A THEORY FOR THE
STRUCTURAL BEHAVIOR OF
REINFORCED CONCRETE PIPE

by
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ABSTRACT

General theoretical methods are developed to investigate elastic stresses, ultimate strength, cracking behavior and deflections of circular reinforced concrete pipe under external loading. Primary attention is directed to the 3-edge bearing load condition, but field load distributions and combined external load and internal pressure conditions are also considered. Pipe with both circular reinforcing and elliptical reinforcing, and both with and without stirrup reinforcing, are considered.

A theoretical analysis based on fundamental laws of mechanics, is developed for each of the above structural criteria. Where such an analysis cannot produce practical results because of the complexity of the structural action, it enables a rational analysis of test results in order to obtain semi-empirical constants in the theoretical solution. Where necessary, such constants are obtained from an experimental program carried out in conjunction with this theoretical investigation.

The procedures which are developed for evaluating elastic stresses, flexural ultimate strength, and diagonal tension ultimate strength are applicable to circular pipe with all types of reinforcing. The concepts developed for ultimate diagonal tension strength, moreover, are also generally applicable to any curved reinforced concrete member. The general method of investigation given for cracking and deformation analysis is valid with all types of reinforcing steel. However,
the quantitative relations suggested for analysis of cracking behavior have semi-empirical constants which apply only to pipe reinforced with certain types of welded wire fabric.

Theoretical analyses of 127 3-edge bearing tests on full size pipe provide good corroboration of the theory. Where test results must be used to obtain semi-empirical constants, other test results are available for checking. An analysis of minimum pipe designs given in the ASTM "Standard Specifications for Reinforced Concrete Culvert, Storm Drain, and Sewer Pipe, C76-51T" and in the "Standard Specifications for Reinforced Concrete Low-Heal Pressure Pipe, C361-59T" indicates some areas of agreement between theoretical and specified designs, but also brings to light a number of inconsistencies and irrational designs in these specifications.

Tentative suggestions are made for revisions to ASTM C76 for pipe reinforced with welded wire fabric. Suggestions are also made for the type of test data needed for the proper evaluation of test results and for the further development of rational specifications for the design of pipe.

Thesis Supervisor: Myle J. Holley, Jr.

Title: Professor of Structural Engineering
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NOMENCLATURE AND UNITS

Distances

\( a \) = Depth of equivalent rectangular stress block at ultimate strength of concrete section, inches

\( b \) = Width of concrete section, inches

\( c_s \) = Distance from neutral axis to tension steel

\( d \) = Depth of section from extreme compressive fiber to center of tensile reinforcement

\( d_1 \) = Depth of section from outside wall surface to center of inner cage reinforcement crown or invert, inches

\( d_2 \) = Depth of section from inside wall surface to center of outer cage reinforcement at springings, inches

\( D_i \) = Inside diameter of pipe, feet

\( e \) = Eccentricity of axial force

\( g \) = Distance from centerline of section to compressive resultant force

\( h \) = Thickness of pipe wall, inches

\( l_o \) = Spacing of longitudinal cross wires, inches

\( q \) = Distance from centerline of section to tension steel

\( r_{av} \) = Average radius of pipe to mid-depth of wall thickness

\( r_i \) = Inside radius of pipe

\( r_n \) = Radius to neutral axis

\( r_o \) = Outside radius of pipe

\( r_s \) = Radius to inner cage steel

\( r_{si} \) = Radius to inner cage steel

\( r_{so} \) = Radius to outer cage steel
NOMENCLATURE AND UNITS (Continued)

**Distances**

- \( s \) = Stirrup spacing, circumferentially, inches
- \( s_l, s_m \) = Longitudinal spacing between circumferential reinforcing, center to center
- \( w_s \) = Width of crack at level of steel, inches
- \( x \) = Crack spacing, inches
- \( \delta_h \) = Horizontal deflection of pipe, inches
- \( \delta_v \) = Vertical deflection of pipe, inches
- \( \phi \) = Diameter of reinforcing wire, inches

**Sectional Properties**

- \( A_c \) = Area of concrete section, in.\(^2\) per foot of length
- \( A_g \) = Gross area of concrete section
- \( A_s \) = Steel area, in.\(^2\) per foot of length
- \( A_{s1} \) = Steel area of inner cage, in.\(^2\) per foot of length
- \( A_{s2} \) = Steel area of outer cage, in.\(^2\) per foot of length
- \( A_t \) = Effective tensile area of concrete
- \( A_V \) = Stirrup steel area, in.\(^2\) per foot of length
- \( I \) = Moment of inertia of cross section
- \( I_c \) = Moment of inertia of gross uncracked concrete section, in.\(^4\) per foot of length
- \( I_{cr} \) = Moment of inertia of average transformed cracked concrete section between cracks, in.\(^4\) per foot of length
- \( I_T \) = Moment of inertia of transformed cracked concrete section at a crack, in.\(^4\) per foot of length
NOMENCLATURE AND UNITS (Continued)

Forces, Loads, Moments

\[ C = \] Compressive force in concrete

\[ (DL)_u = \] Ultimate D-load capacity of pipe, lbs/ft²

\[ (DL)_{uc} = \] Ultimate D-load capacity of pipe wall less effect of stirrups, lbs/ft²

\[ (DL)_{ue} = \] Ultimate D-load capacity with internal pressure, lbs/ft²

\[ (DL)_{um} = \] Ultimate D-load capacity without internal pressure, lbs/ft²

\[ (DL)_{0l} = \] .01 inch crack D-load capacity of pipe, lbs/ft²

\[ (DL)_{0le} = \] .01 inch crack D-load capacity with internal pressure, lbs/ft²

\[ (DL)_{0lm} = \] .01 inch crack D-load capacity without internal pressure, lbs/ft²

\[ H_o = \] Pressure head, feet

\[ H_{cm} = \] Pressure head at first crack, internal pressure only, feet

\[ H_{um} = \] Maximum pressure head at failure, internal pressure only, feet

\[ H_{0lm} = \] Pressure head at .01 inch crack, internal pressure only, feet

\[ M = \] Bending Moment

\[ M_p = \] Ultimate bending moment

\[ M_{p1} = \] Ultimate bending moment at crown and invert

\[ M_{p2} = \] Ultimate bending moment at springings

\[ M_s = \] Bending moment in primary statically determinate structure
NOMENCLATURE AND UNITS (Continued)

Forces, Loads, Moments

\[ m \] = Bending moment due to unit redundant
\[ N \] = Axial Force
\[ P \] = Load on pipe
\[ P_{u} \] = Ultimate load on pipe, test load
\[ P_{fu} \] = Ultimate load on pipe, field loading
\[ P_{w} \] = Pressure from fluid, psi
\[ R_{s} \] = Shrinkage force in pipe wall
\[ T \] = Tension force in circumferential steel
\[ T_{s} \] = Tension force in stirrup steel
\[ V \] = Shear force
\[ V_{c} \] = Shear force taken by concrete
\[ V_{cr} \] = Shear force at section of critical shear
\[ V_{s} \] = Shear force taken by steel
\[ W \] = Weight of pipe, lbs per foot of length
\[ \gamma \] = Unit Weight of fluid, lbs/ft\(^3\)

Stresses (all in psi)

\[ f_{c} \] = Compressive stress in concrete
\[ f'_{c} \] = Ultimate compressive strength of concrete as determined from standard 6x12 cylinders tested at age of pipe test
\[ f_{p} \] = Tensile stress at proportional limit in reinforcement
\[ f_{s} \] = Tensile stress in reinforcement
\[ f_{ss} \] = Stress in steel due to concrete shrinkage
NOMENCLATURE AND UNITS (Continued)

Stresses (all in psi)

\[ f'_s = \text{Ultimate tensile strength of reinforcement} \]
\[ f'_{s1} = \text{Ultimate tensile strength of inner cage reinforcement} \]
\[ f'_{s2} = \text{Ultimate tensile strength of outer cage reinforcement} \]
\[ f_t = \text{Tensile stress in concrete} \]
\[ f_{ts} = \text{Tensile stress in concrete due to shrinkage} \]
\[ f_v = \text{Tensile stress in stirrup steel at 0.2\% offset} \]
\[ t_r = \text{Radial tension stress in concrete} \]
\[ t_\theta = \text{Circumferential tension stress in concrete} \]
\[ u_a = \text{Average bond stress between steel and concrete between cracks} \]
\[ u_m = \text{Maximum bond stress between steel and concrete} \]
\[ v_c = \text{calculated shear stress} \]
\[ v_n = \text{Nominal unit shear stress in concrete} \]
\[ v_t = \text{Test shear stress} \]

Ratios

\[ E_c = \text{Modulus of elasticity of concrete (compression), psi} \]
\[ E_{ct} = \text{Modulus of elasticity of concrete (tension), psi} \]
\[ E_s = \text{Modulus of elasticity of steel (tension), psi} \]
\[ j = \text{Ratio of lever arm of resisting couple to } d \]
\[ k = \text{Ratio of depth of neutral axis to } d, \text{ elastic conditions} \]
\[ k_r = \text{Modified ratio of depth of neutral axis to } d, \text{ including effect of average tension in concrete between cracks} \]
NOMENCLATURE AND UNITS (Continued)

Ratios

\( k_u \) = Ratio of depth of neutral axis to \( d \), ultimate conditions

\( n \) = Ratio of modulus of elasticity of steel to compressive modulus of elasticity of concrete

\( n_t \) = Ratio of compressive modulus of elasticity of steel to tensile modulus of elasticity of concrete

\( p \) = Ratio of area of tension reinforcement to effective area of concrete, bd

\( p_r \) = Modified ratio of area of tension reinforcement to area of concrete, including effect of tension in concrete between cracks

\( r \) = Ratio of area of tension reinforcing to area of concrete, \( A_t \)

\( \varepsilon_f \) = Free Shrinkage strain

\( \varepsilon_{pc} \) = Maximum strain in concrete

\( \varepsilon_s \) = Strain in steel

\( \varepsilon_{sp} \) = Maximum strain in steel

\( \theta \) = Angular rotation at point of maximum moment, radians

Constants

\( A, A_0, A_1, a, B, B_0, B_1, B_{01}, b, c_r, c_v, c, c_1, c_2, c_3, c_4, F, K, K_0, K_1, K_t, B, \lambda_1, \lambda_2, Z \)

Statistical Symbols

\( N \) = Number of tests

\( \sigma \) = Standard deviation
NOMENCLATURE AND UNITS (Continued)

Statistical Symbols

\[ s(y) = \text{Standard deviation estimated from a population} \]
\[ V = \text{Coefficient of variation} \]
\[ \bar{y} = \text{Average value of } y \]
\[ \hat{y} = \text{Estimated value of } y \]
\[ r = \text{Correlation coefficient} \]
I

INTRODUCTION

1.1 Objectives

The primary objective of this thesis is to develop rational procedures for predicting the structural behavior of precast reinforced concrete pipe under various conditions of loading. Such procedures provide a basis for safe and economical structural design of pipe and indicate what characteristics of constituent materials are most desirable. This approach to the design of pipe is based upon the development and extension of modern concepts of ultimate strength, cracking behavior and deformation of reinforced concrete structures for application to circular pipe rings subject to various distributions of loading. These techniques are becoming well established for many types of straight reinforced concrete elements, but have not heretofore been applied to curved ring type elements such as pipe. Instead, it has long been the practice in the pipe industry, both in this country and abroad, to use results of individual "3-edge bearing tests" (20)* on full scale pipe for establishing the validity of empirical pipe designs. Little correlation between pipe behavior and properties of constituent materials has been attempted in a rational manner. This practice has flourished because of the ease of testing these structures. However, with the increasing use of large diameter precast pipe (6 feet to 12 feet internal diameter) and availability of reinforcing steels having a variety of desirable characteristics, it is important to be

*Reference given in Bibliography, Appendix P.
able to establish optimum characteristics for constituent materials and their arrangement in a pipe, prior to confirmation by testing.

At the present time much pipe design in the United States is based on ASTM "Tentative Specifications for Reinforced Concrete Culvert, Storm Drain and Sewer Pipe", C76-61T. This specification includes tables of suggested minimum designs for various classes of pipe. These design tables do not recognize in a rational manner the many properties of the constituent materials in the pipe. The present study is aimed, in part, at providing a basis for the design of pipe which recognizes the significant properties of its constituent materials.

The analytical and experimental work required by this study has also provided an opportunity to gain valuable new insight into some fundamental aspects of the behavior of reinforced concrete structures. The use of carefully controlled methods of manufacture in a precasting plant and the limitation of variables to a few standard pipe sizes, and reinforcing types permits the intensive evaluation of the following aspects of the structural behavior of reinforced concrete elements:

1. Capability of the indeterminate closed pipe ring to redistribute moment effects after plastic yielding in the steel and reach flexural failure as a "mechanism".

2. Effect of limited steel ductility available with the cold drawn wire in welded wire fabric on the ultimate strength of individual sections and on the rotation capacity required for redistribution of moments near ultimate flexural failure.
3. Effect of curvature on ultimate diagonal tension strength.

4. Effect of curvature on requirements of stirrups and ties to resist diagonal tension effects.


1.2 **Criteria for the Structural Design of Pipe**

The structural performance of reinforced concrete pipe is evaluated on the basis of the ultimate strength of the pipe and its cracking behavior under certain distributions of loading. Deformation of the pipe may also be of interest in certain instances. The performance of the pipe is usually evaluated with the aid of a test loading having a known distribution which is simple to apply. Standard specifications for pipe require the evaluation of performance under a "3-edge bearing" test load. This loading is applied by means of a testing machine as a concentrated line load along the crown of the pipe and is reacted by two closely spaced line loads at the invert (20). Results of performance under test load conditions are then suitably modified to indicate expected performance under various field loading conditions.

The ultimate strength of pipe will be limited by its flexural strength (almost always determined primarily by tensile strength of reinforcing, since pipe sections of economical design are "under-reinforced" such that primary compression failure is not likely to occur),
or by its diagonal tension strength. Detailed descriptions of these modes of pipe failure at ultimate load are given later in Chapters 4 and 5.

The cracking behavior of pipe is usually evaluated by observation of the load at which an arbitrarily specified maximum width of crack occurs at the surface of the pipe during a 3-edge bearing test. ASTM C76 defines the .01 inch crack strength as the load which just produces a .01 inch maximum width of crack for a length of one foot, as measured with a standard leaf gage. The details of the gage are given in the C76 specification (20). European specifications limit the maximum crack width in the test to .20 mm (.008 in.) (22,23,24), but use somewhat different methods for arriving at the required 3-edge bearing working capacity for various field conditions.

Recent American practice defines the 3-edge bearing ultimate load and .01 inch cracking load capacities in terms of D-loads. A D-load is the total load per foot of pipe length divided by the inside diameter of the pipe. Thus, pipe of a given class will have the same ultimate D-load (DLu) requirement and the same .01 inch cracking D-load (DL.01) requirement regardless of diameter of pipe.

Deformation of pipe is evaluated from measurement of the total vertical deflection of the crown point toward the invert and the total horizontal movement apart of the springing points on the pipe, under 3-edge bearing load.

Field loading conditions produce lower moments and diagonal tension effects than the 3-edge bearing load, for the same total applied load. The 3-edge bearing load distribution is the most severe
that could occur in the field if the pipe were poorly bedded on hard material such as rock or dense soil. Usually, it is permissible to assume a more favorable distribution of field loading and thus to increase the estimated total load which the pipe can support in the field over that in the 3-edge bearing test by a load factor which varies from 1.1 to 2.0 (1,2). Either the .01 inch cracking load in the test or 2/3 of the ultimate test load, whichever is lower, is taken as the test working load (in American practice). This test working load is multiplied by the load factor to obtain field working load capacity of the pipe. The load factor is determined as follows:

1. When .01 inch cracking load controls: the load factor is the ratio of field .01 inch cracking load to test .01 inch cracking load. It is often assumed to be the ratio of test moment to field moment at the critical point in the pipe (usually the invert) under elastic action of the pipe.

2. When flexural or diagonal tension ultimate strength controls: the load factor is the ratio of field ultimate load capacity to test ultimate load capacity.

Methods for estimating the magnitude and distribution of earth and surcharge loads on pipe are also part of the essential criteria for pipe design. The development of such criteria is primarily a problem in soil mechanics and has not been considered in this thesis or its companion test program.
1.3 **Scope of Investigation**

This investigation of the significant structural characteristics of pipe has been carried out in conjunction with an extensive test program, which was directed by the author (8). This test program provided much insight into the important factors which influence the behavior of pipe and furnished valuable data in cases where purely theoretical developments were not practical and semi-empirical procedures were necessary. The test methods used and the test results have already been reported by the author and Saba (8). No further report on this aspect of the investigation, except for analysis of test results and statistical use of certain data, will be given in this thesis.

The theoretical part of the investigation, which was used, in part, to obtain the conclusions given in the above report (8), will be given herein in its entirety. The following aspects of the structural behavior of pipe will be considered:

1. Flexural action in the elastic range, including distribution of moment, shear and thrust under various distributions of load, and various distributions of cracking.

2. Effect of pipe curvature on internal distribution of stresses in a cracked concrete section in the elastic range.

4. Flexural ultimate strength including consideration of plastification of the cross section, redistribution of moments around the structure, rotation capacity required at plastic hinges, and effect of distribution of loading.

5. Diagonal tension ultimate strength, including effect of curvature on shear stress, influence of radial tension stress, effect of flexural cracking, and location of critical section under various distributions of loading.

6. Requirements for effective use of web reinforcing.

7. Determination of relationship between load and .01 inch crack width. Use of crack criteria for selection of area of circumferential steel.

8. Estimation of deformations of the structure at working loads, including consideration of effect of cracking and creep of concrete.

9. Effect of the addition of internal pressure in pipe subject to the simultaneous action of external loads.

The procedures developed herein to evaluate the above aspects of the structural behavior of pipe are based, in part, on the extension of some well established and also some recently suggested methods for evaluating ultimate strength and cracking behavior of straight reinforced concrete elements; in part, on new theoretical developments by the author; and in part, on the author's statistical analysis of data from the 39 pipe tests reported in
Whenever possible, the results of theoretical and semi-empirical procedures developed in this thesis will be compared with test results reported in references (8), (11), (14), (15), (16).

Finally, an analysis of the several pipe classes established in ASTM C76-61T will be made, using the procedures to be developed herein. Based on these methods, tentative suggestions will be made for the rational design of ASTM C76 Class II, III and IV pipe reinforced with welded wire fabric. Suggestions will also be made for obtaining additional data required to permit similar rational designs for pipe with other types of reinforcing.

This investigation has not included a consideration of the magnitude and distribution of loads on buried pipelines. Treatment of this important subject is limited to consideration of the behavior of pipe for an arbitrary load distribution which is deemed representative of one type of field loading, and comparison of the action of pipe under this type of loading with the 3-edge bearing load condition.

1.4 Acknowledgments

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Thanks are due finally to Miss Marion Curley and Miss Judith Forrest for their careful typing of this thesis.
2 HISTORICAL BACKGROUND

2.1 Development of Methods for the Structural Design of Pipe

Current specifications and design practice for precast reinforced concrete pipe, both in the United States and abroad, are based largely on the experimental results of pipe test programs carried out over the years and on the accumulated experience of pipe manufacturers. An extensive review of American, British, French, German and Spanish technical journals and standard pipe specifications did not reveal a single instance where modern concepts of ultimate strength or cracking behavior of reinforced concrete structures had been applied to evaluate the basic factors which influence the structural behavior of reinforced concrete pipe.

A complete listing of standard American specifications for reinforced concrete pipe is given in reference (1), together with a history of the development of the most important of these specifications. For the most part, these specifications define both minimum acceptable standardized designs for pipe (minimum steel area and arrangement, wall thickness, and minimum steel and concrete properties for various standard sizes and load capacities) and a mandatory 3-edge bearing acceptance test for cracking behavior and ultimate strength. Typical German, French and Spanish specifications, on the other hand, stipulate only the cracking and ultimate test strength requirements for various types of pipe (22, 23, 24).

The author and Saba (8) have summarized recent test
programs (8, 11, 14, 15, 16) which have provided valuable experimental data on the structural action of pipe under 3-edge bearing load. These programs will be analyzed by the methods to be developed herein. Results of many older test programs are of no use for evaluating the methods to be proposed herein because important data on the properties of constituent materials in the pipe, particularly the reinforcing steel, often was not reported. In many instances, the mode of failure was not indicated. Several other test programs of historical importance are discussed in the literature (17, 18, 19). Tables of standard pipe designs in American specifications for pipe are largely based on the above test programs and the experience of the pipe industry.

Theoretical investigations relating to the strength of pipe structures which are available in the literature all refer to the analysis of circular pipe rings for bending moment, shear force and axial force using the elastic theory for indeterminate closed rings. Such solutions are available for 3-edge bearing loading and many other distributions of external loading on the pipe (25, 26, 28, 29, 30 and 31). No comprehensive investigations of the theoretical ultimate strength or cracking behavior of pipe were encountered. Guerrin has presented (4) an extensive discussion of methods for the prediction of initial cracking in reinforced concrete pipe. However, he gives only a brief discussion about the rupture strength of reinforced concrete pipe (in flexure). His suggested semi-empirical ultimate strength procedure does not refer to the effect of redistribution of moment prior to flexural failure; instead the increased strength resulting from redistribution of moment appears to be erroneously ascribed to the contribution
of tensile strength in the concrete at the cracked section.

Much work has been done in the past to determine the magnitude and distribution of static earth loads and moving vehicular loads on buried pipe (1, 2, 3, 7, 26, 27, 28, 29). In spite of all this work, considerable uncertainty still remains about the actual magnitude and distribution of loads on buried structures such as pipelines. This important problem has not been considered within the scope of this thesis.

2.2 **Ultimate Strength of Reinforced Concrete Structures**

A great effort has been expended in recent years to develop methods for predicting the ultimate strength of reinforced concrete structures. This work usually has considered flexural ultimate strength of reinforced concrete sections, redistribution of moments due to inelastic deformation, and shear ultimate strength as separate problems. In general, these ultimate strength methods have been developed for application to continuous frameworks of straight members. Modifications and new developments are required for application to curved structures such as pipe.

Well substantiated methods have been available for many years for accurately predicting the flexural ultimate strength of a reinforced concrete section. Summaries of flexural ultimate strength methods which have been developed by various researchers are available in the literature (32, 33). Many of these methods are widely applied for the practical design of structures at the present time. The ultimate strength method for the design of flexural sections was
accepted as an alternate procedure by the ACI Building Code in 1956. The method is also widely used abroad. Bianchini and Kesler (55) have extended these procedures to analyze flexural ultimate strength of sections reinforced with welded wire fabric.

Another aspect of the flexural ultimate strength of indeterminate structures is the redistribution of bending moments which occurs throughout many structures when points of maximum moment undergo inelastic deformation. So called "limit analysis" procedures have been developed to evaluate this effect. These procedures were first applied to continuous steel structures in a significant research effort during the 1930's by J. F. Baker and his associates in England. Because of the ductility of reinforcing steel, it was evident that such procedures, with suitable precautions, could also be applied to reinforced concrete structures. Professor A. L. Baker of the Imperial College, London, was one of the first to develop "limit analysis" procedures for reinforced concrete structures (37). Summaries of procedures for application of "limit analysis" in reinforced concrete design are available in the literature (34, 35, 36). Few building codes permit full use of "limit analysis" at the present time. However, some codes already permit an arbitrary distribution of bending moment resistance in continuous beams and frames between midspan section and end sections such that statical strength requirements are satisfied without strict conformance to the requirements for compatibility of elastic deformations. This constitutes tacit recognition that some degree of moment redistribution from elastic distribution may occur in a continuous structure.
Baker (37) has developed the requirement for rotation capacity of plastic hinges and ductility required of the steel to permit redistribution of moments. Hajnal-Konyi (38) has found that cold drawn carbon steel, the material used in welded wire fabric, has enough ductility to permit full redistribution of moments in many types of structures.

The above concepts of ultimate flexural strength provided the basis for the flexural ultimate strength theory for pipe given in Chapter 4.

It is only within the last few years that a concerted research effort has been made to develop a widely applicable method for predicting the ultimate shearing or diagonal tension strength of reinforced concrete beams. Joint ACI-ASCE Committee 326 on Shear and Diagonal Tension (39) has suggested a quantitative method for predicting shear strength of beams without web reinforcing under the action of shear force and bending moments. This method appears to be the most reliable procedure available, since it has a firm statistical basis from the evaluation of several hundred tests on straight beams which failed in shear. The Committee (39) has also given a complete history of previous work on shear strength.

The work of Committee 326 has provided valuable insight into some important factors which affect the diagonal tension strength of pipe. This method used by Committee 326 to develop its design formula for straight beams has provided a basis for the further development in this thesis (Chapter 5) of a method of predicting the ultimate diagonal tension strength of curved flexural members. This extension of the Committee's work to include effects of pipe curvature has established
the major variables which affect ultimate diagonal tension strength and, thus, has permitted a rational analysis of pipe test data. The similarity between pipe test results and the above extension of the Committee 326 work provides a valuable link between the pipe tests and the far more numerous beam tests reported in (39).

Committee 326 (39) has also suggested modifications for evaluating shear strength of beams under the combined action of bending, shear and axial force. Application of these modifications to pipe provides a means of comparing effect of various field loading distributions with test load conditions as explained in Chapter 5.

2.3 **Cracking Behavior of Reinforced Concrete Structures**

The influence of width and spacing of cracks on the serviceability of reinforced concrete structures is widely recognized. However, little scientific basis is available for the establishment of rational criteria for the evaluation and control of cracks in such structures. Most codes and specifications for reinforced concrete design provide for control of cracking by the limitation of working stresses in the reinforcing steel. No standard American Specifications or codes, except the pipe specifications referred to previously, permit the use of cracking criteria for the determination of working loads with high strength, non-prestressed, reinforcements. A few European specifications have begun to introduce this practice (41). Recently, however, experimental work has been undertaken in many parts of the world to provide a more rational basis for crack control criteria.

The problem of how to determine the maximum allowable crack
width to limit corrosion of the reinforcement has been discussed in the literature \((41,46,47)\). In several countries, suggestions have been made to limit maximum crack width to about .2 mm or .0079 inches (Europe) and .01 inch (U. S. concrete pipe practice). Certainly, more experimental data is needed to establish the validity of such limits. The problem is further complicated by the phenomenon of autogeneous healing of small cracks in concrete. Peckworth (1) has given a well substantiated discussion of this phenomenon in reinforced concrete structural elements.

The development of procedures for estimating crack spacings and widths requires statistical analysis of large quantities of data from carefully controlled tests, because of the inherent variability associated with the cracking process. Each type of reinforcing steel must be evaluated separately. Research programs which investigated crack width and spacing in reinforced concrete structural elements with various types of reinforcing have been reported in the literature \((40,41,42,43,44,45,48)\). In conjunction with most of these programs semi-empirical methods, with constants obtained from test results, were developed for the prediction of crack widths in practical structures. High variability between predicted and measured results is expected of all such procedures. Some of the concepts suggested in the above references provided a basis for the development in Chapter 7 of criteria for prediction of crack widths in pipe reinforced with welded wire fabric.
2.4 Deformation of Reinforced Concrete Structures

Methods for the evaluation of deformation of reinforced concrete structures are needed in order to control deflections under working load conditions. Such methods usually may be based on well known elastic methods of structural analysis, but require a means for evaluating the average stiffness of cracked concrete structures. The existence of tension in the concrete between cracks greatly affects the average stiffness. Methods for evaluating average stiffness must be closely connected with the investigation of crack width and spacing.

Yu and Winter (50) give methods for prediction of both long term and short term deflections which are relatively simple to apply and give good correlation with test results on straight, simply supported beams. Johnson (51) presented an earlier development of these concepts, with little test data for correlation. Methods for estimating pipe deformation in Chapter 8, which were developed from the cracking theory given in Chapter 7, are somewhat similar to the general methods by the above authors.

2.5 Shrinkage Effects

Generally accepted methods which are available for estimating tensile concrete stresses due to restraint of free shrinkage by the reinforcing steel and by external supports have been summarized by Large (52) together with an extensive list of original source material. These methods are used in Chapter 3 to evaluate the effect of shrinkage and plastic flow in pipe structures.
2.6 **Curvature Effects**

Methods for the elastic analysis of sharply curved beams must recognize the influence of initial curvature on the distribution of strain due to bending. The elastic curved beam theory for analysis of flexural stresses in homogeneous symmetrical sections is given in most textbooks on Strength of Materials. However, few applications of this theory to the elastic analysis of non-homogeneous, reinforced concrete elements are available in the literature. The problem is discussed by Wright (53) and by Topaloff (54), but neither develops results that can be applied to the investigation of steel and concrete stresses in curved pipe rings. Besides, Wright (53) seems to have made approximations which cannot be substantiated. Therefore, a new analysis of the effect of initial curvature on elastic steel and concrete stresses in pipe rings is developed in Chapter 3 by the author.

No work was found in the literature dealing with the effect of curvature on shear and radial tension stresses in reinforced concrete curved members. This development was carried out by the author in conjunction with the work on ultimate diagonal tension strength of pipe in Chapters 5 and 6.

2.7 **Welded Wire Fabric as a Reinforcement for Concrete**

Since the pipe test program (8) which provides the experimental basis for certain sections of this thesis used welded wire fabric reinforcing exclusively, a summary of sources for basic information on this type of reinforcing steel is of special interest. Bianchini and Kesler (55) recently have reported on an extensive program of research on the
use of welded wire fabric as a reinforcement for concrete. Their report includes a comprehensive list of previous related work.

Test data given by Bianchini and Kesler (55) indicates that the usual ultimate strength theory for under-reinforced flexural elements (32) could be used for evaluation of the flexural ultimate strength of sections with welded wire fabric reinforcing. The ultimate strength of the fabric wire, rather than any arbitrarily defined yield point, should be used in the ultimate strength theory, when tensile steel strength, rather than compressive concrete strength, controls the strength of a section. Other tests reported by Bianchini and Kesler indicated the anchorage requirements to develop the full strength of welded wire fabric in pull out tests. An embedment of at least two cross wire spacings was generally sufficient to develop the full strength of most fabric styles tested. These tests indicated that very light cross wires would develop good mechanical bond strength at the intersections of main and cross wires. However, Bianchini and Kesler did not develop quantitative procedures to evaluate the cracking behavior or shear strength of flexural elements reinforced with various styles and quantities of welded wire fabric. Some qualitative information was given on these aspects of the structural behavior of slabs with welded wire fabric reinforcing.
3

BEHAVIOR IN THE ELASTIC RANGE

3.1 Introduction

Working load conditions for a precast reinforced concrete pipe usually occur while all the constituent materials are stressed in what is normally considered their "elastic range." In this range, the stress-strain relationship of the tension and compression steel is well represented by a constant modulus of elasticity for steel, $E_s$, and the compression concrete stress-strain relationship is approximately represented by a constant ratio of stress to strain, $E_c$. Under full working loads, the tension concrete is usually cracked in reinforced concrete pipes. The ratio of compression stress to strain will not be constant with time if loads are not instantaneous (i.e., creep of concrete in compression will occur).

Because of the important effect of cracking on serviceability of pipe, it is necessary to limit the maximum crack width under working load, while all materials are still stressed in their elastic range. Thus, the stress conditions which produce cracks of moderate width under working loads must be examined in detail. Secondly, ultimate failure in diagonal tension often occurs while stress conditions are still elastic. Again, an elastic stress analysis provides information required for the proper evaluation of ultimate strength in diagonal tension. Finally, working load deflections may also be of some significance, although in pipe it is
normally only the effect of order of magnitude of deflection on field
loading which is of much real importance (i.e., whether pipe is rigid
or flexible). Nevertheless, a good understanding of structural
behavior requires a method for predicting deflections under working
loads, which can be based on the elastic behavior of the pipe. Hence,
it is necessary to examine in detail the distribution of moments,
shears and axial forces and their associated stresses, in a pipe under
various types of loading at a stage where all the constituent materials
are still stressed in the elastic range.

3.2 Effect of Initial Curvature on Stress and Stiffness

Since a pipe under arbitrary external load is an indeterminate
structure, in order to obtain a stress analysis for the pipe, it is
first necessary to determine whether the existence of initial curva-
ture in the pipe exerts a sufficiently large effect on stress and
stiffness to require modification of the usual methods for stress
analysis of indeterminate closed ring type structures. Wright (53)
and Topaloff (54) both discuss approaches for the evaluation of the
distribution of flexural stresses in a curved reinforced concrete
member. However, neither of these authors present a satisfactory
method of investigation for application to pipe. Hence, the author
developed the following simple equations for determining the sectional
properties, and the flexural steel and concrete stresses for the inves-
tigation of a curved concrete cracked section (see Appendix B for
derivation):
Case A: Tension on Inside of Curve:

\[
k = \sqrt{\frac{2np}{r_{si}} \left( 1 - 2np \frac{d}{r_{si}} \right) + \left[ np \frac{(r_o + d)}{r_{si}} \right]^2 - np \frac{(r_o + d)}{r_{si}}} \frac{1 - 2np \frac{d}{r_{si}}}{1 - 2np \frac{d}{r_{si}}}
\]  

\[j = 1 - \frac{k}{3} - \frac{1}{2} \frac{k^2}{(r_o - kd)} + \ldots \approx 1 - \frac{k}{3}\]

\[f_c = \frac{2M}{jkbd^2} \frac{(r_o - kd)}{r_o}\]

\[f_s = \frac{M}{A_s jd}\]

Case B: Tension on Outside of Curve:

\[
k = \sqrt{\frac{2np}{r_{so}} \left( 1 + 2npd \frac{d}{r_{so}} \right) + \left[ np \frac{(r_i - d)}{r_{so}} \right]^2 - np \frac{(r_i - d)}{r_{so}}} \frac{1 + 2np \frac{d}{r_{so}}}{1 + 2np \frac{d}{r_{so}}}
\]

\[j \approx 1 - \frac{k}{3}\]
\[ f_c = \frac{2M}{jkbd^2} \left( \frac{r_i+kd}{r_i} \right) \]

\[ f_s = \frac{M}{A_sjd} \]

Investigation of typical precast reinforced concrete pipe (ASTM C76), with \(d/r\) approximately equal to \(1/6\), indicates that use of the above theory based on effect of initial curvature, instead of the usual elastic theory for straight reinforced concrete members, results in an increase in steel tension of less than 1% and decrease in maximum concrete compression stress of less than 10% at the crown and invert of the pipe; likewise at the springings, steel tension is decreased less than 1% and maximum concrete compression stress increased less than 10%.

Because the above relations indicate that stresses in a curved pipe do not vary significantly from values determined from the linear assumptions of the elementary beam theory, it also follows that calculation of deformations in a pipe in the elastic range may be based on the linear elementary beam theory with good accuracy. Since the elastic stress analysis of the indeterminate pipe ring only depends upon satisfaction of the requirements of statics and on the deformation characteristics of the pipe, it follows that the above described effects of the "curved beam" theory will not significantly alter the magnitude and distribution of moments, shears and axial forces in a pipe from that determined by the usual thin ring elastic analysis. Actually, the
effects of uncertain variation of stiffness around the pipe due to cracking of tension concrete introduces far more uncertainty to the elastic stress analysis of the pipe ring, than the effects of non-linear stress variation due to pipe curvature ever would cause.

3.3 Test Load Elastic Stress Analysis

Figure 3-1 summarizes the moments, shears and axial forces in a pipe under the following four elastic stages of 3-edge bearing test load:

(a) Weight of pipe without test load.
(b) Test load - no cracking in the pipe.
(c) Test load - cracking only at crown and invert to extent shown in Fig. 3-1 (c).
(d) Test load - cracking at crown and invert and at springings to extent shown in Fig. 3-1 (d).

In addition, for completeness, the moments, shears and axial forces for the ultimate load condition which will be discussed in Chapter 4 are included in Sketch (e) of Fig. 3-1.

The results of the weight of pipe analysis and of the test loading with no cracking (i.e., constant EI around the pipe circumference) can be calculated using formulas given in reference 30, and are also available with various modifications in references 25 and 26. The results for the two conditions of partial cracking were not available in the literature and were obtained by the writer using the "elastic center" method of stress analysis. An abbreviated derivation
FIG. 3-1  TEST LOADING-DISTRIBUTION OF MOMENTS, SHEARS AND AXIAL FORCES
for the final cracked condition (Fig. 3-1, Sketch d) is given in Appendix C. For this analysis, the relative ratio of stiffness in the cracked and uncracked regions was estimated at 0.3, using the methods presented in Chapter 8. The estimated extent of cracked regions was based on the observed extent of cracking in the M.I.T. tests (see Sketches in Appendix E, reference 8).

It is seen that the distribution of cracking has an important effect on the distribution of moments, but no effect on the shears or axial forces. However, in the final stage of cracking prior to start of inelastic strain in the steel, with cracked regions at the crown and invert, the moment distribution is very similar to the uncracked distribution.

The following expressions for total vertical deflection of crown toward invert and total horizontal deflection of springings (movement apart) were developed for each of the test load cases shown in Fig. 3-1:

Uncracked case (b):

\[ \delta_v = \frac{0.149 P r^3_{av}}{E I_c} \quad ; \quad \frac{\delta_v}{\delta_h} = 1.09 \]  

1st stage cracking (c):

\[ \delta_v = \frac{0.079 P r^3_{av}}{E I_{cr}} = \frac{0.263 P r^3_{av}}{E I_c} \quad ; \quad \frac{\delta_v}{\delta_h} = 1.25 \]
2nd stage cracking (d):

\[ \Delta_v = \frac{0.126 \text{Pr}^3}{EI_{cr}} = \frac{0.420 \text{Pr}^3}{EI_c} ; \quad \frac{\Delta_v}{\Delta_h} = 1.08 \]

Assumptions: \[ I_{cr} \approx 0.3I_c \] and extent of cracking as shown in Fig. 3-1, (c) and (d).

It is evident that deflections increase at a much faster rate under cracked conditions. It should also be noted that the transition between the above assumed stages of cracking is a gradual one. The load deflection curves in Appendix D of reference 8 illustrate the gradually increasing rate of deflection with increase in test load.

Equation 3-8 for the uncracked case (a) may be found in standard reference works (30). Equations 3-9 and 3-10 for the cracked cases were developed by the writer using the method of virtual work in conjunction with the previous "elastic center" solution for moments. A typical derivation for stage 2 cracking is given in Appendix C.

A practical method for calculating the effective stiffness, \( EI_{cr} \), of cracked concrete regions in pipes reinforced with welded wire fabric will be developed in Chapter 8.

3.4 Field Load Elastic Stress Analysis

Figure 3-2 summarizes the moment, shear and axial force.
FIG 3.2 FIELD LOADING: DISTRIBUTION OF MOMENT, SHEAR AND AXIAL FORCE
distributed obtained from elastic stress analysis of pipe for the following three types of field loading with Class C* bedding:

(a) Weight of pipe alone

(b) Weight of water in pipe

(c) Earth load - Elastic conditions with constant moment of inertia.

Since the distributions of moments, shears and axial forces around the pipe did not differ appreciably in the second stage cracking condition from the uncracked distribution for test load conditions see Fig. 3-1 (b) and (d), no separate analysis was made for the cracked stages under field loading.

In addition to the above elastic analyses, the moments, shears and axial forces for the ultimate earth load condition, Class C bedding, which will be discussed in Chapter 4, are included in Sketch (d) of Fig. 3-2 for completeness.

The results for the water load and elastic earth load cases were available from references 25 and 26 and were checked by the writer, using the "elastic center" method. The writer made the analysis for weight of pipe loading under field bedding using the same method.

3.5 Shrinkage Stresses

Tensile stresses due to restraint of the free shrinkage of the concrete by the reinforcing steel exist in most reinforced concrete

*See (1) or (2) for detailed explanation of pipe bedding practice and classification.
structural elements. If additional restraints are imposed due to indeterminacy of the reinforced concrete structure, then additional shrinkage stresses will occur in the structure.

In general, the amount of free shrinkage which is restrained will increase with time, thus tending to produce higher shrinkage tensile strains. The rate of this increase is reduced when hydration of the cement is accelerated by steam curing, or when free water is reduced by a manufacturing process such as that used for centrifugal pipe. However, because of creep effects in the concrete, the effective modulus of elasticity of the tension concrete will show a marked decrease with time, thus tending to reduce the shrinkage tensile stresses. Furthermore, the tensile strength of the concrete increases with the age of the concrete. Hence, it is difficult to determine the age at which the least margin of safety against the effects of shrinkage tensile stresses occurs with reinforced concrete elements. Nevertheless, in reinforced concrete pipe, the existence of a critical age for lowest resistance to first crack, due to the shrinkage phenomena discussed above, is also evident from the experience of several producers of pipe over many years. Pipe which are more than several months old often show higher resistance to cracking than pipes which are only several weeks old.

Current methods of analysis for evaluating shrinkage stresses in reinforced concrete have been summarized elsewhere (52). The methods of shrinkage analysis which have been developed for statically determinate reinforced concrete structures will be shown to be applicable to statically indeterminate pipe structures.
For circular pipe with two circular reinforcing cages of equal or unequal area, the maximum internal force due to restraint of free shrinkage by the reinforcing steel may be estimated by the following equation for shrinkage force in a symmetrically reinforced, statically determinate, element (see reference 52, page 343 for derivation):

$$R_s = \sum f E_s A g \frac{p_s (1-p_s)}{1+ (n_c-1)p_s} \quad 3-11$$

where $$p_s = p_1 + p_2 = \frac{A_s + A_{s2}}{bd} \quad 3-11a$$

This force acts as tension on the concrete and compression on the reinforcing steel. Maximum stress in the concrete (tension) is:

$$f_{ts} = + \frac{R_s}{A_c} \quad 3-12$$

Maximum stress in the steel (compression) is:

$$f_{ss} = - \frac{R_s}{A_s + A_{s2}} \quad 3-13$$

Equation 3-11 was developed by others (52) to evaluate shrinkage stresses in symmetrically reinforced, statically determinate, elements.
It may be shown by symmetry arguments that this expression is approximately valid for a statically indeterminate pipe ring with reinforcing steel located at a constant eccentricity around the ring if the pipe wall is not too thick. Because of symmetry, every section around the ring must be identical, both before and after uniform shrinkage in the concrete. Hence, circumferential strains must be very nearly constant across the thickness of the pipe wall. They would not be exactly constant across the wall thickness unless the radial strain was equal to the circumferential strain. This situation would occur in a plain concrete pipe subject to uniform shrinkage. However, in circularly reinforced pipe rings, radial strains would differ substantially from the circumferential strains, because of entirely different restraints in the radial direction than in the circumferential direction; moreover, radial strains would result from radial stresses due to the curvature effect of the circumferential stresses and from radial contraction (Poisson effect) associated with circumferential stresses. If the radial strain is zero or small relative to the circumferential strain, the relative magnitude of circumferential strains at outside and inside of the ring wall is approximately:

\[
\frac{\varepsilon_o}{\varepsilon_i} = \frac{r_i}{r_o}
\]  

Thus, for standard ASTM wall B pipes, the strain at the outside of the pipe wall would be about 17% less than the strain at the inside. This
means that maximum stress will vary no more than about ± 10% from the average value obtained from the assumption of constant stress over the wall thickness.

Similar reasoning indicates that for a pipe with two circular reinforcing cages, compressive strains in both reinforcing cages will be approximately equal.

On the basis of the "thin ring" approximation discussed above, the "column analogy" method may also be used to show that the bending moment in the pipe wall must be zero at all sections around the pipe (see Appendix D), thereby making the stresses approximately uniform across the wall thickness.

The development of "exact" relations for shrinkage stresses in a thick wall pipe ring was not considered within the scope of this thesis. Hence, no further quantitative evaluation of the true error associated with the above approximate theory is possible.

For circular pipe with a single elliptical reinforcing cage, the effect of restraint of shrinkage by the steel causes a change in curvature of the pipe and hence variable tensile strains across the concrete. An exact analysis for distribution of internal bending moments around the pipe due to variable eccentricity of reinforcing steel and indeterminacy of the pipe is not warranted, because fairly wide variations in steel placement from the assumed elliptical shape will occur in practice. The approximate analysis given in Appendix D assumes that the eccentricity is evenly divided between plus values at the crown and invert and minus values at the springings. Thus, from this elastic analysis, using the "column analogy", it is evident that
the final location of centroid of tensile force will approximately coincide with the location of the steel centroid, the statically determinate condition. Therefore, the tensile force in the concrete may be calculated using the following formula which was developed for eccentricity reinforced, statically determinate, elements (see reference 52, page 350 for derivation):

\[
R_s = \frac{\sum f_{ct}^E}{\frac{1}{A_c} \left(1 + \frac{1}{n_tE} \right) + \frac{e^2}{f_{c}}}
\]

Maximum stress in the concrete (tension) is:

\[
f_{ts} = \frac{R_s}{A_c} \left(1 + \frac{6e}{h}\right)
\]

Maximum stress in the reinforcing (compression) is:

\[
f_{ss} = -\frac{R_s}{A_s}
\]

The above expressions would also hold for an elliptical pipe with circular reinforcing.

In a circular pipe with one or two cages of circular reinforcing and one cage of elliptical reinforcing the separate effects for circular reinforcing and elliptical reinforcing would be additive.

Calculations for shrinkage effects for two typical standard ASTM C76, 72 inch, Class III, Wall B pipe, one with two circular
reinforcing cages and one with a single elliptical cage, were made using the above relations. The results of these calculations indicate the following stresses due to shrinkage at age 1\(\frac{1}{2}\) days in these pipe. The total free shrinkage strain for steam cured pipe at 1\(\frac{1}{2}\) day age was assumed to be .0003 inches per inch for these calculations:

1. Pipe with two circular cages:
   - Uniform tension in the concrete 79 psi
   - Uniform compression in the steel 7600 psi

2. Pipe with a single elliptical cage
   - Maximum tension in the concrete 152 psi
   - Maximum compression in the steel 7950 psi

It is seen that pipe with elliptical reinforcing would be expected to crack under a somewhat lower load than pipe with circular reinforcing. Probably, the higher shrinkage effects in such pipe would cause a slight reduction of the .01 inch cracking load, but it is doubtful if this effect would be of much significance. It will be shown in Chapter 7 that tensile stresses in the concrete make a much greater relative contribution to .01 inch cracking strength of lightly reinforced pipe. Such pipe have lower shrinkage stresses than more heavily reinforced pipe.

In general, the above investigation of shrinkage in concrete pipe indicates that the effects of restraint of shrinkage do not have an important bearing on the .01 inch cracking resistance or ultimate strength of precast concrete pipe. However, they do influence the load under which the first crack is formed in the pipe.
4.1 **Introduction**

The ultimate flexural strength of a reinforced concrete pipe under various distributions of external loading may be predicted with the aid of modern theories for ultimate strength of ductile structures. The development of these theories for application to the investigation of the ultimate strength of pipe is presented in this chapter. Some pipe, of course, may never reach their predicted ultimate flexural strength because of prior diagonal tension failure in the concrete. The development of a method for investigating diagonal tension ultimate strength of pipe will be presented in Chapter 5.

4.2 **Plastification of the Cross Section**

Typical steel areas used in even the highest strength class reinforced concrete pipe result in "under-reinforced" sections. That is, under increasing load, as the stress condition in a pipe goes beyond the elastic range, yielding occurs first in the steel prior to compressive failure in the concrete. Under increasing load, this plastic yielding of the steel causes the neutral axis of the section undergoing yielding to move upward, reducing the concrete compression area, until finally the concrete is crushed. The so-called "Whitney Theory" for ultimate strength of under-reinforced sections, or some modification of the same basic
concept (32, 33), has been widely used to evaluate the ultimate strength of under-reinforced sections.

In order to conform to the assumptions of these methods, the steel must have sufficient ductility to permit this movement of the neutral axis up to the point where crushing of the concrete is complete. With some steels, such as cold drawn wire used in welded wire fabric, the ductility is not always sufficient to exhaust the entire compressive strength of the concrete prior to rupture of the steel. However, from one point of view, at least, this is a desirable situation since the full strength of the steel is developed when the capacity of the section is exhausted. Moreover, even if the concrete has not quite reached its ultimate strength when the steel fails, the error introduced by the use of an arm of resisting moment based on ultimate compression in the concrete is insignificant compared with other variables such as the true value of the steel strength, the precise location of steel, etc. Test results on pipe indicate good agreement with a theory based on the assumption that ultimate steel and concrete strengths are both attained at failure (see Section 4.5). Other investigators have also found that cold drawn wire reinforcement can effectively develop the strength of the concrete as assumed in this theory (37, 38, 39).

On the other hand, with the various types of hot rolled reinforcing steels, failure of the concrete may occur at a steel strain between the initial yield point strain and the strain at ultimate tensile strength. In general, it is difficult to determine how much
steel strength beyond the yield point, due to strain hardening, can be attained prior to crushing of the concrete, and thus it will be difficult to determine a rational estimate for the steel stress at ultimate flexural failure. Hence, with many types of hot rolled reinforcing steels, it is necessary to base ultimate strength predictions and designs on the yield point stress in the steel, rather than on the ultimate strength of the steel. The American Concrete Institute Building Code (ACI 318-56), as well as references 32 and 33 indicate that the yield point stress should be used for evaluating flexural ultimate strength of concrete elements reinforced with deformed hot rolled reinforcing. However, no test results on pipe reinforced with hot rolled reinforcing and proportioned to fail in flexure were available to indicate the actual flexural ultimate behavior of such pipe.

The maximum moment which a ductile reinforced concrete section can carry at ultimate load conditions will be termed the "plastic moment capacity", \( M_p \). The plastic moment capacity of the crown and invert sections of a typical precast concrete pipe under external loads which produce tension on the inside of the pipe is given by the following equation (see Appendix E and Fig. 4-1 for derivation):

\[
M_p = f' P_1 \frac{A}{s_1 s_l} (d - 0.5a_l) \cdot \frac{1}{12} \text{ ft-lbs} \quad 4-1
\]
where: \[ a_1 = \frac{f'_{s_1}}{f'_{c}} A_{s_1} \] 4-2

and: \[ f'_{s_1} = \text{the ultimate strength of cold drawn wire,} \]
\[ \text{or other steels of intermediate* ductility,} \]
\[ \text{inner cage reinforcing.} \]

or: \[ f'_{s_1} = \text{the yield strength for steels of high} \]
\[ \text{ductility with substantial* plastic strain} \]
\[ \text{before onset of strain hardening.} \]

The plastic moment capacity at the springing is significantly
affected by compressive axial force in the pipe at this point. Because
the pipe is under-reinforced (i.e., steel yields prior to crushing
of the concrete), this compressive force increases the moment capacity
at the springing. This moment capacity is given by the following
equation (see Appendix E and Fig. 4-1 for derivation):

\[
M_p = \left[ f'_{s_2} A_{s_2} (d_2 - .5a_2) + .25 (P_u + .5W) d_2 \right] \frac{1}{12} \text{ ft-lbs} \quad 4-3
\]

If the pipe conforms to ASTM C76 as indicated in Appendix E:

\[
M_{p_2} = .75 M_{p_1} + .25 (P_u + .5W) \frac{d_2}{12} \text{ ft-lbs} \quad 4-4
\]

*Note: The effect of stress-strain properties for each type of steel
should be determined from test results on full sized pipe.
A. PLASTIC FAILURE CONDITION OF PIPE RING

CROWN AND INVERT SECTIONS, $M_{P1}$

$$T = f_{s1}'A_{s1}$$

$$T_1 = f_{s1}'A_{s1}$$

$$T_2 = f_{s2}'A_{s2}$$

B. PLASTIC FAILURE CONDITIONS IN PIPE WALL

TEST LOADING-CONDITIONS AT ULTIMATE FLEXURAL FAILURE

FIG. 4-1
Certain pipe sizes below about 66 inches in diameter with two circular cages, standard ASTM C76 wall thickness, and standard depths of cover concrete will have the neutral axis above the second reinforcing cage when the full flexural capacity of the section is developed. Hence, both reinforcing cages will act as tension reinforcing in these pipe, although one cage will be so close to the neutral axis that its contribution to the total sectional moment capacity will be small. For design purposes, the contribution of the second cage could be neglected with no significant loss in economy. However, its effect should be considered in the evaluation of test data. Hence, the plastic moment capacity of crown and invert sections of a typical precast concrete pipe (with outer circular reinforcing area = .75 x inner circular reinforcing cage area, a typical ratio based on ratio of elastic moment at crown to moment at springing in a typical field loading condition) is given by the following equation when both cages carry tension (see Appendix E and Fig. 4-1 for derivation):

$$M' = f' \frac{s_1}{s_1} A \left( d_1 + .8 - .88a \right) \frac{1}{12} \text{ ft-lbs}$$  \hspace{1cm} 4-5

where:  \hspace{0.5cm} a = .175 \frac{f'_s}{f'_c} \frac{A_s}{s_1} \less .8 \text{ inches} \hspace{1cm} 4-6

if \hspace{0.5cm} a \gtr .8 \text{ inches use equation } 4-1.
A question might well be raised regarding the ability of the second reinforcing cage to reach its ultimate strength as assumed in the derivation of equation 4-5. The maximum stress that can be developed in this second cage is related, via the stress-strain diagram for the reinforcing steel, to the maximum strain at this point in the cross section. The maximum strain which can be developed is derived in Appendix E as:

\[
\varepsilon_s^2 = \left( \frac{1.1}{k_u d} - 1 \right) \varepsilon_{pc} \tag{4-7}
\]

\[
\text{also } \varepsilon_s^2 = \left( \frac{1.1}{d} - k_u \right) \varepsilon_s^1 \tag{4-7a}
\]

For typical limiting conditions, we may take:

\[
\varepsilon_{cp} = .002 \text{ (see ref. 37)}
\]

\[
k_u d = \frac{8}{.85} = 2.48 \text{ pd}^2 \tag{4-8}
\]

\[
p = .008 \text{ (inner cage only)}
\]

\[
f'_s = 80,000 \text{ psi}
\]

\[
f'_c = 4,000 \text{ psi}
\]

\[
d = 4.9'' \text{ (60 inch diameter, wall B pipe)}
\]

thus: \( k_u d = .48 \text{ inches} \)

\[
\varepsilon_s^2 = .0026 \text{ in./in.}
\]
A typical stress-strain diagram for welded wire fabric (Fig. A-2) indicates a stress of about 70,000 psi at .0026 strain. Smaller diameter pipe and lower steel quantities would, of course, increase the value of maximum steel strain that could be reached in the second reinforcing cage. It is concluded, thus, that the assumptions about the steel stress in the second cage used in the derivations of equation 4-5 and 4-6 for pipe of 60 inch diameter and smaller have a valid theoretical basis from an analysis of maximum concrete ductility and strain variation across the section.

The moment capacity at the springings is also affected by the presence of the second cage in these small pipes. Here, there is probably insufficient rotation capacity to develop the full strength of the cage closest to the neutral axis. For convenience in developing a relationship between crown moment capacity, $M'_{P_1}$, and springing moment capacity, $M'_{P_2}$, the inner cage at the springing is assumed to reach only 56% of its tensile strength at failure of the entire pipe. This assumption permits the development of the following simple relation between crown and springing moment capacities for pipes of 60 inch diameter and smaller which meet the requirements of C76 as indicated in Appendix E:

$$M'_{P_2} = .75 M'_{P_1} + .25 (P_u + W) \frac{d}{12} \text{ ft-lbs} \quad 4-9$$

4.3 Limit Analysis

Flexural ultimate failure by yielding or rupture of the reinforcing steel involves considerable plastic yielding at the
points of maximum moment. The occurrence of this yielding causes a redistribution of moments around the pipe from the elastic distribution discussed in Chapter 3. As long as sufficient ductility is available at points of maximum moment, the pipe will not fail until plastic "hinges" are active at these points and the structure is deforming as a mechanism. Fig. 4-1 shows the "mechanism" condition at ultimate flexural failure in pipe under test load. Hence, evaluation of ultimate strength of pipe with respect to the flexural mode of failure requires both investigation of plastification of cross sections at points of maximum moments, and use of a "plastic" or "limit" method of analysis for magnitude and distribution of moments, shears and axial force around the pipe.

Methods of limit analysis were originally developed for application to continuous steel structures. The same methods have been applied to continuous reinforced concrete structures with the additional requirement of an approximate investigation of rotation capacity at points of plastic hinge action (34, 35, 36, 37). Many reinforced concrete sections exhibit remarkable ductility. However, overall ductility or rotation capacity is usually limited by the compressive strain in the concrete. Sections with higher amounts of tension reinforcing steel have lower rotation capacity. Occasionally, with reinforcing steels of limited ductility, rotation capacity may be determined by the ductility of the steel.

Methods of limit analysis are used in Appendix E to develop the equations for prediction of ultimate flexural strength of pipe
under both 3-edge bearing test load distribution (see Section 4.5) and field load distribution (Class C bedding—see Section 4.7).

4.4 Rotation Capacity of a Cross Section

The maximum available rotation capacity in a reinforced concrete flexural element may be conservatively estimated with the use of semi-empirical methods developed by Baker and his group at the Imperial College, London. According to Baker the following maximum rotation capacities can be developed in a reinforced concrete flexural element (37):

1. Where concrete strain controls:

allowable rotation, \( \theta_a = \frac{\varepsilon_{cp} d}{k_u d} = \frac{\varepsilon_{cp}}{k_u} \) \hspace{1cm} 4-10

2. Where steel strain controls:

allowable rotation, \( \theta_a = \frac{\varepsilon_{sp} d}{(1-k_u)d} = \frac{\varepsilon_{sp}}{(1-k_u)} \) \hspace{1cm} 4-11

Baker recommends a conservative estimate for maximum plastic strain in the concrete of:

\( \varepsilon_{cp} = .002 \) \hspace{1cm} 4-12

Many investigators (33) have suggested that \( k_u = \frac{a}{0.85} \) where \( a \) is the depth of equivalent rectangular stress block (Fig. 4-1). Using equation 4-2 for "a", the maximum usable rotation capacity based on concrete strain of most pipe reinforced with welded wire...
fabric is given by the following relation (see Appendix E for derivation):

\[
\theta_a = 1.4 \frac{f'_c}{f'_{sp}} \cdot 10^{-3} \text{ radians} \quad 4-13
\]

With steel of intermediate ductility, a check should also be made of maximum allowable rotation capacity using the following equation:

\[
\theta_a = \frac{\varepsilon_{sp}}{1-1.4 \frac{f'_s}{f'_c} \cdot p} \text{ radians} \quad 4-14
\]

where \( \varepsilon_{sp} \) is a conservative estimate of the maximum plastic strain which a given type of steel can develop.

The above limits on available rotation capacity should be checked against an estimate of the required rotation capacity at whichever hinge is subjected to the greatest rotation under a particular distribution of loading on the structure, as discussed in Section 4.5 for test load conditions.

4.5 **Ultimate Strength Equations for Test Load Conditions**

The above considerations are used in Appendix E to develop simple equations for predicting the ultimate flexural strength for pipe loaded in the 3-edge bearing test. For pipe 66 inch in diameter and larger, the ultimate flexural strength is given by the following relation:

\[
P_u = \frac{7.3 \frac{M_p}{D_i}}{D_i} - .5W \quad 4-15
\]
where $M_{pl}$, in ft-lbs, is obtained from equation 4-1, and $D_1$ is in foot units.

Figure 3-1 (e) summarizes the magnitude and distribution of moments, shears, and axial forces around the pipe at ultimate flexural load in the 3-edge bearing test. These results are based on the plastic failure condition of the pipe ring as shown in Fig. 4-1. However, in order to simplify the determination of moments, shears and axial forces around the pipe, and to allow a quicker comparison of the change in moment distribution between the various cracked elastic cases and ultimate flexural conditions, the weight effect was not included in the development of Fig. 3-1 (e). It is seen from equation 4-15, however, that the weight effect could be approximately included by adding one-half the weight of pipe to the applied live loading.

In terms of D-load capacity*, the ultimate flexural strength of pipe is given as:

$$ (DL)_u = \frac{7.3 \ M_{pl}}{D_1^2} - \frac{5W}{D_1} $$

4-16

A modification of equation 4-15 or 4-16 is often required for a careful analysis of test results. The derivation of these equations indicates that in order to state them in a simple form, the following assumptions were made about the physical properties of typical pipe:

* See Section 1.2 for Definition of D-Load.
1. Area of outer reinforcing cage = .75 x area inner cage.

2. Ultimate tensile strength of outer cage = ultimate tensile strength of inner cage.

3. Effective depth to outer cage at springing = effective depth to inner cage at crown and invert.

These assumptions are very nearly correct for typical pipe manufactured in conformance with ASTM C76. However, it is necessary to introduce a correction to equations 4-15 and 4-16 to permit evaluation of slight deviations from these assumptions where full data on physical properties of test pipe and their constituent materials is available. This correction permits a more accurate comparison of test results with the ultimate strength theory given above. Thus, equation 4-15 may be stated in more general terms as follows:

\[
P_u = \frac{7.3 \, cM}{D_1} - .5W
\]

where \( c = \) correction factor = \( .57 \left( 1 + \frac{f_s' A_s}{A} \frac{d_2}{d_1} \right) \)

For pipe smaller than 66 inches in diameter with two circular cages, the effect of width of bearing used at the crown should be taken into account, and the modified plastic moment capacity given by equation 4-5 should be used. The following modified version of equation 4-16 is derived in Appendix E to give a better prediction of ultimate strength for these pipe:
\[(DL)_u = \frac{7.6 \, cM_p}{D_i^2} - \frac{5w}{D_i}\]  \(4-19\)

where \(M_p\) in ft-lbs, is obtained from equation 4-5 and \(c\) normally equals 1, but may be checked by equation 4-18.

The rotation capacity required to reach the above ultimate strength capacities may be estimated from an analysis of deformations of pipe in the statically determinate condition just prior to formation of the final plastic hinges. At this point in the loading history, the limit moment distribution is just on the threshold of attainment. The pipe is assumed to behave elastically everywhere but at the two plastic hinge points at the crown and invert. The results of such an analysis give the following equation for required rotation capacity at the crown and invert points (see Appendix E for derivation):

\[\theta_{\text{req'd.}} = \frac{25 \, M_p \, D_i}{E_c \, I_{cr}}\]  \(\text{radians}\)  \(4-20\)

where \(M_p\), in ft-lbs, is given by equation 4-1, \(D_i\) is in feet, and \(I_{cr}\) is the moment of inertia of the cracked transformed concrete section adjacent to the crown and invert hinge points in inch units. (See Section 8.2 for accurate calculation of \(I_{cr}\).)

Using general expressions for the above quantities and the approximations indicated in Appendix E the following simplified relation is obtained:

\[\theta_{\text{req'd.}} = 7 \, f'c_s \cdot 10^{-8}\]  \(\text{radians}\)  \(4-21\)
In order to develop $f_s' = 80,000$ psi, a typical value for the ultimate strength of welded wire fabric, equation 4-21 indicates that a rotation capacity of $\Theta = .0056$ radians is required.

Pipe that fail in flexure would not often have a steel content greater than a maximum limiting value of about 1%, since even at this steel content, stirrups would be required to prevent diagonal tension failure. With $p = .01$, $f_s' = 80,000$ psi and $f_c' = 4,000$ psi, the allowable rotation capacity from equation 4-13 would be $\theta_a = .0072$ radians. If the maximum steel strain is as low as .01, the allowable rotation capacity from equation 4-14 is .014 radians. Since elongation of steel at failure will certainly exceed 1%, it is evident that concrete strain controls the rotation capacity. Furthermore, assuming the conservative value for maximum allowable concrete strain used above, it is found by equating the required rotation capacity in equation 4-21 to the maximum allowable rotation capacity given by equation 4-13 with $f_s' = 80,000$ psi and $f_c' = 4,000$ psi, that the maximum allowable tension steel content that will still permit the full limit load capacity to be developed is 1.3%. Steel in excess of this amount would rarely be used in a pipe reinforced with welded wire fabric.

Thus, it is seen that a comparison of the theoretically required rotation capacity in typical concrete pipe with the semi-empirically determined (37) maximum allowable rotation capacity indicates that even pipe with relatively high steel content have sufficient rotation capacity to attain the full redistribution of moment (assuming diagonal tension failure is prevented) required for the validity of equation 4-15.
4.6 Analysis of Test Data

Sixteen test pipe in the test program described in reference 8 failed in flexure. Eight of these pipe had web reinforcing and eight had relatively low steel contents and no web reinforcing. Tables A1 and A2 in Appendix A give a complete description of test specimens and reinforcing properties. It should be noted that all of these test pipe had cold drawn welded wire fabric reinforcing. See reference 8 for a complete description of test specimens, test procedure and test results. No test specimens failing in flexure were found in any of the other recent pipe test programs reported in the literature (11, 14, 15, 16) which were available for analysis by the theoretical methods developed herein.

Table 4-1 presents a comparison of predicted flexural ultimate strengths (using equation 4-17 or 4-19 with the measured steel and concrete strength properties reported in Tables A1 and A2) and the measured ultimate loads for the above sixteen test pipe, as reported in Table A3. Fig. 4-2 gives a graphical representation of this comparison. A plot of the cumulative distribution of the ratio $P_u \text{ test} / P_u \text{ calc}$ is given in Fig. 4-3. The average ratio of $P_u \text{ test} / P_u \text{ calc}$ is 1.014 from the 16 tests.

A statistical evaluation of the confidence of the measured average ratio of $P_u \text{ test} / P_u \text{ calc}$ (assuming a normal distribution of test results) indicates that there is a 95% probability that the true average ratio lies between .995 and 1.053. Thus, the analysis of these test results may be summarized as follows:
### TABLE 4-1

**ANALYSIS OF FLEXURAL FAILURES**  
**M.I.T. TEST PROGRAM**

<table>
<thead>
<tr>
<th>Test</th>
<th>Inside Diameter Pipe</th>
<th>$P_u$ test</th>
<th>$P_u$ calc.</th>
<th>$\frac{P_u \text{ test}}{P_u \text{ calc.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-1</td>
<td>108 inches</td>
<td>25.4</td>
<td>23.2</td>
<td>1.09</td>
</tr>
<tr>
<td>Q-2</td>
<td>108 inches</td>
<td>25.4</td>
<td>23.5</td>
<td>1.08</td>
</tr>
<tr>
<td>R-2</td>
<td>108 inches</td>
<td>37.6</td>
<td>33.5</td>
<td>1.12</td>
</tr>
<tr>
<td>G-1</td>
<td>72 inches</td>
<td>26.5 *</td>
<td>27.8</td>
<td>.96</td>
</tr>
<tr>
<td>G-2</td>
<td>72 inches</td>
<td>26.5 *</td>
<td>26.4</td>
<td>1.00</td>
</tr>
<tr>
<td>G-3</td>
<td>72 inches</td>
<td>26.5 *</td>
<td>27.2</td>
<td>.98</td>
</tr>
<tr>
<td>H-4</td>
<td>72 inches</td>
<td>17.5</td>
<td>18.2</td>
<td>.96</td>
</tr>
<tr>
<td>N</td>
<td>72 inches</td>
<td>26.6</td>
<td>27.1</td>
<td>.98</td>
</tr>
<tr>
<td>I-1</td>
<td>48 inches</td>
<td>13.1</td>
<td>12.9</td>
<td>1.01</td>
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<tr>
<td>I-2</td>
<td>48 inches</td>
<td>13.4</td>
<td>13.9</td>
<td>.97</td>
</tr>
<tr>
<td>J-1</td>
<td>48 inches</td>
<td>10.4</td>
<td>10.3</td>
<td>1.01</td>
</tr>
<tr>
<td>J-2</td>
<td>48 inches</td>
<td>10.3</td>
<td>10.5</td>
<td>.98</td>
</tr>
<tr>
<td>L-1</td>
<td>48 inches</td>
<td>10.2</td>
<td>10.2</td>
<td>1.00</td>
</tr>
<tr>
<td>L-2</td>
<td>48 inches</td>
<td>10.4</td>
<td>10.2</td>
<td>1.02</td>
</tr>
<tr>
<td>M-1</td>
<td>48 inches</td>
<td>9.5</td>
<td>9.2</td>
<td>1.03</td>
</tr>
<tr>
<td>M-2</td>
<td>48 inches</td>
<td>9.5</td>
<td>9.2</td>
<td>1.03</td>
</tr>
</tbody>
</table>

**Average**

| 1.014 |

**Standard Deviation**

| .035  |

**Coefficient of Variation**

| 3.5%  |

**Number of Tests**

| 16    |

*Note: These pipes had large deflections, wide cracks at top, bottom and sides but did not quite reach ultimate failure because the testing machine capacity was exceeded.*
16 TESTS 48" 72" 8" M.I.T.  
90 80 70 60 50 40 30 20 10 0 90 95 1.0 1.05 1.10 1.15
% DISTRIBUTION IN %
P u test P u calc
0.035 3.5%

COMPARISON OF MEASURED AND PREDICTED ULTIMATE LOADS - FLEXURAL FAILURE FIG. 4-2

COMBATIVE DISTRIBUTION OF P u test / P u calc - IN FLEXURAL FAILURES FIG. 4-3
number of tests 16
average ratio $\frac{P_{u \text{ test}}}{P_{u \text{ calc}}} = 1.014 \pm .019$ (probability = .95)
standard deviation = .035
coefficient of variation = 3.5%

In addition to the above sixteen test pipe which failed in flexure, nine 72 inch test pipe without stirrups in the same test program (B1, B2, B3, B4, C1, C2, H1, H2, H3) gave visual and electric strain gage evidence that they were close to flexure failure, but actually failed suddenly in diagonal tension (8). For eight of these pipe, application of equation 4-17 indicated that flexure failure should not have occurred until somewhat higher loads than the actual diagonal tension failure load, and thus gave additional corroboration to equation 4-17. For one pipe (H-3), equation 4-17 indicated that the flexural failure load should have been 4% below the actual load at diagonal tension failure. But many signs of imminent flexural failure, such as wide flexural cracks at crown and invert, were present at the time of failure in this pipe.

The results obtained from SR-4 strain gages placed on inner cage reinforcing at the crown and invert and outer cage reinforcing at the springings in some of the above test pipes which failed in flexure gave a good qualitative indication of the progress of redistribution of moments between crown and invert, and springings. Test load vs. steel stress plots for these pipes (Figs. 54 to 58, 61, 62 and 64 in reference 8) were obtained from these strain measurements.
These plots indicate the process of moment redistribution by the marked increase in load with little or no increase in inner cage stress and the large increases in outer cage stress with little increase in load in the vicinity of ultimate load. Finally, at failure both the inner cage steel at the crown and invert and the outer cage steel at springings have measured stresses close to their ultimate strengths as obtained from tension tests on samples from the same size wire and fabric shipment. In all of the tests except the "C" tests, the inner cage wires broke at crown or invert and in some tests, the outer cage also ruptured at the springing. In the "C" series, the capacity of the testing machine was reached just prior to complete ultimate failure, although sizable plastic deformations had already taken place in inner and outer cages.

Thus, the comparisons of measured ultimate loads with predicted values, the results of electric strain gage measurements and the visual observations of behavior at failure all provide strong experimental confirmation of the validity of equation 4-17.

4.7 Field Load Conditions

The crown and invert are still the sections of maximum positive moment (tension on the inside) under field loading conditions. The plastic moment capacity of these sections is given by equation 4-1 (4-5 for 60 inch diameter and smaller pipe). The springings are also still the sections of maximum negative moment (tension on the outside). The plastic moment capacity of these sections is modified only to the extent that total field load, $P_{FU}$, instead of test load, $P_u$, is used for the contribution of compressive load in equations 4-3 and 4-4.
The results of a limit load analysis for a pipe loaded under one typical field load distribution, combined with equations 4-1 and 4-4 (modified with $P_{fu}$) are given in Fig. 3-2 (d). As in the case of Fig. 3-1 (e), the self weight of pipe effect has again been neglected in the figure.

The ultimate load capacity of this particular field loading condition [Class "C" bedding (1, 2)] is given by the following equation (see Appendix E for derivation):

$$P_{fu} = \frac{11.75 \ c \ M_p}{D_1} - .6W$$

where $c$ is normally = 1, but may be checked by equation 4-18 and $M_p^1$ is given by either equation 4-1 or 4-5 depending on pipe diameter. Allowance for self weight of pipe, $W$, is included in the above relation.

The rotation capacity required at crown and invert plastic hinges to reach this ultimate load is much smaller than that required in the 3-edge bearing test, because there is less difference between the elastic and the ultimate moment distributions under field loading [compare Fig. 3-2 (c) and (d)] than under test loading [compare Figs. 3-1 (b) or (d) and (e)].

The ratio of the ultimate flexural capacity under the above field load to the ultimate flexural capacity in the 3-edge bearing test for pipe of 66 inch diameter and larger (equation 4-17) is:
Load Factor-Ultimate Flexural Failure = \( \frac{11.75}{7.3} \) = 1.61

For pipe of 60 inch diameter and smaller (equation 4-19) this ratio is:

Load Factor-Ultimate Flexural Failure = \( \frac{11.75}{7.6} \) = 1.55

These load factors are slightly lower than the ratio of maximum elastic moments of 1.69 [see Fig. 3-1 (b) and 3-2 (c)], but somewhat higher than the recommended load factor of 1.5 for Class C bedding.
5

DIAGONAL TENSION ULTIMATE STRENGTH

5.1 Introduction

The ultimate diagonal tension strength of a reinforced concrete pipe is somewhat more difficult to predict than its ultimate flexural strength. Diagonal tension strength is primarily a function of the tensile strength of the concrete which is subject to much greater variability and inaccuracy of measurement than steel strength properties. Moreover, diagonal tension failure is brittle and, hence, affected by stress concentrations and, to a certain extent, by the chance formation of flexural cracks. However, many typical reinforced concrete pipes fail in diagonal tension, rather than in flexure, so that for rational design of pipe, it is essential to develop a quantitative method for evaluating this mode of failure.

Diagonal tension failure is more complex in curved flexural elements, such as pipe, than in straight elements, because in addition to the usual effects of moment and shear, radial tension stresses result from the curvature of the tension steel. These radial tension stresses significantly reduce the diagonal tension strength of pipe, as compared to a straight element having the same sectional properties and subjected to the same moment and shear forces.

Pictures of diagonal tension failure in 20 test pipes with welded wire fabric reinforcing are included in Appendix F, reference 8. Pictures of typical diagonal tension failure in test pipe reinforced with welded wire fabric are also shown in reference 14 and in test pipe reinforced with hot rolled reinforcing in reference 16.
5.2 Ultimate Strength Equations for Straight Reinforced Concrete Beams

Until very recently, no fully developed theory for predicting shear or diagonal tension strength of straight reinforced concrete flexural members was available. In recent years, however, many test results on straight beams indicated that bending moment, as well as shear force, was an important determinant of ultimate shear strength. Two modes of shear failure were observed in beams without web reinforcing. Beams of larger span to depth ratio failed suddenly, simultaneously with the occurrence of the first diagonal crack. This crack usually formed as an extension of a flexural crack in a region of relatively high shear and moment, not necessarily at the section of maximum shear if moment was low at that point. This type of failure has been termed "diagonal tension failure" (39). Beams of smaller span to depth ratio were still able to carry higher loads after the appearance of the first diagonal crack. After diagonal cracking, the compression zone in these beams had sufficient reserve capacity to carry both shear and compression. In order to make this possible, redistribution of stresses occur in the beam whereby the beam acts more like a shallow tied arch than a beam. Finally, some higher load causes a failure due to combined shear and compression in the concrete, termed "shear-compression failure" (39).

Apparently, Viest was the first to suggest a quantitative relation for predicting the diagonal tension strength (occurrence of first diagonal crack) of straight beams, which gave good correlation with tests covering a wide range of variables and many different test
programs (39). This quantitative relation for diagonal tension strength was obtained from a theoretical analysis of the factors which influence diagonal tension strength and a correlation with a large amount of test data. The correlation of test data was used to determine semi-empirical constants in the theoretical analysis.

As a results of Viest's work and the excellent correlation of his theory with other test results, joint ASCE-ACI Committee 326 has recently suggested equations for predicting the diagonal tension strength of straight beams subject to shear and bending moment and for beams subject to shear, moment and axial force (39). The Committee has also suggested that the strength which some beams exhibit beyond the occurrence of first diagonal crack be disregarded as too unpredictable and subject to chance location of flexural cracks. These recommendations will probably be introduced into the next revision of the ACI Building Code, due in 1962.

Committee 326 has suggested the following equations for predicting the diagonal tension strength of straight reinforced concrete beams (39):

1. Shear and Bending Moment, No Axial Force:

   \[ v = \frac{V}{bd} = 1.9 \sqrt{f'_c} + 2500 \ p \frac{Vd}{M} \]  \hspace{1cm} 5-1

2. Shear, Bending Moment and Axial Force:

   \[ v = \frac{V}{bd} = 1.9 \sqrt{f'_c} + 2500 \ p \frac{Vd}{M + BNd} \]  \hspace{1cm} 5-2

   where \( B = -(1 - \frac{h}{2d} - j) \) and is usually positive. 5-3

   \( N \) is positive if tension, negative if compression.
5.3 Modifications for Curved Reinforced Concrete Beams

The theory which was summarized in the previous section is not applicable to curved flexural members because it does not take into account additional shear stresses and radial tension stresses which exist due to the continually changing direction of the flexural normal stresses in a curved member. However, the method by which the above theory was developed and the semi-empirical constants obtained from the numerous tests which were used in its development provide a valuable starting point for the method proposed herein for evaluating diagonal tension strength of curved reinforced concrete members.

The theoretical shear stress in a curved reinforced concrete flexural element, with axial load also present, differs from the shear stress in a straight flexural member, with or without axial force, because the change in direction of axial force affects shear. This is shown in sketch "a" of Fig. 5-1. The following expression, derived in Appendix F, gives the theoretical shear stress in a curved member:

\[ v = \frac{V}{bd} \left(1 + \frac{g}{r_s}\right) \]  \(5-4\)

Curvature of the normal stresses in a curved reinforced concrete flexural element causes radial tension stresses in the zone between reinforcing steel and compression face. These stresses result from the tendency of the tension steel to straighten. Their disposition in the member is shown in free body sketch "b", Fig. 5-1. The curvature effect of the tension in the steel cannot be equilibrated
by radial compression in the cover concrete inside the steel since this material is cracked and, hence, a constant radial tension must exist up to the neutral axis. Above the neutral axis, this tension drops off due to radial components of the compressive normal stresses and finally becomes zero at the compressive face of the section. The following expression, derived in Appendix F, gives the theoretical maximum value of radial tension stress, assuming uniform longitudinal distribution of stress between circumferential reinforcing elements:

$$t_r = \frac{M + NZ}{bjdr_s}$$  \hspace{1cm} (5-5)

In the vicinity of the crown and invert, $N = 0$ under symmetrical vertical loading.

Appendix F indicates how the above two relations can be used to modify the relations for ultimate nominal shear strength suggested by Committee 326 for straight beams (equation 5-1) to obtain the following expressions for curved flexural elements:

$$v_n = \frac{V}{bd} = \frac{1}{Z} \left( 1.9 \sqrt{f'_c} + 2500 \frac{p}{V_d} \right)$$  \hspace{1cm} (5-6)

$Z = \text{modifying factor for curvature}$

$$Z = \frac{1}{B} \left[ 1 + g/r_s + \frac{C_r}{2c_v Vr_s \sqrt{A^2 + 1}} \right]$$  \hspace{1cm} (5-7)
a.) INTERNAL FORCES ON ELEMENT
CUT FROM PIPE WALL

b.) INTERNAL SHEAR AND RADIAL TENSION STRESSES ON FREE BODY A

c.) INTERNAL SHEAR, RADIAL TENSION, AND FLEXURAL TENSION STRESSES ON ELEMENT B

d.) NOMINAL INTERNAL STRESSES IN ABOVE SKETCHES CAN BE SHOWN TO BE:

\[ V = \frac{v}{b \cdot d} \left( 1 + \frac{g}{r_s} \right) \] & \[ q = d/2 \]

\[ t_r = \frac{M}{b \cdot d \cdot r_s} \]

\[ t_\theta = \frac{C \cdot f_s}{n} \] (assumed)

e.) PRINCIPAL TENSILE STRESS, \( t_p \)

\[ t_p = \frac{t_r + t_\theta}{2} + \sqrt{\left( \frac{t_\theta - t_r}{2v} \right)^2 + 1} \]

THEORETICAL STATE OF INTERNAL TENSION IN PIPE WALL
DUE TO SHEAR AND RADIAL TENSION

FIG. 5-1
A and B depend on ratios of $t_s$ and $t_r$ as given in Appendix F and $C_r$ and $C_v$ are constants which depend on the degree of stress concentration.

As is also indicated in the complete derivation given in Appendix F, there can be little overall error introduced by assuming reasonable values for $C_r/C_v$, A and B since fairly large deviations in these assumptions would not have serious consequences for the final results. Thus, if it is assumed that:

$$\frac{C_r}{C_v} = 1.0$$
$$A = .2$$
$$B = 1.1$$

then
$$Z = .9 \left(1+g/r_s\right) + .45 \frac{M}{V_{rs}}$$

5-8

If axial loads are also present at the critical section for diagonal tension failure, the ultimate nominal shear stress which can exist at the section under investigation can be predicted by replacing the moment term, $M$, in the above formulas by $M + B\,Nd$, where $B$ is given by equation 5-3. Thus, more complete versions of equations 5-6 and 5-7 become:

$$v_n = \frac{v}{bd} = \frac{1}{b} \left[ 1.9 \sqrt{f'_c} + \frac{2500 \, p \, V_d}{(M+B\,Nd)} \right]$$

5-6'  

$$Z = \frac{1}{B} \left[ 1 + g/r_s + \frac{C_r \,(M+B\,Nd)}{2C_v \, V_{rs} \sqrt{A^2+1}} \right]$$

5-7'
Since the value of the ultimate nominal shear stress which a section may carry depends on M, V and N, the critical section for diagonal tension strength will not always be the section of maximum shear. In general, the critical section must be found by trial and error by comparison of the ultimate possible nominal shear stress that each section may carry with the actual nominal shear stress at that point in the structure due to applied loads. The section giving the lowest maximum applied load is the critical section. Such an analysis is made in section 5.9 for pipe under field loading.

Preliminary analyses for typical 3-edge bearing load and field distributed load conditions on pipe which fail in diagonal tension indicate that the above assumed values of \( C_r/C_v \), A, and B are not significantly changed by the practical range of variation of the ratios \( \frac{t_e}{V} \) and \( \frac{t_e}{V} \) due to the above changing load conditions on the pipe. Hence, equation 5-8 may be written more generally as:

\[
Z = 0.9 \left(1 + \frac{g}{r_s}\right) + 0.45 \frac{M + BN_d}{Vr_s} 
\]

5.4 Further Modification for Test Load Conditions - Reinforced Concrete Pipe

For pipe under 3-edge bearing test load conditions, the maximum shear and maximum moment occur simultaneously at the same section, the invert point in the pipe. The invert has a slightly higher shear force and about the same moment as the crown, because the weight of the pipe adds more shear and moment at the invert than at the crown. However, an actual diagonal tension failure may occur at either of these sections since the shear and moment effects are so
nearly alike at both of these points and the concrete strength and steel placement may vary somewhat throughout the pipe.

Committee 326 has found that the actual critical section should be taken a distance equal to the effective depth of beam, d, away from the point of concentrated load or reaction, since right at the load, local compressive stresses cause an increase in shear strength. Thus, for application of the Committee 326 work to pipe, the critical section for diagonal tension failure should be taken a distance "d" away from the lower support points along the centerline arc. The lower support points are specified in ASTM C76 to have a spacing in inches equal to pipe diameter, D_i, in feet. For C76, wall B pipe, d in inches approximately equals D_i, in feet. The critical section, then, is about 13° from the invert of the pipe.

Diagonal tension failure often occurs while the pipe is still elastic. Even if some plastic yield of the steel has occurred, the elastic distribution for moments and shears can still be used with negligible loss in accuracy. Fig. 3-1 indicates that at 13° from the invert, the following relations for shear, moment and axial force may be used:

\[ V = 0.487 \, P + 0.434 \, W \]  
\[ M = 0.208 \, P + 0.135 \, Wr \]  
\[ N = -1.12 \, P - 0.180 \, W \]  
\[ \frac{V}{M} \approx \frac{0.487 \, P}{0.208 \, \frac{P}{r_{av}}} = 2.35 \]  
\[ (\text{may be neglected}) \]  
\[ (\text{Note:} + N \, \text{is tension}) \]
Also, standardized dimensions normally used for C76 pipe permit the following additional simplifications:

\[ r_{av} \approx 1.08 \quad r_i = 0.54 \times 12 \quad D_1 = 6.5 \quad D_i^* \]

also \( r_{av} \approx 1.07 \quad r_s \)

\[ g \approx \frac{d}{2}; \quad \frac{r_i}{12} \approx 0.07 \quad r_s; \quad g/r_s = 0.07 \]

\[ p = \frac{A_{sl}}{12d} \]

Substitution of the above quantities in equation 5-8 gives:

\[ Z = 1.17 \]

The following simplified expression for nominal ultimate shear stress is obtained from equation 5-6 with the above values of \( V/M, p, r_{av} \) and \( Z \):

\[ v_n = \frac{V}{bd} = 1.6 \sqrt{f'_c} + 64 \frac{A_{sl}}{D_1^*} \quad 5-13 \]

If, in addition, equation 5-9 is used for \( V \) with \( (DL)_u = \frac{P}{D_1} \), the following equation in a form suitable for rapid prediction of ultimate diagonal tension strength of pipe results:

\[ (DL)_u = \frac{40d_1}{D_1^*} \sqrt{f'_c} + \frac{1575 \frac{d_1A_{sl}}{D_1^*}}{D_1^*} - \frac{0.07W}{D_1^*} \quad 5-14 \]

5.5 Analysis of Test Data

Predicted values of ultimate nominal shear stress obtained from equations 5-13 were compared with test values based on test shear force from equation 5-9 at failure load for 22 M.I.T. tests which

\[ *D_i \text{ in feet.} \]
failed in diagonal tension. This comparison is given under the appropriate heading in Table 5-1. Test results were divided into two groups. Eighteen group I tests showed fairly consistent behavior and appeared to be "in control." Four group II tests indicated greater variability and may have contained errors in materials control tests for concrete strengths, steel location, etc. It must be expected with so variable a material as reinforced concrete that test cylinders may not always well represent actual in-place concrete and some variability in location of tension steel must be expected.

A description of test specimens and a summary of test results for these tests is given in tables A-1 and A-3, Appendix A. All test pipe were reinforced with welded wire fabric. See reference 8 for a complete description of the test program and test results.

The summarized results of this test analysis using equation 5-13, are given in table 5-1, and indicate good agreement between the theory based on above theoretical modification of Committee 326 equations and the test results for the 18 tests of Group I. The standard deviation of .090, coefficient of variation = 7.9%, is substantially below the 15.1% coefficient of variation obtained from the Committee 326 analysis of straight beam tests (39). The average $v_e/v_c$ is about 14% greater than 1.00, but it should be noted that the Committee's analysis of straight beams produces an average $v_e/v_c$ 9.7% greater than 1.00, and this analysis was the basis of equation 5-13.

Equation 5-13 was also used to predict the ultimate
## Table 5-1

### Analysis of Diagonal Tension Failures - M.I.T. Test Program

<table>
<thead>
<tr>
<th>Test</th>
<th>Pipe D.</th>
<th>V&lt;sub&gt;test&lt;/sub&gt;</th>
<th>√&lt;sub&gt;12&lt;/sub&gt;</th>
<th>A&lt;sup&gt;0.61&lt;/sup&gt;</th>
<th>V&lt;sub&gt;calc&lt;/sub&gt;</th>
<th>V&lt;sub&gt;calc&lt;/sub&gt;/V&lt;sub&gt;c&lt;/sub&gt;</th>
<th>V&lt;sub&gt;calc&lt;/sub&gt;/V&lt;sub&gt;c&lt;/sub&gt;</th>
<th>V&lt;sub&gt;calc&lt;/sub&gt;/V&lt;sub&gt;c&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>in.</td>
<td>psi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I - Tests Used for Development of Ultimate Strength Equation</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-1</td>
<td>108</td>
<td>114</td>
<td>71.9</td>
<td>.138</td>
<td>124</td>
<td>1.16</td>
<td>154</td>
<td>.94</td>
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<tr>
<td>P-2</td>
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<td>154</td>
<td>70.8</td>
<td>.138</td>
<td>122</td>
<td>1.26</td>
<td>152</td>
<td>1.01</td>
</tr>
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<td>A-1</td>
<td>72</td>
<td>136</td>
<td>66.6</td>
<td>.080</td>
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<td>1.21</td>
<td>128</td>
<td>1.05</td>
</tr>
<tr>
<td>A-2</td>
<td>72</td>
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<td>.080</td>
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<td>133</td>
<td>1.06</td>
</tr>
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<td>116</td>
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<td>.080</td>
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<td>1.05</td>
<td>127</td>
<td>.92</td>
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**Average V<sub>c</sub>/V<sub>c</sub> (1)**

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**Number of Tests - 18**

### II - Tests Not Used in Development of Correlation Equation

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**Number of Tests - 22**

### III - Tests - Flexural Failure, Not Close to D.T. Ultimate

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**Number of Tests - 8**

*Note: Diag. crack occurred at failure.*
diagonal tension strength of 46 tests by others on pipe reinforced with welded wire fabric. The results of a comparison of calculated and test values for ultimate nominal shear stress are given under the appropriate heading in Table 5-2. A description of test specimens and summary of test results for these tests is given in summary Tables A-4 and A-5, Appendix B. See references 11 and 14 for a complete description of the test programs and test results.

In the interpretation of these test results, it was necessary to introduce the following two modifications to permit valid comparisons of test nominal shears with predicted nominal shears at ultimate from equation 5-13:

1. The test load per foot at ultimate diagonal tension failure was adjusted for the effect of bell and spigot ends on the pipe. Thus, the total load on the pipe at failure was divided by the nominal length of pipe minus bell length to obtain the test load per foot.

2. $f'_c$ in equation 5-13 was obtained from compression tests on cylinders which were made and cured with the pipe and tested at the same age as the pipe. Where only core strength test results were available, they were reduced by 15% for use in equation 5-13. Comparison of cylinder and core test results for the same pipes in the M.I.T. test program indicated concrete strengths from cylinder tests averaged only
### Table 5-2

**Analysis of Diagonal Tension Failures - Test Programs of Others with Reinforced Shear Rebars**

| Test | Pipe J.D. | \( V_{test} \) (in. \( \times \) psi) | \( \sqrt{V_{calc}^2} \) (psi) | \( \sqrt{V_{calc}^2} \) (in. \( \times \) psi) | \( V_{calc} \) (psi) | \( V_{calc} \) (in. \( \times \) psi) | N.I.T. Test Correlation | N.I.T. Test Correlation | N.I.T. Test Correlation | N.I.T. Test Correlation | N.I.T. Test Correlation |
|------|------------|--------------------------------------|-------------------------------|-----------------------------------------------|---------------------|------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1.   | American Concrete Pipe Association Tests (11,12) | 136 77.7 0.063 | 128 1.06 139 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 | 136 0.96 129 1.05 |
| 2.   | Missouri School of Mines Tests (13) | 134 72.3 1.37 | 128 1.17 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 | 134 0.98 131 1.10 |

**Average \( V_{calc} / V_{calc} \)**

| Average \( V_{calc} / V_{calc} \) | 1.176 | 0.906 | 1.103 |

**Standard Deviation**

| Standard Deviation | 0.167 | 0.106 | 0.139 |

**Coefficient of Variation**

| Coefficient of Variation | 14.2% | 10.8% | 11.8% |

**Number of Tests**

| Number of Tests | 46 |
81% of corrected* core strengths with a standard deviation of only .055 (coefficient of variation of 6.8%). Presumably the Committee 326 equation was based on control tests using standard cylinders so it seems logical to base the comparable pipe strength equations on cylinder tests, rather than core tests. Moreover, it is far more common and economical to use cylinder tests as the basis of materials control in pipe manufacture. It is possible that properly conducted core tests are more representative of the actual material in the pipe. However, if a fairly constant relation exists between core strength and cylinder strength, as indicated above, the difference between use of cylinder strength and actual concrete strength is taken care of by the constant multiplier in the term with $f'_c$ in equation 5-13.

It is seen from the summarized results in Table 5-2 that the variability of $v_t/v_c$ is substantially greater for these tests than in the M.I.T. tests, and the average $v_t/v_c$ is 17.6% greater than 1.00. However, a closer inspection of this comparison reveals that there is a consistent pattern of variation. Pipes with larger amounts of reinforcing steel than were used in the M.I.T. tests have higher strengths than predicted by equation 5-13.

A third series of 42 tests on pipe reinforced with hot rolled reinforcing which failed in diagonal tension was analyzed using equation 5-13 with the results shown in the appropriate column

---

*ASTM C42 correction for $\frac{1}{d}$ of core;
### TABLE 5-3
ANALYSIS OF DIAGONAL TENSION FAILURES - TEST PROGRAM
OF OTHERS WITH NOT ROLLED REINFORCING

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<th>( r_{\text{c}} )</th>
<th>( A_{\phi} / D_1 )</th>
<th>( V_{\text{cal}c.} / V_{A_c} )</th>
<th>( V_{\text{calc}} / V_{\phi} )</th>
<th>( V_{\text{calc}} / V_{\phi} )</th>
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<td></td>
<td></td>
<td>( \text{in} )</td>
<td>( \text{psi} )</td>
<td>( \text{in}^2 / \text{ft}^2 )</td>
<td>( \text{psi} )</td>
<td>( \text{psi} )</td>
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<td>172</td>
<td>1.00</td>
<td>.88</td>
</tr>
<tr>
<td>564835</td>
<td>36</td>
<td>152*</td>
<td>78.3</td>
<td>.182</td>
<td>133</td>
<td>1.14</td>
<td>158</td>
<td>.96</td>
<td>.88</td>
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<tr>
<td>564836</td>
<td>36</td>
<td>164*</td>
<td>71.8</td>
<td>.182</td>
<td>123</td>
<td>1.19</td>
<td>149</td>
<td>.98</td>
<td>.88</td>
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<td></td>
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<td>14.35</td>
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<tr>
<td>Number of Tests - 42</td>
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</table>

*Note: Steel was beyond yield point at D.T. Failure.
of Table 5-3. A description of test specimens and test results for these pipes with hot rolled reinforcing is given in reference 16 for the AREA tests and in reference 14 for the ACPA tests. The same interpretation of test data as described above was used. Discussion of these test results is deferred to the next article. In general, however, they indicate fairly good agreement with equation 5-13, although some show considerably greater test strengths than predicted.

5.6 **Statistical Correlation of Test Data**

The fairly consistent comparison between equation 5-13, which is based on theoretical considerations with empirical constants from straight beam tests, and the test results on pipe indicates that a correct understanding of the major variables which affect diagonal tension strength has probably been achieved. However, in view of the average \( v_t/v_c \) ratios greater than 1.00 and the observation of consistently high values of \( v_t/v_c \) for pipe with larger amounts of welded wire fabric reinforcing, it is evident that a better estimate for the constants in equation 5.13 could be obtained from a statistical correlation of the pipe test data.

The need for re-evaluating these constants is also evident from a close examination of Fig. 5-1 in reference 39. This figure compares the Committee 326 shear equation (equation 5-1 herein) with beam test results. The range of values of \( \frac{1000 \cdot p \cdot V_u}{M \cdot \sqrt{f'_c}} \) for all test pipe lies in a very narrow band between .013 and .041 on the left side of the abscissa in Fig. 5-1, reference 39. The straight line fit
used in reference 39 for the correlation of beam test data only roughly fits the actual beam test data in this narrow region. A greater slope of this line, as will be shown to be indicated by pipe test results, would fit the straight beam test data in this narrow region of reference 39-Fig. 5-1 considerably better than the conservative straight line fit (equation 5-1 herein). If the constants were altered in equation 5-1 for straight beams in this region, they should, of course, be similarly altered in equation 5-13 for pipe.

Hence, new constants were derived on the basis of a linear correlation of test data from the 18 M.I.T. pipe tests designated as Group I in Table 5-1. This correlation was carried out using the same variables as given in equation 5-13. The relationship between these variables may be stated in the following general form:

\[ \frac{V}{\sqrt{f_c}} = A_o + B_o \frac{A_{s_1}}{D_i \sqrt{f'_c}} \]  

5-15

A "least squares" analysis to obtain the best linear correlation between measured values of \[ \frac{V}{\sqrt{f'_c}} \] and the pipe strength parameter, \[ \frac{A_{s_1}}{D_i \sqrt{f'_c}} \] gives (see Appendix G for more details):

\[ A_o = 1.53 \]
\[ B_o = 320 \]

Thus, the following equation for ultimate diagonal tension strength best represents the M.I.T. test results:
\[ v_n = \frac{V}{bd} = 1.53 \sqrt{f'_c} + 320 \frac{A_s}{D_1^*} \] 5-16

In terms of ultimate D-load capacity of a test pipe, this becomes:

\[ (DL)_u = \frac{38.5 \ d \ \sqrt{f' c}}{D_1^*} + \frac{7800 \ \frac{d \ A_s}{D_1^*}}{D_1^{2*}} - \frac{.9W}{D_1^*} \] 5-17

Figure 5-2 compares the M.I.T. test results with calculated shears from both equation 5-13 and 5-16. The appropriate column of Table 5-1 shows the comparison of measured nominal shear stress, \( v_{test} \) (using equation 5-9 for shear force) with the nominal shear stress, \( v_c \), calculated from equation 5-16. This comparison is also shown graphically in Fig. 5-5. For the 18 tests which were used to develop the constants in equation 5-16, the average \( v_t/v_c = .999 \) and the standard deviation is only .061. This indicates a relatively low variability for this type of test. Fig. 5-8 gives the cumulative distribution of the 18 M.I.T. test results which were used to develop the constants in equation 5-16.

Standard statistical methods are available to test for correlation between the assumed variables (58). The "correlation coefficient" for the above "least squares" line is .70. This indicates better than a .999 probability that there is significant correlation between variables. Further analysis shows that there is 95% probability that the best linear correlation for other similar tests, with the same degree of control and no new variables added, would lie in the cross

*\( D_1 \) in feet.
\[ \frac{V}{\sqrt{f_c}} = 1.5 + \frac{320 A_{Sl}}{D_i \sqrt{f_c}} \]
(from I.B.M.I.T. Tests
Equation 5-16)

\[ \frac{V}{\sqrt{f_c}} = 1.5 + \frac{200 A_{Sl}}{D_i \sqrt{f_c}} \]
(Proposed Design
Equation 5-22)

\[ \frac{V}{\sqrt{f_c}} = 1.6 + \frac{64 A_{Sl}}{D_i \sqrt{f_c}} \]
(From Theoretical
Modification of
Committee 326
Formula Equation 5-13)

* Note: Not Used in Data Correlation

95% Confidence Range
of Correlation

90% of Pipes
Estimated to Have
Strength Above
This Limit

DEVELOPMENT OF DESIGN EQUATION FOR DIAGONAL TENSION
ULTIMATE STRENGTH OF PIPE

FIG. 5-2
hatched region shown in Fig. 5-2. Likewise, no more than 10% of these similar tests would be likely to have strengths below the lower limiting line shown in Fig. 5-2. Supporting calculations for the above statistical interpretation of test results are given in Appendix G.

A second correlation of test data was made to check the validity of the assumption that diagonal tension strength varies as the square root of $f'_c$. In this correlation, the second term in equation 5-16, $320 \frac{A_{S1}}{D_1}$, is a minor contributor to the nominal shear strength and is assumed to be correct. The following general relationship among variables was then set up for correlation:

$$ (v_n - 320 \frac{A_{S1}}{D_1}) = a(f'_c)^b $$  \hspace{1cm} 5-18

where 'a' and 'b' are arbitrary constants.

If the logarithm of both sides is taken, this equation becomes:

$$ \log (v_n - 320 \frac{A_{S1}}{D_1}) = \log a + b \log f'_c $$  \hspace{1cm} 5-19

Equation 5-19 is linear in the variables $\log (v_n - 320 \frac{A_{S1}}{D_1})$ and $\log f'_c$. A "least squares" analysis indicates that for best correlation of test data:

$$ (v_n - 320 \frac{A_{S1}}{D_1}) = 5.88 (f'_c)^{3.41} $$  \hspace{1cm} 5-20

Figure 5-3 shows a comparison of test shear strength data.
\[ V_n - 320 \frac{A_s}{D_i} = 5.88 \left( f_{c'}^2 \right)^{0.341} \]
(EQUAT. 5-20)

\[ V_n - 320 \frac{A_s}{D_i} = 1.53 \sqrt{f_{c'}} \]
(EQUAT. 5-16)

EFFECT OF CONCRETE COMpressive STRENGTH ON DIAGONAL TENSION STRENGTH OF PIPE

FIG. 5-3
with equations 5-20 and 5-16. It is evident from this plot that since very low values of $f'_c$ were not used in the tests, there is very little difference between equations 5-16 and 5-20 over the range of $f'_c$ variation used in the tests and normally used in pipe. Hence, equation 5-16 will be adopted as best representing the results of M.I.T. tests for diagonal tension ultimate strength.

The validity of equation 5-16 was further checked by comparing test nominal shear stresses from the 46 tests on pipe reinforced with welded fabric and the 42 pipe reinforced with hot rolled reinforcing previously referred to in article 5.15 with nominal shear stresses calculated from equation 5-16 for these pipe. The test results are plotted in Fig. 5-4 which also shows plots of equations 5-13 and 5-16. Test nominal shear stresses and calculated values are compared in the appropriate columns of Tables 5-2 and 5-3.

The average $v_t/v_c$ for the 46 pipe with welded wire fabric was .986 with a standard deviation of .106 (10.8% coefficient of variation). Fig. 5-9 gives the cumulative distribution for these 46 tests. Fig. 5-6 gives a graphical comparison of $v_{test}$ vs. $v_{calc}$ for these tests. The average $v_t/v_c$ for 42 pipe with hot rolled steel was .969 with a standard deviation of .136 (14.0% coefficient of variation).

These tests give a good corroboration of equation 5-16, which was developed from the M.I.T. tests alone. They show somewhat more variation than the M.I.T. tests, but this would be expected of tests carried out at several different locations with different
Comparison of test results with variables in derived equations for diagonal tension strength of pipe

Fig. 5-4

Key:
- M.I.T. Tests - Welded wire fabric reinforcing
- Tests of others - Welded wire fabric reinforcing
- Tests of others - Hot rolled reinforcing - \( s_x = 2'' \) and greater

Equation 5-13:
\[
\frac{V}{\sqrt{f_c'}} = 1.6 + \frac{64A_s}{D_1\sqrt{f_c'}}
\]

Equation 5-16:
\[
\frac{V}{\sqrt{f_c'}} = 1.53 + \frac{320A_s}{D_1\sqrt{f_c'}}
\]

Equation 5-22:
\[
\frac{V}{\sqrt{f_c'}} = 1.5 + \frac{200A_s}{D_1\sqrt{f_c'}}
\]

Lower limit - 90% confidence for individual test result (from M.I.T. data - does not include \( \Delta \) tests with \( s_x > 2'' \)).
COMPARISON OF MEASURED AND PREDICTED NOMINAL SHEAR STRESSES AT DIAGONAL TENSION FAILURE - M.I.T. TEST PROGRAM

FIG. 5-5

22 Tests
W.W.F.

\[ \text{av} \frac{V_t}{V_c} = 0.997 \]

\[ \sigma = 0.113 \]

\[ V = 11.3\% \]

COMPARISON OF MEASURED AND PREDICTED NOMINAL SHEAR STRESSES AT DIAGONAL TENSION FAILURE - TEST PROGRAMS OF OTHERS WITH HOT ROLLED BAR REINFORCING

FIG. 5-7

42 Tests
H.R. Reinf.

\[ \text{av} \frac{V_t}{V_c} = 1.071 \]

\[ \sigma = 0.114 \]

\[ V = 10.6\% \]

COMPARISON OF MEASURED AND PREDICTED NOMINAL SHEAR STRESSES AT DIAGONAL TENSION FAILURE - TEST PROGRAMS OF OTHERS WITH WELDED WIRE FABRIC REINFORCING

FIG. 5-6

46 Tests
W.W.F.

\[ \text{av} \frac{V_t}{V_c} = 0.986 \]

\[ \sigma = 0.106 \]

\[ V = 10.8\% \]
CUMULATIVE DISTRIBUTION OF \( \frac{V_{\text{test}}}{V_{\text{calc}}} \) - M.I.T. TEST PROGRAM

FIG 5-8

CUMULATIVE DISTRIBUTION OF \( \frac{V_{\text{test}}}{V_{\text{calc}}} \) - TEST PROGRAMS OF OTHERS WITH WELDED WIRE FABRIC REINFORCING

FIG 5-9

CUMULATIVE DISTRIBUTION OF \( \frac{V_{\text{test}}}{V_{\text{calc}}} \) - TEST PROGRAMS OF OTHERS WITH HOT ROLLED BAR REINFORCING

FIG 5-10
personnel, testing equipment, and methods of evaluation of concrete strength. Furthermore, it was necessary to make approximate adjustments for effect of tongue and groove ends and concrete strength measurements from cores, rather than standard cylinders for many of these test pipe. Also, some pipe were made by machine processes which did not permit accurate control of steel placement. The tests include pipe with two circular cages, and with one elliptical cage; pipe with diameters between 36 inches and 84 inches and with fairly wide ranges in steel content. The test pipe with hot rolled steel used both plain and deformed bars with some variation in longitudinal spacings used. In view of the many possibilities for introduction of unknown variations and errors of measurement, the coefficients of variability in the correlation of equation 16 with these test programs of others are not excessive.

A close examination of Table 5-3 for pipe with hot rolled reinforcing indicates that the AREA tests generally indicated test strengths lower than the strengths predicted by equation 5-16, while the ACPA tests indicated somewhat higher test strengths than predicted. It is interesting to note that AREA pipe were cast pipe of wall B thickness while ACPA pipe were centrifugally spun, wall C pipe. Hence, the AREA test pipe had thinner walls and probably somewhat less uniform and well bonded concrete.

Also, in conformance with the previously given rules for application of equation 5-16, the strength estimates for ACPA tests were based on 85% of core test values of $f'_c$, while AREA strength
estimates were based on \( f'_c \) from cylinders, cast with the test pipe. For centrifugally spun pipe, however, it is possible that the full core test strength should be used in estimating \( f'_c \) for equation 5-16. Use of full core strength for the ACPA tests would reduce \( v_t/v_c \) values. Of course it is also possible that small errors in the core test procedure used for this test program resulted in core strengths somewhat lower than representative properly tested cores would have given. Again, if the true core strengths were higher, the \( v_t/v_c \) values in Table 5-3 would be reduced for the ACPA tests.

All pipe in the AREA & ACPA series with hot rolled reinforcing had longitudinal spacings of reinforcing bars greater than 2 inches. Spacings varied from 2 7/16 inches to 4 7/8 inches for AREA tests and from 2 1/4 inches to 2 3/4 inches for ACPA test. It is probable that with larger bars at greater spacings than 2 inches, a greater concentration of radial tension stress occurs, thereby increasing the value of \( C_r/C_v \) (equation 5-7) above 1.0 as assumed in the development of equation 5-8.

On the basis of the AREA test data presented in Table 5-3, it is suggested that for longitudinal reinforcing spacing greater than 2 inches the ultimate shear capacity, given by equation 5-16 or 5-17, be modified by the following empirical factor:

\[
F = 1.15 - .075 \, s_q
\]

5-21

where \( s_q \) may vary from 2 inches minimum to 4 inches maximum.
Thus, equation 5-16 (or 5-17) applies without reduction at 2 inches longitudinal spacing of bars or wires and is reduced by 15% at 4 inch longitudinal spacing of bars or wires.

Strength predictions using equation 5-16, modified by equation 5-21, for pipe with circumferential reinforcing at longitudinal spacing greater than 2 inches are compared with test results in the last column of Table 5-3. A graphical comparison is given in Fig. 5-7.

5.7 Slabbing Failure

Pipe failures in the 3-edge bearing test due to slabbing off of cover concrete have been reported by producers in the pipe industry. None of the test pipe in the various programs described in the literature, and used for evaluation of the theory developed above, exhibited this mode of failure. However, some of the pictures of diagonal failure conditions in the AREA pipe tests showed considerable slabbing of cover concrete beyond the diagonal crack, with some of the diagonal cracks rather flat. In some diagonal failures, moreover, before the crack progressed all the way through to the compression face, it joined a similar crack from the other side of the invert and a large "V-shaped" portion of concrete pulled away from the compression zone at the invert. This seemed to be an indication of the severe radial effects in pipe as the tension steel tries to straighten out.

It is possible that in pipes with large bars at large longitudinal spacings, the stress concentration of radial com-
pressure under the bars and radial tension between bars could cause the cover coat to slab off before general diagonal tension failure. Fig. 5-11 shows a sketch of the radial stresses which have to be distributed longitudinally between reinforcing bars by the cover coat.

One pipe (R-1) in the M.I.T. test program failed essentially by slabbing of the cover coat due to radial tension. This pipe had two inner reinforcing cages and stirrups. Inadvertently, the stirrups were anchored only to the outermost of the inner cages and the innermost inner cage straightened out at a load not too far below the predicted load for flexural failure and somewhat above the diagonal tension strength with no stirrups (8).

In cases where the steel undergoes very large plastic deformations, it is probable that bond around the reinforcing steel in the vicinity of flexural cracks is destroyed and the stress concentration effect increased as shown in Fig. 5-11 (8).

Use of hot rolled reinforcing often means that both larger bar sizes at greater longitudinal spacings, and bars which can undergo extremely large deformations prior to failure are employed. Hence, slabbing failure may sometimes occur before the predicted strength in diagonal tension or in flexure is reached.

Slabbing failure at loads below predicted ultimate strengths can probably be avoided by basing predictions of ultimate flexural strength on yield point, rather than ultimate strength when hot rolled bars of structural and intermediate grade steels are used, and by limiting maximum longitudinal spacing to about 4 inches. However, no
test results were available to corroborate the above hypothesis, or to permit any personal observations of this type of failure by the author.

5.8 **Design Equation for Test Load Conditions**

While equations 5-16 and 5-17 are well suited for predicting the ultimate diagonal tension strength of test pipe, a somewhat more conservative version of these equations should be used for design of pipe, in order to insure that the majority of pipe will pass the ultimate strength test.

A conservative design for diagonal tension ultimate strength can be obtained by designing pipe for a somewhat higher D-load than the specified ultimate D-load. Fig. 5-2 indicates that pipe designed for a 10% increase in D-load capacity above the specified ultimate using equation 5-17 should have about a 90% probability of reaching the specified ultimate D-load.

A conservative design could also be obtained by reducing the coefficients in equations 5-16 and 5-17. The following modified versions of these equations are suggested for design purposes:

\[ V_n = \frac{V}{bd} = \left(1.5 \sqrt{f_c'} + 200 \frac{A_{S_1}}{D_1} \right) F \]  \hspace{1cm} 5-22

or in terms of D-load capacity:

\[ (DL)_u = \left(\frac{37 d \sqrt{f_c'}}{D_1} + \frac{5000 d A_{S_1}}{D_1^2} \right) F - \frac{SW}{D_1} \]  \hspace{1cm} 5-23
a) Full Bond - No Cracks  

b) Partial Bond - Cracked Concrete with Elastic Steel Strain  
c) Partial Bond Failure after Plastic Yield in Steel at Crack

SECTION A - A  
SECTION B - B  
SECTION C - C

STRESSES ON CONCRETE

Case a: \( q = \frac{K_1 T}{br s} \)  
Case b: \( q = \frac{K_1 T}{(b-K_2) r_s} \)  
Case c: \( q = \frac{K_1 K_2 T}{(b-K_2) r_s} \)

DISTRIBUTION OF RADIAL TENSION EFFECTS IN "SLABBING" FAILURE

FIG. 5-11
Equation 5-22 with $F = 1.0$ is plotted in Figures 5-2 and 5-4 for comparison with test data. It is seen that except for some tests with hot rolled bars which should have their predicted strengths reduced in accordance with equation 5-21, few test results lie below this line. The plot of this equation also coincides fairly well with the lower limit line for the strength of 90% of tests as shown in Fig. 5-2. Moreover, equation 5-22 indicates somewhat less increase in shear strength with increase in steel content than equation 5-16, and thus recognizes the possibility of somewhat greater variability of diagonal tension strength at higher steel contents. Also, this reduction in contribution of steel content brings the design equation somewhat closer to equation 5-13, although still above it, for the practical range of values of $\frac{A_s}{D_1\sqrt{f' c}}$ used in pipe.

Comparisons of test nominal shear stresses with predicted shear stresses calculated from equation 5-22 are given in the appropriate columns of Tables 5-1 and 5-2. It is seen that equation 5-22 is less conservative than 5-13, but still gives average values for $\nu_t/\nu_c$ well above 1.00.

A third alternative for conservative design of concrete pipe for diagonal tension strength is to provide a concrete with $f' c$ about 30% higher than the design value assumed with equation 5-16 or 5-17. This would provide about a 10% increase in average diagonal tension strength, the same margin of safety as obtained above.
5.9 **Modification for Field Load Conditions**

Equations 5-6' and 5-8' may be applied to pipe subject to any arbitrary distribution of loading with the aid of the following general relations for moment, shear and axial force around the pipe:

\[ V = c_1 W + c_2 P_f \approx c_2 P_f \]  \hspace{1cm} 5-24

\[ M \approx c_3 P_f r_{av} \]  \hspace{1cm} 5-25

\[ N \approx c_4 P_f \]  \hspace{1cm} 5-26

where \( P_f \) = the total applied load per foot of length.

In addition, the constants in equation 5-6' should be modified in accordance with the analysis of test results discussed in Article 5.6. The following general relation for ultimate diagonal tension strength results from the above considerations (see Appendix F for derivation):

\[ P_f = \frac{12h}{c_2 Z} \left[ 1.8 \sqrt{f_c'} + 160 \frac{A_{sl}}{D_1} \cdot \frac{c_2}{(c_3 - 0.078c_4)} \right] - \frac{c_1}{c_2} W \]  \hspace{1cm} 5-27

where \( Z = .97 + .48 \frac{(c_3 - 0.078c_4)}{c_2} \)  \hspace{1cm} 5-28

For any given load distribution, the ultimate diagonal tension strength of the pipe is best determined by a "cut and try" solution of equation 5-27 for the minimum value of \( P_f \). The "cut and try" process is used with values of \( c_1, c_2, c_3 \) and \( c_4 \) computed at various trial sections until the section which gives the minimum value of \( P_f \) is found. An alternate approach would be to write \( c = f(e) \) for \( c_1, c_2, \)
c_3 and c_4, set \( \frac{dP_r}{de} = 0 \) from equation 5-27 and solve for e to obtain the section of minimum \( P_r \), but the results do not give practical solutions for most conditions of loading.

The "cut and try" process for locating the critical section is facilitated by a diagram showing moment, axial force and shear variation around the pipe, such as Figures 3-1 and 3-2. The critical section for diagonal tension ultimate strength will be between the section of maximum shear and the section of maximum moment, but not closer than a distance, \( d \), to concentrated loads. For typical field loading conditions, this critical section will be on the bedding or invert side of the pipe. The section of maximum shear will be located at the angular distance from the invert to the edge of the bedding distributed load or at 45°, whichever angle is smaller. The point of maximum shear is never beyond a point 45° from the invert for vertical uniformly distributed loads. The point which represents the same moment-shear ratio as the critical section for diagonal tension failure in the 3-edge bearing test (i.e. \( \frac{M}{V} = 0.427r \) at 13° from the invert) may be a good starting point for assumed location of critical section in the "cut and try" solution of equation 5-27.

For Class "C" bedding conditions, the assumed field loading conditions are shown in Fig. 3-2. Sketches under "c" in this figure give the magnitude and distribution of coefficients for moment, axial force and shear in the pipe under the assumed loading condition. For a typical Class IV pipe designed with \( f_c' = 5000 \) psi and
\[ A_{s1}/D_1 \cong 0.1 \], application of equation 5-27 to sections between 150° and 180° shows that \( P_f \) is minimum with the section of critical diagonal tension strength at \( \theta = 158° \), or 22° from the invert. The section of maximum shear, in this case, is at 30° from the invert.

For this particular field loading condition, the estimated ratio between total field load capacity and test load capacity with respect to diagonal tension failure may be obtained by computing \( P_f \) for field loading capacity from equation 5-27 and \( P_t \) for test loading capacity from equation 5-17 with \( P_t = (DL)_u \times D_1 \). For a typical Class IV pipe with \( f'_c = 5000 \text{ psi}, \frac{A_{s1}}{D_1} = 0.1 \) and \( W = 300 \text{ d} \), the "Load Factor" or ratio \( P_f/P_t = 1.58 \) under the assumed Class C bedding conditions. Fortunately, this happens to be about the same as the typical load factor for pipes which fail in flexure as discussed previously in Article 4.6. Hence, the frequently suggested load factor of 1.5 for Class C bedding conditions seems to be logical for both flexural failures or diagonal tension failure.

The above comparison of load factors in flexural and diagonal tension failures also indicates that whatever mode of failure occurs in the test will also occur in Class C field bedding conditions. Thus, when stirrups are needed to meet test ultimate strengths, they would also be required to develop the full field capacity. The above analysis also indicates the approximate region over which the shear reinforcing would be required in order to develop full field capacity, a question which has been the subject of much speculation in the industry.
Load factors for other distributions of field loading could be developed from equation 5-27 in a similar manner to the development given above for Class C bedding. Comparison of ultimate strengths for diagonal tension and flexural failures would indicate whether or not stirrups would be effective for field loaded pipe. The extent of stirrup placement could be determined from the location of critical section in the field loaded pipe.

It should be noted, however, that no test results for pipe under distributed loading were available to check the validity of equation 5-27 given above for predicting location of the critical section under field loading and the field diagonal tension strength. Such tests should be carried out in the future to substantiate or modify the above findings.

It should also be noted that the above equations for diagonal tension strength of pipe under field load distributions are based upon a constant depth to tension steel. In pipe with a single elliptical reinforcing cage, the effective depth to tension steel is reduced at the critical section for diagonal tension under distributed load; however, the steel has a small angle of inclination with the normal to the section which may contribute slightly to diagonal tension resistance. It is probable, however, that under distributed loadings, pipe with a single elliptical reinforcing cage have their critical section for diagonal tension further away from crown and invert and have a lower diagonal tension strength than similar pipe having two circular cages with area of inner cage equal to area of single cage elliptical reinforcing.
STIRRUP REQUIREMENTS FOR PREVENTION
OF DIAGONAL TENSION FAILURE

6.1 Introduction

Diagonal tension failure of pipe may be prevented with the use of stirrups in a radial direction between the inner and outer steel cages of the pipe. These stirrups must resist internal shear force and the internal radial tension which results from the curvature of tensile force in the main reinforcing.

A number of successful arrangements have been used for stirrups in pipe. Figure A-3 shows several arrangements of "comb" type multiple stirrups made from welded wire fabric with main wires at the same longitudinal spacing as pipe circumferential reinforcing and with spacing between cross wires just sufficient to leave one longitudinal cross wire attached at one end of each stirrup when the other end is hooked. The hook may be placed at either side of the pipe wall with the single longitudinal cross wire at the other end of the stirrup. Either the hook or the longitudinal cross wire is used to anchor the inner cage pipe reinforcing to the stirrup system. Tests reported in reference 8 indicate that there is sufficient strength in the weld between stirrup and cross wire to provide the required anchorage strength. However, further tests are needed for cases where the stirrup is stressed up to its ultimate strength.

Another type of stirrup which has been used in the industry consists of individual cold drawn wire stirrups with hooks at each
end secured around the two pipe reinforcing cages. Similar individual hot rolled deformed stirrups are also used. Individual stirrups are usually staggered on adjacent circumferential steel reinforcing to reduce the total number of stirrups required while maintaining a proper average circumferential spacing (60).

6.2 **General Requirements for Effective Structural Action**

In order to be effective, stirrup reinforcing must have the following characteristics:

1. Sufficient strength to resist part of the shear and all of the radial tension effect. The existence of stirrups will prevent the opening of wide diagonal cracks extending up into the compressive region of the concrete. Thus, the concrete can carry an undetermined amount of the total shear force at a critical section. It remains for future research to determine an accurate basis for apportionment of shear between concrete and stirrups.

2. Proper spacing in the circumferential direction to prevent the occurrence of a diagonal crack between adjacent stirrups. Since the combination of shear and radial tension produces a somewhat flatter crack than would be expected in a straight member, maximum stirrup spacing in pipe can be somewhat greater than maximum spacing requirements for beams.
3. Sufficient tie strength at every main circumferential reinforcing element for full effectiveness against radial tension. This means that stirrup spacing longitudinally should be the same as the spacing of the circumferential reinforcing. The anchorage described below on the tension side also ties the circumferential reinforcing.

4. Proper anchorage at upper and lower ends to permit the development of the necessary tensile forces in the stirrup. End anchorage is particularly important when plain stirrups of high tensile cold drawn wire are employed. On the tension side, it can be obtained by hooking the stirrups around main inner-circumferential reinforcing, or around secondary longitudinals (if main reinforcing is welded wire fabric with longitudinals at the proper spacing), or by adding a distribution bar through the stirrup hooks and inside the circumferential steel. On the compression side, anchorage can be obtained by hooking the stirrup around the outer cage; or with stirrups made from welded wire fabric, one longitudinal is left attached to all stirrups in one line and is anchored, in turn, by the outer cage of steel. Hooks must have sufficient straight extension beyond the curve to prevent straightening. See Fig. A-3.
5. Extend over those regions of a pipe where the combination of critical shear and diagonal tension exists. In the 3-edge bearing test, the critical regions exist over only a small length at the crown and invert and a few stirrups at those locations could prevent diagonal tension failure. However, under field loading conditions the critical section is at some distance beyond the invert. It was seen in Article 5.9 that for Class C bedding conditions the critical section is about $22^\circ$ outside the invert. In this case, stirrups should extend a distance beyond the critical section which depends on the total diagonal tension ultimate strength that may be required with the use of stirrups. For typical class IV pipe with stirrups, it would probably be sufficient to extend the stirrups over a region of about $35^\circ$ on either side of the crown and invert for Class C bedding.

For pipes with higher field loads due to higher class of bedding, the critical section could be found with the aid of equation 5-27. It would be a few degrees further out from the invert than for Class C bedding and a somewhat greater extent of stirrups would be required.
6.3 Stirrup Proportioning

Fig. 6-1 shows the internal force system after occurrence of diagonal cracking in a pipe reinforced with stirrups which have a circumferential spacing of $S = jd$. It is not possible to determine the force in the stirrup without first determining how much shear can be carried by the concrete (the contribution of the circumferential steel, $V_S$, can probably be neglected). After a diagonal crack opens, the concrete can still contribute to the shear resistance if there is a redistribution of stresses which enables the section of pipe in the vicinity of the crack to act as a tied arch. This action cannot take place without radial ties to prevent straightening of the steel with attendant loss of its tension carrying ability. Thus, without ties, pipe can have no load carrying capacity after the formation of the first diagonal crack. With adequate ties, it is possible that the concrete may carry the entire shear load as long as the ties can provide the required radial tension forces. More test data is needed before a proper evaluation of the distribution of shear between stirrups and concrete can be made.

As a matter of fact, no such quantitative evaluation of the shear carrying ability of the compression concrete zone in straight beams with web reinforcing has been widely accepted as yet. The best suggestion that Committee 326 could make was that in straight beams, stirrups need be designed to carry only the excess shear above the capacity of the concrete as given by equation 5-1 or 5-2(39). This
recommendation seems totally illogical since the mechanism by which concrete carries shear after diagonal cracking is entirely different than the shear carrying behavior before cracking. The only basis for it given by the Committee is that it produces conservative results, when compared to the limited available test data, and is somewhat similar to the procedure for designing beams with stirrups in the current ACI code which has had a long record of successful use.

If the method for stirrup design in current use by the ACI code (1956 edition) was adapted for pipe, the stirrups in pipe would be proportioned for the full radial tension force plus that portion of the ultimate load which produced excess shear stresses above .03 $f'_c$ (or 90 psi max.) times the factor of safety.

For a factor of safety of 1.5, the shear capacity of the concrete would be 90 psi x 1.5. The 3-edge bearing load capacity of the concrete alone for shear would be:

$$P_c = 2V_c = \frac{2}{\gamma} bjd = 2 \cdot 90 \cdot 1.5 \cdot 12 \cdot .9 \cdot d = 2920 \cdot d$$

$$P_u = (DL)_u D_i$$

for wall B pipe $d'' \approx D_i$

thus $(DL)_u c = 2920 \approx 3000$

The following equation gives the stirrup area required per foot of pipe length per line of circumferentially spaced stirrups when the stirrups are proportioned to carry the radial tension force
If $s \leq jd$:

$$T_s = f_v A_v = \left[ (V + \frac{M}{r_s}) - (V_c + V_s) \right] \frac{s}{jd}$$

INTERNAL FORCE CONDITIONS ASSUMED FOR STIRRUP DESIGN

FIG. 6-1
from the full load plus the shear force from any load in excess of 3000 D (see Appendix H for derivation):

\[
A_v = \frac{.34 (DL)_u D_i s}{f_v d} + \frac{.56 (DL - 3000)_u D_i s}{f_v d}
\]

The first term in the above expression represents the radial tension effect and the second term, the requirements for excess shear. Combining terms, this expression becomes:

\[
A_v = \frac{[.90 (DL)_u - 1600] D_i s}{f_v d} \quad 6-1
\]

with min. \( A_v = \frac{.34 (DL)_u D_i s}{f_v d} \quad 6-2 \)

\( f_v \) should not be taken greater than the 0.2% offset yield stress to prevent wide opening of diagonal cracks with destruction of the concrete shear carrying zone.

For investigation of pipe with a fixed amount of stirrup reinforcing, the above equations may be recast as follows:

\[
f_v = \frac{[.90 (DL)_u - 1600] D_i s}{A_v d} \quad 6-1a
\]

with min. \( f_v = \frac{.34 (DL)_u D_i s}{A_v d} \quad 6-2a \)

A third form for these relations is given in Article 11.2.

Six tests with strain gages on stirrup reinforcing, reported in reference 8, indicated very low stirrup stresses at ultimate flexural failure loads which were well above the diagonal tension
strength without stirrups. Table 6-1 shows a comparison of measured stirrup stresses with two different theoretical assumptions for computing maximum stirrup stresses. It is seen that the measured stresses are somewhat below computed stresses, even for the assumption that the concrete carries all the shear. These strain gage readings certainly indicated that the stirrups carried no more than radial tension effects. Moreover, the measured stirrup stresses often were maximum in a stirrup at the point of maximum moment right at the crown or invert, rather than at the critical section for shear, a distance 'd' on one side of the load points. This is a further indication that stirrups primarily carried radial tension force, rather than shear force. It is important to note, however, that the instrumented stirrups required so much waterproofing around the strain gages that little bond could have existed over the central part of the stirrups. Hence, it is really not certain whether these measured stresses were representative of the true stress in the stirrups or whether the broken bond in the instrumented stirrups caused transfer of part of their load to adjacent stirrups without instrumentation.

There was no indication that stirrups in any of these test pipe were anywhere near their ultimate capacity when the pipe failed in flexure. Future tests should be designed with lighter stirrups to check experimentally the ultimate capacity of the stirrup reinforcing.

Stirrups must be spaced closely enough circumferentially, so that a diagonal crack could not form between adjacent stirrups. On the
other hand, because of the large numbers of stirrups necessary in pipe, it is important to be able to use the maximum possible spacing. The ACI code limitation of maximum spacing to .5d seems too severe for application to relatively thin-walled pipe. The existence of radial tension in pipe causes a somewhat flatter crack than the 45° inclination applicable to straight beams. Moreover, as explained above, the primary function of stirrups in most pipe is to carry radial tension rather than shear.

The very few available test results on maximum stirrup spacing (10) indicate that the maximum effective circumferential spacing of stirrups should be limited as follows:

\[ \text{max. } s = 1.0 \ d \]

6-3

If stirrup spacing is staggered on adjacent circumferential bars or wires, it is not known what maximum spacing on each individual bar could be tolerated for full stirrup effectiveness. Since the stirrups primarily function as radial ties, rather than shear reinforcing, it is probable that the maximum spacing on individual bars should not be very much greater than 1.0d. Tests should be carried out to determine the maximum permissible spacing for ties on an individual bar or wire.

The above equations 6-1 to 6-3 are not recommended for practical design of pipe until they are corroborated with more test results. Instead, for practical design of pipe, it is more conservatively recommended that stirrups should be proportioned to carry two thirds of the shear force plus the full radial tension force, which is
<table>
<thead>
<tr>
<th>Test</th>
<th>Gage No.</th>
<th>Gage Location</th>
<th>Calculated Maximum Stirrup Stresses</th>
<th>Measured Maximum Stirrup Stresses</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Concrete Carries All Shear (psi)</td>
<td>Concrete Carries 1/3 of Shear (psi)</td>
</tr>
<tr>
<td>**</td>
<td>**</td>
<td>**</td>
<td>** psi</td>
<td>** psi</td>
</tr>
<tr>
<td>G-2</td>
<td>21</td>
<td>4&quot; from Invert</td>
<td>27,400</td>
<td>55,000</td>
</tr>
<tr>
<td>G-3</td>
<td>14</td>
<td>Crown</td>
<td>24,400</td>
<td>55,000</td>
</tr>
<tr>
<td>N</td>
<td>22</td>
<td>9&quot; from Invert</td>
<td>27,400</td>
<td>55,000</td>
</tr>
<tr>
<td>Q-1</td>
<td>18</td>
<td>1&quot; from Crown</td>
<td>32,800</td>
<td>66,000</td>
</tr>
<tr>
<td>Q-2</td>
<td>21</td>
<td>Invert</td>
<td>32,800</td>
<td>66,000</td>
</tr>
<tr>
<td>R-1</td>
<td>18</td>
<td>4&quot; from Crown</td>
<td>30,600</td>
<td>71,500</td>
</tr>
<tr>
<td>R-2</td>
<td>18</td>
<td>17&quot; from Invert</td>
<td>47,600</td>
<td>95,500</td>
</tr>
</tbody>
</table>

Notes:  
*See Ref. 8  
**Equation 6-2a  
***Equation 6-4  
****Equation 6-1a
the basis for the following equation (see Appendix H for derivation):

$$A_v = \frac{.7 \ (DL) \ u \ D_i \ s}{f_v \ d} \quad 6-4$$

Also, until more test results are available, the maximum average circumferential spacing should be conservatively limited to:

$$\text{max. } s \approx .75d \quad 6-5$$

6.4 Field Load Conditions

Stirrup proportioning in accordance with any of the above procedures is really based on the requirements for the test load. Under field loading conditions, radial tension at the critical section is lower (since radial tension is almost directly related to moment and the critical section is no longer at the section of maximum moment) and shear is higher. Since stirrups are thought to be needed primarily for resistance to radial tension, it is believed that field load requirements should be less severe than test load conditions. Hence, stirrup proportioning based on test load requirements, with the stirrups extended at constant spacing over the region suggested in article 6.2, Item 5, should be adequate for field load conditions.
LIMITATION OF CRACK WIDTH

7.1 Introduction

The control of cracking is an especially important criterion for the design of pipe. Sewer, culvert, storm drain and other types of non-pressure or low pressure reinforced concrete pipe will normally be cracked due to external service loads from earth pressure and surcharge live loads. Experience and a limited number of tests have shown that, even though such pipe are subject to rather severe corrosion producing conditions, a significant amount of corrosion will not occur in the reinforcing steel if the crack width is not too large. It is also known that autogeneous healing takes place in cracked concrete pipe (1) so that small cracks will often heal themselves with time. However, neither the maximum size of crack that will heal, nor the extent of partial healing that may occur in a crack of given size is well known.

7.2 Maximum Permissible Width of Crack

Little experimental information is available about the maximum size of cracks that may be permitted without significant corrosion of reinforcing steel. Brocard (46) reported tests which indicate a rapid increase in degree of corrosion for crack sizes above 0.2 mm (.0079 in.). A maximum crack width of .2 mm is a widely accepted limiting value in Europe (41,46). However, Kennedy (47) found no significant difference in corrosion resistance with
increasing crack width during exposure tests conducted with specimens stored for two years in sea water. The specimens were made with air entrained concrete.

The pipe industry has long based crack control criteria on a limiting maximum crack width of .01 inch in the 3-edge bearing test, as measured by the insertion of a standard .01 inch leaf gage into the crack for a length of one foot (20). Apparently, long experience with the use of this criteria has been favorable. However, it is not known whether methods for estimating the field design loading may have been so conservative throughout much of this experience that a .01 inch crack seldom occurs in pipe under service load conditions.

Pipe specifications in Germany (22), France (23) and Spain (24) use a maximum crack width of 0.2 mm in the 3-edge bearing test as a crack control criterion. However, there are differences between practice in different countries when converting 3-edge bearing capacity under field load.

7.3 Flexural Cracking of Reinforced Concrete Structural Elements

Flexural cracking of a reinforced concrete flexural element commences when the tensile strength of the concrete is exceeded in the vicinity of a point of maximum bending moment. As the load is increased on the element, new cracks will form at a spacing dependent on the bond stress between steel and concrete and on the tensile strength of the concrete at local points in the member in accordance with the following theoretical relation (40):
\[ x = \frac{A_t}{\xi \pi \phi} \frac{f_t}{U_a} \]  

Where \( A_t \) is the effective tensile area of the concrete  
\( f_t \) = tensile strength of the concrete  
\( U_a \) = average bond between steel and concrete  
\( \xi \pi \phi \) = perimeter of reinforcing steel

At this stage \( U_a \) is determined by the actual bond in the section. However, as the load continues to increase new cracks may form between existing cracks until the minimum crack spacing occurs when \( U_a \) reaches the ultimate possible value of average bond stress between steel and concrete, \( U_{au} \).

If bond stress is assumed to vary by some law from zero at the crack where considerable elongation of the steel along must occur up to \( U_m \), the ultimate bond stress at a point midway between cracks, then \( U_a = u_m / c \), where \( c \) is a constant greater than 1, and the minimum crack spacing is:

\[ x = \frac{cA_t}{\xi \pi \phi} \frac{f_t}{U_m} \]  

The correct value of effective tension concrete area for use in the above expressions cannot be determined from theoretical studies, since the internal transfer of tension from steel to concrete between cracks probably causes a complex distribution of tension stress in the concrete. Also \( c \) cannot be determined theoretically since the distribution of bond between steel and concrete is also
very complex. Thus, the proper value to use for \( cA_t \) could only be
determined from test results on flexural specimens. However, its
determination may be further complicated by dependence on various
dimensional parameters of the flexural specimen. Moreover, the
inherent variability of test results is high, since the actual
final spacing of cracks is very dependent on the location and
spacing of cracks at intermediate stages in the development of a
cracking pattern. The spacing of earlier cracks is subject to great
variability, being due in part to chance locations of local points
of slightly lower tensile strength or chance variations in the bond
distribution between steel and concrete. Later cracks, then, must
fall within the pattern already established, so that the final
crack spacing is greatly affected by the initial crack pattern.

The variability of crack spacing can be greatly reduced
by the presence of "crack initiators" at proper spacing on the
reinforcing steel (41).* Cross wires in welded wire fabric can
serve as such crack initiators to reduce the variability of crack
spacing. However, insufficient experimental information is avail-
able to evaluate fully the factors which determine the proper
spacing of crack initiators with various proportions of reinforcement
and concrete.

Once the minimum crack spacing has been determined, it is
possible to determine the crack width from the product of average strain
in reinforcing steel between cracks and crack spacing, \( x \). If the
steel strain is still within the elastic range, crack width is given as

*See papers by Efsen and Krenchel and by Bohmer in Reference 41.
follows:

\[ v_s = \frac{f_s a v x}{E_s} \]  

7-3

The average stress in the reinforcing steel may be determined from the elastic force in the steel right at a crack:

\[ f_s A_s = \frac{M}{Jd} \]  

7-4

reduced by the average tension force transferred by bond to the concrete between cracks. The magnitude of this concrete tension force is generally considered to be a function of the tensile strength of the concrete and the area of tension concrete (41).* Thus:

\[ v_s = \frac{x}{E_s} \left( f_s - \frac{K f A}{A_s} \right) \]  

7-5

The term, \( f_s - \frac{K f t}{A_s/A_c} \), represents the average stress in the reinforcing steel between cracks.

7.4 Prediction of Crack Spacing for Pipe Reinforced with Welded Wire Fabric

Fortunately, test results (8) have shown that with the typical proportions of welded wire fabric reinforcement used for pipe of 66 inch diameter and larger, average crack spacing often

*See discussion by Thomas in "General Report" Section of Reference 41.
corresponds very closely to the spacing of longitudinal cross wires in the fabric. Thus:

\[ x = \frac{\lambda}{\lambda_0} \]  

This was true for almost every test pipe with four inch, six inch or eight inch spacing of cross wires, single wrap inner and outer cages, and no stirrups. Stirrups at spacings closer than cross wire spacing often forced closer spacing of cracks, as did the use of two wraps for inner cage with cross wire locations offset between the two cages. Most test pipe of four foot diameter, with smaller size reinforcing wires, developed minimum crack spacings of less than the fabric longitudinal cross wire spacing. It is important to note, however, that in those cases where properly spaced cross wires could serve as crack initiators (within the limitations discussed above) the observed crack spacing corresponded to longitudinal cross wire spacing with a very high degree of dependability\(^{(8)}\).

Thus, the major difficulty of finding a simple, accurate method for estimating crack spacing for use with equation 7-5 is overcome for pipe of about 66 inches in diameter and larger, reinforced with welded wire fabric.

7.5 **Prediction of .Ol Inch Crack Load for Pipe**

**Reinforced with Welded Wire Fabric**

With the help of the following additional assumptions, equations 7-5 and 7-6 may be used as a basis for predicting the 3-edge bearing test load which will cause a maximum crack width
of .01 inch in a pipe reinforced with welded wire fabric:

1. At a .01 inch surface crack, \( w_s = .008 \) inches at the level of the reinforcing steel.

2. Steel strain remains in the elastic range at .01 inch crack load: \( E_s = 29 \times 10^6 \) psi.

3. .01 inch crack occurs at the point of maximum moment at the invert of the pipe. The theoretical moment under test load at the invert is slightly smaller than at the crown (see Fig. 3-1), but maintains its maximum value constant over a distance equal to one-twelfth the pipe diameter. Also, the moment due to weight is higher at the invert. Thus, average strain at the invert often is slightly higher than at the crown, although the difference is hardly significant. At the invert:

\[
M = .282 \, Pr_{av} + .202 \, Wr_{av}.
\]

4. Pipe wall dimensions conform to requirements of ASTM C76.

5. Tensile strength of the concrete, \( f_t \), is proportional to the square root of the compressive strength, \( f'_c \), from the standard cylinder test:

\[
f_t = K_t \sqrt{f'_c}
\]

6. Tests results (8) indicate that the quantity, \( K \), in equation 7-5 is not constant in test pipe with welded wire fabric reinforcing. Apparently, the
development of crack spacing due to the presence of crack initiators, rather than due to the ratio of concrete tension to steel bond (equation 7-2) also effects the variables which determine the transfer of tension from steel to concrete between cracks. When crack spacing is determined by equation 7-6, rather than 7-2, the relative area of bond between steel and concrete is an additional determinant of the amount of reduction in steel stress. Thus, it will be assumed that:

$$K \propto \frac{l_0 \pi \rho}{\pi D_i^2} = K_0 \frac{l_0}{\rho}$$  \hspace{1cm} 7-8

7. Rate of loading must be standardized. ASTM C76's requirement for a loading rate of 2000 lbs per foot of pipe length per minute will be assumed.

The .01 inch cracking capacity of the pipe may then be predicted from the following equation which follows from equations 7-5, 7-6 and the above assumptions (see Appendix J for derivation):

$$\begin{align*}
(DL)_{01} &= 1.15 \cdot 10^5 \frac{A_{sl}}{\rho_0 D_i^2} + \frac{K_1 \rho_0 h d \sqrt{f_c'}}{\rho D_i^2} - \frac{72W}{D_i} \\
\end{align*}$$  \hspace{1cm} 7-9

Note on units: \((DL)_{01}\) in lbs/ft\(^2\), \(D_i\) in feet, \(W\) in lbs/ft, all others in pound and inch units. Also applies to equations 7-10 and 7-11.

In the above expression, \(K_1\) is a constant which equals \(K_0 K_t\).
$K_0$ is the constant in equation 7-8 which relates tensile force picked up by the concrete to bonding qualities of the reinforcing and will vary with different types of reinforcing surfaces (i.e., smooth and bright, smooth and rusty, deformed). $K_1$ is the constant in equation 7-7 which relates concrete tensile strength to square root of compressive strength. $K_1$ can best be determined from test results on pipe. Results from the M.I.T. test program (8) indicate that $K_1$ is approximately 0.3 when units noted above are used.

Equation 7-9 thus becomes:

$$\text{(DL)}_{01} = \frac{1.15 \cdot 10^5 A_{s1} d}{\varnothing D_i^2} + \frac{3 \varnothing_{0} h d \sqrt{f'_c}}{\varnothing D_i^2} - \frac{.72W}{D_1} \quad 7-10$$

The following companion equation was developed in order to provide an upper limit to the predicted .01 inch D-load capacity which will insure that the steel stress remains elastic at $(DL)_{01}$ (see Appendix J for derivation):

$$\text{max. (DL)}_{01} = \frac{.49 f A d}{D_i^2} - \frac{.72W}{D_1} \quad 7-11$$

For pipe design, it is desirable to insure that the steel stress remains elastic at the .01 inch crack load, because this criteria is often assumed to be the service load condition of the pipe. Moreover, for loads which produce steel strains beyond the elastic range, crack width opens much more rapidly.

Equations 7-10 and 7-11 may be rearranged in a form suitable for design of pipe as indicated in Section 11.3.
7.6 Analysis of Test Data

Table 7-1 presents a concise summary of all the variables which were believed to have a significant affect on the .01 inch crack behavior for the entire 39 M.I.T. tests (8). The final column in this table gives the steel stresses calculated using equation 7-4, the usual elastic method for calculating nominal steel stresses at a cracked section in a reinforced concrete member.

Table 7-2 gives the same information for 43 tests by the American Concrete Pipe Association (11, 14) and 12 tests by Missouri School of Mines (15). See Appendix A for a complete description of the test specimens and test results in the above tables. Tables 7-1 and 7-2 indicate qualitatively that nominal steel stresses at .01 inch crack load decrease with increasing steel content, increase with higher concrete strengths, increase with closer spacing of longitudinal wires, increase with two wraps of reinforcing steel, and may be affected by the method of manufacture and age of test.

Table 7-3 gives a comparison of calculated nominal steel stresses (using equation 7-4) and steel stresses obtained from strain measurements at various load levels for those specimens in the M.I.T. test program (8) which had SR-4 electric strain gages on the reinforcing steel. The first row for each test gives stresses calculated and measured for a point in the vicinity of the crown and the second row for a point in the vicinity of the springing of the test pipe. Exact locations of these gage points are given in reference 8. The
first two columns in Table 7-3 gives the calculated and measured stress comparisons for the measured .01 inch crack load. Other columns give the same information for ASTM C76 specified .01 inch crack load, for measured ultimate load and for specified ultimate load.

In nearly all cases, measured stresses are lower than calculated stresses. Since most measurements would be at points between cracks, this indicates that the concrete must carry some tension force between cracks. The relative difference between measured and calculated stresses decreases at higher loads, indicating the limited ability of the concrete to carry tension. In general, Table 7-3 gives a fairly good qualitative correlation with the theory developed previously in this chapter. However, no quantitative evaluation of this theory is possible from the strain gage readings for the following reasons:

1. The strain gage results are readings at localized points at or between cracks, not the average strains used to estimate crack widths.

2. The use of waterproofing or box-outs over the gages (6) changes the localized bond conditions in the vicinity of the gages, and thus prevents a really accurate measurement of the typical state of stress in the steel between cracks.

3. The conversion of strain readings to stress results was based on use of an average stress-strain curve for steel in many of the tests (6).
<table>
<thead>
<tr>
<th>Mark</th>
<th>Internal Diameter</th>
<th>D-load Meas. .01&quot; Crack</th>
<th>f'c Cyl.</th>
<th>Mfg. Process</th>
<th>Age at Test Days</th>
<th>Depth to Steel, in.</th>
<th>Steel Ratio $\frac{A_y}{A_o}$</th>
<th>Spacing Long Wire</th>
<th>Spacing Stirrups</th>
<th>Av. Crack Spacing at Invert in.</th>
<th>Calculated Steel Stress at .01&quot; Crack Load psi</th>
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<tbody>
<tr>
<td>F-1</td>
<td>106</td>
<td>2540</td>
<td>5175</td>
<td>Cast</td>
<td>21</td>
<td>7.5</td>
<td>.0115</td>
<td>8</td>
<td>-2 wraps</td>
<td>-</td>
<td>8</td>
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<td>72</td>
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**See crack sketches for individual tests for number and layout of cracks, and for load at which cracks were first visible.
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Notes: *Indicates core, rather than cylinder, strength.
**Indicates depth to steel is estimated assuming approximately
1" clear cover over reinforcing steel.
C indicates 2 circular cages.
E indicates 1 elliptical cage.
<table>
<thead>
<tr>
<th>Material</th>
<th>Finish</th>
<th>Condition</th>
<th>Form</th>
<th>Grade</th>
<th>Shape</th>
<th>Size</th>
<th>Temp.</th>
<th>Cond.</th>
<th>Weight</th>
<th>Heat</th>
<th>Test Data</th>
<th>Specimen</th>
<th>Required</th>
<th>Tested</th>
<th>Results</th>
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*Table 2.1: DataSummary. All data is rounded to 2 decimal places. Additional notes are below.*
### Table 7-3

**Comparison between Measured Stresses and Calculated Nominal Stresses in the Reinforcing Steel**

**M.I.T. Test Program**

<table>
<thead>
<tr>
<th>Test</th>
<th>Gage No.</th>
<th>0.01&quot; Crack Load</th>
<th>0.01&quot; Spec. Crack Load</th>
<th>Ultimate Load</th>
<th>Specified Ultimate Load</th>
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<td>Meas. psi</td>
<td>Calc. psi</td>
<td>Meas. psi</td>
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*Indicates elastic limit of steel exceeded, so that calculated results are not applicable or not too reliable.
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<th>Test Gage No.</th>
<th>0.01&quot; CRACK LOAD</th>
<th>0.01&quot; SPEC. CRACK LOAD</th>
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<td>37,800</td>
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</table>

*Indicates elastic limit of steel exceeded, so that calculated results are not applicable or not too reliable.
4. Some few readings appear to be of the wrong order of magnitude and may include effects of drift in gage resistance and other experimental inaccuracies.

In order to obtain a direct experimental evaluation of the theory developed earlier in this chapter, equations 7-10 or 7-11 were used to predict the .01 inch crack strength of pipe reinforced with welded wire fabric in the M.I.T. test program and in the two ACPA test programs referred to previously. The comparison of predicted and test .01 inch cracking strengths for the M.I.T. tests is given in Table 7-4. Some of the pipe in this table do not meet the limitations for application of equations 7-10 and 7-11 set up in the previous section. Where these limitations were not met, average crack spacing did not always correspond to longitudinal cross wire spacing. In these cases, the actual crack spacing, rather than the spacing of fabric longitudinals, was used with equation 7-10 (or 7-11, whichever gave the lower strength prediction) to obtain predicted .01 inch crack strengths. This approach makes more data available for evaluating the constant, $K_1$, in equation 7-9. Table 7-4 also indicates the summarized average and variability of the test results for pipe of six foot diameter and larger. The variability is actually rather low compared with analyses of cracking theories elsewhere for deformed bar reinforcing (40, 41).

The comparison of predicted and test .01 inch cracking strengths for results from the two ACPA test programs are given in Table 7-5. It is seen that while variability is about the same as
## Table 7-4
### Analyses of .01 Inch Crack Strength
#### M.I.T. Test Program

<table>
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<th>Test Mark</th>
<th>Inside Pipe Diameter</th>
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<th>(DL) .01 calc.</th>
<th>(DL) .01 Test</th>
<th>(DL) .01 calc.</th>
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<td>1950</td>
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<td>2060</td>
<td>2420&lt;sup&gt;TV&lt;/sup&gt;</td>
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<tr>
<td>R-2**</td>
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<td>2360</td>
<td>2420&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>.98</td>
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<td>1.47</td>
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<tr>
<td>Q-2</td>
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<td>1620</td>
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<td>1160</td>
<td>1260</td>
<td>1.16</td>
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Average: 1.017

Standard Deviation: .170

Coefficient of Variation: 16.7%

Number of Tests: 22

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<th>Segment</th>
<th>Inside Pipe Diameter</th>
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<th>(DL) .01 Test</th>
<th>(DL) .01 calc.</th>
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</tr>
<tr>
<td>L-1&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1380</td>
<td>1460</td>
<td>.94</td>
<td></td>
</tr>
<tr>
<td>L-2&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1620</td>
<td>1670&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>.97</td>
<td></td>
</tr>
<tr>
<td>J-1&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1190</td>
<td>1280&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>.93</td>
<td></td>
</tr>
<tr>
<td>J-2&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1250</td>
<td>1310&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>.96</td>
<td></td>
</tr>
<tr>
<td>L-1&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1310</td>
<td>1280&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>L-2&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1310</td>
<td>1280&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>M-1&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1750</td>
<td>1090&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>1.61</td>
<td></td>
</tr>
<tr>
<td>M-2&lt;sup&gt;*&lt;/sup&gt;</td>
<td>48</td>
<td>1440</td>
<td>1090&lt;sup&gt;TV&lt;/sup&gt;</td>
<td>1.32</td>
<td></td>
</tr>
</tbody>
</table>

Average of 48" Pipes: 1.07

### Notes:
1. Observed Crack Spacing differed by more than 1/2 inch from Transverse Wire Spacing. Observed crack spacing was used for (DL)calc. computation.
2. Immediate adjacent cracks did not become visible until well above measured .01 inch crack load. Hence, theoretical crack width formula was not truly applicable.
3. Reinforcing consisted of 2 wraps.
4. Based on limitation of maximum DL by limit of steel elasticity.
the M.I.T. tests, the average .01 inch crack strength in the tests was 28% higher than the predicted .01 inch crack strength. However, predicted strengths were based upon the assumption that crack spacings at the crown and invert were equal to spacings of fabric longitudinals. No information was available on the actual crack spacings or on the condition of the wires (bright or rusty) in these tests. If the fabric was rusty, it is probable that crack spacing was less than assumed, in which case a larger .01 inch crack capacity would be expected.

Figures 7-1 and 7-2 give a graphical representation of the comparative predicted and test strengths given in Tables 7-4 and 7-5, respectively.

Figure 7-3 shows the cumulative distribution of the ratio of test to predicted .01 inch cracking capacity separately for each of these test programs. It is evident that the results from these separate programs fall into separate groups which indicate that some significant constant difference occurred between the two test programs. This difference could have been due to differences in condition of wire surfaces (i.e., rusty steel in ACPA, bright in M.I.T.); different effect of age of pipe between programs; different types of cement giving different relations between true bond and compressive cylinder strength; different techniques of manufacture giving different relations between true bond and compressive cylinder strength; different interpretation of standard leaf gage measurement of .01 inch crack, or to a combination of some of the above factors.
### TABLE 7-5

**ANALYSIS OF .01 INCH CRACK STRENGTH - TEST PROGRAMS**

**OF OTHERS WITH WELDED WIRE FABRIC REINFORCING**

<table>
<thead>
<tr>
<th>Test Mark</th>
<th>Inside Pipe Diameter inches</th>
<th>(DL) &lt;sub&gt;test&lt;/sub&gt;</th>
<th>(DL) &lt;sub&gt;calc.&lt;/sub&gt;</th>
<th>(BL) &lt;sub&gt;test&lt;/sub&gt;</th>
<th>(BL) &lt;sub&gt;calc.&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>GB60-9</td>
<td>84</td>
<td>1740</td>
<td>1160</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>GB60-17</td>
<td>84</td>
<td>1340</td>
<td>1170</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>GB60-10</td>
<td>78</td>
<td>1345</td>
<td>1150</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>GB60-2</td>
<td>78</td>
<td>1345</td>
<td>1150</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>GB60-11</td>
<td>72</td>
<td>1645</td>
<td>1260</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>GB60-3</td>
<td>72</td>
<td>1380</td>
<td>1190</td>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>GB60-8</td>
<td>66</td>
<td>1445</td>
<td>1230</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>GB60-4</td>
<td>66</td>
<td>1370</td>
<td>1270</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td>GB56-1-72</td>
<td>72</td>
<td>1520</td>
<td>1350</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>GB56-2-72</td>
<td>72</td>
<td>2010</td>
<td>1450</td>
<td>1.39</td>
<td></td>
</tr>
<tr>
<td>GB56-3-72</td>
<td>72</td>
<td>2110</td>
<td>1390</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>LZ56-72C</td>
<td>72</td>
<td>2330</td>
<td>1420</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td>LZ56-72A</td>
<td>72</td>
<td>2250</td>
<td>1370</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td>LZ56-721</td>
<td>72</td>
<td>1920</td>
<td>1420</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>LZ56-72A</td>
<td>72</td>
<td>2180</td>
<td>1550</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>LZ56-72A</td>
<td>72</td>
<td>2600</td>
<td>1550</td>
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</tr>
<tr>
<td>LZ56-72A</td>
<td>72</td>
<td>2540</td>
<td>1510</td>
<td>1.68</td>
<td></td>
</tr>
<tr>
<td>AN56-72C</td>
<td>72</td>
<td>1820</td>
<td>1340</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>AM56-72C</td>
<td>72</td>
<td>1550</td>
<td>1500</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>GB56-1-60</td>
<td>60</td>
<td>2820</td>
<td>2000</td>
<td>1.12</td>
<td></td>
</tr>
<tr>
<td>GB56-2-60</td>
<td>60</td>
<td>2450</td>
<td>2000</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>GB56-3-60</td>
<td>60</td>
<td>2310**</td>
<td>1990</td>
<td>1.26</td>
<td></td>
</tr>
<tr>
<td>LZ56-60C1</td>
<td>60</td>
<td>2750**</td>
<td>2040</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>LZ56-60C2</td>
<td>60</td>
<td>2500</td>
<td>2070</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>LZ56-60C3</td>
<td>60</td>
<td>2000</td>
<td>2090</td>
<td>1.44</td>
<td></td>
</tr>
<tr>
<td>AN56-60C2</td>
<td>60</td>
<td>2035</td>
<td>2010</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>AM56-60 C*</td>
<td>60</td>
<td>1890</td>
<td>1580</td>
<td>1.20</td>
<td></td>
</tr>
<tr>
<td>LZ56-60E1</td>
<td>60</td>
<td>2600</td>
<td>2350</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>LZ56-60E2</td>
<td>60</td>
<td>2970</td>
<td>2340</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>LZ56-60E3</td>
<td>60</td>
<td>2920</td>
<td>2350</td>
<td>1.24</td>
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</tr>
<tr>
<td>AN56-60E1</td>
<td>60</td>
<td>2350</td>
<td>2270</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>AM56-60E3</td>
<td>60</td>
<td>2165</td>
<td>2230</td>
<td>.97</td>
<td></td>
</tr>
</tbody>
</table>

**Average**

1.284

**Standard Deviation**

.197

**Coefficient of Variation**

15.3%

**Number of Tests**

32

**Notes:**

1. No information was available about observed crack spacings for any of these tests. If crack spacing was less than transverse wire spacing, theoretical formulas would give low calculated .01 inch B-loads.

2. * Indicates no information about actual wall thickness and depth to steel.

3. ** Indicates failure load where failure in diagonal tension occurred before measured .01 inch crack load was attained.
COMPARISON OF MEASURED AND PREDICTED .01 INCH CRACK STRENGTH—M.I.T. TEST PROGRAM

FIG. 7-1

COMPARISON OF MEASURED AND PREDICTED .01 INCH CRACK STRENGTH—TEST PROGRAMS OF OTHERS WITH WELDED WIRE FABRIC REINFORCING

FIG. 7-2

CUMULATIVE DISTRIBUTION OF $(DL_{o1})_{test} / (DL_{o1})_{calc}$

FIG. 7-3
In order to see if an improved correlation could be attained with the part of the M.I.T. test data which did meet the limitations of the previous section, and to avoid the need for assumption number one in section 7.5, equation 7-9 was recast in the following form (with the unknown constant \( w_s \), replacing its formerly assumed value of .008 inches):

\[
\frac{(DL \cdot 0.1 + 72W)}{D_1} \phi D_1^2 \frac{h}{h_0 \sqrt{f'_c}} = K_1 + \left(1.44 \times 10^7 w_s\right) \frac{A_\phi}{\phi_0^2 \sqrt{f'_c} \cdot h}
\]

7-12

This equation is in the form:

\[
y = A_1 + B_1 x
\]

7-13

Values of \( y \) and \( x \), above, have been computed for the 14 M.I.T. tests which conformed to the limitations established previously, and are plotted in Fig. 7-4.

Fig. 7-4 also presents the results of a "least squares" correlation of the above test data (see Appendix K for derivation). The "least squares" correlation line (heavy black line in Fig. 7-4) is compared with the previous analysis which was based on \( w_s = .008 \) inches (dotted line in Fig. 7-4). It is seen that there is little difference in the two predictions for .01 inch cracking strength. Use of either the original constants of \( A_1 = K_1 = .3 \) and \( B_1 = 1.44 \times 10^2 w_s = 1.15 \), or the "least squares" constants of \( A_1 = .4 \) and \( B_1 = 1.07 \), produces strength predications well within the zone which it may be estimated
with 95% confidence that the true correlation should lay. Hence, on the basis of these few test results, no change seems warranted in the constant proposed in equation 7-10.

Thus, in Section 11.3, equation 7-10, in different form, has been tentatively proposed for design of pipe for .01 inch cracking criteria. On the basis of the M.I.T. test data alone, the use of this equation for design would appear to be unconservative because it represents an average, rather than a lower limit, comparison with test results. Fig. 7-4 indicates the lower limit line for 90% of individual test results (as obtained from standard statistical procedures (58)) considerably below the dotted line representing equation 7-10. However, only one out of the entire 32 tests by the ACPA had a test strength less than predicted by equation 7-10. (See Table 7-5 or Fig. 7-2.) Moreover, the occurrence of the .01 inch crack does not represent a serious structural failure, but merely the attainment of an arbitrary and rather crude limit of serviceability. On the other hand, unless an allowance is made in the acceptance specification for a limited percentage of test pipe falling a limited percentage below specified minimum cracking strength, the M.I.T. test results indicate that use of equation 7-10 for pipe design would result in some 3-edge bearing tests failing to pass the minimum specified .01 inch crack requirement, while the ACPA test results indicate that equation 7-10 is conservative and that nearly all test pipe designed using it would reach the required .01 inch crack strength. Thus, it seems essential to obtain more test data, including observations on crack spacing,
\[ Y = 0.3 + 1.15X \]
(From Rational Development)

\[ Y = 0.4 + 1.07X \]
(From Correlation of M.I.T. Test Data by Least Squares)

90% of Pipes Estimated to Lie Above This Limit

95% Confidence Range of Correlation

\[ X = \frac{A_{51} \phi \times 10^5}{l^2h\sqrt{f_c}} \]

CORRELATION OF TEST DATA ON .01 INCH CRACK STRENGTH OF PIPE FOR 14 TESTS IN M.I.T. PROGRAM

FIG. 7-4
before final recommendations can be made for the selection of steel areas in pipe reinforced with welded wire fabric.

It is hoped that a research program currently underway at M.I.T. under the supervision of the author will furnish much additional information of this nature. Rather than making relatively expensive tests on full pipe rings, this program will use full scale curved beams designed to simulate the invert section of a pipe ring between points of inflection. In this way it is hoped that a fairly large quantity of very useful data can be obtained from the limited funds available for this work.

For the present, equation 7-10 may serve as a basis for tentative estimates of steel requirements in pipe, 66 inches in diameter and larger, reinforced with welded wire fabric.

7.7 Comparative Analysis of Flexural Cracking Investigations By Others

In order to further check the derivation and general validity of equation 7-10, the conclusions and suggested cracking formulae of nine other reinforced concrete cracking investigations were studied and compared with the analysis presented in Section 7.5. In order to facilitate this comparison under the wide variety of reinforcing types and section proportions considered in these investigations, expressions for the average stress in the reinforcing between cracks (i.e., elastic stress at crack reduced by average relative tension force in the concrete between cracks) are compared in Table 7-6.

No comparisons of expressions for prediction of crack spacings are given, since none of the other cracking investigations considered welded wire fabric reinforcing, or used sections with
### Table 7-6

**Comparison of Estimates for Average Steel Stress by Ten Suggested Formulas**

<table>
<thead>
<tr>
<th>Proposed By</th>
<th>Source See Appendix</th>
<th>Type Reinforcing</th>
<th>General Equation for Average Steel Stress</th>
<th>Remarks</th>
<th>Average Steel Stress for a Typical Pipe*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hager</td>
<td>Present Paper Eq. 7-10, See Appendix J</td>
<td>Welded Wire Fabric</td>
<td>$f_s = \left(1 - 0.051 \frac{\kappa b_h d_m}{N_{M_s}} \right)^{1/2} \sqrt{\frac{f_c}{f_y}}$</td>
<td>$\kappa = \frac{1}{x}, \text{ the crack spacing}$</td>
<td>0.69 $f_s$</td>
</tr>
<tr>
<td>Brice</td>
<td>42,41</td>
<td>Plain or Deformed Wire Fabric</td>
<td>$f_s = \left(1 - 1.57 \frac{k_b d_m f_t}{e_b (e_b + 3 s_m)} \right)^{1/2}$</td>
<td>$K = 1 \text{ for plain bars}$, $K = 2 \text{ for rough bars}$, $e_b = \frac{2}{3} f_t$ for width, $b = \text{number of bars}$ in width, $e_m = \frac{2}{3} f_t$ for width, $b = \frac{2}{3} f_t$ for width, $b$</td>
<td>0.70 $f_s$ (plain bars),</td>
</tr>
<tr>
<td>Borges</td>
<td>41</td>
<td>Plain or Deformed Wire Fabric</td>
<td>$f_s = \left(1 - 5.6 \frac{b h d_m}{N_{M_s}} \right)^{1/2}$</td>
<td>$f_c = 4200 \text{ psi}$</td>
<td>0.83 $f_s$</td>
</tr>
<tr>
<td>Odziana</td>
<td>41</td>
<td>Plain Wire Fabric</td>
<td>$f_s = \left(1 - 0.67 \frac{k_b d_m}{e_m} \frac{b h (h-d_m)}{e_m (h-d_m)} \right)^{1/2}$</td>
<td>$e_m = \frac{2}{3} f_t$ for width, $b = \frac{2}{3} f_t$ for width, $b$</td>
<td>0.88 $f_s$</td>
</tr>
<tr>
<td>Rusch &amp; Bahn</td>
<td>41</td>
<td>Plain &amp; Deformed Wire Fabric</td>
<td>$f_s = \left(1 - \frac{0.20 (1 - 90) b d_m}{N_{M_s}} \right)^{1/2}$</td>
<td>$\alpha_f$ varies between 0.75 and 1.1</td>
<td>Coefficient of Variation for $f_s$ var. 35 to 55%</td>
</tr>
<tr>
<td>Johnson</td>
<td>51</td>
<td>Not Stated Wire Fabric</td>
<td>$f_s = \left(1 - \frac{0.20 (1 - 90) b d_m}{N_{M_s}} \right)^{1/2}$</td>
<td></td>
<td>0.77 $f_s$</td>
</tr>
<tr>
<td>Yu &amp; Winter</td>
<td>50</td>
<td>Deformed Wire Fabric</td>
<td>$f_s = \left(1 - \frac{0.1 b h (h-d_m) (e_m)^{1/2}}{f_t} \right)^{1/2}$</td>
<td>$(f_t)^{1/2} \alpha_f$ in psi units</td>
<td>0.90 $f_s$</td>
</tr>
<tr>
<td>Clark</td>
<td>45</td>
<td>Deformed Wire Fabric</td>
<td>$f_s = \left(1 - \frac{56.6 (b d_m a_s) s_m}{N_{M_s}} \right)^{1/2}$</td>
<td>Units must be psi</td>
<td>0.82 $f_s$</td>
</tr>
<tr>
<td>Chi &amp; Kirstein</td>
<td>44</td>
<td>Deformed Wire Fabric</td>
<td>$f_s = \left(1 - \frac{197 b h d_m}{f_t \alpha_f N_{M_s}} \right)^{1/2}$</td>
<td>$r' = \text{ratio effective tensile area of concrete to max. &quot;fully developed&quot; tensile area around a steel bar}$</td>
<td>0.84 $f_s$</td>
</tr>
<tr>
<td>Eftes &amp; Krenchel</td>
<td>40</td>
<td>Deformed Wire Fabric</td>
<td>$f_s = \left(1 - \frac{1.0}{f_t} \right)^{1/2}$</td>
<td>Due to high variability of crack spacing, consideration of the small decrease in $f_s$ was not warranted</td>
<td>1.0 $f_s$</td>
</tr>
</tbody>
</table>

*Note: Properties of the particular pipe selected for illustrative purposes are as follows:

- $D_1 = 6\text{', class III, Wall B pipe}$: $A_s = 0.18 \text{ in.}^2/\text{ft.}$, $b = 0.315\text{',} a_s = 2\text{',} k = 12\text{',}$
- $b/a_s = 6 \text{ wires/ft.}, h = 7\text{',} d = 5.5\text{'}$
- $f_t = 4500, \ k_b = x = 8\text{'} \text{ (welded wire fabric),} f_p = 61,000 \text{ psi}$
- $J = 0.9, K_c = 0.30, L = 0.90, n = 7.2, f_s = 48,000$
- $M = 48,000 x 1.48 x 0.91 x 5.9 = 124,000\text{' lbs.}$

* Assume $f_t = 450 \text{ psi}$
proportions similar to typical pipe. As was shown in Section 7.4, the major determinant of crack spacing in many pipe structures reinforced with welded wire fabric is the cross wire spacing. Hence, the problem of crack spacing is greatly simplified in these structures and no formulae for predicting crack spacing from investigations with other reinforcing types would be applicable to welded wire fabric. However, information on crack spacing from tests on sections reinforced with small diameter plain wire and having similar proportions to typical pipe would be useful in order to establish rational limits wherein the cross wires will function as crack initiators. None of the available test data by others seemed to be suitable for this purpose.

The average steel stress between cracks which was used in the derivation of equation 7-10 (see Appendix J) is given by expression 1 in Table 7-6. The other similar expressions for average steel stress in Table 7-6 were derived by this author from the various equations proposed by others in the nine cracking investigations listed in the table. These expressions were obtained by separating the terms relating to average steel stress from those relating to mean crack spacing in the various equations proposed for prediction of crack widths. Where necessary the relationship, \( M = f_s A_b j d = \frac{f_s}{4} \frac{\Pi b j d^*}{s_m} \), was used by the author to reduce all equations to common terms.

\( *_{b/s_m} = \text{number of bars in width, } b. \)
Comparison of the ten expressions for average steel stress given in Table 7-6 reveals that there are fundamental differences in the variables which appear in the reduction factor applied to the nominal steel stress, $f_s$. Of course, it must be remembered that expressions 2 through 10 are intended for application to bars of uniform surface conditions, either plain or deformed, where the crack spacing, $x$, varies with the steel area, $A_s$, bar diameter, $\phi$, and concrete tensile strength, $f_t$. Hence, many of these expressions do not contain the variables $x$ and $\phi$.

If, in expressions 3, 6, 7 and 8, $x/\phi$ is considered to be constant (i.e., $x \propto \phi$), then these expressions become very similar to expression 1 developed herein. This similarity is an important confirmation of the basis for development of equation 7-10 because expressions 6 and 7 were derived using a rational approach (50, 51) that was somewhat different than the approach used for the derivation of equation 7-10 (basis for expression 1) in Appendix J. Expression 3, based on a specific concrete strength, was developed empirically from analysis of test data on rather deep and very lightly reinforced beam sections which are not very comparable to typical pipe. This expression, however, furnishes additional confirmation of the basic theory for equation 7-10. Expression 8 was also developed empirically and would be comparable to expression 1 if $\frac{xf_t}{\phi}$ is a constant.

Expression 4 has developed from a theoretical interpretation of the data given in reference 49. All this data related to tension
tests on cylinders with a constant ratio of concentric reinforcement to concrete area. Hence, it appears that the important and well substantiated influence of steel quantity on the amount of reduction in steel stress between cracks (41) has not been recognized in this expression. Expression 2 takes into account the amount of reinforcing steel with the term \((1 + \frac{3e}{e_b})\). This term varies only a small amount with large changes in steel content (for typical pipe sections). Hence, expression 2 also does not recognize sufficiently the influence of variations in steel content in typical pipe sections. Likewise, expression 9 does not consider the important influence of steel ratio, except with the term \(r'\), which does not vary at all in typical pipe sections. If \(\frac{A_s}{bd} = \frac{\pi \phi^2}{4s_m d}\) was added to the denominator of the second term in expressions 2, 4, and 9 to allow for the influence of steel content (with suitable revision of constants) these three expressions would be very similar to expression 1 with different constants \(\left[\text{neglecting the terms (}1 + \frac{3e}{e_b}\right] \text{ and } (1 + \frac{nA_s}{2b(n-d)}), \text{ which are of minor importance}\)."

"Expression 10 neglects any reduction in \(f_s\). Efsen and Krenchel (40) feel that possible reductions in \(f_s\) due to concrete tension between cracks are not worth taking into account in practical analysis of crack widths because this reduction is small and masked out by the high variability of crack spacing estimates. This view may be correct for steel quantities over 1% in beam type elements reinforced with deformed bars of the type which were
investigated in reference 40. However, for lightly reinforced pipe
sections with welded wire fabric reinforcing, tests (8) and experi-
ence definitely indicate the important influence of tension in the
concrete between cracks on the reduction of crack widths. This
fact, combined with the low variability of crack spacing in such
structures, permits the use of much higher steel stresses at the
specified cracking strength, than would otherwise be the case.

A comparison is given in Table 7-6 of the actual magnitude
of reduced average stress in the reinforcing steel due to concrete
tension between cracks for the typical pipe described in the foot-
note to the table under a load in the vicinity of the .01 inch
 crack load. The somewhat lower average stress given by expression
1 may be due to the fact that the other expressions were developed,
for the most part, from tests on specimens with larger bar sizes,
larger steel percentages, beam type cross sections (narrow and deep),
and deformed steel with smaller crack spacings. A small crack
spacing may prevent the maximum benefit from concrete tensile
strength between cracks due to slip and low tension force in the
concrete in the region immediately adjacent to the crack location.
Thus, the effect of relatively wide crack spacing between cross
wires in welded wire fabric is somewhat ameliorated by the larger
average reduction in steel stress due to concrete tension between
cracks.

7.8 Cracking Behavior with Reinforcement Other Than Welded Wire Fabric

Types of reinforcement for pipe which do not have such pro-
perly spaced crack initiators as the longitudinal cross wires in welded
wire fabric develop crack spacings in accordance with the qualitative theory of cracking discussed in Section 7.3. Crack spacings are closer with deformed reinforcing than with similar plain reinforcing. Thus, for a given stress level in the reinforcing steel considerably smaller maximum crack widths may be expected with deformed steel, than with similar plain steel.

None of the published test programs on pipe with reinforcing other than welded wire fabric recorded all the data necessary for a rational interpretation of the effect of significant variables on cracking as discussed in Section 7.3. No data was given on crack distribution and spacing which might form the basis of even a tentative quantitative theory for crack load prediction in pipe with reinforcement other than certain types of welded wire fabric. Also, none of the flexural cracking investigations discussed in Section 7.7 furnished sufficient data or conclusions for application to pipe structures. Most of the empirical data in these investigations was obtained from test specimens with very different proportions than typical reinforced concrete pipes.

Thus, a rather extensive test program on the cracking behavior of pipe with each type of reinforcement which is used in the industry should be undertaken to establish a basis for proportioning steel areas to resist .01 inch crack load. The necessary experimental data could probably be obtained from tests on full scale curved beams designed to simulate a section of pipe at the crown or invert between inflection points (9, 10). The tests
should be designed in accordance with the qualitative principles discussed in Section 7.3. Available test results on other types of structural elements (40, 41, 45, 48) should be helpful for the initial design of such a test program on curved beams or full pipe rings.

An important consideration in the evaluation of the crack distribution qualities of a given type of reinforcement is the expected variability of crack spacings at critical sections. It is well established that reinforcing steels without crack initiators (i.e., the longitudinal cross wires in welded wire fabric) show a very high variability of crack spacing (40, 41) and thus are not susceptible to very accurate prediction of maximum crack widths. This indicates that a greater variability of .01 inch cracking capacity of pipe should be expected with such reinforcements, although no experimental results on large numbers of similar specimens were available to check this hypothesis.

Since cracking criteria are usually investigated for working load conditions, rather than ultimate strength conditions, a requirement should also be established that reinforcing steel stresses remain in the elastic range (i.e., below proportional limit stress) under working load conditions, regardless of the maximum width of crack (see Section 7.5). This requirement would probably control the proportioning of reinforcing steel in many pipe reinforced with hot rolled deformed reinforcing steel which produces small crack spacings, but which has a low proportional limit stress.
also. In such cases, the full beneficial effects of good crack distribution at small spacings and widths could not be realized due to the early occurrence of plastic strain in the steel.

7.9 **Field Loading Conditions**

The effect of field loading conditions on the .01 inch crack formation in pipe may be evaluated from the conditions which produce a .01 inch maximum crack width in the 3-edge bearing test. If steel areas in pipe are selected to limit maximum steel stresses at the .01 inch crack load to the elastic range (as suggested in Section 7.5 above), the field load .01 inch crack capacity of a pipe is reached under the same bending moment as existed in the pipe at test 3-edge bearing .01 inch crack capacity. The ratio of field load to test load which produces the same bending moment is easily obtained from a comparison of field and 3-edge bearing elastic moment distributions (see Figs. 3-1 and 3-2).

For Class "C" bedding conditions shown in Fig. 3-2, the ratio of field load to test load or " .01 inch crack load factor" is:

$$\frac{2.80}{1.5} = 1.5$$ (see Fig. 3-1b and 3-2c). Thus, a Class C bedded pipe may be expected to develop a .01 inch maximum crack width under a total load of 1.5 times the 3-edge bearing test .01 inch crack load.

It is evident from a comparison of load factors for ultimate strength given in articles 4.7 and 5.9 with the above load factor that, where .01 inch crack strength controls the design of a pipe, the load factor for field loading conditions is somewhat lower than in the case of designs controlled by either flexural or diagonal tension ultimate strength.
8
DEFLECTIONS

8.1 Introduction

The limitation of deflection at working load is generally not a criteria in the design of relatively rigid precast concrete pipe to meet test or field load conditions. The deflection of such pipe is usually too small to greatly influence the distribution of earth load around the pipe, although undoubtedly there must be some change in the earth load distribution on the pipe after it undergoes deformation. In extremely flexible types of pipe, of course, the reduction in earth load which accompanies large deformations of the pipe is of major significance for the design of the pipe.

Nevertheless, with rigid reinforced concrete pipe it is still desirable to develop a capability for predicting deformations in order to achieve a better understanding of its structural behavior. Moreover, it will be shown later that the deflection of pipe is related to its cracking behavior. An evaluation of deflection at test .01 crack load can be used to further substantiate the theory developed in Chapter 7 for predicting cracking behavior.

8.2 Effect of Cracking Behavior on Flexural Rigidity

with Welded Wire Fabric Reinforcement

The flexural rigidity, EI, of reinforced concrete structural elements in the elastic range is greatly affected by the existence of tension cracks in the concrete. Right at the location of a tensile
crack the flexural rigidity equals $E_c I_T$, where $E_c$ is the modulus of elasticity of the concrete and $I_T$ is the moment of inertia of the cracked transformed section (compression concrete only and steel area transformed to equivalent concrete modulus of elasticity). In between cracks, however, the concrete carries some tension force. This action results in an increase in flexural rigidity between cracks. The interaction between steel and concrete which determines the magnitude of this increase in flexural rigidity is rather complex, involving slip between steel and concrete. However, this interaction was evaluated in Articles 7.3, 7.4, 7.5 and 7.7 for the case of welded wire fabric reinforcement and the conditions which produce the .01 inch crack.

The average flexural rigidity in a cracked region of a reinforced concrete flexural element may be estimated from the average steel stress in this region. Ten different expressions for average steel stress were summarized in Table 7-6, and discussed in Article 7.7. Expression 1 in Table 7-6, which was developed by the author for application to concrete pipe reinforced with welded wire fabric (within the limitations established in Chapter 7), provides a basis for estimating the flexural rigidity of cracked concrete sections.

The "effective moment of inertia", $I_{cr}$, of a cracked concrete region may be evaluated from a comparison of the average steel stress, $f_{sr}$, given by expression 1 in Table 7-6 with the elastic beam theory. Thus:
\[ f_{sr} = \frac{nMc}{I_{cr}} \]  

where \( f_{sr} \) is given by expression 1, Table 7-6, and

\[ M = f_s A_s j \]  

\[ c_s = (1-k_r)d \]  

see Fig. 8-1

Substitution of the above quantities in equation 8-1 gives the following expressions for average stiffness in the cracked regions of a pipe under .01 inch crack conditions at invert and crown (see Appendix L for complete derivation):

\[ (E I_{cr}) \cdot .01 = \frac{(1-k_r) j E_s A_s d^2}{B \cdot .01} \]  

\[ B \cdot .01 = 1 - .33 \frac{f'_c}{\phi} \sqrt{f'_c (\rho L \cdot .01)} \]  

Equation 8-2 will give the flexural rigidity of a cracked concrete section with all tension taken by the steel if \( B = 1 \) and \( k_r = k \). In this case, the internal distribution of stress is shown in Fig. 8-1 (a). Fig. 8-1 (b) shows the assumed average distribution of stress in cracked regions when the effect of concrete tension between cracks is considered. This is not a true distribution of stress at any section, but rather a representation of average effects over the entire cracked region.
As indicated in Fig. 8-1 (b), \( k_r d \) is the depth of concrete compression zone under the modified average cracked condition which takes into account tension in the concrete between cracks. \( k_r \) in equation 8-2 may be found from the following relations (see Appendix L for derivation):

\[
k_r = \sqrt{(p_r n)^2 + 2p_r n} - p_r n \quad \text{8-4}
\]

where

\[
p_r = \frac{A_{s1}}{bd} \cdot \frac{1}{B.01} \quad \text{8-5}
\]

\[
n = \frac{E_c}{E_s} ; \text{ From (62): } E_c = 1,800,000 + 460 f'_c
\]

The above relations have neglected the rather small influence of the second cage acting as compression steel in pipes reinforced with two circular cages. In typical pipe structures, the reinforcing cage which lies on the compression side is located close to the neutral axis and, hence, has very little influence on the short time flexural properties of the section. This compression reinforcing probably has a greater influence under long time loads, because load is transferred to it while the compression concrete is undergoing plastic flow. In this action, the compression steel causes reduction of the increase in deflection with time due to plastic flow.

Equation 8-2 should only be used for evaluation of the deformation of pipe which conform to all the limitations previously established in the development of relations for estimating .01 inch crack load capacity of pipe.
INTERNAL DISTRIBUTION OF STRESS IN A CRACKED CONCRETE

SECTION BETWEEN CRACKS
(b)

SECTION AT A CRACK
(a)

FIG. 8-1
The same general concepts about the contribution of tension between cracks to the average flexural rigidity of cracked reinforced concrete flexural elements that were used to develop equation 8-2, also were used in reference 50 to develop a general theory for short time deflection of reinforced concrete elements. Equation A8 of Method B given in the above reference is similar to equation 8-2 above, although a correction factor somewhat different than the one given by equation 8-3 above is used in equation A8. Also, no provision is made in reference 50 for correcting the location of neutral axis to allow for tension in the concrete between cracks. Equation A8 tacitly implies that kd, the depth of compression zone, is the same as at a fully cracked section. This assumption may be all right for beams with fairly high percentages of hot rolled steel, but is probably not acceptable for relatively lightly reinforced sections where the average tension force in the concrete may be an appreciable portion of the entire tensile force in the section.

The comparative analysis of ten different expressions for reduced average steel stress, taking into account tension between cracks, given in Table 7-6, also applies to the correction factor, B, in equation 8-3. It is seen that the correction factor developed in reference 50 is very similar to the one developed herein (with different constants) though based on a somewhat different theoretical development than the one used herein in Chapter 7. The other expressions in Table 7-6 also corroborate the variables deemed significant in equation 8-3 for the correction factor for effect of concrete tension between cracks.
8.3 Deflection Under .01 Inch Crack Load for Pipe

Reinforced with Welded Wire Fabric

The elastic methods of stress analysis employed in Chapter 3 to determine the distribution of moments, shears and axial forces in pipe rings under various loading conditions may also be used to calculate deflection of pipe in the elastic range. It was shown in Chapter 3 that the location and extent of cracked regions may influence the moment distribution rather substantially (see Fig. 3-1). Location and extent of cracked regions must also be taken into account for accurate estimates of deflection. For working load conditions on pipe structures (.01 inch crack load), much of the pipe is cracked. Only the regions of low moment approximately midway between crown or invert and springing remain uncracked. Since the moments are low and the flexural rigidity is high in this region, the contribution which this part of the pipe makes to the total deflection will be small. Hence, one assumed variation of moment of inertia, I, (see Fig. 3-1) will suffice as a basis for deflection estimates for all pipe designs, if a specific evaluation of average cracked section flexural rigidity is made for each specific design using equation 8-2.

A comparison of typical cracked section and uncracked section flexural rigidities for pipe structures indicates that the uncracked flexural rigidity equals about 3.3 times the cracked section flexural rigidity at the crown and invert sections. The cracked section
flexural rigidity at springings may be estimated as approximately equal to the crown and invert flexural rigidity. Even though tension steel area is 25% smaller at the springings than at the crown, the lower steel stress and smaller crack widths at the springings indicate that the tension force which exists in the concrete between cracks probably contributes more to stiffness at the springings than to stiffness at the crown. For this reason, it is recommended that in the vicinity of the .01 inch crack load, crown and invert cracked section flexural rigidity, as estimated using equation 8-2, be used for all cracked sections in pipe with two circular cages. For pipe with a single elliptical cage, the average flexural rigidity would be reduced since the depth to steel varies in the cracked regions. No test results were available to permit evaluation of flexural rigidities of such pipe.

Pipe test results reported in reference 8 indicated that at the .01 inch crack load condition in the 3-edge bearing tests, cracked regions extended over arc lengths of about 30° at crown and invert and 50° at each springing (see Fig. 3-1c)

Equation 3-10 is re-stated below in terms of standard parameters for load on pipe:

\[
\left( \frac{\sigma_v}{E} \right)_{0.01} = 0.126 \frac{P}{E \cdot I_{cr}} \frac{3}{v_{av}} = \frac{34.4}{E \cdot I_{cr}} \frac{(DL)}{0.01} \frac{D^4}{D_i^4} \quad 8-6
\]

\[
\left( \frac{\sigma_h}{E} \right)_{0.01} = 0.177 \frac{P}{E \cdot I_{cr}} \frac{3}{v_{av}} = \frac{32.0}{E \cdot I_{cr}} \frac{(DL)}{0.01} \frac{D^4}{D_i^4} \quad 8-7
\]

\[
\frac{\sigma_v}{\sigma_h} = 1.08 \quad 8-8
\]
For loads in the vicinity of the .01 inch crack load, 

\[ E_c I_{cr} \]

is given by equation 8-2.

If the entire pipe ring was assumed to be uniformly cracked with a constant flexural rigidity, \( E_c I_{cr} \), the vertical and horizontal deflections would be increased by about 18% each.

Relations similar to equations 8-6 and 8-7 with different coefficients could be developed for field load conditions which produce the .01 inch crack if estimates of deformation of pipe under field conditions are needed. The order of magnitude of field load deflections would be about the same as test load deflections.

Empirical relations developed in reference 50 could be used for estimating long time deflections from the short time values given above. According to Fig. 3 in this reference, the long time deflection of pipe with double circular cages would be about twice the instantaneous deflection. For pipe with only a single elliptical cage the expected long time deflection would be more than 3 times the instantaneous value.

8.4 Analysis of Test Data

The visual observations of crack patterns reported in Appendix E of reference 8 were used to establish the extent of cracked region assumed in the development of equations 8-6 and 8-7.

Measurements of vertical and horizontal deflection at test .01 inch crack load for 24 of the 39 test pipe reported in reference 8 are summarized in Table 8-1. The measured vertical deflections are then compared with the theoretical predication of vertical deflection
using equations 8-2 and 8-6. Table 8-1 does not include any of the 10 tests on 48 inch diameter pipe because crack spacings on this size pipe were irregular, and did not conform to the limitations set up for use of equation 8-2. Tests H1 through H4 likewise are not included because they had 16-inch spacing of longitudinal cross wires, or rusty reinforcing and other irregularities which did not conform to the limitations of equation 8-2. Test Q1 was left out because of inconsistencies between vertical and horizontal measurements. Some tests were included which did not conform to the limitations set up for applicability of equation 8-2, by using the observed crack spacing, rather than the spacing of longitudinal cross wires in the fabric, to calculate the term, \( P_{ct} \), in equation 8-2. In all cases, an average value of the depth to steel, \( d \), was used rather than the measured value at crown or invert.

It is seen from the summary comparison given in Table 8-1 that the method suggested above gives fairly accurate predictions of pipe deflections at the .01 inch crack load. Due to the inherent variability of a heterogeneous material like cracked reinforced concrete, it is doubtful that more refinement in the estimate of effect of variation in moment of inertia around the pipe with various proportions of steel reinforcement would be justified. In fact, the assumption concerning the extent of cracked region also depends on a number of other factors such as the load level for which deflection predictions are being made, and the tensile properties of the concrete. The variability indicated in Table 8-1 for comparisons of measured and
### TABLE 8-1

**ANALYSIS OF DEFLECTIONS AT MEASURED .01 INCH CRACK LOAD**

<table>
<thead>
<tr>
<th>Test</th>
<th>Inside Diameter Pipe inches</th>
<th>$\delta_v$ .01 test</th>
<th>$\delta_v$ .01 test calc.</th>
<th>$\delta_v$ .01 test calc.</th>
<th>$\delta_h$ .01 test</th>
<th>$\delta_h$ .01 test</th>
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<td>A-1</td>
<td>72</td>
<td>.099</td>
<td>.113</td>
<td>.88</td>
<td>.076</td>
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<td>A-2</td>
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<td>.153</td>
<td>.147</td>
<td>1.04</td>
<td>.155</td>
<td>.99</td>
</tr>
<tr>
<td>A-3</td>
<td>72</td>
<td>.178</td>
<td>.153</td>
<td>1.09</td>
<td>.164</td>
<td>1.09</td>
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<tr>
<td>B-1</td>
<td>72</td>
<td>.082</td>
<td>.091</td>
<td>.90</td>
<td>.068</td>
<td>1.20</td>
</tr>
<tr>
<td>B-2</td>
<td>72</td>
<td>.137</td>
<td>.135</td>
<td>1.01</td>
<td>.112</td>
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</tr>
<tr>
<td>B-3</td>
<td>72</td>
<td>.174</td>
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<td>1.06</td>
<td>.152</td>
<td>1.14</td>
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<td>B-4</td>
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<td>.265</td>
<td>.233</td>
<td>1.14</td>
<td>.180</td>
<td>1.14</td>
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<td>C-1</td>
<td>72</td>
<td>.264</td>
<td>.198</td>
<td>1.33</td>
<td>.218</td>
<td>1.21</td>
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<tr>
<td>C-2</td>
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<td>.111</td>
<td>.151</td>
<td>.74</td>
<td>.090</td>
<td>1.23</td>
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<td>.167</td>
<td>1.32</td>
<td>.190</td>
<td>1.16</td>
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<td>.170</td>
<td>.161</td>
<td>1.05</td>
<td>.200</td>
<td>.85</td>
</tr>
<tr>
<td>F-1</td>
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<td>.188</td>
<td>.222**</td>
<td>.85</td>
<td>.170</td>
<td>1.10</td>
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<tr>
<td>F-2</td>
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<td>.205**</td>
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<td>.95</td>
<td>.205</td>
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<td>.368</td>
<td>.96</td>
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<td>.510</td>
<td>.440***</td>
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<td>.500</td>
<td>1.02</td>
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<td>.422</td>
<td>.347***</td>
<td>1.21</td>
<td>.460</td>
<td>1.21</td>
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<tr>
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<td>.348</td>
<td>.384</td>
<td>.91</td>
<td>.327</td>
<td>1.07</td>
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<tr>
<td>Q-2</td>
<td>108*</td>
<td>.426</td>
<td>.415***</td>
<td>1.02</td>
<td>.395</td>
<td>1.08</td>
</tr>
<tr>
<td>R-1</td>
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<td>.540</td>
<td>.460***</td>
<td>1.15</td>
<td>.508</td>
<td>1.06</td>
</tr>
<tr>
<td>R-2</td>
<td>108*</td>
<td>.460</td>
<td>.460***</td>
<td>1.15</td>
<td>.508</td>
<td>1.06</td>
</tr>
</tbody>
</table>

**Average**: 1.016 1.136

**Standard Deviation**: .150 .121

**Coefficient of Variation**: 14.8% 10.7%

**Number of Tests**: 24 23

**Notes:**
- #Wall A Pipe
- **Allowance included for doubled steel area at crown due to splice.
- ***Double wrap cage - 6 inch average crack spacing assumed.
calculated deflection is about the same magnitude as the variability shown in Table 3 of reference 50 for 90 tests on straight beams.

The ratios of measured vertical to horizontal deflection in Table 8-1 should be compared to the theoretical value of 1.08 for the particular variation of moment of inertia around the pipe which was assumed in the theoretical development of equations 8-6 and 8-7. The average value of $\frac{\sigma_v}{\sigma_h}$ of 1.136 is somewhat higher than 1.08 although the sizable variation of this test ratio shows that the average test $\frac{\sigma_v}{\sigma_h}$ does not differ significantly from the theoretical value of 1.08.
BEHAVIOR UNDER COMBINED EXTERNAL LOADING
AND INTERNAL HYDROSTATIC PRESSURE

9.1 Introduction

In American practice, ordinary reinforced concrete pipelines without prestressing or continuous steel cylinders may be designed for hydrostatic pressure conditions up to a maximum head of 100 feet (21). The existence of hydrostatic pressure inside the pipe causes a uniform circumferential tension force in the wall of the pipe plus small bending moments due to the variation of pressure over the height of the pipe (see Fig. 3-2). These effects reduce the ultimate strength and .01 inch cracking strength of the pipe under the combined action of hydrostatic pressure and applied external loads.

A complete basis for predicting behavior and for development of a rational design procedure for reinforced concrete low head pressure pipe requires a study of the requirements for watertight performance in service for such pipe. Because of the complexity of this problem, such a study would require careful correlation with pipe tests under internal hydrostatic pressure alone and under combined external loading and internal hydrostatic pressure. Since no such test results were available, the requirements for watertight performance of low head pressure pipe were not considered in this thesis. Obviously, no cracks extending entirely through the wall could be tolerated under service conditions. However, it is probable
that a certain amount of flexural cracking could exist, without permitting leakage under pressures up to 100 feet of head. No information was found in the literature which considered the question of a suitable criteria for watertight performance of reinforced concrete pressure pipe under the combined action of external load and internal pressure. A procedure for evaluating the resistance of a reinforced concrete pipe to initial cracking, due both to axial tension and to flexure, is given in reference 4, but no correlation with structural or permeability test results is included.

Methods are presented in this chapter for evaluating the ultimate strength and .01 inch cracking strength of pipe under the combined action of external load and internal pressure. It remains for future research to determine the limiting requirements for watertighteners under these conditions.

9.2 Strength Under Uniform Internal Hydrostatic Pressure

The static forces due to uniform internal pressure in a thin walled cylinder are well known. The total circumferential tension in the wall of the cylinder due to a pressure head, \( H \), is:

\[
N = \frac{\sigma H D}{2} \quad 9-1
\]

Reinforced concrete pipes of the usual dimensions (20,21) may be considered as thin-walled cylinders.

The ultimate strength of a reinforced concrete pipe under hydrostatic pressure alone depends on the strength of the circum-
ferential steel. The maximum pressure head required to produce
tension failure of a pipe with two circular cages (outer cages area = 
.75 x inner cage area as specified in ASTM C76) is (see Appendix M
for derivation):

\[ H_{um} = \frac{3.5 A_s f_s'}{\gamma D_1} - \frac{2.9 W}{\gamma D_1} - 3.7 D_1 \]  

where \( \gamma \), \( D_1 \) and \( H_{um} \) are in foot units and \( A_s \) and \( f_s' \) are
in inch units.

The second and third terms in equation 9-2 represent the
approximate reduction of circumferential tension strength of the
pipe due to bending from the weight of pipe and from pressure
variation over the depth of pipe, assuming a single concentrated
support at the invert.

While the pressure required to cause a .01 inch crack
without external load is of no interest by itself, an evaluation of
this pressure becomes useful later in the analysis of combined
external load and pressure effects. Again for pipe with two circular
cages having relative areas as given by C76, the maximum pressure
head which causes .01 inch crack is (see Appendix N for derivation):

\[ H_{.01m} = \frac{8.1 \cdot 10^5 \cdot A_s}{\gamma D_1} + \frac{2.14 \cdot h \cdot f_c^{'1/2}}{\gamma \phi D_1} - \frac{5.2 W}{\gamma D_1} - 4.4 D_1 \]  

The pressure head required to cause initial cracking of the
concrete wall of the pipe when no external load is acting may be
approximately determined on the basis of assumed elastic action
between steel and concrete and uniform distribution of elastic stresses,
as follows:

\[ H_{cm} = \frac{2 \left( F_t - f_{ts} - f_b \right)}{\sqrt{D_1}} \left[ bh + 1.75 \left( n-1 \right) A_{sl} \right] \]

\[ 9-4 \]

\( F_t \) is the tensile strength of the concrete.

\( f_{ts} \) is the tension stress in the pipe wall due to concrete shrinkage. It may be estimated by the methods given in article 3.5. \( f_b \) is the flexural tension stress in the uncracked wall due to bending moments from the weight of the pipe and from the effect of pressure head variation over the depth of the pipe.

See reference 4 for a more extensive discussion of initial cracking of reinforced concrete pipe.

9.3 Flexural Ultimate Strength Under Combined External Load and Internal Pressure

A theoretical method for modifying the 3-edge bearing flexural ultimate strength for the effect of hydrostatic pressure is developed in Appendix M, with the aid of the following assumptions:

1. Axial force due to hydrostatic pressure acts at the centerline of the pipe wall.

2. The distance, \( \frac{h}{2} - \frac{a}{2} \), between resultant axial tension force and resultant compression force in the concrete equals approximately \( d/2 \) (see Fig. 9-1).

3. \( d = D_1/12 \) (i.e., \( d \) in inches = \( D_1 \) in feet - almost correct for wall B pipe).
4. The change in depth of effective stress block, 'a', due to the addition of the axial tension force may be neglected. Since typical pipes are substantially "under-reinforced", 'a' is normally a small quantity in the order of 0.2 d or less. Even a large reduction in 'a' would cause only a small increase in the moment arm. It is conservative to neglect the beneficial effect of this small increase in moment arm.

5. The effect of changes in the geometry of the pipe because of elastic and inelastic deformations is neglected. An externally loaded pipe undergoes inward vertical displacement at the crown and outward horizontal displacement at the springings, which change its shape from circular to an approximate elliptical shape. The outwardly directed hydrostatic pressure inside the pipe produces moments in the wall of the deflected pipe which have opposite signs to the moments due to external load (30). Hence, there is non-linear interaction between external load and internal pressure because of changes in the geometry of the structure due to external load.

The order of magnitude of the deflections at ultimate load in typical precast concrete pipe of several different sizes under 3-edge bearing load only may be obtained from the deflection curves given in Appendix D, reference 8. Since external failure loads will be lower when internal pressure is also present, deflections will be lower than those in
the above curves under combined loading, more or less in proportion to the magnitude of the internal pressure.

An approximate estimate of the moments which result in the deflected pipe from the action of a uniform internal pressure may be obtained from an investigation of moments in an elliptical pipe under uniform pressure (30 - p.166). Such an estimate for several typical pipe under different internal pressures indicates that in a flexural failure, the 3-edge bearing capacity would be increased by an amount which varies between zero and approximately 5% of the 3-edge strength at zero pressure. The actual increase depends on the relative 3-edge bearing strength and the magnitude of internal pressure which acts. This increase is not considered significant in relatively stiff precast pipe for the 0 to 100 foot range of pressure heads permitted in the ASTM specifications for non-prestressed low head pressure pipe. It may be of some significance for the strength of pipe with very high internal pressures and low external loads (lower right portion of Fig. 10-4). It has been conservatively neglected in the developments which follow.

With the use of the above assumptions, the effective flexural ultimate strength in 3-edge bearing modified for the simultaneous action of hydrostatic pressure is (see Appendix M for derivation):

\[(DL)_{ue} = (DL)_{um} - 0.174 \gamma H_o - 0.64 \gamma D_1\]
(DL)_{um} is the maximum D-load capacity with H_o = 0 and may be obtained from equation 4-16. The term, .174 \gamma H_o , represents the portion of steel strength required to resist axial tension due to hydrostatic pressure, and thus, unavailable for moment resistance. For wall A and wall C pipe, this term should be corrected by multiplying it by the factor, \frac{d}{D_1} , (d in inches, D_1 in feet). The term, .54 \gamma D_1 , is the approximate 3-edge bearing D-load equivalent of the bending effect of pressure variation over the height of the pipe.

The order of magnitude of reduction in flexural ultimate 3-edge bearing D-load capacity due to the maximum hydrostatic pressure condition permitted in ASTM C361-59T is:

\begin{align*}
H_o &= 100 \text{ feet max:} \quad .174 \times 62.5 \times 100 = 1090 \text{ D} \\
D_1 &= 8 \text{ feet max:} \quad .64 \times 62.5 \times 8 \quad = 320 \text{ D} \\
\text{Total reduction in normal ultimate} & \quad \text{D-load capacity:} \\
& \quad 1410 \text{ D}
\end{align*}

Another approach that could be used to evaluate the combined effects of 3-edge bearing load and hydrostatic pressure is to develop a theoretical or a semi-empirical "interaction" formula, relating the actual combined load capacity to the flexural ultimate strength under 3-edge bearing without hydrostatic pressure and to the ultimate tension strength under hydrostatic pressure without external loading. This approach is widely used in structural specifications for evaluating combined bending and axial force in columns and other situations involving the combination of different kinds of structural action. The use of a semi-empirical interaction formula has been suggested for
reinforced concrete sewer pipe, (*) and also for prestressed concrete pressure pipe (61). The simplest form of an empirical interaction equation, similar to the approach used in many specifications for the design of members subject to axial load and bending, could be constituted as follows:

\[
\frac{(DL)_{ue}}{(DL)_{um}} + \frac{H_o}{H_{um}} = 1
\]

9-6

where \((DL)_{um}\) is obtained from equation 4-16 and \(H_{um}\) is obtained from equation 9-2.

Equation 9-6 has no basis without the opportunity for comparison with test results. However, equation 9-5 may be recast in a slightly different form (see Appendix M) to obtain the following theoretical interaction formula:

\[
\frac{(DL)_{ue}}{(DL)_{um}} + \frac{H_o}{H_{um}} \left( 1 + 3.7 \frac{D_i}{H_{um}} \right) = 1
\]

9-7

Equation 9-2 for \(H_{um}\) was derived from a direct consideration of the ultimate circumferential tension strength of the pipe (plus approximate allowance for minor bending effects from weight and fluid in depth of pipe). \(H_{um}\) can also be obtained from equation 9-5 with \((DL)_{ue} = 0\). In Appendix M, it is shown that the results of this derivation differ slightly from equation 9-2. This difference occurs because of a slight discrepancy in the assumed location of the line of action of the axial tension force between the two derivations. Equation

(*) Private communication from N.W. Clarke, Eldg. Res. Station, Garston, Eng.
9-2, which is based upon true ultimate strength considerations for
direct tension in the wall of the pipe, should be used for \( H_{um} \).

Fig. 10-4 shows how equation 9-5 may be approximately corrected in
the vicinity of \( H_{um} \). In effect, this correction recognizes the
fact that assumption 4 above is not too good for high pressures and
low moments. However, such combinations of high pressure and low
moments are of no practical interest, since pressure heads in
ordinary reinforced concrete pipe are limited to 100 feet maximum.

Equation 9-7 (or 9-5), rather than 9-6, is recommended
for use in design of pipe subject to combined 3-edge bearing load
and internal pressure. Fig. 10-4 shows an interaction plot of
\((DL)_{ue} vs H_o\) for two typical pipe designs.

The field capacity of a pipe under combined hydrostatic
pressure and earth loading may be estimated as follows:

\[
(P_f)_{u} = (LF)_{u} \cdot (DL)_{ue} \cdot D_i \tag{9-8}
\]

The load factor, \((LF)_{u}\), must be developed in a similar
manner to the procedure given in article 4.7 for pipe without
internal pressure. It probably would have approximately the same value
as the load factor for external load only (given in article 4.7 for
"Class C" bedding).

It should be noted that the existence of hydrostatic pressure
up to 100 feet of head may produce a significant increase in tension
steel stress with a consequent increase the likelihood that the mode
of ultimate strength failure will be in flexure. It will be shown
in the next article that the circumferential tension due to hydrostatic pressure produces almost no change in the resistance of a pipe to diagonal tension failure. Fig. 10-4 clearly shows how the addition of internal pressure may change the mode of ultimate failure from diagonal tension to flexure.

9.4 Diagonal Tension Ultimate Strength Under Combined External Load and Internal Pressure

The ultimate diagonal tension strength of a pipe under the combined action of 3-edge bearing load and internal pressure may be based on modification of the semi-empirical analysis for diagonal tension strength of straight beams subject to combined shear, axial force and bending moment given in reference 39. The ultimate diagonal tension strength of these straight beams is given by equation 5-2. It is evident from this equation that the addition of axial tension reduces the ultimate diagonal tension strength of straight beams. In order to adapt the straight beam equations for application to curved pipe rings, they must be modified for two significant effects of curvature:

1. Additional shear stress results from a change in axial force (for 3-edge bearing loading, see Fig. 5-1). The addition of a uniform circumferential tension force in the pipe wall due to internal pressure causes no change in the internal shear stress in the pipe wall.
PLASTIC FAILURE CONDITIONS IN THE PIPE WALL UNDER COMBINED MOMENT AND AXIAL TENSION

FIG. 9-1

RADIAL TENSION STRESS IN PIPE WALL UNDER COMBINED THREE-EDGE BEARING LOAD AND INTERNAL PRESSURE

FIG. 9-2
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FIG. 9-1

RADIAL TENSION STRESS IN PIPE WALL UNDER COMBINED THREE-EDGE BEARING LOAD AND INTERNAL PRESSURE

FIG. 9-2
2. Radial tension stresses occur in the pipe wall due to the curvature of normal force in the pipe. Fig. 9-2 and Appendix M show that radial tension stress in the pipe wall due to the combined action of 3-edge bearing loading and internal pressure is less than radial tension due to 3-edge bearing stress alone.

Thus, the addition of uniform internal pressure to the external loads on a pipe results in two opposing effects on the diagonal tension ultimate strength of the pipe:

1. A decrease in diagonal tension strength due to higher circumferential tension and greater cracking (similar to effects in straight beams).

2. An increase in diagonal tension strength due to a reduction in radial tension stress in the pipe wall. This reduction in radial tension stress amounts to about one-half the internal pressure in the pipe.

The following equation for effective D-load diagonal tension ultimate strength under combined 3-edge bearing load and internal pressure was developed from the above considerations (see Appendix M for derivation). Note that the constants given in this equation have been modified in conformance with the statistical analysis of test results discussed in article 5.6.

\[ (DL)_{ue} = \frac{38.5 d \sqrt{f_c}}{\lambda_1 D_1} + \frac{7880 d A_s^{1/2}}{\lambda_1 \lambda_2 D_1^2} - \frac{.9W}{D_1} \]  

9-9
where \( \lambda_1 = 1 - \frac{0.03 \gamma H_0}{(DL)_{ue}} \)  
\( \lambda_2 = 1 + \frac{1.85 \gamma H_0}{(DL)_{ue}} \)

The solution of equation 9-9 may be easily obtained by a trial and error procedure. Usually a first trial, based on \( \lambda_1 = \lambda_2 = 1 \), is all that is needed to obtain estimates of \( \lambda_1 \) and \( \lambda_2 \) for a rapid solution of equation 9-9.

The effective D-load capacity under combined load may be compared with the normal D-load capacity in 3-edge bearing alone by recasting equation 9-9 in the following form:

\[
(DL)_{ue} = \frac{1}{\lambda_1} (DL)_{um} - \frac{1}{\lambda_1} (1 - \frac{1}{\lambda_2}) \frac{7880 d A_{sl}}{D_1^2} + (1 - \frac{1}{\lambda_1}) \frac{SW}{D_1} \]

Note that \( \frac{1}{\lambda_1} \) is always slightly greater than 1, \( (1 - \frac{1}{\lambda_2}) \) is always between zero and 1 (usually closer to zero) and \( (1 - \frac{1}{\lambda_1}) \) is slightly greater than 1.

The investigation of the effect of hydrostatic pressure on the 3-edge bearing ultimate diagonal tension strength of some typical pipe designs, using the above equations, indicates that for pressure heads up to 100 feet, the addition of internal pressure has almost no effect on the ultimate diagonal tension strength of the pipe. See Fig. 10-4 for a plot of the interaction relationship between ultimate 3-edge bearing strength both in flexure and in diagonal tension and the applied internal pressure head on the pipe for two typical pipe designs.
The ultimate diagonal tension strength of pipe under field loading conditions may be obtained with equation 9-8, if a suitable load factor for diagonal tension failure is developed. A similar argument to the one developed for combined 3-edge bearing and internal pressure in flexural failures would show that the addition of internal pressure probably would also have little effect on the field load ultimate diagonal tension strength. Under these combined loading conditions, the radial stresses in the wall of the pipe would be reduced. These stresses would probably be changed from tension to a small compression in the region of critical shear stress. Thus, the failure section would probably be moved a few degrees further from the crown and invert. At the same time, the addition of tensile stress in the steel would tend to increase the extent of cracked regions in the pipe and, thus, to reduce diagonal tension strength. It is not known which of the above effects might predominate, since no reliable experimental data is even available on diagonal tension failure under distributed loading without internal pressure. However, it is probable that the addition of hydrostatic pressure, up to a maximum head of 100 feet, will not have a significant effect on the ultimate diagonal tension strength of pipe in the field.

9.5 .01 Inch Cracking Strength Under Combined External Load and Internal Pressure

The effective .01 inch cracking strength of pipe under the combined action of external 3-edge bearing load and internal hydrostatic pressure may be obtained by correcting the .01 inch D-load
capacity given by equation 7-10 for the effect of hydrostatic pressure. This correction is developed in Appendix N with the aid of the following assumptions:

1. Axial force due to hydrostatic pressure acts at the centerline of the pipe wall.

2. The distance, \( \frac{h}{2} - \frac{kd}{3} \), between resultant axial tension force and resultant compression force in the concrete, approximately equals \( d/2 \).

3. \( d = D_1 / 12 \).

4. The change in the arm of resisting moment, \( jd \), due to the addition of the axial tension force may be neglected.

5. The effect of changes in the geometry of the pipe due to deformation under external load is neglected. See the discussion under assumption 5 in article 9.3. Deflections associated with .01 inch crack load are much less than deflections at ultimate load; hence, a theory based on this assumption will not introduce significant reductions in the estimated .01 inch crack strength of pipe under combined external load and internal pressure.

The effective 3-edge bearing capacity corrected for hydrostatic pressure effects is:

\[
(DL)_{01e} = (DL)_{01m} - 0.137 \gamma H_0 - 0.6 \gamma D_1
\]

\( (DL)_{01m} \) may be obtained from equation 7-10 or 7-11. The term, \( 0.137 \gamma H_0 \), represents the effect of circumferential tension on the strain in the tension steel. It should be corrected by the factor \( \frac{d}{D_1} \).
(d in inches, $D_1$ in feet) for wall A and wall C pipe. The term $0.6 \times D_1$ is the approximate 3-edge bearing D-load equivalent of the bending effect of pressure variation over the height of the pipe.

The order of magnitude of the reduction in .01 inch crack 3-edge bearing D-load capacity due to the maximum hydrostatic pressure condition permitted in ASTM C361 - 59T is:

$$H_o = 100 \text{ ft max} \quad 0.137 \times 62.5 \times 100 = 850 \text{ D}$$

$$D_1 = 8 \text{ ft max} \quad 0.6 \times 62.5 \times 8 = 300 \text{ D}$$

Total reduction in normal D-load capacity at .01 inch crack

$$= 1150 \text{ D}$$

Equation 9-13 may be recast to obtain the following interaction equation (see Appendix N for derivation):

$$\frac{(DL) .01e}{(DL) .01m} + \frac{H_o (1 + \frac{4.4 \times D_1}{H_o})}{H_{.01m} (1 + \frac{4.4 \times D_1}{H_{.01m}})} = 1 \quad 9-14$$

where $(DL) .01m$ is obtained from equation 7-10 or 7-11

$H_{.01m}$ is obtained from equation 9-3.

The field load capacity, with respect to .01 inch crack formation, may be estimated from the load capacity under combined 3-edge bearing load and internal pressure as follows:

$$(P_r) .01 = (LF) .01 \times (DL) .01e \times D_1 \quad 9-15$$
The load factor, \( (LF)^{0.01} \), may be obtained by the methods discussed in article 7.9. It probably does not differ significantly from the load factor for comparison of 3-edge bearing load and field load without internal pressure.

9.6 Need For Correlation with Tests

No test results were available to check any of the above theory for the behavior of reinforced concrete pipe under combined 3-edge bearing load and hydrostatic pressure. However, a few test facilities are available in the concrete pipe industry which are capable of making strength tests under the combined action of 3-edge bearing load and hydrostatic pressure (61). It is suggested that the above relations be checked with ultimate strength and .01 inch cracking results from tests on pipe of various designs and sizes under combined loading. Pressure heads in the range of 50 to 100 feet should be used to provide high enough D-load reduction to overshadow random variations in normal 3-edge bearing strength. Results should be correlated with theoretical interaction curves similar to Fig. 10-4 for each pipe design tested.

Test results under combined loading are also badly needed in order to establish a rational basis for designing low head pressure pipe for watertight performance in service under combined external load and internal pressure conditions.
10.1 Ultimate Strength Criteria - C76

The ultimate strength theories given above for predicting the ultimate 3-edge bearing strength of pipe for failure in flexure and failure in diagonal tension were used to analyze the ultimate strength of standard ASTM C76-61T minimum designs for wall B, Class II, III and IV pipe. Two possible types of reinforcing steel were compared in the analysis for flexural failure. One type was welded wire fabric of cold drawn carbon steel with an ultimate strength of 80,000 psi; the other was hot-rolled intermediate grade reinforcing steel with a well defined yield point of 40,000 psi. The results of analyses using equations 4-16 or 4-19 for flexural failure and 5-17 for diagonal tension failure are presented in Fig. 10-1. No allowance has been made in this figure for possible reduction in diagonal tension strength due to tongue and groove end conditions in the pipe. The magnitude of this reduction would depend on the length of pipe sections and the length of tongue and groove. Its order of magnitude would vary between about 3% and 12%.

For any particular pipe size and class, Fig. 10-1 indicates the predicted mode of failure and the ultimate strength. It is evident that for all pipe sizes in Classes II and III, standard pipe with typical welded wire fabric reinforcing steel have predicted ultimate strengths well in excess of the required ultimate D-loads. Moreover,
ULTIMATE STRENGTH ANALYSIS OF STANDARD ASTM C76-61 T PIPE DESIGNS

FIG. 10-1
the predicted mode of failure for all pipe sizes and classes, except class II below about 72 inches in diameter, is by diagonal tension. All designs with welded wire fabric reinforcing meet the specified ultimate strength requirements. However, smaller class IV pipe with specified $f_c' = 4000$ psi are right on the borderline of required ultimate strength. Unless somewhat higher concrete strengths were consistently employed, a substantial number of these pipe would fail to meet the ultimate strength requirements.

It is clear from the figure that with welded wire fabric reinforcing, the required ultimate strength in every class of pipe could be met with very substantial reductions in quantity of reinforcing steel. Moreover, the percentage reductions that would still permit ultimate strength requirements to be met become considerably larger in the pipe of larger diameter.

The figure also indicates that the type of hot rolled steel assumed for purposes of analysis (not the least favorable steel properties permitted by ASTM C76-61T) could not be used to produce standard C76 pipe of acceptable ultimate strength, except for pipe of 96 inch diameter and larger, Classes II and III. It certainly seems probable that in tests where hot rolled steel reinforced pipe with standard steel areas was considered to have met the ultimate strength requirement, the steel had one or more of the following characteristics:

1. Yield strength may have been higher than 40,000 psi.
2. May have been hard grade steel with much less ductility before onset of strain hardening, than with structural
or intermediate grade steel.

3. Area may have been greater than minimum area specified in C76-61T.

10.2 .01 Inch Cracking Criteria - C76

Equation 7-10 was used to predict .01 inch crack load level for standard ASTM C76-61T, wall B, pipe of Class II, III and IV. Only pipe 66 inch in diameter and larger were included and reinforcement was assumed to be welded wire fabric with main circumferential wire at 2 inches o.c. and with longitudinals at 6 inches o.c. or 8 inches o.c., placed in one wrap. Fig. 10-2 shows the results of this analysis and clearly indicates that the specification requirements are conservative to a varying degree for pipe reinforced with the above types of welded wire fabric. The specification is particularly conservative with respect to 102 and 108 inch diameter pipe and does not recognize the beneficial effect of closer spacing of longitudinals, particularly with large diameter and heavily reinforced pipe. However, it should be pointed out again that equation 7-10 is based on the mean value of M.I.T. test data without inclusion of a factor of safety or reliability; however, it was conservative with respect to test results of others. Its validity should be checked by further tests.

Fig. 10-3 shows the computed nominal steel stress, using equation 7-4 for standard pipe designs of the above classes, wall B, at the pipe invert under specified .01 inch crack load. Again, it is evident that larger diameter pipe are required to use disproportionately large amounts of welded wire fabric reinforcing, compared to smaller
.01 INCH CRACKING ANALYSIS OF STANDARD ASTM C76-61T
PIPE DESIGNS FOR PIPES REINFORCED WITH WELDED WIRE FABRIC
FIG. 10-2
INSIDE DIAMETER OF PIPE

COMPARISON OF COMPUTED STRESSES AT SPECIFIED .01 INCH CRACK LOAD FOR STANDARD PIPE DESIGNS WITH COMPUTED STRESS AT MEASURED .01 INCH CRACK LOAD IN VARIOUS PIPE TESTS

FIG. 10-3
diameter pipe. Equations 7-5 and 7-8 indicate that some decrease in nominal steel stress is justified as wire diameter increases; however, the approximately linear drop indicated in Fig. 10-3 for each class of pipe is not in accordance with the theory developed herein. Also, as is shown by equation 7-5, the specification is qualitatively correct to require lower nominal steel stresses at .01 inch crack loads for the more heavily reinforced pipe designed for higher D-loads.

Fig. 10-3 also shows a comparison of nominal steel stresses at specified .01 inch crack loads with nominal steel stresses at measured .01 inch crack load for all the test pipe of both the M.I.T. and "other" test programs. It is seen that stresses in the test pipe at measured .01 inch crack load are substantially above stresses in the standard pipe at specified .01 inch crack load, particularly for the more lightly reinforced pipe.

10.3 Strength Under Combined 3-edge Bearing Load and Internal Pressure - C361

Two typical 72 inch diameter pipe designs specified in ASTM C361-59T were analyzed for combined 3-edge bearing load and internal pressure using the methods developed in Chapter 9. These results are plotted in Fig. 10-4 as an interaction diagram for the combined effects. The shaded region in Fig. 10-4 indicates the region of possible combinations of 3-edge bearing strength and pressure head values which will fall within the predicted limiting .01 inch crack 3-edge bearing strength and the maximum permissible design pressure head of 100 feet.
INTERACTION DIAGRAM FOR TWO TYPICAL ASTM 361 LOW HEAD PRESSURE PIPES UNDER COMBINED THREE EDGE BEARING AND INTERNAL PRESSURE

**FIG. 10 - 4**
Pipe 1 was proportioned in accordance with the standard requirements given in Table I of C361 for use under 100 foot hydrostatic pressure head and nominal external loading. Welded wire fabric reinforcing with longitudinals at 8 inches o.c. and an ultimate strength, $f'_s$, of 80,000 psi was used for this design.

The specification requires that this pipe have a .01 inch cracking strength of 1100D without internal pressure and that no leakage occur under 1.2 times the specified pressure head with no external load at a test pipe age of 28 days. There is no specification requirement for testing under combined loading. Fig. 10-4 shows that the estimated .01 inch crack strength for this pipe for H=0 is 1770D. No theoretical method was available for predicting the pressure at which leakage would first occur. Under a 100 foot head, however, the .01 inch crack 3-edge bearing capacity is still about 850D. With no external load and a 100 foot pressure head, the tension stress in the concrete of an uncracked wall due only to pressure effects is 230 psi. In addition, the estimated tension due to shrinkage effects is 140 psi and the flexural tension due to moments from weight of pipe and water load is about 380 psi.

There is no specification requirement for ultimate strength of this design. Fig. 10-4 shows that the predicted mode of failure is by diagonal tension at a load of about 2700D. There is no change in the predicted ultimate capacity due to addition of internal pressure up to a pressure head of about 185 feet. Beyond this pressure, the ultimate strength is controlled by tension in the reinforcing steel due
to a combination of flexure and axial tension.

Pipe design 2 conforms to C361 requirements for use where appreciable external loading may occur in addition to internal hydrostatic pressure. Under these rather vague conditions, C361 requires the use of an ASTM C76, Class IV, standard pipe design. Hence, pipe design 2 is a C76-61T standard class IV, wall B, pipe with welded wire fabric reinforcing having the same properties given for pipe 1.

The specification requires that this pipe have a .01 inch cracking strength of 2000D without internal pressure and no leakage under 1.2 times the specified pressure head with no external load at 28 day age. There is no specification requirement for testing under combined loading. Fig. 10-4 shows that the estimated .01 inch crack strength of this pipe for H=0 is 2070D. Under a 100 foot head the estimated .01 inch 3-edge bearing capacity is about 1000D. Also under 100 feet head, but no external load, the tensile stress is an uncracked pipe wall is 210 psi. In addition, shrinkage effects may add a maximum of about 120 psi and bending due to weight distribution and variation of water pressure with depth of pipe adds about 300 psi flexural tension.

The predicted ultimate strength mode of failure is by diagonal tension at 3350D, approximately constant for pressure heads up to about 170 feet. Above 170 feet of pressure head, the mode of failure is by excessive tension in the reinforcing steel.

Comparison of the behavior of these two pipe, as plotted in Fig. 10-4, indicates that there is very little difference in the .01
inch cracking capacities of these two designs. Pipe 2 has a somewhat higher cracking D-load capacity, but the difference diminishes under increasing pressure. The difference is not great enough to judge pipe 2 design much superior to pipe 1 for .01 inch crack strength under combined external load and internal pressure. Unfortunately, it is not possible to compare, quantitatively, the estimated resistance to leakage for the two designs under such combined loading. Qualitatively, pipe 2 seems to have somewhat greater resistance to initial cracking than pipe 1 because of its thicker wall and higher minimum concrete strength. Finally, pipe 2 has about a 22% higher ultimate strength than pipe 1 under any pressure head up to 100 feet. All in all, Fig. 10-4 and qualitative reasoning about leakage resistance does not indicate a substantial difference in the ability of these two pipe designs to resist combined external loading and internal pressure.
11
CONCLUSIONS

11.1 General Conclusions

The results of the investigation reported in this thesis provide a new basis for understanding the structural behavior of reinforced concrete pipe. The investigation has provided a much needed link between modern concepts of the structural behavior of straight reinforced concrete elements and empirical knowledge from many years of experience and tests in the pipe industry. It has provided rational explanations and quantitative evaluation procedures for a number of aspects of the structural behavior of pipe which have received only empirical recognition for many years by the pipe industry.

The results of the investigation reported in this thesis provide the following complete or partial solutions to important problems connected with the structural analysis and design of reinforced concrete pipe:

1. A complete procedure is made available for the structural design of precast concrete pipe, 66 inches in diameter and larger, reinforced with welded wire fabric with longitudinal cross wires spaced at 6 inches or 8 inches o.c. This procedure includes consideration of cracking behavior, stiffness, and ultimate strength in both flexure and diagonal tension (see article 11.3 for summary). The procedure is further extended to include the design of
pipe subject to internal fluid pressure, as well as external load. The procedure suggested for analysis or design of pipe under 3-edge bearing load is well substantiated by many available test results.

2. Suggestions are made for extending the above procedure to pipe with other types of reinforcing. However, for some aspects of pipe behavior, new semi-empirical constants will have to be developed for each type of reinforcing used from future tests. In such cases, the present investigation provides methods for designing and interpreting such test programs.

3. A basis is made available for further development of the standard ASTM specifications for reinforced concrete pipe (ASTM C76-61T and C361-59T). The standard tables for minimum required steel areas in these specifications may be tentatively revised and extended for pipe 66 inches in diameter and larger with welded wire fabric reinforcing on the basis of the methods developed herein (see article 11.4 for some tentatively suggested minimum steel areas). More tests on pipe having many different diameters with welded wire fabric and other reinforcing steels, after interpretation in accordance with the rational concepts presented in this investigation, will permit a sounder basis for revision of the specifications. Standard designs, if given at all, must be related to particular properties
of the constituent materials, concrete and reinforcing steel. As an alternate to the use of tables of standard designs, a comprehensive and rational design procedure, such as the one suggested in this thesis for welded wire fabric reinforcing, could be developed for each type of reinforcing steel on the basis of the principles established herein.

4. The methods developed in this thesis for .01 inch crack prediction indicate a definite relationship between crack spacing and crack width. This relationship could be used to establish the measurement of maximum crack spacing at the crown or invert, rather than the direct measurement of crack width, as a very simple criteria for crack control with the standard 3-edge bearing test. The present procedure specified in ASTM C76 for crack measurement by a .01 inch leaf gage is rather crude and depends too much upon the judgment of the testing engineer.

5. Methods are given for special designs of pipe with stirrup reinforcement in order to develop high D-load requirements. Use of rational methods for design of pipe to meet "special conditions" is permitted in the current edition of C76. The use of stirrup reinforcement should be made mandatory by the specification for certain class IV and class V standard pipe.
6. A more accurate design of pipe to resist field loading conditions is facilitated by the development of a rational procedure comparing behavior under 3-edge bearing test load with assumed field load conditions.

7. The new understanding of the fundamental effect of material properties on the structural behavior of reinforced concrete pipe which has been developed in this thesis should be an important aid for the selection of optimum materials for use in precast concrete pipe. In addition, the method of analysis for diagonal tension ultimate strength of pipe may also be applied to evaluate the ultimate strength of other curved reinforced concrete elements. The procedure for investigating cracking behavior of pipe reinforced with welded wire fabric may be applied to other reinforced concrete elements which have steel and concrete proportions similar to typical pipe of 66 inches diameter and larger.

11.2 **Summary of Equations for Evaluation of Structural Behavior of Pipe**

The equations developed in this investigation, presented in a form suitable for direct evaluation of the structural behavior of a pipe ring under 3-edge bearing loading, are collected together in the following summary. See the list of nomenclature and units in front of Chapter 1 for an explanation of symbols and units for each symbol.
1. Flexural Ultimate Strength - 66 Inch Diameter and Larger with Two Circular Cages:

\[
(DL)_{u} = \frac{7.3c M_{p1}}{d_{1}^{2}} - \frac{.5W}{D_{1}}
\]

where \( M_{p1} = f'_{s1} A_{s1} (d_{1} - .5a) \cdot \frac{1}{12} \)

\[
a = \frac{.1 f'_{c} A_{s1}}{f'_{s1} A_{s1}}
\]

\[
c = .57 \left( 1 + \frac{f'_{s2} A_{s1} d_{2}}{f'_{s1} A_{s1} d_{1}} \right)
\]

2. Flexural Ultimate Strength - Below 66 Inch Diameter with Two Circular Cages:

\[
(DL)_{u} = \frac{7.6c M'_{p1}}{d_{1}^{2}} - \frac{.5W}{D_{1}}
\]

\[
M'_{p1} = f'_{s1} A_{s1} (d_{1} + .8 - .88a) \cdot \frac{1}{12}
\]

\[
a = .175 \frac{f'_{s1}}{f'_{c}} A_{s1} < 0.8''
\]

(if \( a > 0.8'' \), use 4-1 for \( M'_{p1} \)

\( c \) from 4-18

3. Flexural Ultimate Strength with Pressure Head:

\[
(DL)_{ue} = (DL)_{um} - 1.74 \gamma H_{0} - .64 \gamma D_{1}
\]

\( (DL)_{um} \) from 4-17 or 4-19
4. Diagonal Tension Ultimate Strength:

\[(DL)_u = \left( \frac{38.5 \text{ d} \sqrt{f'_c}}{D_1} + \frac{7880 \text{ d} A_{s1}}{D_1^2} \right) F - \frac{9W}{D_1} \]  \hspace{1cm} 5-17a

where \( F = 1.15 - .075 \ s_q \)

\( s_q \) between 2 inches and 4 inches

5. Diagonal Tension Ultimate Strength with Pressure Head:

\[(DL)_{ue} = \left( \frac{38.5 \text{ d} \sqrt{f'_c}}{\lambda_1 D_1} + \frac{7880 \text{ d} A_{s1}}{\lambda_1 \lambda_2 D_1^2} \right) F - \frac{9W}{D_1} \]  \hspace{1cm} 9-9a

where \( \lambda_1 = 1 - \frac{.03 \gamma H_0}{(DL)_{ue}} \)  \hspace{1cm} 9-10

\( \lambda_2 = 1 + \frac{.185 \gamma H_0}{(DL)_{ue}} \)  \hspace{1cm} 9-11

6. Diagonal Tension Ultimate Strength with Web Reinforcing:

\[(DL)_u = \frac{1.1 A_{f_v} \text{ d}}{V \ s D_1} + 1800 \]  \hspace{1cm} 6-1b

or max. \( (DL)_u = \frac{3.0 A_{f_v} \text{ d}}{V \ s D_1} \)  \hspace{1cm} 6-2b
7. .01 Inch Crack Strength - 66 Inch Diameter and Larger, Reinforced with Welded Wire Fabric, Longitudinals at 6 inches or 8 inches, o.c. :

\[
(DL)_{01} = \frac{1.15 \cdot 10^5 A_{s1} d}{\phi D_1^2} + \frac{3 \phi h d \sqrt{f_c'}}{\phi D_1^2} - \frac{.72W}{D_1} \tag{7-10}
\]

or max. \( (DL)_{01} = \frac{49 f_p A_{s1} d}{D_1^2} - \frac{.72W}{D_1} \tag{7-11} \)

8. .01 Inch Crack Strength with Pressure Head (same limitations as 7):

\[
(DL)_{01e} = (DL)_{01m} - .137 \frac{H}{\phi} - .6 \frac{\phi d_1}{\phi} \tag{9-13}
\]

\((DL)_{01m}\) from equation 7-10 or 7-11

9. Total Vertical Deflection Between Crown and Invert and Horizontal Deflection at Springings Under the .01 Inch Crack Load for 66 Inch and Larger Pipe Reinforced with Welded Wire Fabric:

\[
(S_v)_{01} = \frac{34.4 (DL)_{01} D_1^4}{(E_{c1} I_{cr})_{01}} \tag{8-6}
\]

\[
(S_h)_{01} = .925 (S_v)_{01} \tag{8-8a}
\]
where \((E_cI_{cr})_{0.01} = \frac{(1-k_f) \, \cdot \, E_sA_{s1}a^2}{B_{0.01}}\)

\(B_{0.01} = 1 - 0.33 \frac{l_0 \sqrt{F_c}}{\phi (D_{L})_{0.01}}\)  

\(k_f = \frac{\sqrt{(p_f n)^2 + 2p_f n - p_f n}}{p_f n}\)

\(p_f = \frac{A_{s1}}{bd} \cdot \frac{1}{B_{0.01}}\)

11.3 Tentative Suggested Design Procedure for Precast Concrete Pipe
Reinforced with Welded Wire Fabric (Applicable to Pipe 66 Inches in Diameter and Larger, Fabric Longitudinals at 6" to 8" o.c.)

The following rational design procedure summarizes the quantita-
tive methods suggested in this thesis in steps suitable for design of pipe. The procedure is set up in accordance with the general require-
ments of ASTM C76. See the list of nomenclature and units for each symbol.

1. Select Standard Class (I, II, III, IV, V) and Wall Thick-
ness (A, B, C) to be used. Refer to ASTM C76-61T for
requirements of various class D-load strengths and
standard wall thicknesses. Disregard provisions of
ASTM C76-61T on concrete strength and steel area require-
ments. These will be set by the procedure which follows.
All other provisions of C76-61T still apply. Pipe diameter
must be 66 inches or larger and reinforcing steel must be
welded wire fabric with longitudinal cross wires spaced at 6" or 8".

2. Select required inside cage area of steel using the following equation. Estimate a value of $f'_c$ (4000 psi to 6000 psi) to be checked in Step 5.

\[
\text{req'd. } A^*_{s1} = \frac{0.87 \cdot 10^{-5} l_0 D^2_{1}(DL)_{01}}{d} + \frac{0.62 \cdot 10^{-5} l_0 D^1 W}{d} - \frac{0.25 \cdot 10^{-5} f'_c l_0^2 h}{\phi} \tag{7-10a}
\]

\[
\text{min. } A^*_{s1} = \frac{3.33 \cdot 10^{-5} D^2_{1}(DL)_{01}}{d} + \frac{2.40 \cdot 10^{-5} D^1 W}{d} \tag{7-11a}
\]

whichever is larger.

Normally equation 7-10a will give the larger required $A_{s1}$ and thus be applicable except where close spacing of longitudinal wires is used. Equation 7-11a is based on equation 7-11 with the assumption that the proportional limit stress of the reinforcing steel, $f_p$, is 61,000 psi. If $f_p$ differs substantially from 61,000 psi, the required minimum area from 7-11a above should be multiplied by $\frac{61,000}{f_p}$.

*Note: With elliptical reinforcing, increase $A_{s1}$ by 10% to allow for poorer control of steel location at critical sections.
3. Where pipe must be designed for the combined action of internal pressure plus 3-edge bearing load, add the following area to that obtained in Step 2.

\[
\Delta A_{s1} = \frac{(0.119 H_o + 0.52D_1) \sigma_{1,0} D_1^2 \cdot 10^{-5}}{d}
\]

4. Select required outside cage area of steel, with two circular cages.

\[
A_{s2} = 0.75 A_{s1}
\]

5. Select required concrete strength for sufficient ultimate strength in diagonal tension (or determine that stirrups will be employed for diagonal tension reinforcing).

\[
\text{req'd. } f_c' = \left[ \frac{D_1 (DL)_u + 0.9W}{37 F d} - \frac{135 A_{s1}}{D_1} \right]^2
\]

or \( f_c' = 4000 \text{ psi} \)

and \( F = 1.15 - 0.075 \frac{s_0}{d} \)

6. Check ultimate strength of pipe with respect to bending failure. See article 11.2, paragraphs 1, 2 and 3.
7. Design stirrup reinforcing, if required, for ultimate strength in diagonal tension. Use a maximum circumferential spacing of \( .75d \) and provide internal stirrup anchorage to each main circumferential reinforcing wire in order to effectively prevent the circumferential reinforcing from straightening.

\[
s \approx .75d \text{ max.}
\]

\[
A_v = \frac{0.7 (DL) D_i s}{f_v d}
\]

11.4 Tentative Suggested Designs for C76 Pipe Reinforced with Welded Wire Fabric

Application of the above procedure to standard requirements for ASTM C76-61T pipe indicates that changes should be made in the recommended steel areas and minimum concrete strengths which are listed in the present specification. Suggested revisions to steel areas and minimum concrete strengths are shown in Fig. 11-1 for Class II, Wall A and B pipe; in Fig. 11-2 for Class III, Walls A, B and C pipe; and in Fig. 11-3 for Class IV, Walls A, B and C pipe. These new suggested steel areas apply only to pipe with welded wire fabric reinforcing with longitudinal cross wires at 6 inch or 8 inch on center.

It is evident from these figures that in most cases this tentative procedure leads to significant reductions in required steel content when welded wire fabric reinforcing is used in C76 pipe. The
TENTATIVE SUGGESTED DESIGN REQUIREMENTS FOR CLASS II CONCRETE PIPE REINFORCED WITH WELDED WIRE FABRIC

FIG. 11-1
TENTATIVE SUGGESTED DESIGN REQUIREMENTS FOR CLASS III CONCRETE PIPE REINFORCED WITH WELDED WIRE FABRIC

FIG. 11-2
TENTATIVE SUGGESTED DESIGN REQUIREMENTS FOR CLASS IV CONCRETE PIPE REINFORCED WITH WELDED WIRE FABRIC

FIG. 11-3
relative reduction in recommended areas increases with increasing pipe diameter. The figures also show the additional reductions in main steel area that can be achieved by using a six inch spacing of longitu-
dinals, rather than eight inch spacing. It is seen that closer
spacing of longitudinals is most effective for higher strength pipe
and pipe of larger diameter.

The present C76 specification suggests that the maximum
number of pipe tested shall not exceed 1% of large orders, or 2% of
smaller orders, after preliminary tests on a maximum of 3 specimens
in each size. Retests on 2 specimens are permitted for each initial
specimen which fails to pass any test. The design procedure suggested
in this thesis will insure that the ultimate strength requirements of
the specification are met with more than the required degree of reli-
ability. However, the design procedure for limitation of crack width
is based on the mean of the M.I.T. test results, with a conservative
comparison with other test results. It has not been related statisti-
cally to the present C76 specification requirements for reliability
of test results. It would probably produce designs which meet these
requirements, but more test data on cracking behavior of pipe with
welded wire fabric reinforcing is needed for a valid statistical
correlation of the specification requirements with test results. Because
the limitation of crack width is a much more arbitrary criteria than
ultimate strength, a lower degree of reliability should be required for
.01 inch crack test strengths than for ultimate strengths.
11.5 **Recommendations for Additional Investigations**

The rational theories for the structural behavior of reinforced concrete pipe developed in this study are generally applicable to pipe with any of the materials used in the industry. However, implementation of these concepts for practical analysis and design in many cases requires the development of new semi-empirical constants from a statistical interpretation of controlled test programs.

The test programs used for development of the constants suggested in this study were conducted using pipe reinforced with welded wire fabric. The semi-empirical constants developed for ultimate diagonal tension strength seem well corroborated by the many available tests. The constants suggested for .01 inch crack strength evaluation, however, need corroboration from further tests.

The following reinforcing materials, other than welded wire fabric, are also currently used or under development by the pipe industry:

1. Cold drawn smooth bright basic wire in coils.
2. Cold drawn deformed bright basic wire in coils.
3. Hot rolled, deformed bars: structural, intermediate, or hard grade.
4. Hot rolled, plain, bars or coils: structural, intermediate, or hard grade.

The theoretical and semi-empirical methods developed in this thesis for **ultimate strength** may be extended to pipe with the above reinforcing types with only the following minor modifications and
requirements for check tests:

1. The ultimate flexural strength may be investigated by the equations given in Chapter 4 if a limiting value of steel strength can be determined. Hot rolled structural and intermediate grade reinforcing will have a well defined yield zone before onset of strain hardening. Total elongation before ultimate strength is reached will probably be much too large to permit the concrete to remain intact until ultimate steel strength is developed. If so, yield strength should be used for ultimate strength predictions with these materials. On the other hand, hard grade steel will have a much smaller ductility to ultimate strength and will probably act much like cold drawn wire with regard to ultimate flexural strength. Tests should be designed to confirm the strength value that should be used with each type of reinforcing in ultimate flexural strength determinations.

2. Ultimate diagonal tension strength may be investigated by the equations given in Chapter 5, including the correction factor for bar spacing given by equation 5-21. More tests are needed to corroborate this correction factor. Also, a better explanation is needed for the discrepancy which is usually found between concrete strength from standard cylinder tests on cylinders made
and cured with the pipe and from core tests, corrected for 1/d, on cores cut from the actual pipe wall.

3. Tests are needed to determine the conditions under which "slabbing" failure may precede the type of diagonal tension failure considered herein.

The determination of .01 inch cracking criteria for reinforcing other than welded wire fabric of the type investigated herein is a much more difficult problem than determination of ultimate strength with such steels. With these reinforcing steels a primary determinant of .01 inch crack strength will be the crack spacing at points of maximum moment in a pipe. As was pointed out in Chapter 7, for reinforcing steel without crack initiators, spacing of flexural cracks will be highly variable. A great deal of experimental data will be needed to establish statistically a reliable semi-empirical design method, similar to the procedure developed in Chapter 7 for welded wire fabric, to insure limitation of crack width at working loads.

For steels with lower yield points (particularly if deformed) tests may show that .01 inch crack widths do not occur until loads close to the ultimate strength are reached. With such materials, ultimate strength rather than cracking criteria would provide the basis for the design of the pipe. Thus, pilot test programs with various types of reinforcing steel are needed to define the scope of the requirements for control of crack widths. Where these programs indicate that it is important to evaluate the conditions which produce .01 inch cracks, a large number of tests with specific reinforcing types, perhaps using
some kind of simplified flexural element which simulates the action of pipe, will be required. A test program is presently being developed, under the author's direction, for testing curved beams made to simulate the section of a pipe between points of inflection on either side of the invert. These curved beam tests are primarily intended to investigate the cracking characteristics of a large number of pipe designs with various types of reinforcing steel.

Additional tests are needed, with all types of reinforcing steel, to corroborate the theory presented in Chapter 5 for the location of critical section and the determination of ultimate strength in diagonal tension under field loading conditions. Again, sufficient tests for statistical interpretation would be required because of the variability in concrete strength that might be expected within the pipe. Tests are also needed under combined action of internal pressure and 3-edge bearing load to check the modifications for combined internal pressure and external load suggested in Chapter 9.

A great deal of valuable experimental data could be obtained from the routine testing carried out each year in the pipe industry, if, through a carefully organized cooperative effort, information sufficient for proper interpretation by the methods developed herein could be made available to a single agency, such as the American Concrete Pipe Association. In addition to the data normally obtained on ultimate strength and .01 inch crack load capacities, all pipe test results should include a record of the mode of failure, a record of the visual inspection of crack spacing around the pipe together with load level for the first
appearance of each crack, and basic compressive strength data on the concrete and tensile strength data on the steel. Of course, a complete description of the pipe specimen should be recorded. The accumulation of such data, in conjunction with the methods developed in this investigation for interpretation of its primary significance, would soon permit a truly rational approach to the selection of economical materials and designs for reinforced concrete pipe.
APPENDIX A

DESCRIPTION OF TEST SPECIMENS AND TEST RESULTS USED FOR DEVELOPMENT AND CORRELATION OF THEORETICAL METHODS

Note: Tables A-1, and A-3, and Figures A-1, A-2, and A-3 in this Appendix are reproduced from Reference 8 with permission of Mr. B. K. Saba.
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<th>Test Series</th>
<th>Inner Cage Reinforcement</th>
<th>Outer Cage Reinforcement</th>
<th>Stirrup Rein.</th>
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<th>Age at Test days</th>
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* Inner and Outer Cage Steel reduced from ASTM C76-57T requirements.
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*Pipe initially cracked due to machine adjustment. No reliable .01" & lat v-rellie crack loads. **Pipes were loaded by discontinuous increments.
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<th>Test Location and Date</th>
<th>Test Mark</th>
<th>Class &amp; Wall C76-60T</th>
<th>Internal Diameter inches</th>
<th>Wall Thickness inches</th>
<th>Type Reinfl. Cage</th>
<th>Req'd. As C76-60T sq.in./ft.</th>
<th>Furnished As C76-60T sq.in./ft.</th>
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*Indicates steel reduced from C76-60T requirements.
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<th>Wall Diameter (inches)</th>
<th>Wall Thickness (inches)</th>
<th>Type of Steel</th>
<th>Furnished As</th>
<th>Wire</th>
<th>Spacing</th>
<th>Unit Weight, As Furnished (lb/ft)</th>
<th>Unit Weight, As Furnished (lb/ft)</th>
<th>Test Code</th>
<th>Notes</th>
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Notes: The table contains data for different classes of steel, wall diameters, wall thicknesses, and types of steel, along with various measurements and notes related to the test specimens. The data includes wire size, spacing, unit weight, and test codes. The notes column provides additional information such as test codes and notes related to the test specimens.
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<th>Test Location and Date</th>
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<th>Pipe Inside Diameter in.</th>
<th>Ratio Measured to ASTM 0.03&quot; Creek</th>
<th>Ratio Measured to ASTM D-Load 0.01&quot; Creek</th>
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TRANVERSE CROSS SECTIONS OF PIPE SPECIMENS
M.I.T. TEST PROGRAM
FIG. A-1
TYPICAL STRESS-STRAIN CURVES FOR WELDED WIRE +ABRIC

FIG. A-2
APPENDIX B

DERIVATION OF RELATIONS FOR EFFECT OF
INITIAL CURVATURE ON STRESS DISTRIBUTION

Elevation:
Differential Element

1. Location of Neutral Axis (Investigation of Given Section) for

Tension on Inside of Curve:

\[ C = \int_0^{kd} f b dy \]

For a curved elastic flexural element:

\[ f \propto \frac{y}{r_n + y} = -\frac{B y/r_n}{1+y/r_n} \quad \text{where } B = \text{a constant} \]

(See Timoshenko, Vol. II - Advanced Strength of Materials or
any similar text for derivation.)

Let \( y/r_n = z \); \( dy = r_n dz \)

\[ f = -\frac{Bz}{1+z} \]
\[ C = \left[ \frac{kd}{r_n} - Bb r_n \frac{zdz}{1+z} \right] = - Bb r_n \left[ \frac{kd}{r_n} - \ln \left( \frac{kd}{r_n} + 1 \right) \right] \]

but \( \ln \left( \frac{kd}{r_n} + 1 \right) = \frac{kd}{r_n} - \frac{1}{2} \left( \frac{kd}{r_n} \right)^2 + \frac{1}{3} \left( \frac{kd}{r_n} \right)^3 \)

Using only first two terms of series (accuracy o.k. for \( \frac{kd}{r_n} \) values common with pipe)

\[ C \approx - \frac{Bb (kd)^2}{2r_n} \]

\[ T = f_s A_s \]

\[ f_s = \frac{B(d-kd)n}{r_n - (d-kd)} = \frac{Bn(d-kd)}{r_{si}} \]

\[ T = \frac{Bn(d-kd)A_s}{r_{si}} \]

\[ C + T = 0 \]

\[ B \left[ - \frac{bkd^2}{2r_n} + \frac{(d-kd)}{r_{si}} nA_s \right] = 0 \]

Note that \( r_n = r_o - kd \)

\[ - \frac{b(kd)^2}{2(r_o-kd)} - \frac{kdnA_s}{r_{si}} + \frac{dnA_s}{r_{si}} = 0 \]

\[ \left( b - \frac{2nA_s}{r_{si}} \right) kd^2 + 2nA_s \frac{(r_o+d)}{r_{si}} kd - 2nA_s \frac{r_o}{r_{si}} d = 0 \]

Let \( p = \frac{A_s}{bd} \)
\[ k = \sqrt{\frac{2np \frac{r_o}{r_{si}} \left( 1 - 2npd \frac{d}{r_{si}} \right) + \left[ np \frac{r_o + d}{r_{si}} \right]^2}{1 - 2np \frac{d}{r_{si}}}} \]  

(3-1)

2. Moment Arm for Tension on Inside

\[ j_d = d - kd + \bar{y} \]

\[ \bar{y} = \frac{\int y fbdy}{\int fbdy} = \frac{Bbr_n^2 \int_{0}^{r_n} \frac{kd}{r_n} \frac{z^2 dz}{1+z}}{Bbr_n \left[ \frac{kd}{r_n} + \ln \left( \frac{kd}{r_n} + 1 \right) \right]} \]

\[ \int_{0}^{r_n} \frac{kd}{r_n} \frac{z^2 dz}{1+z} = \frac{1}{2} \left( \frac{kd}{r_n} \right)^2 - \frac{kd}{r_n} + \ln \left( \frac{kd}{r_n} + 1 \right) \]

\[ \bar{y} = r_n \left[ \frac{\frac{1}{2} \left( \frac{kd}{r_n} \right)^2}{\frac{kd}{r_n} - \ln \left( \frac{kd}{r_n} + 1 \right)} - 1 \right] \]

Using series expansion for \( \ln \left( \frac{kd}{r_n} + 1 \right) \):

\[ \bar{y} = r_n \left[ \frac{\frac{1}{2} \left( \frac{kd}{r_n} \right)^2 - \frac{1}{2} \left( \frac{kd}{r_n} \right)^2 + \frac{1}{3} \left( \frac{kd}{r_n} \right)^3 - \frac{1}{4} \left( \frac{kd}{r_n} \right)^4}{\frac{1}{2} \left( \frac{kd}{r_n} \right)^2 - \frac{1}{3} \left( \frac{kd}{r_n} \right)^3 + \ldots} \right] \]
For typical values of $\frac{kd}{r_n}$ with pipe, sufficient accuracy occurs with:

$$\bar{y} \approx r_n \left[ \frac{\frac{1}{3} \left( \frac{kd}{r_n} \right)^3 - \frac{1}{4} \left( \frac{kd}{r_n} \right)^4}{\frac{1}{2} \left( \frac{kd}{r_n} \right)^2} \right] \approx \frac{kd}{3} - \frac{1}{2} \frac{kd^2}{(r_o - kd)}$$

$$j \approx 1 - \frac{k}{3} - \frac{1}{2} \frac{k^2d}{(r_o - kd)} \approx 1 - \frac{k}{3}$$ (3-2)

with $k$ from equation 3-1.

3. Stress in Concrete with Tension on Inside of Curve:

$$f_c = B \frac{kd}{r_o}$$ from section 1 above

also $C = \frac{Bbd^2}{2r_n}$ or $B = \frac{2r_n C}{bkd}$

$$f_c = \frac{2r_n C}{r_0 bkd}$$

but $C = \frac{M}{Jd}$

$$f_c = \frac{2M}{Jkbd^2} \frac{(r_o - kd)}{r_o}$$ (3-3)
4. Location of Neutral Axis for Tension on Outside of Curve:

\[
C = - Bb r_n \int_0^{-kd/r_n} \frac{zdz}{1+z} = - Bb r_n \left[ - \frac{kd}{r_n} - \ln \left( - \frac{kd}{r_n} + 1 \right) \right]
\]

Using series for ln term:

\[
C = - \frac{Bb(kd)^2}{2r_n}
\]

also \( T = \frac{Bn(d-kd)A_s}{r_{so}} \)

\( r_n = r_i + kd \)

\( C + T = 0 \)

\[
- \frac{Bb(kd)^2}{2(r_i+kd)} + \frac{BnA_s(d-kd)}{r_{so}} = 0
\]

Equation 3-5 results from solution of the above equation for \( k \), in a manner similar to derivation 1 above.

5. Derivations for equations 3-6 and 3-7 for tension on the outside of the curve are similar to derivations 2 and 3 above.
APPENDIX C

ELASTIC MOMENT DISTRIBUTION AND
DEFLECTION - DERIVATIONS FOR STAGE 2 CRACKING

(See Figure 3-1, d)

1. "Column Analogy" Solution for Moments

\[
\frac{I_{cr}}{I_c} = \frac{I_{cr}}{I_{cr}} = 1
\]

Analogous Column Section Primary Structure and \(M_s\) Diagram

\[
M = M_s - \left( \frac{P}{A} + \frac{M_x}{I_y} \right)
\]

\[M_s = Wx = Wr \sin \phi \quad \text{from } \phi = 0 \text{ to } 175.6^\circ\]

\[M_s = Wr \sin \phi + \frac{W}{2} (r \sin 4.4^\circ - r \sin \phi) \quad \text{from } 175.6^\circ \text{ to } 181.4^\circ\]

\[
P = \int \frac{M_s \, ds}{EI} = \int \frac{Wr \sin \phi \, r \, d\phi}{EI}
\]

\[
A = \int \frac{ds}{EI} = \int \frac{rd\phi}{EI}
\]
\[
\frac{P}{A} = \frac{2wr^2 \left[ \int_{0^\circ}^{15^\circ} \sin \phi d\phi + \frac{I_{cr}}{I_c} \int_{15^\circ}^{65^\circ} \sin \phi d\phi + \int_{65^\circ}^{90^\circ} \sin \phi d\phi + \frac{1}{2} \int_{0^\circ}^{4.4^\circ} (0.077 - \sin \phi) d\phi \right]}{4r \left[ \int_{0^\circ}^{15^\circ} d\phi + \frac{I_{cr}}{I_c} \int_{15^\circ}^{65^\circ} d\phi + \int_{65^\circ}^{90^\circ} d\phi \right]}
\]

\[
\frac{P}{A} = \frac{Wr \left[ (-\cos \phi)_{0^\circ}^{15^\circ} + \frac{I_{cr}}{I_c} (-\cos \phi)_{15^\circ}^{65^\circ} + (-\cos \phi)_{65^\circ}^{90^\circ} + (0.077 + \cos \phi)_{0^\circ}^{4.4^\circ} \right]}{2 \left[ (\phi)_{0^\circ}^{15^\circ} + \frac{I_{cr}}{I_c} (\phi)_{15^\circ}^{65^\circ} + (\phi)_{65^\circ}^{90^\circ} \right]}
\]

\[
\frac{P}{A} = \frac{Wr (0.459 + 0.543 \frac{I_{cr}}{I_c})}{0.350 (4.45 \frac{I_{cr}}{I_c})}
\]

\[
M_y = \int \frac{M_y ds}{EI} = 2 \int_{0^\circ}^{90^\circ} \frac{Wr^2 \sin^2 \phi r d\phi}{EI}
\]

\[
I_y = \int \frac{x^2 ds}{EI} = 4 \int_{0^\circ}^{90^\circ} \frac{r^2 \sin^2 \phi r d\phi}{EI}
\]

(Neglecting effect of \( \frac{W}{2} \) from 175.6° to 184.4°.)

\[
\frac{M_y}{I_y} = \frac{Wx}{2} = \frac{Wr \sin \phi}{2}
\]
\[ M = W r \left[ \sin \theta - \left( \frac{0.459 + 0.543 \frac{I_{cr}}{I_c}}{1.40 + 1.75 \frac{I_{cr}}{I_c}} \right) \cdot \frac{\sin \theta}{2} \right] \]

If \( I_{cr} = 0.3 I_c \)

\[ M = W r \left( \frac{\sin \theta}{2} - 0.324 \right) \text{ from } 0^\circ \text{ to } 175.6^\circ \]

or \[ M = W r \left( \sin \theta - 0.286 \right) \text{ from } 175.6^\circ \text{ to } 180^\circ \]

See Fig. 3-1d

2. Virtual Work Solution for Deflection

\[ \delta = \int \frac{M u d s}{E I} \]

a) Vertical Deflection of Crown:

Neglecting Very Small Effect of \( \frac{W}{2} \) Support from \( 175.6^\circ \) to \( 184.4^\circ \):

\[ M = W r \left( \frac{\sin \theta}{2} - 0.324 \right) \]

\[ m = l r \left( \frac{\sin \theta}{2} - 0.324 \right) \]

\[ \delta_v = 4 \int_0^{90^\circ} \frac{W r^2}{E I} \left( \frac{\sin \theta}{2} - 0.324 \right)^2 d\theta \]

\[ I_{cr} = 0.3 I_c \]
\[ \delta_v = \frac{4W_r^3}{EI_{cr}} \left[ \int_{15^\circ}^{90^\circ} 0^\circ,65^\circ (\frac{\sin \phi}{2} - .324)^2 \, d\phi + \frac{I_{cr}}{I_c} \int_{15^\circ}^{65^\circ} (\sin \phi - .324)^2 \, d\phi \right] \]

\[ = \frac{4W_r^3}{EI_{cr}} \left[ (.125\phi - .062\sin2\phi + .324\cos \phi + .105\phi)_{15^\circ,90^\circ} - .3 \left( .125\phi - .062\sin2\phi + .324\cos \phi + .105\phi \right)_{15^\circ} \right] \]

\[ \delta_v = \frac{.126W_r^3}{EI_{cr}} \]

b) Horizontal Deflection of Springings Apart:

Neglecting Very Small Effect of \( \frac{W}{2} \) Support from 175.6° to 184.4°:

\[ M = Wr \left( \frac{\sin \phi}{2} - .324 \right) \]

\( m \): From "Column Analogy" Solution as Follows

\[ m = m_s \left( \frac{P}{A} + \frac{m_{xy}}{I_x} \right) \]
\[
P = 2 \left[ \int_{25^\circ}^{90^\circ} \sin \theta \ d \theta + \frac{I_{cr}}{I_c} \int_{75^\circ}^{25^\circ} \sin \theta \ d \theta \right]
\]
\[
= 2r^2 \left[ (- \cos \theta) \bigg|_{25^\circ}^{90^\circ} + 0.3 (- \cos \theta) \bigg|_{75^\circ}^{25^\circ} \right]
\]
\[
P = 2r^2 (0.5467)
\]
\[
\frac{P}{A} = \frac{2r^2 (0.5467)}{0.70r(5.5)} = 0.285r
\]

\[
\frac{m_x^y}{I_x} = \frac{\sin \theta}{2}
\]

\[
m_s = \sin \theta
\]

\[
m = r \left( \frac{\sin \theta}{2} - 0.285 \right)
\]

but \(\sin \theta = \cos \phi\)

\[
m = r \left( \frac{\cos \phi}{2} - 0.285 \right)
\]

\[
\delta_h = 4 \int_{0^\circ}^{90^\circ} \frac{W_r^3}{EI} \left( \frac{\sin \phi}{2} - 0.324 \right) \left( \frac{\cos \phi}{2} - 0.285 \right) \ d\phi
\]

\[
= \frac{W_r^3}{EI_{cr}} \left[ \int_{15^\circ}^{90^\circ} \left( \frac{\sin \phi}{2} - 0.324 \right) \left( \frac{\cos \phi}{2} - 0.285 \right) \ d\phi \right] + \frac{I_{cr}}{I_c} \int_{65^\circ}^{15^\circ} \left( \frac{\sin \phi}{2} - 0.324 \right) \left( \frac{\cos \phi}{2} - 0.285 \right) \ d\phi
\]
\[ \delta_h = \frac{4Wr^3}{EI_{cr}} \left[ (0.125 \sin^2 \phi - 0.162 \sin \phi + 0.142 \cos \phi + 0.0924 \phi) \right]_{15^\circ, 90^\circ}^{0^\circ, 65^\circ} \\
+ 0.3 \left( (0.125 \sin^2 \phi - 0.162 \sin \phi + 0.142 \cos \phi + 0.0924 \phi) \right)_{65^\circ}^{15^\circ} \]

\[ \bar{\delta}_h = \frac{0.117 Wr^3}{EI_{cr}} \]

\[ \frac{\delta_v}{\delta_h} = \frac{0.126}{0.117} = 1.08 \]
APPENDIX D

DERIVATION FOR ELASTIC MOMENT DISTRIBUTION DUE TO SHRINKAGE EFFECTS

1. Reinforcing Is Two Circular Cages at Constant Location in Circular Pipe

"Column Analogy" Solution for Moments

\[ M = M_0 - \frac{P}{A} + \frac{M_x}{I_y} + \frac{M_y}{I_x} \]

\[ M_0 = R_se = \text{Constant} \]

\[ P = 4 \int_0^{\pi/2} \frac{M_0 ds}{EI} = \frac{4R_se r}{EI} \int_0^{\pi/2} d\phi \]

\[ P = \frac{2\pi r R_se}{EI} \]

\[ A = \frac{2\pi r}{EI} \]

\[ \frac{P}{A} = \frac{R_se}{s} \]
240.

\[ M_y = \int M_y x \, rd\phi = 0 \]
\[ M_x = \int M_x y \, rd\phi = 0 \]

Thus, Final \( M = R_s e - R_s e = 0 \)

2. Reinforcing Is One Elliptical Cage in a Circular Pipe

Assume steel eccentricity varies linearly from crown to springing from \( e_a \) at \( \phi = 0 \) to \( e_1 \) at \( \phi = \frac{\pi}{2} \).

Thus: \( e = e_1 \left( 1 - \frac{4}{\pi} \phi \right) \)

Assume \( R_s \) is approximately constant around the pipe.

Then: \( M_s = R_s e = R_s e_1 \left( 1 - \frac{4}{\pi} \phi \right) \)

\[ P = 4 \int_0^{\pi/2} M_s \, ds = 0 \]
\[ M_y = 2 \int_{-\pi/2}^{\pi/2} M_s x \, ds = 0 \]
\[ M_x = 2 \int_0^{\pi} M_s y \, ds = 0 \]

Therefore: \( M = M_s = R_s e \), the statically determinate moment.
APPENDIX E

DERIVATION OF EQUATIONS FOR FLEXURAL ULTIMATE STRENGTH

I. Test Load - Pipes 66 inch in diameter and larger

A. Limit Analysis (Refer to Fig. 4-1 - Plastic Failure Condition of Pipe Ring)

External Work = \(1.04 D_1 \theta F_u + (0.5 D_1 \theta + 0.207 D_1 \theta) (5W)\)
+ \(.30 D_1 \theta (5W)\)

Internal Work = \(2G M_{p_1} + 6G M_{p_1} + 6G M_{p_2} + 2G M_{p_2} + 2G M_{p_2}\)

External Work = Internal Work

\[
(P_u + 0.48 W) = \frac{4 (M_{p_1} + M_{p_2})}{1.04 D_1}
\]

B. Plastic Moment Capacity of Section (Refer to Fig. 4-1, a)

1. Crown and Invert, \(M_{p_1}\):

\[
M_{p_1} = f'_{s_1} A_{s_1} (d_1 - a/2) \cdot \frac{1}{12} \text{ ft.-lbs.} \tag{4-1}
\]

Neglecting compression steel (which is close to neutral axis):

\[
f'_{s_1} A_{s_1} = 0.85 f'_{c} ab
\]

\[
b = 12''
\]

\[
a = 0.1 \frac{f'_{s_1} A_{s_1}}{f'_{c}} \text{ inches} \tag{4-2}
\]
2. Springing, $M_{p_2}$:

$$M_{p_2} - N \left( \frac{h}{2} - \frac{a}{2} \right) = f'_{s_2} A_{s_2} \left( d_2 - \frac{a}{2} \right)$$

$$\left( \frac{h}{2} - \frac{a}{2} \right) \approx \frac{d_2}{2}$$

$$N = 0.5 \left( P_u + 0.5W \right)$$

$$M_{p_2} = \left[ f'_{s_2} A_{s_2} \left( d_2 - \frac{a}{2} \right) 
+ 0.25 \left( P_u + 0.5W \right) d_2 \right] \frac{1}{12} \quad (4-3)$$

According to ASTM C76:

$$f'_{s_2} = f'_{s_1} ; A_{s_2} = 0.75 A_{s_1} ; d_2 = d_1 = d ; a_2 \approx a_1$$

thus:

$$M_{p_2} = 0.75 M_{p_1} + 0.25 \left( P_u + 0.5W \right) \frac{d}{12} \quad (4-4)$$

C. Predication of Ultimate Strength

$$(P_u + 0.48W) = \frac{7 M_{p_1} + (P_u + 0.5W) \frac{d}{12}}{1.04 D_1}$$

$$d = D_1$$

$$0.48W \approx 0.5W$$

$$P_u + 0.5W = \frac{7 M_{p_1}}{0.92 \cdot 1.04 D_1}$$

$$P_u = \frac{7.3 M_{p_1}}{D_1} - 0.5W \quad (4-15)$$
In terms of D-load capacity with $D_1$ in feet:

$$\text{(DL)}_u = \frac{7.3 \frac{M_p}{D_1^2}}{D_1} - \frac{.5W}{D_1} \quad (4-16)$$

D. Correction if $\frac{f_{s2}A_{s2}}{s_{2}} \neq .75 \frac{f_{s1}A_{s1}}{s_{1}}$

$$M_{p2} = .75 \frac{f_{s2}A_{s2}}{s_{2}} \cdot \frac{d_2}{.75 f_{s1}A_{s1}d_1} + .25 (P_u + .5W) \frac{d}{12}$$

$$P_u + .5W = \left(4 + 3 \frac{f_{s2}A_{s2}}{s_{2}} \frac{d_2}{.75 f_{s1}A_{s1}d_1}\right) \frac{M_p}{.92 \cdot 1.04 D_1}$$

$$= \frac{4.19 \left(1 + \frac{f_{s2}A_{s2}}{s_{2}} \frac{d_2}{f_{s1}A_{s1}d_1}\right) M_p}{D_1}$$

$$= \frac{7.3 \frac{M_p}{D_1}}{D_1} \quad (4-17)$$

where $c = .57 \left(1 + \frac{f_{s2}A_{s2}}{s_{2}} \frac{d_2}{f_{s1}A_{s1}d_1}\right) \quad (4-18)$

II. Test Load - Modification for Pipes, 48 inch to 60 inch diameter.

A. Limit Analysis (Modification - Assume Width of Top Bearing Causes Two Hinges at Crown Similar to Invert Hinges)
External Work = 1.0 \, D_1 \theta \, P_u + (.5 \, D_2 \theta + .2 \, D_1 \theta) (5W) \\
+ .3 \, D_2 \theta (5W) \\

Internal Work = \frac{4 \, M_{p_1}}{P_1} + \frac{4 \, M_{p_2}}{P_2} \\
\frac{P_u + .5W}{D_1} \\

B. Plastic Moment Capacity (Refer to Fig. 25, b)  
(Modification - Both cages act in tension)  

1. Crown and Invert  

\[ M_{p_1} = \left[ f_{s_1} \, A_{s_1} \, (d_1 - a/2) + f_{s_2} \, A_{s_2} \, (1.1 - a/2) \right] \left( \frac{1}{12} \right) \]

ASTM C76: \[ f_{s_2} = f_{s_1} \ ]; \ A_{s_2} = .75 \ A_{s_1} \ ]; \ d_2 = d_1 = d \]

\[ M_{p_1} = f_{s_1} \, A_{s_1} \, (d + .8 - .88a) \left( \frac{1}{12} \right) \text{ ft.-lbs.} \quad (4-5) \]

\[ f_{s_1} \, A_{s_1} + f_{s_2} \, A_{s_2} = .85 \ f_c \, ab \]

\[ f_{s_1} \, A_{s_1} + .75 \ A_{s_1} = .85 \ f_c \quad a \cdot 12 \]

\[ a = .175 \ \frac{f_{s_1} \, A_{s_1}}{f_c} \quad (4-6) \]

Note: \ a \ must \ be \ less \ than \ .8 \ inches. \]
2. Springing

\[
M'_{\mathcal{P}_2} = \left[ f'_{s_2} A_{s_2} \left( d_2 - a/2 \right) + .56 f'_{s_1} A_{s_1} \left( 1.1 - a/2 \right) + .25 (P_u + .5W) d_2 \right] \frac{1}{12}
\]

\[
M'_{\mathcal{P}_2} = \left[ .75 f'_{s_1} A_{s_1} \left( a + .8 - .88a \right) + .25(P_u + .5W) d \right] \frac{1}{12}
\]

\[
M'_{\mathcal{P}_2} = .75 M'_{\mathcal{P}_1} + .25 (P_u + .5W) \frac{d}{12}
\]  \hspace{1cm} (4-9)

*Tension in inner cage assumed at .56 x tensile strength to make \( M'_{\mathcal{P}_2} = .75 M'_{\mathcal{P}_1} \) , a useful simplification.

C. Predatorion of Ultimate Strength

\[
P_u + .5W = \frac{7 M'_{\mathcal{P}_1}}{D_1}
\]

\( \dot{\alpha} = D_1 ; d \) in inches, \( D_1 \) in feet

\[
P_u + .5W = \frac{7 M'_{\mathcal{P}_1}}{.92 D_1}
\]

\[
P_u = \frac{7.6 M'_{\mathcal{P}_1}}{D_1} - .5W
\]

In terms of D-Load Capacity, \( D_1 \) in feet:

\[
(DL)_u = \frac{7.6 M'_{\mathcal{P}_1}}{D_1^2} - \frac{.5W}{D_1}
\]
More generally:

\[
(DL)_{u} = \frac{7.6 \ c \ M}{D_{1}^{2}} \ - \ \frac{.5W}{D_{1}}
\]  

(4-19)

D. Check on Maximum Tensile Strain in Second (Outer) Cage at Failure in the Concrete

\[
\frac{\varepsilon_{s2} + \varepsilon_{cp}}{1.1} = \frac{\varepsilon_{cp}}{k_{u}d}
\]

\[
\varepsilon_{s2} = \frac{1.1 \varepsilon_{cp}}{k_{u}d} - \varepsilon_{cp}
\]  

(4-7)

also

\[
\frac{\varepsilon_{s2}}{\varepsilon_{s1}} = \frac{1.1 - k_{u}d}{d - k_{u}d}
\]

\[
\varepsilon_{s2} = \left(\frac{1.1}{d - k_{u}}\right) \varepsilon_{s1}
\]  

(4-7a)

III. Allowable Rotation Capacity of Typical Pipe Sections

A. If Concrete Strain Controls:

From Ref. 37: \( \theta_{a} = \frac{\varepsilon_{cp}d}{k_{u}d} = \frac{\varepsilon_{cp}}{k_{u}} \)  

(4-10)

and \( \varepsilon_{cp} = .002 \)

From Ref. 33: \( k_{u}d = \frac{a}{.85} \)
From equation 4-2: 
\[ a_1 = 0.1 \frac{f'_{s1} A_{s1}}{f_c} = \frac{1.2 f'_s p d}{f_c} \]

\[ \phi_a = \frac{0.002 \cdot 0.85 f'_c d}{1.2 f'_s p d} = \frac{1.4 f'_c}{f'_s p d} \cdot 10^{-3} \quad (4-13) \]

B. If Steel Strain Controls:

From Ref. 37: 
\[ \phi_a = \frac{\varepsilon_{sp}}{1-k_u} \]

\[ k_u = \frac{1.2 f'_s p d}{0.85 f'_c d} = 1.4 \frac{f'_s}{f'_c} \cdot p \]

\[ \phi_a = \frac{\varepsilon_{sp}}{1 - 1.4 \frac{f'_s}{f'_c} p} \quad (4-14) \]

IV. Required Rotation Capacity of Crown and Invert Pipe Sections for Test Load Ultimate Flexural Strength

A. Using Method of Virtual Work

\[ \phi_a = \int_0^{\pi/2} \frac{M_{uds}}{EI} = \int_0^{\pi/2} \frac{M_{urd}\phi}{EI} \]

\[ M = M_{p_1} - \frac{P_u}{2} r\sin\phi + N_1 (r - r\cos\phi) \]

\[ M = M_{p_1} - \frac{7.3}{2D_1} \left( 0.54D_1 \right) \sin\phi \]

with \( N_1 \) neglected as small

Typical Quadrant at Flexural Ultimate
\[ M = M_p\left(1-1.97\sin\phi\right) \]

\[ m = -1.0 \]

\[ \Phi_a = -\frac{M_p r}{E I_{cr}} \left[ \int_{0,55}^{20,90} (1-1.97\sin\phi) \, d\phi + \frac{I_{cr}}{I_c} \int_{20}^{55} (1-1.97\sin\phi) \, d\phi \right] \]

\[ I_{cr} \approx 0.3 I_c \]

\[ \Phi_a = -\frac{M_p r}{E I_{cr}} \left[ (\phi+1.97\cos\phi)_{0,55}^{20,90} + 0.3 (\phi+1.97\cos\phi)_{0,55}^{20} \right] \]

\[ \Phi_a = -\frac{M_p r}{E I_{cr}} \left[ (0.349 + 1.97 \cdot 0.940 - 0 - 1.97 \cdot 1.0 \right.
\[ + 1.571 + 1.97 \cdot 0.940 - 0.960 - 1.97 \cdot 0.574) \]
\[ + 0.3 (1.571 + 1.97 \cdot 0.574 - 0.349 - 1.97 \cdot 0.940) \right] \]

\[ \Phi_a = +\frac{0.32 M_p r_{av}}{E I_{cr}} \]

If \( M_p \) is in ft-lbs., \( r_{av} = 0.54 \cdot 12 D_1 \) with \( D_1 \) in feet,

and \( E I_{cr} \) in inch-lb. units:

\[ \Phi_a = \frac{0.32 \cdot 144 \cdot 0.54 M_p D_1}{E I_{cr}} = \frac{25 M_p D_1}{E I_{cr}} \]  \hspace{1cm} (4-20)

\[ M_p = f_s A_s \left(d-a/2\right) \cdot \frac{1}{12} \]

\[ I_{cr} \approx \frac{E_s}{E_c} A_s (d-a)^2 + \frac{12a^3}{3} \]
\[ D_1' \approx d'' \]

\[ \theta_a = \frac{25 f'_s A_s (d-a/2) \cdot \frac{1}{12} \cdot d}{E_c \left[ \frac{E_c}{E_s} A_s (d-a)^2 + 4a^3 \right]} \]

Since \( a \) is small, compared to \( d \):

\[ \text{req'd. } \theta_a \approx \frac{2.1 f'_s}{E_s} \approx \frac{2.1 f'_s}{29 \cdot 10^{-6}} \approx 7 f'_s \cdot 10^{-8} \text{ radians} \quad (4-21) \]

V. Field Load - Class C Bedding:

A. Limit Analysis

\[ \text{External Work} = \left( \frac{.54}{2} + .54 \right) D_1 Q P_{fu} \]

\[- \frac{.54}{4} D_1 Q P_{fu} + (.54 + .21 + .33) D_1 \theta \frac{W}{2} - \frac{.54}{4} D_1 \theta W \]

\[ = .675 D_1 \theta \left( P_{fu} + .60W \right) \]

Internal Work = \( 4 \varphi M_{P_1} + 4 \varphi M_{P_2} \)

Let \( P'_{fu} = P_{fu} + .6W \approx P_{fu} + .5W \)
For ASTM C76 Pipe:

\[ M_{p2} = 0.75 M_{p1} + \frac{P_{fu} d}{4} = 0.75 M_{p1} + 0.08 P_{fu} D_1 \]

External Work = Internal Work

\[ 0.675 D_1 P_{fu} = 4M_{p1} + 3M_{p1} + 0.08 P_{fu} D_1 \]

\[ \frac{7M_{p1}}{P_{fu}} = \frac{595D_1}{11.75 M_{p1}} \]

\[ P_{fu} = \frac{11.75 M_{p1}}{D_1} = 0.60W \]
APPENDIX F

DERIVATION OF EQUATIONS FOR DIAGONAL TENSION ULTIMATE STRENGTH

I. General (Refer to Fig. 5-1):

A. Theoretical Expressions for Shear Stress and Radial Tension Stress in Idealized Section with Flexural Cracks Only (Fig. 5-1,a)

1. \[ \sum M_c = 0 \]
   \[ T_{jd} = M - Ng \]
   \[ T = \frac{M}{jd} - \frac{Ng}{jd} \] (a)

2. At Sections Defined by \( \theta \) and \( \theta + d\theta \):
   \[ C - T = N \]
   \[ (C + \overline{dC}) - (T + \overline{dT}) = (N + \overline{dN}) \]
   \[ \overline{dC} - \overline{dT} = \overline{dN} \] (b)

3. \[ \sum F_{\theta} = 0 \]
   \[ \overline{dT} - \overline{dC} - v \overline{d\theta} = 0 \]
   \[ - \overline{dN} - v \overline{d\theta} = 0 \]
   \[ v = - \frac{\overline{dN}}{\overline{d\theta}} \] (c)

4. From (a): \[ \overline{dT} = \frac{\overline{dM}}{jd} - \frac{\overline{dNg}}{jd} \]
   From (c): \[ \overline{dT} = \frac{\overline{dM}}{jd} + \frac{vd\theta}{jd} \] (d)

5. Shear Stress, \( v \): (Refer to Fig. 5-1, b)
   \[ \sum F_{\theta} = 0 \]
   \[ vbds = \overline{dT} \]
From (a): \[ v_{bd} = \frac{dM}{jd} + v \frac{d\theta}{jd} \]

\[ v = \frac{dM}{bd} + \frac{Vg}{bd} \frac{d\theta}{ds} \]

\[ d\theta = r_s \frac{d\theta}{ds} \]

\[ \frac{dM}{d\theta} = V \]

\[ v = \frac{V}{bd} + \frac{V}{bd} \frac{g}{r_s} \]

\[ v = \frac{V}{bd} (1 + g/r_s) \quad (5-4) \]

6. Radial Tension Stress, \( t_r \): (Refer to Fig. 5-1, b)

\[ \phi_{Fr} = 0 \]

\[ t_r \frac{d\theta}{ds} = T \frac{d\theta}{ds} \]

\[ t_r = \frac{T}{b} \frac{d\theta}{d\theta} \]

\[ t_r = \frac{T}{b r_s} \]

From (a): \[ t_r = \frac{M}{bd r_s} + \frac{Ng}{bd r_s} \quad (+ N \text{ is tension}) \quad (5-5) \]

In vicinity of crown and invert, \( N \approx 0 \)

\[ t_r = \frac{M}{bd r_s} \quad (5-5a) \]
B. Relate Effective Measure of "Principal Tension" in Pipe to "Effective Measure of Principal Tension" in Straight Beams

1. Mohr's circle for principal tension, $t_p$

\[
\rho = \sqrt{\frac{v^2}{2} + \left(\frac{t_0 - t_r}{2}\right)^2}
\]

\[
t_p = t_t + \frac{t_0 - t_r}{2} + \rho
\]

\[
t_p = \frac{t_r}{2} + \frac{t_0}{2} + v\sqrt{\left(\frac{t_0 - t_r}{2v}\right)^2 + 1}
\]  \hspace{1cm} (a)

2. Actual stresses at a point:

\[
v = C_v \frac{V}{bd} \left(1 + \frac{g}{r_s}\right) = C_v \frac{V}{bd} \left(1 + \frac{g}{r_s}\right)
\]  \hspace{1cm} (b)
$$t_r = C_r \frac{M}{b j dr_s} = C_r \frac{M}{b dr_s}$$

(c)

assume $$t_\Theta = C_\Theta \frac{f_s}{n}$$ (see Ref. 39)

$$f_s = \frac{M}{A_s j d} = \frac{M}{p j bd^2}$$

$$t_\Theta = C_\Theta \frac{M}{p n bd^2}$$

(d)

3. Relate nominal shear, $$v_n = \frac{V}{bd}$$, to principal tension for straight beams:

$$t_r = 0$$

$$r_s = \infty$$

$$t_p = \frac{t_\Theta}{2} + V \sqrt{\left(\frac{t_\Theta}{2V}\right)^2 + 1}$$

$$V = \frac{t_p - \frac{t_\Theta}{2}}{\sqrt{\left(\frac{t_\Theta}{2V}\right)^2 + 1}}$$

$$v_n = \frac{V}{bd} = \frac{1}{C_v} \frac{t_p - \frac{t_\Theta}{2}}{\sqrt{\left(\frac{t_\Theta}{2V}\right)^2 + 1}}$$

from Ref. 39: $$v_n = 1.9 \sqrt{f_c^1} + 2500 \frac{p j d}{M}$$

(5-1)
\[
\frac{1}{\frac{v}{c_v}} \frac{t_p - \frac{t_0}{2}}{\sqrt{(\frac{t_0 - t_r}{2v})^2} + 1} = 1.9 \sqrt{f'_c} + 2500 \frac{F_{Md}}{M} \tag{e}
\]

4. Relate nominal shear in pipe to equation (e):

From (a):

\[
v + \frac{t_r}{2} \sqrt{\left(\frac{t_0 - t_r}{2v}\right)^2 + 1} = \frac{t_p - \frac{t_0}{2}}{\sqrt{\left(\frac{t_0 - t_r}{2v}\right)^2} + 1}
\]

\[
v_n + \frac{t_r}{2c_v(1+g/r_s)}\sqrt{\left(\frac{t_0 - t_r}{2v}\right)^2 + 1}
\]

\[
= \frac{t_p}{c_v}\sqrt{\left(\frac{t_0}{2v}\right)^2 + 1} \sqrt{\left(\frac{t_0 - t_r}{2v}\right)^2 + 1}
\]

Let \( A = \frac{t_0 - t_r}{2v} \)

Let \( B = \sqrt{\left(\frac{t_0}{2v}\right)^2 + 1} \cdot \sqrt{\left(\frac{t_0 - t_r}{2v}\right)^2 + 1} \)
\[ v_n + \frac{t_r}{2c_v(1+g/r_s) \sqrt{A^2 + 1}} = \left(1.9 \sqrt{f_c'} + 2500 \frac{pV_d}{M} \right) \frac{B}{1+g/r_s} \]

\[ v_n + \frac{c_r M}{2c_v(1+g/r_s)bdr_s \sqrt{A^2 + 1}} = \left(1.9 \sqrt{f_c'} + 2500 \frac{pV_d}{M} \right) \frac{B}{1+g/r_s} \]

For Pipe Assume: \( \frac{c_r}{c_v} = 1.0 \ max. \)

\[ A = 0.2 \]

\[ B = 1.1^* \]

*Note: B is assumed constant for pipe but really would vary with \( t_r \) and \( t_r = 0 \)

\[ v_n \left[ .9(1+g/r_s) + .45 \frac{M}{Vr_s} \right] = 1.9 \sqrt{f_c'} + 2500 \frac{pV_d}{M} \]

Let \( Z = .9(1+g/r_s) + .45 \frac{M}{Vr_s} \) \hspace{1cm} (5-8)

\[ v_n = \frac{V}{bd} = \frac{1}{Z} \left(1.9 \sqrt{f_c'} + 2500 \frac{pV_d}{M} \right) \] \hspace{1cm} (5-6)

II. Development of Simplified Analysis for Pipe Loaded in 3-Edge Bearing. Pipe is Proportioned in Accordance with ASTM C75.

A. Critical Section for Maximum Shear Plus Radial Tension is at a distance, \( d \), from invert support. Since \( d \approx \frac{D_1}{12} \), and the bottom supports are spread apart a distance \( \frac{D_1}{12} \), the critical section is at \( \psi = 13^0 \) from the bottom invert point on the pipe.
B. From Results of Elastic Analysis:

(see Fig. 3-1 at $\psi = 13^\circ$)

$$V = .487P + .434W$$  \hspace{1cm} (5-9)

$$M = .208Pr_{av} + .134WR_{av}$$  \hspace{1cm} (5-10)

$$N = (.112P + .180W) \text{and is neglected}$$

$$r_{av} = (1 + \frac{1}{12})r_s - \frac{1}{2}(\frac{d}{r_s}) \approx 1.07r_s$$

$$\frac{g}{2} \approx \frac{r_1}{12} \approx .07r_s$$

$$g/r_s \approx .07$$

$$Z = .9(1+.07) + .45\frac{.208P \cdot 1.07r_s}{.487P \cdot r_s}$$

$$= (.96 + .21)$$

$$= 1.17$$

$$v_n = \frac{1}{1.17} \left[ 1.9\sqrt{f_c'} + 2500P \frac{.487P \cdot d}{.208P \cdot 1.08r_1} \right]$$

$$= \frac{1}{1.17} \left[ 1.9f_c' + 2500 \frac{A_{sl}}{12d} \cdot \frac{.487P \cdot d}{.208P \cdot .54D_1 \cdot 12} \right]$$

$$v_n = 1.6\sqrt{f_c'} + 64\frac{A_{sl}}{D_1}$$  \hspace{1cm} (5-13)

In terms of D-Load Capacity:

$$\frac{.487(DL)u \cdot D_1 + .434W}{12d} = 1.6\sqrt{f_c'} + 64\frac{A_{sl}}{D_1}$$
\[ (DL)_u = \frac{40 \, d \sqrt{f'^c}}{D_1} + \frac{1575 \, d \cdot A_{s1}}{D_1^2} - \frac{9W}{D_1} \]  

(5.14)

III. Distributed Loading (Field Bedding Conditions)

A. Effect of Axial Load, N

Replace \( M \) with \( \delta + B N_d \)

\[ \delta = - (1 - \frac{h}{2d} - j) \approx - (1 - \frac{7}{11.5} - 0) \approx + .5 \text{ for pipe} \]

\[ d = \frac{D_1}{12} = \frac{r_{av}}{.54/12} = .155 \, r_{av} \]

\[ B N_d = + .5 \, N \cdot .155 \, r_{av} = + .078 \, N_r \]

B. From Equation 5-6' and 5-24, 25, 26:

\[ v_n = \frac{c_1 W + c_2 P_f}{12d} = \frac{1}{2} \left[ 1.9 \sqrt{f'^c} + 2500 \frac{A_{s1} c_2 P_f d}{12d c_3 P_{fr}} - .078 \frac{c_4 P_{fr}}{c_2} \right] \]

\[ P_f = \frac{12a}{c_2^2} \left[ 1.9 \sqrt{f'^c} + 2500 \frac{A_{s1} c_2 P_f d}{12 \cdot .54 \cdot 12 d_1 (c_3 - .078 c_4)} \right] - \frac{c_1 W}{c_2} \]

C. Change Constants in Accordance with

Test Results in Article 5.6:

\[ P_f = \frac{12a}{c_2^2} \left[ 1.9 \cdot 1.53 \sqrt{f'^c} + 32 \cdot \frac{320}{64} \cdot \frac{A_{s1}}{D_1} \cdot \frac{c_2}{c_3 - .078 c_4} \right] - \frac{c_1 W}{c_2} \]

\[ P_f = \frac{12a}{c_2^2} \left[ 1.8 \sqrt{f'^c} + 160 \frac{A_{s1}}{D_1} \frac{c_2}{c_3 - .078 c_4} \right] - \frac{c_1 W}{c_2} \quad (5-27) \]
D. From Equation 5-8', and 5-24, 25, 26:

\[
Z = 0.9 \left(1 + 0.07\right) + 0.45 \frac{(c_3 P_f - 0.078 c_4 P_f)}{c_2 P_f r_s} \cdot 1.07 r_s
\]

\[
Z = 0.97 + 0.48 \frac{(c_3 - 0.078 c_4)}{c_2}
\]

(5-28)
APPENDIX G

STATISTICAL INTERPRETATION OF TEST RESULTS

DIAGONAL TENSION ULTIMATE STRENGTH

(See Reference 58 for more detailed explanation of methods used in this Appendix.)

I. Linear Correlation by "Least Squares"

\[
\frac{v}{\sqrt{f'_c}} = A_o + B_o \frac{A_{s1}}{D_1 \sqrt{f'_c}}
\]  

(5-15)

Let \( y = \frac{v}{\sqrt{f'_c}} \); \( x = \frac{1000 A_{s1}}{D_1 \sqrt{f'_c}} \), \( N \) = number of tests

\[
B_o = \frac{\xi_{xy} - \frac{\xi_x \xi_y}{N}}{\xi_x^2 - \frac{(\xi_x)^2}{N}} = \frac{\xi'_{xy}}{\xi'_x^2}
\]

\[
A_o = \frac{\xi_y}{N} - B_o \frac{\xi_x}{N}
\]

For test data from 18 M.I.T. Tests (Group I, Table 5-1):

\[
B_o = \frac{42.35 - \frac{21.70 \cdot 34.45}{18}}{28.73 - \frac{(21.70)^2}{18}} = .320
\]

\[
A_o = \frac{34.45}{18} - .320 \cdot \frac{21.70}{18} = 1.53
\]
II. Tests for Significance of Linear Correlation:

1. Correlation Coefficient, \( r = \frac{\xi'_{xy}}{(\xi'_{x^2} \cdot \xi'_{y^2})^{1/2}} \)

From Calculations Shown Above: \( \xi'_{xy} = .82, \xi'_{x^2} = 2.56 \)

Also: \( \xi'_{y^2} = \xi y^2 - \frac{(\xi y)^2}{N} = 66.471 - \frac{(34.45)^2}{18} = .534 \)

\( r = \frac{.82}{(0.534 \cdot 2.56)^{1/2}} = .70 \)

2. With 18 tests there are 16 degrees of freedom (two restrictions, slope and intercept, of least squares straight line).

3. Probability Tables (Ref. 58, Table 8.2) indicate less than .001 probability that there is no correlation, or more than .999 probability that there is correlation.

III. 95% Confidence Limits for Estimated Average Values from Least Squares Line

A. Standard Deviation for Estimated Mean \( \frac{\bar{y}}{\sqrt{r_c'}} \) (i.e. \( \bar{y} \)) and for Estimated Slope, \( B_o' \):

1. Variance of average estimated value, \( \bar{y} \), is:

\[
\sigma^2(\bar{y}) = \frac{\xi(y-ar{y})^2}{N-2} = \frac{(1-r^2) \xi'_{y^2}}{N-2} = \frac{(1-.490) \cdot .534}{18-2} = .0170
\]

2. Variance of estimated slope is:

\[
s^2(B_o) = \frac{s^2(\bar{y})}{\xi'_{x^2}} = \frac{.0170}{2.56} = .00664
\]
3. Standard deviation of estimated slope is:
   \[ s(B_0) = 0.0815 \]

4. Variance of estimated mean value, \( \bar{y} \), is:
   \[ s^2(\bar{y}) = \frac{s^2(Y)}{N} = \frac{0.0170}{18} = 0.000944 \]

5. Standard deviation of estimated mean, \( \bar{y} \), is:
   \[ s(\bar{y}) = 0.0307 \]

B. t Value for 95% Confidence Range:

From Ref. 58, Table 6.1:

\[ t_{0.05, 16} = 2.12 \]

C. 95% Confidence Range:

\[ \bar{y} = 1.914 \pm 2.12 \cdot 0.0307 = 1.914 \pm 0.065 \]

\[ B_0 = 0.320 \pm 2.12 \cdot 0.0815 = 0.320 \pm 0.170 \]

(See Figure 5-2 for plotted confidence range - extremities of confidence range are smooth curves through extremities of \( \bar{y} \) values and asymptotic with the above extremities of slope of straight lines through \( \bar{y} = 1.914 \).)

IV. Lower Confidence Limit for 90% of Single
Test Values of \( \frac{v}{\sqrt{\frac{f_c}{n}}} \)

A. Lower Limit Line - General Equation

1. If 90% of test values are above this line, then 80% should be between this lower limit and a similar upper limit. Hence, the 80% confidence range for a single value should be used.

2. From Ref. 58, Table 6.1: \( t_{0.20, 16} = 1.337 \)
3. **Lower Limit Value of** \( \frac{V}{\sqrt{T_C}} \): \( = Y_1 \)

\[
Y_1 = \hat{Y}_1 - 1.34 \cdot s(\hat{Y}_1)
\]

4. **Estimated Value of** \( \hat{Y}_1 \):

\[
\hat{Y}_1 = 1.53 + 0.320 \cdot X_1, \quad \text{the least squares line}
\]

For plotting use \( X_1 = .8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2 \)

**B. Standard Deviation of Any Single Estimated Value, \( \hat{Y}_1 \):**

1. **Variance is:**

\[
s^2(\hat{Y}_1) = s^2(\hat{Y}) \left[ 1 + \frac{1}{N} + \frac{\sum (x - x_1)^2}{\xi \cdot x^2} \right]
\]

\[
= 0.0170 \left[ 1 + \frac{1}{18} + \frac{(1.206 - x_1)^2}{2.56} \right]
\]

\[
= 0.01795 + 0.00664 \cdot (1.206 - x_1)^2
\]

2. **Standard deviation is:**

\[
s(\hat{Y}_1) = \sqrt{s^2(\hat{Y}_1)}
\]

**C. Typical Calculation for** \( X_1 = 1.0 \)

\[
\hat{Y}_1 = 1.530 + 0.320 \cdot 1.0 = 1.850
\]

\[
s^2(\hat{Y}_1) = 0.01823; \quad s(\hat{Y}_1) = 0.135
\]

\[
Y_1 = 1.850 - 1.34 \cdot 0.135 = 1.669 \approx 1.67
\]

See Figure 5-2 - Limit Line for Lowest Strength of 90% of Pipe for Plot of \( Y_1 \) vs. \( X_1 \)
APPENDIX H

DERIVATION OF EQUATIONS FOR
PROPORTIONING STIRRUP REINFORCING

I. Tensile Force Carried by Stirrups
(Refer to Fig. 6-1)

\[ F_v = 0 \]

\[ T_s + V_c + V_s = V + \frac{M}{J_d} \Delta \theta \] \hspace{1cm} (a)

Let \( \Delta c \) = length of crack projected circumferentially at tension steel

\[ \Delta c = r_s \Delta \theta \]

\[ T_s = V + \frac{M}{J_d} \frac{\Delta c}{r_s} - (V_c + V_s) \] \hspace{1cm} (b)

also \( T_s = f_v A_v \frac{\Delta c}{s} \)

\[ f_v A_v = \frac{s}{\Delta c} \left[ V + \frac{M}{J_d} \frac{\Delta c}{r_s} - (V_c + V_s) \right] \]

From tests of pipes without stirrups the diagonal crack formation is somewhat flatter than 45°. These results indicate that \( \Delta c \approx Jd \) and \( s_{\text{max}} \approx Jd \).

If \( \Delta c = Jd \):

\[ f_v A_v = \frac{V_s}{Jd} + \frac{M_s}{r_s Jd} - (V_c + V_s) \frac{s}{Jd} \] \hspace{1cm} (c)
II. Stirrup Proportioning

A. Assume Concrete Carries $90 \cdot 1.5 = 135 \text{ psi}$

\[ V_c = v_c b j d = 135 \cdot 12 \cdot .9 d \approx 1500d \]

if $d \approx D_i$ in feet

\[ V_c = 1500 D_i \]

In 3-edge Bearing Test:

\[ P_{u,c} \approx 2 V_c \approx 3000 D_i \]

\[(DL)_{u,c} \approx 3000 \] (d)

Also in 3-edge Bearing Test:

\[ V_{\text{max}} \approx .5 P_u = .5 (DL)_u D_i \] (e)

(Include weight of pipe as $(DL)_w = .9 W/D_i$)

\[ M_{\text{max}} \approx .3 P_u r_s = .3 (DL)_u D_i r_s \] (f)

(Include weight of pipe as $(DL)_w = .5 W/D_i$)

From c, d, e, and f:

\[ f A_v = .5 (DL)_u D_i \frac{s}{jd} + \frac{.3(DL)_u D_i r_s}{r_s} \frac{j}{jd} - 1500 D_i \frac{s}{jd} \]

If $j \approx .9$

\[ A_v = \frac{.34(DL)_u D_i s}{f_v d} + \frac{.56(DL)_u - 3000) D_i s}{f_v d} \]

\[ A_v = \frac{[.90(DL)_u - 1600]}{f_v d} D_i s \] (6-1)
with min. \( A_v = \frac{\frac{3}{4} (DL) u D_i s}{f_v d} \) \hspace{1cm} (6-2)

B. Assume Concrete Carries 1/3 the Shear

\[ V_c = V/3 \] \hspace{1cm} (g)

From c, e, f and g:

\[ f_v A_v = \frac{2}{3} (0.5 P_u) \frac{s}{0.9 d} + \frac{3 P_u r_g}{r_s} \frac{s}{0.9 d} \]

\[ A_v = \frac{0.7 P_u s}{f_v d} \] \hspace{1cm} (6-4)
APPENDIX J

DERIVATION OF EQUATIONS FOR .01 INCH CRACK STRENGTH WITH WELDED WIRE FABRIC REINFORCING

I. General Expressions and Assumptions:

1. From Reference 41, General Report, p. 54

\[ w_s = \frac{x}{E} \left( f_s - \frac{K_F t}{r} \right) \]  \hspace{1cm} (7-5)

2. From test results on pipes of 72 inch diameter and larger with welded wire fabric reinforcing having longitudinals at 6 inch or 8 inch o.c.:

\[ x = l_o \]  \hspace{1cm} (7-6)

3. Let \( r = \frac{A_s}{bh} \)

Let \( K = K_o \frac{l_0}{\phi} \)  \hspace{1cm} (7-8)

See Discussion in Article 7.5

\[ f_s = \frac{M}{A_s J_d} \]

\[ w_s = \frac{l_0}{E_s} \left( \frac{M}{A_s J_d} - \frac{K_o l_0 b h f_t}{\phi A_s} \right) \]  \hspace{1cm} (a)

II. Application to Pipe

1. For Pipe Subject to .01 Inch Crack Load with Steel Stress Still in Elastic Range Assume:

\[ w_s = .008 \text{ inches at level of steel when the surface crack is .01 inches as measured by standard leaf gage.} \]
\[ E_s = 29 \times 10^6 \text{ psi} \]
\[ j = .9 \]
\[ b = 12 \text{ inches} \]

2. Assume that .01 inch crack occurs at Invert of Pipe, as indicated by M.I.T. Test Results:

From Fig. 3-1

\[ M = .282 \text{ Fr}_{av} + .202 \text{ Wr}_{av} \]  \hspace{1cm} (b)

\[ P = (DL) .01 \cdot D_1 \quad (D_1 \text{ in feet}) \]

\[ r_{av} = \frac{1.08 D_1}{2} \cdot 12 = 6.5 \ D_1 \text{ inches} \]

\[ M = 1.83 (DL) .01 \cdot D_1^2 + 1.31W \cdot D_1 \]  \hspace{1cm} (c)

3. From (a), (c) and above assumptions:

\[ .008 = \frac{10}{29.10^6} \left[ \frac{1.83 (DL) .01 \cdot D_1^2 + 1.31W \cdot D_1}{.9 A_s d} \right. \\
\left. \quad - \frac{12 K \lambda_h f_t}{\phi A_s} \right] \]

\[ (DL) .01 = \frac{1.15 \cdot 10^5 A_s d}{L_0 D_1^2} - \frac{.72W}{D_1} + \frac{5.9 K_{10} \lambda_h f_t}{\phi D_1^2} \]

\[ 5.9 K_{10} f_t = K_1 \sqrt{f_c} \]  \hspace{1cm} (d)

4. From M.I.T. Test Results:

\[ K_1 = .3 \]  \hspace{1cm} (e)
(DL)\_01 = \frac{1.15 \cdot 10^5 A_s d}{l_o D_1^2} - \frac{.72W}{D_1} + .3 \sqrt{f_c'} \frac{1_{hd}}{\phi D_1^2} \tag{7-10}

or req'd. \ A_s = \frac{.87 \cdot 10^{-5} l_o D_1^2(DL)\_01}{d} + \frac{.62 \cdot 10^{-5} l_o D_1^2 w}{d} - \frac{.25 \cdot 10^{-5} l_o^2 h \sqrt{f_c'}}{\phi} \tag{7-10a}

III. To Limit .01 Inch Crack Load Criteria to Elastic Range, Check that Steel Stress at Crack is Not Above Proportional Limit

\[ M = \frac{f_p A_s j d}{p s_1} \]

\[ .282 (DL)\_01 D_1 \cdot 6.5 D_1 + .202w \cdot 6.5 D_1 = .9 f_p A_s d \]

\[
\text{max (DL) } 01 = \frac{.49 f_A d}{p s_1} - \frac{.72W}{D_1} \tag{7-11}
\]

\[
\text{or min } A_s = \frac{.49 (DL)\_01 D_1^2}{p d} + \frac{1.47W D_1}{p} \]

if \ f_p = 61,300 psi

\[
\text{max (DL) } 01 = \frac{30,000 A_s d}{D_1^2} - \frac{.72W}{D_1} \]

\[
\text{or min } A_s = \frac{3.33 \cdot 10^{-5} (DL)\_01 D_1^2}{d} + \frac{2.40 \cdot 10^{-5} w D_1}{d} \tag{7-11a}
\]
IV. Reduced or Average Steel Stress Between Cracks:

From 7-5:  \[ f_{sr} = f_s - \frac{K_f t}{r} \]

\[ = f_s - \frac{K_0 \ell_o b h f_t}{\phi A_s} \]

\[ = f_s \left( 1 - \frac{K_0 \ell_o b h f_t \cdot j d}{\phi M} \right) \]

but from d and e above:

5.9 \[ K_f t = 0.3 \sqrt{f_c'} \]

\[ f_{sr} = f_s \left( 1 - \frac{0.051 \ell_o b d h j \sqrt{f_c'}}{\phi M} \right) \]

(See Table 7-6 - Expression 1.)
APPENDIX K

STATISTICAL INTERPRETATION OF TEST
RESULTS - .01 INCH CRACK STRENGTH

(See Reference 58 for more detailed explanation of methods used in this Appendix.)

I. Linear Correlation by "Least Squares"

\[
\left( \frac{DL_{.01} + \frac{.72W}{D_i}}{Q_{o}h\sqrt{f'_c}} \right) \phi_{D_i}^2 = A_1 + B_1 \frac{A_s\phi}{Q_o^2 h \sqrt{f'_c}}
\]

Let \[ y = \frac{\left( DL_{.01} + \frac{.72W}{D_i} \right) \phi_{D_i}^2}{Q_{o}h\sqrt{f'_c}} \]

\[ x = \frac{A_s\phi}{Q_o^2 h \sqrt{f'_c}} \]

\[ B_1 = \frac{\xi'_{xy}}{\xi'_{x^2}} = \frac{25.92 - \frac{14.59 \cdot 21.31}{14}}{18.68 - \left( \frac{14.59}{14} \right)^2} = 1.07 \]

\[ A_1 = \frac{\xi y}{N} - B_0 \frac{\xi x}{N} = \frac{21.31}{14} - 1.07 \cdot \frac{14.59}{14} = .408 \]

II. Tests for Significance of Linear Correlation

1. \[ r = \frac{\xi'_{xy}}{\left( \xi'_{x^2} \xi'_{y^2} \right)^{1/2}} \]

\[ \xi'_{xy} = 3.715 \text{ from above calculations} \]

\[ \xi'_{x^2} = 3.473 \quad " \quad " \quad " \]
\[ \xi' y^2 = 37.20 - \frac{(21.31)^2}{14} = 4.76 \]

\[ r = \frac{3.715}{3.473 \cdot 4.76} = .91 \]

2. With 14 tests, there are 12 degrees of freedom.

3. Probability table (Ref. 58, Table 8.2) indicates less than .001 probability that there is no correlation, or more than .999 probability that there is correlation.

III. 95% Confidence Limits for Estimated Average Values from Least Squares Line

A. Standard Deviation for Estimated Mean, \( \bar{y} \), and for Estimated Slope, \( B_1 \):

\[ s^2(\bar{y}) = \frac{(1-r^2) \xi' y^2}{N-2} = \frac{(1 - .91^2) \cdot 4.76}{12} = .0655 \]

\[ s^2(B_1) = \frac{s^2(\bar{y})}{\xi' x^2} = \frac{.0655}{3.473} = .0188 \]

Standard Deviation of Estimated Slope: \( s(B_1) = .137 \)

\[ s^2(\bar{y}) = \frac{s^2(\bar{y})}{N} = \frac{.0655}{14} = 46.78 \cdot 10^{-4} \]

Standard Deviation of Estimated Mean: \( s(\bar{y}) = .0684 \)

B. \( t \) Value for 95% Confidence Range:

From Ref. 58, Table 6.1: \( t_{.05,12} = 2.179 \)
C. 95% Confidence Range for Estimated Correlation Line:

\[ \overline{y} = 1.522 \pm 2.18 \cdot .0684 = 1.522 \pm .149 \]

\[ b_1 = 1.070 \pm 2.18 \cdot .137 = 1.070 \pm .299 \]

See Figure 7-4 for plotted confidence range.

IV. Lower Confidence Limit for 90% of Single Test Values of y:

A. Lower Limit Line - General Equation

1. Use 80% Confidence Range

2. From Ref. 58, Table 6.1: \( t_{20,12} = 1.356 \)

3. Lower Limit Value of \( y \) (\( y_1 \))

\[ y_1 = \hat{y}_1 - 1.36 s(\hat{y}_1) \]

4. Estimated Value of \( y \) (\( \hat{y}_1 \)):

\[ \hat{y}_1 = .4 + 1.07 x_1 \], the "Least Squares" line

For Plotting Use \( x_1 = .4, .6, .8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 \)

B. Standard Deviation of Any Single Estimated Value, \( y_1 \):

\[ s^2(\hat{y}_1) = s^2(y) \left[ 1 + \frac{1}{N} + \frac{(x - x_1)^2}{\zeta x^2} \right] \]

\[ = .0655 \left[ 1 + \frac{1}{14} + \frac{(1.042 - x_1)^2}{3.473} \right] \]

\[ = .0702 + .0189 (1.042 - x_1)^2 \]
C. Typical Calculation for $X_1 = 1.0$

\[
\hat{Y}_1 = 0.4 + 1.07 = 1.47
\]

\[
s^2(\hat{Y}_1) = 0.0702 + 0.0189 (1.042 - 1.0)^2
\]

\[
= 0.0702
\]

\[
s(\hat{Y}) = 0.265
\]

\[
Y_1 = 1.47 - 1.356 \cdot 0.265 = 1.11
\]
APPENDIX L

DERIVATION OF EQUATIONS FOR AVERAGE FLEXURAL RIGIDITY IN CRACKED REGIONS OF PIPE WALL

(Refer to Figure 8-1)

I. Flexural Rigidity of Effective Transformed Section Including Effect of Average Tension in Concrete Between Cracks:

In elastic regions from elastic beam theory:

\[
\text{Reduced Steel Stress, } f'_{sr} = \frac{nMc_s}{I_{cr}}
\]  \hspace{1cm} (8-1)

\[
I_{cr} = \frac{nMc_s}{f'_{sr}}
\]

Also \( M = f_s A_{s_1} j_d \)

where \( f_s \) is steel stress and \( j_d \) is effective moment arm right at the crack

\( c_s = (1-k_r)d \)

where \( k_r \) gives the average location of the neutral axis between cracks (see Section II in this Appendix)

\( n = E_s/E_c \)

In vicinity of .01 inch crack load:

\[
f'_{sr} = f_s \left( 1 - \frac{.051 \rho_o b d h \sqrt{f'_{c}}}{\phi_M .01} \right) \]  \hspace{1cm} (see Appendix J-IV)
Let \( B_{.01} = 1 - \frac{0.051 l_o b d h \sqrt{f_c'}}{\phi M_{.01}} \)

\( M_{.01} \approx 0.282 D_L_{.01} D_1 \cdot 0.54 D_1 \cdot 12 \)

\( dh \approx 1.08 D_1^2 ; b = 12'' ; j \approx 0.9 \)

\( B_{.01} \approx 1 - \frac{0.051 l_o \cdot 12 \cdot 1.08 D_1^2 \cdot 0.9 \sqrt{f_c'}}{\phi \cdot 0.282 \cdot 1.08 D_1^2 \cdot 6} \)

\( B_{.01} \approx 1 - \frac{0.33 l_o \sqrt{f_c'}}{\phi D_L_{.01}} \) \( (8-3) \)

Therefore:\n
\[ (E_{c I cr})_{.01} = \frac{E_c E_s f_A d (1-k_r) d}{E_c f_s B_{.01}} \]

\[ (E_{c I cr})_{.01} = \frac{(1-k_r) j E_s A_s d^2}{B_{.01}} \] \( (8-2) \)

II. Average Location of Neutral Axis Between Cracks:

\[ \frac{k_r d}{f_c} \]

\[ \frac{(1-k_r) d}{f_c} \]

\[ \frac{f_s r}{n} \]

From similar triangles:

\[ \frac{f_c}{f_s} = \frac{k_r}{1-k_r} = \frac{f_s B_{.01}}{f_s} \]

From \( \zeta F = 0: \)

\[ C = T \]

\[ \frac{1}{2} f_c b_k d = f_s A_s \]
Therefore: \[ k_r = \frac{2f_A}{bdf_c} = \frac{2f_A n(1-k_r)}{bdf B .01 k_r} \]

Let \( p = \frac{A_s}{bd} \); \( r = \frac{A_s}{B .01 bd} \)

\[ k_r = \frac{2np(1-k_r)}{B .01 k_r} = \frac{2np_r(1-k_r)}{k_r} \]

\[ k_r = \sqrt{(p_r n)^2 + 2p_r n - p_r n} \quad (8.4) \]
APPENDIX M

DERIVATION OF EQUATIONS FOR ULTIMATE STRENGTH UNDER INTERNAL PRESSURE AND COMBINED INTERNAL PRESSURE AND 3-EDGE BEARING LOAD

I. Internal Pressure Alone

\[ N = \frac{p_w D_1}{2} = 0.5 \gamma h_0 D_1 \]  \hspace{1cm} (9-1)

also \[ N_u = f_s' \left( A_{s_1} + A_{s_2} \right) \]

if \[ A_{s_2} = 0.75 A_{s_1} \]

\[ N_u = 1.75 A_{s_1} f_s' \]

\[ \frac{3.5 A_{s_1} f_s'}{D_1} \]

\[ H_{um} = \frac{0.05 A_{s_1} f_s' D_1}{D_1} \]  \hspace{1cm} (9-2)

*Note: Bending effects due to variation in pressure over the depth of the pipe and due to the weight of the pipe are developed later.

See Section II below.

II. Combined Internal Pressure and 3-Edge Bearing Load

Load - Flexural Failure

(Refer to Figure 9-1)

\[ M_{p_1} + N \left( \frac{h}{2} - \frac{a}{2} \right) = f_s A_{s_1} \left( d_1 - \frac{a}{2} \right) \]

Note: \[ + N \] is assumed tension in Figure 9-1.
\[
\frac{h}{2} - \frac{a}{2} \approx \frac{d_1}{2} \approx \frac{D_1}{2}
\] where \( D_1 \) is in feet

Therefore:
\[
M_p = f \frac{s_1 A_1}{\gamma H_0 D_1} (d_1 - a/2) - \frac{ND_1}{2}
\]

\[
N = T = 0.5 \gamma H_0 D_1
\]

And changing from in.-lb. to ft.-lb. units

\[
M_p = f \frac{s_1 A_1}{\gamma H_0 D_1} (d_1 - a/2) \cdot \frac{1}{12} - \frac{0.25 \gamma H_0 D_1}{12}
\]

Neglect effect of \( N \) on "a", since even a substantial change in "a" will have only a small effect on \( M_p \) (since "a" is small compared with \( d_1 \)). Calculate "a" from Equation 4-2.

From Appendix E and above modifications:

\[
M_{p2} = 0.75 M_p + 0.25 (P_u + 0.5W) \frac{d}{12} - \frac{0.25 \gamma H_0 D_1}{12}
\]

Also from Appendix E:

\[
(P_{ue} + 0.5W) = \frac{4(M_p + M_{p2})}{1.04 D_1}
\]

Modify \( P_{ue} \) for Effect of Bending Due to Variation in Fluid Pressure Over the Depth of Pipe as Follows:

Fluid load equivalent to 3-edge bearing effect occurs when the fluid load maximum bending moment equals 3-edge maximum bending
moment - assume elastic ratios of these moments is good enough for an approximate solution:

\[ .318 P_{uw} r_{av} = .75 \chi r_1^2 r_{av} \]

\[ P_{uw} = 2.4 \chi r_1^2 = .6 \chi D_1^2 \]

Therefore:

\[ P_{ue} + .5W + .6 \chi D_1^2 = \frac{7 M_{P_1} + .08(P_{ue} + .5W)}{1.04 D_1} = 8 \cdot .25 \chi H_0 D_1^2 \cdot \frac{1}{12} \]

\[ P_{ue} + .5W + .64 \chi D_1^2 = \frac{7.3 M_{P_1}}{D_1} - .174 \chi H_0 D_1 \]

\[ (DL)_{ue} = \frac{7.3 M_{P_1}}{D_1^2} - \frac{.5W}{D_1} - .174 \chi H_0 - .64 \chi D_1 \]

\[ (DL)_{ue} = (DL)_{um} - .174 \chi H_0 - .64 \chi D_1 \] (9-5)

if \((DL)_{ue} = 0\), \(H_o = H_{um} \):

\[ H_{um} = \frac{7.3 M_{P_1}}{.174 \chi} - \frac{.5W}{.174 \chi D_1} - \frac{.64 \chi D_1}{.174 \chi} \]

\[ H_{um} = \frac{7.3 f' s_1 s_1}{12 \cdot .174 \chi D_1^2} - \frac{2.9 M}{\chi D_1} - 3.7 D_1 \]

if \(d - a/2 \approx .92 D_1\)

\[ H_{um} = \frac{3.2 f' s_1 s_1}{\chi D_1} - \frac{2.9 M}{\chi D_1} - 3.7 D_1 \]
Constant term 3.2 is good check on terms 3.5 obtained in Section I above with somewhat different assumptions.

Divide Equation 9-5 above by (DL)_{um} to obtain an interaction equation

\[
\frac{(DL)_{ue}}{(DL)_{um}} = 1 - \frac{.174\sigma_H + .64\sigma_D_i}{(DL)_{um}}
\]

if \( (DL)_{ue} = 0 \)

\[
(DL)_{um} = .174\sigma_H + .64\sigma_D_i
\]

Therefore:

\[
\frac{(DL)_{ue}}{(DL)_{um}} + \frac{H_o \left(1 + \frac{3.7D_i}{H_o}\right)}{H_{um} \left(1 + \frac{3.7D_i}{H_{um}}\right)} = 1
\]  

(9-7)

III. Combined Internal Pressure and 3-Edge Bearing Load -
Diagonal Tension Failure (Refer to Figure 9-2)

Let \( N_1 \) = variable axial force due to 3-edge bearing load

\[
dN_1 = \text{change in } N_1 \text{ over differential arc length}
\]

\( N_2 = T = \text{constant circumferential tension from internal pressure (see equation 9-1)\)

1. Modification to Radial Tension Stress Due to Addition of Axial Tension:

Taking moments about \( T \):

\[
C\bar{d} = M - (N_1 + N_2) \left( d - \frac{h}{2} \right)
\]
also \( t_r b ds = C d \theta \)

\[ t_r = \frac{C}{b} \frac{d \theta}{ds} = \frac{C}{br_s} \frac{d \theta}{ds} = \frac{C}{br_s} \]

therefore: \( t_r = \frac{M}{br_s j d} - \frac{(N_1 + N_2)}{br_s} \left( \frac{d - \frac{b}{2}}{jd} \right) \)

\[ \frac{d - \frac{b}{2}}{jd} \approx .5 \]

\[ t_r = \frac{M}{br_s j d} - \frac{.5 (N_1 + N_2)}{br_s} \]

at the crown and invert critical sections for diagonal tension strength

\( N_1 \approx 0 ; \quad N_2 = T \quad \text{from Equation 9-1} \)

\[ t_r = \frac{(M - .5 \cdot .9Td)}{br_s j d} = \frac{M - .45Td}{br_s j d} \]

2. No change from derivation in Appendix F for shear stress (Equation 5-4) is required due to addition of \( T \).

3. Change in \( t_Q \) due to addition of axial force, \( T \), requires replacement of "\( M \)" in previous equations for diagonal tension strength by \( (M + .5Td) \) (see Equations 5-2 and 5-3).

4. Therefore, previous derivations for ultimate nominal shear stress in pipe should be modified as follows when internal pressure is added to 3-edge bearing loads:
\[ v_n = \frac{V}{bd} = \frac{1}{Z_t} \left[ 1.9 \sqrt{f_c'} + 2500 \frac{V_d}{(M+.5T_d)} \right] \]

\[ Z' = .9 \left( 1 + \frac{g}{r_s} \right) + .45 \frac{M - .45T_d}{V_s} \]

As before: \( V = .487 \, P \)

\[ M = .208 \, P_{av} = .208 \, P \cdot \frac{1.08 \, D_1}{2} \cdot 12 = 1.35 \, P D_1 \]

\[ T = .5\chi H_0 D_1 \]

\[ d'' \approx D_1 \]

\[ v_n = \frac{1}{Z_t} \left[ 1.9 \sqrt{f_c'} + \frac{2500 \, A_{s_1} \cdot .487 P d}{12d \left( 1.35 P D_1 + .25 \chi H_0 D_1^2 \right)} \right] \]

\[ v_n = \frac{1}{Z_t} \left[ 1.9 \sqrt{f_c'} + \frac{75 \, A_{s_1}}{D_1 \left( 1 + \frac{.185 \chi H_0}{(DL)_{ue}} \right)} \right] \]

also: \( g/r_s = .07 \); \( r_s \approx 1.02 r_1 \approx .51 D_1 \)

\[ Z' = .9(1.07) + .45 \frac{1.35 P D_1 - .23 \chi H_0 D_1 d}{.487 P - .51 D_1 \cdot 12} \]

\[ Z' = 1.17 - \frac{.035 \chi H_0}{(DL)_{ue}} \]

Let \( Z' = 1.17 \lambda_1 \) where \( \lambda_1 = 1 - \frac{.03 \chi H_0}{(DL)_{ue}} \)

also Let \( \lambda_2 = 1 + \frac{.185 \chi H_0}{(DL)_{ue}} \)
\[ v_n = \frac{1.6 \sqrt{f_c}}{\lambda_1} + \frac{64 A_{s1}}{D_1 \lambda_1 \lambda_2} \]

Modify the constant coefficients for pipe without axial tension, 1.6 and 64, in accordance with the M.I.T. test results discussed in Article 5.6:

\[ v_n = \frac{V}{bd} = \frac{1.53 \sqrt{f_c}}{\lambda_1} + \frac{320 A_{s1}}{D_1 \lambda_1 \lambda_2} \]

noting that \[ V = 0.487 \ (DL)_{ue} D_1 + 0.43 \ M \]

\[ b = 12 \]

\[ (DL)_{ue} = \frac{38.5 \ d \sqrt{f_c}}{\lambda_1 D_1} + \frac{7880 \ d \ A_{s1}}{\lambda_1 \lambda_2 D_1^2} = \frac{9M}{D_1} \]

(9-9)
APPENDIX N

DERIVATION OF EQUATIONS FOR .01 INCH CRACK STRENGTH UNDER INTERNAL PRESSURE AND UNDER COMBINED INTERNAL PRESSURE AND 3-EDGE BEARING LOAD

I. Internal Pressure Alone

\[ f_s = \frac{N}{A_{s1} + A_{s2}} = \frac{5\gamma H_{10} \Delta D_i}{1.75 A_{s1}} = 0.286 \frac{H_{10} \Delta D_i}{A_{s1}} \]

From the Derivation in Appendix J

\[ \nu_s = \frac{l_0}{E_s} \left( f_s - \frac{K_o \gamma H_{10} \Delta D_i}{\phi A_{s1}} \right) \]

\[ 0.008 = \frac{l_0}{29.10^6} \left( \frac{0.286 H_{10} \Delta D_i}{A_{s1}} - \frac{1.2K_o \gamma H_{10} \Delta D_i}{\phi A_{s1}} \right) \]

\[ \gamma_{10} = \frac{8.1 \times 10^5 A_{s1}}{\phi \Delta D_i} - \frac{4.2K_o \gamma H_{10} \Delta D_i}{\phi \Delta D_i} \]

If \( K_o \gamma H_{10} \Delta D_i = \frac{3}{5.9} \sqrt{f_c} = 0.51 \sqrt{f_c} \) as in the M.I.T. tests for

for 3-edge bearing only:

\[ \gamma_{10} = \frac{8.1 \times 10^5 A_{s1}}{\phi \Delta D_i} + \frac{2.14 l_0 h \sqrt{f_c}}{\phi \Delta D_i} \]  \( \star \)

*Note: Bending effects due to variation in pressure over the depth of the pipe and due to the weight of the pipe are developed later. See Section II below.
II. Combined Internal Pressure and 3-Edge Bearing Load

Take moments about compression force:

\[ f_s = \frac{M + N \left( \frac{h}{2} - \frac{kd}{3} \right)}{A_s d} \]

Assume that \( j \) does not vary appreciably from its value without \( N \):

also: \( N = .5 \gamma H_o D_i \)

\[ \left( \frac{h}{2} - \frac{kd}{3} \right) \approx .5d \approx .5D_i \quad , \quad D_i \text{ in feet} \]

Therefore: \( N \left( \frac{h}{2} - \frac{kd}{3} \right) \approx .25 \gamma H_o D_i^2 \quad \text{(in.-lbs.)} \)

Also from previous derivation in Appendix J:

\[ M = 1.83 (DL) \cdot \alpha e D_i^2 + 1.31 WD_i \quad \text{in.-lbs.} \]

Add due to pressure variation over depth:

\[ M = .67 \gamma r_2^2 \cdot r_{av} = .09 \gamma D_i^3 \cdot 12 = 1.1 \gamma D_i^3 \]

since \( w_s = \frac{l_o}{E_s} \left( f_s - \frac{K_o l_o b h f_t}{\varphi A_{s_1}} \right) \)

\[ .008 = \frac{l_o}{29 \cdot 10^6} \left[ \frac{1.83(\text{DL}) \cdot \alpha e D_i^2 + 1.31 WD_i + .25 \gamma H_o D_i^2 + 1.1 \gamma D_i^3}{.9 A_{s_1} d} \right] \]

\[ - \frac{12K_o l_o b h f_t}{\varphi A_{s_1}} \]
\[(DL)_{.01e} = \frac{1.15 \cdot 10^5 A_{s_1} d}{\lambda_{o D_1}^2} + \frac{.72W}{D_1} + \frac{5.9 K_o \lambda_{o \Omega} h f_{t} d}{\rho D_1^2} - .137 \chi_{H_o} - .6 \chi_{D_1} \]

\[(DL)_{.01e} = (DL)_{.01m} - .137 \chi_{H_o} - .6 \chi_{D_1} \quad (9-13) \]

If \((DL)_{.01e} = 0\), \(H_o = H_{.01m}\)

\[
\chi_{H_{.01m}} = \frac{1.15 \cdot 10^5 A_{s_1} d}{.137 \lambda_{o D_1}^2} + \frac{.72W}{.137 D_1} + \frac{5.9 K_o \lambda_{o \Omega} h f_{t} d}{.137 \rho D_1^2} - \frac{.6 \chi_{D_1}}{.137} \]

\[
\chi_{H_{.01m}} = \frac{8.4 \cdot 10^5 A_{s_1} d}{\lambda_{o D_1}^2} + \frac{4.3 K_o \lambda_{o \Omega} h f_{t} d}{\rho D_1^2} - \frac{5.2 W}{D_1} - 4.4 \chi_{D_1} \]

Constant terms 8.4 and 4.3 are good checks on terms 8.1 and 4.2 obtained in Section I above with somewhat different assumptions.

Divide Equation 9-13 above by \((DL)_{.01}\) to obtain an interaction equation:

\[
\frac{(DL)_{.01m}}{(DL)_{.01m}} = 1 - \frac{(.137 \chi_{H_o} + .6 \chi_{D_1})}{(DL)_{.01m}} \]

\[
(DL)_{.01m} = .137 \chi_{H_{.01m}} + .6 \chi_{D_1} = \chi_{H_{.01m}} \left(1 + \frac{4.4 D_1}{H_{.01m}}\right) \]

\[
\frac{(DL)_{.01e}}{(DL)_{.01m}} + \frac{H_o \left(1 + \frac{4.4 D_1}{H_o}\right)}{H_{.01m} \left(1 + \frac{4.4 D_1}{H_{.01m}}\right)} = 1 \quad (9-14) \]
APPENDIX O

SAMPLE CALCULATIONS

I. Ultimate Flexural Strength

Used for Table 4-1, Pipe Specimen Q1:

\[ D_1 = 9'0" \]

\[ A_{s_1} = .619 \text{ in.}^2/\text{ft.} \]

\[ f_{s_1} = 85,800 \text{ psi} \]

\[ A_{s_2} = .48 \text{ in.}^2/\text{ft.} \]

\[ f_{s_2} = 72,400 \text{ psi} \]

\[ d_i = 7.82 \text{ in.} \]

\[ f_c' = 5085 \text{ psi} \]

\[ W = 3446 \text{ lbs./ft.} \]

Equat. (4-2): \[ a = .1 \cdot \frac{85,800 \cdot .619}{5085} = 1.06 \text{ in.} \]

Equat. (4-1): \[ M_{p_1} = 85,800 \cdot .619 \cdot (7.82 - .53) \cdot \frac{1}{12} = 32,200 \text{ lbs./ft.} \]

Equat. (4-18): Correction for \[ f_{s_2} A_{s_2} < .75 f_{s_1} A_{s_1} \]

\[ c = .57 \left(1 + \frac{72,400 \cdot .48}{85,800 \cdot .619}\right) = .95 \]

Equat. (4-16) with correction:

\[ (DL)_{u} = \frac{7.3 \cdot 32,200 \cdot .95 - .5 \cdot 3446}{9} = 2580 \]

Test \( (DL)_{u} = 2820 \)
\[
\frac{(DL)_u \text{ test}}{(DL)_u \text{ calc}} = 1.09
\]

II. Ultimate Diagonal Tension Strength

Used for Table 5-1, Pipe Specimen D2:

\[D_i = 6'0"\]
\[d_i = 6.04 \text{ inches}\]
\[A_{sl} = 0.875 \text{ in.}^2/\text{ft.}\]
\[f'_c = 5550 \text{ psi}\]
\[W = 1811 \text{ lbs./ft.}\]

In terms of nominal shear stress:

Equat. (5-16): \[v_n = 1.53 \sqrt{5550} + 320 \cdot \frac{0.875}{6} = 161 \text{ psi}\]

Equat. (5-9): test \[v = 0.486 \cdot 21,800 + 0.434 \cdot 1811 = 11,400 \text{ lbs.}\]

\[\text{test } v = \frac{11,400}{12 \cdot 6.04} = 157 \text{ psi}\]

\[\frac{v_{test}}{v_{calc}} = 0.97\]

In terms of D-Load:

Equat. (5-17): \[(DL)_u = \frac{38.5 \sqrt{5550}}{6} \cdot 6.04 + \frac{7880 \cdot 6.04 \cdot 0.875}{6^2}
- \frac{0.9 \cdot 1811}{6} = 3760\]

\[\text{test } (DL)_u = 3630\]

\[\frac{(DL)_u \text{ test}}{(DL)_u \text{ calc}} = 0.97\]
III. Nominal Steel Stress at Invert at Measured .01 Inch Crack Load
Used for Table 7-1, Pipe Specimen Q1:

\[ M = 0.282 \, P \, r_{av} + 0.202 \, W \, r_{av} \]

\[ = 0.305 \, P \, r_i + 0.218 \, W \, r_i \]

\[ = 0.305 \times 14,850 \times 54 + 0.218 \times 3446 \times 54 = 284,700 \text{ in.-lbs.} \]

\[ f_s = \frac{284,700}{0.619 \cdot 0.9 \cdot 7.82} = 65,500 \text{ psi} \]

IV. .01 Inch D-Load Capacity
Used for Table 7-4, Pipe Specimen Q1:

\[ D_i = 6 \text{ ft.} \]
\[ A_s = 0.619 \text{ in.}^2/\text{ft.} \]
\[ \phi = 0.363 \text{ in.} \]
\[ \lambda_c = 6 \text{ in.} \]
\[ d = 5.8 \text{ in.} \]
\[ f'_c = 4765 \text{ psi} \]
\[ W = 1811 \text{ lbs./ft.} \]

Wall B

Equat. (7-10): 
\[ (DL)_{0.01} = \frac{1.15 \times 10^5 \times 0.619 \times 5.8}{6 \times 6^2} - \frac{0.72 \times 1811}{6} \]
\[ + \frac{0.3 \times 6 \times 7 \times 5.8 \sqrt{4765}}{0.363 \times 30} = 2040 \]

Check Effect of Proportional Limit:
\[ f_p \approx 61,300 \]

Equat. (7-11): 
\[ \text{max (DL)}_{0.01} = \frac{0.49 \times 61,300 \times 0.619 \times 5.8}{6^2} \]
\[ - \frac{0.72 \times 1811}{6} = 2760 \geq 2040 \]
Test (DL).01 = 2000

\[
\frac{(DL).01 \text{ test}}{(DL).01 \text{ calc}} = .98
\]

V. Deflection at .01 Inch Crack Load

Used for Table 8-1, Pipe Specimen Gl:

\[E_c = 1800 + 460 \quad f'_c = 1800 + 460 \cdot 4.765 = 4000 \text{ ksi}\]

\[n = \frac{E_s}{E_c} = \frac{29,000}{4000} = 7.3\]

Equat. (8-3): \[B_{.01} = 1 - \frac{.33 \cdot 6 \sqrt{4765}}{.36 (2000+220)} = .83\]

Equat. (8-5): \[P_r = \frac{.619}{12 \cdot 5.8} \cdot \frac{1}{.83} = .011\]

Equat. (8-4): \[k_r = \sqrt{(.011 \cdot 7.3)^2 + 2 \cdot .011 \cdot 7.3 - .011 \cdot 7.3} = .327\]

\[p = \frac{.619}{12 \cdot 5.8} = .0089\]

\[k = \sqrt{(.0089 \cdot 7.3)^2 - 2 \cdot .0089 \cdot 7.3 - .0089 \cdot 7.3} = .301\]

\[j = 1 - \frac{.301}{3} = .90\]

Equat. (8-2): \[E_c I_{cr} = \frac{(1-.327) \cdot .90 \cdot 29 \cdot 10^6 \cdot .619 \cdot 5.8^2}{.83} = 443 \cdot 10^6\]

Equat. (8-6): \[\left(\sigma_v\right)_{.01} = \frac{34.4 \cdot 2000 \cdot 6^4}{443 \cdot 10^6} = .200 \text{ in.}\]
Test \( (\sigma_v)_{0.01} = 0.195 \text{ in.} \)

\[
\frac{(\sigma_v)_{0.01 \text{ test}}}{(\sigma_v)_{0.01 \text{ calc}}} = \frac{0.195}{0.200} = 0.98
\]

VI. Typical Design Calculations for a Standard ASTM C76 Pipe

Used for Fig. 11-3

1. Design a Class IV, Wall B, 8'0" Diameter Pipe

2. Steel Area in Inner Cage for 2000 D .01 inch crack load

Estimate \( f'_c \) at 6000 psi

Use \( q = 6 \) inches

Estimate \( \phi = 0.43 \) (5-0)

Equat. (7-10a): req'd. \( A_{s_1} = \frac{0.87 \cdot 10^{-5} \cdot 6 \cdot 8^2 \cdot 2000}{7.8} \)

\[+ \frac{0.62 \cdot 10^{-5} \cdot 6 \cdot 8 \cdot 3090}{7.8} \]

\[- \frac{0.25 \cdot 10^{-5} \cdot 6^2 \cdot 9 \cdot \sqrt{6000}}{0.43} \]

\[= 0.855 + 0.117 - 0.146 = 0.83 \text{ in.}^2/\text{ft.} \]

Check Effect of Proportional Limit:

Equat. (7-11a): min \( A_{s_1} = \frac{3.33 \cdot 10^{-5} \cdot 8^2 \cdot 2000}{7.8} \)

\[+ \frac{2.40 \cdot 10^{-5} \cdot 8 \cdot 3090}{7.8} \]

\[= 0.546 + 0.076 = 0.62 \text{ in.}^2/\text{ft.} \]

Provide \( A_{s_1} = 0.83 \text{ in.}^2/\text{ft.} \).
3. Steel Area in Outer Cage

\[ A_{s2} = 0.75 \times 0.83 = 0.62 \text{ in}^2/\text{ft.} \]

4. Required Concrete Strength for 3000 D Ultimate Load

Equat. (5-23a): \[ f_c' = \left( \frac{8 \times 3000 + 0.9 \times 3090 - 135 \times 0.83}{37 \times 1 \times 7.8} \right)^2 \]

= 6250 psi

5. Check Ultimate Strength in Flexure

assume \( f_s' = 80,000 \) psi

Equat. (4-2): \[ a = \frac{1 \times 80,000 \times 0.83}{6250} = 1.06 \text{ in.} \]

Equat. (4-1): \[ M_{p1} = 80,000 \times 0.83 \times (7.8 - 0.53) \times \frac{1}{12} = 40,000 \text{ ft-lbs/ft.} \]

Equat. (4-17): \[ (DL)_u = \frac{7.3 \times 40,000}{82} - \frac{0.5 \times 3090}{8} = 4370 \text{ for bending failure} \]

6. Alternate Design with Stirrups and \( f_c' = 4000 \) psi

\[ A_{s1} = 0.855 + 0.117 - \frac{25 \times 10^{-5} \times 6^2 \times 9 \times \sqrt{4000}}{0.43} \]

\[ A_{s1} = 0.85 \text{ in}^2 \]

\[ A_{s2} = 0.75 \times 0.85 = 0.64 \text{ in}^2 \]

Equat. (6-5): \[ a_{\text{max}} \approx 0.75 \times 7.8 = 5.85 \text{ in.} \]

Use \( s = 6 \) in. (same as spacing of long. cross wires)

\[ f_v = 0.2\% \text{ offset stress} = 70,000 \text{ psi, assumed} \]
Equet. (6-4): \[ A_v = \frac{7 \cdot 3000 \cdot 8 \cdot 6}{70,000 \cdot 7.8} = 0.19 \text{ in}^2/\text{ft. length} \]

Provide 0.19 in²/ft. for each line of comb type welded wire fabric stirrups, combs to be spaced at 6 inches circumferentially for 35° ± on both sides of crown and invert.

VII. D-Load Capacity Under 100 Foot Pressure Head - Same Pipe as Designed in VI Above

A. Ultimate Strength:

Check Flexural Ultimate Strength:

Equet. (9.5): \[(DL)_{ue} = 4370 - 0.174 \cdot 62.4 \cdot 100 - 0.64 \cdot 62.4 \cdot 8 \]
\[= 2970 \]

Check Diagonal Tension Ultimate Strength:

Equet. (9-10): \[ \lambda_1 = 1 - \frac{0.03 \cdot 62.4 \cdot 100}{3000} = 0.94 \]

Equet. (9-11): \[ \lambda_2 = 1 + \frac{0.185 \cdot 62.4 \cdot 100}{3000} = 1.38 \]

Equet. (9-12): \[(DL)_{ue} = \frac{1}{0.94} \cdot 3000 - \frac{5000 \cdot 7.8 \cdot 0.83}{0.94 \cdot 8^2} \left(1 - \frac{1}{1.38}\right) \]
\[+ \frac{0.9 \cdot 3090}{8} \left(1 - \frac{1}{0.94}\right) = 3040 \]

Flexure Controls Ultimate Strength and \((DL)_{ue} = 2970\), a reduction of only 30 from \((DL)_{u}\) in Section VI above.

B. 0.01 Inch Crack Strength:

Equet. (9-13): \[(DL)_{0.01e} = 2000 - 0.137 \cdot 62.4 \cdot 100 - 0.6 \cdot 62.4 \cdot 8 = 850 \]
APPENDIX P

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