SMALL SIGNAL CONTROL OF
MULTITERMINAL DC/AC POWER SYSTEMS

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ABSTRACT

A multiterminal dc (MTDC) network embedded in an interconnected ac power system can be used as an effective control element for damping interarea ac oscillations. This thesis proposes a systematic methodology for the analysis, synthesis and evaluation of small-signal MTDC modulation controllers, with special emphasis on the controllers' stability properties in the face of modeling uncertainties.

The controller-synthesis procedure is designed to be carried out on an aggregate classical-machine model of the power system. Analysis of the ac/MTDC design model is facilitated by techniques of modal decomposition. Methods for interpreting the modal structure are given, and special tools for studying quantitatively the controllability and observability of the system are developed.

The MTDC controller used in this research is a non-adaptive, static, output-feedback compensator which uses the frequency of the commutation buses as measurement signals. The feedback gains are computed in three steps: 1) identification of the interarea modes that are most controllable and observable by the MTDC system, 2) stabilization of these modes using a full-state linear quadratic control law, and 3) finding the output-feedback gains which are least-squares approximations to the linear quadratic gain matrix. The design procedure is easy to use and it allows the user to exploit the physical insights in the design process.

The design methodology is demonstrated on a model of the Western U.S. power system in which a 7-terminal MTDC network is embedded in an ac system with 42 machines. Results show that the controller designed based on a 10-area aggregate model give very similar damping improvements in the interarea modes when the controller is implemented on the 42-machine model. Moreover, the inter-machine modes that were ignored in the design model are not affected appreciably by the feedback controller.
The simulation results of this system for several information structures of varying degrees of decentralization reveal that the centralized feedback design is able to achieve the performance objectives with a minimum of control effort, and the control effort increases with a decrease in communication between dc terminals. The completely decentralized design is found to require significantly more control efforts than the centralized design, and the differences are attributed to the lack of coordination that is essential in an MTDC system operated under the current-margin scheme.

The design procedure is also demonstrated on a 9-machine, 4-terminal ac/MTDC system in which some of the machines are modeled in detail and they are equipped with exciters, power-system stabilizers and turbine governors. The centralized output-feedback design computed based on a classical-machine representation of this system is also shown to give similar damping improvements in the full model, and the unmodeled machine dynamics are not affected appreciably by the controller.

Robustness evaluation techniques are developed for ac/MTDC power systems using some of the recent results in multi-input, multi-output robustness theory. It is shown that the robustness margins of a power system computed at several different points can be interpreted physically in the frequency domain. The applicability of these robustness margins for checking tolerances for shaft torsional dynamics and dc-voltage variations is pointed out.

The robustness evaluation techniques are demonstrated on a 2-machine example and the 42-machine Western-U.S. example. The robustness margins associated with different output-feedback structures of the Western U.S. system show that the centralized design is more robustness (in a certain sense) than the other designs at the frequencies below the interarea frequencies.

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To my parents
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\( a \)  
\( A \)  
\( \text{col}[a_1, a_2, \ldots, a_n] \)  
\( \text{row}[a_1^T, a_2^T, \ldots, a_n^T] \)  
\( \text{diag}[a_1, a_2, \ldots, a_n] \)  
\( I \)  
\( A > 0 \)  
\( A \geq 0 \)  
\( \det(A) \)  
\( \sigma(A) \)  
\( \overline{\sigma}(A) \)  
\( \underline{\sigma}(A) \)  
\( |a| \)  
\( \|a\| \)  
\( \|A\| \)  
\( \text{Re}(\cdot) \)  
\( \text{Im}(\cdot) \)

- \( a \) column vector
- \( A \) matrix
- \( \text{col}[a_1, a_2, \ldots, a_n] \) a matrix whose jth column is \( a_j \)
- \( \text{row}[a_1^T, a_2^T, \ldots, a_n^T] \) a matrix whose jth row is \( a_j^T \)
- \( \text{diag}[a_1, a_2, \ldots, a_n] \) a diagonal matrix
- \( I \) the identity matrix
- \( A > 0 \) matrix \( A \) is positive definite
- \( A \geq 0 \) matrix \( A \) is positive semi-definite
- \( \det(A) \) determinant of matrix \( A \)
- \( \sigma(A) \) singular value of matrix \( A \)
- \( \overline{\sigma}(A) \) maximum singular value of matrix \( A \)
- \( \underline{\sigma}(A) \) minimum singular value of matrix \( A \)
- \( |a| \) magnitude of a scalar \( a \)
- \( \|a\| \) norm of a vector \( a \)
- \( \|A\| \) norm of a matrix \( A \)
- \( \text{Re}(\cdot) \) real part of a complex number or matrix
- \( \text{Im}(\cdot) \) complex part of a complex number or matrix

SUPERSCRIPTS

- \( * \) complex conjugate of a complex matrix, vector or number
- \( T \) transpose of a vector or matrix
- \( H \) complex conjugate transpose of a vector or matrix
CHAPTER 1
INTRODUCTION

1.1 MOTIVATIONS

High-voltage dc power transmission was introduced in 1954 for the purpose of transmitting power from the Swedish mainland to the island of Götland. The Götland DC Project ushered in an era of intense development efforts in applying dc transmission to situations where conventional ac connections had not been technically or economically feasible. These included long-distance point-to-point transmission, asynchronous connections between power systems, and transmission via underground and submarine cables. To date, over 24 2-terminal dc links are in operation or under construction worldwide, with a total capacity of about 30 000 MW [1,2].

Aside from the bulk-power delivery functions, a dc link can be utilized as an extremely effective control element for damping interarea ac oscillations. Interarea oscillations are electromechanical swings in the 0.1-1.0 hz range, involving two or more areas connected by long, relatively weak ties. The 0.3hz oscillation between the Pacific Northwest and the Pacific Southwest power systems is an excellent example. Interarea oscillations are usually stable, but as interconnections between power systems became widespread in the 1960's, some of the interarea oscillations became lightly damped — and even unstable in some systems. The instability problems observed in these systems were traced to interactions between the interarea swings and the generators' controllers.
The utility industry responded by modifying the generators' dynamic characteristics through supplementary feedback elements known as power-system stabilizers. The deployment of power system stabilizers was successful initially. But unstable interarea oscillations recurred in some systems in spite of these corrective measures.

The advent of 2-terminal dc modulation offered a direct means for controlling oscillations between the two areas connected by the dc link. The concept of dc modulation was introduced by Uhlmann in 1964 [3], and was quickly followed by actual implementations at the Pacific DC Intertie [4], the Square Butte Project [5,6], the Nelson River Project [7] and other installations [7,8]. Experiences with dc modulation indicate that low-level control signals superimposed on the steady-state power schedule can indeed effect drastic improvements in ac system stability [4-8].

Successful applications of 2-terminal dc systems suggest that even greater flexibility in power dispatch and enhancement in stability can be realized by a multiterminal dc (MTDC) system where three or more dc converters are interconnected by a common dc network [9]. The work in this thesis is focused on the stability enhancement capability of MTDC systems.

An MTDC system's effectiveness in controlling interarea oscillations stems from the inherent fast response time of the dc converters, and from its ability to influence directly the interarea power-flow pattern with a large number of degrees of freedom. The effectiveness of the controller and the large
geographical dispersion of the sensors and actuators, however, create many design issues that are unique to MTDC systems. Methodologies developed for 2-terminal dc system are, in general, not applicable to MTDC controllers. The reason is that 2-terminal methodologies are based on well-established single-input, single-output design techniques which have no straightforward extension to the multiterminal case. Nevertheless, experiences with 2-terminal dc controllers can be useful for certain aspects of the MTDC controller problem. For instance, recent instability problems at the Pacific DC Intertie [4] and shaft torsional interactions at the Square Butte Project [10] suggest that similar -- if not more serious -- problems can arise in MTDC networks if proper precautions are not taken in the control-system design process.

In designing an MTDC controller, the primary objective is to improve as much as possible the damping of interarea, ac-system oscillations using reliable and readily available measurement signals. Design constraints in this problem take several forms. First, the amplitude limit of the control signals implies that the performance objectives must be achieved with a minimum of dc current fluctuations. Secondly, the large geographical separation of the sensors and actuators favors a feedback scheme that requires a minimum of communication between dc terminals. Finally, the system must be closed-loop stable in the face of modeling errors and changes in operation condition. The combination of these constraints makes the MTDC control problem a rather difficult

INTRODUCTION

1.1 MOTIVATIONS
one.

The purpose of this thesis is to formulate a practical and systematic methodology for designing MTDC small-signal controllers. The term "methodology" here refers to a procedure that is not system specific and is applicable to all MTDC systems. The methodology developed in this research has its foundation on a number of disciplines, ranging from theories of synchronous machines and dc power systems to techniques for multivariable controller design and robustness evaluation. The knowledge in these diverse fields are unified with the objective of understanding the physical characteristics of MTDC/ac power systems and exploiting the physical insights in the controller-synthesis procedures.

Major contributions in this work include new techniques for modeling and analysis of ac/MTDC power systems, and for the synthesis of modulation controllers. A number of important tradeoffs between communication and performance are also uncovered in the course of this research. These results are documented in part I or this thesis. Another major contribution of this thesis is a new conceptual framework for evaluating the stability margins of a multi-machine power system. The development of this framework and its applications to the MTDC problem are presented in part II. The results in these two parts represent the first comprehensive study of MTDC modulation control.

The two parts of this thesis are self-contained and can be read independently. A synopsis of the major results is
given in section 1.3. A more detailed summary of the results and their implications can be found at the end of each chapter.
1.2 RELATIONS WITH PREVIOUS LITERATURE

The research of this thesis must be viewed with respect to a background of literature on MTDC power transmission, multivariable control and robustness theory, and multivariable design methods.

On the area of MTDC power transmission, papers abound on subjects such as current-order coordination, dc fault clearing and system configuration (series vs. parallel). An excellent bibliography on these topics can be found in a recent paper by Reeve [9]. On the subject of MTDC modulation control, two papers are noteworthy. The first is a paper by Carter et al. [11] where the frequency deviation of the dc terminals' commutation voltages are utilized as measurement signals. Unfortunately, the authors did not give any guidelines for selecting feedback gains, nor any justifications for the completely decentralized feedback structure. Sharaf and Mathur, in a more recent paper [12], advocated the use of linear quadratic (LQ) methodology for computing the feedback gains. They concluded on the basis of a 3-machine system that a feedback scheme using the machines' torque angles and shaft speeds is effective in providing ac-system damping. A close examination of this work, however, reveals that the authors did not make use of the physical characteristics of the MTDC/ac power system in their design procedure, and the design example given in the paper was done without any regard to the bandwidth limitation of the system model. A more detailed critique of this method can be found in a technical discussion written by

INTRODUCTION  1.2 RELATIONS WITH PREVIOUS LITERATURE
the present author [13].

The research in this thesis makes use of some of the ideas in these earlier works. For example, the frequency of the commutation buses is selected as the measurement signal because of its advantages in reliability and communication requirements. However, the fact that the LQ technique is used in both this thesis and Sharaf's work [12] is a pure coincidence. The methods in this thesis are radically different from those detailed in Sharaf's paper.

The model simplification technique used to construct the design model is based on the idea of coherency. A number of papers on coherency can be found in the literature (see, for example, the bibliography in [25]). The different coherency methods differ in their criteria for coherency and the techniques for identifying the coherent groups. The slow coherency method due to Winkleman, Chow et al. [24] is chosen for the proposed methodology because it is not disturbance specific, and the result is not a strong function of the operating condition.

A great deal of physical insight is gained by viewing a multi-machine system as a mass-spring system. In this respect, knowledge of elementary concepts from Newtonian mechanics is useful. Physical interpretations are augmented by the method of modal decomposition. Most of the techniques such as Argand diagrams and physical interpretation of eigenvectors are fairly well known [14]. However, quantification of controllability and observability in the modal domain, to

INTRODUCTION 1.2 RELATIONS WITH PREVIOUS LITERATURE
the author's knowledge, has never been documented in detail. The unit-momentum scaling of the eigenvectors for the study of power system controllability and observability is an original contribution of this thesis.

The physical analysis of the system in the modal domain motivates a controller synthesis methodology based on a linear quadratic (LQ) state-feedback design with modal penalties. The LQ theory is well known and can be found in any one of a number of standard textbooks [28]. For the MTDC problem, a modal weighting method due to Solheim [31] is found to be particularly useful. The output feedback designs are computed using the least-squares or minimum-norm approximation to the LQ gain matrix. This minimum-norm approach to output-feedback design with information-structure constraints is due to Kosut [47].

The robustness criteria used in this thesis have their theoretical foundation in the work of Safonov [15]. Specific matrix-norm robustness criteria for linear systems were developed independently by Doyle [16] and later by Barrett [17], but these criteria can be shown to be special cases of Safonov's results. Recently, Lehtomaki [18] achieved some unification of the criteria of Doyle and Barrett in the framework of matrix bilinear fractional transformations. In addition, Lehtomaki showed that certain "directional information" on the destabilizing perturbations can be computed in terms of the singular-value decomposition of $I+G$ and $I+G^{-1}$, (and other functions of $G$) where $G$ is the loop transfer matrix.
The robustness research in this thesis utilizes the results of Lehtomaki [18] in conjunction with the familiar concept of damping and synchronizing torques for synchronous machines. The idea of decomposing the generator's electrical torques into these two orthogonal components appears to have a venerable history, dating back to as far as 1931 [19]. The first use of this concept for designing power-system stabilizers is due to de Mello and Concordia [20], but applications thus far have been largely qualitative in nature and are limited to single-machine studies. The implications of frequency-dependent torques, to the author's knowledge, have never been investigated. The work in this thesis makes precise the notion of synchronous and damping torques for single- and multi-machine settings, and develops stability and robustness tests in terms of these concepts.

The research here represents one of the first attempts in applying the recently developed robustness theory for multi-variable control systems to the power-system problem. The only work that is similar in scope is the research being conducted at Systems Control, Inc. (SCI) [21]. SCI's research is focused on the variability of power-system components subject to changes in operating condition and its impact on the dynamic stability of the power system. One of their important results is the decomposition of power-system components in the form shown in figure 1.1. The elements $G_i(s)$ in this figure are dynamic components that are operating-point invariant, and the elements $K_i(s)$ are static components that are function of
the operating condition. By lumping all the \( G_i(s) \) into a block \( H_G(s) \), and all the \( K_i \) into \( K \), researchers at SCI showed that the system in figure 1.1 can be redrawn in the form shown in figure 1.2. Note that in figure 1.2, only the "feedback" element \( K \) is a function of the operating point. Robustness of this model was determined using a matrix-norm criterion, with the loop broken at the point marked "X" in figure 1.2.

Numerical test on a third-order generator model, however, showed that the robustness criterion was violated even for a 5% change in real- and reactive-power output. In the author's opinion, the conservatism is due in large part to the fact that the system components \( K \) and \( H_G(s) \) contain, by necessity, many blocks of "hard zeros" where perturbations in the form of cross-feed are physically impossible. A system representation of this type is simply not suitable for the matrix-norm robustness criteria, because the robustness margins may be associated with very small destabilizing perturbations which involve cross-feeds at the hard-zero blocks.

The weakness of SCI's approach points out that the applicability and conservatism of robustness tests are dependent to a large degree on the structure of the system model and how the modeling uncertainties are examined. One of the primary objective of this thesis is to overcome the problems of conservatism by examining power-system modeling uncertainties from a physical perspective, and to express stability margins in a way that is familiar to power-system analysts. The research here is not limited to modeling uncertainties due to

INTRODUCTION 1.2 RELATIONS WITH PREVIOUS LITERATURE
operation-point variations, but operating-point induced perturbations are considered as a special case.
Figure 1.1: SCI's power system model

Figure 1.2: Same system as figure 1.1, except for special grouping in blocks K and H_G.

INTRODUCTION 1.2 RELATIONS WITH PREVIOUS LITERATURE
1.3 SUMMARY OF RESULTS

A systematic methodology is formulated in this thesis for the design of MTDC modulation controllers. This methodology includes techniques for constructing the design model, analyzing the system dynamics, synthesizing the controllers and evaluating the closed-loop system's robustness.

1.3.1 Modeling

The model for the ac power system is simplified by aggregating machines into coherent areas (chapter 2). Specifically, the slow coherency method due to Winkleman, Chow, et al. [24] is used to identify the coherent groups, and a singular-perturbation technique is used to approximate the slow subsystem which corresponds to the interarea dynamics of interest. In addition to the slow-coherency method, the inter-machine modes that can be controlled readily by the MTDC system are included by breaking up certain coherent areas into smaller pieces. A method for identifying these inter-machine modes using the residue matrices is developed in this thesis (section 3.5).

Several types of measurement signals for the MTDC controller are evaluated in terms of their reliability and communication requirements (section 2.4). The frequency of the commutation buses is chosen in this research because it is reliable, and the measurements taken at a dc terminal can be transmitted to the other dc terminals via the centralized communication network of the MTDC system.
The dc system is modeled as a set of nodes that injects power into, or taking power out of, the ac network. The dynamics of the dc transmission system are ignored in the design model because its time constants are short compared to a period of interarea oscillation.

1.3.2 Analysis

The design model of the MTDC system is analyzed in the modal domain (chapter 3). For a system with n areas, there are n-1 pairs of oscillatory modes and two real modes. The interpretations of these modes are facilitated by the characteristics of the corresponding right eigenvectors. The oscillatory modes are interpreted as oscillations in which two groups of areas swing against each other. The real mode on the negative, real axis is interpreted as the dynamics of the average frequency of the ac power system, and the mode at the origin is interpreted as an indication of the arbitrariness of the reference for the machine angles.

The mode shape of the oscillatory modes is shown to be a reliable indication of the areas' participation in the oscillations when each component in the right eigenvector is weighted by the inertia of the corresponding area (section 3.3). The components in the weighted right eigenvectors are interpreted as the momentum of the areas.

New techniques are also developed for evaluating quantitatively the controllability and observability of the system (section 3.4). The success of these techniques is based on a special method for scaling the left and right eigenvectors.
The controllability of the \( i \)th mode given by this method can be interpreted as the jump in momentum immediately after implusive forces are applied to the control channels. The observability measure of the \( i \)th mode can be interpreted as the magnitude of the output signal, subject to an initial condition of unit momentum.

The norm of the residue matrix for each mode is found to be a useful indication of the modal controllability and the modal observability (section 3.5). In addition, modes with "large" residue matrices are found to coincide with the modes that are most easily controlled using the class of MTDC feedback controllers proposed in this thesis.

1.3.3 Controller Synthesis

A methodology is developed for designing an output-feedback controller which uses the frequencies at the commutation buses as measurement signals. A static, constant gain feedback controller is chosen for the MTDC problem because it is believed that a controller that is simple in structure is most suitable for controlling a large-scale power system whose design model may contain a significant amount of uncertainty (chapter 4). The controller's amplitude constraints and the existence of unstructured modeling perturbations at low frequencies are coped with by using the controller to stabilize only the oscillatory modes that are most controllable and observable by the MTDC system. It is shown that these mode can be identified using the norm of the associated residue

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1.3 SUMMARY OF RESULTS
matrices (section 3.5).

The feasibility of different feedback laws is investigated in terms of their communication requirements. It is found that regardless of the information structure, a fast communication network linking all the dc terminals to the voltage-setting terminals is necessary for preserving the current margin [37]. This communication requirement makes feasible the centralized feedback scheme in which the control signal at each dc terminal is computed using the measurement signals at all the other terminals. In order to study the tradeoffs between performance and communication, two other information structures are also investigated:

. Distributed Feedback - The control signal at each dc terminal is computed using the local measurement signal and the voltage-setting terminal's measurement signal.

. Decentralized Feedback - The control signal at each dc terminal is computed using the local measurement signal only.

The feedback gain in each of these output-feedback schemes is computed by finding a linear quadratic state-feedback design which stabilizes the modes that are most controllable and observable by the MTDC system, and approximating the state-feedback gain using a least-squares algorithm, subject to structure constraints (section 4.3).

The design methodology is demonstrated using a 10-area equivalent of a 42-machine, 7-terminal model of the Western U.S. system. When the output feedback controllers are implemented on the "full" 42-machine (classical machines) model,
the interarea modes of the closed-loop system are found to be very close to that of the 10-area design model. Moreover, the inter-machines modes that are neglected in the design model are found to be unaffected by the controllers (section 6.3).

A centralized feedback design is also computed for a 9-machine, 4-terminal ac/MTDC example in which two of the machines are modeled in detail, and they are equipped with exciters, power-system stabilizers and turbine governers. The controller is also designed using a classical-machine representation of the 9-machine system (section 6.5). The closed-loop poles of the classical-machine model are again found to be in very good agreement with those of the full model, and the closed-loop poles associated with the unmodeled dynamics (of the design model) are not affected appreciably by the controller.

The favorable results in these two examples are strong indications that an MTDC feedback controller can be designed based on an aggregate, classical-machine model of the interconnected power system.

The performance of the closed-loop system under different output-feedback schemes is evaluated using a linear simulation of the 42-machine Western U.S. example (section 6.4). The results show that while the damping characteristics are similar, the amplitude of the control signal is significantly higher in the decentralized case. The large differences in the amplitude -- and sometimes in sign -- between the control signals in the decentralized case and in the distributed case
make clear the importance of coordinating the control actions in an MTDC system operated under the current-margin scheme. The decentralized feedback is not effective since the control at a dc terminal affects both the power injections locally and at the voltage-setting terminal, but the measurement signal at the voltage-setting terminal is not available to the decentralized controller.

1.3.4 Robustness Evaluation

New techniques for evaluating the robustness of a power-system model is developed using some of the latest results in multi-input, multi-output robustness theory (chapter 7). The new techniques are devised such that the robustness margins and the associated minimum-norm perturbations can be interpreted physically. To achieve this goal, the robustness margins are checked at the torque input to the generator shafts, in addition to the usual practice of checking the robustness margins at the input channels (chapter 8). The robustness margins at the torque input are interpreted as tolerances for perturbations in damping and synchronizing torques, as well as tolerances for unmodeled shaft torsional dynamics. The robustness margins for variations in damping and synchronizing torques are found to be somewhat difficult to apply since no method is yet available for characterizing the variations of these torques in the frequency domain (section 9.2). The interpretation of the robustness margins as tolerances for shaft torsional dynamics is found to be a very promising application since the modeling errors due to shaft
torsional dynamics can be easily characterized (section 9.4). The robustness margins at the input channels are also shown to be a very useful tool for establishing the range of dc-voltage variations for which the closed-loop system is guaranteed to be stable.

The robustness margins are computed for the different output-feedback designs of the Western U.S. example (section 9.5). The results show that the robustness margins of the centralized scheme resemble that of the linear quadratic state-feedback design, and the robustness margins at the physical inputs and at the generator-shaft inputs decrease when the communication between dc terminals is reduced.

The results for the Western U.S. example suggest that the centralized feedback scheme is superior to the distributed and decentralized schemes in terms of the ease of control and robustness considerations.
PART I

ANALYSIS AND DESIGN
CHAPTER 2
SYSTEM MODELING

2.1 OVERVIEW

Modeling is an integral part of the controller design procedure. The modeling issue is of special importance to the MTDC problem since a detailed model of an ac/MTDC power system can be extremely large and complex, and in order to make the controller-synthesis problem tractable using multivariable controller design techniques, it is necessary to find a simplified model which retains the key dynamics of interest. A number of approximations in the ac and MTDC portions of the power system are necessary to arrive at the design model, as will be shown in section 2.2 and 2.3 of this chapter.

The modeling of various types of measurement signals are presented in section 2.4 along with a discussion on their relative merits in terms of the reliability and communication requirements.

In section 2.5, a state-space design model for the MTDC/ac power-system model is formulated.
2.2 DC SYSTEM MODEL

In the approximate model used in this work, the components of the dc system, including current/voltage regulators, smoothing reactors and dc transmission lines are not included. This assumption effectively reduces the dc-system model to a set of algebraic equations.

The question of how much details should be included in the dc-system model is still a topic of much debate (see for example [22] and [23]). Proponents of very detailed dynamic dc models argue that dc transients, depending on their amplitude and time rate of change, may trigger protective mechanisms and lead eventually to large changes in both ac and dc quantities. Therefore, dynamic models of dc transmission lines, current regulators, dc-current transducers, etc., are crucial for an accurate simulation -- especially when the initiating transient originated from the dc network. Others who are interested primarily in the performance of ac/dc systems subject to ac-system disturbances argue that a simple dc model which captures the slower dynamics (due to current regulator and/or smoothing reactors) would suffice. The other dynamics are much faster and have negligible effects on machine swings.

These arguments suggest that the requisite complexity of the dc-system model depends, to a large extent, on the intended application. For the MTDC control problem where low-frequency interarea oscillations are of primary interest, it seems reasonable to ignore the dynamics of the dc system
altogether in the design model. The study of large-scale disturbances initiated by failures in the dc system is beyond the scope of this thesis.

2.2.1 MTDC System Operation

It is instructive to review the operation of a parallel connected MTDC system operated under the current-margin scheme since concepts such as current margin and mode switching are important to the study of MTDC controller design. The description given in this section is brief and emphasizes only those aspects in MTDC operation that impact the control design problem. More details on dc converters and MTDC systems can be found in [44], [9] and [35].

Each dc converter can be modeled as a variable dc voltage source in series with a fictitious resistor. The open-circuit voltage of the dc voltage source is a function of the commutation voltage and the firing angle $\alpha$ or the extinction angle $\gamma$. More precisely, the dc voltage behind the fictitious resistor is $(1.35 E \cos \alpha)$ for a rectifier and $(1.35 E \cos \gamma)$ for an inverter, where $E$ is the rms, line-to-line ac voltage at the commutation bus. The fictitious resistor, $R_C$ for rectifiers and $-R_C$ for inverters, is needed to account for the voltage drop due to overlapping in commutation. The value of $R_C$ is a function of the inductance of the commutation transformer (see [44, ch.3] for more details).

An MTDC system can be operated in a number of ways. The current-margin scheme (also known as the constant-current
scheme) is selected for this research since it is the most widely studied and accepted method of operation. Under the current-margin scheme, the firing and extinction angles of the dc converters are controlled to yield steady-state voltage-current (v-i) characteristics shown in figure 2.1. It can be seen in this figure that the dc converter acts as a constant current source above a certain value of firing or extinction angle, and acts as a constant voltage source in series with an impedance when the firing or extinction angles are "pinned" at their minimum values.

A parallel connected MTDC system, as its name implies, is a system of dc converters connected in parallel by a common dc transmission grid. The method of operating such a system is illustrated using an MTDC system with 3 rectifiers and 2 inverters. The topology of the dc network is not important to the following discussion, since the dc network in the steady state is a resistive network with very low resistance (typical value is 0.03 ohm/km for a bipolar line).

Shown in figure 2.2 are the v-i characteristics of the dc converters. The operating point of the system is determined by the intersection of the v-i curves with a horizontal dotted line which represents the voltage of the dc network (small voltage drop due to the resistance of the network is ignored). The dc voltage is not arbitrary, since at the operating point, the converter currents must satisfy Kirchhoff's current law -- namely, the algebraic sum of all the terminal currents must be zero.
Note in figure 2.2a that the current setpoints (the vertical part of the v-i curve) is identical to the actual dc current at all the terminals except for terminal 4, which operates at minimum extinction angle and determines the voltage of the dc system. At terminal 4, the actual current is different from the setpoint by the amount, \( I_m \), known as the current margin.

It is important to note that terminal 4 controls the dc voltage only because it has the lowest commutation voltage. If another terminal's -- say terminal 1's -- commutation voltage drops to a sufficiently low value, the voltage-setting function is shifted automatically from terminal 4 to terminal 1, as is shown in figure 2.2b. The net effect of a mode shift -- aside from the change in dc voltage -- is a step increase in dc current at terminal 4 equal to the current margin, and a step decrease in dc current at terminal 1 equal to the current margin. Mode switching is important to the small-signal modulation problem in that it disables the current-modulation capability of the new voltage-setting terminal. A solution due to Nozari et al. [37] which maintains the same current modulation at all the dc terminals regardless of the location of the voltage-setting terminal is described in section 5.3.

Current modulation in an MTDC system is accomplished by varying the constant-current setpoint of the dc converters. When the current setpoints are modulated at the dc terminals, it is clear that the voltage-setting terminal's current setpoint must be changed accordingly to preserve the current
margin. This issue will be taken up in section 5.3.

The voltage of the dc system can also be modulated by varying the "minimum-angle" portion of the v-i characteristics at the voltage-setting terminal, but this capability is not utilized in this thesis because of several reasons.

The most important of these is that the modulation in dc voltage does not increase significantly the MTDC system's ability to control interarea oscillations. In a 2-terminal dc system, for example, raising the dc voltage and raising the dc current have the same effect of increasing the ac-power consumption at the rectifier and increasing the ac-power injection at the inverter. However, it should be noted that dc voltage modulation, if coordinated properly with dc current modulation, may be useful for ac voltage support at the commutation buses.

A difficulty in modulating dc voltage is that it affects the power output of all the dc terminals in such a way that the power fluctuation at a terminal can be positive or negative depending on whether the terminal is operating as an inverter or as a rectifier. To overcome this problem would require a feedback design for every possible combination of converter configurations, or \(2^p-2\) designs for an \(p\)-terminal system.

Another potential difficulty is that dc voltage modulations may give rise to mode switching and consequently jumps in converter current. It is conceivable that the dc voltage may be modulated to induce mode switching in a con-

SYSTEM MODELING

2.2 DC SYSTEM MODEL
trolled manner, but this large-signal control problem is beyond the scope of this thesis.
Figure 2.1: Steady state v-i characteristics of dc converters.
Figure 2.2: Operations of an MTDC system under the current-margin scheme.
2.3 AC SYSTEM MODEL

In developing an approximate model for the ac system, the aim is to minimize the order and complexity of the model while retaining the key electromechanical dynamics associated with interarea oscillations. The requirements in model reduction here differ from that of the traditional external-dynamic-equivalent problem in that the latter is concerned primarily with the approximation of the external system; whereas the MTDC problem requires a global approximation of the entire interconnected network.

A global approximation of the ac system is made possible by coherency grouping of generators. The coherency concept is motivated by the observation that certain groups of machines in an ac system tend to swing together during a transient. Different coherency identification techniques differ in their criteria for coherency, and their method for finding the coherent groups [25]. Of the many techniques presently available, the "slow coherency" method due to Winkleman, Chow, et al. [24] appears to be the most suitable for the MTDC problem. The advantages of this technique include the option of specifying a priori the number of coherent areas, and the property that the coherent areas are not disturbance specific and are not a strong function of the operating condition.

The slow-coherency method begins with a classical-machine representation of the original power system. The coherency grouping of machines is done in two steps. In the first step, n reference machines -- where n is the number of areas desired
-- are chosen to span the eigen-space of the slow oscillatory modes. In the second step, other machines are grouped "around" these reference machines to form coherent areas. The grouping matrix found in these steps effectively transforms the state variables of the system into slow variables, \( x_s \), which represent the center of inertia of the areas, and fast variables, \( x_f \), which represent the inter-machine dynamics. The transformed system can be written as follows.

\[
\dot{x}_s(t) = A_{11}x_s(t) + A_{12}x_f(t) + B_{11}u(t) \quad (2.1)
\]

\[
\dot{x}_f(t) = A_{21}x_s(t) + A_{22}x_f(t) + B_{21}u(t) \quad (2.2)
\]

\[
y(t) = C_{11}x_s(t) + C_{12}x_f(t) + D u(t) \quad . \quad (2.3)
\]

As the scalar \( \epsilon \) is driven to zero, the first-order approximation to the slow system is the following.

\[
\dot{x}_s(t) = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_s(t) + (B_{11} - A_{12}A_{22}^{-1}B_{21})u(t) \quad (2.4)
\]

\[
y(t) = (C_{11} - C_{12}A_{22}^{-1}A_{21})x_s(t) + (D - C_{12}A_{22}^{-1}B_{21})u(t) \quad . \quad (2.5)
\]

Equations (2.4) and (2.5) are used in the design of the MTDC controller, except that the "fast correction terms" \( C_{12}A_{22}^{-1}A_{21} \) and \( C_{12}A_{22}^{-1}B_{21} \) are omitted in the output-feedback design procedure. A detailed discussion of the fast-correction-term problem is given later in section 6.2.

In (2.4), the state variables can be interpreted as the angle and shaft speed of the fictitious machines that repre-
sent the slow motion of the n coherent areas. The design model (2.4) and (2.5) is therefore very similar to the linearized model of a classical-machine model of n synchronous machines. For this reason, the analysis techniques in this section are developed based on classical-machine models.

The use of the classical-machine based design model in this work is advantageous for the reason that the model is small enough to be tractable, and the compensators designed based on the simplified model work well when they are implemented in the full model. This design philosophy is verified in two parts. The first part is concerned with the approximation in machine aggregation, and the second part is concerned with unmodeled dynamics due to exciters, governors and other machine regulators. It will be shown in both of these parts that there is a reasonably good match between the closed-loop interarea pole locations in the design model and in the detailed model (see Chapter 6).

2.3.1 Western U.S. Example

The system shown in figure 2.3 will be used throughout this thesis to illustrate the various concepts and methodologies. The detailed model of this system is constructed based on a possible configuration of the 1988 U.S. Western interconnected system. Some model aggregation has already been done by the Los Angeles Department of Water and Power to arrive at this model, but it will be considered as the original system model in this thesis. This system consists of a parallel
connected 7-terminal MTDC network embedded in an ac system with 42 generators, 151 buses and 413 transmission lines.

The coherency identification method is used to reduce the 42-machine system to 10 coherent areas (indicated by closed regions in figure 2.3). The number 10 is a compromise between the accuracy of the response and a number of factors. The most obvious of these is the manageability of the design model.

Another factor in choosing the number of areas is that it is desirable to have at most one dc terminal per area in the design model. Otherwise, the dc system's controllability of certain oscillatory modes can be destroyed when the machines that participate in these modes are aggregated into the same area. This objective, however, cannot always be achieved by the slow-coherency method alone without increasing significantly the dimension of the design model. The sample system is a case in point. In this system, dc terminals 2, 4 and 6 are situated in area 4 which is constituted by a group of very tightly coupled machines. Experiments on this system show that the size of the design model has to increase beyond 20 areas before machines in area 4 are broken down into separate groups where no more than 1 dc terminal is contained within each group of coherent machines.

A possible remedy to this problem is to first find a reasonably small aggregate model using slow coherency, and then identify among the neglected dynamics those modes that are controllable and observable by the MTDC system. These
modes are then taken into account by breaking up certain coherent areas into smaller pieces according to the mode shapes (Argand diagrams). Methods for identifying the controllable and observable modes are developed in sections 3.4 and 3.5.

The 10-area model used in this thesis is found using only the slow-coherency aggregation technique. The resulting controller, therefore, is designed to stabilize only the slow interarea modes among these 10 areas. A controller capable of damping also some of the faster modes can be designed using the same methodology, but the design model has to be enlarged using some method such as the one sketched in the last paragraph.

The eigenvalues of the 42-machine model and of the 10-area model are shown in figure 2.4 (physical interpretations of the eigenvalues will be given in section 3.2). It is apparent in this figure that there is a one-to-one correspondence between the low-frequency modes of the two models. The modes in the 10-area model near 5 rad/sec, however, are seen to be aggregates of two or more modes in the 42-machine model. For this reason, the design model tends to be accurate at the low-frequency end and less accurate near the high-frequency cut-off point. The inaccuracy in modeling can affect adversely the performance -- or even the stability -- of the 42-machine system when it is controlled by the compensator designed using the 10-area model. This problem can be avoided if the boundary between the "interarea" frequencies of the
design model and the "inter-machine" frequencies of the neglected dynamics is picked such that the interarea modes near the boundary are not the modes that are being stabilized by the MTDC network. The analysis technique based on residue matrices (see section 3.5) is ideal for determining which modes in the 42-machine are controllable and observable by the feedback compensator.
Figure 2.3: Location of generators and dc system in the Western U.S. model. Area grouping of generators is indicated by closed regions.
Figure 2.4a: Eigenvalues of the 42-machine and the 10-area open-loop system. The area enclosed by the circle is magnified and shown in figure 2.4b.
Figure 2.4b: Details of figure 2.4a.

SYSTEM MODELING

2.3 AC SYSTEM MODEL
2.4 FEEDBACK SIGNALS

Possible candidates for measurement signals for implementing an MTDC modulation controller include machine shaft speeds, ac line flows and frequency measurements at the commutation buses. All of these quantities contain information on the extent of interarea oscillations.

2.4.1 Shaft-Speeds

Machine shaft speeds give the most direct indication of interarea swings. The author's experience with a 5-terminal, 5-area system [29,30] and the 7-terminal Western-U.S. system in this thesis (figure 2.3) indicates that the shaft speeds are by far the best measurement in terms of performance. However, shaft speed of an area is difficult to obtain, as it requires the shaft speed of all the machines in the area. Although this problem can be overcome partially by approximating the area's speed by the shaft speed of a few large machines, an extensive communication network is still necessary to transmit the machines' shaft speeds to the dc terminals. Moreover, the measurements can be lost if machines are taken out of service.

The problems associated with machines taken out of service can be coped with by modifying the computation of area speeds (provided measurements on other machines in the same area are available). The communication problem, however, is one of economics, and it does not seem economically feasible in the near future to deploy the extensive communication
network required to transmit the shaft-speed measurements to all the dc terminals.

2.4.2 Line-Flows

AC line-flow measurements are most useful for controlling swings between the areas situated at the two ends of the ac tie line. However, in an MTDC system, the controllable modes of oscillations are many, and they are not restricted to those across major tie lines. Moreover, this approach suffers from the drawback that measurements can be lost if lines are taken out of service. An extensive communication network is also required to send line-flow measurements from the major tie lines to the dc terminals.

The communication problem here is basically the same as that for the shaft-speed measurements. To cope with the problem of line outages, however, requires a fairly sophisticated adaptive control strategy. It does not seem possible to the author that a fixed-gain compensator will work satisfactorily using line-flow measurements.

2.4.3 Frequency at Commutation Buses

The frequency measurement at the commutation buses is taken at the output of a small rotating frequency transducer connected to the commutation bus. Mathematically, this measurement contains a linear combination of all the shaft speeds in the system, with those in the immediate vicinity of the dc terminal being the most prominent. In this thesis, the output of the frequency transducer is modeled by passing the commuta-
tion-bus voltage angle through the transfer function $s/((\tau s + 1))$
-- or equivalently, passing the commutation voltage frequency
through a low-pass filter $1/((\tau s + 1))$.

The cut-off frequency $1/\tau$ in this model is a function of
the ac network and the design of the transducer. The cutoff
frequency is assumed to be 20 rad/sec in this thesis. If the
cutoff frequency is relatively high compared to the highest
interarea frequency -- as is the case in the Western U.S.
example, the low-pass portion of the transfer function can be
ignored. However, the entire transfer function is used in the
methodological development of this thesis because

1. the cut-off frequency of the transducer may be near
the interarea frequency of interest in some other systems, or

2. the MTDC system may be used to control certain
inter-machine modes with frequencies near $1/\tau$.

The frequency measurements at the commutation buses are
superior to the shaft-speed and line-flow measurements in
terms of reliability and communication requirements. The
advantages of this type of measurement signals include

1. the measurements are not dependent on the availability
of generator and line outages (barring the traumatic event of
loosing all the lines feeding the dc terminal), and

2. the feedback measurements are taken at the dc termi-
inals, and the measurements may be broadcasted from one
terminal to all the terminals using the communication equip-
ments that may already exist for current coordination (see
Chapter 5).
Because of these advantages, the frequency at the commutation buses is assumed to be the measurement signal in the rest of the thesis.

The only major disadvantage of this type of measurement is that part of the output is influenced by the input in an algebraic way. The physical explanation of this phenomenon is that a sudden change in dc current causes an (almost) instantaneous change in ac bus angle, and consequently lead to an instantaneous change in the frequency of the commutation bus. This problem is discussed in more detail in the next section.
2.5 THE STATE-SPACE DESIGN MODEL

This section shows how the dynamic equations of the MTDC/ac power system are combined with those of the bus-frequency transducers in a state-space design model.

2.5.1 Basic Configuration

The linearized design model of an \((m+1)\)-terminal MTDC/ac power system with \(n\) coherent areas is given by

\[
\begin{bmatrix}
\dot{\theta}(t) \\
\dot{\omega}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \omega_R I \\
-H^{-1}k & -H^{-1}D
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}(t) \\
\dot{\omega}(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
T
\end{bmatrix} v(t) \tag{2.6}
\]

The state variables \(\theta\) and \(\omega\) are the shaft angle and speed of the \(n\) areas, and the control vector, \(v(t)\),

\[v = [ I_2, I_3, \ldots, I_{m+1} ]^T\]

is defined as changes in dc current injection at \(m\) of the dc terminals. Terminal 1 is chosen arbitrarily as the voltage-setting terminal. The current at this terminal cannot be an independent control signal because in a current-margin scheme, Kirchhoff's current law dictates that the current at terminal 1 is equal to the negative sum of all the other terminal's current injections (see section 2.2.1).

The matrix \(T\) in (2.6) is an \(n \times m\) matrix which reflects the impacts of dc-current modulation on the motion of the \(n\) areas. More information on the system matrix in (2.5) can be found in Appendix B.
The actual measurement signal is the output of the bus-frequency transducer, modeled as

\[ f(s) = \frac{20s}{s + 20} \text{I} z(s) \quad (2.7) \]

where \( z(t) \) is the angle of the commutation voltages at all the dc terminals, including the voltage-setting terminal. The variable \( z(t) \) is related to the ac-system variables through

\[ z(t) = [\bar{E} \quad 0] \begin{bmatrix} \dot{\delta}(t) \\ \omega(t) \end{bmatrix} + D y(t) \quad (2.8) \]

The feedback signal \( f(t) \) is seen here as the low-frequency component of the derivative of the commutation bus angle.

2.5.2 Feedthrough Term

The feedthrough term \( D y(t) \) in (2.8) presents some complications in the computation of output-feedback gains, since the output contains not only a linear combination of the states, but also a linear combination of control signals. A possible solution to this problem is to incorporate (2.8) into the dynamic equations by means of state augmentation. In other words, the feedthrough is assumed to be not instantaneous as in (2.8), but is "delayed" by additional filters with very wide bandwidth. This approach proved to be unworkable because the resulting output matrix is not well-suited to the feedback algorithm used in this thesis (see Section 4.7).
A more practical solution is to subtract from the frequency transducer output the component due to the \(Dv(t)\) term. A block diagram showing how this scheme can be implemented in practice is shown in figure 2.5. Implicit in this figure is the fact that the matrix \(D\) is (practically) diagonal, meaning that the output \(f_i(t)\) at terminal \(i\) is affected algebraically only by its own current modulation.

Note that in this scheme, the value of \(d_{ii}\) can be adjusted to minimize the feedthrough component using field data. The proposed procedure is as follows.

1. The current is ramped up or down at a constant rate for a period of time. The feedback channel is left open (i.e. open-loop) and the measurement signal (figure 2.5) is recorded.

2. When the cancellation is not perfect, the feedthrough component appears on the recording as a square pulse (in addition to other signals due to noise and possibly machine swings).

3. The recorded signal is analyzed to identify the height of the square pulse. This information shows how much the value of \(d_{ii}\) has to be changed to effect a better cancellation.

This procedure can be automated and it can be carried out whenever a planned schedule change is made at terminal \(i\).

An alternate way to cope with the feedthrough term is to compute the output feedback gain with the feedthrough term taken into account. This method is illustrated below.
pose that the measurement signal is

\[ \gamma(t) = C x(t) + D \nu(t) \quad (2.9) \]

and the desired feedback law is

\[ \nu(t) = F C x(t) \quad \text{ (2.10)} \]

It can be shown easily that (2.10) is equivalent to the feedback law

\[ \nu(t) = [(I + FD)^{-1} F] \gamma(t) \quad (2.11) \]

(assuming the inverse exists). The author believes that this method is inferior to the first method because in this case, the value of \( d_{ii} \) at all the terminals must be known before the feedback matrix (2.11) can be computed. Moreover, a setup similar to that shown in figure 2.5 is still necessary in the this method to identify \( d_{ii} \) at each dc terminal.

In the rest of this chapter, the cancellation of the feedthrough is assumed to be exact. This is equivalent to substituting (2.8) by

\[ \bar{z}(t) = [F \quad Q] \begin{bmatrix} \xi(t) \\ \omega(t) \end{bmatrix} \quad (2.12) \]

An experiment is performed by the author to check the sensitivity of the closed-loop's stability to errors in cancellation. In this experiment, the D matrix used for cancelling the feedthrough term is assumed to be different from the actual D matrix by a scalar multiplier. When the multiplier is 1, the cancellation is exact. It is found that the

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(centralized output feedback) closed-loop system remains stable for multipliers in the interval [-2,5]. In addition, the closed-loop poles are hardly affected for changes in the range [0.5,1.5]. The results here is a strong indication that the closed-loop performance is not sensitive to errors in cancelling the feedthrough component.

2.5.3 State-Space Representations

A block diagram of the entire system, including (2.6), (2.7) and (2.8), is shown in figure 2.6. A possible state-space representation of this system is

\[
\begin{bmatrix}
\dot{\delta}(t) \\
\dot{\omega}(t) \\
\dot{\xi}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \omega_R \mathbb{I} & 0 \\
-\mathbb{H}^{-1}k & -\mathbb{H}^{-1}d & 0 \\
-20\mathbb{E} & 0 & -20\mathbb{I}
\end{bmatrix}
\begin{bmatrix}
\delta(t) \\
\omega(t) \\
\xi(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} v(t)
\] (2.13)

\[
f(t) =
\begin{bmatrix}
20\mathbb{E} & 0 & 20\mathbb{I}
\end{bmatrix}
\begin{bmatrix}
\delta(t) \\
\omega(t) \\
\xi(t)
\end{bmatrix}
\] (2.14)

where \(\xi(t)\) is the state of the frequency transducer. This system description is found to be unsuitable for output-feedback computation because the positions in the output matrix corresponding to the shaft speeds are zero. A more detailed explanation will be given in Section 4.7 after the output-feedback algorithm is described.

A more suitable design model requires the transducer model be broken into two parts, as shown in figure 2.7. It should be obvious that the system in figures 2.6 and 2.7 are

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identical. In the new system, the output is equal to the pure derivative of the bus angle, or

\[ \mathbf{y}(t) = \dot{z}(t) \]  \hspace{1cm} (2.15)

\[ = \begin{bmatrix} 0 \\ \omega_R^E \end{bmatrix} \begin{bmatrix} \dot{\omega}(t) \\ \omega(t) \end{bmatrix} \] \hspace{1cm} (2.16)

Equation (2.16) implies an alternate system representation shown in figure 2.7. The state-space equations associated with this system are

\[
\begin{bmatrix}
\dot{\phi}(t) \\
\dot{\omega}(t) \\
\dot{y}(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \omega_R^I & 0 \\
-H_1^{-1}K & -H_2^{-1}D & T \\
0 & 0 & -20I
\end{bmatrix}
\begin{bmatrix}
\phi(t) \\
\omega(t) \\
y(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
20I
\end{bmatrix}u(t) \] \hspace{1cm} (2.17)

\[ \mathbf{y}(t) = \begin{bmatrix} 0 \\ \omega_R^E \\ \omega(t) \end{bmatrix} \begin{bmatrix} \phi(t) \\ y(t) \end{bmatrix} \] \hspace{1cm} (2.18)

For a system with \( n \) areas and \( m+1 \) dc terminals, the dimension of the system matrix is \( (2n+m) \times (2n+m) \), the input matrix is \( (2n+m) \times m \), and the output matrix \( (m+1) \times (2n+m) \). Equations (2.17) and (2.18) will be used as a basis for both state- and output-feedback computations. For simplicity, the system matrix, and the input and output matrices will be referred to as \( A, B \) and \( C \).
Figure 2.5: Cancellation of the feedthrough term in actual implementation

Figure 2.6: System configuration

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Figure 2.7: System configuration with the frequency transducer split into two parts.

Figure 2.8: Alternate system representation

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2.6 SUMMARY

Modeling techniques for ac/MTDC systems are presented in this chapter.

The design model is developed based on the need for a simplified model of the ac/MTDC system which represents accurately the interarea machine dynamics of interest. The modeling methodology calls for the identification of coherent areas based on the machines' slow motions. Model simplification is achieved when the slow subsystem which represents the center-of-inertia variables of the coherent areas is used as the design model (section 2.3). The dc system is modeled as time-varying power injections at the ac commutation buses. The dynamics of the dc components are neglected in the design model because of their relatively short time constants (section 2.2). Several considerations that influence the size of the design model are discussed. One of the important considerations is the possibility of obscuring certain high-frequency inter-machine modes that can be controlled readily by the MTDC system (section 2.3).

Three different classes of measurement signals are investigated in section 2.4. The frequency measurement at the commutation buses is believed to be superior to shaft-speed and line-flow measurements in terms of reliability and communication requirements.

The state-space equations for the design model is developed in section 2.5. A physical interpretation of this model is given in the next chapter.

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CHAPTER 3
SYSTEM ANALYSIS

3.1 OVERVIEW

The design model of an MTDC/ac power system (2.17-2.18) is analyzed in this chapter. The aim here is to show how the system dynamics can be interpreted physically. The results of this system analysis will be used to motivate the controller-design methodology in Chapter 4.

The analysis of the system is done in the modal domain. It will be shown in section 3.2 and 3.3 that there are physical interpretations for each of the system modes.

A new method for measuring quantitatively the controllability and observability of an MTDC/ac power system is developed in section 3.4. A key component of this methodology is shown to be the "unit momentum" scaling of left and right eigenvectors.

The residue matrix is introduced in section 3.5 as a tool for measuring the input/output transmission of each mode. Several applications of this analysis technique in modeling and analysis of an MTDC/ac power system are pointed out.

The methods developed in this chapter are demonstrated using the design model of the Western U.S. system.
3.2 EIGENVALUES

The design model (2.17) can be divided into two subsystems: the subsystem associated with the ac system, and the subsystem associated with the sensors. The analysis in this section and in subsequent sections will be concentrated on the ac-system subsystem; the interpretation of the sensor subsystem is obvious.

The eigenvalues (or poles) of the ac system are the roots of the characteristic equation

$$\det(\mathbf{H}s^2 + \mathbf{D}s + \omega_{R,K}) = 0. \quad (3.1)$$

The damping coefficient, $\mathbf{D}$, of a multi-area ac system is usually small; therefore it is instructive to first consider the limiting case where $\mathbf{D}=0$. In this case, the characteristic equation becomes

$$\det(\mathbf{H}s^2 + \omega_{R,K}) = 0, \quad (3.2)$$

which implies that the eigenvalues of the system are equal to the square root of the eigenvalues of $-\omega_{R,H}^{-1}K$. For a purely inductive ac network (which is a good approximation for most systems), this matrix has $n-1$ negative real eigenvalues and one zero eigenvalue (see Appendix B). Consequently, the solution of (3.2) consists of 2 poles at the origin and $(n-1)$ pairs of poles on the imaginary axis. Such a system is only marginally open-loop stable.

With the addition of the damping matrix $\mathbf{D}$, the poles on the imaginary axis are moved into the left half-plane, and one
of the two poles at the origin is moved to the negative real axis. The exact position of the complex poles is primarily a function of the network impedance, machine damping and machine (area) inertias, and, to a certain extent, the system operating point.
3.3 MODE SHAPES

Important physical insights can be derived from the right eigenvectors of the system. It is shown in Appendix A that the components of the right eigenvectors show the state variables' participation in each mode of oscillation. For the MTDC design model, only all the $\delta$'s or all the $\omega$'s need be considered since the relative phase and magnitude of the variables within each set are identical. A diagram showing the components of the right eigenvectors as phasors in the complex plane is referred to in the control literature as the Argand diagram of the right eigenvector, and in the power-system literature as the mode shape of the system mode.

As is mentioned in Appendix A, the interpretation of the right eigenvectors can be affected by the units of the state variables. This is not a problem in the MTDC design model, but care must be exercised in the interpretation to take into account the fact that in a system with a wide range of area inertia, small areas tend to oscillate with much greater speed and amplitude than large areas. Without special scaling that compensates for this fact, the larger components in the right eigenvectors corresponding to the small areas can potentially obscure the large areas' role in the oscillations.

A more realistic picture of system dynamics results if the component corresponding to each area's amplitude or speed is weighted by its inertia. A physical justification of this method of scaling is that it transforms the velocity variables into momentum variables which are sensitive only to the forces.
acting on the area and are not sensitive to the differences in inertia. Incidentally, this method of scaling is related to the "participation factors" developed recently by Pérez-Arriaga et al. [26,53], which are proportional to the energy of the machines (see Appendix I for more detail).

3.3.1 Complex Modes

The mode shape of each of the complex oscillatory modes consists of two groups of vectors spaced 180 degrees apart, indicating that two groups of areas oscillate against each other. An obvious interpretation of these oscillatory modes is that they are interarea electromechanical oscillations. The almost-perfect 180-degree separation is due to the fact that damping is proportional to the shaft speed of each machine and not to the difference in shaft speeds.

The eigenvalues of the sample system are depicted in figure 2.4, and the mode shape for the complex modes in figure 3.1. Normally the components of the right eigenvectors are shown as phasors in the complex plane. In the MTDC/ac power system model, however, instead of showing two groups of vectors 180 degrees apart, the vectors are drawn such that those corresponding to one group of areas are shown to the left of the center line, and those corresponding to the other group to the right of the center line. This method of depicting the mode shapes is done for the sake of clarity. When interpreting the mode shape in figure 3.1, the reader should bear in mind that the lines are phasors that rotate at the
frequency of oscillation.

It is noteworthy that the high-frequency modes are generally associated with oscillations between a small area and one or more large areas; whereas the low-frequency modes are associated with oscillations between two large groups of areas. This is consistent with the notion that the frequency of oscillation of a two-mass-spring system is inversely proportional to the square root of the reduced mass, defined as $m_1m_2/(m_1+m_2)$ where $m_1$ and $m_2$ are the masses (or inertias) of the two groups of areas [27, pp. 397-400].

The mode at 0.26 hz is identified as the well-known Pacific Intertie mode which involves most of the areas in the Pacific Northwest oscillating against most of the areas in the Southwest. The frequency of 0.26 hz is different from the recently observed value of 0.33-0.35 hz [4]. The difference may be due to load and network changes that take place between now and 1988. Another possibility is that the 42-machine system is a poor representation of the Western U.S. system. The exact reason for the discrepancy is not important for the purpose of this research since the goal is not to develop a controller for the Western U.S. system, but merely to use the model for illustrating the concepts. It is also noteworthy that the mode at 0.32 hz is very similar to the Intertie mode, except that area 1 (eastern Montana and Colorado) is dominant and swings in unison with the Northwest.
Figure 3.1: Mode shape of the oscillatory modes

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Figure 3.1 continued.

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3.3 MODE SHAPES
3.3.2 Real Modes

Now attention is turned to the two poles on the real axis. For the pole at the origin, the right eigenvector is

\[ v_o = [1, 1, ..., 1, 0, 0, ..., 0]^T \]

where the 1's correspond to the angle states and the 0's to the frequency states. This implies the existence of an arbitrary phase reference for the angles -- i.e., for any state vector \( x(t) + av_o \), where a scalar "a" is added to each angle, the dynamics of the system remain unchanged. To show this mathematically, simply observe that

\[ \frac{d}{dt}(x(t) + av_o) = \frac{d}{dt} x(t) \]  \hspace{1cm} (3.3)

and

\[ A (x(t) + av_o) = A x(t) \]  \hspace{1cm} (3.4)

where the last equality follows directly from the definition of the eigenvector for the pole at \( s=0 \).

This pole is related to the time error in a clock that is driven by a synchronous motor since a clock can be modeled as a pure integrator whose output (in hours) is depicted in modulo 12. The arbitrariness of the phase -- or of the output of the clock -- means that the power system is oblivious to the accumulated time errors.

The time error is not corrected by the AGC (automatic generation control) system, because AGC is sensitive only to the frequency errors and tie-line interchange errors. The time error is instead corrected periodically (about once a
day) by a central-dispatch operator who temporarily speeds up or slows down all the machines in the system until the time error is reduced to zero.

For the mode \( s = -0.4 \) on the negative real axis, the corresponding left eigenvector has the form

\[ \mathbf{w}_1 = [0, 0, \ldots, 0, a_1, \ldots, a_n]^T. \]

The scalars \( a_1, \ldots, a_n \) can be shown to be directly proportional to the inertia of the \( n \) areas under the assumption that the ac network is purely inductive and the per-unit damping coefficient of all the areas are the same (see Appendix C). Moreover, this mode is related to the motion of the "average frequency", \( \bar{\omega}(t) \) of the power system, defined as

\[ \bar{\omega}(t) = \frac{\sum_{i=1}^{n} a_i \omega_i(t)}{\sum_{i=1}^{n} a_i} \quad (3.5) \]

The average frequency is analogous to the speed of the center of mass of a mechanical mass-spring system (see next section). It is well known that in an undriven mechanical system of this type, the center of mass travels in a straight line, and its speed decays exponentially when Newtonian frictional forces are present. The average frequency defined in (3.5) behaves in exactly the same way as the speed of the center of mass. Its equation of motion (according to (A.7)) is

\[ \ddot{\bar{\omega}}(t) = -0.4 \bar{\omega}(t). \quad (3.6) \]

The time constant associated with the average frequency pole
depends primarily on the damping characteristics of the machines and the dynamics of the turbine governors.

3.3.3 Mechanical Analog

The reader who is familiar with Newtonian mechanics will note the similarities between a classical multi-machine system and a mechanical system where a number of masses on a frictional surface are interconnected by springs. The oscillatory modes of the power system are analogous to mechanical oscillations between groups of masses, and the average-frequency mode is analogous to the motion of the center of mass. The mode at the origin simply means that for the mechanical system, the dynamics are not affected by an equal displacement of all the masses.
3.4 CONTROLLABILITY AND OBSERVABILITY

From an intuitive standpoint, the controllability of a dynamic system refers to the actuators' ability to change the states of the system, and its observability refers to the sensors' ability to see the internal states of the system. Controllability and observability are intrinsic physical properties of a system, and a knowledge of these properties is crucial to the design of a controller.

3.4.1 Formal Definitions

A formal definition of controllability and observability is given in the following. Consider a linear system with m inputs, p outputs and n states:

\[
\dot{x}(t) = A x(t) + B u(t) , \quad x(0) = x_0 \quad (3.7)
\]

\[
y(t) = C x(t) . \quad (3.8)
\]

The solution to this system can be written as

\[
y(t) = \mathcal{H}(x_0) + C \mathcal{L}(u) \quad (3.9)
\]

where \( \mathcal{L}(\cdot) \) and \( \mathcal{H}(\cdot) \) are linear operators defined as

\[
\mathcal{L}(u) = \int e^{A(t-\tau)} B u(\tau) \, d\tau \quad (3.10)
\]

and

\[
\mathcal{H}(x_0) = C e^{At} x_0 . \quad (3.11)
\]

The system (3.7 - 3.8) is said to be completely controllable if
\( R(\mathbf{L}) = \mathbb{R}^n \) \hspace{1cm} (3.12)

and completely observable if

\[ \mathcal{N}(\mathcal{H}) = \{ 0 \} \, , \] \hspace{1cm} (3.13)

where \( R(*) \) and \( \mathcal{N}(*) \) denote the range and null space of a linear operator, respectively. Equation (3.12) implies that the system is controllable if the states can be driven anywhere in \( \mathbb{R}^n \) by some control \( u(\tau) \) applied from \( \tau = 0 \) to \( t \). Analogously (3.13) states that the system is observable if given the output \( y(t) \) and input \( u(\tau) \) for \( \tau = 0 \) to \( t \), the entire state trajectory can be reconstructed.

Controllability and observability of a linear system are traditionally determined by checking conditions (3.12) and (3.13). This approach is not satisfactory in most cases, because the information it provides is qualitative in nature; it can say at best what parts of the system are controllable or observable and what parts are not, but it is not capable of making quantitative statements about the different parts of the system. In addition, the traditional criteria are difficult to determine reliably, for rank determination is known to be a numerically ill-posed problem.

3.4.2 Quantitative Measures

A quantitative measure of observability and controllability is motivated by the following expression for \( y(t) \):

\[ y(t) = \sum_{i=1}^{n} C v_i \left[ w_i^T x_0 + \int_0^t e^{-\lambda_i \tau} a_j \left( \int_0^\tau u_\tau \, d\tau \right) \right] e^{\lambda_i t} \] \hspace{1cm} (3.14)

It is clear from this equation that the extent to which the
input excites the ith mode is determined by the elements of the vector $w_i^T b$, and the extent to which the ith mode appears at the different outputs is determined by the elements of the vector $c_i v_i$. If the matrices $B$ and $C$ are written as

$$B = \text{col}[b_1, b_2, \ldots, b_m],$$

and

$$C = \text{row}[c_1^T, c_2^T, \ldots, c_p^T],$$

then the matrices $WB$ and $CV$ given by

$$WB = \begin{bmatrix}
    w_1^T b_1 & w_1^T b_2 & \cdots & w_1^T b_m \\
    \vdots & \vdots & & \vdots \\
    w_n^T b_1 & \cdots & & w_n^T b_m
\end{bmatrix} \quad (3.15)$$

and

$$CV = \begin{bmatrix}
    c_1^T v_1 & c_1^T v_2 & \cdots & c_1^T v_n \\
    \vdots & \vdots & & \vdots \\
    c_p^T v_n & \cdots & & c_p^T v_n
\end{bmatrix} \quad (3.16)$$

have a useful interpretation: the magnitude of the entry $w_i^T b_j$ of $WB$ measures how much the jth input affects the ith mode, and the magnitude of the entry $c_i^T v_j$ of $CV$ measures how much the jth mode appears in the ith output of $\chi(t)$.

3.4.3 Unit and Scaling Considerations

The interpretation of modal controllability and
observability is sensitive to the choice of units for the input and output variables. In addition, the frequency of the modes can be an important factor -- for example, a highly controllable mode which decays rapidly may have very little impact on the control problem compared to a less controllable slow mode. Fortunately, the interpretation of an MTDC system is not affected by these difficulties because the frequency of all the interarea modes are similar, and the units of all the input variables are the same, and likewise for all the output variables.

A more important consideration in the MTDC application is the scaling of the eigenvectors. Recall that an eigenvector, by itself, is unique only within a scalar constant, and the left and right eigenvectors of the ith mode are constrained by the relation

$$w_i^T v_i = 1$$

(3.17)

The important point to note here is that (3.17) is not sufficient to determine $v_i$ and $w_i$ uniquely, since the vectors $av_i$ and $(1/a)w_i$ (where "a" is a nonzero complex scalar) would also satisfy the equation. It is also important to note that the observability measures (3.16) depend solely on $V$ and the controllability measures (3.15) solely on $W$. Hence by varying the scalar "a", it is possible to make a mode look very controllable but not very observable, or vice versa. The usefulness of the observability and controllability measures, therefore, hinges on a physically meaningful method for scaling the

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left and right eigenvectors.

3.4.4 Unit-Momentum Scaling

The proposed method of scaling makes use of the fact that each oscillatory mode in an ac system involves an oscillation between two groups of areas. For an interarea mode, the areas are divided into two groups according to the mode shape. The right eigenvector is then scaled such that the momentum of one of the groups is equal to unity. The momentum of the other group is automatically scaled to unity because the net momentum of an oscillation is always equal to zero (see Appendix D). This method of scaling will be referred to as the unit-momentum scaling.

This scaling technique is now stated mathematically. Suppose that the inertias of the n areas are $H_1, \ldots, H_n$ (the unit of the $H_i$'s is not important), and let the two groups of machines that swing against each other in mode j be elements of the sets $\Omega_1$ and $\Omega_2$. The right eigenvector $v_j$ is scaled by dividing the entire vector by the momentum of one of the groups. More precisely, the scaled eigenvector is given by

$$v_j / \sum_{i \in \Omega_1} H_i |\omega_i|$$  \hspace{1cm} (3.18)

where the $\omega_i$'s are the shaft-speed components in the vector $v_j$ corresponding to the areas in the set $\Omega_1$. In appendix D, it is proved that the same scaling is achieved by

$$v_j / \sum_{i \in \Omega_2} H_i |\omega_i|$$  \hspace{1cm} (3.19)
since the momentum of both groups of areas are identical.

To see the physical implication of this method, consider the contribution of the \( i \)th mode in the state vector, subject to an input at the \( j \)th input channel:

\[
\mathbf{x}(t) = \mathbf{v}_i \mathbf{w}_i^T \mathbf{b}_j \int_0^t e^{\lambda_i(t-\tau)} \mathbf{u}_j(\tau) \, d\tau
\]  

(3.20)

Suppose that \( \mathbf{x}(0) = 0 \) and the \( j \)th input is an impulse at \( t=0 \). The state vector at \( t=0^+ \) is given by

\[
\mathbf{x}(0^+) = \mathbf{v}_i (\mathbf{w}_i^T \mathbf{b}_j) .
\]  

(3.21)

Under the proposed method of scaling, the momentum of the response at \( t=0^+ \) is equal to the scalar \( \mathbf{w}_i^T \mathbf{b}_j \). This value is related to the controllability of the \( i \)th mode in the sense that, for an impulsive-force input, a large jump in momentum is indicative of good controllability.

The observability measure \( \mathbf{c}_k^T \mathbf{v}_i \) can be interpreted as the kth output signal at \( t=0^+ \), subject to an initial condition \( \mathbf{x}(0) = \mathbf{v}_i \) of unit momentum. More precisely,

\[
\mathbf{y}(0^+) = \mathbf{c}_k^T \mathbf{v}_i (\mathbf{w}_i^T \mathbf{v}_i)
\]

\[
= \mathbf{c}_k^T \mathbf{v}_i .
\]  

(3.22)

This measure of observability is consistent with the intuitive idea that a mode with good observability should produce a large output signal when it is excited.

3.4.5 Alternative Scaling Techniques
Other methods of scaling left and right eigenvectors are also possible. For example, the right and left eigenvectors may be balanced such that

\[ \| v_i \| = \| w_i \| \]  

(3.23)

(Note: Equations (3.23) and (3.17) together do not imply that the norms of \( v_i \) and \( w_i \) are equal to 1). A disadvantage of this method is that (3.23) has no physical justification in most systems.

For MTDC systems, it is decided that the unit-momentum scaling is the most suitable because it gives physically meaningful measures of controllability and observability.

3.4.6 Interpretations and Examples

Regardless of the method of scaling, the matrices \((WB)^T\) and \(CV\) can be analyzed in two different ways. First, the controllability (observability) of a mode at different inputs (outputs) can be compared by scanning a column of \((WB)^T\) (CV) and associate the magnitude of the elements with the degree of controllability (observability). Similarly, the controllability (observability) of different modes at an input (output) can be compared by scanning a row of \((WB)^T\) (CV).

The controllability matrix \((WB)^T\) for the Western-U.S. example is shown graphically in figure 3.2. The following steps are used to construct this figure:

1. Compute \(W\) from the system matrix \(A\) and form the complex matrix \((WB)^T\).

2. The elements in the matrix that are not associated
with the interarea modes are set to zero.

3. Replace each matrix element by its magnitude, and scale the matrix such that the largest element is equal to 1.0.

4. Depict each non-zero element of the matrix by a square whose area is equal to the magnitude of the element. (Instead of showing two identical squares for each complex pair, only one is shown.)

Note that in figure 3.2, each row is associated with an input, and each column with a complex pair of oscillatory mode. The mode numbers are assigned as per table 3.1 below.

<table>
<thead>
<tr>
<th>mode number</th>
<th>eigenvalues</th>
<th>frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.25 ± j 5.24</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>-0.19 ± j 4.60</td>
<td>0.73</td>
</tr>
<tr>
<td>3</td>
<td>-0.19 ± j 4.21</td>
<td>0.67</td>
</tr>
<tr>
<td>4</td>
<td>-0.28 ± j 3.54</td>
<td>0.56</td>
</tr>
<tr>
<td>5</td>
<td>-0.15 ± j 3.28</td>
<td>0.52</td>
</tr>
<tr>
<td>6</td>
<td>-0.19 ± j 2.83</td>
<td>0.45</td>
</tr>
<tr>
<td>7</td>
<td>-0.18 ± j 2.60</td>
<td>0.41</td>
</tr>
<tr>
<td>8</td>
<td>-0.21 ± j 2.05</td>
<td>0.33</td>
</tr>
<tr>
<td>9</td>
<td>-0.25 ± j 1.66</td>
<td>0.26</td>
</tr>
</tbody>
</table>

In interpreting the controllability of an MTDC system, one should bear in mind that the controls applied to terminal \( i \) affect not only the power injection locally, but also the power injection at the voltage-setting terminal. The reason is that in a current-margin scheme, the current fluctuation at the voltage-setting terminal is always equal and opposite to...
the current modulation at terminal i -- with all other terminal currents being the same. This fact will be important to the coordination issue discussed in chapters 5 and 6.

A great deal of information about the system can be gleaned from figure 3.2. In this figure, the relative controllability between different interarea modes can be compared by comparing the size of the elements in different columns. The modes that are most controllable are seen to be modes 1, 2, 4, 7 and 9. In addition, the relative size of the elements in each column is an indication of where the mode can be controlled most readily. For instance, mode 4 is seen to be most controllable from terminals 3 and 5.

The relative controllability of the different inputs can also be compared by comparing the size of the elements in different rows. The results in figure 3.2 indicate that mode 4 is least controllable by terminals 4 and 6.

The physical reason for these results is clear -- in hindsight -- by noting the location of the dc terminals (figure 2.3) and the mode shapes (figure 3.1). It should be noted, however, that the physical reasoning based on the location of the dc terminals and the mode shapes is useful only in a system where the areas are very loosely coupled -- e.g. the Western U.S. model. If the areas are more strongly coupled, an area with no dc terminal may be controllable by terminals in the neighboring areas, and therefore the controllability measure in this case is a much more reliable indicator than visual inspection.
It is also interesting to see that the controllability measures associated with mode 9 are nearly identical for all the inputs. This is explained by

1. Mode 9 involves most of the machines in the Northwest swinging against those in the Southwest, and

2. The voltage-setting terminal is located in the Northwest and all the current-controlled terminals in the Southwest. Therefore this mode can be excited more or less equally by any one of the current-controlled terminals.

The observability measures shown graphically in figure 3.3 are constructed using the same basic method as controllability measures. A general trend that is immediately apparent in this figure is that the size of the squares get larger as one scans from right to left, indicating that the higher frequency modes are more observable than the low-frequency modes. This phenomenon is a direct consequence of the unit-momentum scaling technique, and it is consistent with the observed fact that the higher frequency modes are associated with oscillations of smaller areas and tend to produce output signals of higher amplitude than the slower modes.

Similar to the interpretation of controllability measures, the relative observability between different modes and between different outputs can be compared by comparing the size of the elements in different columns and rows, respectively. The result in figure 3.3 indicates that modes 3, 5, 6 and 7 are the least observable modes. It can be seen in figure 3.2 that modes 3, 5, and 6 are also the least control-
lable modes. The author suspects that there is a strong correlation between the least controllable modes and the least observable modes in most MTDC/ac power systems where the frequency of the commutation buses is used as measurement signals.
Figure 3.2: Controllability measures

<table>
<thead>
<tr>
<th>TERMINAL NUMBER</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td></td>
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<td>5</td>
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<td>7</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**MODE NUMBER**

1 2 3 4 5 6 7 8 9

SYSTEM ANALYSIS 3.4 CONTROLLABILITY AND OBSERVABILITY
Figure 3.3: Observability measures

SYSTEM ANALYSIS 3.4 CONTROLLABILITY AND OBSERVABILITY
3.5 RESIDUE MATRICES

Both the controllability and observability of a mode are important to an output-feedback design. The residue-matrix decomposition

$$y(s) = \sum_{i=1}^{n} \frac{1}{s - \lambda_i} C v_i w_i^T B$$

(3.24)

provides a means for computing a "combined index" of both of these quantities. More precisely, the input-output characteristics of the ith mode is quantified by the entries of the matrix $C v_i w_i^T B$. Note that a mode must be both controllable and observable to allow for good transmission from the input to the output, and the lack of either controllability or observability would result in a zero residue matrix. Note also that the presence of both right and left eigenvectors in (3.24) makes the residue matrix invariant to the scaling of eigenvectors.

The open-loop poles of the Western-U.S. model and the row and column norm of the associated residue matrices are tabulated in Table 3.2 (at the end of this section). Note that only four pairs of oscillatory poles have residues with norms of 90 or more. Another two pairs of oscillatory poles have marginal value of 40, and the rest are considerably lower.

Each of the high-residue modes is found to be an oscillation in where dc terminals are available in two opposite swinging areas that are dominant in the Argand diagram of the associated eigenvector. For example, mode 4 has a high resi-
due because it is an oscillation in which areas 4 and 7 swing against area 8 (see figure 3.1), and dc terminals are available in all these areas (see figure 2.3). In physical terms, the high-residue modes are those on which the MTDC system has direct leverage.

The number of high-residue modes in a system is a function of the location of the dc terminals and the electric coupling between the different areas. The number of high-residue modes decreases when the areas with no dc terminal are very loosely coupled to the rest. To demonstrate this point, consider the case where the coupling is nearly zero. Under this assumption, it is clear that an \((m+1)\)-terminal MTDC system connected to \(m+1\) areas has direct leverage on only \(m\) oscillatory modes involving oscillations between these areas. The modes corresponding to the other areas simply cannot be controlled by the MTDC system because of the weak coupling. The 10-area equivalent of the Western U.S. system appears to be a system of this type. Note that in figure 2.3, the 7 dc terminals are situated in only 5 coherent areas; hence only 4 of the oscillatory modes in this model have large residues. These modes are found to coincide with the modes whose LQ state-feedback gains can be approximated readily by the minimum-norm output-feedback design (see Chapter 4).

The residue matrix has several potential applications in the MTDC problem.

1. Identification of the modes in the full model that can be damped out by the MTDC network using output feedback.

SYSTEM ANALYSIS 3.5 RESIDUE MATRICES
This information is useful in constructing the simplified design model. See also section 2.3.1.

2. Identification of the modes in the design model that can be damped out using output feedback. This application is demonstrated in this chapter.

3. Selection of possible feedback signals or selection of possible sites for the sensors.

4. The knowledge of the "controllable" modes as function of the placement of the dc terminals may serve as a useful guide in the planning stage of an MTDC system. This information, however, is not expected to have a large impact, since, in most cases, the dc terminals are placed according to bulk-power supply/demand considerations.

Table 3.2: Row and column norm of residue matrices

<table>
<thead>
<tr>
<th>mode number</th>
<th>eigenvalues</th>
<th>Residue Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>row norm</td>
</tr>
<tr>
<td>1</td>
<td>-0.25 ± j 5.24</td>
<td>256</td>
</tr>
<tr>
<td>2</td>
<td>-0.19 ± j 4.60</td>
<td>129</td>
</tr>
<tr>
<td>3</td>
<td>-0.19 ± j 4.21</td>
<td>43</td>
</tr>
<tr>
<td>4</td>
<td>-0.28 ± j 3.54</td>
<td>285</td>
</tr>
<tr>
<td>5</td>
<td>-0.15 ± j 3.28</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>-0.19 ± j 2.83</td>
<td>28</td>
</tr>
<tr>
<td>7</td>
<td>-0.18 ± j 2.60</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>-0.21 ± j 2.05</td>
<td>47</td>
</tr>
<tr>
<td>9</td>
<td>-0.25 ± j 1.66</td>
<td>91</td>
</tr>
</tbody>
</table>
3.6 SUMMARY

Analysis techniques for ac/MTDC systems are presented in this chapter, with the goal of utilizing the results for controller synthesis (see Chapter 4).

Analysis of the design model is guided by interpretations of system dynamics in the modal domain. The eigenvalues of an n-machine system are shown to be n-1 pairs of oscillatory modes and 2 modes on the real axis (section 3.2). The physical interpretation of these modes are given using the right eigenvectors whose components are weighted by the machines' inertias. The oscillatory modes are identified as interarea swings involving two groups of areas (section 3.3.1). The mode on the negative real axis is shown to be related to the average frequency of the power system, while the mode at the origin is shown to be related to the arbitrariness of the shaft-angle reference (section 3.3.2).

A method for quantifying the controllability and observability of system modes is developed in section 3.4. A proper interpretation of the controllability and observability measures depends on several factors, and the most important of these in the MTDC problem is believed to be the scaling of the right and left eigenvectors. The proposed method of scaling is based on the momentum of the oscillations. Under this method of scaling, the controllability of the ith mode by the jth input is equal to the jump in momentum immediately after an impulsive force is applied to the jth input. The observability measure of the ith mode at the kth output is equal to
the magnitude of the output signal at $t=0^+$, with an initial condition of unit momentum.

In section 3.5, the residue matrix is used for checking both the controllability and observability of the system modes. The author's experience with output-feedback design (Chapter 6) shows that the residues are excellent indicators for showing which of the interarea modes can be controlled readily by the MTDC network.

The concepts developed in this chapter are demonstrated using a 42-machine model of the Western U.S. system.
CHAPTER 4
CONTROLLER DESIGN METHODOLOGY

4.1 OVERVIEW

The objective of the MTDC modulation controller is to improve the damping of the interarea oscillations using the frequency at the commutation buses as measurement signals. The use of bus frequencies necessitates an output feedback design, since bus frequencies are linear combination of the state variables of the design model.

In developing an output-feedback methodology, two decision must be made at the outset.

1. Which kind of controller (adaptive, nonadaptive, static, dynamic, etc.) should be used?

2. What is the best algorithm for computing the required feedback gains?

Both of these issues are addressed in this chapter.

The guidelines for selecting a class of feedback controllers best suited to the MTDC problem are stated in section 4.2. These guidelines and the overall design philosophy are shown to be a function of the design constraints on robustness and on the amplitude of the control signals.

A number of available output feedback techniques are described in section 4.3. This discussion puts in perspective the proposed method which employs the LQ state-feedback method to design an optimal full-state control law, and the least-squares output-feedback algorithm to approximate the LQ state-feedback gain matrix.
Three different methods for computing the LQ penalty matrices are detailed in section 4.5 after a brief review of the LQ methodology in section 4.4.

The proposed design procedure is described in more detail in sections 4.6 and 4.7. First the choice of closed-loop LQ pole locations is discussed in section 4.6. The least-squares output-feedback procedure is described in section 4.7.
4.2 DESIGN CONSTRAINTS

The design constraints in the MTDC problem include robustness considerations and control-signal amplitude limits.

The control-signal amplitude limit is a function of the physical limitations of the dc converters (see also section 4.5). For a given set of control-signal limits and a given gain matrix, it is still difficult to determine whether the gains are excessive because this question is meaningful only when it is posed with respect to a set of initiating disturbances such as faults and generator outages.

The characterization of the disturbance set is an open question. The basic difficulty here is that a large number of simulations on the full model are necessary to define this set, and it is not clear how one can specify a reasonable set that covers all the important cases. The author feels that research on this topic is worthwhile, but a controller design methodology can be developed in the absence of this information.

The design methodology in this thesis is developed based on the belief that a conservative and simple feedback design is most suitable for a large-scale system whose design model contains a significant amount of uncertainties. The author believes that such a design will also be most acceptable to the electric utility industry. The word "conservative" here refers to an approach in which the controller is designed to stabilize only the modes that are most controllable and observable by the MTDC controller, and the damping of these modes
are improved with the robustness and control-signal-amplitude constraints in mind. The exact amount of damping will depend on bandwidth limitations and simulation results on certain critical contingencies. Some iterations between design and simulation are unavoidable in an actual design.

The desire of keeping the control-signal amplitudes small tend to work hand in hand with the robustness constraint since for a nominal system that is open-loop stable, the tolerance for unmodeled dynamics generally improves with a decrease in gain. The robustness issue is discussed in detail in part II of this thesis.
4.3 OUTPUT FEEDBACK METHODOLOGY

4.3.1 Controller Type

A decision is made not to use adaptive controllers because the stability of this type of controllers is extremely difficult to prove except in a few special cases. Moreover, robustness theory for adaptive controller is nonexistent.

Non-adaptive compensators can be classified as either dynamic or static. A compensator is called "static" if its input-output relations can be described by a constant matrix. A compensator is called "dynamic" otherwise.

A dynamic-compensator methodology that has been tried on the Western U.S. model is the well-known linear-quadratic-Gaussian (LQG) technique [28,ch.4]. In this method, the separation principle allows the poles of the error dynamics and the poles of the plant dynamics to be designed separately. The performance of this design is found to be quite satisfactory for the design model of the Western U.S. example. This design, however, has not been tested in the full 42-machine model. The author suspects that an LQG design may work in the MTDC problem, but some tuning may be necessary to recover the robustness of the LQG regulator [55].

The LQG technique is not further pursued in this research because of its complexity and because of the fact that an LQG controller is inherently a centralized scheme.

The controller used in this thesis is the static output feedback scheme in which the control signals are computed as linear combinations of the available measurement signals.
controller of this type is easy to implement, and the feedback gains for different information structures can be taken into account in the computational algorithm (section 4.7).

4.3.2 Synthesis Techniques

A number of synthesis procedure have been proposed for the static output-feedback compensator. In choosing an output-feedback computational technique for the MTDC problem, the following criteria are felt to be the most important.

1. The computational technique must allow the designer to use his physical insights in the design process. The procedure should be easy to use and requires a minimum of design-evaluation iterations.

2. The computational technique must be able to accommodate feedback structures with varying degrees of decentralization.

3. The resulting output-feedback designs must have good robustness margins.

One class of algorithms that has been tried is the "optimal" output-feedback methodology. Synthesis techniques in this class are based on the necessary conditions for minimizing a quadratic cost functional. The problem statement is as follows. For a linear system (3.7-3.8), the output-feedback matrix $F$ is sought to minimize the quadratic cost

$$
\int_{0}^{\infty} \left( x(t)^{T}Qx(t) + u(t)^{T}Ru(t) \right) dt
$$

(4.1)
where \( Q \) is symmetric and semi-positive definite, and \( R \) is symmetric and positive definite. Levine and Athans [45] showed that the optimal output-feedback gain \( F \) must fulfill the following necessary condition.

\[
F = R^{-1}B^T K L C \left( C L C^T \right)^{-1}
\]

(4.2)

\[
0 = (A-BFC)L + L(A-BFC)^T + P
\]

(4.3)

\[
0 = (A-BFC)^T K + K(A-BFC) + Q + C^T F^T R F C
\]

(4.4)

The matrix \( P \) is the covariance of the initial-condition vector, and the matrices \( L \) and \( K \) are symmetric and positive definite. Unfortunately, no direct method is yet available for solving these coupled Lyapunov equations. Iterative techniques were proposed by Levine and Athans [45] and by Söderström [46], but the convergence properties of these heuristic techniques have never been proven mathematically.

The iterative algorithms in [45] and [46] are tested on the MTDC centralized output-feedback problem using a wide variety of initial starting points and initial-condition covariance matrices. Neither of these techniques is able to give a reasonable solution because the iterative technique diverges in all the cases tried.

Another class of output-feedback algorithm that has been studied and worked successfully can be characterized as weighted least-squares approximations to the state-feedback design. These algorithms can be decomposed into three main steps.
Step 1: The state feedback gain matrix is found using the LQ methodology or other state-feedback synthesis techniques.

Step 2: A vector norm is defined with respect to a certain positive definite weighting matrix.

Step 3: A weighted least-squares problem is solved to find the output-feedback matrix (or output-feedback matrices) whose equivalent state-feedback gain is "closest" to the state-feedback gain found in step 1.

Different techniques in this class differ in the vector norm defined in step 2. The minimum-excitation method due to Kosut [47], for example, employs a weighting matrix given by the solution of a Lyapunov equation. Kosut showed that the solution is optimal in a certain sense, but there seems to be little physical justification for the minimization problem. Another design technique due to Bengtsson and Lindall [48] advocated the use of a modal weighting matrix. The author's experience with the modal-weighting technique shows that it is extremely difficult to correlate the modal weightings to the design objective. Specifically, it is found that all the closed-loop poles are affected in an unpredictable manner when a single modal weight is altered.

Instead of the weighted norms proposed by these methods, the author finds that the usual 2-norm (where the weighting matrix is the identity matrix) gives satisfactory results for the MTDC problem. The output-feedback method in this thesis is based on the least-squares (or minimum-norm) method with
the usual 2-norm for approximating the LQ feedback gain. In the first step of this procedure, the LQ penalty matrices allow a fairly direct translation from the design objectives to the "mathematics" of the control problem (section 4.5). The LQ design is also known to have certain guaranteed robustness margins (section 4.4). It will be shown that the least-squares solution in the centralized case approximates very closely the LQ performance and robustness properties when the LQ design is done with the output-feedback design objective in mind. Another advantage of this technique is that the different feedback structures can be taken into account easily in the output-feedback computational algorithm. The technical aspects of this procedure is described in detail in section 4.6.
4.4 LINEAR QUADRATIC DESIGN

The LQ state feedback technique is well known. For completeness, the basic equations are given below [28, pg. 238]. For a linear, time-invariant system

\[
\dot{x}(t) = A \, x(t) + B \, u(t) \quad (4.5)
\]

the LQ feedback \( u(t) \) is computed to minimize the cost functional

\[
J = \int_{0}^{\infty} (x(t)^T Q \, x(t) + u(t)^T R \, u(t)) \, dt \quad (4.6)
\]

Under the conditions that

1) \( Q = Q^T > 0 \)
2) \( R = R^T > 0 \)
3) \([A, B]\) is stabilizable and \([A, Q^{1/2}]\) is detectable, the optimal control is given by

\[
u(t) = -G \, x(t) = -R^{-1} B^T K \, x(t) \quad (4.7)
\]

where \( K \) is the unique, positive semi-definite solution of the algebraic Riccati equation

\[
A^T K + KA + Q - K R^{-1} B^T K = 0. \quad (4.8)
\]

The closed-loop system using the control (4.7) is guaranteed to be stable.

It is also known that certain robustness properties hold regardless of the choice of the state weighting matrix \( Q \) and diagonal control weighting matrix \( R \) [15, 18, 42]. These guara-
tended stability properties can be interpreted as guaranteed phase and gain margins at all the input channels. This interpretation is demonstrated in figure 4.1, where the perturbations in the design model are represented by the scalars \(a_1, a_2, \ldots, a_m\), which are nominally equal to 1. This feedback system is guaranteed to be stable for simultaneous gain variations satisfying \(0.5 \leq a_i \leq \infty\), and for simultaneous phase variations \(a_i = \exp(j \phi_i)\) satisfying \(|\phi_i| < 60°\), \(i = 1, \ldots, m\). More details on the robustness of LQ controllers can be found in section 7.5.3.
Figure 4.1: Simultaneous perturbations at all the input channels of an LQ regulator.

CONTROLLER DESIGN METHODOLOGY 4.4 LINEAR QUADRATIC DESIGN
4.5 STATE AND CONTROL PENALTY MATRICES

The state weighting matrix \( Q \) for the MTDC controller is selected to penalize only the complex modes associated with interarea oscillations and not to penalize the real modes associated with the average frequency or phase reference. The real modes are not penalized because of the desire to avoid interactions between the MTDC controller and existing mechanisms for correcting generation-load mismatch and clock errors. Three different ways to achieve this design goal are described below.

4.5.1 Penalty on Average-Frequency Deviations

The deviation of an area's shaft speed from the average frequency is analogous to the speed of a mass seen by an observer at the center of mass of a mechanical system. It is intuitively clear that oscillatory motions between masses are impossible if all the masses appear stationary to this observer.

This physical insight motivates the so-called average-frequency-deviation method where the state penalty is set equal to a weighted sum of the square of the areas' speed relative to the average frequency. More precisely,

\[
\begin{align*}
\dot{x}(t)^T Q x(t) &= \sum_{i=1}^{n} a_i (\omega_i(t) - \bar{\omega}(t))^2 \\
&= (4.9)
\end{align*}
\]

where \( \bar{\omega}(t) \) is the average frequency defined in (3.5). In terms of the state variables defined in (2.17), \( Q \) can be
written as

\[ Q = (I - e w_1^T)^T L (I - e w_1^T) \]  \hspace{1cm} (4.10)

where

\[ L = \text{diag}[0, \ldots, 0, \alpha_1, \ldots, \alpha_n, 0, \ldots, 0] \]

\[ e = [1, 1, \ldots, 1]^T \] and

\[ w_1 \] is the left eigenvector associated with the average frequency mode.

Note that this method of constructing \( Q \) does not penalize the real pole at \( s=0 \) since the angle reference is not observable through frequency measurements. The average-frequency pole is not moved since it is being used as a reference and thus not observable in the cost.

4.5.2 Penalty on Modal Variables

The modal variable \( z(t) \) is related to the state variables, \( x(t) \), of (2.17) through

\[ z(t) = W x(t) \] \hspace{1cm} (4.11)

where \( W \) is the matrix of left eigenvectors (Appendix A). The state penalty in this method is written as a weighted sum of the square of the modal variables:

\[ x(t)^T Q x(t) = \sum_{i=1}^{2n+m} \alpha_i z_i^* z_i \] \hspace{1cm} (4.12)

The scalar \( \alpha_i \) is the modal weight for the \( i \)th mode. Equations (4.11) and (4.12) imply that...

CONTROLLER DESIGN METHODOLOGY  \hspace{1cm} 4.5 STATE AND CONTROL PENALTY
\[ Q = W^H \text{diag}[\alpha_1, \ldots, \alpha_{2n+m}] W. \] (4.13)

If the matrix \( W \) is given in real and imaginary parts, i.e.
\[ W = W_R + j W_I, \] (4.14)
then the matrix \( Q \) in (4.13) can be written as
\[ Q = W_R \text{diag}[\alpha_1, \ldots, \alpha_{2n+m}] W_R + W_I \text{diag}[\alpha_1, \ldots, \alpha_{2n+m}] W_I. \] (4.15)

4.5.3 Solheim's Method

Both the average-frequency-deviation and the modal-penalty methods have been tried on a 5-terminal/5-area system [29,30] and a 7-terminal/10-area system (figure 2.3). The average-frequency-deviation method turns out to be rather cumbersome to apply to large systems since there is no one-to-one correspondence between the scalar weights and the oscillatory modes of the system. As a result, repeated trials are necessary to "tune up" a design, but in each trial, there is no way to predict in advance the effects of modifying the scalar weights. The modal-penalty technique suffers from the same basic drawback since the alteration of any one of the modal weights changes the relative penalty between all the modes, and consequently causes all the poles to move in an unpredictable manner. The problems cited above are not very serious for small systems (e.g.,[29]), but for large systems (e.g. the 10-area model), these cut-and-try approaches are not effective.
A method due to Solheim [31] is found to be superior to the two methods mentioned above in that it allows independent movement of the poles. Solheim's method is a multi-step algorithm. In each step, the closed-loop matrix of the previous step is treated as if it were the open-loop matrix, and modal penalties are then used to move only one pole (or one pair of complex poles). The problem of changing the relative weights is avoided since only one pole is moved at a time, and the other poles are not penalized.

Once the final design is obtained, the equivalent Q matrix -- i.e. the Q matrix which yields the final design in one step -- is equal to the sum of the Q matrices used in the multi-step procedure. The same is true with the state-feedback gain matrix.

Solheim's method is well-suited to interactive computations on a digital computer. The following is an outline of a program based on this method.

Step 1: Initialize the overall state penalty matrix $\bar{Q}_s$ and the overall gain matrix $\bar{G}_s$ to zero. Also specify the control-penalty matrix R, and the system and input matrices A and B.

Step 2: Find eigenvalues and left eigenvectors of A.

Step 3: Ask the user which pole he wants moved, and how much modal weight to use. Suppose his reply is i and $\alpha_i$ (for a complex pair, he only has to specify one of the two poles).

Step 4: $Q = \alpha_i w_i w_i^H$ (4.16)

CONTROLLER DESIGN METHODOLOGY 4.5 STATE AND CONTROL PENALTY
Step 5: Solve the algebraic Riccati equation (4.8) using matrices $A$, $B$, $R$ and $Q$. Let $G$ be the resulting gain matrix.

Step 6: Find and display the eigenvalues of $A-BG$.

Step 7: Ask the user if he is satisfied with the closed-loop pole location of mode $i$. If he replies "no", go back to step 3 for another trial.

Step 8: Replace $A$ by $A-BG$, replace $Q_s$ by $Q_s+Q$ and replace $G_s$ by $G_s+G$.

Step 9: Ask the user if he wants more poles to be moved. If he replies "no", then display the equivalent state-penalty matrix $Q_s$ and gain matrix $G_s$ and stop. Otherwise go back to step 2.

4.5.4. Control Penalties

The maximum control-signal amplitude -- or maximum current modulation level -- at a dc converter is primarily a function of its short-term overload capability. Typical values for modulation level range from 10 to 20 percent rated-current for SCR (silicon-controlled-rectifier) valves and 5 to 10 percent for mercury arc valves [8].

The control penalty on a dc terminal is made inversely proportional to its steady-state current rating to penalize current modulations on a per-unit-rating basis. More precisely, the matrix $R$ is made diagonal and the element corresponding to the $i$th current-controlled terminal is set equal to
\( (1/r_i)^2 \), where \( r_i \) is its current rating.

This control weighting has the effect of distributing the control signals among the dc terminals according to their current rating, but it cannot ensure that the currents are within limits at all time. Certain controller-synthesis techniques -- notably the unknown-but-bounded technique [43,ch.4] -- is capable of enforcing the control-magnitude limits for all initial conditions and disturbances that lie within certain sets. However, the unknown-but-bounded technique is still far from being an easy-to-use design tool.

In using the design procedure in this thesis, simulations of the closed-loop design on a number of critical contingencies are necessary to determine whether the modulation signals are excessive. If so, the control gains can be reduced by increasing the control weights -- or equivalently, decreasing the state weights. Experience and engineering judgement are important in this process.
4.6 LQ POLE PLACEMENT

As was mentioned in section 4.2, the objective is to find a conservative design which stabilizes only the modes that are most controllable and most observable by the feedback compensator. The LQ feedback design is guided by this philosophy.

The interarea modes that are both controllable and observable are shown to be the modes with high residues in section 3.5. The design strategy is to stabilize the high-residue poles using Solheim's method of pole placement. The amount of damping improvement that can be achieved is typically from a value of per-unit critical damping of 0.05 to a value of 0.15-0.20. This amount of improvement is considered to be significant by engineers who are familiar with 2-terminal dc modulation controllers. As was mentioned in section 4.2, the amount of improvement achievable in a given system is a function of the closed-loop performance and bandwidth limitations.

Figure 6.1 shows the closed-loop poles of the 10-area/7-terminal system using state feedback. Note that only 4 pairs of oscillatory poles are stabilized and the rest are moved only slightly. To arrive at this design, only 5 of the 6 available input channels are used. The reason for leaving out the input corresponding to terminal 4 will be explained in section 6.2.
4.7 OUTPUT FEEDBACK

The objective of the output-feedback computation is to approximate as closely as possible the state-feedback gains given by the LQ methodology.

The information structure in a given output-feedback design is specified by the makeup of the output matrices $C_i$, $i=1,...,m$. For instance, if the local output and the output at terminal 1 are available at terminal $i$, then $C_i$ is made of two rows corresponding to the two available outputs. For a centralized scheme, all the $C_i$'s are identical.

4.7.1 Basic Algorithm

Let $G=\text{row}[g_1^T,...,g_m^T]$ be the LQ gain matrix. The problem is to find the output matrix $F=\text{row}[f_1^T,...,f_m^T]$ such that the 2-norm of

$$g_i - C_i^T f_i$$

is minimized for each $i$, $i=1,...,m$. Each of these problems can be posed as a system of linear equations with the vector $f_i$ being the unknown. The fact that $C_i^T$ has more rows than columns (i.e., more states than outputs) implies that an exact solution may not exist, because the vector $g_i$ may not lie within the column space of $C_i^T$.

The minimum-error (or least-squares) solution $f_i$ is a vector in the column space of $C_i^T$ which minimizes the distance to $g_i$ [32, ch.3]. This solution may also be visualized geometrically as the projection of $g_i$ on the column space of $C_i^T$.

If all the columns of $C_i^T$ are independent, then the solu-
tion $f_i$ can be found by premultiplying $q_i$ by the left pseudo-inverse of $C_i$:

$$f_i = (C_i C_i^T)^{-1} C_i q_i.$$  \hspace{1cm} (4.18)

However, if the columns of $C_i^T$ are nearly dependent -- as is the case in some MTDC systems, the solution given by (4.18) is not satisfactory. In this case, numerical techniques are required to detect the linear dependence and retain only the most independent columns of $C_i^T$. The output feedback gains used in this thesis are computed using a QR decomposition of $C_i^T$ with optimal column pivoting [33,34].

4.7.2 Output Matrix Structure

In order for this type of least-squares algorithm to work well, the output matrix $C_i$ must be in a form such that $C_i f_i$ can be made reasonably close to $q_i$. This means in mathematical language that the distance between the vector $q_i$ and the subspace spanned by the rows of $C_i$ must be reasonably small. A necessary condition for meeting this requirement is to have the "large" elements in $q_i$ coincide with the "large" elements in the rows of $C_i$. In other words, if $q_i$ contains "large" elements for certain types of variables, it is desirable to have the same general structure in the output matrices. The adjective "large" is put in quotes because it refers to elements in $q_i$ that are most important for the closed-loop performance. These elements need not be large in magnitude compared to the other elements.
The author's experience with state-feedback design of the MTDC problem shows that the large elements in $g_i$ are those associated with the shaft speeds. The elements associated with the shaft angles are small in magnitude and they are not important for the closed-loop performance [29]. The physical explanation is that the machines' damping is improved most easily by shaft-speed feedbacks. A controller that feeds back machine angles tends to alter the frequency of oscillations, but it has little effect on the machines' damping.

The output matrix $C_i$ in the design model, therefore, must have large elements in the shaft-speed positions, and zero or very small elements in the angle positions and elsewhere. The design-model equations (2.17) and (2.18) are formulated with this criterion in mind. The importance of matching the problem formulation to the solution algorithm, and vise versa, cannot be overemphasized.
4.8 SUMMARY

An output-feedback controller is necessary for an MTDC system which relies on the frequency of the commutation buses as measurement signals. A controller with non-adaptive, static gains is chosen for the MTDC problem because it is believed that a controller with a simple structure is most suitable for a large-scale power system whose design model may contain a significant amount of uncertainty (section 4.3.1). It is also emphasized that the controller should be design to stabilize the modes that are most controllable and observable by the MTDC system.

Different synthesis techniques for this type of controller are evaluated in section 4.3.2. These include iterative methods based on the necessary conditions for minimizing a quadratic cost functional, and several minimum-norm type of algorithms. The author's experience with the Western U.S. model indicates that the minimum-norm technique which employs both the LQ technique and the least-squares algorithm is the most promising for the MTDC problem.

The basic properties of LQ controllers are discussed in section 4.4, and several ways of selecting LQ penalty matrices for the MTDC design problem are shown in section 4.5. The emphasis in sections 4.5 is the exploitation of the physical insights in the LQ design process. The multi-step algorithm due to Solheim [31] is shown to be the most easy-to-use method. The details of Solheim's technique are also described in section 4.5.
The guidelines for placing the LQ poles are presented in section 4.6. The basic philosophy is to stabilize only the interarea poles that are most controllable and observable by the MTDC controller.

The method for approximating the LQ feedback gains subject to different structure constraints is described in section 4.7. It is stressed in this section that the output-feedback matrix of the design model must have large elements in the shaft-speed positions and zero or small elements in the other positions in order for the least-squares algorithm to work well.
CHAPTER 5
CONTROLLER STRUCTURE

5.1 OVERVIEW

The information structure of the MTDC output-feedback controller is studied in this chapter.

The selection of different information structures -- e.g. centralized, decentralized, etc. -- is dependent on the performance and robustness of the closed-loop system and the feasibility of the communication network necessary to implement these schemes. This chapter concentrates on the feasibility question, and it will show that the answer depends to a large extent on the basic communication requirements of an MTDC system that is operated under the current-margin scheme. The long-term scheduling of dc current orders and the associated communication issues are discussed in section 5.2, and the problems related to the preservation of current margin are discussed in section 5.3.

The centralized, distributed and decentralized output-feedback structures and the communication system required to implement these schemes are described in section 5.4. The motivations for selecting these structures and their feasibility are discussed.
5.2 BASIC COMMUNICATION REQUIREMENTS

A method for coordinating the dc terminals' current set-points (current orders) is central to an MTDC system operated under the current-margin scheme [9,35]. The coordination of dc current orders in an MTDC system is important for the reason that the current order of the voltage-setting terminal must be adjusted so that it is greater than the negative sum of all the other terminals' currents by the current margin (see section 2.2). As the demand for the transfer of dc power varies due to either planned interchange agreements or unscheduled outage of equipments, the current setpoint of the voltage-setting terminal must be updated to ensure that the current margin is maintained at all time. The coordination of dc current is also important for maintaining the same current modulation control regardless of which terminal is acting as the voltage-setting terminal (see next section).

The Current Reference Balancer (CRB) concept due to Forest is the most widely studied method for enforcing these constraints [36]. In the CRB method, the current orders received from the dc terminals are sum algebraically. The sum of the current orders, if not equal to 0, is redistributed to the current setpoint of all the dc terminals, subject to the maximum-current constraints. The current margin is added to the current order of the voltage-setting terminal, and then the current setpoints are sent back to the dc terminals.

To implement a current coordinator of this type, a centralized communication system capable of 2-way communication...
between the coordinator and all the dc terminals is required. Without current modulation, the speed/bandwidth requirement of the communication network for dc current coordination is not stringent, as most planned schedule changes are relatively infrequent, and most protective mechanisms are designed to act with only local information.

The use of current modulation for stability enhancement changes radically the communication requirements since dc modulation controls act continuously, and the modulation controller at a dc terminal may require measurement signals in remote areas. Reeve et al. showed that the CRB can be used also for systems with current modulation [38], but implicit in their scheme is a communication network with much greater speed and bandwidth than that is required without current modulation.

A consequence of the previous observation is that the communication requirements of an MTDC system should be analyzed in two parts. The first part needed for long-term current-order coordination has been discussed, and it appears that a relatively slow centralized communication network would suffice. The second part needed for current modulation is studied in the following section, with the goal of identifying the minimum high-speed communication requirements for MTDC systems.

CONTROLLER STRUCTURE  5.2 BASIC COMMUNICATION REQUIREMENTS
5.3 CURRENT-ORDER COORDINATION

The preservation of the current margin is one of the primary concerns in MTDC systems with current modulation. The basic problem is that the actual current at the voltage-setting terminal is affected directly by the algebraic sum of the current modulation at all the other terminals, and the current margin -- defined as the difference between the current order of the voltage-controlled terminal and its actual current -- can be lost if the current order of the voltage-setting terminal is not updated properly. A simple solution which requires no additional communication is to limit the current-modulation level at each dc terminal such that the current margin is not exceeded by the algebraic sum of all the modulation currents in the worst case. This simple solution is unfortunately not feasible in most systems, since the required limits are so restrictive that the current modulation controls are rendered useless.

5.3.1 2-Terminal DC Systems

The preservation of the current margin is also a concern for 2-terminal systems where the modulation range is closed to, or exceeds, the current margin. A solution for 2-terminal systems is described in the following since it has a direct extension to the MTDC case. Figure 5.1 shows a 2-terminal system where the rectifier is operated under current control with current setpoint equal to $I_2$, and the inverter is under constant-extinction-angle control with current setpoint equal
to $I_1$. The signal $\Delta I(t)$ is added to the rectifier's current setpoint to effect current modulation. At the same time, the modulation signal is transmitted via a high-speed transmission link to the inverter where it is subtracted from the inverter's current setpoint. The net result is that the inverter's setpoint is always set at $I_m$ away from the actual dc current.

Figure 5.1b shows the system configuration after a mode shift (which can be initiated by a decrease in the rectifier's commutation voltage). In the new mode of operation, the inverter is operated under current control and the rectifier under constant-ignition-angle control. The communication link is unchanged, so that the actual dc current becomes $I_2 - I_m + \Delta I(t)$, or $I_m$ less than the original dc current. Note that the current margin at the rectifier is also held constant at $I_m$ since its current order is greater than the actual current by $I_m$ at all time. Another important point to note is that the current modulation is in effect regardless of the mode of operation.

5.3.2 Multiterminal DC Systems

The current-order coordination scheme for 2-terminal dc systems can be extended to MTDC systems [37]. The method is illustrated on a 5-terminal system shown in figure 5.2. In this system, the current setpoints are assumed to be "balanced" initially, meaning that the current setpoint of the voltage-setting terminal has been adjusted for the current margin, and all the current setpoints are within the current limits of the dc converters. When current modulation is used
in terminals 2 to 5, the modulation signals are transmitted to terminal 1 to modify its current order. The current margin is held constant at terminal 1 since its actual dc current always differs from its current setpoint by $I_0$.

Figure 5.2b shows the operating condition when terminal 4 becomes the voltage-setting terminal. Two important features should be noted in this figure:

1. The current margin is maintained at the new voltage-setting terminal without altering the communication network.

2. The modulation currents at all the terminals remain the same, with current modulation at terminal 4 carried out "implicitly" by the current modulation at terminal 1. This method of current-order coordination is referred to as the implicit modulation scheme [37].

The second feature is crucial from a controller-design standpoint, as it permits the implementation of a fixed-gain compensator which does not depend on the mode of operation.

It is assumed in figure 5.2 that the modulation signals are computed locally, and then the modulation signals are transmitted to terminal 1. The transmission pattern is really unimportant to the implicit modulation scheme; the key feature of implicit modulation is the method used for computing the current setpoint at terminal 1. It will be shown in the next section that other communication schemes using implicit modulation are possible.

The high-speed communication system for current-order coordination is seen to require transmission in only one
direction (with possibly verifications in the opposite direction). The accuracy of the transmitted signal need not be very high in the normal mode of operation (figure 5.2a), since the current margin can fluctuate somewhat without compromising the operation of the system. The transmitted signal in figure 5.2b, however, must be more accurate, since the modulation at terminal 1 depends on the algebraic sum of all the transmitted signals, and an inaccurate signal would result in a "sloppy" modulation current at terminal 4.
Figure 5.1: Modulation coordination in a 2-terminal dc system
(a) Normal Mode of Operation

(b) After Mode Switching

Figure 5.2: Modulation coordination in an MTDC system
5.4 OUTPUT FEEDBACK STRUCTURE

The requirements for fast communication links -- aside from the current-order coordination function described in the last section -- are dependent on the amount of information needed to implement the feedback controller at each dc terminal.

A completely decentralized feedback scheme is suggested by Carter et al. [11]. The method proposed by Reeve et al. [38] is also, in some sense, decentralized, since the current schedule (including the modulation signal) is computed locally in each dc terminal. It should be noted that even though the control signals are computed locally in a decentralized feedback scheme, a centralized communication network is still necessary to implement the implicit modulation (current-order coordination) scheme described in the last section.

The necessity of the high-speed communication network for implicit modulation makes the centralized feedback controller a viable alternative. The communication network in this case is slightly different from the decentralized scheme with implicit modulation, but the difference in terms of cost is believed to be minimal.

A possible implementation of the centralized feedback system is outlined below for a p-terminal MTEC system in which terminal 1 is chosen arbitrarily as the site of the current-order coordinator.

1. The measurement signals at terminals 2 to p are transmitted to terminal 1.
2. The current setpoints for all the dc terminals -- including terminal 1 -- are computed at terminal 1 using all the measurement signals, and the setpoints for terminals 2 to p are transmitted back using the same communication links in the opposite direction. The computation of the current setpoint at terminal 1 is the same as before.

The transmission delay (including verification) for the centralized scheme is expected to be small. Judging from experiences with microwave communication systems presently used for 2-terminal dc links, the communication delay in an MTDC system is estimated conservatively to be 30 ms. This amount of time delay is very small compared to a period of interarea oscillation, which is in the order of 1 sec.

In an effort to understand the performance/communication tradeoffs, a quasi decentralized scheme is considered. In this so-called distributed scheme, the local measurement signal and terminal 1's measurement signal are used for computing each dc terminal's control. The communication requirement of this method is basically the same as that of the centralized scheme.

The distributed structure is motivated by the physical fact that the controls applied to terminal i, i=2,...,p, affect not only the ac power injection locally, but also the ac power injection at terminal 1. A distributed controller implemented with signals at both the local terminal and terminal 1 is, therefore, expected to perform better than the decentralized scheme.

CONTROLLER STRUCTURE

5.4 OUTPUT FEEDBACK STRUCTURE
Simulations in the next chapter using all three of the output-feedback schemes will show the tradeoffs between the information used to implement the controller and its performance. The robustness of the different output-feedback schemes is studied in section 9.5.
5.5 SUMMARY

Different output-feedback structures can be used for implementing the MTDC small-signal controller. This chapter examines the feasibility of the communication system necessary to implement these different feedback schemes.

The basic communication requirements for operating a p-terminal MTDC system are outlined in section 5.2. It is argued in this section that the communication issue can be studied in two parts. The first part needed for long-term current-order scheduling is shown to require a slow centralized communication network, and the second part needed for current-order coordination and implicit modulation is shown to require fast communication links from p-1 terminals to the remaining terminal. The combination of these requirements implies that a high-speed, centralized communication system is required regardless of the information structure of the modulation controller.

The communication system used for current-order coordination (implicit modulation) can also be utilized for transmitting measurement and control signals. This fact makes feasible the following feedback schemes (section 5.4).

1. **DECENTRALIZED** feedback: The control signal at each dc terminal is computed using the local measurement signal only.

2. **DISTRIBUTED** feedback: The control signal at each dc terminal is computed using the local measurement signal and the measurement signal at the voltage-setting terminal.
3. **CENTRALIZED** feedback: The control signal at each dc terminal is computed using all the measurement signals, including the measurement signal at the voltage-setting terminal.

The communication requirements for each of these schemes are discussed in detail in section 5.4. The performance of these output-feedback designs will be evaluated in the next chapter.
CHAPTER 6
RESULTS AND INTERPRETATIONS

6.1 OVERVIEW

The design philosophy and the controller structure for MTDC systems are studied in chapters 4 and 5. The purpose of this chapter is to illustrate the design techniques and to interpret the results obtained on the Western U.S. example and a 9-bus example.

The results for the Western U.S. example are shown after a discussion on some practical aspects of the synthesis procedure (section 6.2). The closed-loop poles of the 10-area design system and that of the 42-machine system using the same output-feedback gains are shown in section 6.3.1. The emphasis up to this point is to show that the feedback gains computed using an aggregated model will indeed yield a satisfactory design for the full classical-machine model.

The time response of the 42-machine closed-loop system subject to a fault at Malin is shown in section 6.3.2. These results for different output-feedback configurations point to communication/performance tradeoffs which must be considered in choosing an output-feedback structure (section 6.4).

A 9-machine 4-terminal ac/MTDC system is used in section 6.5 to demonstrate that an MTDC controller computed using a classical-machine design model works satisfactorily even if the dynamics associated with the exciters, power-system stabilizers and governor are ignored in the computation of the feedback gains.
6.2 PRACTICAL ASPECTS OF OUTPUT FEEDBACK DESIGN

In working with the Western U.S. system, certain practical considerations are found to be necessary to ensure that the controller designed based on the 10-area aggregate model works satisfactorily when it is implemented in the 42-machine model. Two of the most important considerations are described below. These considerations are believed to be important to other MTDC systems.

6.2.1 Nearly Identical Outputs

In the Western-U.S. example, the input/output characteristics of two of the dc terminals -- terminal 2 and terminal 4 -- are very similar, as evident by the similarities in numerical values in the corresponding entries in the input and output matrices, as well as similarities in the controllability and observability measures. The reason can be traced to the electrical proximity of these terminals, and the fact that they are located within the same coherent area (see section 2.3). These facts pose no problem for the state-feedback design, since the control tasks are automatically divided up among these two terminals according to the relative magnitude of the control penalties. However, serious problems in output-feedback computation can arise if both of these nearly identical outputs are used to approximate the state feedback gain matrix.

To clarify this point, consider the hypothetical case in which the rows in the output matrix corresponding to terminals...
2 and 4 are exactly the same. Also suppose that a centralized output-feedback design is sought to approximate the LQ gain at terminal \( i \) which feeds back roughly 10 times the output at terminal 2. A possible output feedback design is, therefore, a gain of 10 for the output at terminal 2 and zero for all other outputs. However, the fact that the output at terminals 2 and 4 are identical implies that the same output-feedback approximation can be achieved by 1000 times the output at terminal 2 minus 990 times the output at terminal 4. There is no difference between these two output-feedback designs as far as the design model is concerned. But the second set of gains in the 42-machine model is almost sure to give disastrous results.

In reality, the rows of \( C \) corresponding to these two terminals are not exactly the same, though the end result in using both rows in the output-feedback approximation can be equally disastrous. This degeneracy problem is avoided in most numerical least-squares algorithms by monitoring the condition number as a new row of \( C \) is brought into the basis, and rejecting those rows of \( C \) which cause the condition number to increase beyond a predetermined threshold. The problem in using these algorithms is that it is difficult to know a priori how to choose the proper threshold for a given system.

The simplest solution to the degeneracy problem is to omit one of these two terminals for both state- and output-feedback computations. The terminal 4 is omitted in this example because it is seen to have less controllability and
observability than terminal 2. If it is desirable to have the current-modulation capability at terminal 4 in the actual implementation (or in a simulation), the gain computed for terminal 2 can be partitioned among itself and terminal 4 according to the relative rating of these terminals.

6.2.2 Fast Correction Term

In finding the design model of the MTDC/ac power system, the singular perturbation technique is used to construct the design model after the state variables are transformed into slow area variables and fast inter-machine variables (see section 2.3 for more details).

The equations (2.4) and (2.5), which represent the first-order approximation to the slow subsystem, are constituted by two kinds of terms: terms related to the slow subsystem, and "correction terms" related to the fast subsystem. For example, the matrix $C_{11}$ in the output matrix comes from the original slow subsystem, and the term $C_{12}A_{22}^{-1}A_{21}$ is the fast correction term which accounts for the steady-state contributions of the neglected fast dynamics.

When designing the output-feedback controller for the Western-U.S. model, it is found that the fast correction term $C_{12}A_{22}^{-1}A_{21}$ in the output matrix give rise to large differences in closed-loop pole locations between the 10-area model and the 42-machine model. The differences in some cases are so large that the 42-machine model becomes unstable when output feedback is implemented.

This problem is avoided when the fast correction terms in
the output matrix are omitted. It does not seem possible to prove mathematically why the output-feedback scheme performs much better when it is computed using the uncorrected output matrix (i.e. $C = C_{11}$). But one can argue on physical grounds that the fast correction term is important primarily for the steady-state response, and it may in fact distort the characteristics of the slow dynamics which are important for the output-feedback controller design.
6.3 RESULTS ON THE WESTERN U.S. MODEL

6.3.1 Closed-Loop Poles

Based on the LQ state-feedback design (section 4.6), the output feedback gains for the centralized, distributed and decentralized cases are computed. The closed-loop poles of these design for the 10-area model are shown in figure 6.1. It can be seen in this figure that the centralized-feedback poles are very close to their LQ counterpart.

The poles of the distributed and decentralized schemes are seen to be either a little more or less damped than the LQ poles. The location of the corresponding poles in these two schemes are also very similar, except for the Intertie mode and the average-frequency pole. The difference in location of the Intertie mode can be explained by the fact that the voltage-setting terminal is located in an area that is dominant in the Intertie mode of oscillation, and consequently the lack of the voltage-setting terminal’s information in the decentralized case affects the approximation to this mode. An implication of this argument is that the difference in pole location between the different feedback structures is dependent to some extent on the assignment of the voltage-setting terminal. In any case, the differences in pole locations seen on figure 6.1 are not serious, and the closed-loop oscillatory poles are always better damped than the corresponding open-loop poles.
Figure 6.1: Closed-loop poles for state- and output-feedback systems. The poles are computed using the 10-area model.
An important test of the use of aggregated models for output-feedback design is that the closed-loop interarea poles in the 42-machine system should agree with those in the 10-area system. Moreover, the high-frequency inter-machine poles should not be affected appreciably by the output feedback.

The closed-loop poles for the 10-area and the 42-machine models are computed and they are shown in figures 6.2, 6.3 and 6.4 for the three output-feedback structures. It is clear from these figures that there is excellent agreement in the closed-loop-pole locations for modes with frequency below 4.5 rad/sec, but the agreement is relatively poor for the two modes near 5 rad/sec. The error in pole location near 5 rad/sec is attributed to the lack of one-to-one correspondence between the modes of the 42-machine system and that of the 10-area aggregated model in this frequency range. This problem and possible solutions were discussed in section 2.3.1. When the inter-machine poles (above 6 rad/sec) in figures 6.2-6.4 are compared to their open-loop positions in figure 2.4, it is found that only one of the poles at 9 rad/sec is affected by the feedback design, and the damping of this pole is increase by the controller.

Overall, the agreement between the two models is good, and this is a strong indication that the output feedback compensator for an MTDC system can be computed using an aggregated design model.
Figure 6.2: Closed-loop poles for the centralized output-feedback system. Poles shown here are computed by using the 10-area and 42-machine systems.

RESULTS AND INTERPRETATIONS 6.3 THE WESTERN U.S. MODEL
Figure 6.3: Closed-loop poles for the distributed output-feedback system. Poles shown here are computed by using the 10-area and 42-machine systems.

RESULTS AND INTERPRETATIONS

6.3 THE WESTERN U.S. MODEL
Figure 6.4: Closed-loop poles for the decentralized output-feedback system. Poles shown here are computed by using the 10-area and 42-machine systems.
6.4 PERFORMANCE/COMMUNICATION TRADEOFFS

6.4.1 42-Machine LQ Design

The output-feedback compensators are found by approximating the gain matrix of the LQ feedback design. It was shown in section 6.3.1 that there is generally good agreement between the LQ feedback design and the output-feedback designs in terms of the closed-loop pole location of the interarea modes. It is also desirable to compare the time simulation of these designs, but a problem is encountered in simulating the LQ design on the 42-machine system since the LQ gain matrix is found for the state variables of the 10-area system.

A possible solution to this problem is to find an LQ feedback design for the 42-machine system whose state and control penalties are equivalent to that of the 10-area design model. More precisely, the state penalty matrix, $Q_{42}$, is computed by

$$Q_{42} = \bar{U}^T Q_{10} \bar{U}$$  \hspace{1cm} (6.1)

where $\bar{U}$ is the grouping matrix which relates the state variables of the 42-machine model, $x_{42}$, to that of the 10-area model, $x_{10}$ -- i.e.,

$$x_{10} = \bar{U} x_{42}$$  \hspace{1cm} (6.2)

The control penalty matrix is the same as before.

The response of this LQ compensator is shown along with the response of the output-feedback schemes in the next section.

RESULTS AND INTERPRETATIONS  6.4 PERF./COM. TRADEOFFS
6.4.2 Simulations

The performance of the feedback controller is evaluated using simulation results on the 42-machine linearized model.

The response of the system immediately after the following set of events is shown in this section.

1) At t=-0.65 sec, the AC Intertie at Malin (near machine 25 in figure 2.3) is faulted.

2) At t=0.0 sec (4 cycles later), the 3-phase-to-ground fault is cleared and the AC Intertie remains intact.

The fault is simulated numerically on a nonlinear model of the 42-machine model (with no dc current modulation). At t=0, the simulation is terminated and the machine angle and shaft speeds at that instant are used as an initial-condition vector for the linear simulation.

The different modes in the systems are excited by this fault to a different extent. Recall from Appendix A that the extent to which a mode is excited by a given initial condition is determined by the inner product between the left eigenvector and the initial-condition vector. This quantity for the AC-Intertie fault is shown in table 6.1 using the unit-momentum scaling of eigenvectors. It can be seen that all but three of the interarea modes are excited by this disturbance.

The shaft speeds of selected machines in the 42-machine model are shown in figures 6.5 to 6.14, and the angles between different pairs of machines are shown in figures 6.15 to 6.19. In addition, plots of current modulation signals are shown in figure 6.20 to 6.25. In each of these plots, the open-loop
response is shown along with the closed-loop response of the LQ, centralized, distributed and decentralized schemes.

<table>
<thead>
<tr>
<th>mode number</th>
<th>open-loop eigenvalues</th>
<th>magnitude of $w^T x_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.25 \pm j 5.24$</td>
<td>0.027</td>
</tr>
<tr>
<td>2</td>
<td>$-0.19 \pm j 4.60$</td>
<td>0.021</td>
</tr>
<tr>
<td>3</td>
<td>$-0.19 \pm j 4.21$</td>
<td>0.108</td>
</tr>
<tr>
<td>4</td>
<td>$-0.28 \pm j 3.54$</td>
<td>0.084</td>
</tr>
<tr>
<td>5</td>
<td>$-0.15 \pm j 3.28$</td>
<td>0.534</td>
</tr>
<tr>
<td>6</td>
<td>$-0.19 \pm j 2.83$</td>
<td>0.179</td>
</tr>
<tr>
<td>7</td>
<td>$-0.18 \pm j 2.60$</td>
<td>0.831</td>
</tr>
<tr>
<td>8</td>
<td>$-0.21 \pm j 2.05$</td>
<td>0.192</td>
</tr>
<tr>
<td>9</td>
<td>$-0.25 \pm j 1.66$</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Table 6.1: Excitation of interarea modes. The values have been scaled such that the largest element is 1.000.
Figure 6.5: Shaft speed of machine 29 in area 1.
Figure 6.6: Shaft speed of machine 13 in area 2.
Figure 6.7: Shaft Speed of machine 14 in area 3.
Figure 6.8: Shaft speed of machine 31 in area 4.

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Figure 6.9: Shaft speed of machine 24 in area 5.

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Figure 6.10: Shaft speed of machine 6 in area 6.

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Figure 6.11: Shaft speed of machine 19 in area 7.

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Figure 6.12: Shaft speed of machine 35 in area 8.
Figure 6.13: Shaft speed of machine 9 in area 9.
Figure 6.14: Shaft speed of machine 39 in area 10.
Figure 6.15: Angle between machine 19 in area 7 and machine 31 in area 4.

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Figure 6.16: Angle between machine 19 in area 7 and machine 35 in area 8.
Figure 6.17: Angle between machines 31 and 33 in area 4.

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Figure 6.18: Angle between machines 19 and 21 in area 7.
Figure 6.19: Angle between machines 13 and 30 in area 2.

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Figure 6.23: Current modulation at terminal 1.

RESULTS AND INTERPRETATIONS 6.4 PERF./COM. TRADEOFFS
Figure 6.21: Current modulation at terminal 2.

RESULTS AND INTERPRETATIONS 6.4 PERF./COM. TRADEOFFS
Figure 6.22: Current modulation at terminal 3.
Figure 6.23: Current modulation at terminal 5.

RESULTS AND INTERPRETATIONS 6.4 PERF./COM. TRADEOFFS
Figure 6.24: Current modulation at terminal 6.

RESULTS AND INTERPRETATIONS 6.4 PERF./COM. TRADEOFFS
Figure 6.25: Current modulation at terminal 7.

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6.4.3 General Observations

The improvement in damping is very clear in the shaft speed of machines 29, 31 and 35. But for other machines, the improvements are not as obvious. The reasons are that

1) the fault did not excited modes 1, 2 and 4, which are stabilized by the controller, (see table 6.1 and figure 6.1)

2) the decaying interarea responses are often obscured by lightly damped inter-machine oscillations.

In figure 6.9, the closed-loop responses look even less damped than the open-loop response, even though all the closed-loop poles are at least as well damped as the open-loop poles. This seemingly paradoxical results is explained by the fact that the open-loop response is a sum of two oscillatory modes that are similar in frequency and are approximately 180 degrees out of phase. The damping of one the modes therefore makes the other mode stand out in the closed-loop response. Machine 24 should be better damped in most cases, since this phenomenon is a function of the initial disturbance.

The improvements in damping can also be seen in the angle difference between two machines which participate in interarea swings being damped out by the MTDC system. The curves shown in figures 6.15 and 6.16 are typical examples. In figure 6.15, the angle separation between the Northwest and the Los Angeles area is shown. The improvement in damping here is a result of the control of the Intertie mode. In figure 6.16, the improvement is a result of the control of the mode at 0.56
hz.

In some instances the error in approximating the LQ gain affects other modes that are neglected in the design model. The slowest "inter-machine" mode between machines 13 and 30 is an example of a mode that is affected favorably by the errors. The angle difference between these two machines shown in figure 6.19 shows that the output-feedback responses are much better damped than the open-loop response. The LQ response shows no improvement in damping because the LQ gains are computed based on the design model, and this mode is not observable in the cost.

The majority of inter-machine oscillations, however, are not affected by the MTDC controller. This fact is apparent in figures 6.17 and 6.18 which show oscillations between two pairs of machines that are very close to dc terminals 2 and 1, respectively.

It is important to note that the amplitude of the first swing is not affected appreciably by any of the controllers. This confirms the general notion that MTDC small-signal modulation is not effective for transient-stability enhancement. However, MTDC systems have the potential of providing the controls needed for holding together areas that are otherwise torn apart by a severe disturbance. The study of MTDC large-signal control is a challenging topic for future research.

6.4.4 Tradeoffs

The performance of the closed-loop system in terms of the damping of the interarea oscillations and the amplitude of the
control signals is seen to be different for different output-feedback schemes.

The LQ and the centralized responses are extremely similar, and they tend to be a little better damped than the other cases. The responses of the distributed and the decentralized schemes are seen to be displaced vertically from the LQ and centralized curves because of the slower average-frequency poles. Overall, the performance of the different feedback structures in terms of damping is quite similar.

The control signals corresponding to these different schemes (figures 6.20 to 6.25), however, are drastically different. They are different not only in amplitude, but their sign is different in many cases. Let's concentrate on the current modulation at terminal 5 (in figure 6.24). In the first second, note that all but the decentralized controller apply negative current changes to effect an increase in power injected in the ac network. The decentralized controller applies positive current changes which have exactly the opposite effect.

The decentralized controller's intention in the first second is to slow down the machines near terminal 5. In the process of decreasing the ac-power injection at terminal 5, however, the ac-power injection at the voltage setting terminal (VST) is increased automatically because of the current margin scheme. What terminal 5 does not realize through the decentralized information structure is that the machines near the VST are over-speeding even more (compare figures 6.11 and
6.12). Consequently, the machines near terminal 2 is slowed down temporarily at the expense of aggravating the over-speeding problem at the VST.

A more coordinated strategy is taken by the other feedback schemes in which each controller have access to the VST's output signal. In these schemes, the controller at terminal 5 recognizes the over-speeding problem at the VST and applies the more effective strategy of slowing down the machines near the VST and speeding up the local areas to reduce the angle and speed differences.

The sign of the control signals are roughly identical for different controllers after a few seconds. The reason is that the machines near the VST have slowed down sufficiently; thus the knowledge about the machines near the VST is no longer as important.

The amplitude of the control signals is seen to vary inversely with the amount of information available to each controller. The control amplitude is the least for the LQ case as expected. The centralized controls are very similar to those of the LQ case, indicating that the centralized feedback is indeed a good approximation to the LQ feedback gain. The distributed controls are about 50 percent larger in amplitude, and the decentralized controls are even larger. The difference in control amplitude in the distributed and decentralized cases confirms the fact that the information on the VST is important for proper coordination.

The tradeoffs between performance and communication in an
MTDC control system is recognized here for the first time. The results in this research show that the communication between terminals is crucial for a coordinated strategy which achieves the objective (of damping interarea oscillations) with a minimum of effort.
6.5 9-MACHINE EXAMPLE

The results on the 42-machine Western U.S. example show that the controller can be designed based on a 10-area aggregate model in which the inter-machine dynamics are ignored. An important issue that has not been addressed is what are the effects of the controller on the unmodeled dynamics associated with the exciters, power-system stabilizers and turbine governors. This issue is felt to be very important because the neglected dynamics in this case are no longer separated in frequency from the interarea dynamics of interest.

An attempt was made to construct a more detailed model of the 42-machine system. But due to many practical problems such as program dimensioning limits, it was decided to perform this study on a 9-machine, 4-terminal MTDC/ac power system model available at the General Electric Company.

In the 9-machine system, two of the generators are modeled with 4 rotor circuits (2 in the direct axis and 2 in the quadrature axis), and they are equipped with IEEE Type-l exciters and 3rd-order power-system stabilizers. The turbine governor in one of the machines is also modeled as a 3rd-order transfer function. The order of this system -- including the frequency tranducers at the dc terminals -- is 44. The linearized model for 5 operating points, obtained by taking lines out and modifying the generation and load patterns, are computed numerically using the POSSIM program at the General Electric Company [52]. The open-loop eigenvalues of this
system (for 5 operating conditions) are shown in figure 6.26. The oscillatory poles that are associated with the machine oscillations are numbered from 1 through 8. The average-frequency pole is located at s = -0.4.

The controller design is performed on a classical-machine representation of the 9-machine system (for one of the five operating conditions). The poles of the design model are plotted with those of the full model in figure 6.27. It can be seen that there is generally good agreement between the oscillatory modes that are associated with machine oscillations. The only exception is mode 8, which is more damped in the full model. The actions of the power-system stabilizers are believed to be responsible for the extra damping in this mode. It is also noteworthy that 6 pairs of the oscillatory modes have no counterpart in the design model, because they are associated with the dynamics of the machine regulators. There are also many real poles on the real axis that are not modeled in the design model.

A centralized output-feedback design is found using the design methodology described in chapter 4. The closed-loop poles of the design model are plotted in figure 6.28. Note that all but 3 of the poles are stabilized, and the average-frequency pole is not affected appreciably.

The centralized design is implemented on the full model, and the closed-loop poles corresponding to the five operating conditions are plotted in figure 6.29. A comparison between the closed-loop poles in this figure and those in figure 6.28
shows that there is again good agreement. Mode 8 in figure 6.29 is more damped than its counterpart in figure 6.28, but the amount of movement from its open-loop position (figure 6.26) is roughly the same as that shown for the design model (figure 6.28).

It can also be observed in figure 6.29 that the mode that are not represented in the design model are not affected by the controller, except for the mode at 5 rad/sec, which shows higher damping in the closed-loop system. The favorable results in this example again support the philosophy of using a classical-machine design model for the control synthesis procedure.
Figure 6.26: Top: Open-loop eigenvalues of the 9-machine example for 5 operating points. Bottom: Same as the top except that the horizontal axis is lengthened.
Figure 6.27: Open-loop eigenvalues of the 44-order model and of the 21-order design model.

Figure 6.28: Open- and closed-loop eigenvalues of the design model.

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Figure 6.29: Top: Closed-loop eigenvalues of the 9-machine example for 5 operating points. Bottom: Same as the top except that the horizontal axis is lengthened.
6.6 SUMMARY

The design methodology developed in this thesis is demonstrated in the first part of this chapter using a 42-machine, 7-terminal model of the Western U.S. power system.

Some practical problems that were encountered in the design process are discussed in section 6.2. These include numerical degeneracy problems caused by nearly identical outputs and the effects of the fast-correction terms given by the singular-perturbation approximation. The author's experience indicates that

1. the degeneracy problem can be avoided by using only one of the two nearly identical outputs in the least-squares approximation, and partitioning the resulting gain among the two controllers afterwards, and

2. the fast correction terms in the output matrix are detrimental to the performance of the controller in the full system, and much better results are obtained without these correction terms.

The results of the Western U.S. example are presented in section 6.3. The closed-loop eigenvalues of the centralized, distributed and decentralized cases in the design model are shown to be very close to those in the full model. Moreover, the inter-machine poles are not affected appreciably by the controller.

Some simulation results of the Western U.S. example are shown in section 6.4. A comparison of the time responses show that even though the damping of the different output-feedback
structures are roughly the same, the decentralized scheme requires significantly more control effort than the centralized scheme. The deficiency of the decentralized scheme is attributed to the lack of coordination between the current-controlled terminals and the voltage-setting terminal.

A 9-machine example is used to study the effects of the controller on the unmodeled dynamics associated with machine regulators such as exciters, power-system stabilizers and turbine governors. A centralized output-feedback design is computed using a classical-machine representation of the 9-machine system (with no machine aggregation), and the closed-loop poles of the design model are compared to those of the full model for 5 operating conditions. Excellent agreements are observed in this comparison for poles that are directly related to machine oscillations. Furthermore, the unmodeled poles are not affected appreciably by the controller, even though many of them are within the frequency band of machine oscillations.
PART II

MODELING UNCERTAINTIES
CHAPTER 7
ROBUSTNESS THEOREMS

7.1 OVERVIEW

A feedback design is said to be robust if the closed-loop system remains stable in spite of the differences between the design model and the actual plant. The evaluation of the robustness of the closed-loop system is an important part of a design methodology.

The motivations for studying the robustness of an MTDC system are given in section 7.2. This section also makes clear the distinction between structured and unstructured uncertainties, and it points out that the recently developed matrix-norm criteria are useful for evaluating the robustness of an MTDC/ac power system which contains both types of modeling errors.

The robustness of a single-input, single-output (SISO) system is the same regardless of where the loop transfer function is computed. This is not true in multi-input, multi-output (MIMO) systems. To facilitate discussion on robustness of MIMO systems at different points, the phrase "breaking the loop at a point" is explained in section 7.3.

The robustness theorems that will be used for studying the robustness of power-system models are developed in section 7.4. A graphical interpretation of these theorems is given in section 7.5. The interpretation based on an SISO system is shown to be useful for MIMO systems if the perturbation matrix is restricted to be diagonal. This section closes with a
discussion on the robustness properties of LQ compensators.

The concept of bandwidth for MIMO system is developed in section 7.6. It will be shown that the maximum bandwidth of the system can be deduced directly from the robustness margins computed using the criterion $1 + G^{-1}$, where $G$ is loop transfer matrix. The results of this chapter will be used in Chapter 8 for evaluating the stability margins of an MTDC/ac power system.
7.2 MOTIVATIONS

Robustness is an important issue in dc controller design because of the uncertainties in system parameters and the modeling errors committed in constructing a simplified design model. Even the 2-terminal dc modulators which are designed through the use of SISO synthesis techniques are not immune to instabilities. The oscillations observed at the Pacific DC Intertie [4] and the generator-shaft torsional interactions with the dc converter observed in the Square Butte Project [10] are excellent examples. At the Pacific Intertie, 0.7-hz oscillations in dc power with amplitude as high as 30 MW (approximately 3% rated power) were detected when the modulation system was first installed [4]. This problem was solved by reducing the feedback gains and modifying the time constants of the compensator. Some intermittent oscillations at 8 hz were observed recently, but the cause of this instability is still under investigation. At the Square Butte Project, unstable generator-shaft torsional oscillations were observed at generators near the dc terminals, and these oscillations were traced to interactions between the dynamics of the shaft and the dynamics of the dc terminals [10]. The solution to this problem consists of reducing the bandwidth of the current controller and modifying the parameters of the frequency-sensitive power controller (i.e. the modulation controller).

The robustness issue is almost certain to be more acute in the MTDC case because of the increase in size of the open-loop plant and the MIMO nature of the control system. The

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7.2 MOTIVATIONS
methods used for 2-terminal systems such as Bode plots [4] or root-locus diagrams [10] are not applicable to MTDC systems. In order to evaluate the robustness of MTDC systems, new methods must be developed. The development of these methods is the goal of this part of the thesis.

7.2.1 Structured and Unstructured Perturbations

Uncertainties in a power system model can be attributed to a wide spectrum of sources. It is important to first make the distinction between structured and unstructured uncertainties. Perturbations in a model are classified as "structured" if they can be attributed to parameter uncertainties in the nominal model, and "unstructured" otherwise. The taxonomy in modeling uncertainties is important in that certain robustness criteria are applicable to only one class of perturbations, while others are applicable to both. As an example, the notion of phase and gain margin in SISO control theory give measures of stability margin for both classes of modeling uncertainties, but root-locus techniques give stability margin for only structured perturbations.

It is clear that robustness criteria based on structured uncertainties are meaningful only if the system model is reasonable accurate. An example of this type of system in power-system analysis is a one-machine-versus-infinite-bus problem where the machine is modeled in great detail. For the MTDC/ac system model, however, both structured and unstructured perturbations are present, and some of these can be
significant near the interarea dynamics of interest.

The recently developed MIMO robustness theory [15-18] is proposed as a tool for assessing the robustness of the MTDC power-system model, as this theory is capable of handling both types of modeling uncertainties. Specifically, the matrix-norm bounds (which include singular-value bounds as special cases) and information on the "smallest" destabilizing perturbations are capable of pointing out any potential changes in phase and gain, as well as coupling between different channels, that destabilize the closed-loop system. The relevant theorems for the robustness-evaluation methodology of MTDC/ac power systems are presented in the remainder of this chapter.
7.3 BREAKING THE LOOP

Unlike SISO systems, the stability margin of an MIMO control system is different at different parts of the systems. For a given type of perturbation, it will be shown in Chapter 8 that the conservatism of the stability tests is dependent on where the loop is broken.

The phrase "breaking the loop at a point" implies the following set of mathematical manipulations. First the loop is cut open at the point. The outgoing channels are treated as inputs, the incoming channels as outputs, and the transfer function corresponding to these input and output channels -- known as the loop transfer matrix $G(s)$ -- is computed. The questions that the robustness theory addresses are how much and what uncertainties can be tolerated in $G(s)$ before the closed-loop system goes unstable.

The stability margins are computed in the frequency domain, and they vary as a function of frequency. To make best use of these results requires a system representation with readily available physical interpretations at the point where the loop is broken. For the power-system application, the damping and synchronizing torque framework of chapter 8 is developed with this purpose in mind.
7.4 ROBUSTNESS THEOREMS

Perturbations of a multiplicative form are considered in this thesis. Specifically, the actual loop transfer function $\hat{G}(s)$ is assumed to be related to the nominal loop transfer function $G(s)$ through the relation

$$\hat{G}(s) = L(s) G(s)$$  \hspace{1cm} (7.1)

or

$$\hat{G}(s) = G(s) L(s)$$  \hspace{1cm} (7.2)

where $L(s)$ is nominally the identity matrix (see figure 7.1). Other ways to relate $G(s)$ and $\hat{G}(s)$ are possible [18], but the multiplicative form is believed to be the most suitable for the MTDC problem. The reason will be apparent later (see section 8.5).

7.3.1 Derivations

The closed-loop transfer function of the nominal system is

$$G_{cl}(s) = G(s) (I + G(s))^{-1}.$$  \hspace{1cm} (7.3)

It can be shown that the characteristic polynomial of the closed-loop system, $\phi_{cl}$, is in the form

$$\phi_{cl} = \phi_{01}(s) \det(I + G(s))$$  \hspace{1cm} (7.4)

where $\phi_{01}(s)$ is the characteristic polynomial of the open-loop system. The closed-loop system is stable if, and only if, all the zeros of the closed-loop characteristic polynomial lie in the left half-plane.

Given the closed-loop characteristic polynomial, the
stability of an MIMO system can be determined via the Nyquist theorem stated below.

**THEOREM 7.1:** An MIMO system with a rational, proper loop transfer matrix $G(s)$ is closed-loop stable if the image of the Nyquist contour (figure 8.2) under the map $\det(I+G(s))$ encircles the origin $P$ times in the counterclockwise direction, where $P$ is the number of unstable poles of the loop transfer matrix $G(s)$ that are enclosed by the Nyquist contour as the radius $R$ of the half circle becomes sufficiently large.

Suppose that the nominal closed-loop system is stable, and $G(s)$ and $\hat{G}(s)$ have the same number of unstable poles. Theorem 7.1 implies that the actual system is stable only if the map under $\det(I+G(s))$ and $\det(I+\hat{G}(s))$ have the same number of encirclement about the origin.

Consider the case where $\hat{G}(s)$ is a continuous deformation of $G(s)$. For a "small" deformation, the number of encirclement is unchanged; consequently the perturbed system is stable. However, if the deformed image of the Nyquist contour passes through the origin at a certain frequency, the number of encirclement about the origin is changed and the perturbed system becomes unstable. The point where the image of the Nyquist contour touches the origin -- or equivalently, $\det(I+\hat{G}(j\omega))=0$ -- therefore, marks the borderline between stability and instability. From this argument, it is reasonable to characterize the stability margin of a system at frequency $\omega$ by the "distance" between the nominal return-

**ROBUSTNESS THEOREMS**

7.4 ROBUSTNESS THEOREMS
difference function $\mathbb{I} + G(j \omega)$ and the nearest singular matrix $\mathbb{I} + \hat{G}(j \omega)$ (whose determinant is zero).

The distance measure in this case must be a scalar function of a complex matrix. The determinant appears to be a reasonable candidate since the determinant of a matrix is zero if, and only if, the matrix is singular. However, the determinant of a non-singular matrix turns out to be an unreliable indicator of the nearness to singularity. It can be shown in a small example that a matrix with a large determinant can become singular when a very small (compared to the determinant) perturbation occurs in one of its elements [42].

It should be noted that the magnitude of the determinant of $\mathbb{I} + G(j \omega)$ in the SISO case is simply the distance between $G(j \omega)$ and the critical point, and this distance is known to be a reliable indicator of the nearness to instability. The familiar concepts of phase and gain margins are, de facto, based on this observation.

In the MIMO case, the matrix norm is a far more reliable measure of the nearness to singularity than the determinant. The following theorem is fundamental to the subsequent arguments [49].

**Theorem 7.2:** A non-singular, complex matrix $A$ differs from a singular matrix by no more in norm than $1/|A^{-1}|$. In other words, given $A$,

$$\{\min ||E||: A + E \text{ is singular} \} = \frac{1}{|A^{-1}|} \quad (7.5)$$

**Robustness Theorems**

7.4 Robustness Theorems
The 2-norm of a matrix is equal to its maximum singular value. The singular-value decomposition of matrix is also useful for giving an explicit expression for the perturbation $E$ which corresponds to the smallest change in 2-norm (see appendix F).

**THEOREM 7.3:** The solution to the minimization problem (7.5) using the 2-norm of a matrix is

$$E = \sigma_n u_n v_n^H$$

(7.6)

and

$$\|E\| = \sigma_n.$$  (7.7)

where $\sigma_n$ is the smallest singular value of $A$, and $v_n$ and $u_n$ are the corresponding left and right singular vectors, respectively.

Before these theorems are applied to the return-difference function, note that the perturbed return-difference function given by (7.1) can be written in two ways. (The argument "s" is omitted for brevity).

$$I + \hat{G} = I + LG$$

$$= L (L^{-1} + G)$$

$$= L \left[ (L^{-1} - I) + (I + G) \right]$$  (7.8)

and

$$I + \hat{G} = I + LG$$

$$= (G^{-1} + L) G$$

$$= \left[ (I + G^{-1}) + (L - I) \right] G$$  (7.9)
Similarly, the perturbed return difference function (7.2) can be written as

\[ I + \hat{G} = I + GL \]
\[ = [(L^{-1} - I) + (I + G)] L \]  \hspace{1cm} (7.10)

and

\[ I + \hat{G} = I + GL \]
\[ = G [(I + G^{-1}) + (L - I)] \]  \hspace{1cm} (7.11)

Note that whether \( \hat{G} \) is expressed as \( GL \) or \( LG \), the quantity inside the square bracket in (7.8) is identical to that in (7.10). The same is true between (7.9) and (7.11). For this reason, the following robustness theorems, which is based on the near-singularity of the quantity inside the square bracket in (7.8-7.11), hold for \( \hat{G} \) in both forms (7.1) and (7.2).

**THEOREM 7.4:** Given a rational, proper transfer matrix \( G(s) \) of a stable closed-loop system, the closed-loop system with a perturbed loop transfer matrix \( \hat{G}(s) \), related to \( G(s) \) through (7.1) or (7.2), is closed-loop stable if

1) \( G(s) \) and \( \hat{G}(s) \) have the same number of unstable poles, and

2) \( L(j\omega) \) has no eigenvalue at 0 or on the negative real axis for all \( \omega \geq 0 \), and

3) \[ ||L^{-1}(j\omega) - I|| < 1 / ||(I + G(j\omega))^{-1}|| \]  \hspace{1cm} (7.12)

for all \( \omega \geq 0 \).

\[ \triangle \]

**THEOREM 7.5:** Given a rational, proper transfer matrix \( G(s) \) of a stable closed-loop system, the closed-loop system with a
perturbed loop transfer matrix \( \hat{G}(s) \), related to \( G(s) \) through (7.1) or (7.2), is closed-loop stable if

1) \( G(s) \) and \( \hat{G}(s) \) have the same number of unstable poles, and

2) \(|L(j \omega) - I| < \frac{1}{|I + G^{-1}(j \omega)|^{-1}}| \). (7.13)

for all \( \omega \geq 0 \).

Strictly speaking, the arguments given in this section are not enough to prove these theorems. A major part that has been omitted is an embedding argument which ensures that the perturbed Nyquist diagram associated with \( I + \hat{G} \) -- where \( \hat{G} \) is given by (7.1) or (7.2) -- can be reached through a continuous deformation of the original Nyquist diagram which preserves the number of encirclements of the origin. Nonetheless, the results given in theorems 7.4 and 7.5 are correct. The reader who is interested in more technical details should consult [18].

When the 2 norm is used in (7.12) and (7.13), these equations can be written as (see Appendix F)

\[
\sigma(L^{-1}(j \omega) - I) < \sigma(I + G(j \omega)) \quad (7.14)
\]

and

\[
\sigma(L(j \omega) - I) < \sigma(I + G^{-1}(j \omega)) \quad (7.15)
\]

Furthermore, it can be shown that when the inequality (7.14) becomes an equality, the "minimum" destabilizing matrix \( L \) can be written as

\[
L(j \omega) = (I + \sigma_n u_n v_n^H)^{-1} \quad (7.16)
\]
where $\sigma_n$ is the minimum singular value of $I+G(j\omega)$ and $v_n$ and $u_n$ are the corresponding singular vectors. Similarly, for (7.15), the "minimum" destabilizing matrix $L$ can be written as

$$L(j\omega) = I + \sigma_n u_n v_n^H$$

(7.17)

where $\sigma_n$ is the minimum singular value of $I+G(j\omega)^{-1}$ and $v_n$ and $u_n$ are the corresponding singular vectors. For a given $G(j\omega)$, the matrices $L$ given by (7.16) and (7.17) are in general not the same. An intuitive explanation is that they represent the minimum-norm solution of different problems: The $L$ in (7.16) makes $I+G$ singular, and $L$ in (7.17) makes $I+G^{-1}$ singular. Since $(I+G)$ and $(I+G^{-1})$ are generally not the same, there is no reason to expect that the matrices $L$ in (7.16) and (7.17) to be identical.
Figure 7.1a: Nominal system.

Figure 7.1b: Perturbed system with loop transfer matrix (7.1).

Figure 7.1c: Perturbed system with loop transfer matrix (7.2).
7.5 GRAPHICAL INTERPRETATIONS

The singular-value inequalities (7.14-7.15) have interesting interpretations in the single-input, single-output setting. It will be shown that, at each frequency, these inequalities constrain \( L \) to lie within a certain region of the complex plane. An important fact to know is that the maximum and minimum singular values of a complex number are the same, and they are equal to its 2 norm.

7.5.1 Stability Margins in terms of I+G

A graphical interpretation is first given for the inequality (7.14). There are 3 cases: \( a < 1 \), \( a = 1 \) and \( a > 1 \), where \( a \) is the minimum singular value of \( I+G(j \omega) \).

CASE 1: In figure 7.2a, the Nyquist diagram of the loop transfer function \( G \) is plotted on the complex plane. For a fixed frequency \( \omega_0 \), a (dotted) circle centered at \((-1,0)\) can be drawn such that it intersects the Nyquist diagram at \( G(j \omega_0) \). The radius of this dotted circle, \( a \), is equal to the norm of \( I+G(j \omega_0) \), and it is assumed to be less than 1.0 in this case. Equation (7.14) then implies that the norm of \( L^{-1}(j \omega_0)-1 \) must be less than \( a \), or equivalently, \( L(j \omega_0) \) must lie inside a circular (shaded) region with center \((1/(1-a^2),0)\) and radius \( a/(1-a^2) \).

CASE 2: The value \( L \) is constrained by (7.14) to lie inside a half-plane, as is shown in figure 7.2b. It is interesting to note that the system is guaranteed to be stable for an infinitely large perturbation, as long as its phase is
within $+90$ degrees.

CASE 3: When $a$ is greater than 1, instead of constraining $L$ to lie inside a closed region, equation (7.14) excludes $L$ from a circle with center $(-1/(a^2-1), 0)$ and radius $a/(a^2-1)$ as shown in figure 7.2c. An additional constraint in theorem 7.4 is that $L$ must be excluded also from the negative real axis. This condition is necessary to prevent an 180-degree phase reversal for large values of $L$.

7.5.2 Stability Margins in terms of $I+G^{-1}$

There are also three cases to consider: $a < 1$, $a = 1$ and $a > 1$, where $a$ is equal to the norm of $1+G^{-1}(j\omega)$.

The graphical construction technique here is similar to that of the last subsection. First the Nyquist diagram of the loop transfer function is drawn, and then for a fixed frequency $\omega_0$, there is a circle with center $(-1/(1-a^2), 0)$ and radius $a/(1-a^2)$ which intersects the Nyquist diagram at $G(j\omega_0)$. Once the value of $a$ is known, $L$ is constrained by (7.15) to lie inside a circular region with center $(1,0)$ and radius $a$. This algorithm applies to all three cases, except that in case 2, the circle that intersects the Nyquist diagram degenerates into a vertical line with real part equal to $-0.5$.

7.5.3 Pure Gain and Phase Variations

A method of characterizing $L$ at each frequency is to ask that if $L$ were constrained to be a real number or a pure phase shift, how much can $L$ vary before the system becomes unstable in the worst case. The answer to this question are defined

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respectively as the gain and phase tolerances.

This characterization of \( L \) is similar to the familiar notion of gain and phase margins. Recall that the phase margin is defined as the angle through which \( G(j\omega) \) must be rotated in order that the unit-magnitude point, \( |G(j\omega)| = 1 \), passes through the critical point \((-1, 0)\), and the gain margin is defined as the reciprocal of the gain \( |G(j\omega)| \) at the frequency at which the phase angle is 180 degrees.

The phase and gain tolerances at each frequency is a function of some distance measure of \( G(j\omega) \). The two distance measures that are of interest here are the minimum singular value of \( I + G(j\omega) \) and \( I + G^{-1}(j\omega) \). Recall from the last section that the stability criteria (7.14) and (7.15) imply that for each value of \( a(I + G^{-1}(j\omega)) \) and \( a(I + G(j\omega)) \), there are certain values of \( L \) for which the closed-loop system with the perturbed loop transfer matrix \( LG \) and \( GL \) are guaranteed to be stable. These values of \( L \) are indicated by the shaded regions in figures 7.2 and 7.3.

The upper and lower limits of the gain tolerance are defined as the smallest and largest real numbers that are within the shaded region. For example, in case 1 of figure 7.2, the gain tolerance is \([1/(1+a), 1/(1-a)]\). The phase tolerance is defined as the largest angle \( \phi_0 \) such that the point

\[
\exp(j\phi)
\]

is within the shaded region for all \( |\phi| < \phi_0 \).

The phase and tolerances as a function of the magnitude

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of $g(I+G)$ and $g(I+G^{-1})$ are tabulated in tables 7.1 and 7.2 below. The formulae used for constructing these tables can be derived through simple geometry. They can be found in Lehtomaki's thesis [18].
Table 7.1: Worst-case gain and phase tolerance as a function of the minimum singular value of $I+G$

<table>
<thead>
<tr>
<th>$\sigma(I+G)$</th>
<th>Gain Tolerance</th>
<th>Phase Tolerance (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9 to 1.1</td>
<td>6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.8 to 1.4</td>
<td>17</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7 to 2.0</td>
<td>29</td>
</tr>
<tr>
<td>0.7</td>
<td>0.6 to 3.3</td>
<td>41</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5 to 10.0</td>
<td>53</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5 to $\infty$</td>
<td>60</td>
</tr>
<tr>
<td>1.1</td>
<td>0.4 to $\infty$</td>
<td>67</td>
</tr>
<tr>
<td>1.3</td>
<td>0.4 to $\infty$</td>
<td>81</td>
</tr>
<tr>
<td>1.5</td>
<td>0.4 to $\infty$</td>
<td>96</td>
</tr>
<tr>
<td>1.7</td>
<td>0.4 to $\infty$</td>
<td>107</td>
</tr>
<tr>
<td>1.9</td>
<td>0.3 to $\infty$</td>
<td>116</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3 to $\infty$</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 7.2: Worst-case gain and phase tolerance as a function of the minimum singular value of $I+G^{-1}$.

<table>
<thead>
<tr>
<th>$\sigma(I+G^{-1})$</th>
<th>Gain Tolerance</th>
<th>Phase Tolerance (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.9 to 1.1</td>
<td>6</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7 to 1.3</td>
<td>17</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5 to 1.5</td>
<td>29</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3 to 1.7</td>
<td>41</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1 to 1.9</td>
<td>53</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0 to 2.0</td>
<td>60</td>
</tr>
<tr>
<td>1.1</td>
<td>-0.1 to 2.1</td>
<td>67</td>
</tr>
<tr>
<td>1.3</td>
<td>-0.3 to 2.3</td>
<td>81</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.5 to 2.5</td>
<td>97</td>
</tr>
<tr>
<td>1.7</td>
<td>-0.7 to 2.7</td>
<td>116</td>
</tr>
<tr>
<td>1.9</td>
<td>-0.9 to 2.9</td>
<td>144</td>
</tr>
<tr>
<td>2.0</td>
<td>-1.0 to 3.0</td>
<td>180</td>
</tr>
</tbody>
</table>

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7.5 GRAPHICAL INTERPRETATIONS
From tables 7.1 and 7.2, two facts are worth noting.

1. The phase tolerance increases with $\sigma(I+G^{-1})$ and equals to 180 degree when $\sigma(I+G^{-1})$ is greater than, or equal to 2. The phase tolerance also increases with $\sigma(I+G)$ and it converges to -- but not including -- 180 degrees as $\sigma(I+G)$ goes to infinity.

2. The gain tolerance in the $I+G^{-1}$ case is always a bounded interval; whereas the gain-increase tolerance goes to infinity when $\sigma(I+G)$ is greater than 1.

The interpretations here have direct extension to the MIMO case when potential destabilizing cross-feeds between channels are excluded and only pure gain and phase perturbations are allowed. Under this assumption, $L$ is a diagonal matrix which represents simultaneous perturbations in all the channels. The criteria (7.14) and (7.15), in this case, represent bounds on the largest element in the diagonal matrices $L^{-1}-I$ and $L-I$, respectively, since the maximum singular value of a diagonal matrix is equal to the magnitude of its largest element.

For linear quadratic regulators, it has been shown that

$$\sigma(I + R^kG(j\omega)R^{-k}) > 1 \tag{7.18}$$

where $G$ is the loop transfer matrix evaluated with the loop broken at the input, and $R$ is the control penalty matrix [18, 42]. Under the assumption of diagonal $R$ and diagonal $L$, it can be shown that the phase and gain tolerances are at least as good as those given for $\sigma(I+G)=1$ in table 7.1.
[15, 18, 42]. More specifically, the phase tolerance is at least \( \pm 60 \) degrees and the gain tolerance is at least \([0.5, \infty)\) at each frequency. This fact was shown graphically in figure 4.1.
Figure 7.3: Graphical interpretation of the singular-value inequality (7.14).
Figure 7.2: Graphical interpretation of the singular-value inequality (7.15).

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7.5 GRAPHICAL INTERPRETATIONS
7.6 MIMO SYSTEM BANDWIDTH

The bandwidth of an SISO system is defined as the highest frequency at which the magnitude of the closed-loop transfer function \((I+G^{-1}(s))^{-1}\) has unity magnitude. (Note: this should not be confused with the crossover frequency, which is defined similarly in terms of the magnitude of the loop transfer function \(G(s)\)). The bandwidth of an MIMO system is more difficult to quantify since there are more than one element in the closed-loop transfer matrix, and the SISO bandwidth of each of the elements may be different.

The norm of the closed-loop transfer matrix matrix, however, give an upper bound on the magnitude of all the elements. The maximum MIMO bandwidth is defined as the highest frequency \(\omega\) at which

\[
\| (I+G^{-1}(j\omega))^{-1} \| = 1. \tag{7.19}
\]

Note that when the 2-norm is used, (7.19) is identical to

\[
\sigma(I+G^{-1}(j\omega)) = 1. \tag{7.20}
\]

and this is simply the point at which the robustness margin (7.15) intersects the 0-db axis.
7.7 SUMMARY

The evaluation of the robustness of an MTDC/ac power system is an important part of the MTDC controller design methodology because of the uncertainties in the power-system model.

In section 7.2.1, modeling uncertainties are classified as structured and unstructured depending on whether they can be attributed to parameter variations. It is stressed in this section that the recently developed matrix-norm stability criteria are capable of handling both types of modeling uncertainties.

The phrase "breaking the loop at a point" is explained in section 7.3 as the process of examining the uncertainties and the stability margins in terms of the loop transfer function evaluated at that point. This term is important because the robustness of a power system will be examined at different points in chapter 8, and it will be shown that the conservatism and the applicability of the robustness criteria are dependent on where the loop is broken.

The robustness theorems that will be used in subsequent chapters are developed in section 7.4. The stability margin of an MIMO system is shown to be related to the singularity of the return difference matrix $I+G$, and the matrix norm is a reliable indicator of the nearness to singularity.

A graphical interpretation of the robustness theorems is given in section 7.5 for an SISO system. The concepts of phase and gain tolerances are also defined in this section as
the pure gain and phase that can be tolerated by the closed-loop system. It is shown that the interpretations for SISO systems have direct extensions to MIMO systems if the matrix $L$ is constrained to be diagonal. The phase and gain tolerances as a function of $\sigma(I+G)$ and $\sigma(I+G^{-1})$ are tabulated in tables 7.1 and 7.2. The robustness properties of LQ compensators are also presented. The minimum guaranteed gain and phase tolerances of an LQ regulator are shown to be $[0.5, \infty)$ and $\pm 60$ degrees at all frequencies.

The concept of bandwidth for MIMO systems is presented in section 7.6. The maximum MIMO bandwidth is defined as the highest frequency at which the matrix norm of the closed-loop transfer matrix is equal to 1. It is pointed out that the maximum bandwidth can be found directly from the robustness margins computed using the minimum singular value of $I+G^{-1}$.

The applications of the robustness theorems for the MTDC/ac power system are demonstrated in the next chapter.
CHAPTER 8
POWER SYSTEM ROBUSTNESS

8.1 OVERVIEW

As was mention in the last chapter, the robustness margins of an MIMO system are dependent on where the loop is broken. In a power system model, the robustness can be examined with the loop broken at a number of points. In System Control Inc.'s approach [21], for example, the loop is broken at the junction between the elements that are functions of the operating point and those that are invariant to changes in the operating conditions (see section 1.2 for a more detailed description). In this thesis, the robustness margins are evaluated at the physical inputs and at the torque inputs to the generator shafts (with and without the compensator).

The breaking of the loop at the generator-torque input is motivated by the need to interpret physically the robustness margins computed using the matrix-norm criteria and the corresponding "smallest" destabilizing perturbations (chapter 7). The physical interpretations are aided by the concepts of damping and synchronizing torques. The definition of damping and synchronizing torques for a multi-machine system is given in section 8.2 along with several stability theorems in terms of these torques.

The justifications for evaluating the robustness at various points of a power-system model are given in section 8.3. The underlying theme in this discussion is that a meaningful application of the robustness results is hinged on
the ability to associate the frequency-domain results directly with the physical attributes of the system.
8.2 DAMPING AND SYNCHRONIZING TORQUES

A synchronous machine in an electric power system is a rotating mass driven by the difference between the mechanical torque provided by the prime mover and the electrical torque originating from electromagnetic interactions between the stator and the rotor circuits. The expression for the electrical torque is generally very complex, as it accounts for the coupling between the machine and the rest of the power system. A very useful concept in the analysis of synchronous machines has been the decomposition of the electrical torque into two orthogonal components: a damping component that is in phase with the speed phasor, and a synchronizing component that is in phase with the angle phasor. In a classical one-machine-versus-infinite-bus system, the damping torque is equal to the product of the shaft speed and the damping coefficient, and the synchronizing torque to the product of the electrical angle and the synchronizing coefficient. The damping torque is so called because the damping coefficient determines the damping of the second-order system. The synchronizing coefficient, on the other hand, is related to the restoring forces on the rotor and thus determines the natural frequency of machine oscillations. It is well known that in such a system, positive damping and synchronizing torques are necessary and sufficient for stability [39,ch.3].

de Mello and Concordia [20] extended the concept of damping and synchronizing torques and the associated stability requirements in a heuristic way to detailed machine models.
where the damping and synchronizing coefficients are no longer constants, but are functions of frequency. The full implications of frequency-dependent damping and synchronizing torques on the stability of a machines, however, have never been investigated in detail. The purpose of this section is to put forth a formal definition of damping and synchronizing torques for single- and multi-machine systems, and to show how this concept is related to the stability properties of a power system.

8.2.1 Definitions

A linearized model of an n-machine system can be represented schematically in the form shown in figure 8.1. No assumption is made here regarding the complexity of the machine or of the network model. The symbols in figure 8.1 are defined in the following:

\[ H = \text{diag}[2H_1, \ldots, 2H_n] \] where \( H_i \) is the inertial constant of the ith machine,

\[ \omega(s) = \text{change in shaft speed}, \]

\[ T_e(s) = \text{change in electrical torque}, \]

\[ T_m(s) = \text{change in mechanical torque (assumed to be zero in this analysis)}, \]

\[ T(s) = \text{a rational, proper transfer function matrix which relates the electrical torques to the shaft speed of the machines}. \]

The damping and synchronizing torques are defined formally in terms of \( T(s) \).
DEFINITION 8.1: The damping matrix $D(s)$ and synchronizing matrix $K(s)$ are defined as nXn, real-valued matrices such that

$$D(s) = \text{Re}[T(s)]$$

$$K(s) = \frac{\text{Re}[sT(s)]}{\omega_R}$$

for all $s$ where $T(s)$ is defined. The constant $\omega_R$ is the rated synchronous speed of the machines.

It should be noted that it is not strictly correct to think of $D(s)$ and $K(s)$ as separate physical entities, since they are merely components of a single transfer matrix $T(s)$.

8.2.2 Stability Properties of a Single-Machine System

For a one-machine-vs-infinite-bus system, the characteristic polynomial is equal to

$$2Hs^2 + D(s)s + \omega_R K(s).$$

(8.3)

It is obvious that if $D(s)$ and $K(s)$ were constants, then the system is stable if, and only if, both $D$ and $K$ are positive. When $D(s)$ and $K(s)$ are functions of frequency, the following theorem gives a sufficient condition for stability:

THEOREM 8.1: A one-machine-vs-infinite-bus system is stable if

1) $T(s)$ has no poles in the closed right half-plane except for a simple pole at 0,
2) $K(0) > 0$,
3) $D(j\omega) > 0$ for all $\omega \in \mathbb{R}$.

POWER SYSTEM ROBUSTNESS 8.2 DAMP. AND SYNCHRON. TORQUES
It is noteworthy that condition 2 is related to the steady-state stability requirement of a machine. It means physically that the restoring force on the rotor must be opposite to the displacement. Condition 3, which requires the damping torque to be positive at all frequencies, is consistent with the time-honored notion that a machine can be stabilized by adding positive damping torques at frequencies where damping is absent or negative.

The proof of Theorem 8.1 is given in the following because it is important to the arguments that follow.

**Proof:** The Nyquist theorem [41] is used to determine the stability of the machine, based on the map of the Nyquist contour under the loop transfer function

\[
((D(s) + \omega R K(s)/s) / (2Hs)) .
\]

The Nyquist contour is shown in figure 8.2. Note that the contour is indented around the double pole at the origin such that none of the open-loop poles are enclosed. According to the Nyquist theorem, the closed-loop system is stable if, and only if, the map of this contour does not encircle the critical point (-1,0).

In the following, the map of the Nyquist contour (or the Nyquist diagram for short) is studied starting from point "a" and moving in the clockwise direction. From "a" to "b" along the imaginary axis, the loop transfer function is

\[
-e_R \frac{K(j\omega)}{2H\omega^2} - j \frac{D(j\omega)}{2H\omega}
\]

(8.5)
It is important to note that the synchronizing torque $K$ is associated with the real part of the map, and the damping torque $D$ the imaginary part of the map. Therefore, if condition 3 of the theorem holds, the map along $a-b$ is inside the first and second quadrants of the complex plane (figure 8.3). Along the semicircle $b-c$ about the origin, the loop transfer function approaches $\omega R K(0)/(2Hs^2)$ where $K(0)$ is a positive constant by condition 2. The map of this semicircle is a complete circle of infinite radius traced out in a clockwise direction. Along the segment $c-d$ the map is merely a mirror image of that of $a-b$. Finally, the map of the large semicircle $d-a$ is a semicircle of infinitesimal radius about the origin (not shown), since the loop transfer function is assumed to be strictly proper.

The theorem is proved by observing in figure 8.3 that it is impossible, under the given assumptions, for the Nyquist diagram to encircle the critical point.

A counter-intuitive result also suggests itself. In cases where $D$ and $K$ are not constants, the damping torque can be negative in some frequencies without compromising the stability of the machine. The Nyquist diagram of one such system is shown in figure 8.4. Note that the critical point is not enclosed even though the contour in the second quadrant crosses into the third quadrant, and vise versa. For this reason, the stability theorem 8.1 is a sufficient, but not necessary, condition for stability.
Figure 8.1: Schematic representation of a power system where \( T(s) \) can be decomposed into damping and synchronizing torques.

Figure 8.2: Nyquist contour
Figure 8.3: The map of the Nyquist contour for a stable system with positive damping torque at all frequencies.

Figure 8.4: The map of the Nyquist contour for a stable system with negative damping torque at some frequencies.
8.2.3 Stability Properties of Multi-Machine Systems

A similar theorem applies to multi-machine systems:

**THEOREM 8.2:** A multi-machine power system is asymptotically stable (except for a pole at the origin) if

1) \( T(s) \) has no poles in the closed right half-plane except for a simple pole at \( s=0 \),

2) \( K(0) + K^T(0) \geq 0 \), and the nullity of \( K \) is 1, and

3) \( D(j\omega) + D^T(-j\omega) - j(K(j\omega) - K^T(-j\omega)) \frac{\omega_R}{\omega} > 0 \)

for all \( \omega \) not equal to 0.

Note: the symbols \( \succ 0 \) and \( \succeq 0 \) mean that the matrix is positive and positive semi-definite, respectively.

The proof of this theorem is given in Appen.G. It is interesting to note that for a system with \( n \) classical machines and a purely inductive transmission network, conditions 2 and 3 can be simplified to

2') \( K \succ 0 \) and the nullity of \( K \) is 1, and

3') \( D \succ 0 \)

since \( K \) is symmetric and \( D \) is diagonal (see Appendix B). A comparison between conditions 2 and 3 of theorem 8.1 and conditions 2' and 3' above shows that the idea of positive damping and synchronizing torque for single-machine systems has a direct multi-machine extension via the concept of positive definiteness.

Theorem 8.2 is also a sufficient but not necessary
condition for stability. This means that a stable multi-machine system need not satisfy all the requirements of this theorem, but a system that satisfies all the requirements are guaranteed to be stable. For this reason, the damping and synchronizing torque framework is not very useful for stability evaluation. Other techniques such as eigenvalue studies in the time domain, or Nyquist theorem in the frequency domain are much more suitable for this task.

The damping and synchronizing torque framework was not intended to be a tool for evaluating the stability of a given machine model. (A recent application of the damping-torque idea for studying the stability of shaft torsional interactions [10] is an exception to this statement, but the methods in [10] do not have straightforward extension to other types of studies). This framework was proposed by de Mello and Concordia as a means for understanding physically the impact of supplementary controls on the damping characteristics of synchronous machines [20].

Similar to the original intention of de Mello and Concordia, the damping and synchronizing torque framework is used here for understanding physically the results of robustness tests on multi-machine systems. This point is clarified in the following section.
8.3 POWER SYSTEM APPLICATIONS

A schematic representation of a power system is shown in figure 8.5. This diagram is identical to figure 8.1 except that a feedback element \( F(s) \) has been added to represent any additional state- or output-feedback compensators (e.g. MTDC modulation controllers).

The robustness of the power-system model is considered with the loop broken at three different points, labeled 1, 2 and 4 in figure 8.5. A graphical representation of the system model with the loop broken at these points is shown in figure 8.6.

When designing a controller, the stability margin is usually checked at point 1 (or inside \( F(s) \), depending on the designation of input channels). It is well known that certain guaranteed stability margins exist at the input channels if the feedback gain is found using the linear quadratic methodology (see section 7.5). However, it will be shown in the next chapter that while the robustness margins at point 1 is useful for checking modeling errors due to dc-voltage variations, they are very conservative for checking perturbations in damping and synchronizing torques. The reason is that with the loop broken at point 1, small changes in \( D(s) \) and \( K(s) \) can lead to large perturbations near the resonant frequencies. When the same perturbation is checked at point 2, the "amplification effect" is much less serious, and therefore the stability margins at this point are in general less conservative.

POWER SYSTEM ROBUSTNESS 8.3 POWER SYSTEM APPLICATIONS
A distinct advantage of examining the perturbations at point 2 is that the destabilizing multiplicative perturbations can be interpreted directly in terms of changes in damping and synchronizing torques. Specifically, if the matrix \( L(j\omega) \) (nominally the identity matrix) is the destabilizing perturbation at point 2, then the perturbed damping and synchronizing matrices \( \hat{D} \) and \( \hat{K} \) can be computed by

\[
\hat{D}(j\omega) = \frac{\omega R}{\omega} \hat{K}(j\omega) = L(j\omega)\left[\hat{D}(j\omega) - \frac{\omega R}{\omega} \hat{K}(j\omega)\right]
\] (8.7)

Conversely, if the perturbed matrices \( \hat{D} \) and \( \hat{K} \) are known, then \( L \) can be computed without knowledge of the rest of the system.

One is also tempted to break the loop at point 3 in figure 8.5. This is indeed possible in a classical-machine system where \( D \) is a constant matrix. But for a general power-system model, it is not legitimate to use the robustness theorem at this point because \( D(s) \) is not a rational transfer matrix, and consequently, the corresponding loop transfer matrix is not rational.

The loop may also be broken at point 4. The robustness margins in this context have direct physical interpretations in terms of shaft torsional dynamics. Specifically, the transfer matrix between the torque and the speed of the shaft can be written as

\[
\frac{1}{s} H^{-1} L(s)
\] (8.8)
where $L(s)$ is a diagonal matrix whose elements are functions of the physical parameters of the shaft. More technical detail is given in Appendix H. It should be noted that point 4 is identical point 2 if $F(s) = 0$ or when $F(s)$ is lumped into $T(s)$.
Figure 8.5: A power system model with feedback controller $F(s)$. 

POWER SYSTEM ROBUSTNESS 8.3 POWER SYSTEM APPLICATIONS
Figure 8.6: Schematic representations of the same power-system model used for computing the robustness margins at points 1, 2 and 4. The portion enclosed by dotted lines is the nominal loop transfer matrix used in each case.
8.4 SUMMARY

The justifications for examining the robustness margins of an MTDC/ac power system at the physical input and at the generator-torque inputs are given in this chapter.

The interpretation of the robustness results with the loop broken at the generator-torque inputs is guided by the concepts of damping and synchronizing torques. The damping torque is defined as the component of the electrical torque that is in phase with the shaft speed phasor, and the synchronizing torque as the component in phase with the electrical angle phasor (section 8.2). A formal definition of this concept for multi-machine systems is given in section 8.2.1.

To provide a better understanding of damping and synchronizing torques, some sufficient conditions for the stability of single- and multi-machine systems are presented in section 8.2.2. Theorem 8.1 shows that if the damping torque is positive at all frequencies and the synchronizing torque is positive at zero frequency, then a single machine connected to an infinite bus is guaranteed to be stable. Theorem 8.2 shows analogous results for multi-machine systems. The idea of positive damping and synchronizing torques in single-machine systems is shown to have a direct analog in the multi-machine setting in the concept of positive definite damping and synchronizing matrices.
CHAPTER 9
INTERPRETATIONS OF POWER-SYSTEM ROBUSTNESS MARGINS

9.1 OVERVIEW

A detailed study of a classical 2-machine system is conducted in the beginning of this chapter to gain a better understanding of the robustness margins and the associated minimum-norm perturbations. A 2-machine system is chosen for this study because it is the smallest system that possesses all the properties of an n-machine system. A one-machine-versus-infinite-bus system is found to be inadequate.

The state-space model of the 2-machine system and its feedback controller are described in section 9.2. In section 9.3, the robustness margins of the 2-machine system are computed at points 1 and 2 (of figure 8.5), and they are compared to the perturbations associated with typical parameter variations in a classical-machine system. The applicability and conservatism of the matrix-norm robustness criteria at points 1 and 2 are discussed.

In section 9.4, the application of the robustness margins at point 4 for evaluating the power system's robustness with respect to neglected shaft torsional dynamics is demonstrated.

The robustness margins of the Western U.S. model is studied in section 9.5. It is shown for this example that the robustness margins of different output-feedback configurations are drastically different. The implications of these findings for the design of MTDC modulation controller are explained.
9.2 2-MACHINE EXAMPLE

In designing the 2-machine example, an effort is made to use typical parameters for interarea oscillations.

The system equation is

\[
\begin{bmatrix}
\dot{\delta}_1(t) \\
\dot{\delta}_2(t) \\
\dot{\omega}_1(t) \\
\dot{\omega}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 377 & 0 \\
0 & 0 & 0 & 377 \\
-3/377 & 3/377 & -0.2 & 0 \\
6/377 & -6/377 & 0 & -0.2
\end{bmatrix}
\begin{bmatrix}
\delta_1(t) \\
\delta_2(t) \\
\omega_1(t) \\
\omega_2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
-1/1000 \\
1/4000
\end{bmatrix} u(t) \tag{9.1}
\]

where \( \delta_i \) and \( \omega_i \) are the angle and speed of the ith machine. The control \( u(t) \) can be thought of as a current-modulation controller on a parallel dc link. An LQ design which enhances the damping of the oscillatory mode is found using the modal penalty techniques described in chapter 4. The gain matrix is approximately

\[
G = \begin{bmatrix} 0 & 0 & -4000c & 4000c \end{bmatrix} \tag{9.2}
\]

where the value of \( c \) is a function of the LQ weights. The characteristic polynomial of the closed-loop matrix

\[
s (s+d) (s^2 + (5c+0.2)s + 9) \tag{9.3}
\]

shows clearly that \( c \) affects only the damping of the oscillatory mode and leaves the other modes fixed. The same property is seen in the LQ design for the Western U.S. system.
9.3 PARAMETER VARIATIONS IN 2-MACHINE EXAMPLE

The modeling uncertainties considered here are results of typical parameter variations in a classical-machine model. More precisely, the perturbed damping and synchronizing matrices are

\[ \hat{K} = K + \frac{a}{377} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \]  \hspace{1cm} (9.4)

and

\[ \hat{D} = D + \begin{bmatrix} d & 0 \\ 0 & d/2 \end{bmatrix} \] \hspace{1cm} (9.5)

where \( a \) and \( d \) are real scalars. For the nominal system (9.1), it is known a priori that the system is stable for \( a \in (-3,\infty) \) and \( d \in (-0.2,\infty) \). The purpose of the following study is to see for a given \( a \) and \( d \) which do not destabilize the system,

1. what is the form of the perturbations \( L-I \) and \( L^{-1}-I \) at points 1 and 2, and
2. how conservative are the matrix-norm stability margins with respect to these perturbations.

9.3.1 Perturbations Considered at Point 2

For a perturbed system, the matrix \( L \) at point 2 can be computed simply by

\[ L(j\omega)(D-j\frac{\omega R}{\omega}K) = \hat{D} - j\frac{\omega R}{\omega}\hat{K} \] \hspace{1cm} (9.6)

Note that the perturbations computed this way is not a function of the feedback gain.

INTERP. OF ROBUSTNESS MARGINS  \hspace{1cm} 9.2 PARAMETER VARIATIONS
When only the matrix $K$ is perturbed, the matrices $L-I$ and $L^{-1}-I$ are given by

$$L(j\omega)-I = \frac{5a}{j\omega+45} \begin{bmatrix} 1 & -2 \\ -1 & 2 \end{bmatrix}$$

(9.7)

and

$$L^{-1}(j\omega)-I = \frac{5a}{j\omega+(45+15a)} \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$$

(9.8)

When only the matrix $D$ is perturbed, the matrices $L-I$ and $L^{-1}-I$ are given by

$$L(j\omega)-I = \frac{5d}{j\omega+45} \begin{bmatrix} 30+j\omega & 30 \\ 15 & 15+j\omega \end{bmatrix}$$

(9.9)

and

$$L^{-1}(j\omega)-I = b \begin{bmatrix} 30d+j\omega (5d+1) & -30d \\ -15d & 15d+j\omega (5d+1) \end{bmatrix}$$

(9.10)

where

$$b = \frac{5d}{(45+225d) + j\omega (25d^2+10d+1)}$$

The norms of (9.5) to (9.8) for $a=0.5$ and $d=-0.1$ are plotted in figures 9.1 to 9.4 as a function of frequency. Also plotted in each of these figures is the relevant stability margins computed with the loop broken at point 2 (see (7.14-7.15)).

For perturbations in $K$, the norms of $L-I$ and $L^{-1}-I$ in figures 9.1 and 9.2 are roughly the same. These curves are flat at low frequencies, and from (9.7) and (9.8), the magnitude of the perturbation at low frequencies is seen to depend

INTERP. OF ROBUSTNESS MARGINS 9.2 PARAMETER VARIATIONS
linearly on the perturbed parameter \( a \). For instance, the size of the perturbation caused by a change in \( K \) is approximately \( |a|/3 \) in figures 9.1 and 9.2. The fact that the size of the perturbation depends linearly on "a" implies that the entire range of stable perturbations, \( a \in (-3, \infty) \), cannot be accommodated by these robustness margins. For the value of \( a=0.5 \), however, the norm of the perturbation in both figures 9.1 and 9.2 is lower than the robustness margins at all frequencies; thus the perturbed closed-loop system is guaranteed to be stable.

For changes in the matrix \( D \), the criterion based on \( L^{-1} - I \) is less conservative than that of \( L - I \). The reason is that the norm of \( L^{-1} - I \) at low frequencies is proportional to \( d^2 \); whereas the norm of \( L - I \) is proportional to \( d \). Therefore even though the robustness margin in figure 9.4 appears to be rather low in the neighborhood of 3 rad/sec, it is still well above the norm of the perturbation.

In figure 9.3, however, the robustness margin is violated by a stable perturbation. This fact implies the existence of destabilizing perturbations with smaller matrix norms than those considered in (9.4) and (9.5). In table 9.1, the nominal damping and synchronizing matrices

\[
D(j\omega) - j\frac{\omega R}{\omega} K(j\omega)
\]

is tabulated along with the perturbed matrix (9.6), where \( L \) is computed using singular vectors (see (7.16-7.17)).

INTERP. OF ROBUSTNESS MARGINS  9.2 PARAMETER VARIATIONS
The perturbed damping and synchronizing matrices in table 9.1 are quite different from the nominal matrices, and the differences cannot be accounted for by parameter variations in the form of (9.4-9.5) alone. This implies that the minimum-norm destabilizing perturbations in the 2-machine example are unstructured perturbations. The usefulness of these results and the conservatism of the corresponding matrix-norm bounds depend on whether these perturbations can actually occur in a physical system. To answer this question requires a frequency-domain characterization of possible perturbations. Such knowledge is unfortunately no precisely known at this time.

The experience with damping and synchronizing torques of single-machine systems, however, offers some guidance in this study. For instance, the damping torque is known to be quite high at low frequency, and it decreases as a function of frequency. At high frequencies, the value of the damping torque is a function of the machine parameters and the design of the exciter and the power-system stabilizer [20]. The synchronizing torque, on the other hand, is similar to that of the classical-machine model in all frequencies. For a multi-machine system, the diagonal terms of the damping- and synchronizing-torque matrices are expected to behave in a similar way. However, a more precise characterization of the perturbations on the diagonal and off-diagonal terms of these matrices and how they relate to the parameters of the system must await future research.

INTERP. OF ROBUSTNESS MARGINS  9.2 PARAMETER VARIATIONS
Returning to table 9.1, the perturbation in synchronizing torque is too large to be realistic. The same applies to damping torques in most cases. For this reason, the matrix-norm bounds are conservative measures of stability margin. Nonetheless, they offer a means of quantifying the close-loop system's nearness to instability, taking into account both structured and unstructured perturbations. Such a tool is available to the power-system analyst for the first time, but some refinements are necessary before it can be useful in this respect.
<table>
<thead>
<tr>
<th>freq rad/sec</th>
<th>nominal matrix</th>
<th>perturbed matrix based on $\mathcal{Z}(I+G)^{-1}$</th>
<th>perturbed matrix based on $\mathcal{Z}(I+G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>$\begin{bmatrix} .2-j30 &amp; j30 \ j30 &amp; .1-j30 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 15.-j17.3 &amp; -10.5+j17.1 \ 10.7+j29.5 &amp; -10.8-j29.5 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -.044-j1.21 &amp; .24+j1.21 \ .484-j0.56 &amp; -.38+j0.56 \end{bmatrix}$</td>
</tr>
<tr>
<td>1.0</td>
<td>$\begin{bmatrix} .2-j3 &amp; j3 \ j3 &amp; .1-j3 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -.41-j2.62 &amp; .43+j1.62 \ -.29+j3.23 &amp; .24-j3.73 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -.04-j.3 &amp; .25+j.29 \ .48+j.36 &amp; -.38-j.36 \end{bmatrix}$</td>
</tr>
<tr>
<td>3.0</td>
<td>$\begin{bmatrix} .2-j1 &amp; j1 \ j1 &amp; .1-j1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -.54-j3.0 &amp; .55+j.004 \ -.38-j1.01 &amp; .38-j1.51 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -.26-j.88 &amp; .46-j1.09 \ -.21+j1.13 &amp; .03-j2.10 \end{bmatrix}$</td>
</tr>
<tr>
<td>10.0</td>
<td>$\begin{bmatrix} .2+j3 &amp; j3 \ j3 &amp; .1+j3 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -.57-j18.11 &amp; .3-j1.82 \ -.15-j3.72 &amp; .4-j1.35 \end{bmatrix}$</td>
<td>$\begin{bmatrix} -3.36+j6.22 &amp; 2.59-j2.65 \ -2.63-j3.70 &amp; 1.81-j2.41 \end{bmatrix}$</td>
</tr>
</tbody>
</table>
Figure 9.1: Perturbation in $K$ considered at point 2.

Figure 9.2: Perturbation in $K$ considered at point 2.

INTERP. OF ROBUSTNESS MARGINS \hspace{1cm} 9.2 PARAMETER VARIATIONS
Figure 9.3: Perturbations in D considered at point 2.

Figure 9.4: Perturbations in D considered at point 2.
9.3.2 Perturbation Considered At Point 1

The perturbations in \( K \) and \( D \) are now examined with the loop broken at the input junction. For this 2-machine example, there is only one input; therefore the loop transfer function is a scalar function of \( s \).

When \( K \) and \( D \) are perturbed as in (9.4) and (9.5), the function \( L \) is given by

\[
L = \frac{s^2 + 0.2s + 9}{s^2 + (0.2+d)s + (9+3a)}
\]  
(9.11)

The values of \( L-I \) and \( L^{-1}-I \) are computed based on this equation and they are

\[
L-I = \frac{-ds - 3a}{s^2 + (0.2+d)s + (9+3a)}
\]  
(9.12)

and

\[
L^{-1}-I = \frac{-ds - 3a}{s^2 + 0.2s + 9}
\]  
(9.13)

In figures 9.5 and 9.6, the norm of (9.12) and (9.13) are plotted for a perturbation in \( K \) (\( a=0.5 \)). Also plotted on the same figures are the relevant stability margins computed at the input junction.

A salient feature in these plots is the peaking of the perturbation near the resonant frequency of 3 rad/sec. The peaking phenomenon can be explained by the deformation of the Nyquist diagram caused by a change in the synchronizing coefficient. Figure 9.7 shows the Nyquist diagram of the nominal
loop transfer function at point 1 and the deformed Nyquist diagram corresponding to $a=0.5$. An important point to note here is that near the resonant frequency, the loop transfer function undergoes a large change in phase and magnitude. Such a change is typical of a lightly damped second-order system. When the synchronizing coefficient is changed, the large change takes place at a slightly different frequency. The shape of the image does not change appreciably, but for a given frequency near 3 rad/sec, the position of the image is quite different from one curve to another. This difference is reflected in the large peak seen in figures 9.5 and 9.6. In a system with more than 1 input, the same phenomenon takes place at each element of the loop transfer matrix, and the peaking is reflected in the norm of the matrix. In figure 9.5, the robustness margin is again violated by a stable perturbed system.

The magnitude of perturbations as a result of a change in $D$ ($d=-0.1$) is shown in figures 9.8 and 9.9. In contrast to perturbation in $K$, the norm of $L-I$ and $L^{-1}I$ is zero at $\omega=0$. The explanation is that damping is unimportant at low frequencies, and therefore changes in $D$ have little effect on the loop transfer function. The peaking in the magnitude of the perturbations in figures 9.7 and 9.8 is explained by the Nyquist diagram shown in figure 9.10. It is interesting to note that a change in damping alters the size of the Nyquist diagram drastically.

Perturbations in $D$ and $K$ are seen to cause the peaking
problem when the loop is broken at the input junction. This problem occurs not only for stable perturbations considered here, but also for other stabilizing and destabilizing perturbations. This observation implies that the robustness margins at point 1 cannot be translated readily to tolerance for changes in damping and synchronizing matrices.

The stability margins at the input junction, however, are useful for assessing the tolerance for modeling errors that are directly related to the actuators. An important application here is the assessment of the effects of dc voltage variations on the stability of the closed-loop system. This application is based on the observation that the effectiveness of the dc-current modulation varies directly with the dc voltage. In other words, for a given change in dc current, the torque "felt" by the machines due to dc modulation varies linearly with dc voltage (with everything else being the same). A change in dc voltage affects all the dc terminals in an MTDC system in the same way; thus the modeling error can be expressed in the form \( L(s) = gI \), where \( g \) is a non-negative scalar with a nominal value of 1, and \( I \) is the identity matrix. Since \( L \) is diagonal and real in this case, the permissible values for \( g \) can be found directly using the gain-tolerance results in section 7.5.

For the 2-machine example, it can be seen from figures 9.5 and 9.6 that the robustness margins \( g(I+g) \) and \( g(I+g^{-1}) \) are greater than 1 (0 db) at all frequencies. The results in table 7.2 imply that the system is guaranteed to be stable for
g between 0.0 and 2.0, and the results in table 7.1 imply that the system is stable for all d greater than 0.5. These results combined give a gain tolerance of [0, ∞). The actual tolerance in a realistic system is perhaps lower since a large change in dc voltage can lead to other perturbations which cannot be modeled by $\mathbf{I}_L = q\mathbf{I}_I$. Nonetheless, this example shows that the robustness margins at the input is indeed useful for estimating the range of dc-voltage variations for which the closed-loop MTDC system is guaranteed to be stable.
Figure 9.5: Perturbations in $\mathbf{K}$ considered at input junction.

Figure 9.6: Perturbations in $\mathbf{K}$ considered at the input junction.

INTERP. OF ROBUSTNESS MARGINS 9.2 PARAMETER VARIATIONS
Figure 9.7: Nyquist diagram of nominal and the perturbed system caused by a change in $K$.
Figure 9.8: Perturbations in $D$ considered at the input junction.

Figure 9.9: Perturbations in $D$ considered at the input junction.
Figure 9.10: Nyquist diagram of the nominal and the perturbed system caused by a change in $D$.

INTERP. OF ROBUSTNESS MARGINS  9.2 PARAMETER VARIATIONS
9.4 TORSIONAL DYNAMICS

In Appendix H, it is shown that the modeling errors at point 4 (figure 8.6c) due to neglected shaft torsional dynamics can be expressed in a relatively simple form. This is in sharp contrast with the difficulties encountered in characterizing the variations in damping and synchronizing torques.

The application of the robustness margins at point 4 is demonstrated below using the 2-machine example. It is assumed that the eigenvalues due to the torsional modes of the two shafts are \(-0.07+j70\) and \(-0.08+j80\). The per-unit critical damping of both shafts is approximately 0.001, and the resonant frequencies are 11 hz and 13 hz, respectively.

From equation (H.6), the perturbations \(L-I\) and \(L^{-1}-I\) for a single shaft are

\[
L-I = \frac{\frac{H}{2} s^2}{\frac{H}{2} s^2 + ds + k}
\]

(9.14)

and

\[
L^{-1}-I = \frac{\frac{H}{2} s^2}{ds + k}
\]

(9.15)

For a multi-machine system, the matrices \(L-I\) and \(L^{-1}-I\) are diagonal and their elements are in the form (9.14) and (9.15). The maximum singular value of these matrices are easy to compute, since it is equal to the magnitude of the largest diagonal element at each frequency.

In theory, both (9.14) and (9.15) can be used with the robustness margins \(|(I+G^{-1})|\) and \(|(I+G)|\), but, in practice, INTERP. OF ROBUSTNESS MARGINS

9.4 TORSIONAL DYNAMICS
(9.15) is not a good measure for this type of modeling errors. The reason is that the magnitude of (9.15) increases with frequency, but the corresponding robustness margin $\sigma(I+G)$ approaches 1 at high frequencies.

The plots of $\sigma(I+G^{-1})$ and $\sigma(L-I)$ for the 2-machine system are shown in figure 9.11. The curve of $\sigma(L-I)$ is seen to be nearly zero at low frequencies and peaks at the resonant frequencies of the two shafts. It is important to emphasize that if the resonant frequency of the two shafts were identical, the results would have been the same as if only one shaft were present. In other words, the errors due to neglected torsional shaft dynamics are not additive. In figure 9.11, the magnitude of the perturbation is seen to be less than the robustness margin at all frequencies; therefore the stability of the closed-loop system is guaranteed.
Figure 9.11: Perturbations due to unmodeled shaft torsional dynamics and the robustness margin at point 4.

INTERP. OF ROBUSTNESS MARGINS 9.4 TORSIONAL DYNAMICS
9.5 ROBUSTNESS OF THE WESTERN U.S. MODEL

The robustness of the different control schemes is investigated in this section using the robustness margins computed for the 10-area model of the Western U.S. example.

Shown in figure 9.12 and 9.13 are the robustness margins of the MTDC system with the loop broken at point 2 of figure 8.2. The curves corresponding to the LQ and centralized schemes are quite similar, and they are larger in magnitude than those corresponding to the distributed and decentralized schemes. It is not strictly correct to conclude from these observations that the LQ and the centralized cases are more robust, since the "smallest" perturbations implicit in these curves may be different. However, the LQ and the centralized cases are more robust in terms of tolerances for phase and gain variations (see section 7.5) at low frequencies. This is true in both figures 9.12 and 9.13.

The stability margin for different control schemes is also checked at the normal input junction (point 1 of figure 8.2). The results are depicted in figures 9.14 and 9.15. At frequencies below 3 rad/sec, the LQ and centralized schemes are also more robust in terms of tolerations for pure gain and phase perturbations in each input channel. At frequency above 3 rad/sec, however, the LQ curve is somewhat lower in magnitude. This is especially pronounce in figure 9.14 where the LQ is seen to have a 20db/decade slope while all the output-feedback schemes have slopes of 40db/decade.

The conclusion in this study is that at the frequencies
below the frequency band of interarea swings, the robustness margins of the Western U.S. example are highest for the centralized output-feedback design, and the robustness margins decrease with the reduction in communication between the dc terminals. It is not clear, however, that the differences in robustness margin is significant in a realistic system. In order to answer this question, a complete characterization of the modeling errors must be known. This difficulty has already been discussed in section 9.3.

It is also unclear at this time that the robustness of a power system can be computed by using its aggregate model—especially when the aggregate model is derived from a classical-machine model of the full system. More research is necessary to answer this question, but it is the author's belief that a more detail model—perhaps an aggregate model with "equivalent" voltage exciters and power-system stabilizers—is more appropriate for this purpose.
Figure 9.12: Stability margin of the 10-area example at point 2.

Figure 9.13: Stability margin of the 10-area example at point 2.

INTERP. OF ROBUSTNESS MARGINS 9.5 THE WESTERN U.S. MODEL
Figure 9.14: Stability margin of the 10-area example at input junction.

Figure 9.15: Stability margin of the 10-area example at input junction.

INTERP. OF ROBUSTNESS MARGINS

9.5 THE WESTERN U.S. MODEL
9.6 SUMMARY

The applicability of the robustness margins is investigated in this chapter using a 2-machine power-system model (section 9.2). The insights gained in this study are used for interpreting the robustness margins of the Western U.S. example.

To evaluate the adequacy of the robustness margins computed at point 1 and point 2 of figure 8.5, these margins are compared to the size of the perturbations associated with some typical parameter variations in the nominal model (section 9.3).

The robustness margins at both points 1 and 2 are found to be conservative measures of robustness, because they are violated by parameter variations that are known to be stable. The reasons for the conservatism are explained in section 9.3.1. It is shown that the "smallest" perturbations represent certain unstructured perturbations which cannot be accounted for by parameter uncertainties alone. The adequacy of the robustness margins for unstructured perturbations in damping and synchronizing torques is not known at this time, since it is not clear how the unstructured modeling uncertainties can be characterized in a power system model.

The robustness margins at the physical input (point 1), however, are shown to be an indicator of the tolerance for dc voltage variations (section 9.3.2). In addition, the robustness margins at point 4 is shown to be very useful for checking the robustness with respect to unmodeled torsional
dynamics.

The robustness of the Western U.S. model is studied in section 9.5. A comparison of the robustness margins for different output feedback designs reveals that the centralized output-feedback design is more robust than the distributed and decentralized designs in terms of tolerations for phase and gain tolerances. The real implication of this finding is still unclear, because it is not known whether the differences are indeed significant in a realistic setting.
CONCLUSION AND RECOMMENDATIONS FOR FUTURE WORK

A methodology for designing small-signal multiterminal dc (MTDC) modulation controllers has been proposed in this thesis. The proposed methodology includes guidelines for constructing the design model and techniques for analyzing the system dynamics, synthesizing the feedback controller and evaluating the robustness of the closed-loop design. The methodology is based in part on known techniques such as the slow-coherency concept, the linear-quadratic state-feedback methodology and the theory on multivariable robustness. The methodology also include many new techniques such as controllability and observability measures and robustness evaluation methodology for power-system models. These methods from a wide range of disciplines are combined into a systematic procedure through which an MTDC controller can be designed starting from a detailed model of the MTDC/ac power system.

Some intrinsic properties of the MTDC control system have also been uncovered in this research. One of the important findings is that the centralized feedback design using the frequencies of the commutation buses as measurement signals is superior to the decentralized case in terms of control efforts and robustness properties. The ease of control in the centralized case is attributed to the coordination of dc-current modulations in a current-margin scheme in which any current variation at a terminal is accompanied by an equal and opposite change at the voltage-setting terminal. The advantage of
the centralized scheme is believed to be a function of the information structure and does not depend on the choice of measurement signals or the method used for computing the feedback gains. The communication requirements of the centralized and decentralized methods are also shown to be similar in complexity because a fast communication network linking all the dc terminals to one of the dc terminals is necessary for current-order coordination regardless of the information structure of the feedback system. For these reasons, the author strongly recommends the use of the centralized feedback scheme in which the control signal at each dc terminal is computed using all the measurement signals.

The controller-design methodology is formulated based on the philosophy of stabilizing only the most controllable and observable interarea modes of oscillation. The static, non-adaptive output-feedback controller proposed in this thesis is shown to be ideal for this purpose.

An area that has not been explored in detail is the use of dynamic, output-feedback compensators such as the Kalman filter. The dynamic-compensator approach was rejected at the outset of this project because of its complexity and the desire to develop a design methodology that is applicable to both centralized and decentralized information structures. The discovery of the inherent advantages of the centralized controller, however, makes the dynamic-compensator approach an attractive alternative. One of the interesting issues in this approach is the tradeoffs in performance, control-signal
amplitude and robustness margins involved in stabilizing the modes that cannot be controlled effectively by the static output-feedback controller.

The development of robustness-evaluation techniques for MTDC/ac power-system models represents one of the first attempts in applying the recently developed robustness theory to a complex, large-scale system. The results show that the application is successful whenever the perturbations of interest can be characterized readily in the frequency domain. The interpretations of the robustness margins as tolerances for unmodeled shaft torsional dynamics and dc-voltage variations are very successful for this reason.

The interpretation of the robustness margins and the associated minimum-norm perturbations as variations in the damping and synchronizing matrices is not as successful, though the author believes that it is an area that deserves additional exploration. An important problem here is that the characterization of the variations in damping and synchronizing torque matrices in a multi-machine system. The research in this direction will also benefit by addition theoretical research in finding "directional information" in robustness tests.

The research in this thesis represents the first comprehensive study of the MTDC small-signal control problem and the first time that an MTDC design methodology is demonstrated on a large-scale design example. The author believes that the methodology can be used on a realistic MTDC/ac power system.
In perspective, this thesis identifies the problem areas that are important in the design of an MTDC modulation controller and presents satisfactory solutions for some of the design issues. But many more problems remain unsolved and alternatives unexplored. Improvements in all aspects of the MTDC design methodology are expected. It is the author's hope that the work in this thesis will stimulate future research in this area.
APPENDIX A

MODAL ANALYSIS AND ARGAND DIAGRAMS

The dynamics of a linear system can be interpreted as a superposition of modes. To facilitate physical interpretation of system modes, the role of right and left eigenvectors is reviewed. Consider an unforced, time-invariant, linear system

\[ \dot{x}(t) = Ax(t); \quad x(0) = x_0 \]  \hspace{1cm} (A.1)

where the eigenvalues of \( A, \lambda_1, \lambda_2, \ldots, \lambda_n \), are assumed to be distinct. Let \( v_i \) and \( w_i \) be respectively the right and left eigenvectors associated with the \( i \)th eigenvalue, and define

\[ V = \text{col}[v_1, v_2, \ldots, v_n] \]
\[ W = \text{row}[w_1^T, w_2^T, \ldots, w_n^T] = V^{-1} \]
\[ \Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_n]. \]

The reason for the terminology "right" and "left" eigenvectors is obvious by noting the position of \( V \) and \( W \) relative to \( A \) in the identities

\[ AV = V \Lambda \]  \hspace{1cm} (A.2)

and

\[ WA = \Lambda W. \]  \hspace{1cm} (A.3)

The matrices \( \Lambda \) and \( \exp(At) \) can be written as dyadic expansions of left and right eigenvectors

\[ \Lambda = \sum_{i=1}^{n} \lambda_i v_i w_i^T \]  \hspace{1cm} (A.4)
\[ e^{At} = \sum_{i=1}^{n} e^{\lambda_i t} v_i w_i^T. \]  
(A.5)

In terms of (A.5), the solution of (A.1) becomes

\[ x(t) = e^{At} x_0 = \sum_{i=1}^{n} (e^{\lambda_i t} v_i^T w_i x_0). \]  
(A.6)

Equation (A.6) shows that \( x(t) \) can be expressed as a linear combination of \( n \) time-varying vectors, or modes, \( \exp(\lambda_i t) v_i \), each weighted by the scalar \( w_i^T x_0 \).

The first term in (A.6), \( \exp(\lambda_i t) v_i \), is a vector whose components rotate at the rate \( \text{Im}(\lambda_i) \), and, at the same time, expand or shrink at the rate \( \text{Re}(\lambda_i) \), depending on whether \( \text{Re}(\lambda_i) \) is positive or negative. At a fixed time, the \( n \) elements of the vector \( \exp(\lambda_i t) v_i \) can be plotted on the complex plane to show the relative extent to which the state variables participate in the \( i \)th mode. Such a snapshot of the modal vector is called the Argand diagram. There is one Argand diagram for each mode, though for a complex pair, the corresponding elements of the two Argand diagrams are mere conjugates of each other.

When interpreting an Argand diagram of a right eigenvector, it is important to keep in mind that the relative magnitude of the elements depends on the units chosen for the state variables. For example, a distance variable in the unit of light years may appear to be small relative to the other components, but this picture may change drastically when the unit of meters is used instead of light years. Unfortunately, there is no universally applicable rule for scaling.
the components of an Argand diagram; the proper interpretation of Argand diagrams will always depend on a good physical understanding of the system.

The second term in (A.6), $W_i^T x_0$, shows that the excitation of the $i$th mode is determined by the relative alignment between the initial-condition vector and the $i$th left eigenvector.

A physical interpretation of the left eigenvector is possible by considering the scalar time function $W_i^T x(t)$. The time derivative of this function,

$$\frac{d}{dt}(W_i^T x(t)) = W_i^T A x(t) = \lambda_i (W_i^T x(t)),$$

(A.7)

reveals that $W_i^T x(t)$ represents a linear combination of states which varies as $\exp(\lambda_i t)$.
APPENDIX B
CLASSICAL MACHINE MODEL

The classical (or constant-voltage-behind-transient-reactance) synchronous machine model has application in many power-system problems [39]. For this model, the following assumptions are required:

1. The mechanical power supplied by the prime mover is constant,
2. Damping torques vary linearly with frequency of the shaft,
3. The machine can be represented electrically by a constant-voltage source in series with the transient reactance,
4. The torque angle of the rotor coincides with the angle of the constant-voltage source.

To construct a classical multi-machine model, the loads in the system are replaced by passive impedances. The nodes with no generators are then eliminated by static equivalencing.

Suppose there are n machines in the system. For each machine, k=1,2,...,n, the equation of motion is given by the swing equation:

$$\frac{2H_k}{\omega_R} \frac{d\omega_k(t)}{dt} + \frac{D_k}{\omega_R} \omega_k(t) = P_{mk}(t) - P_{ik}(t) \quad (B.1)$$

where

$$P_{ek} = \text{electrical power output},$$

APPENDIX B
CLASSICAL MACHINE MODEL
\[ P_{mk} = \text{mechanical power input}, \]
\[ H_k = \text{initial constant}, \]
\[ D_k = \text{damping constant}, \]
\[ \omega_R = \text{synchronous speed of the generator}. \]

The electrical power output is a nonlinear function of the rotor torque angle, \( \delta_k \), and the voltage behind transient reactance, \( E_k \).

\[ P_{ek} = E_k G_{kk} + \sum_{\substack{m=1 \atop m \neq k}}^{n} E_k E_m (B_{km} \sin \delta_{km} + G_{km} \cos \delta_{km}) \quad (B.2) \]

where

\[ \delta_{km} = \delta_k - \delta_m \]
\[ G_{km} + j B_{km} = \text{the \((k,m)\) element of the nodal admittance matrix.} \]

A linear, small-perturbation model about a steady-state solution can be computed from (B.1) and (B.2). The new variables are defined as changes from the steady-state values (denoted by the subscript \( o \)):

\[ \delta_{k\Delta} = \delta_k - \delta_{k0} \quad (B.3) \]
\[ \omega_{k\Delta} = \frac{1}{\omega_R} \frac{d\delta_{k\Delta}}{dt} \quad (B.4) \]

It is straightforward to show that a state-space model is

\[
\begin{bmatrix}
\dot{\delta}_\Delta(t) \\
\dot{\omega}_\Delta(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \omega_R I \\
-H^{-1}K & -H^{-1}D
\end{bmatrix}
\begin{bmatrix}
\delta_\Delta(t) \\
\omega_\Delta(t)
\end{bmatrix}
\quad (B.5)
\]

APPENDIX B . CLASSICAL MACHINE MODEL
where $H$, $K$ and $D$ are $n \times n$ matrices defined as follows:

$$H = \text{diag}[2H_1, 2H_2, \ldots, 2H_n],$$

$$D = \text{diag}[D_1, D_2, \ldots, D_n],$$

$$K = [k_{km}], \quad k_{km} = -E_k E_m (B_{km} \cos \delta_{kmo} - G_{km} \sin \delta_{kmo}).$$

$$k_{kk} = \sum_{m=1}^{n} k_{km}.\quad m \neq k$$

In the text of this thesis, the "Δ" sign is dropped, and all quantities are deviations from the steady-state values unless specified otherwise.

Several properties of the matrix $K$ are worth noting. First, note that this matrix can be decomposed into a symmetric part which depends on the $B$'s, and a skew-symmetric part which depends on the $G$'s. In addition, the matrix $K$ is singular with nullity of 1. This matrix is also positive semi-definite, except in some pathological cases where the resistance of the lines is unusually high.

The positive semi-definiteness of $K$ can be verified immediately via Gershgorin's Theorem since each of the diagonal elements of $K + K^T$ is positive and equal in magnitude to the sum of the off-diagonal elements on the same row. To see that $K$ has an eigenvalue at 0, note that $Kx = 0$ for $x = [1, 1, \ldots, 1]^T$. Finally, to show that the nullity of $K$ is no greater than 1, note that the upper left $(n-1) \times (n-1)$ block of $K$ is diagonal dominant and therefore have full rank.

APPENDIX B

CLASSICAL MACHINE MODEL
For systems where the $G_{km}$'s are zero, the matrix $K$ is symmetric; hence $K$ has $n-1$ positive real eigenvalues and a 0 eigenvalue. The matrix $H^{-1}K$, where $H$ is a positive definite, diagonal matrix, also has $n-1$ positive real eigenvalues and a zero eigenvalue, since the matrix $H^{-1}K$ is similar to the symmetric, semi-positive definite matrix $H^{-1}KH^{-1}$. 
APPENDIX C

AVERAGE FREQUENCY MODE

This appendix shows that under the conditions that
1. the ac lines are purely inductive, and
2. the per-unit damping coefficient of all the areas
   are identical, (Note: this is analogous to assuming
   that the coefficient of friction is the same for all
   the masses in the mechanical analog.)
the left eigenvector associated with the average-frequency
mode has the form

$$w_1 = [0, 0, ... , a_1, ... , a_n]^T,$$

where the scalars $a_1, ... , a_n$ are directly proportional to the
inertia of the $n$ areas.

Mathematically, assumptions 1 and 2 imply, respectively,
that the synchronizing matrix $K$ is symmetric (see Appendix B),
and that the matrix $H^{-1}D$ can be written as $dI$, where $d$ is a
real, positive, scalar constant. Under these assumptions, the
system matrix $A$ becomes

$$A = \begin{bmatrix}
0 & \omega R^T \\
-\omega R^T & -H^{-1}K - dI
\end{bmatrix} \tag{C.1}$$

To show that the vector $w_1$ is a left eigenvector
associated with the real eigenvalue at $s=-d$, note first that

$$w_1^T A = \begin{bmatrix}
[a_1, ... , a_n]H^{-1}K, -da_1, ... , -da_n
\end{bmatrix} \tag{C.2}$$
If the values $a_1, \ldots, a_n$ are proportional to the inertias, then

$$[a_1, \ldots, a_n] H^{-1} = b[1, \ldots, 1]$$  \hspace{1cm} (C.3)

where $b$ is a scalar (proportionality) constant. In addition, it is known from assumption 1 and Appendix B that

$$[1, \ldots, 1] K = K [1, \ldots, 1]^T = [0, \ldots, 0]^T.$$  \hspace{1cm} (C.4)

Therefore, (C.2) can be written as

$$w_1^T A = [0, \ldots, 0, -da_1, \ldots, -da_n]^T = -d w_1^T.$$  \hspace{1cm} (C.5)

Q.E.D.
APPENDIX D

NET MOMENTUM OF OSCILLATORY MODES

This appendix shows that an interarea swing in a power system always has zero net momentum.

Let \( v_i \) be the right eigenvector associated with the \( ith \) oscillatory mode. As was mentioned in Appendix B, this vector consists of two groups of machines swinging 180 degrees out of phase. Also recall from Appendix C that the left eigenvector \( w_i \) associated with the average-frequency mode consists of 0's for the angle states and of the area inertias for the frequency states.

The orthogonality relation

\[
\frac{w_i^T}{w_i v_i} = 0
\]

implies that

\[
\sum_{i \in \Omega_1} H_i \omega_i(t) + \sum_{i \in \Omega_2} H_i \omega_i(t) = 0.
\]

where \( H_i \) is the inertia of an area, and \( \Omega_1 \) and \( \Omega_2 \) are the two sets of areas swinging in opposite directions. Equation (D.2) is interpreted as saying that the momentum of the two groups of machines are equal in magnitude but opposite in sign. Furthermore, the net momentum of all the machines is always equal to zero.
APPENDIX E

MATRIX NORMS

The concept of matrix norm is based on the more familiar concept of vector norms.

The norm of a complex vector \( \mathbf{y} \) is generally a measure of its "size". The commonly used vector norm of a real vector \( \mathbf{y} \),

\[
\| \mathbf{y} \| = (y_1^2 + y_2^2 + \ldots + y_n^2)^{1/2}
\]

(E.1)

is consistent with the usual concept of length -- at least for \( n \) equal to 1, 2 and 3. The definition of a vector norm can be widened to include other real-valued functions on \( \mathbf{x} \) with certain properties.

**DEFINITION E.1:** The norm of a complex n-vector \( \mathbf{x} \), denoted by \( \| \mathbf{x} \| \), is a real number which satisfies the following.

1) \( \| \mathbf{x} \| > 0 \) for \( \mathbf{x} \neq 0 \), and \( \| \mathbf{0} \| = 0 \) implies \( \mathbf{x} = 0 \).

2) \( \| k \mathbf{x} \| = |k| \| \mathbf{x} \| \) for any complex scalar \( k \).

3) \( \| \mathbf{x} + \mathbf{y} \| \leq \| \mathbf{x} \| + \| \mathbf{y} \| \) for any complex n-vector \( \mathbf{y} \).

Based on this definition, one can easily verify that the following are vector norms.

\[
\| \mathbf{x} \|_1 = |x_1| + |x_2| + \ldots + |x_n|
\]

(E.2)

\[
\| \mathbf{x} \|_2 = (|x_1|^2 + |x_2|^2 + \ldots + |x_n|^2)^{1/2}
\]

(E.3)

\[
\| \mathbf{x} \|_\infty = \max |x_i|
\]

(E.4)

These are referred to as 1, 2, and \( \infty \) norms.

These different vector norms are related. First, it is
obvious that

\[ \| \mathbf{x} \|_\infty \leq \| \mathbf{x} \|_2 \leq \| \mathbf{x} \|_1 \]  \hspace{1cm} (E.5)

Moreover, if any one of these norms is known, bounds for the other two norms can be determined. Three of the inequalities are shown below. Others can be derived trivially from these equations.

\[ \| \mathbf{x} \|_\infty \leq \| \mathbf{x} \|_2 \leq \sqrt{n} \| \mathbf{x} \|_\infty \]  \hspace{1cm} (E.6)

\[ \frac{1}{\sqrt{n}} \| \mathbf{x} \|_1 \leq \| \mathbf{x} \|_2 \leq \| \mathbf{x} \|_1 \]  \hspace{1cm} (E.7)

\[ \| \mathbf{x} \|_\infty \leq \| \mathbf{x} \|_1 \leq \sqrt{n} \| \mathbf{x} \|_\infty \]  \hspace{1cm} (E.8)

All these inequalities can be derived from simple algebraic manipulations.

Similar to vector norms, a norm of a matrix \( \mathbf{A} \) is defined to be a real-valued function of \( \mathbf{A} \) with certain properties.

**DEFINITION E.2:** The matrix norm of a square, complex \( n \times n \) matrix \( \mathbf{A} \), denoted by \( \| \mathbf{A} \| \), is a real number which satisfies the following.

a) \( \| \mathbf{A} \| > 0 \) for \( \mathbf{A} = 0 \), and \( \| \mathbf{A} \| = 0 \) implies \( \mathbf{A} = 0 \).

b) \( |k\mathbf{A}| = |k| \| \mathbf{A} \| \) for any complex scalar \( k \).

c) \( \| \mathbf{A} + \mathbf{B} \| \leq \| \mathbf{A} \| + \| \mathbf{B} \| \) for any complex \( n \times n \) matrix \( \mathbf{B} \).

d) \( \| \mathbf{AB} \| \leq \| \mathbf{A} \| \| \mathbf{B} \| \). \hspace{1cm} \( \Delta \)

A matrix norm is often defined in such a way that it is compatible with (consistent with, or subordinate to) a vector...
norm in the sense that

$$\|Ax\| \geq \|A\| \|x\| \tag{E.9}$$

The induced (or associated) matrix norm, defined as

$$\|A\| = \sup_{\|z\|=1} \|Az\| \tag{E.10}$$

is clearly compatible with the vector norm used in its definition. It is also straightforward to show that (E.10) does satisfy all the requirements for matrix norms. When the vector norm in (E.10) is the 1, 2 or $\infty$ norm, the norm of $A$ can be computed easily in terms of its elements.

$$\|A\|_1 = \max_j \sum_{i=1}^n |a_{ij}|$$  \hspace{1cm} \text{the maximum absolute column sum.} \hspace{1cm} \tag{E.11}$$

$$\|A\|_2 = \max_i \sigma_i(A)$$  \hspace{1cm} \text{the maximum singular value of } A. \hspace{1cm} \tag{E.12}$$

$$\|A\|_\infty = \max_i \sum_{j=1}^n |a_{ij}|$$  \hspace{1cm} \text{the maximum absolute row sum.} \hspace{1cm} \tag{E.13}$$

Not all compatible norms are induced norms. For example, the Euclidean norm (or Frobenius norm) of a matrix $A$, defined as

$$\|A\|_E = \left( \sum_{i=1}^n \sum_{j=1}^n |a_{ij}|^2 \right)^{\frac{1}{2}} \tag{E.14}$$

is compatible with the 2 vector norm, but it is not an induced norm. The latter property can be verified by noting that the Euclidean norm of the identity matrix is not equal to 1 for $n$ greater than 1.

APPENDIX E  \hspace{1cm} \text{MATRIX NORMS}
Inequalities similar to those for the vector norms hold for the different matrix norms. First, it can be shown that the three induced norms introduced here are related by a Hölder-type inequality.

\[ \| A \|_2^2 \leq \| A \|_1 \| A \|_{\infty} \]  \hspace{1cm} (E.15)

Moreover, the 2-norm of a matrix is bounded by the other norms as follows.

\[ \frac{1}{\sqrt{n}} \| A \|_1 \leq \| A \|_2 \leq \sqrt{n} \| A \|_1 \]  \hspace{1cm} (E.16)

\[ \frac{1}{\sqrt{n}} \| A \|_{\infty} \leq \| A \|_2 \leq \sqrt{n} \| A \|_{\infty} \]  \hspace{1cm} (E.17)

\[ \frac{1}{\sqrt{n}} \| A \|_E \leq \| A \|_2 \leq \| A \|_E \]  \hspace{1cm} (E.18)

For this reason, an estimate for the 2 norm of a matrix can be found by its 1, \( \infty \), or Euclidean norms. This is a fortunate fact from a computational standpoint, since the work required to compute the 2 norm is proportional to \( n^3 \); whereas the work required to compute the other three matrix norms is proportional to \( n^2 \).
APPENDIX F

SINGULAR-VALUE DECOMPOSITION

For any complex nxn matrix $A$, there is an nxn unitary matrix $U=\text{col}[u_1, \ldots, u_n]$, an nxn unitary matrix $V=\text{col}[v_1, \ldots, v_n]$ and a diagonal matrix $\Sigma=\text{diag}[\sigma_1, \ldots, \sigma_n]$ such that

$$A = U \Sigma V^H$$  \hspace{1cm} (F.1)

$$A = \sum_{i=1}^{n} \sigma_i u_i v_i^H$$  \hspace{1cm} (F.2)

The decomposition (F.1-F.2) is known as the singular-value decomposition (SVD) of $A$. The real, non-negative scalars $\sigma_i$'s are called the singular values of $A$, and the vectors $u_i$ and $v_i$ the left and right singular vectors of $A$, respectively. The terminology here is consistent with that of the eigenvalue decomposition of $A$ (see Appendix A).

The SVD of $A$ is related to the eigen-structure of the matrices $A^HA$ and $AA^H$. Specifically, the singular values of $A$ are equal to the positive square root of the eigenvalues of the matrix $A^HA$ -- or what is the same, of the matrix $AA^H$. The matrix $V$ is equal to the matrix of normalized right eigenvectors of $A^HA$, or

$$A^HA V = V \Sigma^2$$  \hspace{1cm} (F.3)

Similarly, the matrix $U$ is equal to the matrix of normalized right eigenvectors of $AA^H$, or

$$AA^H U = U \Sigma^2$$  \hspace{1cm} (F.4)
It is important to point out that while (F.3) and (F.4) are useful for mathematical manipulations, they are not useful for numerical computations. The reader who is interested in the numerical aspects of SVD should consult [33].

Several facts about singular values that are important to the applications in robustness theory are presented in the following. First the maximum singular value, \( \bar{\sigma}(A) \), and the minimum singular value, \( \underline{\sigma}(A) \), are related to the maximum and minimum "amplification" effects of \( A \) -- namely,

\[
\underline{\sigma}(A) = \min_{\|x\|_2 = 1} \|Ax\|_2 \quad \text{(F.5)}
\]

\[
\bar{\sigma}(A) = \max_{\|x\|_2 = 1} \|Ax\|_2 \quad \text{(F.6)}
\]

The last equation implies that the maximum singular value of the matrix \( A \) is equal to the 2-norm of \( A \). Another important observation is that the inverse of a non-singular matrix \( A \) can be expressed as

\[
A^{-1} = \sum_{i=1}^{n} \frac{1}{\bar{\sigma}_i} v_i u_i^H \quad \text{(F.7)}
\]

and therefore the singular values of \( A^{-1} \) are the reciprocal of those of \( A \). A direct consequence of this fact is

\[
\frac{1}{\|A^{-1}\|} = \frac{1}{\bar{\sigma}(A^{-1})}
\]
According to theorem 7.2 and (F.9), the value $\sigma(A)$ is the norm of the "smallest" additive perturbation which makes $A$ singular. This fact is used in almost all robustness theorems involving singular-value bounds.

Several Weyl-type inequalities on singular values that are useful in robustness theory are the following:

\[
\sigma(A + B) \leq \sigma(A) + \sigma(B) \leq \sigma(A) + \sigma(B) \tag{F.10}
\]

\[
\sigma(A - B) \leq \sigma(A) - \sigma(B) \leq \sigma(A) - \sigma(B) \tag{F.11}
\]

and

\[
\sigma(A - B) \leq \sigma(A) - \sigma(B) \leq \sigma(A) - \sigma(B) \tag{F.12}
\]

\[
\sigma(A - B) \leq \sigma(A) - \sigma(B) \leq \sigma(A) - \sigma(B) \tag{F.13}
\]

For more information on inequalities of this type, see Fan [54].
APPENDIX G

PROOF OF THEOREM 8.2

Theorem 8.2 is proved using the passivity theorem for non-anticipative, linear, time-invariant systems. The passivity theorem in this setting can be stated as follows [40, ch. 6]. For a system with plant \( G(s) \) and feedback compensator \( H(s) \), the closed-loop system is asymptotically stable if \( G(s) \) is passive and \( H(s) \) is strictly passive.

It can be shown [51] that a system represented by a proper transfer matrix \( Z(s) \) with real coefficient is passive if

1. the elements of \( Z(s) \) have no poles in the open right half-plane, and
2. poles on the \( j\omega \) axis are simple and the associated residue matrices are non-negative definite Hermitian.
3. For all \( \omega \), except for the poles on the \( j\omega \) axis,

\[
Z(j\omega) + Z^T(-j\omega) \geq 0 .
\]

(G.1)

The matrix \( Z(s) \) is said to be strictly passive if the residue matrices in condition 2 and equation (G.1) are positive definite Hermitian.

To use the passivity theorem on the power-system model in figure 8.1, the phase reference pole at the origin must be eliminated; otherwise, both the "plant", \( H^{-1}/s \), and the "compensator", \( T(s) \), are only passive. The transformation below are first applied to make machine 1 the reference machine.
\[ U \overset{1}{\frac{1}{s}} H^{-1} U^{-1} \quad \text{(G.2)} \]

\[ U \quad T(s) \quad U^{-1} \quad \text{(G.3)} \]

where

\[
U = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
-1 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 \\
\vdots & & & \\
-1 & 0 & \cdots & 1
\end{bmatrix}
\]

\[ \text{and} \]

\[
U^{-1} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
\vdots & & & \\
1 & 0 & \cdots & 1
\end{bmatrix}
\]

The new plant and compensator models \( H^{-1}/s \) and \( T(s) \) which do not contain the phase reference pole are obtained from (G.2) and (G.3) by deleting the first row and the first column.

It can be shown that \( H^{-1} \) is still diagonal and contains the reciprocal of the inertias for machines 2 through \( n \). The "plant" \( H^{-1}/s \) is therefore passive. Moreover, conditions 1 to 3 of theorem 8.2 guarantee that the "compensator" \( T(s) \) is strictly passive. The stability of the closed-loop system follows from the passivity theorem.
APPENDIX H

SHAFT TORSIONAL DYNAMICS

For a rigid shaft, the equation of motion is

\[ 2H \ddot{x}_1(t) = T(t) \]  \hspace{1cm} (H.1)

where \( x_1 \) is the angular displacement and \( T \) is the torque. The nominal transfer function between \( T(s) \) and the speed of the shaft is

\[ \frac{s x_1(s)}{T(s)} = \frac{1}{2Hs} \]  \hspace{1cm} (H.2)

Torsional oscillations of the shaft become significant at 60 rad/sec or higher, depending on the shaft's physical makeup. Generally, only the first mode of oscillation is important to controller design; other modes are far beyond the bandwidth of most power-system controllers.

A model of the shaft capable of modeling the first torsional mode is shown in figure H.1. The coefficients \( k \) and \( d \) are the torsional spring and damping constants, and \( x_2 \) is the displacement of the "second half" of the shaft. The damping coefficient \( d \) is known to be a function of the loading condition [50]. At light load conditions, the per-unit critical damping of the shaft is typically in the order of .0002, and at heavy load conditions, the per-unit critical damping increases by roughly an order of magnitude. The physical mechanisms for this change is not completely clear to the author.
The equations of motions of the detailed model are

\[ \ddot{H}x_1(t) = k(x_2(t) - x_1(t)) + d(\dot{x}_2(t) - \dot{x}_1(t)) \] (H.3)

\[ \ddot{H}x_2(t) = -k(x_2(t) - x_1(t)) - d(\dot{x}_2(t) - \dot{x}_1(t)) + T(t) \] (H.4)

It can be shown through straightforward algebraic manipulations that the transfer function between the input torque and the output displacements is

\[
\frac{s x_1(s)}{T(s)} = \frac{1}{2Hs} \frac{ds + k}{\frac{H}{2} s^2 + ds + k}
\] (H.5)

Comparing this equation to the nominal transfer function (H.2), it is apparent that \( L \) defined in (5.2) is

\[
L(s) = \frac{ds + k}{\frac{H}{2} s^2 + ds + k}
\] (H.6)

This derivations shows that perturbations due to the neglected torsional dynamics can indeed be represented by

\[
\frac{1}{s} H^{-1} L(s)
\] (H.7)

in the generic power-system model. For an \( n \)-machine system, the matrix \( L \) in figure 8.6c is diagonal, and its elements are given by (H.6).
Figure H.1: A model of the shaft.
APPENDIX I
PARTICIPATION FACTORS

For a linear time-invariant system, let \( v \) and \( w \) be respectively the left and right eigenvectors associated with one of the system modes. The participation factors of this mode are defined by Pérez-Arriaga, Verghese and Schwegge [26,53] as

\[
[|a_1 b_1|, \ldots, |a_n b_n|]
\]

(1.1)

where \( a_i \) is the \( i \)th element of \( v \), and \( b_i \) the \( i \)th element of \( w \). It is claimed in [26] and [53] that the magnitude of the participation factors is a good indicator of the state variables' "significance" in this mode.

The vector of participation factors (1.1) can be interpreted as a weighted right eigenvector where the weighting factors are the corresponding elements of the left eigenvector. The purpose of this appendix is to show that the participation factors in a classical-machine system with purely inductive network and zero machine damping are proportional to the energy of the corresponding machines. This result also applies to lossless spring-mass mechanical systems.

The state-space equation of a classical-machine system with purely inductive network and zero machine damping is

\[
\begin{bmatrix}
\dot{\alpha}(t) \\
\omega(t)
\end{bmatrix} =
\begin{bmatrix}
0 & \omega_R I \\
-H^{-1}K & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\alpha}(t) \\
\omega(t)
\end{bmatrix}
\]

(1.2)
where \( K \) is a symmetric matrix and \( H \) is a diagonal matrix (see appendix B for more details). For an eigenvalue of this 2nx2n system, let the corresponding eigenvectors be partitioned as

\[
\mathbf{w}_i^T = [ \mathbf{w}_{\delta i}^T \, \mathbf{w}_{\omega i}^T ] \tag{I.3}
\]

and

\[
\mathbf{v}_i = \begin{bmatrix} \mathbf{v}_{\delta i} \\ \mathbf{v}_{\omega i} \end{bmatrix} \tag{I.4}
\]

It can be shown through the definition of eigenvalues and eigenvectors that the equations below must hold for both \( \lambda_i \) and its conjugate \( \lambda_i^* \).

\[
\mathbf{w}_{\omega i}^T H^{-1} K = \frac{\lambda_i^2}{\omega R} \mathbf{w}_{\omega i} \tag{I.5}
\]

\[
H^{-1} K \mathbf{v}_{\delta i} = -\frac{\lambda_i^2}{\omega R} \mathbf{v}_{\delta i} \tag{I.6}
\]

The equations for the \( n \) pairs of eigenvalues of (I.2) can be written collectively as

\[
\mathbf{W} H^{-1} K \mathbf{V} = \Lambda \tag{I.7}
\]

where

\[
\mathbf{W} = \text{row } [\mathbf{w}_{\omega 1}^T, \ldots, \mathbf{w}_{\omega n}^T ] \tag{I.8}
\]

\[
\mathbf{V} = \text{col } [\mathbf{v}_{\delta 1}, \ldots, \mathbf{v}_{\delta n} ] = \mathbf{W}^{-1} \tag{I.9}
\]

and

\[
\Lambda = -\frac{1}{\omega R} \text{diag } [ \lambda_1^2, \ldots, \lambda_n^2 ] \tag{I.10}
\]

APPENDIX I

PARTICIPATION FACTORS
(Note: For the double pole at 0, use the generalized eigenvectors of rank 1)

**LEMMA I.1:** The matrices $\mathbf{W}$ and $\mathbf{V}$ defined in (I.8) and (I.9) are related by

$$\mathbf{W}^H = \mathbf{H} \mathbf{V} \quad \triangleq \quad (I.11)$$

Proof: Equation (I.7) can be written as

$$(\mathbf{W} \mathbf{H}^{-\frac{1}{2}}) \mathbf{H}^{-\frac{1}{2}} \mathbf{K} \mathbf{H}^{-\frac{1}{2}} (\mathbf{H}^T \mathbf{V}) = \Lambda \quad (I.12)$$

Note that the eigenvalues of $\mathbf{H}^{-\frac{1}{2}} \mathbf{K} \mathbf{H}^{-\frac{1}{2}}$ are the same as those of $\mathbf{H}^{-1} \mathbf{K}$ (since they are similar), and $\mathbf{W} \mathbf{H}^{-\frac{1}{2}}$ and $\mathbf{H}^T \mathbf{V}$ are the left and right eigenvectors of $\mathbf{H}^{-\frac{1}{2}} \mathbf{K} \mathbf{H}^{-\frac{1}{2}}$. Furthermore, the symmetry of $\mathbf{H}^{-\frac{1}{2}} \mathbf{K} \mathbf{H}^{-\frac{1}{2}}$ implies that the matrices of eigenvectors are orthogonal -- i.e.,

$$(\mathbf{W} \mathbf{H}^{-\frac{1}{2}})^T = \mathbf{H}^{T \frac{1}{2}} \mathbf{V} \quad (I.13)$$

or

$$\mathbf{W}^H = \mathbf{H} \mathbf{V} \quad \triangleq \quad (I.14)$$

**LEMMA I.2:** The right and left eigenvectors associated with $\lambda_i$, $\lambda_i \neq 0$, are related by

$$\mathbf{W}^*_\omega \mathbf{i} = \frac{\omega R}{\lambda_i} \mathbf{H} \mathbf{V}^\omega \mathbf{i} \quad \triangleq \quad (I.15)$$

Proof: The result of Lemma I.1 can be expressed as

$$\mathbf{W}^*_\omega \mathbf{i} = \mathbf{H} \mathbf{V}^\omega \mathbf{i} \quad (I.16)$$

In addition, through the definition of eigenvalues and eigenvectors, the two parts of $\mathbf{V}^\omega \mathbf{i}$ are related by

**APPENDIX I**

**PARTICIPATION FACTORS**
\[ v_{\omega i} = \frac{\lambda_i}{\omega_R} v_{\delta i} \]  \hspace{1cm} (I.17)

Equation (I.15) results when \( v_{\delta i} \) in (I.16) is substituted by (I.17).

**Theorem I.1:** The participation factors of the \( i \)th mode (excluding the modes at 0) corresponding to the shaft speeds are proportional to the kinetic energy of the machines.

**Proof:** If \( v_{\omega i} \) is denoted by

\[ v_{\omega i} = [a_1, \ldots, a_n]^T \]  \hspace{1cm} (I.18)

then (I.15) of Lemma I.2 implies that the vector of participation factors is given by

\[ \frac{\omega_R}{\lambda_i} [\frac{H_1 |a_1|^2}{}, \ldots, \frac{H_n |a_n|^2}{},] \]  \hspace{1cm} (I.19)

The value \( H_j |a_j|^2 \) is directly proportional to the kinetic energy of the \( j \)th machine, since \( a_j \) can be interpreted as the speed of the \( j \)th machine in mode \( i \).

**Corollary I.1:** The participation factors of the \( i \)th mode (excluding the modes at 0) corresponding the shaft angles are proportional to the potential energy of the machines.

**Proof:** This result is a direct consequence of (I.19) and the fact that the shaft speed is a pure derivative of the shaft angle.

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**Appendix I**

**Participation Factors**
REFERENCES


