THE DEMAND FOR HOUSING IN THE UNITED STATES AND WEST GERMANY: A DISCRETE CHOICE ANALYSIS

by

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Submitted to the Department of Economics
In Partial Fulfillment of the Requirements
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ABSTRACT

This thesis applies discrete choice techniques to an analysis of
housing demand in the United States and West Germany. Housing demand
comprises the choices of household formation, tenure, type of
structure, and size and quality of dwelling. We will focus on
peculiarities of housing demand which have found little attention in
the otherwise ample literature on housing demand. Usage of similar
surveys in the United States and West Germany allow us to make
comparisons between the two countries and to identify mechanisms which
are concealed by examining only one country.

The multidimensional heterogeneity of housing demand
possibilities suggests the usage of discrete choice techniques, in
particular hierarchically nested choice models. The theory and
estimation of nested multinomial logit models is reviewed and
extended.

Household formation as a dimension in housing demand renders
household based models inappropriate. A model which simultaneously
determines headship status and conventional housing demand is
developed and estimated for U. S. data. Simulations mimicking the
Experimental Housing Allowance Program underscore the importance of
household formation as a factor in housing demand.

The tenant-landlord relationship over time is examined.
Empirical evidence points to the existence of tenure discounts,
leading to a bias in conventional price specifications. A
microeconomic model highlights the potential misinterpretations in
housing market analysis when tenure discounts are disregarded. As
illustration, we evaluate the German rent and eviction control
legislation.

Finally, we compare housing choices in the United States and West
Germany. We apply a common analytical model on comparable data sets
to examine the differences in tenure choice and size demand. Using
this model, we try to isolate behavioral from institutional
differences.

Thesis Supervisor: Dr. Daniel McFadden,
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ACKNOWLEDGMENTS

It took a good seventh part of my life so far to fulfill what is called "the requirements for the degree of Doctor of Philosophy" on the title page of this thesis. Four years of neglecting wife and child, ignoring the amenities of Boston (almost), leaving my home country with parents and friends. I would not be a proper economist not to ask myself whether it was worth all the trouble, nor would I deserve the appendage Ph.D. not to take this opportunity to philosophize on what I have learned during this time.

Quite a lot. The benefits outweigh the costs. I learned economics, starting pretty much and frustratingly from scratch, in a pace and intensity which I -- finally -- enjoyed. It convinced me that my place is in applied economics rather than in abstract mathematics. And I learned to like my profession. Apart from economics, I enjoyed the company of such a variety of interesting fellow students and teachers as it is only possible in this Cambridgean ivory tower. I am grateful to all my friends for discussions, excursion, and symposiums in which we shared the ups and downs of a great time.

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balance by being a friend in and outside the world of the Economics Department who shared the many incisive events in these four years and supported by word and deed our sometimes shaky two career plus child enterprise.

The four years of M.I.T. life were four years living in a new country. This was an exciting and mind-opening experience which I do not want to miss. I am grateful to the Studienstiftung in West Germany for providing the incentive to sail out to the New World and the generous financial support which made this possible without going through the stage of a dishwasher.

Large benefits do not come without costs. And the costs were borne not only by myself. It is a good and meaningful tradition to dedicate a thesis to the parents. I do this with pleasure and thankfulness. My parents put me on the road I am now pursuing, and always supported me with encouragement even when they had to bear most of the burden when I left Germany four years ago to go through yet another study.

The cost were large for my wife as well. What a time! We left our country, married, got our first child -- all this mixed with constant fickleness about exams, night-long glare at a computer terminal, and the final frenzy to get this thesis finished. It is a wonder that she still talks to me. Believe me, I am grateful for such wonders. After all, we did enjoy these four years, together.
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CHAPTER ONE

INTRODUCTION AND SURVEY
1.1 Substantive Issues

This thesis investigates the demand for housing as a heterogeneous commodity. We will focus on the demand side of the housing market in most of our empirical work and restrict the analysis to partial models, that is, on potential demand under perfect elastic supply. This long run analysis is in tune with the use of two large cross-sectional data sets and their interpretation as steady state equilibria: the Annual Housing Survey in the United States and the One Percent Sample in West Germany. The only deviation from our concentration on demand only will be in the theoretical model of the nature of those steady state equilibria.

To capture the heterogeneity of the commodity housing we will introduce a comprehensive notion of what housing demand consists of: it includes the choices of quality, size, tenure, and headship status. We will not consider choice of location, however, and we will concentrate on large metropolitan areas.

Although housing demand is a well studied field, there are still a host of unresolved substantive issues. Having introduced the notion of the commodity housing as a broad class of different housing categories or alternatives, a general issue is the question of substitutability among these categories. In Joan Robinson's (1933) words, we are looking for the gaps in the chain of substitutes. Do people easily substitute larger dwellings for smaller dwellings in response to price increases, or do they switch tenure? Does the
substitutability among housing alternatives change with the life cycle? Are there differences in behavior between the United States and West Germany?

Furthermore, how does household formation as a dimension of housing demand fit into the chain of substitutes? Is household formation responsive to price changes in the rental and owner markets? Does this response depend on the stage in the life cycle or on demographic characteristics?

Apart from interest in the structure of a comprehensive housing demand per se, we might ask ourselves how this structure is reflected in policy analysis. The tax codes in both countries are asymmetric in their treatment of owner-occupancy versus rental housing, but it is not clear whether all of the observed preferences in the tenure choice can be explained by taxes alone. Would a drastic tax change induce a drastic change in the preference for tenure? A comparison and West Germany and the United States seems to be of particular interest due to their very different proportions of owner-occupancy (1978: U.S.: 65.2 percent, Germany: 36.3 percent). Are there repercussions in the other dimensions of housing choice? What about the response of household formation to changes in the tax code? And, more interesting for the latter dimension, is there a response to direct demand subsidies like housing allowances? Is the Experimental Housing Allowance Program flawed in its complete ignorance of household formation?
A final topic of this thesis and another red thread through the five chapters is the question of what the proper price is for this heterogeneous and durable good. In the last Chapter, we will use hedonic indexes to capture heterogeneity, an at least empirically resolved issue. However, the durability of housing has implications on intertemporal pricing which are not well understood. Is there price dispersion in the housing market? How can it be explained? Can different explanations be empirically tested against each other? How does the existence of price dispersion affect our knowledge of price responsiveness? And finally, do we have to reevaluate policy analysis in the presence of price dispersion? How does normative analysis of rent and eviction control change in a non-Walrasian market with price dispersion?
1.2 Methodological Issues

The comprehensive notion of housing demand as the choice among a collection of heterogeneous alternatives raises many methodological issues. There is the question of the appropriate functional form for housing demand equations which include the qualitative and quantitative components of the commodity housing. We will resolve this question in simply dividing the qualitative dimensions into sufficient of discrete categories, and proceed with large discrete choice models.

However, the specification of large discrete choice models is closely related to the question of substitutability among the choices which was raised in the previous section. Is there a feasible compromise between choice models which are easy to compute but impose strict cross-substitution patterns, and choice models which leave freedom for the cross-substitution effects but are computationally intractable? We will show that nested multinomial logit models (McFadden, 1978) constitute such a compromise in housing demand analysis. The unresolved issues at stake is the efficiency loss of the sequential estimation technique and the viability of full information maximum likelihood. A further theoretical issue is whether the estimation results can be rationalized by a highly structural economic choice model, the random utility hypothesis.

Household formation as a part of housing demand raises the question of how usable our household based surveys are. The entity
"household" is endogenous and we face a potential self selection bias in our estimations. How can we resolve this sample selection problem? Is it possible to avoid a structural model of household formation which is bound to be poorly estimable due to our poor knowledge about this process and will result in a large noise-to-signal ratio? Can we find a reduced form approach with just enough structure to resolve the endogeneity problem?

A final methodological issue is the handling of price dispersion generated by intertemporal processes when only a single cross section of observations is available. How large is the potential bias in estimations ignoring price dispersion? How do we get unbiased results?
1.3 Organization of the Thesis

The remainder of the thesis is organized in four chapters. The first of these chapters is devoted to the microeconomic and econometric underpinnings of our basic tool, the nested multinomial logit demand functions. The microeconomic part includes the compatibility of these demand functions with random utility maximization, the econometric part discusses the use of full information maximum likelihood estimation.

Chapter Three applies the demand model on the joint choice of household formation, tenure, and size of dwelling to three SMSA's in the United States to answer the question of how price responsive our comprehensive housing demand is. Some microsimulation results illustrate public policy implications.

Chapter Four is a digression on the dynamic nature of the rental housing market. This chapter, mainly microeconomic theory, is intended as a motivation and guidance to analyse price dispersion generated by a complicated intertemporal interaction of the demand and supply side.

Finally, Chapter Five applies all the tools we have collected so far on an analytic comparison of housing demand in West Germany and the United States: A common hierarchical choice model for both countries embodies hedonic rent indexes to capture the heterogeneity of the housing stock and corrects for price dispersion in both
countries.

Each chapter contains an introductory section to present the issues at stake, and a conclusion to summarize the results. An appendix lists the FORTRAN source of the full information maximum likelihood estimation program for nested multinomial logit models in random and choice based samples.
CHAPTER TWO

THE BASIC TOOL: NESTED MULTINOMIAL LOGIT DEMAND FUNCTIONS
2.1 Introduction: Discrete Choice Description of Housing Demand

Housing or, more precise, the service stream from a housing unit, is a heterogeneous commodity. Some dimensions, as size or age of structure, are measured on a continuous scale, others, as tenure or type of structure, are discrete properties. Measuring the volume of housing services as housing expenditure essentially ignores this heterogeneity, and for a large number of policy purposes, the distribution of housing consumption into qualitatively different categories is of more interest than an aggregate quantitative measure of housing expenditures alone. The most popular example of the interest in qualitative dimensions is the choice between renting and owning, and the response of this tenure choice to federal income tax treatment. (See Laidler (1969), Rosen (1979), Rosen and Rosen (1980), Henderson and Ioannides (1983).)

We can go one step further: not only the choice of tenure, but also the choices among other continuous or discrete characteristics of a housing unit will be affected by taxes and subsidies. Furthermore, the decision whether to form an autonomous household at all may be dependent on relative prices and income. Thus, housing demand decisions consist of discrete decisions, e. g., concerning headship, tenure, as well as continuous decisions, e. g., size or quality level.

Lee and Trost (1978) and subsequently King (1980) argue that the tenure choice and the choice of size and quality level are made simultaneously. We will point out in Chapter 3 that also the headship
choice is influencing and is in turn influenced by the other two decisions, so that all three choices are made in a joint decision process. This joint decision process constitutes a comprehensive notion of housing demand which will be the focus of this thesis.

The econometric theory of joint discrete/continuous models is well studied, and there exist a variety of applications, e.g. Lee and Trost (1978), King (1980), or Dubin and McFadden (1984). We will not pursue this line of modeling, however, but use consistently a discrete choice framework throughout this work. Sweeney (1974) casts the entire bundle of quality characteristics into discrete categories so that housing units can be arranged in a commodity hierarchy. We will use a similar discretization of the quality space in a finite number of housing alternatives. Discretization of continuous variables has widely been applied in transportation economics, see Chiang, Roberts, and Ben-Akiva (1982) for a model of freight mode and shipment-size, or Small (1981,1982) and Small and Brownstone (1981) for discrete models of trip timing. Ben-Akiva and Watanatada (1981) provide a theoretical analysis of the aggregation of a continuous variable into a finite number of discrete choices. There is good pragmatic reason to do so: it simplifies both the theoretical analysis and the empirical estimation. In addition, for most policy purposes, it suffices to explain or predict shifts among rough categories as "large owner-occupied houses", "low quality rental housing", or "non-headship." Table 2-1 lists the choices we will consider in our comprehensive notion of housing demand.
Table 2-1: Definition of Housing Alternatives

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Housing Alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH</td>
<td>Non Headship: lives as a subnucleus in another household</td>
</tr>
<tr>
<td>O_SF.S</td>
<td>Owner-Occupied, Single-Family-Structure, Small Dwelling</td>
</tr>
<tr>
<td>O_SF.M</td>
<td>Owner-Occupied, Single-Family-Structure, Medium Dwelling</td>
</tr>
<tr>
<td>O_SF.L</td>
<td>Owner-Occupied, Single-Family-Structure, Large Dwelling</td>
</tr>
<tr>
<td>O_2F.S</td>
<td>Owner-Occupied, Two-Family-Structure, Small Dwelling</td>
</tr>
<tr>
<td>O_2F.M</td>
<td>Owner-Occupied, Two-Family-Structure, Medium Dwelling</td>
</tr>
<tr>
<td>O_2F.L</td>
<td>Owner-Occupied, Two-Family-Structure, Large Dwelling</td>
</tr>
<tr>
<td>O_MF.S</td>
<td>Owner-Occupied, Multi-Family-Structure, Small Dwelling</td>
</tr>
<tr>
<td>O_MF.M</td>
<td>Owner-Occupied, Multi-Family-Structure, Medium Dwelling</td>
</tr>
<tr>
<td>O_MF.L</td>
<td>Owner-Occupied, Multi-Family-Structure, Large Dwelling</td>
</tr>
<tr>
<td>R_SF.S</td>
<td>Rental-Housing, Single-Family-Structure, Small Dwelling</td>
</tr>
<tr>
<td>R_SF.M</td>
<td>Rental-Housing, Single-Family-Structure, Medium Dwelling</td>
</tr>
<tr>
<td>R_SF.L</td>
<td>Rental-Housing, Single-Family-Structure, Large Dwelling</td>
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<tr>
<td>R_MF.L</td>
<td>Rental-Housing, Multi-Family-Structure, Large Dwelling</td>
</tr>
</tbody>
</table>
Estimating such a complex joint decision process poses a number of econometric problems: the choice set, that is the set of housing alternatives from which the consumer has to select one, is fairly large and consists of alternatives of which some are close substitutes and others not. The first problem restricts the possible specifications of the functional form of the relation between the choice probabilities and the explanatory variables to functions that have a structure which simplifies the computations involved, e.g., the class of generalized extreme-value functions. On the other hand, the second problem prohibits the use of simplifying assumptions like the Independence of Irrelevant Alternatives which reduces the multinomial decision to binary comparisons. As a viable compromise between computational simplicity and economic complexity, we will use nested multinomial logit models (NMNL) as the basic analytic tool for our empirical research. The remainder of this chapter reviews the microeconomic foundations and the econometrics of NMNL-models and extends the theory of the relation between utility maximization and estimated NMNL-parameters.
2.2 Random Utility Maximization and Hierarchical Choice

2.2.1 Microeconomic Theory

Let us assume the housing market is partitioned into $M$ discrete housing alternatives, e.g., as depicted in Table 2-1. We associate each of these alternatives with an index of desirability, which comprises all advantages and disadvantages for a given consumer into one scalar unit corresponding to the indirect utility function in neoclassical continuous consumer theory. Uncertainty about quality and erratic or irrational valuations introduce a stochastic component into this index. Like the hypothesis of utility maximization under budget restriction, we assume that each household will choose the alternative with the highest index of desirability. Due to the probabilistic nature of the index, we will call this the random utility maximization hypothesis (McFadden, 1981).

In the following, we will give this notion a more precise definition. For each household $t$ we decompose the desirability index $u_{it}$ of the alternative $i$ into a deterministic and a stochastic component:

\begin{equation}
    u_{it} = v_{it} + e_{it}
\end{equation}

The stochastic component $e_{it}$ is drawn from a $M$-dimensional joint distribution characterized by the cumulative distribution function $F(e_1, \ldots, e_M)$ with an associated finite-valued\(^1\) density $f(e_1, \ldots, e_M)$. The deterministic part $v_{it}$ is dependent on the characteristics of the alternative (e.g., price) as well as on the characteristics of the household (e.g., income), and is linear and additive separable\(^2\):
(2.2) \[ v_{it} = \sum_{k} x_{ik}^k \ast D_k + \sum_{l} y_{il}^l \ast a_{il} \]

where \( x_{ik} \) = the k-th characteristic of alternative i for household t,
\( y_{il} \) = the l-th characteristic of household t,
\( a_{il}, b_k \) = weights (to be estimated).

Choices are made by pairwise comparison of utilities. Thus, only \( M-1 \) differences of utilities describe the choice behavior. This implies that household specific variables that are alternative invariant will be irrelevant for the choice among alternatives as long as they do not interact with each alternative. We therefore let the weights of the household characteristics vary by alternative.

In addition to uncertainty and erratic valuations, the stochastic disturbance \( e_{it} \) will pick up deviations of the household t from the weights \( a_{il} \) and \( b_k \) in the population. The different components of \( e_{it} \) can not be identified or only under specific assumptions.

Household t will choose alternative i, if \( u_{it} > u_{jt} \) for all \( j \neq i \). Thus, the probability that household t chooses i among all \( M \) possible alternatives is

\[
P_t(i) = \text{Prob}\{ v_{it} + e_{it} > v_{jt} + e_{jt} \mid j \neq i \} \]

\[
= \int_{\infty}^{\infty} \int_{u_{it} - v_{it}}^{\infty} \cdots \int_{u_{it} - v_{Mt}}^{\infty} dF(e_{1t}, \ldots, e_{Mt})
\]

where \( F \) denotes the joint cumulative distribution function of the errors \( e_{it} \).
Definition (Random Utility Maximization)

Choice probabilities $p_t(i)$ are said to be generated by random utility maximization, if there exists a random utility function (2.1), characterized by a linear, additive separable deterministic utility (2.2) and a distribution function $F$ with a finite-valued density of the stochastic utility, such that (2.3) holds.

Finally, the aggregation

$$f(i) = \frac{1}{T} \sum_{t=1}^{T} p_t(i) \quad i=1, \ldots, M$$

will yield the relative frequencies of alternative $i$ in the population, also called aggregated or market shares of choice $i$, provided the households $t$ are a random sample of the population.
2.2.2 Functional Specification of the Choice Probabilities

This theory has a very important implication: for a given specification of the deterministic utility \( v_{it} \), the choice of a functional form for the relation between the choice probabilities \( p_t(i) \) and the explanatory variables \( x^k_{it} \) and \( y^l_{it} \) is equivalent to the specification of the joint distribution \( F \) of the error terms \( e_{it} \).

The integral formula (2.3) shows the dilemma for this choice. On one hand, the correlation among the \( e_{it} \) should be as flexible as possible to allow different correlations among the choice probabilities. On the other hand, the computational effort of evaluating the multidimensional integral should be minimized, suggesting a distribution function \( F \) where this can be done explicitly. This in particular prohibits the use of a normal distribution for problems with more than four alternatives.

Two families of distribution functions allow easy evaluation of the integral. One leads to a linear functional relation between the choice probabilities and the explanatory variables, and thus does not take account of the adding up and the unity interval restrictions of the choice probabilities. The other family is that of generalized extreme-value distributions, an extension of the logit approach; this is the family we will use to specify the choice probabilities.

A completely free correlation structure of the disturbances implies the estimation of \( M^*(M-1)/2 \) correlation coefficients which is
impractical for most sets of alternatives. Thus, further restrictions are necessary. The most drastic restriction is to postulate the independence of the $e_{it}$. Then the multidimensional integral can be factorized into a product of simple integrals. If in addition the $e_{it}$ are extreme value distributed, the resulting choice probabilities are of the well-known multinomial logit form. An application of the latter specification to the housing market can be found in Quigley (1976).

The assumption of independent $e_{it}$ is known as "Independence of Irrelevant Alternatives" (McFadden, 1973) due to the following necessary and sufficient characterizations:

1. The $e_{it}$ are stochastically independent.
2. The odds of choosing alternative $i$ over alternative $j$ are independent of the attributes of all other alternatives and independent of the existence of any other alternative.
3. The elasticity of the relative frequency $f(i)$ of alternative $i$ with respect to the attributes of any other alternative $j \neq i$ is constant, that is independent of $j$.

Therefore, independence can only be assumed for alternatives that are "equally different," but not for alternatives with different degrees of substitution. The following example translates a classical example (Domencich and McFadden, 1975) into the housing market. For simplicity, consider the tenure choice. Let us assume the relative odds are 1:1 for renting versus owning. Let us introduce a third, new form of tenure (e.g., cooperative) which is a very close substitute for owning. Intuitively, we would expect the new distribution to be something like 50% : 25% : 25%. But condition (2) tells us that the relative odds of renting versus owning have to stay constant, forcing the new distribution to be 33% : 33% : 33%, which is implausible.
because of the similarity of owning individually and owning cooperatively.

The failure to accommodate different degrees of cross-alternative substitution renders the multinomial logit specification inappropriate for such heterogeneous choice sets as depicted in Table 2-1. On the other hand, the possibility of grouping or clustering the alternatives according to their degree of substitution allows us a relatively straightforward way of combining the computational simplicity of the multinomial logit form with a richer substitution pattern: for each cluster, we introduce a parameter that describes the similarity of its alternatives. We can do the same with clusters themselves, and thereby achieve a hierarchical structure of similarities and substitution patterns. Within each cluster and between the clusters, we apply multinomial logit choice probabilities. This approach is called "Nested Multinomial Logit" (NMNL). McFadden (1981) gives a discussion of the development of these models and their relation to other discrete choice approaches.

For the application at hand, let us introduce three steps of clustering. First, we bundle housing alternatives by size and quality, then these clusters by tenure and type of building, and finally by all headship alternatives versus the nonheadship alternative, see Figure 2-2 for a simple example. We can look at NMNL models in two ways: they represent hierarchically grouped clusters of alternatives with a large within group substitutability, and we can interpret them as hierarchical decision processes or decision trees,
FIGURE 2-2: CLUSTERS OF SIMILAR ALTERNATIVES

NH: non-head
R: rented
O: owned
S: small
M: medium
L: large
where each nucleus decides whether to head a household or not, the heads decide about tenure, and owners and renters choose their dwelling size and quality. Of course, this does not necessarily imply a temporal decomposition of the decision process. Figure 2-3 represents this second interpretation graphically, and the equivalence to the representation of Figure 2-2 can be seen in each of the steps.

We can decompose the choice probabilities for a three-level hierarchical decision process into a marginal choice probability at the highest level of the decision tree and conditional probabilities at each lower level (we suppress the index $t$ for the individual household):

\begin{equation}
(2.5) \quad p(i) = p_H(H_i) \times p_T(T_i|H_i) \times p_S(S_i|H_i,T_i)
\end{equation}

where

- $p_H(H_i)$ = probability of the headship choice $H_i$ that is implied by choosing alternative $i$,
- $p_T(T_i|H_i)$ = probability of the tenure choice $T_i$ implied by choosing alternative $i$, given headship choice $H_i$,
- $p_S(S_i|H_i,T_i)$ = probability of the size choice $S_i$ implied by alternative $i$, given headship choice $H_i$ and tenure choice $T_i$.

At each level, the conditional choice probabilities have the multinomial logit form:
FIGURE 2-3: DECISION TREES FOR HOUSING ALTERNATIVES

NH: non-head
R: rented
O: owned
S: small
M: medium
L: large
\[ p_S(S_1|H_1,T_1) = \exp( v(S_1) ) / \sum \exp( v(S_j) ), \]
(summation over all size choices \( S_j \) possible in tenure choice \( T_1 \))

\[ p_T(T_1|H_1) = \exp( u(T_1) ) / \sum \exp( u(T_j) ), \]
(summation over all tenure choices \( T_j \) in headship choice \( H_1 \))

\[ p_H(H_1) = \exp( w(H_1) ) / \sum \exp( w(H_j) ), \]
(summation over all headship choices \( H_j \))

In these choice probabilities, \( v(S_i) \) denotes the utility, a consumer derives from dwelling size \( S_i \), and \( u(T_i) \) \( (w(H_i)) \) the utility from tenure choice \( T_i \) (headship choice \( H_i \), respectively) implied by choosing alternative \( i \).

At the higher levels, we assume that there is no utility per se of either headship or tenure over and above the utility derived from the alternatives underlying each tenure or headship choice. If we aggregate the utility provided by all alternatives \( S_j \) in tenure category \( T_1 \), we obtain (McFadden, 1978):

\[ (2.6) \quad u(T_i) = \log \sum_{j \in T_i} \exp \left( c_j \times v(S_j) \right) \]

and similarly for the aggregated utility of the headship choice \( H_1 \):

\[ (2.7) \quad w(H_i) = \log \sum_{j \in H_i} \exp \left( d_j \times u(T_j) \right) \]
Note that the attributes of the lowest level utility, e.g., income and prices in the size choice, enter recursively, bottom-to-top, the utility of the tenure and headship choices. On the other hand, decisions are clustered in a sequential fashion, top-to-bottom, as Figures 2-2 and 2-3 suggest. Altogether, we have achieved a simultaneous choice of headship, tenure, and dwelling size where prices as well as income are allowed to influence not only the size and tenure choice, but also the headship decision and thus household formation.

The taste weights $c_j$ and $d_j$ have to be estimated. The aggregate utility levels $u(T_i)$ and $w(H_i)$ are called inclusive values of their respective lower level alternatives because they can be interpreted as the surplus generated by these alternatives. The taste weights $c_j$ and $d_j$ are called dissimilarity parameters because they can be interpreted as a measure of the substitutability among the respective lower level alternatives. For $c_j$ and $d_j$ equal to one, the decision tree model collapses to a simple multinomial logit choice model among all alternatives. If they are smaller than one, alternatives in the respective clusters are close substitutes relative to other alternatives.

We can test whether the difference between the simple MNL model and the nested MNL model is significant. Usually, the MNL model has to be estimated in a first stage to obtain initial values for the NMNL estimation. Thus, we have all ingredients of a likelihood ratio test, see Table 5-13 for examples. Furthermore, we can construct a Wald
test based on the estimated dissimilarity coefficients in the WMNL model with their joint covariance matrix. For simple one-dimensional tests we can look at the asymptotic t-statistics around one as reported in the estimation results. Finally, we can calculate a Lagrange multiplier test and evaluate this test at the simple MNL estimates, see McFadden (1983) for the appropriate formula. The asymptotic and small sample properties of this test trinity is examined by Hausman and McFadden (1981). These test are tests of the simple MNL functional specification versus the nested MNL model. Thus, they amount to tests of the independence of irrelevant alternatives property.

The random utility maximization interpretation of the WMNL functional form rested on the integral formula (2.3). However, the cumulative distribution function $F$ is parameterized by the taste weights $a_i$, $b_k$, the dissimilarity coefficients $c_i$, $d_i$, and depends on the data as well. If all similarity parameters are in the unit-interval, the underlying joint distribution of the disturbances is well behaved and consistent with the microeconomic theory outlined at the beginning of this section, independent of the explanatory variables. With similarity parameters outside the unit-interval, this consistency will hold only for a certain range of explanatory variables, and it must be checked, whether this range includes the given data. This check and a reconciliation of such WMNL models with the random utility maximization hypothesis is discussed in the following.
2.2.3 The Relation Between Dissimilarity Parameters and the Random Utility Hypothesis

Let $c_T$ denote the similarity coefficient corresponding to the first-order clusters of elementary alternatives (say, tenure categories), and $d_H$ the similarity coefficient corresponding to the second-order clusters consisting of first-order clusters (say, headship categories). The NMNL functional form specified in (2.5) is then equivalent to the following joint cumulative distribution function of the errors $e_{i_t}$ in (2.1) (McFadden 1978):

$$F(e_1, \ldots, e_M) = \exp \left\{ -G \left[ \exp(-e_1), \ldots, \exp(-e_M) \right] \right\}$$

with

$$G[y_1, \ldots, y_M] = \sum_H \left( \sum_T \left( \sum_S \frac{1}{c_T c_T/d_H d_H} y_S \right) \right)$$

where we sum over the highest-order clusters $H$, the first-order clusters $T$ contained in each cluster $H$, and finally over the elemental alternatives $S$ in each cluster $T$.

Two theorems provide the link between NMNL-models and the random utility maximization hypothesis (RUM). They are global statements in the sense of being independent of the realization of the explanatory variables.
Theorem 1 (Global Sufficiency) (McFadden 1979):
Let $0 < d_H \leq 1$ and $0 < c_T/d_H \leq 1$ for all $T$ and $H$.
Then the NMNL model is consistent with RUM for any data.

Theorem 2 (Global Necessity) (Williams 1977, Daly and Zachary 1979):
Let $d_H > 1$ or $c_T/d_H > 1$ for at least one $T$ or $H$.
Then it is always possible to construct data at which the NMNL model is inconsistent with RUM.

The question arises, whether this possibility of RUM-inconsistent data points is relevant for the data at hand. Theorem 2 leaves the possibility open that for the data given by the application, the NMNL model is consistent with RUM, and that the data points where the inconsistency occurs are insensible for the given application. Thus, the purpose of the remainder of this Section is to construct a discrete choice model that (1) is compatible with RUM, (2) has the same cumulative distribution function $F$ for the given data points, and (3) preserves the choice probabilities of the original NMNL model. We shall give a necessary and sufficient condition under which such a construction is possible.

The failure of the random utility hypothesis occurs because the estimated dissimilarity coefficients prevent $F$ from (2.8) being a cumulative distribution function:
Lemma 1:

Let \( c_T \) and \( d_H > 0 \).

(1) Then \( F \) is differentiable to any order, in particular, all mixed partial derivatives exist.

(2) \( F \to 1 \) for \( e_1 \to \infty \), \( F \to 0 \) for \( e_1 \to -\infty \).

(3) The choice probabilities derived from \( F \) obey

\[
0 \leq p_t(i) \leq 1 \text{ and } p_t(t) = 1.
\]

The Lemma follows from the obvious properties of (2.5), (2.8), and (2.9). Thus, Theorem 2 implies the existence of at least one point in which \( F \) has a negative mixed partial derivative, i.e., a point of negative marginal or joint "density".

We shall illustrate the effect of \( d_H > 1 \) in the simplest case of a three-alternative, two-level NMNL-model in which the first two alternatives constitute a cluster. This model has the cumulative distribution function

\[
F(e) = \exp \left\{ - \left[ \exp(-e_1/d) + \exp(-e_2/d) \right]^d - \exp(e_3) \right\}.
\]

Because of translation invariance (see Footnote 3), we can reduce \( F \) to the two dimensional space of differences without loosing information. Let \( v = e_2 - e_1 \) and \( w = e_3 - e_1 \). Their joint distribution function is

\[
F'(v,w) = \frac{(1 + \exp(-v/d))^{d-1}}{(1 + \exp(-v/d))^d + \exp(-w)}
\]

with density

\[
f'(v,w) = \frac{(1 + \exp(-v/d))^{d-2}}{(1 + \exp(-v/d))^d + \exp(-w)} \ast \exp(-w) \ast \exp(-v/d) \ast \text{discr},
\]

\[
a(n,v,w) = \frac{(1 + \exp(-v/d))^{d-1}}{(1 + \exp(-v/d))^d + \exp(-w)} \ast \exp(-w) \ast \exp(-v/d) \ast \text{discr}.
\]
where the discriminant term

\[ \text{discr} = \frac{2(1+\exp(-v/d))^d - d-1}{(1+\exp(-v/d))^d + \exp(w)} \]

designs the density \( f^* \).

For \( d < 1 \), \( \text{discr} > 0 \). However, for \( d > 1 \),

\[ f^*(v,w) > 0 \iff w > \log(d-1) - \log(d+1) - d \log(1+\exp(-v/d)). \]

This function approaches the constant \( \log(d-1) - \log(d+1) \) for \( v \to \infty \)
and a by this constant shifted 45-degree line for \( v \to -\infty \). Thus, we

can partition the \((v,w)\)-plane in a part with nonnegative and a part

with negative density, see Figure 2-4. Note that the latter part
cannot be contained in any choice probability defining orthant.

If any of our data points is in the (shaded) area \( f^* < 0 \), we will
not be able to explain the data by preference maximization using the
integral formula (2.3) underlying RUM. Moreover, if \( f^*(e) < 0 \) for
some \( e \), then the continuity of \( f^* \) implies the existence of a point \( e \)
with \( F^*(e) > 1 \). This point need not necessarily be in the set
\( \{f^* < 0\} \). Again, we cannot rationalize the data by RUM. Thus,

\textbf{Theorem 3 (Local Necessity):}

Let \( A \) be a set containing all data points.

Let any one of the following conditions be true:

(1) A mixed partial derivative of \( F \) up to order \( M \) is negative
at a point in \( A \).

(2) \( F \) exceeds unity at a point in \( A \).

Then the construction of a RUM-compatible discrete choice model in \( A \)
is not possible\(^5\).
The proof is trivial: $F$ cannot be a c.d.f. if it exceeds unity or any of its associated marginal or the joint densities is negative.

If neither of the conditions of Theorem 3 is raised, we can indeed reconcile a NMNL-model with large dissimilarity coefficients with random utility maximization:

**Theorem 4 (Local Sufficiency):**

Let $A$ be an open interval containing all data points.

Let both of the following conditions be true:

1. All mixed partial derivative of $F$ up to order $M$ are nonnegative in $A$.
2. $F$ does not exceed unity at a point in $A$.

Then for any positive $c$, $d$ exists a continuation $F^c$ of $F$, such that

(a) $F^c$ is a cumulative distribution function,

(b) $F^c$ generates the same choice probabilities as $F$.

We will give the proof by construction. Figure 2-5 illustrates the construction in the case of three alternatives, already underlying Figure 2-4. We first use the translation invariance principle to reduce the dimension of the problem to $M-1$: for any point $y$ in $\mathbb{R}^M$, let $y^*$ be the $M-1$ dimensional vector $(y_1-y_2,\ldots,y_1-y_M)'$. Correspondingly, we introduce $A^*$, $F^*$, and $f^*$ as the $M-1$ dimensional counterparts of $A$, $F$, and $f$. For a set $C$, we use $\text{int}(C)$, $\text{bnd}(C)$, $\text{ext}(C)$, and $\text{clo}(C)$ to denote the interior, the boundary, the exterior,
FIGURE 2-4: THREE ALTERNATIVES: CHOICE PROBABILITIES AND DENSITY

\[ e_2 + y_2 > e_1 + y_1 \]
\[ e_2 + y_2 > e_3 + y_3 \]
\[ \Rightarrow \]
\[ e_2 - e_1 > y_1 - y_2 \]
\[ (e_2 - e_1) - (e_3 - e_1) > (y_1 - y_3) - (y_1 - y_2) \]

\[ P_1: \]
\[ e_1 + y_1 > e_2 + y_2 \]
\[ e_1 + y_1 > e_3 + y_3 \]
\[ \text{or} \]
\[ e_1 - e_2 > y_1 - y_2 \]
\[ e_1 - e_3 > y_1 - y_3 \]

\[ P_2: \]
\[ e_2 + y_2 > e_1 + y_1 \]
\[ e_2 + y_2 > e_3 + y_3 \]
\[ \Rightarrow \]
\[ e_2 - e_1 > y_1 - y_2 \]
\[ (e_2 - e_1) - (e_3 - e_1) > (y_1 - y_3) - (y_1 - y_2) \]

\[ P_3: \]
\[ e_3 + y_3 > e_1 + y_1 \]
\[ e_3 + y_3 > e_2 + y_2 \]
\[ \Rightarrow \]
\[ e_3 - e_1 > y_1 - y_3 \]
\[ (e_3 - e_1) - (e_2 - e_1) > (y_1 - y_3) - (y_1 - y_2) \]

\[ f^*(v,w) > 0 \]

\[ f^*(v,w) < 0 \]
FIGURE 2-5: CHOICE PROBABILITY PRESERVING CHOICE MODELS

\[ A \]

\[ L(y_1) \]

\[ L(y_2) \]

\[ e_2-e_1 \]

\[ e_3-e_2 \]

\[ p_{2}^{\text{min}} \]

\[ p_{1}^{\text{min}} \]

\[ p_{3}^{\text{min}} \]

Data
and the closure of C, respectively.

For each $y^*$, we can partition $\mathbb{R}^n$ according to the $M$ choice probability defining open sets

$$P_i(y^*) = \{ e^* \in \mathbb{R}^n \mid e^*_i + y^*_i > e^*_j + y^*_j, \ j \neq i \}$$

and the $M$ separating half-hyperplanes

$$L_i(y^*) = \{ e^* \in \mathbb{R}^n \mid e^*_i + y^*_i < e^*_j + y^*_j, \ j \neq i \ ; \ e^*_k + y^*_k = e^*_j + y^*_j, \ j, k \neq i \}.$$

The sets $P_i(y^*)$ and $L_i(y^*)$ will be important at the boundary of $A$, and it is convenient to define for $y^*$ in $\text{bnd}(A^*)$:

$$\mathbb{P}_{\text{min}} = \bigcap_{y^* \in A^*} P_i(y^*)$$

and

$$L(y^*) = L_i(y^*)$$

with $i$ such that $L_i(y^*) \cap \text{clo}(A^*)$ is empty.

The $\mathbb{P}_{\text{min}}$ define the smallest choice probability of alternative $i$ attainable in $\text{clo}(A^*)$ where this well-defined minimum occurs for each $i$ at one of $M$ corners of the interval $A^*$. The $L(y^*)$ define the half-hyperplane pointing outward of $A^*$. This is well-defined except for the above mentioned $M$ corners. Here, we define:

$$L(y^*) = \mathbb{P}_{\text{min}}.$$

We now construct a choice model which has identical choice probabilities in $A^*$ by shifting all probability mass outside of $A^*$ onto the boundary of $A^*$ in a way which does not distort the original choice probabilities.
We define
\[ f^*(y) = f^*(y) \text{ for } y \text{ in } A^*, \]
\[ f^*(y) = \int dF^* \text{ for } y \text{ in } \text{bnd}(A^*), \]
\[ L(y^*) \]
\[ f^*(y) = 0 \text{ for } y \text{ in } \text{ext}(A^*). \]

We have to show:

(i) \( f^* \) nonnegative,

(ii) \( f^* \) integrates to one,

(iii) \( P_t(i) = \int dF = \int dF^* \).

\[ P_t(i) \]

To (i):

We only need to show the nonnegativity at the boundary.

Case 1: \( y \) is at one of the \( M \) corners defining \( P^m_{\text{in}}. \)

Then the \( f^*(y) \) is a choice probability which is always nonnegative for positive \( c_T, d_H \) by Lemma 1.

Case 2: \( y \) otherwise.

Then \( \int dF^* = \lim_{h \to 0} \frac{1}{h} \left( \int dF^* - \int dF^* \right) \)
\[ L(y^*) \]
\[ P_j(y^*) \]
\[ P_j(y^* - hz_j) \]

where \( j \) is an arbitrary index other than \( i \) and \( z \) a vector of zeroes except a one at the \( j \)-th component. This, however, defines the \( j \)-th marginal density of \( F^* \) which is nonnegative due to assumption (1).

To (ii):

Decompose \( \int f^*(u)du = \int f^*(u)du + \int f^*(u)du + \int f^*(u)du \)
\[ R^N \]
\[ A^* \]
\[ \text{ext}(A^*) \]
\[ \text{bnd}(A^*) \]

\[ = \int dF^* + \int dF^* + 0 = \int dF^* = 1, \]
\[ A^* \]
\[ R^N \setminus A^* \]
\[ R^N \]

because \( F(e) \to 1 \) for any \( e_i \to \infty \) for positive \( c_T, d_H \) by Lemma 1.
To (iii):
\[ p_1(y) = \int dF^* = \int_{P_1(y) \cap A^*} dF^* + \int_{P_1(y) \cap (R \setminus A^*)} dF^* \]

\[ = \int_{P_1(y) \setminus A^*} dF^* + \int_{P_1(y) \cap \text{bnd}(A^*)} dF^* \]

\[ = \int_{P_1(y) \cap A^*} dF^* + \int_{P_1(y) \cap (R \setminus A^*)} dF^* = \int dF^* \]

This proves Theorem 4.

Theorem 4 extends the usefulness of NMNL-models by reconciling large dissimilarity coefficients with random utility maximization, provided, conditions (1) and (2) hold. In practice, these conditions can be checked by evaluating the density and cumulative distribution function at the corners of an interval containing the data.

Furthermore, the idea behind Theorem 4 can be exploited to construct a large class of RUM-compatible discrete choice models. Any function which is translation invariant in the sense of Footnote 3 and obeys conditions (1) and (2) of the Theorem in an interval containing the data can be extended to a cumulative distribution function generating a probabilistic choice system by equation (2.3).
2.3 Estimation Techniques

2.3.1 Econometric Theory and Numerical Analysis

The likelihood function of the hierarchical choice model are the logarithms of the choice probabilities (2.5), cumulated over all consumers. Thus, we can estimate the model by maximizing over the taste weights \( a_{i1}, b_k \) and the similarity coefficients \( c_i, d_i \). Because the full information maximum likelihood function is highly nonlinear in the similarity parameters, this approach is costly. As an alternative, we can exploit the recursive structure in (2.5) and estimate sequentially by level of clustering. This approach has been applied to a large number of problems in transportation, energy demand, and urban economics, see Domencich and McFadden (1975) or Anas (1982). However, the sequential estimator is inefficient, especially for complex decision trees. Furthermore, it can not embody parameter restrictions across branches/clusters of alternatives.

The first point relates to the flow of information: the sequential estimator uses all the information of the lower branches to estimate the dissimilarity coefficients at the upper levels, but not conversely. Amemiya (1978) noted that the standard errors of the estimated coefficients at the upper levels have to be corrected for the presence of lower level estimated coefficients. McFadden (1981) provides the proper formulae. Evaluating these corrections is expensive and greatly reduces the computational advantages over the FIML estimator, see Small and Brownstone (1982). The second point is more important: if the alternatives in different branches have taste
weights in common, a proportionality constraint has to be fulfilled which is non-trivial for the case of at least two elementary alternatives in at least two branches:

![Diagram of the sequential estimator](image)

The sequential estimator estimates \((b/c)_1\) and \((b/c)_2\) in the first stage choices, then \(c_1\) and \(c_2\) in the choice between the branches. A common \(b\) implies

\[
b = (b/c)_1 \times c_1 = (b/c)_2 \times c_2.
\]

However, imposing this restriction destroys the sequential decomposition and leads us back to full information maximum likelihood. The proportionality constraints could be imposed by an iterative procedure where the \(c_i\) from the sequential estimation as described are used in a second sequential estimate to scale the \((b/c)_i\), as \((b/c)_1\), calculate new \(c_1\), and so on. This iteration will further reduce the computational advantages over FIML estimation in addition to the necessary correction of the standard errors, in particular so in higher level trees.

We therefore prefer joint estimation and use the modified quadratic hill-climbing method developed by Goldfeld and Quandt (1972) with analytical first and numerical second derivatives. This procedure proved computationally fairly efficient compared with BHHH procedures (Berndt, Hall, Hall, Hausman, 1974). Small and Brownstone
(1982) report similar experiences with BHHH as compared to a method of scoring. The reason for the relative poor performance of the BHHH algorithm seems to be the highly nonlinear dependence of the likelihood function on the dissimilarity parameters. In particular, the function and its gradient have a singularity in $c_i$ or $d_i$ at zero. The singularity is well behaved for the likelihood, but is a pole for the gradient. Thus, the outer product of the gradients is ill-behaved and a bad approximation of the hessian for small values of the dissimilarity parameters.

This unpleasant behavior of the likelihood function requires a careful numerical analysis of the algorithms involved. In particular, the data should be normalized to prevent the multiplication of very small with very large numbers and numerical extinction. The appendix lists a FORTRAN program for the full information maximum likelihood estimation of NMNL models which embodies all these considerations.
2.3.2 Elasticities and Goodness-of-Fit Measures

Following the random utility maximization theory and equation (2.2), the estimated parameters represent the taste weights of the respective explanatory variables in the deterministic part of the indirect utility function $v_{it}$. For a more intuitive interpretation of their magnitudes, the taste weights can be transformed into elasticities of the choice probabilities with respect to the various explanatory variables:

\[
\log p(i) \quad (2.10) \quad \frac{\partial \log p(i)}{\partial \log x_{jk}} = a_k \cdot x_{jk} \cdot (\frac{-p(j)}{1/c} + k_S \cdot 1/c + k_T \cdot (1/c-1/d) \cdot Q(S) + k_H \cdot (d-1)/d \cdot Q(S) \cdot Q(T))
\]

where

- $k_S = 0$ if $i$ and $j$ are in the same size category
- $k_T = 0$ if $i$ and $j$ are in the same tenure category
- $k_H = 0$ if $i$ and $j$ are in the same headship category
- $c, d =$ similarity parameters: $c=\text{cr}$ and $d=\text{dH}$
- $Q(S) =$ conditional choice probability $p_S(S_j|H_j,T_j)$
- $Q(T) =$ conditional choice probability $p_T(T_j|H_j)$

These elasticities measure the percentage change of the probability to choose alternative $i$, when the $k$-th attribute of alternative $j$ is changed by one percent. Note that for the cross elasticities the difference between $i$ and $j$ enters only through the "switches" $k_S$, $k_T$, and $k_H$. The structure of the tree is therefore directly reflected in the pattern of cross elasticities. If the dissimilarity parameters $c$
and \( d \) are in the unit interval, equation (2.10) implies descending elasticities with the "distance" in the tree, i.e., elasticities are larger within than between branches. This plausible structure is destroyed for dissimilarity parameters larger than one, hinting to alternative tree specifications.

Derived from a highly non-linear model, elasticities at variable means are generally different from mean individual elasticities, and in interpreting the elasticities, one should keep the absolute level of the choice-probabilities in mind; the elasticities tend to be very high at very low probabilities and vice versa, reflecting saturation effects.

Three scalar measures of performance or fit will be used in the applications of Chapters 3 and 5. Amemiya (1981) provides an extensive review. The straightforward discrete analogy to the continuous \( R^2 \) uses the sum of squared errors:

\[
(2.1.1) \quad 1 - \frac{\sum \sum (y_{it} - P_{it}(b))^2 / P_{it}(b)}{\sum \sum (y_{it} - P_{it}(0))^2 / P_{it}(b)}
\]

where \( P_{it} \) denotes the predicted choice probabilities of alternative \( i \) for consumer \( t \), evaluated at the optimal parameter values \( b \) or at zero, and \( y_{it} \) the actual response\(^6\). However, this measure has little discriminatory power for well specified models. A more satisfactory measure can be constructed from the ratio of the likelihood at the estimated parameters and the likelihood with taste weights at zero and similarity parameters at one. One minus this ratio behaves like the
continuous $R^2$, see McFadden (1973):

\[ \frac{L(b)}{L(0)} = 1 - \frac{L(b)}{L(0)} \]

(2.12) \hspace{1cm} \textit{Domencich and McFadden (1975) give a comparison between these two measures of fit and their discriminatory power. As a third measure of fit, we compare actual with predicted individual choices which is a fairly stringent, though erratic criterion. Note that discrete choice models produce two predictions of the aggregate choice probabilities:}

\[ f(i) = \sum_{t} p_{t}(i) \]

(2.13) \hspace{1cm} \textit{with} \hspace{0.5cm} n(i) = \text{number} \{ t \mid p_{t}(i) = \max_{j=1..M} p_{t}(j) \}

\[ f(i) = \frac{n(i)}{T} \]

(2.14) \hspace{1cm} \text{\textit{T = sample size}}

The erratic nature of the percentage of correct predictions is due the integer constraint in (2.14). We can disaggregate this measure into the form of a success table in which observed and predicted alternatives are crosstabulated and the off-diagonal elements show the mispredictions.
2.3.3 Aggregate Probability Shares

The aggregate probability shares \( f(i), i=1..M \), where \( M \) denotes the number of alternatives, from equation (2.14) can be used for prediction and policy analysis. They should reproduce the aggregate shares in the population \( q(i) \) as close as possible. A multinomial specification with a full set of alternative specific constants will always reproduce the sample shares exactly which can be seen by adding up the first order conditions of the MNL-likelihood function with respect to the alternative specific dummies. This property is not carried over to the nested model. Anas (1982) gives some numerical examples for this bias. However, a full set of alternative specific constants still saturates the model and we can always solve the nonlinear system of \( M-1 \) equations in these constants to adjust the aggregated shares. This suggests the following two stage procedure: first we estimate all parameters freely; then, we solve this nonlinear equation system evaluated at the slope parameters of the first stage. The second step can be achieved by minimizing the sum of squared deviations of fitted to actual aggregated shares. This two stage procedure is consistent, but does not provide efficient parameter estimates. Usually, the adjustment necessary is very small, and so the loss in efficiency.

A more satisfactory approach is to maximize the likelihood function subject to the \( M-1 \) constraints \( f(i)=q(i), i=2..M \). This will yield efficient estimates (Coslett, 1981) where also the slope parameters embody the constraints, not only the constants.
Unfortunately, the nonlinear equation system can not be solved analytically, making a costly constraint maximization necessary which involves additional M-1 nuisance parameters and in general the solution of a saddlepoint problem as opposed to a simple maximization problem. Furthermore, the application of Kuhn-Tucker type algorithms is not possible because the constraints \( f(i) = q(i) \) are highly nonlinear in the alternative specific constants. We will only use the two stage procedure. We apply this procedure in Chapter Five to adjust our baseline estimates before making predictions and policy simulations.
2.3.4 Choice Based Sampling

The problem of fitting the known aggregate sample shares is related to the problems generated by choice based sampling. Choice based samples may arise in two ways: the data may originally be collected by sampling according to the observed choice. This is the case when we interview a fixed number of homeowners and a fixed number of renters and these numbers do not reflect the proportions in the population. Second, we may start from a large random sample. Typically, however, some choices have very low, others very high market shares. To achieve precise estimates for all choices, the overall sample size of a smaller random subsample drawn for estimation has to be large enough that even the smallest cell has a sufficient number of observations. This will yield very large cell counts for the popular choices. We can substantially decrease estimation costs by oversampling the infrequent choices, undersampling the frequent choices, and then treating our subsample as a choice based sample. We will make heavy use of this technique in Chapter Five.

Given a choice based sample, the parameters have to be estimated to predict the population, not the sample shares. Without a correction, the estimates are inconsistent (Heckman, 1979). Thus the efficient full information maximum likelihood estimator is again the Coslett (1981) estimator mentioned in the previous subsection and involves the solution of a saddlepoint problem with \( M-1 \) additional nuisance parameters. Alternative estimators are discussed in Manski and McFadden (1981) of which we mention the two most important.
First, we can compensate for choice based sampling by weighting the observations inversely to the ratio of over or undersampling. This estimator (weighted exogenous sampling maximum likelihood, WESML, Manski and Lerman, 1977) is as cheap to compute as the normal maximum likelihood estimator. Second, we can maximize the likelihood of an endogenously sampled observation conditional on its exogenous characteristics. This estimator (conditional maximum likelihood, CML, Hsieh, Manski, and McFadden, 1983) has a slightly more complicated likelihood function compared to the WESML estimator. Both estimators yield consistent estimates without the introduction of additional nuisance parameters, but there are not efficient compared to the Coslett estimator. However, the efficiency loss seems to be very small as indicated in McFadden, Hsieh, and Manski (1983) or McFadden, Winston, and Boersch-Supan (1984).

Therefore, and for its simplicity, we will only use the WESML estimator. The resulting likelihood function in our case is

\[
L = \sum_{t} \frac{q(i_t)}{f(i_t)} \log p(i,t,b) =: \sum_{t} w(t) \log P_t(b)
\]

where \(i_t\) denotes the chosen alternative of household \(t\),
\(q(i)\) the proportion of alternative \(i\) in population
\(f(i)\) the proportion of alternative \(i\) in the sample
\(p(i,b)\) the choice probability according to (2.5)
\(b\) vector of parameters

The covariance matrix of the estimated \(b\) can be derived by an exact Taylor approximation around the true parameter vector \(b^*\):
where $b^*$ lies on a line segment between $b$ and $b^o$.

Under the appropriate regularity conditions (Manski and McFadden 1981), we can apply a uniform law of large numbers (Jennrich 1969) to show

\begin{equation}
(2.17) \quad H_T \Rightarrow H = E\left( \frac{\partial^2 w(t) \log P(b^*)}{\partial b \partial b'} \right) ,
\end{equation}

and a uniform central limit theorem (Jennrich 1969) to yield

\begin{equation}
(2.18) \quad A_T \Rightarrow N(0, V)
\end{equation}

where

\begin{equation}
(2.19) \quad V = E\left( \frac{\partial w(t) \log P(b^o)}{\partial b} \frac{\partial w(t) \log P(b^*)}{\partial b'} \right).
\end{equation}

Thus,

\begin{equation}
(2.20) \quad \sqrt{T} \left( b - b^* \right) \Rightarrow N(0, H^{-1} V H^{-1}).
\end{equation}

As a consequence,

\begin{equation}
(2.21) \quad f(i) \not\Rightarrow q(i) \Rightarrow H \not\Rightarrow V,
\end{equation}

because $H$ includes the weights linearly, but $V$ quadratically. Thus, the inverse hessian does no longer provide an estimate for the covariance matrix of the estimated $b$. In the estimation, we will use the sample hessian to approximate $H$ and the sample outer product of the gradient to approximate $V$, both evaluated at the optimum.

The likelihood function (2.15) is a special case of the WESML estimator insofar, as we assume independent draws of households $t$, each with one choice of its housing alternative. This deviates from the analysis in Hsieh, Manski, and McFadden (1983) where multiple cell...
counts $m(i,t)$ are observed. In our case, $m(i,t)=1$ for $i=i_0$, the chosen alternative, 0 otherwise. In the case of multiple cell counts, which are distributed multinomially, the negative covariance between $m(i,t)$ and $m(j,s)$ may reduce the variance (2.19). Depending on the way the sample is drawn, either $E( m(i,t) m(i,s) )$ or $E( m(i,t) m(j,t) )$ will have a non-zero contribution.
2.4 Conclusions

This chapter provided the econometric tools for this thesis. We will cast all housing alternatives in a finite set of alternatives, structure them in the form of a hierarchical decision tree, and calculate the nested multinomial logit choice probabilities for each alternative.

These choice probabilities can be rationalized by utility maximization behavior of the consumers, but only under certain parameter restrictions. We developed necessary and sufficient conditions for the consistency of random utility maximization with the nested multinomial logit specification in the case of dissimilarity parameters in and outside the unit interval. If we maintain utility maximization as underlying structural behavior, we can interpret the violation of these conditions (the case of Theorem 3) as a hint to misspecification of either the indirect utility function (2.2) or, more important, the tree structure. Alternative tree structures may be derived from the elasticity patterns created by (2.10).

Because of the inefficiency of sequential estimation procedures and their inability to embody equal utility weights for prices and income across tenure and headship clusters, we use full information maximum likelihood estimation throughout the thesis. This has become possible with the use of sophisticated numerical procedures.
2.5 Footnotes to Chapter 2

(1) The requirement to be finite-valued implies that ties in the pairwise comparison of utilities occur only with probability zero.

(2) We impose linearity and additive separability of the indirect utility function in the definition of RUM. In the language of McFadden (1981), such models are defined as AIRUM-compatible.

(3) Note that the definition of the indirect utility function implies translation invariance of the choice probabilities in the following sense:
\[ p_u(i) = p_{u+c}(i) \text{ for all constants } c \text{ and all utility vectors } u \text{ in } \mathbb{R}^M, \]
or,
\[ G(y+c) = G(y) + c \text{ for all constants } c \text{ and all vectors } y \text{ in } \mathbb{R}^{M*}, \]
where \( G \) denotes the generating function (2.9).

(4) This amounts to including \( M-1 \) alternative specific constants \( D_i \) interacting with \( y_i \) and using a common parameter \( b_1 \) for all alternatives which can be seen by the transformation
\[ y_i \cdot D_i \cdot b_1 = y_i \cdot a_{i1}. \]

(5) The random utility maximization hypothesis as stated in Section 2.2 is only one rationalization of observed choice behavior with the notion of a homo economicus. Failure of RUM does not necessarily preclude the possibility that such a model is rational in an axiomatic sense, i.e., that it fulfills the axiom of stochastically revealed preferences. This is a combinatorial problem of a large dimension. See McFadden and Richter (1979).

(6) The sum of the squared residuals can be weighted in several ways. The natural weights are the true choice probabilities. As the best available estimates, we replace them by their maximum likelihood estimates. See Amemiya (1981) for alternatives.
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CHAPTER THREE

THE INFLUENCE OF HOUSEHOLD FORMATION ON HOUSING DEMAND
3.1 Introduction and Review of Earlier Approaches

Does the current rise in rental housing prices discourage household formation? Was household formation fostered by the falling real rents during the early seventies? This chapter tries to answer these questions to give a rationale for the more comprehensive definition of housing demand outlined in the first section of Chapter 2. New households are formed when existing households split: children decide to leave their parents' home, or marriages are divorced, and families, living together, decide to undouble. Conversely, households cease to exist because of death or merger due to marriage or doubling up. Which of these six mechanisms are affected by economic circumstances, in particular by housing market conditions? This seems implausible for death, but less so for marriage and divorce, and it is worthwhile to shed light on whether doubling up, undoubling, and the timing of the children's leave from home is influenced by the price of housing.

Whereas the influence of housing market conditions on household formation is a rather new topic, the influence of demographic factors on housing demand is well studied. The early literature on household formation, largely written by demographers, concentrated on two aspects: the rise or decline of the total population size, and the distribution of age and marital status within the population (Campbell, 1963, and Kobrin, 1973 and 1976). But a third aspect is as crucial in the determination of housing demand: how the population is
divided into households. This third aspect is closely related to the second, but by no means determines household formation. In particular, much interest has been devoted to the dramatic increase in the number of households with only one person, and the corresponding decline in the proportion of married couples in the total number of households. Both changes are larger than the change in the distribution of age and marital status alone would predict, see George Masnick (1983) or Alonso (1983).

Campbell (1963) used standard demographic techniques to predict headship rates for a given distribution of age and marital status. He then compared these rates with the actual rates and attributed the discrepancy to "the development of a taste for privacy and independence."

The search for an explanation of this development concentrated on the rise in real income during the sixties that made privacy and independence affordable. Beresford and Rivlin (1966) and Carliner (1975) present a verbal discussion of cross-sectional data focusing on this explanation. Maisel (1960) presents a descriptive analysis of aggregated time-series data and links changes in headship rates to cyclical variables like income and unemployment rates.

Three studies apply regression analysis to investigate the quantitative nature of this link: Hickman (1974) estimates an aggregate time-series regression of headship rates and finds a positive income effect. However, he does not report standard errors
of his estimates, so that the significance of his results cannot be assessed. DePamphilis (1977) attempts a time-series regression of headship rates on aggregate income, the number of young adults, and various interest rates, with questionable results. Finally, Michael, Fuchs, and Scott (1980) estimate an aggregate cross-section Berkson Theil regression of the proportions of one-person households across states. Their findings include a significant effect of the income level in the state. They study variables outside the conventional demographic and economic categories as well, e.g., whether the state has adopted a liberal legislation.

The rise of real income slowed considerably down in the mid-seventies, however, the upward trend in household formation even accelerated in this period, see Kitagawa (1981) and Masnick (1983). This puts the role of income as the most important explanation for the growth in household formation over and above demographic changes in some doubt. Observing the falling real rent level in this period\(^2\), an apparent conclusion is to attribute the affordability of privacy and independence not only to the income at disposal, but also to the price of privacy, the price of housing. A look at the recent development underscores the importance of this hypothesis: in the first years of the 1980's we finally observe household formation leveling off. At the same time real rents started to rise again. Does the current rising rent level discourage household formation?

Housing prices as determinants of household headship rates are considered already by Hickman (1974). He estimates a "negligible"
price elasticity and concludes that price effects are not important or cannot be separated from income effects. Michael, Fuchs, and Scott (1980) use this argumentation to exclude price variables a priori.

Hickman's result is confirmed by two studies that use individual cross-sectional data and venture the difficult task of modeling household formation behavior per se. Ermisch (1981) develops a microeconomic model of the determination of household size where the disadvantages of crowding have to be traded off against economies of scale in a multi-person household. He applies his model on data from the General Household Survey in Great Britain. Wial (1983) estimated a multinomial logit choice model among four household types, based on young unmarried men in the 1970 Census. The choices are to stay with the parents, form a independent one-person household, or share a household with one or more other persons. Both studies find insignificant price elasticities.

However, Hickman's hypothesis and the two behavioral models by Ermisch and by Wial are contradicted by two studies that use aggregate time-series data: Rosen and Jaffee (1981), and Smith, Rosen, Markandya, and Ullmo (1982) discover a highly significant influence of the aggregate rent level on headship rates, even after controlling for income.

One purpose of this paper is to add another piece of evidence to the contradictory results in the literature. We will use individual cross-sectional data as Ermisch (1981) and Wial (1983) did, but avoid
modeling the household formation process explicitly. As it turns out, we do in fact estimate highly significant price effects.

The findings of Rosen and Jaffee (1981), Smith et. al. (1982), and of this paper establish a causal relation from the housing market on household formation in the opposite direction of the well studied influence of demographic variables on household formation and housing demand. However, a simultaneous interaction between housing markets and household formation behavior has major implications for estimation and prediction of housing demand. The concept of households as basic sampling units in data collection and econometric estimation is blurred because households may consist of independent members who find together and break off for endogenous reasons. Thus, a sample of households can not be considered a random sample for the purpose of housing market analysis. Estimation results will biased due to a similar kind of sample selection as studied by Heckman (1979). Therefore, prediction and policy analysis will be biased as well. For instance, if privacy as an economic good complements housing, then a housing allowance program may foster household formation rather than induce moves of existing households into higher quality dwellings. Conversely, the current cuts in spending on public housing programs may greatly increase the number of households that "double up".

An analysis without the consideration of household formation may not only mispredict behavior quantitatively, but also qualitatively. Using Sweeney's (1974) commodity hierarchy model of the housing market, conventional analysis will predict an upward demand shift
along the quality/size categories in response to a housing subsidy. However, if housing allowances foster household formation, just the opposite may occur: existing households in large units will split up, and demand is increased in the lower categories of the hierarchy. Note that an accompanying supply program, designed to cushion the excess demand and based on conventional analysis, would fail. The provision of public housing with large units will not only be a waste, but even increase excess demand at the relevant level in the quality/size scale.

How should we design a model of housing demand that is able to answer the question of how price responsive household formation is? First, it should have a smaller decision unit than the household, second, it should have doubling up as an alternative to the conventional types of housing. We will use the concept of the family-nucleus (Pitkin, 1980) to define our decision unit, and we will use discrete choice analysis and include the choice of doubling up.

We will apply such a model to four representative population strata:

1. Young unmarried male and female without children, aged 20-34
2. Married couples with one or two children, aged 35-59
3. Elderly married couples without children, aged 60 and above
4. Widowed, divorced, and separated women without children, aged 60 and above.

The model is estimated for the three Standard Metropolitan Statistical Areas of Albany-Schenectady-Troy, New York; Dallas, Texas; and Sacramento, California, representing the Northeast, the Sunbelt, and the West Coast. The estimates are based on the Annual Housing Survey SMMA cross-sections in 1976 and 1977.
We then apply the estimates to simulate changes in the tax and subsidy structure for housing consumption. In the first case the model simulates the impacts of a simple housing allowance program along the lines of the housing gap formula applied in the Experimental Housing Allowance Program in 1973-79. Second, the local property tax rate is assumed to be a half of its actual rate in 1976-77. Finally the model analyses the effect of reducing the highest marginal tax rate of the federal income tax from 70 percent to 50 percent. For all three changes, we calculate the resulting distribution of the population among the different housing alternatives and study the actual moves that take place in response to these changes with the focus on the response of headship rates to price changes induced by the simulations.
3.2 Household Decomposition into Nuclei

Conventional housing demand analysis is based on households. Households are the sampling units of almost all available housing data bases. However, if household formation is endogenous, the decision unit must be smaller than the household. In fact, the correct decision unit is every housing consumer who could form his own household. We will call this decision unit "nucleus" (Pitkin 1980). A nucleus consists of a married couple or a single individual together with all its own children below a certain age (say, 18 years). Children above this threshold are considered grown-up and, as potential household heads, form a new nucleus, even if they (still) live in their parents' household. Similarly, households that consist of several adults are split up into several nuclei, both when the members are related or unrelated to each other. Examples are elderly parents in the household of their children, or roommates. This construction assumes death, marriage, and divorce as given from the viewpoint of housing market analysis, but allows for the endogeneity of doubling up and undoubling, and it also considers endogenous the decision of adult children to stay at or leave home.

Accordingly, there are five types of households:
(1) households consisting just of one nucleus,
(2) parents with their adult children,
(3) households composed of nuclei with family-relations,
(4) households composed of nuclei without family-relations,
(5) complex households, i.e., combinations of the latter three types.

Pitkin (1980) presents a variety of behavioral hypotheses for these five types of households, and provides a descriptive analysis of

Each nucleus chooses its housing accommodation: it either shares housing within an household composed of several nuclei, or heads its own household. In the latter case, the nucleus has to decide which housing unit in terms of tenure and dwelling size.

The basic idea of this approach is to avoid the difficult task of constructing a behavioral model of the determination of the household size per se (as Ermisch, 1981), or a model of the matching of nuclei (as Wial, 1983). The estimation of the resulting structural form tends to be unsatisfactory because of the high noise to signal ratio given our poor knowledge about these mechanisms. In addition, the approach is applicable to data on the microlevel which allows us to use the rich information provided in surveys and to avoid the typical biases in the use of aggregated data in housing demand studies (see Polinsky and Ellwood, 1979). The split-up of households into their true underlying decision units and the creation of the headship/non-headship dichotomy can be interpreted as a reduced form of the above mentioned behavioral models of household production and formation.

The data base of our empirical research is the Annual Housing Survey by SNSA, cross-sections of 1976 and 1977. Sampling unit of the AHS is the household. However, the composition of each household is well documented. This allows us to detect other adults in the
household with their children, and to create a data record for each of these subnuclei in addition to the head nucleus. The explanatory variables have to be split up according to this partition. All demographic variables and the most important income sources are reported for each household member. Some income sources are only reported for the household as a total, but in very specific income categories. However, with the demographic characteristics of each person, it is possible to employ a very accurate income allocation scheme for these income categories. For housing costs, it is suggestive that the nuclei pay their shares in proportion to the number of adults and children in the nucleus.\textsuperscript{4.5}

This reduced form approach does not come without its costs. Information is lost by splitting up households into independent nuclei and separating them into different strata; e.g., it seems a valuable piece of information, whether an adult child has parents with a large house in town or not. However, this is a supply factor in the provision of non-headship. As we set out in the introduction, our model is designed to describe "potential" housing demand under perfect elastic supply. Furthermore, the housing alternative "non-head" is a single category for a variety of rather different possible multi-nuclei households. As a special problem, the entire concept of headship is blurred in households of roommates, where no clear subordination exists. However, this problem is in so far irrelevant, as in the case of roommates income and demographic characteristics will be very close, and we can thus pick a head at random.
3.3 Specification of the Decision Tree and the Variables

Each nucleus chooses whether to head a household or shelter in an existing household, if one chooses to head a household, then the decisions are whether to rent or own a dwelling, and what quality and size the dwelling should be. As a simple measure of quality and size, we use the number of rooms and the type of the building. A household chooses among three size categories and between single-family detached houses and multi-family houses, in particular apartment buildings. We can arrange the choices in form of a decision tree as depicted in Figure 3-1. "Small" refers to dwellings with up to four rooms, "medium" to dwellings with five or six rooms, and "large" to dwellings with at least seven rooms.

Some of the alternatives are fairly scarce, e.g., renting large apartments or single-family homes, so some alternatives have to be consolidated for a reliable estimation. This consolidation depends on the stratum. Furthermore, no cost data are available for owner-occupied dwellings in multifamily buildings, which forced us to omit these alternatives from the choice set. A more satisfactory approach would be to estimate the cost data for multifamily dwellings by hedonic regressions, or to explicitly model the missing alternatives in the definition of the choice probabilities. But the problem is a minor one for Dallas 1976 and Sacramento 1977, where these alternatives count for only 0.5 and 0.8 percent of all choices; it might bias our results only for Albany, where 6.9 percent of all nuclei chose cooperatively owned multi-family buildings. The final
FIGURE 3-1: BASIC DECISION TREE AND NOTATION OF ALTERNATIVES

Headship:
- HEAD
- NON-HEAD

Tenure:
- OWN
- RENT

Structure:
- SINGLE FAMILY
- MULTI FAMILY

Size:
- SMALL
- LARGE
- MEDIUM

1 2 3 4 5 6 7 8 9 10 11 12 13

Head  Non-head

1 2 3
O_SF_S O_SF_M O_SF_L

7 8 9
O_SF_S O_SF_M O_SF_L

4 5 6
O_SF_S O_SF_M O_SF_L

10 11 12
O_SF_S O_SF_M O_SF_L

Rented

Owned

NH
FIGURE 3-2: DECISION TREES FOR THE DIFFERENT STRATA

(1) YOUNG SINGLES:

(2), (3) FAMILIES AND ELDERLY COUPLES:

(4) WIDOWS:
decision trees are depicted in Figure 3-2. In Chapter 5, we will experiment with other decision trees.

The choice among the alternatives will depend on the following demographic variables: (1) age of head of nucleus, (2) sex of head of nucleus, (3) marital status of head of nucleus, (4) race of head of nucleus, (5) number of children in the nucleus, as well as on financial variables of (6) after-tax user-cost and (7) income. We pursue the approach of de Leeuw (1971) and Quigley (1979) and assume different demand functions for nuclei with different demographic characteristics. Accordingly, we stratify the sample with the first five demographic variables. This approach is equivalent to the use of dummy variables for each of the strata which interact with all regressors to accommodate the unknown nonlinear functional form (see Li, 1977), in which the demographic variables enter the equations for the choice probabilities.

Of all possible strata, this paper examines demand functions for four representative strata:

(1) Young singles: unmarried white male and female without children, aged 20-35,
(2) Middle-aged families: married white couples with one or two children, aged 35-59,
(3) Elderly couples: married white couples without children, aged 60 and above,
(4) Widows: widowed, separated, and divorced white females without children, aged 60 and above.

The financial variables, income and user-cost, enter the demand functions directly. User-cost (UC) must be distinguished for renters and owners. For renters, the user-cost is simply gross rent. For
owners, the user-cost has a number of components (see for example, Hendershott and Hu (1979) or Pollain (1982)):

\[
\text{UC(own)} = \text{maintenance} + \text{insurance} + \text{utility-payments} \\
+ \text{mortgage-rate \times debt} \\
+ \text{property tax rate \times value} \\
- \text{tax savings from federal income tax deductions} \\
+ \text{T-Bill-rate \times equity} \\
- \text{rate of appreciation \times value}
\]

where the tax savings on the federal income tax is the sum of the local property tax and the mortgage interest, multiplied by the appropriate marginal tax rate. Note that federal income tax savings depend through the marginal tax rate on such nucleus characteristics as income and number of children. Furthermore, we assume different interest rates on debt and on equity to account for the effect of inflation on fixed-rate mortgages.

The user-cost of owners consists of two types of cost which the nucleus perceives differently: Maintenance, mortgage costs, property taxes and federal income tax savings are easily perceived as costs, whereas capital gains from appreciation are uncertain and opportunity costs of equity are a rather cloudy concept for non-economists. We therefore split up user-cost in two components:

\[
\text{UC(own)} = \text{OOPOCK(own)} + \text{RETURN(own)},
\]
where the "out-of-pocket cost" is composed of:

\[
\text{OOPOCK(own)} = \text{maintenance} + \text{insurance} + \text{utility-bills} \\
+ \text{local property tax} + \text{mortgage-rate} \cdot \text{debt} \\
- \text{federal income tax savings},
\]

and the return from the asset homeownership is defined as:

\[
\text{RETURN(own)} = \text{rate of appreciation} \cdot \text{value} \\
- \text{T-Bill-rate} \cdot \text{equity}.
\]

For fully rational housing demanders, the coefficients for OOPOCK and RETURN should be of equal magnitude and opposite sign. For renters, we set RETURN to zero.

The Annual Housing Survey gives us rather precise data for out-of-pocket costs. The return variable has to be constructed with external information: appreciation is based on the difference in house values between the Annual Housing Survey 1976/77 and the 1970 Census, converted into yearly rates. This rate varies by SMSA and by type of dwelling. Equity costs are calculated from the value-to-loan ratio in the Annual Housing Survey, multiplied by the interest on five-year U.S. treasury bills. Both appreciation rates and equity costs suffer from serious data problems: loan-to-value ratios are often not reported, making constructed substitutes necessary, and the available appreciation rates vary only by SMSA, but not within SMSA.

Given the cross-sectional data, the choice among the housing
alternatives is a hypothetical one: we observe each nucleus with its chosen alternative and its attributes like user-cost, do not observe the attributes of the alternatives that the nucleus rejected. We take as these attributes the averages in the cross-section confined to recent movers. Underlying is the notion that a household collects its information on prices of other units by skimming through the ads in the newspapers and listening to the experiences of friends and neighbors who just moved. Mean prices of recent movers seem a plausible approximation to the current spot market prices. Note however that the comparison of spot prices with the prices tenants pay after a considerable length of tenure may bias the estimated price response. Specifically, the presence of tenure discounts in the rental market will result in exaggerated price elasticities because we compare the average household with price \( p_0/(1+d_L) \) for the tenure discount \( d_L \) after an average length of tenure \( L \) with the recent mover with price \( p_0 \). Thus, the average price elasticity is overestimated by a factor of \( 1+d_L \). This factor can roughly be estimated by dividing the tenure discounts reported in Table 4-2 by the mean length of tenure. Based on the conservative hedonic estimates by Follain and Malpezzi (1980), this bias amounts in Albany to about 9.5 percent, in Dallas to 1.5 percent, of the estimated price coefficient for \( \delta \). The estimations by Malpezzi, Ozanne, and Thibodeau (1980) predict considerably higher tenure discounts, implying an upward bias of 25 percent in Albany, 5 percent in Dallas, 7.5 percent in Sacramento. Thus, we should be careful to take those numbers in account when interpreting the numerical size of our results. The responses on exogenous price shocks are proportional to the price elasticity, hence
these biases translate in an exaggeration of the predicted reaction to tax and subsidy changes in the order of the above-mentioned percentages. However, in Section 3.5 through 3.7 we will see that these biases do not change our qualitative conclusions. In particular, the most striking results will be found in Dallas and Sacramento where the overestimation is least. Chapter 4 describes the estimation of tenure discounts in more detail and provides some critique of the quoted estimates and a more general discussion about problems related to the description of intertemporal processes by a static model. Finally, in Chapter 5, we explicitly correct housing prices for the presence of tenure discounts.

For the hypothetical loan-to-value ratios, we assume a 20 percent downpayment for young singles, and 99 percent for the elderly households, which takes into account the availability of mortgage loans to the different age groups. This assumption is not critical to the estimates, but is confirmed by cross-tabulations of recent movers.

Finally, income is defined as the current total gross income of all nucleus members. Due to the nature of a choice between discrete alternatives, income enters the demand functions interactively with alternative-specific dummies. Furthermore, income influences the out-of-pocket costs of homeowners because federal income tax savings depend on the marginal tax rate, i.e., on gross income.
3.4 Baseline Estimates

The parameter estimates and summary statistics are tabulated in Tables 3-3 through 3-6 for each of the four strata. The parameters represent the taste weights of the respective explanatory variables in the deterministic part of the indirect utility function. T-statistics are given in brackets, and are evaluated at zero for the taste weights. Note that income \( Y \) interacts with alternative specific dummies, where we use the same mnemonics for the alternatives as in Figure 3-2. The final parameters are the similarity parameters that express the degree of closeness in the respective clusters (See Figure 3-2). T-statistics for the similarity parameters are evaluated at one, using the multinomial logit case as a benchmark. Three scalar measures of performance or fit are used, see Section 4 in Chapter 2 for a discussion.

The model achieves a surprisingly high prediction accuracy in terms of all three measures of fit. This is surprising because of the small number of explanatory variables and the simple specification. The model performs poorest in the strata of young singles and in the family stratum in Albany. The first is not astonishing: a static model can hardly capture changes in housing consumption in this period when the nucleus is establishing its own existence. These strata are also very heterogeneous and include children still living with their parents, student roommates, and singles in their thirties. The poor performance in the case of families with one or two children in Albany may be attributable to the misspecification of the decision tree where
### Table 3-3: NMNL Parameter Estimates

"Young Singles": Unmarried, Age 20-35, No Children

<table>
<thead>
<tr>
<th></th>
<th>ALBANY</th>
<th>DALLAS</th>
<th>SACRAMENTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOPOCK</td>
<td>-0.69598</td>
<td>-1.18246</td>
<td>-1.56947</td>
</tr>
<tr>
<td></td>
<td>(10.04)</td>
<td>( 9.96)</td>
<td>(13.29)</td>
</tr>
<tr>
<td>RETURN</td>
<td>0.13156</td>
<td>0.12316</td>
<td>0.20421</td>
</tr>
<tr>
<td></td>
<td>( 1.55)</td>
<td>( 1.45)</td>
<td>( 2.97)</td>
</tr>
<tr>
<td>Y_NH</td>
<td>-0.02006</td>
<td>-0.13204</td>
<td>-0.14574</td>
</tr>
<tr>
<td></td>
<td>( 1.65)</td>
<td>( 6.88)</td>
<td>( 5.36)</td>
</tr>
<tr>
<td>Y_O</td>
<td>-0.00196</td>
<td>0.00894</td>
<td>0.02125</td>
</tr>
<tr>
<td></td>
<td>( 0.09)</td>
<td>( 0.43)</td>
<td>( 0.65)</td>
</tr>
<tr>
<td>Y_R_SF</td>
<td>0.01213</td>
<td>-0.02375</td>
<td>0.01408</td>
</tr>
<tr>
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<td>( 1.87)</td>
<td>( 1.61)</td>
<td>( 0.62)</td>
</tr>
<tr>
<td>Y_R_MF.S</td>
<td>0.02282</td>
<td>0.04212</td>
<td>0.03539</td>
</tr>
<tr>
<td></td>
<td>( 4.07)</td>
<td>( 3.38)</td>
<td>( 1.59)</td>
</tr>
<tr>
<td>TH_R_MF</td>
<td>0.20422</td>
<td>0.42742</td>
<td>0.45242</td>
</tr>
<tr>
<td></td>
<td>(26.81)</td>
<td>( 8.05)</td>
<td>( 7.08)</td>
</tr>
<tr>
<td>TAU_HEAD</td>
<td>0.14958</td>
<td>0.32322</td>
<td>0.37700</td>
</tr>
<tr>
<td></td>
<td>(28.96)</td>
<td>(13.17)</td>
<td>(12.48)</td>
</tr>
<tr>
<td>LOGLIK</td>
<td>-616.591</td>
<td>-421.180</td>
<td>-457.465</td>
</tr>
<tr>
<td>LOGLIK_0</td>
<td>-1401.82</td>
<td>-925.427</td>
<td>-1199.03</td>
</tr>
<tr>
<td>RHO_SQ</td>
<td>0.56</td>
<td>0.545</td>
<td>0.618</td>
</tr>
<tr>
<td>%CORRECT</td>
<td>82.9%</td>
<td>73.2%</td>
<td>81.2%</td>
</tr>
<tr>
<td>NOBS</td>
<td>871</td>
<td>575</td>
<td>745</td>
</tr>
</tbody>
</table>

In brackets: t-statistics around zero or one

LOGLIK = loglikelihood at optimum
LOGLIK_0 = loglikelihood at zero
RHO_SQ = 1.0 - LOGLIK/LOGLIK_0
%CORRECT = percentage of correct ex post predictions
NOBS = number of observations
Table 3-4: WMNL Parameter Estimates

"Families": Married, Age 35-59, 1-2 Children

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Albany</th>
<th>Dallas</th>
<th>Sacramento</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOPock</td>
<td>-0.92185</td>
<td>-3.55522</td>
<td>-2.81777</td>
</tr>
<tr>
<td></td>
<td>(4.91)</td>
<td>(5.87)</td>
<td>(6.28)</td>
</tr>
<tr>
<td>RETURN</td>
<td>-0.69255</td>
<td>1.07524</td>
<td>0.63690</td>
</tr>
<tr>
<td></td>
<td>(3.55)</td>
<td>(5.08)</td>
<td>(3.87)</td>
</tr>
<tr>
<td>Y_O_SF.S</td>
<td>-0.99077</td>
<td>-0.70474</td>
<td>-0.32266</td>
</tr>
<tr>
<td></td>
<td>(4.35)</td>
<td>(4.69)</td>
<td>(1.91)</td>
</tr>
<tr>
<td>Y_O_SF.M</td>
<td>0.07448</td>
<td>0.05039</td>
<td>0.15820</td>
</tr>
<tr>
<td></td>
<td>(3.12)</td>
<td>(1.20)</td>
<td>(1.21)</td>
</tr>
<tr>
<td>Y_O_SF.L</td>
<td>0.22973</td>
<td>0.20642</td>
<td>0.27165</td>
</tr>
<tr>
<td></td>
<td>(6.19)</td>
<td>(3.56)</td>
<td>(2.08)</td>
</tr>
<tr>
<td>Y_R_SF</td>
<td>-0.00667</td>
<td>-0.02119</td>
<td>0.18802</td>
</tr>
<tr>
<td></td>
<td>(0.37)</td>
<td>(0.63)</td>
<td>(1.49)</td>
</tr>
<tr>
<td>Y_R_MF.S</td>
<td>-0.10296</td>
<td>0.18602</td>
<td>-0.00061</td>
</tr>
<tr>
<td></td>
<td>(1.86)</td>
<td>(1.69)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>TH_R_MF</td>
<td>0.74920</td>
<td>2.25563</td>
<td>1.74344</td>
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<tr>
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<td>(0.71)</td>
<td>(0.86)</td>
<td>(1.06)</td>
</tr>
<tr>
<td>TH_O_SF</td>
<td>5.03934</td>
<td>2.73284</td>
<td>2.20262</td>
</tr>
<tr>
<td></td>
<td>(8.77)</td>
<td>(3.21)</td>
<td>(2.97)</td>
</tr>
<tr>
<td>Loglik</td>
<td>-357.208</td>
<td>-121.134</td>
<td>-127.350</td>
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<tr>
<td>Loglik_0</td>
<td>-627.116</td>
<td>-584.114</td>
<td>-580.530</td>
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<tr>
<td>Rho_SQ</td>
<td>0.43</td>
<td>0.793</td>
<td>0.781</td>
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<tr>
<td>%Correct</td>
<td>78.9%</td>
<td>90.8%</td>
<td>90.9%</td>
</tr>
<tr>
<td>Noobs</td>
<td>350</td>
<td>326</td>
<td>324</td>
</tr>
</tbody>
</table>

In brackets: t-statistics around zero or one
Loglik = loglikelihood at optimum
Loglik_0 = loglikelihood at zero
Rho_sq = 1.0 - Loglik/Loglik_0
%Correct = percentage of correct ex post predictions
Noobs = number of observations
Table 3-5 : NMNL Parameter Estimates

"Elderly Couples": Married, Age 60+, No Children

<table>
<thead>
<tr>
<th></th>
<th>ALBANY</th>
<th>DALLAS</th>
<th>SACRAMENTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOPock</td>
<td>-3.61354</td>
<td>-2.35330</td>
<td>-3.21796</td>
</tr>
<tr>
<td></td>
<td>(8.29)</td>
<td>(6.66)</td>
<td>(6.49)</td>
</tr>
<tr>
<td>Return</td>
<td>0.52908</td>
<td>0.68032</td>
<td>1.02552</td>
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<tr>
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<td>(2.27)</td>
<td>(4.27)</td>
<td>(4.87)</td>
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<td>Y_0_SF.S</td>
<td>-0.26210</td>
<td>-0.19193</td>
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</tr>
<tr>
<td></td>
<td>(4.15)</td>
<td>(2.05)</td>
<td>(2.68)</td>
</tr>
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<td>Y_0_SF.M</td>
<td>0.16994</td>
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<tr>
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<td>(3.96)</td>
<td>(1.02)</td>
<td>(1.11)</td>
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<td>Y_0_SF.L</td>
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<td>0.25455</td>
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<td>(4.94)</td>
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<td>(3.05)</td>
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<td>Y_R_SF</td>
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<td>0.03272</td>
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<td>(0.59)</td>
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<td>(1.61)</td>
</tr>
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<td>-0.08036</td>
<td>0.01262</td>
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<tr>
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<td>(1.17)</td>
<td>(0.15)</td>
<td>(1.01)</td>
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<td>3.39293</td>
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<tr>
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<td>(2.93)</td>
<td>(1.49)</td>
<td>(1.85)</td>
</tr>
<tr>
<td>TH_O_SF</td>
<td>1.41709</td>
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<td>0.47555</td>
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<tr>
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<td>(1.36)</td>
<td>(0.49)</td>
<td>(1.73)</td>
</tr>
<tr>
<td>Loglik</td>
<td>-153.730</td>
<td>-77.926</td>
<td>-62.706</td>
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<tr>
<td>Loglik 0</td>
<td>-582.322</td>
<td>-458.690</td>
<td>-519.610</td>
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<tr>
<td>rho_sq</td>
<td>0.735</td>
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<td>0.88</td>
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<tr>
<td>%Correct</td>
<td>86.8%</td>
<td>90.2%</td>
<td>91.7%</td>
</tr>
<tr>
<td>Nobs</td>
<td>325</td>
<td>256</td>
<td>290</td>
</tr>
</tbody>
</table>

In brackets: t-statistics around zero or one
LOGLIK = loglikelihood at optimum
LOGLIK_0 = loglikelihood at zero
RHO_SQ = 1.0 - LOGLIK/LOGLIK_0
%CORRECT = percentage of correct ex post predictions
NOBS = number of observations
Table 3-6: NMNL Parameter Estimates

"Widows": Widowed, Divorced, Separated, Age 60+, No Children

<table>
<thead>
<tr>
<th></th>
<th>ALBANY</th>
<th>DALLAS</th>
<th>SACRAMENTO</th>
</tr>
</thead>
<tbody>
<tr>
<td>OOPOCK</td>
<td>-2.92313</td>
<td>-1.74357</td>
<td>-3.65622</td>
</tr>
<tr>
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<td>( 9.81)</td>
<td>( 7.17)</td>
<td>( 7.61)</td>
</tr>
<tr>
<td>RETURN</td>
<td>0.28555</td>
<td>0.16035</td>
<td>0.61589</td>
</tr>
<tr>
<td></td>
<td>( 2.65)</td>
<td>( 2.17)</td>
<td>( 4.84)</td>
</tr>
<tr>
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<td>-0.68624</td>
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</tr>
<tr>
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<td>(10.42)</td>
<td>( 4.77)</td>
<td>( 6.47)</td>
</tr>
<tr>
<td>Y_O_SF.S</td>
<td>-0.23088</td>
<td>-0.16355</td>
<td>-0.27639</td>
</tr>
<tr>
<td></td>
<td>( 4.71)</td>
<td>( 1.95)</td>
<td>( 3.12)</td>
</tr>
<tr>
<td>Y_O_SF.M</td>
<td>0.22788</td>
<td>0.09891</td>
<td>0.16234</td>
</tr>
<tr>
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<td>( 6.47)</td>
<td>( 1.36)</td>
<td>( 1.99)</td>
</tr>
<tr>
<td>Y_O_SF.L</td>
<td>0.39921</td>
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</tr>
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<td>( 5.73)</td>
<td>( 2.09)</td>
<td>( 2.88)</td>
</tr>
<tr>
<td>Y_R_SF</td>
<td>0.01295</td>
<td>0.06526</td>
<td>0.01310</td>
</tr>
<tr>
<td></td>
<td>( 0.34)</td>
<td>( 0.89)</td>
<td>( 0.21)</td>
</tr>
<tr>
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</tr>
<tr>
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<td>( 0.63)</td>
<td>( 0.59)</td>
<td>( 0.01)</td>
</tr>
<tr>
<td>TH_O_SF</td>
<td>1.48636</td>
<td>0.60497</td>
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<td>( 1.75)</td>
<td>( 2.10)</td>
<td>( 0.07)</td>
</tr>
<tr>
<td>TH_R_MF</td>
<td>1.48222</td>
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<td>0.97076</td>
</tr>
<tr>
<td></td>
<td>( 1.73)</td>
<td>( 1.12)</td>
<td>( 0.09)</td>
</tr>
<tr>
<td>TAU_HEAD</td>
<td>0.48036</td>
<td>0.45743</td>
<td>0.47043</td>
</tr>
<tr>
<td></td>
<td>( 5.79)</td>
<td>( 6.31)</td>
<td>( 4.21)</td>
</tr>
<tr>
<td>LOGLIK</td>
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<td>-199.974</td>
<td>- 94.920</td>
</tr>
<tr>
<td>LOGLIK_0</td>
<td>1037.17</td>
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<td>-651.880</td>
</tr>
<tr>
<td>RHO_SQ</td>
<td>0.70</td>
<td>0.69</td>
<td>0.85</td>
</tr>
<tr>
<td>%CORRECT</td>
<td>89.0%</td>
<td>88.3%</td>
<td>82.6%</td>
</tr>
<tr>
<td>NOBS</td>
<td>533</td>
<td>332</td>
<td>335</td>
</tr>
</tbody>
</table>

In brackets: t-statistics around zero or one
LOGLIK  = loglikelihood at optimum
LOGLIK_0 = loglikelihood at zero
RHO_SQ  = 1.0 - LOGLIK/LOGLIK_0
%CORRECT = percentage of correct ex post predictions
NOBS    = number of observations
the alternative of owning cooperatively is not included.

Table 3-7 shows the prediction success table for the stratum of unmarried elderly in Sacramento, based on the criterion (2.14). The diagonal dominance reflects the high prediction accuracy. The off-diagonal elements represent the mispredictions: the model has some difficulties in discriminating between small rental housing and non-headship, which may reflect the crude reduced form specification of the household formation process.

The main result is the significance of the price variables in all strata. The out-of-pocket costs are highly significant, while the RETURN variable is somewhat weaker. Note that the hypothesis of rationality -- i.e., equal magnitude and opposite signs for the taste weights of OOPOCK and RETURN -- is rejected; considerably more weight is given to easily perceived out-of-pocket costs as opposed to appreciation minus equity costs. One should keep in mind, however, the difficulties of constructing the RETURN variable. Note furthermore that RETURN is least significant for the young singles, the strata most affected by liquidity constraints, rendering the rationality hypothesis inappropriate and introducing a lot of noise. Note the similarity between the estimates for elderly couples and elderly widows: taking into account the different choice sets (a large proportion of widows live in their children's homes), it reflects the stability of the taste weights during old age.

The income dummies have a threefold function. First, they
Table 3-7: Example of a Success Table and a Full Elasticity Matrix

Stratum: Widowed, Divorced, Separated, Age 60+, No Children
SMSA: Sacramento

PREDICTION SUCCESS TABLE:

<table>
<thead>
<tr>
<th>OBSERVED ALTERNATIVE</th>
<th>NH</th>
<th>O_SF.S</th>
<th>O_SF.M</th>
<th>O_SF.L</th>
<th>R_SF</th>
<th>R_MF.S</th>
<th>R_MF.L</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH</td>
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<td>0</td>
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<td>10</td>
<td>0</td>
</tr>
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<td>O_SF.S</td>
<td>1</td>
<td>32</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>O_SF.M</td>
<td>0</td>
<td>0</td>
<td>77</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>O_SF.L</td>
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<td>0</td>
<td>0</td>
<td>19</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
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<td>0</td>
<td>0</td>
<td>25</td>
<td>1</td>
<td>0</td>
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<tr>
<td>R_MF.S</td>
<td>17</td>
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<td>70</td>
<td>0</td>
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<td>1</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
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</table>

PERCENT CORRECTLY PREDICTED: 88.96%

MEAN INDIVIDUAL ELASTICITIES:

<table>
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<tr>
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<th>ALT.</th>
<th>CHOICE PROBABILITY OF ALTERNATIVE:</th>
</tr>
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<tbody>
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<td></td>
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<td>O_SF.S</td>
</tr>
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<td>OPOCK NH</td>
<td>-2.614</td>
<td>0.739</td>
</tr>
<tr>
<td>OPOCK O_SF.S</td>
<td>0.420-12.898</td>
<td>-4.671</td>
</tr>
<tr>
<td>OPOCK O_SF.M</td>
<td>0.918</td>
<td>0.389-12.540</td>
</tr>
<tr>
<td>OPOCK O_SF.L</td>
<td>0.235</td>
<td>0.195</td>
</tr>
<tr>
<td>OPOCK R_SF</td>
<td>0.287</td>
<td>0.670</td>
</tr>
<tr>
<td>OPOCK R_MF.L</td>
<td>0.103</td>
<td>0.322</td>
</tr>
<tr>
<td>RETURN O_SF.S</td>
<td>-0.200</td>
<td>2.759</td>
</tr>
<tr>
<td>RETURN O_SF.M</td>
<td>-0.429</td>
<td>-0.345</td>
</tr>
<tr>
<td>RETURN O_SF.L</td>
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<tr>
<td>Y_NH</td>
<td>-4.422</td>
<td>0.588</td>
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<tr>
<td>Y_O_SF.S</td>
<td>0.187</td>
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</tr>
<tr>
<td>Y_O_SF.M</td>
<td>-0.330</td>
<td>-0.242</td>
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<tr>
<td>Y_O_SF.L</td>
<td>-0.302</td>
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</tr>
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</tbody>
</table>
reflect the relative price of housing with respect to all other goods. In addition, they indicate the attractiveness of the various alternatives relative to large rented apartments, measured in money terms. In absence of any other alternative-specific dummies, they also pick up all other non-measured advantages and disadvantages of the included alternatives relative to large rented apartments. Thus, one should be careful not to rush to conclusions about pure income effects. Introduction of alternative-specific dummies in several test strata reduces the income parameters, but leaves the price variables virtually constant. Because the focus of the simulations is on relative prices rather than on income, we avoided the costly inclusion of alternative specific dummies.

The attractiveness of the alternatives measured by the taste weights of the income dummies corresponds to a priori assessment. Note that most of the rented single-family houses are small houses, thus their negative weight for families with one or two children.

The last two coefficients in Tables 3-3 through 3-6 are the weights of the inclusive values or similarity coefficients. Note that four of the similarity coefficients are significantly larger than one (at the 5 percent level). This implies that in these strata the compatibility with the underlying microeconomic theory of random utility maximization must be explicitly checked for the given data and is not automatically guaranteed as in the other strata (see appendix). In fact, the test rejects this compatibility. Note that the microeconomic theory described above is based on static utility
maximization. Note furthermore that failure of the test occurs in the strata where people move considerably less frequently than in the strata of young singles, where the similarity parameters are in the unit-interval. The rejection thus could be interpreted as a hint that optimization is done dynamically and that the model in these strata should be interpreted as only a reduced-form description of the steady-state as opposed to a structural static choice model.

To gain some intuition for the magnitudes of the coefficients, we calculate the elasticities corresponding to the parameter estimates of Tables 3-3 through 3-6. Own price elasticities and income elasticities, tabulated in Table 3-8, refer to a change of the probability of choosing alternative $i$, when $OPOCK$ or $RETURN$ in alternative $i$ is changed. The income elasticities in Table 3-8 are the sum over the elasticities of all income dummies. In Table 3-7, the complete matrix of elasticities is presented for the stratum of unmarried elderly in Sacramento. The pattern in the price elasticities across alternatives reflects the structure of the decision tree. Particularly interesting is the first column of price elasticities, referring to the probability to choose non-head status. As closest substitute for this alternative emerges renting small units, as expected. The large cross elasticity of 1.315 once more reflects the price responsiveness of household formation. The cross elasticity with respect to renting large units can be decomposed into two components: a positive component for the substitution effect, and a negative part for a supply effect. Large rental units constitute the supply for non-headship possibilities, therefore, their price will
# Table 3-8: Own Price and Sum of Income Elasticities

<table>
<thead>
<tr>
<th>PROB</th>
<th>NH</th>
<th>O_SF.S</th>
<th>O_SF.M</th>
<th>O_SF.L</th>
<th>R_SF</th>
<th>R_MF.S</th>
<th>R_MF.L</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Albany, Young Singles:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OOPOCK</td>
<td>-0.395</td>
<td>-19.596</td>
<td>-12.973</td>
<td>-2.463</td>
<td>-7.227</td>
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</tr>
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<td>-0.591</td>
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<td></td>
</tr>
<tr>
<td><strong>Albany, Families</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>OOPOCK</td>
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<td>-2.517</td>
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<td>-0.386</td>
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<td>INCOME</td>
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<td>-0.132</td>
<td>0.553</td>
<td>-3.394</td>
<td>-6.250</td>
<td>-3.193</td>
<td></td>
</tr>
<tr>
<td><strong>Albany, Elderly Couples:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RETURN</td>
<td>0.791</td>
<td>0.379</td>
<td>0.653</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
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<td>INCOME</td>
<td>-4.214</td>
<td>0.028</td>
<td>1.333</td>
<td>-1.788</td>
<td>-2.944</td>
<td>-2.574</td>
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</tr>
<tr>
<td><strong>Albany, Widows:</strong></td>
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<td></td>
</tr>
<tr>
<td>RETURN</td>
<td>0.0</td>
<td>0.914</td>
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<td>-0.418</td>
<td>0.0</td>
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<tr>
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<td>-1.284</td>
<td>0.359</td>
<td>0.973</td>
<td>-0.428</td>
<td>-0.737</td>
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<tr>
<td><strong>Dallas, Young Singles:</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>OOPOCK</td>
<td>-0.632</td>
<td>-16.494</td>
<td>-8.392</td>
<td>-3.031</td>
<td>-7.641</td>
<td></td>
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</tr>
<tr>
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<td>-0.710</td>
<td>0.625</td>
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</tr>
<tr>
<td><strong>Dallas, Families:</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>RETURN</td>
<td>1.531</td>
<td>1.335</td>
<td>1.451</td>
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<tr>
<td>INCOME</td>
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<td>-1.343</td>
<td>1.138</td>
<td>-2.536</td>
<td>-4.690</td>
<td>-2.513</td>
<td></td>
</tr>
<tr>
<td><strong>Dallas, Elderly Couples:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OOPOCK</td>
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<td>-5.189</td>
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<td>-4.311</td>
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</tr>
<tr>
<td>RETURN</td>
<td>2.064</td>
<td>1.313</td>
<td>2.497</td>
<td>0.0</td>
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<td></td>
</tr>
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<td>INCOME</td>
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<td>-0.667</td>
<td>2.427</td>
<td>-1.337</td>
<td>-1.676</td>
<td>-1.770</td>
<td></td>
</tr>
<tr>
<td><strong>Dallas, Widows:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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</tr>
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<td>-2.823</td>
<td>-1.958</td>
<td>0.457</td>
<td>1.478</td>
<td>0.341</td>
<td>0.005</td>
<td>-0.164</td>
</tr>
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</table>
Table 5: Own Price and Sum of Income Elasticities (cont'd)

<table>
<thead>
<tr>
<th>PROB</th>
<th>NH</th>
<th>O_SF.S</th>
<th>O_SF.M</th>
<th>O_SF.L</th>
<th>R_SF</th>
<th>R_MF.S</th>
<th>R_MF.L</th>
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</thead>
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<td>Sacramento, Young Singles:</td>
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<td></td>
</tr>
<tr>
<td>OPOCK</td>
<td>-0.728</td>
<td>-16.459</td>
<td>-10.046</td>
<td>-3.563</td>
<td>-9.519</td>
<td></td>
<td></td>
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<tr>
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<td>2.939</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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</tr>
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<td>INCOME</td>
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<td>0.181</td>
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</tr>
<tr>
<td>Sacramento, Families:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>OPOCK</td>
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<td>-2.862</td>
<td>-4.790</td>
<td>-6.922</td>
<td>-5.222</td>
<td>-5.358</td>
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<td>RETURN</td>
<td>1.215</td>
<td>0.844</td>
<td>1.112</td>
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<td>INCOME</td>
<td>-5.563</td>
<td>-0.550</td>
<td>0.632</td>
<td>-0.397</td>
<td>-4.727</td>
<td>-4.719</td>
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</tr>
<tr>
<td>Sacramento, Elderly Couples:</td>
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</tr>
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<td>RETURN</td>
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<td>8.194</td>
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<td>INCOME</td>
<td>-7.813</td>
<td>0.672</td>
<td>5.770</td>
<td>0.765</td>
<td>-2.055</td>
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<td>Sacramento, Widows:</td>
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</tr>
<tr>
<td>RETURN</td>
<td>0.0</td>
<td>2.759</td>
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<td>2.316</td>
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<td>0.0</td>
</tr>
<tr>
<td>INCOME</td>
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<td>0.548</td>
<td>2.146</td>
<td>-0.147</td>
<td>-0.326</td>
<td>-0.320</td>
</tr>
</tbody>
</table>
be negatively correlated to the choice probability of not forming an household. Both effects together seem to cancel out, reflected in the insignificant value of 0.103.

If we compare the own elasticities across strata, the following general pattern emerges: the strata of young singles and elderly widows are the most price-responsive, especially in the owner alternatives, reflecting a priori knowledge of inertia and mobility in the different strata. Return from the asset homeownership exhibits a strong life-cycle behavior, and is thus higher for young people with a long decision horizon than for the elderly. Headship rates are highly responsive to prices for both young singles and elderly widows. Finally, note again that the income elasticities measure not only income but also pure alternative specific effects due to their interaction with alternative specific dummies.

The elasticity pattern is fairly stable across the three SMSAs, in spite of their very different distribution of housing alternatives. This provides some confidence in the robustness of the model. As a general pattern, housing demand reacts most to prices in Sacramento and least in Albany, suggesting the more flexible nature of the housing market in California compared with New England. The pattern holds for both out-of-pocket costs and returns.

Summing up, we observe the following:

- Relative prices significantly determine housing choices for given demographic variables.
o Household formation, in particular, is highly responsive.
  o Out-of-pocket costs have higher taste weights than return from homeownership.
  o Among strata, young singles and widows are more price responsive than the relatively inert strata of families and elderly couples.
  o The sensitivity to RETURN shows the expected life-cycle behavior.
  o The general pattern of elasticities is fairly stable across markets, with Albany behaving least flexibly and Sacramento the most.
3.5 A Housing Allowance Program

The estimation results show a strong responsiveness of household formation to housing prices. It is now interesting what the consequences of endogenous household formation rates are in quantitative terms. Do the price coefficients, highly significant as they are, in fact translate into a reassessment of policy analysis? We will use the estimation results for three comparative static experiments of public intervention in the housing market: housing allowances, local property taxes, and the deduction from the federal income tax. These experiments are exercises in comparative statics and have thus to be interpreted as long-run responses, leading to a new steady state equilibrium. Furthermore, the results are based on the partial analysis of only the demand side in the preceding sections. Thus, we implicitly assume a perfect malleable housing stock and disregard all transitional phenomena like inertia of mobility and transaction costs.

Between 1973 and 1979, the U.S. Department of Housing and Urban Development conducted a large scale Experimental Housing Allowance Program. Kennedy (1980) describes in detail the design of the program, and a good survey of the subsequent discussion and critique is given in Bradbury and Downs (1981). Somewhat surprising is the fact that all components of the Experimental Housing Allowance Program ignored the feedback of housing allowances on household formation. One focal point in this section is the question of how much improvement in housing conditions comes through increased headship
rates over and above moves of existing households into larger dwellings.

The following simulation assumes a so-called housing gap formula for the calculation of the allowances. First, for each family size and site a benchmark rent is calculated, representing the "fair cost of standard housing." Then a minimum standard of quality is established, with only dwellings above this standard eligible for the subsidy. Finally, a linear tax is levied on the allowances in such a way that people with no (adjusted) income will receive the full rent for standard housing, whereas people above a certain income level will receive no allowances at all.

If the minimum standard is measured as a fraction \( a \) of the fair cost of standard housing \( C \), and the upper income limit is a multiple \( b \) of \( C \), then the housing allowance for a household with income \( Y \) and rent \( R \) is:

\[
\begin{align*}
0 & \quad \text{if } R < aC \\
0 & \quad \text{if } Y > bC \\
C-Y/b & \quad \text{otherwise.}
\end{align*}
\]

To perform a realistic experiment, we use the settings \( a=0.7, \ b=4.0 \), and \( C \) from the Experimental Housing Allowances Program, where \( C \) was taken from the Pittsburgh demand experiment and inflated by a yearly as well as inter-SMSA rent index as follows:
Housing allowances introduce nonlinearities in the budget set, see Hausman and Wise (1980) or Venti and Wise (1982). They can be handled fairly elegantly in discrete choice models by changing the prices of the housing alternatives differently rather than by adding the allowances to the income.

Table 3-9 lists the predicted shares of the housing alternatives before and after the introduction of the housing allowance program. Given the static nature of the model, this reflects a change between steady-states. The shares are calculated as means of the individual choice probabilities. Table 3-10, in turn, tabulates the moves according to the individual predictions.

Our main result is the strong impact of housing allowances on headship rates: about half of the people who lived in some sort of shared accommodations created their own households in response to the housing allowance program. Most of these nuclei in the strata of young singles have little or no income, thus their rent net of the housing allowance is virtually zero. More surprising is the strong response in the strata of elderly widows, where the non-head share is far less and the income higher than among the young singles, the share
Table 3-9: AGGREGATED SHARES: HOUSING ALLOWANCES

<table>
<thead>
<tr>
<th>Stra Alt.</th>
<th>ALBANY</th>
<th>DALLAS</th>
<th>SACRAMENTO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>before</td>
<td>after</td>
<td>before</td>
</tr>
<tr>
<td>-----------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>YSL NH</td>
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<td>0.4780</td>
<td>0.5507</td>
</tr>
<tr>
<td>O_SM</td>
<td>0.092</td>
<td>0.092</td>
<td>0.0224</td>
</tr>
<tr>
<td>O_HN</td>
<td>0.0123</td>
<td>0.0159</td>
<td>0.0478</td>
</tr>
<tr>
<td>R_SF</td>
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<td>0.4014</td>
<td>0.3438</td>
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<tr>
<td>R_SM</td>
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<td>0.0928</td>
<td>0.0353</td>
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<tr>
<td>R_LA</td>
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<td>O_SM</td>
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<td>0.3270</td>
<td>0.4439</td>
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<tr>
<td>O_HN</td>
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<td>0.4969</td>
<td>0.4085</td>
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<tr>
<td>R_SF</td>
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<td>0.0572</td>
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<td>0.0207</td>
<td>0.0152</td>
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<tr>
<td>R_LA</td>
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<td>0.0833</td>
<td>0.0244</td>
</tr>
<tr>
<td>ELC NH</td>
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<td>0.0920</td>
<td>0.1443</td>
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<td>0.5446</td>
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<td>0.2683</td>
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<td>0.0193</td>
<td>0.0420</td>
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<td>R_LA</td>
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<td>0.0192</td>
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<tr>
<td>WID NH</td>
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<td>0.2130</td>
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<tr>
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<td>0.1165</td>
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<td>O_HN</td>
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<tr>
<td>O_LA</td>
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<td>0.0909</td>
<td>0.0486</td>
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<td>0.0286</td>
<td>0.0737</td>
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<td>R_SM</td>
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<td>0.3636</td>
<td>0.1842</td>
</tr>
<tr>
<td>R_LA</td>
<td>0.1348</td>
<td>0.2084</td>
<td>0.0496</td>
</tr>
</tbody>
</table>

First column: predicted shares of housing alternatives before housing allowances.

Second column: predicted shares of housing alternatives with housing allowance program in effect (housing gap formula: \( P = C - \frac{Y}{b} \)).
of non-heads is nevertheless drastically reduced in response to the subsidy. We conclude once more that headship rates are important endogenous variables in the housing market.

Within the rental sector, only few moves occur. The mobility rates induced by the housing allowances (Albany 0.047, Dallas 0.057, Sacramento 0.055) are very close to those measured in the demand part of the Experimental Housing Allowance Program by MacMillan (1980) i.e., Pittsburgh 0.045 and Phoenix 0.101. Note again the difference in the price sensitivity between the Northeast and the Southwest.

Unlike the Experimental Housing Allowance Program, our simulation offered allowances for rental housing to everybody in the population, changing the balance in the tenure choice in favor of renting. As a response, we observe a relatively large number of moves from the owner-occupied section into the rental section of the housing market. The mobility rates for the shift from owning to renting induced by the allowances are between 0.125 for Sacramento and 0.189 for Albany. Note the lower rate for Sacramento, reflecting the high valuation of owner-occupancy in the West relative to the Northeast.

Moves from owner-occupancy into the rental market have two important fiscal side-effects: on the federal level, some money given for housing allowances is retrieved through lower mortgage and property tax deductions from the federal income tax. More important is the spill-over effect at the local level of reductions in local property taxes. Table 3-11 lists these fiscal repercussions created
Table 3-10: Predicted Moves in Response to Housing Allowance Program

<table>
<thead>
<tr>
<th>Stratum</th>
<th>from NH</th>
<th>O.S</th>
<th>O.M</th>
<th>O.L</th>
<th>R.SF</th>
<th>R.MS</th>
<th>R.ML</th>
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<td>Albany, Young Singles</td>
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<td>0</td>
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<tr>
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<td>to R_MF.S</td>
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<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>to R_MF.L</td>
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Notes: predicted moves, normalized for 1000 nuclei per stratum
Table 3-11: FISCAL INCIDENCE BETWEEN JURISDICTIONS AND STRATA

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<tr>
<th>Stratum</th>
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<th>Housing Allowances</th>
<th>Property Tax Cut</th>
<th>Fed. Income Tax Change</th>
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Table 9: INCIDENCE: BETWEEN JURISDICTIONS AND STRATA (cont'd)

<table>
<thead>
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<th>Stratum</th>
<th>Level of Government</th>
<th>Housing Allowances</th>
<th>Property Tax Cut</th>
<th>Fed. Income Tax Change</th>
</tr>
</thead>
<tbody>
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<td>Local, lost property tax</td>
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Notes: The table lists the direct subsidy in the case of housing allowances, the indirect subsidy via Federal Income Tax savings due to deduction of interest and local property tax, and the local property tax losses to the local jurisdiction. The unit is $1000 for a normalized stratum of 1000 nuclei, i.e. dollars per nucleus per year. The numbers are based on the predicted moves of Table 8.
by the shift of demand. All amounts are normalized to a stratum of 1000 nuclei to allow for comparisons both among strata and among SMSA. Note that especially for the married strata, the losses in local property taxes are a sizable proportion of the housing allowances paid by the federal government.

We can sum up the results of the housing allowance experiment as follows:

- headship rates are highly responsive to the housing subsidies,
- mobility rates within the rental market are low and of comparable size to the findings of the Experimental Housing Allowance Program,
- greater mobility between renting and owning produces sizable spill-over effects from federal policy to the local level.
3.6 Tax Simulations

3.6.1 Cutting the Local Property Tax By One Half

In recent years, some states have passed legislation that introduces upper ceilings for local property tax rates, e.g., Proposition 13 in California and Proposition 2-1/2 in Massachusetts. These ceilings imply a drastic reduction in local property taxes for given assessment ratios. As a crude approximation of the isolated impact due to a drastic change in the local property tax rate, the following simulation predicts the distribution of nuclei into housing categories assuming a property tax rate of half the level in effect during the estimation period 1976/77.

Effective property taxes (as percentages of the house values reported in the Annual Housing Survey) in this period were 2.2 percent in Albany, 1.3 percent in Dallas, and 1.6 percent in Sacramento. The proportion of property taxes in the out-of-pocket cost varies considerably across strata, mainly due to the variation in mortgage payments in the life cycle, and less so across housing alternatives; the overall proportion is about 10 percent. The impact of the property tax cut is softened by a reduction in the federal income tax deductions proportional to the marginal tax rate of the household. Taking this into account, the simulation reduces the cost of owner-occupancy about 3 percent for the average homeowner. This is a fairly small change in relative prices considering that the property tax rate is lowered by 50 percent.
Table 3-12 lists the distribution of housing alternatives before and after the property tax change, calculated as means of the individual choice probabilities. If we concentrate only on the tenure choice, the share of owner-occupancy increases by:

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Albany</th>
<th>Dallas</th>
<th>Sacramento</th>
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<td>Young Singles</td>
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<td>Families</td>
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<td>Widows</td>
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<td>.0454</td>
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The impact is of course strongest in Albany where the property tax is substantially higher than in Dallas and Sacramento. In addition, the impact is very low for young singles: they have high mortgage payments and the percentage of property taxes in their total out-of-pocket costs is very low. The same reasoning explains why the increase in owner-occupancy is largest for small houses. In addition, smaller houses are attractive for people with low incomes, for whom the offsetting effect of decreasing income tax deductions is least.

Finally, we can see the interjurisdictional fiscal effects in the second column of Table 3-11. In the family strata, the gains for the federal government by smaller deductions are between 23.9 percent and 30.3 percent of the losses in local property taxes. The size of this spill-over effect depends on two factors: it simply reflects the relatively high marginal tax rates for these strata, but the gains are also reduced by the higher share of owner-occupancy in response to the tax change.
Table 3-10: AGGREGATED SHARES: LOCAL PROPERTY TAX CUT

<table>
<thead>
<tr>
<th>Stra Alt.</th>
<th>ALBANY before</th>
<th>DALLAS before</th>
<th>SACRAMENTO before</th>
<th>ALBANY after</th>
<th>DALLAS after</th>
<th>SACRAMENTO after</th>
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<td></td>
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<td>.0637 .0627</td>
<td>.0353 .0350</td>
<td>.0163 .0157</td>
</tr>
</tbody>
</table>

| FAM NH    |               |               |                   |              |              |                 |
| O_SM      | .0182 .0201   | .0440 .0458   | .0320 .0378       |              |              |                 |
| O_ME      | .3310 .3413   | .4439 .4585   | .4772 .4899       |              |              |                 |
| O_LA      | .4997 .5123   | .4085 .4134   | .3852 .3940       |              |              |                 |
| R_SF      | .0546 .0456   | .0639 .0530   | .0780 .0587       |              |              |                 |
| R_SM      | .0161 .0132   | .0152 .0114   | .0219 .0157       |              |              |                 |
| R_LA      | .0804 .0674   | .0244 .0178   | .0056 .0038       |              |              |                 |

| ELC NH    |               |               |                   |              |              |                 |
| O_SM      | .1251 .1745   | .1443 .1529   | .1450 .1617       |              |              |                 |
| O_ME      | .4050 .4173   | .5446 .5503   | .6667 .6728       |              |              |                 |
| O_LA      | .2773 .2826   | .2014 .2023   | .0750 .0770       |              |              |                 |
| R_SF      | .0174 .0137   | .0420 .0374   | .0531 .0436       |              |              |                 |
| R_SM      | .0870 .0543   | .0484 .0412   | .0335 .0246       |              |              |                 |
| R_LA      | .0882 .0576   | .0192 .0159   | .0268 .0203       |              |              |                 |

| WID NH    |               |               |                   |              |              |                 |
| O_SM      | .2757 .2396   | .2130 .2016   | .2745 .2585       |              |              |                 |
| O_ME      | .0694 .1606   | .1165 .1411   | .1105 .1506       |              |              |                 |
| O_LA      | .1602 .1746   | .3144 .3202   | .2281 .2326       |              |              |                 |
| R_SF      | .0985 .1011   | .0486 .0494   | .0562 .0572       |              |              |                 |
| R_SM      | .0273 .0254   | .0737 .0703   | .0729 .0711       |              |              |                 |
| R_LA      | .2340 .1942   | .1842 .1703   | .2437 .2181       |              |              |                 |

First column: predicted shares of housing alternatives under actual 1977 local property taxes.

Second column: predicted shares of housing alternatives under only 50% of the 1977 local property taxes.
We can summarize the results of the property tax experiment as follows:

- The impact of a strong reduction in the local property tax is small in the strata with high mortgage payments and high tax brackets. It is high for the elderly and for small homeowners.
- The spill-over effect to the federal government is sizable. The direct effect through the marginal tax rate is partially offset by the indirect effect of movers into owner-occupancy.
3.6.2 Making the Federal Income Tax Less Progressive

The final simulation concerns the change in the federal income tax law that reduced the highest marginal tax rate from 70 percent to 50 percent. This has two opposing effects on housing consumption: while high-income people pay fewer taxes the deductions for mortgage interest and local property taxes are less worth and thus reduce the tax advantages of ownership. In the following simulation, we isolate the second effect by holding the income level constant and calculate the tax savings in the out-of-pocket costs of homeownership assuming the new tax schedule. We used the federal income tax schedule for 1983 and deflated the tax brackets by the Consumer Price Index to the price and income level of the estimation period.

We can again make the back-on-the-envelope calculation as in the preceding section: for the very rich, deductions lose 20 percent of their value. If we assume that a third of the out-of-pocket costs is deductible, we generate a 7 percent increase in the cost of owner-occupancy. This is an upper limit: people in lower tax brackets face a much smaller increase because below the 50 percent brackets, the marginal tax rates were only very slightly reduced. For the poor, there is no change whatsoever.

It should be noted that the sample includes only few "very rich" people (the 50 percent tax bracket in 1977 was about $40,000) because the selection of strata overrepresents the very young and elderly nuclei. Table 3-13 shows that the change in the marginal tax rate
Table 3-13: AGGREGATE SHARES: FLATTER FEDERAL INCOME TAX SCHEDULE

<table>
<thead>
<tr>
<th>Stra Alt.</th>
<th>ALBANY before</th>
<th>DALLAS before</th>
<th>SACRAMENTO before</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>after</td>
<td>after</td>
<td></td>
</tr>
<tr>
<td>YSL NH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O_SM</td>
<td>.6331</td>
<td>.6333</td>
<td>.5507</td>
</tr>
<tr>
<td>O_ME</td>
<td>.0920</td>
<td>.0888</td>
<td>.0224</td>
</tr>
<tr>
<td>O_LA</td>
<td>.0123</td>
<td>.0124</td>
<td>.0478</td>
</tr>
<tr>
<td>R_SF</td>
<td>.2815</td>
<td>.2817</td>
<td>.3438</td>
</tr>
<tr>
<td>R_SM</td>
<td>.0637</td>
<td>.0638</td>
<td>.0353</td>
</tr>
<tr>
<td>R_LA</td>
<td>.6262</td>
<td>.6269</td>
<td>.5507</td>
</tr>
<tr>
<td></td>
<td>.6269</td>
<td>.5508</td>
<td>.0219</td>
</tr>
<tr>
<td></td>
<td>.0425</td>
<td>.0408</td>
<td>.0478</td>
</tr>
<tr>
<td></td>
<td>.0508</td>
<td>.0509</td>
<td>.3441</td>
</tr>
<tr>
<td></td>
<td>.0163</td>
<td>.0163</td>
<td>.0353</td>
</tr>
<tr>
<td>FAM NH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O_SM</td>
<td>.0182</td>
<td>.0179</td>
<td>.0440</td>
</tr>
<tr>
<td>O_ME</td>
<td>.3310</td>
<td>.3284</td>
<td>.4439</td>
</tr>
<tr>
<td>O_LA</td>
<td>.4997</td>
<td>.4970</td>
<td>.4085</td>
</tr>
<tr>
<td>R_SF</td>
<td>.0546</td>
<td>.0566</td>
<td>.0639</td>
</tr>
<tr>
<td>R_SM</td>
<td>.0161</td>
<td>.0166</td>
<td>.0152</td>
</tr>
<tr>
<td>R_LA</td>
<td>.0804</td>
<td>.0835</td>
<td>.0244</td>
</tr>
<tr>
<td></td>
<td>.0320</td>
<td>.0297</td>
<td>.0427</td>
</tr>
<tr>
<td></td>
<td>.4772</td>
<td>.4734</td>
<td>.4033</td>
</tr>
<tr>
<td></td>
<td>.0780</td>
<td>.0830</td>
<td>.0666</td>
</tr>
<tr>
<td></td>
<td>.0219</td>
<td>.0242</td>
<td>.0168</td>
</tr>
<tr>
<td></td>
<td>.0056</td>
<td>.0064</td>
<td>.0262</td>
</tr>
<tr>
<td>ELC NH</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>O_SM</td>
<td>.1251</td>
<td>.1210</td>
<td>.1443</td>
</tr>
<tr>
<td>O_ME</td>
<td>.4050</td>
<td>.4034</td>
<td>.5446</td>
</tr>
<tr>
<td>O_LA</td>
<td>.2773</td>
<td>.2760</td>
<td>.2014</td>
</tr>
<tr>
<td>R_SF</td>
<td>.0174</td>
<td>.0177</td>
<td>.0420</td>
</tr>
<tr>
<td>R_SM</td>
<td>.0870</td>
<td>.0905</td>
<td>.0484</td>
</tr>
<tr>
<td>R_LA</td>
<td>.0882</td>
<td>.0914</td>
<td>.0192</td>
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<tr>
<td></td>
<td>.1450</td>
<td>.1438</td>
<td>.1440</td>
</tr>
<tr>
<td></td>
<td>.6667</td>
<td>.6645</td>
<td>.5433</td>
</tr>
<tr>
<td></td>
<td>.0750</td>
<td>.0744</td>
<td>.2012</td>
</tr>
<tr>
<td></td>
<td>.0531</td>
<td>.0542</td>
<td>.0425</td>
</tr>
<tr>
<td></td>
<td>.0335</td>
<td>.0352</td>
<td>.0493</td>
</tr>
<tr>
<td></td>
<td>.0268</td>
<td>.0280</td>
<td>.0193</td>
</tr>
<tr>
<td>WID NH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O_SM</td>
<td>.2757</td>
<td>.2760</td>
<td>.2130</td>
</tr>
<tr>
<td>O_ME</td>
<td>.0694</td>
<td>.0676</td>
<td>.1165</td>
</tr>
<tr>
<td>O_LA</td>
<td>.1602</td>
<td>.1596</td>
<td>.3144</td>
</tr>
<tr>
<td>R_SF</td>
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<td>.0984</td>
<td>.0486</td>
</tr>
<tr>
<td>R_SM</td>
<td>.0273</td>
<td>.0274</td>
<td>.0737</td>
</tr>
<tr>
<td>R_LA</td>
<td>.2340</td>
<td>.2351</td>
<td>.1842</td>
</tr>
<tr>
<td></td>
<td>.2745</td>
<td>.2747</td>
<td>.2131</td>
</tr>
<tr>
<td></td>
<td>.1105</td>
<td>.1108</td>
<td>.1163</td>
</tr>
<tr>
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<td>.2281</td>
<td>.2277</td>
<td>.3136</td>
</tr>
<tr>
<td></td>
<td>.0562</td>
<td>.0560</td>
<td>.0486</td>
</tr>
<tr>
<td></td>
<td>.0729</td>
<td>.0728</td>
<td>.0739</td>
</tr>
<tr>
<td></td>
<td>.2437</td>
<td>.2440</td>
<td>.1847</td>
</tr>
<tr>
<td></td>
<td>.0140</td>
<td>.0140</td>
<td>.0498</td>
</tr>
</tbody>
</table>

First column: predicted shares of housing alternatives under actual 1977 Federal Income Tax schedule (highest marginal tax rate: 70%).
Second column: predicted shares of housing alternatives under 1983 Federal Income Tax schedule, deflated by CPI to 1977 levels (highest marginal tax rate: 50%).
results in a slight shift from owning to renting. More comprehensively, the share of renting increases by:

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Albany</th>
<th>Dallas</th>
<th>Sacramento</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Singles</td>
<td>.0005</td>
<td>.0004</td>
<td>.0017</td>
</tr>
<tr>
<td>Families</td>
<td>.0056</td>
<td>.0061</td>
<td>.0081</td>
</tr>
<tr>
<td>Elderly Couples</td>
<td>.0070</td>
<td>.0015</td>
<td>.0040</td>
</tr>
<tr>
<td>Widows</td>
<td>.0026</td>
<td>.0010</td>
<td>.0004</td>
</tr>
</tbody>
</table>

These numbers are very small: not only very few people are affected by the change in the marginal tax rate, but these "very rich" people are also those who are least likely to shift to the rental market.

Within each city, the shifts into rental units basically reflect the tax brackets which can be seen by a look at the yearly mean income before taxes:

<table>
<thead>
<tr>
<th>Stratum</th>
<th>Albany</th>
<th>Dallas</th>
<th>Sacramento</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Singles</td>
<td>$5,200</td>
<td>$6,700</td>
<td>$5,200</td>
</tr>
<tr>
<td>Families</td>
<td>22,200</td>
<td>26,400</td>
<td>23,000</td>
</tr>
<tr>
<td>Elderly Couples</td>
<td>13,900</td>
<td>15,400</td>
<td>13,700</td>
</tr>
<tr>
<td>Widows</td>
<td>5,300</td>
<td>5,600</td>
<td>6,000</td>
</tr>
</tbody>
</table>

But mean income will not tell the entire story because the picture is complicated by distributional differences within each stratum and among SMSAs - both in terms of the income distributions and in terms of mortgage payments. This might explain the large shift to rental units among elderly couples in Albany.

Finally, the spill-over effects induced by the few moves in the
rental market are calculated from the predicted moves in a stratum of 1000 nuclei (see the last column of Table 3-11). Note that the already mentioned problems with the small number of affected people are compounded by the erratic nature of the individual forecasts. The predicted changes in local property tax payments might therefore be unreliable. Aggregated over the three SMSAs and over all strata, the spill-over effect in lost property taxes is about 15 percent of the income tax deductions saved by the federal government. The latter are measured after the tax change: the percentage in terms of the direct effect is lower because the moves into the rental market partially offset the savings in income tax deductions.

We sum up the Federal Income Tax experiment as follows:

- Flattening the income tax schedule affects relatively few people and the changes in the aggregate are therefore small. Too few sample nuclei are affected to allow a reliable simulation.
- The pure price effect makes the federal income tax deductions worth less at high marginal tax rates. The resulting shift in the rental market is very small because the "very rich" people that are affected by the change are the least likely nuclei to switch to renting.
3.7 Conclusions

The main conclusion from the baseline estimates and from the housing allowance experiment is the strong response of headship rates to relative housing prices. Headship rates can not be treated as exogenous variables. The second conclusion concerns fiscal federalism: in all three fiscal changes, the spill-over effects from federal fiscal action to the local level and vice versa are of sizable magnitudes.

Taken as a descriptive device, the model performs well in terms of fit and prediction accuracy. Simulation results give a fairly stable pattern across SMSAs. In the case where the simulations coincide with other published experiments, the results were very close. All this gives us some confidence in the robustness of the model and its forecasts.

However, one caveat should be made which leads us to the next chapter: it concerns the interpretation of the cross-sectional data as a steady state, especially with using the housing prices as they are reported in the Annual Housing Survey. The approach ignores all intertemporal effects that might produce price dispersion or disequilibria. Spurious price elasticities may come from the fact that many sitting tenants receive tenure discounts: if we compare the rent of their actual unit with the hypothetical prices of those not chosen (measured as the prices paid by recent movers), the existence of tenure discounts will give us a larger price response than if we
compare the prices with the tenure discounts subtracted. The same argument holds for other kinds of factors producing price dispersion in the housing market, e.g., search equilibria, explicit or implicit long run contract agreements, and rent control. In the following chapter, we will develop a theoretical model of idiosyncratic exchange that produces tenure discounts even in the absence of rent control. We will also find empirical evidence for tenure discounts: hedonic regressions produce significant negative coefficients for length of tenure, indicating an upward bias of the price coefficient. Estimations by Follain and Malpezzi (1980) translate in a bias of about 9.3 percent in Albany, only 1.5 percent in Dallas. Estimates by Malpezzi, Ozanne, and Thibodeau (1980) indicate much larger tenure discounts, yielding an upward bias of about 25 percent in Albany, 8 percent in Sacramento, and 4.9 percent in Dallas. However, even after subtracting 25 percent of the price coefficients, the main conclusion of this chapter still holds: household formation is highly responsive to housing prices, with elasticities and simulation responses smaller by 25 percent which remains still very large.
Footnotes to Chapter 3

(1) To take an extreme position: even suicide rates and health status may depend on housing market conditions as pending eviction or urban blight.


(3) This assumption contradicts Hu (1980), who considers marriage as the crucial link between economic factors and household formation. See the discussion of the six mechanisms in household formation at the outset of this paper - of those seem marriage and divorce (and of course death) the least likely to be price responsive.

(4) This sharing scheme is realistic for roommates, less so for adult children living in their parents' household. However, they incur non-monetary cost in the form of household help etc.

(5) Note that for a common price for all housing units this relation establishes an identity between the prices of non-heads and heads. In this case, the price coefficients would not be identified in certain functional forms of the demand equations. One sufficient condition for identification independent of the functional form is the presence of economies of scale in the formation of larger households. In fact, these economies are likely to exist and seem to be a major attraction to share accommodations.

(6) These categories are still fairly large and do not distinguish quality levels within each size category. This holds for the non-head category as well where we do not differentiate nuclei according to the number of nuclei per household. As a consequence for marginal analysis, OPOCKX measures expenditure rather than price within each size category. However, the confusion between prices and expenditure vanishes between the discrete categories of our demand model: in each category, OPOCK can be interpreted as the price of a standard bundle in the category. Note that one reason to use discrete choice rather than marginal analysis is the assumption that these standard bundles are qualitatively different and do not only group housing according to some scalar measure of housing services. In Chapter 5, the standard bundle of each category is computed and priced as a Lancastrian commodity, using hedonic regressions.

(7) However, the accommodating supply shifts are ignored in this analysis. The fiscal side-effects between jurisdictions may well be offset by increased property-taxes paid by landlords. If landlords overassess their buildings, the shifts in tax revenues depicted in Table 3-11 are even overcompensated.
CHAPTER FOUR

DYNAMIC ASPECTS IN THE HOUSING MARKET EQUILIBRIUM
4.1 Introduction: Market Imperfections and Government Intervention

So far, our empirical results were derived from a single cross section of the Annual Housing Survey. Implicitly, we assumed that the observed market outcome represents a stable steady state with a unique price for each housing alternative. Thus, we could ignore the time dimension. In addition, we assumed perfect elastic long run housing supply. In this chapter, we will examine how violations of those heroic assumptions affect housing demand estimation. In particular, we try to shed some light on the existence of and causes for price dispersion. We will concentrate on the rental housing market, where the above mentioned assumptions seem most likely unrealistic.

Market imperfections abound in the rental housing market. First, housing is a durable good where prices are not necessarily defined by one period spot market conditions alone. Second, high monetary and nonmonetary transactions cost are involved with changing consumption by moving. These two conditions create intertemporal externalities. Third, property rights of the rental unit are given up only temporarily, giving the seller a strong incentive to care who the buyer is. However, there is uncertainty: the tenants' characteristics will be revealed only after some time. These two conditions create an interpersonal externality. We will show that these externalities create rental price dispersion.

There are other mechanisms which create a sustained price
dispersion: costly search for an appropriate housing unit may create a partition of the market in expensive units which are easy to find, and well hidden inexpensive units. Prices may vary according to implicit contracts including maintenance on the tenants side. These contracts may be correlated with such characteristics as landlord living in the unit or the landlord being a small private owner.

In addition to these deviations from perfect market assumptions still within the competitive market context, government intervention regulates competition in the housing market. Government intervention takes place both on the demand and the supply side of the market, in form of housing allowances, tax subsidies, public housing provision, and rent control. Again, we will show that government intervention is able to generate price dispersion in the housing market, intertwined with the above mentioned market imperfections.

Empirical analysis of government intervention in West Germany faces the problem of the impossibility of a with and without analysis, due to the fact that housing market intervention is basically a federal function, depriving researchers from regional variation, and that there is only poor time series data available to exploit the temporal variation in government policies. However, we will be able to draw empirical conclusions by comparing evidence in West Germany with evidence from the United States where we observe a variety of local housing policies as well as the total absence of intervention.

This chapter proceeds as follows: First, we will model
mechanisms leading to market imperfection, in particular price
dispersion, and collect empirical evidence for this. Second, we try
to disentangle the effects of government intervention and the effects
of intrinsic housing market imperfections. Often, in particular in
the discussion about rent control, these two sources of inefficiency
are confused, and policy recommendations are easily victims of the
mistake to propose first best solutions in this second best
environment. As a case in point, we will study a highly controversial
piece of legislation that regulates the rental housing market in the
Federal Republic of Germany, the "2. Wohnungsraumkuendigungsschutzgesetz" (WKSchG), or law for the protection of tenants from
arbitrary eviction. Finally, in the conclusions, we will propose some
ad hoc remedies which will allow estimation of the price elasticities
unbiased from the price dispersion, even if no panel data is available
to explicitly model the underlying intertemporal processes.
4.2 Empirical Evidence for Price Dispersion

4.2.1 Tenure Discounts

The first example of price dispersion in the rental housing market are so-called tenure discounts. Tenure discounts are the phenomenon of a gap between spot market rent and actual rent that increases with length of tenure. They can be measured as the difference in rent paid for comparable units by households moved in at different times. Units are kept comparable by controlling for housing quality and neighborhood characteristics as well as tenants' and landlords' characteristics. This is achieved by applying hedonic regression techniques where a function of the form

\[ R = f(Q,N,T,L,t) \]

is estimated. Here \( R \) denotes the rent, and \( Q, N, T, \) and \( L \) vectors of housing quality, neighborhood, tenant, and landlord characteristics. Finally, the length of tenure (denoted by \( t \)) enters this hedonic rent index. The following estimated tenure discounts in West Germany are calculated from hedonic regressions by Behring and Goldrian (1983), based on about one percent of all West German households in 1978. Behring and Goldrian use a semilogarithmic functional form, where the length of tenure enters linearly, quadratically, and in form of a dummy variable for very long length of tenure. This generates the nonlinear time profile of Table 4-1.

Already after one year of tenure, tenants pay two percent less rent than new tenants in comparable units. The discounts increase until they level off for very long lengths of tenure where they amount
TABLE 4-1: TENURE DISCOUNTS IN WEST GERMANY

<table>
<thead>
<tr>
<th>Length of Tenure (Years)</th>
<th>Areas with High Population Density</th>
<th>Low Population Density</th>
<th>Rural Area</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>City</td>
<td>Fringe</td>
<td>Environ</td>
</tr>
<tr>
<td>1</td>
<td>2%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>13</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>14</td>
<td>19</td>
<td>25</td>
<td>27</td>
</tr>
</tbody>
</table>

Source: Behring and Goldrian (1982).

to savings of up to more than a quarter of the rent which a new resident would have to pay for the same unit.

We use the same methodology to measure tenure discounts in the United States, but, in addition, we categorize SMSAs by their rent control legislation. Tables 4-2 and 4-3 present estimated tenure discounts for fifty-nine Standard metropolitan Statistical Areas (SMSA), calculated from hedonic regressions by Follain and Malpezzi (1980) and Malpezzi, Ozanne, and Thibodeau (1980), based on the Annual Housing Surveys by SMSA 1974-1977. Follain and Malpezzi (1980) estimate tenure discounts as a linear function of the length of tenure only, whereas Malpezzi, Ozanne, and Thibodeau (1980) use the same nonlinear specification as Behring and Goldrian (1980). The large difference in the size of the discounts is disturbing. It might be attributable to these specification differences. If the true relation between the discounts and length of tenure is concave, the linear specification of Follain and Malpezzi will be biased downwards for
### TABLE 4-2: TENURE DISCOUNTS IN THE UNITED STATES 1974-76

<table>
<thead>
<tr>
<th>Standard Metropolitan Statistical Area</th>
<th>Rent Control Status</th>
<th>Tenure Discounts after 10 Years</th>
<th>Average Length of Tenure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average Length of Tenure</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Albany, NY</td>
<td>*</td>
<td>13.5%</td>
<td>36.4%</td>
</tr>
<tr>
<td>Anaheim, CA</td>
<td>*</td>
<td>18.3</td>
<td>25.0</td>
</tr>
<tr>
<td>Atlanta, GA</td>
<td></td>
<td>6.9</td>
<td>21.3</td>
</tr>
<tr>
<td>Boston, MA</td>
<td>**</td>
<td>13.4</td>
<td>30.2</td>
</tr>
<tr>
<td>Chicago, IL</td>
<td></td>
<td>7.7</td>
<td>17.9</td>
</tr>
<tr>
<td>Cincinnati, OH</td>
<td></td>
<td>10.5</td>
<td>27.2</td>
</tr>
<tr>
<td>Colorado Springs, CO</td>
<td></td>
<td>9.3</td>
<td>18.1</td>
</tr>
<tr>
<td>Columbus, OH</td>
<td></td>
<td>8.6</td>
<td>25.3</td>
</tr>
<tr>
<td>Dallas, TX</td>
<td></td>
<td>4.1</td>
<td>13.5</td>
</tr>
<tr>
<td>Detroit, MI</td>
<td></td>
<td>9.7</td>
<td>25.5</td>
</tr>
<tr>
<td>Fort Worth, TX</td>
<td></td>
<td>7.1</td>
<td>18.6</td>
</tr>
<tr>
<td>Hartford, CT</td>
<td></td>
<td>9.2</td>
<td>18.8</td>
</tr>
<tr>
<td>Kansas City, KS/MO</td>
<td></td>
<td>10.1</td>
<td>22.0</td>
</tr>
<tr>
<td>Los Angeles, CA</td>
<td>**</td>
<td>10.8</td>
<td>19.0</td>
</tr>
<tr>
<td>Madison, WI</td>
<td></td>
<td>5.6</td>
<td>14.1</td>
</tr>
<tr>
<td>Memphis, TN</td>
<td></td>
<td>8.7</td>
<td>18.4</td>
</tr>
<tr>
<td>Miami, FL</td>
<td>**</td>
<td>8.4</td>
<td>13.7</td>
</tr>
<tr>
<td>Milwaukee, WI</td>
<td></td>
<td>13.2</td>
<td>23.7</td>
</tr>
<tr>
<td>Minneapolis, MN</td>
<td>**</td>
<td>7.6</td>
<td>13.3</td>
</tr>
<tr>
<td>Newark, NJ</td>
<td>**</td>
<td>9.1</td>
<td>18.9</td>
</tr>
<tr>
<td>New Orleans, LA</td>
<td></td>
<td>9.9</td>
<td>24.1</td>
</tr>
<tr>
<td>Newport News, VA</td>
<td></td>
<td>10.1</td>
<td>23.8</td>
</tr>
<tr>
<td>Orlando, FL</td>
<td></td>
<td>4.7</td>
<td>25.5</td>
</tr>
<tr>
<td>Paterson, NJ</td>
<td>**</td>
<td>10.3</td>
<td>19.8</td>
</tr>
<tr>
<td>Philadelphia, PE</td>
<td></td>
<td>7.4</td>
<td>24.4</td>
</tr>
<tr>
<td>Phoenix, AZ</td>
<td></td>
<td>18.6</td>
<td>22.5</td>
</tr>
<tr>
<td>Pittsburgh, PE</td>
<td></td>
<td>8.2</td>
<td>33.0</td>
</tr>
<tr>
<td>Portland, OR</td>
<td></td>
<td>11.0</td>
<td>22.0</td>
</tr>
<tr>
<td>Rochester, NY</td>
<td>*</td>
<td>12.5</td>
<td>22.6</td>
</tr>
<tr>
<td>Salt Lake City, UT</td>
<td></td>
<td>12.3</td>
<td>31.0</td>
</tr>
<tr>
<td>San Antonio, TX</td>
<td></td>
<td>8.9</td>
<td>22.3</td>
</tr>
<tr>
<td>San Bernadino, CA</td>
<td>*</td>
<td>12.9</td>
<td>28.9</td>
</tr>
<tr>
<td>San Diego, CA</td>
<td>*</td>
<td>11.6</td>
<td>29.5</td>
</tr>
<tr>
<td>San Francisco, CA</td>
<td>**</td>
<td>11.3</td>
<td>22.2</td>
</tr>
<tr>
<td>Spokane, WA</td>
<td></td>
<td>11.9</td>
<td>34.2</td>
</tr>
<tr>
<td>Springfield, MA</td>
<td>*</td>
<td>6.6</td>
<td>26.3</td>
</tr>
<tr>
<td>Tacoma, WA</td>
<td></td>
<td>8.1</td>
<td>23.5</td>
</tr>
<tr>
<td>Washington, DC</td>
<td>**</td>
<td>7.5</td>
<td>18.6</td>
</tr>
<tr>
<td>Wichita, WA</td>
<td></td>
<td>11.6</td>
<td>26.3</td>
</tr>
</tbody>
</table>

Source: (1) Follain and Malpezzi (1980),
       (2) Malpezzi, Ozanne, and Thibodeau (1980).

Two asterisks denote a strict measure of rent and/or eviction control
One asterisk denotes a weak measure of rent and/or eviction control
TABLE 4-3: TENURE DISCOUNTS IN THE UNITED STATES 1976-77

<table>
<thead>
<tr>
<th>Standard Metropolitan Statistical Area</th>
<th>Rent Control Status</th>
<th>Tenure Discounts after .... Years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Allentown, PA</td>
<td></td>
<td>4.1%</td>
</tr>
<tr>
<td>Baltimore, MD</td>
<td>* *</td>
<td>2.4</td>
</tr>
<tr>
<td>Birmingham, AL</td>
<td></td>
<td>3.4</td>
</tr>
<tr>
<td>Buffalo, NY</td>
<td>*</td>
<td>4.1</td>
</tr>
<tr>
<td>Cleveland, OH</td>
<td></td>
<td>2.2</td>
</tr>
<tr>
<td>Denver, CO</td>
<td></td>
<td>3.4</td>
</tr>
<tr>
<td>Grand Rapids, MI</td>
<td></td>
<td>3.8</td>
</tr>
<tr>
<td>Honolulu, HI</td>
<td>*</td>
<td>2.9</td>
</tr>
<tr>
<td>Houston, TX</td>
<td></td>
<td>4.1</td>
</tr>
<tr>
<td>Indianapolis, IN</td>
<td></td>
<td>3.4</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
<td></td>
<td>1.9</td>
</tr>
<tr>
<td>Louisville, KY</td>
<td></td>
<td>3.4</td>
</tr>
<tr>
<td>New York, NY</td>
<td>* *</td>
<td>3.9</td>
</tr>
<tr>
<td>Oklahoma City, OK</td>
<td></td>
<td>3.8</td>
</tr>
<tr>
<td>Omaha, NE</td>
<td></td>
<td>2.3</td>
</tr>
<tr>
<td>Providence, RI</td>
<td>*</td>
<td>4.6</td>
</tr>
<tr>
<td>Raleigh, NC</td>
<td></td>
<td>3.2</td>
</tr>
<tr>
<td>Sacramento, CA</td>
<td>* *</td>
<td>2.8</td>
</tr>
<tr>
<td>St. Louis, MI</td>
<td></td>
<td>2.4</td>
</tr>
<tr>
<td>Seattle, WA</td>
<td></td>
<td>3.3</td>
</tr>
</tbody>
</table>

Source: Malpezzi, Ozanne, and Thibodeau (1980).
Two asterisks denote a strict measure of rent and/or eviction control
One asterisk denotes a weak measure of rent and/or eviction control

TABLE 4-4: AVERAGE TENURE DISCOUNTS BY RENT CONTROL STATUS

<table>
<thead>
<tr>
<th>Rent Control Status</th>
<th>Discounts for a 10-Year Tenure (1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controlled according to strict measure</td>
<td>10.0%</td>
<td>20.3%</td>
<td>24.0%</td>
</tr>
<tr>
<td></td>
<td>(2.2)</td>
<td>(5.0)</td>
<td>(6.1)</td>
</tr>
<tr>
<td>Controlled according to weak measure</td>
<td>11.3</td>
<td>23.9</td>
<td>28.6</td>
</tr>
<tr>
<td></td>
<td>(3.1)</td>
<td>(6.2)</td>
<td>(6.3)</td>
</tr>
<tr>
<td>Uncontrolled Market</td>
<td>9.3</td>
<td>22.7</td>
<td>23.4</td>
</tr>
<tr>
<td></td>
<td>(2.9)</td>
<td>(5.3)</td>
<td>(5.5)</td>
</tr>
</tbody>
</table>

Computed from: (1) Follain and Malpezzi (1980), 1974-76,
(2) Malpezzi, Ozanne, and Thibodeau (1980), 1974-76,

Standard deviation in brackets.
small and medium tenures. The estimates by Malpezzi, Ozanne, and Thibodeau may show an upward bias because of collinearity with their identically specified concave age of dwelling variable so that some impact of old structures pollutes the tenure discounts. Goodman and Kawai (1982) produced linear estimates for 19 SMSAs in the Annual Housing Survey of 1977-78 which are almost identical to those reported in Table 4-2. Barnett (1979), Noland (1980), and Lowery (1981) in turn reproduce estimates similar to the nonlinear specification of Table 4-2. Guasch and Marshall (1983) mention the possibility of an upward bias in all these estimates due to sample selection adverse to movers in response to low tenure discounts. Their empirical results, however, were inconclusive\(^1\). For this Chapter, we are less interested in the actual size of the discounts as their relation with rent control. As we will see in Table 4-4, our conclusions hold for both the linear and the nonlinear specification.

In the United States, rent and eviction control legislation is at the discretion of the state or even municipal level of jurisdiction. Information about the presence of controls is collected from Braid (1980), Thibodeau (1981), and the National Multi Housing Council (1982). We use two measures to assess whether the market was influenced by rent or eviction control. The stricter measure includes all SMSAs, in which rent control was in effect in at least one jurisdiction. These SMSAs are marked in Tables 4-2 and 4-3 by two asterisks. The weaker measure is denoted by one asterisk. It marks an SMSA where state legislation made rent control easy to introduce. Included into this category are also SMSAs, in which rent control was
a "big issue" and was rejected only by a small margin. Thus the presence of at least one asterisk indicates a SMSA where landlords faced or perceived an incentive to restrain rent increases and eviction.

Does rent control affect tenure discounts? Even in SMSAs where no jurisdiction ever had some kind of rent control, e.g., Phoenix and Milwaukee, we observe substantial tenure discounts. Table 4-4 lists the average tenure discounts and their corresponding standard deviations, by rent control status, time period, and specification. Average tenure discounts tend to be higher under effective or likely rent control, but none of the differences is statistically significant: the hypothesis that tenure discounts are not affected by rent control at all cannot be rejected. Note that this result holds under both specifications of the hedonic index and in both time periods of Tables 4-2 and 4-3.
4.2.2 Landlord Characteristics and Search

Price dispersion in the rental market may not only be generated by the intertemporal process in which the landlord grants tenure discounts to the tenant, but by all mechanisms that create some form of partition of the rental market and therefore produce a possibility for sustained cross sectional price variation.

The first partition to be discussed in this subsection concerns landlord characteristics: landlords with only a few units to lease or landlords who live in the building where they rent out the remaining units may behave quite differently from large scale landlords who administer their units by a house manager. Second, private landlords may have a different behavior than institutions like the public sector housing companies in England or West Germany. Confirming evidence is presented in Tables 4-6 and 4-7.

On average over the 59 SMSA's in the survey of Malpezzi, Ozanne, and Thibodeau (1980), we find a rent reduced by 2.8 percent ceteris paribus when the landlord is present in the building. The evidence, however, is weak: it is significant only in 21 of the SMSA's, insignificant in 37, and has a significant but reversed sign in one SMSA. In our estimations, reported in Section 5.5.1 in Chapter Five, we present similar evidence with a significant discount in three out of four SMSA's. The pattern of discounts granted by a private landlord in Germany is much clearer, see Table 4-7. In cities situated in regions with high population density comparable to U.S.
### Table 4-6: Discounts, When Landlord is Present in Building

<table>
<thead>
<tr>
<th>Location</th>
<th>Mean Over 59 SMSA's in the U.S. 1974-1977</th>
<th>T-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean over 59 SMSA's in the U.S. 1974-1977</td>
<td>-0.028</td>
<td></td>
</tr>
<tr>
<td>Boston 1977</td>
<td>-0.039</td>
<td>(2.59)</td>
</tr>
<tr>
<td>Dallas 1977</td>
<td>-0.056</td>
<td>(1.66)</td>
</tr>
<tr>
<td>Los Angeles 1977</td>
<td>-0.042</td>
<td>(2.04)</td>
</tr>
<tr>
<td>Minneapolis/St. Paul 1977</td>
<td>-0.0012</td>
<td>(0.06)</td>
</tr>
</tbody>
</table>

### Table 4-7: Discounts, When Landlord is Private Person

<table>
<thead>
<tr>
<th>Location</th>
<th>West Germany 1978</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High Density Region, City</td>
</tr>
<tr>
<td></td>
<td>Fringe</td>
</tr>
<tr>
<td></td>
<td>Environ</td>
</tr>
<tr>
<td></td>
<td>Low Density Region, City</td>
</tr>
<tr>
<td></td>
<td>Fringe</td>
</tr>
<tr>
<td></td>
<td>Rural Region</td>
</tr>
<tr>
<td>Boston 1977</td>
<td>-0.127</td>
</tr>
<tr>
<td>Dallas 1977</td>
<td>-0.139</td>
</tr>
<tr>
<td>Los Angeles 1977</td>
<td>-0.097</td>
</tr>
<tr>
<td>Minneapolis/St. Paul 1977</td>
<td>-0.144</td>
</tr>
<tr>
<td></td>
<td>-0.084</td>
</tr>
<tr>
<td></td>
<td>-0.174</td>
</tr>
</tbody>
</table>

### Table 4-8: Premium for Movers from Outside SMSA

<table>
<thead>
<tr>
<th>Location</th>
<th>Boston 1977</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>Dallas 1977</td>
</tr>
<tr>
<td></td>
<td>Los Angeles 1977</td>
</tr>
<tr>
<td></td>
<td>Minneapolis/St. Paul 1977</td>
</tr>
</tbody>
</table>

Coefficients represent hedonic regression coefficients of a semi-logarithmic form, i.e., percentage discounts or premiums on the rent. T-statistics in brackets.

Source: (1) Malpezzi, Oztan, and Thibodeau (1980),
         (2) Own Hedonic Estimations in Section 5.5.1,
         (3) Behring and Goldrian (1983),
         (4) Own Hedonic Estimations in Section 5.5.1. based on recent movers only.
SMSA's, we measure a discount as large as 13 percent and highly significant.

All these hedonic coefficients measure discounts on the rent over and above the tenure discounts already corrected for as reported in Tables 4-1 through 4-3. However, they may be related to the emergence of tenure discounts as an amplifying factor. The line of reasoning follows Williamson's (1979) notion of idiosyncratic exchange: the landlord-tenant relation will be more affected by idiosyncrasies of either tenant or landlord if the latter is present in the building or a private person. It would be interesting to study the interaction of the intertemporal phenomenon of tenure discounts with the cross sectional phenomenon of "landlord-present-discounts". However, this would require panel data which is not available, so that we cannot pursue this topic further.

A second partition of the rental market which is able to produce price dispersion is related to search. Finding a new housing unit is a costly process and people will trade off housing characteristics including price with search costs. This argument follows Loikannen's (1982) suggestion that people searching under time pressure choose suboptimal housing bundles. This would imply on average a rent differential between people with high and with low search cost. In particular, we might partition the rental market of recent movers in those who move from outside the SMSA and those who move within the SMSA. The latter group will have lower search cost due to more information than the former. One might even think of inter-SMSA moves
as a two stage process where under time pressure a suboptimal unit is chosen for a short initial period. Then, from this base, a search for the optimal unit is started, with no time pressure and accumulated information. The following simple model illustrates our point.

Let us assume a fixed supply of $N$ vacant housing units in the SMSA, where a fraction $c$ are good values ("bargains"), and $(1-c)\cdot N$ units are lemons. However, at the time of the search, lemons and bargains cannot be distinguished by the searcher.

For simplicity, we assume an equal number of $N$ movers, out of which a fraction $g$ ("greenhorns") come from outside the SMSA, and the remaining $(1-g)\cdot N$ persons are intra-SMSA movers. The movers from outside the SMSA have one period to find a place to live, then they stay further $b$ periods in the city, then die or migrate. We will consider a steady state: a stable distribution of greenhorns and intra-city movers implies $g=1/b$. If a greenhorn picks a bargain, he stays there for the remainder of his life. Otherwise, he will start his second stage search for a better value. Thus, we have $b$ vintages of people:

1: greenhorns,
2: insider with one chance to recontract,
...  
b: insider with $b-1$ chances to recontract.

For stationarity, we assume that each vintage has the same size. In addition, let us assume for the beginning that all searchers face the same probability to pick a lemon. Then $(1-c)^i$ persons of vintage
i still live in lemons. In a cross section of all movers and sitting tenants, the proportion of people living in bargains is

\[
p = 1 - \frac{1}{b} \sum_{i=0}^{b-1} (1-c)^{i+1}
\]

\[
= \frac{c}{b} \sum_{i=0}^{b-1} (b-i)(1-c)^{i} > c
\]

This steady state "price dispersion" between new movers and all sitting tenants just reflects the accumulated chance to recontract. This dispersion will be even greater if the probability to pick a lemon is larger for greenhorns than for insiders.

This simplistic model implies a division of the market into two categories: A segment with long-run leases, relatively low rents for a standard unit, occupied by a majority of tenants already living a long time in the SMSA; and a segment with short-run leases, higher rents, and a majority of tenants moved in from outside the SMSA.

This hypothesis can be substantiated by evidence from hedonic estimation, see Table 4-8. Using the functional specification of Section 5.5.1 in Chapter Five, we include an indicator variable for movers from outside the SMSA, and restrict the estimation to recent movers (within twelve month). We estimate a premium charged to movers from outside the SMSA which ranges from 1.7 percent in Los Angeles to 4.1 percent in Boston, the latter significant at the 99 percent level.
4.3 Rent and Eviction Control in West Germany

What implications have the deviations from textbook economics listed in the preceding section on the analysis of rent control? As a case in point, we will study a highly controversial piece of legislation that regulates the rental housing market in the Federal Republic of Germany, the "2. Wohnungsraumkuendigungsschutzgesetz" (WKSchG), or law for the protection of tenants from arbitrary eviction. This law is in effect since January 1975\(^2\). The law consists of two-stylized-provisions: eviction of tenants is prohibited, and the rent is indexed once the tenant moved in. However, when a new tenant moves in, the rent can be set freely\(^3\).

More precisely, eviction is only permitted, if (1) the tenant breaches his contract (e.g., does not pay his rent), if (2) the landlord himself or a close relative wants to move into the unit, or if (3) the landlord is substantially inhibited in the appropriate economic usage of the lot (e.g., conversion into office space in areas assigned by zoning laws as a business district)\(^4\). The rent regulation permits the landlord to pass on cost increases and some of the cost of upgrading. In total, he may raise the rent up to the level of a standard rent that is defined by the average rent of comparable units, allowing him in addition to passing costs also to skim off some of the appreciation. However, a time-consuming formal procedure is required for any rent change\(^5\). Thus, the rent level for sitting tenants can be described as lagged average rent of comparable units.
The law is highly controversial: proponents argue, it is necessary to counterbalance the weak position of tenants in a sellers' market by an regulated pricing scheme. Specifically, it is claimed, that without the price regulation a landlord can exploit the exit barriers of high moving costs giving him a sort of local monopoly power. We will argue that the existence of tenure discounts contradicts this claim by showing that also the landlord faces exit barriers in terms of costs of uncertainty. A second point concerns the eviction control: eviction, arbitrary in the sense of justified or unjustified discrimination, inflicts high moving costs on the tenant. The legislation would eliminate or at least restrict the landlords' discriminatory behavior, reduces moving costs, and therefore makes tenants better off. On the hand, opponents argue, that crucial property rights - the right to evict an unpleasant tenant - are only given up against compensation for the money value of those rights in terms of higher rents and depressed supply, which, as they claim, will ultimately reduce tenants' utility. We will show that the claim of higher rents and depressed supply holds. However, the latter conclusion is not necessarily correct, because the balance between the value of reduced moving for the tenants and the value of restricted property rights for the landlords is affected by the externalities enumerated above. In fact, rent and eviction control can make both tenants and landlords better off by reducing the distorting effects of those externalities.

We will first examine the part of the German tenants' protection legislation that controls the rent level. As was pointed out in the
introduction, though the rent for sitting tenants is regulated, the initial rent can be set freely. Eekhoff (1981) shows that the primary effect of this price regulation is a heavily front-loaded payment schedule, depicted in Figure 4-5, to keep profits at the pre-legislation level. Losses (B) in the second phase of the lease are compensated by profits (A) from the high initial rent in the first phase.

![Figure 4-5: Effect of the Price Regulation. (Source: Eekhoff 1981).](image)

This brings us to the topic of tenure discounts: the pricing scheme of Figure 4-5 implies that the rent of sitting tenants in terms of the initial rent is a falling, then constant function of the length of tenure. Thus, the price regulation in the German tenants' protection legislation seems to be a perfect explanation for price dispersion in the form of tenure discounts. This, however, seems rushing to a wrong conclusion: significant tenure discounts are also present in metropolitan areas in the United States, even where no rent and
eviction control ever has been in effect. In turn, if tenure discounts indeed exist independently of rent control and are as substantial as indicated by the evidence of Tables 4-1 through 4-3, then the analysis of rent control must take into account the mechanisms that produce those discounts.

The second part of the West German tenants' protection legislation, the eviction control provision, stirs most of the heat of the emotional debate on either side, obviously, because it concerns the subtle balance between two vital rights. On one side, there is the tenant's right of the invulnerability of his dwelling, sometimes interpreted as the right to stay in the dwelling, along the lines of positive prescription. Its economic value is expressed in the psychological and monetary moving costs inflicted on the tenant when forced to move. On the other side, we have the landlord's right to dispose of his property. Its economic value is expressed in the increased psychological and monetary maintenance costs inflicted on the landlord, when forced to keep a costly or unpleasant tenant. An economic analysis of the tenants' protection legislation must take into account the economic values of those rights and how they are affected by the legislation.

Obviously, the restriction of the landlord's property rights only matters, because tenants are different, but do not reveal these differences at the time when the rental starts. Eviction is the landlords' instrument to cope with that uncertainty. Eviction control deprives the landlord of this instrument. Thus, a model to analyze
the rental market and eviction control must include heterogeneous landlords and tenants, and it must include uncertainty about the type of tenant/landlord in the initial period.

Finally, the interest of landlords for a matching tenant, and the interest of tenants not to move too often, coincides, once tenant and landlord realized that they are matching. This provides a basis for an externality: the tenant will better maintain the unit, reducing the costs for the landlord. The landlord in turn will want to keep the tenant, for instance by giving him tenure discounts, reducing his probability to move out. The landlord has two incentives to do so: first, he will reward the tenant's good care for the unit, second, he saves the uncertainty of gambling for a new good tenant. Eviction control yields a positive probability for the landlord to still have a bad tenant in the second period. Thus, tenure discounts might become an even more important instrument for the landlord, once he is not permitted to evict.

In the following section, we will construct a model around those hypotheses, concentrating on the link between tenure discounts, heterogeneous tenants and landlords, and moving costs.
4.4 A Microeconomic Model of Landlord-Tenant Relations

Let us assume a rental housing market with four different agents:

(1) Landlords of type A,
(2) Landlords of type B,
(3) Tenants of type A,
(4) Tenants of type B.

We will consider two time periods. The landlords supply $N$ housing units, where $N$ is large, but fixed over this two periods. The proportion of type A tenants on the market in period 1 and 2 will be denoted by $a_1$ and $a_2$, respectively, and the corresponding proportions of type A landlords by $b_1$ and $b_2$. Though the number of housing units is fixed, the service stream generated by each unit is variable and reflects the quality of the dwelling unit. The housing stock is durable in the sense that once the landlord has decided on the quality level of his unit, this level will stay put over both periods. Similarly, we assume that the tenant, once having chosen his optimal quality level, will not change his mind in the second period. The difference between both periods will become clear later. We denote the quality level or service stream of a housing unit by $h$. The landlord recieves profits $P(h)$ from the dwelling he is leasing, and the tenant enjoys utility $U(h,x)$ from the stream of housing services $h$ and from the consumption of other goods, denoted by $x$, which he can afford after having spent his income $y$ on housing.

The difference between the types of landlords and tenants consist in the monetary and non-monetary maintenance cost of the dwelling unit. If landlord and tenant match these costs will be lower as compared to the case of a mismatch. A landlord of type A faces costs
supply a unit of quality $h$ to a type A tenant, $c_{AB}(h)$ to a type B tenant. Analogously, a landlord of type B has costs $c_{BA}(h)$ and $c_{BB}(h)$ for type A and type B tenants, respectively. We will view the world of our model through the eyes of a type A landlord and tenant, and assume complete symmetry for type B persons. Then, we simply denote the costs in a match by $c_A(h) = c_{AA}(h) = c_{BB}(h)$, and the costs in a mismatch by $c_B(h) = c_{AB}(h) = c_{BA}(h) > c_A(h)$. Tenants are assumed to be indifferent with respect to the type of their landlord.

The two periods are short enough to keep our assumptions of a fixed stock, durable quality, and lasting preferences sensible. Furthermore, we are not interested in the division of consumption across the periods and assume that landlords and tenants have a common discount rate which is equal to the interest rate. The only purpose of dividing the short run horizon into two periods is the information available to the agents in the market. At the beginning of the first period, landlords and tenants are unable to identify the other party's type. Only after the first period, the types are revealed.

Rental contracts are made at the beginning of the first period. A contract $(r,h)$ lays down the rent $r$ for a dwelling with quality $h$. Both parties may breach the contract after the types are revealed at the beginning of the second period: the landlord is allowed to evict the tenant, and we will assume that he will do so in the case of a mismatch. A tenant does not care about match or mismatch, but he may breach the contract for some exogenous reason, say, to migrate because of a job offer. We will denote the probability of such an event by
However, the tenant will actually move only if the attractiveness of the job offer outweighs the opportunities he may have in his current housing unit. This is the point where tenure discounts enter: the landlord may reduce the rent in the second period by an amount $t$, and this tenure discount will negatively influence the probability of moving. We will denote this moving probability by $p_m(t)$ with $p_m(0) = p_0$ and $p_m'(t) < 0$. This tenure discount is entirely at the discretion of the profit maximizing landlord. A priori, it can be positive or negative.

The rental contract is a contingent contract in the sense that the rent-quality relation is determined taking into account the possible events at the transition between the two periods. Table 4-9 gives a survey of all possible events with their respective probabilities. Competition among the large number of landlords and tenants will produce an equilibrium rent level $r$ corresponding to an equilibrium quality level $h^9$. Once the quality of the match is revealed, landlord and tenant form a sort of bilateral monopoly. As we defined the contract $(r,h)$, this situation is anticipated in the competitive bargaining process before the initial period starts and the rent level adjusted accordingly. Therefore, the bilateral monopoly does not result in a bargaining game in the second period when match or mismatch is revealed, and we can avoid specifying bargaining rules with their arbitrariness$^{10}$.

The contingent market approach does not resolve the market failure intrinsic in the model constructed so far. The tenant type is
### TABLE 4-9: SPECIFICATION OF EVENTS FOR TYPE A LANDLORDS AND TENANTS

<table>
<thead>
<tr>
<th>Case</th>
<th>Tenant in Period</th>
<th>Move</th>
<th>Probability</th>
<th>Rent in Period</th>
<th>Costs in Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A A stays</td>
<td></td>
<td>(a_1(1-P_m(t)))</td>
<td>(r \ \ \ r-t)</td>
<td>(c_A \ \ c_A)</td>
</tr>
<tr>
<td>2</td>
<td>A A moves</td>
<td></td>
<td>(a_1P_m(t)a_2)</td>
<td>(r \ \ \ r)</td>
<td>(c_A \ \ c_A)</td>
</tr>
<tr>
<td>3</td>
<td>A B moves</td>
<td></td>
<td>(a_1P_m(t)(1-a_2))</td>
<td>(r \ \ \ r)</td>
<td>(c_A \ \ c_B)</td>
</tr>
<tr>
<td>4</td>
<td>B B stays</td>
<td></td>
<td>((1-a_1)(1-sP_m(0)))</td>
<td>(r \ \ \ r)</td>
<td>(c_B \ \ c_B)</td>
</tr>
<tr>
<td>5</td>
<td>B A moves</td>
<td></td>
<td>((1-a_1)sP_m(0)a_2)</td>
<td>(r \ \ \ r)</td>
<td>(c_B \ \ c_A)</td>
</tr>
<tr>
<td>6</td>
<td>B B moves</td>
<td></td>
<td>((1-a_1)sP_m(0)(1-a_2))</td>
<td>(r \ \ \ r)</td>
<td>(c_B \ \ c_B)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Landlord in Period</th>
<th>Move</th>
<th>Probability</th>
<th>Rent in Period</th>
<th>Moving Expen.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A A stays</td>
<td></td>
<td>(b_1(1-P_m(t)))</td>
<td>(r \ \ \ r-t)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A A moves</td>
<td></td>
<td>(b_1P_m(t)b_2)</td>
<td>(r \ \ \ r)</td>
<td>(m)</td>
</tr>
<tr>
<td>3</td>
<td>A B moves</td>
<td></td>
<td>(b_1P_m(t)(1-b_2))</td>
<td>(r \ \ \ r)</td>
<td>(m)</td>
</tr>
<tr>
<td>4</td>
<td>B B stays</td>
<td></td>
<td>((1-b_1)(1-sP_m(0)))</td>
<td>(r \ \ \ r)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>B A moves</td>
<td></td>
<td>((1-b_1)sP_m(0)b_2)</td>
<td>(r \ \ \ r)</td>
<td>(m)</td>
</tr>
<tr>
<td>6</td>
<td>B B moves</td>
<td></td>
<td>((1-b_1)sP_m(0)(1-b_2))</td>
<td>(r \ \ \ r)</td>
<td>(m)</td>
</tr>
</tbody>
</table>

\(a_i\): Proportion of tenants type A in period \(i\),  
\(b_i\): Proportion of landlords type A in period \(i\),  
\(P_m(t)\) with \(P_0=P_m(0)\): Moving probability,  
\(s\): Rent and eviction control parameter (\(s=1/p_0 \leftrightarrow\) no control, \(s=1\) \leftrightarrow\) regulated market)
revealed only after the first period and only to his landlord. There will be no market emerging for the information of match or mismatch because other landlords are unable to identify both the tenant's type and the type of their fellow landlord. A second external effect is introduced by the moving probability \( p_m(t) \). The expected utility of the tenant is a function of the probability density of the possible future states of the world, and this density depends via \( p_m(t) \) on the tenure discounts \( t \) granted at the discretion of the landlord.

Both externalities create a "second best environment" in which the classical welfare theorems do not hold, and it is this environment in which we want to analyse regulation of the rental housing market. Rent and eviction control will be introduced in Section 4.5. We define rent and eviction control as follows:

1. The landlord is not permitted to breach the rental contract.
2. The landlord is not permitted to charge a rent premium in the second period, that is, negative tenure discounts are unlawful.

We can combine the regulated and unregulated market setting by introducing a rent and eviction control parameter \( s \). If eviction is at the landlord's discretion, this parameter is set to \( 1/p_0 \), otherwise \( s=1 \). With this definition, the tenant's moving probability in the case of a mismatch can be written \( s p_0 \). In the case of a match, regulation has no influence on the tenant's probability to leave the unit \( p_m(t) \).

The final ingredient of the model are the expenses which occur to the tenant if he moves. We will denote the moving expenses by \( m \), and
assume that they are a lumpsum amount and independent of the cause of the move which may be the new job or eviction.

We can enumerate all possible events at the transition from period 1 to period 2:

(1) There is either a match or a mismatch in the first period;
(2) The tenant either moves after the first period or stays in the unit;
(3) If he moves, there will be either a match or a mismatch in the second period.

The probabilities for the events and the corresponding realization of rents, maintenance costs, and moving expenses are given in Table 4-9.

Two asymmetries may be worth noting. First it is not necessary to introduce the possibility of a rent premium in the case of a mismatch: if the market is unregulated, the landlord will evict the tenant; under rent and eviction control, such a premium is unlawful. Second, we did not introduce physical turnover costs for the landlord, say the cost of a vacant unit. However, the difference between the certain maintenance costs for the current tenant and the expected costs with an unknown tenant can be interpreted as turnover costs. For \(0 < a_2 < 1\) this difference is positive for a matching, negative for a non-matching tenant. Thus, in case of a match, this difference constitutes an "exit-barrier" for the landlord as do the moving costs for the tenant. The introduction of physical moving costs will only strengthen results which we will derive from this exit-barriers and is thus unnecessary. In case of a mismatch, the cost difference is negative and thus provides an incentive for the landlord to breach the
contract. Turnover costs will weaken the assumption that all mismatches will be severed in unregulated markets. To put results in the proper perspective, we can interpret the second period costs of a good tenant \( c_A \) as including turnover costs. The exit-barriers -- the cost difference \( c_A - c_B \) for the landlord and the moving expenses \( m \) for the tenant -- are the main parameters for the interpretation of the model and will play a key role when we evaluate the welfare gains and losses from rent and eviction control.

In the following Sections, we will study the landlord's and the tenant's optimization problem in more detail, and then define a suitable equilibrium concept for the two period model.
4.4.1 The Landlord

We will examine the landlord's behavior first, and look at a landlord of type A who considers tenants of type A as good, tenants of type B as bad tenants. At the beginning of the first period, he maximizes his expected profits with respect to the quality level \( h \) and the tenure discounts \( t \) which he will grant in case the tenant turns out to be a good tenant:

\[
\text{max}_{h,t} \ E \left[ P(h,t) \right]
\]

From Table 4-9, we compute the expected profit function

\[
P_E(h,t) = a_1 (1-p_m(t)) \left[ (1+e)rh - (1+e)c_A(h) - eth \right]
+ a_1 p_m(t) a_2 \left[ (1+e)rh - (1+e)c_A(h) \right]
+ a_1 p_m(t) (1-a_2) \left[ (1+e)rh - (1+e)c_A(h) - e(c_B(h)-c_A(h)) \right]
+ (1-a_1) (1-s_p c_A) \left[ (1+e)rh - (1+e)c_B(h) \right]
+ (1-a_1) s_p c_A \left[ (1+e)rh - (1+e)c_B(h) + e(c_B(h)-c_A(h)) \right]
\]

where \( e = 1/(1+i) \) the discount factor for interest rate \( i \).

Due to the linearity of the profit function, (4.4) reduces to

\[
P_E(h,t) = r_1^E h - c_1^E(h) + e (r_2^E(t)h - c_2^E(h,t))
\]

with the expected values of rents and costs in period 1 and 2:

\[
r_1^E = r
\]

\[
r_2^E(t) = r - a_1 \left( 1 - p_m(t) \right) t
\]

\[
c_1^E(h) = c_B(h) + a_1 \left( c_A(h) - c_B(h) \right)
\]

\[
c_2^E(h,t) = c_B(h) + p_A(t) \left( c_A(h) - c_B(h) \right)
\]
where

\[ p_A(t) = (a_1(1-p_m(t)) + a_1p_m(t)a_2 + (1-a_1)sp_m(0)a_2) \]

Maximization of (4.5) with respect to \( h \) and \( t \) yields the first order conditions:

(4.10) \[ r + e r^F_2(t) = c^F_1(h) + e c^F_2(h;t) \]

(4.11) \[ (1-p_m(t)) - p_m'(t)t = p_m'(t) (1-a_2) (c_A(h)-c_B(h))/h \]

As a first result, we can characterize the tenure discounts by equation (4.11).

**Theorem 1:**

If the cost differences between tenant types are large enough, the landlord will grant positive tenure discounts to keep a good tenant. More precisely, the optimal tenure discounts are given in implicit form by

(4.12) \[ t = (1-a_2) (c_B(h)-c_A(h))/h + (1-p_m(t))/p_m'(t) \]

Note that the first term in (4.12) is positive, the second negative by our assumption. The landlord faces a trade-off between losing money by granting tenure discounts on one hand and by risking to get a bad tenant in the second period on the other hand. A large cost difference in the first term will outweigh the second term and generate positive tenure discounts. However, if the probability of a bad tenant in period two is very small and the moving probability insensitive to tenure discounts, the landlord is better off by discouraging the old good tenant to gamble for a new good tenant, whom
he does not have to pay tenure discounts.

To simplify the analysis and to be able to solve (4.12) for the discounts \( t \), we assume a linear functional form for the moving probability \( p_m(t) \):

\[
(4.13) \quad p_m(t) = p_0 - kt \quad \text{for} \; t < p_0/k \quad (k > 0) \\
= 0 \quad \text{otherwise}
\]

Next, we examine the structure of the cost functions. Obviously, \( c_A(0) = c_B(0) = c_0 \), the cost of a vacant unit. We have already defined the type of tenant by \( c_A(h) < c_B(h) \) for \( h > 0 \). For a well behaved cost function, we postulate \( c_A' > 0, c_B' > 0, c_A'' > 0, c_B'' > 0 \). In addition, it is convenient to assume that the difference between the costs for different tenure types increases linearly with the housing unit service stream: \( c_B(h) - c_A(h) = c_D h > 0 \).

With the linear specifications of \( p_m \) and \( c_B - c_A \), the tenure discounts are given by:

\[
(4.14) \quad t = (1-a_2)/2 \cdot c_D - (1-p_0)/2k,
\]

hence positive for

\[
(4.15) \quad c_D > \frac{(1-p_0)}{(1-a_2)k}.
\]

We can substitute this into the first order condition (4.10) to obtain the following implicit reduced form equation in the landlord's.
For the comparative static analysis in Section 4.5 we compute the following partial derivatives of $F$ with respect to rent $r$, the regulation parameter $s$, and the quality level $h$:

(4.17) \[ F_r = (1+e) > 0 \]

(4.18) \[ F_s = e(1-a_1)a_2p_0c_D > 0 \]

(4.19) \[ F_h = -(1+e)c''_B(h) < 0 \]

Applying the implicit function theorem, we obtain an monotonously upward sloping supply function in the initial rent level $r$, with the supply elasticity given by

(4.20) \[ E_S = - \frac{r F_r}{h F_h} = \frac{r/h}{c''_B(h)} > 0. \]

Finally, we define the elasticity of supply of housing services with respect to the regulation parameter $s$ which falls from 1 to $1/p_0$ when rent and eviction control is introduced:

(4.21) \[ R_S = - \frac{s F_s}{h F_h} = \frac{e(1-a_1)a_2p_0c_D}{c''_B(h)} > 0. \]

Note that this elasticity is positive and proportional to the cost differential $c_D$ between good and bad tenants.
4.4.2 The Tenant

We turn now to the demand side. We will look at a tenant of type A. He chooses housing quality h and consumption of other goods x at the beginning of period 1 as to maximize his expected utility subject to a budget constraint. For simplicity, we assume a separable utility function \( U(x,h) = u(x) + v(h) \). Because we are not interested in the intertemporal distribution of consumption, we define \( x = x_1 + e x_2 \) as two period consumption and \( y = y_1 + e y_2 \) as two period income with \( e = 1/(1+i) \). Rent is denoted by \( r \), moving expenses by \( m \), and other goods are normalized to have unit price. With these conventions, the tenants maximization problem can be written as

\[
\max_{h,x} \mathbb{E} [ u(x) + v(h) ] \quad \text{s. t.} \quad \mathbb{E} [ y - x - rh - m ] = 0.
\]

The lower part of Table 4-9 specifies the outcomes and probabilities for all possible events for a tenant of type A. There are three different outcomes according to the receipt of tenure discounts and whether the tenant moves. Note that depending on the imposition of rent and eviction control one of the three outcomes will not occur. The maximand of the tenant expands to

\[
v(h) + b_1 (1-p_m(t)) u(y-(1+e)rh+eth)
+ (1-b_1)(1-s_p) u(y-(1+e)rh)
+ (b_1 p_m(t)+(1-b_1)s_p) u(y-(1+e)rh-em).
\]

We will use the simplifying notation
to denote the probabilities of the different events for a tenant, and

\[ \bar{x} = y - (1+e)rh \]

\[ w(\bar{x}) = q_1 u(\bar{x}+eth) + q_2 u(\bar{x}) + q_3 u(\bar{x}-em) \]

for "normal" consumption and the expected utility from consumption. The first order condition of (4.23) with respect to h is

\[ G = v'(h) - w'(\bar{x})(1+e)r - q_1 u'(\bar{x}+eth)e(1+e)th = 0 \]

and defines the implicit reduced form of the tenant's demand for housing quality. This gives us the following partial derivatives for comparative static analysis:

\[ G_r = w''(\bar{x})(1+e)^2 rh - w'(\bar{x})(1+e) - q_1 u''(\bar{x}+eth)e(1+e)th < 0 \]

\[ G_s = (1-b_1)p_0(1+e)r (u'(\bar{x}) - u'(\bar{x}-em)) < 0 \]

\[ G_h = v''(h) + w''(\bar{x})(1+e)^2 r^2 - q_1 u''(\bar{x}+eth)(e^2 t^2 - 2e(1+e)rt < 0 \]

where the signs are implied by the assumption of well behaved neoclassical utility functions with \( v' > 0 \), \( u' > 0 \), \( v'' < 0 \), and \( u''' < 0 \). In particular, we have a monotonously downward sloping demand curve with a demand elasticity of

\[ E_D = - \frac{r G_r}{h G_h} < 0. \]
In analogy to the supply side, we define the elasticity of demand for housing services with respect to the regulation parameter $s$ as

\begin{equation}
R_D = - \frac{s}{h} \frac{G_s}{G_n} < 0.
\end{equation}

Note that this elasticity is negative and roughly proportional to the moving expenses $m$, which can be seen from (4.28) when the bracketed difference in marginal utilities is approximated by $u''(\bar{x})em$. 
4.4.3 Steady State Market Equilibrium

A steady state equilibrium is characterized by three conditions:

\[(4.32) \quad a_1 N^T = b_1 N^L \quad \text{and} \quad (1-a_1) N^T = (1-b_1) N^L\]

\[(4.33) \quad a_1 D^A(r) + (1-a_1) D^B(r) = b_1 S^A(r) + (1-b_1) S^B(r)\]

\[(4.34) \quad a_1 = a_2 \quad \text{and} \quad b_1 = b_2\]

The first condition assures that the number of housing units demanded equals the number of housing units supplied, for both types of landlords and tenants. This equilibrium condition of the extensive margin of course implies \(a_1 = b_1\) and \(N^T = N^L = N\), the number of landlords and tenants. The second condition equalizes demand and supply of housing service streams at the intensive margin. \(S^A(r)\) denotes the optimal housing service supplied by a landlord of type A at initial rent \(r\), that is \(S^A = h\) given by equation (4.16). \(D^A(r)\) denotes the corresponding demand, given by equation (4.26). Finally, the third condition characterizes the steady state by a stable distribution of tenant and landlord types.

Note the distinction between the discrete problem of matching the number of units (=landlords) and tenants, and the continuous problem of determining the equilibrium housing service level. The latter condition (4.33) is the substantive equilibrium condition of this model, the first holds by assumption. The slope of the demand and
supply functions, given by (4.20) and (4.30), and their monotonicity guaranty that the continuous housing service equilibrium condition (4.33) yields a well defined and unique equilibrium rent level \( r \). If the discrete equilibrium condition (4.32) is violated, vacencies or homeless will emerge, and we have to specify rules for this disequilibrium. We will not consider this problem in this paper.

The violation of the steady state condition (4.34) can be interpreted as a kind of adverse selection problem. Let \( N \) denote the population in the housing market under examination, and \( M \) denote the net migration to another housing market, with \( g_1 \) the proportion of type A migrants. After one period, \( p_m(t) a_1 N \) is the number of type A tenants moving within the given housing market. In addition, \( g_1 M \) type A tenants will move from outside into this market. For type B tenants, we have \( sp_0 (1-a_1) N \) intra-city movers and \( (1-g_1) M \) inter-city movers. Thus,

\[
(4.35) \quad a_2 = \frac{p_m(t) a_1 N + g_1 M}{(p_m(t) a_1 + sp_0 (1-a_1)) N + M}
\]

is the second period share of type A tenants on the market. In steady state, \( a_1 = a_2 \). Solving (4.35) for \( a_1 = a_2 \) yields a quadratic equation

\[
(4.36) \quad a_1^2 - \frac{M}{(p_m(t) - sp_0) N} \quad a_1 - g_1 M/N = 0.
\]

Thus, in the absence of migration \((M=0)\), we obtain the corner solutions \( a_1 = 0 \) or \( a_1 = 1 \) as steady state shares to which any other
initial distribution $0 < a_1 < 1$ will converge. This is due to the unbalanced shares of movers in each period, $p_m(t)/sp_0$, which creates a selection of bad tenants among movers adverse for the landlords.

We will assume a steady migration of tenants to produce a steady state distribution $0 < a_1 < 1$. For simplicity and without loss of generality, we set $a_1 = 0.5$. Now all reactions of type A agents are mirror-images of their type B counterparts, and it is sufficient to examine only half of the market.
4.5 Analysis of Rent and Eviction Control

The eviction and rent control as defined in Section 4.1 is modelled along the lines of the tenants' protection legislation in West Germany and has two effects in our model: (1) landlords are not permitted to evict tenants of a different type, and (2) landlords are prohibited from charging tenants a premium on top of their second period rent. Note that the initial rent level is unrestricted and free to move according to the competitive market equilibrium (4.33). We model the two effects simultaneously by changing the parameter $s$ in the specification of the probabilities in Table 4-9. Setting $s = 1/p_m(0)$ corresponds to a housing market without intervention: mismatches will be severed with probability one, and Case 4 in the Table will never occur. In turn, setting $s = 1$ corresponds to a housing market under eviction and rent control, where the bad tenant will only move with probability $p_m(0)$ and pays the contract rent $r$ in period 2. A decrease in $s$ thus corresponds to a step in direction of rent and eviction control.

Total differentiation\textsuperscript{12} of the equilibrium condition (4.33) yields

\[ dr G_s F_h - G_r F_s = - \frac{G_r F_h - G_h F_r}{G_s F_h - G_r F_s} < 0 \]  

where the pieces can be collected from equations (4.17), (4.18), (4.19); and (4.27), (4.28), and (4.29). In terms of the elasticities
(4.21), (4.22), and (4.31), and (4.32), we can write the impact of a step in direction of regulation on the rent level as

\[
\text{(4.38)} \quad dr = \frac{- r/s \left( R_D - R_S \right)}{\left( E_D - E_S \right)} \, ds
\]

Thus, we can describe the first effect of the intervention:

**Theorem 2:**

The initial rent level for a new lease will rise in response to the rent and eviction control.

**Proof:**

The signs in (4.37) and (4.38) follow from the quoted equations, and imposition of rent and eviction control is equivalent to ds < 0.

Note that we can decompose the price change dr in two terms. The first term includes \( R_D \) and is roughly proportional to the moving expenses m, see (4.31). The second term includes \( R_S \) and is proportional to the difference in maintenance costs \( C_D \) according to (4.21). Thus, the price change reflects the sharing of the burden and the gain from eviction and rent control between landlord and tenant, the gain expressed in the moving expenses saved by the tenant and the burden in the cost difference inflicted on the landlord.

How does the landlord respond to the imposition of rent and eviction control?

**Theorem 3:**

(1) The landlord's supply for housing services will fall in response to the rent and eviction control.

(2) Under linear cost differences, the tenure discounts will remain
unchanged.

Proof: We computed the supply effect when defining the elasticity $R_s$ in (4.21) by applying the implicit function on (4.16):

$$
(4.39) \quad dh = -\frac{e^{(1-a_1)} a_2 P_0 C_D}{(1+e) c''_B(h)} \frac{dS}{ds} < 0
$$

The absence of a tenure discount effect is entirely due to the linear cost difference $c_B(h) - c_A(h) = C_D = \text{constant}$ which can be seen from equation (4.14) where all right hand side items are constants.

Supply will be depressed, because the legislation imposes higher costs on the landlord: he looses the economic value of his right to evict. To adjust optimal profits under a convex cost function, output has to go down. The change in tenure discounts for a general cost specification can be obtained from equation (4.12):

$$
(4.40) \quad dt = (1-a_2)/2h^2 \left[ (c_B'(h)-c_A'(h))h - (c_B(h)-c_A(h)) \right] \frac{dh}{ds}
$$

The landlord will grant larger discounts in response to the legislation if the cost difference increases more than linearly with the housing service stream, and less discounts for a concave cost difference. He will do so to counterbalance the changes in cost differences with a changed moving probability $p_m(t)$: if this difference decreases due to (4.39) and a convex cost difference, he is more interested to keep the tenant, and will give him larger discounts.

The effect for profits is given by the envelope theorem.
\[
(4.41) \quad dP = (1+e) - h \, ds + e \,(l-a) \, a_2 \, p_0 \, c_0 \, ds
\]

and consists of two effects of opposite signs. The second term in (4.41) is negative: the landlord looses because he is forced to keep a bad tenant, if this tenant decides to stay. This effect is proportional to the cost difference. On the other hand, he is able to regain some of the losses through rent increases. As we have seen in (4.38), we can interpret the first effect as appropriation of some part of the tenant's advantage from the legislation in the form of lower expected moving expenses.

We now turn to the impact of rent and eviction control on the demand side of our rental housing market model. We evaluate the effect on the tenant's utility by applying the envelope theorem on (4.23):

\[
(4.42) \quad du = - \, w' (\bar{x}) \, (1+e) - h \, ds - (1-b_1) p_0 (u(\bar{x}) - u(\bar{x}-em)) \, ds
\]

The first effect in the tenant's utility (4.42) is the mirror image of the first effect in the landlord's profits (4.41). Tenants suffer from an utility loss due to the increase in in the initial rent level. This price increase is converted by the marginal utility \( w'(\bar{x}) \) as defined in (4.25) into utility units. The second effect reflects the increase in utility due to the decreased likelihood of moving, that is lower expected moving expenses. This utility increase is approximately \( w'(\bar{x})em \), thus

\[
(4.43) \quad du/w'(\bar{x}) = - (1+e) - h \, ds - (1-b_1) \, p_0 \, em \, ds
\]
If we analyze the relative weights of the opposing effects in (4.41) and (4.42), we obtain the main result of this Section:

**Theorem 4:**

1. If moving expenses are small relative to the cost differential between good and bad tenants, both landlords and tenants are worse off by eviction and rent control.
2. If moving expenses are large relative to the cost differential between good and bad tenants, both landlords and tenants are better off by eviction and rent control.
3. For a given ratio of moving expenses and differential maintenance costs, Case (2) is the more relevant the smaller the likelihood is of a second period match after the severance of a mismatch.

**Proof:**

We express the utility change (4.42) locally as

\[ du = (G_r \frac{dr}{ds} + G_s) \Delta ds \]

and the profit change (4.41) correspondingly as

\[ dP = (F_r \frac{dr}{ds} + F_s) \Delta ds \]

To obtain a utility (profit) increase in response to the imposition of the legislation (ds<0) the bracketed expressions have to be negative. Substitute dr/ds from (4.37). Then

\[ du > 0 \iff -G_r \frac{GrF_r - GhF_s}{G_rF_h - GhF_r} + G_s < 0, \]

and

\[ dP > 0 \iff -F_r \frac{GsF_h - GsF_s}{G_rF_h - GhF_r} + F_s < 0. \]

Thus,
\[ du > 0 \iff dP > 0 \iff G_s/G_r > F_s/F_r. \]

From (4.19), (4.27), and (4.43):

\[
F_s/F_r = \frac{e (1-a_1) a_2 P_0}{l+e} \quad c_D > 0,
\]

and

\[
G_s/G_r = \frac{(1-a_1) P_0 e w'\prime (x)}{w''(x) rh(l+e) - w'(x) - q_1 u''(x+eth)eth} \quad m > 0.
\]

Hence,

\[ du > 0 \iff dP > 0 \iff m > a_2 K c_D \]

with

\[
K = \frac{w''(x) rh(l+e) - w'(x) - q_1 u''(x+eth)eth}{u''(x) (1+e)r} > 0
\]

which proves the Theorem.

The result is an example how the intuition from a first best environment can be misleading in a second best environment. Market failures occur in the rental housing market because of two externalities: a missing market for the information of landlord and tenant type; and an expected utility function of the tenant which is responsive to the tenure discounts. The fundamental welfare theorems are not valid in an environment with external effects. In Case (2), interference with the market in form of rent and eviction control proved to be Pareto superior to laissez faire: the tenants' protection legislation did not only improve the tenant's welfare but also the profits of the landlords.
4.6 Conclusions

Using empirical evidence on the existence of tenure discounts in West Germany and the United States we screened the arguments pro and contra rent and eviction control, specifically the West German tenants' protection legislation. A model was build based on the mechanisms identified as important for the working of the rental housing market: tenant and landlord idiosyncrasy, potential tenure discounts, and high moving costs.

As a first result, the model indeed predicted positive tenure discounts. Second, the comparative static analysis of rent and eviction control showed, that with low moving costs and a large difference in costs between good and bad tenants rent and eviction control makes both tenants and landlords worse off. However, with high moving costs and only little difference in costs between tenant types, the intervention increases the utility of all participants, of landlords as well as of tenants. The conclusion of many analyses that the German tenants' protection legislation will not only reduce the profits of landlords, but even harm the tenants which should have been protected by the legislation is at least premature.

This result of an intervention Pareto superior to laissez faire is due to the externalities between landlord and tenant and an example of how first best analysis misleads policy evaluation in a second best environment.
The second conclusion concerns estimating housing demand. As discussed in the end of Chapter Three, positive tenure discounts bias the estimated price responses upward. How can we correct for that? Obviously, the proper solution is to take all units of a given cross section, but look at their spot market rent rather than their actual rent. The spot market rent is observed only at the time of moving. With a panel, we could trace all units back to this period. In absence of a panel, however, we can use the hedonic estimates of average discounts quoted in Tables 4-1 through 4-3 and add these discounts to the actual rent to achieve approximations of the spot market rent. Note that this procedure avoids the bias which is introduced when confining the estimation only to recent movers. We will use this procedure in the next chapter, when we compare housing demand in the United States and in West Germany.
Footnotes to Chapter 4

(1) Guasch and Marshall (1983) argue that the hedonic regression suffers from self selection bias, because tenants with low or no tenure discounts will move out of their expensive units and therefore have less probability to be in the sample. Our model in Section 4 will have the same implication. However, their empirical findings about the size of this effect and the remaining "true" discounts are inconclusive. Correction for the selection bias in three different samples changes the estimated tenure discounts only little and in either direction. Their standard errors, though, increase as much as to render the discounts statistically insignificant at all. It is not clear whether this result is due to the specific sample they used or to an inefficient estimation procedure for their relatively small sample size.

(2) The law was modified in 1983, but not changed in its substance.

(3) Only usury is prohibited by §5 WiStG.

(4) §564d BGB.

(5) §2 MiethoehG. This procedure can only be waived if the tenant agrees to it.

(6) This might be a frivolous choice of words. We want to distinguish discrimination due to objectively higher costs from discrimination due to taste or prejudice.

(7) Compare Goodman and Kawai (1982) for a sketch of a similar model.

(8) We assume that the expected costs of a bad tenant staying are always larger than the expected costs of an unknown new tenant. For $0 < a_2 < 1$, this is basically a statement about small turnover costs. See the discussion at the end of this Section.

(9) A suitable equilibrium concept is developed in Section 4.4.3.

(10) Eckart, Schulz, and Stahl (1983) consider a model of voluntary and involuntary exchange in a housing market with implicit contracts. The landlord-tenant relationship in our model resembles an implicit contract: the tenant takes the discounts in the second period in account when maximizing his expected utility over the two periods.

(11) We assume each landlord supplies only one housing unit. This includes landlords with more than one housing unit, as long as each of the units is supplied by maximizing its own profit.

(12) We keep $a_2$ constant. In fact, $a_2$ is a function of the tenure discounts, thus affected by the change of $s$. However, the linear specification of costs and moving probability keeps $a_2$ unchanged, as it will turn out in (4.40).
CHAPTER FIVE

AN ANALYTIC COMPARISON OF HOUSING DEMAND DECISIONS IN THE UNITED STATES AND WEST GERMANY
5.1 Introduction: Idea and Scope of an Analytic Comparison

This last chapter of the thesis uses all the analytical tools we have developed so far to compare housing demand in the United States and West Germany. In both countries housing surveys exist with comparable scope of questions asked, sample sizes, and sampling procedures. This is the Annual Housing Survey in the United States which we already used in Chapter Three, and the One Percent Sample in West Germany, named after its sampling ratio.

The existence of parallel data sets in the two countries allows us to use the same analytical model, i.e., functional specification of the demand equations and specification of the explanatory price, and income, and demographic variables. In this sense, we will carry out an analytical rather than a descriptive comparison.

Though the scope of questions asked is roughly the same in both surveys, due to confidentiality restrictions in West Germany the data available to us has little information on household composition, prohibiting the decomposition of households into nuclei as described in Chapter Three. Thus, our comparison will be confined to population strata in which household formation can safely be considered exogenous to the housing market: married couples.

Second, we do not want to stress the comparison to areas where the two countries are structured very differently. We will exclude guestworkers in West Germany and non-whites in the United States from
our comparison. With similar reasoning, we confine the analysis to metropolitan areas because the density pattern of rural settlements are completely different in the two countries. In Germany, we sample only from counties which are classified as highly densely populated. The United States are represented by the Standard Metropolitan Statistical Areas of Boston, Dallas, Los Angeles, and Minneapolis/St. Paul from the Annual Housing Survey by SMSA.

Though we pool the data across cities in both countries, we will keep city centers and suburbs apart, and we will further stratify the households (married couples) according to three age groups (age below 35, age between 35 and 50, and age above 50). This yields six strata for each country. In addition, we pool all data to explore a variety of functional forms -- decision trees -- and different specifications of the explanatory variables. These pooled samples contain 8035 white married couples in West Germany and 8139 in the United States. We will refer to these two samples as our "basic samples".

This chapter begins with a short enumeration of descriptive statistics to outline the differences in housing consumption in the two countries. In addition, we will compare summary statistics from the entire nation with summary statistics from our basic sample to obtain a sense of how the samples represent the countries. Section Three briefly sketches the differences in the tax treatment of owner-occupancy between the countries.

This descriptive part is intended to set the stage for the
analytical comparison: how much of the observed differences in housing consumption can be explained by differences in the exogenous variables, and how much has to be attributed to preference differences. Sections Four through Seven specify the common demand model, estimate the components — permanent income and hedonic prices — and finally the NMNL-demand equations. Then, we compare the price and income responsiveness and optimal tree structures. This will give us insight in the differences between the preferences in West Germany and the United States. The last section attempts to separate those from the effects of the tax differences: we forecast each country's housing consumption first at the other country's preferences and then at the other country's tax and subsidy system.
5.2 A Brief Descriptive Comparison

The most striking difference in housing consumption between West Germany and the United States is the difference in the tenure choice: of all households in the U.S. 1978, 65.2 percent lived in owner-occupied housing\(^1\), only 36.3 percent in Germany\(^2\). This pattern is as striking in our basic samples of white married couples in urban areas: compared to the entire population, this ratio rises to 77.0 percent in the U.S.; but only to 42.2 percent in West Germany. Table 5-1 gives a more detailed decomposition of housing demand for the two countries, based on the entire population (Annual Housing Survey 1977, National Sample, and the West German One Percent Sample 1978, respectively) and our basic samples. We consider eight housing alternatives, generated by three dimensions: owner-occupied versus rental housing, single-family homes and duplexes versus multi-family structures, and one-to-four room dwelling (plus kitchen) versus dwellings with more than five rooms.

The differences in tenure choice are echoed in the differences between structure types: whereas in Germany single-family structures (including duplexes) and multi-family structures have almost equal shares, single-family homes constitute the overwhelming share of structures in the United States. Note, that our sample differs from the population in two ways: we consider only married couples with a higher likelihood of owner-occupancy, and only high density urban areas with a lower propensity to own. The ownership ratio for married couples all over Germany is 43.2 percent, whereas the ownership for
### TABLE 5-1: MARKET SHARES OF HOUSING ALTERNATIVES

---------------------------------------------------------------
<table>
<thead>
<tr>
<th>All Households</th>
<th>Rental Housing</th>
<th>Owner-Occupied</th>
<th>Unit Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>23,067,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1-4 Rooms</td>
<td>5+ Rooms</td>
<td>1-4 Rooms</td>
</tr>
<tr>
<td></td>
<td>10.5%</td>
<td>5.7%</td>
<td>8.0%</td>
</tr>
<tr>
<td>1-2 Units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3+ Units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>38.7%</td>
<td>8.8%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Tenure Choice</td>
<td>49.2%</td>
<td>14.4%</td>
<td>11.2%</td>
</tr>
<tr>
<td>Size Choice</td>
<td>60.4%</td>
<td></td>
<td>36.3%</td>
</tr>
</tbody>
</table>

### WEST GERMANY: REGIONS WITH HIGH DENSITY

---------------------------------------------------------------
| Married, German  | Rental Housing | Owner-Occupied | Unit Choice |
| 8,019            |                |                |             |
|                  | 1-4 Rooms      | 5+ Rooms       | 1-4 Rooms   | 5+ Rooms   |
| 1-2 Units        | 10.2%          | 2.5%           | 21.1%       | 15.9%      | 47.7%     |
| 3+ Units         | 41.0%          | 4.2%           | 4.6%        | 0.6%       | 50.3%     |
| Tenure Choice    | 51.2%          | 6.7%           | 25.7%       | 16.5%      |           |
| Size Choice      | 76.9%          |                | 23.1%       |           |           |

### UNITED STATES: ANNUAL HOUSING SURVEY 1977, NATIONAL SAMPLE

---------------------------------------------------------------
| All Households | Rental Housing | Owner-Occupied | Unit Choice |
| 75,280,000     |                |                |             |
|                | 1-4 Rooms      | 5+ Rooms       | 1-4 Rooms   | 5+ Rooms   |
| 1-2 Units      | 11.5%          | 60.0%          | 71.5%       |
| 3+ Units       | 30.2% + 5.0%   | 27.7% + 37.1%  |             |
| Tenure Choice  | 35.2%          | 64.8%          |             |
| Size Choice    | 57.9%          |                | 42.1%       |

### UNITED STATES: BOSTON, DALLAS, LOS ANGELES, MINNEAPOLIS/ST.PAUL SMSA

---------------------------------------------------------------
| White Married   | Rental Housing | Owner-Occupied | Unit Choice |
| 8,139           |                |                |             |
| 1-2 Units       | 7.2%           | 24.5%          | 85.5%       |
| 3+ Units        | 11.9%          | 1.1%           | 14.5%       |
| Tenure Choice   | 19.1%          | 3.9%           | 51.4%       |
| Size Choice     | 44.7%          | 55.3%          |             |
all households in urban areas in the United States (SMSA's) is 61.0 percent.

The difference between rural and urban areas in the two countries are most striking in the market shares of small versus large dwellings. On the national scale, they are very close: 57.9 percent of all dwellings have less than five rooms in the United States, 60.4 percent in West Germany. However in urban areas, this ratio falls in the United States to 44.7 percent, but rises in Germany to 76.9 percent. This reflects a greater degree of suburbanization in the United States with both urban sprawl and a concentration of higher income households in the suburbs as compared to rural areas.

So far, we have compared the endogenous variables of housing demand. Table 5-2 compares the patterns of exogenous variables and their reflection in the tenure choice. Germany has a substantially higher proportion of elderly households (aged over 65) both on national scale and in our sample of couples in urban areas. In both countries our sample has a more centered age distribution, here, the married couple effect and the urban area effect accumulate. The ownership ratios show the familiar life cycle pattern, but with considerable differences between the countries. The peak in Germany is about ten years earlier than in the United States and the decline in homeownership among the elderly is much more pronounced.

The second part of Table 5-2 lists the annual gross income in Dollars at prices of 1977. Median income is about equal in West
TABLE 5-2: EXOGENOUS VARIABLES: AGE, INCOME, AND PRICES

(1) AGE DISTRIBUTION AND TENURE CHOICE:

<table>
<thead>
<tr>
<th>Age</th>
<th>United States</th>
<th>West Germany</th>
<th>Ownership Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All HH(^a)</td>
<td>Sample(^b)</td>
<td>All HH(^a)</td>
</tr>
<tr>
<td>&lt; 25</td>
<td>6.4%</td>
<td>5.3%</td>
<td>12.6%</td>
</tr>
<tr>
<td>25-29</td>
<td>11.6</td>
<td>11.3</td>
<td>&quot;</td>
</tr>
<tr>
<td>30-34</td>
<td>12.4</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>35-39</td>
<td>20.0</td>
<td>11.2</td>
<td>&quot;</td>
</tr>
<tr>
<td>40-44</td>
<td>&quot;</td>
<td>9.5</td>
<td>18.8</td>
</tr>
<tr>
<td>45-49</td>
<td>35.5</td>
<td>10.3</td>
<td>&quot;</td>
</tr>
<tr>
<td>50-64</td>
<td>&quot;</td>
<td>27.5</td>
<td>23.2</td>
</tr>
<tr>
<td>&gt; 65</td>
<td>14.2</td>
<td>12.4</td>
<td>26.7</td>
</tr>
</tbody>
</table>

Income is yearly gross household income in Thousand 1977 Dollars

(2) INCOME DISTRIBUTION AND TENURE CHOICE:

<table>
<thead>
<tr>
<th>Income</th>
<th>United States</th>
<th>West Germany</th>
<th>Ownership Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All HH</td>
<td>Sample</td>
<td>All HH</td>
</tr>
<tr>
<td>&lt; 4.0</td>
<td>13.1%</td>
<td>2.1%</td>
<td>9.7%</td>
</tr>
<tr>
<td>&lt; 6.0</td>
<td>8.9</td>
<td>3.2</td>
<td>14.9</td>
</tr>
<tr>
<td>&lt; 8.0</td>
<td>10.2</td>
<td>4.9</td>
<td>18.4</td>
</tr>
<tr>
<td>&lt;10.0</td>
<td>6.6</td>
<td>4.9</td>
<td>14.9</td>
</tr>
<tr>
<td>&lt;12.5</td>
<td>10.7</td>
<td>8.0</td>
<td>16.0</td>
</tr>
<tr>
<td>&lt;15.0</td>
<td>7.7</td>
<td>7.6</td>
<td>9.7</td>
</tr>
<tr>
<td>&lt;20.0</td>
<td>14.7</td>
<td>18.3</td>
<td>10.3</td>
</tr>
<tr>
<td>&lt;25.0</td>
<td>28.0</td>
<td>16.4</td>
<td>6.1</td>
</tr>
<tr>
<td>&gt;25.0</td>
<td>&quot;</td>
<td>34.6</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

(3) HOUSING PRICES

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>West Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>City Center</td>
<td>Suburbs</td>
</tr>
<tr>
<td>Gross Rent</td>
<td>2,816</td>
<td>2,981</td>
</tr>
<tr>
<td>Hedonic Rent</td>
<td>2,742</td>
<td>2,935</td>
</tr>
<tr>
<td>Out-Pocket-Cost</td>
<td>4,108</td>
<td>4,300</td>
</tr>
<tr>
<td>Hedonic Rent</td>
<td>3,428</td>
<td>3,546</td>
</tr>
<tr>
<td>Value</td>
<td>49,238</td>
<td>48,443</td>
</tr>
</tbody>
</table>

Yearly rents and values in 1977 Dollars
Germany ($13,361) and the United States ($13,444)\textsuperscript{3}. However, the income distribution is very different. In both countries income is approximately evenly distributed across all categories up to median income, but in Germany only the small fraction of 6.1 percent earns more than $20,000, whereas in the United States this percentage is as large as 28.0 percent. A similar pattern holds in our sample: mean income is similar (United States: $22,020, West Germany: $23,420), but the income among couples in U.S. cities has somewhat fatter tails on either side: the low income category (below $8,000) accounts for 10.2 percent as compared to 3.0 percent in German cities, and the high income category (above $35,000) 34.6 percent as compared to 29.8 percent. In both countries ownership is strongly correlated with income where the ownership ratio reaches 95.8 percent for an income above $45,000 in the U.S. sample, and 58.8 percent in the German sample. Interestingly, the pattern is reversed for the very low income groups. A more detailed cross-tabulation reveals that most of these households are elderly or very young couples. This may reflect reporting errors in transfer income of the young and extremely low out-of-pocket cost due to paid-off mortgages among the elderly.

The most striking observation is the difference between the income of the renter households and the homeowners. In the United States, median income for homeowners is about twice as high as among renter households ($17,100 versus $8,800), but in West Germany it is only about 30 percent higher ($15,750 versus $12,000). The same discrepancy holds in our sample: the mean income in the U.S. sample is $24,030 for homeowners and $15,410 for renters, in West Germany $
Finally, the third part of Table 5-2 lists average housing prices for our two samples, stratified by city center and suburbs. Again, there are very pronounced differences: gross rents in Germany are substantially lower (about 55 percent)\(^4\), but prices of single-family houses including the lot are drastically more expensive (more than twice). Note, that this does not translate into high mean out-of-pocket-costs. This is basically due to a much lower loan-to-value ratio in Germany where a large downpayment is made from a building society's savings contract ("Bausparkassenvertrag"). The mean hedonic rent for owner-occupied housing is about 40 percent higher than for rental housing in Germany, only about 25 percent percent in the United States. Thus, a part of the differential in house prices can be explained by better quality dwellings. Second, in 1977/78, about 40 percent of the German house prices in high density areas are paid for the lot\(^5\), only about 20 percent in the U.S. SMSA's\(^6\). This translates into a 4.5 times higher lot price in Germany than in the United States and only 80 percent higher structure costs.
5.3 Tax Treatment of Owner Occupancy in the Two Countries

We will argue that a substantial part of the different housing consumption patterns can be explained by the different tax laws. This Section gives a brief description of how the out-of-pocket costs of homeownership are reduced by tax deductions from the personal income tax in the two countries. Compared with the United States, the tax provisions for homeowners in Germany are confusing and complicated, and we present only a simplified version.

Table 5-3 gives a stylized survey of the tax laws. The important tax tool in the United States is the deduction of mortgage interest, whereas in West Germany the basic mechanism is the allowance for the depreciation of the structure. There are three depreciation schedules in Germany: an "accelerated schedule" with a cap on time and value, a "degressive schedule" applicable only to new structures, and the omnibus linear depreciation schedule. In the United States, imputed rental income is not taxed; in Germany, it is taxed except in the case of an accelerated depreciation schedule. Germany has only a negligible property tax, whereas in the United States the property tax is substantial even after deduction from personal income tax. The German tax law has an additional twist in its special treatment of two-family homes. In this case, mortgage interest can be deducted in addition to depreciation allowances as well as all maintenance and a percentage of modernization expenses, only partially offset by the taxation of imputed rent. This is insofar relevant to our demand analysis, as the law did not exclude the rentless "lease" of the
## FIGURE 5-3: STYLIZED TAX-TREATMENT OF HOMEOWNERSHIP

<table>
<thead>
<tr>
<th>United States 1977</th>
<th>West Germany 1978</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single-Family</td>
<td>Two-Family</td>
</tr>
</tbody>
</table>

### (1) Property Tax:

<table>
<thead>
<tr>
<th>United States</th>
<th>West Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>varies locally, about 2% of value</td>
<td>varies locally, about 0.1% of value</td>
</tr>
</tbody>
</table>

### (2) Imputed Rent Taxed as Personal Income:

<table>
<thead>
<tr>
<th>United States</th>
<th>West Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>yes, if schedule B</td>
</tr>
<tr>
<td>no, if schedule A</td>
<td>yes</td>
</tr>
</tbody>
</table>

### (3) Deductions from Personal Income Tax:

<table>
<thead>
<tr>
<th>United States</th>
<th>West Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Property Tax</td>
<td>yes</td>
</tr>
<tr>
<td>Mortgage Interest</td>
<td>yes</td>
</tr>
<tr>
<td>Depreciation</td>
<td>no</td>
</tr>
<tr>
<td>Maintenance</td>
<td>no</td>
</tr>
<tr>
<td>Modernization</td>
<td>no</td>
</tr>
</tbody>
</table>

### Notes:

- **Schedule A ("accelerated")**: 8 years 5% of structure costs, capped at $3,000. Thereafter none.
- **Schedule B ("degressive")**: 12 years 3.5% of structure costs, 20 years 2%, finally 10 years 1%. Only applicable for new buildings.
- **Schedule C ("linear")**: 50 years 2%. No restrictions.
- **Imputed Rent**: Effectively ca. 1% of structure costs.
second unit to a family member. Not too surprisingly, this "fake two-family house" became very popular (1977: 17.3 percent on national scale, 13.0 percent in our sample, that is about 40 percent of the one or two unit owner-occupied houses) until the loop-hole was closed. This, too, is the main reason to include two-family homes into the same category as single-family homes in the definition of the eight housing alternatives: apart from the required extra kitchen and bath, the "fake two-family home" is virtually indistinguishable from a single-family house.

Tax advantages from homeownership are much more favorable in the United States apart from this odd loop-hole for two-family homes in Germany. Table 5-4 lists the value of the tax savings from single-family homeownership in both countries for various income levels, house values, and equity ratios in the first few years of a new home. Note, that the German tax write-offs depend on the value of the structure only, not on the value of the land, and that they are independent of the loan-to-value ratio, tilting the symmetry between borrowing and lending (interest income is taxable in Germany). The tax savings in Germany do not increase strictly monotonically with value due to the cap on the accelerated depreciation schedule long before the degressive schedule becomes attractive.

A final note concerns the subsidy given to renters in form of housing allowances ("Wohngeld"). They depend on the rent paid, income, and family-size similar to the housing gap formula applied in Chapter Three. For a couple with two children paying the median rent
### TABLE 5-4: YEARLY TAXSAVINGS FROM OWNER-OCCUPANCY IN GERMANY VS. USA

<table>
<thead>
<tr>
<th>EQUITY: 50%, STRUCTURE: 80%</th>
<th>Value</th>
<th>Yearly Gross Income in 1977 Dollars:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ 1000</td>
<td>8,000</td>
</tr>
<tr>
<td></td>
<td>GER: USA:</td>
<td>GER: USA:</td>
</tr>
<tr>
<td>30.0</td>
<td>$ 260</td>
<td>$ 260</td>
</tr>
<tr>
<td>50.0</td>
<td>440</td>
<td>440</td>
</tr>
<tr>
<td>80.0</td>
<td>540</td>
<td>690</td>
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<tr>
<td>120.0</td>
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<tr>
<td>160.0</td>
<td>540</td>
<td>700</td>
</tr>
<tr>
<td>200.0</td>
<td>540</td>
<td>880</td>
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</table>

<table>
<thead>
<tr>
<th>EQUITY: 50%, STRUCTURE: 60%</th>
<th>Value</th>
<th>Yearly Gross Income in 1977 Dollars:</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>$ 1000</td>
<td>8,000</td>
</tr>
<tr>
<td></td>
<td>GER: USA:</td>
<td>GER: USA:</td>
</tr>
<tr>
<td>30.0</td>
<td>$ 200</td>
<td>$ 200</td>
</tr>
<tr>
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<tr>
<td>200.0</td>
<td>540</td>
<td>690</td>
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<table>
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<th>Value</th>
<th>Yearly Gross Income in 1977 Dollars:</th>
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<tr>
<td></td>
<td>GER: USA:</td>
<td>GER: USA:</td>
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<td>$ 260</td>
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<tr>
<td>80.0</td>
<td>540</td>
<td>690</td>
</tr>
<tr>
<td>120.0</td>
<td>540</td>
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<tr>
<td>160.0</td>
<td>540</td>
<td>700</td>
</tr>
<tr>
<td>200.0</td>
<td>540</td>
<td>880</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EQUITY: 20%, STRUCTURE: 60%</th>
<th>Value</th>
<th>Yearly Gross Income in 1977 Dollars:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ 1000</td>
<td>8,000</td>
</tr>
<tr>
<td></td>
<td>GER: USA:</td>
<td>GER: USA:</td>
</tr>
<tr>
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<td>$ 200</td>
<td>$ 200</td>
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<tr>
<td>50.0</td>
<td>330</td>
<td>330</td>
</tr>
<tr>
<td>80.0</td>
<td>540</td>
<td>530</td>
</tr>
<tr>
<td>120.0</td>
<td>540</td>
<td>690</td>
</tr>
<tr>
<td>160.0</td>
<td>540</td>
<td>690</td>
</tr>
<tr>
<td>200.0</td>
<td>540</td>
<td>690</td>
</tr>
</tbody>
</table>
in an urban area the income cap is at $7,000 yearly after-tax income. At an income of $4,000, about 60 percent of the rent will be reimbursed. In 1977, 7.4 percent of all households in West Germany received housing allowances.

Thus, all things considered, the tax and subsidy structure in Germany is much less favorable to homeownership than in the United States.
5.4 Specification of the Demand Equations

In Table 5-1, we already introduced the eight housing alternatives of the dependent variable:

- **O_S0_14**: Owner-Occupied, Single or Two-Family Home, 1-4 Rooms
- **O_S0_5+**: Owner-Occupied, Single or Two-Family Home, 5+ Rooms
- **O_MF_14**: Owner-Occupied, Multi-family Structure, 1-4 Rooms
- **O_MF_5+**: Owner-Occupied, Multi-family Structure, 5+ Rooms
- **R_S0_14**: Rental Housing, Single or Two-Family Home, 1-4 Rooms
- **R_S0_5+**: Rental Housing, Single or Two-Family Home, 5+ Rooms
- **R_MF_14**: Rental Housing, Multi-family Structure, 1-4 Rooms
- **R_MF_5+**: Rental Housing, Multi-family Structure, 5+ Rooms

They are generated by three dimensions of choice (tenure, structure type in number of units, dwelling size in number of rooms) and a simple binary set of possibilities in each choice. This symmetric set-up allows us to estimate a variety of functional forms: there are six ways to order the three dimensions in a three stage hierarchical choice model. The six trees are depicted in Figure 5-5 and are denoted by the order of the dimensions (T = tenure choice, U = choice of number of units, S = size choice). In addition to the six trees, we will estimate the simple multinomial logit model.

We use a similar set of explanatory variables as we already introduced in Chapter Three:

1. **YP**: Permanent income before taxes,
2. **HEDON**: Hedonic index as a measure of user cost,
3. **RETURN**: Return from equity minus its opportunity costs,
4. **CROWDS**: Squared deviation of optimal from actual dwelling size,
5. **AGE**: Age of household head.

Most of the differences to Chapter Three are motivated by making the cross country comparison as meaningful as possible. Unfortunately, the German survey only reports net income which is endogenous in this
FIGURE 5-5: DECISION TREES AND HOUSING ALTERNATIVES

T-U-S:
======

- Own
- Rent

O_SF_14  O_SF_5+  O_MF_14  O_MF_5+  R_SF_14  R_SF_5+  R_MF_14  R_MF_5+

T-S-U:
======

- Own
- Rent

1-4 Rooms  5+ Rooms  1-4 Rooms  5+ Rooms
O_SF_14  O_MF_14  O_SF_5+  O_MF_5+  R_SF_14  R_MF_14  R_SF_5+  R_MF_5+

U-S-T:
======

- Single-Fam.
- Multi-Fam.

1-4 Rooms  5+ Rooms  1-4 Rooms  5+ Rooms
O_SF_14  R_SF_14  O_SF_5+  R_SF_5+  O_MF_14  R_MF_14  O_MF_5+  R_MF_5+

U-T-S:
======

- Single-Fam.
- Multi-Fam.

Owner-Occ.  Rental  Owner-Occ.  Rental
O_SF_14  O_SF_5+  R_SF_14  R_SF_5+  O_MF_14  O_MF_5+  R_MF_14  R_MF_5+

S-T-U:
======

- 1-4 Rooms
- 5+ Rooms

Owner-Occ.  Rental  Owner-Occ.  Rental
O_SF_14  O_MF_14  R_SF_14  R_MF_14  O_SF_5+  O_MF_5+  R_SF_5+  R_MF_5+

S-U-T:
======

- 1-4 Rooms
- 5+ Rooms

O_SF_14  R_SF_14  O_MF_14  R_MF_14  O_SF_5+  R_SF_5+  O_MF_5+  R_MF_5+
housing demand equation because after-tax income depends on the housing choice and its implications on tax deductions. Gross income is calculated using stepwise polynomial approximations of the tax schedule, given imputed tax deductions from homeownership and average other deductions by age, household size, social status, and other demographic variables. This necessary calculation, though made as careful as possible, is rough and noisy.

The use of permanent income rather than current income reflects the common wisdom (Quigley, 1979) that housing decisions are long run decisions and should not be influenced by transitory income fluctuations. Cleaning the explanatory variables from transitory components is in particular important when comparing two countries where the transitory components are more likely to be different. In particular, it may help to minimize the noise introduced by the calculation of gross income in West Germany. Section 5.6 will discuss the estimation of permanent income in both countries. Because income varies by household, but is not alternative specific, it has to interact with a set of dummies. Instead of using a full set of seven alternative specific dummies, we exploit the symmetric tree structure and let income interact with a dummy for each dimension of choice.

Age of the household head is included in the equation over and above the stratification of the sample into age categories to take account of the differences in the life cycle pattern of housing consumption within each stratum, discovered in Table 5-2. Age enters the equation linearly and quadratically, and both variables interact
with the dummies for each dimension of choice in the same way income does.

We use two price variables, one measuring the out-of-pocket user cost, and the other measuring the less tangible return from equity invested in an owner-occupied home. The latter variable is defined as

\[
(5.1) \quad \text{RETURN} = (\text{APPR} - \text{TBILLR} \times (1 - \text{MARGTR}) \times (1 - \text{LOANRA})) \times \text{VALUE}
\]

where
- APPR: rate of appreciation
- TBILLR: alternative interest rate
- MARGTR: marginal tax rate of household
- LOANRA: loan-to-value ratio

Note that the capital gains accrued by appreciation are not taxed, whereas the alternative investment (in the U.S.: in treasury bills, in Germany in "festverzinsliche Wertpapiere") is subject to personal income tax. This reflects the tax treatment of home sales in both countries where the capital gains essentially evade taxation.

To compute user cost, we could in principle proceed as in Chapter Three. However, the data provided in both surveys is quite different, in particular, house values have to be imputed from external information because they are not reported in the German survey. Therefore, it is likely that user cost of homeownership are not comparable across the two countries, and we rather use a hedonic index. Furthermore, this allows us to include condominiums into the choice set. Condominiums are an important housing alternative in West Germany (5.5 percent in the sample) and Boston (5.1 percent in the sample), less so in the other U.S. SMSA's (Dallas 0.6 percent, Los Angeles 1.6 percent, Minneapolis/St.Paul 0.8 percent). A discussion
of the hedonic estimation follows in the next section.

As a second demographic variable, we introduce a variable to measure household size which boils down to the number of children in our sample of married couples. As most useful a variable turned out which measures the crowding of a dwelling in terms of the deviation of an optimal from the actual dwelling size:

\[ \text{CROWDS} = (\text{NROPT} - \text{NRACT})^2 \]

where \( \text{NRACT} \): actual number of rooms
\( \text{NROPT} \): optimal number of rooms
\( \text{NROPT} = \text{NADULTS} + 1 + (\text{NKIDS} - 1)/2 \)
\( \text{NADULTS} \): number of adults in household
\( \text{NADULTS} \): number of children in household

Finally, the attributes of hypothetical, i.e., not chosen alternatives are imputed as sample averages evaluated at the household's housing independent characteristics. The treatment differs from that in Chapter Three: we keep the chosen number of rooms constant when the household hypothetically moves to another tenure or structure category. This way, we can more clearly separate the choices on each level of the tree.

We did a fair number of experiments with alternative specifications for the exogenous variables in the chosen and hypothetical alternatives, the functional form of the decision trees, and the number of alternatives. Some sensitivity results are reported in Section 5.7.
5.5 Hedonic Price Indices

Hedonic rent indexes are estimated from the renter subsample in each country, then evaluated at each dwelling. This yields an imputed rent for owner-occupied homes and condominiums, where no value and out-of-pocket cost data is available in both countries. Methodological underpinnings of hedonic estimation can be found in Lancaster (1966), Rosen (1974), Muellbauer (1974), and Murray (1978). Mean gross rents and hedonic rents are reported at the bottom of Table 5-2. In both countries, we use a semilogarithmic functional form to accommodate interaction terms but avoid using nonlinear specifications.

5.5.1 United States

In the United States, we mimic the specification of Malpezzi, Ozanne, and Thibodeau (1980), in the following denoted by KOT. The estimates are readjusted for our sample and the sampling period (1977-78 instead of 1974-75), and the specification was slightly changed to be applicable for both renter and owner-occupied units. We estimated four sets of coefficients, one for each SMSA, to accommodate regional differences in the housing stock and its evaluation in each geographical region due to, e.g., climatic differences, as well as in the general level of rents due to market conditions.
The estimated coefficients are reported in Table 5-6. Very little changes compared to the original MOT results. The number of rooms has the expected significance as the most important quality measure. Structure type performs weakly, as it does in the MOT estimations for 1974-76, and, as we will see, in the German hedonic regressions. Of the building attributes, the informal ratings are unreliable. Age enters in a S-shaped nonlinear fashion. The rental unit depreciates fast in the first years after construction; this rate slows down after the initial period; but increases again when the building becomes very old. Interestingly, the pattern is different in Boston where old buildings even have a positive contribution to the hedonic value. Almost all dwelling attributes have their expected signs or are insignificant.

We measure strong tenure discounts with a concave relationship to the length of tenure. Ten years after moving in, tenants receive a discount of 22.1 percent in Boston, 16.1 percent in Dallas, 24.5 percent in Los Angeles, and 10.7 percent in Minneapolis/St.Paul, compared to those of MOT quoted in Table 4-2. The presence of the landlord in the building implies a significant (at the 90 percent level) rent reduction in all SMSA's except Minneapolis/St.Paul, which supports our reasoning in Section 4.2.2. Tenant characteristics may proxy neighborhood characteristics not picked up in the coefficients at the bottom of Table 5-6. Finally, we control for inclusions in the gross rent and inflation during the lengthy interview period.

The Boston rental market emerges as distinctly different from the
TABLE 5-6  HEDONIC REGRESSION COEFFICIENTS (USA)

<table>
<thead>
<tr>
<th>VARIABLE:</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN/ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>0.90199</td>
<td>0.94663</td>
<td>1.33857</td>
<td>0.83656</td>
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(1) NUMBER OF ROOMS:

<table>
<thead>
<tr>
<th>NUMBER OF ROOMS</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN/ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+1/2 BATHS</td>
<td>0.13285</td>
<td>0.10007</td>
<td>0.11486</td>
<td>0.12840</td>
</tr>
<tr>
<td>TWO BATHS</td>
<td>0.21148</td>
<td>0.16076</td>
<td>0.11136</td>
<td>0.13190</td>
</tr>
<tr>
<td>THREE BATHS</td>
<td>0.50431</td>
<td>0.37019</td>
<td>0.21600</td>
<td>0.11278</td>
</tr>
<tr>
<td>ONE ROOM</td>
<td>-0.10873</td>
<td>-0.16168</td>
<td>-0.14698</td>
<td>-0.14291</td>
</tr>
<tr>
<td>TWO ROOMS</td>
<td>-0.05875</td>
<td>-0.12015</td>
<td>-0.10482</td>
<td>-0.04869</td>
</tr>
<tr>
<td>FOUR ROOMS</td>
<td>0.07249</td>
<td>0.05947</td>
<td>0.20443</td>
<td>0.04485</td>
</tr>
<tr>
<td>FIVE+ ROOMS</td>
<td>0.02473</td>
<td>-0.01625</td>
<td>0.04412</td>
<td>0.06999</td>
</tr>
<tr>
<td>NO BEDROOM</td>
<td>-0.21798</td>
<td>-0.20446</td>
<td>-0.27271</td>
<td>-0.21347</td>
</tr>
<tr>
<td>TWO BEDROOMS</td>
<td>0.11596</td>
<td>0.12006</td>
<td>0.19344</td>
<td>0.13746</td>
</tr>
<tr>
<td>THREE BEDROOMS</td>
<td>0.22820</td>
<td>0.26933</td>
<td>0.35732</td>
<td>0.29278</td>
</tr>
<tr>
<td>FOUR+ BEDROOMS</td>
<td>0.06679</td>
<td>0.08153</td>
<td>0.10280</td>
<td>0.06839</td>
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</tbody>
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(2) STRUCTURE TYPE:

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<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN/ST.P.:</th>
</tr>
</thead>
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<tr>
<td>ONE FAM., ATT.</td>
<td>-0.00398</td>
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<td>-0.07958</td>
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<tr>
<td>ONE FAM., DET.</td>
<td>0.09331</td>
<td>0.01735</td>
<td>-0.01601</td>
<td>0.08406</td>
</tr>
<tr>
<td>DUPLEX</td>
<td>0.03736</td>
<td>0.05303</td>
<td>-0.00146</td>
<td>0.03988</td>
</tr>
<tr>
<td>FIFTY+ UNITS</td>
<td>0.06505</td>
<td>-0.02114</td>
<td>0.09605</td>
<td>0.05409</td>
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</table>

(3) ATTRIBUTES OF BUILDING:

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<tr>
<th>ATTRIBUTE</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN/ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUILDING AGE</td>
<td>0.03099</td>
<td>-0.02530</td>
<td>-0.03452</td>
<td>-0.02873</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>-0.00262</td>
<td>0.00101</td>
<td>0.00170</td>
<td>0.00135</td>
</tr>
<tr>
<td>AGE CUBED</td>
<td>0.00054</td>
<td>-0.00012</td>
<td>-0.00030</td>
<td>-0.00024</td>
</tr>
<tr>
<td>PRIOR 1940</td>
<td>0.02558</td>
<td>-0.22387</td>
<td>-0.35393</td>
<td>-0.28725</td>
</tr>
<tr>
<td>ELEVATOR</td>
<td>0.12151</td>
<td>0.37430</td>
<td>0.10561</td>
<td>0.10681</td>
</tr>
<tr>
<td>BAD HALLWAY</td>
<td>-0.00373</td>
<td>-0.04854</td>
<td>0.02274</td>
<td>0.00021</td>
</tr>
<tr>
<td>LEAKS, CRACKS...</td>
<td>0.00888</td>
<td>-0.00778</td>
<td>0.00075</td>
<td>0.00403</td>
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(4) ATTRIBUTES OF DWELLING:

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<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN/ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROOM HEATER</td>
<td>-0.07710</td>
<td>-0.03693</td>
<td>-0.04243</td>
<td>-0.05421</td>
</tr>
<tr>
<td>STEAM HEAT</td>
<td>0.00216</td>
<td>0.02376</td>
<td>-0.08911</td>
<td>-0.01782</td>
</tr>
<tr>
<td>ELECTRIC HEAT</td>
<td>-0.00169</td>
<td>-0.03554</td>
<td>0.01642</td>
<td>0.00553</td>
</tr>
<tr>
<td>ROOM AIRCOND.</td>
<td>0.04912</td>
<td>0.15800</td>
<td>0.01314</td>
<td>0.03727</td>
</tr>
<tr>
<td>CENTRAL AIRCO.</td>
<td>0.17119</td>
<td>0.09890</td>
<td>0.10926</td>
<td>0.08094</td>
</tr>
<tr>
<td>NO RADIATORS</td>
<td>-0.03860</td>
<td>-0.02579</td>
<td>-0.05037</td>
<td>-0.02954</td>
</tr>
<tr>
<td>POOR PLUMBING</td>
<td>-0.15616</td>
<td>-0.16900</td>
<td>-0.13973</td>
<td>-0.07503</td>
</tr>
<tr>
<td>NO PRIVACY</td>
<td>-0.04803</td>
<td>-0.04793</td>
<td>-0.06610</td>
<td>-0.09208</td>
</tr>
<tr>
<td>NO OUTLETS</td>
<td>-0.06982</td>
<td>-0.17764</td>
<td>-0.06638</td>
<td>0.00217</td>
</tr>
<tr>
<td>COOK WITH ELEC.</td>
<td>0.06503</td>
<td>0.07872</td>
<td>-0.00012</td>
<td>0.05554</td>
</tr>
</tbody>
</table>
### TABLE 5-6 HEDONIC REGRESSION COEFFICIENTS (CONT'D)

<table>
<thead>
<tr>
<th>VARIABLE:</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN/ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5) TENURE DISCOUNTS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LENGTH OF TEN.</td>
<td>-0.02841 *</td>
<td>-0.02671 *</td>
<td>-0.03469 *</td>
<td>-0.01335 *</td>
</tr>
<tr>
<td>L.O.T. SQUARED</td>
<td>0.00063 *</td>
<td>0.00106 *</td>
<td>0.00101 *</td>
<td>0.00027</td>
</tr>
<tr>
<td>MOVE PRIOR 1950</td>
<td>-0.24097 *</td>
<td>-0.18788 +</td>
<td>-0.30103 *</td>
<td>-0.37291 *</td>
</tr>
<tr>
<td>LANDLDR PRESENT</td>
<td>-0.03875 +</td>
<td>-0.05625 -</td>
<td>-0.04171 +</td>
<td>0.00118</td>
</tr>
<tr>
<td>(6) TENANT CHARACTERISTICS:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BLACK</td>
<td>-0.02188</td>
<td>-0.15769 *</td>
<td>-0.17172 *</td>
<td>0.04533 -</td>
</tr>
<tr>
<td>SPANISH</td>
<td>-0.09366 *</td>
<td>-0.06463 +</td>
<td>-0.14289 *</td>
<td>0.07447</td>
</tr>
<tr>
<td>CHINESE</td>
<td>0.04310</td>
<td>-0.07988 -</td>
<td>-0.07150 -</td>
<td>-0.03224</td>
</tr>
<tr>
<td>PERSONS/ROOM</td>
<td>0.04012 -</td>
<td>0.05058 +</td>
<td>0.04329 +</td>
<td>0.09127 *</td>
</tr>
<tr>
<td>(7) CONTROLS FOR GROSS RENT:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UTILITIES INCL.</td>
<td>0.20532 *</td>
<td>0.06374 *</td>
<td>0.11750 *</td>
<td>0.18198 *</td>
</tr>
<tr>
<td>HEAT INCL.</td>
<td>-0.00491</td>
<td>0.34012 +</td>
<td>0.23630 -</td>
<td>0.20027 +</td>
</tr>
<tr>
<td>PARKING INCL.</td>
<td>0.10299 -</td>
<td>0.03587</td>
<td>0.03850</td>
<td>0.13858 *</td>
</tr>
<tr>
<td>FURNITURE INCL.</td>
<td>-0.01085</td>
<td>0.00889</td>
<td>-0.00394 -</td>
<td>-0.03753 -</td>
</tr>
<tr>
<td>INTERVIEW DATE</td>
<td>-0.00086</td>
<td>-0.00220</td>
<td>-0.00414 +</td>
<td>-0.00120</td>
</tr>
<tr>
<td>DATE * HEAT INCL</td>
<td>0.00395 -</td>
<td>0.01973</td>
<td>0.00747</td>
<td>-0.00695</td>
</tr>
<tr>
<td>(8) NEIGHBORHOOD CHARACTERISTICS:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXCELLENT</td>
<td>0.07622 *</td>
<td>0.03738 -</td>
<td>0.10468 *</td>
<td>0.09294 *</td>
</tr>
<tr>
<td>GOOD</td>
<td>0.04408 *</td>
<td>0.00768</td>
<td>0.04550 *</td>
<td>0.05350 *</td>
</tr>
<tr>
<td>POOR</td>
<td>-0.02213</td>
<td>-0.06031 -</td>
<td>0.00591</td>
<td>-0.02192</td>
</tr>
<tr>
<td>ABANDONED STRUC</td>
<td>-0.07983 *</td>
<td>0.05323 -</td>
<td>-0.06336 +</td>
<td>-0.00985</td>
</tr>
<tr>
<td>LITTER</td>
<td>0.01417</td>
<td>-0.02959 -</td>
<td>-0.01269 -</td>
<td>-0.01083</td>
</tr>
<tr>
<td>NO SHOPS</td>
<td>0.02936 -</td>
<td>-0.00515</td>
<td>-0.02996 -</td>
<td>-0.01104</td>
</tr>
<tr>
<td>CENTER CITY</td>
<td>-0.02243 -</td>
<td>0.01051</td>
<td>0.04379 *</td>
<td>0.05219 *</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>GROSS RENT/MONTH: $ 225</th>
<th>$ 190</th>
<th>$ 198</th>
<th>$ 201</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN DEP. VARIABLE</td>
<td>0.995</td>
<td>0.825</td>
<td>0.863</td>
</tr>
<tr>
<td>STANDARD DEV.:</td>
<td>0.366</td>
<td>0.397</td>
<td>0.412</td>
</tr>
<tr>
<td>STANDARD ERROR:</td>
<td>0.262</td>
<td>0.244</td>
<td>0.263</td>
</tr>
<tr>
<td>OBSERVATIONS:</td>
<td>1714</td>
<td>1368</td>
<td>1818</td>
</tr>
<tr>
<td>R-SQUARED:</td>
<td>0.509</td>
<td>0.636</td>
<td>0.606</td>
</tr>
</tbody>
</table>

* = SIGNIFICANT AT 99%
+ = SIGNIFICANT AT 95%
- = SIGNIFICANT AT 80%
other three SMSA's: the mean monthly rent is considerably higher; the explained fraction of rent variation lower; and the age pattern, discussed above, is reversed. This is consistent with the MOT findings and parallels the other SMSA's in the North-East considered in their investigation.
5.5.2 West Germany

For West Germany, we use the hedonic regression results by Behring, Goldrian, et. al. (1983) reported in Table 5-7. These indexes vary by city center versus suburbs, but are pooled across West Germany. To make up for intercity differences, we calculated average rents by housing alternative for each city and adjusted the level of the hedonic index accordingly. The variables used are similar to those in the specification of Malpezzi, Ozanne, and Thibodeau (1980).

The sample for the estimation is huge and produces very large t-statistics. As in the U.S. estimates, number of rooms has the strongest impact on hedonic values. For a given number of rooms, the average size of the rooms contributes positively to the rent. This variable is measured as the deviation of average room size from the standard room size of 135 square feet.

The coefficients for structure type are weak, and the negative value for two-family structures may indicate the German peculiarity of the "fake two-family homes" where the rent in the second unit is substantially reduced in some form of gentlemens' agreement as mentioned in Section 5.3. Note, however, that Behring and Goldrian excluded a unit from the sample if the tenant did not pay rent at all or explicitly stated that his rent was reduced due to an agreement with the landlord.

The age of the building is measured linearly and enters
TABLE 5-7 HEDONIC REGRESSION COEFFICIENTS (GERMANY)

Dependent Variable: Logarithm of Monthly Rent Net of Utilities

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CITY CENTERS</th>
<th>SUBURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>5.451 (402.90)</td>
<td>5.451 (248.93)</td>
</tr>
</tbody>
</table>

(1) NUMBER OF ROOMS:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CITY CENTERS</th>
<th>SUBURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE ROOM</td>
<td>-0.289 (47.08)</td>
<td>-0.320 (24.24)</td>
</tr>
<tr>
<td>THREE ROOMS</td>
<td>0.215 (51.06)</td>
<td>0.212 (27.41)</td>
</tr>
<tr>
<td>FOUR ROOMS</td>
<td>0.389 (69.60)</td>
<td>0.367 (38.85)</td>
</tr>
<tr>
<td>FIVE+ ROOMS</td>
<td>0.103 (79.98)</td>
<td>0.094 (44.52)</td>
</tr>
<tr>
<td>MEAN ROOM SIZE</td>
<td>0.0325 (79.68)</td>
<td>0.0317 (43.94)</td>
</tr>
<tr>
<td>NO KITCHEN</td>
<td>-0.261 (40.14)</td>
<td>-0.320 (27.02)</td>
</tr>
<tr>
<td>NO BATH</td>
<td>-0.172 (29.56)</td>
<td>-0.129 (13.12)</td>
</tr>
<tr>
<td>TWO+ BATHROOMS</td>
<td>0.114 (20.00)</td>
<td>0.184 (20.69)</td>
</tr>
</tbody>
</table>

(2) STRUCTURE TYPE:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CITY CENTERS</th>
<th>SUBURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ONE FAMILY HOME</td>
<td>0.005 (0.49)</td>
<td>0.071 (5.45)</td>
</tr>
<tr>
<td>TWO FAMILY HOME</td>
<td>-0.082 (10.11)</td>
<td>-0.065 (7.99)</td>
</tr>
<tr>
<td>5+ F., 4- FLOORS</td>
<td>0.029 (5.58)</td>
<td>0.070 (8.79)</td>
</tr>
<tr>
<td>5+ F., 5+ FLOORS</td>
<td>0.059 (10.29)</td>
<td>0.150 (11.68)</td>
</tr>
</tbody>
</table>

(3) ATTRIBUTES OF BUILDING:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CITY CENTERS</th>
<th>SUBURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUILDING AGE</td>
<td>-0.0082 (21.81)</td>
<td>-0.0075 (12.54)</td>
</tr>
<tr>
<td>PRIOR 1949</td>
<td>-0.330 (41.93)</td>
<td>-0.239 (19.13)</td>
</tr>
<tr>
<td>GENTRIFIED</td>
<td>-0.046 (17.86)</td>
<td>-0.039 (6.22)</td>
</tr>
</tbody>
</table>

(4) ATTRIBUTES OF DWELLING:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>CITY CENTERS</th>
<th>SUBURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTRAL HEAT</td>
<td>0.216 (54.62)</td>
<td>-0.163 (22.72)</td>
</tr>
<tr>
<td>WARM WATER</td>
<td>0.137 (17.81)</td>
<td>-0.045 (3.13)</td>
</tr>
<tr>
<td>BALCONY</td>
<td>0.037 (24.81)</td>
<td>0.065 (14.39)</td>
</tr>
<tr>
<td>DOUBLE WINDOWS</td>
<td>0.007 (1.81)</td>
<td>0.031 (4.64)</td>
</tr>
</tbody>
</table>
### TABLE 5-7 HEDONIC REGRESSION COEFFICIENTS (CONT'D)

<table>
<thead>
<tr>
<th>VARIABLE:</th>
<th>CITY CENTERS</th>
<th>SUBURBS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5) TENURE DISCOUNTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LENGTH OF TENURE</td>
<td>-0.020 (8.20)</td>
<td>-0.022 (5.59)</td>
</tr>
<tr>
<td>L.O.T. SQUARED</td>
<td>0.0007 (3.32)</td>
<td>0.0007 (2.06)</td>
</tr>
<tr>
<td>MOVE PRIOR 1955</td>
<td>-0.189 (30.70)</td>
<td>-0.249 (21.44)</td>
</tr>
<tr>
<td>LANDL.D PRESENT</td>
<td>-0.127 (31.99)</td>
<td>-0.139 (15.85)</td>
</tr>
<tr>
<td>(6) TENANT CHARACTERISTICS:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GUESTWORKER</td>
<td>0.008 (1.14)</td>
<td>-0.011 (0.98)</td>
</tr>
<tr>
<td>(7) CONTROLS FOR GROSS RENT:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMPANY DWELLING</td>
<td>-0.083 (10.20)</td>
<td>-0.062 (4.51)</td>
</tr>
<tr>
<td>(8) NEIGHBORHOOD CHARACTERISTICS:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOISE</td>
<td>-0.005 (2.10)</td>
<td>-0.018 (4.51)</td>
</tr>
<tr>
<td>PUBLIC TRANSP.</td>
<td>0.022 (5.95)</td>
<td>0.016 (3.76)</td>
</tr>
<tr>
<td>SHOPPING</td>
<td>0.003 (1.28)</td>
<td>-0.002 (0.57)</td>
</tr>
<tr>
<td>PUBLIC PARKS</td>
<td>0.015 (6.86)</td>
<td>0.009 (2.11)</td>
</tr>
<tr>
<td>NET NORMAL RENT:</td>
<td>$100</td>
<td>$100</td>
</tr>
<tr>
<td>OBSERVATIONS:</td>
<td>31207</td>
<td>11952</td>
</tr>
<tr>
<td>R-SQUARED:</td>
<td>0.692</td>
<td>0.665</td>
</tr>
</tbody>
</table>

* t-statistics in brackets.
negatively, approaching a discount of a third of the normal rent in city centers, and of a quarter in suburbs for pre-1949 structures. An indicator variable for gentrification obviously measures the cause rather than the effect because it depresses the rent significantly. The tenure discounts are those reported in Table 4-1. In addition, the landlord's presence in the building has a significant influence on the rent, reducing it more than 12 percent. An indicator variable for guestworkers is included to test for discrimination, however, this variable can not be measured with any precision. Finally, a group of neighborhood variables is included with the expected sign pattern, but weak t-statistics.

The interviews in the One Percent Sample were conducted within a single week, so no adjustment for inflation was necessary. There are very little differences in the valuation of the hedonic attributes between city centers and suburbs. Buildings depreciate faster in cities, highrises add a substantially higher amount to the hedonic value in suburbs. Unfortunately, Behring and Goldrian do not report the means of the dependent variable. However, the explanatory variables are constructed in a way that the constant measures "normal rent" of a standard German rental unit. This rent is the monthly rent net of utilities and is approximately $100 for suburbs and city centers, which is considerably higher than the average in non-urban areas, $77 per month. For the demand equation in Section 5.7, we will add utility bills to this number as reported in Behring and Goldrian (1983). The average of this gross rent in our sample is reported at the bottom of Table 5-2.
5.5.3 Comparison

Using the same functional specification and similar explanatory variables, we can make a meaningful comparison between the results of the hedonic estimations in both countries. Note first the stunning difference in the rent levels as already pointed out in Table 5-2. The regression fit is somewhat tighter in the German estimation in spite of a more parsimonious specification which might be due to a smaller spread of rents in Germany. In both countries, the hedonic regressions do not discriminate well among structure types which indicates some trouble in predicting hedonic rents for owner-occupied single or two-family homes and comparing hedonic rents across tenure. For this reason, we will use the hedonic rents in the demand equations only interacting with a tenure choice dummy.

We already compared tenure discounts in Chapter Four. We also refer to this chapter for a discussion of measurement problems pertaining to this variable. The discounts for a private landlord are substantial in Germany. Many of these landlords live in the building they rented out. If we compare the discounts with the landlord-present-discounts measured for the American SMSA's, we may conclude that the relationship between landlord and tenant is more idiosyncratic as compared to the United States.
5.6 Permanent Income Estimates

As we argued in Section 5.3, the decision for a durable commodity like housing should depend on transitory income components. Thus, we use the following age specific permanent income

\[ Y_{curr} = Y_{perm}(age, \text{human and nonhuman wealth}) + Y_{trans} \]

where the transitory income is defined as the residual of a regression of current income on the determinants of permanent income, thus uncorrelated to the latter. In both countries, we face data problems to measure all the components determining permanent income. The estimation heavily relies on the puritan proposition that human and nonhuman wealth are strongly correlated, and missing data in one of the wealth categories is predicted by the other. However, structural interpretation of the estimated coefficients should be only made with care.

5.6.1 United States

The Annual Housing Survey has very little information on human wealth as measured by professional status, social position, or job training. The only variable in this direction is the household head's education given by the highest degree received. The survey, however, has some information on non-human wealth in form of indicator variables of asset income, rental income, the number of cars and trucks, and the possession of a second home. To accommodate interaction patterns, we stratify the estimation by age groups and use
a semilogarithmic form. To cope with inter-city differences, we estimate each set of regressions for each city separately. The results are presented in Table 5-8.

The regression fits are fairly low, most satisfactory in the elderly strata. Average gross income varies considerably among strata, reflecting life cycle earning patterns, and among SMSA's, with Minneapolis/St.Paul at the top and Boston at the bottom of the income scale. Only for the youngest stratum, Los Angeles has the lowest average income. The cross sectional variation in income is increasing with age in all strata, lowest in Minneapolis/St.Paul.

Not surprisingly, the employment status in the week of the interview has in almost all regressions a large and significant influence on permanent income. Similarly, the impact of additional wage and salary incomes in the household is positive, though measured with less precision. Among the coefficients of non-human wealth, the presence of assets and the number of cars play the most important role to predict permanent income. They are highly significant and of large magnitude in every regression. The presence of rental income has a negative impact which may be explained by the lack of other income sources.

The household characteristics perform very weakly. This is partially due to the stratification by age which takes care of most of the age generated income variation. In the pooled sample, all age variables are highly significant. The number of dependents is of
TABLE 5-8: PERMANENT INCOME ESTIMATION (USA)

Dependent Variable: Yearly Gross Income in 1000 Dollars

MARRIED COUPLES, HEAD AGED UNDER 35

<table>
<thead>
<tr>
<th>VARIABLE:</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>-0.80973</td>
<td>1.59981</td>
<td>0.14282</td>
<td>-0.42008</td>
</tr>
</tbody>
</table>

(1) INCOME SOURCES:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMPLOYED</td>
<td>0.51998 *</td>
<td>0.17746</td>
<td>0.19068</td>
<td>0.25783 *</td>
</tr>
<tr>
<td>RETIRED</td>
<td>-0.03709</td>
<td>0.09544</td>
<td>0.05716</td>
<td>0.26918 *</td>
</tr>
<tr>
<td>NUMBER OF WAGES</td>
<td>0.12955</td>
<td>0.05445</td>
<td>0.19286</td>
<td>0.05011</td>
</tr>
</tbody>
</table>

(2) NONHUMAN WEALTH:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASSETS PRESENT</td>
<td>0.19084 *</td>
<td>0.13198 *</td>
<td>0.14937 *</td>
<td>0.12817 *</td>
</tr>
<tr>
<td>RENTAL INCOME</td>
<td>-0.07496</td>
<td>-0.01337</td>
<td>-0.30735</td>
<td>-0.17260 *</td>
</tr>
<tr>
<td>SECOND HOME</td>
<td>0.04524</td>
<td>0.00428</td>
<td>-0.18293</td>
<td>0.15247 -</td>
</tr>
<tr>
<td>NUMBER OF CARS</td>
<td>0.20459 *</td>
<td>0.17218 *</td>
<td>0.16886</td>
<td>0.15266 *</td>
</tr>
<tr>
<td>TRUCKS</td>
<td>0.15078</td>
<td>-0.13096</td>
<td>0.19457</td>
<td>0.08854 *</td>
</tr>
</tbody>
</table>

(3) HOUSEHOLD CHARACTERISTICS:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGE OF HEAD</td>
<td>0.20867</td>
<td>-0.01180</td>
<td>0.08496</td>
<td>0.12986 +</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>-0.00308</td>
<td>0.00037</td>
<td>-0.00125</td>
<td>-0.00203 -</td>
</tr>
<tr>
<td>AGE OF SPOUSE</td>
<td>-0.05026</td>
<td>0.03911</td>
<td>0.06507</td>
<td>- 0.06350 -</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>0.00094</td>
<td>-0.0030</td>
<td>-0.00085</td>
<td>- 0.00076 -</td>
</tr>
<tr>
<td>HEAD FEMALE</td>
<td>-0.53898</td>
<td>-0.27547 +</td>
<td>-0.06889</td>
<td>-0.46801 *</td>
</tr>
<tr>
<td>HEAD SPANISH</td>
<td>-0.29875</td>
<td>-0.13915</td>
<td>-0.16636</td>
<td>0.12459</td>
</tr>
<tr>
<td>DEPENDENTS</td>
<td>-0.07036</td>
<td>-0.02233</td>
<td>-0.03441</td>
<td>-0.05588 *</td>
</tr>
</tbody>
</table>

(4) EDUCATION OF HEAD:

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADE 0-7</td>
<td>-0.17824</td>
<td>-0.24808 +</td>
<td>-0.29891</td>
<td>-0.04451</td>
</tr>
<tr>
<td>GRADE 8</td>
<td>0.15895</td>
<td>-0.21486</td>
<td>-0.19937</td>
<td>-0.14088</td>
</tr>
<tr>
<td>GRADE 9-11</td>
<td>-0.03026</td>
<td>-0.18188 *</td>
<td>-0.22314</td>
<td>-0.19088 +</td>
</tr>
<tr>
<td>TWO Y. COLLEGE</td>
<td>0.14623</td>
<td>-0.00325</td>
<td>0.01650</td>
<td>-0.00028</td>
</tr>
<tr>
<td>FOUR Y. COLL.</td>
<td>0.21300 +</td>
<td>0.17191</td>
<td>0.14929</td>
<td>0.10726 *</td>
</tr>
<tr>
<td>GRAD. SCHOOL</td>
<td>0.25401 *</td>
<td>0.16361 *</td>
<td>0.26262</td>
<td>0.05295</td>
</tr>
</tbody>
</table>

GROSS INCOME: $16,729 $16,413 $14,490 $18,758
MEAN DEP. VAR.: 2.790 2.798 2.673 2.932
STANDARD DEV.: 0.653 0.517 0.622 0.439
STANDARD ERROR: 0.551 0.425 0.488 0.368
OBSERVATIONS: 435 674 493 782
R-SQUARED: 0.323 0.347 0.412 0.316

* = SIGNIFICANT AT 99%, + = SIGNI. AT 95%, - = SIGNIFICANT AT 80%
TABLE 5-8: PERMANENT INCOME ESTIMATION (CONT'D)

Dependent Variable: Yearly Gross Income in 1000 Dollars

MARRIED COUPLES, HEAD AGED 35 TO 50

<table>
<thead>
<tr>
<th>VARIABLE:</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>3.36395</td>
<td>-2.00465</td>
<td>-0.81824</td>
<td>4.66697 *</td>
</tr>
</tbody>
</table>

(1) INCOME SOURCES:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMPLOYED</td>
<td>0.59880 *</td>
<td>0.14077</td>
<td>0.29925 *</td>
<td>0.45921 *</td>
</tr>
<tr>
<td>RETIRED</td>
<td>0.17779 +</td>
<td>-0.01350</td>
<td>0.05632</td>
<td>0.05910</td>
</tr>
<tr>
<td>NUMBER OF WAGES</td>
<td>0.06008 *</td>
<td>0.02934</td>
<td>0.03601 -</td>
<td>0.03577 +</td>
</tr>
</tbody>
</table>

(2) NONHUMAN WEALTH:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASSETS PRESENT</td>
<td>0.20699 *</td>
<td>0.21666 *</td>
<td>0.15666 *</td>
<td>0.11054 *</td>
</tr>
<tr>
<td>RENTAL INCOME</td>
<td>-0.06914</td>
<td>-0.14468 -</td>
<td>-0.16673 -</td>
<td>-0.07303</td>
</tr>
<tr>
<td>SECOND HOME</td>
<td>-0.00404</td>
<td>-0.05242</td>
<td>-0.00915</td>
<td>0.13155 *</td>
</tr>
<tr>
<td>NUMBER OF CARS</td>
<td>0.18490 *</td>
<td>0.13613 *</td>
<td>0.15155 *</td>
<td>0.09574 *</td>
</tr>
<tr>
<td>TRUCKS</td>
<td>0.00954</td>
<td>0.10244 +</td>
<td>0.11271 +</td>
<td>0.07375 +</td>
</tr>
</tbody>
</table>

(3) HOUSEHOLD CHARACTERISTICS:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGE OF HEAD</td>
<td>-0.00953</td>
<td>-0.04111</td>
<td>0.17083 -</td>
<td>-0.13516 +</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>0.00013</td>
<td>0.00049</td>
<td>-0.00200 -</td>
<td>0.00154 -</td>
</tr>
<tr>
<td>AGE OF SPOUSE</td>
<td>-0.05746 -</td>
<td>-0.02400</td>
<td>-0.00396</td>
<td>0.03509</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>0.00070 -</td>
<td>0.00026</td>
<td>0.00009</td>
<td>-0.00041</td>
</tr>
<tr>
<td>HEAD FEMALE</td>
<td>-0.05133</td>
<td>-0.32602 -</td>
<td>-0.068750 *</td>
<td>-0.26308 -</td>
</tr>
<tr>
<td>HEAD SPANISH</td>
<td>-0.08615</td>
<td>-0.11953</td>
<td>-0.10984 -</td>
<td>0.04132</td>
</tr>
<tr>
<td>DEPENDENTS</td>
<td>-0.00412</td>
<td>-0.02729</td>
<td>-0.00806 -</td>
<td>-0.01937 -</td>
</tr>
</tbody>
</table>

(4) EDUCATION OF HEAD:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADE 0-7</td>
<td>-0.20554 -</td>
<td>-0.49628 *</td>
<td>-0.44321 *</td>
<td>-0.09350</td>
</tr>
<tr>
<td>GRADE 8</td>
<td>-0.13150</td>
<td>-0.06132</td>
<td>-0.22485 -</td>
<td>-0.10471</td>
</tr>
<tr>
<td>GRADE 9-11</td>
<td>-0.31727 *</td>
<td>-0.27937 *</td>
<td>-0.22982 *</td>
<td>-0.03883</td>
</tr>
<tr>
<td>TWO Y. COLLEGE</td>
<td>0.05834</td>
<td>0.00526</td>
<td>0.13788 +</td>
<td>0.10378 +</td>
</tr>
<tr>
<td>FOUR Y. COLL.</td>
<td>0.33085 *</td>
<td>0.16639 *</td>
<td>0.19509 *</td>
<td>0.24956 *</td>
</tr>
<tr>
<td>GRAD. SCHOOL</td>
<td>0.45139 *</td>
<td>0.23079 *</td>
<td>0.24461 *</td>
<td>0.36131 *</td>
</tr>
</tbody>
</table>

GROSS INCOME: $ 20,795   $ 22,727   $ 21,170   $ 24,210
MEAN DEP. VAR.: 3.035   3.124   3.053   3.187
STANDARD DEV.: 0.633   0.648   0.640   0.466
STANDARD ERROR: 0.502   0.571   0.508   0.409
OBSERVATIONS: 566   753   552   818
R-SQUARED: 0.396   0.245   0.396   0.251

* = SIGNIFICANT AT 99%, + = SIGNI. AT 95%, - = SIGNIFICANT AT 80%
TABLE 5-8: PERMANENT INCOME ESTIMATION (CONT'D)

Dependent Variable: Yearly Gross Income in 1000 Dollars

MARRIED COUPLES, HEAD AGED ABOVE 65

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>0.83725</td>
<td>2.89141</td>
<td>3.81292</td>
<td>4.44491</td>
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</tbody>
</table>

(1) INCOME SOURCES:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMPLOYED</td>
<td>0.56220</td>
<td>0.42915</td>
<td>0.52288</td>
<td>0.46670</td>
</tr>
<tr>
<td>RETIRED</td>
<td>-0.03779</td>
<td>-0.16794</td>
<td>-0.00646</td>
<td>-0.00477</td>
</tr>
<tr>
<td>NUMBER OF WAGES</td>
<td>0.10064</td>
<td>0.05139</td>
<td>0.02891</td>
<td>0.05000</td>
</tr>
</tbody>
</table>

(2) NONHUMAN WEALTH:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASSETS PRESENT</td>
<td>0.17069</td>
<td>0.30942</td>
<td>0.22881</td>
<td>0.21248</td>
</tr>
<tr>
<td>RENTAL INCOME</td>
<td>-0.12446</td>
<td>-0.11814</td>
<td>-0.25154</td>
<td>-0.11196</td>
</tr>
<tr>
<td>SECOND HOME</td>
<td>0.07390</td>
<td>0.15351</td>
<td>0.00349</td>
<td>0.07766</td>
</tr>
<tr>
<td>NUMBER OF CARS</td>
<td>0.14855</td>
<td>0.15342</td>
<td>0.20605</td>
<td>0.12445</td>
</tr>
<tr>
<td>TRUCKS</td>
<td>0.06069</td>
<td>0.04032</td>
<td>0.11191</td>
<td>-0.00938</td>
</tr>
</tbody>
</table>

(3) HOUSEHOLD CHARACTERISTICS:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGE OF HEAD</td>
<td>0.02659</td>
<td>0.01631</td>
<td>-0.01261</td>
<td>-0.03608</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>-0.00022</td>
<td>-0.00019</td>
<td>0.00006</td>
<td>0.00015</td>
</tr>
<tr>
<td>AGE OF SPOUSE</td>
<td>0.01521</td>
<td>-0.02337</td>
<td>-0.02787</td>
<td>-0.01469</td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>-0.00012</td>
<td>0.00019</td>
<td>0.00025</td>
<td>-0.00013</td>
</tr>
<tr>
<td>HEAD FEMALE</td>
<td>-0.14404</td>
<td>0.05405</td>
<td>-0.38831</td>
<td>-0.35599</td>
</tr>
<tr>
<td>HEAD SPANISH</td>
<td>-0.20846</td>
<td>-0.39402</td>
<td>-0.05994</td>
<td>-0.03418</td>
</tr>
<tr>
<td>DEPENDENTS</td>
<td>0.01323</td>
<td>-0.03779</td>
<td>-0.03355</td>
<td>-0.03570</td>
</tr>
</tbody>
</table>

(4) EDUCATION OF HEAD:

<table>
<thead>
<tr>
<th></th>
<th>BOSTON:</th>
<th>DALLAS:</th>
<th>LOS ANGELES:</th>
<th>MINN./ST.P.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRADE 0-7</td>
<td>-0.13966</td>
<td>-0.38534</td>
<td>-0.04978</td>
<td>-0.36015</td>
</tr>
<tr>
<td>GRADE 8</td>
<td>-0.09756</td>
<td>-0.22214</td>
<td>-0.14171</td>
<td>-0.22988</td>
</tr>
<tr>
<td>GRADE 9-11</td>
<td>0.00421</td>
<td>-0.25738</td>
<td>-0.11925</td>
<td>-0.15364</td>
</tr>
<tr>
<td>TWO Y. COLLEGE</td>
<td>0.13113</td>
<td>0.09931</td>
<td>0.14367</td>
<td>0.05274</td>
</tr>
<tr>
<td>FOUR Y. COLL.</td>
<td>0.30181</td>
<td>0.12521</td>
<td>0.16261</td>
<td>0.09221</td>
</tr>
<tr>
<td>GRAD. SCHOOL</td>
<td>0.39379</td>
<td>0.21239</td>
<td>0.37758</td>
<td>0.13384</td>
</tr>
</tbody>
</table>

GROSS INCOME: $ 16,183  $ 17,329  $ 16,793  $ 17,489
MEAN DEP. VAR.:  2.784  2.852   2.821   2.862
STANDARD DEV.:   0.732  0.769   0.767   0.672
STANDARD ERROR:  0.534  0.559   0.590   0.468
OBSERVATIONS:    813   694    670    880
R-SQUARED:       0.481  0.487   0.427   0.526

* = SIGNIFICANT AT 99%, + = SIGNI. AT 95%, - = SIGNIFICANT AT 80%
importance only in the strata of young couples where it reduces the expected normal income.

The dummy variables for the highest degree have the expected sign pattern and are in general measured with satisfactory precision. Comparing the three strata, we discover an interesting life cycle pattern: the magnitude of the education coefficients are largest in the strata of medium age, less so among the couples over fifty, and smallest in the young couples' stratum. Here, it may reflect the lag before the educational achievements are translated into money income. Among the people over fifty years old, it may indicate the supersession of education by experience and on-the-job-training.

It is very unfortunate that no information about the latter group of variables is contained in the Annual Housing Survey. We are left with a less than satisfactory determination of permanent income, but still prefer the poorly measured correct explanatory variable to current income. Section 5.7.3 discusses the sensitivity of the demand estimations when we replace permanent income by current income.
5.6.2 West Germany

With the German One Percent Sample we face the opposite data problem: the sample contains rich information on human wealth of both household head and spouse, but no information on asset holdings. We use the regression results by Schneider, Stahl, and Struyk (1983) which were estimated from a large pooled sample across West Germany. This sample includes rural areas as well as other demographic strata apart from married couples, so the mean income is lower than the mean income in our sample. Summary statistics and coefficients are reported in Table 5-9. Not presented are the coefficients for marital status, guestworkers, and rural area dummies which are irrelevant for our basic sample. Furthermore, the constant and the city center dummy are adjusted for the base case of a suburban location. Finally, the original specification of Schneider, Stahl, and Struyk included an indicator variable for homeownership on the right hand side. The inclusion of this variable would introduce a simultaneity bias in the demand equations. The corresponding coefficients were barely significant and of small magnitude (0.035 for single-family homeowners, -0.048 for two-family homeowners, with t-statistics of 3.36 and 2.97, respectively, in a sample of 29,017 observations). We excluded these coefficients.

The dependent variable is net income adjusted for tax advantages of homeownership as discussed in Section 5.4. We will briefly discuss the coefficients of the explanatory variables estimated by Schneider, Stahl, and Struyk. The age variables draw the familiar first
TABLE 5-9: PERMANENT INCOME ESTIMATION (GERMANY)

Dependent Variable: Logarithm of Monthly Net Income

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>HOUSEHOLD</th>
<th>HEAD:</th>
<th>SPOUSE:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>7.42 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) DEMOGRAPHIC CHARACTERISTICS:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGE OF HEAD</td>
<td>0.016 *</td>
<td>0.002 *</td>
<td></td>
</tr>
<tr>
<td>AGE SQUARED</td>
<td>-0.002 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HEAD MALE</td>
<td>0.189 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GUESTWORKER</td>
<td>0.051 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) EDUCATION:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NO SCHOOL DEGREE</td>
<td>0.085 *</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>MIDDLE SCHOOL (MITTLERE REIFE)</td>
<td>0.194 *</td>
<td>0.035 *</td>
<td></td>
</tr>
<tr>
<td>HIGH SCHOOL (ABITUR)</td>
<td>0.268 *</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>COLLEGE (VORDIPLOM)</td>
<td>0.237 *</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td>UNIVERSITY (DIPLOM)</td>
<td>0.353 *</td>
<td>0.128 *</td>
<td></td>
</tr>
<tr>
<td>(4) PROFESSIONAL STATUS:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NUMBER OF WAGES</td>
<td>0.303 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD. JOB AS FARMER</td>
<td>-0.404 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>APPRENTICESHIP</td>
<td>0.109 *</td>
<td>0.050 *</td>
<td></td>
</tr>
<tr>
<td>MASTER CRAFTSMAN</td>
<td>0.137 *</td>
<td>0.035</td>
<td></td>
</tr>
<tr>
<td>SELF EMPLOYED</td>
<td>0.237 *</td>
<td>0.020</td>
<td></td>
</tr>
<tr>
<td>WHITE COLLAR</td>
<td>0.141 *</td>
<td>0.290 *</td>
<td></td>
</tr>
<tr>
<td>BLUE COLLAR</td>
<td>-0.001 *</td>
<td>0.228 *</td>
<td></td>
</tr>
<tr>
<td>UNEMPLOYED</td>
<td>-0.177 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RETIRED</td>
<td>-0.378 *</td>
<td>-0.087</td>
<td></td>
</tr>
<tr>
<td>STUDENT</td>
<td>-0.234 *</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>(5) CONTROLS:</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>COMPANY HOUSING</td>
<td>-0.087 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HOUSING ALLOWANCES</td>
<td>-0.046 *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CITY CENTER</td>
<td>0.036 *</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

YEARLY NET INCOME: $9,322
MEAN DEP. VAR.: 7.48
STANDARD ERROR: 0.381
OBSERVATIONS: 29,017
R-SQUARED: 0.592

* = SIGNIFICANT AT 99%
increasing, then decreasing pattern of life cycle earnings. Education of the head performs as expected, both significant and with increasing magnitude for the improving educational status. Education of the spouse is measured with mixed results, mostly insignificant. Furthermore, human capital is indicated by the professional status. The number of wage and salary earners in the household contributes significantly to the household income, each additional wage at about 30 percent, whereas the combination of an industrial job at day with a second job as farmer ("moonlight farmer") has a sharp negative impact. Self employed or white collar status expectedly enhance; blue collar, unemployed, retired, or student status decrease permanent income. Again, the professional status of the spouse yields mixed, mostly unprecise results. Finally, Schneider, Stahl, and Struyk control for subsidies by receiving housing allowances or company housing, as well as location: the income differential between city center and suburbs is significant at about 3.6 percent.
5.6.3 Comparison

The regressions in both countries suffer from the lack of data, and we will only compare the predictions generated by the regressions. The third column of each block in Table 5-10 lists the distribution of actual current income versus predicted permanent income for both countries. Both incomes are measured yearly, before taxes, and in 1977 Dollars. By construction, permanent income has a substantially lower variance. The distributional differences between West Germany and the United States, already discussed for current income in Section 5.2, remain the same for permanent income: the fraction of households with a yearly permanent income $8,000 is 7.4 percent in the United States and only 3.2 percent in Germany. On the other hand, in the middle income classes ($8,000 to $25,000) the distribution is more even in Germany.

It is interesting to compare the correlation between ownership and income for both income concepts. In both countries, permanent income has a stronger impact on tenure choice than current income, as can be seen in the lower (greater) ownership ratio of permanent income at the bottom (top) of the income scale, compared with current income. This confirms the common wisdom of higher permanent than current income elasticities and the inclusion of permanent rather than current income into the demand analysis.
TABLE 5-10: DISTRIBUTION OF PERMANENT AND CURRENT INCOME

(1) CURRENT INCOME VERSUS OWNERSHIP:

<table>
<thead>
<tr>
<th>Income Class</th>
<th>UNITED STATES SAMPLE</th>
<th>WEST GERMAN SAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Income Distribution</td>
<td>Ownership Total Ratio</td>
</tr>
<tr>
<td>&lt; 3.0</td>
<td>1.04%</td>
<td>52 61.18%</td>
</tr>
<tr>
<td>&lt; 4.0</td>
<td>1.09</td>
<td>52 58.43</td>
</tr>
<tr>
<td>&lt; 5.0</td>
<td>1.33</td>
<td>55 50.93</td>
</tr>
<tr>
<td>&lt; 6.0</td>
<td>1.86</td>
<td>79 52.32</td>
</tr>
<tr>
<td>&lt; 7.0</td>
<td>2.36</td>
<td>101 52.60</td>
</tr>
<tr>
<td>&lt; 8.0</td>
<td>2.51</td>
<td>111 54.41</td>
</tr>
<tr>
<td>&lt; 10.0</td>
<td>4.88</td>
<td>228 57.43</td>
</tr>
<tr>
<td>&lt; 12.5</td>
<td>8.04</td>
<td>394 60.24</td>
</tr>
<tr>
<td>&lt; 15.0</td>
<td>7.63</td>
<td>393 63.29</td>
</tr>
<tr>
<td>&lt; 20.0</td>
<td>18.26</td>
<td>1129 75.98</td>
</tr>
<tr>
<td>&lt; 25.0</td>
<td>16.43</td>
<td>1106 82.72</td>
</tr>
<tr>
<td>&lt; 35.0</td>
<td>19.18</td>
<td>1395 89.37</td>
</tr>
<tr>
<td>&lt; 45.0</td>
<td>7.81</td>
<td>582 91.51</td>
</tr>
<tr>
<td>&lt; 55.0</td>
<td>7.59</td>
<td>592 95.79</td>
</tr>
<tr>
<td>&gt; 55.0</td>
<td>0.00</td>
<td>0 0.00</td>
</tr>
</tbody>
</table>

(2) PERMANENT INCOME VERSUS OWNERSHIP:

<table>
<thead>
<tr>
<th>Income Class</th>
<th>UNITED STATES SAMPLE</th>
<th>WEST GERMAN SAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Income Distribution</td>
<td>Ownership Total Ratio</td>
</tr>
<tr>
<td>&lt; 3.0</td>
<td>0.00%</td>
<td>0 0.0%</td>
</tr>
<tr>
<td>&lt; 4.0</td>
<td>0.07</td>
<td>2 33.3</td>
</tr>
<tr>
<td>&lt; 5.0</td>
<td>0.33</td>
<td>16 59.3</td>
</tr>
<tr>
<td>&lt; 6.0</td>
<td>1.47</td>
<td>58 48.3</td>
</tr>
<tr>
<td>&lt; 7.0</td>
<td>2.20</td>
<td>79 44.1</td>
</tr>
<tr>
<td>&lt; 8.0</td>
<td>3.34</td>
<td>133 48.9</td>
</tr>
<tr>
<td>&lt; 10.0</td>
<td>7.95</td>
<td>349 53.9</td>
</tr>
<tr>
<td>&lt; 12.5</td>
<td>13.05</td>
<td>663 62.4</td>
</tr>
<tr>
<td>&lt; 15.0</td>
<td>15.00</td>
<td>866 70.9</td>
</tr>
<tr>
<td>&lt; 20.0</td>
<td>29.34</td>
<td>2016 84.4</td>
</tr>
<tr>
<td>&lt; 25.0</td>
<td>17.95</td>
<td>1360 93.1</td>
</tr>
<tr>
<td>&lt; 35.0</td>
<td>9.12</td>
<td>713 96.1</td>
</tr>
<tr>
<td>&lt; 45.0</td>
<td>0.17</td>
<td>14 100.0</td>
</tr>
<tr>
<td>&lt; 55.0</td>
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<td>0 0.0</td>
</tr>
<tr>
<td>&gt; 55.0</td>
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</tbody>
</table>

Yearly gross current and permanent income in thousand 1977 Dollars.
The totals are based on a sample size of 8035 married couples in West German Urban Areas and 8139 white married couples in U.S. SMSA's.
5.7 Estimation of the Demand Equations

With the preliminary estimations of permanent income and hedonic indexes completed, we can proceed to estimate the NMNL-demand equations. The extensive preliminary work provided us with the basis for a common model for the United States and West Germany with directly comparable variables and units. We estimated four sets of regressions, two in each country: for the first two sets, we draw a choice based sample of the pooled population for each country, and estimated all six trees depicted in Figure 5-5 in addition to the multinomial logit model. For the second two sets, we stratified the basic sample of each country according to the six strata of age and location:

(1) CC,YO: City center, young married couples, head aged under 35
(2) CC,ME: City center, married couples, head aged between 35 and 50
(3) CC,EL: City center, elderly married couples, head aged over 50
(4) SU,YO: Suburbs, young married couples, head aged under 35
(5) SU,ME: Suburbs, married couples, head aged between 35 and 50
(6) SU,EL: Suburbs, elderly married couples, head aged over 50

Within each stratum, we selected a smaller choice based subsample for estimation with the optimal tree discovered in the first regression sets. This procedure allowed us to use relatively small sample sizes (about 400 observations) without losing precision on the housing choices with small market shares. To make up for the endogenous sampling, we weighted the observations inversely to their sampling ratio and proceeded as if we had an exogenous sample. This estimation technique (WESML, Manski and Lerman, 1977) and the necessary correction for standard errors is described in Chapter Two.
5.7.1 Pooled Sample: Optimal Tree Structure

Table 5-11 reports the estimation results for the pooled sample of the United States; Table 5-12 for those of West Germany. The performance of each tree is indicated by the value of the likelihood function presented in the first row in the tables. This likelihood is evaluated relative to the null model at the bottom of the table ("RHO_SQ") as well as the percentage of correct ex post predictions. Section 2.3.2 discusses the interpretation of these statistics. The estimated coefficients represent the taste weights in the linear indirect utility function (2.2) and can be transformed into elasticities according to (2.10). Finally, the dissimilarity coefficients translate into the inter-alternative substitution effects as discussed in Section 2.2.3.

Three observations strike the eye: in both countries, the same hierarchical decision tree dominates; in Germany, the transition from simple multinomial logit analysis to this optimal nested model is more rewarding; however, all results are more precise in the American sample.

Figure 5-13 shows the pattern of performance and significance graphically. All trees can be distinguished statistically from the MNL-model by a likelihood test which is distributed chi-squared with six degrees of freedom, the number of dissimilarity parameters. In turn, all trees can be interpreted as restrictions of a common generalized extreme value model (GEV, McFadden, 1981) of the form (2.8)
TABLE 5-11: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (USA)

<table>
<thead>
<tr>
<th>TREE:</th>
<th>MNL</th>
<th>T-U-S</th>
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<th>S-T-U</th>
<th>S-U-T</th>
<th>U-T-S</th>
<th>U-S-T</th>
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<td>-272.82</td>
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(1) ALTERNATIVE SPECIFIC VARIABLES:

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<th>U-T-S</th>
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TABLE 5-11: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (USA)

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(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

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TABLE 5-12: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (GERMANY)

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(1) ALTERNATIVE SPECIFIC VARIABLES:

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<td>(0.10)</td>
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<td>(-0.36)</td>
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<td>(1.30)</td>
<td>(1.65)</td>
<td>(1.32)</td>
<td>(0.50)</td>
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### TABLE 5-12: POOLED CHOICE BASED SAMPLE, WESML-ESTIMATES (GERMANY)

(3) ALTERNATIVE SPECIFIC DUMMIES:

|         | Coefficient | Standard Error | t-Statistic | Pr(>|t|) |
|---------|-------------|----------------|-------------|---------|
| **O SF 14** | -1.0219     | 4.5272         | 2.5825      | 0.18    |
|         | (-0.43)     | (0.30)         | (0.27)     | (0.18)  |
| **O SF 5+** | -0.9606     | 12.5258        | 4.5345      | 0.4808  |
|         | (-0.51)     | (0.68)         | (0.54)     | (0.25)  |
| **O MF 14** | -0.6503     | 12.2380        | 5.5270      | 1.4559  |
|         | (-0.34)     | (0.82)         | (0.58)     | (0.66)  |
| **O MF 5+** | -3.0975     | 8.2310         | 2.5823      | -1.7037 |
|         | (-2.37)     | (0.51)         | (-1.25)    | (-2.13) |
| **R SF 14** | 1.6966      | 13.0461        | 4.7581      | 1.1652  |
|         | (0.88)      | (0.86)         | (0.48)     | (0.57)  |
| **R SF 5+** | 0.3387      | 10.0395        | 0.6173      | -0.1822 |
|         | (0.27)      | (0.53)         | (0.10)     | (-0.10) |
| **R MF 14** | 2.6783      | 16.2191        | 8.4049      | 2.7091  |
|         | (1.79)      | (1.10)         | (1.02)     | (1.77)  |

(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

|         | Coefficient | Standard Error | t-Statistic | Pr(>|t|) |
|---------|-------------|----------------|-------------|---------|
| **TH 0101** | 1.0000      | 11.9006        | 7.1650      | 1.7605  |
|         | (0.00)      | (1.96)         | (3.42)      | (2.26)  |
| **TH 0102** | 1.0000      | 6.3372         | 7.3321      | 1.5579  |
|         | (0.00)      | (0.63)         | (3.74)      | (1.13)  |
| **TH 0201** | 1.0000      | 8.4397         | 5.1846      | 2.6473  |
|         | (0.00)      | (2.84)         | (3.39)      | (2.22)  |
| **TH 0202** | 1.0000      | 12.5118        | 7.8569      | 2.5257  |
|         | (0.00)      | (1.27)         | (3.20)      | (2.10)  |
| **TAU 01**  | 1.0000      | 8.1912         | 5.2373      | 0.4488  |
|         | (0.00)      | (2.85)         | (2.09)      | (2.10)  |
| **TAU 02**  | 1.0000      | 4.4291         | 3.7735      | 0.7231  |
|         | (0.00)      | (3.13)         | (1.09)      | (1.07)  |

---

NOBS: 442 442 442 442 442 442
RHO SQ: 0.466 0.545 0.539 0.513 0.480 0.513
CORRECT: 67.57% 65.36% 67.32% 68.13% 68.48% 64.88%
FIGURE 5-13: CHI-SQUARED DISTANCES OF TREES

UNITED STATES:

-GEV-

tenure choice at top tenure choice at bottom tenure choice in middle


-21.96 21.46 16.08 15.58 13.56 11.72

**** *** ** **

MNL: -280.61

WEST GERMANY:

-GEV-

tenure choice at top tenure choice in middle tenure choice at bottom


145.20 134.22 87.34 87.30 25.50 23.18

**** *** *** *** ***

MNL: -490.99

Note:

*** = significant at the 99.5% level
** = significant at the 95.0% level
* = significant at the 92.5% level
6 degrees of freedom
where $G$ in (2.19) is replaced by a less restrictive function.

All differences between MNL and the various NMNL-models are significant at the 99.5 percent level in West Germany, whereas in the United States only the trees with tenure choice at the uppermost level meet this criterion. However, of the remaining trees, all except one are significant at the 95 percent level, none has a chi-squared distance to the MNL-model below 92.5 percent. The trees can be divided into three groups according to the position of tenure choice. In the West Germany, the pattern is clear and simple: the further down the tenure choice, the worse the trees perform. In the American sample, the differences between positions of tenure choice are less pronounced and the clear order characterizing the German hierarchies is broken up. Once tenure choice is positioned, the trees can not be distinguished statistically in terms of dominance of size or structure choice.

We draw two conclusions: unambiguously in both countries, the tenure-structure-size choice sequence produces the most plausible hierarchical decision structure, and we will use this tree for our stratified estimations in the following Section. Second, however, the general dominance of tenure choice is less clear in Germany. On one side, once tenure choice is not at the top of the hierarchy, the dominance of tenure choice continues in Germany, but not in the United States. However, a closer look at the parameters in Table 5-12 reveals that the dissimilarity coefficients in West Germany are not contained in the unit interval, indicating a more complicated
microeconomic interpretation than the theory outlined in the beginning of Section 2.2. In particular, the estimated relation can be interpreted as a mixture of different underlying trees, some with tenure choice at the top, other lower in the hierarchy. Because of the extent of this task, we were not able to identify a stratification of the sample which produces well behaved unique but different trees in each stratum. The large standard errors associated with the dissimilarity coefficients indicate such a mixture of models rather than a failure of the utility maximization hypothesis per se.

In both countries and in all specifications, we observe significant coefficients for the hedonic prices, consistently larger for renters than for owners. We introduced the interaction of hedonic rents with tenure choice for two reasons: as mentioned in Section 5.5, the hedonic regressions (on the renter subsamples) did not pick up well the differences in structure type, that is the attractiveness of single-family homes. Second, even if there were no measurement differences, out-of-pocket cost may be perceived differently by owners and renters. The general attractiveness of owning versus renting an object, e. g., the freedom of disposition, may interact with the price of the object.

The return variable is insignificant, pointing to offsetting effects of opportunity costs and appreciation, or poor specification. Unfortunatly, the global appreciation rates used here lack a differentiation by location within a city which is not reported in the two samples.
Surprisingly, the household size variable is significant in the United States but not in Germany. This result is in contrast to earlier studies, see Section 5.7.4.

The income and age pattern can be calculated by addition of different combinations of choice dimension dummies for each alternative. This yields the following preference ordering for tree T-U-S:

Germany: 
(1) R_MF_5+  
(2) R_SF_5+  
(3) O_MF_5+  
(4) O_SF_5+  
(5) R_MF_14  
(6) R_SF_14  
(7) O_MF_14  
(8) O_SF_14

United States: 
(1) O_SF_5+  
(2) R_SF_5+  
(3) O_MF_5+  
(4) R_MF_5+  
(5) O_SF_14  
(6) R_SF_14  
(7) O_MF_14  
(8) R_MF_14

Clearly, in both countries an additional Dollar of permanent income is spent first on more size. But again, tenure choice is differently perceived in Germany than in the United States where more income always implies a higher tendency to own.

This Section was primarily intended to highlight the differences in the valuation of tenure choice between the two countries. We will discuss the numerical values of the coefficients translated into elasticities in the next Section where we stratify the samples as discussed above.
5.7.2 Stratified Sample: Age and Location

Having discovered the tenure-structure-size choice hierarchy as the optimal functional description of housing demand within our NMNL-framework, we now stratify the sample according to location within the SMSA (city center versus suburb) and into three age categories (below 35, between 35 and 50, and over 50). The latter stratification takes account of a possible interaction between age and all other variables, that is a shift in preferences through the life cycle (de Leeuw 1971), whereas the separation of the sample by location stratifies by social status and income class as well as supply factors. Results are reported in Tables 5-14 for the United States and 5-15 for West Germany; compare the out-set of this Section for an explanation for the symbols used to denote the strata.

Stratification considerably improves the fit measured by the likelihood ratio ("RHO-SQ") or the percentage of correctly predicted choices in the strata of the city center and the couples over 50. For the young and middle aged couples in the suburbs we have relatively poor results. This holds for both countries. Except for those two strata, the German model now performs as well as the American model and has an even tighter fit for the oldest strata.

The results of the previous Section pertaining to the tree structure carry over to all separate strata. The bottom rows of Tables 5-14 and 5-15 show the performing of the corresponding MNL-models. The differences are all highly significant (at the 99.5
TABLE 5-14: SAMPLE STRATIFIED ACCORDING TO AGE AND LOCATION (USA)

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<tr>
<th>STRATUM: POOL</th>
<th>CC,YO</th>
<th>CC,ME</th>
<th>CC,EL</th>
<th>SU,YO</th>
<th>SU,ME</th>
<th>SU,EL</th>
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<td>-232.22</td>
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(1) ALTERNATIVE SPECIFIC VARIABLES:

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<th>-2.2485</th>
<th>-1.9020</th>
<th>-3.3301</th>
<th>-1.0870</th>
<th>-0.8438</th>
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<td>(1.52)</td>
<td>(3.99)</td>
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<td>-3.3064</td>
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<td>-1.7023</td>
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<td>(7.22)</td>
<td>(3.46)</td>
<td>(5.73)</td>
<td>(4.84)</td>
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<td>(3.44)</td>
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<td>0.0033</td>
<td>0.2337</td>
<td>-0.1951</td>
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<td>(4.05)</td>
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<td>-0.1997</td>
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<td>(3.63)</td>
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(2) HOUSEHOLD SPECIFIC VARIABLES:

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### (4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

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**TREE:**

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<td>346</td>
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<td>434</td>
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<td>0.661</td>
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<td>78.40%</td>
<td>79.59%</td>
<td>80.35%</td>
<td>82.59%</td>
<td>74.38%</td>
<td>77.52%</td>
<td>79.14%</td>
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**MNL-PERFORMANCE FOR COMPARISON:**

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<td>CORRECT:</td>
<td>76.89%</td>
<td>79.27%</td>
<td>78.93%</td>
<td>82.50%</td>
<td>74.43%</td>
<td>76.23%</td>
<td>78.37%</td>
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### TABLE 5-15: SAMPLE STRATIFIED ACCORDING TO AGE AND LOCATION (GERMANY)

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<th>STRATUM: POOL</th>
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<th>CC, ME</th>
<th>CC, EL</th>
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<th>SU, ME</th>
<th>SU, EL</th>
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</thead>
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(1) ALTERNATIVE SPECIFIC VARIABLES:

- **HEDONO**:
  
  -11.1418  
  -7.1768  
  -7.1136  
  -8.3649  
  -10.8740  
  -6.0120  
  -5.4039  
  (-5.53) (-6.25) (-6.84) (-5.61) (-4.11) (-5.86) (-4.79)

- **HEDONR**:
  
  -13.2527  
  -6.6116  
  -9.2795  
  -11.5718  
  -13.1639  
  -7.5246  
  -8.1463  
  (-4.00) (-4.93) (-5.40) (-5.35) (-3.31) (-4.42) (-5.93)

- **RETURN**:
  
  0.0594  
  -0.0653  
  -0.0615  
  -0.1301  
  -0.1125  
  -0.0946  
  -0.1482  
  (0.94) (-1.13) (-1.62) (-2.41) (-1.68) (-1.91) (-2.41)

- **CROWDS**:
  
  0.0242  
  0.0684  
  -0.6622  
  -0.1659  
  -1.2894  
  -1.2236  
  -0.3242  
  (0.06) (0.18) (-5.21) (-0.61) (-3.96) (-5.26) (-2.43)

(2) HOUSEHOLD SPECIFIC VARIABLES:

- **INC_OWN**:
  
  -0.1685  
  0.2095  
  -0.0685  
  0.0424  
  -0.0257  
  0.0814  
  0.0238  
  (-1.06) (2.25) (-1.11) (0.49) (-0.23) (0.95) (0.21)

- **INC_SFM**:
  
  -0.0859  
  -0.3019  
  0.1372  
  -0.1078  
  -0.1066  
  -0.1516  
  -0.2231  
  (-0.39) (-2.49) (1.33) (-0.83) (-0.29) (-1.16) (-2.44)

- **INC_14R**:
  
  -0.7135  
  -0.3249  
  -0.2581  
  -0.3104  
  -0.2982  
  -0.3624  
  -0.4449  
  (-1.28) (-1.19) (-3.25) (-2.35) (-1.94) (-2.65) (-3.16)

- **AGE_OWN**:
  
  1.1970  
  -17.4918  
  14.7392  
  -0.7486  
  26.1760  
  3.2150  
  2.6209  
  (0.72) (-4.69) (24.29) (-0.46) (28.75) (0.81) (0.78)

- **AGE_SFM**:
  
  -0.2824  
  35.1461  
  -12.2237  
  -5.0913  
  18.7470  
  8.6190  
  -4.5715  
  (-0.09) (4.65) (24.84) (-0.64) (28.75) (0.81) (0.78)

- **AGE_14R**:
  
  2.2728  
  -40.0533  
  -17.9722  
  1.5316  
  -22.0170  
  6.9028  
  -3.9208  
  (0.29) (-2.25) (-9.57) (0.52) (-2.95) (1.26) (-0.35)

- **AG2_OWN**:
  
  -0.5486  
  32.2272  
  -17.2238  
  1.0017  
  -40.0000  
  -2.5016  
  -2.0388  
  (-0.36) (4.65) (-16.1) (0.82) (-18.8) (-0.52) (-0.82)

- **AG2_SFM**:
  
  1.0275  
  -64.0541  
  13.1820  
  4.1835  
  -25.1000  
  -8.1032  
  2.6955  
  (0.32) (-10.1) (1.57) (0.72) (-1.49) (-1.56) (0.37)

- **AG2_14R**:
  
  -0.3025  
  57.8748  
  21.6473  
  -1.4120  
  36.6000  
  -6.6932  
  2.9590  
  (-0.04) (1.50) (7.44) (-0.60) (2.48) (-0.94) (0.34)
(3) ALTERNATIVE SPECIFIC DUMMIES:

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<th>MF 14</th>
<th>MF 5+</th>
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<td>(-4.43)</td>
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<td>(-1.62)</td>
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(4) DISSIMILARITY PARAMETERS (T-STATISTICS AROUND 1.0):

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<td>(1.23)</td>
<td>(2.91)</td>
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<td>(2.64)</td>
<td>(2.64)</td>
<td>(2.64)</td>
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NOBS: 442 302 443 361 396 403 389
RHO_SQ: 0.545 0.634 0.560 0.710 0.497 0.511 0.640
CORRECT: 65.36% 74.29% 69.22% 81.52% 57.11% 64.39% 79.19%

MNL-PERFORMANCE FOR COMPARISON:
RHO_SQ: 0.545 0.634 0.560 0.710 0.497 0.511 0.640
CORRECT: 67.57% 72.42% 68.69% 82.80% 55.55% 64.39% 79.19%
percent level) with the exception of the young couples in American suburbs where this difference only passes the 90 percent mark. Most of the individual dissimilarity parameters exceed one, but are measured with little precision, telling us that we have not discovered the correct specification to avoid estimating mixtures of hierarchies.

The taste coefficients for price and income can most easily be compared using the elasticity tabulations of Table 5-16. The elasticities refer to the percentage change of the probability to choose a housing alternative, when the price of this alternative or the household income is changed by one percent. The formula for the elasticities is given in (2.10). For each variable, the model produces an entire matrix of elasticities which describes the change in each choice probability $i$ in response to a change of this variable in alternative $j$. For the pooled sample, the elasticity matrix for the hedonic rent and permanent income is presented in Table 5-17. All elasticities are evaluated at sample means and weighted according to the population shares to offset the effects of choice based sampling.

We will first study the complete matrices in Table 5-17 for the pooled samples. The hierarchy of the tree is reflected in the block structure of equal and unequal elasticities. The corresponding MNL model would have equal cross elasticities entirely within each row. The nested model does away with this pattern: if the price of any renter alternative goes up, all other renter alternatives are also less likely to be chosen. Of course, the own effect dominates the cross effects. This pattern holds in both countries. It can be
TABLE 5-16: OWN PRICE AND SUM OF INCOME ELASTICITIES OF MARKET SHARES

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<th>(1) GERMANY:</th>
<th>MS(1)</th>
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<th>MS(3)</th>
<th>MS(4)</th>
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<th>MS(6)</th>
<th>MS(7)</th>
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<td>-5.92</td>
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<td>-5.83</td>
<td>-5.52</td>
<td>-8.05</td>
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<th>MS(4)</th>
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<th>MS(6)</th>
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<td>-3.33</td>
</tr>
<tr>
<td>Stratum</td>
<td>Sum of Income Elasticities:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POOLED</td>
<td>-0.22</td>
<td>1.54</td>
<td>-3.65</td>
<td>-2.03</td>
<td>-2.36</td>
<td>-1.37</td>
<td>-3.75</td>
<td>-1.59</td>
</tr>
<tr>
<td>CC_YO</td>
<td>0.21</td>
<td>2.49</td>
<td>0.03</td>
<td>1.38</td>
<td>-0.17</td>
<td>0.37</td>
<td>-1.49</td>
<td>-0.09</td>
</tr>
<tr>
<td>CC_ME</td>
<td>-1.14</td>
<td>1.81</td>
<td>-1.41</td>
<td>1.40</td>
<td>-2.11</td>
<td>-1.55</td>
<td>-6.01</td>
<td>-0.60</td>
</tr>
<tr>
<td>CC_EL</td>
<td>0.29</td>
<td>1.27</td>
<td>-1.10</td>
<td>-0.43</td>
<td>-1.59</td>
<td>-1.18</td>
<td>-3.85</td>
<td>-2.15</td>
</tr>
<tr>
<td>SU_YO</td>
<td>-0.06</td>
<td>1.94</td>
<td>n.i.</td>
<td>n.i.</td>
<td>-2.24</td>
<td>-1.50</td>
<td>-2.24</td>
<td>-0.88</td>
</tr>
<tr>
<td>SU_ME</td>
<td>-3.25</td>
<td>1.73</td>
<td>-0.19</td>
<td>1.06</td>
<td>-4.10</td>
<td>-3.44</td>
<td>-4.65</td>
<td>-3.79</td>
</tr>
<tr>
<td>SU_EL</td>
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<td>1.09</td>
<td>-0.25</td>
<td>2.42</td>
<td>-3.28</td>
<td>-1.86</td>
<td>-2.47</td>
<td>-1.60</td>
</tr>
</tbody>
</table>

Indices of Alternatives:
1: O_SF_14 O_SF_5+ O_MF_14 O_MF_5+ 2: R_SF_14 R_SF_5+ 3: R_MF_14 R_MF_5+
TABLE 5-17: MATRIX OF PRICE AND INCOME ELASTICITIES OF MARKET SHARES

(1) POOLED SAMPLE GERMANY:

<table>
<thead>
<tr>
<th>Variable Alt.</th>
<th>MS(1)</th>
<th>MS(2)</th>
<th>MS(3)</th>
<th>MS(4)</th>
<th>MS(5)</th>
<th>MS(6)</th>
<th>MS(7)</th>
<th>MS(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEDONO 3</td>
<td>-1.679</td>
<td>-1.679</td>
<td>-5.922</td>
<td>-0.612</td>
<td>1.553</td>
<td>1.553</td>
<td>1.553</td>
<td>1.553</td>
</tr>
<tr>
<td>HEDONO 4</td>
<td>-0.274</td>
<td>-0.274</td>
<td>-0.100</td>
<td>-6.937</td>
<td>0.253</td>
<td>0.253</td>
<td>0.253</td>
<td>0.253</td>
</tr>
<tr>
<td>HEDONR 5</td>
<td>2.480</td>
<td>2.480</td>
<td>2.480</td>
<td>2.480</td>
<td>-5.834</td>
<td>-2.942</td>
<td>-0.841</td>
<td>-0.841</td>
</tr>
<tr>
<td>HEDONR 6</td>
<td>0.932</td>
<td>0.932</td>
<td>0.932</td>
<td>0.932</td>
<td>-1.106</td>
<td>-5.523</td>
<td>-0.316</td>
<td>-0.316</td>
</tr>
<tr>
<td>HEDONR 8</td>
<td>1.369</td>
<td>1.369</td>
<td>1.369</td>
<td>1.369</td>
<td>-0.464</td>
<td>-0.464</td>
<td>-0.906</td>
<td>-3.509</td>
</tr>
</tbody>
</table>

(2) POOLED SAMPLE USA:

<table>
<thead>
<tr>
<th>Variable Alt.</th>
<th>MS(1)</th>
<th>MS(2)</th>
<th>MS(3)</th>
<th>MS(4)</th>
<th>MS(5)</th>
<th>MS(5)</th>
<th>MS(7)</th>
<th>MS(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEDONO 1</td>
<td>-2.363</td>
<td>0.446</td>
<td>1.562</td>
<td>1.562</td>
<td>1.415</td>
<td>1.415</td>
<td>1.415</td>
<td>1.415</td>
</tr>
<tr>
<td>HEDONO 2</td>
<td>1.581</td>
<td>-3.247</td>
<td>5.542</td>
<td>5.542</td>
<td>5.020</td>
<td>5.020</td>
<td>5.020</td>
<td>5.020</td>
</tr>
<tr>
<td>HEDONO 3</td>
<td>0.097</td>
<td>0.097</td>
<td>-6.445</td>
<td>-2.905</td>
<td>0.088</td>
<td>0.088</td>
<td>0.088</td>
<td>0.088</td>
</tr>
<tr>
<td>HEDONO 4</td>
<td>0.085</td>
<td>0.085</td>
<td>-2.523</td>
<td>-7.189</td>
<td>0.077</td>
<td>0.077</td>
<td>0.077</td>
<td>0.077</td>
</tr>
<tr>
<td>HEDONR 6</td>
<td>0.786</td>
<td>0.786</td>
<td>0.786</td>
<td>0.786</td>
<td>-2.387</td>
<td>-9.107</td>
<td>-1.126</td>
<td>-1.126</td>
</tr>
<tr>
<td>HEDONR 7</td>
<td>2.059</td>
<td>2.059</td>
<td>2.059</td>
<td>2.059</td>
<td>-2.950</td>
<td>-10.701</td>
<td>-0.344</td>
<td>-0.344</td>
</tr>
<tr>
<td>HEDONR 8</td>
<td>0.133</td>
<td>0.133</td>
<td>0.133</td>
<td>0.133</td>
<td>-0.191</td>
<td>-0.191</td>
<td>-0.022</td>
<td>-0.452</td>
</tr>
</tbody>
</table>

Elasticities at Sample Means and Frequencies, WESML-Weighted.
Parameters from Nested Model T-U-S.
MS(i) denotes the market share of housing alternative i.

Indices of Alternatives:

1: O_SF_14  2: O_SF_5+  3: O_MF_14  4: O_MF_5+  5: R_SF_14  6: R_SF_5+  7: R_MF_14  8: R_MF_5+
interpreted in the following way: if rental housing prices of one category go up, substitution goes more likely into owner-occupancy than into other rental housing. In owner-occupied housing, we observe different patterns: As contrasted to the United States, cross substitution within the owner market occurs in Germany. The magnitudes of the elasticities are similar, the Germans are more price responsive as homeowners, the Americans as renters, reflecting the higher propensity of Americans to choose an owner-occupied home even at a slightly higher price.

The income elasticities are composed of the elasticities with respect to each of the three income/choice-dimension interactions. They are tabulated at the bottom of each block in Table 5-17. The income elasticities reiterate the "American Dream" interpretation just given for the price elasticities and they, of course, reflect the preference pattern for an additional dollar income discovered above. A rise in permanent income in Germany will increase the consumption of large units independent of tenure and structure type, whereas in the United States the proportion of renters will decrease unambiguously and the income will be spent on large single-family homes. Income is measured against all other goods. Thus, the sum of all income elasticities reveals that housing as a composite commodity is a normal good in Germany, but an inferior good in the United States.

We now compare the basic price and income responses across strata, using the elasticities tabulated in Table 5-16. The differences between the two countries which we detected in the pooled
sample hold also in each of the strata, giving confidence in our results. In general, the suburban households are less price responsive than those living in the city centers, but more income elastic. The differential in price responsiveness is very strong in both countries, the difference in income sensitivity only in the United States. All estimates show a life cycle dependence with again reversed roles of price and income. The young and elderly strata are the most price responsive, but the least income sensitive age group. These age and location patterns are remarkably similar for both countries.
5.7.3 Sensitivity Analysis

The results presented in this section rest on a fairly extensive set of intermediate calculations with numerous assumptions. Except for age, none of the variables used in the final demand equation are taken directly from the raw data. Thus, it is appropriate to briefly report some results from alternative specifications which are displayed in Table 5-18.

The use of current instead of permanent income slightly improves the fit in Germany, slightly worsens the fit in the United States. The price and crowding variables are virtually unaffected. Income coefficients sharply decrease in the United States. This is the expected direction, but the magnitude is unreasonably large. In Germany, there is little change. No change occurs in the order of preferences. All things considered, the use of current versus permanent income does not change our results qualitatively, and the quantities of the hedonic rent variables are robust to this change.

We performed another series of experiments to test the models' sensitivity with respect to the technique of imputing the attributes of hypothetical alternatives. Instead of constructing a not chosen alternative with the same number of rooms as the chosen one, we simply assigned it the mean attributes of this choice, including the mean number of rooms. This improves the fit and increases the magnitudes of most price and income coefficients, see Table 5-18. These changes are small. However, there is a danger of creating spurious
TABLE 5-18: SENSITIVITY ANALYSIS

MNL-Estimates, Pooled Samples.

(1) CURRENT VERSUS PERMANENT INCOME:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Germany:</th>
<th>USA:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YPERM</td>
<td>YCURR</td>
</tr>
<tr>
<td>HEDONO</td>
<td>-2.07</td>
<td>-2.12</td>
</tr>
<tr>
<td>HEDONR</td>
<td>-2.42</td>
<td>-2.49</td>
</tr>
<tr>
<td>RETURN</td>
<td>-0.055</td>
<td>-0.053</td>
</tr>
<tr>
<td>CROWDS</td>
<td>-0.098</td>
<td>-0.095</td>
</tr>
<tr>
<td>INC_OWN</td>
<td>-0.0045</td>
<td>0.041</td>
</tr>
<tr>
<td>INC_SFM</td>
<td>-0.032</td>
<td>-0.059</td>
</tr>
<tr>
<td>INC_14R</td>
<td>-0.089</td>
<td>-0.063</td>
</tr>
</tbody>
</table>

(2) CONSTANT ROOM VERSUS AVERAGE NUMBER OF ROOMS:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Germany:</th>
<th>USA:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CONSTANT</td>
<td>AVERAGE</td>
</tr>
<tr>
<td>HEDONO</td>
<td>-2.07</td>
<td>-2.22</td>
</tr>
<tr>
<td>HEDONR</td>
<td>-2.42</td>
<td>-2.51</td>
</tr>
<tr>
<td>RETURN</td>
<td>-0.055</td>
<td>-0.060</td>
</tr>
<tr>
<td>CROWDS</td>
<td>-0.098</td>
<td>0.015</td>
</tr>
<tr>
<td>INC_OWN</td>
<td>-0.0045</td>
<td>0.029</td>
</tr>
<tr>
<td>INC_SFM</td>
<td>-0.032</td>
<td>-0.044</td>
</tr>
<tr>
<td>INC_14R</td>
<td>-0.089</td>
<td>-0.069</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Germany:</th>
<th>USA:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RHO SQ</td>
<td>%CORRECT</td>
</tr>
<tr>
<td></td>
<td>0.466</td>
<td>67.57%</td>
</tr>
<tr>
<td></td>
<td>0.476</td>
<td>68.21%</td>
</tr>
<tr>
<td></td>
<td>0.642</td>
<td>76.89%</td>
</tr>
<tr>
<td></td>
<td>0.630</td>
<td>75.31%</td>
</tr>
</tbody>
</table>
elasticities when we specify hypothetical alternatives unlikely to be relevant for the given household. In particular, the average number of rooms may be too large for a small, and too large for a small household.

This relates to the issue of what the correct discrete categories are. There are obviously more than eight different housing choices, and picking a relatively small number of alternatives means an implicit aggregation procedure of the underlying elemental alternatives. In a related study by Behring, Goldrian, et. al. (1983), we started from a large set of elementary alternatives and used different sets of aggregated alternatives and different aggregation weights. The results turned out to be very sensitive to the pooling of unrelated alternatives, in particular, spurious elasticities can be created by posing irrelevant hypothetical alternatives. The variable CROWDS is particular sensitive to the bundling of alternatives which is obvious given its definition depending on the number of rooms. In theory, there is a unique correct aggregation. This is the extension of the tree down to all elemental alternatives, and the corresponding estimation of the dissimilarity parameters bundling the elemental alternatives provides the correct weights for aggregation. In practice, the number of elemental alternatives is much too large to make this approach feasible. Given clear cut aggregates, however, the results were fairly robust to the choice of aggregation weights.
5.8 Simulations with Tax Laws and Preferences

The discussion of descriptive statistics in Section 5.2 revealed substantial differences in the market shares of housing alternatives between the United States and West Germany, markedly in the tenure choice. How can they be explained? The endogenous variables -- predicted market shares for each tenure-structure-size combination -- are generated by the estimated parameters and the set of explanatory variables. We discussed the differences in the exogenous variables between the two countries in Section 5.2: the income and age distribution differ; a most stunning discrepancy occurs in the prices due to a greater land scarcity in Germany; and, a very advantageous tax treatment of homeownership in the United States. Given these differences in the explanatory variables, we still discovered a variety of differences in the estimates of the taste parameters, that is in the preferences of the German and the American households for different housing choices.

The normalization of all data to a common standard allows us to predict each country at the other countries tax laws or preferences. Thus, we can distinguish the consequences of differences in preferences from those in taxes or other exogenous variables as age and income. Table 5-19 presents the results. The first column denotes the baseline market shares of each of our eight housing alternative. These shares coincide with the actual shares in the population, because the saturation of the demand model allows us to produce a perfect fit of the market shares. This procedure is
TABLE 5-19: PREDICTIONS: PREFERENCES AND TAX LAWS

(1) GERMAN DATA

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>O_SF_14</td>
<td>0.2108</td>
<td>0.0680</td>
<td>0.3685</td>
<td>0.3352</td>
</tr>
<tr>
<td>O_SF_5+</td>
<td>0.1591</td>
<td>0.3298</td>
<td>0.3901</td>
<td>0.3540</td>
</tr>
<tr>
<td>O_MF_14</td>
<td>0.0462</td>
<td>0.0005</td>
<td>0.0969</td>
<td>0.0925</td>
</tr>
<tr>
<td>O_MF_5+</td>
<td>0.0058</td>
<td>0.0005</td>
<td>0.0236</td>
<td>0.0221</td>
</tr>
<tr>
<td>OWN:</td>
<td>0.4219</td>
<td>0.3988</td>
<td>0.8791</td>
<td>0.8038</td>
</tr>
<tr>
<td>R_SF_14</td>
<td>0.1016</td>
<td>0.1177</td>
<td>0.0124</td>
<td>0.0245</td>
</tr>
<tr>
<td>R_SF_5+</td>
<td>0.0250</td>
<td>0.0731</td>
<td>0.0015</td>
<td>0.0036</td>
</tr>
<tr>
<td>R_MF_14</td>
<td>0.4096</td>
<td>0.3858</td>
<td>0.1018</td>
<td>0.1592</td>
</tr>
<tr>
<td>R_MF_5+</td>
<td>0.0420</td>
<td>0.0247</td>
<td>0.0053</td>
<td>0.0090</td>
</tr>
<tr>
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<td>0.5782</td>
<td>0.6012</td>
<td>0.1209</td>
<td>0.1962</td>
</tr>
</tbody>
</table>

(1) US-AMERICAN DATA:

<table>
<thead>
<tr>
<th>Alternative:</th>
<th>US- Baseline:</th>
<th>German Preferences:</th>
<th>German Tax Laws:</th>
<th>German Tax Laws/Equity:</th>
</tr>
</thead>
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<tr>
<td>O_SF_14</td>
<td>0.2453</td>
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<td>0.2387</td>
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<tr>
<td>O_SF_5+</td>
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<td>0.1352</td>
<td>0.4462</td>
<td>0.4460</td>
</tr>
<tr>
<td>O_MF_14</td>
<td>0.0112</td>
<td>0.2990</td>
<td>0.0102</td>
<td>0.0102</td>
</tr>
<tr>
<td>O_MF_5+</td>
<td>0.0073</td>
<td>0.0223</td>
<td>0.0100</td>
<td>0.0099</td>
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<tr>
<td>OWN:</td>
<td>0.7701</td>
<td>0.7945</td>
<td>0.7052</td>
<td>0.7048</td>
</tr>
<tr>
<td>R_SF_14</td>
<td>0.0720</td>
<td>0.0760</td>
<td>0.0964</td>
<td>0.0964</td>
</tr>
<tr>
<td>R_SF_5+</td>
<td>0.0321</td>
<td>0.0225</td>
<td>0.0416</td>
<td>0.0416</td>
</tr>
<tr>
<td>R_MF_14</td>
<td>0.1193</td>
<td>0.0999</td>
<td>0.1436</td>
<td>0.1437</td>
</tr>
<tr>
<td>R_MF_5+</td>
<td>0.0063</td>
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<td>0.0132</td>
<td>0.0132</td>
</tr>
<tr>
<td>RENT:</td>
<td>0.2299</td>
<td>0.2055</td>
<td>0.2948</td>
<td>0.2952</td>
</tr>
</tbody>
</table>

All predictions with NMNL-model T-U-S, adjusted to fit aggregate shares exactly. Shares may not add up because of rounding.
discussed in Section 2.3.

We performed three experiments: first, we predicted the German data with American preferences; second with the U.S. tax code; finally with the U.S. tax code in combination with the loan-to-value ratios observed in the United States. Then, we performed the analogous experiments on the American data base. These experiments represent a drastic interference with the steady state given by the cross sectional data. Therefore, they provide only qualitative guidelines and the quantities should not be taken literally.

Predicting either country with the other country's preferences does not change the tenure choice very much, nor are the proportions within the renter alternatives affected. However, there is a strong shift within the owner categories: large single-family homes dominate the American taste, whereas German preferences give a large share to condominiums and small one or two-family units. We may interpret this finding as a confirmation that the "American Dream" in fact is an important element of housing preferences in the United States.

The next experiments predict each country's housing consumption at its own preferences, but the other country's tax code. This produces a drastic shift into ownership in Germany, and only a much smaller corresponding shift towards rental housing in the United States. This can be explained by the peculiarities of the tax laws and by the discrepancy in the price of land. Germany has very high land prices in addition to higher structure costs. With the U.S. tax
code, Germans can deduct a much higher proportion of their income than Americans do. In turn, the German tax code is most unfavorable to high house values relative to the American tax code, but the tax advantages are comparable to the United States for low priced structures and lots which are dominant in the United States. Thus, the change here is much smaller than the corresponding change in Germany.

Finally, we may argue that the loan-to-value ratios are endogenous and respond to the tax changes introduced in the previous paragraph. Predicting each country at the other country's tax code and equity pattern produces the last column of Table 5-19. The German tax advantages from homeownership are independent of the loan-to-value ratio, so for the Americans living under German tax law little changes. The only changes come through the change in opportunity costs of equity. However, as estimated for the Americans, the RETURN coefficient is small and insignificant. In Germany, we observe two effects. Raising the loan-to-value ratio makes the American tax deductions even more valuable. However, the same time we decrease the return from homeownership by increasing the opportunity costs. Both effects depend on the marginal tax rate. If this is zero, the additional deductions have no value, but the opportunity costs are counted fully, leading to a strong effect away from homeownership. If on the other extreme the marginal tax rate is one, all additional interest can be deducted, but the return from an alternative investment would be taxed away, making investment into housing very attractive. We estimated a positive coefficient of RETURN in the
German sample. With the high proportion of elderly homeowners with a low marginal tax rate in Germany the first mechanism outweighs the second, and we observe a decline in homeownership as compared to U.S. tax laws combined with German equity patterns.
5.9 Conclusions

The main conclusion from both estimation and simulation is the confirmation of the often quoted "American Dream": Americans have a strong preference for large single-family homes, independent from the advantages granted by the U.S. tax code. Germans do not share this dream in this extent: they prefer owning to renting, but they consider large rental units attractive as well.

Second, Americans are highly responsive to price changes in rental housing, less so in owner-occupied housing. Germans have the reverse pattern: their elasticity of choosing owner-occupied housing with respect to prices exceeds the corresponding elasticity to rent housing units.

Third, the market shares of housing alternatives can neither be explained by the preferences nor by the tax laws alone.

Fourth, an American-style tax law would have drastic effects in Germany because such a tax code has especially favorable consequences for the after tax costs of owner-occupancy, given the high land prices and structure costs in Germany.

Finally, we observe remarkable similarities as well: the pattern of price and income responsiveness by location and age neatly coincides in the two countries. The strata of the suburbs and of the middle aged couples are more income responsive but less price
sensitive than households in the city centers or of younger and older age who react more on price changes relative to income changes. In both countries, the same hierarchical choice pattern describes the actual behavior best. Tenure choice is the primary dimension to categorize housing choice. The choice of structure type and dwelling size follow with no clear order in both countries.

On a more technical level, we conclude that in both countries the nesting of choices and the stratification of the sample is a rewarding method to achieve good estimation results. We succeeded in similar satisfactory ex post prediction accuracies and high likelihood ratio statistics in the United States and West Germany.
5.10 Footnotes

(1) These and all following summary statistics for nationwide U.S. housing consumption are quoted from Annual Housing Survey 1977, Part A: General Housing Characteristics, United States and Regions, U.S. Departments of Commerce and of Housing and Urban Development.

(2) These and all following summary statistics on housing choice, income, and age distribution are quoted from %Wohnungsstichprobe 1978, Heft 5: Wohnungsversorgung der Haushalte und Familien, Statistisches Bundesamt, Wiesbaden.


(4) These and all following summary statistics on rent levels are quoted from %Wohnungsstichprobe 1978, Heft 5: Wohnungsversorgung der Haushalte und Familien, Statistisches Bundesamt, Wiesbaden.

(5) Own tabulations from the One Percent Sample.

(6) The Urban Land Institute, Land Use Digest, Volume 14, No. 12, December 1981 (Single Family Improved Lot Price Median), and unpublished data, provided by the National Association of Home Builders.

(7) Unpublished disaggregate tabulations for 1978, provided by the West German Bundesministerium fuer Finanzen.

(8) See equation (2.2) in Chapter Two.
APPENDIX

FORTRAN Program for Nested Multinomial Logit Models
\[ p(TIB, L) = \exp(X(T, B, L) \times BETA / THETA(B, L)) / \exp(INC(B, L)) \]

\[ p(BIL) = \exp(INC(B, L) \times THETA(B, L) / TAU(L)) / \exp(INC(L)) \]

This program analyses a nested multinomial logit model or hierarchical decision tree up to three levels.

As an example, consider the following "tree":

```
               LEVEL 0 (STEM)
                +-------------+
                |             |
                |             |
                LEVEL 1 (LIMBS)
                ++-------------+
                |             |
                |             |
                LEVEL 2 (BRANCHES)
                ++++-------------+
                | | | | | | | |
                | | | | | | | |
                LEVEL 3 (TWIGS)
```

Characterized by the likelihood function

\[ L = \log(p(T|B,L) \times p(B|L) \times p(L)) \]

Where:

- \( T \) = chosen twig, \( B \) = chosen branch, \( L \) = chosen limb

\[ p(T|B,L) = \exp(X(T,B,L) \times BETA / THETA(B,L)) / \exp(INC(B,L)) \]

with: \( INC(B,L) = \log \sum \exp(X(T',B,L) \times BETA / THETA(B,L)) \)

\[ p(B|L) = \exp(INC(B,L) \times THETA(B,L) / TAU(L)) / \exp(INC(L)) \]

with: \( INC(L) = \log \sum \exp(INC(B',L) \times THETA(B',L) / TAU(L)) \)

\[ p(L) = \exp(INC(L) \times TAU(L)) / \exp(INC(S)) \]

with: \( INC(S) = \log \sum \exp(INC(L') \times TAU(L')) \)

In particular, the program
- defines the tree structure
- reads the data, calculates sample means, and generates dummies
- weights observations if choice based sampled (WEMSML)
- maximizes the likelihood function (*)
- predicts shares and computes prediction success table
THE PROGRAM-STRUCTURE WITH ITS LEVELS IS AS FOLLOWS:

0  GQTREE   MAIN: DEFINES DIMENSIONALITY
1  TASKS    CALLING PROGRAM FOR TASKS OF GQTREE
1,3 UPDATE UPDATES PARMS IN COMMON: THETA, TAU, PMASK
2  INDATA   READS DATA IN AND CALCULATES SAMPLE STATISTICS
2  PUNCH,CNTR, OTHER GQOPT-Routines
       OPTMOV,MATEV2
2  TRAFO,AIRUM, PARAMETER TRANSFORMATION, AIRUM-CHECK,
   LMSTAT,ELAST,  LM-STATISTIC, ELASTICITIES,
   WESML  WESML-COVARIANCE CALCULATION
2  FDIFF,SDIFF FIRST AND SECOND FINITE DIFFERENCES
3  INV      MATRIX-INVERSION
2  OPT      OPTIMIZER BY GOLDFELD/QUANDT
3,2 FUNC,FP,SP EVALUATION OF LIKELIHOOD, GRADIENT, HESSIAN
4,2 CONT    CONTRIBUTION OF THE N-TH OBS TO LIKELIHOOD
4,3 LSFIT   LEAST-SQUARES FIT OF AGGR. PROB. SHARES
5,4 SSQDEV,FPSSQD EVALUATION OF PROB. SHARES, THEIR GRADIENTS
6,5 CONTPS  CONTRIBUTION OF THE N-TH OBS TO PROB. SHARES

THERE ARE TWO INPUT FILES:

"STRATUM":
FI 7  DATA (DEPENDENT AND EXPLANATORY VARIABLES BY ALTERNATIVE)

"INPARMS":
FI 20 DIMENSIONS, TREE STRUCTURE, PARAMETER-LABELS, INITIAL VALUES

(THE TERMINAL WILL PROMPT YOU FOR THE ACTUAL FILE-NAMES)

INPUT FILE 7:

READING AND FORMATTING IS DONE IN SUBROUTINE INDATA.
DATA1 contains the alternative-specific variables, which vary by household and by alternative.
DATA2 contains the agent-specific variables, which vary only by household. They have to interact with a set of alternative-specific dummies.

Example with standard set of dummies:
1) The left-hand-side variable C, which takes only the values 1 for chosen, 0 otherwise.
2) Two alternative-specific expl. variables P and R.
3) One agent-specific variable Y, which will interact with four alternative specific dummies.

For NOBS=2 and NALT=5:

--- DATA1 ---

- C11 P11 R11 Y1
- C12 P12 R12
- C13 P13 R13
- C14 P14 R14
- C15 P15 R15

--- DATA2 ---

- C21 P21 R21 Y2
- C22 P22 R22
- C23 P23 R23
- C24 P24 R24
- C25 P25 R25

Note: The excluded dummy is always the last alternative.

Example with standard set of dummies:
1) The left-hand-side variable C, which takes only the values 1 for chosen, 0 otherwise.
2) Two alternative-specific expl. variables P and R.
3) One agent-specific variable Y, which will interact with four alternative specific dummies.

Input file 20:

---

This file contains the functional form of the decision-tree, i.e., the number and arrangement of alternatives in the tree, the number of exogenous variables, and the initial values for the parameters.

A) Elemental alternatives:

A1) NALT, NALTFR, RELSIZ

Where NALT = number of elemental alternatives
NALTFR = number of alternatives in a larger frame, from which NALT alternatives were consolidated.
If irrelevant, set to zero.
RELSIZ = ratio of main sample size over auxiliary sample size. Set to zero for exogenous sampling.
TO ONE IF AUXILIARY INFORMATION IS EXACT.

A2) ONLY FOR NALTFR>NALT:
(IAKTIV(I),I=1,NALTFR)
(PROBFR(I),I=1,NALTFR)
(JAKTIV(J),J=1,NALT)

WHERE IAKTIV=INDEX OF NALT ALTERNATIVES IN NALTFR
PROBFR=RELATIVE WEIGHTS OF ALTERNATIVES
JAKTIV=INVERS OF IAKTIV

A3) ONLY FOR RELSIZ > 0.0
(WEIGHT(J),J=1,NALT)

WHERE WEIGHT=RATIO OF AUXILIARY SAMPLE SHARE OVER MAIN
SAMPLE SHARE OF RESP. ALTERNATIVE

B) EXPLANATORY VARIABLES:

B1) NX,NXD,NXM,NXA

WHERE NX = # OF ALTERNATIVE-SPECIFIC VARIABLES AT LEVEL 3
(MIMICRY LEVEL 1 AND 2 VARIABLES AT LEVEL 3)
NXD= # OF AGENT-SPECIFIC VARIABLES
NXM= SET OF DUMMIES FOR THE AGENT-SPECIFIC VARIABLES
NXA= SET OF ALTERNATIVE SPECIFIC CONSTANTS

THERE ARE TWO DIFFERENT POSSIBILITIES TO DUMMY AGENT-SPECIFIC
VARIABLES AND TO SET PURE ALTERNATIVE-SPECIFIC CONSTANTS:

(1) "STANDARD-DUMMIES"
NALT-1 DUMMIES WILL BE ASSIGNED TO EACH ELEMENTAL ALTERNATIVE
EXCEPT THE LAST. SEE EXAMPLE ABOVE.

(2) "TREE-DUMMIES"
FOR EACH LEVEL OF THE TREE, NL1-1, NL2-1, NL3-1 DUMMIES
WILL BE ASSIGNED. NOTE: THIS IS ONLY POSSIBLE FOR A SYMMETRIC
UNDERLYING TREE (i.e., NL2=CONSTANT, NL3=CONSTANT,
SEE BELOW). THIS IMPLIES AN ASSIGNMENT OF LESS THAN NALT-1
DUMMIES TO THE ALTERNATIVES, WHERE IN TURN EACH ALTERNATIVE
CAN BE ASSIGNED UP TO THREE DUMMIES. THIS IS CONTROLLED BY
THE MAP IN B2.

EIGHT COMBINATIONS ARE POSSIBLE:

(1) AGENT-SPECIFIC VARIABLES INTERACT WITH STANDARD-DUMMIES,
NO ALTERNATIVE SPECIFIC CONSTANTS:
SET NXD > 0, NXM = 0, NXA = 0

(2) AGENT-SPECIFIC VARIABLES INTERACT WITH STANDARD-DUMMIES,
FULL SET OF NALT-1 ALTERNATIVE SPECIFIC CONSTANTS:
SET NXD > 0, NXM = 0, NXA = 1
(3) AGENT-SPECIFIC VARIABLES INTERACT WITH TREE-DUMMIES,
NO ALTERNATIVE SPECIFIC CONSTANTS:
SET NXD > 0, NXM = # OF DUMMIES, NXA = 0

(4) AGENT-SPECIFIC VARIABLES INTERACT WITH TREE-DUMMIES,
FULL SET OF NALT-1 ALTERNATIVE SPECIFIC CONSTANTS:
SET NXD > 0, NXM = # OF DUMMIES, NXA = 1

(5) AGENT-SPECIFIC VARIABLES INTERACT WITH TREE-DUMMIES,
SET OF ALTERNATIVE SPECIFIC CONSTANTS AS TREE-DUMMIES:
SET NXD > 0, NXM = # OF DUMMIES, NXA = 2

(6) NO AGENT-SPECIFIC VARIABLES,
NO ALTERNATIVE SPECIFIC CONSTANTS:
SET NXD = 0, NXM = 0, NXA = 0

(7) NO AGENT-SPECIFIC VARIABLES,
FULL SET OF NALT-1 ALTERNATIVE SPECIFIC CONSTANTS:
SET NXD = 0, NXM = 0, NXA = 1

(8) NO AGENT-SPECIFIC VARIABLES,
SET OF ALTERNATIVE SPECIFIC CONSTANTS AS TREE-DUMMIES:
SET NXD = 0, NXM = # OF DUMMIES, NXA = 2

IF NXA IS SET TO -1 RATHER THAN TO 1, STANDARD-DUMMIES ARE NOT
ESTIMATED FREELY TO MAXIMIZE THE LIKELIHOOD-FUNCTION, BUT ARE
SET AFTER EACH ITERATION TO MATCH THE AGGREGATE SAMPLE SHARES.
THE PROCEDURE, AS IMPLEMENTED, IS NOT GUARANTEED TO BE
NUMERICALLY STABLE AND IS NOT NECESSARILY CONVERGENT.
HOWEVER, IF THE PROCEDURE DOES CONVERGE, THE ESTIMATES ARE
CONSISTENT AND EFFICIENT, BEING EQUIVALENT TO COSLETT'S
CONCENTRATED ESTIMATOR. THE SAMPLE SHARES CAN BE MATCHED BY A
NONLINEAR LEAST SQUARES PROCEDURE AFTER SUCCESSFUL LIKELIHOOD
MAXIMIZATION; FOR THIS, SET NXA = 1 AND SELECT THE ADJUST TASK.

THE CORRESPONDENCE BETWEEN ALTERNATIVES AND A SET OF
TREE-DUMMIES IS DEFINED BY THE FOLLOWING MAP:

B2) ONLY FOR NXM > 0:
(MAP( 1 ,J),J=1,3)
...
(MAP(NALT,J),J=1,3)

WHERE MAP(I,J)=M IMPLIES THAT TREE DUMMY M IS PRESENT IN
ELEMENTAL ALTERNATIVE I. UP TO J=3 DUMMIES
CAN BE PRESENT. IF LESS THAN 3 DUMMIES ARE
ASSIGNED, FILL THE REMAINING MAP(I,J) WITH
ZEROES.

C) TREE STRUCTURE

C1) NL1
NL2(I),I=1,NL1
NL3( 1 ,J),J=1,NL2( 1 )
... 
NL3(NL1,J),J=1,NL2(NL1) 

WHERE NL1 = # OF LIMBS (E.G., SEE FIGURE AT TOP: 2), 
NL2 = # OF BRANCHES FOR EACH LIMB (E.G.: 1,2), 
NL3 = # OF TWIGS FOR EACH BRANCH OF EACH LIMB, 
(E.G.: 2 FOR LIMB 1 AND 4,3 FOR LIMB 2) 

C2) NTAU, NTAU1, (LOCTAU(I), I=1,NTAU+NTAU1) 

WHERE NTAU = # OF LEVEL 1 DISS. PARAMETERS (TAU), 
TO BE OPTIMIZED OVER ("FREE"), 
NTAU1 = # OF LEVEL 1 DISS. PARAMETERS (TAU), 
TO BE EQUALITY CONSTRAINED, 
LOCTAU = RELATIVE LOCATION (LIMB) OF THE TAU'S. 

C3) NTH , NTH1 , (LOCLIM(I), LOCBRA(I), I=1,NTH+NTH1) 

WHERE NTH = # OF LEVEL 2 DISS. PARAMETERS (THETA), 
TO BE OPTIMIZED OVER ("FREE"), 
NTH1 = # OF LEVEL 2 DISS. PARAMETERS (THETA), 
TO BE EQUALITY CONSTRAINED, 
(LOCLIM,LOCBRA) = RELATIVE LOCATION (LIMB,BRANCH) 
OF THE THETA'S. 

NOTE: THE LOCATIONS OF FREE DISS.PARMS HAVE TO APPEAR FIRST IN 
LOCTAU, LOCLIM, AND LOCBRA, FOLLOWED BY THE LOCATIONS OF THE 
EQUALITY CONSTRAINT DISS.PARMS. 
THERE IS NO PROVISION IN THE CONT-ROUTINE FOR INTER-NODE EQUA-
LITY CONSTRAINTS. 

NOTE: THE PROGRAM CHECKS THE LEVEL OF THE TREE BY CHECKING 
NTAU+NTAU1 AND NTH+NTH1. 
IF BOTH ARE ZERO, THE TREE COLLAPSES TO A SIMPLE MNL MODEL. 
IF ONLY NTAU+NTAU1 IS ZERO, THE TREE COLLAPSES TO TWO LEVELS. 
IN EITHER CASE, SIMPLER FORMULAS ARE USED IN SUBROUTINE CONT. 

NOTE: ALL INPUTS IN A) THROUGH C) ARE IN *-FORMAT 

D) INITIAL VALUES 

D1) NAME(1) PARM(1) 
...
NAME(NP+NP1) PARM(NP+NP1) 

WHERE: NAME(I) = LABEL OF THE I'TH PARAMETER 
PARM(I) = INITIALVALUES OF THE I'TH PARAMETER 
NP = TOTAL NUMBER OF FREE PARAMETERS 
NP1 = TOTAL NUMBER OF CONSTRAINT PARAMETERS 

D2) A BLANK LINE TO INDICATE END OF INPUT 

THE FINAL CARDS OF FILE 20 CONTAIN IN (A8,F15.9)-FORMAT THE 
LABELS AND INITIALVALUES OF ALL THOSE PARAMETERS, OVER WHICH
Either will be optimized or which are equality constraint, in the following order:

1) Coefficients of explanatory variables on level 3 (Beta's)
   A) Alternative-specific
   B) Agent-specific
   C) Pure dummies (free or constraint)

2) Diss. parameters of level 2 (Theta's) (Free)

3) Diss. parameters of level 1 (Tau's) ("")

4) Diss. parameters of level 2 (Theta's) (Eq. Constr.)

5) Diss. parameters of level 1 (Tau's) ("")

Apart from output on the terminal, the program generates the following output-files:

"OutPars":
FI 19 Tree structure and estimation results. This file can be used as new tree-input file 20. Parts A) through C) are copied from "InPars", in D), the initial values are replaced by the estimated parameters, incl. standard-errors and t-statistics. Appended are summary-statistics.

"Predict":
FI 9 Data with predicted choices rather than observed. This file can be used as new data-input file 7. The independent variables are copied from file 7, the dependent variable is replaced by its prediction according to the maximum probability principle.

(The terminal will prompt you for file-names. If the files already exist, they will be overwritten. If you enter blank names, the above default names will be used.)

FI 18 is used internally as a virtual scratch-file for DFP.

The terminal will prompt you for options and iteration parameters as they are explained in the GQopt-Manual.

There is a menu-display before each operation.

Note: Certain operations can only be done in a fixed order:
(1) Calculation of the average individual elasticities needs the probability-shares from the prediction-operation.
(2) For the transformation to MNL-format the covariance-matrix has to be provided.
THE PROGRAM ALWAYS USES ANALYTICAL FIRST DERIVATIVES.

THE CHOICE FOR SECOND DERIVATIVES IS GOVERNED BY IDIFF2. THEY CAN BE APPROXIMATED ACCORDING TO BHHH, BY SIMPLE FINITE DIFFERENCES, OR BY SYMMETRIC FINITE SECOND DIFFERENCES. ONLY FOR THE SIMPLE MNL-CASE, ANALYTIC SECOND DERIVATIVES ARE IMPLEMENTED.

THE PACKAGE ALLOWS FOR THE MAXIMAL DIMENSIONS:

PARAMETER

* (MAXOBS = NUMBER OF OBSERVATIONS + 1 FOR MEANS,
* MAXNP = TOTAL NUMBER OF PARAMETERS TO OPTIMIZE OVER,
* MAXTH = 1ST ORDER DISSIMILARITY PARAMETERS AT LEVEL 2,
* MAXTAU = 2ND ORDER DISSIMILARITY PARAMETERS AT LEVEL 1,
* MAXNX = ALTERNATIVE SPECIFIC EXPLANATORY VARIABLES,
* MAXNXD = AGENT SPECIFIC EXPLANATORY VARIABLE,
* MAXNXM = NUMBER OF TREE-DUMMIES,
* MAXLEV = NUMBER OF LEVELS,
* MAXLIM = LIMBS AT LEVEL 1,
* MAXBRA = BRANCHES AT LEVEL 2 FOR EACH LIMB AT LEVEL 1,
* MAXALT = ELEMENTAL ALTERNATIVES,
* MAXFRA = ELEMENTAL ALTERNATIVES IN AN UNDERLYING LARGER FRAME)

ALL DIMENSIONS CAN EASILY BE CHANGED BY CHANGING THESE PARAMETER SETTINGS IN ALL ROUTINES THROUGHOUT THE PACKAGE.

IMPLICIT REAL*8 (A-H, O-Z)

PARAMETER

* (MAXOBS = 2001,
* MAXNP = 50,
* MAXTH = 10,
* MAXTAU = 5,
* MAXNX = 10,
* MAXNXD = 5,
* MAXNXM = 20,
* MAXLEV = 3,
* MAXLIM = 5,
* MAXBRA = 5,
* MAXALT = 20,
* MAXFRA = 61,
* MAXNXY = MAXNX + 1, MAXNXR = MAXNX + MAXNXD,
* MAXNPS = MAXNP * (MAXNP + 1) / 2, MAXALS = MAXALT * (MAXALT + 1) / 2,
* MAXSTK = 5 * MAXNP * MAXNP + 6 * MAXNP + MAXALT * MAXALT + 8 * MAXALT)

C
REAL*8 PARM(MAXNP), GRAD(MAXNP), HESS(MAXNP, MAXNP),
  * GRAD2(MAXNP), SCRA(MAXNP, MAXNP), MBHH(MAXNPS), HESS2(MAXNPS),
  * THETA(MAXLIM, MAXBRA), TAU(MAXLIM),
  * WORK(MAXNP), ZBAR(MAXNP), ZIZBAR(MAXALT, MAXNP),
  * HINV(MAXNP, MAXNP), AAUX(MAXNP, MAXALT), BAUX(MAXALT, MAXALT),
  * SCRAUX(MAXNP, MAXALT), QAUX(MAXALT)
REAL*8 EXB(MAXALT), EZG(MAXALT), EYA(MAXLIM),
  * EINC2(MAXLIM, MAXERA), EINC1(MAXLIM),
  * XB(MAXALT), ZG(MAXALT), YA(MAXLIM),
  * INC2(MAXLIM, MAXBRA), INC1(MAXLIM),
  * DER2B(MAXLIM, MAXBRA, MAXNP), DER1B(MAXLIM, MAXNP),
  * DER2T(MAXLIM, MAXBRA, MAXTH), DER1T(MAXLIM, MAXTH),
  * DER1U(MAXLIM, MAXTAU)
REAL*8 PKASKY(MAXNXD, MAXALT), PMASKD(MAXALT),
  * PDUMMY(MAXALT), PNAME(MAXNP), NDUMMY(MAXALT),
  * PROB1(MAXALT), PROB2(MAXALT), PROBS(MAXALT),
  * DERIV1(MAXALT, MAXALT), DERIV2(MAXALT, MAXALT),
  * PSUM1(MAXALT), Q3(MAXALT),
  * PSUM2(MAXLIM, MAXBRA), Q2(MAXLIM, MAXBRA)
REAL*4 DATA1(MAXALT, MAXNXD, MAXOBS), DATA2(MAXNXD, MAXOBS),
  * PSHARE(MAXALT, MAXOBS), PROBFR(MAXFRA),
  * PROB(MAXALT), FREQ(MAXALT), ACT1(MAXALT), ACT2(MAXALT, MAXALT),
  * ELAS(MAXALT, MAXALT, MAXXR), ELAS2(MAXALT, MAXALT, MAXXR)
INTEGER MAP(MAXALT, MAXLEV), MAPLEN(MAXNXM), NAPTR(MAXNXM, MAXALT),
  * MS3(MAXLIM, MAXBRA), IAKTIV(MAXFRA), JAKTIV(MAXALT),
  * NCT1(MAXALT), NCT2(MAXALT, MAXALT),
  * IF1(MAXLIM), IF2(MAXLIM, MAXBRA),
  * ACTTAU(MAXLIM), ACTTH(MAXLIM, MAXBRA)
REAL*4 FCNTR(20, 20)
LOGICAL*1 FCNTR(20, 20)
LOGICAL LEVEL1, LEVEL2, LMAP(MAXNXM, MAXALT), STDDUM, NOALT, NOAGE,
  * STDDUM, STDDUM, NALSTD, NAGSTD, NALTRE, NAGTRE, UNCON
CHARACTER*21 VERS10
CHARACTER*20 TRFILE

COMMON / BSTACK / AINT(MAXSTK)
COMMON / BPRINT / IPT, NPRINT, NDIG, NPUNCH
COMMON / DPARM / NP, PARM, GRAD, HESS
COMMON / DIMEN / NALT, NALT1, NX, NXD, NXX, NXR, MTH, MTAU, NOBS
COMMON / DDISS / LOCTAU(MAXTAU), LOCLIM(MAXTH), LOCBRA(MAXTH),
  * NL1, NL2(MAXLIM), NL3(MAXLIM, MAXBRA)
COMMON / DATA1 / DATA1
COMMON / DATA2 / DATA2
COMMON / DCOVM / MBHH
COMMON / DHESS / HESS2
COMMON / DCONT / EXB, EZG, EYA, EINC2, EINC1,
  * XB, ZG, YA, INC2, INC1,
  * DER2B, DER1B, DER2T, DER1T, DER1U,
  * ZBAR, ZIZBAR, UPP, LOW,
  * MS3K(MAXTH), ML3K(MAXTH), MS2K(MAXLIM)
COMMON / DSSQD / PROB1, PROB2, DERIV1, DERIV2
COMMON / DCNTR / FCNTR, PCNTR
COMMON / DMAIN / KANALS, LEVCHG, PNAME, NTAU1, NTH1, NXX2,
  * NALTFR, PROBFR, IAKTIV, JAKTIV,
* NCT1,NCT2,ACT1,ACT2,IP1,IP2,ACTTAU,ACTTH,
* GRAD2,SCRA,PSHARE,ELAS,ELAS2,PROB,FREQ

COMMON / DWORK / WORK,PROBS
COMMON / DELAS / PSUM1,PSUM2,Q2,Q3
COMMON / DTREE / NTH,NTAU,THEMA,TAU,LEVEL1,LEVEL2
COMMON / DMAPP / NXM,NAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
COMMON / DDUMM / STDDUM,STDUMA,STLDUM,NDUMA,NDUMD,
* NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
* NALSTD,NALTRE,NAGSTD,NAGTRE
COMMON / DCONST / UNCON,PDUMMY,NDUMMY
COMMON / DWESML / RELSZ,WEIGHT(MAXALT)
COMMON / DWESTK / HINV,AAUX,BAUX,QAUX,SCRAUX

START DIALOG

CALL START(NPRINT,KANAL5,KANAL6,UPP,LOW,VERSIO)
WRITE (KANAL6,1) VERSIO
1 FORMAT (' GQTREE [VERSION ',A21,']'
* /' ======================================
* /' ENTER TREE-INPUT-FILE >')
READ (KANAL5,'(A20)') TRFILE
READ INPUT-FILE 20 "INPARMS"
OPEN (UNIT=20,STATUS='OLD',FILE=TRFILE)
READ IN NUMBER OF ALTERNATIVES AND NAP OF ALTERNATIVES, IF ANY
READ (20,*) NALT,NALTFR,RELSIZ
NALT1=NALT-1
IF (NALT.GE.NALTFR) GOTO 5
READ (20,*) (IAKTIV(I),I=1,NALTFR)
READ (20,*) (PROBFR(I),I=1,NALTFR)
READ (20,*) (JAKTIV(J),J=1,NALT)
5 IF (RELSIZ.GT.0.0) THEN
READ (20,*) (WEIGHT(I),I=1,NALT)
ELSE
DO 6 I=1,NALT
6 WEIGHT(I)=1.0
END IF
READ IN NUMBER OF EXOGENOUS VARIABLES
READ (20,*) NX,NXD,NXM,NXA
NXR=NX+NXD
NXA2=NXA
IF (NXA2.GE.2.AND.NXM.EQ.0) .OR. NXA.LT.-2)
* WRITE (KANAL6,*) 'WARNING: CONFLICTING CHOICE OF NXA AND NXM'
UNCON=.TRUE.
IF (NXA.EQ.-1) THEN
NXA=0
UNCON=.FALSE.
SWITCHES FOR DUMMIES

IF (NXA.EQ.0) THEN
   NO PURE DUMMIES TO BE ESTIMATED
   NOALT=.TRUE.
   NALTRE=.TRUE.
   NALSTD=.TRUE.
   STDUMA=.FALSE.
   NDUMA=0
ELSE
   NOALT=.FALSE.
ENDIF

IF (NXD.EQ.0) THEN
   NO AGENT-SPECIFIC VARIABLES PRESENT
   NOAGE=.TRUE.
   NAGTRE=.TRUE.
   NAGSTD=.TRUE.
   STDUMD=.FALSE.
   NDUMD=0
ELSE
   NOAGE=.FALSE.
ENDIF

IF (NXA.GE.2) THEN
   PURE TREE-DUMMIES
   NALTRE=.FALSE.
   NALSTD=.TRUE.
   STDUMA=.FALSE.
   NDUMA=NXM
   NXA=1
ELSE
   PURE STD.-DUMMIES
   STDDUM=.TRUE.
   STDUMA=.FALSE.
   NALTRE=.TRUE.
   NALSTD=.FALSE.
   NDUMA=NALT1
ENDIF

IF (NXM.GT.0) THEN
   AGENT SPEC. VAR. * TREE-DUMMIES
   NAGTRE=.FALSE.
   NAGSTD=.TRUE.
   STDUMD=.FALSE.
   NDUMD=NXM
ELSE
   AGENT SPEC. VAR. * STD.-DUMMIES
   STDDUM=.TRUE.
   STDUMD=.TRUE.
   NAGSTD=.FALSE.
   NAGTRE=.FALSE.
   NDUMD=NALT1
ENDIF
INDEX FOR AGENT SPECIFIC VARIABLES IN PARM-VECTOR

IF (.NOT.NOAGE) THEN
   KK=NX
   DO 10 K=1,NXD
   DO 10 I=1,NDUMD
      KK=KK+1
      INDAGE(K,I)=KK
   END IF

READ IN MAP OF TREE-DUMMIES, IF ANY

IF (NXM.EQ.0) GOTO 12
DO 11 I=1,NALT
   READ (20,*) (MAP(I,K),K=1,3)

READ IN TREE STRUCTURE

READ (20,*) NL1
READ (20,*) (NL2(I),I=1,NL1)
DO 13 I1=1,NL1
   ML2=NL2(I1)
   READ (20,*) (NL3(I1,I2),I2=1,ML2)

READ IN NUMBER AND LOCATION OF DISSIMILARITY PARAMETERS

READ (20,*) NTAU,NTAU1,(LOCTAU(J), J=1,NTAU1+NTAU)
READ (20,*) NTH ,NTH1 ,,(LOCLIM(J),LOCBRA(J),J=1,NTH +NTH1 )

READ IN INITIAL VALUES

N1=NX+NXD*NDUMD+NXA*NDUMA+NTH+NTH1+NTAU+NTAU1
IF (.NOT.UNCON) N1=N1+NALT1
DO 15 I=1,N1
   READ (20,16,END=17) PNAME(I),PARM(I)
16 FORMAT(A8,F15.9)
15 CONTINUE

CLOSE (UNIT=20)

REARRANGE PARM-VECTOR FOR CONSTRAINT ESTIMATION

IF (UNCON) GOTO 29
N1=NX+NXD*NDUMD
DO 22 I=1,NALT1
   PDUMMY(I)=PARM (N1+I)
   NDUMMY(I)=PNAME(N1+I)
N2=N1+NTH+NTAU+NTH1+NTAU1
DO 24 I=N1+1,N2
   PARM (I)=PARM (NALT1+I)
24 PNAME(I)=PNAME(NALT1+I)
29 CONTINUE
C LEVEL OF TREE
LEVChG=0
30 IF (NTAU+NTAU1.EQ.0) THEN
    LEVEL2=.TRUE.
    IF (NTH+NTH1.EQ.0) THEN
        LEVEL1=.TRUE.
        ELSE
            LEVEL1=.FALSE.
    END IF
    ELSE
        LEVEL2=.FALSE.
        LEVEL1=.FALSE.
    END IF

C ACTUAL DIMENSIONS OF PROBLEM

MXA = NX + NXD*NDUMD
MTH = MXA + MXA*NDUMA
MTAU = MTH + NTH
NP = MTAU + NTAU

C CHECK DIMENSIONS

IF (NX.GT.HAXNX) THEN
    WRITE (KANAL6,*) 'NX=',NX,' > MAXNX=',MAXNX
    GOTO 9999
END IF
IF (NXD.GT.MAXNXD) THEN
    WRITE (KANAL6,*) 'NXD=',NXD,' > MAXNXD=',MAXNXD
    GOTO 9999
END IF
IF (NXM.GT.MAXNXM) THEN
    WRITE (KANAL6,*) 'NXM=',NXM,' > MAXNXM=',MAXNXM
    GOTO 9999
END IF
IF (NTAU.GT.MAXTAU) THEN
    WRITE (KANAL6,*) 'NTAU=',NTAU,' > MAXTAU=',MAXTAU
    GOTO 9999
END IF
IF (NTH.GT.KAXTH) THEN
    WRITE (KANAL6,*) 'NTH=',NTH,' > MAXTH=',MAXTH
    GOTO 9999
END IF
IF (NP.GT.HAXNP) THEN
    WRITE (KANAL6,*) 'NP=',NP,' > MAXNP=',MAXNP
    GOTO 9999
END IF
IF (NALT.GT.MAXALT) THEN
    WRITE (KANAL6,*) 'NALT=',NALT,' > MAXALT=',MAXALT
    GOTO 9999
END IF
IF (NALTFR.GT.KAXFRA) THEN
    WRITE (KANAL6,*), WARNING: FRAME TOO LARGE
    NALTFR=KAXFRA
END IF
IF (NL1.GT.MAXLIM) THEN
WRITE (KANAL6,*), 'NL1=',NL1,' > MAXLIM=',MAXLIN
GOTO 9999
END IF

NBRA=0
DO 31 I1=1,NL1
   IF (NL2(I1).GT.NBRA) NBRA=NL2(I1)
   CONTINUE
   IF (NBRA.GT.MAXBRA) THEN
      WRITE (KANAL6,*), 'NL2=',NBRA,' > MAXBRA=',MAXBRA
      GOTO 9999
   END IF
C
C INITIALIZE TAU AND THETA ARRAYS
C
DO 32 I1=1,NL1
   TAU(I1)=1.0
   ML2=NL2(I1)
   DO 32 I2=1,ML2
      THETA(I1,I2)=1.0
C
C UPDATE FREE DISS. PARAMETERS
C
CALL UPDATE(PARM,NP,*33)
C
C UPDATE EQUALITY-CONSTRAINT DISS. PARAMETERS
C
DO 33 K=1,NTH1
   KK=NTH+K
   IF (LOCLIM(KK).EQ.0) GOTO 35
      THETA(LOCLIM(KK),LOCBRA(KK)) = FARM(NP+K)
   CONTINUE
35 MTAU1=NP+NTH1
DO 36 K=1,NTAU1
   KK=NTAU+K
   IF (LOCTAU(KK).EQ.0) GOTO 40
      TAU(LOCTAU(KK)) = FARM(MTAU1+K)
   CONTINUE
C
C THESE ARRAYS HELP SPEEDING UP THE LOOPS IN CONT
C
40 IB=0
IT=0
DO 41 I1=1,NL1
   START FOR TAU-LOOPS IN CONT
      MS2K(I1)=IB
      IB=IB+NL2(I1)
   DO 42 K=1,NTAU
      DER1U(I1,K)=0.D0
   DO 43 K=1,NTH
      DER1T(I1,K)=0.D0
   DO 41 I2=1,ML2(I1)
   C
   START FOR THETA-LOOPS IN CONT
      MS3(I1,I2)=IT
      IT=IT+NL3(I1,I2)
DO 41 K=1,NTH
   DER2T(I1,I2,K)=0.DO
DO 44 K=1,NTH
   NT11=LOC1IM(K)
   NT12=LOCBRA(K)
C START FOR THETA-LOOPS
   MS3K(K)=MS3(NT11,NT12)
C LENGTH OF THETA-LOOPS
   ML3K(K)=NL3(NT11,NT12)
C C DRIVER CALLS SUPERVISOR SUBROUTINE
   CALL TASKS(PARM,GRAD,HESS,NP)
C C -----------------------------
C C NORMAL EXIT
C IF (LEVCHG.EQ.0) STOP
C C C LEVEL-CHANGE
C CHANGE LOCATION AND ACTIVENESS OF DISS.PARMS
C USEFUL FOR LEVELWISE ESTIMATION AND FREEZING OF DISS. PARMS
C 200 WRITE (KANAL6,251)
251 FORMAT(/' ENTER NEW NTH, NTH! '>')
READ (KANALS,*) NTH,NTH!
IF (NTH+NTH!.EQ.0) GO TO 250
C IF (NTH.EQ.0) GOTO 265
WRITE (KANAL6,261)
261 FORMAT( /' ENTER NEW LIMB,BRANCH-LOCATIONS FOR FREE THETA''S '>')
READ (KANALS,EQ.0) (LOCLIM(I),LOCBRA(I),I=1,NTH)
WRITE (KANAL6,EQ.0) 'INDEX AND INITIALVALUES OF FREE THETA''S:'
262 WRITE (KANAL6,209) I,PNAME(MTH+I),PARM(MTH+I)
C IF (NTH1.EQ.0) GOTO 270
WRITE (KANAL6,266)
266 FORMAT( /' ENTER NEW LIMB,BRANCH-LOCATIONS FOR CONSTR. THETA''S '>')
READ (KANALS,EQ.0) (LOCLIM(I),LOCBRA(I),I=NTH+1,NTH+NTH1)
WRITE (KANAL6,EQ.0) 'INDEX AND INITIALVALUES OF CONSTR. THETA''S:'
267 WRITE (KANAL6,209) I,PNAME(MTH+I),PARM(MTH+I)
C 270 WRITE (KANAL6,221)
READ (KANALS,EQ.0) K,ANAME,APARM
IF (K.EQ.0) GOTO 250
   PNAME(MTH+K)=ANAME
   PARM (MTH+K)=APARM
GO TO 270
C
WRITE (KANAL6,205)
205 FORMAT(/' ENTER NEW NTAU, NTAU1 >')
READ (KANAL5,*) NTAU,NTAU1
IF (NTAU+NTAU1.EQ.0) GOTO 290
209 FORMAT(I3,' = ',F9.4)
C
IF (NTAU.EQ.0) GOTO 215
WRITE (KANAL6,211)
211 FORMAT(' ENTER NEW LIMB-LOCATIONS FOR FREE TAU''S >')
READ (KANAL5,*) (LOCTAU(I),I=1,NTAU)
WRITE (KANAL6,*) 'INDEX AND INITIALVALUES OF FREE TAU''S:'
MTH=MTH+NTAU
DO 212 I=1,NTAU
212 WRITE (KANAL6,209) I,PNAKE(MTH+I),PARM(MTH+I)
C
IF (NTAU1.EQ.0) GOTO 220
WRITE (KANAL6,216)
216 FORMAT(' ENTER NEW LIMB-LOCATIONS FOR CONSTRAINT TAU''S >')
READ (KANAL5,*) (LOCTAU(I),I=NTAU+1,NTAU+NTAU1)
WRITE (KANAL6,*) 'INDEX AND INITIALVALUES OF CONSTRAINT TAU''S:'
DO 217 I=NTAU+1,NTAU+NTAU1
217 WRITE (KANAL6,209) I,PNAME(MTAU+I),PARM(MTAU+I)
C
WRITE (KANAL6,221)
221 FORMAT(' ENTER INDEX, NEW NAME, AND NEW INITIALVALUE FOR CHANGE' *      ' (ZEROES IF OK) >')
READ (KANAL5,*) K,ANAME,APARM
IF (K.EQ.0) GOTO 290
   PNAME(MTAU+K)=ANAME
   PARM (MTAU+K)=APARM
GOTO 220
C
290 GOTO 30
C
C ERROR EXIT
C
9999 WRITE (KANAL6,*) 'ERROR: ACTUAL DIMENSIONS EXCEED MAXIMUM'
9998 FORMAT(/' NUMBER OF PARAMETERS : ')
   / ' NX+(NXD*NDUMD)+(NXA*NDUMA)+NTH+NTAU = NP'
   / 1X,12,15,316,315)
WRITE (KANAL6,9998) NX,NXD,NDUMD,NXA,NDUMA,NTH,NTAU,NP
STOP
END

SUBROUTINE TASKS(PARM,GRAD,HESS,NP)

SUPERVISOR PROGRAM FOR THE TASKS OF GQTREE.

AXEL BOERSCH-SUPAN       VERSION  MARCH 22, 1984

NOTE: DYNAMIC DIMENSION NP FOR USE IN GQOPT-ROUTINES.
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PARM(NP),GRAD(NP),HESS(NP,NP)

PARAMETER
* (MAXOBS =2001,
* MAXNP =50,
* MAXTH = 10,
* MAXTAU =5,
* MAXNX = 10,
* MAXNXD =5,
* MAXNXM =20,
* MAXLEV =3,
* MAXLIM =5,
* MAXBRA =5,
* MAXALT =20,
* MAXFRA =61,
* MAXNX1=MAXNX+1, MAXNXR=MAXNX+MAXNXD,
* MAXNPS=MAXNP*(MAXNP+1)/2,
* MAXDA1=MAXALT*MAXNX1*MAXOBS, MAXDA2=MAXNXD*MAXOBS)

REAL*8 WORK(MAXNP), SCRA(MAXNP,MAXNP),
= MBHHH(MAXNPS),THETA(MAXLIM,MAXBRA), TAU (MAXLIM),
= PMASKY (MAXNXD,MAXALT),PMASKD (MAXALT),
= GRAD2 (MAXNP), PROBS (MAXALT), PDMUNY (MAXALT)
REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS), DATA2 (MAXNXD,MAXOBS),
= FDATA1 (MAXDA1), FDATA2 (MAXDA2),
= PSHERE (MAXALT, MAXOBS), PROBFR (MAXFRA),
= ELAS (MAXALT,MALXT,MAXNXR), ELAS2 (MAXALT,MALXT,MAXNXR),
= PROB (MAXALT), FREQ (MAXALT), CH,
= ACT1 (MAXALT), ACT2 (MAXALT, MAXALT)
INTEGER MAP (MAXALT,MAXLEV), MAPLEN (MAXNXM,MAXALT),
= NLMS (5), NCT1 (MAXALT), NCT2 (MAXALT, MAXALT),
= IP1 (MAXLIM), IP2 (MAXLIM, MAXBRA),
= IAKTIV (MAXFRA), JAKTIV (MAXALT),
= ACTTAU (MAXLIM), ACTTH (MAXLIM, MAXBRA)
LOGICAL LEVEL1, LEVEL2, LMAP (MAXNXM, MAXALT),
= STDUMA, STDUMD, STDUDH, NOALT, NOAGE, UNCON,
= NALSTD, NALTRE, NAGSTD, NAGTRE

C

REAL*8 PNAME(MAXNP), NDUMMY (MAXALT), BLANK/8H
CHARACTER*1 TR1(100),TR2(100),TR3(100),TR7(100),TR8(100),TR9(100),
= BL/ ' /, VB/ ' | /, CR/ ' + /, DA/ ' - /, FK/ ' F /, CK/ ' C /,
= DG(0:9)/'0', '1', '2', '3', '4', '5', '6', '7', '8', '9'/
CHARACTER*8 HENAME(7)/ ' DA. FL. PO', ' QUADHILL', ' BHHH-COV', ' SIMP-COV',
= ' SYMM-COV', ' ANAL-COV', ' ADJUSTED/',
= SUMME/ ' SUM '/
CHARACTER*14 CHVER(6)/ ' FLETCHER ', ' DFP-ORIGINAL ',
= ' BROYDEN ', ' RANK-1-CORR. ',
= ' QUADR. HILLCL. ', ' MODIF. HILLCL. '/
CHARACTER*5 CHIST(0:3)/ ' NONE ', ' DLNSR ', ' LNSR ', ' STRCH '/
CHARACTER*4 CHDF2(4)/ ' BHHH ', ' SIMP ', ' SYMM ', ' MNL '/
CHARACTER*8 CHSTOP(0:1)/ ' ENABLED ', ' DISABLED '/
CHARACTER*18 CWESML(2)/ ' ', ' (WESML-WEIGHTED)' /
CHARACTER*20 TITLE,EMPTY/*','*/
CHARACTER*4 DASH4(MAXALT)/MAXALT*'-----'/
CHARACTER*7 DASH7(MAXALT)/MAXALT*'--------'/

C

COMMON / BSTACK / AINT(1)
COMMON / BSTAK / NQ,NSTOP
COMMON / BREAD / NREAD
COMMON / BSTOP / NPSTOP,ISTOP(3)
COMMON / BINPUT / INFLG
COMMON / BOPT / IVAR,LT,IPP,ISP,NLOOP1,IST,ILoop1
COMMON / BOPT2 / TOL,RM1,PM1,IVALFU,ITERL,ITERC,MAX,IER
COMMON / BSPD / ISPD
COMMON / BPRINT / IRT,KANAL6,NDIG,NPUNCH

COMMON / DIMEN / NALT,NALT1,NX,NXD,NXA,WXR,WXA,MTA,MTAU,NOBS
COMMON / DDISS / LOCTAU(MAXTAU),LOCLEM(MAXTH),LOCBRA(MAXTH),
* N1,N2,N3(MAXLIM),NL3(MAXLIM,MAXBRA)
COMMON / DATA1 / DATA1
COMMON / DATA2 / DATA2
COMMON / DCOV1 / MBHHH
COMMON / DMAIN / KANAL5,LEVCHG,PNAME,NTAU1,NTH1,NXA2,
* NALT1,PROBFR,IAKTIV,IAKTIV,
* NCT2,ACT1,ACT2,IP1,IP2,ACTTAU,ACTTH,
* GRAD2,SCRA,PSHARE,ELAS,ELAS2,PROB,FREQ
COMMON / DWORK / WORK,PROBS
COMMON / DTREE / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
COMMON / DMAPP / NTH,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
COMMON / DDUMM / STDDUM,STDDUM,STDDUM,NDUMA,NDUMl,
* NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),
CLOSE
COMMON / DIFF2 / IDIFF2
COMMON / DCONST / UNCON,PDUMMY,NDUMMY
COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)

C

EXTERNAL FUNC,GRADX,DFP,SSQDEV
C

EQUIVALENCE (DATA1,FDATA1),(DATA2,FDATA2)
C

C *** INITIALIZATIONS ***
C

ISTRUC=0
ISHARE=0
ICOVAR=0
IDATA=0
IF (LEVCHG.EQ.1) THEN
IDATA=1
LEVCHG=0
END IF
C

C *** ECHO TREE STRUCTURE, PARAMETER LABELS, INITIALVALUES ***
C

940 WRITE (KANAL6,*)'ENTER TREE-OUTPUT-FILE >'
READ (KANAL5,'(A20)') TITLE
IF (TITLE.EQ.EMPTY) TITLE='OUTPARKS'
OPEN (UNIT=19,STATUS='NEW',FILE=TITLE,ERR=9402)
GOTO 9405
OPEN (UNIT=19,STATUS='OLD',FILE=TITLE)
CLOSE(UNIT=19,STATUS='DELETE')
GOTO 9401

9406 FORMAT(8(12I6 /))
9407 FORMAT(8(12F6.3/))
9410 FORMAT(25I3)
9411 FORMAT(213,F10.6)

WRITE (19,9411) NALT,NALTFR,RELSIZ
IF (NALT.GE.NALTFR) GOTO 9420
WRITE (19,9406) (IAKTIV(I),I=1,NALTFR)
WRITE (19,9407) (PROBFR(I),I=1,NALTFR)
WRITE (19,9410) (JAKTIV(J),J=1,NALT)

IF (RELSIZ.EQ.0.) GOTO 9440
WRITE (19,9407) (WEIGHT(I),I=1,NALT)

WRITE (19,9410) NX,NXD,NXM,NXA2
IF (NXM.EQ.0) GOTO 9442
DO 9441 I=1,NALT
WRITE (19,9410) (MAP(I,K),K=1,3)

WRITE (19,9410) NL1
WRITE (19,9410) (NL2(I),I=1,NL1)
WRITE (KANAL6,942) NALT,NL1,(NL2(I),I=1,NL1)
WRITE (KANAL6,943)
FORMAT(' */' HIERARCHICAL CHOICE TREE WITH ',I2,' ALTERNATIVES:'
* /' ----------------------------------'
*/ THE STEM HAS',I2,' LIMBS, EACH HAS'/913)

DO 9441 I=1,NL1
ML2=NL2(I)
WRITE (19,9410) (NL3(Il,I2),I2=1,ML2)
WRITE (KANAL6,944) (NL3(Il,I2),I2=1,ML2)

FORMAT(' ',5I3)
WRITE (19,9410) NTAU,NTAU1,(LOCTAU(J), J=1,NTAU+NTAU1)
WRITE (19,9410) NTH ,NTH1 ,(LOCLIM(J),LOCBRA(J),J=1,NTH+NTH1 )
WRITE (KANAL6,945) NTAU,(LOCTAU(J),J=1,NTAU)
WRITE (KANAL6,946) NTH ,(LOCLIM(J),LOCBRA(J),J=1,NTH)
WRITE (KANAL6,947) NTAU1,(LOCTAU(J),J=NTAU+1,NTAU+NTAU1)
WRITE (KANAL6,948) NTH1 ,(LOCLIM(J),LOCBRA(J),J=NTH+1,NTH+NTH1)

FORMAT(/13,' FREE TAU''S':,' AT ', 5('(',I1,')':))
FORMAT( 13,' FREE THETA''S':,' AT ',10('(',I1,')':))
FORMAT( 13,' CONSTRAINT TAU''S':,' AT ', 5('(',I1,')':))
FORMAT( 13,' CONSTRAINT THETA''S':,' AT ',10('(',I1,')':))

ACTIVE (= FREE AND EQUALITY CONSTRAINT) DISS. PARAMETERS

DO 9750 I1=1,NL1
ACTTAU(Il)=0
DO 9750 I2=1,NL2(I1)
ACTTH(I1,I2)=0
C
DO 9751 K=1,NTH
ACTTH(LOCLIM(K),LOCBRA(K))=1
DO 9752 K=NTH+1,NTH+NTH1
ACTTH(LOCLIM(K),LOCBRA(K))=2
C
DO 9755 K=1,NTAU
ACTTAU(LOCTAU(K))=1
DO 9756 K=NTAU+1,NTAU+NTAU1
ACTTAU(LOCTAU(K))=2
C
PAINT THE TREE
C
IC=0
I=0
IP1(1)=2
IP2(1,1)=2
DO 9700 II=1,ML1
IF (II.EQ.1) GOTO 9701
IC=IC+1
TR7(IC)=BL
TR8(IC)=BL
TR9(IC)=BL
IC=IC+1
TR7(IC)=BL
TR8(IC)=BL
TR9(IC)=VB
IP1(II)=IC+2
9701 ML2=NL2(II)
DO 9700 I2=1,ML2
IC=IC+1
TR7(IC)=BL
TR8(IC)=BL
TR9(IC)=BL
IF (II.NE.1.OR.I2.NE.1) IP2(II,I2)=IC+1
ML3=NL3(II,I2)
DO 9700 I3=1,ML3
I=I+1
IN=MOD(I,10)
IC=IC+1
TR7(IC)=CR
TR8(IC)=VB
TR9(IC)=DG(IN)
CONTINUE
9700 IFINIS=IC
C
WRITE (KANAL6,9790) CR
9790 FORMAT(//' STRUCTURE : ','A1,' (F,C DENOTE DISS. PARAMETERS)'))
WRITE (KANAL6,9791) VB
9791 FORMAT(14X,100A1)
C
IEND=IP1(NL1)
DO 9710 IC=2,IEND
TR1(IC)=DA
TR2(IC)=BL
TR3(IC)=BL
9710 CONTINUE
DO 9715 I1=1,NL1
IC=IP1(I1)
TR1(IC)=CR
TR2(IC)=VB
TR3(IC)=VB
IF (ACTTAU(I1).EQ.1) TR3(IC)=FK
IF (ACTTAU(I1).EQ.2) TR3(IC)=CK
9715 CONTINUE
WRITE (KANAL6,9791) (TR1(IC),IC=2,IEND)
WRITE (KANAL6,9791) (TR2(IC),IC=2,IEND)
WRITE (KANAL6,9791) (TR3(IC),IC=2,IEND)
C
IEND=IP2(NL1,NL2(NL1))
DO 9720 IC=2,IEND
TR1(IC)=DA
TR2(IC)=BL
TR3(IC)=BL
9720 CONTINUE
DO 9725 I1=1,NL1
DO 9725 I2=1,NL2(I1)
IC=IP2(I1,I2)
TR1(IC)=CR
TR2(IC)=VB
TR3(IC)=VB
IF (ACTTH(I1,I2).EQ.1) TR3(IC)=FK
IF (ACTTH(I1,I2).EQ.2) TR3(IC)=CK
9725 CONTINUE
DO 9729 I1=2,NL1
IBL1=IP2(I1-1,NL2(I1-1))+1
IBL2=IP2(I1 ,1 )-1
DO 9729 IC=IBL1,IBL2
TR1(IC)=BL
9729 CONTINUE
WRITE (KANAL6,9791) (TR1(IC),IC=2,IEND)
WRITE (KANAL6,9791) (TR2(IC),IC=2,IEND)
WRITE (KANAL6,9791) (TR3(IC),IC=2,IEND)
C
WRITE (KANAL6,9792) (TR7(IC),IC=1,IFINIS)
WRITE (KANAL6,9792) (TR8(IC),IC=1,IFINIS)
WRITE (KANAL6,9793) (TR9(IC),IC=1,IFINIS)
9792 FORMAT(13X,100A1)
9793 FORMAT('CLUSTERING :',100A1)
WRITE (KANAL6,'(A1)') BL
C
IF (LEVEL2) THEN
IF (LEVEL1) THEN
WRITE (KANAL6,* ) 'TREE IS COLLAPSED TO ONE LEVEL'
ELSE
WRITE (KANAL6,* ) 'TREE IS COLLAPSED TO TWO LEVELS'
END IF
ELSE
WRITE (KANAL6,*) 'THIS IS A FULL THREE LEVEL TREE'
END IF

DISPLAY OF ALTERNATIVES AND DUMMIES
WRITE (KANAL6,9800)
9800 FORMAT(/' ALTERNATIVES AND DUMMIES:'
* /' INDEX LABEL LIMB BRANCH TWIG'/)
DO 9801 I=1,NALT
  IF (.NOT.UNCON) ANAME=NDUMMY(I)
  IF (UNCON.AND.NXA.NE.1) ANAME=BLANK
  IF (UNCON.AND.NXA.EQ.1) ANAME=PNAME(NXA+I)
  IF (I.EQ.NALT) ANAME=BLANK
  WORK(1)=BLANK
  WORK(2)=BLANK
  WORK(3)=BLANK
  IF (NXM.EQ.O) GOTO 9801
  IF (MAP(I,1).GT.O) WORK(1)=PNAME(NX+MAP(I,1))
  IF (MAP(I,2).GT.O) WORK(2)=PNAME(NX+MAP(I,2))
  IF (MAP(I,3).GT.O) WORK(3)=PNAME(NX+MAP(I,3))
  WRITE (KANAL6,9a05) I,ANAME,(WORK(J),J=1,3)
FORMAT(LX,I3,4X,A8,2X,AB,2X,A8,2X,AB)
WRITE (KANAL6,951) NX,NXD,NDUMD,NXA,NDUHA,NTH,NTAU,NP
951 FORMAT(/' NUMBER OF PARAMETERS '/I' ----------------------,/'
* /' NX+(NXD*NDUMD)+NTH+NTAU = NP'
* /' INITIALVALUES :'/I' ----------------------')
DO 952 I=1,NP
  WRITE (KANAL6,953) PNAME(I),PARM(I)
  CONTINUE
952 FORMAT(' ',A8,F15.9)
  IF (UNCON.AND.NTAU1+NTH1.EQ.O) GOTO 9620
WRITE (KANAL6,955)
955 FORMAT(/' CONSTRAINT :'/I' ----------------------')
  IF (UNCON) GOTO 957
DO 956 I=1,NALT1
  WRITE (KANAL6,953) NDUMMY(I),PDUMMY(I)
  CONTINUE
956 CONTINUE
957 N1=NP
  IF (.NOT.UNCON) N1=N1+NALT1
DO 958 I=1,NTH1
  KK=NTH+I
  WRITE (KANAL6,953) PNAME(N1+I),THETA(LOCCLIM(KK),LOCBRA(KK))
  CONTINUE
958 CONTINUE
DO 959 I=1,NTAU1
  KK=NTAU1+I
  WRITE (KANAL6,953) PNAME(N1+NTH1+I),TAU(LOCTAU(KK))
959 CONTINUE

MAPS FOR INDICES OF NON-STANDARD DUMMIES:

(1) LMAP(K,I) , I=1,NALT , K=1,NXM : LOGICAL MAP
   RETURNS .TRUE., IF DUMMY K APPEARS IN ALTERNATIVE I
   .FALSE., IF NOT

(2) HAP(I,N) , I=1,NALT , N=1,N1 : ORIGINAL HAP
   RETURNS UP TO N1 DUMMIES K PER ALTERNATIVE I
   N1=3 FOR THREE LEVEL TREE

(3) MAPTR(K,N) , K=1,NXM , N=1,N2 : TRANSPOSED MAP
   RETURNS UP TO N2 ALTERNATIVES I PER DUMMY K
   N2=MAPLEN(K) : # OF APPEARANCES OF DUMMY K

9620 IF (NXM.EQ.0) GOTO 969

   DO 9621 K=1,NXM
   MAPLEN(K)=0
   DO 9625 I=1,NALT
      M1=MAP(I,1)
      M2=MAP(I,2)
      M3=MAP(I,3)
      IF (M1.NE.0) THEN
         MAPLEN(M1)=MAPLEN(M1)+1
         MAPTR(M1,MAPLEN(M1))=I
      END IF
      IF (M2.NE.0) THEN
         MAPLEN(M2)=MAPLEN(M2)+1
         MAPTR(M2,MAPLEN(M2))=I
      END IF
      IF (M3.NE.0) THEN
         MAPLEN(M3)=MAPLEN(M3)+1
         MAPTR(M3,MAPLEN(M3))=I
      END IF
   DO 9625 K=1,NXM
   IF (K.EQ.M1 .OR. K.EQ.M2 .OR. K.EQ.M3) THEN
      LMAP(K,I)=.TRUE.
   ELSE
      LMAP(K,I)=.FALSE.
   END IF

9625 continue

FINISHED WITH PRELIMINARY WORK ON TREE-SPECIFICATION

969 ISTRUC=1

*** DATA AND NUMBER OF OBSERVATIONS ***

900 IF (IDATA.EQ.1) GOTO 920
   CALL INDATA(KANALS,KANAL6)
   NOBS1=NOBS+1
   IWESML=1
903 WRITE (KANAL6,904) NOBS,NALT,CWESKL(IWESML)
904 FORMAT(/' IDCASE = ',I5,' ', IDALT = ',I4, 
         */' SAMPLE FREQUENCIES AND MEANS :',A18
DO 906 I=1,NALT
906 WRITE (KANAL6,911) DATA1(I,1,NOBS1),
* (DATA1(I,J+1,NOBS1),J=1,NX),
* (DATA2(J,NOBS1),J=1,NXD)
911 FORMAT(' ',F6.4,12F10.4)
C
IF (RELSIZ.EQ.0.0 .OR. IWESML.EQ.2) GOTO 920
C
IWESML=2
NOBS1=NOBS+2
GOTO 903
C
920 IDATA=1
NOBS1=NOBS+1
C
*** MENU ****
C
1 WRITE (KANAL6,3)
3 FORMAT(' HIT ENTER FOR MENU, 7 FOR STOP >')
READ (KANAL5,4) MENU
4 FORMAT(I1)
IF (MENU.NE.0) GOTO 8
WRITE (KANAL6,2)
2 FORMAT(' SELECT FROM THE MENU :
* /* 1=ESTIMATION, 2=PREDICTION, 3=ELASTICITIES, 4=Covariance,'
* /* 5=LEVEL CHANGE, 6=DERIVATIVE CHECK, 7=STOP, 8=PRINT DATA,'
* /* 9=LM-TEST, 10=TRANSFORM, 11=PLOT, 12=AIRUM, 13=SELECT DATA,'
* /* 14=ADJUST SHARES >')
READ (KANAL5,* ) MENU
8 GOTO (100,200,300,400,9,500,99,600,700,780,650,790,620,800),MENU
( 1 2 3 4 5 6 7 8 9 10 11 12 13 14)
C
5 WRITE (KANAL6,6)
6 FORMAT(' IMPROPER ORDER OF OPERATIONS')
7 FORMAT(' INPUT ERROR')
GOTO 1
9 LEVCHG=1
GOTO 99
C
*** TASKS OF GQTREE ***
C
********************
* ESTIMATION *
********************
C
** ITERATION PARAMETERS **
C
100 WRITE (KANAL6,101)
101 FORMAT(' TYPE 1 TO OVERRIDE DEFAULT ITERATION PARAMETERS >')
READ (KANAL5,'(I1)') IQUERY
IF (IQUERY.EQ.1) GOTO 105
ITER=NP/4*2
TOL =0.001
METHOD=2
IVER=2
IF (LEVEL1) IVER=1
IST=1
IDIFF2=2
IF (LEVEL1) IDIFF2=4
ISPD=1
IF (NP.EQ.20) ISPD=2
ISTOP(1)=0
ISTOP(2)=0
ISTOP(3)=1
GOTO 110

105 WRITE (KANAL6,106)

106 FORMAT(' ITERATION : LIMIT >'
* 2/ ' : ACCURACY >'
* 2/ ' : METHOD : 1 = DFP, 2 = GRADX >'
* 2/ ' : VERSION : 1-4 IN DFP, 1-2 IN GRADX >'
* 2/ ' : LINE-SEARCH : 0-2 IN DFP, 0-3 IN GRADX >'
* 2/ ' : 2ND DERIVATIVES: 1 = BHHH, 2 = SIMPLE, 3 = SYM..,'
* 2/ ' : 4 = MNL >'
* 2/ ' : FREQUENCY OF UPDATING >'
* 2/ ' : CONV.-CRITERION: PARM (0=ENABLE,1=DISABLE) >'
* 2/ ' : GRAD (0=ENABLE,1=DISABLE) >'
* 2/ ' : LIKL (0=ENABLE,1=DISABLE) >'
READ (KANAL5,*) ITER,TOL,METHOD,IVER,IST,IDIFF2,ISPD,ISTOP
IF (.NOT.LEVEL1.AND.IDIFF2.EQ.4) THEN
   WRITE (KANAL6,*),'NO ANALYTIC 2ND DERIVATIVES IMPLEMENTED'
END IF
IVER2=IVER
IF (METHOD.EQ.2) IVER2=IVER+4
WRITE (KANAL6,115) ITER,TOL,MENAME(METHOD),CWESML(IWESML),
* CHVER(IVER2),CHIST(IST),CHDIF2(IDIFF2),ISPD,
* CHSTOP(ISTOP(1)),CHSTOP(ISTOP(2)),CHSTOP(ISTOP(3))

110 FORMAT(/' ',68(''-'))

* 2/ ' : LIMIT=' '13',' ACCURACY=' 'F8.6
* 2/ ' : METHOD : ' 'A8,A18
* 2/ ' : VERSION : ' 'A14
* 2/ ' : LINE-SEARCH : ' 'A5
* 2/ ' : 2ND DERIVATIVES: ' 'A4',' UPDATE EVERY' 'I2,' ITERATION'
* 2/ ' : CONV.-CRITERION: PARM=' 'A8,' GRADIENT=' 'A8,
* 2/ ' : LOGLIK=' 'A8
* 2/ ' : ' '68(''-'))/

116 MATPRT=0
NLOOP1=20
INFLG=0
MAX=1
NPSTOP=NP-NTH-NTAU
IF (METHOD.EQ.1) THEN
   NQ = NP*NP + 8*NP
   NPUNCH=18
   NREAD=18
   OPEN (UNIT=18,STATUS='SCRATCH')
ELSE
   NQ = 4*NP*NP + 5*NP
IF (IVER.EQ.2) NQ = 5*NP*NP + 6*NP
END IF
IF (.NOT.UNCON) NQ = NQ + NALT1*NALT1 + 8*NALT1

C ** ITERATION **

ITERAL=0
NEVALF=0
120 CONTINUE
IF (METHOD.EQ.1)
  * CALL OPT(PARM,NP,XLF, DFP ,ITER,MAX,IER,TOL,FUNC,PNAME)
IF (METHOD.EQ.2)
  * CALL OPT(PARM,NP,XLF,GRADX,ITER,MAX,IER,TOL,FUNC,PNAME)
ITERAL=ITERAL+ITERC
NEVALF=NEVALF+IVALFU
WRITE (KANAL6,122) IER
122 FORMAT(//' *** OPTIMIZATION TERMINATED *** IER=',I3
  ' /: TYPE NR OF MORE ITERATIONS OR 0 TO STOP >')
READ (KANAL5,*IQUERY
IF (IQUERY.EQ.0) GOTO 140
ITER=IQUERY
IF (METHOD.NE.1) GOTO 120
CALL PUNCH(PARM,NP)
REWIND NPUNCH
INFLG=1
GOTO 120

C ** PRINT PARAMETERS, STD ERRORS AND T-STATS **
C NOTE: COVARIANCE-MATRIX = -INV(HESSIAN)
C
140 IF (IER.GT.0) GOTO 145
C RECALCULATE COVARIANCES
MATPRT=2
IREDO=1
GOTO 420

C USE GQOPT-COVARIANCES
C
145 LK=NP*NP
CALL OPTMOV(1,HESS,LK)
CALL OPTMOV(3,GRAD,NP)
CALL WESML(PARM,GRAD,HESS,NP,SCRA,FUNC)
ICOVAR=1

C
150 XLFO = NOBS * DLOG(1.DO/DBLE(NALT))
DOF = NOBS * NP
AMSQGR = DOTV(GRAD,NP,GRAD)/DOF
AMSQGR = DSQRT(AMSQGR)
WRITE (KANAL6,151)
151 FORMAT(' ENTER TITLE >')
READ (KANAL5,152) TITLE
152 FORMAT(A20)
WRITE (KANAL6,153) MENAME(METHOD)

153 FORMAT(' * ** OUTPUT OF RESULTS **/* ',A8,' ESTIMATE' 
* 
* , STD-ERR T-STAT*/)

N1=NP
IF (.NOT.UNCON) N1=N1+NALT1
I=0
DO 160 II=1,N1
   IF (.NOT.UNCON.AND.II.GT.MXA.AND.II.LE.MXA+NALT1) GOTO 165
   I=I+1
   ZZZ = - HESS(I,I)
   IF (ZZZ.LE.0.) THEN
      STDERR = 0.0
      TSTAT = 0.0
   ELSE
      STDERR = DSQRT(ZZZ)
      TSTAT = PARM(I)/STDERR
      IF (I.GT.MTH) TSTAT=(PARM(I)-1.0)/STDERR
   END IF
   WRITE (KANAL6,161) PNAME(I),PARM(I),STDERR,TSTAT

161 FORMAT(' ,A8,2X,F8.4,2X,F7.3,2X,F7.3)

WRITE (19,162) PNAME(I),PARM(I),STDERR,TSTAT

162 FORMAT(A8,3F15.9)

GOTO 160

165 WRITE (KANAL6,161) NDUMMY(II-MXA),PDUMMY(II-MXA)

WRITE (19,162) NDUMMY(II-MXA),PDUMMY(II-MXA)

160 CONTINUE

DO 170 I=1,NTH1
   KK=NTH+I
   WRITE (KANAL6,161) PNAME(N1+I),THETA(LOCLIM(KK),LOCBRA(KK))

170 CONTINUE

DO 175 I=1,NTAU1
   KK=NTAU+I
   WRITE (KANAL6,161) PNAME(N1+NTH1+I),TAU(LOCTAU(KK))

175 CONTINUE

WRITE (19,180) TITLE,XLF,XLFO,ITERAL,NEVALF,NOBS,IER,
               AMSQGR,(TR9(IC),IC=1,IFINIS)

WRITE (KANAL6,180) TITLE,XLF,XLFO,ITERAL,NEVALF,NOBS,IER,
               AMSQGR,(TR9(IC),IC=1,IFINIS)

180 FORMAT('// TITLE : ',A20 
*  
*  // LOGLIKELIHOOD : ',F10.4 
*  
*  // LOGLIKELIHOOD AT ZERO : ',F10.4 
*  
*  // TOTAL NUMBER OF ITERATIONS : ',I10 
*  
*  // TOTAL NUMBER OF EVALUATIONS: ',I10 
*  
*  // NUMBER OF OBSERVATIONS : ',I10 
*  
*  // MEAN SQUARE GRADIENT (',I2,'): ',D10.4 
*  
*  // CLUSTERING : ',65A1//)

C.

IF (MATPRT.EQ.3) GOTO 430

GOTO 1

C

******************************************************************************
C  * PREDICTION SUCCESS TABLE AND PROBABILITY SHARES  *
C  ************************************************************
C
200 IF (IDATA.EQ.0) GOTO 5
WRITE (KANAL6,202)
202 FORMAT (' TYPE 1 TO CREATE DATA-SET WITH PREDICTED CHOICES >')
READ (KANALS,'(I1)') IWRITE
IF (IWRITE.EQ.1) THEN
WRITE (KANAL6,203)
203 FORMAT (' ENTER NEW DATA-SET NAME >')
READ (KANALS,'(A20)') TITLE
IF (TITLE.EQ.EMPTY) TITLE='PREDICT'
204 OPEN (UNIT=9,STATUS='NEW',FILE=TITLE,ACCESS='SEQUENTIAL'
      ,ERR=206)
      *  
GOTO 205
205 CONTINUE
END IF
XLFO = NOBS * DLOG(1.DO/DBLE(NALT))
XLF = 0.DO
XLP = 0.DO
PCP = 0.DO
UTIL = 0.DO
Z4 = 0.DO
Z5 = 0.DO
NFAIL= 0
DO 201 K=1,NALT
  ACT1(K)=0.0
  PROB(K)=0.0
  FREQ(K)=DATA1(K,1,NOBS+1)*WEIGHT(K)
DO 201 J=1,NALT
  ACT2(K,J)=0.0
C  
NN1=1
NN2=1
INCREM=MAXALT*MAXNXY
DO 210 IOBS=1,NOBS
C
C  ACTUAL CHOICE
C
I=0
DO 211 I1=1,NL1
  ML2=NL2(I1)
  DO 211 I2=1,ML2
    ML3=NL3(I1,I2)
    DO 211 I3=1,ML3
      I=I+1
      IF (DATA1(I,1,IOBS).EQ.1.0) IC=I
      CONTINUE
    WT=WEIGHT(IC)
C
C  PREDICTED CHOICE
C

NF=NFAIL
CALL CONT(FDATA1(NN1),FDATA2(NN2),
  Z,PARM,NP,XLFI,WORK,PROBS,NFAIL)

NN1=NN1+INCREM
NN2=NN2+MAXNXD
IF (NFAIL.GT.NF) THEN
  WRITE (KANAL6,212) IOBS,IC
  FORMAT(' CONT: FAILURE AT IOBS=',I5)
  IMAX=IA
  GOTO 222
END IF

XLF=XLF+XLFI
UTIL=UTIL+WORK(1)*WT
PMAX=0.DO
DO 221 IA=1,NALT
  PIA=PROBS(IA)
  PSHARE(IA,IOBS)=PIA
  PROB(IA)=PROB(IA)+PIA*WT
  IF (PIA.LE.PMAX) GOTO 221
    PMAX=PIA
    IMAX=IA
221 CONTINUE
C
COMPARISON ACTUAL VS. PREDICTED CHOICE
C
IF (IC.EQ.IMAX) PCP=PCP+WT
ACT2(IC,IMAX)=ACT2(IC,IMAX)+WT
NCT2(IC,IMAX)=ACT2(IC,IMAX)+0.5
ACT1(IMAX)=ACT1(IMAX)+WT
IF (PMAX.LE.0.DO) GO TO 215
XLP=XLP+DLOG(PMAX)*WT
Z4 =Z4 +PMAX*WT
Z5 =Z5 +PMAX/FREQ(IMAX)*WT
C
WRITE DATASET WITH PREDICTED CHOICES
C
222 IF (IWRITE.EQ.1) THEN
  DO 223 J=1,NALT
    IF (J.EQ.IHAX) THEN
      CH=1.0
    ELSE
      CH=0.0
    END IF
    WRITE (9) CH,(DATA1(J,K+1,IOBS),K=1,NX),
      (DATA2(K,IOBS),K=1,NXD)
  END IF
210 CONTINUE
C
IF (IWRITE.EQ.1) CLOSE(UNIT=9)
PCP=PCP/NOBS*100.0
WRITE (KANAL6,230) CWESHL(IWESHL),(J,J=1,NALT)
230 FORMAT(//' PREDICTION SUCCESS TABLE',A18

* // ' OBSERVED PREDICTED ALTERNATIVE'
* // ALT. | ',30I4)
WRITE (KANAL6,231) (DASH4(I),I=1,NALT)
231 FORMAT ( '-------',30A4)
DO 232 I=1,NALT
232 WRITE (KANAL6,233) I,(NCT2(I,J),J=1,NALT)
233 FORMAT ( I6 ',' | ',30I4)
WRITE (KANAL6,231) (DASH4(I),I=1,NALT)
WRITE (KANAL6,235) PCP,XLF0,XLF,XLP,UTIL
235 FORMAT (/* PERCENT CORRECTLY PREDICTED : ',F10.2,' %'
/* LIKELIHOOD AT ZERO : ',F12.4'
/* LIKELIHOOD AT ACTUAL CHOICES : ',F12.4'
/* LIKELIHOOD AT PREDICTED CHOICES : ',F12.4'
/* UTILITY AT PREDICTED CHOICES : ',F12.4')
WRITE (KANAL6,240) CWESM(ICESML)
240 FORMAT (/* PROBABILITY-SHARES', A18
* / ' ALT. ACTUAL DISCRETE CONTINUOUS'
*/ ------------------------------------------------------,)
DO 245 I=1,NALT
MOBSV=FREQ(I)*NOBS+0.5
ADISC=ACT1(I)/NOBS
MDISC=ACT1(I)+0.5
ACONT=PROB(I)/NOBS
245 WRITE (KANAL6,246) I,FREQ(I),MOBSV,ADISC,MDISC,ACONT,PROB(I)
246 FORMAT(I6,F9.4,I6,F10.4,I6,F10.4,F8.2)
C
C R-SQUARE-EQUIVALENTS (SEE DOMENICHI/MCFADDEN P.123)
C
Z1=0.0D0
Z2=0.0D0
Z3=0.0D0
S1=0.0D0
S2=0.0D0
S3=0.0D0
E1=0.0D0
E2=0.0D0
E3=0.0D0
Z7=0.0D0
NFAIL=0
ALT=1.0D0/NALT
DO 250 IOBS=1,NOBS
DO 251 I=1,NALT
IF (DATA1(I,1,IOBS).EQ.1) WT=WEIGHT(I)
CONTINUE
251 DO 250 J=1,NALT
FIJ = DATA1(J,1,IOBS)
FJ = PSHARE(J,IOBS)
PJ = FREQ(J)
IF (PIJ.LT.0.0D0 .OR. PIJ.GT.1.0D0)
* WRITE (KANAL6,252) PIJ,IOBS,J
252 FORMAT(' ERROR: PSHARE=',F10.4,' IN OBS=',I5,' AND ALT=',I3)
IF (PIJ.EQ.0) PIJ = PJ
IF (PIJ.EQ.0) NFAIL = NFAIL + 1
R1 = (FIJ-PIJ)**2
R2 = (FIJ-ALT)**2
R3 = (FIJ-PJ)**2
Z1 = Z1 + R1*WT
Z2 = Z2 + R1/PJ*WT
Z3 = Z3 + R1/PIJ*WT
S1 = S1 + R2*WT
S2 = S2 + R2/PJ*WT
S3 = S3 + R2/PIJ*WT
E1 = E1 + R3*WT
E2 = E2 + R3/PJ*WT
E3 = E3 + R3/PIJ*WT
Z7 = Z7 + FIJ*(1.0-ALT)/PJ*WT
CONTINUE
C
R1 = 1.0 - Z1/S1
R2 = 1.0 - Z2/S2
R3 = 1.0 - Z3/S3
C
R4 = 1.0 - Z1/E1
R5 = 1.0 - Z2/E2
R6 = 1.0 - Z3/E3
C
R7 = 1.0 - XLF/XLFO
C
R8 = Z4/NOBS
R9 = Z5/Z7
C
WRITE (KANAL6,253) CWESML(IWESML),R1,R2,R3,R4,R5,R6,R7,R8,R9
253 FORMAT( '//', /* R-SQUARE-EQUIVALENTS:',A18
* /', =*=*=*=*=*=*=*****
* /', WEIGHTS  :  NONE  FREQ  PROBS'
* /', SIMPLE  :', 3F10.5
* /', EFRON''S  :', 3F10.5
* /', MCFADDEN''S  :', F10.5
* /', SUCCESSES  :', 2F10.5 )
IF (NFAIL.GT.0) WRITE (KANAL6,*)) '# PROBS REPL. BY FREQ: ',NFAIL
C
ISHARE=1
GOTO 1
C
************************
C
* ELASTICITIES *
C
************************
300 IF (ISTRUC.EQ.0) GOTO 5
C
WRITE (KANAL6,302) CWESML(IWESML)
302 FORMAT( '//', ELASTICITIES AT SAMPLE MEANS AND FREQUENCIES:',A18//)
DO 3020 I=1,NALT
3020   FREQ(I)=DATA1(I,1,NOBS+IWESML)
      CALL ELAST(PARM,NP,FREQ,ELAS,NOBS+IWESML)
      WRITE (KANAL6,303) (I,I=1,NALT)
303 FORMAT(' VARIABLE ALT. 1',16(' P(',I2,'),':'))
      WRITE (KANAL6,301) (I,I=1,NALT)
301 FORMAT( ' ',14('-'),'+',16A7 )
      DO 304 K=1,NX
         WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
      DO 304
      WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,K),I=1,NALT)
      FORMAT( ' ',A8, ' ',I3, ' ',16F7.3)
      IF (NOAGE) GOTO 3061
      DO 306 K1=1,NXD
         WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
      DO 307 I=1,NALT
         PROBS(I)=0.DO
      DO 308 J=1,NDUMD
         K=NX+(K1-1)*NDUMD+J
         WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,NX+K1),I=1,NALT)
      DO 309 I=1,NALT
         PROBS(I)=PROBS(I)+ELAS(I,J,NX+K1)
      CONTINUE
      WRITE (KANAL6,306) SUMME,K1,(PROBS(I),I=1,NALT)
      CONTINUE
      WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
      IF (ISHARE.EQ.0) THEN
         WRITE (KANAL6,* )
*   'FOR INDIVIDUAL ELASTICITIES, RUN PREDICTION FIRST'
         GOTO 1
      END IF
      WRITE (KANAL6,310) CWESML(IWESML)
310 FORMAT(// ' AVERAGE INDIVIDUAL ELASTICITIES: ',A18//)
      DO 311 I=1,NALT
         DO 311 J=1,NALT
            DO 311 K=1,NXR
               ELAS(I,J,K)=0.0
            C
      DO 312 IOBS=1,NOBS
         DO 313 J=1,NALT
            IF (DATA1(J,1,IOBS).EQ.1.0) WT=WEIGHT(J)
            PROB(J)=PSHARE(J,IOBS)
         CALL ELAST(PARM,NP,PROB,ELAS2,IOBS)
         DO 314 I=1,NALT
            DO 314 J=1,NALT
               ELAS(I,J,K)=ELAS(I,J,K)+ELAS2(I,J,K)*WT
            CONTINUE
         CONTINUE
      DO 320 I=1,NALT
         DO 320 J=1,NALT
            DO 320 K=1,NXR

320 ELAS(I,J,K)=ELAS(I,J,K)/NOBS
WRITE (KANAL6,303) (I,I=1,NALT)
DO 321 K=1,NX
WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
DO 321 J=1,NALT
WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,K),I=1,NALT)
IF (NOAGE) GOTO 326
DO 322 K1=1,NXD
WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
DO 323 I=1,NALT
323 PROBS(I)=0.DO
DO 324 J=1,NDUMD
   K=NX+(K1-1)*NDUMD+J
WRITE (KANAL6,305) PNAME(K),J,(ELAS(I,J,NX+K1),I=1,NALT)
DO 325 I=1,NALT
325 PROBS(I)=PROBS(I)+ELAS(I,J,NX+K1)
324 CONTINUE
WRITE (KANAL6,305) SUMME,K1,(PROBS(I),I=1,NALT)
322 CONTINUE
326 WRITE (KANAL6,301) (DASH7(I),I=1,NALT)
GOTO 1

C ********************************************
C * COVARIANCE MATRIX *
C ********************************************

400 IF (IDATA.EQ.0) GOTO 5
IRED0=0
IER=0
WRITE (KANAL6,401)
401 FORMAT(' COVARIANCES : 1=PRINT, 2=RECALCULATION, 3=BOTH' '*/ RECALCULATION: 1=BHflH, 2=SIMPLE, 3=SYM., 4=MNL >1)
READ (KANAL5,*) MATPRT,IDIFF2
IF (.NOT.LEVEL1 .AND.IDIFF2.EQ.4) IDIFF2=2
IF (MATPRT .EQ. 1) GO TO 430
CALL FUNC(PARM,NP,XLF,*440)

C RECALCULATION OF COVARIANCE-MATRIX
C NOTE: COV = -INV(HESS) = -INV(MBHHH) = -INV(-GRAD*GRAD')

420 CALL FP(PARM,NP,XLF,GRAD,FUNC)
CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
C
C NOTE : IF IDIFF2=1, FP RECALCULATES MBHHH = -GRAD*GRAD'
C SP REFILLS HESS
C IF IDIFF2=2,3 FP CALCULATES GRAD
C SP CALCULATES FINITE DIFFERENCES OF GRAD
C IF IDIFF2=4, FP CALCULATES EXACT DERIVATIVES FOR MNL
C SP REFILLS HESS
C
C INVERSION OF HESSIAN AND CONTROL OF EIGENVALUES
C IERINV=1
CALL MATEV2(HESS,SCRA,NP,NP,MAXNP,IERINV)

.
IF (IERINV.GT.1) IER=-8
WRITE (KANAL6,422)
422 FORMAT(/' EIGENVALUES OF HESSIAN:'
* /' ----------------------------------')
DO 423 I=1,NP
    WORK(I)=HESS(I,I)
    IF (WORK(I).NE.O.O) GOTO 423
    WORK(I)=O.D0
    IER=-9
    IERINV=2
423 WRITE (KANAL6,'(I3,2X,D10.4)') I,WORK(I)
    IF (IERINV.EQ.2) WRITE (KANAL6,421)
421 FORMAT(' COV-MATRIX NOT INVERTIBLE')
DO 425 I=1,NP
    DO 425 J=1,I
        ZZZ = O.D0
        DO 426 K=1,NP
            ZZZ = ZZ+SCRA(I,K)*SCRA(J,K)/WORK(K)
        CONTINUE
        HESS(I,J) = ZZ
        HESS(J,I) = ZZ
    CONTINUE
CALL WESML(PARM,GRAD,HESS,NP,SCRA,FUNC)
C C PRINT RESULTS ANEW C C
500 IF (IREDO.EQ.1) GOTO 150
METHOD=2+IDIFF2
ITERAL=0
NEVALF=0
GOTO 150
C C PRINT COVARIANCE-MATRIX C C
430 WRITE (KANAL6,432)
432 FORMAT(' COVARIANCE-MATRIX ://')
DO 435 I = 1,NP
    DO 435 J = 1,I
        ZZZ = -HESS(I,J)
        WRITE (KANAL6,436) PNAME(I),PNAME(J),ZZZ
    CONTINUE
436 FORMAT(3X,A8,3X,A8,3X,G20.7)
435 CONTINUE
GOTO 1
440 WRITE (KANAL6,441)
441 FORMAT(' INADMISSIBLE INITIALVALUES')
GOTO 1
C C *******************************************
C * DERIVATIVE CHECK *
C *******************************************
C 500 IF (IDATA.EQ.0) GOTO 5
CALL FUNC(PARM,NP,XLF,*530)
WRITE (KANAL6,501) XLF
DERIVATIVE CHECK: LIKELIHOOD = \( F_{10.4} \)
* HIT ENTER TO QUIT, IDIFF2=1-4 IF SECOND DERIVATIVES, 2 IF ONLY FIRST DERIVATIVES, 5 FOR CHECK OF SSQDEV >
READ (KANAL5,'(Il)') IDIFF2
IF (IDIFF2.EQ.0) GOTO 1
IF (IDIFF2.EQ.5) GOTO 540
WRITE (KANAL6,502)

PARAMETER ANALYTIC DIFFERENCE
CALL FP(PARM,NP,XLF,GRAD,FUNC)
CALL FDIFF(PARM,NP,XLF,GRAD2,FUNC)
DO 510 I=1,NP
  WRITE (KANAL6,511) PNAME(I),GRAD(I),GRAD2(I)
  511 FORMAT (1X,A8,10X,2D13.5)
WRITE (KANAL6,512)

HIT ENTER TO QUIT, IDDIFF2=1-4 FOR SECOND DERIV. >
READ (KANAL5,4) IDIFF2
IF (IDIFF2.EQ.0) GOTO 1
IF (IDIFF2.EQ.2.OR.IDIFF2.EQ.3.OR.(IDIFF2.EQ.4.AND..NOT.LEVEL1)) THEN
  WRITE (KANAL6,*) 'ERROR: IDIFF2=',IDIFF2
  GOTO 1
END IF
CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
CALL SDIFF(PARM,NP,XLF,GRAD,HESS,FUNC)
( NOTE: MBHHH IS NOT ALTERED BY CALLING SDIFF)
C
  KK=0
  DO 520 I=1,NP
    DO 520 J=1,I
      KK=KK+l
      ZZZ=MBHHH(KK)
  520 WRITE (KANAL6,521) PNAME(I),PNAME(J),ZZZ,HESS(I,J)
  521 FORMAT (1X,A8,1X,A8,1X,2D13.5)
GOTO 1
  530 WRITE (KANAL6,531)
  531 FORMAT ( ' INADMISSIBLE INITIALVALUES')
  GOTO 1
  C
  IF (NXA.NE.1) GOTO 549
  N1=NX+NXD*NDUMD
  DO 542 I=1,NALT1
    PDUMMY(I)=PARAM(N1+I)
  542 NDUMMY(I)=PNAME(N1+I)
  CALL SSQDEV(PDUMMY,NALT1,SSD,*530)
  CALL FPSSQD(PDUMMY,NALT1,SSD,PROBS,PROBS,SSQDEV)
  ACC=0.00001
  WRITE (KANAL6,543) SSD
  543 FORMAT ( ' SUM OF SQUARED DEVIATIONS =',F10.4)
  WRITE (KANAL6,502)
  DO 545 I=1,NALT1
    PD=PDUMMY(I)
    PDUMMY(I)=PD+ACC
    CALL SSQDEV(PDUMMY,NALT1,SSD2,*530)
    PDUMMY(I)=PD
    SQDIFF=(SSD2-SSD)/ACC
WRITE (KANAL6,511) NDMY(I),PROBS(I),SQDIFF
GOTO 1
WRITE (KANAL6,*), 'INADMISSIBLE NXA SETTING'
GOTO 1
C
C ********************
C  * PRINT DATA  *
C ********************
C
IF (IDATA.EQ.0) GOTO 5
WRITE (KANAL6,601)
FORMAT(//' FIRST AND LAST OBSERVATION >')
READ (KANAL5,*), IOBS1,IOBS2
DO 606 IOBS=IOBS1,IOBS2
DO 606 J=1,NALT
WRITE (KANAL6,911) DATA1(I,J,IOBS),
* (DATA1(I,J+1,IOBS),K=1,NX),(DATA2(K,IOBS),K=1,NXD)
GOTO 1
C
C ********************
C  * ELIMINATE SINGLE DATA POINTS  *
C ********************
C
WRITE (KANAL6,*), 'ENTER NUMBER OF OBSERVATIONS TO BE ELIMINATED >'
READ (KANAL5,*), NSKIP
WRITE (KANAL6,*), 'NOW ENTER OBSERVATIONS IN DESCENDING ORDER >'
DO 621 ISKIP=1,NSKIP
WRITE (KANAL6,*), 'IOBS >'
READ (KANAL5,*), IOBS
DO 625 IOBS=IOBS+1,NOBS+1
DO 626 I=1,NXR
DATA1(I,K,IOBS-1)=DATA1(I,K,IOBS)
DO 627 K=1,NXD
DATA2(K,IOBS-1)=DATA2(K,IOBS)
CONTINUE
NOBS=NOBS-1
CONTINUE
GOTO 1
C
C ********************
C  * PLOT OF CONTOUR LINES  *
C ********************
C
IF (IDATA.EQ.0) GOTO 5
WRITE (KANAL6,*), 'SELECT TWO VARIABLES TO PLOT, REST IS CONSTANT'
DO 652 K=1,NP
WRITE (KANAL6,653) K,PNAME(K)
FORMAT(',I3,=',A8)
WRITE (KANAL6,654)
FORMAT(//' ENTER (1) TWO INDICES FOR THE TWO VARIABLES TO PLOT,'
* /' (2) RANGE OF FIRST VARIABLE (XMIN,XMAX) >'
* /' (3) RANGE OF SECOND VARIABLE (YMIN,YMAX) >')
READ (KANAL5,*) IND1,IND2,XMIN,XMAX,YMIN,YMAX
CALL FP(PARM,NP,XLF,GRAD,FUNC)
CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
CALL LMSTAT(NP,NP1,MTH,NLMST,GRAD,HESS,TEST)

XLABEL=PNAME(IND1)
YLABEL=PNAME(IND2)
CALL CNTR(PARM,NP,IND1,IND2,XMIN,XMAX,YMIN,YMAX,
* FUNC,XLABEL,YLABEL)
GOTO 1

C

***************
C * LM-STATISTIC *
C

700 IF (IDATA.EQ.0) GOTO 5
WRITE (KANAL6,701)
701 FORMAT(' 2ND DERIVATIVES: 1=BHHH, 2=SIMPLE, 3=SYMM. >')
READ (KANAL5,*) IDIFF2
WRITE (KANAL6,702)
702 FORMAT(' DEGREES OF FREEDOM (= # OF DISS.PARMS TO BE TESTED) >')
READ (KANAL5,*) NP1
IF (NP1.GT.5) THEN
  WRITE (KANAL6,*) 'ERROR: NP1 > 5'
  GOTO 1
END IF
WRITE (KANAL6,703)
703 FORMAT(' INDICES OF DISS. PARAMETERS IN PARM >')
READ (KANAL5,*) (NLMST(I),I=1,NP1)

C

CALL FP(PARM,NP,XLF,GRAD,FUNC)
CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
CALL LMSTAT(NP,NP1,MTH,NLMST,GRAD,HESS,TEST)

C

WRITE (KANAL6,709) TEST
709 FORMAT(//' LM-STATISTIC : ',F15.5/' ',30(('-'»

***********-****************************************************
TRANSFORM PARAMETERS TO BE COMPATIBLE WITH MNL (OBSOLETE)
~***************************************************************

IF (ISTRUC.EQ.0 .OR. NTH.EQ.0 .OR. ICOVAR.EQ.0) GaTO 5
WRITE (KANAL6,781)
781 FORMAT(///' TRANSFORMED VARIABLES :
* / PARAMETER / SIM.COEFF MEAN STD.ERROR T-STAT'/)
DO 782 I=1,NX
DO 782 K=1,NTH
  KP=MTH+K
  CALL TRAFO(PARM,NP,I,KP,HESS,C,SC,W)
  WRITE (KANAL6,783) PNAHE(I),PNAME(KP),C,SC,W
WRITE (21,783) PNAME(I),PNAME(KP),C,SC,W
782 FORMAT(' ',AS,' / ',AS,' :',2X,F8.4,2X,F7.3,2X,F7.3)
IF (NOAGE.AND.NOALT) GOTO 1
IF (.NOT.STDDUM) GOTO 1
K=MTH
1=0
DO 784 I1=1,NL1
  ML2=NL2(I1)
DO 784 I2=1,ML2
   ML3=ML3(I1,I2)
   IF (THETA(I1,I2).NE.1.D0) GOTO 785
   I=I+ML3
   GOTO 784
785   K=K+1
   DO 788 I3=1,ML3
       I=I+1
       IF (I.EQ.NALT) GOTO 1
       IF (NOAGE) GOTO 786
       DO 787 M=1,NXD
           KK=(M-1)*(NALT-1)+NX+I
           CALL TRAFO(PARM,NP,KK,K,HESS,C,SC,W)
           WRITE (KANAL6,783) PNAME(KK),PNAME(K),C,SC,W
           WRITE (21,783) PNAME(KK),PNAME(K),C,SC,W
       CONTINUE
       CONTINUE
       GOTO 1
   CONTINUE
C
C  *********************************************************************************
C  * AIRUM-CHECK (NOTE: SPECIAL CASE ONLY) * 
C  *********************************************************************************
C
790   CALL AIRUM(PARM,NP,KANAL6)
   GOTO 1
C
C  *********************************************************************************
C  * ADJUST AGGREGATED SAMPLE SHARES WITH LEAST SQUARES  * 
C  *********************************************************************************
C
800   IF (NXA.NE.1) GOTO 840
C
   ISTOP(1)=0
   ISTOP(2)=0
   ISTOP(3)=0
   IERLS=2*NALT
   WRITE (KANAL6,811)
     811 FORMAT(/' TYPE 1 TO OVERRIDE DEFAULT ITERATION PARAMETERS >'
     READ (KANAL5,'(I1)') IQUERY
     IF (IQUERY.EQ.1) THEN
       WRITE (KANAL6,812)
       812 FORMAT(' ITERATION LIMIT >'
     * /' CONV. CRITERION (PARM,GRAD,SSD; 0=ENABLE,1=DISABLE) >'
     * READ (KANAL5,*) IERLS,ISTOP
     END IF
   NQ=NALT1*NALT1+8*NALT1
C
   M1=NX+NXD*NDUMD
   DO 820 I=1,NALT1
820 PDUMMY(I) = PARM(N1+I)
CALL LSQFIT(PDUMMY,NALT1,SSD,IERLS,SSQDEV,ITERLS)
WRITE (KANAL5,821) SSD,IERLS,ITERLS
821 FORMAT('/' ALTERNATIV SPECIFIC DUMMIES ADJUSTED:'
  /* SUM OF SQUARED DEVIATIONS:',F11.5
  /* DA.FL.PO. ITERATIONS ('I2','):',I11
  /* DUMMY INITIALVALUE ADJUSTED VALUE')
DO 822 I=1,NALT1
  WRITE (KANAL5,823) PNAME(N1+I),PARM(N1+I),PDUMMY(I)
823 FORMAT(' ',A8,3X,F9.4,9X,F9.4)
822 PARM(N1+I)=PDUMMY(I)
C
CALL FUNC(PARM,NP,XLF,*830)
DO 825 I=1,NP
  GRAD(I)=0.0
  HESS(I,I)=0.0
825 ITERAL=ITERLS
NEVALF=0
IER=IERLS
METHOD=7
MATPRT=0
GOTO 150
C
830 WRITE (KANAL5,831)
831 FORMAT(' ERROR: FUNCTION UNDEFINED AT ADJUSTED DUMMIES')
GOTO 1
C
840 WRITE (KANAL5,841) NXA
841 FORMAT(' INCORRECT SETTING FOR ALTERNATIVE SPECIFIC DUMMIES:'
  /* NXA = ',I1',.NE. 1 FOR CORRECT INITIALIZATION')
GOTO 1
C
C ************
C * EXIT *
C ************
C
99 RETURN
END

SUBROUTINE START(NPRINT,KANAL5,KANAL6,UPP,LOW,VERSIO)
------------------------------------------------------------------
REAL*8 UPP,LOW
CHARACTER*21 VERSIO
C
IBM
KANAL5=5
IMPLICIT REAL*8 (A-H,O-Z)

PARAMETER
* (MAXOBS =2001,
* MAXNX =10,
* MAXNXD =5,
* MAXALT =20,
* MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD)

REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS),DATA2(MAXNXD,MAXOBS)
CHARACTER*20 DAFILE
COMMON / DIMEN / NALT,NALT1,NX,NXD,NXR,MXA,MTH,MTAU,NOBS
COMMON / DATAX / DATA1
COMMON / DATAY / DATA2
COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)

STORAGE FOR MEANS : NDW = MAXOBS, THEN NOBS+2
ND1 = MAXOBS-1, THEN NOBS+1

NDD=MAXOBS-2
ND1=MAXOBS-1
NDW=MAXOBS

WRITE (KANAL6,10)
10 FORMAT(// ' ENTER DATA-INPUT-FILE >')
READ (KANAL5,'(A20)') DAFILE
OPEN (UNIT=7,STATUS='OLD',FILE=DAFILE,ACCESS='SEQUENTIAL')
WRITE (KANAL6,12)
12 FORMAT(' ENTER MAX. NUMBER OF OBSERVATIONS >')
READ (KANAL5,*) NNOBS
IF (NNOBS.EQ.0) NNOBS=NDD
IF (NNOBS.GT.NDD) THEN
   WRITE(KANAL6,*) 'WARNING: NOBS SET TO ',NDD
   NNOBS=NDD
END IF

C INITIALIZATIONS FOR MEANS AND COUNT
C
NOBS=0
DO 20 I=1,NALT
   DATA1(I,1,ND1)=0.0
   DATA1(I,1,NDW)=0.0
   DO 20 J=1,NX
      DATA1(I,J+1,ND1)=0.0
      DATA1(I,J+1,NDW)=0.0
20 CONTINUE
DO 20 J=1,NXD
   DATA2(J,ND1)=0.0
20 CONTINUE

C LOOP THROUGH DATA
C
DO 100 IOBS=1,NNOBS
   IC=0
   DO 120 I=1,NALT
      READ (7,END=200) DATA1(I,1,IOBS),
         *(DATA1(I,J+1,IOBS),J=1,NX),(DATA2(J,IOBS),J=1,NXD)
      IF (DATA1(I,1,IOBS).EQ.1.) THEN
         IC=I
         WT=WEIGHT(IC)
      END IF
   120 CONTINUE
   IF (IC.EQ.0) WRITE (KANAL6,130) IOBS
   130 FORMAT(' ERROR: NO ALTERNATIVE CHOSEN IN OBSERVATION',I6)
   IF (ABS(DATA1(I,J+1,IOBS)).GT.10000.0) WRITE (1,141) J+1,IOBS,DATA1(I,J+1,IOBS)
      141 FORMAT(' DATA CHECK: X(KALT,K,IOBS)=',3I3,G20.10)
   DATA1(IC,1,ND1) = DATA1(IC,1,ND1) + 1.0
   DATA1(IC,1,NDW) = DATA1(IC,1,NDW) + WT
   DO 140 I=1,NALT
      DO 140 J=1,NX
         IF (ABS(DATA1(I,J+1,IOBS)).GT.10000.0) WRITE (1,141) J+1,IOBS,DATA1(I,J+1,IOBS)
            141 FORMAT(' DATA CHECK: X(KALT,K,IOBS)=',3I3,G20.10)
      DATA1(I,J+1,ND1) = DATA1(I,J+1,ND1) + DATA1(I,J+1,IOBS)
      DATA1(I,J+1,NDW) = DATA1(I,J+1,NDW) + DATA1(I,J+1,IOBS)*WT
   DO 145 J=1,NXD
      IF (ABS(DATA2(J,IOBS)).GT.10000.0) WRITE (1,146) J,IOBS,DATA2(J,IOBS)
         146 FORMAT(' DATA CHECK: Y(K,IOBS)=',2I3,G20.10)
      DATA2(J,ND1) = DATA2(J,ND1) + DATA2(J,IOBS)
DATA2(J,NDW) = DATA2(J,NDW) + DATA2(J,IOBS)*WT
C
NOBS=NOBS+1
100 CONTINUE
C
SUMMARY
C
200 CLOSE (UNIT=7)
IF (NOBS.EQ.0) THEN
  WRITE (KANAL6,*) 'ERROR: DATA-FILE EMPTY'
  RETURN
END IF
SUM=O.DO
DO 220 I=1,NALT
  SUM=SUM+DATA1(I,1,ND1)*WEIGHT(I)
  DATA1(I,1,NOBS+1)=DATA1(I,1,ND1)/NOBS
  DATA1(I,1,NOBS+2)=DATA1(I,1,NDW)/NOBS
DO 220 J=1,NX
  DATA1(I,J+1,NOBS+1)=DATA1(I,J+1,ND1)/NOBS
  DATA1(I,J+1,NOBS+2)=DATA1(I,J+1,NDW)/NOBS
DO 221 J=1,NXD
  DATA2(J,NOBS+1)=DATA2(J,ND1)/NOBS
  DATA2(J,NOBS+2)=DATA2(J,NDW)/NOBS
C
IF (DABS(SUM-NOBS).GT.1) WRITE (KANAL6,229) SUM,NOBS
229 FORMAT(' ERROR: WEIGHTS DO NOT SUM UP TO NOBS:G,G20.10,10)
C
RETURN
END
SUBROUTINE UPDATE(PARM,NP,*)
C----------------------------------------------------------------------
C UPDATES THE PARAMETERS IN COMMON BLOCKS:
C (1) DISSIMILARITY PARAMETERS
C (2) PARAMETERS FOR TREE DUMMIES
C----------------------------------------------------------------------
REAL*8 PARM(NP)
C
PARAMETER
* (MAXNXD =5, *
* MAXNXM =20, *
* MAXTH =10, *
* MAXTAU =5, *
* MAXLEV =3, *
* MAXLIM =5, *
* MAXBRA =5, *
* MAXALT =20)
C
REAL*8 THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
* PMASKY(MAXNXD,MAXALT),PMAKD(MAXALT),P
INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT)
LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDUM,NOALT,NOAGE,
* L1,L2,L3,STDUMA,STDUMD,NALSTD,NALTRE,NAGSTD,NAGTRE
COMMON / DHEN / NALT,NALT1,NX,NXD,NXA,NXR,NXH,MTH,MTAU,NOBS
COMMON / DDISS / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
* \text{COMMON / DTREE / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2}
\text{COMMON / DMAPP / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP}
\text{COMMON / DDUMM / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,}
* \text{NOALT,NOAGE,INDAGE(MAXNXD,MAXALT),}
* \text{NALSTD,NALTRE,NAGSTD,NAGTRE}

C
NFAIL=0
IF (NTAU+NTH.EQ.0) GOTO 20

C
IF (NTH.EQ.0) GOTO 9
C
DO 8 K=1,NTH
IF (PARM(MTH+K).LT.0.001) THEN
PRINT 91,K,PARM(MTH+K)
NFAIL=NFAIL+1
END IF
THETA(LOCLIM(K),LOCBRA(K))=PARM(MTH+K)
CONTINUE

C
9 IF (NTAU.EQ.0) GOTO 20
C
DO 10 K=1,NTAU
IF (PARM(MTAU+K).LT.0.001) THEN
PRINT 92,K,PARM(MTAU+K)
NFAIL=NFAIL+1
END IF
TAU(LOCTAU(K))=PARM(MTAU+K)
CONTINUE

C
20 IF (NXM.EQ.0) GOTO 90
C
DO 25 I=1,NALT
M1=MAP(I,1)
M2=MAP(I,2)
M3=MAP(I,3)
IF (M1.NE.0) THEN
L1=.TRUE.
ELSE
L1=.FALSE.
END IF
IF (M2.NE.0) THEN
L2=.TRUE.
ELSE
L2=.FALSE.
END IF
IF (M3.NE.0) THEN
L3=.TRUE.
ELSE
L3=.FALSE.
END IF
IF (NOALT) GOTO 26
P=0.DO
IF (L1) P=P+PARM(MAXA+M1)
IF (L2) P=P+PARM(MXA+M2)
IF (L3) P=P+PARM(MXA+M3)
PMASKD(I)=P
26 IF (NGAE) GOTO 25
DO 27 K=1,NXD
  P=0.DO
  IF (L1) P=P+PARM(INDAGE(K,M1))
  IF (L2) P=P+PARM(INDAGE(K,M2))
  IF (L3) P=P+PARM(INDAGE(K,M3))
  PMASKY(K,I)=P
27 CONTINUE
25 CONTINUE
C
90 IF (NFAIL.GT.0) RETURN 1
91 FORMAT(' ATTEMPT THETA(',I2,')','=D12.4)
92 FORMAT(' ATTEMPT TAU(',I1,')','=D12.4)
RETURN
END
C-----SUBROUTINE PACKAGE: FUNC,FP,SP,FDIFF,SDIFF---------------------
C
C AXEL BOERSCH-SUPAN SEP 06 , 1983
C
C----------------------------------------------------------------------
SUBROUTINE FUNC(PARM,NP,XLF,*)
C-------------------------~--------------------------------------------
C
C ** THIS SUBROUTINE EVALUATES AND ACCUMULATES THE LIKELIHOOD FUNCTION
C ** FOR UNCON=.FALSE. IT ADJUSTS AFTER SUCCESSFUL EVALUATION THE
C ALTERNATIVE SPECIFIC DUMMIES TO FIT THE AGGREGATE SAMPLE SHARES
C----------------------------------------------------------------------
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PAIDI(NP)
PARAMETER *
  (MAXOBS =2001,
   MAXNP =50,
   MAXNX =10,
   MAXNXD =5,
   MAXALT =20,
   MAXNXY=MAXNX+1,
   MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS,
   MAXALS=MAXALT*(MAXALT+1)/2)
REAL*8 WORK(MAXNP),PROBS(MAXALT),PDUMMY(MAXALT),NDUMMY(MAXALT)
REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)
LOGICAL UNCON
COMMON / DIMEN / NALT,NALT1,NPARM(7),NOBS
COMMON / DATAX / FDATA1
COMMON / DATAY / FDATA2
COMMON / DCONST / UNCON,PDUMMY,NDUMMY
COMMON / DWORK / WORK,PROBS
COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH
EXTERNAL SSQDEV

C
ILOOP=1
NFAIL=0
NOBSF=NOBS/10
CALL UPDATE(PARM,NP,*10)

C
C EVALUATION OF LIKELIHOOD

C
10 NN1=1
NN2=1
INC=MAXALT*MAXNXD
XLF=0.DO
DO 100 IOBS = 1,NOBS
NFAIL1=NFAIL
CALL CONT(FDATA1(NN1),FDATA2(NN2),
*       ILOOP,PARM,NP,XLFI,WORK,PROBS,NFAIL)
NN1=NN1+INC
NN2=NN2+MAXNXD
IF (NFAIL.GT.NFAIL1) IOBF=IOBS
IF (NFAIL.GT.NOBSF) GOTO 97
IF (NFAIL.GT.NFAIL1) GOTO 95
XLF = XLF + XLFI
CONTINUE

C
C SOLVE LEAST SQUARES PROBLEM TO FIT SAMPLE-SHARES

C
IF (UNCON) GOTO 90
ITERLS=2*NALT1
CALL LSFIT(PDUMMY,NALT1,SSD,IER,SSQDEV,ITERLS)
IF (IER.LT.0) GOTO 93
GOTO 90

C
C ERROR HANDLING, EXIT

C
RETURN
90 RETURN
93 WRITE (KANAL6,94) SSD,IER,ITERLS
94 FORMAT(' SHARES NOT EXACT: SSD=',D10.4,', IER=',I2,', ITERLS=',I2)
RETURN
95 IF (NFAIL.GT.0) WRITE (KANAL6,96) NFAIL,IOBF
96 FORMAT(' FUNC: NFAIL =',I2,', LAST OBS =',I4)
RETURN 1
97 WRITE (KANAL6,98) NFAIL
98 FORMAT(' FUNC: ABORT B/O MORE THAN ',I4,' FAILURES')
99 RETURN 1
END

C---------------------------------------------------------------------
SUBROUTINE FP(PARM,NP,XLF,GRAD,FUNC)
C---------------------------------------------------------------------

**THIS SUBROUTINE EVALUATES THE GRADIENT OF THE LIKELIHOOD-FUNCTION**

(However, it will NOT and MUST NOT update XLF)

**FOR IDIFF2=1: THE BHHH APPROXIMATION OF THE HESSIAN IS PROVIDED**

[NOTE: HESSIAN = -GRAD*GRAD' = - INV(COV)]

**FOR IDIFF2=4: THE EXACT HESSIAN IS CUMULATED**

IMPLICIT REAL*8 (A-H,O-Z)

REAL*8 PARM(NP),GRAD(NP)

PARAMETER
* (MAXOBS =2001,
  * MAXNP =50,
  * MAXNX =10,
  * MAXNXD =5,
  * MAXALT =20,
  * MAXNNY=MAXNX+1, MAXNPS=MAXNP*(MAXNP+1)/2,
  * MAXDA1=MAXALT*MAXNNY*MAXOBS, MAXDA2=MAXNXD*MAXOBS)

REAL*8 WORK(MAXNP),PROBS(MAXALT),HESS(MAXNPS),HESS2(MAXNPS)
REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)

COMMON / DIMEN / NPARM(9),NOBS
COMMON / DATAX / FDATA1
COMMON / DATAY / FDATA2
COMMON / DWORK / WORK,PROBS
COMMON / DCOV/M / HESS
COMMON / DHESS / HESS2
COMMON / DDIFF2 / IDIFF2
COMMON / DBOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
COMMON / DBPRINT / IPT,KANAL6,NDIG,NPUNCH
EXTERNAL FUNC

ILOOP=0
NFAIL=0
CALL UPDATE(PARM,NP,*92)

EVALUATION OF GRADIENT (AND HESSIAN)

DO 5 I=1,NP
  GRAD(I)=0.DO
  IF (IDIFF2.EQ.2.OR.IDIFF2.EQ.3) GOTO 12
  LK = NP*(NP+1)/2
  DO 10 I=1,LK
    HESS(I)=0.DO
  10 NN1=1
  NN2=1
  INC=MAXALT*MAXNNY
  DO 100 IOBS = 1,NOBS
    NFAIL1=NFAIL
    CALL CONT(FDATA1(NN1),FDATA2(NN2),
      * ILOOP,PARAM,NP, XLFI, WORK, PROBS, NFAIL)
    NN1=NN1+INC
NN2=NN2+MAXNXD
IF (NFAIL.GT.NFAIL1) IOBF=IOBS
DO 30 J=1,NP
  GRAD(J) = GRAD(J) + WORK(J)
C
IF (IDIFF2.EQ.1) GOTO 50
IND=0
DO 40 J=1,NP
DO 40 K=1,J
  IND=IND+1
  HESS(IND) = HESS(IND) - WORK(J) * WORK(K)
GOTO 100
C
50 IF (IDIFF2.EQ.4) GOTO 100
DO 55 K=1,LK
  HESS(K) = HESS(K) + HESS2(K)
C
100 CONTINUE
C
IVAL=IVAL+2
IF (NFAIL.GT.0) WRITE (KANAL6,97) NFAIL,IOBF
97 FORMAT(' FP: NFAIL=',I4,' LAST OBS=',I5)
RETURN
92 WRITE (KANAL6,*),' FP: DISS. PARM < 0.01'
RETURN
END

-------------------------------------------------------------------
SUBROUTINE SP(PARM,NP,XLF,GRAD,SDERIV,FUNC)
-------------------------------------------------------------------

ARRAY OF SECOND DERIVATIVES:

** IDIFF2=1 : FILLS BHHH-TRIANGLE MBHHH ON ARRAY SDERIV
** IDIFF2=2 : COMPUTES SIMPLE FINITE SECOND DIFFERENCES
** IDIFF2=3 : COMPUTES SYMMETRIC FINITE SECOND DIFFERENCES
** IDIFF2=4 : FILLS TRIANGLE WITH ANAL. DERIVS ON SDERIV

FOR FINITE DIFFERENCES, GRAD HAS TO BE SUPPLIED

-------------------------------------------------------------------
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PARM(NP),GRAD(NP),SDERIV(NP,NP)

PARAMETER *
  (MAXNP =50,
  MAXNPS=MAXNP*(MAXNP+1)/2)
REAL*8 MBHHH(MAXNPS)

COMMON / DCOVM / MBHHH
COMMON / DIMEN / NPARM(9),NOBS
COMMON / DIFF2 / IDIFF2
COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
COMMON / BSTACK/ A(1)
COMMON / BOPT1 / NPARS,JH,JFP,JFP1,JSP,JSP1,JA1,JS
COMMON / BFIDIF/ FDFRAC,FDMIN
EXTERNAL FUNC

C
IF (IDIFF2.EQ.2 .OR. IDIFF2.EQ.3) GOTO 40
C
REFILL OF SDERIV (IDIFF2=1,4)
C
IND=0
DO 35 J=1,NP
DO 35 K=1,J
   IND=IND+1
   ZZZ=MBHHH(IND)
   SDERIV(J,K)=ZZZ
   SDERIV(K,J)=ZZZ
35
RETURN
C
COMPUTATION OF FINITE DIFFERENCES (IDIFF2=2,3)
C
40 DO 90 I=1,NP
   AI=PARM(I)
   EI=DHAX1(DABS(AI)*FDFRAC,FDMIN)
   PARM(I)=AI+EI
   CALL FP (PARM,NP,XLF2,A(JA1),FUNC)
   JA2I=JA1+I-1
   IF (IDIFF2.EQ.2) GO TO 475
C
SYMMETRIC FIRST DIFFERENCES OF GRAD
C
   PARM(I)=AI-EI
   CALL FP (PARM,NP,XLF1,A(JA1+NP),FUNC)
   JA1I=JA2I+NP
   SDERIV(I,I)=(A(JA2I)-A(JA1I))/(2.DO*EI)
   DO 410 J=1,NP
      IF (J.EQ.I) GO TO 410
      JA2J=JA1+J-1
      SDERIV(I,J)=(A(JA2J)-A(JA2J+NP))/(2.DO*EI)
      IF (J.GT.I) GO TO 410
      SDERIV(I,J)=(SDERIV(I,J)+SDERIV(J,I))/2.0DO
      SDERIV(J,I)=SDERIV(I,J)
410 CONTINUE
   GO TO 90
C
SIMPLE FIRST DIFFERENCES OF GRAD
C
475 SDERIV(I,I)=(A(JA2I)-GRAD(I))/EI
   K=I-1
   IF (K.EQ.0) GO TO 90
   DO 476 J=1,K
      SDERIV(I,J)=(A(JA1+J-1)-GRAD(J))/EI
      SDERIV(J,I)=SDERIV(I,J)
476 CONTINUE
90 PARM(I)=AI
C
RETURN
SUBROUTINE FDIFF(AO,NP,AFUO,FPD,FUNC)
C------------------------------------------------------------------
C FIRST ORDER SYMMETRIC DIFFERENCE QUOTIENT (CENTRAL APPROX.)
C------------------------------------------------------------------
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AO(NP),FPD(NP)
COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
DO 90 I=1,NP
   AI=AO(I)
   EI=DMAX1(DABS(AI)*.1D-3,.1D-5)
   AO(I)=AI+EI
   CALL FUNC(AO,NP,AFU2,*85)
   IVAL=IVAL+1
   AO(I)=AI-EI
   CALL FUNC(AO,NP,AFU1,*85)
   IVAL=IVAL+1
   FPD(I)=(AFU2-AFU1)/(2.DO*EI)
GO TO 90
85 EI=EI/10.DO
IF (EI.LE.1.D-10) GOTO 130
GOTO 11
90 AO(I)=AI
RETURN
130 AO(I)=AI
IER=-2
RETURN
END

SUBROUTINE SDIFF(AO,NP,AFUO,FPD,SPD,FUNC)
C--------------------------------------------~----------------------
C SECOND ORDER SYMMETRIC DIFFERENCE QUOTIENT
C
C STAR: AFUll AFU21
C AFU1 AFU0 AFU2
C AFU12 AFU22
C------------------------------------------~----------------------
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AO(NP),SPD(NP,NP),FPD(NP)
COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
DO 90 I=1,NP
   AI=AO(I)
   EI=DMAX1(DABS(AI)*.1D-3,.1D-5)
   AO(I)=AI+EI
   CALL FUNC(AO,NP,AFU2,*85)
   IVAL=IVAL+1
   AO(I)=AI-EI
   CALL FUNC(AO,NP,AFU1,*85)
   IVAL=IVAL+1
   SPD(I,I)=(AFU2-2.DO*AFUO+AFU1)/(EI*EI)
   IF (I.EQ.1) GOTO 90
   K=I-1
   DO 80 J=1,K
   SPD(I,J)=(AFU2-2.DO*AFU0+AFU1)/(EI*EI)
   GO TO 90
80 SPD(I,J)=(AFU2-2.DO*AFU0+AFU1)/(EI*EI)
GO TO 90
90 AO(I)=AI
RETURN
130 AO(I)=AI
IER=-2
RETURN
END
THREE LEVEL NESTED LOGIT MODEL

\[
\begin{align*}
AJ &= A0(J) \\
EJ &= \text{DMAX1(DABS}(AJ) \cdot 1D-3, 1D-5) \\
A0(J) &= AJ + EJ \\
A0(I) &= AI + EI \\
\text{CALL FUNC(A0, NP, AFU21, *70)} \\
IVAL &= IVAL + 1 \\
A0(I) &= AI - EI \\
\text{CALL FUNC(A0, NP, AFU11, *70)} \\
IVAL &= IVAL + 1 \\
A0(J) &= AJ - EJ \\
\text{CALL FUNC(A0, NP, AFU12, *70)} \\
IVAL &= IVAL + 1 \\
A0(I) &= AI + EI \\
\text{CALL FUNC(A0, NP, AFU22, *70)} \\
IVAL &= IVAL + 1 \\
SPD(I, J) &= (AFU21 - AFU11 - AFU22 + AFU12) / (4 \cdot D0 \cdot EI \cdot EJ) \\
SPD(J, I) &= SPD(I, J) \\
\text{GOTO 80} \\
EJ &= EJ / 10.D0 \\
\text{IF (EJ.LE.1.D-10) GOTO 125} \\
\text{GOTO 30} \\
A0(J) &= AJ \\
GOTO 90 \\
A0(I) &= AI \\
\text{RETURN} \\
\text{IER} &= -2 \\
\text{RETURN} \\
\text{END}
\end{align*}
\]

SUBROUTINE CONT(DATA1, DATA2, ILOOP, PARM, NP, XLFI, WORK, PROBS, NFAIL)

THREE LEVEL NESTED LOGIT MODEL

CONTRIBUTION OF THE N-TH OBSERVATION
- TO THE LIKELIHOOD (XLFI, ILOOP=0,1,2)
- TO ITS DERIVATIVE (WORK, ILOOP=0)
- TO THE HESSIAN (HESS, ILOOP=0 AND IDIFF2=4, MNL ONLY)
- PROBABILITY-SHARE (PROBS, ILOOP=2)
EXCEPT ILOOP=0, WORK(1) CARRIES UTILITY-LEVEL INC

NORMALIZED VERSION OCT 31, 1983 23.00 AXEL BOERSCH-SUPAN

TREE STRUCTURE:

LEVEL: DATA: PARMS: EXP(INCL.VALUE): DISSIMILARITY-PARMS:
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PARM(NP)
REAL*4 DATA1(*),DATA2(*)

PARAMETER
* (MAXNP =50,
* MAXTH =10,
* MAXTAU =5,
* MAXNX =10,
* MAXNXD =5,
* MAXNXM =20,
* MAXLEV =3,
* MAXLIM =5,
* MAXBRA =5,
* MAXALT =20,
* MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXD, MAXNPS=MAXNP*(MAXNP+1)/2

REAL*8 UPP,LOW,ONE/1.DO/,ZERO/0.DO/
INTEGER IZERO/0/

REAL*8 THETA(MAXLIK,MAXBRA),TAU(MAXLIM)IWORK(MAXNP),
* EXB(MAXALT),EZG(MAXALT),EYA(MAXALT),
* XB(MAXALT),ZG(MAXALT),YA(MAXALT),
* EINC2(MAXALT,MAXBRA),EINC1(MAXALT),EINCO,
* INC2(MAXLIM,MAXBRA),INC1(MAXLIM),INC0,INC2C,INC1C,
* DER2B(MAXLIM,MAXBRA,MAXNP),DER1B(MAXLIM,MAXNP),
* DER2T(MAXLIM,MAXBRA,MAXTH),DER1T(MAXLIM,MAXTH),
* DER1U(MAXLIM,MAXTAU),PROBS(MAXALT),
* PDUMMY(MAXALT),NDUMMY(MAXALT),
* PMASKY(MAXNXD,MAXALT),PMASKD(MAXALT),
* ZBAR(MAXNP),ZIZBAR(MAXALT,MAXNP),HESS(MAXNPS)
INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM),MAPTR(MAXNXM,MAXALT)
LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDDUM,NOALT,NOAGE,DUMP,
* LDUMMY,STDUMA,STDUMD,NDUXA,NDUMD,

COMMON / DIMEN / NALT,NALT1,NX,NXD,NXA,NXR,MTA,MTH,MTAU,NOBS
COMMON / DDISS / LOCTAU(MAXTAU),LOCLIN(MAXTH),LOCBRA(MAXTH),
* NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
COMMON / DCONT / XB,ZG,YA,INC2,INC1,
* EXB,EZG,EYA,EINC2,EINC1,
* DER2B,DER1B,DER2T,DER1T,DER1U,
* ZBAR,ZIZBAR,UPP,LOW,
* MS3K(MAXTH),ML3K(MAXTH),MS2K(MAXLIM)
COMMON / DTREE / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
COMMON / DMAPP / NMX,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
COMMON / DDUMM / STDDUM,STDUMA,STDUMD,NDUXA,NDUMD,
COMMON / DHESS / HESS
COMMON / DIFF2 / IDIFF2
COMMON / DCONST / UNCON,PDUMMY,NDUMMY
COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)
COMMON / BPRINT / IPT,KAVAL,NDIG,NPUNCH

DUMP=.FALSE.

INNER PRODUCTS XB(I)

IALT=IZERO
XBM=ZERO
DO 9 I=1,NALT
   IF (DATA1(I).EQ.1) IALT=I
   XBC = ZERO
   NN1 = MAXALT
   DO 7 K=1,NX
      XBC = XBC + DATA1(NN1+I)*PARM(K)
   NN1 = NN1 + MAXALT
   IF (NAGTRE) GOTO 3
   DO 4 K=1,NXD
      XBC = XBC + DATA2(K)*PMASKY(K,I)
   3 IF (NALTRE) GOTO 2
      XBC = XBC + PMASKD(I)
   2 IF (I.EQ.NALT) GOTO 5
   IF (NAGSTD) GOTO 6
   DO 1 I=-1,NXD
   1 XBC = XBC + DATA2(K)*PARM(INDAGE(K,I))
   6 IF (NALSTD) GOTO 8
      XBC = XBC + PARM(MXA+I)
   8 IF (UNCON) GOTO 5
      XBC = XBC + PDUMMY(I)
   5 XB(I)=XBC
   XBM=XBM+XBC
9 CONTINUE

CHECK AND MEAN OF XB(I)

IF (IALT.EQ.0) GOTO 9005
XBM=XBM/NALT

INCLUSIVE VALUES

I=IZERO
IB=IZERO
EINCO=ZERO
DO 10 I=1,NL1
   ML2=ML2(I1)
   TA=TAU(I1)
   DINC1=ZERO
10 DO 20 I2=1,ML2
   IB =IB+1
ML3 = ML3(I1, I2)
TH = THETA(I1, I2)
THT = TH/TA
DINC2 = ZERO
DO 30 I3 = 1, ML3
   I = I + 1
   CHOICE
   IF (I .NE. IALT) GOTO 40
   IC1 = I1
   IC2 = I2
   IBPA = IB
   INNER PRODUCT XB/TH
   /* NORMALIZE BY SUBTRACTION OF XB */
   40 XB1 = (XB(I) - XBM) / TH
   IF (XB1 .GT. UPP) GOTO 9001
   XB(I) = XB1
   ZZZ = DEXP(XB1)
   EXB(I) = ZZZ
   DINC2 = DINC2 + ZZZ
   CONTINUE
   LOWER LEVEL INCLUSIVE VALUE INC2(I1, I2)
   IF (LEVEL1) GOTO 21
   IF (DINC2 .LT. LOW) GOTO 9002
   EINC2(I1, I2) = DINC2
   DINC2 = DLOG(DINC2)
   INC2(I1, I2) = DINC2
   DINC2 = THT * DINC2
   IF (DINC2 .GT. UPP) GOTO 9002
   ZG(IB) = DINC2
   DINC2 = DEXP(DINC2)
   EZG(IB) = DINC2
   21 DINC1 = DINC1 + DINC2
   CONTINUE
   HIGHER LEVEL INCLUSIVE VALUE DLOG(INC1(I1))
   EINC1(I1) = ONE
   IF (LEVEL2) GOTO 11
   IF (INC1(I1) .LT. LOW) GOTO 9003
   EINC1(I1) = DINC1
   DINC1 = DLOG(DINC1)
   INC1(I1) = DINC1
   DINC1 = TA * DINC1
   IF (INC1(I1) .GT. UPP) GOTO 9003
   YA(I1) = DINC1
   DINC1 = DEXP(DINC1)
   EYA(I1) = DINC1
   11 EINCO = EINCO + DINC1
10 CONTINUE
IF (EINCO.LT.LOW) GOTO 9004
INCO=DLOG(EINCO)
C
\begin{verbatim}
XBC  =XB(IALT)
INC2C=INC2(IC1,IC2)
INC1C=INC1(IC1)
C
LOG LIKELIHOOD
C
XLFI = XBC - INCO
WORK(1) = INCO
C WRITE (KANAL6,9900) XLFI
C9900 FORMAT(' $X$ MNL-XLFI=',D14.5)
IF (LEVEL1) GOTO 51
THC  = THETA(IC1,IC2)
TAC  = TAU(IC1)
TH1  = THC/TAC-ONE
XLFI = XLFI + TH1*INC2C
IF (LEVEL2) GOTO 50
TA1  = TAC-ONE
XLFI = XLFI + TA1*INC1C
C WRITE (KANAL6,9901) XLFI
C9901 FORMAT(' $X$ NMNL-XLFI=',D14.5)
C
50     IF (ILOOP.EQ.1) GOTO 5000
IF (ILOOP.EQ.0) GOTO 70
C
PROBABILITY-SHARES
C
51     DO 52 I=1,NALT
PROBS(I)=EXB(I)/EINCO
CONTINUE
IF (LEVEL1) GOTO 69
I=IZERO
DO 60 I1=1,NL1
ML2=NL2(I1)
IF (LEVEL2) GOTO 61
DINC1=(TAU(I1)-ONE)*INC1(I1)
CONTINUE
DO 60 I2=1,ML2
ML3=NL3(I1,I2)
DINC2=(THETA(I1,I2)/TAU(I1)-ONE)*INC2(I1,I2)
DO 60 I3=1,ML3
I=I+1
IF (LEVEL2) GOTO 62
PROBS(I)=PROBS(I)*DEXP(DINC1+DINC2)
GOTO 60
62     PROBS(I)=PROBS(I)*DEXP(DINC2)
CONTINUE
69 IF (ILOOP.EQ.0) GOTO 4000
GOTO 5000
C
C SWITCHES FOR DUMMY
\end{verbatim}
C
70  IF (IALT.EQ.NALT) THEN
    LDUMMY=.FALSE.
    ELSE
    LDUMMY=.TRUE.
    END IF

C
C  DERIVATIVES OF INCLUSIVE VALUES LEVEL 2
C
C  W.R.T. BETA
C
I=IZERO
DO 80 I1=1,NL1
   ML2=NL2(I1)
DO 80 I2=1,ML2
   ML3=NL3(I1,I2)
   DINC2=EINC2(I1,I2)*THETA(I1,I2)
   NN1=MAXALT
   DO 82 K=1,NX
      ZZ=ZERO
      DO 81 I3=1,ML3
         I=I+1
         ZZ=ZZ + EXB(I) * DATA1(NN1+I)
         81 CONTINUE
         I=I-ML3
         DER2B(I1,I2,K) = ZZ / DINC2
      C WRITE (KANAL6,9907) I1,I2,K,DER2B(I1,I2,K)
      C9907 FORMAT(' $X$ DER2B(',3I2,')=',D18.4)
      NN1=NN1+MAXALT
   82 CONTINUE
   C
   IF (NALTRE) GOTO 823
   DO 821 K=1,NXM
      ZZ=ZERO
      DO 822 I3=1,ML3
         I=I+1
         IF (LMAP(K,I)) ZZ=ZZ + EXB(I)
         822 CONTINUE
         I=I-ML3
         DER2B(I1,I2,K+MXA) = ZZ / DINC2
      C WRITE (KANAL6,9907) I1,I2,K+MXA,DER2B(I1,I2,K+MXA)
   821 CONTINUE
   C
   823 IF (NAGTRE) GOTO 826
   DO 824 K=1,NXM
      DO 824 Kl=.l,NXD
         KK=INDAGE(Kl,K)
         DO 825 I3=1,ML3
            I=I+1
            IF (LMAP(K,I)) ZZ=ZZ + EXB(I)*DATA2(K1)
            825 CONTINUE
            I=I-ML3
            DER2B(I1,I2,KK) = ZZ / DINC2
WRITE (KANAL6,9907) I1,I2,KK,DER2B(I1,I2,KK)

CONTINUE

IF (NAGSTD) GOTO 829

DO 827 I3=1,ML3
   I=I+1
   IF (I.EQ.NALT) GOTO 828
   DO 827 K1=1,NXD
      ZZ=EXB(I) * DATA2(K1)
      KK=INDAGE(K1,I)
      DER2B(I1,I2,KK) = ZZ / DINC2
   C
   WRITE (KANAL6,9907) I1,I2,KK,DER2B(I1,I2,KK)
   CONTINUE

827
   I=I-ML3
C
828
   I=I-ML3
C
829
   IF (NALSTD) GOTO 83

DO 830 I3=1,ML3
   I=I+1
   IF (I.EQ.NALT) GOTO 831
   DER2B(I1,I2,MXA+I) = EXB(I) / DINC2
   C
   WRITE (KANAL6,9907) I1,I2,MXA+I,DER2B(I1,I2,MXA+I)
   CONTINUE

830
   I=I-ML3
C
831
   I=I-ML3
C
80
   CONTINUE
C
W.R.T. THETA
C
IF (NTH.EQ.0) GOTO 89
C
DO 85 K=1,NTH
   NT11=LOCLIM(K)
   NT12=LOCBRA(K)
   I=MS3K(K)
   ML3=ML3K(K)
   ZZ=ZERO
   DO 86 I3=1,ML3
      I=I+1
      ZZ=ZZ + EXB(I) * XB(I)
      DER2T(NT11,NT12,K) = -ZZ / EINC2(NT11,NT12) / THETA(NT11,NT12)
   C
   WRITE (KANAL6,9921) NT11,NT12,K,DER2T(NT11,NT12,K)
   C9921
      FORMAT( ' $X$ DER2T(',312,')=',D18.6)
   CONTINUE
C
85
CONTINUE
C
89
CONTINUE
C
C
DERIVATIVES OF INCLUSIVE VALUES LEVEL 1
C
W.R.T. BETA
C
DO 93 K=1,NTH
   I=IZERO
DO 93 I1=1,ML1
   ML2=NL2(I1)
   ZZ=ZERO
   DO 94 I2=1,ML2
      I=I+1
54     ZZ = ZZ + EZG(I) * DER2B(I1,I2,K) * THETA(I1,I2)
   DER1B(I1,K) = ZZ / EINC1(I1) / TAU(I1)
   WRITE (KANAL6,9908) I1,K,DER1B(I1,K)
C9908   FORMAT( ' $X$ DER1B(',I2,')=',D18.4)
   CONTINUE
C
C W.R.T. THETA
C
   IF (NTH.EQ.0) GOTO 99
C
DO 95 K=1,NTH
   NT11=LOCLIM(K)
   NT12=LOCBRA(K)
   ZZZZ = EINC2(NT11,NT12) ** ( THETA(NT11,NT12)/TAU(NT11) )
   ZZ = ZZZZ * ( EINC2(NT11,NT12)
   + THETA(NT11,NT12) * DER2T(NT11,NT12,K) )
   DER1T(NT11,K) = ZZ / EINC1(NT11) / TAU(NT11)
   WRITE (KANAL6,9922) NT11,K,DER1T(NT11,K)
C9922   FORMAT( ' $X$ DER1T(',I2,')=',D18.6)
   CONTINUE
C
C W.R.T. TAU
C
   IF (NTAU.EQ.0) GOTO 920
C
DO 921 K=1,NTAU
   NT1=LOCTAU(K)
   I =MS2K(NT1)
   ML2=NL2(NT1)
   ZZZ=ZERO
   DO 922 I2=1,ML2
      I=I+1
922     ZZZZ=ZZZ + EZG(I) * ZG(I)
   DER1U(NT1,K) = - ZZZ / EINC1(NT1) / TAU(NT1)
   WRITE (KANAL6,9211) NT1,K,DER1U(NT1,K)
C9211   FORMAT( ' DER1U(',I2,')=',D14.5)
   CONTINUE
C
920   CONTINUE
C
C DERIVATIVES OF THE LOG LIKELIHOOD (THREE LEVEL TREE)
C
C IF (LEVEL2) GOTO 200
C
C W.R.T. BETA
C
   NN1 = MAXALT
   DO 120 K=1,NX
      ZZ=ZERO
DO 121 I1=1,NL1
   ZZ = ZZ + EYA(I1) * DER1B(I1,K) * TAU(I1)
ZZZ = DATA1(NN1+IALT)/THC - ZZ/EINCO
   + TH1 * DER2B(IC1,IC2,K) + TA1 * DER1B(IC1,K)
   NN1 = NN1 + MAXALT
   WORK(K) = ZZZ
C
IF (NALTRE) GOTO 1223
DO 1221 K=1,NXH
   K1=K+HXA
   ZZ=ZERO
   DO 1222 I1=1,NL1
      ZZ = ZZ + EYA(I1)*DER1B(I1,K1)*TAU(I1)
   ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K1) + TA1 * DER1B(IC1,K1)
   IF (LMAP(K,IALT)) ZZ = ZZ + ONE/THC
   WORK(K1)=ZZ
   CONTINUE
C
1223 IF (NAGTRE) GOTO 122
DO 1224 K=1,NXH
   DO 1224 K2=1,NXD
      K1=INDAGE(K2,K)
      ZZ=ZERO
      DO 1225 I1=1,NL1
         ZZ = ZZ + EYA(I1)*DER1B(I1,K1)*TAU(I1)
      ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K1) + TA1 * DER1B(IC1,K1)
      IF (LMAP(K,IALT)) ZZ = ZZ + DATA2(K2)/THC
      WORK(K1)=ZZ
      CONTINUE
C
122 IF (NAGSTD) GOTO 123
K=NX
   DO 125 K1=1,NXD
      DO 125 K2=1,NALT1
         K=K+1
         ZZ=ZERO
         DO 126 I1=1,NL1
            ZZ = ZZ + EYA(I1) * DER1B(I1.K) * TAU(I1)
         ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K) + TA1 * DER1B(IC1,K)
         WORK(K) = ZZ
         IF (.NOT.LDUMHY) GOTO 126
      WORK(INDAGE(K,IALT)) = WORK(INDAGE(K,IALT)) + DATA2(K)/THC
C
123 IF (NALSTD) GOTO 129
NX1=MXA+1
   DO 127 K=NX1,MTH
      ZZ=ZERO
      DO 128 I1=1,NL1
         ZZ = ZZ + EYA(I1) * DER1B(I1,K) * TAU(I1)
      ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K) + TA1 * DER1B(IC1,K)
      WORK(K) = ZZ
      IF (LDUMHY) WORK(MXA+IALT) = WORK(MXA+IALT) + ONE/THC
      CONTINUE
W.R.T. THETA

IF (NTH.EQ.0) GOTO 139

DO 130 K=1,NTH
  NT11=LOCLIM(K)
  NT12=LOCBRA(K)
  ZZZ = -EYA(NT11) * DER1T(NT11,K) * TAU(NT11)/ EINCO
  IF (IC1.NE.NT11) GOTO 131
    ZZZ = ZZZ + TAU * DER1T(IC1,K)
  IF (IC2.NE.NT12) GOTO 131
    ZZZ = ZZZ + INC2C/TAC + TH1*DER2T(IC1,IC2,K) - XBC/THC
  131 WORK(MTH+K) = ZZZ
  130 CONTINUE

W.R.T. TAU

IF (NTAU.EQ.0) GOTO 149

DO 140 K=1,NTAU
  NT1=LOCTAU(K)
  ZZZ = -EYA(NT1) * ( INC1(NT1)+TAU(NT1)*DER1U(NT1,K) ) / EINCO
  IF (IC1.NE.NT1) GOTO 140
    ZZZ = ZZZ + INC1C + TA1*DER1U(IC1,K) - INC2C*THC/TAC/TAC
  WRITE (KANAL6,9914) K,ZZZ
  WORK(MTAU+K) = ZZZ
  140 CONTINUE

CONTINUE

DERIVATIVES OF THE LOG LIKELIHOOD (TWO LEVEL TREE)

W.R.T. BETA

DO 220 K=1,NX
  ZZ=ZERO
  DO 221 I1=1,NL1
    ZZ = ZZ + DER1B(I1,K)
  221 ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K)

220 WORK(K) = ZZ

IF (NALTRE) GOTO 2223

DO 2221 K=1,NXM
  K1=K+MXA
  ZZ=ZERO
  DO 2222 I1=1,NL1
    ZZ = ZZ + DER1B(I1,K1)
  2222 ZZ = -ZZ/EINCO + TH1 * DER2B(IC1,IC2,K1)
  IF (LMAP(K,IALT)) ZZ = ZZ + ONE/THC
WORK(K1)=ZZ
2221 CONTINUE
C
2223 IF (NAGTRE) GOTO 222
DO 2224 K=1,NXM
DO 2224 K2=1,NXD
K1=INDAGE(K2,K)
ZZ=ZERO
DO 2225 I1=1,NL1
2225 ZZ=ZZ+DERIB(I1,K1)
ZZ=-ZZ/EINCO+TH1*DER2B(IC1,IC2,K1)
IF (LMAP(K,IALT)) ZZ = ZZ + DATA2(K2)/THC
WORK(K1)=ZZ
2224 CONTINUE
C
222 IF (NAGSTD) GOTO 223
K=NX
DO 225 K1=1,NXD
DO 225 K2=1,NALT1
K=K+1
ZZ=ZERO
DO 226 I1=1,NL1
226 ZZ=ZZ+DERIB(I1,K)
ZZ=-ZZ/EINCO+TH1*DER2B(IC1,IC2,K)
225 WORK(K)=ZZ
IF (.NOT.LDUMMY) GOTO 223
DO 224 K=1,NXD
224 WORK(INDAGE(K,IALT)) = WORK(INDAGE(K,IALT)) + DATA2(K)/THC
C
223 IF (NALSTD) GOTO 229
NX1=MXA+1
DO 227 K=NX1,MTH
ZZ=ZERO
DO 228 I1=1,NL1
228 ZZ=ZZ+DERIB(I1,K)
ZZ=-ZZ/EINCO+TH1*DER2B(IC1,IC2,K)
227 WORK(K)=ZZ
IF (LDUMMY) WORK(MXA+IALT) = WORK(MXA+IALT) + ONE/THC
229 CONTINUE
C
C W.R.T. THETA
C
C IF (NTH.EQ.0) GOTO 239
C
DO 230 K=1,NTH
NT11=LOCLIM(K)
NT12=LOCBRA(K)
ZZZ=-DERIT(NT11,K)/EINCO
IF ((IC2.NE.NT12) GOTO 231
ZZZ=ZZZ+INC2C+TH1*DER2T(IC1,IC2,K)-XBC/THC
C WRITE (KANAL6,9923) K,ZZZ
C9923 FORMAT( ' $X$ WORK(MTH+',I2,')=',D14.5)
231 WORK(MTH+K) = ZZZ
230 CONTINUE
C
239 GOTO 5000
C
C DERIVATIVES OF THE LOG LIKELIHOOD (SIMPLE MNL-VERSION)
C
4000 NN1 = MAXALT
DO 4020 K=1,NX
   ZZ = ZERO
   ZB = ZERO
   DO 4021 I=1,NALT
      DA = DATA1(NN1+I)
      ZP = PROBS(I) * DA
      ZZ = ZZ + DATA1(I) * DA - ZP
      ZB = ZB + ZP
   4021 CONTINUE
   NN1 = NN1 + MAXALT
   ZBAR(K) = ZB
4020 WORK(K) = ZZ
C
IF (NALTRE.AND.NAGTRE) GOTO 4039
DO 4036 K=NX+1,MTH
   ZBAR(K) = ZERO
   WORK(K) = ZERO
4036 CONTINUE
DO 4031 K=1,NXM
   K1 = K + MXA
   DO 4032 M=1,MAPLEN(K)
      I = MAPTR(K,M)
      ZZZ = DATA1(I) - PROBS(I)
      IF (NALTRE) GOTO 4033
      WORK(K1) = WORK(K1) + ZZZ
      ZBAR(K1) = ZBAR(K1) + PROBS(I)
4033 IF (NAGTRE) GOTO 4032
4034 INDEX = INDAGE(K2,K)
   ZBAR(INDEX) = ZBAR(INDEX) + PROBS(I) * DATA2(K2)
4034 WORK(INDEX) = WORK(INDEX) + ZZZ * DATA2(K2)
4032 CONTINUE
4031 CONTINUE
C
4039 IF (NAGSTD) GOTO 4040
   K=NX
   DO 4030 K1=1,NXD
   DO 4030 K2=1,NALT1
      K=K+1
      WORK(K) = ( DATA1(K2) - PROBS(K2) ) * DATA2(K1)
      ZBAR(K) = PROBS(K2) * DATA2(K1)
4030 CONTINUE
C
4040 IF (NALSTD) GOTO 4100
   DO 4041 K=1,NALT1
      WORK(K+MXA) = DATA1(K) - PROBS(K)
      ZBAR(K+MXA) = PROBS(K)
4041 CONTINUE
C
C EXACT SECOND DERIVATIVES (SIMPLE MNL-VERSION)

4100 IF (IDIFF2.NE.4) GOTO 5000
   NN1=IZERO
   DO 4105 K=1,NX
      NN1=NN1+MAXALT
   DO 4105 I=1,NALT
      ZIZBAR(I,K)=DATA1(NN1+I)-ZBAR(K)
   CONTINUE

4110 IF (NAGTRE) GOTO 4120
   K=NX
   DO 4115 K1=1,NXD
      DO 4115 K2=1,NXM
         K=K+1
      DO 4115 I=1,NALT
         DA=ZERO
         IF (LMAP(K2,I)) DA=DATA2(K1)
         ZIZBAR(I,K)=DA-ZBAR(K)
      CONTINUE
   C

4120 IF (NALTRE) GOTO 4130
   K=MXA
   DO 4125 K2=1,NXM
      K=K+1
   DO 4125 I=1,NALT
      DA=ZERO
      IF (LMAP(K2,I)) DA=ONE
      ZIZBAR(I,K)=DA-ZBAR(K)
   CONTINUE
   C

4130 IF (NAGSTD) GOTO 4140
   K=NX
   DO 4135 K1=1,NXD
      DO 4135 K2=1,NALT1
         K=K+1
      DO 4135 I=1,NALT
         DA=ZERO
         IF (K2.EQ.I) DA=DATA2(K1)
         ZIZBAR(I,K)=DA-ZBAR(K)
      CONTINUE
   C

4140 IF (NALSTD) GOTO 4200
   K=MXA
   DO 4145 K2=1,NALT1
      K=K+1
   DO 4145 I=1,NALT
      DA=ZERO
      IF (K2.EQ.I) DA=ONE
      ZIZBAR(I,K)=DA-ZBAR(K)
   CONTINUE
   C
   C HESSIAN
   C
4200  KK=IZERO
   DO 4205 J=1,NP
   DO 4205 K=1,J
      KK=KK+1
      ZZ=ZERO
      DO 4210 I=1,NALT
         ZZ = ZZ + ZIZBAR(I,J)*PROBS(I)*ZIZBAR(I,K)
   4210 CONTINUE
   HESS(KK)=-ZZ
   CONTINUE
C
C APPLY WESML WEIGHTS TO LIKELIHOOD, GRADIENT, AND HESSIAN
C
5000 IF (RELSIZ.EQ.0.0) GOTO 9999
C
   WT=WEIGHT(IALT)
   XLFI=XLFI*WT
   IF (ILOOP.EQ.1) GOTO 9999
C
   DO 5001 I=1,NP
5001   WORK(I)=WORK(I)*WT
   IF (IDIFF2.NE.4) GOTO 9999
C
   KK=1
   DO 5004 J=1,NP
   DO 5004 K=1,J
      KK=KK+1
5004   HESS(KK)=HESS(KK)*WT
   GOTO 9999
C
C FAILURE DUE TO UNDER- OR OVERFLOW
C
9001 CONTINUE
   WRITE (KANAL6,*) 'XBI',XBI,' IALT',IALT
   GOTO 9000
9002 CONTINUE
   WRITE (KANAL6,*) 'DINC2',DINC2,' IALT',IALT
   GOTO 9000
9003 CONTINUE
   WRITE (KANAL6,*) 'DINC1',DINC1,' IALT',IALT
   GOTO 9000
9004 CONTINUE
   WRITE (KANAL6,*) 'EINCO',EINCO,' IALT',IALT
   GOTO 9000
9005 CONTINUE
   WRITE (KANAL6,*) 'CH=0,IALT',IALT
C
9000 IF (.NOT.DUMP) GOTO 9998
C9000 WRITE (KANAL6,*) 'DUMP OF LAST RECORD:'
   DO 9091 I=1,NALT
      WRITE (KANAL6,*) 'CH',I,DATA1(I)
   9091 NN1=MAXALT
      DO 9092 K=1,NX
         WRITE (KANAL6,*) 'X ',DATA1(NN1+I),PARM(K)
9092        NN1=NN1+MAXALT
   DO 9093 K=1,NXD
9093       WRITE (KANAL6,*) 'XD',DATA2(K),PMASKY(K,I)
       WRITE (KANAL6,*) 'DU',PARM(MXA+I)
       WRITE (KANAL6,*) 'XBI',XBI(I)
9091       CONTINUE
       WRITE (KANAL6,*) 'TH',(THETA(LOCLIM(I),LOCBRA(I)),I=1,NTH)
       WRITE (KANAL6,*) 'TAU', (TAU(LOCTAU(I)),I=1,NTAU)

9998 XLFI=-1.D10
       NFAIL=NFAIL+1
C
9999       CONTINUE
       RETURN
END
SUBROUTINE WESML(PARM,GRAD,HESS,NP,SCRA,FUNC)

C-------------------------------------------------------------------
C             Calculates correct covariance for WESML estimator
C
C COVM = INV(HESS) * ( BHHH + ABA' ) * INV(HESS)
C
C RELSIZ = 0.0: Exogenous sampling - no correction necessary
C RELSIZ < 1.0: Choice based sampling with estimated shares
C (RELSIZ=Main sample size/Auxiliary sample size)
C RELSIZ = 1.0: Choice based sampling with known shares
C WEIGHT = Vector of Aux. sample shares/Main sample shares
C
C THE INVERSE HESSIAN HAS TO BE SUPPLIED IN HESS
C
C AXEL BOERSCH-SUPAN      MARCH 21, 1984
C
C-------------------------------------------------------------------
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PARM(NP),GRAD(NP),HESS(NP,NP)
C
PARAMETER
* (MAXOBS=2001,
  * MAXNP =50,
  * MAXNX =10,
  * MAXALT=20,
  * MAXNXY=MAXNX+1)
C
REAL*8 SCRA(MAXNP,MAXNP),HINV(MAXNP,MAXNP),AAUX(MAXNP,MAXALT),
  * BAX(MAXALT,MAXALT),QAUX(MAXALT),SCRAUX(MAXNP,MAXALT)
REAL*4 DATA1(MAXALT,MAXNXY,MAXOBS)
EXTERNAL FUNC
C
COMMON / DIMEN / NALT,NALT1,NX,NXD,NXA,NXR,MXA,MTH,MTAU,NOBS
COMMON / DIFF2 / IDIFF2
COMMON / DATAX / DATAI
COMMON / DWESKL / RELSIZ,WEIGHT(MAXALT)
COMMON / DWESTK / HINV,AAUX,BAX,QAUX,SCRAUX
IF (RELSIZ.EQ.0) RETURN

SAVE INVERSE HESSIAN IN "HINV"

DO 10 I=1,NP
DO 10 J=1,NP
   HINV(I,J)=HESS(I,J)
10

CALCULATE BHHH MATRIX IN "HESS"

ISAVE=IDIFF2
IDIFF2=1
CALL FP(PARM,NP,XLF,GRAD,FUNC)
CALL SP(PARM,NP,XLF,GRAD,HESS,FUNC)
IDIFF2=ISAVE

ADD CONTRIBUTION OF AUXILIARY SAMPLE SHARE ABA TO BHHH MATRIX

IF (RELSIZ.EQ.1.0) GOTO 200

- GET AUXILIARY SAMPLE SHARE AND POOL FP WITH DIFFERENT WEIGHTS

DO 100 I=1,NALT
   QAUX(I) =WEIGHT(I)*DATA1(I,1,NOBS+1)
100
   WEIGHT(I)=WEIGHT(I)/(QAUX(I)*NOBS)

- ASSEMBLE A AND B

DO 110 I=1,NALT
   CALL FP(PARM,NP,XLF,GRAD,FUNC)
   DO 112 K=1,NP
      AAUX(K,I)=GRAD(K)
   112
   DO 113 J=1,NALT
      BAUX(I,J)=-RELSIZ*QAUX(I)*QAUX(J)
      IF (I.EQ.J) BAUX(I,I)=BAUX(I,I)+RELSIZ*QAUX(I)
   113
110  CONTINUE

- RESET WEIGHTS AND GRADIENT

DO 120 I=1,NALT
   WEIGHT(I)=WEIGHT(I)*QAUX(I)*NOBS
120 CALL FP(PARM,NP,XLF,GRAD,FUNC)

- MATRIX PRODUCT ABA'

CALL MATP (AAUX,NP,MAXNP,NALT,BAUX,MAXALT,NALT,SCRAUX,MAXNP)
CALL MATPT(SCRAUX,NP,MAXNP,NALT,AAUX,MAXNP,NALT,SCRA,MAXNP)

- MATRIX ADDITION BHHH + ABA'

DO 130 I=1,NP
DO 130 J=1,NP
HESS(I,J) = HESS(I,J) + SCRA(I,J)

PRE- AND POST-MULTIPLY INV. HESSIAN TO BHHH+ABA', STORE IN HESS

CALL MATP(HINV, NP, MAXNP, NP, HESS, NP, NP, SCRA, MAXNP)
CALL MATP(SCRA, NP, MAXNP, NP, HINV, MAXNP, NP, HESS, NP)

RETURN
END

SUBROUTINE ELAST(PARM, NP, PROB, ELAS, IOBS)

CALCULATES ARRAY OF ELASTICITIES IN THE THREE-LEVEL-TREE
FOR EACH INDEPENDENT VARIABLE

AXEL BOERSCH-SUPAN    MARCH 29, 1984

IMPLICIT REAL*8 (A-H, O-Z)
REAL*8 PARM(NP)

PARAMETER
  * (MAXOBS = 2001,
  *    MAXNP = 50,
  *    MAXNX = 10,
  *    MAXNXD = 5,
  *    MAXNXM = 20,
  *    MAXTAU = 5,
  *    MAXTH = 10,
  *    MAXLEV = 3,
  *    MAXLIM = 5,
  *    MAXBRA = 5,
  *    MAXALT = 20,
  *    MAXNXY = MAXNX + 1, MAXNXR = MAXNX + MAXNXD)

REAL*8 THETA(MAXLIM; MAXBRA), TAU(MAXLIM),
  PSUM1(MAXALT), Q3(MAXALT),
  PSUM2(MAXLIM, MAXBRA), Q2(MAXLIM, MAXBRA),
  PMASKY(MAXNXD, MAXALT), PMASKD(MAXALT)
REAL*4 DATA1(MAXALT, MAXNXY, MAXOBS), DATA2(MAXNXD, MAXOBS),
  ELAS(MAXALT, MAXALT, MAXNXR), PROB(MAXALT)
INTEGER MAP(MAXALT, MAXLEV), MAPLEN(MAXNXM), MAPTR(MAXNXM, MAXALT)
LOGICAL LEVEL1, LEVEL2, NOALT, NOAGE, NALTRE, NAGTRE, NALSTD, NAGSTD,
  STDUMA, STDUMD, STDERR, LMAP(MAXNXM, MAXALT)
COMMON / DIMEN / NALT, NALT1, NX, NXD, NXX, MTH, MTAU, NOBS
COMMON / DATA / DATA1
COMMON / DATAY / DATA2
COMMON / DTREE / NTH, NTAU, THETA, TAU, LEVEL1, LEVEL2
COMMON / DMAPP / NXX, MAP, MAPLEN, MAPTR, PMASKY, PMASKD, LMAP
COMMON / DDISS / LOC(THETA), LOC1(MAXALT), LOC2(MAXALT),
  NL1, NL2(MAXALT), NL3(MAXALT, MAXBRA)
COMMON / DDUMM / STDERR, STDUMA, STDUMD, NDUMA, NDUMD,
  NOALT, NOAGE, INDAGE(MAXNXD, MAXALT),
  NALSTD, NALTRE, NAGSTD, NAGTRE
COMMON / DELAS / PSUM1,PSUM2,Q2,Q3
COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH

SUMMATION OF ELEMENTAL PROBABILITIES TO SECOND LEVEL

I=0
DO 10 I1=1,NL1
   ML2=NL2(I1)
   DO 10 I2=1,ML2
      PSUM2(I1,I2)=0.DO
      ML3=NL3(I1,I2)
      DO 11 I3=1,ML3
         I=I+1
      11   PSUM2(I1,I2)=PSUM2(I1,I2)+PROB(I)
      IF (PSUM2(I1,I2).LE.0.0) GO TO 90
   10 CONTINUE

SUMMATION TO FIRST LEVEL

DO 20 I1=1,NL1
   PSUM1(I1)=0.DO
   ML2=NL2(I1)
   DO 21 I2=1,ML2
      PSUM1(I1)=PSUM1(I1)+PSUM2(I1,I2)
   20 CONTINUE

NORMALIZE TO GET TRANSITION PROBABILITIES

I=0
DO 30 I1=1,NL1
   ML2=NL2(I1)
   DO 30 I2=1,ML2
      Q2(I1,I2)=PSUM2(I1,I2)/PSUM1(I1)
      ML3=NL3(I1,I2)
      DO 30 I3=1,ML3
         I=I+1
      30   Q3(I)=PROB(I)/PSUM2(I1,I2)
   CONTINUE

ARRAY OF ELASTICITIES

DO 40 JM=1,NXM
   DO 40 K1=1,NXD
      DO 40 I=1,NALT
         ELAS(I,JM,NX+K1)=0.0
   40   CONTINUE

I=0
J=0
DO 50 J1=1,NL1
   TA=TUM(J1)
   XLI=(TA-1.DO)/TA
   ML2=NL2(J1)
   DO 50 J2=1,ML2
      ATW=1.DO/THETA(J1,J2)
   50 CONTINUE
XBR=1.DO/TA-ATW
ML3=NL3(J1,J2)
DO 50 J3=1,ML3
   J=J+1
   ABR=XBR*Q3(J)
   ALI=XLI*Q3(J)*Q2(J1,J2)
   I=0
   DO 60 I1=1,NL1
      LL2=NL2(I1)
      DO 60 I2=1,LL2
         LL3=NL3(I1,I2)
         DO 60 I3=1,LL3
            I=I+1
            EL=-PROB(J)
            IF (I1.NE.J1) GOTO 70
               EL=EL+ALI
            IF (I2.NE.J2) GOTO 70
               EL=EL+ABR
            IF (I.EQ.J) EL=EL+ATW
         70
c
         DO 72 K=1,NX
            ELAS(I,J,K)=PARM(K)*DATA1(J,K+1,IOBS)*EL
            IF (NAGTRE) GOTO 75
         72
c
         DO 74 K=1,NXD
            DO 74 JM=1,NXM
               IF (.NOT.LMAP(JM,J)) GOTO 74
               K1=INDAGE(K,JM)
               ADD=PARM(K1)*DATA2(K,IOBS)*EL
               ELAS(I,JM,NX+K)=ELAS(I,JM,NX+K)+ADD
            74      CONTINUE
         74
c
         IF (NAGSTD) GOTO 60
         IF (J.EQ.NALT) GOTO 60
         DO 76 K=1,NXD
            K1=INDAGE(I,J)
            ELAS(I,J,NX+K)=PARM(K1)*DATA2(K,IOBS)*EL
         76      CONTINUE
      60     CONTINUE
   50     CONTINUE

RETURN
C
90 WRITE (KANAL6,91) I1,I2,PSUM2(I1,I2)
91 FORMAT(' PSUM2(',213,')=',D15.5)
RETURN
END

SUBROUTINE LSFIT(PDUMMY,NALT1,SSD,IERLS,SSQDEV,ITERLS)

C FIT AGGREGATE SAMPLE SHARES BY LEAST SQUARES
C CAN BE CALLED AS INNER MINIMIZATION LOOP WITHIN AN
C OUTER MAXIMIZATION OF THE LIKELIHOOD.
INPUT:  PDUMMY(NALT1)  INITIAL ALTERNATIVE SPECIFIC DUMMIES
        ITERLS  MAXIMUM NUMBER OF ITERATIONS

OUTPUT: PDUMMY(NALT1)  ADJUSTED ALTERNATIVE SPECIFIC DUMMIES
        ITERLS  NUMBER OF ITERATIONS USED
        IERLS  RETURN CODE FROM DFP-ALGORITHM
        SSD  SUM OF SQUARED DEVIATIONS OF SHARES

AXEL BOERSCH-SUPAN  MARCH 28, 1984

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION PDUMMY(NALT1)
EXTERNAL FUNC,DFP,SSQDEV

COMMON / BSTACK / A(l)
COMMON / BOPT / IVER,LT,IFP,ISP,NLOOP1,IST,ILOOP1
COMMON / BOPT1 / NPARS,JH,JFP,JFP1,JSP,JSP1,JA1,JS
COMMON / BOPT2 / ACC,R,PM1,IVAL,ITERL,ITERC,MX,IER
COMMON / BOPT3 / JSMP,JF,JSPD
COMMON / BSTAK / NQ,NTOP
COMMON / BPRINT / IPT,NFILE,NDIG,NPUNCH
COMMON / BLNSR / STEP1,STPACC,NLNSR
COMMON / BSTOP / NVAR1,ISTOP(3)
COMMON / BSTR / ST2,F1,ST3,F3,IEDGE
COMMON / BINPUT / INFLG
COMMON / BDFP / STPMIN,FMIN

C STORE PARAMETERS OF LIKELIHOOD MAXIMIZATION

IVER7=IVER
IFP7=IFP
ILOOP7=ILOOP1
NPARS7=NPARS
JH7=JH
JFP7=JFP
JFP17=JFP1
JSP7=JSP
JSP17=JSP1
JA17=JA1
JS7=JS
ACC7=ACC
R7=R
PM17=PM1
ITERL7=ITERL
ITERC7=ITERC
MX7=MX
IER7=IER
JSMP7=JSMP
JF7=JF
JSPD7=JSPD
IPT7=IPT
STEP17=STEP1
STPAC7=STPACC
NLNSR7=NLNSR
NVAR17=NVAR1
ST27=ST2
F17=F1
ST37=ST3
F37=F3
IEDGE7=IEDGE
RSMAL7=RSMALL
ABSMA7=ABSMAL
INFLG7=INFLG
STPMI7=STPMIN
FMIN7=FMIN

C
C RESET PARAMETERS WHERE NECESSARY
C

NPARS=NALT1
IER=0
IFP=4
MX=2
STPACC=ACC
NVAR1=NALT1
INFLG=0
PM1=FLOAT(3-2*MX)
ITERC=0
LASTOP=NTOP

C
C CALL MINIMIZATION ALGORITHM
C

IVER=1
ACC=0.01
ITERL=ITERLS
ITERC=0
IPT=2
CALL DFP(PDUMMY,NALT1,SSD,SSQDEV)
ITERLS=ITERC
IERLS=IER

C
C RESTORE PARAMETERS OF LIKELIHOOD MAXIMIZATION
C

NTOP=LASTOP
IVER=IVER7
IFP=IFP7
ILOOP1=ILOOP7
NPARS=NPARS7
JH=JH7
JFP=JFP7
JFP1=JFP17
JSP=JSP7
JSP1=JSP17
JA1=JA17
JS=JS7
ACC=ACC7
R=R7
PM1=PM17
ITERL=ITERL7
ITERC=ITERC7
MX=MX7
IER=IER7
JSMP=JSMP7
JF=JF7
JSPD=JSPD7
IPT=IPT7
STEP1=STEP17
STPAC=STPAC7
NLNSR=NLNSR7
NVAR1=NVAR17
ST2=ST27
F1=F17
ST3=ST37
F3=F37
IEDGE=IEDGE7
RSMALL=RSMALL7
ABSMAL=ABSMAL7
INFLG=INFLG7
STPMIN=STPMIN
FMIN=FMIN7

RETURN
END

SUBROUTINE SSQDEV(PDUMMY,NALT1,SSDf~)

C------------~------------------------~~-------------------------------

C SUM OF SQUARED DEVIATIONS OF AGGREGATE PROBABILITY-SHARES FROM
C OBSERVED SAMPLE-SHARES
C
C AXEL BOERSCH-SUPAN MARCH 27, 1984
C
C---------------------------------------------------------~------------

IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PDUMMY(NALT1)

PARAMETER
* (MAXOBS =2001,
  MAXNP =50,
  MAXNX =10,
  MAXNXD =5,
  MAXALT =20,
  MAXNXY=MAXNX+1,
  MAXDA1=MAXALT*MAXNXY*MAXOBS, MAXDA2=MAXNXD*MAXOBS,
  MAXALS=MAXALT*(MAXALT+1)/2)

REAL*8 PROB1(MAXALT),PROB2(MAXALT),
  DERIV1(MAXALT,MAXALT),DERIV2(MAXALT,MAXALT),
  PARM(MAXNP),GRAD(MAXNP),HESS(MAXNP,MAXNP)
REAL*4 FDATA1(MAXDA1),FDATA2(MAXDA2)
COMMON / DIMEN / NALT,NPARM(8),NOBS
COMMON / DATAX / FDATA1
COMMON / DATAY / FDATA2
COMMON / DSSQD / PROB1,PROB2,DERIV1,DERIV2
COMMON / DPARM / NP,PARM,GRAD,HESS
COMMON / DWESML / RELSIZ,WEIGHT(MAXALT)

C
ILOOP=1
NFAIL=0
CALL UPDATE(PARM,NP,*90)
DO 10 I=1,NALT
   PROB2(I)=0.DO
10
C
EVALUATION OF PROBABILITY-SHARES
C
NN1=1
NN2=1
INC=MAXALT*MAXNXY
DO 100 IOBS = 1,NOBS
   CALL CONTPS(FDATA1(NN1),FDATA2(NN2),
               ILOOP,PARM,NP,PDUMMY,NALT1,PROB1,DERIV1,NFAIL)
   DO 15 I=1,NALT
      IF (FDATA1(NN1+I-1).EQ.1.0) WT=WEIGHT(I)
15 CONTINUE
   NN1=NN1+INC
   NN2=NN2+MAXNXD
   DO 16 I=1,NALT
      PROB2(I)=PROB2(I)+PROB1(I)*WT
16 CONTINUE
C
IF (NFAIL.GT.0) GOTO 90
C
SUM OF SQUARED DEVIATIONS IN PERCENTAGES
C
NN=NN1+INC
SSD=0.DO
DO 20 I=1,NALT
   ZZZ = (PROB2(I)/NOBS - FDATA1(NN+I-1))*100.0
   SSD = SSD + ZZZ*ZZZ
20 CONTINUE
C
RETURN
90 PRINT 91,NFAIL
91 FORMAT(' ERROR IN SSQDEV:',I5,' FAILURES IN EVALUATION')
RETURN 1
END
C---------------------------------------------------------------------
SUBROUTINE FPSSQD(PDUMMY,NALT1,SSD,DERIV,SDERIV,SSQDEV)
C---------------------------------------------------------------------
C
GRADIENT OF THE SUM OF SQUARED DEVIATIONS
C
C---------------------------------------------------------------------
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PDUMMY(NALT1), DERIV(NALT1), SDERIV(NALT1)

PARAMETER
* (MAXOBS = 2001,  
* MAXNP = 50,  
* MAXNX = 10,  
* MAXNXD = 5,  
* MAXALT = 20,  
* MAXNXY = MAXNX + 1, MAXNPS = MAXNP*(MAXNP+1)/2,  
* MAXDA1 = MAXALT*MAXNXY*MAXOBS, MAXDA2 = MAXNXD*MAXOBS)

REAL*8 PROB1(MAXALT), PROB2(MAXALT),  
* DERIV1(MAXALT, MAXALT), DERIV2(MAXALT, MAXALT),  
* PARM(MAXNP), GRAD(MAXNP), HESS(MAXNP, MAXNP)

REAL*4 FDATA1(MAXDA1), FDATA2(MAXDA2)

COMMON / DIMEN / NALT, NPARM(8), NOBS
COMMON / DATAX / FDATA1
COMMON / DATAY / FDATA2
COMMON / DPARM / NP, PARM, GRAD, HESS
COMMON / DSSQD / PROB1, PROB2, DERIV1, DERIV2
COMMON / DWESML / RELSZ, WEIGHT(MAXALT)
COMMON / BOPT2 / ACC, R, PM1, IVAL, ITERL, ITERC, MX, IER
EXTERNAL SSQDEV

ILOOP = 0
NFAIL = 0

CALL UPDATE(PARM, NP, *90)

DO 10 I = 1, NALT
  DO 10 K = 1, NALT
    PROB2(I) = 0.0
    DERIV2(I, K) = 0.0
  10

EVALUATION OF PROBABILITY-SHARES AND THEIR DERIVATIVES

NN1 = 1
NN2 = 1
INC = MAXALT*MAXNXY

DO 100 IOBS = 1, NOBS
  CALL CONTPS(FDATA1(NN1), FDATA2(NN2),  
    * ILOOP, PARM, NP, PDUMMY, NALT1, PROB1, DERIV1, NFAIL)
  DO 15 I = 1, NALT
    IF (FDATA1(NN1+I-1).EQ.1.0) WT = WEIGHT(I)
    CONTINUE
  15
  NN1 = NN1 + INC
  NN2 = NN2 + MAXNXD

  DO 16 I = 1, NALT
    PROB2(I) = PROB2(I) + PROB1(I)*WT
    DO 16 K = 1, NALT
      DERIV2(I, K) = DERIV2(I, K) + DERIV1(I, K)*WT
    16
  100

IVAL = IVAL + 1

IF (NFAIL.GT.0) GOTO 90
SUM OF SQUARED DEVIATIONS IN PERCENTAGES

\[ NN = NN_1 + INC \]

\[ SSD = 0.0 \]

DO 20 I = 1, NALT
   \[ PROB1(I) = \frac{PROB2(I)}{NOBS} - FDATA1(NN + I - 1) \] * 100
   SSD = SSD + PROB1(I) * PROB1(I)
20 CONTINUE

ITS GRADIENT W.R.T. ALTERNATIVE SPECIFIC DUMMIES

DO 30 K = 1, NALT1
   ZZZ = 0.0
   DO 31 I = 1, NALT
      ZZZ = ZZZ + 2.0 * PROB1(I) / NOBS * 100.0 * DERIV2(I, K)
30 DERIV(K) = ZZZ
31
RETURN

PRINT 91, NFAIL
91 FORMAT(' ERROR IN FPSSQD:', I5, ' FAILURES IN EVALUATION')
RETURN

SUBROUTINE CONTPS(DATA1, DATA2, ILOOP, PARM, NP, PDUMMY, NALT1, PROB1, DERIV1, NFAIL)

---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PARM(NP), PDUMMY(NALT1)
REAL*4 DATA1(*), DATA2(*)
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
---------------------------------------------------------------------
PARAMETER
* (MAXOBS =2001,
* MAXNP =50,
* MAXTH =10,
* MAXTAU =5,
* MAXNX =10,
* MAXNXd =5,
* MAXNXM =20,
* MAXLEV =3,
* MAXLIM =5,
* MAXBRA =5,
* MAXALT =20,
* MAXNXY=MAXNX+1, MAXNXR=MAXNX+MAXNXd, MAXNPS=MAXNP*(MAXNP+1)/2)

C

REAL*8 UPP,LOW,ONE/1.DO/,ZERO/0.DO/
INTEGER IZERO/0/

C

REAL*8 PROB1(MAXALT),DERIV1(MAXALT,MAXALT),
* THETA(MAXLIM,MAXBRA),TAU(MAXLIM),
* EXB(MAXALT),EZG(MAXALT),EYA(MAXALT),
* XB(MAXALT),ZG(MAXALT),YA(MAXALT),
* EINC2(MAXLIM,MAXBRA),EINC1(MAXLIM),EINCO,
* INC2(MAXLIM,MAXBRA),INC1(MAXLIM),INCO,INC2C,INC1C,
* DER2B(MAXLIM,MAXBRA,MAXNP),DER1B(MAXLIM,MAXNP),
* DER2T(MAXLIM,MAXBRA,MAXTH),DER1T(MAXLIM,MAXTH),
* DER1U(MAXLIM,MAXTAU),
* PMASKY(MAXNXd,MAXALT),PMASKD(MAXALT),
* ZBAR(MAXNP),ZIZBAR(MAXALT,MAXNP)
INTEGER MAP(MAXALT,MAXLEV),MAPLEN(MAXNXM,MAXALT)
LOGICAL LEVEL1,LEVEL2,LMAP(MAXNXM,MAXALT),STDDUM,NOALT,NOAGE,DUMP,
* LDUMMY,STDUMA,STDUMD,NALSTD,NALTRE,NAGSTD,NAGTRE

C

COMMON / DIMEN / NALT,NALTDU,NX,NXD,NXA,NXR,MTH,MTAU,NOBS
COMMON / DDISS / LOCTAU(MAXTAU),LOCLIM(MAXTH),LOCBRA(MAXTH),
* NL1,NL2(MAXLIM),NL3(MAXLIM,MAXBRA)
COMMON / DCONT / XB,ZG,YA,INC2,INC1,
* EXB,EZG,EYA,EINC2,EINC1,
* DER2B,DER1B,DER2T,DER1T,DER1U,
* ZBAR,ZIZBAR,UPP,LOW,
* MS3K(MAXTH),ML3K(MAXTH),MS2K(MAXLIM)
COMMON / DTREE / NTH,NTAU,THETA,TAU,LEVEL1,LEVEL2
COMMON / DMAPP / NXM,MAP,MAPLEN,MAPTR,PMASKY,PMASKD,LMAP
COMMON / DDUMM / STDDUM,STDUMA,STDUMD,NDUMA,NDUMD,
* NOALT,NOAGE,INDAGE(MAXNXd,MAXALT),
* NALSTD,NALTRE,NAGSTD,NAGTRE
COMMON / BPRINT / IPT,KANAL6,NDIG,NPUNCH

INNER PRODUCT XBETA, INCLUSIVE VALUES

I=IZERO
IB=IZERO
EINCO=ZERO
DO 10 I1=1,NL1
ML2=ML2(I1)
TA=TAU(I1)
DINC1=ZERO
DO 20 I2=1,ML2
   IB=IB+1
   ML3=ML3(I1,I2)
   TH=THETA(I1,I2)
   THT=TH/TA
   INC2=ZERO
DO 30 I3=1,ML3
   I=I+1
C
C INNER PRODUCT XB/TH
C
31 XBI=ZERO
NN1=MAXALT
DO 40 K=1,NX
   XBI=XBI+DATA1(NN1+I)*PARM(K)
40   NN1=NN1+MAXALT
IF (NAGTRE) GOTO 42
DO 41 K=1,NXD
   XBI=XBI+DATA2(K)*PMASKY(K,I)
42 IF (I.EQ.NALT) GO TO 49
   XBI=XBI+PDUMMY(I)
43 IF (NAGSTD) GOTO 49
49 CONTINUE
XBI=XBI/TH
IF (XBI.GT.UPP) GOTO 9001
XB(I)=XBI
ZZZ=DEXP(XBI)
EXB(I)=ZZZ
DINC2=DINC2+ZZZ
30 CONTINUE
C
C LOWER LEVEL INCLUSIVE VALUE INC2(I1,I2)
C
IF (LEVEL1) GOTO 21
IF (DINC2.LT.LOW) GOTO 9002
EINC2(I1,I2)=DINC2
DINC2=DLOG(DINC2)
INC2(I1,I2)=DINC2
DINC2=THT*DINC2
IF (DINC2.GT.UPP) GOTO 9002
ZG(IB)=DINC2
EZG(IB)=DINC2
21 DINC1=DINC1+DINC2
20 CONTINUE
C
C HIGHER LEVEL INCLUSIVE VALUE DLOG(INC1(I1))
C
EINC1(I1)=ONE
IF (LEVEL2) GOTO 11
IF (DINC1.LT.LOW) GOTO 9003
EINC1(I1)=DINC1
DINC1=DLOG(DINC1)
INC1(I1)=DINC1
DINC1=TAU*DINC1
IF (DINC1.GT.UPP) GOTO 9003
YA(I1)=DINC1
DINC1=DEXP(DINC1)
EYA(I1)=DINC1
11 EINCO=EINCO+DINC1
10 CONTINUE
IF (EINCO.LT.LOW) GOTO 9004
INCO=DLOG(EINCO)

C
C PROBABILITY-SHARES
C
51 DO 52 I=1,NALT
   PROB1(I)=EXB(I)/EINCO
52 CONTINUE
IF (LEVEL1) GOTO 69
I=IZERO
DO 60 I1=1,NL1
   ML2=NL2(I1)
IF (LEVEL2) GOTO 61
   DINC1=(TAU(I1)-ONE)*INC1(I1)
61 CONTINUE
DO 60 I2=1,ML2
   ML3=NL3(I1,I2)
   DINC2=(THETA(I1,I2)/TAU(I1)-ONE)*INC2(I1,I2)
60 CONTINUE
69 IF (ILOOP.EQ.1) GOTO 9999
IF (LEVEL1) GOTO 4000
C
C DERIVATIVES OF INCLUSIVE VALUES LEVEL 2 W.R.T. DUMMIES
C
I=IZERO
DO 80 I1=1,NL1
   ML2=NL2(I1)
DO 80 I2=1,ML2
   ML3=NL3(I1,I2)
   DINC2=EINC2(I1,I2)*THETA(I1,I2)
80 CONTINUE
DO 80 I3=1,ML3
   I=I+1
   IF (I.EQ.NALT) GOTO 80
   DER2B(I1,I2,MXA+I) = EXB(I) / DINC2
80 WRITE (KANAL6,9907) I1,I2,MXA+I,DER2B(I1,I2,MXA+I)
C9907 FORMAT(' $X$ DER2B(',312,')=',D18.4)
CONTINUE

DERIVATIVES OF INCLUSIVE VALUES LEVEL 1 W.R.T. DUMMIES

DO 93 K=MXA+1, MXA+NALT1
I=IZERO
DO 93 I1=1, NL1
   ML2=ML2(I1)
   ZZ=ZERO
   DO 94 I2=1, ML2
      I=I+1
      ZZ = ZZ + EZG(I) * DER2B(I1, I2, K) * THETA (I1, I2)
   END
   DERIB(I1, K) = ZZ / EINC1(I1) / TAU(I1)
WRITE (KANAL6, 9908) I1, K, DER1B(I1, K)
CONTINUE

DERIVATIVES OF THE PROBABILITIES (THREE LEVEL TREE)

IF (LEVEL2) GOTO 200

DO 127 KK=1, NALT1
   K=KK+MXA
   ZZ=ZERO
   DO 128 I1=1, NL1
      ZZ = ZZ + EYA(I1) * DER1B(I1, K) * TAU(I1)
   END
   ZZ = -ZZ/EINC0
   I=0
   DO 127 I1=1, NL1
      ML2=ML2(I1)
      TA1=TAU(I1)-ONE
      DO 127 I2=1, ML2
         ML3=NL3(I1, I2)
         TH1=(THETA(I1, I2)/TAU(I1)-ONE)
      END
      DO 127 I3=1, ML3
         I=I+1
         ZZZ = ZZ + TH1 * DER2B(I1, I2, K) + TA1 * DER1B(I1, K)
         IF (I.EQ.KK) ZZZ = ZZZ + ONE/THETA(I1, I2)
         DERIV(I, KK) = ZZZ * PROB1(I)
      END
   END
GOTO 9999

DERIVATIVES OF THE PROBABILITIES (TWO LEVEL TREE)

DO 227 KK=1, NALT1
   K=KK+MXA
   ZZ=ZERO
   DO 228 I1=1, NL1
      ZZ = ZZ + DER1B(I1, K)
   END
   ZZ = -ZZ/EINC0
   I=0
   DO 227 I1=1, NL1
      ML2=ML2(I1)
      DO 227 I2=1, ML2
      END
ML3 = NL3(I1, I2)
TH1 = THETA(I1, I2) - ONE
DO 227 I3 = 1, ML3
    I = I + 1
    ZZZ = ZZZ + TH1 * DER2B(I1, I2, K)
    IF (I.EQ.KK) ZZZ = ZZZ + ONE / THETA(I1, I2)
    DERIV1(I, KK) = ZZZ * PROB1(I)
    CONTINUE
GOTO 9999
C
C DERIVATIVES OF THE PROBABILITIES (SIMPLE MNL-VERSION)
C
4000 DO 4041 K = 1, NALT1
      DO 4041 I = 1, NALT
      ZZ = ZERO
      IF (I.EQ.K) ZZ = ONE
      DERIV1(I, K) = PROB1(I) * (ZZ - PROB1(K))
  4041 CONTINUE
GOTO 9999
C
C FAILURE DUE TO UNDER- OR OVERFLOW
C
9001 CONTINUE
WRITE (KANAL6, *) 'XBI', XBI
GOTO 9000
9002 CONTINUE
WRITE (KANAL6, *) 'DINC2', DINC2
GOTO 9000
9003 CONTINUE
WRITE (KANAL6, *) 'DINC1', DINC1
GOTO 9000
9004 CONTINUE
WRITE (KANAL6, *) 'EINCO', EINCO
C
9000 XLFI = -1.D10
NFAIL = NFAIL + 1
C
9999 CONTINUE
RETURN
END
SUBROUTINE LMSTAT(NP, NP1, NP2, ND, FPD, SPD, TEST)
C---------------------------------------------------------------
C CALCULATES LM STATISTIC
C-------------------------------~-------------------------------
PARAMETER (MAXNP = 50,
             MAXNPS = MAXNP*(MAXNP+1)/2)
REAL*8 FPD(NP), SPD(NP, NP)
REAL*8 DL(5), DLB(5, MAXNP), DLL(5, 5), DBB(MAXNPS), TEST
INTEGER ND(5)
C
C FILL FIRST AND SECOND DERIV W.R.T. THETA : DL, DLL
C
DO 560 I = 1, NP1
    DL(I) = FPD(ND(I))
DO 560 J=1,NP1
   DLL(I,J)=SPD(ND(I),ND(J))
C
C FILL MIXED SECOND DERIV : DLB
C
DO 561 I=1,NP1
DO 561 J=1,NP2
561   DLB(I,J)=SPD(ND(I),J)
C
C FILL SECOND DERIV W.R.T. BETA ON TRIANGLE : DBB
C
   IND=0
   DO 562 I=1,NP2
      DO 562 J=1,I
         IND=IND+1
         DBB(IND)=-SPD(I,J)
         (NOTE: NEED POS. DEF. MATRIX)
C
C INVERT DBB
C
   LK2=NP2*(NP2+1)/2
   CALL INV(DBB,NP2,LK2,IER)
   IF (IER.NE.0) PRINT 565
565   FORMAT(' DBB SINGULAR')
C
C REFILL INVERS OF DBB ON ARRAY
C
   IND=0
   DO 568 I=1,NP2
      DO 568 J=1,I
         IND=IND+1
         SPD(I,J)=-DBB(IND)
         SPD(J,I)=-DBB(IND)
C
C MATRIX PRODUCT : DLB * DBB(-1) * DLB'
C
   DO 570 I=1,NP1
      IND =(I-1)*NP2
      DO 570 J=1,NP2
         TEST=0.DO
         DO 571 K=1,NP2
            TEST=TEST+DLB(I,K)*SPD(K,J)
371            DBB(IND+J)=TEST
      DO 575 I=1,NP1
         IND =(I-1)*NP2
         DO 575 J=1,NP1
            TEST=0.DO
            DO 576 K=1,NP2
               TEST=TEST+DBB(IND+K)*DLB(J,K)
576            SPD(I,J)=TEST
C
C FILL DLL - DLB*DBB(-1)*DLB' ON TRIANGLE
C
   IND=0
DO 580 I=1,NP1
DO 580 J=1,I
IND=IND+1
580 DBB(IND)=DLL(I,J)-SPD(I,J)

C INVERT THIS MATRIX

LK1=NP1*(NP1+1)/2
CALL INV(DBB,NP1,LK1,IER)
IF (IER.NE.0) PRINT 582
582 FORMAT(' DLL-DLB*DBB(-1)*DLB SINGULAR')

C REFILL THIS INVERSE ON ARRAY

IND=0
DO 585 I=1,NP1
DO 585 J=1,I
IND=IND+1
SPD(I,J)=DBB(IND)
585 SPD(J,I)=DBB(IND)

C FINAL MATRIX PRODUCT DL \times [DLL-DLB*DBB(-1)*DBL](-1) \times DL' C

DO 590 I=1,NP1
DBB(I)=0.DO
DO 590 J=1,NP1
590 DBB(I)=DBB(I)+SPD(I,J)*DL(J)
TEST=0.DO
DO 595 I=1,NP1
595 TEST=TEST+DBB(I)*DL(I)
C RETURN
END

SUBROUTINE AIRUM(PARM,NP,NFILE)
C---------------------------------------------~-----------~-----------
C
C
C C---------------------------------------------------------------------IMPLICIT REAL*8 (A-H,O-Z)
PARAMETER
*  (MAXOBS=2001,
*  MAXALT=20,
*  MAXLIM=5,
*  MAXBRA=5,
*  MAXNX =10,
*  MAXNXD=5,
*  MAXNXY=MAXNX+1)
REAL*8 PARM(NP), THETA(MAXLIM, MAXBRA), TAU(MAXLIM),
*    VMIN(9), VMAX(9), VMEA(9), V(9)
CHARACTER*3 C(2),'MAX', 'MIN'/
REAL*4 DATA1(MAXALT, MAXNXY, MAXOBS), DATA2(MAXNXD, MAXOBS)
LOGICAL LEVEL1, LEVEL2
COMMON / DIMEN / NALT, NALT1, NX,NXD, NXR, NWA, MTH, MTAU, NOBS
COMMON / DATA1 / DATA1
COMMON / DATA2 / DATA2
COMMON / DTREE / NTH, NTAU, THETA, TAU, LEVEL1, LEVEL2
C
C INITIALIZATION - NOTE SPECIAL CASE
C
IF (NALT.NE.7) RETURN
NFILE=1
DO 10 I=1, NALT
    VMEA(I)= 0.D0
    VMIN(I)= 1.D99
10     VMAX(I)=-1.D99
T3=TAU(1)
T2=THETA(3,1)
WRITE (NFILE,300) T3, T2
300 FORMAT ('COMPATIBILITY-CHECK'//' =======7==========='
C
C LOOP ON DATA FOR VMIN, VMEA, CHECK AT DATA
C
NEG=0
DO 1000 N=1, NOBS
C
BENCHMARK: ALTERNATIVE 4
C
XB4=0.D0
DO 20 K=1, NX
20     XB4 = XB4 + DBLE( DATA1(4,K+1,N) ) * PARM(K)
IF (NXD.EQ.0) GOTO 24
DO 21 K1=1, NXD
    K3 = NX+(K1-1)*NALT+4
21     XB4 = XB4 + DATA2(K1,N)*PARM(K3)
24     IF (NXA.EQ.0) GOTO 29
XB4 = XB4 + PARM(NXA+4)
C
OTHER ALTERNATIVES
C
29     DO 30 IT=1, NALT
30     IF (IT.EQ.4) GOTO 30
    XB=0.D0
    DO 40 K=1, NX
40     XB = XB + DBLE( DATA1(IT,K+1,N) ) * PARM(K)
    IF (IT.EQ.NALT) GOTO 49
    IF (NXD.EQ.0) GOTO 44
    DO 41 K1=1, NXD
        K3 = NX+(K1-1)*NALT+IT
41     XB = XB + DATA2(K1,N)*PARM(K3)
    IF (NXA.EQ.0) GOTO 49
\[
XB = XB + PARM(MXA+IT)
\]

```
C
C RELATIVE UTILITY
C
49   XB = XB - XB4
   V(IT) = XB
   VMEA(IT) = VMEA(IT) + XB
   IF (XB.GT.VMAX(IT)) VMAX(IT) = XB
   IF (XB.LT.VMIN(IT)) VMIN(IT) = XB
C
C CHECK AT DATA
C
   CALL GEV(V(1),V(2),V(3),V(5),V(6),T3,T2,G,DENS)
   IF (DENS.LT.0.0) NEG = NEG + 1
C
1000 CONTINUE
C
C CHECK AT MEAN
C
   DO 31 I = 1, NALT
   31 V(I) = VMEA(I)/NOBS
   CALL GEV(V(1),V(2),V(3),V(5),V(6),T3,T2,G,DENS)
   WRITE (NFILE,301) NOBS, NEG, DENS
301 FORMAT(' NUMBER OF OBSERVATIONS:', I13
   *   /' WITH NEGATIVE DENSITY :', I13
   *   /' DENSITY AT MEAN : ', D13.4/
C
C ESTABLISH INTERVAL
C
   DO 35 I = 1, NALT
   35 WRITE (NFILE,302) I, VMIN(I), VMAX(I)
302 FORMAT(' INTERVAL V(', I1,I) '-V(', I1,') : [', F10.4, ',', F10.4, ']' )
   WRITE (NFILE,303)
303 FORMAT(' CORNER: SIGN: G-FUNCTION DENSITY'
   *   ' ', 55('-' )
C
C CHECK ON ALL CORNERS, ACCUMULATE PARTS OF INTEGRAL
C
   AMASS = 0.0
   DO 50 I6 = 1, 2
   IF (I6 .EQ. 1) THEN
      A6 = VMAX(6)
   ELSE
      A6 = VMIN(6)
   END IF
   DO 50 I5 = 1, 2
   IF (I5 .EQ. 1) THEN
      A5 = VMAX(5)
   ELSE
      A5 = VMIN(5)
   END IF
   DO 50 I3 = 1, 2
   IF (I3 .EQ. 1) THEN
```
A3 = VMAX(3)
ELSE
A3 = VMIN(3)
END IF

DO 50 I2 = 1, 2
  IF (I2 .EQ. 1) THEN
    A2 = VMAX(2)
  ELSE
    A2 = VMIN(2)
  END IF

DO 50 I1 = 1, 2
  IF (I1 .EQ. 1) THEN
    A1 = VMAX(1)
  ELSE
    A1 = VMIN(1)
  END IF

C
CALL GEV(A1, A2, A3, A5, A6, T3, T2, G, DENS)
SIGN = (-1)**(I1 + I2 + I3 + I5 + I6 - 5)
AMASS = AMASS + SIGN/G
C
WRITE (NFILE, 51) C(I1), C(I2), C(I3), C(I5), C(I6), SIGN, G, DENS
51 FORMAT( ' ', 3(A3, ', '), '0', 2(' ', ', , A3), 2X, F4.1, 2X, 2D1.3, .4)
50 CONTINUE
C
WRITE (NFILE, 52) AMASS
52 FORMAT( ' ', 55('-')/' MASS OVER THE INTERVAL: ', F10.4)
RETURN
END
SUBROUTINE GEV(A1, A2, A3, A5, A6, T3, T2, G, DENS)
C---------------------------------------------------------------~-------
C EVALUATES G=1/(JOINT CDF) AND JOINT DENSITY FOR GEV MODEL
C--------------------------------------------------------------~----~---
IMPLICIT REAL*8 (A-H,O-Z)
C
G-FUNCTION
C
Y1 = DEXP(-A1/T3)
Y2 = DEXP(-A2/T3)
Y3 = DEXP(-A3/T3)
Y5 = DEXP(-A5/T2)
Y6 = DEXP(-A6/T2)
H3 = Y1 + Y2 + Y3
H2 = Y5 + Y6
G = H3**T3 + 1.DO + H2**T2
C
DERIVATIVES OF G-FUNCTION
C
H31 = H3**(T3-1.DO)
H21 = H2**(T2-1.DO)
H32 = H3**(T3-2.DO)
H22 = H2**(T2-2.DO)
H33 = H3**(T3-3.DO)
T31 = (T3-1.DO)/T3
T21 = (T2-1.DO)/T2
T32 = T31*(T3-2.DO)/T3
G1 = -H31*Y1
G2 = -H31*Y2
G3 = -H31*Y3
G5 = -H21*Y5
G6 = -H21*Y6
G12 = T31*H32*Y1*Y2
G23 = T31*H32*Y2*Y3
G13 = T31*H32*Y1*Y3
G56 = T21*H22*Y5*Y6
G123 = -T32*H33*Y1*Y2*Y3
G5G6 = G5*G6
GG56 = G*G56
GSUM = G12*G3+G23*G1+G13*G2
GPRO = G1*G2*G3

POWERS OF THE G-FUNCTION

GTO4 = G**4
GTO5 = GTO4*G
GTO6 = GTO5*G

DENSITY

S3 = (GG56-3.DO*G5G6)/GTO4
S4 = (GG56-4.DO*G5G6)/GTO5
S5 = (GG56-5.DO*G5G6)/GTO6
DENS = 24.DO*GPRO*S5 - 6.DO*GSUM*S4 + 2.DO*G123*S3

RETURN
END

SUBROUTINE TRAFO(PARM,NP,K,IND,HESS,C,SC,W)

TRANSFORM PARAMETERS TO BE COMPATIBLE WITH MNL

IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 PARM(NP),HESS(NP,NP)

A = PARM(K)
SA = -HESS(K,K)
B = PARM(IND)
SAB = -HESS(IND,K)
SB = -HESS(IND,IND)
C = A/B
SC = (SA - 2.*C*SAB + C*C*SB)/B/B
IF (SC.LE.0.) SC=1.DO
SC = DSQRT( SC )
W = C/SC

RETURN
END


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