SECONDARY FLOW HEAT TRANSFER PHENOMENA
IN BRANCH PIPING

by

Luis A. Levy

SUBMITTED TO THE DEPARTMENT OF MECHANICAL ENGINEERING
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREES OF

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

and

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MAY 1983

(c) Luis Levy, 1983

The author hereby grants to MIT permission to reproduce and to distribute copies of this thesis document in whole or part.

Signature of Author: ..........................................................

Chairman, Departmental Graduate Committee

Accepted by .................................................................

Warren M. Rohsenow

Archives

MASSACHUSETTS INSTITUTE
OF TECHNOLOGY

JUN 23 1983
SECONDARY FLOW HEAT TRANSFER PHENOMENA
IN BRANCH PIPING

by

Luis A. Levy

Submitted to the Department of Mechanical Engineering
on February 1, 1983 in partial fulfillment of
of the requirements for the degrees of
Master of Science in Mechanical Engineering, and
Bachelor of Science in Mechanical Engineering

ABSTRACT

A run/branch tee assembly was designed and built to
investigate secondary flow heat transfer phenomena in branch
piping. The heat transfer was treated in two parts: (i)
Characterization of the branch fluid temperature response
(to a run temperature transient), and (ii) Characterization
of the film coefficients. Run diameter, branch diameter,
branch orientation (horizontal or vertical), temperature
transient direction (hot to cold, or cold to hot), and run
flow Reynolds number were varied to provide comprehensive
information.

The branch thermal propagation velocity is 6.8% to 2.4%
of the bulk run fluid velocity in the branch region L/D = 2.
In this same region, the branch heat transfer coefficients
are 44% to 25% of the run heat transfer coefficients. The
effect on secondary flow temperature response and film
coefficients of the run Re for the range $10^4 \leq Re \leq 10^5$ is
small.

From the experimental results, design curves for use in
secondary flow thermal stress analysis were generated.

Thesis Supervisor: Borivoje B. Mikic
Title: Professor of Mechanical Engineering

Company Supervisor: Daniel A. Van Duyne
ACKNOWLEDGEMENTS

I am indebted to my company supervisor, Daniel A. Van Duyne, for defining the thesis and making it an interesting, manageable project. His help and guidance throughout all stages of the project were invaluable.

I gratefully acknowledge my thesis advisor, Professor Borivoje B. Mikic, for his suggestions and advice; and for taking time to proof read and edit the thesis.

I would like to express my appreciation to Mr. Lenard Myatt for his help in defining the design aspects of this project. Special thanks must be extended to the rest of my friends at Stone & Webster, Boston who made my stay with them so enjoyable. Similarly I would like to thank my friends at the MIT Heat Transfer office for their assistance and good sense of humor.

For their encouragement and support, I would like to thank my family and Eva.

I am much obliged to Stone & Webster Engineering Corporation for its support of this thesis work as part of the MIT Engineering Internship Program.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE PAGE</td>
<td>1</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>2</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>3</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>4</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>6</td>
</tr>
<tr>
<td>1.0 INTRODUCTION AND BACKGROUND</td>
<td>8</td>
</tr>
<tr>
<td>2.0 BRANCH THERMAL RESPONSE</td>
<td></td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td></td>
</tr>
<tr>
<td>2.1.1 Objective</td>
<td>11</td>
</tr>
<tr>
<td>2.1.2 Assumptions</td>
<td>12</td>
</tr>
<tr>
<td>2.2 Experimental Apparatus</td>
<td></td>
</tr>
<tr>
<td>2.2.1 List and Specification of Equipment</td>
<td>13</td>
</tr>
<tr>
<td>2.2.2 Design</td>
<td>14</td>
</tr>
<tr>
<td>2.3 Experimental Procedure</td>
<td></td>
</tr>
<tr>
<td>2.3.1 Calibration</td>
<td>27</td>
</tr>
<tr>
<td>2.3.2 Test Procedure and Parameter Variation</td>
<td>28</td>
</tr>
<tr>
<td>2.3.3 Data Reduction</td>
<td></td>
</tr>
<tr>
<td>2.3.3.1 Branch Temperature Response</td>
<td>30</td>
</tr>
<tr>
<td>2.3.3.2 Propagation Velocity</td>
<td>33</td>
</tr>
<tr>
<td>2.4 Experimental Results and Discussion</td>
<td></td>
</tr>
<tr>
<td>2.4.1 Observations</td>
<td></td>
</tr>
<tr>
<td>2.4.1.1 Secondary Flow Pattern From Dye Test</td>
<td>35</td>
</tr>
<tr>
<td>2.4.1.2 Parameter Variation</td>
<td>39</td>
</tr>
<tr>
<td>2.4.2 Temperature Time Branch Characterization</td>
<td>48</td>
</tr>
<tr>
<td>2.4.3 Dimensionless Branch Velocity Characterization</td>
<td>54</td>
</tr>
<tr>
<td>2.4.4 Discussion of Mechanisms</td>
<td></td>
</tr>
<tr>
<td>2.4.4.1 Natural Convection</td>
<td>56</td>
</tr>
<tr>
<td>2.4.4.2 Viscous Dissipation and Thermal Capacitance</td>
<td>60</td>
</tr>
<tr>
<td>2.4.5 Accuracy and Experimental Error</td>
<td></td>
</tr>
<tr>
<td>2.4.5.1 Accuracy</td>
<td>69</td>
</tr>
<tr>
<td>2.4.5.2 Experimental Error</td>
<td>70</td>
</tr>
<tr>
<td>2.5 Design Implications</td>
<td></td>
</tr>
<tr>
<td>2.5.1 Motivation</td>
<td>72</td>
</tr>
<tr>
<td>2.5.2 Design Temperature Response Characterization</td>
<td>72</td>
</tr>
</tbody>
</table>
2.5.3 Design Curve of Dimensionless Velocity.................................78
2.6 Conclusion for Branch Thermal Response.........................81

3.0 BRANCH HEAT TRANSFER COEFFICIENTS

3.1 Introduction
3.1.1 Objective.................................................................83
3.1.2 Assumptions.................................................................83
3.2 Experimental Apparatus
3.2.1 List and Specification of Equipment.................85
3.2.2 Experimental Design..................................................86
3.3 Experimental Procedure
3.3.1 Calibration.................................................................95
3.3.2 Test Procedure and Parameter Variation........97
3.3.3 Data Reduction.........................................................99
3.4 Experimental Results and Discussion
3.4.1 Film Coefficient Calibration.........................100
3.4.2 Effects of Parameter Variation....................103
3.4.3 Accuracy and Experimental Error
   3.4.3.1 Correspondence to Actual System
         Events..............................................................110
   3.4.3.2 Experimental Error.............................................111
3.5 Design Implications
3.5.1 Motivation.............................................................113
3.5.2 Design Curves..........................................................113
3.6 Conclusion for Branch Heat Transfer
   Coefficients............................................................122

4.0 CONCLUDING REMARKS AND RECOMMENDATIONS.....................124

Bibliography.........................................................127
APPENDIX A. Representative Visicorder Traces.
APPENDIX B. Dimensionless Branch Velocity.
APPENDIX C. Quantification of Instrument Error.
APPENDIX D. Film Coefficient Calculation Method.
APPENDIX E. Heater Unit Calibration Data.
APPENDIX F. Raw Branch Film Coefficient Data.
APPENDIX G. Calibrated Film Coefficients used
   in Data Reduction.
APPENDIX H. Design Implications: Dimensionless
   Branch Film Coefficients Hb/Hr.
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PVC Run/branch Tee Assembly</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>General Layout</td>
<td>17</td>
</tr>
<tr>
<td>3(a)</td>
<td>Helical Flow Pattern in Branch</td>
<td>19</td>
</tr>
<tr>
<td>3(b)</td>
<td>Honeycomb Flow Straightener</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>Tee Cross Sections</td>
<td>21</td>
</tr>
<tr>
<td>5</td>
<td>Eliminating Sharp Corners</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>Thermocouple Placement</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>Thermal Response Signal Processing</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>Amplifier-Filter Circuit</td>
<td>26</td>
</tr>
<tr>
<td>9(a)</td>
<td>Raw Data Trace</td>
<td>32</td>
</tr>
<tr>
<td>9(b)</td>
<td>Real Time Plot</td>
<td>32</td>
</tr>
<tr>
<td>9(c)</td>
<td>Virtual Time Plot</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>Dimensionless Branch Propagation Velocity</td>
<td>34</td>
</tr>
<tr>
<td>11(a)</td>
<td>Entering Flow Pattern (D = 2x2)</td>
<td>36</td>
</tr>
<tr>
<td>11(b)</td>
<td>Entering Flow Pattern (D = 1x1)</td>
<td>36</td>
</tr>
<tr>
<td>11(c)</td>
<td>Entering Flow Pattern (D = 2x1)</td>
<td>36</td>
</tr>
<tr>
<td>12</td>
<td>Two Dimensional Flow Cavity</td>
<td>38</td>
</tr>
<tr>
<td>13</td>
<td>Branch &quot;sees&quot; Different Run Areas</td>
<td>38</td>
</tr>
<tr>
<td>14(a)</td>
<td>Temperature Response. D = 1x1, Horizontal</td>
<td>41</td>
</tr>
<tr>
<td>14(b)</td>
<td>Temperature Response. D = 2x2, Horizontal</td>
<td>41</td>
</tr>
<tr>
<td>14(c)</td>
<td>Temperature Response. D = 2x2, Vertical</td>
<td>42</td>
</tr>
<tr>
<td>15(a)</td>
<td>Temperature Response. D = 2x2, Horizontal</td>
<td>43</td>
</tr>
<tr>
<td>15(b)</td>
<td>Temperature Response. D = 2x2, Horizontal</td>
<td>43</td>
</tr>
<tr>
<td>16(a)</td>
<td>Temperature Response. D = 1x1, Horizontal</td>
<td>45</td>
</tr>
<tr>
<td>16(b)</td>
<td>Temperature Response. D = 1x1, Horizontal</td>
<td>45</td>
</tr>
<tr>
<td>17(a)</td>
<td>Temperature Response. D = 2x2, Horizontal</td>
<td>47</td>
</tr>
<tr>
<td>17(b)</td>
<td>Temperature Response. D = 2x2, Vertical</td>
<td>47</td>
</tr>
<tr>
<td>18(a)</td>
<td>Average Temperature Response. D = 2x2, Horizontal</td>
<td>50</td>
</tr>
<tr>
<td>18(b)</td>
<td>Average Temperature Response. D = 2x2, Vertical</td>
<td>50</td>
</tr>
<tr>
<td>18(c)</td>
<td>Average Temperature Response. D = 1x1, Horizontal</td>
<td>51</td>
</tr>
<tr>
<td>18(d)</td>
<td>Average Temperature Response. D = 1x1, Vertical</td>
<td>51</td>
</tr>
<tr>
<td>18(e)</td>
<td>Average Temperature Response. D = 2x1, Horizontal</td>
<td>52</td>
</tr>
<tr>
<td>18(f)</td>
<td>Average Temperature Response. D = 2x1, Vertical</td>
<td>52</td>
</tr>
<tr>
<td>19</td>
<td>Dimensionless Branch Velocity as a Function of L/D</td>
<td>55</td>
</tr>
<tr>
<td>20(a)</td>
<td>Branch Propagation Velocity Vb/Vr (D/s/ft/s) as a Function of L/D. D = 2x2</td>
<td>57</td>
</tr>
<tr>
<td>20(b)</td>
<td>Branch Propagation Velocity Vb/Vr (D/s/ft/s) as a Function of L/D. D = 1x1</td>
<td>57</td>
</tr>
<tr>
<td>20(c)</td>
<td>Branch Propagation Velocity Vb/Vr (D/s/ft/s) as a Function of L/D. D = 2x1</td>
<td>58</td>
</tr>
<tr>
<td>21</td>
<td>Branch Propagation Velocity Vb/Vr (D/s/ft/s). All configurations.</td>
<td>59</td>
</tr>
<tr>
<td>22(a)</td>
<td>Suggested Natural Convection Flow (Hot-Cold). Horizontal.</td>
<td>61</td>
</tr>
</tbody>
</table>
22(b). Suggested Vertical Natural Convection Flow Pattern (Hot-cold) 61
23. Temperature Response Mechanisms. Size on size 64
25. Temperature Response Interpretation. 68
26(a). Response to Actual Temperature Input 74
26(b). Scaled Response to Simulated Step Input 74
27(a). D = 2x2. Maximum Response Curves with Step Simulation. Horizontal 75
27(b). D = 2x2. Maximum Response Curves with Step Simulation. Vertical 75
27(c). D = 2x1. Maximum Response Curves with Step Simulation. Horizontal 76
27(d). D = 2x1. Maximum Response Curves with Step Simulation. Vertical 76
27(e). D = 1x1. Maximum Response Curves with Step Simulation. Horizontal 77
27(f). D = 1x1. Maximum Response Curves with Step Simulation. Vertical 77
28. Design Curve: Vb/Vr as a Function of L/D 79
29. Design Curve: Vb/Vr (D/s/ft/s) vs L/D 79
30. Hotwat Band Heaters 87
31. Metal Ring and PVC Pipe Assembly 88
32. Groove on Metal Ring 90
33. Complete Heater Unit Assembly 91
34. Signal Flow Diagram 93
35(a). 2" Heater Unit Calibration Data 101
35(b). 1" Heater Unit Calibration Data 101
36. Hb/Hr as a Function of L/D. D = 2x2 104
37. Hb/Hr as a Function of L/D. D = 1x1 105
38. Hb/Hr as a Function of L/D. D = 2x1 106
39. Hb/Hr as a Function of L/D. All configurations 109
40. Condensed Branch Film Coefficient Data as a Function of L/D 114
41. Hb/Hr as a Function of L/D. All Configurations 117
42. Hb/Hr as a Function of Absolute Distance 118
43. Tee Crotch: Discontinuity Stress Consideration. 119
44. Design Curve: Hb/Hr as a Function of L/D 121

Table 1. Procedure Matrix 29
1.0 INTRODUCTION AND BACKGROUND

Thermal fatigue analysis of nuclear piping systems involves identification of temperature transients and accompanying heat transfer characteristics such as film coefficients and flow rates. The piping systems in a nuclear plant that must undergo fatigue analysis are those identified as ASME Class 1 and include those lines in the primary pressure boundary. Examples are: Primary Coolant Loop, Feedwater, Reactor Core Injection Coolant, Residual Heat Removal, and chemical and volume control lines. Due to the variety of thermal transients on these systems situations arise where the heat transfer characteristics and configurations are uncertain. These characteristics are especially uncertain at so called "secondary flow" zones. The most frequent occurrence of a secondary flow zone is at a tee or fabricated branch connection where the branch pipe off the main line (the run) has zero net flow. Say there is a valve on the branch pipe which is closed during some prescribed mode. A secondary flow will be established in the branch driven by the flow in the run. Any thermal transients in the run pipe will be transported to the branch through the
secondary flow microstructure. Thermal stresses along the branch pipe are a function of the heat transfer phenomena associated with this secondary flow. The problem with secondary flow branch piping thermal fatigue analysis is thus identification of the temperature transients, and the heat transfer characteristics (i.e., film coefficients) in the branch piping.

Current practice at Stone & Webster is to define certain heat-affected zones and subregions in the branch pipe based on limited information gathered from flow studies, and engineering judgement. Estimates of film coefficients and temperature transients in these branch regions as a function of the main flow are made. These estimates provide inputs to finite element models of the branch wall at chosen locations; and thermal analysis is thus performed.

A more comprehensive and rigorous body of information, however, is still needed concerning the branch heat transfer phenomena. In particular two things are needed: (i) characterization of the temperature-time response at different locations as a function of run transient, including effects of branch and run pipe diameter sizes, and run flow rate; (ii) characterization of film coefficients at branch locations as a function of the run pipe film coefficients.

Current literature concerning secondary flow
phenomena in branch piping is limited. Haugen and Dhanak[6,7] performed experiments on flow over a surface cavity, where the cavity depth to width ratio was varied. Their studies, however, are not completely applicable: the flow pattern they observed is of 2-dimensional air flow; the heat transfer considerations are for heat transfer normal to the cavity towards the main flow.

Similarly, Fox's[4] investigations on heat transfer and air flow in rectangular notches are only for 2-dimensional air flow.

This project is to construct a working model of a run/branch tee arrangement; and to characterize (1) the temperature-time response at branch locations as a function of run transient, and (2) the film coefficients at branch locations as a function of run film coefficients. Knowing the temperature and film coefficient profiles of the branch fluid the total heat transfer can be described.

This thesis presents the branch thermal response characterization in section 2.0, and the branch film coefficient characterization in section 3.0. Each section includes a discussion of design implications for its use in thermal analysis. Suggestions for future work are included in section 4.0.
2.0 BRANCH THERMAL RESPONSE

2.1 INTRODUCTION

2.1.1 Objective

This section examines the branch secondary flow temperature behavior as a function of temperature changes in the run pipe flow. An experiment was designed to investigate temperature-time response at different branch locations to run pipe temperature transients. A step in the run temperature was used as input to the branch secondary flow. Three different tee configurations were used to provide comprehensive information of the branch secondary flow thermal response:

1. a 2" diameter run pipe on a 2" diameter branch pipe (D = 2x2).
2. a 2" run pipe on a 1" branch pipe (D = 2x1).
3. a 1" run pipe on a 1" branch pipe (D = 1x1).

The influence of different parameters—such as branch orientation (horizontal or vertical), temperature transient
direction (down transient or up transient), run flow Reynolds number, Re—was also investigated. Run flow pressure was kept constant; its influence on the heat transfer phenomena was not investigated.

2.1.2 Assumptions

The tee arrangements constructed for these experiments are assumed to be representative models of existing piping tees in nuclear power plants. Although the temperature transients experienced by piping systems range from gradual ramps to steps, we assume that branch thermal response to a step input is information that can be applied to most real transient conditions.

An important assumption is that the run thermal transient is a function of the step change in temperature alone. In this connection, the velocity of the run flow is assumed constant during the run transient. Measures were taken to provide for this.
2.2 EXPERIMENTAL APPARATUS

2.2.1 List and specification of equipment


2. Omega Copper-Constantan thermocouple wire. Type T. Gauge 24.

3. Honeywell Visicorder. Model 1508A.


5. Omega Engineering Digital Thermocouple thermometer Model 199.

6. Textronix 511 Storage oscilloscope.


8. Digitec 26c Multimeter

2.2.2 Design

The first design objective was construction of a suitable run/branch tee arrangement. Commercial Polyvinyl Chloride (PVC) piping and tees were purchased for three reasons: PVC is relatively inexpensive compared to metal tubing, it is transparent, and it is easy to machine. Three different tee sizes were obtained—(2"x2"), (1"x1"), (2"x1")—to enable construction of the three configurations under investigation. The PVC tees are socket fitted so the the PVC piping simply slips into a shoulder, (see Fig. 1). Connections did not have to be glued; a small application of vacuum grease on the socket prevents water leakage.

The second design objective was to provide the run flow with a temperature regulator that would not disturb the run flow rate. Because we are trying to achieve maximum run flow rates with Re = 1x10^5 it was necessary to use flow rates as high as 46 GPM on the 2" run line. The scheme used to impose a step temperature change on 46 GPM of at least 10°F compromised versatility and expense. A 1" steam line with steam at 400°F, and 200 psia was connected to a 4" water line with water at 60°F, and 45 psia across two control valves. See Figure 2. The steam mixes with the cold water to increase the run bulk water temperature; yet because the steam line is small compared
Fig. 1  PVC Run-Branch Tee Assembly.
to water line, the mean flow rate remains unaffected. The biggest constraint on the experimentation was that the flow rate and temperature step size are coupled. As the flow rate increases the temperature step size decreases. To monitor the main line flow temperature, a precaution against overheating the apparatus, a digital thermocouple readout was installed upstream of the PVC tee arrangement, (see Figure 2).

A third design objective was proper measurement of the run pipe flow rate. For this purpose a rotameter was installed upstream of the tee, (Fig. 2). The rotameter was rated for a maximum flow of 37 GPM; to extend this measurement capability a 50 gallon weigh tank was placed at the system discharge. The weigh tank sat on a weighing scale; discharge from the weigh tank, when its valve was open, was dumped to a discharge tunnel underneath the lab floor. The system was capable of sustaining measured flow rates as high as 50 GPM.

Movement of the branch about the y-axis (see Fig. 2) was another design objective. Information regarding the branch secondary flow phenomena in different orientations is needed. Two orientations were chosen: branch in horizontal position, and branch in vertical position. Rotation of the PVC tee arrangement was made possible at the socket connections between the PVC and the copper tubing. Unfortunately, the support structure securing the
Fig. 2 General Layout

cold water line

honeycomb flow straightener

run pipe (PVC)

branch (PVC)

discharge tunnel

weighing scale

copper tubing

digital thermometer

rotameter

steam line

weigh tank
piping system in its position prevented the branch from rotating to a vertical down position. Consequently experimentation was performed only on orientations: horizontal, and vertical up. Rotation of the branch pipe about the x-axis was made possible at the tee socket.

Preliminary observations and test runs on the system led to one significant modification and one minor modification of the system. A Potassium Permanganate dye solution was used to make observations of the branch secondary flow. Injected at various locations—upstream of the tee, and in the branch— the dye revealed a helical motion, an asymmetric screw motion in the branch, (see Fig. 3(a)). This observation was made for all three tee configurations. To ensure that the helical motion was not due to secondary flow upstream of the tee, such as due to elbows and the rotameter, a honeycomb flow straightener was installed immediately after the last elbow upstream of the tee, (see Fig. 3(b)). The test tee was 18" downstream of the flow straightener: 9 diameters on the 2" line and 18 diameters on the 1" line. Subsequent dye observations revealed the persistance of this helical motion. Discussion of these observations is covered in Section 2.4.1.

During preliminary system testing a minor problem arose regarding the existence of air bubbles along the tee connections. At the connection of the PVC piping( both
(a). Helical flow pattern in branch.

(b). Honeycomb flow straightener.

Fig. 3 Investigating the branch helical flow pattern.
run and branch) and the socket fitted tee, a lip was formed. Air bubbles from the main cold water line were being trapped as they passed through the tee at these lips, (see Fig. 4). This was especially pronounced during steam-water mixing which produced a high density mixture of entrained bubbles. To smooth out these corners two things were done: on the 2"x2" configuration epoxy was applied at the connection, (see Fig. 5(a)); in the other two configurations the lips were machined down, (see Fig. 5(b)). Both methods worked equally well. It is assumed these methods have the same effect on the bulk flow.

Measurement of the branch temperature response as a function of the run transient was accomplished by designing a thermocouple array on the PVC tee configuration, (see Fig. 6). Six thermocouple locations were chosen. The first, located on the run pipe centerline 1 diameter(1D) downstream of the tee, monitored the run pipe temperature transient. This measurement serves as a reference for all response tests in the branch. The remaining five thermocouples were chosen at 2D, 4D, 6D, 8D, and 12D. For the 2"x1" configuration the last location was at 10D. Copper-Constantan thermocouple wire was used. The beads were argon arc welded individually using a thermocouple welder. Since the temperature response of the secondary flow is on the order
Fig. 6 Thermocouple placement.
of 30 seconds and above, differences in thermocouple time constants are negligible. We decided to monitor the branch temperature response near the branch wall. This would provide more information than temperature responses at the branch centerline. The 2" diameter branch thermocouples were placed roughly 3/8" from the wall; the 1" diameter branch thermocouples were placed roughly 2/8" from the wall. Epoxy glue was used to secure the thermocouples in place. This method of installation allowed for easy replacement of thermocouples, and furthermore provided good water sealing properties.

The thermocouples were all connected to an ice-bath reference junction. Steps were taken to ensure proper immersion of thermocouples in the ice-bath, and maintenance of an ice-point temperature. Voltage signals from the thermocouples, referenced to ice-point, were sent to a 4 channel Honeywell Visicorder via a 4 channel Accudata amplifier, (see Fig. 7). The visicorder was chosen because of its response capabilities and because the output, produced on light sensitive paper, is easy to handle and provides immediate visual feedback—the response can be seen in real time (as opposed to storing the data on tape for later use on a computer). The 4 channel recording limitation meant that only 4 thermocouples could be monitored during any given run.

The average run temperature step produced by mixing
the steam and water was on the order of 20°F at 20 GPM. A 20°F temperature change results in a Cu-C thermocouple voltage change of 0.4 mv. A six channel amplifier circuit was built to boost the low voltage signal to a 1 V signal range. The Accudata amplifier would provide the remaining gain necessary to produce satisfactory signal amplitudes on the visicorder. The amplifier circuit was designed with high input impedance, low output impedance (output buffer) for proper impedance matching, (see Fig. 8(a)). A low pass filter attenuates noise above 50 Hz, (see Fig. 8(b)). Because relative and not absolute temperatures are important, differences between signal channels (i.e. dc offsets) do not affect the data. As long as each channel registers the same voltage difference for a given temperature difference the data is valid.
(a). Design (one channel).

(b). Circuit frequency response.

Fig. 8 Amplifier-filter circuit.
2.3 EXPERIMENTAL PROCEDURE

2.3.1 Calibration

To ensure accurate flow measurements the rotameter and weigh tank were calibrated against each other. The rotameter is assumed to be calibrated. Good correlation was found between the two measuring devices. For flow rates of 25 GPM and less, they measured flow rates to within 0.4% of each other. For higher flow rates, the weigh tank produced measurements 5% higher than the rotameter.

A more crucial issue is calibration of the thermocouples. It is important that for a given temperature difference all thermocouples produce the same voltage difference at the visicorder output. A two point calibration was performed. First the system was emptied and dried. The visicorder light beams were zeroed with the reference being room temperature. The system was filled with running water at 65 °F. After the thermocouples settled, the displacement on the trace was measured. It was found that all thermocouples were calibrated to each other to within reading accuracy of the visicorder trace--5%. Relative calibration is the important issue. On the visicorder trace the conversion
is 10 °F to 1cm. During the experimental runs, a digital thermometer measured absolute temperature levels of the run pipe temperature transient. The digital thermometer was calibrated to an ice-point reference.

2.3.2 Test Procedure and Parameter Variation

From the onset certain parameters were defined as important contributors to the branch thermal response. Variation of these parameters became the focus of the experimental procedure as depicted by the procedure matrix, Table 1. These parameters are: (i) size configuration—3 sizes were used; (ii) branch orientation—horizontal and vertical; (iii) temperature transient direction—cold to hot, and hot to cold; (iv) and run Re range—as defined by range of flow rates. Due to time and system limitations the run temperature step size, ΔT, was not varied independently of flow rate. The average run temperature step size was 20 °F in the temperature range of 120 °F to 60 °F. Undoubtedly higher temperatures can have an effect on the flow pattern when natural convection plays an important role.

Each experimental run began with choice of thermocouple locations, branch orientation, Re for flow rate, and temperature step direction according to
### TABLE 1 Procedure Matrix

(1) Three different configurations:
\[ D = 2 \times 2, \ 1 \times 1, \ 2 \times 1 \]

(ii) For each configuration: at locations \( L/D = N \) \( N = 2, 4, 6, 8, 12 \)

A. Horizontal Position.

1. Run temperature transient Cold to Hot.
   a. Thermocouples on branch top.
   b. Thermocouples on branch bottom.

2. Run temperature transient Hot to Cold.
   a. Thermocouples on branch top.
   b. Thermocouples on branch bottom.

B. Vertical-up Position.

1. Run temperature transient Cold to Hot.

2. Run temperature transient Hot to Cold.

Over the range of flow rates: \( 2 \times 10^4 \leq \text{Re} \leq 10^5 \)

(i.e. Run velocity range:
   2" run pipe: 1.88 fps - 4.6 fps
   1" run pipe: 3.00 fps - 13.5 fps)
procedure matrix, Table 1. The tee system was filled with either cold water (prior to an up transient) or hot water (prior to a down transient) until the temperature throughout the system was constant; this would be the initial temperature, $T_i$. To help the thermocouple readouts stabilize at $T_i$, the branch was rotated about the run pipe up and down. This "cranking" action enhanced mixing at the far end of the branch where secondary flow intensity is small. With the thermocouples stabilized, the visicorder output was zeroed and hot water (for an up transient) or cold water (for a down transient) was allowed to flow through the run pipe. To obtain data on all branch thermocouple locations using only the 4 channel recorder combinations were made. For example, one run would include thermocouple locations: run, $L/D=2, 4, 6$. A second run would include thermocouple locations: run, $L/D= 6, 8, 12$. With all test runs the run thermocouple location served as a reference. A total of 68 tests were performed.

2.3.3 Data Reduction

2.3.3.1 Branch Temperature Response.

Experimental data takes the form of temperature
response as a function of time, at different thermocouple locations (L/D's), recorded on a visicorder trace. The first step in data reduction dimensionalizes the temperature. Because the run pipe transients were not perfect steps (average time to steady state was 25 seconds) two interpretations of the data are possible. Both interpretations are used, each provides added information regarding the branch thermal response. The first interpretation considers that at any time, t, the temperature change at a branch location, \( \Delta T_1 \), is a percentage of the temperature change in the run, \( \Delta T_3 \), (see Fig. 9(a)). The ratio \( \Delta T_1 / \Delta T_3 \) is a dimensionless temperature that can be plotted in "real time" as shown in Figure 9(b). This "real time" plot includes lapse time information at each thermocouple location. A second interpretation considers each L/D response to have its own origin, the point at which it begins to experience a temperature change. This approach expresses the branch temperature change, \( \Delta T_1 \), as a function of \( \Delta T_2 \), the run flow temperature change that was originally transmitted to the branch \( t' \) seconds ago, (see Fig. 9(a)). This "local" point of view expresses the dimensionless temperature \( \Delta T_1 / \Delta T_2 \) plotted in "virtual time," (see Fig. 9(c)). Plots of \( \Delta T_1 / \Delta T_2 \) provide information regarding the relative temperature responses at branch locations independent of lapse times. These \( \Delta T_1 / \Delta T_2 \) plots are
Fig. 9(a) Raw Data Trace

Fig. 9(b) Real Time Plot

Fig. 9(c) Virtual Time Plot
eventually the most useful form the results will take.

In both plots the response of the run is expressed 

\[(T - T_i)/(T_f - T_i)\]; so it goes from zero to unity.

2.3.3.2 Propagation Velocity

Direct measurement of local velocities at branch locations was too difficult and expensive to perform. The propagation velocity, the velocity of temperature propagation along the branch obtained from thermal response data, provides the next best piece of information. From branch temperature response curves plotted in "real time," lapse times at each branch L/D and for each run pipe Re can be obtained. These lapse times represent the time for the temperature disturbance, initiated in the run, to propagate along the branch, (see Fig. 10). This propagation velocity, \(V_b\), is some function of the local velocity at the branch wall. \(V_b\) is obtained by considering the distance traveled by a run temperature disturbance to a thermocouple location during a period of time defined by the difference in lapse times. A dimensionless branch velocity \((V_b/V_r)\) can then be plotted as a function of L/D, the dimensionless branch distance.
\[ V_{bl} = \frac{L}{t} = \frac{2D}{\Delta t_1} \quad \text{for } i = 2,4,6,8 \]

\[ V_{b12} = \frac{4D}{\Delta t_{12}} \]

Lapse times:
- \( \Delta t_2 = t_{L2} \)
- \( \Delta t_4 = t_{L4} - t_{L2} \)
- \( \Delta t_6 = t_{L6} - t_{L4} \)
- \( \Delta t_8 = t_{L8} - t_{L6} \)
- \( \Delta t_{12} = t_{L12} - t_{L8} \)

\((V_b/V_r)_i = \text{Dimensionless branch propagation velocity, at locations } i = 2,4,6,8,12\)

**Fig. 10** Dimensionless branch propagation velocity \( V_b/V_r \)
2.4 EXPERIMENTAL RESULTS AND DISCUSSION

2.4.1 Observations

2.4.1.1 Secondary Flow Pattern from dye experiments

Inclusion of the honeycomb flow straightener eliminated secondary flow effects of elbows and equipment upstream of the tee. Subsequent dye experiments revealed the persistence of a helical motion in the branch secondary flow. This "screw" motion is characterized by fluid moving into the branch along one side of the screw contour, and fluid moving out along the other side. This exchange satisfies continuity: as much fluid leaves the branch as goes into the branch. In both the D=2x2 and D=1x1 configurations, rotational direction of the incoming fluid is clockwise along the x-axis, (see Figures 11(a) and 11(b)). The cause of this asymmetric flow pattern is probably the same in both configurations. In the D=2x1 configuration the rotational direction showed no preference; half the time incoming flow rotated clockwise, and half the time it rotated counterclockwise (see Fig. 11(c)).

Existence of this helical flow pattern means that different locations in the branch see different fluid
Fig. 11(a) Entering Flow Pattern (D = 2x2)

Fig. 11(b) Entering Flow Pattern (D = 1x1)

Fig. 11(c) Entering Flow Pattern (D = 2x1)
motion. At some locations the fluid is moving into the branch and at other locations the fluid is moving out of the branch. The flow into the branch has more of the nature and structure of the main(run) flow, and is probably more dependent on the run Re. The flow out of the branch is probably more damped, as it is pushed out from the fluid deep in the branch.

Results of dye tests conducted by Haugen and Dhanak[6,7] on 2-dimensional air flow past a cavity revealed secondary flow in the shape of propagating figure-8's. See Figure 12. These figure-8's were a 2-dimensional pattern. The dye tests suggest that 2-dimensional channel flow experiments do not apply directly to pipe flow. Perhaps the branch pipe helical secondary flow pattern is a result of a third dimension flow component. In configurations D=2x1, and D=2x2 the first cell in the branch flow is relatively 2-dimensional. Subsequent cells resemble 3-dimensional stretched out and twisted figure-8's. For the D=2x2 configuration all the cells resembled twisted figure-8's.

That configurations D=2x2, and D=1x1 have a preferred branch rotational direction and configuration D=2x1 does not, suggests that the branch can "see" different run pipe flow. Figure 13 shows that for a tee with branch and run pipes of same diameter(size-on-size) the branch "sees" the entire run bulk flow. If the branch is half the size, as
Fig. 12 Two Dimensional Cavity Flow

Fig 13 Branch "sees" Different Run Areas
in $D=2\times 1$, the branch "sees" one-quarter as much (ratio of the areas). As such the $D=2\times 1$ branch may be more sensitive to local flow fluctuations near the entrance. This may explain the fluctuations in rotational direction that were seen in the $D=2\times 1$ branch.

In addition, dye tests indicate that secondary flow intensity dies off dramatically past $L/D=4$. By $L/D=12$ stratification of the dye became pronounced. In configuration $D=2\times 1$ stratification occurred at $L/D=10$; for this reason the last thermocouple location for $D=2\times 1$ in the thermal response experiment was $L/D=10$.

2.4.1.2 Parameter variation: effect on thermal response

The influence of four parameters on the branch thermal response was examined. One important influence on the branch temperature response is the run temperature transient direction. Erratic temperature fluctuations and suppressed temperature response occur at the top of a horizontal branch for a down transient (hot to cold), and at the bottom of a horizontal branch for an up transient (cold to hot). These fluctuations and suppressed responses are not present in opposite situations. Appendix A contains representative visicorder output traces for two experimental thermal response tests.
illustrating this phenomena. For a vertical branch the fluctuations are much less pronounced, but nevertheless suppressed responses occur during a down transient (hot to cold). Figures 14(a, b, c), plots of dimensionless temperature $\Delta T_1/\Delta T_2$ vs L/D, illustrate this behavior. These observations suggest the existence of buoyancy and natural convection effects. Heat is convected to the top of a horizontal branch and tends to remain there. For a down transient the top will cool down slower, see Fig. 14(a). The reverse is true of the bottom of a horizontal branch (Fig. 14(b)). A down transient produces roughly the same effect in a vertical-up branch as in the horizontal bottom case (see Fig. 14(c)). It was observed that increases in the run Re smooth out and increase the branch temperature response for these particular cases. The increased turbulence enhances overall mixing of affected region (top or bottom) and works against the natural convection; the top and bottom of a horizontal branch will see similar temperature responses as the run Re increases. Figure 15(a) shows branch temperature response curves for a run with Re = $3.31 \times 10^4$; note the temperature fluctuations at L/D = 2 and the suppressed responses at L/D = 4.6. Figure 15(b) shows the same configuration for a run with Re = $7.76 \times 10^4$; note the smooth response at L/D = 2 and enhanced responses at L/D = 4.6. The data suggests that an upper limit for this
Fig. 14(a) Temperature Response. D = 1x1, Horizontal, TC Top, Hot-Cold, \( V_r = 3.32 \) fps, \( Re = 2.81 \times 10^4 \)

Fig. 14(b) Temperature Response. D = 2x2, Horizontal, TC Bottom, Cold-Hot, \( V_r = 2.64 \) fps, \( Re = 4.5 \times 10^4 \)
Fig. 14(c) Temperature Response. D = 2x2, Vertical, Hot-Cold, 
Vr = 4.6 fps, Re = 7.77x10^4
Fig. 15(a) Temperature response. \( D = 2 \times 2 \). Horizontal branch, TC on top, Hot-Cold, \( V_r = 1.96 \text{ fps} \), \( Re = 3.31 \times 10^4 \)

Fig. 15(b) Temperature response. \( D = 2 \times 2 \). Horizontal branch, TC on top, Hot-Cold, \( V_r = 4.5 \text{ fps} \), \( Re = 7.6 \times 10^4 \)
enhancement in temperature response is the response of the other side of the branch. For example, during an up transient in a horizontal branch, natural convection effects attenuate the response at the bottom. Increases in Re will enhance the bottom response until its response is similar to that of the branch top. In general it is difficult to quantify these natural convection effects because of uncertainty and data scatter. As such, these effects are presented in qualitative fashion only.

The effect of run Re on branch thermal response, over the range \(2 \times 10^4 \leq \text{Re} \leq 11.0 \times 10^4\) was investigated. Except for those abovementioned situations where natural convection plays an important role, the Re had no noticeable effect on the branch temperature responses. Figures 16(a) and 16(b) illustrate two typical runs where in one case (16(a)) \( \text{Re} = 2.5 \times 10^4 \), and in the other (16(b)) \( \text{Re} = 8.9 \times 10^4 \); the responses at all locations are similar to within 5%, which is within experimental error of 10%. Assuredly there must be some lower bound (i.e., laminar flow) where the responses will attenuate; that bound was not investigated.

The effect on the branch response of branch orientation, the branch either horizontal or vertical-up with respect to the run pipe, was also considered. In general for the \( D=1\times1 \) configuration, branch orientation had little effect on the responses. For both the \( D=2\times2 \),
Fig. 16(a) Temperature response. \( D = 1 \times 1 \). Horizontal branch, TC on top, Cold-Hot, \( V_r = 3.01 \) fps, \( Re = 2.54 \times 10^4 \)

Fig. 16(b) Temperature response. \( D = 1 \times 1 \). Horizontal branch, TC on top, Cold-Hot, \( V_r = 10.63 \) fps, \( Re = 8.98 \times 10^4 \)
and D=2x1 configurations, however, the horizontal branch had faster temperature response curves than the vertical branch case. Figures 17(a) and 17(b) illustrate this. Both runs are of an up transient with Re = (3.2 - 3.8) x 10^4; in Figure 17(a) the branch is horizontal, and in Figure 17(b) the branch is vertical. At all locations the horizontal branch thermal response curves are faster.

Data for D=2x1 configuration with the branch in vertical-up position is not completely reliable. During transient conditions entrained bubbles in the main pipe would enter the branch. The bubbles would start displacing branch fluid. Experimental runs were usually terminated if the water level went below L/D = 10. Branch secondary flow must have been affected in several runs.

The effect of size configuration on the branch thermal response was also investigated. Comparison of branch thermal response configurations for D=2x2, and D=1x1 provides information on the effect of pipe size. Comparison of branch thermal response for D=2x2 and D=2x1 provides information on the effect of a non size-on-size run/branch arrangement. These comparisons are best made by taking the average of the response curves for different categories of data. This will be covered extensively in the next section, which presents the temperature time characteristic curves.
Fig. 17(a) Temperature Response, $D = 2 \times 2$. Horizontal branch, TC on top, Cold-Hot, $Vr = 1.33$ fps, $Re = 3.2 \times 10^4$

Fig. 17(b) Temperature response, $D = 2 \times 2$. Vertical branch, Cold-Hot, $Vr = 2.26$ fps, $Re = 3.82 \times 10^4$
The effects of the aforementioned parameters (Re, orientation, temperature transient direction) on the dimensionless branch velocity ($V_b/V_r$) can be quickly summarized. The dimensionless branch velocity is independent of Re (i.e., if the run velocity doubles, the propagation velocity doubles) and of branch orientation. Furthermore, for the D=1x1 configuration, the branch velocity is independent of the temperature transient direction. For the D=2x2 and D=2x1 configurations ($V_b/V_r$) exhibits the same dependence on the temperature direction as the thermal response. For a given transient that side of a horizontal branch that sees attenuated thermal response also sees slower branch velocity, and vice versa. This is not surprising since the dimensionless branch velocity is based on thermal response lapse time data. In this regard, there should be close agreement between thermal response information and propagation velocity information. As with the thermal response data, comparison of branch velocity for different configurations is best done with average values. These will be covered in Section 2.4.3.

2.4.2 Temperature Time Branch Characterization
Characterization of the branch temperature time response as a function of run transient is performed by taking the average of all the response curves for different categories. This averaging is done by taking numerical averages at three time points, \( t = 10, 20, 60 \) seconds, and fitting the rest of the curves by visual comparison of the data. The response curves plot \( \Delta T_1/\Delta T_2 \) versus time and \( L/D \). These response curves are in "virtual" time and do not consider the lapse time to each branch location. For each configuration, averages were obtained for horizontal and vertical branch orientations. They are shown in Figures 18(a,b,c,d,e,f). Since Re has little effect on temperature response, where natural convection effects are negligible, it does not enter in the categorization. Those response curves attenuated by natural convection effects were ignored in the averaging process. As the Re increases, top and bottom of the horizontal branch become similar. In other words for a given transient the averaging process selected those cases where natural convection enhanced the response and ignored those cases where natural convection attenuated the responses.

Comparison of these characteristic response curves reveals important information regarding the effect of pipe size. In the horizontal case, for branch locations near
Fig. 18(a) Average Temperature Response. \( D = 2 \times 2 \),
Horizontal Branch, \( Re = (3.2-7.8) \times 10^4 \)

Fig. 18(b) Average Temperature Response. \( D = 2 \times 2 \),
Vertical Branch, \( Re = (3.5-7.7) \times 10^4 \)
Fig. 18(c) Average Temperature Response. \( D = 1 \times 1 \),
Horizontal Branch, \( \text{Re} = (2.5-11.5) \times 10^4 \)

Fig. 18(d) Average Temperature Response. \( D = 1 \times 1 \),
Vertical Branch, \( \text{Re} = (3.18-9.62) \times 10^4 \)
Fig. 13(e) Average Temperature Response. $D = 2 \times 1$, Horizontal Branch, $Re = (2.18-8.98) \times 10^4$

Fig. 18(f) Average Temperature Response. $D = 2 \times 1$, Vertical Branch, $Re = (1.9-2.3) \times 10^4$
the tee, $L/D < 6$, configurations $D=2 \times 2$ (Fig. 18(a)) and $D=2 \times 1$ (Fig. 18(e)) have similar temperature response to within 4%. Away from the branch, $L/D > 6$, $D=2 \times 2$ has greater response than $D=2 \times 1$ by 20%. Comparison of configurations $D=2 \times 2$ and $D=1 \times 1$ in horizontal orientation (Fig. 18(a) and Fig. 18(c)) suggests that they have similar response characteristics, to within 4%, everywhere except at $L/D = 4.6$ where $D=1 \times 1$ has a faster response by about 12%. In the vertical orientation $D=1 \times 1$ (Fig. 18(d)) has faster response at all locations than configurations $D=2 \times 1$ (Fig. 18(f)), and $D=2 \times 2$ (Fig. 18(b)) by about 20%. Note that $D=2 \times 1$ vertical data may have been affected by air bubbles entering the branch. In any case for the vertical orientation $D=2 \times 2$ and $D=2 \times 1$ have similar responses to within 10%. The characteristic curves follow the trends observed in the previous sub-section.

Note that in the vertical cases for configurations $D=2 \times 2$, and $D=1 \times 1$ averaging is performed for $L/D=2,4,6$ only. One run for $L/D=8,12$ in the $D=1 \times 1$ configuration was not included; it will be considered in plots of maximum responses in the section on Design Implications, Section 2.5.
2.4.3 Dimensionless branch velocity characterization

Characterization of the dimensionless branch velocity ($V_b/V_r$) along the branch is performed by taking the average of all the time lapse data for each configuration. See Appendix B for raw data. Figure 19 is a plot of dimensionless branch velocity ($V_b/V_r$) versus dimensionless branch distance ($L/D$) for the three configurations under investigation. The branch velocity is a measure of the amount of momentum transfer into the branch for a given turbulence in the run. In this connection, comparison of data for $D=2\times2$, and $D=1\times1$ suggests that the larger diameter branch provides greater momentum and turbulence transfer from the run to the branch. The smaller diameter branch has higher viscous forces; dissipation is higher along branch. Comparison of data for $D=2\times2$, and $D=2\times1$ substantiates this hypothesis. In the limit as the branch diameter decreases, say to capillary size, the momentum transfer should drop to zero. Note that the general shape of the three curves is the same. This shape suggests an exponential decay of fluid momentum due to viscous damping forces.

To take into account the branch diameter, the dimensionless branch velocity ($V_b/V_r$) can be transformed into a ratio ($V_b/V_r$) in [D/s/ft/s] where the thermal
Fig. 19 Dimensionless branch velocity as a function of L/D
propagation velocity, $V_b$, is expressed in units of diameters per second and the run velocity, $V_r$, is left in terms of feet per second. Figures 20(a,b,c) are plots of this velocity ratio as a function of $L/D$. (50% of the data falls between the hashed lines). Plotted together, Figure 21, the data correlates better. It is not clear that more physics is afforded with this correlation.

2.4.4 Discussion of Mechanisms

The author suggests the following mechanisms to account for the thermal response results. Evidence for these mechanisms is tentative; more investigation is needed to provide support.

2.4.4.1 Natural convection

Results from temperature response data indicated the presence of natural convection effects in the branch temperature response. Momentum transfer to the branch takes the form of forced convection at least within the region $L/D < 2$. Within this region natural convection effects were not observed. In the region $L/D > 2$ the fluid will have a radial temperature distribution during
Fig. 20 Branch propagation velocity $V_b/V_r$ (D/s/ft/s) as a function of $L/D$.  
(a) For configuration $D = 2 \times 2$.  
(b) For configuration $D = 1 \times 1$
Fig. 20(c) Branch propagation velocity $V_b/V_r$ (D/s/ft/s) as a function of $L/D$. For configuration $D = 2\times 1$. 

mean values
50% of values
Fig. 21  Branch propagation velocity \( \frac{V_b}{V_r} \) (D/s/ft/s). All configurations.
the thermal transient, and natural convection begins to have an effect; the flow is defined by combined free and forced convection. Suggested flow patterns for the natural convection, superimposed on the forced convection, are presented in Figures 22(a,b). So that for an up transient the top of a horizontal branch is enhanced by natural convection; for down transient the bottom is enhanced. As with all mixed forced and natural convection flows, increases in the Re will decrease effects of natural convection. In the branch secondary flow, increases in the run Re enhance forced convection and work to make the top and bottom response similar.

2.4.4.2 Viscous Dissipation and Thermal Capacitance

Observations of temperature response data suggests the existence of two mechanisms in controlling the thermal response of different size piping arrangements: viscous dissipation and thermal capacitance. Momentum pumped into the dead end branch decays exponentially along the branch due to the viscous dissipation and damping; this is clearly seen in the dimensionless branch velocity profile of Figure 19.
Fig. 22(a) Suggested Natural Convection Flow (Hot to Cold). Horizontal Branch.

Fig. 22(b) Suggested Vertical Natural Convection Flow Pattern (Hot to Cold).
As the branch pipe diameter decreases the viscous forces increase, and the momentum transfer into the branch decreases. Consequently the $D=1\times 1$ configuration, with a 1" diameter branch, would have less momentum transfer along the branch than the $D=2\times 2$ configuration which has a 2" diameter branch. This explains why the dimensionless branch velocity is greater along the branch for configurations $D=2\times 2$ than configurations $D=1\times 1$, and $D=2\times 1$. In the limit as the branch diameter decreases in size the dimensionless branch velocity should approach zero.

Temperature transients are transported to the branch by the branch momentum transfer. As such, viscous forces act to attenuate the temperature response along the branch. Viscous forces would tend to attenuate smaller diameter branch temperature response characteristics more than larger diameter branch characteristics at each location.

A second mechanism governing the branch temperature response is related to the thermal capacitance of the fluid along the branch. The rate of energy stored in a volume of fluid is given by $Q_s$:

$$Q_s = mC_p \frac{dT}{dt} \quad (i)$$

where: $\dot{Q}_s =$ power stored in fluid volume.

$m =$ mass of unit volume.
\[ C_p = \text{specific heat, constant volume.} \]
\[ T = \text{fluid temperature.} \]
\[ t = \text{time.} \]

The thermal capacitance, \( C \), is defined by \( C = mC_p \). So that a larger diameter branch pipe has more mass per unit length, and consequently more thermal capacitance. Higher capacitance increases the fluid temperature response time, (i.e., makes the response slower and more gradual).

From the point of view of different diameter branch configurations, these mechanisms—viscous damping and thermal capacitance—act against each other. In relative terms, a smaller diameter branch pipe will experience more viscous dissipation, and consequently more attenuated temperature response. At the same time, however, the smaller branch will have less thermal capacitance.

Branch temperature-time response characteristics (Fig. 18(a-e)) can be explained using these mechanisms. Figure 23 illustrates the roles of these mechanisms in accounting for the horizontal size-on-size data (i.e., \( D=2 \times 2 \), and \( D=1 \times 1 \)). In region 1, close to the tee (\( L/D < 2 \)) high viscous dissipation at the branch entrance offsets the thermal capacitance effects: thermal response characteristics for \( D=2 \times 2 \) and \( D=1 \times 1 \) are the same at \( L/D < 2 \). In this region the response is a function of \( L/D \)
Region 1: Viscous forces and thermal capacitance effects offset.
Region 2: Thermal capacitance effects dominate.
Region 3: Viscous forces and thermal capacitance effects offset.

Fig. 23 Temperature Response mechanisms. Size-on-size configuration.
only. In a middle region, $2 < \frac{L}{D} < 6$, thermal capacitance effects begin to dominate the response. As a result, temperature response curves for $D=1\times1$ are greater than those of $D=2\times2$. In this region temperature response is probably some function of distance, $X$, and of $L/D$. Further down the branch viscous effects begin to offset thermal capacitance effects; and by $L/D=6$ temperature response for both $D=2\times2$ and $D=1\times1$ is the same. So in this region the response is a function of $L/D$.

Figure 24 illustrates the role of the governing mechanisms in accounting for non size-on-size data (i.e. configuration $D=2\times1$). Comparison of temperature time response characteristics of $D=2\times2$ and $D=2\times1$ suggests that in a region, $L/D < 6$, viscous and thermal capacitance effects offset each other: temperature response for $D=2\times2$, and $D=2\times1$ is the same for $L/D=2, 4, 6$. In this region the temperature response of a non size-on-size is a function of $L/D$ with respect to the response of a size-on-size. In region 2, $L/D > 6$, viscous forces begin to dominate; this results in smaller thermal response for $D = 2\times1$ than for $D = 2\times2$.

Branch temperature time characteristics for the vertical orientation shows: (1) the response for configuration $D=1\times1$ is greater than for both $D=2\times2$, and $D=2\times1$; and (2) the difference between horizontal and
Region 1: Viscous Dissipation and Thermal Capacitance Effects Offset.
Region 2: Viscous Effects Dominate.

Fig. 24 Temperature Response Mechanisms. Non Size-on-size Configuration.
vertical temperature response was much less for D=1x1. The reason for these two observations is that the wall of a horizontal branch "sees" only a certain part of the entire fluid cross-sectional area, while the wall of a vertical branch "sees" the entire fluid cross-sectional (see Figure 25). In other words natural convection effects will enhance thermal response on one side of the horizontal branch while attenuating the response of the other side. The temperature response averaging process used to generate the characteristic curves was weighted toward the response enhanced by natural convection; instead of being an average of top and bottom responses for a given transient. In this sense the horizontal branch "sees" only a part of the cross section. On the other hand the vertical branch fluid does not have the same natural convection flow pattern. The branch wall "sees" the entire cross section. So horizontal branch response is greater than vertical branch response.

The D=1x1 configuration has greater thermal response than the D=2x2 configuration because both are more dependent on the branch fluid mass (the vertical case "sees" the entire cross-sectional area). Consequently the vertical branch response is more dependent on fluid thermal capacitance. A larger diameter branch will have
Fig. 25 Temperature Response Interpretation.
more thermal capacitance; its response will be slower.

It should be emphasized again that these mechanisms are tentative descriptions of the thermal response phenomena. More work has to be carried out to provide verification.

2.4.5 Accuracy and Experimental error

2.4.5.1 Accuracy.

Considerations of how closely the experimental tee-branch prototypes simulate real and existing tees are presented qualitatively. The experimental run step inputs only approximated ideal temperature steps; however, I suspect temperature transients in real piping systems are no more severe. Heat loss from the experimental PVC branch arrangements to the environment is probably less than in real cases. (PVC has lower conductivity than metal by a factor of 1000). As such during a temperature transient the branch approximates adiabatic boundary conditions.

In general, because of the variety of piping systems that experience secondary flow phenomena it would be fair to conclude that the experimental tee arrangements are a
good model of existing piping configurations.

2.4.5.2 Experimental error

Experimental error due to instrumentation, and measurement performance (i.e., hand curve fitting, visicorder trace reading) for the branch temperature response characterization is estimated at 10%. See Appendix C(I) for quantification of the error. Experimental error in determining the dimensionless branch velocity (i.e., rotameter values, and visicorder traces) is estimated at 5.8%. The aforementioned conclusions regarding the data were made with these experimental error values in mind.

Some remarks on lab technique are worth noting. As is suggested in the temperature time characteristics for the D=2x1 configuration in vertical orientation, entrained bubbles sometimes entered the vertical branch. Displacement of the branch fluid would occur to varying degrees. For the D=2x1 configuration this displacement was severe. Most experimental runs were terminated after 90 seconds.
To commence a temperature transient test, it is necessary to have the entire tee system at a steady state temperature $T_i$. Sometimes it was difficult to determine when such a state was reached for locations far from the tee crotch (i.e., $L/D=8,12$). Since secondary flow is considerably less at these branch locations the fluid takes a long time to reach the initial temperature. One or two experimental runs were made that did not have this problem; the conclusions and temperature time characterization should not change.

One last remark, most of the experimental error lies in reading the visicorder traces, and data reduction via hand curve fitting methods.
2.5 DESIGN IMPLICATIONS

2.5.1 Motivation

A set of branch temperature time response characteristic curves for use in thermal fatigue analysis is needed. These response characteristics must lead to conservative thermal stress analysis so that safety factors can be maintained.

Large temperature gradients at the branch pipe wall will usually lead to large thermal stresses. Therefore, maximum response curves from all the data at each location should be used as conservative temperature characteristics. In addition to large temperature gradients, those branch locations near the tee entrance will experience higher stresses if the lapse times for the temperature transient to arrive are small. The quicker the branch wall receives the thermal transient the more severe the thermal shock. With these considerations in mind, maximum values for the dimensionless branch velocity at each location should be used.

2.5.2 Design Temperature Response Characteristics
Maximum temperature response curves from all experimental data, at each branch location and for each tee configuration, were obtained. The temperature response curves are independent of lapse time; in most cases far from the branch entrance, lapse time information is not needed. An additional measure to ensure conservative results is to alter the response curves by simulating a temperature step response. For example, Figure 26(a) depicts typical temperature response curves with a run that takes tss seconds to reach steady state. The step response simulation assumes that if the run were a step input the branch response would be quicker, linearly scaled up, for the time period t < tss. Figure 26(b) illustrates this step response simulation. This method is performed on the maximum temperature time response characteristics. The results are shown in Figures 27(a-f) for the various configurations tested.

For use in thermal analysis with secondary flow, the following procedure is recommended: (1) if the piping system is the same size as the experimental tee branch system -- D=2"x2", D=1"x1", or D=2"x1" -- the design curves corresponding to those sizes (Fig. 27(a-f)) should be used; (2) if the piping is larger than 2" diameter pipe and of different tee configuration (say D=1x1/2) then D=1x1 design curves, Fig. 27(e,f), should be used. The
Fig. 26(a) Response to actual temperature input

\[ \frac{\Delta T_1}{\Delta T_2} \]

Fig. 26(b) Scaled response to simulated step input

\[ \frac{\Delta T_1}{\Delta T_2} \text{ step} = \frac{\Delta T_{\text{branch}}}{\Delta T_{\text{run}}} \times 1.0 \]

\[ = \frac{0.5}{0.8} = 0.63 \]
Fig. 27(a) \( D = 2 \times 2 \). Maximum response curves with step simulation. Horizontal branch. \( \text{Re} = (3.2 - 7.8) \times 10^4 \)

Fig. 27(b) \( D = 2 \times 2 \). Maximum response curves with step simulation. Vertical branch. \( \text{Re} = (3.5 - 7.7) \times 10^4 \)
Fig. 27(c)  \( D = 2 \times 1 \). Maximum response curves with step simulation. Horizontal branch.  \( \text{Re} = (2.18 - 8.9) \times 10^4 \)

Fig. 27(d)  \( D = 2 \times 1 \). Maximum response curves with step simulation. Vertical branch.  \( \text{Re} = (1.8 - 2.3) \times 10^4 \)
Fig. 27(e) D = 1x1. Maximum response curves with step simulation. Horizontal branch. \( \text{Re} = (2.5 - 11.5) \times 10^4 \)

Fig. 27(f) D = 1x1. Maximum response curves with step simulation. Vertical branch. \( \text{Re} = (3.18 - 9.62) \times 10^4 \)
reason for the latter recommendation is, as discussed in Section 2.4.4.2, that although larger diameter branches will have less viscous dissipation effects they will have more thermal capacitance effects. Response characteristics for D=1x1 are assumed to be conservative for larger diameter pipes.

Thermal response tests were not performed for a secondary flow branch in the vertical down position. It is assumed that the down orientation and the up orientation are symmetric with respect to secondary flow thermal response, except that natural convection attenuation will occur in opposite temperature directions. As such it is suggested that the vertical up design curves be use for both vertical orientations adjusting for natural convection as judged appropriate.

2.5.3 Design Curve of Dimensionless Branch Velocity

Maximum experimental values of dimensionless branch velocity enveloping all three configurations were obtained. (See Appendix B for the raw data). The results are plotted in Figure 28. This curve can readily be used for analysis of 2", and 1" diameter piping systems; it is unclear, however, that the curve is conservative for much
Fig. 28 Design curve: Dimensionless branch velocity $V_b/V_r$ as a function of $L/D$. (Maximum values). For use with branch size: $1" \leq D \leq 2"$.

Fig. 29 Design curve: Branch velocity $V_b/V_r$ (D/s/ft/s) as a function of $L/D$. For use with branch sizes: $D \geq 2"$.
larger pipe diameters, say 24" lines. The data, plots of \((V_b/V_r)\) versus \(L/D\) for different configurations, suggests that as the branch diameter increases the dimensionless branch velocity also increases due to decreasing viscous forces. Undoubtedly some limit will be reached. That limit was not investigated. One suggestion is to use a plot of the ratio \((V_b/V_r)\) in \([D/s/ft/s]\) versus \(L/D\); Figure 21 showed that this function correlates the data better. With this in mind the curve in Figure 28 can be converted (using \(D=2''/12\)) into a plot of \(V_b/V_r\) \([D/s/ft/s]\) versus \(L/D\), (see Appendix B). The design curve, Figure 29, is recommended for use in thermal analysis where branch pipe sizes are larger than 2". The lapse time at each branch location is obtained simply:

\[
L(t_{L(i-2)-(i)}) = \frac{2D}{[V_b/V_r]_i} \times V_r
\]  

(2)

where:
- \(t_L\) = lapse time between locations \([\text{sec}]\)
- \(V_b/V_r\) = velocity ratio \([D/s/ft/s]\).
- \(V_r\) = run velocity \([\text{ft/s}]\).
- \(V_b\) = branch thermal propagation velocity \([D/s]\).
- \(D\) = branch diameter \([\text{ft}]\).
- \(i\) = locations 2, 4, 6, 8, ... etc
2.6 CONCLUSIONS FOR BRANCH THERMAL RESPONSE

An experimental secondary flow branch-tee arrangement was built. Results of branch thermal response tests indicate the presence of 3 governing secondary flow mechanisms. Natural convection attenuates the temperature response at the top of a horizontal branch during a down transient, the bottom of a horizontal branch during an up transient, and a vertical up branch during a down transient. As would be expected, natural convection effects decrease as the run Re increases.

The branch secondary flow phenomenon can be thought of as fluid being pumped into an infinite cavity; the propagating momentum decays exponentially along the branch due to viscous damping at the branch wall shear layer. As the branch diameter decreases, viscous forces increase; less momentum is transferred to the branch from the main line flow. This tends to attenuate the branch temperature response with respect to a larger diameter branch. At the same time, however, a decrease in diameter decreases the fluid thermal capacitance along the branch. This tends to enhance branch temperature response with respect to a larger diameter branch. These two mechanisms act against each other. Near the branch entrance, \( L/D < 2 \), the
mechanisms offset each other and the temperature response is a function of L/D only. From then on thermal capacitance effects begin to show up; the extent of these effects and the region in which they appear depends on whether the tee is a size-on-size or a non size-on-size tee arrangement.

Design temperature response and velocity propagation curves based on maximum data values were generated for use in branch thermal fatigue analysis. These are found in Figures 27, 28, 29. The temperature response curves define branch temperature transients as a function of run temperature transient. These are presented independent of lapse times. Lapse time information is provided in the branch velocity design curve.
3.0 BRANCH HEAT TRANSFER COEFFICIENTS

3.1 INTRODUCTION

3.1.1 Objective

Characterization of branch film coefficients is needed to complete the description of the branch piping heat transfer. To this end a heater unit was designed that could be moved along the branch pipe. A known heat flux was introduced through the heater unit to the fluid. Thermocouple probes monitored the temperature difference between fluid and pipe wall; and bulk film coefficients were determined for two locations along the branch pipe. The influence of such parameters as run flow Reynolds number Re, branch orientation (horizontal or vertical) was investigated. The run fluid temperature and flow rate were kept constant for a given experimental test.

3.1.2 Assumptions
A major assumption underlying this experimental approach is that during real transient conditions the secondary flow film coefficients are primarily velocity dependent. The film coefficients are assumed to exist prior to any temperature transient as a function of the secondary flow structure itself. As such the effects of friction, due to pipe material, and temperature were not investigated.

The film coefficients determined by use of the heater units are local coefficients that describe a developing thermal boundary layer. It is assumed that information from these coefficients is applicable to real transient conditions.

The heater units introduce a known heat flux to the branch secondary flow. Changes in the secondary flow microstructure due to temperature gradients are assumed to be representative of real transient conditions.
3.2 EXPERIMENTAL APPARATUS

3.2.1 List and Specification of Equipment


2. Omega Type T thermocouple wire. Gauge 30.


4. Digitec 26c Multimeter.


7. Hotwat Band Heaters:  
   1. 2" Diameter heater:  
      1. Rated Power = 350 W.  
      2. Rated Voltage = 120 V.  
   2. 1" Diameter heater:  
      1. Rated Power = 85 W.  
      2. Rated Voltage = 110 V.  
   3. Power Tolerance = 5%.

8. Precision Thermometer. Ton 11257. Range = 40-108 C. Scale 0.03 C.


10. Superior Electric Powerstat(Variac). Type 116. Output voltage = 0-140 V. Max. Amp. = 7.5 amps.
3.2.2 Experimental Design

The first objective in designing the heater units was the development of a heating source that could generate a steady, uniform, and measurable heat flux to the secondary flow. For this purpose commercial Hotwat Band heaters were purchased. They were the most inexpensive and convenient pipe heaters available. An alternative, construction of a heating coil, would have led to complicated calibration procedures and overrun time schedules. Two sizes were obtained: (1) a 2" diameter, 2" long heater for use with the D = 2x2 configuration; and (2) a 1" diameter, 1" long heater for use with both D = 2x1 and D = 1x1 configurations. See Figure 30. The Band heaters wrap around the outside surface of a pipe and are secured by tightening the connection bolts.

It was decided to wrap the Band heaters around a metal ring rather than the PVC piping directly. The metals used, Copper for the 1" unit and Cartridge Brass for the 2" heater unit, have conductivities 100-1000 times greater than PVC. As such, less power is needed to provide the same temperature difference. Temperature measurement is thus more reliable. Figure 31 illustrates the metal ring and PVC assembly designed for this purpose. The metal rings were machined for close contact between
Fig. 30 Hotwat Band Heaters

1" Band Heater (One Piece)

2" Band Heater (Two Piece)
Fig. 31 Metal Ring And PVC Pipe Assembly

- Epoxy cement
- Brass ring for 2" pipe
- Copper ring for 1" pipe
- PVC piping
their outside surfaces and the Band heaters. A high thermal conductivity grease was applied to the contact surface to enhance heat transfer. Epoxy cement was applied to the ring-pipe socket connection. Use of epoxy allowed for easy disassembly of the units; consequently the units could be moved easily to different branch locations.

Proper measurement of ring inside surface and fluid temperatures was another design objective. Two thin grooves were milled on the outside surfaces of the metal rings, (see Fig. 32). Thermocouple beads were silver soldered to the middle of the ring. The trailing wires were strung out through the grooves. Two thermocouples, at the top and bottom of the rings, were used to provide adequate information regarding film coefficients at the top and bottom of the branch. Knowing the outside ring temperature, the inside surface temperature can easily be calculated. Measurement of the fluid temperature was performed by use of a moveable probe, through which the thermocouple wire was threaded, (see Fig. 33). There are three thermocouple probes altogether for measuring the temperature drop between surface wall and fluid—TC1, TC2, TC3.
Fig. 32 Grooves on Metal Ring
Fig 33 Complete Heater Unit Assembly

- TC3 fluid temperature thermocouple
- Band Heaters outside surface thermocouple:
  - TC4 leads from Variac
  - TC2 ring bottom surface thermocouple
- Glass wool insulation
- Flow direction
- Ring top surface thermocouple TC1
A layer of glass wool, wrapped around the outside of the Band heaters, provided insulation against heat loss from the units, (Fig. 33). To account quantitatively for heat loss due to natural convection from a heater unit to the environment, a thermocouple was soldered to the outside surface of the Band heaters (TC4). Temperature measurements from TC4 provide information necessary for heat loss calculations.

The "in-line" heater units did not interfere with rotation of the branch pipe into the vertical and horizontal positions as used in the thermal response experiments. Thus information regarding orientation could be investigated.

To make the film coefficient information compatible with the thermal response information the heater units were placed such that their mid points lay on the \( L/D = N \) line, (see Fig. 34). This way we are assured that the local film coefficients correspond to the same branch locations as the temperature response data.

A Variac was used as power source to the Band heaters. As such, the heat flux could be varied. A voltmeter was attached to the Variac leads to ensure accurate measurement of the Band heater voltages. The four thermocouple voltage signals were sent to a
potentiometer via a multiple switch bus. Since the temperature measurements were for steady-state conditions there was no concern with reading the potentiometer quickly from TC1 through to TC4.
3.3 EXPERIMENTAL PROCEDURE

3.3.1 Calibration

To ensure accurate temperature measurements all five thermocouples TC1, TC2, TC3, TC4, and TC5 (upstream of the tee configuration) were calibrated against a precision thermometer. The precision thermometer itself was calibrated to an ice point reference. Each metal ring, with its two soldered thermocouples TC1 and TC2, was immersed in a bath of heated aluminum pellets. Contained inside a large insulated flask the aluminum pellets cooled off slowly. The precision thermometer was placed next to the metal ring. A transient calibration of TC1 and TC2 was performed for half an hour. It was found the TC1 and TC2 (for both heater units) were calibrated against the precision thermometer to within reading accuracy of the potentiometer—1 °F. A steady-state calibration was performed between TC1, TC2 and TC3, TC4, TC5 with the system emptied and dried and then with the system full of water at a constant temperature. It was found that TC3, TC4 and TC5 were calibrated to TC1, TC2 to within accuracy of the potentiometer.
An important issue is calibration of the experimental heater units. For this purpose each heater unit was placed in the run pipe. Experimental tests over a range of Re were performed. With this information two concerns can be addressed. First, the nature of the secondary flow suggests that film coefficients at the branch top and bottom may not be the same. It is important to know how much of this discrepancy is due to the flow and how much is due to error in the experimental heater units themselves. Film coefficients in the run pipe should be the same top and bottom, thus a calibration can be performed. A second concern is that the two heater units, the 2" and the 1" units, may not be calibrated to one another. In other words, the two units may predict different film coefficients given the same flow Re. Film coefficients in turbulent, fully developed pipe flow are well known. The author uses McAdams[9] correlation, eq. 3, in providing a calibration standard for the heater units.

\[ h_{TH} = \frac{K}{D} 0.023 \ Re^{0.8} \ Pr^{0.4} \]

(3)

where:  
\( h_{TH} \) = Theoretical film coefficient.  
\( K \) = Fluid conductivity.  
\( D \) = pipe diameter[ft].  
\( Re \) = Reynolds number.  
\( Pr \) = Prandtl number.
The experimental film coefficients will be larger than $h_{TH}$ because they describe a developing thermal boundary layer. If the scaling factors due to the developing thermal layer are the same for both heaters then we conclude the heater units are calibrated to one another. The results of this calibration procedure are presented in results Section 3.4.

3.3.2 Test Procedure and Parameter Variation

The experimental test procedure focused on four parameters: (i) size configuration--3 sizes were used; (ii) run flow Re; (iii) branch orientation(vertical or horizontal); and (iv) branch location. Two branch locations were chosen for the film coefficient investigation; these are $L/D = 2, 4$. The branch dimensionless velocity curves of the Section 2.0 can provide information regarding the phenomena at $L/D > 4$. Each experimental test began with choice of branch location, branch orientation, and run flow rate for a given tee configuration. The piping system was filled with running water; its temperature recorded at TC5. The Variac was turned on and tuned so that the reading at TC4(soldered to the Band heater's outside surface) did not exceed 180 °F. The concern here was that the epoxy cement
or the PVC might overheat and melt. TC1 and TC2 were prevented from exceeding 130 F, also for this purpose. Once all five thermocouple readouts stabilized their temperatures were recorded. These five temperature values are necessary for calculation of the film coefficients. The voltage at the Variac lead connection was recorded; it provides a measure of the heat flux generated by the Band heater. The piping system was emptied after completion of each run. And the procedure would begin anew for each test. There was a concern that if more than two tests were performed consecutively the secondary flow would reach a point where it would not be able to get rid of the heat as fast as it gained it; the increase in temperature could adversely affect the secondary flow microstructure. Occasionally air bubbles from the run flow would run along the branch top. The bubbles' added heat resistance is undesirable. When bubble accumulation became pronounced the test was aborted. For each tee configuration, branch location, and orientation a range of Re was investigated: $2.5 \times 10^4 \leq Re \leq 10 \times 10^4$. To ensure reproducibility each test was exactly repeated a minimum of two times. A total of 106 runs were performed.

The fluid probe controlling TC3 could be moved up and down between top and bottom branch surfaces. Three readings were typically taken: at the branch center line,
1/4 of the diameter from the top, and 1/4 of the diameter from the bottom. In the final film coefficient calculation the centerline fluid temperature was used.

3.3.3 Data Reduction

The five thermocouple temperature measurements and Variac voltage measurement from a given experimental run are used to calculate the branch film coefficients. The film coefficient calculation method is detailed in Appendix D. From this calculation method two film coefficients are determined; one corresponding to the branch top surface, the other corresponding to the bottom surface. Discrepancies between these two coefficient values are calibrated using the run experimental film coefficients. A dimensionless branch film coefficient \( \frac{h_b}{h_r} \) is formed by dividing the experimental branch coefficient \( h_b \) by the experimental run coefficient \( h_r \). The information is presented as a function of dimensionless branch distance \( L/D \).
3.4 EXPERIMENTAL RESULTS AND DISCUSSION

3.4.1 Film Coefficient Calibration

Experimental run film coefficient data is used to calibrate the heater units, and subsequently the branch film coefficient data. See Appendix E for the raw run film coefficient data. Figures 35(a,b) are plots of the run film coefficient as a function of Re; superimposed on each plot is a curve representing the theoretical run film coefficient, $h_{TH}$, as a function of Re based on equation 3. Because the heater units have a developing heat flux length of $L/D = 1/2$ the experimental film coefficients are a function of a developing thermal boundary layer. As such, these experimental coefficients are expected to be larger than the theoretical coefficients based on a fully developed thermal layer, (eq. 3). This is seen in the 2" heater unit data Figure 35(a). Figure 35(b), however, indicates that the top film coefficient corresponding to TC1, is consistent with respect to this entrance effect, but the bottom film coefficient, corresponding to TC2, is not. This must be due to instrument error in the 1" heater unit. Therefore the 1" branch film coefficient data is calibrated such that only data corresponding to TC1 is used. Figure 35(a) indicates experimental scatter
Fig. 35(a) 2" Heater Unit Calibration Data. (Unit in run pipe).

Fig. 35(b) 1" Heater Unit Calibration Data. (Unit in run pipe)
of 26% between TC1(top) and TC2(bottom). This discrepancy is attributed to instrument error. The 2" branch film coefficient data is calibrated such that where the discrepancy between TC1 and TC2 is less than 26% the average of TC1 and TC2 is used. A discrepancy greater than 26% is attributed to the secondary flow structure. Recall that at some branch locations the fluid is moving into the branch, this fluid has more of the structure of the run flow and is more dependent on run Re; and at other locations the fluid is moving out of the branch, this fluid is more damped and less dependent on run Re. To have different film coefficients at top and bottom is not a surprise. Appendix F contains all the raw (uncalibrated) branch film coefficient data. For a situation where the discrepancy between TC1 and TC2 is greater than 26% that side of the heater unit (TC1 or TC2) which shows more dependence on the run Re is used in the data reduction.

Having calibrated the branch data for discrepancies between the two thermocouple measurements (TC1 and TC2) the next step is presentation of a dimensionless local film coefficient \((h_b / h_r)\) as a function of dimensionless distance \((L/D)\). For the size on size data \((D = 2\times 2, D = 1\times 1)\) this amounts to dividing the experimental values of branch and run film coefficients at
the appropriate Re. This procedure eliminates the heater unit entrance effect due to the developing thermal layer. Note that the entrance effect scaling factor for the 2" heater unit is \( h_r / h_{TH} = 3-5 \), while the scaling factor for the 1" heater unit is \( h_r / h_{TH} = 1.3-1.6 \), (see Figs. 35(a,b)). For \( D = 2x2 \) and \( D = 1x1 \) this discrepancy in scaling factor washes out; for the \( D = 2x1 \) data, however, this discrepancy must be taken into account. For \( D = 2x1 \) data the scaling factor, at the appropriate Re, of the 1" unit is used to obtain a correct interpretation of \( h_b / h_r \). Appendix G contains all the reduced experimental run and branch film coefficient data.

3.4.2 Effects of Parameter Variation

The reduced and calibrated branch film coefficient data is presented in the form of a dimensionless branch film coefficient \( (h_b / h_r) \) as a function of a dimensionless branch distance \( (L/D) \). This data is illustrated in Figures 36(a,b) for \( D = 2x2 \), Fig. 37(a,b) for \( D = 1x1 \), and Fig. 38 for \( D = 2x1 \).

Over the Re range investigated (essentially one decade: \( 10^4 \leq Re \leq 10^5 \)) the effect of run Re on the dimensionless film coefficient is small. In other words,
Fig. 36 Dimensionless branch film coefficient (Hb/Hr) as a function of L/D. D = 2×2.
(a) Horizontal branch, (b) Vertical Branch. $3.2 \times 10^4 \leq Re \leq 8.1 \times 10^4$. 
Fig. 37 Dimensionless film coefficient $H_b/H_r$ as a function of $L/D$. $D = 1 x 1$. (a) Horizontal branch. (b) Vertical Branch. $2.5 \times 10^5 \leq \text{Re} \leq 10 \times 10^4$. 

*data points*}

*least square line*
Fig. 38 Dimensionless branch film coefficient $H_b/H_r$ as a function of $L/D$. $D = 2x1$. Horizontal branch.
as the run Re increases, the branch film coefficient, $h_B$, and the run film coefficient, $h_r$, increase proportionally so that the ratio $h_B/h_r$ remains the same. Since the film coefficients are functions of velocity this suggests that as the run velocity increases, the local branch velocity increases proportionally. This result is consistent with the behavior of the dimensionless propagation velocity($V_b/V_r$) of Section 2.0. Over the same Re range ($V_b/V_r$) is independent of Re at each branch location.

The effect of branch orientation on the dimensionless branch film coefficient is also small. For both the $D = 2 \times 2$ and $D = 1 \times 1$ configurations the difference between horizontal and vertical coefficients is roughly 8%, (see. Fig. 36 and 37). Data for the $D = 2 \times 1$ vertical case is incomplete. The temperatures of TC1 and TC2 never reached steady-state. The secondary flow was probably unable to get rid of the heat introduced by the heater units fast enough to prevent the units from overheating (i.e. ever reaching a steady-state). That the branch film coefficient is relatively independent of branch orientation is consistent with results of the branch propagation velocity($V_b/V_r$). It too is independent of branch orientation.
There are two major results regarding the effect of size configuration on the dimensionless branch film coefficient. Figures 39(a,b) present the data for all three configurations on two convenient plots (for horizontal and vertical orientations). The slope of the film coefficient curves for the D = 2x1 and D = 1x1 are similar and steeper than the slope for the D = 2x2 curve. This steepness of slope is a measure of the local velocity decay rate along the branch due to viscous dissipation. The data indicates a higher velocity decay rate for the 1" branch than for the 2" branch. This is consistent with a result from Section 2.0: smaller diameter branch will have higher viscous forces than a larger branch.

A second result concerns the absolute magnitudes of the branch film coefficients. Figure 39(a) indicates that the coefficients for D = 2x1 and D = 1x1 are similar to within 11%. This is consistent with the results of (Vb/Vr) from Section 2.0. The coefficients for D = 1x1, however, are greater than for D = 2x2 by 35% at L/D = 2. From Fig. 19, note that the dimensionless propagation velocity (Vb/Vr) for D = 2x2 is greater than for D = 2x1, and D = 1x1 at all branch locations. Recall, (Vb/Vr) is a measure of the propagation along the branch of a thermal disturbance. (H_b / h_r) is a measure of the local secondary flow intensity as a function of run intensity. The
Fig. 29  Dimensionless Branch Film Coefficient as a function of I/D.
(a) Horizontal Branch, (b) Vertical Branch.
results in Figures 39(a,b) suggest that the local secondary flow intensity close to the tee crotch, in terms of L/D, is greater for a smaller diameter branch. At L/D = 4 the film coefficients for D = 2x2 are greater by 11% than those for D = 1x1. This additional result suggests that the branch film coefficient is a function of absolute branch distance (as opposed to L/D) near the tee crotch; further away say L/D = 4 the branch coefficient is a function of L/D. More information, however, is needed to resolve this issue.

3.4.3 Accuracy and Experimental Error

3.4.3.1 Correspondence to actual system events

The tests performed in this project were for branch-tee sizes in the 1" and 2" diameter range, 2.5x10^4 \( \leq \text{Re} \leq 10^5 \), and run flow at a constant temperature of 65 °F-75 °F. Real conditions in power plant piping systems involve Re ranges of 10^4 \( \leq \text{Re} \leq 10^5 \), pipe diameters of 1" < D < 20", and temperature transients as high as \( \Delta T = 350 \) °F. How well does the data here apply to those real conditions? The effect of run Re on \( h_b / h_r \) was found to be small; so it would be a fair hypothesis to say that above \( \text{Re} = 10^5 \) the effect remains small.
Increasing branch diameter size was found to decrease $h_b/h_f$ at $L/D = 2$. Large temperature transients will make the film coefficients more dependent on temperature and less on fluid velocity. Natural convection will be enhanced in these cases, adding another effect to the secondary flow structure. It is not clear whether the data presented here applies in cases with extreme temperature conditions. Information regarding temperature dependence is needed to resolve these issues.

3.4.3.2 Experimental error

Experimental error due to measurement performance (e.g., thermocouple temperature readings, Variac voltage readings) for this film coefficient characterization is estimated at 7%. See Appendix C(II) for error quantification.

The film coefficient calculation method (Appendix D) is rigorous, but nevertheless does make assumptions regarding heat loss surface areas, and heat flux uniformity. These assumptions introduce some uncertainty to the data reduction.
The heater units were carefully machined and assembled. The Hotwat heaters, however, are a source of uncertainty. Faith must be placed on the manufacturer's specifications that advertise uniform heat flux, and accuracy to within 5%. Experimental scatter (from run film coefficient data) provides evidence of additional uncertainties. The two piece 2″ Band heater, for example, has to be tightened properly onto the ring assembly. A high conductivity grease was applied between ring outside surface and heater inside surface. Possible deformation of the heater upon tightening the connection bolts can lead to an irregular heat flux distribution. It would seem most of the experimental error lies in the abovementioned heater unit irregularities.
3.5 DESIGN IMPLICATIONS

3.5.1 Motivation

A characterization of the branch secondary flow film coefficients is needed for use in thermal analysis. This characterization must apply to the branch region \(0 \leq L/D \leq 12\). In addition, the characteristic curve generated for this purpose must lead to conservative thermal stress analysis in order that safety factors be maintained.

3.5.2 Design Curves

Results of the film coefficient experiments only provide film coefficient data (for the three configurations) at \(L/D = 2, 4\). This information was presented in Figures 39(a,b) for the horizontal and vertical orientations. Figure 40 condenses that horizontal and vertical data into one set of curves. (See Appendix H(II) for design implication numbers.) This set of curves is used in the extrapolation procedure to generate a design characteristic curve.
Fig. 40 Condensed branch film coefficient data as a function of L/D
The dimensionless branch velocity curves (Fig. 19: \( \frac{V_b}{V_r} \text{ vs } L/D \)) are used together with the dimensionless branch film coefficient data (Fig. 40) to extrapolate the film coefficient information into the region \( L/D > 4 \). The author assumes an exponential relationship exists between film coefficient and branch velocity of the form:

\[
\left( \frac{h_b}{h_r} \right) = a \left( \frac{V_b}{V_r} \right)^b
\]

A power curve fit is performed using data at \( L/D = 2, 4 \), (see Appendix H(III)). This relationship is assumed to exist at branch locations \( L/D > 4 \). Figure 41 illustrates the extrapolated film coefficient curves as a function of dimensionless distance (L/D). The branch region \( L/D < 2 \) is extrapolated by performing a linear regression from data points at \( L/D = 2, 4 \). This is done because no branch velocity information is available in this region. Note that the \( D = 2 \times 2 \) curve is very flat. One might question whether the same power relationship really holds in the region \( L/D > 8 \) as it does in the region \( 2 \leq L/D \leq 4 \).
As suggested in Section 3.4, the branch film coefficient \( \frac{h_b}{h_r} \) may be a function of absolute branch distance rather than dimensionless branch distance. Figure 42 investigates this hypothesis. The suggestion is that near the crotch, say \( L/D < 4 \), the coefficients are more a function of absolute distance, say in inches. At \( L/D > 4 \) the relationship might be a function of \( L/D \). So that at \( L/D = 1 \) for the 2" branch, which is 2" in absolute distance, \( \frac{h_b}{h_r} \) might be around 0.4. Whether \( \frac{h_b}{h_r} \) is a function of absolute distance or dimensionless distance is not clear from Figures 41 and 42. Clearly more information is needed to decide one way or another.

Construction of a design film coefficient curve using the extrapolated coefficient results is based on two considerations. Near the tee crotch the definition of conservative branch film coefficients depends on the relative wall thicknesses of the run and branch pipes (see Fig. 43) coupled with the \( h_b/h_r \) ratio. For example, when the branch and run thicknesses are the same, the run pipe will always respond more quickly than the branch because it has the higher film coefficient. However, if the run pipe is thicker than the branch \( (tr > tb) \), there could be a very small thermal discontinuity stress, or in fact the branch could respond more quickly than the run. If the branch pipe is thicker than the run \( (tb > tr) \), the run
Fig. 41 Dimensionless branch film coefficient $\frac{H_b}{H_r}$ as a function of $L/D$. For all configurations.
Fig. 42 Dimensionless branch film coefficient $H_b/H_r$ as a function of absolute distance from tee. All configurations.
Fig. 43 Tee crotch: Discontinuity stress considerations.
pipe will always respond more quickly, but it is still important to evaluate the effects of various branch film coefficients due to possible higher through the wall gradient thermal stresses in the thicker branch. The thermal stress analyst must use these branch heat transfer coefficients with proper engineering judgement.

Figure 44 presents the final branch film coefficient design curve, (see Appendix H(iv) for numerical values). In the region $L/D < 4$ two cases, A and B, are presented. Case A is an average of the $D = 1x1$ and $D = 2x1$ data. Case B is from the $D = 2x2$ data. All three cases were averaged for the region $L/D > 4$. New values at $L/D = 0$ were obtained by performing a linear regression from new values at $L/D = 2, 4$. Note the author plots the design curve as a function of $L/D$. 
Fig. 44  Design curve: Dimensionless branch film coefficient $H_b/H_r$ as a function of dimensionless branch distance $L/D$.
3.6 CONCLUSION FOR BRANCH HEAT TRANSFER COEFFICIENTS

An experimental heater unit was designed and built to determine branch secondary flow film coefficients at locations L/D = 2, 4. Results of film coefficient tests on three tee configurations indicate that the dimensionless branch film coefficient is fairly insensitive to run Re(for the range $2 \times 10^4 \leq \text{Re} \leq 10^5$) and branch orientation. The effect of branch size confirmed the existence of a viscous damping mechanism suggested in Section 2.0. Furthermore, comparison of film coefficients for the three configurations suggests that near the tee crotch the coefficients may be a function of absolute distance rather than dimensionless distance L/D. More information, however, is needed to confirm this hypothesis.

A design branch film coefficient curve was generated for the region $0 \leq \text{L/D} \leq 12$ using the experimental film coefficient data and the dimensionless branch velocity ($V_b/V_r$) data. It is presented in Figure 44 as a ratio of film coefficient in the branch ($h_b$) to the run ($h_r$) for various L/D positions in the branch. See also Figure 42 for a plot of this ratio as a function of actual distance from the tee crotch. Recommendations were made regarding
relative run and branch thicknesses to ensure conservative thermal fatigue analysis.

Application of the results presented in this paper to real transient conditions must be made with knowledge of the underlying assumptions. The most important assumption is that the branch film coefficients are primarily a function of velocity. As such, the effects of large temperature gradients on the coefficients was not investigated. The motivation here was to keep the approach simple and direct.
4.0 CONCLUDING REMARKS AND RECOMMENDATIONS

A run-branch tee assembly was designed and built to investigate secondary flow heat transfer phenomena in branch piping. Description of the heat transfer was divided into two parts: (1) characterization of the temperature response (to a step input), and (2) characterization of the film coefficients. Based on experimental results from these two parts, design curves for use in thermal analysis were generated.

See Section 2.6 for specific conclusions related to branch thermal response, and Section 3.6 for specific conclusions related to branch heat transfer coefficients.

Recommendations for future work in both areas can be made.

4.1 Thermal Response Characterization

The temperature transient generated in the run flow was an approximation of a true step input due to limitations of available equipment. As such, the data interpretation was not as clean as one might like. A temperature control system that could decouple the flow rate and temperature step size would generate larger, more effective temperature steps. This would make data
interpretation (and reduction) more straightforward. In this connection, investigation of temperature ramp inputs could provide valuable information.

A suggestion for work that tries to repeat what was done here: Data acquisition through use of a micro-computer would make data reduction much more efficient. Data could be stored on disk and later transferred onto magnetic tape. Then available software could easily be used for numerical curve fitting and accurate statistical data treatment. Experimental error would be greatly reduced.

The results in this paper serve as stepping stones towards the development of a model that describes the secondary flow phenomena and accounts for the experimental data. An attempt was made by the author to model the secondary flow temperature response as a diffusion process. The model failed to account for the data. Other approaches should be taken.

4.2 Film Coefficient Characterization

Experimental scatter in the heater unit data suggests the need for a design improvement. Rigorous calibration of each heater unit must accompany a new design to ensure satisfactory heater unit performance.
To resolve the issue of whether $h_b/h_r$ is a function of absolute distance or dimensionless distance, film coefficient data at $L/D = 1, 6, 8, 12$ must be taken. This data would also confirm the accuracy of the design extrapolation procedure that used $V_b/V_r$ information.

The effects of flow rate and temperature extremes on the branch film coefficients remain a question. What happens to $h_b/h_r$ for run flows with $Re = 10^6$? How does $h_b/h_r$ change with the existence of large temperature transients in run flow? A temperature transient could be included in determining the branch film coefficients. In the same manner, effects of friction (due to pipe material) on $h_b/h_r$ might be investigated.

The local secondary flow velocity was never measured directly— instrumentation for this purpose is very complicated. The branch film coefficient and branch propagation velocity results provide indirect information on the behavior of the local velocity. Nevertheless valuable information would be gained from direct measurement of secondary flow local velocity.
BIBLIOGRAPHY


APPENDIX A. REPRESENTATIVE VISICORDER TRACES.

Output from the Honeywell Visicorder for thermal response tests takes the form of temperature as a function of time on a UV sensitive trace. The conversion factors for the trace are: 10 °F for every centimeter, on the temperature axis, and 10 seconds per every inch on the time axis.

Figures A-1 and A-2 are copies of representative thermal response traces. (Note these copies have been reduced in scale.)
Both tests are for an up transient horizontal branch with a run $Re = 3.8 \times 10^4$. In Figure A-1 the top of the horizontal branch is monitored. In Figure A-2 the bottom of the same horizontal branch is monitored. Both temperature step sizes are about 34°F. Note the erratic temperature fluctuations at $L/D = 4$ for the bottom locations compared to the top locations. Furthermore note the suppressed temperature responses at $L/D = 6$ for the bottom case compared to the top case.
APPENDIX B. DIMENSIONLESS BRANCH VELOCITY DATA.

\[(V_b/V_r)_o = V_o = \text{Dimensionless Branch Velocity.}\]
\[(V_b/V_r)d = V_d = \text{Propagation ratio(D/s/ft/s).}\]
\[SD = \text{Standard Deviation.}\]
\[P(E) = \text{Probable error (i.e. 0.675xSD).}\]

The following are average values from all the data:

\[D = 2 \times 2\]

<table>
<thead>
<tr>
<th>L/D</th>
<th>V_o</th>
<th>SD</th>
<th>P(E)</th>
<th>V_d</th>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.066</td>
<td>0.033</td>
<td>0.022</td>
<td>0.396</td>
<td>0.264</td>
<td>0.530</td>
</tr>
<tr>
<td>4</td>
<td>0.033</td>
<td>0.023</td>
<td>0.016</td>
<td>0.198</td>
<td>0.102</td>
<td>0.290</td>
</tr>
<tr>
<td>6</td>
<td>0.026</td>
<td>0.019</td>
<td>0.013</td>
<td>0.156</td>
<td>0.078</td>
<td>0.230</td>
</tr>
<tr>
<td>8</td>
<td>0.033</td>
<td>0.039</td>
<td>0.027</td>
<td>0.198</td>
<td>0.036</td>
<td>0.360</td>
</tr>
<tr>
<td>12</td>
<td>0.017</td>
<td>0.024</td>
<td>0.016</td>
<td>0.102</td>
<td>0.006</td>
<td>0.198</td>
</tr>
</tbody>
</table>

\[D = 1 \times 1\]

<table>
<thead>
<tr>
<th>L/D</th>
<th>V_o</th>
<th>SD</th>
<th>P(E)</th>
<th>V_d</th>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.024</td>
<td>0.009</td>
<td>0.006</td>
<td>0.288</td>
<td>0.216</td>
<td>0.360</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
<td>0.012</td>
<td>0.008</td>
<td>0.180</td>
<td>0.084</td>
<td>0.280</td>
</tr>
<tr>
<td>6</td>
<td>0.009</td>
<td>0.003</td>
<td>0.002</td>
<td>0.108</td>
<td>0.084</td>
<td>0.132</td>
</tr>
<tr>
<td>8</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
<td>0.720</td>
<td>0.036</td>
<td>0.108</td>
</tr>
<tr>
<td>12</td>
<td>0.004</td>
<td>0.002</td>
<td>0.001</td>
<td>0.048</td>
<td>0.036</td>
<td>0.060</td>
</tr>
</tbody>
</table>

\[D = 2 \times 1\]

<table>
<thead>
<tr>
<th>L/D</th>
<th>V_o</th>
<th>SD</th>
<th>P(E)</th>
<th>V_d</th>
<th>MIN</th>
<th>MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.038</td>
<td>0.011</td>
<td>0.007</td>
<td>0.463</td>
<td>0.379</td>
<td>0.547</td>
</tr>
<tr>
<td>4</td>
<td>0.024</td>
<td>0.018</td>
<td>0.013</td>
<td>0.288</td>
<td>0.132</td>
<td>0.444</td>
</tr>
<tr>
<td>6</td>
<td>0.011</td>
<td>0.011</td>
<td>0.008</td>
<td>0.132</td>
<td>0.036</td>
<td>0.228</td>
</tr>
<tr>
<td>8</td>
<td>0.018</td>
<td>0.019</td>
<td>0.013</td>
<td>0.216</td>
<td>0.060</td>
<td>0.372</td>
</tr>
<tr>
<td>12</td>
<td>0.006</td>
<td>0.007</td>
<td>0.005</td>
<td>0.072</td>
<td>0.012</td>
<td>0.132</td>
</tr>
</tbody>
</table>

The following are maximum values from all the data:

<table>
<thead>
<tr>
<th>L/D</th>
<th>V_o</th>
<th>V_d (using D = 2&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.148</td>
<td>0.888</td>
</tr>
<tr>
<td>4</td>
<td>0.088</td>
<td>0.528</td>
</tr>
<tr>
<td>6</td>
<td>0.066</td>
<td>0.396</td>
</tr>
<tr>
<td>12</td>
<td>0.054</td>
<td>0.324 (Fig. 28)</td>
</tr>
</tbody>
</table>
APPENDIX C. QUANTIFICATION OF
INSTRUMENT ERROR.

I. Estimated Error for Thermal Response characterization.

(1). Temperature Response.

(A). Due to reading temperature values from trace.

Accurate up to 0.1 cm.
Average of 15 F x 1 cm/ 10 F = 1.5 cm.

\[ \varepsilon_{\theta r} = \frac{0.1}{1.5} = 7\% \]

(B). Due to reading time scale from trace.

Accurate up to 0.1 inches.
Average of 12 inches.

\[ \varepsilon_t = \frac{0.1}{12.0} = 1\% \]

(C). Due to curve fitting the temperature curves.

Accurate to 1 division on graph paper.
Average of 15 divisions.

\[ \varepsilon_c = \frac{1.0}{15.0} = 7\% \]

and so estimated error:

\[ \varepsilon_\theta = \sqrt{\varepsilon_{\theta r}^2 + \varepsilon_t^2 + \varepsilon_c^2} = 10\% \]

(2). Velocity Propagation calculation.

(A). Rotameter reading.

Accurate to 3% of 37 gpm = 1.11 gpm.
Average to 18.5 gpm.

\[ \varepsilon_r = \frac{1.11}{18.5} = 6\% \]

(B). Reading time scale.

Accurate to 0.1 inches.
Average of 3 inches.

\[ \varepsilon_t = \frac{0.1}{3.0} = 3.3\% \]

and so estimated error:

\[ \varepsilon_v = \sqrt{\varepsilon_r^2 + \varepsilon_t^2} = 6.8\% \]
II. Estimated Error for Film Coefficient Characterization.

since:

\[ h = \frac{q}{A \Delta T} \]

\[ \varepsilon_h = \sqrt{\varepsilon_q^2 + \varepsilon_A^2 + \varepsilon_\theta^2} \]

where:
- \( \varepsilon_q \) = Probable error in h.
- \( \varepsilon_A \) = Error due to q.
- \( \varepsilon_\theta \) = Error due to \( \Delta T \).

(A). Error due to power measurement from multimeter accurate up to 0.5 V.

\[
\begin{align*}
\bar{P} &= 350(0.5/110) = 0.007 \text{ W} \\
\bar{P}_{\text{max}} &= 318.0 \text{ W}
\end{align*}
\]

\[ \varepsilon_{q_1} = \frac{0.007}{318.0} = 1\% \]

(B). Error due to Hotwat heater = 5%. (From Manufacturer's Specifications.)

\[ \varepsilon_{q_2} = 5\% \]

(C). Error due to machining the heat transfer area (A).

\[ \varepsilon_A = 1\% \]

(D). Error due to temperature measurement from potentiometer reading.

Accurate to 0.02 mv.

Average = 20 °F, \( V = 0.4 \text{ mv.} \)

\[ \varepsilon_\theta = \frac{0.02}{0.4} = 4.4\% \]

and so:

\[ \varepsilon_h = \sqrt{(\varepsilon_{q_1} + \varepsilon_{q_2})^2 + \varepsilon_A^2 + \varepsilon_\theta^2} = 7\% \]
APPENDIX D
FILM COEFFICIENT CALCULATION METHOD

I. Thermocouples

TC1 = Outside surface of metal ring (Top).
TC2 = Outside surface of metal ring (Bottom).
TC3 = Fluid probe.
TC4 = Outside surface of band heater.
TC5 = Fluid probe upstream of tee.

Fig. D-1
II. Heat Losses

Fig. D-2

A. Conduction to PVC shoulder. \( q_{\text{Li}} \)

\[ q_{\text{Li}} = 2 \times AK \Delta T / L \]

\[ A = \pi D t + \pi (D + 2t)L \]

\[ K = \text{PVC Conductivity} \]

\[ = 0.17 \text{ BTU/ hr ft } ^\circ\text{F} \]

\[ \Delta T = \max(TC1, TC2) - TC5 \]

\[ L = 0.25 \text{ inches}. \]

Fig. D-3
B. Natural convection from outside band heater surface to atmosphere. \( \dot{q}_{BL2} \)

\[ \dot{q}_{BL2} = \frac{2\pi L (TC4 - T_{Air})}{\left( \frac{\ln(r_4/r_3)}{K_{Air}} + \frac{\ln(r_5/r_4)}{K_{INS}} + \frac{1}{r_5 h_{NC}} \right)} \]

where:
- \( K_{Air} \) = Air conductivity.
- \( K_{INS} \) = Insulation conductivity.
- \( r_3 \) = \( \xi \) to outside band heater surface.
- \( r_4 \) = \( \xi \) to insulation.
- \( r_5 \) = \( \xi \) to outside of insulation.
- \( h_{NC} \) = Natural convection film coefficient
  \[ = 0.25(\Delta T/ D)^m \]
  where \( \Delta T = TC4 - T_{Air} \)
C. Total heat loss: $(\dot{q}_{\text{loss}})$

$$\dot{q}_{\text{loss}} = \dot{q}_{L1} + \dot{q}_{L2}$$

III. Heat transfer to fluid: $(\dot{q}_o)$

$$\dot{q}_o = \dot{q}_{\text{rated}} - \dot{q}_{\text{loss}}$$

WHERE: \(\dot{q}_{\text{rated}} = \dot{P}_{\text{rated}} \times \left(\frac{\text{Volts}}{\text{Rated Voltage}}\right)^2\)

IV. Inside Surface Temperature:
For both top(Tsi) and bottom(Ts2).

$$T_{si} = T_{ci} - \frac{\dot{q}_o \ln(r_2/r_1)}{2\pi L K_{\text{ring}}}$$  \(i = 1, 2\)

V. Film coefficients: (h)

$$h = \frac{\dot{q}_o}{A \Delta T}$$

where: \(A = \pi DL\)

\(\Delta T = T_{si} - T_{C3}\)  (For both Tsi and Ts2).
APPENDIX E
HEATER UNIT CALIBRATION DATA

I. 1" Heater Raw Film Coefficient Data.

<table>
<thead>
<tr>
<th>Re</th>
<th>Experimental $H_r$ [btu/hr ft$^2$ F]</th>
<th>Theoretical $H_{TH}$ [btu/hr ft$^2$ F]</th>
<th>$H_r/H_{TH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.5 \times 10^4$</td>
<td>(top) 1054.9</td>
<td>665.0</td>
<td>1.60</td>
</tr>
<tr>
<td></td>
<td>(bot) 603.3</td>
<td></td>
<td>0.90</td>
</tr>
<tr>
<td>$3.8 \times 10^4$</td>
<td>(top) 1430.4</td>
<td>930.0</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>(bot) 756.8</td>
<td></td>
<td>0.80</td>
</tr>
<tr>
<td>$6.4 \times 10^4$</td>
<td>(top) 2010.9</td>
<td>1410.0</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>(bot) 913.7</td>
<td></td>
<td>0.60</td>
</tr>
<tr>
<td>$10 \times 10^4$</td>
<td>(top) 2624.3</td>
<td>2016.0</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>(bot) 1071.7</td>
<td></td>
<td>0.50</td>
</tr>
</tbody>
</table>

II. 2" Heater Raw Film Coefficient Data.

<table>
<thead>
<tr>
<th>Re</th>
<th>Experimental $H_r$ [btu/hr ft$^2$ F]</th>
<th>Theoretical $H_{TH}$ [btu/hr ft$^2$ F]</th>
<th>$H_r/H_{TH}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.2 \times 10^4$</td>
<td>(top) 2186.2</td>
<td>405.0</td>
<td>5.40</td>
</tr>
<tr>
<td></td>
<td>(bot) 1644.9</td>
<td></td>
<td>4.10</td>
</tr>
<tr>
<td>$5.1 \times 10^4$</td>
<td>(top) 2868.9</td>
<td>588.0</td>
<td>4.90</td>
</tr>
<tr>
<td></td>
<td>(bot) 2003.8</td>
<td></td>
<td>3.40</td>
</tr>
<tr>
<td>$6.4 \times 10^4$</td>
<td>(top) 3196.9</td>
<td>705.0</td>
<td>4.53</td>
</tr>
<tr>
<td></td>
<td>(bot) 2376.3</td>
<td></td>
<td>3.40</td>
</tr>
</tbody>
</table>

Where: $H_{TH} = \frac{K}{D} 0.023 \ Re^{0.8} \ Pr^{0.4}$

$K = 0.352$ Btu/hr ft$^2$°F
$D =$ Diameter[ft].
$Pr = 6.2$
## APPENDIX F
### RAW BRANCH FILM COEFFICIENT DATA

### I. Configuration \( D = 2 \times 2 \), \( H_b \) [btu/hr ft\(^2\) F]

<table>
<thead>
<tr>
<th>Re(x10)</th>
<th>( L/D = 2 )</th>
<th>( L/D = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HORIZONTAL</td>
<td>VERTICAL</td>
</tr>
<tr>
<td></td>
<td>Top</td>
<td>Bot</td>
</tr>
<tr>
<td>3.2</td>
<td>492.9 279.2</td>
<td>527.4 432.7</td>
</tr>
<tr>
<td></td>
<td>Top</td>
<td>Bot</td>
</tr>
<tr>
<td></td>
<td>510.2 356.0</td>
<td>24.4 108.5</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>398.5 293.8</td>
<td>622.3 847.8</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>611.0 581.2</td>
<td>19.2 237.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma )</td>
<td>538.0 546.6</td>
<td>746.5 910.0</td>
</tr>
<tr>
<td>SD</td>
<td>19.4 147.1</td>
<td>99.0 290.9</td>
</tr>
<tr>
<td>5.1</td>
<td>553.6 373.0</td>
<td>607.0 498.5</td>
</tr>
<tr>
<td></td>
<td>581.0 581.0</td>
<td>746.5 910.0</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>567.3 477.0</td>
<td>676.8 704.3</td>
</tr>
<tr>
<td>SD</td>
<td>19.4 147.1</td>
<td>99.0 290.9</td>
</tr>
<tr>
<td>6.4</td>
<td>641.9 268.5</td>
<td>624.8 364.2</td>
</tr>
<tr>
<td></td>
<td>665.6 650.0</td>
<td>667.9 677.9</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>654.0 459.3</td>
<td>624.8 364.2</td>
</tr>
<tr>
<td>SD</td>
<td>16.8 269.8</td>
<td>0 0</td>
</tr>
<tr>
<td>7.42</td>
<td>700.6 436.8</td>
<td></td>
</tr>
<tr>
<td>7.87</td>
<td></td>
<td>705.7 774.0</td>
</tr>
<tr>
<td>8.1</td>
<td></td>
<td>827.8 354.1</td>
</tr>
</tbody>
</table>
II. Configuration D = 1x1. $H_b$ [btu/hr ft$^2\cdot$F]

<table>
<thead>
<tr>
<th>$Re \times 10^4$</th>
<th>L/D = 2</th>
<th>L/D = 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HORIZONTAL</td>
<td>VERTICAL</td>
</tr>
<tr>
<td></td>
<td>Top</td>
<td>Bot</td>
</tr>
<tr>
<td>2.5</td>
<td>399.2</td>
<td>322.0</td>
</tr>
<tr>
<td></td>
<td>411.3</td>
<td>309.8</td>
</tr>
<tr>
<td></td>
<td>405.2</td>
<td>315.9</td>
</tr>
<tr>
<td>SD</td>
<td>8.5</td>
<td>8.6</td>
</tr>
<tr>
<td>3.8</td>
<td>528.3</td>
<td>378.0</td>
</tr>
<tr>
<td></td>
<td>528.8</td>
<td>374.9</td>
</tr>
<tr>
<td></td>
<td>528.6</td>
<td>376.5</td>
</tr>
<tr>
<td>SD</td>
<td>0.4</td>
<td>2.2</td>
</tr>
<tr>
<td>6.4</td>
<td>804.6</td>
<td>434.1</td>
</tr>
<tr>
<td></td>
<td>794.7</td>
<td>434.1</td>
</tr>
<tr>
<td></td>
<td>799.7</td>
<td>434.1</td>
</tr>
<tr>
<td>SD</td>
<td>7.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10.0</td>
<td>1085.2</td>
<td>539.9</td>
</tr>
<tr>
<td></td>
<td>1051.8</td>
<td>538.2</td>
</tr>
<tr>
<td></td>
<td>1068.5</td>
<td>539.1</td>
</tr>
<tr>
<td>SD</td>
<td>23.6</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Note: $\sigma$ = Mean.  
SD = Standard Deviation.
### III. Configuration D = 2xl. \( H_b \) [btu/hr ft\(^2\) F]

<table>
<thead>
<tr>
<th>Re(x10)</th>
<th>HORIZONTAL</th>
<th>( L/D = 2 )</th>
<th>VERTICAL</th>
<th>( L/D = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top</td>
<td>Bot</td>
<td>TC1</td>
<td>TC2</td>
</tr>
<tr>
<td>3.2</td>
<td>279.8</td>
<td>231.0</td>
<td>319.9</td>
<td>220.1</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>283.8</td>
<td>240.8</td>
<td>331.2</td>
<td>229.8</td>
</tr>
<tr>
<td>SD</td>
<td>5.6</td>
<td>13.9</td>
<td>15.9</td>
<td>13.6</td>
</tr>
<tr>
<td>4.14</td>
<td></td>
<td></td>
<td>396.7</td>
<td>274.8</td>
</tr>
<tr>
<td>(\sigma)</td>
<td></td>
<td></td>
<td>393.9</td>
<td>271.0</td>
</tr>
<tr>
<td>SD</td>
<td></td>
<td></td>
<td>1.8</td>
<td>29.5</td>
</tr>
<tr>
<td>5.1</td>
<td>391.2</td>
<td>314.3</td>
<td>456.5</td>
<td>302.0</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>393.4</td>
<td>293.4</td>
<td>444.3</td>
<td>297.1</td>
</tr>
<tr>
<td>SD</td>
<td>1.8</td>
<td>29.5</td>
<td>17.3</td>
<td>6.9</td>
</tr>
<tr>
<td>6.4</td>
<td>431.1</td>
<td>327.3</td>
<td>487.4</td>
<td>332.9</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>413.1</td>
<td>301.1</td>
<td>476.8</td>
<td>329.4</td>
</tr>
<tr>
<td>SD</td>
<td>25.5</td>
<td>37.1</td>
<td>14.9</td>
<td>4.9</td>
</tr>
<tr>
<td>8.3</td>
<td>534.6</td>
<td>373.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.4</td>
<td>521.8</td>
<td>499.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX G
CALIBRATED FILM COEFFICIENTS USED IN DATA REDUCTION

I. Branch film coefficients. \( H_b \) [btu/hr ft²°F]

\[ D = 2 \times 2 \]

<table>
<thead>
<tr>
<th>Re ((x10))</th>
<th>L/D = 2 HORIZONTAL</th>
<th>L/D = 2 VERTICAL</th>
<th>L/D = 4 HORIZONTAL</th>
<th>L/D = 4 VERTICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>510.2</td>
<td>566.9</td>
<td>476.0</td>
<td>501.1</td>
</tr>
<tr>
<td>4.1</td>
<td>596.1</td>
<td></td>
<td>552.1</td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>567.3</td>
<td>690.6</td>
<td>536.2</td>
<td>530.2</td>
</tr>
<tr>
<td>6.4</td>
<td>654.0</td>
<td>624.8</td>
<td>672.9</td>
<td>666.7</td>
</tr>
<tr>
<td>7.42</td>
<td>700.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.87</td>
<td></td>
<td>739.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10</td>
<td></td>
<td></td>
<td>827.8</td>
<td></td>
</tr>
</tbody>
</table>

\[ D = 1 \times 1 \]

<table>
<thead>
<tr>
<th>Re ((x10))</th>
<th>L/D = 2 HORIZONTAL</th>
<th>L/D = 2 VERTICAL</th>
<th>L/D = 4 HORIZONTAL</th>
<th>L/D = 4 VERTICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>405.2</td>
<td>453.7</td>
<td>200.4</td>
<td>401.2</td>
</tr>
<tr>
<td>3.8</td>
<td>528.6</td>
<td>598.6</td>
<td>210.9</td>
<td>292.2</td>
</tr>
<tr>
<td>6.4</td>
<td>799.7</td>
<td>758.1</td>
<td>375.0</td>
<td>366.3</td>
</tr>
<tr>
<td>10.0</td>
<td>1068.5</td>
<td>971.5</td>
<td>483.7</td>
<td>485.3</td>
</tr>
</tbody>
</table>
\[ D = 2 \times 1 \]

<table>
<thead>
<tr>
<th>( \text{Re}(x10^4) )</th>
<th>( \text{HORIZONTAL} )</th>
<th>( \text{VERTICAL} )</th>
<th>( \text{HORIZONTAL} )</th>
<th>( \text{VERTICAL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>283.8</td>
<td>331.2</td>
<td>167.3</td>
<td></td>
</tr>
<tr>
<td>4.14</td>
<td></td>
<td>393.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>393.4</td>
<td>444.3</td>
<td>178.6</td>
<td></td>
</tr>
<tr>
<td>6.4</td>
<td>413.1</td>
<td>476.8</td>
<td>192.4</td>
<td></td>
</tr>
<tr>
<td>8.3</td>
<td>534.6</td>
<td></td>
<td></td>
<td>204.7</td>
</tr>
<tr>
<td>8.49</td>
<td></td>
<td></td>
<td></td>
<td>204.7</td>
</tr>
<tr>
<td>9.4</td>
<td>521.8</td>
<td></td>
<td></td>
<td>204.7</td>
</tr>
</tbody>
</table>

II. Run film coefficients. \( H_r [\text{btu/hr ft}^2 F] \).

\[ D = 2 \times 2 \]

<table>
<thead>
<tr>
<th>( \text{Re} \times 10^4 )</th>
<th>( \text{Experimental H}_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2 \times 10^4</td>
<td>1915.6</td>
</tr>
<tr>
<td>4.1 \times 10^4</td>
<td>2161.6*</td>
</tr>
<tr>
<td>5.1 \times 10^4</td>
<td>2436.3</td>
</tr>
<tr>
<td>6.4 \times 10^4</td>
<td>2786.6</td>
</tr>
<tr>
<td>7.4 \times 10^4</td>
<td>3060.3*</td>
</tr>
<tr>
<td>7.87 \times 10^4</td>
<td>3188.3*</td>
</tr>
<tr>
<td>8.1 \times 10^4</td>
<td>3250.9*</td>
</tr>
</tbody>
</table>

* = Linear Regression Estimates.
\[ D = 1 \times 1 \]

<table>
<thead>
<tr>
<th>Re</th>
<th>Experimental Hr</th>
<th>Scale Factor ((h_r/h_{TH}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 \times 10^4</td>
<td>1054.9</td>
<td>1.586</td>
</tr>
<tr>
<td>3.8 \times 10^4</td>
<td>1430.4</td>
<td>1.538</td>
</tr>
<tr>
<td>6.4 \times 10^4</td>
<td>2010.9</td>
<td>1.426</td>
</tr>
<tr>
<td>10 \times 10^4</td>
<td>2624.3</td>
<td>1.302</td>
</tr>
</tbody>
</table>

\[ D = 2 \times 1 \]

<table>
<thead>
<tr>
<th>Re</th>
<th>Estimated Scale Factor ((h_r/h_{TH}))</th>
<th>Theoretical (H_{TH})</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2 \times 10^4</td>
<td>1.557</td>
<td>405</td>
</tr>
<tr>
<td>4.14 \times 10^4</td>
<td>1.522</td>
<td>405</td>
</tr>
<tr>
<td>5.10 \times 10^4</td>
<td>1.485</td>
<td>538</td>
</tr>
<tr>
<td>6.4 \times 10^4</td>
<td>1.435</td>
<td>705</td>
</tr>
<tr>
<td>8.3 \times 10^4</td>
<td>1.363</td>
<td>868</td>
</tr>
<tr>
<td>8.49 \times 10^4</td>
<td>1.356</td>
<td>884</td>
</tr>
<tr>
<td>9.4 \times 10^4</td>
<td>1.321</td>
<td>959</td>
</tr>
</tbody>
</table>

So that:

\[
\frac{h_b}{h_r} = \frac{h_b}{h_{TH}} \times \frac{h_{TH}}{h_r} = \frac{h_b}{h_{TH}} \times \frac{1}{\text{SCALE FACTOR}}
\]
III. Dimensionless Film Coefficients \( (h_b / h_r) \).

**Configuration \( D = 2 \times 2 \). (Fig. 36)**

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( Re \times 10^3 )</th>
<th>( \mu )</th>
<th>( L/D = 2 )</th>
<th>( \mu )</th>
<th>( L/D = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal</td>
<td>Vertical</td>
<td>HORIZONTAL</td>
<td>Vertical</td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>0.226</td>
<td>0.296</td>
<td>0.248</td>
<td>0.262</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>0.276</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.1</td>
<td>0.233</td>
<td>0.283</td>
<td>0.220</td>
<td>0.218</td>
<td></td>
</tr>
<tr>
<td>6.4</td>
<td>0.235</td>
<td>0.224</td>
<td>0.241</td>
<td>0.239</td>
<td></td>
</tr>
<tr>
<td>7.4</td>
<td>0.229</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.87</td>
<td>0.261</td>
<td></td>
<td>0.232</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.10</td>
<td>0.262</td>
<td></td>
<td>0.255</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \sigma \)  \( 0.248 \)  \( 0.269 \)  \( 0.239 \)  \( 0.243 \)

\( SD \)  \( 0.017 \)  \( 0.032 \)  \( 0.014 \)  \( 0.019 \)

**Regression Analysis: Least Square Line.**

**Horizontal:** \( (h_b / h_r) = 0.256 - 0.004(L/D) \) \( (r = 0.54) \)

\( (L/D) = 2 \)

\( (L/D) = 4 \)

\( (h_b / h_r) = 0.2476 \)

\( (h_b / h_r) = 0.239 \)

**Vertical:** \( (h_b / h_r) = 0.296 - 0.013(L/D) \) \( (r = 0.71) \)

\( (L/D) = 2 \)

\( (L/D) = 4 \)

\( (h_b / h_r) = 0.2697 \)

\( (h_b / h_r) = 0.2435 \)

(Note: \( r \) = Correlation Coefficient)
### Configuration D = 2x1. (Fig. 38)

<table>
<thead>
<tr>
<th>Re(x10)</th>
<th>$\mu$</th>
<th>L/D = 2</th>
<th>HORIZONTAL</th>
<th>VERTICAL</th>
<th>L/D = 4</th>
<th>HORIZONTAL</th>
<th>VERTICAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>0.450</td>
<td>0.525</td>
<td>0.265</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.14</td>
<td></td>
<td></td>
<td>0.524</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.10</td>
<td>0.450</td>
<td>0.509</td>
<td>0.209</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.40</td>
<td>0.408</td>
<td>0.471</td>
<td>0.190</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.30</td>
<td>0.452</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.49</td>
<td></td>
<td></td>
<td></td>
<td>0.171</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.40</td>
<td>0.412</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.434</td>
<td>0.507</td>
<td>0.208</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>0.022</td>
<td>0.025</td>
<td>0.41</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Regression Analysis: Least Square Line.

Horizontal: \[(h_b/h_r) = 0.660 - 0.113(L/D) \quad (r = 0.98)\]

\[(L/D) = 2 \quad \quad \quad (h_b/h_r) = 0.434 / 4 \quad \quad \quad 0.208\]

(Note: $r$ = Correlation Coefficient)
Configuration D = 1x1 (Fig. 37)

<table>
<thead>
<tr>
<th>$Re(\times 10)$</th>
<th>$L/D = 2$</th>
<th>$L/D = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HORIZONTAL</td>
<td>VERTICAL</td>
</tr>
<tr>
<td>2.5</td>
<td>0.384</td>
<td>0.430</td>
</tr>
<tr>
<td>3.8</td>
<td>0.369</td>
<td>0.418</td>
</tr>
<tr>
<td>6.4</td>
<td>0.397</td>
<td>0.377</td>
</tr>
<tr>
<td>10.0</td>
<td>0.407</td>
<td>0.370</td>
</tr>
<tr>
<td>$\varnothing$</td>
<td>0.389</td>
<td>0.398</td>
</tr>
<tr>
<td>SD</td>
<td>0.016</td>
<td>0.029</td>
</tr>
</tbody>
</table>

Regression Analysis: Least Square Line.

**Horizontal:** \( (h_b/h_r) = 0.602 - 0.106(L/D) \) \( (r = 0.99) \)

\[
(L/D) = \frac{2}{4} \quad (h_b/h_r) = \frac{0.389}{0.176}
\]

**Vertical:** \( (h_b/h_r) = 0.559 - 0.081(L/D) \) \( (r = 0.89) \)

\[
(L/D) = \frac{2}{4} \quad (h_b/h_r) = \frac{0.398}{0.2378}
\]

(Note: \( r \) = Correlation Coefficient)
### APPENDIX H

DESIGN IMPLICATIONS:

DIMENSIONLESS BRANCH FILM COEFFICIENTS $H_b/H_r$.

### I. Raw data (from Appendix G). $H_b/H_r$.

<table>
<thead>
<tr>
<th>L/D</th>
<th>2x2 Horiz.</th>
<th>Vert.</th>
<th>1x1 Horiz.</th>
<th>Vert.</th>
<th>2x1 Horiz.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.248</td>
<td>0.269</td>
<td>0.389</td>
<td>0.398</td>
<td>0.434</td>
</tr>
<tr>
<td>4</td>
<td>0.239</td>
<td>0.435</td>
<td>0.176</td>
<td>0.238</td>
<td>0.208</td>
</tr>
</tbody>
</table>

### II. Condensed Film Coefficient results. $H_b/H_r$.

<table>
<thead>
<tr>
<th>L/D</th>
<th>2x2</th>
<th>1x1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.259</td>
<td>0.394</td>
</tr>
<tr>
<td>4</td>
<td>0.241</td>
<td>0.207</td>
</tr>
</tbody>
</table>

### III. Extrapolated results using $V_b/V_r = V_o$ data.

<table>
<thead>
<tr>
<th>L/D</th>
<th>$V_o$</th>
<th>2x2 $H_b/H_r$</th>
<th>1x1 $H_b/H_r$</th>
<th>2x1 $H_b/H_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>--</td>
<td>0.277</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>0.066</td>
<td>0.259</td>
<td>0.024</td>
<td>0.0386</td>
</tr>
<tr>
<td>4</td>
<td>0.033</td>
<td>0.241</td>
<td>0.015</td>
<td>0.024</td>
</tr>
<tr>
<td>6</td>
<td>0.026</td>
<td>0.235</td>
<td>0.009</td>
<td>0.011</td>
</tr>
<tr>
<td>8</td>
<td>0.021</td>
<td>0.229</td>
<td>0.006</td>
<td>0.009</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>0.006</td>
</tr>
<tr>
<td>12</td>
<td>0.017</td>
<td>0.225</td>
<td>0.004</td>
<td>0.034</td>
</tr>
</tbody>
</table>
(i) Curve fit using HP-33C: For L/D > 4.

D = 2x2 \hspace{1cm} hb/hr = 0.343 \hspace{1cm} \text{Vo}^{0.1039}

D = 1x1 \hspace{1cm} hb/hr = 65.1 \hspace{1cm} \text{Vo}^{1.369}

D = 2x1 \hspace{1cm} hb/hr = 68.86 \hspace{1cm} \text{Vo}^{1.548}

(ii) For L/D = 0 performed least squared regression form data points at L/D = 2,4.

III. Final Design Values \hspace{1cm} hb/hr. \hspace{1cm} (Fig. 44)

<table>
<thead>
<tr>
<th>L/D</th>
<th>CASE A</th>
<th>CASE B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.610</td>
<td>0.300</td>
</tr>
<tr>
<td>2</td>
<td>0.414</td>
<td>0.259</td>
</tr>
<tr>
<td>4</td>
<td>0.218</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.133</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.094</td>
<td></td>
</tr>
</tbody>
</table>