where

\[
NL = 10 \log S_N \quad \text{dB re } 1 \mu Pa, 1 \text{ Hz}
\]

\[
NLD = 10 \log S_{ND} \quad \text{dB re } 1 \mu Pa
\]
In the case of isotropic noise, the array gain has a simpler expression

$$AG = 10 \log \frac{S_f d\Omega}{S_f B^2(\Omega) d\Omega} = 10 \log \frac{4\pi}{|B^2(\Omega) d\Omega|} = DI$$

(2.5-8)

where DI is the directivity index.

In practice, noise is not isotropic. Its various sources, as illustrated in Figure 2-5, include principally the vehicle carrying the receiving sonar (self noise) and the natural or man-made phenomena generating sound at a distance in the sea and surrounding the receiver (ambient noise). These two kinds of noise add incoherently

$$NLD = NLS \oplus NLA = 10 \log_{10} \left( \frac{NLS}{10} + \frac{NLA}{10} \right)$$

(2.5-9)

Figure 2-5. Interrelationships of various sonar noise sources (Ref. [16]).
2.5.3 Self Noise

The post-array self noise NLS is a determining parameter in sonar design. It is minimized by a silencing of the self noise sources: machinery and propellers, flow around the sonar dome. The array gain is increased outside the frequencies and directions of the self noise, for instance, by inserting a baffle between the sonar and the noise sources, and by steering the receiver physically or electronically towards preferable directions. Self noise is introduced and discussed in Urick [1], Chapter 11. A more extensive physical analysis is carried in Ross [15].

If the submarine carrying the receiver is known, then the best models for NLS is experimental measurements done at various speeds and depths. Actually, efforts toward submarine quietness will bring NLS several decibels below the ambient noise. A reasonable procedure consists therefore in neglecting NLS with respect to NLA.

2.5.4 Ambient Noise

The ambient noise itself is not isotropic (Axelrod [17]). Its two main components correspond to the surface noise, NLSS, coming from ray angles maximized in the vertical, and, in the deep ocean situations typically considered, the shipping noise, NLSH, which comes from distant sources along the long range travel paths, coming from ray angles maximized at or near the horizontal. Again these two types of noise add in levels

\[
NLA = NLSS \oplus NLSH \quad \text{dB re } 1 \mu Pa, 1 \text{ Hz} \quad (2.5-10)
\]

2.5.5 Shipping Noise

Shipping noise fills the entire long-range propagation solid angle, \( \Omega_n \). The array gain for the shipping noise is recalling Eq. (2.5-6)

\[
AG = 10 \log_{10} \frac{S_{NSH}}{S_{NDSH}}
\]

where \( L \) has been omitted for ease of notation.
The horizontal distribution of the shipping noise may be assessed, given the receiver's position with respect to dense shipping routes (Wenz [18] and Dyer [19]). For the purpose of this study, the shipping noise is assumed horizontally isotropic, and

\[
S_{\text{NSH}} = \frac{1}{4\pi} \int S_{\text{SH}}(f', \Omega) \, d\Omega
\]

\[
= \frac{1}{2} S_{\text{SHO}}(f_0) \int_{-\pi/2}^{\pi/2} S_{\text{SH}}(\theta) \cos \theta \, d\theta \quad (2.5-12)
\]

where

\[
S_{\text{SHO}}(f_0) = \text{omnidirectionally measured shipping noise}
\]

\[
S_{\text{SH}}(\theta) = \text{shipping noise vertical directionality, assumed independent of frequency}
\]

Omnidirectional shipping and surface noise spectra are proposed in Figure 2-6.

![Figure 2-6. Average deep water ambient noise spectra \( S_{\text{SHO}} \) and \( S_{\text{SSO}} \) from Urick [1].](image)
The array gain, $AG$, may be found by numerical integration of the beam pattern and the vertical directionality of the shipping noise.

$$AG = 10 \log \frac{\frac{1}{2} \int_{-\pi/2}^{\pi/2} S_{SH}(\theta) \cos \theta \, d\theta}{\frac{1}{4\pi} \int_{-\pi/2}^{\pi/2} S_{SH}(\theta) \int_{0}^{2\pi} B^2(\theta, \phi) \cos \theta \, d\phi \, d\theta} \quad (2.5-13)$$

Defining the array equivalent azimuth angle $\phi_e$ by

$$\phi_e B^2(\theta) = \int_{0}^{2\pi} B^2(\theta, \phi) \, d\phi \quad (2.5-14)$$

the array gain becomes

$$AG = 10 \log \frac{2\pi \int_{-\pi/2}^{\pi/2} S_{SH}(\theta) \cos \theta \, d\theta}{\phi_e \int_{-\pi/2}^{\pi/2} S_{SH}(\theta) B^2(\theta) \cos \theta \, d\theta} \quad (2.5-15)$$

In Axelrod [17], measured noise directionality may be found, although the shipping noise has not been isolated from the other ambient noise sources. A statistical model based on propagation between randomly positioned dipole-like sources near the surface (the ships) and the receiver could provide results approximated by Figure 2-7.

A further simplification consists in replacing the integrals by equivalent angles, given the relative directions of the incoming noise solid angle $\Omega_{SH}$ and of the "listening" receiver solid angle $\Omega_e$. Assume $\Omega_e$ is steered in the horizontal direction, and is included in $\Omega_{SH}$. The "co-pointed" array gain is then

$$AG = 10 \log \frac{\Omega_{SH}}{\Omega_e} \quad (2.5-16)$$

with

$$\Omega_{SH} = \Omega_n = 2\pi \int_{-\theta'}^{\theta'} \cos \theta \, d\theta = 4\pi \sin \theta' \quad (2.5-17)$$
Figure 2-7. Estimated ambient noise density in the vertical plane, for abyssal waters from Dyer [20].
where \( \theta' \) is the angle limiting the long-range propagation. The post-array shipping noise level is then

\[
NLSH = 10 \log S_{\text{SHO}}(f_0) + 10 \log W - 10 \log \frac{4\pi \sin \theta'}{\Omega e} \tag{2.5-18}
\]

2.5.6 Surface Noise

The surface noise, also assumed horizontally isotropic, comes into the array at angles distributed around the vertical. The corresponding array gain could be integrated, knowing the vertical beam pattern \( B^2(\theta) \), on the basis of surface noise vertical directionalities as shown in Figure 2-7

\[
AG = 10 \log \frac{\int S_{SS}(\Omega) \, d\Omega}{\int S_{SS}(\Omega)B^2(\Omega) \, d\Omega} = 10 \log \frac{2\pi \int_{-\pi/2}^{\pi/2} S_{SS}(\theta) \cos \theta \, d\theta}{\Phi e^{-\pi/2} \int_{-\pi/2}^{\pi/2} S_{SS}(\theta)B^2(\theta) \cos \theta \, d\theta} \tag{2.5-19}
\]

Assuming again the array steered in the horizontal, the equivalent angle approximation is more complicated since \( S_{SS}(\theta) \) and \( B^2(\theta) \) do not reach their maximum values simultaneously ("contra-pointed" situation). The two components of the integral at the denominator of \( AG \) are:

1. Within the noise solid angle, \( \Omega_{SS} \), where the array beam has an average level of \( B_n^2 \), is called side lobe level.

2. Within the array main lobe, \( \Omega_e \), where the noise has an average level of \( S_a \)

\[
\int S_{SS}(\Omega)B^2(\Omega) \, d\Omega = S_a \Omega_e + S_n B_n^2 \Omega_{SS} \tag{2.5-20}
\]

and

\[
AG = 10 \log \frac{S_0 \Omega_{SS}}{S_a \Omega_e + S_n B_n^2 \Omega_{SS}} \tag{2.5-21}
\]

These results are from Dyer [20].
The post-array surface noise level is then

\[ NLSS = 10 \log S_{SS}(f_0) + 10 \log W - AG \]

with the spectral level \( S_{SS}(f_0) \) estimated from an average spectrum (Figure 2-6) or provided by measurement and/or a computer noise model as referenced in Urick [11].

In the ice-covered ocean, the surface noise frequency and angular spectra depend strongly on the type of ice cover, the temperature rising or dropping, the wind speed. As explained in Urick [1] and illustrated in Figure 2-8, under-ice noise prediction, as for sea surface noise, will then rely on probabilistic models of atmospheric and oceanographic conditions.

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![Image of Figure 2-8](image_url)

**Figure 2-8.** Spectra of ambient noise observed under ice (see Urick [1], page 201, for references).
2.6 Target Strength

The target strength measures how well the target acts as a secondary source when struck by the sonar pulse. Its value is

\[ TS = 10 \log \frac{I_r}{I_i} \text{ db re } 1 \text{ m} \quad (2.6.1) \]

where

- \( I_r \) = intensity of reradiated sound at unit distance from the target acoustic center, in the direction of the receiver, and
- \( I_i \) = intensity of incident wave from the distant source

In the bistatic case, target strength is thus a function of both the incident direction and the direction of the receiver. Monostatic, or back-scattering strengths have been calculated for fixed, rigid bodies of simple shape (Urick [1], page 274), as well as for a large class of more complicated bodies.

Define, following Clay and Medwin [21], the scattering function \( \mathcal{S} \) as the dimensionless angular and frequency dependence of the reradiated sound

\[ 4\pi I_r = I_i \mathcal{S} (\theta, \phi, \theta_i, \phi_i, f) A(\theta_i, \phi_i) \quad (2.6.2) \]

where \( \theta, \phi, \theta_i, \phi_i \) are the directions of the radiated and incident sound, respectively; \( A \) is the cross section of the scatterer viewed from the source, \( f \) is the frequency, and \( I_r \) is measured at 1 m from the target.

The product \( \mathcal{S} \cdot A \) is called the differential scattering cross section and is designated \( \sigma_d \). If the target is assumed as an omnidirectional secondary source, the reradiated intensity \( I_r \) is independent of angle and integration of \( \sigma_d \) over all angles yields the total scattering cross section, which is the ratio of the total scattered power to the incident intensity.
\[ \sigma_t = \frac{\int I_x \, dS}{I_i} = \int_0^{4\pi} \frac{I_x}{I_i} \, d\Omega = \sigma_d \]  
\hspace{1cm} (2.6.3)

When the source is not omnidirectional, referring to Dyer [20], the bistatic target strength may be expressed in terms of \( \sigma_t \) and an angle dependent term

\[ TS = 10 \log \frac{\sigma_d}{4\pi} = 10 \log \frac{\sigma_t}{4\pi} + 10 \log \frac{B_t^2}{d_t}, \text{ dB re } 1 \text{ m} \]  
\hspace{1cm} (2.6.4)

where \( B_t^2 \) is the beam pattern of the target considered as a secondary source, in the direction of the receiver, and \( d_t \) is the target directivity factor defined by

\[ d_t = \frac{1}{4\pi} \int B_t^2 \, d\Omega \]  
\hspace{1cm} (2.6.5)

The angle dependent term may be expressed in terms of the scattering function

\[ \frac{B_t^2}{d_t} = \frac{4\pi \cdot I_x}{\sigma_t \cdot I_i} = \frac{\mathcal{P}_A}{\sigma_t} \]  
\hspace{1cm} (2.6.5)

The scattering function \( \mathcal{P} \) depends mainly on the size of the obstacle with respect to the wavelength, on the ratios of its elasticity and density to the medium elasticity and density (see Rayleigh [22] and Anderson [23]), and on details of the structural design.

In Varadan [24], sound scattering by rigid and elastic obstacles is compared, with the main conclusions that elasticity has a significant effect when the wavelength is on the order of or smaller than the target's largest half dimension, e.g., for a 50 m submarine, at frequencies larger than 100 Hz. The bistatic angle dependence can be accounted for by resonance phenomena which are current subjects of research.

For rigid bodies, the scattering may be explained on the basis of creeping waves and is less sensitive to the bistatic angle.

A method of estimating the bistatic target strength when the monostatic value is known, is presented in Ruck [25], and mentioned by
Urick [1], page 274, as the *bistatic theorem*. Referring to Figure 2-9, the bistatic target strength of a large smooth object is the same as the monostatic value in a direction OP along the bisector of the bistatic angle SOR, as long as this angle is much less than 180 degrees (forward scattering).

![Bistatic Geometry Diagram]

**Figure 2-9.** The bistatic geometry.

As mentioned in Urick [1], the lack of experimental data should induce caution; for instance, in the monostatic case, approximating a submarine by a rigid prolate spheroid is valid in the beam aspect (TS = 24 dB re 1 m typically) but is wrong in the bow aspect, where more complex structural features come into account.

The design procedure remains open to any bistatic target strength model, whether approximate (based on simple rigid shapes within the sound pulse) and based on the bistatic theorem, or experimental, or more theoretical, as in [24]. In any case, the actual size and orientation of the target is not known in advance. The bistatic strength is typically of the order of the monostatic strength, and several values, within the range of possible values (5 to 25 dB re 1 m) must be considered for input in the absence of more precise data.
2.7 Reverberation Level

As stated by L. M. Harvey in [26], the term most affected by the bistatic geometry is the reverberation level. Reverberation is the unwanted signal reradiated by inhomogeneities within the volume and on the boundaries of the sea illuminated by the sonar pulse.

In the modeling of volume reverberation as well as surface reverberation, the bistatic angle comes into effect in the scattering cross section of the scatterers and in the evaluation of the domain (surface or volume) enclosing the scatterers.

2.7.1 Volume Reverberation

Recall the target strength expression for one scatterer, Eq. (2.6.4) is

\[ TS = 10 \log \left( \frac{\sigma_d}{4\pi} \right) \text{ dB re } 1 \text{ m} \]  \hspace{1cm} (2.7.1)

where \( \sigma_d \) is the differential scattering cross section.

Assume a volume density \( n_v \) of scatterers, and that the volume illuminated by one ping is \( V \). The total number of scatterers interfering with the target is

\[ N = n_v V \]  \hspace{1cm} (2.7.2)

Assume that the scatterers are isotropic, have the same differential scattering cross section and are independent. Individual target strengths may then be added incoherently and yield an equivalent volume target strength.

\[ T_v = 10 \log \left( \frac{N n_v \sigma_d}{4\pi} \right) \]

\[ = 10 \log \left( \frac{n_v \sigma_d}{4\pi} \right) + \log V \text{ dB re } 1 \text{ m} \]  \hspace{1cm} (2.7.3)

where \( n_v \) is measured in \( m^{-3} \), \( \sigma_d \) in \( m^2 \) and \( V \) in \( m^3 \).
The differences between the bistatic scattering section $\sigma$ and the more extensively studied monostatic backscattering cross section $\sigma_{bs}$ are mentioned in Section 2.6. The variability of $\sigma_{bs}$ with depth and time is linked input with the diurnal migrations of the main deep scattering layer (DSL), as described in Clay and Medwin [21], page 236. The frequency dependence is also shown to be a function of the type of scatterer encountered. In Figure 2-10, the peak at about 3.5 kHz for the night data probably represents the resonance frequency of the swim bladder of a prominent species of fish.

![Graph showing column scattering strength as a function of frequency and depth](image)

**Figure 2-10.** Column scattering strength as a function of frequency and depth, giving three different integration depths $d$ for day and for night. Measurements at 36°41'N, 65°00'W (from Chapman [27]).

The bistatic reverberation volume is computed in Appendix D.
2.7.2 Surface Reverberation

Defining in the same manner a reverberation surface, $S$, and a surface density, $n_s$, the total number $N$ of surface scatterers distributed on either the sea surface or the bottom is

$$N = n_s S$$

The surface $S$ is the intersection of the reverberation volume introduced above, by the surface bearing the scatterers (see Appendix D). The equivalent surface target strength is, assuming the incoherent summation of the echoes of scatterers of same scattering cross section $\sigma_d$

$$T_s = 10 \log \frac{n_s \sigma_d S}{2\pi} = 10 \log \frac{n_s \sigma_d}{2\pi} + 10 \log S \text{ dB re } 1 \text{ m} \quad (2.7.5)$$

Here $\sigma_d$ is very strongly dependent on the coherence of the rough surface, through the Rayleigh parameter $R = 2k a \sin \theta$, where $k$ is the wave number, $a$ is the rms surface amplitude, and $\theta$ the grazing angle. Theoretical studies of surface scattering are abundant, as surveyed in Fortuin [28], with some experimental results, for the monostatic case. A solution of the scattering of a CW sound at a time-dependent, randomly rough surface is developed in Clay and Medwin [21], page 230 and Appendix A10.

2.7.3 Treatment of Reverberation Levels

The reverberation level $RL$ is the level at the receiver of the scattered part of the sound waveform. The surface and volume reverberation arriving independently at the receiver, the equivalent surface and volume target strengths add incoherently.

$$RL = SL - TL_1 - TL_2 + \{T_s \oplus T_v\} \text{ dB re } 1 \text{ mPa}$$

The presence of the transmission loss explains why reverberation models are included with propagation models. Several monostatic models are listed in Urick [11], page 3.5. Bistatic prediction models are developed in Seegal [29] and Shale [30].
Although in the case of reverberation-limited conditions, the signal to background ratio has a simple expression

$$\text{SNR} = \text{TS} - (\text{T}_s \oplus \text{T}_v) \quad \text{dB re 1 \, \mu Pa}$$

RL must be calculated independently for comparison with the post-array noise level NLD. The range dependence of RL is due to the transmissison loss and the scattering volume and surface terms, as analyzed by W. S. Burdick [2], page 372.

A design rule based on the reverberation limitation is proposed by Urick [1], page 327 for the monostatic sonar. The source level should be increased until the reverberation level is equal to a level of the noise background at the maximum useful range of the system.

Several ways to free the signal from reverberation, such as Doppler shifting and linear FM slide are introduced in W. C. Knight [5].

2.8 Detection Threshold

To perform its function the sonar signal processor needs a minimum signal to noise ratio called detection threshold (DT) for detection. For other functions such as range, bearing, and speed estimation, a parameter estimation threshold (PET) can also be defined.

DT and PET can be computed, as developed in Burdick [2], Chapter 13, if assumptions are made on the characteristics of the arriving waveform, on the statistics of the noise and on the structure of the receiver. DT is a function of the required probability of false alarm and the probability of detection. PET depends on the required variance of the parameter estimate.

The design of the waveform at the source and the receiver signal processor and the characteristics of the propagation medium all influence the computation of DT's and PET's. The bandwidth W and the pulse duration T define, given the medium time spread L and Doppler spread B, and the signal diversity are key parameters (see Section 2.2 and further developments in Knight [5]).
For instance in the case of an unknown signal in a background of Gaussian noise, in an underspread channel, following Urick [1], page 350, the detection index of the receiver operating characteristic (ROC) curves is

\[ d = WT \left( \frac{S}{N} \right)^2 \]

where

\[ S = \text{signal power in bandwidth } W \]
\[ N = \text{noise power in bandwidth } W \]

Solving for the required signal to noise ratio \( S/N \) and using the 1 Hertz band noise power \( N_0 \), we obtain

\[ \frac{S}{N_0} = \frac{SW}{N} = (\frac{dW}{T})^{1/2} \]

and the corresponding detection threshold is

\[ DT_{Hz} = 10 \log \frac{S}{N_0}, \text{ dB re 1 Hz} \]

Since \( S/N_0 \) is used extensively in the DT and PET formulations, the option of setting \( W = 1 \) Hz in the ambient noise computations (Section 2.5), is appropriate.
CHAPTER 3

A FLEXIBLE DESIGN PROCEDURE

3.1 Introduction

A computer program is developed to support the sonar design procedure. Models for each term of the sonar equation are linked by a main routine which is designed to be as independent as possible of the models used. A sketch of the central routine called SE (for sonar equation) and the diverse satellite models is given in Figure 3-1.

![Diagram](image)

Figure 3-1. Conceptual sketch for a sonar design procedure.
Another issue of flexibility concerns the form of the outputs which shall be as much as possible, the choice of the user. The program is implemented in FORTRAN 77 and executable on a TI 990/12 minicomputer. This also imposes a limit on the memory size, which includes the executable code and the data storage. With regard to the form of the inputs the user is given the choice between an interactive run where all parameters are prompted at the terminal, and a "batch" run where the inputs are read from an input file. In Section 3.2, the inputs of each model are listed and a distinction is made between particular and standard (or shared) model inputs.

In Section 3.3, the solution of the sonar equation for various system parameters is investigated. Section 3.4 proposes a set of first-order models. Section 3.5 presents the logical structure of the central routine SE. Section 3.6 provides a few applications to illustrate the use of the procedure.

3.2 Model Inputs

Each term of the sonar equation depends on several system design parameters. The system parameters which are inputs to more than one term will be called standard inputs. Parameters which do not interact with other terms are called particular inputs.

Following is a brief list of various model inputs for each term of the sonar equation.

- Source level:

\[ SL = 10 \log P + DI + 171 \quad \text{dB re } 1 \mu Pa, 1 \text{ m} \]

The acoustic power (expressed in watts) is a particular input. The directivity index should depend on frequency; however, it can be assumed that the source design will adapt the array size to the frequency in order to produce a given directivity index DI. In that case DI is a particular input.

- Transmission loss:

\[ TL = TL_1 + TL_2 \quad \text{dB re } 1 \text{ m} \]
The ranges \( R_1 \) and \( R_2 \) are both standard inputs (they also affect RL, DT). The frequency \( f \) is also a standard input (it affects most, if not all, the terms of the sonar equation). The sound speed profile could be considered as affecting NLD through the distant shipping noise propagation mode. For simplicity we assume it is particular to the transmission loss model. The same consideration goes into the source, target and receiver depths which will be assumed particular.

- **Target strength**

\[
TS = 10 \log A(\theta_i, \phi_i) \mathcal{S}(\theta, \phi, \theta_i, \phi_i, f) \cdot \frac{1}{4\pi}
\]

When the bistatic configuration is constrained in a horizontal plane

\[
TS = 10 \log A(\phi_i) \mathcal{S}(\phi, \phi_i, f) \cdot \frac{1}{4\pi}
\]

The aspect angles \( \phi_i, \phi \) relative to the source and the receiver depend on the ranges \( R_1, R_2 \), on the source - receiver baseline \( R_b \), and on the relative bearing \( \phi_t \) of the target (see Figure 3-2). With a proper model for the bistatic scattering function and for the target cross section, a list of standard inputs is hence: \( R_1, R_2, R_b \). The relative bearing \( \phi_t \) is a particular input. The frequency also affects the scattering function.

\( \phi \) and \( \phi_i \) may be found from the law of cosines

\[
R_1^2 = R_2^2 + R_b^2 + 2R_2R_b \cos (\phi_t + \phi)
\]

\[
R_2^2 = R_1^2 + R_b^2 - 2R_1R_b \cos (\phi_t + \phi_i)
\]
Figure 3-2. The bistatic configuration.

A range of values for $\phi_t$ and $A(\phi_i)$ as well as reasonable margins on the accuracy of the model for the scattering function could lead to specifying TS directly. TS then becomes a particular parameter by itself (see Section 2.6).

• Post Array Noise Level

$$\text{NLD} = \text{NLS} \Theta (\text{NL} + 10 \log W - AG) \Theta (\text{NL} + 10 \log W - AG)_{\text{Surf}}$$

Frequency $f$ is needed in the omnidirectional spectral densities $\text{NL}_s$ and $\text{NL}_{\text{Surf}}$ as well as in the array gains, through the wavelength dependency. Self noise also could depend on frequency and bandwidth, but it is assumed as an independent parameter. The bandwidth $W$ is a standard input (but may be set to 1 Hz here, see Section 2.5). Shipping density and sea state are assumed particular, although the sea state affects surface losses ($\alpha_g$) and surface scattering (RL). The array gains depend on frequency, array size and beam forming, these last inputs being particular.
• Reverberation Level

\[ RL = SL - TL + \left( 10 \log \frac{n_v \sigma_V}{4\pi} + 10 \log \frac{n_s \sigma_s}{2\pi} \right) \text{ dB re } 1 \mu Pa \]

SL and TL are used in RL, but we consider here only the last term, called reverberation strength RS, expressed in decibels. The range and angles are inputs to the reverberation volume and surface calculations, as well as the pulse length T. The scatterers densities \(n_v, n_s\) and cross sections \(\sigma\) depend on environmental parameters which could be standard (e.g., sea state) or particular (e.g., thickness of the Deep Scattering Layer).

• Detection Threshold (DT)—Parameter Estimation Threshold (PET)

The detection threshold is a function of bandwidth \(W\), pulse length \(T\), Doppler spread \(B\) (which depends on frequency), the time spread \(L\) (which depends on maximum range, associated with the baseline range \(R_b\)). The required probabilities of detection and false alarm, \(P_D\) and \(P_{Fa}\), are particular inputs for DT. Likewise, the required variances \(\sigma_t^2, \sigma_f^2, \sigma_\psi^2\) of the target parameters time delay, frequency shift and bearing, are particular inputs for the PET.

Table 3-1 summarizes these interdependencies. The list of inputs in this table is not exhaustive. For instance relative motions of the source, target and receiver, including a characterization of the propulsion/navigation capabilities of the source are important in regard of the decorrelation time \(\tau_m\) associated with these motions (see Section 2.2.5), and the corresponding effect on Doppler spread and velocity estimation capabilities.
Table 3-1. Interdependencies between inputs of the sonar equation models. All the intermediary variables are introduced corresponding sections of Chapter 2.

<table>
<thead>
<tr>
<th>SL</th>
<th>TL</th>
<th>TS</th>
<th>NLD</th>
<th>RL</th>
<th>DT (PET)</th>
<th>Standard/Particular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic Power</td>
<td>$10 \log P$</td>
<td>$\lambda + DI$</td>
<td>$\alpha, \alpha_s, \alpha_b$</td>
<td>$\mathcal{J}$</td>
<td>$S_N, AG$</td>
<td>(TL), $\sigma$</td>
</tr>
<tr>
<td>Frequency</td>
<td>$\lambda + DI$</td>
<td>$\alpha, \alpha_s, \alpha_b$</td>
<td>$\mathcal{J}$</td>
<td>$S_N, AG$</td>
<td>(TL), $\sigma$</td>
<td>B, S</td>
</tr>
<tr>
<td>Source Array Size &amp; Beamforming</td>
<td>$DI$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ranges, $R_1, R_2$</td>
<td>$10 \log R_1 R_2$</td>
<td>$\phi, \phi_1 + \mathcal{J}$</td>
<td>$(TL), V, S$</td>
<td>L</td>
<td>S</td>
<td></td>
</tr>
<tr>
<td>Sound Speed Profile</td>
<td>Propag. Mode</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SRC, TRGT, RCVR Depths</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline $R_b$</td>
<td>$\phi, \phi_1 + \mathcal{J}$</td>
<td>$\sigma$</td>
<td>$\mathcal{J}$</td>
<td>$S_N$</td>
<td>(L)</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Target Size</td>
<td>$A(\phi_1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shipping Intensity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sea State</td>
<td>$(\alpha_s)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RCVR Array Size $H_x, L_x$ &amp; Beamforming $\phi_e, \phi_e$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pulse Length $T$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bandwidth $W$</td>
<td>$10 \log W$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required Probabilities</td>
<td>DT (PET)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_D$ and $P_{FA}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required Parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variances $\sigma_t^2, \sigma_F^2, \sigma_\psi^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The main distinction between standard and particular inputs, is that if an input is particular, it may be requested in a specific way by the corresponding model and is "transparent" to the central routine. For instance the shipping density is not used elsewhere than in the computation of the noise level. A first programming rule is that associated with each model is an input routine prompting for the parameters particular to that routine. To avoid long subroutine argument lists, the data are filled into a common storage space, shared by the corresponding input routine and model.

3.3 **Solvable Parameters**

Having assessed the inputs to all the terms of the sonar equation but one, this last one may be solved for. If the unknown is a particular parameter to that term, a solution can be found, provided there exists an explicit inverse model for that parameter. For example assume that all terms but SL are known. Then

\[ SL = TL - TS + NLD + DT \]

From \( SL = 10 \log P + DI + 171 \), the required acoustic power \( P \) may be found, for a source of given directivity index.

When the dependency is multivalued (such as that of range on TL in convergence zone propagation), a set of points or entire margins may be found. For the sake of intelligibility, all results should be supported by a graphical representation of the multivalued dependency.

The first type of problem (solution for particular inputs) should be implemented by an inverse model associated with the initial one, e.g.

\[ P = 10^{((SL-DI-171)/10)} \]

Again the particular input parameters will be in common storage and "transparent" to the main routine. The parameters for which such a solution is not expected (for instance shipping density, sea state) may
be considered as standard if necessary. Referring to Table 3-1, the "weak" or neglected dependencies shown in parenthesis could be reconsidered, turning the corresponding particular inputs (e.g., sea state, sound speed profile) to standard inputs.

Another problem, which is more interesting is the solution of the sonar equation for standard parameters (such as frequency). The equation in that parameter may be so complex, and possibly nonanalytic (when numerical tables are used), that again the solution will have to be iterative and supported by graphical output.

The iterative solution for a standard parameter is necessarily implemented in the central routine.

3.4 First-Order Models

Following is a list of approximate models, which are implemented in the computer-based procedure. These models are totally arbitrary. They can and should be replaced by more rigorously derived models, according to the system designer's choices and specifications. Their only purpose is to illustrate the operation of the design procedure.

3.4.1 Source Level

\[ SL = 10 \log P + DI + 171 \text{ dB re } 1 \mu Pa \]

Assume the directivity index corresponds to a horizontally omnidirectional source, with its main lobe filling the entire long range propagation vertical angular extent which is limited by \( \theta_L = \cos^{-1} \left( \frac{C_S}{C_D} \right) \), as explained in Section 2.3. Then

\[ DI = 10 \log \frac{1}{\theta_L} \text{ dB} \]

An optional model is to assume a source array of fixed vertical dimension \( L \). The directivity index depends then on frequency through the formula

\[ DI = 10 \log \frac{2L}{\lambda} \text{ dB} \]
valid for a uniform line array. See Urick [1], Table 3.2 for other simple transducer directivity indices. Inverse models solving for P or L are obvious but may lead to unachievable designs.

3.4.2 Transmission Loss

Tables of transmission losses as a function of range and frequency for several environmental conditions are used. See Section 2.4 and Appendix B.

3.4.3 Target Strength

We use a formula based on the bistatic theorem. Assume a typical target of length \( L = 50 \text{ m} \), with a target strength of 25 dB re 1 m in beam aspect and 12 dB in bow aspect. Fitting the target strength of an ellipsoidal body to these numbers yields

\[
TS = 20 \log \left[ 0.025 L (3.75 \sin^2 \phi_b + 0.25) \left( \frac{1}{2} + \frac{1}{2} \cos \frac{\psi}{2} \right) \right]
\]

where \( \phi_b \) is the target aspect along the bisector of the source and receiver directions (see Figure 3.2)

\[
\phi_b = \frac{1}{2} (\phi + \phi_i).
\]

and \( \psi \) is the bistatic angle, \( \psi = \phi - \phi_i \)

3.4.4 Post-Array Noise Level

\[
NLD = NLS + NLSH + NLSS
\]

We assume the self noise level well under the ambient noise: \( NLS = -\infty \text{ dB} \), unless otherwise specified by the user. Shipping and surface spectral densities are read from average curves in Urick [1] and Ross [14]. The array gains calculations can be done in several ways.

- A beam pattern \( B^2(\theta) \) is specified by the user, along with the noise directionality, for integration (Eqs. (2.5-13) and (2.5-15)). The frequency dependence is not taken into account.
• Equivalent beamwidths $\phi_e$, $\theta_e$ are specified, also with no
  frequency dependence. A frequency dependence could be added
  in the form

  $$ \phi_e(f) = \phi_e(f_0) \frac{f_0}{f} $$

  where $f_0$ is the frequency at which $\phi_e$ is specified. A
  sidelobe level $B^2_n$ must also be provided, as well as the ratios
  of vertical to horizontal average level for the surface noise.

• A third option is to specify the receive array dimensions,
  which provide $\phi_e$ and $\theta_e$ as a function of frequency, and
  the type of sidelobe tapering, represented by $B^2_n$.

3.4.5 Reverberation Level

$$ RL = SL - TL + (T_s \oplus T_v) \quad \text{dB re 1 \, \mu Pa} $$

The reverberation strength may be drawn partly from monostatic
results, assuming that the scatterers are isotropic, and that the bistatic
configuration affects only the calculation of the reverberating
volume and surface. Recalling Eq. (2.7-3) and (2.7-5)

$$ T_v = 10 \log \frac{n_v \sigma_d}{4\pi} + 10 \log V \quad \text{dB re 1 \, m} $$

$$ T_s = 10 \log \frac{n_s \sigma_d}{2\pi} + 10 \log S \quad \text{dB re 1 \, m} $$

The first term of each of these two equations, denoted $S_b$ and
$S_v$ in Urick [1], are the surface and volume backscattering strengths
in the monostatic configuration. From Urick[1], Section 8.12, the sea
surface reverberation may be modelled by

$$ S_{sb} = -36 + 40 \log \tan \theta_s $$

where $\theta_s$ is the grazing angle, assumed on average equal to half the
long-range propagation limiting angle

$$ \theta_s = \frac{1}{2} \cos^{-1} \frac{C_s}{C_b} $$

55
In long-range propagation, the bottom-reflecting rays are neglected, and the scattering strength is taken for a zero grazing angle. From Urick [1], Figure 8.27, we choose $S_{sb} = -35$dB. Other estimates are rational, including $S_{sb} = -\infty$.

The volume backscattering strength $S_v$ is associated with the deep scattering layer (DSL). The variable average depth and thickness of the DSL should be taken into account in the calculation of the reverberation volume. From Urick [1], Figure 8.13, the night DSL is approximately between 250 and 400 meters of depth, with an average value of $S_v = -75$dB.

From Appendix D the bistatic reverberation area of the sea surface is, for long ranges

$$S = C \pi R_2^2 \frac{R_1^3 + 4R_1^2R_2 + 5R_1R_2^2 + 2R_2^3 - r_b^2(R_1 + 2R_2)}{[(R_1 + R_2)^2 - r_b^2]^2}$$

At long ranges, the bottom bistatic reverberation surface is the same. The reverberation volume is, given a thickness of $H = 200$ meters for the DSL,

$$V = SH$$

Hence, the overall reverberation level is

$$RL = SL - TL + ((S_v + 10 \log H) \oplus S_{ss} \oplus S_{sb}) + 10 \log S$$

3.4.6 Detection Threshold

Several options, depending on the function the system is to perform, are proposed.
Detection: the following system characteristics are assumed:

The sonar is noise limited.
The sound channel is time spread, and the receiver is not matched to the incoming pulse length.
The noise and signal + noise probability density functions are Gaussian. Then, from Urick [1], Section 12.9

\[ \frac{\bar{D}_{\text{Hz}}}{T} = 5 \log \frac{\text{BW}}{T} + \left| 5 \log \frac{\text{L}}{L} \right| \]

where \( \text{BW} \) is the bandwidth, \( T \) the average time, \( L \) the received pulse length, and \( d \) is the detection index read on receiving operating characteristic curves such as in Urick [1], Figure 12.6. It depends on the probabilities of detection and false alarm. When independent Gaussian statistics are assumed for the noise and the signal + noise spectra, then

\[ d^{1/2} = \sqrt{2} \left[ \text{erfc}^{-1}(2P_{\text{FA}}) - \text{erfc}^{-1}(2P_{D}) \right] \]

where

\[ \text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_{z}^{\infty} e^{-t^2} \, dt \]

These assumptions are poor ones in practice, but the resulting values of detection threshold serve to indicate the manner in which DT parameters enter the problem.

3.4.7 Parameter Estimation Threshold

For ranging, the variance of the estimate of time of arrival \( (\sigma^2_t) \) is linked to the rms bandwidth of the sonar signal \( (W) \), the noise bandwidth at the receiver inputs \( (W_N) \), the integration period \( (T) \), and the effective signal to noise ratio \( (\text{SNR}) \) in the receiver bandwidth. The Cramer-Rao lower bound prediction is
\[ \sigma_t^2 \geq \frac{1}{W^2 W_N W T \text{SNR}} \]  

(seconds squared)

Hence the minimum SNR required

\[ \text{SNR} = \frac{1}{W^3 T_0^2} \]

where the noise is assumed computed in the receiver bandwidth. If the relative uncertainty on sound speed (\( \varepsilon_c \)) leads to a computed range estimate variance greater than that resulting from the measured time of arrival, then SNR only needs to be

\[ \text{SNR} = \frac{1}{W^3 T \left( \frac{r}{c} \varepsilon_c \right)^2} \]

where

- \( r \) = total travelled range
- \( c \) = sound speed

In that case the range estimate is limited by \( \varepsilon_c \).

For bearing estimation, the variance of the estimate could be after Burdick [2], Chapter 13

\[ \sigma^2_{\psi} = \frac{1}{L_o^2 \left( \frac{r}{\lambda} \right) \text{SNR} \cdot 2T} \]

where \( L_o \) is a rms array length based on the receiver beamforming, \( T \) is the integration time and \( \lambda \) the wavelength. For a uniform array of length \( L \), the rms length is

\[ L_o = \frac{\pi L}{\sqrt{3}} \]
Hence the required signal-to-noise ratio

\[
\text{SNR} = \frac{3 \lambda^2}{\pi^2 L \sigma^2 \psi_2 \cdot 2T}
\]

where the noise is in a 1 Hz band.

For a receiver of fixed horizontal equivalent beamwidth \( \Phi_e \) (Option 2 in model DT), an alternate expression could be

\[
\text{SNR} = \frac{3 \Phi_e^2}{\pi^2 \sigma^2 \psi_2 \cdot 2T}
\]

3.5 **Logical Structure of the Central Routine**

An immediate decomposition of the routine SE is

<Inputs>

<Computations and Outputs>

3.5.1 **Organization of the Inputs**

As was seen in Section 3.4, several design options might be chosen at the beginning. For instance, one might decide that the source array has a fixed size and that the projector directivity index is thus dependent on frequency. Or, one might decide that the directivity index has an assigned value, whatever the frequency. In the same way, a choice is ordered between frequency dependent and frequency independent array gain models in the computation of the noise level.

The first task of the system input is thus to prompt for all options. For flexibility, the prompting for model options is carried by subroutines attached to each term of the sonar equation. According to the option, a specific list of parameters is proposed for input. Each parameter is defined the form

\[ n, \ nam, u, v \]
where

\[ n = \text{integer indicating if the parameter is unknown (} n = -1), \]
\[ \text{specified (} n = 1), \text{ or to be scanned (} n = \text{number of steps}) \]
\[ \text{nam} = \text{a 6-character word representing the parameter} \]
\[ u = \text{the value of this specified parameter, or its minimum value, if scanned} \]
\[ v = \text{the maximum value, if the parameter is scanned} \]

Once the input routines of each term have been called, the routine SE takes care of the standard inputs in a similar manner: for each parameter in the list of the standard inputs, n, nam, u, v are read. However, because we assume the sonar equation cannot be solved explicitly for a standard input, n must be 1 (specified) or greater than 1 (scanned).

The input algorithm is sketched in Figure 3-3. This input structure contains the form of the calculations, i.e., how the sonar equation is to be solved and what parameter is to be scanned.

![Figure 3-3. Input algorithm.](image)
3.5.2 Checking and Equation Structuring

Because of the main design constraint that only one equation links all the system parameters, only one unknown is allowed for one run. (Multiple scans are avoided for sake of simplicity. A loop over the entire calculation set can always be inserted.)

The first operation is thus to check that a maximum of one explicit unknown has been specified. Also, only one "scanned" unknown is allowed.

When an explicit unknown is specified in the model TERM (TERM is any one of SL, TL, NLD, RL, TS, DT) then the sonar equation is solved for TERM

\[ Y_{\text{TERM}} = SE_{\text{TERM}} \]

and using the inverse model, the unknown is found

\[ X = \text{TERM}^{-1}(Y_{\text{TERM}}) \]

The scan over another unknown may be allowed, thus allowing the creation of curves such as the required DI as a function of frequency.

When no explicit unknown is specified, the echo excess is calculated

\[ EE = SL - TL + TS - (NLD \oplus RL) - DT \]

3.5.3 Computations and Outputs

The kernel of the computations is either one of the equations introduced above.

Kernel: \[ <X = \text{TERM}^{-1}(Y_{\text{TERM}}) or EE = SL - TL + TS - (NLD \oplus RL) - DT> \]

If a parameter XS is to be scanned, these two equations are inserted in the corresponding loops

\[ <\text{for XS} = X_{\text{min}} \text{ to } X_{\text{max}}, N_X \text{ steps, perform KERNEL and output results (XS, X) or (XS, EE) }> \]
3.5.4 **Issues on Bandwidth**

Choice is given to the user, at the input of the noise parameters, to compute the noise as a spectral density in a 1 Hz band, or as the total noise power in the receiver bandwidth $W$. This option has to be respected in the detection threshold computations, which may depend on the total signal to noise power ratio $\text{SNR}$ or on the signal to noise density ratio $\text{SNR}_0$.

An indicator of the noise level band is hence initialized at the input of the noise data, and carried through function argument lists via the central routine $\text{SE}$, to the direct and inverse $\text{DT}$ models.

3.5.5 **Issues on Background**

The fact that the levels $\text{NLD}$ and $\text{RL}$ add incoherently imposes a constraint on the solution of the sonar equation for terms within $\text{NLD}$ and $\text{RL}$. If the background is dominated by reverberation, for instance, then the noise level cannot be solved from the sonar equation because it is only a small and unknown fraction of the background. Similarly, if the noise dominates, neither the reverberation level nor any of its related parameters can be solved from the sonar equation.

A constraint of the same nature arises on the solution of $\text{SL}$ and $\text{TL}$ from the sonar equation. The reverberation level is a function of source level and transmission loss, as stated in the equation

$$RL = SL - TL + RS$$

where $RS$ is the reverberation strength and is modeled separately. To solve the sonar equation for $\text{SL}$ or $\text{TL}$, one must assume that the noise dominates the signal at the receiving array. For instance

$$SL = DT + TL + NLD - TS$$
If the reverberation dominates, the sonar equation degenerates to

\[ DT = TS - RS \]

which implies that any result on SL or TL will be totally unreliable.

Hence the practice to assume the noise limitation, solve for SL or TL as required, and then check that the resulting reverberation level is effectively below the noise level.

3.5.6 **Algorithm**

Table 3-2 is the flowchart of the design procedure. Let ** represent any one of SL, TL, TS, NL, RS, or DT in the following explanation.

In the first half of the program, the data, the model options, and the definition of the design problem required by the user are specified.

Each of the **IN routines reads the option (a number n) and branches to the parameter input routine PARIN with the corresponding list of arguments. Each model (**n in the second half of the program) requires a specific list of particular parameters. These parameters are by definition not used in other terms of the sonar equation. They are arranged in a common storage named /COM**/ which will be used by the model **n. Each particular parameter is read by the routine PARIN, along with indicators, as illustrated in the input file in Table 3-3, in the form:

\[ NSC, NAM, V, VMAX \]

where \( NSC = -1 \) if the parameter is unknown and to be solved for, \( NSC = 1 \) if the parameter is specified a value \( V \), \( NSC > 1 \) if the parameter is to be scanned \( NSC \) values between \( V \) and \( VMAX \). \( NAM \) is the six-character identifier of the parameter. Note that \( V \) is necessarily real; the conversion to integer parameter values, if required, is done by the corresponding model.
Table 3-2. Flowchart.
Table 3-3. Input file.

```
RUN #1: COMPUTE -TL, SCAN XT,YT
0
1,SL 0.
1,
1,RECDZ 1.
1,SRCDZ 1.
1,TUTDZ 1.
1,SEATY 1.
1,SEASO 1.
1,FROM 1.
1,REFL 1.
0
1,ST 0.
1
0
1,NLD 0.
0
1,RS 0.
0
1,DT 0.
60,XT -30..90.
30,YT -20..60.
1,F 1000.
1,RB 30.
1,T 0.1
1,W 50.
1,PHIE .0873
1,THETA .0873
1,RECL 3.
1,RECN 2.
1,TOTAL .26
1
```

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The identifier NAM is used by the routine PARIN to keep track of which parameter has been specified to be scanned (NAMX), and which parameter has been specified as an unknown (NAMY). Tests forbidding two unknowns or two scanned parameters are performed gradually as the inputs are read. The same rule and format apply to the standard inputs which are monitored by SE directly. An exception on the single-loop rule for the coordinates of the target XT, YT allows plotting contour lines over a whole region about the source and receiver.

The second half of the program performs a loop over the parameter identified by NAMX. The scanned value X is assigned to the corresponding variable, standard as well as particular (which explains the presence of NAMX and X as argument of the direct models). The direct models are evaluated for all terms except the one where NPUN** is 1. The unknown indicator NPUN** is a flag returned to SE by the input routine PARIN when an unknown has been specified in the model **.

Structuring the equations is also based on the flags NPUN**. For instance if NPUNSL = 1 then the proper equation is (assuming noise limited sonar)

\[ YSL = YDT + YTL - YTS + YNL \]

In each case of NPUN** the corresponding sonar equation is solved and the inverse model **INV is called to solve for the particular unknown designated by NAMY.

The verifications mentioned in Section 3.5.5 are done at the same time.

The output is in the form of a table showing the scanned input NAMX, the six terms of the sonar equation, as calculated SL, TL, TS, NL, RL, DT, and the solved unknown NAMY.

Plots of NAMY as a function of NAMX are also made possible.
In the particular case where the model **n needs the entire tables, which are assumed to be provided on other input files, an intermediary subroutine PAR**n is assigned the task of reading the table(s) in question into a separate common storage and of calling, in turn, PARIN to input the other particular parameters. This distinction between tabular inputs—read by PAR**n—and particular parameters—read by PARIN—frees the user from the heavy format NSC, NAM, V, VMAX, when part of the data definitely will never be scanned or unknown.

These routines are

\[ \text{PARTL1} \text{ which reads typical transmission loss tables used in TL1 (see appendix B).} \]

\[ \text{PARTL2} \text{ which reads typical transmission loss tables provided by the user and used in TL2.} \]

\[ \text{PARNL1} \text{ which reads the receiver vertical beam pattern } B^2(\theta), \text{ the sea surface, and shipping noise vertical directionalities, to be used in NLAG1.} \]

\[ 3.6 \quad \text{Applications} \]

Following are examples of the use of design procedure. The variety of the examples should prove the versatility of the procedure with respect to the definition of the problem by the user, and to the facility with which one model may be replace by another.

\[ 3.6.1 \quad \text{Study of Transmission Loss in the Horizontal Bistatic Configuration} \]

As indicated in the input file reproduced in Table 3-3, to compute one of the terms of the sonar equation as a function of its own parameters, the user must still solve the sonar equation and set all other terms to zero. Here the echo excess EE is computed

\[ EE = SL - TL + TS - (NLD \odot RL) - DT \]

and with

\[ SL = TS = NLD = RS = DT = 0 \]
the reverberation level is simply

\[ RL = SL - TL + RS = -TL \]

hence, the background is dominated by noise

\[ NLD \oplus RL = NLD = 0 \]

which yields

\[ EE = -TL \]

In this example, the position of the target is scanned over a rectangle 120 km long in the source-receiver axis, and 80 km wide, as illustrated in Figure 3-4.

The source is at \((30 \text{ km}, 0)\) and the receiver at \((0, 0)\). Since this transmission loss model considers no depth effect, the figure is roughly symmetrical about the perpendicular bisector of the source-receiver axis, here the line \(x = 15 \text{ km}\). The otherwise complex shape of the contour liner is partly due to the converging zones, and partly due to the finite steps in the largest displacement (2 kilometers). Basins and hills are clearly represented in the perspective view of the surface \(z = TL(x, y)\). They could be baptized respectively bistatic shadow and convergence zones.

3.6.2 Frequency Dependence of a System of Fixed Dimensions

This run (Tables 3-4, 3-5 and Figure 3-5) illustrates the study of the system frequency dependence for several geometric configurations. For this particular case, another capability was added to the tabular transmission loss model. As explained in Appendix B, Section B-2, for frequencies outside the tabulated data, an approximate model based on volumetric absorption is used, rather than a mere linear extrapolation.
Figure 3-4. Run #1: transmission loss contour lines.
Z: $-\text{TL (dB re 1 m)}$; Y, Y: target position (km).
Figure 3-5. Run #2: signal-to-noise ratio vs frequency.
SNR in dB.
Table 3-4. Outputs for Run #2.

```plaintext
### BISTATIC SONAR DESIGN PROCEDURE ###
RUN # 2:  COMPUTE DT=SNR, SCAN F
RANGE UNITS CHOSEN: (KM)
SOURCE LEVEL MODEL OPTION: 1 (0=SL IS SPECIFIED OR UNKNOWN)
                           (1=FIXED SIZE VERT.UNIFORM LINE ARRAY)
                           (2=FIXED DIRECTIVITY INDEX)
   P     =   1000.0
   H     =   2.0000
T.L. MODEL OPTION: 1 (TL SPECIFIED OR UNKNOWN=0)
                    (TYPICAL TABLES=1, USER-PROVIDED=2)
   4 FREQUENCIES: 100.0  500.0  1000.  3500.
NUMBER OF TABLES IN TL INPUT FILE:  3
*** ITAB= 2, NT= 65
*** ITAB= 4, NT= 79
*** ITAB= 6, NT= 64
RECDZ     =   1.0000
SRCZ     =   1.0000
TOGTZ     =   1.0000
SEATY     =   1.0000
SEASU     =   1.0000
FORM     =   1.0000
REPL     =   1.0000
TARGET SIZE/THICK OPTION: 1 (0=TS SPECIFIED OR UNKNOWN, 1=NORMAL MODEL)
   PHIT     =   .00000
   TINV     =   50.000
NOISE BAND OPTION: 1 (0= SPECTRUM DENSITY, 1 HZ BAND)
                    (1=LEVEL IN BAND W)
   NOISE MODEL OPTION: 3 (0=NL SPECIFIED OR UNKNOWN)
                    (1=INTEGRATE THE BEAM PATTERN)
                    (2= FIXED SOLID ANGLES)
                    (3= FIXED ARRAY DIMENSIONS)
   NLS     =   20.000
   SEAST    =   2.0000
   SHIP     =   2.0000
   B2SIDE   =   -12.000
   TETAS    =   .10000
   HVRS     =   .10000
REVERBERATION MODEL OPTION: 2 (0= RS SPECIFIED OR UNKNOWN )
                           (1= FIXED REC, HORIZ. SOLID ANGLE)
                           (2= FIXED RECEIVER HORIZ. SIZE)
   DSLH     =   200.00
D.T. MODEL OPTION: 0 (0= SPECIFIED OR UNKNOWN DT )
                   (1= FIXED REC, VERT, SOLID ANGLE)
                   (2= FIXED RECEIVER VERT, SIZE)
*** SE WILL BE SOLVED FOR THE PARAMETER DT
   XT     =   30.000
   YT     =   15.000
*** F WILL BE SCANNED: 20 VALUES FROM   .50E+03 TO   .10E+05
   RB     =   30.000
   T      =   .10000
   W      =   50.000
   PHIE   =   .87300E-01
   TETAE  =   .87300E-01
   RECL   =   3.0000
   RECH   =   2.0000
   TETAL  =   .26000
PLOT OPTION: 1 (1=PLOTS)
```

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Table 3-5. Results for run #2.

<table>
<thead>
<tr>
<th>F</th>
<th>SL</th>
<th>TL</th>
<th>TS</th>
<th>NLD</th>
<th>RL</th>
<th>DT</th>
<th>DT</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>.20E+03</td>
<td>.17E+03</td>
<td>10</td>
<td>75</td>
<td>67</td>
<td>-33</td>
<td>-33.1</td>
</tr>
<tr>
<td>1.100E+04</td>
<td>.21E+03</td>
<td>.17E+03</td>
<td>10</td>
<td>66</td>
<td>64</td>
<td>-26</td>
<td>-25.6</td>
</tr>
<tr>
<td>1.150E+04</td>
<td>.21E+03</td>
<td>.18E+03</td>
<td>10</td>
<td>58</td>
<td>60</td>
<td>-22</td>
<td>-22.1</td>
</tr>
<tr>
<td>2.200E+04</td>
<td>.21E+03</td>
<td>.18E+03</td>
<td>10</td>
<td>53</td>
<td>55</td>
<td>-21</td>
<td>-20.6</td>
</tr>
<tr>
<td>2.250E+04</td>
<td>.21E+03</td>
<td>.19E+03</td>
<td>10</td>
<td>49</td>
<td>51</td>
<td>-20</td>
<td>-19.8</td>
</tr>
<tr>
<td>3.300E+04</td>
<td>.21E+03</td>
<td>.19E+03</td>
<td>10</td>
<td>46</td>
<td>46</td>
<td>-20</td>
<td>-19.7</td>
</tr>
<tr>
<td>3.350E+04</td>
<td>.21E+03</td>
<td>.19E+03</td>
<td>10</td>
<td>44</td>
<td>42</td>
<td>-20</td>
<td>-19.8</td>
</tr>
<tr>
<td>4.400E+04</td>
<td>.21E+03</td>
<td>.20E+03</td>
<td>10</td>
<td>42</td>
<td>41</td>
<td>-19</td>
<td>-18.9</td>
</tr>
<tr>
<td>4.450E+04</td>
<td>.21E+03</td>
<td>.20E+03</td>
<td>10</td>
<td>40</td>
<td>40</td>
<td>-18</td>
<td>-18.2</td>
</tr>
<tr>
<td>5.500E+04</td>
<td>.21E+03</td>
<td>.20E+03</td>
<td>10</td>
<td>38</td>
<td>38</td>
<td>-18</td>
<td>-17.8</td>
</tr>
<tr>
<td>5.550E+04</td>
<td>.21E+03</td>
<td>.20E+03</td>
<td>10</td>
<td>37</td>
<td>36</td>
<td>-18</td>
<td>-17.6</td>
</tr>
<tr>
<td>6.600E+04</td>
<td>.21E+03</td>
<td>.20E+03</td>
<td>10</td>
<td>36</td>
<td>34</td>
<td>-18</td>
<td>-17.6</td>
</tr>
<tr>
<td>6.650E+04</td>
<td>.21E+03</td>
<td>.21E+03</td>
<td>10</td>
<td>35</td>
<td>32</td>
<td>-18</td>
<td>-17.9</td>
</tr>
<tr>
<td>7.700E+04</td>
<td>.21E+03</td>
<td>.21E+03</td>
<td>10</td>
<td>34</td>
<td>30</td>
<td>-18</td>
<td>-18.4</td>
</tr>
<tr>
<td>7.750E+04</td>
<td>.21E+03</td>
<td>.21E+03</td>
<td>10</td>
<td>33</td>
<td>27</td>
<td>-19</td>
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<td>8.800E+04</td>
<td>.21E+03</td>
<td>.21E+03</td>
<td>10</td>
<td>32</td>
<td>25</td>
<td>-20</td>
<td>-20.4</td>
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<tr>
<td>8.850E+04</td>
<td>.21E+03</td>
<td>.22E+03</td>
<td>10</td>
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<td>22</td>
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<td>9.900E+04</td>
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<td>10</td>
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<td>-23.6</td>
</tr>
<tr>
<td>9.950E+04</td>
<td>.22E+03</td>
<td>.22E+03</td>
<td>10</td>
<td>29</td>
<td>16</td>
<td>-26</td>
<td>-25.6</td>
</tr>
<tr>
<td>1.000E+05</td>
<td>.22E+03</td>
<td>.22E+03</td>
<td>10</td>
<td>29</td>
<td>12</td>
<td>-28</td>
<td>-27.9</td>
</tr>
</tbody>
</table>

The same run, solving the sonar equations for the detection threshold, i.e., computing the signal-to-noise ratio DT = SNR as a function of frequency, between 500 Hz and 10 kHz, is repeated for several configurations, yielding the three curves of Figure 3-5. Referring to this figure, the fact that the continuous curve corresponding to the longest configuration (XT = RB = 80 km) crosses the midrange configuration (XT = RB = 40 km) can be interpreted in two ways. A probable reason is that either the mid-range configuration lies in a "bistatic shadow zone" or that the long-range configuration lies in a "bistatic convergence zone". This could be confirmed by a run of the program similar to Run #1, with the baseline RB taken at 40 or 80 kilometers.

Another valid explanation is that the results are provided by very approximate models (especially concerning the frequency dependence of RL and SL), and should definitely not be considered as reliable.

Nevertheless, qualitative and relative conclusions may be drawn, such as the existence of the approximate position of an optimum frequency.

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CHAPTER 4.

SUMMARY AND CONCLUSIONS

4.1 **Summary**

In order to develop a flexible design procedure based on the bistatic sonar system, the interrelationships of the main sonar system parameters have been explored in Chapter 2. In this chapter, equations were introduced and references to the underwater acoustic research and sonar engineering literature were made, mainly with the purpose of illustrating the main parameters of the sonar equation and their dependence upon environmental or design conditions.

This led, in Chapter 3, to the central distinction between standard and particular parameters. Subsequently, an input format was developed to allow the user to specify any possible unknown, i.e., any particular parameter for which the corresponding model could be solved. All possible manipulations of the bistatic sonar equation were considered, with special attention given to background, which is, to first order, the incoherent sum of the reverberation level and the ambient noise. Verifications were developed, to ensure that the results remain consistent with the background limitation. For instance, there is no sense in solving the sonar equation for the reverberation strength when the system noise is limited. The particular parameters for which there is no inverse model, and the standard (shared) parameters can still be explored by specifying a calculation loop on that parameter.

Two applications were exposed to show the two possibilities of graphical output. A double loop on the target x and y-coordinates in the horizontal plane allows, with the help of another program, CONGRID, to plot curves of equal transmission loss or signal-to-noise ratio, and
to draw a perspective view of the surface $z = TL(x, y)$. (Figure 3-4). A single loop on frequency, performed for three different configurations, allows one to draw comparative and qualitative conclusions on the system frequency dependence (Figure 3-5).

4.2 Further Work, Recommendations

The computer procedure developed here is really the frame for further design work on a bistatic sonar system. It is meant, thanks to its modular and standardized programming, to be modified in several ways, according to the system designer's needs.

The first modification will be to implement relevant models for each term of the sonar equation. Even if the system to be designed is almost completely unspecified, for instance, with regard to operating range, acoustic power, source size, and autonomy, any good quantitative assessment of the system operation should rely on accurate and reliable models provided by the designer. The new model derivation should follow the same approach as in Section 3.4, and be inserted in the program according to the formats stated in Appendix C, Section C.4. As stated earlier, the best model is necessarily provided by experimental measurements done on a system identical or very similar to the one to be designed, or on parts thereof.

A second type of development from the actual procedure would be in the analysis of problems not directly related to the bistatic sonar equation. In the fields of waveform and signal processor design, a bistatic sonar sensitivity analysis could be done, based on the calculations of the actual procedures in conjunction with a relevant source navigation model. More in the point of view of operations research, and again, provided a relevant source-receiver navigation model, simulations could be done, allowing the analysis of cost function based on the source size, power, and the system performance in terms of detection on parameter estimation. See Appendix C, Section C.4 for a consistent way of implementing these added features.
4.3 Conclusions

The bistatic sonar equation has appeared to be a very convenient way to formalize the travel of acoustic energy between the source, the target, and the receiver. Although each term of this equation is a complex and unknown function of system and environmental parameters, which varies somewhat unpredictably in time and space, some approximate models have been postulated, thus allowing us to analyze the interrelationships of various system parameters.

The bistatic configuration is in this outcome more an issue of the prediction of particular terms, e.g., reverberation level and target strength, which were not directly addressed here. Globally, it adds one dimension and several angle parameters in the design problem. Rather than range and bearing, the target position in cartesian coordinates with respect to the receiver-source baseline, and the length of the baseline itself, must be considered.

One limitation on the manipulations of the sonar equation is the interdependency of several terms: the reverberation level with the source level and the transmission loss, and the background level with the noise and the reverberation levels. This imposed verifications on the noise or reverberation limitation of the system. However, the "modularity" of the sonar equation was a very practical and conceptually efficient basis for the development of a structured computer-based procedure.
REFERENCES


REFERENCES (Cont'd)

REFERENCES (Cont'd)

APPENDIX A

USER'S GUIDE TO THE PROGRAM RAYTRC

The program RAYTRC gives information on the propagation of sound in a horizontally stratified ocean (sound speed is a function of depth only). Within the assumptions of ray theory it uses a set of geometrical rules to plot the trace of sound rays from a source of given depth to a maximum range.

A.1 The Inputs

The program is partially interactive in the way that some of the input is read from a data file (Unit 10) and the other parameters are asked for input at the terminal. The user is first provided a choice for the units in which the inputs and the outputs shall be expressed.

The data file contains the sound velocity profile (see Table A-1), namely:

\[ \text{NSP} \] the number of points describing the profile; arrays in the program have been dimensioned so that NSP must not exceed 100.

\[ \{\text{DEPTH}(I), \text{SPEED}(I)\} \] \( I = 1, \text{NSP} \) A set of profile points, the depth in meters (or feet) positive downwards, and the sound speed in meters (or feet) per second.

The points must be chosen so that between each point the profile to be modeled is very close to a straight line (constant vertical gradient). Characteristic sound profiles can be found in Urick [1], pp. 111, 113, 114.
Table A-1.

SOUND VELOCITY PROFILE

<table>
<thead>
<tr>
<th>POINT</th>
<th>DEPTH (M)</th>
<th>SPEED (M/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>1472.00</td>
</tr>
<tr>
<td>2</td>
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<table>
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<th>RMAX KM</th>
<th>ANG</th>
<th>DE M</th>
<th>ANGE</th>
<th>TT S</th>
<th>ST KM</th>
<th>NSB</th>
<th>NBB</th>
<th>NTP</th>
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<td>197.500</td>
<td>-4.64</td>
<td>136.361</td>
<td>200.1214</td>
<td>0</td>
<td>0</td>
<td>20</td>
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<tr>
<td>200.00</td>
<td>200.00</td>
<td>2.950</td>
<td>203.625</td>
<td>3.024</td>
<td>136.669</td>
<td>200.2364</td>
<td>0</td>
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<td>20</td>
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The data read from the terminal are in order:

RS, DS The range and depth of the source. RS, in kilometers (or nautical miles), is usually taken to be zero. DS is in meters (or feet); positive downwards. See Figure A-1.

RMAX The range of the receiver, in kilometers or nautical miles.

ANG The angle at which the ray leaves the source, in degrees measured positively downwards from the horizontal, as shown in Figure A-1. If RMAX is greater than RS, then ANG must be in the interval $\left(0^\circ, 90^\circ\right].$ Degrees.

![Figure A-1. Geometry orientation and units.](image_url)
An inner loop on the angle parameter allows the user to iterate manually and select for plotting the rays with satisfactory output characteristics.

An outer loop on RS, DS, RMAX, ANG allows to generate a new plot with a different setup.

A.2 The Outputs

Results are displayed at the terminal before a ray is inserted in the plot file. The outputs are as illustrated in Table A-1.

The sound velocity profile is a recall of the input data file; for each ray, the source depth DS, the maximum range RMAX, and the angle are recalled, and the following results are displayed:

- **DE**: the depth of the ray at RMAX, in meters positive downwards
- **TT**: the travel time in seconds
- **ST**: the distance traveled
- **NSB**: the number of surface bounces
- **NBB**: the number of bottom bounces
- **NTP**: the number of turning points

Only the results of the rays selected for plotting are copied onto the print file (Unit 20).

The plot files, to be taken as input for future plotting using the PLOT 10 package, are located in (Unit 11, Unit 12), (Unit 13, Unit 14), and so on, with as many pairs of files as created plots.

A.3 Results and Interpretation

Within the assumptions of ray theory and a horizontally stratified ocean, the plots give a good physical image of underwater sound propagation. The program does not solve for rays with a specified receiver depth (DE). This problem of finding the eigenrays between a source and a receiver has to be solved by manual iteration, based on criteria such as NSB, NBB, NTP, and DE. A ray is assumed to hit the receiver when its arrival depth DE is within some vertical distance of
the receiver. The frequency-dependent angular width of a ray is not computed in this simple ray tracing program.

With most of the profiles, the propagation is of the refracted surface reflected (RSR) type, see the plots in Figures A-2 and A-3. As illustrated in Figure A-3, a set of eigenrays between a given source and receiver, having the same number of turning points (here, 4 turning points at 3000 m), can contain no more than 4 rays, with angles symmetrical about the horizontal, and NSB = NTP ± 1.

The time spreading, which is the difference in travel time between the slowest ray and the fastest ray not hitting the bottom, can be evaluated to assess the duration of the impulse response. In general the slowest ray will be the most horizontal and the fastest will be the steepest. In the example given in Figure A-4, the time spreading is 136.66 - 135.37 = 1.29 seconds for a range of 200 kilometers.

Figure A-2. The ray angles and characteristics are listed in Table A-1.
Figure A-3.
Figure A-4.
APPENDIX B

TRANSMISSION LOSS GRAPHICAL MODEL

B.1 The Parameters of Transmission Loss

Transmission loss is mainly a function of range (through geometrical spreading) and of frequency (through volumetric absorption).

To a lesser extent, it depends on the sound speed profile, the source and receiver depths, and the conditions at the sea surface bottom. Of course, at a given range and frequency, one may find as many values of TL, theoretical or measured, as there are possible descriptions of the ocean environment. But the experience of many measurement campaigns, as well as computational models, allows one to restrict this indetermination to a set of typical tables, based on the evidence of well-known propagation modes associated with a type of sound speed profile. In fact, data banks such as those listed in Urick [11], Chapter 3, show that one can expect, at a given place and a given time of the year, sound speed profiles with some degree of predictability. Limiting this analysis to abyssal waters, a practical approach, following the presentation in Dyer [20], Appendix G-2, is to postulate two global types of sound speed profile, one for open oceans and one for Mediterranean or polar waters, to define three possible depth zones for the source and the receiver with respect to the sound speed profile. Transmission losses are precomputed and presented in charts to which the user is referenced, according to the source and receiver depth zone and to the season. When relevant, corrections to the converging source range

85
width, and to the plotted losses are added, according to the source depth and to the formation of the sound channel (measured by the depth excess defined in Figure 2-3). Between the convergence zones, a portion of the sound coming from bottom propagation is measured, hence, the use of loss tables for bottom bounce propagation.

A practical list of parameters for the use of precomputed transmission losses is thus:

The receiver depth zone: Shallow, Intermediate or Deep -
RECDZ = 1, 2, 3

The source depth zone: Shallow, Intermediate or Deep -
SRCDZ = 1, 2, 3

The sea types: Mediterranean, Polar, or Open Ocean -
SEATY = 1, 2, 3

The season: Summer or Winter -
SEASO = 1, 2

The sound channel formation: Good or Poor -
FORM = 1, 2

The bottom reflection: Good or Poor
REFL = 1, 2

B.2 Algorithm

Transmission losses are presented in [20] versus range at four frequencies: 100, 500, 1000, and 3500 Hz. Eight charts corresponding to the various possibilities of sound propagation can be numerized (see Section B.3) and referred to according to the logical tree of Table B-1, based on the parameters stated above.

In the computation of the bistatic transmission loss, according to the leg number, a target depth zone TGTDZ is substituted to the receiver on the first leg (source-target), and to the source on the second leg (target-receiver).
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<th>FORM</th>
<th>CHART</th>
<th>Loss Correction (dB)</th>
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Add 8 dB of loss if FORM = 2 (Poor form)
Notice that this tree is not symmetrical: transmission losses between Source and Receiver at S and D so not equal the transmission loss from D to S.

The graphs, drawn from [20], are as follows:

Chart B1: Open Oceans, deep source, 100% convergence zone width correction from chart B4

B2 Bottom bounce propagation, good reflection
B3 Bottom bounce propagation, poor reflection
B4 Open Oceans, shallow and intermediate source
B5 Open Oceans, deep receiver
B6 Mediterranean
B7 Half channel: Polar and Mediterranean waters
B8 Mediterranean, deep source, 100% convergence zone width correction from chart B6

The charts B2 through B7 correspond to the Figures A-2 through A-7 of Reference [20]. The charts B2, B4, and B6 have been numerized and are represented in Figures B-1, B-2, and B-3.

Transmission loss at a given range and frequency is interpolated from these numerized charts. When the range and/or the frequency lies outside the precomputed model, linear interpolation doesn't hold. At long range and high frequency, the transmission loss is primarily due to the volumetric absorption which increases as the square of the frequency and exponentially with range. Instead of extrapolation, a correction based on a formula for the volumetric absorption $\alpha$, borrowed from Fisher [12], is done.
Figure B-1. Chart B2: bottom bounce transmission loss (dB re 1 m).

Figure B-2. Chart B4: open oceans transmission loss (dB re 1 m).
Figure B-3. Chart B6: Mediterranean transmission loss dB re 1 m.

This approach applies for the entirely refracted paths (charts B3 through B7) as well as the bottom bounce propagation modes (charts B2, B3), and the two modes are added to yield the total transmission loss. In fact, the summation is the incoherent summation of mean square pressures. If TLS represents the water-borne propagation loss and TLB the bottom bounce loss, then the total transmission loss is

\[
TL = -10 \log \left[ 10^{\frac{-TLS}{10}} + 10^{\frac{-TLB}{10}} \right] \text{ dB re } 1 \text{ m}
\]

The corresponding model with the same branching tree and the interpolations on range and frequency is programmed and inserted in the sonar design procedure as the function routine TL1.
Table B.2. Input file for the program LOSSTA.

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### Table B-3. Output file of program LOSSTA to be used as tabular input for model TL1.

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</tbody>
</table>

etc... (79 lines)
B.3 Numerization of Transmission Loss Tables

A small routine to make the numerization of these tables easier, called LOSSTA, is available. It reads from unit 10 a set of points

\[
\begin{array}{ccccc}
  x_1 & y_{11} & y_{12} & y_{13} & y_{14} \\
  \cdots & \cdots & \cdots & \cdots & \cdots \\
  x_n & y_{n1} & y_{n2} & y_{n3} & y_{n4}
\end{array}
\]

where each line \(i\) is constituted of one range \(x_i\) and the 4 corresponding transmission losses \(y_{ij}\), \(j = 1, 4\). For the sake of rapidity of input these points are measured on the charts and expressed in centimeters with the origin at \(x = 0\), \(TL = 120\), and formatted in 5 fields \(F*0.1\).

The program LOSSTA, aside from the conversion from centimeters into kilometers and decibels, fits a smooth curve through the input points. The logic of this curve fitting is such that all points where a sharp angle is really intended should be repeated, and all vertical lines should have their upper and lower points also repeated. See Table B-2 as an example of an input file and Table B-3 as an example of an output file.

B-4. Transmission Loss User's guide

As input to the model TL1, the tabular data are read by the routine PARTL1 (see Section 3.5.6) in the format

\[NF, [FF(I) , I = 1, NF]\]
where

\[ NF = \text{the number of frequencies at which the transmission loss tables are given (maximum of 4)} \]

\[ FF = \text{the array of these frequencies} \]

\[ \text{NTAB the number of tables in the input file (maximum of 8)} \]

\[ \text{ITAB the index of the table, within } (1, \ldots, 8) \]

\[ \text{NT the number of lines = ranges in the table (maximum 100)} \]

\[ x, (\text{TAB(I)}, I = 1, NF) \text{ the range and the corresponding transmission loss values} \]

\[ \text{NTAB NT TIMES TIMES} \]

The "inner block" of this input file is one of the type of Table B.3, generated by the program LOSSTA.

The other particular inputs, listed in Section B.1 and in Appendix C-2, namely RECDZ, SRCDDZ, TGTDZ, SEATY, SEASO, FORM, and REFL, are taken care of by the routine PARIN, as explained in Section 3.5.6. Note that they are read as real numbers and converted to integers only as they are used in TL1.
APPENDIX C

SE DESIGN PROCEDURE USTRS' GUIDE

C.1 Introduction

This appendix has two purposes. An immediate one is to describe all the possible inputs and outputs of the design procedure in the form in which it is implemented on the TI990 minicomputer of Division 15L, CSDL. See Sections C.2 and C.3.

The second purpose is to assist the future system designer in implementing his/her own prediction model within the standards and formats established in Chapter 3. See Section C.4.

C.2 The Inputs of the Design Procedure

A helpful feature of the procedure is the possibility to be assisted by the program itself in the flow of inputs. This interactive mode, however "user-friendly" for the inexperienced user, is laborious, and might fail as soon as a format error is made at the input from the terminal. A faster and safer input mode, comparable to a batch run, is to provide the data in an input file. In the "batch mode" several numbers are nevertheless prompted from the terminal: they correspond to the mainstream input and output file unit numbers, and to the unit numbers of the tabular data (required for instance by the TL1, TL2, NLAG1).

The interactive input mode differs from the batch mode in two other aspects: the data, systematically echoed to the output file (see Table 3.4) are also rearranged into the batch input file, so that subsequent runs can be done in batch mode. Also, the format of any particular input is not NV, NAM, VMIN, VMAX, but rather NV, VMIN, VMAX, since
there is no error possible as to which input is to be provided, as it is
prompted by the program.

In the batch mode, the order of the inputs is strictly fixed; an
error in the order of the inputs gives rise to an error message and the
program interruption. The order is as follows (with Fortran format
field in parenthesis)

<table>
<thead>
<tr>
<th>Field</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTER</td>
<td>interactive flag (read from the terminal)</td>
</tr>
<tr>
<td>LIN, LOUT</td>
<td>input and output file, logical unit numbers (read from the terminal)</td>
</tr>
<tr>
<td>TITLE</td>
<td>title for the run, read in LIN</td>
</tr>
<tr>
<td>IRAN</td>
<td>choice of range units in the input of transmission loss data and the output of XT, YT, RB</td>
</tr>
</tbody>
</table>

Source Level Model: 1 + kilometers 2 + nautical miles

Model Option:

0 + SL is specified or unknown
1 + fixed size uniform line array model, with frequency dependence
2 + fixed directivity index, independent of frequency

If NOP = 0

NV, NAM, V1, V2 (I3, A6, 2E15.5), where NAM must be 'SL', (six characters), corresponding to the source level, its value Vi expressed in dB re 1μPa at 1 m.

If NOP = 1

The acoustic power 'P' expressed in watts, same format as above
The array height 'H' in meters
If NOP = 2

The acoustic power 'P' in watts
The directivity index 'DI' in dB

Transmission Loss:

NOP

Model Option:

0 + SL is specified or unknown
1 + typical tables (with depth effect)
2 + user-provided table

If NOP = 0

The transmission loss 'TL' in dB re 1 m, assumed equal on legs 1 and 2 of the bistatic path (0 dB, for instance)

If NOP = 1

LUTL (I3) logical unit number for the transmission on data file (read down from terminal)

Read on unit LUTL are the tables of transmission loss numerized from Reference [20], Appendix G.2 (see Appendix B). From unit LUTL, the transmission loss data is read as follows

NF, (FF(1), I = 1, NF) (I3 4F10.4) the number (max 4) of frequencies on which are based the transmission loss tables, and their values, in Hz

NTAB (I3) the number of tables in input file (max 8)

for I = 1, NTAB

ITAB (I3) the index of the chart in the graphical model

NT (I3) the number of lines (ranges) in that table

for J = 1, NT

RR (J, I) (TABTL (JF, J, I), JF = 1, NF) (5F10.4)
For each range RR, the (max 4) values of the transmission loss, in dB re 1 m, corresponding to the frequencies defined above. The range is in the chosen unit (kilometers or nautical miles). Coming back to the mainstream input file #LIN, the remaining particular data are read.

The receiver depth zone 'RECDZ' (1. = Shallow) 
(2. = Intermediate) 
(3. = Deep)

The three zones S, I, D are defined with respect to the sound speed profile according to Figure C-1.

Figure C-1. Definition of depth zones S, I, D as input to a graphical transmission loss model.

The source depth zone 'SRCDS'

The target depth zone 'TGTDZ'

The sea type 'SEATY'
(1. = Mediterranean) 
(2. = Polar Waters) 
(3. = Open Oceans)

The season 'SEASON' 
(1. = Summer) 
(2. = Winter)
C

FUNCTION SUM(A,B)
C INCOHERENT SUM OF TWO LEVELS
C INITIALLY IN DECIBELS
C
C C=0.1*LOG10(10.)
C SUM=10.*LOG10(EXP(A*C)+EXP(B*C))
RETURN
END

FUNCTION DIF(A,B)
C INCOHERENT DIFFERENCE OF TWO LEVELS
C INITIALLY IN DECIBELS
C
C C=0.1*LOG10(10.)
C DIF=10.*LOG10(EXP(A*C)-EXP(B*C))
RETURN
END

SUBROUTINE HQD(I,NAM)
CHARACTER*6 NAM
CHARACTER*40 TEXT(6)
INTEGER*4 LAPP,WPPC
COMMON /COM10/TEXT,LIN,LOUT
C
DATA TEXT/'PLEASE NO MORE THAN ONE SCANNED INPUT',
& 'IRRELEVANT INPUT',
& 'IRRELEVANT UNKNOWN',
& 'INPUT NOT ALLOWED TO BE SCANNED',
& 'IMPOSSIBLE, ALREADY ONE UNKNOWN',
& 'STANDARD INPUTS CANNOT BE UNKNOWN'/
C
LAPP=WPPC() WRITE(LOUT,70) LAPP
90 FORMAT(' LAPP=',I8)
C
WRITE(LOUT,100) TEXT(I),NAM
100 FORMAT('!! ',A40,'; NAME=',A6)
STOP
END

112
SUBROUTINE TO INPUT A LIST OF PARAMETERS

IN:
NPAR: NUMBER OF PARAMETERS IN THE LIST
ANAM: ARRAY OF THE NAMES OF THESE PARAMETERS
NAM: CURRENT NAME OF SCANNED INPUT
NAMX: CURRENT NAME OF PARTICULAR UNKNOWN

OUT:
APAR: ARRAY OF THE VALUES OF THESE PARAMETERS
IPAR: INDEX OF THE SCANNED PARAMETER IN THAT LIST
NSIN: NUMBER OF STEPS OF THE SCANNED INPUT
NM: THE NAME OF THE SCANNED INPUT
MIN.XMAX: ITS BOUNDS
NPUN: = 1 IF A PARTICULAR SOLVABLE UNKNOWN IS SPECIFIED IN THAT MODEL
NAMX: THEN NAME OF THAT UNKNOWN

COMMON /COMO/INTER,LIN,LOUT

SUBROUTINE PARIN(NPAR,ANAM,APAR,IPAR,
& NSIN,NAMX,XMIN,XMAX,IPUN,NAMX)
CHARACTER*(6,ANAM,NAMX,ANAM,IPAR,IPAR)
REAL APAR(NPAR)
COMMON /COMO/INTER,LIN,LOUT

DO 10 I=1,NPAR

C--- SCANNING CHECK AND GLOBAL VARIABLE ASSIGNMENT
IF (NV.GT.1, SCAN IF NV.EQ.1, UNKNOWN

C

IF (INTER.EQ.1) THEN
WRITE(6,80) ANAM(I)
FORMAT(' FOR PARAMETER ','A6', ' INPUT MV.V1.V2')
READ(5,90) MV.V1.V2
90 FORMAT(I3,2E15.5)
WRITE(LIN,99) MV,ANAM(I),V,V2
99 FORMAT(I3,6A,E15.5)
ELSE
READ(LIN,100) MV,NAM.XMX, V.V2
100 FORMAT(I3,2A,2E15.5)
WRITE(LOUT,110) MV,NAM.XMX, V.V2
C10 FORMAT(' MV,NAM.XMX, V.V2', 'I3,6A,E15.5)
C--- CHECK RELEVANT INPUT
IF (NAMX.EQ.ANAM(I)) CALL MSO(2,NAM)
ENDIF
NAMX=ANAM(I)
IF (NV.GT.1) THEN
C--- SCANNED INPUT
IF (NAMX.EQ.' ') CALL MSO(1,NAMX)
NSIN=NV
NAMX=ANAM
IPAR=I
XMIN=V
XMAX=V2
WRITE(LOUT,120) NAMX,NSIN,XMIN,XMAX

113
120  &  ***,.A6., WILL BE SCANNED:***
   &  ***,.VALUES FROM */.G10.2., TO */.G10.2.***
   & ELSE IF (MV.EQ.-1) THEN
   & C-- PARTICULAR UNKNOWN
   & IF (NAMY.EQ.,'EE',.) CALL MSG(6,NAMY)
   & IF (NAMY.NE.,'EE') CALL MSG(5,NAMY)
   & NAMY=NAMY
   & NPUN=1
   & WRITE(LOUT,120) NAMY

130  & ***,.SE WILL BE SOLVED FOR THE PARAMETER *,.A6.,***
   & ELSE
   & C-- SPECIFIED PARAMETER
   & WRITE(LOUT,140) NAM,V

140  & FORMAT(11,.A6.,'=',.015.5)
   & APAR(I)=V
   & ENDIF

10  CONTINUE
   RETURN
   END
APPENDIX D

THE BISTATIC REVERBERATION VOLUME AND SURFACE

The reverberation volume $V$ is the intersection of the receiver equivalent solid angle $\Omega_e$, with the volume between two prolate spheroids, defined by

$$r_1 + r_2 = R_1 + R_2 \pm \frac{CT}{2} \quad \text{(D-1)}$$

Referring to Figure D-1, $V$ can be computed by Huygens' theorem.

Figure D-1. The reverberating volume.
\[ V = A R_2 \theta_e \]  \hspace{1cm} (D-2)

with

\[ R_2 = \text{horizontal target-receiver distance} \]

\[ \theta_e = \text{equivalent elevation angle of the receiver} \]

\[ A = \text{area of the intersection of the equivalent azimuth} \]
\[ \text{angle } \phi_e \text{ by the ellipsoidal annulus defined by} \]
\[ \text{Eq. (D-1) (Shaded area of Figure D-2)} \]

At long ranges, due to refractive propagation, this definition has to be modified. First assume that the entire water column is inscribed, with a time stretching of the pulse proportional to range, as developed in Section 2.2.4. If the pulse length at the source is \( \tau_0 \), convolution with the channel impulse response results approximately in pulse lengths of \( \tau_1 = \tau_0 + \beta R_1 \) at the target and \( \tau = \tau_1 + \beta R_2 \) at the receiver. As developed in 2.2.6, an approximation for the time stretching is

\[ \beta = \frac{\theta^2}{\xi/6C_s} \]

The time range within which the scatterers will be included is \( \tau \). The volume extends then on the entire water column \( H \), yielding

\[ V = A H \]  \hspace{1cm} (D-3)

whether (D-2) or (D-3) is used, the calculation of \( A \) is the same, and is based on the following figure.
A is calculated by a double integral on range and azimuth

\[ A = \int_\phi \int_r r \, \text{d}r \, \text{d}\phi \]

where

\[ \phi = \phi_t \pm \frac{1}{2} \phi_e \]

\[ r_{\text{min}} < r < r_{\text{max}} \]

\( \phi_t \) is the target azimuth, \( \phi_e \) the receiver equivalent azimuth angle. For each angle \( \phi \), \( r_{\text{min}} \) and \( r_{\text{max}} \) are found from Eq. (D-1) which can be rewritten

\[ r + r_1 = R + \varepsilon \quad \text{(D-4)} \]

with

\[ R = R_1 + R_2 \]

\[ \varepsilon = \pm \frac{C_T}{2} \]

The law of cosines in Figure D-2 yields

\[ r_1^2 = r^2 + r_b^2 - 2r \, r_b \cos \phi \quad \text{(D-5)} \]
From Eq. (D-3)

\[ r_1^2 = (R + \varepsilon)^2 + r^2 + r^2 - 2r(R + \varepsilon) \]

which substituted into Eq. (D-5), gives

\[ r = \frac{1}{2} \frac{(R + \varepsilon)^2 - r_b^2}{R + \varepsilon - r_b \cos \phi} \] (D-6)

Developing and neglecting \( \varepsilon^2 \) with respect to \( R^2 \) in the long range, short pulse approximation

\[ r = \frac{1}{2} \frac{R^2 - r_b^2 + 2\varepsilon R}{R - r_b \cos \phi + \varepsilon} \]

Hence for each azimuth \( \phi \), \( r_{\text{min}} \) and \( r_{\text{max}} \) are

\[ r_{\text{min}} = \frac{1}{2} \frac{R^2 - r_b^2 - CR}{2R - r_b \cos \phi + \frac{1}{2} C} \] (D-7)

\[ r_{\text{max}} = \frac{1}{2} \frac{R^2 - r_b^2 + CR}{2R - r_b \cos \phi - \frac{1}{2} C} \]

The integration over \( r \) yields

\[ A = \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \frac{1}{2} (r_{\text{max}}^2 - r_{\text{min}}^2) \, d\phi \] (D-8)

Define

\[ a = R^2 - r_b^2 \]
\[ b = CR \]
\[ d = R - r_b \cos \phi \]
\[ e = \frac{1}{2} C \]

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so that (D-7) is rewritten

\[ r_{\text{min}} = \frac{1}{2} \frac{a - b}{d + e}, \quad r_{\text{max}} = \frac{1}{2} \frac{a + b}{d - e} \]  \hspace{1cm} (D-9)

Substitution of Eq. (D.9) into the integral (D-8) yields

\[ A = \frac{1}{2} \int \frac{a^2de + abe^2 + abd^2 + b^2de}{(d^2 - e^2)^2} d\phi \]

But in the long range, short pulse, several terms may be dropped

\[ e^2 \ll d^2 \text{ and } b^2 \ll a^2 \text{ yield} \]

\[ A = \frac{1}{2} \int \frac{a^2de + abd^2}{d^4} d\phi = \frac{1}{2} (R^2 - r_b^2) \text{ct} \int \frac{1}{2} \frac{(R^2 - r_b^2) + R(R - r_b \cos \phi)}{(R - r_b \cos \phi)^3} \]

Assume \( \phi_e \) is small. The integral may be replaced by its average.

Hence

\[ A = \frac{1}{4} (R^2 - r_b^2) \text{ct} \frac{3R^2 - r_b^2 - 2Rr_b \cos \phi}{(R - r_b \cos \phi)^3} \phi_e \]  \hspace{1cm} (D-10)

The angle \( \phi \) may be expressed in terms of distances, using Eq. (D-5)

\[ r_b \cos \phi = \frac{R^2 + r_b^2 - R_1^2}{2R_2} \]

which, substituted into (D-10), yields

\[ A = 2\text{ct} \phi_e \frac{R_1^2 + 4R_1R_2 + 5R_1^2}{2R_2} + 2R_2^3 - r_b(R_1 + 2R_2)}{(R^2 - r_b^2)^2} \]

And \( V = AR_2\theta_e \) or \( V = AH \), depending on whether the system is in short or long-range configuration. When the volume scatterers are within a layer, the expression \( V = AH \) holds for long ranges, provided \( H \) represents the scattering layer thickness. In the same manner, for long ranges and small grazing angles, \( A \) represents the bistatic reverberation surface.