A FAST RESPONSE INSTRUMENT TO DIRECTLY READ THE RATIO OF TWO ELECTRICAL SIGNALS

BY

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Submitted to the department of Aeronautics and Astronautics on May 20, 1960, in partial fulfillment of the requirements for the degree of Bachelor of Science.

ABSTRACT

Initial specifications imposed on the design of an instrument to directly read the ratio of two electrical signals were an accuracy of 1% or better, a range of 30 milliamps and a frequency of 30 cps. An instrument meeting these specifications will be applicable not only to electrical signals measurements, but, using high accuracy transducers, will also be capable of measurements of pressure, flow, force, and other parameters that may be represented by electrical signals.

The proposed instrument consists of a torque summing shaft acted on by two Microsyn torque generators. A nulling signal, feedback to one of torque generators from a Microsyn signal generator mounted on the shaft, provides torque balance on the shaft. This feedback signal is directly proportional to the ratio of two of the torque generator excitation currents, one from each of the torque generators.

Experimental results show that the instrument has an accuracy better than 1% over a range of 29 milliamps, and a natural frequency up to 40 cps. The experimental results are based on an instrument not specifically designed for this application and it is felt that with an instrument specifically designed for this problem, much better results can be obtained.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to Professor Y.T. Li for his invaluable guidance as thesis supervisor, and to Mr. Philip J. Gilinson, Jr., Electromagnetic Coordinator for the M.I.T. Instrumentation Laboratory, for his aid throughout the development and experimental stages of this thesis.

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LIST OF ILLUSTRATIONS

INTRODUCTION

One of the parameters present in the determination of many measurements is the ratio of two similar but independent quantities. The thrust of a jet or rocket engine, for example, is a function of the ratio of the chamber pressure to the free stream pressure. Another aerodynamic example is the measurement of the strength of a shock wave by the pressures fore and aft of the shock. Besides these aerodynamic examples dependent on the ratio of two pressures, the ratio of forces, flows, displacements, and electrical signals assume a similar role in structures, control and guidance systems, and electronics. Other ratio measurements are present in other fields and studies and place an interest upon the development of an instrument that will directly reproduce the desired ratio.

In recent years tranducer design has made it possible to represent, with high accuracy, most desired quantities, such as pressure, flow, force, and displacement, by an electrical signal. An Instrument capable of measuring the ratio of electrical signals is then applicable for the measurement of ratios of most quantities. With this in mind, investigation was made to find a system that would accurately reproduce directly the ratio of two electrical signals.

Initial specifications were accuracy of at least 1%, a range of 30 units, applicable to the particular quantity being measured for each signal, and as fast a response time as it was possible to achieve. The system proposed, meets these specifications and is significantly better than anticipated in almost all respects. Accuracy is significantly better than 1% for a range of 29 units for each signal and for a manufacted model the frequency could easily be 30 cps.

GENERAL DESIGN THEORY

The design of a ratio measuring instrument necessitates the use of devices with outputs dependent upon the product of two quantities. Restricting the design to a fast response instrument immediately excludes any hydraulic or pneumatic systems and focuses an interest on either an electrical circuit or electromagnetic system. Several possible systems were investigated before one was decided upon. (See Appendices A and B)

The system used consists of a torque summing shaft acted upon by two Microsyn torque generators (see Figure 1) which produce a torque proportional to the product of the current in their two coils. (Reference 1) Balancing the torques on the shaft by means of a nulling feedback signal from a Microsyn signal generator, placed on the same torque summing shaft, the ratio of two of the currents, one from each torque generator, is directly proportional to the feedback signal.

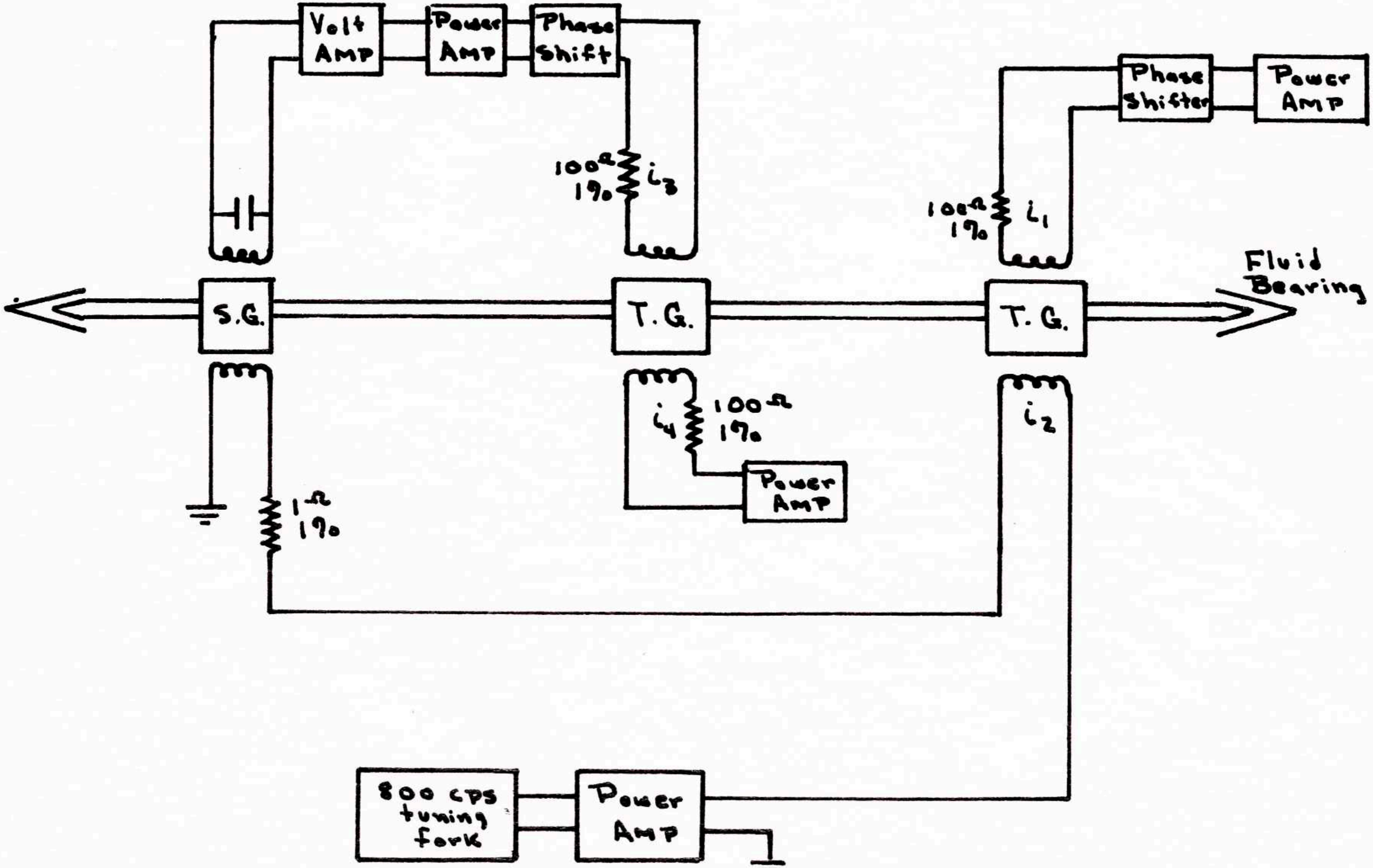

Volt AMP
Power AMP
Phase Shift
100Ω 1%
i_3
Phase Shifter
Power AMP
100Ω 1%
i_1
Fluid Bearing
S.G.
T.G.
T.G.
i_4
100Ω 1%
Power AMP
i_2
1Ω 1%
800 cps tuning fork
Power Amp

Fig. 1

DIFFERENTIAL EQUATION OF THE TORQUE SUMMING SHAFT

Basically the system can be represented by the following block diagram.

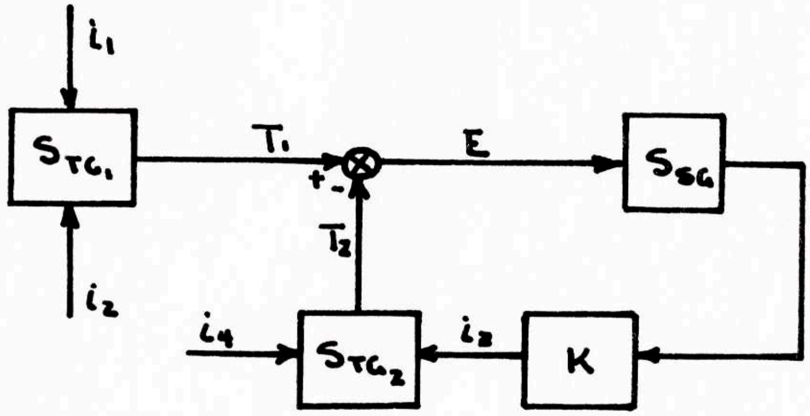


Figure 2

i_1, i_2, i_4 = applied currents to coils of the respective torque generators.

i_3 = feedback current from the signal generator.

S_{TG} = sensitivity of the respective torque generators $= T/\ i_a i_b$

S_{SG} = sensitivity of the signal generator $= \frac{mv}{mr}$

K = gain of the feedback amplifier.

Looking at the torque summing shaft the differential equation takes the following form:

$$I\ddot{A} + C\dot{A} + KA = T$$

where A = angular rotation of the shaft.

$$T_1 = S_{TG1}\ i_1\ i_2$$

$$T_2 = S_{TG2}\ i_3\ i_4 = S_{TG2} i_4\ (AS_{SG}K)$$

$$i_3 = AS_{SG}K$$

$$\ddot{A} + \frac{C}{I}\dot{A} + \frac{S_{TG2}S_{SG}K}{I}\ i_4\ A = \frac{S_{TG1}}{I}\ i_1\ i_2$$

Homogeneous solution of this equation is of form $A_c e^{pt}$ where:

$$p = \frac{1}{2}\left(-\frac{c}{I} \pm \sqrt{\left(\frac{c}{I}\right)^2 - 4\frac{S_{TG2}S_{SG}K}{I}\, i_4}\right)$$

Particular solution:

$$A = \frac{S_{TG1}i_1i_2}{S_{TG2}S_{SG}K\, i_4}$$

$$A = A_c e^{-\frac{1}{2}\frac{c}{I}t} \cos\left\{ \frac{1}{2}\sqrt{-\left(\frac{c}{I}\right)^2 + 4\,\frac{S_{TG2}S_{SG}K}{I}\, i_4}\right\} t + \frac{S_{TG1}i_1i_2}{S_{TG2}S_{SG}K\, i_4}$$

Using initial conditions solve for A_c at $t = o$ the shaft is balanced, $T_1 = T_2$ & $A = 0$

$$0 = A_c + \frac{S_{TG1}i_1i_2}{S_{TG2}S_{SG}K i_4}$$

$$A = \frac{S_{TG1}i_1i_2}{S_{TG2}S_{SG}K i_4}\left(1-\exp\left(-\frac{1}{2}\frac{c}{I}t\right) \cos\left\{\frac{1}{2}\sqrt{-\left(\frac{c}{I}\right)^2 + 4 i_4 \frac{S_{TG2}S_{SG}K}{I}}\right\} T\right)$$

But $i_3 = KS_{SG}A = \frac{S_{TG1}i_1i_2}{S_{TG2}i_4}$ (same quantity as before)

If $S_{TG1} = S_{TG2}$ and $t \to \infty$, transients have disappeared.

$$i_3 = i_2\left(\frac{i_1}{i_4}\right)$$

Making i_2 = constant then i_3 is a direct reading of the ratio $\frac{i_1}{i_4}$.

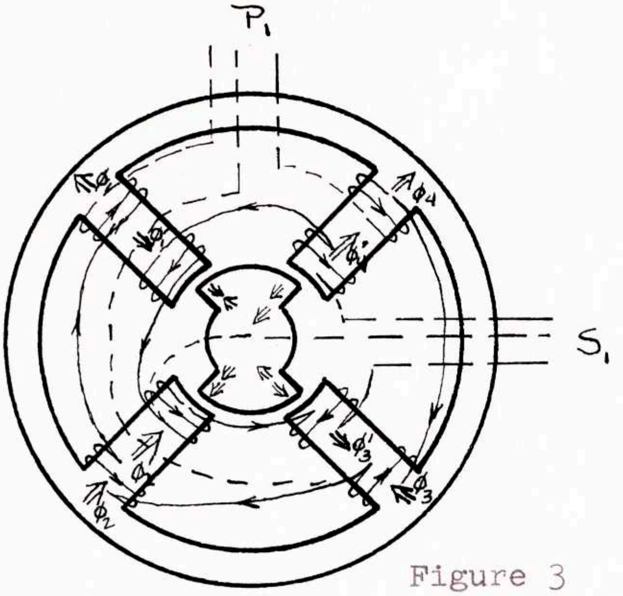
The damped natural frequency of the system, $w = \frac{1}{2}\sqrt{4i_4\frac{S_{TG2}S_{SG}K}{I} - \left(\frac{c}{I}\right)^2}$, is the parameter that the specifications require to be large. Of the six variables present, S_{TG2}, S_{SG}, i_4, K, C, AND I, the first two are restricted by the range of the torque generator and the signal generator and i_4 is an independent variable. Satisfying the specification of a fast response system then requires that K be large, and C and I small, with C I.

It must be remembered that this is a theoretical calculation based on perfect components and neglecting any errors. A complete error analysis is covered later in the paper.

MICROSYN UNITS (Reference 1,2,and 3)

All Microsyn units have basically a four pole stator and a magnetized rotor. For a torque generating unit each pole has two coils, these coils being connected in two series circuits around the stator. Each of the coil circuits is energized and the resulting torque on the rotor is proportional to the product of these two coil currents. A similar arrangement is used for the signal generating microsyn except that one circuit is used as a primary, or excitation circuit and the other is used as a secondary, or output circuit. With an A.C. excitation current the output current is proportional in magnitude to the angular displacement of the rotor with respect to the stator. A slightly different circuitry using one series coil arrangement for an excitation circuit, results in the elastic restraint generator with a restraining torque proportional to the angular rate of the rotor. The torque generator and signal generator are used on the torque summing shaft and will be analysed more thoroughly.

FUNDAMENTAL THEORY OF THE TORQUE GENERATOR



P_1 - excitation circuit 1

S_1 - excitation circuit 2

Figure 3

$\phi_1, \phi_2, \phi_3, \phi_4$ - Reference flux polarity due to first set of coils

$\phi_1', \phi_2', \phi_3', \phi_4'$ - Reference flux polarity due to second set of coils.

Small arrows on rotor indicate induced change of voltage

$$e = K\frac{d\phi'}{dt} = L\frac{di'}{dt}$$

$$e' = K\frac{d\phi}{dt} = L\frac{di}{dt}$$

Electrical power of the system $= \sum ei$

$$= K\sum\left[\phi\frac{d\phi'}{dA} + \phi'\frac{d\phi}{dA}\right]\frac{dA}{dt}$$

Mechanical power of the system $= KM\frac{dA}{dt}$

Equating electrical and mechanical power:

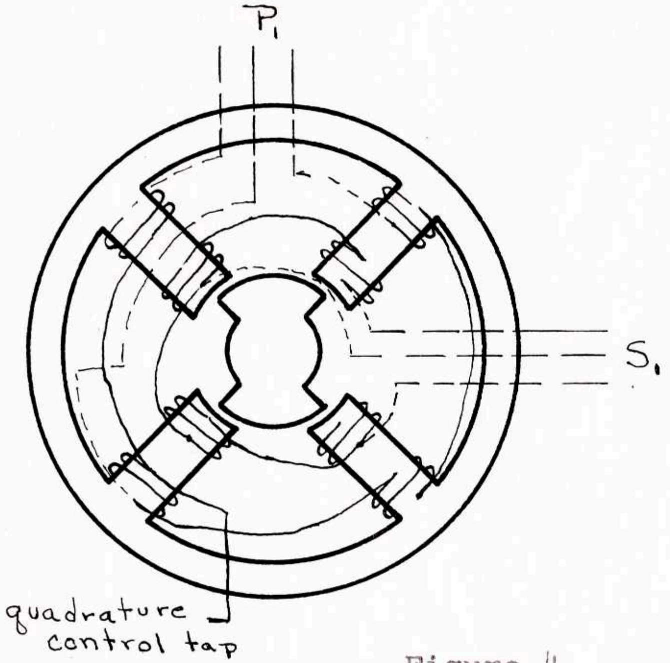
$$M = \phi\frac{d\phi'}{dt} + \phi'\frac{d\phi}{dt}$$

From electromagnetic theory $\phi = Ki$

$$M = K\,ii'$$

Sensitivity of the torque generator $S_{TG} = \frac{\text{Torque}}{i\,i'}$

FUNDAMENTAL THEORY OF SIGNAL GENERATOR



P_1 = excitation circuit

S_1 = output circuit

Figure 4

With an A.C. excitation an induced voltage is generated in the secondary circuit. Faraday's Induction Law states the $E_{induced} = - N \frac{d\Phi}{dt} = - \frac{d}{dt}(N\Phi)$ where

$$\Phi = \int B_n dS \qquad B = \mu H = \mu \oint Hds$$

For the stator coils which are similar to a solenoid:

$\oint Hds = nli$ n = turns per unit length
or $H = ni$ i = current in the coils

$B = \mu ni$ S = cross sectional area of the solenoid

$\Phi = uni\,S$ for the solenoid

Inside of the stator is the rotor which turns with the shaft and varies the area of the rotor which is exposed to Φ. This varying area, proportional to the angular rotation, changes the magnetic permiability between the two opposite poles which in turn varies the induced voltage across the secondary. Therefore $E_{induced} = S_{SG} A$

A = angular rotation of the rotor.

One of the problems inherent in a signal generator is an output voltage with no angular rotation of the rotor. This voltage arises from quadrature effects, phase differences between the four coils of the secondary circuit, and can be eliminated by introduction of a resistor in the secondary circuit between two adjoining poles. This acts as a phase shifter and results in a null voltage with no angular rotation.

To vary the sensitivity of the signal generator and to smooth the output voltage, a capacitor is placed across the output terminals in parallel with the secondary coils.

SUPPORT OF TORQUE SUMMING SHAFT

One of the important design criterions is the use of low friction bearings to support the torque summing shaft. For a static instrument the bearings need only to provide radial support, but for an instrument subjected to arbitrary forces, such as those encountered in an aircraft, the bearing must provide axial as well as radial support. Two possibilities, both meeting the requirements for a high precision instrument, are the pressurized fluid bearing (Reference 4) and the magnetic suspension. (Reference 3) Recent trends are toward the use of the magnetic suspension but, because of its availability, the pressurized fluid bearing was used for experimental purposes.

ERROR ANALYSIS

Using the basic block diagram as before, a complete error analysis can be made.

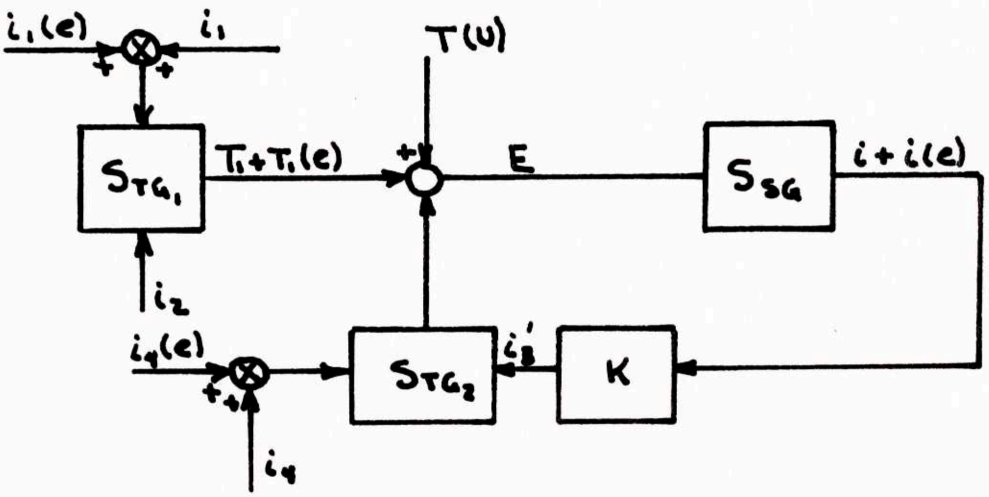


Figure 5

$i_1(e)$ & $i_4(e)$ = error in the signals that are being measured. These could arise due to the errors introduced by the transducers being used.

$i(e)$ = error introduced by the signal generator. Due to the high accuracy of the signal generator, (accurate to .01%) this will be small and can be neglected.

$i_3' = KS_{SG}A \quad K(U)S_{SG}A$

$K(U)$ = uncertainty in the feedback amplifier mostly due to the drift of the amplifier. Becuase this error is in the feedback loop, it introduces no error in the system.

$T(U)$ = torque uncertainty acting directly on the shaft due to mass unbalance. This will be the largest possible error acting on the system but with proper balancing of the shaft, it can be reduced to practically zero.

$T_1(e)$ & $T_2(e)$ = torque error from the torque generators. Like the signal generator, the torque generators are accurate to 01% and these errors can be neglected.

The differential equation of the shaft now becomes:

$$\ddot{A} + \frac{C}{I}\dot{A} + \frac{S_{TG2}S_{SG}K}{I} i_4 A + \frac{S_{TG2}S_{SG}K}{I} i_4(e) A = \frac{1}{I}\left[S_{TG1}i_1 i_2 + S_{TG1}i_1(e)i_2 + T(u)\right]$$

Measurement of i_3, the signal ratio is then, in a steady state.

$$i_3 = \frac{1}{S_{TG2}\left[i_4 + i_4(e)\right]}\left[S_{TG1}i_1 i_2 + S_{TG1}i_1(e)i_2 + T(u)\right]$$

Under the assumption that high accuracy signals will be used, i.e., $i_1(e) \ll i_1$ & $i_4(e) \ll i_4$, the largest error in the system will be that introduced by the mass unbalance of the torque summing shaft.

$$i_3 = i_2 \left(\frac{i_1}{i_4}\right) + \frac{T(u)}{S_{TG} i_4}$$

This indicates the use of a high torque generator sensitivity and, in the extreme case, the amplification of i_4 before it is fed into the torque generator.

RANGE OF THE INSTRUMENT

The restraints governing the range of the instrument are imposed by the signal and torque generators.

For the signal generator, the angular rotation of the rotor must be less than - 10°. Above this angle nonlinearity results. The linear range of the torque generator requires the torque to be less than 25,000 dyne cm.

To meet the signal generator specifications:

Constraint 1.

$$\frac{i_1}{i_4} < \frac{A1 S_{SG} K}{i_2}$$

where A1 = angle limit of the signal generator = - 10°

To meet the torque generator specifications:

Constraint 2.

$$i_1 \, i_2 \, S_{TG} < 25{,}000 \text{ dyne cm.}$$

Constraint 3.

$$i_3 \, i_4 \, S_{TG} < 25,000 \text{ dyne cm.}$$

Requiring the system to have a fast response time indicates the use of a high gain in the feedback loop. Constraints (1) and (3) are dependent upon this gain and considerations will have to be given to reach a compromise between the response time of the system and the range of the signals to be used. For most practical uses the range of the system is greater than will be required.

EXPERIMENTAL RESULTS

The set up used for experimental purposes is pictured on the following page. This shows the basic components of the system as well as the meters used for measurements and the oscilloscopes necessary to check the phase angle in different parts of the system.

(1) Microsyn signal generator.

(2) (not seen) Potentiometer for quadrature adjustment of the signal generator.

(3) Torque summing shaft.

(4) Mass balance adjustment.

(5) Both Microsyn torque generators enclosed in one housing.

(6) Pressurized fluid bearing.

(7) Pressure gauge for the fluid bearing shown at operating point, 150 psi.

(8) Amplifiers used to produce input signals and the feedback loop amplifier.

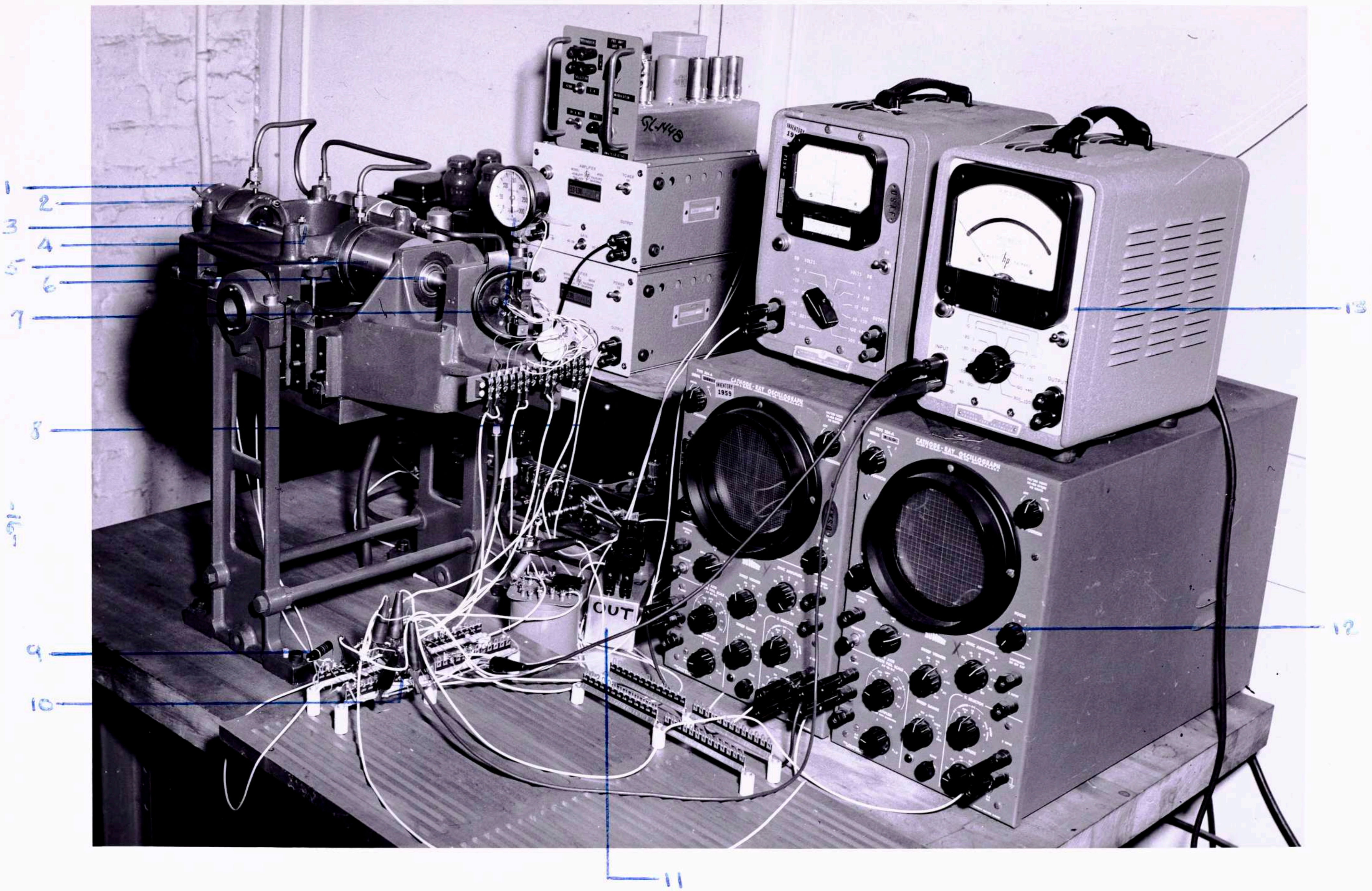
(9) Capacitor across the signal generator secondary.

(10) Bank of resistors used for measuring purposes.

(11) Phase shifter used in the feedback loop.

(12) Oscilloscopes used to check the phase between the primary and secondary coils of the torque generators.

(13) Vacumn tube volt meters for measurement.


1
2
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GL-448
INVENTORY 1959
AMPLIFIER
CATHODE-RAY OSCILLOGRAPH
OUT

Fig. 6

Many families of curves could be plotted to show the system response with varying parameters. One set of values will be used to illustrate the accuracy of the system, the effect of mass unbalance, and the lower operating limit. One input current, i_1, is varying while the other input current, i_4, and i_2, the proportional field current, are held constant. (Figure 7) From $i_1 = .006$ to $i_1 = .035$ the output current is an exact reading of the ratio $\frac{i_1}{i_4}$. Above and below these values a small error is present in the system, probably due to shaft unbalance which was not entirely eliminated. At $i_1 = .05$ amps the output has an error of 6.5% which, for certain applications requiring less accuracy and a larger range, may be within operating specifications.

Figures 8, 9, and 10, are the transient response of the system subject to a step change in i_4 with different value of the feedback gain. These transient responses illustrate the change of the frequency of the system with the change of two of the parameters, K & i_4. $W = \frac{1}{2}\sqrt{4 i_4 \frac{S_{TG2} S_{SG} K}{I} - \left(\frac{C}{I}\right)^2}$ and increases with an increase of either K or i_4. The variance in W due to K is apparent from a comparison of Figures 8, 9, and 10.

The change in W with i_4 is observed as either a positive or negative step in i_4 is applied to the system. From the fundamental equation, $i_3 = i_2 \frac{i_1}{i_4}$, it is seen that a positive step decreases i_3 and a negative step increases i_3. In turn a positive step in i_4, a decrease in i_3, should result in a larger W. From the experimental results this is seen to be the case. Initial specifications were to design a system with a natural frequency of 30 c.p.s. With a high feedback gain, just under the value for the system to oscillate, a frequency of approximately 40 cps was obtained.

Figure 11 illustrates the transient response of i_3 after the torque summing shaft has been subjected to a large impulse disturbance torque.

Checking the transient response of the system it can be calculated that $\zeta = 0.05$ which accounts for the many oscillations before reaching the final value. The specification for time requires the system to reach steady state in the shortest possible time. To satisfy this specification the system should have a large natural frequency and $\zeta = 0.7$.

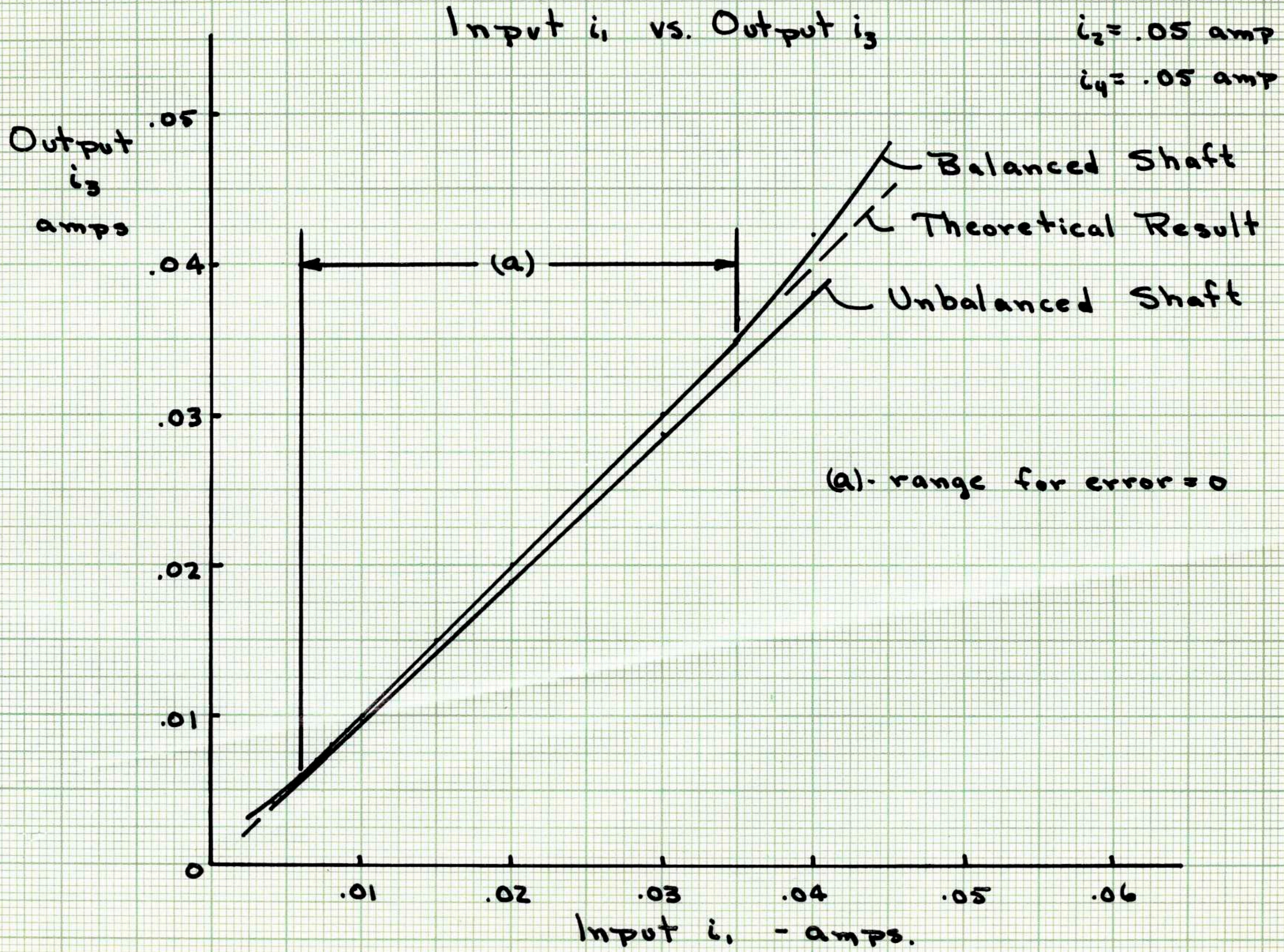
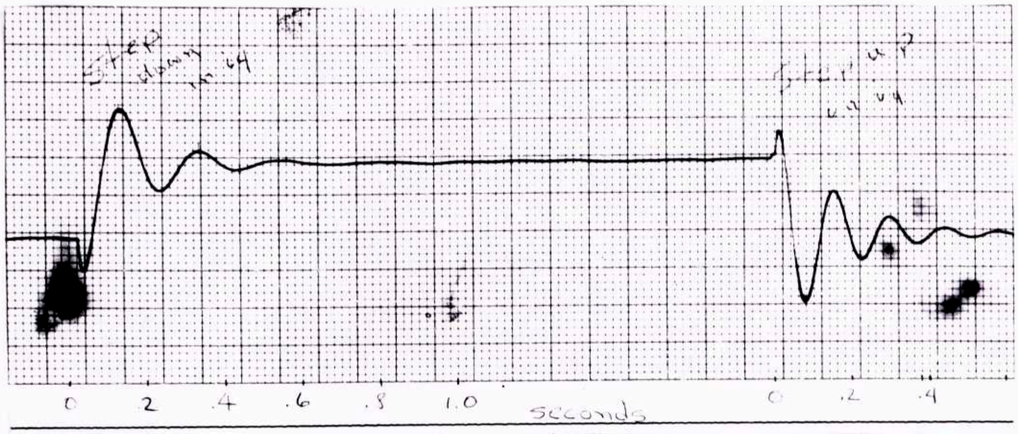

Input i_1 vs. Output i_3
i_2 = .05 amp
i_4 = .05 amp
Output i_3 amps
.05
.04
.03
.02
.01
0
.01
.02
.03
.04
.05
.06
(a)
Balanced Shaft
Theoretical Result
Unbalanced Shaft
(a) - range for error = 0
Input i_1 - amps.

Fig. 7

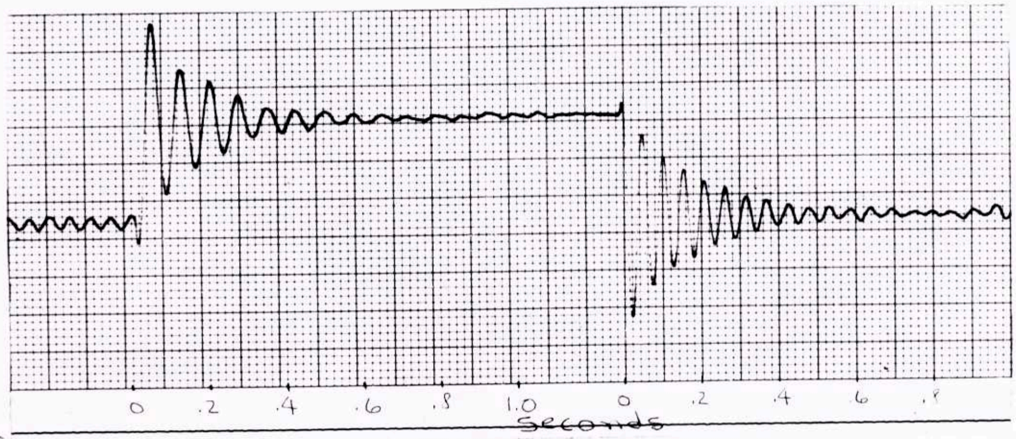
Transient Response

Step Change in input i_4



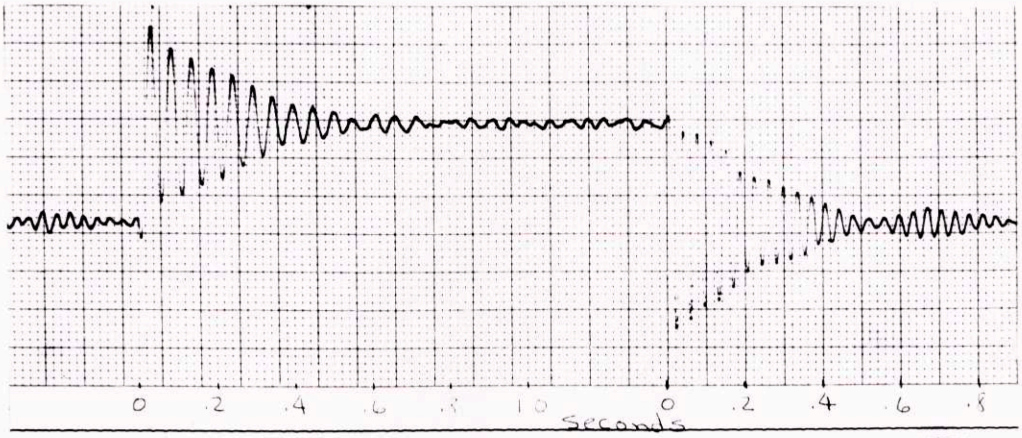
Low Feedback Gain

Fig. 8

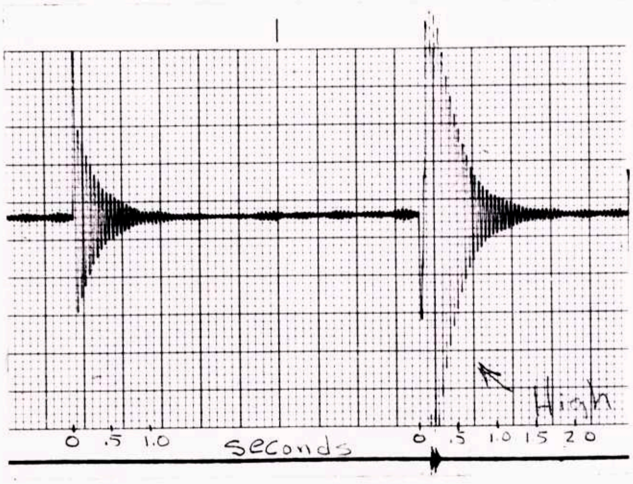


Intermediate Feedback Gain

Fig. 9



High Feedback Gain
Fig. 10



Large Impulse Torque Disturbance
High Feedback Gain
Fig. 11

CONCLUSIONS AND RECOMMENDATIONS

From the experimental results the suthor feels that the proposed instrument is worthy of further investigation pointing toward a packaged unit. Accuracy of 1% over a range of 29 units on the experimental set up used indicates a greater degree of accuracy possible on a specially designed unit. The response time of the instrument indicates its adability for use in missile and aircraft applications as well as commercial use where such a fast response time is not necessary.

For a package unit the author recommends the following considerations:

1) Use of a transistorized amplifier in the feedback loop and the possibility of a one transistor amplifier here.
2) Magnetic suspension of the torque summing shaft for a system necessitating an extremely fast response time and high accuracy. For missile or aircraft applications this would be a necessary specification.
3) For commercial use, where the response time would not be as fast and the desired accuracy not as stringent, the possibility of ball bearings for economic reasons. With the use of ball bearings a high torque generator sensitivity should be used to avoid errors due to the sticking friction of the bearings.

APPENDIX A

ONE TORQUE GENERATOR

One of the possible solutions considered was to use the same basic system with one torque generator replacing the two that were used. The nature of the torque generator, a four pole stator with two coils on each stator, suggests the use of four excitation currents to produce the torque. All aspects of the system that was used are present, i.e., the differential equation of the torque summing remains the same, and the problem was to see if the torque generator will operate on the same basic principles yielding a torque proportional to the product of the two excitation currents.

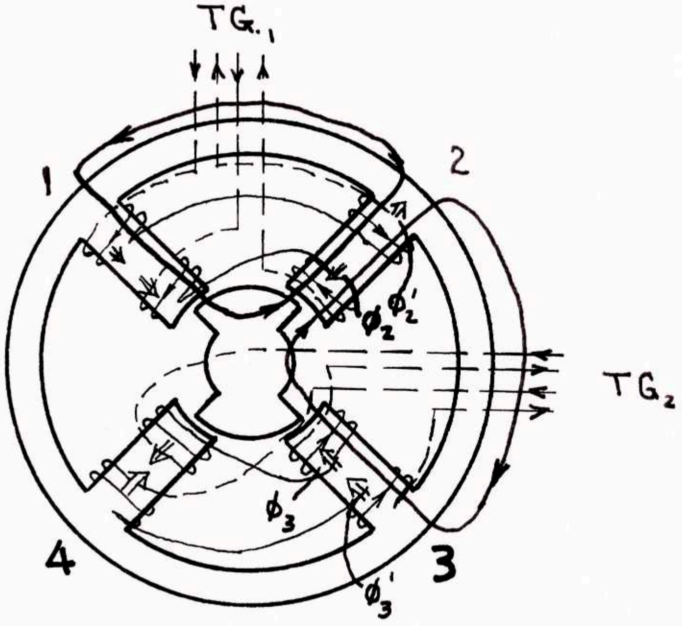


Figure 12

Poles 1 and 2 will serve as one torque generator and poles 3 and 4 will serve as a second torque generator. Observing the flux path from one coil of pole 2 it can be seen to follow two separate paths, one through pole 1 and the other through pole 3. The induced voltage in the outer coil of pole 3 is then due not only to the inner coil on pole 3 but also to the outer coil on pole 2. This cross coupling effect between the two torque generators can be eliminated by exciting them with different frequencies.

The electrical energy of the outer coil of pole 3 = $K\left[\phi_3 \left(\frac{d\phi_2'}{dt} + \frac{d\phi_3'}{dt} + \frac{d\phi_4'}{dt}\right)\right]$. The necessary requirement is to eliminate the product $\phi_3 \frac{d\phi_2'}{dt}$. From the law of orthogonality of functions the $\int_{w_1}^{w_2} \cos w_1 t \cos w_2 t \, dt = 0$ for $w_1 - w_2 = \frac{K}{t_1}$, where $K = 0, 1, 2, 3 \ldots$ and t_1 is any specified increment of time.

Although this is a solution that satisfies the requirements of the system the use of two independent power supplies, operating at different frequencies, would involve considerable expense and complications. The use of one power supply with two torque generators is a more satisfying solution.

APPENDIX B

CURRENT BALANCING DESIGN

The following circuit is another solution to the problem.

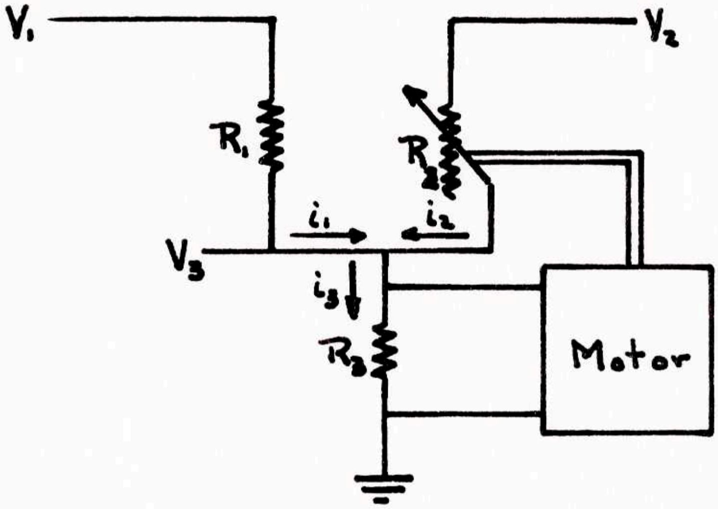


Figure 13

To prove that when $V_3 = 0$ then $R_2 = -R_1 \frac{V_2}{V_1}$

Where V_2 & V_1 are the given signals.

The following relations are apparent from the circuit:

$$V_3 = i_3 R_3 = R_3(i_1 + i_2) \quad (1)$$

$$i_1 = \frac{V_1}{R_1} \; (2) \qquad \& \; i_2 = \frac{V_2}{R_3} \; (3)$$

Substituting (2) & (3) into (1)

$$V_3 = R_3 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

When $V_3 = 0$, a situation satisfied when $i_1 = -i_2$

then $\frac{V_1}{R_1} = -\frac{V_2}{R_2}$ or $R_2 = -R_1 \frac{V_2}{V_1}$

R_2 would be a potentiometer, operated by a motor in the feedback loop, to null the voltage V_3. Although this system is a solution to the signal ratio problem it was felt that the friction drag of the potentiometer would result in a slow response system.

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