# THE DEMAND FOR HOUSING IN THE UNITED STATED AND WEST GERMANY:

# A DISCRETE CHOICE ANALYSIS

by

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(1980)

Submitted to the Department of Economics
In Partial Fulfillment of the Requirements
For the Degree of

Doctor of Philosophy

at the

Massachusetts Institute of Technology

June 1984



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ABSTRACT

This thesis applies discrete choice techniques to an analysis of housing demand in the United States and West Germany. Housing demand comprises the choices of household formation, tenure, type of structure, and size and quality of dwelling. We will focus on peculiarities of housing demand which have found little attention in the otherwise ample literature on housing demand. Usage of similar surveys in the United States and West Germany allow us to make comparisons between the two countries and to identify mechanisms which are concealed by examining only one country.

The multidimensional heterogeneity of housing demand possibilities suggests the usage of discrete choice techniques, in particular hierarchically nested choice models. The theory and estimation of nested multinomial logit models is reviewed and extended.

Household formation as a dimension in housing demand renders household based models inappropriate. A model which simultaneously determines headship status and conventional housing demand is developed and estimated for U. S. data. Simulations mimicking the Experimental Housing Allowance Program underscore the importance of household formation as a factor in housing demand.

The tenant-landlord relationship over time is examined. Empirical evidence points to the existence of tenure discounts, leading to a bias in conventional price specifications. A microeconomic model highlights the potential misinterpretations in housing market analysis when tenure discounts are disregarded. As illustration, we evaluate the German rent and eviction control legislation.

Finally, we compare housing choices in the United States and West Germany. We apply a common analytical model on comparable data sets to examine the differences in tenure choice and size demand. Using this model, we try to isolate behavioral from institutional differences.

Thesis Supervisor: Dr. Daniel McFadden,
Elisabeth and James R. Killian Class of 1926 Professor of Economics

AXEL HELMUT BOERSCH-SUPAN

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## ACKNOWLEDGMENTS

It took a good seventh part of my life so far to fulfill what is called "the requirements for the degree of Doctor of Philosophy" on the title page of this thesis. Four years of neglecting wife and child, ignoring the amenities of Boston (almost), leaving my home country with parents and friends. I would not be a proper economist not to ask myself whether it was worth all the trouble, nor would I deserve the appendage Ph.D. not to take this opportunity to philosophize on what I have learned during this time.

Quite a lot. The benefits outweigh the costs. I learned economics, starting pretty much and frustratingly from scratch, in a pace and intensity which I -- finally -- enjoyed. It convinced me that my place is in applied economics rather than in abstract mathematics. And I learned to like my profession. Apart from economics, I enjoyed the company of such a variety of interesting fellow students and teachers as it is only possible in this Cambridgean ivory tower. I am grateful to all my friends for discussions, excursion, and symposiums in which we shared the ups and downs of a great time.

I own many thanks to Dan McFadden, my mentor and thesis advisor, on both accounts. He taught me the art of combining solid and well-founded econometrics with the interest in substantive but less palpable since real life issues. And he taught me where to eat the best Peking Duck in town. He contributed essentially to my positive

balance by being a friend in and outside the world of the Economics Department who shared the many incisive events in these four years and supported by word and deed our sometimes shaky two career plus child enterprise.

The four years of M.I.T. life were four years living in a new country. This was an exciting and mind-opening experience which I do not want to miss. I am grateful to the Studienstiftung in West Germany for providing the incentive to sail out to the New World and the generous financial support which made this possible without going through the stage of a dishwasher.

Large benefits do not come without costs. And the costs were borne not only by myself. It is a good and meaningful tradition to dedicate a thesis to the parents. I do this with pleasure and thankfulness. My parents put me on the road I am now pursuing, and always supported me with encouragement even when they had to bear most of the burden when I left Germany four years ago to go through yet another study.

The cost were large for my wife as well. What a time! We left our country, married, got our first child -- all this mixed with constant fickleness about exams, night-long glare at a computer terminal, and the final frenzy to get this thesis finished. It is a wonder that she still talks to me. Believe me, I am grateful for such wonders. After all, we did enjoy these four years, together.

Finally, these acknowledgments must include all those who contributed to this thesis by commenting on earlier drafts when the work was still in the snips and thoughts stage. Among those are Jerry Rothenberg and Bill Wheaton in my thesis committee, Wolfgang Eckart and Konrad Stahl in Dortmund, John Pitkin at the Joint Center, and Ray Struyk at the Urban Institute. The Ifo-Institut fuer Wirtschaftsforschung in Munich granted permission to quote from an unpublished interim report and supported the German part of my research.

The work was exciting and rewarding, but it was the acquaintance with and the support of so many people which made this time worth a considerable part of my life.

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## CHAPTER ONE

## INTRODUCTION AND SURVEY

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## 1.1 Substantive Issues

This thesis investigates the demand for housing as a heterogeneous commodity. We will focus on the demand side of the housing market in most of our empirical work and restrict the analysis to partial models, that is, on potential demand under perfect elastic supply. This long run analysis is in tune with the use of two large cross-sectional data sets and their interpretation as steady state equilibria: the Annual Housing Survey in the United States and the One Percent Sample in West Germany. The only deviation from our concentration on demand only will be in the theoretical model of the nature of those steady state equilibria.

To capture the heterogeneity of the commodity housing we will introduce a comprehensive notion of what housing demand consists of: it includes the choices of quality, size, tenure, and headship status. We will not consider choice of location, however, and we will concentrate on large metropolitan areas.

Although housing demand is a well studied field, there are still a host of unresolved substantive issues. Having introduced the notion of the commodity housing as a broad class of different housing categories or alternatives, a general issue is the question of substitutability among these categories. In Joan Robinson's (1933) words, we are looking for the gaps in the chain of substitutes. Do people easily substitute larger dwellings for smaller dwellings in response to price increases, or do they switch tenure? Does the substitutability among housing alternatives change with the life cycle? Are there differences in behavior between the United States and West Germany?

Furthermore, how does household formation as a dimension of housing demand fit into the chain of substitutes? Is household formation responsive to price changes in the rental and owner markets? Does this response depend on the stage in the life cycle or on demographic characteristics?

Apart from interest in the structure of a comprehensive housing demand per se, we might ask ourselves how this structure is reflected in policy analysis. The tax codes in both countries are asymmetric in their treatment of owner-occupancy versus rental housing, but it is not clear whether all of the observed preferences in the tenure choice can be explained by taxes alone. Would a drastic tax change induce a drastic change in the preference for tenure? A comparison and West Germany and the United States seems to be of particular interest due to their very different proportions of owner-occupancy (1978: U.S.: 65.2 percent, Germany: 36.3 percent). Are there repercussions in the other dimensions of housing choice? What about the response of household formation to changes in the tax code? And, more interesting for the latter dimension, is there a response to direct demand subsidies like housing allowances? Is the Experimental Housing Allowance Program flawed in its complete ignorance of household formation?

A final topic of this thesis and another red thread through the five chapters is the question of what the proper price is for this heterogeneous and durable good. In the last Chapter, we will use hedonic indexes to capture heterogeneity, an at least empirically resolved issue. However, the durability of housing has implications on intertemporal pricing which are not well understood. Is there price dispersion in the housing market? How can it be explained? Can different explanations be empirically tested against each other? How does the existence of price dispersion affect our knowledge of price responsiveness? And finally, do we have to reevaluate policy analysis in the presence of price dispersion? How does normative analysis of rent and eviction control change in a non-walrasian market with price dispersion?

## 1.2 Methodological Issues

The comprehensive notion of housing demand as the choice among a collection of heterogeneous alternatives raises many methodological issues. There is the question of the appropriate functional form for housing demand equations which include the qualitative and quantitative components of the commodity housing. We will resolve this question in simply dividing the qualitative dimensions into sufficient of discrete categories, and proceed with large discrete choice models.

However, the specification of large discrete choice models is closely related to the question of substitutability among the choices which was raised in the previous section. Is there a feasible compromise between choice models which are easy to compute but impose strict cross-substitution patterns, and choice models which leave freedom for the cross-substitution effects but are computationally intractable? We will show that nested multinomial logit models (McFadden, 1978) constitute such a compromise in housing demand analysis. The unresolved issues at stake is the efficiency loss of the sequential estimation technique and the viability of full information maximum likelihood. A further theoretical issue is whether the estimation results can be rationalized by a highly structural economic choice model, the random utility hypothesis.

Household formation as a part of housing demand raises the question of how usable our household based surveys are. The entity "household" is endogenous and we face a potential self selection bias in our estimations. How can we resolve this sample selection problem? Is it possible to avoid a structural model of household formation which is bound to be poorly estimable due to our poor knowledge about this process and will result in a large noise-to-signal ratio? Can we find a reduced form approach with just enough structure to resolve the endogeneity problem?

A final methodological issue is the handling of price dispersion generated by intertemporal processes when only a single cross section of observations is available. How large is the potential bias in estimations ignoring price dispersion? How do we get unbiased results?

## 1.3 Organization of the Thesis

The remainder of the thesis is organized in four chapters. The first of these chapters is devoted to the microeconomic and econometric underpinnings of our basic tool, the nested multinomial logit demand functions. The microeconomic part includes the compatibility of these demand functions with random utility maximization, the econometric part discusses the use of full information maximum likelihood estimation.

Chapter Three applies the demand model on the joint choice of household formation, tenure, and size of dwelling to three SMSA's in the United States to answer the question of how price responsive our comprehensive housing demand is. Some microsimulation results illustrate public policy implications.

Chapter Four is a digression on the dynamic nature of the rental housing market. This chapter, mainly microeconomic theory, is intended as a motivation and guidance to analyse price dispersion generated by a complicated intertemporal interaction of the demand and supply side.

Finally, Chapter Five applies all the tools we have collected so far on an analytic comparison of housing demand in West Germany and the United States: A common hierarchical choice model for both countries embodies hedonic rent indexes to capture the heterogeneity of the housing stock and corrects for price dispersion in both countries.

Each chapter contains an introductory section to present the issues at stake, and a conclusion to summarize the results. An appendix lists the FORTRAN source of the full information maximum likelihood estimation program for nested multinomial logit models in random and choice based samples.

# CHAPTER TWO

## THE BASIC TOOL: NESTED MULTINOMIAL LOGIT DEMAND FUNCTIONS

## 2.1 Introduction: Discrete Choice Description of Housing Demand

Housing or, more precise, the service stream from a housing unit, is a heterogeneous commodity. Some dimensions, as size or age of structure, are measured on a continuous scale, others, as tenure or type of structure, are discrete properties. Measuring the volume of housing services as housing expenditure essentially ignores this heterogeneity, and for a large number of policy purposes, the distribution of housing consumption into qualitatively different categories is of more interest than an aggregate quantitative measure of housing expenditures alone. The most popular example of the interest in qualitative dimensions is the choice between renting and owning, and the response of this tenure choice to federal income tax treatment. (See Laidler (1969), Rosen (1979), Rosen and Rosen (1980), Henderson and Ioannides (1983).)

We can go one step further: not only the choice of tenure, but also the choices among other continuous or discrete characteristics of a housing unit will be affected by taxes and subsidies. Furthermore, the decision whether to form an autonomous household at all may be dependent on relative prices and income. Thus, housing demand decisions consist of discrete decisions, e. g., concerning headship, tenure, as well as continuous decisions, e. g., size or quality level.

Lee and Trost (1978) and subsequently King (1980) argue that the tenure choice and the choice of size and quality level are made simultaneously. We will point out in Chapter 3 that also the headship choice is influencing and is in turn influenced by the other two decisions, so that all three choices are made in a joint decision process. This joint decision process constitutes a comprehensive notion of housing demand which will the focus of this thesis.

The econometric theory of joint discrete/continuous models is well studied, and there exist a variety of applications, e. g. Lee and Trost (1978), King (1980), or Dubin and McFadden (1984). We will not pursue this line of modeling, however, but use consistently a discrete choice framework throughout this work. Sweeney (1974) casts the entire bundle of quality characteristics into discrete categories so that housing units can be arranged in a commodity hierarchy. We will use a similar discretization of the quality space in a finite number of housing alternatives. Discretization of continuous variables has widely been applied in transportation economics, see Chiang, Roberts, and Ben- Akiva (1982) for a model of freight mode and shipment-size, or Small (1981,1982) and Small and Brownstone (1981) for discrete models of trip timing. Ben Akiva and Watanatada (1981) provide a theoretical analysis of the aggregation of a continuous variable into a finite number of discrete choices. There is good pragmatic reason to do so: it simplifies both the theoretical analysis and the empirical estimation. In addition, for most policy purposes, it suffices to explain or predict shifts among rough categories as "large owner-occupied houses", "low quality rental housing", or "non-headship." Table 2-1 lists the choices we will consider in our comprehensive notion of housing demand.

Table 2-1: Definition of Housing Alternatives

| Symbol | Housing Alternative |
|---|---|
| NH | Non Headship: lives as a subnucleus in another household |
| O_SF.S | Owner-Occupied, Single-Family-Structure, Small Dwelling |
| O_SF.M | Owner-Occupied, Single-Family-Structure, Medium Dwelling |
| O_SF.L | Owner-Occupied, Single-Family-Structure, Large Dwelling |
| O_2F.S | Owner-Occupied, Two-Family-Structure, Small Dwelling |
| O_2F.M | Owner-Occupied, Two-Family-Structure, Medium Dwelling |
| O_2F.L | Owner-Occupied, Two-Family-Structure, Large Dwelling |
| O_MF.S | Owner-Occupied, Multi-Family-Structure, Small Dwelling |
| O_MF.M | Owner-Occupied, Multi-Family-Structure, Medium Dwelling |
| O_MF.L | Owner-Occupied, Multi-Family-Structure, Large Dwelling |
| R_SF.S | Rental-Housing, Single-Family-Structure, Small Dwelling |
| R_SF.M | Rental-Housing, Single-Family-Structure, Medium Dwelling |
| R_SF.L | Rental-Housing, Single-Family-Structure, Large Dwelling |
| R_2F.S | Rental-Housing, Two-Family-Structure, Small Dwelling |
| R_2F.M | Rental-Housing, Two-Family-Structure, Medium Dwelling |
| R_2F.L | Rental-Housing, Two-Family-Structure, Large Dwelling |
| R_MF.S | Rental-Housing, Multi-Family-Structure, Small Dwelling |
| R_MF.M | Rental-Housing, Multi-Family-Structure, Medium Dwelling |
| R_MF.L | Rental-Housing, Multi-Family-Structure, Large Dwelling |

Estimating such a complex joint decision process poses a number of econometric problems: the choice set, that is the set of housing alternatives from which the consumer has to select one, is fairly large and consists of alternatives of which some are close substitutes and others not. The first problem restricts the possible specifications of the functional form of the relation between the choice probabilities and the explanatory variables to functions that have a structure which simplifies the computations involved, e. g., the class of generalized extreme-value functions. On the other hand, the second problem prohibits the use of simplifying assumptions like the Independence of Irrelevant Alternatives which reduces the multinomial decision to binary comparisons. As a viable compromise between computational simplicity and economic complexity, we will nested multinomial logit models (NMNL) as the basic analytic tool for our empirical research. The remainder of this chapter reviews the microeconomic foundations and the econometrics of NMNL-models and extends the theory of the relation between utility maximization and estimated NMNL-parameters.

## 2.2 Random Utility Maximization and Hierarchical Choice

### 2.2.1 Microeconomic Theory

Let us assume the housing market is partitioned into M discrete housing alternatives, e.g., as depicted in Table 2-1. We associate each of these alternatives with an index of desirability, which comprises all advantages and disadvantages for a given consumer into one scalar unit corresponding to the indirect utility function in neoclassical continuous consumer theory. Uncertainty about quality and erratic or irrational valuations introduce a stochastic component into this index. Like the hypothesis of utility maximization under budget restriction, we assume that each household will choose the alternative with the highest index of desirability. Due to the probabilistic nature of the index, we will call this the random utility maximization hypothesis (McFadden, 1981).

In the following, we will give this notion a more precise definition. For each household t we decompose the desirability index $u_{it}$ of the alternative i into a deterministic and a stochastic component:

(2.1) $u_{it} = v_{it} + e_{it}$

The stochastic component $e_{it}$ is drawn from a M-dimensional joint distribution characterized by the cumulative distribution function $F(e_1,...,e_M)$ with an associated finite-valued[1] density $f(e_1,...,e_M)$. The deterministic part $v_{it}$ is dependent on the characteristics of the alternative (e. g., price) as well as on the characteristics of the household (e. g., income), and is linear and additive separable[2]:

(2.2) $$v_{it} = \sum_k x^k_{it} * b_k + \sum_l y^l_t * a_{il}$$

where $x^k_{it}$ = the k-th characteristic of alternative i for household t,
$y^l_t$ = the l-th characteristic of household t,
$a_{il}$, $b_k$ = weights (to be estimated).

Choices are made by pairwise comparison of utilities. Thus, only M-1 differences of utilities describe the choice behavior[3]. This implies that household specific variables that are alternative invariant will be irrelevant for the choice among alternatives as long as they do not interact with each alternative. We therefore let the weights of the household characteristics vary by alternative[4].

In addition to uncertainty and erratic valuations, the stochastic disturbance $e_{it}$ will pick up deviations of the household t from the weights $a_{il}$ and $b_k$ in the population. The different components of $e_{it}$ can not be identified or only under specific assumptions.

Household t will choose alternative i, if $u_{it} > u_{jt}$ for all $j \neq i$. Thus, the probability that household t chooses i among all M possible alternatives is

$$p_t(i) = \text{Prob}\{ v_{it}+e_{it} > v_{jt}+e_{jt} \mid j \neq i \}$$

(2.3) $$= \int_{e_{it}=-\infty}^{\infty} \int_{e_{1t}=-\infty}^{u_{it}-v_{1t}} \cdots \int_{e_{Mt}=-\infty}^{u_{it}-v_{Mt}} dF(e_{1t},\ldots,e_{Mt})$$

where F denotes the joint cumulative distribution function of the errors $e_{it}$.

Definition (Random Utility Maximization)

Choice probabilities $p_t(i)$ are said to be generated by random utility maximization, if there exists a random utility function (2.1), characterized by a linear, additive separable deterministic utility (2.2) and a distribution function F with a finite-valued density of the stochastic utility, such that (2.3) holds.

Finally, the aggregation

$$f(i) = \frac{1}{T} \sum_{t=1}^{T} p_t(i) \qquad i=1,\ldots,M \tag{2.4}$$

will yield the relative frequencies of alternative i in the population, also called aggregated or market shares of choice i, provided the households t are a random sample of the population.

## 2.2.2 Functional Specification of the Choice Probabilities

This theory has a very important implication: for a given specification of the deterministic utility $v_{it}$, the choice of a functional form for the relation between the choice probabilities $p_t(i)$ and the explanatory variables $x^k_{it}$ and $y^l_t$ is equivalent to the specification of the joint distribution F of the error terms $e_{it}$.

The integral formula (2.3) shows the dilemma for this choice. On one hand, the correlation among the $e_{it}$ should be as flexible as possible to allow different correlations among the choice probabilities. On the other hand, the computational effort of evaluating the multidimensional integral should be minimized, suggesting a distribution function F where this can be done explicitly. This in particular prohibits the use of a normal distribution for problems with more than four alternatives.

Two families of distribution functions allow easy evaluation of the integral. One leads to a linear functional relation between the choice probabilities and the explanatory variables, and thus does not take account of the adding up and the unity interval restrictions of the choice probabilities. The other family is that of generalized extreme-value distributions, an extension of the logit approach; this is the family we will use to specify the choice probabilities.

A completely free correlation structure of the disturbances implies the estimation of M*(M-1)/2 correlation coefficients which is impractical for most sets of alternatives. Thus, further restrictions are necessary. The most drastic restriction is to postulate the independence of the $e_{it}$. Then the multidimensional integral can be factorized into a product of simple integrals. If in addition the $e_{it}$ are extreme value distributed, the resulting choice probabilities are of the well-known multinomial logit form. An application of the latter specification to the housing market can be found in Quigley (1976).

The assumption of independent $e_{it}$ is known as "Independence of Irrelevant Alternatives" (McFadden, 1973) due to the following necessary and sufficient characterizations:

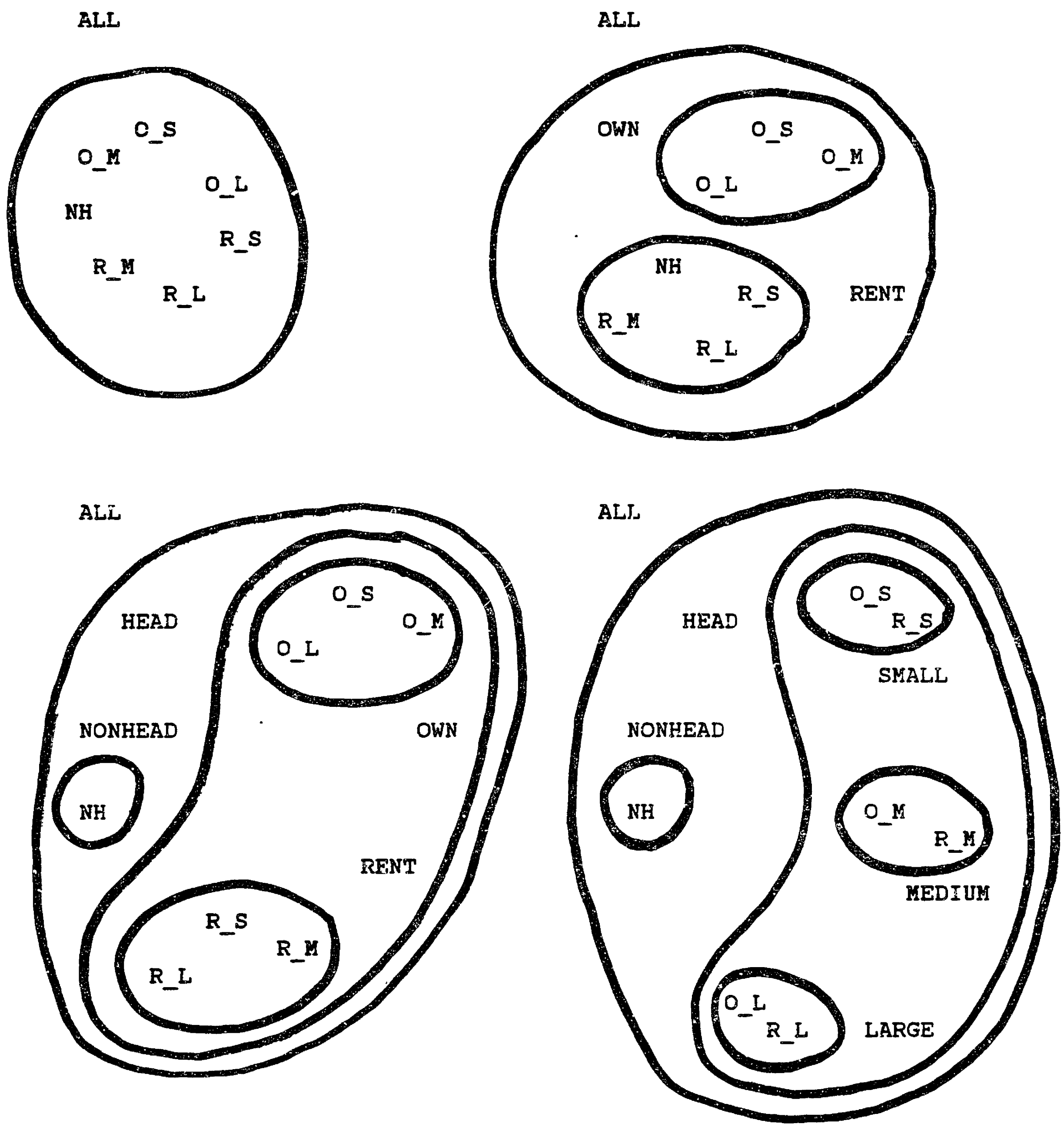
(1) The $e_{it}$ are stochastically independent.
(2) The odds of choosing alternative i over alternative j are independent of the attributes of all other alternatives and independent of the existence of any other alternative.
(3) The elasticity of the relative frequency f(i) of alternative i with respect to the attributes of any other alternative $j \neq i$ is constant, that is independent of j.

Therefore, independence can only be assumed for alternatives that are "equally different," but not for alternatives with different degrees of substitution. The following example translates a classical example (Domencich and McFadden, 1975) into the housing market. For simplicity, consider the tenure choice. Let us assume the relative odds are 1:1 for renting versus owning. Let us introduce a third, new form of tenure (e.g., cooperative) which is a very close substitute for owning. Intuitively, we would expect the new distribution to be something like 50% : 25% : 25%. But condition (2) tells us that the relative odds of renting versus owning have to stay constant, forcing the new distribution to be 33% : 33% : 33%, which is implausible because of the similarity of owning individually and owning cooperatively.

The failure to accommodate different degrees of cross-alternative substitution renders the multinomial logit specification inappropriate for such heterogeneous choice sets as depicted in Table 2-1. On the other hand, the possibility of grouping or clustering the alternatives according to their degree of substitution allows us a relatively straightforward way of combining the computational simplicity of the multinomial logit form with a richer substitution pattern: for each cluster, we introduce a parameter that describes the similarity of its alternatives. We can do the same with clusters themselves, and thereby achieve a hierarchical structure of similarities and substitution patterns. Within each cluster and between the clusters, we apply multinomial logit choice probabilities. This approach is called "Nested Multinomial Logit" (NMNL). McFadden (1981) gives a discussion of the development of these models and their relation to other discrete choice approaches.

For the application at hand, let us introduce three steps of clustering. First, we bundle housing alternatives by size and quality, then these clusters by tenure and type of building, and finally by all headship alternatives versus the nonheadship alternative, see Figure 2-2 for a simple example. We can look at NMNL models in two ways: they represent hierarchically grouped clusters of alternatives with a large within group substitutability, and we can interpret them as hierarchical decision processes or decision trees,

FIGURE 2-2: CLUSTERS OF SIMILAR ALTERNATIVES




NH: non-head
R: rented
O: owned
S: small
M: medium
L: large

where each nucleus decides whether to head a household or not, the heads decide about tenure, and owners and renters choose their dwelling size and quality. Of course, this does not necessarily imply a temporal decomposition of the decision process. Figure 2-3 represents this second interpretation graphically, and the equivalence to the representation of Figure 2-2 can be seen in each of the steps.

We can decompose the choice probabilities for a three-level hierarchical decision process into a marginal choice probability at the highest level of the decision tree and conditional probabilities at each lower level (we suppress the index t for the individual household):

$$(2.5) \quad p(i) = p_H(H_i) * p_T(T_i|H_i) * p_S(S_i|H_i,T_i)$$

where

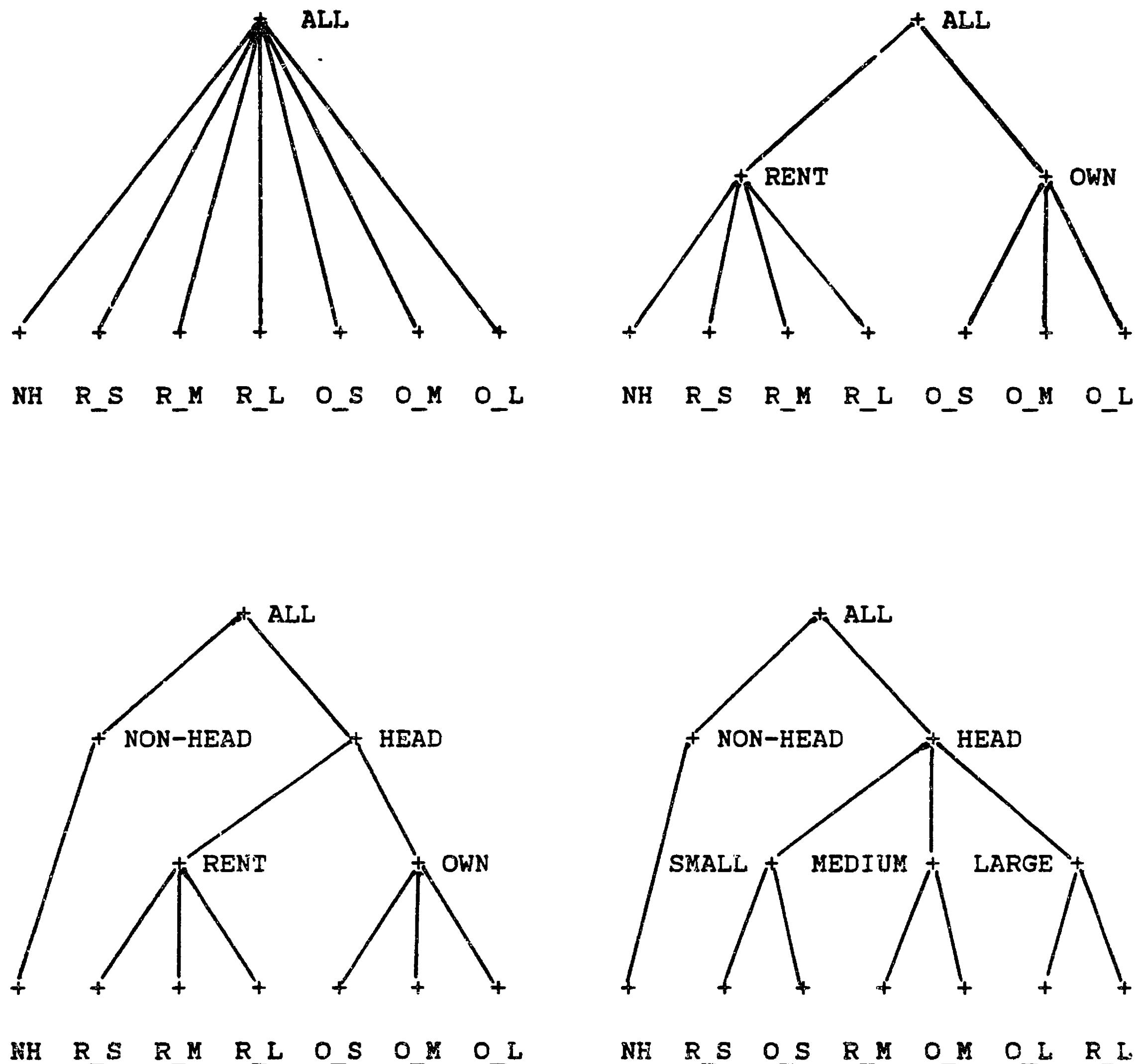
$p_H(H_i)$ = probability of the headship choice $H_i$ that is implied by choosing alternative i,

$p_T(T_i|H_i)$ = probability of the tenure choice $T_i$ implied by choosing alternative i, given headship choice $H_i$,

$p_S(S_i|H_i,T_i)$ = probability of the size choice $S_i$ implied by alternative i, given headship choice $H_i$ and tenure choice $T_i$.

At each level, the conditional choice probabilities have the multinomial logit form:

FIGURE 2-3: DECISION TREES FOR HOUSING ALTERNATIVES




NH: non-head
R: rented
O: owned
S: small
M: medium
L: large

$$p_S(S_i | H_i, T_i) = \exp(\ v(S_i)\ ) \Big/ \sum \exp(\ v(S_j)\ ),$$

(summation over all size choices $S_j$ possible in tenure choice $T_i$)

$$p_T(T_i | H_i) = \exp(\ u(T_i)\ ) \Big/ \sum \exp(\ u(T_j)\ ),$$

(summation over all tenure choices $T_j$ in headship choice $H_i$)

$$p_H(H_i) = \exp(\ w(H_i)\ ) \Big/ \sum \exp(\ w(H_j)\ ),$$

(summation over all headship choices $H_j$)

In these choice probabilities, $v(S_i)$ denotes the utility, a consumer derives from dwelling size $S_i$, and $u(T_i)$ ( $w(H_i)$ ) the utility from tenure choice $T_i$ (headship choice $H_i$, respectively) implied by choosing alternative i.

At the higher levels, we assume that there is no utility per se of either headship or tenure over and above the utility derived from the alternatives underlying each tenure or headship choice. If we aggregate the utility provided by all alternatives $S_j$ in tenure category $T_i$ we obtain (McFadden, 1978):

$$(2.6) \quad u(T_i) = \log \sum_{j \in T_i} \exp\ (\ c_j * v(S_j)\ )$$

and similarely for the aggregated utility of the headship choice $H_i$:

$$(2.7) \quad w(H_i) = \log \sum_{j \in H_i} \exp\ (\ d_j * u(T_j)\ )$$

Note that the attributes of the lowest level utility, e. g., income and prices in the size choice, enter recursively, bottom-to-top, the utility of the tenure and headship choices. On the other hand, decisions are clustered in a sequential fashion, top-to-bottom, as Figures 2-2 and 2-3 suggest. Altogether , we have achieved a simultaneous choice of headship, tenure, and dwelling size where prices as well as income are allowed to influence not only the size and tenure choice, but also the headship decision and thus household formation.

The taste weights $c_j$ and $d_j$ have to be estimated. The aggregate utility levels $u(T_i)$ and $w(H_i)$ are called inclusive values of their respective lower level alternatives because they can be interpreted as the surplus generated by these alternatives. The taste weights $c_j$ and $d_j$ are called dissimilarity parameters because they can be interpreted as a measure of the substitutability among the respective lower level alternatives. For $c_j$ and $d_j$ equal to one, the decision tree model collapses to a simple multinomial logit choice model among all alternatives. If they are smaller than one, alternatives in the respective clusters are close substitutes relative to other alternatives.

We can test whether the difference between the simple MNL model and the nested MNL model is significant. Usually, the MNL model has to estimated in a first stage to obtain initial values for the NMNL estimation. Thus, we have all ingredients of a lekilihood ratio test, see Table 5-13 for examples. Furthermore, we can construct a Wald test based on the estimated dissimilarity coefficients in the NMNL model with their joint covariance matrix. For simple one-dimensional tests we can look at the asymptotic t-statistics around one as reported in the estimation results. Finally, we can calculate a Lagrange multiplier test and evaluate this test at the simple MNL estimates, see McFadden (1983) for the appropriate formula. The asymptotic and small sample properties of this test trinity is examined by Hausman and McFadden (1981). These test are tests of the simple MNL functional specification versus the nested MNL model. Thus, they amount to tests of the independence of irrelevant alternatives property.

The random utility maximization interpretation of the NMNL functional form rested on the integral formula (2.3). However, the cumulative distribution function F is parameterized by the taste weights $a_{il}$, $b_k$, the dissimilarity coefficients $c_i$, $d_i$, and depends on the data as well. If all similarity parameters are in the unit-interval, the underlying joint distribution of the disturbances is well behaved and consistent with the microeconomic theory outlined at the beginning of this section, independent of the explanatory variables. With similarity parameters outside the unit-interval, this consistency will hold only for a certain range of explanatory variables, and it must be checked, whether this range includes the given data. This check and a reconciliation of such NMNL models with the random utility maximization hypothesis is discussed in the following.

### 2.2.3 The Relation Between Dissimilarity Parameters and the Random Utility Hypothesis

Let $c_T$ denote the similarity coefficient corresponding to the first-order clusters of elementary alternatives (say, tenure categories), and $d_H$ the similarity coefficient corresponding to the second-order clusters consisting of first-order clusters (say, headship categories). The NMNL functional form specified in (2.5) is then equivalent to the following joint cumulative distribution function of the errors $e_{it}$ in (2.1) (McFadden 1978):

$$(2.8) \qquad F(e_1,\ldots,e_M) = \exp \{ -G [ \exp(-e_1),\ldots,\exp(-e_M) ] \}$$

with

$$(2.9) \qquad G[y_1,\ldots,y_M] = \sum_{H} ( \sum_{T \in H} ( \sum_{S \in T} y_S^{1/c_T} )^{c_T/d_H} )^{d_H}$$

where we sum over the highest-order clusters H, the first-order clusters T contained in each cluster H, and finally over the elemental alternatives S in each cluster T.

Two theorems provide the link between NMNL-models and the random utility maximization hypothesis (RUM). They are global statements in the sense of being independent of the realization of the explanatory variables.

<u>Theorem 1 (Global Sufficiency) (McFadden 1979):</u>

Let $0 < d_H \leq 1$ and $0 < c_T/d_H \leq 1$ for all T and H.

Then the NMNL model is consistent with RUM for any data.

<u>Theorem 2 (Global Necessity) (Williams 1977, Daly and Zachary 1979):</u>

Let $d_H > 1$ or $c_T/d_H > 1$ for at least one T or H.

Then it is always possible to construct data at which the NMNL model is inconsistent with RUM.

The question arises, whether this possibility of RUM-inconsistent data points is relevant for the data at hand. Theorem 2 leaves the possibility open that for the data given by the application, the NMNL model is consistent with RUM, and that the data points where the inconsistency occurs are insensible for the given application. Thus, the purpose of the remainder of this Section is to construct a discrete choice model that (1) is compatible with RUM, (2) has the same cumulative distribution function F for the given data points, and (3) preserves the choice probabilities of the original NMNL model. We shall give a necessary and sufficient condition under which such a construction is possible.

The failure of the random utility hypothesis occurs because the estimated dissimilarity coefficients prevent F from (2.8) being a cumulative distribution function:

Lemma 1:

Let $c_T$ and $d_H > 0$.

(1) Then F is differentiable to any order, in particular, all mixed partial derivatives exist.

(2) $F \to 1$ for $e_i \to \infty$ , $F \to 0$ for $e_i \to -\infty$.

(3) The choice probabilities derived from F obey

$0 \le p_t(i) \le 1$ and $p_i(t) = 1$.

The Lemma follows from the obvious properties of (2.5),(2.8), and (2.9). Thus, Theorem 2 implies the existence of at least one point in which F has a negative mixed partial derivative, i. e., a point of negative marginal or joint "density".

We shall illustrate the effect of $d_H > 1$ in the simplest case of a three-alternative, two-level NMNL-model in which the first two alternatives constitute a cluster. This model has the cumulative distribution function

$F(e) = \exp \{ - [ \exp(-e_1/d) + \exp(-e_2/d) ]^d - \exp(e_3) \}$.

Because of translation invariance (see Footnote 3), we can reduce F to the two dimensional space of differences without loosing information. Let $v = e_2-e_1$ and $w = e_3-e_1$. Their joint distribution function is

$$F^*(v,w) = \frac{( 1+\exp(-v/d) )^{d-1}}{( 1+\exp(-v/d) )^d + \exp(-w)}$$

with density

$$f^*(v,w) = \frac{( 1+\exp(-v/d) )^{d-2}}{( 1+\exp(-v/d) )^d + \exp(-w)} * \exp(-w) * \exp(-v/d) * \text{discr},$$

where the discriminant term

$$\text{discr} = \frac{2(\ 1+\exp(-v/d)\ )^{d}}{(\ 1+\exp(-v/d)\ )^{d} + \exp(w)} - \frac{d-1}{d}$$

signs the density $f^*$.

For $d < 1$, discr $> 0$. However, for $d > 1$,

$f^*(v,w) > 0 \iff w > \log(d-1)-\log(d+1) - d \log(1+\exp(-v/d))$.

This function approaches the constant $\log(d-1)-\log(d+1)$ for $v \to \infty$ and a by this constant shifted 45-degree line for $v \to -\infty$. Thus, we can partition the (v,w)-plane in a part with nonnegative and a part with negative density, see Figure 2-4. Note that the latter part cannot be contained in any choice probability defining orthant.

If any of our data points is in the (shaded) area $f^* < 0$, we will not be able to explain the data by preference maximization using the integral formula (2.3) underlying RUM. Moreover, if $f^*(e) < 0$ for some e, then the continuity of $f^*$ implies the existence of a point e with $F^*(e) > 1$. This point need not necessarily be in the set $\{f^* < 0\}$. Again, we cannot rationalize the data by RUM. Thus,

<u>Theorem 3 (Local Necessity):</u>

Let A be a set containing all data points.

Let any one of the following conditions be true:

(1) A mixed partial derivative of F up to order M is negative at a point in A.

(2) F exceeds unity at a point in A.

Then the construction of a RUM-compatible discrete choice model in A is not possible[5].

The proof is trivial: F cannot be a c.d.f. if it exceeds unity or any of its associated marginal or the joint densities is negative.

If neither of the conditions of Theorem 3 is raised, we can indeed reconcile a NMNL-model with large dissimilarity coefficients with random utility maximization:

<u>Theorem 4 (Local Sufficiency):</u>

Let A be an open interval containing all data points.

Let both of the following conditions be true:

(1) All mixed partial derivative of F up to order M are nonnegative in A.

(2) F does not exceed unity at a point in A.

Then for any positive $c_T$, $d_H$ exists a continuation $F^c$ of F, such that

(a) $F^c$ is a cumulative distribution function,

(b) $F^c$ generates the same choice probabilities as F.

We will give the proof by construction. Figure 2-5 illustrates the construction in the case of three alternatives, already underlying Figure 2-4. We first use the translation invariance principle to reduce the dimension of the problem to M-1: for any point y in $R^M$, let $y^*$ be the N=M-1 dimensional vector $(y_1-y_2,\dots,y_1-y_M)'$. Correspondingly, we introduce $A^*$, $F^*$, and $f^*$ as the M-1 dimensional counterparts of A, F, and f. For a set C, we use int(C), bnd(C), ext(C), and clo(C) to denote the interior, the boundary, the exterior,

FIGURE 2-4: THREE ALTERNATIVES: CHOICE PROBABILITIES AND DENSITY

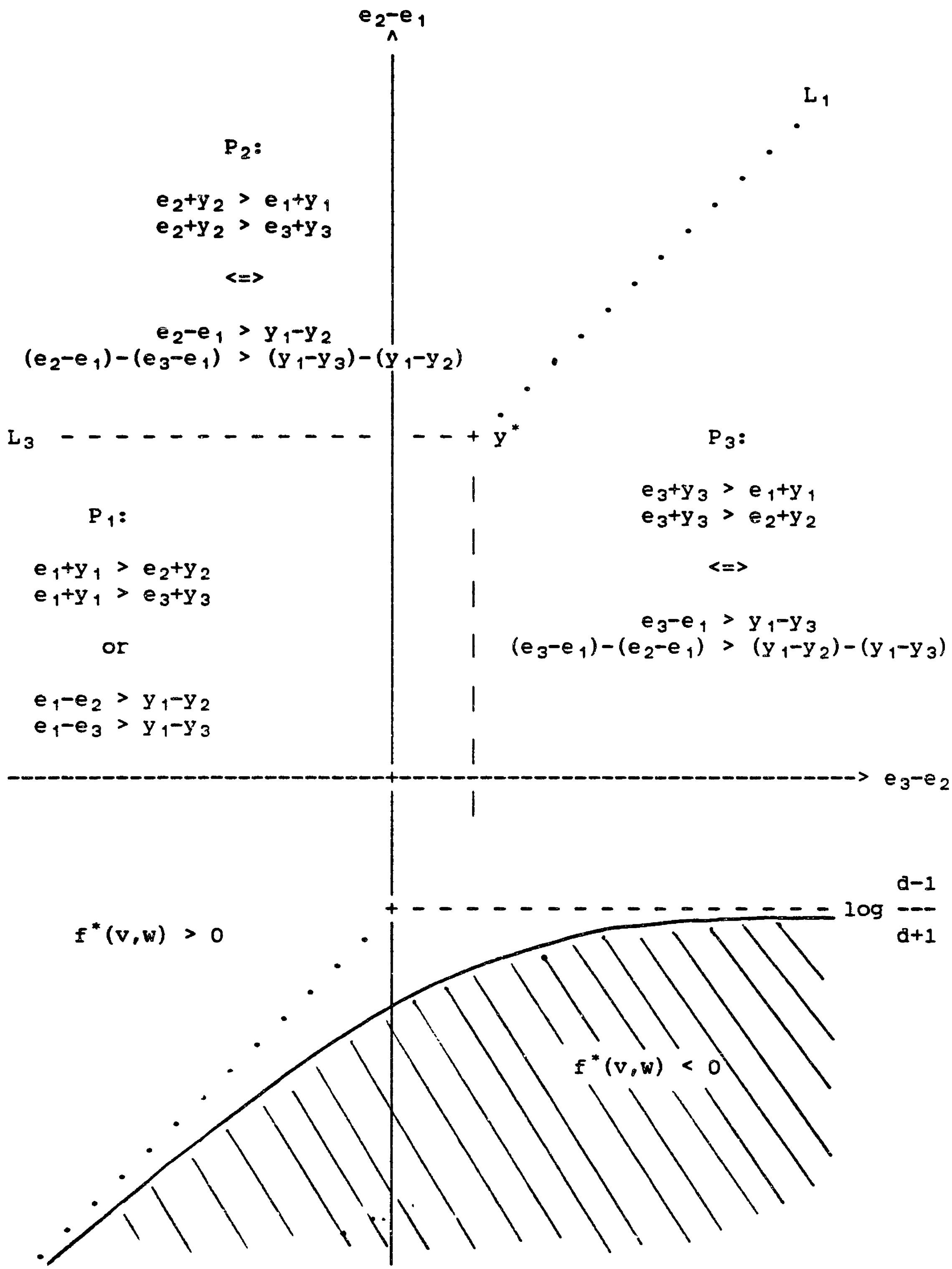
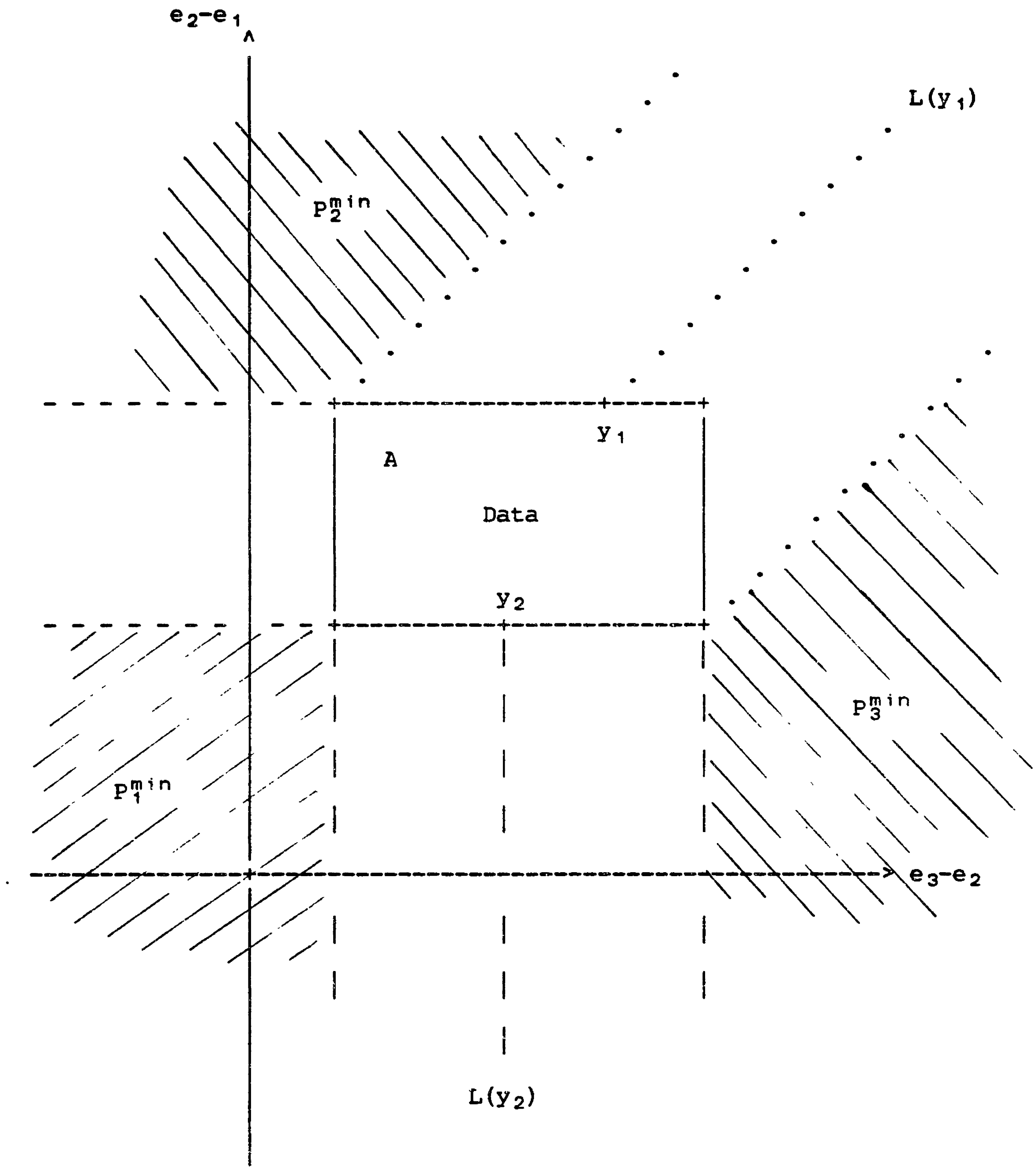


FIGURE 2-5: CHOICE PROBABILITY PRESERVING CHOICE MODELS



and the closure of C, respectively.

For each $y^*$, we can partition $R^N$ according to the M choice probability defining open sets

$P_i(y^*) = \{ e^* \text{ in } R^N \mid e_i+y_i > e_j+y_j \ , \ j\neq i \}$

and the M separating half-hyperplanes

$L_i(y^*) = \{ e^* \text{ in } R^N \mid e_i+y_i < e_j+y_j \ , \ j\neq i \ ; \ e_k+y_k = e_j+y_j \ , \ j,k\neq i \}$.

The sets $P_i(y^*)$ and $L_i(y^*)$ will be important at the boundary of A, and it is convenient to define for $y^*$ in bnd($A^*$):

$$P_i^{min} = \bigcap_{y^* \text{ in } A^*} P_i(y^*)$$

and

$L(y^*) = L_i(y^*)$ with i such that $L_i(y^*) \cap$ clo($A^*$) is empty.

The $P_i^{min}$ define the smallest choice probability of alternative i attainable in clo($A^*$) where this well-defined minimum occurs for each i at one of M corners of the interval $A^*$. The $L(y^*)$ define the half-hyperplane pointing outward of $A^*$. This is well-defined except for the above mentioned M corners. Here, we define:

$L(y^*) = P_i^{min}$.

We now construct a choice model which has identical choice probabilities in $A^*$ by shifting all probability mass outside of $A^*$ onto the boundary of $A^*$ in a way which does not distort the original choice probabilities.

We define

$$f^{*c}(y) = f^{*}(y) \quad \text{for} \quad y \text{ in } A^{*},$$

$$f^{*c}(y) = \int_{L(y^{*})} dF^{*} \quad \text{for} \quad y \text{ in } \mathrm{bnd}(A^{*}),$$

$$f^{*c}(y) = 0 \quad \text{for} \quad y \text{ in } \mathrm{ext}(A^{*}).$$

We have to show:

(i) $f^{*c}$ nonnegative,

(ii) $f^{*c}$ integrates to one,

(iii) $p_t(i) = \int_{P_t(i)} dF = \int_{P_t(i)} dF^{c}$.

To (i):

We only need to show the nonnegativity at the boundary.

Case 1: y is at one of the M corners defining $P_i^{min}$.

Then the $f^{*c}(y)$ is a choice probability which is always nonnegative for positive $c_T$, $d_H$ by Lemma 1.

Case 2: y otherwise.

Then $\int_{L_i(y^{*})} dF^{*} = \lim_{h \to 0} 1/h \left( \int_{P_j(y^{*})} dF^{*} - \int_{P_j(y^{*}-hz_j)} dF^{*} \right)$

where j is an arbitrary index other than i and z a vector of zeroes except a one at the j-th component. This, however, defines the j-th marginal density of $F^{*}$ which is nonnegative due to assumption (1).

To (ii):

Decompose $\int_{R^N} f^{*c}(u)du = \int_{A^{*}} f^{*c}(u)du + \int_{\mathrm{ext}(A^{*})} f^{*c}(u)du + \int_{\mathrm{bnd}(A^{*})} f^{*c}(u)du$

$$= \int_{A^{*}} dF^{*} + \int_{R^N \backslash A^{*}} dF^{*} + 0 = \int_{R^N} dF^{*} = 1,$$

because $F(e) \to 1$ for any $e_i \to \infty$ for positive $c_T$, $d_H$ by Lemma 1.

To (iii):

$$P_i(y) = \int_{P_i(y^*)} dF^* = \int_{P_i(y^*)\cap A^*} dF^* + \int_{P_i(y^*)\cap(R^N\setminus A^*)} dF^*$$

$$= \int_{P_i(y^*)\cap A^*} dF^{*c} + \int_{P_i(y^*)\cap \mathrm{bnd}(A^*)} dF^{*c}$$

$$= \int_{P_i(y^*)\cap A^*} dF^{*c} + \int_{P_i(y^*)\cap(R^N\setminus A^*)} dF^{*c} = \int_{P_i(y^*)} dF^{*c}$$

This proves Theorem 4.

Theorem 4 extends the usefulness of NMNL-models by reconciling large dissimilarity coefficients with random utility maximization, provided, conditions (1) and (2) hold. In practice, these conditions can be checked by evaluating the density and cumulative distribution function at the corners of an interval containing the data.

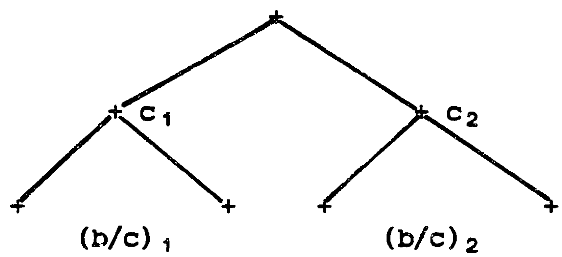
Furthermore, the idea behind Theorem 4 can be exploited to construct a large class of RUM-compatible discrete choice models. Any function which is translation invariant in the sense of Footnote 3 and obeys conditions (1) and (2) of the Theorem in an interval containing the data can be extended to a cumulative distribution function generating a probabilistic choice system by equation (2.3).

## 2.3 Estimation Techniques

### 2.3.1 Econometric Theory and Numerical Analysis

The likelihood function of the hierarchical choice model are the logarithms of the choice probabilities (2.5), cumulated over all consumers. Thus, we can estimate the model by maximizing over the taste weights $a_{ij}$, $b_k$, and the similarity coefficients $c_i$, $d_i$. Because the full information maximum likelihood function is highly nonlinear in the similarity parameters, this approach is costly. As an alternative, we can exploit the recursive structure in (2.5) and estimate sequentially by level of clustering. This approach has been applied to a large number of problems in transportation, energy demand, and urban economics, see Domencich and McFadden (1975) or Anas (1982). However, the sequential estimator is inefficient, especially for complex decision trees. Furthermore, it can not embody parameter restrictions across branches/clusters of alternatives.

The first point relates to the flow of information: the sequential estimator uses all the information of the lower branches to estimate the dissimilarity coefficients at the upper levels, but not conversely. Amemiya (1978) noted that the standard errors of the estimated coefficients at the upper levels have to be corrected for the presence of lower level estimated coefficients, McFadden (1981) provides the proper formulae. Evaluating these corrections is expensive and greatly reduces the computational advantages over the FIML estimator, see Small and Brownstone (1982). The second point is more important: if the alternatives in different branches have taste weights in common, a proportionality constraint has to be fulfilled which is non-trivial for the case of at least two elementary alternatives in at least two branches:




The sequential estimator estimates $(b/c)_1$ and $(b/c)_2$ in the first stage choices, then $c_1$ and $c_2$ in the choice between the branches. A common b implies

$b = (b/c)_1 * c_1 = (b/c)_2 * c_2$.

However, imposing this restriction destroys the sequential decomposition and leads us back to full information maximum likelihood. The proportionality constraints could be imposed by an iterative procedure where the $c_i$ from the sequential estimation as described are used in a second sequential estimate to scale the $(b/c)_i$ as $(b/c_i)_i$, calculate new $c_i$, and so on. This iteration will further reduce the computational advantages over FIML estimation in addition to the necessary correction of the standard errors, in particular so in higher level trees.

We therefore prefer joint estimation and use the modified quadratic hill-climbing method developed by Goldfeld and Quandt (1972) with analytical first and numerical second derivatives. This procedure proved computationally fairly efficient compared with BHHH procedures (Berndt, Hall, Hall, Hausman, 1974). Small and Brownstone

(1982) report similar experiences with BHHH as compared to a method of scoring. The reason for the relative poor performance of the BHHH algorithm seems to be the highly nonlinear dependence of the likelihood function on the dissimilarity parameters. In particular, the function and its gradient have a singularity in $c_i$ or $d_i$ at zero. The singularity is well behaved for the likelihood, but is a pole for the gradient. Thus, the outer product of the gradients is ill-behaved and a bad approximation of the hessian for small values of the dissimilarity parameters.

This unpleasant behavior of the likelihood function requires a careful numerical analysis of the algorithms involved. In particular, the data should be normalized to prevent the multiplication of very small with very large numbers and numerical extinction. The appendix lists a FORTRAN program for the full information maximum likelihood estimation of NMNL models which embodies all these considerations.

## 2.3.2 Elasticities and Goodness-of-Fit Measures

Following the random utility maximization theory and equation (2.2), the estimated parameters represent the taste weights of the respective explanatory variables in the deterministic part of the indirect utility function $v_{it}$. For a more intuitive interpretation of their magnitudes, the taste weights can be transformed into elasticities of the choice probabilities with respect to the various explanatory variables:

$$(2.10)\quad \frac{\partial \log p(i)}{\partial \log x_{jk}} = a_k * x_{jk} * ( -p(j) + k_S * 1/c + k_T * (1/c-1/d) * Q(S) + k_H * (d-1)/d * Q(S) * Q(T) )$$

where $k_S = 0$ if i and j are in the same size category
$= 1$ otherwise

$k_T = 0$ if i and j are in the same tenure category
$= 1$ otherwise

$k_H = 0$ if i and j are in the same headship category
$= 1$ otherwise

c,d = similarity parameters: $c=c_T$ and $d=d_H$

Q(S) = conditional choice probability $p_S(S_j|H_j,T_j)$

Q(T) = conditional choice probability $p_T(T_j|H_j)$

These elasticities measure the percentage change of the probability to choose alternative i, when the k-th attribute of alternative j is changed by one percent. Note that for the cross elasticities the difference between i and j enters only through the "switches" $k_S$, $k_T$, and $k_H$. The structure of the tree is therefore directly reflected in the pattern of cross elasticities. If the dissimilarity parameters c and d are in the unit interval, equation (2.10) implies descending elasticities with the "distance" in the tree, i. e., elasticities are larger within than between branches. This plausible structure is destroyed for dissimilarity parameters larger than one, hinting to alternative tree specifications.

Derived from a highly non-linear model, elasticities at variable means are generally different from mean individual elasticities, and in interpreting the elasticities, one should keep the absolute level of the choice-probabilities in mind; the elasticities tend to be very high at very low probabilities and vice versa, reflecting saturation effects.

Three scalar measures of performance or fit will be used in the applications of Chapters 3 and 5. Amemiya (1981) provides an extensive review. The straightforward discrete analogy to the continuous $R^2$ uses the sum of squared errors:

$$(2.11) \qquad 1 - \frac{\sum\sum (y_{it}-P_{it}(b))^2 \;/\; P_{it}(b)}{\sum_i\sum_t (y_{it}-P_{it}(0))^2 \;/\; P_{it}(b)}$$

where $P_{it}$ denotes the predicted choice probabilities of alternative i for consumer t, evaluated at the optimal parameter values b or at zero, and $y_{it}$ the actual response[6]. However, this measure has little discriminatory power for well specified models. A more satisfactory measure can be constructed from the ratio of the likelihood at the estimated parameters and the likelihood with taste weights at zero and similarity parameters at one. One minus this ratio behaves like the continuous $R^2$, see Mcfadden (1973):

(2.12) $$1 - \frac{L(b)}{L(0)}$$

Domencich and McFadden (1975) give a comparison between these two measures of fit and their discriminatory power. As a third measure of fit, we compare actual with predicted individual choices which is a fairly stringent, though erratic criterion. Note that discrete choice models produce two predictions of the aggregate choice probabilities:

(2.13) $$f(i) = \frac{1}{T} \sum_t p_t(i)$$

(2.14) $$f(i) = n(i)/T$$

with $n(i) = \text{number} \{ t \mid p_t(i) = \max_{j=1..M} p_t(j) \}$

$T$ = sample size

The erratic nature of the percentage of correct predictions is due the integer constraint in (2.14). We can disaggregate this measure into the form of a success table in which observed and predicted alternatives are crosstabulated and the off-diagonal elements show the mispredictions.

### 2.3.3 Aggregate Probability Shares

The aggregate probability shares f(i), i=1..M, where M denotes the number of alternatives, from equation (2.14) can be used for prediction and policy analysis. They should reproduce the aggregate shares in the population q(i) as close as possible. A multinomial specification with a full set of alternative specific constants will always reproduce the sample shares exactly which can be seen by adding up the first order conditions of the MNL-likelihood function with respect to the alternative specific dummies. This property is not carried over to the nested model. Anas (1982) gives some numerical examples for this bias. However, a full set of alternative specific constants still saturates the model and we can always solve the nonlinear system of M-1 equations in these constants to adjust the aggregated shares. This suggests the following two stage procedure: first we estimate all parameters freely; then, we solve this nonlinear equation system evaluated at the slope parameters of the first stage. The second step can be achieved by minimizing the sum of squared deviations of fitted to actual aggregated shares. This two stage procedure is consistent, but does not provide efficient parameter estimates. Usually, the adjustment necessary is very small, and so the loss in efficiency.

A more satisfactory approach is to maximize the likelihood function subject to the M-1 constraints f(i)=q(i), i=2..M. This will yield efficient estimates (Coslett, 1981) where also the slope parameters embody the constraints, not only the constants.

Unfortunately, the nonlinear equation system can not be solved analytically, making a costly constraint maximization necessary which involves additional M-1 nuisance parameters and in general the solution of a saddlepoint problem as opposed to a simple maximization problem. Furthermore, the application of Kuhn-Tucker type algorithms is not possible because the constraints f(i)=q(i) are highly nonlinear in the alternative specific constants. We will only use the two stage procedure. We apply this procedure in Chapter Five to adjust our baseline estimates before making predictions and policy simulations.

### 2.3.4 Choice Based Sampling

The problem of fitting the known aggregate sample shares is related to the problems generated by choice based sampling. Choice based samples may arise in two ways: the data may originally be collected by sampling according to the observed choice. This is the case when we interview a fixed number of homeowners and a fixed number of renters and these numbers do not reflect the proportions in the population. Second, we may start from a large random sample. Typically, however, some choices have very low, others very high market shares. To achieve precise estimates for all choices, the overall sample size of a smaller random subsample drawn for estimation has to be large enough that even the smallest cell has a sufficient number of observations. This will yield very large cell counts for the popular choices. We can substantially decrease estimation costs by oversampling the infrequent choices, undersampling the frequent choices, and then treating our subsample as a choice based sample. We will make heavy use of this technique in Chapter Five.

Given a choice based sample, the parameters have to be estimated to predict the population, not the sample shares. Without a correction, the estimates are inconsistent (Heckman, 1979). Thus the efficient full information maximum likelihood estimator is again the Coslett (1981) estimator mentioned in the previous subsection and involves the solution of a saddlepoint problem with M-1 additional nuisance parameters. Alternative estimators are discussed in Manski and McFadden (1981) of which we mention the two most important.

First, we can compensate for choice based sampling by weighting the observations inversely to the ratio of over or undersampling. This estimator (weighted exogenous sampling maximum likelihood, WESML, Manski and Lerman, 1977) is as cheap to compute as the normal maximum likelihood estimator. Second, we can maximize the likelihood of an endogenously sampled observation conditional on its exogenous characteristics. This estimator (conditional maximum likelihood, CML, Hsieh, Manski, and McFadden, 1983) has a slightly more complicated likelihood function compared to the WESML estimator. Both estimators yield consistent estimates without the introduction of additional nuisance parameters, but there are not efficient compared to the Coslett estimator. However, the efficiency loss seems to be very small as indicated in McFadden, Hsieh, and Manski (1983) or McFadden, Winston, and Boersch-Supan (1984).

Therefore, and for its simplicity, we will only use the WESML estimator. The resulting likelihood function in our case is

$$(2.15)\quad L = \sum_t \frac{q(i_t)}{f(i_t)} \log p(i_t,b) \;=:\; \sum_t w(t) \log P_t(b)$$

where $i_t$ denotes the chosen alternative of household t
$q(i)$ the proportion of alternative i in population
$f(i)$ the proportion of alternative i in the sample
$p(i,b)$ the choice probability according to (2.5)
b vector of parameters

The covariance matrix of the estimated b can be derived by an exact Taylor approximation around the the true parameter vector $b^*$:

$$(2.16) \quad 0 = \underbrace{\frac{1}{\sqrt{T}} \sum w(t) \frac{\partial \log P(b^*)}{\partial b}}_{A_T} + \underbrace{\left(\frac{1}{T} \sum w(t) \frac{\partial^2 \log P(b^\#)}{\partial b \; \partial b'}\right)}_{H_T} \sqrt{T}(b-b^*)$$

where $b^\#$ lies on a line segment between b and $b^*$.

Under the appropriate regularity conditions (Manski and McFadden 1981), we can apply a uniform law of large numbers (Jennrich 1969) to show

$$(2.17) \qquad H_T \xrightarrow{P} H = E\left( \frac{\partial^2 w(t) \log P(b^*)}{\partial b \quad \partial b'} \right) ,$$

and a uniform central limit theorem (Jennrich 1969) to yield

$$(2.18) \qquad A_T \xrightarrow{a.d.} N ( 0 , V )$$

where

$$(2.19) \qquad V = E\left( \frac{\partial w(t) \log P(b^*)}{\partial b} \; \frac{\partial w(t) \log P(b^*)}{\partial b'} \right).$$

Thus,

$$(2.20) \qquad \sqrt{T} ( b - b^*) \longrightarrow N ( 0 , H^{-1} V H^{-1} ).$$

As a consequence,

$$(2.21) \qquad f(i) \neq q(i) \quad \Rightarrow \quad H \neq V,$$

because H includes the weights linearly, but V quadratically. Thus, the inverse hessian does no longer provide an estimate for the covariance matrix of the estimated b. In the estimation, we will use the sample hessian to approximate H and the sample outer product of the gradient to approximate V, both evaluated at the optimum.

The likelihood function (2.15) is a special case of the WESML estimator insofar, as we assume independent draws of households t, each with one choice of its housing alternative. This deviates from the analysis in Hsieh, Manski, and McFadden (1983) where multiple cell counts $m(i,t)$ are observed. In our case, $m(i,t)=1$ for $i=i_t$, the chosen alternative, 0 otherwise. In the case of multiple cell counts, which are distributed multinomially, the negative covariance between $m(i,t)$ and $m(j,s)$ may reduce the variance (2.19). Depending on the way the sample is drawn, either $E(\ m(i,t)\ m(i,s)\ )$ or $E(\ m(i,t)\ m(j,t)\ )$ will have a non-zero contribution.

## 2.4 Conclusions

This chapter provided the econometric tools for this thesis. We will cast all housing alternatives in a finite set of alternatives, structure them in the form of a hierarchical decision tree, and calculate the nested multinomial logit choice probabilities for each alternative.

These choice probabilities can be rationalized by utility maximization behavior of the consumers, but only under certain parameter restrictions. We developed necessary and sufficient conditions for the consistency of random utility maximization with the nested multinomial logit specification in the case of dissimilarity parameters in and outside the unit interval. If we maintain utility maximization as underlying structural behavior, we can interpret the violation of these conditions (the case of Theorem 3) as a hint to misspecification of either the indirect utility function (2.2) or, more important, the tree structure. Alternative tree structures may be derived from the elasticity patterns created by (2.10).

Because of the inefficiency of sequential estimation procedures and their inability to embody equal utility weights for prices and income across tenure and headship clusters, we use full information maximum likelihood estimation throughout the thesis. This has become possibl. the use of sophisticated numerical procedures.

## 2.5 Footnotes to Chapter 2

(1) The requirement to be finite-valued implies that ties in the pairwise comparison of utilities occur only with probability zero.

(2) We impose linearity and additive separability of the indirect utility function in the definition of RUM. In the language of McFadden (1981), such models are defined as AIRUM-compatible.

(3) Note that the definition of the indirect utility function implies translation invariance of the choice probabilities in the following sense:
$p_u(i) = p_{u+c}(i)$ for all constants c and all utility vectors u in $R^M$,
or,
$G(y+c) = G(y) + c$ for all constants c and all vectors y in $R^{M+}$,
where G denotes the generating function (2.9).

(4) This amounts to including M-1 alternative specific constants $D_i$ interacting with the $y_t^1$ and using a common parameters $b_1$ for all alternatives which can be seen by the transformation
$y_t^1 \cdot D_i \cdot b_1 = y_t^1 \cdot a_{i1}$.

(5) The random utility maximization hypothesis as stated in Section 2.2 is only one rationalization of observed choice behavior with the notion of a homo economicus. Failure of RUM does not necessarily preclude the possibility that such a model is rational in an axiomatic sense, i. e., that it fulfills the axiom of stochastically revealed preferences. This is a combinatorial problem of a large dimension. See McFadden and Richter (1979).

(6) The sum of the squared residuals can be weighted in several ways. The natural weights are the true choice probabilities. As the best available estimates, we replace them by their maximum likelihood estimates. See Amemiya (1981) for alternatives.

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P. 59-62

# CHAPTER THREE

# THE INFLUENCE OF HOUSEHOLD FORMATION ON HOUSING DEMAND

## 3.1 Introduction and Review of Earlier Approaches

Does the current rise in rental housing prices discourage household formation? Was household formation fostered by the falling real rents during the early seventies? This chapter tries to answer these questions to give a rationale for the more comprehensive definition of housing demand outlined in the first section of Chapter 2. New households are formed when existing households split: children decide to leave their parents' home, or marriages are divorced, and families, living together, decide to undouble. Conversely, households cease to exist because of death or merger due to marriage or doubling up. Which of these six mechanisms are affected by economic circumstances, in particular by housing market conditions? This seems implausible for death[1], but less so for marriage and divorce, and it is worthwhile to shed light on whether doubling up, undoubling, and the timing of the childrens' leave from home is influenced by the price of housing.

Whereas the influence of housing market conditions on household formation is a rather new topic, the influence of demographic factors on housing demand is well studied. The early literature on household formation, largely written by demographers, concentrated on two aspects: the rise or decline of the total population size, and the distribution of age and marital status within the population (Campbell, 1963, and Kobrin, 1973 and 1976). But a third aspect is as crucial in the determination of housing demand: how the population is divided into households. This third aspect is closely related to the second, but by no means determines household formation. In particular, much interest has been devoted to the dramatic increase in the number of households with only one person, and the corresponding decline in the proportion of married couples in the total number of households. Both changes are larger than the change in the distribution of age and marital status alone would predict, see George Masnick (1983) or Alonso (1983).

Campbell (1963) used standard demographic techniques to predict headship rates for a given distribution of age and marital status. He then compared these rates with the actual rates and attributed the discrepancy to "the development of a taste for privacy and independence."

The search for an explanation of this development concentrated on the rise in real income during the sixties that made privacy and independence affordable. Beresford and Rivlin (1966) and Carliner (1975) present a verbal discussion of cross-sectional data focusing on this explanation. Maisel (1960) presents a descriptive analysis of aggregated time-series data and links changes in headship rates to cyclical variables like income and unemployment rates.

Three studies apply regression analysis to investigate the quantitative nature of this link: Hickman (1974) estimates an aggregate time-series regression of headship rates and finds a positive income effect. However, he does not report standard errors of his estimates, so that the significance of his results cannot be assessed. DePamphilis (1977) attempts a time-series regression of headship rates on aggregate income, the number of young adults, and various interest rates, with questionable results. Finally, Michael, Fuchs, and Scott (1980) estimate an aggregate cross-section Berkson Theil regression of the proportions of one-person households across states. Their findings include a significant effect of the income level in the state. They study variables outside the conventional demographic and economic categories as well, e. g., whether the state has adopted a liberal legislation.

The rise of real income slowed considerably down in the mid-seventies, however, the upward trend in household formation even accelerated in this period, see Kitagawa (1981) and Masnick (1983). This puts the role of income as the most important explanation for the growth in household formation over and above demographic changes in some doubt. Observing the falling real rent level in this period[2], an apparent conclusion is to attribute the affordability of privacy and independence not only to the income at disposal, but also to the price of privacy, the price of housing. A look at the recent development underscores the importance of this hypothesis: in the first years of the 1980's we finally observe household formation leveling off. At the same time real rents started to rise again. Does the current rising rent level discourage household formation?

Housing prices as determinants of household headship rates are considered already by Hickman (1974). He estimates a "negligible" price elasticity and concludes that price effects are not important or cannot be separated from income effects. Michael, Fuchs, and Scott (1980) use this argumentation to exclude price variables a priori.

Hickman's result is confirmed by two studies that use individual cross-sectional data and venture the difficult task of modeling household formation behavior per se. Ermisch (1981) develops a microeconomic model of the determination of household size where the disadvantages of crowding have to be traded off against economies of scale in a multi-person household. He applies his model on data from the General Household Survey in Great Britain. Wial (1983) estimated a multinomial logit choice model among four household types, based on young unmarried men in the 1970 Census. The choices are to stay with the parents, form a independent one-person household, or share a household with one or more other persons. Both studies find insignificant price elasticities.

However, Hickman's hypothesis and the two behavioral models by Ermisch and by Wial are contradicted by two studies that use aggregate time-series data: Rosen and Jaffee (1981), and Smith, Rosen, Markandya, and Ullmo (1982) discover a highly significant influence of the aggregate rent level on headship rates, even after controlling for income.

One purpose of this paper is to add another piece of evidence to the contradictory results in the literature. We will use individual cross-sectional data as Ermisch (1981) and Wial (1983) did, but avoid modeling the household formation process explicitly. As it turns out, we do in fact estimate highly significant price effects.

The findings of Rosen and Jaffee (1981), Smith et. al. (1982), and of this paper establish a causal relation from the housing market on household formation in the opposite direction of the well studied influence of demographic variables on household formation and housing demand. However, a simultaneous interaction between housing markets and household formation behavior has major implications for estimation and prediction of housing demand. The concept of households as basic sampling units in data collection and econometric estimation is blurred because households may consist of independent members who find together and break off for endogenous reasons. Thus, a sample of households can not be considered a random sample for the purpose of housing market analysis. Estimation results will biased due to a similar kind of sample selection as studied by Heckman (1979). Therefore, prediction and policy analysis will be biased as well. For instance, if privacy as an economic good complements housing, then a housing allowance program may foster household formation rather than induce moves of existing households into higher quality dwellings. Conversely, the current cuts in spending on public housing programs may greatly increase the number of households that "double up".

An analysis without the consideration of household formation may not only mispredict behavior quantitatively, but also qualitatively. Using Sweeney's (1974) commodity hierarchy model of the housing market, conventional analysis will predict an upward demand shift along the quality/size categories in response to a housing subsidy. However, if housing allowances foster household formation, just the opposite may occur: existing households in large units will split up, and demand is increased in the lower categories of the hierarchy. Note that an accompanying supply program, designed to cushion the excess demand and based on conventional analysis, would fail. The provision of public housing with large units will not only be a waste, but even increase excess demand at the relevant level in the quality/size scale.

How should we design a model of housing demand that is able to answer the question of how price responsive household formation is? First, it should have a smaller decision unit than the household, second, it should have doubling up as an alternative to the conventional types of housing. We will use the concept of the family-nucleus (Pitkin, 1980) to define our decision unit, and we will use discrete choice analysis and include the choice of doubling up.

We will apply such a model to four representative population strata:

(1) Young unmarried male and female without children, aged 20-34
(2) Married couples with one or two children, aged 35-59
(3) Elderly married couples without children, aged 60 and above
(4) Widowed, divorced, and separated women without children, aged 60 and above.

The model is estimated for the three Standard Metropolitan Statistical Areas of Albany-Schenectady-Troy, New York; Dallas, Texas; and Sacramento, California, representing the Northeast, the Sunbelt, and the West Coast. The estimates are based on the Annual Housing Survey SMSA cross-sections in 1976 and 1977.

We then apply the estimates to simulate changes in the tax and subsidy structure for housing consumption. In the first case the model simulates the impacts of a simple housing allowance program along the lines of the housing gap formula applied in the Experimental Housing Allowance Program in 1973-79. Second, the local property tax rate is assumed to be a half of its actual rate in 1976-77. Finally the model analyses the effect of reducing the highest marginal tax rate of the federal income tax from 70 percent to 50 percent. For all three changes, we calculate the resulting distribution of the population among the different housing alternatives and study the actual moves that take place in response to these changes with the focus on the response of headship rates to price changes induced by the simulations.

## 3.2 Household Decomposition into Nuclei

Conventional housing demand analysis is based on households. Households are the sampling units of almost all available housing data bases. However, if household formation is endogenous, the decision unit must be smaller than the household. In fact, the correct decision unit is every housing consumer who could form his own household. We will call this decision unit "nucleus" (Pitkin 1980). A nucleus consists of a married couple or a single individual together with all its own children below a certain age (say, 18 years). Children above this threshold are considered grown-up and, as potential household heads, form a new nucleus, even if they (still) live in their parents' household. Similarly, households that consist of several adults are split up into several nuclei, both when the members are related or unrelated to each other. Examples are elderly parents in the household of their children, or roommates. This construction assumes death, marriage, and divorce[3] as given from the view-point of housing market analysis, but allows for the endogeneity of doubling up and undoubling, and it also considers endogenous the decision of adult children to stay at or leave home.

Accordingly, there are five types of households:
(1) households consisting just of one nucleus,
(2) parents with their adult children,
(3) households composed of nuclei with family-relations,
(4) households composed of nuclei without family-relations,
(5) complex households, i.e., combinations of the latter three types.

Pitkin (1980) presents a variety of behavioral hypotheses for these five types of households, and provides a descriptive analysis of trends in household composition. Pitkin and Masnick (1980) use the nucleus approach for housing projections in the United States.

Each nucleus chooses its housing accommodation: it either shares housing within an household composed of several nuclei, or heads its own household. In the latter case, the nucleus has to decide which housing unit in terms of tenure and dwelling size.

The basic idea of this approach is to avoid the difficult task of constructing a behavioral model of the determination of the household size per se (as Ermisch, 1981), or a model of the matching of nuclei (as Wial, 1983). The estimation of the resulting structural form tends to be unsatisfactory because of the high noise to signal ratio given our poor knowledge about these mechanisms. In addition, the approach is applicable to data on the microlevel which allows us to use the rich information provided in surveys and to avoid the typical biases in the use of aggregated data in housing demand studies (see Polinsky and Ellwood, 1979). The split-up of households into their true underlying decision units and the creation of the headship/non-headship dichotomy can be interpreted as a reduced form of the above mentioned behavioral models of household production and formation.

The data base of our empirical research is the Annual Housing Survey by SMSA, cross-sections of 1976 and 1977. Sampling unit of the AHS is the household. However, the composition of each household is well documented. This allows us to detect other adults in the household with their children, and to create a data record for each of these subnuclei in addition to the head nucleus. The explanatory variables have to be split up according to this partition. All demographic variables and the most important income sources are reported for each household member. Some income sources are only reported for the household as a total, but in very specific income categories. However, with the demographic characteristics of each person, it is possible to employ a very accurate income allocation scheme for these income categories. For housing costs, it is suggestive that the nuclei pay their shares in proportion to the number of adults and children in the nucleus[4,5].

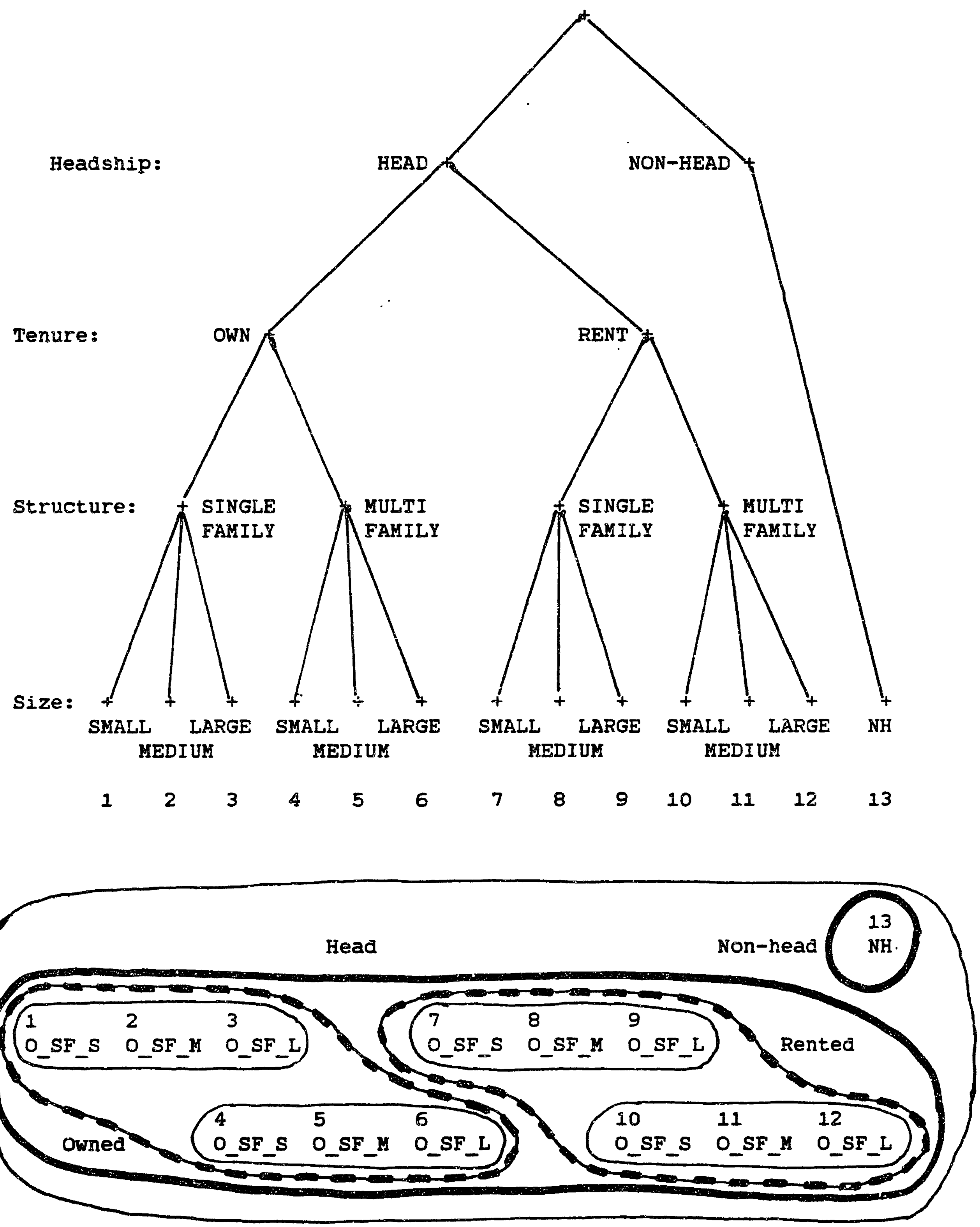
This reduced form approach does not come without its costs. Information is lost by splitting up households into independent nuclei and separating them into different strata; e. g., it seems a valuable piece of information, whether an adult child has parents with a large house in town or not. However, this is a supply factor in the provision of non-headship. As we set out in the introduction, our model is designed to describe "potential" housing demand under perfect elastic supply. Furthermore, the housing alternative "non-head" is a single category for a variety of rather different possible multi-nuclei households. As a special problem, the entire concept of headship is blurred in households of roommates, where no clear subordination exists. However, this problem is in so far irrelevant, as in the case of roommates income and demographic characteristics will be very close, and we can thus pick a head at random.

## 3.3 Specification of the Decision Tree and the Variables

Each nucleus chooses whether to head a household or shelter in an existing household, if one chooses to head a household, then the decisions are whether to rent or own a dwelling, and what quality and size the dwelling should be. As a simple measure of quality and size, we use the number of rooms and the type of the building. A household chooses among three size categories and between single-family detached houses and multi-family houses, in particular apartment buildings. We can arrange the choices in form of a decision tree as depicted in Figure 3-1. "Small" refers to dwellings with up to four rooms, "medium" to dwellings with five or six rooms, and "large" to dwellings with at least seven rooms[6].

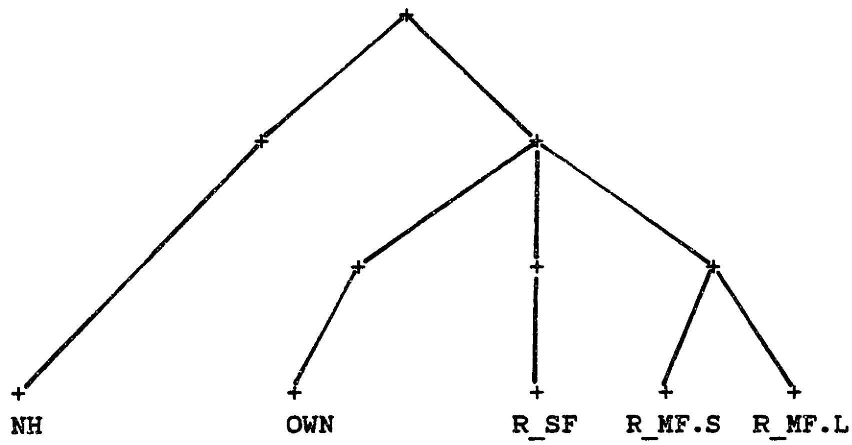
Some of the alternatives are fairly scarce, e.g., renting large apartments or single-family homes, so some alternatives have to be consolidated for a reliable estimation. This consolidation depends on the stratum. Furthermore, no cost data are available for owner-occupied dwellings in multifamily buildings, which forced us to omit these alternatives from the choice set. A more satisfactory approach would be to estimate the cost data for multifamily dwellings by hedonic regressions, or to explicitly model the missing alternatives in the definition of the choice probabilities. But the problem is a minor one for Dallas 1976 and Sacramento 1977, where these alternatives count for only 0.5 and 0.8 percent of all choices; it might bias our results only for Albany, where 6.9 percent of all nuclei chose cooperatively owned multi-family buildings. The final

FIGURE 3-1: BASIC DECISION TREE AND NOTATION OF ALTERNATIVES

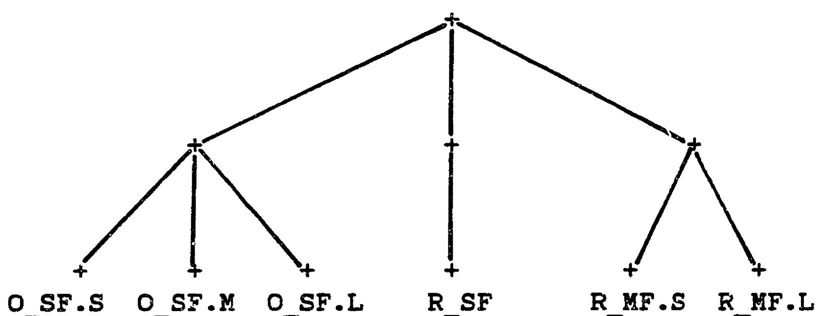


## FIGURE 3-2: DECISION TREES FOR THE DIFFERENT STRATA

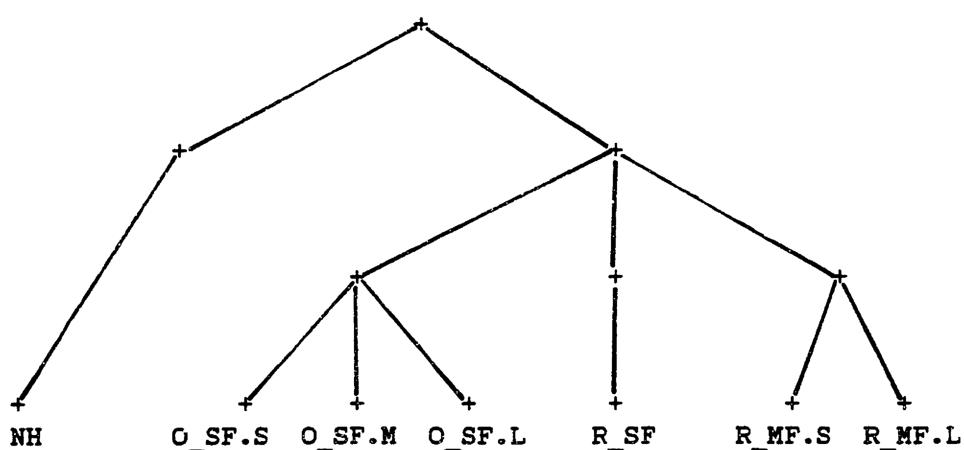
### (1) YOUNG SINGLES:




### (2),(3) FAMILIES AND ELDERLY COUPLES:




### (4) WIDOWS:



decision trees are depicted in Figure 3-2. In Chapter 5, we will experiment with other decision trees.

The choice among the alternatives will depend on the following demographic variables: (1) age of head of nucleus, (2) sex of head of nucleus, (3) marital status of head of nucleus, (4) race of head of nucleus, (5) number of children in the nucleus, as well as on financial variables of (6) after-tax user-cost and (7) income. We pursue the approach of de Leeuw (1971) and Quigley (1979) and assume different demand functions for nuclei with different demographic characteristics. Accordingly, we stratify the sample with the first five demographic variables. This approach is equivalent to the use of dummy variables for each of the strata which interact with all regressors to accommodate the unknown nonlinear functional form (see Li, 1977), in which the demographic variables enter the equations for the choice probabilities.

Of all possible strata, this paper examines demand functions for four representative strata:

(1) Young singles: unmarried white male and female without children, aged 20-35,
(2) Middle-aged families: married white couples with one or two children, aged 35-59,
(3) Elderly couples: married white couples without children, aged 60 and above,
(4) Widows: widowed, separated, and divorced white females without children, aged 60 and above.

The financial variables, income and user-cost, enter the demand functions directly. User-cost (UC) must be distinguished for renters and owners. For renters, the user-cost is simply gross rent. For owners, the user-cost has a number of components (see for example, Hendershott and Hu (1979) or Follain (1982)):

UC(own ) = maintenance + insurance + utility-payments
+ mortgage-rate * debt
+ property tax rate * value
- tax savings from federal income tax deductions
+ T-Bill-rate * equity
- rate of appreciation * value

where the tax savings on the federal income tax is the sum of the local property tax and the mortgage interest, multiplied by the appropriate marginal tax rate. Note that federal income tax savings depend through the marginal tax rate on such nucleus characteristics as income and number of children. Furthermore, we assume different interest rates on debt and on equity to account for the effect of inflation on fixed-rate mortgages.

The user-cost of owners consists of two types of cost which the nucleus perceives differently: Maintenance, mortgage costs, property taxes and federal income tax savings are easily perceived as costs, whereas capital gains from appreciation are uncertain and opportunity costs of equity are a rather cloudy concept for non-economists. We therefore split up user-cost in two components:

UC(own) = OOPOCK(own) + RETURN(own),

where the "out-of-pocket cost" is composed of:

OOPOCK(own) = maintenance + insurance + utility-bills
+ local property tax + mortgage-rate * debt
- federal income tax savings,

and the return from the asset homeownership is defined as:

RETURN(own) = rate of appreciation * value
- T-Bill-rate * equity.

For fully rational housing demanders, the coefficients for OOPOCK and RETURN should be of equal magnitude and opposite sign. For renters, we set RETURN to zero.

The Annual Housing Survey gives us rather precise data for out-of-pocket costs. The return variable has to be constructed with external information: appreciation is based on the difference in house values between the Annual Housing Survey 1976/77 and the 1970 Census, converted into yearly rates. This rate varies by SMSA and by type of dwelling. Equity costs are calculated from the value-to-loan ratio in the Annual Housing Survey, multiplied by the interest on five-year U.S. treasury bills. Both appreciation rates and equity costs suffer from serious data problems: loan-to-value ratios are often not reported, making constructed substitutes necessary, and the available appreciation rates vary only by SMSA, but not within SMSA.

Given the cross-sectional data, the choice among the housing alternatives is a hypothetical one: we observe each nucleus with its chosen alternative and its attributes like user-cost, do not observe the attributes of the alternatives that the nucleus rejected. We take as these attributes the averages in the cross-section confined to recent movers. Underlying is the notion that a household collects its information on prices of other units by skimming through the ads in the newspapers and listening to the experiences of friends and neighbors who just moved. Mean prices of recent movers seem a plausible approximation to the current spot market prices. Note however that the comparison of spot prices with the prices tenants pay after a considerable length of tenure may bias the estimated price response. Specifically, the presence of tenure discounts in the rental market will result in exaggerated price elasticities because we compare the average household with price $p_O/(1+d_L)$ for the tenure discount $d_L$ after an average length of tenure L with the recent mover with price $p_O$. Thus, the average price elasticity is overestimated by a factor of $1+d_L$. This factor can roughly be estimated by dividing the tenure discounts reported in Table 4-2 by the mean length of tenure. Based on the conservative hedonic estimates by Follain and Malpezzi (1980), this bias amounts in Albany to about 9.5 percent, in Dallas to 1.5 percent. of the estimated price coefficient for OOPOCK. The estimations by Malpezzi, Ozanne, and Thibodeau (1980) predict considerably higher tenure discounts, implying an upward bias of 25 percent in Albany, 5 percent in Dallas, 7.5 percent in Sacramento. Thus, we should be careful to take those numbers in account when interpreting the numerical size of our results. The responses on exogenous price shocks are proportional to the price elasticity, hence these biases translate in an exaggeration of the predicted reaction to tax and subsidy changes in the order of the above-mentioned percentages. However, in Section 3.5 through 3.7 we will see that these biases do not change our qualitative conclusions. In particular, the most striking results will be found in Dallas and Sacramento where the overestimation is least. Chapter 4 describes the estimation of tenure discounts in more detail and provides some critique of the quoted estimates and a more general discussion about problems related to the description of intertemporal processes by a static model. Finally, in Chapter 5, we explicitly correct housing prices for the presence of tenure discounts.

For the hypothetical loan-to-value ratios, we assume a 20 percent downpayment for young singles, and 99 percent for the elderly households, which takes into account the availability of mortgage loans to the different age groups. This assumption is not critical to the estimates, but is confirmed by cross- tabulations of recent movers.

Finally, income is defined as the current total gross income of all nucleus members. Due to the nature of a choice between discrete alternatives, income enters the demand functions interactively with alternative-specific dummies. Furthermore, income influences the out-of-pocket costs of homeowners because federal income tax savings depend on the marginal tax rate, i.e., on gross income.

## 3.4 Baseline Estimates

The parameter estimates and summary statistics are tabulated in Tables 3-3 trough 3-6 for each of the four strata. The parameters represent the taste weights of the respective explanatory variables in the deterministic part of the indirect utility function. T-statistics are given in brackets, and are evaluated at zero for the taste weights. Note that income Y interacts with alternative specific dummies, where we use the same mnemonics for the alternatives as in Figure 3-2. The final parameters are the similarity parameters that express the degree of closeness in the respective clusters (See Figure 3-2). T-statistics for the similarity parameters are evaluated at one, using the multinomial logit case as a benchmark. Three scalar measures of performance or fit are used, see Section 4 in Chapter 2 for a discussion.

The model achieves a surprisingly high prediction accuracy in terms of all three measures of fit. This is surprising because of the small number of explanatory variables and the simple specification. The model performs poorest in the strata of young singles and in the family stratum in Albany. The first is not astonishing: a static model can hardly capture changes in housing consumption in this period when the nucleus is establishing its own existence. These strata are also very heterogeneous and include children still living with their parents, student roommates, and singles in their thirties. The poor performance in the case of families with one or two children in Albany may be attributable to the misspecification of the decision tree where

Table 3-3 : NMNL Parameter Estimates

"Young Singles": Unmarried, Age 20-35, No Children

| | ALBANY | DALLAS | SACRAMENTO |
|---|---|---|---|
| OOPOCK | -0.69598 | -1.18246 | -1.56947 |
| | (10.04) | ( 9.96) | (13.29) |
| RETURN | 0.13156 | 0.12316 | 0.20421 |
| | ( 1.55) | ( 1.45) | ( 2.97) |
| Y_NH | -0.02006 | -0.13204 | -0.14574 |
| | ( 1.65) | ( 6.88) | ( 5.36) |
| Y_O | -0.00196 | 0.00894 | 0.02125 |
| | ( 0.09) | ( 0.43) | ( 0.65) |
| Y_R_SF | 0.01213 | -0.02375 | 0.01408 |
| | ( 1.87) | ( 1.61) | ( 0.62) |
| Y_R_MF.S | 0.02282 | 0.04212 | 0.03539 |
| | ( 4.07) | ( 3.38) | ( 1.59) |
| TH_R_MF | 0.20422 | 0.42742 | 0.45242 |
| | (26.81) | ( 8.05) | ( 7.08) |
| TAU_HEAD | 0.14958 | 0.32322 | 0.37700 |
| | (28.96) | (13.17) | (12.48) |
| LOGLIK | -616.591 | -421.180 | -457.465 |
| LOGLIK_0 | -1401.82 | -925.427 | -1199.03 |
| RHO_SQ | 0.56 | 0.545 | 0.618 |
| %CORRECT | 82.9% | 73.2% | 81.2% |
| NOBS | 871 | 575 | 745 |

In brackets: t-statistics around zero or one
LOGLIK = loglikelihood at optimum
LOGLIK_0 = loglikelihood at zero
RHO_SQ = 1.0 - LOGLIK/LOGLIK_0
%CORRECT = percentage of correct ex post predictions
NOBS = number of observations

Table 3-4 : NMNL Parameter Estimates

**"Families": Married, Age 35-59, 1-2 Children**

| | ALBANY | DALLAS | SACRAMENTO |
|---|---|---|---|
| OOPOCK | -0.92185 | -3.55522 | -2.81777 |
| | ( 4.91) | ( 5.87) | ( 6.28) |
| RETURN | -0.69255 | 1.07524 | 0.63690 |
| | ( 3.55) | ( 5.08) | ( 3.87) |
| Y_O_SF.S | -0.99077 | -0.70474 | -0.32266 |
| | ( 4.35) | ( 4.69) | ( 1.91) |
| Y_O_SF.M | 0.07448 | 0.05039 | 0.15820 |
| | ( 3.12) | ( 1.20) | ( 1.21) |
| Y_O_SF.L | 0.22973 | 0.20642 | 0.27165 |
| | ( 6.19) | ( 3.56) | ( 2.08) |
| Y_R_SF | -0.00667 | -0.02119 | 0.18802 |
| | ( 0.37) | ( 0.63) | ( 1.49) |
| Y_R_MF.S | -0.10296 | 0.18602 | -0.00061 |
| | ( 1.86) | ( 1.69) | ( 0.00) |
| TH_R_MF | 0.74920 | 2.25563 | 1.74344 |
| | ( 0.71) | ( 0.86) | ( 1.06) |
| TH_O_SF | 5.03934 | 2.73284 | 2.20262 |
| | ( 8.77) | ( 3.21) | ( 2.97) |
| LOGLIK | -357.208 | -121.134 | -127.350 |
| LOGLIK_O | -627.116 | -584.114 | -580.530 |
| RHO_SQ | 0.43 | 0.793 | 0.781 |
| %CORRECT | 78.9% | 90.8% | 90.9% |
| NOBS | 350 | 326 | 324 |

In brackets: t-statistics around zero or one
LOGLIK = loglikelihood at optimum
LOGLIK_O = loglikelihood at zero
RHO_SQ = 1.0 - LOGLIK/LOGLIK_O
%CORRECT = percentage of correct ex post predictions
NOBS = number of observations

Table 3-5 : NMNL Parameter Estimates

"Elderly Couples": Married, Age 60+, No Children

| | ALBANY | DALLAS | SACRAMENTO |
|---|---|---|---|
| OOPOCK | -3.61354 | -2.35330 | -3.21796 |
| | ( 8.29) | ( 6.66) | ( 6.49) |
| RETURN | 0.52908 | 0.68032 | 1.02552 |
| | ( 2.27) | ( 4.27) | ( 4.87) |
| Y_O_SF.S | -0.26210 | -0.19193 | -0.21797 |
| | ( 4.15) | ( 2.05) | ( 2.68) |
| Y_O_SF.M | 0.16994 | 0.08080 | 0.07719 |
| | ( 3.96) | ( 1.02) | ( 1.11) |
| Y_O_SF.L | 0.30284 | 0.25677 | 0.25455 |
| | ( 4.94) | ( 2.67) | ( 3.05) |
| Y_R_SF | 0.03169 | 0.03272 | 0.11807 |
| | ( 0.59) | ( 0.42) | ( 1.61) |
| Y_R_MF.S | -0.08036 | 0.01262 | -0.14165 |
| | ( 1.17) | ( 0.15) | ( 1.01) |
| TH_R_MF | 3.02968 | 2.08399 | 3.39293 |
| | ( 2.93) | ( 1.49) | ( 1.85) |
| TH_O_SF | 1.41709 | 0.87605 | 0.47555 |
| | ( 1.36) | ( 0.49) | ( 1.73) |
| | | | |
| LOGLIK | -153.730 | - 77.926 | - 62.706 |
| LOGLIK_0 | -582.322 | -458.690 | -519.610 |
| RHO_SQ | 0.735 | 0.83 | 0.88 |
| %CORRECT | 86.8% | 90.2% | 91.7% |
| NOBS | 325 | 256 | 290 |

In brackets: t-statistics around zero or one
LOGLIK = loglikelihood at optimum
LOGLIK_0 = loglikelihood at zero
RHO_SQ = 1.0 - LOGLIK/LOGLIK_0
%CORRECT = percentage of correct ex post predictions
NOBS = number of observations

Table 3-6 : NMNL Parameter Estimates

"Widows": Widowed, Divorced, Separated, Age 60+, No Children

| | ALBANY | DALLAS | SACRAMENTO |
|---|---|---|---|
| OOPOCK | -2.92313 | -1.74357 | -3.65622 |
| | ( 9.81) | ( 7.17) | ( 7.61) |
| RETURN | 0.28555 | 0.16035 | 0.61589 |
| | ( 2.65) | ( 2.17) | ( 4.84) |
| Y_NH | -0.68624 | -0.49553 | -0.83475 |
| | (10.42) | ( 4.77) | ( 6.47) |
| Y_O_SF.S | -0.23088 | -0.16355 | -0.27639 |
| | ( 4.71) | ( 1.95) | ( 3.12) |
| Y_O_SF.M | 0.22788 | 0.09891 | 0.16234 |
| | ( 6.47) | ( 1.36) | ( 1.99) |
| Y_O_SF.L | 0.39921 | 0.20989 | 0.43735 |
| | ( 5.73) | ( 2.09) | ( 2.88) |
| Y_R_SF | 0.01295 | 0.06526 | 0.01310 |
| | ( 0.34) | ( 0.89) | ( 0.21) |
| Y_R_MF.S | -0.02053 | 0.04518 | -0.00102 |
| | ( 0.63) | ( 0.59) | ( 0.01) |
| TH_O_SF | 1.48636 | 0.60497 | 1.03269 |
| | ( 1.75) | ( 2.10) | ( 0.07) |
| TH_R_MF | 1.48222 | 1.48958 | 0.97076 |
| | ( 1.73) | ( 1.12) | ( 0.09) |
| TAU_HEAD | 0.48036 | 0.45743 | 0.47043 |
| | ( 5.79) | ( 6.31) | ( 4.21) |
| LOGLIK | -311.114 | -199.974 | - 94.920 |
| LOGLIK_0 | -1037.17 | -646.042 | -651.880 |
| RHO_SQ | 0.70 | 0.69 | 0.85 |
| %CORRECT | 89.0% | 88.3% | 82.6% |
| NOBS | 533 | 332 | 335 |

In brackets: t-statistics around zero or one
LOGLIK = loglikelihood at optimum
LOGLIK_0 = loglikelihood at zero
RHO_SQ = 1.0 - LOGLIK/LOGLIK_0
%CORRECT = percentage of correct ex post predictions
NOBS = number of observations

the alternative of owning cooperatively is not included.

Table 3-7 shows the prediction success table for the stratum of unmarried elderly in Sacramento, based on the criterion (2.14). The diagonal dominance reflects the high prediction accuracy. The off-diagonal elements represent the mispredictions: the model has some difficulties in discriminating between small rental housing and non-headship, which may reflect the crude reduced form specification of the household formation process.

The main result is the significance of the price variables in all strata. The out-of-pocket costs are highly significant, while the RETURN variable is somewhat weaker. Note that the hypothesis of rationality -- i.e., equal magnitude and opposite signs for the taste weights of OOPOCK and RETURN -- is rejected; considerably more weight is given to easily perceived out-of-pocket costs as opposed to appreciation minus equity costs. One should keep in mind, however, the difficulties of constructing the RETURN variable. Note furthermore that RETURN is least significant for the young singles, the strata most affected by liquidity constraints, rendering the rationality hypothesis inappropriate and introducing a lot of noise. Note the similarity between the estimates for elderly couples and elderly widows: taking into account the different choice sets (a large proportion of widows live in their children's homes), it reflects the stability of the taste weights during old age.

The income dummies have a threefold function. First, they

Table 3-7: Example of a Success Table and a Full Elasticity Matrix

Stratum: Widowed, Divorced, Separated, Age 60+, No Children
SMSA: Sacramento

PREDICTION SUCCESS TABLE:

| OBSERVED ALT. | PREDICTED ALTERNATIVE NH | O_SF.S | O_SF.M | O_SF.L | R_SF | R_MF.S | R_MF.L |
|---|---|---|---|---|---|---|---|
| NH | 73 | 1 | 0 | 0 | 0 | 10 | 0 |
| O_SF.S | 1 | 32 | 0 | 0 | 0 | 1 | 0 |
| O_SF.M | 0 | 0 | 77 | 0 | 0 | 1 | 0 |
| O_SF.L | 0 | 0 | 0 | 19 | 0 | 0 | 0 |
| R_SF | 2 | 0 | 0 | 0 | 25 | 1 | 0 |
| R_MF.S | 17 | 0 | 0 | 0 | 1 | 70 | 0 |
| R_MF.L | 1 | 0 | 0 | 0 | 0 | 1 | 2 |

PERCENT CORRECTLY PREDICTED : 88.96 %

MEAN INDIVIDUAL ELASTICITIES:

| VARIABLE | ALT. | CHOICE PROBABILITY OF ALTERNATIVE: NH | O_SF.S | O_SF.M | O_SF.L | R_SF | R_MF.S | R_MF.L |
|---|---|---|---|---|---|---|---|---|
| OOPOCK | NH | -2.814 | 0.739 | 0.739 | 0.739 | 0.739 | 0.739 | 0.739 |
| OOPOCK | O_SF.S | 0.420 | -12.898 | -4.671 | -4.671 | 1.876 | 1.876 | 1.876 |
| OOPOCK | O_SF.M | 0.918 | 0.389 | -12.540 | 0.389 | 1.968 | 1.968 | 1.968 |
| OOPOCK | O_SF.L | 0.235 | 0.195 | 0.195 | -23.002 | 0.502 | 0.502 | 0.502 |
| OOPOCK | R_SF | 0.287 | 0.670 | 0.670 | 0.670 | -19.015 | 0.670 | 0.670 |
| OOPOCK | R_MF.S | 1.315 | 4.097 | 4.097 | 4.097 | 4.097 | -10.477 | -3.161 |
| OOPOCK | R_MF.L | 0.103 | 0.322 | 0.322 | 0.322 | 0.322 | -0.317 | -10.964 |
| RETURN | O_SF.S | -0.200 | 2.759 | 0.892 | 0.892 | -0.634 | -0.634 | -0.634 |
| RETURN | O_SF.M | -0.429 | -0.345 | 1.711 | -0.345 | -0.915 | -0.915 | -0.915 |
| RETURN | O_SF.L | -0.120 | -0.112 | -0.112 | 2.316 | -0.256 | -0.256 | -0.256 |
| Y_NH | | -4.422 | 0.588 | 0.588 | 0.588 | 0.588 | 0.588 | 0.588 |
| Y_O_SF.S | | 0.187 | -2.059 | -0.453 | -0.453 | 0.452 | 0.452 | 0.452 |
| Y_O_SF.M | | -0.330 | -0.242 | 0.702 | -0.242 | -0.704 | -0.704 | -0.704 |
| Y_O_SF.L | | -0.302 | -0.281 | -0.281 | 2.261 | -0.642 | -0.642 | -0.642 |
| Y_R_SF | | -0.005 | -0.012 | -0.012 | -0.012 | 0.155 | -0.012 | -0.012 |
| Y_R_MF.S | | 0.001 | 0.004 | 0.004 | 0.004 | 0.004 | -0.009 | -0.003 |
| Y_SUM | | -4.871 | -2.002 | 0.548 | 2.146 | -0.147 | -0.326 | -0.320 |

reflect the relative price of housing with respect to all other goods. In addition, they indicate the attractiveness of the various alternatives relative to large rented apartments, measured in money terms. In absence of any other alternative-specific dummies, they also pick up all other non-measured advantages and disadvantages of the included alternatives relative to large rented apartments. Thus, one should be careful not to rush to conclusions about pure income effects. Introduction of alternative-specific dummies in several test strata reduces the income parameters, but leaves the price variables virtually constant. Because the focus of the simulations is on relative prices rather than on income, we avoided the costly inclusion of alternative specific dummies.

The attractiveness of the alternatives measured by the taste weights of the income dummies corresponds to a priori assessment. Note that most of the rented single-family houses are small houses, thus their negative weight for families with one or two children.

The last two coefficients in Tables 3-3 through 3-6 are the weights of the inclusive values or similarity coefficients. Note that four of the similarity coefficients are significantly larger than one (at the 5 percent level). This implies that in these strata the compatibility with the underlying microeconomic theory of random utility maximization must be explicitly checked for the given data and is not automatically guaranteed as in the other strata (see appendix). In fact, the test rejects this compatibility. Note that the microeconomic theory described above is based on static utility maximization. Note furthermore that failure of the test occurs in the strata where people move considerably less frequently than in the strata of young singles, where the similarity parameters are in the unit-interval. The rejection thus could be interpreted as a hint that optimization is done dynamically and that the model in these strata should be interpreted as only a reduced-form description of the steady-state as opposed to a structural static choice model.

To gain some intuition for the magnitudes of the coefficients, we calculate the elasticities corresponding to the parameter estimates of Tables 3-3 through 3-6. Own price elasticities and income elasticities, tabulated in Table 3-8, refer to a change of the probability of choosing alternative i, when OOPOCK or RETURN in alternative i is changed. The income elasticities in Table 3-8 are the sum over the elasticities of all income dummies. In Table 3-7, the complete matrix of elasticities is presented for the stratum of unmarried elderly in Sacramento. The pattern in the price elasticities across alternatives reflects the structure of the decision tree. Particularly interesting is the first column of price elasticities, referring to the probability to choose non-head status. As closest substitute for this alternative emerges renting small units, as expected. The large cross elasticity of 1.315 once more reflects the price responsiveness of household formation. The cross elasticity with respect to renting large units can be decomposed into two components: a positive component for the substitution effect, and a negative part for a supply effect. Large rental units constitute the supply for non-headship possibilities, therefore, their price will

Table 3-8: Own Price and Sum of Income Elasticities

| PROB | NH | O_SF.S | O_SF.M | O_SF.L | R_SF | R_MF.S | R_MF.L |
|---|---|---|---|---|---|---|---|
| Albany, Young Singles: | | | | | | | |
| OOPOCK | -0.395 | | -19.596 | | -12.973 | -2.463 | -7.227 |
| RETURN | 0.0 | | 3.675 | | 0.0 | 0.0 | 0.0 |
| INCOME | -0.083 | | -0.591 | | -0.101 | 0.238 | -0.343 |
| Albany, Families | | | | | | | |
| OOPOCK | | -0.209 | -0.686 | -0.898 | -2.460 | -2.517 | -2.169 |
| RETURN | | -0.468 | -0.386 | -0.415 | 0.0 | 0.0 | 0.0 |
| INCOME | | -4.835 | -0.132 | 0.553 | -3.394 | -6.250 | -3.193 |
| Albany, Elderly Couples: | | | | | | | |
| OOPOCK | | -3.765 | -6.222 | -11.998 | -10.187 | -4.517 | -5.160 |
| RETURN | | 0.791 | 0.379 | 0.653 | 0.0 | 0.0 | 0.0 |
| INCOME | | -4.214 | 0.028 | 1.333 | -1.788 | -2.944 | -2.574 |
| Albany, Widows: | | | | | | | |
| OOPOCK | -2.430 | -6.984 | -7.529 | -11.861 | -16.940 | -5.979 | -6.016 |
| RETURN | 0.0 | -0.914 | -0.292 | -0.418 | 0.0 | 0.0 | 0.0 |
| INCOME | -3.694 | -1.284 | 0.359 | 0.973 | -0.428 | -0.737 | -0.664 |
| Dallas, Young Singles: | | | | | | | |
| OOPOCK | -0.632 | | -16.494 | | -8.392 | -3.031 | -7.641 |
| RETURN | 0.0 | | 2.165 | | 0.0 | 0.0 | 0.0 |
| INCOME | -0.669 | | -0.037 | | -0.710 | 0.625 | -0.030 |
| Dallas, Families: | | | | | | | |
| OOPOCK | | -2.167 | -3.366 | -6.141 | -8.548 | -4.786 | -8.590 |
| RETURN | | 1.531 | 1.335 | 1.451 | 0.0 | 0.0 | 0.0 |
| INCOME | | -7.663 | -1.343 | 1.138 | -2.536 | -4.690 | -2.513 |
| Dallas, Elderly Couples: | | | | | | | |
| OOPOCK | | -4.581 | -5.189 | -18.934 | -5.798 | -4.316 | -4.311 |
| RETURN | | 2.064 | 1.313 | 2.497 | 0.0 | 0.0 | 0.0 |
| INCOME | | -5.462 | -0.667 | 2.427 | -1.337 | -1.676 | -1.770 |
| Dallas, Widows: | | | | | | | |
| OOPOCK | -1.431 | -5.806 | -8.680 | -24.426 | -8.964 | -5.162 | -4.867 |
| RETURN | 0.0 | 0.816 | 0.682 | 1.016 | 0.0 | 0.0 | 0.0 |
| INCOME | -2.823 | -1.958 | 0.457 | 1.478 | 0.341 | 0.005 | -0.164 |

Table 5: Own Price and Sum of Income Elasticities (cont'd)

| PROB | NH | O_SF.S | O_SF.M | O_SF.L | R_SF | R_MF.S | R_MF.L |
|---|---|---|---|---|---|---|---|
| **Sacramento, Young Singles:** | | | | | | | |
| OOPOCK | -0.728 | | -16.459 | | -10.046 | -3.563 | -9.519 |
| RETURN | 0.0 | | 2.939 | | 0.0 | 0.0 | 0.0 |
| INCOME | -0.513 | | 0.279 | | 0.181 | 0.468 | 0.064 |
| **Sacramento, Families:** | | | | | | | |
| OOPOCK | | -2.570 | -2.862 | -4.790 | -6.922 | -5.222 | -5.358 |
| RETURN | | 1.215 | 0.844 | 1.112 | 0.0 | 0.0 | 0.0 |
| INCOME | | -5.563 | -0.550 | 0.632 | -0.397 | -4.727 | -4.719 |
| **Sacramento, Elderly Couples:** | | | | | | | |
| OOPOCK | | -13.100 | -9.361 | -42.173 | -8.112 | -4.318 | -5.485 |
| RETURN | | 5.479 | 2.490 | 8.194 | 0.0 | 0.0 | 0.0 |
| INCOME | | -7.813 | 0.672 | 5.770 | 0.765 | -2.055 | -1.484 |
| **Sacramento, Widows:** | | | | | | | |
| OOPOCK | -2.814 | -12.898 | -12.540 | -23.002 | -19.015 | -10.477 | -10.964 |
| RETURN | 0.0 | 2.759 | 1.711 | 2.316 | 0.0 | 0.0 | 0.0 |
| INCOME | -4.871 | -2.002 | 0.548 | 2.146 | -0.147 | -0.326 | -0.320 |

be negatively correlated to the choice probability of not forming an household. Both effects together seem to cancel out, reflected in the insignificant value of 0.103.

If we compare the own elasticities across strata, the following general pattern emerges: the strata of young singles and elderly widows are the most price-responsive, especially in the owner alternatives, reflecting a priori knowledge of inertia and mobility in the different strata. Return from the asset homeownership exhibits a strong life-cycle behavior, and is thus higher for young people with a long decision horizon than for the elderly. Headship rates are highly responsive to prices for both young singles and elderly widows. Finally, note again that the income elasticities measure not only income but also pure alternative specific effects due to their interaction with alternative specific dummies.

The elasticity pattern is fairly stable across the three SMSAs, in spite of their very different distribution of housing alternatives. This provides some confidence in the robustness of the model. As a general pattern, housing demand reacts most to prices in Sacramento and least in Albany, suggesting the more flexible nature of the housing market in California compared with New England. The pattern holds for both out-of-pocket costs and returns.

Summing up, we observe the following:

o Relative prices significantly determine housing choices for given demographic variables.

- o Household formation, in particular, is highly responsive.
- o Out-of-pocket costs have higher taste weights than return from homeownership.
- o Among strata, young singles and widows are more price responsive than the relatively inert strata of families and elderly couples.
- o The sensitivity to RETURN shows the expected life-cycle behavior.
- o The general pattern of elasticities is fairly stable across markets, with Albany behaving least flexibly and Sacramento the most.

## 3.5 A Housing Allowance Program

The estimation results show a strong responsiveness of household formation to housing prices. It is now interesting what the consequences of endogenous household formation rates are in quantitative terms. Do the price coefficients, highly significant as they are, in fact translate into a reassessment of policy analysis? We will use the estimation results for three comparative static experiments of public intervention in the housing market: housing allowances, local property taxes, and the deduction from the federal income tax. These experiments are exercises in comparative statics and have thus to be interpreted as long-run responses, leading to a new steady state equilibrium. Furthermore, the results are based on the partial analysis of only the demand side in the preceding sections. Thus, we implicitly assume a perfect malleable housing stock and disregard all transitional phenoma like inertia of mobility and transaction costs.

Between 1973 and 1979, the U.S. Department of Housing and Urban Development conducted a large scale Experimental Housing Allowance Program. Kennedy (1980) describes in detail the design of the program, and a good survey of the subsequent discussion and critique is given in Bradbury and Downs (1981). Somewhat surprising is the fact that all components of the Experimental Housing Allowance Program ignored the feedback of housing allowances on household formation. One focal point in this section is the question of how much improvement in housing conditions comes through increased headship rates over and above moves of existing households into larger dwellings.

The following simulation assumes a so-called housing gap formula for the calculation of the allowances. First, for each family size and site a benchmark rent is calculated, representing the "fair cost of standard housing." Then a minimum standard of quality is established, with only dwellings above this standard eligible for the subsidy. Finally, a linear tax is levied on the allowances in such a way that people with no (adjusted) income will receive the full rent for standard housing, whereas people above a certain income level will receive no allowances at all.

If the minimum standard is measured as a fraction a of the fair cost of standard housing C, and the upper income limit is a multiple b of C, then the housing allowance for a household with income Y and rent R is:

$0$ if $R < aC$

$0$ if $Y > bC$

$C-Y/b$ otherwise.

To perform a realistic experiment, we use the settings a=0.7, b=4.0, and C from the Experimental Housing Allowances Program, where C was taken from the Pittsburgh demand experiment and inflated by a yearly as well as inter-SMSA rent index as follows:

| FAIR MONTHLY RENTS NO. OF PERSONS: | PITTSBG 75 | DALLAS 77 | ALBANY 77 | SACRAM 76 |
|---|---|---|---|---|
| 1 | $ 115 | $ 150 | $ 130 | $ 140 |
| 2 | 130 | 180 | 160 | 170 |
| 3-4 | 150 | 200 | 180 | 190 |
| 5-6 | 170 | 225 | 205 | 215 |
| 7+ | 205 | 275 | 245 | 260 |

Housing allowances introduce nonlinearities in the budget set, see Hausman and Wise (1980) or Venti and Wise (1982). They can be handled fairly elegantly in discrete choice models by changing the prices of the housing alternatives differently rather than by adding the allowances to the income.

Table 3-9 lists the predicted shares of the housing alternatives before and after the introduction of the housing allowance program. Given the static nature of the model, this reflects a change between steady-states. The shares are calculated as means of the individual choice probabilities. Table 3-10, in turn, tabulates the moves according to the individual predictions.

Our main result is the strong impact of housing allowances on headship rates: about half of the people who lived in some sort of shared accommodations created their own households in response to the housing allowance program. Most of these nuclei in the strata of young singles have little or no income, thus their rent net of the housing allowance is virtually zero. More surprising is the strong response in the strata of elderly widows, where the non-head share is far less and the income higher than among the young singles, the share

## Table 3-9: AGGREGATED SHARES: HOUSING ALLOWANCES

| Stratum | Alt. | ALBANY before | ALBANY after | DALLAS before | DALLAS after | SACRAMENTO before | SACRAMENTO after |
|---|---|---|---|---|---|---|---|
| YSL | NH | .6331 | .4780 | .5507 | .3226 | .6262 | .3067 |
| | O_SM | | | | | | |
| | O_ME | .0092 | .0092 | .0224 | .0219 | .0425 | .0409 |
| | O_LA | | | | | | |
| | R_SF | .0123 | .0159 | .0478 | .0913 | .0508 | .0772 |
| | R_SM | .2815 | .4041 | .3438 | .5046 | .2642 | .5396 |
| | R_LA | .0637 | .0928 | .0353 | .0596 | .0163 | .0356 |
| FAM | NH | | | | | | |
| | O_SM | .0182 | .0149 | .0440 | .0272 | .0320 | .0210 |
| | O_ME | .3310 | .3270 | .4439 | .4370 | .4772 | .4682 |
| | O_LA | .4997 | .4969 | .4085 | .4046 | .3852 | .3831 |
| | R_SF | .0546 | .0572 | .0639 | .0694 | .0780 | .0772 |
| | R_SM | .0161 | .0207 | .0152 | .0299 | .0219 | .0394 |
| | R_LA | .0804 | .0833 | .0244 | .0320 | .0056 | .0111 |
| ELC | NH | | | | | | |
| | O_SM | .1251 | .0920 | .1443 | .1174 | .1450 | .1191 |
| | O_ME | .4050 | .3830 | .5446 | .5316 | .6667 | .6418 |
| | O_LA | .2773 | .2683 | .2014 | .2004 | .0750 | .0746 |
| | R_SF | .0174 | .0193 | .0420 | .0517 | .0531 | .0595 |
| | R_SM | .0870 | .1225 | .0484 | .0686 | .0335 | .0613 |
| | R_LA | .0882 | .1149 | .0192 | .0302 | .0268 | .0437 |
| WID | NH | .2757 | .1300 | .2130 | .1005 | .2745 | .1068 |
| | O_SM | .0694 | .0387 | .1165 | .0562 | .1105 | .0946 |
| | O_ME | .1602 | .1398 | .3144 | .2875 | .2281 | .2232 |
| | O_LA | .0985 | .0909 | .0486 | .0453 | .0562 | .0558 |
| | R_SF | .0273 | .0286 | .0737 | .0953 | .0729 | .0777 |
| | R_SM | .2340 | .3636 | .1842 | .3039 | .2437 | .4160 |
| | R_LA | .1348 | .2084 | .0496 | .1133 | .0140 | .0259 |

First column : predicted shares of housing alternatives before housing allowances.

Second column: predicted shares of housing alternatives with housing allowance program in effect (housing gap formula: $P = C - Y/b$).

of non-heads is nevertheless drastically reduced in response to the subsidy. We conclude once more that headship rates are important endogenous variables in the housing market.

Within the rental sector, only few moves occur. The mobility rates induced by the housing allowances (Albany 0.047, Dallas 0.057, Sacramento 0.055) are very close to those measured in the demand part of the Experimental Housing Allowance Program by MacMillan (1980) i.e., Pittsburgh 0.045 and Phoenix 0.101. Note again the difference in the price sensitivity between the Northeast and the Southwest.

Unlike the Experimental Housing Allowance Program, our simulation offered allowances for rental housing to everybody in the population, changing the balance in the tenure choice in favor of renting. As a response, we observe a relatively large number of moves from the owner-occupied section into the rental section of the housing market. The mobility rates for the shift from owning to renting induced by the allowances are between 0.125 for Sacramento and 0.189 for Albany. Note the lower rate for Sacramento, reflecting the high valuation of owner-occupancy in the West relative to the Northeast.

Moves from owner-occupancy into the rental market have two important fiscal side-effects: on the federal level, some money given for housing allowances is retrieved through lower mortgage and property tax deductions from the federal income tax. More important is the spill-over effect at the local level of reductions in local property taxes. Table 3-11 lists these fiscal repercussions created

Table 3-10: Predicted Moves in Response to Housing Allowance Program

| Stratum | from | NH | O.S | O.M | O.L | R.SF | R.MS | R.ML |
|---|---|---|---|---|---|---|---|---|
| Albany, | to R_SF | 1 | | 0 | | - | 0 | 0 |
| Young Singles | to R_MF.S | 309 | | 0 | | 0 | - | 0 |
| | to R_MF.L | 3 | | 0 | | 0 | 0 | - |
| Albany, | to R_SF | | 0 | 0 | 0 | - | 0 | 0 |
| Families | to R_MF.S | | 9 | 3 | 0 | 3 | - | 3 |
| | to R_MF.L | | 0 | 0 | 3 | 0 | 3 | - |
| Albany, | to R_SF | | 0 | 0 | 0 | - | 0 | 0 |
| Elderly Couples | to R_MF.S | | 24 | 28 | 3 | 0 | - | 0 |
| | to R_MF.L | | 6 | 0 | 0 | 0 | 0 | - |
| Albany, | to R_SF | 0 | 0 | 0 | 0 | - | 0 | 0 |
| Widows | to R_MF.S | 156 | 11 | 26 | 11 | 0 | - | 4 |
| | to R_MF.L | 0 | 0 | 0 | 2 | 0 | 2 | - |
| Dallas, | to R_SF | 4 | | 0 | | - | 0 | 0 |
| Young Singles | to R_MF.S | 426 | | 0 | | 0 | - | 0 |
| | to R_MF.L | 2 | | 0 | | 0 | 0 | - |
| Dallas, | to R_SF | | 3 | 0 | 0 | - | 0 | 0 |
| Families | to R_MF.S | | 18 | 6 | 0 | 3 | - | 0 |
| | to R_MF.L | | 0 | 0 | 0 | 0 | 0 | - |
| Dallas, | to R_SF | | 8 | 0 | 0 | - | 0 | 0 |
| Elderly Couples | to R_MF.S | | 35 | 0 | 0 | 4 | - | 0 |
| | to R_MF.L | | 0 | 0 | 0 | 0 | 0 | - |
| Dallas, | to R_SF | 18 | 0 | 0 | 0 | - | 0 | 0 |
| Widows | to R_MF.S | 172 | 33 | 12 | 3 | 12 | - | 0 |
| | to R_MF.L | 0 | 0 | 0 | 0 | 0 | 0 | - |
| Sacramento, | to R_SF | 9 | | 0 | | - | 0 | 0 |
| Young Singles | to R_MF.S | 525 | | 1 | | 3 | - | 1 |
| | to R_MF.L | 1 | | 0 | | 0 | 0 | - |
| Sacramento, | to R_SF | | 3 | 0 | 0 | - | 0 | 0 |
| Families | to R_MF.S | | 16 | 6 | 3 | 9 | - | 0 |
| | to R_MF.L | | 0 | 0 | 0 | 0 | 0 | - |
| Sacramento, | to R_SF | | 7 | 0 | 0 | - | 0 | 0 |
| Elderly Couples | to R_MF.S | | 31 | 17 | 0 | 3 | - | 0 |
| | to R_MF.L | | 0 | 0 | 0 | 0 | 0 | - |
| Sacramento, | to R_SF | 0 | 0 | 0 | 0 | - | 0 | 0 |
| Widows | to R_MF.S | 194 | 3 | 3 | 0 | 3 | - | 0 |
| | to R_MF.L | 0 | 0 | 0 | 0 | 0 | 0 | - |

Notes: predicted moves, normalized for 1000 nuclei per stratum

Table 3-11: FISCAL INCIDENCE BETWEEN JURISDICTIONS AND STRATA

| Stratum | Level of Government | Housing Allowances | Property Tax Cut | Fed. Income Tax Change |
|---|---|---|---|---|
| Albany, Young Singles: | | | | |
| | Federal, direct subsidy | 638.60 | 0.0 | 0.0 |
| | Federal, tax-subsidy | 0.0 | -1.02 | -0.91 |
| | Local, lost property tax | 0.0 | 5.20 | 0.0 |
| Albany, Families | | | | |
| | Federal, direct subsidy | 50.16 | 0.0 | 0.0 |
| | Federal, tax-subsidy | 0.0 | -110.4 | -33.73 |
| | Local, lost property tax | 12.30 | 433.9 | 1.94 |
| Albany, Elderly Couples | | | | |
| | Federal, direct subsidy | 60.52 | 0.0 | 0.0 |
| | Federal, tax-subsidy | -0.98 | -77.54 | -26.83 |
| | Local, lost property tax | 44.31 | 357.57 | 9.05 |
| Albany, Widows | | | | |
| | Federal, direct subsidy | 274.65 | 0.0 | 0.0 |
| | Federal, tax-subsidy | -0.33 | -14.24 | -6.28 |
| | Local, lost property tax | 39.37 | 80.87 | 3.81 |
| Dallas, Young Singles | | | | |
| | Federal, direct subsidy | 823.08 | 0.0 | 0.0 |
| | Federal, tax-subsidy | 0.0 | -0.79 | -0.63 |
| | Local, lost property tax | 0.0 | 3.73 | 0.0 |
| Dallas, Families | | | | |
| | Federal, direct subsidy | 46.24 | 0.0 | 0.0 |
| | Federal, tax-subsidy | -0.91 | -87.48 | -43.63 |
| | Local, lost property tax | 10.93 | 286.66 | 3.73 |
| Dallas, Elderly Couples | | | | |
| | Federal, direct subsidy | 58.22 | 0.0 | 0.0 |
| | Federal, tax-subsidy | 0.0 | -47.80 | -16.80 |
| | Local, lost property tax | 12.12 | 220.49 | 0.0 |
| Dallas, Widows | | | | |
| | Federal, direct subsidy | 480.17 | 0.0 | 0.0 |
| | Federal, tax-subsidy | 0.0 | -10.31 | -4.21 |
| | Local, lost property tax | 19.58 | 123.76 | 0.0 |

Table 9: INCIDENCE: BETWEEN JURISDICTIONS AND STRATA (cont'd)

| Stratum | Level of Government | Housing Allowances | Property Tax Cut | Fed. Income Tax Change |
|---|---|---|---|---|
| Sacramento, Young Singles | | | | |
| Federal, direct subsidy | | 982.60 | 0.0 | 0.0 |
| Federal, tax-subsidy | | -0.04 | -3.71 | -1.60 |
| Local, lost property tax | | 1.92 | 9.72 | 0.67 |
| Sacramento, Families | | | | |
| Federal, direct subsidy | | 45.74 | 0.0 | 0.0 |
| Federal, tax-subsidy | | -2.77 | -80.80 | -38.42 |
| Local, lost property tax | | 13.70 | 338.09 | 7.31 |
| Sacramento, Elderly Couples | | | | |
| Federal, direct subsidy | | 70.02 | 0.0 | 0.0 |
| Federal, tax-subsidy | | -1.33 | -56.72 | -41.30 |
| Local, lost property tax | | 27.71 | 289.92 | 6.66 |
| Sacramento, Widows | | | | |
| Federal, direct subsidy | | 299.39 | 0.0 | 0.0 |
| Federal, tax-subsidy | | 0.0 | -13.78 | -0.71 |
| Local, lost property tax | | 2.96 | 101.39 | 0.0 |

Notes: The table lists the direct subsidy in the case of housing allowances, the indirect subsidy via Federal Income Tax savings due to deduction of interest and local property tax, and the local property tax losses to the local jurisdiction. The unit is $ 1000 for a normalized stratum of 1000 nuclei, i.e. dollars per nucleus per year. The numbers are based on the predicted moves of Table 8.

by the shift of demand[7]. All amounts are normalized to a stratum of 1000 nuclei to allow for comparisons both among strata and among SMSA. Note that especially for the married strata, the losses in local property taxes are a sizable proportion of the housing allowances paid by the federal government.

We can sum up the results of the housing allowance experiment as follows:

- o headship rates are highly responsive to the housing subsidies,
- o mobility rates within the rental market are low and of comparable size to the findings of the Experimental Housing Allowance Program,
- o greater mobility between renting and owning produces sizable spill-over effects from federal policy to the local level.

## 3.6 Tax Simulations

### 3.6.1 Cutting the Local Property Tax By One Half

In recent years, some states have passed legislation that introduces upper ceilings for local property tax rates, e.g., Proposition 13 in California and Proposition 2-1/2 in Massachusetts. These ceilings imply a drastic reduction in local property taxes for given assessment ratios. As a crude approximation of the isolated impact due to a drastic change in the local property tax rate, the following simulation predicts the distribution of nuclei into housing categories assuming a property tax rate of half the level in effect during the estimation period 1976/77.

Effective property taxes (as percentages of the house values reported in the Annual Housing Survey) in this period were 2.2 percent in Albany, 1.3 percent in Dallas, and 1.6 percent in Sacramento. The proportion of property taxes in the out-of-pocket cost varies considerably across strata, mainly due to the variation in mortgage payments in the life cycle, and less so across housing alternatives; the overall proportion is about 10 percent. The impact of the property tax cut is softened by a reduction in the federal income tax deductions proportional to the marginal tax rate of the household. Taking this into account, the simulation reduces the cost of owner-occupancy about 3 percent for the average homeowner. This is a fairly small change in relative prices considering that the property tax rate is lowered by 50 percent.

Table 3-12 lists the distribution of housing alternatives before and after the property tax change, calculated as means of the individual choice probabilities. If we concentrate only on the tenure choice, the share of owner-occupancy increases by:

| Stratum | Albany | Dallas | Sacramento |
|---|---|---|---|
| Young Singles | .0050 | .0024 | .0119 |
| Families | .0249 | .0213 | .0273 |
| Elderly Couples | .0670 | .0151 | .0249 |
| Widows | .1080 | .0333 | .0454 |

The impact is of course strongest in Albany where the property tax is substantially higher than in Dallas and Sacramento. In addition, the impact is very low for young singles: they have high mortgage payments and the percentage of property taxes in their total out-of-pocket costs is very low. The same reasoning explains why the increase in owner-occupancy is largest for small houses. In addition, smaller houses are attractive for people with low incomes, for whom the offsetting effect of decreasing income tax deductions is least.

Finally, we can see the interjurisdictional fiscal effects in the second column of Table 3-11. In the family strata, the gains for the federal government by smaller deductions are between 23.9 percent and 30.3 percent of the losses in local property taxes. The size of this spill-over effect depends on two factors: it simply reflects the relatively high marginal tax rates for these strata, but the gains are also reduced by the higher share of owner-occupancy in response to the tax change.

Table 3-10: AGGREGATED SHARES: LOCAL PROPERTY TAX CUT

| Stratum | Alt. | ALBANY before | ALBANY after | DALLAS before | DALLAS after | SACRAMENTO before | SACRAMENTO after |
|---|---|---|---|---|---|---|---|
| YSL | NH | .6331 | .6322 | .5507 | .5501 | .6262 | .6227 |
| | O_SM | | | | | | |
| | O_ME | .0092 | .0142 | .0224 | .0248 | .0425 | .0544 |
| | O_LA | | | | | | |
| | R_SF | .0123 | .0119 | .0478 | .0477 | .0508 | .0494 |
| | R_SM | .2815 | .2791 | .3438 | .3424 | .2642 | .2578 |
| | R_LA | .0637 | .0627 | .0353 | .0350 | .0163 | .0157 |
| FAM | NH | | | | | | |
| | O_SM | .0182 | .0201 | .0440 | .0458 | .0320 | .0378 |
| | O_ME | .3310 | .3413 | .4439 | .4585 | .4772 | .4899 |
| | O_LA | .4997 | .5123 | .4085 | .4134 | .3852 | .3940 |
| | R_SF | .0546 | .0456 | .0639 | .0530 | .0780 | .0587 |
| | R_SM | .0161 | .0132 | .0152 | .0114 | .0219 | .0157 |
| | R_LA | .0804 | .0674 | .0244 | .0178 | .0056 | .0038 |
| ELC | NH | | | | | | |
| | O_SM | .1251 | .1745 | .1443 | .1529 | .1450 | .1617 |
| | O_ME | .4050 | .4173 | .5446 | .5503 | .6667 | .6728 |
| | O_LA | .2773 | .2826 | .2014 | .2023 | .0750 | .0770 |
| | R_SF | .0174 | .0137 | .0420 | .0374 | .0531 | .0436 |
| | R_SM | .0870 | .0543 | .0484 | .0412 | .0335 | .0246 |
| | R_LA | .0882 | .0576 | .0192 | .0159 | .0268 | .0203 |
| WID | NH | .2757 | .2396 | .2130 | .2016 | .2745 | .2585 |
| | O_SM | .0694 | .1606 | .1165 | .1411 | .1105 | .1506 |
| | O_ME | .1602 | .1746 | .3144 | .3202 | .2281 | .2326 |
| | O_LA | .0985 | .1011 | .0486 | .0494 | .0562 | .0572 |
| | R_SF | .0273 | .0254 | .0737 | .0703 | .0729 | .0711 |
| | R_SM | .2340 | .1942 | .1842 | .1703 | .2437 | .2181 |
| | R_LA | .1348 | .1046 | .0496 | .0450 | .0140 | .0120 |

First column : predicted shares of housing alternatives under actual 1977 local property taxes.

Second column: predicted shares of housing alternatives under only 50% of the 1977 local property taxes.

We can summarize the results of the property tax experiment as follows:

- o The impact of a strong reduction in the local property tax is small in the strata with high mortgage payments and high tax brackets. It is high for the elderly and for small homeowners.
- o The spill-over effect to the federal government is sizable. The direct effect through the marginal tax rate is partially offset by the indirect effect of movers into owner-occupancy.

### 3.6.2 Making the Federal Income Tax Less Progressive

The final simulation concerns the change in the federal income tax law that reduced the highest marginal tax rate from 70 percent to 50 percent. This has two opposing effects on housing consumption: while high-income people pay fewer taxes the deductions for mortgage interest and local property taxes are less worth and thus reduce the tax advantages of ownership. In the following simulation, we isolate the second effect by holding the income level constant and calculate the tax savings in the out-of-pocket costs of homeownership assuming the new tax schedule. We used the federal income tax schedule for 1983 and deflated the tax brackets by the Consumer Price Index to the price and income level of the estimation period.

We can again make the back-on-the-envelope calculation as in the preceding section: for the very rich, deductions lose 20 percent of their value. If we assume that a third of the out-of-pocket costs is deductible, we generate a 7 percent increase in the cost of owner-occupancy. This is an upper limit: people in lower tax brackets face a much smaller increase because below the 50 percent brackets, the marginal tax rates were only very slightly reduced. For the poor, there is no change whatsoever.

It should be noted that the sample includes only few "very rich" people (the 50 percent tax bracket in 1977 was about $ 40,000) because the selection of strata overrepresents the very young and elderly nuclei. Table 3-13 shows that the change in the marginal tax rate

Table 3-13: AGGREGATE SHARES: FLATTER FEDERAL INCOME TAX SCHEDULE

| Stratum | Alt. | ALBANY before | ALBANY after | DALLAS before | DALLAS after | SACRAMENTO before | SACRAMENTO after |
|---|---|---|---|---|---|---|---|
| YSL | NH | .6331 | .6333 | .5507 | .5508 | .6262 | .6269 |
| | O_SM | | | | | | |
| | O_ME | .0092 | .0088 | .0224 | .0219 | .0425 | .0408 |
| | O_LA | | | | | | |
| | R_SF | .0123 | .0124 | .0478 | .0478 | .0508 | .0509 |
| | R_SM | .2815 | .2817 | .3438 | .3441 | .2642 | .2651 |
| | R_LA | .0637 | .0638 | .0353 | .0353 | .0163 | .0163 |
| FAM | NH | | | | | | |
| | O_SM | .0182 | .0179 | .0440 | .0427 | .0320 | .0297 |
| | O_ME | .3310 | .3284 | .4439 | .4400 | .4772 | .4734 |
| | O_LA | .4997 | .4970 | .4085 | .4077 | .3852 | .3833 |
| | R_SF | .0546 | .0566 | .0639 | .0666 | .0780 | .0830 |
| | R_SM | .0161 | .0166 | .0152 | .0168 | .0219 | .0242 |
| | R_LA | .0804 | .0835 | .0244 | .0262 | .0056 | .0064 |
| ELC | NH | | | | | | |
| | O_SM | .1251 | .1210 | .1443 | .1440 | .1450 | .1438 |
| | O_ME | .4050 | .4034 | .5446 | .5433 | .6667 | .6645 |
| | O_LA | .2773 | .2760 | .2014 | .2012 | .0750 | .0744 |
| | R_SF | .0174 | .0177 | .0420 | .0425 | .0531 | .0542 |
| | R_SM | .0870 | .0905 | .0484 | .0493 | .0335 | .0352 |
| | R_LA | .0882 | .0914 | .0192 | .0193 | .0268 | .0280 |
| WID | NH | .2757 | .2760 | .2130 | .2131 | .2745 | .2747 |
| | O_SM | .0694 | .0676 | .1165 | .1163 | .1105 | .1108 |
| | O_ME | .1602 | .1596 | .3144 | .3136 | .2281 | .2277 |
| | O_LA | .0985 | .0984 | .0486 | .0486 | .0562 | .0560 |
| | R_SF | .0273 | .0274 | .0737 | .0739 | .0729 | .0728 |
| | R_SM | .2340 | .2351 | .1842 | .1847 | .2437 | .2440 |
| | R_LA | .1348 | .1359 | .0496 | .0498 | .0140 | .0140 |

First column : predicted shares of housing alternatives under actual 1977 Federal Income Tax schedule (highest marginal tax rate: 70%).
Second column: predicted shares of housing alternatives under 1983 Federal Income Tax schedule, deflated by CPI to 1977 levels (highest marginal tax rate: 50%).

results in a slight shift from owning to renting. More comprehensively, the share of renting increases by:

| Stratum | Albany | Dallas | Sacramento |
|---|---|---|---|
| Young Singles | .0005 | .0004 | .0017 |
| Families | .0056 | .0061 | .0081 |
| Elderly Couples | .0070 | .0015 | .0040 |
| Widows | .0026 | .0010 | .0004 |

These numbers are very small: not only very few people are affected by the change in the marginal tax rate, but these "very rich" people are also those who are least likely to shift to the rental market.

Within each city, the shifts into rental units basically reflect the tax brackets which can be seen by a look at the yearly mean income before taxes:

| Stratum | Albany | Dallas | Sacramento |
|---|---|---|---|
| Young Singles | $ 5,200 | $ 6,700 | $ 5,200 |
| Families | 22,200 | 26,400 | 23,000 |
| Elderly Couples | 13,900 | 15,400 | 13,700 |
| Widows | 5,300 | 5,600 | 6,000 |

But mean income will not tell the entire story because the picture is complicated by distributional differences within each stratum and among SMSAs - both in terms of the income distributions and in terms of mortgage payments. This might explain the large shift to rental units among elderly couples in Albany.

Finally, the spill-over effects induced by the few moves in the rental market are calculated from the predicted moves in a stratum of 1000 nuclei (see the last column of Table 3-11). Note that the already mentioned problems with the small number of affected people are compounded by the erratic nature of the individual forecasts. The predicted changes in local property tax payments might therefore be unreliable. Aggregated over the three SMSAs and over all strata, the spill-over effect in lost property taxes is about 15 percent of the income tax deductions saved by the federal government. The latter are measured after the tax change: the percentage in terms of the direct effect is lower because the moves into the rental market partially offset the savings in income tax deductions.

We sum up the Federal Income Tax experiment as follows:

o Flattening the income tax schedule affects relatively few people and the changes in the aggregate are therefore small. Too few sample nuclei are affected to allow a reliable simulation.

o The pure price effect makes the federal income tax deductions worth less at high marginal tax rates. The resulting shift in the rental market is very small because the "very rich" people that are affected by the change are the least likely nuclei to switch to renting.

## 3.7 Conclusions

The main conclusion from the baseline estimates and from the housing allowance experiment is the strong response of headship rates to relative housing prices. Headship rates can not be treated as exogenous variables. The second conclusion concerns fiscal federalism: in all three fiscal changes, the spill-over effects from federal fiscal action to the local level and vice versa are of sizable magnitudes.

Taken as a descriptive device, the model performs well in terms of fit and prediction accuracy. Simulation results give a fairly stable pattern across SMSAs. In the case where the simulations coincide with other published experiments, the results were very close. All this gives us some confidence in the robustness of the model and its forecasts.

However, one caveat should be made which leads us to the next chapter: it concerns the interpretation of the cross-sectional data as a steady state, especially with using the housing prices as they are reported in the Annual Housing Survey. The approach ignores all intertemporal effects that might produce price dispersion or disequilibria. Spurious price elasticities may come from the fact that many sitting tenants receive tenure discounts: if we compare the rent of their actual unit with the hypothetical prices of those not chosen (measured as the prices paid by recent movers), the existence of tenure discounts will give us a larger price response than if we compare the prices with the tenure discounts subtracted. The same argument holds for other kinds of factors producing price dispersion in the housing market, e.g., search equilibria, explicit or implicit long run contract agreements, and rent control. In the following chapter, we will develop a theoretical model of idiosyncratic exchange that produces tenure discounts even in the absence of rent control. We will also find empirical evidence for tenure discounts: hedonic regressions produce significant negative coefficients for length of tenure, indicating an upward bias of the price coefficient. Estimations by Follain and Malpezzi (1980) translate in a bias of about 9.3 percent in Albany, only 1.5 percent in Dallas. Estimates by Malpezzi, Ozanne, and Thibodeau (1980) indicate much larger tenure discounts, yielding an upward bias of about 25 percent in Albany, 8 percent in Sacramento, and 4.9 percent in Dallas. However, even after subtracting 25 percent of the price coefficients, the main conclusion of this chapter still holds: household formation is highly responsive to housing prices, with elasticities and simulation responses smaller by 25 percent which remains still very large.

Footnotes to Chapter 3

(1) To take an extreme position: even suicide rates and health status may depend on housing market conditions as pending eviction or urban blight.

(2) See Statistical Abstract of the United States 1980, Table 819: Indexes of Residential Rents in Selected SMSAs: 1970-1980, divided by Table 811: Consumer Price Indexes - Selected Cities and SMSAs: 1960-1979.

(3) This assumption contradicts Hu (1980), who considers marriage as the crucial link between economic factors and household formation. See the discussion of the six mechanisms in household formation at the outset of this paper - of those seem marriage and divorce (and of course death) the least likely to be price responsive.

(4) This sharing scheme is realistic for roommates, less so for adult children living in their parents' household. However, they incur non-monetary cost in the form of household help etc.

(5) Note that for a common price for all housing units this relation establishes an identity between the prices of non-heads and heads. In this case, the price coefficients would not be identified in certain functional forms of the demand equations. One sufficient condition for identification independent of the functional form is the presence of economies of scale in the formation of larger households. In fact, these economies are likely to exist and seem to be a major attraction to share accommodations.

(6) These categories are still fairly large and do not distinguish quality levels within each size category. This holds for the non-head category as well where we do not differentiate nuclei according to the number of nuclei per household. As a consequence for marginal analysis, OOPOCK measures expenditure rather than price within each size category. However, the confusion between prices and expenditure vanishes between the discrete categories of our demand model: in each category, OOPOCK can be interpreted as the price of a standard bundle in the category. Note that one reason to use discrete choice rather than marginal analysis is the assumption that these standard bundles are qualitatively different and do not only group housing according to some scalar measure of housing services. In Chapter 5, the standard bundle of each category is computed and priced as a Lancastrian commodity, using hedonic regressions.

(7) However, the accommodating supply shifts are ignored in this analysis. The fiscal side-effects between jurisdictions may well be offset by increased property-taxes paid by landlords. If landlords overassess their buildings, the shifts in tax revenues depicted in Table 3-11 are even overcompensated.

# CHAPTER FOUR

# DYNAMIC ASPECTS IN THE HOUSING MARKET EQUILIBRIUM

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## 4.1 Introduction: Market Imperfections and Government Intervention

So far, our empirical results were derived from a single cross section of the Annual Housing Survey. Implicitly, we assumed that the observed market outcome represents a stable steady state with a unique price for each housing alternative. Thus, we could ignore the time dimension. In addition, we assumed perfect elastic long run housing supply. In this chapter, we will examine how violations of those heroic assumptions affect housing demand estimation. In particular, we try to shed some light on the existence of and causes for price dispersion. We will concentrate on the rental housing market, where the above mentioned assumptions seem most likely unrealistic.

Market imperfections abound in the rental housing market. First, housing is a durable good where prices are not necessarily defined by one period spot market conditions alone. Second, high monetary and nonmonetary transactions cost are involved with changing consumption by moving. These two conditions create intertemporal externalities. Third, property rights of the rental unit are given up only temporarily, giving the seller a strong incentive to care who the buyer is. However, there is uncertainty: the tenants' characteristics will be revealed only after some time. These two conditions create an interpersonal externality. We will show that these externalities create rental price dispersion.

There are other mechanisms which create a sustained price dispersion: costly search for an appropriate housing unit may create a partition of the market in expensive units which are easy to find, and well hidden inexpensive units. Prices may vary according to implicit contracts including maintenance on the tenants side. These contracts may be correlated with such characteristics as landlord living in the unit or the landlord being a small private owner.

In addition to these deviations from perfect market assumptions still within the competitive market context, government intervention regulates competition in the housing market. Government intervention takes place both on the demand and the supply side of the market, in form of housing allowances, tax subsidies, public housing provision, and rent control. Again, we will show that government intervention is able to generate price dispersion in the housing market, intertwined with the above mentioned market imperfections.

Empirical analysis of government intervention in West Germany faces the problem of the impossibility of a with and without analysis, due to the fact that housing market intervention is basically a federal function, depriving researchers from regional variation, and that there is only poor time series data available to exploit the temporal variation in government policies. However, we will be able to draw empirical conclusions by comparing evidence in West Germany with evidence from the United States where we observe a variety of local housing policies as well as the total absence of intervention.

This chapter proceeds as follows: First, we will model mechanisms leading to market imperfection, in particular price dispersion, and collect empirical evidence for this. Second, we try to disentangle the effects of government intervention and the effects of intrinsic housing market imperfections. Often, in particular in the discussion about rent control, these two sources of inefficiency are confused, and policy recommendations are easily victims of the mistake to propose first best solutions in this second best environment. As a case in point, we will study a highly controversial piece of legislation that regulates the rental housing market in the Federal Republic of Germany, the "2. Wohnungsraumkuendigungs-schutzgesetz" (WKSchG), or law for the protection of tenants from arbitrary eviction. Finally, in the conclusions, we will propose some ad hoc remedies which will allow estimation of the price elasticities unbiased from the price dispersion, even if no panel data is available to explicitly model the underlying intertemporal processes.

## 4.2 Empirical Evidence for Price Dispersion

### 4.2.1 Tenure Discounts

The first example of price dispersion in the rental housing market are so-called tenure discounts. Tenure discounts are the phenomenon of a gap between spot market rent and actual rent that increases with length of tenure. They can be measured as the difference in rent paid for comparable units by households moved in at different times. Units are kept comparable by controlling for housing quality and neighborhood characteristics as well as tenants' and landlords' characteristics. This is achieved by applying hedonic regression techniques where a function of the form

(4.1) $R = f(Q,N,T,L;t)$

is estimated. Here R denotes the rent, and Q, N, T, and L vectors of housing quality, neighborhood, tenant, and landlord characteristics. Finally, the length of tenure (denoted by t) enters this hedonic rent index. The following estimated tenure discounts in West Germany are calculated from hedonic regressions by Behring and Goldrian (1983), based on about one percent of all West German households in 1978. Behring and Goldrian use a semilogarithmic functional form, where the length of tenure enters linearly, quadratically, and in form of a dummy variable for very long length of tenure. This generates the nonlinear time profile of Table 4-1.

Already after one year of tenure, tenants pay two percent less rent than new tenants in comparable units. The discounts increase until they level off for very long lengths of tenure where they amount

TABLE 4-1: TENURE DISCOUNTS IN WEST GERMANY

| Length of Tenure (Years) | Areas with High Population Density | | | Low Population Density | | Rural Area |
|---|---|---|---|---|---|---|
| | City | Fringe | Environs | City | Fringe | |
| 1 | 2% | 2% | 2% | 2% | 2% | 2% |
| 5 | 8 | 9 | 11 | 10 | 10 | 10 |
| 10 | 13 | 15 | 20 | 16 | 19 | 17 |
| 14 | 19 | 25 | 27 | 19 | 25 | 26 |

Source: Behring and Goldrian (1982).

to savings of up to more than a quarter of the rent which a new resident would have to pay for the same unit.

We use the same methodology to measure tenure discounts in the United States, but, in addition, we categorize SMSAs by their rent control legislation. Tables 4-2 and 4-3 present estimated tenure discounts for fifty-nine Standard metropolitan Statistical Areas (SMSA), calculated from hedonic regressions by Follain and Malpezzi (1980) and Malpezzi, Ozanne, and Thibodeau (1980), based on the Annual Housing Surveys by SMSA 1974-1977. Follain and Malpezzi (1980) estimate tenure discounts as a linear function of the length of tenure only, whereas Malpezzi, Ozanne, and Thibodeau (1980) use the same nonlinear specification as Behring and Goldrian (1980). The large difference in the size of the discounts is disturbing. It might be attributable to these specification differences. If the true relation between the discounts and length of tenure is concave, the linear specification of Follain and Malpezzi will be biased downwards for

TABLE 4-2: TENURE DISCOUNTS IN THE UNITED STATES 1974-76

| Standard Metropolitan Statistical Area | Rent Control Status | Tenure Discounts after 10 Years (1) | (2) | Average Length of Tenure |
|---|---|---|---|---|
| Albany, NY | * | 13.5% | 36.4% | 6.9 |
| Anaheim, CA | * | 18.3 | 25.0 | 3.3 |
| Atlanta, GA | | 6.9 | 21.3 | 4.4 |
| Boston, MA | * * | 13.4 | 30.2 | 6.9 |
| Chicago, IL | | 7.7 | 17.9 | 6.0 |
| Cincinnati, OH | | 10.5 | 27.2 | 5.2 |
| Colorado Springs, CO | | 9.3 | 18.1 | 3.5 |
| Columbus, OH | | 8.6 | 25.3 | 4.6 |
| Dallas, TX | | 4.1 | 13.5 | 3.6 |
| Detroit, MI | | 9.7 | 25.5 | 5.3 |
| Fort Worth, TX | | 7.1 | 18.6 | 3.7 |
| Hartford, CT | | 9.2 | 18.8 | 6.0 |
| Kansas City, KS/MO | | 10.1 | 22.0 | 4.8 |
| Los Angeles, CA | * * | 10.8 | 19.0 | 4.6 |
| Madison, WI | | 5.6 | 14.1 | 4.0 |
| Memphis, TN | | 8.7 | 18.4 | 4.9 |
| Miami, FL | * * | 8.4 | 13.7 | 4.4 |
| Milwaukee, WI | | 13.2 | 23.7 | 5.5 |
| Minneapolis, MN | | 7.6 | 13.3 | 4.3 |
| Newark, NJ | * * | 9.1 | 18.9 | 6.6 |
| New Orleans, LA | | 9.9 | 24.1 | 5.7 |
| Newport News, VA | | 10.1 | 23.8 | 4.5 |
| Orlando, FL | | 4.7 | 25.5 | 3.5 |
| Paterson, NJ | * * | 10.3 | 19.8 | 6.6 |
| Philadelphia, PE | | 7.4 | 24.4 | 6.3 |
| Phoenix, AZ | | 18.6 | 22.5 | 3.6 |
| Pittsburgh, PE | | 8.2 | 33.0 | 6.5 |
| Portland, OR | | 11.0 | 22.0 | 4.4 |
| Rochester, NY | * | 12.5 | 22.6 | 4.5 |
| Salt Lake City, UT | | 12.3 | 31.0 | 4.3 |
| San Antonio, TX | | 8.9 | 22.3 | 4.7 |
| San Bernadino, CA | * | 12.9 | 28.9 | 4.0 |
| San Diego, CA | * | 11.6 | 29.5 | 3.9 |
| San Francisco, CA | * * | 11.3 | 22.2 | 5.4 |
| Spokane, WA | | 11.9 | 34.2 | 4.4 |
| Springfield, MA | * | 6.6 | 26.3 | 7.1 |
| Tacoma, WA | | 8.1 | 23.5 | 3.8 |
| Washington, DC | * * | 7.5 | 18.6 | 5.5 |
| Wichita, WA | | 11.6 | 26.3 | 4.3 |

Source: (1) Follain and Malpezzi (1980),
(2) Malpezzi, Ozanne, and Thibodeau (1980).
Two asterisks denote a strict measure of rent and/or eviction control
One asterisk denotes a weak measure of rent and/or eviction control

TABLE 4-3: TENURE DISCOUNTS IN THE UNITED STATES 1976-77

| Standard Metropolitan Statistical Area | Rent Control Status | Tenure Discounts after .... Years | | | |
|---|---|---|---|---|---|
| | | 1 | 5 | 10 | 14 |
| Allentown, PA | | 4.1% | 18.6% | 32.4% | 40.1% |
| Baltimore, MD | * * | 2.4 | 11.5 | 21.1 | 27.4 |
| Birmingham, AL | | 3.4 | 14.7 | 23.2 | 25.6 |
| Buffalo, NY | * | 4.1 | 18.3 | 31.2 | 37.7 |
| Cleveland, OH | | 2.2 | 9.8 | 16.7 | 20.1 |
| Denver, CO | | 3.4 | 15.3 | 26.6 | 32.8 |
| Grand Rapids, MI | | 3.8 | 16.9 | 28.6 | 34.0 |
| Honolulu, HI | * | 2.9 | 15.6 | 33.7 | 49.9 |
| Houston, TX | | 4.1 | 18.2 | 30.8 | 36.9 |
| Indianapolis, IN | | 2.4 | 10.9 | 18.5 | 22.2 |
| Las Vegas, NV | | 1.9 | 8.3 | 14.4 | 17.5 |
| Louisville, KY | | 3.4 | 15.0 | 25.1 | 29.7 |
| New York, NY | * * | 3.9 | 17.9 | 31.0 | 38.2 |
| Oklahoma City, OK | | 3.8 | 16.5 | 26.9 | 30.7 |
| Omaha, NE | | 2.3 | 10.5 | 17.9 | 21.7 |
| Providence, RI | * | 4.6 | 20.5 | 34.3 | 40.6 |
| Raleigh, NC | | 3.2 | 14.3 | 24.6 | 30.0 |
| Sacramento, CA | * * | 2.8 | 12.2 | 19.9 | 22.9 |
| St. Louis, MI | | 2.4 | 10.8 | 18.2 | 21.5 |
| Seattle, WA | | 3.3 | 14.5 | 24.2 | 28.4 |

Source: Malpezzi, Ozanne, and Thibodeau (1980).
Two asterisks denote a strict measure of rent and/or eviction control
One asterisk denotes a weak measure of rent and/or eviction control

TABLE 4-4: AVERAGE TENURE DISCOUNTS BY RENT CONTROL STATUS

| Rent Control Status | Discounts for a 10-Year Tenure | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Controlled according to strict measure | 10.0%<br>(2.2) | 20.3%<br>(5.0) | 24.0%<br>(6.1) |
| Controlled according to weak measure | 11.3<br>(3.1) | 23.9<br>(6.2) | 28.6<br>(6.3) |
| Uncontrolled Market | 9.3<br>(2.9) | 22.7<br>(5.3) | 23.4<br>(5.5) |

Computed from: (1) Follain and Malpezzi (1980), 1974-76,
(2) Malpezzi, Ozanne, and Thibodeau (1980), 1974-76,
(3) Malpezzi, Ozanne, and Thibodeau (1980), 1976-77.
Standard deviation in brackets.

small and medium tenures. The estimates by Malpezzi, Ozanne, and Thibodeau may show an upward bias because of collinearity with their identically specified concave age of dwelling variable so that some impact of old structures pollutes the tenure discounts. Goodman and Kawai (1982) produced linear estimates for 19 SMSAs in the Annual Housing Survey of 1977-78 which are almost identical to those reported in Table 4-2. Barnett (1979), Noland (1980), and Lowery (1981) in turn reproduce estimates similar to the nonlinear specification of Table 4-2. Guasch and Marshall (1983) mention the possibility of an upward bias in all these estimates due to sample selection adverse to movers in response to low tenure discounts. Their empirical results, however, were inconclusive[1]. For this Chapter, we are less interested in the actual size of the discounts as their relation with rent control. As we will see in Table 4-4, our conclusions hold for both the linear and the nonlinear specification.

In the United States, rent and eviction control legislation is at the discretion of the state or even municipal level of jurisdiction. Information about the presence of controls is collected from Braid (1980), Thibodeau (1981), and the National Multi Housing Council (1982). We use two measures to assess whether the market was influenced by rent or eviction control. The stricter measure includes all SMSAs, in which rent control was in effect in at least one jurisdiction. These SMSAs are marked in Tables 4-2 and 4-3 by two asterisks. The weaker measure is denoted by one asterisk. It marks an SMSA where state legislation made rent control easy to introduce. Included into this category are also SMSAs, in which rent control was a "big issue" and was rejected only by a small margin. Thus the presence of at least one asterisk indicates a SMSA where landlords faced or perceived an incentive to restrain rent increases and eviction.

Does rent control affect tenure discounts? Even in SMSAs where no jurisdiction ever had some kind of rent control, e. g., Phoenix and Milwaukee, we observe substantial tenure discounts. Table 4-4 lists the average tenure discounts and their corresponding standard deviations, by rent control status, time period, and specification. Average tenure discounts tend to be higher under effective or likely rent control, but none of the differences is statistically significant: the hypothesis that tenure discounts are not affected by rent control at all can not be rejected. Note that this result holds under both specifications of the hedonic index and in both time periods of Tables 4-2 and 4-3.

### 4.2.2 Landlord Characteristics and Search

Price dispersion in the rental market may not only be generated by the intertemporal process in which the landlord grants tenure discounts to the tenant, but by all mechanisms that create some form of partition of the rental market and therefore produce a possibility for sustained cross sectional price variation.

The first partition to be discussed in this subsection concerns landlord characteristics: landlords with only a few units to lease or landlords who live in the building where they rent out the remaining units may behave quite differently from large scale landlords who administer their units by a house manager. Second, private landlords may have a different behavior than institutions like the public sector housing companies in England or West Germany. Confirming evidence is presented in Tables 4-6 and 4-7.

On average over the 59 SMSA's in the survey of Malpezzi, Ozanne, and Thibodeau (1980), we find a rent reduced by 2.8 percent ceteris paribus when the landlord is present in the building. The evidence, however, is weak: it is significant only in 21 of the SMSA's, insignificant in 37, and has a significant but reversed sign in one SMSA. In our estimations, reported in Section 5.5.1 in Chapter Five, we present similar evidence with a significant discount in three out of four SMSA's. The pattern of discounts granted by a private landlord in Germany is much clearer, see Table 4-7. In cities situated in regions with high population density comparable to U.S.

TABLE 4-6: DISCOUNTS, WHEN LANDLORD IS PRESENT IN BUILDING

| | | |
|---|---|---|
| Mean over 59 SMSA's in the U.S. 1974-1977[1] | -0.028 | |
| Boston 1977[2] | -0.039 | (2.59) |
| Dallas 1977 | -0.056 | (1.66) |
| Los Angeles 1977 | -0.042 | (2.04) |
| Minneapolis/St.Paul 1977 | -0.0012 | (0.06) |

TABLE 4-7: DISCOUNTS, WHEN LANDLORD IS PRIVATE PERSON

| | | | |
|---|---|---|---|
| West Germany 1978[3]: | High Density Region, City | -0.127 | (31.99) |
| | Fringe | -0.139 | (15.85) |
| | Environs | -0.097 | ( 5.54) |
| | Low Density Region, City | -0.144 | (16.06) |
| | Fringe | -0.084 | ( 8.27) |
| | Rural Region | -0.174 | (15.41) |

TABLE 4-8: PREMIUM FOR MOVERS FROM OUTSIDE SMSA

| | | |
|---|---|---|
| Boston 1977[4] | 0.041 | (2.77) |
| Dallas 1977 | 0.026 | (1.80) |
| Los Angeles 1977 | 0.017 | (1.14) |
| Minneapolis/St.Paul 1977 | 0.030 | (2.15) |

---

Coefficients represent hedonic regression coefficients of a semi-logarithmic form, i. e. percentage discounts or premiums on the rent. T-statistics in brackets.

Source: (1) Malpezzi, Ozanne, and Thibodeau (1980),
(2) Own Hedonic Estimations in Section 5.5.1,
(3) Behring and Goldrian (1983),
(4) Own Hedonic Estimations in Section 5.5.1, based on recent movers only.

SMSA's, we measure a discount as large as 13 percent and highly significant.

All these hedonic coefficients measure discounts on the rent over and above the tenure discounts already corrected for as reported in Tables 4-1 through 4-3. However, they may be related to the emergence of tenure discounts as an amplifying factor. The line of reasoning follows Williamson's (1979) notion of idiosyncratic exchange: the landlord-tenant relation will be more affected by idiosyncrasies of either tenant or landlord if the latter is present in the building or a private person. It would be interesting to study the interaction of the intertemporal phenomenon of tenure discounts with the cross sectional phenomenon of "landlord-present-discounts". However, this would require panel data which is not available, so that we cannot pursue this topic further.

A second partition of the rental market which is able to produce price dispersion is related to search. Finding a new housing unit is a costly process and people will trade off housing characteristics including price with search costs. This argument follows Loikannen's (1982) suggestion that people searching under time pressure choose suboptimal housing bundles. This would imply on average a rent differential between people with high and with low search cost. In particular, we might partition the rental market of recent movers in those who move from outside the SMSA and those who move within the SMSA. The latter group will have lower search cost due to more information than the former. One might even think of inter-SMSA moves as a two stage process where under time pressure a suboptimal unit is chosen for a short initial period. Then, from this base, a search for the optimal unit is started, with no time pressure and accumulated information. The following simple model illustrates our point.

Let us assume a fixed supply of N vacant housing units in the SMSA, where a fraction c are good values ("bargains"), and $(1-c)*N$ units are lemons. However, at the time of the search, lemons and bargains can not be distinguished by the searcher.

For simplicity, we assume an equal number of N movers, out of which a fraction g ("greenhorns") come from outside the SMSA, and the remaining $(1-g)*N$ persons are intra-SMSA movers. The movers from outside the SMSA have one period to find a place to live, then they stay further b periods in the city, then die or migrate. We will consider a steady state: a stable distribution of greenhorns and intra-city movers implies $g=1/b$. If a greenhorn picks a bargain, he stays there for the remainder of his life. Otherwise, he will start his second stage search for a better value. Thus, we have b vintages of people:

1: greenhorns,
2: insider with one chance to recontract,
...
b: insider with b-1 chances to recontract.

For stationarity, we assume that each vintage has the same size. In addition, let us assume for the beginning that all searchers face the same probability to pick a lemon. Then $(1-c)^i$ persons of vintage i still live in lemons. In a cross section of all movers and sitting tenants, the proportion of people living in bargains is

$$(4.2)\quad p = 1 - 1/b \sum_{i=0}^{b-1} (1-c)^{i+1}$$

$$= c/b \sum_{i=0}^{b-1} (b-i)(1-c)^{i} > c$$

This steady state "price dispersion" between new movers and all sitting tenants just reflects the accumulated chance to recontract. This dispersion will be even greater if the probability to pick a lemon is larger for greenhorns than for insiders.

This simplistic model implies a division of the market into two categories: A segment with long-run leases, relatively low rents for a standard unit, occupied by a majority of tenants already living a long time in the SMSA; and a segment with short-run leases, higher rents, and a majority of tenants moved in from outside the SMSA.

This hypothesis can be substantiated by evidence from hedonic estimation, see Table 4-8. Using the functional specification of Section 5.5.1 in Chapter Five, we include an indicator variable for movers from outside the SMSA, and restrict the estimation to recent movers (within twelve month). We estimate a premium charged to movers from outside the SMSA which ranges from 1.7 percent in Los Angeles to 4.1 percent in Boston, the latter significant at the 99 percent level.

## 4.3 Rent and Eviction Control in West Germany

What implications have the deviations from textbook economics listed in the preceding section on the analysis of rent control? As a case in point, we will study a highly controversial piece of legislation that regulates the rental housing market in the Federal Republic of Germany, the "2. Wohnungsraumkuendigungsschutzgesetz" (WKSchG), or law for the protection of tenants from arbitrary eviction. This law is in effect since January 1975[2]. The law consists of two - stylized - provisions: eviction of tenants is prohibited, and the rent is indexed once the tenant moved in. However, when a new tenant moves in, the rent can be set freely[3].

More precisely, eviction is only permitted, if (1) the tenant breaches his contract (e. g., does not pay his rent), if (2) the landlord himself or a close relative wants to move into the unit, or if (3) the landlord is substantially inhibited in the appropriate economic usage of the lot (e. g., conversion into office space in areas assigned by zoning laws as a business district)[4]. The rent regulation permits the landlord to pass on cost increases and some of the cost of upgrading. In total, he may raise the rent up to the level of a standard rent that is defined by the average rent of comparable units, allowing him in addition to passing costs also to skim off some of the appreciation. However, a time consuming formal procedure is required for any rent change[5]. Thus, the rent level for sitting tenants can be described as lagged average rent of comparable units.

The law is highly controversial: proponents argue, it is necessary to counterbalance the weak position of tenants in a sellers' market by an regulated pricing scheme. Specifically, it is claimed, that without the price regulation a landlord can exploit the exit barriers of high moving costs giving him a sort of local monopoly power. We will argue that the existence of tenure discounts contradicts this claim by showing that also the landlord faces exit barriers in terms of costs of uncertainty. A second point concerns the eviction control: eviction, arbitrary in the sense of justified or unjustified[6] discrimination, inflicts high moving costs on the tenant. The legislation would eliminate or at least restrict the landlords' discriminatory behavior, reduces moving costs, and therefore makes tenants better off. On the hand, opponents argue, that crucial property rights - the right to evict an unpleasant tenant - are only given up against compensation for the money value of those rights in terms of higher rents and depressed supply, which, as they claim, will ultimately reduce tenants' utility. We will show that the the claim of higher rents and depressed supply holds. However, the latter conclusion is not necessarily correct, because the balance between the value of reduced moving for the tenants and the value of restricted property rights for the landlords is affected by the externalities enumerated above. In fact, rent and eviction control can make both tenants and landlords better off by reducing the distorting effects of those externalities.

We will first examine the part of the German tenants' protection legislation that controls the rent level. As was pointed out in the introduction, though the rent for sitting tenants is regulated, the initial rent can be set freely. Eekhoff (1981) shows that the primary effect of this price regulation is a heavily front-loaded payment schedule, depicted in Figure 4-5, to keep profits at the pre-legislation level. Losses (B) in the second phase of the lease are compensates by profits (A) from the high initial rent in the first phase.

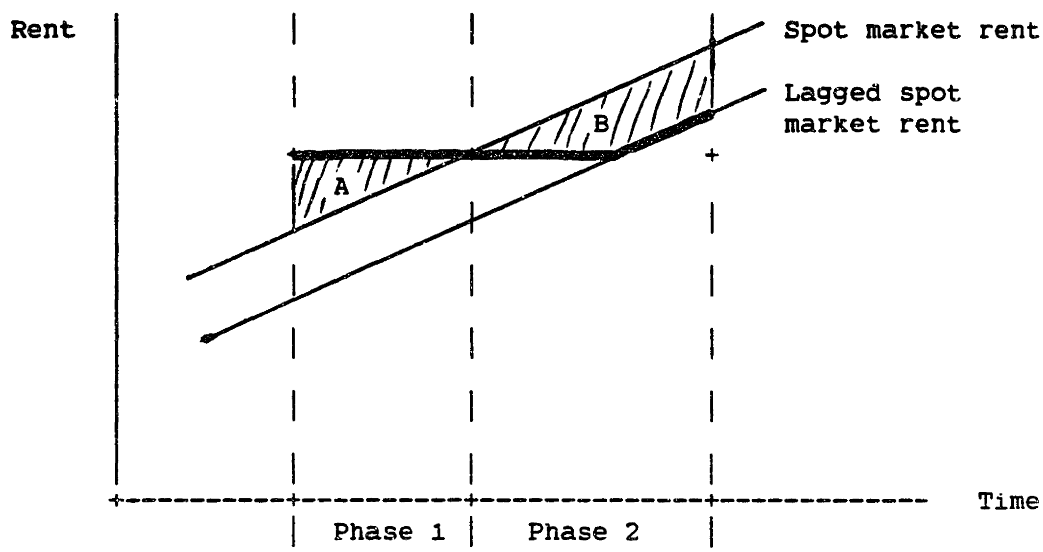


Figure 4-5: Effect of the Price Regulation. (Source: Eekhoff 1981).

This brings us to the topic of tenure discounts: the pricing scheme of Figure 4-5 implies that the rent of sitting tenants in terms of the initial rent is a falling, then constant function of the length of tenure. Thus, the price regulation in the German tenants' protection legislation seems to be a perfect explanation for price dispersion in the form of tenure discounts. This, however, seems rushing to a wrong conclusion: significant tenure discounts are also present in metropolitan areas in the United States, even where no rent and eviction control ever has been in effect. In turn, if tenure discounts indeed exist independently of rent control and are as substantial as indicated by the evidence of Tables 4-1 through 4-3, then the analysis of rent control must take into account the mechanisms that produce those discounts.

The second part of the West German tenants' protection legislation, the eviction control provision, stirs most of the heat of the emotional debate on either side, obviously, because it concerns the subtle balance between two vital rights. On one side, there is the tenant's right of the invulnerability of his dwelling, sometimes interpreted as the right to stay in the dwelling, along the lines of positive prescription. Its economic value is expressed in the psychological and monetary moving costs inflicted on the tenant when forced to move. On the other side, we have the landlord's right to dispose of his property. Its economic value is expressed in the increased psychological and monetary maintenance costs inflicted on the landlord, when forced to keep a costly or unpleasant tenant. An economic analysis of the tenants' protection legislation must take in account the economic values of those rights and how they are affected by the legislation.

Obviously, the restriction of the landlord's property rights only matters, because tenants are different, but do not reveal these differences at the time when the rental starts. Eviction is the landlords' instrument to cope with that uncertainty. Eviction control deprives the landlord of this instrument. Thus, a model to analyze the rental market and eviction control must include heterogeneous landlords and tenants, and it must include uncertainty about the type of tenant/landlord in the initial period.

Finally, the interest of landlords for a matching tenant, and the interest of tenants not to move too often, coincides, once tenant and landlord realized that they are matching. This provides a basis for an externality: the tenant will better maintain the unit, reducing the costs for the landlord. The landlord in turn will want to keep the tenant, for instance by giving him tenure discounts, reducing his probability to move out. The landlord has two incentives to do so: first, he will reward the tenant's good care for the unit, second, he saves the uncertainty of gambling for a new good tenant. Eviction control yields a positive probability for the landlord to still have a bad tenant in the second period. Thus, tenure discounts might become an even more important instrument for the landlord, once he is not permitted to evict.

In the following section, we will construct a model around those hypotheses, concentrating on the link between tenure discounts, heterogeneous tenants and landlords, and moving costs[7].

## 4.4 A Microeconomic Model of Landlord-Tenant Relations

Let us assume a rental housing market with four different agents:

(1) Landlords of type A,
(2) Landlords of type B,
(3) Tenants of type A,
(4) Tenants of type B.

We will consider two time periods. The landlords supply N housing units, where N is large, but fixed over this two periods. The proportion of type A tenants on the market in period 1 and 2 will be denoted by $a_1$ and $a_2$, respectively, and the corresponding proportions of type A landlords by $b_1$ and $b_2$. Though the number of housing units is fixed, the service stream generated by each unit is variable and reflects the quality of the dwelling unit. The housing stock is durable in the sense that once the landlord has decided on the quality level of his unit, this level will stay put over both periods. Similarly, we assume that the tenant, once having chosen his optimal quality level, will not change his mind in the second period. The difference between both periods will become clear later. We denote the quality level or service stream of a housing unit by h. The landlord recieves profits P(h) from the dwelling he is leasing, and the tenant enjoys utility U(h,x) from the stream of housing services h and from the consumption of other goods, denoted by x, which he can afford after having spent his income y on housing.

The difference between the types of landlords and tenants consist in the monetary and non-monetary maintenance cost of the dwelling unit. If landlord and tenant match these costs will be lower as compared to the case of a mismatch. A landlord of type A faces costs

$c_{AA}(h)$ to supply a unit of quality h to a type A tenant, $c_{AB}(h)$ to a type B tenant. Analogously, a landlord of type B has costs $c_{BA}(h)$ and $c_{BB}(h)$ for type A and type B tenants, respectively. We will view the world of our model through the eyes of a type A landlord and tenant, and assume complete symmetry for type B persons. Then, we simply denote the costs in a match by $c_A(h) = c_{AA}(h) = c_{BB}(h)$, and the costs in a mismatch by $c_B(h) = c_{AB}(h) = c_{BA}(h) > c_A(h)$. Tenants are assumed to be indifferent with respect to the type of their landlord.

The two periods are short enough to keep our assumptions of a fixed stock, durable quality, and lasting preferences sensible. Furthermore, we are not interested in the division of consumption across the periods and assume that landlords and tenants have a common discount rate which is equal to the interest rate. The only purpose of dividing the short run horizon into two periods is the information available to the agents in the market. At the beginning of the first period, landlords and tenants are unable to identify the other party's type. Only after the first period, the types are revealed.

Rental contracts are made at the beginning of the first period. A contract (r,h) lays down the rent r for a dwelling with quality h. Both parties may breach the contract after the types are revealed at the beginning of the second period: the landlord is allowed to evict the tenant, and we will assume that he will do so in the case of a mismatch[8]. A tenant does not care about match or mismatch, but he may breach the contract for some exogenous reason, say, to migrate because of a job offer. We will denote the probability of such an event by

$p_0$. However, the tenant will actually move only if the attractiveness of the job offer outweighs the opportunities he may have in his current housing unit. This is the point where tenure discounts enter: the landlord may reduce the rent in the second period by an amount t, and this tenure discount will negatively influence the probability of moving. We will denote this moving probability by $p_m(t)$ with $p_m(0) = p_0$ and $p_m'(t) < 0$. This tenure discount is entirely at the discretion of the profit maximizing landlord. A priori, it can be positive or negative.

The rental contract is a contingent contract in the sense that the rent-quality relation is determined taking in account the possible events at the transition between the two periods. Table 4-9 gives a survey of all possible events with their respective probabilities. Competition among the large number of landlords and tenants will produce an equilibrium rent level r corresponding to an eqiulibrium quality level h[9]. Once the quality of the match is revealed, landlord and tenant form a sort of bilateral monopoly. As we defined the contract (r,h), this situation is anticipated in the competitive bargaining process before the initial period starts and the rent level adjusted accordingly. Therefore, the bilateral monopoly does not result in a bargaining game in the second period when match or mismatch is revealed, and we can avoid specifying bargaining rules with their arbitrariness[10].

The contingent market approach does not resolve the market failure intrinsic in the model constructed so far. The tenant type is

TABLE 4-9: SPECIFICATION OF EVENTS FOR TYPE A LANDLORDS AND TENANTS

(1) LANDLORD:

| Case | Tenant in Period 1 | Tenant in Period 2 | Move | Probability | Rent in Period 1 | Rent in Period 2 | Costs in Period 1 | Costs in Period 2 |
|---|---|---|---|---|---|---|---|---|
| 1 | A | A | stays | $a_1(1-p_m(t))$ | r | r-t | $c_A$ | $c_A$ |
| 2 | A | A | moves | $a_1p_m(t)a_2$ | r | r | $c_A$ | $c_A$ |
| 3 | A | B | moves | $a_1p_m(t)(1-a_2)$ | r | r | $c_A$ | $c_B$ |
| 4 | B | B | stays | $(1-a_1)(1-sp_m(0))$ | r | r | $c_B$ | $c_B$ |
| 5 | B | A | moves | $(1-a_1)sp_m(0)a_2$ | r | r | $c_B$ | $c_A$ |
| 6 | B | B | moves | $(1-a_1)sp_m(0)(1-a_2)$ | r | r | $c_B$ | $c_B$ |

(2) TENANT:

| Case | Landld in Period 1 | Landld in Period 2 | Move | Probability | Rent in Period 1 | Rent in Period 2 | Moving Expen. Period 2 |
|---|---|---|---|---|---|---|---|
| 1 | A | A | stays | $b_1(1-p_m(t))$ | r | r-t | |
| 2 | A | A | moves | $b_1p_m(t)b_2$ | r | r | m |
| 3 | A | B | moves | $b_1p_m(t)(1-b_2)$ | r | r | m |
| 4 | B | B | stays | $(1-b_1)(1-sp_m(0))$ | r | r | |
| 5 | B | A | moves | $(1-b_1)sp_m(0)b_2$ | r | r | m |
| 6 | B | B | moves | $(1-b_1)sp_m(0)(1-b_2)$ | r | r | m |

$a_i$: Proportion of tenants type A in period i,
$b_i$: Proportion of landlords type A in period i,
$p_m(t)$ with $p_0=p_m(0)$: Moving probability,
s: Rent and eviction control parameter ($s=1/p_0$ <=> no control,
$s=1$ <=> reguated market)

revealed only after the first period and only to his landlord. There will be no market emerging for the information of match or mismatch because other landlords are unable to identify both the tenant's type and the type of their fellow landlord. A second external effect is introduced by the moving probability $p_m(t)$. The expected utility of the tenant is a function of the probability density of the possible future states of the world, and this density depends via $p_m(t)$ on the tenure discounts t granted at the discretion of the landlord.

Both externalities create a "second best environment" in which the classical welfare theorems do not hold, and it is this environment in which we want to analyse regulation of the rental housing market. Rent and eviction control will be introduced in Section 4.5. We define rent and eviction control as follows:

(1) The landlord is not permitted to breach the rental contract.

(2) The landlord is not permitted to charge a rent premium in the second period, that is, negative tenure discounts are unlawful.

We can combine the regulated and unregulated market setting by introducing a rent and eviction control parameter s. If eviction is at the landlords discretion, this parameter is set to $1/p_O$, otherwise s=1. With this definition, the tenant's moving probability in the case of a mismatch can be written $sp_O$. In the case of a match, regulation has no influence on the tenant's probability to leave the unit $p_m(t)$.

The final ingredient of the model are the expenses which occur to the tenant if he moves. We will denote the moving expenses by m, and assume that they are a lumpsome amount and independent of the cause of the move which may be the new job or eviction.

We can enumerate all possible events at the transtion from period 1 to period 2:

(1) There is either a match or a mismatch in the first period;

(2) The tenant either moves after the first period or stays in the unit;

(3) If he moves, there will be either a match or a mismatch in the second period.

The probabilities for the events and the corresponding realization of rents, maintenance costs, and moving expenses are given in Table 4-9.

Two asymmetries may be worth noting. First it is not necessary to introduce the possibility of a rent premium in the case of a mismatch: if the market is unregulated, the landlord will evict the tenant; under rent and eviction control, such a premium is unlawful. Second, we did not introduce physical turnover costs for the landlord, say the cost of a vacant unit. However, the difference between the certain maintenance costs for the current tenant and the expected costs with an unknown tenant can be interpreted as turnover costs. For $0 < a_2 < 1$ this difference is positive for a matching, negative for a non-matching tenant. Thus, in case of a match, this difference constitutes an "exit-barrier" for the landlord as do the moving costs for the tenant. The introduction of physical moving costs will only strengthen results which we will derive from this exit-barries and is thus unnecessary. In case of a mismatch, the cost difference is negative and thus provides an incentive for the landlord to breach the contract. Turnover costs will weaken the assumption that all mismatches will be severed in unregulated markets. To put results in the proper perspective, we can interpret the second period costs of a good tenant $c_A$ as including turnover costs. The exit-barriers -- the cost difference $c_A$-$c_B$ for the landlord and the moving expenses $m$ for the tenant -- are the main parameters for the interpretation of the model and will play a key role when we evaluate the welfare gains and losses from rent and eviction control.

In the following Sections, we will study the landlord's and the tenant's optimization problem in more detail, and then define a suitable equilibrium concept for the two period model.

### 4.4.1 The Landlord

We will examine the landlord's behavior first, and look at a landlord of type A who considers tenants of type A as good, tenants of type B as bad tenants. At the beginning of the first period, he maximizes his expected profits with respect to the quality level h and the tenure discounts t which he will grant in case the tenant turns out to be a good tenant:

$$(4.3) \quad \max_{h,t} E\,[\,P(h,t)\,]$$

From Table 4-9, we compute the expected profit function

$$\begin{aligned}
P^E(h,t) = {} & a_1\,(1-p_m(t))\quad [\,(1+e)rh - (1+e)c_A(h) - eth\,] \\
& + a_1 p_m(t)\,a_2\quad [\,(1+e)rh - (1+e)c_A(h)\,] \\
(4.4) \qquad & + a_1 p_m(t)(1-a_2)\quad [\,(1+e)rh - (1+e)c_A(h) - e(c_B(h)-c_A(h))] \\
& + (1-a_1)(1-sp_0a_2)\quad [\,(1+e)rh - (1+e)c_B(h)\,] \\
& + (1-a_1)\,sp_0a_2\quad [\,(1+e)rh - (1+e)c_B(h) + e(c_B(h)-c_A(h))]
\end{aligned}$$

where $e = 1/(1+i)$ the discount factor for interest rate i.

Due to the linearity of the profit function, (4.4) reduces to

$$(4.5) \quad P^E(h,t) = r_1^E h - c_1^E(h) + e\,(r_2^E(t)h - c_2^E(h,t))$$

with the expected values of rents and costs in period 1 and 2:

$$(4.6) \quad r_1^E = r$$

$$(4.7) \quad r_2^E(t) = r - a_1\,(\,1 - p_m(t)\,)\,t$$

$$(4.8) \quad c_1^E(h) = c_B(h) + a_1\,(\,c_A(h) - c_B(h)\,)$$

$$(4.9) \quad c_2^E(h,t) = c_B(h) + p_A(t)\,(\,c_A(h) - c_B(h)\,)$$

where

$$p_A(t) = (\ a_1(1-p_m(t)) + a_1 p_m(t) a_2 + (1-a_1) s p_m(0) a_2\ )$$

Maximization of (4.5) with respect to h and t yields the first order conditions:

$$(4.10) \quad r + e\ r_2^{\xi}(t) = c_1^{\xi\prime}(h) + e\ c_2^{\xi\prime}(h;t)$$

$$(4.11) \quad (1-p_m(t)) - p_m'(t)t = p_m'(t)\ (1-a_2)\ (c_A(h)-c_B(h))/h$$

As a first result, we can characterize the tenure discounts by equation (4.11).

<u>Theorem 1:</u>

If the cost differences between tenant types are large enough, the landlord will grant positive tenure discounts to keep a good tenant. More precisely, the optimal tenure discounts are given in implicit form by

$$(4.12) \quad t = (1-a_2)\ (c_B(h)-c_A(h))/h + (1-p_m(t))/p_m'(t)$$

Note that the first term in (4.12) is positive, the second negative by our assumption. The landlord faces a trade-off between losing money by granting tenure discounts on one hand and by risking to get a bad tenant in the second period on the other hand. A large cost difference in the first term will outweigh the second term and generate positive tenure discounts. However, if the probability of a bad tenant in period two is very small and the moving probability insensitive to tenure discounts, the landlord is better off by discouraging the old good tenant to gamble for a new good tenant, whom he does not have to pay tenure discounts.

To simplify the analysis and to be able to solve (4.12) for the discounts t, we assume a linear functional form for the moving probability $p_m(t)$:

(4.13) $p_m(t) = p_O - kt \quad \text{for } t < p_O/k \quad ( k>0 )$

$\qquad = 0 \quad \text{otherwise}$

Next, we examine the structure of the cost functions. Obviously, $c_A(0) = c_B(0) = c_O$, the cost of a vacant unit. We have already defined the type of tenant by $c_A(h) < c_B(h)$ for $h > 0$. For a well behaved cost function, we postulate $c_A' > 0$, $c_B' > 0$, $c_A'' > 0$, $c_B'' > 0$. In addition, it is convenient to assume that the difference between the costs for different tenure types increases linearly with the housing unit service stream: $c_B(h) - c_A(h) = c_D h > 0$.

With the linear specifications of $p_m$ and $c_B$-$c_A$, the tenure discounts are given by:

(4.14) $t = (1-a_2)/2 * c_D - (1-p_O)/2k$,

hence positive for

(4.15) $$c_D > \frac{(1-p_O)}{(1-a_2)\ k} .$$

We can substitute this into the first order condition (4.10) to obtain the following implicit reduced form equation in the landlord's housing supply h:

$$(4.16) \quad F = r(1+e) - ea_1(1-p_m(t))t - (1+e)c_B'(h) - (a_1+ep_A(t))c_D = 0$$

with $p_A$ from (4.9).

For the comparative static analysis in Section 4.5 we compute the following partial derivatives of F with respect to rent r, the regulation parameter s, and the quality level h:

$$(4.17) \quad F_r = (1+e) > 0$$

$$(4.18) \quad F_s = e\ (1-a_1)\ a_2\ p_O\ c_D > 0$$

$$(4.19) \quad F_h = -(1+e)\ c''_B(h) < 0$$

Applying the implicit function theorem, we obtain an monotonously upward sloping supply function in the initial rent level r, with the supply elasticity given by

$$(4.20) \quad E_S = -\frac{r\ F_r}{h\ F_h} = \frac{r/h}{c''_B(h)} > 0.$$

Finally, we define the elasicity of supply of housing services with respect to the regulation parameter s which falls from 1 to $1/p_O$ when rent and eviction control is introduced:

$$(4.21) \quad R_S = -\frac{s\ F_s}{h\ F_h} = \frac{e(1-a_1)a_2p_Oc_D}{c''_B(h)} > 0\ .$$

Note that this elasticity is positive and proportional to the cost differential $c_D$ between good and bad tenants.

### 4.4.2 The Tenant

We turn now to the demand side. We will look at a tenant of type A. He chooses housing quality h and consumption of other goods x at the beginning of period 1 as to maximize his expected utility subject to a budget constraint. For simplicity, we assume a separable utility function $U(x,h) = u(x) + v(h)$. Because we are not interested in the intertemporal distribution of consumption, we define $x = x_1 + ex_2$ as two period consumption and $y = y_1 + ey_2$ as two period income with $e=1/(1+i)$. Rent is denoted by r, moving expenses by m, and other goods are normalized to have unit price. With these conventions, the tenants maximization problem can be written as

$$(4.22) \qquad \max_{h,x} \; E\,[\,u(x) + v(h)\,] \quad \text{s. t.} \quad E\,[\,y - x - rh - m\,] = 0.$$

The lower part of Table 4-9 specifies the outcomes and probabilities for all possible events for a tenant of type A. There are three different outcomes according to the receipt of tenure discounts and whether the tenant moves. Note that depending on the imposition of rent and eviction control one of the three outcomes will not occur. The maximand of the tenant expands to

$$(4.23) \qquad \begin{aligned} & v(h) + b_1\,(1-p_m(t))\;\; u(y-(1+e)rh+eth) \\ & \quad + (1-b_1)(1-sp_O)\;\; u(y-(1+e)rh) \\ & \quad + (b_1p_m(t)+(1-b_1)sp_O)\;\; u(y-(1+e)rh-em). \end{aligned}$$

We will use the simplifying notation

$$q_1 = b_1\,(1-p_m(t))$$

$$(4.24)\quad q_2 = (1-b_1)(1-sp_o)$$

$$q_3 = (b_1 p_m(t)+(1-b_1)sp_o)$$

to denote the probabilities of the different events for a tenant, and

$$(4.25)\quad \begin{aligned} \bar{x} &= y - (1+e)rh \\ w(\bar{x}) &= q_1\,u(\bar{x}+eth) + q_2\,u(\bar{x}) + q_3\,u(\bar{x}-em) \end{aligned}$$

for "normal" consumption and the expected utility from consumption. The first order condition of (4.23) with respect to h is

$$(4.26)\quad G = v'(h) - w'(\bar{x})(1+e)r - q_1 u'(\bar{x}+eth)et = 0$$

and defines the implicit reduced form of the tenant's demand for housing quality. This gives us the following partial derivatives for comparative static analysis:

$$(4.27)\quad G_r = w''(\bar{x})(1+e)^2 rh - w'(\bar{x})(1+e) - q_1 u''(\bar{x}+eth)e(1+e)th < 0$$

$$(4.28)\quad G_s = (1-b_1)p_o(1+e)r\,(\,u'(\bar{x}) - u'(\bar{x}-em)\,) < 0$$

$$(4.29)\quad G_h = v''(h) + w''(\bar{x})(1+e)^2 r^2 - q_1 u''(\bar{x}+eth)(e^2t^2-2e(1+e)rt < 0$$

where the signs are implied by the assumption of well behaved neoclassical utility functions with $v' > 0$, $u' > 0$, $v'' < 0$, and $u'' < 0$. In particular, we have a monotonously downward sloping demand curve with a demand elasticity of

$$(4.30)\quad E_D = -\frac{r\,G_r}{h\,G_h} < 0.$$

In analogy to the supply side, we define the elasicity of demand for housing services with respect to the regulation parameter s as

$$(4.31) \qquad R_D = - \frac{s\ G_s}{h\ G_h} < 0.$$

Note that this elasticity is negative and roughly proportional to the moving expenses m, which can be seen from (4.28) when the bracketed difference in marginal utilities is approximated by $u''(\bar{x})em$.

### 4.4.3 Steady State Market Equilibrium

A steady state equilibrium is characterized by three conditions:

(4.32) $a_1N^T = b_1N^L$ and $(1-a_1)N^T = (1-b_1)N^L$

(4.33) $a_1D^A(r) + (1-a_1)D^B(r) = b_1S^A(r) + (1-b_1)S^B(r)$

(4.34) $a_1 = a_2$ and $b_1 = b_2$

The first condition assures that the number of housing units demanded equals the number of housing units supplied, for both types of landlords and tenants. This equilibrium condition of the extensive margin of course implies $a_1 = b_1$ and $N^T = N^L = N$, the number of landlords[11] and tenants. The second condition equalizes demand and supply of housing service streams at the intensive margin. $S^A(r)$ denotes the optimal housing service supplied by a landlord of type A at initial rent r, that is $S^A = h$ given by equation (4.16). $D^A(r)$ denotes the corresponding demand, given by equation (4.26). Finally, the third condition characterizes the steady state by a stable distribution of tenant and landlord types.

Note the distinction between the discrete problem of matching the number of units (=landlords) and tenants, and the continuous problem of determining the equilibrium housing service level. The latter condition (4.33) is the substantive equilibrium condition of this model, the first holds by assumption. The slope of the demand and supply functions, given by (4.20) and (4.30), and their monotonicity guaranty that the continuous housing service equilibrium condition (4.33) yields a well defined and unique equilibrium rent level r. If the discrete equilibrium condition (4.32) is violated, vacencies or homeless will emerge, and we have to specify rules for this disequilibrium. We will not consider this problem in this paper.

The violation of the steady state condition (4.34) can be interpreted as a kind of adverse selection problem. Let N denote the population in the housing market under examination, and M denote the net migration to another housing market, with $g_1$ the proportion of type A migrants. After one period, $p_m(t)a_1N$ is the number of type A tenants moving within the given housing market. In addition, $g_1M$ type A tenants will move from outside into this market. For type B tenants, we have $sp_O(1-a_1)N$ intra-city movers and $(1-g_1)M$ inter-city movers. Thus,

$$(4.35) \qquad a_2 = \frac{p_m(t)\ a_1\ N \ + \ g_1\ M}{(\ p_m(t)a_1 + sp_O(1-a_1)\ )\ N \ + \ M}$$

is the second period share of type A tenants on the market. In steady state, $a_1 = a_2$. Solving (4.35) for $a_1 = a_2$ yields a quadratic equation

$$(4.36) \qquad a_1^2 - \left( 1 - \frac{M}{(p_m(t)-sp_O)\ N} \right) a_1 - g_1\ M/N = 0.$$

Thus, in the absence of migration (M=0), we obtain the corner solutions $a_1 = 0$ or $a_1 = 1$ as steady state shares to which any other initial distribution $0 < a_1 < 1$ will converge. This is due to the unbalanced shares of movers in each period, $p_m(t)/sp_O$, which creates a selection of bad tenants among movers adverse for the landlords.

We will assume a steady migration of tenants to produce a steady state distribution $0 < a_1 < 1$. For simplicity and without loss of generality, we set $a_1 = 0.5$. Now all reactions of type A agents are mirror-images of their type B counterparts, and it is sufficient to examine only half of the market.

## 4.5 Analysis of Rent and Eviction Control

The eviction and rent control as defined in Section 4.1 is modelled along the lines of the tenants' protection legislation in West Germany and has two effects in our model: (1) landlords are not permitted to evict tenants of a different type, and (2) landlords are prohibited from charging tenants a premium on top of their second period rent. Note that the initial rent level is unrestricted and free to move according to the competitive market equilibrium (4.33). We model the two effects simultaneously by changing the parameter s in the specification of the probabilities in Table 4-9. Setting s = $1/p_m(0)$ corresponds to a housing market without intervention: mismatches will be severed with probability one, and Case 4 in the Table will never occur. In turn, setting s = 1 corresponds to a housing market under eviction and rent control, where the bad tenant will only move with probability $p_m(0)$ and pays the contract rent r in period 2. A decrease in s thus corresponds to a step in direction of rent and eviction control.

Total differentiation[12] of the equilibrium condition (4.33) yields

$$(4.37)\qquad \frac{dr}{ds} = - \frac{\overset{(-)}{G_s}\overset{(-)}{F_h} - \overset{(-)}{G_h}\overset{(+)}{F_s}}{\underset{(-)}{G_r}\underset{(-)}{F_h} - \underset{(-)}{G_h}\underset{(+)}{F_r}} < 0$$

where the pieces can be collected from equations (4.17), (4.18), (4.19); and (4.27), (4.28), and (4.29). In terms of the elasticities

(4.21), (4.22); and (4.31), and (4.32), we can write the impact of a step in direction of regulation on the rent level as

$$(4.38)\quad dr = - r/s \; ( \overset{(-)}{R_D} - \overset{(+)}{R_S} ) \; / \; ( \overset{(-)}{E_D} - \overset{(+)}{E_S} ) \; ds$$

Thus, we can describe the first effect of the intervention:

Theorem 2:

The initial rent level for a new lease will rise in response to the rent and eviction control.

Proof:

The signs in (4.37) and (4.38) follow from the quoted equations, and imposition of rent and eviction control is equivalent to $ds < 0$.

Note that we can decompose the price change dr in two terms. The first term includes $R_D$ and is roughly proportional to the moving expenses m, see (4.31). The second term includes $R_S$ and is proportional to the difference in maintenance costs $c_D$ according to (4.21). Thus, the price change reflects the sharing of the burden and the gain from eviction and rent control between landlord and tenant, the gain expressed in the moving expenses saved by the tenant and the burden in the cost difference inflicted on the landlord.

How does the landlord respond to the imposition of rent and eviction control?

Theorem 3:

(1) The landlord's supply for housing services will fall in response to the rent and eviction control.

(2) Under linear cost differences, the tenure discounts will remain

unchanged.

Proof: We computed the supply effect when defining the elasticity $R_S$ in (4.21) by applying the implicit function on (4.16):

$$(4.39) \quad dh = - F_s/F_h = \frac{e\ (1-a_1)\ a_2\ p_O\ c_D}{(1+e)\ c''_B(h)}\ ds \ < \ 0$$

The absence of a tenure discount effect is entirely due to the linear cost difference $c_B(h) - c_A(h) = c_D$ = constant which can be seen from equation (4.14) where all right hand side items are constants.

Supply will be depressed, because the legislation imposes higher costs on the landlord: he looses the economic value of his right to evict. To adjust optimal profits under a convex cost function, output has to go down. The change in tenure discounts for a general cost specification can be obtained from equation (4.12):

$$(4.40) \quad dt = (1-a_2)/2h^2\ [\ (c_B'(h)-c_A'(h))h - (c_B(h)-c_A(h))\ ]\ \frac{dh}{ds}\ ds$$

The landlord will grant larger discounts in response to the legislation if the cost difference increases more than linearly with the housing service stream, and less discounts for a concave cost difference. He will do so to counterbalance the changes in cost differences with a changed moving probability $p_m(t)$: if this difference decreases due to (4.39) and a convex cost difference, he is more interested to keep the tenant, and will give him larger discounts.

The effect for profits is given by the envelope theorem

$$\text{(4.41)}\quad dP = (1+e)\,\frac{dr}{ds}\,h\,ds + e\,(1-a_1)\,a_2\,p_0\,c_D h\,ds$$

and consists of two effects of opposite signs. The second term in (4.41) is negative: the landlord looses because he is forced to keep a bad tenant, if this tenant decides to stay. This effect is proportional to the cost difference. On the other hand, he is able to regain some of the losses through rent increases. As we have seen in (4.38), we can interpret the first effect as appropriation of some part of the tenant's advantage from the legislation in the form of lower expected moving expenses.

We now turn to the impact of rent and eviction control on the demand side of our rental housing market model. We evaluate the effect on the tenant's utility by applying the envelope theorem on (4.23):

$$\text{(4.42)}\quad du = -\,w'(\bar{x})\,(1+e)\,\frac{dr}{ds}\,h\,ds - (1-b_1)p_0(u(\bar{x})-u(\bar{x}-em))\,ds$$

The first effect in the tenant's utility (4.42) is the mirror image of the first effect in the landlord's profits (4.41). Tenants suffer from an utility loss due to the increase in in the initial rent level. This price increase is converted by the marginal utility $w'(\bar{x})$ as defined in (4.25) into utility units. The second effect reflects the increase in utility due to the decreased likelihood of moving, that is lower expected moving expenses. This utility increase is approximately $w'(\bar{x})em$, thus

$$\text{(4.43)}\quad du/w'(\bar{x}) = -\,(1+e)\,\frac{dr}{ds}\,h\,ds - (1-b_1)\,p_0\,em\,ds$$

If we analyze the relative weights of the opposing effects in (4.41) and (4.42), we obtain the main result of this Section:

Theorem 4:

(1) If moving expenses are small relative to the cost differential between good and bad tenants, both landlords and tenants are worse off by eviction and rent control.

(2) If moving expenses are large relative to the cost differential between good and bad tenants, both landlords and tenants are better off by eviction and rent control.

(3) For a given ratio of moving expenses and differential maintenance costs, Case (2) is the more relevant the smaller the likelihood is of a second period match after the severance of a mismatch.

Proof:

We express the utility change (4.42) locally as

$$du = ( G_r\, dr/ds + G_s )\, \tilde{x}\, ds$$

and the profit change (4.41) correspondingly as

$$dP = ( F_r\, dr/ds + F_s )\, h\, ds$$

To obtain a utility (profit) increase in response to the imposition of the legislation (ds<0) the bracketed expressions have to be negative. Substitute dr/ds from (4.37). Then

$$du > 0 \iff -G_r \frac{G_sF_h - G_hF_s}{G_rF_h - G_hF_r} + G_s < 0,$$

and

$$dP > 0 \iff -F_r \frac{G_sF_h - G_hF_s}{G_rF_h - G_hF_r} + F_s < 0.$$

Thus,

$du > 0 \iff dP > 0 \iff G_s/G_r > F_s/F_r.$

From (4.19), (4.27), and (4.43):

$$F_s/F_r = \frac{e\ (1-a_1)\ a_2\ p_0}{1+e}\ c_D > 0,$$

and

$$Gs/Gr = \frac{(1-a_1)\ p_0\ er\ w''(\bar{x})}{w''(\bar{x})rh(1+e) - w'(\bar{x}) - q_1u''(\bar{x}+eth)eth}\ m > 0.$$

Hence,

$$du > 0 \iff dP > 0 \iff m > a_2\ K\ c_D$$

with

$$K = \frac{w''(\bar{x})rh(1+e) - w'(\bar{x}) - q_1u''(\bar{x}+eth)eth}{u''(\bar{x})\ (1+e)r} > 0$$

which proves the Theorem.

The result is an example how the intuition from a first best environment can be misleading in a second best environment. Market failures occur in the rental housing market because of two externalities: a missing market for the information of landlord and tenant type; and an expected utility function of the tenant which is responsive to the tenure discounts. The fundamental welfare theorems are not valid in an environment with external effects. In Case (2), interference with the market in form of rent and eviction control proved to be Pareto superior to laissez faire: the tenants' protection legislation did not only improve the tenant's welfare but also the profits of the landlords.

## 4.6 Conclusions

Using empirical evidence on the existence of tenure discounts in West Germany and the United States we screened the arguments pro and contra rent and eviction control, specifically the West German tenants' protection legislation. A model was build based on the mechanisms identified as important for the working of the rental housing market: tenant and landlord idiosyncrasy, potential tenure discounts, and high moving costs.

As a first result, the model indeed predicted positive tenure discounts. Second, the comparative static analysis of rent and eviction control showed, that with low moving costs and a large difference in costs between good and bad tenants rent and eviction control makes both tenants and landlords worse off. However, with high moving costs and only little difference in costs between tenant types, the intervention increases the utility of all participants, of landlords as well as of tenants. The conclusion of many analyses that the German tenants' protection legislation will not only reduce the profits of landlords, but even harm the tenants which should have been protected by the legislation is at least premature.

This result of an intervention Pareto superior to laissez faire is due to the externalities between landlord and tenant and an example of how first best analysis misleads policy evaluation in a second best environment.

The second conclusion concerns estimating housing demand. As discussed in the end of Chapter Three, positive tenure discounts bias the estimated price responses upward. How can we correct for that? Obviously, the proper solution is to take all units of a given cross section, but look at their spot market rent rather than their actual rent. The spot market rent is observed only at the time of moving. With a panel, we could trace all units back to this period. In absence of a panel, however, we can use the hedonic estimates of average discounts quoted in Tables 4-1 through 4-3 and add these discounts to the actual rent to achieve approximations of the spot market rent. Note that this procedure avoids the bias which is introduced when confining the estimation only to recent movers. We will use this procedure in the next chapter, when we compare housing demand in the United States and in West Germany.

Footnotes to Chapter 4

(1) Guasch and Marshall (1983) argue that the hedonic regression suffers from self selection bias, because tenants with low or no tenure discounts will move out of their expensive units and therefore have less probability to be in the sample. Our model in Section 4 will have the same implication. However, their empirical findings about the size of this effect and the remaining "true" discounts are inconclusive. Correction for the selection bias in three different samples changes the estimated tenure discounts only little and in either direction. Their standard errors, though, increase as much as to render the discounts statistically insignificant at all. It is not clear whether this result is due to the specific sample they used or to an inefficient estimation procedure for their relatively small sample size.

(2) The law was modified in 1983, but not changed in its substance.

(3) Only usury is prohibited by §5 WiStG.

(4) §564b BGB.

(5) §2 MiethoeheG. This procedure can only be waived if the tenant agrees to it.

(6) This might be a frivolous choice of words. We want to distinguish discrimination due to objectively higher costs from discrimination due to taste or prejudice.

(7) Compare Goodman and Kawai (1982) for a sketch of a similar model.

(8) We assume that the expected costs of a bad tenant staying are always larger than the expected costs of an unknown new tenant. For $0 < a_2 < 1$, this is basically a statememt about small turnover costs. See the discussion at the end of this Section.

(9) A suitable equilibrium concept is developed in Section 4.4.3.

(10) Eckart, Schulz, and Stahl (1983) consider a model of voluntary and involuntary exchange in a housing market with implicit contracts. The landlord-tenant relationship in our model resembles an implicit contract: the tenant takes the discounts in the second period in account when maximizing his expected utility over the two periods.

(11) We assume each landlord supplies only one housing unit. This includes landlords with more than one housing unit, as long as each of the units is supplied by maximizing its own profit.

(12) We keep $a_2$ constant. In fact, $a_2$ is a function of the tenure discounts, thus affected by the change of s. However, the linear specification of costs and moving probability keeps $a_2$ unchanged, as it will turn out in (4.40).

# CHAPTER FIVE

# AN ANALYTIC COMPARISON OF HOUSING DEMAND DECISIONS IN THE UNITED STATES AND WEST GERMANY

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## 5.1 Introduction: Idea and Scope of an Analytic Comparison

This last chapter of the thesis uses all the analytical tools we have developed so far to compare housing demand in the United States and West Germany. In both countries housing surveys exist with comparable scope of questions asked, sample sizes, and sampling procedures. This is the Annual Housing Survey in the United States which we already used in Chapter Three, and the One Percent Sample in West Germany, named after its sampling ratio.

The existence of parallel data sets in the two countries allows us to use the same analytical model, i. e., functional specification of the demand equations and specification of the explanatory price, and income, and demographic variables. In this sense, we will carry out an analytical rather than a descriptive comparison

Though the scope of questions asked is roughly the same in both surveys, due to confidentiality restrictions in West Germany the data available to us has little information on household composition, prohibiting the decomposition of households into nuclei as described in Chapter Three. Thus, our comparison will be confined to population strata in which household formation can safely be considered exogenous to the housing market: married couples.

Second, we do not want to stress the comparison to areas where the two countries are structured very differently. We will exclude guestworkers in West Germany and non-whites in the United States from our comparison. With similar reasoning, we confine the analysis to metropolitan areas because the density pattern of rural settlements are completely different in the two countries. In Germany, we sample only from counties which are classified as highly densely populated. The United States are represented by the Standard Metropolitan Statistical Areas of Boston, Dallas, Los Angeles, and Minneapolis/St.Paul from the Annual Housing Survey by SMSA.

Though we pool the data across cities in both countries, we will keep city centers and suburbs apart, and we will further stratify the households (married couples) according to three age groups (age below 35, age between 35 and 50, and age above 50). This yields six strata for each country. In addition, we pool all data to explore a variety of functional forms -- decision trees -- and different specifications of the explanatory variables. These pooled samples contain 8035 white married couples in West Germany and 8139 in the United States. We will refer to these two samples as our "basic samples".

This chapter begins with a short enumeration of descriptive statistics to outline the differences in housing consumption in the two countries. In addition, we will compare summary statistics from the entire nation with summary statistics from our basic sample to obtain a sense of how the samples represent the countries. Section Three briefly sketches the differences in the tax treatment of owner-occupancy between the countries.

This descriptive part is intended to set the stage for the analytical comparison: how much of the observed differences in housing consumption can be explained by differences in the exogenous variables, and how much has to be attributed to preference differences. Sections Four through Seven specify the common demand model, estimate the components -- permanent income and hedonic prices -- and finally the NMNL-demand equations. Then, we compare the price and income responsiveness and optimal tree structures. This will give us insight in the differences between the preferences in West Germany and the United States. The last section attempts to separate those from the effects of the tax differences: we forecast each country's housing consumption first at the other country's preferences and then at the other country's tax and subsidy system.

## 5.2 A Brief Descriptive Comparison

The most striking difference in housing consumption between West Germany and the United States is the difference in the tenure choice: of all households in the U. S. 1978, 65.2 percent lived in owner-occupied housing[1], only 36.3 percent in Germany[2]. This pattern is as striking in our basic samples of white married couples in urban areas: compared to the entire population, this ratio rises to 77.0 percent in the U. S.; but only to 42.2 percent in West Germany. Table 5-1 gives a more detailed decomposition of housing demand for the two countries, based on the entire population (Annual Housing Survey 1977, National Sample, and the West German One Percent Sample 1978, respectively) and our basic samples. We consider eight housing alternatives, generated by three dimensions: owner-occupied versus rental housing, single-family homes and duplexes versus multi-family structures, and one-to-four room dwelling (plus kitchen) versus dwellings with more than five rooms.

The differences in tenure choice are echoed in the differences between structure types: whereas in Germany single-family structures (including duplexes) and multi-family structures have almost equal shares, single-family homes constitute the overwhelming share of structures in the United States. Note, that our sample differs from the population in two ways: we consider only married couples with a higher likelihood of owner-occupancy, and only high density urban areas with a lower propensity to own. The ownership ratio for married couples all over Germany is 43.2 percent, whereas the ownership for

## TABLE 5-1: MARKET SHARES OF HOUSING ALTERNATIVES

### WEST GERMANY: ONE PERCENT SAMPLE 1977, NATIONAL SAMPLE

| All Households 23,067,000 | Rental Housing 1-4 Rooms | 5+ Rooms | Owner-Occupied 1-4 Rooms | 5+ Rooms | Unit Choice |
|---|---|---|---|---|---|
| 1-2 Units | 10.5% | 5.7% | 8.0% | 22.9% | 47.1% |
| 3+ Units | 38.7% | 8.8% | 3.2% | 2.1% | 52.9% |
| | 49.2% | 14.4% | 11.2% | 25.0% | |
| Tenure Choice | 63.7% | | 36.3% | | |
| Size Choice | 60.4% | | | 39.6% | |

### WEST GERMANY: REGIONS WITH HIGH DENSITY

| Married, German 8,019 | Rental Housing 1-4 Rooms | 5+ Rooms | Owner-Occupied 1-4 Rooms | 5+ Rooms | Unit Choice |
|---|---|---|---|---|---|
| 1-2 Units | 10.2% | 2.5% | 21.1% | 15.9% | 47.7% |
| 3+ Units | 41.0% | 4.2% | 4.6% | 0.6% | 50.3% |
| | 51.2% | 6.7% | 25.7% | 16.5% | |
| Tenure Choice | 57.8% | | 42.2% | | |
| Size Choice | 76.9% | | | 23.1% | |

### UNITED STATES: ANNUAL HOUSING SURVEY 1977, NATIONAL SAMPLE

| All Households 75,280,000 | Rental Housing 1-4 Rooms | 5+ Rooms | Owner-Occupied 1-4 Rooms | 5+ Rooms | Unit Choice |
|---|---|---|---|---|---|
| 1-2 Units | 11.5% | | 60.0% | | 71.5% |
| | 30.2% + | 5.0% + | 27.7% + | 37.1% | |
| 3+ Units | 24.6% | | 3.9% | | 28.5% |
| Tenure Choice | 35.2% | | 64.8% | | |
| Size Choice | 57.9% | | | 42.1% | |

### UNITED STATES: BOSTON, DALLAS, LOS ANGELES, MINNEAPOLIS/ST.PAUL SMSA

| White Married 8,139 | Rental Housing 1-4 Rooms | 5+ Rooms | Owner-Occupied 1-4 Rooms | 5+ Rooms | Unit Choice |
|---|---|---|---|---|---|
| 1-2 Units | 7.2% | 3.2% | 24.5% | 50.6% | 85.5% |
| 3+ Units | 11.9% | 0.6% | 1.1% | 0.7% | 14.5% |
| | 19.1% | 3.9% | 25.6% | 51.4% | |
| Tenure Choice | 23.0% | | 77.0% | | |
| Size Choice | 44.7% | | | 55.3% | |

all households in urban areas in the United States (SMSA's) is 61.0 percent.

The difference between rural and urban areas in the two countries are most striking in the market shares of small versus large dwellings. On the national scale, they are very close: 57.9 percent of all dwellings have less than five rooms in the United States, 60.4 percent in West Germany. However in urban areas, this ratio falls in the United States to 44.7 percent, but rises in Germany to 76.9 percent. This reflects a greater degree of suburbanization in the United States with both urban sprawl and a concentration of higher income households in the suburbs as compared to rural areas.

So far, we have compared the endogenous variables of housing demand. Table 5-2 compares the patterns of exogenous variables and their reflection in the tenure choice. Germany has a substantially higher proportion of elderly households (aged over 65) both on national scale and in our sample of couples in urban areas. In both countries our sample has a more centered age distribution, here, the married couple effect and the urban area effect accumulate. The ownership ratios show the familiar life cycle pattern, but with considerable differences between the countries. The peak in Germany is about ten years earlier than in the United States and the decline in homeownership among the elderly is much more pronounced.

The second part of Table 5-2 lists the annual gross income in Dollars at prices of 1977. Median income is about equal in West

TABLE 5-2: EXOGENOUS VARIABLES: AGE, INCOME, AND PRICES

(1) AGE DISTRIBUTION AND TENURE CHOICE:

| Age | Age Distribution | | | | Ownership Ratio | | | |
|---|---|---|---|---|---|---|---|---|
| | United States | | West Germany | | United States | | West Germany | |
| | All HH[a] | Sample[b] | All HH[a] | Sample[b] | All HH | Sample | All HH | Sample |
| < 25 | 6.4% | 5.3% | 12.6% | 3.2% | 35.3% | 24.6% | 10.4% | 5.1% |
| 25-29 | 11.6 | 11.3 | " | 8.5 | 56.6 | 53.3 | " | 15.1 |
| 30-34 | 12.4 | 12.7 | 18.5 | 10.1 | 74.5 | 74.6 | 32.6 | 28.8 |
| 35-39 | 20.0 | 11.2 | " | 14.5 | 82.5 | 83.5 | " | 40.3 |
| 40-44 | " | 9.5 | 18.8 | 12.5 | " | 86.2 | 48.4 | 54.2 |
| 45-49 | 35.5 | 10.3 | " | 9.9 | 86.8 | 86.6 | " | 54.1 |
| 50-64 | " | 27.5 | 23.2 | 21.3 | " | 87.7 | 47.6 | 52.4 |
| > 65 | 14.2 | 12.4 | 26.7 | 20.1 | 83.2 | 79.2 | 33.6 | 43.2 |

a: Nationwide, see Footnote 1, b: White Married Couples in Urban Areas

(2) INCOME DISTRIBUTION AND TENURE CHOICE:

| Income | Income Distribution | | | | Ownership Ratio | | | |
|---|---|---|---|---|---|---|---|---|
| | United States | | West Germany | | United States | | West Germany | |
| | All HH | Sample | All HH | Sample | All HH | Sample | All HH | Sample |
| < 4.0 | 13.1% | 2.1% | 9.7% | 0.4% | 44.3% | 59.8% | 23.9% | 42.8% |
| < 6.0 | 8.9 | 3.2 | 14.9 | 0.9 | 48.5 | 51.7 | 25.2 | 44.7 |
| < 8.0 | 10.2 | 4.9 | 18.4 | 1.7 | 52.1 | 53.5 | 29.5 | 39.6 |
| <10.0 | 6.6 | 4.9 | 14.9 | 9.4 | 54.1 | 57.4 | 34.7 | 38.2 |
| <12.5 | 10.7 | 8.0 | 16.0 | 8.2 | 58.2 | 60.2 | 37.7 | 39.5 |
| <15.0 | 7.7 | 7.6 | 9.7 | 8.8 | 66.5 | 63.3 | 42.3 | 38.3 |
| <20.0 | 14.7 | 18.3 | 10.3 | 25.7 | 72.3 | 76.0 | 50.6 | 40.8 |
| <25.0 | 28.0 | 16.4 | 6.1 | 15.1 | 85.0 | 82.7 | 61.6 | 38.8 |
| >25.0 | " | 34.6 | " | 29.8 | " | 91.3 | " | 48.2 |

Income is yearly gross household income in Thousand 1977 Dollars

(3) HOUSING PRICES

| | United States | | West Germany | |
|---|---|---|---|---|
| | City Center | Suburbs | City Center | Suburbs |
| Gross Rent | 2,816 | 2,981 | 1,627 | 1,628 |
| Hedonic Rent | 2,742 | 2,935 | 1,980 | 2,010 |
| Out-Pocket-Cost | 4,108 | 4,300 | 2,741 | 2,667 |
| Hedonic Rent | 3,428 | 3,546 | 2,738 | 3,014 |
| Value | 49,238 | 48,443 | 116,694 | 103,724 |

Yearly rents and values in 1977 Dollars

Germany ($ 13,361) and the United States ($ 13,444)[3]. However, the income distribution is very different. In both countries income is approximately evenly distributed across all categories up to median income, but in Germany only the small fraction of 6.1 percent earns more than $ 20,000, whereas in the United States this percentage is as large as 28.0 percent. A similar pattern holds in our sample: mean income is similar (United States: $ 22,020, West Germany: $ 23,420), but the income among couples in U.S. cities has somewhat fatter tails on either side: the low income category (below $ 8,000) accounts for 10.2 percent as compared to 3.0 percent in German cities, and the high income category (above $ 35,000) 34.6 percent as compared to 29.8 percent. In both countries ownership is strongly correlated with income where the ownership ratio reaches 95.8 percent for an income above $ 45,000 in the U.S. sample, and 58.8 percent in the German sample. Interestingly, the pattern is reversed for the very low income groups. A more detailed cross-tabulation reveals that most of these households are elderly or very young couples. This may reflect reporting errors in transfer income of the young and extremely low out-of-pocket cost due to paid-off mortgages among the elderly.

The most striking observation is the difference between the income of the renter households and the homeowners. In the United States, median income for homeowners is about twice as high as among renter households ($ 17,100 versus $ 8,800), but in West Germany it is only about 30 percent higher ($ 15,750 versus $12,000). The same discrepancy holds in our sample: the mean income in the U.S. sample is $ 24,030 for homeowners and $ 15,410 for renters, in West Germany $

25,260 and $ 22,080, respectively.

Finally, the third part of Table 5-2 lists average housing prices for our two samples, stratified by city center and suburbs. Again, there are very pronounced differences: gross rents in Germany are substantially lower (about 55 percent)[4], but prices of single-family houses including the lot are drastically more expensive (more than twice). Note, that this does not translate into high mean out-of-pocket-costs. This is basically due to a much lower loan-to-value ratio in Germany where a large downpayment is made from a building society's savings contract ("Bausparkassenvertrag"). The mean hedonic rent for owner-occupied housing is about 40 percent higher than for rental housing in Germany, only about 25 percent percent in the United States. Thus, a part of the differential in house prices can be explained by better quality dwellings. Second, in 1977/78, about 40 percent of the German house prices in high density areas are paid for the lot[5], only about 20 percent in the U.S. SMSA's[6]. This translates into a 4.5 times higher lot price in Germany than in the United States and only 80 percent higher structure costs.

## 5.3 Tax Treatment of Owner Occupancy in the Two Countries

We will argue that a substantial part of the different housing consumption patterns can be explained by the different tax laws. This Section gives a brief description of how the out-of-pocket costs of homeownership are reduced by tax deductions from the personal income tax in the two countries. Compared with the United States, the tax provisions for homeowners in Germany are confusing and complicated, and we present only a simplified version.

Table 5-3 gives a stylized survey of the tax laws. The important tax tool in the United States is the deduction of mortgage interest, whereas in West Germany the basic mechanism is the allowance for the depreciation of the structure. There are three depreciation schedules in Germany: an "accelerated schedule" with a cap on time and value, a "degressive schedule" applicable only to new structures, and the omnibus linear depreciation schedule. In the United States, imputed rental income is not taxed; in Germany, it is taxed except in the case of an accelerated depreciation schedule. Germany has only a negligible property tax, whereas in the United States the property tax is substantial even after deduction from personal income tax. The German tax law has an additional twist in its special treatment of two-family homes. In this case, mortgage interest can be deducted in addition to depreciation allowances as well as all maintenance and a percentage of modernization expenses, only partially offset by the taxation of imputed rent. This is insofar relevant to our demand analysis, as the law did not exclude the rentless "lease" of the

## FIGURE 5-3: STYLIZED TAX-TREATMENT OF HOMEOWNERSHIP

| | United States 1977 | West Germany 1978 Single-Family | West Germany 1978 Two-Family |
|---|---|---|---|
| **(1) Property Tax:** | | | |
| | varies locally, about 2% of value | varies locally, about 0.1% of value | |
| **(2) Imputed Rent Taxed as Personal Income:** | | | |
| | no | yes, if schedule B<br>no, if schedule A | yes |
| **(3) Deductions from Personal Income Tax:** | | | |
| Property Tax | yes | no | yes |
| Mortgage Interest | yes | no | yes |
| Depreciation | no | schedule A or schedule B | any schedule |
| Maintenance | no | no | yes |
| Modernization | no | no | yes, 15% |

**Notes:**

Schedule A ("accelerated"): 8 years 5% of structure costs, capped at $ 3,000. Thereafter none.

Schedule B ("degressive"): 12 years 3.5% of structure costs, 20 years 2%, finally 10 years 1%. Only applicable for new buildings.

Schedule C ("linear"): 50 years 2%. No restrictions.

Imputed Rent: Effectively ca. 1% of structure costs.

second unit to a family member. Not too surprisingly, this "fake two-family house" became very popular (1977: 17.3 percent on national scale, 13.0 percent in our sample, that is about 40 percent of the one or two unit owner-occupied houses) until the loop-hole was closed. This, too, is the main reason to include two-family homes into the same category as single-family homes in the definition of the eight housing alternatives: apart from the required extra kitchen and bath, the "fake two-family home" is virtually indistinguishable from a single-family house.

Tax advantages from homeownership are much more favorable in the United States apart from this odd loop-hole for two-family homes in Germany. Table 5-4 lists the value of the tax savings from single-family homeownership in both countries for various income levels, house values, and equity ratios in the first few years of a new home. Note, that the German tax write-offs depend on the value of the structure only, not on the value of the land, and that they are independent of the loan-to-value ratio, tilting the symmetry between borrowing and lending (interest income is taxable in Germany). The tax savings in Germany do not increase strictly monotonically with value due to the cap on the accelerated depreciation schedule long before the degressive schedule becomes attractive.

A final note concerns the subsidy given to renters in form of housing allowances ("Wohngeld"). They depend on the rent paid, income, and family-size similar to the housing gap formula applied in Chapter Three. For a couple with two children paying the median rent

TABLE 5-4: YEARLY TAXSAVINGS FROM OWNER-OCCUPANCY IN GERMANY VS. USA

EQUITY: 50%, STRUCTURE: 80%

| Value in $ 1000 | Yearly Gross Income in 1977 Dollars: 8,000 GER: | 8,000 USA: | 12,000 GER: | 12,000 USA: | 16,000 GER: | 16,000 USA: | 25,000 GER: | 25,000 USA: |
|---|---|---|---|---|---|---|---|---|
| 30.0 | $ 260 | 240 | $ 260 | $ 370 | $ 260 | $ 440 | $ 470 | $ 560 |
| 50.0 | 440 | 420 | 440 | 610 | 440 | 710 | 770 | 920 |
| 80.0 | 540 | 420 | 690 | 930 | 690 | 1,080 | 1,180 | 1,420 |
| 120.0 | 540 | 420 | 690 | 1,150 | 690 | 1,550 | 1,180 | 2,060 |
| 160.0 | 540 | 420 | 700 | 1,150 | 700 | 1,930 | 1,200 | 2,650 |
| 200.0 | 540 | 420 | 880 | 1,150 | 880 | 1,930 | 1,490 | 3,170 |

EQUITY: 50%, STRUCTURE: 60%

| Value in $ 1000 | Yearly Gross Income in 1977 Dollars: 8,000 GER: | 8,000 USA: | 12,000 GER: | 12,000 USA: | 16,000 GER: | 16,000 USA: | 25,000 GER: | 25,000 USA: |
|---|---|---|---|---|---|---|---|---|
| 30.0 | $ 200 | 240 | $ 200 | $ 370 | $ 200 | $ 440 | $ 350 | $ 560 |
| 50.0 | 330 | 420 | 330 | 610 | 330 | 710 | 580 | 920 |
| 80.0 | 530 | 420 | 630 | 930 | 530 | 1,080 | 910 | 1,420 |
| 120.0 | 540 | 420 | 690 | 1,150 | 690 | 1,550 | 1,180 | 2,060 |
| 160.0 | 540 | 420 | 690 | 1,150 | 690 | 1,930 | 1,180 | 2,650 |
| 200.0 | 540 | 420 | 690 | 1,150 | 690 | 1,930 | 1,180 | 3,170 |

EQUITY: 20%, STRUCTURE: 80%

| Value in $ 1000 | Yearly Gross Income in 1977 Dollars: 8,000 GER: | 8,000 USA: | 12,000 GER: | 12,000 USA: | 16,000 GER: | 16,000 USA: | 25,000 GER: | 25,000 USA: |
|---|---|---|---|---|---|---|---|---|
| 30.0 | $ 260 | 360 | $ 260 | $ 530 | $ 260 | $ 610 | $ 470 | $ 790 |
| 50.0 | 440 | 420 | 440 | 840 | 440 | 970 | 770 | 1,270 |
| 80.0 | 540 | 420 | 690 | 1,150 | 690 | 1,490 | 1,180 | 1,970 |
| 120.0 | 540 | 420 | 690 | 1,150 | 690 | 1,930 | 1,180 | 2,790 |
| 160.0 | 540 | 420 | 700 | 1,150 | 700 | 1,930 | 1,200 | 3,510 |
| 200.0 | 540 | 420 | 880 | 1,150 | 880 | 1,930 | 1,490 | 4,100 |

EQUITY: 20%, STRUCTURE: 60%

| Value in $ 1000 | Yearly Gross Income in 1977 Dollars: 8,000 GER: | 8,000 USA: | 12,000 GER: | 12,000 USA: | 16,000 GER: | 16,000 USA: | 25,000 GER: | 25,000 USA: |
|---|---|---|---|---|---|---|---|---|
| 30.0 | $ 200 | 360 | $ 200 | $ 530 | $ 200 | $ 610 | $ 350 | $ 790 |
| 50.0 | 330 | 420 | 330 | 840 | 330 | 970 | 580 | 1,270 |
| 80.0 | 540 | 420 | 530 | 1,150 | 530 | 1,490 | 910 | 1,970 |
| 120.0 | 540 | 420 | 690 | 1,150 | 690 | 1,930 | 1,180 | 2,790 |
| 160.0 | 540 | 420 | 690 | 1,150 | 690 | 1,930 | 1,180 | 3,510 |
| 200.0 | 540 | 420 | 690 | 1,150 | 690 | 1,930 | 1,180 | 4,100 |

in an urban area the income cap is at $ 7,000 yearly after-tax income. At an income of $ 4,000, about 60 percent of the rent will be reimbursed. In 1977, 7.4 percent of all households in West Germany received housing allowances.

Thus, all things considered, the tax and subsidy structure in Germany is much less favorable to homeownership than in the United States.

## 5.4 Specification of the Demand Equations

In Table 5-1, we already introduced the eight housing alternatives of the dependent variable:

| | | |
|---|---|---|
| O_SF_14 | : | Owner-Occupied, Single or Two-Family Home, 1-4 Rooms |
| O_SF_5+ | : | Owner-Occupied, Single or Two-Family Home, 5+ Rooms |
| O_MF_14 | : | Owner-Occupied, Multi-family Structure, 1-4 Rooms |
| O_MF_5+ | : | Owner-Occupied, Multi-family Structure, 5+ Rooms |
| R_SF_14 | : | Rental Housing, Single or Two-Family Home, 1-4 Rooms |
| R_SF_5+ | : | Rental Housing, Single or Two-Family Home, 5+ Rooms |
| R_MF_14 | : | Rental Housing, Multi-family Structure, 1-4 Rooms |
| R_MF_5+ | : | Rental Housing, Multi-family Structure, 5+ Rooms |

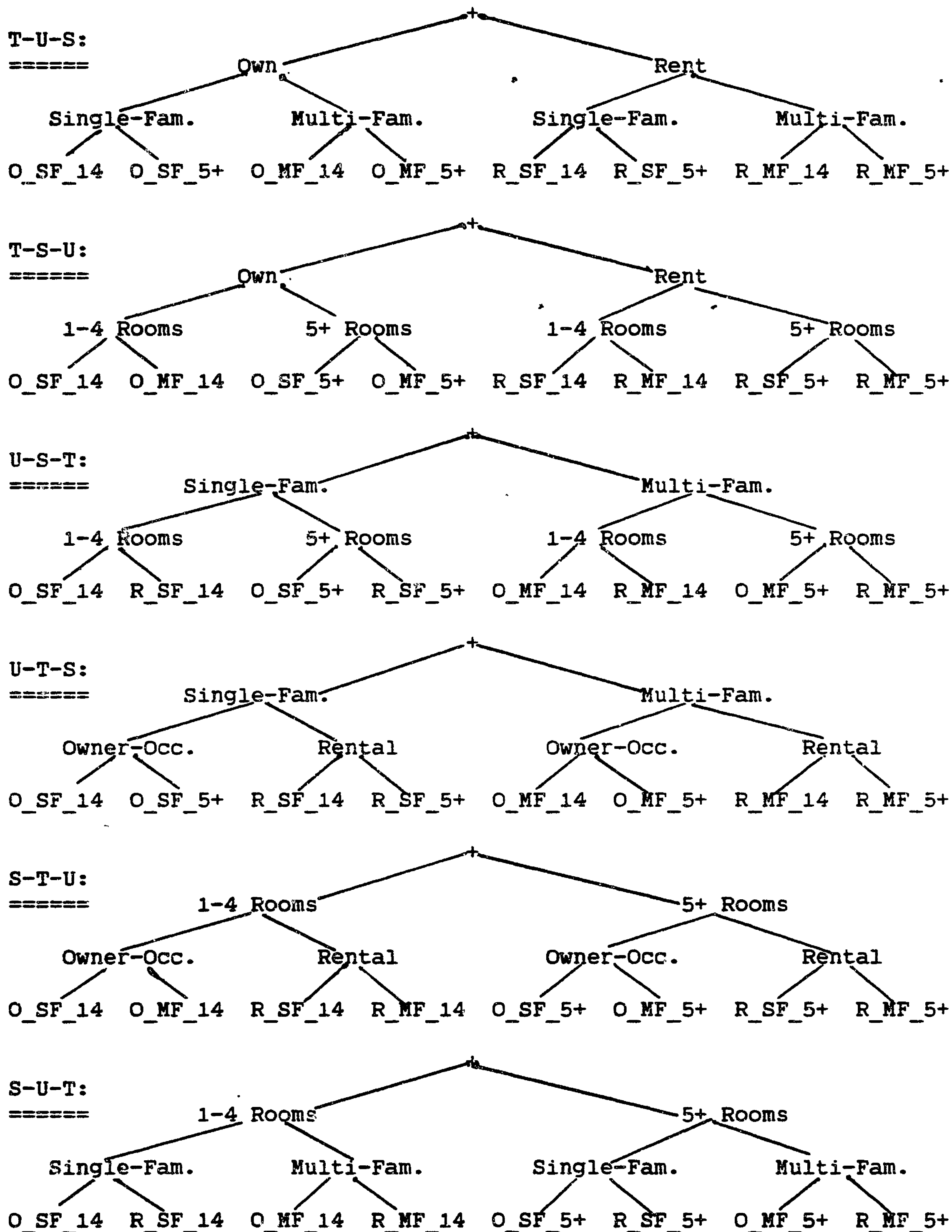
They are generated by three dimensions of choice (tenure, structure type in number of units, dwelling size in number of rooms) and a simple binary set of possibilities in each choice. This symmetric set-up allows us to estimate a variety of functional forms: there are six ways to order the three dimensions in a three stage hierarchical choice model. The six trees are depicted in Figure 5-5 and are denoted by the order of the dimensions (T = tenure choice, U = choice of number of units, S = size choice). In addition to the six trees, we will estimate the simple multinomial logit model.

We use a similar set of explanatory variables as we already introduced in Chapter Three:

(1) YP: Permanent income before taxes,
(2) HEDON: Hedonic index as a measure of user cost,
(3) RETURN: Return from equity minus its opportunity costs,
(4) CROWDS: Squared deviation of optimal from actual dwelling size,
(5) AGE: Age of household head.

Most of the differences to Chapter Three are motivated by making the cross country comparison as meaningful as possible. Unfortunately, the German survey only reports net income which is endogenous in this

FIGURE 5-5: DECISION TREES AND HOUSING ALTERNATIVES



housing demand equation because after-tax income depends on the housing choice and its implications on tax deductions. Gross income is calculated using stepwise polynomial approximations of the tax schedule, given imputed tax deductions from homeownership and average other deductions by age, household size, social status, and other demographic variables[7]. This necessary calculation, though made as careful as possible, is rough and noisy.

The use of permanent income rather than current income reflects the common wisdom (Quigley, 1979) that housing decisions are long run decisions and should not be influenced by transitory income fluctuations. Cleaning the explanatory variables from transitory components is in particular important when comparing two countries where the transitory components are more likely to be different. In particular, it may help to minimize the noise introduced by the calculation of gross income in West Germany. Section 5.6 will discuss the estimation of permanent income in both countries. Because income varies by household, but is not alternative specific, it has to interact with a set of dummies[8]. Instead of using a full set of seven alternative specific dummies, we exploit the symmetric tree structure and let income interact with a dummy for each dimension of choice.

Age of the household head is included in the equation over and above the stratification of the sample into age categories to take account of the differences in the life cycle pattern of housing consumption within each stratum, discovered in Table 5-2. Age enters the equation linearly and quadratically, and both variables interact with the dummies for each dimension of choice in the same as way income does.

We use two price variables, one measuring the out-of-pocket user cost, and the other measuring the less tangible return from equity invested in an owner-occupied home. The latter variable is defined as

(5.1) RETURN = ( APPR - TBILLR * (1-MARGTR) * (1-LOANRA) ) * VALUE

where APPR: rate of appreciation
TBILLR: alternative interest rate
MARGTR: marginal tax rate of household
LOANRA: loan-to-value ratio

Note that the capital gains accrued by appreciation are not taxed, whereas the alternative investment (in the U.S.: in treasury bills, in Germany in "festverzinsliche Wertpapiere") is subject to personal income tax. This reflects the tax treatment of home sales in both countries where the capital gains essentially evade taxation.

To compute user cost, we could in principle proceed as in Chapter Three. However, the data provided in both surveys is quite different, in particular, house values have to be imputed from external information because they are not reported in the German survey. Therefore, it is likely that user cost of homeownership are not comparable across the two countries, and we rather use a hedonic index. Furthermore, this allows us to include condominiums into the choice set. Condominiums are an important housing alternative in West Germany (5.5 percent in the sample) and Boston (5.1 percent in the sample), less so in the other U.S. SMSA's (Dallas 0.6 percent, Los Angeles 1.6 percent, Minneapolis/St.Paul 0.8 percent). A discussion of the hedonic estimation follows in the next section.

As a second demographic variable, we introduce a variable to measure household size which boils down to the number of children in our sample of married couples. As most useful a variable turned out which measures the crowding of a dwelling in terms of the deviation of an optimal from the actual dwelling size:

(5.2) $CROWDS = ( NROPT - NRACT)^2$

where NRACT: actual number of rooms
NROPT: optimal number of rooms
NROPT = NADULTS + 1 + (NKIDS-1)/2
NADULTS: number of adults in household
NADULTS: number of children in household

Finally, the attributes of hypothetical, i. e., not chosen alternatives are imputed as sample averages evaluated at the household's housing independent characteristics. The treatment differs from that in Chapter Three: we keep the chosen number of rooms constant when the household hypothetically moves to another tenure or structure category. This way, we can more clearly separate the choices on each level of the tree.

We did a fair number of experiments with alternative specifications for the exogenous variables in the chosen and hypothetical alternatives, the functional form of the decision trees, and the number of alternatives. Some sensitivity results are reported in Section 5.7.

## 5.5 Hedonic Price Indices

Hedonic rent indexes are estimated from the renter subsample in each country, then evaluated at each dwelling. This yields an imputed rent for owner-occupied homes and condominiums, where no value and out-of-pocket cost data is available in both countries. Methodological underpinnings of hedonic estimation can be found in Lancaster (1966), Rosen (1974), Muellbauer (1974), and Murray (1978). Mean gross rents and hedonic rents are reported at the bottom of Table 5-2. In both countries, we use a semilogarithmic functional form to accommodate interaction terms but avoid using nonlinear specifications.

### 5.5.1 United States

In the United States, we mimicky the the specification of Malpezzi, Ozanne, and Thibodeau (1980), in the following denoted by MOT. The estimates are readjusted for our sample and the sampling period (1977-78 instead of 1974-75), and the specification was slightly changed to be applicable for both renter and owner-occupied units. We estimated four sets of coefficients, one for each SMSA, to accommodate regional differences in the housing stock and its evaluation in each geographical region due to, e. g., climatic differences, as well as in the general level of rents due to market conditions.

The estimated coefficients are reported in Table 5-6. Very little changes compared to the original MOT results. The number of rooms has the expected significance as the most important quality measure. Structure type performs weakly, as it does in the MOT estimations for 1974-76, and, as we will see, in the German hedonic regressions. Of the building attributes, the informal ratings are unreliable. Age enters in a S-shaped nonlinear fashion. The rental unit depreciates fast in the first years after construction; this rate slows down after the initial period; but increases again when the building becomes very old. Interestingly, the pattern is different in Boston where old buildings even have a positive contribution to the hedonic value. Almost all dwelling attributes have their expected signs or are insignificant.

We measure strong tenure discounts with a concave relationship to the length of tenure. Ten years after moving in, tenants receive a discount of 22.1 percent in Boston, 16.1 percent in Dallas, 24.5 percent in Los Angeles, and 10.7 percent in Minneapolis/St.Paul, compared to those of MOT quoted in Table 4-2. The presence of the landlord in the building implies a significant (at the 90 percent level) rent reduction in all SMSA's except Minneapolis/ St.Paul, which supports our reasoning in Section 4.2.2. Tenant characteristics may proxy neighborhood characteristics not picked up in the coefficients at the bottom of Table 5-6. Finally, we control for inclusions in the gross rent and inflation during the lengthy interview period.

The Boston rental market emerges as distinctly different from the

TABLE 5-6 HEDONIC REGRESSION COEFFICIENTS (USA)

Dependent Variable: Logarithm of Yearly Gross Rent

| VARIABLE: | BOSTON: | DALLAS: | LOS ANGELES: | MINN/ST.P.: |
|---|---|---|---|---|
| CONSTANT | 0.90199 * | 0.94663 * | 1.33857 * | 0.83656 * |
| (1) NUMBER OF ROOMS: | | | | |
| 1+1/2 BATHS | 0.13285 * | 0.10007 * | 0.11486 * | 0.12840 * |
| TWO BATHS | 0.21148 * | 0.16076 * | 0.11136 * | 0.13190 * |
| THREE BATHS | 0.50431 * | 0.37019 * | 0.21600 * | 0.11278 |
| ONE ROOM | -0.10873 * | -0.16168 * | -0.14698 * | -0.14291 * |
| TWO ROOMS | -0.05875 * | -0.12015 * | -0.10482 * | -0.04869 * |
| FOUR ROOMS | 0.07249 * | 0.05947 - | 0.20443 * | 0.04485 - |
| FIVE+ ROOMS | 0.02473 + | -0.01625 - | 0.04412 * | 0.06999 * |
| NO BEDROOM | -0.21798 * | -0.20446 * | -0.27271 * | -0.21347 * |
| TWO BEDROOMS | 0.11596 * | 0.12006 * | 0.19344 * | 0.13746 * |
| THREE BEDROOMS | 0.22820 * | 0.26933 * | 0.35732 * | 0.29278 * |
| FOUR+ BEDROOMS | 0.06679 * | 0.08153 * | 0.10280 * | 0.06839 * |
| (2) STRUCTURE TYPE: | | | | |
| ONE FAM.,ATT. | -0.00398 | 0.02405 | -0.07958 + | 0.12123 * |
| ONE FAM.,DET. | 0.09331 + | 0.01735 | -0.01601 | 0.08406 + |
| DUPLEX | 0.03736 - | 0.05303 - | -0.00146 | 0.03988 - |
| FIFTY+ UNITS | 0.06505 - | -0.02114 | 0.09605 * | 0.05409 + |
| (3) ATTRIBUTES OF BUILDING: | | | | |
| BUILDING AGE | 0.03099 + | -0.02530 * | -0.03452 * | -0.02873 * |
| AGE SQUARED | -0.00262 + | 0.00101 - | 0.00170 + | 0.00135 + |
| AGE CUBED | .000054 * | -.000012 | -.000030 + | -.000024 - |
| PRIOR 1940 | 0.02558 | -0.22387 * | -0.35393 * | -0.28725 * |
| ELEVATOR | 0.12151 * | 0.37430 * | 0.10561 + | 0.10681 * |
| BAD HALLWAY | -0.00373 | -0.04854 + | 0.02274 - | 0.00021 |
| LEAKS,CRACKS... | 0.00888 | -0.00778 | 0.00075 | 0.00403 |
| (4) ATTRIBUTES OF DWELLING: | | | | |
| ROOM HEATER | -0.07710 - | -0.03693 | -0.04243 + | -0.05421 |
| STEAM HEAT | 0.00216 | 0.02376 | -0.08911 - | -0.01782 |
| ELECTRIC HEAT | -0.00169 | -0.03554 | 0.01642 | 0.00553 |
| ROOM AIRCOND. | 0.04912 * | 0.15800 * | 0.01314 | 0.03727 + |
| CENTRAL AIRCO. | 0.17119 * | 0.09890 * | 0.10926 * | 0.08094 * |
| NO RADIATORS | -0.03860 - | -0.02579 | -0.05037 * | -0.02954 |
| POOR PLUMBING | -0.15616 * | -0.16900 * | -0.13973 * | -0.07503 - |
| NO PRIVACY | -0.04803 + | -0.04793 + | -0.06610 * | -0.09208 * |
| NO OUTLETS | -0.06982 - | -0.17764 * | -0.06638 - | 0.00217 |
| COOK WITH ELEC. | 0.06503 * | 0.07872 * | -0.00012 | 0.05554 * |

TABLE 5-6 HEDONIC REGRESSION COEFFICIENTS (CONT'D)

| VARIABLE: | BOSTON: | DALLAS: | LOS ANGELES: | MINN/ST.P.: |
|---|---|---|---|---|
| (5) TENURE DISCOUNTS | | | | |
| LENGTH OF TEN. | -0.02841 * | -0.02671 * | -0.03469 * | -0.01335 * |
| L.O.T. SQUARED | 0.00063 * | 0.00106 * | 0.00101 * | 0.00027 |
| MOVE PRIOR 1950 | -0.24097 * | -0.18788 + | -0.30103 * | -0.37291 * |
| LANDLD PRESENT | -0.03875 + | -0.05625 - | -0.04171 + | 0.00118 |
| (6) TENANT CHARACTERISTICS: | | | | |
| BLACK | -0.02188 | -0.15769 * | -0.17172 * | 0.04533 - |
| SPANISH | -0.09236 * | -0.06463 + | -0.14289 * | 0.07447 |
| CHINESE | 0.04310 | -0.07988 - | -0.07150 * | -0.03224 |
| PERSONS/ROOM | 0.04012 - | 0.05058 + | 0.04329 + | 0.09127 * |
| (7) CONTROLS FOR GROSS RENT: | | | | |
| UTILITIES INCL. | 0.20532 * | 0.06374 * | 0.11750 * | 0.18198 * |
| HEAT INCL. | -0.00491 | 0.34012 + | 0.23630 - | 0.20027 + |
| PARKING INCL. | 0.10299 - | 0.03587 | 0.03850 | 0.13858 * |
| FURNITURE INCL. | -0.01085 | 0.00889 | -0.00394 | -0.03753 - |
| INTERVIEW DATE | -0.00086 | -0.00220 | -0.00414 + | -0.00120 |
| DATE * HEAT INCL | 0.00395 | -0.01973 | 0.00747 | -0.00695 |
| (8) NEIGHBORHOOD CHARACTERISTICS: | | | | |
| EXCELLENT | 0.07622 * | 0.03738 - | 0.10468 * | 0.09294 * |
| GOOD | 0.04408 * | 0.00768 | 0.04550 * | 0.05350 * |
| POOR | -0.02213 | -0.06031 - | 0.00591 | -0.02192 |
| ABANDONED STRUC | -0.07983 * | 0.05323 - | -0.06336 + | -0.00985 |
| LITTER | 0.01417 | -0.02959 - | -0.01269 | -0.01083 |
| NO SHOPS | 0.02936 - | -0.00515 | -0.02996 - | -0.01104 |
| CENTER CITY | -0.02243 - | 0.01051 | 0.04379 * | 0.05219 * |
| GROSS RENT/MONTH: | $ 225 | $ 190 | $ 198 | $ 201 |
| MEAN DEP. VARIABLE | 0.995 | 0.825 | 0.863 | 0.880 |
| STANDARD DEV.: | 0.368 | 0.397 | 0.412 | 0.328 |
| STANDARD ERROR: | 0.262 | 0.244 | 0.263 | 0.206 |
| OBSERVATIONS: | 1714 | 1368 | 1818 | 1196 |
| R-SQUARED: | 0.509 | 0.636 | 0.606 | 0.624 |

* = SIGNIFICANT AT 99%
+ = SIGNIFICANT AT 95%
\- = SIGNIFICANT AT 80%

other three SMSA's: the mean monthly rent is considerably higher; the explained fraction of rent variation lower; and the age pattern, discussed above, is reversed. This is consistent with the MOT findings and parallels the other SMSA's in the North-East considered in their investigation.

## 5.5.2 West Germany

For West Germany, we use the hedonic regression results by Behring, Goldrian, et. al. (1983) reported in Table 5-7. These indexes vary by city center versus suburbs, but are pooled across West Germany. To make up for intercity differences, we calculated average rents by housing alternative for each city and adjusted the level of the hedonic index accordingly. The variables used are similar to those in the specification of Malpezzi, Ozanne, and Thibodeau (1980).

The sample for the estimation is huge and produces very large t-statistics. As in the U.S. estimates, number of rooms has the strongest impact on hedonic values. For a given number of rooms, the average size of the rooms contributes positively to the rent. This variable is measured as the deviation of average room size from the standard room size of 135 square feet.

The coefficients for structure type are weak, and the negative value for two-family structures may indicate the German peculiarity of the "fake two-family homes" where the rent in the second unit is substantially reduced in some form of gentlemens' agreement as mentioned in Section 5.3. Note, however, that Behring and Goldrian excluded a unit from the sample if the tenant did not pay rent at all or explicitly stated that his rent was reduced due to an agreement with the landlord.

The age of the building is measured linearly and enters

TABLE 5-7 HEDONIC REGRESSION COEFFICIENTS (GERMANY)

Dependent Variable: Logarithm of Monthly Rent Net of Utilities

| VARIABLE: | CITY CENTERS | | SUBURBS | |
|---|---|---|---|---|
| CONSTANT | 5.451 | (402.90) | 5.451 | (248.93) |
| (1) NUMBER OF ROOMS: | | | | |
| ONE ROOM | -0.289 | (47.08) | -0.320 | (24.24) |
| THREE ROOMS | 0.215 | (51.06) | 0.212 | (27.41) |
| FOUR ROOMS | 0.389 | (69.60) | 0.367 | (38.85) |
| FIVE+ ROOMS | 0.103 | (79.98) | 0.094 | (44.52) |
| MEAN ROOM SIZE | 0.0325 | (79.68) | 0.0317 | (43.94) |
| NO KITCHEN | -0.261 | (40.14) | -0.320 | (27.02) |
| NO BATH | -0.172 | (29.56) | -0.129 | (13.12) |
| TWO+ BATHROOMS | 0.114 | (20.00) | 0.184 | (20.69) |
| (2) STRUCTURE TYPE: | | | | |
| ONE FAMILY HOME | 0.005 | ( 0.49) | 0.071 | ( 5.45) |
| TWO FAMILY HOME | -0.082 | (10.11) | -0.065 | ( 7.99) |
| 5+ F., 4- FLOORS | 0.029 | ( 5.58) | 0.070 | ( 8.79) |
| 5+ F., 5+ FLOORS | 0.059 | (10.29) | 0.150 | (11.68) |
| (3) ATTRIBUTES OF BUILDING: | | | | |
| BUILDING AGE | -0.0082 | (21.81) | -0.0075 | (12.54) |
| PRIOR 1949 | -0.330 | (41.93) | -0.239 | (19.13) |
| GENTRIFIED | -0.046 | (17.86) | -0.039 | ( 6.22) |
| (4) ATTRIBUTES OF DWELLING: | | | | |
| CENTRAL HEAT | 0.216 | (54.62) | -0.163 | (22.72) |
| WARM WATER | 0.137 | (17.81) | -0.045 | ( 3.13) |
| BALCONY | 0.037 | (24.81) | 0.065 | (14.39) |
| DOUBLE WINDOWS | 0.007 | ( 1.81) | 0.031 | ( 4.64) |

TABLE 5-7 HEDONIC REGRESSION COEFFICIENTS (CONT'D)

| VARIABLE: | CITY CENTERS | | SUBURBS | |
|---|---|---|---|---|
| (5) TENURE DISCOUNTS | | | | |
| LENGTH OF TENURE | -0.020 | ( 8.20) | -0.022 | ( 5.59) |
| L.O.T. SQUARED | 0.0007 | ( 3.32) | 0.0007 | ( 2.06) |
| MOVE PRIOR 1955 | -0.189 | (30.70) | -0.249 | (21.44) |
| LANDLD PRESENT | -0.127 | (31.99) | -0.139 | (15.85) |
| (6) TENANT CHARACTERISTICS: | | | | |
| GUESTWORKER | 0.008 | ( 1.14) | -0.011 | ( 0.98) |
| (7) CONTROLS FOR GROSS RENT: | | | | |
| COMPANY DWELLING | -0.083 | (10.20) | -0.062 | ( 4.51) |
| (8) NEIGHBORHOOD CHARACTERISTICS: | | | | |
| NOISE | -0.005 | ( 2.10) | -0.018 | ( 4.51) |
| PUBLIC TRANSP. | 0.022 | ( 5.95) | 0.016) | ( 3.76) |
| SHOPPING | 0.003 | ( 1.28) | -0.002 | ( 0.57) |
| PUBLIC PARKS | 0.015 | ( 6.56) | 0.009 | ( 2.11) |
| NET NORMAL RENT: | $ 100 | | $ 100 | |
| OBSERVATIONS: | 31207 | | 11952 | |
| R-SQUARED: | 0.692 | | 0.665 | |

t-statistics in brackets.

negatively, approaching a discount of a third of the normal rent in city centers, and of a quarter in suburbs for pre-1949 structures. An indicator variable for gentrification obviously measures the cause rather than the effect because it depresses the rent significantly. The tenure discounts are those reported in Table 4-1. In addition, the landlord's presence in the building has a significant influence on the rent, reducing it more than 12 percent. An indicator variable for guestworkers is included to test for discrimination, however, this variable can not be measured with any precision. Finally, a group of neighborhood variables is included with the expected sign pattern, but weak t-statistics.

The interviews in the One Percent Sample were conducted within a single week, so no adjustment for inflation was necessary. There are very little differences in the valuation of the hedonic attributes between city centers and suburbs. Buildings depreciate faster in cities, highrises add a substantially higher amount to the hedonic value in suburbs. Unfortunately, Behring and Goldrian do not report the means of the dependent variable. However, the explanatory variables are constructed in a way that the constant measures "normal rent" of a standard German rental unit. This rent is the monthly rent net of utilities and is approximately \$ 100 for suburbs and city centers, which is considerably higher than the average in non-urban areas, \$ 77 per month. For the demand equation in Section 5.7, we will add utility bills to this number as reported in Behring and Goldrian (1983). The average of this gross rent in our sample is reported at the bottom of Table 5-2.

## 5.5.3 Comparison

Using the same functional specification and similar explanatory variables, we can make a meaningful comparison between the results of the hedonic estimations in both countries. Note first the stunning difference in the rent levels as already pointed out in Table 5-2. The regression fit is somewhat tighter in the German estimation in spite of a more parsimonious specification which might be due to a smaller spread of rents in Germany. In both countries, the hedonic regressions do not discriminate well among structure types which indicates some trouble in predicting hedonic rents for owner-occupied single or two-family homes and comparing hedonic rents across tenure. For this reason, we will use the hedonic rents in the demand equations only interacting with a tenure choice dummy.

We already compared tenure discounts in Chapter Four. We also refer to this chapter for a discussion of measurement problems pertaining to this variable. The discounts for a private landlord are substantial in Germany. Many of these landlords live in the building they rented out. If we compare the discounts with the landlord-present-discounts measured for the American SMSA's, we may conclude that the relationship between landlord and tenant is more idiosyncratic as compared to the United States.

## 5.6 Permanent Income Estimates

As we argued in Section 5.3, the decision for a durable commodity like housing should depend on transitory income components. Thus, we use the following age specific permanent income

$Y_{curr} = Y_{perm}(\text{age, human and nonhuman wealth}) + Y_{trans}$

where the transitory income is defined as the residual of a regression of current income on the determinants of permanent income, thus uncorrelated to the latter. In both countries, we face data problems to measure all the components determining permanent income. The estimation heavily relies on the puritan proposition that human and nonhuman wealth are strongly correlated, and missing data in one of the wealth categories is predicted by the other. However, structural interpretation of the estimated coefficients should be only made with care.

### 5.6.1 United States

The Annual Housing Survey has very little information on human wealth as measured by professional status, social position, or job training. The only variable in this direction is the household head's education given by the highest degree received. The survey, however, has some information on non-human wealth in form of indicator variables of asset income, rental income, the number of cars and trucks, and the possession of a second home. To accommodate interaction patterns, we stratify the estimation by age groups and use a semilogarithmic form. To cope with inter-city differences, we estimate each set of regressions for each city separately. The results are presented in Table 5-8.

The regression fits are fairly low, most satisfactory in the elderly strata. Average gross income varies considerably among strata, reflecting life cycle earning patterns, and among SMSA's, with Minneapolis/St.Paul at the top and Boston at the bottom of the income scale. Only for the youngest stratum, Los Angeles has the lowest average income. The cross sectional variation in income is increasing with age in all strata, lowest in Minneapolis/St.Paul.

Not surprisingly, the employment status in the week of the interview has in almost all regressions a large and significant influence on permanent income. Similarly , the impact of additional wage and salary incomes in the household is positive, though measured with less precision. Among the coefficients of nun-human wealth, the presence of assets and the number of cars play the most important role to predict permanent income. They are highly significant and of large magnitude in every regression. The presence of rental income has a negative impact which may be explained by the lack of other income sources.

The household characteristics perform very weakly. This is partially due to the stratification by age which takes care of most of the age generated income variation. In the pooled sample, all age variables are highly significant. The number of dependents is of

TABLE 5-8: PERMANENT INCOME ESTIMATION (USA)

Dependent Variable: Yearly Gross Income in 1000 Dollars

MARRIED COUPLES, HEAD AGED UNDER 35

| VARIABLE: | BOSTON: | DALLAS: | LOS ANGELES: | MINN./ST.P.: |
|---|---|---|---|---|
| CONSTANT | -0.80973 | 1.59981 + | 0.14282 | -0.42008 |
| (1) INCOME SOURCES: | | | | |
| EMPLOYED | 0.51998 * | 0.17746 - | 0.19068 - | 0.25783 * |
| RETIRED | -0.03709 | 0.09544 | 0.05716 | 0.26918 * |
| NUMBER OF WAGES | 0.12955 | 0.05445 | 0.19286 * | 0.05011 |
| (2) NONHUMAN WEALTH: | | | | |
| ASSETS PRESENT | 0.19084 * | 0.13198 * | 0.14937 * | 0.12817 * |
| RENTAL INCOME | -0.07496 | -0.01337 | -0.30735 + | -0.17260 * |
| SECOND HOME | 0.04524 | 0.00428 | -0.18293 | 0.15247 - |
| NUMBER OF CARS | 0.20459 * | 0.17218 * | 0.16886 * | 0.15266 * |
| TRUCKS | 0.15078 - | 0.13096 * | 0.19457 * | 0.08854 * |
| (3) HOUSEHOLD CHARACTERISTICS: | | | | |
| AGE OF HEAD | 0.20867 - | -0.01180 | 0.08496 | 0.12986 + |
| AGE SQUARED | -0.00308 - | 0.00037 | -0.00125 | -0.00203 - |
| AGE OF SPOUSE | -0.05026 | 0.03911 | 0.06507 - | 0.06350 - |
| AGE SQUARED | 0.00094 | -0.00030 | -0.00085 - | -0.00076 - |
| HEAD FEMALE | -0.53898 - | -0.27547 + | -0.06889 | -0.46801 * |
| HEAD SPANISH | -0.29875 + | -0.13915 + | -0.16636 * | 0.12459 |
| DEPENDENTS | -0.07036 + | -0.02233 - | -0.03441 - | -0.05588 * |
| (4) EDUCATION OF HEAD: | | | | |
| GRADE 0-7 | -0.17824 | -0.24808 + | -0.29891 * | -0.04451 |
| GRADE 8 | 0.15895 | -0.21486 - | -0.19937 | -0.14088 |
| GRADE 9-11 | -0.03026 | -0.18188 * | -0.22314 * | -0.19088 + |
| TWO Y. COLLEGE | 0.14623 - | -0.00325 | 0.01650 | -0.00028 |
| FOUR Y. COLL. | 0.21300 + | 0.17191 * | 0.14929 - | 0.10726 * |
| GRAD. SCHOOL | 0.25401 * | 0.16361 * | 0.26262 * | 0.05295 |
| GROSS INCOME: | $ 16,729 | $ 16,413 | $ 14,490 | $ 18,758 |
| MEAN DEP. VAR.: | 2.790 | 2.798 | 2.673 | 2.932 |
| STANDARD DEV.: | 0.653 | 0.517 | 0.622 | 0.439 |
| STANDARD ERROR: | 0.551 | 0.425 | 0.488 | 0.368 |
| OBSERVATIONS: | 435 | 674 | 493 | 782 |
| R-SQUARED: | 0.323 | 0.347 | 0.412 | 0.316 |

* = SIGNIFICANT AT 99%, + = SIGNI. AT 95%, - = SIGNIFICANT AT 80%

TABLE 5-8: PERMANENT INCOME ESTIMATION (CONT'D)

Dependent Variable: Yearly Gross Income in 1000 Dollars

MARRIED COUPLES, HEAD AGED 35 TO 50

| VARIABLE: | BOSTON: | DALLAS: | LOS ANGELES: | MINN./ST.P.: |
|---|---|---|---|---|
| CONSTANT | 3.36395 - | 4.29046 + | -0.81824 | 4.66697 * |
| (1) INCOME SOURCES: | | | | |
| EMPLOYED | 0.59880 * | 0.14077 | 0.29925 * | 0.45921 * |
| RETIRED | 0.17779 + | -0.01350 | 0.05632 | 0.05910 |
| NUMBER OF WAGES | 0.06008 * | 0.02934 | 0.03601 - | 0.03577 + |
| (2) NONHUMAN WEALTH: | | | | |
| ASSETS PRESENT | 0.20699 * | 0.21668 * | 0.15666 * | 0.11054 * |
| RENTAL INCOME | -0.06914 | -0.14468 - | -0.16673 - | -0.07303 |
| SECOND HOME | -0.00404 | -0.05242 | -0.00915 | 0.13155 * |
| NUMBER OF CARS | 0.18490 * | 0.13613 * | 0.15155 * | 0.09574 * |
| TRUCKS | 0.00954 | 0.10244 + | 0.11271 + | 0.07375 + |
| (3) HOUSEHOLD CHARACTERISTICS: | | | | |
| AGE OF HEAD | -0.00953 | -0.04111 | 0.17083 - | -0.13516 + |
| AGE SQUARED | 0.00013 | 0.00049 | -0.00200 - | 0.00154 - |
| AGE OF SPOUSE | -0.05746 - | -0.02400 | -0.00396 | 0.03509 |
| AGE SQUARED | 0.00070 - | 0.00026 | 0.00009 | -0.00041 |
| HEAD FEMALE | -0.05133 | -0.32602 - | -0.86750 * | -0.26408 - |
| HEAD SPANISH | -0.08615 | -0.11953 | -0.10984 - | 0.04132 |
| DEPENDENTS | -0.00412 | -0.02729 | -0.00806 | -0.01937 - |
| (4) EDUCATION OF HEAD: | | | | |
| GRADE 0-7 | -0.20554 - | -0.49628 * | -0.44321 * | -0.09350 |
| GRADE 8 | -0.13150 | -0.06132 | -0.22485 - | -0.10471 |
| GRADE 9-11 | -0.31727 * | -0.27937 * | -0.22982 * | -0.03883 |
| TWO Y. COLLEGE | 0.05834 | 0.00526 | 0.13788 + | 0.10378 + |
| FOUR Y. COLL. | 0.33085 * | 0.16639 * | 0.19509 * | 0.24956 * |
| GRAD. SCHOOL | 0.45139 * | 0.23079 * | 0.24461 * | 0.36131 * |
| GROSS INCOME: | $ 20,795 | $ 22,727 | $ 21,170 | $ 24,210 |
| MEAN DEP. VAR.: | 3.035 | 3.124 | 3.053 | 3.187 |
| STANDARD DEV.: | 0.633 | 0.648 | 0.640 | 0.466 |
| STANDARD ERROR: | 0.502 | 0.571 | 0.508 | 0.409 |
| OBSERVATIONS: | 566 | 753 | 552 | 818 |
| R-SQUARED: | 0.396 | 0.245 | 0.396 | 0.251 |

* = SIGNIFICANT AT 99%, + = SIGNI. AT 95%, - = SIGNIFICANT AT 80%

TABLE 5-8: PERMANENT INCOME ESTIMATION (CONT'D)

Dependent Variable: Yearly Gross Income in 1000 Dollars

MARRIED COUPLES, HEAD AGED ABOVE 65

| VARIABLE: | BOSTON: | DALLAS: | LOS ANGELES: | MINN./ST.P.: |
|---|---|---|---|---|
| CONSTANT | 0.83725 | 2.89141 + | 3.81292 * | 4.44491 * |
| (1) INCOME SOURCES: | | | | |
| EMPLOYED | 0.56220 * | 0.42915 * | 0.52288 * | 0.46670 * |
| RETIRED | -0.03779 | -0.16794 * | -0.00646 | -0.00477 |
| NUMBER OF WAGES | 0.10064 * | 0.05139 - | 0.02891 | 0.05000 + |
| (2) NONHUMAN WEALTH: | | | | |
| ASSETS PRESENT | 0.17069 * | 0.30942 * | 0.22881 * | 0.21248 * |
| RENTAL INCOME | -0.12446 + | -0.11814 - | -0.25154 * | -0.11196 + |
| SECOND HOME | 0.07390 | 0.15351 + | 0.00349 | 0.07766 - |
| NUMBER OF CARS | 0.14855 * | 0.15342 * | 0.20605 * | 0.12445 * |
| TRUCKS | 0.06069 | 0.04032 | 0.11191 + | -0.00938 |
| (3) HOUSEHOLD CHARACTERISTICS: | | | | |
| AGE OF HEAD | 0.02659 | 0.01631 | -0.01261 | -0.03608 |
| AGE SQUARED | -0.00022 | -0.00019 | 0.00006 | 0.00015 |
| AGE OF SPOUSE | 0.01521 | -0.02337 | -0.02787 - | -0.01469 |
| AGE SQUARED | -0.00012 | 0.00019 | 0.00025 - | 0.00013 |
| HEAD FEMALE | -0.14404 | 0.05405 | -0.38831 + | -0.35599 * |
| HEAD SPANISH | -0.20846 | -0.39402 * | -0.05994 | -0.03418 |
| DEPENDENTS | 0.01323 | -0.03779 | -0.03355 | -0.03570 |
| (4) EDUCATION OF HEAD: | | | | |
| GRADE 0-7 | -0.13966 - | -0.38534 * | -0.04878 | -0.36015 * |
| GRADE 8 | -0.09756 | -0.22214 + | -0.14171 - | -0.22988 * |
| GRADE 9-11 | 0.00421 | -0.25738 * | -0.11925 - | -0.15364 * |
| TWO Y. COLLEGE | 0.13113 + | 0.09931 - | 0.14367 + | 0.05274 |
| FOUR Y. COLL. | 0.30181 * | 0.12521 - | 0.16261 - | 0.09221 - |
| GRAD. SCHOOL | 0.39379 * | 0.21239 * | 0.37758 * | 0.13384 + |
| GROSS INCOME: | $ 16,183 | $ 17,329 | $ 16,793 | $ 17,489 |
| MEAN DEP. VAR.: | 2.784 | 2.852 | 2.821 | 2.862 |
| STANDARD DEV.: | 0.732 | 0.769 | 0.767 | 0.672 |
| STANDARD ERROR: | 0.534 | 0.559 | 0.590 | 0.468 |
| OBSERVATIONS: | 813 | 694 | 670 | 880 |
| R-SQUARED: | 0.481 | 0.487 | 0.427 | 0.526 |

* = SIGNIFICANT AT 99%, + = SIGNI. AT 95%, - = SIGNIFICANT AT 80%

importance only in the strata of young couples where it reduces the expected normal income.

The dummy variables for the highest degree have the expected sign pattern and are in general measured with satisfactory precision. Comparing the three strata, we discover an interesting life cycle pattern: the magnitude of the education coefficients are largest in the strata of medium age, less so among the couples over fifty, and smallest in the young couples' stratum. Here, it may reflect the lag before the educational achievements are translated into money income. Among the people over fifty years old, it may indicate the supersession of education by experience and on-the-job-training.

It is very unfortunate that no information about the latter group of variables is contained in the Annual Housing Survey. We are left with a less than satisfactory determination of permanent income, but still prefer the poorly measured correct explanatory variable to current income. Section 5.7.3 discusses the sensitivity of the demand estimations when we replace permanent income by current income.

## 5.6.2 West Germany

With the German One Percent Sample we face the opposite data problem: the sample contains rich information on human wealth of both household head and spouse, but no information on asset holdings. We use the regression results by Schneider, Stahl, and Struyk (1983) which were estimated from a large pooled sample across West Germany. This sample includes rural areas as well as other demographic strata apart from married couples, so the mean income is lower than the mean income in our sample. Summary statistics and coefficients are reported in Table 5-9. Not presented are the coefficients for marital status, guestworkers, and rural area dummies which are irrelevant for our basic sample. Furthermore, the constant and the city center dummy are adjusted for the base case of a suburban location. Finally, the original specification of Schneider, Stahl, and Struyk included an indicator variable for homeownership on the right hand side. The inclusion of this variable would introduce a simultaneity bias in the demand equations. The corresponding coefficients were barely significant and of small magnitude (0.035 for single-family homeowners, -0.048 for two-family homeowners, with t-statistics of 3.36 and 2.97, respectively, in a sample of 29,017 observations). We excluded these coefficients.

The dependent variable is net income adjusted for tax advantages of homeownership as discussed in Section 5.4. We will briefly discuss the coefficients of the explanatory variables estimated by Schneider, Stahl, and Struyk. The age variables draw the familiar first

TABLE 5-9: PERMANENT INCOME ESTIMATION (GERMANY)

Dependent Variable: Logarithm of Monthly Net Income

| VARIABLE: | HOUSEHOLD: | HEAD: | SPOUSE: |
|---|---|---|---|
| CONSTANT | 7.42 * | | |
| (1) DEMOGRAPHIC CHARACTERISTICS: | | | |
| AGE OF HEAD | | 0.016 * | 0.002 * |
| AGE SQUARED | | -.0002 * | |
| HEAD MALE | | 0.189 * | |
| GUESTWORKER | | 0.051 * | |
| (2) EDUCATION: | | | |
| NO SCHOOL DEGREE | | 0.085 * | 0.048 |
| MIDDLE SCHOOL (MITTLERE REIFE) | | 0.194 * | 0.036 * |
| HIGH SCHOOL (ABITUR) | | 0.268 * | 0.031 |
| COLLEGE (VORDIPLOM) | | 0.237 * | 0.048 |
| UNIVERSITY (DIPLOM) | | 0.353 * | 0.128 * |
| (4) PROFESSIONAL STATUS: | | | |
| NUMBER OF WAGES | 0.303 * | | |
| SD. JOB AS FARMER | | -0.404 * | |
| APPRENTICESHIP | | 0.109 * | 0.050 * |
| MASTER CRAFTSMAN | | 0.137 * | 0.035 |
| SELF EMPLOYED | | 0.237 * | 0.020 |
| WHITE COLLAR | | 0.141 * | 0.290 * |
| BLUE COLLAR | | -0.001 * | 0.228 * |
| UNEMPLOYED | | -0.177 * | |
| RETIRED | | -0.378 * | -0.087 |
| STUDENT | | -0.234 * | 0.006 |
| (5) CONTROLS: | | | |
| COMPANY HOUSING | -0.087 * | | |
| HOUSING ALLOWANCES | -0.046 * | | |
| CITY CENTER | 0.036 * | | |
| YEARLY NET INCOME: | $ 9,322 | | |
| MEAN DEP. VAR.: | 7.48 | | |
| STANDARD ERROR: | 0.381 | | |
| OBSERVATIONS: | 29,017 | | |
| R-SQUARED: | 0.592 | | |

* = SIGNIFICANT AT 99%

increasing, then decreasing pattern of life cycle earnings. Education of the head performs as expected, both significant and with increasing magnitude for the improving educational status. Education of the spouse is measured with mixed results, mostly insignificant. Furthermore, human capital is indicated by the professional status. The number of wage and salary earners in the household contributes significantly to the household income, each additional wage at about 30 percent, whereas the combination of an industrial job at day with a second job as farmer ("moonlight farmer") has a sharp negative impact. Self employed or white collar status expectedly enhance; blue collar, unemployed, retired, or student status decrease permanent income. Again, the professional status of the spouse yields mixed, mostly unprecise results. Finally, Schneider, Stahl, and Struyk control for subsidies by receiving housing allowances or company housing, as well as location: the income differential between city center and suburbs is significant at about 3.6 percent.

### 5.6.3 Comparison

The regressions in both countries suffer from the lack of data, and we will only compare the predictions generated by the regressions. The third column of each block in Table 5-10 lists the distribution of actual current income versus predicted permanent income for both countries. Both incomes are measured yearly, before taxes, and in 1977 Dollars. By construction, permanent income has a substantially lower variance. The distributional differences between West Germany and the United States, already discussed for current income in Section 5.2, remain the same for permanent income: the fraction of households with a yearly permanent income $ 8,000 is 7.4 percent in the United States and only 3.2 percent in Germany. On the other hand, in the middle income classes ($ 8,000 to $ 25,000) the distribution is more even in Germany.

It is interesting to compare the correlation between ownership and income for both income concepts. In both countries, permanent income has a stronger impact on tenure choice than current income, as can be seen in the lower (greater) ownership ratio of permanent income at the bottom (top) of the income scale, compared with current income. This confirms the common wisdom of higher permanent than current income elasticities and the inclusion of permanent rather than current income into the demand analysis.

TABLE 5-10: DISTRIBUTION OF PERMANENT AND CURRENT INCOME

(1) CURRENT INCOME VERSUS OWNERSHIP:

| Income Class | UNITED STATES SAMPLE Income Distribution | Ownership Total | Ratio | WEST GERMAN SAMPLE Income Distribution | Ownership Total | Ratio |
|---|---|---|---|---|---|---|
| < 3.0 | 1.04% | 52 | 61.18% | 0.07% | 4 | 66.67% |
| < 4.0 | 1.09 | 52 | 58.43 | 0.27 | 8 | 36.36 |
| < 5.0 | 1.33 | 55 | 50.93 | 0.91 | 34 | 46.58 |
| < 6.0 | 1.86 | 79 | 52.32 | 0.04 | 0 | 0.00 |
| < 7.0 | 2.36 | 101 | 52.60 | 1.56 | 48 | 38.40 |
| < 8.0 | 2.51 | 111 | 54.41 | 0.11 | 5 | 55.56 |
| < 10.0 | 4.88 | 228 | 57.43 | 9.42 | 289 | 38.18 |
| < 12.5 | 8.04 | 394 | 60.24 | 8.19 | 260 | 39.51 |
| < 15.0 | 7.63 | 393 | 63.29 | 8.81 | 271 | 38.28 |
| < 20.0 | 18.26 | 1129 | 75.98 | 25.66 | 841 | 40.79 |
| < 25.0 | 16.43 | 1106 | 82.72 | 15.10 | 470 | 38.75 |
| < 35.0 | 19.18 | 1395 | 89.37 | 13.24 | 445 | 41.82 |
| < 45.0 | 7.81 | 582 | 91.51 | 9.82 | 390 | 49.43 |
| < 55.0 | 7.59 | 592 | 95.79 | 2.30 | 100 | 54.05 |
| > 55.0 | 0.00 | 0 | 0.00 | 4.49 | 221 | 61.22 |

(2) PERMANT INCOME VERSUS OWNERSHIP:

| Income Class | UNITED STATES SAMPLE Income Distribution | Ownership Total | Ratio | WEST GERMAN SAMPLE Income Distribution | Ownership Total | Ratio |
|---|---|---|---|---|---|---|
| < 3.0 | 0.00% | 0 | 0.0% | 0.00% | 0 | 0.0% |
| < 4.0 | 0.07 | 2 | 33.3 | 0.04 | 0 | 0.0 |
| < 5.0 | 0.33 | 16 | 59.3 | 0.42 | 1 | 2.9 |
| < 6.0 | 1.47 | 58 | 48.3 | 0.56 | 1 | 2.2 |
| < 7.0 | 2.20 | 79 | 44.1 | 0.65 | 7 | 13.5 |
| < 8.0 | 3.34 | 133 | 48.9 | 1.54 | 64 | 51.6 |
| < 10.0 | 7.95 | 349 | 53.9 | 17.47 | 591 | 42.1 |
| < 12.5 | 13.05 | 663 | 62.4 | 16.73 | 500 | 37.2 |
| < 15.0 | 15.00 | 866 | 70.9 | 15.88 | 552 | 43.3 |
| < 20.0 | 29.34 | 2016 | 84.4 | 21.18 | 684 | 40.2 |
| < 25.0 | 17.95 | 1360 | 93.1 | 12.01 | 389 | 40.3 |
| < 35.0 | 9.12 | 713 | 96.1 | 9.99 | 441 | 54.9 |
| < 45.0 | 0.17 | 14 | 100.0 | 2.45 | 84 | 42.6 |
| < 55.0 | 0.0 | 0 | 0.0 | 0.77 | 54 | 87.1 |
| > 55.0 | 0.0 | 0 | 0.0 | 0.30 | 18 | 75.0 |

Yearly gross current and permanent income in thousand 1977 Dollars. The totals are based on a sample size of 8035 married couples in West German Urban Areas and 8139 white married couples in U.S. SMSA's.

## 5.7 Estimation of the Demand Equations

With the preliminary estimations of permanent income and hedonic indexes completed, we can proceed to estimate the NMNL-demand equations. The extensive preliminary work provided us with the basis for a common model for the United States and West Germany with directly comparable variables and units. We estimated four sets of regressions, two in each country: for the first two sets, we draw a choice based sample of the pooled population for each country, and estimated all six trees depicted in Figure 5-5 in addition to the multinomial logit model. For the second two sets, we stratified the basic sample of each country according to the six strata of age and location:

(1) CC,YO: City center, young married couples, head aged under 35
(2) CC,ME: City center, married couples, head aged between 35 and 50
(3) CC,EL: City center, elderly married couples, head aged over 50
(4) SU,YO: Suburbs, young married couples, head aged under 35
(5) SU,ME: Suburbs, married couples, head aged between 35 and 50
(6) SU,EL: Suburbs, elderly married couples, head aged over 50

Within each stratum, we selected a smaller choice based subsample for estimation with the optimal tree discovered in the first regression sets. This procedure allowed us to use relatively small sample sizes (about 400 observations) without losing precision on the housing choices with small market shares. To make up for the endogenous sampling, we weighted the observations inversely to their sampling ratio and proceeded as if we had an exogenous sample. This estimation technique (WESML, Manski and Lerman, 1977) and the necessary correction for standard errors is described in Chapter Two.

### 5.7.1 Pooled Sample: Optimal Tree Structure

Table 5-11 reports the estimation results for the pooled sample of the United States; Table 5-12 for those of West Germany. The performance of each tree is indicated by the value of the likelihood function presented in the first row in the tables. This likelihood is evaluated relative to the null model at the bottom of the table ("RHO_SQ") as well as the percentage of correct ex post predictions. Section 2.3.2 discusses the interpretation of these statistics. The estimated coefficients represent the taste weights in the linear indirect utility function (2.2) and can be transformed into elasticities according to (2.10). Finally, the dissimilarity coefficients translate into the inter-alternative substitution effects as discussed in Section 2.2.3.

Three observations strike the eye: in both countries, the same hierarchical decision tree dominates; in Germany, the transition from simple multinomial logit analysis to this optimal nested model is more rewarding; however, all results are more precise in the American sample.

Figure 5-13 shows the pattern of performance and significance graphically. All trees can be distinguished statistically from the MNL-model by a likelihood test which is distributed chi-squared with six degrees of freedom, the number of dissimilarity parameters. In turn, all trees can be interpreted as restrictions of a common generalized extreme value model (GEV, McFadden, 1981) of the form (2.8)

TABLE 5-11: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (USA)

| TREE: | MNL | T-U-S | T-S-U | S-T-U | S-U-T | U-T-S | U-S-T |
|---|---|---|---|---|---|---|---|
| LIK: | -280.61 | -269.63 | -269.88 | -273.83 | -272.82 | -274.75 | -272.57 |
| (1) ALTERNATIVE SPECIFIC VARIABLES: | | | | | | | |
| HEDONO | -1.3764 | -2.0677 | -1.9741 | -1.3271 | -1.3922 | -1.5606 | -1.6066 |
| | (-5.25) | (-4.26) | (-4.55) | (-3.96) | (-3.96) | (-4.65) | (-5.22) |
| HEDONR | -2.5573 | -4.8441 | -4.5636 | -2.9145 | -3.0677 | -3.0102 | -3.4092 |
| | (-8.11) | (-5.22) | (-5.16) | (-4.41) | (-4.87) | (-4.98) | (-4.81) |
| RETURN | 0.0535 | -0.0312 | -0.0097 | 0.0382 | 0.0621 | 0.0369 | 0.0589 |
| | ( 0.82) | (-0.27) | (-0.09) | ( 0.52) | ( 0.80) | ( 0.49) | ( 0.75) |
| CROWDS | -0.1886 | -0.3790 | -0.3432 | -0.1734 | -0.1745 | -0.2334 | -0.1972 |
| | (-3.44) | (-3.37) | (-3.10) | (-2.01) | (-2.28) | (-4.51) | (-3.43) |
| (2) HOUSEHOLD SPECIFIC VARIABLES: | | | | | | | |
| INC_OWN | 0.1759 | 0.0507 | 0.0997 | 0.1020 | 0.1766 | 0.1368 | 0.1721 |
| | ( 3.14) | ( 0.60) | ( 1.18) | ( 0.83) | ( 1.60) | ( 1.72) | ( 1.69) |
| INC_SFM | 0.0671 | 0.2316 | 0.1553 | 0.0848 | 0.0596 | 0.1058 | 0.0745 |
| | ( 1.01) | ( 1.88) | ( 1.32) | ( 1.30) | ( 0.81) | ( 1.42) | ( 0.98) |
| INC_14R | -0.1815 | -0.3117 | -0.2993 | -0.1647 | -0.1627 | -0.2073 | -0.1816 |
| | (-2.29) | (-1.82) | (-1.80) | (-2.11) | (-2.07) | (-1.85) | (-1.80) |
| AGE_OWN | -0.5434 | -0.1771 | -0.3322 | -0.0855 | -1.1320 | -0.8134 | -1.2329 |
| *10 | (-0.90) | (-0.23) | (-0.42) | (-0.17) | (-1.44) | (-1.17) | (-1.49) |
| AGE_SFM | 1.3477 | 1.1049 | 1.4628 | 1.0142 | 1.6491 | 1.4462 | 1.7425 |
| | ( 2.16) | ( 1.47) | ( 1.59) | ( 2.09) | ( 2.30) | ( 2.00) | ( 2.20) |
| AGE_14R | 0.0589 | -0.3321 | -0.0608 | -0.1359 | -0.2663 | -0.1777 | -0.3572 |
| | ( 0.06) | (-0.18) | (-0.03) | (-0.11) | (-0.24) | (-0.14) | (-0.26) |
| AG2_OWN | 1.2423 | 0.9336 | 1.0823 | 0.5839 | 1.8189 | 1.5042 | 2.0017 |
| *1000 | ( 1.91) | ( 1.10) | ( 1.29) | ( 0.92) | ( 1.88) | ( 1.91) | ( 2.08) |
| AG2_SFM | -1.6818 | -1.3697 | -1.7155 | -1.1665 | -1.9571 | -1.7429 | -2.0991 |
| | (-2.73) | (-1.82) | (-1.83) | (-2.31) | (-2.56) | (-2.40) | (-2.63) |
| AG2_14R | 0.0800 | 0.6597 | 0.3716 | 0.2961 | 0.4183 | 0.3348 | 0.5134 |
| | ( 0.08) | ( 0.36) | ( 0.21) | ( 0.24) | ( 0.38) | ( 0.25) | ( 0.37) |

TABLE 5-11: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (USA)

(3) ALTERNATIVE SPECIFIC DUMMIES:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| O_SF_14 | -7.5473 | -13.0095 | -9.6823 | -4.6048 | -8.4654 | -7.7687 | -8.5669 |
| | (-1.93) | (-2.27) | (-1.47) | (-0.69) | (-1.64) | (-1.74) | (-1.84) |
| O_SF_5+ | -5.1823 | -9.7826 | -6.1457 | -2.6792 | -6.6825 | -5.4892 | -6.5924 |
| | (-2.72) | (-3.45) | (-1.67) | (-0.75) | (-2.42) | (-2.32) | (-2.57) |
| O_MF_14 | -7.2345 | -11.0102 | -7.8277 | -3.1124 | -8.1853 | -6.9033 | -8.0642 |
| | (-2.22) | (-2.07) | (-1.30) | (-0.47) | (-1.52) | (-1.72) | (-1.94) |
| O_MF_5+ | -6.0281 | -9.0941 | -6.2969 | -2.9078 | -6.2275 | -5.7448 | -6.5197 |
| | (-4.15) | (-3.69) | (-2.06) | (-0.89) | (-2.40) | (-2.59) | (-2.50) |
| R_SF_14 | -2.6049 | -4.2363 | -0.9959 | 1.3766 | -3.2005 | -2.7521 | -3.0756 |
| | (-0.80) | (-0.88) | (-0.18) | ( 0.28) | (-0.86) | (-0.70) | (-0.74) |
| R_SF_5+ | -1.0764 | -2.7777 | 0.0476 | 1.1024 | -4.2145 | -2.1269 | -4.3937 |
| | (-0.76) | (-1.13) | ( 0.02) | ( 0.61) | (-2.28) | (-1.17) | (-2.05) |
| R_MF_14 | 1.9062 | 2.5031 | 5.4261 | 5.5839 | 1.7697 | 2.4580 | 2.4728 |
| | ( 0.77) | ( 0.56) | ( 1.13) | ( 1.29) | ( 0.61) | ( 0.73) | ( 0.69) |

(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TH_0101 | 1.0000 | 2.0533 | 0.9181 | 0.4866 | 0.6603 | 1.3184 | 0.7170 |
| | ( 0.00) | ( 1.42) | (-0.15) | (-2.27) | (-1.90) | ( 0.68) | (-1.10) |
| TH_0102 | 1.0000 | 2.2287 | 1.1359 | 1.0598 | 1.4883 | 2.1746 | 2.3410 |
| | ( 0.00) | ( 1.53) | ( 0.40) | ( 0.19) | ( 0.86) | ( 1.67) | ( 1.64) |
| TH_0201 | 1.0000 | 3.6476 | 1.7588 | 0.7479 | 2.1029 | 1.3742 | 1.6029 |
| | ( 0.00) | ( 2.28) | ( 1.39) | (-1.23) | ( 1.94) | ( 0.73) | ( 1.48) |
| TH_0202 | 1.0000 | 1.6663 | 5.5940 | 3.6865 | 1.1668 | 1.0697 | 1.2285 |
| | ( 0.00) | ( 1.43) | ( 2.07) | ( 1.41) | ( 0.44) | ( 0.23) | ( 0.56) |
| TAU_01 | 1.0000 | 0.9259 | 1.8699 | 0.6832 | 0.9618 | 0.8115 | 1.2074 |
| | ( 0.00) | (-0.27) | ( 1.33) | (-1.03) | (-0.11) | (-0.72) | ( 0.59) |
| TAU_02 | 1.0000 | 2.2673 | 2.5172 | 1.4645 | 0.7482 | 1.1697 | 0.9229 |
| | ( 0.00) | ( 2.20) | ( 2.32) | ( 0.94) | (-1.35) | ( 0.57) | (-0.26) |
| NOBS: | 377 | 377 | 377 | 377 | 377 | 377 | 377 |
| RHO_SQ: | 0.642 | 0.656 | 0.656 | 0.651 | 0.652 | 0.650 | 0.652 |
| CORRECT: | 76.89% | 78.40% | 78.40% | 76.56% | 76.48% | 77.14% | 75.80% |

TABLE 5-12: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (GERMANY)

| TREE: | MNL | T-U-S | T-S-U | S-T-U | S-U-T | U-T-S | U-S-T |
|---|---|---|---|---|---|---|---|
| LIK: | -490.99 | -418.39 | -423.88 | -447.32 | -478.24 | -447.34 | -479.40 |
| (1) ALTERNATIVE SPECIFIC VARIABLES: | | | | | | | |
| HEDONO | -2.0699 | -11.1418 | -9.3458 | -2.5215 | -2.5256 | -2.8547 | -2.3419 |
| | (-7.87) | (-5.53) | (-4.19) | (-5.91) | (-5.50) | (-11.1) | (-8.67) |
| HEDONR | -2.4206 | -13.2527 | -10.5991 | -2.8063 | -3.2451 | -3.3627 | -2.9411 |
| | (-6.46) | (-4.00) | (-3.23) | (-5.60) | (-5.82) | (-8.26) | (-6.30) |
| RETURN | -0.0546 | 0.0594 | 0.0540 | -0.0085 | -0.0693 | -0.0057 | -0.0532 |
| | (-2.53) | ( 0.94) | ( 0.92) | (-0.43) | (-2.21) | (-0.40) | (-2.19) |
| CROWDS | -0.0977 | 0.0242 | -0.4716 | -0.1888 | -0.1877 | 0.0259 | -0.1496 |
| | (-0.94) | ( 0.06) | (-1.35) | (-2.74) | (-2.47) | ( 1.36) | (-1.79) |
| (2) HOUSEHOLD SPECIFIC VARIABLES: | | | | | | | |
| INC_OWN | 0.0407 | -0.1685 | 0.0076 | 0.0416 | 0.0582 | -0.0363 | 0.0478 |
| | ( 1.71) | (-1.06) | ( 0.05) | ( 1.12) | ( 1.23) | (-1.30) | ( 1.22) |
| INC_SFM | -0.0587 | -0.0859 | -0.2165 | -0.1130 | -0.0841 | -0.0595 | -0.0733 |
| | (-1.89) | (-0.39) | (-1.12) | (-2.07) | (-1.97) | (-1.50) | (-1.85) |
| INC_14R | -0.0632 | -0.7135 | -0.2352 | -0.0634 | -0.0658 | -0.1545 | -0.0826 |
| | (-1.90) | (-1.28) | (-1.01) | (-1.60) | (-1.75) | (-2.48) | (-1.45) |
| AGE_OWN | 0.9027 | 1.1970 | 0.1233 | 0.2656 | 0.7224 | 0.4444 | 0.5317 |
| *10 | ( 2.24) | ( 0.72) | ( 0.10) | ( 0.56) | ( 1.13) | ( 1.11) | ( 0.96) |
| AGE_SFM | 0.0807 | -0.2824 | 0.4862 | 0.5011 | 0.3907 | 0.2891 | 0.2807 |
| | ( 0.15) | (-0.09) | ( 0.17) | ( 0.76) | ( 0.56) | ( 0.46) | ( 0.46) |
| AGE_14R | -0.8051 | 2.2728 | -2.6354 | -0.9136 | -0.7072 | -0.1082 | -0.7904 |
| | (-1.55) | ( 0.29) | (-1.13) | (-1.61) | (-1.18) | (-0.11) | (-1.20) |
| AG2_OWN | -0.6556 | -0.5486 | 0.2077 | -0.1643 | -0.4207 | -0.1810 | -0.2789 |
| *1000 | (-1.64) | (-0.36) | ( 0.17) | (-0.36) | (-0.68) | (-0.47) | (-0.51) |
| AG2_SFM | 0.0012 | 1.0275 | 0.0771 | -0.3487 | -0.2657 | -0.1105 | -0.1873 |
| | ( 0.00) | ( 0.32) | ( 0.03) | (-0.55) | (-0.40) | (-0.18) | (-0.32) |
| AG2_14R | 0.8679 | -0.3025 | 3.0544 | 0.9197 | 0.7751 | 0.4782 | 0.8530 |
| | ( 1.66) | (-0.04) | ( 1.30) | ( 1.65) | ( 1.32) | ( 0.50) | ( 1.31) |

TABLE 5-12: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (GERMANY)

(3) ALTERNATIVE SPECIFIC DUMMIES:

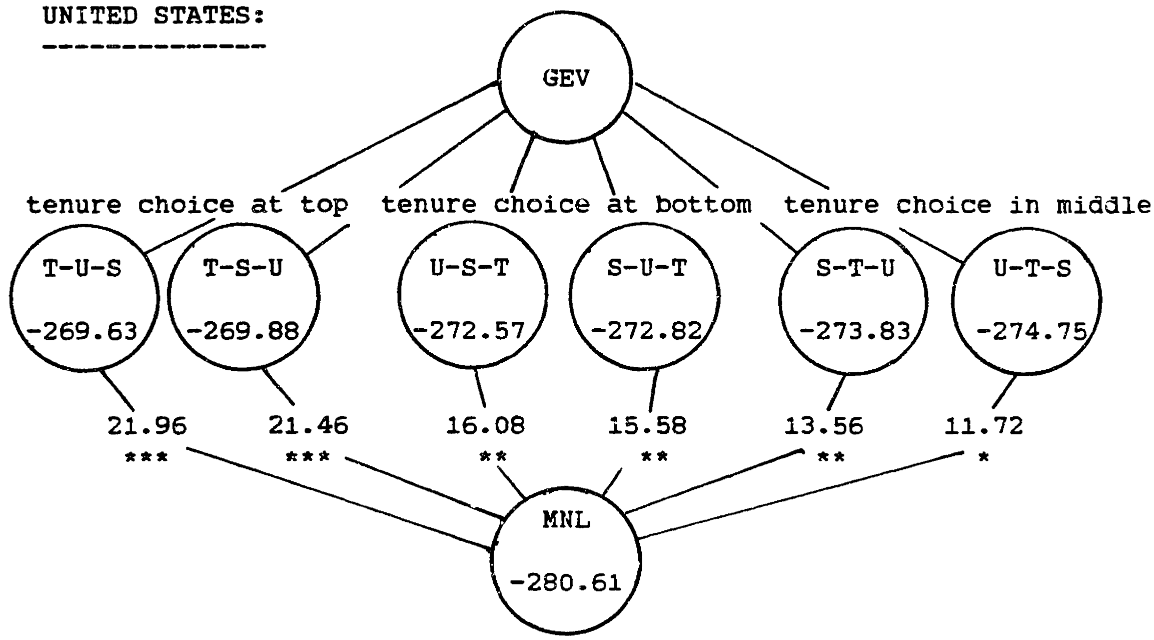
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| O_SF_14 | -1.0219 | 4.5272 | 2.5825 | 0.4180 | -2.4752 | 1.8103 | -0.4063 |
| | (-0.43) | ( 0.30) | ( 0.27) | ( 0.18) | (-0.82) | ( 0.50) | (-0.12) |
| O_SF_5+ | -0.9606 | 12.5258 | 4.5345 | 0.4808 | -1.4490 | 2.2662 | -0.3293 |
| | (-0.51) | ( 0.68) | ( 0.54) | ( 0.25) | (-0.62) | ( 0.92) | (-0.13) |
| O_MF_14 | -0.6503 | 12.2380 | 5.5270 | 1.4559 | -2.1118 | 4.3126 | -0.1059 |
| | (-0.34) | ( 0.82) | ( 0.58) | ( 0.66) | (-0.78) | ( 1.50) | (-0.03) |
| O_MF_5+ | -3.0975 | 8.2310 | 2.5823 | -1.7037 | -4.2900 | 0.5331 | -4.0728 |
| | (-2.37) | ( 0.51) | ( 0.36) | (-1.25) | (-2.13) | ( 0.31) | (-1.91) |
| R_SF_14 | 1.6966 | 13.0461 | 4.7581 | 1.1652 | 0.6535 | 4.2933 | 2.0388 |
| | ( 0.88) | ( 0.86) | ( 0.48) | ( 0.57) | ( 0.29) | ( 1.45) | ( 0.66) |
| R_SF_5+ | 0.3387 | 10.0395 | 0.6173 | -0.1822 | -1.1810 | 2.0400 | -0.3158 |
| | ( 0.27) | ( 0.53) | ( 0.10) | (-0.10) | (-0.62) | ( 1.00) | (-0.17) |
| R_MF_14 | 2.6783 | 16.2191 | 8.4049 | 2.7091 | 2.2445 | 5.5084 | 3.2308 |
| | ( 1.79) | ( 1.10) | ( 1.02) | ( 1.77) | ( 1.36) | ( 2.20) | ( 1.21) |

(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

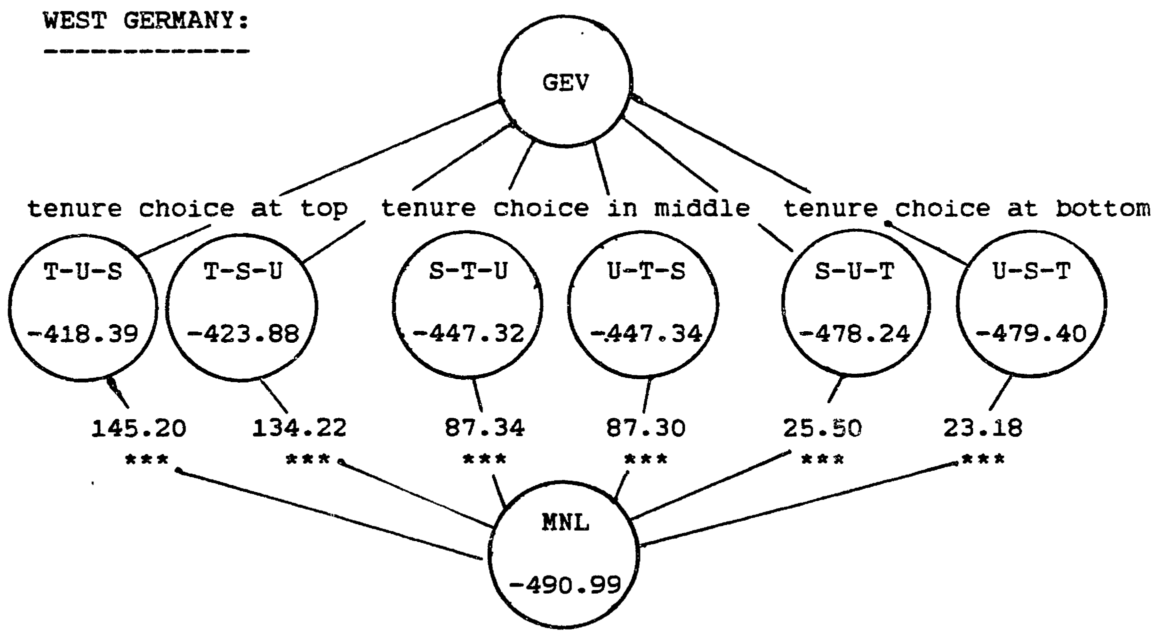
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TH_0101 | 1.0000 | 11.9006 | 7.1650 | 1.7605 | 0.9247 | 2.8748 | 0.8075 |
| | ( 0.00) | ( 1.96) | ( 3.42) | ( 2.26) | (-0.30) | ( 2.57) | (-1.16) |
| TH_0102 | 1.0000 | 6.3372 | 7.3321 | 1.5579 | 1.5054 | 2.3697 | 2.1988 |
| | ( 0.00) | ( 0.63) | ( 3.74) | ( 1.13) | ( 1.23) | ( 3.96) | ( 2.68) |
| TH_0201 | 1.0000 | 8.4397 | 5.1846 | 2.6473 | 2.6438 | 2.2335 | 1.3246 |
| | ( 0.00) | ( 2.84) | ( 3.39) | ( 2.22) | ( 1.88) | ( 1.87) | ( 1.04) |
| TH_0202 | 1.0000 | 12.5118 | 7.8569 | 2.5257 | 2.3853 | 2.9437 | 2.1199 |
| | ( 0.00) | ( 1.27) | ( 3.20) | ( 2.10) | ( 1.89) | ( 3.05) | ( 1.97) |
| TAU_01 | 1.0000 | 8.1912 | 5.2373 | 0.4488 | 1.2882 | 0.2715 | 1.1079 |
| | ( 0.00) | ( 2.85) | ( 2.09) | (-4.38) | ( 1.02) | (-9.17) | ( 0.44) |
| TAU_02 | 1.0000 | 4.4291 | 3.7735 | 0.7231 | 1.4458 | 1.2329 | 1.4341 |
| | ( 0.00) | ( 3.13) | ( 1.09) | (-1.24) | ( 1.07) | ( 0.79) | ( 0.55) |
| NOBS: | 442 | 442 | 442 | 442 | 442 | 442 | 442 |
| RHO_SQ: | 0.466 | 0.545 | 0.539 | 0.513 | 0.480 | 0.513 | 0.478 |
| CORRECT: | 67.57% | 65.36% | 67.32% | 68.13% | 68.48% | 64.88% | 68.03% |

FIGURE 5-13: CHI-SQUARED DISTANCES OF TREES

UNITED STATES:




WEST GERMANY:




Note:

*** = significant at the 99.5% level
** = significant at the 95.0% level
* = significant at the 92.5% level
6 degrees of freedom

where G in (2.19) is replaced by a less restrictive function.

All differences between MNL and the various NMNL-models are significant at the 99.5 percent level in West Germany, whereas in the United States only the trees with tenure choice at the uppermost level meet this criterion. However, of the remaining trees, all except one are significant at the 95 percent level, none has a chi-squared distance to the MNL-model below 92.5 percent. The trees can be divided into three groups according to the position of tenure choice. In the West Germany, the pattern is clear and simple: the further down the tenure choice, the worse the trees perform. In the American sample, the differences between positions of tenure choice are less pronounced and the clear order characterizing the German hierarchies is broken up. Once tenure choice is positioned, the trees can not be distinguished statistically in terms of dominance of size or structure choice.

We draw two conclusions: unambiguously in both countries, the tenure-structure-size choice sequence produces the most plausible hierarchical decision structure, and we will use this tree for our stratified estimations in the following Section. Second, however, the general dominance of tenure choice is less clear in Germany. On one side, once tenure choice is not at the top of the hierarchy, the dominance of tenure choice continues in Germany, but not in the United States. However, a closer look at the parameters in Table 5-12 reveals that the dissimilarity coefficients in West Germany are not contained in the unit interval, indicating a more complicated microeconomic interpretation than the theory outlined in the beginning of Section 2.2. In particular, the estimated relation can be interpreted as a mixture of different underlying trees, some with tenure choice at the top, other lower in the hierarchy. Because of the extent of this task, we were not able to identify a stratification of the sample which produces well behaved unique but different trees in each stratum. The large standard errors associated with the dissimilarity coefficients indicate such a mixture of models rather than a failure of the utility maximization hypothesis per se.

In both countries and in all specifications, we observe significant coefficients for the hedonic prices, consistently larger for renters than for owners. We introduced the interaction of hedonic rents with tenure choice for two reasons: as mentioned in Section 5.5, the hedonic regressions (on the renter subsamples) did not pick up well the differences in structure type, that is the attractiveness of single-family homes. Second, even if there were no measurement differences, out-of-pocket cost may be perceived differently by owners and renters. The general attractiveness of owning versus renting an object, e. g., the freedom of disposition, may interact with the price of the object.

The return variable is insignificant, pointing to offsetting effects of opportunity costs and appreciation, or poor specification. Unfortunatly, the global appreciation rates used here lack a differentiation by location within a city which is not reported in the two samples.

Surprisingly, the household size variable is significant in the United States but not in Germany. This result is in contrast to earlier studies, see Section 5.7.4.

The income and age pattern can be calculated by addition of different combinations of choice dimension dummies for each alternative. This yields the following preference ordering for tree T-U-S:

| | Germany: | | United States: |
|---|---|---|---|
| (1) | R_MF_5+ | (1) | O_SF_5+ |
| (2) | R_SF_5+ | (2) | R_SF_5+ |
| (3) | O_MF_5+ | (3) | O_MF_5+ |
| (4) | O_SF_5+ | (4) | R_MF_5+ |
| (5) | R_MF_14 | (5) | O_SF_14 |
| (6) | R_SF_14 | (6) | R_SF_14 |
| (7) | O_MF_14 | (7) | O_MF_14 |
| (8) | O_SF_14 | (8) | R_MF_14 |

Clearly, in both countries an additional Dollar of permanent income is spent first on more size. But again, tenure choice is differently perceived in Germany than in the United States where more income always implies a higher tendency to own.

This Section was primarily intended to highlight the differences in the valuation of tenure choice between the two countries. We will discuss the numerical values of the coefficients translated into elasticities in the next Section where we stratify the samples as discussed above.

5.7.2 Stratified Sample: Age and Location

Having discovered the tenure-structure-size choice hierarchy as the optimal functional description of housing demand within our NMNL-framework, we now stratify the sample according to location within the SMSA (city center versus suburb) and into three age categories (below 35, between 35 and 50, and over 50). The latter stratification takes account of a possible interaction between age and all other variables, that is a shift in preferences through the life cycle (de Leeuw 1971), whereas the separation of the sample by location stratifies by social status and income class as well as supply factors. Results are reported in Tables 5-14 for the United States and 5-15 for West Germany; compare the out-set of this Section for an explanation for the symbols used to denote the strata.

Stratification considerably improves the fit measured by the likelihood ratio ("RHO-SQ") or the percentage of correctly predicted choices in the strata of the city center and the couples over 50. For the young and middle aged couples in the suburbs we have relatively poor results. This holds for both countries. Except for those two strata, the German model now performs as well as the American model and has an even tighter fit for the oldest strata.

The results of the previous Section pertaining to the tree structure carry over to all separate strata. The bottom rows of Tables 5-14 and 5-15 show the performing of the corresponding MNL-models. The differences are all highly significant (at the 99.5

TABLE 5-14: SAMPLE STRATIFIED ACCORDING TO AGE AND LOCATION (USA)

| STRATUM: | POOL | CC,YO | CC,ME | CC,EL | SU,YO | SU,ME | SU,EL |
|---|---|---|---|---|---|---|---|
| LIK: | -269.63 | -230.75 | -232.65 | -232.22 | -359.53 | -276.25 | -297.05 |
| (1) ALTERNATIVE SPECIFIC VARIABLES: | | | | | | | |
| HEDONO | -2.0677 | -2.2485 | -1.9020 | -3.3301 | -1.0870 | -0.8438 | -0.9044 |
| | (-4.26) | (-4.02) | (-3.61) | (-6.31) | (-8.67) | (-1.52) | (-3.99) |
| HEDONR | -4.8441 | -4.2474 | -2.9094 | -4.0330 | -3.3064 | -2.1411 | -1.7023 |
| | (-5.22) | (-7.22) | (-3.46) | (-5.73) | (-4.84) | (-2.56) | (-3.44) |
| RETURN | -0.0312 | -0.2246 | 0.0033 | 0.2337 | -0.1951 | -0.1798 | -0.0303 |
| | (-0.27) | (-3.31) | ( 0.05) | ( 4.05) | (-3.94) | (-2.63) | (-0.81) |
| CROWDS | -0.3790 | -0.2374 | -0.1099 | -0.2934 | -0.1997 | -0.0001 | -0.1412 |
| | (-3.37) | (-3.24) | (-2.88) | (-3.63) | (-4.07) | (-0.00) | (-2.74) |
| (2) HOUSEHOLD SPECIFIC VARIABLES: | | | | | | | |
| INC_OWN | 0.0507 | 0.0679 | 0.1838 | 0.1456 | 0.2575 | 0.1738 | 0.2210 |
| | ( 0.60) | ( 0.70) | ( 2.71) | ( 2.68) | ( 2.82) | ( 1.10) | ( 2.89) |
| INC_SFM | 0.2316 | 0.1479 | 0.1210 | 0.2192 | -0.0196 | -0.0552 | -0.0882 |
| | ( 1.88) | ( 1.27) | ( 1.04) | ( 2.38) | (-0.17) | (-0.67) | (-0.80) |
| INC_14R | -0.3117 | -0.2308 | -0.2321 | -0.1224 | -0.1672 | -0.3457 | -0.1563 |
| | (-1.82) | (-1.89) | (-2.44) | (-1.33) | (-5.51) | (-1.47) | (-3.26) |
| AGE_OWN | -0.1771 | -9.3476 | -0.2771 | -0.3844 | -5.6479 | -15.4512 | 0.8244 |
| | (-0.23) | (-2.64) | (-0.04) | (-0.63) | (-3.36) | (-8.34) | ( 1.50) |
| AGE_SFM | 1.1049 | 6.4182 | -5.8375 | -4.6315 | -3.1839 | 29.7665 | 2.5541 |
| | ( 1.47) | ( 0.72) | (-0.27) | (-1.53) | (-0.11) | (17.09) | ( 1.95) |
| AGE_14R | -0.3321 | -2.4372 | 0.1134 | -3.1698 | -4.9824 | 3.9929 | 1.9218 |
| | (-0.18) | (-0.13) | ( 0.00) | (-0.72) | (-8.60) | ( 1.72) | ( 0.53) |
| AG2_OWN | 0.9336 | 17.3716 | 0.5852 | 0.5070 | 11.2519 | 17.9575 | -0.0804 |
| | ( 1.10) | ( 2.61) | ( 0.08) | ( 1.01) | ( 3.36) | (11.77) | (-0.17) |
| AG2_SFM | -1.3697 | -13.0038 | 6.9317 | 3.4476 | 7.7779 | -35.3570 | -2.4791 |
| | (-1.82) | (-0.80) | ( 0.27) | ( 1.53) | ( 0.15) | (-26.6) | (-2.57) |
| AG2_14R | 0.6597 | 3.8766 | 0.3978 | 2.8129 | 9.1134 | -4.6284 | -1.5555 |
| | ( 0.36) | ( 0.12) | ( 0.01) | ( 0.81) | (23.70) | (-1.67) | (-0.57) |

(3) ALTERNATIVE SPECIFIC DUMMIES:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| O_SF_14 | -13.0095 | 1.4334 | 5.7648 | 17.4194 | 9.7461 | -20.1911 | -13.6451 |
| | (-2.27) | ( 0.04) | ( 0.06) | ( 0.84) | ( 0.25) | (-4.48) | (-0.77) |
| O_SF_5+ | -9.7826 | 1.0910 | 9.3856 | 16.3250 | 5.1413 | -13.4647 | -6.4494 |
| | (-3.45) | ( 0.08) | ( 0.27) | ( 1.48) | ( 0.13) | (-3.05) | (-1.03) |
| O_MF_14 | -11.0102 | 6.4831 | -8.7359 | 2.2890 | 2.7310 | 12.5749 | -13.5431 |
| | (-2.07) | ( 0.23) | (-0.06) | ( 0.17) | (13.72) | ( 0.02) | (-0.98) |
| O_MF_5+ | -9.0941 | 3.9687 | -9.0644 | -2.4649 | -3.3324 | 20.0976 | -8.1960 |
| | (-3.69) | ( 0.88) | (-0.71) | (-0.79) | (-38.0) | ( 0.03) | (-2.22) |
| R_SF_14 | -4.2363 | -4.7114 | 10.5676 | 20.3636 | 10.6962 | -50.6275 | -6.6906 |
| | (-0.88) | (-0.14) | ( 0.13) | ( 0.99) | ( 0.28) | (-4.61) | (-0.41) |
| R_SF_5+ | -2.7777 | -13.2731 | 4.0935 | 9.1275 | 6.4102 | -50.8433 | -0.1667 |
| | (-1.13) | (-1.06) | ( 0.08) | ( 0.86) | ( 0.17) | (-3.34) | (-0.03) |
| R_MF_14 | 2.5031 | 7.6133 | 1.4742 | 9.1840 | 9.4563 | 12.7632 | -0.4020 |
| | ( 0.56) | ( 0.30) | ( 0.01) | ( 0.67) | ( 5.74) | ( 1.87) | (-0.03) |

(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TH_0101 | 2.0533 | 0.9267 | 1.0671 | 1.4208 | 0.8422 | 1.0073 | 0.9197 |
| | ( 1.42) | (-0.26) | ( 0.20) | ( 1.34) | (-1.00) | ( 0.01) | (-0.39) |
| TH_0102 | 2.2287 | 1.5557 | 1.1187 | 2.0356 | 0.0010 | 4.0122 | 0.7073 |
| | ( 1.53) | ( 0.81) | ( 0.27) | ( 2.12) | ( 0.00) | ( 0.04) | (-1.10) |
| TH_0201 | 3.6476 | 3.8981 | 5.6619 | 3.3670 | 2.2662 | 7.6243 | 1.3270 |
| | ( 2.28) | ( 2.31) | ( 1.36) | ( 1.96) | ( 0.37) | ( 1.87) | ( 0.31) |
| TH_0202 | 1.6663 | 1.5150 | 0.5811 | 0.8088 | 1.2399 | 5.8375 | 2.1675 |
| | ( 1.43) | ( 1.49) | (-1.94) | (-0.72) | ( 0.44) | ( 3.21) | ( 1.72) |
| TAU_01 | 0.9259 | 1.7742 | 1.9535 | 1.5887 | 1.3979 | 5.7624 | 1.6498 |
| | (-0.27) | ( 1.35) | ( 2.26) | ( 1.16) | ( 1.64) | ( 0.03) | ( 1.58) |
| TAU_02 | 2.2673 | 1.2685 | 0.7581 | 1.2421 | 1.2495 | 1.0075 | 1.4213 |
| | ( 2.20) | ( 1.01) | (-0.61) | ( 0.76) | ( 0.83) | ( 0.02) | ( 1.02) |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TREE: | T-U-S | T-U-S | T-U-S | T-U-S | T-U-S | T-U-S | T-U-S |
| NOBS: | 377 | 343 | 346 | 398 | 434 | 381 | 422 |
| RHO_SQ: | 0.656 | 0.676 | 0.677 | 0.719 | 0.602 | 0.651 | 0.661 |
| CORRECT: | 78.40% | 79.59% | 80.35% | 82.59% | 74.38% | 77.52% | 79.14% |

MNL-PERFORMANCE FOR COMPARISON:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| LIK: | -280.61 | -243.70 | -253.30 | -242.10 | -365.20 | -286.80 | -299.80 |
| RHO_SQ: | 0.642 | 0.658 | 0.648 | 0.707 | 0.595 | 0.638 | 0.658 |
| CORRECT: | 76.89% | 79.27% | 78.93% | 82.50% | 74.43% | 76.23% | 78.37% |

TABLE 5-15: SAMPLE STRATIFIED ACCORDING TO AGE AND LOCATION (GERMANY)

| STRATUM: | POOL | CC,YO | CC,ME | CC,EL | SU,YO | SU,ME | SU,EL |
|---|---|---|---|---|---|---|---|
| LIK: | -418.39 | -230.08 | -405.44 | -217.55 | -414.63 | -409.63 | -291.59 |
| (1) ALTERNATIVE SPECIFIC VARIABLES: | | | | | | | |
| HEDONO | -11.1418 | -7.1768 | -7.1136 | -8.3649 | -10.8740 | -6.0120 | -5.4039 |
| | (-5.53) | (-6.25) | (-6.84) | (-5.61) | (-4.11) | (-5.86) | (-4.79) |
| HEDONR | -13.2527 | -6.6116 | -9.2795 | -11.5718 | -13.1639 | -7.5246 | -8.1463 |
| | (-4.00) | (-4.93) | (-5.40) | (-5.35) | (-3.31) | (-4.42) | (-5.93) |
| RETURN | 0.0594 | -0.0653 | -0.0615 | -0.1301 | -0.1125 | -0.0946 | -0.1482 |
| | ( 0.94) | (-1.13) | (-1.62) | (-2.41) | (-1.68) | (-1.91) | (-2.41) |
| CROWDS | 0.0242 | 0.0684 | -0.6622 | -0.1659 | -1.2894 | -1.2236 | -0.3242 |
| | ( 0.06) | ( 0.18) | (-5.21) | (-0.61) | (-3.96) | (-5.26) | (-2.43) |
| (2) HOUSEHOLD SPECIFIC VARIABLES: | | | | | | | |
| INC_OWN | -0.1685 | 0.2095 | -0.0685 | 0.0424 | -0.0257 | 0.0814 | 0.0238 |
| | (-1.06) | ( 2.25) | (-1.11) | ( 0.49) | (-0.23) | ( 0.95) | ( 0.21) |
| INC_SFM | -0.0859 | -0.3019 | 0.1372 | -0.1078 | -0.1066 | -0.1516 | -0.2231 |
| | (-0.39) | (-2.49) | ( 1.33) | (-0.83) | (-0.29) | (-1.16) | (-2.44) |
| INC_14R | -0.7135 | -0.3249 | -0.2581 | -0.3104 | -0.2982 | -0.3624 | -0.4449 |
| | (-1.28) | (-1.19) | (-3.25) | (-2.35) | (-1.94) | (-2.65) | (-3.16) |
| AGE_OWN | 1.1970 | -17.4918 | 14.7392 | -0.7486 | 26.1760 | 3.2150 | 2.6209 |
| | ( 0.72) | (-4.69) | (24.29) | (-0.46) | (28.75) | ( 0.81) | ( 0.78) |
| AGE_SFM | -0.2824 | 35.1461 | -12.2237 | -5.0913 | 18.7470 | 8.6190 | -4.5715 |
| | (-0.09) | ( 9.32) | (-1.76) | (-0.64) | ( 2.17) | ( 2.07) | (-0.46) |
| AGE_14R | 2.2728 | -40.0533 | -17.9722 | 1.5316 | -22.0170 | 6.9028 | -3.9208 |
| | ( 0.29) | (-2.25) | (-9.57) | ( 0.52) | (-2.95) | ( 1.26) | (-0.35) |
| AG2_OWN | -0.5486 | 32.2272 | -17.2238 | 1.0017 | -40.0000 | -2.5016 | -2.0388 |
| | (-0.36) | ( 4.65) | (-16.1) | ( 0.82) | (-18.8) | (-0.52) | (-0.82) |
| AG2_SFM | 1.0275 | -64.0541 | 13.1820 | 4.1835 | -25.1000 | -8.1032 | 2.6955 |
| | ( 0.32) | (-10.1) | ( 1.57) | ( 0.72) | (-1.49) | (-1.56) | ( 0.37) |
| AG2_14R | -0.3025 | 57.8748 | 21.6473 | -1.4120 | 36.6000 | -6.6932 | 2.9590 |
| | (-0.04) | ( 1.50) | ( 7.44) | (-0.60) | ( 2.48) | (-0.94) | ( 0.34) |

(3) ALTERNATIVE SPECIFIC DUMMIES:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| O_SF_14 | 4.5272 | 45.0187 | 25.2199 | 9.6873 | -43.6420 | -48.3300 | 27.6633 |
| | ( 0.30) | ( 4.30) | ( 2.10) | ( 0.50) | (-4.60) | (-4.43) | ( 2.78) |
| O_SF_5+ | 12.5258 | -32.5944 | -5.5253 | 18.8074 | -65.9939 | -24.3661 | 16.4734 |
| | ( 0.68) | (-2.15) | (-0.40) | ( 0.79) | (-9.39) | (-6.22) | ( 0.38) |
| O_MF_14 | 12.2380 | 97.4145 | 4.0762 | -1.4141 | -2.7157 | -29.3015 | 8.0982 |
| | ( 0.82) | ( 5.48) | ( 0.95) | (-0.11) | (-0.28) | (-1.70) | ( 0.29) |
| O_MF_5+ | 8.2310 | 6.8571 | -33.3569 | 0.9284 | -30.5816 | -12.2768 | -8.8480 |
| | ( 0.51) | ( 0.20) | (-11.9) | ( 0.15) | (-5.21) | (-1.25) | (-0.72) |
| R_SF_14 | 13.0461 | 25.5750 | 55.6639 | 10.3353 | 0.3641 | -39.1544 | 38.1278 |
| | ( 0.86) | ( 2.48) | ( 4.50) | ( 0.43) | ( 0.02) | (-3.84) | ( 4.50) |
| R_SF_5+ | 10.0395 | -49.8680 | 21.6928 | 15.5875 | -26.3631 | -19.4744 | 19.3552 |
| | ( 0.53) | (-6.20) | ( 1.51) | ( 0.56) | (-2.85) | (-2.06) | ( 0.59) |
| R_MF_14 | 16.2191 | 74.8717 | 35.6858 | -1.3449 | 36.2376 | -17.7430 | 17.7709 |
| | ( 1.10) | ( 5.58) | ( 9.74) | (-0.16) | ( 3.39) | (-1.62) | ( 0.48) |

(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TH_0101 | 11.9006 | 17.4521 | 3.8013 | 3.5932 | 10.5537 | 4.5294 | 2.6661 |
| | ( 1.96) | ( 1.08) | ( 3.07) | ( 2.31) | ( 2.05) | ( 3.21) | ( 2.76) |
| TH_0102 | 6.3372 | 11.4852 | 2.4881 | 2.1644 | 2.7304 | 2.1706 | 1.5683 |
| | ( 0.63) | ( 0.56) | ( 2.02) | ( 1.74) | ( 1.89) | ( 1.58) | ( 1.00) |
| TH_0201 | 8.4397 | 5.0174 | 6.1321 | 3.5413 | 4.2833 | 6.0452 | 4.0094 |
| | ( 2.84) | ( 1.33) | ( 3.48) | ( 2.16) | ( 1.23) | ( 3.91) | ( 2.45) |
| TH_0202 | 12.5118 | 3.9098 | 4.5335 | 5.0244 | 5.7484 | 5.2970 | 3.9442 |
| | ( 1.27) | ( 1.39) | ( 3.85) | ( 2.91) | ( 1.93) | ( 2.82) | ( 2.64) |
| TAU_01 | 8.1912 | 4.7120 | 5.4953 | 3.5688 | 8.2979 | 5.1430 | 2.5888 |
| | ( 2.85) | ( 2.81) | ( 3.89) | ( 3.42) | ( 2.75) | ( 2.66) | ( 3.29) |
| TAU_02 | 4.4291 | 3.1176 | 4.0467 | 3.5663 | 11.4965 | 4.5360 | 1.6553 |
| | ( 3.13) | ( 2.34) | ( 4.61) | ( 4.13) | ( 2.30) | ( 3.61) | ( 1.85) |

---

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TREE: | T-U-S | T-U-S | T-U-S | T-U-S | T-U-S | T-U-S | T-U-S |
| NOBS: | 442 | 302 | 443 | 361 | 396 | 403 | 389 |
| RHO_SQ: | 0.545 | 0.634 | 0.560 | 0.710 | 0.497 | 0.511 | 0.640 |
| CORRECT: | 65.36% | 74.29% | 69.22% | 81.52% | 57.11% | 64.39% | 79.19% |

MNL-PERFORMANCE FOR COMPARISON:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| LIK: | -490.99 | -248.20 | -445.10 | -242.40 | -482.30 | -455.03 | -314.80 |
| RHO_SQ: | 0.466 | 0.605 | 0.517 | 0.677 | 0.414 | 0.457 | 0.611 |
| CORRECT: | 67.57% | 72.42% | 68.69% | 82.80% | 55.55% | 65.86% | 78.98% |

percent level) with the exception of the young couples in American suburbs where this difference only passes the 90 percent mark. Most of the individual dissimilarity parameters exceed one, but are measured with little precision, telling us that we have not discovered the correct specification to avoid estimating mixtures of hierarchies.

The taste coefficients for price and income can most easily be compared using the elasticity tabulations of Table 5-16. The elasticities refer to the percentage change of the probability to choose a housing alternative, when the price of this alternative or the household income is changed by one percent. The formula for the elasticities is given in (2.10). For each variable, the model produces an entire matrix of elasticities which describes the change in each choice probability i in response to a change of this variable in alternative j. For the pooled sample, the elasticity matrix for the hedonic rent and permanent income is presented in Table 5-17. All elasticities are evaluated at sample means and weighted according to the population shares to offset the effects of choice based sampling.

We will first study the complete matrices in Table 5-17 for the pooled samples. The hierarchy of the tree is reflected in the block structure of equal and unequal elasticities. The corresponding MNL model would have equal cross elasticities entirely within each row. The nested model does away with this pattern: if the price of any renter alternative goes up, all other renter alternatives are also less likely to be chosen. Of course, the own effect dominates the cross effects. This pattern holds in both countries. It can be

TABLE 5-16: OWN PRICE AND SUM OF INCOME ELASTICITIES OF MARKET SHARES

(1) GERMANY:

| Market Share: | MS(1) | MS(2) | MS(3) | MS(4) | MS(5) | MS(6) | MS(7) | MS(8) |
|---|---|---|---|---|---|---|---|---|
| Stratum | Own Price Elasticities: | | | | | | | |
| POOLED | -6.80 | -8.90 | -5.92 | -6.93 | -5.83 | -5.52 | -8.05 | -3.51 |
| CC_YO | -6.52 | -7.42 | -8.89 | -4.26 | -3.64 | -4.66 | -1.73 | -4.46 |
| CC_ME | -6.16 | -9.10 | -6.98 | -10.97 | -4.17 | -5.48 | -4.86 | -5.93 |
| CC_EL | -6.45 | -11.22 | -8.58 | -14.82 | -6.25 | -10.85 | -5.63 | -6.60 |
| SU_YO | -7.52 | -11.11 | -7.42 | -15.04 | -3.81 | -7.08 | -4.76 | -5.03 |
| SU_ME | -2.55 | -4.44 | -4.26 | -9.64 | -4.33 | -4.06 | -5.80 | -3.88 |
| SU_EL | -2.84 | -5.21 | -6.12 | -12.55 | -8.07 | -5.64 | -8.49 | -4.92 |
| Stratum | Comprehensive Income Elasticities: | | | | | | | |
| POOLED | -0.50 | 0.24 | -0.55 | 0.84 | -0.08 | 0.96 | 0.09 | 0.80 |
| CC_YO | 1.07 | 1.31 | 1.70 | 2.06 | -1.31 | -0.51 | -0.14 | 0.89 |
| CC_ME | 0.40 | 1.38 | 0.07 | 1.57 | 0.05 | 0.66 | -0.52 | 0.30 |
| CC_EL | -0.09 | 0.86 | 0.18 | 1.76 | -0.49 | 0.48 | -0.14 | 0.55 |
| SU_YO | -0.06 | 0.28 | 0.01 | 1.29 | -0.23 | 0.59 | -0.06 | 0.55 |
| SU_ME | -0.02 | 1.02 | 0.25 | 2.43 | -1.33 | -0.55 | -0.95 | -0.06 |
| SU_EL | -0.57 | 1.14 | 0.21 | 3.13 | -0.98 | 0.16 | 0.36 | 1.52 |

(2) USA:

| Market Share: | MS(1) | MS(2) | MS(3) | MS(4) | MS(5) | MS(6) | MS(7) | MS(8) |
|---|---|---|---|---|---|---|---|---|
| Stratum | Own Price Elasticities: | | | | | | | |
| POOLED | -2.36 | -3.25 | -6.44 | -7.19 | -7.59 | -9.11 | -10.70 | -12.56 |
| CC_YO | -5.10 | -8.50 | -3.93 | -6.23 | -7.36 | -6.50 | -7.53 | -12.34 |
| CC_ME | -4.14 | -3.28 | -3.91 | -6.33 | -6.59 | -7.51 | -11.63 | -21.00 |
| CC_EL | -5.50 | -6.89 | -6.55 | -9.63 | -8.44 | -8.22 | -11.94 | -21.33 |
| SU_YO | -2.26 | -3.20 | n.i. | n.i. | -6.52 | -8.65 | -8.29 | -11.25 |
| SU_ME | -1.71 | -1.22 | -0.64 | -0.80 | -4.13 | -4.67 | -6.70 | -2.08 |
| SU_EL | -1.88 | -2.21 | -3.12 | -5.53 | -4.24 | -6.59 | -4.68 | -3.33 |
| Stratum | Sum of Income Elasticities: | | | | | | | |
| POOLED | -0.22 | 1.54 | -3.65 | -2.03 | -2.36 | -1.37 | -3.75 | -1.59 |
| CC_YO | 0.21 | 2.49 | 0.03 | 1.38 | -0.17 | 0.37 | -1.49 | -0.09 |
| CC_ME | -1.14 | 1.81 | -1.41 | 1.40 | -2.11 | -1.55 | -6.01 | -0.60 |
| CC_EL | 0.29 | 1.27 | -1.10 | -0.43 | -1.59 | -1.18 | -3.85 | -2.15 |
| SU_YO | -0.06 | 1.94 | n.i. | n.i. | -2.24 | -1.50 | -2.24 | -0.88 |
| SU_ME | -3.25 | 1.73 | -0.19 | 1.06 | -4.10 | -3.44 | -4.65 | -3.79 |
| SU_EL | -0.96 | 1.09 | -0.25 | 2.42 | -3.28 | -1.86 | -2.47 | -1.60 |

Indices of Alternatives:

| 1: | 2: | 3: | 4: | 5: | 6: | 7: | 8: |
|---|---|---|---|---|---|---|---|
| O_SF_14 | O_SF_5+ | O_MF_14 | O_MF_5+ | R_SF_14 | R_SF_5+ | R_MF_14 | R_MF_5+ |

TABLE 5-17: MATRIX OF PRICE AND INCOME ELASTICITIES OF MARKET SHARES

(1) POOLED SAMPLE GERMANY:

| Variable | Alt. | MS(1) | MS(2) | MS(3) | MS(4) | MS(5) | MS(6) | MS(7) | MS(8) |
|---|---|---|---|---|---|---|---|---|---|
| HEDONO | 1 | -6.794 | -5.083 | -4.641 | -4.641 | 4.293 | 4.293 | 4.293 | 4.293 |
| HEDONO | 2 | -6.151 | -8.896 | -5.617 | -5.617 | 5.195 | 5.195 | 5.195 | 5.195 |
| HEDONO | 3 | -1.679 | -1.679 | -5.922 | -0.612 | 1.553 | 1.553 | 1.553 | 1.553 |
| HEDONO | 4 | -0.274 | -0.274 | -0.100 | -6.937 | 0.253 | 0.253 | 0.253 | 0.253 |
| HEDONR | 5 | 2.480 | 2.480 | 2.480 | 2.480 | -5.834 | -2.942 | -0.841 | -0.841 |
| HEDONR | 6 | 0.932 | 0.932 | 0.932 | 0.932 | -1.106 | -5.523 | -0.316 | -0.316 |
| HEDONR | 7 | 9.392 | 9.392 | 9.392 | 9.392 | -3.185 | -3.185 | -8.051 | -6.218 |
| HEDONR | 8 | 1.369 | 1.369 | 1.369 | 1.369 | -0.464 | -0.464 | -0.906 | -3.509 |
| INC_OWN | 1 | -1.199 | -1.199 | -1.199 | -1.199 | 0.875 | 0.875 | 0.875 | 0.875 |
| INC_SFM | 2 | -0.418 | -0.418 | -0.289 | -0.289 | 0.107 | 0.107 | 0.346 | 0.346 |
| INC_14R | 3 | 1.121 | 1.859 | 0.942 | 2.327 | -1.063 | -0.022 | -1.129 | -0.427 |
| SUM INCOME | | -0.496 | 0.242 | -0.546 | 0.840 | -0.081 | 0.960 | 0.091 | 0.793 |

(2) POOLED SAMPLE USA:

| Variable | Alt. | MS(1) | MS(2) | MS(3) | MS(4) | MS(5) | MS(6) | MS(7) | MS(8) |
|---|---|---|---|---|---|---|---|---|---|
| HEDONO | 1 | -2.363 | 0.446 | 1.562 | 1.562 | 1.415 | 1.415 | 1.415 | 1.415 |
| HEDONO | 2 | 1.581 | -3.247 | 5.542 | 5.542 | 5.020 | 5.020 | 5.020 | 5.020 |
| HEDONO | 3 | 0.097 | 0.097 | -6.445 | -2.905 | 0.088 | 0.088 | 0.088 | 0.088 |
| HEDONO | 4 | 0.085 | 0.085 | -2.523 | -7.189 | 0.077 | 0.077 | 0.077 | 0.077 |
| HEDONR | 5 | 1.109 | 1.109 | 1.109 | 1.109 | -7.588 | -3.367 | -1.589 | -1.589 |
| HEDONR | 6 | 0.786 | 0.786 | 0.786 | 0.786 | -2.387 | -9.107 | -1.126 | -1.126 |
| HEDONR | 7 | 2.059 | 2.059 | 2.059 | 2.059 | -2.950 | -2.950 | -10.701 | -0.344 |
| HEDONR | 8 | 0.133 | 0.133 | 0.133 | 0.133 | -0.191 | -0.191 | -0.022 | -12.564 |
| INC_OWN | 1 | 0.135 | 0.135 | 0.135 | 0.135 | -0.452 | -0.452 | -0.452 | -0.452 |
| INC_SFM | 2 | 0.392 | 0.392 | -2.506 | -2.506 | -0.433 | -0.433 | -1.617 | -1.617 |
| INC_14R | 3 | -0.744 | 1.015 | -1.281 | 0.339 | -1.470 | -0.480 | -1.685 | 0.482 |
| SUM INCOME | | -0.217 | 1.542 | -3.652 | -2.031 | -2.356 | -1.366 | -3.754 | -1.587 |

Elasticities at Sample Means and Frequencies, WESML-Weighted.
Parameters from Nested Model T-U-S.
MS(i) denotes the market share of housing alternative i.

Indices of Alternatives:

| 1: | 2: | 3: | 4: | 5: | 6: | 7: | 8: |
|---|---|---|---|---|---|---|---|
| O_SF_14 | O_SF_5+ | O_MF_14 | O_MF_5+ | R_SF_14 | R_SF_5+ | R_MF_14 | R_MF_5+ |

interpreted in the following way: if rental housing prices of one category go up, substitution goes more likely into owner-occupancy than into other rental housing. In owner-occupied housing, we observe different patterns: As contrasted to the United States, cross substitution within the owner market occurs in Germany. The magnitudes of the elasticities are similar, the Germans are more price responsive as homeowners, the Americans as renters, reflecting the higher propensity of Americans to choose an owner-occupied home even at a slightly higher price.

The income elasticities are composed of the elasticities with respect to each of the three income/choice-dimension interactions. They are tabulated at the bottom of each block in Table 5-17. The income elasticities reiterate the "American Dream" interpretation just given for the price elasticities and they, of course, reflect the preference pattern for an additional dollar income discovered above. A rise in permanent income in Germany will increase the consumption of large units independent of tenure and structure type, whereas in the United States the proportion of renters will decrease unambiguously and the income will be spent on large single-family homes. Income is measured against all other goods. Thus, the sum of all income elasticities reveals that housing as a composite commodity is a normal good in Germany, but an inferior good in the United States.

We now compare the basic price and income responses across strata, using the elasticities tabulated in Table 5-16. The differences between the two countries which we detected in the pooled sample hold also in each of the strata, giving confidence in our results. In general, the suburban households are less price responsive than those living in the city centers, but more income elastic. The differential in price responsiveness is very strong in both countries, the difference in income sensitivity only in the United States. All estimates show a life cycle dependence with again reversed roles of price and income. The young and elderly strata are the most price responsive, but the least income sensitive age group. These age and location patterns are remarkably similar for both countries.

5.7.3 Sensitivity Analysis

The results presented in this section rest on a fairly extensive set of intermediate calculations with numerous assumptions. Except for age, none of the variables used in the final demand equation are taken directly from the raw data. Thus, it is appropriate to briefly report some results from alternative specifications which are displayed in Table 5-18.

The use of current instead of permanent income slightly improves the fit in Germany, slightly worsens the fit in the United States. The price and crowding variables are virtually unaffected. Income coefficients sharply decrease in the United States. This is the expected direction, but the magnitude is unreasonably large. In Germany, there is little change. No change occurs in the order of preferences. All things considered, the use of current versus permanent income does not change our results qualitatively, and the quantities of the hedonic rent variables are robust to this change.

We performed another series of experiments to test the models' sensitivity with respect to the technique of imputing the attributes of hypothetical alternatives. Instead of constructing a not chosen alternative with the same number of rooms as the chosen one, we simply assigned it the mean attributes of this choice, including the mean number of rooms. This improves the fit and increases the magnitudes of most price and income coefficients, see Table 5-18. These changes are small. However, there is a danger of creating spurious

TABLE 5-18: SENSITIVITY ANALYSIS

MNL-Estimates, Pooled Samples.

(1) CURRENT VERSUS PERMANENT INCOME:

| | GERMANY: | | USA: | |
|---|---|---|---|---|
| Variable: | YPERM | YCURR | YPERM | YCURR |
| HEDONO | -2.07 | -2.12 | -1.38 | -1.18 |
| HEDONR | -2.42 | -2.49 | -2.56 | -2.57 |
| RETURN | -0.055 | -0.053 | 0.054 | -0.012 |
| CROWDS | -0.098 | -0.095 | -0.19 | -0.18 |
| INC_OWN | -0.0045 | 0.041 | 0.176 | 0.025 |
| INC_SFM | -0.032 | -0.059 | 0.067 | 0.0048 |
| INC_14R | -0.089 | -0.063 | -0.182 | -0.053 |
| RHO_SQ | 0.466 | 0.476 | 0.642 | 0.630 |
| %CORRECT | 67.57% | 68.21% | 76.89% | 75.31% |

(2) CONSTANT ROOM VERSUS AVERAGE NUMBER OF ROOMS:

| | GERMANY: | | USA: | |
|---|---|---|---|---|
| Variable: | CONSTANT | AVERAGE | CONSTANT | AVERAGE |
| HEDONO | -2.07 | -2.22 | -1.38 | -1.02 |
| HEDONR | -2.42 | -2.51 | -2.56 | -2.31 |
| RETURN | -0.055 | -0.060 | 0.054 | 0.015 |
| CROWDS | -0.098 | 0.015 | -0.19 | -0.11 |
| INC_OWN | -0.0045 | 0.029 | 0.176 | 0.035 |
| INC_SFM | -0.032 | -0.044 | 0.067 | -0.010 |
| INC_14R | -0.089 | -0.069 | -0.182 | -0.052 |
| RHO_SQ | 0.466 | 0.482 | 0.642 | 0.577 |
| %CORRECT | 67.57% | 70.52% | 76.89% | 68.90% |

elasticities when we specify hypothetical alternatives unlikely to be relevant for the given household. In particular, the average number of rooms may be too large for a small, and too large for a small household.

This relates to the issue of what the correct discrete categories are. There are obviously more than eight different housing choices, and picking a relatively small number of alternatives means an implicit aggregation procedure of the underlying elemental alternatives. In a related study by Behring, Goldrian, et. al. (1983), we started from a large set of elementary alternatives and used different sets of aggregated alternatives and different aggregation weights. The results turned out to be very sensitive to the pooling of unrelated alternatives, in particular, spurious elasticities can be created by posing irrelevant hypothetical alternatives. The variable CROWDS is particular sensitive to the bundling of alternatives which is obvious given its definition depending on the number of rooms. In theory, there is a unique correct aggregation. This is the extension of the tree down to all elemental alternatives, and the corresponding estimation of the dissimilarity parameters bundling the elemental alternatives provides the correct weights for aggregation. In practice, the number of elemental alternatives is much too large to make this approach feasible. Given clear cut aggregates, however, the results were fairly robust to the choice of aggregation weights.

## 5.8 Simulations with Tax Laws and Preferences

The discussion of descriptive statistics in Section 5.2 revealed substantial differences in the market shares of housing alternatives between the United States and West Germany, markedly in the tenure choice. How can they be explained? The endogenous variables -- predicted market shares for each tenure-structure-size combination -- are generated by the estimated parameters and the set of explanatory variables. We discussed the differences in the exogenous variables between the two countries in Section 5.2: the income and age distribution differ; a most stunning discrepancy occurs in the prices due to a greater land scarcity in Germany; and, a very advantageous tax treatment of homeownership in the United States. Given these differences in the explanatory variables, we still discovered a variety of differences in the estimates of the taste parameters, that is in the preferences of the German and the American households for different housing choices.

The normalization of all data to a common standard allows us to predict each country at the other countries tax laws or preferences. Thus, we can distinguish the consequences of differences in preferences from those in taxes or other exogenous variables as age and income. Table 5-19 presents the results. The first column denotes the baseline market shares of each of our eight housing alternative. These shares coincide with the actual shares in the population, because the saturation of the demand model allows us to produce a perfect fit of the market shares. This procedure is

TABLE 5-19: PREDICTIONS: PREFERENCES AND TAX LAWS

(1) GERMAN DATA

| Alternative: | German Baseline: | US-Preferences: | US-Tax Laws: | US-Tax Laws/Equity: |
|---|---|---|---|---|
| O_SF_14 | 0.2108 | 0.0680 | 0.3685 | 0.3352 |
| O_SF_5+ | 0.1591 | 0.3298 | 0.3901 | 0.3540 |
| O_MF_14 | 0.0462 | 0.0005 | 0.0969 | 0.0925 |
| O_MF_5+ | 0.0058 | 0.0005 | 0.0236 | 0.0221 |
| OWN: | 0.4219 | 0.3988 | 0.8791 | 0.8038 |
| R_SF_14 | 0.1016 | 0.1177 | 0.0124 | 0.0245 |
| R_SF_5+ | 0.0250 | 0.0731 | 0.0015 | 0.0036 |
| R_MF_14 | 0.4096 | 0.3858 | 0.1018 | 0.1592 |
| R_MF_5+ | 0.0420 | 0.0247 | 0.0053 | 0.0090 |
| RENT: | 0.5782 | 0.6012 | 0.1209 | 0.1962 |

(1) US-AMERICAN DATA:

| Alternative: | US-Baseline: | German Preferences: | German Tax Laws: | German Tax Laws/Equity: |
|---|---|---|---|---|
| O_SF_14 | 0.2453 | 0.3380 | 0.2388 | 0.2387 |
| O_SF_5+ | 0.5063 | 0.1352 | 0.4462 | 0.4460 |
| O_MF_14 | 0.0112 | 0.2990 | 0.0102 | 0.0102 |
| O_MF_5+ | 0.0073 | 0.0223 | 0.0100 | 0.0099 |
| OWN: | 0.7701 | 0.7945 | 0.7052 | 0.7048 |
| R_SF_14 | 0.0720 | 0.0760 | 0.0964 | 0.0964 |
| R_SF_5+ | 0.0321 | 0.0225 | 0.0416 | 0.0416 |
| R_MF_14 | 0.1193 | 0.0999 | 0.1436 | 0.1437 |
| R_MF_5+ | 0.0063 | 0.0071 | 0.0132 | 0.0132 |
| RENT: | 0.2299 | 0.2055 | 0.2948 | 0.2952 |

All predictions with NMNL-model T-U-S, adjusted to fit aggregate shares exactly. Shares may not add up because of rounding.

discussed in Section 2.3.

We performed three experiments: first, we predicted the German data with American preferences; second with the U.S. tax code; finally with the U.S. tax code in combination with the loan-to-value ratios observed in the United States. Then, we performed the analogous experiments on the American data base. These experiments represent a drastic interference with the steady state given by the cross sectional data. Therefore, they provide only qualitative guidelines and the quantities should not be taken literally.

Predicting either country with the other country's preferences does not change the tenure choice very much, nor are the proportions within the renter alternatives affected. However, there is a strong shift within the owner categories: large single-family homes dominate the American taste, whereas German preferences give a large share to condominiums and small one or two-family units. We may interpret this finding as a confirmation that the "American Dream" in fact is an important element of housing preferences in the United States.

The next experiments predict each countries housing consumption at its own preferences, but the other country's tax code. This produces a drastic shift into ownership in Germany, and only a much smaller corresponding shift towards rental housing in the United States. This can be explained by the peculiarities of the tax laws and by the discrepancy in the price of land. Germany has very high land prices in addition to higher structure costs. With the U.S. tax code, Germans can deduct a much higher proportion of their income than Americans do. In turn, the German tax code is most unfavorable to high house values relative to the American tax code, but the tax advantages are comparable to the United States for low priced structures and lots which are dominant in the United States. Thus, the change here is much smaller than the corresponding change in Germany.

Finally, we may argue that the loan-to-value ratios are endogenous and respond to the tax changes introduced in the previous paragraph. Predicting each country at the other country's tax code and equity pattern produces the last column of Table 5-19. The German tax advantages from homeownership are independent of the loan-to-value ratio, so for the Americans living under German tax law little changes. The only changes come through the change in opportunity costs of equity. However, as estimated for the Americans, the RETURN coefficient is small and insignificant. In Germany, we observe two effects. Raising the loan-to-value ratio makes the American tax deductions even more valuable. However, the same time we decrease the return from homeownership by increasing the opportunity costs. Both effects depend on the marginal tax rate. If this is zero, the additional deductions have no value, but the opportunity costs are counted fully, leading to a strong effect away from homeownership. If on the other extreme the marginal tax rate is one, all additional interest can be deducted, but the return from an alternative investment would be taxed away, making investment into housing very attractive. We estimated a positive coefficient of RETURN in the

German sample. With the high proportion of elderly homeowners with a low marginal tax rate in Germany the first mechanism outweighs the second, and we observe a decline in homeownership as compared to U.S. tax laws combined with German equity patterns.

## 5.9 Conclusions

The main conclusion from both estimation and simulation is the confirmation of the often quoted "American Dream": Americans have a strong preference for large single-family homes, independent from the advantages granted by the U.S. tax code. Germans do not share this dream in this extent: they prefer owning to renting, but they consider large rental units attractive as well.

Second, Americans are highly responsive to price changes in rental housing, less so in owner-occupied housing. Germans have the reverse pattern: their elasticity of choosing owner-occupied housing with respect to prices exceeds the corresponding elasticity to rent housing units.

Third, the market shares of housing alternatives can neither be explained by the preferences nor by the tax laws alone.

Fourth, an American-style tax law would have drastic effects in Germany because such a tax code has especially favorable consequences for the after tax costs of owner-occupancy, given the high land prices and structure costs in Germany.

Finally, we observe remarkable similarities as well: the pattern of price and income responsiveness by location and age neatly coincides in the two countries. The strata of the suburbs and of the middle aged couples are more income responsive but less price sensitive than households in the city centers or of younger and older age who react more on price changes relative to income changes. In both countries, the same hierarchical choice pattern describes the actual behavior best. Tenure choice is the primary dimension to categorize housing choice. The choice of structure type and dwelling size follow with no clear order in both countries.

On a more technical level, we conclude that in both countries the nesting of choices and the stratification of the sample is a rewarding method to achieve good estimation results. We succeeded in similar satisfactory ex post prediction accuracies and high likelihood ratio statistics in the United States and West Germany.

## 5.10 Footnotes

(1) These and all following summary statistics for nationwide U.S. housing consumption are quoted from Annual Housing Survey 1977, Part A: General Housing Characteristics, United States and Regions, U.S. Departments of Commerce and of Housing and Urban Development.

(2) These and all following summary statistics on housing choice, income, and age distribution are quoted from 1%-Wohnungsstichprobe 1978, Heft 5: Wohnungsversorgung der Haushalte und Familien, Statistisches Bundesamt, Wiesbaden.

(3) We first deflate 1978 DM into 1977 DM, then apply the exchange rate between DM and Dollars of 1977 as reported in the Statistical Abstract of the United States, 101st. Edition (1980), pages 488 and 927.

(4) These and all following summary statistics on rent levels are quoted from 1%-Wohnungsstichprobe 1978, Heft 5: Wohnungsversorgung der Haushalte und Familien, Statistisches Bundesamt, Wiesbaden.

(5) Own tabulations from the One Percent Sample.

(6) The Urban Land Institute, Land Use Digest, Volume 14, No. 12, December 1981 (Single Family Improved Lot Price Median), and unpublished data, provided by the National Association of Home Builders.

(7) Unpublished disaggregate tabulations for 1978, provided by the West German Bundesministerium fuer Finanzen.

(8) See equation (2.2) in Chapter Two.

## APPENDIX

## FORTRAN Program for Nested Multinomial Logit Models

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     ******        ******
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   *******          *******
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```
C-----GQTREE-----------------------------------------------------------
C
C     GRADX+DFP FOR THREE LEVEL TREE
C     ================================
C     ©
C     AXEL BOERSCH-SUPAN       VERSION MARCH 1984
C
C----------------------------------------------------------------------
C
C     THIS PROGRAM ANALYSES A NESTED MULTINOMIAL LOGIT MODEL OR
C     HIERARCHICAL DECISION TREE UP TO THREE LEVELS.
C
C     AS AN EXAMPLE, CONSIDER THE FOLLOWING "TREE" :
C
C                       |                 LEVEL 0   (STEM)
C                 +-----+-----+
C                 |           |           LEVEL 1   (LIMBS)
C               +-+      +----+----+
C               |        |         |      LEVEL 2   (BRANCHES)
C              +-+    +-+-+-+     +-+-+
C              | |    | | | |     | | |   LEVEL 3   (TWIGS)
C
C     CHARACTERIZED BY THE LIKELIHOOD FUNCTION
C
C     L = LOG( P(T|B,L) * P(B|L) * P(L) )
C
C     WHERE:
C
C     T=CHOSEN TWIG, B=CHOSEN BRANCH, L=CHOSEN LIMB
C
C     P(T|B,L) = EXP( X(T,B,L)*BETA/THETA(B,L) ) / EXP( INC(B,L) )
C
C        WITH: INC(B,L) = LOG SUM EXP( X(T',B,L)*BETA/THETA(B,L) )
C                             T'C B
C
C     P(B|L) = EXP( INC(B,L)*THETA(B,L)/TAU(L) ) / EXP( INC(L) )
C
C        WITH: INC(L) = LOG SUM EXP( INC(B',L)*THETA(B',L)/TAU(L) )
C                           B'C L
C
C     P(L) = EXP( INC(L)*TAU(L) ) / EXP( INC(S) )
C
C        WITH: INC(S) = LOG SUM EXP( INC(L')*TAU(L') )
C                           L'C S
C
C---------------------------------------------------------------------
C
C     IN PARTICULAR, THE PROGRAM
C
C     - DEFINES THE TREE STRUCTURE
C     - READS THE DATA, CALCULATES SAMPLE MEANS, AND GENERATES DUMMIES
C     - WEIGHTS OBSERVATIONS IF CHOICE BASED SAMPLED (WESML)
C     - MAXIMIZES THE LIKELIHOOD FUNCTION (*)
C     - PREDICTS SHARES AND COMPUTES PREDICTION SUCCESS TABLE
```

```
C     - CALCULATES THE COVARIANCE MATRIX, CORRECTS FOR WESML ESTIMATION
C     - ADJUSTS ALTERNATIVE SPECIFIC DUMMIES TO FIT AGGREGATE SHARES (*)
C     - PLOTS 2-DIM. PARAMETER SPACES (*)
C     - CALCULATES ELASTICITIES
C     - TRANSFORMS PARAMETERS INTO MNL-FORMAT
C     - CHECKS DERIVATIVES
C     - CALCULATES LM-STATISTICS OF THE TREE PARAMETERS
C     - CHECKS COMPATIBILITY WITH RANDOM UTILITY MAXIMIZATION
C
C     FOR (*) THE PROGRAM USES THE GOLDFELD/QUANDT GQOPT-PACKAGE
C
C-----------------------------------------------------------------------
C
C     THE PROGRAM-STRUCTURE WITH ITS LEVELS IS AS FOLLOWS:
C
C     0   GQTREE           MAIN: DEFINES DIMENSIONALITY
C     1   TASKS            CALLING PROGRAM FOR TASKS OF GQTREE
C     1,3 UPDATE           UPDATES PARMS IN COMMON: THETA, TAU, PMASK
C     2   INDATA           READS DATA IN AND CALCULATES SAMPLE STATISTICS
C     2   PUNCH,CNTR,      OTHER GQOPT-ROUTINES
C         OPTMOV,MATEV2
C     2   TRAFO,AIRUM,     PARAMETERTRANSFORMATION, AIRUM-CHECK,
C         LMSTAT,ELAST,    LM-STATISTIC, ELASTICITIES,
C         WESML            WESML-COVARIANCE CALCULATION
C     2   FDIFF,SDIFF      FIRST AND SECOND FINITE DIFFERENCES
C     3   INV              MATRIX-INVERSION
C
C     2   OPT              OPTIMIZER BY GOLDFELD/QUANDT
C     3,2 FUNC,FP,SP       EVALUATION OF LIKELIHOOD, GRADIENT, HESSIAN
C     4,2 CONT             CONTRIBUTION OF THE N-TH OBS TO LIKELIHOOD
C
C     4,3 LSFIT            LEAST-SQUARES FIT OF AGGR. PROB. SHARES
C     5,4 SSQDEV,FPSSQD    EVALUATION OF PROB. SHARES, THEIR GRADIENTS
C     6,5 CONTPS           CONTRIBUTION OF THE N-TH OBS TO PROB. SHARES
C
C-----------------------------------------------------------------------
C
C     THERE ARE TWO INPUT FILES:
C
C     "STRATUM":
C     FI 7    DATA (DEPENDENT AND EXPLANATORY VARIABLES BY ALTERNATIVE)
C
C     "INPARMS":
C     FI 20   DIMENSIONS, TREE STRUCTURE, PARAMETER-LABELS,
C             INITIALVALUES
C
C     (THE TERMINAL WILL PROMPT YOU FOR THE ACTUAL FILE-NAMES)
C
C-----------------------------------------------------------------------
C
C     INPUT FILE 7:
C     -------------
C
C     READING AND FORMATTING IS DONE IN SUBROUTINE INDATA.
```

```
C
C     DATA1 CONTAINES THE ALTERNATIVE-SPECIFIC VARIABLES, WHICH VARY BY
C           HOUSEHOLD AND BY ALTERNATIVE,
C     DATA2 CONTAINES THE AGENT-SPECIFIC VARIABLES, WHICH VARY ONLY BY
C           HOUSEHOLD. THEY HAVE TO INTERACT WITH A SET OF ALTERNATIVE
C           SPECIFIC DUMMIES.
C
C     EXAMPLE WITH STANDARD SET OF DUMMIES:
C     1)  THE LEFT-HAND-SIDE VARIABLE C, WHICH TAKES ONLY THE VALUES 1
C         FOR CHOSEN, 0 OTHERWISE.
C     2)  TWO ALTERNATIVE-SPECIFIC EXPL. VARIABLES  P AND R.
C     3)  ONE AGENT-SPECIFIC VARIABLE Y, WHICH WILL INTERACT WITH FOUR
C         ALTERNATIVE SPECIFIC DUMMIES.
C
C     FOR NOBS=2 AND NALT=5:
C         ----DATA1----  DATA2        WILL BE USED AS :
C         C11  P11  R11   Y1          C11  P11  R11  Y1   0    0    0
C         C12  P12  R12               C12  P12  R12  0    Y1   0    0
C         C13  P13  R13               C13  P13  R13  0    0    Y1   0
C         C14  P14  R14               C14  P14  R14  0    0    0    Y1
C         C15  P15  R15               C15  P15  R15  0    0    0    0
C         C21  P21  R21   Y2          C21  P21  R21  Y2   0    0    0
C         C22  P22  R22               C22  P22  R22  0    Y2   0    0
C         C23  P23  R23               C23  P23  R23  0    0    Y2   0
C         C24  P24  R24               C24  P24  R24  0    0    0    Y2
C         C25  P25  R25               C25  P25  R25  0    0    0    0
C
C     NOTE: THE EXCLUDED DUMMY IS ALWAYS THE LAST ALTERNATIVE
C
C     NOTE: THE PACKAGE USES THE IDALT/IDCASE-STRUCTURE OF QUAIL.
C     NALT REFERS TO QUAIL IDALT, NOBS REFERS TO QUAIL IDCASE.
C
C-----------------------------------------------------------------------
C
C     INPUT FILE 20:
C     --------------
C
C     THIS FILE CONTAINS THE FUNCTIONAL FORM OF THE DECISION-TREE,
C     I. E., THE NUMBER AND ARRANGEMENT OF THE ALTERNATIVES IN THE TREE,
C     THE NUMBER OF EXOGENOUS VARIABLES, AND THE INITIALVALUES FOR THE
C     PARAMETERS.
C
C
C     A) ELEMENTAL ALTERNATIVES:
C
C        A1) NALT, NALTFR, RELSIZ
C
C            WHERE NALT  =NUMBER OF ELEMENTAL ALTERNATIVES
C                  NALTFR=NUMBER OF ALTERNATIVES IN A LARGER FRAME, FROM
C                         WHICH NALT ALTERNATIVES WERE CONSOLIDATED.
C                         IF IRRELEVANT, SET TO ZERO.
C                  RELSIZ=RATIO OF MAIN SAMPLE SIZE OVER AUXILIARY SAMPL
E
C                         SIZE. SET TO ZERO FOR EXOGENOUS SAMPLING. SET
```

```
C                           TO ONE IF AUXILIARY INFORMATION IS EXACT.
C
C          A2) ONLY FOR NALTFR>NALT:
C              (IAKTIV(I),I=1,NALTFR)
C              (PROBFR(I),I=1,NALTFR)
C              (JAKTIV(J),J=1,NALT)
C
C              WHERE IAKTIV=INDEX OF NALT ALTERNATIVES IN NALTFR
C                    PROBFR=RELATIVE WEIGHTS OF ALTERNATIVES
C                    JAKTIV=INVERS OF IAKTIV
C
C          A3) ONLY FOR RELSIZ > 0.0
C              (WEIGHT(J),J=1,NALT)
C
C              WHERE WEIGHT=RATIO OF AUXILIARY SAMPLE SHARE OVER MAIN
C                           SAMPLE SHARE OF RESP. ALTERNATIVE
C
C
C       B) EXPLANATORY VARIABLES:
C
C          B1) NX,NXD,NXM,NXA
C
C          WHERE NX = # OF ALTERNATIVE-SPECIFIC VARIABLES AT LEVEL 3
C                      (MIMICRY LEVEL 1 AND 2 VARIABLES AT LEVEL 3)
C                NXD= # OF AGENT-SPECIFIC VARIABLES
C                NXM= SET OF DUMMIES FOR THE AGENT-SPECIFIC VARIABLES
C                NXA= SET OF ALTERNATIVE SPECIFIC CONSTANTS
C
C          THERE ARE TWO DIFFERENT POSSIBILITIES TO DUMMY AGENT-SPECIFIC
C          VARIABLES AND TO SET PURE ALTERNATIVE-SPECIFIC CONSTANTS:
C
C          (1) "STANDARD-DUMMIES"
C              NALT-1 DUMMIES WILL BE ASSIGNED TO EACH ELEMENTAL ALTER-
C              NATIVE EXCEPT THE LAST. SEE EXAMPLE ABOVE.
C
C          (2) "TREE-DUMMIES"
C              FOR EACH LEVEL OF THE TREE, NL1-1, NL2-1, NL3-1 DUMMIES
C              WILL BE ASSIGNED. NOTE: THIS IS ONLY POSSIBLE FOR A SYM-
C              METRIC UNDERLYING TREE (I.E., NL2=CONSTANT, NL3=CONSTANT,
C              SEE BELOW). THIS IMPLIES AN ASSIGNMENT OF LESS THAN NALT-1
C              DUMMIES TO THE ALTERNATIVES, WHERE IN TURN EACH ALTERNATIVE
C              CAN BE ASSIGNED UP TO THREE DUMMIES. THIS IS CONTROLLED BY
C              THE MAP IN B2.
C
C          EIGHT COMBINATIONS ARE POSSIBLE:
C
C          (1) AGENT-SPECIFIC VARIABLES INTERACT WITH STANDARD-DUMMIES,
C              NO ALTERNATIVE SPECIFIC CONSTANTS:
C              SET   NXD > 0, NXM = 0, NXA = 0
C
C          (2) AGENT-SPECIFIC VARIABLES INTERACT WITH STANDARD-DUMMIES,
C              FULL SET OF NALT-1 ALTERNATIVE SPECIFIC CONSTANTS:
C              SET   NXD > 0, NXM = 0, NXA = 1
C
```

```
C        (3) AGENT-SPECIFIC VARIABLES INTERACT WITH TREE-DUMMIES,
C            NO ALTERNATIVE SPECIFIC CONSTANTS:
C            SET   NXD > 0, NXM = # OF DUMMIES, NXA = 0
C
C        (4) AGENT-SPECIFIC VARIABLES INTERACT WITH TREE-DUMMIES,
C            FULL SET OF NALT-1 ALTERNATIVE SPECIFIC CONSTANTS:
C            SET   NXD > 0, NXM = # OF DUMMIES, NXA = 1
C
C        (5) AGENT-SPECIFIC VARIABLES INTERACT WITH TREE-DUMMIES,
C            SET OF ALTERNATIVE SPECIFIC CONSTANTS AS TREE-DUMMIES:
C            SET   NXD > 0, NXM = # OF DUMMIES, NXA = 2
C
C        (6) NO AGENT-SPECIFIC VARIABLES,
C            NO ALTERNATIVE SPECIFIC CONSTANTS:
C            SET   NXD = 0, NXM = 0, NXA = 0
C
C        (7) NO AGENT-SPECIFIC VARIABLES,
C            FULL SET OF NALT-1 ALTERNATIVE SPECIFIC CONSTANTS:
C            SET   NXD = 0, NXM = 0, NXA = 1
C
C        (8) NO AGENT-SPECIFIC VARIABLES,
C            SET OF ALTERNATIVE SPECIFIC CONSTANTS AS TREE-DUMMIES:
C            SET   NXD = 0, NXM = # OF DUMMIES, NXA = 2
C
C        IF NXA IS SET TO -1 RATHER THAN TO 1, STANDARD-DUMMIES ARE NOT
C        ESTIMATED FREELY TO MAXIMIZE THE LIKELIHOOD-FUNCTION, BUT ARE
C        SET AFTER EACH ITERATION TO MATCH THE AGGREGATE SAMPLE SHARES.
C        THE PROCEDURE, AS IMPLEMENTED, IS NOT GUARANTEED TO BE
C        NUMERICALLY STABLE AND IS NOT NECESSARILY CONVERGENT.
C        HOWEVER, IF THE PROCEDURE DOES CONVERGE, THE ESTIMATES ARE
C        CONSISTENT AND EFFICIENT, BEING EQUIVALENT TO COSLETT'S
C        CONCENTRATED ESTIMATOR. THE SAMPLE SHARES CAN BE MATCHED BY A
C        NONLINEAR LEAST SQUARES PROCEDURE AFTER SUCCESSFUL LIKELIHOOD
C        MAXIMIZATION; FOR THIS, SET NXA = 1 AND SELECT THE ADJUST TASK.
C
C        THE CORRESPONDENCE BETWEEN ALTERNATIVES AND A SET OF
C        TREE-DUMMIES IS DEFINED BY THE FOLLOWING MAP:
C
C        B2) ONLY FOR NXM > 0:
C            (MAP(  1  ,J),J=1,3)
C            ...
C            (MAP(NALT,J),J=1,3)
C
C            WHERE MAP(I,J)=M IMPLIES THAT TREE DUMMY M IS PRESENT IN
C                             ELEMENTAL ALTERNATIVE I. UP TO J=3 DUMMIES
C                             CAN BE PRESENT. IF LESS THAN 3 DUMMIES ARE
C                             ASSIGNED, FILL THE REMAINING MAP(I,J) WITH
C                             ZEROES.
C
C     C) TREE STRUCTURE
C
C        C1) NL1
C            NL2(I),I=1,NL1
C            NL3( 1 ,J),J=1,NL2( 1 )
```

```
C           ...
C           NL3(NL1,J),J=1,NL2(NL1)
C
C           WHERE NL1 = # OF LIMBS (E.G., SEE FIGURE AT TOP: 2),
C                 NL2 = # OF BRANCHES FOR EACH LIMB (E.G.: 1,2),
C                 NL3 = # OF TWIGS FOR EACH BRANCH OF EACH LIMB,
C                       (E.G.: 2 FOR LIMB 1 AND 4,3 FOR LIMB 2)
C
C       C2) NTAU, NTAU1, (LOCTAU(I), I=1,NTAU+NTAU1)
C
C           WHERE NTAU    = # OF LEVEL 1 DISS. PARAMETERS (TAU),
C                           TO BE OPTIMIZED OVER ("FREE"),
C                 NTAU1   = # OF LEVEL 1 DISS. PARAMETERS (TAU),
C                           TO BE EQUALITY CONSTRAINED,
C                 LOCTAU = RELATIVE LOCATION (LIMB) OF THE TAU'S.
C
C       C3) NTH , NTH1 , (LOCLIM(I), LOCBRA(I), I=1,NTH+NTH1)
C
C           WHERE NTH     = # OF LEVEL 2 DISS. PARAMETERS (THETA),
C                           TO BE OPTIMIZED OVER ("FREE"),
C                 NTH1    = # OF LEVEL 2 DISS. PARAMETERS (THETA),
C                           TO BE EQUALITY CONSTRAINED,
C                 (LOCLIM,LOCBRA) = RELATIVE LOCATION (LIMB,BRANCH)
C                           OF THE THETA'S.
C
C       NOTE: THE LOCATIONS OF FREE DISS.PARMS HAVE TO APPEAR FIRST IN
C       LOCTAU, LOCLIM, AND LOCBRA, FOLLOWED BY THE LOCATIONS OF THE
C       EQUALITY CONSTRAINT DISS.PARMS.
C       THERE IS NO PROVISION IN THE CONT-ROUTINE FOR INTER-NODE EQUA-
C       LITY CONSTRAINTS.
C
C       NOTE: THE PROGRAM CHECKS THE LEVEL OF THE TREE BY CHECKING
C       NTAU+NTAU1 AND NTH+NTH1.
C       IF BOTH ARE ZERO, THE TREE COLLAPSES TO A SIMPLE MNL MODEL.
C       IF ONLY NTAU+NTAU1 IS ZERO, THE TREE COLLAPSES TO TWO LEVELS.
C       IN EITHER CASE, SIMPLER FORMULAS ARE USED IN SUBROUTINE CONT.
C
C       NOTE: ALL INPUTS IN A) THROUGH C) ARE IN *-FORMAT
C
C    D) INITIAL VALUES
C
C       D1) NAME(1)        PARM(1)
C           ...
C           NAME(NP+NP1)   PARM(NP+NP1)
C
C           WHERE: NAME(I) = LABEL OF THE I'TH PARAMETER
C                  PARM(I) = INITIALVALUES OF THE I'TH PARAMETER
C                  NP      = TOTAL NUMBER OF FREE PARAMETERS
C                  NP1     = TOTAL NUMBER OF CONSTRAINT PARAMETERS
C
C       D2) A BLANK LINE TO INDICATE END OF INPUT
C
C       THE FINAL CARDS OF FILE 20 CONTAIN IN (A8,F15.9)-FORMAT THE
C       LABELS AND INITIALVALUES OF ALL THOSE PARAMETERS, OVER WHICH
```

```
C         EITHER WILL BE OPTIMIZED OR WHICH ARE EQUALITY CONSTRAINT,
C         IN THE FOLLOWING ORDER:
C
C         1)  COEFFICIENTS OF EXPLANATORY VARIABLES ON LEVEL 3 (BETA'S)
C             A)  ALTERNATIVE-SPECIFIC
C             B)  AGENT-SPECIFIC
C             C)  PURE DUMMIES (FREE OR CONSTRAINT)
C         2)  DISS. PARAMETERS OF LEVEL 2 (THETA'S)  (FREE)
C         3)  DISS. PARAMETERS OF LEVEL 1 (TAU'S)    (  " )
C         4)  DISS. PARAMETERS OF LEVEL 2 (THETA'S)  (EQ. CONSTR.)
C         5)  DISS. PARAMETERS OF LEVEL 1 (TAU'S)    (  "         )
C
C-----------------------------------------------------------------
C
C      APART FROM OUTPUT ON THE TERMINAL, THE PROGRAM GENERATES THE
C      FOLLOWING OUTPUT-FILES:
C
C      "OUTPARMS":
C      FI 19     TREE STRUCTURE AND ESTIMATION RESULTS. THIS FILE CAN BE
C                USED AS NEW TREE-INPUT FILE 20. PARTS A) THROUGH C) ARE
C                COPIED FROM "INPARMS", IN D), THE INITIALVALUES ARE RE-
C                PLACED BY THE ESTIMATED PARAMETERS, INCL. STANDARD-
C                ERRORS AND T-STATISTICS. APPENDED ARE SUMMARY-STATISTICS.
C
C      "PREDICT":
C      FI 9      DATA WITH PREDICTED CHOICES RATHER THAN OBSERVED. THIS
C                FILE CAN BE USED AS NEW DATA-INPUT FILE 7. THE INDEPEN-
C                DENT VARIABLES ARE COPIED FROM FILE 7, THE DEPENDENT
C                VARIABLE IS REPLACED BY ITS PREDICTION ACCORDING TO THE
C                MAXIMUM PROBABILITY PRINCIPLE.
C
C      (THE TERMINAL WILL PROMPT YOU FOR FILE-NAMES. IF THE FILES
C       ALREADY EXIST, THEY WILL BE OVERWRITTEN. IF YOU ENTER BLANK
C       NAMES, THE ABOVE DEFAULT NAMES WILL BE USED)
C
C-----------------------------------------------------------------
C
C      FI 18 IS USED INTERNALLY AS A VIRTUAL SCRATCH-FILE FOR DFP.
C
C-----------------------------------------------------------------
C
C      THE TERMINAL WILL PROMPT YOU FOR OPTIONS AND ITERATION PARAMETERS
C      AS THEY ARE EXPLAINED IN THE GQOPT-MANUAL.
C
C      THERE IS A MENU-DISPLAY BEFORE EACH OPERATION.
C
C      NOTE: CERTAIN OPERATIONS CAN ONLY BE DONE IN A FIXED ORDER:
C            (1) CALCULATION OF THE AVERAGE INDIVIDUAL ELASTICITIES
C            NEEDS THE PROBABILITY-SHARES FROM THE PREDICTION-OPERATION.
C            (2) FOR THE TRANSFORMATION TO MNL-FORMAT THE COVARIANCE-
C            MATRIX HAS TO BE PROVIDED.
C
C-----------------------------------------------------------------
C
```

```
C     THE PROGRAM ALWAYS USES ANALYTICAL FIRST DERIVATIVES.
C
C     THE CHOICE FOR SECOND DERIVATIVES IS GOVERENED BY IDIFF2. THEY
C     CAN BE APPROXIMATED ACCORDING TO BHHH, BY SIMPLE FINITE
C     DIFFERENCES, OR BY SYMMETRIC FINITE SECOND DIFFERENCES.
C     ONLY FOR THE SIMPLE MNL-CASE, ANALYTIC SECOND DERIVATIVES ARE
C     IMPLEMENTED.
C
C-----------------------------------------------------------------------
C
C     THE PACKAGE ALLOWS FOR THE MAXIMAL DIMENSIONS:
C
C-----------------------------------------------------------------------
C
C      PARAMETER
C     *  (MAXOBS = NUMBER OF OBSERVATIONS + 1 FOR MEANS,
C     *   MAXNP  = TOTAL NUMBER OF PARAMETERS TO OPTIMIZE OVER,
C     *   MAXTH  = 1ST ORDER DISSIMILARITY PARAMETERS AT LEVEL 2,
C     *   MAXTAU = 2ND ORDER DISSIMILARITY PARAMETERS AT LEVEL 1,
C     *   MAXNX  = ALTERNATIVE SPECIFIC EXPLANATORY VARIABLES,
C     *   MAXNXD = AGENT SPECIFIC EXPLANATORY VARIABLE,
C     *   MAXNXM = NUMBER OF TREE-DUMMIES,
C     *   MAXLEV = NUMBER OF LEVELS,
C     *   MAXLIM = LIMBS AT LEVEL 1,
C     *   MAXBRA = BRANCHES AT LEVEL 2 FOR EACH LIMB AT LEVEL 1,
C     *   MAXALT = ELEMENTAL ALTERNATIVES,
C     *   MAXFRA = ELEMENTAL ALTERNATIVES IN AN UNDERLYING LARGER FRAME)
C
C-----------------------------------------------------------------------
C
C     ALL DIMENSIONS CAN EASILY BE CHANGED BY CHANGING THESE
C     PARAMETER SETTINGS IN ALL ROUTINES THROUGHOUT THE PACKAGE.
C
C-----------------------------------------------------------------------
C
      IMPLICIT REAL*8 (A-H,O-Z)
C
    . PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXTH  =10,
     *   MAXTAU =5,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXNXM =20,
     *   MAXLEV =3,
     *   MAXLIM =5,
     *   MAXBRA =5,
     *   MAXALT =20,
     *   MAXFRA =61,
     *   MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD,
     *   MAXNPS=MAXNP*(MAXNP+1)/2, MAXALS=MAXALT*(MAXALT+1)/2,
     *   MAXSTK=5*MAXNP*MAXNP+6*MAXNP+MAXALT*MAXALT+8*MAXALT)
C
```

```
      REAL*8 PARM(MAXNP),GRAD(MAXNP),HESS(MAXNP,MAXNP),
     *       GRAD2(MAXNP),SCRA(MAXNP,MAXNP),MBHHH(MAXNPS),HESS2(MAXNPS),
     *       THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
     *       WORK(MAXNP),ZBAR(MAXNP),ZIZBAR(MAXALT,MAXNP),
     *       HINV(MAXNP,MAXNP),AAUX(MAXNP,MAXALT),BAUX(MAXALT,MAXALT),
     *       SCRAUX(MAXNP,MAXALT),QAUX(MAXALT)
      REAL*8 EXB(MAXALT),EZG(MAXALT),EYA(MAXLIM),
     *       EINC2(MAXLIM,MAXBRA),EINC1(MAXLIM),
     *       XB(MAXALT),ZG(MAXALT),YA(MAXLIM),
     *       INC2(MAXLIM,MAXBRA),INC1(MAXLIM),
     *       DER2B(MAXLIM,MAXBRA,MAXNP),DER1B(MAXLIM,MAXNP),
     *       DER2T(MAXLIM,MAXBRA,MAXTH),DER1T(MAXLIM,MAXTH),
     *       DER1U(MAXLIM,MAXTAU)
      REAL*8 PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT),
     *       PDUMMY(MAXALT),PNAME(MAXNP),NDUMMY(MAXALT),
     *       PROB1(MAXALT),PROB2(MAXALT),PROBS(MAXALT),
     *       DERIV1(MAXALT,MAXALT),DERIV2(MAXALT,MAXALT),
     *       PSUM1(MAXALT),Q3(MAXALT),
     *       PSUM2(MAXLIM,MAXBRA),Q2(MAXLIM,MAXBRA)
      REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS),DATA2(MAXNXD,MAXOBS),
     *       PSHARE(MAXALT,MAXOBS),PROBFR(MAXFRA),
     *       PROB(MAXALT),FREQ(MAXALT),ACT1(MAXALT),ACT2(MAXALT,MAXALT),
     *       ELAS(MAXALT,MAXALT,MAXNXR),ELAS2(MAXALT,MAXALT,MAXNXR)
      INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT),
     *       MS3(MAXLIM,MAXBRA),IAKTIV(MAXFRA),JAKTIV(MAXALT),
     *       NCT1(MAXALT),NCT2(MAXALT,MAXALT),
     *       IP1(MAXLIM),IP2(MAXLIM,MAXBRA),
     *       ACTTAU(MAXLIM),ACTTH(MAXLIM,MAXBRA)
      REAL *4   FCNTR(20,20)
      LOGICAL*1 PCNTR(20,20)
      LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDDUM,NOALT,NOAGE,
     *       STDUMA,STDUMD,NALSTD,NAGSTD,NALTRE,NAGTRE,UNCON
      CHARACTER*21 VERSIO
      CHARACTER*20 TRFILE
C
      COMMON / BSTACK / AINT(MAXSTK)
      COMMON / BPRINT / IPT,NPRINT,NDIG,NPUNCH
      COMMON / DPARM  / NP,PARM,GRAD,HESS
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DDISS  / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
     *                  NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
      COMMON / DATAX  / DATA1
      COMMON / DATAY  / DATA2
      COMMON / DCOVM  / MBHHH
      COMMON / DHESS  / HESS2
      COMMON / DCONT  / EXB,EZG,EYA,EINC2,EINC1,
     *                  XB,ZG,YA,INC2,INC1,
     *                  DER2B,DER1B,DER2T,DER1T,DER1U,
     *                  ZBAR,ZIZBAR,UPP,LOW,
     *                  MS3K(MAXTH),ML3K(MAXTH),MS2K(MAXLIM)
      COMMON / DSSQD  / PROB1,PROB2,DERIV1,DERIV2
      COMMON / DCNTR  / FCNTR,PCNTR
      COMMON / DMAIN  / KANAL5,LEVCHG,PNAME,NTAU1,NTH1,NXA2,
     *                  NALTFR,PROBFR,IAKTIV,JAKTIV,
```

```
     *                 NCT1,NCT2,ACT1,ACT2,IP1,IP2,ACTTAU,ACTTH,
     *                 GRAD2,SCRA,PSHARE,ELAS,ELAS2,PROB,FREQ
      COMMON / DWORK  / WORK,PROBS
      COMMON / DELAS  / PSUM1,PSUM2,Q2,Q3
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
      COMMON / DMAPP  / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
      COMMON / DDUMM  / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
     *                 NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
     *                 NALSTD,NALTRE,NAGSTD,NAGTRE
      COMMON / DCONST / UNCON,PDUMMY,NDUMMY
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
      COMMON / DWESTK / HINV,AAUX,BAUX,QAUX,SCRAUX
C
C
C     START DIALOG
C
C
      CALL START(NPRINT,KANAL5,KANAL6,UPP,LOW,VERSIO)
      WRITE (KANAL6,1) VERSIO
1     FORMAT(//' GQTREE [VERSION ',A21,']'
     *       /' ======================================'
     *      //' ENTER TREE-INPUT-FILE >')
      READ (KANAL5,'(A20)') TRFILE
C
C     READ INPUT-FILE 20 "INPARMS"
C
      OPEN (UNIT=20,STATUS='OLD',FILE=TRFILE)
C
C     READ IN NUMBER OF ALTERNATIVES AND MAP OF ALTERNATIVES, IF ANY
C
      READ (20,*) NALT,NALTFR,RELSIZ
      NALT1=NALT-1
      IF (NALT.GE.NALTFR) GOTO 5
         READ (20,*) (IAKTIV(I),I=1,NALTFR)
         READ (20,*) (PROBFR(I),I=1,NALTFR)
         READ (20,*) (JAKTIV(J),J=1,NALT)
5     IF (RELSIZ.GT.0.0) THEN
         READ (20,*) (WEIGHT(I),I=1,NALT)
         ELSE
         DO 6 I=1,NALT
6           WEIGHT(I)=1.0
         END IF
C
C     READ IN NUMBER OF EXOGENOUS VARIABLES
C
      READ (20,*) NX,NXD,NXM,NXA
      NXR=NX+NXD
      NXA2=NXA
      IF ( (NXA.GE.2.AND.NXM.EQ.0) .OR. NXA.LE.-2)
     *   WRITE (KANAL6,*) 'WARNING: CONFLICTING CHOICE OF NXA AND NXM'
      UNCON=.TRUE.
      IF (NXA.EQ.-1) THEN
                      NXA=0
                      UNCON=.FALSE.
```

```
                  END IF
C
C     SWITCHES FOR DUMMIES
C
      STDDUM=.FALSE.
      IF (NXA.EQ.0) THEN
C                   NO PURE DUMMIES TO BE ESTIMATED
                       NOALT=.TRUE.
                       NALTRE=.TRUE.
                       NALSTD=.TRUE.
                       STDUMA=.FALSE.
                       NDUMA=0
                    ELSE
                       NOALT=.FALSE.
                       IF (NXA.GE.2) THEN
C                                    PURE TREE-DUMMIES
                                        NALTRE=.FALSE.
                                        NALSTD=.TRUE.
                                        STDUMA=.FALSE.
                                        NDUMA=NXM
                                        NXA=1
                                     ELSE
C                                    PURE STD.-DUMMIES
                                        STDDUM=.TRUE.
                                        STDUMA=.TRUE.
                                        NALSTD=.FALSE.
                                        NALTRE=.TRUE.
                                        NDUMA=NALT1
                                     END IF
                    END IF
      IF (NXD.EQ.0) THEN
C                   NO AGENT-SPECIFIC VARIABLES PRESENT
                       NOAGE=.TRUE.
                       NAGTRE=.TRUE.
                       NAGSTD=.TRUE.
                       STDUMD=.FALSE.
                       NDUMD=0
                    ELSE
                       NOAGE=.FALSE.
                       IF (NXM.GT.0) THEN
C                                    AGENT SPEC.VAR. * TREE-DUMMIES
                                        NAGTRE=.FALSE.
                                        NAGSTD=.TRUE.
                                        STDUMD=.FALSE.
                                        NDUMD=NXM
                                     ELSE
C                                    AGENT SPEC.VAR. * STD.-DUMMIES
                                        STDDUM=.TRUE.
                                        STDUMD=.TRUE.
                                        NAGSTD=.FALSE.
                                        NAGTRE=.TRUE.
                                        NDUMD=NALT1
                                     END IF
                    END IF
```

```
C
C     INDEX FOR AGENT SPECIFIC VARIABLES IN PARM-VECTOR
C
      IF (.NOT.NOAGE) THEN
                        KK=NX
                        DO 10 K=1,NXD
                        DO 10 I=1,NDUMD
                           KK=KK+1
10                         INDAGE(K,I)=KK
                     END IF
C
C     READ IN MAP OF TREE-DUMMIES, IF ANY
C
      IF (NXM.EQ.0) GOTO 12
      DO 11 I=1,NALT
11       READ (20,*) (MAP(I,K),K=1,3)
C
C     READ IN TREE STRUCTURE
C
12    READ (20,*) NL1
      READ (20,*) (NL2(I),I=1,NL1)
      DO 13 I1=1,NL1
         ML2=NL2(I1)
13       READ (20,*) (NL3(I1,I2),I2=1,ML2)
C
C     READ IN NUMBER AND LOCATION OF DISSIMILARITY PARAMETERS
C
      READ (20,*) NTAU,NTAU1,(LOCTAU(J),          J=1,NTAU+NTAU1)
      READ (20,*) NTH ,NTH1 ,(LOCLIM(J),LOCBRA(J),J=1,NTH +NTH1 )
C
C     READ IN INITIAL VALUES
C
      N1=NX+NXD*NDUMD+NXA*NDUMA+NTH+NTH1+NTAU+NTAU1
      IF (.NOT.UNCON) N1=N1+NALT1
      DO 15 I=1,N1
         READ (20,16,END=17) PNAME(I),PARM(I)
16       FORMAT(A8,F15.9)
15       CONTINUE
C
17    CLOSE (UNIT=20)
C
C     REARRANGE PARM-VECTOR FOR CONSTRAINT ESTIMATION
C
20    IF (UNCON) GOTO 29
      N1=NX+NXD*NDUMD
      DO 22 I=1,NALT1
         PDUMMY(I)=PARM (N1+I)
22       NDUMMY(I)=PNAME(N1+I)
      N2=N1+NTH+NTAU+NTH1+NTAU1
      DO 24 I=N1+1,N2
         PARM (I)=PARM (NALT1+I)
24       PNAME(I)=PNAME(NALT1+I)
29    CONTINUE
C
```

```
C     LEVEL OF TREE
C
      LEVCHG=0
30    IF (NTAU+NTAU1.EQ.0) THEN
                            LEVEL2=.TRUE.
                            IF (NTH+NTH1.EQ.0) THEN
                                                 LEVEL1=.TRUE.
                                                ELSE
                                                 LEVEL1=.FALSE.
                                                END IF
                           ELSE
                            LEVEL2=.FALSE.
                            LEVEL1=.FALSE.
                           END IF
C
C     ACTUAL DIMENSIONS OF PROBLEM
C
      MXA  = NX   + NXD*NDUMD
      MTH  = MXA + NXA*NDUMA
      MTAU = MTH  + NTH
      NP   = MTAU + NTAU
C
C     CHECK DIMENSIONS
C
      IF (NX.GT.MAXNX) THEN
                       WRITE (KANAL6,*) 'NX=',NX,' > MAXNX=',MAXNX
                       GOTO 9999
                       END IF
      IF (NXD.GT.MAXNXD) THEN
                       WRITE (KANAL6,*) 'NXD=',NXD,' > MAXNXD=',MAXNXD
                       GOTO 9999
                       END IF
      IF (NXM.GT.MAXNXM) THEN
                       WRITE (KANAL6,*) 'NXM=',NXM,' > MAXNXM=',MAXNXM
                       GOTO 9999
                       END IF
      IF (NTAU.GT.MAXTAU) THEN
                       WRITE (KANAL6,*) 'NTAU=',NTAU,' > MAXTAU=',MAXTAU
                       GOTO 9999
                       END IF
      IF (NTH.GT.MAXTH) THEN
                       WRITE (KANAL6,*) 'NTH=',NTH,' > MAXTH=',MAXTH
                       GOTO 9999
                       END IF
      IF (NP.GT.MAXNP) GOTO 9999
      IF (NALT.GT.MAXALT) THEN
                       WRITE (KANAL6,*) 'NALT=',NALT,' > MAXALT=',MAXALT
                       GOTO 9999
                       END IF
      IF (NALTFR.GT.MAXFRA) THEN
                       WRITE (KANAL6,*) 'WARNING: FRAME TOO LARGE'
                       NALTFR=MAXFRA
                       END IF
      IF (NL1.GT.MAXLIM) THEN
```

```
                    WRITE (KANAL6,*) 'NL1=',NL1,' > MAXLIM=',MAXLIM
                    GOTO 9999
                    END IF
      NBRA=0
      DO 31 I1=1,NL1
         IF (NL2(I1).GT.NBRA) NBRA=NL2(I1)
31       CONTINUE
      IF (NBRA.GT.MAXBRA) THEN
                    WRITE (KANAL6,*) 'NL2=',NBRA,' > MAXBRA=',MAXBRA
                    GOTO 9999
                    END IF
C
C     INITIALIZE TAU AND THETA ARRAYS
C
      DO 32 I1=1,NL1
         TAU(I1)=1.0
         ML2=NL2(I1)
         DO 32 I2=1,ML2
32          THETA(I1,I2)=1.0
C
C     UPDATE FREE DISS. PARAMETERS
C
      CALL UPDATE(PARM,NP,*33)
C
C     UPDATE EQUALITY-CONSTRAINT DISS. PARAMETERS
C
33    DO 34 K=1,NTH1
         KK=NTH+K
         IF (LOCLIM(KK).EQ.0) GOTO 35
            THETA(LOCLIM(KK),LOCBRA(KK)) = PARM(NP+K)
34       CONTINUE
35    MTAU1=NP+NTH1
      DO 36 K=1,NTAU1
         KK=NTAU+K
         IF (LOCTAU(KK).EQ.0) GOTO 40
            TAU(LOCTAU(KK)) = PARM(MTAU1+K)
36       CONTINUE
C
C     THESE ARRAYS HELP SPEEDING UP THE LOOPS IN CONT
C
40    IB=0
      IT=0
      DO 41 I1=1,NL1
C        START FOR TAU-LOOPS IN CONT
         MS2K(I1)=IB
         IB=IB+NL2(I1)
         DO 42 K=1,NTAU
42          DER1U(I1,K)=0.D0
         DO 43 K=1,NTH
43          DER1T(I1,K)=0.D0
         DO 41 I2=1,NL2(I1)
C           START FOR THETA-LOOPS IN CONT
            MS3(I1,I2)=IT
            IT=IT+NL3(I1,I2)
```

```
            DO 41 K=1,NTH
41             DER2T(I1,I2,K)=0.D0
      DO 44 K=1,NTH
         NT11=LOCLIM(K)
         NT12=LOCBRA(K)
C        START FOR THETA-LOOPS
         MS3K(K)=MS3(NT11,NT12)
C        LENGHT OF THETA-LOOPS
44       ML3K(K)=NL3(NT11,NT12)
C
C
C     DRIVER CALLS SUPERVISOR SUBROUTINE
C
      CALL TASKS(PARM,GRAD,HESS,NP)
C     -----------------------------
C
C     NORMAL EXIT
C
      IF (LEVCHG.EQ.0) STOP
C
C
C     LEVEL-CHANGE
C
C     CHANGE LOCATION AND ACTIVENESS OF DISS.PARMS
C     USEFUL FOR LEVELWISE ESTIMATION AND FREEZING OF DISS. PARMS
C
200   WRITE (KANAL6,251)
251   FORMAT(/' ENTER NEW NTH, NTH1 >')
      READ (KANAL5,*) NTH,NTH1
      IF (NTH+NTH1.EQ.0) GOTO 250
C
      IF (NTH.EQ.0) GOTO 265
      WRITE (KANAL6,261)
261   FORMAT(' ENTER NEW LIMB,BRANCH-LOCATIONS FOR FREE THETA''S >')
      READ (KANAL5,*) (LOCLIM(I),LOCBRA(I),I=1,NTH)
      WRITE (KANAL6,*) 'INDEX AND INITIALVALUES OF FREE THETA''S:'
      DO 262 I=1,NTH
262      WRITE (KANAL6,209) I,PNAME(MTH+I),PARM(MTH+I)
C
265   IF (NTH1.EQ.0) GOTO 270
      WRITE (KANAL6,266)
266   FORMAT(' ENTER NEW LIMB,BRANCH-LOCATIONS FOR CONSTR. THETA''S >')
      READ (KANAL5,*) (LOCLIM(I),LOCBRA(I),I=NTH+1,NTH+NTH1)
      WRITE (KANAL6,*) 'INDEX AND INITIALVALUES OF CONSTR. THETA''S:'
      DO 267 I=NTH+1,NTH+NTH1
267      WRITE (KANAL6,209) I,PNAME(MTH+I),PARM(MTH+I)
C
270   WRITE (KANAL6,221)
      READ (KANAL5,*) K,ANAME,APARM
      IF (K.EQ.0) GOTO 250
         PNAME(MTH+K)=ANAME
         PARM (MTH+K)=APARM
         GOTO 270
C
```

```
250   WRITE (KANAL6,205)
205   FORMAT(/' ENTER NEW NTAU, NTAU1 >')
      READ (KANAL5,*) NTAU,NTAU1
      IF (NTAU+NTAU1.EQ.0) GOTO 290
209   FORMAT(I3,': ',A8,' = ',F9.4)
C
      IF (NTAU.EQ.0) GOTO 215
      WRITE (KANAL6,211)
211   FORMAT(' ENTER NEW LIMB-LOCATIONS FOR FREE TAU''S >')
      READ (KANAL5,*) (LOCTAU(I),I=1,NTAU)
      WRITE (KANAL6,*) 'INDEX AND INITIALVALUES OF FREE TAU''S:'
      MTAU=MTH+NTH
      DO 212 I=1,NTAU
212      WRITE (KANAL6,209) I,PNAME(MTAU+I),PARM(MTAU+I)
C
215   IF (NTAU1.EQ.0) GOTO 220
      WRITE (KANAL6,216)
216   FORMAT(' ENTER NEW LIMB-LOCATIONS FOR CONSTRAINT TAU''S >')
      READ (KANAL5,*) (LOCTAU(I),I=NTAU+1,NTAU+NTAU1)
      WRITE (KANAL6,*) 'INDEX AND INITIALVALUES OF CONSTRAINT TAU''S:'
      DO 217 I=NTAU+1,NTAU+NTAU1
217      WRITE (KANAL6,209) I,PNAME(MTAU+I),PARM(MTAU+I)
C
220   WRITE (KANAL6,221)
221   FORMAT(' ENTER INDEX, NEW NAME, AND NEW INITIALVALUE FOR CHANGE'
     *       ,' (ZEROES IF OK) >')
      READ (KANAL5,*) K,ANAME,APARM
      IF (K.EQ.0) GOTO 290
         PNAME(MTAU+K)=ANAME
         PARM (MTAU+K)=APARM
         GOTO 220
C
290   GOTO 30
C
C     ERROR EXIT
C
9999  WRITE (KANAL6,*) 'ERROR: ACTUAL DIMENSIONS EXCEED MAXIMUM'
9998  FORMAT(/' NUMBER OF PARAMETERS : '
     *        /' NX+(NXD*NDUMD)+(NXA*NDUMA)+NTH+NTAU = NP'
     *        /  1X,I2,I5,3I6,3I5)
      WRITE (KANAL6,9998) NX,NXD,NDUMD,NXA,NDUMA,NTH,NTAU,NP
      STOP
      END
      SUBROUTINE TASKS(PARM,GRAD,HESS,NP)
C-----------------------------------------------------------------------
C
C     SUPERVISOR PROGRAM FOR THE TASKS OF GQTREE.
C
C     AXEL BOERSCH-SUPAN           VERSION  MARCH 22, 1984
C
C     NOTE: DYNAMIC DIMENSION NP FOR USE IN GQOPT-ROUTINES.
C
C-----------------------------------------------------------------------
C
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP),GRAD(NP),HESS(NP,NP)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXTH  =10,
     *   MAXTAU =5,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXNXM =20,
     *   MAXLEV =3,
     *   MAXLIM =5,
     *   MAXBRA =5,
     *   MAXALT =20,
     *   MAXFRA =61,
     *   MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD,
     *   MAXNPS=MAXNP*(MAXNP+1)/2,
     *   MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS)
C
      REAL*8 WORK(MAXNP),SCRA(MAXNP,MAXNP),
     *       MBHHH(MAXNPS),THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
     *       PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT),
     *       GRAD2(MAXNP),PROBS(MAXALT),PDUMMY(MAXALT)
      REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS),DATA2(MAXNXD,MAXOBS),
     *       FDATA1(MAXDA1),FDATA2(MAXDA2),
     *       PSHARE(MAXALT,MAXOBS),PROBFR(MAXFRA),
     *       ELAS(MAXALT,MAXALT,MAXNXR),ELAS2(MAXALT,MAXALT,MAXNXR),
     *       PROB(MAXALT),FREQ(MAXALT),CH,
     *       ACT1(MAXALT),ACT2(MAXALT,MAXALT)
      INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT),
     *       NLMST(5),NCT1(MAXALT),NCT2(MAXALT,MAXALT),
     *       IP1(MAXLIM),IP2(MAXLIM,MAXBRA),
     *       IAKTIV(MAXFRA),JAKTIV(MAXALT),
     *       ACTTAU(MAXLIM),ACTTH(MAXLIM,MAXBRA)
      LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),
     *       STDUMA,STDUMD,STDDUM,NOALT,NOAGE,UNCON,
     *       NALSTD,NALTRE,NAGSTD,NAGTRE
C
      REAL*8      PNAME(MAXNP),NDUMMY(MAXALT),BLANK/8H        /
C
      CHARACTER*1 TR1(100),TR2(100),TR3(100),TR7(100),TR8(100),TR9(100),
     *            BL/' '/,VB/'|'/,CR/'+'/,DA/'-'/,FK/'F'/,CK/'C'/,
     *            DG(0:9)/'0','1','2','3','4','5','6','7','8','9'/
      CHARACTER*8 MENAME(7)/'DA.FL.PO','QUADHILL','BHHH-COV','SIMP-COV',
     *                      'SYMM-COV','ANAL-COV','ADJUSTED'/,
     *                SUMME/'SUM     '/
      CHARACTER*14 CHVER(6)/'FLETCHER      ','DFP-ORIGINAL  ',
     *                      'BROYDEN       ','RANK-1-CORR.  ',
     *                      'QUADR. HILLCL.','MODIF. HILLCL.'/
      CHARACTER*5  CHIST(0:3)/'NONE ','DLNSR','LNSR ','STRCH'/
      CHARACTER*4  CHDIF2(4)/'BHHH','SIMP','SYMM','MNL '/
      CHARACTER*8  CHSTOP(0:1)/'ENABLED ','DISABLED'/
      CHARACTER*18 CWESML(2)/' ','  (WESML-WEIGHTED)'/
```

```
      CHARACTER*20 TITLE,EMPTY/' '/
      CHARACTER*4  DASH4(MAXALT)/MAXALT*'----'/
      CHARACTER*7  DASH7(MAXALT)/MAXALT*'-------'/
C
      COMMON / BSTACK / AINT(1)
      COMMON / BSTAK  / NQ,NTOP
      COMMON / BREAD  / NREAD
      COMMON / BSTOP  / NPSTOP,ISTOP(3)
      COMMON / BINPUT / INFLG
      COMMON / BOPT   / IVER,LT,IFP,ISP,NLOOP1,IST,ILOOP1
      COMMON / BOPT2  / TOL,RM1,PM1,IVALFU,ITERL,ITERC,MAX,IER
      COMMON / BSPD   / ISPD
      COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
C
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DDISS  / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
     *                  NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
      COMMON / DATAX  / DATA1
      COMMON / DATAY  / DATA2
      COMMON / DCOVM  / MBHHH
      COMMON / DMAIN  / KANAL5,LEVCHG,PNAME,NTAU1,NTH1,NXA2,
     *                  NALTFR,PROBFR,IAKTIV,JAKTIV,
     *                  NCT1,NCT2,ACT1,ACT2,IP1,IP2,ACTTAU,ACTTH,
     *                  GRAD2,SCRA,PSHARE,ELAS,ELAS2,PROB,FREQ
      COMMON / DWORK  / WORK,PROBS
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
      COMMON / DMAPP  / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
      COMMON / DDUMM  / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
     *                  NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
     *                  NALSTD,NALTRE,NAGSTD,NAGTRE
      COMMON / DIFF2  / IDIFF2
      COMMON / DCONST / UNCON,PDUMMY,NDUMMY
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
C
      EXTERNAL FUNC,GRADX,DFP,SSQDEV
C
      EQUIVALENCE (DATA1,FDATA1),(DATA2,FDATA2)
C
C *** INITIALIZATIONS ***
C
      ISTRUC=0
      ISHARE=0
      ICOVAR=0
      IDATA=0
      IF (LEVCHG.EQ.1) THEN
                       IDATA=1
                       LEVCHG=0
                       END IF
C
C *** ECHO TREE STRUCTURE, PARAMETER LABELS, INITIALVALUES ***
C
940   WRITE (KANAL6,*) 'ENTER TREE-OUTPUT-FILE >'
      READ (KANAL5,'(A20)') TITLE
      IF (TITLE.EQ.EMPTY) TITLE='OUTPARMS'
```

```
9401  OPEN (UNIT=19,STATUS='NEW',FILE=TITLE,ERR=9402)
      GOTO 9405
9402  OPEN (UNIT=19,STATUS='OLD',FILE=TITLE)
      CLOSE(UNIT=19,STATUS='DELETE')
      GOTO 9401
C
9406  FORMAT(8(12I6  /))
9407  FORMAT(8(12F6.3/))
9410  FORMAT(25I3)
9411  FORMAT(2I3,F10.6)
C
9405  WRITE (19,9411) NALT,NALTFR,RELSIZ
      IF (NALT.GE.NALTFR) GOTO 9420
         WRITE (19,9406) (IAKTIV(I),I=1,NALTFR)
         WRITE (19,9407) (PROBFR(I),I=1,NALTFR)
         WRITE (19,9410) (JAKTIV(J),J=1,NALT)
9420  IF (RELSIZ.EQ.0.0) GOTO 9440
         WRITE (19,9407) (WEIGHT(I),I=1,NALT)
C
9440  WRITE (19,9410) NX,NXD,NXM,NXA2
      IF (NXM.EQ.0) GOTO 9442
      DO 9441 I=1,NALT
9441     WRITE (19,9410) (MAP(I,K),K=1,3)
C
9442  WRITE (19,9410) NL1
      WRITE (19,9410) (NL2(I),I=1,NL1)
      WRITE (KANAL6,942) NALT,NL1,(NL2(I),I=1,NL1)
      WRITE (KANAL6,943)
942   FORMAT(
     *//' HIERARCHICAL CHOICE TREE WITH ',I2,' ALTERNATIVES:'
     * /' -----------------------------------------------'
     * /' THE STEM HAS',I2,' LIMBS, EACH HAS'/9I3)
943   FORMAT(' BRANCHES, WHICH HAVE THE FOLLOWING TWIGS :')
      DO 941 I1=1,NL1
         ML2=NL2(I1)
         WRITE (19,9410) (NL3(I1,I2),I2=1,ML2)
941      WRITE (KANAL6,944) (NL3(I1,I2),I2=1,ML2)
944      FORMAT(' ',5I3)
      WRITE (19,9410) NTAU,NTAU1,(LOCTAU(J),          J=1,NTAU+NTAU1)
      WRITE (19,9410) NTH ,NTH1 ,(LOCLIM(J),LOCBRA(J),J=1,NTH+NTH1  )
      WRITE (KANAL6,945) NTAU,(LOCTAU(J),J=1,NTAU)
      WRITE (KANAL6,946) NTH ,(LOCLIM(J),LOCBRA(J),J=1,NTH)
      WRITE (KANAL6,947) NTAU1,(LOCTAU(J),J=NTAU+1,NTAU+NTAU1)
      WRITE (KANAL6,948) NTH1 ,(LOCLIM(J),LOCBRA(J),J=NTH+1,NTH+NTH1)
945   FORMAT(/I3,' FREE TAU''S':,' AT ', 5('(',I1,')':))
946   FORMAT( I3,' FREE THETA''S':,' AT ',10('(',I1,',',I1,')':))
947   FORMAT( I3,' CONSTRAINT TAU''S':,' AT ', 5('(',I1,')':))
948   FORMAT( I3,' CONSTRAINT THETA''S':,' AT ',10('(',I1,',',I1,')':))
C
C     ACTIVE (= FREE AND EQUALITY CONSTRAINT) DISS. PARAMETERS
C
      DO 9750 I1=1,NL1
         ACTTAU(I1)=0
         DO 9750 I2=1,NL2(I1)
```

```
9750        ACTTH(I1,I2)=0
C
      DO 9751 K=1,NTH
9751     ACTTH(LOCLIM(K),LOCBRA(K))=1
      DO 9752 K=NTH+1,NTH+NTH1
9752     ACTTH(LOCLIM(K),LOCBRA(K))=2
C
      DO 9755 K=1,NTAU
9755     ACTTAU(LOCTAU(K))=1
      DO 9756 K=NTAU+1,NTAU+NTAU1
9756     ACTTAU(LOCTAU(K))=2
C
C     PAINT THE TREE
C
      IC=0
      I=0
      IP1(1)=2
      IP2(1,1)=2
      DO 9700 I1=1,NL1
         IF (I1.EQ.1) GOTO 9701
         IC=IC+1
         TR7(IC)=BL
         TR8(IC)=BL
         TR9(IC)=BL
         IC=IC+1
         TR7(IC)=BL
         TR8(IC)=BL
         TR9(IC)=VB
         IP1(I1)=IC+2
9701     ML2=NL2(I1)
         DO 9700 I2=1,ML2
            IC=IC+1
            TR7(IC)=BL
            TR8(IC)=BL
            TR9(IC)=BL
            IF (I1.NE.1.OR.I2.NE.1) IP2(I1,I2)=IC+1
            ML3=NL3(I1,I2)
            DO 9700 I3=1,ML3
               I=I+1
               IN=MOD(I,10)
               IC=IC+1
               TR7(IC)=CR
               TR8(IC)=VB
               TR9(IC)=DG(IN)
9700           CONTINUE
      IFINIS=IC
C
      WRITE (KANAL6,9790) CR
9790  FORMAT(//' STRUCTURE  : ',A1,'   (F,C DENOTE DISS. PARAMETERS)')
      WRITE (KANAL6,9791) VB
9791  FORMAT(14X,100A1)
C
      IEND=IP1(NL1)
      DO 9710 IC=2,IEND
```

```
         TR1(IC)=DA
         TR2(IC)=BL
         TR3(IC)=BL
9710     CONTINUE
      DO 9715 I1=1,NL1
         IC=IP1(I1)
         TR1(IC)=CR
         TR2(IC)=VB
         TR3(IC)=VB
         IF (ACTTAU(I1).EQ.1) TR3(IC)=FK
         IF (ACTTAU(I1).EQ.2) TR3(IC)=CK
9715     CONTINUE
      WRITE (KANAL6,9791) (TR1(IC),IC=2,IEND)
      WRITE (KANAL6,9791) (TR2(IC),IC=2,IEND)
      WRITE (KANAL6,9791) (TR3(IC),IC=2,IEND)
C
      IEND=IP2(NL1,NL2(NL1))
      DO 9720 IC=2,IEND
         TR1(IC)=DA
         TR2(IC)=BL
         TR3(IC)=BL
9720     CONTINUE
      DO 9725 I1=1,NL1
         DO 9725 I2=1,NL2(I1)
            IC=IP2(I1,I2)
            TR1(IC)=CR
            TR2(IC)=VB
            TR3(IC)=VB
            IF (ACTTH(I1,I2).EQ.1) TR3(IC)=FK
            IF (ACTTH(I1,I2).EQ.2) TR3(IC)=CK
9725     CONTINUE
      DO 9729 I1=2,NL1
         IBL1=IP2(I1-1,NL2(I1-1))+1
         IBL2=IP2(I1  ,1        )-1
         DO 9729 IC=IBL1,IBL2
            TR1(IC)=BL
9729        CONTINUE
      WRITE (KANAL6,9791) (TR1(IC),IC=2,IEND)
      WRITE (KANAL6,9791) (TR2(IC),IC=2,IEND)
      WRITE (KANAL6,9791) (TR3(IC),IC=2,IEND)
C
      WRITE (KANAL6,9792) (TR7(IC),IC=1,IFINIS)
      WRITE (KANAL6,9792) (TR8(IC),IC=1,IFINIS)
      WRITE (KANAL6,9793) (TR9(IC),IC=1,IFINIS)
9792  FORMAT(13X,100A1)
9793  FORMAT(' CLUSTERING :',100A1)
      WRITE (KANAL6,'(A1)') BL
C
      IF (LEVEL2) THEN
         IF (LEVEL1) THEN
                     WRITE (KANAL6,*) 'TREE IS COLLAPSED TO ONE LEVEL'
                    ELSE
                     WRITE (KANAL6,*) 'TREE IS COLLAPSED TO TWO LEVELS'
                    END IF
```

```
                  ELSE
                   WRITE (KANAL6,*) 'THIS IS A FULL THREE LEVEL TREE'
                  END IF
C
C     DISPLAY OF ALTERNATIVES AND DUMMIES
C
      WRITE (KANAL6,9800)
9800  FORMAT(/' ALTERNATIVES AND DUMMIES:'
     *       /' -------------------------'
     *       /' INDEX   LABEL     LIMB      BRANCH    TWIG'/)
      DO 9801 I=1,NALT
         IF (.NOT.UNCON) ANAME=NDUMMY(I)
         IF (UNCON.AND.NXA.NE.1) ANAME=BLANK
         IF (UNCON.AND.NXA.EQ.1) ANAME=PNAME(MXA+I)
         IF (I.EQ.NALT) ANAME=BLANK
         WORK(1)=BLANK
         WORK(2)=BLANK
         WORK(3)=BLANK
         IF (NXM.EQ.0) GOTO 9801
         IF (MAP(I,1).GT.0) WORK(1)=PNAME(NX+MAP(I,1))
         IF (MAP(I,2).GT.0) WORK(2)=PNAME(NX+MAP(I,2))
         IF (MAP(I,3).GT.0) WORK(3)=PNAME(NX+MAP(I,3))
9801     WRITE (KANAL6,9805) I,ANAME,(WORK(J),J=1,3)
9805     FORMAT(1X,I3,4X,A8,2X,A8,2X,A8,2X,A8)
C
C     NUMBER AND INITIALVALUES OF PARAMETERS
C
      WRITE (KANAL6,951) NX,NXD,NDUMD,NXA,NDUMA,NTH,NTAU,NP
951   FORMAT(/' NUMBER OF PARAMETERS : '/' ----------------------'
     *       /' NX+(NXD*NDUMD)+(NXA*NDUMA)+NTH+NTAU = NP'
     *       /  1X,I2,I5,3I6,3I5
     *      //' INITIALVALUES :'/' ---------------')
      DO 952 I=1,NP
         WRITE (KANAL6,953) PNAME(I),PARM(I)
952      CONTINUE
953   FORMAT(' ',A8,F15.9)
      IF (UNCON.AND.NTAU1+NTH1.EQ.0) GOTO 9620
      WRITE (KANAL6,955)
955   FORMAT(//' CONSTRAINT :'/' ------------')
      IF (UNCON) GOTO 957
      DO 956 I=1,NALT1
         WRITE (KANAL6,953) NDUMMY(I),PDUMMY(I)
956      CONTINUE
957   N1=NP
      IF (.NOT.UNCON) N1=N1+NALT1
      DO 958 I=1,NTH1
         KK=NTH+I
         WRITE (KANAL6,953) PNAME(N1+I),THETA(LOCLIM(KK),LOCBRA(KK))
958      CONTINUE
      DO 959 I=1,NTAU1
         KK=NTAU+I
         WRITE (KANAL6,953) PNAME(N1+NTH1+I),TAU(LOCTAU(KK))
959      CONTINUE
C
```

```
C     MAPS FOR INDICES OF NON-STANDARD DUMMIES:
C
C     (1) LMAP(K,I)     , I=1,NALT , K=1,NXM : LOGICAL MAP
C           RETURNS .TRUE.,  IF DUMMY K APPEARS IN ALTERNATIVE I
C                   .FALSE., IF NOT
C     (2) MAP(I,N)    , I=1,NALT , N=1,N1  : ORIGINAL MAP
C           RETURNS UP TO N1 DUMMIES K PER ALTERNATIVE I
C                   N1=3 FOR THREE LEVEL TREE
C     (3) MAPTR(K,N) , K=1,NXM   , N=1,N2  : TRANSPOSED MAP
C           RETURNS UP TO N2 ALTERNATIVES I PER DUMMY K
C                   N2=MAPLEN(K) : # OF APPEARANCES OF DUMMY K
C
9620  IF (NXM.EQ.0) GOTO 969
C
      DO 9621 K=1,NXM
9621     MAPLEN(K)=0
      DO 9625 I=1,NALT
         M1=MAP(I,1)
         M2=MAP(I,2)
         M3=MAP(I,3)
         IF (M1.NE.0) THEN
                      MAPLEN(M1)=MAPLEN(M1)+1
                      MAPTR(M1,MAPLEN(M1))=I
                      END IF
         IF (M2.NE.0) THEN
                      MAPLEN(M2)=MAPLEN(M2)+1
                      MAPTR(M2,MAPLEN(M2))=I
                      END IF
         IF (M3.NE.0) THEN
                      MAPLEN(M3)=MAPLEN(M3)+1
                      MAPTR(M3,MAPLEN(M3))=I
                      END IF
         DO 9625 K=1,NXM
            IF (K.EQ.M1 .OR. K.EQ.M2 .OR. K.EQ.M3) THEN
                                                   LMAP(K,I)=.TRUE.
                                                   ELSE
                                                   LMAP(K,I)=.FALSE.
                                                   END IF
9625        CONTINUE
C
C     FINISHED WITH PRELIMINARY WORK ON TREE-SPECIFICATION
C
969   ISTRUC=1
C
C *** DATA AND NUMBER OF OBSERVATIONS ***
C
900   IF (IDATA.EQ.1) GOTO 920
      CALL INDATA(KANAL5,KANAL6)
C
      NOBS1=NOBS+1
      IWESML=1
903   WRITE (KANAL6,904) NOBS,NALT,CWESML(IWESML)
904   FORMAT(/' IDCASE = ',I5,' ,  IDALT = ',I4,
     *       /' SAMPLE FREQUENCIES AND MEANS :',A18
```

```
     *        /' -----------------------------')
      DO 906 I=1,NALT
906      WRITE (KANAL6,911) DATA1(I,1,NOBS1),
     *                     (DATA1(I,J+1,NOBS1),J=1,NX),
     *                     (DATA2(J,NOBS1),J=1,NXD)
911      FORMAT(' ',F6.4,12F10.4)
C
      IF (RELSIZ.EQ.0.0 .OR. IWESML.EQ.2) GOTO 920
C
      IWESML=2
      NOBS1=NOBS+2
      GOTO 903
C
920   IDATA=1
      NOBS1=NOBS+1
C
C *** MENU ****
C
1     WRITE (KANAL6,3)
3     FORMAT(/' HIT ENTER FOR MENU, 7 FOR STOP >')
      READ (KANAL5,4) MENU
4     FORMAT(I1)
      IF (MENU.NE.0) GOTO 8
      WRITE (KANAL6,2)
2     FORMAT(' SELECT FROM THE MENU :'
     * /' 1=ESTIMATION, 2=PREDICTION, 3=ELASTICITIES, 4=COVARIANCE,'
     * /' 5=LEVEL CHANGE, 6=DERIVATIVE CHECK, 7=STOP, 8=PRINT DATA,'
     * /' 9=LM-TEST, 10=TRANSFORM, 11=PLOT, 12=AIRUM, 13=SELECT DATA,'
     * /' 14=ADJUST SHARES >')
      READ (KANAL5,*) MENU
8     GOTO (100,200,300,400,9,500,99,600,700,780,650,790,620,800),MENU
C          ( 1   2   3   4  5  6   7  8   9   10  11  12  13  14)
5     WRITE (KANAL6,6)
6     FORMAT(' IMPROPER ORDER OF OPERATIONS')
7     FORMAT(' INPUT ERROR')
      GOTO 1
9     LEVCHG=1
      GOTO 99
C
C *** TASKS OF GQTREE ***
C
C     ******************
C     *   ESTIMATION   *
C     ******************
C
C **  ITERATION PARAMETERS   **
C
100   WRITE (KANAL6,101)
101   FORMAT(/' TYPE 1 TO OVERRIDE DEFAULT ITERATION PARAMETERS >')
      READ (KANAL5,'(I1)') IQUERY
      IF (IQUERY.EQ.1) GOTO 105
         ITER=NP/4*2
         TOL =0.001
         METHOD=2
```

```
      IVER=2
      IF (LEVEL1) IVER=1
      IST=1
      IDIFF2=2
      IF (LEVEL1) IDIFF2=4
      ISPD=1
      IF (NP.GE.20) ISPD=2
      ISTOP(1)=0
      ISTOP(2)=0
      ISTOP(3)=1
      GOTO 110
105   WRITE (KANAL6,106)
106   FORMAT(' ITERATION         : LIMIT >'
     *      /'                   : ACCURACY >'
     *      /' METHOD            : 1 = DFP, 2 = GRADX >'
     *      /' VERSION           : 1-4 IN DFP, 1-2 IN GRADX >'
     *      /' LINE-SEARCH       : 0-2 IN DFP, 0-3 IN GRADX >'
     *      /' 2ND DERIVATIVES: 1 = BHHH, 2 = SIMPLE, 3 = SYMM.,'
     *      ,                    ' 4 = MNL >'
     *      /'                   : FREQENCY OF UPDATING >'
     *      /' CONV.-CRITERION: PARM  (0=ENABLE,1=DISABLE) >'
     *      /'                   : GRAD  (0=ENABLE,1=DISABLE) >'
     *      /'                   : LIKL  (0=ENABLE,1=DISABLE) >')
      READ (KANAL5,*) ITER,TOL,METHOD,IVER,IST,IDIFF2,ISPD,ISTOP
      IF (.NOT.LEVEL1.AND.IDIFF2.EQ.4) THEN
         WRITE (KANAL6,*) 'NO ANALYTIC 2ND DERIVATIVES IMPLEMENTED'
         IDIFF2=2
         END IF
110   IVER2=IVER
      IF (METHOD.EQ.2) IVER2=IVER+4
      WRITE (KANAL6,115) ITER,TOL,MENAME(METHOD),CWESML(IWESML),
     *      CHVER(IVER2),CHIST(IST),CHDIF2(IDIFF2),ISPD,
     *      CHSTOP(ISTOP(1)),CHSTOP(ISTOP(2)),CHSTOP(ISTOP(3))
115   FORMAT(//' ',68('-')
     *       /' ITERATION        : LIMIT=',I3,', ACCURACY=',F8.6
     *       /' METHOD           : ',A8,A18
     *       /' VERSION          : ',A14
     *       /' LINE-SEARCH      : ',A5
     *       /' 2ND DERIVATIVES: ',A4,', UPDATE EVERY',I2,' ITERATION'
     *       /' CONV.-CRITERION: PARMS=',A8,', GRADIENT=',A8,
     *                                  ', LOGLIK=',A8
     *       /' ',68('-')/)
116   MATPRT=0
      NLOOP1=20
      INFLG=0
      MAX=1
      NPSTOP=NP-NTH-NTAU
      IF (METHOD.EQ.1) THEN
                        NQ = NP*NP + 8*NP
                        NPUNCH=18
                        NREAD=18
                        OPEN (UNIT=18,STATUS='SCRATCH')
                       ELSE
                        NQ = 4*NP*NP + 5*NP
```

```
                          IF (IVER.EQ.2) NQ = 5*NP*NP + 6*NP
                        END IF
      IF (.NOT.UNCON) NQ = NQ + NALT1*NALT1 + 8*NALT1
C
C **  ITERATION  **
C
      ITERAL=0
      NEVALF=0
120   CONTINUE
      IF (METHOD.EQ.1)
     *   CALL OPT(PARM,NP,XLF, DFP ,ITER,MAX,IER,TOL,FUNC,PNAME)
      IF (METHOD.EQ.2)
     *   CALL OPT(PARM,NP,XLF,GRADX,ITER,MAX,IER,TOL,FUNC,PNAME)
      ITERAL=ITERAL+ITERC
      NEVALF=NEVALF+IVALFU
      WRITE (KANAL6,122) IER
122   FORMAT(//' *** OPTIMIZATION TERMINATED *** IER=',I3
     *        /' TYPE NR OF MORE ITERATIONS OR 0 TO STOP >')
      READ (KANAL5,*) IQUERY
      IF (IQUERY.EQ.0) GOTO 140
      ITER=IQUERY
      IF (METHOD.NE.1) GOTO 120
         CALL PUNCH(PARM,NP)
         REWIND NPUNCH
         INFLG=1
         GOTO 120
C
C  ** PRINT PARAMETERS, STD ERRORS AND T-STATS **
C     NOTE: COVARIANCE-MATRIX = -INV(HESSIAN)
C
140   IF (IER.GT.0) GOTO 145
C
C     RECALCULATE COVARIANCES
C
      MATPRT=2
      IREDO=1
      GOTO 420
C
C     USE GQOPT-COVARIANCES
C
145   LK=NP*NP
      CALL OPTMOV(1,HESS,LK)
      CALL OPTMOV(3,GRAD,NP)
      CALL WESML(PARM,GRAD,HESS,NP,SCRA,FUNC)
      ICOVAR=1
C
150   XLF0    = NOBS * DLOG(1.D0/DBLE(NALT))
      DOF     = NOBS * NP
      AMSQGR = DOTV(GRAD,NP,GRAD)/DOF
      AMSQGR = DSQRT(AMSQGR)
      WRITE (KANAL6,151)
151   FORMAT(' ENTER TITLE >')
      READ (KANAL5,152) TITLE
152   FORMAT(A20)
```

```
      WRITE (KANAL6,153) MENAME(METHOD)
153   FORMAT(//' **** OUTPUT OF RESULTS ****'///' ',A8,'  ESTIMATE'
     *         ,'  STD-ERR   T-STAT'/)
      N1=NP
      IF (.NOT.UNCON) N1=N1+NALT1
      I=0
      DO 160 II=1,N1
         IF (.NOT.UNCON.AND.II.GT.MXA.AND.II.LE.MXA+NALT1) GOTO 165
         I=I+1
         ZZZ = - HESS(I,I)
         IF (ZZZ.LE.0.) THEN
                          STDERR = 0.0
                          TSTAT  = 0.0
                        ELSE
                          STDERR = DSQRT(ZZZ)
                          TSTAT  = PARM(I)/STDERR
                          IF (I.GT.MTH) TSTAT=(PARM(I)-1.0)/STDERR
                        END IF
         WRITE (KANAL6,161) PNAME(I),PARM(I),STDERR,TSTAT
161      FORMAT(' ',A8,2X,F8.4,2X,F7.3,2X,F7.3)
         WRITE (19,162) PNAME(I),PARM(I),STDERR,TSTAT
162      FORMAT(A8,3F15.9)
         GOTO 160
165      WRITE (KANAL6,161) NDUMMY(II-MXA),PDUMMY(II-MXA)
         WRITE (19,162) NDUMMY(II-MXA),PDUMMY(II-MXA)
160      CONTINUE
      DO 170 I=1,NTH1
         KK=NTH+I
         WRITE (KANAL6,161) PNAME(N1+I),THETA(LOCLIM(KK),LOCBRA(KK))
         WRITE (19,162) PNAME(N1+I),THETA(LOCLIM(KK),LOCBRA(KK))
170      CONTINUE
      DO 175 I=1,NTAU1
         KK=NTAU+I
         WRITE (KANAL6,161) PNAME(N1+NTH1+I),TAU(LOCTAU(KK))
         WRITE (19,162) PNAME(N1+NTH1+I),TAU(LOCTAU(KK))
175      CONTINUE

      WRITE (19,180) TITLE,XLF,XLF0,ITERAL,NEVALF,NOBS,IER,
     *               AMSQGR,(TR9(IC),IC=1,IFINIS)
      WRITE (KANAL6,180) TITLE,XLF,XLF0,ITERAL,NEVALF,NOBS,IER,
     *                  AMSQGR,(TR9(IC),IC=1,IFINIS)
180   FORMAT(/' TITLE                         : ',A20
     *       /' LOGLIKELIHOOD                 : ',F10.4
     *       /' LOGLIKELIHOOD AT ZERO         : ',F10.4
     *       /' TOTAL NUMBER OF ITERATIONS : ',I10
     *       /' TOTAL NUMBER OF EVALUATIONS: ',I10
     *       /' NUMBER OF OBSERVATIONS      : ',I10
     *       /' MEAN SQUARE GRADIENT   (',I2,'): ',D10.4
     *       //' CLUSTERING : ',65A1//)
C
      IF (MATPRT.EQ.3) GOTO 430
      GOTO 1
C
C     *******************************************************
```

```
C     *   PREDICTION SUCCESS TABLE AND PROBABLITY SHARES     *
C     *******************************************************
C
200   IF (IDATA.EQ.0) GOTO 5
      WRITE (KANAL6,202)
202   FORMAT(' TYPE 1 TO CREATE DATA-SET WITH PREDICTED CHOICES >')
      READ (KANAL5,'(I1)') IWRITE
      IF (IWRITE.EQ.1) THEN
         WRITE (KANAL6,203)
203      FORMAT(' ENTER NEW DATA-SET NAME >')
         READ (KANAL5,'(A20)') TITLE
         IF (TITLE.EQ.EMPTY) TITLE='PREDICT'
204      OPEN (UNIT=9,STATUS='NEW',FILE=TITLE,ACCESS='SEQUENTIAL'
     *            ,ERR=206)
         GOTO 205
206      OPEN (UNIT=9,STATUS='OLD',FILE=TITLE,ACCESS='SEQUENTIAL')
         CLOSE(UNIT=9,STATUS='DELETE')
         GOTO 204
205      CONTINUE
         END IF
      XLF0 = NOBS * DLOG(1.D0/DBLE(NALT))
      XLF  = 0.D0
      XLP  = 0.D0
      PCP  = 0.D0
      UTIL = 0.D0
      Z4   = 0.D0
      Z5   = 0.D0
      NFAIL= 0
      DO 201 K=1,NALT
         ACT1(K)=0.0
         PROB(K)=0.0
         FREQ(K)=DATA1(K,1,NOBS+1)*WEIGHT(K)
         DO 201 J=1,NALT
201         ACT2(K,J)=0.0
C
      NN1=1
      NN2=1
      INCREM=MAXALT*MAXNXY
      DO 210 IOBS=1,NOBS
C
C        ACTUAL CHOICE
C
         I=0
         DO 211 I1=1,NL1
            ML2=NL2(I1)
            DO 211 I2=1,ML2
               ML3=NL3(I1,I2)
               DO 211 I3=1,ML3
                  I=I+1
                  IF (DATA1(I,1,IOBS).EQ.1.0) IC=I
211               CONTINUE
         WT=WEIGHT(IC)
C
C        PREDICTED CHOICE
```

```
C
         NF=NFAIL
         CALL CONT(FDATA1(NN1),FDATA2(NN2),
     *              2,PARM,NP,XLFI,WORK,PROBS,NFAIL)
         NN1=NN1+INCREM
         NN2=NN2+MAXNXD
         IF (NFAIL.GT.NF) THEN
215         WRITE (KANAL6,212) IOBS,IC
212         FORMAT(' CONT: FAILURE AT IOBS=',I5)
            IMAX=IA
            GOTO 222
            END IF
         XLF=XLF+XLFI
         UTIL=UTIL+WORK(1)*WT
         PMAX=0.DO
         DO 221 IA=1,NALT
            PIA=PROBS(IA)
            PSHARE(IA,IOBS)=PIA
            PROB(IA)=PROB(IA)+PIA*WT
            IF (PIA.LE.PMAX) GOTO 221
               PMAX=PIA
               IMAX=IA
221        CONTINUE
C
C        COMPARISON ACTUAL VS. PREDICTED CHOICE
C
         IF (IC.EQ.IMAX) PCP=PCP+WT
         ACT2(IC,IMAX)=ACT2(IC,IMAX)+WT
         NCT2(IC,IMAX)=ACT2(IC,IMAX)+0.5
         ACT1(IMAX)=ACT1(IMAX)+WT
         IF (PMAX.LE.0.DO) GOTO 215
         XLP=XLP+DLOG(PMAX)*WT
         Z4 =Z4 +PMAX*WT
         Z5 =Z5 +PMAX/FREQ(IMAX)*WT
C
C        WRITE DATASET WITH PREDICTED CHOICES
C
222      IF (IWRITE.EQ.1) THEN
            DO 223 J=1,NALT
               IF (J.EQ.IMAX) THEN
                  CH=1.0
               ELSE
                  CH=0.0
               END IF
223            WRITE (9) CH,(DATA1(J,K+1,IOBS),K=1,NX),
     *                    (DATA2(K,IOBS),K=1,NXD)
            END IF
C
210   CONTINUE
C
      IF (IWRITE.EQ.1) CLOSE(UNIT=9)
      PCP=PCP/NOBS*100.0
      WRITE (KANAL6,230) CWESML(IWESML),(J,J=1,NALT)
230   FORMAT(//'  PREDICTION SUCCESS TABLE',A18
```

```
     *        /'  ======================='
     *       //' OBSERVED                  PREDICTED ALTERNATIVE'
     *        /' ALT.    |',30I4)
      WRITE (KANAL6,231) (DASH4(I),I=1,NALT)
 231  FORMAT  (' -------+',30A4)
      DO 232 I=1,NALT
 232     WRITE (KANAL6,233) I,(NCT2(I,J),J=1,NALT)
 233     FORMAT( I6 ,'   |',30I4)
      WRITE (KANAL6,231) (DASH4(I),I=1,NALT)
      WRITE (KANAL6,235) PCP,XLF0,XLF,XLP,UTIL
 235  FORMAT(
     */'  PERCENT CORRECTLY PREDICTED     : ',F10.2,' %'
     */'  LIKELIHOOD AT ZERO              : ',F12.4
     */'  LIKELIHOOD AT ACTUAL CHOICES    : ',F12.4
     */'  LIKELIHOOD AT PREDICTED CHOICES : ',F12.4
     */'  UTILITY AT PREDICTED CHOICES    : ',F12.4)
      WRITE (KANAL6,240) CWESML(IWESML)
 240  FORMAT(//'  PROBABILITY-SHARES',A18
     *        /'  ==================='
     *//'  ALT.   ACTUAL           DISCRETE         CONTINUOUS'
     */'  -------------------------------------------------------')
      DO 245 I=1,NALT
         MOBSV=FREQ(I)*NOBS+0.5
         ADISC=ACT1(I)/NOBS
         MDISC=ACT1(I)+0.5
         ACONT=PROB(I)/NOBS
 245     WRITE (KANAL6,246) I,FREQ(I),MOBSV,ADISC,MDISC,ACONT,PROB(I)
 246     FORMAT(I6,F9.4,I6,F10.4,I6,F10.4,F8.2)
C
C     R-SQUARE-EQUIVALENTS (SEE DOMENCICH/MCFADDEN P.123)
C
      Z1=0.D0
      Z2=0.D0
      Z3=0.D0
      S1=0.D0
      S2=0.D0
      S3=0.D0
      E1=0.D0
      E2=0.D0
      E3=0.D0
      Z7=0.D0
      NFAIL=0
      ALT=1.D0/NALT
      DO 250 IOBS=1,NOBS
         DO 251 I=1,NALT
            IF (DATA1(I,1,IOBS).EQ.1) WT=WEIGHT(I)
 251        CONTINUE
         DO 250 J=1,NALT
            FIJ = DATA1(J,1,IOBS)
            PIJ = PSHARE(J,IOBS)
            PJ  = FREQ(J)
            IF (PIJ.LT.0.D0 .OR. PIJ.GT.1.D0)
     *         WRITE (KANAL6,252) PIJ,IOBS,J
 252        FORMAT(' ERROR: PSHARE=',F10.4,' IN OBS=',I5,' AND ALT=',I3)
```

```
            IF (PIJ.EQ.0) PIJ=PJ
            IF (PIJ.EQ.0) NFAIL=NFAIL+1
            R1 = (FIJ-PIJ)**2
            R2 = (FIJ-ALT)**2
            R3 = (FIJ-PJ )**2
            Z1 = Z1 + R1*WT
            Z2 = Z2 + R1/PJ*WT
            Z3 = Z3 + R1/PIJ*WT
            S1 = S1 + R2*WT
            S2 = S2 + R2/PJ*WT
            S3 = S3 + R2/PIJ*WT
            E1 = E1 + R3*WT
            E2 = E2 + R3/PJ*WT
            E3 = E3 + R3/PIJ*WT
            Z7 = Z7 + FIJ*(1.D0-ALT)/PJ*WT
250         CONTINUE
C
      R1 = 1.0 - Z1/S1
      R2 = 1.0 - Z2/S2
      R3 = 1.0 - Z3/S3
C
      R4 = 1.0 - Z1/E1
      R5 = 1.0 - Z2/E2
      R6 = 1.0 - Z3/E3
C
      R7 = 1.0 - XLF/XLF0
C
      R8 =  Z4/NOBS
      R9 =  Z5/Z7
C
      WRITE (KANAL6,253) CWESML(IWESML),R1,R2,R3,R4,R5,R6,R7,R8,R9
253   FORMAT(
     * '  ------------------------------------------------------'
     *///'  R-SQUARE-EQUIVALENTS:',A18
     *  /'  ====================='
     *  /'  WEIGHTS      :     NONE       FREQ       PROBS'
     *  /'  SIMPLE       :', 3F10.5
     *  /'  EFRON''S      :',3F10.5
     *  /'  MCFADDEN''S  :', F10.5
     *  /'  SUCCESSES    :', 2F10.5 )
      IF (NFAIL.GT.0) WRITE (KANAL6,*) '# PROBS REPL. BY FREQ: ',NFAIL
C
      ISHARE=1
      GOTO 1
C
C     ********************
C     *   ELASTICITIES   *
C     ********************
C
300   IF (ISTRUC.EQ.0) GOTO 5
C
      WRITE (KANAL6,302) CWESML(IWESML)
302   FORMAT(//' ELASTICITIES AT SAMPLE MEANS AND FREQUENCIES:',A18//)
C
```

```
      DO 3020 I=1,NALT
3020     FREQ(I)=DATA1(I,1,NOBS+IWESML)
      CALL ELAST(PARM,NP,FREQ,ELAS,NOBS+IWESML)
      WRITE (KANAL6,303) (I,I=1,NALT)
303   FORMAT(' VARIABLE ALT. |',16('  P(',I2,')':))
301   FORMAT(' ',14('-'),'+',16A7 )
      DO 304 K=1,NX
         WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
         DO 304 J=1,NALT
304         WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,K),I=1,NALT)
305         FORMAT(' ',A8,' ',I3,'  |',16F7.3)
      IF (NOAGE) GOTO 3061
      DO 306 K1=1,NXD
         WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
         DO 307 I=1,NALT
307         PROBS(I)=0.D0
         DO 308 J=1,NDUMD
            K=NX+(K1-1)*NDUMD+J
            WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,NX+K1),I=1,NALT)
            DO 309 I=1,NALT
309            PROBS(I)=PROBS(I)+ELAS(I,J,NX+K1)
308         CONTINUE
         WRITE (KANAL6,305) SUMME,K1,(PROBS(I),I=1,NALT)
306      CONTINUE
3061  WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
C
      IF (ISHARE.EQ.0) THEN
         WRITE (KANAL6,*)
     *          'FOR INDIVIDUAL ELASTICITIES, RUN PREDICTION FIRST'
         GOTO 1
         END IF
C
      WRITE (KANAL6,310) CWESML(IWESML)
310   FORMAT(//' AVERAGE INDIVIDUAL ELASTICITIES:',A18//)
C
      DO 311 I=1,NALT
      DO 311 J=1,NALT
      DO 311 K=1,NXR
311      ELAS(I,J,K)=0.0
C
      DO 312 IOBS=1,NOBS
         DO 313 J=1,NALT
            IF (DATA1(J,1,IOBS).EQ.1.0) WT=WEIGHT(J)
313         PROB(J)=PSHARE(J,IOBS)
         CALL ELAST(PARM,NP,PROB,ELAS2,IOBS)
         DO 314 I=1,NALT
         DO 314 J=1,NALT
         DO 314 K=1,NXR
314         ELAS(I,J,K)=ELAS(I,J,K)+ELAS2(I,J,K)*WT
312      CONTINUE
C
      DO 320 I=1,NALT
      DO 320 J=1,NALT
      DO 320 K=1,NXR
```

```
320        ELAS(I,J,K)=ELAS(I,J,K)/NOBS
      WRITE (KANAL6,303) (I,I=1,NALT)
      DO 321 K=1,NX
         WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
         DO 321 J=1,NALT
321         WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,K),I=1,NALT)
      IF (NOAGE) GOTO 326
      DO 322 K1=1,NXD
         WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
         DO 323 I=1,NALT
323         PROBS(I)=0.D0
         DO 324 J=1,NDUMD
            K=NX+(K1-1)*NDUMD+J
            WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,NX+K1),I=1,NALT)
            DO 325 I=1,NALT
325            PROBS(I)=PROBS(I)+ELAS(I,J,NX+K1)
324         CONTINUE
         WRITE (KANAL6,305) SUMME,K1,(PROBS(I),I=1,NALT)
322      CONTINUE
326   WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
      GOTO 1
C
C     *************************
C     *   COVARIANCE MATRIX   *
C     *************************
C
400   IF (IDATA.EQ.0) GOTO 5
      IREDO=0
      IER=0
      WRITE (KANAL6,401)
401   FORMAT(' COVARIANCES  : 1=PRINT, 2=RECALCULATION, 3=BOTH'
     *      /' RECALCULATION: 1=BHHH, 2=SIMPLE, 3=SYMM., 4=MNL >')
      READ (KANAL5,*) MATPRT,IDIFF2
      IF (.NOT.LEVEL1.AND.IDIFF2.EQ.4) IDIFF2=2
      IF (MATPRT .EQ. 1) GO TO 430
      CALL FUNC(PARM,NP,XLF,*440)
C
C     RECALCULATION OF COVARIANCE-MATRIX
C     NOTE: COV = -INV(HESS) = -INV(MBHHH) = -INV(-GRAD*GRAD')
C
420   CALL FP(PARM,NP,XLF,GRAD,FUNC)
      CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
C
C     NOTE : IF IDIFF2=1,  FP RECALCULATES MBHHH = -GRAD*GRAD'
C                          SP REFILLS HESS
C            IF IDIFF2=2,3 FP CALCULATES GRAD
C                          SP CALCULATES FINITE DIFFERENCES OF GRAD
C            IF IDIFF2=4,  FP CALCULATES EXACT DERIVATIVES FOR MNL
C                          SP REFILLS HESS
C
C     INVERSION OF HESSIAN AND CONTROL OF EIGENVALUES
C
      IERINV=1
      CALL MATEV2(HESS,SCRA,NP,NP,MAXNP,IERINV)
```

```
      IF (IERINV.GT.1) IER=-8
      WRITE (KANAL6,422)
422   FORMAT(/' EIGENVALUES OF HESSIAN:'
     *       /' ----------------------')
      DO 423 I=1,NP
         WORK(I)=HESS(I,I)
         IF (WORK(I).NE.0.0) GOTO 423
             WORK(I)=0.1D-99
             IER=-9
             IERINV=2
423      WRITE (KANAL6,'(I3,2X,D10.4)') I,WORK(I)
      IF (IERINV.EQ.2) WRITE (KANAL6,421)
421   FORMAT(' COV-MATRIX NOT INVERTIBLE')
      DO 425 I=1,NP
      DO 425 J=1,I
         ZZZ = 0.D0
         DO 426 K=1,NP
            ZZZ = ZZZ + SCRA(I,K)*SCRA(J,K)/WORK(K)
426         CONTINUE
         HESS(I,J) = ZZZ
         HESS(J,I) = ZZZ
425      CONTINUE
      CALL WESML(PARM,GRAD,HESS,NP,SCRA,FUNC)
C
C     PRINT RESULTS ANEW
C
      IF (IREDO.EQ.1) GOTO 150
      METHOD=2+IDIFF2
      ITERAL=0
      NEVALF=0
      GOTO 150
C
C     PRINT COVARIANCE-MATRIX
C
430   WRITE (KANAL6,432)
432   FORMAT('  COVARIANCE-MATRIX :'//)
      DO 435 I = 1,NP
      DO 435 J = 1,I
         ZZZ = -HESS(I,J)
         WRITE (KANAL6,436) PNAME(I),PNAME(J),ZZZ
436      FORMAT(3X,A8,3X,A8,3X,G20.7)
435      CONTINUE
      GOTO 1
440   WRITE (KANAL6,441)
441   FORMAT(' INADMISSIBLE INITIALVALUES')
      GOTO 1
C
C     ************************
C     *   DERIVATIVE CHECK   *
C     ************************
C
500   IF (IDATA.EQ.0) GOTO 5
      CALL FUNC(PARM,NP,XLF,*530)
      WRITE (KANAL6,501) XLF
```

```
501   FORMAT(/' DERIVATIVE CHECK : LIKELIHOOD = ',F10.4
     *      //' HIT ENTER TO QUIT, IDIFF2=1-4 IF SECOND DERIVATIVES,'
     *       /' 2 IF ONLY FIRST DERIVATIVES, 5 FOR CHECK OF SSQDEV >')
      READ (KANAL5,'(I1)') IDIFF2
      IF (IDIFF2.EQ.0) GOTO 1
      IF (IDIFF2.EQ.5) GOTO 540
      WRITE (KANAL6,502)
502   FORMAT(//' PARAMETER            ANALYTIC     DIFFERENCE')
      CALL FP(PARM,NP,XLF,GRAD,FUNC)
      CALL FDIFF(PARM,NP,XLF,GRAD2,FUNC)
      DO 510 I=1,NP
510      WRITE (KANAL6,511) PNAME(I),GRAD(I),GRAD2(I)
511      FORMAT(1X,A8,10X,2D13.5)
      WRITE (KANAL6,512)
512   FORMAT(/' HIT ENTER TO QUIT, IDIFF2=1-4 FOR SECOND DERIV. >')
      READ (KANAL5,4) IDIFF2
      IF (IDIFF2.EQ.0) GOTO 1
      IF (IDIFF2.EQ.2.OR.IDIFF2.EQ.3.OR.(IDIFF2.EQ.4.AND..NOT.LEVEL1))
     *   THEN
          WRITE (KANAL6,*) 'ERROR: IDIFF2=',IDIFF2
          GOTO 1
         END IF
      CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
      CALL SDIFF(PARM,NP,XLF,GRAD,HESS,FUNC)
C     (NOTE: MBHHH IS NOT ALTERED BY CALLING SDIFF)
      KK=0
      DO 520 I=1,NP
      DO 520 J=1,I
         KK=KK+1
         ZZZ=MBHHH(KK)
520      WRITE (KANAL6,521) PNAME(I),PNAME(J),ZZZ,HESS(I,J)
521      FORMAT(1X,A8,1X,A8,1X,2D13.5)
      GOTO 1
530   WRITE (KANAL6,531)
531   FORMAT(' INADMISSIBLE INITIALVALUES')
      GOTO 1
C
540   IF (NXA.NE.1) GOTO 549
      N1=NX+NXD*NDUMD
      DO 542 I=1,NALT1
         PDUMMY(I)=PARM(N1+I)
542      NDUMMY(I)=PNAME(N1+I)
      CALL SSQDEV(PDUMMY,NALT1,SSD,*530)
      CALL FPSSQD(PDUMMY,NALT1,SSD,PROBS,PROBS,SSQDEV)
      ACC=0.00001
      WRITE (KANAL6,543) SSD
543   FORMAT(/' SUM OF SQUARED DEVIATIONS =',F10.4)
      WRITE (KANAL6,502)
      DO 545 I=1,NALT1
         PD=PDUMMY(I)
         PDUMMY(I)=PD+ACC
         CALL SSQDEV(PDUMMY,NALT1,SSD2,*530)
         PDUMMY(I)=PD
         SQDIFF=(SSD2-SSD)/ACC
```

```
545      WRITE (KANAL6,511) NDUMMY(I),PROBS(I),SQDIFF
      GOTO 1
549   WRITE (KANAL6,*) 'INADMISSIBLE NXA SETTING'
      GOTO 1
C
C     ******************
C     *   PRINT DATA   *
C     ******************
C
600   IF (IDATA.EQ.0) GOTO 5
      WRITE (KANAL6,601)
601   FORMAT(//' FIRST AND LAST OBSERVATION >')
      READ (KANAL5,*) IOBS1,IOBS2
      DO 606 IOBS=IOBS1,IOBS2
      DO 606 J=1,NALT
606      WRITE (KANAL6,911) DATA1(J,1,IOBS),
     *          (DATA1(J,K+1,IOBS),K=1,NX),(DATA2(K,IOBS),K=1,NXD)
      GOTO 1
C
C     ************************************
C     *   ELIMINATE SINGLE DATA POINTS   *
C     ************************************
C
620   WRITE (KANAL6,*) 'ENTER NUMBER OF OBSERVATIONS TO BE ELIMINATED >'
      READ (KANAL5,*) NSKIP
      WRITE (KANAL6,*) 'NOW ENTER OBSERVATIONS IN DESCENDING ORDER >'
      DO 621 ISKIP=1,NSKIP
         WRITE (KANAL6,*) 'IOBS >'
         READ (KANAL5,*) IOBSS
         DO 625 IOBS=IOBSS+1,NOBS+1
            DO 626 K=1,NXR
            DO 626 I=1,NALT
626            DATA1(I,K,IOBS-1)=DATA1(I,K,IOBS)
            DO 627 K=1,NXD
627            DATA2(K,IOBS-1)=DATA2(K,IOBS)
625         CONTINUE
         NOBS=NOBS-1
621      CONTINUE
      GOTO 1
C
C     *****************************
C     *   PLOT OF CONTOUR LINES   *
C     *****************************
C
650   IF (IDATA.EQ.0) GOTO 5
      WRITE (KANAL6,*) 'SELECT TWO VARIABLES TO PLOT, REST IS CONSTANT'
      DO 652 K=1,NP
652      WRITE (KANAL6,653) K,PNAME(K)
653      FORMAT(' ',I3,' = ',A8)
      WRITE (KANAL6,654)
654   FORMAT(/' ENTER (1) TWO INDICES FOR THE TWO VARIABLES TO PLOT,'
     *       /'       (2) RANGE OF FIRST VARIABLE (XMIN,XMAX) >'
     *       /'       (3) RANGE OF SECOND VARIABLE (YMIN,YMAX) >')
      READ (KANAL5,*) IND1,IND2,XMIN,XMAX,YMIN,YMAX
```

```
      XLABEL=PNAME(IND1)
      YLABEL=PNAME(IND2)
      CALL CNTR(PARM,NP,IND1,IND2,XMIN,XMAX,YMIN,YMAX,
     *          FUNC,XLABEL,YLABEL)
      GOTO 1
C
C     ********************
C     *   LM-STATISTIC   *
C     ********************
C
700   IF (IDATA.EQ.0) GOTO 5
      WRITE (KANAL6,701)
701   FORMAT(' 2ND DERIVATIVES: 1=BHHH, 2=SIMPLE, 3=SYMM. >')
      READ (KANAL5,*) IDIFF2
      WRITE (KANAL6,702)
702   FORMAT(' DEGREES OF FREEDOM (= # OF DISS.PARMS TO BE TESTED) >')
      READ (KANAL5,*) NP1
      IF (NP1.GT.5) THEN
                      WRITE (KANAL6,*) 'ERROR: NP1 > 5'
                      GOTO 1
                     END IF
      WRITE (KANAL6,703)
703   FORMAT(' INDICES OF DISS. PARAMETERS IN PARM >')
      READ (KANAL5,*) (NLMST(I),I=1,NP1)
C
      CALL FP(PARM,NP,XLF,GRAD,FUNC)
      CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
      CALL LMSTAT(NP,NP1,MTH,NLMST,GRAD,HESS,TEST)
C
      WRITE (KANAL6,709) TEST
709   FORMAT(//' LM-STATISTIC : ',F15.5/' ',30('-'))
      GOTO 1
C
C     ****************************************************************
C     *   TRANSFORM PARAMETERS TO BE COMPATIBLE WITH MNL (OBSOLETE)  *
C     ****************************************************************
C
780   IF (ISTRUC.EQ.0 .OR. NTH.EQ.0 .OR. ICOVAR.EQ.0) GOTO 5
      WRITE (KANAL6,781)
781   FORMAT(///' TRANSFORMED VARIABLES :'
     *         /' PARAMETER / SIM.COEFF :    MEAN  STD.ERROR T-STAT'/)
      DO 782 I=1,NX
      DO 782 K=1,NTH
         KP=MTH+K
         CALL TRAFO(PARM,NP,I,KP,HESS,C,SC,W)
         WRITE (KANAL6,783) PNAME(I),PNAME(KP),C,SC,W
782      WRITE (21,783) PNAME(I),PNAME(KP),C,SC,W
783      FORMAT(' ',A8,' / ',A8,' :',2X,F8.4,2X,F7.3,2X,F7.3)
      IF (NOAGE.AND.NOALT) GOTO 1
      IF (.NOT.STDDUM) GOTO 1
      K=MTH
      I=0
      DO 784 I1=1,NL1
         ML2=NL2(I1)
```

```
          DO 784 I2=1,ML2
             ML3=NL3(I1,I2)
             IF (THETA(I1,I2).NE.1.D0) GOTO 785
                I=I+ML3
                GOTO 784
785          K=K+1
             DO 788 I3=1,ML3
                I=I+1
                IF (I.EQ.NALT) GOTO 1
                IF (NOAGE) GOTO 786
                   DO 787 M=1,NXD
                      KK=(M-1)*(NALT-1)+NX+I
                      CALL TRAFO(PARM,NP,KK,K,HESS,C,SC,W)
                      WRITE (KANAL6,783) PNAME(KK),PNAME(K),C,SC,W
787                   WRITE (21,783) PNAME(KK),PNAME(K),C,SC,W
786             IF (NOALT) GOTO 788
                   KK=MXA+I
                   CALL TRAFO(PARM,NP,KK,K,HESS,C,SC,W)
                   WRITE (KANAL6,783) PNAME(KK),PNAME(K),C,SC,W
                   WRITE (21,783) PNAME(KK),PNAME(K),C,SC,W
788             CONTINUE
784          CONTINUE
      GOTO 1
C
C     *********************************************
C     *   AIRUM-CHECK  (NOTE: SPECIAL CASE ONLY)  *
C     *********************************************
C
790   CALL AIRUM(PARM,NP,KANAL6)
      GOTO 1
C
C     **********************************************************
C     *   ADJUST AGGREGATED SAMPLE SHARES WITH LEAST SQUARES   *
C     **********************************************************
C
800   IF (NXA.NE.1) GOTO 840
C
      ISTOP(1)=0
      ISTOP(2)=0
      ISTOP(3)=0
      ITERLS=2*NALT
      WRITE (KANAL6,811)
811   FORMAT(/' TYPE 1 TO OVERRIDE DEFAULT ITERATION PARAMETERS >')
      READ (KANAL5,'(I1)') IQUERY
      IF (IQUERY.EQ.1) THEN
         WRITE (KANAL6,812)
812      FORMAT(' ITERATION LIMIT >'
     *         /' CONV. CRITERION (PARM,GRAD,SSD; 0=ENABLE,1=DISABLE) >')
         READ (KANAL5,*) ITERLS,ISTOP
         END IF
      NQ=NALT1*NALT1+8*NALT1
C
      N1=NX+NXD*NDUMD
      DO 820 I=1,NALT1
```

```
820        PDUMMY(I)=PARM(N1+I)
      CALL LSFIT(PDUMMY,NALT1,SSD,IERLS,SSQDEV,ITERLS)
      WRITE (KANAL6,821) SSD,IERLS,ITERLS
821   FORMAT(//' ALTERNATIV SPECIFIC DUMMIES ADJUSTED:'
     *         /' SUM OF SQUARED DEVIATIONS:',F11.5
     *         /' DA.FL.PO. ITERATIONS (',I2,'):',I11
     *        //' DUMMY      INITIALVALUE    ADJUSTED VALUE')
      DO 822 I=1,NALT1
         WRITE (KANAL6,823) PNAME(N1+I),PARM(N1+I),PDUMMY(I)
823      FORMAT(' ',A8,3X,F9.4,9X,F9.4)
822      PARM(N1+I)=PDUMMY(I)
C
      CALL FUNC(PARM,NP,XLF,*830)
      DO 825 I=1,NP
         GRAD(I)=0.0
825      HESS(I,I)=0.0
      ITERAL=ITERLS
      NEVALF=0
      IER=IERLS
      METHOD=7
      MATPRT=0
      GOTO 150
C
830   WRITE (KANAL6,831)
831   FORMAT(' ERROR: FUNCTION UNDEFINED AT ADJUSTED DUMMIES')
      GOTO 1
C
840   WRITE (KANAL6,841) NXA
841   FORMAT(/' INCORRECT SETTING FOR ALTERNATIVE SPECIFIC DUMMIES:'
     *        /' NXA = ',I1,' .NE. 1 FOR CORRECT INITIALIZATION')
      GOTO 1
C
C     ************
C     *   EXIT   *
C     ************
C
99    RETURN
      END
      SUBROUTINE START(NPRINT,KANAL5,KANAL6,UPP,LOW,VERSIO)
C-----------------------------------------------------------------
C
C     START UP ROUTINE FOR GQTREE
C
C     1) FORTRAN UNITS FOR TERMINAL IN/OUTPUT
C     2) LOG(LARGEST) AND SMALLEST POSITIVE REAL*8 NUMBER
C     3) DATE, TIME, AND WEEKDAY OF LAST CHANGE
C
C-----------------------------------------------------------------
      REAL*8 UPP,LOW
      CHARACTER*21 VERSIO
C
C     IBM
C
      KANAL5=5
```

```
      KANAL6=6
      UPP=173.D0
      LOW=1.D-74
C
C     PRIME
C
      KANAL5=1
      KANAL6=1
      UPP=22622.D0
      LOW=1.D-9824
C
C     GQOPT OUTPUT
C
      NPRINT=KANAL6
C
C     LAST UPDATE
C
      VERSIO='03/30/84.15:43:52.Fri'
C
      RETURN
      END
      SUBROUTINE INDATA (KANAL5,KANAL6)
C-----------------------------------------------------------------------
-
C     DATA INPUT FOR GQTREE, CALCULATES WEIGHTED AND UNWEIGHTED MEANS
C     CHECKS DATA AND MEANS FOR CONSITENCY
C     KANAL5
C-----------------------------------------------------------------------
-
      IMPLICIT REAL*8 (A-H,O-Z)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD)
C
      REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS),DATA2(MAXNXD,MAXOBS)
      CHARACTER*20 DAFILE
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DATAX  / DATA1
      COMMON / DATAY  / DATA2
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
C
C     STORAGE FOR MEANS : NDW = MAXOBS,   THEN NOBS+2
C                         ND1 = MAXOBS-1, THEN NOBS+1
C
      NDD=MAXOBS-2
      ND1=MAXOBS-1
      NDW=MAXOBS
C
      WRITE (KANAL6,10)
10    FORMAT(//' ENTER DATA-INPUT-FILE >')
```

```
      READ (KANAL5,'(A20)') DAFILE
      OPEN (UNIT=7,STATUS='OLD',FILE=DAFILE,ACCESS='SEQUENTIAL')
      WRITE (KANAL6,12)
12    FORMAT(' ENTER MAX. NUMBER OF OBSERVATIONS >')
      READ (KANAL5,*) NNOBS
      IF (NNOBS.EQ.0) NNOBS=NDD
      IF (NNOBS.GT.NDD) THEN
         WRITE(KANAL6,*) 'WARNING: NOBS SET TO ',NDD
         NNOBS=NDD
         END IF
C
C     INITIALIZATIONS FOR MEANS AND COUNT
C
      NOBS=0
      DO 20 I=1,NALT
         DATA1(I,1,ND1)=0.0
         DATA1(I,1,NDW)=0.0
         DO 20 J=1,NX
            DATA1(I,J+1,ND1)=0.0
20          DATA1(I,J+1,NDW)=0.0
      DO 21 J=1,NXD
         DATA2(J,ND1)=0.0
21       DATA2(J,NDW)=0.0
C
C     LOOP THROUGH DATA
C
      DO 100 IOBS=1,NNOBS
         IC=0
         DO 120 I=1,NALT
            READ (7,END=200) DATA1(I,1,IOBS),
     *            (DATA1(I,J+1,IOBS),J=1,NX),(DATA2(J,IOBS),J=1,NXD)
            IF (DATA1(I,1,IOBS).EQ.1.) THEN
               IC=I
               WT=WEIGHT(IC)
               END IF
120         CONTINUE
C
         IF (IC.EQ.0) WRITE (KANAL6,130) IOBS
130      FORMAT(' ERROR: NO ALTERNATIVE CHOSEN IN OBSERVATION',I6)
C
         DATA1(IC,1,ND1) = DATA1(IC,1,ND1) + 1.0
         DATA1(IC,1,NDW) = DATA1(IC,1,NDW) + WT
         DO 140 I=1,NALT
         DO 140 J=1,NX
            IF (ABS(DATA1(I,J+1,IOBS)).GT.10000.0)
     *         WRITE (1,141) J+1,IOBS,DATA1(I,J+1,IOBS)
141            FORMAT(' DATA CHECK: X(IALT,K,IOBS)=',3I3,G20.10)
            DATA1(I,J+1,ND1) = DATA1(I,J+1,ND1) + DATA1(I,J+1,IOBS)
140         DATA1(I,J+1,NDW) = DATA1(I,J+1,NDW) + DATA1(I,J+1,IOBS)*WT
         DO 145 J=1,NXD
            IF (ABS(DATA2(J,IOBS)).GT.10000.0)
     *         WRITE (1,146) J,IOBS,DATA2(J,IOBS)
146            FORMAT(' DATA CHECK: Y(K,IOBS)=',2I3,G20.10)
            DATA2(J,ND1) = DATA2(J,ND1) + DATA2(J,IOBS)
```

```
145         DATA2(J,NDW) = DATA2(J,NDW) + DATA2(J,IOBS)*WT
C
         NOBS=NOBS+1
100      CONTINUE
C
C     SUMMARY
C
200   CLOSE (UNIT=7)
      IF (NOBS.EQ.0) THEN
         WRITE (KANAL6,*) 'ERROR: DATA-FILE EMPTY'
         RETURN
         END IF
      SUM=0.D0
      DO 220 I=1,NALT
         SUM=SUM+DATA1(I,1,ND1)*WEIGHT(I)
         DATA1(I,1,NOBS+1)=DATA1(I,1,ND1)/NOBS
         DATA1(I,1,NOBS+2)=DATA1(I,1,NDW)/NOBS
         DO 220 J=1,NX
            DATA1(I,J+1,NOBS+1)=DATA1(I,J+1,ND1)/NOBS
220         DATA1(I,J+1,NOBS+2)=DATA1(I,J+1,NDW)/NOBS
      DO 221 J=1,NXD
         DATA2(J,NOBS+1)=DATA2(J,ND1)/NOBS
221      DATA2(J,NOBS+2)=DATA2(J,NDW)/NOBS
C
      IF (DABS(SUM-NOBS).GT.1) WRITE (KANAL6,229) SUM,NOBS
229      FORMAT(' ERROR: WEIGHTS DO NOT SUM UP TO NOBS:',G20.10,I10)
C
      RETURN
      END
       SUBROUTINE UPDATE(PARM,NP,*)
C----------------------------------------------------------------------
C      UPDATES THE PARAMETERS IN COMMON BLOCKS:
C      (1) DISSIMILARITY PARAMETERS
C      (2) PARAMETERS FOR TREE DUMMIES
C----------------------------------------------------------------------
      REAL*8 PARM(NP)
C
      PARAMETER
     *  (MAXNXD =5,
     *   MAXNXM =20,
     *   MAXTH  =10,
     *   MAXTAU =5,
     *   MAXLEV =3,
     *   MAXLIM =5,
     *   MAXBRA =5,
     *   MAXALT =20)
C
      REAL*8  THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
     *        PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT),P
      INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT)
      LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDDUM,NOALT,NOAGE,
     *        L1,L2,L3,STDUMA,STDUMD,NALSTD,NALTRE,NAGSTD,NAGTRE
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DDISS  / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
```

```
     *                    NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
      COMMON / DMAPP  / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
      COMMON / DDUMM  / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
     *                    NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
     *                    NALSTD,NALTRE,NAGSTD,NAGTRE
C
      NFAIL=0
      IF (NTAU+NTH.EQ.0) GOTO 20
C
      IF (NTH.EQ.0) GOTO 9
C
      DO 8 K=1,NTH
         IF (PARM(MTH+K).LT.0.001) THEN
            PRINT 91,K,PARM(MTH+K)
            NFAIL=NFAIL+1
            END IF
         THETA(LOCLIM(K),LOCBRA(K))=PARM(MTH+K)
8        CONTINUE
C
9     IF (NTAU.EQ.0) GOTO 20
C
      DO 10 K=1,NTAU
         IF (PARM(MTAU+K).LT.0.001) THEN
            PRINT 92,K,PARM(MTAU+K)
            NFAIL=NFAIL+1
            END IF
         TAU(LOCTAU(K))=PARM(MTAU+K)
10       CONTINUE
C
20    IF (NXM.EQ.0) GOTO 90
C
      DO 25 I=1,NALT
         M1=MAP(I,1)
         M2=MAP(I,2)
         M3=MAP(I,3)
         IF (M1.NE.0) THEN
                      L1=.TRUE.
                      ELSE
                      L1=.FALSE.
                      END IF
         IF (M2.NE.0) THEN
                      L2=.TRUE.
                      ELSE
                      L2=.FALSE.
                      END IF
         IF (M3.NE.0) THEN
                      L3=.TRUE.
                      ELSE
                      L3=.FALSE.
                      END IF
         IF (NOALT) GOTO 26
         P=0.D0
         IF (L1) P=P+PARM(MXA+M1)
```

```
          IF (L2) P=P+PARM(MXA+M2)
          IF (L3) P=P+PARM(MXA+M3)
          PMASKD(I)=P
26        IF (NOAGE) GOTO 25
          DO 27 K=1,NXD
             P=0.D0
             IF (L1) P=P+PARM(INDAGE(K,M1))
             IF (L2) P=P+PARM(INDAGE(K,M2))
             IF (L3) P=P+PARM(INDAGE(K,M3))
             PMASKY(K,I)=P
27           CONTINUE
25        CONTINUE
C
90    IF (NFAIL.GT.0) RETURN 1
91    FORMAT(' ATTEMPT THETA(',I2,') = ',D12.4)
92    FORMAT(' ATTEMPT TAU(',I1,') = ',D12.4)
      RETURN
      END
C-----SUBROUTINE PACKAGE : FUNC,FP,SP,FDIFF,SDIFF---------------------
C
C     AXEL BOERSCH-SUPAN   SEP 06 , 1983
C
C--------------------------------------------------------------------
      SUBROUTINE FUNC(PARM,NP,XLF,*)
C--------------------------------------------------------------------
C
C  ** THIS SUBROUTINE EVALUATES AND ACCUMULATES THE LIKELIHOOD FUNCTION
C
C  ** FOR UNCON=.FALSE. IT ADJUSTS AFTER SUCCESSFUL EVALUATION THE
C     ALTERNATIVE SPECIFIC DUMMIES TO FIT THE AGGREGATE SAMPLE SHARES
C
C--------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1,
     *   MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS,
     *   MAXALS=MAXALT*(MAXALT+1)/2)
C
      REAL*8 WORK(MAXNP),PROBS(MAXALT),PDUMMY(MAXALT),NDUMMY(MAXALT)
      REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)
      LOGICAL UNCON
C
      COMMON / DIMEN  / NALT,NALT1,NPARM(7),NOBS
      COMMON / DATAX  / FDATA1
      COMMON / DATAY  / FDATA2
      COMMON / DCONST / UNCON,PDUMMY,NDUMMY
      COMMON / DWORK  / WORK,PROBS
```

```
      COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
      EXTERNAL SSQDEV
C
      ILOOP=1
      NFAIL=0
      NOBSF=NOBS/10
      CALL UPDATE(PARM,NP,*10)
C
C     EVALUATION OF LIKELIHOOD
C
10    NN1=1
      NN2=1
      INC=MAXALT*MAXNXY
      XLF=0.D0
      DO 100 IOBS = 1,NOBS
         NFAIL1=NFAIL
         CALL CONT(FDATA1(NN1),FDATA2(NN2),
     *             ILOOP,PARM,NP,XLFI,WORK,PROBS,NFAIL)
         NN1=NN1+INC
         NN2=NN2+MAXNXD
         IF (NFAIL.GT.NFAIL1) IOBF=IOBS
         IF (NFAIL.GT.NOBSF ) GOTO 97
         IF (NFAIL.GT.NFAIL1) GOTO 95
         XLF = XLF + XLFI
100      CONTINUE
C
C     SOLVE LEAST SQUARES PROBLEM TO FIT SAMPLE-SHARES
C
      IF (UNCON) GOTO 90
      ITERLS=2*NALT1
      CALL LSFIT(PDUMMY,NALT1,SSD,IER,SSQDEV,ITERLS)
      IF (IER.LT.0) GOTO 93
      GOTO 90
C
C     ERROR HANDLING, EXIT
C
90    RETURN
C
93    WRITE (KANAL6,94) SSD,IER,ITERLS
94    FORMAT(' SHARES NOT EXACT: SSD=',D10.4,', IER=',I2,', ITERLS=',I2)
      RETURN
C
95    IF (NFAIL.GT.0) WRITE (KANAL6,96) NFAIL,IOBF
96    FORMAT(' FUNC: NFAIL =',I2,', LAST OBS =',I4)
      RETURN 1
C
97    WRITE (KANAL6,98) NFAIL
98    FORMAT(' FUNC: ABORT B/O MORE THAN ',I4,' FAILURES')
99    RETURN 1
      END
C-----------------------------------------------------------------------
      SUBROUTINE FP(PARM,NP,XLF,GRAD,FUNC)
C-----------------------------------------------------------------------
C
```

```
C  ** THIS SUBROUTINE EVALUATES THE GRADIENT OF THE LIKELIHOOD-FUNCTION
C     (HOWEVER, IT WILL NOT AND MUST NOT UPDATE XLF)
C
C  ** FOR IDIFF2=1: THE BHHH APPROXIMATION OF THE HESSIAN IS PROVIDED
C                   [ NOTE: HESSIAN = -GRAD*GRAD' = - INV(COV) ]
C  ** FOR IDIFF2=4: THE EXACT HESSIAN IS CUMULATED
C
C---------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP),GRAD(NP)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1, MAXNPS=MAXNP*(MAXNP+1)/2,
     *   MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS)
C
      REAL*8 WORK(MAXNP),PROBS(MAXALT),HESS(MAXNPS),HESS2(MAXNPS)
      REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)
C
      COMMON / DIMEN  / NPARM(9),NOBS
      COMMON / DATAX  / FDATA1
      COMMON / DATAY  / FDATA2
      COMMON / DWORK  / WORK,PROBS
      COMMON / DCOVM  / HESS
      COMMON / DHESS  / HESS2
      COMMON / DIFF2  / IDIFF2
      COMMON / BOPT2  / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
      COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
      EXTERNAL FUNC
C
      ILOOP=0
      NFAIL=0
      CALL UPDATE(PARM,NP,*92)
C
C     EVALUATION OF GRADIENT (AND HESSIAN)
C
      DO 5 I=1,NP
 5       GRAD(I)=0.D0
      IF (IDIFF2.EQ.2.OR.IDIFF2.EQ.3) GOTO 12
      LK = NP*(NP+1)/2
      DO 10 I=1,LK
10       HESS(I)=0.D0
12    NN1=1
      NN2=1
      INC=MAXALT*MAXNXY
      DO 100 IOBS = 1,NOBS
         NFAIL1=NFAIL
         CALL CONT(FDATA1(NN1),FDATA2(NN2),
     *             ILOOP,PARM,NP,XLFI,WORK,PROBS,NFAIL)
         NN1=NN1+INC
```

```
         NN2=NN2+MAXNXD
         IF (NFAIL.GT.NFAIL1) IOBF=IOBS
         DO 30 J=1,NP
30          GRAD(J) = GRAD(J) + WORK(J)
C
         IF (IDIFF2.NE.1) GOTO 50
         IND=0
         DO 40 J=1,NP
         DO 40 K=1,J
            IND=IND+1
40          HESS(IND) = HESS(IND) - WORK(J) * WORK(K)
         GOTO 100
C
50       IF (IDIFF2.NE.4) GOTO 100
         DO 55 K=1,LK
55          HESS(K) = HESS(K) + HESS2(K)
C
100   CONTINUE
C
      IVAL=IVAL+2
      IF (NFAIL.GT.0) WRITE (KANAL6,97) NFAIL,IOBF
97    FORMAT(' FP: NFAIL=',I4,' LAST OBS=',I5)
      RETURN
92    WRITE (KANAL6,*) ' FP: DISS. PARM < 0.01'
      RETURN
      END
C---------------------------------------------------------------
      SUBROUTINE SP(PARM,NP,XLF,GRAD,SDERIV,FUNC)
C---------------------------------------------------------------
C
C     ARRAY OF SECOND DERIVATIVES:
C
C     ** IDIFF2=1 : FILLS BHHH-TRIANGLE MBHHH ON ARRAY SDERIV
C     ** IDIFF2=2 : COMPUTES SIMPLE FINITE SECOND DIFFERENCES
C     ** IDIFF2=3 : COMPUTES SYMMETRIC FINITE SECOND DIFFERENCES
C     ** IDIFF2=4 : FILLS TRIANGLE WITH ANAL. DERIVS ON SDERIV
C
C     FOR FINITE DIFFERENCES, GRAD HAS TO BE SUPPLIED
C
C---------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP),GRAD(NP),SDERIV(NP,NP)
C
      PARAMETER
     *  (MAXNP  =50,
     *   MAXNPS=MAXNP*(MAXNP+1)/2)
C
      REAL*8 MBHHH(MAXNPS)
C
      COMMON / DCOVM / MBHHH
      COMMON / DIMEN / NPARM(9),NOBS
      COMMON / DIFF2 / IDIFF2
      COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
      COMMON / BSTACK/ A(1)
```

```
      COMMON / BOPT1 / NPARS,JH,JFP,JFP1,JSP,JSP1,JA1,JS
      COMMON / BFIDIF/ FDFRAC,FDMIN
      EXTERNAL FUNC
C
      IF (IDIFF2.EQ.2 .OR. IDIFF2.EQ.3) GOTO 40
C
C     REFILL OF SDERIV (IDIFF2=1,4)
C
      IND=0
      DO 35 J=1,NP
      DO 35 K=1,J
         IND=IND+1
         ZZZ=MBHHH(IND)
         SDERIV(J,K)=ZZZ
35       SDERIV(K,J)=ZZZ
      RETURN
C
C     COMPUTATION OF FINITE DIFFERENCES (IDIFF2=2,3)
C
40    DO 90 I=1,NP
         AI=PARM(I)
         EI=DMAX1(DABS(AI)*FDFRAC,FDMIN)
         PARM(I)=AI+EI
         CALL FP (PARM,NP,XLF2,A(JA1),FUNC)
         JA2I=JA1+I-1
         IF (IDIFF2.EQ.2) GO TO 475
C
C        SYMMETRIC FIRST DIFFERENCES OF GRAD
C
         PARM(I)=AI-EI
         CALL FP (PARM,NP,XLF1,A(JA1+NP),FUNC)
         JA1I=JA2I+NP
         SDERIV(I,I)=(A(JA2I)-A(JA1I))/(2.D0*EI)
         DO 410 J=1,NP
            IF (J.EQ.I) GO TO 410
            JA2J=JA1+J-1
            SDERIV(I,J)=(A(JA2J)-A(JA2J+NP))/(2.D0*EI)
            IF (J.GT.I) GO TO 410
            SDERIV(I,J)=(SDERIV(I,J)+SDERIV(J,I))/2.0D0
            SDERIV(J,I)=SDERIV(I,J)
 410        CONTINUE
         GO TO 90
C
C        SIMPLE FIRST DIFFERENCES OF GRAD
C
 475     SDERIV(I,I)=(A(JA2I)-GRAD(I))/EI
         K=I-1
         IF (K.EQ.0) GO TO 90
         DO 476  J=1,K
            SDERIV(I,J)=(A(JA1+J-1)-GRAD(J))/EI
 476        SDERIV(J,I)=SDERIV(I,J)
 90      PARM(I)=AI
C
      RETURN
```

```
      END
C-----------------------------------------------------------------
      SUBROUTINE FDIFF(AO,NP,AFUO,FPD,FUNC)
C-----------------------------------------------------------------
C     FIRST ORDER SYMMETRIC DIFFERENCE QUOTIENT (CENTRAL APPROX.)
C-----------------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AO(NP),FPD(NP)
      COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
      DO 90 I=1,NP
         AI=AO(I)
         EI=DMAX1(DABS(AI)*.1D-3,.1D-5)
11       AO(I)=AI+EI
         CALL FUNC(AO,NP,AFU2,*85)
         IVAL=IVAL+1
         AO(I)=AI-EI
         CALL FUNC(AO,NP,AFU1,*85)
         IVAL=IVAL+1
         FPD(I)=(AFU2-AFU1)/(2.DO*EI)
         GO TO 90
 85         EI=EI/10.DO
            IF (EI.LE.1.D-10) GOTO 130
            GOTO 11
 90      AO(I)=AI
      RETURN
 130  AO(I)=AI
      IER=-2
      RETURN
      END
C-----------------------------------------------------------------
      SUBROUTINE SDIFF(AO,NP,AFUO,FPD,SPD,FUNC)
C-----------------------------------------------------------------
C     SECOND ORDER SYMMETRIC DIFFERENCE QUOTIENT
C
C     STAR:   AFU11          AFU21
C             AFU1   AFUO    AFU2
C             AFU12          AFU22
C-----------------------------------------------------------------
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AO(NP),SPD(NP,NP),FPD(NP)
      COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
      DO 90 I=1,NP
         AI=AO(I)
         EI=DMAX1(DABS(AI)*.1D-3,.1D-5)
11       AO(I)=AI+EI
         CALL FUNC(AO,NP,AFU2,*85)
         IVAL=IVAL+1
         AO(I)=AI-EI
         CALL FUNC(AO,NP,AFU1,*85)
         IVAL=IVAL+1
13       SPD(I,I)=(AFU2-2.DO*AFUO+AFU1)/(EI*EI)
         IF (I.EQ.1) GOTO 90
         K=I-1
         DO 80 J=1,K
```

```
               AJ=AO(J)
               EJ=DMAX1(DABS(AJ)*.1D-3,.1D-5)
 30            AO(J)=AJ+EJ
               AO(I)=AI+EI
               CALL FUNC(AO,NP,AFU21,*70)
               IVAL=IVAL+1
               AO(I)=AI-EI
               CALL FUNC(AO,NP,AFU11,*70)
               IVAL=IVAL+1
               AO(J)=AJ-EJ
               CALL FUNC(AO,NP,AFU12,*70)
               IVAL=IVAL+1
               AO(I)=AI+EI
               CALL FUNC(AO,NP,AFU22,*70)
               IVAL=IVAL+1
               SPD(I,J)=(AFU21-AFU11-AFU22+AFU12)/(4.D0*EI*EJ)
               SPD(J,I)=SPD(I,J)
               GOTO 80
 70            EJ=EJ/10.D0
               IF (EJ.LE.1.D-10) GOTO 125
               GOTO 30
 80            AO(J)=AJ
            GOTO 90
 85         EI=EI/10.D0
            IF (EI.LE.1.D-10) GOTO 130
            GOTO 11
 90         AO(I)=AI
         RETURN
 125     AO(J)=AJ
 130     AO(I)=AI
         IER=-2
         RETURN
         END
         SUBROUTINE CONT(DATA1,DATA2,ILOOP,PARM,NP,XLFI,WORK,PROBS,NFAIL)
C-----------------------------------------------------------------------
C
C        THREE LEVEL NESTED LOGIT MODEL
C        ==============================
C
C        CONTRIBUTION OF THE N-TH OBSERVATION
C        - TO THE LIKELIHOOD (XLFI , ILOOP=0,1,2)
C        - TO ITS DERIVATIVE (WORK , ILOOP=0)
C        - TO THE HESSIAN    (HESS , ILOOP=0 AND IDIFF2=4, MNL ONLY)
C        - PROBABILITY-SHARE (PROBS, ILOOP=2)
C        EXCEPT ILOOP=0, WORK(1) CARRIES UTILITY-LEVEL INCO
C
C        NORMALIZED VERSION  OCT 31 , 1983   23.00    AXEL BOERSCH-SUPAN
C
C-----------------------------------------------------------------------
C
C        TREE STRUCTURE :
C
C        LEVEL:  DATA:  PARMS:  EXP(INCL.VALUE):  DISSIMILARITY-PARMS:
C
```

```
C       0                           EINCO
C       1      (Y)     (ALPHA)      EINC1                TAU
C       2      (Z)     (GAMMA)      EINC2                THETA
C       3       X       BETA
C
C     FOR ARRAY DIMENSIONS, SEE MAIN
C
C---------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP)
      REAL*4 DATA1(*),DATA2(*)
C
      PARAMETER
     *  (MAXNP  =50,
     *   MAXTH  =10,
     *   MAXTAU =5,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXNXM =20,
     *   MAXLEV =3,
     *   MAXLIM =5,
     *   MAXBRA =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD, MAXNPS=MAXNP*(MAXNP+1)/2)
C
      REAL*8 UPP,LOW,ONE/1.D0/,ZERO/0.D0/
      INTEGER IZERO/0/
C
      REAL*8 THETA(MAXLIM,MAXBRA),TAU(MAXLIM),WORK(MAXNP),
     *        EXB(MAXALT),EZG(MAXALT),EYA(MAXLIM),
     *        XB(MAXALT),ZG(MAXALT),YA(MAXLIM),
     *        EINC2(MAXLIM,MAXBRA),EINC1(MAXLIM),EINCO,
     *        INC2(MAXLIM,MAXBRA),INC1(MAXLIM),INCO,INC2C,INC1C,
     *        DER2B(MAXLIM,MAXBRA,MAXNP).DER1B(MAXLIM,MAXNP),
     *        DER2T(MAXLIM,MAXBRA,MAXTH),DER1T(MAXLIM,MAXTH),
     *        DER1U(MAXLIM,MAXTAU),PROBS(MAXALT),
     *        PDUMMY(MAXALT),NDUMMY(MAXALT),
     *        PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT),
     *        ZBAR(MAXNP),ZIZBAR(MAXALT,MAXNP),HESS(MAXNPS)
      INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT)
      LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDDUM,NOALT,NOAGE,DUMP,
     *        LDUMMY,STDUMA,STDUMD,NALSTD,NALTRE,NAGSTD,NAGTRE,UNCON
C
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DDISS  / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
     *                  NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
      COMMON / DCONT  / XB,ZG,YA,INC2,INC1,
     *                  EXB,EZG,EYA,EINC2,EINC1,
     *                  DER2B,DER1B,DER2T,DER1T,DER1U,
     *                  ZBAR,ZIZBAR,UPP,LOW,
     *                  MS3K(MAXTH),ML3K(MAXTH),MS2K(MAXLIM)
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
      COMMON / DMAPP  / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
      COMMON / DDUMM  / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
```

```
     *                   NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
     *                   NALSTD,NALTRE,NAGSTD,NAGTRE
      COMMON / DHESS  / HESS
      COMMON / DIFF2  / IDIFF2
      COMMON / DCONST / UNCON,PDUMMY,NDUMMY
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
      COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
C
      DUMP=.FALSE.
C
C     INNER PRODUCTS XB(I)
C
      IALT=IZERO
      XBM=ZERO
      DO 9 I=1,NALT
         IF (DATA1(I).EQ.1) IALT=I
         XBC = ZERO
         NN1 = MAXALT
         DO 7 K=1,NX
            XBC = XBC + DATA1(NN1+I)*PARM(K)
7           NN1 = NN1 + MAXALT
         IF (NAGTRE) GOTO 3
         DO 4 K=1,NXD
4           XBC = XBC + DATA2(K)*PMASKY(K,I)
3        IF (NALTRE) GOTO 2
            XBC = XBC + PMASKD(I)
2        IF (I.EQ.NALT) GOTO 5
         IF (NAGSTD) GOTO 6
         DO 1 K=1,NXD
1           XBC = XBC + DATA2(K)*PARM(INDAGE(K,I))
6        IF (NALSTD) GOTO 8
            XBC = XBC + PARM(MXA+I)
8        IF (UNCON) GOTO 5
            XBC = XBC + PDUMMY(I)
5        XB(I)=XBC
         XBM=XBM+XBC
9        CONTINUE
C
C     CHECK AND MEAN OF XB(I)
C
      IF (IALT.EQ.0) GOTO 9005
      XBM=XBM/NALT
C
C     INCLUSIVE VALUES
C
      I=IZERO
      IB=IZERO
      EINCO=ZERO
      DO 10 I1=1,NL1
         ML2=NL2(I1)
         TA =TAU(I1)
         DINC1=ZERO
         DO 20 I2=1,ML2
            IB =IB+1
```

```
            ML3=NL3(I1,I2)
            TH =THETA(I1,I2)
            THT=TH/TA
            DINC2=ZERO
            DO 30 I3=1,ML3
               I=I+1
C
C              CHOICE
C
               IF (I.NE.IALT) GOTO 40
                  IC1=I1
                  IC2=I2
                  IBPA=IB
C
C              INNER PRODUCT XB/TH
C
C              /* NORMALIZE BY SUBTRACTION OF XBM */
40             XBI = (XB(I)-XBM) / TH
               IF (XBI.GT.UPP) GOTO 9001
               XB(I)=XBI
               ZZZ=DEXP(XBI)
               EXB(I)=ZZZ
               DINC2=DINC2+ZZZ
30             CONTINUE
C
C           LOWER LEVEL INCLUSIVE VALUE  INC2(I1,I2)
C
            IF (LEVEL1) GOTO 21
            IF (DINC2.LT.LOW) GOTO 9002
            EINC2(I1,I2)=DINC2
            DINC2=DLOG(DINC2)
            INC2(I1,I2)=DINC2
            DINC2=THT*DINC2
            IF (DINC2.GT.UPP) GOTO 9002
            ZG(IB)=DINC2
            DINC2=DEXP(DINC2)
            EZG(IB)=DINC2
21          DINC1=DINC1+DINC2
20          CONTINUE

C        HIGHER LEVEL INCLUSIVE VALUE  DLOG(INC1(I1))
C
         EINC1(I1)=ONE
         IF (LEVEL2) GOTO 11
         IF (DINC1.LT.LOW) GOTO 9003
         EINC1(I1)=DINC1
         DINC1=DLOG(DINC1)
         INC1(I1)=DINC1
         DINC1=TA*DINC1
         IF (DINC1.GT.UPP) GOTO 9003
         YA(I1)=DINC1
         DINC1=DEXP(DINC1)
         EYA(I1)=DINC1
11       EINC0=EINC0+DINC1
```

```
10          CONTINUE
       IF (EINCO.LT.LOW) GOTO 9004
       INCO=DLOG(EINCO)
C
       XBC  =XB(IALT)
       INC2C=INC2(IC1,IC2)
       INC1C=INC1(IC1)
C
C      LOG LIKELIHOOD
C
       XLFI = XBC - INCO
       WORK(1) = INCO
C       WRITE (KANAL6,9900) XLFI
C9900   FORMAT(' $X$ MNL-XLFI=',D14.5)
       IF (LEVEL1) GOTO 51
          THC  = THETA(IC1,IC2)
          TAC  = TAU(IC1)
          TH1  = THC/TAC-ONE
          XLFI = XLFI + TH1*INC2C
          IF (LEVEL2) GOTO 50
             TA1  = TAC-ONE
             XLFI = XLFI + TA1*INC1C
C             WRITE (KANAL6,9901) XLFI
C9901         FORMAT(' $X$ NMNL-XLFI=',D14.5)
C
50     IF (ILOOP.EQ.1) GOTO 5000
       IF (ILOOP.EQ.0) GOTO 70
C
C      PROBABILITY-SHARES
C
51     DO 52 I=1,NALT
          PROBS(I)=EXB(I)/EINCO
52        CONTINUE
       IF (LEVEL1) GOTO 69
       I=IZERO
       DO 60 I1=1,NL1
          ML2=NL2(I1)
          IF (LEVEL2) GOTO 61
          DINC1=(TAU(I1)-ONE)*INC1(I1)
61        CONTINUE
          DO 60 I2=1,ML2
             ML3=NL3(I1,I2)
             DINC2=(THETA(I1,I2)/TAU(I1)-ONE)*INC2(I1,I2)
             DO 60 I3=1,ML3
                I=I+1
                IF (LEVEL2) GOTO 62
                PROBS(I)=PROBS(I)*DEXP(DINC1+DINC2)
                GOTO 60
62              PROBS(I)=PROBS(I)*DEXP(DINC2)
60              CONTINUE
69     IF (ILOOP.EQ.0) GOTO 4000
       GOTO 5000
C
C      SWITCHES FOR DUMMY
```

```
C
70    IF (IALT.EQ.NALT) THEN
                        LDUMMY=.FALSE.
                        ELSE
                        LDUMMY=.TRUE.
                        END IF
C
C     DERIVATIVES OF INCLUSIVE VALUES LEVEL 2
C
C     W.R.T. BETA
C
      I=IZERO
      DO 80 I1=1,NL1
         ML2=NL2(I1)
         DO 80 I2=1,ML2
            ML3=NL3(I1,I2)
            DINC2=EINC2(I1,I2)*THETA(I1,I2)
            NN1=MAXALT
            DO 82 K=1,NX
               ZZ=ZERO
               DO 81 I3=1,ML3
                  I=I+1
                  ZZ=ZZ + EXB(I) * DATA1(NN1+I)
81                CONTINUE
               I=I-ML3
               DER2B(I1,I2,K) = ZZ / DINC2
C               WRITE (KANAL6,9907) I1,I2,K,DER2B(I1,I2,K)
C9907           FORMAT(' $X$ DER2B(',3I2,')=',D18.4)
               NN1=NN1+MAXALT
82             CONTINUE
C
            IF (NALTRE) GOTO 823
            DO 821 K=1,NXM
               ZZ=ZERO
               DO 822 I3=1,ML3
                  I=I+1
                  IF (LMAP(K,I)) ZZ=ZZ + EXB(I)
822               CONTINUE
               I=I-ML3
               DER2B(I1,I2,K+MXA) = ZZ / DINC2
C              WRITE (KANAL6,9907) I1,I2,K+MXA,DER2B(I1,I2,K+MXA)
821            CONTINUE
C
823         IF (NAGTRE) GOTO 826
            DO 824 K=1,NXM
            DO 824 K1=1,NXD
               ZZ=ZERO
               KK=INDAGE(K1,K)
               DO 825 I3=1,ML3
                  I=I+1
                  IF (LMAP(K,I)) ZZ=ZZ + EXB(I)*DATA2(K1)
825               CONTINUE
               I=I-ML3
               DER2B(I1,I2,KK) = ZZ / DINC2
```

```
C               WRITE (KANAL6,9907) I1,I2,KK,DER2B(I1,I2,KK)
824             CONTINUE
C
826          IF (NAGSTD) GOTO 829
             DO 827 I3=1,ML3
                I=I+1
                IF (I.EQ.NALT) GOTO 828
                DO 827 K1=1,NXD
                   ZZ=EXB(I) * DATA2(K1)
                   KK=INDAGE(K1,I)
                   DER2B(I1,I2,KK) = ZZ / DINC2
C                  WRITE (KANAL6,9907) I1,I2,KK,DER2B(I1,I2,KK)
827                CONTINUE
828          I=I-ML3
C
829          IF (NALSTD) GOTO 83
             DO 830 I3=1,ML3
                I=I+1
                IF (I.EQ.NALT) GOTO 831
                DER2B(I1,I2,MXA+I) = EXB(I) / DINC2
C               WRITE (KANAL6,9907) I1,I2,MXA+I,DER2B(I1,I2,MXA+I)
830             CONTINUE
831          I=I-ML3
C
83           I=I+ML3
80           CONTINUE
C
C      W.R.T. THETA
C
       IF (NTH.EQ.0) GOTO 89
C
       DO 85 K =1,NTH
          NT11=LOCLIM(K)
          NT12=LOCBRA(K)
          I   =MS3K(K)
          ML3 =ML3K(K)
          ZZ=ZERO
          DO 86 I3=1,ML3
             I=I+1
86           ZZ=ZZ + EXB(I) * XB(I)
          DER2T(NT11,NT12,K) = -ZZ / EINC2(NT11,NT12) / THETA(NT11,NT12)
C          WRITE (KANAL6,9921) NT11,NT12,K,DER2T(NT11,NT12,K)
C9921      FORMAT(' $X$ DER2T(',3I2,')=',D18.6)
85        CONTINUE
C
89     CONTINUE
C
C
C      DERIVATIVES OF INCLUSIVE VALUES LEVEL 1
C
C      W.R.T. BETA
C
       DO 93 K=1,MTH
       I=IZERO
```

```
      DO 93 I1=1,NL1
         ML2=NL2(I1)
         ZZ=ZERO
         DO 94 I2=1,ML2
            I=I+1
94          ZZ = ZZ + EZG(I) * DER2B(I1,I2,K) * THETA (I1,I2)
         DER1B(I1,K) = ZZ / EINC1(I1) / TAU(I1)
C         WRITE (KANAL6,9908) I1,K,DER1B(I1,K)
C9908     FORMAT(' $X$ DER1B(',2I2,')=',D18.4)
93       CONTINUE
C
C     W.R.T. THETA
C
      IF (NTH.EQ.0) GOTO 99
C
      DO 95 K=1,NTH
         NT11=LOCLIM(K)
         NT12=LOCBRA(K)
         ZZZ = EINC2(NT11,NT12) ** ( THETA(NT11,NT12)/TAU(NT11) )
         ZZ  = ZZZ * (   INC2(NT11,NT12)
     *                 + THETA(NT11,NT12) * DER2T(NT11,NT12,K) )
         DER1T(NT11,K) = ZZ / EINC1(NT11) / TAU(NT11)
C         WRITE (KANAL6,9922) NT11,K,DER1T(NT11,K)
C9922     FORMAT(' $X$ DER1T(',2I2,')=',D18.6)
95       CONTINUE
C
C     W.R.T. TAU
C
99    IF (NTAU.EQ.0) GOTO 920
C
      DO 921 K=1,NTAU
         NT1=LOCTAU(K)
         I  =MS2K(NT1)
         ML2=NL2(NT1)
         ZZZ=ZERO
         DO 922 I2=1,ML2
            I=I+1
922         ZZZ=ZZZ + EZG(I) * ZG(I)
         DER1U(NT1,K) = - ZZZ / EINC1(NT1) / TAU(NT1)
C        WRITE (KANAL6,9211) NT1,K,DER1U(NT1,K)
9211     FORMAT(' DER1U(',2I2,')=',D14.5)
921      CONTINUE
C
920   CONTINUE
C
C     DERIVATIVES OF THE LOG LIKELIHOOD (THREE LEVEL TREE)
C
      IF (LEVEL2) GOTO 200
C
C     W.R.T. BETA
C
      NN1 = MAXALT
      DO 120 K=1,NX
         ZZ=ZERO
```

```
         DO 121 I1=1,NL1
121         ZZ = ZZ + EYA(I1) * DER1B(I1,K) * TAU(I1)
         ZZZ = DATA1(NN1+IALT)/THC - ZZ/EINCO
     *        + TH1 * DER2B(IC1,IC2,K) + TA1 * DER1B(IC1,K)
         NN1 = NN1 + MAXALT
120      WORK(K) = ZZZ
C
      IF (NALTRE) GOTO 1223
      DO 1221 K=1,NXM
         K1=K+MXA
         ZZ=ZERO
         DO 1222 I1=1,NL1
1222        ZZ=ZZ + EYA(I1)*DER1B(I1,K1)*TAU(I1)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K1) + TA1 * DER1B(IC1,K1)
         IF (LMAP(K,IALT)) ZZ = ZZ + ONE/THC
         WORK(K1)=ZZ
1221     CONTINUE
C
1223  IF (NAGTRE) GOTO 122
      DO 1224 K=1,NXM
      DO 1224 K2=1,NXD
         K1=INDAGE(K2,K)
         ZZ=ZERO
         DO 1225 I1=1,NL1
1225        ZZ=ZZ + EYA(I1)*DER1B(I1,K1)*TAU(I1)
         ZZ = -ZZ/EINCO + TH1 *DER2B(IC1,IC2,K1) + TA1 *DER1B(IC1,K1)
         IF (LMAP(K,IALT)) ZZ = ZZ + DATA2(K2)/THC
         WORK(K1)=ZZ
1224     CONTINUE
C
122   IF (NAGSTD) GOTO 123
      K=NX
      DO 125 K1=1,NXD
      DO 125 K2=1,NALT1
         K=K+1
         ZZ=ZERO
         DO 126 I1=1,NL1
126         ZZ = ZZ + EYA(I1) * DER1B(I1,K) * TAU(I1)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K) + TA1 * DER1B(IC1,K)
125      WORK(K) = ZZ
      IF (.NOT.LDUMMY) GOTO 123
      DO 124 K=1,NXD
124      WORK(INDAGE(K,IALT)) = WORK(INDAGE(K,IALT)) + DATA2(K)/THC
C
123   IF (NALSTD) GOTO 129
      NX1=MXA+1
      DO 127 K=NX1,MTH
         ZZ=ZERO
         DO 128 I1=1,NL1
128         ZZ = ZZ + EYA(I1) * DER1B(I1,K) * TAU(I1)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K) + TA1 * DER1B(IC1,K)
127      WORK(K) = ZZ
      IF (LDUMMY) WORK(MXA+IALT) = WORK(MXA+IALT) + ONE/THC
129   CONTINUE
```

```
C
C     W.R.T. THETA
C
      IF (NTH.EQ.0) GOTO 139
C
      DO 130 K=1,NTH
         NT11=LOCLIM(K)
         NT12=LOCBRA(K)
         ZZZ = -EYA(NT11) * DER1T(NT11,K) * TAU(NT11)/ EINCO
         IF (IC1.NE.NT11) GOTO 131
            ZZZ = ZZZ + TA1 * DER1T(IC1,K)
            IF (IC2.NE.NT12) GOTO 131
               ZZZ = ZZZ + INC2C/TAC + TH1*DER2T(IC1,IC2,K) - XBC/THC
131      WORK(MTH+K) = ZZZ
130      CONTINUE
C
139   CONTINUE
C
C     W.R.T. TAU
C
      IF (NTAU.EQ.0) GOTO 149
C
      DO 140 K=1,NTAU
         NT1=LOCTAU(K)
         ZZZ= -EYA(NT1) * ( INC1(NT1)+TAU(NT1)*DER1U(NT1,K) ) / EINCO
         IF (IC1.NE.NT1) GOTO 140
            ZZZ = ZZZ + INC1C + TA1*DER1U(IC1,K) - INC2C*THC/TAC/TAC
C         WRITE (KANAL6,9914) K,ZZZ
C9914     FORMAT(' $X$ WORK(MTAU+',I2,')=',D14.5)
140      WORK(MTAU+K) = ZZZ
C
149   GOTO 5000
C
C     DERIVATIVES OF THE LOG LIKELIHOOD (TWO LEVEL TREE)
C
C     W.R.T. BETA
C
200   NN1 = MAXALT
      DO 220 K=1,NX
         ZZ=ZERO
         DO 221 I1=1,NL1
221         ZZ = ZZ + DER1B(I1,K)
         ZZZ = DATA1(NN1+IALT)/THC - ZZ/EINCO + TH1*DER2B(IC1,IC2,K)
         NN1 = NN1 + MAXALT
220      WORK(K) = ZZZ
C
      IF (NALTRE) GOTO 2223
      DO 2221 K=1,NXM
         K1=K+MXA
         ZZ=ZERO
         DO 2222 I1=1,NL1
2222        ZZ=ZZ + DER1B(I1,K1)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K1)
         IF (LMAP(K,IALT)) ZZ = ZZ + ONE/THC
```

```
         WORK(K1)=ZZ
2221     CONTINUE
C
2223  IF (NAGTRE) GOTO 222
      DO 2224 K=1,NXM
      DO 2224 K2=1,NXD
         K1=INDAGE(K2,K)
         ZZ=ZERO
         DO 2225 I1=1,NL1
2225        ZZ=ZZ + DER1B(I1,K1)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K1)
         IF (LMAP(K, IALT)) ZZ = ZZ + DATA2(K2)/THC
         WORK(K1)=ZZ
2224     CONTINUE
C
222   IF (NAGSTD) GOTO 223
      K=NX
      DO 225 K1=1,NXD
      DO 225 K2=1,NALT1
         K=K+1
         ZZ=ZERO
         DO 226 I1=1,NL1
226         ZZ = ZZ + DER1B(I1,K)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K)
225      WORK(K) = ZZ
      IF (.NOT.LDUMMY) GOTO 223
      DO 224 K=1,NXD
224      WORK(INDAGE(K,IALT)) = WORK(INDAGE(K,IALT)) + DATA2(K)/THC
C
223   IF (NALSTD) GOTO 229
      NX1=MXA+1
      DO 227 K=NX1,MTH
         ZZ=ZERO
         DO 228 I1=1,NL1
228         ZZ = ZZ + DER1B(I1,K)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K)
227      WORK(K) = ZZ
      IF (LDUMMY) WORK(MXA+IALT) = WORK(MXA+IALT) + ONE/THC
229   CONTINUE
C
C     W.R.T. THETA
C
      IF (NTH.EQ.0) GOTO 239
C
      DO 230 K=1,NTH
         NT11=LOCLIM(K)
         NT12=LOCBRA(K)
         ZZZ = -DER1T(NT11,K) / EINCO
            IF (IC2.NE.NT12) GOTO 231
               ZZZ = ZZZ + INC2C + TH1 * DER2T(IC1,IC2,K) - XBC/THC
C         WRITE (KANAL6,9923) K,ZZZ
C9923     FORMAT(' $X$ WORK(MTH+',I2,')=',D14.5)
231      WORK(MTH+K) = ZZZ
230      CONTINUE
```

```
C
239   GOTO 5000
C
C     DERIVATIVES OF THE LOG LIKELIHOOD (SIMPLE MNL-VERSION)
C
4000  NN1 = MAXALT
      DO 4020 K=1,NX
         ZZ = ZERO
         ZB = ZERO
         DO 4021 I=1,NALT
            DA  = DATA1(NN1+I)
            ZP  = PROBS(I) * DA
            ZZ  = ZZ  +  DATA1(I) * DA - ZP
            ZB  = ZB  +  ZP
4021        CONTINUE
         NN1 = NN1 + MAXALT
         ZBAR(K) = ZB
4020     WORK(K) = ZZ
C
      IF (NALTRE.AND.NAGTRE) GOTO 4039
      DO 4036 K=NX+1,MTH
         ZBAR(K) = ZERO
4036     WORK(K) = ZERO
      DO 4031 K=1,NXM
         K1 = K + MXA
         DO 4032 M=1,MAPLEN(K)
            I   = MAPTR(K,M)
            ZZZ = DATA1(I) - PROBS(I)
            IF (NALTRE) GOTO 4033
            WORK(K1) = WORK(K1) + ZZZ
            ZBAR(K1) = ZBAR(K1) + PROBS(I)
4033        IF (NAGTRE) GOTO 4032
            DO 4034 K2=1,NXD
               INDEX = INDAGE(K2,K)
               ZBAR(INDEX) = ZBAR(INDEX) + PROBS(I) * DATA2(K2)
4034           WORK(INDEX) = WORK(INDEX) + ZZZ      * DATA2(K2)
4032        CONTINUE
4031     CONTINUE
C
4039  IF (NAGSTD) GOTO 4040
      K=NX
      DO 4030 K1=1,NXD
      DO 4030 K2=1,NALT1
         K=K+1
         WORK(K) = ( DATA1(K2) - PROBS(K2) ) * DATA2(K1)
         ZBAR(K) = PROBS(K2) * DATA2(K1)
4030     CONTINUE
C
4040  IF (NALSTD) GOTO 4100
      DO 4041 K=1,NALT1
         WORK(K+MXA) = DATA1(K) - PROBS(K)
         ZBAR(K+MXA) = PROBS(K)
4041     CONTINUE
C
```

```
C     EXACT SECOND DERIVATIVES (SIMPLE MNL-VERSION)
C
4100  IF (IDIFF2.NE.4) GOTO 5000
      NN1=IZERO
      DO 4105 K=1,NX
         NN1=NN1+MAXALT
         DO 4105 I=1,NALT
            ZIZBAR(I,K)=DATA1(NN1+I)-ZBAR(K)
4105        CONTINUE
C
4110  IF (NAGTRE) GOTO 4120
      K=NX
      DO 4115 K1=1,NXD
      DO 4115 K2=1,NXM
         K=K+1
         DO 4115 I=1,NALT
            DA=ZERO
            IF (LMAP(K2,I)) DA=DATA2(K1)
            ZIZBAR(I,K)=DA-ZBAR(K)
4115        CONTINUE
C
4120  IF (NALTRE) GOTO 4130
      K=MXA
      DO 4125 K2=1,NXM
         K=K+1
         DO 4125 I=1,NALT
            DA=ZERO
            IF (LMAP(K2,I)) DA=ONE
            ZIZBAR(I,K)=DA-ZBAR(K)
4125        CONTINUE
C
4130  IF (NAGSTD) GOTO 4140
      K=NX
      DO 4135 K1=1,NXD
      DO 4135 K2=1,NALT1
         K=K+1
         DO 4135 I=1,NALT
            DA=ZERO
            IF (K2.EQ.I) DA=DATA2(K1)
            ZIZBAR(I,K)=DA-ZBAR(K)
4135        CONTINUE
C
4140  IF (NALSTD) GOTO 4200
      K=MXA
      DO 4145 K2=1,NALT1
         K=K+1
         DO 4145 I=1,NALT
            DA=ZERO
            IF (K2.EQ.I) DA=ONE
            ZIZBAR(I,K)=DA-ZBAR(K)
4145        CONTINUE
C
C     HESSIAN
C
```

```
4200  KK=IZERO
      DO 4205 J=1,NP
      DO 4205 K=1,J
         KK=KK+1
         ZZ=ZERO
         DO 4210 I=1,NALT
            ZZ = ZZ + ZIZBAR(I,J)*PROBS(I)*ZIZBAR(I,K)
4210        CONTINUE
         HESS(KK)=-ZZ
4205     CONTINUE
C
C     APPLY WESML WEIGHTS TO LIKELIHOOD, GRADIENT, AND HESSIAN
C
5000  IF (RELSIZ.EQ.0.0) GOTO 9999
C
      WT=WEIGHT(IALT)
      XLFI=XLFI*WT
      IF (ILOOP.EQ.1) GOTO 9999
C
      DO 5001 I=1,NP
5001     WORK(I)=WORK(I)*WT
      IF (IDIFF2.NE.4) GOTO 9999
C
      KK=1
      DO 5004 J=1,NP
      DO 5004 K=1,J
         KK=KK+1
5004     HESS(KK)=HESS(KK)*WT
      GOTO 9999
C
C     FAILURE DUE TO UNDER- OR OVERFLOW
C
9001  CONTINUE
      WRITE (KANAL6,*) 'XBI',XBI,' IALT',IALT
      GOTO 9000
9002  CONTINUE
      WRITE (KANAL6,*) 'DINC2',DINC2,' IALT',IALT
      GOTO 9000
9003  CONTINUE
      WRITE (KANAL6,*) 'DINC1',DINC1,' IALT',IALT
      GOTO 9000
9004  CONTINUE
      WRITE (KANAL6,*) 'EINCO',EINCO,' IALT',IALT
      GOTO 9000
9005  CONTINUE
      WRITE (KANAL6,*) 'CH=0,IALT',IALT
C
9000  IF (.NOT.DUMP) GOTO 9998
C9000  WRITE (KANAL6,*) 'DUMP OF LAST RECORD:'
      DO 9091 I=1,NALT
         WRITE (KANAL6,*) 'CH',I,DATA1(I)
         NN1=MAXALT
         DO 9092 K=1,NX
            WRITE (KANAL6,*) 'X ',DATA1(NN1+I),PARM(K)
```

```
9092        NN1=NN1+MAXALT
         DO 9093 K=1,NXD
9093        WRITE (KANAL6,*) 'XD',DATA2(K),PMASKY(K,I)
         WRITE (KANAL6,*) 'DU',PARM(MXA+I)
         WRITE (KANAL6,*) 'XBI',XB(I)
9091     CONTINUE
      WRITE (KANAL6,*) 'TH',(THETA(LOCLIM(I),LOCBRA(I)),I=1,NTH)
      WRITE (KANAL6,*) 'TAU',(TAU(LOCTAU(I)),I=1,NTAU)
C
9998  XLFI=-1.D10
      NFAIL=NFAIL+1
C
9999  CONTINUE
      RETURN
      END
      SUBROUTINE WESML(PARM,GRAD,HESS,NP,SCRA,FUNC)
C-------------------------------------------------------------------
C
C     CALCULATES CORRECT COVARIANCE FOR WESML ESTIMATOR
C
C     COVM = INV(HESS) * ( BHHH + ABA' ) * INV(HESS)
C
C     RELSIZ = 0.0: EXOGENOUS SAMPLING - NO CORRECTION NECESSARY
C     RELSIZ < 1.0: CHOICE BASED SAMPLING WITH ESTIMATED SHARES
C                   (RELSIZ=MAIN SAMPLE SIZE/AUXILIARY SAMPLE SIZE)
C     RELSIZ = 1.0: CHOICE BASED SAMPLING WITH KNOWN SHARES
C     WEIGHT = VECTOR OF AUX. SAMPLE SHARES/MAIN SAMPLE SHARES
C
C     THE INVERSE HESSIAN HAS TO BE SUPPLIED IN HESS
C
C
C     AXEL BOERSCH-SUPAN       MARCH 21, 1984
C
C-------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP),GRAD(NP),HESS(NP,NP)
C
      PARAMETER
     *    (MAXOBS=2001,
     *     MAXNP =50,
     *     MAXNX =10,
     *     MAXALT=20,
     *     MAXNXY=MAXNX+1)
C
      REAL*8 SCRA(MAXNP,MAXNP),HINV(MAXNP,MAXNP),AAUX(MAXNP,MAXALT),
     *       BAUX(MAXALT,MAXALT),QAUX(MAXALT),SCRAUX(MAXNP,MAXALT)
      REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS)
      EXTERNAL FUNC
C
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DIFF2  / IDIFF2
      COMMON / DATAX  / DATA1
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
      COMMON / DWESTK / HINV,AAUX,BAUX,QAUX,SCRAUX
```

```
C
      IF (RELSIZ.EQ.0) RETURN
C
C     SAVE INVERSE HESSIAN IN "HINV"
C
      DO 10 I=1,NP
      DO 10 J=1,NP
10       HINV(I,J)=HESS(I,J)
C
C     CALCULATE BHHH MATRIX IN "HESS"
C
      ISAVE=IDIFF2
      IDIFF2=1
      CALL FP(PARM,NP,XLF,GRAD,FUNC)
      CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
      IDIFF2=ISAVE
C
C     ADD CONTRIBUTION OF AUXILIARY SAMPLE SHARE ABA TO BHHH MATRIX
C
      IF (RELSIZ.EQ.1.0) GOTO 200
C
C     -  GET AUX. SAMPLE SHARE AND FOOL FP WITH DIFFERENT WEIGHTS
C
      DO 100 I=1,NALT
         QAUX(I)   =WEIGHT(I)*DATA1(I,1,NOBS+1)
100      WEIGHT(I)=WEIGHT(I)/(QAUX(I)*NOBS)
C
C     -  ASSEMBLE A AND B
C
      DO 110 I=1,NALT
         CALL FP(PARM,NP,XLF,GRAD,FUNC)
         DO 112 K=1,NP
112         AAUX(K,I)=GRAD(K)
         DO 113 J=1,NALT
            BAUX(I,J)=-RELSIZ*QAUX(I)*QAUX(J)
            IF (I.EQ.J) BAUX(I,I)=BAUX(I,I)+RELSIZ*QAUX(I)
113         CONTINUE
110      CONTINUE
C
C     -  RESET WEIGHTS AND GRADIENT
C
      DO 120 I=1,NALT
120      WEIGHT(I)=WEIGHT(I)*QAUX(I)*NOBS
      CALL FP(PARM,NP,XLF,GRAD,FUNC)
C
C     -  MATRIX PRODUCT ABA'
C
      CALL MATP (AAUX,NP,MAXNP,NALT,BAUX,MAXALT,NALT,SCRAUX,MAXNP)
      CALL MATPT(SCRAUX,NP,MAXNP,NALT,AAUX,MAXNP,NALT,SCRA,MAXNP)
C
C     - MATRIX ADDITION BHHH + ABA'
C
      DO 130 I=1,NP
      DO 130 J=1,NP
```

```
130       HESS(I,J)=HESS(I,J)+SCRA(I,J)
C
C     PRE- AND POST-MULTIPLY INV. HESSIAN TO BHHH+ABA', STORE IN HESS
C
200   CALL MATP(HINV,NP,MAXNP,NP,HESS,NP,NP,SCRA,MAXNP)
      CALL MATP(SCRA,NP,MAXNP,NP,HINV,MAXNP,NP,HESS,NP)
C
      RETURN
      END
      SUBROUTINE ELAST(PARM,NP,PROB,ELAS,IOBS)
C---------------------------------------------------------------
C
C     CALCULATES ARRAY OF ELASTICITIES IN THE THREE-LEVEL-TREE
C     FOR EACH INDEPENDENT VARIABLE
C
C     AXEL BOERSCH-SUPAN      MARCH 29, 1984
C
C---------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXNXM =20,
     *   MAXTAU =5,
     *   MAXTH  =10,
     *   MAXLEV =3,
     *   MAXLIM =5,
     *   MAXBRA =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD)
C
      REAL*8 THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
     *       PSUM1(MAXALT),Q3(MAXALT),
     *       PSUM2(MAXLIM,MAXBRA),Q2(MAXLIM,MAXBRA),
     *       PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT)
      REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS),DATA2(MAXNXD,MAXOBS),
     *       ELAS(MAXALT,MAXALT,MAXNXR),PROB(MAXALT)
      INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT)
      LOGICAL LEVEL1,LEVEL2,NOALT,NOAGE,NALTRE,NAGTRE,NALSTD,NAGSTD,
     *       STDUMA,STDUMD,STDDUM,LMAP(MAXNXM,MAXALT)
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DATAX  / DATA1
      COMMON / DATAY  / DATA2
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
      COMMON / DMAPP  / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
      COMMON / DDISS  / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
     *                  NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
      COMMON / DDUMM  / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
     *                  NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
     *                  NALSTD,NALTRE,NAGSTD,NAGTRE
```

```
      COMMON / DELAS  / PSUM1,PSUM2,Q2,Q3
      COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
C
C     SUMMATION OF ELEMENTAL PROBABILITIES TO SECOND LEVEL
C
      I=0
      DO 10 I1=1,NL1
         ML2=NL2(I1)
         DO 10 I2=1,ML2
            PSUM2(I1,I2)=0.DO
            ML3=NL3(I1,I2)
            DO 11 I3=1,ML3
               I=I+1
11             PSUM2(I1,I2)=PSUM2(I1,I2)+PROB(I)
            IF (PSUM2(I1,I2).LE.0.0) GOTO 90
10          CONTINUE
C
C     SUMMATION TO FIRST LEVEL
C
      DO 20 I1=1,NL1
         PSUM1(I1)=0.DO
         ML2=NL2(I1)
         DO 21 I2=1,ML2
21          PSUM1(I1)=PSUM1(I1)+PSUM2(I1,I2)
20       CONTINUE
C
C     NORMALIZE TO GET TRANSITION PROBABILITIES
C
      I=0
      DO 30 I1=1,NL1
         ML2=NL2(I1)
         DO 30 I2=1,ML2
            Q2(I1,I2)=PSUM2(I1,I2)/PSUM1(I1)
            ML3=NL3(I1,I2)
            DO 30 I3=1,ML3
               I=I+1
               Q3(I)=PROB(I)/PSUM2(I1,I2)
30             CONTINUE
C
C     ARRAY OF ELASTICITIES
C
      DO 40 JM=1,NXM
      DO 40 K1=1,NXD
      DO 40 I=1,NALT
40       ELAS(I,JM,NX+K1)=0.0
C
      I=0
      J=0
      DO 50 J1=1,NL1
         TA=TAU(J1)
         XLI=(TA-1.DO)/TA
         ML2=NL2(J1)
         DO 50 J2=1,ML2
            ATW=1.DO/THETA(J1,J2)
```

```
      XBR=1.D0/TA-ATW
      ML3=NL3(J1,J2)
      DO 50 J3=1,ML3
         J=J+1
         ABR=XBR*Q3(J)
         ALI=XLI*Q3(J)*Q2(J1,J2)
         I=0
         DO 60 I1=1,NL1
            LL2=NL2(I1)
            DO 60 I2=1,LL2
               LL3=NL3(I1,I2)
               DO 60 I3=1,LL3
                  I=I+1
                  EL=-PROB(J)
                  IF (I1.NE.J1) GOTO 70
                              EL=EL+ALI
                  IF (I2.NE.J2) GOTO 70
                              EL=EL+ABR
                  IF (I.EQ.J) EL=EL+ATW
C
70                DO 72 K=1,NX
72                   ELAS(I,J,K)=PARM(K)*DATA1(J,K+1,IOBS)*EL
C
                  IF (NAGTRE) GOTO 75
                  DO 74 K=1,NXD
                  DO 74 JM=1,NXM
                     IF (.NOT.LMAP(JM,J)) GOTO 74
                     K1=INDAGE(K,JM)
                     ADD=PARM(K1)*DATA2(K,IOBS)*EL
                     ELAS(I,JM,NX+K)=ELAS(I,JM,NX+K)+ADD
74                   CONTINUE
C
75                IF (NAGSTD) GOTO 60
                  IF (J.EQ.NALT) GOTO 60
                  DO 76 K=1,NXD
                     K1=INDAGE(K,J)
76                   ELAS(I,J,NX+K)=PARM(K1)*DATA2(K,IOBS)*EL
C
60                CONTINUE
50          CONTINUE
C
      RETURN
C
90    WRITE (KANAL6,91) I1,I2,PSUM2(I1,I2)
91    FORMAT(' PSUM2(',2I3,')=',D15.5)
      RETURN
      END
      SUBROUTINE LSFIT(PDUMMY,NALT1,SSD,IERLS,SSQDEV,ITERLS)
C-------------------------------------------------------------------
C
C     FIT AGGREGATE SAMPLE SHARES BY LEAST SQUARES
C
C     CAN BE CALLED AS INNER MINIMIZATION LOOP WITHIN AN
C     OUTER MAXIMIZATION OF THE LIKELIHOOD.
```

```
C
C     INPUT:  PDUMMY(NALT1)  INITIAL ALTERNATIVE SPECIFIC DUMMIES
C             1TERLS         MAXIMUM NUMBER OF ITERATIONS
C
C     OUTPUT: PDUMMY(NALT1)  ADJUSTED ALTERNATIVE SPECIFIC DUMMIES
C             ITERLS         NUMBER OF ITERATIONS USED
C             IERLS          RETURN CODE FROM DFP-ALGORITHM
C             SSD            SUM OF SQUARED DEVIATIONS OF SHARES
C
C     AXEL BOERSCH-SUPAN     MARCH 28, 1984
C
C-------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION PDUMMY(NALT1)
      EXTERNAL FUNC,DFP,SSQDEV
C
      COMMON / BSTACK / A(1)
      COMMON / BOPT   / IVER,LT,IFP,ISP,NLOOP1,IST,ILOOP1
      COMMON / BOPT1  / NPARS,JH,JFP,JFP1,JSP,JSP1,JA1,JS
      COMMON / BOPT2  / ACC,R ,PM1,IVAL,ITERL,ITERC,MX,IER
      COMMON / BOPT3  / JSMP,JF,JSPD
      COMMON / BSTAK  / NQ,NTOP
      COMMON / BPRINT / IPT,NFILE,NDIG,NPUNCH
      COMMON / BLNSR  / STEP1,STPACC,NLNSR
      COMMON / BSTOP  / NVAR1,ISTOP(3)
      COMMON / BSTR   / ST2,F1,ST3,F3,IEDGE
      COMMON / BPREC  / RSMALL,ABSMAL
      COMMON / BINPUT / INFLG
      COMMON / BDFP   / STPMIN,FMIN
C
C     STORE PARAMETERS OF LIKELIHOOD MAXIMIZATION
C
      IVER7=IVER
      IFP7=IFP
      ILOOP7=ILOOP1
      NPARS7=NPARS
      JH7=JH
      JFP7=JFP
      JFP17=JFP1
      JSP7=JSP
      JSP17=JSP1
      JA17=JA1
      JS7=JS
      ACC7=ACC
      R7=R
      PM17=PM1
      ITERL7=ITERL
      ITERC7=ITERC
      MX7=MX
      IER7=IER
      JSMP7=JSMP
      JF7=JF
      JSPD7=JSPD
      IPT7=IPT
```

```
      STEP17=STEP1
      STPAC7=STPACC
      NLNSR7=NLNSR
      NVAR17=NVAR1
      ST27=ST2
      F17=F1
      ST37=ST3
      F37=F3
      IEDGE7=IEDGE
      RSMAL7=RSMALL
      ABSMA7=ABSMAL
      INFLG7=INFLG
      STPMI7=STPMIN
      FMIN7=FMIN
C
C     RESET PARAMETERS WHERE NECESSARY
C
      NPARS=NALT1
      IER=0
      IFP=4
      MX=2
      STPACC=ACC
      NVAR1=NALT1
      INFLG=0
      PM1=FLOAT(3-2*MX)
      ITERC=0
      LASTOP=NTOP
C
C     CALL MINIMIZATION ALGORITHM
C
      IVER=1
      ACC=0.01
      ITERL=ITERLS
      ITERC=0
      IPT=2
      CALL DFP(PDUMMY,NALT1,SSD,SSQDEV)
      ITERLS=ITERC
      IERLS=IER
C
C     RESTORE PARAMETERS OF LIKELIHOOD MAXIMIZATION
C
      NTOP=LASTOP
      IVER=IVER7
      IFP=IFP7
      ILOOP1=ILOOP7
      NPARS=NPARS7
      JH=JH7
      JFP=JFP7
      JFP1=JFP17
      JSP=JSP7
      JSP1=JSP17
      JA1=JA17
      JS=JS7
      ACC=ACC7
```

```
      R=R7
      PM1=PM17
      ITERL=ITERL7
      ITERC=ITERC7
      MX=MX7
      IER=IER7
      JSMP=JSMP7
      JF=JF7
      JSPD=JSPD7
      IPT=IPT7
      STEP1=STEP17
      STPACC=STPAC7
      NLNSR=NLNSR7
      NVAR1=NVAR17
      ST2=ST27
      F1=F17
      ST3=ST37
      F3=F37
      IEDGE=IEDGE7
      RSMALL=RSMAL7
      ABSMAL=ABSMA7
      INFLG=INFLG7
      STPMIN=STPMI7
      FMIN=FMIN7
C
      RETURN
      END
      SUBROUTINE SSQDEV(PDUMMY,NALT1,SSD,*)
C-----------------------------------------------------------------------
C
C     SUM OF SQUARED DEVIATIONS OF AGGREGATE PROBABILITY-SHARES FROM
C     OBSERVED SAMPLE-SHARES
C
C     AXEL BOERSCH-SUPAN    MARCH 27, 1984
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PDUMMY(NALT1)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1,
     *   MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS,
     *   MAXALS=MAXALT*(MAXALT+1)/2)
C
      REAL*8 PROB1(MAXALT),PROB2(MAXALT),
     *       DERIV1(MAXALT,MAXALT),DERIV2(MAXALT,MAXALT),
     *       PARM(MAXNP),GRAD(MAXNP),HESS(MAXNP,MAXNP)
      REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)
C
```

```
      COMMON / DIMEN  / NALT,NPARM(8),NOBS
      COMMON / DATAX  / FDATA1
      COMMON / DATAY  / FDATA2
      COMMON / DSSQD  / PROB1,PROB2,DERIV1,DERIV2
      COMMON / DPARM  / NP,PARM,GRAD,HESS
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
C
      ILOOP=1
      NFAIL=0
      CALL UPDATE(PARM,NP,*90)
      DO 10 I=1,NALT
10       PROB2(I)=0.D0
C
C     EVALUATION OF PROBABILITY-SHARES
C
      NN1=1
      NN2=1
      INC=MAXALT*MAXNXY
      DO 100 IOBS = 1,NOBS
         CALL CONTPS(FDATA1(NN1),FDATA2(NN2),
     *               ILOOP,PARM,NP,PDUMMY,NALT1,PROB1,DERIV1,NFAIL)
         DO 15 I=1,NALT
            IF (FDATA1(NN1+I-1).EQ.1.0) WT=WEIGHT(I)
15          CONTINUE
         NN1=NN1+INC
         NN2=NN2+MAXNXD
         DO 16 I=1,NALT
16          PROB2(I)=PROB2(I)+PROB1(I)*WT
100      CONTINUE
C
      IF (NFAIL.GT.0) GOTO 90
C
C     SUM OF SQUARED DEVIATIONS IN PERCENTAGES
C
      NN=NN1+INC
      SSD=0.D0
      DO 20 I=1,NALT
         ZZZ = (PROB2(I)/NOBS - FDATA1(NN+I-1))*100.0
         SSD = SSD + ZZZ*ZZZ
20       CONTINUE
C
      RETURN
90    PRINT 91,NFAIL
91    FORMAT(' ERROR IN SSQDEV:',I5,' FAILURES IN EVALUATION')
      RETURN 1
      END
C-----------------------------------------------------------------------
      SUBROUTINE FPSSQD(PDUMMY,NALT1,SSD,DERIV,SDERIV,SSQDEV)
C-----------------------------------------------------------------------
C
C     GRADIENT OF THE SUM OF SQUARED DEVIATIONS
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      REAL*8 PDUMMY(NALT1),DERIV(NALT1),SDERIV(NALT1)
C
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1, MAXNPS=MAXNP*(MAXNP+1)/2,
     *   MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS)
C
      REAL*8 PROB1(MAXALT),PROB2(MAXALT),
     *       DERIV1(MAXALT,MAXALT),DERIV2(MAXALT,MAXALT),
     *       PARM(MAXNP),GRAD(MAXNP),HESS(MAXNP,MAXNP)
      REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)
C
      COMMON / DIMEN  / NALT,NPARM(8),NOBS
      COMMON / DATAX  / FDATA1
      COMMON / DATAY  / FDATA2
      COMMON / DPARM  / NP,PARM,GRAD,HESS
      COMMON / DSSQD  / PROB1,PROB2,DERIV1,DERIV2
      COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
      COMMON / BOPT2  / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
      EXTERNAL SSQDEV
C
      ILOOP=0
      NFAIL=0
      CALL UPDATE(PARM,NP,*90)
      DO 10 I=1,NALT
         PROB2(I)=0.D0
         DO 10 K=1,NALT1
10          DERIV2(I,K)=0.D0
C
C     EVALUATION OF PROBABILITY-SHARES AND THEIR DERIVATIVES
C
      NN1=1
      NN2=1
      INC=MAXALT*MAXNXY
      DO 100 IOBS = 1,NOBS
         CALL CONTPS(FDATA1(NN1),FDATA2(NN2),
     *               ILOOP,PARM,NP,PDUMMY,NALT1,PROB1,DERIV1,NFAIL)
         DO 15 I=1,NALT
            IF (FDATA1(NN1+I-1).EQ.1.0) WT=WEIGHT(I)
15          CONTINUE
         NN1=NN1+INC
         NN2=NN2+MAXNXD
         DO 16 I=1,NALT
            PROB2(I)=PROB2(I)+PROB1(I)*WT
            DO 16 K=1,NALT1
16             DERIV2(I,K)=DERIV2(I,K)+DERIV1(I,K)*WT
100      CONTINUE
C
      IVAL=IVAL+1
      IF (NFAIL.GT.0) GOTO 90
```

```
C
C     SUM OF SQUARED DEVIATIONS IN PERCENTAGES
C
      NN=NN1+INC
      SSD=0.D0
      DO 20 I=1,NALT
         PROB1(I) = ( PROB2(I)/NOBS - FDATA1(NN+I-1) ) * 100
         SSD = SSD + PROB1(I)*PROB1(I)
20       CONTINUE
C
C     ITS GRADIENT W.R.T. ALTERNATIVE SPECIFIC DUMMIES
C
      DO 30 K=1,NALT1
         ZZZ = 0.D0
         DO 31 I=1,NALT
31          ZZZ = ZZZ + 2.0 * PROB1(I)/NOBS*100.0 * DERIV2(I,K)
30       DERIV(K) = ZZZ
C
      RETURN
90    PRINT 91,NFAIL
91    FORMAT(' ERROR IN FPSSQD:',I5,' FAILURES IN EVALUATION')
      RETURN
      END
      SUBROUTINE CONTPS(DATA1,DATA2,ILOOP,PARM,NP,
     *                  PDUMMY,NALT1,PROB1,DERIV1,NFAIL)
C----------------------------------------------------------------------
C
C     THREE LEVEL NESTED LOGIT MODEL
C     ==============================
C
C     CONTRIBUTION OF THE N-TH OBSERVATION
C     - TO THE PROBABILITY-SHARE (PROB1 , ILOOP=0,1)
C     - AND TO ITS DERIVATIVE    (DERIV1, ILOOP=0)
C
C     VERSION  MARCH 27, 1984     AXEL BOERSCH-SUPAN
C
C----------------------------------------------------------------------
C
C     TREE STRUCTURE :
C
C     LEVEL:  DATA:  PARMS:  EXP(INCL.VALUE):  DISSIMILARITY-PARMS:
C
C       0                        EINC0
C       1      (Y)    (ALPHA)    EINC1               TAU
C       2      (Z)    (GAMMA)    EINC2               THETA
C       3       X      BETA
C
C     FOR ARRAY DIMENSIONS, SEE MAIN
C
C----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP),PDUMMY(NALT1)
      REAL*4 DATA1(*),DATA2(*)
C
```

```
      PARAMETER
     *  (MAXOBS =2001,
     *   MAXNP  =50,
     *   MAXTH  =10,
     *   MAXTAU =5,
     *   MAXNX  =10,
     *   MAXNXD =5,
     *   MAXNXM =20,
     *   MAXLEV =3,
     *   MAXLIM =5,
     *   MAXBRA =5,
     *   MAXALT =20,
     *   MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD, MAXNPS=MAXNP*(MAXNP+1)/2)
C
      REAL*8 UPP,LOW,ONE/1.D0/,ZERO/0.D0/
      INTEGER IZERO/0/
C
      REAL*8 PROB1(MAXALT),DERIV1(MAXALT,MAXALT),
     *       THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
     *       EXB(MAXALT),EZG(MAXALT),EYA(MAXLIM),
     *       XB(MAXALT),ZG(MAXALT),YA(MAXLIM),
     *       EINC2(MAXLIM,MAXBRA),EINC1(MAXLIM),EINC0,
     *       INC2(MAXLIM,MAXBRA),INC1(MAXLIM),INC0,INC2C,INC1C,
     *       DER2B(MAXLIM,MAXBRA,MAXNP),DER1B(MAXLIM,MAXNP),
     *       DER2T(MAXLIM,MAXBRA,MAXTH),DER1T(MAXLIM,MAXTH),
     *       DER1U(MAXLIM,MAXTAU),
     *       PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT),
     *       ZBAR(MAXNP),ZIZBAR(MAXALT,MAXNP)
      INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT)
      LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDDUM,NOALT,NOAGE,DUMP,
     *       LDUMMY,STDUMA,STDUMD,NALSTD,NALTRE,NAGSTD,NAGTRE
C
      COMMON / DIMEN  / NALT,NALTDU,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DDISS  / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
     *                  NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
      COMMON / DCONT  / XB,ZG,YA,INC2,INC1,
     *                  EXB,EZG,EYA,EINC2,EINC1,
     *                  DER2B,DER1B,DER2T,DER1T,DER1U,
     *                  ZBAR,ZIZBAR,UPP,LOW,
     *                  MS3K(MAXTH),ML3K(MAXTH),MS2K(MAXLIM)
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
      COMMON / DMAPP  / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
      COMMON / DDUMM  / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
     *                  NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
     *                  NALSTD,NALTRE,NAGSTD,NAGTRE
      COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
C
C     INNER PRODUCT XBETA, INCLUSIVE VALUES
C
C
      I=IZERO
      IB=IZERO
      EINC0=ZERO
      DO 10 I1=1,NL1
```

```
      ML2=NL2(I1)
      TA =TAU(I1)
      DINC1=ZERO
      DO 20 I2=1,ML2
         IB =IB+1
         ML3=NL3(I1,I2)
         TH =THETA(I1,I2)
         THT=TH/TA
         DINC2=ZERO
         DO 30 I3=1,ML3
            I=I+1
C
C           INNER PRODUCT XB/TH
C
31          XBI = ZERO
            NN1 = MAXALT
            DO 40 K=1,NX
               XBI = XBI + DATA1(NN1+I)*PARM(K)
40             NN1 = NN1 + MAXALT
            IF (NAGTRE) GOTO 42
            DO 41 K=1,NXD
41             XBI = XBI + DATA2(K)*PMASKY(K,I)
42          IF (I.EQ.NALT) GOTO 49
               XBI = XBI + PDUMMY(I)
            IF (NAGSTD) GOTO 49
            DO 43 K=1,NXD
43             XBI = XBI + DATA2(K)*PARM(INDAGE(K,I))
49          CONTINUE
            XBI = XBI / TH
            IF (XBI.GT.UPP) GOTO 9001
            XB(I)=XBI
            ZZZ=DEXP(XBI)
            EXB(I)=ZZZ
            DINC2=DINC2+ZZZ
30          CONTINUE
C
C        LOWER LEVEL INCLUSIVE VALUE  INC2(I1,I2)
C
         IF (LEVEL1) GOTO 21
         IF (DINC2.LT.LOW) GOTO 9002
         EINC2(I1,I2)=DINC2
         DINC2=DLOG(DINC2)
         INC2(I1,I2)=DINC2
         DINC2=THT*DINC2
         IF (DINC2.GT.UPP) GOTO 9002
         ZG(IB)=DINC2
         DINC2=DEXP(DINC2)
         EZG(IB)=DINC2
21       DINC1=DINC1+DINC2
20       CONTINUE

C     HIGHER LEVEL INCLUSIVE VALUE  DLOG(INC1(I1))
C
      EINC1(I1)=ONE
```

```
         IF (LEVEL2) GOTO 11
         IF (DINC1.LT.LOW) GOTO 9003
         EINC1(I1)=DINC1
         DINC1=DLOG(DINC1)
         INC1(I1)=DINC1
         DINC1=TA*DINC1
         IF (DINC1.GT.UPP) GOTO 9003
         YA(I1)=DINC1
         DINC1=DEXP(DINC1)
         EYA(I1)=DINC1
11       EINC0=EINC0+DINC1
10       CONTINUE
      IF (EINC0.LT.LOW) GOTO 9004
      INC0=DLOG(EINC0)
C
C     PROBABILITY-SHARES
C
51    DO 52 I=1,NALT
         PROB1(I)=EXB(I)/EINC0
52       CONTINUE
      IF (LEVEL1) GOTO 69
      I=IZERO
      DO 60 I1=1,NL1
         ML2=NL2(I1)
         IF (LEVEL2) GOTO 61
         DINC1=(TAU(I1)-ONE)*INC1(I1)
61       CONTINUE
         DO 60 I2=1,ML2
            ML3=NL3(I1,I2)
            DINC2=(THETA(I1,I2)/TAU(I1)-ONE)*INC2(I1,I2)
            DO 60 I3=1,ML3
               I=I+1
               IF (LEVEL2) GOTO 62
               PROB1(I)=PROB1(I)*DEXP(DINC1+DINC2)
               GOTO 60
62             PROB1(I)=PROB1(I)*DEXP(DINC2)
60             CONTINUE
69    IF (ILOOP.EQ.1) GOTO 9999
      IF (LEVEL1) GOTO 4000
C
C     DERIVATIVES OF INCLUSIVE VALUES LEVEL 2 W.R.T. DUMMIES
C
      I=IZERO
      DO 80 I1=1,NL1
         ML2=NL2(I1)
         DO 80 I2=1,ML2
            ML3=NL3(I1,I2)
            DINC2=EINC2(I1,I2)*THETA(I1,I2)
            DO 80 I3=1,ML3
               I=I+1
               IF (I.EQ.NALT) GOTO 80
               DER2B(I1,I2,MXA+I) = EXB(I) / DINC2
c               WRITE (KANAL6,9907) I1,I2,MXA+I,DER2B(I1,I2,MXA+I)
c9907           FORMAT(' $X$ DER2B(',3I2,')=',D18.4)
```

```
80              CONTINUE
C
C     DERIVATIVES OF INCLUSIVE VALUES LEVEL 1 W.R.T. DUMMIES
C
      DO 93 K=MXA+1,MXA+NALT1
      I=IZERO
      DO 93 I1=1,NL1
         ML2=NL2(I1)
         ZZ=ZERO
         DO 94 I2=1,ML2
            I=I+1
94          ZZ = ZZ + EZG(I) * DER2B(I1,I2,K) * THETA (I1,I2)
         DER1B(I1,K) = ZZ / EINC1(I1) / TAU(I1)
c         WRITE (KANAL6,9908) I1,K,DER1B(I1,K)
c9908     FORMAT(' $X$ DER1B(',2I2,')=',D18.4)
93       CONTINUE
C
C     DERIVATIVES OF THE PROBABILITIES (THREE LEVEL TREE)
C
      IF (LEVEL2) GOTO 200
C
      DO 127 KK=1,NALT1
         K=KK+MXA
         ZZ=ZERO
         DO 128 I1=1,NL1
128         ZZ = ZZ + EYA(I1) * DER1B(I1,K) * TAU(I1)
         ZZ = -ZZ/EINC0
         I=0
         DO 127 I1=1,NL1
            ML2=NL2(I1)
            TA1=TAU(I1)-ONE
            DC 127 I2=1,ML2
               ML3=NL3(I1,I2)
               TH1=(THETA(I1,I2)/TAU(I1)-ONE)
               DO 127 I3=1,ML3
                  I=I+1
                  ZZZ = ZZ + TH1 * DER2B(I1,I2,K) + TA1 * DER1B(I1,K)
                  IF (I.EQ.KK) ZZZ = ZZZ + ONE/THETA(I1,I2)
                  DERIV1(I,KK) = ZZZ * PROB1(I)
127               CONTINUE
      GOTO 9999
C
C     DERIVATIVES OF THE PROBABILITIES (TWO LEVEL TREE)
C
200   DO 227 KK=1,NALT1
         K=KK+MXA
         ZZ=ZERO
         DO 228 I1=1,NL1
228         ZZ = ZZ + DER1B(I1,K)
         ZZ = -ZZ/EINC0
         I=0
         DO 227 I1=1,NL1
            ML2=NL2(I1)
            DO 227 I2=1,ML2
```

```
               ML3=NL3(I1,I2)
               TH1=THETA(I1,I2)-ONE
               DO 227 I3=1,ML3
                  I=I+1
                  ZZZ = ZZ + TH1 * DER2B(I1,I2,K)
                  IF (I.EQ.KK) ZZZ = ZZZ + ONE/THETA(I1,I2)
                  DERIV1(I,KK) = ZZZ * PROB1(I)
227               CONTINUE
      GOTO 9999
C
C     DERIVATIVES OF THE PROBABILITIES (SIMPLE MNL-VERSION)
C
4000  DO 4041 K=1,NALT1
      DO 4041 I=1,NALT
         ZZ = ZERO
         IF (I.EQ.K) ZZ = ONE
         DERIV1(I,K) = PROB1(I) * ( ZZ - PROB1(K) )
4041     CONTINUE
      GOTO 9999
C
C     FAILURE DUE TO UNDER- OR OVERFLOW
C
9001  CONTINUE
      WRITE (KANAL6,*) 'XBI',XBI
      GOTO 9000
9002  CONTINUE
      WRITE (KANAL6,*) 'DINC2',DINC2
      GOTO 9000
9003  CONTINUE
      WRITE (KANAL6,*) 'DINC1',DINC1
      GOTO 9000
9004  CONTINUE
      WRITE (KANAL6,*) 'EINCO',EINCO
C
9000  XLFI=-1.D10
      NFAIL=NFAIL+1
C
9999  CONTINUE
      RETURN
      END
      SUBROUTINE  LMSTAT(NP,NP1,NP2,ND,FPD,SPD,TEST)
C---------------------------------------------------------------
C     CALCULATES LM STATISTIC
C---------------------------------------------------------------
      PARAMETER (MAXNP=50,
     *           MAXNPS=MAXNP*(MAXNP+1)/2)
      REAL*8 FPD(NP),SPD(NP,NP)
      REAL*8 DL(5),DLB(5,MAXNP),DLL(5,5),DBB(MAXNPS),TEST
      INTEGER ND(5)
C
C     FILL FIRST AND SECOND DERIV W.R.T. THETA : DL,DLL
C
      DO 560 I=1,NP1
         DL(I)=FPD(ND(I))
```

```
        DO 560 J=1,NP1
560        DLL(I,J)=SPD(ND(I),ND(J))
C
C     FILL MIXED SECOND DERIV : DLB
C
      DO 561 I=1,NP1
      DO 561 J=1,NP2
561      DLB(I,J)=SPD(ND(I),J)
C
C     FILL SECOND DERIV W.R.T. BETA ON TRIANGLE : DBB
C
      IND=0
      DO 562 I=1,NP2
      DO 562 J=1,I
            IND=IND+1
562         DBB(IND)=-SPD(I,J)
C           (NOTE: NEED POS. DEF. MATRIX)
C
C     INVERT DBB
C
      LK2=NP2*(NP2+1)/2
      CALL INV(DBB,NP2,LK2,IER)
      IF (IER.NE.0) PRINT 565
565   FORMAT(' DBB SINGULAR')
C
C     REFILL INVERS OF DBB ON ARRAY
C
      IND=0
      DO 568 I=1,NP2
      DO 568 J=1,I
         IND=IND+1
         SPD(I,J)=-DBB(IND)
568      SPD(J,I)=-DBB(IND)
C
C     MATRIX PRODUCT : DLB * DBB(-1) * DLB'
C
      DO 570 I=1,NP1
    . IND =(I-1)*NP2
      DO 570 J=1,NP2
         TEST=0.D0
         DO 571 K=1,NP2
571         TEST=TEST+DLB(I,K)*SPD(K,J)
570      DBB(IND+J)=TEST
      DO 575 I=1,NP1
      IND =(I-1)*NP2
      DO 575 J=1,NP1
         TEST=0.D0
         DO 576 K=1,NP2
576         TEST=TEST+DBB(IND+K)*DLB(J,K)
575      SPD(I,J)=TEST
C
C     FILL DLL - DLB*DBB(-1)*DLB' ON TRIANGLE
C
      IND=0
```

```
      DO 580 I=1,NP1
      DO 580 J=1,I
         IND=IND+1
580      DBB(IND)=DLL(I,J)-SPD(I,J)
C
C     INVERT THIS MATRIX
C
      LK1=NP1*(NP1+1)/2
      CALL INV(DBB,NP1,LK1,IER)
      IF (IER.NE.0) PRINT 582
582   FORMAT(' DLL-DLB*DBB(-1)*DBL SINGULAR')
C
C     REFILL THIS INVERS ON ARRAY
C
      IND=0
      DO 585 I=1,NP1
      DO 585 J=1,I
         IND=IND+1
         SPD(I,J)=DBB(IND)
585      SPD(J,I)=DBB(IND)
C
C     FINAL MATRIX PRODUCT DL * [DLL-DLB*DBB(-1)*DBL](-1) * DL'
C
      DO 590 I=1,NP1
         DBB(I)=0.D0
         DO 590 J=1,NP1
590         DBB(I)=DBB(I)+SPD(I,J)*DL(J)
      TEST=0.D0
      DO 595 I=1,NP1
595      TEST=TEST+DBB(I)*DL(I)
C
      RETURN
      END
      SUBROUTINE AIRUM(PARM,NP,NFILE)
C-----------------------------------------------------------------------
C
C     COMPATIBILITY-CHECK WITH AIRUM                  |
C     ==============================            +----+---+
C                                               |    |   |
C     - DENSITY NONNEGATIVE ?                 +-+-+  |  +-+
C     - MASS OVER DATA < 1  ?                 | | |  |  | |
C
C     VERSION  MAR 17 , 1983   23.00     AXEL BOERSCH-SUPAN
C
C-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER
     *   (MAXOBS=2001,
     *    MAXALT=20,
     *    MAXLIM=5,
     *    MAXBRA=5,
     *    MAXNX =10,
     *    MAXNXD=5,
     *    MAXNXY=MAXNX+1)
```

```
      REAL*8 PARM(NP),THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
     *       VMIN(9),VMAX(9),VMEA(9),V(9)
      CHARACTER*3 C(2)/'MAX','MIN'/
      REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS),DATA2(MAXNXD,MAXOBS)
      LOGICAL LEVEL1,LEVEL2
      COMMON / DIMEN  / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
      COMMON / DATAX  / DATA1
      COMMON / DATAY  / DATA2
      COMMON / DTREE  / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
C
C   INITIALIZATION - NOTE SPECIAL CASE
C
      IF (NALT.NE.7) RETURN
      NFILE=1
      DO 10 I=1,NALT
         VMEA(I)= 0.D0
         VMIN(I)= 1.D99
10       VMAX(I)=-1.D99
      T3=TAU(1)
      T2=THETA(3,1)
      WRITE (NFILE,300) T3,T2
300   FORMAT(//' COMPATIBILITY-CHECK'/' ==================='
     *       //' TH_OWNER = ',F10.4,' , TH_RENTER = ',F10.4)
C
C   LOOP ON DATA FOR VMIN  VMEA, CHECK AT DATA
C
      NEG=0
      DO 1000 N=1,NOBS
C
C           BENCHMARK: ALTERNATIVE 4
C
            XB4=0.D0
               DO 20 K=1,NX
20                XB4 = XB4 + DBLE( DATA1(4,K+1,N) ) * PARM(K)
               IF (NXD.EQ.0) GOTO 24
               DO 21 K1=1,NXD
                  K3 = NX+(K1-1)*NALT1+4
21                XB4 = XB4 + DATA2(K1,N)*PARM(K3)
24             IF (NXA.EQ.0) GOTO 29
            XB4 = XB4 + PARM(MXA+4)
C
C           OTHER ALTERNATIVES
C
29          DO 30 IT=1,NALT
               IF (IT.EQ.4) GOTO 30
               XB=0.D0
               DO 40 K=1,NX
40                XB = XB + DBLE( DATA1(IT,K+1,N) ) * PARM(K)
               IF (IT.EQ.NALT) GOTO 49
               IF (NXD.EQ.0) GOTO 44
               DO 41 K1=1,NXD
                  K3 = NX+(K1-1)*NALT1+IT
41                XB = XB + DATA2(K1,N)*PARM(K3)
44             IF (NXA.EQ.0) GOTO 49
```

```
             XB = XB + PARM(MXA+IT)
C
C         RELATIVE UTILITY
C
49           XB        = XB - XB4
             V(IT)     = XB
             VMEA(IT) = VMEA(IT) + XB
             IF (XB.GT.VMAX(IT)) VMAX(IT)=XB
             IF (XB.LT.VMIN(IT)) VMIN(IT)=XB
30           CONTINUE
C
C         CHECK AT DATA
C
          CALL GEV(V(1),V(2),V(3),V(5),V(6),T3,T2,G,DENS)
          IF (DENS.LT.0.DO) NEG=NEG+1
C
1000  CONTINUE
C
C  CHECK AT MEAN
C
      DO 31 I=1,NALT
31       V(I)=VMEA(I)/NOBS
      CALL GEV(V(1),V(2),V(3),V(5),V(6),T3,T2,G,DENS)
      WRITE (NFILE,301) NOBS,NEG,DENS
301   FORMAT(//' NUMBER OF OBSERVATIONS:',I13
     *        /' WITH NEGATIVE DENSITY :',I13
     *        /' DENSITY AT MEAN       :',D13.4//)
C
C  ESTABLISH INTERVAL
C
      DO 35 I=1,NALT
35       WRITE (NFILE,302) I,VMIN(I),VMAX(I)
302      FORMAT(' INTERVAL V(',I1,')-V(4) : [',F10.4,',',F10.4,']')
      WRITE (NFILE,303)
303   FORMAT(//' CORNER:                SIGN:    G-FUNCTION    DENSITY'
     *       /' ',55('-'))
C
C  CHECK ON ALL CORNERS, ACCUMULATE PARTS OF INTEGRAL
C
      AMASS=0.DO
      DO 50 I6=1,2
         IF (I6.EQ.1) THEN
                      A6=VMAX(6)
                      ELSE
                      A6=VMIN(6)
                      END IF
      DO 50 I5=1,2
         IF (I5.EQ.1) THEN
                      A5=VMAX(5)
                      ELSE
                      A5=VMIN(5)
                      END IF
      DO 50 I3=1,2
         IF (I3.EQ.1) THEN
```

```
                     A3=VMAX(3)
                     ELSE
                     A3=VMIN(3)
                     END IF

      DO 50 I2=1,2
         IF (I2.EQ.1) THEN
                     A2=VMAX(2)
                     ELSE
                     A2=VMIN(2)
                     END IF
      DO 50 I1=1,2
         IF (I1.EQ.1) THEN
                     A1=VMAX(1)
                     ELSE
                     A1=VMIN(1)
                     END IF
C
         CALL GEV(A1,A2,A3,A5,A6,T3,T2,G,DENS)
         SIGN = (-1)**(I1+I2+I3+I5+I6-5)
         AMASS= AMASS + SIGN/G
C
         WRITE (NFILE,51) C(I1),C(I2),C(I3),C(I5),C(I6),SIGN,G,DENS
51       FORMAT(' ',3(A3,','),'0',2(',',A3),2X,F4.1,2X,2D13.4)
50       CONTINUE
C
      WRITE (NFILE,52) AMASS
52    FORMAT(' ',55('-')/' MASS OVER THE INTERVAL: ',F10.4)
      RETURN
      END
      SUBROUTINE GEV(A1,A2,A3,A5,A6,T3,T2,G,DENS)
C------------------------------------------------------------------------
C     EVALUATES G=1/(JOINT CDF) AND JOINT DENSITY FOR GEV MODEL
C------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
C
C        G-FUNCTION
C
         Y1    = DEXP(-A1/T3)
         Y2    = DEXP(-A2/T3)
         Y3    = DEXP(-A3/T3)
         Y5    = DEXP(-A5/T2)
         Y6    = DEXP(-A6/T2)
         H3    = Y1 + Y2 + Y3
         H2    = Y5 + Y6
         G     = H3**T3 + 1.D0 + H2**T2
C
C        DERIVATIVES OF G-FUNCTION
C
         H31   = H3**(T3-1.D0)
         H21   = H2**(T2-1.D0)
         H32   = H3**(T3-2.D0)
         H22   = H2**(T2-2.D0)
         H33   = H3**(T3-3.D0)
```

```
          T31  = (T3-1.DO)/T3
          T21  = (T2-1.DO)/T2
          T32  = T31*(T3-2.DO)/T3
          G1   = -H31*Y1
          G2   = -H31*Y2
          G3   = -H31*Y3
          G5   = -H21*Y5
          G6   = -H21*Y6
          G12  = T31*H32*Y1*Y2
          G23  = T31*H32*Y2*Y3
          G13  = T31*H32*Y1*Y3
          G56  = T21*H22*Y5*Y6
          G123 = -T32*H33*Y1*Y2*Y3
          G5G6 = G5*G6
          GG56 = G*G56
          GSUM = G12*G3+G23*G1+G13*G2
          GPRO = G1*G2*G3
C
C         POWERS OF THE G-FUNCTION
C
          GTO4 = G**4
          GTO5 = GTO4*G
          GTO6 = GTO5*G
C
C         DENSITY
C
          S3 = (GG56-3.DO*G5G6)/GTO4
          S4 = (GG56-4.DO*G5G6)/GTO5
          S5 = (GG56-5.DO*G5G6)/GTO6
          DENS = 24.DO*GPRO*S5 - 6.DO*GSUM*S4 + 2.DO*G123*S3
C
          RETURN
          END
      SUBROUTINE TRAFO(PARM,NP,K,IND,HESS,C,SC,W)
C-------------------------------------------------------
C *** TRANSFORM PARAMETERS TO BE COMPATIBLE WITH MNL ***
C-------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*8 PARM(NP),HESS(NP,NP)
C
      A   = PARM(K)
      SA  = -HESS(K,K)
      B   = PARM(IND)
      SAB = -HESS(IND,K)
      SB  = -HESS(IND,IND)
      C   = A/B
      SC  = (SA - 2.*C*SAB + C*C*SB)/B/B
      IF (SC.LE.0.) SC=1.DO
      SC  = DSQRT( SC )
      W   = C/SC
C
      RETURN
      END
```

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