UNSTEADY AIRFOIL PRESSURES
INDUCED BY
PERTURBATION OF THE TRAILING EDGE FLOW

by

Peter Frederick Lorber
B.S., Cornell University
(1979)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
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Signature of Author

Signature redacted

Department of Aeronautics
and Astronautics
1981

Certified by

Signature redacted

Eugene E. Covert
Thesis Supervisor

Accepted by

Signature redacted

Harold Y. Wachman
Chairman, Departmental Graduate Committee

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Submitted to the Department of Aeronautics and Astronautics on February, 1981, in partial fulfillment of the requirements for the Degree of Master of Science in Aeronautics and Astronautics.

ABSTRACT

An experimental study was made of the unsteady pressures induced on the surface of a two-dimensional airfoil section by the rotation of an elliptical cylinder located behind and beneath the airfoil trailing edge. The experiment was conducted in a low Mach number flow at reduced frequencies, $K$, based on airfoil semichord of 0.5 to 6.4.

Mean airfoil difference pressures, oscillating amplitudes and phase lags were measured and compared with predictions based upon thin airfoil theory and measurements of the upwash induced by the rotating cylinder. The comparison showed good agreement for the means and qualitative agreement for the unsteady difference pressures away from the trailing edge, where an observed increase in phase lag was not predicted. This difference, and an overprediction of the unsteady amplitude were attributed to the assumptions of potential flow, a cusped trailing edge, and an unnecessarily stringent condition on the trailing edge velocity.
Unsteady airfoil pressures were measured on both surfaces, with two types of behavior being observed. The differences between the high frequency \((K>1.5)\) and low frequency \((K<1.5)\) behaviors were seen primarily in the phase behavior near the leading edge and the unsteady amplitude behavior near the trailing edge.

Amplitudes were found to increase smoothly and phases to be reasonably uniform over the rear suction surface of the airfoil.

Thesis Supervisor:  Dr. Eugene E. Covert  
Title:  Professor of Aeronautics and Astronautics
ACKNOWLEDGEMENTS

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Professor Eugene E. Covert developed the basic concept for the experiment and has provided guidance throughout the research. Dr. Charles W. Haldeman, Alexander Kanevsky and Richard Cervisi were responsible for the construction of much of the experimental apparatus. Mr. Frank H. Durgin and the staff of the Wright Brothers Wind Tunnel were of much assistance in conducting the actual tests. Finally, thanks are due to Mrs Patricia R McSweeney for typing this report.
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LIST OF SYMBOLS

\begin{align*}
a &\quad \text{airfoil leading edge radius} \\
b &\quad \text{airfoil semichord, } c/2 \\
c &\quad \text{airfoil chord} \\
C(k) &\quad \text{Theodorsen function} \\
c_p &\quad \text{pressure coefficient, } (P-P_\infty)/(1/2\rho U_\infty^2) \\
\Delta c_p &\quad \text{difference pressure coefficient, pressure surface-suction surface} \\
f &\quad \text{frequency, Hertz} \\
g &\quad \text{acceleration of gravity} \\
H &\quad \text{strength of source} \\
i &\quad \sqrt{-1} \\
k &\quad \text{reduced frequency, } \omega c/2U_\infty \\
M &\quad \text{Mach number} \\
P &\quad \text{static pressure} \\
P_\infty &\quad \text{static pressure at freestream conditions} \\
q &\quad \text{velocity} \\
q_r,q_\theta &\quad \text{radial, tangential velocity components in computational plane} \\
Q &\quad \text{wake integral} \\
Re &\quad \text{Reynolds number, } U_\infty c/\nu \\
t &\quad \text{time} \\
u,v &\quad \text{horizontal, vertical perturbation velocity components in physical plane} \\
U &\quad \text{horizontal mean velocity along airfoil chord} \\
U_{\text{inf}},U_\infty &\quad \text{freestream velocity} \\
v_a &\quad \text{vertical velocity along position of airfoil chord} \\
x,y &\quad \text{horizontal, vertical coordinates in physical plane} \\
X,Y &\quad \text{horizontal, vertical coordinates in computational plane} \\
y_c &\quad \text{vertical position of airfoil camber line}
\end{align*}
LIST OF SYMBOLS (continued)

\( \alpha \)  
flow angle

\( \alpha_{\text{mean}} \)  
mean flow angle

\( \alpha_{\text{fluct}} \)  
amplitude of fluctuating flow angle

\( \alpha_{\text{else}} \)  
remainder of flow angle

\( \Delta \alpha_A, \Delta \alpha_B \)  
subharmonic deviation flow angle parameters

\( \gamma_W \)  
wake vorticity

\( \Gamma_0 \)  
bond vorticity

\( \varepsilon \)  
added velocity fraction due to blockage

\( \zeta \)  
wake coordinate

\( \Theta, \text{ELL} \)  
elliptical cylinder angle of rotation from horizontal

\( \Theta_{t,r} \)  
polar coordinates in computational plane

\( \nu \)  
kinematic viscosity

\( \rho \)  
density

\( \phi \)  
phase lag relative to elliptical cylinder

\( \Delta \phi \)  
pressure system phase delay

\( \phi \)  
disturbance velocity potential

\( \phi_{\text{UP}}, \phi_{\text{LO}} \)  
upper and lower computational surfaces velocity potentials

\( \omega \)  
radian frequency

\( \frac{D}{Dt} \)  
substantive derivative, \( \frac{\partial \phi}{\partial t} + \mathbf{U} \cdot \nabla \phi \)
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Upper Surface Pressure Power Spectral Density, $U_{inf}=30$ mps, $K=1.0$
Chapter I

INTRODUCTION

In order to make accurate measurements upon a turbulent boundary layer in an oscillatory adverse pressure gradient it is simpler if one can hold a surface stationary and oscillate the external flow. In this manner boundary layer velocity profiles and skin friction measurements may be readily obtained. An experimental arrangement was devised that perturbs the flow about a stationary two-dimensional airfoil by means of a rotating elliptical cylinder located behind and beneath the airfoil trailing edge. This report is a description of the high reduced frequency unsteady airfoil surface pressures that result from such a perturbation.

Previous studies of unsteady airfoil pressures have involved many diverse techniques for producing the unsteady flow and analyzing the resulting pressures. They have also covered a wide range of operating velocities and reduced frequencies, defined here as radian frequency times semi-chord divided by free stream velocity. Some of the more recent techniques included the following.

Commerford and Carta used the natural shed vorticity of a cylinder to induce unsteady pressures of reduced frequency $K=3.9$ upon an airfoil located above and behind it.
Pressures were recorded at five chordwise locations and integrated to give the unsteady lift, both of which were then compared to several thin airfoil theories (1).

Sagena, Fejer and Morkovin rotated shutters downstream of a stationary airfoil to produce perturbations at reduced frequencies of 0.18 and 0.9. Instantaneous pressure distributions were reported (2). Satyanarayana and Davis oscillated their airfoil about the one-quarter chord position. Unsteady pressures were obtained for reduced frequencies ranging between 0.05 and 1.2 at 5 chordwise locations, primarily in the trailing edge region (3).

In contrast to the above low velocity (less than \( M=.3 \)) tests, Davis and Malcom studied airfoil vertical displacement (plunging) and rotation (pitching) at up to transonic velocities. Unsteady and static pressures were measured at 20 chordwise location for reduced frequencies ranging from \( K=.025 \) to .25. Averaged pressure distributions were reported for the various high subsonic and transonic flow regimes (4).

Further, Fleeter studied unsteady pressures near the trailing edge of fixed stators perturbed by the rotor wakes in a large low speed compressor. The behavior for cambered and uncambered blades at various incilence angles was studied at fundamental reduced frequencies near 8.0 (5).
As can be seen from the above and from the summary in Table 1, the diverse range of situations studied has resulted in a varied set of conclusions being drawn. Overall, however, two trends were noticed. First, in all but one of the studies discussed, the unsteady difference pressure approached zero at the airfoil trailing edge. The exception was the result in Reference 5. Here, difference pressure amplitudes increased from x/c=.90 to x/c=.97. However, the suction surface transducers were located on a different airfoil in the cascade from the pressure surface transducers, possibly introducing phase differences or other discrepancies.

The second trend was a lack of quantitative agreement between thin airfoil theory predictions and experimental results for difference pressure amplitudes and phases as reduced frequencies increased beyond K=0.5 (1,3,4,5).

The inclusion of the present study in the table demonstrates the comparatively wide range of chordwise pressure measuring locations and reduced frequencies studied here. It was hoped that the wide coverage would lessen the change of missing significant effects by concentrating on a more limited region.

Previous experimentation involving the present concept consisted of measuring steady state parameters. Kanevsky (6) first measured the pressure induced on a two-dimensional airfoil by circular cylinders with diameters
ranging from 0.063 to 0.250 of the airfoil chord, located at various downstream locations from x/c=1.175 to x/c=1.300 with y/c between -.275 and -.500. Based upon this information the locus of cylinder positions giving maximum effect on the airfoil was determined. Next an elliptical cylinder was designed and constructed so as to simulate, depending upon its orientation, the various sized circular cylinders. Steady pressure distributions were then determined along the airfoil surface, for the various elliptical cylinder orientations. Cervisi (7) continued this work and also measured turbulent boundary layer velocity profiles at two locations on the rear of the upper (suction) surface of the airfoil.

Mechanical Apparatus

The experimental measurements were performed at M.I.T.'s Wright Brothers Memorial Wind Tunnel, a low-speed facility with an elliptical test section having axes 2.3m x 3.05m (7.5 ft x 10 ft). Currently the wind tunnel is capable of being operated continuously at atmospheric pressure at velocities up to 62.5 mps (200 ft/sec). The typical free-stream turbulence level varies between 0.5 and 1.0% of the mean velocity. In addition to turbulent fluctuations, noise generated by the wind tunnel fan at harmonics of its blade passing frequencies are present. This noise can be significant for tests done at freestream velocities of 9 mps (30 ft/sec) or less. The mean freestream velocity is
measured using a Prandtl-design pitot static probe connected to a slant-tube alcohol manometer.

The current experiment occupied the space between two vertical sideboards 50.8 cm (20 in) apart. These sideboards took up the 208 cm (80 in) height of the test section, and were 122 cm (48 in) in length, or 2.4 airfoil chord lengths. The sideboards diverged in the downstream direction to compensate for their increasing boundary layer displacement thicknesses. The static pressure along the sideboards was measured and the divergence angle was modified so as to minimize the change in static pressure with streamwise position. Tape and clay were used to seal all joints between the airfoil and the sideboards to eliminate secondary flows and encourage two-dimensionality.

A N.A.C.A. 0012 basic thickness airfoil of 51 cm (20 in) chord could be mounted at fixed angle of attack between the sidewalls. This section was selected for its well understood steady behavior and smooth pressure gradients along the rear suction surface. The airfoil was machined in two sections from aluminum, providing ample interior space for instrumentation. Reynolds numbers based on the chord ranged from about 300,000 for velocities of 9 mps to 1 million for 30 mps.

The unsteady perturbation was provided by rotating an elliptical cylinder behind and beneath the airfoil trailing edge. The dimensions of the axes of the cylinder were
6.50 x 13.77 cm (2.56 x 5.42 in). As the cylinder was to be spun at up to 3000 rpm, accurate balancing of this aluminum element was required. Power for the rotation was transmitted from a 2 hp D.C. motor via a 2:1 ratio belt drive. A flywheel was attached to the cylinder shaft to reduce fluctuations in the rotational speed. Finally, in order to delay separation on the cylinder and thereby increase perturbation amplitudes, #120 grit was applied to roughen the surface.

Figure 1 shows a typical test section front view, with the key elements identified. Figure 2 shows a side view, indicating the dimensions and positions of the components.

**Instrumentation**

The first series of unsteady measurements involved the velocity perturbation created by the rotating elliptical cylinder. In order to examine separately the effect of this element, airfoil was removed from the test section, as shown in Figure 2. Velocity measurements were then made along the position of the centerline of the airfoil from the position of the leading edge to 10% of chord downstream of the trailing edge position. A cross hot wire was operated using Flow Corporation Series 900 components, including constant temperature control circuitry output linearizers, and a high frequency sum and difference amplifier. Platinum wires of 12 micron diameter and length
approximately 0.25 cm set at ± 45° angles. The sum of the two outputs was related to the horizontal velocity and the difference to the vertical velocity. The quotient of the difference over the sum was calibrated in terms of flow angles. Stiffeners and support wires were installed onto the probe body as needed to reduce mechanical vibrations caused by turbulence and by the cylinder drive system.

In order to measure the absolute flow angle accurately at varied horizontal positions a means of determining a reference angle was required. The expected flow angle produced by a stationary elliptical cylinder oriented with its major axis horizontal, the minimum perturbation position, was calculated using conformal mapping onto a circular cylinder. These calculated angles were relatively small, being 2.0 degrees at the trailing edge position. Prior to collecting each set of flow angle data, the hot wire balance was adjusted to give zero output at the reference configuration. During data reduction, the calculated flow perturbation was added to the measurements to produce corrected absolute angles.

The second series of unsteady measurements involved airfoil surface pressures at zero angle of attack. The upper and lower surfaces were each fitted with 17 pressure taps, a 35th tap being located at the leading edge. The most rearward taps were placed at x/c=0.98. Figure 3 shows the tap locations. The taps were connected to 25 to 28 cm long, 1.5 mm diameter tubing, fitted with individually sized
and positioned yarn inserts to avoid resonance problems in the frequency band of interest. The tubes leading from the taps on each surface were connected to 48 port DS448 scanivalves. Pressures were converted to voltages using two Setra model 237 capacitive transducers, also located within the airfoil. These transducers are good for both mean and unsteady pressures of up to $+3800 \text{ Nt/m}^2 (+.50 \text{ psi})$. This arrangement allows upper and lower surface pressures at any location to be simultaneously measured, allowing determination of instantaneous difference pressures.

Calibration of the frequency response of this pressure system was done for both amplitude and phase. The procedure was to compare the response to an acoustic sinusoidal signal of two transducers located on a 10 cm by 6 cm cylindrical cavity. One transducer was mounted with its bare diaphragm exposed. The other was connected using the operational system of tubing, yarn and scanivalve. Since the response of the bare diaphragm transducer was known to be flat up to well above the 1000 Hz maximum frequency of present interest, it served as a reference. The difference between the two transducer outputs was assumed to represent the effect of the operational system.

The results of the calibration were as follows. Figure 4 shows the amplitude ratio for input frequencies from 20 to 2000 Hz. The response is flat up to 600 Hz, following which there is some variation of up to 9 percent
in the range 600 to 800 Hz. Above 800 Hz the ratio drops rapidly, reaching 60 percent at 950 Hz. Figure 5 illustrates the lag in phase of the system as compared to the bare diaphragm transducer. Over the range shown a least-squares fit of the calibration data results in

$$\Delta \phi = 0.35 f^{1.03}$$

with $f$ in Hertz and $\Delta \phi$ in degrees.

For a transmission line with constant propagation velocity at all frequencies the relation would be

$$\Delta \phi = 360 \times \text{length} \times \text{frequency} / \text{velocity}$$

A comparison between the measured delay times and the delay time that would result in the least squares fit to the phase lag is shown in Figure 6. For a typical delay time of 1.1 ms the propagation velocity based upon a 25 cm length is 230 mps. The difference between this velocity and the tube's speed of sound can be accounted for by both additional delays in the scannivalve and, more importantly, by the non-linear character of the flow in the tubing plus yarn system. The primary calibration was done at a peak-to-peak amplitude of 120 mv, with verification of amplitude independence at 50 and 300 mv. This covered the range of unsteady pressure signals typically encountered.
Pressures were non-dimensionalized using the mean freestream dynamic pressure, as measured by the pitot-static probe located between the sidewalls approximately one chord length above the airfoil. In addition to being measured with the slant tube manometer, the dynamic pressure was also read using the wing-mounted transducers. These output voltages were used to produce the non-dimensional cp's, thus avoiding complications that might result from shifting transducer gains if the dynamic pressures were measured using separate transducers from those that were used to measure the surface pressures.

**Electronics and Data Acquisition**

A photoelectric sensor was installed viewing the elliptical cylinder shaft. By covering all but a thin strip of the shaft with non-reflecting tape a pulse was generated each time the shaft reached position \( \theta = 0 \), when the major axis was horizontal. This pulse train served two purposes. First, the use of an averaging counter to measure the period gave the frequency of rotation. Second, the pulses provided a constant phase trigger for the instrumentation.

A generalized block diagram of this instrumentation is shown in Figure 7. Once the hot wire signals have left the sum/difference amplifier and the pressure signals the transducer amplifiers, their paths were quite similar. Mean values were determined by passing the signals through an
R.C. low pass filter with a one second time constant and reading the result on a United Systems Corporation Model 268 D.C. millivoltmeter. Root mean square signal amplitudes were obtained using Hewlett-Packard Model 3400A RMS voltmeters. Selected signals were recorded using a Hewlett-Packard Model 3960 four channel instrumentation recorder.

The primary means for analyzing the unsteady signals was by means of phase lock averaging. This was done using a Princeton Applied science model THD-9 wave form eductor, which drew out the periodic signal from the noise and turbulence which masked it. Following the pulse trigger, the period of revolution of the cylinder was divided into 100 equal intervals. An average value of the signal during each interval was then determined over a preset time constant. This time constant was typically of the order of 50 to 100 revolutions. As long as there were no signals present with period greater than one cylinder revolution this procedure was equivalent to the classical ensemble averaging, where a series of identical experiments are performed simultaneously. The result of the phase lock averaging was a smooth signal which contained the frequency components of the input signal that are harmonics of the cylinder revolution frequency. All other frequencies were eliminated over long averaging times.
This output signal was displayed in two manners. First, it, together with the raw or unaveraged signal, was shown on a Tektronix Model 5103N dual beam storage oscilloscope. This gave a check on the behavior of the experiment and the operation of the waveform eductor. Second, the phase lock averaged signal was plotted using a Mosely Model 135 x-y recorder. These plots were later measured for amplitudes and an estimate of the phase lag relative to the cylinder was made. Phase lag is here defined as the delay in the position of the minimum of a quantity from the horizontal orientation of the cylinder. One revolution of the cylinder corresponds to 720 degrees of phase.

Pressure signals were also analyzed using a Federal Scientific Co. Model UA14 Ubiquitous Spectrum Analyzer and Model 1014 Spectrum Averager. These instruments were set to produce the power spectral densities of their inputs over selected ranges of from 0-100 Hz to 0-2000 Hz, depending on the high frequency content of the signal. Spectra were also recorded using the chart recorder.

Test Conditions Studied

Velocity perturbation measurements were to be taken at freestream velocities of 9, 20 and 30 mps (30, 65 and 100 ft/sec). For each freestream velocity four elliptical cylinder rotation rates were chosen. These rates were based on reduced frequencies of 1.0, 2.0, 4.0 and 6.0. Because of the symmetry of the elliptical cylinder the fundamental
perturbation frequency is twice the elliptical cylinder rotation frequency.

In addition to these cases, pressure data were to be taken at reduced frequencies of 0.5 and 1.5 for velocity 20 mps. These conditions were added in order to examine regions of transition in the flow character. In order to verify previous work on the quasi-steady behavior of the wing-elliptical cylinder system, pressure data were also taken for fixed cylinder orientations of \( \theta = 0, 45, 90 \) and 135 degrees.
Chapter II

RESULTS OF INDUCED VELOCITY TESTS

As discussed above, the initial unsteady experimentation involved the measurement of the velocities induced by the rotating elliptical cylinder with no airfoil present. At first the cylinder was rotated in the counterclockwise direction, so as to produce a mean circulatory downwash upstream of the cylinder. At this time no roughness had been applied. A qualitative evaluation of the induced velocity at the position of the airfoil trailing edge over the desired velocity and frequency ranges was then made. Roughness was added and the survey repeated. Phase lock averaged data was next recorded at 8 stations going upstream to the position \(x/c=0\). The cylinder was then spun in the opposite direction so as to produce a mean upwash, and the measurements repeated.

Several general trends became apparent. First, when compared to the smooth case, the rough surfaced cylinder produced generally higher mean and fluctuating upwash amplitudes and more regular velocity wave forms. Increased regularity refers to the reduction of frequency components not equal to the fundamental (twice the rotation rate). The increase in amplitude was thought to be a result of delayed separation on the roughened cylinder surface,
principally at orientations where the angle of attack of the major axis was low. A reduction in the separation region would decrease the effective obstruction area, defined as the projected area of the cylinder plus the separation region. Thus the difference in obstruction area between cylinder angles of attack of $\theta=0$ and $90$ degrees would increase, since there was likely to be little change in the separation region on the bluff shape seen near $\theta=90$ degrees. The increased regularity of the waveforms was assumed to be a result of the roughness producing a more regular definition of the separation position on the cylinder surface. Also, the roughness and resulting thicker boundary layers would tend to mask any irregularities in the cylinder surface itself.

The second trend was improved behavior when the cylinder direction of rotation was reversed. Because of symmetry the induced velocities should have the same character at the centerline of the cylinder and at equivalent points above and below this centerline. Since the measurements were performed along the airfoil chord line, 14 cm (5.5 in) above the ellipse centerline, there is no reason for symmetry to result in the same flow behavior at this point. Mean upwash amplitudes were increased largely due to the fact that the circulation induced velocity was now in the same direction as the velocity induced by having to direct the flow around some mean obstruction area.
Fluctuating amplitudes and waveform regularity were also increased. Clockwise rotation resulted in flow acceleration on the upper portion of the cylinder, whereas counterclockwise rotation decelerated that portion of the flow nearest to the measuring stations. The acceleration due to rotation would tend to counteract adverse pressure gradients on the cylinder upper surface, delaying separation and improving the behavior in the region above the cylinder centerline.

Because of these considerations the operational configuration for the elliptical cylinder was chosen to be a roughened surface rotating in the clockwise direction. Typical samples of horizontal and vertical velocity fluctuations before signal processing are illustrated in Figures 8 and 9. Both were taken at reduced frequency $K=1.0$, the first at the trailing edge position and at $U_{\text{in}} = 20$ mps; the second at midchord position and 9 mps. Largely due to the increased perturbation amplitude with respect to the mean velocity, the trailing edge data have significantly lower relative noise levels, a trend that was even more marked for the leading edge measurements, where it was hard to see the periodic signal without using phase lock averaging. For some measurements at the leading edge positions, the induced flow velocities had become so small that mechanical vibrations due to the cylinder drive were observed in the output signal.
Another noticeable feature is the difference in phase between the vertical and horizontal velocities. This difference was also seen during a check of the quasi-steady behavior, performed by fixing the cylinder in various orientations and measuring mean flow velocity components at the trailing edge position. Here the horizontal maximum led the vertical maximum by about 60 degrees while the minimum led by about 90 degrees. These valves are consistent with the unsteady results and imply that the phase difference is primarily a quasi-steady phenomenon. The phase difference could probably be deduced from a proper modelling of the steady elliptical cylinder as a velocity source and sink combination at various orientations and calculating the velocities induced at the measurement station. For present purposes this was not done, since the linear theory used to predict airfoil difference pressures only accounted for vertical velocities, approximating the horizontal velocity by a constant in time. This reflects a guiding philosophy in the study of the elliptical cylinder. Wherever possible it should be viewed as a tool for producing unsteady airfoil pressures, not as the primary object of interest.

The next set of figures compare phase lock averaged upwash data for one cylinder revolution with sinusoidal curves of the same amplitude and frequency. Figure 10 shows the case of speed 20 mps (65 fps) and reduced frequency 1.0.
The two halves of the cycle are seen to be quite similar, indicating that for these conditions the expected symmetry holds. The increasing amplitude portions are much steeper than the decreasing regions. This is presumably a result of the fact that in the steep forward portion the elliptical cylinder is at $0 < \theta < 90^\circ$, or positive angle of attack, and thus separation is likely to occur over the upper rear portion. Once separation has occurred, the obstruction area increases so that the maximum occurs before the $\theta = 90^\circ$ position is reached. On the other hand, during the $90 < \theta < 180^\circ$ portion, the elliptical cylinder is at negative angle of attack, and the upper surface flow remains attached. The obstruction area, therefore, follows the ellipse position in a more gradual fashion.

Figure 11(a) illustrates the same situation at a higher reduced frequency $K = 6.4$. At this higher frequency there is not as much time available for the separation during $0 < \theta < 90^\circ$ to occur, and therefore the velocity perturbations follow the cylinder position more faithfully, giving the waveform a more sinusoidal appearance. However, another undesirable phenomenon has begun to be seen. This is the difference between the two halves of the cycle, referred to here as subharmonic deviation. Figure 11(b) shows a somewhat more extreme case for the same frequency with velocity 9 mps (30 fps).
Subharmonic Deviation

The subharmonic deviation is perhaps the most unwanted and least understood effect seen in this study. It was planned to measure velocity perturbations for 3 velocities at frequencies $K=1.0$, 2.0, 4.0 and 6.0. However, operation at some of the resulting velocity-frequency combinations resulted in irregular waveforms. The subharmonic deviation was a common form of irregularity.

Two parameters may be defined to quantify this phenomenon (Figure 12(a)). $\Delta \alpha_1$ is the difference between the two amplitudes of the increasing portion of the waveform. $\Delta \alpha_2$ is the difference between the two maxima or between the two minima, whichever is greater. For reduced frequencies $K<2$ these parameters were less than or equal to 5 percent of the fluctuating peak-to-peak amplitude $\alpha$. For some higher frequency-velocity combinations they grew to 70% or greater of $\alpha$.

By adjusting the rotational frequency a considerable reduction in the $\Delta \alpha$'s could be achieved. For example, changing $K$ from 4.0 to 3.9 at velocity 20 mps (65 fps) resulted in reducing the $\Delta \alpha$'s to between 10 and 20 percent near the trailing edge. More dramatically, an increase in $K$ from 6.0 to 6.4 and 6.3 for velocities 20 mps and 9 mps (65 fps and 30 fps) reduced the $\Delta \alpha$'s from over 70 percent to near 10 percent in the trailing edge region (Figures 12(b) and 12(c)). Because of motor speed limitations the reduced frequency for the 30 mps (100 fps) case could not be increased
above \( K=5.7 \), so that a reduction to \( K=5.1 \) was needed to get the \( \Delta \alpha \)'s down to 30 percent of \( \alpha \).

Over the more forward positions the behavior was similar, with less than 10 percent subharmonic deviations for \( K<2 \), and higher, up to 80 to 90 percent valves for the worst high speed-frequency combinations.

Two classes of waveform were observed at speed-frequency combinations displaying subharmonic deviation. During testing, fluctuating signals were displayed in real time using the storage tube oscilloscope. Most often the sweep rate was set so as to show a time corresponding to one period of the rotating cylinder. This gave maximum resolution of the waveform and matched the phase lock averaged data from the eductor. However, upon occasion a series of sequential cycles were displayed.

It was observed that the subharmonic deviation waveforms repeated themselves with period equal to the cylinder period, as expected from the eductor results. But, strings of from 5 to 20 cycles were seen which had much lower deviations than those of the typical waveform at that operating point. This behavior is illustrated in Figure 13. Both traces were obtained at the same air speed and frequency. It should be noted that both waveforms have gone through the eductor, using an averaging time constant of 5 seconds, so that the low deviation waveform was significantly longer.
lived than usual, 5 seconds corresponding for this frequency to about 150 periods.

In most cases, however, the high deviation case was by far the dominant state, with the more desirable waveform appearing only briefly. Thus, it usually had little effect on the phase-lock averaged output.

Examination of Figure 13 reveals several characteristics which may aid in understanding the subharmonic deviation. First note that the two plots were made at the same scale and at about the same time, so that the reference voltage of the eductor, indicated by the horizontal lines, should not have drifted significantly, as it was sometimes prone to do. Relative to this reference, the first maximum of the higher deviation state has dropped somewhat from its low deviation position, while the second maximum has increased. The more sizeable difference between the two states, however, was in the minima. The first minimum was larger for the high deviation state while the second minimum was much lower.

One interpretation of this phenomena is the following. Assume that the earlier that separation occurred on the elliptical cylinder, the larger the obstruction area, and therefore the larger the induced flow angle that was observed. For elliptical cylinder orientations near $\theta=90$ degrees the separation point was relatively fixed, since the
pressure gradient grew rapidly near this relatively sharp corner. Thus any large shift in the separation point at this orientation was unlikely, which agrees with Figure 13, since the maxima have not shifted by any great extent.

For orientations near the minimum $\theta=0$ degree orientation, the pressure gradient had a more gradual increase, so that a significant shift in the separation point position would seem to be more likely. If the separation position near $\theta=0$ degrees were to move forward on the first half cycle and aft on the second half, a velocity perturbation waveform such as Figure 13 could be produced.

This explanation has, of course, only succeeded in moving the location of the unknown factor backward one level, since the cause of the periodic oscillation in the separation position is still undetermined. Also, the above interpretation is in essence quasi-steady, whereas the phenomena in question occurred primarily at the higher frequency operating conditions, where quasi-steady behavior was not found in general. Subharmonic deviation will be discussed further in connection with its appearance in the surface pressure waveforms.
Induced Velocity Distributions

The results of the velocity tests that are of primary importance for the present purposes are the plots of the mean upwash and the amplitude of the fundamental frequency component as functions of horizontal distance $x/c$. Note that the cylinder axis is located at $x/c=1.175$. For the mean case the time average of the vertical velocity was divided by the time average of the horizontal velocity and interpreted as a local mean flow angle in radians. Calibration corrections to the angle were applied at this point. Fluctuating amplitudes were found by measuring the graphical phase-lock averaged results, dividing by the local mean horizontal velocity, applying calibration corrections and interpreting the result as a fluctuating flow angle amplitude in radians. Therefore the local flow angle would be

$$\alpha(x,y=0,t) = \alpha_{\text{mean}}(x)+\alpha_{\text{fluct}}(x)\sin(\omega t-\phi(x))+\alpha_{\text{else}}(x,t)$$

where $\phi(x)$ is the phase lag and $\alpha_{\text{else}}(x,t)$ is the remaining turbulence, fan noise, harmonics and subharmonics. The elliptical cylinder major axis orientation in these terms is

$$\theta(t) = 1/2 \cdot \omega t$$
Figures 14 to 18 plot the fluctuating amplitudes for the cases $K=1.0$, 2.0, 4.0 and 3.9, 5.1, 6.3 and 6.4. Each figure contains data from all freestream velocities measured at each given reduced frequency. It can be seen that a strong similarity exists at each reduced frequency, with the variation with freestream velocity being typically less than or of the order of the estimated scatter of the measurements. Due to the combination of the errors due to the actual hotwire measurements, the phase lock averaging process, the application of steady calibration data for unsteady measurements and the hard analysis of the output, this probable error is estimated to be about 0.1 degree (.002 radians) in amplitude.

The two most important qualitative features about the fluctuating amplitudes are the following. First the amplitude was much greater at the rear, near the rotating cylinder, as would be expected. Typically, the amplitude was 8 to 10 times greater at the trailing edge position than at the 20 percent chord position. Second, the amplitude decreased markedly with increasing induced frequency, amplitudes at $K=6.4$ being approximately 3 times smaller than amplitudes at $K=1.0$. This appears understandable, since the same changes in geometry produced a force that resulted in an acceleration of the fluid $Du/Dt$. As reduced frequency was increased, the time scale was decreased, requiring the velocity scale to also decrease at a similar
rate. The actual process was more complicated, of course, but the fact that the relative decrease in velocity amplitude appears to be about one-half of the relative increase in reduced frequency seems to indicate that the basic concept is not completely in error.

Figures 19 to 21 plot the mean local flow angles as functions of the horizontal position. Here each figure contains the data for an individual freestream velocity at all four studied reduced frequencies. Increasing curve number labels correspond to increasing frequency as stated on each figure. For the mean velocity the expected error is greater than for the fluctuating components; the estimate is 0.4 degrees, or .007 radians. This is due to several factors. First, observed and not completely compensated drift in the hot wire baseline voltage due in part to temperature changes in the wind tunnel. Second, misalignment of the probe and/or the position of the elliptical cylinder when it is used in zeroing the hot wires. Third, high noise and turbulence levels, particularly at lower velocities, made reaching equilibrium in the time averaging circuitry occasionally uncertain. Variations in the mean are most obvious for the low velocity runs, where such errors are a larger fraction of the true mean.
One major qualitative feature of these plots was again an increase in induced angle as the trailing edge was approached. However, some turning over was seen beyond the trailing edge, which is not unreasonable, since the elliptical cylinder itself begins at from \( x/c = 1.036 \) to \( x/c = 1.061 \), depending on its orientation. The flow should have begun turning over as it passed over the cylinder. A second feature was the general increase in mean flow angle with reduced frequency. This was principally the result of the increased circulation generated by increasing the rotation rate. Finally, there was a good correlation between the various freestream velocities at the same reduced frequency, although not in general as good as was observed for the fluctuating portions.

The final characteristic of the induced velocity field to be discussed is the phase lag. The general features were the following. The lag in phase of the vertical velocity behind the cylinder was a maximum at the trailing edge, where it attained a value of between 25 and 30 degrees for higher frequency \( K \geq 2 \) cases, and between 5 and 20 degrees for the \( K = 1 \) situations. The phase then typically dropped off steadily until around the 70% of chord position, at which point it was about 15 to 20 degrees less than at the trailing edge. Further reductions were relatively modest, and were often lost in the low amplitudes seen near the leading edge position.
The larger phase lags at higher frequencies were perhaps the result of a finite propagation time for the perturbation, combined with the decreasing period. The decrease in phase lag with distance away from the source is less easy to understand, but it may be a result of the differing dependence on distance of the various components of the unsteady signal. For example, if an in-phase component had a radial dependence similar to a source, inversely proportional to distance, while an out-of-phase component behaved similar to a doublet, inversely proportional to the square of the distance, as distance increased the in-phase component would increase in relative importance, thus reducing the phase lag.

The procedure which was used to obtain the phase was based upon a visual matching of the phase-lock averaged waveform with a pure sinusoid of equal frequency and similar amplitude. Qualitative matching criteria included positions of the maxima and minima, the crossing of the mean ('zero') value, and equalizing the areas above and below the sinusoid. Once the best match was achieved the difference in time origin between the curves was measured, and converted into a phase lag in degrees.
The probable error in the phase determinations are relatively large, with the standard deviation in the hand measurements being from 2 to 3 degrees. Further difficulties arise because of the problem of estimating phases for dissimilar waveforms, since the vertical velocity does not maintain the same shape at all positions along the chord. Also, since measurements were taken for all speed-frequency combinations at a particular chordwise location followed by a move to a new position, any drifts in time could show up as trends along the wing. Therefore, phase measurements to within 5 degrees of the true values of the fundamental harmonic are probably all that can be expected from this technique.

Equivalent Camber and Angle of Attack

The stationary airfoil-rotating elliptical cylinder system may be compared to a system with no rotating cylinder, but with an airfoil that rotates and changes its camber. That is, the actual situation, a fixed airfoil in an unsteady non-uniform flow field, is modelled as a moving airfoil in a uniform flow field. In a quasi-steady situation, where all changes are assumed to be taking place at a rate slow enough so that the instantaneous flow is the same as that of a steady system with the
identical physical configuration, the relation is straightforward. The measured unsteady upwash is simply integrated from the leading edge position to give the equivalent airfoil chord line.

\[ y_c(x,t) = \int_0^x \frac{v(x',y=0,t)}{U(x)} \, dx' \]

The equivalent angle of attack is then defined as

\[ \alpha(t) = \arctan \left( \frac{y_c(c,t)}{c} \right) \]

The equivalent camber line of the airfoil then becomes

\[ \bar{c}(x,t) = y_c(x,t) - x \tan(\alpha(t)) \]

Figure 22 shows the distribution of equivalent camber for the case of freestream velocity 9 mps (30 ft/sec) at a reduced frequency of \( K=1.0 \). Plotted are camber lines corresponding to the minimum, mean and maximum conditions. Also listed on this figure are the respective values for the equivalent angles of attack. The same information is shown in Figure 23 for the case of reduced frequency \( K=4.0 \). One noticeable feature of these figures is that the position of maximum camber occurred at approximately 60 percent of chord, with a steep fall off in camber over the final third
of the airfoil. This is a result of the fact that the maximum upwash occurred over this region. Also apparent is the increase in mean camber and angle of attack with increasing frequency, while the fluctuating components decline. This follows from the induced angle results. Maximum values of the equivalent camber reached 2 to 2.5 percent, in the same range as many low speed airfoil cambers.

It must be emphasized that this approach is only quantitatively valid for a quasi-steady situation. The upwash is actually related to the equivalent chord line by means of the substantive derivative:

\[
v(x, y=0, t) = \frac{\partial y_c}{\partial t} + \bar{U}(x) \frac{\partial y_c}{\partial x}
\]

The present model neglects the time derivative term, which can be shown on dimensional grounds to be of order K compared to the spatial derivative term. Thus quantitative use of the quasi-steady approximation is questionable for K=1.0 and almost certainly wrong for K=4.0.

If the assumption is made that \( \bar{U}(x) = U_{\infty} \)

\[
v(x, y=0, t) = U_{\infty} \left( \sum_{n=0}^{\infty} a_n \cos n\pi x \cos \omega t \right)
\]

a solution for \( y_c \) becomes
\[ y_c(x,t) = y_0 \cos(\omega t - 2Kx) + c \sum_{n=0}^{\infty} \frac{\alpha_n \sin(\omega t + n\pi x/c)}{n\pi + 2K} \]

Imposing the condition \( y_c = 0 \) if \( v = 0 \) requires \( y_0 = 0 \). Thus

\[ y_c(x,t) = c \sum_{n=0}^{\infty} \frac{\alpha_n \sin(\omega t + n\pi x/c)}{n\pi + 2K} \]

As \( K \) (and therefore \( \omega \)) approaches zero, this expression reduces to the quasi-steady result for the same Fourier series representation for \( v(x,y=0,t) \). This approach was not pursued any further, since the information of primary importance, the induced airfoil difference pressures is found by using the upwash directly.

**Prediction of Difference Pressures using Measured Upwash**

Linearized thin airfoil theory was used to attempt to predict the unsteady difference pressures on the airfoil based upon the measured unsteady upwash. Theodorsen's theory as presented by Bisplinghoff, Ashley and Halfman was used (8,9). This theory was used instead of the Sears theory since the perturbation here is fixed in the airfoil frame of reference, rather than being converted downstream past the airfoil. The maximum velocity is always located at the trailing edge, never anywhere else, as would be the case if gusts were considered. The Sears theory (10,11) considers converted disturbances of the form \( \exp(i\omega(t-x/U_{int})) \), while the
Theodorsen theory considers airfoil fixed disturbances of the form exp(iωt). The second case most closely approximates the present situation.

The derivation will not be reproduced in detail here, but the essential features will be summarized. The problem is to find a solution to the Laplace equation for the disturbance velocity potential, \( \nabla^2 \phi' = 0 \), subject to the boundary conditions,

\[
v(x, y=0, t) = v_a(x, t); \quad -b < x < b
\]

Also to be applied is a trailing edge condition, that the velocities and pressures at the trailing edge \( x=b \) be finite and continuous. The procedure is to insert a series of sources and sinks above and below \( y=0 \) to obtain noncirculatory solution satisfying the boundary condition. The trailing edge condition is satisfied by introducing bound vorticity along the airfoil chord, together with image vorticity being convected downstream in the wake.

The slit airfoil is first mapped into a circle of radius \( b/2 \) using a Joukowski conformal transformation,

\[
x + iy = X + iY + b^2/4(X + iY)
\]

The wake is assumed to lie along the \( x \) axis,

\[
x = X + b^2/4X; \quad y = Y = 0
\]

The velocity components are related on the upper surface by
\( q_\theta = -2u' \sin \theta_t; \quad q_r = 2v \sin \theta_t; \quad 0 < \theta < \pi \)

Figure 24a and b show these physical and computational planes.

The noncirculatory solution is produced by two-dimensional sources of strength \( H \) on the upper half of the circle with corresponding sinks in the opposite position. Application of the condition

\[
v(x, 0^+, t) = \frac{\partial \phi}{\partial y} \bigg|_{x, 0^+, t}
\]

gives \( H = 2v_a \).

Summing the contribution of all sources results in

\[
g_\theta(\theta_t, t) = \frac{2}{\pi} \int_0^\pi \frac{V_a(\phi, t) \sin^2 \phi \, d\phi}{(\cos \phi - \cos \theta)}
\]

Applying the definition of the potential, and specifying \( \phi = 0 \) at the leading edge \( \theta_t = \pi \), results in

\[
\phi'_{up}(\theta_t, t) = -\frac{b}{\pi} \int_{0}^{\theta_t} \int_{0}^{\pi} \frac{V_a(\phi, t) \sin^2 \phi \, d\phi \, d\theta'}{(\cos \phi - \cos \theta')}
\]

\[
\phi'_{L0}(-\theta_t, t) = \phi'_{up}(\theta_t, t)
\]
The linearized Bernouilli equation is now used to obtain difference pressures

\[ P - P_\infty = -\rho U_\infty u' - \rho \frac{\partial \phi'}{\partial t} \]

\[ P_{UP} - P_{LO} = -2\rho \left( \frac{\partial \phi'}{\partial t} - \frac{U_\infty}{b \sin \theta_t} \frac{\partial \phi'}{\partial \theta_t} \right) \]

Examination of the disturbance velocity \( u' \) at the trailing edge \( \theta_t = 0 \) for this noncirculatory solution yields

\[ |u'| = |g_\theta| / |2 \sin \theta_t| \]

The condition that is now applied is that \( g_\theta = 0 \) at \( \theta_t = 0 \) to maintain \( u' \) as a finite number. This is accomplished by adding a circulatory solution. This solution is composed of a series of bound vortices \( \Gamma_0 \) within the circle in the computational plane together with their images \( -\Gamma_0 \) along the positive \( X \) axis, at distance \( \bar{X} \) from the origin. Each pair produces a contribution

\[ g_\theta = \frac{\Gamma_0}{\pi \overline{b}} \frac{\bar{X}^2 - (b/2)^2}{\bar{X}^2 + (b/2)^2 - \bar{X} \cos \theta_t} \]
The vortices are now assumed to be convected with mean velocity $U_\infty$, thus the contribution of each wake element $X=\zeta_i; b<\zeta<\infty$, with $d\zeta/dt=U_\infty$, to the velocity potential and to the pressure become:

$$
\phi'_{UP}(\Theta,t) = \int_0^{\pi} g_\Theta \cdot b/2 \cdot d\Theta = \frac{U_\infty}{\pi} \tan^{-1} \frac{\sqrt{(\zeta-b)(1+\cos\Theta)}}{(\zeta+b)(1-\cos\Theta)}
$$

$$
(P_{UP} - P_{LO}) = -2\rho \int_0^{\pi} \frac{\partial \phi'_{UP}}{\partial t} - \frac{U_\infty}{b \sin\Theta} \frac{\partial \phi'_{UP}}{\partial \Theta}
$$

$$
= -\frac{\rho U_\infty}{\pi b \sin\Theta} \int_0^\infty (\zeta + b \cos\Theta) \frac{d\zeta}{\sqrt{\zeta^2 - b^2}}
$$

Evaluating $g_\Theta$ at the trailing edge, summing both circulatory and noncirculatory contributions:

$$
g_\Theta(0,t) = \frac{2}{\pi} \int_0^{\pi} \frac{V_a(\phi,t) \sin^2\phi d\phi}{\cos\phi - 1} + \frac{1}{\pi b} \int_b^\infty \sqrt{\frac{\zeta+b}{\zeta-b}} \gamma_w(\zeta,t) d\zeta
$$

$$
= 0,
$$

where $\gamma_w(\zeta,t)$ is the wake vortex strength.

Defining the first integral as $2Q$, and integrating along the wake, the difference pressure becomes
\[ P_{UL} = P_{UL}^{non-circulatory} - 2\rho U_\infty Q (\cot \theta + \ldots) \]

\[
\frac{1 - \cos \theta}{\sin \theta} \int_b^\infty \frac{\zeta}{\sqrt{\zeta^2 - b^2}} \gamma_w d\zeta/ \int_b^\infty \sqrt{\zeta} \gamma_w d\zeta
\]

Next, harmonic time behavior and an uncurved, uniformly convected wake are postulated,

\[
V_a = \overline{V}_a (x) \exp(i\omega t), \quad \zeta = \zeta_0 + Ut,
\]

resulting in \( \gamma_w = \gamma_w \exp(i\omega t - i\xi/b) \).

The ratio of wake integrals above now depends only upon \( k = \omega b/U_\infty \), and is referred to as the Theodorsen function, \( C(k) = F(k) + iG(k) \). This function is tabulated in Reference 8, and approximated by Figure 24c.

Finally, in order to account for the effect of the rounded leading edge a correction given by Van Dyke was applied (12). In the vicinity of the nose the airfoil is approximated by a parabola with the same radius as the leading edge radius \( a \). The surface speed on the parabola is then calculated as

\[
q = U_i \sqrt{x/(x+a/2)}
\]

with \( U_i \) being the maximum speed on the parabola. Expanding for small \( a/x \) gives
"g" = \( U_i (1-a/4x + ...) \),

where the " " refers to a result of thin airfoil theory. The ratio of these two values is said\( (12) \) to be a multiplicative correction factor for thin airfoil theory,

\[
\bar{g} = \sqrt{x/(x+a/2)/(1-a/4x)} "g".
\]

Expanding again,

\[
g/U = \sqrt{x/(x+a/2)} ("g"/U + a/4x).
\]

In terms of difference pressures this result, valid to second order in thickness, becomes

\[
\Delta C_p = \sqrt{x/(x+a/2)} ("\Delta C_p" - a/4x)
\]

The key assumptions in the Theodorsen approach used here are therefore the following: potential flow exists everywhere. Wake vorticity is convected downstream at the mean freestream velocity. Linearization of the upwash boundary conditions and of the Bernouilli equation are permissible. Most importantly, the perturbation velocity at the trailing edge approaches zero fast enough so that \( u \cdot x/(x-c) \) also approaches zero. This final condition is a result of requiring \( g_0 \) to be zero at the
trailing edge. This condition is sufficient but perhaps not necessary to ensure that the velocities at the trailing edge remain finite.

The following procedure was used to apply the Theodorsen theory to the present case. First, the upwash is assumed to be composed of two components, a mean time independent part and a fluctuating part oscillating harmonically at the fundamental frequency. The variation in phase of the upwash along the chord is ignored, with the phase being set at a constant value for each velocity-frequency representing a value near the trailing edge position. This position corresponds to the maximum upwash amplitude.

The mean and fluctuating components are then decomposed into Fourier cosine series in the variable
\[ \Theta_t = \arccos \left( \frac{2x}{c-1} \right) \]

\[ v(x, y=0, t) = v_a(\Theta_t, t) \]

\[ = \sum_{n=0}^{\infty} \left( a_n \cos \left( \frac{n \pi x}{c} \right) + b_n \sin \left( \frac{n \pi x}{c} \right) \right) \cos n\Theta \]

In most cases the inclusion of terms up to \( \cos 3\Theta \) was sufficient to represent the data to within experimental accuracy, although the digital computer analysis routines were designed to utilize the series up to \( \cos 6\Theta \) terms.
Figure 25 illustrates this process. The curve labels refer to the result of including more terms of the Fourier series. The data curve is hidden under the Fourier curves for n=4 and above.

For upwash $v_\alpha$ having spatial dependence $\cos(n\theta)$ the various integrals involved in the calculation of the difference pressure reduce after numerous trigonometric substitutions to sums of integrals of the well known Glauert type,

$$\int_0^\pi \frac{\cos n\phi d\phi}{\cos \phi - \cos \theta} = \pi \frac{\sin n\theta}{\sin \theta}$$

Therefore, evaluation of the pressure became a matter of substituting the coefficients for the Fourier series of the upwash into the terms appropriate to the value of n, applying the correct tabulated values for the Theodorsen function, and evaluating the sum. This somewhat tedious process was done using a digital computer, with the results being returned in the form of mean difference pressure, in and out of phase components of the fluctuating difference pressure, and fluctuating difference pressure phases and amplitudes. All of these are given as functions of chordwise distance x/c.
Chapter III

RESULTS OF PRESSURE TESTS

Mean Airfoil Surface Pressures

The first set of pressure measurements were taken with the airfoil alone, without the elliptical cylinder being present. Upper and lower mean pressures were obtained using the time averaging filter and compared. The angle of attack of the airfoil was then adjusted to minimize the difference pressures and thus compensate for any departure in the incoming flow from horizontal. The results of this process are shown as Figure 26. Mean pressures for upper and lower surfaces at the three test velocities are presented together with the pressure distribution expected for a N.A.C.A. 0012 airfoil as computed from Reference 13. The agreement between both the various cases and with the expected results seemed quite acceptable.

Pressures were next measured with the elliptical cylinder at fixed orientations $\theta=0, 45, 90$ and 135 degrees. Figures 27 to 34 illustrate the mean and difference pressure distributions obtained for these cases. Agreement between the pressures measured at the various velocities is in general quite good, the principal differences being two $U_{\text{inf}}=9$ mps, $\theta=90^\circ$ points that appeared to be a few standard deviations off. The other 178 measured points seem to
match much better. The only regular difference between the velocities occurred for the θ=45 and 135° cases on the lower airfoil surface. Here the results for $U_{\text{inf}}=20$ mps were uniformly greater in $c_p$ by approximately 0.05 than the $U_{\text{inf}}=30$ mps results. The difference might be due to the effect of Reynolds number on the elliptical cylinder separation point.

All four cases examined exhibited smooth increasing pressure gradients along the rear upper surface of the airfoil, indicating that the quasi-steady boundary layer there should be expected to be reasonably well behaved. Further, comparison of the results for the $\theta=0$ and $\theta=90°$ cases revealed that a sizeable difference in pressure existed, demonstrating that for the quasi-steady situation at least, the desired oscillatory pressure field could be achieved. These results essentially confirm the results of References 6 and 7, with most differences being probably a result of the fact that a smooth wooden elliptical cylinder was used previously, while the present study utilizes an aluminum, grit roughened surface. Separation characteristics on these two surfaces may be expected to differ to some extent.

Figures 35 to 48 illustrate mean surface and difference pressure data taken for the rotating cases with reduced frequency $K$ ranging from 0.5 to 6.4. For those frequencies where there were data for different freestream
velocities the agreement seems excellent; with the exception of only a very few points, the scatter seen among the varied velocities was approximately the same as seen in the steady cases.

As reduced frequency is increased, the mean difference pressure was seen to increase. This agreed with the upwash measurements, and with the interpretation of this increase being due to increased circulation generated by the rotating cylinder. In general the mean pressure distributions for the rotating case resembled most closely the steady pressure distributions seen for the cases \( \theta = 45^\circ \) and \( \theta = 135^\circ \). This agreed with the quasi-steady notion of the mean for the rotational case being an average of the steady values. The extreme cases \( \theta = 0^\circ \) and \( \theta = 90^\circ \) tended to cancel out, leaving the more moderate cases remaining to characterize the average.

The key features for the mean surface pressure distributions in the rotational cases were thus the following. First, reduced frequency was the preeminent parameter characterizing the results. Second, mean pressure gradients along the rear upper surface were smooth and steadily increasing. Third, qualitative agreement existed between the mean unsteady distributions and the averaged quasi-steady results, if rotation induced circulation was accounted for.
Unsteady Difference Pressures

During this phase of the pressure testing mean and phase lock averaged fluctuating difference pressures were measured for the 12 speed-frequency combinations for which induced upwash data were available. Comparisons were then made between the data and the predictions based upon application of the Theodorsen thin airfoil theory.

Figures 49 to 60 show this comparison for the mean difference pressures. For 9 of the 12 cases the agreement was very good in the region from \( x/c = 0.05 \) to 0.95, with differences being of the order of the variation seen among pressure coefficients calculated for varied velocities. Over the rear 5 percent the pressure data tended to approach zero more gradually than the upwash based predictions, but the differences were not great. The question of trailing edge behavior will be discussed at greater length below.

The three cases which do not support such close comparison were \( K=2.0 \) at velocities 9 and 20 mps, and \( K=1.0 \) at 20 mps. In all three cases the predicted curve was greater than the measured difference pressure, the typical value being \( \Delta c_p = 0.05 \). Since the difference pressure data matched well with their counterparts at the other velocities, the cause of this error was probably in the velocity data.
It was possible that small errors such as in positioning the hot wire probe and compensating for drift accumulated for these cases in a manner sufficient to unnecessarily increase the Fourier series representation for the upwash. Since these are all lower frequency lower velocity combinations, the relative error accumulated in the flow angle calculations could be large enough to produce this problem.

The next set of figures, 61 to 72, compare the amplitudes for the fundamental frequency of the oscillating difference pressure with the predictions for this quantity. The general features were again primarily dependent upon the reduced frequency. On the broadest qualitative scale the results had the same form. There was a peak within 5% of the leading edge, the amplitude then dropping before steadily increasing to reach a maximum at near 80% of the chord. A rapid decline then occurred, difference pressure amplitudes apparently approaching zero at the trailing edge.

The edge peak moved closer to the nose as reduced frequency increased, with the region of minimum amplitude moving from about 20% of chord for \( \kappa=1.0 \) to about 8% at \( \kappa=2.0 \), finally disappearing into the gap between the leading edge tap and the 0.5% tap for the highest frequency \( \kappa>6 \) cases. The position of the rearward maximum remained relatively constant with frequency. The amplitude at this
maximum declined from over $\Delta c_p = 0.16$ for the $K=1$ cases to from 0.08 to 0.10 at $K=2.0$. Amplitude then increased back to around $\Delta c_p = 0.13$ as reduced frequency reached 6.3 and above. Figure 73 shows the amplitude versus frequency response for upper, lower and difference amplitudes at $x/c=0.70$, illustrating this behavior.

The comparison with the Theodorsen theory predictions was also chiefly dependent upon reduced frequency. Qualitative shapes for the amplitude distributions were all reasonable. This refers to the positions of the maxima and minima, the trailing edge behavior, and the leading edge behavior. However, the predictions consistently overshot the measurements by a significant amount. Agreement was best for the lower frequency $K=1$ situation, where the pressure measurements were between 20 and 30% below the predictions. The discrepancies increased with reduced frequency, reaching approximately 60% undershoots at the high frequency $K>6$ cases. An attempt to examine some reasons for this difference will be made after the phase results have been discussed.

Some further mention must first be made here of the question of the behavior of difference pressure amplitudes as the trailing edge was approached. As mentioned above, the present study, involving measurements back to 98% of chord saw no evidence that the fundamental amplitude did not approach a value of $\Delta c_p = 0$ at the trailing edge.
Measurements were also made of the root mean square difference pressure amplitude, non-dimensionalized by the freestream dynamic pressure, near the trailing edge for reduced frequencies from $K=0.5$ to 6.4. The rms values include the fundamental, its harmonics, turbulence and wind tunnel fan noise. Figure 74 shows these data. Values for freestream velocities of 20 and 30 mps definitely approach zero while for some frequencies the lower speed, 9 mps values, do not clearly approach zero. This is probably due to the high relative fan noise level at this velocity, which became more apparent as the other components of the rms declined near the trailing edge. No convincing evidence was found, therefore, to support any non-zero unsteady difference pressure at the trailing edge.

Figures 75 to 86 present measurements and upwash based predictions for the phase lag of the difference pressure. The phase measurements were made using the same procedure used for the induced upwash phase, with probable errors again being on the order of 3 to 5 degrees. The typical features characterizing the data were the following. Near the leading edge there was a phase lag behind the elliptical cylinder ranging from 15 to 35 degrees. The phase lag decreased steadily over the front half of the airfoil, reaching values typically of from -10 to -40 degrees over the region from 50 to 80% of chord. An increase in phase then occurred near the trailing edge. This
was much more pronounced for the higher reduced frequency cases, where lags over 200 degrees at 98% of chord was observed (Figures 84-86).

The data and predictions over the front 90% of chord will be discussed first. Overall, the qualitative features were predicted fairly well, including the maximum at the nose, the decline in phase lag until midchord, and the relatively constant phase over the next 20 to 30% of chord. It must here be mentioned that the comparison procedure included some freedom to add a constant phase to the prediction. This was a result of two factors, the assumption of constant phase upwash velocity in the theory, and the uncertain phase calibration of the pressure data at low frequency. The phases were corrected for the time delay in the tubing using an extrapolation of the calibration data down to the region below 30 Hz, where the calibration technique was inaccurate. The predicted phases used a constant phase equal to a representative value of the upwash phase near the trailing edge position. An uncertainty of between 10 and 20 degrees, therefore, existed between the reference phases of the data and the predictions, which allowed some vertical translation to improve the comparison.

Over the final 10% of the chord severe discrepancies between prediction and data exist. The predicted difference pressure phase totally missed the phase increase, and in
fact had a sharp drop to approach \(-90\) degrees with respect to the reference phase. The increase in phase at the trailing edge seen in the data appeared to be more dependent on the actual cylinder rotation frequency than upon reduced frequency. For example, a phase lag of 120 degrees at \(x/c=0.98\) was reached at \(K=3\) for a velocity of 30 mps, at \(K=4.5\) for 20 mps, but not until \(K=8\) for 9 mps. This difference was much greater than the 10 to 20 degrees difference possible due to uncertainty in low frequency phase calibration. For positions forward of \(x/c=0.98\), such differences were not observed.

As the rotation frequency was increased the difference pressure waveform at 98% behaved in a unique manner. From \(K=0.5\) to a reduced frequency that varied from \(K=2\) for 30 mps to \(K=4\) for 9 mps the amplitude declined steadily, with the waveform shape remaining regular. Over this region in reduced frequency the phase lag was still small, and similar to that observed at 95% of chord. Above this frequency a transition region was seen, with amplitudes \(\Delta c_p \approx 0.02\) or less. Here the waveform deteriorated, losing regularity and in some cases being characterized by odd frequency harmonics. Following this transition period, which lasted until \(K=3\) for 30 mps and \(K=6\) for 9 mps, the amplitude began to increase again, reaching \(\Delta c_p = 0.04\) at high frequency. This new waveform was again regular, but characterized by a positive phase lag, increasing with
frequency. Beyond a belief that this behavior was a result of the strong perturbation of the wake just behind the trailing edge, no explanation of this effect can be given (see Appendix 2).

The failure of the theory to correctly predict behavior near the trailing edge should not be surprising if the differences between its assumptions and the experimental conditions are considered. Many of these assumptions are included in the trailing edge condition that was applied, the familiar Kutta-Joukowski condition. This condition formally requires "steady incompressible potential flow around a two-dimensional airfoil having a cusped trailing edge" (14). The trailing edge singularity is removed by placing a rear stagnation point at the position in the computational plane corresponding to the cusp, thus fixing the circulation and maintaining a finite velocity.

Of the five requirements for the formal Kutta-Joukowski trailing edge condition, only two dimensionality and incompressibility can reasonably be assumed to apply. Two-dimensionality should be valid in the region near the airfoil centerline where the measurements were made. As discussed in Reference 15, incompressibility requires three conditions to be satisfied. The first is $M^2 << 1$. Since $M = 0.09$ at 30 mps at test conditions, this condition
was met. The second is $gL^2/n^2 \ll 1$, with $L$ being a length scale. This requirement is also satisfied since $L$ must be greater than 8 km to violate this condition in air. The third requirement reduces to $K^2M^2/(2\pi)^2 \ll 1$. Since $K/2\pi \leq 1$ in these tests this condition is also satisfied for the fundamental harmonic.

With regard to the trailing edge geometry the N.A.C.A. 0012 used here had a finite thickness trailing edge with included angle equal to 16 degrees. Since it has been found (16) that increasing the trailing edge angle decreases the lift curve slope, the lower response than predicted to the induced upwash may have been at least in part due to this significant departure from the assumed cusped edge.

The assumption of potential flow near the trailing edge is also far from reality. Displacement thicknesses, measures of the thickness of the viscous boundary layer, were found at 94% of chord to be 0.25 cm, larger than the airfoil thickness at this point. The viscous wake, in which the perturbation by the rotating elliptical cylinder occurred, was also a highly non-potential flow region.

In addition to these problems with applying the standard trailing edge conditions, the other assumptions in the thin airfoil theory must also be considered in judging the correspondence between the measurements and predictions. These included linearity, negligible thickness
effects, and steady convection of the wake vorticity. In view of all this, the remarkable fact may not be why the predictions were incorrect near the trailing edge, but that they were even this accurate. As mentioned in the Introduction, other investigators have also noted discrepancies at the trailing edge of airfoils at high reduced frequency, some even reporting non-zero trailing edge difference pressures.

A final comment involving the unsteady difference pressures involves Figures 87 and 88. They present in and out-of-phase pressure amplitudes calculated from the upwash for reduced frequencies of 1.0 and 4.0. The key feature was the large value for the out-of-phase component over the rear of the airfoil for the high-frequency case. This high out-of-phase portion accounted for much of both the excess amplitude and the high negative phase seen in the trailing edge region of the predictions. It came about from the fact that the Theodorsen theory involves a linearization which assumes certain terms proportional to reduced frequency are small. This assumption becomes less valid as K becomes much greater than one. The effect of these terms proportional to K was seen primarily in the out-of-phase component, resulting in behavior such as Figure 88.
Fluctuating Surface Pressures

The actual pressures seen on the airfoil surface are the most important quantities that influence the unsteady boundary layers, as these pressures are the forcing function that determines their growth, properties and possible separation. With that importance in mind, Figures 89 to 102 present the amplitude of the fundamental harmonic of the phase-lock averaged pressures on the upper and lower airfoil surfaces as a function of chordwise distance $x/c$. For each freestream velocity-reduced frequency combination four sets of data points are plotted. The amplitudes corresponding to the two fundamental periods contained in each elliptical cylinder revolution are shown for each surface. This points out the importance for certain cases of the subharmonic deviation. The average of these two values would therefore approximate the actual amplitude of the fundamental frequency.

With respect to this average amplitude the reduced frequency was again seen to be the key determinant for the behavior of the system. For reduced frequencies $K < 1.0$ the upper surface pressure amplitude had a maximum of $\Delta c_p = .15$ at $x/c=0.005$, declined rapidly to $c_p = .04$ at 15% of chord and then steadily increased, reaching $c_p = .20$ at 98% of chord. The maximum on the lower surface did not occur until $x/c= .025$, after which it followed the same trend as the upper surface pressure, with the difference in amplitude
decreasing steadily giving the appearance for K=1 of coming to a relatively sharp corner near the trailing edge. For these low frequencies the amplitude of the subharmonic deviation was small, but still greater in some cases than the estimated standard deviation in measuring pressure amplitude at any one tap, a $c_p$ of 0.002 (Figures 89-92).

Following an intermediate state at K=1.5, the qualitative features at K=2.0 have assumed what was found to be a 'high frequency' character. On the upper surface a much lower maximum of $c_p=.04$ was still located at $x/c=.005$, with the minimum still being located near 15% of chord. However, on the after portion of the airfoil the pressure amplitude did not increase as steadily, there being a slower increase until the final 10% of chord, where a rapid increase to $c_p=.09$ occurred. On the lower surface the behavior of the front 85% of the chord was similar to the lower frequency situation. The maximum of $c_p=.14$ was reached at this point, followed by a rounded fall to the lower surface value near the trailing edge. The rounded character of the plots in the after portion was the key feature distinguishing the high frequency behavior from the sharper low frequency shape. Again subharmonic deviations were low, similar to the K=1 cases (Figures 94-96).
Significant subharmonic deviations were seen at $K=4$. The average, or true fundamental, still adhered to the 'high frequency' behavior pattern, with reduced maximum amplitudes of $c_p = 0.08$ on the upper, and $c_p = 0.12$ on the lower surface. However, for this reduced frequency only the case with velocity 20 mps has a low deviation. The other two velocities have a very large subharmonic component. It can be noticed that over one-half of the cycle the amplitude follows the low frequency sharp character, while the second half follows the rounded high-frequency behavior. In order for this variation to survive the phase-lock averaging, it must repeat itself regularly upon each revolution. This was confirmed by the real time observation of many cycles using the storage tube oscilloscope (Figures 97-99).

The case of reduced frequency $K=5.1$, velocity 30 mps exhibited the high frequency character with maximum amplitudes of $c_p = 0.14$ on the upper, and $c_p = 0.20$ on the lower surface. Subharmonic deviation was seen to a moderate extent over the rear of the airfoil, being more pronounced on the upper surface (Figure 100).

For reduced frequency $K=6.4$ the rounded behavior was still seen, with maximum amplitudes being $c_p = 0.18$ on the low and $c_p = 0.14$ on the upper surface. Deviations were again moderate, but noticeable (Figures 101-102).
The relationship between the subharmonic deviation seen here in the pressures and that seen in the induced upwash velocities was not completely clear. The highest pressure deviations occurred at $K=4.0$ at velocities at 9 and 30 mps, with less deviations at the other high frequency cases. Upwash deviations were also concentrated among the high frequency cases, but the highest deviations occurred here for all three speeds at $K=4$ and for the $K=5.1$, 30 mps case. Thus while there was fair correlation, it is not as complete as would be expected if the deviation in the pressures were solely a result of the deviation in the upwash that produced them.

If the induced upwash produced by the rotating cylinder is viewed as an input that is operated upon by the airfoil to produce the pressures, some useful information may be found. First, if the mean difference pressure is divided by the mean induced angle for various positions along the chord, a fan-shaped curve is obtained with average value of approximately 70 near the leading edge, dropping to about 0.4 near the trailing edge.

The width of this fan, determined by the 12 considered speed-frequency combinations, was typically 40% of the average value. The average shape may be predicted from quasi-steady considerations, since difference pressure has a strong peak at the leading edge for even a constant
angle of attack. If the same procedure is followed for a few phase lock averaged plots, dividing $\Delta c_p$ by $\alpha$ for various times, sinusoidal type curves are again obtained. The main result that came out of this process was the fact that curves for high deviation cases were similar to those of the low deviation cases; that is, much of the pressure deviation was correlated with the upwash deviation. This again seemed to indicate that the pressure subharmonic deviation was primarily a result of the deviation introduced by the rotating cylinder, an interpretation of the origin of which was given earlier.

One further test of this relationship was an attempt to treat the subharmonic deviation amplitudes as an upwash at one half the fundamental frequency and apply the Theodorsen theory to them. The resulting difference pressures were then compared to the actual measured deviations. In some cases the magnitudes were similar, but most often the comparison was not very good, the actual deviations being a good deal less than the predictions. This would seem to indicate that either the assumption that the deviations could be treated as a sine wave of the given amplitude was not accurate, or that the pressure deviations were not, in fact, primarily a result of the upwash deviation. The first assumption will be checked later in the project, when it becomes possible to fourier transform the waveform, getting the actual
phases and amplitudes for the subharmonic components. Until then the question remains unsettled.

The phase of the fundamental harmonic of the surface pressure is plotted for the various speed-frequency combinations in Figures 103 to 116. These phase measurements were made in the same manner as with the upwash velocity and pressure difference phase lag measurements, and were subject to the same estimated probable errors of 3 to 5 degrees. This error refers to phases measured along the chord at one frequency.

For reduced frequencies $K < 1.5$ the phase behavior was quite similar for all speed frequency combinations. The phase lag was approximately 200 degrees for $K < 1.0$, 230 degrees for $K = 1.5$. Proceeding aft along the upper surface the phase dropped steadily, until about $x/c = .40$ was reached, at which point it leveled off at between 70 and 100 degrees, depending upon position and case. Beyond 80% of chord the phase decreased to reach its final value of between 25 and 50 degrees near the trailing edge. This was also the value obtained on the lower surface at this chordwise position, which was necessary for the difference pressure to go to zero at the trailing edge, as it had been shown to do. Moving forward along the lower surface, the phase lag dropped over the final 20% of chord until it reached a level near zero, or in phase with the rotating cylinder. This level was maintained
until the front 15% of the chord was reached. This behavior appeared reasonable since the lower surface was located on the same side as the cylinder, and therefore felt its direct influence more strongly. The upper surface had the metal between itself and the source of the perturbation, requiring a more complex propagation.

The fact that a significant difference in phase existed between the surfaces is also comprehensible in terms of quasi-steady concepts. If the induced circulation from the cylinder is pictured as a set of vortices located along the chordline, at any given time one surface would experience an increase in velocity, while the other surface would see a decrease. In the uniform case this would imply a 180 degree phase difference. The observed difference ranges from about this value near the leading edge to about 80 degrees in the nearly constant phase region to zero degrees near the trailing edge. The variation is due to the fact that the actual phase (and amplitude) at any location is a sum of the contributions from the vorticity all along the chord, each with different phase and magnitude, as was demonstrated by the difference pressure data.

As the leading edge was approached along the lower surface there was a slight increase in phase to near 10 degrees over the front 10% of chord. This value was
maintained until $x/c=0.005$. However, as mentioned above, the leading edge phase agreed with the upper surface value of near 200 degrees, implying a large jump over this small 1/2 per cent of chord. Note that the leading edge tap was read by the lower surface pressure system, so that this effect appears to be real and not an artifact of some difference between the pressure systems. This jump can be made somewhat more palatable by remembering that for the mean induced upwash seen here, the mean stagnation point would be located on the lower surface, so that it was likely that this phase jump occurred at the mean stagnation point.

Using the pressure distributions obtained for fixed cylinder orientations corresponding to 0, 90, 180 and 270 degrees of phase, an estimate was made of the resulting quasi-steady phase distribution. Since resolution was $\pm$ 45 degrees, this was necessarily a rough estimate. However, within these limitations the quasi-steady result agreed with the above described low frequency behavior. That is, about 180 degrees on the upper surface front, dropping to about zero degrees on the rear, with lower surface pressure phases remaining at about zero degrees.
For reduced frequencies $K > 2$ the phase behavior was quite different. When $K = 2.0$ the phase along the rear 60% of the chord was qualitatively similar to the lower frequency case, with lower surface phases of zero degrees to $-10$ degrees and upper surface phases of nearly 50 degrees, approaching a common value of about 25 degrees near the trailing edge. The lower surface behavior near the front was also familiar, with somewhat of an increase to 10 to 20 degrees on the front 60% of chord, until $x/c = 0.005$. Again there was a large jump between this position and the leading edge, but instead of increasing to 200 degrees, it dropped to around $-80$ degrees. The upper surface phase started out at near $-100$ degrees and then steadily increased, reaching the 50 degree lag level around midchord.

This behavior was continued for $4 < K < 5.1$, the primary changes being located near the leading edge. The first change was to shrink the region of increasing phase to the first 10 to 20 percent of chord. Second, the minimum phase was reduced from $-100$ degrees to $-50$. The behavior over the aft 80% of chord was similar to the lower frequency situation (Figures 111-114).

In the highest frequency $K = 6.4$ cases, this qualitative trend was maintained, the only significant change being a general downward translation of nearly 20 degrees downward in phase.
No convincing explanation has yet been found for this shift in phase behavior, but several comments may still be made. First, the shift in phase characteristics occurred at the same point, between $K=1.0$ and $2.0$, as the shift in amplitude shape. This indicated that it was all a result of one shift from a low frequency, almost quasi-steady phenomenon to a high frequency one near the dimensionally logical region $K=1$. Second, the large shift in phase on an unsteady airfoil was not a unique observation. Franke and Henderson showed such a large shift on the suction surface of an airfoil at reduced frequency $5.0$ at $17$ degree incidence (17).

**Surface Pressure Frequency Spectra**

The final class of surface pressure measurements were power spectral densities. Figures 117 to 121 show these results for reduced frequency $K=1.0$ and velocity $30$ mps for varied chordwise locations. Frequencies shown range from $0$ to $200$ Hz, surveys at higher frequencies having shown little energy to have been present above this frequency. Averages were made over 64 individual spectra. Amplitudes were squared and plotted logarithmically, referenced to an arbitrary full-scale value. Thus, absolute amplitudes are not given, the relative values being the key information.
For $x/c=0.05$, Figure 117, there were three largely independent spectra present. The first was the largely continuous spectrum due to the background turbulence, which faded away by 70 Hz. The second spectrum consisted of the fundamental perturbation frequency and its second and third harmonics. The final spectrum was made up of the wind tunnel fan blade passing frequency and its third harmonic.

The components of the spectra were similar for the other chordwise positions. The background turbulence spectrum largely vanished for these locations, since the pressure taps were now oriented nearly horizontal, rather than at the more vertical positioning of the $x/c=0.05$ tap, which collected some of the dynamic pressure fluctuations. The other spectra, the cylinder fundamental, the blade passing, and their harmonics still accounted for all of the major peaks. The fundamental cylinder frequency was, however, accompanied by more harmonics, up to the sixth near the trailing edge.

Spectra for other speed-frequency combinations were similar, and composed of the same three elements. As mentioned earlier, the lower the velocity the higher the fan blade passing frequency power became compared to the cylinder frequency power. For some locations at 9 mps this noise actually overshadowed the desired frequency.
Chapter IV

CONCLUSIONS

The conclusions drawn from this study may be divided into two categories. The first involves the comparison between the unsteady difference pressure results and the predictions based upon the measurements of the upwash induced by the rotating elliptical cylinder. The second category deals with the utility of this experimental arrangement as a means of producing unsteady adverse pressure gradients for the study of turbulent boundary layers.

Three classes of results were compared to upwash based predictions: mean difference pressures, amplitudes of fluctuating difference pressure fundamental frequencies and the phase lag of this fundamental. Mean difference pressures were found to agree reasonably well with the predictions of thin airfoil theory. Regions of disagreement were noted along the front and rear 5% of the airfoil chord. The small leading edge difference was felt to result from imperfect corrections for the leading edge geometry. The trailing edge disagreement in the mean pressure was thought to be due to the finite trailing edge angle of the airfoil and to wake curvature effects.
Fluctuating difference pressure amplitudes agreed qualitatively with the predictions, but the measurements were significantly lower in magnitude. This difference was found to increase with reduced frequency. Assumptions implicit in the unsteady incompressible thin airfoil theory regarding the trailing edge condition, potential flow and uniform wake convection were considered to be the cause of this difference.

Phases were quantitatively similar to the predictions over the front 90% of the chord, while trailing edge behavior showed gross dissimilarities. Again, differences with the theoretical assumptions were felt to be responsible.

In comparing the present results to other work on unsteady difference pressures many similarities were found. The unsteady difference pressures were seen to approach zero at the trailing edge in accord with most previous studies using sharp trailing edge airfoils (1,4,5). The discrepancies seen here between measurements and thin airfoil theory predictions for reduced frequencies of 1.0 and above have also been noted previously (1,4,6). However, while all of these previous studies reported discrepancies, the form of the disagreements between theory and experiment varied considerably (Table 1).
With regard to the other category of conclusions, the present experimental arrangement of producing a two-dimensional unsteady flow by means of a rotating elliptical cylinder near the trailing edge was found to produce satisfactory unsteady adverse pressure gradients for the study of turbulent boundary layers.

The mean surface pressure distributions were quite smooth and were found to depend almost exclusively upon the reduced frequency. In the key region of interest, the rear suction surface, the mean adverse pressure gradient was characterized by a steady and regular increase as the trailing edge was approached.

Fluctuating pressure distributions on the rear suction surface of the airfoil were also reasonably well behaved. Phases were fairly uniform from around mid-chord until the final 10% of chord, at which point the phase lag tended to decrease somewhat so as to match with the lower (pressure) surface phase. Amplitudes of the fluctuating pressure increased steadily over the rear suction surface, with maximum of between $c_p = 0.05$ and $0.20$, depending primarily upon reduced frequency.

Two general types of surface pressure behaviors were observed, a low frequency ($K<1$) form and a high frequency ($K>1$) form. The low frequency behavior was similar to the
quasi-steady results, with a large positive phase lag on the suction surface near the leading edge and a jump at the stagnation point to near zero phase lag over the pressure surface. Low frequency amplitudes were characterized by a rounded plot of the junction between the values on the two surfaces near the trailing edge. The high frequency cases eliminated the phase peak near the leading edge and had a much sharper junction between trailing edge amplitudes; that is, the suction and pressure surface amplitudes took on similar values at more forward locations along the chord than for the lower frequency cases.

The only significant problem that should be encountered in using this technique to study the turbulent boundary layer is the necessity to avoid those air velocity-perturbation frequency combinations that produce high sub-harmonic components in the unsteady pressure. Even with these combinations eliminated, there should still be many possible operating velocities and frequencies available in the range of reduced frequencies of 0.5 to 6.5.
The close match seen in Figure 26 between the measured mean pressure coefficient distribution for the airfoil alone and previous results (13) was a confirmation of the correctness of the placement of the pitot-static probe. The probe was placed above and ahead of the airfoil, between the sideboards, as seen in Figure 1. By placing it there, rather than ahead of the sideboards, the effect of the blockage of the models and their wakes on the velocity field could be accounted for.

A conservative estimate of blockage effects was made based upon Thom's solid blocking criterion for two-dimensional testing (18). The difference between the freestream velocity measured in a test section from that seen in free flight is given by

\[ \varepsilon = 0.74 \times \frac{\text{(Model Volume)}}{\text{(Test Section Area)}}^{1.5} \]

It was assumed that the airfoil wake blockage was less than that of a solid volume of height twice the boundary layer thickness at 98% of chord, about 0.5 cm. Similarly, the wake blockage of the elliptical cylinder was assumed to be less than that of a solid body of height 9.2 cm. Using
these very conservative estimates, the blockage effect on the freestream velocity was found to be less than 1.7%.

The velocity measured by the pitot static probe included this effect, which would not have been observed by a pitot static probe located forward of the side walls.

Streamline curvature effects due to the presence of the test section walls added an apparent angle of attack to the airfoil. In the present case this angle was small, less than 0.05 degrees, much less than the adjustments needed to correct for the non-horizontal flow in the test section. The procedure for obtaining this apparent angle can be found in Reference 18.

Finally, there was an effect of the presence of the test section walls on the unsteady pressures. A modification must be made to the Theodorsen function to account for this (19). In the present case, where the wind tunnel test section height was nine times the airfoil semichord, and reduced frequencies were greater than or equal to 1.0, the magnitude of the modification was less than or equal to 1%.
One step that may be taken in order to increase the understanding of a new phenomenon is the determination of dimensionless parameters which describe it. Since the phase lag of the fundamental harmonic of the difference pressure at the chordwise position $x/c = 0.98$ was not found to depend primarily upon the reduced frequency, a new dimensionless parameter was needed.

The first physical quantity that must be included in this new parameter is the radian frequency. Since it was felt that the behavior near the trailing edge was strongly influenced by boundary layer and wake effects, the other physical quantities involved in the parameter should include these viscous effects.

Therefore the parameter $\omega \delta_2^2/\nu$ was proposed as a dimensionless determinate of the near trailing edge phase lag. The estimate for $\delta_2$ used was an extrapolation of the measured momentum thicknesses at $x/c = 0.94$. The measurements included those of Cervisi (7) for steady conditions and unpublished unsteady work of the author. Based upon this work a rough approximation of the momentum thickness in inches was
\[ \delta_2 = 0.063 - 4 \times 10^{-9} \text{(Re)} + 5.3 \times 10^{-4} \text{(K)} \]

Figure 122 shows the results of this process, the phase lag of the difference pressure fundamental harmonic as a function of \( \omega \delta_2^2/\nu \). Fairly good agreement was seen, with phase lags of -20 to 0 degrees for \( \omega \delta_2^2/\nu < 40 \), of greater than 140 degrees for \( \omega \delta_2^2/\nu > 60 \), with intermediate values in the transition region.

This agreement did not prove the uniqueness of this parameter, since based upon the available data, many other length scales that had relatively small variation over the range of data taken would probably have done as well. Further investigation is clearly required.
REFERENCES


<table>
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<tr>
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<td>NACA 64A010</td>
<td>NACA 64A010</td>
<td>NACA 65 Series</td>
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<td>Downstream Shutters</td>
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<td>Pitching &amp; Plunging Airfoil</td>
<td>Rotating Elliptical Cylinder</td>
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<td>No</td>
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<td>0.89 - 0.973</td>
<td>0.3 - 0.94</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0 for flat plate in cascade &amp; isolated $\neq 0$ for cambered cascade</td>
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Unsteady Difference Pressure & Theory:

| Amplitude | Experiment Greater | --- | Experiment Loss, More Agreement for Subsonic | "Good" Agreement for Subsonic | Experiment Greater, esp. for Cambered Cascade | Experiment Less, Differences Increased with $V$ Cascade |
| Phase | Chordwise Variation Greater Than Theory | --- | Experiment Less, More Agreement for Subsonic | "Good" Agreement for Subsonic | Fair | Fair Away From Trailing Edge |

TABLE 1
1. Rotating Elliptical Cylinder
2. X Hot Wire Probe
3. Pitot Static Probe
4. 2 HP DC Motor
5. Flywheel
6. Photoelectric Phase Sensor
7. Sidewalls and Supports

Fig. 1
CONFIGURATION FOR
INDUCED UPWASH TEST

DIMENSIONS IN CM

C = 50.8
8.9 x 6.1
# Position of Pressure Taps

![Diagram of pressure taps](image)

## Table of Taps

<table>
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<th>TAP #</th>
<th>POSITION, X/C</th>
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</table>

*Figure 3*
MEASURED PRESSURE TO ACTUAL PRESSURE
AMPLITUDE RATIO

FIGURE 4
FIGURE 5

\[ \Delta \phi = 0.35 F^{1.034} \text{ DEG} \]
PRESSURE SYSTEM TIME DELAY

FIG. 6

T = 0.97 F\(^{0.035}\) ms.

FREQUENCY, F (HZ)
INSTRUMENTATION DIAGRAM

FIGURE 7
UNPROCESSED VELOCITY PERTURBATION ELLIPSE ALONE $U_\infty = 9 \text{ MPS}$

$K = 1.0 \quad X/C = 1.0$

FIG. 8

VERTICAL

MEAN 1.1

PEAK-PEAK 1.9

HORIZONTAL

9.1

2.1 MPS

TIME →
UNPROCESSED VELOCITY PERTURBATION

ELLIPSE ALONE

$K = 1.0$  

$U_\infty = 20$ MPS  

$X/C = 0.5$

FIG. 9

MEAN

VERTICAL

0.6

HORIZONTAL

19.8

PEAK-PEAK

1.1

0.9  MPS
UPWASH AT TRAILING EDGE.

$K = 1.0 \quad U_{\infty} = 20 \text{MPS}$

**Fig. 10**
UPWASH AT TRAILING EDGE

FIG. 11

K = 6.4  UINF = 20 MPS

K = 6.3  UINF = 9 MPS

VELOCITY

TIME
SUBHARMONIC DEVIATION WAVFORMS

A) DEFINITION OF PARAMETERS

\[ \bar{\alpha} = \frac{1}{2} (\bar{\alpha}_1 + \bar{\alpha}_2) \]
\[ \Delta \bar{\alpha}_1 = |\bar{\alpha}_1 - \bar{\alpha}_2| \]
\[ \Delta \bar{\alpha}_2 = \text{MAX}(\Delta \bar{\alpha}_A, \Delta \bar{\alpha}_B) \]

EFFECT OF CHANGING K

U_{INF} = 20 MPS \quad X/C = 1.0

B) \quad K = 5.99

C) \quad K = 6.28

FIG. 12
DEVIATION WAVEFORM SHAPES

FIG. 13
FIGURE 14

$K = 1.0$

<table>
<thead>
<tr>
<th>CURVE #</th>
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<tbody>
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<td>1</td>
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<tr>
<td>5</td>
<td>20 MPS</td>
</tr>
<tr>
<td>9</td>
<td>30 MPS</td>
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**FIGURE 15**

$k = 2.0$

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<tr>
<td>6</td>
<td>20 MPS</td>
</tr>
<tr>
<td>10</td>
<td>30 MPS</td>
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FIGURE 16

\( k = 3.9, 4.0 \)

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<th>CURVE #</th>
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<tr>
<td>3</td>
<td>9 MPS</td>
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<td>7</td>
<td>20 MPS</td>
</tr>
<tr>
<td>11</td>
<td>30 MPS</td>
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Fluctuating Induced Angle Amplitude, RAD.

\( X / C \)
Figure 17

$K = 5.1$

$U_{\text{INF}} = 30 \text{ m/s}$
FIGURE 18

$K = 6, 3, 6, 4$

<table>
<thead>
<tr>
<th>CURVE #</th>
<th>$U_{INF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>9 MPS</td>
</tr>
<tr>
<td>8</td>
<td>20 MPS</td>
</tr>
</tbody>
</table>

FLUCTUATING INDUCED ANGLE AMPLITUDE, RAD.

$X / C$
FIGURE 19

$U_{\text{INF}} = 9 \text{ MPS}$

<table>
<thead>
<tr>
<th>CURVE #</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
</tr>
<tr>
<td>3</td>
<td>4.0</td>
</tr>
<tr>
<td>4</td>
<td>6.3</td>
</tr>
</tbody>
</table>
FIGURE 20

$V_{\text{INF}} = 20$ MPS

<table>
<thead>
<tr>
<th>CURVE #</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>6</td>
<td>2.0</td>
</tr>
<tr>
<td>7</td>
<td>3.9</td>
</tr>
<tr>
<td>8</td>
<td>6.4</td>
</tr>
</tbody>
</table>

MEAN INDUCED ANGLE, RADIANS

$X / C$

0.0 0.2 0.4 0.6 0.8 1.0 1.2
FIGURE 21

$U_{\text{INF}} = 30 \text{ MPS}$

<table>
<thead>
<tr>
<th>CURVE #</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>2.0</td>
</tr>
<tr>
<td>11</td>
<td>4.0</td>
</tr>
<tr>
<td>12</td>
<td>5.1</td>
</tr>
</tbody>
</table>
FIGURE 22

EQUIVALENT CAMBER

INDUCED ANGLE OF ATTACK

MIN  1.35  0.0068
MEAN 2.81  0.0155
MAX  4.30  0.0243

CAMBER

X/C
FIGURE 23

EQUIVALENT CAMBER

\( u_{\text{INF}} = 9 \text{ MPS} \)

\( k = 4.0 \)

INDUCED ANGLE OF ATTACK

<table>
<thead>
<tr>
<th>MIN</th>
<th>3.56</th>
<th>0.0163</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEAN</td>
<td>3.99</td>
<td>0.0195</td>
</tr>
<tr>
<td>MAX</td>
<td>4.42</td>
<td>0.0223</td>
</tr>
</tbody>
</table>

MAXIMUM CAMBER

\( x/c \)

\( \text{CAMBER} \)
Theodorsen Model

a) Physical Plane

b) Computational Plane

c) Theodorsen Function

FIGURE 24
FIG. 25

MEAN INDUCED FLOW ANGLE, RAD.

NUMBER OF FOURIER TERMS

DATA

CHORDWISE DISTANCE, THETA

K = 1.0

0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8

0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35

1 2 3 6 5

FIG. 25
FIGURE 26

ABBOT AND VON DOENHOFF

PRESSURE MEASUREMENTS

U INFINITY UPPER SURFACE LOWER SURFACE

9 MPS × △
20 MPS * +
30 MPS △ ◊

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; NO ELLIPSE
FIGURE 27

PRESSURE MEASUREMENTS

U INFINITY  UPPER SURFACE  LOWER SURFACE
9 MPS     ×     
20 MPS    *     
30 MPS    △     

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; ELL = 0 DEG

-1.5 -1.0 -0.5 0.0 0.5 1.0

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
FIGURE 28

PRESSURE MEASUREMENTS

U INFINITY

9 MPS ▲
20 MPS □
30 MPS ○

MEAN DIFFERENCE PRESSURE COEFFICIENT, Cp

DISTANCE ALONG CHORD, X/C; ELL = 0 DEG
FIGURE 29

PRESSURE MEASUREMENTS
U INFINITY UPPER SURFACE LOWER SURFACE
20 MPS × ▼
30 MPS ★ +

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; ELL= 045DEG
FIGURE 30

PRESSURE MEASUREMENTS
U INFINITY
20 MPS ▲
30 MPS □

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; ELL = 045DEG
Figure 31

Pressure measurements

U infinity, upper surface, lower surface

9 mps
20 mps
30 mps

Mean pressure coefficient, CP

Distance along chord, X/C; ELL = 90deg
FIGURE 32

PRESSURE MEASUREMENTS

U INFINITY

9 MPS
20 MPS
30 MPS

DISTANCE ALONG CHORD, X/C; ELL = 90DEG
FIGURE 33

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; ELL= 135DEG

PRESSURE MEASUREMENTS
U INFINITY  UPPER SURFACE  LOWER SURFACE
20 MPS     X
30 MPS     △
FIGURE 34

PRESSURE MEASUREMENTS

U INFINITY

20 MPS ▲
30 MPS □

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; ELL = 135DEG
FIGURE 35

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 0.5

PRESSURE MEASUREMENTS
U INFINITY UPPER SURFACE LOWER SURFACE
20 MPS
Figure 36

Pressure Measurements

U Infinity
20 mps

Mean difference pressure coefficient, C_p

Distance along chord, x/c; K = 0.5
FIGURE 37

PRESSURE MEASUREMENTS

U INFINITY UPPER SURFACE LOWER SURFACE
9 MPS × △
20 MPS * +
30 MPS △ ♦

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 1.0
FIGURE 38

PRESSURE MEASUREMENTS

U INFINITY

9 MPS  ▲
20 MPS  □
30 MPS  ○

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C;  K = 1.0
FIGURE 39

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C: K = 1.5

PRESSURE MEASUREMENTS
U INFINITY UPPER SURFACE LOWER SURFACE
20 MPS
FIGURE 40

PRESSURE MEASUREMENTS

U INFINITY 20 MPS

K = 1.5

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 1.5
FIGURE 41

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K=2.0

PRESSURE MEASUREMENTS
U INFINITY UPPER SURFACE LOWER SURFACE
9 MPS ✗ ✕
20 MPS ✫ +
30 MPS △ ◊
FIGURE 42

PRESSURE MEASUREMENTS

U INFINITY

9 MPS ▲
20 MPS □
30 MPS ○

MEAN DIFFEREN CE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 2.0
FIGURE 43

PRESSURE MEASUREMENTS

U INFINITY UPPER SURFACE LOWER SURFACE

9 MPS × ▲
20 MPS ★ +
30 MPS △ ◊

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 4.0
FIGURE 44

PRESSURE MEASUREMENTS
U INFINITY
9 MPS
20 MPS
30 MPS

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 4.0
MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 5.1

PRESSURE MEASUREMENTS
U INFINITY UPPER SURFACE LOWER SURFACE
30 MPS 

FIGURE 45
FIGURE 46

PRESSURE MEASUREMENTS
U INFINITY
30 MPS

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 5.1
FIGURE 47

PRESSURE MEASUREMENTS
U INFINITY UPPER SURFACE LOWER SURFACE
9 MPS × ✔
20 MPS ★ +

MEAN PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 6.4
FIGURE 48

PRESSURE MEASUREMENTS

U INFINITY

9 MPS

20 MPS

MEAN DIFFERENCE PRESSURE COEFFICIENT, CP

DISTANCE ALONG CHORD, X/C; K = 6.4
Figure 49

Pressure measurements and prediction from upwash

Mean difference pressure, delta CP

X/C; UINF = 0.09 MPS, K = 1.0
FIGURE 51

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

MEAN DIFFERENCE PRESSURE, DELTA CP

X/C: UINF = 0.30 MPS, K = 1.0
FIGURE 52

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

MEAN DIFFERENCE PRESSURE, DELTA CP

X/C; UINF = 0.09 MPS, K = 2.0
Figure 53

Pressure measurements and prediction from upwash for mean difference pressure, deltap, at X/C = Uinf = 0.20 MPS, K = 2.0.
FIGURE 54

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

X/C: UINF = 030 MPS, K = 2.0
Figure 55

Pressure Measurements
Prediction from Upwash

Mean Difference Pressure, Delta CP

X/C; UINF = 0.09 MPS, K = 4.0
Figure 56

Pressure Measurements
Prediction from Upwash

Mean Difference Pressure, Delta CP

X/C; UINF = 020 MPS, K = 3.9
FIGURE 57

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

MEAN DIFFERENCE PRESSURE, DELTA CP

X/C: UINF = 030 MPS, K = 4.0
FIGURE 58

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

MEAN DIFFERENCE PRESSURE, DELTA CP

X/C; UINF = 0.30 MPS, K = 5.1
FIGURE 59

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

MEAN DIFFERENCE PRESSURE, DELTA CP

X/C: UINF = 0.09 MPS, K = 6.3
FIGURE 60

PRESSURE MEASUREMENTS

PREDICTION FROM UPWASH

X/C: UINF = 0.20 MPS, K = 6.4
FIGURE 61

- PRESSURE MEASUREMENTS
- PREDICTION FROM UPWASH

FLUCTUATING DIFFERENCE PRESSURE AMPL.

X/C; UINF = 0.09 MPS, K = 1.0
FIGURE 62
PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

X/C; UINF = 0.20 MPS, K = 1.0
FIGURE 63

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

FLUCTUATING DIFFERENCE PRESSURE AMPL.

X/C; UINF = 030 MPS, K = 1.0
Figure 64

Pressure measurements and prediction from upwash

Fluctuating difference pressure ampl.

X/C; UINF = 0.09 MPS, K = 2.0
FIGURE 65

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

FLUCTUATING DIFFERENCE PRESSURE AMPL.

X/C; UINF = 0.20 MPS, K = 2.0
FIGURE 66

FLUCTUATING DIFFERENCE PRESSURE AMPL.

PRESSURE MEASUREMENTS

PREDICTION FROM UPWASH

X/C; UINF = 0.30 MPS, K = 2.0
FIGURE 67

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

FLUCTUATING DIFFERENCE PRESSURE AMPL.

X/C; UINF = 009 MPS, K = 4.0
Figure 68

Fluctuating Difference Pressure Amplitude

Pressure Measurements
Prediction from Upwash

X/C; UINF = 0.20 MPS, K = 3.9
FIGURE 69

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

X/C; UINF = 0.30 MPS, K = 4.0
Figure 70

Pressure Measurements

Prediction from Upwash

Fluctuating difference pressure ampl.

X/C; UINF = 0.30 MPS, K = 5.1
FIGURE 71

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

FLUCTUATING DIFFERENCE PRESSURE AMPL.

X/C: UINF = 0.09 MPS, K = 6.3
FIGURE 72

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

FLUCTUATING DIFFERENCE PRESSURE AMPL.

X/C: UINF = 0.20 MPS, K = 6.4
FLUCTUATING PRESSURE AMPLITUDE

$U_{\text{INF}} = 20 \text{ MPS}$  $X/C = 0.70$

**FIG. 73**

<table>
<thead>
<tr>
<th>CP</th>
<th>REDUCED FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

**CP UP**

**CP LO**

**CP DELTA**

REDUCED FREQUENCY

REDUCED FREQUENCY

REDUCED FREQUENCY
RMS DIFFERENCE PRESSURE NEAR AIRFOIL TRAILING EDGE

$U_{INF} (\text{MPS}): +0.9, \Delta 20, \circ 30$

$\Delta CP$ RMS

$X/C$ vs $\Delta CP$ RMS for different $K$ values:

- $K=0.5$
- $K=1.0$
- $K=4.0$
- $K=64$

FIG. 74
Figure 75

Pressure measurements
Prediction from upwash

Phase lag for delta cp, deg

x/c: Uinf = 0.09 MPS, k = 1.0
FIGURE 76

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C: UINF = 0.20 MPS, K = 1.0
FIGURE 77

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C; UINF = 0.30 MPS, K = 1.0
FIGURE 78

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C: UINF = 0.09 MPS, K = 2.0
FIGURE 79

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C; UINF = 020 MFS, K = 2.0
FIGURE 80

PRESSURE MEASUREMENTS
FREQUENCY FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C; U_{INF} = 0.30 MPS, K = 2.0
FIGURE 81

PHASE LAG FOR DELTA CP, DEG

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

X/C; UINF = 0.09 MPS, K = 4.0
FIGURE 82

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C; UINF = 020 MPS, K = 3.9
FIGURE 83

PRESSURE MEASUREMENTS  
PREDSICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C; UINF = 0.30 MPS, K = 4.0
FIGURE 84

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEC

X/C; UINF = 0.30 MPS, K = 5.1
FIGURE 85

PRESSURE MEASUREMENTS
PREDICTION FROM UPWASH

PHASE LAG FOR DELTA CP, DEG

X/C: UINF = 009 MPS, K = 6.3
Figure 86

Pressure measurements vs. prediction from upwash

Phase lag for delta CP, deg

X/C; U∞ = 0.20 m/s, K = 6.4
FIGURE 87

PREDICTION FROM UPWASH

DELTA CP: 4=IN, 5=OUT OF PHASE COMPONENTS

X/C; UINF = 0.09 MPS, K = 1.0
FIGURE 88
PREDICTION FROM UPWASH

DELTA CP: 4=IN, 5=OUT OF PHASE COMPONENTS

X/C: UINF = 009 MPS, K = 4.0
FIGURE 89

FLUCTUATING SURFACE PRESSURE AMPLITUDE

X/C; K = 0.5, U INFINITY = 020 MPS
Figure 90

Fluctuating surface pressure amplitude vs. X/C for K = 1.0, U Infinity = 0.09 MPS.

- Triangle: Upper surface
- Plus: Lower surface
FIGURE 91

X/C; K = 1.0, U INFINITY = 020 MPS

FLUCTUATING SURFACE PRESSURE AMPLITUDE

V \ UPPPER SURFACE
+
\ LOWER SURFACE
FIGURE 92

\[ x/c; k = 1.0, u \text{ INFINITY} = 030 \text{ MPS} \]
FIGURE 93

FLUCTUATING SURFACE PRESSURE AMPLITUDE

X/C: K= 1.5, U INFINITY = 020 MPS
FIGURE 95

FLUCTUATING SURFACE PRESSURE AMPLITUDE

\[ I = 3.83 \]

\[ L = 0.20 \]

\[ M_{\infty} = 0.020 \]

\[ V_{UPPER\ SURFACE} = V_{LOWER\ SURFACE} \]

\[ X/C; K = 2.0, U_{\infty} = 020\ MPS \]
FIGURE 96

- Upper Surface
- Lower Surface

FLUCTUATING SURFACE PRESSURE AMPLITUDE

X/C; K = 2.0, U INFINITY = 030 MPS
FIGURE 97

FLUCTUATING SURFACE PRESSURE AMPLITUDE

X/C; K= 4.0, U INFINITY = 009 MPS
FIGURE 98

V A UPPER SURFACE
+  O LOWER SURFACE

FLUCTUATING SURFACE PRESSURE AMPLITUDE

X/C; K= 3.9, U INFINITY = 020 MPS
Figure 99

Fluctuating Surface Pressure Amplitude

Upper Surface

Lower Surface

X/C; K = 4.0, U Infini ty = 030 MPS
FIGURE 100

FLUCTUATING SURFACE PRESSURE AMPLITUDE

UPPER SURFACE
LOWER SURFACE

X/C; K = 5.1, U INFINITY = 030 MPS
FIGURE 101

FLUCTUATING SURFACE PRESSURE AMPLITUDE

\( X/C; k = 6.3, U \text{ INFINITY} = 0.09 \text{ MPS} \)
FIGURE 102

FLUCTUATING SURFACE PRESSURE AMPLITUDE

< UPPER SURFACE
+ SQUARE LOWER SURFACE

X/C; K = 6.4, U INFINITY = 020 MPS
**FIGURE 103**

- **UPPER SURFACE**
- **LOWER SURFACE**

\[ \frac{y}{c}; \quad K = 0.5, \quad \text{U \ INFINITY} = 020 \text{ MPS} \]
Figure 104

$X/C; K = 1.0, U_{\infty} = 0.09$ MPS

FLUCTUATING SURFACE PRESSURE PHASE, DEG

- * Upper Surface
- ▲ Lower Surface
Figure 105

* UPPER SURFACE

△ LOWER SURFACE

Fluctuating Surface Pressure Phase, deg

X/C: k = 1.0, \( U_{\text{infinity}} = 020 \) MPS
FIGURE 106

FLUCTUATING SURFACE PRESSURE PHASE, DEG

X/C; K = 1.0, U INFINITY = 030 MPS
FIGURE 107

* UPPER SURFACE

\[ X/C; K = 1.5, U \text{ INFINITY} = 020 \text{ MPS} \]
FIGURE 108

* UPPER SURFACE
△ LOWER SURFACE

X/C; K = 2.0, U INFINITY = 009 MPS
FIGURE 109

- UPPER SURFACE
- LOWER SURFACE

FLUCTUATING SURFACE PRESSURE PHASE, DEG

X/C; K = 2.0, U INFINITY = 020 MPS
FIGURE 110

- UPPER SURFACE
- LOWER SURFACE

FLUCTUATING SURFACE PRESSURE PHASE, DEG

X/C; K = 2.0, U INFINITY = 030 MPS
FIGURE 111

* UPPER SURFACE

△ LOWER SURFACE

FLUCTUATING SURFACE PRESSURE PHASE, DEG

X/C; K = 4.0, U INFINITY = 009 MPS
FIGURE 112

- UPPER SURFACE
- LOWER SURFACE

X/C: K = 3.9, U INFINITY = 020 MPS
FIGURE 113

* UPPER SURFACE
Δ LOWER SURFACE

X/C; K = 4.0, U INFINITY = 030 MPS
Figure 114

Fluctuating Surface Pressure Phase, Deg

Upper Surface

Lower Surface

X/C; K = 5.1, U Infinity = 300 MPS
FIGURE 115

* UPPER SURFACE
△ LOWER SURFACE

X/C; K = 6.3, U INFINITY = 009 MPS
FIGURE 116

FLUCTUATING SURFACE PRESSURE PHASE, DEG

X/C; K= 6.4, U INFINITY = 020 MPS
Figure 117

\( \kappa = 1.0 \)

\( u_{\text{INF}} = 30 \text{ MPS} \)

\( x/c = 0.05 \)

Frequency, Hertz

\( \log(\text{pressure}) \), DB
UPPER SURFACE PRESSURE POWER SPECTRAL DENSITY

FIGURE 118

\( k = 1.0 \)

\( u_{\text{INF}} = 30 \text{ MPS} \)

\( x/c = 0.20 \)

<table>
<thead>
<tr>
<th>FUNDAMENTAL</th>
<th>2X FUNDAMENTAL</th>
<th>3X FUNDAMENTAL</th>
<th>4X FUNDAMENTAL</th>
<th>FAN BLADES</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
<td>80</td>
<td>120</td>
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<td>-70</td>
<td>-80</td>
<td>-90</td>
<td>-100</td>
</tr>
</tbody>
</table>

FREQUENCY, HERTZ
FIGURE 119

$\kappa = 1.0$

$u_{\text{INF}} = 30 \text{ m/s}$

$x/c = 0.60$

FUNDAMENTAL

2X FUNDAMENTAL

FAN BLADES

4X FUNDAMENTAL

3X FUNDAMENTAL

3X FAN

5X FUND

LOG (PRESSURE $^2$), DB

FREQUENCY, HERTZ
Figure 120

$k = 1.0$

$U_{\text{INF}} = 30 \text{ MPS}$

$x/c = 0.80$
FIGURE 121

$K = 1.0$

$U_{\text{INF}} = 30 \text{ MPS}$

$x/c = 0.95$
FIGURE 122
DIFFERENCE PRESSURE PHASE
X/C = 0.98

PHASE LAG, DEG

VELOCITY
○ 9 MPS
△ 20 MPS
□ 30 MPS

ωδ^2/γ