ANalytical techniques for determining temperatures,
thermal strains, and residual stresses
during welding

by

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ANALYTICAL TECHNIQUES FOR DETERMINING TEMPERATURES, THERMAL STRAINS, AND RESIDUAL STRESSES DURING WELDING

by

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ABSTRACT

New and modifications of existing analytical and numerical techniques have been developed for predicting temperatures, thermal strains and residual stresses due to welding of high strength steels, especially HY-130.

A closed form solution was found for predicting temperatures during welding of thin plates using a new three-dimensional heat source model, improving upon the already existing ones based on the conventional point and line heat sources. Comparisons with experimental results show good correlation.

Using ADINAT, a nonlinear finite element model for solving the heat transfer problem during welding was devised. The obtained results were compared with experiments performed on medium thickness GMA welded HY-130 plates.

A model for taking into account the phase transformation strains during welding quenched and tempered steels was developed and incorporated into the finite element program ADINA. The analytical predictions were then compared with the above mentioned experiments yielding a good correlation.

Thesis Supervisor: Dr. Koichi Masubuchi.
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To Nada
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CHAPTER I

INTRODUCTION

The welding processes have been and are extensively used for the fabrication of various structures ranging from bridges and machinery to all kinds of seagoing vessels to nuclear reactors and space vehicles. This is the case because of the many advantages it offers compared to other fabrication techniques — excellent mechanical properties, air and water tightness, good joining efficiency, to name but a few.

At the same time, however, welding creates various problems of its own that have to be solved. In the past these problems have been tackled through experimental investigations. Since low-carbon steel was the primary material used in the first two decades after World War II, extensive experimental parametric analyses were performed in an effort to solve these problems and hence produce satisfactory welded structures. As a consequence, a substantial amount of data has been collected over the years on welding low-carbon steel using conventional arc techniques.

A shortcoming of these types of experimental investigation, though, is the large amount of manpower required and the high cost. The question thus arises of what to do when conventional welding techniques are to be applied to other materials, such as high strength quenched and tempered steels, aluminum, and titanium or when modern welding techniques, such as electron beam and laser, are to be used. Fortunately enough, sinc
advent of the computer, a new tool became available which, combined with limited experimental efforts, could give solutions to many of the problems due to welding.

One of the problems suitable for computer analysis is the one involving residual stresses and distortion due to welding. The uneven temperature distribution produced during welding gives rise to incompatible strains which in turn result in self equilibrating residual stresses and distortion that remain in the structure after it has cooled down to ambient temperature. The phenomena involved are very complex necessitating the use of numerical techniques and hence computers for their solution.

The next logical question coming to one's mind is why should one be concerned with residual stresses and distortion. The answer is that the effect they have on the service behavior of the welded structure can be detrimental. Brittle fracture can occur earlier because the presence of the residual stresses, combined with any external loading, can substantially decrease the critical flow size of the weldment. Compressive residual stresses in the regions located some distance away from the weld line can also substantially decrease the critical buckling stress of a structure, especially if it is composed of thin plates, causing structural instabilities. Similar effects occur in the areas of fatigue fracture, stress corrosion cracking, hydrogen embrittlement, etc.

It becomes, thus, evident from the above discussion that the existence of a methodology capable of predicting the residual stresses after the structure has cooled down is highly
desirable. The designer could then change the design or use different materials or welding processes in an effort to minimize the residual stresses and distortion and hence their adverse effects. This thesis is concerned with modifying the already available techniques in an effort to improve their predictive capabilities.

1.1 Sources of Residual Stresses

Residual stresses are those stresses that would exist in a body if all external loads were removed. If a material is heated uniformly, it expands uniformly and no thermal stress is produced. On the other hand, if the material is heated unevenly, thermal stresses are produced. Residual stresses will also be produced when unevenly distributed non-elastic strains, such as plastic strains exist. In the case of welding such plastic strains do occur in an area around the weld centerline caused primarily by the dramatic decrease of the material's yield stress at very high temperatures. As a consequence residual stresses are always produced in welded structures [1].

Three different sources of residual stresses due to welding can be identified according to Macherauch and Wohlfahrt [2], all contributing to the inelastic strain. One is the difference in shrinkage of differently heated and cooled areas of a welded joint. The weld metal, originally subjected to the highest temperatures, tends upon cooling to contract more than all other areas. This contraction is hindered by the other parts of the joint, thus resulting in the formation of high longitudinal stresses, \( \sigma_x \), in the weld metal. Similar tensile
stresses, $\sigma_y$, arise in the transverse direction, too, but of smaller magnitude.

A second source is the uneven cooling in the thickness direction of the weld. Surface layers of the weld and the highly heated areas close to it usually cool more rapidly than the interior, especially in the case of thick plates. Thermal stresses thus arise over a cross-section which can lead to heterogenous plastic deformations and hence to residual stresses. These "quenching" residual stresses are expected to be compressive at the surface of the highly heated areas and to self-equilibrate with the tensile ones in the inner regions.

The final source of residual stresses will come from the phase transformations of austenite to ferrite, bainite, or martensite occurring during cooling. These transformations are accompanied by an increase in specific volume, causing the material being transformed (in the weld metal and the HAZ) to want to expand. But this expansion is hindered by the cooler material not being transformed, resulting in compressive stresses of the transformed material and tensile in the other regions.

The total residual stresses will thus be a combination of the above. In most previous analyses, though, only the first two sources were considered. This study will try to incorporate all three, in an effort to find the actual impact of phase transformations on the magnitude and distribution of transient and residual stresses.
1.2 **Methodology**

To accurately solve the welding problem, or for this matter any problem involving both thermal and stress analysis, one should start from first principles, which in this case is the first law of thermodynamics. This would mean, however, that one would have to solve a problem containing mechanical and thermal coupling, a fact that makes the analysis extremely complicated, if not impossible with the present state of knowledge. Mróz [3-5] has made initial attempts at formulating and investigating the problem of coupled thermoplasticity, but the whole process is still in its initial stages.

It becomes, therefore, necessary to uncouple the thermal and mechanical parts of the welding problem and solve each of them separately. Hibbitt [6] has examined the assumptions that have to be made in order to uncouple the problem and found them to be acceptable for the case of high strength quenched and tempered steels, such as HY-130.

The same approach was followed in this study. First, the problem of heat flow during welding was solved and then a strain and stress analysis was performed using as input, among others, the previously calculated temperature distribution.

1.3 **Problem Characteristics**

Figure 1.1 shows the physical phenomenon to be examined. A welding arc is travelling with a speed $v$ between two plates causing them to coalesce by providing filler metal.
FIGURE 1.1 Weldment Configuration
The four parts of a weld that are subjected to different thermal histories are shown in Figure 1.2. Part 1 constitutes the filler metal which is deposited molten and later solidifies as cooling begins. Part 2 is the part of the joint that melts and later resolidifies during cooling. Both these parts define the weld metal. Part 3 is the heat affected zone (HAZ) defined as that part of the joint in which the maximum temperature reached is above the $A_1$ but below the solidus temperature. Finally, part 4 is the base metal.

Interest will be focused in this study in the first three parts. These are the ones in which the largest plastic deformations, causing the occurrence of residual stresses, are exhibited. All phase transformations also occur in this region.

1.4 Objectives

This study is primarily concerned with the development of analytical and numerical techniques capable of predicting temperatures, thermal strains, transient strains, and residual stresses due to welding high strength quenched and tempered steels, especially the HY-130 steel. More specifically, the objectives are as follows:

1. Development of a closed form solution for predicting temperature distributions due to a moving three-dimensional heat source taking into account temperature dependent material properties and surface heat losses.

2. Comparison of the above solution with other closed-form ones and with experiments.
FIGURE 1.2 The Four Parts of a Weld Subjected to Different Thermal Histories.

1, 2  Weld Metal
3  Heat Affected Zone
4  Base Metal
3. Establishment of the transformation characteristics and the temperature dependence of material properties for the HY-130 steel.

4. Measurement of temperatures and transient strains during gas metal arc welding of 1 in (25.4 mm) thick HY-130 plates.

5. Modelling of the heat transfer problem during welding using the finite element method.

6. Comparison of the numerical results with the experimentally obtained ones.

7. Assessment of the accuracy of the one-dimensional stress analysis through comparisons with experiments.

8. Incorporation of phase transformation effects in the finite element nonlinear stress analysis program ADINA.


10. Comparison of the results obtained in the previous step with experimental data.
CHAPTER II

ANALYTICAL TREATMENT OF HEAT FLOW DURING WELDING

The importance of accurately predicting the temperature distribution during welding has been recognized for many years by both scientists and engineers working with welding problems. This importance stems from the fact that most of the phenomena subsequently encountered, such as residual stresses, distortion, metallurgical changes, etc. have their origin in the uneven temperature distribution and the fast heating and cooling rates that occur during the welding operation.

All of the early attempts at solving the problem of heat flow during welding were analytical in nature since they were performed before the advent of the computer. As a consequence several simplifying assumptions had to be made to allow for the solution of the highly nonlinear governing partial differential equation and the accompanying boundary conditions. Starting in the mid-sixties, however, several investigators from around the world have used the computer to numerically predict the thermal history during welding with much greater accuracy, as it will be discussed in Chapter 4.

This chapter will exclusively deal with analytical solutions of the problem which, though not as accurate as the numerical ones, provide for the establishment of the general laws and thus facilitate a better understanding of the phenomena involved. A thorough literature review will first be presented dealing primarily with point and line heat source
solutions to the problem. A new model will then be developed based on a finite heat source and linear variation of the material thermal conductivity. Finally, comparisons will be made between the new model and the old ones, including some modifications of them, as well as with experimental results.

2.1 Literature Review

2.1.1 Fundamental Solutions

The first exact analytical solutions of the problem of heat flow during welding were obtained by Rosenthal [7 - 9] in the late 1930s and early 1940s, although a particular case was considered independently at around the same time by Boulton and Lance Martin [10]. Rosenthal solved the conventional heat conduction differential equation for constant point, line, and plane heat sources moving at a constant speed with respect to a fixed Cartesian coordinate system. To facilitate easier handling of the problem, he assumed that welding was performed over a sufficient length so that the temperature distribution around the heat source would not change if viewed from a coordinate system moving with the heat source. This phenomenon is called quasi-stationary or quasi-steady state. Additional assumptions were made as follows:

1. The physical properties of the conducting medium are constant.

2. The heat losses through the surface of the conducting medium to the surrounding atmosphere are neglected.

3. Heat created in electric welding by the Joule effect is negligible.
4. The phase changes and the accompanying absorption or release of latent heat in the conducting medium are neglected.

5. The conducting medium is infinitely large in the two-dimensional case (line heat source) and semi-infinitely large in the three dimensional case (point source).

Based on the above assumptions Rosenthal developed the following exact solutions for the two- and three-dimensional cases respectively (see Figures 1.1 and 2.1):

\[ \theta - \theta_0 = \frac{Q}{2\pi kH} \cdot e^{-\lambda v \xi} \cdot K_0(\lambda vr) \]  \hspace{1cm} (2.1)

\[ \theta - \theta_0 = \frac{Q}{2\pi k} \cdot e^{-\lambda v \xi} \cdot \frac{e^{-\lambda v R}}{R} \]  \hspace{1cm} (2.2)

where

\( \theta_0 \) = initial temperature
\( Q \) = total heat input
\( H \) = plate thickness
\( \xi \) = \( x - vt \)
\( v \) = welding (or arc travel) speed
\( t \) = time

\((x,y,z)\) = fixed Cartesian coordinate system
\( r = (\xi^2 + y^2)^{1/2} \)
\( R = (\xi^2 + y^2 + z^2)^{1/2} \)
\( k \) = thermal conductivity
\( 1/2\lambda = k/\rho c=\kappa = \text{thermal diffusivity} \)
(a) Two-Dimensional Heat Flow: Thin Plate

(b) Three-Dimensional Heat Flow: Semi-infinite Plate

FIGURE 2.1 Schematic Diagram of Heat Flow Patterns
\( \rho = \text{density} \)
\( c = \text{specific heat} \)

\( K_0(x) = \) modified Bessel function of second kind and zero order.

For the case of thin plates (two-dimensional solution) Rosenthal [8] and other investigators have suggested that heat losses through the surface to the surrounding atmosphere might have to be taken into account by replacing in the Bessel function of eqn. (2.1) the factor \( \lambda v \) by

\[
\left[ \frac{\lambda^2}{(\lambda v) + \frac{h_1 + h_2}{kH}} \right]^{1/2}
\]

(2.3)

where \( h_1 \) and \( h_2 \) are the heat transfer coefficients, assumed constant, at the top and bottom of the plate respectively.

Furthermore, by using the so-called "method of images" or "image source method" [8], one can get solutions for the cases of large but finite thickness and/or finite breadth plates. For example, a three-dimensional solution for laying a weld bead on a finite thickness plate with adiabatic boundary conditions can be obtained by adding an infinite series to eqn. (2.2), yielding:

\[
\theta - \theta_0 = \frac{Q}{2\pi k} \cdot e^{-\lambda v \xi} \cdot \left\{ \frac{e^{-\lambda v R}}{R} + \sum_{n=1}^{\infty} \left( \frac{e^{-\lambda v R_n}}{R_n} + \frac{e^{-\lambda v R'_n}}{R'_n} \right) \right\}
\]

(2.4)
where

\[ R_n = \left( \xi^2 + y^2 + (2nH-z)^2 \right)^{1/2} \]

\[ R'_n = \left( \xi^2 + y^2 + (2nH+z)^2 \right)^{1/2} \]

An extension of this concept to the case of multipass welding of thick plates will be described in Section 2.5.

2.1.2 Experimental Verifications

Following these initial developments many investigators tested the validity of the proposed equations experimentally. Although Myers et al [11] provide a thorough exposition of these works in their review paper, some of the most important research efforts will be cited here for completeness.

Rosenthal and Schmerber [12] tested the two-dimensional solution, eqn. (2.1), experimentally and reported satisfactory agreement of results only for distances far from the weld line. In the neighborhood of the arc the computed isotherms were further away from the arc than the measured ones, a fact that can be attributed to the simplified, but unrealistic, assumptions of a concentrated heat source and constant material properties.

In 1941, Mahla and co-workers [13] proposed a new solution for the case of a point source moving on the top surface of a semi-infinite conducting medium which is adiabatic along its boundaries. By integrating an appropriate Green's function they found the following equation:
\[ \theta - \theta_0 = \frac{Q}{2\pi kR} \cdot e^{-\lambda v (\xi + R)} \cdot (1 - \hat{\gamma}) \]  \hspace{1cm} (2.5)

where

\[ \hat{\gamma} = \frac{\beta}{\lambda v R} \left( 1 + \frac{1}{\lambda v R} \right) \]

\[ \beta = \frac{(\lambda v)^2}{24} \left( 4p^2 + d^2 \right) \]

\[ p = \text{weld penetration} \]
\[ d = \text{weld width} \]

Despite the fact, however, that the introduced correction factor, \( \hat{\gamma} \), tends to reduce the temperatures compared to the ones predicted by Rosenthal's solution, the formula was still found to yield too high maximum temperatures and excessive cooling rates when correlated with experimental results.

A somewhat different approach was taken by Wells [14] who developed an equation relating the width of the melting isotherm to the amount of heat input based on the moving line heat source theory. He reports good agreement with data from Jackson and Shrubsall [15]. Apps and Milner, however, during their investigation of heat flow during argon-arc welding [16] found that Wells' equation does not correlate very well with experimental measurements of weld pool contours.

To test the infinite plate assumption made in most
analytical solutions, Roberts and Wells [17] examined the
effect of bounding planes on the temperature distribution due
to welding using Rosenthal's solution, Wells' correlation,
and the "image source method". They concluded that a plate
with half-width greater than ten times the width of the fusion
zone approaches the thermal condition of an infinite plate.

Various other investigators looked at comparisons between
predicted and measured values of peak temperatures and cooling
rates, two parameters crucial for the determination of the
metallurgical structure of the base metal after the welding
operation. Adams [18 - 19], among others, derived engineer-
ing relationships, on the basis of extensive experimental
results, giving peak temperatures and cooling rates directly
as functions of welding, thermal, and geometric variables.
He concluded that cooling rates at a constant temperature are
comparatively independent of distance from the weld center-
line within the heat affected zone region and that cooling
rates can be predicted within 25% if the above mentioned
variables are known with sufficient accuracy. The computed
values, however, are always higher than the experimental
ones, a fact that can be attributed to the omission of the
latent heat effects near the heat source region.

Finally, in another investigation Christensen and
co-workers [20 - 21] found good agreement only with trends
for the width, depth, and cross-sectional area of the molten
zone in the three-dimensional case. They concluded that the
transfer of heat from the welding arc to the base metal is a
complicated process making it difficult to perform accurate calculations.

2.1.3 Variable Material Properties

Looking at data on thermal conductivity, specific heat, and density one finds that all these parameters are highly dependent upon temperature, thus making the constant properties assumption of the analytical solutions unrealistic, especially for the regions close to the heat source where the material exhibits very high temperatures. It is desirable to predict the high temperature region as accurately as possible, however, since it is this region that is directly related to the size of the plastic zone and the accompanying residual stresses and distortion.

Grosh, et al., [22 - 23] have formulated an analytical expression for the temperature distribution during welding assuming that thermal conductivity, k, and the product of density and specific heat, \( \rho c \), vary in the same manner with temperature so that their ratio, the thermal diffusivity, remains constant. For the simple, special case of two-dimensional temperature distribution with both \( k \) and \( \rho c \) varying linearly with temperature, the solution is

\[
\theta = \frac{1}{\gamma} \left[ \left( \frac{\gamma_0}{\pi k_0 H} \cdot e^{-\lambda \nu \xi} \cdot K_0 (\lambda \nu r) + (1 + \gamma \theta_0)^2 \right)^{1/2} - 1 \right] \quad (2.6)
\]

where

\[
k = k_0 \cdot (1 + \gamma \theta)
\]

\[
\rho c = (\rho c)_0 \cdot (1 + \gamma \theta)
\]
This linearity assumption can be valid in some aluminum alloys. For other materials $k$ and $\rho c$ cannot be assumed to vary according to the same law, making the analytical treatment very complex.

Researchers at the Massachusetts Institute of Technology [24 - 26] adopted the iteration method to take into account the temperature dependence of material properties. The fundamental heat source solution with material properties at some temperature, say 300°C, provides the first approximate solution at a particular point. This temperature is compared with the initial guess and if the two temperatures disagree by more than 0.5°C, new properties are found for a temperature halfway inbetween. These new values are used to obtain a new temperature estimate. The process is repeated until convergence is reached. It should be pointed out that although this iteration method generally gives good predictions outside the fusion zone, there is no guarantee that it will converge to the correct solution since the approximation used may not satisfy the energy conservation law.

2.2 Heat Input Modeling

Most investigators agree that perhaps the most critical input required for the welding thermal analysis is the power, $Q$, that enters the plate or section being welded. It is customary to express this total heat input by the formula

$$Q = \eta_a \cdot V \cdot I \quad (2.7)$$

where $V$ and $I$ are the arc voltage and current respectively, their product giving the electric arc power. The other
parameter in the above equation, $\eta_a$, is called arc efficiency; it represents the ratio of the power introduced by the arc into the metal to the total electric arc power. In other words it provides a semi-empirical way of taking into account the various heat losses that occur through electrode tip heating, radiation to the surrounding atmosphere, metal spatter, etc.

Arc efficiency, $\eta_a$, is heavily dependent on the welding process used, the penetration achieved, the shielding gas and many other factors that make it very difficult to predict. It is therefore measured experimentally using the so-called calorimetry method which consists of measuring the temperature rise of a water bath after immersion of a just welded metal specimen. Christensen and co-workers [20 - 21] made extensive measurements using this approach and found approximate ranges of the arc efficiency for various welding processes. Table 2.1 is a summary of the values proposed by Christensen. Included in the same table are also values proposed by Rykalin and co-workers [27 - 28].

Several other investigators have tried a different approach to the problem of estimating the arc efficiency. Among the latest efforts is that of Tsai [29] who tried to correlate the arc efficiency with the weld width in the case of thin plates. Dividing the conducting medium into a solid and liquid region, he derived an equation of the form:

$$Q = 2\pi k_H \cdot f(d)$$

(2.8)
<table>
<thead>
<tr>
<th></th>
<th>Christensen</th>
<th>Rykalin</th>
<th>Tsai</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMAW</td>
<td>-</td>
<td>0.65-0.85</td>
<td>-</td>
</tr>
<tr>
<td>Mild Steel</td>
<td>0.65-0.70</td>
<td>-</td>
<td>0.80-0.90</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.70-0.85</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>SAW</td>
<td>0.90-0.99</td>
<td>0.90-0.99</td>
<td>0.85-0.98</td>
</tr>
<tr>
<td>SMAW</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mild Steel, ac</td>
<td>0.66-0.85</td>
<td>0.65-0.85</td>
<td>0.55-0.90</td>
</tr>
<tr>
<td>GTAW</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Mild steel, ac</td>
<td>0.22-0.48</td>
<td>0.20-0.50</td>
<td>-</td>
</tr>
<tr>
<td>Mild steel, dc</td>
<td>0.36-0.46</td>
<td>0.45-0.75</td>
<td>-</td>
</tr>
<tr>
<td>Aluminum, ac</td>
<td>0.21-0.43</td>
<td>0.20-0.50</td>
<td>-</td>
</tr>
</tbody>
</table>

**Nomenclature**

GMAW = Gas Metal Arc Welding  
SAW = Submerged Arc Welding  
SMAW = Shielded Metal Arc Welding  
GTAW = Gas Tungsten Arc Welding
Then, by experimentally measuring the weld width, d, he was able to calculate the total heat input, Q, and then estimate the arc efficiency using eqn. (2.7). His estimates are also shown in Table 2.1, the values being similar to those of the other investigators with the exception of GMAW where he predicts higher arc efficiencies.

Of equal importance to the magnitude of the total heat input is its distribution. At the solutions presented in Section 2.1 a point or line heat source was assumed. As Rykalin [28 - 30] and other investigators report, however, a more realistic approach is to assume a Gaussian radial heat flux distribution of the form (see Fig. 2.2)

\[ q(r) = q_0 \cdot e^{-Cr^2} \quad (2.9) \]

where

- \( q_0 \) = maximum heat flux at the center of the heat spot, [W/cm²]
- \( C \) = heat flux concentration coefficient, [cm⁻²]
- \( r \) = radial distance from the center of the heat spot, [cm]

A stationary source with such a heat flux distribution may be called normal circular. The concentration coefficient C evaluates the source concentration: the higher C is, the more concentrated a heat source is and the smaller the diameter \( d_h \) of the heat spot is.

The distance \( r_h = d_h/2 \) can be defined for all practical
FIGURE 2.2 Distribution of Heat Flux Across the Hot Spot for a Normal Circular Source.
purposes as this at which the heat flux \( q(r_h) \) falls to 0.05 of the maximum heat flux \( q_o \). Then

\[
q(r_h) = q_o \cdot e^{-Cr_h^2} = 0.05 \ q_o
\]

from which

\[
e^{-Cr_h^2} = 0.05
\]

or

\[
Cr_h^2 = 3.0
\]

The conventional diameter of the heat spot

\[
d_h = 2 \cdot \frac{\sqrt{3.0/C}}{\sqrt{C}} = \frac{3.46}{\sqrt{C}}
\]

(2.10)

is thus inversely proportional to the square root of the concentration coefficient \( C \), as previously mentioned.

Furthermore, if the total power absorbed by the workpiece, \( Q \), is known, the distribution function, eqn. (2.9), may be integrated to give

\[
Q = \int_0^\infty q(r) \cdot 2\pi r \cdot dr
\]

\[
= 2\pi q_o \int_0^\infty r \cdot e^{-Cr^2} \ dr
\]

or

\[
Q = \pi q_o / C
\]

(2.11a)
from which the maximum heat flux at the center of the heat
spot can be calculated to be

\[ q_0 = \frac{CQ}{\pi} \quad (2.11b) \]

where \( Q \) is given by eqn. (2.7).

Experimental data on ranges of \( C \) for the various welding
processes are shown in Table 2.2 after Rykalin and Nikolaev
[28].

The above equations are valid for a stationary arc.
High speed cinematography reveals, however, that during weld-
ing, when the arc is moving, the arc column is not radially
symmetric but rather distorted backwards. This observation
led Tsai [29] to propose the following equation for the arc
heat distribution instead of eqn. (2.9):

\[ q(r, \xi) = q_0 \cdot e^{-Cr^2 - \lambda v \xi} \quad (2.12) \]

where all the symbols have been previously defined.

Equation (2.12) can also be written in terms of a
moving cylindrical coordinate system as

\[ q(r, \psi) = q_0 \cdot e^{-Cr^2 - \lambda r \cos \psi} \quad (2.13) \]

where \( \xi = r \cos \psi \).
This last expression, if calculated at a constant radial dis-
tance, \( r \), from the center of the arc column, clearly
represents the backward distortion of the arc heat distribu-
<table>
<thead>
<tr>
<th>Welding Process</th>
<th>$C, [\text{cm}^{-2}]$</th>
<th>Polarity, Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>GMAW</td>
<td>1.26-1.32</td>
<td>ac, 550-1100A</td>
</tr>
<tr>
<td>SAW</td>
<td>1.00-2.50</td>
<td>ac, 550-1200A</td>
</tr>
<tr>
<td>SMAW</td>
<td>1.26-1.32</td>
<td>ac, 550-1100A</td>
</tr>
<tr>
<td>GTAW, steel</td>
<td>6.00-14.00</td>
<td>dcsp, 40-200A</td>
</tr>
<tr>
<td>Carbon electrode</td>
<td>1.00-3.50</td>
<td>dcsp, 95-2500A</td>
</tr>
</tbody>
</table>
tion: more heat enters the plate behind the arc
\((\pi/2 < \psi < 3\pi/2)\) than ahead of it \((0 < \psi < \pi/2)\) and \(3\pi/2 < \psi < 2\pi\).
Furthermore, for the special case of the weld centerline
\((y = 0, r = |\xi|)\), the maximum arc heat flux can be found from eqn. (2.12) to be located at

\[ \xi = -\frac{\lambda v}{2C} \]

i.e. behind the center of the arc, and to have a value equal to

\[ q_{\text{max}} = q_o \cdot e^{-(\lambda v)^2 / 4C} \]

i.e. greater than \(q_o\).

The two unknowns, \(q_o\) and \(C\), appearing in eqns. (2.12) and (2.13) can be found by employing two conditions. The first one uses the total heat input to the plate in a similar manner that produced eqn. (2.11a):

\[ Q = 2q_o \int_0^r \int_0^{\pi} e^{-Cr^2 - \lambda v r \cos \psi} r dr d\psi \quad (2.14) \]

The second condition stems from the heat input model proposed by Apps and Milner [16]. According to them, the heat input from an argon arc, \(Q\), can be expressed as the sum of the concentrated intense heat, \(Q_c\), uniformly distributed in the cylindrical arc column of radius \(r_a\), and the diffused heat, \(Q_d\), dispersed around the arc column and over the heat spot due to radiation, convection, and conduction from the
arc column to the plate. The following formulae were proposed:

\[ Q_c = a \cdot I \]  \hspace{1cm} (2.15a)

\[ Q_d = b \cdot l_{ac} \cdot I \]  \hspace{1cm} (2.15b)

where \( l_{ac} \) is the arc length and \( a \) and \( b \) are constants. For an argon arc in air \( a/b \) was reported to be equal to 1.59.

Accepting this model, one can then write the following

\[
\frac{Q_c}{Q_d} = \frac{\int_0^{\pi} \int_0^{r_{a}} e^{-Cr^2 - \lambda vr \cos \psi} \cdot r \cdot d\phi \cdot dr}{\int_0^{\pi} \int_0^{r_{h}} e^{-Cr^2 - \lambda vr \cos \psi} \cdot r \cdot d\phi \cdot dr - \int_0^{\pi} \int_0^{r_{a}} e^{-Cr^2 - \lambda vr \cos \psi} \cdot r \cdot d\phi \cdot dr}
\]

\[ = \frac{a}{b} \cdot \frac{1}{l_{ac}} \]  \hspace{1cm} (2.16)

Equations (2.14) and (2.16), if solved numerically, give values of \( q_o \) and \( C \) if all other parameters are known. Figure 2.3 shows an example of the ensuing arc heat distribution assuming that \( a/b \) is 1.59 cm and \( v \) is 91 cm (0.06 cm/sec).

Given the fact, however, that several of the required parameters, e.g. \( a/b \), \( l_{ac} \), \( r_{a} \), are not generally known with accuracy, only eqn. (2.14) will be used in later calculations for the determination of \( q_o \) (see Appendix A); the concentration coefficient \( C \) will be estimated from eqn. (2.10).

2.3 Three-Dimensional Finite Heat Source Model

The discussion of the previous two sections leads to
\[ q_0 = 320.0 \text{ kJ/sec.cm}^2 \]
\[ C = 87.3 \text{ cm}^{-2} \]

FIGURE 2.3 Arc Heat Distribution for a Moving Finite Heat Source.
the conclusion that in order to more accurately describe the temperature distribution and cooling rates in the region close to the weld more realistic assumptions should be employed. In this section a model will be developed under the following general assumptions:

1. Quasi-stationary state

2. The heat input will be provided by a moving three-dimensional skewed normally distributed heat source (eqn. 2.12), instead of a line or point source.

3. The thermal conductivity of the material will be assumed to be a linear function of temperature.

4. The thermal diffusivity of the material will be assumed to be constant.

5. Convective and radiation boundary heat losses will be taken into account through a constant average "effective" heat transfer coefficient.

6. Phase transformation and Joule heating effects will be neglected.

7. The initial temperature of the plate, $\theta_o$, can be different from the environmental (ambient) temperature, $\theta_e$, to allow for preheating.

2.3.1 Governing Equation

The governing equation of heat flow during welding is the partial differential equation (P.D.E.) of heat flow in a conducting medium [11, 31], namely

$$\frac{\partial}{\partial x} \left( k \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial \theta}{\partial z} \right) = \rho c \frac{\partial \theta}{\partial t} - W_i \quad (2.17)$$
where $w_i$ is the internal heat generation rate per unit volume and the other variables as previously defined.

Letting

$$T = \theta - \theta_o$$

(2.18)

where $\theta_o$ is the initial temperature, eqn. (2.18) can be written as

$$\frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) = \rho c \frac{\partial T}{\partial t} - w_i$$

(2.19)

Using the so-called Kirchhoff's transformation [31] the new variable $u$ can be defined

$$u = \frac{1}{k_o} \int_0^T k(\eta) \, d\eta$$

(2.20)

where $k_o$ is the value of thermal conductivity at $T = 0$ ($\theta = \theta_o$), and $k(\eta)$ is the function describing the temperature dependence of the material thermal conductivity. Assuming that this function is linear

$$k(T) = k_o (1 + \gamma T)$$

(2.21a)

or

$$k(\theta) = k_o [1 + \gamma (\theta - \theta_o)]$$

(2.21b)
equation (2.20) can be solved to give

$$\theta = \theta_0 + \frac{1}{\gamma} \left( \sqrt{1 + 2\gamma u} - 1 \right)$$  \hspace{1cm} (2.22)

Furthermore, noting that

$$\frac{\partial u}{\partial t} = \frac{k}{k_0} \cdot \frac{\partial T}{\partial t}$$

$$\nabla u = \frac{k}{k_0} \nabla T$$

eqn. (2.19) can be written as

$$\nabla^2 u = \frac{1}{k} \cdot \frac{\partial u}{\partial t} - \frac{1}{k_0} \cdot w_1$$  \hspace{1cm} (2.23)

where $\kappa = k/\rho c$ is the thermal diffusivity, assumed constant (recall that, as mentioned in section 2.1.3, this assumption is particularly valid for aluminum alloys) and $\nabla^2$ is the Laplacian operator.

Equation (2.23) was derived with respect to a stationary Cartesian coordinate system $(x,y,z)$. A coordinate system moving with the welding arc can now be defined through the transformation

$$\xi = x - vt, \hspace{0.5cm} y = y, \hspace{0.5cm} z = z$$

and noting that

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi} \cdot \frac{\partial \xi}{\partial x} = \frac{\partial u}{\partial \xi}$$

$$\frac{\partial u}{\partial t} \bigg|_{SS} = \frac{\partial u}{\partial \xi} \cdot \frac{\partial \xi}{\partial t} + \frac{\partial u}{\partial t} \bigg|_{MS} = -v \cdot \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial t} \bigg|_{MS}$$
where SS denotes the stationary and MS the moving system, eqn. (2.23) can be written in the new system as

\[
\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial z^2} = -2\lambda v \frac{\partial u}{\partial \xi} + 2\lambda \frac{\partial u}{\partial t} \bigg|_{MS} - \frac{1}{k_0} W_i
\]  

(2.24)

where 2\lambda = 1/\kappa.

Given the fact that a solution in the quasi-stationary state is sought, the time dependence can be dropped; moreover, since it is assumed that no heat is generated internally, i.e. that both the Joule and transformation effects can be neglected, the last term of the above equation can also be dropped. Equation (2.24) therefore becomes

\[
\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} + \frac{\partial^2 u}{\partial z^2} + 2\lambda v \frac{\partial u}{\partial \xi} = 0
\]  

(2.25)

To further simplify this equation it is appropriate at this point to briefly mention a type of boundary conditions that will be fully investigated in the next section. The reference is to the heat losses from the plate's surfaces through natural convection and radiation which are expressed in equations having terms that include the quantity (θ-θ_e), where θ_e is the environmental temperature. This temperature can be smaller or equal to the plate's initial temperature θ_o depending on whether a plate preheating operation has taken place or not respectively. It should be mentioned that it is important for the solution to be able to handle the preheating case since the operation is used extensively when welding high
strength quenched and tempered steels to prevent cracking and also as a measure to reduce distortion. Based on the considerations just described, a new variable is introduced by

\[ \hat{u} = u + (\theta_o - \theta_e) \]  

(2.26)

where the operation is legitimate since the variable \( u \) has, according to eqn. (2.20), the same dimensions as temperature. Note that eqn. (2.25) is then transformed into

\[ \frac{\partial^2 \hat{u}}{\partial \xi^2} + \frac{\partial^2 \hat{u}}{\partial y^2} + \frac{\partial^2 \hat{u}}{\partial z^2} + 2 \lambda \nu \cdot \frac{\partial \hat{u}}{\partial \xi} = 0 \]  

(2.25a)

Furthermore, the simple solutions presented in section 2.1.1 and experimental data show that the temperature distribution, as viewed by an observer travelling with the welding arc, is steeper ahead of than behind the arc. This behavior can be simulated by invoking the function \( \exp(-\lambda \nu \xi) \) through the following transformation

\[ \hat{u} = e^{-\lambda \nu \xi} \cdot \phi(\xi, y, z) + (\theta_o - \theta_e) \]  

(2.27a)

under which eqn. (2.25a) becomes

\[ \frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} - (\lambda \nu)^2 \cdot \phi = 0 \]  

(2.28a)

Using cylindrical coordinates \((r, \psi, z)\) the last two equations can be written as

\[ \hat{u} = e^{-\lambda \nu \xi} \cdot \phi(r, z) + (\theta_o - \theta_e) \]  

(2.27b)

\[ \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} - (\lambda \nu)^2 \cdot \phi = 0 \]  

(2.28b)
where \( r^2 = \xi^2 + y^2 \)

\[ \xi = r \cos \psi \]

The angle dependence has been dropped from the function \( \phi \) because of the symmetrical form of eqn. (2.28a) with respect to \( \xi \) and \( y \) and the circularity of the boundary conditions as it will become evident in the next section.

Equation (2.28b) is the final equation whose solution is sought.

2.3.2 Boundary Conditions

Let \( r_h \) be the radius of the heat input circle on the top surface of the plate, \( z = H \) (see Figure 2.4). The solution to equation (2.28b) will be divided into an inner solution \( \phi^i(r,z) \) for \( r < r_h \) and an outer solution \( \phi^o(r,z) \) for \( r > r_h \) which will be matched on the common boundary \( r = r_h \). Boundary conditions will accordingly be divided.

First, the boundary conditions will be expressed in the original variable \( \theta \) as follows:

for \( r \leq r_h \)

\[
 k(\theta^i) \cdot \frac{\partial \theta^i}{\partial z} - \frac{\hat{h}_2}{\hat{h}_1} \cdot (\theta^i - \theta_e) = 0 \text{ at } z = 0, \text{ all } r \leq r_h \quad (2.29a)
\]

\[
 -k(\theta^i) \cdot \frac{\partial \theta^i}{\partial z} + q_o \cdot e^{-Cr^2 - \lambda v \xi} \cdot \frac{\hat{h}_2}{\hat{h}_1} \cdot (\theta^i - \theta_e) = 0 \text{ at } z = H, \text{ all } r \leq r_h \quad (2.29b)
\]

\[
 \frac{\partial \theta^i}{\partial r} = 0 \quad \text{ at } r = 0, \text{ all } z \quad (2.29c)
\]

for \( r \geq r_h \)

\[
 k(\theta^o) \cdot \frac{\partial \theta^o}{\partial z} - \frac{\hat{h}_2}{\hat{h}_1} \cdot (\theta^o - \theta_e) = 0 \text{ at } z = 0, \text{ all } r \geq r_h \quad (2.29d)
\]
FIGURE 2.4 Schematic Representation of Geometry for Boundary Value Problem.
\[-k(\theta^0) \cdot \frac{\partial \theta^0}{\partial z} - \hat{h} \cdot (\theta^0 - \theta_e) = 0 \quad \text{at } z = H, \text{ all } r \geq r_h \quad (2.29e)\]

\[\theta^0 \to \theta_0 \quad \text{as } r \to \infty, \text{ all } z \quad (2.29f)\]

for \( r = r_h \)

\[\theta^i = \theta^0 \quad \text{at } r = r_h, \text{ all } z \quad (2.29g)\]

\[\frac{\partial \theta^i}{\partial r} = \frac{\partial \theta^0}{\partial r} \quad \text{at } r = r_h, \text{ all } z \quad (2.29h)\]

where \( \hat{h}_1 \) and \( \hat{h}_2 \) are the heat transfer coefficients at the top and bottom surfaces of the plate that take into account the heat losses to the surrounding atmosphere through convection and radiation. They are defined as:

\[\hat{h} = h_C + \sigma \cdot \varepsilon \cdot A \cdot (\theta^2 + \theta_e^2) \cdot (\theta + \theta_e) \quad (2.30)\]

where \( h_C \) = temperature dependent convection coefficient

\( \sigma \) = Stefan-Boltzmann constant

\( \varepsilon \) = emissivity of the surface

\( A \) = shape function

\( \theta_e \) = environmental (ambient) temperature

The boundary conditions as expressed above are non-linear, owing to the temperature dependence of the coefficients \( k, \hat{h}_1, \) and \( \hat{h}_2 \). As a first step towards linearizing them the assumption will be made that the heat transfer coefficients \( \hat{h}_1 \) and \( \hat{h}_2 \) are constant and equal to an average value over the temperature history of the plate; these average values will be denoted \( h_1 \) and \( h_2 \). As a second step use will have to be made of equations (2.18), (2.20), (2.26), and (2.27) to transform the boundary conditions to ones having the variable \( \phi \) and to
linearize them wherever necessary. As an example of this process eqn. (2.29a) will be worked on.

Using transformation (2.18), eqn. (2.29a) can be written

$$k(T^i) \cdot \frac{\partial T^i}{\partial z} - h_2 \cdot [T^i + (\theta_i - \theta_e)] = 0 \quad (2.31)$$

From the Kirchhoff transformation (2.20) and noting that

$$\frac{\partial T}{\partial z} = \frac{k_0}{k} \cdot \frac{\partial u}{\partial z},$$

eqn. (2.31) can be further written

$$k(T^i) \cdot \frac{k_0}{k(T^i)} \cdot \frac{\partial u}{\partial z} - h_2 \cdot [T^i(u) + (\theta_i - \theta_e)] = 0$$

or

$$k_0 \cdot \frac{\partial u}{\partial z} - h_2 \cdot [T^i(u) + (\theta_i - \theta_e)] = 0 \quad (2.32)$$

To find an approximation for $T^i(u)$, $k(\eta)$ is expanded in a Taylor series around 0

$$k(\eta) = k(0) + k'(0) \cdot \eta + k''(0) \cdot \frac{\eta^2}{2} + O(\eta^3)$$

or

$$k(\eta) = k_0 + \eta \cdot k'(0) + \frac{1}{2} \cdot \eta^2 \cdot k''(0) + O(\eta^3)$$

and substituted into eqn. (2.20) to give

$$u = T + \frac{k'(0)}{2k_0} \cdot T^2 + \frac{k''(0)}{6k_0} \cdot T^3 + O(T^4)$$

Thus, to a first approximation

$$u = T + O(T^2)$$

or

$$u = T$$
Eqn. (2.32) can then be written to a first approximation:

\[ k_0 \cdot \frac{\partial u^i}{\partial z} - h_2 \cdot [u^i + (\theta_0 - \theta_e)] = 0 \]  \hspace{1cm} (2.33)

Finally, invoking eqns. (2.26) and (2.27b), the above expression becomes

\[ \frac{\partial \phi_i}{\partial z} = \frac{h_2}{k_0} \cdot \phi_i \hspace{1cm} z = 0, \ r < r_h \]  \hspace{1cm} (2.34a)

Working similarly on the other boundary conditions the following equations were derived:

\[ \frac{\partial \phi_i}{\partial z} = \frac{\phi_0}{k_0} \cdot e^{-Cr^2} - \frac{h_1}{k_0} \cdot \phi_i, \ z = H, \ r < r_h \]  \hspace{1cm} (2.34b)

\[ \frac{\partial \phi_i}{\partial r} = 0, \hspace{1cm} r = 0, \ \text{all} \ z \]  \hspace{1cm} (2.34c)

\[ \frac{\partial \phi^o}{\partial z} = \frac{h_2}{k_0} \cdot \phi^o, \hspace{1cm} z = 0, \ r > r_h \]  \hspace{1cm} (2.34d)

\[ \frac{\partial \phi^o}{\partial z} = -\frac{h_1}{k_0} \cdot \phi^o, \hspace{1cm} z = H, \ r > r_h \]  \hspace{1cm} (2.34e)

\[ \phi^o \rightarrow 0, \hspace{1cm} r \rightarrow \infty, \ \text{all} \ z \]  \hspace{1cm} (2.34f)

\[ \phi^o = \phi_i, \hspace{1cm} r = r_h, \ \text{all} \ z \]  \hspace{1cm} (2.34g)

\[ \frac{\partial \phi^o}{\partial r} = \frac{\partial \phi_i}{\partial r}, \hspace{1cm} r = r_h, \ \text{all} \ z \]  \hspace{1cm} (2.34h)

2.3.3 Solution

The governing equation (2.28b) will be solved using the method of separation of variables [32]. Let

\[ \phi(r,z) = R(r) \cdot Z(z) \]  \hspace{1cm} (2.35)
Substituting the above equation into eqn. (2.28b) and re-grouping the similar terms, the following two ordinary differential equations can be derived:

\[
\frac{1}{R(r)} \cdot \left[ R''(r) + \frac{1}{r} \cdot R'(r) \right] - (\lambda \nu)^2 = \pm \omega^2 \tag{2.36}
\]

\[- \frac{1}{Z(z)} \cdot Z''(z) = \pm \omega^2 \tag{2.37}
\]

where the sign of \( \omega^2 \) will be chosen such that the boundary-value problem of the homogeneous direction leads to a characteristic-value problem [ 33 ]. Constant \( \omega \) will be determined by the boundary conditions.

A. **Outer solution.** For the outer solution, \( r \geq r_h \), the above equations become

\[
R^O''(r) + \frac{1}{r} \cdot R^O'(r) - \left[ \omega^2 + (\lambda \nu)^2 \right] \cdot R^O(r) = 0 \tag{2.38a}
\]

\[
Z^O''(z) + \omega^2 \cdot Z^O(z) = 0 \tag{2.38b}
\]

and the boundary conditions (2.34d) - (2.34f)

\[
Z^O'(0) - \frac{h_2}{k_O} \cdot Z^O(0) = 0 \tag{2.39a}
\]

\[
Z^O'(H) + \frac{h_1}{k_O} \cdot Z^O(H) = 0 \tag{2.39b}
\]

\[
R^O(r) \rightarrow 0 \text{ as } r \rightarrow \infty \tag{2.39c}
\]

Note that \( +\omega^2 \) was selected so that the orthogonal trigonometric functions will be the characteristic functions of the homogeneous \( z \)-direction; \( -\omega^2 \) would have resulted in hyperbolic functions which are not orthogonal.
Letting
\[ \zeta^2 = \omega^2 + (\lambda v)^2 \quad (2.40) \]
the general solutions to eqns. (2.38) can be written as
\[ R^0(r) = A_1 \cdot I_\zeta(\zeta \cdot r) + A_2 \cdot K_\zeta(\zeta \cdot r) \quad (2.41a) \]
\[ Z^0(z) = B_1 \cdot \sin(\omega \cdot z) + B_2 \cdot \cos(\omega \cdot z) \quad (2.41b) \]
where \( A_1, A_2, B_1, B_2 \) are arbitrary constants and \( I_\zeta(x) \) and \( K_\zeta(x) \) are the modified Bessel functions of the first and second kind respectively and of zero order.

Invoking eqn. (2.39c) it can be found that \( A_1 = 0 \).

Furthermore, using boundary conditions (2.39a) and (2.39b), eqn. (2.41b) gives
\[ (k_\zeta \cdot \omega) \cdot B_1 - h_2 \cdot B_2 = 0 \quad (2.42a) \]
and
\[ \left[ \frac{h_1}{k_\zeta} \cdot \sin(\omega H) + \omega \cdot \cos(\omega H) \right] B_1 + \left[ \frac{h_1}{k_\zeta} \cdot \cos(\omega H) - \omega \cdot \sin(\omega H) \right] B_2 = 0 \quad (2.42b) \]
respectively. To get nontrivial solutions for \( B_1 \) and \( B_2 \), the determinant of the above homogeneous system of equations should vanish, providing an equation for the characteristic values \( \omega_n, n = 0,1,2\ldots \) of the \( z \)-direction solution
\[ (k_\zeta \cdot \omega_n^2 - h_1 \cdot h_2) \cdot \tan(\omega_n H) = k_\zeta \cdot \omega_n \cdot (h_1 + h_2) \quad (2.43) \]
For the case of adiabatic boundary conditions \( (h_1 = h_2 = 0) \), the characteristic values will be given by
\[ \omega_n = \frac{n\pi}{H}, \quad n = 0,1,2,\ldots \quad (2.43a) \]
Note that both eqns. (2.43) and (2.43a) give \( \omega_0 = 0 \). The question thus arises whether this characteristic value is acceptable in one and/or both cases. To resolve it, use will be made of energy considerations and in particular of the heat that flows out of a cylindrical surface with radius \( r, r \geq r_h \),

\[
Q(r) = 2 \int_0^H \int_0^\pi e^{-\lambda rv} \cos \psi \cdot R_0^\prime(r) \cdot Z^\circ(z) \cdot r \cdot d\psi \cdot dz
\]

which, by making the appropriate substitutions, can be written as

\[
Q(r) = -2 k_0 \int_0^H \int_0^\pi e^{-\lambda rv} \cos \psi \cdot R_0^\prime(r) \cdot Z^\circ(z) \cdot r \cdot d\psi \cdot dz
\]

(2.44)

or

\[
Q(r) = -2 k_0 \cdot r \cdot R_0^\prime(r) \cdot I \cdot P
\]

(2.45)

where

\[
I = \int_0^H Z^\circ(z) \cdot dz
\]

(2.46)

\[
P = \int_0^\pi e^{-\lambda rv} \cos \psi \cdot d\psi
\]

(2.47)

Interest here will be focused on \( I \) which, using eqn. (2.38b), integrating by parts and then using eqns. (2.39a), (2.39b), and (2.41b), may be written as
\[ I = \frac{1}{k_0 \omega^2} \cdot [h_2 \cdot B_2 + h_1 \cdot B_1 \cdot \sin(\omega H) + h_1 \cdot B_2 \cdot \cos(\omega H)] \]

or, making the appropriate substitution from eqns. (2.42),

\[ I = B_1 \cdot H \cdot \frac{1 - \cos(\omega H)}{\omega H} + B_2 \cdot H \cdot \frac{\sin(\omega H)}{\omega H} \] \hspace{1cm} (2.48)

For the general case of non-adiabatic boundary conditions, \( h_1 \cdot h_2 \neq 0 \), the constant \( B_2 \) can be expressed from eqn. (2.42a) as

\[ B_2 = \frac{k_0 \omega}{h_2} \cdot B_1 \]

Equation (2.48) then gives

\[ I = B_1 \cdot H \cdot \left[ \frac{1 - \cos(\omega H)}{\omega H} + \frac{k_0 \cdot \omega}{h_2} \cdot \frac{\sin(\omega H)}{\omega H} \right] \] \hspace{1cm} (2.49a)

so that \( I = 0 \) when \( \omega = 0 \) for finite \( B_1 \). In other words, the characteristic value \( \omega_0 = 0 \) and its associated characteristic function do not contribute to the transfer of energy and can thus be neglected. As a consequence, the characteristic functions can be written from eqns. (2.41b) and (2.42a) as

\[ \frac{h_2}{k_0 \cdot \omega_n} \cdot \sin(\omega_n z) + \cos(\omega_n z), \text{ } n=1,2,3,... \]

The outer solution is thus expressed as

\[ \phi^0(r,z) = \sum_{n=1}^{\infty} C_n \cdot K_0(\zeta_n r) \cdot \left[ \frac{h_2}{k_0 \cdot \omega_n} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right] \] \hspace{1cm} (2.50)

where \( C_n = A_{2n} \cdot B_{2n} \), \( n = 1,2,3,... \)

For the case of adiabatic boundary conditions, \( h_1 = h_2 = 0 \), eqn. (2.42a) gives \( B_1 = 0 \) so that eqn. (2.48)
can be written as

\[ I = B_2 H \cdot \frac{\sin(\omega H)}{\omega H} \quad (2.49b) \]

When \( \omega = 0 \) the integral \( I \) is \( B_2 H \), that is finite and nonzero, making it necessary to include the zero characteristic value in the solution. The characteristic functions now are \( \cos(n\pi z/H) \), \( n = 0,1,2,... \) and the outer solution can be expressed as

\[ \phi^0(r,z) = C_0 K_0(\lambda vr) + \sum_{n=1}^{\infty} C_n K_0(\zeta_n r) \cdot \cos \left( \frac{n\pi z}{H} \right) \quad (2.50a) \]

where again \( C_n = A_2n \cdot B_2n \), \( n = 1,2,3,... \). To find the constant \( C_0 \), eqn. (2.45) will be applied at \( r = r_h \). Since there are no heat losses from the surface \( Q(r_h) = Q \), so after some algebra

\[ C_0 = \frac{Q}{2k_0 \cdot \lambda v \cdot r_h \cdot H \cdot K_1(\lambda vr_h) \cdot P} \quad (2.51) \]

where \( Q \) is the total heat input, \( K_1(x) \) is the modified Bessel function of second kind and order one, and \( P \) is given by eqn. (2.47). The other constants \( C_n \), \( n = 1,2,3,... \) will be found from the ones of the non-adiabatic solution by setting \( h_1 = h_2 = 0 \).

The situation previously described can be intuitively explained as follows. In the adiabatic case, and concentrating on the "frequency domain", one sees that the zeros of the energy integral \( I \), eqn. (2.49b), are also the characteristic values \( \omega_n \), \( n = 1,2,3,... \). So each time the terms of the series...
expansion of the solution attempt to make a contribution to the energy integral they become zero, the only exception being the first term where \( \omega_0 = 0 \) is not a zero of \( I \). In other words the dominant term of the outer solution is the first one with the others playing a modulation role. This could be expected due to the simplicity of the adiabatic boundary conditions.

On the other hand in the general case, \( h_1 \cdot h_2 \neq 0 \), the zeros of the energy integral \( I \), eqn. (2.49a), are different from the characteristic values \( \omega_n \), \( n = 1, 2, 3, \ldots \) given by eqn. (2.43). All terms of the series expansion of the outer solution contribute therefore to the energy flown through the cylindrical surface, making the solution more complicated, as expected.

**B. Inner solution.** For the inner domain one observes that there are two homogeneous and two non-homogeneous boundary conditions, the latter ones at \( r = r_h \) and \( z = H \), so that the method of separation of variables cannot be applied. Fortunately, however, the fact that the problem is linear allows the application of the superposition principal, leading to the separation of the inner solution into two parts (see Fig. 2.5),

\[
\phi^i(r,z) = \phi_1^i(r,z) + \phi_2^i(r,z) \quad (2.52)
\]

each one having only one non-homogeneous boundary condition.

**B1.** In part I the homogeneous direction is the \( r \)-direction. The \(-\omega^2\) constant is thus chosen in eqns. (2.36) and (2.37) in order to get the orthogonal Bessel functions as
FIGURE 2.5  Superposition Used for Inner Solution.
the characteristic functions for the $r$-equation. The governing equations thus are

\[ R_I^{i''}(r) + \frac{1}{r} \cdot R_I^{i'}(r) + \delta^2 \cdot R_I^i(r) = 0 \quad (2.53a) \]

\[ Z_I^{i''}(z) - \chi^2 \cdot Z_I^i(z) = 0 \quad (2.53b) \]

where $\chi$ stands for the constant $\omega$ in eqns. (2.36) and (2.37) and

\[ \delta^2 = \chi^2 - (\lambda v)^2 \quad (2.54) \]

The boundary conditions, according to eqns. (2.34a) - (2.34c) and Fig. 2.5, can be expressed as

\[ R_I^{i'}(0) = 0 \quad (2.55a) \]

\[ R_I^i(r_h) = 0 \quad (2.55b) \]

\[ Z_I^{i'}(0) - \frac{h^2}{k_0} \cdot Z_I^i(0) = 0 \quad (2.55c) \]

The general solutions of eqns. (2.53) can then be written as

\[ R_I^i(r) = A_1^i \cdot J_\delta (\delta_n r) + A_2^i \cdot Y_\delta (\delta_n r) \quad (2.56a) \]

\[ Z_I^i(r) = B_1^i \cdot \sinh(\chi_n z) + B_2^i \cdot \cosh(\chi_n z) \quad (2.56b) \]

where $A_1^i, A_2^i, B_1^i, B_2^i$ are arbitrary constants (the primes do not denote differentiation) and $J_\delta(x)$ and $Y_\delta(x)$ are the Bessel functions of first and second kind respectively and of zero order.

By differentiating eqn. (2.56a) and using condition (2.55a) it can be found that $A_2^i = 0$. Moreover, using condition
(2.55b), eqn. (2.56a) gives

\[ J_0(\delta_n \cdot r_h) = 0, \ n=1,2,3, \ldots \]  \hspace{1cm} (2.57)

i.e. the characteristic values (eigenvalues) \( \delta_n \) can be found from the zeros of \( J_0(x) \). Finally, the use of boundary condition (2.55c) in conjunction with eqn. (2.56b) results in the following relation between constants \( B'_1 \) and \( B'_2 \)

\[ B'_1 = \frac{h_2}{\chi \cdot k_0} \cdot B'_2 \]  \hspace{1cm} (2.58)

The first part of the inner solution can now be written as

\[ \phi^i(r,z) = \sum_{n=1}^{\infty} D_n \cdot J_0(\delta_n r) \cdot \left[ \frac{h_2}{\chi_n \cdot k_0} \cdot \sinh(\chi_n z) + \cosh(\chi_n z) \right] \]  \hspace{1cm} (2.59)

To find constants \( D_n \) the fourth boundary condition of the first part of the inner solution

\[ \frac{\partial \phi^i(r,H)}{\partial z} = \frac{q_o}{k_0} \cdot e^{-Cr^2} - \frac{h_1}{k_0} \cdot \phi^i(r,H) \]  \hspace{1cm} (2.60)

will be invoked (see Fig. 2.5), to give

\[ \sum_{n=1}^{\infty} D_n \cdot J_0(\delta_n r) \cdot \chi_n \left[ \frac{h_2}{\chi_n \cdot k_0} \cdot \cosh(\chi_n H) + \sinh(\chi_n H) \right] \]

\[ = \frac{q_o}{k_0} \cdot e^{-Cr^2} - \frac{h_1}{k_0} \cdot \sum_{n=1}^{\infty} D_n \cdot J_0(\delta_n r) \cdot \left[ \frac{h_2}{\chi_n \cdot k_0} \cdot \sinh(\chi_n H) + \cosh(\chi_n H) \right] \]  \hspace{1cm} (2.61)

Multiplying both sides of the above equation by \( r \cdot J_0(\delta_m r) \), integrating from 0 to \( r_h \), and utilizing the following orthogonality property of Bessel functions
\[ \int_0^{r_h} r \cdot J_0(\delta_n r) \cdot J_0(\delta_m r) \, dr = 0 \quad \text{if} \quad \delta_m \neq \delta_n \]
\[ = \frac{r_h^2}{2} \cdot J_1^2(\delta_n r_h) \quad \text{if} \quad \delta_m = \delta_n \]

where \( J_1(x) \) is the Bessel function of first kind and order one, the constant \( D_n \) can be found as

\[
D_n = \frac{2 \cdot q_o \cdot K_n}{k_o \cdot J_1^2(\delta_n r_h) \cdot \left[ \left( \frac{h_1 + h_2}{k_o} \right) \cosh(\chi_n H) + \left( \chi_n + \frac{h_1 h_2}{k_o^2} \right) \sinh(\chi_n H) \right]} \quad (2.62)
\]

where

\[
K_n = \int_0^{1} e^{-C \cdot r_h \cdot x^2} \cdot J_0(\delta_n r_h \cdot x) \, dx \quad (2.63)
\]

The first part of the inner solution is therefore

\[
\phi^{1}_I(r,z) = \sum_{n=1}^{\infty} \frac{2 \cdot q_o \cdot K_n \cdot J_0(\delta_n r) \cdot [h_2 \cdot \sinh(\chi_n z) + k_o \cdot \chi_n \cdot \cosh(\chi_n z)]}{J_1^2(\delta_n r_h) \cdot [k_o \cdot \chi_n \cdot (h_1 + h_2) \cdot \cosh(\chi_n H) + (k_o^2 + h_1 h_2) \cdot \sinh(\chi_n H)]} \quad (2.64)
\]

Note, that in the case of adiabatic boundary conditions \( (h_1 = h_2 = 0) \) the above solution becomes

\[
\phi^{1}_I(r,z) = \sum_{n=1}^{\infty} \frac{2 \cdot q_o \cdot K_n \cdot J_0(\delta_n r) \cdot \cosh(\chi_n z)}{k_o \cdot \chi_n \cdot J_1^2(\delta_n r_h) \cdot \sinh(\chi_n H)} \quad (2.64a)
\]

B2. For part II of the inner solution note that the boundary conditions in the z-direction are similar to those of the outer solution. The general solutions should thus be similar to eqns. (2.41), namely

\[
R^{1}_{II}(r) = A_1'' \cdot I_o(\zeta \cdot r) + A_2'' \cdot K_o(\zeta \cdot r) \quad (2.65a)
\]
\[ Z_{II}^{i}(z) = B''_{1} \cdot \sin(\tilde{\omega}.z) + B''_{2} \cdot \cos(\tilde{\omega}.z) \quad (2.65b) \]

where \( A''_{1}, A''_{2}, B''_{1}, B''_{2} \), and \( \tilde{\omega} \) are constants to be determined and

\[ \bar{\zeta}^2 = \tilde{\omega}^2 + (\lambda v)^2 \quad (2.66) \]

The boundary conditions can be expressed, according to Fig. 2.5, as

\[ R_{II}^{i'}(0) = 0 \quad (2.67a) \]

\[ Z_{II}^{i'}(0) - \frac{h_2}{k_o} \cdot Z_{II}^{i}(0) = 0 \quad (2.67b) \]

\[ Z_{II}^{i'}(H) + \frac{h_1}{k_o} \cdot Z_{II}^{i}(H) = 0 \quad (2.67c) \]

Condition (2.67a) demands that \( A''_{2} = 0 \). Furthermore, using the other two boundary conditions and following an approach similar to the one for the outer solution it can be found that

\[ B''_{1} = \frac{h_2}{\tilde{\omega}.k_o} \cdot B''_{2} \quad (2.68) \]

and

\[ \tan(\tilde{\omega}_n.H) = \frac{k_o.\tilde{\omega}_n.(h_1+h_2)}{k_o.\tilde{\omega}_n - h_1.h_2} \quad n = 1, 2, 3, \ldots \quad (2.69) \]

Comparing the above equation with eqn. (2.43) it can be concluded that

\[ \tilde{\omega}_n = \omega_n \quad , \quad n = 1, 2, 3, \ldots \quad (2.70) \]

and thus \( \tilde{\zeta}_n = \zeta_n \quad , \quad n = 1, 2, 3, \ldots \)

One can therefore obtain the second part of the inner solution in the following series form
\[ \phi_{\text{II}}^i(r,z) = \sum_{n=1}^{\infty} E_n \cdot I_o(\zeta_n r) \cdot \left[ \frac{h_2}{\omega_n k_o} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right] \]  \hspace{1cm} (2.71)

or, for the case of adiabatic boundary conditions

\[ (h_1 = h_2 = 0) \]

\[ \phi_{\text{II}}^i(r,z) = E_0 \cdot I_o(\lambda vr) + \sum_{n=1}^{\infty} E_n \cdot I_o(\zeta_n r) \cdot \cos\left(\frac{n\pi z}{H}\right) \]  \hspace{1cm} (2.71a)

C. **Matching region.** To complete the solution, that is to find the constants \( C_n \) and \( E_n \), the boundary conditions at the interface \( r = r_h \) will have to be invoked.

The first condition, stating that the inner and outer solutions should match on the interface, can be expressed according to Fig. 2.5 as

\[ \phi^0(r_h, z) = \phi_{\text{II}}^i(r_h, z) \]  \hspace{1cm} (2.72a)

or, using eqns. (2.50) and (2.71),

\[ \sum_{n=1}^{\infty} C_n \cdot K_o(\zeta_n r_h) \cdot \left[ \frac{h_2}{\omega_n k_o} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right] = \sum_{n=1}^{\infty} E_n \cdot I_o(\zeta_n r_h) \cdot \left[ \frac{h_2}{\omega_n k_o} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right] \]  \hspace{1cm} (2.72b)

Since the above equation must hold for all \( z \), it is concluded that

\[ E_n = \frac{K_o(\zeta_n r_h)}{I_o(\zeta_n r_h)} \cdot C_n, \quad n = 1, 2, 3, \ldots \]  \hspace{1cm} (2.73)
Based on the above and eqns. (2.52), (2.64), and (2.71) the complete inner solution can now be written as

\[
\phi^i(r,z) = \sum_{n=1}^{\infty} \left[ \frac{2 q_o \cdot K_n \cdot J_0(\delta_n r) \cdot [h_2 \cdot \sinh(\chi_n z) + k_o \cdot \chi_n \cdot \cosh(\chi_n z)]}{J_1(\delta_n r_1) \cdot [k_o \cdot (h_1 h_2) \cdot \cosh(\chi_n H) + (k_o^2 + h_1 h_2) \cdot \sinh(\chi_n H)]} \right]
\]

\[
+ C_n \cdot K_n(\delta_n r_n) \cdot I_0(\zeta_n r_n) \cdot \left[ \frac{h_2}{\omega_n k_o} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right]
\]

(2.74)

For the case of adiabatic boundary conditions \((h_1 = h_2 = 0)\) the inner solution can be similarly written

\[
\phi^i(r,z) = \frac{Q}{2 k_o \cdot \lambda \nu r_h H P} \cdot \frac{K_0(\lambda \nu r_h)}{K_1(\lambda \nu r_h)} \cdot \frac{I_0(\lambda \nu r)}{I_0(\lambda \nu r) H}
\]

\[
+ \sum_{n=1}^{\infty} \left[ \frac{2 q_o \cdot K_n \cdot J_0(\delta_n r) \cdot \cosh(\chi_n z)}{\omega_n k_o^2} \cdot \frac{I_0(\zeta_n r_n)}{I_0(\zeta_n r_n)} \cdot \cos(\omega_n z) \right]
\]

(2.74a)

The second condition at the interface states that

\[
\frac{\partial \phi^i(r_h, z)}{\partial r} = \frac{\partial \phi^o(r_h, z)}{\partial r}
\]

(2.75)

Substituting the outer and inner solutions from eqns. (2.50) and (2.74) respectively and using the orthogonality conditions

\[
\int_0^H \left[ \frac{h_2}{\omega_n k_o} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right] \cdot \left[ \frac{h_2}{\omega_m k_o} \cdot \sin(\omega_m z) + \cos(\omega_m z) \right] dz = 0
\]

for \(\omega_n \neq \omega_m\), and

\[
\int_0^H \left[ \frac{h_2}{\omega_n k_o} \cdot \sin(\omega_n z) + \cos(\omega_n z) \right]^2 dz = \frac{H}{2} \left( 1 + \frac{h_2^2}{\omega_n^2 k_o} \right)
\]
\[ + \frac{1}{4\omega_n} \left( 1 - \frac{h_2^2}{\omega_n^2 k_0^2} \right) \sin(2\omega_n H) + \frac{h_2}{\omega_n^2 k_0} \sin^2(\omega_n H) \]

one can find the following expression for the constants \( C_n \)

\[ C_n = \frac{1}{a_n} \sum_{m=1}^{\infty} \frac{2 q_o K_m \delta_m [r_{mm} \tanh(\chi_m H) + s_{mm}]}{(\chi_m^2 + \omega_n^2) \cdot (J_1(\delta_m r_m) \cdot [d_m + e_m \tanh(\chi_m H)]}, \quad n=1,2,3,\ldots \quad (2.76) \]

where

\[ a_n = b_n \cdot c_n \quad (2.77a) \]

\[ b_n = \zeta_n \cdot \left[ k_1(\zeta_n r_n) + k_o(\zeta_n r_n) \cdot \frac{I_1(\zeta_n r_n)}{I_0(\zeta_n r_n)} \right] \quad (2.77b) \]

\[ c_n = \frac{h_2^2}{2} \left( 1 + \frac{h_2^2}{\omega_n^2 k_0^2} \right) + \frac{1}{4\omega_n} \left( 1 - \frac{h_2^2}{\omega_n^2 k_0^2} \right) \sin(2\omega_n H) + \frac{h_2}{\omega_n^2 k_0} \sin^2(\omega_n H) \quad (2.77c) \]

\[ d_m = k_o \cdot \chi_m \cdot (h_1 + h_2) \quad (2.77d) \]

\[ e_m = k_o^2 \cdot \chi_m^2 + h_1 + h_2 \quad (2.77e) \]

\[ r_{mn} = h_2 \left( \omega_n + \frac{\chi_m^2}{\omega_n} \right) \sin(\omega_n H) + \left( k_o \chi_m - \frac{h_2^2}{k_o} \right) \cos(\omega_n H) \quad (2.77f) \]

\[ s_{mn} = \chi_m \left( k_o \omega_n + \frac{h_2^2}{k_o \omega_n} \right) \sin(\omega_n H) \quad (2.77g) \]

In the case of adiabatic boundary conditions \((h_1 = h_2 = 0)\),

the constants \( C_n \), \( n = 1,2,3,\ldots \) are given by

\[ C_n = (-1)^n \cdot \frac{4 q_o}{k_o \cdot H \cdot b_n} \cdot \sum_{m=1}^{\infty} \frac{K_m \delta_m}{(\chi_m^2 + \omega_n^2) \cdot J_1(\delta_m r_m)} \quad (2.76b) \]
The final solution can be written as

\[ \theta = \theta_0 + \frac{1}{\gamma} \left( \sqrt{1+2\gamma e^{-\lambda \nu G}} \cdot \phi(r,z) - 1 \right) \]  \hspace{1cm} (2.78)

where

\[ \phi(r,z) = \phi^i(r,z) \cdot [u_{-1}(r)-u_{-1}(r-r_h)] + \phi^o(r,z) \cdot u_{-1}(r-r_h) \]  \hspace{1cm} (2.79)

with \( u_{-1}(r-a) \) being the unit step function and \( \phi^i(r,z) \), \( \phi^o(r,z) \) being given by eqns. (2.50) and (2.74). All other variables have been previously defined.

A computer program has been written in the FORTRAN IV language to perform the necessary calculations. Appendix A provides some details of the program.

2.4 Parametric Investigation

The computer program developed for the three-dimensional finite heat source solution discussed in the previous section was used to perform a parametric investigation in an effort to study the effects the arc efficiency, \( \eta_a \), the concentration coefficient, \( C \), and the average heat loss coefficient, \( h \), have on the predicted temperature distribution.

As a case study, the laying of a bead on the top surface of a 1 in (25.4 mm) thick HY-130 plate using the GMA welding process was chosen. The plate measured 24in x 24in (610mm x 610mm) so that the quasi-stationary state could be assumed to hold in the mid-length region of the weld. Two arc efficiencies, 0.50 and 0.60, were assumed. For the concentration coefficient \( C \) the values of 39.50 and 8.2 in\(^{-2} \) (6.12 and 1.28
cm⁻²) were investigated corresponding to a heat spot radius \( r_h \) of 0.275 and 0.600 in (7. and 15.2 mm) respectively. The average heat loss coefficient for both surfaces ranged from 0. to 0.0006 Btu/sec.in².°F corresponding to adiabatic boundary conditions and to heat losses at high temperatures respectively. Finally the cases of constant thermal conductivity (\( \gamma=0. \)) and of linearly varying thermal conductivity (\( \gamma=0.0001 \)) were examined, the latter value being computed from data for the HY-130 steel (see Section 3.3.1) using least squares approximation.

The effect of heat losses from the surface at a point 0.5 in (12.7 mm) from the weld centerline is shown in Figure 2.6. As expected, the adiabatic case exhibits unreasonably high temperatures and slower cooling rate than the ones for finite heat losses. In Figure 2.7 the case of variable versus constant thermal conductivity is depicted. At lower temperatures the difference between the two assumptions is not great, whereas at higher temperatures the variable \( k \) case tends to predict somewhat lower temperatures.

Figures 2.8 through 2.11 show the temperature distribution at 1, 0.75, 0.50, and 0.284 in (25.4, 19.1, 12.7, and 7.2 mm) from the weld centerline for \( \eta_a = 0.60 \) and various values of \( C, \gamma, \) and \( h \). It is observed that the smaller concentration coefficient (larger heat spot radius) predicts higher temperatures at all points, if all other parameters are kept constant, as expected. The effect of \( \gamma \) and \( h \) is also similar
Figure 2.6 Effect of Heat Losses on Finite Heat Source Solution.
$C = 39.5 \text{ in}^{-2}$

$\eta_a = 0.60$

$\gamma = 0.$
- $h = 0.$
- $h = 0.0006$

$\gamma = 0.0001$
- $h = 0.$
- $h = 0.0006$

0.5 in from weld line

**FIGURE 2.7** Effect of Variation of Thermal Conductivity on Finite Heat Source Solution.
FIGURE 2.8 Parametric Investigation of Finite Heat Source Solution (1.0 in).
FIGURE 2.9 Parametric Investigation of Finite Heat Source Solution (0.75 in).
FIGURE 2.10 Parametric Investigation of Finite Heat Source Solution (0.5 in).
FIGURE 2.11 Parametric Investigation of Finite Heat Source Solution (0.284 in).
to the one explained in the previous paragraphs.

The effect of the arc efficiency on the temperature history of the same four points is illustrated in Figures 2.12 through 2.15. The curves shown were all calculated using the same C and γ. Two values of the heat loss coefficient were used. Again the same observation can be made with the additional one that larger arc efficiencies result in higher temperatures, all other things being equal.

In conclusion thus it can be said that the impact the various parameters have on the temperature distribution during welding is as follows:

1. Larger arc efficiencies result in higher temperatures.

2. The smaller the concentration coefficient (i.e the larger the heat spot radius) the higher the temperatures. This effect is more pronounced at the high temperature range.

3. The inclusion of linearly varying thermal conductivity does not alter by much the temperature distribution predicted by constant thermal conductivity calculation. Only in high temperature regions (inside the HAZ) is the effect pronounced, resulting in lower temperatures.

4. The average heat loss coefficient (from convection and radiation) has the largest impact on the temperature distribution with the adiabatic boundary conditions resulting in unreasonably high temperatures.

2.5 Comparison with Other Solutions and Experiments

The conventional point heat source closed form solution
C = 39.5, \gamma = 0.

1. in from weld line

\[ \eta_a = 0.60 \quad h = 0.0001 \]
\[ \quad \circ \quad h = 0.0003 \]

\[ \eta_a = 0.50 \quad \quad h = 0.0001 \]
\[ \quad \quad \quad \quad \boxdot \quad h = 0.0003 \]

**FIGURE 2.12**: Effect of Heat Input on Finite Heat Source Solution (1.0 in).
FIGURE 2.13 Effect of Heat Input on Finite Heat Source Solution (0.75 in).
$C = 39.5$, $\gamma = 0$.

0.5 in from weld line

\[\eta_a = 0.60 \quad h = 0.0001\]
\[\bullet \quad h = 0.0003\]

\[\eta_a = 0.50 \quad \quad h = 0.0001\]
\[\textbullet \quad h = 0.0003\]

**FIGURE 2.14** Effect of Heat Input on Finite Heat Source Solution (0.5 in).
FIGURE 2.15 Effect of Heat Input on Finite Heat Source Solution (0.284 in).
was modified in this study to be able to simulate multipass welds. This modification, explained in Appendix B, involves the possibility of the point source being located at any point within the plate thickness as well as the inclusion of the effect the joint shape has on the temperature distribution. The point source can thus be located on the geometric center of each weld pass and not just on the top surface of the plate as was the case with the conventional solutions.

Both the conventional and modified point source solutions were applied to the same case as the one for the three-dimensional heat source model. Figures 2.16 through 2.19 depict the results obtained for four points located at various distances from the weld centerline. Shown are the cases of constant properties with the point source on the top surface of the plate and at a distance 0.4375 in (11.1 mm) from the top (simulating the first pass of the welding experiments described in detail in Section 4.4) and the case of variable properties using the iterative approach outlined in Section 2.1.3.

The results show clearly the overestimation of temperatures for points located on the top surface of the plates if the conventional point source solution is applied even if the temperature variation of material properties is considered. This overestimation is even more pronounced in the high temperature regions close to the weld centerline. At the same time the experimental results (see Section 4.4) seem to have a good correlation with the ones obtained using the modified approach.
\[ T_a = 0.60 \]

1. in from weld line

Heat Source on Top Surface
- variable properties
- constant properties

Heat Source in Center of First Pass
--- constant properties

FIGURE 2.16 Investigation of Point Heat Source Solutions (1.0 in from weld line).
\( \gamma_a = 0.60 \)

0.75 in from weld line

Heat Source on Top Surface
- variable properties
- constant properties

Heat Source in Center of First Pass
- constant properties

FIGURE 2.17 Investigation of Point Heat Source Solutions (0.75 in from weld line).
\[ \theta [^\circ F] \]

\[ \eta_a = 0.60 \]

0.5 in from weld line

Heat Source on Top Surface
--- variable properties
• constant properties

Heat Source in Center of First Pass
--- constant properties

FIGURE 2.18 Investigation of Point Heat Source Solutions (0.5 in from weld line).
\[ \theta^{\circ}F \]

\[ n_a = 0.60 \]

0.284 in from weld line

Heat Source on Top Surface
--- variable properties
○ constant properties

Heat Source in Center of First Pass
--- constant properties

FIGURE 2.19 Investigation of Point Heat Source Solutions (0.284 in from weld line).
Furthermore, a comparison of the modified point heat source results with the ones obtained using the three dimensional finite heat source approach (see Figures 2.12 through 2.15) reveals that the latter predicts too high cooling rates if the maximum temperatures reached are matched for the two solutions. The situation is even worse if the results for the first pass of the multipass welding experiments are compared.

These results should not be of a surprise, however. The assumptions involved in the development of the three-dimensional heat source model make its use more appropriate in the cases of thin plates. A better correlation with experimental results should therefore be expected in these situations.

To test this proposition an analysis of experimental results obtained by Tsai [29] during laying a bead on a 0.125 in (3.2 mm) low carbon steel plate was attempted. The welding speed was 9 ipm and the arc power 5000 J/sec. Figure 2.20 shows the comparison between experiments and analysis at two points located 0.4 and 0.7 in (10.2 and 17.8 mm) away from the weld centerline. As postulated, the three dimensional heat source model gives good results in this thin plate test case.

In summary, the new finite heat source model developed for predicting temperature distributions during welding can be applied when welding thin plates, preferably made of materials exhibiting a linear temperature dependence of the thermal conductivity (such as aluminum).
FIGURE 2.20 Comparison of Finite Heat Source Solution with Experimental Data for a 0.125 in thick Welded Plate.
CHAPTER III

MATERIAL CHARACTERISTICS OF HY-130 STEEL

This study is primarily concerned with temperature, transient strain, and residual stress distributions due to welding of high strength quenched and tempered steels, and in particular of HY-130 steel. It is thus appropriate to devote a chapter to the characterization of this material.

The first section deals with general characteristics of the HY-130 steel, including its composition, desired mechanical properties, welding consumables and the problems encountered during welding.

This is followed by a section reviewing the available experimental data on the transformation kinetics of HY-130. Isothermal (TTT) and cooling (CCT) transformation diagrams are presented for the base and weld metal.

The third section details an effort towards a rational determination of the temperature dependence of the HY-130 mechanical and physical properties, in the absence of any available experimental data. Such an effort was deemed necessary because the properties are required input to both the thermal and stress analyses.

The final section, returning to the subject of transformation kinetics, deals with proposed methods for the analytical derivation of the CCT diagram on the basis of the experimentally available isothermal one. These methods will be applied and commented upon for the case of HY-130 steel.
3.1 **General Characteristics**

HY-130 was developed as an upgrading of the HY-80 class of Naval hull-construction alloys. The developmental effort started in the early sixties at Westinghouse Electric Corporation [34, 35]. In 1962 a contract was placed with the U.S. Steel Co. by the U.S. Navy for the systematic development of a steel with a minimum yield strength in the 130 to 150ksi (895 to 1030 MPa) range and of associated welding procedures and consumables [36]. The steel development program was actually carried out in the U.S. Steel Co. laboratories, whereas the consumable development was subcontracted to the McKay Company and the Arcos Corporation for shielded metal arc electrode development, and to the Air Reduction Company (Airco) primarily for the development of GMA welding consumables [37].

The steel developed at the U.S. Steel Co. is based on 5% nickel and contains chromium, molybdenum and vanadium. In going through the literature of the 1960s and before the final standarization of the alloy's name to HY-130, one encounters several other designations such as 5Ni-Cr-Mo-V, HY-150, HY-140, HY-130/150, and HY-130(T), all being used to describe the same alloy in different stages of development [38]. These designations reflect the progressive lowering of the desired minimum yield strength from 150 to 130ksi (from 1030 to 895 MPa) through the realization of the inherent fracture toughness limitations of this class of alloy steel at high yield strengths.
Table 3.1 provides the compositional ranges of the HY-130 steel and compares them with those of the HY-80 alloy [37]. Both of them are intended for use in the quenched-and-tempered condition, in which the microstructure is primarily tempered martensite. More about the transformation kinetics will be discussed in a later section.

The mechanical properties of HY-130 in the as-received quenched and tempered condition are typically as follows [38, 39]:

(a) Yield strength of at least 130ksi (895MPa) at the center of a 4 in. (101.6mm) plate — it should be noted that yield stress of as high as 160ksi (1100MPa) has been reported in heats of 1.5 in (38mm) plates [40].

(b) Elongation in 2 in. (50mm) of 15 to 20% for 1 in (25.4mm) thick plates.

(c) Reduction of area equal to 50-64% transversely and 70% through thickness.

(d) A Charpy-V-notch impact energy absorption of 60ft-lb (81.3J) at 0°F (-17.8°C) in the ductile fracture region ("shelf").

Other general properties of the HY-130 steel are presented in Table 3.2 as reported in a 1964 paper by Manganello et al [41]; some of these constants could not be found in the more recent literature. Nevertheless the values connected with transformation kinetics will be discussed in another section.

Consumables for welding HY-130 have been developed for
TABLE 3.1  Compositional Ranges of HY-80 and HY-130 steels (weight, %) [37]

<table>
<thead>
<tr>
<th>Element</th>
<th>HY-80</th>
<th>HY-130</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.18 max</td>
<td>0.08-0.12</td>
</tr>
<tr>
<td>Mn</td>
<td>0.10-0.40</td>
<td>0.60-0.90</td>
</tr>
<tr>
<td>Si</td>
<td>0.15-0.35</td>
<td>0.20-0.35</td>
</tr>
<tr>
<td>Ni</td>
<td>2.00-3.25</td>
<td>4.75-5.25</td>
</tr>
<tr>
<td>Cr</td>
<td>1.00-1.80</td>
<td>0.40-0.70</td>
</tr>
<tr>
<td>Mo</td>
<td>0.20-0.60</td>
<td>0.30-0.65</td>
</tr>
<tr>
<td>V</td>
<td>0.03 max</td>
<td>0.05-0.10</td>
</tr>
<tr>
<td>S</td>
<td>0.025 max</td>
<td>0.010 max</td>
</tr>
<tr>
<td>P</td>
<td>0.025 max</td>
<td>0.010 max</td>
</tr>
<tr>
<td>S+P</td>
<td>0.045 max</td>
<td>-</td>
</tr>
<tr>
<td>Ti</td>
<td>0.02 max</td>
<td>0.02 max</td>
</tr>
<tr>
<td>Cu</td>
<td>0.25 max</td>
<td>0.25 max</td>
</tr>
<tr>
<td>Property</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------</td>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td>$A_c^1$ Temperature</td>
<td>1210°F (654°C)</td>
<td></td>
</tr>
<tr>
<td>$A_c^3$ Temperature</td>
<td>1415°F (768°C)</td>
<td></td>
</tr>
<tr>
<td>$M_s$ Temperature</td>
<td>715°F (379°C)</td>
<td></td>
</tr>
<tr>
<td>Recommended final austenitizing temperature</td>
<td>1500°F (815°C)</td>
<td></td>
</tr>
<tr>
<td>Microstructure (as quenched, mid-thickness)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5in (12.2mm) plate</td>
<td>100% martensite</td>
<td></td>
</tr>
<tr>
<td>4.0in (101mm) plate</td>
<td>60-75% martensite, remainder bainite</td>
<td></td>
</tr>
<tr>
<td>Recommended tempering temperature range</td>
<td>1000-1150°F (538-621°C)</td>
<td></td>
</tr>
<tr>
<td>Magnetic properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_{sat}$</td>
<td>20,520 G (2.052 T)</td>
<td></td>
</tr>
<tr>
<td>$H_{sat}$</td>
<td>2,190 O (1.7x10^5 A/m)</td>
<td></td>
</tr>
<tr>
<td>$\mu_{max}$</td>
<td>630</td>
<td></td>
</tr>
</tbody>
</table>
both the shielded metal arc (SMA) and gas metal arc (GMA) welding [42, 43]. Table 3.3 provides the compositional ranges for the three available consumables: the Navy specifications for a low-hydorgen type E14018 electrode to be used with SMAW; the AX-140 GMAW wire as developed by Airco; and the Navy specifications for the 140S GMAW wire. Comparing Tables 3.1 and 3.3 it can be seen that the carbon content of the consumables is below that of the base metal because of the carbon's tendency to cause cracking. Similarly, manganese content is relatively high because it imparts toughness; nickel content is lowered to avoid cracking known to be caused by high nickel contents; and chromium content is raised for increased hardenability.

Generally speaking hydrogen induced HAZ cold cracking, hot tearing, and lamellar tearing are not considered to be a problem with HY-130 welds. Weld metal solidification cracking could be a high risk for HY-130 welds; it is believed, however, that the reduced phosphorus and sulfur contents necessary for good weld metal toughness are adequate for avoiding the problem [37]. Many investigators [36, 38, 43] have reported that weld metal hydrogen cracking is perhaps the most serious problem facing the production of HY-130 welds. A compilation of the various recommendations proposed for avoiding this type of cracking, such as preheating, moisture control in electrodes, and maximum heat input, can be found in the Welding Institute review paper by Shackleton [37].
TABLE 3.3 Analysis of Consumables for Welding HY-130 Type Steels

<table>
<thead>
<tr>
<th>Element</th>
<th>MIL-14018&lt;sup&gt;1&lt;/sup&gt;</th>
<th>AX-140&lt;sup&gt;2&lt;/sup&gt;</th>
<th>MIL-140S&lt;sup&gt;3&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.10 max</td>
<td>0.07-0.11</td>
<td>0.12 max</td>
</tr>
<tr>
<td>Mn</td>
<td>0.75-1.35</td>
<td>1.70-2.00</td>
<td>1.50-2.00</td>
</tr>
<tr>
<td>Si</td>
<td>0.65 max</td>
<td>0.25-0.45</td>
<td>0.30-0.50</td>
</tr>
<tr>
<td>S</td>
<td>0.010 max</td>
<td>~ 0.007</td>
<td>0.010 max</td>
</tr>
<tr>
<td>P</td>
<td>0.013 max</td>
<td>~ 0.006</td>
<td>0.010 max</td>
</tr>
<tr>
<td>Cr</td>
<td>0.35-1.20</td>
<td>0.85-1.20</td>
<td>0.65-1.05</td>
</tr>
<tr>
<td>Mo</td>
<td>0.30-0.90</td>
<td>0.50-0.60</td>
<td>0.40-1.00</td>
</tr>
<tr>
<td>Ni</td>
<td>3.10-3.90</td>
<td>2.00-2.50</td>
<td>1.95-3.10</td>
</tr>
<tr>
<td>Al</td>
<td>-</td>
<td>~ 0.018</td>
<td>0.04 max</td>
</tr>
<tr>
<td>Ti</td>
<td>-</td>
<td>~ 0.01</td>
<td>0.04 max</td>
</tr>
<tr>
<td>Zr</td>
<td>-</td>
<td>-</td>
<td>0.04 max</td>
</tr>
<tr>
<td>Cu</td>
<td>-</td>
<td>-</td>
<td>0.15 max</td>
</tr>
</tbody>
</table>

Notes: 1. MI-30-CE/3 specification requirements; electrode El4018 manufactured by McKay Co.

2. Technical data provided by Air Reduction Co.

3. MI-30-BE/1 specification requirements; wire Linde 140S manufactured by the Union Carbide Co.
3.2 Experimentally Derived Transformation Kinetics

3.2.1 Isothermal Kinetics

The isothermal transformation characteristics for the base HY-130 steel could only be found in a 1967 U.S. Steel Co. report compiled by Manganello, et al [44]. For lack of other data and although these characteristics pertain to the alloy still in its developmental stages, the time-temperature-transformation (TTT) diagram is reproduced here in Figure 3.1 in the hope that the basic attributes have been carried over to the finalized alloy. The TTT diagram shown has been experimentally determined from a heat having the following composition: 0.09% C, 0.75% Mn, 4.98% Ni, 0.61% Cr, 0.54% Mo, and 0.07% V; the steel was austenitized at 1500°F (815°C) and had an ASTM grain size number between 7 and 8.

The three major transformation temperatures were determined to be as follows (see also Table 3.2):

\[ \begin{align*}
A_{c3} &= 1415°F (768°C) \\
A_{c1} &= 1210°F (654°C) \\
M_s &= 715°F (379°C)
\end{align*} \]

One can compare these temperatures with the ones predicted by the empirical formulae proposed by Andrews [45], since the HY-130 composition is within the ranges for which the formulae can be used. The predicted values for the \( M_s \) and \( A_{c1} \) temperatures are 713°F (378.5°C) and 1204°F (651°C) respectively, i.e. very close to the experimentally obtained ones; the \( A_{c3} \) temperature was predicted to be 1430°F (777°C), 15°F (8°C) higher than the experimental one.
FIGURE 3.1 Isothermal Transformation Diagram for HY-130 Steel.

- A = Austenite
- F = Ferrite
- C = Cementite
- B = Bainite

Temperature vs. Time

- Temperatures: 650, 700, 800, 900, 1000, 1100, 1200°F
- Times: 5, 10, 50, 100, 500, 5000 seconds

- Ms point indicated
During the isothermal study it was noted that transformation to ferrite, at temperatures just below the $A_1$ temperature, were suppressed to times in excess of $10^4$ seconds. Given the fact that such time scales at so high temperatures do not occur during the welding cycle, it follows that this allotropic transformation is of no interest to the welding problem and it is therefore not shown in Fig. 3.1. Furthermore, the TTT diagram shows that transformation to lower bainite can occur in as short a time as 20 seconds at temperatures just above the $M_s$ temperature. As a consequence, heavy plates may end up having a bainitic microstructure around their mid-thickness as compared to the tempered martensitic structure prevailing in the remaining volume.

A limited isothermal transformation study of the HY-130 weld metal welded using 1/16 inch (1.5mm) Airco AX-140 GMA filler wire was conducted as a part of a larger investigation by Holsberg [46]. In the study heating was controlled to reach peak temperature in 8 seconds at a linear rate; the peak was maintained 5 seconds before cooling. Free cooling was then used up to the preselected isothermal temperature and after the desired hold time the specimens were freely cooled to ambient temperature. These thermal cycles were produced in a Duffer's Gleeble apparatus [47,48]. Note that the described thermal cycles are not the same as the conventional ones used in isothermal studies, but rather ones reminiscent of the actual welding cycle. In particular the 5 seconds during which the specimen is held at the peak temperature might not
be enough for full austenitization to take place.

In all cases a martensitic transformation was observed during free cooling from the isothermal temperature, even after long hold times. This is in agreement with the base metal isothermal study referred to above. Holsberg reports an $M_s$ temperature of 760°F (405°C) and notes that this temperature was depressed (see Table 3.4) whenever a bainitic transformation preceded the martensitic one, owing to segregation of alloying elements into the remaining austenite. Table 3.4 reproduces the results of the study.

3.2.2 Continuous Cooling Kinetics

Although isothermal transformation diagrams, as the one for HY-130 shown in Fig. 3.1, are easily obtained experimentally, they are not very useful in predicting microstructures during practical applications. The reason is that TTT diagrams show in an approximate way the transformation kinetics over time for constant temperatures, whereas in practical applications, such as casting and welding, the temperature does not remain constant but changes with time. The need thus arises for diagrams showing the transformation kinetics in non-isothermal cases. Such diagrams, called continuous cooling transformation (CCT) diagrams, although difficult and time consuming to experimentally obtain, have been derived for a limited number of alloys under conditions of constant cooling rates and/or of previously specified cooling patterns [49].

A few cooling transformation studies have been conducted, mostly at the DTNSRDC, for the HY-130 steel and its weld metal,
<table>
<thead>
<tr>
<th>Peak Temperature °F</th>
<th>Isothermal Temperature °F</th>
<th>Hold Time min</th>
<th>Ms Temperature °F</th>
<th>Ms Temperature °C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>900</td>
<td>60</td>
<td>710</td>
<td>380</td>
</tr>
<tr>
<td>2000</td>
<td>825</td>
<td>360</td>
<td>575</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>60</td>
<td>ND</td>
<td>ND</td>
</tr>
<tr>
<td>1600</td>
<td>900</td>
<td>5</td>
<td>715</td>
<td>380</td>
</tr>
<tr>
<td></td>
<td>750</td>
<td>60</td>
<td>585</td>
<td>305</td>
</tr>
</tbody>
</table>

ND - not determined

In all these studies the peak temperature was reached in 8 seconds, held there for 5 seconds, and then the specimen was allowed to cool down according to predetermined cooling rates. The cooling rates from 1000° to 800°F (538° to 427°C) ranged between 67 and 4°F/sec (37 and 2°C/sec) for the 1600°F (870°C) and between 55 and 2°F/sec (31 and 1°C/sec) for the 2000°F (1090°C) peak temperature. Hardness measurements were taken in all cases; in addition, Schreitz and Thompson [50] measured the yield strength and the Charpy V-notch impact toughness of the specimens after the imposition of the thermal cycles.

Figure 3.2 shows the continuous cooling transformation diagram for the HY-130 steel as obtained experimentally for both peak temperature cases [50]. Figure 3.3 presents similar diagrams for the Airco AX-140 and Linde 140S GMAW weld metals for a peak temperature of 2000°F (1090°C) [46,50,51]. It should also be noted that the CCT diagram for the McKay 14018 SMAW metals, not shown here, is similar to the Airco AX-140 GMAW metal at slow cooling rates and the Linde 140S metal at rapid cooling rates [50].

A careful observation of the two figures indicates that
FIGURE 3.2 Continuous Cooling Transformation Diagrams for HY-130 Steel.

PEAK TEMPERATURE

$1600\,^\circ F$ (870$^\circ C$)

$2000\,^\circ F$ (1090$^\circ C$)

TIME FROM PEAK (sec)

$T_f$

$B_S$

$M_S$

$[^\circ F]$
FIGURE 3.3 Continuous Cooling Transformation Diagrams for GMA Welded HY-130 Weld Metal (from 2000°F).
at high cooling rates a wholly martensitic structure is always the end result. Bainite starts forming at slower cooling rates, typically below 15°F/sec (8°C/sec), and at temperatures ranging from about 900°F (480°C) to the $M_s$ temperature; greater amounts of bainite are formed, as expected, with a decrease in the cooling rate. Furthermore, a comparison of the HY-130 and weld metal CCT diagrams reveals that bainite is formed sooner in the weld metal, i.e. at higher cooling rates. More specifically, in the case of HY-130 bainite starts forming at 14°F/sec (8°C/sec) for the 1600°F (870°C) peak and at 4.5°F/sec (2.5°C/sec) for the 2000°F (1090°C) peak; in the case of the AX-140 weld metal this happens at 24°F/sec (13°C/sec) for the 1600°F peak and at 15°F/sec (8°C/sec) for the 2000°F peak; and in the case of the 140S weld metal bainite starts forming at 15°F/sec (8°C/sec) for the 2000°F peak. Given the fact that typical production welding conditions for 1 in. (25mm) thick HY-130 plates involve cooling rates in excess of 25°F/sec (14°C/sec) around the weld area, it can be safely assumed that the final microstructure will be wholly martensitic in this region.

Regarding the martensite start transformation temperature, $M_s$, Table 3.5 was compiled using data from the previously mentioned studies. For the HY-130 base metal it can be seen that the isothermal studies report a somewhat lower $M_s$ temperature as compared to the continuous cooling one. Since such a discrepancy is not theoretically justifiable because
<table>
<thead>
<tr>
<th></th>
<th>$M_s$</th>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Isothermal</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HY-130</td>
<td>715</td>
<td>379</td>
<td></td>
</tr>
<tr>
<td>AX-140</td>
<td>760</td>
<td>405</td>
<td></td>
</tr>
<tr>
<td><strong>Continuous Cooling</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HY-130</td>
<td>720±10</td>
<td>382±5</td>
<td></td>
</tr>
<tr>
<td>AX-140</td>
<td>760±25</td>
<td>405±15</td>
<td></td>
</tr>
<tr>
<td>140S</td>
<td>770±10</td>
<td>410±5</td>
<td></td>
</tr>
</tbody>
</table>
of the athermal nature of the martensitic transformation -
assuming the same material and the same stress conditions -
it is believed that the difference can be due either to
experimental error or to small differences in the material
chemical composition, the latter being known as heavily in-
fluencing $M_s$. For the purpose of this study, which is rather
macroscopic in nature, it is felt that a reasonable approxi-
mation for the $M_s$ temperature of the HY-130 base metal can be
set at the 720°F (380°C) level.

With respect to the $M_s$ temperatures of the Airco AX-140
and Linde 140S GMA weld metals, Table 3.5 reveals much higher
values compared to the HY-130 one. It is believed that these
differences can be attributed to the differences in the base
and weld metal chemical compositions (compare Tables 3.1 and
3.3) which tend to suppress the base metal's $M_s$ temperature
(see also the various empirical formulae for predicting $M_s$
based on chemical composition, as the one proposed by
Andrews [45]).

One further note is in order at this point. The various
$M_s$ temperatures discussed above pertain to the cases where the
cooling rates are so high so as to exclusively form martensite
in the specimen. At lower cooling rates, when bainite is also
formed, the $M_s$ temperature cannot be accurately determined.
This is due to the fact that the bainitic reaction also pro-
duces material expansion tending to mask the expansion of the

*More about it will be discussed in the appropriate chapter.
martensitic reaction. For the purposes of this study, however, it will be assumed that the same, constant $M_s$ temperature exists for all cooling rates regardless of the prior formation or no formation of bainite.

A recent study by Stoop and Metzbower [52] looked into the metallurgical characterization of HY-130 steel welds made using the shielded metal arc (SMA) with El4018 electrodes, gas metal arc (GMA) with 140S wire, electron beam (EB), and laser beam (LB) welding processes. Specimens having 1/4 in. (6.35mm) and 1/2 in. (12.7mm) thicknesses were used. The microstructures of the SMA and GMA weldments were found to be similar, consisting primarily of a large percentage of acicular ferrite with somewhat small percentages of bainite and martensite. The EB and LB weldments were also similar in microstructure, composed mostly of martensite with smaller amounts of bainite.

3.3 Temperature Dependence of Material Properties

The analytical and numerical study of temperature, thermal strain and residual stress distributions due to welding requires the knowledge of the temperature dependence of both the physical and mechanical properties of the material under consideration. More specifically, in the heat transfer analysis the temperature variations of thermal conductivity, $k$, specific heat, $c$, and density, $\rho$, are used, whereas for the stress analysis the variations of the thermal expansion coefficient, $\alpha$, virgin yield stress, $\sigma_{yy}$, Young's modulus, $E$, tangent modulus, $E_T$, and Poisson's ratio, $\nu$, with temperature are
required. This is due to the fact that during the welding cycle the material experiences temperatures ranging from room to melting and it is well established that all material properties exhibit severe changes within that range.

In so much, however, as the temperature dependence of material properties is such a critical input for the accurate calculation of temperatures, strains, and stresses due to welding, a quick look through the available literature shows a rather disappointing situation. Tall [53] in his pioneering 1964 study on residual stresses in welded plates, as well as many other investigators after him, including this author, have observed a great variation in the temperature dependence of properties among materials differing only slightly in chemical composition. On top of that, most available data on the properties do not include the high side of the temperature range of interest, the reason being the great difficulties encountered in the measurement of properties at elevated temperatures.

The combination of the above facts has thus left analysts with no other alternative but to approximate the elevated temperature properties through careful extrapolations, or even worse, to approximate the non-available properties of some materials from the available ones of materials with similar chemical compositions through an averaging process. It is obvious, though, that such or similar techniques do not only lack adequate accuracy but can, in some cases, lead to significant errors. This situation tends to improve in recent years
especially with respect to materials widely used in critical structures, such as stainless steels in nuclear reactors, and where accurate analyses are of paramount importance from the safety standpoint.

As far as the welding problem is concerned, however, particularly in the case of high strength quenched and tempered steels, such as HY-80 and HY-130, the situation has not improved. The availability of elevated temperature properties of these materials is at best scant, if not nonexistent. The problem is further complicated by the presence of the molten pool, becoming the weld metal when solidified, which exhibits different properties than the base metal, something due to both chemical composition differences and differences in the heat treating it is subjected to during the welding cycle. Most investigators, though, have chosen to neglect this latter effect not because it complicates the problem but because of the high uncertainty involved; in other words the assumption of same material properties for the weld and base metals is as good as any in the absence of concrete information.

Having made the above observations, an effort will be undertaken in the next two subsections to approximate the temperature dependence of the HY-130 properties in the best possible way. A similar effort made by Schrodt [39] in 1974 will be utilized as the starting point coupled with information that has become available since then, especially on the material's transformation kinetics.
3.3.1 Mechanical Properties

An estimated stress-strain curve at room temperature for HY-130 is shown in Fig. 3.4 as developed by the author using data from Willner and Salive [54] and Schrod [39], and pertaining to tension. In the case of compression Flax et al [38] report that the $\sigma$-$\varepsilon$ curve is displaced by about 10-15 \textsuperscript{'}si (70-105 MPa) higher; for the purposes of this study, however, it will be assumed that the same curve can represent both tension and compression.

The figure reveals that at room temperature HY-130 has a Young's modulus, $E$, equal to $30 \times 10^3$ksi ($2 \times 10^5$ MPa) and that the 0.2\% offset yield stress (point A) is equal to 143 ksi (986 MPa).

At this point some notes on the modeling to be used for the stress-strain relationships are in order. During the welding thermal history part of the material undergoes plastic deformation at high temperatures accompanied by conditions of loading and unloading reminiscent of cases involving cyclic stress-strain responses. For these kinds of responses, it is a well established fact [55,57] that they are highly dependent on past history of the material — for example, in uniaxial cyclic tests it was found that the $\sigma$-$\varepsilon$ curves depend strongly on the strain range. It would be thus impossible to realistically model such cases in an analysis, especially since no such experimentally confirmed $\sigma$-$\varepsilon$ curves exist nor do proven multiaxial theories that can adequately predict the nonlinear, history-dependent material behavior under arbitrary no radial
FIGURE 3.4  Typical Stress-Strain Diagram for HY-130 Steel at Room Temperature.
loading.

As a way out researchers at the Oak Ridge National Laboratory [57] have proposed the use of an approximating theory based on the bilinear representation of the uniaxial stress-strain curve for the virgin material in the case of initial loading; their similar proposal for subsequent loading cycles are of no relevance to the welding problem. According to this approach, which is claimed to be conceptually and mathematically consistent for general non-proportional loadings, the elastic segment of the bilinear curve is represented by a straight line determined by the initial response of the material, i.e. a line having a slope equal to the material's Young's modulus; and the elastic-plastic segment of the curve is represented by a straight line connecting the stress point at the maximum expected elastic plus plastic strain, $\varepsilon_{\text{max}}'$ to the stress point, at strain $\varepsilon_{\text{max}}/2^*$. The intersection of these two lines will then determine a point defined as the virgin yield point, $\sigma_{yv}$; furthermore, the slope of the elastic-plastic segment defines the material tangent modulus, $E_T$.

Following the procedure described above and assuming, based on experimental data, a maximum strain equal to 1.2%, the bilinear representation of the HY-130 uniaxial stress-

\*Note that in the case of multiaxial strain state the maximum strain should be assessed on the basis of the effective strain, $\bar{\varepsilon}$, defined by

$$
\bar{\varepsilon} = \left( \frac{2}{3} \varepsilon_{ij} \varepsilon_{ij} \right)^{1/2}
$$
strain curve at room temperature was constructed as shown in Fig. 3.4. The room temperature values of the virgin yield stress, \( \sigma_{YV} \), (point B) and tangent modulus, \( E_T \), can then be read as 138 ksi (950 MPa) and \( 1.6 \times 10^3 \) ksi (11 \( \times 10^3 \) MPa) respectively.

**Yield stress.** The approximate effect of temperature on the virgin yield stress is shown in Fig. 3.5 as developed from available data on T-1 steel (a commercially available quenched and tempered high strength steel with mechanical properties close to those of HY-80) [58] and a low alloy high strength Ni-Cr-Mo-V steel [59]. Note that although the available data pertained to the 0.2\% offset yield stress, it was assumed that a similar behavior would be expected for the virgin yield stress, \( \sigma_{YV} \). Furthermore, most of the elevated temperature data run up to about 1200°F (650°C) making it necessary to extrapolate for higher temperatures and up to the solidus; this was done bearing in mind that at very high temperatures the material loses most of its strength, becoming unable to sustain any strength in its molten state.

A comparison of Fig. 3.5 with similar ones proposed by Hibbitt [6,60] and Cacciatore and Morante [61] for the HY-130 steel reveals many qualitative similarities, although the latter ones pertain to the 0.2\% offset yield strength and have been idealized by straight line approximations. Quantitative differences include a faster reduction in yield stress with temperature as compared to Fig. 3.5 and higher values in
FIGURE 3.5 Variation of Virgin Yield Stress with Temperature for HY-130.
the transformation temperatures, depicted in the diagrams by changes in the curves' slopes.

Young's modulus. Figure 3.6 presents the approximate effect of temperature on the Young's modulus for HY-130. It was developed using again data from the T-1 steel [58]. The observations made above with respect to the virgin yield stress hold here as well for the most part.

Tangent modulus. Looking at most uniaxial stress-strain curves one observes that the tangent (or plastic or work-hardening) modulus, represented by the slope at any point of the elastic-plastic segment of the stress-strain curve, changes with the stress level; and that for the case of work-hardenable materials it decreases with increasing stress level. This is not the case, however, for the bilinear representation of the stress-strain curve where a unique tangent modulus is defined at each temperature.

Having said that, the question arises on how to evaluate the temperature dependence of the tangent modulus, given the fact that nowhere in the literature does such data exist and that the investigators who have studied the HY-130 welding response have assumed an elastic-perfectly plastic material [6,60,61]. The answer can be given by assuming a proportionality between the tangent and Young's moduli at all temperatures which leads to the following equation:

\[ E_T(\theta) = \frac{E_T(\theta_e)}{E(\theta_e)} \cdot E(\theta) \]
FIGURE 3.6 Variation of Young's Modulus with Temperature for HY-130.
where \( E_T(\theta_e) \) and \( E(\theta_e) \) are the values of the tangent and Young's moduli at room temperature, and \( E_T(\theta) \) and \( E(\theta) \) the corresponding values at any other temperature.

Based on the above assumption and Fig. 3.6, Fig. 3.7, showing the approximate variation of tangent modulus with temperature, was constructed.

**Stress-strain diagram.** On the basis of the temperature variation of \( \sigma_{YV} \), \( E \), and \( E_T \) as earlier depicted, Fig. 3.8 was made showing the bilinear approximations of the HY-130 stress-strain diagrams at various temperatures ranging from room to 2000°F (1090°C). The figure clearly depicts the effects of a temperature change on the material. A decrease in the yield strength and the elastic region is a consequence of a temperature increase, making it possible for plastic flow to occur under constant stress even if the material was fully elastic at lower temperatures. This phenomenon is even more pronounced at very high temperatures where plastic flow can be initiated at very low stress levels. Such a behavior is obviously of paramount importance in welding and is the cause of residual stress formation. Moreover, a temperature increase results in a reduction of the tangent modulus, which means that for a specimen under constant applied stress and already exhibiting plastic flow, the further increment of plastic flow corresponding to a given temperature increment will decrease as the temperature level is increased. On the other hand, a decrease in temperature under constant stress will not result in any plastic strain decrease (since plastic strain is
Figure 3.7 Variation of Tangent Modulus with Temperature for HY-130.
FIGURE 3.8 Bilinear Stress-Strain Diagrams for HY-130 at Various Temperatures.
irrecoverable) but rather in a small elastic strain decrease, owing to an increase in the Young's modulus.

**Poisson's ratio.** The variation of Poisson's ratio with temperature is shown in Fig. 3.9 as proposed by Hibbitt [6]. A linear variation is shown from a value of 0.30 to 0.454 as temperature varies from room temperature to 2600°F (1430°C); in other words, Poisson's ratio tends to a value of one half as temperature increases, but it does not reach it in the melting region because of volume change considerations (at 0.50 there would be no volume change in the molten pool).

Friedman [62], in a welding thermomechanical analysis involving the Iconel Alloy 600, reports that a better choice for the Poisson's ratio's temperature variation would be one which, when coupled with the Young's modulus' variation, would always provide for a constant bulk modulus, \( B = E/(1-2\nu) \), equal to its room temperature value. He claims that by following this approach possible ill-behavior of the solution can be avoided and convergence facilitated. This proposition was not verified in the present study since the solution converged using the assumed variations outlined above.

To summarize the variation of the mechanical properties of HY-130 with temperature, Tables 3.6a and 3.6b were prepared and presented here.

### 3.3.2 Physical Properties

The effect of temperature on the thermal conductivity, \( k \), specific heat, \( c \), average thermal expansion coefficient, \( \alpha \), and density, \( \rho \), will be discussed in this section.
FIGURE 3.9 Variation of Poisson's Ratio with Temperature for HY-130.
### TABLE 3.6a Summary of Mechanical Properties for HY-130 (Anglo-Saxon units)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>70</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1,000</th>
<th>1,200</th>
<th>1,400</th>
<th>1,600</th>
<th>1,800</th>
<th>2,000</th>
<th>2,200</th>
<th>2,400</th>
<th>2,600</th>
<th>2,700</th>
<th>2,800</th>
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<tbody>
<tr>
<td>$\sigma_{yv}$</td>
<td>136</td>
<td>133.5</td>
<td>126.7</td>
<td>117</td>
<td>106</td>
<td>85</td>
<td>49</td>
<td>24.5</td>
<td>11</td>
<td>5.8</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E$</td>
<td>30</td>
<td>29.6</td>
<td>28.8</td>
<td>27.7</td>
<td>26</td>
<td>22.7</td>
<td>14</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1.2</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E_T$</td>
<td>1.60</td>
<td>1.58</td>
<td>1.54</td>
<td>1.48</td>
<td>1.39</td>
<td>1.21</td>
<td>0.75</td>
<td>0.27</td>
<td>0.21</td>
<td>0.16</td>
<td>0.11</td>
<td>0.06</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.300</td>
<td>0.308</td>
<td>0.321</td>
<td>0.334</td>
<td>0.346</td>
<td>0.359</td>
<td>0.372</td>
<td>0.384</td>
<td>0.397</td>
<td>0.410</td>
<td>0.422</td>
<td>0.435</td>
<td>0.447</td>
<td>0.454</td>
<td>0.454</td>
<td>0.454</td>
</tr>
</tbody>
</table>

**Nomenclature**

$\theta$ = Temperature (°F)

$\sigma_{yv}$ = Virgin Yield Stress (ksi)

$E$ = Young's Modulus (ksi x 10^{-3})

$E_T$ = Tangent Modulus (ksi x 10^{-3})

$\nu$ = Poisson's Ratio
**TABLE 3.6b** Summary of Mechanical Properties for HY-130 (SI units)

<table>
<thead>
<tr>
<th>θ</th>
<th>20</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1,000</th>
<th>1,100</th>
<th>1,200</th>
<th>1,300</th>
<th>1,400</th>
<th>1,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{yv}$</td>
<td>950</td>
<td>872</td>
<td>820</td>
<td>751</td>
<td>648</td>
<td>472</td>
<td>248</td>
<td>124</td>
<td>69</td>
<td>41</td>
<td>27.6</td>
<td>17.2</td>
<td>8.3</td>
<td>3.2</td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>206</td>
<td>200</td>
<td>192</td>
<td>183</td>
<td>167</td>
<td>132</td>
<td>56.5</td>
<td>33.1</td>
<td>25.5</td>
<td>20</td>
<td>14.5</td>
<td>9.6</td>
<td>2.8</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>$E_T$</td>
<td>11</td>
<td>10.7</td>
<td>10.3</td>
<td>9.8</td>
<td>8.9</td>
<td>7</td>
<td>3</td>
<td>1.8</td>
<td>1.4</td>
<td>1.1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.15</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>ν</td>
<td>0.300</td>
<td>0.320</td>
<td>0.331</td>
<td>0.342</td>
<td>0.353</td>
<td>0.364</td>
<td>0.374</td>
<td>0.385</td>
<td>0.396</td>
<td>0.407</td>
<td>0.418</td>
<td>0.429</td>
<td>0.440</td>
<td>0.451</td>
<td>0.462</td>
</tr>
</tbody>
</table>

**Nomenclature**

θ = Temperature (°C)

$\sigma_{yv}$ = Virgin Yield Stress (MPa)

E = Young's Modulus (MPa x 10^{-3})

$E_T$ = Tangent Modulus (MPa x 10^{-3})

ν = Poisson's Ratio
Thermal conductivity. In the absence of any information regarding the temperature variation of the HY-130 steel's thermal conductivity, Fig. 3.10 was constructed based on data for a low alloy 3.4 Ni-Fe steel [63]. These data were available up to a temperature of 1400°F (760°C); from this temperature and up to the solidus (2600°F or 1430°C) line a linearly increasing variation was assumed based on the fact that an increase is expected with temperature*. Note that this assumption is different from that made by other investigators who usually treat the thermal conductivity as constant from the highest temperature for which data exists and up to the solidus temperature. The adequacy of either assumption is of course open to question, and only the experimental determination of the required high temperature data can serve to test their validity.

A much more formidable problem is the one concerning the characteristics of heat transfer in the liquid metal. Although this problem will be addressed in some detail in the next chapter, a few words are in order here as far as the thermal conductivity is concerned. Tsai [29], Hibbitt [6], and other investigators have assumed that the thermal conductivity in the weld molten pool is much lower than the one of the solid metal, sometimes an order of magnitude lower. This assumption,

*Heat is conducted through metals by two distinct mechanisms: one involves the transfer of thermal energy by the free electrons in the metal, and the other results from the coupling together of the vibrations of the atoms in the crystal [49]. Both contributions are expected to result in an increase in thermal conductivity at high temperatures.
FIGURE 3.10 Variation of Thermal Conductivity with Temperature for HY-130.
although consistent with the fact that metal liquids have smaller thermal conductivity than the corresponding solid metals, does not take into account the question regarding the actual heat transfer mechanism in the weld pool, a mixture of conduction and convection. Although this mechanism has not been adequately modeled yet, several researchers, Friedman [64] among them, have maintained that the molten metal thermal conductivity, an equivalent and not a true one, can be assumed to realize a value much higher than the solid metal one. In other words, the convective part of the heat transfer mechanism is expected to increase the heat flow to a level above the one explained by pure conduction.

This latter approach was followed in the present study. Details of the actual values of \( k \) tested will be presented at the appropriate places in the next chapter.

**Specific heat.** The same difficulties, as the ones described for the thermal conductivity, were encountered in the case of the temperature variation of HY-130 steel's specific heat, c. Figure 3.11 was constructed on the basis of a low-alloy carbon steel, having a carbon content of 0.10 percent [63]. The graph shown includes a high peak of 0.354 Btu/lbm.\(^\circ\)F (1.48kJ/kg.\(^\circ\)C) at 1400°F (760°C) signifying the inclusion of the latent heat (16.4 Btu/lbm) in the allotropic phase transformation that takes place at the temperature range around this temperature (ferrite to austenite upon heating and vice versa upon cooling) under equilibrium conditions. It is understood that this inclusion of the latent heat is very approximate
FIGURE 3.11  Variation of Specific Heat with Temperature for HY-130.
since it does not take into account the nonequilibrium transformations that take place during the cooling stage of the welding cycle (austenite to bainite and/or martensite); in the absence of any other data, however, this is the best one can do.

As far as the high temperature region (close to the solidus line) is concerned, a constant specific heat was assumed owing again to the absence of data. Beyond the solidus temperature, where a solid-liquid transformation takes place, the latent heat effect is not shown as an equivalent increase in the specific heat, but is treated explicitly in the numerical calculations, as discussed in the next chapter. The latent heat value chosen is the one proposed by Hibbitt [67] for both the HY-130 and HY-80 steels and is equal to 118 Btu/lbm (275 kJ/kg).

**Average thermal expansion coefficient.** The temperature variation of the average thermal expansion coefficient, \( \alpha \), for HY-130 is shown in Fig. 3.12 and is based on values of the same low-alloy carbon steel used for the specific heat's temperature dependency [63]. Note the abrupt reversal in slope that takes place at around 1400°F (760°C) which is due to the allotropic phase transformation occurring under equilibrium conditions upon heating or cooling.

It should be emphasized at this point that Fig. 3.12 pertains to equilibrium conditions only, i.e. it assumes similar allotropic transformations occurring in both the heating and cooling stages of a temperature cycle. This is not the case,
FIGURE 3.12 Variation of Average Thermal Expansion Coefficient with Temperature for HY-130.
however, for welding where nonequilibrium conditions prevail and where the allotropic phase transformations upon heating (ferrite or martensite to austenite) and cooling (austenite to bainite and/or martensite) are different and take place within different temperature intervals. As a consequence, Fig. 3.12 cannot be strictly applied. Instead, a more fundamental approach based on first principles should be devised. Such an approach is outlined in detail in Chapter 5.

Density. Figure 3.13 shows the variation with temperature of the density, \( \rho \), of HY-130. It was calculated on the basis of a measured density of 0.284 lbm/in\(^3\) (7.86 \times 10^3 kg/m\(^3\)) at 70°F (21°C) and the thermal expansion data presented in Fig. 3.12.

To summarize the variation of the physical properties of HY-130 with temperature, Tables 3.7a and 3.7b were prepared and presented here.

3.4 **Computed Continuous Transformation Diagram from Isothermal Data**

As mentioned in Section 3.2.2, continuous cooling transformation (CCT) diagrams are more appropriate than isothermal transformation (TTT) diagrams for predicting the steel microstructures during the cooling stage of the welding cycle. Today, experimentally derived CCT diagrams exist for several steel alloys (see for example Cavanagh [65] for an extended bibliography), thus making such predictions possible. There are, however, alloys -- the HY-130 steel being
FIGURE 3.13 Variation of Density with Temperature for HY-130.
TABLE 3.7a Summary of Physical Properties for HY-130 (Anglo-Saxon units)

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>70</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
<th>1,000</th>
<th>1,200</th>
<th>1,400</th>
<th>1,600</th>
<th>1,800</th>
<th>2,000</th>
<th>2,200</th>
<th>2,400</th>
<th>2,600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>20.03</td>
<td>20.9</td>
<td>21.3</td>
<td>21.03</td>
<td>20.7</td>
<td>19.5</td>
<td>17.6</td>
<td>14.9</td>
<td>15.3</td>
<td>15.7</td>
<td>16.0</td>
<td>16.4</td>
<td>16.7</td>
<td>17.0</td>
</tr>
<tr>
<td>$c$</td>
<td>0.102</td>
<td>0.110</td>
<td>0.120</td>
<td>0.132</td>
<td>0.148</td>
<td>0.166</td>
<td>0.190</td>
<td>0.354</td>
<td>0.168</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
<td>0.156</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.2839</td>
<td>0.2830</td>
<td>0.2816</td>
<td>0.2802</td>
<td>0.2788</td>
<td>0.2773</td>
<td>0.2759</td>
<td>0.2749</td>
<td>0.2747</td>
<td>0.2747</td>
<td>0.2747</td>
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<td>0.2747</td>
<td>0.2747</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>--</td>
<td>6.55</td>
<td>6.97</td>
<td>7.35</td>
<td>7.75</td>
<td>8.06</td>
<td>8.23</td>
<td>8.15</td>
<td>6.95</td>
<td>7.18</td>
<td>7.30</td>
<td>7.45</td>
<td>7.50</td>
<td>7.50</td>
</tr>
</tbody>
</table>

Nomenclature

$\theta$ = Temperature ($^\circ$F)

$k$ = Thermal Conductivity (Btu/hr.ft.$^\circ$F)

$c$ = Specific Heat (Btu/lb$_m$. $^\circ$F)

$\rho$ = Density (lb$_m$/in$^3$)

$\alpha$ = Average Thermal Expansion Coefficient (in/in$^\circ$F $\times$ 10$^{-6}$)
TABLE 3.7b Summary of Physical Properties for HY-130 (SI units)

<table>
<thead>
<tr>
<th>θ</th>
<th>20</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>760</th>
<th>900</th>
<th>1,000</th>
<th>1,100</th>
<th>1,200</th>
<th>1,300</th>
<th>1,400</th>
<th>1,500</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>35.1</td>
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<td>36.8</td>
<td>36.2</td>
<td>34.6</td>
<td>32.0</td>
<td>28.5</td>
<td>25.8</td>
<td>26.6</td>
<td>27.2</td>
<td>27.8</td>
<td>28.4</td>
<td>29.0</td>
<td>29.6</td>
<td>--</td>
</tr>
<tr>
<td>c</td>
<td>0.427</td>
<td>0.502</td>
<td>0.548</td>
<td>0.602</td>
<td>0.665</td>
<td>0.741</td>
<td>0.481</td>
<td>0.686</td>
<td>0.653</td>
<td>0.653</td>
<td>0.653</td>
<td>0.653</td>
<td>0.653</td>
<td>0.653</td>
<td>--</td>
</tr>
</tbody>
</table>

Nomenclature

θ = Temperature (°C)
k = Thermal Conductivity (W/m.°C)
c = Specific Heat (kJ/kg.°C)
ρ = Density (kg/m³ x 10⁻³)
α = Average Thermal Expansion Coefficient (m/m.°C)
one of them -- for which CCT diagrams are not available. In such instances the analyst would like to be able to compute a schematic CCT diagram based on available isothermal data and resembling as closely as possible the experimentally obtained one.

Bain [66] is credited with the first such schematic cooling diagrams; it indicated the beginning of transformation and the structures produced by several typical cooling rates for a eutectoid steel.

Grange and Kiefer [67] were among the first to theoretically compute a CCT diagram from a TTT one. The essence of their method consists of representing any stage of the cooling by a point on the isothermal diagram which indicates, by its position, the equivalent amount of transformation that has occurred on cooling to that temperature at the specified rate. In other words, on cooling through a limited temperature range, the amount of transformation is assumed equal to that indicated with an isothermal curve at an average temperature and for an equal time interval and transformation extent. More details on the method will be included in Appendix C. Their method predicted curves possessing the same general shape but lying to the left and above the measured curves.

Although the above method resulted in reasonably close agreement between experimental and calculated CCT curves for a Ni-Cr-Mo eutectoid steel, Manning and Lorig [68] were re-
luctant to apply the same technique to any steel composition, claiming it to be entirely empirical. Working with five steels of varying Cr content, they instead upheld the validity of the additivity concept when restricted to one austenitic decomposition mechanism at a time. This concept, first proposed by Scheil [69], states that the time spent at a particular temperature divided by the time required for beginning transformation isothermally at that temperature represents a fraction of the incubation requirement. Transformation thus begins when the sum of these fractions reaches unity. In mathematical terms this rule of additivity can be stated as follows: Suppose that an isothermal reaction has reached a certain fraction of completion, \( f_0 \), at time \( \tau \) which is a function of temperature. Then on continuous cooling, the fraction completed will be \( f_0 \) at that time \( t \) and temperature \( \theta \) at which

\[
\int \frac{dt}{\tau} = \int \frac{d\theta}{\tau \left( \frac{df}{dt} \right)} = \int \frac{df}{\tau \left( \frac{df}{dt} \right)} = 1
\]

The results obtained by Manning and Lorig using this technique (described in more detail in Appendix C) were of the same general nature as the ones previously derived by Grange and Kiefer.

A question thus arose regarding the validity of the additivity rule. Cahn [70] in 1956 finally eliminated much of the confusion by supporting its application if one of
two cases existed:

(a) the transformation process is "isokinetic", i.e., the nucleation and growth rates are proportional over a range of temperature

(b) the nucleation is restricted to the beginning of the transformation product formation, i.e., the transformation is growth-dependent which usually means temperature-dependent.

The second case exists at relatively high cooling rates, which is the situation encountered in the heat affected zone (HAZ) of welds.

Since the advent of the computer several other investigations, including Markowitz and Richman [71] and Kennon [72], tried to improve upon the calculated CCT curves by using the methods previously described but with much smaller time increments. The discrepancy they found with the experimental curves was usually smaller and in the reverse direction.

3.4.1 Application in the Case of HY-130 Steel

Given the fact that no comprehensive CCT diagram existed for the HY-130 steel, it was decided to apply the two methods, Grange-Kiefer and Manning-Lorig, in an effort to calculate such a diagram based on the isothermal one described in Section 3.2.1 and depicted in Figure 3.1.

One important note has, however, first to be made. The CCT diagram is heavily dependent not only on the type of alloy considered, but also, for the same alloy, on the cool-
ing rate. All of the methods described previously utilize linear cooling rates, although the authors point out that the procedures can be modified to encompass any type of cooling rate. On the other hand, looking at the temperature curves during welding (see Chapters 2 and 4), it is obvious that the cooling rates encountered are far from linear. Plotting them, however, on a semilogarithmic graph (linear scale for temperatures, logarithmic scale for time), the emerging picture can be adequately represented by straight lines (the cooling stage of the welding temperature cycle only).

Based on the above observations a modification of the two approaches was made to enable them to handle log-linear cooling curves. The details of these modifications are described in Appendix C.

Figure 3.14 shows the schematic CCT diagrams calculated using both linear and log-linear cooling curves. In the linear case both the Grange-Kiefer and Manning-Lorig approaches predict almost identical bainite start, $B_s$, transformation curves. This was expected and confirms results obtained by previous investigators. Comparing the obtained curves with the experimental ones shown in Figure 3.2 and 3.3 (although the latter ones are not CCT curves but have been measured using log-linear cooling curves), it is observed that the schematic curves lie below and to the right of the experimental ones, again confirming the more recent investigations
FIGURE 3.14  Schematic CCT Diagrams for HY-130 Steel
(Bainite Start Transformation Curves).
[71].

Regarding the log-linear case, the curves derived from both approaches lie above and to the left of the linear ones, with the one predicted using the Manning-Lorig method being slightly further away. As a consequence, both log-linear curves are much closer to the experimentally obtained ones.

It can therefore be concluded that the application of the methods using log-linear cooling curves results in schematic CCT diagrams that are closer to the ones appropriate for predicting microstructures in the HAZ and the weld metal of a weldment. It should be pointed out, however, that an even better approximation can be obtained if the microstructure prediction is based on rate equations. Efforts in this direction have been made for a very few steel alloys, primarily of eutectoid composition (see for example McCullough [73]), and much more research in the materials science field is needed for the development of similar rate equations for more complicated alloys, like the quenched and tempered HY-130 steel. Until such developments are made the above described methods with the proposed modifications are thought to be adequate in describing the phase transformations from an applied mechanics (macroscopic) point of view.

The previous discussion pertained to the bainite start, $B_s$, transformation temperature and can be extended to include any similar nucleation and growth transformation, such as the ferritic one. As far as the martensite transformation
is concerned, the situation is different. Austenite transforms to martensite athermally (no thermal activation is involved). This means that once the martensite start transformation temperature, $M_s$, is known, the extent of the transformation is dependent only on the amount of undercooling below the $M_s$ temperature (Burke [74]). It can therefore be assumed that the constant $M_s$ temperature found in the TTT diagram can be carried over to the CCT diagram. As Cavanagh [65] and others have pointed out, however, with the precipitation of ferrite and/or bainite, the build-up of carbon and/or alloy elements in the remaining austenite will alter its $M_s$ temperature, resulting in slightly depressed $M_s$ temperature at slower cooling rates. Although such effects are recognized here, it is felt that due to the uncertainty of the amount of depression and for the purposes of this study, they can be neglected. Similar considerations have been taken into account for the martensite finish, $M_f$, transformation temperature.
CHAPTER IV

NUMERICAL SOLUTION OF HEAT FLOW
DURING WELDING

In Chapter II some analytical solutions to the problem of heat flow during welding were discussed and analyzed. The general conclusion was that these closed form solutions, however complicated they may be, cannot very accurately predict the temperature distribution during welding, especially close to the weld line (i.e., in the weld metal and the HAZ). This is primarily due to the highly nonlinear nature of the problem. Nevertheless, such solutions are very useful in providing insight to the problem and allow for extensive parametric analyses due to their low cost.

On the other hand, there are occasions when an accurate description of the temperature is necessary, as for example when a metallurgical characterization of the weld metal and HAZ is needed, or when a subsequent stress analysis is required to determine the transient strains and residual stresses. In these cases a numerical analysis using the finite difference or finite element method is more appropriate.

In this chapter concern will be focused on the finite element method for solving the heat transfer problem during welding. After a brief literature survey, the model will be discussed in some detail and an assessment of its limitations
will be made.

The results obtained using this model will then be compared with experimentally obtained data. The experiments themselves will also be briefly discussed.

4.1 **Literature Survey**

Since the advent of the electronic digital computer several serious efforts have been made to numerically solve field, and in particular heat transfer, problems. And although the finite difference method had initially the edge, the advantages of the finite element method, especially if coupled with thermal stress analysis, are more and more recognized today.

Many finite element programs have thus been developed over the years capable of performing heat transfer analyses. Several of these codes can take various nonlinearities into account in a more or less sophisticated manner. One of the most sophisticated ones, and the one actually chosen in this study to solve the welding heat flow problem, is ADINAT (Automatic Dynamic Incremental Nonlinear Analysis for Temperatures) developed by Bathe and co-workers [75 - 77] over a period of years. Some details of this program will be discussed later. It suffices here to mention that ADINAT can take into account temperature dependent material properties as well as nonlinear convection and radiation boundary conditions.

The discussion so far dealt with multipurpose heat trans-
fer finite element method (FEM) programs. Concurrently, however, several investigators concerned with the welding problem have developed similar programs. Hibbitt and Marcal [6, 60] developed such a program in 1973 which was later used by other investigators including Hsu [78] and Nickell [79]. At the Massachusetts Institute of Technology a team headed by Masubuchi has also developed similar programs in the early 1970s [24, 80-83]. Friedman [62, 64, 84, 85] has also made substantial contributions in the case of GTA welding working at the Westinghouse Electric Corporation's Bettis Atomic Power Laboratory.

The way each of the above individuals approached the various aspects of the welding problem will be discussed at the appropriate places in the next sections where the way in which the current model was developed will be detailed.

4.2 Finite Element Procedure

As mentioned previously, the welding heat flow analysis was performed using ADINAT. The governing incremental isoparametric finite element equations for this problem have been derived by Bathe [77] and are

\[
(t_K^k + t_K^c + t_K^r) \Delta \theta(i) = t+\alpha\Delta t \Omega \\
+ t+\alpha\Delta t \Omega c(i-1) + t+\alpha\Delta t \Omega r(i-1) - t+\alpha\Delta t \Omega k(i-1)
\]  
(4.1a)

or
\[ t_K \Delta \theta(i) = t + \alpha \Delta t \hat{Q}(i-1) \] (4.1b)

where \( t_K \) is the effective conductivity matrix at time \( t \) consisting of the conductivity, nonlinear convection, and radiation matrices; \( t + \alpha \Delta t \hat{Q} \) is the heat flow vector including the effects of surface heat flow inputs, internal heat generation and temperature-dependent heat capacity; \( t + \alpha \Delta t \hat{Q}(i-1) \) is the effective heat flow vector; and \( \Delta \theta(i) \) is the increment in nodal-point temperatures in iteration \( i \),

\[ t + \alpha \Delta t \hat{\theta}(i) = t + \alpha \Delta t \hat{\theta}(i-1) + \Delta \theta(i) \] (4.2)

Equations (4.1) represent the heat flow equilibrium of the body at time \( t + \alpha \Delta t \), where \( 0 \leq \alpha \leq 1 \) and \( \alpha \) is chosen to obtain optimum stability and accuracy in the solution. Furthermore, the solution using eqn. 4.1 corresponds to a modified Newton-Raphson iteration.

**Boundary conditions.** Convection and radiation boundary conditions are taken into account by including the matrices \( t_K^C \) and \( t_K^R \) and the vectors \( t + \alpha \Delta t \hat{Q}^C \) and \( t + \alpha \Delta t \hat{Q}^R \) in eqn. (4.1a). Additional external heat flow input on the boundary is specified in \( t + \alpha \Delta t \hat{Q} \) as surface heat flow input. Prescribed temperature conditions can also be specified.

**Time integration schemes.** A family of one-step methods [86] is considered for the time integration using the para-
meter \( \alpha \). The scheme was found [77] to be unconditionally stable for \( \alpha \geq 1/2 \) and to generally give better solution accuracy when \( \alpha = 1 \) (Euler backward method). Finally the modified Newton iteration is guaranteed to converge provided the time step \( \Delta t \) is small enough.

4.3 Weld Model

In this section all the necessary considerations for the modelling of a multipass weldment between two relatively thick plates -- thickness on the order of 1 inch (25 mm) -- will be discussed. They include the two-dimensionality of the problem, the boundary conditions except the heat input, the material properties, the weld metal and the heat input.

4.3.1 Two-Dimensionality of Problem

The heat flow pattern resulting from a moving welding arc is generally three-dimensional; temperature gradients exist through the weldment's thickness direction, as well as in directions parallel and transverse to the arc travel (welding) one. A closer examination of the physics of the process, however, reveals that the necessary assumptions for the transformation of the problem from a three- to a two-dimensional one are reasonable.

As a starting point and referring to the fixed coordinate system \((x,y,z)\) depicted in Figure 4.1, let's consider the general heat flow equation for a differential element inside the conducting medium:
FIGURE 4.1 Weldment Configuration
\[ \frac{\partial}{\partial x} \left( k \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial \theta}{\partial z} \right) + w_i = \rho c \frac{\partial \theta}{\partial t} \] (4.3)

where all parameters have been previously defined.

For the case of a stationary arc situated above the origin 0 of the coordinate system, the above three-dimensional equation can be solved numerically with no simplifications. Closer examination of the process, however, reveals that the stationary arc (being modeled either as a point source or as a bell shaped function in space) results in heat flow being supplied axisymmetrically to the weldment. Therefore, if the weldment is geometrically regular (a flat plate for example), heat transfer in the weld can be idealized as radially symmetric in the high temperature region of interest -- or everywhere if the dimensions in the xy-plane are large compared to the thickness so as to be for all practical purposes infinite. As a consequence, the welding thermal cycle can be analyzed using a two-dimensional, axisymmetric heat conduction model. This method of analysis was actually used by Friedman [64] in the case of a stationary gas tungsten arc, providing much useful data and insight on the effects of the heat input magnitude and distribution on weld penetration and fusion zone shape.

The problem becomes complicated, however, when the more realistic moving welding arc is considered. A first simpli-
fication of the problem can be attained by assuming that the welding heat source (arc) is moving at a constant speed on a regular path (i.e., a straight line in a planar weld, or a circle in an axisymmetric weld) and that end effects resulting from either initiation or termination of the heat source can be neglected. Under these assumptions quasi-steady state conditions are attained for the fixed coordinate system \((x,y,z)\). Alternatively, the problem can be treated as a steady state one with respect to the moving coordinate system \((\xi=x-\nu t, y,z)\), where \(\nu\) is the speed of the heat source. To be more precise, introducing the new variable \(\tau = t - x/\nu\), equation (4.3) can be transformed into

\[
\frac{\partial}{\partial y} \left( k \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial \theta}{\partial z} \right) + \frac{1}{\nu^2} \frac{\partial}{\partial \tau} \left( k \frac{\partial \theta}{\partial \tau} \right) + W_i = \rho C \frac{\partial \theta}{\partial \tau}
\] (4.4)

With this formulation, the time dependent solution at any section, say \(x=0\), normal to the welding direction is expressed as a function of the variables \(y,z\), and \(\tau\). Knowing this temperature distribution, the temperature can be evaluated directly for any other section using the transformation

\[
\theta(x,y,z,t) = \theta(0,y,z, \tau + \frac{x}{\nu})
\] (4.5)

The problem is, therefore, reduced to finding the two-dimensional, unsteady temperature field at a section normal to the weld line (e.g., at the section \(x=0\)). A planar analysis may
be used for this purpose when the weld speed is sufficiently high, relative to a characteristic diffusion rate for the material. In this case, the net heat flow across any infinitesimally thin slice of the weldment normal to the weld line is assumed to be negligible relative to the heat being conducted within the slice itself. If this assumption is justified, and it is, then the third term in the left hand side of equation (4.4) can be dropped, resulting into the final equation:

$$\frac{\partial}{\partial y} \left( k \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial \theta}{\partial z} \right) + \dot{W}_1 = \rho c \frac{\partial \theta}{\partial t}$$  (4.6)

Friedman [85] notes that the restriction of high weld speed can be relaxed, accounting for net heat flow in the x-direction by approximating the third term of equation (4.4) and including it in the formulation as an equivalent internal heat generation term. The two-dimensional character of the formulation can thus be retained for any welding speed.

It should be mentioned at this point that attempts for the solution of the three-dimensional moving arc problem using the finite element method have been undertaken by several investigators (Muraki [83], Nishida [25], etc.) but with little success, due mainly to the prohibitively high computational cost. Tsai [29] was able, however, to develop a three-dimensional formulation with the finite-difference method using a semi-empirical iterative approach by eliminating the
molten pool area through an inner boundary condition.

Finally, two dimensional finite element programs have been developed [83] for use when welding thin plates. In these programs the problem is formulated in the x-y plane, by assuming that the temperature gradient in the thickness direction is zero.

In conclusion, the approximation of the three-dimensional character of the problem by a two-dimensional one is rational for relatively high welding speeds.

4.3.2 Boundary Conditions

The boundary conditions relevant to the welding problem will be discussed in this section. An exception will be made of the heat input which will be described separately.

Prescribed temperatures. It is rare that the need for specifying temperatures at points and/or surfaces of a weldment arises. An exception can be the presence of a large heat sink in contact with the weldment, but such a case will not be considered in this study.

Convection. Heat is lost from the surface of the weldment by two mechanisms, one of which is natural (or, more rarely, forced) convection -- the other is radiation discussed separately. Linear Newton convection cooling is assumed in the model according to

\[ q^s = h \cdot (\theta_e - \theta^s) \]  \hspace{1cm} (4.7)
where \( h \) is the temperature dependent convection coefficient, \( \theta_e \) the environmental temperature, and \( \theta^S \) the surface temperature at the point under consideration. Note that slag formation is not considered in the thermal model, possibly causing overestimation of the heat losses through natural convection at the lower temperatures at which slag is formed. It is believed, however, that the error involved is small.

The estimation of the convection coefficient, \( h \), is not an easy task. This is particularly true if the temperature dependence of \( h \) is to be taken into account, something that most previous investigators did not. Hibbitt and Marcal [60], for example, in their analysis of gas metal arc axisymmetric welds assumed a constant \( h \) equal to \( 2 \times 10^{-6} \) Btu/in\(^2\)sec\(^\circ\)F (5.88 W/m\(^2\)K) for most part of the weld history, except for the weld-side surface during the weld deposit operation when they took \( h \) to be equal to \( 1.5 \times 10^{-5} \) Btu/in\(^2\)sec\(^\circ\)F (44.1 W/m\(^2\)K) to take some account of the additional heat removal due to the shielding gas flow. On the other hand, Andersson [87], analyzing submerged arc welds, assumed a constant \( h \) equal to \( 4.1 \times 10^{-6} \) Btu/in\(^2\)sec\(^\circ\)F (12 W/m\(^2\)K). Finally, Friedman [64] in his investigation of stationary GTA welds utilized an empirical correlation equation for natural convection over horizontal plates with the hot surface facing up, yielding \( h \) equal to \( 1.566 (\theta - 480)^{1/4} \) W/m\(^2\)K.

In this study use was made of a temperature dependent convection coefficient based on semi-empirical studies made
by Tsai [29]. Table 4.1 provides three different sets of values used. Discussion of the obtained results will be deferred until Section 4.5.

**Radiation.** In the vicinity of the weld metal radiation heat losses are significant because of the large difference between the surface and environmental temperatures. These losses are modeled according to the quartic Stefan-Boltzmann law

\[ q^S = \sigma \varepsilon A (\theta_r^4 - \theta^S) \quad (4.8) \]

where
- \( \sigma \) = Stefan-Boltzmann constant
- \( \varepsilon \) = emissivity of the surface
- \( A \) = shape function
- \( \theta_r \) = sink temperature
- \( \theta^S \) = surface temperature

The above equation can also be written as

\[ q^S = \kappa (\theta_r - \theta^S) \quad (4.9a) \]

where

\[ \kappa = \sigma \varepsilon A (\theta_r^2 + \theta^S^2)(\theta_r + \theta^S) \quad (4.9b) \]

Radiation heat losses have been taken into account by many of the previously mentioned investigators. In this study the shape factor \( A \) was taken to be unity and the emis-
sivity coefficient \( \varepsilon \) 0.8. The Stefan-Boltzman constant is 0.33044 \( \times 10^{-14} \) Btu/sec/in\(^2\)°R \(^4\) (5.67 \( \times 10^{-8} \) W/m\(^2\)°K\(^4\)).

### TABLE 4.1 Temperature Dependence of Heat Convection Coefficient

<table>
<thead>
<tr>
<th>( \theta^S - \theta_e ) [°R]</th>
<th>( h ) [Btu/sec/in(^2)°R]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_e ) [°R]</td>
<td>Case 1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>( .2 \times 10^{-5} )</td>
</tr>
<tr>
<td>500</td>
<td>( .5 \times 10^{-5} )</td>
</tr>
<tr>
<td>1000</td>
<td>( .1 \times 10^{-4} )</td>
</tr>
<tr>
<td>5000</td>
<td>( .58 \times 10^{-3} )</td>
</tr>
<tr>
<td>50000</td>
<td>( .12 \times 10^{-2} )</td>
</tr>
</tbody>
</table>

Note: \( \theta^S \) = surface temperature  
\( \theta_e \) = environmental temperature

#### 4.3.3 Material Properties

The temperature dependence of the thermal conductivity, \( k \), the specific heat, \( c \), and the density, \( \rho \), of the HY-130 steel was described in detail in Section 3.3.2. The values reported there were utilized as input in the finite element heat transfer analysis.
4.3.4 Weld Metal Modelling

The heat transfer mechanism in the weld metal, when molten, is extremely complex and its physics are not well understood as of today. These complexities arise not only from the difficulty involved in modelling the welding arc heat flux correctly, but also from the behaviour of the convective motion of the molten metal; the thermal properties of the molten metal (including the phase transformations that take place during melting and solidification), the electric heating due to the current flow in the base metal; the boundary conditions for heat losses, etc. Many investigators have recognized these difficulties and have tried to find ways to approximate the phenomena involved.

A look at the available literature reveals that there are generally three ways for handling the problem. The first one, still at its developing stages, tries to understand and subsequently mathematically model the physical phenomena involved through sophisticated experimental and analytical techniques. Since no conclusive results are yet available, this approach will not be discussed any further.

A second way of handling the problem stems from the following thoughts: since the physics involved in the weld pool are not well understood, the problem will be divided into two parts. In the first part, an effort will be undertaken to predict the shape of the molten pool, and then in the second part the energy equations will be solved numerically in the
solid metal using the melting isotherm as a boundary condition. This approach was first hinted by Apps and Milner [16], was first applied by Pavelic et al. [88] to calculate temperature distributions when Gas Tungsten Arc welding thin plates, and was further expanded by Tsai [29] in the underwater welding case. All investigators have reported good correlation between experiments and analysis. The method works especially when the temperature distribution in the heat affected zone is required in order to predict the resulting microstructures in this area.

Some aspects and limitations of this approach are:

1. The shape of the molten pool is determined semi-empirically by measuring the weld bead width and the ripple length.

2. There is no need to estimate an arc efficiency for the welding process since the two weld bead shape parameters previously mentioned can be correlated to the heat input into the welded plate.

3. All investigations have been confined to thin plates only (less than 1 inch thick).

4. The case of multipass welding has not been examined.

The third and final way by which the problem seems to be handled in the literature is by assuming thermal properties of the molten metal such that its behaviour can be best approximated [6, 78, 79, 84]. For example, most investigators [64] use a value for the thermal conductivity of the molten
metal an order of magnitude higher than that of the material at the solidus temperature, in an effort to simulate the convective heat transfer mechanism and electromagnetically induced circulation in the puddle. On the other hand, other investigators use a value for the thermal conductivity of the molten metal equal to one half of that for the solid material [6]. Such inconsistencies arise because of the lack of data for the thermal properties at such high temperatures. As mentioned in Section 3.3.2, however, it is believed that the higher thermal conductivity better approximates the actual phenomena.

In this study the ultimate goal is the successful prediction of transient strains and residual stresses in weldments. It is therefore necessary to know the temperature distribution not only outside the weld metal but also inside it, since most plastic deformations that cause the formation of residual stresses take place in that region. As a consequence, the second approach mentioned above cannot be utilized. Instead, the third approach was applied using as thermal conductivity for the molten metal a value an order of magnitude higher than the one for the solid metal just below the solidus temperature.

Finally, since the cases analyzed involved multipass welding, it was necessary to find a way to model the laying of the various beads during the welding cycle. This was made possible by the element birth-and-death capabilities of the
code used. In other words, the program ADINAT [76] is capable of giving birth to a predetermined number of elements at predetermined time instances, thus enabling one to model the laying of a bead by specifying the appearance of the elements representing it at the time it physically appears.

4.3.5 Heat Input

The general modelling of heat input during welding was extensively discussed in Section 2.2, where both its magnitude and distribution were described. It was then pointed out that the arc efficiency, $\eta_a$, is one of the most important parameters involved. In this study where a gas metal arc welded specimen is analyzed, three arc efficiencies were tested, namely 0.60, 0.65, and 0.70. Discussion of the obtained results is delayed until Section 4.5.

As far as the distribution of heat input is concerned, a consistent formulation was adopted [89] assuming uniform distribution in space over the top of each weld bead.

4.4 Description of Experiments

A description of the welding experiments performed on HY-130 steel to measure temperature histories and transient strains will be undertaken in this Section. The experimental results regarding temperatures will be compared with the ones obtained by the finite element analysis in the next section, whereas the transient strains will be discussed in Chapter 5. Note that all experimental data will be shown in the appro-
appropriate places as mentioned above.

4.4.1 Specimen Geometry

Two experiments using the GMA welding process on HY-130 steel plates were performed. Similar experiments using the EB and LB welding processes on identical specimens were performed by Coneybear [90], Rogalsky [91], Coumis [92], and Lipsey [93] as part of the same study.

In both experiments each plate measured 12" x 12" x 1" (305 mm x 610 mm x 25.4 mm) resulting after welding in a 24" x 24" x 1" (610 x 610 x 25.4 mm) configuration as shown in Figure 4.2. The weld length was chosen to be larger than 18 inches (457.2 mm) because previous experimental experience has shown that in such lengths the maximum stresses are developed and kept constant around the middle of the plate. Moreover, in such lengths the end effects can be neglected as far as the mid-length region of the weld is concerned, so that a quasi-stationary state can be assumed for the heat transfer analysis -- in other words it can be assumed that any cross-section in the vicinity of the middle of the plate experiences the same temperature history.

Figure 4.3 shows the weld joint configuration. It consists of a double-V groove with a 60° included angle in accordance with U.S. Navy specifications. This configuration was made by machining.

The test plate support arrangement is shown in Figure 4.4.
FIGURE 4.2 Test Plate Arrangement

FIGURE 4.3 Weld Joint Configuration
FIGURE 4.4 Test Plate Support Arrangement
It is the same one used by Lipsey [93] and consists of knife edge supports located through the whole specimen length at 5 and 8 inches (127 and 203.2 mm) from the weld centerline on either side. This way the experiments were performed with the specimens completely unrestrained.

4.4.2 Instrumentation

Temperatures on the top surface of the specimens were measured using Chromel/Alumel adhesive bonded thermocouples referenced at 32°F (0°C). Transient strains were also measured on the top surface of the specimens using 90° Rosette electric resistance strain gages which were cemented on the plates' surface. Table 4.2 provides the gages' specifications and Figure 4.5 shows the apparent strain curve supplied by the manufacturer, necessary to perform the temperature compensations on the raw data [94].

Temperatures and strains were simultaneously read out on a twelve channel, continuous recording Visicorder.

<table>
<thead>
<tr>
<th>TABLE 4.2 Strain Gage Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gage</td>
</tr>
<tr>
<td>Designation</td>
</tr>
<tr>
<td>Manufacturer</td>
</tr>
<tr>
<td>Grid dimensions</td>
</tr>
<tr>
<td>Temperature range</td>
</tr>
<tr>
<td>Resistance</td>
</tr>
<tr>
<td>Gage factor</td>
</tr>
<tr>
<td>Cement</td>
</tr>
</tbody>
</table>
FIGURE 4.5 Effect of Temperature on Apparent Strain (SR-4 Strain Gage).
Figures 4.6 and 4.7 show the locations of the thermocouples and strain gages on the two specimens, I and II. In both cases the strain gages were located at the midlength of the weld (12 inches or 305 mm from the plates' edge) and at the transverse distances from the weld centerline shown in the figures. The thermocouples were located 1/2 inch (12.7 mm) ahead of the strain gages, assuming that the same temperature history was to be experienced at both locations with a time lag equal to the time spent by the arc to travel the two locations. This time lag was taken into account when reducing the data.

4.4.3 Welding Procedure

Welding on both tests was performed by the semiautomatic GMA method, using a SVI-300 power supply and associated controls manufactured by the Linde Division of the Union Carbide Corporation. Arc travel speed, voltage, and amperage were set prior to the start of each welding test so as to obtain the desired heat input (less than 45 kJ/in according to U.S. Navy specifications) and hence a sound weld. Preheating was also applied by resistance heaters, monitored by the installed thermocouples so as to be as uniform as possible and not exceed 175°F (80°C). Interpass temperature was also monitored.

The filler wire was 0.045 in (1.14 mm) in diameter and of the Linde-140s type described in Section 3.1. A mixture of argon plus 2% oxygen was used as the shielding gas.
**FIGURE 4.6** Thermocouple and Strain Gage Location on HY-130 Specimen I.

**FIGURE 4.7** Thermocouple and Strain Gage Location on HY-130 Specimen II.
Six welding passes were required to fill the top half of the joint. The specimens were then turned over and six more passes were made to complete the joining.

Table 4.3 provides a summary of the welding conditions used.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weld type</td>
<td>Butt</td>
<td>Butt</td>
</tr>
<tr>
<td>Process</td>
<td>GMA</td>
<td>GMA</td>
</tr>
<tr>
<td>Arc voltage</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Polarity</td>
<td>d.c.r.p.¹</td>
<td>d.c.r.p.</td>
</tr>
<tr>
<td>Travel speed (ipm)</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Heat input (kJ/in)</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>Filler wire</td>
<td>0.045&quot; Linde 140s</td>
<td></td>
</tr>
<tr>
<td>Shielding gas</td>
<td>Argon + 2% O₂</td>
<td></td>
</tr>
<tr>
<td>Number of passes</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Preheat temperature (°F)</td>
<td>150-175</td>
<td>150-175</td>
</tr>
<tr>
<td>Interpass temperature (°F)</td>
<td>150-175</td>
<td>150-175</td>
</tr>
</tbody>
</table>

¹direct current reverse polarity

4.5 Comparison of Numerical and Experimental Results

The experiments described in the previous section were analyzed using the finite element program ADINAT [76] and the modelling procedures described in Section 4.3. Due to cost considerations and in order to perform a limited parametric analysis, a rather crude mesh was used. Four runs were totally made analyzing the first two weld passes.
Figures 4.8a and 4.8b show the finite element mesh used at a cross section of the plate in its mid-length. Note that Figure 4.8a shows the leftmost 11.5 inches (292.1 mm) of the plate with the rightmost 11.5 inches (292.1 mm) being exactly symmetric, whereas Figure 4.8b shows, in different scale, the middle 1 inch (25.4 mm) that includes the weld metal and the heat affected zone.

A total of 37 isoparametric elements were used divided into two groups of 30 and 7 elements, the second group representing the weld metal. Referring to Figure 4.8b, elements 3, 4, and 5 model the first welding pass, elements 1 and 2 the second, and elements 6 and 7 the third.* It is once more emphasized that during the analysis of each pass the elements representing the next passes remained inactive using the birth-and-death option of the program. Finally, a total of 72 nodes were used.

It should also be mentioned at this point that due to the fact that only the first welding pass is symmetric, the total cross-section, and not half of it, had to be modelled with finite elements.

The heat input was calculated using a consistent formulation as described in Section 4.3.5. It was then distrib-

*The general shape of each welding pass was found by cross-sectioning each specimen after all measurements have been made and etching the surface to reveal the weld pass boundaries.
FIGURE 4.8a Finite Element Mesh Used in Heat Transfer Analysis (rightmost 11.5 in of cross-section)
FIGURE 4.8b Finite Element Mesh Continued (center 1. in).
ted over 5.5 sec, as shown in Figure 4.9 for the case $\eta_a = 0.70$, to simulate the passing of the arc traveling at 12 in/min (0.2 in/sec) over the 1 in thickness of the cross-section.

Table 4.4 summarizes the four analyses performed. Two parameters were varied, the arc efficiency, $\eta_a$, and the temperature variation of the convection coefficient, $h$, given the fact that some uncertainty exists regarding their true values. All other variables were kept constant in the four cases, assuming the values referred to in previous Sections.

Figures 4.10 and 4.11 show results for the first welding pass. In Fig. 4.10 the temperature history at a point on the plate's top surface 0.5 in (12.7 mm) away from the weld centerline as predicted by cases A1 and A2 (refer to Table 4.4) is depicted. With the only difference between the two cases being the arc efficiency (i.e. amount of heat input), the curve for case A1 ($\eta_a = 0.65$) shows, as expected, higher temperatures than case A2 ($\eta_a = 0.60$). The cooling rates are about the same, although both predict too slow cooling if compared with the experimental data. This latter observation means that the values chosen for the heat convection coefficient are probably low.

Cases A2, A3, and A5 are compared with the experimentally obtained results in Figure 4.11 for the same point as the one of the previous figure. Considering the same heat input ($\eta_a = 0.60$) little difference is found between cases A2 and A3. This is due to the fact that although higher values for the
\[ Q = n_a V I = (0.70)(25)(395) = 6.552 \text{ Btu/sec} \]

\[ H = \frac{Q}{v} = \frac{6.552}{0.2} = 32.76 \text{ Btu/in} \]

\[ \text{Area} = 2(0.5)(6.552) + (4.5)(6.552) = 32.76 \text{ Btu/in} \]

FIGURE 4.9 Heat Input Distribution (example).
### TABLE 4.4 Summary of Analyses

<table>
<thead>
<tr>
<th>Analysis</th>
<th>$\eta_a$</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.65</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0.60</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>0.60</td>
<td>2</td>
</tr>
<tr>
<td>A4</td>
<td>0.70</td>
<td>3</td>
</tr>
</tbody>
</table>

**Notes:**
1. $\eta_a$ is the arc efficiency utilized.
2. Refers to the cases presented in Table 4.1.
FIGURE 4.10 Comparison of Finite Element Results with Experimental Data (weld pass no. 1)
FIGURE 4.11 Comparison of Finite Element Results with Experimental Data (weld pass no. 1).

PASS #1
0.5 in from weld line
convection coefficient were chosen in analysis A3, these values were not high enough to significantly alter the heat losses from the top and bottom surfaces of the plate and consequently the temperature distribution. A substantial increase in the convection coefficient was therefore chosen for case A4; at the same time, however, an increase in the arc efficiency was made to partially compensate for the higher h and thus to obtain a good estimate of the maximum temperature reached. As seen in Fig. 4.11 the combination of values used in this latter analysis succeeded in bringing the cooling rate much closer to the experimental one. At the same time a 4% overprediction of the maximum temperature is observed. The difference, however, was very small so that any further analysis was not felt necessary.

Similar results were found at other points of the plate. Case A4 came always closest at matching the experimentally obtained results.

Temperature distributions for the second welding pass are shown in Figures 4.12 through 4.14. Results for three points on the plate's surface located 0.5, 1.0, and 1.5 in (12.7, 25.4, and 38.1 mm) away from the weld centerline as predicted by the four cases analyzed are compared with experimental data. As was the case with the first welding pass, analysis A4 comes again closest at matching the experimentally obtained temperature histories.

The temperature histories calculated from analysis A4 were subsequently used as input to the stress analysis
FIGURE 4.12 Comparison of Finite Element Results with Experimental Data (weld pass no. 2)
FIGURE 4.13 Comparison of Finite Element Results with Experimental Data (pass no. 2).
FIGURE 4.14  Comparison of Finite Element Results with Experimental Data (pass no. 2).
discussed in Chapter 5. It should be noted that when this stress analysis failed to give accurate results using the finite element mesh shown in Figures 4.8a and 4.8b, the heat transfer analysis for the first welding pass was repeated with the finer mesh shown in Figures 5.7a and 5.7b. The same conditions as the ones for case A4 were used. No significant difference was observed between the temperatures calculated on this finer mesh and the ones with the more crude one.

Finally, some thoughts regarding the parametric analyses performed should be expressed. To many this would look like an exercise in curve-fitting, trying to match experiments and analysis. In reality, the problem is much more fundamental and has to do with the uncertainties involved in estimating several parameters required as input to the solution. The many complexities and the nonlinearity of the welding problem combined with the state-of-the-art in arc physics and heat transfer, do not allow at present the accurate calculation of, among others, the heat input to the plates being welded and the temperature dependence of the heat convection coefficient. Therefore, and until further developments in these areas are made, some parametric analyses will be necessary to better estimate the temperature distribution during welding. It is nevertheless hoped that analyses similar to the ones performed in this study would help future investigators in better selecting appropriate values for uncertain parameters.
CHAPTER V

TRANSIENT STRAINS, TRANSIENT STRESSES
AND RESIDUAL STRESSES DUE TO WELDING

Using the temperature distributions predicted based on the techniques described in Chapters II and IV, one can calculate the transient strains, transient stresses and residual stresses due to welding since, under the conditions described in the Introduction, the problem was assumed to be uncoupled. The calculation of strains and stresses, however, poses a much more formidable problem than the one encountered in the heat transfer analysis, making the use of numerical techniques a necessity. These difficulties stem from the complicated thermal-elastic-plastic state developed in and around the weld metal during welding.

Two general techniques have been developed to solve the problem. One is a simple one-dimensional analysis and the other a more sophisticated one based on the finite element method. Both will be considered here, although emphasis will be placed on the latter.

After a brief literature survey, the one-dimensional analysis will be considered and applied to several cases. Based on the obtained results, the advantages and limitations of this method will be discussed.

The finite element method as applied to the welding problem will then be described. Particular emphasis here will be
given to the development of a model, incorporated in the finite element analysis, which can take into account the effects on strains and stresses produced by the allotropic phase transformations. These effects can be quite pronounced, as it will be seen, when considering quenched and tempered steels, like, for example, the HY-130 steel.

Finally, the results obtained by the finite element analysis will be compared with the experiments described in the previous Chapter.

5.1 Literature Survey

Two review articles, by Tall [95] and Masubuchi [82], provide the general picture as far as developments in the area of calculation of residual stresses is concerned. In this Section, part of their discussions will be borrowed, although much of what follows is relatively new.

The one-dimensional model for calculating stresses parallel to the weld line only was first developed in 1964 by Tall [53]. Using this model, Masubuchi, Simons, and Monroe [96] developed a computer program which was later modified and improved both at M.I.T. and elsewhere [26,97,98].

The complexities involved, however, in performing simple analytical studies of welding stresses and distortions for various weld configurations -- in particular, the complex effects of inelastic material response and material loading and unloading -- did not allow for the further development of similar studies. Instead, attention was focused on numerical
methods, predominantly the finite element method, which could
be applied to the highly nonlinear, inelastic behavior of
welded structures.

Fundamental aspects of the finite element method are dis-
cussed in many books and articles [89, 99]. Nonlinear thermo-
elastic-plastic procedures are described by Bathe and co-wor-
kers [100, 103], Inoue and Tanaka [104] and others.

One of the first applications of the FEM to weld problems
was presented by Hibbitt and Marcal [60], who considered a
thermomechanical model for the welding and subsequent loading
of a fabricated structure. Their model simulates GMA welding
processes and accounts for temperature dependent material pro-
perties. This model has been subsequently used by other in-
vestigators, including Nickell and Hibbitt [79], Hsu [78],
Lobitz and co-workers [105], and others. Friedman [62, 85]
also developed finite element analysis procedures for calcu-
lating stresses and distortions in longitudinal butt welds.
These procedures are applicable to planar or axisymmetric
welds. Rybicki and co-workers [106-108] have developed simi-
lar procedures. At the Massachusetts Institute of Technology
a team headed by Masubuchi has also developed two-dimensional
finite element programs capable of performing plane strain
and plane stress analyses [80-83].

Japanese investigators have also been very active in the
field. Ueda and co-workers [109-113], Satoh et al. [114],
and more recently Fujita and Nomoto [115-117] have all
developed models to calculate transient strains and residual stresses due to welding based primarily on the initial strain method.

Efforts to include the effects of phase transformation on transient strains and residual stresses have been undertaken by some investigators, including Inoue and Tanaka [118], Toshioka [119-120], Jones and Alberry [121], and Andersson [87]. By and large their approach consists of dividing the area where phase transformations are expected (the weld metal and HAZ) into a number of regions, typically three, and assuming that the material exhibits a constant cooling rate in each of these regions. Based on that, they estimate one dilatational curve for each region and use it as the thermal strain curve for that region. A more fundamental approach has been developed in this study and will be extensively discussed in Section 5.4.

As a general assessment of the models developed so far, it can be said that they give reasonable results for the cases reported as compared to experiments, although the deviation from experimental data varies considerably.

5.2 One-Dimensional Analysis

The basic assumption inherent in the one-dimensional stress analysis is that the only stress present is the one parallel to the weld line, $\sigma_x$, and that this stress is a function of the transverse distance from the weld centerline only. As a consequence, the equilibrium conditions are not
satisfied. Despite this, however, it appears that the obtained solutions correlate reasonably well with experimental data in certain cases, as will be further explored.

The analysis lends basically the procedure originally proposed by Tall [53]; the computer code implementing the solution with the latest modifications can be found in [26].

The algorithm for solving the problem is based on the method of successive elastic solutions as proposed by Mendelson [122]. The program can take into account the temperature dependence of all material properties, any type of strain hardening and can solve bead-on-plate, edge, and butt welds of flat plates with finite width. One of the input requirements, the temperature distribution, can also be calculated if desired by the same program using the line heat source solution. The output at each time step consists of the temperature, total strain, mechanical strain, plastic strain, and stress at each of the predetermined points located at various transverse distances from the weld centerline.

Previous applications of the program have shown that it gives good results in the case of thin plates. This happens because in thin plates all stresses, except $\sigma_x$, are very small, sometimes an order of magnitude smaller than $\sigma_x$.

In the present study an analysis was undertaken of the experiments performed by Coneybear [90] on HY-130 specimens welded using the Electron Beam (EB) welding process, as well as of the GMA welded specimens described in Section 4.4. The
EB specimens had the same geometric configuration as the GMA welded ones, the only difference being the joint shape which was square butt in the case of EB welding.

Figure 5.1 shows the comparison between experiment and analysis of the longitudinal transient strain history, \( \varepsilon_X \), in the EB specimen at two points 0.67 and 1 inch (17 and 25.4 mm) away from the weld centerline. Despite the thickness of the specimens (1 inch or 25.4 mm) the results show a remarkably good correlation of analysis and experiment. This can be attributed to the physics of the EB welding process which result in a relatively uniform through thickness temperature distribution and a narrow heat affected zone. As a consequence, stresses \( \sigma_Y \), \( \tau_{XY} \), and \( \sigma_Z \) are small compared to \( \sigma_X \), a fact substantiated by the experimental data (see Coneybear [90]). The assumption of the one-dimensional theory that \( \sigma_X \) is the only stress present is, therefore, approximately valid. It should be emphasized once more, however, that these results apply for the midlength of a relatively long plate (longer than 18 inches or 457.2 mm), where the maximum possible stresses have been developed and where the end effects can be neglected.

The comparison between experimental data and analytical results for the longitudinal transient strain history in the GMA welded specimens at two points 1.0 and 2.25 inches (25.4 and 57.2 mm) away from the weld centerline is shown in Figure 5.2. Here the story is completely different. The correlation
FIGURE 5.1 Longitudinal Transient Strain Histories for EB Welded Specimens (Experiment and Analysis)

FIGURE 5.2 Longitudinal Transient Strain Histories for GMA Welded Specimens (Experiment and Analysis)
is not good, especially for the closest point; at around 45 sec from the commencement of welding, for example, the experimental data show a maximum positive longitudinal strain of about 0.9, whereas the analysis predicts a negative longitudinal strain 0.4 in magnitude. It is thus apparent that the assumptions involved in the one-dimensional analysis break down in the case of multipass GMA welds of thick plates. Experimental data actually confirm this by showing that the transverse strains are of the same order of magnitude as the longitudinal ones.

In conclusion, the one-dimensional program seems to be appropriate for analyzing thin plates or medium thickness plates welded by one of the high energy density processes (Electron Beam or Laser Beam). For the case of multipass welds the basic assumptions of the method do not hold and hence a more sophisticated technique, like the finite element method, seems to be necessary.

5.3 Finite Element Formulation

The thermal-elastic-plastic and creep constitutive model (with the phase transformation modification discussed in the next Section) of the multipurpose finite element program ADINA (Automatic Dynamic Incremental Nonlinear Analysis) will be used for predicting the transient strains, transient stresses, and residual stresses due to welding. The model was developed by Snyder and Bathe [103] in 1980. Some features of this model will be described in this Section
for completeness.

The governing incremental finite element equations for the problem can be written as [100],

$$
\tau_K \Delta u^{(i)} = t + \Delta t_R - t + \Delta t_F^{(i-1)}
$$

(5.1)

where $\tau_K$ is the tangent stiffness matrix corresponding to time $t$; $t + \Delta t_R$ is the nodal-point external force vector at time $t + \Delta t$; $t + \Delta t_F^{(i-1)}$ is a vector of nodal-point forces that are equivalent, in the virtual work sense, to the internal element stresses at time $t + \Delta t$ and iteration $i - 1$.

$$
t + \Delta t_F^{(i-1)} = \int_V B_L^T \tau t + \Delta t_G^{(i-1)} dv
$$

(5.2)

and $\Delta u^{(i)}$ is the increment in nodal-point displacement in iteration $i$.

$$
t + \Delta t_u^{(i)} = t + \Delta t_u^{(i-1)} + \Delta u^{(i)}
$$

(5.3)

The solution using eq. (5.1) corresponds to the modified Newton-Raphson iteration procedure which is helpful in improving the solution accuracy and in many cases in preventing the development of numerical instabilities. The convergence of the iteration can be accelerated using the Aitken method or, in complex material nonlinear cases (like the welding problem), improved using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) matrix updating method [102].

In the thermo-elastic-plastic and creep model, and
assuming infinitesimal strains, the total strain at time $\tau$, $\tau_{e_{ij}}$, is assumed to be given by

$$
\tau_{e_{ij}} = \tau_{e_{ij}}^E + \tau_{e_{ij}}^P + \tau_{e_{ij}}^C + \tau_{e_{ij}}^{TH}
$$

(5.4)

where $\tau_{e_{ij}}^E$ = elastic strain
$\tau_{e_{ij}}^P$ = plastic strain
$\tau_{e_{ij}}^C$ = creep strain
$\tau_{e_{ij}}^{TH}$ = thermal strain

so that at any time $\tau$ during the response the stress is given by the constitutive law for an isotropic thermo-elastic material

$$
\tau_{\sigma_{ij}} = \tau_{C_{ijrs}} (\tau_{e_{rs}}^E - \tau_{e_{rs}}^P - \tau_{e_{rs}}^C - \tau_{e_{rs}}^{TH})
$$

(5.5)

with $\tau_{C_{ijrs}}$ denoting a component of the elastic constitutive tensor.

The thermal strains are

$$
\tau_{e_{rs}}^{TH} = \tau_{\alpha_m} (\tau_{\theta - \theta_R}) \delta_{rs}
$$

(5.6)

where $\tau_{\alpha_m}$ is the average thermal expansion coefficient, $\theta_R$ is the reference temperature, and $\delta_{rs}$ the Kronecker delta. It is this term of eqn. (5.5) that will be modified in the next section to take into account the effects of phase transformation.
The creep strains, $\tau^{C}_{rs}$, are determined using the approach proposed by researchers at the Oak Ridge National Laboratory [56, 57]. Given that in the welding problem the time intervals at high temperatures are short, however, creep will not be considered in this study.

For the plastic strains, $\tau^{P}_{rs}$, the situation is more complicated. Although the classical theory of isothermal plasticity is a well tested one [122,123], extension of the theory to non-isothermal cases is difficult to experimentally substantiate. Several investigators have proposed modifications — Prager [124], Ziegler [125,126], Naghdi [127], Cyr and Teter [128], Sharifi and Yates [129], Chang [130] — but very few experiments have been performed. Corum et al. [57] and Phillips [55] have reported relatively good agreement between theory and experiments, but for temperatures up to about $1000^\circ F$ only. During welding, though, the temperatures rise to above the $A_1$ temperature ($1210^\circ F$ for steel) inside the HAZ-base metal boundary and above the liquidus temperature in the weld metal. For lack of any alternative, however, the same non-isothermal theory of plasticity will be used throughout the temperature range encountered in welding problems.

The general form of the yield or loading function for multiaxial stress conditions is

$$\tau_F = \tau_F(\tau_{\sigma_{ij}}, \tau_{\alpha_{ij}}, \tau_{\sigma_Y})$$  \hspace{1cm} (5.7)

where $\tau_{\alpha_{ij}}$ and $\tau_{\sigma_Y}$ are functions of the history of plastic
deformation and temperature. For elastic behavior, $\tau_F < 0$, and for plastic behavior $\tau_F = 0$. As a consequence of Drucker's postulate for stable plastic materials, $\tau_F$ defines a convex surface in the stress-temperature space. It is also assumed that the isothermal normality condition remains valid, so that

$$\tau_{eP}^{rs} = \tau_\lambda \left( \frac{\partial \tau_F}{\partial \sigma_{ij}} \right)$$

(5.8)

where $\tau_\lambda$ is a positive scalar. The selection of a hardening rule is also required for the calculation of $\tau_\lambda$. In ADINA either isotropic or kinematic hardening can be assumed. Because cyclic plasticity is expected in the welding problem, the kinematic hardening mechanism, thought by many to better model the phenomena involved, was chosen. The assumptions involved in this mechanism are that the size of the yield surface depends on the temperature only, whereas the translation rate of the yield surface in the stress space depends on the plastic strain rate.

A very important aspect of the analysis is the accurate integration of stresses [103] at time $t+\Delta t$ and iteration $i$

$$t+\Delta t_{\sigma}(i) = t+\Delta t_{CE}(i) + t+\Delta t_P(i) - t+\Delta t_{TH}(i)$$

(5.9)

where

$$t+\Delta t_{eP}(i) = t_{eP} + \int_{t}^{t+\Delta t(i)} d \tau_{eP}$$

(5.10)
and where it was assumed that creep strains are zero. The integration of eqn. (5.10) is performed using the α-method where α = 0 and 1 correspond to the Euler forward (explicit) and Euler backward (implicit) methods, respectively. The implicit scheme can display considerably better stability characteristics but is more costly, requiring equilibrium iterations. Furthermore, the accuracy of the integration is enhanced with the use of subincrements in each time step, an important feature considering the rapidity with which some of the solution variables and material parameters vary with time during welding.

5.4 Phase Transformation Effects

In most previous stress analyses of the welding problem no consideration was given to the phenomenon of allotropie phase transformations. At the same time in cases where poor agreement was observed between experimental data and analytical results, the discrepancy was attributed exactly to the fact that phase transformations were not taken into account [6].

The limited number of efforts made by investigators to include the phase transformation effects have been outlined in Section 5.1. All of these efforts, however, were based on using dilatational curves and did not concentrate on the fundamentals of the phenomena involved. In this Section such an effort towards a more fundamental approach will be outlined.

The model developed starts with the prediction of the
history of microstructure formation during the cooling stages of the welding cycle using the CCT diagram. It should be understood that this is also an approximation of reality although more rational than the previously used ones. An even better approach would have made use of the rate equations for transformation kinetics. Such equations have not yet been adequately developed, however, for complex alloys, like the high strength quenched and tempered HY-130 steel.

Using the predicted microstructure history and the transformation strains for each allotropic phase change as well as the average thermal expansion coefficients appropriate for each phase, the combined transformation and thermal strains are then estimated. Finally, these calculated strains are incorporated into the finite element program ADINA replacing the conventional thermal strains, $\tau_{ij}^{TH}$.

Details of the model are described in the following subsections.

5.4.1 Microstructure Prediction

The usefulness of the CCT diagrams for predicting the microstructures during cooling from temperatures higher than the $A_l$ temperature has been discussed in Section 3.2.2. At the same time, methods for calculating schematic CCT diagrams from isothermal data have been outlined and applied in the case of HY-130 steel in Section 3.4.

An idealization of the CCT diagram is shown in Figure 5.3. The basic assumption involved is that the bainite start,
B_s', and bainite finish, B_f', transformation temperatures are constant, that is independent of the cooling rate, the maximum temperature (above the A_1 temperature) reached before cooling commences, and the time held at this maximum temperature. The last two assumptions require that full austenitization has taken place before, upon cooling, the A_1 temperature is reached and is thought to be reasonable for the welding case.

As far as the constant B_s temperature is concerned, a closer look of Figure 3.14 reveals that the approximation for the case of HY-130 steel is not unrealistic. Although some difference in the B_s temperatures is observed for different cooling rates, the range is not large. Moreover, the cooling rates observed in the weld metal and HAZ of a weld are relatively high, thus diminishing the range of possible B_s temperatures towards the lower size. In the present study the B_s temperature was taken equal to 840°F (449°C).

Furthermore, Figure 5.3 shows the B_f temperature to be in general different than the martensite start, M_s', one. For the case of HY-130, however, the two temperatures can be assumed to be equal. Finally, the assumption of constant M_s and M_f temperature is, as discussed in Section 3.4, a completely acceptable one, owing to the athermal nature of the transformation. Values of M_s and M_f were taken to be 715°C and 415°F (379°C and 213°C) respectively.

To predict the microstructures during the cooling stage of the welding process an incremental strategy is involved so
that the model can be compatible with the step-by-step solution of the nonlinear stress analysis using the finite element program ADINA. A procedure has therefore to be established that will enable one to calculate the proceeding of each transformation. The following equation, proposed by Toshioka [119], has been used for this purpose:

\[ f = f_F \cdot \left[ 1 - \left( \frac{\theta - \theta_f}{\theta_s - \theta_f} \right)^2 \right] \]  \hspace{1cm} (5.11)

where \( f \) is the fraction of the product transformed at temperature \( \theta \), \( f_F \) the fraction of the product to be transformed at the transformation finish temperature, \( \theta_f \), and \( \theta_s \) the start transformation temperature. Equation (5.11) is applied separately for each allotropic transformation, namely for the martensite to austenite one upon heating and for the austenite to bainite and austenite to martensite ones upon cooling.

For the calculation of \( f_F \) the following guidelines have been applied. Referring to Figure 5.4, the martensite to austenite transformation during the heating stage of the welding cycle takes place progressively within the temperature range bounded by the \( A_1 \) and \( A_3 \) temperatures. Since full austenitization is assumed to occur as soon as the \( A_3 \) temperature is exceeded, \( f_F \) for this transformation has been taken to be equal to unity regardless of the heating rate.

The situation is somewhat different in the case of the austenite to bainite phase transformation, which takes place
FIGURE 5.4 Typical Temperature History Inside the HAZ of a Weld.
between the $B_s$ and $M_s$ temperatures. A major parameter is the cooling rate. If it exceeds a critical value then, as shown in Figure 5.3, no bainite is formed. Alternatively, if for a particular cooling curve the time elapsed between the instance the curve passes the $A_1$ temperature and the instance it passes the $B_s$ temperature is less than the precipitation start time for bainite, $t_{B_s}^*$, the bainite reaction will not take place. This latter criterion was the one used in this study. Furthermore, if the bainite reaction does occur, its extent will depend again on the cooling rate, with slower cooling rates resulting in the final formation of larger fractions of bainite, $f_F$. The occurrence of this phenomenon has been modeled by calculating $f_F$ using

$$f_F = (\Delta t)_B \cdot (\log_{10} t_{B_s}^* - t_{B_s})$$

(5.12)

where $(\Delta t)_B$ is the time interval over which the bainite reaction can take place (see Fig. 5.3), $t_{B_s}^*$ is the time elapsed between the instances the cooling curve passes $A_1$ and $B_s$ temperatures, and $t_{B_s}$ has been previously defined. Note that $f_F$ has to be smaller or equal to 1.

For the case of the martensite reaction things are simpler. Since this is an athermal transformation, it is not dependent on the cooling rate. As a consequence, $f_F$ is equal to the remaining austenite fraction (the one that has not been already transformed to bainite). For the case in which no bainite reaction has taken place (cooling rate higher than the
critical one), it is obvious that $f_p$ is equal to unity.

Finally, it should be mentioned that in the previous discussion it was assumed that full austenitization has taken place, i.e. that the maximum temperature reached was higher than the $A_3$ temperature. If, on the other hand, the maximum temperature lied between $A_1$ and $A_3$, in which case only partial austenitization took place, all equations giving $f_p$ will have to be multiplied by the fraction of austenite formed as given by eqn. (5.11).

5.4.2 Calculation of Transformation and Thermal Strains

Based on the microstructure predictions calculated by the methodology outlined in the previous subsection, the transformation and thermal strains developed during the welding cycle can be calculated. The importance of including these transformation strains has been recognized by several authors, including Linnert [131], especially for thick weldments. This importance stems from the expansion associated with allotropic phase changes, as opposed to the contraction due to cooling.

To qualitatively show the phenomena involved, the cooling of a thick section in air will be considered. The surface of the section cools quicker than the interior, thus reaching the transformation start temperature (for the bainite or martensite reaction) quicker. As a consequence the surface expands. To accommodate this surface expansion the metal in the center region, which is still austenitic and much hotter,
is pulled towards the surface causing the ends of the section to be forced inwards and thus resulting in a concave shape. As the cooling proceeds, the center also reaches the transformation start temperature and starts expanding. The surface layers must, therefore, also expand. At this stage, however, the surface is contracting due to thermal strains, so that the surface expansion can take place by plastic flow only. After the center has gone through the transformation, it again starts contracting but at a higher rate than the surface (owing to the higher temperature of the former). This contraction difference tends to pull the surface inwards. For self-equilibrating purposes this tensile pull of the center must be balanced by the compressive strength of the surface. At room temperature the section thus ends up with compressive residual stresses at the surface and tensile ones in the center.

To calculate these expansions and contractions, the combined thermal and transformation strain, $e_{rs}^{TH}$, can be written as

$$e_{rs}^{TH} = (e_1 + e_2 + e_3) \cdot \delta_{rs} \quad (5.13)$$

where $e_1$ is the thermal strain in a mixture of phases, $e_2$ the transformation strain, $e_3$ the thermal strain in a single phase, and $\delta_{rs}$ the Kronecker delta.

The estimation of the thermal strains $e_1$ and $e_3$ requires knowledge of the microstructures present and of the average thermal expansion coefficients of austenite, $\alpha_A$, bainite, $\alpha_B$, etc.
and martensite, $a_M$. Similarly, to calculate the transformation strain, $e_2$, one has to also know the transformation strains for each separate phase change, namely from martensite to austenite, $e^{TR}_{M\rightarrow A}$, from austenite to bainite, $e^{TR}_{A\rightarrow B}$, and from austenite to martensite, $e^{TR}_{A\rightarrow M}$. Table 5.1 shows the values used in this study, taken from a variety of sources [119,131].

The equations used for calculating each term in eqn. (5.13) depend on the temperature history the considered location has undergone. Appendix D gives in detail these equations for all possible cases encountered during the welding cycle.

To show the impact the inclusion of phase transformation effects has on the thermal strain of a section which is free to expand or contract, Figure 5.5 is constructed. The solid line depicts for a specific temperature history the thermal strain during the heating stage of this history, including the phase transformation effects (phase change from martensite to austenite). This is the usual curve assumed as input in previous stress analyses using the finite element method. Upon cooling, the same curve was usually assumed, thus neglecting the phase changes from austenite to bainite and/or martensite. By contrast, the intermittent line shows clearly the effect the expansion accompanying these transformations has on the thermal strain.

It becomes evident from the previous discussion that the model developed captures the main characteristics of the phase transformation effects. The accuracy of the analysis will, of
FIGURE 5.5 Typical Dialatational Curve Including Phase Transformation Effects.
TABLE 5.1 Values Used for the Calculation of Transformation Strains

**Transformation temperatures:**

<table>
<thead>
<tr>
<th></th>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>1210</td>
<td>654</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1415</td>
<td>768</td>
</tr>
<tr>
<td>$B_s$</td>
<td>840</td>
<td>449</td>
</tr>
<tr>
<td>$M_s$</td>
<td>715</td>
<td>379</td>
</tr>
<tr>
<td>$M_f$</td>
<td>415</td>
<td>213</td>
</tr>
</tbody>
</table>

**Thermal expansion coefficients:**

<table>
<thead>
<tr>
<th></th>
<th>in/in. °F</th>
<th>mm/mm. °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_A$</td>
<td>$8.9 \times 10^{-6}$</td>
<td>$4.9 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_M$</td>
<td>$7.5 \times 10^{-6}$</td>
<td>$4.2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$\alpha_B$</td>
<td>$6.7 \times 10^{-6}$</td>
<td>$3.7 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

**Transformation strains:**

<table>
<thead>
<tr>
<th></th>
<th>$e_{TR}^{M \to A}$</th>
<th>$e_{TR}^{A \to B}$</th>
<th>$e_{TR}^{A \to M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.10 \times 10^{-2}$</td>
<td>$0.25 \times 10^{-2}$</td>
<td>$0.38 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
course, depend on the correct evaluation of the required input.

5.5 **Weld Model Using the FEM**

In this section all the necessary considerations for the modelling of a multipass weldment between two relatively thick plates will be discussed. They include the geometry and boundary conditions, the mechanical properties of the material, and some thoughts on the solution strategy.

5.5.1 **Geometry - Boundary Conditions**

A cross-section of the weldment in its midlength was used to calculate the transient strains, transient stresses, and residual stresses due to welding. This is rationalized by the fact that for relatively long plates the maximum stresses are developed in this region.

Furthermore, the plane strain assumption was used. This implies that all sections normal to the weld line remain plane during the entire welding process. Though this assumption is probably adequate in regions somewhat removed from the weld puddle, it may not be valid in the neighborhood of the molten metal. The approach employed therefore prevents calculation of distortions ahead of the welding arc, and may inhibit accurate computations of deformations in the immediate vicinity of the weld puddle. Because of the various other assumptions involved in the material modelling of the weld metal (as outlined in Section 5.5.2), however, the plane strain conditions should be adequate. Furthermore, the method enables one to determine
complete transient strain, transient stress, and residual stress distributions, transverse distortions, and estimates of the damage (permanent plastic strains) accumulated during the welding process.

To be able to characterize the longitudinal bending distortion as well one can utilize three-dimensional analysis. It can be noted, however, that by introducing three additional degrees of freedom per node, in other words by resorting to the generalized plastic strain approach, one could conceivably obtain similar results. Such an approach would let the longitudinal strain, $\varepsilon_{xx}$, be a linear function of the longitudinal extension and the two rotations about the y and z axes of Figure 4.1. It is recommended that an effort towards this direction be undertaken by future investigators.

The boundary conditions used in the analysis should be such so as to allow free expansion of the weldment in the transverse direction as well as bending. At the same time the structure should be properly restrained to eliminate all possible modes of rigid body motion; otherwise the stiffness matrix will not be positive definite. The exact boundary conditions used will be discussed when the finite element mesh used is introduced.

5.5.2 Material Properties

The temperature dependence of the Young's modulus, $E$, Poisson's ratio, $\nu$, virgin yield stress, $\sigma_{vy}$, and strain hardening modulus, $E_T$, are all required input for the thermo-
elastic-plastic and creep material model incorporated in ADINA. This dependence has been explained and evaluated in Section 3.3.1 for the case of HY-130 and the values obtained there were used in this analysis. The temperature dependence of the average thermal expansion coefficient, \( \alpha \), is also a required input, but it has been overridden in this study by the phase transformation model discussed in the previous Section.

A word of caution is appropriate at this point as far as the material properties above the liquidus temperature is concerned. The material does not have any strength when molten since all its mechanical properties are zero. But due to numerical considerations, zero properties cannot be entered as input to the program. Hence to avoid any instabilities (or even divisions by zero) very small values for \( E \), \( \sigma_{vy} \), and \( E_t \) should be used above the liquidus temperature.

Another point is the accumulation of plastic strains in the regions that become molten during the welding cycle. When the temperature reaches the liquidus these plastic strains are physically relieved, starting to accumulate again when the metal solidifies. The presence of nonzero material properties above the liquidus, however, would cause the plastic strains not only to continue accumulating but also to reach artificially high values owing to the very low magnitude of the mechanical properties. It was therefore necessary to modify ADINA by imposing a total relief of plastic strains when the material melts.
5.5.3 Solution Strategy

One of the most important decisions an analyst has to make when performing a nonlinear incremental stress analysis is the solution strategy to be followed, because the accuracy and the convergence characteristics of the solution depend very much on it. This is especially true for complex situations involving highly nonlinear material behavior like the one encountered in the welding problem. Bathe and Cimento [102] provide some guidelines to be followed in such cases. Based on this article and on experience gained in making simpler nonlinear analyses, a discussion will be made on the solution strategy used in analyzing the welding problem.

As mentioned in Section 5.3 the solution of the incremental equations can be made using either the modified Newton-Raphson iterative technique (with or without the Aitken acceleration) or the BFGS matrix updating method. Both were tried for the initial steps of the analysis (the first 100 steps). The Newton-Raphson technique did not exhibit, however, good behavior signaling on several occasions increase in the out-of-balance loads owing to the sudden stiffening of the structure during elastic unloading from the plastic stage at several integration points. On the contrary, using BFGS updates no such behavior was observed. This last method was thus used for the entire analysis.

As far as the displacement and force convergence tolerances are concerned, the default value of $\varepsilon_p = 0.1$ was used
for the latter, whereas one hundredth of the default value, namely \( \varepsilon_D = 1 \times 10^{-5} \), was used for the former because of the softening involved in the problem (plasticity). Iterations and stiffness reformation were required at each time step because of the complex material behavior.

Table 5.2 provides a summary of the solution strategy. Also shown are the parameters used in integrating the stresses at each time step.

**TABLE 5.2 Solution Parameters Used with ADINA**

<table>
<thead>
<tr>
<th>Equilibrium Iteration Parameters</th>
<th>Stress Integration Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>METH\text{\textO}D = 2</td>
<td>ALPHA = 0.0</td>
</tr>
<tr>
<td>IATKEN = 0</td>
<td>XISUBM = 150.</td>
</tr>
<tr>
<td>DT\text{\textO}L = 0.00001</td>
<td>XNITE = 15.</td>
</tr>
<tr>
<td>RT\text{\textO}L = 0.1</td>
<td>XNALG = 2.</td>
</tr>
<tr>
<td>ST\text{\textO}L = 0.5</td>
<td>T\text{\textO}LIL = 0.005</td>
</tr>
<tr>
<td></td>
<td>T\text{\textO}LPC = 0.1</td>
</tr>
</tbody>
</table>

**Note:** The above symbols refer to the ones used for the variables in ADINA [100].

5.6 Results

The same finite element mesh as the one used for the heat transfer analysis and shown in Figures 4.8a and 4.8b was initially utilized to analyze the strains and stresses developed during the welding experiments described in Section 4.4. Figure 5.6 shows the constraints imposed; nodes 6, 68, and 70 are
FIGURE 5.6 Constraints Applied to the Finite Element Model.
not allowed to move in the z-direction, whereas node 4 is not allowed to move at all. It is thus possible for the weld to freely expand in the y-direction and also to bend transversly.

This case was analyzed for three seconds before and three seconds after the arc center passed over the middle of the cross-section. The obtained results were not satisfactory, however, owing to the crudeness of the mesh around the weld metal. Based on these facts and considering the long computer time and large cost required for the analysis, it was decided to analyze only the first welding pass (of the twelve involved) using a finer mesh to more accurately capture the rapid variation of the problem's variables close to the weld centerline, while at the same time taking advantage of the geometric symmetry of the first welding pass. Figures 5.7a and 5.7b show the finer mesh used in the right half part of the cross-section. In the first figure the rightmost 11.5 in are shown, whereas in the second one the 0.5 in to the right of the weld centerline are depicted in different scale.

Four- to six-node isoparametric elements were used; special care was given not to include any triangular elements in the mesh. A total of 77 nodes and 47 elements were utilized. Nodes 2 and 4 were restrained in the z-direction, and nodes 74 through 77 in the y-direction.

Figure 5.8 shows the accumulation of the effective plastic strain, $\varepsilon^p$, at three points on the midthicknes of the plate up to four seconds after the center of the arc has passed
FIGURE 5.7a  Finer Finite Element Mesh Used in Stress Analysis of Welding Problem.
FIGURE 5.7b  Finer Finite Element Mesh, cont. (0.5 in to the right of weld centerline).
FIGURE 5.8

Plate's midthickness

Distance from weld line
- 0.052 in
- 0.114 in
- 0.424 in
the middle of the cross section. The rapid accumulation of $e^P$ at the two points closest to the weld centerline is evident. Also shown is the relaxation of these strains as soon as melting occurs. On the other hand, for the point located 0.424 in (10.8 mm) away from the welding centerline, the accumulation is much slower. This is the pattern observed for the whole HAZ region.

Figure 5.9 depicts the longitudinal transient stress, $\sigma_x$, history at the same points and for the same time interval. These stresses are compressive as would have been expected since at the instances shown the material expansion is hindered by the relatively colder bulk of the metal located outside the HAZ. For the two points closest to the weld centerline these stresses become negligibly small as soon as melting occurs.

The transverse transient strain, $e_{yy}$, history for two points on the top surface of the plate are compared with experimental measurements in Figure 5.10. Good correlation is observed.

A more complete history of the accumulated plastic strain, the longitudinal stress and the transverse strain is shown in the next three figures. Plots of accumulated effective plastic strain versus time at various locations on the plate's mid-thickness are shown in Figure 5.11. In the weld metal zone, plastic strain is built up rapidly as the material yields in compression. At the melting region this plastic strain is completely relieved, to rapidly reach again a level in close pro-
FIGURE 5.9 Transient Stress History at Various Distances from Weld Centerline (initial 5 sec).
FIGURE 5.10 Comparison of Transverse Transient Strain Histories - Experimental Data vs. Analytical Predictions (initial 4 sec).
FIGURE 5.11 Accumulated Effective Plastic Strain History at Various Distances from the Weld Center-line.
ximity to that existing prior to melting upon solidification. As cooling continues and yielding in tension progresses, the plastic strain steadily increases but at a smaller rate than before. In the heat-affected zone, plastic strain is accumulated during heat-up in much the same way as in the weld metal zone but not quite as rapidly. As the material cools, it yields in tension and further plastic strain is accumulated.

Figure 5.12 shows calculated transient longitudinal stress distributions. Compressive stresses exist in the weld metal prior to melting. When the metal is in its molten stage, negligible compressive or tensile stresses were calculated.* As cooling commences, tensile stresses start appearing in the weld metal. These stresses then build up to the residual stress pattern when ambient temperature is reached. For self-equilibrating purposes, compressive stresses exist in areas removed from the weld centerline. Note in Figure 5.12 the effect of phase transformation, causing a sudden decrease in the tensile stresses.

Comparison of the experimentally measured transverse transient strain history at a point located on the top surface of the plate is made with the numerically obtained results in Figure 5.13. The correlation is relatively good if one takes

*Note that in reality the molten metal has no strength. For numerical reasons, however, a very small amount of strength had to be assumed as explained in previous Sections.
FIGURE 5.12 Longitudinal Stress Distribution at Several Time Instances.
FIGURE 5.13 Comparison of Experimentally Measured Transverse Transient Strains on the Plate's Top Surface with Numerical Predictions.
into account the various assumptions involved in modeling the complex welding problem. A delay in the transition from tensile to compressive and again from compressive to tensile strains is observed in the analysis. The same delay as far as the occurrence of the maximum strain is also exhibited. It is believed that this phenomenon is primarily due to the relative coarseness of the finite element mesh, and the complex loading history present in the welding problem.

Summarizing it can be said that the finite element model developed based on ADINA modified for phase transformation effects captures most of the important aspects of the welding stress analysis. Although some discrepancies have been observed between the obtained results and experimental data, considerations of cost and the assumptions made do not allow much more sophisticated analyses at present. It is expected, however, that the dramatically decreasing computer costs, together with developments in the areas of coupled thermoplasticity, will enable investigators to perform more accurate analyses in the future.
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

A closed-form analytical solution capable of predicting temperature distributions in a finite thickness plate caused by a moving three-dimensional finite heat source on its top surface has been developed in this study. The linear temperature dependence of the material's thermal conductivity and heat losses from the top and bottom surfaces of the plate have been taken into account in the analysis. A parametric investigation of the effects the magnitude of the heat input, the concentration of the heat source and the heat losses have on the results obtained by using this model was undertaken. Comparison of the model's predictions with experimental results show good correlation in the cases of thin-plate welding and bead-on-plate welding.

The classical point heat source solution was also modified to enable it to predict temperatures during multipass welding. The obtained results showed good agreement with experimental data.

Since the high strength quenched and tempered HY-130 steel was the material used in this investigation, a thorough examination of its transformation kinetics and of the temperature dependence of its mechanical and physical properties was made. In the absence of available data on the continuous cooling transformation kinetics, existing methods for estimating
them on the basis of isothermal data were modified and applied in the case of the HY-130 steel. The modifications involved the use of log-linear cooling curves instead of the linear ones presently used.

Plate specimens made of HY-130 steel and measuring 24 in x 12 in x 1 in were welded using the multipass GMA process. Temperatures and transient strains were measured during each welding pass. The collected data were compared with numerical predictions as discussed in the following paragraphs.

Using the multipurpose nonlinear finite element program ADINAT, the heat transfer of the above mentioned experiments was modelled. The temperature variation of the material properties, nonlinear convection and radiation heat losses, and the appearance of each welding pass through the birth-and-death option, all features of ADINAT, were taken into consideration. Four different analyses were performed to better establish the magnitude of the heat input and the temperature dependence of the convection coefficient. Very good agreement of the results obtained was observed upon comparison with experimental data.

Finally, a stress analysis of the welding problem was made using the thermo-elastic-plastic and creep material model of the nonlinear finite element program ADINA. This model was further developed by including the effects of phase transformation, being of particular importance when welding quenched and tempered steels such as the HY-130 one. These effects were calculated by first evaluating the microstructure history ba-
sed on an idealization of the CCT diagram previously derived; then the dilatational curve for the cooling curve of each integration point in the finite element mesh was found using data for each of the phases present at any time during the welding cycle. These dilatational curves were then used to replace the conventional thermal strains of the program's model.

Comparison of the obtained results with experimental data were good confirming the importance of phase transformation effects.

6.2 Recommendations for Future Research

Based on the findings and observations made in this study the following topics for future research can be recommended:

1. One of the basic assumptions underlying all efforts towards analyzing the temperature and stress fields during welding is the uncoupling of the thermal and mechanical problems. This assumption may be good in most cases but there is always the question regarding its validity in and around the molten pool. The development of a coupled thermo-plasticity theory, currently in its initial stages, should therefore be further pursued so that a better understanding of the phenomena involved and consequently a test of the uncoupling postulate can be accomplished.

2. In the heat transfer analysis using numerical techniques, such as the finite element method, the assumption is made that the heat transfer mechanism is pure conduction even when the weld metal is in a molten stage. The combined conduction
and convection mechanisms that are actually present in the li-
quid metal are treated by an apparent increase in the value of
the material thermal conductivity. Although this method has in
general yielded satisfactory results, it is recommended that a
more fundamental modelling of the phenomena be undertaken by
including the convection heat transfer mechanism in the analy-
sis as well. It is of course understood that such an objective
will be a long range one because of the complexities and uncer-
tainties present inside the weld fusion zone.

3. An initial effort towards including the effects of pha-
se transformation in the stress analysis of a welded structure
has been undertaken in this study. The model developed made use
of the CCT diagrams to predict the microstructure history in
the weld metal and the HAZ. A more fundamental approach would
have made use of the rate equations of transformation kinetics.
Such equations, however, are not available in general; the ones
already developed pertain to very simple alloys, usually steels
of the eutectoid composition. It is therefore recommended that
research in the direction of developing rate equations for more
complicated alloys, high strength quenched and tempered steels
for example, be undertaken.

4. One of the inputs required in both the heat transfer
and stress analyses of welded structures is the temperature
dependence of the physical and mechanical properties of the
material under consideration. The available data on this depend-
ence are, however, scarce and whenever present do not cover the
whole temperature range encountered during welding, i.e. from room to above liquidus temperature. At the same time slight variations in alloy composition can have a drastic impact on the material properties. As a consequence, a significant amount of rational judgement is involved in estimating them, with usually uncertain results regarding the accuracy of the obtained values. It is therefore recommended that an experimental program be undertaken to measure the temperature variation of these properties, at least for some materials. The obtained results could then be compared with the statistical thermodynamic theories that have been developed over the years for estimating these values. This effort could conceivably improve these theories and hence provide the analyst with a more reliable tool for calculating the material properties.

5. The plane strain material nonlinear finite element analysis performed in this study to calculate the strains and stresses during welding was adequate for our purposes. To study the longitudinal bending distortion of a welded structure, though, either a three-dimensional or a generalized plane strain analysis should be performed. The latter analysis is preferable due to cost considerations. It is thus recommended that efforts should be undertaken to modify the multipurpose finite element program ADINA by including the generalized plane strain case, not currently present.
REFERENCES


42. Heuschkel, J., "Composition Controlled, High-Strength, Ductile, Tough, Steel Weld Metals", Welding Journal, 43 (8), 1964, pp. 361s-384s.


APPENDICES
APPENDIX A

COMPUTER IMPLEMENTATION OF THE THREE-DIMENSIONAL
FINITE HEAT SOURCE SOLUTION

Some notes on the computer implementation of the three-dimensional finite heat source solution derived in Section 2.3.3 will be presented in this Appendix. The computer program has been written in the FORTRAN IV language using double precision arithmetic. Runs of the program have been made in both the Digital Equipment Co.'s VAX and IBM's 370 systems at the Joint Computer Facilities and the Information Processing Services respectively at M.I.T.

A.1 Solution of the Transcendental Equation (2.43)

The characteristic values of the outer solution, $\omega_n'$, were found in part A of Section 2.3.3 to be the roots of the following equation:

$$(k_0' \cdot \omega_n^2 - h_1' \cdot h_2') \cdot \tan(\omega_n H) = k_0' \cdot \omega_n' \cdot (h_1' + h_2'), \quad n=1,2,3,... \quad (2.43)$$

which can be written as

$$(A \cdot \lambda_n^2 - B) \cdot \tan(\lambda_n) = C \cdot \lambda_n', \quad n = 1,2,3,... \quad (A.1)$$

where

$$\lambda_n = \omega_n \cdot H$$

$$A = (k_0'/H)^2$$

$$B = h_1' \cdot h_2'$$

$$C = k_0' \cdot (h_1' + h_2')/H$$

with all constants $A$, $B$, and $C$ being non-negative; $A$ is always positive, whereas $B$ and $C$ are zero only under the adiabatic
boundaries assumption \( h_1 = h_2 = 0 \).

Letting

\[
f(\lambda_n) = (A \cdot \lambda_n^2 - B) \cdot \tan(\lambda_n), \quad n = 1, 2, 3, \ldots \quad (A.2)
\]

it can be seen that the roots of this function are \((B/A)^{1/2}\) and \(\lambda \pi, \lambda = 0, 1, 2, \ldots\), where the first root is such that

\[
(i-1) \cdot \frac{\pi}{2} < \left( \frac{B}{A} \right)^{1/2} < i \cdot \frac{\pi}{2}, \quad i = 1, 2, 3, \ldots \quad (A.3)
\]

Based on the above and a thorough investigation of the analytical properties of eqn. (A.2), it was concluded that the roots of equation

\[
g(\lambda_n) = f(\lambda_n) - C \cdot \lambda_n \quad (A.4)
\]

could be pinpointed as follows, depending on the value of \(i\):

1. For \(i\) odd, \(g(\lambda_n)\) has

   (a) \((i-1)/2\) number of roots \(\lambda_n < (i-1)\pi/2\), one in each of the intervals

   \[
   \left( \frac{(j-2)\pi}{2}, \frac{(j-1)\pi}{2} \right), \quad j = 3, 5, 7, \ldots, i
   \]

   (b) one root in the interval

   \[
   \left( \frac{(i-1)\pi}{2}, \frac{i\pi}{2} \right)
   \]

   (c) the remaining roots \(\lambda_n > i\pi/2\), one in each of the intervals

   \[
   \left( \frac{i\pi}{2} + (2j-1)\frac{\pi}{2}, \frac{i\pi}{2} + j\pi \right), \quad j = 1, 2, 3, \ldots
   \]
2. For \( i \) even, \( g(\lambda_n) \) has

(a) \( \frac{i}{2} - 1 \) number of roots \( \lambda_n < (i-1)\pi/2 \), one in each of the intervals

\[
\left( \frac{(j-3)\pi}{2}, \frac{(j-2)\pi}{2} \right), \quad j = 4, 6, 8, \ldots, i
\]

(b) one root in the interval

\[
\left( \frac{(i-1)\pi}{2}, \frac{i\pi}{2} \right)
\]

(c) the remaining roots \( \lambda_n > i\pi/2 \), one in each of the intervals

\[
\left( \frac{i\pi}{2} + (j-1)\pi, \frac{i\pi}{2} + (2j-1)\pi \right), \quad j = 1, 2, 3, \ldots
\]

These roots were calculated using the so-called half-interval method [132]. As it will be later explained, only twenty roots need to be found.

In the computer program the necessary calculations for the determination of the characteristic values \( \omega_n \) are performed in subroutines OMEGA and ROOT and the function FUNC.

A.2 Numerical Calculation of Integral (2.63)

In the expressions giving the inner solution, \( \phi^i(r,z) \), and the constants, \( C_n \), the integral

\[
K_n = \int_0^1 e^{\frac{-Cr^2}{h}x^2} \cdot J_0(\delta_n r_h x) \cdot dx, \quad n = 1, 2, 3, \ldots \quad (2.63)
\]

is involved and since no closed-form solution exists, it must be calculated numerically. Letting

\[
z_n = \delta_n \cdot r_h \quad \text{and} \quad y = z_n \cdot x
\]
the above equation can be written as

\[ K_n = \frac{1}{z_n^2} \int_0^{z_n} f_n(y) \cdot dy \]  \hspace{1cm} (A.5)

where

\[ f_n(y) = y \cdot \exp(- \frac{C}{n} y^2) \cdot J_0(y) \]  \hspace{1cm} (A.6)

or, equivalently

\[ K_n = \frac{1}{z_n^2} \sum_{i=1}^{n} \left\{ \int_{z_{i-1}}^{z_i} f_n(y) \cdot dy \right\} \]  \hspace{1cm} (A.7)

where \( z_0 = 0 \).

Applying the trapezoidal numerical integration scheme \([132]\), each of the integrals in the series can be approximated as

\[ \int_{z_{i-1}}^{z_i} f_n(y) \cdot dy \approx \frac{h_i}{2} \left[ f_n[y(i-1) \cdot N+1] + f_n[y(i) \cdot N+1] \right] \]  \hspace{1cm} (A.8)

where

\[ h_i = \frac{z_i - z_{i-1}}{N}, \quad i = 1, 2, 3, \ldots, n \]

are the lengths of the \( N \) equal subintervals into which each interval \([z_{i-1}, z_i]\), \( i = 1, 2, \ldots, n \) is divided and \( y, \quad l = 1, 2, \ldots, nN+1 \) are the resulting \( nN+1 \) values of the parameter \( y \). Note that due to the special construction of
the scheme (see Fig. A.1)

\[ Y(i-1)_{N+1} = z_{i-1}, \quad i = 1, 2, 3, \ldots, n+1 \]

and

\[ Y(i-1)_{N+k+1} = z_{i-1} + k h_i, \quad i = 1, 2, 3, \ldots, n \]

Furthermore, recalling that \( z_n = \delta_n \cdot r_n \) are the zeros of \( J_0(x) \), it can be seen that the function \( f_n(y) \) vanishes at the limits of each interval. Equation (A.8) can thus be written as

\[ \int_{z_{i-1}}^{z_i} f_n(y) \, dy = h_i \cdot \sum_{j=1}^{N-1} f_n[Y(i-1)_{N+j+1}] \]

and going back to eqn. (A.7)

\[ K_n = \frac{1}{z_n^2} \cdot \sum_{i=1}^{n} \left( h_i \cdot \sum_{j=1}^{N-1} f_n[Y(i-1)_{N+j+1}] \right) \]  \hspace{1cm} (A.9)

This last equation is evaluated numerically in subroutine KAPPA and function WAX, with each interval \([z_{i-1}, z_i]\) divided into ten subintervals (\( N=10 \)).

A.3 Calculation of Maximum Heat Flux

The maximum heat flux, \( q_o \), can be found as outlined in section 2.2 from

\[ Q = 2q_o \int_0^\pi \int_0^r \exp(-C.r^2 - \lambda v r \cos\psi) \, r \, d\psi \, dr \]  \hspace{1cm} (2.14)

where

\[ Q = \eta_a \cdot V \cdot I \]  \hspace{1cm} (2.7)
FIGURE A.1 Representative Interval Used in Numerical Integration.
Using again the trapezoidal scheme for the numerical integration, it can be found that

\[
Q = q_o \cdot \frac{r_h}{N} \cdot \left[ \sum_{n=2}^{N-1} r_n \cdot \exp(-C \cdot r_n^2) \cdot g(r_n) + r_h \cdot \exp(-C \cdot r_h^2) \cdot g(r_h) \right]
\]  
(A.10)

where

\[
g(r) = \frac{\pi}{2M} \cdot \left[ e^{-\lambda vr} + e^{\lambda vr} + 2 \cdot \sum_{m=2}^{M-1} e^{-\lambda vr \cdot \cos \psi_m} \right]
\]

\[
\psi_m = (m-1) \cdot \frac{\pi}{M}, \quad m = 1, 2, 3, \ldots, M
\]

\[
r_n = (n-1) \cdot \frac{r_h}{N}, \quad n = 1, 2, 3, \ldots, N
\]

and M, N are the number of subintervals into which the intervals [0, π] and [0, r_h] are divided respectively.

The calculations are performed in subroutine QMAX.

A.4 Calculation of Bessel Functions

The Bessel and modified Bessel functions appearing in the various equations of the solution have been calculated using the well known approximate product formulae [133]. Several subroutines have been written for this purpose as follows (the Bessel function calculated is shown in parenthesis): JLBES (J_1(x)), J0BES (J_0(x)), I1BES (I_1(x)), I0BES, (I_0(x)), KLBES (K_1(x)), and K0BES (K_0(x)).
Finally, to calculate the characteristic values, $\delta_n'$ of the first part of the inner solution the zeros of $J_0(x)$ have to be found according to eqn. (2.57). The first twenty zeros can be found tabulated in many mathematical handbooks and are given in the program in BLOCK DATA. In case, however, more than twenty zeros are required, i.e. if more than twenty terms of the series solutions must be added for convergence, the following expansion can be used for the $m^{th}$ zero, $m \geq 20$

$$x_m \sim \frac{\pi}{4}(4m-1) + \frac{1}{2\pi(4m-1)} - \frac{1}{6\pi^3(4m-1)^3} + \frac{3779}{15\pi^5(4m-1)^5} - \ldots \quad (A.11)$$

with an accuracy of at least five digits [134].

A.5 Program Listing

A listing of the developed program is given in the following pages. Enough comment statements have been inserted into the program to facilitate easier understanding.
PROGRAM MAIN

THIS PROGRAM CALCULATES THE TEMPERATURE DISTRIBUTION IN A PLATE OF INFINITE LENGTH, FINITE WIDTH, AND FINITE THICKNESS DUE TO A THREE-DIMENSIONAL GAUSSIAN DISTRIBUTED HEAT SOURCE MOVING ALONG ITS SURFACE.

PROGRAM DEVELOPED BY Y. J. PAPAZOGLU

JULY 1979

0061 IMPLICIT REAL*8(A-H,O-Z)
0062 COMMON/S1/GBU(100)
0063 COMMON/S2/UC
0064 COMMON/S1/A,B,S
0065 DIMENSION X(100), Y(100), Z(100), TEM(100)
0066 DIMENSION U(100), P(100), CH(100), ZET(100)
0067 DIMENSION p(100), C(100)
0068 DIMENSION CK(6)
0069 DIMENSION A(20)
0070 DATA CK/.002515,.005879,.010624,.021896,.07832,1.253314/
0071
0072 C
0073 * * * INPUT DATA * * *
0074
0075 REAL HEAD(5,490) (ANAM(1), I=1,20)
0076 REAL HEAD(5,500) AV,A1,V,UC,EFF
0077 REAL HEAD(5,510) CKO,G,ALPHA,H1,H2
0078 REAL HEAD(5,520) H,T,TI,T1,L0,MJ,MJ,NI,NI,NCON
0079 REAL HEAD(5,530) (X(N), Y(I), Z(I), I=1,NN)
0080
0081 REAL FORMAT(6F10.5)
0082 REAL FORMAT(4F10.5,1)
0083 FORMAT(2(F9.4))
0084 FORMAT(20A4)
0085 FORMAT(6,1000) (ANAM(I), I=1,20)
0086 FORMAT(6,1050)
0087 FORMAT(6,1100) AV,A1,V,UC,EFF
0088 FORMAT(6,1150) CKO,G,ALPHA,H1,H2
0089 FORMAT(6,1200) H,T,TI,T1,L0
0090 FORMAT(6,1250) (L,N), Y(I), Z(I), I=1,NN)
0091 FORMAT(6,2000)
0092 FORMAT(20A4)
0093 FORMAT(11,21,31,41,51,61,71,81,91)
0094 FORMAT(11,21,31,41,51,61,71,81,91)
0095 FORMAT(21,31,41,51,61,71,81)
0096 FORMAT(21,31,41,51,61,71,81)
0097 FORMAT(21,31,41,51,61,71,81)
0098 FORMAT(21,31,41,51,61,71,81)
0099 FORMAT(21,31,41,51,61,71,81)
0100 FORMAT(20A4)
0101 FORMAT(6,1000) (ANAM(I), I=1,20)
0102 FORMAT(6,1050)
0103 FORMAT(6,1100) AV,A1,V,UC,EFF
0104 FORMAT(6,1150) CKO,G,ALPHA,H1,H2
0105 FORMAT(6,1200) H,T,TI,T1,L0
0106 FORMAT(6,1250) (L,N), Y(I), Z(I), I=1,NN)
0107 FORMAT(6,2000)
0108 FORMAT(20A4)
0109 FORMAT(11,21,31,41,51,61,71,81,91)
0110 FORMAT(11,21,31,41,51,61,71,81,91)
0111 FORMAT(21,31,41,51,61,71,81)
0112 FORMAT(21,31,41,51,61,71,81)
0113 FORMAT(21,31,41,51,61,71,81)
0114 FORMAT(21,31,41,51,61,71,81)
0115 FORMAT(20A4)
0116 FORMAT(6,1000) (ANAM(I), I=1,20)
0117 FORMAT(6,1050)
0118 FORMAT(6,1100) AV,A1,V,UC,EFF
0119 FORMAT(6,1150) CKO,G,ALPHA,H1,H2
0120 FORMAT(6,1200) H,T,TI,T1,L0
0121 FORMAT(6,1250) (L,N), Y(I), Z(I), I=1,NN)
0122 FORMAT(6,2000)
0123 FORMAT(20A4)
0124 FORMAT(11,21,31,41,51,61,71,81,91)
0125 FORMAT(11,21,31,41,51,61,71,81,91)
0126 FORMAT(21,31,41,51,61,71,81)
0127 FORMAT(21,31,41,51,61,71,81)
0128 FORMAT(21,31,41,51,61,71,81)
0129 FORMAT(21,31,41,51,61,71,81)
0130 FORMAT(20A4)
0131 FORMAT(6,1000) (ANAM(I), I=1,20)
0132 FORMAT(6,1050)
0133 FORMAT(6,1100) AV,A1,V,UC,EFF
0134 FORMAT(6,1150) CKO,G,ALPHA,H1,H2
0135 FORMAT(6,1200) H,T,TI,T1,L0
0136 FORMAT(6,1250) (L,N), Y(I), Z(I), I=1,NN)
0137 FORMAT(6,2000)
0138 FORMAT(20A4)
0139 FORMAT(11,21,31,41,51,61,71,81,91)
0140 FORMAT(11,21,31,41,51,61,71,81,91)
0141 FORMAT(21,31,41,51,61,71,81)
0142 FORMAT(21,31,41,51,61,71,81)
0143 FORMAT(21,31,41,51,61,71,81)
0144 FORMAT(21,31,41,51,61,71,81)
0145 FORMAT(20A4)
0146 FORMAT(6,1000) (ANAM(I), I=1,20)
0147 FORMAT(6,1050)
0148 FORMAT(6,1100) AV,A1,V,UC,EFF
0149 FORMAT(6,1150) CKO,G,ALPHA,H1,H2
0150 FORMAT(6,1200) H,T,TI,T1,L0
0151 FORMAT(6,1250) (L,N), Y(I), Z(I), I=1,NN)
0152 FORMAT(6,2000)
0153 FORMAT(20A4)
4/EX, 'CONVECTION COEFFICIENT-TOP (DU/SEC.IN.FP) ****** F12.5/ // 0031
5/EX, 'CONVECTION COEFFICIENT-BOTTOM (DU/SEC.IN.FP) ****** F12.5/ // 0035
1200 FORMAT('9.4', ' PLATE THICKNESS (IN) ****** F10.5/// 0039
12X, 'INITIAL TEMPERATURE (F) ****** F10.5/// 0043
22X, 'ENVIRONMENTAL TEMPERATURE (F) ****** F10.5/// 0047
32X, 'MAXIMUM TEMPERATURE (F) ****** F10.5/// 0051
0632
06X, 'COORDINATES (X,Y,Z) /// 0055
1(X,Y,Z,J15,***,JF10.4))\} 0059
0033
2000 FORMAT('11', '****** RESULTS ******) 0063

C
0034
PI=3.141592
0038
C=1.//(2.*ALPHA)
0042
H0=1.73/DSCWT(W)
0046
KMAX=MAKO(NOUT,NIN)
0050
LMAX=MAKO(NIN,NCON)

C
C CALCULATE CHARACTERISTIC VALUES

C
A5=CKO/H
0054
A=A5**2
0058
B=H1**2
0062
S=A5*(H1+H2)
0066
PAE=(C-V)**2
0070
IF(H1.EQ.0.O.AND.H2.EQ.0.O) GO TO 30
0074
CALL OMEGA(A,B,H0,KMAX)
0078
GO TO 50
0082
30 CONTINUE
0086
DO 20 I=1,KMAX
0090
20 O(I)=DFLOAT(I)*PI/H
0094
50 CONTINUE
0098
DO 10 I=1,KMAX
0102
10 ZEP(I)=DFLOAT(I)**2+PAE
0106
IF(LMAX.LE.20) GO TO 15
0110
DO 25 J=1,LMAX
0114
CMF=(4.*DFLOAT(J)-1.)*PI
0118
25 ZERO(J)=CMF/4.**1./(2.*CMF-J7./((6.**CMF)**3)*3779./((15.**CMF)**5)
0122
15 CONTINUE
0126
DO 35 J=1,LMAX
0130
35 CHI(J)=DFloat(J)**2+PAE
0134
C
C CALCULATE INTEGRAL KN

C
C CALL KAPPA(D,P,LMAX)

C
C CALCULATE MAXIMUM HEAT FLUX

C
Q=EFF*AV*AL/1055.
0353
C
C CALCULATE COEFFICIENTS CN OF SERIES SOLUTION

C
IF(H1*H2.NE.0.O) GO TO 700
CIVA=CL*Y*W
CALL KBES(CLVA, BEK1)
C=1/(2.*CKO*CLVA*W*BEK1*PINT)
CONTINUE
DO 200 W=1,KMAX
ZB=ZET(N)*W
CALL KOBES(ZRC, BK0)
CALL KBES(ZRC, BK1)
CALL 106ES(ZCO, B10)
CALL 105ES(ZCO, B11)
DENS*ZET(N)*(EK1*W*BI/B10)
IF(U1.EQ.0.0.AND.U2.EQ.0.0) GO TO 710
V1=H2/ZCO/0(N)**2
V2=H2*V1/ZC0
V3=0(N)**H
DEN2=.5*H*(1.+V2)+(1.-V2)*DSIN(2.*V3)/(4.*D0(N))+V1*DSIN(V3)**2
GO TO 715
710 V3=0(N)**H
DENS=0.5**H
715 DENS=0.5**H
U=CKO*0(N)
SINO=DSIN(V3)
COS0=DCOS(V3)
SU=0.0
DU=1000 M=1,MW
CH2=CHI(M)**2
V1=CKO*CHI(M)
B1=H2*(C(N)+CH2/0(N))**SINO
B2=CKO*CH2*H2**2/CX0)**COS0
D1=B1*H2
D2=CHI(M)**2*U**2*H2)**SINO
D3=U1***(H1+H2)
D4=U1+U1**2/2
W=CHI(M)**2
D5=U1*W**4
D6=U1**2*(H1+H2)
D7=U1**2*(H1+H2)
D8=U1+1*(H1+H2)
IF(W2.GT.90.0) GO TO 70
EX1=DEXP(W2)
GO TO 70
70 EX1=1.0E+J5
80 EX2=1.0/EX1
HYPT=(EX1-EX2)/(EX1*EX2)
CALL J12ES(W3, B11)
U1=2.*UG*U1**0(N)**2*U1+HYPT+B2
VCH=CH2*O(N)**2
U2=VCH*BJ1*(D3+D4*HYPT)
100 SUM=SUM+U1/U2
111 C(N)=SUM/DEN
112 200 CONTINUE
C INPUT
C TH=TIME AT WHICH TEMPERATURES ARE TO BE CALCULATED
C ISTOP=INDEX TO TERMINATE CALCULATIONS
C EQ.0 CONTINUE
C    EQ. 1 STOP
C
0113  800  READ(5,520) TR,ISTOP
C
0114  520  FORMAT(F10.4,15)
C
0115  IF(ISTOP,N.E.0) GO TO 999
C
0116  DO 400  I=1,NN
C
0117  XI=XI(I)-V*TH
C
0118  RAD1=1+2*Y(I)**2
C
0119  RAD1=DSQRT(RAD1)
C
0120  CLY=CLY*XI
C
0121  CLY=CLY*RADI
C
0122  IF(RADI.LT.80) GO TO 350
C
C    SOLUTION FOR OUTER REGION
C
C
0123  SUM=0.0
C
0124  IF(H1.NE.0.0) GO TO 40
C
C    FOR H1=H2=0, CALCULATE THE ZEROTH TERM OF SERIES SOLUTION
C
0125  IF(CLVR.LT.170.0) GO TO 42
C
0126  XS=2./CLVR
C
0127  SK=4.00053208
C
0128  DO 45  L=1,6
C
0129  45  SK=SK*XS+CK(L)
C
0130  EKR=SK/DSQRT(CLVR)/DLXP(CLVR+CLVR)
C
0131  GO TO 43
C
0132  CALL KODES(CLVR,BK)
C
0133  EKR=0.0
C
0134  IF(CLVR.GE.15.0) GO TO 43
C
0135  EKR=BK/DLXP(CLVR)
C
0136  43  SUM=SUM+EKR*CC
C
0137  CONTINUE
C
C    CALCULATE THE SERIES TERMS FROM 1 TO NOUT
C
0138  DO 220  L=1,NOUT
C
0139  US=0.0+SN(I)
C
0140  FACTOR=H2*D US b(U5)/O(N)/CKO+DCUS(U5)
C
0141  US=ZET(H)*RADI
C
0142  IF(UO=U-2.) 240,240,266
C
0143  CALL KODES(U6,BK)
C
0144  FN=SK/DLXP(CLVR)
C
0145  GO TO 230
C
0146  240  C1=CLVR*U6
C
0147  IF(C1.LE.-170.0) GO TO 270
C
0148  IF(C1.GE.170.0) GO TO 310
C
0149  CEX=DLXP(C1)
C
0150  GO TO 320
C
0151  310  CEX=1.0+60
C
0152  320  XS=2./U6
C
0153  SK=0.0053208
C
0154  DO 260  L=1,6
C
0155  260  SK=SK*XS+CK(L)
C
0156  FN=SK/CEX/DSQRT(U6)
C
0157  GO TO 230
C
0158  270  FN=1.08*60
C SOLUTION FOR INNER REGION
C
C 350 SUM=0.0
C
C IF(H1*H2,NE.0.0) GO TO 90
C
C FOR H1=H2=0, CALCULATE ZERO TH TERM OF SERIES SOLUTION
C CALL KOBES(CLVA,BK3)
C
C CALL JOBES(CLVA,B13)
C CALL JOBES(CLVA,B14)
C SUM=SUM+COB3*BI4/B13
C
C 90 CONTINUE
C
C CALCULATE THE SERIES TERMS FROM 1 TO NOUT
C
C DO 360 N=1,NIN
C U7=U(N)*FAU
C U8=CHI(N)*ZH(I)
C U9=U(J)*ZH(I)
C U10=D(N)*RS
C CALL JOBES(U7,UJO)
C CALL JIBES(U10,UJ)
C IF(U8.GT.170.)GO TO 120
C E10=DXEP(U0)
C GO TO 130
C
C 120 E10=1.0E+60
C
C 130 E11=1./E10
C COSHUU=5.*(E10+E11)
C SINHUU=5.*(E10-E11)
C FACT1=P(N)*BJO*(H2*SIHUB8+CBO*CHI(N)*COSHUB8)
C U11=CHI(N)*H
C U12=CBO*CHI(N)
C IF(U11.GT.170.)GO TO 160
C E15=DXEP(U11)
C GO TO 170
C
C 160 E15=1.0E+60
C
C 170 E16=1./E15
C COSHUU = 5.*(E15+E16)
C SINHUU = 5.*(E15-E16)
C FACT2=BJ*2.*((U12*(H1+H2))*COSH11+(U12**2+H1*H2)*SINH11)
C U13=ZET(N)*K0
C U14=ZET(N)*RAD1
C CALL KOBES(U11,BK3)
C CALL JOBES(U14,B12)
C CALL JOBES(U13,B13)
C FACT3=C(N)*BK2*BI2/BI*((U2*DSIN(U9)/CBO/G(N)*DCOS(U9))
C
C 360 SUM=SUM+3.*V0*FACT1/FACT2*FACT3
C PHI=SUM
C UT=PHI/DEXP(CLVA)+ (TO-TE)
C
C CALCULATE THE TEMPERATURES
C
0204       600 IF(UT.GT.1.0E+20) GO TO 420
0205               IF(GEQ.0.0) GO TO 450
0206               G1=DGIN(1.0G+(UT-TO)/TH)
0207                   IF(0.0.LT.G1) GO TO 430
0208                       TEMP(I)=TO+(G1+1.0)/G
0209                       GO TO 480
0210                      430 TEMP(I)=TO-(G1+1.0)/G
0211                      GO TO 480
0212                      420 TEMP(I)=1.0E+30
0213                      GO TO 480
0214                      450 TEMP(I)=UT
0215                      480 IF(TMP(I).GE.TLIQ) TEMP(I)=TLIQ
0216                      400 CONTINUE

C
C PRINT RESULTS
C
0217       WRITE(6,2100) TM
0218               WRITE(6,2200) (I,TEMP(I),I=1,NW)
0219               2100 FORMAT(////,TIME=,'F10.4)
0220               2200 FORMAT('00',', TEMPERATURE DISTRIBUTION ( AT THE POINTS IN CONSIDERATION)
0221               //',I7(I,3X,F9.2)))
0222                      GO TO 900
0223                      999 CONTINUE
0224                      STOP
0225                      END
SUBROUTINE OMEGA(A, B, N, O, NS1)
C
C THIS SUBROUTINE CALCULATES THE CHARACTERISTIC VALUES OMEGA
C
IMPLICIT REAL*8(A-H,O-Z)
REAL R
DIMENSION O(100)
PI=3.141593
N=5*DSQRT(W/A)/PI
I=IFIX(W)*1
IF(I,EQ.1) GO TO 40
K=2
L=3
10 IF(1.EQ.K) GO TO 100
IF(I,EQ.L) GO TO 200
K=K+2
L=L+2
GO TO 10
40 CONTINUE
DO 20 J=1,KMAX
L=DFLOAT(J)-1.*PI
CALL ROOT(ZL,U)
20 O(J)=U/H
RETURN
100 K=0
DO 120 J=4,1,2
ZL=DFLOAT(J)-3.*PI/2.
CALL RCCT(ZL,U)
120 O(M)=U/H
ZL=DFLOAT(I)-1.*PI/2.
CALL RCCT(ZL,U)
M=I/2
O(M)=U/H
K=KMAX-1/2
DO 150 N=1,K
ZL=DFLOAT(I)/2.*DFLOAT(N)-1.5*PI
CALL RCCT(ZL,U)
M=I/2+N
150 O(M)=U/H
RETURN
200 L=0
DO 220 J=3,1,2
ZL=DFLOAT(J)-2.*PI/2.
CALL RCCT(ZL,U)
220 O(M)=U/H
ZL=DFLOAT(I)-1.*PI/2.
CALL RCCT(ZL,U)
M=(J-1)/2
O(M)=U/H
L=KMAX-(I+1)/2
M=(I+1)/2
O(M)=U/H
L=KMAX-(I+1)/2
DO 250 N=1,L
ZL=.5*(DFLOAT(I)*2.*DFLOAT(N)-1.5)/PI
250 CALL RCCT(ZL,U)
RETURN
SUBROUTINE ROOT(W,Z)

THIS SUBROUTINE FINDS THE ROOT OF A FUNCTION

by the half-interval method

IMPLICIT REAL*8(A-H,O-Z)

!T=ALOG(1.1*1.E-6)/ALOG(2.)*1.

ZLE=W-.001

!IF (FUNC(ZLE)*FUNC(ZLE+.1).LT.0.) GO TO 3

ZLE=ZLE+.1

GO TO 1

!Z=ZLE+.1

!FZL=FUNC(ZLE)

DO 6 J=1,ITER

ZHALF=(ZLE+FZL)/2.

FZHALF=FUNC(ZHALF)

IF (FZHALF+FZL.LE.0.) GO TO 5

ZLE=ZHALF

!FZL=FZHALF

GO TO 6

Z=ZHALF

!CONTINUE

Z=(ZLE+FZL)/2.

RETURN

END
SUBROUTINE KAPPA(D,P,LMAX)
C
C THIS SUBROUTINE CALCULATES THE INTEGRAL KN
C USING THE TRAPEZOIDAL RULE
C
IMPLICIT REAL*8(A-H,O-Z)
COMMON/S1/ZERO(100)
DIMENSION P(100),H(100),Y(1000),BJ(1000),D(100)
NINTG=10
H(1)=ZERO(1)/DFLOAT(NINTG)
DO 5 L=2,LMAX
5 N=(ZERO(L)-ZERO(L-1))/DFLOAT(NINTG)
I=1
Y(I)=D(I)
10 CALL JOBES(Y(I),B)
BJ(I)=B
I=I+1
IF(I.EQ.N*NINTG+1) GO TO 20
Y(I)=Y(I-1)+H(I)
GO TO 10
20 Y(I)=ZERO(N)
BJ(I)=0.
N=N+1
IF(N.GT.LMAX) GO TO 30
I=I+1
Y(I)=Y(I-1)+H(I)
GO TO 10
30 CONTINUE
N=N*NINTG-1
DO 100 K=1,LMAX
ATHR=0.0
DO 290 K=1,N
N=(K-1)*NINTG+1
SUM=0.0
DO 332 L=1,K
J=K+L
50 SUM=SUM+MAX(Y(J),D(N),BJ(J))
290 CONTINUE
ATHR=ATHR+H(K)*SUM
DO 316 K=1,L
P(K)=ATHR/ZERO(N)**2
316 RETURN
END
FUNCTION WAX(A,B,C)
IMPLICIT REAL*8(A-H,O-Z)
COMMON/SZ/UC
K=-UC*A**2/B**2
IF (K.01.170.) GO TO 5
WAX=A*C*DEXP(K)
GO TO 10
5 WAX=1.0E+70
10 RETURN
END
SUBROUTINE QMAX(Q, QC, CL, V, RO, PINT, U)

THIS SUBROUTINE CALCULATES THE HEAT FLUX AT THE CENTER OF THE GAUSSIAN DISTRIBUTION

IMPLICIT REAL*8(A-H,O-Z)
PI=3.141592
CV=CL*V
IN=30

DM=PI/DFLOAT(IN)
DN=RO/DFLOAT(IN)
SUM1=0.0
RN=0.0

DO 200 N=2, IN
RN=RN+DN
F=CV*RN
THM=THM+DM

DO 160 M=2, IN
THM=THM+DM

E1=DEXP(F*CTH)
E2=DEXP(F)

PF=0.5*(E2+1.0/E2+2.0/SUM2)

E3=DEXP(VC*MM**2)

SUB1=SUM1+MM*PF/E3

CLVA=CV*RO
ATHR=0.0
TH=0.0

DO 300 L=2, IN
TH=TH+DM

COSTH=DCOS(TH)

EX1=DEXP(CLVA*COSTH)

ATHM=ATHM+1.0/EX1

EX4=DEXP(CLVA)

PINT=0.5*DN*(EX4+2.0*ATHM+1.0/EX4)

E4=DEXP(UC*RO**2)

U0=Q/(DN*(2.0*SUM1+RO*PINT/E4))

RETURN
END
SUBROUTINE JIBES(X, W)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION C(6), CF(6), CTH(6)
DATA C/-.00J3176, -.004332, -.0395429, -.2109357, -.5628998, .5/
DATA CF/-.0011365, -.0024951, -.0001711, -.0165967, -.0000016, .7978846/
DATA CTH/-.0007992, -.0007435, -.0063708, -.0000565, .1249961, -2.3561945/

10 X=X
20 IF(X-3.) 30, 50, 50
30 XS=X**2/9.
10 S1=0.00001109
10 DO 35 L=1, 6
35 SI=SI+XS+C(L)
10 B=X*S1
10 RETURN
50 XS=3./X
80 SF=-.0020033
10 STH=-.00029166
10 DO 55 L=1, 6
55 SF=SF*XT+C(L)
10 STH=STH+XS+CTH(L)
10 Z=SF*DCGS(I+STH)/DSQRT(X)
10 RETURN
END
SUBROUTINE JOBES(X,D)

IMPLICIT REAL*8(A-H,O-Z)

DIMENSION C(6),CF(6),CTH(6)

DATA C/-.0039444,-.0449479,-.3163866,1.2656208,-2.2949997,1./

DATA CF/-.00072805,-.0013724,-.0000951,-.0055274,-.00000077,
1.7978826/

DATA CTH/-0.662933,-0.0054125,-0.0265730,-0.0000395,
0.04166397,-.78539816/

IF(X) 10,20,20

10 X=-X

20 IF(X-.3) 30,50,50

30 XS=X**2/9

SI=.00021

DO 55 L=1,6

55 SI=SI*XS+C(L)

RETURN

D=31

RETURN

50 LS=-3./X

SF=.00014476

STD=.00013585

DO 55 L=1,6

55 SF=SF*XS+CF(L)

STM=STD*XS+CTH(L)

J=SF*DCGS(X+STM)/DSQ8T(X)

RETURN

END

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SUBROUTINE I1BES(X,B)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION C(6),D(8)
DATA C/,0.00301532,0.02656733,1.5084934,51498069,8789059,50/
DATA D/0.0170765,-0.0269531,0.029297,-.0103155,.0016380,
-0.006202,-.0350802,.3989423/
IF(X) 10,20,20
10 X=X
20 IF(X<3.75) 10,50,50
30 X=X**2/14.062
50 IF(X.GT.170.) GO TO 60
60 XBE=DEXP(X)/DSQRT(X)
RETURN
END
SUBROUTINE IOBES(X, B)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION C(6), D(8)
DATA C/0.36076, 2659732, 1.205792, 1.0899424, 3.5156229, 1.7 /
DATA D/-0.01647633, 0.0263554, -0.0205771, 0.0091628, -0.0015756, /
1.0022532, 0.0132859, 0.3909423/
10 X=-X
20 IF(X<3.75) 30, 50, 50
30 XS=X**2/14.0625
40 SI=.0045813
50 DO J=1,6
60 SI=Si*XS+C(L)
70 N=SI
80 RETURN
90 GO TO 50
XRE=DEXP(X)/DSQRT(X)
10 XS=3.75/X
11 SI=.00392377
12 DO J=1,6
13 SI=SI*XS+D(L)
14 B=XRE*SI
15 RETURN
16 B=1.08+70
17 RETURN
18 END
SUBROUTINE K1BES(X,B)

IMPLICIT REAL*8(A-H,O-Z)

DIMENSION C(6),CI(6),CK(6)

DATA CI/.003235315,-.00707815,.015040926,.0365562,.02349862,1.2533141/

DATA CI/.0010153,.02358711,.1508493,.5149887,.0789059,.5/

DATA CI/.0010153,.02658711,.1508493,.5149887,.0789059,.5/

DO 010 X=-10,10,20

IF(X)10,10,20

10 B=1.0E+70

RETURN

X=X

IF(X>-1.)30,50,50

30 XL=X*LOG(1.5*X)

XSK=X**2/4.

XI=X**2/14.0625

SI=-0.00032411

SK=-0.00004869

DO 35 L=1,6

35 SK=SK*SK*CK(L)

B=(SK*XL*X*SI)/X

RETURN

50 IF(X-170.)60,60,70

60 XKE=DSCWRT(X)*DEEP(Z)

XSK=X**2/4.

SK=-0.00068245

DO 65 L=1,6

65 SK=SK*SK*CK(L)

U=SK*XKE

RETURN

70 U=1.0E-70

RETURN

END
SUBROUTINE KOBES(X,B)
IMPLICIT REAL*8(A-H,O-Z)

DIMENSION C(6),CI(6),CK(6)

DATA C/-.0025154,.00587887,-.0106244,.0216956,-.0783235,
11.2533141/ DATA CI/0.360768,.2659732,1.2067492,3.0899424,3.5156229,1./
DATA CK/0.001075,.0026269,.0348899,.2306975,.4227942,-.577215/ IF(X) 11,10,20

10 B=1.0E*70 RETURN

11 X=-X

20 IF (X-2.) 30,50,50

30 X=X=0.5*X

40 ISK=X**2/4.

50 ISI=X**2/14.0625

60 SI=.0045813

70 SK=.0030074

80 DO 35 L=1,6

35 SI=SI*SK*CK(L)

40 SK=SK*ISK*CI(L)

60 B=SK-SI*XL

50 RETURN

20 IF (X-170.) 60,60,70

60 X2=SQRT(X)*DEXP(X)

70 X=2./X

80 SK=.00053208

90 DO 55 L=1,6

55 SK=SK*X*CI(L)

80 B=SK/X2

90 RETURN

70 B=1.0E-70

20 RETURN

END
BLOCK DATA
IMPLICIT REAL*(A-H,O-Z)

COMMON/S1/ZEN(100)
DATA ZERO/2.40482,5.52007,8.65372,11.79153,14.93091,18.07106,
121.21163,24.35247,27.49347,30.63460,33.77582,36.91709,40.05942,
243.1979,46.14118,49.48260,52.62405,55.76551,58.90698,
362.04896/
END
APPENDIX B

MODIFICATION OF THE CONVENTIONAL
POINT HEAT SOURCE SOLUTION

The solution to the problem of a point heat source moving on the top surface of a finite thickness plate was given in Section 2.1.1 as eqn. 2.4. It was there pointed out that the method of images had to be applied to satisfy the adiabatic boundary conditions on the top and bottom surfaces of the plate. This solution has been found to be satisfactory by previous investigators in the case of bead-on-plate welds, especially if the temperature dependence of material properties was taken into account through an iterative approach.

For the multipass welding case, however, the solution failed to give good correlation with experimental results owing to the location of the point source on the top surface of the plates. To accommodate this possibility, a modification of the solution was accomplished in this study enabling one to locate the point source at any point through the plate's thickness. It thus becomes possible to simulate each welding pass by positioning a point source at the center of the pass.

Figure B.1 shows the superposition of heat sources necessary to obtain the desired solution. At point 0 the welding heat source WS is situated. To satisfy the adiabatic boundary conditions on the top and bottom surfaces of the plate, two imaginary heat sources $T_1$ and $B_1$ are required, being the mirror images of WS with respect to the two surfaces. These new sour-
FIGURE B.1 Superposition of Heat Sources.
ces, however, produce heat transfer through the two surfaces. Consequently new image sources of them should be taken, etc. ad infinitum. For practical purposes around 30 total imaginary sources are required to obtain the solution.

This solution can now be expressed by the following equation:

$$\theta = \theta_0 + \frac{Q}{4\pi k} \cdot e^{-\lambda \nu \xi} \cdot \left\{ \frac{e^{-\lambda \nu R}}{R} + \sum_{n=1}^{\infty} \left[ \frac{e^{-\lambda \nu R_n}}{R_n} + \frac{e^{-\lambda \nu R_n'}}{R_n'} \right] \right\}$$

(B.1)

where

$$R_n = \sqrt{\xi^2 + y^2 + (OT_n + z)^2}$$

$$R_n' = \sqrt{\xi^2 + y^2 + (OB_n - z)^2}$$

$$OT_n = OB_{n-1} + 2.F$$

$$OB_n = OT_{n-1} + 2.G$$

and all other quantities have been previously defined or can be found in Figure B.1.

A computer program has been written in FORTRAN to implement this solution. In addition, the program can take into account the temperature variation of the material properties as well as the effect of each pass on the weld joint shape.
B.1 Program Listing

A listing of the computer program developed to implement the point heat source solution is given in the following pages.
C QUASI-STATIONARY STATE
C TEMPERATURE DISTRIBUTION DUE TO A MOVING POINT HEAT SOURCE
C ON A PLATE OF FINITE THICKNESS
C USE OF ITERATION METHOD FOR TEMPERATURE DEPENDENCE OF
C MATERIAL PROPERTIES
C
C COMPILED BY V. J. PAPAZOGLOU
C DECEMBER 1980

C
C IMPLICIT REAL*8 (A-H,O-Z)
C DIMENSION T(200),XY(100),IZ(100),Z(100),TY(100),BAT(20),TNEW(200)
C DIMENSION TEMP(10),COND(10),CAP(10),RHO(10)
C DIMENSION X(20),Y(20),Z(20,100),TYPH(20,100)
C DIMENSION STATE(2)
C DATA STATE /8HUCCONSTANT,8HVARIANLE/
C
C * * * INPUT PHASE * * *
C
C
C 870 READ(5,100) VOLT,AMP,THICK
C IF(VOLT.EQ.0.) GO TO 999
C READ(5,110) (BAT(I),K=1,20)
C READ(5,120) (TEMP(I),I=1,10)
C READ(5,120) (COND(I),I=1,10)
C READ(5,120) (CAP(I),I=1,10)
C READ(5,120) (RHO(I),I=1,10)
C READ(5,130) MFASS,MPHC,NSMAC,NPHOP
C DO 3 I=1,NPASS
C 3 READ(5,120) (TYPH(I,J),J=1,NPNC)
C READ(5,150) (F(I),G(I),I=1,NPNC)
C READ(5,160) TMAX,TMIN,TIX
C READ(5,160) T1,T2,T3,T4,T5,TLAST
C READ(5,160) T1,T2,T3,T4,T5,T6,T7
C READ(5,160) (XY(N),IZ(N),N=1,NPNC)
C
C 100 FORMAT(5F10.4)
C 120 FORMAT(10E8.3)
C 130 FORMAT(4I5)
C 150 FORMAT(8F10.4)
C 160 FORMAT(8F10.4)
C
C WRITE IDENTIFICATION INFORMATION
C
C #WRITE(6,200)
C 200 FORMAT(161,48,'POINT HEAT SOURCE/30X,CALCULATION OF TEMPERATURE/'
C 14 DISTRIBUTION DURING WELDING/49X,WELDING CONDITIONS/
C #WRITE(6,200) VOLT,AMP,EFF,THICK
C 205 FORMAT(2X01,5HVOLT=F5.1,5 AMP=F5.1,5 EFF=F4.2,7 SPEED=F7.5,
C 11 THICKNESS=F5.1,F3.1)
C NPHOP=NPASS+1
C #WRITE(6,210) BAT(K),K=1,23,STATE(NPNC)
C 210 FORMAT(10X,20E4,*'(A8,2X,'PROPERTIES)')
C #WRITE(6,215)
C 215 FORMAT(4X,'MATERIAL PROPERTIES VS. TEMPERATURE/')
FORTAN IV G1 RELEASE 2.0

WRITE (6,220) (TEMP(I),I=1,10)

220 FORMAT (2X,'TEMPERATURE',4X,10(2X,F4.1))

WRITE (6,221) (COND(I),I=1,10)

221 FORMAT (2X,'CONDUCITIVITY',4X,10(2X,F8.6))

WRITE (6,222) (CAP(I),I=1,10)

222 FORMAT (2X,'CAPACITANCE',4X,10(2X,F8.6))

WRITE (6,223) (RHO(I),I=1,10)

223 FORMAT (2X,'DENSITY',4X,10(2X,F8.6))

WRITE (6,224) TRAX,TIX,TREF

224 FORMAT (2X,'TRAX',4X,'TIX',4X,'TREF')

WRITE (6,225) (TST(I),I=1,10)

225 FORMAT (2X,'TST',4X,10(2X,F8.6))

WRITE (6,226) T1,T2,T3,T4,T5

226 FORMAT (2X,5X,'T1','T2','T3','T4','T5')

WRITE (6,230) NPASS,TLAST,NPNC,NSMAX

230 FORMAT ('NPASS','TLAST','NPNC','NSMAX')

ICH PASS=1,F70.1/1X,

231 FORMAT ('NPASS','TOUT','NUMER OUTPUT POINTS','TOUT','TOUT')

10.1,13)

WRITE (6,231) (I,IF(I),I=1,10)

231 FORMAT (10X,'I','F7.1')

WRITE (6,232)

232 FORMAT ('CHANGE OF JOINT SHAPE BY EFFECT OF MULTIPASS')

WRITE (6,233) K(1,1),1,1,1

233 FORMAT (10X,'K1','K2','K3','K4')

DO 10 K=1,10

10 CONTINUE

WRITE (6,240) (X(I),I=1,10)

240 FORMAT (10X,'X')

DEFINITION OF TIME INSTANCES

T1=TI1

N=1

40 IF (T(M).GE.T1) T1=T12

IF (T(M).GE.T2) T1=T13

IF (T(M).GE.T3) T1=T14

IF (T(M).GE.T4) T1=T15

IF (T(M).GE.T5) GO TO 41

T(M+1)=T(M)+T1

N=N+1

GO TO 40

41 CONTINUE

B=N+1

T=M=TLAST

POI000920

POI000930

POI000940

POI000950

POI000960

POI000970

POI000980

POI000990

POI010000

POI010010

POI010020

POI010030

POI010040

POI010050

POI010060

POI010070
TEMPERATURE DUE TO ACTUAL POINT SOURCE

C1 = V * CA * RH / 2 / CON
C2 = P / 12.56637 / CON
CH = C1 * (IT + B)
IF (CH < LT - 0.1) CEX = 0.
IF (CH < CT - 0.1) CEX = 1.0 * E + JG
IF (DABS (CH) < LT < 170.) CEX = DB2 * CH
IF (CEX < 0.) GO TO 11
TP0 = 1. / CEX / H

TEMPERATURE DUE TO IMAGE SOURCES

TP1 = 0.
TP2 = 0.
L = 1
ON1 = 2. * G (K)
ON2 = 2. * F (K)
30 Z1 = ON1 - 2 (J)
Z2 = ON2 + 2 (J)
H1 = DQ0T (RO2 + 21 * Z1)
H2 = DQ0T (RO2 + 22 * Z2).
CR1 = C1 * (XT + B1)
CR2 = C1 * (XT + B2)
IF (CH < LT - 0.) CEX = 0.
IF (CR < CT - 0.) CEX = 0.
IF (CR < CT - 0.) CEX = 1.0 * E + JG
IF (CR2 < CT - 0.) CEX = 1.0 * E + JG
IF (DABS (CR1) < LT < 170.) CEX1 = DB2 * CR1
IF (DABS (CR2) < LT < 170.) CEX2 = DB2 * CR2
IF (CEX < 0. or CEX1 < 0. or CEX2 < 0.0) GO TO 11
TP1 = TP1 + 1. / CEX1 / H1
TP2 = TP2 + 1. / CEX2 / H2
L = L + 1
IF (L < GT - 0.5 MAX) GO TO 80
ON1 = ON1 - 2. * G (K)
ON2 = ON2 - 2. * F (K)
GO TO 30

FINAL TEMPERATURE

80 TP = C2 * (TP0 + TP1 + TP2)
TH = TP + TREFK (K, J)
IF (TH < TMAX) TH = TMAX
IF (NPBOP < 0.) GO TO 15

CHECK FOR CONVERGENCE

AD = DABS (TH - T0)
IF (AD < LE - 2.0) GO TO 15
TH = 0.5 * (TO + TN)
IF (IT < GT - 30) GO TO 998
TO = TH
C NEW PROPERTIES
0160 CON=FiLiN(TM,TEMP,COND,10)  POI02130
0161 CA=FiLiN(TM,TEMP,CAP,10)  POI02140
0162 BN=FiLiN(TM,TEMP,HNO,10)  POI02150
0163 GO TO 10  POI02160
0164 11 TN=TMAX  POI02170
0165 15 TY(J)=TN  POI02180
0166 GO TO 688  POI02190
0167 590 WRITE(6,290) J,TM,TU,TN  POI02200
0168 290 FORMAT(1X,22TEMP DOES NOT CONVERGE,J=",12," TM,TU,TN=",F9.2)  POI02210
0169 *** TM IS USED ***
0170 TY(J)=TH
0171 880 CONTINUE  POI02220
0172 WRITE(4,295) (J,TY(J)),J=1,NPN  POI02230
0173 295 FORMAT(1U0,92,2,1J,1,3X))  POI02240
0174 WRITE(56) THW(I),TY  POI02250
0175 777 CONTINUE  POI02260
0176 DO 667 J=1,NINC  POI02270
0177 667 TFEN(K+1,J)=TY(J)  POI02280
0178 GO 350 1=1,N  POI02290
0179 350 THW(I)=THW(I)+TLAST  POI02300
0180 CONTINUE  POI02310
0181 GO TO 870  POI02320
0182 999 CONTINUE  POI02330
0183 STOP  POI02340
0184 END  POI02350
FUNCTION FILLIN(X, AB, OR, NO)
C *** FILLIK *** LINEAR INTERPOLATION
C FINDS Y(X) FROM TABLE OF
C AB(N) AND OR(N) CONTAINING NO POINTS.
0002 IMPLICIT REAL*8(A-H,O-Z)
0003 DIMENSION AB(N), OR(NO)
0004
C IF(X.GE.AB(1)) GO TO 10
0005 WRITE(6,3001)
3001 FORMAT('ERROR TEMPERATURE OUTSIDE RANGE OF MATERIAL PROPERT')
18TH TEMPERATURES (FUNCTION FILLIN)
0007 STOP
C DETERMINE INTERPOLATION FACTOR
C
0604 10 L=0
0605 DO 20 K=2, NO
0606 L=L+1
0607 DUM=AB(K)
0608 IF(X.EQ.NO) DUM=AB(NO)
0609 IF(X.GT.DUM) GO TO 20
0610 GO TO 25
0611 20 CONTINUE
0612 WRITE(6,3001)
0613 STOP
C
0618 25 XRATIO=(X-AB(L))/(AB(L+1)-AB(L))
C IF(XRATIO.GT.1.0) XRATIO=1.0
0620 IF(XRATIO.LT.0.0) XRATIO=0.0
C
C INTERPOLATE THE MATERIAL PROPERTY
C
0621 FILLIN=OR(L)+XRATIO*(OR(L+1)-OR(L))
0622 RETURN
0623 END
APPENDIX C

DERIVATION OF SCHEMATIC CONTINUOUS COOLING TRANSFORMATION DIAGRAMS

A brief description of the methods proposed by Grange and Kiefer [67] and Manning and Lorig [68] for the derivation of schematic CCT diagrams based on isothermal data are presented in this Appendix. The necessary modifications to enable one to use log-linear as well as the traditional linear cooling curves are also outlined. Finally, the computer implementation (made on an IBM 370 machine) is discussed.

C.1 The Grange-Kiefer Approach

A part of the isothermal transformation diagram showing the starting curve for an allotropic transformation (say austenite to bainite) is shown in Figure C.1. A cooling curve is superimposed on the diagram starting from the $A_\lambda$ (eutectoid) temperature assuming that the transformation product is bainite*. The cooling curve intersects the isothermal starting curve at the point $x$ corresponding to a temperature $\theta_x$ and a time $t_x$ (time zero is defined as the time at which the cooling curve intersects the $A_\lambda$ temperature). An arbi-

*Note that the starting temperature should be $A_3$ if the primary product is ferrite and $A_{cm}$ if proeutectoid cementite.
FIGURE C.1 Schematic Representation of the Grange-Kiefer Method.
trary lower point 0 is taken on the cooling curve corresponding to temperature \( \theta_0 \) and time \( t_0 \). To describe what happens between points \( x \) and 0, two assumptions are made:

1. The extent of transformation of the austenite at the instant it cools to the intersection point \( x \) is not substantially different than it would have been if quenched instantly to \( \theta_x \).

2. On cooling through the limited temperature range \( \theta_x \) to \( \theta_0 \), the amount of transformation is approximately equal to the amount indicated by the isothermal diagram at the mean temperature \( \theta_* = \frac{\theta_x + \theta_0}{2} \) after a time interval \( t_* = t_0 - t_x \).

Based on the second assumption, point 0 will be on the starting curve of the CCT diagram if point * is on the starting curve of the TTT diagram. It is therefore evident that a number of iterations are required for the determination of a point on the starting curve of the CCT diagram, a fact that makes the process ideally suited for computer implementation. Furthermore, the whole starting CCT curve can be obtained by joining several points obtained using different cooling curves.

C.2 The Manning-Lorig Approach

This approach is based on the rule of additivity. The cooling curve is approximated by a step curve so that for a time interval \( \Delta t_i \) the temperature is assumed to be constant and equal to \( \theta_i \). This time interval \( \Delta t_i \) is then divided by
the time $t_1$ required for the transformation to begin isothermally at temperature $\theta_1$ (see Figure C.2), and the resulting quantity is considered to represent a fraction of the total nucleation time required. When the sum of a number of such fractions equals one, that is, when

$$\sum_{t=0}^{t_1} \frac{\Delta t_i(\theta)}{t_1(\theta)} = 1$$

then transformation commences. The point on the cooling curve at which this occurs is thus considered to be a point on the starting curve of the CCT diagram. By employing several cooling curves, the starting CCT curve can be adequately described.

It is again evident that the nature of the method makes it appropriate for computer implementation. Furthermore, the note regarding the $A_1$ temperature (which can be $A_3$ or $A_{cm}$ depending on the nature of the transformation) mentioned in the Grange-Kiefer approach holds true here as well.

Finally, it should be mentioned that when the sum of the fractions does not reach 1 before the $M_S$ temperature is encountered, then only the martensitic transformation takes place. This will happen when the cooling rates are relatively high, exceeding the so-called critical cooling rate.
FIGURE c.2 Schematic Representation of the Manning-Lorig Method.
C.3 Log-linear Cooling Rates

As noted in Section 3.4.1, the cooling stage of the welding temperature history can be approximated by a linear curve in a \( \theta - \log_{10} t \) graph, and thus represented by an equation of the form

\[
\dot{\theta} = A \log_{10} t + B
\]

where \( A \) and \( B \) are parameters chosen to curve-fit representative welding temperature curves using the least-squares method (see Table C.1).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1000.0</td>
<td>2000.0</td>
</tr>
<tr>
<td>- 780.0</td>
<td>1960.0</td>
</tr>
<tr>
<td>- 700.0</td>
<td>1980.0</td>
</tr>
<tr>
<td>- 650.0</td>
<td>2050.0</td>
</tr>
<tr>
<td>- 535.0</td>
<td>1985.0</td>
</tr>
<tr>
<td>- 500.0</td>
<td>2000.0</td>
</tr>
<tr>
<td>- 463.3</td>
<td>2000.0</td>
</tr>
<tr>
<td>- 425.1</td>
<td>2000.0</td>
</tr>
<tr>
<td>- 384.3</td>
<td>2000.0</td>
</tr>
</tbody>
</table>

Note: The parameters \( A \) and \( B \) pertain to the log-linear equation \( \theta = A \log_{10} t + B \).
From the discussion of the two previous sections it is moreover evident that cooling curves of this type are appropriate and compatible with both methods, the only difficulty being lengthier calculations, a fact that does not pose any problems when a computer is used.

C.4 Computer Implementation

A double precision computer program was written using the FORTRAN IV language to implement the two methods of calculating schematic CCT diagrams. Some notes on the program follow:

1. The TTT diagram is expressed in terms of N data points. Since the program consequently approximates the transformation curve with linear segments, care should be taken in spacing the input data points closer in the regions where abrupt slope changes occur, as for example, near the nose of the diagram.

2. The log-linear cooling curve is approximated by a piecewise linear curve when the Grange-Kiefer approach is used so that its intersection with the TTT curve can be calculated.

3. About 10 cooling curves are adequate for determining the CCT diagram.

4. The time step size, \( \Delta t \), in the case of linear cooling curves, and the temperature step size, \( \Delta \theta \), in the case of log-linear ones, should be selected according to the trans-
formation times depicted in the TTT diagram. For example, short times require relatively small step sizes. In the case of HY-130 Δt was chosen to be from 0.2 to 2.0 sec., increasing with decreasing cooling rates; whereas Δθ was chosen to be from 5.0 to 2.0°F respectively.

5. The maximum number of iterations allowed for the selection of point 0 in the Grange-Kiefer approach was set to 100.

6. The convergence criterion in the Grange-Kiefer approach for the selection of point 0, i.e., the distance between $t_*$ and the time of the point on the TTT curve at temperature $\theta_*$, was set equal to 1 sec. Use of a smaller convergence criterion will usually require a larger maximum number of iterations allowed.
APPENDIX D

EVALUATION OF COMBINED THERMAL
AND TRANSFORMATION STRAIN

The equations developed for the evaluation of the combined thermal and transformation strain will be discussed in this Appendix. As mentioned in Section 5.4.2 this strain was used to replace the conventional thermal strain, $e_{TH}^{rs}$, in the thermo-elastic-plastic and creep constitutive model incorporated into the finite element stress analysis program ADINA.

The heating and cooling stages of the welding cycle will be treated separately. Furthermore, and given the incremental nature of the solution process, a time step will be considered that brings the solution from time $t$ to time $t+\Delta t$. Finally, for each of the three terms in equation (5.13), namely $e_1$ for the thermal strain in a mixture of phases, $e_2$ for the transformation strain, and $e_3$ for the thermal strain in a single phase, a separate equation will be established.

D.1 Heating Stage

Depending on the relative position of the temperatures $t_0$ and $t+\Delta t_0$ with respect to $A_{C1}$ and $A_{C3}$, the following cases are possible.

D.1.1 $t_0 < t + \Delta t_0 < A_{C1}$

No transformation has started taking place, so the
structure is still martensitic. Therefore:

\[ t + \Delta t_{e_1} = 0 \quad (D.1a) \]

\[ t + \Delta t_{e_2} = 0 \quad (D.1b) \]

\[ t + \Delta t_{e_3} = \alpha_M (t + \Delta t_\theta - \theta_R) \quad (D.1.c) \]

where \( \alpha_M \) is the thermal expansion coefficient of martensite and \( \theta_R \) the ambient temperature.

**D.1.2** \( A_{C1} < t + \Delta t_\theta \leq A_{C3} \)

Austenitization has started at the \( A_{C1} \) temperature, and has already produced a fraction \( f^A \) of austenite with the remaining \( 1-f^A \) still being martensite. Using eqn. (5.11) for the calculation of \( f^A \), we have

\[
t + \Delta t_{e_1} = \alpha_A \cdot \int_{A_{C1}}^{t + \Delta t_\theta} f^A \cdot d\theta + \alpha_M \cdot \int_{A_{C1}}^{t + \Delta t_\theta} (1-f^A) \cdot d\theta
\]

or

\[
t + \Delta t_{e_1} = \left[ \alpha_A - \left( \frac{A_{C3}}{A_{C3} - A_{C1}} \right)^2 (\alpha_A - \alpha_M) \right] \cdot (t + \Delta t_\theta - A_{C1})
\]

\[
- (\alpha_A - \alpha_M) \cdot \frac{(t + \Delta t_\theta^3 - A_{C1}^3)}{3} - \frac{(t + \Delta t_\theta^2 - A_{C1}^2)}{A_{C3} - A_{C1}^2} \cdot A_{C3}
\]

\[
t + \Delta t_{e_2} = \left\{ 1 - \left[ \frac{A_{C3} - t + \Delta t_\theta}{A_{C3} - A_{C1}} \right]^2 \right\} \cdot e^{TR}_{M+A} \quad (D.2b)
\]
\[ t + \Delta t_{e3} = \alpha_M (A_{cl} - \theta_R) \] (D.2c)

where all variables have been defined in Section 5.4.

D.1.3 \[ A_{c3} < t + \Delta t \theta \]

Full austenitization has taken place when the temperature is above \( A_{c3} \). One therefore obtains

\[ t + \Delta t_{e1} = \alpha_A \int_{A_{cl}}^{A_{c3}} f^A d\theta + \alpha_M \int_{A_{cl}}^{A_{c3}} (1-f^A) d\theta \]

or

\[ t + \Delta t_{e1} = \left[ \alpha_A - \left( \frac{A_{c3}}{A_{c3} - A_{cl}} \right)^2 (\alpha_A - \alpha_M) \right] \cdot (A_{c3} - A_{cl}) \]

\[ - (\alpha_A - \alpha_M) \cdot \frac{(A_{c3} - A_{cl})/3 - A_{c3} \cdot (A_{c3}^2 - A_{c3}^2)}{[A_{c3} - A_{cl}]^2} \] (D.3a)

\[ t + \Delta t_{e2} = e_{TR}^{TR} \]

\[ t + \Delta t_{e3} = \alpha_M (A_{cl} - \theta_R) + \alpha_A (t + \Delta t \theta - A_{c3}) \] (D3.b)

D.2 Cooling Stage

The equations for the cooling stage of the temperature cycle depend on the maximum temperature reached, \( \theta_{\text{max}} \), during the heating phase. They have been derived in such a way that the total thermal and transformation strain can be written
\[ t+\Delta t_{e_{rs}} = (e_{\text{max}} + e_1 + e_2 + e_3) \cdot \delta_{rs} \tag{D.4} \]

where \( e_{\text{max}} \) is the total strain at the maximum temperature reached and all other variables have been previously defined.

### D.2.1 \( \theta_{\text{max}} \leq A_c1 \)

Cooling started before the temperature of the point under consideration has exceeded the \( A_c1 \) temperature. As a consequence, no austenitization took place, so

\[ t+\Delta t_{e_1} = 0 \tag{D.5a} \]

\[ t+\Delta t_{e_2} = 0 \tag{D.5b} \]

\[ t+\Delta t_{e_3} = \alpha_M \cdot (t+\Delta t_{\theta}-\theta_{\text{max}}) \tag{D.5c} \]

### D.2.2 \( A_c1 < \theta_{\text{max}} \)

Since the maximum temperature has exceeded the \( A_c1 \) one, at least partial austenitization has taken place. The austenite can therefore (under the conditions described in Section 5.4.1) be transformed upon cooling into bainite and/or martensite. To study the various possibilities, each of the three terms \( e_1 \), \( e_2 \), and \( e_3 \) will be considered separately.

#### D.2.2.1 Thermal Expansion of the Untransformed Martensite.

Depending on the relative magnitude of \( \theta_{\text{max}} \) with respect to the \( A_c3 \) temperature, two subcases are possible.
(1) $A_{C3}^{\theta_{\text{max}}}$: Full austenitization has taken place upon heating, so

$$t + \Delta t_{e_3} = 0 \quad (D.6a)$$

(2) $A_{C1}^{\theta_{\text{max}}} \leq A_{C3}$: Since only partial austenitization took place upon heating, there is a fraction of untransformed martensite equal to $1 - f^A$ giving

$$t + \Delta t_{e_3} = (1 - f^A) \cdot \alpha_M \cdot (t + \Delta t_{\theta - \theta_{\text{max}}}) \quad (D.6b)$$

D.2.2.2 Transformation Expansion

Four subcases are considered for the calculation of the transformation expansion.

(1) $B_S \leq t + \Delta t_{\theta}$: The temperature has not yet reached the bainite start transformation one upon cooling, therefore

$$t + \Delta t_{e_2} = 0 \quad (D.7a)$$

(2) $M_S \leq t + \Delta t_{\theta} < B_S$: The existing fraction of austenite, $f^A$, can be transformed into bainite within this temperature region if the precipitation starting time is exceeded (refer to Section 5.4.1), so

$$t + \Delta t_{e_2} = f^A \cdot f^B_P \cdot \left[ 1 - \left( \frac{t + \Delta t_{\theta - M_S}}{B_S - M_S} \right)^2 \right] \cdot e_{T_R}^{A+B} \quad (D.7b)$$

where $f^B_P$ is the final fraction of bainite to be transformed at the $M_S$ temperature for the cooling curve under consideration.
(3) $M_f \leq t + \Delta t_0 < M_s$: Any remaining austenite (not previously transformed into bainite) will be transformed to martensite in this temperature region, the transformation extent being a function of the undercooling below the $M_s$ temperature. It can thus be written

$$t + \Delta t \epsilon_2 = f_A \left\{ f_B e^{T_R} \left[ (1-f_B) \left[ 1 - \left( \frac{(t + \Delta t_0 - M_f)}{M_s - M_f} \right) \right] e^{T_R}_{A\to M} \right] \right\}$$

(D.7c)

(4) $t + \Delta t_0 < M_f$: All transformations have already been made and there may still be some remaining austenite. Eqn. (D.7c) also holds for this subcase.

D.2.2.3 Thermal Expansion of Mixed Region

The same four subcases considered in Section D.2.2.2 are distinguished here as well.

(1) $B_s \leq t + \Delta t_0$: Only the transformed fraction of austenite is present, so

$$t + \Delta t \epsilon_1 = f_A \alpha_A (t + \Delta t_0 - \theta_{max})$$

(D.8a)

Note that the possibly present untransformed martensite has already been taken into account in eqn. (D.6).

(2) $M_s \leq t + \Delta t_0 < B_s$: To arrive at this temperature from $\theta_{max}$ the point under consideration went through a temperature region (from $\theta_{max}$ to $B_s$) where only the transformed austenite
was present and a region (from \( B_s \) to \( t+\Delta t_\theta \)) in which the fraction of bainite was continuously increasing accompanied by a concurrent decrease in austenite. Therefore

\[
t+\Delta t e_l = f^A \left\{ \alpha_A \cdot (B_s - \theta_{\text{max}}) \right. \\
\left. + \int_{B_s} \left[ \alpha_B f^B + \alpha_A (1-f^B) \right] \cdot d\theta \right\}
\]

or, using eqn. (5.11)

\[
t+\Delta t e_l = f^A \left\{ \alpha_A \cdot (B_s - \theta_{\text{max}}) \right. \\
\left. + \left[ \frac{f_B}{f_F} \cdot (\alpha_B - \alpha_A) + \alpha_A - \frac{f_B}{(B_s - M_s)^2} \cdot \left( \frac{1}{3} \cdot (t+\Delta t_\theta^2 + t+\Delta t_\theta \cdot B_s + B_s^2) + M_s \cdot (M_s - t+\Delta t_\theta - B_s) \right) \right] \right\} \\
\left( t+\Delta t_\theta - B_s \right)
\]

\[ (D.8b) \]

(3) \( M_f \leq t+\Delta t_\theta < M_s \): Similar thoughts to the ones described in the previous subcase lead to the following equation

\[
t+\Delta t e_l = f^A \left\{ \alpha_A \cdot (B_s - \theta_{\text{max}}) \right. \\
\left. + \int_{B_s} ^{M_s} \left[ \alpha_B f^B + \alpha_A (1-f^B) \right] \cdot d\theta \right. \\
\left. + \int_{B_s} ^{M_s} \left[ \alpha_B f^B + \alpha_A (1-f^B) \right] \cdot d\theta \right\}
\]
\[ + f_F^B \alpha_B \cdot (t + \Delta t_{\theta - M_S}) \]

\[ + \int_{M_S}^{t + \Delta t_{\theta}} \left[ \alpha_M f^M + \alpha_A (1 - f_F^B - f^M) \right] \cdot d\theta \]

where \( f^M \) is the fraction of transformed martensite and is given by

\[ f^M = (1 - f_F^B) \left[ 1 - \left( \frac{\theta - M_f}{M_S - M_f} \right)^2 \right] \]

After some algebra one finds

\[ t + \Delta t_{e_1} = f_A \left\{ \alpha_A \cdot (B_S - \theta_{\text{max}}) \right. \]

\[ + \left[ \frac{2}{3} f_F^B (\alpha_B - \alpha_A + \alpha_A) \cdot (M_S - B_S) \right. \]

\[ + \left. \alpha_B f_F^B \cdot (t + \Delta t_{\theta - M_S}) \right. \]

\[ + \left. (1 - f_F^B) \left[ \alpha_M + \frac{\alpha_A - \alpha_M}{(M_S - M_f)^2} \cdot (t + \Delta t_{\theta})^2 + \right. \right. \]

\[ \left. \left. t + \Delta t_{\theta} \cdot \frac{M_S + M_S^2}{3} + M_f \cdot (M_f - t + \Delta t_{\theta - M_S}) \right] \right] \cdot \left( t + \Delta t_{\theta - M_S} \right) \]

\[ \right\} \quad \text{(D.8c)} \]

(4) \( t + \Delta t_{\theta < M_f} \): In this subcase eqn. (D.8c) also holds if the variable \( t + \Delta t_{\theta} \) is replaced by \( M_f \) and the additional
term

\[ f^A \alpha_M (1 - f^B_F) (t + \Delta t_\theta - M_F) \]

is taken into account.