Kinematic Chain Modelling of Rotary System as Applied to a Selectric Typewriter

by

Joshua Mitchell

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Authored by: Joshua Mitchell

Department of Mechanical Engineering

August 11, 2023

Certified by: Daniel Frey

Professor of Mechanical Engineering

Thesis supervisor

Accepted by: Kenneth Kamrin

Associate Professor of Mechanical Engineering

Undergraduate Officer

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ABSTRACT

Kinematic chains were created for the rotation and tilting mechanisms of an IBM Selectric II Typewriter. The chain for the rotation mechanism was used to create a homogeneous transform matrix system that allowed the position of the printing character in a local coordinate system to be represented in the global coordinate system through a series of shape and joint matrix transforms. This allowed the position and rotation of the character of interest to be calculated based on a commanded motion at a certain joint. A drive error in that joint, representing a spring losing stiffness, was also included in the transform system and influenced the final values of position and rotation. A particular error value of .01745 radians was used along with a commanded motion of .07667 radians. This resulted in a final rotation of the printhead of .3079 radians, which is an increase of .0233 over the value obtained with no drive error. This same technique could be applied to determine the feasibility of future rotary manufacturing systems.

Thesis supervisor: Daniel Frey

Title: Professor of Mechanical Engineering

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1. Introduction

An error budget is a way of modelling a manufacturing process that can be used to predict the amount of variation in that process. To create an error budget, the machine and fixtures of the process must be broken down into a series of rigid bodies and joints with attached local coordinate systems. Those coordinate systems are then related to the base of the machine and then to each other in a kinematic chain of transform matrices. With a complete system of transforms, specific errors, such as a drive error in a motor or a deformation in a machine arm, can be input and the transforms will carry the effect of those errors through the process into the workpiece and show the final variation it causes (Frey, 1997). When combined with the product's specification and tolerances, this knowledge of the variation allows a manufacturer to determine whether a given process is capable of producing the product in a satisfactory manner. Rather than relying on statistical methods of process capability that are calculated using information from the production runs, constructing an error budget allows changes to be made before the process is set up. This leads to savings in both manpower and capital, and therefore can be a valuable step in the planning stages of a manufacturing process.

One such proposed manufacturing system as conceived by Professor Daniel Frey is a modular pressboard embosser (Frey, 2022). The concept is composed of drums with a number of discrete characters about the surface and relies on motors to actuate the drums into position to crease a desired character into the pressboard as it passes on a conveyor. Constructing an error budget for this proposed system would give an idea for its feasibility.

However, the physical incarnation of this system is far from finalized, so another system with similar fundamentals was used: that of an IBM Selectric typewriter.

2. Background

2.1 IBM Selectric

The IBM Selectric is an electric typewriter introduced in 1961. This typewriter features an innovative single element print head. Every character of the typeface is located on this truncated sphere rather than each character occupying its own typebar. The characters are arranged into eleven columns and four rows on each half of the sphere. In the home position, the print head is aligned with the middle of these columns and the topmost row in place to print, meaning pressing a key will rotate the ball up to 5 units either clockwise or counterclockwise and tilt the ball up to three units upward to reach the character that was depressed. Pressing the shift key rotates the ball 180 degrees so that another set of characters (capital letters and alternate symbols) are available in the same fours and eleven columns. Figure 1 below shows the locations of each character on the print head (Palmer, 1957).

CODE CHART

2 @ R-5 T0	3 # R-4 T0	4 $ R+4 T0	5 % R+5 T0	6 ¢ R-5 T1	7 & R-4 T1	8 * R+4 T1
9 (R+5 T1	0) R+4 T2	– – R+4 T3	½ ¼ R+5 T3	: ; R-4 T2	I II R-5 T2	, , R-3 T1
· · R-3 T0	? ! R+3 T2	a A R0 T2	b B R+1 T2	c C R-3 T2	d D R+2 T2	e E R0 T0
f F R-2 T2	g G R+2 T3	h H R+1 T0	i I R-1 T1	j J R-3 T3	k K R+2 T1	l L R+1 T1
m M R-2 T1	n N R-1 T0	o O R0 T3	p P R+1 T3	q Q R+3 T3	r R R-1 T3	s S R-1 T2
t T R0 T1	u U R-2 T0	v V R+3 T0	w W R+2 T0	x X R+3 T1	y Y R-2 T3	z Z R+3 T2
a TP R-4 T3	= + R-5 T3					

Figure 1: Code chart showing position of each character on the printhead. The R value indicates rotation in either the clockwise (positive) or counterclockwise (negative) direction, while the T value indicates tilt. Characters on the right of each box are accessed using the shift key.

Pressing a key powers a number of mechanisms that cause the spherical typehead to simultaneously tilt and rotate to the correct character. This rotary method of selection is similar to that of the proposed pressboard embossing machine previously discussed and therefore prompts the interest in this typewriter. This thesis focuses on creating an error budget specifically for the linkages that actuate the tilting and rotating of the typehead.

2.2. Homogeneous Transformation Matrices and Local Coordinate Systems

Using robust homogonous transformation matrices (HTMs) allows all motions and geometries of a manufacturing machine to be modelled, whether rotary or linear. A generalized HTM takes the form below. (Frey, 1997.)

$$\begin{bmatrix} 1 & 0 & 0 & X \\ 0 & 1 & 0 & Y \\ 0 & 0 & 1 & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta_x & -\sin\Theta_x & 0 \\ 0 & \sin\Theta_x & \cos\Theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\Theta_y & 0 & \sin\Theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta_y & 0 & \cos\Theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\Theta_z & -\sin\Theta_z & 0 & 0 \\ \sin\Theta_z & \cos\Theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is a matrix with six degrees of freedom, which accounts for all possible motions in three dimensions, along with their associated error motions. The directions referenced in the HTM are defined in a local coordinate system (LCS). LCSs are assigned to each rigid body of a manufacturing machine and allow that rigid body to be located in space in relation to a global coordinate system fixed to the base of the machine. “Shape” matrices relate the position and orientation of an LCS to the one before it. The orientation of the LCS in a shape matrix is in its “home” position in which its commanded rotations and translations are zero.

A second HTM at each LCS, known as a “joint” matrix, describes the movement of the joint in the machine that the rigid body is attached to. Joint matrices can be simplified from the generalized form, however, depending on the geometry and relevant errors of that particular joint. A rotary joint, for example, can be modelled with a simplified HTM of the form:

$$\begin{bmatrix} \varepsilon_z + \cos\Theta_z & -\sin\Theta_z & \varepsilon_y & \delta_x \\ \sin\Theta_z & \varepsilon_z + \cos\Theta_z & -\varepsilon_x & \delta_y \\ \varepsilon_x \sin\Theta_z - \varepsilon_y \cos\Theta_z & \varepsilon_x \cos\Theta_z + \varepsilon_y \sin\Theta_z & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A kinematic chain is formed by creating a system of HTMs that relate the LCSs from the base coordinate system to the end effector, which in this case is the character that will be printing against the platen. Starting from the base, the shape matrix to the next LCS is multiplied by the joint matrix of the associated joint. This multiplication continues until the last set of shape and joint matrices are reached. The final form of the kinematic chain therefore is:

$$ {}^{0}\mathbf{T}_{n} = {}^{0}\mathbf{S}_{1} \cdot \mathbf{J}_{1} \cdot {}^{1}\mathbf{S}_{2} \cdot \mathbf{J}_{2} \cdot \mathrm{K}\ \mathbf{J}_{n-1}\, {}^{n-1}\mathbf{S}_{n} $$

The matrix T allows the mapping of coordinates from the final coordinate system (p_x, p_y, p_z) to the base coordinate system (p'_x, p'_y, p'_z) according to:

$$ \begin{Bmatrix} p'_x \\ p'_y \\ p'_z \\ 1 \end{Bmatrix} = {}^{0}\mathbf{T}_{n} \begin{Bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{Bmatrix} $$

3. Kinematic Chain Modeling

3.1 Printhead Rotation

As discussed previously, the Selectric typehead has characters arranged into eleven rows. Depressing a key in the keyboard actuates a key lever which is connected through a series of levers and interposers to a mechanism known as a whiffletree.

This is a series of links connected asymmetrically to perpendicular supports, as shown in figure 2.

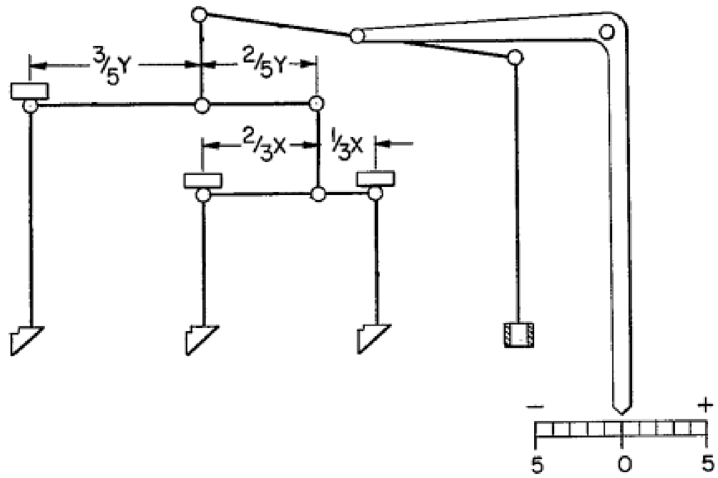

Figure 2: Schematic of the rotation whiffletree mechanism. The X and Y distances are placeholders; the ratio of the distances is what is critical.

This asymmetry means that pulling on certain combinations of the links can result in several discrete multiples of displacement. Each key, then, is connected to a number of these links through an interposer, and when pressed causes the whiffletree crank to rotate 1, 2, 3, 4, or 5 units either clockwise or counterclockwise. The crank causes a force on a rigid body fixed about a pivot, and the chain from this pivot to the printing character is one of the systems the thesis is focused on. This simplification avoids the complicated geometry of the linkages from the key press to the whiffletree, while modelling the system input as a rotation about the pivot and creating the kinematic chain to the printhead will still allow the errors to be introduced in the HTM and show the effect on character position those errors cause.

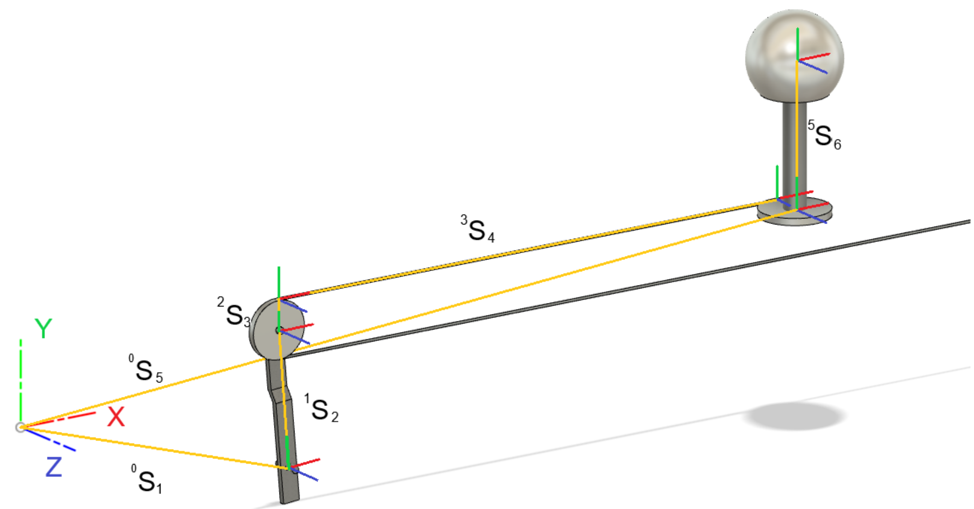

Figure 3: Simplified drawing showing rigid bodies associated with printhead rotation. The large axes represent the base coordinate system at bottom of machine.

In order to represent the rotation of the printhead, there are two kinematic chains extending from the base of the machine. The first LCS is located on the pivot point of the lever connected to the whiffletree. This LCS is also rotated .175 radians about the Z axis relative to the base coordinate system so that in the neutral pose of the machine, the next LCS at the center of the pulley is in alignment with the local Y axis. To model the rotation of the lever as caused by the whiffletree mechanism, this pivot is treated as a rotary joint with commanded rotation about the Z-axis (Θ_Z). The rest of this first kinematic chain is composed only of shape matrices: no other motions are commanded in any joints, so all movement is a result of the rotation about J_1. The shape matrix from LCS_1 to LCS_2 returns the coordinate system to the same orientation as the base using the transpose of the two rotation matrices used in ${}^{0}S_1$ and J_1. Since the carrier holding the ball moves along the base X-axis as the user types, the distance used for ${}^{3}S_4$ is variable. The pulley tape is attached to the other end of the carrier, meaning the rotation about J_1 will extend the tape the same amount no matter what X-position the carrier is in. The end point of the first kinematic chain, LCS_4, is located at the tangent of the second pulley. The full HTM for this kinematic chain is:

$$\begin{bmatrix} 1 & 0 & 0 & 1\,in \\ 0 & 1 & 0 & .5\,in \\ 0 & 0 & 1 & 4.85\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(.175) & -\sin(.175) & 0 & 0 \\ \sin(.175) & \cos(.175) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(\Theta_Z) & -\sin(\Theta_Z) & 0 & 0 \\ \sin(\Theta_Z) & \cos(\Theta_Z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} 1 & 0 & 0 & 0\,in \\ 0 & 1 & 0 & 1.95\,in \\ 0 & 0 & 1 & 0\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(.175) & -\sin(.175) & 0 & 0 \\ \sin(.175) & \cos(.175) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T * \begin{bmatrix} \cos(\Theta_Z) & -\sin(\Theta_Z) & 0 & 0 \\ \sin(\Theta_Z) & \cos(\Theta_Z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$* \begin{bmatrix} 1 & 0 & 0 & 0\,in \\ 0 & 1 & 0 & .4125\,in \\ 0 & 0 & 1 & 0\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 8\,in \\ 0 & 1 & 0 & 0\,in \\ 0 & 0 & 1 & 0\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^0T_4$$

A second kinematic chain is required because the second pulley, on which LCS_5 is located, is fixed to the machine, and does not change position with the rotation of J_1. The two chains are linked, however, because the pulleys at LCS_2 and LCS_5 are connected by the tape. LCS_4 at zero commanded rotation Θ_z is located tangent to pulley 2, so when LCS_4 is translated by Θ_z, the extension of the tape causes pulley 2 to rotate. This rotation is calculated by determining the change in the x-position of LCS_4. The x-position is found by using equation …. LCS_4 and LCS_5 have the same initial X-position, so the change in X is equal to:

$$\Delta X = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} * {}^0S_5 * \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} * {}^0T_4 * \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

As the tape extends by ΔX, it pulls pulley 2 by an equivalent arc length which causes a particular rotation, Φ. With a pulley diameter of 1 inch, rotation can be calculated according to:

$$\phi = \frac{\Delta X}{0.5}$$

The calculated rotation is a key value in the operation of the typewriter. Since there are 11 possible rotation values per half of the ball, Φ must be a multiple of .2856 radians in order for the characters to properly interface with the paper mounted to the platen. If Φ is off due to error, then the desired character will not print completely as it will not have rotated to face the paper correctly.

With the rotation of the second pulley calculated, it is possible to continue the second kinematic chain from LCS_5 to the center of the printing ball. J_5 is modelled as a rotary joint about the Y axis of LCS_5 with commanded rotation Φ. A shape matrix then follows the local y-axis to the center of the printing ball, where local coordinates to any point on the ball can be converted to global coordinates. The HTM is shown below.

$$\begin{bmatrix} 1 & 0 & 0 & 8.4953\ in \\ 0 & 1 & 0 & 2.7961\ in \\ 0 & 0 & 1 & 5.35\ in \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(\Phi) & 0 & \sin(\Phi) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\Phi) & 0 & \cos(\Phi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2.125\ in \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}^{0}T_6$$

3.2 Printhead Tilt

The method of tilting the ball to utilize the four rows of characters is similar to the method of rotation. The keys are also interposed into a second whiffletree mechanism that also exerts force on a pivoting pulley arm, and so a similar HTM system can be created for the tilting of the printhead.

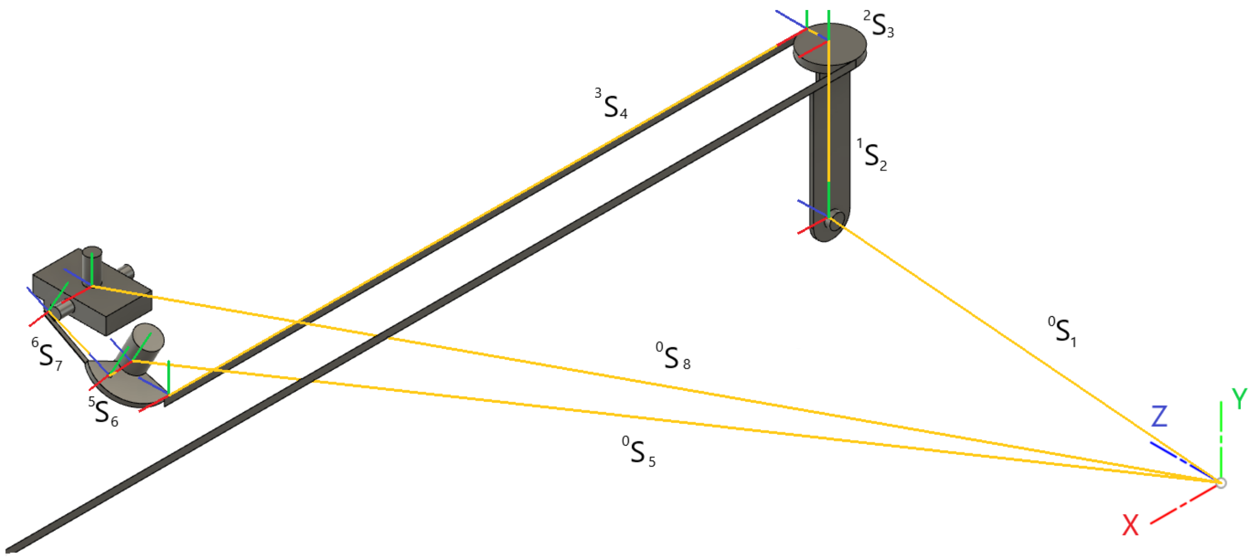

Figure 4: Simplified model of tilt mechanism. The large coordinate axis represents the global coordinate system at the base of the machine. The printhead is not pictured for clarity but would be centered on top of the post that LCS_8 is centered on.

Compared to the rotation HTM system, the tilt system is more complex with three separate kinematic chains and eight local coordinate systems. LCS_5 and LCS_8 are centers of rotation that are fixed to the machine. The rotations of both can be modelled as a commanded rotation about the local Y-axes, which are calculated from the movement of LCS_4 and LCS_7 respectively. It is the rotation of LCS_8 about the X-axis that is the actual tilt value of the printhead. The full HTM system was not calculated for the tilt system, but using the same methods as on the rotation system it is possible to combine the shape and joint matrices for the LCSs defined in Figure 4 to determine the effect of a commanded rotation about joint 1 on the tilting of the printhead.

4. Introducing Error

With a completed system of HTMs, it is now possible to determine both the rotation of the printhead and the global coordinates of any point on it based on the rotation of the joint J_1. By introducing error terms into these matrices, the effect of those error sources on rotation and global coordinates can also be determined.

One example of an error to be introduced is a drive error in J_1. This could be caused by a loss of stiffness in the spring attached to the pulley arm, meaning the discrete increments of force applied by the whiffletree now result in a larger rotation about J_1. With the error implemented, J_1 now takes the following form, with ε_z representing the error.

$$\begin{bmatrix} \cos(\Theta_Z + \varepsilon_z) & -\sin(\Theta_Z) & 0 & 0 \\ \sin(\Theta_Z) & \cos(\Theta_Z + \varepsilon_z) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

One still uses the system of HTMs as laid out before but substituting this new form of J_1. As an example, consider a desired Φ value of .2856 radians. This corresponds to a rotation of one column of characters. Using the HTM system with no errors as described in the previous section, it can be determined that an input Θ_Z of .07667 radians produces this desired Φ value. An error value is assigned, in this case .01745 radians. Using the following HTM therefore allows you to see the new value of Φ and compare it to the desired value of .2856.

$${}^{0}T_4 = \begin{bmatrix} 1 & 0 & 0 & 1\,in \\ 0 & 1 & 0 & .5\,in \\ 0 & 0 & 1 & 4.85\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(.175) & -\sin(.175) & 0 & 0 \\ \sin(.175) & \cos(.175) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} \cos(.07667 + .01745) & -\sin(.07667) & 0 & 0 \\ \sin(.07667) & \cos(.07667 + .01745) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0\,in \\ 0 & 1 & 0 & 1.95\,in \\ 0 & 0 & 1 & 0\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} \cos(.175) & -\sin(.175) & 0 & 0 \\ \sin(.175) & \cos(.175) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T * \begin{bmatrix} \cos(.09412) & -\sin(.07667) & 0 & 0 \\ \sin(.07667) & \cos(.09412) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T *$$

$$\begin{bmatrix} 1 & 0 & 0 & 0\,in \\ 0 & 1 & 0 & .4125\,in \\ 0 & 0 & 1 & 0\,in \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 8\,in \\ 0 & 1 & 0 & 0\,in \\ 0 & 0 & 1 & 0\,in \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi_{error} = ([1 \quad 0 \quad 0 \quad 0] * {}^0S_5 * \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} - [1 \quad 0 \quad 0 \quad 0] * {}^0T_4 * \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix})/0.5$$

By placing these HTMs into a numerical software like MATLAB, it is possible to calculate that the new $\phi_{error} =$ .3079 radians. This is a difference of .0223 radians attributable to the error in the commanded rotation. Such a difference would mean that the desired character would not print fully onto the paper as it is misaligned. Other sources of error, such as pitch or parallelism, can also be inserted into the HTM system and calculated to show the effect on either the rotation of the printhead or the global coordinates of a character on its face.

5. Conclusion

The completed transformation matrix for the rotation of the Selectric printhead shows the ability of kinematic chain modelling to determine a desired position value in a system based on a commanded input motion and any error motions of interest. Further study would show the same ability with the tilting system. In operation, the tilting and rotation of the Selectric printhead occurs simultaneously to position the correct character in place to print. With both HTM systems completed, they could be combined to show the true global position of the printhead character based on the input motions and errors. The flexibility of the kinematic chain modelling then is a significant benefit, as it allows a system model to be either simplified or expanded as required.

This demonstration of the effectiveness of kinematic chain modelling in a rotary system shows the potential it has for creating an error budget for a proposed manufacturing process. A well-constructed model allows key positions to be determined arithmetically using the matrix chains and inputs, meaning it is possible to develop an idea of whether a particular amount or type of error results in a suitable product before investing the time and money into setting up an actual process. Ensuring the modelling is rigorous and correct is very important though, as an inaccurate model will not give any useful data. It is also important to be thorough in the input of error into the system. There are many types of error that can be present in a system, and it may not be required to include all of them if they are either not present to a large extent or have a small impact in the system being modelled; however, care must be taken to ensure that no sources of error are missed.

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