A PARAMETRIC STUDY OF THE STRESS DISTRIBUTION IN A FOUR-CELLED BOX BEAM MODEL OF THE ASR-21 CLASS CATAMARAN CROSS-STRUCTURE

by

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ABSTRACT

The objectives of this thesis are:

(a) to attempt to analyze the stress distribution in the cross structure of the ASR-21 Class Catamaran by finite element analysis utilizing the ICES STRUDL-II Program of the Massachusetts Institute of Technology, Department of Civil Engineering.

(b) to develop a set of plating effectiveness curves for a doubly symmetrical model of the catamaran cross structure based on output from a finite element analysis, and to compare the values obtained from the actual cross structures to these curves.

(c) to attempt to draw some basic conclusions concerning the design of the actual cross structures.

This study is three dimensional in nature. Various geometric parameters are varied for the model to analyze what effect they produce on the resultant stresses in the cross structure. The breadth to depth (B/D), the length to breadth (L/B), and the flange area to web area (A_{A}/A_{W}) ratios are the primary variables considered. A total of twenty-one (21) geometric variations are performed.

The catamaran cross structure and models are assumed to be cut at the inboard side of each hull and removed for proper loading. The three loadings used are symmetric bending moment, antisymmetric bending moment and shear, and torsional moment.

The stress distribution obtained from the model indicates that there is very little variation in the effectiveness at various B/D or A_f/A_W ratios for a given L/B ratio. There is a slight increase in average stress across a set of elements as A_f/A_W increases. The effectiveness of the various element sections of the actual ASR-21 catamaran cross structures is generally higher than that predicted by the model effectiveness curves. The effective breadth used for the design of the ASR-21 Class catamaran cross structures is very conservative when compared to the actual effective breadths determined in the thesis. Linearity in the stress distribution response to various loading conditions was observed.

An effort should be made to study additional structural cross sections planned for use in future catamaran cross structures, and to examine the plating effectiveness at the higher L/B ratios proposed for them. A satisfactory analytical solution of the cross structure using the stress function should be conducted.

Thesis Supervisor: Alaa Mansour Title: Assistant Professor of Naval Architecture

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TABLE OF CONTENTS

																							Page
TITLE P.	AGE .	• •	•		•		•	0	٠	•			•	•	•	•	•			٠	•		l
ABSTRAC	г	• •				0				•					•			٠		٥	•	0	2
ACKNOWL	EDGEM	ENT.	0	•		٠	0								۰	۰					0	٠	4
TABLE O	F CON	TENTS	5.				0		٠	۰			٠	•	•								5
LIST OF	FIGU	RES.						•				٠					٠						6
LIST OF	TABL	ES .		•			0			۰		•	•	•	÷					•			9
INTRODU	CTION	• •							•							٠			•		0	٠	11
PROCEDU	RE		•				•		٠														18
Lo	ading	Anal	lys	sis	5.					•	•		•			٠						•	18
Fi	nite	Eleme	ent		100	le]	lli	ing	у. •														21
Ra	nge o	f Pai	ran	net	cer	?S	6			•	•					٠					0	۰	26
Mo	del L	oadiı	ng						•					•		•	•	•	•			٠	27
Pl	ating	Effe	ect	ci.	/er	ies	SS	۰					٠								0	9	37
RESULTS			•				•	•	•	•			•					9		0	0	•	40
DISCUSS	ION O	F RES	SUI	TS	5.			0									•	•		0			73
CONCLUS	IONS.						0		٠							٠		•	•		•		83
RECOMME	NDATI	ONS.		•					•			٠		•		۰	٠	0		0	0	۰	86
APPENDI	CES .	• •	ē				•		•	•	•					0	•					•	87
Α.	Anal	ytica	al	To	ors	sic	ona	al	Lc	bad	lir	ıg						0	0	0			88
В.	Comm	ents	Oľ	ı F	Rel	lat	cir	ng	tł	ne	Сс	omp	out	cer	: I	Res	sul	ts	s t	0			
0	EXIS	ung	'I'r	ie (orj	0	•	0	٠	•	•	•	٠	٠		٠	٠	0		٠	•		94
C.	Data	Tab.	те	SI	ımr	nai	.,⊺€	es	٠	٠	٠	•	•	•	0	٠	۰	*	٠	•	۰	•	98
D.	пете	rence	es																				190

LIST OF FIGURES

Number Title T FORWARD CROSS STRUCTURE OF ASR-21 CATAMARAN AFTER CROSS STRUCTURE OF ASR-21 CATAMARAN TT INBOARD PROFILE, PORT HULL, OF ASR-21 CATAMARAN III THE LOADED BOX BEAM ΤV FINITE ELEMENT MODEL OF ASR-21 FORWARD CROSS V STRUCTURE FINITE ELEMENT MODEL OF ASR-21 AFTER CROSS VI STRUCTURE CROSS STRUCTURE MODEL WITH QUARTER STRUCTURE VII DETAILED QUARTER STRUCTURE FINITE ELEMENT MODEL ILLUSTRATING VIII NODE, ELEMENT AND COORDINATE LOCATION IX FULL AND QUARTER STRUCTURE NODAL FORCES FOR THE SYMMETRICAL BENDING MOMENT LOADING FULL AND QUARTER STRUCTURE NODAL LOADS FOR THE Χ SHEAR FORCES QUARTER STRUCTURE NODAL LOADS FOR TORSIONAL XI MOMENT (B/D=3.0) TOP PLATE EFFECTIVENESS DEFINITION XTT SYMMETRICAL BENDING MOMENT LONGITUDINAL STRESS XIII DISTRIBUTION OF QUARTER STRUCTURE (L/B=.7, B/D=3.0, $A_{f}/A_{w}=3.0$) ANTISYMMETRICAL BENDING MOMENT AND SHEAR XIV LONGITUDINAL STRESS DISTRIBUTION OF QUARTER STRUCTURE (L/B=.7, B/D=3.0, $A_{f}/A_{w}=3.0$) COMBINED LONGITUDINAL STRESS DISTRIBUTION OF XV QUARTER STRUCTURE (L/B=.7, B/D=3.0, Af/Au=3.0) COMBINED LONGITUDINAL STRESS DISTRIBUTION FOR THE XVI FORWARD ASR-21 CROSS STRUCTURE COMBINED LONGITUDINAL STRESS DISTRIBUTION FOR THE XVII AFTER ASR-21 CROSS STRUCTURE

LIST OF FIGURES (Cont'd.)

Number

Title

- XVIII SYMMETRICAL BENDING MOMENT GIRTH STRESS DISTRIBUTION OF QUARTER STRUCTURE (L/B=.7, B/D=3.0, A_f/A_W=3.0)
 - XIX ANTISYMMETRIC BENDING MOMENT AND SHEAR GIRTH STRESS DISTRIBUTION OF QUARTER STRUCTURE (L/B=.7, B/D=3.0, $A_{f}/A_{w}=3.0$)
 - XX COMBINED GIRTH STRESS DISTRIBUTION OF QUARTER STRUCTURE (L/B=.7, B/D=3.0, A_f/A_w=3.0)
 - XXI COMBINED GIRTH STRESS DISTRIBUTION FOR THE FORWARD ASR-21 CROSS STRUCTURE
 - XXII COMBINED GIRTH STRESS DISTRIBUTION FOR THE AFTER ASR-21 CROSS STRUCTURE.
- XXIII SYMMETRICAL BENDING MOMENT SHEAR STRESS DISTRIBUTION $(L/B=.7, B/D=3.0, A_f/A_w=3.0)$
 - XXIV ANTISYMMETRIC BEND MOMENT SHEAR STRESS DISTRIBUTION (L/B=.7, B/D=3.0, A_f/A_w=3.0)
 - XXV COMBINED SHEAR STRESS DISTRIBUTION OF QUARTER STRUCTURE
 - XXVI COMBINED SHEAR STRESS DISTRIBUTION FOR THE FORWARD ASR-21 CROSS STRUCTURE
- XXVII COMBINED SHEAR STRESS DISTRIBUTION FOR THE AFTER ASR-21 CROSS STRUCTURE
- XXVIII TORSIONAL MOMENT SHEAR STRESS DISTRIBUTION OF QUARTER STRUCTURE (B/D=3.0, $A_f/A_w=2.5$; B/D=2.5, $A_f/A_w=3.0$; B/D=3.5, $A_f/A_w=3.0$)
 - XXIX EFFECTIVENESS (ρ_1) VS L/B FOR SYMMETRIC BENDING MOMENT
 - XXX EFFECTIVENESS (ρ₁) VS L/B FOR ANTISYMMETRIC BENDING MOMENT AND SHEAR
 - XXXI EFFECTIVENESS (p,) VS L/B FOR THE COMBINED LOADING
- XXXII EFFECTIVENESS (p2) VS L/B FOR SYMMETRIC BENDING MOMENT
- XXXIII EFFECTIVENESS (p₂) VS L/B FOR ANTISYMMETRIC BENDING MOMENT AND SHEAR

LIST OF FIGURES (Cont'd.)

Number

Title

- XXXIV EFFECTIVENESS (p,) VS L/B FOR COMBINED LOADING
- XXXV DISPLACEMENT OF QUARTER STRUCTURE DUE TO TORSIONAL MOMENT
- XXXVI DISPLACEMENT OF QUARTER STRUCTURE DUE TO ANTI-SYMMETRIC BENDING MOMENT
- XXXVII DISPLACEMENT OF QUARTER STRUCTURE DUE TO SYMMETRIC BENDING MOMENT
- XXXVIII DISPLACEMENT OF QUARTER STRUCTURE DUE TO COMBINED LOADING
 - XXXIX AVERAGE STRESS VS L/B FOR THE QUARTER STRUCTURE
 - XL EFFECTIVE BREADTH ASSOCIATED WITH CENTER TRANSVERSE BULKHEAD VS LONGITUDINAL POSITION ALONG THE CROSS STRUCTURE
 - XLI THE FIVE ELEMENT MODEL NUMBERING SEQUENCE
 - XLII PARABOLIC FIT VERIFICATION
 - XLIII SYMMETRIC AND ANTISYMMETRIC LONGITUDINAL STRESS DISTRIBUTIONS AT y=3L/8
 - XLIV COMBINED LOAD LONGITUDINAL STRESS DISTRIBUTIONS AT y=3L/8
 - A-I CROSS SECTION OF A FOUR-CELLED BOX BEAM
 - C-I TORSIONAL MODEL ELEMENT NUMBERING SEQUENCE

LIST OF TABLES

Number	Title
1	PREDICTED CATAMARAN SEA LOADINGS (FOOT-TONS)
2	CHARACTERISTICS OF THE ASR-21 CATAMARAN CROSS STRUCTURES
3	FLANGE AND WEB PLATING THICKNESS
4	FULL MODEL AND QUARTER STRUCTURE NODAL LOADINGS FOR SYMMETRIC BENDING MOMENT (M _{x2} =1500 FOOT-TONS) AND VARIOUS B/D RATIOS
5	FULL MODEL AND QUARTER STRUCTURE NODAL LOADINGS FOR ANTISYMMETRIC BENDING MOMENT (M _{x3} =500 FOOT-TONS) AND VARIOUS B/D RATIOS
6	QUARTER STRUCTURE NODAL SHEAR FORCES (V _z =M _{x1} /L= 1000/L TONS) FOR VARIOUS LENGTHS
7	QUARTER STRUCTURE NODAL LOADINGS FOR TORSIONAL MOMENT (M_=1000 FOOT-TONS) AND VARIOUS B/D RATIOS
8	TOP PLATING EFFECTIVENESS LOCATORS FOR THE ASR-21 CROSS STRUCTURES
9	EFFECTIVE BREADTHS OF THE MODEL AND ACTUAL CROSS STRUCTURES
C-1	ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)
C-2	ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)
C-3	NORMAL STRESS (σ) IN KSI FOR SYMMETRIC BENDING MOMENT (LONGITUDINAL STRESSES)
C-4	NORMAL STRESSES (σ_{yy}) IN KSI FOR ANTISYMMETRIC BENDING MOMENT AND SHEAR (LONGITUDINAL STRESSES)
C-5	NORMAL STRESSES (σ_{xx}) FOR SYMMETRIC BENDING MOMENT (GIRTH STRESSES)
С-б	NORMAL STRESSES (σ _{xx}) FOR ANTISYMMETRIC BENDING MOMENT + SHEAR (GIRTH STRESSES)
C-7	SHEAR STRESSES ($\sigma_{\rm Xy}$) FOR SYMMETRIC BENDING MOMENT
C-8	SHEAR STRESSES (σ_{xy}) FOR ANTISYMMETRIC BENDING MOMENT + SHEAR

-10-

LIST OF TABLES (Cont'd.)

Number

Title

- C-9 COMBINED RESULTS FOR LONGITUDINAL STRESS (σ_{yy}) IN KSI
- C-10 COMPUTER CALCULATED TORSION RESULTS (FOR 1/8 STRUCTURE) M₊=1000 FOOT-TONS
- C-11 CALCULATED SHEAR STRESSES (M_t=1000 FOOT-TONS) FOR TORSION CASE
- C-12 AVERAGE LONGITUDINAL STRESS (σ_{YY}) AVG FOR SYMMETRIC BENDING MOMENT AND ANTISYMMETRIC BENDING MOMENT AND SHEAR CASES AND EFFECTIVENESS
- C-13 COMBINED RESULTS OF AVERAGE LONGITUDINAL STRESSES AND EFFECTIVENESS
- C-14 EXTRAPOLATION & AVERAGE STRESS VALUES AND PLATING EFFECTIVENESS FOR THE ACTUAL STRUCTURES

INTRODUCTION

Since 1965 there has been a great surge of interest in the catamaran ship. The large deck area and favorable stability of this ocean vehicle make it ideal for numerous ocean engineering tasks. Serious efforts have been channeled towards two areas in the analysis of the catamaran: the hydrodynamics of the hulls, and the statistical response of the ship to limiting sea conditions that allows one to determine the maximum loads on the ship's structural members.

References (1) through (8) have dealt almost exclusively with the problem of predicting the maximum loads that the new catamaran submarine rescue ship (ASR-21 Class) will experience throughout its life cycle. Of particular interest has been the analysis of the loadings on the cross structures which connect the demi-hulls, and the fabrication problems associated with joining the cross structures to the demi-hulls.

The variations in the predicted loadings which have resulted from the analyses are interesting. This is not because of their absolute values but rather because of the variations evidenced in the calculations of these quantities. This indicates that some degree of uncertainty still exists in the selection of the most proper procedural method for analysis.

It is generally accepted by all the authors that three distinct conditions of maximum loading exist. Lankford (3) describes these conditions for the catamaran when it is at zero

-11-

speed in beam seas, when it is at zero speed in quartering seas, and finally when it is in a condition of either grounding or docking. The beam seas-zero speed case produces vertical bending moments, a smaller torsional moment and slight vertical shear forces. The vertical bending moments tend to roll the hulls, and the vertical shear forces tend to heave the hulls differentially. The quartering seas-zero speed case produces a twisting moment on the hulls about the center of torsion and a slightly lower bending moment. The grounding-docking case considered primarily by Lankford illustrates the most severe torsional loading to which the cross structure could be subjected. It is assumed for this case that one hull is supported forward and one hull is supported aft. Table 1 is a summary of the loadings that various researchers have determined for the ASR-21 catamaran. These figures do not include the catamaran cross structure weight.

These variations in loading point to the difficulty associated with an analysis of the stress and displacement distribution which are produced by these loads on the cross structures. When an analysis is performed on the structures, the results are a direct function of the applied loads. Calculations for a range of loadings could be costly, cumbersome and time consuming.

As long as the loads are assumed to give results that correspond to the linear elastic range of the utilized construction materials, then the use of finite element analysis appears to be the most versatile method to use. Because of the linearity that exists between the applied loadings and the

-12-

TABLE 1

PREDICTED CATAMARAN SEA

LOADINGS (FOOT-TONS)

		Lankford (3)	Scott (8)	Schade (4)	Dinsenbacher (4)
1.	Beam Seas	55,000	26,572	60,900	53,400
	(Maximum Bending Moment)				
2.	Quartering Seas	27,120	23,495	73,500	16,500
	(Maximum Torsional Moment)				
3.	Grounding-Docking	102,790			
	(Torsional Moment)				

-13-



FORWARD CROSS STRUCTURE OF ASR-21 CATAMARAN



AFTER CROSS STRUCTURE OF ASR-21 CATAMARAN

TABLE 2

CHARACTERISTICS OF THE ASR-21 CATAMARAN

CROSS STRUCTURES

	Forward		A	fter
Length (feet)	35			35
Breadth (feet	56			52
Depth (feet)	18			18
Flange Thicknesses (inches)				
Main Deck	5/16			5/16
Ol Level	9/16			9/16
02 Level	3/8			3/8
Web Thicknesses (inches)				
Frame 21	1/2	Frame	84	3/8
Frame 37	3/8	Frame	96	3/8
Frame 49	3/8	Frame	110	1/2
L/B	.62			.671
B/D	3.11		ć	2.89
A _f /A _W	2.82			3.05

output displacements, structural analysis can be performed for standard loads, and by scaling the results of these loads either up or down, and assuring oneself that he remains within the linear range, results for any reasonable loading can be quickly calculated.

The objectives of this thesis are three-fold. An attempt is made to analyze the stress distribution in the cross structures of the ASR-21 Class Catamaran by finite element analysis utilizing the ICES STRUDL-II program of the Massachusetts Institute of Technology, Department of Civil Engineering (Ref. 9). The forward and after cross structures and their locations relative to the ship itself are indicated in Figures I, II and III. Development of a set of curves for the top plating effectiveness similar to those presented by Schade (Refs. 12, 13) and Mansour (Ref. 11) are calculated for a range of length to breadth, breadth to depth, and flange area to web area ratios. These calculations are based on a finite element analysis of a doubly symmetrical model of the cross structure. By plotting the effectiveness of the top plating of the actual catamaran cross structures on the previously determined effectiveness curves, conclusions are drawn about the design of the actual ASR-21 cross structures.

Loading conditions utilized in this analysis approximate as closely as possible the various loadings deemed most critical by the investigators previously mentioned. The characteristics of the actual ASR-21 cross structures are included here in Table 2.

-17-

PROCEDURE

Loading Analysis

In order to apply the desired loads to the cross structure the sections of the cross members between the hulls of the catamaran were cut at the inboard side of each hull and removed. The vertical bending moment, shear and torsional moment were then applied to the cross structure as one would apply them to a free body diagram of a beam (Fig. IVa). The beam in this instance is a closed, thin-walled, multicelled box beam.

Conceivably, the cross structure can be loaded simultaneously to some degree by all of the sea loading conditions specified in the Introduction. By dividing the loads into three separate loading conditions, insuring equilibrium, and analyzing each, the effect of the individual loadings on the cross structure can be examined. Equations (1) to (3) are applicable for determining the relationships between the fully loaded beam and the individual loading cases.

$$\frac{\text{Shear Load}}{\text{L}} \qquad \qquad V_{z} = \frac{M_{xl}}{L} \qquad \qquad (1)$$

(L is the hull to hull spacing)

$$\frac{\text{Antisymmetric}}{\text{Bending Moment}} \qquad M_{x3} = \frac{V_z L}{2} = \frac{M_{x1}}{2} \qquad (2)$$

$$\frac{\text{Symmetric}}{\text{Bending Moment}} \qquad M_{x2} = \frac{M_{x1} + M_{x3}}{2} \qquad (3)$$

The beam seas case then becomes the superposition of the symmetric bending moment loading (Fig. IVc), a smaller



FIGURE III INBOARD PROFILE, PORT HULL, OF ASR-21 CATAMARAN WITH CROSS STRUCTURES (HEAVILY OUTLINED)

-19-



torsional moment (Fig. IVb) and the antisymmetric bending moment and shear loadings (Fig. IVd). The quarter seas case is primarily the torsional loading (Fig. IVb) with a smaller bending moment also operating on the structure.

Finite Element Modelling

The forward and after cross structures (Figs. I, II) were subdivided into a set of elements and nodes in accordance with reference (9) for use with the ICES-STRUDL II finite element program. As can be observed in Figures (V) and (VI) there are 116 elements/130 nodes and 112 elements/125 nodes for the forward and after structures, respectively. Additionally, the forward cross structure has no planes of symmetry, and the after cross structure has only one plane of symmetry. To run numerous analyses with these two structures with varying L/B, B/D and A_f/A_W ratios would be extremely expensive because of the computer time required to calculate the stiffness matrices. For the two runs made with the actual structures the computer calculation time required to solve the partitioned matrix was 316.76 seconds and 341.47 seconds. Therefore, a doubly symmetrical model was constructed (Fig. VII). With this model it was possible to reduce the computer costs significantly by taking advantage of the double symmetry, and completing the analysis for only one quarter of the structure with 35 elements/45 nodes (Fig. VIII). In fact, only 24.47 seconds of computer time was required to solve the partitioned



FINITE ELEMENT MODEL OF ASR-21 FORWARD CROSS STRUCTURE



Scale: 1/8'' = 1'-0''

FIGURE VI FINITE ELEMENT MODEL OF ASR-21 AFTER CROSS STRUCTURE



FIGURE VII CROSS STRUCTURE MODEL WITH QUARTER STRUCTURE DETAILED

-24-



QUARTER STRUCTURE FINITE MODEL ILLUSTRATING NODE, ELEMENT AND COORDINATE LOCATION -25-

matrix for the quarter structure. Hence, all computer analysis involving the variance of length to breadth, breadth to depth, and flange area to web area ratios were performed on the quarter structure. Single runs were made with each of the actual cross structures.

Range of Parameters

The ranges of selected parameter values utilized in this thesis were established by the actual catamaran cross structure parameter values. Table (2) indicates the catamaran's cross structure length to breadth, breadth to depth, and flange area to web area ratios. In order to observe the effect of these ratios on the stress level and the effectiveness of the plating, the parameters were varied both above and below the values of the actual structure. The following ratio ranges were utilized in this thesis:

$$\cdot 3 \leq L/B \leq 1.5$$

2.5 $\leq B/D \leq 3.5$
2.5 $\leq A_f/A_w \leq 3.5$

The last parameter, $A_{\rm f}/A_{\rm W},$ is actually a function of the second parameter, B/D, and the ratio of plating thicknesses, $t_{\rm f}/t_{\rm W}.$

$$A_{f}/A_{W} = (B/D) (t_{f}/t_{W})$$
(4)

-26-

It can be observed from equation (4) that given the specific ranges of the two parameters, a unique set of plating thickness ratios can be obtained. To facilitate the analysis it was assumed that the thickness of the flange, t_f , was held constant at a value of .38 inches. Another goemetric descriptor that was held constant in the thesis was the cross structure breadth, B. A breadth of 50 feet was used. Table (3) lists the values of the plate thicknesses assumed for the analysis.

TABLE 3

	B/D	2.5	3.0	3.5
A _f /A _W				
2.5			$t_{f} = .38$ $t_{W} = .45$	
3.0		t _f = .38 t _W = .3167	t _f = .38 t _W = .38	t _f = .38 t _w = .444
3.5			$t_{f} = .38$ $t_{W} = .32$	

FLANGE AND WEB PLATING THICKNESS

Model Loading

For this thesis the standard moment, M_{xl} , was arbitrarily set equal to 1000 foot-tons. From equations (1) to (3) this implies that

 $M_{x3} = 500$ foot-tons $M_{x2} = 1500$ foot-tons $V_z = 1000/L$ tons Results for different loads can be obtained by scaling the stresses obtained from these standard loads provided the yield strength of the material is not exceeded.

Applying the loads to the finite element model is probably the major departure from theoretical analysis. Essentially, this statement can be reduced to the fact that a uniformly applied moment must be approximated by individual concentrated forces which are applied to the nodes. An increase in the number of nodes and elements will produce a better approximation, but only at the expense of time and money.

Division of the applied moment into nodal forces was based on the assumption that each transverse bulkhead will carry the load associated with each flange. Since the center transverse bulkhead has adjacent flange areas on both sides, it was assumed that the center bulkhead carried twice the load that the end transverse bulkheads carried. Nodal forces for the full structure under symmetrical bending moment loading were then determined by equations (5) through (10). The moment distribution was assumed to be linear in the vertical direction: zero load at nodes 1, 3, 14, half the maximum load at nodes 13, 4, 9 and maximum at nodes 5, 8, 12.

$M_{x2} = 1500 \text{ foot-tons}$	(5)
$F_{13}(D/4) + F_{12}(D/2) = 375 \text{ foot-tons}$	(6)
F_9 (D/4) + F_8 (D/2) = 750 foot-tons	(7)
F_4 (D/4) + F_5 (D/2) = 375 foot-tons	(8)
$F_4 = F_{13} = 1/2 F_{12} = 1/2 F_5$	(9)
$F_9 = F_5 = F_{12}$	(10)

-28-

To obtain the forces for the quarter structure the forces at the center transverse bulkhead were divided in half (The plate thicknesses are also divided in half where the quarter structure is removed from the full structure). The application of the nodal forces to the full and quarter structures are illustrated in Figure IX. Symmetrical bending moment nodal forces for the various B/D ratios utilized in this thesis are presented in Table 4. Similar results for the antisymmetric bending moment case with M_{x3} = 500 foot-tons are included in Table 5.

TABLE 4

FULL AND QUARTER STRUCTURE NODAL LOADINGS FOR SYMMETRIC BENDING MOMENT

 $(M_{x2} - 1500 \text{ foot-tons})$ AND VARIOUS B/D RATIOS

Full Structure

Nodal Forces (tons)

Node Number	B/D = 2.5	$\underline{B/D} = 3.0$	B/D = 3.5
8, 120	-60.	-72.	-84.4
5, 9, 12, 117, 121, 124	-30.	-36.	-42.2
4, 13, 116, 125	-15.	-18.	-21.1
20, 108	60.	72.	84.4
17, 21, 24, 105, 109, 112	30.	36.	42.2
16, 25, 104, 113	15.	18.	21.1



FIGURE IX FULL AND QUARTER STRUCTURE NODAL FORCES FOR THE SYMMETRICAL BENDING MOMENT LOADING

TABLE 4 (Cont'd.)

Quarter Structure

Nodal Forces (tons)

Node Number	B/D = 2.5	B/D = 3.0	$\underline{B/D} = 3.5$
5,8	-30.	-36.	-42.2
4,9	-15.	-18.	-21.1
41, 44	30.	36.	42.2
40, 45	15.	18.	21.1

TABLE 5

FULL AND QUARTER STRUCTURE NODAL LOADINGS FOR ANTISYMMETRIC BENDING MOMENT ($M_{x3} = 500$ foot-tons) AND VARIOUS B/D RATIOS

Full Structure

Nodal Forces (tons)

Node Number	B/D = 2.5	$\underline{B/D} = 3.0$	$\underline{B/D} = 3.5$
8, 108	20.	24.	28.13
5, 9, 12, 105, 109, 112	10.	12.	14.06
4, 13, 104, 113	5.	б.	7.03
20, 120	-20.	-24.	-28.13
17, 21, 24, 117, 121, 124	-10.	-12.	-14.06
16, 25, 116, 125	-5.	-6.	-7.03

TABLE 5 (Cont'd.)

Quarter Structure

Nodal Forces (tons)

Node Number	$\underline{B/D} = 2.5$	B/D = 3.0	B/D = 3.5
5, 8, 41, 44	10.	12.	14.06
4, 9, 40, 45	5.	6.	7.03

The total shear load for the end cross section must be divided in much the same way as the bending moments. In addition to distributing the shear so that one half the total shear is applied to the center transverse bulkhead and one half is distributed equally between the other two transverse bulkheads, the shear must also be divided so that each node carries its share of the load. For this procedure it was assumed that any node having neighboring nodes on both sides would carry a full proportion of the load at that particular bulkhead. Any node that had only one neighboring node (either the top or bottom nodes) would carry one half the load of a node with two neighbors. Figure X shows how the total shear (q) is applied to the nodes of the full cross structure and the quarter structure. A summary of calculations for the nodal shear forces for various lengths of the cross structure, and M_{x1} = 1000 foot-tons is presented in Table 6.

TOTAL SHEAR = q





¶q/32

-33-

TABLE 6

QUARTER STRUCTURE NODAL SHEAR FORCES

 $(V_Z = M_{x1}/L = 1000/L \text{ tons})$ FOR VARIOUS LENGTHS

Nodal Forces (tons)

<u>Node Number</u>	<u>L=15</u>	L=25	<u>L=35</u>	<u>L=45</u>	L=55	L=65	L=75
Total Shear of Full Struc.	66.67	40.	28.6	22.2	18.2	15.4	13.34
4,9	4.17	2.5	1.79	1.39	1.14	.96	.835
1, 3, 5, 8	2.08	1.25	.895	.695	.57	.48	.4175
40, 45	-4.17	-2.5	-1.79	-1.39	-1.14	96	835
37, 39, 41, 44	-2.08	-1.25	895	695	57	48	4175

The nodal loading due to the torsional moment, M_y , arises from the assumption that the center reinforcing bulkhead and deck do not carry any load, and from the assumptions implied by the use of the well known Bredt Formula (Ref. 19).

My = 2AQ (11)
Q = Shear Flow
A = Area enclosed by the
 perimeter of the section

The first assumption is verified by the application of the general analytical solution method involving shear flow about the individual cells of the cross structure (See Appendix A). The shear flows in the center bulkhead and deck cancel because of the double symmetry resulting in a zero stress in the center reinforcing membranes. This statement does not hold true for unsymmetrical cases such as the actual ASR-21 cross structures.

Equation (11) can be further modified by defining the shear flow as the force per unit length of cross structure.

- $Q = F/c \tag{12}$
- $F = M_y c/2A$ (13)
- c = The length between node
 midpoints
- F = nodal force

The loading for the quarter structure is shown in Figure (XI), and values for the nodal forces for an applied moment, M_y , of 1000 foot-tons are specified in Table 7.

TABLE 7

QUARTER STRUCTURE NODAL LOADINGS

FOR TORSIONAL MOMENT ($M_v = 1000$ foot-tons)

AND VARIOUS B/D RATIOS

Nodal Forces (tons)

Node Number	B/D = 2.5	B/D = 3.0	$\underline{B/D} = 3.5$
3, 5 (positive z direction)	1.5	1.25	1.07
4	3.0	2.5	2.14
5 (negative y direction)	-2.5	-2.5	-2.5
6,7	-5.0	-5.0	-5.0
8	-2.5	-2.5	-2.5




Plating Effectiveness

The concept of effective plating width and breadth resulted from work done by Von Karman and Schnadel in the 1920's. The aircraft industry referred to it as the plating efficiency in the late 1930's. Significant work was performed in this area by Schade (Ref. 12 and 13) in the early 1950's, and most recently by Mansour (Ref. 11). Both terms, effectiveness and efficiency, aid in graphically illustrating the fact that shear lag exists in stiffened plating. Because of the shear lag, stress peaks occur at the stiffening bulkheads, and much lower stresses exist in the remainder of the plating. This phenomenon means that only a small width of the plating is being stressed to any significant degree. Hence, there is some effective width of the plate that is carrying the load.

Equations (14) and (15) define the effectiveness, and Figure XII shows how these definitions are applied to the catamaran cross structure.

$$\rho_{1} = \sigma avg/\sigma max$$
(14)
$$\rho_{2} = \sigma avg/\sigma max 2$$
(15)

Because the finite element solution gives the stress resultant at the center of an element, extrapolation of the data was necessary to obtain the maximum stresses at the plating edges. Reference (28) assumes that the shear lag has a parabolic distribution in the transverse direction for box beams. Since data was obtained for three transverse elements across the top plating, the parabolic assumption was

-37-



FIGURE XII TOP PLATE EFFECTIVENESS DEFINITION used to fit the data, and to calculate the maximum stresses at each edge of the plate. Integration of this distribution over the entire width of the plating was performed to obtain the average value of the stress. (Equation 16)

$$\sigma_{yavg} = 1/x_{\circ} \int_{0}^{x} \sigma_{y}(x) dx \qquad (16)$$

- x_o = The overall width of
 plating
- $\sigma_y(x)$ = The longitudinal stress in the y direction as a function of plate width

From equation (16) and the extrapolated maximums the effectiveness of the plating can be calculated using equations (14) and (15).

RESULTS

1. Tabulated computer results of the longitudinal, girth and shear stress distribution in the forward and after ASR-21 Catamaran cross structures are presented in Tables C-1 and C-2. Symmetric bending moment, antisymmetric bending moment and shear, and combined loadings are included.

2. The stress results for the cross structure model under symmetric and antisymmetric loadings are included in Tables C-3 through C-9.

3. The computer results for the stresses resulting from the torsional loads are included in Table C-10; the calculated results are in Table C-11.

4. Average longitudinal stresses and plating effectiveness calculations are summarized:

(a) in Table C-12 for the symmetric and antisymmetric bending moment loadings of the quarter structure model.

(b) in Table C-13 for the combined loading case of the quarter structure model.

(c) in Table C-14 for all the loading conditions of the ASR-21 Catamaran cross structures.

5. Typical longitudinal stress distributions for the cross structures are illustrated:

(a) in Figure XIII for the symmetric bending moment case of the quarter structure.

(b) in Figure XIV for the antisymmetricbending moment case of the quarter structure.(c) in Figure XV for the combined loadingcase of the quarter structure.

(d) in Figure XVI and XVII for the combined loading case of the actual forward and after ASR-21 cross structure.

6. Girth stress distributions are plotted:

(a) in Figure XVIII for the symmetric
bending moment case of the quarter structure.
(b) in Figure XIX for the antisymmetric
bending moment case of the quarter structure.
(c) in Figure XX for the combined loading
case of the quarter structure.

(d) in Figures XXI and XXII for the combined loading case of the actual forward and after ASR-21 cross structures.

7. Shear stress distributions are shown:

(a) in Figure XXIII for the symmetric bending moment case of the quarter structure.

(b) in Figure XXIV for the Antisymmetric bending moment case of the quarter structure.

(c) in Figure XXV for the combined loading case of the quarter structure.

(d) in Figures XXVI and XXVII for the combined loading case of the actual ASR-21 cross structures.

(e) in Figure XXVIII for the torsional loading of the quarter structure.

8. Plating effectiveness as a function of the length to breadth ratio (L/B) for all the loading conditions are plotted in Figures XXIX to XXXIV. Table 8 which precedes the graphs shows the numbering sequence one uses to locate the effectiveness of the actual structures.

9. Average stress along the top of the quarter structure is plotted versus the L/B ratio for the combined loading case in Figure XXXIX.

10. The qualitative effect of the various loading conditions on the displacement of the quarter structure is shown in Figures XXXV to XXXVIII.

11. The effective breadths of the model and the actual cross structures at positions along the length of the structures are presented in Table 9 and Figure XL.



Stress Results for

L/B=.7 B/D=3.0 A_f/A_W=3.0





ANTISYMMETRICAL BENDING MOMENT + SHEAR LONGITUDINAL STRESS DISTRIBUTION OF QUARTER STRUCTURE



COMBINED LONGITUDINAL STRESS DISTRIBUTION OF QUARTER STRUCTURE



-46-



-47-



-48-



GIRTH STRESS DISTRIBUTION OF QUARTER STRUCTURE

-49-



COMBINED GIRTH STRESS DISTRIBUTION OF QUARTER STRUCTURE



-51-

FIGURE XXI COMBINED GIRTH STRESS DISTRIBUTION FOR THE FORWARD ASR-21 CROSS STRUCTURE



-52-

FIGURE XXII COMBINED GIRTH STRESS DISTRIBUTION FOR THE AFTER ASR-21 CROSS STRUCTURE



-53-





-55-

FIGURE XXV COMBINED SHEAR STRESS DISTRIBUTION OF QUARTER STRUCTURE











FIGURE XXVIII TORSIONAL MOMENT SHEAR STRESS DISTRIBUTION OF QUARTER STRUCTURE

TABLE 8

TOP PLATING EFFECTIVENESS LOCATORS FOR THE ASR-21 CROSS STRUCTURES

Points plotted on Figures XXIX to XXXIV and designated by the following set of numbers indicate the effectiveness of the actual ASR-21 cross structures.

Number	Cross Structure	Y-Coordinate	Element Numbers
l	Forward	L/8	9,113,13
2		3L/8	10,114,14
3		5L/8	11,115,15
4		7L/8	12,116,16
5		L/8	49,45,41
6		3L/8	50,46,42
7		5L/8	51,47,43
8	Ļ	7L/8	52,48,44
9	After	L/8	9,13,17
10		3L/8	10,14,18
11		5L/8	11,15,19
12		7L/8	12,16,20
13		L/8	49,45,41
14		3L/8	50,46,42
15		5L/8	51,47,43
16	¥	7L/8	52,48,44













FORM 1 HG

40 MASS. AVE., CAMBRIDGE, MASS.



FIGURE XXXV TORSIONAL MOMENT DISPLACEMENT (1/8 STRUCTURE)



FIGURE XXXVI ANTISYMMETRIC BENDING MOMENT + SHEAR DISPLACEMENT



FIGURE XXXVII SYMMETRIC BENDING MOMENT DISPLACEMENT



FIGURE XXXVIII COMBINED LOADING DISPLACEMENT



TABLE 9

EFFECTIVE BREADTHS OF THE MODEL AND

ACTUAL CROSS STRUCTURES

	Effective B With Center	readth Ass Bulkhead	oc. Ef Wi	fective Br th Side Bu	Breadth Assoc. Bulkheads		
	Fwd. Cross Aft Struc. S		s Fw	d. Cross Struc.	Aft. Stri	Cross uc.	
Model							
y=L/8	8.1'	8.45'	8.45' 8.75'				
y=3L/8	13.5'	14.2'				-	
y=5L/8	9.75	10.3	12.75		13.0	13.6	
y=7L/8	7.05	7.25	8.45		8.75		
ASR-21			Left Blkhd.	Right Blkhd.	Left Blkhd.	Right Blkhd.	
y=L/8 Top Plate	8.17	8.53	8.72	9.05	9.61	8.93	
Bot. Plate	8.48	8.15	9.57	10.7	10.1	9.31	
y=3L/8 Top Plate	17.0	19.3					
Bot. Plate	20.82	19.97					
<mark>y=5L/8</mark> Top Plate	7.67	11.2	8.29	8.64	11.4	11.2	
Bot. Plate	8.07	10.9	9.06	9.74	12.5	11.8	
y=7L/8 Top Plate	10.8	7.96	11.4	11.3	9.15	8.5	
Bot. Plate	11.05	7.60	12.0	12.0	9.67	8.85	

Effective breadth used in the actual ASR-21 design was 4 feet (Ref. 3).


DISCUSSION OF RESULTS

From any finite element program there usually results a large amount of output. This was the case for this thesis, and in order to reduce the extensive volume of this information, sections of this output were either left out or reduced. In particular, the array of displacements that each computer run produced was not included in Appendix C. It was decided that since this thesis concentrated on the stress distribution in the structure, the presentation of the displacements in toto would be unnecessary and unwarranted. To fill this gap Figures XXXV to XXXVIII were included to show qualitatively what effect the separate loadings have on the displacement of the structure. As can be seen in the figures the structure responds as anticipated to a specific type of loading. Also, the stress results for the model contained in Tables C-3 to C-8 are only half of the stress output. Elemental stresses for the near end of the cross structure (y=L/8 and y=3L/8) are included. The stresses for the far end (y=5L/8 and y=7L/8) are omitted. The value of these stresses produced by the symmetric bending moment loading are equal to the stresses of the near end; the stresses for the far end of the beam with antisymmetric bending moment and shear loading are the negative of the stresses of the near The torsional moment loading stresses at the far end end. are also the negative of the near end.

It was mentioned in the effectiveness section part of

-73-

the Procedure that a parabolic fit to the data was used to extrapolate to the plating edges for the purpose of determining the maximum stresses. The parabolic fit was then used to calculate the average stresses and the effectiveness of the section. The use of the parabolic fit resulted from investigation of the work done by Hildebrand and Reissner on shear lag (Ref. 28) in which they utilized a parabolic distribution of stresses in a box beam. The requirement to use the parabolic distribution was also occasioned by a need conserve funds. For instance, the addition of two to elements to the top plating of the cross structure (xdirection) of Figure VIII results in a computer cost increase of 28.8%. A check was made to determine what effect the use of five instead of three elements would have on the stress levels and consequently the average stress and effectiveness. Figure XLI shows the quarter structure with the added elements, and Figure XLII is a graph of the longitudinal stress level for both the three and five element cases. As can be observed the parabolic fit using three elements is only slightly different, and the difference between the average stresses was 1.8%. Thus, all the results in Tables C-12 to C-14 are based on the parabolic assumption. These results are then reflected in the effectiveness curves Figures XXVIII to XXXIV.

The effectiveness curves were plotted only for B/D=3.0and $A_f/A_w=3.0$. The reasons for this result from the calculations summarized in Tables C-12 to C-14. The variation of the effectiveness with respect to the B/D and $A_{\rm f}/A_{\rm w}$ ratios is very small, and because of this were not plotted. It must be noted that in the model and actual cross structures at y=3L/8 for all loading cases the longitudinal stress distribution changes significantly in shape as L/B increases. Figures XLIII and XLIV show this. Specifically, in all loading cases there is a point where the edge stress or the second maximum indicated by σ_{max2} in Figure XII drops below the average stress and causes ρ_2 to be greater than 1.0. As L/B increases further the curvature of the stress distribution reverses so that the maximum stresses in the plate occurs nearer the middle of the plate. Thus, for the longitudinal stresses at y=3L/8 a change in the definition of effectiveness as shown in Figure XII must be made. For distributions where the second maximum (σ_{max2}) drops below the average stress the plating is considered to be fully effective (i.e., ρ_2 =1.0). For stress distributions where the curvature reverses and the maximum occurs nearer the center, $\rho_{\rm l}$ is calculated using this maximum, and ρ_{2} is considered equal to 1.0. These changes are reflected in the effectiveness curves plotted on Figures XXVIII to XXXIV. The computer results using the original definition are included in Tables C-12 to C-14 for comparison.

The values of the effectiveness of the actual structures are plotted on Figures XXVIII to XXXIV. Table 8 defines the locations of the effectiveness on the cross structures.

The effectiveness graphs can be very helpful to the designer. Given a cross structural shape and an allowable

-75-



FIGURE XLI THE FIVE ELEMENT MODEL NUMBERING SEQUENCE

-76-

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average stress permitted for the structure, the designer can predict the maximum stresses in the structure. The apparent lack of dependence upon B/D and A_f/A_W in the ranges studied in this thesis show these curves to be of even greater importance. When varying the plating thicknesses of this structure, however, the designer must observe very carefully his average and maximum stresses. He could very easily have two structures that have the same effectiveness but because of the difference in plating thickness may have one structure that has stresses that exceed a reasonable level.

The torsional results are very interesting. It was found from several computer runs that the shear stresses did not vary along the length of the beam. Instead the shear stress remained constant, as one might predict them from the Bredt Formula (Equation 10) and the appropriate plate thicknesses. Additionally, it was also noted that in Table C-10 the longitudinal and girth stresses produced by the torsional moment were several orders of magnitude smaller than the same stresses produced by the bending moment loadings, and thus did not have any effect on the total longitudinal and girth stresses in the structure. These small values are actually a verification of the theory discussed by Oden (Ref. 20) and Venkatraman and Patel (Ref. 22). As a result of this, the remaining parametric cases were calculated by known analytical methods. (Appendix A and Table C-11).

The longitudinal stress resultants for all the loadings (Tables C-1 through C-4 and C-9, and Figures XIII through XVII)

-80-

describe quite graphically the shear lag effect. The standard assumption in elasticity that plane sections remain plane is no longer true, and, hence, the simple theory of bending is not applicable. The main cause of the distortion of the cross sections is shear strain in the flange area of the box beams. Considerable effort was expended to relate the longitudinal stress results of this thesis to some applicable shear lag theory. Not only the loading but also the boundary conditions and geometry made the cross structure quite different from the single-celled box beams for which a suitable theory exists. Additional comments on this subject can be found in Appendix B.

All the figures showing the qualitative stress results of the structure are based on the data for L/B=.7, B/D=3.0, and A_f/A_W =3.0. This particular case was the closest model to the actual cross structures.

The average stresses that were calculated show only a small increase in value as A_f/A_W increases for given L/B and B/D ratios. The average stress change is very small for the range of L/B values (Figure XXXIX).

A check was performed on the linearity of the stress response of the structure to a set of scaled loadings, and it was found that the response was linear as expected by theory.

Based on the effectiveness curves and the L/B ratios of the actual cross structures Table 9 and Figure XL were generated to illustrate the effective breadths of plating in the cross structures. These were obtained from the standard

-81-

definition that the effective breadth is equal to the actual breadth times the effectiveness. Note should be taken that a four-foot effective breadth was utilized by the designers of the ASR-21 Catamaran.

CONCLUSIONS

1. The effect of varying the breadth to depth (B/D) and the flange area to web area (A_f/A_W) ratios over the ranges of this thesis on the plating effectiveness of a structure with a given L/B ratio is negligible. This means that the type of loading, and the length and the breadth of the plating determine the structural plating effectiveness.

2. Average stress in the top plating of the cross structure model varies by only a small amount as L/B increases for given B/D and A_f/A_w ratios, which is simply a substantiation of the simple beam formula.

3. The highest stress level in both the model and the actual cross structures occurs at the plane y=7L/8 and for the combined loading case.

4. The actual ASR-21 Catamaran cross structures in general tend to be higher in effectiveness than the model curves for equal L/B ratios. The length of the ASR cross structures are constrained to 35 feet because of the requirement of the ASR to handle the DSRV, and to reduce the deleterious effect of wave action upon the hydrodynamics of the hulls and the strength of the cross structures. This leads one to conclude that the breadth of the ASR cross structures can be decreased at least to the point where it equals the model curves. The advantage of this would be a decrease in structural weight. 5. The maximum longitudinal web stresses occur in the top of the center web at the location y=7L/8 in the combined load-

-83-

ing case.

6. The maximum shear stress occurs at the top of the web and at the edge of the plating at y=L/8.

7. The longitudinal stresses are by far the most predominant stresses in the structure. In all sections of the structure the shear lag effect is graphically visible causing the longitudinal stresses to peak at the web stiffeners of the top and bottom plating. By using the effectiveness curves the designer can calculate the maximum stress in the cross structure flanges and the webs. This is, given a desired average stress for a given structure, the designer can calculate the maximum longitudinal stress in the structure.
8. The forward ASR cross structure has a cut out section at its forward end between the 0-2 and 0-1 levels. The remaining structure from the 0-1 level to the main deck serves very little purpose except to accommodate the interior arrangements of the vessel. Additionally, it carries very little

load and adds significantly to the total weight of the structure. Structually the cross structure can do without its forward section, be just as effective and would be much lighter without it.

9. It is significant to note that the A_f/A_W ratio for the catamaran cross structures is nearly two times the same ratio for normal ship girders, which leads one to believe again that the structures are too heavy.

10. The torsional moment produces longitudinal and girth stresses that are orders of magnitude below those produced

-84-

by the bending moment and, hence, do not have any appreciable effect on the structure.

11. Existing torsional techniques for computing shear stresses in a closed, multicell, box beam are preferable to calculate these stresses than a finite element solution. Essentially, they are faster and much cheaper to use.

RECOMMENDATIONS

Although this thesis has concentrated on the ASR-21 Catamaran cross structures, the information is applicable to any four-celled box beam cross structure which satisfies the same boundary conditions. However, this thesis is but a small piece of the picture. With new catamaran designs being conceived for both military and industrial uses that will have both higher L/B ratios and different cross structural shapes the need exists for more detailed and expanded studies to be conducted. An effort should be made to examine these new designs in much the same manner, and possibly try to optimize the size, shape and weight of the cross structure.

A satisfactory analytical stress solution for a doubly symmetric four-celled box beam should be developed utilizing the stress function approach as used by Hildebrand (Ref. 29).

APPENDICES

-87-

APPENDIX A

ANALYTICAL TORSIONAL ANALYSIS

The analysis of a four-celled box beam subjected to a torsional moment has many facets which could lead to problems in handling the analytical solution of its loading. Some basic assumptions must then be made to extend existing methods of torsional analysis to the catamaran cross structure. It is assumed that a uniform shear stress exists across the thickness of the web and flanges of the cross structure. Additionally, shear stresses are assumed to be directed tangent to the boundary curve of the box beam. Normal stresses are assumed to be negligible. The product of the shear stress (σ_{xy}) and the plate thickness (t) is constant at all points along the perimeter of a cell. Finally, it is assumed that there is no in plane distortion so that the angle of twist (θ) of cell (i) is identically equal to the angle of twist of cell (i+1) and so forth. (ie. $\theta_i = \theta_{i+1} \dots = \theta_n$).

For the general four-celled box beam pictured in Figure A-1 the equation of equilibrium (equation 1) relates the twisting moment to the shear flows.

$$\begin{split} M_t &= 2 \sum_{j=1}^{k} q_j A_j \eqno(1) \\ q_j &= \text{shear flow in cell } j \\ A_j &= \text{area enclosed by the} \\ perimeter of a cell \\ M_t &= \text{the applied twisting moment} \\ \end{split}$$
 The rate of twist for a cell (j) is indicated by equation (2).

$$\theta_{j} = \frac{q_{j}}{2GA_{j}} \oint_{sj} \frac{ds}{t}$$

$$t = \text{thickness of the boundary}$$

$$ds = \text{an incremental distance along}$$
(2)

For the case represented by Figure (A-1) where adjacent cells and in particular the neighboring boundary affect the total twist, the shear flows in these adjacent cells must be taken into consideration. Applying equation (2) around the boundary of a cell (j), results in equation (3).

the perimeter

$$2GA_{j}\theta = (q_{j} \phi_{sj} \frac{ds}{t} - q_{i} \int_{s_{ji}} \frac{ds}{t} - q_{k} \int_{s_{jk}} \frac{ds}{t}) \quad (3)$$

For simplification in this Appendix the following definitions are utilized.

$$\delta_{ji} = -\frac{1}{G} \int_{s_{ji}} \frac{ds}{t}$$
(4a)

$$\delta_{jk} = -\frac{1}{G} \int_{s_{jk}} \frac{ds}{t}$$
(4b, c)

$$\delta_{jj} = \frac{1}{G} \phi_{s_j} \frac{ds}{t}$$

Thus equation (3) can be rewritten as

$$\theta = \frac{1}{2A_{j}} \left(q_{j} \delta_{jj} + q_{i} \delta_{ji} + q_{k} \delta_{jk}\right) (5)$$

If equation (5) is now applied to each cell of the four-celled box beam, four equations with four unknowns $(i_q, j=1, 2, 3, 4)$ will be generated.

$$\delta_{11}q_{1}+\delta_{12}q_{2}+\delta_{13}q_{3} - 2A_{1}\theta = 0$$

$$\delta_{21}q_{1}+\delta_{22}q_{2}+\delta_{23}q_{4} - 2A_{2}\theta = 0$$

$$\delta_{31}q_{1}+\delta_{34}q_{4}+\delta_{33}q_{3} - 2A_{3}\theta = 0$$

$$\delta_{42}q_{2}+\delta_{43}q_{3}+\delta_{44}q_{4} - 2A_{4}\theta = 0$$

(6a-d)

Equation (6) can be much more clearly expressed as a matrix equation (7). Note the coefficient matrix ∇ is symmetric with $\delta_{ji} = \delta_{ij}$.

$$\begin{bmatrix} \delta_{11} & \delta_{12} & \delta_{13} & 0 & q_{1} \\ \delta_{21} & \delta_{22} & 0 & \delta_{24} & q_{2} \\ \delta_{31} & 0 & \delta_{33} & \delta_{34} & q_{3} \\ 0 & \delta_{42} & \delta_{43} & \delta_{44} & q_{4} \end{bmatrix} = 2^{\theta} \begin{bmatrix} A_{1} \\ A_{2} \\ A_{3} \\ A_{4} \end{bmatrix}$$
(7)

 $\nabla \qquad Q = 2\theta \qquad A \qquad (8)$

Utilizing the generalized dimensions of Figure (A-1) in equations 4a-c, and substituting them into the matrix ∇ gives the generalized ∇^1 (Equation 9) which can be augmented with 20Å to solve for the q_j .



FIGURE A-I CROSS SECTION OF FOUR-CELLED BOX BEAM

Equation (9)

$$\begin{split} & \nabla^{1} = \frac{1}{G} \begin{bmatrix} L_{1} \left(\frac{t_{2}}{t_{1}} \frac{t_{1}}{t_{2}} \right)^{+L_{3}} \left(\frac{t_{5}^{+t_{4}}}{t_{4}} \frac{t_{5}}{t_{5}} \right) & -L_{3} \left(\frac{1}{t_{5}} \right) & -L_{1} \left(\frac{1}{t_{2}} \right) \\ & -L_{3} \left(\frac{1}{t_{5}} \right) & L_{2} \left(\frac{t_{2}^{+t_{1}}}{t_{1}} \frac{t_{2}}{t_{2}} \right)^{+L_{3}} \left(\frac{t_{6}^{+t_{5}}}{t_{5}} \frac{t_{6}}{t_{6}} \right) & 0 & -L_{2} \left(\frac{1}{t_{2}} \right) \\ & -L_{3} \left(\frac{1}{t_{2}} \right) & L_{2} \left(\frac{t_{2}^{+t_{1}}}{t_{1}} \frac{t_{2}}{t_{2}} \right)^{+L_{3}} \left(\frac{t_{6}^{+t_{5}}}{t_{5}} \frac{t_{6}}{t_{6}} \right) & 0 & -L_{2} \left(\frac{1}{t_{2}} \right) \\ & -L_{1} \left(\frac{1}{t_{2}} \right) & 0 & L_{1} \left(\frac{t_{3}^{+t_{2}}}{t_{3}} \frac{t_{2}}{t_{2}} \right)^{+L_{4}} \left(\frac{t_{5}^{+t_{4}}}{t_{4}} \frac{t_{5}}{t_{5}} \right) & -L_{4} \left(\frac{1}{t_{5}} \right) \\ & 0 & -L_{2} \left(\frac{1}{t_{2}} \right) & -L_{4} \left(\frac{1}{t_{5}} \right) & L_{2} \left(\frac{t_{3}^{+t_{2}}}{t_{3}} \frac{t_{2}}{t_{2}} \right)^{+L_{4}} \left(\frac{t_{6}^{+t_{5}}}{t_{5}} \frac{t_{6}}{t_{6}} \right) \\ \end{array}$$

-92-

Therefore, given a twisting moment and solving for the shear flows in equation (7) as functions of G and θ then the quantity G θ can be determined by substituting into equation (1). The shear stresses can be calculated by dividing the shear flow by the appropriate thickness.

$$q_{j} = f_{j}(G\theta)$$
(10)

$$M_{t} = 2\sum_{j=1}^{2} q_{j}A_{j} = 2\sum_{j=1}^{2} f_{j}(G\theta) A_{j}$$
(11)

$$\sigma_{x_{z}} = \frac{q_{j}}{t_{i}} \quad j = 1,4$$
 (12)

The calculations in Table C-XI were performed for the representative cases analyzed in this thesis. Because computer tests of the torsional loading (Table C-X) showed that the only stresses of importance in the structure could be obtained by the simple analytical method outlined above, the calculations in Table C-XI were done in lieu of additional computer runs for the Torsional Loading.

APPENDIX B

COMMENTS ON RELATING THE COMPUTER RESULTS TO EXISTING THEORY

In attempting to verify the computer results of this thesis by existing theory a significant effort was made to determine the applicable theory. Attempts at correlating both the stresses and the effectivenesses obtained from the computer results with theory proved futile. The major reasons for this are varied, but essentially all result from the fact that the catamaran cross structural shape has not been analyzed for the application of end bending moments. Sufficient theory exists to calculate the effects of torsional moments (Appendix A) and shear in the webs (Ref. 20). In addition to the geometrical shape differences, loading and boundary conditions vary significantly from existing box beam or box girder analyses.

Lankford (Ref. 3) and Dinsenbacher (Ref. 4) assumed "the cross structure bulkheads to be fixed ended beams undergoing a settlement of supports." Applying boundary conditions for a closed solution to "a settlement of supports" might be very difficult. The versatility of the finite element method allows one to solve problems with multiple degrees of indeterminancy and theoretically impossible boundary conditions. This makes the analysis of the cross structures relatively easy with the finite element method, and rather difficult to check in cases where a theoretical solution is only an approximation, or does not even exist.

When the first computer results were checked, it was obvious that some shear lag analysis must be used to verify the results. The extensive works on shear lag by Reissner (Refs. 23 and 24), Reissner and Hildebrand (Ref. 28), Hildebrand (Ref. 29), Kuhn (Refs. 25, 26 and 27), Smith (Ref. 34), and Yuille (Ref. 32) dealt principally with two types of structures, the box beam and stiffened plating.

Hildebrand and Reissner assumed a parabolic distribution of longitudinal stresses in the box beam in which the shear lag causes the stresses to vary around the stresses obtained by the standard beam formula (My/I). The analysis was done for single-cell box beams that were cantilevered, completely built-in, and simply supported. Some of the loading conditions that they examined were uniform lateral pressure and concentrated lateral loads. Hildebrand repeated many of the same boundary conditions and loadings for the identical geometry, but approached the shear lag solution by the use of an infinite series stress function whose variables were the geometric and material properties of the beams. Since neither of these two approaches could satisfy either the geometry of the four-celled box beam or the proper loading, it was not possible to use them.

Kuhn and others considered only the cantilevered box beam case with end moments and concentrated forces acting at the tip through the webs. Kuhn's interest was strictly the aircraft wing in all its geometric variations, and while

-95-

interesting to read his results were not helpful to the analysis of the four-celled cross structure.

If one removes the top plate from the cross structure, and considers it as stiffened plating, it might very well be possible to compare the shear lag in this case to the shear lag for the cross structure. Smith and Yuille performed shear lag analysis on multiply stiffened plating. However, the only loads examined are uniform, concentrated, and sinusoidal loads in the plane of the plating which are highly applicable to the bottom plating of ships, but not to the cross structure. One of the difficult problems in solving the plating problem with edge loads in both flanges and webs is the difficulty encountered in convergence of the infinite series solution for this type of loading.

Schade (Refs. 12 and 13), Mansour (Ref. 11), Winter (Ref. 35) and Reissner (Ref. 24) either discuss or present curves of effectiveness (or as Reissner calls it "efficiency") and effective breadth. It was thought that a comparison might be made with some of their results. The problems of comparison are many. The effective breadth as defined by Schade is for lateral loads which cause the plate or panel to bend out of its original plane, and for which all of Schade's work was performed. This type of analysis is again very dependent upon loading. Neither Schade nor Mansour examined the loading of this thesis.

Various expressions exist for determining effective breadth of plating and beam flanges. The most common are

-96-

listed below:

Reissner:

beff = b(1/3 2/3(Stress at Plate Center)
beff = Effective breadth
b= One half the plate breadth (from
 stiffener to midpoint)

Winter:

$$b_{eff} = \frac{0 \int b(stress) dy}{Max Stress} = \frac{\sigma a v g}{\sigma max}$$

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
l	.00076	0109	0115
2	00338	0214	0047
3	00338	0214	.0047
4	.0076	0109	.0115
5	.00437	0097	.006
6	0700	0214	.00191
7	0700	0214	00191
8	.00437	0097	006
9	.0466	.5312	.3396
10	00399	.4632	.1061
11	00399	.4632	1061
12	.0466	.5312	3396
13	1418	.0173	.0623
14	.1845	.298	.0454
15	.1845	.298	0454
16	1418	.0173	0623
17	0862	.4658	1812
18	.1288	.4971	0672
19	.1288	.4971	.0672
20	0862	.4658	.1812
21	0928	832	516
22	.01778	7115	1578
23	.01778	7115	.1578

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
24	0928	832	.516
25	.1919	024	088
26	267	4399	0722
27	267	4399	.0722
28	.1919	024	.088
29	.1185	6483	.2516
30	1936	6702	.0839
31	1936	6702	0839
32	.1185	6483	2516
33	.00079	9948	.0100
34	00277	0189	.00434
35	00277	0189	00434
36	.00079	9948	0100
37	.00418	00757	00533
38	00616	0169	0019
39	00616	0169	.0019
40	.00418	00757	.00533
41	.0422	.5048	2996
42	0178	.4181	1047
43	0178	.4181	.1047
44	.0422	.5048	.2996
45	1492	03095	0501
46	.1736	.2069	0415

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
47	.1736	.2069	.0415
48	1492	03095	.0501
49	0879	.4294	.1605
50	.1124	.4548	.07408
51	.1124	.4548	07408
52	0879	.4294	1605
53	0871	792	.4538
54	0376	6467	.153
55	0376	6467	153
56	0871	792	4538
57	.1964	.0456	.631
58	246	303	.06308
59	246	303	06308
60	.1964	.0456	631
61	.166	5692	2143
62	1656	5769	0909
63	1656	5769	.0909
64	.166	5692	.2143
65	.1273	.5852	2197
66	09348	.1702	0498
67	09348	.1702	.0498
68	.1273	.5852	.2197
69	.0995	1.078	.3407

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
70	0651	.5261	.1121
71	0651	.5261	1121
72	.0995	1.078	3407
73	.0478	.3076	0421
74	0068	.1727	0216
75	0068	.1727	.0216
76	.0478	.3076	.0421
77	.03223	.7711	.1133
78	.0088	.5441	.0570
79	.0088	.5441	0570
80	.03223	.7711	1133
81	.0298	.2563	0120
82	.01221	.177	0159
83	.01221	.177	.0159
84	.0298	.2563	.0120
85	.0177	.7047	.0673
86	.0243	.5549	.04314
87	.0243	.5549	04314
88	.0177	.7047	0673
89	0851	6247	2897
90	03013	3382	10142
91	03013	3382	.10142
92	0851	6247	.2897

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

SYMM.	BEND.	MOM.

Element	Girth Stress	Longitudinal Stress	Shear Stress
93	055	-1.398	.1787
94	.01636	812	.03912
95	.01636	812	03912
96	055	-1.398	1787
97	.0237	3436	0777
98	07102	29579	0504
99	07102	29579	.0504
100	.0237	3436	.0777
101	00432	9798	.0065
102	043	7345	.01498
103	043	7345	01498
104	00432	9798	0065
105	.0262	2876	0381
106	0741	2794	0379
107	0741	2794	.0379
108	.0262	2876	.0381
109	.0044	8725	0171
110	0523	7088	.0108
111	0523	7088	0108
112	.0044	8725	.0171

TABLE C-I

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
l	.00148	.00509	.00506
2	.0039	.0141	00298
3	0039	0141	00298
4	00148	00509	.00506
5	00015	.0038	0018
6	.00489	.0119	.00392
7	00489	0119	.00392
8	.00015	0038	0018
9	0319	2054	1049
10	03845	2812	.0738
11	.03845	.2812	.0738
12	.0319	.2054	1049
13	0149	.00796	0199
14	0701	.0331	0111
15	.0701	0331	0111
16	.0149	00796	0199
17	.00742	1738	.0387
18	0783	2633	0698
19	.0783	.2633	0698
20	00742	.1738	.0387
21	.0616	.3147	.157
22	.0554	.4045	1014
23	0554	4045	1014

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
24	0616	3147	.157
25	.03017	0128	.0278
26	.1007	0416	.0281
27	1007	.0416	.0281
28	03017	.0128	.0278
29	0052	.2342	0472
30	.1039	.3388	.1033
31	1039	3388	.1033
32	.0052	2342	0472
33	.00147	.0046	00417
34	.00375	.0139	.00239
35	00375	0139	.00239
36	00147	0046	00417
37	.00010	.00273	.00142
38	.00397	.00875	00319
39	00397	00875	00319
40	00010	00273	.00142
41	0319	1962	.0884
42	03769	2717	0617
43	.03769	.2717	0617
44	.0319	.1962	.0884
45	0173	.0253	.0137
46	0583	.0530	.0129

-105-

TABLE C-I (Cont'd.)

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
47	.0583	0530	.0129
48	.0173	0253	.0137
49	.00528	1601	0283
50	0704	2422	.0550
51	.0704	.2422	.0550
52	00528	.1601	0283
53	.06141	.301	1333
54	.0549	.3906	.0852
55	0549	3906	.0852
56	06141	301	1333
57	.0335	03649	0167
58	.0832	.0678	03215
59	0832	0678	03215
60	0335	.03649	0167
61	0018	.2053	.0317
62	.0889	.298	0797
63	0889	298	0797
64	.0018	2053	.0317
65	0553	2187	1345
66	.0281	0346	1272
67	0281	.0346	1272
68	.0553	.2187	1345
69	02605	4149	2774
70	2067	3608	2664

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
71	.0267	.3608	2664
72	.02605	.4149	2774
73	02736	1227	0837
74	.00131	0294	0285
75	00131	.0294	0285
76	.02736	.1227	0837
77	0104	3057	0988
78	04549	3333	1436
79	.04549	•3333	1436
80	.0104	.3057	0988
81	0185	10333	07836
82	00455	02949	0176
83	.00455	.02949	0176
84	.0185	.10333	07836
85	00068	2799	07236
86	0489	3184	125
87	.0489	.3184	125
88	.00068	.2799	07236
89	00014	.2407	1017
90	.0216	.1015	0605
91	0216	1015	0605
92	.00014	2407	1017

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
93	.0030	.5415	1925
94	.0671	.5112	2518
95	0671	5112	2518
96	0030	5415	1925
97	0037	.141	0698
98	0306	.08224	.0111
99	.0306	08224	.0111
100	.0037	141	0698
101	00271	.3866	0487
102	.07137	.4298	1402
103	07137	4298	1402
104	.00271	3866	0487
105	005	.1184	0689
106	.0297	.073	.01279
107	0297	073	.01279
108	.005	1184	0689
109	0093	.3443	0338
110	.0692	.3939	1236
111	0692	3939	1236
112	.0093	3443	0338

TABLE C-I

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

COMBINED

Element	Girth Stress	Longitudinal Stress	Shear Stress
l	.00224	00589	00647
2	.00053	00724	0077
3	00729	0355	.00175
4	00072	0161	.0166
5	.0042	00588	.00419
6	0021	0095	.00584
7	0119	0333	.00201
8	.00453	0135	00790
9	.01467	.3258	.2347
10	0424	.1819	.1799
11	.03445	.7445	0323
12	.07866	.7367	4446
13	1568	.0253	.04238
14	.1144	.3312	.03427
15	.2547	.265	0565
16	1269	.00936	0823
17	0787	.292	1424
18	.05004	.2338	1371
19	.2077	.7605	00265
20	09363	.6398	.220
21	03119	51778	3592
22	.0732	3069	2592
23	0377	-1.116	.05637
ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
24	1546	-1.147	.6742
25	.2221	03696	06011
26	1664	4816	04407
27	3678	3983	.1004
28	.1618	01121	.1159
29	.1133	4141	.2044
30	08977	3314	.1872
31	2976	-1.009	.01936
32	.1239	8826	2989
33	.00227	00534	.00586
34	.000974	00506	.00674
35	00653	03289	00195
36	000676	0146	0142
37	.00429	00484	00391
38	00219	00823	00513
39	01014	02574	00127
40	.00408	0103	.00675
41	.01035	.3086	2112
42	05557	.1464	1666
43	.0198	.68988	.0430
44	.07423	.7012	.3881
45	1666	0056	0364
46	.1152	.260	0286

-110-

TABLE C-I (Cont'd.)

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
47	.2320	.1539	.0545
48	1318	0563	.0638
49	0827	.2693	.1322
50	.04199	.2126	.1291
51	.1828	.6971	01905
52	0933	.5895	1889
53	02576	4911	.3205
54	.09257	2561	.2383
55	01735	-1.037	06774
56	1486	-1.093	5872
57	.2301	.00918	.0463
58	1628	3709	.03183
59	3293	2351	0943
60	.1629	.0822	07992
61	.1142	364	1826
62	0766	2789	1707
63	2546	8750	.0111
64	.1178	7745	.2461
65	.07198	.3665	3542
66	06531	.1356	1771
67	1216	.2049	0774
68	.1827	.8040	.0852
69	.0735	.6630	.0633

-111-

TABLE C-I (Cont'd.)

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
70	0924	.1653	1544
71	0389	.8871	3786
72	.1256	1.493	618
73	.02049	.18496	1259
74	005497	.1433	05012
75	00812	.2022	00689
76	.0752	.4304	0416
77	.02174	.4654	.0145
78	03667	.2108	0865
79	.0543	.8775	2007
80	.0427	1.0769	2121
81	.01131	.1529	09042
82	.00766	.1476	0336
83	.01677	.2066	00161
84	.0483	• 3597	0663
85	.01702	.4248	00502
86	0246	.2365	08188
87	.07330	.8734	1682
88	.01839	.9847	1397
89	00866	3840	3915
90	00849	2367	1620
91	0518	4398	.04087
92	00838	8654	.1881

ACTUAL AFTER CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
93	05198	8564	02377
94	.0835	301	2128
95	05082	-1.323	291
96	05804	-1.940	3612
97	.01996	203	1476
98	0404	2135	0393
99	1016	378	.0615
100	.0274	485	.0078
101	00704	593	0422
102	.0284	305	1252
103	1144	-1.164	1552
104	001608	-1.366	0553
105	.0212	1692	107
106	04439	2064	0251
107	1038	3524	.05074
108	.03121	4062	03078
109	0049	5283	05101
110	.0169	315	1129
111	1215	-1.103	1345
112	.0137	-1.217	01672

TABLE C-II

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
l	00358	0102	0084
2	.0053	0179	0049
3	.0053	0179	.0049
4	00358	0102	.0084
5	.00646	.00445	0072
6	0047	.01715	0056
7	0047	.01715	.0056
8	.00646	.00445	.0072
9	.0433	.543	.3308
10	0026	.459	.101
11	0026	.459	101
12	.0433	.543	3308
13	0824	.447	1746
14	.1231	.465	065
15	.1231	.465	.065
16	0824	.447	.1746
17	.0139	.709	.0474
18	.0228	.5198	.0275
19	.0228	.5198	0275
20	.0139	.709	0474
21	0792	7457	4138
22	.0504	5963	1548
23	.0504	5963	.1548

-114-

TABLE II (Cont'd.)

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
24	0792	7457	.4138
25	.2164	.0979	0638
26	2452	2122	05976
27	2452	2122	.05976
28	.2164	.0979	.0638
29	.1268	6108	.2084
30	1557	6248	.1032
31	1557	6248	1032
32	.1268	6108	2084
33	0027	0118	.01009
34	0007	0218	.00343
35	0007	0218	00343
36	0027	0118	01009
37	.00163	0025	0028
38	00508	0058	.00042
39	00508	0058	00042
4 O	.00163	0025	.0028
41	.0384	.5435	3399
42	.00146	. 459	108
43	.00146	.459	.108
44	.0384	.5435	.3399
45	152	.0134	0634
46	.192	.3051	0446

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
47	.192	.3051	.0446
48	152	.0134	.0634
49	0906	• 5	.1917
50	.1305	.525	.0733
51	.1305	.525	0733
52	0906	• 5	1917
53	0987	815	.513
54	.0223	715	.154
55	.0223	715	154
56	0987	815	513
57	.1812	0283	.0851
58	2576	4298	.0717
59	2576	4298	0717
60	.1812	0283	0851
61	.1134	606	2393
62	1899	6346	0779
63	1899	6346	.0779
64	.1134	606	.2393
65	.1263	.5848	221
66	0919	.168	0514
67	0919	.168	.0514
68	.1263	.5848	.221
69	.1001	1.09	.3379

-116-

TABLE II (Cont'd.)

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
70	0657	.5217	.1138
71	0657	.5217	1138
72	.1001	1.09	3379
73	.00602	.0106	.0126
74	00076	.0122	.0100
75	00076	.0122	0100
76	.00602	.0106	0126
77	.00816	.2629	0474
78	.02869	.2146	0275
79	.02869	.2146	.0275
80	.00816	.2629	.0474
81	.0398	.3403	06108
82	.00456	.20913	0253
83	.00456	.20913	.0253
84	.0398	.3403	.06108
85	.0308	.833	.1074
86	.0134	.585	.0544
87	.0134	.585	0544
88	.0308	.833	1074
89	00958	624	288
90	0284	339	0993
91	0284	339	.0993
92	00958	624	.288

-117-

TABLE II (Cont'd.)

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
93	0542	-1.37	.1719
94	.0162	816	.0369
95	.0162	816	0369
96	0542	-1.37	1719
97	.00337	3286	0181
98	0633	3077	0285
99	0633	3077	.0285
100	.00337	3286	.0181
101	00227	9928	.0181
102	0576	8403	.0285
103	0576	8403	0285
104	00227	9928	0181
105	.01426	3139	0634
106	0574	2566	0469
107	0574	2566	.0469
108	.01426	3139	.0634
109	00687	912	0170
110	03629	6855	.0178
111	03629	6855	0178
112	00687	912	.0170
113	138	.0131	.0554
114	.1793	.2898	.0432
115	.1793	.2898	0432
116	138	.0131	0554

-118-

TABLE C-II (Cont'd.)

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
l	.0235	.00813	00715
2	.0128	.0239	02059
3	0128	0239	02059
4	0235	00813	00715
5	.04747	.01299	02918
6	00261	.00579	.00143
7	.00261	00579	.00143
8	04747	01299	02918
9	03198	2092	106
10	0385	2866	.0748
11	.0385	.2866	.0748
12	.03198	.2092	106
13	.00653	1667	.0369
14	075	2486	068
15	.075	.2486	068
16	00653	.1667	.0369
17	.00186	2804	0657
18	04912	3139	1194
19	.04912	.3139	1194
20	00186	.2804	0657
21	0446	.2824	.1105
22	.0526	•3789	078
23	0526	3789	078

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
24	.0446	2824	.1105
25	.0262	0584	.02239
26	.0769	1096	.0101
27	0769	.1096	.0101
28	0262	.0584	.02239
29	0113	.2586	04063
30	. 1174	.4251	.0731
31	1174	4251	.0731
32	.0113	2586	04063
33	.00988	.00596	00624
34	.00836	.0175	00374
35	00836	0175	00374
36	00988	00596	00624
37	.00307	.0011	0009
38	.00454	.00554	00907
39	00454	00554	00907
40	00307	0011	0009
4 l	0318	2082	.1069
42	03918	2858	0758
43	.03918	.2858	0758
44	.0318	.2082	.1069
45	01458	.00878	.0218
46	073	.0347	.0093

-120-

TABLE C-II (Cont'd.)

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
47	.073	0347	.0093
48	.01458	00878	.0218
49	.00822	1876	0416
50	0833	2802	.0728
51	.0833	.2802	.0728
52	00822	.1876	0416
53	.0452	.3095	150
54	.0523	.4030	.1068
55	0523	4030	.1068
56	0452	3095	150
57	.02096	0119	0219
58	.0955	0148	02138
59	0955	.0148	02138
60	02096	.0119	0219
61	.0081	.2215	.0491
62	.0999	.3279	0924
63	0999	3279	0924
64	0081	2215	.0491
65	0529	2192	1323
66	.0271	0331	1244
67	0271	.0331	1244
68	.0529	.2192	1323
69	0248	4183	2739

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
70	0282	3667	2647
71	.0282	.3667	2647
72	.0248	.4183	2739
73	0526	.0386	.0652
74	136	03578	.0479
75	.136	.03578	.0479
76	.0526	0386	.0652
77	00858	1047	0665
78	0104	0363	0129
79	.0104	.0363	0129
80	.00858	.1047	0665
81	016	132	0843
82	0042	0395	028
83	.0042	.0395	028
84	.016	.132	0843
85	00171	328	1025
86	0508	354	15
87	.0508	.354	15
88	.00171	.328	1025
89	0022	.2404	1046
90	.02098	.1034	0628
91	02098	1034	0628
92	.0022	2404	1046

-122-

TABLE C-II (Cont'd.)

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
93	.00378	.5391	195
94	.0668	.5103	254
95	0668	5103	254
96	00378	5391	195
97	.0147	.0609	159
98	0023	0809	0805
99	.0023	.0809	0805
100	0147	0609	159
101	0111	.419	1409
102	.0889	.5509	2204
103	0889	5509	2204
104	.0111	419	1409
105	.0038	.1318	0819
106	.0258	.0716	0026
107	0258	0716	0026
108	0038	1318	0819
109	00607	.3635	0589
110	.0703	.4158	146
111	0703	4158	146
112	.00607	3635	0589
113	01512	.0077	018
114	0689	.0319	01313
115	.0689	0319	01313
116	.01512	0077	018

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
l	.01992	00207	01555
2	.0181	.006	02549
3	0075	0418	01569
4	02708	01833	.00125
5	.05393	.01744	03638
6	00731	.02294	00417
7	00209	.01136	.00703
8	04101	00854	02198
9	.01132	.3341	.2248
10	0411	.1729	.1758
11	.0359	.7521	0262
12	.07528	.7449	4368
13	07587	.2805	1377
ユ 4	.0481	.2164	133
15	.1981	.6139	003
16	08893	.7136	.2115
17	.01576	.4286	0183
18	02632	.2059	0919
19	.07192	.8337	1469
20	.01204	.9894	1131
21	1238	464	3033
22	.1030	2171	2328
23	0022	-1.028	.0768

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
24	0346	9749	.5243
25	.2426	.0395	04141
26	1683	.3218	04966
27	3221	.1563	.06986
28	.1902	1026	.08619
29	.1155	3523	.16777
30	0383	1998	.1763
31	2731	8695	0301
32	.1381	-1.6498	2490
33	.00718	00588	.00385
34	.00766	0043	00031
35	00906	0393	00717
36	01258	01772	01633
37	.00476	0014	0037
38	00054	00026	00865
39	00962	01134	00949
40	00144	0036	.0021
41	.0066	•3353	233
42	03772	.1735	1838
43	.04064	.7517	.0322
44	.0702	.7451	.4468
45	1666	.0222	0416
46	.119	.3398	0353

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
47	.265	.0046	.0539
48	1374	.2704	.0852
49	08238	.3124	.1501
50	.0472	.2448	.1461
51	.2138	.6876	0005
52	09882	.8052	2333
53	0535	5055	+.363
54	.0746	312	.2608
55	0300	-1.1245	0472
56	1439	-1.118	663
57	.2022	0402	.0632
58	1421	4698	.05032
59	3531	0164	09308
60	.1602	3862	1070
61	.1215	3850	1902
62	0900	3068	1703
63	2898	828	0145
64	.1053	9626	.2884
65	.0734	.3656	3533
66	0648	.1349	1758
67	1190	.2011	073
68	.1792	.8040	.0887
69	.0753	.6717	.064

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

Element	Girth Stress	Longitudinal Stress	Shear Stress
70	0939	.1550	1509
71	0375	.8884	3785
72	.1249	1.5083	6118
73	04658	.0492	.0778
74	13676	02358	.0579
75	+.13524	.04798	.0379
76	.05862	0280	.0526
77	00042	.1582	1139
78	.01829	.1783	404
79	.03909	.2509	+.0146
80	.01674	.3676	0191
81	.0238	.2083	14538
82	.00036	.16963	0533
83	.00876	.24863	0027
84	.0558	.4723	02322
85	.02909	.505	.0049
86	0374	.231	0956
87	.0642	.939	2644
88	.03251	1.161	2099
89	01178	3836	3926
90	00742	2356	1621
91	04938	4424	.0365
92	00738	8644	.1834

ACTUAL FORWARD CATAMARAN CROSS STRUCTURE STRESSES (KSI)

		COMBINED	
Element	Girth Stress	Longitudinal Stress	Shear Stress
93	05042	8309	0231
94	.0830	3057	2171
95	0506	-1.3263	2909
96	05798	-1.9091	3669
97	.01807	2677	1771
98	0656	3886	109
99	0610	2268	0520
100	01133	3895	1409
101	01337	5738	1228
102	.0313	2892	1919
103	1465	-1.3912	2489
104	+.0088	-1.4118	159
105	.01806	1821	1453
106	0316	185	0495
107	0832	3282	.0443
108	.01046	4457	0185
109	01294	5485	0759
110	.03401	2697	1282
111	1066	-1.1013	1638
112	0008	-1.2755	0419
113	15312	.0209	.0374
114	.1104	.3217	.0301
115	.2482	.0055	0563
116	- 1229	.2579	0734

TABLE C-III

NORMAL STRESSES FOR SYMM. BEND. MOM. (σ yy) in KSI

L/B	• 3	• 3	• 3	• 5	• 5	.5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			SYMM. B	END. MOM.			
Left	3.772	4.554	5.371	3.81	3.99	4.14	3.31
9	1.45	1.761	2.081	1.60	1.67	1.73	1.515
10	477	57	670	252	266	281	.0104
11	1.20	1.441	1.683	1.310	1.405	1.495	1.28
Right	3.342	4.074	4.777	3.35	3.69	3.80	2.95
Left	3.27	3.999	4.776	2.70	2.82	2.91	1.94
12	1.376	1.69	2.009	1.422	1.475	1.52	1.295
13	205	248	294	.366	.384	• 399	.792
14	1.165	1.401	1.641	1.314	1.405	1.509	1.33
Right	2.956	3.569	4.198	2.54	2.71	2.89	1.99

NORMAL STRESSES FOR SYMM. BEND. MOM (σ yy) in KSI

L/B	• 3	• 3	• 3	• 5	• 5	•5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
Ar/Aw	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			SYMM. B	END. MOM.			
21	1.698	1.943	2.178	1.33	1.585	1.860	1.132
22	3.291	3.872	4.458	2.940	3.27	3.61	2.57
23	1.351	1.439	1.526	.796	.861	.924	.533
24	2.844	3.27	3.704	2.075	2.19	2.29	1.48
29	•799	.943	1.094	.744	.865	1.01	.687
30	2.229	2.663	3.106	2.182	2.41	2.66	1.97
31	.764	.893	1.025	.657	.715	.776	.521
32	2.118	2.522	2.936	1.860	2.000	2.15	1.49

-129-

NORMAL STRESSES FOR SYMM. BEND. MOM. (σ yy) in KSI

L/B	• 7	• 7	• 9	• 9	• 9	1.1	l.l
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			SYMM. B	END. MOM.			
Left	3.48	3.61	2.93	3.08	3.20	2.63	2.76
9	1.585	1.64	1.45	1.52	1.57	1.40	1.46
10	.00638	00218	.209	.214	.218	.359	.372
11	1.372	1.46	1.245	1.33	1.415	1.205	1.29
Right	3.16	3.43	2.61	2.80	2.97	2.33	2.51
Left	1.995	2.04	1.545	1.57	1.59	1.34	1.365
12	1.335	1.365	1.245	1.28	1.30	1.225	1.255
13	.829	.861	1.031	1.075	1.115	1.15	1.198
14	1.415	1.490	1.315	1.39	1.452	1.29	1.351
Right	2.12	2.24	1.65	1.74	1.825	1.44	1.505

NORMAL STRESSES FOR SYMM. BEND. MOM. (oyy) in KSI

L/B	• 7	• 7	.9	. 9	• 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			SYMM. B	END MOM.			
21	1.133	1.54	.99	1.15	1.315	.88	.974
22	2.84	3.10	2.29	2.51	2.73	2.07	2.18
23	•55	.564	.394	.398	.394	.328	.314
24	1.52	1.55	1.16	1.175	1.18	1.000	.976
29	.789	.905	.634	.719	.815	.585	.636
30	2.16	2.37	1.78	1.95	2.13	1.62	1.710
31	.557	.59	.427	.45	.47	.368	.371
32	1.58	1.675	1.24	1.31	1.37	1.085	1.10

-131-

NORMAL STRESSES FOR SYMM. BEND. MOM. (oyy) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _W	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			SYMM. B	END. MOM.			
Left	2.88	2.123	2.39	2.51	2.62	2.90	2.308
9	1.52	1.194	1.355	1.415	1.47	1.64	1.378
10	.382	.414	.473	.493	.509	.564	.584
11	1.37	1.072	1.168	1.25	1.325	1.418	1.212
Right	2.66	1.938	2.11	2.26	2.41	2.567	2.059
Left	1.38	1.063	1.24	1.26	1.28	1.463	1.205
12	1.28	1.053	1.212	1.25	1.28	1.443	1.238
13	1.24	1.064	1.215	1.26	1.30	1.446	1.278
14	1.41	1.116	1.260	1.315	1.365	1.509	1.283
Right	1.585	1.159	1.305	1.36	1.455	1.563	1.27

NORMAL STRESSES FOR SYMM. BEND. MOM. (σ_{yy}) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			SYMM. B	END. MOM.			
21	1.14	.829	.789	.891	1.000	.964	.804
22	2.44	1.795	1.875	2.04	2.19	2.290	1.867
23	.316	.177	.294	.289	.282	• 335	.272
24	1.01	.785	.92	.921	.926	1.075	.882
29	.737	•534	.540	.600	.671	.668	.554
30	1.925	1.405	1.480	1.61	1.75	1.816	1.485
31	.396	.295	.330	.342	.350	.388	.415
32	1.18	.878	.987	1.030	1.060	1.176	.959

TABLE C-IV

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (σ yy) in KSI

L/B	• 3	• 3	• 3	• 5	.5	• 5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			ANTISYM. H	B.M. + SHEAP	{ *		
Left	-1.242	-1.465	-1.672	-1.269	-1.411	- 1.444	-1.172
9	465	546	622	520	559	591	523
10	+.179	+.213	+.245	+.129	+.138	+.147	+.054
11	389	466	541	353	38	806	342
Right	-1.128	-1.345	-1.554	-1.019	-1.33	-1.167	883
Left	-2.704	-3.209	-3.691	-1.799	-1.902	-1.99	-1.591
12	-1.19	-1.409	-1.622	828	8754	914	782
13	+.315	+.378	+.432	+.152	+.1602	+.167	+.0611
14	211	252	293	149	141	173	129
Right	-1.236	-1.4734	-1.697	785	830	941	611

*Note: For Asym. B.M. + Shear the signs of the stress are opposite when considering the opposite end of the -134-

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (oyy) in KSI

L/B	• 3	• 3	• 3	• 5	.5	•5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			ANTISYM. B	.M. + SHEAN	R *		
21	378	5019	6329	487	575	669	466
22	-1.859	-2.25	-2.655	-1.308	-1.434	-1.567	-1.120
23	+.073	+.0255	048	133	149	166	178
24	-1.737	-2.121	-2.521	-1.194	-1.266	-1.341	-1.078
29	211	251	291	210	248	294	203
30	699	833	968	646	717	793	590
31	0776	092	108	075	084	094	0674
32	259	309	360	234	255	278	20

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (σ yy) in KSI

L/B	• 7	• 7	• 9	.9	.9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			ANTISYM. B	.M. + SHEAR	*		
Left	-1.249	-1.316	-1.048	-1.11	-1.164	932	984
9	557	585	5104	5402	564	493	519
10	+.0575	+.0608	0218	0233	024	0868	092
11	305	381	3156	3400	364	307	331
Right	946	-1.017	755	813	864	652	699
Left	-1.679	-1.753	-1.396	-1.596	-1.534	-1.23	-1.294
12	825	861	7368	7766	8091	694	731
13	+.0634	+.065	0236	0263	0286	093	0992
14 14	129	149	119	128	136	116	124
Right	651	687	 471	499	525	363	382

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (oyy) in KSI

L/B	• 7	• 7	.9	. 9	.9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			ANTISYM. B.	M. + SHEAR*			
21	537	611	417	475	534	365	413
22	-1.22	-1.315	96	-1.042	-1.125	833	900
23	189	198	192	200	205	1925	198
24	-1.035	-1.182	962	-1.01	-1.05	861	900
29	235	272	189	215	246	173	194
30	655	718	529	582	639	472	506
31	073	079	055	0585	0613	0435	0449
32	216	233	161	173	1825	1285	134

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (oyy) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			ANTISYM. B.	.M. + SHEAR [*]	÷		
Left	-1.027	825	835	878	915	-1.128	886
9	541	496	477	501	521	673	569
10	096	168	139	146	153	223	246
11	354	289	302	324	359	385	336
Right	744	516	478	612	654	696	537
Left	-1.37	-1.575	-1.106	-1.162	-1.208	-2.007	-1.68
12	761	928	664	698	726	-1.191	-1.04
13	104	165	146	155	162	208	255
14	131	114	118	124	131	177	141
Right	403	355	286	301	328	483	335

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (yy) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			ANTISYM. B.I	M. + SHEAR*			
21	447	379	320	359	398	441	363
22	965	794	731	79	843	-1.009	804
23	201	228	189	193	195	370	402
24	931	-1.076	782	816	845	-1.201	-1.198
29	22	158	155	176	197	198	161
30	564	411	422	461	502	532	426
31	0457	033	0341	0345	0342	045	029
32	 140	098	1035	107	110	- .148	093

TABLE C-V

NORMAL STRESSES FOR SYMM. BEND. MOM. ($^{\sigma}xx$) in KSI

L/B	• 3	• 3	• 3	• 5	• 5	.5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			SYMM. H	BEND. MOM.			
9	.1761	.212	.250	.180	.186	.190	.0847
10	3441	417	492	485	514	542	432
11	 1747	210	247	275	293	310	241
12	1761	212	250	183	188	193	0855
13	.3441	.417	.492	.484	.522	.546	.435
14	.1747	.210	.247	.280	.298	.315	.243
21	.252	.302	.311	.162	.217	.278	.1168
22	.279	.307	.316	.177	.233	.294	.1375
23	252	302	311	162	221	282	1180
24	279	307	316	177	237	299	1395
29	.0339	.038	.0393	.0312	0495	.074	.033
30	.0385	.0427	.0443	.0418	.0611	.086	.0464
31	0339	038	0393	0318	0506	075	0334
32	0385	0427	0443	0424	0621	0874	0469

-140-

NORMAL STRESSES FOR SYMM. BEND. MOM. ($\sigma_{{\tt X}{\tt X}}$) in KSI

L/B	• 7	• 7	.9	. 9	. 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
Af ^A w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			SYMM. BI	END. MOM.			
9	.0834	.0798	.0168	.0111	.00356	0143	0209
lO	456	479	351	371	390	277	284
11	254	266	186	196	204	142	143
12	084	0806	0169	0111	00351	.0144	.0211
13	.462	.485	+.354	.371	.393	.279	.286
14	.257	.269	+.188	.197	.206	.142	·144
21	.151	.189	.085	.109	.134	.065	.0815
22	.1735	.213	.108	.1332	.1595	.086	.104
23	153	1915	0867	110	1352	0654	0823
24	176	216	109	1344	162	0866	1055
29	.0454	.0615	.029	.038	.0492	.0243	.0312
30	.0605	.0777	.0428	.0535	.0665	.0374	.0457
31	0463	0623	0293	0384	0495	0246	0314
32	0612	0787	0432	054	0672	0375	0461

NORMAL STRESSES FOR SYMM. BEND. MOM. (σ_{xx}) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			SYMM. BI	END. MOM.			
9	0304	0298	023	0303	039	0291	0284
10	309	198	217	23	243	263	1819
11	1535	095	107	112	116	129	0863
12	.0306	.0298	.0232	.0306	.039	.0291	.0284
13	.311	.198	.218	.232	.244	.263	.1819
14 1	.155	.095	.108	.112	.117	.129	.0863
21	.099	.0704	.0508	.063	.076	.0582	.0502
22	.124	.0882	.0697	.0837	.0982	.0821	.0687
23	100	0704	0510	0634	0768	0582	0502
24	125	0882	0702	0844	0992	0821	0687
29	.0394	.0278	.0203	.0256	.032	.0243	.0211
30	.0559	.0396	.0319	.0386	.0469	.0389	.0328
31	0398	0278	0207	0257	0322	0242	0211
32	0564	0396	0320	0389	0472	0389	0328

TABLE C-VI

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (σ yy) in KSI

L/B		.3	• 3	• 3	• 5	.5	.5	• 7
B/D		2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A	W	3.0	3.0	3.0	2.5	3.0	3.5	2.5
				ANTISYM.	B.M. + SHEA	R*		
	9	5.14	6.167	7.196	+1.447	+1.45	+1.45	+.766
	10	3.03	3.637	4.294	+ .854	+.854	+.855	+.457
	11	.999	1.198	1.398	+.275	+.273	+.274	+.145
	12	1.347	1.619	1.893	+.289	+.284	+.280	+.103
	13	1.009	1.210	1.414	+.193	+.188	+.182	+.0207
	14	.318	.381	.445	+.0604	+.058	+.055	+.00232
	21	27	227	1916	075	0938	1144	0414
	22	22	169	123	0428	0597	079	0117
	23	105	132	163	0377	0255	0101	0202
	24	365	429	496	179	1742	165	1299
	29	134	116	1019	0398	0482	0582	0235
	30	149	142	1199	0501	0591	0697	0339
	31	0185	0128	0098	+.0011	+.0062	+.0131	+.0056
	32	020	0141	0109	+.0014	+.0063	+.01303	+.0074
		*Note:	Signs shift	at the opr	osite end c	of the quart	er structure	model

NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (Gyy) in KSI

L/B	• 7	. 7	.9	.9	.9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
AfAw	3.0	3.5	2.5	3.0	3.5	2.5	3.0
		-1	ANTISYM. B.	M. + SHEAR*			
9	+.768	+.769	+.476	+.476	+.479	+.326	+.328
10	+.459	+.460	+.29	+.291	+.293	+.204	+.204
11	+.145	+.145	+.091	+.091	+.091	+.0627	+.626
12	+.0992	+.096	+.03	+.0267	+.0235	00396	00736
13	+.013	+.0063	048	0555	063	0741	0811
14 1	00597	0095	0285	0321	0355	0372	0391
21	0528	0653	0266	0344	043	0183	0240
22	0215	0326	00035	0067	014	+.0048	+.00034
23	0089	+.0044	0124	00326	+.00778	0094	00205
24	135	127	1149	1112	1048	0983	0956
29	0285	0346	0186	0204	0247	0133	0160
30	0384	0451	0254	0296	0346	0213	0247
31	+.0103	+.0164	+.00839	+.0124	+.0178	+.0096	+.0133
32	+.0121	+.0184	+.0118	+.0163	+.022	+.0143	+.0185
	*Note: Si	gns shift at	t the oppos	ite end of	the quarter	structure	model
NORMAL STRESSES FOR ANTISYM. B.M. + SHEAR (Gyy) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
		1	ANTISYM. B.I	M. + SHEAR*			
9	+.329	.341	+.242	+.243	+.242	.450	.318
10	+.205	.205	+.153	+.154	+.155	.276	.192
11	+.626	.064	+.0461	+.046	+.046	.088	.059
12	0103	0411	0204	0235	0265	0457	054
13	0875	083	0802	0865	0925	1106	095
14	0432	039	038	0405	0428	0535	043
21	0303	0165	0128	0166	021	008	0049
22	0049	+.0209	+.0073	+.0040	+.000058	+.0390	+.0323
23	+.0067	0181	0093	0034	+.0035	0255	0289
24	0907	1422	0879	0862	0826	1857	1584
29	0193	0139	0108	0129	0155	0135	0108
30	0287	0211	0188	0217	0250	0215	0194
31	+.0177	+.0125	+.0097	+.0128	+.0166	+.0176	+.0117
32	+.0236	+.0176	+.0152	+.01904	+.0236	+.0195	+.0180
	*Note:	Signs shift	at the opp	osite end of	the quarte	r structure	model

TABLE C-VII

SHEAR STRESS (σxy) in KSI

L/B	• 3	• 3	• 3	• 5	. 5	.5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
ELEMENT			SYMMET.	BEND. MOM.			
9	.600	.732	.8698	.882	.921	.955	.926
10	.00756	.0045	.00104	.113	.125	.137	.147
11	201	242	283	408	434	462	500
12	.243	.297	.352	.336	.351	.365	.293
13	.438	.051	.0587	.112	.119	.125	.1065
14	118	142	1678	231	245	258	251
21	405	516	608	418	562	726	426
22	.405	.516	.608	.418	.562	.726	.726
23	203	242	262	151	195	244	110
24	.203	.242	.262	.151	.195	.244	.110
29	0496	0636	0746	0718	121	184	106
30	.0496	.0635	.0746	.0718	.121	.184	.106
31	0276	0338	0374	0406	0564	0766	047
32	.0276	.0337	.0374	.0406	.0564	.0766	.047

SHEAR STRESS (σxy) in KSI

J/B	• 7	. 7	. 9	.9	. 9	1.1	1.1
3/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
LEMENT			SYMMET.	BEND. MOM.			
9	.965	.995	.850	.881	.905	.761	.788
10	.157	.167	.133	.139	.145	.112	.116
11	528	555	50	53	555	473	502
12	.303	.312	.204	.208	.210	.126	.125
13	.110	.114	.0698	0704	0694	+.038	.036
14	264	276	216	217	237	172	180
21	56	707	39	506	634	353	455
22	.56	.707	. 39	.506	.634	.353	. 455
23	134	158	066	0742	0817	0318	0308
24	.134	.158	.066	.0742	.0817	.0318	.0308
29	153	214	114	157	212	114	153
30	.153	.214	.114	.157	.212	.114	.153
31	057	0693	038	0434	049	0258	0278
32	.057	.0693	.038	.0434	.049	.0258	.0278

-147-

SHEAR STRESS (oxy) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
3/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
Ar/Aw	3.5	3.0	2.5	3.0	3.5	3.0	3.0
ELEMENT			SYMMET. E	BEND. MOM.			
9	.81	.594	.681	.705	.725	.818	.635
10	.118	.081	.0957	.0971	.0975	.114	.085
11	527	398	439	465	49	530	426
12	.124	.054	.0684	.064	.0604	.076	.0222
13	+.0328	.009	.0172	.0138	.00935	.019	.0013
14	187	116	129	135	140	155	097
21	565	395	320	408	508	426	374
22	.565	.395	.320	.408	.508	.426	.374
23	0285	0225	0705	00061	00742	.019	.0204
24	.0285	.0225	.0705	.00061	.00742	019	0204
29	202	139	111	146	189	154	139
30	.202	.139	.111	.146	.189	.154	.139
31	0292	0219	0146	0141	0129	0061	003
32	.0292	.0219	.0146	.0141	.0129	.00604	.0027

TABLE C-VIII

SHEAR STRESS (σ_{xy}) in KSI

L/B	• 3	• 3	• 3	• 5	.5	•5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
ELEMENT			ANTISYMM H	B.M. + SHEAD	2		
9	.596	.718	.841	.138	.126	.119	0024
10	.6657	.797	.928	.310	.312	.318	.226
11	.6099	.732	.854	.284	.286	.292	.244
12	1.231	1.475	1.718	.655	.669	.686	.573
13	1.162	1.396	1.630	.582	.481	.485	.346
14 1	1.218	1.460	1.704	.508	.500	.510	.328
21	2793	330	387	286	376	484	218
22	598	547	489	145	145	- .144	0945
23	.1031	.155	.2108	494	566	650	346
24	981	-1.031	-1.088	+.063	.045	.0231	.0337
29	0823	0817	0818	185	226	279	142
30	3565	357	356	0351	035	0344	0178
31	0056	0060	0061	213	246	287	146
32	433	432	432	0073	0151	0262	0133

SHEAR STRESS (^oxy) in KSI

L/B	• 7	• 7	• 9	• 9	.9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
ELEMENT			ANTISYMM H	B.M. + SHEAF	2		
9	0113	0226	0681	0821	094	104	117
10	.228	.229	.169	.169	.169	.13	.13
11	.249	.253	.225	.231	.236	.214	.22
12	.586	.596	.511	.524	.535	.465	.479
13	.344	.345	.271	.271	.271	.229	.229
14	.324	.319	.214	.208	.203	.145	.1395
21	29	372	183	243	310	164	215
22	0875	0775	0558	0462	0341	0312	0209
23	392	439	249	279	311	189	211
24	.0137	+.0102	+.0095	00995	0326	0067	025
29	174	214	117	144	176	103	125
30	0149	0107	0051	00116	00436	00261	+.0071
31	167	191	104	118	134	078	0885
32	0219	0333	0185	0269	0378	022	0301

SHEAR STRESS (σ_{xy}) in KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
ELEMENT			ANTISYMM B	.M. + SHEAR			
9	128	129	120	1325	143	168	159
10	.130	.151	.104	.104	.103	.199	.148
11	.225	.251	.202	.208	.213	• 337	.277
12	.495	.550	.430	.442	.453	.716	.604
13	.231	.271	.202	.204	.204	.360	.297
14	.134	.180	.105	.0994	.094	.234	.168
21	272	043	146	191	24	0355	032
22	0846	159	131	00242	.0103	166	156
23	235	.0028	147	164	181	007	0162
24	0464	205	0134	0299	0486	195	172
29	154	.0082	0905	110	1345	.0118	.00999
30	+.00129	109	.00844	.01325	.0194	113	104
31	100	028	060	0675	0765	337	034
32	0402	0728	0223	0294	0387	0671	060

TABLE C-IX

COMBINED RESULTS FOR LONGITUDINAL STRESS (σyy) KSI

L/B	• 3	• 3	• 3	• 5	.5	•5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.0	3.0	2.5	3.0	3.5	2.5
*Left	2.524	3.089	3.696	2.538	2.628	2.562	2.131
9	.987	1.214	1.459	1.08	1.111	1.139	.992
10	295	358	425	123	128	134	0644
11	.813	.975	1.144	•957	1.025	.689	.938
*Right	2.263	2.731	3.22	2.353	2.498	1.886	2.050
Left	.568	.792	1.057	.903	.918	.742	.346
12	.186	.277	.387	.594	.5996	.506	.513
13	.111	.126	.139	.518	.5442	.566	.8531
14	•954	1.148	1.348	1.165	1.264	1.336	1.201
Right	1.720	2.099	2.50	1.760	1.915	1.99	1.378

*Values extrapolated to plating edge.

-152-

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\circ}yy$) KSI

L/B	• 7	• 7	.9	• 9	. 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
*Left	2.478	2.309	1.876	1.962	2.038	1.697	1.774
9	1.028	1.055	.9396	.9798	1.006	.907	.941
10	.0639	.05862	.1872	.1967	.194	.2722	.280
11	1.067	1.079	.9294	.99	1.051	.898	.959
*Right	2.306	2.345	1.861	1.978	2.105	1.684	1.801
Left	.323	.290	.142	.106	.0559	.115	.0689
12	.510	.504	.5082	.5034	.4909	.531	.524
13	.8924	.926	1.0074	1.0487	1.0864	1.057	1.099
14	1.286	1.341	1.196	1.262	1.316	1.174	1.227
Right	1.487	1.546	1.174	1.244	1.293	1.079	1.123

*Values extrapolated to plating edge.

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}yy$) KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
*Left	1.859	1.281	1.554	1.627	1.697	1.774	1.423
9	.979	.6986	.878	.914	.949	.967	.809
10	.286	.246	.334	.347	.356	.341	.338
11	1.016	.742	.866	.926	.966	1.032	.876
*Right	1.915	1.346	1.536	1.645	1.722	1.871	1.523
Left	.0328	514	.1195	.100	.068	45162	472
12	.519	.125	.548	.552	.554	.251	.2008
13	1.136	.899	1.069	1.105	1.147	1.213	1.022
14	1.279	1.002	1.142	1.191	1.234	1.584	1.142
Right	1.173	.802	1.011	1.059	1.088	1.5487	.939

*Values extrapolated to plating edge.

COMBINED RESULTS FOR LONGITUDINAL STRESS(σyy) KSI

L/B	• 3	. 3	• 3	• 5	.5	• 5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.0	3.0	2.5	3.0	3.5	2.5
*Left	5.975	7.206	8.439	4.50	4.708	4.904	3.528
15	2.565	3.097	3.63	2.25	2.3504	2.434	2.077
16	520	623	726	.214	.2234	.232	.7309
17	1.376	1.653	1.934	1.463	1.546	1.682	1.459
*Right	4.192	5.040	5.895	3.32	3.50	3.776	2.601
Left	5.01	6.019	7.047	5.075	5.468	5.750	4.484
18	1.918	2.307	2.703	2.12	2.29	2.321	2.038
19	6535	783	9156	381	404	428	0436
20	1.591	1.907	2.225	1.663	1.785	2.301	1.622
Right	4.519	5.419	6.33	4.389	4.711	5.720	3.860

*Values extrapolated to plating edge.

-155-

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}yy$) KSI

L/B	• 7	• 7	.9	. 9	• 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.5	2.5	3.0	3.5	2.5	3.0
*Left	3.653	3.793	2.935	3.049	3.121	2.572	2.655
15	2.15	2.226	1.9818	2.0566	2.1091	1.919	1.986
16	.7656	.796	1.0546	1.1013	1.1436	1.243	1.297
17	1.544	1.639	1.434	1.518	1.588	1.406	1.475
*Right	2.744	2.913	2.114	2.241	2.339	1.802	1.889
Left	4.707	4.941	3.972	4.192	4.366	3.559	3.738
18	2.141	2.225	1.9604	2.0602	2.134	1.893	1.979
19	05112	06298	.2308	.2373	.242	.4458	.464
20	1.677	1.841	1.5606	1.67	1.779	1.512	1.621
Right	4.011	4.365	3.373	3.607	3.833	2.988	3.201

*Values extrapolated to plating edge.

-156-

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}yy$) KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
*Left	2.725	2.638	2.333	2.423	2.506	3.399	2.885
15	2.041	1.98	1.876	1.948	2.006	2.58	2.275
16	1.344	1.229	1.361	1.415	1.453	1.642	1.532
17	1.541	1.23	1.378	1.439	1.496	1.638	1.425
*Right	1.975	1.512	1.586	1.660	1.741	1.986	1.61
Left	3.912	2.95	3.221	3.384	3.537	4.009	3.288
18	2.06	1.69	1.832	1.916	1.991	2.298	1.997
19	.478	.583	.612	.639	.662	.776	.830
20	1.724	1.36	1.470	1.574	1.684	1.788	1.549
Right	3.408	2.455	2.678	2.871	3.077	3.244	2.616

*Values extrapolated to plating edge.

-157-

COMBINED RESULTS FOR LONGITUDINAL STRESS (σ yy) KSI

L/B	• 3	• 3	• 3	.5	.5	• 5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
21	1.31	1.441	1.545	.843	1.01	1.191	.666
22	1.432	1.621	1.802	1.632	1.836	2.043	1.450
23	1.424	1.465	1.478	.663	.712	.758	.355
24	1.107	1.149	1.183	.881	.924	.949	.402
25	1.2	1.413	1.574	.929	1.01	1.090	.711
26	4.582	5.391	6.225	3.269	3.456	3.631	2.558
27	2.087	2.445	2.811	1.817	2.16	2.529	1.598
28	5.15	6.124	7.113	4.248	4.704	5.177	3.690

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}$ yy) KSI

L/B	• 7	• 7	.9	.9	. 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
21	.596	.929	.573	.675	.781	.515	.561
22	1.62	1.785	1.33	1.468	1.605	1.237	1.28
23	.361	.366	.202	.198	.189	.136	.116
24	.485	.368	.198	.165	.130	.139	.076
25	•739	.762	.586	.598	.599	.52	.512
26	2.555	2.732	2.122	2.185	2.23	1.861	1.876
27	1.67	2.151	1.407	1.625	1.849	1.245	1.387
28	4.06	4.415	3.25	3.552	3.855	2.903	3.08

-159-

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}yy$) KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _w	3.5	3.0	2.5	3.0	3.5	3.0	3.0
21	.639	.449	.469	.532	.602	.522	.440
22	1.475	1.001	1.144	1.25	1.347	1.28	1.063
23	.115	.021	.105	.096	.087	0357	0302
24	.079	291	.138	.105	.081	378	315
25	.517	.478	.483	.482	.477	.696	.574
26	1.941	1.861	1.702	1.737	1.771	2.481	2.08
27	1.587	1.208	1.109	1.25	1.398	1.404	1.166
28	3.405	2.589	2.606	2.83	3.033	3.286	2.672

-160-

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}yy$) KSI

L/B	• 3	• 3	• 3	• 5	.5	.5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
29	.583	.692	.803	.534	.617	.716	.484
30	1.532	1.830	2.139	1.536	1.693	1.867	1.38
31	.686	.800	.917	.582	.631	.682	.4536
32	1.858	2.212	2.58	1.626	1.745	1.872	1.29
33	.842	.986	1.134	.732	.799	.870	.5884
34	2.377	2.83	3.296	2.094	2.255	2.428	1.69
35	1.005	1.194	1.386	.954	1.113	1.304	.890
36	2.927	3.496	4.074	2.828	3.127	3.453	2.56

COMBINED RESULTS FOR LONGITUDINAL STRESS (σ_{yy}) KSI

L/B	• 7	• 7	. 9	. 9	. 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
29	.554	.633	.445	.504	.569	.412	.442
30	1.505	1.652	1.251	1.368	1.491	1.148	1.204
31	.484	.511	.372	.3915	.4087	.3245	.3261
32	1.364	1.442	1.079	1.137	1.188	.9565	.966
33	.630	.669	.482	.5085	.5313	.4115	.4159
34	1.796	1.908	1.401	1.483	1.552	1.2135	1.234
35	1.024	1.177	.823	.934	1.061	.758	.830
36	2.815	3.088	2.309	2.532	2.769	2.092	2.21

-162-

COMBINED RESULTS FOR LONGITUDINAL STRESS ($^{\sigma}yy$) KSI

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _W	3.5	3.0	2.5	3.0	3.5	3.0	3.0
29	.517	.375	.385	.424	.474	.470	.392
30	1.361	.994	1.058	1.149	1.248	1.284	1.059
31	.3503	.262	.296	.308	.316	.343	.286
32	1.04	.780	.884	.923	.950	1.028	.866
33	.4417	.328	.364	.377	.384	.422	.344
34	1.32	.976	1.091	1.137	1.17	1.284	1.053
35	.957	.692	.695	.776	.868	.864	.716
36	2.489	1.816	1.902	2.071	2.252	2.335	1.911



FIGURE C-I TORSION MODEL ELEMENT NUMBERING SEQUENCE (1/8 STRUCTURE)

2

			TABLE C-X	
	COMPUTER	CALCULATED	TORSION RESULTS (FOR 1,	/8 STRUCTURE)
			$M_t = 1000 \text{ft-tons}*$	
			GIRTH STRESSES	
L/B		• 5	1.3	1.3
B/D		3.0	3.0	3.0
A _f /	Aw	3.5	3.0	3.5
Eler	ment			
	l	186x10-7	.157x10-4	.167x10 ⁻¹
	2	415x10-5	.495xl0-4	. 54x10-1
	3	.588x10-4	.126x10-3	.135x10-3
	4	126x10-4	.482x10-4	.517x10 ⁻¹
	5	.413x10-5	.187x10-3	. 2x10-3
	6	. 18x10-3	.376x10-3	. 4x10-3
	7	98x10-5	.745x10-4	.798x10-4
	8	.663x10-6	.264x10-3	.284x10-3
	9	.314x10-3	.635x10-3	. 68x10-3
	10	62x10-5	239x10-5	.146x10-5
	11	156x10 ⁻⁴	525x10-5	238x10 ⁻⁵
	12	108x10-4	55x10-5	295x10-5
	13	758x10-5	221x10-4	264x10 ⁻⁴
	14 14	.164x10 ⁻⁴	686x10-5	95x10-5
	15	.984x10-5	- 11x10-5	- 114×10-5

*See Figure C-1 for Element Numbering Sequence.

COMPUTER CALCULATED TORSION RESULTS (FOR 1/8 STRUCTURE)

$M_t = 1000 ft - tons *$

GIRTH STRESSES

L/B	•5	1.3	1.3
B/D	3.0	3.0	3.0
Ar/Aw	3.5	3.0	3.5

Element

16	.645x10-4	.102x10-4	+. llx10-3
17	.455x10-4	.614x10-4	+.657x10-4
18	. 18x10-4	.204x10-4	.219x10-4
19	0	0	0

*See Figure C-1 for Element Numbering Sequence.

-166-

COMPUTER CALCULATED TORSION RESULTS (FOR 1/8 STRUCTURE)

M_t -1000ft-tons*

LONGITUDINAL STRESSES

L/B	• 5	1.3	1.3
B/D	3.0	3.0	3.0
A _f /A _w	3.5	3.0	3.5

Element

l	101x10 ⁻⁵	377x10-4	405x10 ⁻⁴
2	8x10-4	985x10-4	107x10-4
3	753x10-4	646x10-4	692x10-4
4	688x10-5	.145x10-4	.157x10-4
5	.437x10-5	.384x10-4	.406x10-4
6	.787x10-5	. 94x10-5	. 91x10-5
7	125x10-5	.271x10-4	. 29x10-4
8	.159x10 ⁻⁴	.764x10-3	.814x10-4
9	.727x10-4	.115x10-3	.123x10-3
10	.975x10-4	.148x10-3	.159x10-3
11	.484x10-4	.826x10-4	.886x10-4
12	.151x10-4	.263x10-4	.287x10-4
13	.266x10-3	.407x10-3	.433x10-3
14	.151x10-3	.257x10-3	.273x10-3
15	.454xl0 ⁻⁴	.867x10 ⁻⁴	.926x10 ⁻⁴

*See Figure C-1 for Element Numbering Sequence.

COMPUTER CALCULATED TORSION RESULTS (FOR 1/8 STRUCTURE)

$M_t = 1000 ft - tons*$

LONGITUDINAL STRESSES

L/B	• 5	1.3	1.3
B/D	3.0	3.0	3.0
A _f /A _w	3.5	3.0	3.5
Flement			

16	.442x10-3	.716x10-3	.767x10-3
17	.204x10-3	.405x10-3	.434x10-3
18	.616x10-4	.131x10-3	.141x10-3
19	0	0	0
20			

*See Figure C-1 for Element Numbering Sequence.





COMPUTER CALCULATED TORSION RESULTS (FOR 1/8 STRUCTURE)

$M_t = 1000 ft - tons *$

SHEAR STRESSES

L/B	• 5	1.3	1.3
B/D	3.0	3.0	3.0
A _f /A _w	3.5	3.0	3.5

Element



*See Figure C-1 for Element Numbering Sequence.

-170-

TABLE C-XI

CALCULATED SHEAR STRESSES (M_t =1000ft-tons) FOR TORSION CASE*

	B/D=3.0	B/D=3.0	B/D=3.0	B/D=2.5	B/D=3.5	After	Fwd
δ _{ij}	$A_{f}/A_{W}=2.5$	A _f /A _W =3.0	A _f /A _w =3.5	A _f /A _w =3.0	A _f /A _W =3.0	Struc.	Struc.
⁸ ll] (2024)	<u>l</u> (2106)	1 G(2206)	1 (2212)	<u>l</u> (2032)	<u>l</u> (1973)	<u>l</u> (1856)
δ ₂₁ =δ ₁₂	- <u>l</u> (222)	$-\frac{1}{G}(264)$	$-\frac{1}{G}(313)$	- <u>l</u> (263)	$-\frac{1}{G}(226)$	$-\frac{1}{G}(288)$	$-\frac{1}{G}(288)$
δ ₃₁ =δ ₁₃	$-\frac{1}{G}(789)$	$-\frac{1}{G}(789)$	$-\frac{1}{G}(789)$	$-\frac{1}{G}(789)$	$-\frac{1}{G}(789)$	$-\frac{1}{G}(896)$	$-\frac{1}{G}(768)$
δ41=δ14	0	0	0	0	0	0	0
δ ₂₂	<u>⊥</u> (2024)	<u>l</u> (2106)	<u>l</u> (2206)	<u>l</u> (2212)	<u>1</u> (2032)	<u>1</u> (1856)	<u>1</u> (1784)
δ ₃₂ =δ ₂₃	0	0	0	0	0	0	0
δ ₄₂ =δ ₂₄	$-\frac{1}{G}(789)$	$-\frac{1}{G}(789)$	$-\frac{1}{G}(789)$	- <u>1</u> (789)	$-\frac{1}{G}(789)$	$-\frac{1}{G}(768)$	$-\frac{1}{G}(768)$
⁶ 33	<u>l</u> (2024)	<u>l</u> (2106)	<u>l</u> (2206)	<u>1</u> (2212)	<u>1</u> (2032)	<u>1</u> (2451)	$\frac{1}{G}(2266)$
δ ₄₃ =δ ₃₄	$-\frac{1}{G}(222)$	$-\frac{1}{G}(264)$	- <u>l</u> (313)	- <u>l</u> (263)	- <u>1</u> (226)	$-\frac{1}{G}(288)$	$-\frac{1}{G}(288)$
δ ₄₄	$\frac{1}{G}(2024)$	<u>l</u> (2106)	$\frac{1}{G}(2206)$	<u>1</u> (2212)	+ <u>1</u> (2032)	$\frac{1}{G}(2266)$	<u>1</u> (2757
A _l (ft. ²)	208.325	208.325	208.325	250	178.75	252	216
A ₂	\downarrow				Ļ	216	216

*For details See Appendix A.

	CALCULATE	D SHEAR STR	ESSES (M _t =1	000ft-tons)	FOR TORSIO	N CASE*	
	B/D=3.0	B/D=3.0	B/D=3.0	B/D=2.5	B/D=3.5		
	A _f /A _w =2.5	A _f /A _w =3.0	A _f /A _w =3.5	A _f /A _W =3.0	A _f /A _w =3.0	After Struc.	Fwd. Struc.
A 3						252	216
Aц	\downarrow	Ļ	Ļ	ł	\downarrow	216	288
$Q_i \frac{\text{tons}}{\text{ft.}}$	60	60	60	50	70	602	585
						552	596
						509	494
						467	476
Top Plate	263	263	263	219	307 Le	ft178	174
Shear Stress					R	t164	176
KSI							
Left Side	0	0	0	0	0	200	260
Rt. Side	222	263	3125	219	307	24	199
Center						0222	+.0049

*For Details See Appendix A.

-172-

TABLE C-XII

AVERAGE LONGITUDINAL STRESS (σ_{yy})avg. AND EFFECTIVENESSES

L/B	• 3	• 3	• 3	• 5	.5	•5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			ANTISYM.	B.M. + SHEA	R		
End Elements							
Avg Stress	276	326	374	291	320	337	305
Pl	.222	.223	.224	.229	.227	.233	.260
ρ2	.244	.243	.241	.286	.241	.288	.346
Center Elements							
Avg Stress	447	528	609	329	347	374	326
ρl	.165	.165	.165	.183	.183	.187	.205
P2	.361	.358	.359	.420	.419	.398	.534

AVERAGE LONGITUDINAL STRESS (oyy)avg. AND EFFECTIVENESSES

L/B	• 7	• 7	. 9	. 9	• 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _W	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			ANTISYM.	B.M. + SHEA	R		
End Elements							
Avg Stress	326	341	314	336	356	319	341
ρl	.261	.260	.300	.302	.306	.343	.347
ρ2	.345	.336	.416	.413	.412	.490	.489
Center Elements							
Avg Stress	346	362	324	347	363	328	342
ρ _l	.206	.206	.232	.235	.237	.267	.265
P2	.532	.527	.689	.693	.692	.903	.896

AVERAGE LONGITUDINAL STRESS (oyy)avg. AND EFFECTIVENESSES

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _W	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			ANTISYM.	B.M. + SHEA	R		
End Elements							
Avg Stress	359	336	327	346	364	453	401
ρ _l	.350	.407	.392	.394	• 397	.401	.453
°2	.483	.651	.571	.565	.556	.650	.748
Center Elements							
Avg Stress	363	432	329	347	362	570	506
ρ _l	.270	.274	.298	.299	.300	.284	.301
ρ2	.902	1.216	1.149	1.152	1.151	1.18	1.51

TABLE C-XII

AVERAGE LONGITUDINAL STRESS (oyy)avg. AND EFFECTIVENESSES FOR SYMM. B.M. AND ANTISYM. B.M. + SHEAR CASES

L/B	• 3	. 3	• 3	.5	• 5	•5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
			SYMM. B	END. MOM.			
End Elements							
Avg Stress	.877	1.058	1.244	1.02	1.092	1.13	1.045
ρl	.233	.232	.232	.267	.273	.272	.315
ρ2	.258	.260	.260	.304	.304	.297	.354
Center Elements							
Avg Stress	.901	1.0958	1.295	1.11	1.18	1.24	1.185
ρl	.276	.274	.273	.412	.418	.425	.595
P 2	.305	.307	.308	.437	.434	.428	.611

AVERAGE LONGITUDINAL STRESS $(\sigma yy)avg$. AND EFFECTIVENESSES

L/B	• 7	. 7	.9	.9	.9	1.1	1.1
			2 2		2 0	2 0	2 0
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
			SYMM. E	END. MOM.			
End Elements							
Avg Stress	1.115	1.19	1.04	1.17	1.165	1.07	1.13
ρl	.321	.329	.355	.380	.364	.406	.409
P2	.354	.347	.398	.418	.392	.458	.452
Center Elements							
Avg Stress	1.24	1.24	1.215	1.27	1.31	1.235	1.282
ρl	.584	.563	.735	.731	.717	.857	.852
P 2	.620	.607	.788	.809	.821	.920	.940

AVERAGE LONGITUDINAL STRESS $(\sigma yy)avg$. AND EFFECTIVENESSES

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _W	3.5	3.0	2.5	3.0	3.5	3.0	3.0
			SYMM. B	END. MOM.			
End Elements							
Avg Stress	1.191	.953	1.065	1.122	1.175	1.288	1.117
ρl	.415	.449	.446	.447	.449	.444	.484
⁶ 2	.448	.492	.505	.498	.490	.502	.542
Center Elements							
Avg Stress	1.33	1.079	1.230	1.278	1.315	1.469	1.265
ρl	.838	.932	.941	.936	.930	.940	.993
ρ2	.961	1.02	.993	1.011	1.029	1.002	1.049

TABLE C-XIII

COMBINED RESULTS OF AVERAGE LONGITUDINAL STRESSES (KSI) AND EFFECTIVENESS

L/B	• 3	• 3	• 3	• 5	• 5	.5	• 7
B/D	2.5	3.0	3.5	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.0	3.0	2.5	3.0	3.5	2.5
Plane Y= ^L /8 Avg Stress	.601	.731	.870	•733	.769	.652	.740
٩	.238	.237	.235	.289	.293	.254	.347
P2	.266	.268	.270	.312	.308	.346	.361
Plane Y= ^{3L} /8 Avg Stress	.455	.566	.685	.789	.835	.832	.856
рŢ	.265	.270	.274	.448	.436	.419	.6212
ρ2	.802	.714	.648	.874	.909	1.121	2.4747
Plane Y= ^{5L} /8 Avg Stress	1.348	1.625	1.905	1.446	1.517	1.601	1.508
рl	.226	.226	.226	.321	.322	.326	.428
P2	.322	.322	.323	.436	.433	.424	.580
Plane Y=7L/8 Avg Stress	1.152	1.384	1.619	1.323	1.427	1.626	1.361
ρl	.230	.230	.230	.261	.261	.283	.304
ρ2	.255	.255	.256	.301	.303	.284	.353

COMBINED RESULTS OF AVERAGE LONGITUDINAL STRESSES (KSI) AND EFFECTIVENESS

L/B	• 7	• 7	. 9	.9	. 9	1.1	1.1
B/D	3.0	3.0	3.0	3.0	3.0	3.0	3.0
A _f /A _w	3.0	3.5	2.5	3.0	3.5	2.5	3.0
Plane Y= ^L /8 Avg Stress	.802	.815	.748	.788	.820	.745	.782
ρ _l	• 348	.347	.398	.398	.389	.439	.434
P2	.357	• 353	.402	.401	.402	.442	.441
Plane Y= ^{3L} /8 Avg Stress	.896	.923	.891	.924	.949	.904	.931
ρ _l	.603	.597	.759	.743	.734	.837	.829
°2	2.776	3.18	6.268	8.697	16.967	7.882	13.521
Plane Y= ^{5L} /8 Avg Stress	1.576	1.648	1.544	1.6156	1.672	1.557	1.622
ρ _l	.432	.434	.526	.530	.536	.606	.611
P2	.574	.566	.731	.721	.715	.864	.858
Plane Y= ^{7L} /8 Avg Stress	1.419	1.509	1.378	1.458	1.528	1.388	1.466
ρŢ	.301	.305	.347	.348	.350	. 39	.392
ρ2	.354	.346	.408	.404	.398	.465	.458

-180-
COMBINED RESULTS OF AVERAGE LONGITUDINAL STRESSES (KSI) AND EFFECTIVENESS

L/B	1.1	1.3	1.3	1.3	1.3	1.3	1.5
B/D	3.0	2.5	3.0	3.0	3.0	3.5	3.0
A _f /A _W	3.5	3.0	2.5	3.0	3.5	3.0	3.0
Plane Y= ^L /8 Avg Stress	.820	.602	•737	•777	.807	.835	.716
ρl	<mark>.</mark> 428	.447	.475	.472	.469	.446	.470
ρ2	.441	.470	.480	.477	.476	.471	.503
Plane Y=3L/8 Avg Stress	.958	.647	.901	•930	•957	.99128	.759
ρl	.817	.802	.892	.878	.880	.640	.808
ρ2	29.257	-1.26	7.539	9.263	14.128	-2.195	-1.605
Plane Y= ^{5L} /8 Avg Stress	1.679	1.511	1.560	1.624	1.676	1.992	1.77
ρ _l	.616	.573	.669	.670	.669	.586	.614
ρ2	.850	•999	.984	.978	.963	1.003	1.099
Plane Y= ^{7L} /8 Avg Stress	1.538	1.289	1.391	1.468	1.543	1.726	1.537
ρl	.393	.437	.432	.434	.436	.431	.468
P2	.451	.525	.519	.511	.502	.532	.588

TABLE C-XIV

	Top Pla	te (Right)	Top Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		SYMM. BEND. MOM.		
Extrapolated Left y=L/8	1.170	1.149	1.190	1.1465
Extrapolated Right y=L/8	1.026	1.0504	1.1246	1.033
Average Stress	.3746	.3780	.3946	.343
ρl	.3203	.329	.332	.2988
Р ₂	.3652	.360	.351	.3316
Extrapolated Left y=3L/8	.6742	.6824	.6770	.6959
Extrapolated Right y-3L/8	.68205	.7331	.7752	.7511
Average Stress	.4192	.4345	.4453	.3791
ρl	.6146	.5927	.5745	.5047
P2	.6217	.6368	.6581	.545

	Bottom Pla	ate (Right)	Bottom Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		SYMM. BEND. MOM.		
Extrapolated Left y=L/8	-1.750	-1.774	-1.7201	-1.756
Extrapolated Right y=L/8	-1.548	-1.497	-1.407	-1.421
Average Stress	484	5611	5401	4989
ρl	.277	.316	.314	.284
ρ2	.313	.375	.384	.351
Extrapolated Left y=3L/8	-1.0866	-1.035	-1.0424	-1.051
Extrapolated Right y=3L/8	-1.1298	973	9219	946
Average Stress	5108	6280	6133	535
ρl	.4521	.6065	.588	.509
6.5	.4701	.6452	.665	.565

	Top Plate (Right)		Top Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		ANTISYMM. B.M. + SH	EAR	
Extrapolated Left y=L/8	4640	4496	4714	3962
Extrapolated Right y=L/8	4006	4028	4408	342
Average Stress	1389	142	1461	 140
ρl	.2994	.316	.310	.353
ρ ₂	.347	.353	.332	.408
Extrapolated Left y=3L/8	6694	5841	6843	665
Extrapolated Right y=3L/8	6132	5571	6759	622
Average Stress	1925	2122	2036	179
ρl	.288	.363	.297	.269
ρ2	.314	.381	.301	.288

	Bottom Pl	late (Right)	Bottom Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		ANTISYMM. B.M. + SH	EAR	
Extrapolated Left y=L/8	.699	.6939	.6782	.6864
Extrapolated Right y=L/8	.664	.5731	.5462	.543
Average Stress	.1881	.203	.1961	.181
ρ _l	.269	.292	.289	.2635
⁰ 2	.283	• 354	.359	.333
Extrapolated Left y=3L/8	1.0068	•9375	.9308	.928
Extrapolated Right y=3L/8	1.076	.839	.8182	.7898
Average Stress	.274	.268	.264	.2410
ρl	.255	.286	.283	.2598
ρ ₂	.272	.320	.322	.3052

	Top Plate (Right)		Top Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		COMBINED LOAD		
Extrapolated Left y=L/8	.7055	.6888	.7181	.6865
Extrapolated Right y=L/8	.6251	.6381	.6837	.6271
Average Stress	.2357	.238	.2484	.2152
ρ _l	.3341	.346	.346	.313
ρ ₂	.377	.372	.363	. 343
Extrapolated Left y=3L/8	.00321	.015	00764	.029
Extrapolated Right y=3L/8	.06846	.093	.0993	.128
Average Stress	.2264	.2388	.2418	.1995
ρ _l	.706	.719	.712	.767
р 2				

	Bottom Plate (Right)		Bottom Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		COMBINED LOAD		
Extrapolated Left y=L/8	-1.0515	-1.080	-1.0419	-1.0686
Extrapolated Right y=L/8	8839	924	8612	8779
Average Stress	2962	359	3439	3182
ρl	.282	332	.330	.298
Р ₂	.335	.388	.399	.362
Extrapolated Left y=3L/8	8842	0971	1128	121
Extrapolated Right y=3L/8	8583	133	105	155
Average Stress	239	360	349	293
ρl	.745	.745	.744	.79
P2	419 AUX 100 Aux			100 000 000 000

	Top Pla	te (Right)	Top Plate (Left)		
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure	
		COMBINED LOAD			
Extrapolated Left y=5L/8	1.634	1.350	1.662	1.362	
Extrapolated Right y=5L/8	1.426	1.374	1.565	1.373	
Average Stress	.514	.631	.541	.5585	
ρ _l	.314	.4589	.3255	.4068	
P2	.360	.467	.3455	.4099	
Extrapolated Left y=7L/8	1.3419	1.609	1.361	1.6058	
Extrapolated Right y=7L/8	1.295	1.464	1.451	1.439	
Average Stress	.6113	.518	.6489	.4701	
٥l	.4556	.322	.447	.293	
P2	.4721	.3541	.477	. 327	

	Bottom Plate (Right)		Bottom Plate (Left)	
	Forward Cross Structure	After Cross Structure	Forward Cross Structure	After Cross Structure
		COMBINED LOAD		
Extrapolated Left y=5L/8	-2.4489	-1.973	-2.398	-1.979
Extrapolated Right y=5L/8	-2.211	-1.813	-1.954	-1.74
Average Stress	6724	 896	7362	776
ρl	.2746	.454	.307	.391
ρ2	.304	.494	.377	.447
Extrapolated Left y=7L/8	-2.0933	-2.47	-1.974	-2.442
Extrapolated Right y=7L/8	-2.206	-2.072	-1.741	-1.9647
Average Stress	785	764	8767	6797
ρ _l	.356	.3096	• 4 4 4	.278
ρ2	.375	.369	.503	.346

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