THE PULSE AMPLIFIER

in

THEORY AND EXPERIMENT

by

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Introduction

In the field of radioactivity and nuclear physics, most processes and reactions are studied by means of ionizing particles accompanying them. These particles are, in general, photons, electrons, or atomic nuclei, and they may be detected by a variety of methods. The pulse amplifier or as it is sometimes called, the linear amplifier, is an instrument for the detection of particles which produce relatively large ionization such as atomic nuclei, and this investigationis an attempt to improve it through a detailed study of its operation.

The two most commonly observed atomic nuclei that produce a density of ionization large enough to be detected with this instrument are the nuclei of hydrogen and helium, usually called protons and alpha particles respectively. Such particles, with sufficient energy, ionize the gas through which they pass, and these gas ions are swept to the electrodes maintained at fixed potentials. The passage of these ions to the electrodes constitute a minute electrical current which is amplified through the use of several ordinary vacuum tubes. The pulse of ionization due to an alpha particle is roughly four times as alrge as that due to a proton, so that these may be distinguished if care is taken in the experiment. Furthermore, the above two particles may easily be distinguished from photons or electrons passing through the gas since the ionization pulses due to these particles is of the order of one one-hundredth that of the proton or alpha particle. Thus, alpha particles and protons may be detected in the presence of considerable photon or electron activity.

Historical

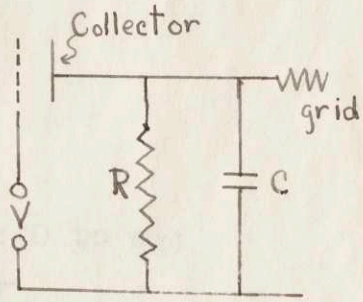
The first type of recording instrument for this purpose was the electrometer. Due to its inherent period, it is in general difficult to record particles which pass at intervals of less than, say, five seconds. In many experiments this is a distinct disadvantage especially when strong sources of radiation are available giving thousands of particles per hour. When such great numbers of particles are incident and it is necessary to actuate an oscillograph or a counting device with a low resolving time, a particle must be made to give a signal of such power that one of these instruments will record it. If an ionization chamber of the type with simple collecting electrodes is used, no such power is available. Greinacher[1] conceived that the voltage pulse formed by the collection of the ions in the chamber could be impressed upon the grid of a vacuum tube and the voltage amplification of the tube used to operate a less sensitive instrument than the usual electrometer. Several years later Ortner and Stetter[2] constructed a multistage amplifier and presented curves showing the effects of transformer and condenser coupling without, however, stating the manner in which these results were obtained. Wynn- Williams and Ward[3] and M.C.Henderson[4] were the first to construct amplifiers of practical design. The best designed amplifier as well as the most satisfactory discussion of the problem is presented by Dunning[5].

The Problem

As it presents itself today, the limit of observation by means of the linear amplifier is determined by the necessity of using a resistance to impress the voltage across the ionization chamber upon the grid of a vacuum tube. This resistance may be an external one or as is more usual in the case of the linear amplifier it is the internal grid resistance of the tube. J.B.Johnson[6] discovered that after all external sources of inteference were removed, there remained a "noise" in a high gain amplifier which could be attributed only to the input resistance of the first tube. As might be inferred, the "noise" is due to a random fluctuation of voltage with a specific frequency spectrum dependent on the input impedance of the amplifier. Theoretical considerations[7] showed that this "noise" may be attributed to the random thermal agitation of the electrons in the resistance. Hence the input circuit itself generates a signal and any voltage change to be observed must be greater than the random input signal voltage. With a view to reduce the relative value of this random input "noise", an investigation of the fundamental properties of the amplifier and ionization chamber is presented.

The Pulse

In the simple Wynn-Williams type of chamber there is a high collecting voltage impressed across the plates of a parallel plate condenser which serves to collect the ions and the resulting impulse of voltage is impressed upon the grid element of a vacuum tube as shown in the diagram. The effective resistance to ground is R and the effective capacity to ground of the grid, chamber and leads is C . The collection of the ions is assumed to be equivalent to the passage of a current through the capacitor i = Q/T where Q is the total collected charge and T is the time of collection. This assumption of the passage of a uniform current is not wholly justified for several reasons. First, the original column of ions is not of uniform density for a particle ionizes according to the familiar Bragg ionization curve for a single particle; second, the positive and negative ions formed are not of the same mobility so that there are strictly speaking two ion columns of non-uniform density (ions per unit length) which are collected at different rates; third, there is the point of recombination since the ions last collected have a greater chance of recombining. Since it is easier to handle analitically, it will be assumed that the ion



column is of uniform density, speed, and collection probability. Bearing these conditions in mind, the following is carried out in the usual manner . If e_g be the grid voltage at time t, then;

$$i = Q/T = e_g/R + C\,de_g/dt$$
$$dt/C = de_g/(Q/T - e_g/R)$$

integrating from time 0 to t and voltage 0 to e_g;

$$e_g = \frac{RQ}{T}\left(1 - e^{-\frac{t}{RC}}\right) \qquad 0 \leq t \leq T.$$

At time t = T there is a discontinuity in the current for all the ions are collected and i drops immediately to zero and from the above relation the grid voltage is

$$e_g = \frac{RQ}{T}\left(1 - e^{-\frac{T}{RC}}\right). \qquad (1)$$

Since the current is zero the differential equation becomes:

$$-\frac{e_g}{R} = C\frac{de_g}{dt}$$

and integrating this from time T to time t and the voltage from $\frac{RQ}{T}\left(1 - e^{-T/RC}\right)$ to e_g:

$$e_g = \frac{RQ}{T}\left(e^{\frac{T}{RC}} - 1\right)e^{-t/RC} \qquad t \geq T \qquad (2)$$

Now the assumed form of the pulse is known explicitly and it is only necessary to determine a frequency distribution. To employ a Fourier series has the disadvantage that a definite period must be assumed, however the Fourier integral requires no period and is also an exact solution hence the analysis will be in the form of a Fourier integral. The conditions of the problem state that the voltage on the grid is zero up to zero time and from then on the voltage is described by the above expressions. For such an event it it is necessary to use both parts of the integral for the specific condition is that the function is zero at zero time. In some problems one part may be neglected for the time axis may be arbitrarily shifted. The form to be used is:

$$\phi(t) = \int_{-\infty}^{\infty} \left[A(\omega)\cos\omega t + B(\omega)\sin\omega t \right] d\omega \qquad (3)$$

$$B(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \phi(t)\sin\omega t \, dt \qquad (4)$$

$$A(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \phi(t)\cos\omega t \, dt \, . \qquad (5)$$

$\phi(t)$ is explicitly e_g. Substituting the expressions for e_g (1),(2) into the integral to determine $A(\omega)$, it is found that:

$$A(\omega) = \frac{RQ}{\pi T}\left\{ \int_{-\infty}^{0} 0 \times \sin\omega t \, dt + \int_0^T \cos\omega t \, dt - \int_0^T \cos\omega t \, e^{-t/RC} \, dt + \int_T^{\infty} \left(e^{\frac{T}{RC}} - 1\right) \cos\omega t \, e^{-t/RC} \, dt \right\}$$

$$= \frac{RQ}{\pi T}\left\{ \left[\frac{\sin\omega t}{\omega} - \frac{e^{-t/RC}}{\omega^2 + \frac{1}{R^2C^2}}\left(-\frac{1}{RC}\cos\omega t + \omega\sin\omega t\right)\right]_0^T + \left(e^{T/RC} - 1\right)\left[\frac{e^{-t/RC}}{\omega^2 + 1/R^2C^2}\left(-\frac{1}{RC}\cos\omega t + \omega\sin\omega t\right)\right]_T^{\infty} \right\}$$

$$= \frac{RQ}{\pi T}\left\{ \left[\frac{\sin\omega t}{\omega} - \frac{e^{-T/RC}}{\omega^2 + 1/R^2C^2}\left(-\frac{1}{RC}\cos\omega T + \omega\sin\omega T - \frac{1}{RC}\right)\right] - \left(e^{\frac{T}{RC}} - 1\right)\left[\frac{e^{-T/RC}}{\omega^2 + 1/R^2C^2}\left(-\frac{1}{RC}\cos\omega T + \omega\sin\omega T\right)\right] \right\}$$

$$= \frac{RQ}{\pi T}\left\{ \frac{\sin\omega T}{\omega} - \frac{R^2C^2\left(-\frac{1}{RC}\cos\omega T + \omega\sin\omega T\right)}{1 + R^2C^2\omega^2} + \frac{1}{\omega^2 + \frac{1}{R^2C^2}} \right\}$$

$$A(\omega) = \frac{RQ}{\pi T}\left\{ \frac{\sin\omega T}{\omega} - \frac{1}{\omega^2 + \frac{1}{R^2C^2}}\left(\omega\sin\omega T + \frac{1}{RC}\left(1 - \cos\omega T\right)\right) \right\} \qquad (6)$$

In a similar manner B(ω) is found. $\phi(t)$ is the same as are the limits of integration. Hence:

$$B(\omega) = \frac{RQ}{\pi T}\left\{ \int_0^T \left(\sin\omega t - \sin\omega t\, e^{-t/RC}\right) dt + \int_T^\infty \left(e^{T/RC} - 1\right)\sin\omega t\, e^{-t/RC}\, dt \right\}$$

$$= \frac{RQ}{\pi T}\left\{ \left[-\frac{\cos\omega t}{\omega} - \frac{e^{-t/RC}}{\omega^2 + 1/R^2C^2}\left(-\frac{1}{RC}\sin\omega t - \omega\cos\omega t\right)\right]_0^T \right.$$

$$\left. + \left[\left(e^{\frac{T}{RC}} - 1\right)\frac{e^{-t/RC}}{\omega^2 + \frac{1}{R^2C^2}}\left(-\frac{1}{RC}\sin\omega t - \omega\cos\omega t\right)\right]_T^\infty \right\}$$

$$= \frac{RQ}{\pi T}\left\{ \frac{1-\cos\omega t}{\omega} + \frac{e^{-T/RC}}{\omega^2 + 1/R^2C^2}\left(\sin\omega T/_{RC} + \omega\cos\omega T\right) - \frac{\omega}{\omega^2 + 1/R^2C^2} \right.$$

$$\left. + \left[e^{T/RC} - 1\right]\frac{e^{T/RC}}{\omega^2 + 1/R^2C^2}\left(\frac{\sin\omega t}{RC} + \omega\cos\omega t\right) \right\}$$

$$B(\omega) = \frac{RQ}{\pi T}\left\{ \frac{1-\cos\omega T}{\omega} + \frac{1/_{RC}\sin\omega T + \omega\cos\omega T - \omega}{\omega^2 + 1/R^2C^2} \right\}. \qquad (7)$$

The substitution of these Fourier coeficients into the expression for $\phi(t)$ (3) would give the input pulse, but this is not of greatest interest for all that is seen is the output of the amplifier. This means that the coefficients must be weighted by a function whichgives the relative respon se of the amplifier as a function of the frequency. Let this function be Y(ω). The output voltage is therefore

$$\phi_o(t) = \int_{-\infty}^{\infty} Y(\omega)\left[A(\omega)\cos\omega t + B(\omega)\sin\omega t\right] d\omega. \qquad (8)$$

To integrate this directly would be, in general, extremely difficult. Fortunately, the time t at which we are interested in the pulse is the time at which it is a maximum. An examination of the first form of $\phi(t)$ i.e. e_g shows that it has a definite maximum at $t = T$. However this may not be true of $\phi_0(t)$ but by plotting the derivitive of the input pulse with respect to frequency for many values of RC, it was found to be a maximum for almost all values of ω at time $t = T$ and hence its integral $\phi(t)$ is also a maximum. Further the final working range of ω selected was found to lie in the region where $\frac{d\phi(t)}{d\omega}$ was a maximum at $t = T$. This is demonstrated in figure (1). Since this is self consistent it will be assumed a correct solution.

Input resistance "noise"

The Johnson effect mentioned before may be stated explicitly in the following manner:

$$\overline{\phi_R^2} = 4kT_0 \int_0^{\infty} R(\omega) \, |Y(\omega)|^2 \, d\omega. \qquad (9)$$

$\overline{\phi_R^2}$ is the average value of the square of the "noise" voltage across the terminals of the resistance R. whose real component of impedance is $R(\omega)$. k is Boltzman's constant and T_0 is the Kelvin temperature, $Y(\omega)$ is the amplifier function as before. In the case under consideration

$$R(\omega) = \frac{R}{1 + R^2C^2\omega^2}. \qquad (10)$$

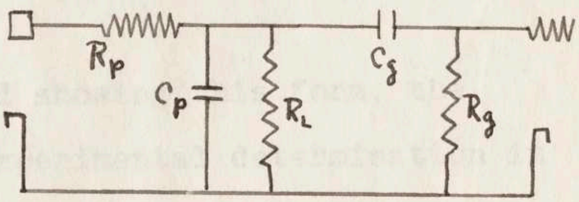
Ratio of Input Signal to Input "Noise" at the Output of the Amplifier

Let the ratio of the input signal to the input "noise" be defined as $\rho \equiv \frac{\phi_0(t)}{\sqrt{\overline{\phi_R^2}}}$. Since only the voltage at time t = T is to be considered

$$\rho^2 \equiv \frac{\phi_0^2(T)}{\overline{\phi_R^2}} = \frac{\left[\int_0^\infty Y(\omega)\left[A(\omega)\cos\omega T + B(\omega)\sin\omega T\right]d\omega\right]^2}{4kT_0\int_0^\infty R(\omega)\,Y^2(\omega)\,d\omega}. \quad (11)$$

The Amplifier Function $Y(\omega)$

In the construction of a linear amplifier it has been customary to use interstage coupling of the resistance-capacity type rather than transformer coupling as Ortner and Stetter have shown that the latter introduces a relaxation pulse. In the following analysis only the resistance capacity case will be considered. Schematically the interstage coupling is as shown in the figure. R_p is the internal plate resistance, C_p is the interelectrode plate capacity plus any external capacity, R_l is the plate load reistor, C_g is the coupling capacity and R_g is the grid resistance. The grid capacity is neglected as



Note: I am indedted to Mr.CM Neitzert for the reistance-capacitance network analysis for the interstage coupling.

it is small enough to have an impedance much larger than R_g. If there is a voltage generated at the plate E_p of frequency ω then the ratio of the transmitted grid voltage E_g to E_p is given by:

$$\frac{E_g}{E_p} = \frac{R_g C_g \omega}{\sqrt{1 + R_o^2 C_p^2 \omega^2}\sqrt{1 + R_g^2 C_g^2 \omega^2}} \qquad (12)$$

where $R_o = R_p R_1 / (R_p + R_1)$. The amplifier function is determined by the number of stages since each stage will pass only a certain fraction of the impressed voltage. If the Capacities and resistances for each network is known the complete function is the product of the separate ones.

It is readily seen that to use the above expression as $Y(\omega)$, even in the simple case of one stage, will be difficult. By trial it was found that $Y(\omega)$ could be reasonably described by the general expression

$$Y(\omega) = \frac{\omega^n}{\omega_o^n} e^{-\alpha\omega} \qquad (13)$$

Comparitive graphs are presented showing this form, the exact expression (12), and an experimental determination in figure (2).

The Determination of ρ

By combining expressions (3), (6), and (7) the value of the input pulse at time T is found to be

$$\phi(t) = \frac{RQ}{\pi T}\int_0^\infty \left[\frac{\sin\omega T}{\omega} + \frac{(1-\cos\omega T)\frac{1}{RC} - \omega\sin\omega T}{\omega^2 + 1/R^2C^2}\right] d\omega. \quad (14)$$

Suppose the amplifier response $Y(\omega)$ is of the form

$$Y(\omega) = \frac{\omega^2 + 1/R^2C^2}{\omega_0^2} e^{-\alpha\omega}, \quad (15)$$

then the amplifier output at time T is

$$\phi_0(T) = \frac{RQ}{\pi T}\int_0^\infty \left(\frac{\sin\omega T}{\omega} + \frac{(1-\cos\omega T)\frac{1}{RC} - \omega\sin\omega T}{(\omega^2 + 1/R^2C^2)}\right)\frac{(\omega^2 + 1/R^2C^2)e^{-\alpha\omega}}{\omega_0^2} d\omega$$

$$= \frac{RQ}{\pi T\omega_0^2}\int_0^\infty \left(\omega\sin\omega T + \frac{\sin\omega T}{\omega R^2C^2} + (1-\cos\omega T)\frac{1}{RC} - \omega\sin\omega T\right)e^{-\alpha\omega} d\omega$$

$$= \frac{RQ}{\pi T\omega_0^2}\int_0^\infty \left[\frac{\sin\omega T}{\omega R^2C^2} + (1-\cos\omega T)\frac{1}{RC}\right]e^{-\alpha\omega} d\omega. \quad (16)$$

Let $m(T)$ be defined in the following manner:

$$m(T) = \int_0^\infty \frac{\sin\omega T}{\omega} e^{-\alpha\omega} d\omega \quad (17)$$

$$m'(T) = \int_0^\infty \cos\omega T\, e^{-\alpha\omega} d\omega = \frac{\alpha}{\alpha^2 + T^2}$$

from which it is seen ~~that~~

$$m(T) = \tan^{-1}\frac{T}{\alpha} + \text{constant}. \quad (18)$$

In integral (17) when T is zero $m(T)$ is also zero and in (18) when T is zero $\tan^{-1}\frac{T}{\alpha}$ is zero so ~~that~~ the constant in (18) must be zero and (18) becomes

$$m(T) = \tan^{-1}\frac{T}{\alpha}.$$

The integration of (16) gives

$$q_0(T) = \frac{Q}{\pi C T \omega_0^2}\left[\frac{1}{RC}\tan^{-1}\frac{T}{\alpha} + \frac{T^2}{\alpha(\alpha^2+T^2)}\right]. \qquad (19)$$

which is the output signal with the amplifier function given by (15).

The output resistance noise is obtained by substituting (15) into (9) giving

$$\overline{q_R^2} = 4kT_0\int_0^\infty \frac{R\,d\omega}{1+R^2C^2\omega^2}\,\frac{\left(\omega^2+\frac{1}{R^2C^2}\right)^2}{\omega_0^4}\,e^{-2\alpha\omega}. \qquad (20)$$

Suppose ~~that~~ $R^2C^2\omega^2 >> 1$ in the region of maximum output and that as $RC\omega \to 1$ the integrand is negligible relative to its value in the region where $R^2C^2\omega^2 >> 1$, then

(20) becomes

$$\bar{\varphi}_R^2 = \frac{4kT_0}{\omega_0^4 RC^2}\int_0^\infty \omega^2 e^{-2\alpha\omega}\, d\omega$$

$$= \frac{4kT_0}{\omega_0^4 RC^2}\,\frac{1}{4\alpha^3}\,. \tag{21}$$

Forming the ratio of signal to noise, ρ (11) is

$$\rho^2 = \frac{4\alpha^3 Q^2 R}{\pi^2 T^2\, 4kT_0}\left[\frac{1}{RC}\tan^{-1}\frac{T}{\alpha} + \frac{T^2}{\alpha(\alpha^2+T^2)}\right]^2.$$

Define the parameters u and m by

$\alpha \equiv uT$ and $RC \equiv mT$, then

$$\rho^2 = \frac{4Q^2}{\pi^2\, 4kT_0 C}\cdot u^3 m\left[\frac{1}{m}\tan^{-1}\frac{1}{u} + \frac{1}{u(u^2+1)}\right]^2.$$

T is always much less than unity so that m must be $\gg 1$ to make valid the assumption $R^2C^2\omega^2 \gg 1$. If such be the case, then

$$\rho^2 = \frac{Q^2}{\pi^2\, 4kT_0}\cdot\frac{R}{\pi}\,\frac{4u}{(u^2+1)^2}\,. \tag{22}$$

The fraction $\frac{u}{(u^2+1)^2}$ has a maximum at $u = \frac{1}{3}\sqrt{3}$ and the fraction is equal to $\frac{3}{16}\sqrt{3}$. The relation between α and T at this value of u is simply $\alpha = \frac{1}{3}\sqrt{3}\, T = .577 T$. In equation (22), $Q^* = Ne = N \times 1.59 \times 10^{-14}$ coulombs, where N is the number of ion pairs collected; $4kT_0 = 1.64\ 10^{-20}$. Suppose the collection time is $T = 2 \times 10^{-4}$ seconds and the input resistance $R = 10^{10}$ ohms, then (22) reduces to:

$$\rho^2 = 1.02\ 10^{-5} N^2.$$

To be observable, a pulse should be about twice the background voltage so the minimum observable number of ions under these conditions is about 1000.

Under the amplifier assumptions (15) becomes (since $RC\omega \gg 1$):

$$Y(\omega) = \frac{\omega^2}{\omega_0^2} e^{-\alpha\omega}. \qquad (23)$$

This function has a maximum value when $\omega = 2/\alpha$ and from the relation between α and T, it is seen the frequency of maximum response is $1.73\ 10^4$. In cycles per second the value of the peak of response is $f = \frac{\omega}{2\pi} =$ 2750 c.p.s.

The Determination of ρ Continued

The following functions of $Y(\omega)$ will be demonstrated briefly and they will be so normalized that the relative merits of the functions may easily be determined:

$\frac{\omega}{\omega_0} e^{-\beta\omega}$; $\frac{\omega^3}{\omega_0^3} e^{-\gamma\omega}$; and $\frac{\omega^n}{\omega_0^n} e^{-\varepsilon\omega}$.

It is assumed that $R^2C^2\omega^2 << 1$ so that (14) is

$$\phi_a(T) = \frac{RQ}{\pi T}\int_0^\infty \frac{1-\cos\omega T}{RC\omega^2}\,d\omega \qquad (24)$$

A) $\frac{\omega}{\omega_0} e^{-\beta\omega}$

$$\phi_o(T) = \frac{RQ}{\pi T\omega_0}\int_0^\infty \frac{1-\cos\omega T}{RC\omega} e^{-\beta\omega}\,d\omega$$

$$= \frac{Q}{\pi T C\omega_0}\,\frac{1}{2}\ln\frac{T^2+\beta^2}{\beta^2}$$

The noise voltage is given by (9)

$$\overline{\phi_R^2} = \frac{4kT_0}{RC^2\omega_0^2}\int_0^\infty e^{-2\beta\omega}\,d\omega$$

$$\overline{\phi_R^2} = \frac{4kT_0}{RC^2 w_0^2} \cdot \frac{1}{2\beta}$$

The signal to "noise" ratio is from (11)

$$\rho^2 = \frac{RQ^2}{\pi^2 T^2 4kT_0} \quad \frac{\beta}{2}\left[\ln \frac{T^2+\beta^2}{\beta^2}\right]^2.$$

allowing $\beta \equiv bT$,

$$\rho^2 = \frac{RQ^2}{\pi^2 T\, 4kT_0} \quad \frac{b}{2}\left[\ln \frac{1+b^2}{b^2}\right]^2.$$

The maximum value of the fraction $\frac{b}{2}\left[\ln \frac{1+b^2}{b^2}\right]^2$ is at the point $b \sim .17$ giving it the value of 1.08 which makes ρ:

$$\rho^2 = \frac{RQ^2}{\pi^2 T\, 4kT_0} \cdot 1.08 \qquad (25)$$

B) $\frac{w^3}{w_0^3} e^{-rw}$

Substituting directly into (24)

$$\phi_0(T) = \frac{Q}{\pi T C w_0^3} \int_0^\infty (1-\cos wT)\, w e^{-rw}\, dw$$

$$= \frac{Q}{\pi T C w_0^3} \left\{ \frac{1}{r^2} - \frac{r^2 - T^2}{(r^2+T^2)^2} \right\}$$

The "noise" voltage is obtained from (9')

$$\overline{\phi_R^2} = \frac{4kT_0}{\pi(RC^2\omega_0^6)}\int_0^\infty \omega^4 e^{-2r\omega}\,d\omega$$

$$= \frac{4kT_0}{\pi(RC^2\omega_0^6)}\cdot\frac{3}{4r^5}.$$

And the signal to "noise" ratio is (11)

$$\rho^2 = \frac{RQ^2}{\pi^2T^2\,4kT_0}\;\frac{4r^5}{3}\left\{\frac{1}{r^2}-\frac{r^2-T^2}{(r^2+T^2)^2}\right\}^2.$$

If the usual substitution $r = gT$ is made,

$$\rho^2 = \frac{RQ^2}{\pi^2T\,4kT_0}\cdot\frac{4}{3}g^5\left\{\frac{1}{g^5}-\frac{g^2-1}{(g^2+1)^2}\right\}^2,$$

This has a maximum at $g = 1$ giving the value of

$$\rho^2 = \frac{RQ^2}{\pi^2T\,4kT_0}\times 1.33 \qquad (26)$$

c) $\frac{\omega^n}{\omega_0^n}e^{-c\omega}$

In the usual manner

$$\phi_0(T) = \frac{Q}{\pi T \omega_0^n} \int_0^\infty (1-\cos\omega T)\,\omega^{n-2} e^{-\varepsilon\omega}\, d\omega$$

$$= \frac{Q}{\pi T \omega_0^n}\, \Gamma(n-1)\left[\frac{1}{\varepsilon^{n-1}} - \frac{\cos\left[(n-1)\tan^{-1}\frac{T}{\varepsilon}\right]}{(\varepsilon^2+T^2)^{\frac{n-1}{2}}}\right]$$

Also the "noise" appears as

$$\overline{\phi_R^2} = \frac{4kT_0}{RC^2\omega_0^{2n}} \int_0^\infty \omega^{2n-2} e^{-2\varepsilon\omega}\, d\omega$$

$$= \frac{4kT_0}{RC^2\omega_0^{2n}}\, \frac{\Gamma(2n-1)}{(2\varepsilon)^{2n-1}}.$$

The merit ratio ρ is, where $\varepsilon = rT$

$$\rho^2 = \frac{RQ^2}{\pi^2 T\, 4kT_0}\, \frac{(2r)^{2n-1}\left[\Gamma(n-1)\right]^2 \left\{\frac{1}{r^{n-1}} - \frac{\cos\left[(n-1)\tan^{-1}\frac{1}{r}\right]}{(r^2+1)^{\frac{n-1}{2}}}\right\}^2}{\Gamma(2n-1)}.$$

It is now seen that a variation in the characteristics of the amplifier does not seriously affect the limit of observation as determined by ρ . This is expressed by $\rho^2_{(24)} : \rho^2_{(22)} : \rho^2_{(25)}$ :: 1.08 : 1;33 : 1;30. An inspection of expression (22) shows that the experimenter has at his disposal only the two variables, T and R, which are related to ρ by ρ^2 = constant x R/T. R is controlled by the particular tube chosen and there are no tubes available with a high enough input resistance as well as low plate noise. T is controlled by the collecting space field and the nature of the gas. It may be increased until the breakdown voltage of the gas is reached or by allowing the the particles to enter the chamber parallel to the plates instead of the usual practice of normal incidence.

The Input Resistance R

Apparently there is no simple method described in the literature for the determination of the input resistance R of the floating grid. Since the "noise" component of the resistance is the the factor of interest, it may be determined in the following manner. Usually the grid "noise" resistance is of the order of 10^9 ohms or greater and its floating input capacity is about 6 x 10^{-12} farads. If the 'average' frequency of the amplifier is about ten-thousand then RCω

is the order of 60 and $R^2C^2\omega^2$ is in the region where it is very much greater than one. On the other hand if the grid is grounded through a one meg-ohm resistor, C is about 10^{-11} farads due to the added capacity of the resistance and now $R^2C^2\omega^2$ is in the region where it is much less than one.

If the amplifier output voltage is measured with the megohm resistor in place, equation (9) may be written:

$$\overline{V^2_{R_1}} = 4kT_0R_1\int_0^\infty Y^2(\omega)\,d\omega$$

Removing this resistor and allowing the resistor to the grid to be the internal reistance of the grid then equation (9) becomes:

$$\overline{V^2_{R_2}} = \frac{4kT_0}{R_2C^2}\int_0^\infty Y^2(\omega)\,\frac{d\omega}{\omega^2}$$

Taking the ratio of these two "noise" voltages, it is found that:

$$\frac{\overline{V^2_{R_1}}}{\overline{V^2_{R_2}}} = R_1R_2C^2\,\frac{\int_0^\infty Y^2(\omega)\,d\omega}{\int_0^\infty Y^2(\omega)\,\frac{d\omega}{\omega}}$$

Suppose $Y(\omega)$ is given by $\frac{\omega}{\omega_1}e^{-\alpha\omega}$ then the ratio is:

$$\frac{\overline{V^2_{R_1}}}{\overline{V^2_{R_2}}} = \frac{R_1R_2C^2}{2\alpha^2}$$

or

$$R_2 = \frac{2\alpha^2}{C^2R_1}\cdot\frac{\overline{V^2_{R_1}}}{\overline{V^2_{R_2}}} \qquad (27)$$

Tube "Noise"

Unfortunately there is no extensive study of tube "noises" in the literature. Pearson[8] gives some experimental values for a selected list of tubes; a tube similar to the 38 has less noise at higher audio frequencies than at lower frequencies. This would indicate that the higher frequencies are the best from the standpoint of general tube "noise" which is fortunate since this is the conclusion of the earlier part of this paper.

Pearson[9] has also studied grid shot noises and in general they are small compared to the input resistance noise. Qualitatively, the effect of grid currents on the observational limit may be considered as follows. Suppose the grid current, positive and negative, is of the order of 10^{-11} amperes which corresponds to about 6 x 10^7 electrons per second. In an observational interval of .0002 seconds, about 12,000 electrons will arrive at the grid(strictly, half will be positive ions). The statistical fluctuations in this number of particles will be its square-root ,110. For a pulse of ions to be observed above this variation, it must contain at least several hundred ions.

Selection of the Tubes

Since standard tubes are to be used in the amplifier, the type 38 is probably the best. Johnson and Neitzert [10] have given the optimum operating characteristics of this for operation at low levels of "noise". The conditions given are 300,000 ohms plate load resistance, -1.5 volts grid bias, 6 volts screen potential and 28½ volts plate supply. Under these conditions they reported a plate current of 60 micro-amperes while a dozen 1936 38-tubes gave an average of 80 micro-amperes under the same conditions.

The Circuit

With the assumed network and the 38 tube operating under the above conditions, the circuit design of the amplifier is fairly well defined. The complete circuit is shown in figure (3) and it is to be noted that the last two tubes are in parallel. The reason for this particular arrangement is that two different observations of the pulse may be made at the same time quite independent of each other.

The Collecting Chamber

The first consideration in the design of the chamber is that its capacity should be a minimum. Although the capacity of the chamber does not enter into the calculated value of ρ , a high value of the capacity will contribute "noise" on small pulses for the amplification must be so high that second order noise effects enter.

A second important point is that the background count of the chamber should be as small as possible for this determines the limit of observation in number of particles per unit time. Experience in the operation of a large area ionization chamber(11) has shown that it is desirable to have the chamber lined in such a manner that the surfaces in the chamber which may eject contamination particles may be easily removed and cleaned. The best method of cleaning appears to be a rinse in concentrated nitric acid.

Amplifier Housing

The general principles of amplifier design require complete shielding of each stage from each other and from the outside. This is accomplished by the use of copper interstage shielding throughout. The internal copper shield is insulated from the external aluminum cabinet which gives double shielding. Since the first stage is separated from the main body of the amplifier, it is necessary to transmit energy to the first stage and pulses back. This is accomplished by the use of double shielded high quality cable provided through the kindness of Mr. Stoddard of the Simplex Wire and Cable Company. The total capacity of the plate return lead, which is two meters long, is about 46 micro-micro farads at 1000 c.p.s. The external cable shields are bolted to the cabinet and the internal ones are soldered to the internal copper shield. When the amplifier was constructed it was not know whether or not it was necessary to shield the filament storage battery. To to determine its relative importance the battery terminal is suddenly disconnected and grounded while the amplifier is in operation. Since this does not change the noise level by an observable amount, the heater type tubes having enough thermal inertia to keep up the emission for a short time, it is unnecessary to shield the battery.

The output high voltage condensors for the collecting potential are non-inductive; each leaf is brought out and soldered to its neighbor instead of being coiled and having leads brought out by occasional tabs.

Auxiliary Equipment

At present a cathode ray oscillograph is the only available direct reading instrument. An attempt to measure "noise" voltages by the oscillograph clearly demonstrates the futility of estimating the voltage. It is evident that an indicator giving the RMS of the AC output is essential. The usual method is to couple a transformer and thermal meter to the output tube. Since neither of these are available, another type of recording meter is used. The only available AC meter is a two milliampere rectifier type meter and the following circuit is used coupled with this meter. As shown in figure (4) it is a balanced triode amplifier using a type 53 with two triodes in the same envelope and a 53 twin amplifier inverter. In operation this device proves to be surprisingly linear and its sensitiveness immediately suggestes another use besides the general standardization of the operation of the amplifier with standard resistances as sources of constant AC potential.

At times it is desirable to record alpha rays in an intense beta or gamma ray background and to know the value of this background. Since the latter gives rise to a current through the chamber which is statistical in nature, it is possible to measure the current. The statistical variation in the current which is proportional to the square root of the number of particles is a measurable AC component. The meter reading is proportional to the square root of the number of particles and therefore a resistance is an ideal standard for the meter reading is also proportional to the square root of the resistance.

Experimental Results

It is unfortunate that the shop was so busy that they were unable to complete this apparatus in time to present the present theories to experiment. However the amplifier proper was constructed in time to make a spectrum analysis and a "noise"analysis, the former is shown in a comparative graph (figure (2)) and the latter is summarized as follows. With one meg-ohm on the first grid, atypical reading on the "noise" meter is 1.6, with the grid grounded the reading is .3 and when it is allowed to float it is .6. In the first case with the grid at ground and tha one meg-ohm resistor, the meter reading is proportional to the square root of the grid resistance which indicates a grounded grid "noise " equivalent to about 30,000 ohms. In the second, case with the use of equation (27) and a value of alpha of .000085, the noise due to the first tube with floating grid is equivalent to a resistance of about four thousand meg-ohms.

Conclusions

The results of this present analysis indicate that the amplifier frequency characteristics do not affect the signal to "noise" ratio very critically and that the peak of response of the amplifier is determined by the collection time. The only available method at the control of the experimenter in increasing this ratio, aside from improving the characteristics of the first tube as outlined, appears to be that the collection time should be decreased.

The assistance of many in the Physics department is greatly appreciated. In particular, Messers Shremp and Schiff helped in the analytical work, Mr Robes ably constructed the apparatus, Dr. Gingrich helped edit the thesis, and Professor Evans suggested the problem and criticized it in its developement.

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(8)	Pearson	Bell Lab Rec	14 (Oct 1934)
(10)	Johnson and Neitzert	R.S.I.	5 198 (1934)
(11)	Finney and Evans	Phys Rev	48 503 (1935)
(12)	E.A.Johnson	N.Y. Meet. Amer Phys Soc Feb 1936 abst. 6	

Note: the last paper is covered by an abstract which was published during the course of this work and appears to cover much the same ground, however no details are presented.

GRAPHS, DRAWINGS, and PICTURES

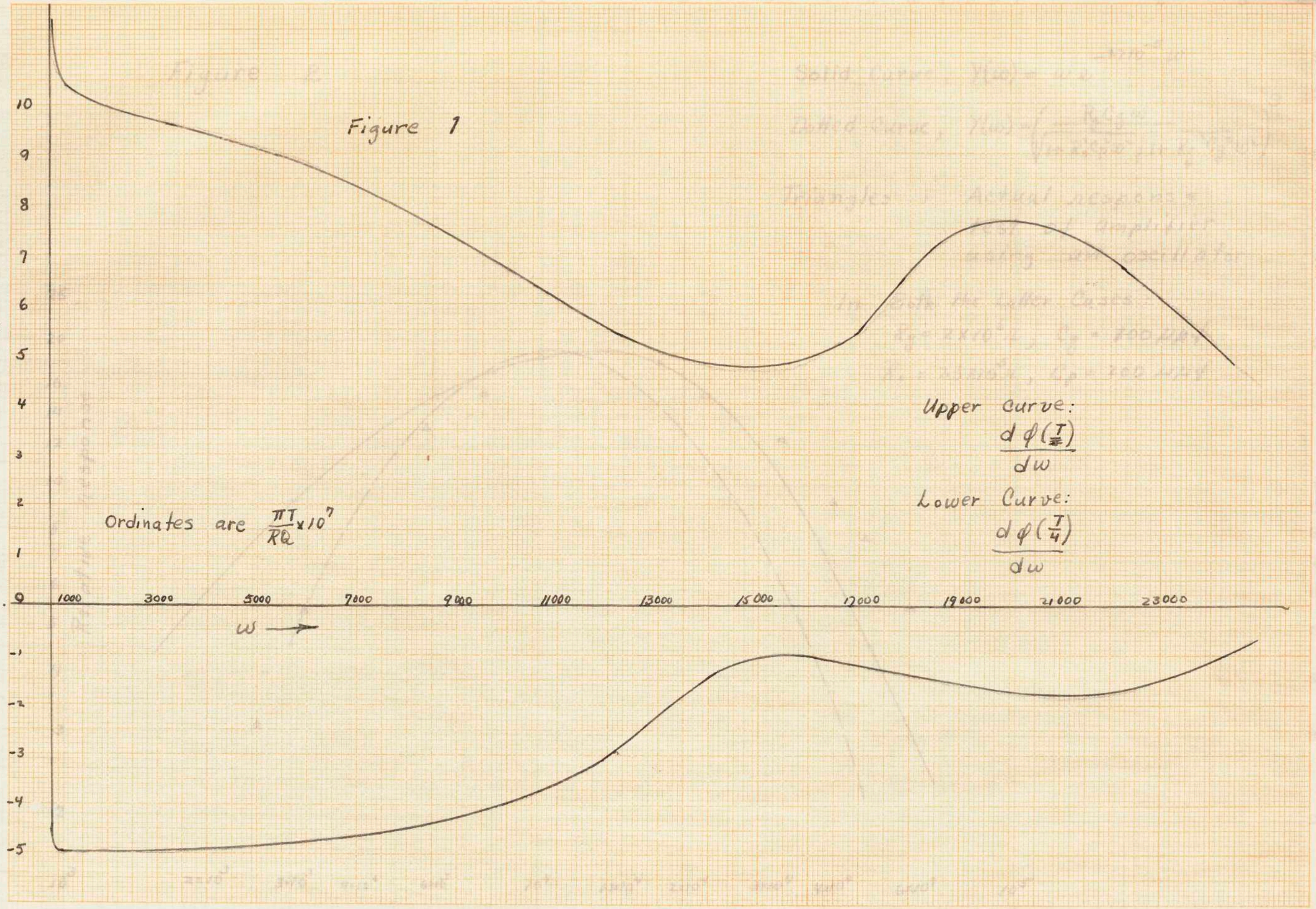
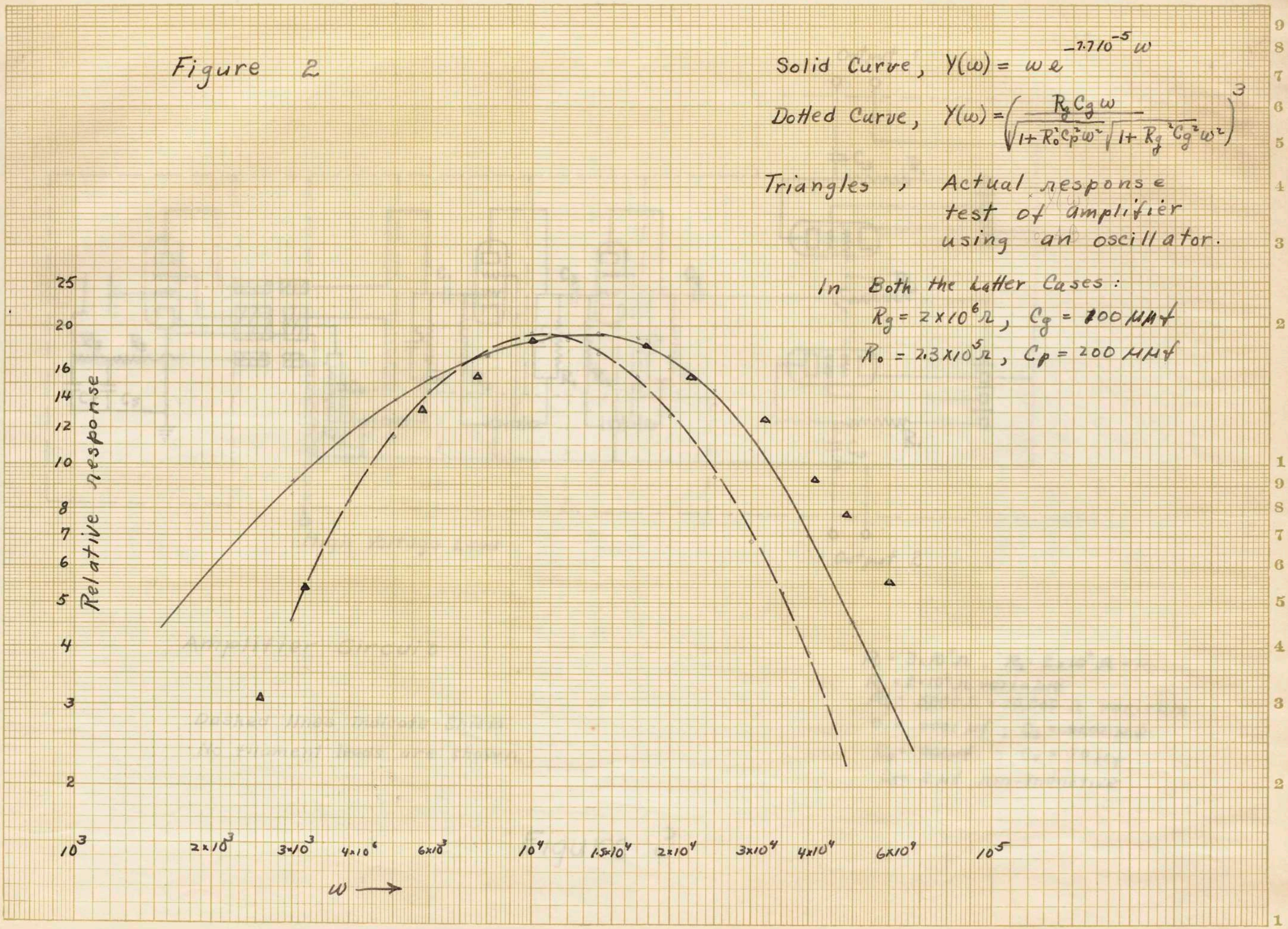
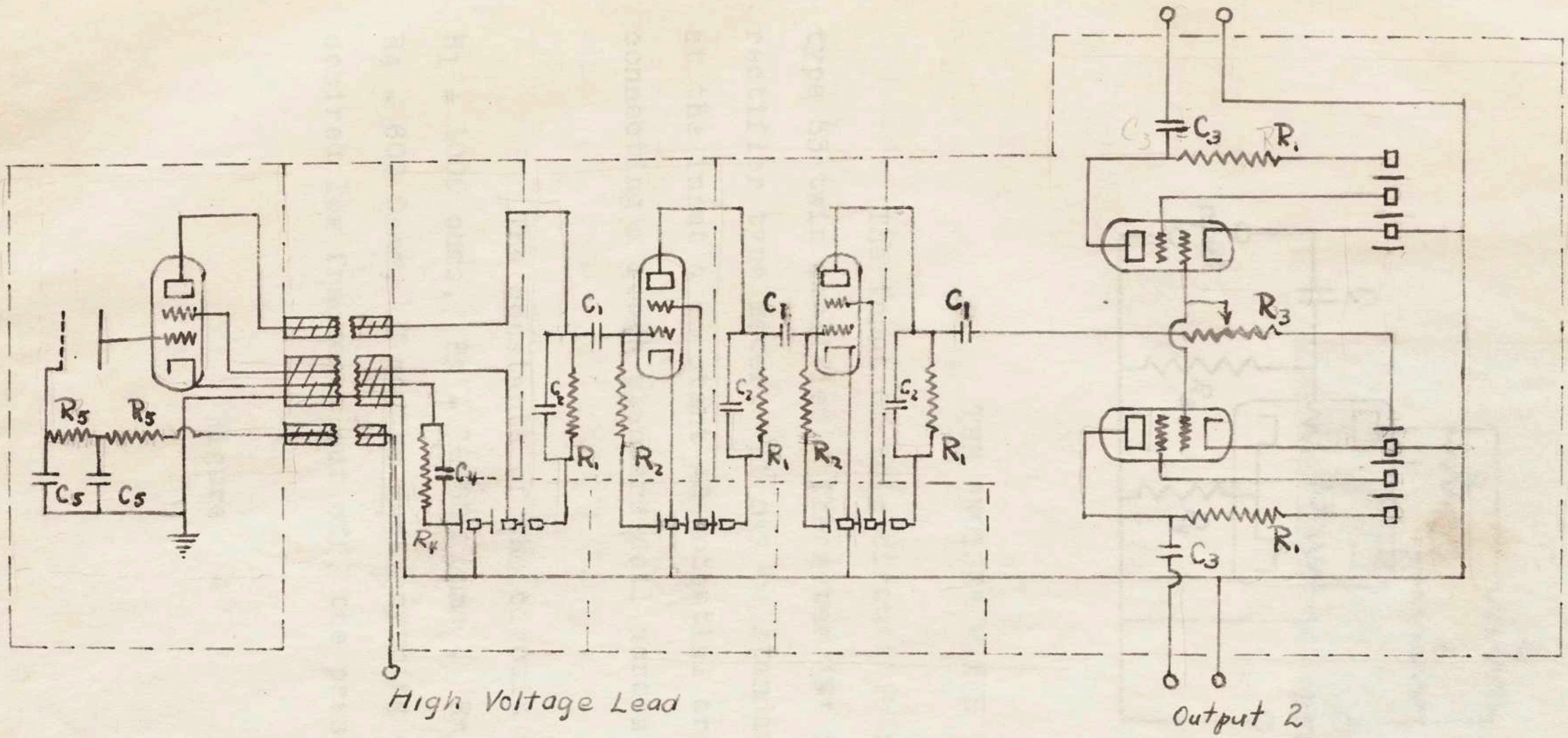


Figure 1
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
1000
3000
5000
7000
9000
11000
13000
15000
17000
19000
21000
23000
ω →
Ordinates are $\frac{\pi T}{RQ} \times 10^7$
Upper Curve:
$\frac{d\phi(\frac{T}{\pi})}{d\omega}$
Lower Curve:
$\frac{d\phi(\frac{T}{4})}{d\omega}$



Solid Curve, $Y(\omega) = \omega e^{-7.7 \cdot 10^{-5} \omega}$
Dotted Curve, $Y(\omega) = \left(\frac{R_g C_g \omega}{\sqrt{1 + R_o^2 C_p^2 \omega^2}\sqrt{1 + R_g^2 C_g^2 \omega^2}} \right)^3$
Triangles, Actual response test of amplifier using an oscillator.
In Both the Latter Cases:
$R_g = 2 \times 10^6 \Omega$, $C_g = 100 \mu\mu f$
$R_o = 2.3 \times 10^5 \Omega$, $C_p = 200 \mu\mu f$
Relative response
25
20
16
14
12
10
8
7
6
5
4
3
2
10^3
2×10^3
3×10^3
4×10^6
6×10^3
10^4
1.5×10^4
2×10^4
3×10^4
4×10^4
6×10^4
10^5
$\omega \longrightarrow$

Figure 2

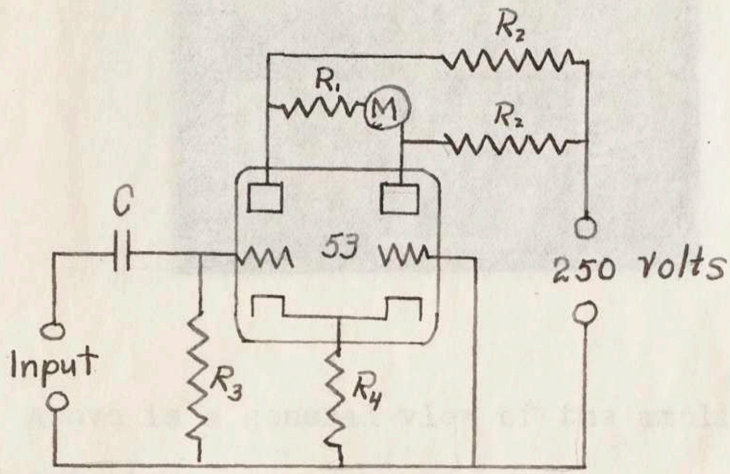


Amplifier Circuit

Dashed lines indicate Shield.
No filament leads are shown.

$R_1 = 3 \times 10^5 \Omega$; $R_2 = 2 \times 10^6 \Omega = R_5$
$R_3 = 2 \times 10^6 \Omega$ variable
$R_4 = 5{,}000 \Omega + 10{,}000 \Omega$ variable
$C_1 = .0001 \mu f.$; $C_2 = .0002 \mu f.$
$C_3 = .001 \mu f.$; $C_4 = 50 \mu f.$
$C_5 = 2 \mu f$ non-inductive

Figure 3.



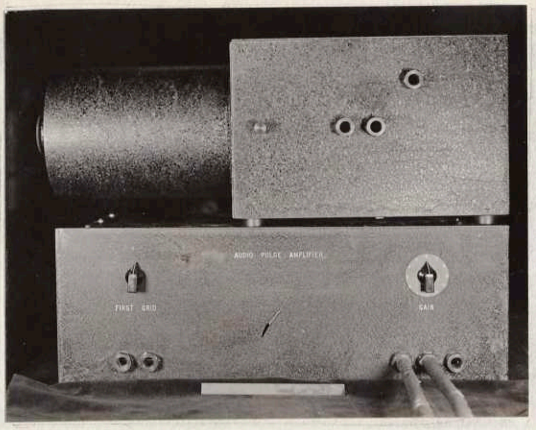
THE "NOISE" METER

The input of this circuit is fed by a conventional type 53 twin amplifier. The meter 'M' is a two milliampere rectifier type meter. It may be standardized by placing at the input a constant AC potential or better by connecting a rough standard cell across R_3.

The constants of the circuit are:

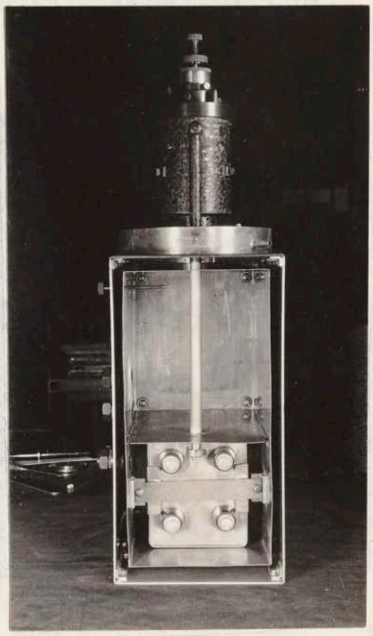
R_1 = 1000 ohms, R_2 = 10,000 ohms , R_3 = 200,000 ohms, R_4 = 600 Ohms, C may be most anything depending on the desired low frequency cut off, the present value is $.01 \mu f$.

Figure 4



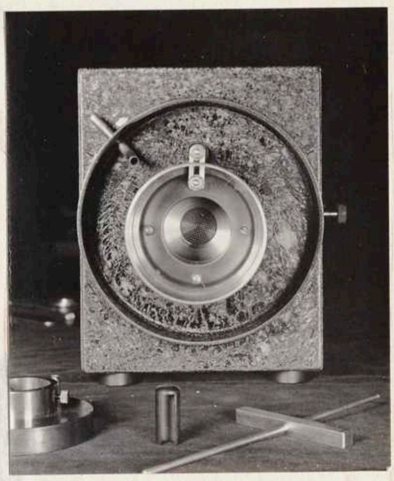
Above is a general view of the amplifier proper and the first stage.

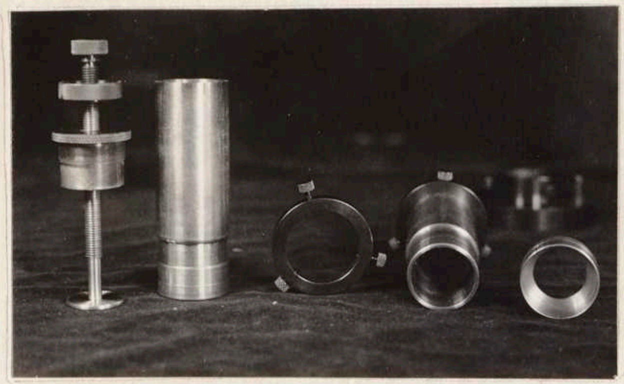
The photograph on the right shows the first stage looking down into the empty tube compartment. The four terminals are those of the non-inductive high-voltage smoothing condensors. The cylinder above is the container for the ionization chamber and the long tube leading to it is the high-voltage lead. The inner copper shield may easily be seen.



This is a plan view of the amplifier with the cover removed. The batteries in the compartment on the right are those for the first stage.and the compartment on the left contains the twin amplifier which makes up the last stage.

Below is a head on view of the ionization chamber with the front cover removed which is lying in the left foreground. Behind the wire gauze which is the entrance window for the particles and the high voltage electrode may be seen the sheen of the collector. In the center foreground is a bakalite cap which fits over the link just over the chamber. The device on the right ia a gauge to measure the chamber depth.





The thread on the long object on the left has a millimeter pitch and is used for making rough absorption measurements. It is held in place by the cylinder to the right or the cylinder may itself be used to hold the source by means of the cap at the bottom. The ring in the center is used to keep the cylinder in place. On the right is a similar cylinder and cap.

The picture below is a view of the high voltage electrode, the protruding rod is the terminal. The three pieces to the rightare,respectively, the collector, its amber insulater with air-flow holes, and the guard ring.