RANDOM PHASE CODES FOR DOPPLER WEATHER RADARS

by

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DEDICATION

To my mother and father, Anne and Sidney Siggia,
who encouraged and supported my love of science
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Master of Science in Electrical Engineering and Computer Science

ABSTRACT

Phase modulation as a means of alleviating range ambiguity in a meteorological pulsed Doppler radar is studied. With a properly chosen phase code and clutter filter, two otherwise range aliased echoes are distinguishable as long as their relative power does not differ by more than 6 dB. With the addition of adaptive filtering, this limitation can be relaxed to a 30 dB difference. Applications include simultaneous measurement of target range and Doppler over large unambiguous intervals.

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CHAPTER I
INTRODUCTION

Microwave radars have found widespread use in the detection and monitoring of weather events. A ground-based meteorological radar typically surveys a 200 kilometer (km) radius with .2 km range resolution, and detects hydrometeors as small as .1 mm in size. This scale of measurement lies between that of local ground weather stations and that of orbiting satellites. For this reason, weather radar has been successfully applied for many years now to severe storm detection, air traffic routing, and watershed management [1].

To be most useful, a weather radar should provide estimates of storm reflectivity, velocity, and range. Furthermore, these estimates should be provided as frequently as possible, and with minimal uncertainty. Unfortunately, such requirements place conflicting demands on the radar system; in particular, it is difficult to achieve high certainty in both velocity and range via a single measurement. The problem stems from fundamental ambiguities inherent in the radar measurement of velocity and range.

Consider a typical pulsed Doppler radar operating at a wavelength of \( \lambda = 10 \) cm, and with a pulse repetition time (PRT) of \( T = .001 \) seconds. For such a radar operated in a simple burst mode we can define an ambiguous velocity and range as:

\[
V_{amb} = \frac{\lambda}{2T} = 50 \text{ m/sec}
\]

\[
R_{amb} = \frac{CT}{2} = 150 \text{ km}
\]

where \( C \) is the velocity of light. These quantities are important in that radar estimates of velocity and range are correct only to within an
(unknown) additive integer multiple of $V_{amb}$ and $R_{amb}$. Thus, a storm located at 100 km from the radar is not readily distinguished from another located at 250 km. Similarly, a storm moving at 50 m/sec would be indistinguishable from a stationary target.

Such ambiguities would not be serious if both $V_{amb}$ and $R_{amb}$ were large in comparison to true target parameters, for we could then assume that the radar always provides the correct "baseband" estimate. For weather radar, reasonable bounds are $V_{amb} > 80$ m/sec and $R_{amb} > 400$ km. These bounds encompass the typical range of winds and storm heights which are encountered throughout the year. Writing the constraints:

$$\frac{\lambda}{2T} > 80 \text{ m/sec}$$
$$\frac{CT}{2} > 400 \text{ km}$$

we arrive at

$$T > 2.6 \text{ msec}$$
$$\lambda > 42 \text{ cm}$$

Unfortunately, this value of $\lambda$ is much larger than any practical radar will allow. Large $\lambda$ would necessitate a large and costly radar antenna, as well as a more powerful transmitter to compensate for the decreased scattering efficiency from hydrometeors. Thus, a practical weather radar system must overcome range/velocity ambiguities in some other way.

Perhaps the most direct solution is provided by a dual frequency radar. Such a system employs two independent transmitters and receivers, each operated at a different PRF [16]. The low-rate channel is used to unambiguously assess range, and the high-rate channel to assess velocity. Two objections immediately arise to this approach. One is economic --
although some system components are shared between the two channels, a dual frequency radar is largely two radars, and thus is quite expensive. The second objection is that, because of a problem known as "dominance", the dual frequency approach does not successfully solve the velocity-ambiguity problem.

Suppose that the high-rate channel is observing a weather target at range \( r_0 \), upon which target is mixed a range-aliased return from a second storm at \( r_0 + R_{\text{amb}} \). The low-rate channel will correctly identify the two targets at their respective ranges. However, the high-rate channel is faced with the task of somehow separating the velocity information from two superimposed echoes. Because there is no simple way to achieve this separation, velocity estimates for range-aliased returns are available only when one echo sufficiently dominates the other in signal power. In that case we obtain a velocity estimate for the stronger target, but no additional information for the weaker target. Range aliasing thus poses a restriction on the availability of velocity data in a dual frequency system. For this reason, such systems by themselves do not offer a complete solution to ambiguity problems.

Techniques collectively known as "batch PRT" have been investigated [3]. In these methods the radar pulses are not transmitted with uniform spacing, but rather in batches of pulses, with each batch having a different inter-pulse period. Two periods, one long and one short, are chosen, the first to de-alias range, and the second to de-alias velocity. This sounds promising, but unfortunately this technique too suffers from problems of dominance. The short PRT echoes are frequently aliased in range, and we are unable to sort out the accompanying velocity information.
Another problem with batch PRT is that overall surveillance time is less than optimal since the same volume must be investigated by each of the two pulse batches.

Other attempts to resolve ambiguity in weather data have focused on the constraints imposed by the weather itself [4]. For instance, velocity ambiguities within a complete atmospheric scan can be resolved by recognizing that winds are continuous at all points within the volume. The additional constraints thus imposed are often sufficient to arrive at a unique assignment of values to the otherwise ambiguous velocities. In a related technique, external knowledge of mean wind vs. height is used to assign a maximally likely velocity to each point of the radar scan. Note that in both techniques the radar is operated with a PRT sufficiently long to preclude range aliasing. We are then confident that the measured velocities, in spite of being "folded", at least correspond uniquely to the actual targets. In this way problems of echo dominance are prevented. Note too that the first technique requires enormous computation and probably is not suitable for real-time use. The second approach, while computationally fairly simple, requires external knowledge which may not always be available. Finally, both methods suffer a "low-pass" type of distortion; i.e., true velocity anomalies are assumed to be velocity folds, and are smoothed out by the processing. Often it is just these anomalies which are the most crucial to detect (e.g., a tornado embedded within a larger system), and we would not want to artificially limit our chances of finding them.

Geotis [5] has suggested a velocity unfolding technique which is as yet untested. Two fairly low PRF's, $f_1$ and $f_2$, are selected such that $nf_1 \neq mf_2$, for $n$ and $m$ integers less than perhaps 4. Here, the inequality
should be taken to mean that for all such m,n the minimum of \( |nf_1-mf_2| \) is maximized. We then transmit alternate batches of pulses at the rates \( f_1 \) and \( f_2 \). Since \( f_1 \) and \( f_2 \) are small, there will be no range folding within each batch measurement. Of course, velocities will be severely folded, but the ambiguities within each batch will be relative to two different \( V_{amb} \) 's. We then use the relative primeness of \( f_1 \) and \( f_2 \) to determine the true velocity which led to the two ambiguous measurements. Possible problems with this technique are (1) the low PRF will necessitate a slow scan, since confidence in the measurements is related to the number of pulses available in one antenna beam width, and (2) small uncertainties in the measured velocities could cause the deduced velocity to jump unpredictably. However, the technique certainly deserves further study.

Recently, phase-coding techniques have been suggested for the simultaneous de-aliasing of range and velocity [6,7]. Phase coding as a means of pulse compression has been used with radar and sonar for a long time [8], but ours is a new application. We are not concerned with improving the range resolution (as in compression), but rather with distinguishing echoes from superimposed targets. In the context of radar meteorology such techniques are referred to as "multiple coherent interval" methods.

Suppose that our radar is capable of varying the phase of the microwave energy contained in each transmitted pulse. It is not required that the phase vary within the pulse, but only that successive pulses be tagged with a phase which is held constant for the entire pulse duration. Because the phase varies only once per inter-pulse period, this type of modulation adds negligible bandwidth to the original radar waveform. For this reason it is simple, in general, to retrofit a pulse-phase modulator
to an existing Doppler radar. Alterations are required only to the radar's local oscillator circuitry, but not to the receivers, filters, or high power amplifier.

What can such a modulation accomplish? We have seen that echo dominance is a recurring problem in many of the previous unfolding techniques. If a phase-modulated radar is operated at a high repetition rate (to measure velocity unambiguously), we still encounter the usual range-aliasing effects. However, superimposed echoes can now be distinguished from each other because the phase applicable to each ambiguous range interval will affect each target separately. Thus, the Doppler waveform from range-aliased targets is the sum of the individual target-Doppler waveforms, each multiplied by a different phase sequence.

The idea behind multiple-coherent-interval processing is to employ a phase sequence which makes possible the recovery of individual Doppler waveforms from such an overall sum. In Chapter II it is argued that the sequence which accompanies this will exhibit a maximally flat power spectrum. Because of the similarity to the theoretical power spectrum of a white-noise process, the technique is justifiably, and more commonly, referred to as "random phase" processing.

There are many possible arrangements of the radio-frequency (RF) components in a random-phase-processing system; each involving different arrangements of modulators and receivers. The configuration in this thesis is the simplest one possible: one modulator and one receiver. It was felt best to keep the radar RF requirements as few as possible, and to use digital post-processing to recover all of the information which we might otherwise have obtained using more than one receiver.
The specific system under study is one in which the 30-MHz local coherent oscillator (COHO) phase is varied just prior to transmission of each pulse. The COHO then runs undisturbed for the period of time in which first-trip echoes are expected to return, i.e., until the next transmitted pulse. Because this same oscillator is used as reference for the radar receiver, the receiver is said to be "first-trip coherent". Evidently, first-trip echoes are "seen" as if no phase modulation were applied at all. However, range-aliased targets are seen with the phase code impressed upon them.

The dominance problem still exists for a phase-coded radar, but it takes on a much less restrictive character than before. In the uncoded systems, Doppler data was available only from the dominant aliased target. In a coded system data remains available for the weaker target, insofar as that target is not too strongly dominated by the other. Depending on the post-processing techniques used, this can mean that even as much as 30 dB of dominance will not preclude recovery of Doppler data from the weaker target. Thus, with phase coding we can handle a much larger class of range-aliased echoes than before. We will always recover the dominant signal, and will almost always recover the weaker one.

The random-phase technique is a viable means of de-aliasing meteorological radar data. The consistently high PRF leads to a rapid scan, and permits the use of continuously running digital filters (rather than ones which must be initialized for each batched PRF). The additional signal processing which is required by the technique is not excessive and is suitable for real-time implementation. Of course, random-phase systems are not totally free of problems, one objection being that low reflectivity
data are sometimes unavailable within the "clutter ring" which marks the beginning of the second ambiguous-range interval. However, with suitable digital post processing, this objection is not of great practical concern.

Chapter II will present an analysis of random-phase systems, and the types of digital signal processing which make them useful. Chapter III will describe the conversion of an existing 10-cm radar to a phase-coded radar, and data collected by that system will be discussed in Chapter IV.
CHAPTER II
ANALYSIS OF PHASE CODING TECHNIQUES

The analysis of a complete phase-coded Doppler radar system would have to account for system-dependent quantities such as receiver nonlinearity, antenna rotation effects, local oscillator stability, etc. However, in order to focus attention on the key enhancements which are provided by phase coding, it will be assumed that its implementation is within the context of a more or less ideal radar.

Interpretation via Ambiguity Functions

Consider a transmitted pulse train $x(t)$, consisting of $N$ pulses of duration $\delta$, separated by an interval $I$. For the moment, assume that the pulses simply represent gated RF energy, i.e., no coding. The matched filter receiver response to a point target with delay $\tau$ and frequency offset $\nu$ is given by the ambiguity function

$$\chi(\tau,\nu) = \int_{-\infty}^{\infty} x^*(t) x(t+\tau) e^{j2\pi\nu t} dt$$

For the given $x(t)$, the ambiguity function takes on the familiar "bed of nails" appearance shown in Figure 2.1. The energy in $\chi(\tau,\nu)$ lies primarily under the regularly spaced peaks centered at the origin and separated by $I$ on the $\tau$ axis, and by $1/I$ on the $\nu$ axis. From Figure 2.1 it is clear that the peaks for which $\tau \neq 0$ (i.e., the range aliased echoes) are still capable of resolution along the $\nu$ axis. This observation gives an alternate view of something which is already known: for the simple pulse train,
range-aliased targets can make a systematic contribution to first-interval Doppler measurements.

Now suppose that the pulses of \( x(t) \) are coded according to a sequence \( \{\phi_i\}, i = 0, 1, \ldots N-1, 0 < \phi_i < 2\pi \). Here \( \phi_i \) represents the absolute phase of the \( i \)th transmitted pulse. The ambiguity function for the waveform no longer has a closed form solution as before. However, the small-\( \nu \) dependence along \( \tau = LI \), where \( L \) is an integer between 0 and \( N-1 \), can be reduced (see Appendix I) to

\[
\chi(LI,\nu) = \left| \sum_{m=0}^{N-L-1} e^{j(\phi_m-\phi_m+L)} e^{2\pi\nu Im} \right|
\]

which is recognized as the discrete Fourier transform (DFT) of the \( N-L \) point sequence

\[
e^{j(\phi_i-\phi_i+L)} \quad i = 0 \ldots N-L-1.
\]

This ambiguity function is shown graphically in Figure 2.2. Along the \( \nu \) axis, where \( L = 0 \), the coded and uncoded functions are identical. Thus, first trip Doppler measurements are unaffected by the code. However, for multiple trip returns, the phases \( \phi_i \) can be employed to tailor the response of \( \chi(LI,\nu) \). In particular, we wish to minimize frequency resolution of the range aliased targets, which can be done by choosing \( \phi_i \) so as to make \( \chi(LI,\nu) \) as flat as possible in \( \nu \). It is seen then that the intuitive notion that \( \phi_i \) should whiten the power content of aliased targets can be justified through an ambiguity-type argument. Note also that \( L=1 \) is usually the case in question, and occasionally \( L=2 \), but that weather radars rarely detect targets at delays at greater than \( 3I \).* For this reason the

* Typically \( I > 1 \) ms.
remaining analysis and experiments will be confined to singly range-aliased targets (L=1).

Linear Algebraic Approach*

Consider now in detail the problem of separating the Doppler information from two targets aliased in range. Let \( Z_1 \overset{\Delta}{=} S_1 + C_1 + N_1 \) be an \( N \)-vector of complex radar receiver samples from the first-trip target, where

- \( S_1 \): \( N \)-vector of samples of first trip weather
- \( C_1 \): \( N \)-vector of samples of first trip ground clutter
- \( N_1 \): \( N \)-vector of samples of first trip white noise

Similarly, let \( Z_2 \overset{\Delta}{=} S_2 + C_2 + N_2 \) be defined for the second trip aliased target.** If we define an \( N \times N \) diagonal matrix \( \Phi \) as

\[
\Phi_{mn} = \begin{cases} 
0 & \text{if } m \neq n \\
\Delta & \text{if } m = n,
\end{cases}
\]

then the modulation process results in the received signal vector

\[
Z_r \overset{\Delta}{=} Z_1 + \Phi Z_2
\]

Therefore, the problem is to somehow recover both \( S_1 \) and \( S_2 \) from \( Z_r \) alone.

In this context, "signal recovery" should be taken to mean the isolation of original weather signal components, up to the possible addition of

* The symbols and notation for this section are similar to that used in [3].

** Since \( N_1 \) and \( N_2 \) contain receiver noise as well as target noise, the two vectors are not statistically independent. This will not affect the remaining discussion.
white noise terms. For the recovery to be useful, the accompanying noise should be as small as possible, lest the effective SNR of the weather target be severely worsened. SNR degradation tends to be the limiting factor in the performance of phase coded systems. When comparing alternative algorithms for signal restoration, an SNR test is therefore a reasonable basis for choosing one algorithm over another.

Consider first the recovery of $S_1$ from $Z_r$. It is expected that some type of clutter filter will be required, and it is convenient to use a square $N \times N$ high pass filter impulse response matrix, $G$, for this purpose. Although this square format cannot represent arbitrary high pass filters, it does have the nice property that there are $N$ filtered data points output for every $N$ points of input. This would not be true of, for example, a convolution filter. A good clutter filter will have the property that

$$G(S+N) = S + N + \delta \quad |\delta| \ll |S+N|$$

$$GC = \epsilon \quad |\epsilon| \ll |C|$$

Suppose $Z_r$ is passed through $G$, obtaining

$$\hat{S}_1 \overset{\Delta}{=} GZ_r = GZ_1 + G\varrho Z_2$$

$$\overset{\Delta}{=} S_1 + \{N_1 + \epsilon_1 + \delta_1 + G\varrho Z_2\}$$

It is assumed that the $\{\phi_i\}$ have been chosen in accordance with the results of the previous section, so that DFT($\phi_i$) has maximally flat magnitude and uncorrelated complex argument. It follows, then, that $\varrho Z_2$ will look like samples of a white noise process, and thus the entire quantity in brackets represents a noise term added to $S_1$. The actual power in this noise is approximately $|N_1| + |Z_2|$, i.e., the original noise power in $S_1$, plus the total power in the second target. Because of range effects, second
target power is expected to be less than $|S_1|$ on the average. Therefore, if $\hat{S}_1$ is used to estimate $S_1$, a significant average loss in recovered SNR is not expected.

It is tempting to copy this last procedure and try to recover $S_2$ via

$$\hat{S}_2 = G\varphi^{-1}Z_r = G\varphi^{-1}(Z_1 + \varphi Z_2)$$

$$= G\varphi^{-1} Z_1 + GZ_2$$

$$= (G\varphi^{-1} Z_1 + N_2 + \delta_2 + \epsilon_2) + S_2.$$  

This corresponds to first "recohering" the received signal vector, and then filtering the residual second trip clutter. As before, the term in brackets is white; but it now has an expected power much greater than $|S_2|$. If $\hat{S}_2$ were used to estimate $S_2$, it would generally give poor results; clearly, a different approach is needed.

Suppose that the clutter power in $Z_1$ is eliminated before recohering, i.e., define

$$\hat{S} = G\varphi^{-1} GZ_r = G\varphi^{-1} G(Z_1 + \varphi Z_2)$$

$$= G\varphi^{-1} G(S_1 + N_1 + C_1) + G\varphi^{-1} G\varphi Z_2$$

$$= (G\varphi^{-1} (S_1 + N_1)) + G(\varphi^{-1} G\varphi)Z_2.$$  

The bracketed term is again additive noise, but its power content has been reduced by approximately $|C_1|$ over the previous case. Since first trip clutter is frequently the strongest component of the received radar signal, this reduction is a significant step toward improving the recovered SNR. But another problem is introduced: the right-hand term of $V_2$ is not quite what was desired, i.e., $GZ_2 = S_2$. Rather, an additional term, $(\varphi^{-1} G\varphi)$, has been inserted. The extent to which $(\varphi^{-1} G\varphi)$ differs from the identity $I$
determines the degree of additional "self interference" noise generated by the extra term.

The origin of self interference noise can be intuitively appreciated by looking into the frequency content of the processed signals. The power from $Z_2$ is spread out by the coding operation, and contributes uniformly to all spectral components of $Z_r$. When $Z_r$ is operated on with the high pass filter $G$, a certain fraction of the "evidence" that $Z_r$ is present is removed. The amount of $Z_2$ destroyed is governed by the stop band width of the filter, as well as by the filter's pass band phase response. Since filters typically have either constant or linear pass band phase, it is the stop band width that most influences the self noise. Appendix III gives an example of the self noise produced by three different high pass filters.

To a lesser extent the self noise is also dependent on the particular match of filter and phase sequence, i.e., given a collection of similar filters and similar phase sequences, certain combinations will exhibit less self noise than others. Fine tuning of the match is unpredictable, and is best done by numerical techniques. It does suggest, however, that for best results the filter and phase sequence should be chosen as a pair.

The signal recovery operations are summarized in the block diagram in Figure 2.3. Note that the initial application of $G$ serves double duty in that it supplies both $\hat{S}_1$ and the first stage of processing for $\hat{S}_2$.

**Extension to Adaptive Filtering**

The previous section has shown that both $S_1$ and $S_2$ can be recovered from $Z_r$, but that a certain amount of additive white noise is introduced in the process. The amount of power in this noise is approximately the sum of
the signal power and noise power in the accompanying signal. Therefore, if either the first or second echo dominates we will not be able to recover the other target. As a rule of thumb, meteorological signals are difficult to use when the SNR is less than -6 dB.* This implies that phase coding would be able to restore two aliased echoes only if their relative powers differ by less than 6 dB. This is a very restrictive condition for a meteorological radar in which target intensities span a range of 80 dB or more.

Recall that in recovering $S_2$ from $Z_r$ it was advantageous to first remove the first trip clutter from $Z_r$. Although the removal introduced a certain amount of self noise, this was far better than whitening the clutter by immediate application of $\phi^{-1}$. However, the remaining first trip weather was still whitened by $\phi^{-1}$, and would still cause trouble when $|S_1| >> |S_2|$. This suggests the idea of adaptive filtering: before applying $\phi^{-1}$, process $Z_r$ with a filter specially adapted to remove the first trip weather in addition to the first trip clutter.

Figure 2.4 shows a block diagram for an adaptive filter system used to recover $S_2$ with improved SNR. The frequency content of the signals are also shown at each state of the diagram. At the upper left are seen two original target echoes sampled by the radar to form the received signal $Z_r$, which is then fed to the processing system. First trip clutter is immediately filtered from $Z_r$ as before. An adaptive filter is then synthesized "on the fly" and applied to notch out the first trip weather. It is expected that this step will introduce yet more self noise, but it is hoped

* Since rapid scan precludes long integration time
hoped that the final SNR will improve because of the large amount of first trip power that is removed. The adaptive filter output will have an essentially flat power spectrum. When this output is recohered by $\phi^{-1}$ and filtered by $G$, a good estimate of $S_2$ should result.

Adaptive filtering can be employed in an analogous way to recover $S_1$ from $Z_r$ in those cases where $|S_2| \gg |S_1|$. All that is required is that $\phi^{-1}G$ be applied first, just as if the second trip were being recovered in the usual way. This output is then run through an adaptive filter, from which $S_1$ is recovered by a final application of $\phi$. Because of this analogous procedure, attention may be confined without loss of generality, to the recovery of $S_2$ when $|S_1| \gg |S_2|$. Adaptive filter recohering systems will still exhibit a fundamental limitation, independent of the details of the implementation. That is, if most of the power in $S_1$ is noise, rather than band-limited weather echo, this noise will be indistinguishable from $\phi Z_2$. It will therefore be impossible to remove this first trip noise through any adaptive filtering scheme. This suggests that the best signal to noise ratio that can be achieved in the recovered second trip echo is

$$2^{\text{nd-Trip}} \quad \text{SNR}_{\text{best}} = 10 \log_{10} \frac{|S_2|}{|N_2| + |N_1|}$$

This limitation should not cause too many problems in a meteorological radar, since strong noise sources do not exist in the atmosphere. However, undersampled and highly turbulent storm regions will occasionally exhibit Doppler spectra which are essentially white.
There are a variety of adaptive filter realizations to choose from [9]. However, their use in the context of radar range de-aliasing is new. In this case it is a whitening filter that is needed, i.e., a filter which, when applied to the radar signal, yields an output whose autocorrelation function is maximally impulsive. Note that this is the inverse of the usual tasks to remove noise and thus "darken" the spectrum. The remainder of this section will discuss results obtained with two such filters. One is a maximum entropy method (MEM) inverse filter, and the other is an ad-hoc technique based on preserving only the phase of the Discrete Fourier Transform of the input.

The MEM method of spectral estimation uses estimates of the autocorrelation function of a signal \( x[n] \), and models the \( Z \) transform of \( x[n] \) as an \( N \)th order all-pole system

\[
\overline{X}(Z) = \frac{b_0}{\sum_{k=0}^{N} a_k z^{-k}}
\]

The \( \{a_k\} \) are determined from the correlation values by solving a system of equations known as the "autocorrelation normal equations" [10,11]. Given this model, the inverse filter \( \overline{X}^{-1}(z) \) may be used to whiten \( x[n] \). Note that \( \overline{X}^{-1}(z) \) is simply an \( N \)th-order finite impulse response (all zero) filter. For \( N=1 \) the solution of the normal equations is particularly simple: \( a_0 = 1, \ a_1 = -r_1^*/r_0 \), where \( r_0 \) and \( r_1 \) are the zero and first-lag autocorrelation estimates of \( x[n] \), and "*" denotes complex conjugation. For higher order models the solution rapidly becomes more involved. Since we are tacitly seeking an algorithm for real-time use, only first order MEM was considered. A block diagram of the MEM whitening filter is shown in Figure 2.5.
The second filter to be considered is one which preserves the phase, but not the amplitude, of the DFT of its output. The input signal $x[n]$ is first windowed, and then the DFT computed (e.g., via a Fast Fourier Transform). Each complex DFT component is then divided by its modulus; i.e., the components are converted to unit vectors in which only the original angle is preserved. The unit vectors are then run through an inverse DFT to produce the filter output. A block diagram is shown in Figure 2.6. Appropriately, this technique will be called the "unit vector" (UV) method.

The intention behind the unit vector filter is as follows: $\Phi$ has been designed so that $|\text{DFT}(\Phi Z_2)|$ is maximally flat. This is analogous to saying that all of the information in $\text{DFT}(\Phi Z_2)$ is contained in its phase, and very little in its magnitude. The unit vector filter should, therefore, pass $\Phi Z_2$ with little distortion. On the other hand, a signal having a non-flat power spectrum would be attenuated. Since the unit vector filter is highly nonlinear, it does not immediately follow that $UV(Z_1 + \Phi Z_2) = \Phi Z_2$. However, the simulations will show that this indeed is the case.

**Simulation**

In order to simulate the performance of each adaptive filter, it was necessary to artificially synthesize the two radar signals $Z_1$ and $Z_2$. A Gaussian model employing both Poisson and white noise was used as the signal source [12]. This model has been used successfully to synthesize weather-like radar signals for some time. The received signal $Z_r$ was obtained as the sum of $Z_1$ with a phase-modulated and weighted version of
Z₂. The weighting served to vary the relative contributions of the first trip and second trip signals. The individual signal to noise ratios of Z₁ and Z₂ were fixed at 25 dB.

The adaptive filtering simulation involved synthesizing many different Z_r, each having a different ratio of first trip power to second trip power. Each Z_r was passed through an adaptive filter to remove the first trip signal, and then recohered by \( \varphi^{-1} \) to recover S₂. For simplicity, clutter signals were not used in the simulation, and thus the high pass filter G was not needed. Clutter was omitted because (1) it is difficult to model very accurately, and (2) clutter filtering is well understood and good clutter filters are already documented and available [13].

Figure 2.7 shows the simulation set-up. The sequence \( \{\phi_i\} \) was the eight-phase pseudo-random code derived by the numerical optimization procedure detailed in Appendix II. As a figure of merit of the system's performance, a signal-to-noise ratio estimate based on two autocorrelation lags was used [14]. This particular estimator works well when the actual SNR of its input lies between -8 and +15 dB. The simulation proceeded by stepping the first-to-second trip power in Z_r in 2 dB increments from -10 to +40 dB, and then plotting the effective SNR of the recovered second trip signal. The results, shown in Figure 2.8, compare the performance of three different filters: First Order Maximum Entropy, Unit Vector (128 point), and the identity (i.e., no filtering).

Let us look first at the results when no filter is used. In that case, the noise in the recovered second trip is due almost entirely to whitened first trip signal power. We therefore expect the effective recohered SNR to be the negative (in dB) of the actual first-to-second trip
power ratio. This reasoning is indeed confirmed by the simulation, as shown by the line of slope -1 in the plot. The identity filter has been included in the plot so that the performance of the other more interesting filters can be gauged. Those points lying above and to the right of the identity line represent adaptive filter performance which is better than doing nothing; points below and to the left are worse.

The maximum entropy filter consistently performs at least as well as the identity. When there is very little first trip power the two work equally well. This is because there simply is not very much power available to be filtered out. As first trip power increases, its removal by the MEM filter becomes apparent as the plot breaks away from the identity line. Eventually, there is more power than can be removed by the first order filter, and the second trip signal cannot be recovered. At the point of breaking down, MEM has added approximately 10 dB of useful range to the unfiltered system.

The unit vector filter behaves rather differently. When first trip power is small, it is actually worse than the identity. This can be understood by realizing that $\zeta_2$ does not have a perfectly flat DFT magnitude, and thus a little bit of information always gets destroyed by the unit vector process. This is similar to the self interference problems which arise when first trip clutter is removed. The problem is not serious for two reasons: (1) even with the self interference, the effective SNR is acceptable ($>4$ dB) and (2) in the numerical procedure for choosing $\phi$, the degradation by the UV filter could be taken into account, and a somewhat better $\phi$ could undoubtedly be found.
The redeeming feature of unit vector filtering is its outstanding performance at large first trip power levels. At the point where MEM breaks down, the UV filter still recovers the second trip signal to 0 dB SNR. The technique eventually fails when first trip power is 30 dB in excess of second trip power. This represents a 23 dB improvement over the identity and 12 dB over MEM. The improvement over MEM is explained by the high selectivity of the UV method. Whereas first order MEM has only two parameters at its disposal, UV has, in some sense, as many parameters as there are DFT points.* This gives the UV filter the ability to reject first trip signals without disturbing very much of the underlying second trip power.

These simulations suggest that a fairly simple adaptive filtering scheme can greatly extend the usable range of a phase coded radar system. Without such filtering, signal-to-noise restrictions would limit recovery to only those aliased echoes having relative powers within 6 dB of each other.

* Although, depending on the window used, only about half of these points are statistically independent.
Figure 2.1  Ambiguity Diagram of an Uncoded Burst

Figure 2.2  Ambiguity Diagram of a Phase Coded Burst
Figure 2.3  Block Diagram of the Basic Random-Phase Signal Recovery Technique

Figure 2.5  Block Diagram of a First Order Maximum Entropy Whitening Filter
2.4 Synthesis of two-target test signal, from which the second target is recovered with good SNR via adaptive filtering.
2.6 Unit Vector Whitening Filter

2.7 Simulation of the Adaptive Filter
Recohering System
2.8 Comparison to abilities of three different adaptive filters in recovering second trip parameters from a mixed received signal. The abscissa shows the true ratio of first-to-second trip signal power in dB. The ordinate shows the derived SNR of the recovered waveform.
CHAPTER III
EXPERIMENTAL PROCEDURES

Radar measurements of actual precipitation targets were made using the facilities of the MIT Weather Radar Laboratory in the Department of Meteorology and Physical Oceanography, Cambridge, Massachusetts. The observations were carried out during the summer months of 1982 and were intended to test the efficacy of random-phase processing on real data. Because intense thunderstorms can be detected even at far ranges, range-aliasing problems tend to be most severe during the summer. Consequently, it is during this season that the best data can be gathered for the study.

Appendix IV summarizes the characteristics of the MIT-WR66 radar. The radar is built around a coherent Klystron transmitter and had already been set up for making routine Doppler measurements. However, several modifications were needed in order to make random-phase measurements. Changes were required in both the radar RF chain, and in the digital signal processor (DSP), which reduces the incoming radar data [15].

A block diagram of the modified radar system is shown in Figure 3.1. The trigger and phase control signals are generated within the signal processor and applied to the radar RF section. Timing of these signals is such that the phase is selected 3 microseconds prior to triggering the transmitter pulse. This phase is then held steady for one interpulse period, after which a new phase is selected in preparation for the next pulse. Because the phase is selected just before transmission, the radar receiver is always operating coherently with the most recent pulse. On the other hand, range aliased echoes are received with respect to a different COHO phase, and thus will be altered by the modulation.
One should keep in mind that the actual transmitted phase sequence is not the sequence \( \{\phi_i\} \), but rather the "integrated" sequence \( \{\phi'_i\} \), defined as

\[
\phi'_i = \sum_{k=0}^{i} \phi_k, \quad i = 0, 1, \ldots, N-1
\]

When \( \{\phi'_i\} \) is transmitted, the net phase shift of the \( i \)th received pulse from a second trip target will be

\[
\phi'_i - \phi'_{i-1} = \sum_{k=0}^{i} \phi_k - \sum_{k=0}^{i-1} \phi_k = \phi_i.
\]

Thus, by transmitting \( \{\phi'_i\} \) we effectively modulate second-trip echoes by \( \{\phi_i\} \), as was required in the original analysis.

The COHO modulator which was built and installed for the study is an 8-phase, 30-MHz, digitally controlled device designed around a commercially available "block". The block contains various combinations of 90° and 180° shifters, as well as power splitters and combiners which can be switched together to provide the desired net phase shift. A small amount of digital hardware was needed to interface the modulator with the signal processor. Relative phase accuracy between any two settings is ±1 degree.

While testing the modulator, we happened upon a novel scheme to adjust the quadrature receiver for exactly orthogonal response.* Using the phase sequence

\[
\phi_i = \frac{2\pi}{8} \sum_{k=0}^{i} k,
\]

* Lacking, as we were, an (expensive) phase-locked signal generator.
a fixed frequency will be added to the nominal Doppler frequency of second
trip targets. By observing second trip ground clutter, which is a good
source of zero Doppler, we thus obtain a good off-zero source whose image
in Doppler can be observed and thereby nulled via an FFT of the "I,Q" time
series. In this way, we were able to correct a long standing maladjustment
(of about 6 degrees) in our quadrature receiver.

The Doppler signal processor was reprogrammed to collect raw "I" and
"Q" timeseries samples, which were directly transferred to the minicomputer
and to magnetic tape for later analysis. Although the signal processor
could have carried out the random phase calculations by itself in real
time, it was much simpler to develop and test the new algorithms using
higher level language programs on the minicomputer. Since the minicomputer
could not process the data in real time it was necessary to first buffer
the unprocessed time series onto magnetic tape. These series could then be
used as input to the algorithms which were being evaluated. In spite of
this two-step procedure, data could still be viewed within a few minutes of
the time it was taken.

In preparation for collecting random phase data, the radar was first
operated in the normal surveillance mode. A low PRF was used during these
scans to ensure that none of the echoes would be aliased in range. The
reflectivity pattern throughout a 256-km radius was then displayed on a
color monitor, and was checked for the sorts of configurations which are
conducive to range aliasing. Specifically, any single ray containing
echoes which are separated by 150-250 km in range is likely to give aliased
returns when the radar is operated at its typical PRF. This includes rays
which are completely filled with echoes, as often happens when viewing
widespread precipitation at a low elevation angle.
Based on the surveillance scan, the antenna was then manually positioned to a direction appropriate for observing second-trip echoes. A live display of target reflectivity and velocity was available from the signal processor so that the echoes along the ray could be observed. The PRF was then increased slowly, starting from its initial low value, until range aliasing was evident by the display. It is a simple matter to spot an aliased echo which is mixed in with the other data on the display, since its apparent range will change with varying PRF. First trip echoes, of course, remain stationary with varying PRF.

Having chosen an antenna bearing and radar PRF such that range aliasing is known to be present, we are ready to acquire and store the Doppler time-series data. Two sets of data were collected in each instance: one with phase modulation turned on, and the other with it turned off. The two sets were sampled within a few seconds of each other, during which time negligible change would have occurred in the observed storms. Both types of data are needed in order to determine the effectiveness of the random-phase-recovery algorithms, i.e., we must have a second data set to which our estimates can be compared.

It is important to remember that the unmodulated data set does not by itself define the radar echoes unambiguously. Rather, it does so only in the context of the extra information provided in the original surveillance scan. In other word, by combining the surveillance scan with external knowledge (such as measured surface winds), we deduce the true reflectivity and velocity along the ray. This truth is then used to manually sort out the aliased power in the unmodulated data sets. Without the surveillance scan it would be impossible to make this determination. Thus, to be
strictly correct, the data that was recorded in each instance consisted of the two data sets, plus a hand-written account of the "truth".

In all, 52 data sets were collected and stored on magnetic tape during a one-month period. All part-processing was then performed on the lab's TI 980 minicomputer. An effort was made to collect data on as many different configurations of storms as possible.
3.1 Block diagram of Doppler Radar system, modified to make random phase measurements.
CHAPTER IV

DISCUSSION

The data collected with the modified MIT 10-cm Doppler radar were used to verify that the results of Chapter II, which relied on synthetic data, do indeed carry over to actual radar data. Two data sets will be discussed in detail, these being from an early evening storm which occurred on 2 September 1982. The two observations were made within a few minutes of each other, but at different antenna bearings.

The first measurement, shown in Figure 4.1 involved two range-aliased echoes of approximately equal power. The diagram shows the power spectra of the received radar signals in decibels v.s. normalized frequency. The spectra were computed using a 128-point Hamming-weighted discrete Fourier transform. Four such transforms were averaged with 50% overlap to produce the plots shown [16]. At the top of Figure 4.1 we see the Doppler spectrum obtained without phase modulation. Two peaks are apparent, corresponding to the different velocities of the two range aliased targets. With the help of an earlier surveillance scan it was concluded that the right hand peak corresponds to the first-trip echo. Thus, when modulation is switched on, as shown in the bottom of Figure 4.1, the left hand target "disappears". However, closer examination reveals its continued presence in the form of an increased noise floor - a jump from 27 dB to 37 dB. Note that although has first-trip target is still easily discernable in the lower plot, it has nonetheless suffered a genuine loss in SNR.

When the phase-modulated data is multiplied by $\phi^{-1}$, we obtain the spectrum of Figure 4.2. We now see the second trip target standing out clearly from the noise floor established by the whitened first-trip signal,
but once again, the effective SNR has decreased. This data set demonstrates that simple phase recohering can be used in cases where one echo does not dominate the other by more than a few dB. In such cases the noise floor is not raised to the point of obscuring the coherent echo, and we are able to manage a complete recovery of Doppler data from both targets.

Let us now consider the second data set. Doppler spectra, both with and without modulation, are shown in Figure 4.3. At the top we see a very strong first-trip echo on the right, accompanied by a weak second-trip echo on the left. The strong echo dominates the weak one by approximately 11 dB. In the bottom plot, for which modulation is enabled, we see the first-trip echo standing clearly on its own.

When we attempt to recover the second-trip target via a simple multiplication by $\sigma^{-1}$, we obtain the plot of Figure 4.4 (top). Evidently, the attempt was a complete failure; the echo is totally masked by the whitened first-trip power. We thus have a case for which adaptive filter recohering is essential to successful recovery. The lower plot shows the result when a first order MEM filter is applied to the received data prior to application of $\sigma^{-1}$. We now see the second-trip echo standing out clearly, and with essentially undiminished SNR. The adaptive filter has thus enabled the recovery of an otherwise inaccessible target. Incidentally, the actual MEM filter output spectrum is shown in Figure 4.5. This should be compared with the bottom plot of Figure 4.3, in order to appreciate the large amount of first-trip power which the filter has removed.

In practice, it is not likely for range-aliased echoes to have nearly equal received power. There are two reasons for this. First, the dynamic range of reflectivities from meteorological targets is typically 60 dB;
thus, given two storms chosen at random, they are unlikely to exhibit equal returned power. Secondly, even under conditions of uniform, wide-spread precipitation, range effects will attenuate the absolute power received from the more distant scatterers. Thus, we expect equal power from two range-aliased targets only when the physical magnitudes of the storms exactly compensate their respective range attenuations.

These observations suggest that adaptive filtering will be essential to the use of phase-coding techniques in weather radar systems. If we were confined to use only direct recohering, then most of the time we would lose information on the weaker target. This is not to condemn the simple scheme, for it does add 12 dB of usable relative signal power when compared with an uncoded radar. Rather, it simply does not take full advantage of the information contained in the radar signals. Moreover, the added cost of the digital processing for the adaptive filter is amply compensated by the system's greater accommodation of targets.

There are several variations on the adaptive filtering technique which are necessary to an operational weather radar. We have described the case in which first-trip echo power dominates second-trip power, but what about the reverse situation? A strong second-trip target will make it impossible to recover the first-trip signal by the method used thus far, i.e., by direct examination of the receiver waveform. Instead, we must first use $\phi^{-1}$ to recover the second signal, then apply the adaptive filter, and finally use $\phi$ to reconstruct the first signal. The new procedure is analogus to the old one, if we imagine the initial application of $\phi^{-1}$ as simulating a receiver which is coherent with the second, rather than the first, ambiguous-range interval.
Since we do not know \textit{a priori} which aliased echo is dominant, we do not immediately know which of the two filtering methods to use. However, we can try them both and use a SNR estimator on the final products to determine which method was in fact appropriate, i.e., we choose the technique which produced the waveforms having the greatest SNR. It may seem wasteful to run both methods, only to throw away half the results, but many of the computations can be overlapped so that the actual computational requirements are not far above those of the single method. For instance, the first application of $\varphi^{-1}$ is carried out identically in both approaches.

This "two-way" processing is robust in that it applies the optimal recohering strategy to each target combination. We thus add the benefits of the adaptive filter to both ends of the range of usable relative target power. For the unit-vector filter, this would lead to an additional 46 dB of operational flexibility. In view of these large benefits, it is felt that adaptive-filtering algorithms should be included as part of any meteorological phase-recohering system. The Maximum-Entropy filter would be the simplest to retrofit to an existing time-domain signal processor, while the unit-vector filter is most appropriate for processors equipped with FFT hardware.
Figure 4.1  Doppler spectra of two range-aliased targets having nearly equal power. Phase modulation disabled (top) and enabled (bottom).
Figure 4.2  Second trip waveform recovered from 4.1 (bottom) by direct application of $\phi^{-1}$
Figure 4.3  Doppler spectra of two range-aliased targets having dissimilar power. Phase modulation disabled (top) and enabled (bottom)
Figure 4.4 Recovery of weak second trip target from 4.3 (bottom). Direct application of $\omega^{-1}$ (top), and adaptive filter restoration (bottom).
Figure 4.5 Frequency content of MEM filter output when applied to 4.3 (bottom)
APPENDIX I

Ambiguity Function of a Phase-Coded Burst

Let \( x(t) \) represent a burst of \( N \) pulses of width \( \delta \) and separation \( I \), originating at \( t=0 \), and having an absolute phase at the \( i \)th pulse of \( \phi_i \). Let \( L \) be an integer in the range \(-N < L < N\). The matched filter response in the interval \(-\delta < |\tau-LI| < \delta\) to \( x(t) \) is given by the ambiguity function

\[
|\chi(\tau,\nu)| = \left| \int_{-\infty}^{\infty} x^*(t) x(t+\tau) e^{j2\pi\nu t} dt \right|
\]

\[
= \sum_{m=0}^{N-L-1} \delta - |\tau-LI| \int_{t=0}^{N-L-1} e^{j(\phi_m - \phi_{m+L})} dT
\]

\[
= \sum_{m=0}^{N-L-1} e^{j(\phi_m - \phi_{m+L})} e^{j2\pi\nu m} \left| \frac{\sin \pi\nu(\delta - |\tau-LI|)}{\pi\nu\delta} \right|
\]

(1)

If \( x(t) \) represents the transmitted waveform of a typical phase coded radar, then \( \delta \ll I \) (e.g., \( \delta = 1 \mu\text{sec}, I = 1 \text{ms} \)). For this reason, the \( \nu \)-dependence of \( \chi(\tau,\nu) \) is determined mainly by the first term of (1) as long as \( \nu \ll 1/\delta \). It turns out that this range of \( \nu \) still encompasses the expected range of Doppler shifts produced by meteorological targets. Therefore, if we approximate the second term of (1) by 1 we get

\[
|\chi(LI,\nu)| = \left| \sum_{m=0}^{N-L-1} e^{j(\phi_m - \phi_{m+L})} e^{j2\pi\nu m} \right|
\]

\[
= |\text{DFT} \{ e^{j(\phi_m - \phi_{m+L})}, m=0 \ldots N-L-1 \} |
\]

that is, the discrete Fourier transform of the \( (N-L) \) point sequence of phase differences.
APPENDIX II

Numerical Optimization Procedure for Choosing \( \{\phi_i\} \)

We seek a phase sequence \( \{\phi_i\} \), \( i = 0, 1, \ldots, N-1 \), consisting of elements of the set

\[
\left\{ \frac{2\pi k}{M}, \ k = 0 \ldots M-1 \right\}
\]

such that

\[
|X(K)| = |\text{DFT}\{e^{j\phi_i}\}|
\]

\[
= \sum_{i=0}^{N-1} e^{j\phi_i} e^{-j\frac{2\pi ki}{N}}
\]

is maximally flat. Here \( N \) is the length of \( \phi_i \) and \( M \) is the number of equi-spaced phase angles available for use. \( N \) is constrained by the available target dwell time, and \( M \) by the modulation hardware.

The "maximally flat" criterion must be rephrased in algebraic terms. We are concerned with both mean square deviation and maximum deviation from the mean level. Mean fourth power deviation has been used in similar contexts to capture the combined effect of mean square and maximum deviation [17,18]. However, because of computational limitations of the available minicomputer, a different figure of merit was devised:

\[
Q = \sum_{k=0}^{N-1} \left| |X(K)| - \bar{X} \right| + \lambda \max_k \left\{ |X(K)| - \bar{X} \right\}
\]
Here $X$ is the mean of $|X(K)|$, and $\lambda$ is a penalty coefficient against the maximum deviation of $|X(K)|$ from $X$. For $M = 128$, $\lambda$ was taken to be 12, i.e., the maximum deviation is penalized 12 times more heavily than any other deviation. Our objective now is to choose $\{\phi_i\}$ so as to minimize $Q$.

The problem as phrased here has no direct algebraic solution, so numerical methods had to be used. The procedure used starts with a random uniformly distributed phase assignment for $\{\phi_i\}$. We then choose an integer, $j$, at random in the range $0 \ldots N-1$, and evaluate $Q$ for each of the $M$ possible phases which might be assigned to $\phi_j$. We keep the assignment which gave the lowest value of $Q$, and repeat the procedure. This continues until no further reduction of $Q$ can be attained.

In practice this procedure requires modification for good results. One problem is that it tends to get "stuck", i.e., a given sequence may be far from optimal, yet no single phase change will improve $Q$. The remedy is to choose two numbers, $j_1$ and $j_2$, in the range $0 \ldots N-1$, and to try all of the $M^2$ different phase assignments to $\phi_{j_1}$ and $\phi_{j_2}$. Again, we keep the pair which gives the lowest value for $Q$, and iterate the procedure.

A second problem is this: if we run the entire method several times, some of the sequences produced will be better than others. The minimum attainable $Q$ depends on the initial pattern of $\{\phi_i\}$ as well as the order in which the changes were made. Thus, the algorithm must be run in its entirety several times, and then we choose as our best sequence the one with the smallest $Q$.

The numerical procedure was also used to assess the virtue of polyphase codes ($M>2$) over binary codes ($M=2$). Binary codes are usually simple to implement, both in the radar RF and in the digital signal processing
algorithms, but perhaps there is something to be gained by an augmented code. The results of these optimizations indicate that approximately 15% improvement in Q is had for M=4 vs. M=2, and for M=8 vs. M=4. Thus, although polyphase codes do seem to work better than binary codes, the added expense may not always be justifiable.

The actual code used throughout this thesis is the 128-point, 8-phase sequence listed below. The phases have been summarized as integers between 0 and 7. The corresponding DFT magnitude is also shown.

6, 7, 2, 3, 5, 6, 3, 5, 6, 3, 5, 2, 2, 7, 3, 7
1, 3, 7, 0, 3, 4, 7, 6, 5, 4, 7, 6, 0, 3, 3, 5
4, 5, 2, 7, 3, 7, 4, 3, 1, 4, 5, 0, 3, 3, 0, 2
7, 2, 2, 0, 6, 4, 2, 4, 7, 5, 1, 1, 3, 2, 6, 2
7, 1, 1, 5, 0, 4, 4, 7, 3, 2, 7, 3, 7, 5, 5, 3
4, 3, 1, 0, 1, 1, 5, 3, 4, 2, 4, 7, 5, 7, 5, 4
1, 7, 3, 3, 1, 4, 3, 2, 6, 2, 6, 5, 4, 4, 6, 6
7, 2, 3, 3, 7, 6, 7, 6, 7, 2, 0, 7, 7, 2, 2, 1
Figure II.1  DFT magnitude of the 128-point pseudo-random phase sequence
APPENDIX III

Case Study of the Self-Interference Term $\varphi^{-1}G\varphi$

In the recovery of second-trip weather using a phase rotation matrix $\varphi$ and clutter filter $G$, it is found that a spurious "self-interference" term of the form $\varphi^{-1}G\varphi$ is introduced. The frequency distortion introduced by $\varphi^{-1}G\varphi$ will ultimately limit the resolution in Doppler of the recohered second trip. To estimate this distortion a case study was carried out using the phase sequence derived in Appendix II, and three different clutter filters. A purely sinusoidal $N$-vector was input to $\varphi^{-1}G\varphi$, and the frequency content of the distorted output signal was plotted. Figure III.1 shows on the left the frequency response magnitude of the three filters $G$, and on the right the output power spectrum of the distorted sinusoid.

If there were no self interference, then the output power spectrum would consist of a single line at the normalized frequency of $+.35$. Instead, we see that $\varphi^{-1}G\varphi$ has broadened the line into a peak, and has also introduced a white noise floor. The peak-to-floor power is 28, 35, and 43 dB, respectively for the wide, typical and narrow clutter filters. Thus noise is introduced in direct correspondence with the stop-band width of $G$. However, in no case was the frequency location of the peak altered.

These results suggest that $\varphi^{-1}G\varphi$ will preserve mean Doppler measurements of the recohered second-trip signal, but that signal-to-noise ratio and spectral-width measurements will be affected.
Figure III.1 Frequency response of three filters, and the self-interference produced by each filter on a pure sinusoid
Appendix IV

Characteristics of the MIT WR66 Radar

Antenna

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>18 ft diameter</td>
</tr>
<tr>
<td>Gain</td>
<td>42 dB</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>1.4°</td>
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<tr>
<td>Polarization</td>
<td>Horizontal linear</td>
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Transmitter (FPS-18)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power</td>
<td>600 kW</td>
</tr>
<tr>
<td>Frequency</td>
<td>2700-2800 MHz</td>
</tr>
<tr>
<td>Pulse width</td>
<td>1.0 μsec</td>
</tr>
<tr>
<td>Pulse repetition rate</td>
<td>100-1300 Hz</td>
</tr>
<tr>
<td>COHO modulation</td>
<td>Octant phase modulator (accuracy ±1°)</td>
</tr>
</tbody>
</table>

Receivers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>TYPE</td>
<td>Both linear and logarithmic</td>
</tr>
<tr>
<td>Noise figure</td>
<td>8 dB</td>
</tr>
<tr>
<td>Dynamic range</td>
<td>80 dB</td>
</tr>
<tr>
<td>Minimum detectable signal</td>
<td>-105 dBM</td>
</tr>
</tbody>
</table>
References


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